## ADVANCED MODULATION FORMATS FOR FLEXIBLE NETWORKING



## Master Thesis

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#### Abstract

This master thesis has been developed during an academic research in optical communication systems field in the Department of Electronics and Telecommunications at Politecnico di Torino.

In the present, user's demand of data transmission rates increases exponentially, this represent a challenge and a reason of study for telecommunications area in terms of how to migrate to improvements in implemented systems without compromise their complexity. First assumption to consider is that if standard modulation formats (PM-QPSK,PM-16QAM) are used optical communications are limited with respect to their flexibility. Moreover, single-carrier approach results in a channel with higher symbol-rate in contrast with a multisubcarrier approach that allows to work with lower symbol-rates, which is an optimal solution.

The objective of the master thesis is analyze and implement subcarrier-multiplexing (SCM) in conjunction with Frequency Domain Hybrid Modulation Formats (FDHMF) as advanced modulation format to improve the performance in the optical communication channel considering the impact of filtering due to the introduction of Wavelength Selective Switches at optical nodes. This aims to obtain performance results in terms of BER and required SNR for scenarios in which optical routing adds tight filtering, as well as system's behavior, when different Forward Error Correction approaches and equalization are implemented.

Simulation results demonstrate that the proposal previously presented allows to obtain optimal results at net-bit rate of 200 GHz . SCM with FDHMF give advantages against filtering in optical data transmission due to the possibility of allocate low-cardinality modulation formats at edges making the transmitted signal more robust. Furthermore, the use of higher number of subcarriers not just improve the performance against filtering also gives possibility of achieve a desirable transmission bit rate and increase the flexibility by mixing low- and high cardinality modulation formats.


To my Father, Rubén Fernández.

## Contents

1. Introduction ..... 1
1.1 Overview ..... 1
1.2 Problem ..... 1
1.3 Approach ..... 2
1.4 Documentation Overview ..... 2
2. Optical Communication systems description ..... 3
2.1 Optical Communication Systems wavelengths ..... 4
2.2 Optical Fibers ..... 5
2.3 Optical Amplification ..... 6
2.4 Optical Routing ..... 6
3. Advanced modulation formats ..... 8
3.1 Advanced techniques in optical communication systems. ..... 8
3.1.1 Forward Error Correction ..... 8
3.1.2 Wavelength Selective Switch ..... 9
3.1.3 Equalization ..... 10
3.2 Hybrid modulation formats ..... 11
3.2.1 Basics ..... 11
3.2.2 Multi-subcarrier with Hybrid Modulation Format implementation ..... 13
3.2.3 BER analysis on HMF ..... 14
3.2.4 Power allocation on HMF ..... 15
4. FDHMF design and simulation ..... 17
4.1 Simulation Performance ..... 18
4.2 SCM with FDHMF implementation ..... 23
4.2.1 FDHMF with SCM cases ..... 25
4.2.2 Comparison theory and simulation ..... 27
4.2.3 Comparison between different number of subcarriers ..... 29
4.2.4 Comparison between best cases ..... 35
CONCLUSIONS ..... 38
APPENDIX ..... 39
REFERENCES ..... 44

## List of Figures

Figure 2.1: General representation of a Communication System. ............................................................................. 3
Figure 2.2: General Optical Communication System. ................................................................................................ 3
Figure 2.3: Wavelength Bands [2]. ............................................................................................................................... 4
Figure 2.4: Optic fiberS [3]. .......................................................................................................................................... 5
Figure 2.5: Erbium Doped Fiber A mplifier [4]........................................................................................................... 6
Figure 2.6: Wavelength routing [6]............................................................................................................................... 7
Figure 2.7: Wavelength Selective Switch [5].............................................................................................................. 8
Figure 3.1: WSS filter spectrum. ................................................................................................................................. 10
Figure 3.2: Constellation diagrams of higher-order modulation formats [7]......................................................... 11
Figure 3.3: Scalability of optical commun ication systems [7]. ................................................................................ 12
Figure 3.4: example of multi-subcarrier hybrid modulation format. ....................................................................... 13
Figure 3.5: SC configuration vs MSC with HMF configuration. ............................................................................ 14
Figure 4.1: Strategies for power allocation. .............................................................................................................. 19
Figure 4.2: Equalization impact in terms of BER. .................................................................................................... 20
Figure 4.3: Impact in the spectrum of using WSS in cascade .................................................................................. 21
Figure 4.4 Impact of using WSS in cascade.............................................................................................................. 22
Figure 4.5: Impact of FEC strategy in the spectrum. ................................................................................................ 22
Figure 4.6: Spectrum of single-carrier approach .......................................................................................................... 23
Figure 4.7: Spectrum of 4-subcarrier approach......................................................................................................... 24
Figure 4.8: Spectrum of 8-subcarrier approach.................................................................................................................... 24
Figure 4.9: Spectrum of 16-subcarrier approach .......................................................................................................... 24
Figure 4.10: Spectrum of 32-subcarrier approach ..................................................................................................... 25
Figure 4.11: Theory vs simulation for single carrier. ........................................................................................................... 28
Figure 4.12: Theory vs Simulation for 4-subcarriers - No equalization .................................................................. 28
Figure 4.13: Theory vs Simulation for 4-subcarriers - equalization....................................................................... 29
Figure 4.14: Theory vs Simulation for 32-subcarriers .............................................................................................. 29
Figure 4.15: Cases for 4-subcarriers, without Equalization..................................................................................... 30
Figure 4.16: Cases for 4-subcarriers, with Equalization. ......................................................................................... 30
Figure 4.17: Cases for 8-subcarriers, without Equalization..................................................................................... 31
Figure 4.18: Cases for 8-subcarriers, with Equalization. ......................................................................................... 31
Figure 4.19: Cases for 16-subcarriers, without Equalization. .................................................................................. 32
Figure 4.20: Cases for 16-subcarriers, with Equalization. ....................................................................................... 32
Figure 4.21: Cases for 32 -subcarriers......................................................................................................................... 33
Figure 4.22: Performance of a PM-16QAM using different number of subcarriers - without Equalization ...... 34
Figure 4.23: Performance of a PM-16QAM using different number of subcarriers - with Equalization ........... 35
Figure 4.24: Best cases for all subcarriers approach @200G, without equalization ............................................. 36
Figure 4.25: Best cases for all subcarriers approach @200G, with equalization. ................................................. 37

## List of Tables

Table 4.1: SD-FEC and HD-FEC characteristics. ..... 17
Table 4.2: equalization impact in terms of BER. ..... 20
Table 4.3: FDHMF configurations for 4-Subcarriers @200G. ..... 25
Table 4.4: FDHMF configurations for 8-subcarriers @ 200G ..... 26
Table 4.5: FDHMF configurations best cases for 16-subcarriers @ 200G ..... 26
Table 4.6: FDHMF configurations best cases for 32 -subcarriers @ 200G ..... 27

## 1. INTRODUCTION

This master thesis deals with the implementation and simulation of advanced modulation formats for flexible networking in optical communications, during a work in the Department of Electronics and Telecommunications in the Politecnico di Torino.

### 1.1 OVERVIEW

Presently, the world of telecommunications is rapidly advancing and evolving, caused in part by an increase in customer demands for services. Optical communication systems assume an important place in this development, revolutionizing the industry by replacing conventional wired communications. This obeys to improvements in capacity, achieving higher data transfer rates. Several techniques have been implemented in optical communication to achieve better performance. These include the implementation of advanced modulation formats that increase network flexibility, in terms of bit-rate granularity.

Frequency-domain hybrid modulation formats (FDHMF) have been proposed to achieve flexible networking where low-cardinality and high-cardinality modulations are combined to reach desirable transmission bit-rates. Also a factor, is the introduction of digital subcarrier multiplexing (SCM), by which the overall channel is divided in several low symbol-rate channels. This technique addresses nonlinear propagation and transmission reach limitations. In addition, it has been proposed that the subcarrier multiplexed hybrid QAM scheme supports high bit-rate optical channels that are more robust to optical filtering effects by allocating low-cardinality constellations in edge subcarrier and high-cardinality in the center. In summary, these techniques allow the introduction of advanced modulation format schemes that achieve higher flexibility in data-rate and tolerance against filtering.

### 1.2 PROBLEM

The increasing demand in the performance of optical communications in terms of flexibility is limited if standard modulation formats (PM-QPSK, PM-16QAM) are used. Besides the implementation of single-carrier approach results in a channel with higher symbol-rate in contrast with a multi-subcarrier approach that allows to work with lower symbol-rates, which is optimal for standard systems. So, hybrid modulation with a multi-subcarrier approach where a proper mix of high and low-cardinality constellations helps to provide more flexibility in the optical communications, mitigating nonlinear propagations and the impact of tight optical filtering.

This master thesis provides the analysis and implementation of Multi-subcarrier Frequency Domain Hybrid Modulation Formats as advanced modulation format to improve the
performance in the optical communication channel considering the impact of filtering due to the introduction of Wavelength Selective Switches at optical nodes.

### 1.3 APPROACH

This thesis proposes a study of a communication channel simulator specific for optical communications. This is to be used in conjunction with FDHMF, with either a single-carrier or a multi-subcarrier approach. This aims to obtain performance results in terms of BER and SNR, required for scenarios in which optical routing adds tight filtering, as well as system's behavior, when different Forward Error Correction approaches and equalization are implemented.

The simulation object of this thesis uses MATLAB (matrix laboratory) as platform. MATLAB is a fourth-generation programming language that supports vector and matrix manipulation easily and intuitively, while providing for the plotting of functions and data.

Several optical signals have been designed under a single-carrier approach and under a multisubcarrier approach, with $4,8,16$ and 32 subcarriers in an optical channel, operating at a netbit rate of 200 G . To this respect, the simulation models and compares two FEC approaches: SD-FEC and HD-FEC, with and without equalization.

### 1.4 DOCUMENTATION OVERVIEW

This thesis reviews the considerations for developing an optical communication channel simulator and the design and implementation of multi-subcarrier hybrid modulation formats, based on the modeling and simulation of relevant scenarios. The work supporting this thesis' conclusions is organized in five chapters that focus on the validation of the mentioned simulation's models and associated results.

Chapter 1 introduces the investigation field. Chapter 2 gives a theoretical overview of the Optical Communications System. Chapter 3 describes the advanced modulation format proposed in this study, the design of the HMF, the estimation of BER, and the power allocation on HMF. It also addresses other advanced techniques implemented in the optical channel, present in the simulation model, as elements of the performance study. Chapter 4 describes the modeling and simulation of the optical channel, and provides simulated performance and results data, based on the FDHMF. Chapter 5 provides final considerations, recommendations and conclusions.

## 2. OPTICAL COMMUNICATION SYSTEMS DESCRIPTION



Figure 2.1: General representation of a Communication System.

In telecommunications, very generally the main elements that play a role in the process of transmission and propagation of data are shown in Figure 2.1, can be observed the presence of a transmitter (set of transmitters), a receiver (set of receivers) and the communication channel, being the last one, the medium by which a signal is propagated. Two types of media can be distinguished by which electrical signals travel, guided media (coaxial cable, cooper, optical fiber, etc.) and unguided media (air).

Entering in the study of the optical communication systems, the most relevant characteristic to introduce is that the communication channel is typically the optical fiber. This assumption is represented in Figure 2.2.


Figure 2.2: General Optical Communication System.

Usually, the communication channel is a scarce resource that have to be shared between signals, so in communications is introduced one technique that recombine the signals and
allows to share the medium, called multiplexing. In optics, the multiplexing technique used is Wavelength Division Multiplexing.

Wavelength division multiplexing is a kind of frequency division multiplexing - a technique where optical signals with different wavelengths are combined, transmitted together, and separated again. It is mostly used for optical fiber communications to transmit data in several (or even many) channels with slightly different wavelengths. In this way, the transmission capacities of fiber-optic links can be increased strongly, so that most efficient use is made not only of the fibers themselves but also of the active components such as fiber amplifiers [1].

### 2.1 OPTICAL COMMUNICATION SYSTEMS WAVELENGTHS

Typically, optical communication systems operate in a wavelength region. A direct relation exists between frequency and wavelength, this give us the wavelength at which an electromagnetic wave would be propagated in vacuum at a determined frequency. Fiber optic systems have been developed for longer distances, higher speed and WDM, based on that those systems have an associated wavelength band, where fiber and transmission equipment can operate more efficiently.

These bands are a subdivision of the second and third "Telecom Windows" shown in Figure 2.3:

| Optical band |  | Wavelengths |
| :--- | :--- | :--- |
| O | (Original)-Band | $1260 \mathrm{~nm}-1360 \mathrm{~nm}$ |
| E | (Extended)-Band | $1360 \mathrm{~nm}-1460 \mathrm{~nm}$ |
| S | (Short)-Band | $1460 \mathrm{~nm}-1530 \mathrm{~nm}$ |
| C | (Conventional)-Band | $1530 \mathrm{~nm}-1565 \mathrm{~nm}$ |
| L | (Long)-Band | $1565 \mathrm{~nm}-1625 \mathrm{~nm}$ |
| U | (Ultralong)-Band | $1625 \mathrm{~nm}-1675 \mathrm{~nm}$ |

Figure 2.3: Wavelength Bands [2].

At the beginning a first window at $800-900 \mathrm{~nm}$ was used but due to high losses and problems to develop proper fiber amplifiers this window is just used for short-distance transmission.

With the second telecom window wavelengths are around $1.3 \mu \mathrm{~m}$ with low losses and weak chromatic dispersion, however fiber amplifiers (like praseodymium-doped glass) are not as good as those used on $1.5 \mu \mathrm{~m}$ based on erbium.

Presently, the third telecom window with wavelengths around $1.5 \mu \mathrm{~m}$ is widely used. The mentioned C-Band losses in the optical fiber are lowest and the available Erbium-Doped fiber amplifiers offer very high performances.

C-Band is expressed in the wavelength range between $1530 \sim 1565 \mathrm{~nm}$ and in terms of frequency in the range $191 \sim 196 \mathrm{THz}$.

In terms of attenuation, modern fiber optical communications can guarantee less than $\approx 0.2 \mathrm{~dB} / \mathrm{km}$ for a single mode silica fibers, so that means that can achieve tens of kilometers without amplification.

### 2.2 OPTICAL FIBERS

In Fiber-Optic technology, light (typically IR) is used as primary medium to carry information and is guided through optical fibers by reflecting off at the core-cladding interface, being the most cost-effective way to move huge amounts of information.

Optical fibers allow to have long distance communications, are composed by a core and a cladding, usually made of ultra-pure glass; some fibers are all plastic and some have glass cores and plastic claddings. The glass used is usually SiO 2 (Silicon dioxide).


Figure 2.4: Optic fibers [3].

Optical fibers have a standard diameter (core + cladding) of $125 \mu \mathrm{~m}$, represented in Figure 2.4, also standardized is the fact that the core refractive index must have a refractive index larger than the cladding one. Two types of optical fibers; Multi-mode Optical Fibers and Single-mode Optical Fibers exist, where high-performance is achieve with Single-mode, reason why they are widely used in telecommunication systems. The most used is the ITU-T Standard G. 625.

### 2.3 OPTICAL AMPLIFICATION

Optical amplifier is a vital technology for optical communication networks, since it can directly amplify an input optical signal without needing to transform it first to an electronic signal. As we know, there are three common types of optical amplifier: EDFA (Erbium Doped Fiber Amplifier), Raman amplifier and semiconductor optical amplifier [4].


Figure 2.5: Erbium Do ped Fiber Amplifier [4].

The most used optical amplifiers in telecommunications are EDFA, especially if the working band is the C-band due to their high performance when they are introduced in the network. EDFA presents some advantages with respect to other optical amplifiers as: reduction of transmission losses when the light signal is transmitted (even on far distance), is practically transparent to digital signal format and data rate, has low figure noise allowing the use of multiple amplifiers in cascade (for long haul applications) and can amplify many wavelengths simultaneously.

### 2.4 OPTICAL ROUTING

Nowadays, second generation optical networks are capable of perform also switching/routing in the optical domain and introduce the wavelength routing, an example is shown in Figure 2.6, where transparent or opaque optical circuits, called lightpaths, are used to connect network nodes. Lightpaths may span one or more hops in the physical topology, and may cross switching elements in the optical layer.


Figure 2.6: Wavelength ro uting [6].

In WDM optical communications networks, important configurations as Optical add-drop multiplexers (OADM) are present, this allow to provide a better performance when data transmission take place. In optical networks signals are directed to destination by optical nodes which typically consist of multi-degree reconfigurable optical add drop multiplexers (ROADMs) based on wavelength selective switches (WSSs)

The properties of dynamic reconfigurability and channel add/drop in the optical domain make them suitable candidates for the evolving meshed optical networks. Several implementations of ROADM are possible using optical devices including MUX/DMUX, optical splitters/combiners, wavelength blockers and Wavelength Selective Switches (WSSs) [13].

Wavelength Selective Switches (WSS) are used in this type of networks to route (switch) signals between optical fibers on a per-wavelength basis and are also able to perform monitoring in the system. A better definition is reported:

The WSS is an active component that performs the actual wavelength switching and monitoring. It can dynamically route, block and attenuate wavelengths within a network node. And, the output patterns can be changed or reassigned to several output fibers through electrical interface. In order to vary the fiber connectivity between the transceiver and networks access ports for a given direction, directionless extra WSSs were deployed [5].


Figure 2.7: Wavelength Selective Switch [5].

## 3. ADVANCED MODULATION FORMATS

### 3.1 ADVANCED TECHNIQUES IN OPTICAL COMMUNICATION SYSTEMS.

### 3.1.1 Forward Error Correction

In actual optical communication systems that work to improve the spectral efficiency and demand for flexibility, in combination with advanced modulation formats, others methods as coherent detection, polarization multiplexing and error control coding are being implemented.

Today's high-speed electronics enables very sophisticated signal processing and coding to applied, even at extremely high data rates. FEC (forward error correction) systems for fiber typically apply or adapt known binary error-correcting codes, in a manner that is essentially independent of other subsystems (e.g., the modulation) [7].

Demanding improvements in the capacity of the links are translate in the ability of support lower BERs, that today can move between $10 \mathrm{e}-12$ and $10 \mathrm{e}-15$, FEC is used by almost all applications as an alternative to achieve and satisfy a desired BER in the system with low cost.

When FEC is integrated, an ECC (Error Control Coding) is applied to add redundancy, so an information sequence is encoded to a longer sequence, this means that an information sequence of k bits or symbols is converted in a code-word of n bits or symbols ( $\mathrm{n}>\mathrm{k}$ ). This lies in a relation between the redundancy and the information length called overhead.

$$
\begin{equation*}
o h=\frac{n-k}{k}=\frac{1}{R}-1 \tag{15}
\end{equation*}
$$

Where R is the ratio between the bit rate without FEC and bit rate with FEC, called code rate. As can be seen the overhead affects the net data rate.

The use of FEC method enhance the system providing some coding gain and net coding gain that are strictly related with the desired BER performance in the system:

The definitions of CG and NCG can be found in ITU-T G.975.1 [20]. CG means the improvement of received optical sensitivity by using FEC, without considering penalty by bit rate increasing. NCG has bit rate increasing penalty considered in addition to the CG. As the code rate R is less than 1 , a NCG is always smaller than the corresponding CG. NCG is characterized by both the code rate R , and the maximum allowable BER ( Bin ) of the input signal to the FEC decoder that can be reduced to a reference output BER (Bout $=$ Bref) by applying the FEC algorithm the more NCG is, the better error correcting capability the code provides. Yet, the bit rate penalty is of great importance in real systems [7].

In the study of FEC, different approaches can be used depending on the results in terms of desirable BER performance, this considering that a better performance is translate in the capability of provide higher NCG, at cost of introduce some overhead in the system. Some approaches of FECs can be implemented in optical communications systems: Soft-Detection FEC and Hard Detection FEC, this changing the overhead and the target BER at which the system should work to achieve a fix net-bit rate. Approaches that will be extended in the simulation part to analyze their behavior.

### 3.1.2 Wavelength Selective Switch

When an HFM is being implemented there are some considerations related to the system that have to be taken into account to properly adapt the assignment of modulations formats. One of these is the impact of the filter-narrowing effect induced by a WSSs at an optical node that are integrated in ROADMs present in the optical communication network. This tight optical filtering induces penalties to the performance of the system and the through traffic is degraded, this effect increases in a meshed optical network as several WSSs are cascaded.

Current flex-grid WSSs are capable of configuring the bandwidth of each channel with a resolution of 12.5 GHz or less and directing each of them to one of the N available output ports. Due to the tight optical filtering of the WSSs inside the ROADM, each node induc es filtering penalty on the optical signal. In long-haul and meshed regional transmission links, the optical signal may pass through a cascade of ROADMs before being detected. Due to the ROADM cascade, the net 3-dB bandwidth keeps decreasing and consequently a filtering penalty arises, setting a limit on the maximum number of ROADMs a signal can pass before regeneration is required [13]

Each WSS acts a bandpass filter, the optical spectrum narrowed by a WSS in the routing process is define in Eq. 16:

$$
\begin{align*}
& S(f)=\frac{1}{2} * \sigma * \sqrt{2 \pi} *\left[\operatorname{erf}\left(\frac{\frac{B}{2}-f}{\sqrt{2} * \sigma}\right)-\operatorname{erf}\left(\frac{-\frac{B}{2}-f}{\sqrt{2} * \sigma}\right)\right]  \tag{16}\\
& \sigma=\frac{B_{W, O T F}}{2 * \sqrt{2 * \ln 2}} \tag{17}
\end{align*}
$$

Where B is the bandwidth of the frequency slot in which the WSS is placed, $\sigma$ is the standard deviation of the Gaussian optical transfer function (OTF) that depends of the 3-dB bandwidth.

In order to show the properties of WSS channel, Figure 3.1 is plotted with a fixed bandwidth, $\mathrm{B}=37.5 \mathrm{GHz}$ and Bw, otF $=10.4 \mathrm{GHz}$.


Figure 3.1: WSS FILTER SPECTRUM.

### 3.1.3 Equalization

Signal transmission over the channel is affected by many distortions that result in both amplitude and phase fluctuations. As established in previous section, the introduction of WSSs induce penalties to the system, as a result of bandwidth narrowing, signal power at the edge of channel spectrum is attenuate, resulting in significant intersymbol interference (ISI) in the received signal, which is one of the major obstacles to reliable and high-speed data transmission, because the performance of the system is affected leading in consequences as an increasing of the required SNR for a target BER. To compensate this attenuation of spectrum due to use of WSS, equalization is introduce with the objective to reverse this effect and reduce ISI to allow recovery of the transmit symbols.

To achieve the goal of compensation the equalizer uses an estimate of the channel frequency response, however in optical communication system the channel is consider varies, therefore adaptive equalization must be introduced where the equalizer adapts to the channel variations, so it updates its parameters as it processes the data, to be able to continuously mitigate the negative effect of the channel.

In general, the training of the equalizer is done by sending a fixed-length known bit sequence over the channel and used it to determine the optimum tap weights for the equalizer. Nowadays equalization techniques fall into linear and nonlinear categories, where the tradeoff is between complexity and performance, generally nonlinear are more complex than linear equalization techniques but with higher performance.

In this study a MIMO equalizer is implemented, based on least mean square (LMS) algorithm as adaptation strategy that determines the optimal set of filter coefficients that minimize MLSE. With the introduction of the equalizer in the system the expectative is notice improvements in terms of BER and SNR in the overall simulation system.

### 3.2 HYBRID MODULATION FORMATS

### 3.2.1 Basics

Optical Transmission Systems have been evolving through years, this imply also an evolution to a higher-order modulation formats that enable communications at higher bit rates.

High order modulation format is a modulation format that maps m bits to a symbol, where m is usually 4 or more. In Figure 2.1 are shown some constellation diagrams of high-order modulation formats of Phase Shift-Keying (PSK) and Quadrature Amplitude Modulation (QAM). From left to right these modulation formats are quadrature phase shift keying (QPSK), 8-ary PSK (8PSK), 16-ary PSK (16PSK), 16-ary QAM (16QAM), 32-ary QAM (32QAM) and 64-ary QAM (64QAM).


FIGURE 3.2: CONSTELLATION DIAGRAMS OF HIGHER-O RDER MODULATION FORMATS [7].

Figure 3.2 shows the evolution of optical transmission system where WDM technology is combine with modulation formats to allow systems to scale to a higher capacity. Nowadays, research on advanced modulation format and denser WDM transmission has managed to increase the capacity beyond $100 \mathrm{~Gb} / \mathrm{s}$.


FIGURE 3.3: SCALABILITY O F O PTICAL CO MMUNICATIO N SYS TEMS [7].

Nowadays, more challenging optical communication systems ask more spectral efficiency and data-rate flexibility. At the beginning standard m-QAM modulation over a single-carrier (SC) transmission in the channel was used, but in terms of signal design this provides limited bit-rate granularity to systems. To enable more flexibility in terms of data-rate a change into Multi-subcarrier (MSC) transmission have been proposed.

Multi-subcarrier (MSC) transmission, in which the signal is split into closely spaced low Baudrate subcarriers, has been widely addressed in recent papers due to its increased robustness against nonlinearities, resulting from the phenomenon of symbol-rate optimization (SRO) [8].

So the Hybrid Modulation Formats can be introduced for modulating data signals using the MSC approach to make the use of bandwidth more flexible, this means that low-cardina lity and high-cardinality formats are mixed to achieve any desirable transmission bit-rate. In order to improve performance, high-cardinality modulations are the central ones in the Hybrid scheme and low-cardinality are allocated in edge subcarriers, this due to the fact that
the second ones suffer less to optical filtering. Another fact that is important to consider is that the power allocation is not uniform between subcarriers.

### 3.2.2 Multi-subcarrier with Hybrid Modulation Format implementation

In order to have a channel that implements a HMF, it is necessary to set up some parameters in the design. First, establish the net-bit rate at which the system should work, e.g. 200 GHz . To achieve every net-bit rate, is possible to use different combinations of the standard modulation formats, distributed properly (low-cardinality at edges and high-cardinality in the middle) between the subcarriers. Also, different number of subcarriers can be used, taking into account that this number cannot be too much in order to improve feasibility, e.g. with a net-bit rate of 200 G with eight subcarriers, $50 \%$ QPSK, $50 \% 64-\mathrm{QAM}$ configurations can be used as shows Figure 3.4.


FIGURE 3.4: EXAMPLE O F MULTI-SUBC ARRIER HYBRID MO DULATION FO RMAT.

The type of modulation format PM-mQAM will change the average bit-per-symbol among all subcarriers:

$$
\begin{equation*}
k=\log _{2}(m) \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
\overline{B p S}=2 * k \tag{2}
\end{equation*}
$$

With the net-bit rate definition, is necessary also define the symbol rate that will be allocated in the channel, e.g. 32 GBaud. This symbol rate is then divided between the numbers of subcarriers to have the symbol rate on each subcarrier.

$$
\begin{equation*}
R_{S, S C}=\frac{R_{S, M S C}}{n S C} \tag{3}
\end{equation*}
$$

And the net-bit rate is obtained, taking into account the total overhead ( OH total), e.g. 28\%:

$$
\begin{equation*}
R_{b, \text { net }}=\frac{2 *\left[\sum_{i=1}^{n S C} R_{S, S C i} * \log _{2}\left(m_{i}\right)\right]}{1+\text { OH total }} \tag{4}
\end{equation*}
$$

A square root raised-cosine (SRRC) with a certain roll-off (e.g. $\rho=0.05$ ) is applied to each carrier, so the subcarrier spacing is:

$$
\begin{equation*}
\Delta f=(1+\rho) * R_{s, S C} \tag{5}
\end{equation*}
$$



Figure 3.5: SC CONFIGURATIO N VS MSC WITH HMF CONFIGURATION.

In Figure 3.5 can be seen the difference in terms of spectrum between a channel with a singlecarrier approach and a channel with a multi-subcarrier. Both of them give us the same netbit rate (e.g. 200G), advantages given by the second approach will be detailed below.

### 3.2.3 BER analysis on HMF

Nowadays, optical networks are more demanding with the data transmission asked going up and the performance of the capacity of the network increasing an improvement on the supported bit error rate (BER) is required. Today is possible talk about values around $10^{-12}$ and $10^{-15}$, which are lower than the reference BER from decades ago.

When HMF are present, frames of M symbols characterize them. M1 symbols can be set for the first modulation format (F1) and M2 symbols for the second modulation format (F2). Both F 1 and F 2 are characterized by a bit-per-symbol $\mathrm{BpS} 1, \mathrm{BpS} 2$, by the individual average
power P1, P2 and individual SNRi, given the overall average transmission power Ptx, the power ratio $P R$ and the format ratio $F R$, the overall bit-error-rate (BER) can be define [11]:

$$
\begin{equation*}
B E R=\frac{1}{B p S} *\left((1-F R) * B p S_{1} * B E R_{Q A M-1}+F R * B p S_{2} * B E R_{Q A M-2}\right) \tag{6}
\end{equation*}
$$

Where the missing parameters are define below:

$$
\begin{align*}
& F R=\frac{N_{s c, 2}}{N_{s c}}  \tag{7}\\
& P R=\frac{P_{2}}{P_{1}}  \tag{8}\\
& P_{T X}=(1-F R) * P_{1}+F R * P_{2}  \tag{9}\\
& \overline{S N R}=(1-F R) * S N R_{1}+F R * S N R_{2}  \tag{10}\\
& \overline{B p S}=(1-F R) * B p S_{1}+F R * B p S_{2} \tag{11}
\end{align*}
$$

PR can be also written in as:

$$
\begin{equation*}
P R_{d B}=10 * \log _{10}\left(\frac{S N R_{1}}{S N R_{2}}\right) \tag{12}
\end{equation*}
$$

$B E R_{Q A M-1}$ and $B E R_{Q A M-2}$ are the BER values for a QAM format, that can be approximated by analytical estimation equations:

$$
\begin{equation*}
B E R_{m Q A M}=2 * \frac{\sqrt{m}-1}{\sqrt{m} * \log _{2}(m)} * \operatorname{erfc}\left(\sqrt{\frac{3 * S N R}{2 *(m-1)}}\right) \tag{13}
\end{equation*}
$$

Equation (13) can be applied for a square QAM formats but also a definition for an approximated BER for non-square QAM formats can be set:

$$
\begin{equation*}
B E R_{\text {mQAM }}=\left(1+\frac{1}{\sqrt{2 * m}}+\frac{1}{3 * m}\right) * \frac{4-\frac{6}{\sqrt{2 * m}}}{\log _{2}(m)} * \frac{1}{2} * \operatorname{erfc}\left(\sqrt{\frac{48 * S N R}{31 * m-32}}\right) \tag{14}
\end{equation*}
$$

### 3.2.4 Power allocation on HMF

Another aspect relevant to introduce in HFM is how the power is distributed on each subcarrier. In a single carrier approach the information is spread over the occupied bandwidth, in a multi-subcarrier approach exists the opportunity to spectrally allocate power and information, this means that the power allocation is not uniform. After the definitions presented above, the following step is set the strategy for the operation of the transmitter to
establish the power ratio PR , so the power ratio between subcarriers is calculated to guarantee a given strategy. This power ratio calculation is applied to multi-subcarriers signals with FDHMF.

### 3.2.4.1 "Min BER" strategy

The strategies can have different purposes and parameters to set that allow to calculate the PR as: all subcarriers are at the same power level, all subcarriers are operating at the same BER for a given target BER, etc.

The use of the "Min BER" strategy is implemented, where PR is obtaining minimizing the average BER among all subcarriers for a pre-specified operating SNR (target SNR).

Although "Min BER" strategy is a possibility of power allocation between subcarriers, on the back-to-back analysis the system may suffer due to filtering when WSSs are introduced, so parameters as the SNR are affected. The PR allocation using "Min BER" strategy can be optimized.

### 3.2.4.2 Optimization through Power Loading strategy

Different parameter optimization algorithms can be implemented and influence the performance of the communication system. So, the PR allocation can be improved by using adaptive loading algorithms on each individual subcarrier as: power loading, bit loading, water filling, that assign bits and electrical power to subcarriers according to the SNR experienced by the subcarrier.

Through negotiations between the transmitter and the receiver, all the adaptive loading algorithms can thus be implemented according to the total channel BER and each individual subcarrier BER [10].

In this study, Power Loading strategy is implemented:
An optimized power loading of a MSC signal is performs in a channel with different SNR per subcarrier, so according to the system frequency response of a specific channel (in simulation part the channel response is assumed to be frequency-flat within each SC for simplicity), a maximum possible signal modulation format is taken on all the subcarriers, and each individual subcarrier power is optimized to ensure that the individual subcarrier BERs detected in the receiver are almost uniformly distributed among all the subcarriers and that the corresponding total channel BER is less than $1.0 \times 10^{-3}[10]$.

## 4. FDHMF DESIGN AND SIMULATION

Generally, the simulation scenario describes a generic B2B optical transmission system composed of transmitter, optical filtering due to WSS, noise loading and coherent receiver, including digital signal processing (DSP) subsystems. As output the bit-error-rate (BER) and the signal-to-noise ratio (SNR) are studied. Simulation programs also support subcarrier multiplexing that is combined with frequency-domain hybrid modulation formats (FDHMF) to achieve flexible modulation formats. Is important to consider that the noise loading is performed after the WSS filter, as a worst-case scenario.

Considering a channel with a bandwidth of 37.5 GHz per WSS. Two cases in terms of the aggregate symbol rate are studies at which the channel runs, first case is SD-FEC that provides a symbol rate of 32 GBaud, the second case is HD-FEC that provides a symbol rate of 28 GBaud. They work to achieve different target BERs and with a proper selection of the overhead both allow to achieve the same net-bit rate.

|  | SD-FEC | HD-FEC |
| :---: | :---: | :---: |
| Target BER | $2 \mathrm{e}-2$ | $3.8 \mathrm{e}-3$ |
| Symbol Rate MSC | 32 e 9 | 28 e 9 |
| Overhead FEC | 0.2 | 0.07 |
| Overhead total | 0.28 | 0.12 |

TABLE 4.1: SD-FEC AND HD-FEC CHARACTERISTICS.

The channel studied is a MSC solution where several low symbol-rate non-overlapping Nyquist filtered channels generated by digital to analog converters (DACs) are implemented with a square root raised-cosine spectral shaping with $\rho=0.05$ to each subcarrier, so the subcarrier spacing is determined by equation (5). FDHMF enables a net-bit rate granularity of up to $12.5 \mathrm{~Gb} / \mathrm{s}$, using a MSC approach larger number of possible FDHMF configurations can be analyzed with the implementation of different permutations of modulation format assignment to each subcarrier.

The analysis of different FDHMF configurations for a different number of subcarriers was done, cases for $4,8,16$ and 32 subcarriers for all cases were simulated allowing to achieve a net-bit rate of 200 G , also single carrier was studied in order to do the confront, make the comparison and see the advantages of using a MSC approach.

Signal passes through a filtering process due to the presence of WSSs, a loop in the simulation is implemented in order to quantify performance evolution due to this process when a cascade of WSS is present, and more exactly impact for 0 up to 10 WSS was quantified.

After routing through ROADMs, the target signal is received and detected with a digital coherent receiver, signal demodulation is then performed by standard DSP and the BER can be calculated, then an interpolation method is used to find the SNRs required for a given BER target, depending on FEC strategy. Another consideration contemplated this study was the impact of equalization, simulation were done taking into account when an adaptive equalizer is present or not.

### 4.1 SIMULATION PERFORMANCE

Theoretically, when a MSC with FDHMF is used the power between subcarriers is not uniform and is necessary to define a strategy at the transmitter side to properly allocate the power in order to achieve a desirable performance. At the beginning "Min BER" strategy is used in order to have a PR that minimizes the average BER among all subcarriers for a given operating SNR that is correlated to a given BER target. Is important to establish that the work consider two possible values of BER target, depending on FEC strategy, with a value of $3.8 \mathrm{e}-$ 3 for HD-FEC and a value of $2 \mathrm{e}-2$ for a SD-FEC.

However results show that the performance of the system can be optimize in terms of power allocation by applying adaptive loading algorithms on each individual subcarriers, according to the SNR experienced, in the study is implemented Power Loading algorithm to optimize the PR allocation, this is performed in the simulation study by fminsearch function provided by Matlab.

Results are shown in Figure 4.1 for two FEC strategies. First can be observed that either using SD-FEC or HD-FEC; SNR values are lower when Power Loading is implemented instead of Min BER, even in presence of filtering. This tells that the adaptive algorithm optimizes the PR allocation between subcarriers leading in better values of required SNRs.


FIGURE 4.1: STRATEGIES FOR POWER ALLOCATION.

Also with respect to Figure 4.1, is important to detail that either using "Min BER" or "Power Loading", simulations results are above theory values. Theory values represent an ideal case and can be set as lower-bound to analyze the good performance of the simulation system. Another important point is to see the impact of using or not an adaptive equalizer, where can be seen that using it simulated values are closer to theoretical ones.

Continuing with the equalization analys is, here casually selection of a case is introduced to see the difference before and after using the equalizer, in this case 4 -subcarriers signal is
introduced. Table 4.2 shows the results, simulations state that the average BER between subcarriers is lower when simulations are done using an equalizer, which reveals the advantage of equalization in these types of optical systems.

| Theoretical BER | Simulated BER - No Equalization | Simulated BER - Equalization |
| :---: | :---: | :---: |
| $1.745 \mathrm{e}-04$ | $5.70 \mathrm{e}-2$ | $5.786 \mathrm{e}-03$ |
| TABLE 4.2: EQ UALIZATION IMPACT IN TERMS OF BER. |  |  |

Figure 4.2 reflects graphically the results given on Table 4.2, as established BER values using equalization are lower than BER values without equalization, is important to clarify that achieve lower BERs is translate in better system performance.


FIGURE 4.2: EQ UALIZATION IMPACT IN TERMS OF BER.

Another relevant point in the analysis of the system performance is the impact of filtering due to the presence of either one WSS or a cascaded of them. In order to show the properties of WSS channel, plot in Figure 4.3 reflects the impact in the spectrum and how it is reduced when they are allocate in cascade, this leading to worst BER at the receiver and worst performance in the system, situation that can be observed in Figure 4.4.


Figure 4.3: IMPACT IN THE SPECTRUM OF USING WSS IN CASCADE

Selecting one of the study cases allows to see the impact of an increasing number of WSS in cascade. Figure 4.4 presents a BER vs SNR curve, an interpolation method can be used to find the SNRs required for a given BER target. This can be applied for any FDHFM configuration and allows to make a comparison between the SNRs required to study which configuration suffers more due to filtering.

The case of $W S S^{0}$ is an ideal condition, where filtering due to the presence of a WSS is not consider and can be allocated all the bandwidth for the subcarriers, but this is not true in real conditions. The simulation takes into account the impact of WSS introduction, the signal suffers because the spectrum is cut and the final bandwidth that can be used is decreasing, this is observed in Figure 4.3. So, for an increasing number of WSS in cascade, for example $W S S^{5}$ or $W S S^{10}$ shown in Figure 4.4, the BER curve as a function of the SNR values moves out. For a given target BER the required SNR to achieve it is going to be higher for the case of $W S S^{10}$ with respect to the required SNR for the ideal case $W S S^{0}$.


Figure 4.4 Impact of using WSS in cascade.

The proposal is also analyze between two different perspectives in terms of FEC strategies, with the purpose of evaluate the performance against filtering. Taking one case with 8subcarriers and a fix number of WSS in cascade equal to 5 , can be observed that the main difference is in terms of spectrum because HD-FEC has less spectrum to be used, no matter the number of subcarriers. This occurs due to the symbol rate of the overall system that allows transmission of 32 Gbaud in SD-FEC and 28 Gbaud in HD-FEC, that is subdivided depending on the number of subcarriers, so SD-FEC strategy will suffer more than HD-FEC due to the impact of WSS because the spectrum is more cut at edges as set in Figure 4.5.


Figure 4.5: IMPACT OF FEC STRATEGY IN THE SPEC TRUM.

### 4.2 SCM WITH FDHMF IMPLEMENTATION

Thesis' proposal consists in use a SCM with hybrid modulation formats technique, the idea is to make a comparison and an analysis when the number of subcarriers used change in the communication channel and the hybrid configurations allowed use: PM-QPSK, PM-8QAM, PM-16QAM, PM-32QAM, PM-64QAM, always to achieve a net-bit rate of 200G and for two different symbol rate of 32 Gbaud and 28 Gbaud. The purpose is to study and analyze the impact of filtering for all cases, choose the cases with best performance and make the comparison to establish the advantage of work with SCM solution. Also, is important to investigate the performance of the system when the techniques explained before: equalization and FEC strategy are implemented.
First important aspect to set is how the spectrum is used when different number of subcarriers are implemented, considering the following cases: single carrier, 4 -subcarriers, 8 -subcarriers, 16 -subcarriers and 32 -subcarriers. Figures 4.6 up to 4.10 show the results obtained in simulation for the described cases. Is relevant to set that these figures reflect the situation when just one WSS is implemented, if WSSs are cascaded the behavior changes, this translate into a narrower spectrum in all cases, that will influence the performance of the system, due to the cut of the spectrum BER values will undoubtedly increase. Another relevant aspect in this figure is related to FEC strategy where for all cases using HD-FEC can be easily seen how the subcarriers are narrow in terms of spectrum.



Figure 4.7: Spec trum of 4-S UbC ARrier appro ach


FIGURE 4.9: SPEC TRUM OF 16-SUBCARRIER APPRO ACH


FIGURE 4.10: SPEC TRUM OF 32-SUBC ARRIER APPRO ACH

### 4.2.1 FDHMF with SCM cases.

To do an exhaustive analysis, simulation comprises all configurations that allow a combination between different standard modulation formats PM-QPSK, PM-8QAM, PM16QAM, PM-32QAM and PM-64QAM for the different number of subcarriers. All cases are designed considering some limitations as: the net-bit rate of 200 GHz to achieve and the position of the modulation formats in the configurations, where low-cardinality are allocated at edges and high-cardinality in the middle. Allocating in this manner the formats enhance the performance of the overall system because low-cardinality behaves better against filtering than high-cardinality. All these assumptions gives us a finite number of cases to study that vary depending on the number of subcarriers (e.g. for higher number of subcarriers as 32 are possible 177 cases).

The purpose of this grouping is study the behavior of all the cases when are in presence of tight filtering, with the influence of zero WSS up to 10 , based on this a criteria is established to evaluate best cases among all for each number of subcarrier. First is important to set that for a single carrier approach only one option is able to achieve a net-bit rate of 200 GHz in this simulation scenario and corresponds with a pure PM-16QAM. Other cases of study are described below on Table 4.3 up to 4.7 .

|  | CONFIGURATIONS |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Case 1 | 16 | 16 | 16 | 16 |
| Case 2 | 4 | 64 | 64 | 4 |
| Case 3 | 8 | 32 | 32 | 8 |

TABLE 4.3: FDHMF CONFIGURATIONS FOR 4-SUBCARRIERS@200G
As can be observed in Table 4.3 only just 3 configurations are possible due to the limited number of subcarriers, instead for 8-subcarrier approach described in Table 4.48 configurations are found, where cases colored in blue represent best cases in terms of their behavior against filtering.

|  |
| :---: |
|  |
| Case 1 16 16 16 16 16 16 16 16 |
| Case 2 |
| Case 3 | 4


|  | CONFIGURATIONS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case 1 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| Case 11 | 4 | 4 | 8 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 8 | 4 | 4 |
| Case 14 | 4 | 4 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 4 | 4 |
| Case 22 | 4 | 8 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 8 | 4 |
| Case 24 | 4 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 4 |
| Case 33 | 8 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 8 |

TABLE 4.5: FDHMF CONFIGURATIONS BESTCASES FOR16-SUBCARRIERS@ 200G
In cases as 16 -subcarriers and 32 -subcarriers a more limited information is given in order to synthetize the results presented, either if all cases were simulated ${ }^{1}$ choosing only best cases against filtering to be presented. For 16 -subcarrier were evaluated a total of 33 cases, best cases are described in Table 4.5.

For 32 -subcarriers were evaluated a total of 177 cases, best cases are described in Table 4.6, where a subdivision of best cases is done depending on FEC-strategy. In this approach many cases provide different alternatives of best cases working with SD-FEC or HD-FEC, so the subdivision must be done.

[^0]| Cases | CONFIGURATIONS SD-FEC |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| 53 | 4 | 4 | 4 | 4 | 8 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 8 | 4 | 4 | 4 | 4 |
| 58 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 4 | 4 | 4 | 4 |
| 86 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 4 | 4 | 4 |
| 117 | 4 | 4 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 4 | 4 |
| 148 | 4 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 4 |


| Cases | CONFIGURATIONS HD-FEC |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| 82 | 4 | 4 | 4 | 8 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 8 | 4 | 4 | 4 |
| 86 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 4 | 4 | 4 |
| 114 | 4 | 4 | 8 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 8 | 4 | 4 |
| 146 | 4 | 8 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 8 | 4 |
| 177 | 8 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 8 |

### 4.2.2 Comparison theory and simulation

Is relevant to set the behavior of our simulation with respect to the theoretical values, last ones give us a lower-bound to evaluate properly if the results are congruent and the simulation model is working. This is done by establishing a comparison between theory and simulation results. In previous steps, also the impact of equalization, FEC strategy and behavior of configurations are studied, when tight filtering is present and different number of subcarriers are applied.

In Figure 4.11 results for a pure PM-16QAM are shown, the model simulation developed for during the investigation is congruent because simulated values are always above with respect to theory. Single carrier approach improves importantly when equalization is applied, can be observed how without equalization Fig 4.11(a) simulation curves for SD-FEC and HD-FEC, first to reach $W S S^{3}$ get worst, with a highly increase in the required SNR; different from case (b) where until $W S S^{5}$ penalties are around 3 dB and lower for HD-FEC. Can be set that HD-FEC is more robust against filtering, with lower penalties in terms of required SNR and good improvement when equalization is used.


FIGURE 4.11: THEORY VS SIMULATION FOR SINGLE CARRIER.
Figures 4.12 and 4.13 show results for the three possible cases given for a 4 -subcarrier approach, again is proved that the model simulation developed for this approach is congruent because simulated values are always above with respect to theory, equal as single carrier. Moreover, behavior against equalization is confirm. Also, in these cases where system behavior improves importantly when an adaptive equalizer is applied, HD-FEC results are closer to theoretical ones even trespassing six WSSs in cascade. This tells that equalization in addition with a SCM scheme tend to improve the results in terms of SNR vs an increase number of WSS in cascade.


FIGURE 4.12: THEORY VS SIMULATIO N FOR 4-SUBCARRIERS - NO EQUALIZATIO N


FIGURE 4.13: THEO RY VS SIMULATIO N FO R 4-S UBCARRIERS - EQ UALIZATIO N
Similar behavior repeats when higher number of subcarriers (e.g. 8, 16, 32), where the impact of equalization and the fact of increasing number of subcarrier are translated in simulated values closer to theoretical ones. Actually, when 32-subcarriers is used an important study emerges and can be seen in Figure 4.14, the selection of 3 cases among of best cases for 32subcarrier allow to notice that simulated values are almost similar to theoretical ones, even when equalization is not employed. This fact is a gain in terms of computational time and complexity, because we can use the theoretical values (do not take into account equalization) to properly describe and study the behavior of the system with 32 -subcarriers.


FIGURE 4.14: THEO RY VS SIMULATIO N FO R 32-SUBCARRIERS

### 4.2.3 Comparison between different number of subcarriers

Frequency Domain Hybrid Modulation Formats that work with Subcarrier Multiplexing are been propose as an advanced modulation formats that allow flexibility in the network by introducing a mix between low- and high cardinality formats to yield any desirable transmission bit-rate, the proper allocation of low-cardinality formats at edges make this advanced modulation robust against optical filtering.

So as was introduced several configurations work based on these principles, now is consider the improvement and advantages of work with FDHMF when severe penalties are caused as effect of optical routing.


Figure 4.16: CASES FOR 4-SUBCARRIERS, with Equalization.

Figure 4.15 and Figure 4.16 show the results obtained for a SCM system with 4 -subcarriers, where just three configurations combining standard QAM formats are possible, represented in Table 4.3, graphics represent the required SNR values as a function of the number of cascaded WSS, for both SD-FEC and HD-FEC. Also a comparison between the values without equalization and with equalization to see how the adaptive equalizer improves the behavior of the system in terms of SNR values when the signal is in presence of filtering. As can be observed, Case 1 composes just by PM-16QAM formats in all subcarriers, starts being the best case with the lowest required SNR, without any WSS influence but this behavior is not constant as filtering increases with more WSS in cascade. Indeed, values of SNR required
highly increase and this case finishes as worst configuration for a number of WSS in cascade equal to 10 , no matter the system scheme in terms of equalization and FEC strategy.

Moreover, Case 2 with an PM-8QAM formats at edge starts as second best and behaves in some cases as best configuration for some values of WSS in cascade but at the end also Case 2 is improved for Case 3 which is the case with the lowest constellation at edges (PM-QPSK) that finishes being the best case in terms of performance against optical filtering. Just in Fig. 4.16 (b) can be observed that when the equalizer and HD-FEC strategy work together, enhance the behavior of cases with higher constellation at edges.


FIgure 4.17: CASES FOR 8-SUBCARRIERS, without Equalization.


Figure 4.17 and Figure 4.18 give the results for best configurations when a FDHMF with SCM system is implemented using 8 -subcarriers. Cases that are present are taken as the three best cases of 8 possible, given by a communication system with these specifications. With some difference the system performs almost equal to the case of 4 -subcarriers where configurations with low-cardinality at edges behaves better than the others against filtering.

The comparison between the simulated results for Case 1,5 and 8 , given on Table 4.4, allows to analyze the fact that a pure 16-QAM with 8 -subcarriers (Case 1) behaves worse than hybrid
configurations (Case 2 and 3 ) when filtering start to increase. Actually, Case 3 has the lowest cardinality modulation format at edges and can be seen that improves its behavior with respect to the other two cases, this is because low-cardinality used at edges in Case 3 is more robust against filtering. Without an equalizer acting, after WSS2 for SD-FEC and WSS3 for HD-FEC, Case 3 improves with very low penalties, whit equalization after WSS 3 and WSS 4, for SD-FEC and HD-FEC respectively.


FIGURE 4.19: CASES FOR 16-SUBCARRIERS, WITHOUT EQUALIZATION.


The analysis of the behavior for a 16-subcarrier approach continues, results are given in Figure 4.19 and Figure 4.20, for best configurations. Cases studied are taken as the six best cases of 33 possible, given by a communication system with these specifications. Can be observed in Table 4.5. The behavior changes in this situation with respect to situations previously discussed, because more subcarriers are possible and in such away more possible configurations are allowed, this is reflected in more cases that behave as best cases.

One similitude with the other two cases can be established, Case 1 made by a pure 16-QAM with 16 -subcarriers starts as best case but behaves worse than hybrid configurations when filtering start to increase. Cases related to this study are design by the combination of three or four different standard modulations which include PM-QPSK, PM-8QAM, PM-16QAM. Actually, four of these six cases have allocated PM-QPSK at edges: Case 11, Case 14, Case 22, and Case 24. These cases have a behavior flatter than other two cases, this means lower penalties when the number of cascaded WSS increase.


FIgure 4.21: CASES FOR 32-SUBCARRIERS.

As a final situation, the evaluation of best configurations is done when a FDHMF with SCM system is implemented using 32 -subcarriers. Same analysis as 16 -subcarriers can be done, as the number of subcarriers increase more possible configurations to achieve specification as net-bit rate equal to 200 GHz are given, this translates in more cases to evaluate as best cases against filtering. Is important to set that for this situation, with 177 possible configurations, best cases for SD-FEC are not the same for HD-FEC, this in opposition with previous cases. For HD-FEC can be seen that 32 -subcarriers approach is robust when optical filtering is affecting and can be improved with the use of low-cardinality constellations allocated at edges. Moreover, also for SD-FEC, this situation of 32 -subcarriers seems to give good performance in terms of required SNRs as function of number of cascaded WSS.

Simulated results with 32 -subcarrier without equalization, were very close to theoretical results, so this cases presented in Figure 4.21 are based on theoretical results improving computational time and complexity in simulation programs.

Finally, the behavior of the performance when we introduce SCM is studied and also a comparison with a single carrier system. Here selection for all number of subcarrier signals compose just by PM-16QAM is done (not hybrid solution,) in order to generally analyze the influence of the number of subcarrier. Results are shown in Figure 4.22 and 4.23.


When equalization is not present in the system, results of Figure 4.22 are obtained. Figures give the required SNR for a target BER as function of number of cascaded WSS. As can seen either using SD-FEC or HD-FEC, same order in terms of performance for the different configurations are given. Can be seen that best performance, lower penalties against filtering is given by 32 -subcarrier approach and worst performance for 4 -subcarriers. Moreover, single carrier configuration performs better just with respect to 4 -subcarriers, without equalization it cannot achieve better results that system with an increasing number of subcarriers: 8, 16, 32.

Previous analysis is not equal when equalization is used, as can be observed in Figure 4.23, single carrier system is better option against SCM cases and this shows that equalizer enhances even more the performance of single-carrier system, allowing to achieve lower required SNRs when filtering is introduced. The equalizer is more capable to fight against ISI for a single carrier system.

For SCM systems the behavior is the same, higher number of subcarriers perform better than lower ones, this for the case of all subcarriers using PM-16QAM modulation.


### 4.2.4 Comparison between best cases

Until now, best cases separately have been analyzed. An important point in this study is to know which scheme between single carrier, multi-subcarrier and hybrid multi-subcarrier is more robust to be implemented in the channel, taking into account that in the communication process optical routing is introduced and generates severe penalties due to filtering.

A confront selecting all the best cases previously presented for each subcarrier is done and plotted together to identify which one performs better. Must be mentioned that for all number of subcarriers, configurations that result as best cases are compose by a PM-QPSK at edges and a mixture of PM-8QAM, PM-16QAM and PM-32QAM in others subcarriers. Higher constellation tested as PM-64QAM does not appear in best cases, this confirms that higher constellation formats are more penalized due to smaller Euclidean distance of PM-64QAM constellation points, effects of ISI induced by optical filtering are more pronounced.


FIGURE 4.24: BESTCASES FOR ALL SUBCARRIERS APPROACH @200G, WITHOUTEQUALIZATION

When equalization is not introduced, simulation gives the results shown in Figure 4.24. As can be seen black curves corresponding with cases for 32 -subcarriers are best cases, the ir performance is better than other cases with lower required SNR for an increasing number of WSS, either using SD-FEC or HD-FEC.

In Figure 4.25 can be observed the simulation values for best cases when equalization is using in the system. Interesting results are shown where at the beginning, single carrier approach performs as best case among all cases, until two number of WSS in cascade for SD-FEC and five for HD-FEC. After this the behavior previously discussed repeats and cases corresponding to 32 -subcarriers respond better against filtering. Even if single carrier can be consider as best case is important to set that SCM is proposed because allows flexibility and exploits all advantages of SRO by using low symbol rates.


FIGURE 4.25: BEST CASES FOR ALL SUBCARRIERS APPROACH @200G, WITH EQUALIZATION.

## CONCLUSIONS

This thesis deals with the design and implementation of an optical communication channel to be optimized and analyzed when advanced modulation formats are used. The proposal in the design of the advanced modulation is make a conjunction between SCM approach and frequency-domain hybrid modulation approach, first one as a solution to get all advantages of symbol-rate optimization (SRO) and deal with optical filtering, and second one to allow flexibility in the network. Analysis also consider the implementation of techniques that enhance system's performance as forward error correction and equalization.
When SCM and FDHMF are applied PR between each subcarrier must be allocated properly, the study proves that PR can be optimized according to the SNRs experienced by using Power Loading strategy as adaptive loading algorithms on each individual subcarrier.

Investigation also has allowed to observe an important behavior when equalization is implemented because as expected, the equalizer allowed to improve the performance and counteracts the impact of filtering, given results closer to theoretical ones. Especially it helps to improve signals' behave composed by lower number of subcarriers as 4 -subcarriers and single carrier that suffer high penalties against filtering. However an important aspect must be described, when 32 -subcarriers is used computing time can be very high, nevertheless in this thesis was demonstrated that simulated results were similar to theoretical ones even without equalization in this case, this allowed to make the analysis using theoretical values and decrease computational time and complexity in the code.

Furthermore, results in terms of FEC strategy reflect that against filtering derived from optical routing HD-FEC suffers less than SD-FEC, this due to the fact that using HD-FEC the spectrum is lower and less bits are penalized.

Finally, simulation process proved that FDHMF in conjunction with SCM give advantages against filtering in optical data transmission at net-bit rate of 200G because allow to allocate low-cardinality modulation formats at edges making the transmitted signal more robust. Indeed, dominant cases use a PM-QPSK format at edges. This proposal not just behaves better when filtering is applied also gives possibility of achieve a desirable transmission bit rate, increasing the flexibility by mixing low- and high cardinality modulation formats.

Besides, implementation of SCM is an alternative against single-carrier system, was proven that for increasing number of WSS in cascade, cases with 32 -subcarriers and hybrid formats were better solution also in terms of computational time because as set before, theoretical values can be used to estimate the behavior or the system. System using SCM approach can reach lower symbol-rates which is an optimal solution.

Recommendations to extend investigations in this field include moving this proposal to yield others net-bit rates and compare behaviors against filtering when they are increased. Furthermore study and compare others advanced formats as Probabilistic Shaping approach.

## APPENDIX

1. All cases of FDHMF for 16 -subcarriers.

|  | CONFIGURATIONS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case 1 | 4 | 4 | 4 | 4 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 4 | 4 | 4 | 4 |
| Case 2 | 4 | 4 | 4 | 8 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 8 | 4 | 4 | 4 |
| Case 3 | 4 | 4 | 4 | 16 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 16 | 4 | 4 | 4 |
| Case 4 | 4 | 4 | 4 | 16 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 16 | 4 | 4 | 4 |
| Case 5 | 4 | 4 | 4 | 32 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 32 | 4 | 4 | 4 |
| Case 6 | 4 | 4 | 8 | 8 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 8 | 8 | 4 | 4 |
| Case 7 | 4 | 4 | 8 | 8 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 8 | 8 | 4 | 4 |
| Case 8 | 4 | 4 | 8 | 16 | 16 | 32 | 64 | 64 | 64 | 64 | 32 | 16 | 16 | 8 | 4 | 4 |
| Case 9 | 4 | 4 | 8 | 16 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 16 | 8 | 4 | 4 |
| Case 10 | 4 | 4 | 8 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 8 | 4 | 4 |
| Case 11 | 4 | 4 | 16 | 16 | 16 | 16 | 64 | 64 | 64 | 64 | 16 | 16 | 16 | 16 | 4 | 4 |
| Case 12 | 4 | 4 | 16 | 16 | 16 | 32 | 32 | 64 | 64 | 32 | 32 | 16 | 16 | 16 | 4 | 4 |
| Case 13 | 4 | 4 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 4 | 4 |
| Case 14 | 4 | 8 | 8 | 8 | 8 | 64 | 64 | 64 | 64 | 64 | 64 | 8 | 8 | 8 | 8 | 4 |
| Case 15 | 4 | 8 | 8 | 8 | 16 | 32 | 64 | 64 | 64 | 64 | 32 | 16 | 8 | 8 | 8 | 4 |
| Case 16 | 4 | 8 | 8 | 8 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 8 | 8 | 8 | 4 |
| Case 17 | 4 | 8 | 8 | 16 | 16 | 16 | 64 | 64 | 64 | 64 | 16 | 16 | 16 | 8 | 8 | 4 |
| Case 18 | 4 | 8 | 8 | 16 | 16 | 32 | 32 | 64 | 64 | 32 | 32 | 16 | 16 | 8 | 8 | 4 |
| Case 19 | 4 | 8 | 8 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 8 | 8 | 4 |
| Case 20 | 4 | 8 | 16 | 16 | 16 | 16 | 32 | 64 | 64 | 32 | 16 | 16 | 16 | 16 | 8 | 4 |
| Case 21 | 4 | 8 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 8 | 4 |
| Case 22 | 4 | 16 | 16 | 16 | 16 | 16 | 16 | 64 | 64 | 16 | 16 | 16 | 16 | 16 | 16 | 4 |
| Case 23 | 4 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 4 |
| Case 24 | 8 | 8 | 8 | 8 | 8 | 32 | 64 | 64 | 64 | 64 | 32 | 8 | 8 | 8 | 8 | 8 |
| Case 25 | 8 | 8 | 8 | 8 | 16 | 16 | 64 | 64 | 64 | 64 | 16 | 16 | 8 | 8 | 8 | 8 |
| Case 26 | 8 | 8 | 8 | 8 | 16 | 32 | 32 | 64 | 64 | 32 | 32 | 16 | 8 | 8 | 8 | 8 |
| Case 27 | 8 | 8 | 8 | 8 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 8 | 8 | 8 | 8 |
| Case 28 | 8 | 8 | 8 | 16 | 16 | 16 | 32 | 64 | 64 | 32 | 16 | 16 | 16 | 8 | 8 | 8 |
| Case 29 | 8 | 8 | 8 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 8 | 8 | 8 |
| Case 30 | 8 | 8 | 16 | 16 | 16 | 16 | 16 | 64 | 64 | 16 | 16 | 16 | 16 | 16 | 8 | 8 |
| Case 31 | 8 | 8 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 8 | 8 |
| Case 32 | 8 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 8 |
| Case 33 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |

## 2. All cases of FDHMF for 32-subcarriers

| Cases | CONFIGURATIONS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| 2 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 8 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 8 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 16 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 16 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 16 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 16 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 6 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 7 | 4 | 4 | 4 | 4 | 4 | 4 | 8 | 8 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 8 | 8 | 4 | 4 | 4 | 4 | 4 | 4 |
| 8 | 4 | 4 | 4 | 4 | 4 | 4 | 8 | 8 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 8 | 8 | 4 | 4 | 4 | 4 | 4 | 4 |
| 9 | 4 | 4 | 4 | 4 | 4 | 4 | 8 | 16 | 16 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 16 | 16 | 8 | 4 | 4 | 4 | 4 | 4 | 4 |
| 10 | 4 | 4 | 4 | 4 | 4 | 4 | 8 | 16 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 16 | 8 | 4 | 4 | 4 | 4 | 4 | 4 |
| 11 | 4 | 4 | 4 | 4 | 4 | 4 | 8 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 8 | 4 |  |  | 4 | 4 | 4 |


| 12 | 4 | 4 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 16 | 16 | 16 | 4 | 4 | 4 | 4 | 4 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | 4 | 4 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 16 | 16 | 16 | 4 | 4 | 4 | 4 | 4 | 4 |
| 14 | 4 | 4 | 4 | 4 | 4 | 4 | 16 | 16 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 16 | 16 | 4 | 4 | 4 | 4 | 4 | 4 |
| 15 | 4 | 4 | 4 | 4 | 4 | 4 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 4 | 4 | 4 | 4 | 4 | 4 |
| 16 | 4 | 4 | 4 | 4 | 4 | 4 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 4 | 4 | 4 | 4 | 4 | 4 |
| 17 | 4 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 64 | 64 | 6 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 8 | 8 | 8 | 8 | 4 | 4 | 4 | 4 | 4 |
| 18 | 4 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 16 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 16 | 8 | 8 | 8 | 4 | 4 | 4 | 4 | 4 |
| 19 | 4 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 8 | 8 | 8 | 4 | 4 | 4 | 4 | 4 |
| 20 | 4 | 4 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 16 | 16 | 8 | 8 | 4 | 4 | 4 | 4 | 4 |
| 21 | 4 | 4 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 16 | 16 | 8 | 8 | 4 | 4 | 4 | 4 | 4 |
| 22 | 4 | 4 | 4 | 4 | 4 | 8 | 8 | 16 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 16 | 8 | 8 | 4 | 4 | 4 | 4 | 4 |
| 23 | 4 | 4 | 4 | 4 | 4 | 8 | 8 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 8 | 8 | 4 | 4 | 4 | 4 | 4 |
| 24 | 4 | 4 | 4 | 4 | 4 | 8 | 16 | 16 | 16 | 16 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 16 | 16 | 16 | 16 | 8 | 4 | 4 | 4 | 4 | 4 |
| 25 | 4 | 4 | 4 | 4 | 4 | 8 | 16 | 16 | 16 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 16 | 16 | 16 | 8 | 4 | 4 | 4 | 4 | 4 |
| 26 | 4 | 4 | 4 | 4 | 4 | 8 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 8 | 4 | 4 | 4 | 4 | 4 |
| 27 | 4 | 4 | 4 | 4 | 4 | 8 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 8 | 4 | 4 | 4 | 4 | 4 |
| 28 | 4 | 4 | 4 | 4 | 4 | 8 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 8 | 4 | 4 | 4 | 4 | 4 |
| 29 | 4 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 16 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 16 | 16 | 16 | 16 | 16 | 4 | 4 | 4 | 4 | 4 |
| 30 | 4 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 4 | 4 | 4 | 4 | 4 |
| 31 | 4 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 4 | 4 | 4 | 4 | 4 |
| 32 | 4 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 4 | 4 | 4 | 4 | 4 |
| 33 | 4 | 4 | 4 | 4 | 4 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 4 | 4 | 4 | 4 | 4 |
| 34 | 4 | 4 | 4 | 4 | 4 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 4 | 4 | 4 | 4 | 4 |
| 35 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 8 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 8 | 8 | 8 | 8 | 8 | 4 | 4 | 4 | 4 |
| 36 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 16 | 16 | 6 | 64 | 64 | 64 | 64 | 64 | 64 | 6 | 64 | 64 | 64 | 64 | 16 | 16 | 8 | 8 | 8 | 8 | 4 | 4 | 4 | 4 |
| 37 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 16 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 16 | 8 | 8 | 8 | 8 | 4 | 4 | 4 | 4 |
| 38 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 6 | 64 | 64 | 32 | 32 | 32 | 32 | 8 | 8 | 8 | 8 | 4 | 4 | 4 | 4 |
| 39 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 16 | 16 | 16 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 16 | 16 | 16 | 8 | 8 | 8 | 4 | 4 | 4 | 4 |
| 40 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 16 | 16 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 16 | 16 | 8 | 8 | 8 | 4 | 4 | 4 | 4 |
| 41 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 16 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 16 | 8 | 8 | 8 | 4 | 4 | 4 | 4 |
| 42 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 8 | 8 | 8 | 4 | 4 | 4 | 4 |
| 43 | 4 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 16 | 16 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 16 | 16 | 16 | 16 | 8 | 8 | 4 | 4 | 4 | 4 |
| 44 | 4 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 16 | 16 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 16 | 16 | 16 | 16 | 8 | 8 | 4 | 4 | 4 | 4 |
| 45 | 4 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 8 | 8 | 4 | 4 | 4 | 4 |
| 46 | 4 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 8 | 8 | 4 | 4 | 4 | 4 |
| 47 | 4 | 4 | 4 | 4 | 8 | 8 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 8 | 8 | 4 | 4 | 4 | 4 |
| 48 | 4 | 4 | 4 | 4 | 8 | 8 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 8 | 8 | 4 | 4 | 4 | 4 |
| 49 | 4 | 4 | 4 | 4 | 8 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 8 | 4 | 4 | 4 | 4 |
| 50 | 4 | 4 | 4 | 4 | 8 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 8 | 4 | 4 | 4 | 4 |
| 51 | 4 | 4 | 4 | 4 | 8 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 8 | 4 | 4 | 4 | 4 |
| 52 | 4 | 4 | 4 | 4 | 8 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 8 | 4 | 4 | 4 | 4 |
| 53 | 4 | 4 | 4 | 4 | 8 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 8 | 4 | 4 | 4 | 4 |
| 54 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 4 | 4 | 4 | 4 |
| 55 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 4 | 4 | 4 | 4 |
| 56 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 4 | 4 | 4 | 4 |
| 57 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 4 | 4 | 4 | 4 |
| 58 | 4 | 4 | 4 | 4 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 4 | 4 | 4 | 4 |
| 59 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 8 | 8 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 8 | 8 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |
| 60 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 8 | 8 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 8 | 8 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |


| 61 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 8 | 16 | 16 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 16 | 16 | 8 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |
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| 62 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 8 | 16 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 16 | 8 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |
| 63 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 8 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 8 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |
| 64 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 16 | 16 | 16 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 16 | 16 | 16 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |
| 65 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 16 | 16 | 16 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 16 | 16 | 16 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |
| 66 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 16 | 16 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 16 | 16 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |
| 67 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |
| 68 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 8 | 8 | 8 | 8 | 4 | 4 | 4 |
| 69 | 4 | 4 | 4 | 8 | 8 | 8 | 16 | 16 | 16 | 16 | 16 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 16 | 16 | 16 | 16 | 16 | 8 | 8 | 8 | 4 | 4 | 4 |
| 70 | 4 | 4 | 4 | 8 | 8 | 8 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 8 | 8 | 8 | 4 | 4 | 4 |
| 71 | 4 | 4 | 4 | 8 | 8 | 8 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 8 | 8 | 8 | 4 | 4 | 4 |
| 72 | 4 | 4 | 4 | 8 | 8 | 8 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 8 | 8 | 8 | 4 | 4 | 4 |
| 73 | 4 | 4 | 4 | 8 | 8 | 8 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 8 | 8 | 8 | 4 | 4 | 4 |
| 74 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 64 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 8 | 8 | 4 | 4 | 4 |
| 75 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 8 | 8 | 4 | 4 | 4 |
| 76 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 64 | 64 | 64 | 64 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 16 | 8 | 8 | 4 | 4 | 4 |
| 77 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 64 | 64 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 16 | 8 | 8 | 4 | 4 | 4 |
| 78 | 4 | 4 | 4 | 8 | 8 | 16 | 16 | 16 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 16 | 16 | 16 | 8 | 8 | 4 | 4 | 4 |
| 79 | 4 | 4 | 4 | 8 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 32 | 64 | 64 | 64 | 64 | 64 | 64 | 32 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 8 | 4 | 4 | 4 |
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[^0]:    ${ }^{1}$ All cases are presented in Appendix.

