POLITECNICO DI TORINO

Master of Science in Communications and Computer Networks Engineering

Master's Degree Thesis

Data transmission in multi-hop wireless network under uncertainty: Prospect theory approach

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Contents

Chapter 1

Introduction

1.1 Motivation

Our world today is characterized by great transformations and new dynamics are affecting the context in which we live and work every day. The new society that is being formed is characterized by new needs, which did not exist or were not considered fundamental some time ago. The epicenter of this transformation lies in *connectivity* that is becoming always more simplified and easily accessible. Wireless network services, in this sense, are intended to play a role of primary importance in the immediate future.

As was clearly to be expected, the number of mobile devices has significantly increased in the last few years. This means that the traditional centralized, fixed networks will no longer satisfy the growth in demand for making wireless connections faster in an ever-increasing mobility. New concepts related to infrastructure-less networks, therefore, are attracting a lot of attention nowadays. The reason is that they allow to get a more scalable and flexible wireless networking, efficient power usage and robust connections when fixed network infrastructures are not available. For instance, *Device-to-Device* (D2D) Communication [1] networks aim to implement a self-organizing and self-configuring multi-hop wireless network where the nodes composing it are free to move randomly. Since these nodes are guided by forward-looking goals, they are able to converge on-the-fly to arbitrary form graphs that allow them to provide wireless services without requiring a centralized control entity. Due to the dynamic environment, the vulnerability of the wireless medium, the amount of control traffic and power constraints, these networks are becoming increasingly complex system to analyze and design.

An interesting problem arises from the fact that the participants do not necessarily have an incentive to cooperate with each other. Because of that, the *selfish* behavior of each node in the network has made it very difficult to find a solution with traditional mathematical analysis. In order to deal with such a problem, the idea of modeling the topology from a game theoretical perspective has become particularly attractive. This is because game theory provides the tools to model individual, independent decision-makers, whose actions potentially affect all other participants. Thus it resulted to be remarkably effective in selecting the best node of the network to connect to and thereby achieving excellent performance.

The dominating framework behind such model is *Expected Utility Theory* (EUT) [3], where decision-makers are strictly guided by accepted mathematical outcomes coming from utility notions. Here a function is defined, which assigns a number to every possible action of the player so that higher utilities represent more desirable outcomes. This mechanism led us in having always rational choices, i.e. optimal regarding the analytical and mathematical aspects, but totally uninfluenced by real-life perceptions.

In the analysis of decision making under uncertainty, the foregoing model worked properly in case a decision is taken by "Passive Users". That means the choice is made only by the engineered system design, while human beings as end-users do not interfere with such design. Since the end-user's ability to control devices is going to steadily increase over time, as well as freedom in the latter's configuration, they will start to play an important role in decision making. In this case we are talking about "Active Users", that means users could make decisions that influence the underlying design of various algorithms and impact the performance of the overall system [4]. Motivated by this growing phenomenon, it was relevant to consider the scenario where players follow the principles of *Prospect Theory* (PT) [5] to explain that real-life decisions often deviate from the behavior expected under EUT. To understand the implications of these issues and how they can be related to real life perception, an utility function has been appropriately defined to obtain efficient topology control for a non-cooperative game. This has been done in conjunction with the impact analysis of mobility and human interference on the network design.

1.2 Purposes of the project

This project focuses on developing a game theoretical framework aimed at minimizing the transmit power in a multi-hop wireless broadcast network under dynamic network scenario. A common message has to be sent from a fixed source to all the nodes of the network in a multi-hop manner. A typical application can be found in the context of vehicular content caching and distribution, e.g. a video with a specific quality and the same length, in a video streaming scenario, or an emergency information, such as traffic collision warning, which can be disseminated to drivers by multi-hop wireless communications. In applications of D2D communications, message dissemination is an essential function for information distribution and sharing among mobile devices, which are mostly driven by human. As a consequence, device mobility (i.e., human mobility) introduces the major challenges for message dissemination in D2D communication networks, including rapid network topology changes, frequent link and path breaks, and intermittent network connectivity [2].

Firstly, we wanted to address two main issues that arise in the considered network scenario. One concerns the fact that nodes can join and leave the system at any time, which means that the connection conditions can continuously change over time. This generates uncertainty about the complete data reception from dynamic source of information. For this purpose, a probability has been introduced for a node to remain in the network that depends on the mobility of the environment, which allowed to evaluate when it is convenient to choose reception from a dynamic or static node. The second is that in a multi-hop network an incentive is very important for intermediate nodes, because if they have to relay a large amount of information, a pricing mechanism may be required for forwarding and cooperation. A cost has been so designed, which represents the core of our research, through appropriate cost sharing function so that the receiving nodes pay for the service provided to them.

Finally, an important observation to do is that many related works consider all nodes to be empty (did not download any data) when the system in analyzed. On the contrary, in reality, some nodes could start receiving and at the same time other devices could join the network. One of the goal is thus to find an energy efficient way of disseminating the information for a network in which one node arrives when there is an already going on transmission. In our work we will consider those crucial points and we will see how an incentive mechanism works if a decision is made under uncertainty. To model the preference with which a node decides the service provider from which to receive the message, two models have been chosen: EUT and PT. Based on these we will see how the network should be formed when the information is transferred within a limited contact duration.

1.3 Thesis organization

The thesis begins with a brief overview of game theory and methods for decision making under uncertainty in Chapter 2. A literature review of game theoretical models for data dissemination in wireless D2D networks is presented in Chapter 3. In Chapter 4 we focus on appropriately model the user's preference relationships, which is one of the most challenging point and also the core aspect for the design of our algorithm. Initially, it is developed by considering the simplest case where only one decision-maker is added to the system. Then it is extended in Chapter 5 to two players (action of an agent influences both participants) and a pricing mechanism is presented for minimizing the total energy consumption in the network. In Chapter 6, several simulations of our algorithm are performed so that explained. Finally, Chapter 7 concludes this dissertation and discusses the future work.

Chapter 2

Preliminaries

This chapter aims to provide readers with the necessary tools they need to understand and participate in this work. Before talking about game theory, it is worth to introduce the theories of decision making under uncertainty. In fact, for many aspects, game theory is nothing more than a decision theory under interaction conditions. Once it becomes clear what an utility is, it will be possible to represent mathematically an agent's preference and thus be prepared to deal formally with game theory itself.

2.1 Decision Making under uncertainty

The evolution of decision theory under uncertainty can be fundamentally divided into two main parts: *Expected Utility Theory* (EUT) formulated by von Neumann and Morgenstern (1944) [6], and *Prospect Theory* (PT) [5], proposed by Kahnemann and Tversky (1979). While the first is a *normative* theory intended to provide an ideal behavior model of choice under risk, the second instead aimed to provide a descriptive and explanatory scheme of decision-making processes in people. The Expected Utility Theory is therefore bound to some principles that are those of rational behavior and axioms. Over time, we wanted to understand how this theory contradicts the natural behavior of humans. Regarding this matter, the expected utility has been criticized and subject to a few paradoxes [?]. The Prospect Theory tried to give an alternative to the interpretation of this phenomenon. It provides a new decision-making metric by introducing a value function and a probability weighting function associated with each outcome. Thus, by outlining the Prospect Theory, it is possible to explain how often decisions of real life deviate from the expected behavior of the EUT. This presentation work has been done by using the original texts of the theories, accompanied by various scientific texts and articles on the subject.

2.1.1 Expected Utility Theory

To explain the choices made by individuals under risk conditions, economists used as a main framework the expected utility model of von Neumann and Morgenstern. This theory, accepted and applied as the economic pattern for humans behavior, considers men as a rational and predictable beings, studying then their preferences. The expected utility theory is a deductive process based on some axioms that define the concept of *rationality* and its requirements. The choices and preferences of the subjects are then tied to this concept.

When an individual has to choose among several alternatives, without knowing with certainty which of them will come true, according to the theory he chooses the one associated with a higher utility. To do this, he has to know the probability distribution of the various alternatives. In other words it can be said that the expected utility theory is a criterion that allows the individual to select the choice in a state of uncertainty, under stochastic conditions.

With the term "utility" we represent the actual user's preference, which correspond to the level of "satisfaction" that the subject obtains by achieving a specific goal. In general, such a preference is represented by a *utility function*. It is a function that associates with each possible choice a corresponding utility measure. Specifically, for representing the user's preference, this function matches a numeric value with every possible outcome. A subject that aims to maximize its utility, will assign higher utilities to more desirable outcomes.

Axioms were defined in order to model the rational behavior of agents in making choices. These are intended to further specify the preferences of individuals in order to obtain sufficiently general and analytically manageable utility functions. However, they will not be covered in this chapter and the reader is referred to the original text in case he is particularly interested. It is thus important to know that the fundamental idea behind this theory, in respect of axioms, is that the individuals act *rationally*. This means that the decision-maker chooses what is most useful for himself whenever a decision is taken.

To sum it up we can say that, if axioms are verified, it is possible to construct an utility function such that if choice X is preferred to choice Y, then the utility function of X will be greater than the one of Y and in the end write that :

$$U(X) \geq U(Y).$$

Once the utility function is defined, it will be possible to compute the *Expected Utility* (EU) of the alternative by summing the utility of each outcome weighted with their probability.

The *expected utility function* of an alternative A is so defined as:

$$EU(A) = \sum_{i=1}^{N} p_i \, U(o_i) \tag{2.1}$$

where $o_i \in O$ is the value of each outcome, measured by a real number representing the user's benefit, being O the finite set of possible outcomes composed by a finite positive number $N \in \mathbb{Z}$ of elements. The realization probability and the utility for each outcome o_i are represented by p_i and $U(o_i)$ respectively.

At this point it is good to introduce a simple example, in our case related to the telecommunications field, in order make the concepts outlined above clearer.

Example 2.1. Suppose that a customer has to make a choice between two Internet providers. He is so in front to two alternatives A_1 and A_2: the first one is a service that ensure a data-rate of 50 Mb/s with the risk that for 3 month in a year it may slow down to 40 Mb/s, while the second is a service that ensure a speed of 100Mb/s with the risk that for 6 months it could have problems and get a data-rate of 10 Mb/s. The probabilities regarding A_1 are therefore $p_{A_1} = 75\%$ to go at higher speed $U_{A_1}(o_H)$ and a complementary probability of $(1 - p_{A_1}) = 25\%$ to get a lower bit-rate $U_{A_1}(o_L)$. In the same way for A_2 we have $p_{A_2} = 50\%$ for $U_{A_2}(o_H)$ and $(1 - p_{A_2}) = 50\%$ for $U_{A_2}(o_L)$. Computing now the two Expected Utility for both the alternatives with the formula (2.1), it will be possible to figure out which one is preferable to the other:

$$\begin{aligned} EU(A_1) &= p_{A_1}U_{A_1}(o_H) + (1 - p_{A_1})U_{A_1}(o_L) = \\ &= 0.75 \cdot 50 + 0.25 \cdot 40 = \\ &= 47.5 \end{aligned}$$

$$\begin{aligned} EU(A_2) &= p_{A_2}U_{A_2}(o_H) + (1 - p_{A_2})U_{A_2}(o_L) = \\ &= 0.50 \cdot 100 + 0.50 \cdot 10 = \\ &= 55 \end{aligned}$$

As can be seen, the second alternative A_2 is preferable to the first A_1 even if it has a greater risk in having low performance during one year. This is due to the fact that, from a mathematical point of view, A_2 provides on average an higher rate.

It is interesting to notice that in the above case the expected utility criterion coincides with that of the expected value

$$EU[U(X)] = E(X)$$

where X is a Random Variable with a finite number of outcomes $x_1, x_2, \ldots, x_n$ occurring with probabilities $p_1, p_2, \ldots, p_n$ respectively. This phenomenon depends on how the utility function is designed. As from the theory, the utility has to be a strictly increasing function and so it can be concave, convex, or linear. Depending on the shape of the utility function, three main individual attitudes towards risk can be outlined.

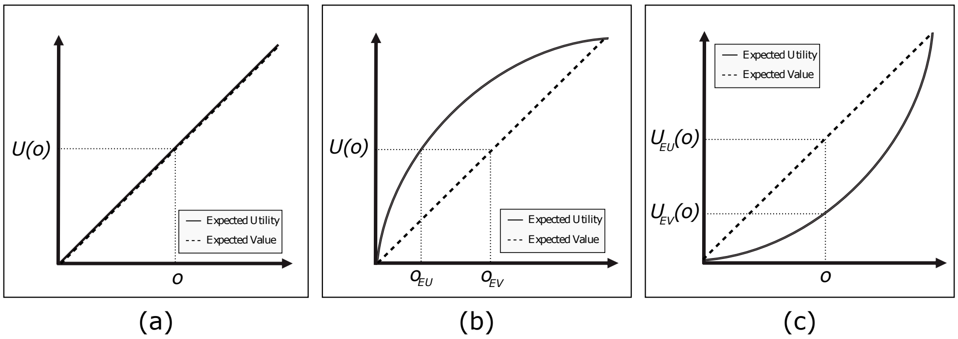


Figure 2.1: (a): Individual attitude as Risk Neutral. (b): Individual attitude as Risk Averse. (c): Individual attitude as Risk Seeking.

An individual is said to be:

- *risk neutral* when he is indifferent between the alternative of certainty and that of uncertainty. This means that for the subject is equivalent to obtaining a gain represented by the outcome o either through the Expected Utility or Expected Value. The level of satisfaction $U(o)$ would be exactly the same. The expected utility under risk neutrality conditions can be so represented on a Cartesian coordinate system as a straight line that coincides with the expected value (figure 2.1 (a)). This is the situation that has been discussed during the previous example.

- *risk averse* when the Utility Function that has been designed is concave. In this case the subject can get the same level of satisfaction that he would get with the expected value, but by means of a lower profit. In fact, relating to figure 2.1 (b), it is possible to see that the same utility $U(o)$ is obtained from both the outcomes o_{EU} and o_{EV}, where the first is smaller and than the second. The difference between o_{EV} and o_{EU} is the price that the individual is willing to pay in order not to be exposed to risk.

  An example of Utility Function for risk aversion could be the square root: $U(o) = \sqrt{o}$. This is because it is going to give greater utility for smaller values of o compared to the one which would be obtained for higher numbers. Applying this utility to the Internet provider choice discussed above we see that:

$$\begin{aligned} EU(A_1) &= p_{A_1}U_{A_1}(o_H) + (1 - p_{A_1})U_{A_1}(o_L) = \\ &= 0.75 \cdot \sqrt{50} + 0.25 \cdot \sqrt{40} = \\ &= 6.88 \end{aligned}$$

$$\begin{aligned}EU(A_2) &= p_{A_2}U_{A_2}(o_H) + (1 - p_{A_2})U_{A_2}(o_L) = \\ &= 0.50 \cdot \sqrt{100} + 0.50 \cdot \sqrt{10} = \\ &= 6.58\end{aligned}$$

In this case the customer is going to select the more steady alternative A_1 even if he is never going to reach the high speed of 100Mb/s. Thus the individual has more satisfaction with a lower expected value and he tries to avoid A_2 that has greater risk of having low performance.

- *risk seeking* when he achieves greater pleasure in obtaining higher outcomes. In this case the Utility Function is represented on the on a Cartesian coordinate system by a convex curve. Figure 2.1 (c) illustrates that for the same outcome o the subject gets a level of utility U_{EV} with the expected value and U_{EU} with the expected utility. As it is easily to observe from the graph, for the same gain o, the U_{EU} utility level (more risky) is higher than the U_{EV} utility level (less risky).

Despite the various models to describe the inclination of individuals seen so far, this theory is at the center of several criticisms. Disapproval is mainly due to incoherence between the choices proposed by the model and those of individuals. The behavior of the latter is often incompatible with the principles of rationality on which the expected utility is based.

Example 2.2. In [4], the authors provided a variation of the Allais' paradox as in Table 2.1, which shows how EUT contradicts people's real-life decisions.

Prospect / *Problem*	*A*	*B*
1	\$2500 *with probability* 0.33 \$2400 *with probability* 0.66 \$0 *with probability* 0.01	\$2400 *with certainty*
2	\$2500 *with probability* 0.33 \$0 *with probability* 0.67	\$2400 *with probability* 0.34 \$0 *with probability* 0.66

Table 2.1: An example of EUT violation

There were two problems in the experiment and for each problem, the respondents were asked to choose between two prospects (A or B). For example, in Table 2.1, the respondent had two prospects in problem 1. If she chose A, she would win 2500 dollars with probability 0.33 or 2400 dollars with probability 0.66 or nothing with probability 0.01. If she chose B, she would win 2400 dollars for sure. It was found that a majority of the respondents (61 per cent) chose B for problem 1 and A for problem 2. According to the definition of EUT (2.1), a respondent would evaluate a prospect, e.g., problem 1A, as the expectation of all the prospect's outcomes, e.g., $0.33 \cdot U(2500) + 0.66 \cdot U(2400) + 0.01 \cdot U(0)$. Thus, a preference of 1B over 1A implies $0.33 \cdot U(2500) + 0.66 \cdot U(2400) + 0.01 \cdot U(0) < U(2400)$ that is equivalent to $0.34 \cdot U(2400) > 0.33 \cdot U(2500)$. Meanwhile, the choice of 2A over 2B implies $0.34 \cdot U(2400) < 0.33 \cdot U(2500)$. Thus, these two results produce a paradox. This has led to consider this theory inadequate to evaluate decisions made by humans.

One of the first opponents of this theory is the Nobel Prize winner Herbert Simon, who argues that people act by *bounded rationality* and therefore their aim is not to look for the optimum, but rather to look for *satisfying solutions* [7]. However, there are cases where the use of Expected Utility Theory is appropriated: for example in the presence of structured problems, with a large information base and when decision makers are automated computers. Another factor is given by experience, learning from mistakes the agent is driven to adopt a behavior that is increasingly approaching to rationality.

In conclusion, the expected utility theory refers to an "ideal" subject to which rationality is attributed by respecting some axioms. It does so without taking into account characteristics and limitations of human rationality. Moreover, it does not consider any emotional aspects that can influence the actual individual choices. It is clear that this model is inappropriate and its behavior does not reflect what actually reality is. An alternative theory that tries to shape the non-rational aspects of people is Prospect Theory, which will be discussed in the next section.

2.1.2 Prospect Theory

The *Prospect Theory* (PT) is a model for describing more accurately the decisions of individuals under risk. It shows that preferences are not absolute, but that they depend on the context in which they are taken. Prospect means looking ahead and, in this theory, it is intended as "getting an idea in advance". The term *prospect* was chosen to emphasize the highly introspective nature with which agents imagine and analyze alternatives. Prospect Theory is not in contradiction with Expected Utility Theory, but aims to integrate it:

- EUT provides a theoretical model of how people *should* act to make optimal decisions.
- PT provides a theoretical model related to real decision-making processes that *induce* people to make suboptimal decisions.

In fact, the EUT becomes the benchmark against which to judge the goodness of choices made by people. However, very often, it is impossible to overcome this benchmark. It is the classic problem of the models that try to achieve the optimum, that is, they are usually unrealistic. To make it short, the EUT shows "how it should be", while the PT shows "how it is".

The greatest contribution of PT is to consider two aspects of choice that are not treated appropriately in conventional behavioral paradigms:

I) The first problem is that people do not evaluate risk decisions according to the expected utility, but they over-weight the importance of unlikely events.

II) The second concerns the framing consideration, which means that equivalent results are treated differently depending on the way they are described and on situation in which the decision is made.

To take these two phenomena into account, Prospect Theory bases the choice on two functions used to evaluate, subjectively, the outcomes and the probabilities associated with them. The first one is the *Probability Weighting Function* and the second is the *Asymmetrical Value Function* that are illustrated below.

Probability Weighting Function

It is revealed in PT that people use their subjective probabilities rather than objective probabilities to weigh the values of possible outcomes. The probability weighting function $w(p)$ models the fact that people tend to over-weigh low probability outcomes and under-weigh moderate and high probability outcomes [9]. A common choice of probability distortion function (e.g., [10], [11], [12]) is

$$w(p) = \exp(-(-\ln p)^{\alpha}), \qquad 0 < \alpha \leq 1 \tag{2.2}$$

where p is the objective probability of realizations and $w(p)$ corresponds to the *subjective* probability. The probability distortion parameter α reveals how a person's subjective evaluation distorts the objective probability, where a smaller α means a larger distortion.

From the graph shown in figure 2.2 it is possible to see that, at the highest point of the 45° line representing the objective probability ($\alpha = 1$), a certain gain becomes an *almost* certain gain. In other words, the figure shows that here the preference for almost certain alternative is reduced if compared to normative behavior. Thus, high probability is underestimated with respect to objective calculations.

At the lowest points instead, coherently with assumptions of prospect theory, there is an aversion to uncertainty in the presence of events with a low probability of occurrence. In fact, the graph shows that the probability of unlikely events is overestimated (i.e. $w(p) > p$ for small values of p). In other words, low probability is over-weighted with respect to objective calculations.

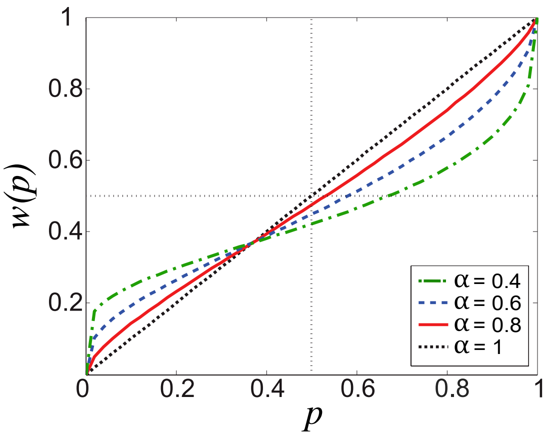


Figure 2.2: the probability weighting function $w(p)$ in PT.

Asymmetrical Value Function

As we have seen, the probability weighting function shows how humans react to probability, but the PT differs from the EUT also for the fact that, in theory formulation, the concept of *value* replaces the notion of *utility*. It is not just a change of terms, it is a real change of perspective in determining the judgment basis for taking a decision. It can be said that the utility is traditionally considered in terms of achievable net welfare. The value, instead, is defined in terms of gains and losses, which means referring to positive or negative returns with respect to a certain position assumed as a neutral *reference point*. An example will clarify the concept and how it affects individual behaviors.

Example 2.3. Suppose that a company employee receives a salary of 100 € per month. At one point his office manager decides to increase the employee's salary of 100 € per month. This means that his wage, after his boss made the choice, has doubled to 200 €. Suppose now that there is a second employee who receives 10000 € per month. Again its employer says that will increase its salary by 100 € per month, that means the total wage will be of 10100 €.

The question at this point is: how happy is the first employee compared to the second one? Obviously the first is much happier than the second although in both cases the increase in salary is the same and equal to 100 €. It is seen, therefore, that what people feel is different. The value V of 100 € for the person that has 100 € as monthly income is much higher than that of the person who has 10000 € entry (figure 2.3). This effect is called *diminishing effect*, that is, when you go further the slope of the curve reduces.

As we have see, possible alternatives are therefore evaluated not with the usual utility function, but with the value function that is not related to the final position of the subject, but to the variations of his wealth. It is interesting to note that the

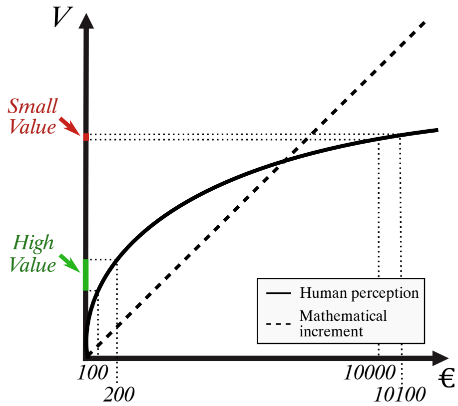


Figure 2.3: Value function for gains

function for gains follows the same principle on which the risk averse EUT is based, where the benefits are represented by a strictly increasing and concave curve.

What is even more remarkable and makes the PT particularly innovative is to give less importance to earnings than losses. This explains the peculiar characteristic of the value function in relation to the utility function: the *asymmetry*. Asymmetry means that losing and earning a certain amount is subjectively perceived in a different way from what should be done objectively.

From a psychological point of view, a win and a loss of the same amount not cancel each other out. The individual perceives the final net result as a loss. Losses create about twice a pain than a pleasure aroused by the winnings. For this reason people are reluctant to accept a bet like the following: 50% chance to win 1000 € and 50% chance of losing 1000 €. The psychological weight of the possible loss exceeds that of the possible winnings and the bet is perceived as unfair.

With reference to the example 2.3, suppose that the employer, instead of raising the salary, decreases it by 50 € to both employees. It can therefore be assumed that the loss of this money for the employee who earned 100 € will have a greater psychological impact than the case when the salary was doubled. For the second worker, however, the impact will be of little relevance also in this case.

Having made these considerations, we expect that the concept of asymmetry will lead to: a convex and relatively steeper function that refers to losses, while a concave and less steep function that refers gains. A common choice of value function [9] is:

$$v(x) = \begin{cases} x^{\beta} & \text{if } x \geq 0, \\ -\Lambda(-x)^{\gamma} & \text{if } x < 0, \end{cases} \tag{2.3}$$

where $\Lambda > 1$, $0 < \beta < 1$, and $0 < \gamma < 1$. The parameter Λ is the *loss penalty parameter*, where a larger Λ indicates that the virtual operator is more concerned of loss, and hence is more risk-averse. The parameters β and γ are the *risk parameters*, where the value function of the gain part is more concave (i.e., the virtual operator is more risk-averse) when β approaches zero, and the value function of the loss part is more convex (i.e., the virtual operator is more risk-seeking) when γ approaches zero. The impact of β and γ can be interpreted by the risk-seeking behavior in loss and risk-averse behavior in gain. The plot of such a function is shown in figure 2.4. We note that EUT is a special case if we choose $\Lambda = 1$ and $\gamma = \beta = 1$.

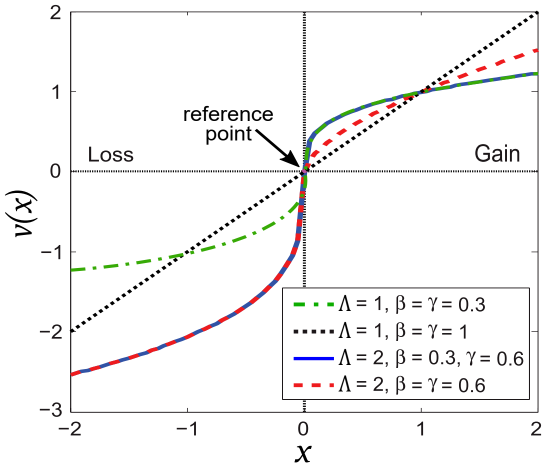


Figure 2.4: Value function

The first thing that catches the eye into seeing the graph of the value function is the presence of the *reference point*. The reference point is a benchmark to evaluate the payoff, where $x \geq 0$ means a gain, while $x < 0$ means a loss. The central idea of this fundamental reformulation is that the utility function needs to explain real behaviors and for this purpose its arguments should be *changes in states* (or events) and not simply the states. Since change is important, then the value that individuals attribute to states depends on the relationship between the state and the reference point. The value, consequently, depends on the starting point from which the user moves because of the change. Summing up, the value function proposed by Kahneman and Tversky [10] has three fundamental features:

- Outcomes are assessed in relation to a reference point and are categorized as gains or losses.
- In both quadrants (gains and losses), the function is characterized by a decrease in sensitivity to change.
- In the loss side the function is steeper than in the gain side.

To conclude, because of the aversion to loss and the tendency to codify outcomes in gains and losses, people are more able to make comparative evaluations rather than absolute evaluations. Do not change the state in which the decision maker is, also has the effect of making him feel less guilty (less sorry) if anything goes wrong. The behavior illustred in PT is clearly not standard, since from an objective point of view you should always strive to achieve the highest possible outcome. At the same time it turns out to be a powerful tool able to model the user's satisfaction when he is driven by a bounded rationality.

2.2 Introduction to Game Theory

This section describes the game theory in general, introducing the main notations, terminologies and the mathematical model adopted in the study of conflict situations. Only the topics considered to be particularly relevant for modeling this project will be studied in deep. Since there has been a great deal of interest in the application this theory to wireless communications, it has been considered to be of fundamental importance formalizing and analyzing the methods on which the analysis proposed by the game theory is based. Indeed, an D2D network is a self-configuring, multi-hop network in which there is no central authority and so Game Theory is particularly suitable for studying the interaction of autonomous agents (i.e. nodes of the network). This mathematical model describes the interaction between players assuming that each of them can be influenced by the actions of all other players.

Two important classes of games can be identified:

- **non-cooperative games**, in which players cannot enter into agreements with each other, regardless of their goals. Non-cooperative games are also called, in a totally equivalent way, *competitive games*.
- **cooperative games**, in which players pursue a common goal, at least for the duration of the game, tending to collaborate for improving their profit.

This project is focused on one of the two particular classes of non-cooperative games, where no binding agreements between players are permitted: *Strategic Form Games*. They represent, through one-turn games, situations in which all players choose their own strategy at the same time. The second class, which is not covered in this chapter, is about *Extended Form Games*, which add to the first class a sequential structure of successive moves for several players. Basically, in strategic games, strategies are selected once for all players at the beginning of the game, while in an extended game they can be reconsidered and varied during the evolution of the game.

The work of this thesis was done on dynamic D2D networks, where nodes can join and leave the network at any time. About this, the second class, which specifically relates to Repeated and Markov Games, was not considered for the following reason:

in these networks, once a connection has been established, it is difficult to meet again the same player a second time. Therefore, it would be difficult to reformulate the strategy through a sufficiently expressive temporal language able to represent the sequential structure of the game.

Next, the most important concept of Game Theory will be developed: the Nash's equilibrium. It will be seen that this equilibrium will play a crucial role in the analysis of a strategic game, in particular it will have major implications in searching the optimal solution for each individual player. In particular, the Nash Equilibrium for *mixed strategies* will be explained in more detail, with the help of some examples, as it was the concept used to find a solution to the model developed in Chapter 5.

2.2.1 What is game theory?

> Game theory is a bag of analytical tools designed to help us
> understand the phenomena that we observe
> when decision-makers interact.
>
> - *Martin Osborne and Ariel Rubinstein* [13]

Game theory is born with the aim of providing a unitary mathematical environment for the analysis of situations in which more rational individuals interact. It proposes to found a decision theory in conditions of interaction, based on the decision theory (simple), which was previously described. In addition, it is intended to predict what might (and perhaps what should) happen when a game is played.

There are many examples of game situations. The term game is not only meant in a playful sense, but with it are meant situations of real conflict where the scheme of game theory can be an important tool for interpreting the situation. It should be mentioned, in this regard, that the Nobel prize of 1994 was awarded to three game theory scholars: John F. Nash of Princeton University, John C. Harsanyi of the University of California at Berkeley and Reinhard Selten of the University of Bonn. At that time, concepts such as Nash equilibrium, prisoner's dilemma [18] became time-honored topic in newspapers.

A game is made up of three basic components: a set of players, a set of actions, and a set of preferences.

- The **players** are the decision makers in the modeled scenario.
- The **actions** are the alternatives available to each player. When each player chooses an action, the resulting "action profile" determines the outcome of the game.
- Finally, a **preference** relationship for each player represents that player's evaluation of all possible outcomes.

In many cases, the preference relationship is represented by a utility function, which assigns a number to each possible outcome, with higher utilities representing more desirable outcomes. Appropriately modeling these preference relationships is one of the most challenging aspects of the application of game theory [14]. Thus, in a game theory problem, each decision-maker has its own objective function, which will be the usual benefit, a utility if has to be maximized or a cost has to be minimized.

The most important feature is that the result for the single player depends not only on the choice made by her (In general, to refer to generic players in a game are adopted female pronouns as game-theoretic convention), but also on the choice made by others. She alone will not determine the result that will get, but she along with all the others will determine the result for herself.

The two fundamental hypotheses that have been made (and from this moment on will always be considered valid) are that the game participants are *rational* and *intelligent*. Rational means that an agent is able to sort his preferences on a set of results. This is to say that such preferences must satisfy a set of axioms (or beliefs) that are reasonable from his point of view. Intelligent, on the other hand, is to indicate the player's logical ability to recognize the actions necessary to maximize her own profit. Consequently, the solution to a game is a systematic description of the results that may emerge in a given model, compatible with the hypotheses of intelligence and rationality of the players [16].

As stated so far, a game theory problem presents a certain number of players and each player has her decision-making variables available. Then, it can be assumed that the theory takes place in a continuous domain, where each player has at their disposal continuous decision-making variables. For the purposes of this thesis there is no interest in this area, so we imagine that each player has a finite number of choices (discrete) and possible alternatives available. For instance, if we imagine a problem related to playing the stock market, we can suppose that any decision-maker (player that makes up our scheme) can buy or sell, only one of these two possibilities. Moreover, let us assume also for the moment that the players participating in the game are only two. This is because if they are more than two, the problem complicates only in terms of calculations, but not conceptually.

Example 2.4. Consider two players, each of them having a finite number of alternatives and hence the outcome of each of them depends on her choice together with the choice of the other player. It means that results can be imagined, for both one and the other, as matrices: a matrix for player A and another for player B. Matrices because the end result is dependent on the choices of both participants and both have to work with a limited, finite number of possible choices. So the situation is similar to what appears in a Table 2.2.

Player A has available a finite number of possible alternatives $a_1, a_2, \ldots, a_n$. Player B also has in turn a finite number of alternatives that are $b_1, b_2, \ldots, b_m$. At this point, in correspondence to the fact that the row and column players have made

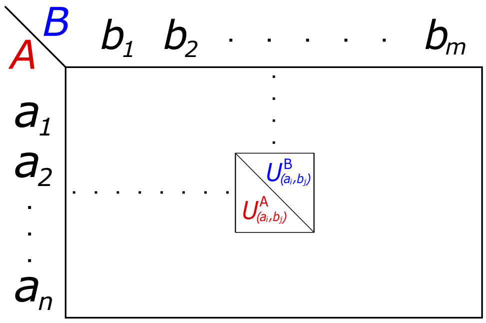


Table 2.2: An example of payoff matrix

their choice, some results are obtained. The values that players get are called *payoff*, both for player A and player B. So what we see in the Table 2.2, synthesized in a single matrix, are actually two matrices. Red values refer to player A, blue values instead refer to player B. Then we say that for every player there is a payoff matrix. If the participants are two, there will be two pay off matrices. However, we can briefly illustrate both matrices in a single table, so that for each box there are two values. In addition, it is necessary to determine if these values are benefits or costs. Here we can simply say that the red and blue values ($U^{\mathrm{A}}_{(a_i,b_j)}$ and $U^{\mathrm{B}}_{(a_i,b_j)}$) are the rewards for player A and B, respectively.

There are several types of games that will not be covered in this work, but that are very interesting. One of these is zero-sum games. They are interesting, compared to the general one, because it is a scheme in which it is not possible to cooperate. If the decision makers are in two, the amount that wins one loses the other and there is no way to make them agree. The possibility that both have benefits is inexistent at all. Here will be treated games whose sum is not always equal to zero, i.e. there is a possibility that in specific situations both players have advantages. It is assumed that in game represented by these tables the values inside them are known to both players. At this stage makes sense to define the concept of strategy, which will give to the player an answer in any situation arises.

2.2.2 Strategic Form Games

This section presents the definition of a strategic form game. A strategic game is a model of interaction with a single move, where players simultaneously choose an action to be performed within known sets of admissible actions. At the same time, it is not meant in a temporal sense: it refers to the fact that leak of information about the choices of the various players, before everyone has made their choice, should not occur. The outcome of the game is completely determined once every player has made her choice.

Definition 2.1. A *strategic form game* $G = \langle N, S_i, \prec_i \rangle$ consists of:

- a finite set of players $N = \{1, 2, \ldots, n\}$;
- a non-empty strategy set S_i for every player $i \in N$, from which a strategy $s_i \in S_i$ (denoted as scalar) is selected by every participant in order to achieve maximum benefits;
- a preference relationship $\prec_i$ on the space of strategy profiles $S = \times_{i \in N} S_i$, defined as the Cartesian product of the individual strategy sets S_i , for every player $i \in N$ (the i-th player's preferences on the set of outcomes).

It is called a strategy profile the vector $\mathbf{s} \in S$ containing the strategies of all players: $\mathbf{s} = (s_i)_{i \in N} = (s_1, s_2, \ldots, s_n)$. It is denoted by $\mathbf{s}_{\text{-}i}$ the collective strategies of all players except player i. Similarly, S_{-i} represent the space of strategy profiles of all participants excluding player i.

The first result for numerical representation of preferences is now illustrated. A preference relationship can be expressed by means of a utility function which characterizes each player's sensitivity to everyone's actions: $u_i(\mathbf{s}) : S \to \mathbb{R}$

Theorem 2.1. *If $\prec$ is a preference relationship, then in S there exist a function $u :\to \mathbb{R}$ such that*

$$x \prec y \Leftrightarrow u(x) < u(y) \quad \forall \; x, y \in S$$

Moreover, such u is unique except for strictly increasing changes in the variable.

The fact that decision-makers' preferences are defined on S and not on their own S_i is exactly what distinguishes a game from a decision-making problem under risk condition: interaction with other players is not negligible. Typically, as previously mentioned, the preference relationship of each player in set S can be expressed through a utility function (or payoff), associated with each player, which matches higher values with better results. Thus, we can associate the player with a function $u_i(\mathbf{s})$, defined on S, which expresses the utility for the player deriving from the $\mathbf{s}$ strategy profile. For the sake of concreteness, let us look at an example taken from [14].

Example 2.5. Consider a game of resource sharing in peer-to-peer networks. In such a situation there may be free-riders, who take advantage of files made available by others but never contribute to the network. Clearly, if all users decide to free-ride, the system will not work because no one will make any files available to other users. Modeling these networks in a game theoretic sense allows us to better understand the incentive structures needed to support resource sharing. In this simple model, each player i can decide whether to share her resources with others (we denote that strategy as $s_i = 1$) or to refrain from sharing ($s_i = 0$). The joint strategy space is $S = \{0,1\}^N$, where N is the number of peers connected to the network. Let us assume the cost for each user of making her own resources available for sharing to be 1.5 units (arbitrary positive value), and that the user benefits by 1 unit for each other user in the network who decides to make her resources available. For $N = 3$, this game can be represented in strategic form in Table 2.3, showing the utilities of each player for every possible strategy profile.

$s_3 = 0$

	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	(0,0,0)	(1,−1.5,1)
$s_1 = 1$	(−1.5,1,1)	(−0.5,−0.5,2)

$s_3 = 1$

	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	(1,1,−1.5)	(2,−0.5,−0.5)
$s_1 = 1$	(−0.5,2,−0.5)	(0.5,0.5,0.5)

Table 2.3: Strategic form representation of a Game of Resource Sharing with three players. The numbers in parentheses represent the Utilities accrued by players 1, 2, and 3, Respectively, for each possible Strategy Profile.

Note that the strategy profile that maximizes the aggregate utility, a possible indication of social welfare from the network point of view, is (1,1,1). It is not clear, though, that there are intrinsic incentives for players to arrive at that strategy. As you may expect, we are interested in determining which of these joint actions is the most likely outcome of the game.

From now on, the issue of how to determine the most likely outcome of a game will be addressed. About this, the most important and most well-known solution concepts of game theory will so be discussed. The first concept of all is the iterative deletion of *dominated strategies*. As we will see, that will be a sufficient predictor only for some games, hence it allows to completely solve a game in a limited number of cases. Next, the most common game theoretic solution concept is considered: the Nash equilibrium. It can be shown that *every* finite strategic-form game has a *mixed-strategy* Nash equilibrium and this is one of the reasons why the latter strategy will be the one applied in this project.

2.2.3 Dominated Strategies

This section presents the basic notions of dominated strategies and the iterated deletion of dominated strategies. Starting from the hypothesis of rationality and intelligence of the players, it is possible to predict in some games a consistent solution. This type of strategy is based on these two hypotheses and thus excludes any type of choice that no rational player would choose. There is no common technique that guarantees to be able to predict the evolution of a general game. In some situations it is not possible to predict any result, so such a solution might not exist. To give a more formal description of this model, the following definition is presented.

Definition 2.2. Given a strategic form game, consider a player i having two strategies s_i and s'_i both elements of S_i. Be $\mathbf{s_{\text{-}i}}$ an array containing the other $N-1$ players' strategies. A pure strategy s_i *strictly* dominates all other strategies of player i if its payoff $u_i(s_i, \mathbf{s_{\text{-}i}})$ is strictly greater than the payoff of any other alternative available to player i, regardless of the choices made by the other players. Indicating with u_i the payoff calculated using the utility function, we get then:

$$u_i(s_i, \mathbf{s_{\text{-}i}}) > u_i(s'_i, \mathbf{s_{\text{-}i}}) \quad \forall \ \mathbf{s_{\text{-}i}} \in \mathbf{S_{\text{-}i}}$$

Furthermore, a pure strategy s_i *weakly* dominates all other strategies of player i if its payoff is greater or equal than any other alternative's payoff available to player i, regardless of the choices made by the other players:

$$u_i(s_i, \mathbf{s_{\text{-}i}}) \geq u_i(s'_i, \mathbf{s_{\text{-}i}}) \quad \forall \ \mathbf{s_{\text{-}i}} \in \mathbf{S_{\text{-}i}}$$

If a strategy is dominated (weakly or strictly) by another, we will say that this strategy is therefore *dominated* (weakly or strictly). A strategy that dominates (weakly or strictly) all the other is called (weakly or strictly) *dominant.* Obviously, if there is a strictly dominant strategy, this is unique. On the other hand, many weak dominant strategies may exist. If each player has a dominant strategy, then it is said that the game has a solution in the dominated strategy. A strategy that is not dominated by anybody else is said, on the contrary, non-dominated.

To make the concept clearer, an example (taken from [14]) is going to be discussed to see if it has dominant strategies and how the iterated deletion of dominated strategies technique is applied.

Example 2.6. Let us now consider the game in Table 2.4. Player 1 can choose between moving to the left, to the right, or staying in the middle ($S_1 = L, M, R$), while player 2 can choose between moving to the left and moving to the right ($S_2 = L, R$). Notice that, regardless of what player 2 does, it is never a good idea for player 1 to select $s_1 = R$: we say that this strategy is (strictly) dominated by the other two strategies in player 1's strategy set. Assuming, as always, that player 1 is rational, we can eliminate that row from our considerations of the likely outcome of this game. Once we do that, we notice that strategy $s_2 = R$ dominates strategy $s_2 = L$, and therefore it is reasonable for player 2 to select the former. Finally, if player 2 selects strategy $s_2 = R$, we expect player 1 to select $s_1 = M$. By iterative deletion of dominated strategies, we predict the outcome of this game to be strategy profile (M, R).

	$s_2 = L$	$s_2 = R$
$s_1 = L$	(1,1)	(0.5,1.5)
$s_1 = M$	(2,0)	(1,0.5)
$s_1 = R$	(0,3)	(0,2)

Table 2.4: Left/Middle/Right Game: an illustration of Dominated Strategies.

The procedure described in this example takes the name of iterative deletion of dominated strategies. So if that process allows us to come up with only one pair of strategies, these are the solution to the game. However, the same identical considerations could have been repeated in the case of weak dominance, but with a substantial difference: the profile that survives the iterative deletion may, in this case, depend on the order in which the eliminations are performed. it is easy to show that this circumstance can not occur in the case of strictly dominated strategies, in which case the order of elimination is of no importance.

This technique provides the first set of approximate solution for a game. Thus, if rational players are participating in the game, it is clear that they should never choose a strategy that would be eliminated by the iterated deletion of dominated strategies. Unfortunately, for most games we need a stronger predictive notion. We therefore move to the broader concept of Nash equilibria. First, though, mixed strategies are introduced.

2.2.4 Mixed Strategies

This section defines the concept of mixed strategies. Definition 2.2 considers a strategy dominated if there is at least another strategy, for the same player, who always provides a value of utility not lower (and in at least one case, strictly larger). We now see an interesting and important alternative to strictly dominated strategy, which arises from a different approach. For simplicity, we illustrate this in the case of two players only, but the definition is of utterly general validity.

Suppose a player, instead of picking a single strategy, randomizes the choice over her strategy set. This means a generic player i could decide to adopt a strategy s_i with probability $0 < \sigma_i < 1$. In this case it is said that a *mixed strategy* has been adopted.

In a strategic form game, a mixed (or probabilistic) strategy σ_i is a probability distribution defined on all possible player i strategies. In particular we denote with $\sigma_i(s_i)$ the probability that σ_i assigns to $s_i \in S_i$. According to probability laws, the underlying relationship must be respected:

$$\sum_{s_i \in S_i} \sigma_i(s_i) = 1.$$

The space of mixed strategies for player i is denoted with Σ_i. Similarly to what has been done so far, the mixed strategy profile is defined as:

$$\sigma = (\sigma_i, \sigma_2, \dots, \sigma_N)$$

where N is the number of player participating in the game. The Cartesian product of all the Σ_i generates the mixed strategy space Σ. At this stage, the expected utility function (or expected payoff) of a player i for mixed strategy can be expressed by

$$u_i(\sigma) = \sum_{\boldsymbol{s} \in \boldsymbol{S}} \left(\prod_{j=1}^{N} \sigma_j(s_j) \right) u_i(\mathbf{s}) \tag{2.4}$$

which represents the sum of the payoffs of each profile, weighed on their probabilities. For instance, in the case of $N = 2$, suppose that a Player A and a player B have available a finite number of alternatives $S_A = (a_1, a_2, \dots, a_n)$ and $S_B = (b_1, b_2, \dots, b_m)$ respectively. If A chooses to play a given strategy a_i with $i \in \mathbb{Z} : 1 \leq i \leq n$ and denoting with $\sigma_j(b_j)$, where $j \in \mathbb{Z} : 1 \leq j \leq m$, the probability that player B chooses b_j, then her expected utility would be

$$u_A(a_i) = \sigma_1(b_1)u_A(a_i, b_1) + \sigma_2(b_2)u_A(a_i, b_2) + \cdots + \sigma_m(b_m)u_A(a_i, b_m).$$

A rational player will try to maximize her expected utility, thus A will choose the strategy a_i such that $u_A(a_i) = \max_{a_k \in S_A} u_A(a_k)$. Of course, if the probabilities $\sigma_j(b_j)$ were known, the decision problem for player A would be easily solved: it is enough to calculate the expected utility of each possible strategy and then choose the strategy which corresponds to the highest utility. The fact that probabilities are not known a priori justifies the effort that has been made over the years to define methods to compute such probabilities in order to reach a solution for the game. A formal definition of mixed strategy can now be provided:

Definition 2.3. Given a finite set of actions S_i available to player i, a *mixed strategy* for player i is a probability distribution on such a set. That is, a mixed strategy is a probability vector $(\sigma_1(s_1), \sigma_2(s_2), \dots, \sigma_n(s_n))$ with $s_i \in S_i : \sigma_i(s_i) > 0$, $i \in \mathbb{Z} : 1 \leq i \leq n$ on the possible actions the player can take.

The literal interpretation of a mixed strategy therefore requires a player to take her decision by deliberately introducing a stochastic element, and this may seem irrational or even bizarre. However, if we accept the interpretation that a game models a situation that may occur many times, the mixed strategy can be interpreted as the *frequency* with which the player selects the different strategies, which are called *pure strategies*, to distinguish them from mixed ones. Of course, a pure strategy s_i is a degenerate case of a mixed strategy σ_i , where $\sigma_i(s_i) = 1$.

There are numerous games where no pure strategy can be justified (more precisely, where there are no equilibria in pure strategies), and where the logical course of action is to randomize over pure strategies. Let us take as an example [15].

Example 2.7. Consider the well-known children's game "Rock-Sciccor-Paper". In each contest, a player must adopt one of these "strategies" in advance: then Rock blunts Scissors, Scissors cuts Paper, and Paper wraps Rock. Suppose that the winner of each contest receives one dollar from the loser; if both adopt the same strategy, no money changes hands. The payoff matrix is shown in Table 2.5. Clearly, a player adopting the pure strategy "Rock" will lose in the long run, because his opponent will catch on and play "Paper". A player adopting the mixed strategy $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ will break even. Thus, the strategy for this game is to randomize among the three pure strategies, assigning a probability of $\frac{1}{3}$ to each.

	s_2 = Paper	s_2 = Rock	s_2 = Scissors
s_1 = Paper	(0,0)	(1,−1)	(−1,1)
s_1 = Rock	(−1,1)	(0,0)	(1,−1)
s_1 = Scissors	(1,−1)	(−1,1)	(0,0)

Table 2.5: Strategic form representation of Paper–Scissors–Rock game.

It is interesting to notice how the above example could not have any solution if we did not randomize. The application of mixed strategies has led to the conclusion that it would not otherwise be possible. Now we are going to face the issue of how to predict the evolution of the game for more general cases and more accurately. In some circumstances, the concept of iterative deletion of dominated strategies is not a sufficient predictor of the outcome of the game. We are therefore going to discuss the most common game-theoretic solution, which will be valid for most games: Nash's equilibrium.

2.3 Nash Equilibrium

The solution concept most significant and important in game theory is that of Nash equilibrium. This concept, essentially, shapes a sort of "stationary state" where no player has an interest in unilaterally deviating for improving their utility. It should be noted that game theory does not deal with the mechanisms by which one achieves this state.

A Nash equilibrium is a stable equilibrium, since no player has an interest in changing her decision (strategy). Each player draws the maximum benefit possible from their choices, considering the best choice of the other player. Any strategic change could only lead to worse payoff (or utility). Nash's equilibrium represents a well-balanced situation of a non-cooperative game, which ensures both the best possible outcome for each player (individual optimum) and the best collective equilibrium (social optimum).

A non-cooperative game can have multiple Nash equilibria. Even in the presence of multiple equilibria, each Nash equilibrium of the game is still a stable equilibrium, since from its position (local equilibrium) any other choice leads to a worse condition for each player. A possible issue regarding this solution concept is the possibility that the conditions for determining it will be completely absent. Many games have no Nash equilibrium. Another important problem is that the non-cooperative game could converge towards a stable but not optimal equilibrium. This means that, even if the equilibrium exists, it may not bring the maximum benefits to the players.

Initially, pure strategies will be studied, and then we will extend the analysis to mixed strategies that will be of crucial importance for finding the final solution to the project presented here.

2.3.1 Nash Equilibrium in Pure Strategy

For now only pure strategies will be considered, therefore, given a game in strategic form, a strategy profile $\boldsymbol{s} \in \boldsymbol{S}$ is a vector containing the strategies of all players $(s_1, s_2, \ldots, s_N)$.

Definition 2.4. A strategy profile $\boldsymbol{s}^* \in \boldsymbol{S}$ is a *Nash equilibrium* (NE) in pure strategies if

$$u_i(\boldsymbol{s}^*) \geq u_i(s_i, \boldsymbol{s}^*_{\text{-}\boldsymbol{i}}) \qquad \forall\ s_i \in \boldsymbol{S_i}\ , \forall\ i \in N$$

where $\boldsymbol{s}^*_{\text{-}\boldsymbol{i}}$ indicates the strategies of all the other $N-1$ players.

Thus, in a Nash equilibrium, no player can change strategy, once the actions of other players are fixed, without getting a worse outcome. It is useful to consider an equivalent formulation of the Definition 2.4.

Definition 2.5. For every player i, consider a strategy profile for the others $N-1$ participants $\boldsymbol{s}_{\text{-}\boldsymbol{i}} \in \boldsymbol{S}_{\text{-}\boldsymbol{i}}$ and define the set

$$B(\boldsymbol{s}_{\text{-}\boldsymbol{i}}) = \{\hat{s}_i \in S_i \mid u_i(\boldsymbol{s}_{\text{-}\boldsymbol{i}}, \hat{s}_i) \geq u_i(\boldsymbol{s}_{\text{-}\boldsymbol{i}}, s_i)\ \ \forall\ s_i \in S_i\}$$

as the set of *best reply* that player i can make with respect to the decision taken by other players. A strategy profile $\boldsymbol{s}^*$ is a Nash equilibrium if and only if

$$s_i^* \in B(\boldsymbol{s}^*_{\text{-}\boldsymbol{i}}) \qquad \forall\ i \in N, i = 1, 2, \ldots, N.$$

How can we directly draw from the mathematical definition of NE, if a player selects any strategy other than $\hat{s}_i$, he could only worsen his payoff or, at most, leave it unchanged. So if players reach a NE, no one could improve their outcome by thinking about unilaterally changing their strategy. As this applies to all players, we deduce that if there is a NE and this is unique, it would represent the solution of the game, as none of the players would have any incentive to move away from the equilibrium.

The most important contribution given by John F. Nash to game theory is the invention (and subsequent mathematical proof of existence) of this equilibrium [17]. However, it is not said that the equilibrium is unique: in particular, in games where individuals adopt exclusively pure strategies, a Nash equilibrium may not exist. Now we are ready to see what are the Nash equilibrium for the previous examples:

Example 2.8. Consider the resource sharing example [14] in Table 2.3. Joint strategy (0,1,0) is not an equilibrium because player 2 can improve her payoff by unilaterally deviating, thereby getting a payoff of 0 (greater than -1.5). Systematically analyzing all eight possible joint strategies, we can see that the only one where no player can benefit by unilaterally deviating is (0,0,0). This is the only Nash equilibrium for this game. Note that joint strategy (1,1,1) would yield higher payoff than strategy (0,0,0) for every player; however, it is not an equilibrium, since each individual player would benefit from unilaterally deviating. The Nash equilibrium for this game is clearly inefficient, a direct result of independent decisions without coordination among players.

The example discussed above is simply a variation of the famous Prisoner's Dilemma [18]. Regarding the Left/Middle/Right Game in Table 2.4, the Nash equilibrium coincides with the result obtained by iterative deletion of dominated strategies. Instead, for the paper–scissors–rock there is no Nash equilibrium in pure strategies, but as we will see it exists in mixed strategies. It is therefore necessary to consider the case where a mixed strategy can represent a Nash equilibrium for a game.

2.3.2 Nash Equilibrium in Mixed Strategy

Starting from Nash's equilibrium, many other solving concepts can be found, both for trying to solve some of the problems mentioned in the previous examples and for finding a substitute in cases where Nash equilibrium does not exist. The search for a solution changes substantially if we broaden our horizon of interest to include mixed strategies. Suppose that each player has, as usual, a finite number of pure strategies at their disposal, but here he can decide to implement any mixed strategy defined on them. The simplest resolution concepts based on Nash equilibrium is therefore introduced, namely mixed strategies and an interesting interpretation in terms of convictions is given.

Definition 2.6. A mixed strategy profile $\sigma^* \in \Sigma$ is a Nash equilibrium if

$$u_i(\sigma^*) \geq u_i(s_i, \sigma^*_{\text{-}\boldsymbol{i}}) \qquad \forall\ s_i \in \boldsymbol{S_i}\ , \forall\ i \in N$$

where $\sigma^*_{\text{-}\boldsymbol{i}}$ represents the collective mixed strategies of all players except player i.

The above-mentioned statement generalizes the notion of NE given earlier in Definition 2.4. As it has been done before, we can extend the concept of best reply:

Definition 2.7. The best reply correspondence in mixed strategies from player i to other players' strategies is the set

$$B(\sigma_{\text{-}\boldsymbol{i}}) = \{\hat{\sigma}_i \in \Sigma_i \mid u_i(\sigma_{\text{-}\boldsymbol{i}}, \hat{\sigma}_i) \geq u_i(\sigma_{\text{-}\boldsymbol{i}}, \sigma_i) \ \ \forall\ \sigma_i \in \Sigma_i\}$$

and a mixed strategy profile σ^* is a Nash equilibrium if

$$\sigma^*_i \in B(\sigma^*_{\text{-}\boldsymbol{i}}) \qquad \forall\ i \in N, i = 1, 2, \ldots, N \tag{2.5}$$

Since the basic premises have been presented, we are now ready to present the most important theorem for the purpose of the thesis. The following result is of great interest in the whole theory and the importance of mixed strategies is further strengthened by it.

Theorem 2.2. (Nash) *Every strategic form game with a finite number of players, each of which with a finite strategy set, has at least one Nash Equilibrium in mixed strategies.*

The theorem has been demonstrated by Nash in [19] and is an application of the Kakutani's fixed point theorem. The Nash Theorem 2.2 does not describe a method to find the Nash Equilibrium. It is still a considerable result as it provides a *sufficient* condition for a game to have at least one NE in mixed strategies: in fact, it is sufficient that the game is finished, that is,

$$|N| < +\infty, \ |S_i| < +\infty, \quad \forall\ i \in N$$

This does not contradict what is said for a NE in pure strategies: if the game is finite, the NE may not exist in pure strategies, but according to the Theorem 2.2, there is always at least one NE in mixed strategies.

Example 2.9. We are now ready resume the the "Rock-Sciccor-Paper" in Table 2.5 in order to explain how a NE in mixed strategy can be found. A mixed strategy is represented by a vector $(\sigma_1, \sigma_2, \sigma_3)$ that indicates the probability of playing each of the possible strategies. A mixed strategy profile that is Nash equilibrium requires that each strategy is the best answer of all mixed strategies of a player. To do this, it is enough to show that the expected value (the gain with the mixed strategy) is greater than or equal to the gain you would get by playing any pure strategy. The Nash equilibrium in this case is for both players $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$: fixed this strategy for one of the two players, the other is *indifferent* to play any pure strategy that will give zero gain as well as the equilibrium mixed strategy. Viceversa for the other player. The fact that earning is the same for pure strategies does not imply anything: if they were played, it would break the equilibrium as the other player would have an incentive to move and as we have seen the game in pure strategies has no equilibrium.

Since, as already mentioned above, the theory does not provide us with any procedure for finding the equilibrium, it is crucial to introduce at least one methodology to find it. In this regard we are going to introduce the Battle of the Sexes game. This is done because what will be explained soon will be the method for finding the solution used in the research on D2D networks presented here. It is therefore advisable for the reader to pay particular attention to the mathematical steps that will be illustrated and how the concept of *indifference* is central to the formulation of the problem.

Example 2.10. (Battle of the sexes [20])

A husband and a wife decided to spend the evening together going to the movies. In this regard, the two are debating heatedly. Her (player 1) was trying to convince him to go to watch "Wondrous Love (WL)", while he (Player 2) would prefer to bring her to watch "Lethal Weapon (LW)". Both, however, prefer to go out together rather than stay separate: loneliness would make even the most enjoyable show unattractive.

We can then represent the situation as shown in Table 2.6.

		Husband	
		s_2 = LW	s_2 = WL
Wife	s_1 = LW	(2, 1)	(0, 0)
	s_1 = WL	(0, 0)	(1, 2)

Table 2.6: Battle of the Sexes game.

Looking at the table, it can be immediately seen that the game has two NE in pure strategy: (LW,LW) and (WL,WL). In fact, in both the situation were they choose the same movie we can check that whenever one of the players plays the given (pure) strategy, the other player would only lose by deviating. Are these the only Nash equilibria? The answer is no; although they are indeed the only pure-strategy equilibria, there is also another mixed-strategy equilibrium.

What will be described below is a sort of starting point to find an equilibrium which is enough that it works in small games. It is possible to turn this in a general algorithm, but not necessarily the most efficient or insightful way of finding an equlibria.

It will be shown here that this computational problem is easy when we know (or can determine) the *support* of the equilibrium strategies, particularly so in this small game. A support is the set of pure strategies that receives positive probabilities under the mixed strategy of the players. In other words, an equilibrium support is a set of that actions that occur with positive probabilities. So, for battle of the sexes let us assume that the support of the equilibrium is made by all the actions available to the players. Try then to reason about what the NE may be given that support.

First of all is better to introduce some notation in order to proceed with the calculation. Assume that both players randomize and that husband's strategy is to play LW with probability q_2 and WL with probability $1 - q_2$. Now, if the wife has to best respond to this mixed strategy, we need that player 2 (husband) must have set those two probabilities $(q_2, 1-q_2)$ in a way that makes player 1 (wife) *indifferent* between her own actions LW and WL. This is an important point in reasoning about how mixed strategies work. The reason why player 1 needs to be indifferent is that she is going to play a mixed strategy as well, which means some of the time she's playing LW and some other time WL. This is because such two strategies are both in the support and so they both get played with nonzero probability. If the player is not indifferent, then she could get even more utility by reducing the amount of probability she put on the lower outcome and increasing the amount of probability she puts on the higher outcome. In an extreme case she could get the most utility by putting absolutely no utility on the lower payoff in order to choose always the higher. This is to say that the only way she would actually want to play a mixed strategy is in the case it is the same to play LW and WL. That means we can reason that player 2 has set his probabilities q_2 and $1-q_2$ in such a way that makes player 1 indifferent.

In this way, thanks to the last observations, it is actually possible to write that down in math. Thus we can say the utility for player 1 of playing LW is equal to the utility of player 1 of playing WL given that player2 plays $(q_2, 1 - q_2)$. Then we can write the following equation:

$$u_1(LW) = u_1(WL)$$

$$2 \cdot q_2 + 0 \cdot (1 - q_2) = 0 \cdot q_2 + 1 \cdot (1 - q_2)$$
$$q_2 = \frac{1}{3}.$$

From the equalities we see that if the wife is indifferent between the two strategies, then that means when she plays LW she gets 2 with probability q_2 and 0 withe $1 - q_2$. In case she chooses WL she gets 0 with probability q_1 and 1 with $1 - q_2$.

Solving the simple equation in 1 variable, we end up concluding that the only way such that player 1 can be indifferent between playing LW an WL is $q_2 = \frac{1}{3}$.

In the same way, if player 2 (husband) was randomizing, which we had just assumed he was, then player 1 must make him indifferent. Why is player 1 willing to randomize? because she is simultaneously being made indifferent by player 2. So now assume that player 1 plays LW and WL with probabilities q_1 and $1 - q_1$ respectively. At this stage, the same computation of the previous case can be done:

$$u_2(LW) = u_2(WL)$$
$$1 \cdot q_1 + 0 \cdot (1 - q_1) = 0 \cdot q_1 + 2 \cdot (1 - q_2)$$
$$q_1 = \frac{2}{3}.$$

The important thing to notice here, happening in the game, is that player 1 and player 2 were both willing to randomize (Table 2.7), so we ended up getting out numbers (representing probabilities) that make sense, since q_1 and q_2 are both between 0 and 1. That means it is actually possible to set them in such a way that makes both the players indifferent. If the payoffs were different, we might have gotten out with number out of the probability range [0,1]. That would really be telling us that there is no way of making the other player indifferent, so no equilibrium would be achievable with that support.

		(q_2)	$(1 - q_2)$
		s_2 = LW	s_2 = WL
(q_1)	s_1 = LW	(2, 1)	(0, 0)
$(1 - q_1)$	s_1 = WL	(0, 0)	(1, 2)

Table 2.7: Battle of the Sexes game with the introduction of probabilities.

Concluding, the mixed strategy $(\frac{2}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{2}{3})$ are Nash equilibrium, since these distributions make the other player indifferent, and so they are both willing to play the mixed strategies.

There are several interpretations and, consequently "justifications" for the use of mixed strategies. The interesting descriptive interpretation of the use of mixed strategies that has been made is also a good example of how deep (and complicated) is the use of beliefs in the foundations of game theory.

The positive side of this approach are many, but there is a need to define a way that players consider and know each other's beliefs. From a networking point of view, this approach resulted particularly suitable, since we are in presence of programmable nodes that may be interested in maximizing their own utility so as to improve the network performance.

Chapter 3

Energy-efficiency Optimization: State of the Art Review

This chapter contains a brief summary of the state of the art for Energy-Efficient optimization in wireless D2D networks. It is not purported to be an exhaustive review, but a compendium of the most interesting models found in the literature that are related to the work of this thesis.

As first, networks schemes for data dissemination are illustrated. Those methods are designed to find the minimum cost path among nodes in a network. Such a cost, in our case, is represented by the transmit power required for distributing a message in a single source wireless D2D network. Some example are presented to allow the reader to have a better comprehension of the basic graph search algorithm that are used in this field. We will see how, over time, such mechanisms became suitable for wireless communications, incorporating features as the wireless multicast advantage, which takes into account the fact that a transmitter may communicate with multiple receivers if they are within its communication range. The paradigm that has been followed to achieve the best results is the centralized networking. It assumes the that networks parameters are known a priori to all participants with the help of a central infrastructure, which is not realistic in scenarios where the latter is not available. Because of this, the only way for finding a solution in the D2D wireless scenario is to move toward a decentralized approach.

Next, we will see how the introduction game theory has supported distributed solutions, allowing to achieve considerable results in decentralized environments. Moreover, the incentive mechanisms for data replication will be discussed. This has been thought to support nodes that have to relay, and so spend their energy, the information to other nodes of the network. Pricing schemes are so characterized to accomplish the will to pay for the service that a user receives.

Having considered those studies, the main challenge lies in determining the utility function of these games, which is not trivial and it is still an open problem. We focus specifically on a utility definition that represents the trade-off between energy conservation and network throughput. In the current literature, such a technique has been mainly applied to resource allocation for wireless communication, but not to topology construction mechanisms. In our project, we are going to see how these principles can be exploited to determine a procedure for energy-efficient data dissemination in D2D networks. The most innovative aspect, beside the use of game theory with the possibility of end-user interaction, lies in the introduction of dynamicity in the network, so that nodes may join and leave it at any time.

3.1 The Minimum-Energy Broadcasting Problem

In the literature, many schemes have been proposed for data dissemination in D2D wireless networks. Energy conservation is a critical issue for network life, because nodes are powered by batteries only. The most intuitive approach for saving energy in such a networks is to route a communication session along the paths that require the lowest total energy consumption. This optimization problem is known as *minimum-energy broadcasting*. In [21–23], regarding this issue, a performance analysis of the greedy heuristics centralised algorithms MST (minimum spanning tree) and SPT (shortest-path tree) is proposed.

- In MST, the requirement is to reach each vertex once (create graph tree) and to do this is required the minimum collective cost for obtaining, among all possible combinations, the minimum weight connected graph with no cycles. It is based on the Prim's algorithm [24] which starts creating an empty list of visited nodes and it is used to keep track of nodes that we have touched. The next step is to pick an arbitrary node from which the algorithm is going to start. For the sake of clarity, it is better to look at the scenario proposed the example 3.1.

  Example 3.1. With reference to Figure 3.1, suppose that the algorithm starts from node A. The first thing to do is to add A to the visited list: $Visited = \{A\}$. Next, examine all vertices reachable from A. They are the set composed by the nodes $\{B, C, D\}$. Prim's is a greedy algorithm, so we are going to choose the smallest edge that connects to an unvisited node. In this case A to B ($A \to B$). The minimum spanning tree is then started and for now it is composed only of these two nodes. The visited list can be so updated: $Visited = \{A, B\}$. Notice that B has been added to the list. We now look at all nodes reachable from A and B and this is the neighborhood $\{C, D, E\}$. Three edges all have a weight of 3. Pick one of these randomly. Here it has been chosen $A \to C$. Continue in this manner each time picking the smallest edge that connects to an unvisited node: $C \to E$, $A \to D$. Observe at this point the edge between B and E within weight of 3 is the smallest edge, but both vertices are already in the MST, so we cannot consider it. Instead, we will choose to add F to the three: $C \to F$. The only unvisited node remaining is G and also it will be added the the MST: $F \to G$. All the nodes are now connected in a three. Note that if edge weights are distinct, this minimum spanning three will be unique.

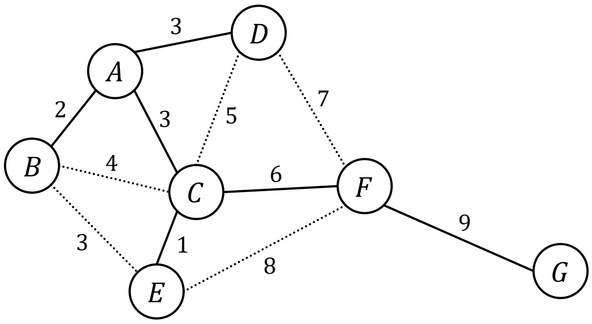


Figure 3.1: Minimum Spanning Tree application: non-dashed edges represent the final shape of the tree

– The SPT, instead, is focused on reaching a destination vertex starting from a source of the data and this is required to be done with the lowest possible cost (shortest weight). Such a mechanism does not have the goal of including every node into the broadcast tree, but only focuses on source and destination vertices. The SPT applies Dijkstra's algorithm [25] which tells the shortest distance to one node to every other node in the graph. It differs from [24] which results in a MST. Let us use the next representation as example (Figure 3.2).

Example 3.2. It is define also here a set of unvisited nodes. At the beginning in such a list are obviously present all the nodes. The first step is to pick arbitrarily the starting node. Referring to Figure 3.2, suppose this to be node A and the set of unvisited node should be upgraded removing the starting node: $UnvisitedNodes = \{B, C, D, E\}$. In addition to the previous method, we will use a table that keeps track of distances. More precisely, we are measuring those from the starting node A. Originally, the distance between A and itself is set to 0, while all the others for reaching each node of the network are set to infinity, as we have not visited them yet: $Distances = \{A : 0,\ B : \infty, C : \infty,\ D : \infty,\ E : \infty\}$. The next step is to examine the edges leaving A. It is possible to see that B and C are so reachable. We can here update the chart with the corresponding costs: $Distances = \{\ A : 0,\ B : 4,\ C : 2,\ D : \infty,\ E : \infty\}$. Next we look at the chart and pick the smallest edge of which the vertex has not been chosen: in this case C. Let us remove C from the unvisited node list $UnvisitedNodes = \{B, D, E\}$. The graph can be updated accordingly by selecting the route $A \to C$. Node B is now reachable from A with the cost of 3 by traveling through C so we can replace the old cost with this new one and establish the connection $C \rightarrow B$. Again, due to the connection of the latter, also D and E become reachable for the first time. At this step, the same thing done before should be performed. Upgrade the distances $Distances = \{A : 0,\ B : 3, C : 2,\ D : 6,\ E : 7\}$ and choose the smallest path among the not selected yet. This time is B, thus $UnvisitedNodes = \{D, E\}$. We repeat the process examining the edges leaving B and updating the cost getting $B \rightarrow D$ and $B \rightarrow E$: $Distances = \{A : 0,\ B : 3,\ C : 2,\ D : 5,\ E : 6\}$. Now we choose D and this time there is no updates to our table as there are no edges leaving D and $UnvisitedNodes = \{E\}$. Finally we select E and again there are no updates, but this time because the edges leaving E does not result in a shorter path. All the edges in the graph have now been visited and are closed.

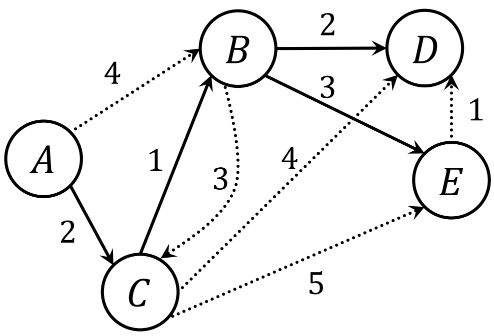


Figure 3.2: Dijkstra's algorithm application: non-dashed edges represent the shortest-path tree.

The algorithms that have been studied above are for the link-based, wired environment. In such fully-connected LANs, since there is single-hop connectivity among all the nodes, the multicasting problem is trivial. Link-based models are appropriate for wired applications, because they do not take into consideration the broadcast-based nature of wireless communications.

– In [26–29] a performance analysis is performed for comparing the link-based algorithms described so far with the Broadcast Incremental Power (BIP) algorithm. This new mechanism aims to a multicast tree construction in infrastructureless, all-wireless applications. The performance metric used to evaluate broadcast and multicast trees is energy-efficiency. Results demonstrate that BIP provide better performance than conventional link-based schemes over a scenario composed by wireless agents only. Their objective is to form a minimum-energy tree, rooted at the source, that reaches all of the desired destinations. The main difference that lies in this method is that the wireless channel is characterized by its broadcast nature. When an antenna is transmitting, this transmission can be received not only by the destination node, but by every receiver located its communication range. Consequently, if in the neighborhood of the transmitting device there ares multiple nodes, a single transmission is sufficient for reaching all these receivers in multicast mode.

In the previous unicast applications it is best (from the perspective of transmission energy consumption) to transmit at the lowest possible power level, even though doing so requires multiple hops to reach the destination. However, in multicast this solution may not be efficient, along with interference problems that may rise in case of too many transmission. For this reason in wireless environments the use of higher power may permit simultaneous connectivity to a sufficiently large number of nodes, so that the total energy required to reach all members of the multicast group may be actually reduced. From this work it is interesting to derive the "wireless multicast advantage" property, which makes multicasting an excellent tool for energy conservation:

- A node is capable of reaching another node if the latter is within communication range, which means that the received signal-to-noise ratio exceeds a given threshold and the receiving nodes have allocated receiver resources for this purpose.
- The total power required to reach a set of other nodes is simply the maximum required to reach any of them individually.

By contrast, in wired models, to reach two different node the total cost of the transmission will be always the sum of the two costs, because there is a wire or cable link connecting them.

Example 3.3. The basic operation of BIP for tree construction will be described taking as reference the scenario shown in Figure 3.3, where the axes give a reference regarding distances. Basically, a broadcast tree is computed from a source node, in our case Node 10, by adding nodes one at time. At each step, the less expensive action to add a node is selected, either by increasing the radius of an already transmitting node, or by creating a new emission from a passive one.

Step 1. Figure 3.3(a) shows a ten-node network, in which Node 10 is the Source. Initially, the tree consists of only the Source. We begin by determining the node that the Source can reach with minimum expenditure of power, i.e., the Source's nearest neighbor, which is Node 9. This node is added to the tree. Thus, at this point, two nodes are included in the tree, namely Node 10 and Node 9 ($10 \rightarrow 9$).

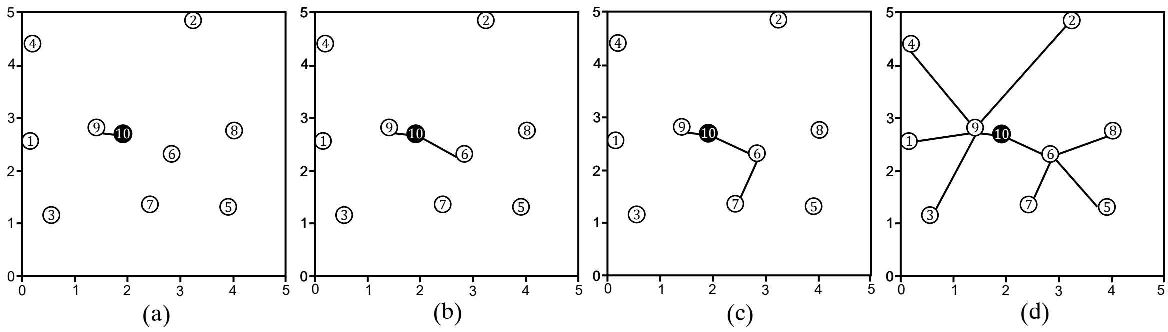


Figure 3.3: Example of tree construction using BIP: (a) step 1: 10 → 9; (b) step 2: 10 → 6; (c) step 3: 6 → 7; (d) final tree.

Step 2. We then determine which "new" node can be added to the tree at *minimum additional cost.* There are two alternatives. Either Node 10 can increase its power to reach a second node, or Node 9 can transmit to its nearest neighbor that is not already in the tree. In this example, Node 10 increases its power level to reach Node 6 (Figure 3.3(b)). Here is the crucial point: the concept of *incremental cost.* It means that Node 10 increases its power from a level sufficient to reach Node 9 to a level sufficient to reach Node 6. Suppose the cost of a transmission between Node 10 and Node 9 to be, in terms of power, $p_{10,9}$, while between Node 10 and Node 6 is $p_{10,6}$. Then the incremental cost associated with adding Node 6 to the preliminary tree consisting of Node 9 and Node 10 is $p_{10,6} - p_{10,9}$. We exploited in this way the multicast advantage of the wireless media, because both Node 6 and Node 9 can be reached when Node 10 transmits with sufficient power to reach Node 6.

Step 3. There are now three nodes in the tree, namely Node 6, Node 9, and Node 10. For each of these nodes, we determine the incremental cost to reach a new node. The node that can be added to the is Node 7 from Node 6 (6 → 7). Since Node 6 was not transmitting, its incremental power is equal to a full transmission power, but it is worth to do it because the cost would lower than the case where Node 10 further increments its power (Figure 3.3(c)).

Continue. The procedure goes on until all the nodes are included in the tree, as shown in Figure 3.3(d). The order in which the nodes were added is: 6 → 8, 6 → 5, 9 → 1, 9 → 3, 9 → 4, 9 → 2.

At the end, the total energy of the broadcast tree is simply the sum of the energy expended at each of the transmitting nodes in the tree; leaf nodes (which do not transmit) do not contribute to this quantity. The implementation of the BIP algorithm is based on Prim's algorithm as well, but with a fundamental difference: instead of having as inputs the link costs, the BIP updates at each step the networks parameters in order to have as reference the incremental cost.

All the algorithms presented so far are defined as centralized. In general, in such methods, there is a central unit that has all the information about the network and so is able to to coordinate the other nodes in order to achieve an optimal configuration. In fact, we can notice that those methods do not say who chooses the path to follow and this may be one focus of control. In fewer words, in centralized problems we have an entity that is responsible for the whole network, which get the information from all the connected devices, i.e. knows all the link parameters for all the link between every node, and makes know to everyone what to do. However, they result not suitable for D2D networks. In fact, the latter are decentralized and due to the lack of a central unit that guarantees a spread view of the network scenario is more difficult to obtain an optimal topology. Nevertheless, the fact remains that the only way for finding a solution in the D2D wireless scenario is to move toward a decentralized approach.

In distributed algorithms every node decide by its own, which makes the network more scalable and opens the way towards dynamic environments. In addition to that, controlling and managing the network results to be easier. There are, however, also drawbacks in such approach: their performance is poor compared to the well-known centralized algorithms. This is mainly due to the limited knowledge of the network that each of the nodes has, if compared to that of the central unit which would allow everyone to act in the best way.

In support of the above argument, taking as benchmark the BIPSW centralized algorithm proposed in [26], we see that decentralized solutions, as the broadcast decremental power (BDP) algorithm [30], are not able outperform the centralized solution.

The BIPSW is an appropriate modification of the BIP algorithm and improvements are obtained by reducing the transmission energy requirements of some nodes. The decentralized alternative BDP it is shown to behave better than the original BIP, bot not of its subsequent ameliorations. In this regard, we will see how the introduction game theory has supported distributed solutions, allowing to achieve considerable results in decentralized environments.

3.2 Game Theoretic Approaches

Game theory is introduced to solve the problem of having performance close to the centralized systems. The center of this new approach is to decide in a strategic situation what is the better solution for a participant in a game, where the node decision affect the others and viceversa.

Note that in distributed algorithms, as those for which the game theory is applied, each node may have also a global view of the network, e.g. energy levels, communication load or neighbor degree. The indispensable requirement is that each of them have to decide on their own. When such a spread view is provided to the node the game is said to be a *complete game*: you have global information about the reference scenario and an agent can use the information for local decision processes, thus no external entity is involved.

The game theory allows to improve flexibility by reorganizing to increase local control. No more communication towards a mandatory centralized point. A lot of effort is being made to study how GT can be good for dynamic environments that we will see will be the fundamental concept of this research. This theory may help also in reducing the amount of information that has to be exchanged among all the participants, but what matters is being able to decide for themselves to which nodes to connect.

– The first game theoretical mechanism we are going to describe is the one proposed in [31]. This algorithm is designed based on a potential game model, which is a game where all agents are maximizing (utility) or minimizing (cost) the same function. Generally, in this game the participants have aligned interests and depending on the form of the potential function it is possible to verify some properties as the existence af a Nash Equilibrium. Such a mechanism has been demonstrated to be capable to overcome other conventional decentralized algorithms like BDP [30] and DynaBIP [32] in terms of total energy expenditure. Moreover, it has been shown as this method can also outperform centralized algorithms such as the BIPSW [26] when the network is not dense.

  From this moment on the game theoretic algorithm [31] is presented. The system model provides that in the network there are multiple nodes including a source. The source has a message that has to bee received by all the other nodes in the network. Such a message should be forwarded in a multi-hop manner, which means that the information initialized by a source is distributed with the help of some node located between the source and the destination. Hence, the source's message has to be forward by other nodes in the network. This work aims at minimizing the total transmit power required for the information to be received by everyone.

  In this game, the nodes in the network are modeled as rational players, i.e. each node nodes rationally decide to minimize their own costs on each iteration, and the weakly dominant strategy, illustrated in section 2.2.3, is exploited. The network scenario is defined as follows:

- $N + 1$ wireless nodes with random locations are forming the network: a source node S and a set of N other nodes numbered from 1 to N;
- every node has a maximum transmit power $p^{\max}$;
- the data dissemination is modeled as a graph with a tree structure. This tree is called *broadcast tree*;
- for a single hop point to point transmission in the tree, node j as a transmitter and node i as a receiver are called the parent node and the child node, respectively. In order to benefit from the broadcast nature of wireless channel, a parent node may serve multiple child nodes and the set of child nodes served by a parent node j is denoted by $\mathcal{M}_j$.

The game is designed in a way that minimizing the cost at each individual node minimizes the total transmit power. It is played iteratively such that at each iteration, a node makes decision given the decision of the other nodes. In addition, it is important to specify that the game is *child-driven*, that is, a node as child selects a parent with minimum cost. The action set for each player has been defined in a way that cycle must not occur.

In the proposed algorithm the nodes are incentivized to choose a common parent, so that multicast transmission are preferred. To do this, *Marginal Contribution* (MC) is used as cost sharing concept that will be specifically explained in **section xxx**. Following this approach, the cost of a child i when his action a_i is to connect to a parent j has been defined as:

$$C_i(j, \mathbf{a}_{\text{-}\boldsymbol{i}}) = p_j^{\mathrm{Tx}}(\mathcal{M}_j) - p_j^{\mathrm{Tx}}(\mathcal{M}_j \setminus \{i\}) \tag{3.1}$$

where $\mathbf{a}_{\text{-}\boldsymbol{i}}$ is the action profile representing the actions of all players except the i-th one. The multicast transmit power of a parent node j correspond to p_j^{Tx}, which is dominated by the highest required unicast power $p_{i,j}^{\mathrm{uni}}$ of its child nodes and is given by

$$p_j^{\mathrm{Tx}}(\mathcal{M}_j) = \max_{i \in \mathcal{M}_j} \{p_{i,j}^{\mathrm{uni}}\}. \tag{3.2}$$

The assigned positive cost (3.1) is equal to the difference between the highest and second highest required unicast powers in $\mathcal{M}_j$.

Since we are interested to the total power in the network, that we want to minimize, it will be equal to

$$p^{\text{net}} = \sum_{j=1}^{N+1} p_j^{\text{Tx}} \tag{3.3}$$

and is computed as the sum of all the powers, including the one spent by the source, needed to forward the message. Obviously, if a generic node j is not transmitting its p_j^{Tx} is equal to 0.

The Nash Equilibrium (NE) is considered as the solution concept of this game. Since for every node the number of its neighboring nodes is limited (from 1 up to a maximum of N), then the number of iteration for every participant is limited as well. When the number of iteration is finished, there would be a situation in which not dominant actions are available to the player. Thus, there will be a point where no gain can be achieved for all agents by unilaterally deviating and the nodes will stop updating their actions.

It is time, reached this stage, to introduce an example of the last game theoretic algorithm, so that the steps necessary for its operation can be clearly illustrated.

Example 3.4. Consider a network composed of 5 nodes. The scenario to refer to is that shown in Figure 3.4. A source node named S has a message for all other 4 nodes which are indicated with the numbers from 1 to 4. We now see the steps leading to the formation of the *broadcast tree*.

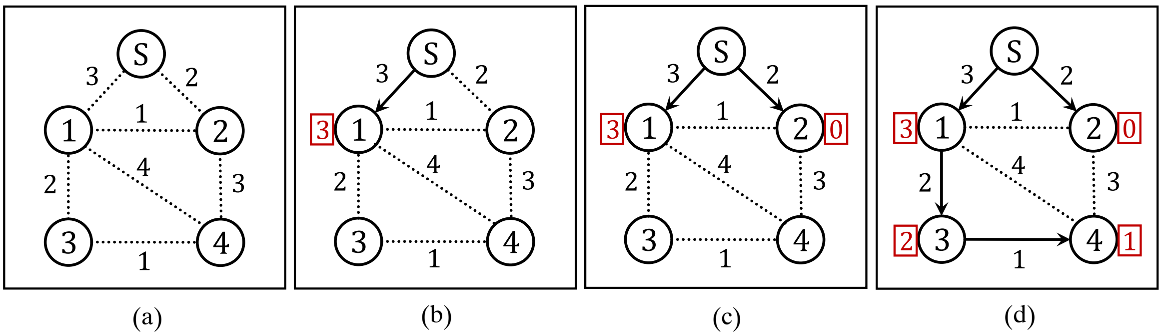


Figure 3.4: Mechanism Design with Potential Game for Energy-Efficient Data Dissemination in D2D Networks: (a) step 1: analyzing network scenario; (b) step 2: costs computation and $S \to 1$ association; (c) step 3: $S \to 2$; (d) final broadcast tree, $1 \to 3$ and $3 \to 4$.

Step 1. At the beginning the tree is composed by only the Source. No other node belongs to the tree, thus *Broadcast Tree* $= \{S\}$. We have therefore to start considering the neighbors of the nodes already belonging to the acyclic structure and these are node 1 and node 2 which are close to S. In figure 3.4 (a) we see that node 2 and node 1 are within the area that can be covered by the transmitting power of S. At this point we choose randomly one of these two reachable nodes. In our example we are going to pick node 1. The action set of the the selected node would be then

$$A_1 = \{S\}$$

because node 1 can choose only S, being the latter the unique node that is already part of the propagation structure. We can now proceed with the computation of the cost associated to the decision made by node 1 of choosing node S as its parent node. This mean that the child node 1 will receive the information from its parent and eventually it will be in charge of further propagate the message.

Step 2. In this phase we see the evaluation of the cost associated to the choice of a parent by a child node. The cost is computed by considering the formula (3.1). In a simplified view, this formula states that $C_{i,p} = C_p - C_{p-i}$, which means that: the cost $C_{i,p}$ for a child i of choosing the parent p is equal to the cost for connecting to the parent C_p (basically the highest unicast transmit power when child i is connected to parent p) minus the cost of the parent's communications already going on (corresponding to the highest unicast required power when child i is not connected to parent p yet). In case of node 1, we have simply

$$\begin{aligned} C_{1,S} &= C_S - C_{S-1} \\ C_{1,S} &= \;\; 3 \;\; - \;\; 0 \;\; = 3. \end{aligned}$$

As we can see C_S here corresponds simply to the weight associated to the edge between node S and node 1. Moreover, since the source was not transmitting so far, the cost related to the power for existing connections from S before this new link is zero. Once the cost is computed and we established the new connection, we can associate a weight to the node just added to the tree. As we see in the figure 3.4 (b), such a weight is represented by the number 3 within the red square beside node 1.

Step 3. The same procedure, as for the node 1, must now be repeated for another node near those who already belong to the broadcast tree. For this purpose let us pick node 2. Its action set will be different from the one seen for node 1. In fact, here the nodes belonging to the propagation structure are two (*Broadcast Tree* $= \{S, 1\}$) and they correspond exactly to the two choices available at node 2. This is because the latter, as we see in the figure 3.4 (a), is reachable from both the source and node 1. Consequently, the action set of node 2 is

$$A_2 = \{S,1\}.$$

In order to make the selected node choose among the two possible actions, the cost function has to be evaluated for both alternatives. The one that will provide us with the lowest cost value will be chosen as parent node for the child node 2. The cost for association to node 1 is

$$\begin{aligned} C_{2,1} &= C_1 - C_{1-2} \\ C_{2,1} &= \;\; 1 \; - \; 0 \;\; = \; 1. \end{aligned}$$

Again, node 1 has not on going transmission, so the cost is simply equal to the one associated to the wireless link between them. Instead, results more interesting the calculation for connecting to the source, which shows the peculiar aspect of this algorithm:

$$\begin{aligned} C_{2,S} &= C_S - C_{S-2} \\ C_{2,S} &= \;\; 3 \; - \; 3 \;\; = \; 0. \end{aligned}$$

The cost for connecting node 2 directly to the source is 0. This is because the source is already transmitting with a power of 3 to which must be subtracted the value associated to the connections already in place when the node 2 is not connected. Thus, the broadcast tree is updated with the connection from the source to node 2 and the latter is associated with a cost of 0 (red value next to node 2 in figure 3.4 (c)).

Continue. The same procedure has to be applied for the remaining nodes still not belonging to the acyclic structure that now is $Broadcast\ Tree = \{S,1,2\}$. Considering node 3, its action set is composed only of node 1, being the unique neighbor appertaining to the tree: $A_3 = \{1\}$. The cost that can be computed is just the one related to node 1, which is the parent from whom the child node 3 will receive the information:

$$\begin{aligned} C_{3,1} &= C_1 - C_{1-3} \\ C_{3,1} &= \;\; 2 \; - \; 0 \;\; = \; 2. \end{aligned}$$

Such a value is then the weight representing the addition of the new node, illustrated within the rectangle beside node 3 (figure 3.4 (d)). The last step to perform is the connection of node 4 to one of the nodes forming its action set $A_4 = \{1,2,3\}$. To select the parent let us compute the three costs and see which will be the minimum:

$$C_{4,1} = C_1 - C_{1-4} = 4 - 2 = 2$$
$$C_{4,2} = C_2 - C_{2-4} = 3 - 0 = 3$$
$$C_{4,3} = C_3 - C_{3-4} = 1 - 0 = 1.$$

Note that $C_{1-4} \neq 0 = 2$ because node 1 is already forwarding the information to node 3, so the difference between the highest and second highest required unicast power is 2. For the other two computation, the second highest required unicast power is 0, since node 2 and node 3 are not forwarding any information before the eventual connection of child node 4. The final broadcast structure is the one represented by non-dashed arrow in figure 3.4 (d).

As a last stage, the total power in the network can be evaluated using the formula (3.3), which gives us the result: $p^{\text{net}} = 3 + 2 + 1 = 6$. Looking at figure 3.4 (d) the last result corresponds simply to the sum of the cost values we computed in the previous steps (numbers inside the red rectangles) and we see how the benefit of multicasting for wireless network is exploited for $S \rightarrow 1$ and $S \rightarrow 2$.

It is left to the reader the computation of the total power on the scenario in figure 3.4 with the Dijkstra algorithm, which we can anticipate will be equal to 7. The reason is that $S \rightarrow 2$ and $2 \rightarrow 1$ links are preferred to those previously selected. Furthermore, by applying the BIP to the same situation, the result was seen to be equal the one evaluated with the last algorithm studied so far, that is, 6. We see then how the same performance of the centralized solution has been obtained in a totally decentralized manner.

– A further game-theoretic framework [33] is developed to study the problem of selecting neighbors for selfish nodes in a wireless D2D network. Here is proposed a Neighbor Selection Method (NSG) to optimize energy consumption by the help of game theory modeling. In this method, each node tries to select its own neighbor in a selfish way by the help of local and global connectivity. But it has a limitation that it cannot consider other parameters of the network. Hence, some of the network metrics are not optimized. For instance it adopts the extreme traffic model, without considering the general case, and they model the problem as static game, without considering the dynamic nature of the game. They just analyze the convergence result, but don't consider the problem of how to optimize the system performance from the system view, either. To sum up, such a work on non-cooperative game just considers the case with strong assumption on network traffic, which assumes wireless nodes always have packets to send. Such extreme assumption is obviously impractical for actual wireless network and is not suitable for the mobility of applications without intelligent spectrum management.

- Currently, most work application of game theory focus on improve the performance of networks, such as [34] where is introduced a game theoretic method, called *forwarding dilemma game* (FDG). The performance of the whole system depends on the forwarding willingness of the devices, but this last aspect will be further analyzed in the next section. The FDC is a game theoretic approach that introduce a *forwarding probability* for strategy selection based on the mixed strategy Nash equilibrium of the game. It limits the number of redundant broadcasts in dense networks while still allowing connectivity. The game is composed of the number of players receiving the packet, the forwarding cost and the network gain factor and it offers primarily two strategies: forwarding or dropping the packet. Using a mixed strategy Nash equilibrium, discussed above in 2.3.2, the authors derived the forwarding probability that leaded to a mitigated broadcast storm mechanisms. Since the the sending probability will allow to forward the message only to a subset (winners of the game) of the total payers set, the number of nodes participating in the route discovery process is reduced. The selected devices resulted to be at the end those who will spread the information with the lowest cost for the network.

- Other problems that have been formulated in terms of potential game for broadcasting are [35] [36]. They focus on a distributed potential game-based algorithm that addresses the *minimum transmission broadcast* (MTB) problem in wireless networks, especially for the many-to-all scenario. The authors of [35] proposed a decentralized algorithm able to construct a broadcast tree with a minimum number of hops. The main issue of this study lies in the power control mechanism tha has been applied. Hence, it provides that parent nodes always send information at the maximum power, which distances us from a sufficiently approximate calculation of the actual power used, along with the possible inappropriate exploitation of the advantages of wireless multicasting.

An analogy between the last algorithms found in the literature and the project proposed in this thesis, which we will be studying in subsequent chapters, can be made. In fact, in our distributed game theoretic algorithm we will introduce a probability that a node may disappear at any time from the network, in order to have a dynamic environment. This can be seen as a variation of the forwarding probability and the link reception probability concepts described in [34] and [35], respectively.

Meanwhile, existed schemes only consider static game, which indicates the mobile terminals never leave the network. Such assumption indicates that these schemes are unable to support mobile applications, as well as the majority of those works that will be presented later.

3.2.1 Incentive mechanisms for Data Replication

In D2D wireless networks, in order to transmit in a multi hop manner, an incentive is very important for the node in the middle to relay the data. If a node wants to relay a huge amount of data to another one, it needs an incentive for spending its energy. The forwarding node requires then some reward for forwarding and cooperation. For this reason, the receiving nodes that get the service in D2D network must pay something for the service they receive.

Thus, it is really important how they decide:

- if a node wants to pay;
- how much would be the price.

In many works, cooperation is induced by payments toward the node that relays the information and such credit is used to encourage those nodes to cooperate. Middle agents usually decide independently the extent of their cooperation with the network, depending on their energy status and reputation they could get based on their previous behaviors.

– A pricing and payment-based power control scheme was presented in [37]. This non-cooperative game has the goal of attaining a fair energy share among wireless D2D networks users without a central billing system. Such a payment-based scheme, to alleviate selfishness problem, controls the power that each user should use by providing different compensation paid for their own share of performance. The authors treat the problem of lack of motivation among users by giving them the due share through the use of an adaptive pricing function. The latter generates values basing the calculation on the interference generated by network users. In order to achieve the Nash Equilibrium of the game as efficient as possible, the pricing function has be designed such that the greater is the interference generated by a user transmitting at high power level, the greater will the value of pricing it will be pay. From this we can deduce that the price is strictly increasing with power. The mechanism provides that each user announces a set of price coefficients that reflects different compensations that should be payed by other users. To do that, an intense inter-node signaling scheme and massaging control is required. It thus ends up in much additional overhead for finding the Nash Equilibrium and so the service preferences, also because the environment conditions may change really fast. At the end, the increase in fairness and the the reduction of transmission power have not been radically improved.

– The study [38] shows a pricing and payment technique to obtain an optimal path among spatially distributed autonomous agents. The proposed algorithm is based on Dijkstra's algorithm in order to minimize the transmission cost for routing the information. Moreover, it is able to improve lifetime of network and load distribution using game theory. To do that four important parameters are optimized: the distance between two nodes, the remained energy of the network, the network load traffic, and as a further matter the path reliability. The mechanism is designed such that each data packet transmission has a cost for each node that participates in the route. This cost is a function of the four previously mentioned parameters. Nodes wanting to maximize their profit, will accept to be part of the path if its profit is not negative. The process of checking the positivity of utility function is added to the algorithm. As said before, the output of this algorithm is the optimized path and the results of their simulation analysis show the achievement of the objectives set. However, the mechanism resulted unable to maximize the utility for all participant and so not everyone has benefited from being willing to cooperate. In addiction to that, a simple problem arises form the use of a graph search algorithm which not considers the wireless nature of communications.

– Among the latest works, in [39], to stimulate selfish users to participate in forwarding information, a game theoretical incentive pattern based on the relay selection is proposed. In mobile D2D networks, the propagation process of data needs resources (bandwidth, energy, buffer, etc.), which are limited in reality. Therefore, to mitigate the selfish behavior of participants, which may not be active to forward data to others or receive data from their neighbors, a credit-based incentive scheme with the use of virtual currency is applied. This mechanism allows a node to pay for the relay service it receives and this helped in reducing the selfish negative effect on data propagation. In this method, a message carrier selects next intermediate nodes based on its available resources, such as those mentioned above. Although, this work has made a lot of efforts for the resource allocation, even though with some flaws in the management of the latter. Moreover, the characteristics necessary for the interaction in mobility conditions have not been sufficiently considered.

So far, different approaches and mathematical methods have been used to characterize the pricing problem. However, the main challenge is represented in the trade-off between energy conservation and network throughput, which makes the problem of energy conservation more complicated. Unfortunately, determining the pricing function and utility function in these games is non-trivial and requires a great deal of effort. Nevertheless, finding the "optimum" pricing and utility function is still an open problem. After outlining how the throughput can be improved by means of a specific definition of the utility function, works that applies Prospect Theory to wireless communications scenarios will be presented.

To conclude, the survey [40], which gives a taxonomy of games applied to wireless networks, would be of precious help for any researcher that wants to start contributing to this area. It is very extensive and capable of filling a need in the current literature. It reviews also latest works using economic and pricing models, showing how those became very useful in wireless D2D networks.

3.2.2 Utility Design for Wireless Applications

An overview of game-theoretic approaches is presented in which energy-efficient utility design in wireless networks has been identified [43–45]. Generally, in a non-cooperative (distributed) games each user seeks to choose a strategy that maximizes its own utility while satisfying its quality of service (QoS) requirements. Hence, an efficient transmission power control method is necessary for wireless networks. However, high transmission power can improve the signal strength as well as signal quality at the receiving end, but at the same time it can cause more interference and consume of energy. On the other hand, lower transmission power can reduce the interference signal level, but may not satisfy user needs. Based on this discussions, the choice of the utility function has a great impact on the nature of the game and how the users choose their actions.

When energy efficiency is the main concern, a good choice for the utility function is the one that is considered in the following works, which measures the number of reliable bits transmitted per joule of energy consumed. It has been showed to be particularly suitable for energy-constrained networks. The tradeoff between a higher signal-to-interference-plus-noise ratio (SINR) level at the output of the receiver, which means having a lower bit error rate and hence higher throughput, and lower SIR, which lead to an extended battery life and low interference among users, has been so formulated as:

$$u_k = \frac{T_k}{p_k}. \tag{3.4}$$

The nominator is the *Throughput* that here is the net number of information bits that are transmitted without error per unit time (this sometimes is referred to as *goodput*) and the denominator is the transmit power. Both the magnitudes are referred to user k and their ratio represents the utility that the user achieves selecting a specific strategy. As has been said in the previous explanation of game theory, such a payoff is going to measure the degree of "happiness" of the player. This utility function, has we can deduce from the formula, has units of $bits/joule$.

The main motivation behind formulating a game with the above utility function is the large interdependence between the actions of the network nodes due to two main factors:

- limited amount of energy available to participants;
- interference across wireless nodes.

As we will see, the formulation (3.4) captures very well the tradeoff between throughput and battery life and is particularly suitable for applications where energy efficiency is more important than achieving a high throughput. Along with this need, the actions available to each user in trying to maximize its own utility are not only the choice of the transmit power, but depending on the situation [42], the user may also be able to choose its transmission rate, modulation, packet size, multiuser receiver, multi-antenna processing algorithm, or carrier allocation strategy.

– Paradigmatic examples of this approach can be found in the broad field of wireless communication, in which power control games are used to mitigate the co-channel interference and improve the system throughput. In wireless networks, one popular application is the *uplink* power-control game provided in [43] for code-division multiple-access (CDMA) networks. It considers a single cell of a wireless CDMA network where user equipments compete with each other to access the channel using orthogonal codes simultaneously. The main goal was to allow users to control their uplink transmit power in a distributed manner so that their QoS would be optimized. All this had to be done in presence of the mutual interference that occurs among the users. Game theory has been adopted to solve the issue of how the resources should have been allocated, in this specific case the transmit power. To do this the utility has been designed as a function of the power consumed by the users and the SINR they attain. Thus, through the formula (3.4), individual payoff were provided to each player, who tries to maximize it. The strategies of the game were in fact the transmit power values and it has been shown that the algorithm had good performances in terms of energy efficiency. A constraint on the total received power has been imposed and since user's interference depends on his own power allocation, it can make the problem non-convex. As we have seen, here the pricing of transmit powers is a linear function of the transmit power and this method leaded to improvements of the non-cooperative power control game. As conclusion, this work illustrated us an innovative view of distributed game theoretic algorithms, but there is neither a proof of convergence nor a theoretical explanation of the performance improvement due to pricing.

– A game theoretic approach [44] to energy-efficient power allocation in multicarrier systems is presented in here. This paper studies power control for *uplink* transmission of a cellular system (like the one described above) and an algorithm for resource allocation is suggested, which leads to the best subcarrier selection. Motivated by the facts that mobile terminals have a limited battery lifetime, this mechanism results more suitable for scenarios where the main concern is not the transmission rate. The game considers a CDMA system in which the information is sent on multiple parallel stream by the players. The strategy of each node is to choose its transmission power. It differs from the previous research by considering a receiver design where the realized SINR on each subcarrier is the same. However, such multi-carrier power control game is more complicated than power control game introduced early, and so it may be possible not to achieve any Nash Equilibrium. Moreover, it assumes the spectral utilization information is known as a priori to all participants with the help of a centralized infrastructure, which is not realistic in scenarios where an infrastructure is not available. These works that considers radio resource management in a single wireless access network problems are nevertheless predisposed for heterogeneous wireless environments.

– In a similar manner to the last two works presented above, the authors of [45] generalized the game further by considering effect of modulation on energy efficiency. In particular, a non-cooperative game is proposed in which each user can choose its modulation level (e.g., 16-QAM or 64-QAM) as well as its transmit power and transmission rate. Like the other cases, each user chooses its strategy in order to maximize its own utility while satisfying its QoS constraints. To do that, the utility function applied in this work has been measured with formula (3.4), but further specifications for other parameters are introduced, so to manage, besides the transmit power, transmit symbol rate and constellation size. Through the proposed game-theoretic framework, the effect of coding on energy efficiency is studied and quantified, along with the tradeoff between energy and spectral efficiency. Since possible variations and bad conditions of the channel may occur, the degradation of service quality for every node cannot be avoided. This strategy is then different from the one obtained when maximizing simply throughput, because an user can switch to a higher-order modulation scheme improving the spectral efficiency, but with the drawback of degrading its energy efficiency. The main important thing that can be noticed so far is that the problem of energy-efficiency and rate control for dynamic wireless network has not been studied yet.

As can be seen, in the current literature there is no application of this well-understood method for system optimization to topology construction problems. In this thesis we take inspiration from the model presented in this section, which is used mainly for resource allocation, to provide the decision-maker with an utility function capable of evaluating the neighbor node to which it is better to connect. This will be done in dynamic conditions, that is, a node could leave or join the network at any time.

It is worth to remark that an important assumption has been made in the last three works: a transmission is assumed to be successful if a fixed minimum SINR requirement is met. We are going to keep this assumption to reduce complexity and make math procendimers more intuitive. Moreover, the pricing terms, as a heuristic to improve the performance of a NE in power control games, do not need to be necessarily linear as seen here. However, this choice has been made to allow the cost function to respect and provide meaning from the physical point of view.

3.3 User Interference with Protocols

Traditional game theory assumes that all the players in the game are rational and uninfluenced by real-life perception. This is due to the fact that in most existing game theoretic studies participants make decisions according to their expected utilities (EUT). To address this issue, prospect theory (PT) has been proposed to provide a subjective probabilities that models how the user perception can deviate from the conventional, rational norm set by game theory. Thi has been done thanks to the three main characteristics of prospect theory discussed in 2.1.2:

- **Probability weighting function**: A decision maker tends to overweight small probability events, but underweight medium and large probability outcomes (figure 2.2).
- **Asymmetrical value function:** A decision maker tends to be risk averse, since he strongly prefers avoiding losses than achieving gains. This function is then steeper for losses than for gains (figure 2.4).
- **Impact of reference point:** A decision maker evaluates the various options available to him with respect to a reference point (the zero point). Only at this point he can quantify the gain and losses, thus the reference point significantly affects the valuation.

In literature, PT has been applied mostly for resource allocation in wireless D2D network. Thus, its applications have not been sufficiently studied in the construction of the network topology when an end user it interferes with the nodes selection. Here we present some work that gives an idea on how PT is increasingly becoming an important framework for making decision.

- One of the first paper that applies PT to understand wireless networking is [41], where the authors compared the equilibrium strategies of a two-user random access game under EUT and PT. This has been done to study channel access between two subjective end-users in wireless networks. At this early stage of PT application, considering all the three characteristics listed above was challenging. Indeed, only the probability distortion was considered keeping the value function linear. Basically, in this non -cooperative game selfish players adjust their transmission probabilities over a collision channel according to rewards received for successful transmission. At the end, they proved the existence of a unique Nash Equilibrium under PT.

– Many other studies [46–49], in particular the one proposed in [48], applied PT to solve problems in the social science. Such a study suggested a routing policy choice model, based on prospect theoretic model, to predict travelers' route adaptation to to real-time information and risk attitudes. The approach showed that travelers can make strategic route choices and path prediction where predicted under EUT and PT models. Using simulated choice data has been revealed significant differences in path share predictions under two behavioral paradigms. In fact has been showed how PT provides a better framework than EUT for a routing policy choice model conditioned by end users.

– The research [4], which considers behavioral economics (PT in particular) to understand user decisions in networking, is an extension of the work proposed in [41] where only two homogeneous player has been considered in the wireless random access game. Here, the authors have added the comparison of both two player heterogeneous and N-player homogeneous games where general random access channel model can be applied, that is, packet reception probability can range within $[0,1]$. A linear value function is supposed also here with the addiction of probability distortion. To analyze the impact of end users deviation from EUT in a more practical scenario, a data pricing in the network has been introduced. As a conclusion, where the users do not objectively evaluate their probability of successfully accessing the channel, the impact on evaluation if a service is worth the purchase resulted in degradation of system throughput and energy consumption.

– Authors in [50] proposed a prospect pricing mechanism under game theory to improve the utilization of radio resources in wireless networks. This spectrum allocation algorithm improved the will of relay nodes to provide service in the presence of subjective users. This research includes a more detailed mapping techniques to normalize objective and subjective measurements, taking into account the probability that a node accepts the offered bandwidth. The equilibrium strategies of the adaptive decision-making optimization is compared under EUT and PT. Prospect pricing resulted indispensable to combat the under-weighting effect by the end-users that moves PT away from the ideal performance of EUT. Thus, there is still the need better design of optimal pricing schemes and networking algorithms considering PT for wireless environments to manage the ever increasing demand for connections.

After analyzing some of the work that has been done on the use of prospect theory in wireless networks, two main issues have emerged from the literature:

- under EUT in most of the case has been achieved an optimal performance
- in general, deviation from EUT resulted in a loss of performance for the overall system and pricing has been adopted in order to reduce such a loss.

It must point out in the end that we are still at the very initial stage of this new research for the identification of the possible impact of end-user behavior on wireless systems.

Chapter 4

Utility Design for Dynamic Environments: Single User case

In D2D networks, wireless devices, simply called nodes, have limited transmission range. Therefore, each node can directly communicate with only those within its transmission range (i.e. its neighbors) and requires other nodes to act as relays in order to communicate with out-of-range destinations. We assume that a source nodes has a message for all the other nodes of the network and due to the transmit power constraint at the nodes, the data should be disseminated in a multi-hop manner. In the considered system model a fundamental assumption has been made: the network is dynamic. A probability of leaving the network for each node has been so defined based on the mobility of the environment. This is one of the main innovations that our project introduces compared to the game theoretic framework found in the literature.

The dissemination mechanism of a message common to all users composing the network has been so developed in order to support changes in numbers and density of participants. We decided to start from a very simple model, in fact there is only one new agent who has to choose whether to receive information from a dynamic node or a static node, taking into account the risks to which the new user is exposed according to the decision taken. For this reason, in this chapter the game theory will not be addressed, because there is no competition between new nodes that need to receive the information, as there is only one new agent in the network.

As a design factor, we decide to fix the data rate among the nodes involved in communications. This has been done in order to reduce the number of parameters (variables) in the networks and for helping us to get more intuition into mathematical procedures. Another particularly innovative aspect is the fact that, at the time of connection of the new agent, the dynamic node has already received part of the message. There is therefore an ongoing transmission between the two possible service providers and the new user must take into account this issue, because as long as the relay does not receive the entire message, the latter cannot leave the network.

We wanted to examine how the network topology (broadcast tree) construction would change when one agent knows that the node it would like to connect has a certain probability to leave the network, which also depends on the total amount of data that the potential parent node has received.

The further knowledge provided to each child node about the possibility that a node may leave for sure will influence the preference among all the parent nodes available to connect. At this point, the determination of the utility function representing the preference of each individual player, which is the key point of our research, has become crucial. Conditions under which a child node should choose either the dynamic node as a parent, which perhaps can lead to more efficient energy consumption, or the static node, which will certainly not cause failures in transfers since it always remains in the system, will be therefore defined. The analysis of the impact of mobility on the performance for multiple users will be then addressed in the next chapters.

4.1 System model

In the initial stage of the work, a network composed of three wireless nodes with random locations is considered: a source node s that wants to transmit the information to all the other nodes, a relay node r already connected to the source and a new node i that wants to connect to the network (figure 4.1).

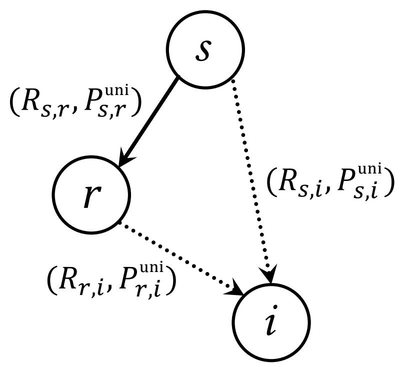


Figure 4.1: Network scenario, including edges characterizations, to which reference is made: Solid and dashed arrows show current and possible connections, respectively.

The message dissemination from the source node to the other nodes can be modeled as a graph with a tree structure called *broadcast tree*. For a single hop point to point transmission, a node already belonging to the broadcast tree and a node that wants to join the tree are called *parent* node and *child* node, respectively. In this scenario, the set of parents nodes is composed by node s and node r, since they own at least part of the information that can be sent to others, while the only child is represented by node i who has to select from which node to receive the message. In this regard, we will refer to this situation by saying that the game is child-driven, that is, a node as child selects a parent with minimum cost in order to minimize the total transmit power in the network.

The main important feature that has been introduced in this project, with respect to other related works studied in chapter 3, is the introduction of *dynamicity*. Whit this we are stating that a node may leave the network with a certain probability after that it received the whole information. To implement this, a *probability of stay* in the network p_j^{st} is associated to every node $j = \{s, r, i\}$. Moreover, in order to simplify the issue, here we just assumed that a node suddenly turns off depending on the level of mobility of the wireless environment. The source of the message is always assumed to be under no mobility condition so that $p_s^{\text{st}} = 1$ in any case. We also see in figure 4.1 that every edge between two couples of nodes, for instance from a generic transmitter node j and a receiver node k, is characterized by:

- a Rate $R_{j,k}$ that simply represent the number of bits that are conveyed per unit of time over the single hop channel;
- a transmit Power $P_{j,k}$ that corresponds to the one needed in a unicast transmission to have a minimum signal to noise ratio (SNR) to the receiver from parent node j to child node k. This allows to have all the bits successful received.

The last assumption will be better explained in the next section where is described the *packet success rate* (PSR). We want it always equal to 1, which translates in having no error during a transmission.

As most of the basic graph search algorithm, the source always belongs tho the propagation structure, from the very first moment. Moreover, here it is assumed that a connection between the source and the relay node has been already established at the time when the situation is being analyzed. That means the relay node is already receiving data form the source and thus r can contribute to the dissemination of the message.

In a broadcast tree, a child node has one parent node, but in order to benefit from the multicast nature of wireless channel, a parent node may serve multiple child nodes. In this situation, where a simple D2D network with three nodes is considered, basically two type of transmission may take place:

- **Unicast**: it refers to a one-to-one transmission between a sender and a receiver. Doing that, the transmitter node establishes a different unicast connection for each of the destinations. The data must therefore be duplicated and transmitted on each of the individual connections. This solution is shown

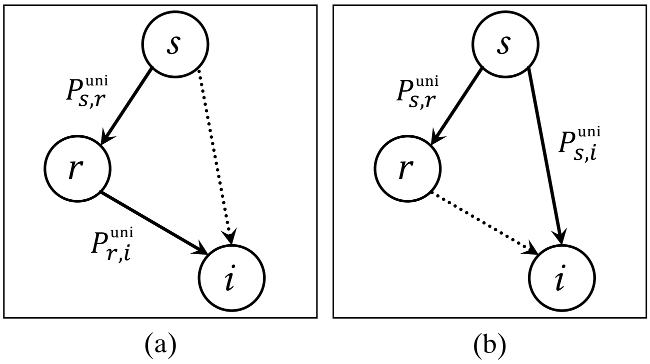


Figure 4.2: Possible connection available to communicate with node i: (a) unicast solution; (b) multicast solution.

in figure 4.2 (a). Here we see that two unicast connection are established between node s and node i where node r relay the information. The total power expenditure of the network is therefore equal to

$$P^{\text{tot}} = P_{s,r}^{\text{uni}} + P_{r,i}^{\text{uni}}$$

- **Multicast**: in this case the distribution of the information occurs simultaneously to a group of receiving nodes. This means that it is possible to transmit the same message to multiple end-devices without the need to duplicate the information to be disseminated (figure 4.2 (b)). The total power the parent has to spend for spreading the data to all its child nodes, considering that it does not have to communicate with each of them individually, is simply equal to the higher required unicast power and is given by

$$P^{\text{tot}} = \max(P_{s,r}^{\text{uni}}, P_{r,i}^{\text{uni}})$$

In this network we presuppose that an incentive for forwarding node is provided. However, such a mechanism is not implemented in this work, but as we have seen in section 3.2.1, many are the scheme proposed in other papers concerning this matter. Here it is only supposed a reward, in the general sense, to be given to the node that relays the message to other devices, so that it is always willing to spend its energy and resources for this purpose. This has been done to obtain the best in terms of total transmit power and interference in the network.

4.2 Problem Statement

Nearly all the studies analyzed in the previous chapter for data transmission in wireless D2D network consider that, since there is a source having a set of packet to transmit, all the receiving nodes are empty (they did not receive any packet). So, when they want to receive the information, they start receiving it at the same time. In reality, instead, some node may start receiving the message sent by the source and after other nodes may join the network. In this regard, we study exactly the case where one node already started receiving the message (node r), then an other (node i) comes and starts receiving from scratch. This situation is shown in figure 4.3.

We see that the source has a message of length L bits that has to be received by all the other nodes. The relay node r has already downloaded part of the information to which we have been making clear reference calling it $L_{s,r}^{\mathrm{dwn}}$. As a consequence, there is still a part to be received that corresponds to the remaining data to complete the transfer. This has been denoted with $L_{s,r}^{\mathrm{rem}}$ and during the time needed to receive such an amount of information we are sure the relay will not leave the network. Only after receiving all the data r will be able to leave the network, but before that happens it will remain an additional time that will depend on its probability of staying.

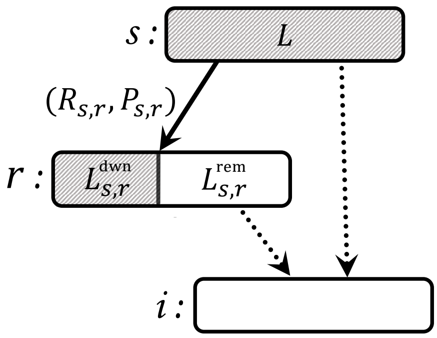


Figure 4.3: Message propagation with the relay that has already received part of the data.

The problem statement can be synthesized in three main points. So, we want to find an energy efficient way able to define the network topology for a scenario where:

1. A nodes arrives in the middle of an already going on transmission.

2. Such a node has to choose the relay node from which it has to receive the data in order to minimize the total energy consumption in the network. Here the alternatives available are either the source s or the relay r.

3. This is important because, analyzing the two situation, we see that:

   – if node i chooses the relay, the latter may leave after the reception of all the data. In this case, if node i is unable to receive the whole information before r leaves, it has it has to get all the data from the source by initializing a new connection from scratch. This may lead to a waste of energy due to the failure of the first transfer.

– if node i chooses the source, it is going to receive surely all the data without any failure given by the disappearance of the parent node. Nevertheless, this may be not the optimal way for minimize the energy consumption, because if the relay has an high probability of staying node i should choose r. In this case, therefore, there may be a loss of energy in the situation where the power required to receive data from the source is greater than that required to communicate with the relay.

From what has been described above, it is evident that we are looking for a compromise on which node to choose in order to receive the message: always to get the source without risk (perhaps not the most efficient solution) or take the risk (in the worst case of spending almost twice the energy needed for the transfer) and choose the relay.

At this point it is necessary to introduce some parameters that we may refer to later on during the work of this thesis.

4.2.1 Parameters

Network parameters are the key for the evaluation of a utility that will lead a node in selecting the best action. With reference to figure 4.3, we have already introduced three important parameters, which are:

L : Length of the message in bit that has to be disseminated by the source;

$L_{s,r}^{\text{dwn}}$: Amount of information received (downloaded) by the relay at the time when the node i joins the network;

$L_{s,r}^{\text{rem}}$: Amount of information remaining to the relay to complete its reception at the time when the node i joins the network. It can be easy calculated by subtracting from L the amount of already received data $L_{s,r}^{\text{dwn}}$:

$$L_{s,r}^{\text{rem}} = L - L_{s,r}^{\text{dwn}}.$$

Having such values available, it is possible to divide those different amount of bits by the rate of the channel between the two nodes involved in the transmission. Doing this, we can compute, for now in a general form, the *transmission time* between a transmitter node j and the receiver node k as

$$t_{j,k}^{\text{TX}} = \frac{b_{j,k}}{R_{j,k}} \tag{4.1}$$

where the numerator represents quantity of information (bit) on which we are interested in knowing the transmission time and the denominator is simply the bit-rate, known a priori, of the wireless medium between j and k. Obviously, the unit of measurement is expressed in seconds, as we have that $\frac{[bit]}{[bit/sec]} = [sec]$.

Referring now to our model, therefore by considering the source node s and the relay node r, the rate of interest to be put in the equation (4.1) is $R_{s,r}$. Furthermore, by keeping this rate at the denominator, we can substitute $b_{j,k}$ at the nominator with the values expressed by L, $L_{s,r}^{\text{dwn}}$ and $L_{s,r}^{\text{rem}}$. In doing so, i.e. replacing these quantities in formula (4.1) as in the order of writing, we obtain the parameters listed below:

$t_{s,r}^{\text{tot}}$: Total time required by the relay node r to receive the whole information L from the source s;

$t_{s,r}^{\text{dwn}}$: Time corresponding to the one that was needed for the relay node r to receive the amount $L_{s,r}^{\text{dwn}}$ of data from the source s;

$t_{s,r}^{\text{rem}}$: Time required by the relay node r to complete the reception of the message from the source. This is clearly the time needed to get the remaining data $L_{s,r}^{\text{rem}}$.

For the sake of clarity, these parameters are shown in figure 4.4 where it is studied the case in which the child node i selects the relay node r as its parent node. In the figure it is possible to notice the introduction of other parameters indispensable for calculating the utility. We are now going to analyze, separately, which variables should be taken into account for each of the two alternatives available to node i to receive the information. These are the choice of the source node s or the relay node r under dynamic conditions.

I) Choose the relay

As a first option, let us assume that node r is chosen. It was previously described that the relay will surely remain in the network until it receives the entire message. This means that, by using the parameters just introduced, it will not go away for a time equal to $t_{s,r}^{\text{rem}}$. Calculating that time was the main reason for applying the formula (4.1). Moreover, by making the inverse of this equation, we can calculate the amount of bits received during a given time. Indeed, it is now possible to obtain how much information will be received by node i from the relay r during the period $t_{s,r}^{\text{rem}}$ by simply doing

$$L_{r,i}^{\text{dwn}} = t_{s,r}^{\text{rem}} \cdot R_{r,i}.$$

We are interested in this because such an amount of information is the one that certainly will be received by i before that r leaves.

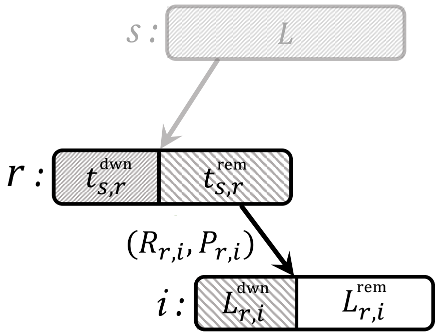


Figure 4.4: Selection of the relay r for receiving the message. The source is present in the network, but it is put in the background because node i does not receive information from s.

Once the received quantity $L_{r,i}^{\text{dwn}}$ has been calculated, we can derive the amount of data remaining to node i to receive the entire message. This can be straightforwardly computed by subtracting from L the percentage of data already downloaded $L_{r,i}^{\text{dwn}}$ by node i so that we have $L_{r,i}^{\text{rem}} = L - L_{r,i}^{\text{dwn}}$. To recap, the new parameters that have been introduced in the last step are:

$L_{r,i}^{\text{dwn}}$: Amount of information that node i would receive with certainty from the relay node r;

$L_{r,i}^{\text{rem}}$: Amount of information that would remain to node i to complete its reception from the relay r.

Having got this far, we can follow the same reasoning applied earlier to calculate the transmission times with formula (4.1) on the last two quantities we found. Then, by considering the rate between the relay r and node i, that is $R_{r,i}$, we can compute

$$t_{r,i}^{\text{dwn}} = \frac{L_{r,i}^{\text{dwn}}}{R_{r,i}} \qquad \text{and} \qquad t_{r,i}^{\text{rem}} = \frac{L_{r,i}^{\text{rem}}}{R_{r,i}}. \tag{4.2}$$

These last two times, as we see in figure 4.5, correspond to:

$t_{r,i}^{\text{dwn}}$: Time corresponding to the one that would be needed to node i to receive the amount $L_{r,i}^{\text{dwn}}$ of data from the relay node r;

$t_{r,i}^{\text{rem}}$: Time required by node i to complete the data reception from the relay node r. This would be the time needed to get the remaining data $L_{r,i}^{\text{rem}}$ after node r has finished downloading the message and is therefore ready to leave the network with probability $(1 - p_r^{\text{st}})$.

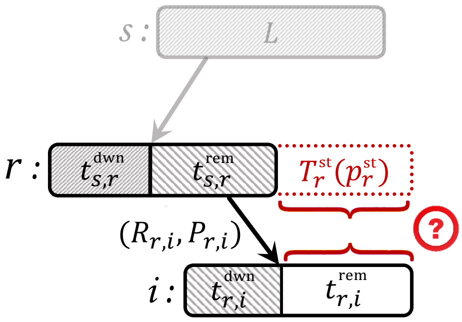


Figure 4.5: Transmission times between node r and node i. Red indicates the additional probabilistic time the relay remains in the network after its full reception. The question mark indicates the uncertainty over the duration of that time.

This would mean that node i, to be able to receive successfully the remaining data $L_{r,i}^{\text{rem}}$, it needs the relay r to remain in the network an additional time (after the the transmission from s ro r is completed) at least equal to $t_{r,i}^{\text{rem}}$. Such an additional time, as stated so far, is stochastic and so depends on the probability p_r^{st} that the relay node r stays in the network. We refer to this probabilistic period (red colored in figure 4.5) by denoting it with:

$T_r^{\text{st}}(p_r^{\text{st}})$: Expected time during which the relay node r will stay in the network after finishing its reception.

Thus, in this simplified model, analyzing the situation form the node i point of view, the time that the relay node r remains in the network is given by the sum of two quantities: a *deterministic time* $t_{r,i}^{\text{rem}}$ and a *random time* $T_r^{\text{st}}(p_r^{\text{st}})$, which will be analyzed more in detail when the utility that node i gets, based on the choice made, will be designed.

What is important to perceive from this model is that the smaller the amount of data $L_{s,r}^{\text{dwn}}$ received by r when node i wants to join the network, the greater will be the quantity of information $L_{r,i}^{\text{dwn}}$ surely received by node i during the deterministic time $t_{r,i}^{\text{rem}}$. As a consequence of this, the shorter would be the probabilistic time that node r has to stay in the network after its entire reception to allow node i to download the remaining portion $L_{r,i}^{\text{rem}}$ of the message. This results in a greater chance for node i of receiving all the information from the relay, since r may leave only during $T_r^{\text{st}}(p_r^{\text{st}})$ that, in this case, would be relatively short. Viceversa, the greater $L_{s,r}^{\text{dwn}}$ (smaller $t_{r,i}^{\text{dwn}}$ and therefore the deterministic amount $L_{r,i}^{\text{dwn}}$), the greater $t_{r,i}^{\text{rem}}$, which means that most of the information should be received during the stochastic time $T_r^{\text{st}}(p_r^{\text{st}})$. Accordingly to that, the possibility of a failure increases. This is because node r might disappear (leave the network) before that the large remaining portion of the message is downloaded by node i. One of the main purposes of our work has been that of modeling, according to mathematical procedures, the circumstances just illustrated, that is, when not all nodes begin to communicate from an empty state. It is good at this point to overlook the complexity of partial information management at the nodes of the network and focus on the two cases that basically can happen in a situation like the one presented here. Furthermore, there is still the need of properly specifying what happens when a failure occurs and what countermeasures are applied. Essentially, observing the scenario shown in figure 4.5, what can happen is:

- if $T_r^{\text{st}}(p_r^{\text{st}}) \geq t_{r,i}^{\text{rem}}$, then the reception of the entire message from the ralay node r to node i would be successfully completed (figure 4.6). This is because the parent node r did stay in the network a time large enough to compensate for the time needed for node i to get the remaining part of the message.

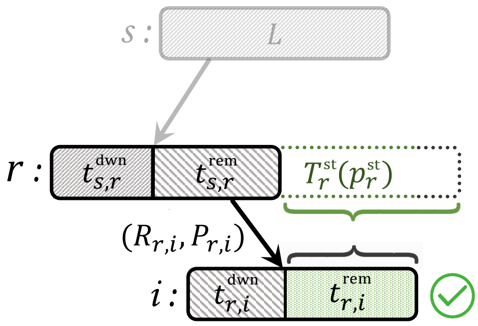
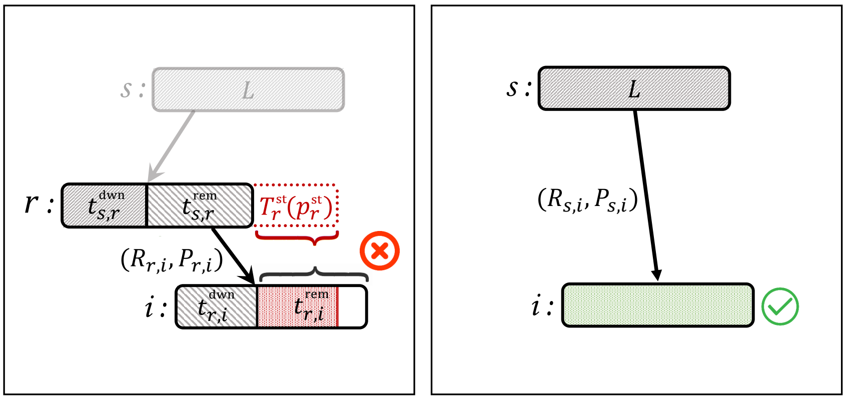


Figure 4.6: Success: node r did not leave the network before the time needed to accomplish the whole transmission with node i.

- if $T_r^{\text{st}}(p_r^{\text{st}}) < t_{r,i}^{\text{rem}}$, then the parent node r leaves the network before that child node i could receive everything it needs (figure 4.7 (a)). When this happen, the consequences are based on the *worst case* scenario: all the amount of data received during the time $t_{r,i}^{\text{dwn}} + T_r^{\text{st}}(p_r^{\text{st}})$ is lost. Thus, it has been supposed that the failure involves the entire loss of the information downloaded by node i until the moment in which the departure of the relay node r occurs. As a result, those data become useless and will only increase the network overhead.

  In this situation, since all that has been received is discarded, node i must start receiving the whole message from the source s again. We can see that happening in figure 4.7 (b) where the relay node r has already left the network. To let the message be disseminated, a new connection with the source is established. Neglecting the additional overhead required to set up the new connection, we are sure that in this case no more failures can occur during the transmission. The reason behind this is that the source is not subject to mobility and so all the message will be surely received. Obviously, when the new connection takes place, network parameters will no longer be those used during the first transfer from r to i that has not been successful, but those characterizing the wireless medium between s and i which are the last two remaining nodes in the network.



(a) (b)

Figure 4.7: Failure: (a) node r leaves the network before the time needed to accomplish the whole transmission with node i; (b) a new connection with the source s must be established to allow the message to be received.

Logically, the hope of the child node i is that the parent node r does not leave the network for the period $t_{r,i}^{\text{rem}}$, since in this case the relay has been chosen for receiving the message. It needs to be pointed out that such a decision is based on a well-defined criterion, that is, the utility that node i would get by choosing the relay as its parent node, which would bring greater benefits to the network even taking some risk.

All of the parameters needed for utility design have finally been introduced. The only missing step is a brief analysis of the simplest case that come to be present in the proposed scenario: the direct connection to the source without passing through the relay node. As there is nothing casual in the latter, the problem is reduced to calculating the transmission time of the entire message between the source and the node i. What will be significant in the part we are about to study, however, is the presentation of the formula for calculating the energy required to put in place the the transmissions discussed so far and it is precisely on such a point that the work will focus later on.

II) Direct connection to the source

The second option available to the child node i for receiving the message is to choose the source s as its parent. In this case, all issues related to mobility are not present. The source sends the data that will be fully received with certainty. This is because, as already mentioned, node s does not leave the network. The time to receive the whole message is therefore deterministic and depends on the rate that characterizes the channel between the two involved nodes. Such transmission time is

$$t_{s,i}^{\text{tot}} = \frac{L}{R_{s,i}}$$

and it is simply calculated applying the formula (4.1) that has been used so far to determine the other periods of interest. By giving it a more formal definition, we refer to it as:

$t_{s,i}^{\text{tot}}$: Total time required by the new node i to receive the whole information L from the source s;

Considering for a moment the previous case, and precisely the one shown in the figure 4.7 (b), the time just expressed is exactly what is needed to retransmit the message after the failure caused by the departure of the relay node r.

In the case of direct connection to the source, instead, the situation is that illustrated in figure 4.8. What we can see in the last scenario is that a new connection

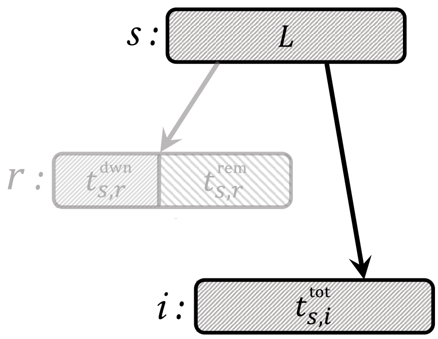


Figure 4.8: Selection of the source for receiving the message. The relay is present in the network, but it is put in background because it does not participate in the dissemination.

with node i is established from the source and, in the meanwhile, the data transfer between node s and node r is in progress. To make this situation happen, a new unicast connection needs to be set up. It is not possible to exploit a multicast communication from the parent to the two child nodes, because one of them was already going on. The power used is therefore not the maximum between $P_{s,r}$ and $P_{s,i}$, but an average of the two until the relay has finishes downloading everything. The source needs so, for the moment, to begin a different unicast connection with node i (e.g. by picking a different frequency from the one it is already using, or on the same transmitting medium by assigning cyclically to each of these predetermined time intervals). To ensure a simultaneous propagation of the message to two nodes without the need to duplicate the information, we have to wait for a new empty participant, besides a node i, that wishes to connect to the network at the same time node i does. This will be addressed in the next chapter, but what's important here is that to make the child node choose the best solution is not trivial at all.

Now that both of the verifiable situations of our model have been analyzed in the last two paragraphs, we are ready to explain why there has been so much interest in calculating the transmission times. Having this knowledge it is possible to compute the *energy required* for a transmission as the product between the power required to transmit the data from a source of information j to a receiver k multiplied by the time of the transmission:

$$E_{j,k} = P_j, k \cdot t_{j,k} \tag{4.3}$$

The goal of this work is then to minimize the total transmit energy in the network, which is given simply by the sum of all the actual expended energy after that all the parent are selected so that the topology is defined. We aim to a achieve that by trying to avoid failures given by the relay departure and providing the new node that wants to join the network with an analytical model for making the best choice for the good of itself and others in the network. All of this is based on an appropriate design of the utility function which is the core of our optimization process.

4.3 Utility Formulation under Fixed Rate conditions

In this section the utility function will be designed and all the necessary elements for its formulation will be thoroughly studied. For now, the player of the game we are interested in providing a methodology for a preference evaluation is only node i (figure 4.3). We know that the latter has to choose between receiving the message from the relay (with the risk that the last leaves, but perhaps with the possibility to get better performance) or from the source (no risk of failure during he transmission given by the parent departure, but perhaps not the optimum solution in terms of energy expenditure). It is assumed the relay to be a *full duplex* device, i.e. device that can transmit and receive simultaneously, by supporting paired and unpaired spectrum using frequency (FDD) and time (TDD) division duplexing operations as a technical solution [51]. The ability to assign resources simultaneously for downlink and uplink transmissions will allow an efficient utilization of the available spectrum. However, this approach increase the interference because of the simultaneous transmissions to different directions. Thus, there is a need of a tradeoff between spectral efficiency and area coverage that makes such a technology particular suitable for small networks (as in our case) given practical self-interference cancellation limits [52]. We have so, in the proposed scenario, a child node that has to make a decision between two parent nodes. Following on from the game theory fundamentals, it has been discussed in chapter 2 that a preference relationship (i.e. the ability of a player to evaluate and compare the consequences associated with an action that has to be selected within the player's action set) is represented by an utility function. Such a function has, therefore, the task of assigning a number, called utility, to each option available to the player with the aim of being able to select the one that brings greater benefits (higher utility value) to the decision maker and consequently to the network.

The main purpose of this research part was on appropriately modeling these preference relationships finding at the end an analytical model able to justify the action of each player. To do this, a probabilistic utility function has been defined as a performance metric for a wireless packet-switched data user, which measures the average information throughput over the air link powered by each unit of the mobile node energy $[bits/Joule]$. It interesting to notice that, achieving a high SINR level requires the user terminal to transmit at a high power, which in turn results in low battery life. From this last consideration it can be easily seen that the two magnitudes are in conflict. A trade-off can be quantified (as in [43–45], by further specifying the formula (3.4)) by defining the utility function, of a child node k that choses a node j as its parent node, to be the ratio of throughput achievable over the link between them and the transmit power from node j toward node k, i.e.,

$$u_{j,k} = \frac{T_{j,k}}{P_{j,k}}. \tag{4.4}$$

Throughput is defined as the net number of information bits that are transmitted without error per unit time (sometimes referred to as *goodput*). It can be expressed as

$$T_{j,k} = \frac{L}{M} R_{j,k} f(\gamma_{j,k}) \tag{4.5}$$

where M and L are the number of information bits and the total number of bits in a packet (comprising the various overhead introduced by the OSI layered model) to be transmitted, respectively; $R_{j,k}$ and $\gamma_{j,k}$ are the transmission rate and the signal-to-interference-plus-noise ratio (SINR) of the transmitted signal from parent node j and received by child node k, respectively; and $f(\gamma_{j,k})$ is the *efficiency function* representing the packet success rate (PSR), which is portrayed in figure 4.9 as Γ, i.e., the probability that a packet is received without an error.

The assumptions that has been made are: the first is that L and M are the same, which translates in having all the packet of length L made by only information bits

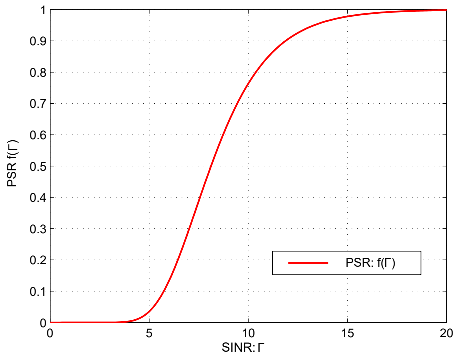


Figure 4.9: Packet Success Rate as a function of SINR

(the overhead introduced by the network communication layered model is neglected); the second is that $f(\gamma_{ij}) \simeq 1$, which means having a minimum power $P_{j,k}$ between the two communicating nodes that ensures a SINR such as to assert the last hypothesis. The power needed to do that is exactly the one that characterizes the edges among the nodes described in our system model. This, then, leads us to an ideal scenario where every bit is successfully received by the receiver and as shown in 4.9 this can be achieved assuming a $\gamma_{ij} \geq 10$. Having done these assumptions, the throughput is going to coincide directly with the rate of the single hop link between the two nodes j and k so that

$$T_{j,k} = R_{j,k}. \tag{4.6}$$

and such a rate is precisely the one we have found specifying the wireless medium among the nodes in our simplified scenario.

Indubitably, all these suppositions make our environment clearly unreal, although it is well-known that in wireless transmissions the number of errors given by the air channel is relatively high. However, as a first step it was very worthwhile pursue this approach, so that we could focus on the main innovative aspect which is the introduction of probabilities which determine the dynamism of the system.

It is possible to notice that formula (4.6) is derived from formula (4.5) where L and M simplify each other and the multiplication by $f(\gamma_{ij})$ can be simply omitted since it is assumed to be approximately equal to 1. Having done so, the formula (4.4) can be rewritten as follows:

$$u_{j,k} = \frac{R_{j,k}}{P_{j,k}} = \frac{L}{P_{j,k} \cdot t_{j,k}^{\mathrm{TX}}} \tag{4.7}$$

where the rate has been simply substituted with $L/t_{j,k}^{\mathrm{TX}}$ and such a ratio is derived from formula (4.1) used to evaluate the transmission time for the message to be transmitted from the parent node j to child node k. Since L and $P_{j,k}$ are fixed, the only parameter on which we will operate is precisely the transmission time $t_{j,k}^{\mathrm{TX}}$.

It is important because such a time will be conceptually different for the two cases addressed in this project: certain reception when there is a connection to the source and possible partial reception due to the mobility of the relay node.

The scenario is repurposed in figure 4.10 where we find the introduction of the probability of staying for every node of the network. The probability related to the source has been omitted in the illustration being always constant and equal to 1. The reason is that, as we know, the source is not subject to mobility. It is clear now that the maximum rate achievable on every channel is for the case $f(\gamma_{ij}) \simeq 1$.

We have seen earlier that in the moment in which we are going to analyze the system, the connection between the source s and the relay r has already been established. Obviously, this has been done to allow the node i to choose between a static node (source) or dynamic node (relay).

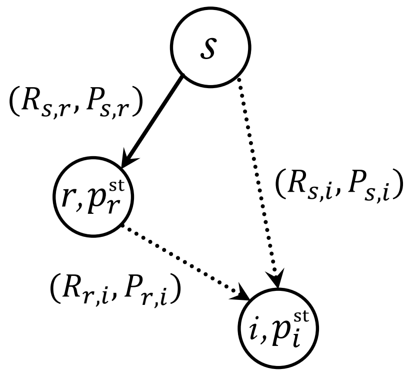


Figure 4.10: System model with the introduction of the probability that a node stays in the network.

To give an idea of how the situation we are analyzing has been reached, we can image that, from a game theoretic perspective, in a first step node r, alone, wanted to joint the network. The latter has a certain probability of "turning off" (suddenly disappears from the network) represented by $(1 - p_r^{\text{st}}) \in [0,1]$. We can see that it has only node s as a parent node belonging to the broadcast tree, then the action set of node r is composed only by $A_r = \{s\}$. Having only one choice in its action set, no preference evaluation is needed. It will select surely the only option available to it which will provide an associated utility $u_{s,r}$. At this point node r belongs to the broadcast tree, which means that a connection can be established between the two nodes. After that, a new node i wishes to join the network. Here the situation is different, because it can choose between two parents already connected to the broadcast tree, since node i can be reached in a radium less or equal to the maximum power of both node s and r. The action set, the choice of the node in order to join the network, will be clearly composed by the two alternatives to i available: $A_r = \{s, r\}$. Also here, each choice is associated with a utility, which are

$u_{s,i}$ and $u_{r,i}$ for node s and r respectively. The player's evaluation of the two utility is so really important, since node i is going to select the bigger value among them. It is necessary to say that probability p_i^{st} is useless at this time, because node i has just to receive and not relay the information. We know that probability comes into play only when you have to forward the message once you have received it all. Here node i not to children to look after, so p_i^{st} could also be omitted in the last figure.

The formulation of the utility function is therefore made for both alternatives available to node i, which, according to the decision taken, will obtain:

- $u_{r,i}$: the utility associated with choosing the *dynamic* node, that is, the relay node r. This could mean achieving better performance because, being they closer in the proposed model than the distance between i and s, the denominator of the general formulation of the utility (4.5) would be smaller. The statement on the distance among nodes has been made because, as we will see in the chapter of simulations and results, node r is always positioned in an area that is in the middle of i and s, otherwise it would make sense to always choose the source. However, making this decision would lead to take the risk that the relay leaves the network before the transfer completion with i and consequently to lose the whole data received up to that moment. As we said, in this last case node i has to establish a new connection toward the source restarting again the transfer with the drawback of increasing the overhead of the system.

- $u_{s,i}$: the utility associated with choosing the s*tatic* node, that is, the source node s. Such a connection would lead to greater power consumption (larger denominator in formula (4.5)), but being sure to complete the transfer.

We are therefore starting the steps for the definition of both, by analyzing the two cases again separately as was done during the definition of the parameters. The connection to the static node (source node s), the simplest, will be analyzed first. We will then pass to the dynamic node (relay node r).

As a first step for the utility formulation, it is assumed that all the rate among the nodes in the simplified scenario of figure 4.10 are fixed and equal, which means

$$R_{s,r} = R_{s,i} = R_{r,i}.$$

It has been decided to do that in order to reduce the number of parameters (variables) in the networks and for helping us to get more intuition into mathematical procedures.

The case of variable rate will be also discussed in the future work direction, given that part of the work for this circumstance has been made during this project. For now it is worth to keep going considering the constant and equal transmission speed for each wireless channel, so as to address more clearly the problem of the utility definition associated with the choice of a node.

4.3.1 Connection to the Static Node

The case where child node i chooses as its parent node the source s for receiving the message is now examined. Since the source node s has probability of staying in the network $p_s^{\mathrm{st}} = 1$, it will never leave the system. This means that once the connection among the two nodes is established and the data transfer starts, the latter will end successfully in a deterministic time equal to $t_{s,i}^{\mathrm{tot}}$. Knowing the size of the message L, its transfer time $t_{s,i}^{\mathrm{tot}}$, and the power required for it to be successfully received without errors, we can calculate the utility associated with choosing the source as the parent node. This utility is the one introduced earlier, namely $u_{s,i}$, and is calculated using the formula (4.7) so to get:

$$u_{s,i} = \frac{L}{P_{s,i} \cdot t_{s,i}^{\mathrm{tot}}}. \tag{4.8}$$

The transmission time at the denominator here is simply the one needed to node i to receive the whole information L from the source s.

Observing the denominator carefully we see that it is the same as the product defined in the formula (4.3) where the energy required for a transmission has been introduced. Thus, thanks to that formula we can substitute at the denominator the multiplication of the power by the transmission time and the utility can be rewritten as:

$$u_{s,i} = \frac{L}{E_{s,i}^{\mathrm{tot}}} \tag{4.9}$$

where $E_{s,i}^{\mathrm{tot}}$ is the energy required for transmitting the entire data from the source node s to the receiving node i.

Concluding, $u_{s,i}$ is the numeric value that node i will get by choosing the source. It will then be compared with the one it would obtain by choosing the relay so that the largest of them may indicate to node i from which device to receive the information.

4.3.2 Connection to the Dynamic Node

In the model proposed in this work it was said that the relay node leaves the network after receiving the entire message from the source. This happens not immediately at the moment when the reception is completed, but after a stochastic time $T_r^{\mathrm{st}}(p_r^{\mathrm{st}})$ that depends on the probability p_r^{st} that node r stays in the network. In this case we are so studying the situation in which node i chooses as its parent node the dynamic node r, which will leave the network after a random time once it has received all the information.

Thus, as we see in figure 4.11, in modeled scenario the time that the relay node r remains in the network is given so by the sum of two quantities:

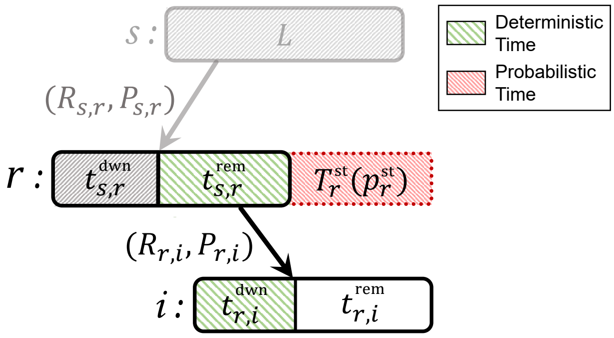


Figure 4.11: Transmission times between node r and node i. Red indicates the additional probabilistic time the relay remains in the network after its full reception. Green indicates the time needed by node r to complete its reception, which correspond to the period it will surely remain in the system.

1) **Deterministic Time:** it corresponds to effective time remaining to terminate a data transfer from the source node s to its child node r. Referring to the parameters introduced in section 4.2.1, we know that when node i wants to join the system, the relay node r has already downloaded a part $L_{s,r}^{\text{dwn}}$ of the complete message L. To download such a portion of data the relay needed a time $t_{s,r}^{\text{dwn}}$, which depends on the rate between the source and the relay. Having this knowledge, it is possible to evaluate the *deterministic time* that node r will stay certainly in the network to finish receiving the message, which corresponds to the finite time $t_{s,r}^{\text{rem}}$. During the deterministic time, if node r it is chosen as a parent by node i, the child is going to receive for sure a portion of information $L_{r,i}^{\text{dwn}}$ that depends on the rate between this two last nodes. This allows to compute, as we have seen in formula (4.2), the time $t_{r,i}^{\text{rem}}$ remaining to node i to finish receiving the message, which requires node r to stay in the system at least a time equal to the last one to avoid a failure. Note that $t_{s,r}^{\text{rem}}$ is always equal to $t_{r,i}^{\text{dwn}}$, but the quantity of information received by node r and i may be different depending on $R_{s,r}$ and $R_{r,i}$. Having considered in the assumption that the rates of those two hops are equal, we will see that in this simplest scenario the amount of information $L_{s,r}^{\text{dwn}}$ is equal to $L_{r,i}^{\text{dwn}}$. What is important to predict is how likely node r is going to stay in the network to allow node i receiving the portion of data out of the deterministic conditions.

2) **Probabilistic Time:** Once the transfer between the source node s and the relay node r has been completed, the latter will remain in the network for a further time $T_r^{\text{st}}(p_r^{\text{st}})$ that is probabilistic. In fact, such a *probabilistic time* depends on the probability p_r^{st} that node r stays in the network. As we have seen before, we need that $T_r^{\text{st}}(p_r^{\text{st}}) > t_{r,i}^{\text{rem}}$ to allow the node i receiving the full message avoiding a failure, which will cause the entire message retransmission

from node s.

Thus, for the child node is extremely important predict how likely the parent node will remain in the network after that the deterministic time expires. Also because the part of the data received surely during the first time represents the potential overhead it is going to add in the network if the transmission during the second time does not conclude successfully.

As a first step for the formulation of the utility $u_{r,i}$ associated to the choice of the relay node r by node i, we try to define the *reception time* of child i when it is receiving data from parent r. This can be done, with the support of figure 4.11, as:

$$t^{\text{rec}}_{r,i} = t^{\text{rem}}_{s,r} + p^{\text{st}}_r \cdot t^{\text{rem}}_{r,i} \tag{4.10}$$

which tell us that the first part of information $L^{\text{dwn}}_{r,i}$ will be downloaded for sure during the time $t^{\text{rem}}_{s,r}$, while the remaining data $L^{\text{rem}}_{r,i}$ are going to be received with probability p^{st}_r during the time $t^{\text{rem}}_{r,i}$ needed to complete the transfer from node r to node i. Indeed, if we take $p^{\text{st}}_r \to 0$, which means that the relay r goes to leave the network as soon as its reception its completed, the time $t^{\text{rec}}_{r,i}$ in formula (4.10) results to be simply equal to $t^{\text{rem}}_{s,r}$. If this happens, node i is going to receive only the deterministic amount of information and then it will download only a part of the message. Such a situation would lead to a failure (all the data $L^{\text{dwn}}_{r,i}$ would be lost) and a to the consequent retransmission of the whole message from the source. Instead, if $p^{\text{st}}_r \to 1$, which means that the relay node r will likely stay in the network, we have that $t^{\text{rec}}_{r,i} = t^{\text{rem}}_{s,r} + t^{\text{rem}}_{r,i} = t^{\text{tot}}_{r,i}$ and so all the data are gong to be received by i from the forwarding node r. In this case the dissemination takes place successfully, so as to obtain the best performance for the network.

Having seen how times can be related to each other, we are ready now to define the utility $u_{r,i}$ in terms of number of reliable bits transmitted per joule of energy consumed:

$$u_{r,i} = \frac{L}{P_{r,i} \cdot t^{\text{rec}}_{r,i} + (1 - p^{\text{st}}_r) P_{s,i} \cdot t^{\text{tot}}_{s,i}}. \tag{4.11}$$

The last formula defines at the denominator energy that would be spent in case the relay node r is chosen by node i to receive the message of length L which is in the numerator. Observing carefully the denominator, we can distinguish to main components: a first energy $P_{r,i} \cdot t^{\text{rec}}_{r,i}$ that is the one that would be expended during the reception from node r to node i, to which it is added a second term, again an energy, that would decrease the utility value if the relay node leaves the network with probability $(1 - p^{\text{st}}_r)$. Indeed, the second component is exactly the energy $E^{\text{tot}}_{s,i} = P_{s,i} \cdot t^{\text{tot}}_{s,i}$ we found in formula (4.9), which is the one that would be consumed, in case of failure, for the total retransmission of the message from the source s to node i. In the event of this last circumstance, all the energy used for the first transfer attempt, that is during time $t^{\text{rec}}_{r,i}$, would be wasted.

To give an example of how the newly defined utility works, it is worth to play again with the probability p_r^{st} that the node r remains on the network. Before doing so, it is necessary to substitute the reception time of node i (equation (4.10)) in the utility defined in formula (4.11) in order to obtain:

$$u_{r,i} = \frac{L}{P_{r,i}(t_{s,r}^{\mathrm{rem}} + p_r^{\mathrm{st}} \cdot t_{r,i}^{\mathrm{rem}}) + (1 - p_r^{\mathrm{st}})P_{s,i} \cdot t_{s,i}^{\mathrm{tot}}}. \tag{4.12}$$

Consider now the two extreme case for $p_r^{\mathrm{st}} \to 1$ and $p_r^{\mathrm{st}} \to 0$ to see what happen:

$$u_{r,i} = \begin{cases} \dfrac{L}{P_{r,i}(t_{s,r}^{\mathrm{rem}} + t_{r,i}^{\mathrm{rem}})} & \text{for } \; p_r^{\mathrm{st}} \to 1, \\ \dfrac{L}{P_{r,i}(t_{s,r}^{\mathrm{rem}}) + P_{s,i} \cdot t_{s,i}^{\mathrm{tot}}} & \text{for } \; p_r^{\mathrm{st}} \to 0. \end{cases} \tag{4.13}$$

The utility then provides different values depending on the probability. To give an exhaustive explanation of the results achieved, the two cases are going to be discussed below:

- The first term represents the utility for the case where the relay node r does not leave the network. In fact, we can see that the energy consumed for transmitting the message of length L is equal to the one needed for the communication between node i and the relay without taking into account the possibility of a connection to the source. Such an energy therefore results to be the product of the transmit power $P_{r,i}$ by the sum of two times: the deterministic time $t_{s,r}^{\mathrm{rem}}$ remaining to node r to complete the reception form the source and the time $t_{r,i}^{\mathrm{rem}}$ remaining to node i for receiving the missing portion of the data from node r after that the latter finished its reception. This is because, when the first time of the sum expires, the remaining part of the message has to be received during the second time that in this case is going to cover for sure the entire reception, being $p_r^{\mathrm{st}} \to 1$.

- The second ratio, instead, shows the utility that would be obtained in case the relay leaves the network strictly after its entire reception. Wee see here that the first term of the sum $P_{r,i}(t_{s,r}^{\mathrm{rem}})$ represent the energy that is wasted because of the data certainty received by node i during the deterministic time $t_{s,r}^{\mathrm{rem}}$. Unfortunately, as soon as the latter expires, the relay node r leaves the network and so a new transmission is going to be established with the source node r to receive the whole message. This corresponds to an additional expenditure of energy that is represented by the second term of the sum $P_{s,i} \cdot t_{s,i}^{\mathrm{tot}}$. Indeed, the last is exactly the energy $E_{s,i}^{\mathrm{tot}}$ required for the total transmission of the message form the source to node i. Of course, the situation in which such a situation may occur should be avoided, because does not make sense to choose to receive data from a parent node that will not ensure you a full reception. Thus, this additional term at the denominator will decrease the value resulting as utility in order to make a more wiser decision in the moment when the alternatives are compared.

Everything presented so far in this section will be clearer when the utilities associated with the choices of the parent node by the child node will be graphically represented. Before doing so, however, it is good to provide the reader with a brief summary of the topics addressed so far. This is done by resuming the parameters of section 4.2.1 again and placing them in a wider view given by the knowledge of the utility formulas. Resuming, at the moment when a new node i wishes to join the network:

1. A relay node r as received part of the message $L_{s,r}^{\text{dwn}}$ from a source of information, which has a message of length L that has to be received by all the nodes of the network.

2. The remaining portion of data to be received from the relay is so equal to $L_{s,r}^{\text{rem}} = L - L_{s,r}^{\text{dwn}}$. This means that the relay r for finishing receive the message needs a time $t_{s,r}^{\text{rem}} = L_{s,r}^{\text{rem}}/R_{s,r}$ that depends on the rate of the channel between it and the source. This is the time during which the relay will certainty stay on the network and after that such a period expires node r will remain in the network an additional probabilistic time in which it may leave the network with a certain probability p_r^{st}.

3. If the child node i chooses to receive the information form the relay r (dynamic node), it will download during the time $t_{s,r}^{\text{rem}}$ surely a portion of information $L_{r,i}^{\text{dwn}}$. Then to complete the transmission, node i requires a time equal to $t_{r,i}^{\text{rem}} = (L - L_{r,i}^{\text{dwn}})/R_{r,i} = L_{r,i}^{\text{rem}}/R_{r,i}$ that starts at the moment when node r finishes to receive the message. Then, if the relay stays in the network an additional probabilistic time (after completing its reception from the source) at least equal to $t_{r,i}^{\text{rem}}$ the message will received successfully by node i. Otherwise a failure occurs and node i has to receive the full message form the source, wasting the energy consumed for the first communication with the relay. To this choice is associated an utility $u_{r,i}$ described by the formula (4.12).

   If the child node i chooses to receive the information form the source s (static node), then all the information will be received for sure since the source never leaves the network. To this alternative is associated the utility $u_{s,i}$ described by the formula (4.8). Thus, the best choice will depend on the network conditions, like transmit powers, probability p_r^{st}, data-rate and especially how much information the relay r has already downloaded.

We are now ready to study under which network conditions the child node will choose the static node as a parent rather than the dynamic node and viceversa. This will be done by comparing the two utilities $u_{s,i}$ and $u_{r,i}$ for the two specific cases in order to understand which choice must be taken by a node that wants to join the system. In addition, a graphic illustration of the trend of the utilities will be provided according to the parameters imposed by the network so that we can distinctly see and discuss the two cases.

4.3.3 Decision Criteria Analysis

The utility function, as widely discussed, associates a numerical value to each of the possible alternative available to a given agent. This function, in case the dynamic node is chosen by the new node of the network for receiving the message, depends on the probability that the parent node remains in the network after finishing its reception. Therefore, before going to investigate how the decision criteria has been addressed, we need to analyze the dynamics regarding the departure process of a node in order to be able to evaluate the probability of staying in the network of the relay node r. This has been done by designing such a probability in a way that a generic node j leaves the network after a finite time exponentially distributed with departing rate λ_j, so that

$$T_j^{\mathrm{st}} \sim exp(\lambda_j) = \lambda_j \; e^{(-\lambda_j \; t)}$$

where T_j^{st} represents the further time that node j will stay in the network after receiving the whole message.

As was mentioned at the beginning of this chapter, it has been just assumed that a generic node j, instead of going away from the network, suddenly turns off when the probabilistic time T_j^{st} expires. This has been done in order to simplify the dynamicity issue so as to have instantaneous disappearances of the nodes and therefore without the need of considering power decay that would take place if a node moved away from the system. The moment in which this phenomenon occurs, that is to say, the disappearance of a node j after a finite time exponentially distributed with parameter λ_j, depends on the level of mobility of the wireless environment. Under no mobility condition, all links will stay active during simulation time ($\lambda_j = 0$). With medium mobility, there will be a certain percentage of links between nodes that will fail at certain time ($\lambda_j > 0$). With high mobility, there will be a large number of these links between nodes failing during simulation time ($\lambda_j \gg 0$). The source node s of the message is always assumed to be under no mobility condition which means having $\lambda_s = 0$.

Considering the specific case of this work, i.e. the scenario shown in figure 4.11, the hope of the child node i, when it chooses the relay node r for getting the message, is that the parent node (dynamic in this case) would not leave the network for a period of time at least equal to $t_{r,i}^{\text{rem}}$. This corresponds to the probability of having no departure events of the parent node during this last time frame. Such a condition con be modeled, by assuming that the period the relay node r stays in the network after its complete reception $T_r^{\text{st}}(p_r^{\text{st}})$ to be exponentially distributed with departing rate λ_r, as a Poisson process. In fact, this type of stochastic process will count the number of departures of the the relay node r during the time interval $[0, t_{r,i}^{\text{rem}}]$. With the 0 point of the interval, in which we are going to count the number of leaving occurrences of the parent, we refer to the instant where the parent node r completes its data reception. Therefore, we are considering the period of time starting from the moment when the deterministic time expires and, consequently, the probabilistic time begins.

We define then a random variable $N(t)$, with $t > 0$, counting the total number of events (departures of a parent node in our specific case) that have happened up to and including time t. Referring once again to a generic node j of the network, a Poisson counting process with rate λ_j has the property that $N(0) = 0$. The probability of the random variable $N(t)$ being equal to n, which means having n as number of events in any interval of length t, is given by

$$P\{N(t) = n\} = \frac{(\lambda_j t)^n}{n!} e^{-\lambda_j t}.$$

By applying last definition of the counting process $N(t)$ to our scenario, we want to count the number of departing events of the relay node r, if the latter would be chosen by the child node i as a parent node, during the period of time of length $t_{r,i}^{\text{rem}}$. This can be modeled, as

$$P\{N(t_{r,i}^{\text{rem}}) = n\} = \frac{(\lambda_r t_{r,i}^{\text{rem}})^n}{n!} e^{-\lambda_r t_{r,i}^{\text{rem}}}. \tag{4.14}$$

which represents the probability of having n departure events of the relay node r during the time required by node i to finish receiving the message. Obviously, we want no parent's departure to take place during such a time and so we are interested only in the case where $n = 0$. Thus, the probability of having no departure of the parent node r during the time interval of length $t_{r,i}^{\text{rem}}$ can be expressed as:

$$P\{N(t_{r,i}^{\text{rem}}) = 0\} = e^{-\lambda_r t_{r,i}^{\text{rem}}}. \tag{4.15}$$

This last result is simply obtained by substituting $n = 0$ in the formula (4.14). Thinking carefully about the outcome coming from equation (4.14) we see that the probability that the node r does not leave the network for a time $t_{r,i}^{\text{rem}}$ has exactly the meaning of the probability that the relay stays in the network during such a period of time. We can so agree on the fact that such a result coincides with p_r^{st} and so we finally obtain that

$$p_r^{\text{st}} = e^{-\lambda_r t_{r,i}^{\text{rem}}}. \tag{4.16}$$

Having such a result allows us to redefine the total reception time of the child i when it is receiving data from parent node r that we have seen in formula (4.10) in order to get

$$\begin{aligned} t_{r,i}^{\text{rec}} &= t_{s,r}^{\text{rem}} + p_r^{\text{st}} \cdot t_{r,i}^{\text{rem}} = \\ &= t_{s,r}^{\text{rem}} + P\{N(t_{r,i}^{\text{rem}}) = 0\} \cdot t_{r,i}^{\text{rem}} = \\ &= t_{s,r}^{\text{rem}} + e^{-\lambda_r t_{r,i}^{\text{rem}}} \cdot t_{r,i}^{\text{rem}} \end{aligned} \tag{4.17}$$

It is possible to verify now that if $\lambda_r \to 0$ (absence of mobility condition, relay node r is going to stay always in the network) $p_r^{\text{st}} \to 1$, so as to have the message completely received by the relay. In this circumstance, by looking at formula (4.12), there are no contributions in the denominator given by the connection with the source because the failure in this limit case does not occur and $(1 - p_r^{\text{st}}) \to 0$.

In the opposite case, for high environment dynamicity, it has been seen that $\lambda \gg 1$. For the limit case $\lambda \to \infty$ we have that the connection with the source tends to happen almost with certainty being $(1 - p_r^{\text{st}}) \to 1$. Regarding the amount of data that the child node could receive from the relay, we have instead that $t_{r,i}^{\text{rem}} \to 0$ and consequently it would be lost because the message is incomplete. The situation is so completely identical to what has been previously illustrated in the relations (4.13), the only difference is that in this case the parameter to play with is λ, which has a direct impact on the remaining probability p_r^{st} of the dynamic node in the network. As we have seen in the formula (4.12), the numerator of the utility function is fixed. This allows to say that to achieve greater utility it is sufficient to minimize the denominator, which will subsequently named as the price that a new agent has to pay to take advantage of the service offered by the parent node. Having said that, the child node can select the utility value such as to have the lowest value of energy in the denominator.

At this point it is possible to graphically analyze the results obtained in terms of utility in function of the amount of data received (Figure 4.12) and the probability of remaining in the network (Figure 4.13), and therefore of the λ parameter related to environment mobility, of the node relay r. The utility value on the y axis is obtained by placing the relay at a shorter distance from the new agent with respect to the source, otherwise there would have been no advantage in choosing the dynamic node since, at the same cost, the new user would have preferred always the static node to avoid risks. The transmission power required at the two possible parent nodes to have the message received without error, packet success rate (PSR) equal to 1, at the receiver node i was set to $P_{r,i} = 80\ mW$ and $P_{s,i} = 200\ mW$ for the dynamic and static node, respectively. The channel rate was set at 54 Mb/s and the message size was chosen as $L = 10\ Mb$. For the moment, the choice of parameters for the numerical calculation of the utility is not the point to dwell on, since this analysis has been done in order to have a clearer interpretation of what a new user might face when it has to take a decision.

By looking at Figure 4.12, it is possible to identify two main behaviors: the first concerns the choice of the source, whose utility is constant and independent of the percentage of data received from the dynamic node, while the second one is the utility value that would be obtained by choosing the relay, which decreases as the data received from this potential parent node increases. What we see is perfectly in line with what has been described when the utility was defined, in fact we know that the source will always remain in the network and therefore generate a value of constant utility (blue curve) and independent of the introduction of the concept of mobility. As for the relay, however, we have that it provides a greater utility in the left side of the graph, so when it received a small part of the total amount L. The reward then becomes lower when, at the moment of connection of the new node i in the network, the node r has already received almost all the message. The explanation of this phenomenon is given by the assumption made in the model, which provides that a vertex of the graph cannot leave the network before having received all the information. In fact, when the amount downloaded is small, it will surely remain in the network for a longer time. This means that the deterministic time is greater and therefore the probability of failure decreases. As a consequence, the amount of data that must be received during the stochastic time is smaller and since this period is shorter the probability of having a departure event is lower.

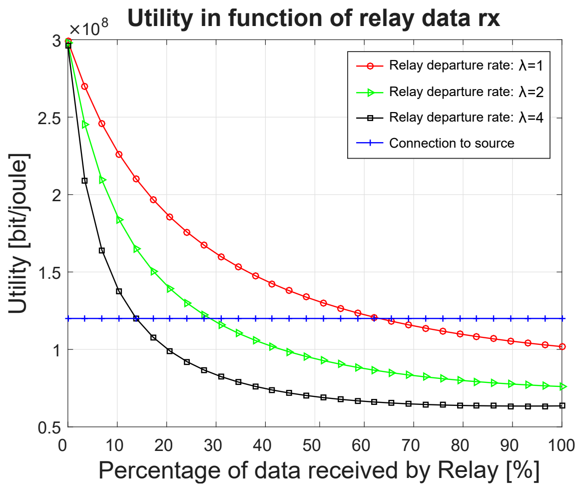


Figure 4.12: Utility that the new node i would get in function of the data received by the relay node r. The straight line represents the utility that the child node i would get by selecting the source s as its parent node.

Summarizing, it follows that the less information has downloaded the relay, the more it will remain in the network to complete the transfer. Thus the new node will be able to receive a good part of the information during the secure permanence time of the relay and only a small part remaining when the parent node will be in the process of disappearing. If the relay has already received a good part of the message, instead, it is better to choose the source because the amount of data to be received in the stochastic time is greater. A possible failure in this case would be more likely to happen, which consists in having to receive, immediately after the negative event, the whole message from the source. For this reason, under these conditions, the new agent tends directly to receive the message from the node not affected by mobility.

A further analysis aimed to verify the correctness of utility function is given by the three curves obtained for three different values of the λ parameter. We see that if the mobility of the environment is high (lambda large, black curves in figure 4.12), the new node that wants to join the network tends to connect to the source for a smaller amount of data received by the relay. This is because the new agent aims to receive as much data as possible during the secure staying time of the parent node, knowing that in this case the latter will leave the system with higher probability. The opposite occurs when the mobility of the environment is less (lambda smaller, red curves in figures 4.12). Here the node prefers to connect to the relay even when the latter has received more than half of the message. From an energy point of view this is convenient, because as explained above, in this scenario the relay is placed between the source and the new agent. Since the intermediate node in this case is more likely to remain in the network after having completed its reception, the child node is more willing to receive part of the message during the stochastic time of its dynamic parent node.

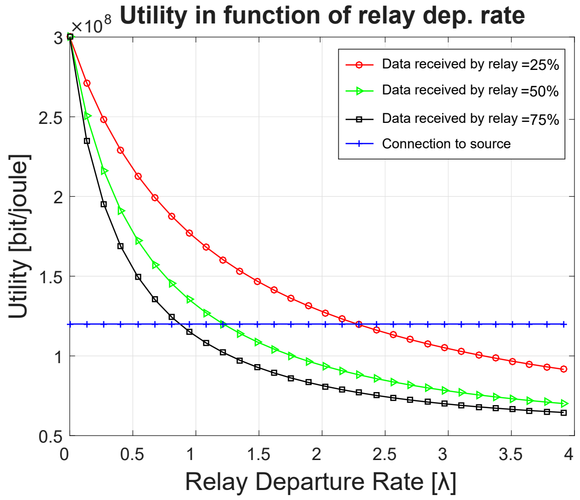


Figure 4.13: Utility that the new node i would get in function the departure rate of the relay node r. The straight line represents the utility that the child node i would get by selecting the source s as its parent node.

A similar behavior can be seen in Figure 4.13, where the curve representing the utility has been obtained by varying the λ parameter. Here we see that as the environment becomes more dynamic, the new node tends to prefer a connection to the source so as to receive the message. Obviously, also in this case the curve showing the utility for choosing the source is constant, since it is always present in the network. It is therefore confirmed that the greater si the departure rate that characterizes the environment (i.e. the lower the probability that the relay remains in the network after receiving the whole message), the greater is the preference of the new agent joining the network of establishing a connection with the source. As always, it is preferred to receive a larger amount of data possible during the deterministic time, that is the period required by the intermediate node to complete its reception. The quantity of packets that the child node will surely receive may be smaller if the staying probability of the dynamic node network is greater (may be tolerated a good portion of data already received by the dynamic node). On the contrary, the amount of information downloaded by the relay has to be small when it is known this potential parent node can quickly disappear after completing its operations (when the mobility of the environment is high).

Also in this case the utility curve obtained for the choice of the relay as parent node was calculated for three cases, specifically for three percentages of data downloaded from it when the new agent wants to join the network. The values chosen, with respect to the total amount L, are 25% (red curve), 50% (green curve) and 75% (black curve). We see that the threshold characterizing the change of the choice between source and relay, corresponding to the point of intersection between the decreasing and the constant curves, moves gradually to the left as the number of packets received from node r increases. This is because, as already explained in the analysis of Figure 4.12, the time that a node surely remains in the network decreases when it has received many packets. So, for the same mobility value, if the relay received a small percentage of the message, the amount of data that the new agent will have to receive during the stochastic time decreases. Consequently, also the time necessary for node i to complete the transfer, once the node r has received the totality of data, is less and therefore also the probability that the parent node leaves the network while the transfer is in progress decreases. To avoid failure, the value of utility associated with the choice of the relay is greater when the latter has to remain more time, with certainty, in the network. In conclusion, we have seen how the values relating to the percentage of data received by the relay and the mobility of the environment affect in a decisive way the formation of the network topology.

4.4 Final remarks

In this chapter, the decision criteria for a node have been defined, that is to say the utility value to be associated with each of the choices available in the action set of a player. Of course, here we focused only on one agent and so we have not faced the competition between selfish players when they want to connect to the network. However, the tool for obtaining the preferences has been defined, i.e. the utility function, which will be used to achieve the best energy consumption in the network. Later, in the non-cooperative game, we will see that the value of utility that a player obtains will not depend only on its choice to connect to the source or to the relay, and so from the only conditions of the network environment, but also from the choices made by the other agents who want to join system. The formulation described in this chapter concludes one of the core topics of this thesis, that is the generation of an analytical model aimed to find the best conditions for an efficient energy consumption.

Chapter 5

Interaction among Multiple Users

In this chapter, we examine the scenario where more than one player is going to join the dynamic network. Until now we have considered the simple case where only one node had to join the network. Now we have, instead, that two nodes want to connect at the same time to the vertices already part of the graph. To model this conflict situation, where more than one child node has to select a parent node simultaneously to build the network topology, game theory has been applied. The choice of this tool was given by the fact that the decision taken at the moment of the connection of a specific node inevitably influences the utility obtained by the other participants. The utility, as already widely discussed, allows an agent to take a decision, which will always look for finding the one that brings greater benefits to itself. In particular, it was calculated for two different types of users: the first where the nodes are reactive, in which panning tasks are tackled through a myopic optimization-based approaches; the second is represented by the case in which the nodes are proactive, then through the rules established by Expected Utility Theory (EUT) and Prospect Theory (PT) they are able to take into account possible future events.

In this game players are represented by the new devices that are added to the system. The alternatives available to each of them will be represented by the choice of connecting to the information source (static node) or to the a relay (mobile node). As we have seen, the evaluation of the possible alternatives that will lead to the construction of the topology depend on the mobility of the environment, the power needed to establish a new connection, the rates of the wireless channels and the amount of information already downloaded by the relay at the moment of the new agents connection. In this section we will see that the choice of the single node, aimed at minimizing the total energy of the network, will also be strongly influenced by the decision taken by the other players who simultaneously wants to receive the message.

Mechanisms for cost sharing have been introduced in order to find a solution in the case more than one player has to share the price (energy expenditure) for the information forwarded by a parent node, which may be the relay r or the source s in our case. This price is paid to the service provider in order to encourage it to participate in the dissemination process of the common message, so that the best energy-efficient solution can be available. With this we want to avoid that a vertex already part of the propagation structure refuses to spend its energy for spreading the information when that may be the most efficient choice.

As a solution concept for the decentralized model proposed in this project, the Nash Equilibrium (NE) has been used. Precisely, it was decided to apply a methodology to research the NE for mixed strategies, which steps will then be illustrated in detail.

5.1 Two players scenario

Consider a network composed of four nodes. The reference scenario is similar to that illustrated in the previous chapter in Figure 4.3. A source node$s$ has to send a message to all devices forming the network. Connected to it there is a relay node r that, at the moment when the system is analyzed, has already received part of the information from the source. In addition to a child node i, which as we know wants to join the network, in the case we are going to study there is also another child node j that has to choose a parent node for receiving the information. It is also assumed that the two new nodes are synchronized (access to the communication medium at the same time) and therefore time is divided into slots [53] [54]. They will begin to receive the message simultaneously and this generates a conflict situation.

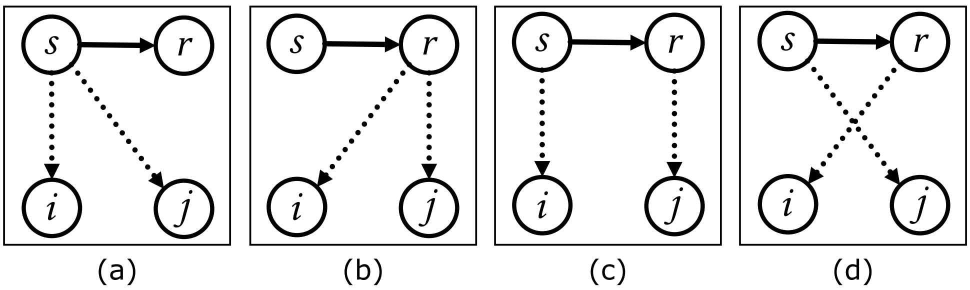


Figure 5.1: System model with all possible transmission scenarios for the two new vertices i and j. (a): multicast connection from the source node s; (b): multicast connection from the relay node r; (c) and (d): unicast connections when two different parent nodes are selected.

The choice of each of the two nodes to connect either to the source or to the relay defines the type of transfer that will be established. In fact the latter can be multicast, in the event where both the two new nodes choose the same parent node, or unicast when they select two different service providers for receiving the message.

This decision has so a direct impact on the topology that will be formed, as well as on the power levels necessary to realize the data transfers. The situation is therefore that shown in Figure 5.1.

As we can see from the last illustration, we have four different transmission combinations, which lead to four different power levels needed for disseminating the message. A multicast transmission is established if both nodes, when they access the system, choose either the source or the relay. In this case, the power needed to realize the transfer is given by the maximum power level at the parent node to communicate with node i and node j, which is $max(P_{k,i}^{\text{uni}}, P_{k,j}^{\text{uni}})$ where $k = \{s,r\}$ is either the source or the relay node. On the contrary, if the two new agents, when they join the network, choose to receive the message from two different parent nodes, they are going to generate two unicast communications. In this case the power imposed by adding the two child nodes i and j is equal to the sum of the two powers $P_{k,r}^{\text{uni}} + P_{k,i}^{\text{uni}}$, with $k = \{s,r\}$, needed to establish the two unicast transmissions.

In this game theoretic framework, the action sets are $A_i = \{u_{s,i}, u_{r,i}\}$ and $A_j = \{u_{s,j}, u_{r,j}\}$ for player i and j respectively and represent, in both cases, the benefit that each participant gets by selecting as its parent node the source s or the relay node r. What happens is that, for example, if the player i decides to receive the message from the static node, while the other player j for the same purpose prefers the dynamic node, they will get different utility with respect to the case in which both choose the same parent node (the transmission power required for the two cases are different). For this reason, and as previously stated, the choice of each player inevitably influences the utility value of the other.

Since in the considered scenario the relay node may forward the message that it is receiving from the source to other devices, motivate it to participate in the forwarding process is of fundamental importance. In such a network, in order to incentivize the relay to act as a parent node for a data transfer, and therefore to consume its energy, towards the child nodes that are added to the system, the forwarding node is paid by its respective receivers. In fact, each vertex of the graph must pay the cost for receiving the message that will be sent to it. The scenario can so be seen as that of a network with different message providers to whom a price has to be paid, e.g. by a virtual currency [55], for the reception of information. The main objective of this section is to distribute the source message through devices that act as intermediaries through a fair cost allocation mechanism. By this we mean that each child node pays a price, which for the objectives of this project coincides with the energy that the parent node spends for the transmission of the message, which must be as low as possible, so as to minimize the total expenditure of energy in the network.

In order to measure how much power is imposed by adding the two new devices to the network, which corresponds to the total cost to be paid by the two players for receiving the message, three cost sharing functions have been used. Precisely, the chosen functions are: Equal Sharing, Marginal Contribution and Shapley Value. They will be analyzed in the next paragraph. It will be seen how they have been of fundamental importance in the calculation of the utility values necessary for the definition of a preference among the actions available to a player, which subsequently allowed to generate the payoff matrix.

5.2 Cost sharing functions

In the multi-hop data dissemination model for wireless networks considered so far, we deal with selfish users. As already mentioned, we want the source's message to be received by all the other nodes of the network. Due to the limits given by the maximum power that a node can achieve in the realization of a transfer, there is the need for intermediate nodes to resend the message toward devices that are not one-hop-reachable. To incentivize the relays, in our case the node r only, to forward the message, a *forwarding cost* is paid to the parent node by its respective child nodes who want to receive the data. The price to pay is defined in this project as the energy that the intermediate nodes spend for the retransmission of the message [56]. Therefore, to reduce the energy required for the dissemination of the message, the child nodes are required to make a decision that would make the parent node consume (pay) as little energy as possible to forward the information. To achieve this goal, such a cost must be shared between the receivers by means of *cost sharing function* (CS) and this is possible only when the established transmission is multicast. In fact, in multicast connections, a forwarding node has more than one receiver and consequently the price to be paid to the relay can be divided among all the devices that receive the message.

The non-cooperative game proposed here, aimed at reducing the total energy consumption, requires that a child node i, when choosing the parent k from which to receive the data , knows the power level $P_{k,i}^{\text{uni}}$ that the service provider should use to establish the new connection. The level of energy required must be such as to have an acceptable SNR at the receiver. In our case, since several child nodes can have different power level requests to get the information with as few errors as possible, the fairness of the shared cost is a key issue for the receiving nodes. Since the price to pay can be different according to the project design (e.g. share the expenses equally, proportionally or in other ways), in order to choose the most appropriate sharing function $f_i^{\text{CS}}(P_{k,i}^{\text{uni}})$ for the user i requiring the service from node k, two cost allocation properties were considered the most suitable for our case:

- **Budget Balancing:** an allocation is budget balancing if the sum of the allocated costs equals total cost [57] [58]. In other words, this property means that the service provider (parent node) recovers from its customers (child nodes) the entire cost of providing the service (message forwarding). Formalizing is obtained that, if the total number of agents benefiting from the service is M_k, which belong to the $\mathcal{M}_k$ set of nodes served by the same service provider k (in our case either the source s or the relay node r), and the total price that the service provider has paid is $C_k(\mathcal{M}_k)$, then:

$$\sum_{i \in \mathcal{M}_k} c_{k,i} = C_k(\mathcal{M}_k) \tag{5.1}$$

where $c_{k,i} = f_i^{\mathrm{CS}}(P_{k,i}^{\mathrm{uni}})$ il the price paid by the single user i to the service provider k, which comes from the application of the cost sharing function on the resource required to establish the connection. As we can see, the sum of all the contributions paid by the M_k agents requesting the service compensates the entire cost necessary for its fruition.

- **Fairness:** the notion of fairness has been defined within this project as a principle of equity [59] that comes out from the fact that if a node consumes more resources it has to pay a higher price. Some participants may complain about an unfair sharing if the obtained solution, at the end of the decision-making process, is achieved at the expense of some players. The price to pay for each child node that wants to receive the message from its parent node providing the service has to take into account the the actual utilization of the resources that each of the new devices, to be added to the system, requires. It is therefore defined a utilization criterion that takes into account the resources necessary to be able to add a new agent to the system. The cost to be paid $c_{k,i}$ should therefore be proportional to the amount of resources $P_{k,i}^{\mathrm{uni}}$ that are used by a parent node k to send the message to a child node i. This means that the cost has to be distributed in function to the stand-alone worths of each member [60] forming the coalition. A coalition is a group of users willing to share the cost of a service. One of the players has not incentive to join the group if, when it participates, pays more than when it is alone. In game theory, when a node always stays in the group it is said to be in the *core*. Formalizing, if the use of the resources $P_{k,i}^{\mathrm{uni}}$ of one of the agents i is greater than the use $P_{k,j}^{\mathrm{uni}}$ of another user j to be served by the same service provider k, i.e. $P_{k,i}^{\mathrm{uni}} > P_{k,j}^{\mathrm{uni}}$, then the cost values obtained from the application of the sharing function must be such as to have $f_i^{\mathrm{CS}}(P_{k,i}^{\mathrm{uni}}) = c_{k,i} > f_i^{\mathrm{CS}}(P_{k,j}^{\mathrm{uni}}) = c_{k,j}$. So, if a user consumes more, for correctness, he has to pay more and viceversa. With this property the perception of every single player about fairness is satisfied.

As previously anticipated, three cost functions have been analyzed in the non-cooperative game developed here: Equal Sharing, Marginal Contribution and Shapley Value. At this stage, where the two of the most important sharing properties have been defined, which will allow us to optimize the total power consumption of the system by assigning the price to be paid for each new agent joining the network, we are going to see which of them are satisfied in the three approaches considered.

5.2.1 Equal Sharing

The first cost sharing function considered is the Equal Sharing (ES). It is a classic sharing rule where every player who receives a service pays the same amount to the service provider. Therefore, the cost to be paid according to this criterion is equally divided between users who choose the same source of information. Some variants of this approach [61] introduce a threshold that a user has to overcome in order to pay the same price as the other users of the service. This is because it is not fair to become part of a group where there may be agents that require a far greater amount of resources than others and the price to pay is still the same. In fact, as we will see in the next example, one of the major limits of this rule lies in the fairness concept. However, we chose not to introduce any threshold in our model to participate in cost sharing, so each node can freely choose a service provider knowing that, regardless of the amount of resources required, will pay the same price as all the other agents with whom it shares the service.

In order to formalize this concept, let $N = \{1, \dots, n\}$ denote the set of all players in a game. Given the broadcast nature of the wireless channel, we know that a forwarding node can transmit the message to a set of receiving devices. We thus denote with $\mathcal{M}_k$ the set of child nodes that receive the message from the same parent node k. The total number of entries in the $\mathcal{M}_k$ set is indicated with M_k. The total price $C_k(\mathcal{M}_k)$ that the M_k agents must pay to the service provider k to get the data is equal to, since the channel is broadcast, the highest transmission power required from its child nodes in $\mathcal{M}_k$ formula (3.2). Being $p_j^{\mathrm{Tx}}(\mathcal{M}_k)$ the total cost to be paid to the parent node k to provide the service, according to the ES rule, a child node i must pay a price $c_{k,i} = f_i^{\mathrm{ES}}(P_{k,i}^{\mathrm{uni}})$ shared with the other users in the same set $\mathcal{M}_k$ equal to [62]:

$$c_{k,i} = \begin{cases} \dfrac{C_k(\mathcal{M}_k)}{M_k} = \dfrac{p_k^{\mathrm{Tx}}(\mathcal{M}_k)}{M_k} & \text{if} \quad i \in \mathcal{M}_k, \\ 0 & \text{otherwise.} \end{cases} \tag{5.2}$$

So, if one players i of the total set of players N decides to take advantage of the service provided by the parent node k, i.e. becomes part of the sub-set $\mathcal{M}_k \subset N$, then must pay the same price as all the other nodes that have made the same decision (cost equally shared among all the M_k nodes of the coalition).

We now apply this rule to the specific case of only two players. In the following example, such a situation will be addressed, also because this specific number of agents coincides exactly with the number of devices added to the system in our scenario. In addition, it will be possible to verify the satisfaction of the budget balancing and fairness properties that are particularly important for achieving the minimum overall energy expenditure.

Example 5.1. Consider the system model shown in figure 5.1. Two child nodes i and j have to receive a message from a parent node that can be either the source s or the relay node r. We are interested in the case where both the two new agents choose the same source of information. This is because, in the opposite case, two unicast connections would be established and both $\mathcal{M}_s$ and $\mathcal{M}_r$ sets of nodes served by the two service providers would be composed by only one user. Observing the formula (5.2), being only one the node served, it will simply pay the entire cost (denominator equal to one), not obtaining any sharing advantage.

Now let us focus on the most interesting case for our study: suppose that both nodes i and j choose to connect to the same parent node k (which can be either r or s in our model), so as to have $M_k = 2$. The transmission that is established is, according to these assumptions, multicast. We have therefore that the total cost $C_k(\mathcal{M}_k)$ at the parent k in order to transmit the message to the two children composing the set $\mathcal{M}_k$ is given by, according to formula (3.2), $p_k^{\text{Tx}}(\mathcal{M}_k) = max(P_{k,i}^{\text{uni}}, P_{k,j}^{\text{uni}})$. The values $P_{k,i}^{\text{uni}}$ and $P_{k,j}^{\text{uni}}$ represent the power levels required by node i and node j, respectively, to have an acceptable SNR at the reception. In other words, they are the amount of resources needed by each child node (power that would needed if a unicast communication is established between two communicating parts, as if each of them connects alone to the network).

Assume to be $P_{k,i}^{\text{uni}} < P_{k,j}^{\text{uni}}$, with $P_{k,i}^{\text{uni}}, P_{k,j}^{\text{uni}} > 0$, which means that node i requires a smaller amount of resources with respect to node j in order to receive the information. Applying the Equal Sharing formula (5.2), such that $c_{k,i} = f_i^{\text{ES}}(P_{k,i}^{\text{uni}})$ and $c_{k,j} = f_j^{\text{ES}}(P_{k,j}^{\text{uni}})$, we obtain that:

$$c_{k,i} = c_{k,j} = \frac{C_k(\mathcal{M}_k)}{M_k} = \frac{p_k^{\text{Tx}}(\mathcal{M}_k)}{2}.$$

The two new nodes therefore pay the same price even if the resources needed to communicate with i are less than those needed to communicate with j. This is unfair, because as we see the allocation cost for a player is not proportional to the real use of the resources necessary for its communication. This is negative aspect because can generate tendencies in child nodes to disrupt the sharing concept, so that and the end the total energy expenditure is more. The fairness property is not so satisfied.

On the other hand, by applying the formula (5.1), we obtain that:

$$c_{k,i} + c_{k,j} = \frac{p_k^{\text{Tx}}(\mathcal{M}_k)}{2} + \frac{p_k^{\text{Tx}}(\mathcal{M}_k)}{2} = p_k^{\text{Tx}}(\mathcal{M}_k) = C_k(\mathcal{M}_k).$$

We see that the Budget Balancing property is met and so the parent node recovers, from its child nodes, the entire cost of providing the service. This allows to incentivize a node located in an intermediate position between two other devices to forward the message, with the consequent benefit of reducing the energy necessary for the dissemination of a message common to all the nodes of the network.

5.2.2 Marginal Contribution

Another method for cost allocation among the players of a games that allow side payments to be made among the players is Marginal Contribution (MC). As before, let N denote the set of all players and $\mathcal{M}_k \subset N$ be the set of those who share a service provided by the same parent node k, with cardinality M_k. The Characteristic value $C_k(\mathcal{M}_k)$ gives the maximum cost incurred by the set of users being part of $\mathcal{M}_k$. As we already know, it is equal to the highest transmission power requested by the parent node to communicate with the child nodes who decided to receive the message from it. According to the MC rule, the price to pay $c_{k,i} = f_i^{\mathrm{MC}}(P_{k,i}^{\mathrm{uni}})$ for a node i in order to share the total cost with the other users served by the same forwarding node is [63]:

$$c_{k,i} = \begin{cases} C_k(\mathcal{M}_k \cup i) - C_k(\mathcal{M}_k) & \text{if} \quad i \in \mathcal{M}_k, \\ 0 & \text{otherwise} \end{cases} \tag{5.3}$$

where $C_k(\mathcal{M}_k \cup i)$ represents the new set of nodes that would be formed if the agent i is added to those who chose to share the cost of a service given by the parent node k. From this amount has to be subtracted the maximum cost $C_k(\mathcal{M}_k)$ that the service provider would pay if node i was not part of the group served by it.

Also for this new sharing concept, it is good to provide the reader with an example of its application in the case of only two players who compete to reduce their price to pay for joining the system. The verification of the fairness and budget balancing properties will be carried out, in order to highlight the substantial differences with the previously studied cost function.

Example 5.2. Consider again the case in which two new devices want to join the network formed by two possible sources of information: a static and a dynamic node (Figure 5.1). As we have seen in the previous case, it is not of particular interest, from a cost sharing perspective, the case in which the two child nodes choose to receive the message from two different parent nodes, i.e. one chooses the source s and the other the relay r. In such a case two unicast transmissions would be established. This means that the set of nodes served by the same service provider is composed of only one node, thus there is no possibility of sharing the cost between multiple agents (each player pays the entire cost of the received service). For this reason, let us consider directly the case in which the two child nodes choose the same parent node $k = \{s, r\}$ for getting the data, so that a multicast transmission is formed.

Also assume in this example that $P_{k,i}^{\mathrm{uni}} < P_{k,j}^{\mathrm{uni}}$, with $P_{k,i}^{\mathrm{uni}}, P_{k,j}^{\mathrm{uni}} > 0$, where these two quantities represent the power levels required by each of the two nodes to receive the common message with an acceptable SNR. Applying the formula (5.3) of the MC we get, according to the resources actually used, that the cost $c_{k,i} = f_i^{\mathrm{MC}}(P_{k,i}^{\mathrm{uni}})$

to pay for the user i is:

$$c_{k,i} = P_{k,j}^{\text{uni}} - P_{k,j}^{\text{uni}} = 0$$

being the maximum transmission power requested at the parent node k from the child nodes in $\mathcal{M}_k$ that of the node j, even when node i is added to the set. So, if a node requires a lower power level than others in the sharing group, it has the possibility, within a multicast connection, to receive the message without paying the cost of the service. This turns out to be a fundamental principle for the reduction of total energy expenditure in a system: given that the parent node to communicate with both nodes must spend the energy required by j, it might as well not require any additional cost to node i in order to receive the same information.

The price to pay $c_{k,j} = f_j^{\text{MC}}(P_{k,j}^{\text{uni}})$ for the other agent instead is:

$$c_{k,j} = P_{k,j}^{\text{uni}} - P_{k,i}^{\text{uni}} > 0,$$

being the required power level to disseminate the information, when node j participates in cost sharing, greater than the case in which to receive the message from the parent node k is the user i only.

We say this taxing scheme to be incentive-compatible, which means that if a node requires less resources to be able to join the system, it has to pays less. In fact we have that $c_{k,i} < c_{k,j}$. Thanks to this result, a node is more incentivized to share the costs for receiving a service in common with other users, which could bring benefits in terms of total energy expenditure for the dissemination of the message. The fairness property is so satisfied, the players perceive a correctness given by the proportionality of the assigned costs with respect to the actual resources needed for adding a device to the system. However, this does not happen for the budget balancing property, which is not met: the total amount of tax collected is usually less than $C_k(\mathcal{M}_k)$ [64]. In fact, by applying the formula (5.1) we obtain that

$$c_{k,i} + c_{k,j} = 0 + P_{k,j}^{\text{uni}} - P_{k,i}^{\text{uni}} < P_{k,j}^{\text{uni}} = p_k^{\text{Tx}}(\mathcal{M}_k) = C_k(\mathcal{M}_k).$$

Thus, the forwarding node does not recover the full cost of the service that it has provided. This could lead to a non-participation of the k node in the data dissemination process. As a result, it may be necessary to establish multiple unicast connections, which may not be energy efficient, since the broadcast nature of the wireless channel is not exploited and perhaps the multicast solution would have been the best.

We are therefore in the opposite case of what was examined in the ES rule, in fact in the MC the properties have been respected in a reverse way with respect to previous sharing rule. Each of the two cost functions studied so far has therefore advantages and disadvantages: the first (ES) allows the parent node to get the total cost of the service, but does not divide the expenses equally among the users; the second one (MC), instead, allows the to share the costs for the use of resources proportionally, but it may not motivate a potential forwarding node to participate in the data dissemination. We will see if with the third proposed approach it is possible to combine the positive aspects of the two sharing methods described so far, so as to compensate for the expenses of the intermediate node and to rightly distribute the cost among the participants.

5.2.3 Shapley Value

The last solution concept used to allocate the cost that an agent has to pay in order to take advantage of a service provided by a forwarding node is Shapley Value (SV) [65]. It is based on the marginal contribution of each player. This means that the price that a player pays varies depending on the number of agents who choose the same service provider. In our case, it will depend on the number of child nodes that decide to receive the message from the same parent node. SV aims to distribute the total cost of a service in proportion to the use of resources allocated for a user. It therefore meet the fairness concept. It also has the goal, in our case, to totally compensate for the cost paid by the forwarding node to deliver the message to those users who have to receive it.

In wireless networks it is known that a device can send the message to more than one receiver. As in the two previous cases, the set of nodes that selected the parent node k so as to receive the information is indicated by $\mathcal{M}_k$, with a total number of M_k entries. So, to a child node i that selects parent node k for getting the data, SV assigns a cost $c_{k,i}$ to the new agent of the system that is dependent on the power required by the child node i to receive the message with the fewest transmission errors. This transmission power is denoted with $P_{k,i}^{\text{uni}}$. Given the broadcast nature of the wireless channel, the transmission power $p_k^{\text{Tx}}(\mathcal{M}_k)$ that a parent node uses to communicate with all its child nodes it is simply equal, in case of a multicast transmission, to the highest power required by the nodes in $\mathcal{M}_k$. By sorting the requested powers of the nodes in $\mathcal{M}_k$ in the form of $P_{k,1_k}^{\text{uni}}, \ldots, \leq P_{k,M_k}^{\text{uni}}$ we obtain that the cost $c_{k,i} = f_i^{\text{SV}}(P_{k,i}^{\text{uni}})$ of a node i within the set $\mathcal{M}_k$ of the agents served by the parent node k, using the SV rule can be calculated as [56]:

$$c_{k,i} = \begin{cases} \displaystyle\sum_{j=1}^{i} \frac{P_{k,j}^{\text{uni}} - P_{k,j-1}^{\text{uni}}}{M_k + 1 - k} & \text{if} \quad i \in \mathcal{M}_k, \\ 0 & \text{otherwise.} \end{cases} \tag{5.4}$$

for which $P_{k,0}^{\text{uni}} = 0$.

As previously done, to verify which of the budget balancing and fairness properties are satisfied, it is convenient to make an application of this approach in the specific case of only two nodes that go to share the cost to be paid to the service provider.

Example 5.3. Suppose again that $P^{\text{uni}}_{k,i} < P^{\text{uni}}_{k,j}$, with $P^{\text{uni}}_{k,i}, P^{\text{uni}}_{k,j} > 0$, that is to say the power level necessary at the parent node k to communicate with i is less than the one needed to communicate with j. The system model in consideration is always the one in figure 5.1. The set of nodes served by the service provider $k = \{s, r\}$, as we said $\mathcal{M}_k$, is formed by both nodes i and j in the case considered for the calculation of the $c_{k,i} = f^{\text{SV}}_i(P^{\text{uni}}_{k,i})$ and $c_{k,j} = f^{\text{MC}}_j(P^{\text{uni}}_{k,j})$, which means that the case interesting for us is when the transmission established is multicast. SV would coincide with the unicast power required at the parent node if they choose different forwarding nodes. For this reason, we are always interested in the multicast case where there is an effective cost-sharing. Applying the SV formula (5.4) we obtain that the costs for the two nodes are:

$$c_{k,i} = \frac{P^{\text{uni}}_{k,i}}{2}$$

$$c_{k,j} = \frac{P^{\text{uni}}_{k,i}}{2} + (P^{\text{uni}}_{k,j} - P^{\text{uni}}_{k,i})$$

where, since the transmission power $p^{\text{Tx}}_k(\mathcal{M}_k)$ at the parent node k is equal to the highest required from its child nodes in $\mathcal{M}_k$, we have that $p^{\text{Tx}}_k(\mathcal{M}_k) = P^{\text{uni}}_{k,j}$ corresponding to the total cost $C_k(\mathcal{M}_k)$ to be collected by the forwarding node to provide the service.

We now evaluate which of the two properties, the most important for our project, are met in this cost-sharing solution. Consider, as first, the budget balancing property. Applying the formula 5.1 we see that:

$$c_{k,i} + c_{k,j} = \frac{P^{\text{uni}}_{k,i}}{2} + \frac{P^{\text{uni}}_{k,i}}{2} + (P^{\text{uni}}_{k,j} - P^{\text{uni}}_{k,i}) = P^{\text{uni}}_{k,j} = p^{\text{Tx}}_k(\mathcal{M}_k) = C_k(\mathcal{M}_k).$$

Since the sum of the individual costs of the child nodes benefiting from the service provided by the parent node k is equal to the total cost paid by the latter to satisfy their requests, the budget balancing property is satisfied. With the SV solution, an agent is encouraged to forward the common message (to be a parent node) that must be received by all the network devices. This positive aspect, which consists in the forwarding availability of intermediate nodes, may allow a more efficient energy expenditure, since a voluntary participation of the relay node in the dissemination process is not excluded.

Regarding the fairness property, we see that it is also satisfied. In fact, the cost that node i goes to pay $c_{k,i}$ to receive the message from the parent node k is less than $c_{k,j}$, which is the one paid by j for the same service. This is because the resources that would actually be allocated for the addition of node i to the system, i.e. $P^{\text{uni}}_{k,i}$, are less than those necessary for communicating with node j, i.e. $P^{\text{uni}}_{k,j}$. What we have seen is not particularly surprising, since, as shown in [66], SV is known as a fair sharing method in cost allocation games.

Now that three of the most known sharing cost functions have been analyzed, we can compare the results obtained in terms of satisfaction of the two properties considered most interesting for the purpose of this project.

The first cost function studied was ES, which, as we have seen, was able to make recover to the forwarding the entire cost of the dissemination service, but at the same time did not rightly distribute the cost of the operations among the customers of the service. This could lead to a refusal by new devices that are added to the network of the will to connect to the same service provider, as the costs are not shared proportionally to the actual resources needed by each of them for the communication. Such a situation is disadvantageous from an energetic point of view, because when there is a tendency to select the same node for receiving the information, guaranteed by the fairness property, the connections that form a network are mostly multicast. Since it is convenient in wireless networks many times for a message to be propagated via a multicast transmission, due to the broadcast nature of the transmission medium, the first approach analyzed may not be the best.

In the second approach, namely MC, the multicast connections could lead to free-riding nodes, i.e. some nodes can receive the information without paying the cost of the service. This could lead to better use of resources and therefore to an energy efficiency improvement, but, as we have seen, the forwarding node could be unwilling to participate in the propagation process. The reason is that the property of budget balancing in this case is not satisfied. Consequently, there may be cases in which the optimal solution is not available given to the refusal of the forwarding node in providing the service, since it will not receive the full cost for its delivery.

Given the limit shown by this second approach, a final solution has also been analyzed. The SV in fact goes to compensate for the deficiencies shown by the two previous cost sharing functions. In fact we have seen that, in addition to recovering the total cost of the service provided by the parent node, it allocates proportionally the individual costs of its child nodes based on the resources actually used by them. All players are so incentivized to share a service. Consequently, the choice a parent node that establishes a unicast connection with the child node, of which the latter must pay the full cost, generates no more interest. For this reason, the last solution turns out to be, in wireless a environment, more efficient from an energy optimization perspective. Given the two advantages seen in the analysis of the last approach, when we will discuss the results obtained at the end of the work presented in this thesis, the SV will be chosen as a cost function to perform the simulations.

5.3 Payoff matrix derivation

As we have seen in Chapter 2, whenever a game is proposed to us, we can build a payoff matrix. It contains all the rewards for the various choices that players have in their strategy sets. In our specific case of 2 players, we will have two matrices.

Each entry of the structure will refer to one of the four topology combinations that can be formed in the proposed network scenario. In this regard, referring to the system model in figure 5.1, the first payoff can be the one obtained in case both the players i and j choose the source s; the second can be the reward that a player gets when it chooses the static node s and the other the dynamic node r; the third is so the one that the participant gets when it chooses the relay r and the other selects the source s; while the last gain refers to the case where both players choose the dynamic node r in order to receive the message.

In game theory, the two payoff matrices, one for each player, are merged together in one payoff matrix. By doing this, we obtain a single structure where each entry is composed of two values, called utilities, which correspond to the benefit obtained in case a specific choice is made: one for the row player, which is node i given the model considered up to now, and one for the column player, node j in our case.

What has been briefly said can therefore be summarized in table 5.1 (a) where we see the strategy sets of the two players $\mathcal{A}_i = \{s, r\}$ and $\mathcal{A}_j = \{s, r\}$, for player i and j respectively, to be composed by the possibility of selecting the source s or the relay r to receive the message. To each decision made by them is associated a utility value $u_l(a_l, \mathbf{a}_{-l})$, which depends (section 2.2.2) on the strategy $a_l \in \mathcal{A}_l$ picked up by one player $l = \{i, j\}$ and the action $\mathbf{a}_{-l}$ selected by the other participant. The rewards values that make up the matrix come out from the utility function designated in Chapter 4 for a single device that wants to connect to the network. It will now be adapted for the case in which two players participate to the game, i.e the two nodes i and j that want to join the system.

Player i \ Player j	s	r
s	$u_i(s,s), u_j(s,s)$	$u_i(s,r), u_j(s,r)$
r	$u_i(r,s), u_j(r,s)$	$u_i(r,r), u_j(r,r)$

(a)

Player i \ Player j	s	r
s	$c_i(s,s), c_j(s,s)$	$c_i(s,r), c_j(s,r)$
r	$c_i(r,s), c_j(r,s)$	$c_i(r,r), c_j(r,r)$

(b)

Table 5.1: A two row, two column strategic game. (a) Payoff matrix: each entry is the utility that would be obtained (preferably the highest) according to the strategies selected by the two players. (b) Cost matrix: each entry is the cost that would be paid (preferably the lowest) according to the strategies selected by the two players.

The fundamental step in this process of obtaining the utilities for the two-player model is made by introducing in its calculation the cost sharing functions (CS) illustrated in the previous section. It is therefore defined $f_l^{\mathrm{CS}} = \{f_l^{\mathrm{ES}}, f_l^{\mathrm{MC}}, f_l^{\mathrm{SV}}\}$ the cost function applied to the power level $P_{k,l}^{\mathrm{uni}}$ required by a node l to receive the information from a forwarding node k, which as we have seen correspond to the price to be paid for the service, and it can be Equal Sharing (ES), Marginal Contribution

(MC) and Shapley Value (SV). It is not important for the time being which is specifically used in the calculation of utilities, but it is important to know that the values on which it is applied come out of one of the three previously discussed cost sharing methods.

Referring to the utility function defined in formula (4.7), which represents the starting point to make a new agent choose the forwarding node, we see that the numerator is always constant. In fact, the amount of data to be sent is always the same. For this reason, from this moment on we will only refer to the denominator of this function defining it as a cost, on which the cost sharing function will be applied, so as to obtain $c_l(a_l, \mathbf{a}_{-l})$. It so represents the price to pay for a node $l = \{i, j\}$ to access the network that depends on the strategies picked up by the two players and minimize it corresponds to obtaining greater benefits for the selfish player. The choice of an agent, at this point, will be made not by trying to maximize the utility obtained when an action is selected, but in order to minimize the costs that it entails. Similarly to the payoff matrix, the cost matrix (Table 5.1 (b)) has been defined, which reports the price to be paid for every possible action available to the players. In this case the decision that will bring greater benefits to the agent will be the one corresponding to the lower cost value.

In addition to the introduction of the cost sharing functions for the definition of the analytical model that allows a node to make a decision, i.e. the cost (from now on the cost matrix will be considered), two classes of users have been defined. The price to pay must therefore be calculated based on the type of player that is added to the system. The two classes of agents considered are: *reactive user* and *proactive user*, which will then be individually investigated. They have been defined so as to compare the total energy required to disseminate the message based on the decisions that a node can make for the creation of the network topology.

5.3.1 Reactive user

Reactive users are those who, when selecting an action available in their action set, do not take into account any future failure that may occur once the decision has been made. They are therefore reactive, which means they will be forced to react to the occurrence of a negative event that had not previously been taken into account. The type of approach to the problem is defined in the literature as myopic. A user who follows myopic rules looks only at the current state of the system and not at the possible evolutions to which the latter may face.

The cost function defined in this case, referring to the system model illustrated in figure 5.1, does not take into account a possible disappearance of the dynamic node r. At the time of the decision of the parent node to which the child node must connect, the myopic user simply searches for the lowest cost, i.e. the power required to receive the message, to be paid at the current time. Thus, nodes i and j will look primarily at the respective $P^{\text{uni}}_{k,i}$ and $P^{\text{uni}}_{k,j}$, respectively, where k is the forwarding node they want to connect to, that may be either the static node s or the dynamic node r. Basically, the two nodes see the network as static, since it does not consider the mobility of the environment and the consequent probability that a device other than the source remains in the network.

In order to define the cost, consider the well known utility function (formula (4.4)) found in the literature that for a unique child node i to be added to the network is expressed as $u_{k,i} = \frac{T_{k,i}}{P_{k,i}}$ when it chooses k as forwarding node. It can be rewritten in the form seen in formula (4.7) where it is clearly seen that the numerator is constant ($u_{k,i} = \frac{L}{P^{\text{uni}}_{k,i} \cdot t^{\text{TX}}_{k,i}}$) and for this reason we will no longer refer to it. To obtain the cost that has to be shared among the two user, we apply the CS function on the price to be paid that in our case is equal to the transmission power requested at the parent node k present in the denominator of the utility function, so we have:

$$c_l(a_l, \mathbf{a}_{-l}) = f^{\text{CS}}_l(P^{\text{uni}}_{k,l}) \cdot t^{\text{tot}}_{k,l} \tag{5.5}$$

where $t^{\text{tot}}_{k,l}$ would be the time needed to transfer the whole data without failures. We so defined the cost $c_l(a_l, \mathbf{a}_{-l})$ that a child node $l = \{i, j\}$ (can be agent i or j when both simultaneously choose to join the network) has to pay to receive the message from a parent node k, which depends on the decision made by both the participants.

As we know two main situations can occur: either both choose the same parent node (multicast transmission) or they choose different information sources (two unicast transmissions). In the study of the CS functions we have seen that they make sense only when the parent node k serves more than one child node at the same time, otherwise the cost to be paid is not shared among multiple entities, being the set of agents served by the parent node $\mathcal{M}_k$ composed of a single node. For this reason, the CS function is actually working when the connection is multicast, which means when both players choose the same parent node ($c_l(s, s)$ or $c_l(r, r)$, with $l = \{i, j\}$).

We must not confuse the value $f^{\text{CS}}_i(P^{\text{uni}}_{k,i})$ and $f^{\text{CS}}_j(P^{\text{uni}}_{k,j})$ with the real power consumption in the network. The CS function is applied only in order to obtain the cost value that influences the choice of a player, who aims at the smallest possible. The power actually used, as well known, when choosing the same forwarding node, is $p^{\text{Tx}}_k(\mathcal{M}_k) = max(P^{\text{uni}}_{k,i}, P^{\text{uni}}_{k,j})$. Thus, the formula (5.5) provide costs shared between the two users only when the decision they made leads to the establishment of a multicast connection.

In the opposite case, when the two agents choose two different service providers ($c_l(s, r)$ or $c_l(r, s)$, with $l = \{i, j\}$), the cost function loses its meaning so as to have $f^{\text{CS}}_l(P^{\text{uni}}_{k,l}) = P^{\text{uni}}_{k,l}$ and the price to pay for a user l is simply $c_l(a_l, \mathbf{a}_{-l}) = P^{\text{uni}}_{k,l} \cdot t^{\text{TX}}_{k,l}$. Since here there is no tax sharing to obtain the service, the power value that appears in the formula coincides with the resource actually spent for the addition of a single node in the system.

In conclusion, it is now possible to see clearly that the calculations for obtaining the entries forming the cost matrix have been made not taking into account the probability of a node to remain in the network. This is because, as mentioned before, a myopic user does not consider the event that the service provider may leave. It only aims to minimize the energy spent at the connection time.

The behavior of these users reflects most of those seen in the literature (Chapter 3), where most of the networks taken into consideration were static. It was decided to consider in this situation so as to have a benchmark to compare how much energy can be saved when a node, at the time of connection, already considers the possibility of some future negative event. In the subsequent case study, in fact, the nodes are proactive and the cost formula designed for them will therefore take into account the dynamism of the environment.

5.3.2 Proactive user

When proactive users are added to the system, the dynamism of the network environment becomes the key for the calculation of the costs associated with the choice of the single agent. They, therefore, instead of myopically overreacting to a negative event that may occur in the future, take into account such a possibility already at the time when the selection of the node from which to receive the message is taken. According to this model, child nodes consider, within the calculation of the cost associated with a specific action available to them, the possibility that the forwarding node may suddenly leave the network.

For what concerns the connection to the source, there will not be substantial differences compared to the case seen in the Reactive User, since they see the environment as static and the source in any case never leaves the network. Indeed we have that, when both players choose the static source s, the cost for the user $l = \{i, j\}$ is simply (from formula (5.5)):

$$c_l(s, s) = f_l^{\mathrm{CS}}(P_{k,l}^{\mathrm{uni}}) \cdot t_{k,l}^{\mathrm{tot}}.$$

Instead, when a user chooses the source s and the other selects the relay r, there is no longer a share to be done on the cost. Under this condition, the price to pay for the user who decides to receive the message directly from the source s is

$$c_{k,i} = \begin{cases} c_i(s, r) = P_{s,i}^{\mathrm{uni}} \cdot t_{s,i}^{\mathrm{tot}} \\ c_j(r, s) = P_{s,j}^{\mathrm{uni}} \cdot t_{s,j}^{\mathrm{tot}} \end{cases}$$

where the first cost is obtained for the case in which such user is i (as we see j selects the dynamic node), while the second one is for user j (and i chooses the relay r). Note that the source s is at the first term in the cost brackets if the player who chooses it is i. The reason is that it is the row player. For the column player j the choice is represented by the second term in brackets, which allows to address the price to be paid into the cost matrix.

The main difference concerns the connection to the dynamic node, which, as we know, has a certain probability of remaining in the network and may not be able to transfer the whole message before its disappearance. For this reason, the starting formula from which the cost that a child node must pay is the one described in formula (4.12) for the dynamic forwarding node (relay r). Being the numerator also in this case constant, the cost is defined as the denominator of that formula, which is reported below for a unique user who wants to join the system:

$$c_{r,i} = P_{r,i}^{\text{uni}}(t_{s,r}^{\text{rem}} + p_r^{\text{st}} \cdot t_{r,i}^{\text{rem}}) + (1 - p_r^{\text{st}})P_{s,i}^{\text{uni}} \cdot t_{s,i}^{\text{tot}} \tag{5.6}$$

The value $c_{r,i}$ represents the price that a new child node i, if it is the only node to be added to the network, must pay to the relay node r for receiving the message.

We are now interested, obviously, in the case where the agents who have to compete in order to get the service are two. To this end, the cost sharing mechanism that has been formulated in the previously studied CS functions must be integrated into the last formula. So, as for the case of reactive users, also here the price for the service can be shared when both new users choose the same service provider (musticast transmission).

Contrary to the previous case, however, it is not possible to apply the cost sharing function directly to the formula (5.6), because a further subdivision of users has to be done when they are proactive. In fact, it is possible to identify two classes of agent that do not address the problem myopically: *passive user* and *active user*. The main difference between the two lies in the fact that active users interfere with the choices made by the underlying system design, while the passive ones do not affect such a decision. By this we mean that the model to follow for the choice of a passive user is the Expected Utilty Theory (EUT), while the decision taken by an active user, who has an unbalanced perception between losses and gains at the time of decision, is based on the Prospect Theory (PT). The cost function, which provides the values that make up the matrix with which a user can evaluate his preferences, has therefore been redefined for these two types of users.

I) Passive user

The approach to be taken in the case of a passive user for the cost definition, on which the player has to make an assessment in order to come to a decision, is fundamentally the one used in Chapter 4. In fact, according to the EUT, the decision weight depends linearly from the probability of each outcome occurring. In our case, the numerical value associated with the alternatives that a player has available depends on the probability that the relay node leaves the network, without this expectation being altered somehow by the user's perception.

The weight attributed by the individual to the probability of occurrence of the negative event, i.e. the relay disappears so as to receive the message from the source from scratch, reflects a linear function, which allows the rational agent to make optimal decisions.

Consider as first the case in which both the players i and j decide to receive the message from the relay node r. In this case we define the cost that a passive user $l = \{i, j\}$ has to pay to the forwarding node r as:

$$c_l(r,r) = f_l^{\mathrm{CS}}(P_{r,l}^{\mathrm{uni}})(t_{s,r}^{\mathrm{rem}} + p_r^{\mathrm{st}} \cdot t_{r,l}^{\mathrm{rem}}) + (1 - p_r^{\mathrm{st}}) f_l^{\mathrm{CS}}(P_{s,l}^{\mathrm{uni}}) \cdot t_{s,l}^{\mathrm{tot}} \tag{5.7}$$

where the CS function chosen to assign the price to be shared among the users receiving the message from the same forwarding node, i.e. the power requested at the parent node for the service to be provided, can be ES, MC and SV. We can see that the $c_l(r,r)$ cost, with $l = \{i, j\}$, has been simply obtained by applying the CS function on formula (5.6) and this was possible because in the previous chapter the approach used to obtain the utility function was, as in this circumstance, EUT. It is interesting to notice that the CS function is also applied if the two nodes fail and have receive the message from the source ($f_l^{\mathrm{CS}}(P_{s,l}^{\mathrm{uni}})$). This is because if the communication with the relay fails, both must at the same time receive the message from the static node and this allows, since they are synchronized, to share the new cost with the source.

As we see the CS application makes sense only when the two players choose to get the data from the same parent node, because if they connect to two different information sources (no sharing possible) we simply have for the two player i and j that

$$\begin{cases} c_i(r,s) = P_{r,i}^{\mathrm{uni}}(t_{s,r}^{\mathrm{rem}} + p_r^{\mathrm{st}} \cdot t_{r,i}^{\mathrm{rem}}) + (1 - p_r^{\mathrm{st}}) P_{s,i}^{\mathrm{uni}} \cdot t_{s,i}^{\mathrm{tot}}, \\ c_j(s,r) = P_{r,j}^{\mathrm{uni}}(t_{s,r}^{\mathrm{rem}} + p_r^{\mathrm{st}} \cdot t_{r,j}^{\mathrm{rem}}) + (1 - p_r^{\mathrm{st}}) P_{s,j}^{\mathrm{uni}} \cdot t_{s,j}^{\mathrm{tot}}. \end{cases}$$

We have thus seen how the cost function for a passive user is obtained according to the principles derived by the Expected Utility Theory.

II) Active user

An active user acts by limited rationality and therefore its purpose is not to look for the optimum, as it was for the case of the passive user, but rather is to look for satisfactory solutions. It interferes with the decisions made by the underlying system design, in fact it tends to overestimate the small probabilities and to underestimate the medium or high ones.

The relationship between the probability of occurrence of an event and the weight attributed by the individual to such a value is not linear in this case. The model that allows the definition of a cost for this specific type of user is the Prospect

Theory (PT) (section 2.1.2), which induces the agent to a decision-making process that does not necessarily lead to the highest utility (or lower cost as in our case).

In order to assign a subjective weight to the expectation of a user about the realization of an event, PT uses a Probability Weighting Function (PWF) defined as $w(p) = \exp(-(-\ln p)^\alpha)$. Because of this function an option is not multiplied by the probability of occurrence of the option itself (as stated by the EUT), but for the value obtained through the PWF, which overestimates the unlikely events and underestimates the likely ones.

In addition to the distortion introduced by the PWF, a further Asymmetric Value Function (AVF) is used to define the cost that the player must pay to participate in the game. Such a function allows the passive user to be risk averse, which means the value of an option is not calculated in absolute terms, but compared to a reference point that is the basis for understanding whether the decision taken leads to gains or losses. In the PT, the AVF makes a player to overestimate the losses (it makes them feel amplified) compared to the gains, so that the user tends to avoid the most unfavorable situations. We will so refer only the the loss side of the AVF, defining it as $v(x) = \Lambda(x)^\gamma$.

To calculate the cost that an active user has to pay for getting the service, it is not enough in this case only to apply the CS function to the formula (5.6), but we also need to consider how the PWR and the AVF influence the decision-making process. If two vertices have to be added to the graph, the cost that a new agent l (active user, which can be either player i or j) pays to the relay r to receive the message, in case the other child node also selected the dynamic node as its parent node, is:

$$\begin{aligned} c_l(r,r) =& f_l^{\mathrm{CS}}(P_{r,l}^{\mathrm{uni}})\ t_{s,r}^{\mathrm{rem}} + w(p_r^{\mathrm{st}}) \cdot v(f_l^{\mathrm{CS}}(P_{r,l}^{\mathrm{uni}})\ t_{r,l}^{\mathrm{rem}}) + \\ &+ w(1-p_r^{\mathrm{st}}) \cdot v(f_l^{\mathrm{CS}}(P_{s,l}^{\mathrm{uni}})\ t_{s,l}^{\mathrm{tot}}) = \\ =& f_l^{\mathrm{CS}}(P_{r,l}^{\mathrm{uni}})\ t_{s,r}^{\mathrm{rem}} + e^{[-(-\ln\ p_r^{\mathrm{st}})^\alpha]} \cdot \Lambda[f_l^{\mathrm{CS}}(P_{r,l}^{\mathrm{uni}})\ t_{r,l}^{\mathrm{rem}}]^\gamma + \\ &+ e^{\{-[-\ln\ (1-p_r^{\mathrm{st}})]^\alpha\}} \cdot \Lambda[f_l^{\mathrm{CS}}(P_{s,l}^{\mathrm{uni}})\ t_{s,l}^{\mathrm{tot}}]^\gamma\ . \end{aligned} \tag{5.8}$$

As was widely discussed in section 2.1.2, the parameters introduced by the PT model for the calculation of costs are: $0 < \alpha \leq 1$ is the probability distortion parameter, which reveals how the subjective evaluation of the user distorts the objective probability (smaller α means stronger distortion); $\Lambda > 1$ is the loss penalty parameter, which is going to amplify loss perception of the active user (larger Λ mens that it is more risk-averse); $0 < \gamma < 1$ is the risk parameter, which makes the loss side of the value function more convex (i.e., the virtual operator is more risk-seeking) when gamma approaches to zero and viceversa.

Also in this case, as it was for the previous types of user, the application of the CS function only makes sense if the connection established between the parent node and the two child nodes is multicast. As a result, if the two players choose to receive the message from two different service providers, the price can not be shared and the costs result to be:

$$\begin{cases} c_i(r,s) = P^{\text{uni}}_{r,i} \cdot t^{\text{rem}}_{s,r} + w(p^{\text{st}}_r) \cdot v(P^{\text{uni}}_{r,i} \cdot t^{\text{rem}}_{r,i}) + w(1-p^{\text{st}}_r) \cdot v(P^{\text{uni}}_{s,i} \cdot t^{\text{tot}}_{s,i}), \\ c_j(s,r) = P^{\text{uni}}_{r,j} \cdot t^{\text{rem}}_{s,r} + w(p^{\text{st}}_r) \cdot v(P^{\text{uni}}_{r,j} \cdot t^{\text{rem}}_{r,j}) + w(1-p^{\text{st}}_r) \cdot v(P^{\text{uni}}_{s,j} \cdot t^{\text{tot}}_{s,j}). \end{cases}$$

which means that two unicast connections are established. Finally, we got how PT can be used to model the subjective perception of the active user that influences the decision-making process for building the network topology.

The procedures for obtaining the values that form the cost matrix have therefore been illustrated. It must emphasized that those costs are only needed to make a player select the action that could bring the highest benefits to the selfish players. It is true that the cost is defined as the power that a parent node must spend to serve its child nodes, but in the decision-making process it is seen only as a price to be paid, not as the real power for resource allocation. Only after the decision has been made, and consequently the network topology is formed, it is possible to proceed with the calculation of the total energy spent for the dissemination of the common message.

In conclusion, the decision driven by wanting to minimize the costs that have been defined in this section will allow the child nodes to establish the actual connection with the respective parent node, which, thanks to this incentive, is motivated to forward the message as well. This decision-making process was modeled in order to achieve the most efficient energy consumption once the topology has been defined.

5.4 Solution of the game

The most important solution concept for a non-cooperative game is the Nash equilibrium (NE). As we have seen in the study of game theory (section 2.2), in some cases it is not possible to determine a NE in pure strategies. However, it has been shown that (theorem 2.2) every game with a finite number of players who have a finite set of possible actions to be selected has a mixed NE. For this reason, in the model developed in this project, where the participants in the game are two and each of them has a total number of actions equal to two, a NE has been found on the basis of a stochastic behavior. In this way there is the certainty of reaching a solution, so as to have the basis to be able subsequently to carry out simulations aimed at showing the results achieved by means of the model illustrated so far. It is therefore necessary to identify the probabilities that indicate how often a player decides to perform an action instead of an other, so that the NE can be expressed through mixed strategies. In fact, formally a mixed strategy corresponds to a probability distribution on the strategies available to the player.

In order to simplify the notation in the mathematical passages that will subsequently be presented, from this moment on we refer to the two new agents i and j that want to join the system as player 1 and player 2, respectively.

The procedure followed to obtain the frequency with which a player tends to select a specific action is shown in Example 2.10 (battle of sexes). The strategy profile $\{q_1, (1-q_1)\}, \{q2, (1-q_2)\}$ so represents the NE in mixed strategy. The values of q_1 and q_2 represent the probabilities with which the first and second players respectively choose the first action (in our case the preference of receiving the message from the static source s). Accordingly to that, $(1-q_1)$ and $(1-q_2)$ are the frequency with which player 1 and 2 choose the second action (that is, the reception of the message from the dynamic node r).

To be able to compute q_2 and $(1-q_2)$, player 2 must set its probabilities in such a way as to make player 1 indifferent between his own two actions, namely the connection of to the source or relay. This is obtained, with reference to the payoff matrix in table 5.1 (a), following the mathematical steps below:

$$u_1(s, a_2) = u_1(r, a_2)$$
$$u_1(s, s)q_2 + u_1(s, r)(1-q_2) = u_1(r, s)q_2 + u_1(r, r)(1-q_2)$$
$$q_2 = \frac{u_1(r, r) - u_1(s, r)}{u_1(s, s) + u_1(r, r) - u_1(r, s) - u_1(s, r)}$$

where $a_2 \in \mathcal{A}_2 = \{s, r\}$ is the action selected by column player 2, which can select either the source or the relay as its parent node.

The same has to be done by the other player as well. Indeed, to calculate q_1 and $(1-q_1)$, player 1 has to make player 2 one indifferent among the actions at its disposal. The action selected by the row player is denoted with $a_1 \in \mathcal{A}_1 = \{s, r\}$. Similarly to what has be done for player 2, the frequency with which player 1 selects the first or second option is obtained as:

$$u_2(a_1, s) = u_2(a_1, r)$$
$$u_2(s, s)q_1 + u_2(r, s)(1-q_1) = u_2(s, s)q_1 + u_2(r, r)(1-q_1)$$
$$q_1 = \frac{u_2(r, r) - u_2(r, s)}{u_2(s, s) + u_2(r, r) - u_2(r, s) - u_2(s, r)}.$$

The procedure illustrated so far for obtaining q_1 and q_2 is valid when the two players want to get the greatest benefits by choosing the highest utility that a certain choice entails. Since in our case the players have to pick up the strategy aimed at minimizing the price to be paid to the forwarding node (the lowest possible value in the cost matrix), the values of q_l and $(1-q_l)$, with $l = \{1,2\}$ have to be interchanged. This simply means that q_1, once obtained as shown above, becomes the probability with which the player 1 chooses the second action (connection to the relay, before was the source) and $(1-q_1)$, consequently, will be the frequency with which player 1 is going to select the first action (connection to the source, before was the relay). The reason for this a change is simple, because if before a player was selecting, for example, with a higher frequency the action providing the highest utility, now such a probability must be applied at the choice that provides the lowest cost, since it is a question of expenses rather than earnings. Thus, a utility chosen with frequency q_1, referring to player 1, becomes in our case a cost chosen with probability $(1-q_1)$ and viceversa. The same is true for the probabilities q_2 and $(1-q_2)$ with reference to player 2, which has to be interchanged in the transition from choosing of greater utility to selecting the lower cost.

The cost matrix, with the respective frequencies with which the players decide to select the actions, is represented in table 5.2. It is possible to see in this representation that in the case of the cost matrix the probability of choosing the strategies of the row and column players have been swapped with respect to the payoff matrix seen in the Example 2.10 (battle of sexes).

		$(\mathbf{1}-\boldsymbol{q}_2)$	$\boldsymbol{q}_2$
	Player 1 \ *Player 2*	s	r
$(\mathbf{1}-\boldsymbol{q}_1)$	s	$c_1(s,s)\,,c_2(s,s)$	$c_1(s,r)\,,c_2(s,r)$
$\boldsymbol{q}_1$	r	$c_1(r,s)\,,c_2(r,s)$	$c_1(r,r), c_2(r,r)$

Table 5.2: Cost matrix for the two-player game with the introduction of probabilities.

An issue, however, has been found in the probabilities computation: sometimes it was found to be $q_l < 0$ or $q_l > 1$, with $l = \{1,2\}$. This of course is not possible, since to be a probability the values found must be $q1, q2 \in [0,1]$. This situation occurs when a player is not willing to randomize such as to make indifferent the other participant in the game. Therefore, if $q_l < 0$, the player l will never choose the action associated with that probability, since it is strictly dominated by the option to which the associated probability is $(1-q_l)$. In order to solve the problem, due to the fact that a player would never pick up a strategy strictly dominated by another, it was imposed that if $q_l > 1$ then $q_l = 1$ and if $q_l < 0$ then $q_l = 0$, with $l = \{1,2\}$. Such a solution has been possible since it is known, from the theory, that a pure strategy can be seen as a degenerative case of a mixed strategy and in the issue we have dealt with we seemed to be under this condition.

Now that the two-player model aimed at optimizing the total energy consumption for the dissemination of a common message has been defined, we are ready to see its practical feedback. Various simulations will therefore be carried out in the next chapter so as to see the results obtained from its application in a real situation.

Chapter 6

Simulation Results

In this chapter the simulations of the model discussed in chapter 5 are illustrated and explained. The software that has been chosen to perform the simulations is Matlab. Although nowadays simulation environments are efficient and powerful, they can never take into account all the problems that can be found in a real physical environment. Indeed, in a real network, there are many factors that can affect performance. To name a few: the climatic conditions (if we consider an external environment), the presence of obstacles that a signal can encounter while it is transmitted, the proximity of other possible wireless networks and other factors that influence the distance between host, as in the case of dynamic networks. Therefore, the results obtained with the simulations can only come close to what in reality may happen, but it will never be what actually occurs. Firstly, the scenario in which the simulations are carried out will be presented and described, including the parameters of the network, the magnitudes of interest for obtaining the results and the model that allowed the positioning of the vertices composing the graph. Finally, the results collected at the end of the simulations are shown. A first part will focus on the performances shown by the system in terms of energy expenditure for the two types of users described in the previous chapter: reactive users (myopic approach) and proactive users, who take into account future events and can be further divided into passive (user does not interfere with the optimal choice researched by the EUT) and active (user influences the decisions made by the underlying system design, through its perception of the situation provided by the PT). These results will be widely discussed and represented by plots, so as to have a clear vision of the objectives achieved by this research.

6.1 Simulation parameters

The simulation scenario involves a network node, which can act as a relay for a common message that has to be disseminated, interposed between the fixed source of such information and the two new nodes that have to join the system. The scheme is illustrated in figure 6.1. Referring to this representation, we see that in the considered network the dynamic relay r is so located closer to the static source s than the other two new agents i and j who have to decide which node to receive the message from. The intermediate node r is located randomly in a range from 1 to 15 meters (green area) away from the fixed information source, while the other two devices i and j are always randomly placed in a range from 15 to 50 meters (purple area). The source, as already stated, is fixed in a point that coincides with the center of the circular quadrant with a radius of 50 meters.

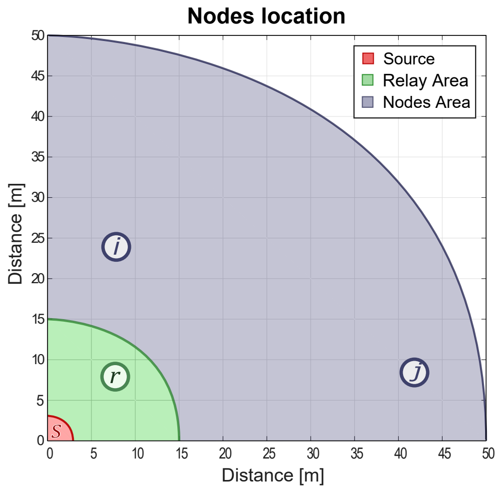


Figure 6.1: Network scenario with the respective positioning areas of the nodes.

The scenario considered is dynamic, which means that a node can leave the network at any time. For simplicity, it has been assumed that a device, instead of gradually moving away from the network by varying its connection conditions, suddenly disappears. This simplification avoids the use of complicated algorithms for tracking the movement of devices within an D2D wireless network. Moreover, in this way it is possible to carry out a clearer analysis whose main purpose is to provide a first solution to a non-competitive game for a dynamic environments, leaving a more in-depth study to other forums.

Summarizing, to experimentally evaluate our approach, we simulated a region of $50m \times 50m$, in which three nodes are randomly located in the areas delimited for them. A fourth node is also present in the network, which is fixed and is the source of the message. The simulation is based on the Monte Carlo approach. Samples were collected at each simulation in order to obtain estimates on the choices made by the players and the overall energy spent in the described scenario.

One of the fundamental parameters on which the model considered in the simulation is based, as already seen in Chapter 4, is the probability of stay in the network that has been associated to the relay node, which remains in the system an additional stochastic time after receiving the whole message. This probability allowed to compute the cost that a proactive user has to pay to the service provider, which incentivized the latter to participate in the dissemination process. In fact, through this parameter it was possible to evaluate the possibility of a future failure in the communication with a dynamic node, which can suddenly switch off depending on its probability of remaining in the network.

The communication channel between two generic nodes of the network is based on the *pathloss model*, which means a channel gain equal to $|h_{k,l}|^2 = 1/d_{k,l}^{\alpha}$, where $d_{k,l}$ represents the Euclidean distance between two communicating parts k and l. The exponent has been set equal to $\alpha = 3$. The minimum required signal to noise ratio (SNR) at the generic receiver node l, in order to successfully decode the data sent form its parent node k, is considered as $\gamma = 10$ dB. Therefore, for a unicast transmission, the transmit power at node k, in order to guarantee at least SNR of γ to the child node l, is calculated as:

$$p_{k,l}^{\text{uni}} = \frac{\gamma \cdot \sigma^2}{|h_{k,l}|^2}$$

where the noise power has been set to $\sigma^2 = -90$ dBm.

The rate $R_{k,l}$ of the single hop channel, which simply represent the number of bits that are conveyed per unit of time over the wireless medium from parent node k to child node l, has been set to 54 Mb/s. Regarding the common message to be disseminated from the source to all users of the network, it has been set to $L = 10$ Mb.

To incentivize an intermediate node to forward the message to other devices, a pricing mechanism has been introduced. Thus, the child nodes, in order to receive the message from their respective service provider, must pay a cost that has been defined as the energy that the parent node spends for the transmission of the information. In order to pay the lower price, which will allow an efficient overall energy expenditure in the network, cost-sharing functions have been applied to this game theoretic framework. In our algorithm, we use the Shapley Value (SV) as a fair cost allocation method, to determine the cost share of each receiving node. This is because it allows the service provider to receive the full price of the service (is budget balanced). Moreover, SV guarantees a right division of costs between the various nodes that receive the message, i.e. it is fair and for this reason it encourages consumers to form coalitions. These groups of users who choose to receive the message from the same parent will actually have the opportunity to share the cost of the service. If they choose, in the opposite case, different service providers (fairness property not satisfied), the coalition would be formed by only one node and therefore this unique agent has to pay the whole service price. In wireless networks, the principle of fairness is crucial because it allows to establish multicast connections that exploit the broadcast nature of wireless channels, so as to save energy for sending the same message to multiple receivers. In this respect, SV has provided better results in terms of energy consumption and number of failures given by the departure of a dynamic node, compared to the other two cost functions, namely ES and MC. To summarize, in table 6.1 the parameters used to perform the simulations are presented.

Unicast transmit power	$p_{k,l}^{\text{uni}} = \gamma\sigma^2/\|h_{k,l}\|^2$
Channel gain	$\|h_{k,l}\|^2 = 1/d_{k,l}^3$
SNR	$\gamma = 10\text{dB}$
Noise power	$\sigma^2 = -90$ dBm
Data-rate	54 Mb/s
Data size	10 Mb
Network size	$50m \times 50m$
Cost sharing function	Shapley Value

Table 6.1: Network information.

In order to obtain results as accurate as possible, a large number of simulations have been performed by varying other parameters that do not appear in Table 6.1, which are the departure rate and the percentage of data received by the relay node r. A further analysis was then carried out to understand at what distances a new agent that wants to join the network tends to prefer a connection to a dynamic node or to a static node. The data collected at the end of the simulations were analyzed and then arranged in the form of descriptive plots, which will be discussed in the next section.

6.2 Results analysis

In this section we analyze the graphs obtained from the simulations in order to evaluate the performance of the game theoretic algorithm developed within this project. The total transmit power in the network is considered as the performance measure. The reference scenario is simple, in fact we have that only two new agents have to be added to a network made up of only two devices. We therefore have that the broadcast tree, at the moment when the algorithm is performed, is composed only of node s and r. Moreover, is assumed that there is a communication in progress between these two parts and in the meanwhile two other nodes want to join the system. The relay node r has already received part of the message and may leave the network after receiving the total amount of data.

The algorithm takes into account the fact that, in order to make a decision about the parent node from which to receive the information, not all nodes are in the same initial condition as the two new nodes, i.e. did not downloaded any data (not all receiving buffers are empty at the moment of the connection of the two new nodes). Along with the fact that the nodes are not in the same condition when the algorithm is executed, which is one of the most innovative points of our model, we wanted to focus on the introduction of mobility in the network. This is because in the literature a large part of multi-hop data dissemination mechanisms based on game theory have been developed for static environments. In this regard, the energy spent by the reactive user (myopic, as it sees the network static) was chosen as performance evaluation benchmark, in order to see how the utility defined in this thesis for reactive users (they are aware of the dynamism of the environment) can bring benefits in terms of energy expenditure. Reactive users have been divided into two classes: passive users, who follow the principles dictated by the EUT, and active users, who make their choices according to the PT.

As extensively explained in chapter 2, PT introduces, with respect to the EUT, two functions (PWF and AVF) to model the subjective perception of the user. These functions have coefficients that must be quantified, in fact, referring to the cost defined for the active user in formula 5.8, we see the presence of three coefficients that, in all the realizations, were set as $\alpha = 0.6$, $\Lambda = 1.5$ and $\gamma = 0.9$.

In all the graphs that will be discussed later, the points that make up the curves derive from the average value obtained from multiple simulations. Precisely, we have that for each input value to the simulator (x-axis of the plots), the scenario has been repeated for ten thousand times. This high number has been chosen to have the most reliable results possible. At each execution, the nodes were randomly placed in the areas of their competence, which changed the power required for the transmissions and consequently the decision that the two new agents make in order to minimize the cost of the service.

Figure 6.2 shows the total energy expenditure in the network as a function of the percentage of data received by the relay (figure 6.2 (a), obtained by fixing the departure rate of the relay to $\lambda_r = 1$ departure per second) and the departure rate of the relay (figure 6.2 (b), obtained by assuming that the relay already downloaded half of the packets).

The energy required for a transmission is calculated as the product between the power required to transmit the data from the service provider to the receiver node and the transmission time (formula (4.3)). In both plots we see that the actual energy consumption to disseminate the message from the source to all the node of the network is normalized by the energy that would have been spent if both the two new nodes i and j of the network had selected the source s to receive the information. Thus, the values on the y-axis derive from the total transmit power in the network divided by the multicast transmit power from the node s to the two new agents i and j, i.e. $max(P^{\text{uni}}_{s,i}, P^{\text{uni}}_{s,j})$. For a better analysis of the results it is good to focus on one chart at a time.

In figure 6.2 (a) we see that energy consumption increases as the relay node

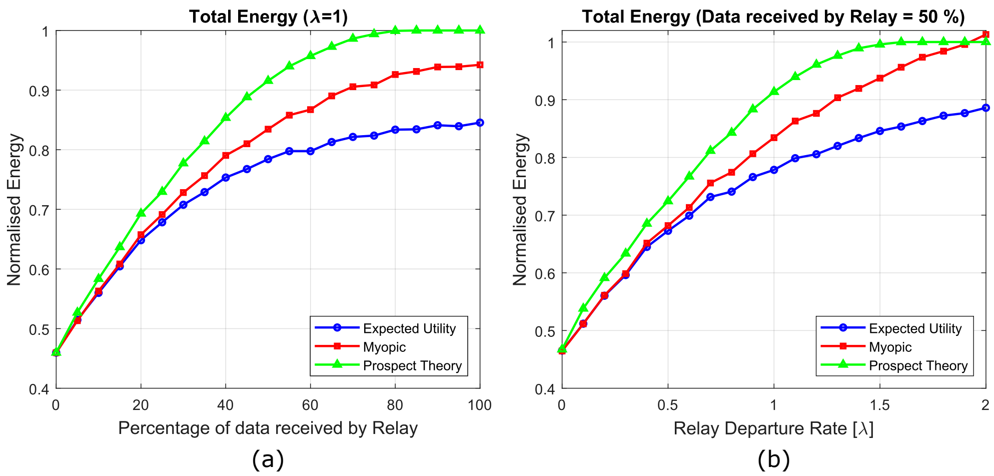


Figure 6.2: Total transmit energy in the network normalized by the energy spent if both the new agent select the fixed source as parent node.

receives more information. This is easily explained because when the relay node has received a small amount of data, it certainly remains more in the network so as to be able to send a large part of the message to the child node during the deterministic time. The two new users can therefore choose to receive the data from the dynamic node with a low risk of failure, as most of the information will be transmitted while the parent node is receiving its own message. A lower transmission power is a consequence of the fact that the relay r is located closer to the two new agents with respect to the source s. Therefore, the cost is less when the dynamic node has received a small amount of the total data because it guarantees reliability for the reception of nodes i and j and at the same time ensures a lower energy expenditure as it is located at a shorter distance than the source s.

By observing the three types of users (the three curves in the graph) they in the left part of the representation consume a similar energy amount. This is because the reactive user, regardless of situations, tends to almost always choose the relay as a parent node since, being at a shorter distance, it requires less transmission energy at the decision time. The other two users, the active and the passive, select the dynamic node as well, since in such circumstance there is a low risk of failure given by the departure of the parent node.

When we move towards the right side of the graph we see that the energy needed for the data dissemination increases. The reasons is mainly that the dynamic node becomes less reliable in forwarding the information and the different energy trends depend on the type of player who participates in the game. In the case of the myopic user, the highest energy consumption is given by the larger number of failures that occur between the dynamic parent node and its two child nodes. In fact, the reactive users, not being able to foresee a possible future event, continue to almost always choose the relay node r as a service provider, which, having received a large part of the message, tends to remain a short time in the network. As explained in chapter 4, the greater the amount of data received by the relay, the less deterministic time it will remain in the system. Consequently, the agents served by it have to receive most of the message during the stochastic time in which the failure may happen. For this reason, the two types of proactive users try to avoid a connection to the relay r in these uncertain conditions. However, we note a substantial difference in the energy needed to spread the message to active users (green line on the graph, obtained by applying the PT) and to passive users (blue line on the graph, obtained by applying the EUT). The curve that describes the energy trend with the cost calculated following the EUT provides the best result compared to the other types of users. On the contrary, the user who seeks more satisfactory solutions (through the subjective perception of the occurrence possibility of an event provided by the PT) results in a worse energy consumption than the other two types of users. The reason for these behaviors is as follows: the passive user (based on EUT) requires a lower overall transmission power than the reactive user because he is able to avoid the failures that the myopic agent inevitably meets. Being proactive, the passive user takes into account the dynamism of the environment and can evaluate when a future failure has a high probability of occurring. In this case the action to be taken is to select the source that, although is more distant and requires more power to transmit the message, saves the energy that would be lost as soon as the relay r leaves the network. As we will see in the following analysis, the reactive user tends in almost all cases to choose the close dynamic node as a father, which will induce to a consistent amount of data lost because of the failures. The active user (based on PT) also tries to avoid unfavorable events that may occur in the future, but does so by looking for the least risky solution possible. This leads the user to choose most of the times as parent node the static source which, not being able to leave the network, guarantees a sure reception of the information. The price to pay for this decision is a higher energy consumption and consequently also the higher price that the two active users have to pay for the satisfaction given by the absence of failures.

The EUT thus provides the best tradeoff between the choice of receiving the message from the source s or from the relay r, while the PT, to avoid any risk, tends to choose several times the static node which requires a higher transmission power level.

What has been said so far is also valid for the graph where the energy is varied according to the departure rate of the dynamic node (figure 6.2 (b)). Briefly we have that when the environment mobility is low, the choice of the relay brings more benefits. In fact, it appears to be reliable because it will remain in the network a time long enough to allow the almost complete message reception to two new users. As the rate increases, the proactive users begin to choose the source as the parent node in order to avoid possible failures, since the time that the dynamic node will remain in the network at the end of its reception progressively decreases. However, the reactive user will continue to choose the option that provides the highest current benefit, i.e. connection to the relay (closer than the source), since it does not look to the future. The difference between the two types of proactive user remains. The passive, in fact, looks for the best tradeoff between choosing an option with unlikely failure and avoiding one where the negative event most likely happens, while the passive is willing to pay more as long as the service runs smoothly.

By observing the obtained results, it could therefore be said that a reactive user may require less energy consumption than a proactive user, precisely the active user who assigns a weight to the occurrence of an event that is non-linear with the probability that the latter occurs, despite such an agent is aimed at avoiding the risk of possible future failures.

Before drawing hasty conclusions, it is good to observe the curves obtained considering the total number of failures that occurred, as a percentage of the total number of transfers, for the three types of users (figure 6.3).

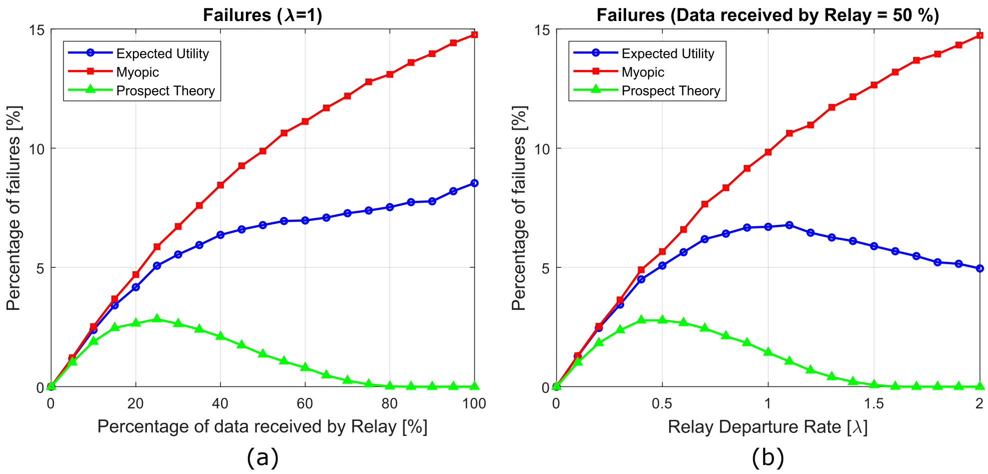


Figure 6.3: Number of failures occurred with respect to the total number of transfers.

In this case we see that the percentage of failures occurred when reactive users are added to the system far exceeds that achieved by the other two types of user. This is valid for both the graphs in figure 6.3 (a) and figure 6.3 (b), where we count the percentage of transmission failures as the amount of data received from the dynamic node and its departure rate change, respectively. Obviously, the more uncertain the permanence of the relay r in the network, the greater the number of failures that may occur in the transmissions. Looking at the red curve, this behavior can be clearly seen.

When the PT is applied, however, we see a peak in the percentage of failures in an area where reception from the dynamic node should be fairly secure. In fact, when the relay has received a little amount of data or its departure rate is low, the active user is able to choose the dynamic node as service provider, given the almost secure reception. For this reason, only in that case failures can occur, because as the uncertainty increases (we move towards the right of the graphs), the active user prefers to receive the message from the source so that the failures are reduced.

In this circumstance the EUT finds itself in the middle of the other two behaviors, i.e. not taking risks into account as the myopic does and avoiding them as much as possible as the active user does. The passive user has indeed a much lower percentage of failures than the reactive user, especially when the staying probability of the dynamic node decreases. However, it results in a far greater number of retransmissions than the active user and the difference in terms of energy may be different from what was observed in figure 6.2 if the cost for retransmission was also considered.

It is important to notice that our model does not take into account the *transition costs*, which means that when a failure occurs and there is a need to connect to another service provider, in the system there is an additional consumption of energy to set up the new connection. In our case, if the relay r leaves the network while a transmission is in progress, the new agent must receive the message from the source, which means to exchange additional packages with the new parent node in order to establish a new communication before that any useful data can be transferred.

We have not considered this additional energy consumption, but if it had been taken into account for each new connection that the reactive user needs, which as we have seen has the highest number of failures, the total energy trend would have been much worse for the myopic user. The same worsening would have occurred also for the passive user, who, as is clearly visible, requires a substantially greater number of retransmissions than the active user.

From the failures point of view, the PT is the model with which the best results have been achieved. Such a behavior, although from the previous analysis was not the one with greater energy efficiency, would save a fair amount of energy that would be consumed to establish new connections caused by the dynamic node departure, as well as providing greater user satisfaction.

The study of the percentage of failed connections over the total number of transmissions is not the only circumstance in which PT achieves the best results. By looking at figure 6.4, we note that the percentage of energy wasted (compared to total energy consumption) due to unsuccessful data transfers is much lower when active users are added to the system. This behavior is a direct consequence of the number of failures that occur for the three types of users, because the greater the number of negative events, the greater the amount of energy loss.

It is not surprising that the reactive user wastes a larger amount than the other two types of agent, as well as the fact that the passive user exceeds the active user regarding this comparison metric. The same behavior is seen both in the graph as

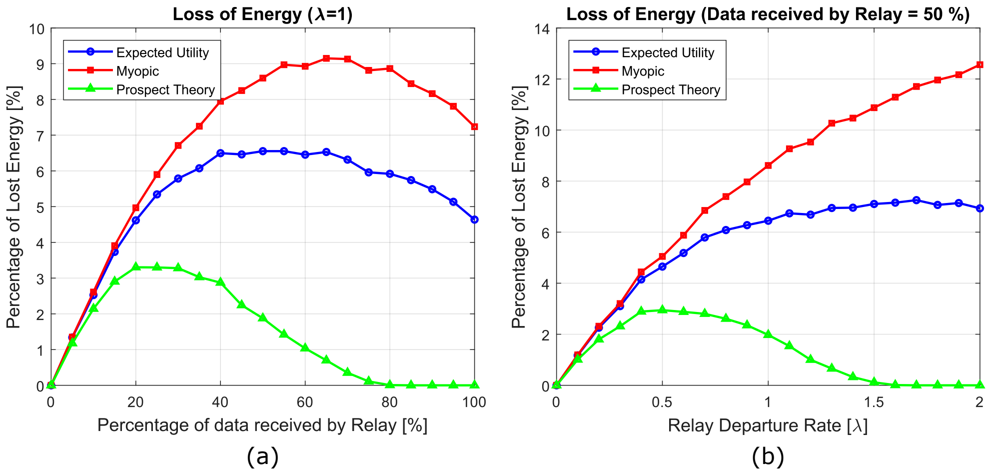


Figure 6.4: Energy loss, caused by the relay node departure, with respect to the total transmit energy in the network.

a function of the percentage of data received by the relay (figure 6.4 (a)) and in the graph where the departure rate of the relay node varies (figure 6.4 (b)).

The hill shape, i.e. the progressive increase in the amount of energy loss and the subsequent decrease of the latter at the right side of the graph, is mainly given by the following reason: if the probability of permanence of the dynamic node in the network is greater, then a new agent can receive a substantial part of the message from the parent node, then the wasted energy will be greater if a failure occurs under such conditions. As a consequence, the lower the probability that the parent node remains in the network, the less the energy loss because the transmission lasted for a very short time. Obviously with the PT the only possible failures can occur when the permanence of the service provider in the network is almost sure (left of the graph), because otherwise it chooses the static source to receive the message.

As a last observation, and as already explained for the results obtained from the study of the failures, the wasted energy would have been very high if the transition cost was considered for each retransmission occurred during the simulations. This would have generated a greater separation between the curves obtained for the reactive, passive and active user, where the first would have shown a behavior far worse than that of the other two, whereas the last could have saved much more energy because of the non-necessity to re-establish a connection.

A further analysis was made by observing how many times the two new agents choose to connect to the source or relay node r. It has been done as a function of the distance from the source, so as to be able to investigate their behavior at diffeerent network locations. The decisions taken by the two nodes i and j when they are located at a wide distance from the source and when they, on the contrary, are very close to the two vertices already part of the broadcast structure turn out to be of great interest. The analysis was performed for the three types of user considered so far.

Before showing the obtained results, the expectations regarding the behavior of the nodes are: when they move away a lot from the source, since the difference in distance between the two new agents and the two potential information sources is small, selecting the static or dynamic node involves a similar cost. Being both far away, the user prefers not to take the risk of a future failure and since the price changes slightly we should see, in these circumstances, a preference in choosing the static source s as a parent node. On the contrary, when the two new agents are close to the two possible service providers, the difference in distance for a connection to the static node and the dynamic node becomes more marked. For this reason we expect that in this situation the relay r should be preferred to the source s.

All the figures that will be subsequently discussed are formed by two histograms. On the left are shown the results obtained by fixing the departure rate of the relay node as $\lambda_r = 1$ and by varying the percentage of data received by it from 0 to 100. Whereas, on the right, we can find the results of the simulations where the relay node received half of the total amount of data and its departure rate varies from $\lambda_r = 1$ to $\lambda_r = 2$.

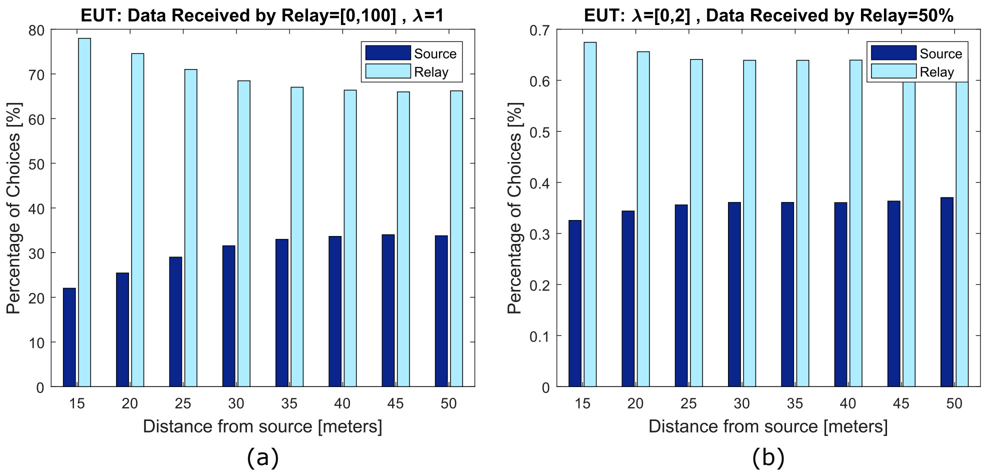


Figure 6.5: Number of times the passive users (decision based on EUT) select node s or node r for receiving the message in function of the distance from the fixed source.

The first investigated case is that concerning the passive user, who, as is well known, makes its choices based on the rules derived from the EUT. In figure 6.5 it is possible to see the percentage of times the two proactive users choose the static node or the dynamic node as service provider.

As we can see, in both cases shown in figure 6.5 (a) and figure 6.5 (b), passive users prefer to receive the message from the dynamic node. The reason is that such a vertex is located in the middle of the network and therefore the transmission power required at the relay node r to reach the two new agents is always less than or equal to the one needed if the transfer had started from the source s.

It is good to notice that there are cases in which the two new devices are randomly located at a very similar distance from the dynamic and static potential parent nodes. Referring to the figure 6.1 and imagining it as a Cartesian coordinate system, it can happen that the relay node r is located near the point (15; 0) with the other two nodes positioned around (0; 15). In this situation, the transmission power required at the relay node r is not less than that required at the source s, so there may be cases in which it is not convenient to choose only the dynamic node as information source in order to save resources. In most cases, the choice of the dynamic node allows the total energy spent in the network to be as low as possible even if, in a considerable number of times, the the passive users prefer to receive the message form the source s. This is because, in order to avoid failures, the rules of the EUT provide the user with the tools to find an optimal compromise between the two choices. Since the goal is to minimize the costs necessary for the dissemination of the common message, it is not surprising that the closest dynamic node is chosen a greater number of times.

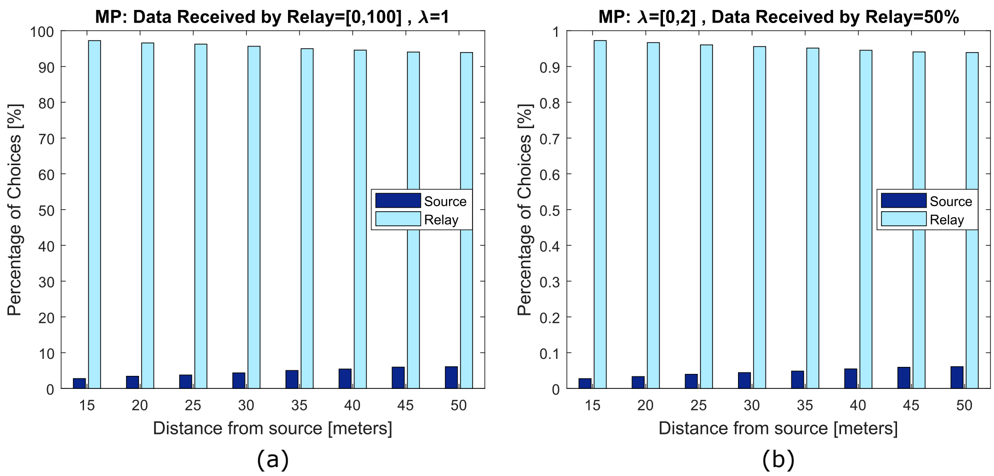


Figure 6.6: Number of times the reactive users (myopic approach) select node s or node r for receiving the message in function of the distance from the fixed source.

Another result that reflects our expectations is to see that when the two players move away from the source node s, the latter is preferred more times with respect to the situation in which the two new nodes are located close to this potential parent node. The reason for this behavior is, as expected, that when the distance increases there is no big difference between the power values needed to receive the message from the static node or the dynamic node. Therefore, the two players prefer not to take the risk and choose the fixed information source s.

The same analysis is performed for the reactive user. The results of this experiment are shown in figure 6.6. In this case, the dynamic node is chosen in almost all the simulations, in fact the number of times the source has been selected is very small. These results are not surprising since the myopic agent, not minding about the future, takes into account only the lowest price to pay to the service provider at the time of connection. Obviously, this choice is addressed most of the time to the relay that is generally closer to the two agents and, as we have seen, this decision will cause the number of transmission failures to be quite high.

Similarly to the previous analysis, the source begins to be slightly more preferred when the two users are long distance from it. The reason for this behavior is the same as in the case of the passive user, in fact at wide distances the difference in cost associated with the choice of the dynamic and static node becomes less marked so that the latter can sometimes be preferred.

The last study reported is that concerning the decisions made by the two active users. The results obtained in this experiment are shown in figure 6.7. We can see

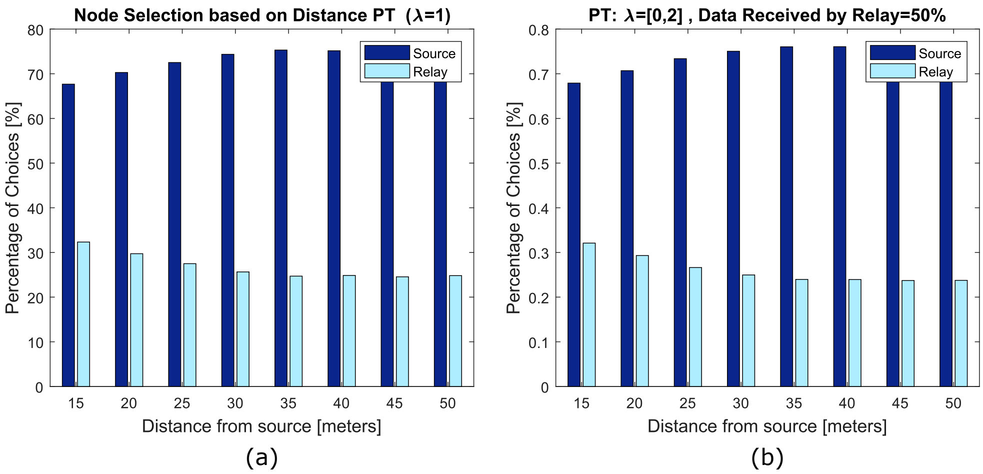


Figure 6.7: Number of times the active users (decision based on PT) select node s or node r for receiving the message in function of the distance from the fixed source.

that in this case the two agents prefer to receive the message from the fixed source. The reason for this behavior lies in the fact that active users, being risk-averse, avoid as much as possible the possibility of a failure occurring.

Obviously, choosing as the parent node, in most cases, the source s can lead to not achieving an efficient energy consumption since it is usually at a greater distance than the relay node r. However, the advantage of this decision-making policy lies in obtaining a much lower number of transmission failures than the other two types of users (active and myopic) who, in a completely different way, preferred to receive the message from the dynamic node. An active user is therefore satisfied if the service is provided without any unexpected events, even at the cost of spending a little more. However, this price could be even lower with respect to what the passive and reactive user spend if the transition cost is considered.

The common behavior of all types of users is that of preferring the source s as the parent node when the two new agent are long distance from it. In fact, we see that even in this case the number of transmissions from the static node is greater when the two nodes are moving away from it. Thus, we saw that when they are very far away form the two potential information sources the difference in distance (and cost) between node s and r is less significant and the decision made by the two players, through the designed cost function, have met our expectations.

Chapter 7

Conclusion

In this research we designed a mechanism to reduce the total energy consumption for the dissemination of a common message sent from a fixed source to all the other nodes of the network. We focused on the study of D2D multi-hop networks composed of wireless intelligent devices, with selfish behaviors, which have the capacity to self-organize following a decentralized paradigm. For this purpose the game theory has been used where every new user, child node, of the network chooses the most advantageous option available to it in order to select the forwarding node, parent node, from which to receive the message. The core part of this study is represented by the definition of the analytical model for a user's preference relationship, that is to say the utility function that assigns a payoff to a player's action of this child driven game theoretic framework.

One fundamental peculiarity of the proposed scenario, for which our mechanism has been developed, is that it is dynamic. A node can thus leave the network at any moment. In order to realize this characteristic, a probability of remaining in the system has been associated with each node that composes the network. For simplicity we just assumed that a node suddenly turns off depending on the level of mobility of the wireless environment, so as to avoid the use of further algorithms for tracking the movement of devices within the system. It allowed to reduce the number of parameters (variables) in the network by helping us to get more intuition into mathematical procedures.

Another important aspect is that when the system is analyzed, the nodes are in different initial conditions. Indeed, when new agents have to select the service provider, there may be a transmission in progress among the nodes already part of the propagation structure. It is so assumed that a node cannot leave the network before receiving the whole information from the source. Only at the end of the transmission it will be able to leave the network, but before this happens it remains an additional time which depends on its probability of staying.

The system model for which the algorithm have been developed was very simple, in fact at the beginning the broadcast tree is composed of only two nodes: the static source of information and a dynamic relay node. As a first step, only one new node has to be added to the network. A pricing mechanism has therefore been defined to allow for the new agent to decide whether to take the risk of receiving information from a relay node that can leave the network after the end of its reception or whether to obtain such a message, perhaps paying a higher price, from the source without risk, since the latter is static. In the first case, a possible failure can be caused by the parent node departure and as a consequence the child node has to download the information from the fixed source (everything that has received up to that point is lost). In order to have a successful transmission, the forwarding node needs to remain in the network at least a time equal to the one needed by the new agent for receiving the entire message. This period depends on how much information has been downloaded by the relay node, as well as on its probability of remaining in the network, when the new user arrives.

Some parameters were presented in order to define the energy required for a transmission, which allowed to design the cost to be paid based on the choice made by the child node. This was done by trying to maximize the well known utility function obtained as the ratio between the number of bits transmitted and the power required for the transmission. In our work we considered the message to be of fixed length and this allowed to convert the concept of benefit to be maximized in cost to be minimized (denominator of the utility function) in order to reduce the power needed for the data dissemination.

As a step after the definition of the cost to be paid for receiving the information, a network consisting of four nodes was considered. In this second scenario, two new users want to connect to the network and the decision of one of them also determines the cost of the other player. To model this situation we used game theory, where players are the nodes that are added to the system. The solution concept has been given by the Nash Equilibrium for mixed strategies, which is a prediction of what will happen when the game is played. In this way, there was the certainty of reaching a solution, so as to have the basis to subsequently carry out simulations aimed at showing the results found on the basis of a stochastic behavior.

We had so different combinations of power depending on the choices made by the two participants. To measure the power imposed by each of them in the network, we used cost sharing functions. Specifically, we considered three of them: Equal Sharing (ES), Marginal Contribution (MC) and Shapley Value (SV). They allowed to compute the price that each node has to pay to receive the service. In the end, SV was chosen to run the simulation because it provided the forwarding node with the full price of the service, which was defined as the energy that the intermediate node spends for relaying the message. Furthermore, we had with SV that the two nodes were willing to form coalition, which led to a greater number of multicast transmissions with respect to ES and MC. It allowed to exploit the broadcast nature of the wireless channel and to save energy that would be wasted if many unicast transmissions were established.

We have seen that, according to the model used for the cost definition, two different classes of users have been identified: the first is the passive user, where the decision is taken without the end-user interaction. In this case the decisions were modeled with the Expected Utility Theory (EUT), which allowed to achieve optimum results (rational user), but totally uninfluenced by the real life perception. The second approach that has been used was Prospect Theory (PT), which allowed to define the active user, which means that the end-user interfere with the decision taken by the underlying system design. In this case the decisions are guided by a bounded rationality, since people do not look for the optimum, but for satisfying solutions. The active user, being risk-averse, tended to overweight unlikely events and underestimate the likely ones. Moreover, the impact of a loss resulted to be higher than that of a win for the same absolute value.

To show all the possible cost that the child nodes have to pay by selecting the parent node from which to receive the message, we defined a cost matrix. Having used mixed strategies, we found at the end the frequency with which each players selects each of the two option available to it. Once the decisions were made, it was possible, through simulations, to calculate the total energy expenditure in the network. We then compared the values obtained for the two classes of user (active and passive) with a third type of agent used as a benchmark for the evaluation of benefits obtained by performing our algorithm: the reactive user (myopic). It makes the most convenient decision by looking only at the current time and not at the possible future events. Basically it sees the network as static and does not take into account the fact that a parent node can leave the network while it is forwarding information.

In the simulation scenario, the relay node has been placed closer to the source than the two new nodes of the network, so that there may be an advantage in choosing the dynamic node for forwarding the message. The final results showed that, as the mobility of the environment and the percentage of data received by the relay node vary, the total energy required in the network to disseminate the message was found to be lower in the case of a passive user. In fact, it found the best tradeoff between the choice of the static and dynamic information source. In the other two study cases, i.e. those concerning the total number of transmission failures and the wasted energy given by the departure of the dynamic forwarding node, the active user showed the best results. This is because it prefers the satisfactory solution that has the lowest risk of failure, which was also confirmed by the analysis of its choices in function of the distance from the fixed source. In fact, while the reactive and passive user have selected most of the time to receive information from the dynamic relay node (lower cost, but risky), the active user preferred the static source (no risk, but higher cost). However, the behavior shared by all types of users was that as they move away from the static source, the latter begins to be preferred. This is due to the fact that the advantage given by the choice of relay node (far away too) becomes less consistent. The cost for a retransmission was not considered, which would have certainly led to worse performance for the reactive user (greater number of failures) and for the passive user (willing to take the risk when, according to the EUT, it is worth it).

In conclusion, we have seen that a deviation from the behaviors expected from the application of the EUT can lead to a degradation of the total energy spent in the network. However, in a dynamic environment, recovery from a failure can definitely require additional resources and according to the rules of the PT this circumstance is successfully avoided. A scenario has therefore been analyzed in which end-users interfere with the protocols in order to reach satisfactory solutions, which has been compared with the most widely used and known models found in the literature.

7.1 Future research directions

A possible improvement for the dissemination mechanism of a common message sent from an information source to all the nodes of the network has already been partially performed during the development period of the algorithm presented in this dissertation. The utility function was in fact modeled, but not tested, also for the case in which the data-rates between the nodes composing the network are different. The user's preference definition for this case requires the message to be divided into packets. Moreover, it needs that the new agent, who wants to join the network, also knows the speed of the communication in progress between the two communicating nodes that form the broadcast tree (static source and dynamic relay node) when the system is analyzed. Given the excessive amount of information and parameters necessary to make a decision under these conditions, we chose to focus only on the scenario having the same data-rate for all wireless channels.

For the future, the mechanism described in this thesis can be extended to more than two nodes that join the system, considering not only departures, but also arrivals. It can be also improved by considering more than one hop between the new nodes that have to receive the message and the fixed source. For this specific scenario, would be interesting to apply Markov Games, since they result to be more suitable for generalized cases.

There is certainly a need for further research, perhaps by involving real users and wireless devices, in order to confirm and improve the mechanism studied so far for the data dissemination in a dynamic environment.

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