Polytechnic University Of Turin
Department Of Mechanical And Aerospace Engineering

Master Degree Thesis In Automotive Engineering

Accurate GPS Positioning For Autonomous Vehicles

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Abstract:

Global Positioning System (Navigation System) receiver is an essential and indispensable part in every real-time localization and position technology, especially in the Autonomous vehicle. Within the GPS systems, the high-performance Differential Global Positioning System (DGPS) receiver with real-time kinematics can provide more accurate absolute localization for a driverless car.However, not only is it suspected to user equivalent range errors (UERE) such as Multi-path effect, but it is incapable of error correction to fulfill the precise requirement of autonomous vehicle positioning in a wide range of driving area. In order to provide and guarantee high-level of safety and confidence for the vehicles occupants which is of prime importance in the Autonomous Land Vehicle (ALV), accurate positioning information is a must.Therefore, in this dissertation, the basic concepts of the probability and statistics theories in addition to the random signals (Noise) which are a common type of error sources in the sensors data fusion are also described. Then, the working principles of Global Positing Systems with all suspected errors contained in its provided data will be elaborated.Furthermore, different types of data collection sensor and their principles, along with the theories of Kalman Filter and Extended Kalman Filter will be explained in more details. Having addressed the common problems in real-time position estimation of Autonomous Land Vehicle, the dynamics equation of the 2D vehicle model with assumption of traveling in constant velocity and turn rate will be derived. In fact, due to lack of reliability of low performance GPS signal (UERE) in some cases and measurement errors due to the presence of measurement noises, considering high nonlinearity of the system equations deriven from the vehicle dynamics, the simple combination of Gps/Imu may not lead to a satisfactory result.Therefore, the problem of GPS systems with introduction to the GPS and Inertial Measurement Unit (GPS+IMU) using data fusion technique is described through the theory of Extended Kalman Filter in which the GPS and IMU data coming from real tests on a land vehicle in real-time.Subsequently, the performance of this approach is evaluated by comparison of the results (estimated variables) with the unprocessed real-time measured data. In the last case study, a further step is taken by presenting an overview on Robot Operating System (ROS) and different possible fusion techniques available in this realm.Finally, a more comprehensive case of the fusion technique with a fusion of data from three sensors (IMU+GPS+Odometry) in association with ROS extended Kalman Filter package ($Robot\_localization\_Ekf$) with a discussion on the results will be demonstrated.

Keywords :GPS,navigation,DGPS,Localization,Autonomous,Fusion,differential,Multipath,Extended Kalman Filter,sensor,IMU,Odometry,UERE,ALV,ROS ,turn rate.

Chapter 1

Introduction and Literature review

1.1 Navigation

Navigation for thousands of years has been presenting in some forms. Almost every being in nature must be able to navigate from one point in space to another such as birds, the bees, and ants [1].For people, navigation had originally included using the sun and stars which means the sun and star were the only references to their navigation. The methods and technologies for determining the time-varying position and attitude of a moving object by the measurement called Navigation. The time variable function of the Position, velocity, and attitude, are called navigation states because they include all navigation information that is essential for georeference the moving object at that moment of time.

Navigation system

A navigation system is a system or device (usually electronic) whose aim is to assists in navigation. Navigation system in a vehicle or vessel may be fully on-boarded and in the ship they installed on ships or it may be located elsewhere to be able to receive strongest signals ,it usually communicates via radio or other types of electro-magnetic signals with a vehicle or vessel, or it might use a combination of these methods[2]. Navigation systems may be adequate of:

- With information from different sources via sensors, maps, or external sources determine a vehicle or vessel's location
- Containing maps, this could be displayed in text or in a graphical format which is readable for the human.
- Providing suggested directions by means of text or voice to a human in charge of a vehicle.
- Directly able to provide the information to an autonomous vehicle such as a robotic probe or guided missile.
- Providing emergency information to the user, such as the presence of on nearby vehicles or vessels, or other hazards or obstacles.
- Providing information for traffic management and depends on traffic conditions could suggest an alternative direction or directions

Types of the Navigation system

Due to the abundant and significant application of navigation system, currently different types of this system are available which could be listed as the following:

- Automotive navigation system:This type of navigation system designed for Automotive application
- Marine navigation system: This type of navigation system as its name implies is designed for utilizing in the boat, ship, and different maritime application :
- Global Positioning System: A group of satellites placed on the earth orbit which are able to provide information to any receiver that is compatible to this technology such as the person, vessel, or vehicle via a GPS receiver
- GPS navigation device: a device which is able to receive GPS signals and its purpose is the device's location determining and is possible suggests or gives directions.
- Surgical navigation system: A system that in relation to patient images such as CT or MRI scans determines the position of surgical instruments.
- Inertial guidance system: a system without the need for external reference is able to continuously determine the position, orientation, and velocity (direction and speed of movement) of a mobile object.
- Robotic mapping: The problem of learning maps is an important problem in mobile robotics. Models of the environment are needed for a series of applications such as transportation, cleaning, rescue, and various other service robotic tasks.the methods and equipment by which an autonomous robot is able to construct (or use) a map or floor plan in order to localize itself within it.

1.1.1 Automotive Positioning:

The Automotive positioning is determining the position of the vehicle on the road or on moving area by means of some techniques or technologies.So that here in this section we are going to briey express its importance and some previous research which have been carried out :

Introduction:

To introductory Automotive positioning being expressed, we are going to state an overview of fusion techniques which has been applied to land vehicle's navigation systems for determining its position on the road. A topic which especially in these last few years has become a very important research area is the Automotive Positioning and localization.Generally speaking, the prime importance of safety concern is accurate positioning information in road transportation. Actually, to fulfill this important it is necessary and required to know where we are (vehicle position) and also it is very important and required to locate obstacles and other objects/vehicles in the vicinity and adjacent environment of our own vehicle.So for increasing safety, it is required to an approach to precisely localize our vehicle and evaluate its environment on the road[3]. Apart from safety concerns, the next generation of vehicles will allow the driver and passengers to have access to a diverse range of services, which are based on information technologies, telecommunications, and telematics such as path planning, navigation, guidance, and tracking. Next-generation vehicles probably are like mobile offices, information centers which are constructed on the wheel, or e-nodes which are connected to the web and other networks [4]. To supply those function and services and to provide the suitable and valuable information contents to the driver and passengers, it is mandatory and necessary to be determined the vehicles position as accurately and efficiently as possible in real-time. This function is an essential part of an integrated navigation information system (INIS). An INIS embedded in a vehicle is basically composed of a geographic information system (GIS), a database combined of roadmaps, cartographic and geo-referenced data, a positioning module, a human-machine interface (HMI) as well as computing and telecommunication capabilities. It delivers helpful functionalities to the driver such as path planning, guidance, digital map and points of interest directory [5]. The guidance module uses a trip which planned by the driver to indicate the driver which route to take. To avoid giving wrong suggestion and damaging the driving safety, the navigation system relies on a positioning module to understand and recognize precisely and continuously the localization of the vehicle. In this chapter, we will focus on the positioning module of INISs.

Autonomous positioning:

Autonomous navigation is one of the most key and essential technologies for autonomous driving and driverless cars. Accurate and precise estimate positioning and orientation of vehicles is commonly considered as the basis for the number of sophisticated and complex planes such as environmental perception, path planning, and autonomous decision-making of driverless cars under sophisticated and complicate urban scenarios. In this system, a navigation sensor such as Global Positioning System (GPS) measures quantity related to one or more elements of the navigation state. A navigation system can be built up with the combination of all required sensors which are able to determine all navigation states such as Inertial Navigation System (INS).A sensor that only is able to supplies a partial information on the navigation states or that is used as a constraint on some of the states will be card Navaid (such as odometers). [6].Inertial navigation is a system which is able to determine the position of a vehicle within the implementation of inertial sensors. It works based on the principle that an object will remain in smooth motion except it is disturbed by an external force. This is the force which in turn generates acceleration on the object. If this acceleration can be measured and then be integrated mathematically, consequently the velocity and position change of the object with respect to an initial condition can be determined. A sensor such as an accelerometer is an inertial sensor which is able to measure the acceleration. In addition to the acceleration sensor for measurement of the attitude of the vehicle, an inertial sensor which is called a gyroscope is required.This sensor (gyroscope) measures angular velocity and if this data is integrated mathematically can provide the change in angle with respect to an initial known angle. The combination of the accelerometers and gyros sensor allows the determination of the pose of the vehicle. If it is required to be measured the full vehicle's behavior an inertial navigation system usually composed of three accelerometers are required which they are commonly mounted with their sensitive axes perpendicular to one another. The accelerometer working theory is based on Newtons laws. So navigation is important with respect to inertial reference frame so it is necessary to keep track the direction of which the accelerometers are pointing out.so far we expressed the liner directional motion for navigation so for measuring rotational motion of the body with respect to the inertial reference frame being used gyroscopic sensors and used to determine all the time the orientation of the accelerometers or in another word the vehicle bodys orientation. Then with this information as an input from inertial navigation sensors, it makes possible the accelerations be transformed into the desired frame before the integration process takes place. At each time step of the system's clock, this quantity timely integrated by the computer navigation to get the body's velocity vector. In order to evaluate or determine the position, the velocity vector is then time integrated. Hence, inertial navigation is the operation by which the information delivered by gyroscopes and accelerometers are used to determine the position of the vehicle where they are installed. Consequently, these two sets of measurements make feasible determining the translational motion of the vehicle and consequently to calculate its position within the inertial reference frame.

1.2 Literatures review

In this part, we have tried to present the most significant researches have been done by some researchers since start until the recent years .
Unlike from stand-alone GPS that is extensively a popular navigation system; an enhanced differential GPS (DGPS) receiver with phase carrier signal measurements may run in operating modes of real-time kinematics (RTKDGPS), which has the highest absolute position accuracy of up to a few centimeters. In DGPS, GPS device which is mobile, continuously receives correction data from ground-based reference station over transmitter of shorter range, in order to compensate location inaccuracies [7].
DGPS systems are able to operate under complex urban scenarios, however, occasionally loss of broadcasted signals and probably receiving of inaccurate localization data due to many unpredictable factors such as buildings obstruction, signal attenuation, and a diversity of electronic interference. In general, it works quite well in a limited range in terms of pseudo range correction principle. In addition, atmospheric visibility of satellite, potential environmental effects, and multipath effect may have the negative effect on precision and reliability of GPS itself [8]. For improving the accuracy of the GPS estimation two widely used multipath mitigation methods, ,high-resolution correlator (HRC) and multipath mitigation technique (MMT), and a new coupled amplitude delay lock loops (CADLL) method, which is based on multipath signal amplitude, code phase, and carrier phase, are evaluated by Chen,Dovis and colleagues[9].these methods and techniques under dynamic multipath scenario or when multipath is stronger than line-of-sight (LOS) may be failed.Except for GPS which is independent of vehicle dynamics, the DR (Dead Recognition) that employs vehicle kinematic model and incremental measurements of wheel encoder often seen to play a very important and crucial role in the precise short-term navigation of driverless cars [10]. The DR technique as one of the autonomous relative navigations is capable of continually providing position information.However, a significant disadvantage of using DR for navigation is that it typically endure from accumulated error which caused by the wheel slippage and wheel imperfection [11]. It means that actually, the data which provided by DR, the localization accuracy can maintain only within a very short range. In order to be improved long-term precision and robustness through slip estimation, substantial efforts have been made [12]. A number of supplementary navigation systems, including GPS, IMU, and DR, are commonly combined through a variety of information fusion methods, typically such as Kalman lter (KF) [13].
In fact, GPS or GPS+IMU is able to provide absolute position and orientation, even if it contains discontinuous data and/or random drifts. Contrarily, as a local navigation system, DR is able to conduct accurate localization within a certain distance or duration. However, position errors will be accumulated with the increase of distance. Undoubtedly, an appropriate sensors integration can be presented by integration of GPS+IMU and DR which are able to accurately navigate the driverless car. In the recent decades, many multimodal data fusion methods for meeting reliable, robust, and decimeter-level requirements for driverless cars, the extended Kalman lter (EKF) and the unscented Kalman lter (UKF), has emerged. The EKF simplies nonlinear ltering and is used for state estimation in nonlinear systems [14-23]. Kalman filters are employed extensively for sensor fusion and can be a suitable option for linear systems. These Kalman filters by combining the data from a variety of different sensors can produce estimates of the states of a system. The result of state estimation, in this case, maybe more accurate than those that would be produced without sensor fusion. To develop the Kalman filter for integration of data from Global Positioning System (GPS) and Inertial Measurement Unit (IMU) sensors a theory suitably established by [30-31]. Meanwhile, one of the significant challenges to produce a reliable Kalman filter is tuning of its parameters adequately [32].For tuning of the parameters the trial and error is used and found out that which can result in satisfactory Kalman filter performance in some applications when educated guesses are used; however, this approach is not very suitable and is a time-consuming and unreliable approach[33].

A technique which is more common and by which dependable parameters will be produced is an adaptive Kalman filtering and offline stochastic metaheuristic searches. Parameters will be tuned online by Adaptive Kalman filters and by Using this technique, parameters are rectified according to how accurately the internal equations of the filter can predict the measurements of the system [34]. Fuzzy adaptive Kalman filters have been found in order to perform appropriately in experiments [35-36]. In order to be compensated the EKF based navigation system several types of additional sources of information, including onboard motion sensors, cameras or LiDAR vision systems, and road map databases, are adopted. References [14, 17] present the improvement of accuracy of localization by integration of different navigation systems such as IMU, GPS, and DR. Ma et al. [18] combination of stereo-camera sensor, IMU, and leg odometry by virtue of EKF has been investigated in this research. Also in the [19,20] improve location accuracy by integration of both accurate digital map and camera has been performed. Moreover, four EKF-based state estimation architectures are evaluated in [21], including nonlinear model (NLM) [22] and error model (ERM) [23], each with/without a supplementary lter [24,25]. The experimental results show that NLM with a supplementary lter has higher localization performance. Unlike the EKF, the UKF employs unscented transform to address approximation issues of the EKF, which is also extensively exploited in multimodal data fusions [26,28].
Actually, there still exist some problems even if the above two kinds of methods have been widely applied. The deciencies of the KFs including EKF and UKF were specically pointed out in[29]. For example, considering that there are uncertainties or unknown statistical characteristics for process and/or measurement noises, it is very hard to perform reliable multimodal data fusion. Hence, the above-mentioned fusion methods are not sufcient to establish robust and accurate state estimation. As we briefly have reported the previous tasks and researches which have carried out by serval researchers, we are going to state our investigation in this dissertation by means of multi-data fusion with EKF and Robot-localization-EKF in Robot Operating System. If briefly the overview of this thesis going to be stated is as follows:
In the First chapter, the probabilities and basic concepts along to random signals would be demonstrated, In the second chapter, we will explain step by step, the Kalman Filter, the Extended Kalman Filter method and the theories behind of them for dealing with unknown noisy data.In addition to them in the Third Chapter will be familiarized with Navigation Sytems, especially the Global Positioning System which is the main data source for our case of study, Autonomous vehicle positioning.
In the chapter-fourth, we will demonstrate the other types of data collection sensors, which are odometry and IMU sensor.In addition, different principles, technologies behind of these sensors, and their pros and cons will be discussed.
In chapter-fifth which is the main chapter of our investigation, the derivation of the 2D vehicle dynamics equations in constant velocity and turn rate, and simulation by Kalman filter in detail will be demonstrated.Moreover the accuracy of result with GPS data, IMU data will be compared and discussed.
In chapter-sixth, we will have a general overview of ROS and its operation principle and finally, data fusion along with results of simulation in this excellent environment will be demonstrated.

Chapter 2

Probability and Basics Concepts

2.1 Introduction

In this section is going to be stated, one the most important theory which is called probability.We will later use this theory and its concepts to accomplish the Kalman filter and the Extended Kalman filter. Actually, without knowledge of probabilities, it is impossible accomplishment and simulation of vehicle position estimation with Kalman theory.

2.2 Probability

Probability is something directly related to the random occurrence of an event in sample space so we use this keyword to assign the how likely will a random event accrues and so we assign the measure of probability $P(A)$ to an event which could be between 0 and 1.The $P(A)$ closer to 0 means less probable this event will occur and in other hands, $P(A)$ closer to 1 means that the more likely this event will occur. The main aim of probability theory is to develop tools and techniques to calculate probabilities of different events. So the probability could be stated as follows:

$$P(A) = \frac{possiblie\ outcome\ of\ anevent\ (A)}{Total\ possible\ events} \tag{2.2.1}$$

For example, consider a flipped coin, the probability that outcome is head or tail is equal P(Head=Tail)=$\frac{1}{2}$ if the probability of the events are independent or disjoint, which means that the outcomes are independent of other, then outcome probability of either A or B can be stated as follows:

$$P(A \cup B) = P(AorB) = P(A) + P(B) \tag{2.2.2}$$

If the probability of two outcomes are independent(disjoint) which means the occurrence one not affects the other ones, then the probability of both occurrence is the product of their individual probabilities:

$$P(A \cap B) = P(AandB) = P(A, B) = P(A).P(B|A) \tag{2.2.3}$$

Again we consider the coin example but in this case, we flipped two coins at the same time, in this case, the probability that outcome is Head is $1/2 * 1/2 = 1/4$ and the probability that head or Tail is $(1/2 + 1/2) = 1$ And finally going to define the *conditionalprobability* which is very important in our survey and can define the probability of an event outcome A when the occurrence and outcome B is given and can be expressed as follows;

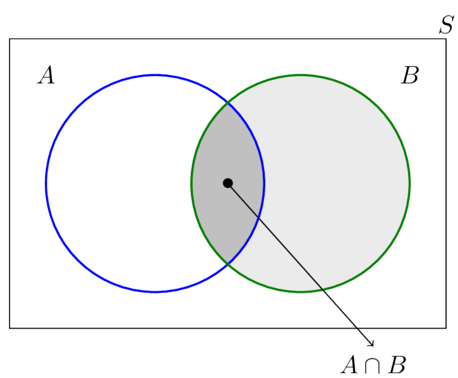
$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{2.2.4}$$

let's give an example for clarification of conditional probability; if we roll a die.Let A be the event that the outcome is an odd number,$A = \{1, 3, 5\}$ and also let $B = \{1, 2, 3\}$ what is the probability of A,$P(A)$? And what is the probability of A given B , $P(A|B)$? Solution: we know that have 6 numbers on the die so the probability of A can be

$$P(A) = \frac{|\ A\ |}{|\ S\ |} = \frac{|\ \{1, 3, 5\}\ |}{6} = \frac{1}{2}$$

Now let's find the conditional probability of A respecting that the B accrued.If we know that the B accrued which must be within $\{1, 2, 3\}$.For A to also happen the outcome must be in $A \cap B = \{1, 3\}$.since all die roll are equally likely then the $P(A|B)$ must be equal to :

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$$



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Figure 2.1: Conditional probability in S space

2.3 Random Variables

In the probability and statistic, a random variable can be stated as a variable whose possible values are a numerical consequence of a random phenomenon.There are two types of random variables, discrete and continuous.

2.3.1 Discret Random Variable:

Definition :A random variable X can be a discrete random variable if:
there is the possibility of the finite number of outcomes of X or there are a countably infinite number of the possible outcomes of X.For example, a rolling of the fair die which has 6 numbers and possible outcomes can be $\{1, 2, 3, 4, 5, 6\}$

2.3.2 Continuouse Random Variable:

Definition: Continuous random variables X is the random variable, whose support S contains an infinite interval of possible outcomes. For example, if we assume that X indicates the height (in meters) of a randomly selected maple tree, then, in this case, the X will be a continuous random variable.

2.4 Expected Value

In the theory of probabilities, the expected value of a random variable, directly, is the long-time running, the average value of repetitions of the experiment which it represents. For instance, the expected value for the rolling of a six-sided dice is 3.5, because the average of all the outcome numbers in an extremely large number of outcome is close to 3.5.the law of large numbers expresses that the mean values almost certainly converge to the expected value as the number of repetitions approaches to infinity. The expected value is known also as the expectation, mathematical expectation, average, mean value or first moment. More practically, for a discrete random variable, the expected value is the averagely weighted probability of all values which are possible. In other words, each possible value which can be assumed for a random variable is multiplied by its probability of occurrence, and the produced result is summed together to produce the expected value.The same principle can be applied to a continuous random variable, except that the integral of the variable with respect to its probability density function, rather than the sum is used.Also, these principles work for distributions which are not discrete or continuous; the value of expectation a random variable is given by the integration of the random variable respect to its probability measure.

2.5 Univariate discrete random variable, finite case

Assume that the random variable X can take value $x1$ and value $x2$ with probability p_1 and probability p_2 respectively , and so on, up to the value x_k with probability p_k. Then for this random variable X the expectation is defined as:

$$E[X] = x_1p_1 + x_2p_2 + x_3p_3 + ... + x_kp_k. \tag{2.5.1}$$

Since summation of all probabilities become equal to one (p_1 +p_2 + ... +p_k= 1), the expected value can be considered as the weighted average, with p_is being the weights:

$$E[X] = \frac{x_1p_1 + x_2p_2 + x_3p_3 + ... + x_kp_k}{1} = \frac{x_1p_1 + x_2p_2 + x_3p_3 + ... + x_kp_k}{p_1 + p_2 + ... + p_k}. \tag{2.5.2}$$

2.6 Univariate continuous random variable

If the probability distribution of X has a probability density function $f(x)$, then the expected value can be computed as:

$$E(X) = \int_{-\infty}^{+\infty} xf(x)\,dx \tag{2.6.1}$$

2.7 Probability Density Function

As already we've stated the discrete random variable, the value of which this variable could take are finite but for the continuous random variable the situation is different and we deal with infinite value.so that is needed a function to present all its possible values. In theory of probabilities and stochastic, a probability density function (PDF),or a continuous random variable density,is a function, whose value at each given point (or sample) in the sample space (the group of values which are the random variable can take) can be stated as presenting a relative likelihood that the random variable value would be equal in that sample or in other words,(whereas numerous set of possible values exist to start with) for a continuous random variable the absolute likelihood to obtain any particular value is 0.The PDF value at two different samples can be used for guessing that, in any particular draw of the random variable, how much more probable it is that the random variable would equal one sample compared to the other sample. In a more accurate significant, the PDF is used to determine the probability of the random variable falling within a particular interval or range of values, as opposed to taking on any one value. then the integral of this variables PDF over that range gives the probability of that sample and it is given by the area under the density function in the interval between the lowest and greatest values. The probability density function is positive everywhere, and its integral over the entire space is equal to one.

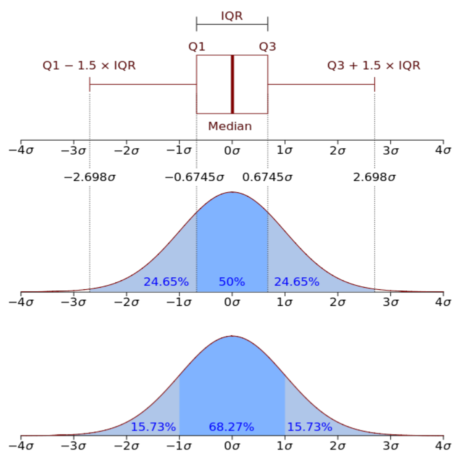


Figure 2.2: probability density function of a normal distribution $N(0, \sigma^2)$.

2.8 Signal:

In signal processing, the signal can be divided into two categories:

2.8.1 Deterministic Signal

Deterministic signals are a type of signal, whose values are completely known or specified at any given time. Thus, a deterministic signal can be modeled by a specified function of time.For instance, a signal which is harmonic and can be modeled with harmonic or harmonics function.

2.8.2 Random Signal

A random signal is a type of signal which also called non-deterministic signal is a kind of signal that gets random values at any given time and must be specified and characterized statistically.It means that this kind of signal could not be presented with a specified function.

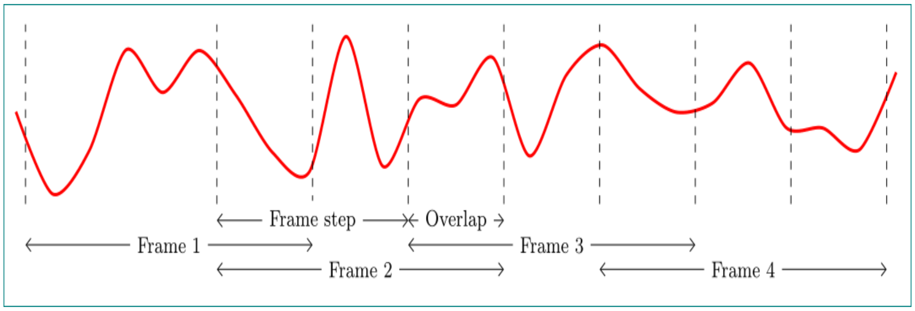


Figure 2.3: Random signal

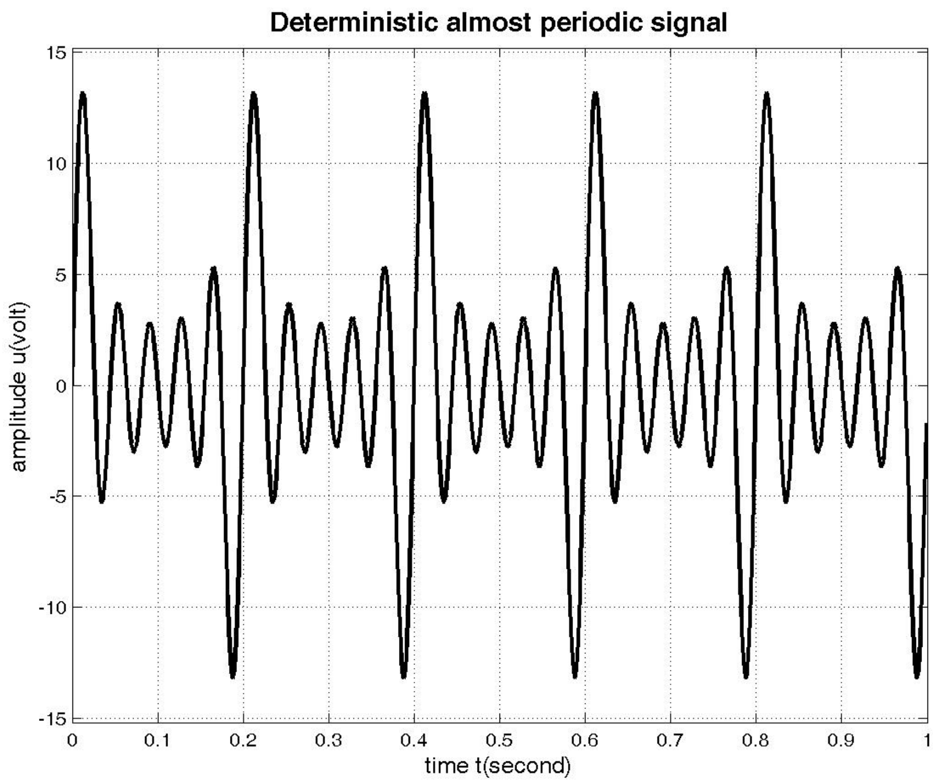


Figure 2.4: Deterministic signal

2.9 Guassin Noise:

Gaussian noise, which is also known as the Gaussian distribution is a type of statistical noise which has a probability density function (PDF) similar to the normal distribution.[1][2] In other words, the values that this type of noise can get are distributed Gaussian. The probability density function p of a Gaussian random variable z is given by:

$$p(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{z-\mu}{2\sigma^2}} \tag{2.9.1}$$

where z represents the grey level, μ is the mean value and σ is the standard deviation.[3] A special case of Gaussian noise is white Gaussian noise, in which the values at any pair of times are identically distributed and statistically independent (and hence uncorrelated).

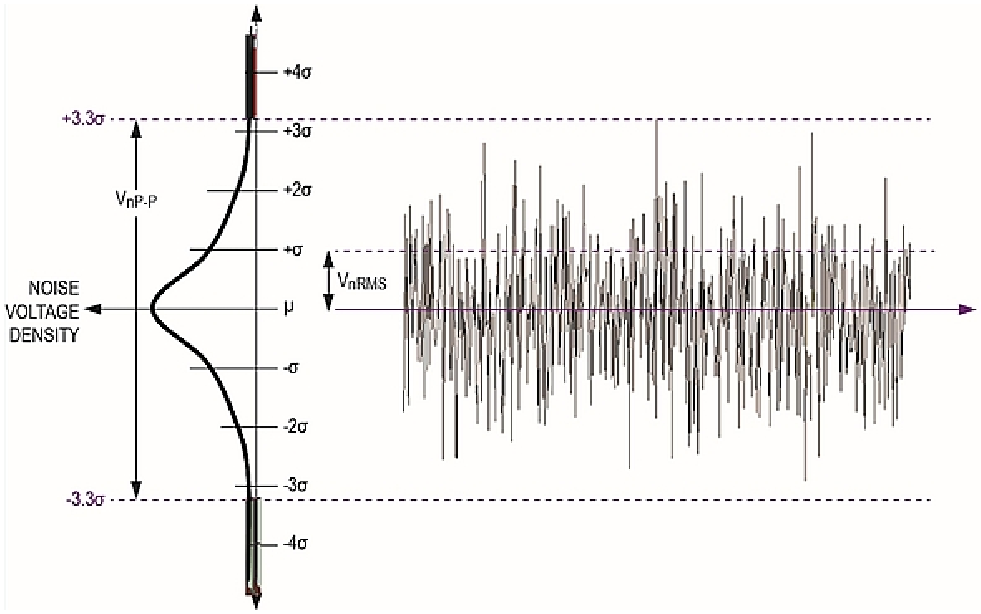


Figure 2.5: Guasian noise distribution.

2.10 White noise :

In signal processing, white noise is a kind of random signal which has uniform power at different frequencies, then consequently it has a constant power spectral density[1].This or similar meaning is being used in different scientific and technical application, including physics, acoustic engineering, telecommunications, statistical forecasting, and so on. White noise is a statistical model of signals and signal sources, rather than to any specific signal.

2.11 Variance:

In theory of probabilities and statistics, the expectation of the squared deviation of a random variable from its mean called variance and it determines roughly how far a group or set of numbers (random numbers) are distributed out from their mean. The variance has an important role in statistics. As the statistic can be applied to the different science of area and in the different numerical methods, so this importance makes it a central value or quantity in numerous fields such as physics, biology, chemistry, cryptography, economics, and finance.The variance has the different definition and can be stated as a square of the standard deviation, central moment of a distribution, and the random variable covariance with itself, and often it is stated by σ^2, Var(X).The covariance of the random variable X is the squared deviation the expected value from the mean of X,μ=$E[X]$:

$$Var(X) = E[(X - \mu)^2] \tag{2.11.1}$$

This definition belongs to random variables that are generated by processes which are discrete, continuous, neither, or combination of these two. The variance can also be considered of as the covariance of a random variable with itself as is expressed below;

$$Var(X) = Cov(X, X) \tag{2.11.2}$$

The expression for the variance can be expanded:

$$\begin{aligned} Var(X) &= E[(X - E[X])^2] \\ &= E[X^2 - 2XE[X] + E[X]^2] \\ &= E[X]^2 - 2E[X]E[X] + E[X]^2 \\ &= E[X^2] - E[X]^2 \end{aligned} \tag{2.11.3}$$

2.12 Continuous random variable

If we consider random variable X that states samples which generated by a continuous distribution that has probability density function $f(\ x\ )$, then the population variance is given by:

$$Var(X) = \sigma^2 = \int (x-\mu)^2 f(x)\,dx \ = \int x^2 f(x)\,dx \ - 2\mu \int x f(x)\,dx \ + \int \mu^2 f(x)\,dx \ = \int x^2 f(x)\,dx \ - \mu^2 \tag{2.12.1}$$

2.13 Discrete random variable

If the generator of random variable X is discrete with probability mass function $x_1{\mapsto}p_1 x_2{\mapsto}p_2,...,x_n{\mapsto}p_n$ then:

$$Var(X) = \sum_{i=1}^{n} p_i.(x_i - \mu)^2 \tag{2.13.1}$$

or equivalently

$$Var(X) = \sum_{i=1}^{n} p_i x_i^2 - \mu^2 \tag{2.13.2}$$

where μ is the average value

$$\mu = \sum_{i=1}^{n} p_i.x_i \tag{2.13.3}$$

2.14 Covariance:

In the theory of probabilities and statistics, the value of the dependency variability of two random variables is called covariance which means that can be found a relation between the variation of the variables.[1] If in the one variable the greater values of that variable correspond with the greater values of in the other variable, and the same retain for the lesser values, these two variables will show similar behavior, and the covariance is positive.[2] While in the reverse case, when the bigger values of one variable intensively correspond to the smaller values of the other then the variables will show opposite behavior and in this case the covariance is negative.The covariance's sign, therefore, displays the propensity or trend in the linear relationship between the variables. The size or measure of the covariance is not simple to interpret and be perceived.However, the correlation coecient and normalized or dimensionless version of the covariance shows the strength of the linear relation by its magnitude and measure.The covariance between two jointly distributed(probability distributions of two or more random variables) real-valued random variables$X$ and$Y$ with finite second moments is defined as the following relationship expressed as the expectation of the product of their deviations from their individual expected values:

$$Cov(X,Y) = E[(X - E[X])(Y - E[Y])] \tag{2.14.1}$$

Where $E[X]$ is the expected value of variable X, also known as the mean value of variable X.The covariance is also sometimes expressed "σ", as an indication of variance. By using the expectations' linearity property of covariance, this can be simplified to the expectation value of their product minus the product of their expected values:

$$\begin{aligned} Cov(X,Y) &= E[(X - E[X])(Y - E[Y])] \\ &= E[XY - XE[X] - E[X]Y + E[X]E[Y]] \\ &= E[XY] - E[X]E[Y] - E[X]E[Y] + E[X]E[Y] \\ &= E[XY] - E[X]E[Y]. \end{aligned} \tag{2.14.2}$$

For random vectors $X \in R^n$ and $Y \in R^n$,the $m \times n$ cross covariance matrix (also known as dispersion matrix or variancecovariance matrix,[5] or simply called covariance matrix) is equal to :

$$\begin{aligned} Cov(X,Y) &= E[(X - E[X])(Y - E[Y])] \\ &= E[XY^T] - E[X]E[Y]^T \end{aligned} \tag{2.14.3}$$

The covariance which can be stated $Cov(X_i, Y_j)$ is represented by the$(i,j)th$ member in this matrix (covaraince matrix)which is the covarivance between the $(i)th$ scalar component of X and the $(j)th$ scalar component of Y .Actually the $Cov(Y,X)$ is The transpose of $Cov(X,Y)$. For a vector X=$[X_1, X_2, ..., X_m]^T$ which has m jointly distributed random variables with nite second moment,its covaraince matrix is dened as :

$$\sum(X) = Cov(X,X) \tag{2.14.4}$$

If the Random variables whose covariance become zero are called uncorrelated and also for random vector can have similar definition the vector whose covariance matrix is zero in every entry outside the main diagonal are called uncorrelated.

2.15 Discrete variables

If every variable has a finite set of equal-probability values,x_i and y_i respectively for i=$1, ..., n$ and j=$1, ..., k$,then equivalently can be written the covariance of these variables in terms of means $E(X)$and$E(Y)$ as:

$$Cov(X,Y) = \frac{1}{nk}\sum_{i=1}^{n}\sum_{j=1}^{k}(x_i - E(X))(y_j - E(Y)) \tag{2.15.1}$$

2.16 Covariance Matrix

In theory of probabilities and statistics, a matrix of covariance of random vector (also called as a variance-covariance matrix) is a matrix that the covariance between the i_{th} and j_{th} elements of a random vector is the i,j positioned element in this matrix. A random variable with multiple dimensions is also called a random vector and every element of the random vector is a scalar random variable in which each of this element has either a nite or innite number of potential values or nite number of observed experimental values.The potential values are determined by a theoretical joint probability distribution. practically, the covariance matrix generalizes the concept of variance for multiple dimensions. As an example, the variation in a collection of random points in two-dimensional space cannot be characterized fully by a single number, and also would not have the single number contains all of the necessary information of variances in the x and y directions,then a 2*2 matrix would be necessary to the two-dimensional variation be fully characterized. each element on the principal diagonal of the covariance matrix is the variance of one of the random variables Because the covariance of the ith random variable with itself is simply that random variables variance.Take into account that every covariance matrix is symmetric because the covariance of the ith random variable with the jth one is the same thing as the covariance of the jth random variable with the ith one and In addition, every covariance matrix is positive semi-denite.

2.17 Definition

Consider that X and Y are the random vectors which are used, and X_i and Y_i being used to refer to random scalars if entries, it means elements in the column vector such as the following presentation:

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \tag{2.17.1}$$

are random variables, each of them with nite variance, then the covariance matrix $\sum$ is the matrix that whose (i,j) components are the covariance and is expressed as the following :

$$\sum\nolimits_{ij} = cov(X_i, X_j) = E[(X_i - \mu_i)(X_j - \mu_j)] = E[(X_i X_j] - \mu_i \mu_j \tag{2.17.2}$$

where $\mu_i = E(X_i)$ is the expected value of the $i\ th$ entry in the vector$X$.In another words:

$$\sum\nolimits_{ij} = \begin{bmatrix} E[(X_1 - \mu_1)(X_1 - \mu_1)] & E[(X_1 - \mu_1)(X_2 - \mu_2)] & \cdots & E[(X_1 - \mu_1)(X_n - \mu_n)] \\ E[(X_2 - \mu_2)(X_1 - \mu_1)] & E[(X_2 - \mu_2)(X_2 - \mu_2)] & \cdots & E[(X_2 - \mu_2)(X_n - \mu_n)] \\ \vdots & \vdots & \ddots & \vdots \\ E[(X_n - \mu_n)(X_1 - \mu_1)] & E[(X_n - \mu_n)(X_2 - \mu_2)] & \cdots & E[(X_n - \mu_n)(X_n - \mu_n)] \end{bmatrix} \tag{2.17.3}$$

the $\sum^{-1}$ represents The inverse of the above-expressed matrix and if it exists, it will represent the inverse covariance matrix, also known as the concentration matrix of the precision matrix which precision is inversely proportional to variance.the less variance the higher the precision and vice-versa. The definition above is equivalent and correspond to the matrix equality.

$$\Sigma = E\left[(X - E[X])(X - E[X])^T\right] \tag{2.17.4}$$

2.18 Properties

The special case of the covariance is variance in which the two variables are identical (the case that in which one variable always takes the same value like the other variable):

$$Cov(X, X) = Var(X) = \sigma^2(X) = \sigma_X^2 \tag{2.18.1}$$

2.19 A more general identity of covariance matrices

Here we are going to state some property of covariance matrix. Let's assume that have a random vector X which its covariance matrix is $\sum(X)$, and also assume we have a A which is a matrix that can act on this variable X.Consequently, the covariance matrix of the matrix AX is:

$$\sum(AX) = E[AXX^TA^T] - E[AX]E[X^TA^T] = A\sum(X)A^T \tag{2.19.1}$$

2.20 Block Matrices

The joint mean $\mu_{X,Y}$ and joint covariance matrix $\Sigma_{X,Y}$ of X and Y can be written in block form:

$$\mu_{X,Y} = \begin{bmatrix} \mu_X \\ \mu_Y \end{bmatrix}, \Sigma_{X,Y} = \begin{bmatrix} \Sigma_{XX} & \Sigma_{XY} \\ \Sigma_{YX} & \Sigma_{YY} \end{bmatrix} \tag{2.20.1}$$

Where Σ_{XX}=$Var(X)$, Σ_{YY}=$Var(Y)$,and Σ_{XY}= Σ_{XY}^T=$cov(X,Y)$. Σ_{XX} and Σ_{YY} can be identified as the variance matrices of the marginal distributions for X and Y respectively.If X and Y are jointly normal distributed:

$$x, y \sim N(\mu_{X,Y}, \Sigma_{X,Y}). \tag{2.20.2}$$

2.21 Mean Square Error:

The mean square error is the value which tells you how much your estimated value is close to true value which must be estimated and it is an essential tool in probability and statistics.
In the probabilities this value the mean squared error (MSE) or also called mean squared deviation (MSD) of an estimator which is used in estimating or predicting of unobserved quantities such as we encounter in identification system for observer design, determines the average of the squared of the errors(error, in this case, means the difference between estimated and the expected) or deviations which addresses or defines the difference between the estimator and what is estimated. Also, the mean squared error (MSE) is a risk function because it expresses the squared error loss of estimator which not considered some information or due to the randomness of information. The MSE value also expresses the measure of the quality of an estimator if this value closer to zero are better.Later this concept will be more cleared.

2.22 Definition and basic properties

The MSE evaluates the quality and accuracy of an estimator (it is expressed as a mathematical function which maps sample of data to a parameter of the population from which the data is sampled) or a predictor (it is expressed as a function which maps arbitrary inputs to a sample of values of some random variable).The MSE definition according to whether one is describing an estimator or a predictor could differ.

2.23 Predictor

If $\hat{Y}$ is a vector which obtained by the n predictions of predictor and Y is the vector of observed values corresponding to the inputs as must be exactly applied as an input to predictor function, then the MSE of the predictor can be estimated by

$$\boldsymbol{MSE} = \frac{1}{n}\sum_{i=1}(\hat{Y_i} - Y_i)^2) \tag{2.23.1}$$

as the above expression stated the MSE value, in this case, is the quality of predictor because it determines the error of predicted values. The $\boldsymbol{MSE}$ is the mean($\frac{1}{n}\sum_{i=1}$) of the square of error $(\hat{Y_i} - Y_i)^2$).This is an easily computable quantity for a particular sample (and hence is sample-dependent).

2.24 Estimator

let's have the n random variables $X_1, X_2, X_3, ...X_n$ and $\hat{\boldsymbol{\theta}}$ be the estimator of the unknown parameter from these sample variables then it's deviation from true value of θ mesures the quality of the estimator and express $(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})$ and consequently the mean squred error(MSE) of an estimator $\hat{\boldsymbol{\theta}}$ with respect to an unknown parameter $\boldsymbol{\theta}$ is defined as

$$\boldsymbol{MSE(\hat{\theta})} = E\left[(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^2\right] \tag{2.24.1}$$

The MSE is a property of an estimator,which in this case as already expressed, the MSE measures the average squared dierence between the estimator$\hat{\boldsymbol{\theta}}$ and the true parameter θ. The MSE can be expressed as the sum of the variance of the estimator and the squared bias of the estimator and with this way of expression providing a useful way to calculate the MSE and implying that in the case of unbiased estimators, the MSE and variance are equivalent.[2]

$$\boldsymbol{MSE(\hat{\theta})} = Var_\theta(\hat{\boldsymbol{\theta}}) + Bias_\theta(\hat{\boldsymbol{\theta}}, \boldsymbol{\theta})^2 \tag{2.24.2}$$

In the MSE which has expressed before we can see the $Variance$ and $bias$, the concept of variance already explained but here we just going to define what is the bias.

2.24.1 Bias:

The bias of an estimator $\hat{\boldsymbol{\theta}}$ of a parameter θ is the difference between the expected value of $\hat{\boldsymbol{\theta}}$ and true value o of the parameter θ which stated the accuracy of the estimator.if the bias of an estimator is equal 0 it means the estimator is unbiased.

2.25 Proof of variance and bias relationship

$$\begin{aligned}
\boldsymbol{MSE(\hat{\theta})} &= E_\theta\left[(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^2\right] \\
&= E_\theta\left[(\hat{\boldsymbol{\theta}} - E_\theta[\hat{\boldsymbol{\theta}}] + E_\theta[\hat{\boldsymbol{\theta}}] - \boldsymbol{\theta})^2\right] \\
&= E_\theta\left[(\hat{\boldsymbol{\theta}} - E_\theta[\hat{\boldsymbol{\theta}}])^2 + 2(\hat{\boldsymbol{\theta}} - E_\theta[\hat{\boldsymbol{\theta}}])(E_\theta[\hat{\boldsymbol{\theta}}] - \boldsymbol{\theta}) + (E_\theta[\hat{\boldsymbol{\theta}}] - \boldsymbol{\theta})^2\right] \\
&= E_\theta\left[(\hat{\boldsymbol{\theta}} - E_\theta[\hat{\boldsymbol{\theta}}])^2\right] + E_\theta\left[2(\hat{\boldsymbol{\theta}} - E_\theta[\hat{\boldsymbol{\theta}}])(E_\theta[\hat{\boldsymbol{\theta}}] - \boldsymbol{\theta})\right] + E_\theta\left[(E_\theta[\hat{\theta}] - \boldsymbol{\theta})^2\right] \\
&= E_\theta\left[(\hat{\boldsymbol{\theta}} - E_\theta[\hat{\boldsymbol{\theta}}])^2\right] + 2(E_\theta[\hat{\boldsymbol{\theta}}] - \boldsymbol{\theta})E_\theta\left[(\hat{\boldsymbol{\theta}} - E_\theta[\hat{\boldsymbol{\theta}}]\right] + (E_\theta[\hat{\theta}] - \boldsymbol{\theta})^2 \\
&= E_\theta\left[(\hat{\boldsymbol{\theta}} - E_\theta[\hat{\boldsymbol{\theta}}])^2\right] + 2(E_\theta[\hat{\boldsymbol{\theta}}] - \boldsymbol{\theta})(E_\theta[\hat{\boldsymbol{\theta}}] - E_\theta[\hat{\boldsymbol{\theta}}]) + (E_\theta[\hat{\theta}] - \boldsymbol{\theta})^2 \\
&= E_\theta\left[(\hat{\boldsymbol{\theta}} - E_\theta[\hat{\boldsymbol{\theta}}])^2\right] + (E_\theta[\hat{\theta}] - \boldsymbol{\theta})^2 \\
&= Var_\theta(\hat{\boldsymbol{\theta}}) + Bias_\theta(\hat{\boldsymbol{\theta}}, \boldsymbol{\theta})^2
\end{aligned} \tag{2.25.1}$$

2.26 Bibliography

[1]://en.wikipedia.org/wiki/

Chapter 3

Kalman Filter And Extended Kalman Filter's Theories

3.1 Kalman filter :

Before introducing the Kalman Filter's principles, let's clarify some keywords which are fundamental for our consideration:

3.1.1 Stochastic Estimation

Since different application-specific approaches have been used in computing (estimating) an unknown state from a set of process measurements, many of these methods do not inherently take into account the typical noisy nature of the measurement(s).While the requirements for the tracking information varies with application, the fundamental source of information is almost the same, for instance, pose estimates information are obtained from noisy measurements of mechanical, inertial, acoustic or magnetic sensors and etc... This noise is typically statistical in nature (or can be effectively modeled as stated in the previous chapter), which guides and leads us to stochastic methods for addressing and dealing with these types of problems. Here, we are going to demonstrate a very basic introduction to the subject aiming to be prepared for next chapters.

3.1.2 State-Space Models:

This is a commonly used term in the subject of systems control. By definition, a model that describes a system by a set of first-order differential equations or difference equations using state variables is known as the state-space model.The state space model uses the first order equations rather thann_th-order differential equation which means that an n_th-order differential equation can be broken up into n first-order differential equation.It is a very effective model for control and estimation problems. Here, we are going to demonstrate it with the help of an example to make the reader understand the concept. Consider a dynamic system or process which is described by a n-th order differential equation (like a differential equation) of the following form:

$$y_{i+1} = a_{0,i}y_i + ... + a_{n-1,i}y_{i-n+1} + u_i, \qquad for \qquad i \geq 0, \tag{3.1.1}$$

Where in Equation(3.1.1), u_i, presents a type of process noise, white (spectrally) random which has zero-mean (statistically) with autocorrelation as the following:

$$E(u_i, u_j) = R_u = Q_i\delta_{ij} \tag{3.1.2}$$

And initial values are$\{y_0, y_{-1}, ..., y_{-n+1}\}$ zero-mean random variables with a known $n \times n$

$$P_0 = E(y_j, y_k) \qquad j,k \in \{0, n-1\} \tag{3.1.3}$$

Also assume that:

$$E(u_i, y_i) = 0 \qquad for \qquad n+1 \leq j \leq 0 \quad and\, i \geq 0, \tag{3.1.4}$$

Refer to [1] for more certainty,and correlation between variables and noise could be present as the following:

$$E(u_i, y_i) = 0 \qquad i \geq j \geq 0 \tag{3.1.5}$$

In other words, the Equ(3.1.4 and 3.1.5) are expressing that, the noises are independent statistically from the process to be estimated, under some other basic conditions which can be found in [1], the difference equation(3.1.1) can be re-written as Equation(3.1.6)

$$x_{i+1} = \begin{bmatrix} y_{i+1} \\ y_i \\ y_{i-1} \\ \vdots \\ y_{i-n+2} \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & \dots & a_{n-2} & a_{n-1} \\ 1 & 0 & \dots & 0 & 1 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} y_i \\ y_{i-1} \\ y_{i-2} \\ \vdots \\ y_{i-n+1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_i \tag{3.1.6}$$

Where the Equ(3.1.6) is the state-space model of the dynamic system equation(3.1.1) and can be expressed in the compact form Eq(3.1.7)

$$x_{i+1} = Ax_i + Gu_i \tag{3.1.7}$$

$$y_i = \begin{bmatrix} 1 & 0 & \dots & 0 & x_i \end{bmatrix} \tag{3.1.8}$$

or the more general form

$$x_{i+1} = Ax_i + Gu_i \tag{3.1.9}$$

$$y_i = H_i x_i \tag{3.1.10}$$

Equation (3.1.9) express the way a new state x_{i+1} which is modeled as a linear combination of both the previous statex_i and some process noise which in this case is ui. Equation (3.1.10) demonstrates the method in which the process measurements or *observations* y_i are derived from the internal statex_i. These two equations, as from control theory, are referred to as the *process model* and the *measurement model* respectively, and they are fundamental in virtual modeling of all linear estimation methods, such as the Kalman filter which will be explained in the following sections.

3.1.3 The Observer Design Problem:

In the area of linear systems theory, there is a relevant general problem usually called the observer design problem. The basic problem in this system is to determine (estimate) the internal states of a linear system with access only to the systems outputs which called measured variables although access to the systems control inputs is also considered, but we are not going to consider it and we neglect that aspect here. This problem is exactly something like what people usually think of as the black box problem where you can see some signals coming from the box (the outputs) but you can not directly observe what is inside. To deal with this basic problem, many approaches, typically based on the state-space model have been demonstrated, such as the one we have expressed in the previous section. There is typically a process model that models the transformation of the process state, which means transformation from current stance to the next stance. This can usually be demonstrated as a linear stochastic difference equation similar to equation (3.1.9):

$$x_k = Ax_{k-1} + Bu_k + w_{k-1} \tag{3.1.11}$$

Moreover, there is some form of a measurement model that describes the relationship between the process state and the measurement variables.This can usually be represented with a linear expression similar to equation (3.1.10)

$$z_k = Hx_k + v_k \tag{3.1.12}$$

The terms w_k and v_k are random variables describing the process and measurement noise respectively. Note that in equation (3.1.12) we have changed the measured variable to z_k instead of y_k as in equation (10).The rationale is to reinforce the concept that the measurements do not have to be of elements of the state specifically but can be any linear combination of the state elements.

3.1.4 Measurement and Process Noise:

Noises are the main error source in control systems and based on that, here we will be investigating the noisy sensor measurement which commonly exists. In reality, many sources of noise in the measurements of variables (parameters) could be found.This issue can be related to the type and capabilities of the sensors.For instance, each type of sensor has fundamental limitations related to the associated physical medium, and if somehow it overpasses from the limitation of sensor capability, the signals and the measured parameters are typically degraded.Moreover, some amount of random electrical noise is combined with the signal via the sensor and the electrical circuits.In fact, the quality and quantity of the information are always and continuously affected by the time-varying ratio of “pure” signal- to-electrical noise.The result is that the information obtained from any sensor must be qualified as it is explicated as part of an overall sequence of estimates and analytical measurement models which typically synthesize some notion of random measurement noise or uncertainty as shown above in Equ(3.1.12).

3.1.5 Kalman Filter:

In the recent years, a number of methods and techniques have been used for noisy data analysis.Within the fundamental and effective tools for stochastic estimation from noisy sensor measurement can be used is called Kalman lter, which is one of the most well-known and reputed tools for state estimation. Its name is derived from its inventor Rudolf E. Kalman when he published his popular paper demonstrating a recursive solution to the discrete-data linear altering problem[4]. The most basic data fusion method for localization is based on the Kalman lter.

The Kalman Filter is a set of mathematical equation that provides the optimal computational tool for estimating the state of a process very effectively and impressively in a way that minimizes the mean square error. This means that the concept behind the working of a Kalman filter is minimum square theory and this concept will be discussed later.This filter from the different point of views is very powerful and has versatile functionalities:

it supports estimation of past which means use information of past step, present, and even able to support future states and it can even identify the system in which the exact and precise nature of the modeled system is unknown. Thus, this tool is very much suitable for the identification system. In case of Autonomous vehicles along with other applications, the Kalman filter is considered as an optimal linear estimator which uses the past information on the sensor noise source which called prior information,the vehicle dynamics and the kinematic equations to recursively compute an optimal position while minimizing the error based on MSE theory[5]. The filter is optimal when the process noise and the measurement noise can be modeled and assumed by white Gaussian process noise.The recursive implementation of Kalman filters is well suited to the fusion of data from different sources at different times in a statistically optimum manner. Many other filter designs can be demonstrated to be equivalent to the performance of Kalman filter under several constraints.The recursive sequence involves prediction and update steps, which means at the same time,it can update its estimation.The prediction step uses a dynamics model that represents the relationship between variables over time due to the time-dependance of the dynamic system. A statistical model of this dynamic process is also necessary.A prediction is usually done to estimate the variable at each of the measurements, as well as in between measurements when an estimate is required.The updated measurement combines the historical data that is passed through the dynamics model with the new information in an optimal procedure base on MSE which will be discussed later. The Kalman filter aims at directing the general issue of estimating the state $x \in \Re^n$ of the discrete-time controlled process that is governed by the linear stochastic equation.Since the time of its introduction, the Kalman filter has been used in comprehensive research and broad applications, particularly in the area of autonomous or assisted navigation.In fact, anywhere the estimate is spoken of, the footprint of Kalman Filter can be seen. The huge progressing of computer sciences and digital computing and the relative simplicity and robust nature of the filter itself have made the use of the filter practically without any limitation.

3.1.6 The Discrete Kalman Filter

In this section, we are going to illustrate the original formulation of the Kalman Filter as presented by Kalman [4] where the occurrence of measurements and state estimation at discrete points in time are considered.

The Process to be Estimated

The Kalman filter endeavors to direct the problem of estimation state $x \in \Re^n$ of controlled process which is very general and common problem in the discrete-time systems governed by linear stochastic difference equation. It could be expressed as the following[4]:

$$x_k = Ax_{k-1} + Bu_k + w_{k-1} \tag{3.1.13}$$

with a measurement $z \in \Re^m$ that is

$$z_k = Hx_k + v_k \tag{3.1.14}$$

Where in the equation of (3.1.13) and (3.1.14) the random variables of w_k and v_k are representative of the process and measurement noise respectively. They are assumed to be white noise,independent of each other and with normal probability distributions.To fulfilled these conditions the noises could have following properties.

$$P(w) \sim N(0, Q) \tag{3.1.15}$$

$$P(v) \sim N(0, R) \tag{3.1.16}$$

Notice that, the covariances matrices of process noise Q and measurement noise R might not be practically constant, and might change with each time step or measurement although here, for simplifying of investigation, we suppose that they are constant.In the Equ(3.1.13) the states at the previous time step $k-1$ to the state at the current step k are linked by $n \times n$ matrix A(System matrix).Note that here also A might change with each time step in practice, and is not necessarily fixed constant matrix but here, in this case, it is considered to be constant.The $n \times 1$-Dimensional matrix, B joints the control input $u \in \Re^l$ to the state x.The $m \times n$-Dimensional matrix H in the measurement equation (3.1.14) linkes the state variables to the measurement variables z_k which gives the relation of the internal states with measurement states.As A and u, in practice, the H might be constant or might change with each time step or measurement step, but here we assume it is constant.

3.2 Computational Origins of the Filter:

We define $\hat{x}_k^- \in \Re^n$ (the *superminus* indictaes the prior) is our a prior state estimate at step k if the information of the process prior (before) to step k was given, and $\hat{x}_k \in \Re^n$ is our a posterior state estimate at step k if the measurement z_k was given.Consequently, We are able to define a prior and posterior estimate errors as following equations(3.2.1 and 3.2.2)

$$e_k = x_k - \hat{x}_k \tag{3.2.1}$$

and

$$\bar{e}_k = x_k - \hat{x}_k^- \tag{3.2.2}$$

The a prior estimate error covariance is the

$$P^- = E[e_k^- e^{-T}_k] \tag{3.2.3}$$

and the a posterior estimate error covariance is,

$$P = E[e_k e_k^T] \tag{3.2.4}$$

To derive the equations for the Kalman filter, we will start with the goal to find an equation that calculates a *posterior* state estimate $\hat{x}_k$ as a linear combination of an a prior estimate $\hat{x}_k^-$ and a weighted difference between an actual measurement z_k and a measurement prediction $H\hat{x}^-$.Consequently the equation (3.2.5) is derived, which fulfills all the mentioned requirement. Some rationalization for equation (3.2.5) is given in The Probabilistic principles of the Filter which can be observed below.

$$\hat{x}_k = \hat{x}^- + K(z_k - H\hat{x}^-) \tag{3.2.5}$$

The difference $z_k - H\hat{x}^-$ in equation (3.2.5) is called the residual or the measurement innovation.As it is obvious, the residual reflects the discrepancy or difference between the predicted measurement $H\hat{x}^-$ and the actual measurementz_k values .If the residual becomes zero, it means that the two (predicted and measured values) are in complete compromise.So that the target with this discovery can be clarified, and will be the minimization of residual.To reach to this aim, the key is in the hand of Kalman Gain. The $n \times m$ matrix K in equation (3.2.5) is the gain or combining factor to be chosen to minimize the posterior error covariance equation (3.2.4). This minimization can be demonstrated by first substituting equation (3.2.5) into the above-explained for e_k Equ(3.2.1), substituting that into equation (3.2.4), performing the indicated expectations and then with performing the derivative of the trace of the obtained result of previous operation(substitutions) with respect to K, setting equal to zero of the final operation result, and then solving it for K.With this operations the optimum points will be found and make the values which make minimize the residual are the desired values of Kalman gains. For more details please refer to these articles [2][7].One common form of computing the K which minimizes the residual (equal to MSE_{min}) equation (3.2.4) is given by:

$$\begin{aligned} K_k &= P_k^- H^T (H P_k^- H^T + R)^{-1} \\ &= \frac{P_k^- H^T}{H P_k^- H^T + R)} \end{aligned} \tag{3.2.6}$$

Looking at the equation (3.2.5) we could say that, as the measurement covariance error R comes closer to zero,which means that the posterior estimat error comes close to zero, pertaining to Equ.(1 and 4), the gain K weights or relies on the residual (the difference between actual and predicted) more heavily. Specifically the Equ(3.2.6) converted to Equ(3.2.7) only depends to the inverse of H:

$$\lim_{R_k \to 0} K_k = H^{-1} \tag{3.2.7}$$

On the other hand, as the prior estimate error covariance P_k^- comes closer to zero, which means the prior estimat error comes close to zero, regarding to Equ.(3.2.2 and 3.2.3) the gain K weights or relies on the residual(the difference between actual and predicted measurements) less heavily. Specifically:

$$\lim_{P_k^- \to 0} K_k = 0 \tag{3.2.8}$$

Another way of thought or perception about the weighting by K is that as the error covariance of measurements R comes close to zero, the actual measurements which are the measured by sensors z_k are more trusted while the predicted measurement which is estimated by filter $H\hat{x}^-$ is less and less trusted.On the other side, when the reverse case of earlier mentioned happens, as the a *prior estimate error covariance* P_k^- comes close to zero, the actual measurement z_k is less and less trusted, while the predicted measurement $H\hat{x}^-$ is more and more trusted.

The Discrete Kalman Filter Algorithm

Already stated briefly, the discrete Kalman filter provides the best estimate and subsequently, a demonstration of the algorithm of this powerful tools is done.To clarify the algorithm, we will start this part with a comprehensive overview, covering the "high-level" operation of one form of the discrete Kalman filter (refer to the previous section). After we have described this high-level view, the investigation will be confined focusing on the specific equations and their use in this version of the filter. A careful examination of the Kalman Filter would show that it is using a type of feedback control to estimates a process In fact, the filter estimates the state variables at the same time and then measures feedback in the form of noisy measurements.The time update equations are responsible for projecting forward the current states and calculating of the error covariance estimate, obtaining the prior sates estimate for next step.While the responsibility for the feedback is the measurement update equations, to consolidate a new measurement into the prior estimate to obtain an improved a posterior estimate(Improving the estimated states of the first group equations). The time update equations are responsible for projecting forward the current states and calculation of the error covariance estimate and obtaining the prior sates estimate for next step.While the responsibility for the feedback is the measurement update equations, it is required to consolidate a new measurement into the prior estimate to obtain an improved posterior estimate(Improving the estimated states of the first group equations). The time updated equations which already have been illustrated can also be considered as predictor equations, while the measurement update equations can be considered as corrector equations.In fact, the final estimation algorithm is similar to that of a predictor-corrector algorithm for solving numerical problems as shown below in Figure 3.1.

Again refer to the time updated equations in the table (3.1) and how the state and covariance estimates from time step $k-1$ to step k forwardly are being projected. A and B are from equation (3.1.13), while Q is from equation (3.1.15).In practice the initial condition is considered arbitrary and we consider arbitrary too, but there are some methods that deal with initial conditions definition of state and covariance.(Initial conditions for Kalman filtering: prior knowledge specification) The first task during the measurement update is to compute the Kalman gainK_k, taking into account the equations have given here as equation (3.2.11) is similar to equation

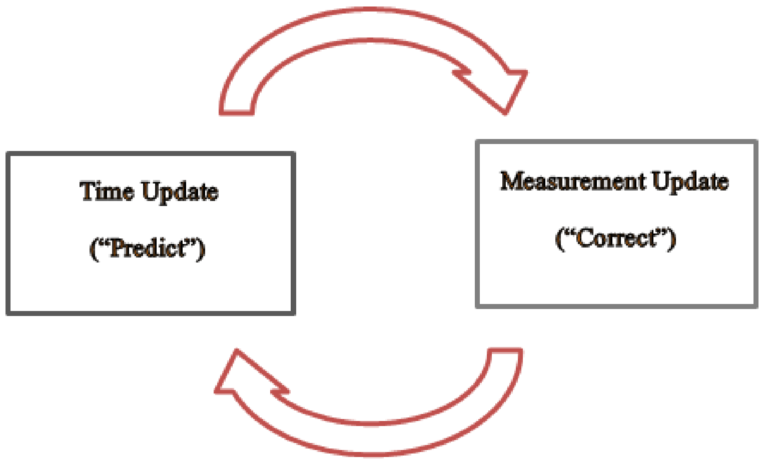


Figure 3.1: The ongoing discrete Kalman filter cycle. The time-updated projects the current state estimate ahead of time. The measurement update adjusts the projected estimate by an actual measurement at that time.

Table 3.1: Discrete Kalman filter time update equations.

$$x_k^- = Ax_{k-1}^- + Bu_k \tag{3.2.9}$$
$$P_k^- = AP_{k-1}A^T + Q \tag{3.2.10}$$

Table 3.2: Discrete Kalman filter measurement update equations

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1} \tag{3.2.11}$$
$$\hat{x}_k = \hat{x}^- + K(z_k - H\hat{x}^-) \tag{3.2.12}$$
$$P_k = (I - K_k H)P_k^- \tag{3.2.13}$$

(3.2.6). The next step is to actually measure the process to obtain z_k, and then to generate a posterior state estimate by consolidating the measurement as in equation (3.2.12). Again equation (3.2.12) is simply equation (3.2.5) repeated here for completeness. The final step is to obtain a posterior error covariance estimate via equation (3.2.13). After each time step and measurement update, the process is repeated recursively with the previous posterior estimates to project forward or predict a new prior-estimates as presented in equation(3.2.9). This recursive nature is one of the very attractive features of the Kalman filter. It makes practical implementations of the Kalman filter much more feasible than the implementation of other kinds of filters[3] which is designed to operate on all of the data directly for each estimate. The Kalman filter recursively conditions the current estimate on all of the past measurements instead. Fol-

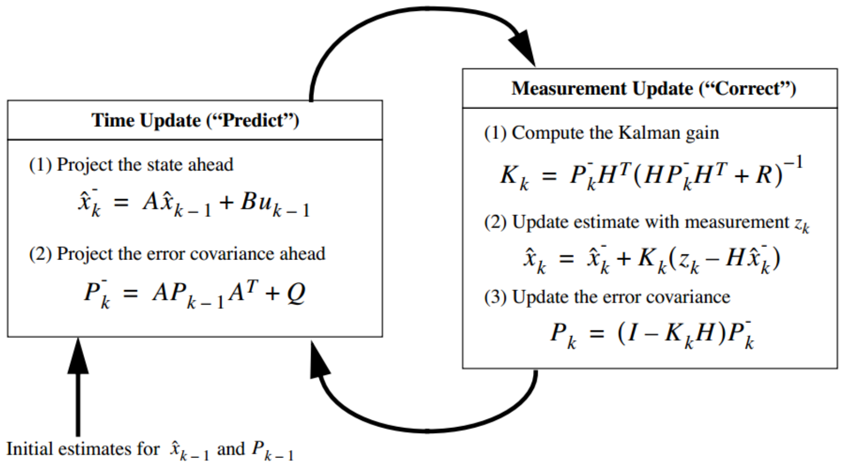


Figure 3.2: A complete picture of the operation of the Kalman filter,incorporating the high-level diagram of Fig.1, with the equations from table(1) and table(2).

lowing Fig.2 represents a complete picture of the operation of Kalman the filter incorporating the high-level diagram of Figure 1 with the equations from table (1) and table(2).

3.3 The Extended Kalman Filter (EKF)

3.3.1 The Process to be Estimated:

As explained and described above in section of [$The process to be estimated$], the Kalman filter directs the popular and common problem for finding or estimation of the state $x \in \Re^n$ of a controlled process which is discrete-time and that is governed by a linear stochastic difference equation (similar to the differential equation) or in another word, the linear systems. But what will happen if the process to be estimated (state variables) and (or) the relationships between measurement with the process is non-linear(nonlinear system), which means that, the differential equation which describes system dynamics is not any more linear?.This is the point where the capability of the Kalman filter comes evident. One of the most attractive and significant applications of Kalman filter is dealing with nonlinearity of systems. A type of Kalman filter is a filter, which is known or called extended Kalman filter (EKF), is able to overcome the nonlinearity of the systems with linearizing of the system equations about the current mean and covariance.

In the case of encountering with non-linear relationships, we can linearize the estimation around the current estimate by means of Tylor series which partially derives the process and measurement functions around the current estimate to be able to compute estimation.To perform this, some of the expression, which has already demonstrated in the previous section [$the\ discrete\ Kalman\ filter$], must be modified so we start modifying from this point. Let us assume that our states or process again has a state vector $x \in \Re^n$, but noticing that the process is now governed by the non-linear stochastic difference equation as expressed in following:

$$x_k = f(x_{k-1}, u_k, w_{k-1}) \tag{3.3.1}$$

with a measurement $z \in \Re^m$ that is

$$z_k = h(x_k, v_k) \tag{3.3.2}$$

where the random variables w_k and v_k demonstrate the process and measurement noise respectively as in equation (3.1.15) and equation (3.1.16). In this case, we do not have the constant matrix any more but we have the non-linear function in the difference equation, the equation (3.3.1) which links or connects the state at the previous time step k_1 to the state at the current time step k. It includes the parameters, driving function (control input) u_k which could be any function, and the zero-mean process noise w_k. The non-linear function h in the measurement equation,the equation (3.3.1) links the state x_k to the measurement z_k.

Considering these equations, it is obvious that one equation does not know the values of the noise w_k and v_k at each time step individually which can reduce the efficiency of Kalman Filter. However, another one can approximate the state and measurement vector without them as follows.

$$\tilde{x}_k = f(\hat{x}_{k-1}, u_k, 0) \tag{3.3.3}$$

and

$$\tilde{z}_k = h(\tilde{x}_k, 0) \tag{3.3.4}$$

where $\hat{x}_k$ is a posterior estimate of the state (from a previous time step k).

It is important to take into account that a substantial deficiency of the EKF is that the various random variable distributions (or densities in the continuous case) do not remain normal any more after undergoing their respective nonlinear transformations.Some interesting work has been done by Julier [8] using methods that keep the normal distributions all over the non-linear transformations in developing a variation to the EKF[8].

3.3.2 The Computational Origins of the Filter

In the section(3.2) is described the computational origion of the Kalman filter and as already mentioned, here we are going to describe the origion of Extended Kalman filter which relies on the linearization of a non-linear system. Thus, we must be able to estimate the process (state variables) governed by non-linear equations, starting with new sets of governing equation which linearize an estimate about equation (3.3.3) and equation (3.3.4).Consequently the linearized model of the nonlinear model, Equ. (3.3.21) and Equ.(3.3.2) can be rewritten as the following:

$$x_k \approx \tilde{x}_k + A(x_{k-1} - \hat{x}_{k-1}) + W w_{k-1} \tag{3.3.5}$$
$$z_k \approx \tilde{z}_k + H(x_k - \tilde{x}_k) + V v_k \tag{3.3.6}$$

where

- x_k and z_k are the actual state and measurement vectors.
- $\tilde{x}_k$ and $\tilde{z}_k$ are the approximate state and measurement vector from equation (3.3.3) and equation(3.3.4).
- $\hat{x}_k$ is a posterior estimate of state at step k.
- the random variable w_k and v_k are representative of the process and measurement noise as like in the equation(3.1.15) and equation(3.1.16).

- A is the jacobian matrix of partial derivative of f with respect to x(states), that is

$$A_{[i,j]} = \frac{\partial f_{[i]}}{\partial x_{[j]}}(\hat{x}_{k-1}, u_k, 0), \tag{3.3.7}$$

- W is the Jacobian matrix of partial derivatives of f with respect to w,

$$W_{[i,j]} = \frac{\partial f_{[i]}}{\partial w_{[j]}}(\hat{x}_{k-1}, u_k, 0), \tag{3.3.8}$$

- H is the Jacobian matrix of partial derivatives of h with respect to x,

$$H_{[i,j]} = \frac{\partial h_{[i]}}{\partial x_{[j]}}(\tilde{x}_{k-1}, 0), \tag{3.3.9}$$

- V is the Jacobian matrix of partial derivative of h with respect to v,

$$V_{[i,j]} = \frac{\partial h_{[i]}}{\partial v_{[j]}}(\tilde{x}_{k-1}, 0), \tag{3.3.10}$$

considering that for simpleness in the symbolization, we do not use the time step subscript with the Jacobians A,W,H,V and, even though they are actually different at each time step.

Now we assign a new notation for the prediction error,

$$\tilde{e}_{x_k} \cong x_k - \tilde{x_k} \tag{3.3.11}$$

and the measurement residual,

$$\tilde{e}_{z_k} \cong z_k - \tilde{z_k} \tag{3.3.12}$$

It must be kept in mind that, in practice, we do not have access to x in equation (3.3.11). It is the actual(real) state vector,because it is the internal state of systemand and we try to estimate the quantity one. On the other side, we have access to z_k in equation (3.3.12) which are the actual measurements from sensors or other measurement device that, we are using to estimate the desired state. Using equation (3.3.11) and equation (3.3.12) and substuted to equations (3.3.5) , (3.3.6), we are able to write governing equations for a *error process* like :

$$\tilde{e}_{x_k} \approx A(x_{k-1} - \hat{x}_{k-1}) + \epsilon_k \tag{3.3.13}$$

$$\tilde{e}_{z_k} \approx H\tilde{e_{x_k}} + \eta_k \tag{3.3.14}$$

where ϵ_k and η_k represent new independent random variables which have zero mean and covariance matrix WQW^T and VRV^T .With Q and R as in (3.1.15),and(3.1.16) respectively.

Notice that the equations of (3.3.13) and ((3.3.14) are linear and they closely simulate the equations of difference (3.1.13) and measurement (3.1.14) from discrete Kalman filter respectively. This helps us to use the actual measurement residual $\tilde{e}_{z_k}$ in equation (3.3.12) and a second (hypothetical) Kalman filter to estimate the prediction error $\tilde{e}_{x_k}$ given by (3.3.13).This estimate, let's call it $\hat{x_k}$, could then be used along with equation (3.3.11) to obtain the $posterior\, state\, estimates$ for the original non-linear process like :

$$\hat{x_k} = \tilde{x}_k + \hat{e}_k \tag{3.3.15}$$

The random variables of equation (3.3.13) and equation (3.3.14) have approximately the following probability distributions (as presented previously):

$$P(\tilde{e}_{x_k}) \sim N(0, E[(\tilde{e}_{x_k}\tilde{e}_{x_k}^T]) \quad (3.3.16)$$
$$P(\epsilon_k) \sim N(0, WQ_kW^T) \quad (3.3.17)$$
$$P(\eta_k) \sim N(0, VR_kV^T) \quad (3.3.18)$$

Given these approximations and letting the predicted value of $\hat{e}_k$ be zero, the Kalman filter equation used to estimate $\hat{e}_k$ is:

$$\hat{e}_k = K_k\tilde{e}_{z_k} \quad (3.3.19)$$

By substituting equation (3.3.19) back into equation (3.3.15) and making use of equation (3.3.12) we see that we do not actually need the second (hypothetical) Kalman filter:

$$\begin{aligned}\hat{x}_k &= \tilde{x}_k + K_k\tilde{e}_{z_k} \\ &= \tilde{x}_k + K_k(z_k - \tilde{z}_k)\end{aligned} \quad (3.3.20)$$

Equation (3.3.20) in the extended Kalman filter can now be used for the measurement update, with $\tilde{x}_k$ and $\tilde{z}_k$ coming from equation (3.3.3) and equation (3.3.4), and the Kalman gain K_k coming from equation (3.2.11) with the appropriate substitution for the measurement error covariance.

The complete set of EKF equations presented as shown in table(3) and table(4). Notice that, we have substituted $\hat{x}_k^-$ for $\tilde{x}_k$ to remain consistent with the earlier super minus for a prior notation, and that we now attach the subscript k to the Jacobians$A$,W,H,and V,to reinforce the notion that they are different at (and therefore must be recomputed) each time step.

Table 3.3: EKF time update equations.

$$\hat{x}_k^- = f(\hat{x}_{k-1}, u_k, 0) \quad (3.3.21)$$
$$P_k^- = A_kP_{k-1}A_k^T + W_kQ_{k-1}W_k^T \quad (3.3.22)$$

As stated in the section of the basic discrete Kalman filter, the state and covariances of estimation are projected forward from the previous time step $k-1$ to the current time step$k$ by the time update equations in table 3. .Here The f in equation (3.3.21) comes from equation (3.3.3),A_k and W_k are the process Jacobians at step k, and Q_k is the process noise covariance equation (3.1.15) at step k

Table 3.4: EKF measurement update equations..

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V R_k V^T)^- 1 \qquad (3.3.23)$$

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - h(\hat{x}_k^-, 0)) \qquad (3.3.24)$$

$$P_k = (I - K_k H_k) P_k^- \qquad (3.3.25)$$

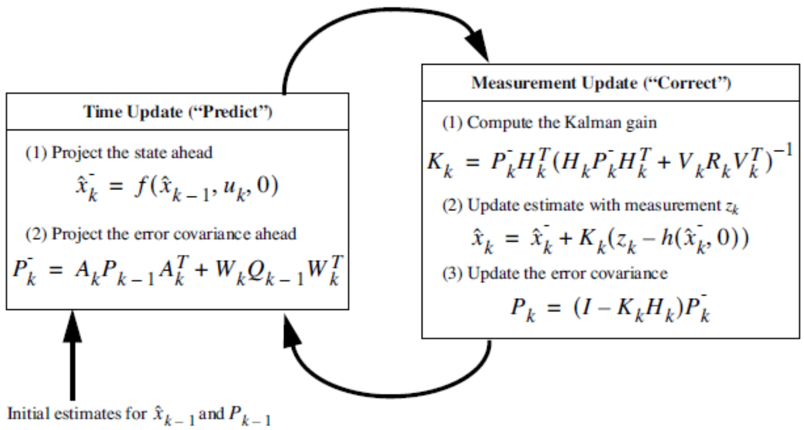


Figure 3.3: A picture of complete operation of the extended Kalman filter, incorporating the high-level diagram of Figure 1 with the equations from table 3 and table 4.

As demonstrated by means of the basic discrete Kalman filter and as can be seen in table(4), the measurement update equations by means of the measurement z_k can correct or in other word can modify the state and covariance of the estimates.Again we mention that h in equation(3.3.23) comes from equation (3.3.4), and the Jacobians of measurements are presented by H_k and V at step k,and covariance equation of measurement noise(3.1.16) is presented by R_k at step k. (Notice that now we subscript R allowing it to change with each measurement.)

The fundamental and the preliminary operation of the EKF is akin to the linear discrete Kalman filter as shown in Figure (1).The Figure(3) as presented in following, offers a complete and comprehensive picture of the EKF's operations, which present the combination of the diagram of Figure 1 with the equations from table 3 and table 4.

A very significant property of the EKF is hidden behind the Kalman gain equation K_k,in which the Jacobian H_k serves to properly and correctly magnify or propagate only the measurement information's components which are appropriate. For instance, if not a one-to-one mapping doesn't exists between the measurement z_k and the state via h, which means that might not all state variable are involved in measurement, in this case the Jacobian H_k modify the Kalman gain and consequently it only propagates the portion of the residual $z_k - h(\hat{x}_k, 0)$ that does affect the state.It is obvious that if one-to-one mapping does not exist between the measurement z_k and the state via h,then as you can anticipate that, the filter will rapidly diverge. In this case, the processor, in other words, the state is *unobservable*.

As a conclusion for this chapter, so far, we have demonstrated the proper tools with the concept of different terms and notations to model and simulate our objective which is the estimation of the vehicle position in noisy environments more accurate.

3.4 Bibliography

[1] Kailath, T., Sayed, A. H., and Hassibi, B. Linear estimation, Prentice-Hall, Englewood Cliffs, NJ, 2000.
[2] Stochastic model,simulation and control,Maybeck 1979.
[3]An introduction to Kalman filter,Brown and Hwang1996
[4] Kalman, R. E. 1960. A New Approach to Linear Filtering and Prediction Problems, Transaction of the ASMEJournal of Basic Engineering, pp. 35-45 (March 1960).
[5]Gelb, A. 1974. Applied Optimal Estimation, MIT Press, Cambridge, MA.
[6] Brown, R. G. and P. Y. C. Hwang. 1992. Introduction to Random Signals and Applied Kalman Filtering, Second Edition, John Wiley and Sons, Inc.
[7]Jacobs93 Jacobs, O. L. R. 1993. Introduction to Control Theory, 2nd Edition. Oxford University Press.
[8]Julier, Simon and Jeffrey Uhlman. "A General Method of Approximating Nonlinear Transformations of Probability Distributions," Robotics Research Group, Department of Engineering Science, University of Oxford.

Chapter 4

Global Position system(GPS) and Priciples

4.1 Introduction

Two types of sensors are able to provide position of mobile vehicles :Absolute Position sensors (GPS , Radar) which take their information from outside of mobile vehicle in the environment and get the position of the mobile vehicle in an absolute reference frame and dead Recognition that called, DR sensor or Odometry sensor,which take their information from mobile vehicle itself by using the latest information (point) and with integration in time difference interval of current and previous time evaluates the current position of mobile vehicle so therefore positioning error is time dependent and drifting with time. At the present, the basic component of a navigation or land positioning system is global positioning system or GPS.

4.1.1 What is Global Position System

The GPS (global positioning system) is a constellation of approximately of 30 (or 24) well-spaced satellites (in general a satellite is anything that orbits around something else, for instance, the moon that orbits around the earth.In communications technology a satellite is a specialized wireless receiver and the transmitter that is sent to space by a rocket and is placed in orbit around the earth) that orbits around the earth that are precisely tracked from the ground station. In order to satellite be updated, the updated information sent to the satellite by each ground station which has a precisely known location. consequently, each satellite transmits and sends its location to the receiver and makes it possible for people with ground receivers to determine their geographical location.Actually, the GPS satellite composed of computer and receiver along with the satellites which are able to determine the latitude, altitude, and longitude of a user or receiver in space or earth by calculation the time difference from transmitted to and received a signal from the receiver. The accuracy of location determining of GPS is variable from 100 to 10 meters for most equipment.Although it is possible to reach the accuracy of centimeter In a differential mode. However, the lack of credibility or reliability of GPS in some cases, due to multipath or mask effect or when the satellites signal not available such as in underpass or tunnel leads to overcoming the problem or at least being decreased this erroneous of GPS data with combination to other sensors.[1]

GPSs equipment is widely used in science, traffic management with real-time data and industrial application such as freight management, agricultural and in advanced and innovation cases in the autonomous vehicles which is our motivation and interest.in the following picture

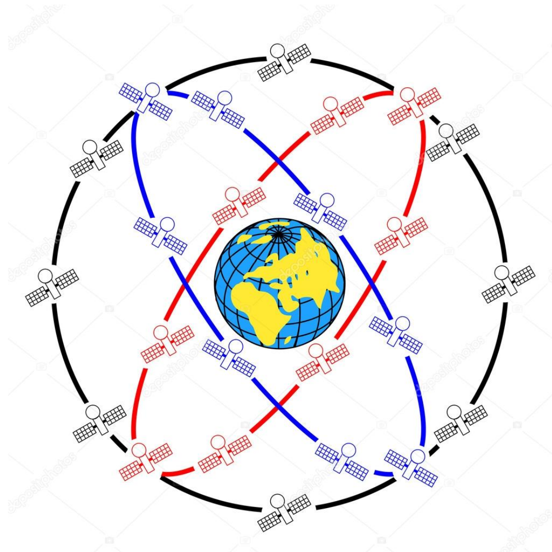


Figure 4.1: Spaced Satellites around the Earth

is shown the location of the satellite around the earth.

4.1.2 How GPS determines the position of user:

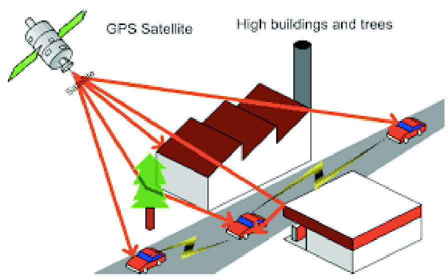
GPS, as it is clear, must use the time to determine and measure the distance from each satellite to the receiver.The time required for the signal to travel from the satellite (a known position) to the receiver (an unknown position) is computed by the receiver. The distance (pseudo range) of each satellite to the receiver is calculated by following formula: $distance = rate * time$.Since we know the speed of light almost is fixed and approximately (186,000 miles per second), and what is here determinative is the time.Actually, variation in time causes a variation in distance. A GPS receiver is able to determine the range to the satellite by measuring the time elapsed for a signal to broadcast from a satellite to a receiver and multiplying it by the speed of light. [2]. The receiver's location is determined by solving three(x,y, $time$) or preferably four (x,y,z,$time$) 6 unknowns. These unknowns are solved using three equations with three unknowns or four equations with four unknowns. Three satellites are enough to determine a location.To have more accurate location information is better to have four satellites rather than three satellite because another unknown variable is calculated by additional satellite which is elevation (z). As already stated the information is received from GPS system are not accurate due to signal loss, multipath eect and mask eect and, etc. Here these eects going to be brie y explained which they called $UserEquivalentRangeErrors$.

User Equivalent Range Errors:

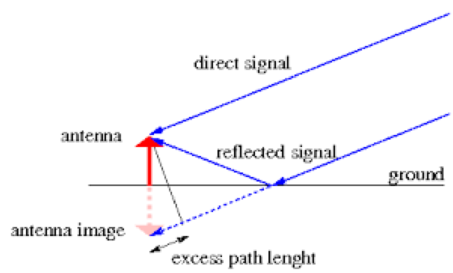
User Equivalent Range Errors (UERE) are the errors which are related to the timing and path readings of the satellites due to anomalies in the hardware in GPS device or interference from the atmosphere.An almost comprehensive list of the sources of User Equivalent Range Errors(UERE), regarding their contribution to total error, is as the following :

- **Signal loss:** GPS is a line-of-sight sensor and therefore the GPS measurements are subjecting to signal outages. To estimate the expected position of a receiver or in our case mobile vehicle, at least four satellites are needed to be in view to a receiver for a 3D position (altitude, latitude, and longitude) or at least 3 satellites for 2D positioning. If it cannot see the four satellites in view the measured position is not accurate and it is called signal loss.

- **Multipath effect:** Multipath is the phenomenon where a GPS signal arrives at the receivers antenna of the user via more than one paths. In the GPS the multipath effect is a common error source that has to be taken into account whether in static or dynamic precise positioning. As you can see in the following picture the receiver is getting an indirect signal which is reflected from building or ground so obviously the position being estimated by GPS receiver included to an error.As the satellites move and their position are not fixed, so the indirect path is obviously dependent on the reflecting surface and the satellite position it means that can be delt just by considering of the receiver side.The reflecting surface is usually a static one related to the receiver, although the satellite moves with time. Therefore, the multipath effect is also a variable of time as stated that satellites are mobile. Antennas are designed to minimize interference from signals reflected from below, but the elimination of receiving signals reflected from above is a more complicated task.One technique for minimizing multipath errors is to track only those satellites that are at least 15 above the horizon, this is a threshold angle which is called the "Mask angle" in the following pictures is shown the effect of multipath.



(a) multipath signal recivieving to user after being reflected.



(b) excess path length of signal after being reflected.

- **Satellite clock:** GPS is able to calculate the position, as expressed before depending on measuring transmission time of a signal from the satellite to the receiver; in fact, this depends on knowledge of the time on both ends. NAVSTAR satellites commonly use atomic clocks, which are very accurate but they can drift-up to a millisecond (which is enough to make an accuracy dierence). by monitoring of the stations and calculation of clock correction These errors are minimized and with transmitting the clock corrections together with the GPS signal in order to the two side of GPS receivers and transmitter appropriately be outfitted.[3]

- **Upper atmosphere (ionosphere):** The GPS satellites are located in space at the ionosphere so that the GPS signals must pass through the upper atmosphere (the ionosphere 50-1000km above the surface).consequently of changing the environment and The ionosphere density variation, the signals are delayed and deflected.; thus, signals are delayed more in some places than others.note that the delay also depends on the location distance of satellites from this surface and how close the satellites are to this surface(where distance that the signal travels through the ionosphere is least).this issue can be dealt and error can be minimized by modeling ionosphere characteristics. and GPS monitoring stations can calculate and transmit corrections to the satellites, which in turn transmit these corrections along to receivers.It has been proven that by scientists, removing of this error completely is impossible but only about three-quarters of the bias can be removed, however, this error considered as the second largest contributor to the GPS error which could be categorized in the User Equivalent Range Error.[3]

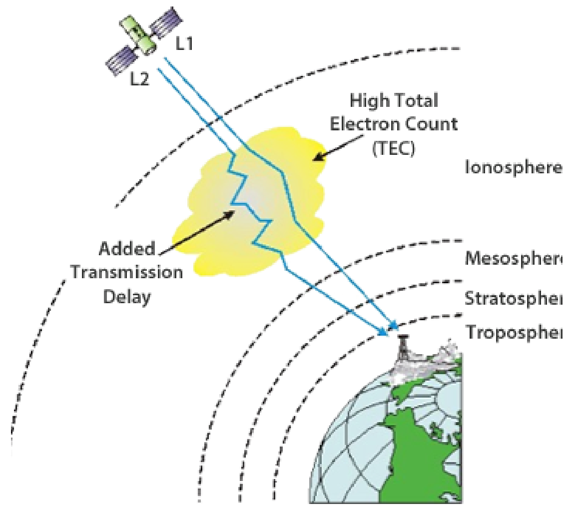


Figure 4.3: Effect of ionosphere refraction on transmited and recieved signal

- **Receiver clock:** GPS receivers clocks are different from satellite clocks which are atomic clocks. GPS receivers are equipped with quartz crystal clocks that are less accurate and stable than the atomic clock which is used in NAVSTAR satellites.The error which is caused by receiver clocks can be removed by comparing the arrival time of a signal from two satellites that whose transmission time are known exactly.

- **Satellite orbit:** As we know satellites are orbiting around the earth but the shape of orbit that they are orbiting around is not constant.in order to the GPS receivers calculate the position they must calculate coordinates relative to the known locations of satellites in space.but as stated the shape is not constant and a complicated task is knowing the shape of a satellite orbit as well as their velocities.This task is conducted by GPS Control Segment with monitoring all times the satellite locations in order to calculate orbit eccentricities and compiles these deviations in documents called ephemerides.GPS receivers that are able to process ephemerides receives this compiled document which is transmitted with GPS signal from satellite and it can compensate for some orbital errors

- **Lower Atmosphere:** This error is related to the effect of a lower layer of atmosphere on GPS signal, these lower layers of the atmosphere are the layers which extend t an altitude of about 50 km from Earth's surface.these layers are three layers of the atmosphere which are called troposphere, tropopause, and stratosphere.The effect of the lower atmosphere on GPS signals is a delay of the transmitted signal to the receiver and consequently led to adding slightly to the distance which is calculated between satellite and receivers. Actually, the Signals from satellites close to the horizon due to passing through the most atmospheres experience the most delay.[3]

4.1.3 How Gps works:

Up to know we have expressed the short functionality of GPS and its errors and here on will be mathematically stated how the GPS computes and estimate the position of a user or mobile device. GPS systems composed of three segments:

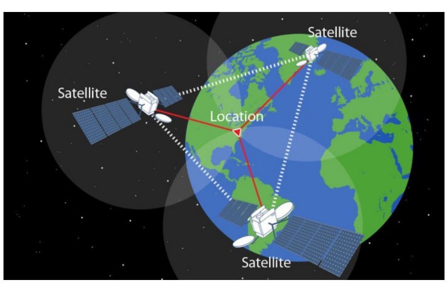
1. **Space satellite:** which composed of 24 satellites constellation.
2. **Control segment:** -tracking station: continuously monitors the orbital data -Master station: data processing, updating data and time scale -Up-loading station: transmits the updated data to satellites
3. **User or receiver segments:** receivers determining their own position and velocity and time.

   The user segment is made of a wide range of different receivers with different performance level as already was mentioning of user equivalent range error. The receiver estimates the position of the user on the basis of the signal transmitted by the satellites the functionalities are common to any kind of receivers and can be summarized as :

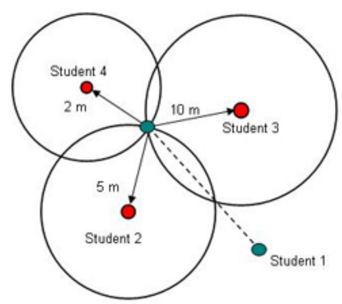
   - **Identification of the satellite in view**
   - **Estimation of the distance user-satellite**
   - **Triangulation**

4.1.4 Triangulation

The receiver position is the intersection point of the spheres centered on three visible satellites which are called triangulation.Additional visible satellites improve the triangulation accuracy. The minimum for 3D estimation is four satellites and for 2D are 3 satellites.the following pictures show the in-sight satellites for triangulation. As already explained the distance estimation od



(a) triangulation of three satellites in sapce view.



(b) triangulation of three satellite in 2 dimensional view

receiver and satellite is provided by transmitting a signal from the satellite and receiving of it with delayτ from the receiver. The distance D between transmitter (TX) and receiver (RX) can be estimated as:

$$D = c.\tau \tag{4.1.1}$$

Where c is the speed of light ($186,000$ miles per second) as already explained and the accuracy of the measurement depends on the accuracy in the estimation of delay τ as we have stated the clock synchronous of satellites and receivers.As already expressed it is impossible to have user clock aligned with the satellite time scale at low cost and complexity, thus the distance is affected by error due to the misalignment of the user clock respect to satellite clock.The measured distance R (with timing error δt_u) is referred to measurement with the pseudo-range error.So that the following formula shows the pseudo-range error :

$$R = c.\tau + c.\delta t = D + c.\delta t_u \tag{4.1.2}$$

Regarding stated above equation, just the distance of the satellite from the user is measured and for being determined the position of receiver the following equation must be established and solved.

$$\rho_k = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} - b_r \tag{4.1.3}$$

Then the four unknown variables which are the East, North, and height or(longitude, latitude, altitude) in addition to time delay (error) must be determined. In order to fully be determined the all four unknown variables, it is needed to four satellites be the light-of-sight of the receiver so consequently, the following set of equations will be established:

$$\begin{aligned}
\rho_1 &= \sqrt{(x_{s1} - x_u)^2 + (y_{s1} - y_u)^2 + (z_{s1} - z_u)^2} - c.\delta t_u \\
\rho_2 &= \sqrt{(x_{s2} - x_u)^2 + (y_{s2} - y_u)^2 + (z_{s2} - z_u)^2} - c.\delta t_u \\
\rho_3 &= \sqrt{(x_{s3} - x_u)^2 + (y_{s3} - y_u)^2 + (z_{s3} - z_u)^2} - c.\delta t_u \\
\rho_4 &= \sqrt{(x_{s4} - x_u)^2 + (y_{s4} - y_u)^2 + (z_{s4} - z_u)^2} - c.\delta t_u
\end{aligned} \tag{4.1.4}$$

In the set equation(4) the ρ_1,ρ_2,ρ_3,ρ_4 are the distance of the receiver from the first, second, third and fourth satellite respectively and x_s ,y_s,z_s,t_u represent the position of each satellite on constellation in fixed reference frame and the signal time error the position with lower case index of u represent the unknown parameters of receivers.in the equation the distance and position of each satellite are known but the position of receiver is unknown in addition the time error is unknown. Note that the above equation estimates the position of receiver inaccurately, as already explained due to user equivalent range error which the clock's asynchronous of receiver and satellite was one of that. So that here will be represented the complete pseudo range with all parameters considered and established as following:

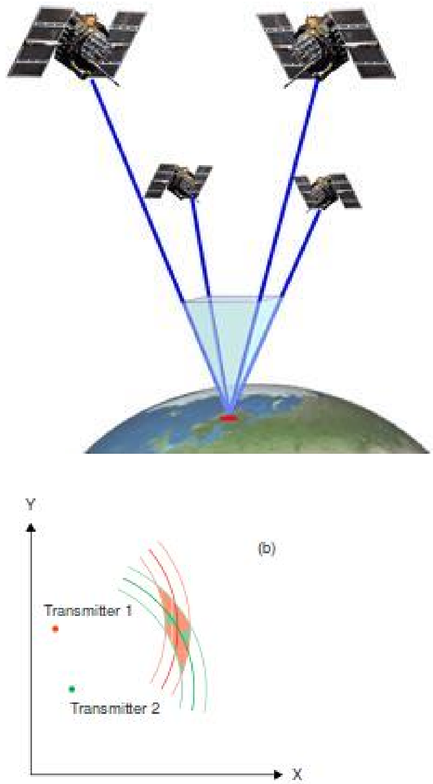
$$\rho_1 = \sqrt{(x_{sj} - x_u)^2 + (y_{sj} - y_u)^2 + (z_{sj} - z_u)^2} - c.\delta t_u + c.\delta t_a + E_j + \eta \tag{4.1.5}$$

Where:

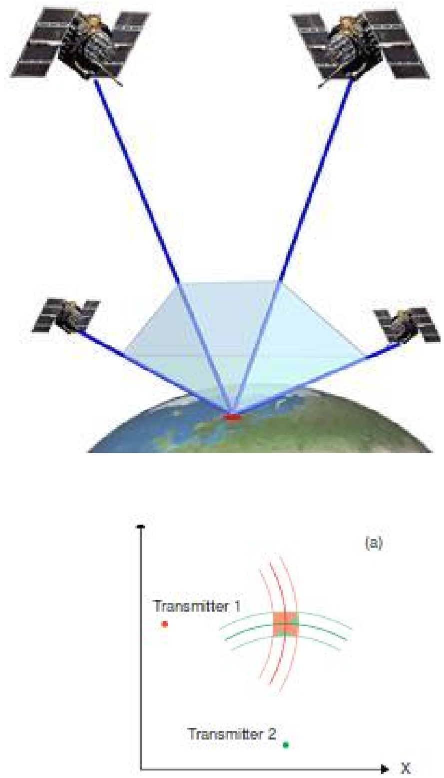
- δt_a is due to the propagation in the ionosphere and troposphere
- E_j is the ephemeris error for the j_{th} satellite
- represents the other errors such as multipath, receiver noise

4.2 Geometrical interpretation of Dilution of Precision (DOP)

In order to be able to present the relative three-dimensional positioning accuracy of GPS receivers and GPS satellites an indicator is used which is called DOP. In order to dene a geometrical interpretation to this, we use a term called geometrical dilution of precision or GDOP. The following Figure explains in a simple way the interpretation for this. A close distribution or poorly spaced satellites gives very poor GDOP value whereas well-distributed satellites yield good GDOP.The ideal situation which gives the best or in other words lowest GDOP is that the satellites overhead of each other spaced at the equal horizontal angle.In this case, the satellites will give the most accurate position estimation, the best GDOP. As you can see the well-spaced



(a) Poor spaced satellites with high GDOP



(b) Well spaced satellites with low GDOP

satellite respect to receiver reduces the error due to low GDOP figure(a) and vice versa the poorly spaced satellite causes a high error due to large or high GDOP figure(b).

Up to know we explained the GPS system Principles and the errors which are the effect the position estimation of GPS system. The errors which are involved in GPS estimation must be modeled so here we will establish modeling of the user equivalent error which is necessary for future consideration.

4.3 Mathematical Modeling Of User Equivalent Range Error:

The pseudo-range error can be modeled as a random variable with the following properties:

- Gaussian with zero mean and variance
- Identically distributed
- Independent

Then this variable with above properties can be modeled as random Gaussian noise with zero-mean which the variance of this variable is expressed as a following:

$$\sigma_{UERE} = \sqrt{\Sigma_j {\sigma_j}^2} \tag{4.3.1}$$

4.4 conclusion

As a conclusion of GPS system, we would like to express that the accuracy of estimation depends on the different factors and the low-cost GPS cannot be very reliable in the crucial scenario for position estimation which in our case of interest is in the autonomous vehicle that whose real-time and accurate position is so crucial.So that we will discuss in next chapters about the accuracy improving with combination of GPS sensor with fusion of different sensor such as GPS/IMU,Wheel Odometry,etc.. .

4.5 Bibliography

[1]://people.uwec.edu [2]:(Mirsa, 1996, p.60) [3]://e-education.psu.edu

Chapter 5

Sensors and their principles

5.1 Sensors:

The complexity of road traffic is being increased day by day and makes great demands on drivers. Driver assistance systems reduce the driver's driving stress and provide and optimize safety on the road. Therefore nowadays modern driver assistance systems are part of the standard equipment in almost all new cars in Europe and US. To make feasible implementation of this equipment in the vehicle, vehicle electronics play a principal role and it is a key factor in all comfort and safety features. The vehicle electronic which is bases of the control unit in the vehicle provide the optimal interaction of complex electronic systems that guarantees flawless function of the vehicle and thus improves traffic safety. The sensors provide and support the complex and intelligent data communication of the electronic vehicle system between all sensors In relation to driving safety, which are implement in the vehicle today, wheel speed sensors and IMU sensors are of particular importance and are used in numerous applications in various vehicle systems. In driver assistance systems such as ABS, TCS, ESP or ACC, motor control units use these sensors to determine the wheel speed and acceleration and orientation of the vehicle. Via bus the wheel speed and IMU information from different driving assistant system such the Anti-Lock Brake System (ABS) and ESP are also provided to other systems such (engine management, gearbox and chassis control systems and navigation systems) to provide the more reliable data provided to driver and for controlling of vehicle more appropriately which in case of fault functioning cause to crucial problem on the roads. Due to the variety of sensors applications, wheel speed sensors and IMU sensors provide a direct contribution to driving dynamics, driving safety, driving comfort and reduced fuel consumption and emissions. Beside of all necessity of sensor application which is implemented for increasing the driving and road safety, we would like to point that of their application in an autonomous vehicle and in our case determination of vehicle position on the roads. This chapter is going to be stated the different type of sensors which are used in data collection for position estimating which are fused into Kalman filter for position estimating of the vehicle.[1]

5.1.1 GPS Sensor:

The GPS functionality and the way of this system uses has been stated in the previous chapter and here just briefly going to be stated. The GPS sensor is a device which determines the position of a mobile device which could be a robot or which an airplane in world reference frame (the frame of the origin of the earth with longitude and latitude of zero).this device communicates with the satellites in sight to evaluate its position.This position as already expressed is evaluated with triangulation in fixed or in other world word frame. The outputs of a GPS sensor is position in 3D space which are latitude, longitude, and attitude and they can be presented by conversation formulas in a Cartesian coordinate system. The indirect measurements of GPS sensor could be the velocity and orientation(head) of the vehicle which is provided in the local reference frame or in another word in GPS device frame itself. All GPS satellite use a reference frame in order to describe it's and GPS device's location and within them, the Cartesian Coordinate frame of reference which is used in GPS/GLONASS called Earth-Centered, Earth-Fixed (ECEF).In order to be described the location of a GPS user or satellite, three-dimensional XYZ coordinates (in meters) is used by ECEF.Since in fact the origin of the axis (0,0,0) is located at the mass center of gravity and with consideration of this fact this frame is called Earth-Centered (determined through years of tracking satellite trajectories)[2].Also, the word Earth-Fixed point that the axes are xed with respect to the earth which means they rotate with the earth and there is no any relative displacement. The Z-axis points the North Pole, and the XY-axis defines the equatorial plane (Figure 1).

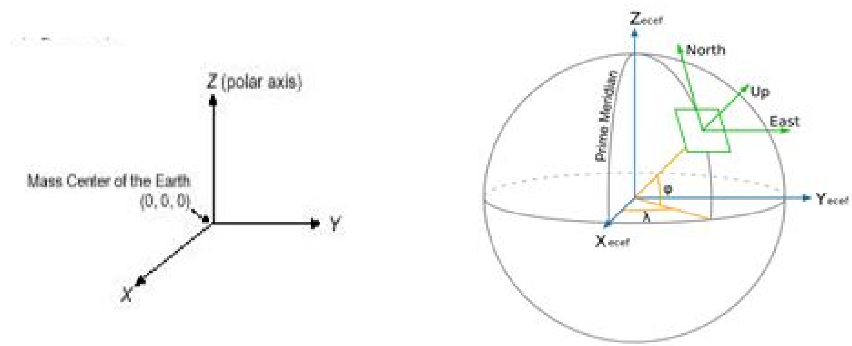


Figure 5.1: Gps Coordinate Fram.

As you see the center of the earth is considered as origin with geographic location of (0,0,0) and the position of device or user calculated respect to this point .

5.1.2 Inertial Measurement Unit (IMU) Sensor:

Inertial Measurement Unit (IMU) is an electronic device which is able to measure angular velocities about 3 axes and linear acceleration along 3 axes and then these data are sent to the main processor.Actually, the IMU sensor is composed of two set of sensors, the acceleration sensor, and gyroscopic sensor. The accelerometer sensor measures the acceleration, in three directions (along with local axis of the device) and the gyroscopic sensor measures the angular velocity of these axes.In the following picture the schematic of the IMU sensor is shown.as you see it has 6 degrees of freedom which are 3 angular and 3 translation.Thanks to Micro-electro-mechanical system (MEMS) these six degrees of freedom are achievable which provides translational movement sensing in three directions which are three perpendicular axes, surge(About y axis),heave(About z axis),sway(About x axis) and rotational movement about this three perpendicular axes (roll, pitch, yaw). Since this three-movement and three rotation along the three axes are independent of each other, such motion is said to have "six degrees of freedom. Depending on the application types,a very wide variety of IMUs exists with performance ranging:

- from 0.1°/s to 0.001°/s for gyroscope
- from 100 mg to 10 μg for accelerometers

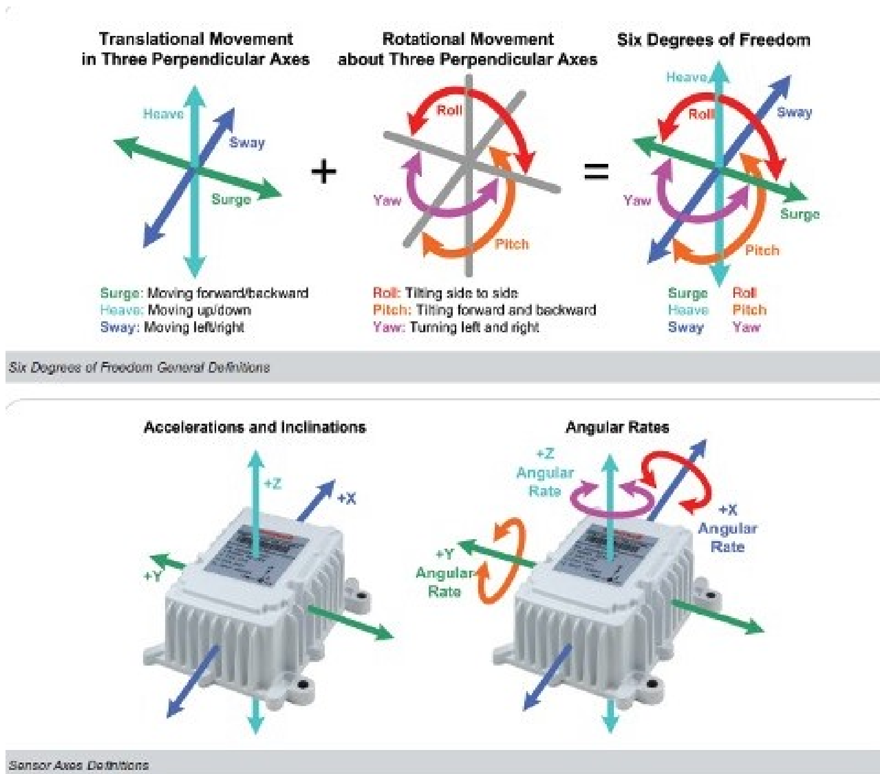


Figure 5.2: IMU sensor with presentation of Six degrees of freedom .

Both sub sensors of IMU sensors is going to be described for better understanding of its functionalities.

5.1.3 Acceleration sensor:

Accelerator sensor is one of the most popular inertial sensors.That is a dynamic sensor which had the capability of a wide range of sensing.Accelerometers, as their name implies, are able to provide the measure of acceleration in single or multi-dimensional, one, two, or three orthogonal axes that means along the frame axes. They are typically used in one of three modes:

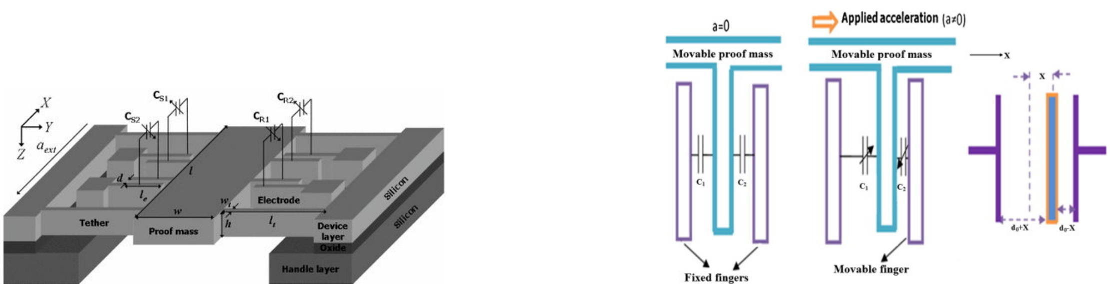
- As an inertial measurement of velocity and position which is used in IMU sensor.
- they can be exploited as a sensor of inclination, tilt, or orientation in *two* or *three* dimensions, as referenced from the acceleration of gravity ($1g$=$9.8m/s^2$);
- As a vibration or impact (shock) sensor such as used in Human Injury Criteria or seat comforts.

Principles of Operation
Most accelerometers which are used today are Micro-Electro-Mechanical Sensors (MEMS) due to high accuracy can be expected from these sensors. The basic but fundamental operating principle of MEMS accelerometer is relied on to the displacement of a small proof mass which is etched into a silicon surface of an integrated circuit and suspended by a small beam. According to Newtons second law of motion (F = ma), when an acceleration is applied to the device as a consequence vehicles maneuvering, a force develops as a result of acceleration which displaces the mass. The support beams here in this device due to elastic property act as a spring, and the fluid (usually air) trapped inside the IC due to energy dissipation effect acts as a damper.so that this

eystem can be analyzed and resulting in a second-order lumped physical system.Consequently that with measuring the displacement of suspended mass which is obtained with second order equation, being able to evaluate the acceleration.
Types of Accelerometer:
The principles that can be used are very different, which based on an analog accelerometer can be built. Two types of accelerometer sensor which are very popular, utilizing capacitive sensing and the piezoelectric effect for determining or sensing the displacement of the proof mass (the mass implemented to exploit the newton force) proportional to the applied acceleration.

5.1.4 Capacitive Accelerometer:

A capacitive accelerometer as its name implies exploits capacitive principle in which the output voltage is dependent on the distance between two planar surfaces.One or two of these surfaces are charged by electrical current.As we know the capacity of the capacitor is dependent on the distance of plates .when the gap between two plates is changing the electrical capacity of the system is changed which can be measured as a voltage output.This method of sensing is populated for its high accuracy and stability.Capacitive accelerometers typically dissipate less power and also less prone to noise and variation with temperature.Due to internal feedback circuity, they can have a larger bandwidth. The following picture shows the principle of the capacitive acceleration sensor.[Elwenspoek-1993]



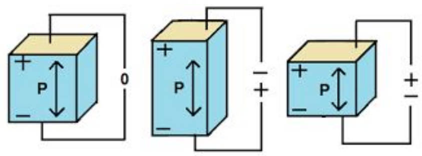
(a) Capacitive Acceleromtere one

(b) Capacitive Acceleromtere one

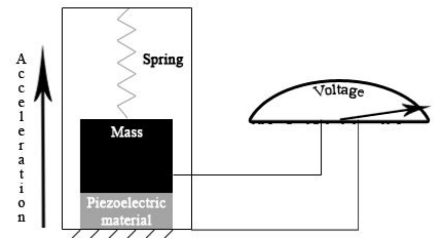
Figure 5.3: Capacitive sensor schematic and priciple.

5.1.5 Piezoelectric Accelerometer:

Piezoelectricity is the characteristic of some kind of material and it also can be described as the electric charge that accumulates in certain solid materials in response to applied mechanical stress.So that with exploiting this phenomenon we can state the piezo acceleration. As we know acceleration is directly proportional to force regarding newtons second low so it is natural to measure the accelerating. Piezoelectricity is the property of certain types of crystal such as Quartz and when these certain types of crystal are subject to compression or extension, charges of opposite polarity accumulate on opposite sides of the crystal which is known as the piezoelectric eect. In a piezoelectric accelerometer, charge which accumulates on the crystal due to an external force (Acceleration) is translated and amplied into either an output current or voltage and consequently, this voltage level is proportional to acceleration. Piezoelectric accelerometers only respond to AC phenomenon such as vibration or shock which means the subjecting loads must be variable with time.They have a large dynamic range and depending on their quality can be expensive.[Elwenspoek-1993]



(a) Piezoelectric crystal

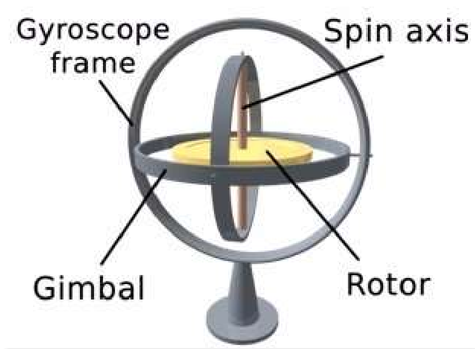


(b) Piezoelectric accelerometer principle

Figure 5.4: Piezoelectric accelerometer

5.1.6 Gyroscope Sensor:

A gyroscope is a device that could be mechanical or electronic which has been used for navigation and measurement of angular velocity.Nowadays Gyroscopes are available that able to measure angular velocity in one, two or three directions.As already has been stated, the 3-axis gyroscopes are usually implemented with a 3-axis accelerometer to provide a full 6 degree-of-freedom (DOF) motion tracking system. In the following picture can be seen the mechanical and electronic type gyro sensor.



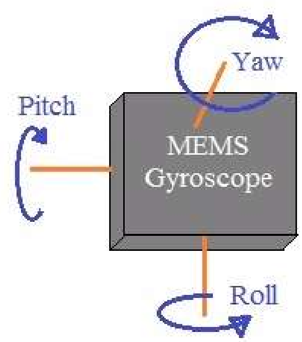


Figure 5.5: Mechanical and MEMS gyroscop

Either the mechanical and electronic gyroscope uses the conservation principle of angular momentum to measure the angular rate which means in the case of the mechanical type the spinning wheel will spin with constant angular speed if the torque applied to a direction of rotation is zero. With exploiting of this principle in the gyroscope the measurement of angular velocity is possible.

5.1.7 Classical or Mechanical Gyroscope

The classical or mechanical gyroscope is a device that uses the angular momentum conservation law, which says that if the resultant external torque which is applied to the system, is zero, the total angular momentum of the system both in direction and magnitude is constant. This gyroscope consists of a rotor in order to spin about one axis.The journal of the rotor is installed in inner gimbal and inner gimbal is installed in outer gimbal and the outer gimbal is pivoted about an axis in its own plane. All mounting axis of rotor and gimbals are perpendicular to each other.As you can see in classical gyroscope, an additional degree of freedom provided by each gimbal and consequently they allow the rotor to spin independently, without applying any external net torque to the gyroscope.So that as a consequence of gyroscopic effect as long as the gyroscope is spinning it will maintain a constant orientation. Actually, what makes possible the measurement of angular velocity in gyroscope device is the result of phenomena of precession which is occurred when external torques or rotations about a given axis are present in these devices, orientation can be maintained and measurement of angular velocity can be measured due to this phenomenon. Let us state the precession phenomenon; usually, precession occurs when an external torque applied in the direction perpendicular to the spinning axis of an object which is spinning.As we know in a rotational system the angular momentum vector is along the spin axis and when net external torque is present this vector will move in the direction of the applied torque vector.As a result of the applied torque, the spin axis rotates about an axis that is perpendicular to both the input axis(applied torque axis) and spin axis (called the output axis). In gyroscope device then this rotation about the output axis can be sensed and be fed back to apply in the opposite direction torque equal to sensed torque about the input axis for cancelling out of the precession of gyroscope to maintain its orientation This cancellation can also be accomplished with two gyroscopes oriented at right angles to one another.[3]

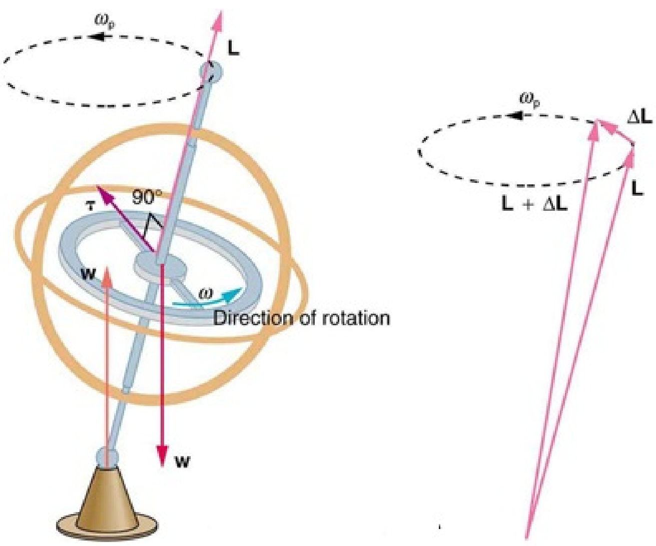
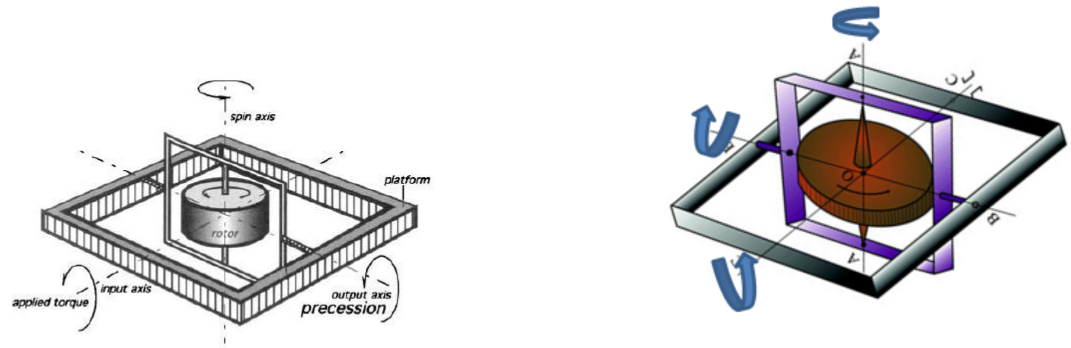


Figure 5.6: Precession phenomenon in Gyroscope.

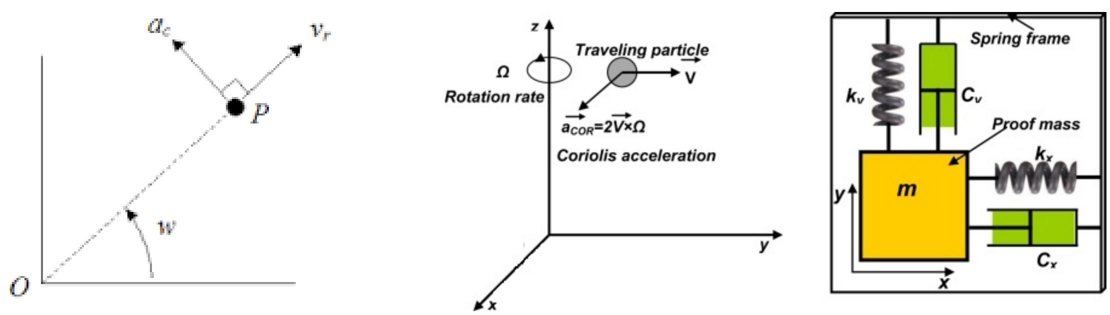
As already mentioned the main application of the gyroscopic device is measuring the angular rate or angular velocity.So that for measuring the rotation rate, in regular time interval a torque is pulsed which is called counteracting torque for precession.Each of this pulse represents a fixed angular rotation $\delta\theta$ and if be able to count the number pulse in a fixed period of time interval t which is proportional to the net angel range θ in that period of time, the angular velocity can be measured which is proportional to the applied counteracting torque . Today classical or rotary gyroscopes are mainly used in stabilization applications and there cannot be used in the automotive application. As already stated the classical gyroscope is mechanical and it is composed of moving parts such as gimbal and rotor and mechanical parts.The presence of these moving parts means the wearing due to friction and dealing with this problem for minimizing the wear and chance for jamming in these gyroscopes a number of bearing types have been developed.Another negative effect of moving parts is a restriction in size of this device which makes limits how small these gyroscopes can be. So that due to these effects and problems the rotary gyroscopes are commonly used today in military and naval environments application which are subject to shock and severe vibration, and where the size of this device is not a critical issue and for commercially application These units are therefore not available.



(a) (b)

Figure 5.7: Classical Gyroscope with spin, input and output axis with precession

For understanding the Coriolis acceleration,consider a particle P which is moving in $2D$ space with velocity of v_r in the radial direction and if in this case a rotation movement applied to this particle the acceleration applies to it will be equal to $2{*}v{*}w$ which is called Coriolis acceleration. The direction of this acceleration is perpendicular to both velocity direction and angular velocity axis.



(a) (b)

Figure 5.8: Schematic of the vibratory gyroscope a illustration of Coriolis effect

The principle for the understanding of MEMS Gyroscope is the understanding of physic Coriolis force. As we know In a rotating system, every point in this system rotates with the same rotational velocity. the rotation velocity or in other words the angular velocity as approaching to the system's rotational axis, remains the same because is not dependent to distance from rotation axis,but in turn the speed in the direction perpendicular to the axis of rotation decreases regarding to relation of $Velocity{=}\omega{*}distance\ from\ rotation\ axis$.if particle would travel in straight line toward or away from of rotation axis while the system is rotating,need and external force to compensate this deviation from straight line in order to the same relative angular position (longitude) on the body be maintained,this force can be provide by increasing and decreasing the lateral speed of particle which is perpendicular to radial velocity.Actually, the act of reduction or increasing of the velocity means the acceleration and the Coriolis force is this acceleration with multiplying to the mass of particle whose longitude is going to be maintained. The Coriolis force is a force which is proportional to both the angular velocity and the radial velocity of the particle and the relation of this force is $F_{Coriolis}{=}2{*}\Omega{*}V_r$ After being stated the Coriolis force concept, let's explain the structure of vibrating gyroscope, Fig(9).

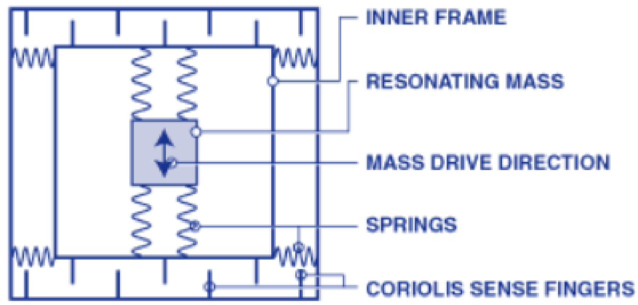
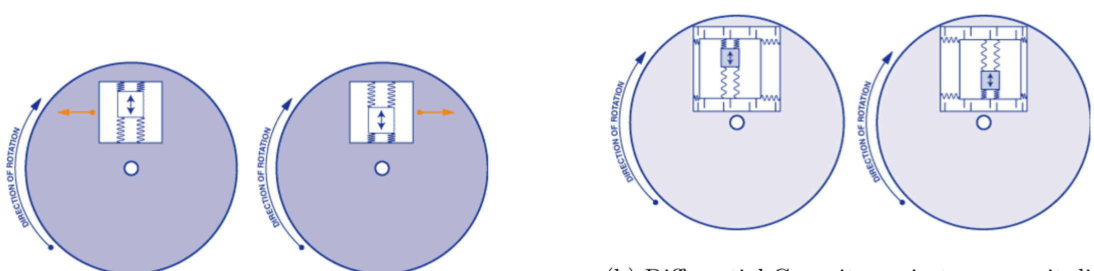


Figure 5.9: Schematic of MEMS Gyroscope sensor

This type of gyroscope contain a micro mass which is connected to another housing which is located outer side of this mass.The connection of this mass to this housing is provided by a set of spring.This outer housing is connected in turn to another housing which called fixed circuit by another set of orthogonal springs. As we already stated for exploding the Coriolis phenomena, the mass is continuously driven sinusoidally along in direction of first set springs.so that any rotation of the system will produce Coriolis acceleration which is applied to micro mass. Consequently this force pushes the mass in direction of second orthogonal springs. The direction this force is dependent to direction of driving mass, if the mass driven away from rotation axis the force applied to second of spring is in opposite direction when the mass is driven toward to rotation axis.as it can be seen in following picture for detecting or measuring the Coriolis force ,some capacitor fingers is used and their capacity is proportional to Coriolis force. The orientation of these finger are along with housing mass and the rigid structure. When the Coriolis force pushes the mass, the capacity will be changed as a consequence of this force and due to the differential capacitance which is detected by these finger the measurement is provided. For concerning the direction of the force different set of sense finger came closer and hereby the direction sensing will be possible.



(a) Direction of coriolis Force

(b) Differential Capacitance in two opposit direction

Figure 5.10: Schematic of the vibratory gyroscope a illustration of Coriolis effect

As it is obvious in the figure(10), depends on the mass driven direction the Coriolis force's direction is changed and consequently the sensing finger movement has been changed. As has been observed and explained the gyroscopic and acceleration sensor are independently able to measure some parameters accelerating and angular velocity independently so that if we put together these two sensors features we will have the 6 degrees of freedoms inertial measurement unit sensor which is called IMU sensor figure(2).

5.1.8 Odometery Sensor:

Odometry sensor is the essential sensor of mobile robots, which indicates and determines the travel of robot based on the amount of wheel, has traveled.In another world can be stated that the odometry is the use of data from the motion sensor to estimate the change in position over time.Such measurement is often not very accurate due to wheel slippage, surface imperfection, and modeling errors and also this method is sensitive to errors due to the integration of velocity measurements over time to give position estimates. Nowadays motion sensor which is known a speed sensor can be found In driver assistance systems such as ABS, TCS, and ESP, motor control units use these sensors to determine the wheel speed.For clarifying the subject consider that robot has a rotary encoder (speed sensor) on its wheel.It drives forward for a time interval and going to know how far it has traveled. The speed sensor can measure the rotation of the wheel and with knowing the circumference of the wheel can calculate the traveled distance.If we want to state mathematically this scenario can be presented in the following relation:

$$D = 2\pi Rn \Rightarrow distance\,traveled \tag{5.1.1}$$

In this relation, the n is the number of wheel rotation and R is the wheel radius and D is the distance traveled by the wheel. If the vehicle is moving in forwarding direction the differential distance can be presented as the difference between the last distance and the previous distance.

$$D' = \pi 2Rn' \Rightarrow new\ distance\ traveled \tag{5.1.2}$$

$$\Delta D = D' - D \Rightarrow\ difference\ distance\ traveled\ of\ two\ measuremnet \tag{5.1.3}$$

I suppose that the robot is turning the left in this condition the right wheel travels more than the left wheel so that the turning radius for left wheel is smaller than the right one. If D_L is the left wheel travel and the D_r is the Right wheel travel the distance traveled to the center of the robot is the average of left and right. and the previous distance

$$D_c = \frac{D_L + D_r}{2} \tag{5.1.4}$$

In the following picture it is clearly demonstrated the turing of robot to the left and difference distance traveled of two wheels.

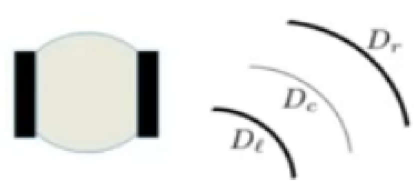


Figure 5.11: Schematic of turing of robot and wheel distance traveled

If we want to compute the new position state and heading we can demonstrate the following equation .

$$x' = x + D_c * cos(\phi) \tag{5.1.5}$$
$$y' = x + D_c * sin(\phi) \tag{5.1.6}$$
$$\phi' = \phi + \frac{D_r - D_L}{L} \tag{5.1.7}$$

In above equation, the new position of the vehicle is calculated with knowledge of vehicle parameters which is the wheelbase (L), wheel radius and knowing of the previous position of the vehicle.The parameter shown by prim is a new position and new angle which corresponds to yaw angle.That is the principle of encoder base of dead recognition and the accuracy is not very high and with increasing the time the error accumulates.
what is explained and demonstrated above is related to the robot and if we want to demonstrate for a real vehicle these relationships are so simple and the kinematic and dynamics of the vehicle must be considered.

5.2 Speed Sensor:

Wheel speed sensor is an electric or electronic device which measures the wheel speed. Design and function of wheel speed sensors: Wheel speed sensors could be categorized differently based on the mode of function or principles which are used to measure parameters.[1] Based on the functioning modes, wheel speed sensors into active and passive sensors are classified: If a sensor becomes "active" only when a power supply is connected to it and if it then generates an output signal, it is called "active".
If a sensor works without an additional power supply, then it is called "passive". in the following would be more clarified of the Active and Passive definition.

Inductive Passive Sensor:

Passive inductive sensors do not need a separate power supply from the control unit. A type of this sensors is the Variable reluctant speed sensor.

Signal Processing:

Actually, in this type of sensor by analyzing the output signal from the sensor the speed can be measured.In the following picture can be seen the Inductive passive sensor with the signal output from this sensor.

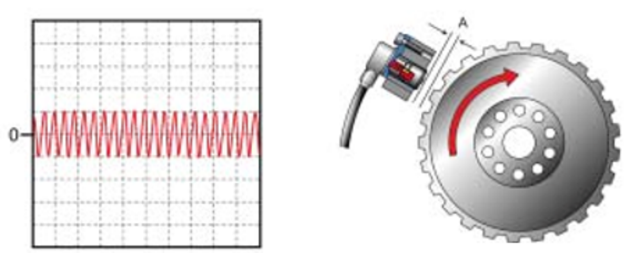


Figure 5.12: Inductive speed sensor with it's output signal

Variable Reluctance Sensors:

As already categorized, this sensor is a type of passive sensor since it does not need external energy supply source for operation. Variable Reluctance (VR) sensors as its name say, works based on magnetic flux variable.this sensor actually converts mechanical motion to an electric signal without direct contacts with a regularly moving device which must be measured its speed.This device is positioned near a gear, shaft, rotor, or other regularly moving the device.The output signal which is the consequence of flux variation can be fed by an electronic circuit.The sensor is quite accurate and can provide a reliable, simple and inexpensive transducer for highly complex control systems. Variable Reluctance sensor is composed of a coil of wire which is wound around a magnetic material usually is cylindrical and ferrous material and is referred to as a pole piece.To create a magnetic field through the coil and core material a permanent magnet is usually used which is attached to behind of the pole piece.This magnetic field which is created by permanent magnet can be extended till to tip of the pole piece which also known as the sensor tip.

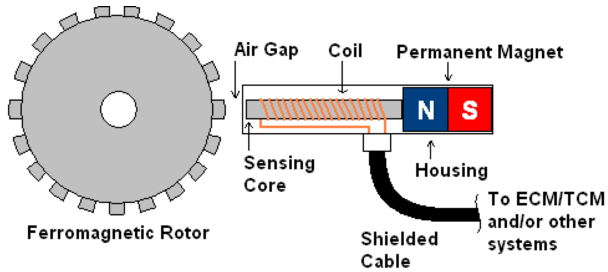


Figure 5.13: complate assembly of Reluctance variable sensor and it's output

when the toothed wheel such as gear teeth or other rotating parts pass by sensor tip, the magnetic field will be interrupted.consequently, the magnetic flux which generated by the permanent magnet will varies which is passing through to the coil and finally due to time variation of this flux a voltage will be induced in the coil. actually, the flux variation is dependent to the variation of the gap between the rotation part and sensor tip.when the gap between sensor tip and movable part is increased the magnetic flux will be increased and vice versa when the gap has decreased the flux consequently will be decreased.so that the motion of the target feature results in a time-varying flux that induces a proportional voltage in the coil. for measuring the speed of the wheel or another rotating target The amplitude and frequency of the induced voltage must be processed which they are proportional to speed. This voltage is fed as a signal to an electronic circuit for being processed and this circuit sends this signal in the desired format as output. these type of sensors have a wide industrial application such as measurement of gear tooth's speed and turbine speed measurement in a jet engine.

Advantages

- The Variable reluctance sensors due to not requirement of external power sources are passive sensors.
- these sensors composed of very inexpensive elements and parts which make these types of senor very low-cost.
- They are light weight, robust and can work in harsh (high temperature and high vibration) environments

Disadvantages

- Target material must be ferrous only.
- It is difficult to design a circuit suitable for low speed as these sensor measures the speed with the magnitude of the induced voltage.So that there is a lower limit for these sensor for measuring the speed.
- Although sensors themselves are cheap but the requirement for additional electronic circuits for processing low ampltude signal causes of cost offset.

The second type of sensor in this category is the Active sensor.As already in the definition of this category stated the active sensor is a type of sensor that needs the external power supply for operating.The Method which active sensor is operating is like proximity sensor with an electronic circuit that integrated to the sensor and which is supplied by a voltage from ABS control unit.Here in this sensor a multi-polar ring rather than a simple pulse wheel is used which is positioned in a sealing ring of the wheel bearing. Magnets with alternating poles are installed in the sealing ring.The magneto-resistors integrated into the sensor electronics detect a rotating magnetic field can be detected by the magneto-resistors in the sensor electronics when the multipolar ring rotates. The electronics system in this sensor converts the sinusoidal signal of the sensor into a digital signal (Fig.14).The sensor is connected to control unit with a two-wire electronic cable which transmits the signal output of the sensor to control unit in the form of a signal which is used pulse with modulation.These two wires are one as a signal transmitter and the other ones are used as an earth for the sensor.In addition to magneto-resistor sensor elements, Hall effect sensors are also used today permit wider air gaps and react to the smallest changes in the magnetic field.the operation of this sensor wheel is explained in following.If instead of the multipolar ring a steel pulse wheel is installed in the vehicle then a magnet must be installed on the sensor element.the consequence of pulse wheel rotating is changed in the magnetic field in the sensor.Signal processing and the IC are the same as in the case of magneto-resistive sensors.

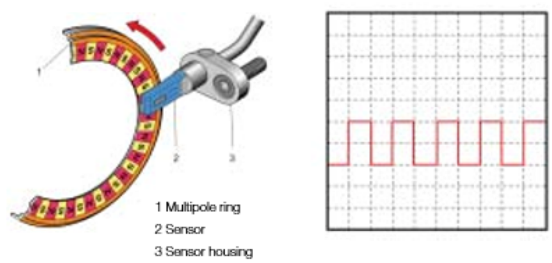


Figure 5.14: Proximity Active sensor with the output signal from this sensor.

5.2.1 Hall Effect Sensors:

Hall Effect sensor is a type of active speed sensor which uses the hall effect principle

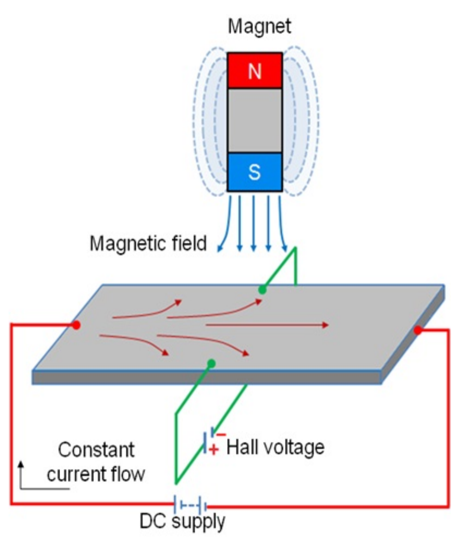


Figure 5.15: Hall Effect Phenomena and Hall Effect Voltage.

Hall Effect sensor as briefly stated in the previous section uses the magnet element on the sensor side and simple ferrous pulse wheel in the sealed part of the wheel.Hall Effect speed sensor actually is a transducer that its output varies with changing the magnetic field. As in variable reluctance sensors, the induced flux as a consequence of target movement is detected. But, a Hall effect transducer is sensitive only to the magnitude of flux; it does not sense its rate of change.The big disadvantage of Variable reluctance sensor was the disability of measuring of low speed but Hall Effect sensor smartly overcomes this problem of VR sensor.Hall Effect speed sensors are able to detect targets moving at very slow speeds, or even can detect the presence or absence of non-moving targets. It is also able to sense zero speed and the other feature of this sensor is that the target material can be either ferrous or magnetic.
Advantages

- These sensors are able to operate in wide range of temperature from -40° to 150°
- The most Hall Effect sensor provides the output which is comparable with digital logic and this characteristic of this sensor makes them be highly immune from-electromagnetic interference induced malfunction and failures.
- Signal processing electronic system is integrated to sensor transducer.

5.2.2 Eddy Current Speed Sensor:

Eddy current sensor is a non-contact device which is able to measure the position and change of any conductive target with high precision.This sensor also called inductive sensor and due to high precision make this sensor indispensable in todays modern industrial operation in dirty environments. Eddy current sensors operate with a magnetic field. Eddy Currents are the induced currents which are closed loops and circulating in the plane which is perpendicular to magnetic flux. An alternating current applied and fed to the coil winding which consequently induces a primary alternating magnetic field. This Primary magnetic field causes inducing eddy currents in the electrically conducting material (in the vicinity of the coil). Eddy currents which are induced by the primary magnetic field, in turn, induces secondary magnetic field which resists the field being generated by the coil winding in opposite direction of the primary induced magnetic field. The interaction of the magnetic field is dependent on the distance between conductive material and winding coil. Presence or absence or in another word the variation of the gap between conductive material and winding coil alters the secondary field and in turn, the coil impedance. As the distance changes the electronic system can detect this variation and produce the output voltage which is proportional to the gap between the winding coil and the conductive material.

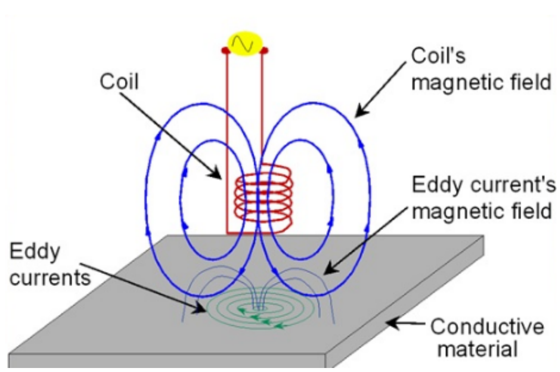


Figure 5.16: A schematic of eddy current senso.

Eddy Current principle like the variable reluctance or Hall Effect principles is exploited in speed sensors. However, they need a preamplifier or a signal conditioner to operate. When a target is present or the distance changed between the probe coil and conductive material eddy currents are formed causing a decrease in signal amplitude.The preamplifier demodulates the signal, detects the changes in voltage and produces an output whose frequency is directly proportional to the target speed, the number of blades, teeth, etc. Eddy current sensors are the position measuring devices which their output is dependent on distance and their output define the gap between the winding coil and conductive target.
Advantages:

- near zero speed response,
- no magnetic drag
- Relatively large air gaps
- Ability to sense non-ferrous metals as well as ferrous metals.

5.3 Bibliography

[1]://cars245.net [2]://www.satsleuth.com/ [3]://en.wikipedia.org/wiki/Gyroscope

Chapter 6

Modeling and derivation of the vehicle dynamics equations

6.1 System equations or vehicle dynamics model

As already presented in the previous chapter, the Kalman filter is used to improve GPS measured(estimated) position of the vehicle more precisely and accurately.Therefore, we need the dynamic model of vehicle as much as possible comprehensive to incorporate more vehicle parameters for improving the estimation of vehicle's position on the road that is more reliable, especially when satellite GPS signal is not available such as underground, tunnel or is subjected to different phenomena which have been demonstrated in GPS chapter. Here we will consider the vehicle is moving in the plane or in other words in 2D space so we just consider all dynamic parameters related to 2D dynamics of the vehicle, such as yaw rate, yaw angle, movement in x, y direction, velocity, and acceleration.The vehicle dynamics (Kinematics) model considered, is moving in constant turn rate and constant velocity magnitude model(CTRV). So the first step is to derive and express the dynamic or systems equation related to state variables. here we again rewrite the state transition (system matrix) and observation models.

$$x_k = g(x_{k-1}, u_{k-1}) + w_{k-1} \tag{6.1.1}$$

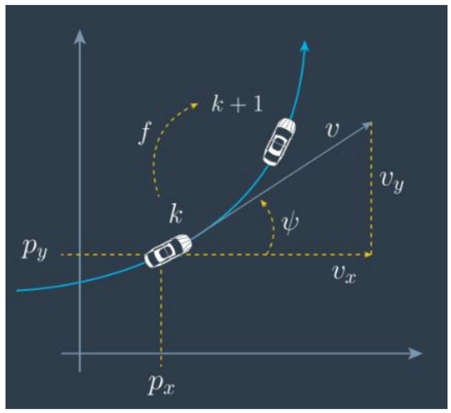
$$z_k = h(x_k) + v_k \tag{6.1.2}$$

By carefully looking at these equations, it can be seen that these equations are exactly the eqautions (3.3.1) and (3.3.2) which are presented in the chapter 3, Where w_k and v_k are the process and measurement noises respectively as already described in the previous chapter (Kalman Filter chapter) in which both are supposed to be zero mean Multivariate Gaussian noises with covariance matrix Q and R respectively. Function g which presents the dynamics of the system can be used to compute the predicted state from the previous estimate(at previous filter step) and similarly the function h which links the state variables to measurement variables, can be used to compute the predicted measurement from the predicted state or in other word from prior estimate. However, g and h due to nonlinearity cannot be applied to the covariance operation directly. Instead, a matrics of partial derivatives (the Jacobian matrix) is computed. As already has been presented at each time step, the Jacobian is computed with current predicted states.These matrices can be used in the Kalman filter equations.Infact, This process essentially linearizes the non-linear function around the current estimate. The system equation or dynamic matrix and state variable are drived as follows: As we know the system's equations are function of the control input, the state variables and time:

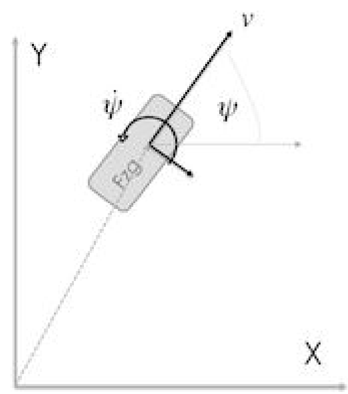
$$\frac{dx}{dt} = f(x, t, u) \tag{6.1.3}$$

$$x_{k+1} = g(x_k, t, u_k) \tag{6.1.4}$$

with assumption of control input u=0 and integration of this equation we will have: with considering of CTRV-car model as following: the state variables can be considered as x state



(a) constant turn rate and constant velocity car model with presenting of current and previuse position of vehicle



(b) constant turn rate and constant velocity car model

vectoer:

$$x_{k+1} = x_k + \int_{t_k}^{t_{k+1}} \begin{bmatrix} \dot{X}(t) \\ \dot{Y}(t) \\ \dot{\psi}(t) \\ \dot{v}(t) \\ \ddot{\psi}(t) \end{bmatrix} dt \tag{6.1.5}$$

if it is assumed that, yaw rate and velocity are constant $\ddot{\psi}(t)$=$\dot{v}(t)$=0 then we will have:

$$x_{k+1} = x_k + \begin{bmatrix} \int_{t_k}^{t_{k+1}} v(t).cos(\psi(t))dt \\ \int_{t_k}^{t_{k+1}} v(t).sin(\psi(t))dt \\ 0 \\ \dot{\psi}_k \Delta(t) \\ 0 \end{bmatrix} \tag{6.1.6}$$

(6.1.7)

(6.1.8)

$$x_{k+1} = x_k + \begin{bmatrix} \frac{v}{\dot{\psi}}(sin(\psi_k + \dot{\psi}_k \Delta t) - sin(\psi_k) \\ \frac{v}{\dot{\psi}}(-cos(\psi_k + \dot{\psi}_k \Delta t) - sin(\psi_k) \\ 0 \\ \dot{\psi}_k \Delta(t) \\ 0 \end{bmatrix} \tag{6.1.9}$$

and finally the states at the time step $x_{k+1} = g$ will be as following matrix:

$$g = \begin{bmatrix} X + \frac{v}{\psi^{\cdot}}(-sin(\psi) + sin(T\psi^{\cdot} + \psi)) \\ Y + \frac{v}{\psi^{\cdot}}(-cos(\psi) + cos(T\psi^{\cdot} + \psi)) \\ T\psi^{\cdot} + \psi \\ v \\ \psi^{\cdot} \end{bmatrix} \tag{6.1.10}$$

Since matrix g is the matrix which directly relates to the vehicle dynamics and links the estimated state variable of previous time step(filter step) to the current time step. It obviousely consists of a set of nonlinear equations and consequently the state varibale is expressed as the following vector:

$$x = \begin{bmatrix} X \\ Y \\ \psi \\ v \\ \psi^{\cdot} \end{bmatrix} = \begin{bmatrix} PositionX \\ PositionY \\ Heading \\ Velocity \\ YawRate \end{bmatrix} \tag{6.1.11}$$

where the vehicle speed (v) in heading direction (ψ) and a yaw rate sensor ($\dot{\psi}$) which are provided by velocity sensor which all have to be fused with the position (x , y) from a GPS sensor after being converted from the latitutde and longitude, together provide the information of mesurements. Here again we notice that the system is nonlinear and it's covariance not directly computable,so that it is needed to be linearised about state at previous time step. So, the jacobian of dynamic matrix with respect to state variables will be::

$$J_g = \frac{\partial g}{\partial x} = \begin{bmatrix} \frac{\partial g_{11}}{\partial X} & \frac{\partial g_{12}}{\partial Y} & \frac{\partial g_{13}}{\partial \psi} & \frac{\partial g_{14}}{\partial v} & \frac{\partial g_{15}}{\partial \dot{\psi}} \\ \frac{\partial g_{21}}{\partial X} & \frac{\partial g_{22}}{\partial Y} & \frac{\partial g_{23}}{\partial \psi} & \frac{\partial g_{24}}{\partial v} & \frac{\partial g_{25}}{\partial \dot{\psi}} \\ \frac{\partial g_{31}}{\partial X} & \frac{\partial g_{32}}{\partial Y} & \frac{\partial g_{33}}{\partial \psi} & \frac{\partial g_{34}}{\partial v} & \frac{\partial g_{35}}{\partial \dot{\psi}} \\ \frac{\partial g_{41}}{\partial X} & \frac{\partial g_{42}}{\partial Y} & \frac{\partial g_{43}}{\partial \psi} & \frac{\partial g_{44}}{\partial v} & \frac{\partial g_{45}}{\partial \dot{\psi}} \\ \frac{\partial g_{51}}{\partial X} & \frac{\partial g_{52}}{\partial Y} & \frac{\partial g_{53}}{\partial \psi} & \frac{\partial g_{54}}{\partial v} & \frac{\partial g_{55}}{\partial \dot{\psi}} \end{bmatrix} \tag{6.1.12}$$

$$= \begin{bmatrix} 1 & 0 & \frac{v}{\dot{\psi}}(-cos(\psi) + cos(T\dot{\psi} + \psi)) & \frac{1}{\dot{\psi}}(-sin(\psi) + sin(T\dot{\psi} + \psi)) & \frac{Tv}{\dot{\psi}}cos(T\dot{\psi} + \psi) - \frac{v}{\dot{\psi}^2}(-sin(\psi) + sin(T\dot{\psi} + \psi \\ 0 & 1 & \frac{v}{\dot{\psi}}(-sin(\psi) + sin(T\dot{\psi} + \psi)) & \frac{1}{\dot{\psi}}(cos(\psi) - cos(T\dot{\psi} + \psi)) & \frac{Tv}{\dot{\psi}}cos(T\dot{\psi} + \psi) - \frac{v}{\dot{\psi}^2}(cos(\psi) - sin(T\dot{\psi} + \psi) \\ 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.1.13}$$

The jacobian matrix must be computed at every time step. Now the initial state, initial state covariance(uncertainty $P0$) and the noise covariance matrix(Q) must be defined :

6.1.1 Initial States Uncertainty $P0$

As already in detail has been illustraited the variables are uncorrelated to each other which means that the covariance matrix is diagonal so it will be presented as following:

$$P0 = \begin{bmatrix} 1000 & 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 1000 \end{bmatrix} \tag{6.1.14}$$

The states uncertainty model, simulates or models the disturbances which excite the linear system .Conceptually, it estimates how bad things can get when the system is run open loop for a given period of time[1].Consequently, we will use this information to be applied to our linearised model. Thus, if we suppose that the following parameters: Maximum acceleration $a = 8.8\frac{m}{s^2}$ and maximum turn rate acceleration $\ddot{\psi} = 1.0\frac{rad}{s^2}$ and assume that they are forcing the vehicle, and then after first integrating of acceleration with respect to time to obtain velocity, and the same procedure to obtain the yaw rate and subsequently with double integration of the acceleration and turn rate acceleration respectively with respect the to time displacement and yaw angle, velocity and turn rate uncertainty will be presented with the following:

$$\begin{aligned} Gps_{uncertainty} &= \frac{1}{2}(8.8).\Delta t^2 \\ Heading_{uncertainty} &= 0.1.\Delta t \\ Velocity_{uncertainty} &= 8.8.\Delta t \\ Yaw_{uncertainty} &= 1.0.\Delta t \end{aligned}$$

so the the process noise covariance matrix as already explained will be diagonal with the values equal to squared standard deviation and consequently the the covariance process noise matrix will be as following:

$$Q = \begin{bmatrix} (Gps_{uncertainty})^2 & 0 & 0 & 0 & 0 \\ 0 & (Gps_{uncertainty})^2 & 0 & 0 & 0 \\ 0 & 0 & (Heading_{uncertainty})^2 & 0 & 0 \\ 0 & 0 & 0 & (Velocity_{uncertainty})^2 & 0 \\ 0 & 0 & 0 & 0 & (Yaw_{uncertainty})^2 \end{bmatrix} \tag{6.1.15}$$

Measurement Function definition : The H matrix links the measured parameters to state variables and depends on the desired output the H matrix is defined based upon.If assuming that the outputs are vehicle position(x,y) ,vehicle speed(v) and yaw rate ($\dot{\psi}$) we will have:

$$Measuredparameters = \begin{bmatrix} x \\ y \\ v \\ \dot{\psi} \end{bmatrix} \tag{6.1.16}$$

and Measurment matrix H will be :

$$H = \begin{bmatrix} x & 0 & 0 & 0 & 0 \\ 0 & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v & 0 \\ 0 & 0 & 0 & 0 & \dot{\psi} \end{bmatrix} \tag{6.1.17}$$

If the same procedure as the one applied to state-transition function is applied to the H matrix, we will have that jacopian of maesurement function with respect to sate variables, J_H will be as following :

$$J_H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.1.18}$$

This matrix represents that when all measurements are available except GPS measurements due to low measurements frequency with respect to IMU sensor,the corresponding values in J_H matrix must be set to zero.

6.1.2 Measurement Noise Covariance R

The R matrix is the covariance of measurement noise that is a consequence of or depends on the sensor's characteristic.It is not important that, uncertainty is absolutely correct but note that it must be relatively consistent across the all model. Usually more accurate sensors have less measurement error and inversely, less accurate sensors have high value of measurement error.Thus, the measurement covariance matrix R can be presented as following:

$$\begin{aligned} Gps_{variance} &= 6.0_{standard\ deviation\ of\ GPS\ measurement} \\ Velocity_{variance} &= 1_{standard\ deviation\ of\ the\ speed\ measurement} \\ Yawrate_{variance} &= 0.1_{standarddeviation\ of\ the\ yaw\ rate\ measurement} \end{aligned} \tag{6.1.19}$$

and measurement nois covariance is :

$$R = \begin{bmatrix} {Gps_{variance}}^2 & 0 & 0 & 0 & 0 \\ 0 & {Gps_{variance}}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & {Velocity_{variance}}^2 & 0 \\ 0 & 0 & 0 & 0 & {Yawrate_{variance}}^2 \end{bmatrix} \tag{6.1.20}$$

And finally the identity matrix must be efined as an initial value input, which is:

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.1.21}$$

Now after defining all the input data, we are able to start our Kalman filter and finally evaluate the test result of Kalman filter compared to the data obtained by Gps alone. notice that at the first all data must be converted and compatible with each other from the unit point of view. They are vehicle's position coordinate, velocity, yaw and yaw rate which the units are $meter, \frac{meter}{second}$ $,radiant, \frac{radiant}{second}$ respectively.

6.1.3 Gps Data Preparing:

As already explained in previous chapter the Global Positioning System (GPS) provides the data in 3D dimensional as longitude ,latitude, altitude and in 2D plane as longitude and latitude which are not neither desired for us nor can be fused to Kalman Filter too.So these data must be converted (prepared) to be suitable for fusing to Kalman filter. Here we are going to illustrate the GPS data conversion and plotting of these data in 2D plan which could be compared with output data from Kalman filter in next.

- **Longitude:** Longitude (λ) is a geographic coordinate that ascertains the east-west position of a point on the surface of Earth. It is an angular measurement. Meridians (lines that move from the North Pole to the South Pole) link points with the same longitude. By agreement, one of these, the Prime Meridian, which goes through the Royal Observatory, Greenwich, England, was assigned the position of zero degrees longitude.
- **latitude:** latitude (Φ) is a geographic coordinate that determines the north-south position of a point on the on the surface of Earth. Latitude is an angle (in the following picture is shown) which ranges from $0°$ at the Equator to $90°$ (North or South) at the poles. Lines of constant latitude, or parallels, go Eastwest as circles parallel to the equator. Latitude together with longitude is used to determine the precise location of features on the surface of the Earth.

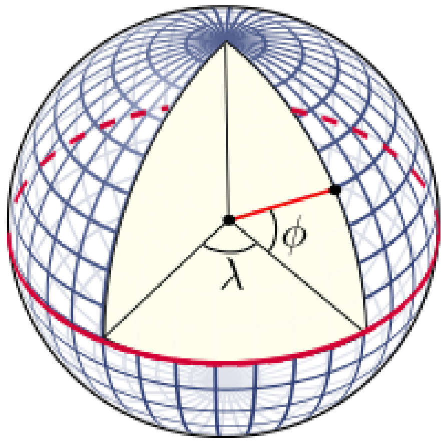


Figure 6.2: The definition of latitude (Φ) and longitude (λ) on an Earth.

After defining the latitude and longitude, the coordinate or position of our vehicle will be defined by simply conversion; thanks to algebra. Subsequently, we will have the following conversation formula which is available in *pyhton* and other programming language package so we will have:

$$EASTING - NORTHING = utm.from - latlon(Gps - lat[i], Gps - lon[i]) \tag{6.1.22}$$

wherein the presented relation which is provided by the UTM package in Python the $East$ means the position of the vehicle in X axis and $North$ means the position of the vehicle on Y axis in meter and i represents the current time measurement. Hence, the position of the vehicle in real time is converted to cartesian coordinate in the 2D plane where the real-time data are provided by GPS sensor. Notice that Positive latitude is above the equator (N), and negative latitude is below the equator (S). Positive longitude is east of the prime meridian, while negative longitude is west of the prime meridian.We plot the data provided by GPS sensor we will have the following figure which shows the vehicle path from starting point to stop piont where vehicle stop:

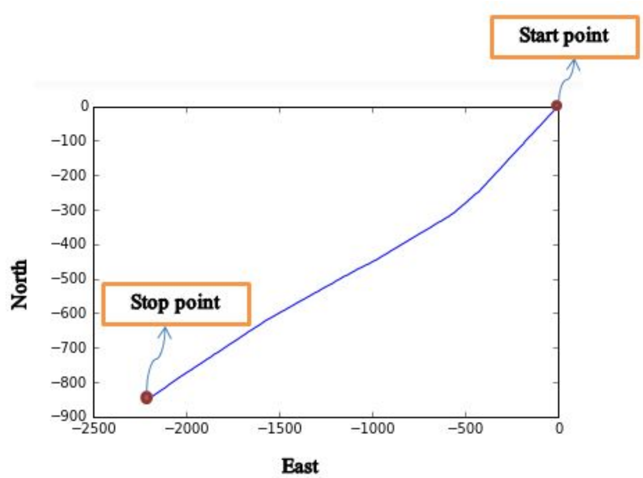


Figure 6.3: The vehcle position in cartesian coordinate(East-North) with highlited of start and stop vehicle point

6.1.4 Quaternion angle conversion:

As we know from rigid body dynamics, every rigid body can be transformed to any orientation with 3 consecutive angles which these three angles called Euler(Yaw, Pitch, Roll) angles. These angles also are required to present instantly the orientation of vehicle where the gyroscopic sensor in the IMU unit exploits this principle for vehicle orientation but due to the problem of gimbal lock in gyroscope which is responsible for rigid body rotation measurement, the Quaternion angles are used. The measured angles in the (IMU sensor) are the quaternion angle in order to overcome the problem of gimbals lock and these angles must be presented again in the form of Euler angle to be fused in Kalman filter. Following figure(4) represents the gimbal lock's phenomena in a gyroscopic sensor.

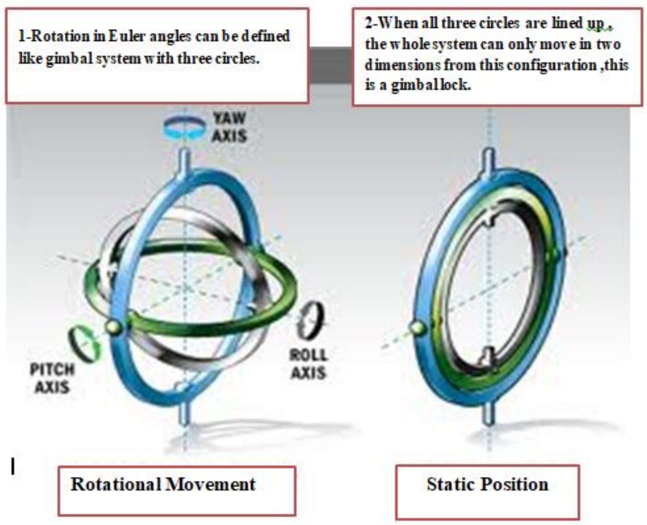


Figure 6.4: Gimbal lock incident in Gyroscop

Again thanks to algebra tools in Python Package, we are able to transform easily these two sets of angles to each other. Thus we will have the following relation for converting:
$q = [xsense\ orientation_X[i], xsense\ orientation_Y[i], xsense\ orientation_Z[i], xsense\ orientation - W[i]]$
$Euler\ angle = tf.transformations.euler\ from\ quaternion(q)$
In above illustrated relations the q represents the quaternion rotation vector in (4D space) at current time where elements in this vector presents the roation about axes which are x, y, z, w respectively, the second relation represents the Euler angle after being converted to the quaternion angles at curent time which is fused in real time to Kalman filter. In order to initiate the Kalman filter after being provided all data compatible with each other in term of dimensional and definition initial uncertainty of process and measurement noise, the initial state of variables must be provided. So at this step, we are going to determine these states as follows.

6.1.5 Initial states definition:

The initial state must be determined with consideration of all sensor information in starting time or in other word the data from starting point of the vehicle, including all data from position(x,y), velocity(v), heading(ψ)and yaw rate($\dot{\psi}$). So the initial state will be stated as the following vector:

$$initial\ state(state\ at\ (t_0)) = \begin{bmatrix} PositionX[t0] \\ PositionY[t0] \\ Heading[t0] \\ Velocity[t0] \\ YawRate[t0] \end{bmatrix} \tag{6.1.23}$$

6.2 Starting the Kalman filter and result

As Kalman Filter was already illustrated mathematically and theoretical with all consideration, here we are going to initiate practically the Kalman filter over the vehicle dynamics equations with following of all principles behind this filter with respect to prior and posterior steps.The stage after being defined all the initial conditions and prerequisite parameters which are necessary for Kalman filter simulation, is prediction of the prior state have illustrated the relationship with other parameters in the previous chapter(refer to chater 3).
Therefore, it will be again represented as following:

$$x_k^- = Ax_{k-1}^- + Bu_k \tag{6.2.1}$$

$$P_k^- = J_A P_{k-1} {J_A}^T + Q \tag{6.2.2}$$

The equstions (6.2.1) and (6.2.2) are exactly similar to the equations(3.3.21) and (3.3.22) respectively in which, $A = g$, $B = 0$ and x_k^- is prior state predicted at current filter step and in the second relation, P_k^- is prior predicted states Covariance matrix and J_A is the jacobean of system matrix and Q is the covariance matrix of process noise. As mentioned earlier, this stage is the first step of Kalman filter running, the state covariance matrix, states, and process covariance matrix must be as initial values in above equations. Notice that all data available to Kalman filter are provided by sensors in the real time. the second stage of Kalman filter processing is the Kalman gain computation according to the set equation of (3.3.24),(3.3.25) and (3.3.26) are rewritten here as the following:

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1} \tag{6.2.3}$$

$$\hat{x}_k = \hat{x}_k^- + K(z_k - H\hat{x}^-) \tag{6.2.4}$$

$$P_k = (I - K_k H)P_k^- \tag{6.2.5}$$

Note that these equations (6.2.3).(6.2.4) and (6.2.5) are exactly equal to the equations(3.3.24),(3.3.25) and (3.3.26) respectively,wherein set of equations K_k is the Kalman gain matrix in current filter step, H is the measurement matrix,R is the measurement noise covariance matrix, $\hat{x}_k$ is the posterior state (updated states) variables, z_k is the measurement vector variables including the noise . As is obvious in these set equations, after prior states at current filter step predicted, the Kalman gain must be computed by Equation(6.2.3) but take in to account the H matrix which is representative of measurement matrix,will not remain same in each filter step dut to difference measurement frequency of GPS sensor and IMU sensor which are, in this case, $GpsFrequency = 1HZ$,$IMU_{Frequency} = 400HZ$. The different measurement frequency means that the number of measurement of two sensors are not same at unit of time.In this case the IMU sensor measures 400 time more than GPS sensor per time unit. At the end of each filter step, the error covariance matrices will be updated (P_k) and together with the updated state $\hat{x}_k$ applied to prior prediction equation for next filter step until the filter stops.

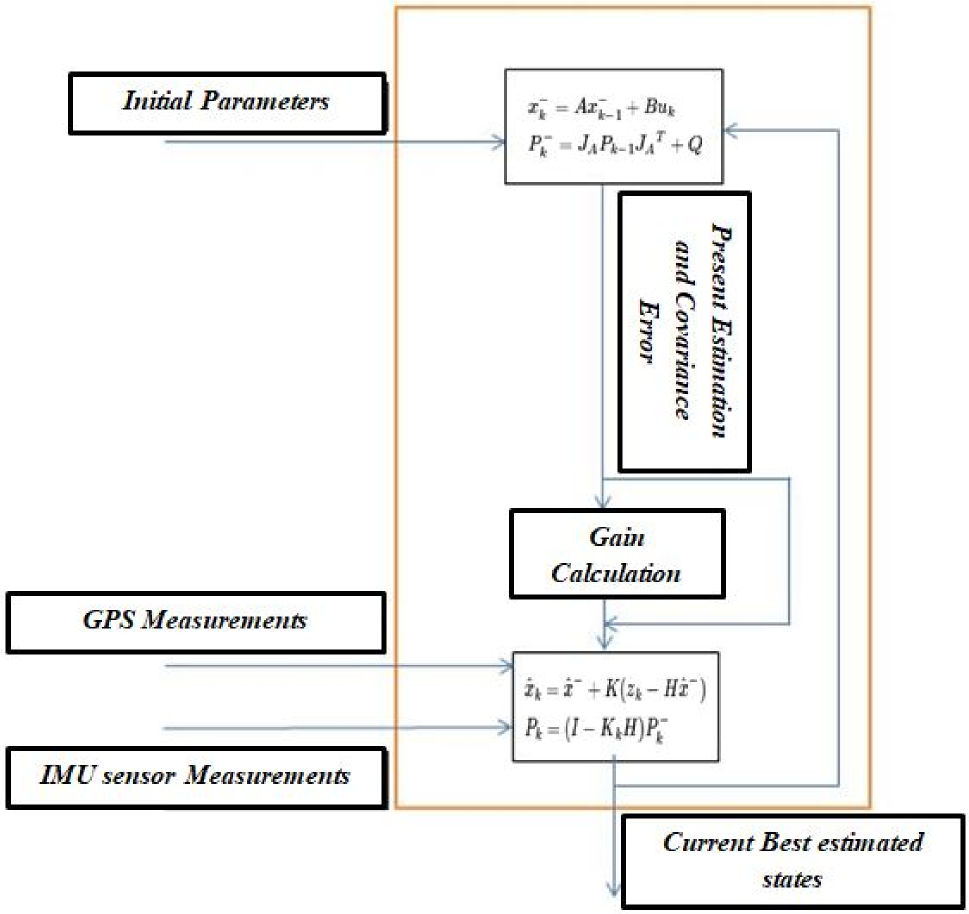


Figure 6.5: The Extended Kalman Filter flow diagram

Here again the following diagram (Ekf flow-chart), fig(6.5) clearly presents the Extended Kalman stages, as you see the flow of best estimation is started with initial conditions and goes ahead for computing the prior estimation, Kalman gain computation and finally with receiving of data from sensors, it will be updated. Consequently, This updated data will be exploited or fused as an input data to prdeiction equation for next filter step.

6.2.1 Results

After simulating and compiling the Kalman filter as presented in the previous flow-chart for entire steps for all data, the results are presented as follows.

Uncertainty(states covariances)

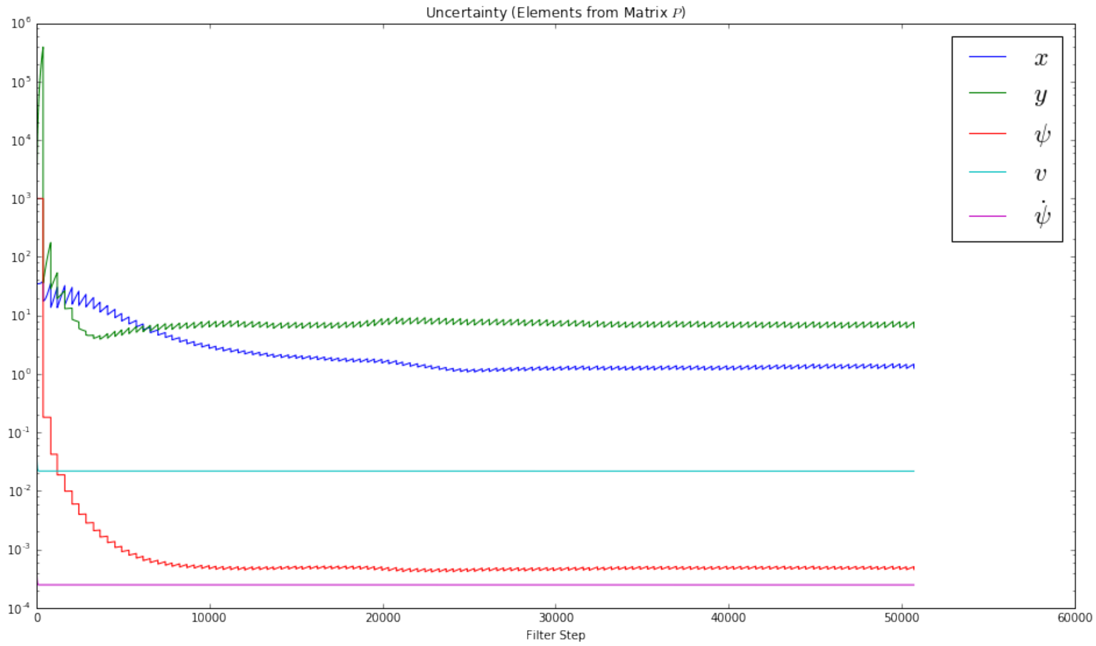


Figure 6.6: The values of uncertainty for each state at each filter step.

Figure(6.6) illustrates the state variables covariance values at each filter step and variation of these values for the entire filter running steps. As it is observed and what is desirable for us, the values of all the state variables after certain filter step converge to constant values, and as a consequence of convergence, the results or estimated values will be more reliable. Also this plot states that, the covariances which are representing the uncertainty of estimations, converge to a minimum value as the filter step goes forward, which in turn implies more accuracy of the estimation by EKF in higher filter steps.

Kalman Gains

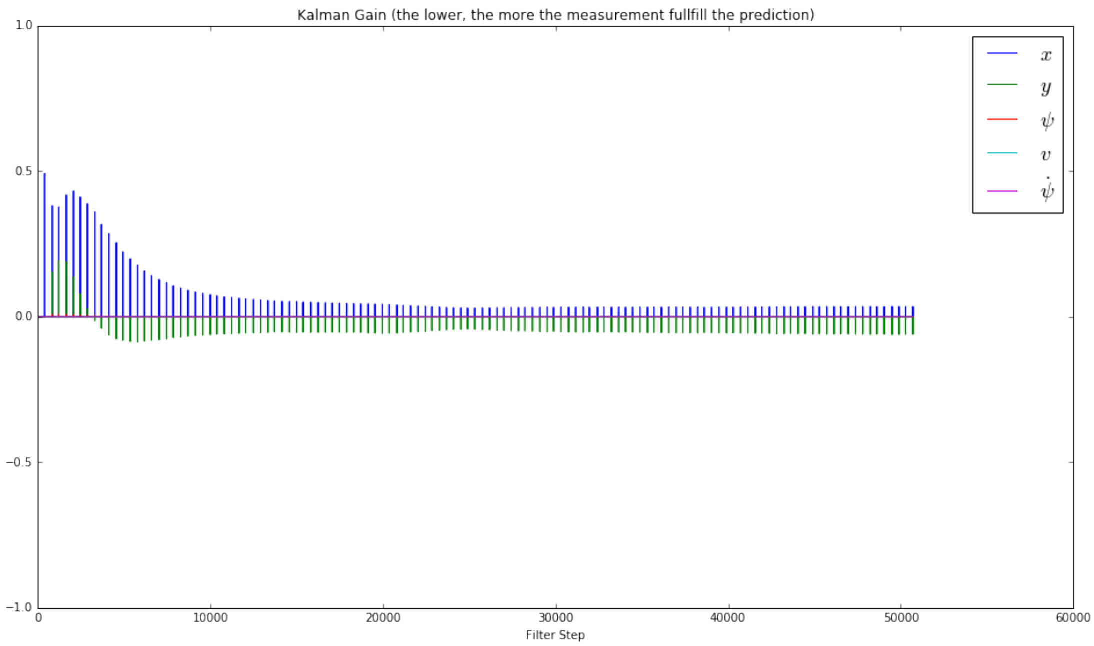


Figure 6.7: The values of Kalman Gain for each state at each filter step

This figure(6.7) demonstrates that the Kalman gain values of the state variables at each filter steps and variation of them for the entire filter steps. The trend of gain values variation are consistent with what already demonstrated in Fig (6.6) is the low value of uncertainty in the prediction means the low values of Kalman gain. Also, it shows that when the Kalman gain values are higher, means that the predicted states are less reliable and vice versa. As you observe the higher filter steps, the lower Kalman gain and the estimated values are more reliable which is consistent with previous figure (6.6) demonstration. Finally, The lower covariance values of estimated and predicted states lead to the lower values of Kalman gain which is applied to measured value or in other word to residual.

States estimation

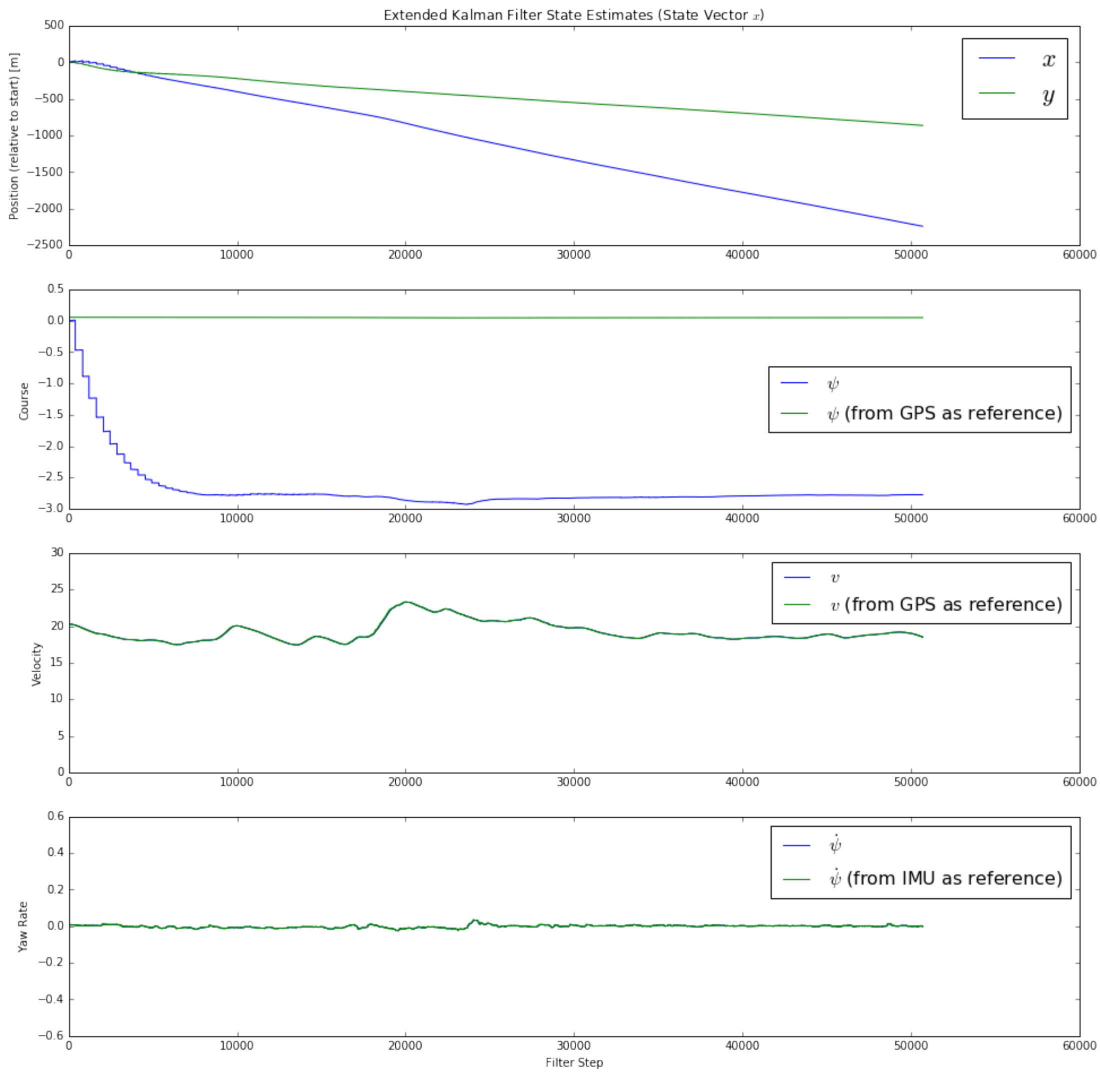


Figure 6.8: The values of state estimation at each filter step

This set of figures Fig.(6.8) represents the updated estimation or the best estimation of states in each filter step and comparison of them with the measured sensor values itself. The aim is to compare the results of EKF with sensor measured values for all state variables and to evaluate the Extended Kalman Filter accuracy in the state estimation.
For the position state values, just comparison has been carried out by previous values and since these two states are our desired states, they will be demonstrated later on, separately. For the rest of state variables, comparison has carried out with sensors measured values itself (GPS measured values for the yaw angle and velocity, while for yaw rate the comparison has been carried out with IMU sensor measured values). As clearly illustrated for the case of yaw angle (course)the difference between sensor's meaurement and estimated values are notable which are emerging the weakness of course evaluation by GPS sensor. Consequentely, as mentioned in the section of dynamics equation derivation, the position of vehicle is directly related to vehicle attitude (heading) and poor evaluation of vehicle heading, leads to inaccurate vehicle positioning

determination on the road. For the two other state variable, if the sensor measurement values assumed as reference values, the accuracy of estimation is quite considerable and will be observed that both velocity and yaw rate estimation are consistent with sensors measured values.

GPS Measured values vs Estimated values

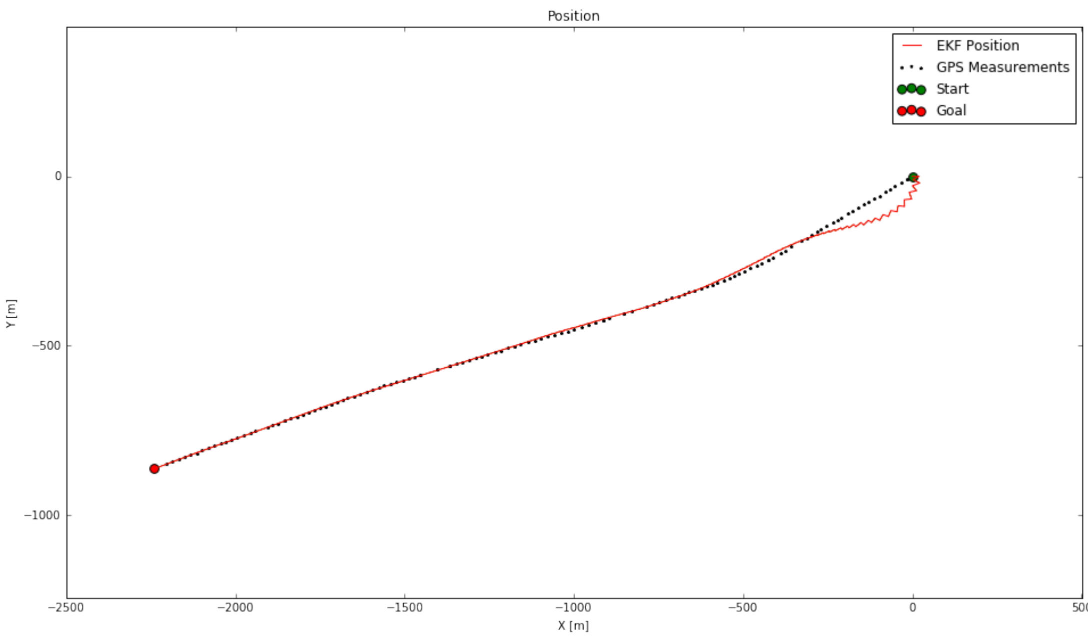


Figure 6.9: The values of estimated states for each state at each filter step vs Gps measured values

After being explained all parameters, such as Kalman gain, uncertainty,etc .., which individually affect the state estimation of EKF, and the accuracy and convergence of Extended Kalman Filter estimation while the filter goes forward, we will arrive to the point where the filter's outputs, which are postion state variables must be compared to GPS sensor self-estimated position values. In the "Global Positioning System" Chapter, we explained the weakness of GPS estimated vehicle position, the factors that influence the accuracy of GPS measurements, which motivated us to implement EKF. So that, the EKF filter's outputs,which in our case of interest are the vehicle positions being compared, to evaluate the inaccuracy of GPS measurements and effectiveness of Extended Kalman Filter. Consequently, the figure(6.9) with entire filter step running established, which shows the vehicle traveled path from starting to stoping point.In this figure,it is evident, how much the GPS values are different from the Kalman filter estimated values.

Conclusion

As a conclusion, it would be claimed that the EKF is a adequate and powerful tool for improving the vehicle positioning estimation, especially for the Autonomous vehicle and it is able to provide more accurate position estimation than the GPS sensors itself, on the road and underground situation (EUER). It has been clarified, the strength of EKF dealing with nonlinearity of vehicle dynamic equations referring to results and recursive operation of this amzing tool for the identifying and prediction of the states even if the reliable or noisy data from GPS sensor are not available.

6.3 Bibliography

[1]:A 3D State Space Formulation of a Navigation Kalman Filter for Autonomous Vehicles,Alonzo Kelly,1994.

Chapter 7

Data Fusion and Simulation

7.1 Introduction

It was mentioned in previous chapters that, each sensor has its own strengths and weakness from different point of views, such as cost or accuracy, etc, and they couldn't provide a suitable performance without any negative effect on the cost. In case of Autonomous vehicle positioning, we need as much accurate positioning data without too much influence on the cost of devices. For instance, a high-performance differential global positioning system receiver with real-time kinematics is able to provide absolute localization for an autonomous vehicle but it increases the cost too much which is not suitable for the autonomous car. However, this sensor is subjected to different effects as mentioned in the previous chapter, such as multipath effect and also not being able to fulfill the precision error correction in the wide range of driving areas or the Odometry sensor typically suffers from accumulate error which leads to highly error position estimation on the long vehicle traveled path. hence, has been decided to implement data fusion technique, by considering both cost and performance.
Let's explain what is literally the definition of data fusion. Data fusion is the process of integrating multiple data sources to produce more consistent, accurate, and useful information than that provided by any individual data source.
In this work, the fusion method in which the information data are fused is Extended Kalman Filter(EKF). Already, its principle and the mathematics operation behind of this technique in detailed accomplished. In this chapter, different contribution of sensor data with different techniques of EKF in the ROS, are considered and finally, the experimental results will be discussed. In the first contribution, we consider such a case, which was already described in the chapter of vehicle modeling, where the procedures of EKF method was established step by step with the Python script. Actually, in that case, two sets of data sources from IMU and GPS sensors fused into Extended Kalman filter. However, there was not fusion possibility of more data sources with the same variable to EKF and that was the biggest weakness of EKF followed by just Python script.
The second case is no more being implemented in Python script's EKF, but the `Robot_position_Ekf` provided package of ROS will be implemented and finllay in the third case the `Robot_localization_Ekf` package is implemented which is commpletely different from the `Robot_position_Ekf`.
lets before defining the simulation and data fusion in ROS, give an introduction of this interesting enviornment (ROS).

7.2 Robot Operating System (ROS)

What is ROS? Actually, there is not a specific definition of a ROS and all the definition could be found are almost correct. In one definition: The Robot Operating System (ROS) is a flexible framework for writing robot software. It has the collection of many useful packages and libraries to provide a tool which simplifies the task the task of creating complex and robust robot behavior across a wide variety of robotic platforms.(*www.ros.org*) **Why?** As it is obvious, creating a robot software which is truly robust and general-purpose so sophisticated.From the robots perspective, problems that seem trivial to humans often vary widely between instances of tasks and environments. Dealing with these variations is so hard that no single individual, laboratory, or institution can hope to do it on their own[1]

In another definition could be defined as the following:
What is ROS? ROS (Robot Operating System) is a BSD (Berkeley System Distribution)-licensed system for controlling robotic components from a PC. A ROS system consists of many independent nodes that are able to communicate with each other using a publish/subscribe messaging model. Keep in mind that just nodes are able to communicate.For instance, consider a particular sensor driver might be implemented as a node, which publishes sensor data in the form of messages which is streaming from. In order to use this message from that sensor node, these messages must be subscribed by other nodes, so that it could be (consumed) by any number of other nodes, including filters, loggers, and also higher-level systems such as guidance, pathfinding, etc.
Why ROS? What makes the ROS flexible is that nodes in ROS do not have to be on the same system (multiple computers) or even of the same architecture. It means that different system with different architecture is able to communicate to each other in ROS environment. For example, you could have an Arduino publishing message, a laptop subscribing to them, and an Android phone driving motor, and what makes the ROS adoptable is that it is maintained by many people thanks to being open source.
Here we are going to describe some general concepts of ROS which help to capture the technical concepts easily.

7.2.1 General Concepts

Nodes: Node is not something more than an executable file .it uses the ROS to be able to communicate to other Nodes. It is a necessary element for system parts communication.
Topics: Topics actually are names which can be dedicated to messaging names.In fact, Nodes can publish messages to a topic as well as subscribe to a topic to receive messages.Nodes are not aware of who they are communicating with instead, they are interstate into data with relevant topics.it means they are concerning of topic names, not Node names.
Message: we already stated the Node and topic definitions and said that the node has the task of publishing and subscribing message to the topic or topics.In order to the Nodes communicate with each other they need information which they publish or subscribe to topics in form of message.Actually, a message is a simple data structure, comprising typed fields. Standard primitive types (integer, floating point, Boolean, etc.) are supported, as are arrays of primitive types. Messages can include arbitrarily nested structures and arrays.
After the definition of some general concepts Lets look at the ROS system from a very high-level view. Actually, ROS starts with ROS Master, which means without a master, the ROS will not be compiled.

What is Master in ROS?
The ROS Master as the name implies is the master of all other nodes in the ROS system.Actually, naming and registration services provided by ROS master to the rest of the nodes in the ROS system. In ROS system different nodes must publish and subscribe different topic and tracking of these publishers and subscribers to the topics as well as the services are provide by ROS master.In fact, the Master has the role to enable individual ROS nodes to locate one another.Actually, they are able to communicate with each other peer-to-peer, once they have located each other by ROS master. ROS starts with the ROS Master.
The Master in ROS has the task of registration services and naming of all the nodes in the ROS system. Actually, the Master allows all other ROS pieces of software (Nodes) to nd and talk to each other by locating of each other.To clarify its task considers the following scenario:
Lets consider that for instance, we have two Nodes; a Camera node and a `Image_viewer` Node.As it is obvious, a typical sequence of events would start with Camera notifying the master that it wants to publish images on the topic `images`: like the following scenario:

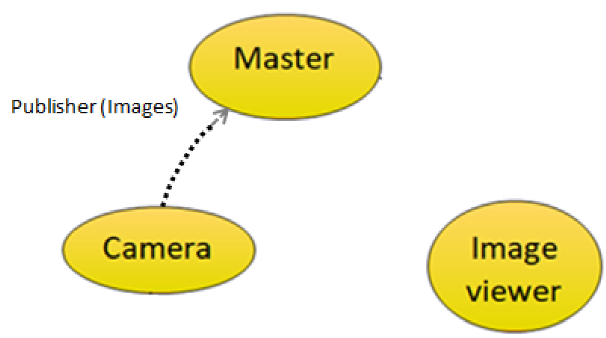


Figure 7.1: Messages publication of a Node(Camera) to ROS Master in ROS.

Now, Camera publishes images to the `images` topic, but nobody is subscribing to that topic yet so no data is actually sent. Now,`Image_viewer` node wants to subscribe to the topic`images` to see if there's maybe some images there:

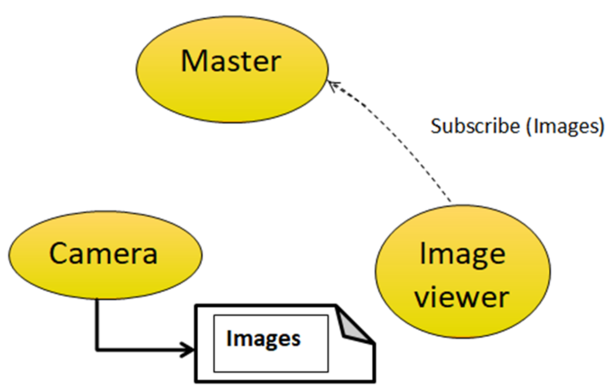


Figure 7.2: Messages subcribtion of a Node(Image viewer) from ROS Master in ROS.

Now that the `images` topic has both a publisher and a subscriber, the master node notifies Camera and `Image_viewer` about each others existence so that they can start transferring images to one another:

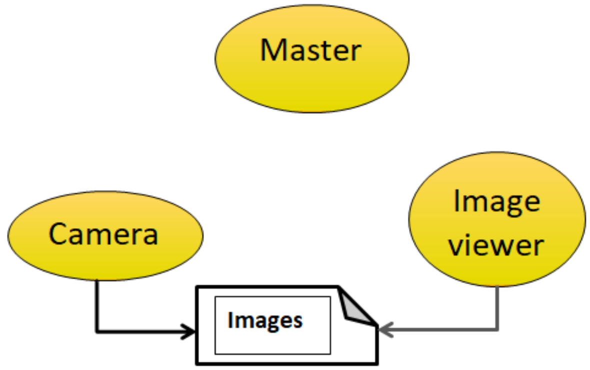


Figure 7.3: Publisher and Subcriber Nods Communication.

Lets now consider a more comprehensive situation with three nodes as a the following scenario: In this case, we have three nodes and these three nodes would communicate to each other. So here again as you see the Master plays a significant role allowing the nodes to be able to communicate by naming and registration. In fact, nodes do this task with publishing and subscribing as already in the first example accomplished.

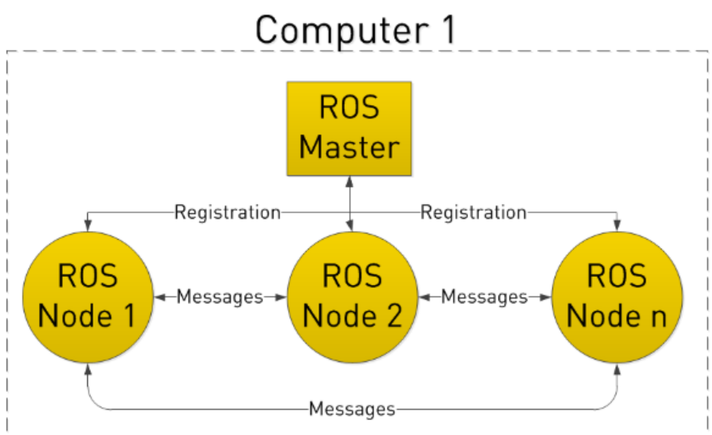


Figure 7.4: Nodes Registration by ROS master and Nodes communication in ROS

Now consider that like the first example we have a camera on our robot which can send data to camera node which subscribes the data from the camera. Our motivation is to be able to see the image both in robot and on another laptop, So that we will have an image processing Node on the robot that processes the image data, and an Image Display Node that displays image on the screen. In order to do that all nodes must be registered by MASTER. Actually, the Master tracks the location of nodes for communication. Here the laptop is considered as a node.[2]

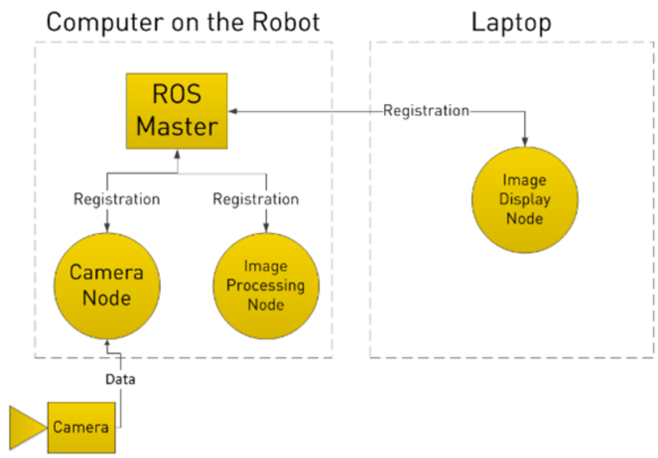


Figure 7.5: External system Registration by ROS master as a Node in ROS

In registering with the ROS Master, the Camera Node states what type of topic is going to be published and informs the ROS Master that it will publish a Topic called `/image_data`. The other Nodes which are the clients of this topics register that they are subscribed to the/ `image_data`.Thus, once the Camera Node receives some data from the Camera, it sends the `/image_data` message directly to the other two nodes. (Through what is essentially TCP/IP)
What is TCP/IP?: The NODES in ROS communicate by means of TCP/IP protocols which are two type of protocols in network and communication. the TCP (Transmission Control Protocol) is a protocol by which a transmitting messages or files being divided to packets that are transmitted over the internet and in the other side at destination these messages will be resembled. The internet protocol(IP) has the task of addressing the transmitted packets which must be sent to correct destination.TCP/IP functionality is divided into four layers, each with its own set of agreed-upon protocols:

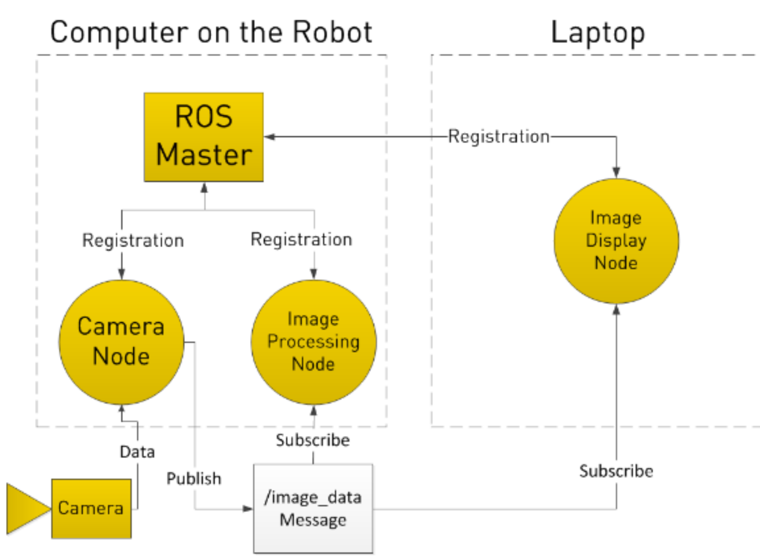


Figure 7.6: External Node (image display)communication with Camera Node

Lets consider a situation that image processing node requesting data from the camera node at the specific time.This task can be conducted by implement services. A Node can register a specific service with the ROS Master, just as it registers its messages.The following picture demonstrates this functionality of ROS system. The Image Processing Node first sends requests `/image_data`, the Camera Node collects the data from the Camera, and then sends the reply following to the Image Processing Node request.

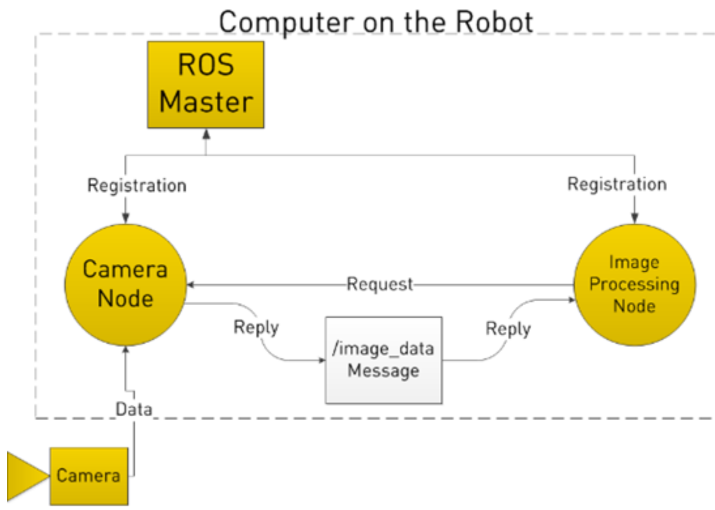


Figure 7.7: Request of and Reply to image display Node with Camera Node

So far has been stated an introduction and the general concepts of Robot Operating Systems and the ways by which nodes communicate in this open source system. These concepts have been accomplished to be captured easier what we are going to state later. We have stressed that for improving the vehicle position estimation, the different fusion techniques and methods can be used, such as data fusion methods by KF, UK, EKF, ARMA, and neural networks etc and in our investigation, we consider data fusion (multi-sensor fusion) with Extended Kalman filter which is quite accurate for the nonlinear systems. To evaluate the vehicle positioning, different combination of sensor or data provide by them are considered:

- The combination of IMU-GPS and comparison of result by GPS itself
- The combination of IMU-GPS-Odometry and comparison of result with GPS itself and IMU-GPS

In the first case(Fig.8) we investigate the vehicle positioning just by implementation of extended Kalman filter which has been accomplished by Python script (refer to cahpter3 and 6) where the data fused are provided just by two sensors, GPS and IMU. This employment is the simplest possible investigation with the minimum number of data sources which can be fused. Two type of investigation could be considered, in the first case data provided by sensors (IMU and GPS) fused to Kalman filter script which is written in python. In the second investigation, the data are fused in the ROS environment. As mentioned earlier already in "introduction" section,to be able to work with the sensors data in the ROS environment,it is necessary to IMU and GPS sensor be considered as a ROS Node and these nodes must be able to publish the measured data to the suitable topic in form of desired message. Beside these nodes, the presence of another Node is necessary to subscribe these messages which is called EKF Node. This node which is created by python script is able to subscribe both messages, provided by sensors and republishes the outputs which are desired for our consideration. The configuration of EKF with the sensors data fusion from GPS and IMU is exhibited the following architecture:

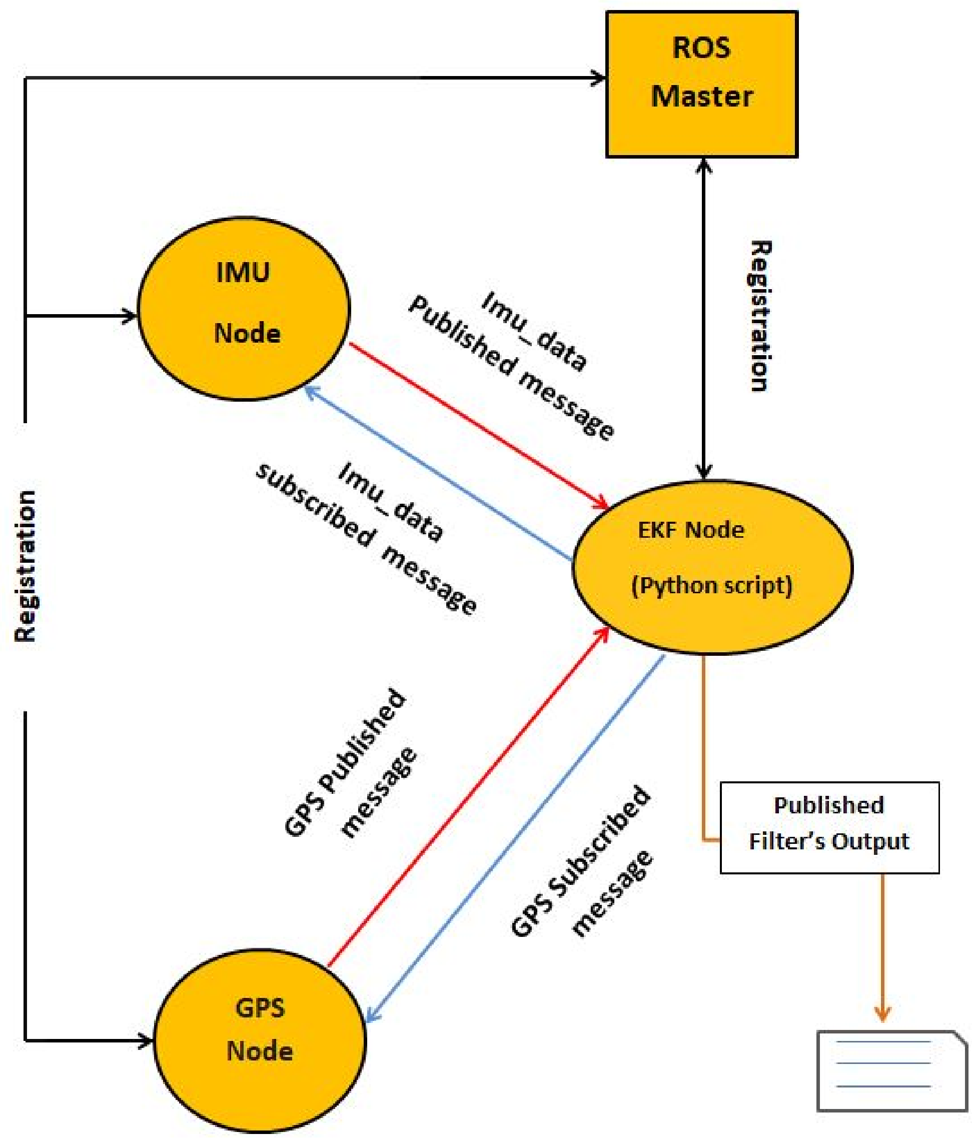


Figure 7.8: Fusion of IMU and GPS sensors data to EKF(Python script) in ROS

In the second investigation refererring to Fig (7.10) and Fig.(7.12) we employ the ROS capabilities and the available EKF packages for the purpose of data Fusion, while in turn it could be considered, in two subcases Fig(10) and Fig(7.12). In the first subcase, (Fig 7.10) the data fused to or in other world subscribed by `robot_pose_ekf` package. This package is able to subscribe three types of message `/odometry` type messages, `/IMU` type message and `/visual odometry` type messages which could be provided by three sensors IMU,GPS,and Odometry Sensors. These packages which act as a node makes possible activation and de-activation of fusing data coming from different sensors and provides real-time vehicle positioning which is desired in Autonomous vehicles. Keep in mind that the weakness of this package is that, it is just able to fuse once each type of three above mentioned message types. Another disadvantage s of this package which should be considered is that there is no parameters option for assigning the covariance values to each measured states by sensors which could be led to insufficient results. The second subcase investigation is the fusing of data to `robot_localoization_ekf` node by

```
<launch>
 <node pkg="robot_pose_ekf" type="robot_pose_ekf" name="robot_pose_ekf">
   <param name="output_frame" value="odom"/>
   <param name="freq" value="30.0"/>
   <param name="sensor_timeout" value="1.0"/>
   <param name="odom_used" value="true"/>
   <param name="imu_used" value="true"/>
   <param name="vo_used" value="true"/>
   <param name="debug" value="false"/>
   <param name="self_diagnose" value="false"/>
 </node>
</launch>
```

Figure 7.9: Robot_pose_ekf launch file example.

means of `robot_localoization_ekf` package.This package like the `robot_pose_ekf` is able to subscribe three types of messages Fig(7.11), which are odometry type messages, IMU type message, and visual odometry type message. This package which includes a node that makes it possible to enable and disable the of fusing data coming from different sensors, same as the `robot_pose_ekf` package but the difference is that this ROS's EKF package is able to fuse any number of sensors which publishe three types of above-mentioned messages, just by consecutive numbering naming of nodes which subscribe same type message Fig(7.11). This ability is provided; thanks to parameters which are able to be set to true or false in the launch file.As a result, data fusion by means of this package provides very high freedom in numbers of sensors and fusion data sources which could deliver the higher accuracy of estimated vehicle localization to obtain the desired results. Besides the all useful features of this package, the two capabilities which are not deniable,are the transformation of different sensor's coordinate frame to Earth coordinate frame and considering the reliability of input data by comparing of assigned covariance value of that variable from the individual sensors. All these features make this package distinct and Powerful comapred to other available packages in the ROS.

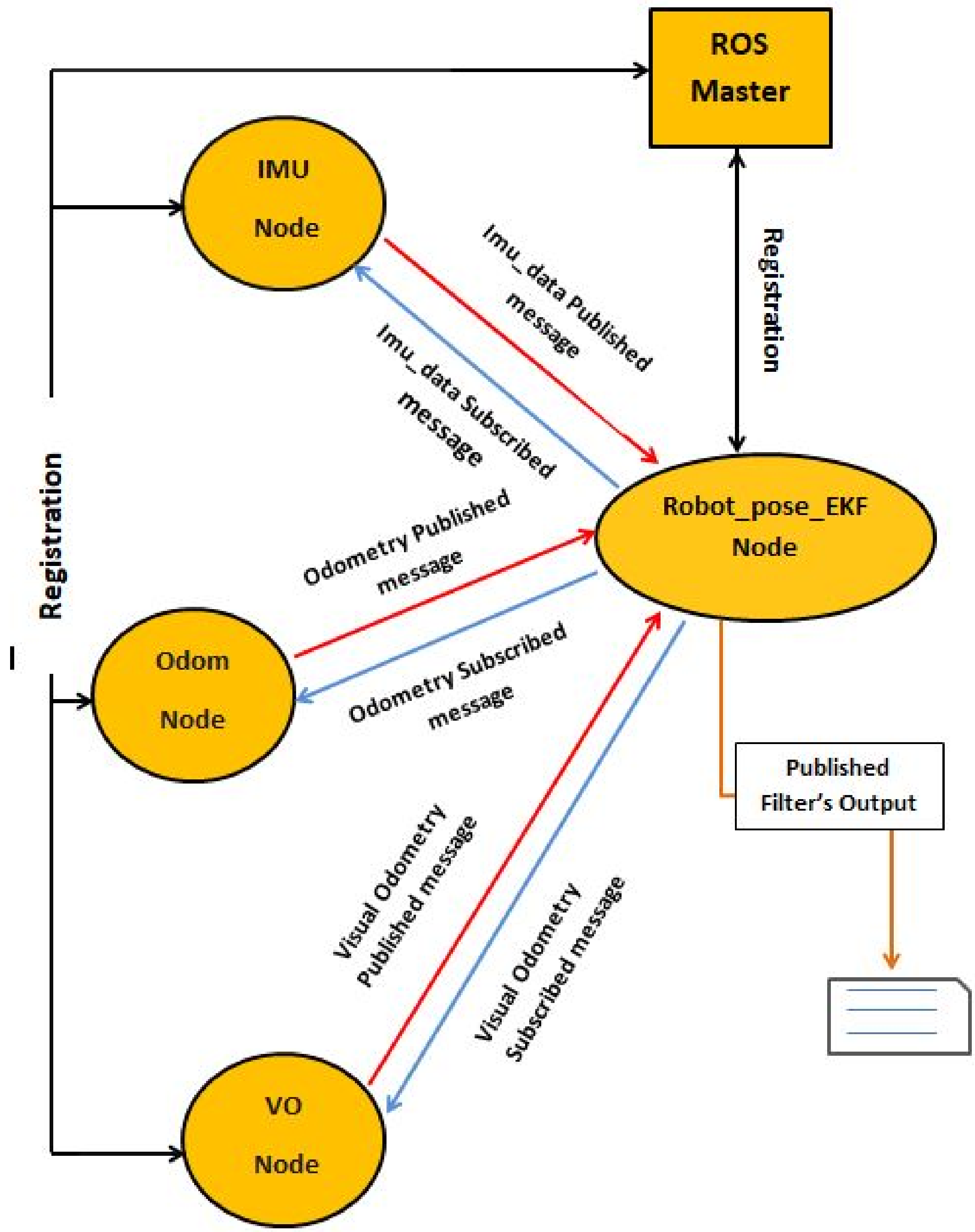


Figure 7.10: Data Fusion with Robot_pose_ekf Package.

```
<launch>

    <node pkg="robot_localization" type="ekf_localization_node" name="ekf_localization" clear_
params="true">
      <param name="frequency" value="10"/>
      <param name="sensor_timeout" value="0.2"/>
      <param name="two_d_mode" value="true"/>

      <param name="map_frame" value="map"/>
      <param name="odom_frame" value="odom"/>
      <param name="base_link_frame" value="base_link"/>
      <param name="world_frame" value="odom"/>

      <param name="odom0" value="odom"/>
      <param name="imu0" value="/imu/data"/>

      <rosparam param="odom0_config">[true,  true,  false,
                                      false, false, true,
                                      true, true, false,
                                      false, false, true,
                                      false, false, false]</rosparam>

      <rosparam param="imu0_config">[false, false, false,
                                     true,  true,  true,
                                     false, false, false,
                                     true,  true,  true,
                                     true, true, true]</rosparam>

      <param name="odom0_differential" value="true"/>
      <param name="imu0_differential" value="true"/>

      <param name="imu0_remove_gravitational_acceleration" value="false"/>

      <rosparam param="process_noise_covariance">[LARGE ...
```

Figure 7.11: Robot_localization_ekf launch file example.

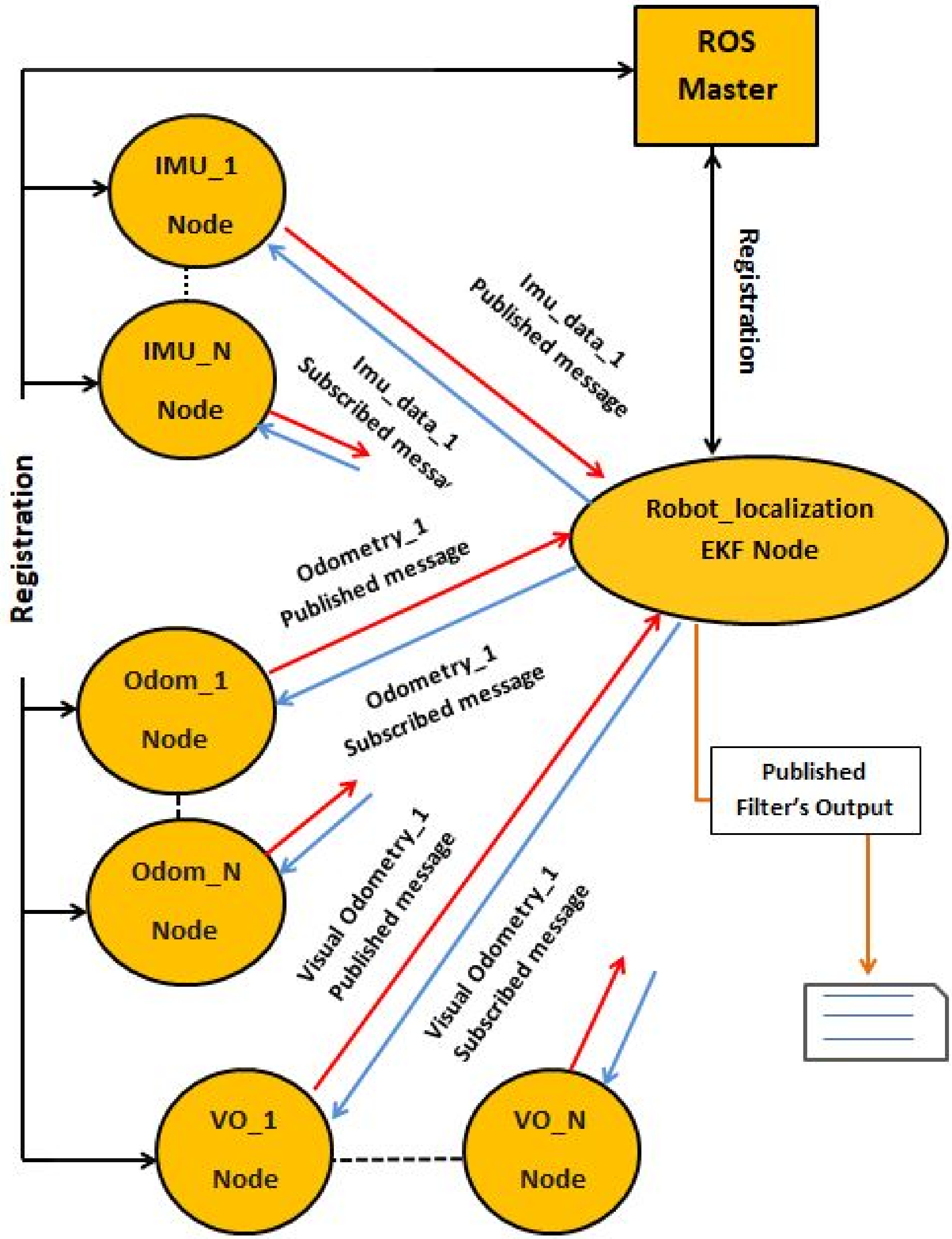


Figure 7.12: Data Fusion with Robot_localization_ekf Package.

7.3 Simulation and Experimental results

So far,it has been described and illustrated, all possible combination of the sensors data and the fusion methods of these data in ROS environment. Whitin these methods and packages which are available in ROS for data fusion,a package which has been already described Fig(7.12), is `robot_localoization_ekf` Package. This package makes possible, the fusion of different sensors's data, such as IMU,GPS and Odometry sensors.In case of vehicle localization or in another word the estimation of Autonomous vehicle positioning on the road, the all three inputs data which are provided by IMU+GPS+Odometry has been fused simultaneousely to EKF(`robot_pose_ekf`) and the result will be presented as follows: Figure 7.13 demonstrates that

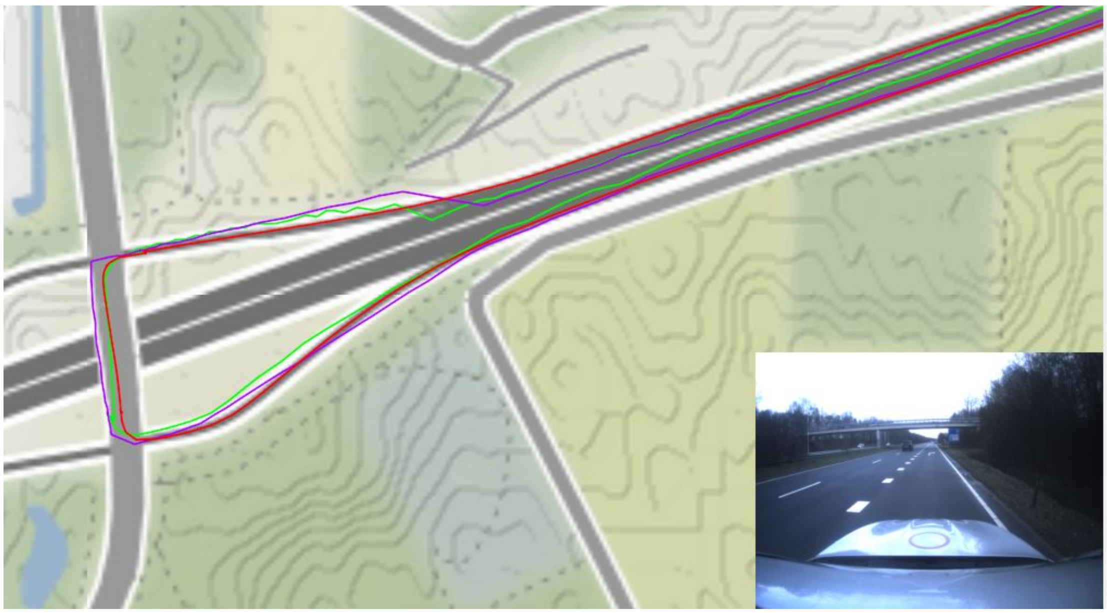


Figure 7.13: Real time Vehicle position estimation by Robot_localization_ekf in ROS.

the Fusion results we achieved are almost between ground truth and raw GPS data and almost in some cases coincide with ground truth and even in a curved segment where the vehicle changes the path and in the cornering maneuver, provides smooth and steady vehicle position estimation. Compared to ground truth, the fusion provided data in most cases are much closer to actual localization going with the driverless car.Although some diffrence can be observed between fusion results compared to raw GPS and actual data(ground-truth),but this is due to parameters set-up in the launch file specific to this package and certainly can be resolved.

7.4 Bibliography

[1]://www.ros.org [2]://www.clearpathrobotics.com

Chapter 8

Conclusion:

In this thesis, we have surveyed and demonstrated the classical and conventional methods of vehicle positioning Global Positioning System on the Autonomous Land Vehicles(ALV) and the main suspected errors of this system which have an inuence on its estimated position.To address this problem, we have proposed a multi-sensor data fusion technique for accurate navigation of autonomous vehicles based on Extended Kalman Filter.This technique has state-of-the-art features such as:(1) The fusion of multi-sensor data and the best estimation of vehicle position based on Minimization of Squared Error with consideration of all the noisy and erroneous data.(2) Dealing with non-linearity of dynamics model and providing the result which is almost close to the actual system being another powerful feature of the EKF technique.(3) In addition to these, this technique helps in determining the accurate Realtime vehicle positioning with less eect on the cost.To evaluate the estimation performance of proposed fusion technique, we employed dierent combinations of sensor data which were collected on the road in real-time by the sensors mounted on the vehicle.The results have demonstrated that our data fusion technique not only provides very accurate results comparable to ground-truth but also provides smooth and steady performance during cornering maneuvers where the low-cost GPS data shows jumping in a similiar situation.Despite all the achievements, the `robot_localoization_ekf` in the ROS provides unlimited number of sensor combinations such as IMU+GPS+Odometry which are almost impossible to implement in other techniques. The unique feature of this package is that it delivers sucient results even in the case of GPS signal loss and underground conditions(UERE).However, the accumulated position error caused by Odometry sensor is considerable which is reduced by employing it in this technique; it assigns the covariance values to the state variables measured by this sensor.Finally, with all the considerations and valid results obtained and with the current evolution of automotive technologies, EKF(Embedded) positioning systems will become more and more feasible and easily incorporable at lower cost with robust and reliable performance.

References

Chapter1
[1]Low Cost IMU GPS Integration Using Kalman Filtering for Land Vehicle Navigation Application Othman Maklouf, Abdurazag
Ghila, Ahmed Abdulla, Ameer Yousef
[2] $/wiki/Navigation_system.$
[3]Drane, C. & Rizos C. (1997). Positionng systems in intelligent transportation systems. Artech House, Boston, Ma., USA.
[4] Kim, S.-B, Lee, S.-Y.,. Choi J. al. (2003). Multi-Sensor data fusion for future telematics applications. Journal Astronomical Space Sciences 20(4), pp.359364.
[5] Farrell, J.& Barth, M. (1999). The Global Positioning System and Inertial Navigation. NY, USA McGraw-Hill.
[6] Mohander S. Grewal, Lawrance R. Weill, Angus P. Anderws Global positioning system inertial navigation system and integration, Copyright 2001 A JohnWiley&Sons, Inc.
[7],Bouvet ,D; Garcia, G. Improving the accuracy of dynamic localization systems using RTK GPS by Identifying the GPS latency. In Proceedings of the IEEE International Conference on the Robotic and Automation (ICRA), San Francisco, CA, USA, 2428 April 2000; Volume 3, pp.
[8], Karim Al-Jewari, Y.H.; Ahmad, R.B.; AlRawi, A.A.A. Impact of multipath interference and change of velocity on the reliability and precision of GPS. In Proceedings of the 2nd International Conference on the Electronic electronic Design (ICED), Penang, Malaysia, 1921 August 2014; p
[9], chen, X.; Dovis, F.; Peng, S.; Morton, Y. Comparative studies of GPS multipath mitigation methods performance. IEEE Trans. Aerosp. Electron. Syst. 2013, 49, 15551568.
[10], Oryschuk, P.; Salerno, A.; Al-Husseini, A.M.; Angeles, J. Experimental validation of an underactuated two-wheeled mobile robot. IEEE ASME Trans. Mechatron. 2009, 14, 252257.
[11], Kleeman, L. Optimal estimation of position and heading for mobile robots using ultrasonic beacons and dead-reckoning. In Proceedings of the IEEE International Conference on the Robotics and Automation, Nice, France, 1214 May 1992; Volume 3, pp. 25822587.
[12]. Yi, J.; Zhang, J.; Song, D.; Jayasuriya, S. IMU-based localization and slip estimation for skid-steered mobile robots. In Proceedings of the IEEE/RSJ International Conference on the Intelligent Robots and Systems (IROS), San Diego, CA, USA, 29 October2 November 2007; pp. 28452850.
[13] Jo, K.; Chu, K.; Sunwoo, M. GPS-bias correction for precise localization of autonomous vehicles.In Proceedings of the IEEE Intelligent Vehicles Symposium (IV), Gold Coast, Australia, 2326 June 2013;pp. 636641.
[14]. Aghili, F.; Salerno, A. Driftless 3-D attitude determination and positioning of mobile robots by integration of IMU with two RTK GPSs. IEEE ASME Trans. Mechatron. 2013, 18, 2131. [CrossRef]
[15]. Arsenault, A.; Velinsky, S.A.; Lasky, T.A. A low-cost sensor array and test platform for automated roadside mowing. IEEE ASME Trans. Mechatron. 2011, 16, 592597. [CrossRef]
[17]. Salmon, D.C.; Bevly, D.M. An exploration of low-cost sensor and vehicle model solutions for ground vehicle navigation. In Proceedings of the IEEE/ION Position Location and Naviga-

tion Symposium (PLANS), Monterey, CA, USA, 58 May 2014; pp. 462471.
[18]. Ma, J.; Susca, S.; Bajracharya, M.; Matthies, L.; Malchano, M.;Wooden, D. Robust multi-sensor, day/night 6-DOF pose estimation for a dynamic legged vehicle in GPS-denied environments. In Proceedings of the IEEE International Conference on the Robotics and Automation (ICRA), Saint Paul, MN, USA, 1418 May 2012; pp. 619626.
[19]. Tao, Z.; Bonnifait, P.; Fremont, V.; Ibanez-Guzman, J. Mapping and localization using GPS, lane markings and proprioceptive sensors. In Proceedings of the IEEE/RSJ International Conference on the Intelligent Robots and Systems (IROS), Tokyo, Japan, 37 November 2013; pp. 406412.
[20]. Gruyer, D.; Belaroussi, R.; Revilloud, M. Map-aided localization with lateral perception. In Proceedings of the IEEE Intelligent Vehicles Symposium, Dearborn, MI, USA, 811 June 2014; pp. 674680.
[21]. Simanek, J.; Reinstein, M.; Kubelka, V. Evaluation of the EKF-based estimation architectures for data fusion in mobile robots. IEEE ASME Trans. Mechatron. 2015, 20, 985990. [CrossRef]
[22]. Nemra, A.; Aouf, N. Robust INS/GPS sensor fusion for UAV localization using SDRE nonlinear filtering. IEEE Sens. J. 2010, 10, 789798. [CrossRef]
[23]. Shin, E.-H. Estimation Techniques For Low-Cost Inertial Navigation; UCGE Reports Number 20219; Department of Geomatics Engineering, University of Calgary: Calgary, AB, Canada, 2005.
[24]. Kubelka, V.; Reinstein, M. Complementary filtering approach to orientation estimation using inertial sensors only. In Proceedings of the IEEE International Conference on the Robotics and Automation (ICRA), Saint Paul, MN, USA, 1418 May 2012; pp. 599605.
[25]. Vasconcelos, J.F.; Cardeira, B.; Silvestre, C.; Oliveira, P.; Batista, P. Discrete-time complementary filters for attitude and position estimation: design, analysis and experimental validation. IEEE Trans. Control Syst. Technol. 2011, 19, 181198. [CrossRef]
[26]. Brink, K.; Soloviev, A. Filter-based calibration for an IMU and multi-camera system. In Proceedings of the IEEE/ION Position Location and Navigation Symposium (PLANS), Myrtle Beach, SC, USA, 2326 April 2012; pp. 730739.
[28]. Vincenzo Angelino, C.; Baraniello, V.R.; Cicala, L. High altitude UAV navigation using IMU, GPS and camera. In Proceedings of the 16th International Conference on the Information Fusion (FUSION), Istanbul, Turkey, 912 July 2013; pp. 647654.
[29]. Taylor, C.N. An analysis of observability-constrained Kalman filtering for vision-aided navigation. In Proceedings of the IEEE/ION Position Location and Navigation Symposium (PLANS), Myrtle Beach, SC, USA, 2326 April 2012; pp. 12401246.
[30] Silverio Bolognani, Luca Tubiana and Mauro Zigliotto. Extended Kalman filter tuning in sensorless PMSM drives. IEEE Transactions on Industry Applications, 39(6):1741-1747, November/ December 2003.
[31] Jay Farrell and Matthew Barth. The Global Positioning System and Inertial Navigation. McGraw-Hill, New York, 1998.
[32] Mohinder S. Grewal, Lawrence Weill and Angus P. Andrews. Global Positioning Systems, Inertial Navigation, and Integration. John Wiley, New York, 2001.
[33] Congwei Hu, Wu Chen, Yongqi Chen and Dajie Liu. Adaptive Kalman filtering for vehicle navigation. Journal of Global Positioning Systems, 2(1): 42-47, November 2003.
[34] D. Loebis, R. Sutton and J. Chudley. A fuzzy Kalman filter optimized using a multi-objective genetic algorithm for enhanced autonomous underwater vehicle navigation. Proceedings of the Institution of Mechanical Engineers Part M: Journal of Engineering for the Maritime Environment, 218(1):53-69, February 2004.
[35] Thomas D. Powell. Automated tuning of an extended Kalman filter using the downhill simplex algorithm. Journal of Guidance, Control and Dynamics, 25(5):901-908, September/ October 2002.
[36] Zhilan Xiong, Yanling Hao, Jinchen Wei and Lijuan Li. Fuzzy adaptive Kalman filter for

marine INS/GPS navigation. Proceedings of ICMA 2005, IEEE International Conference on Mechanics and Automation, 747-751, 2005
Chapter2.
[1]://en.wikipedia.org/wiki
Chapter3
[1] Kailath, T., Sayed, A. H., and Hassibi, B. Linear estimation, Prentice-Hall, Englewood Cliffs, NJ, 2000.
[2] Stochastic model,simulation and control,Maybeck 1979.
[3]An introduction to Kalman filter,Brown and Hwang1996.
[4] Kalman, R. E. 1960. A New Approach to Linear Filtering and Prediction Problems, Transaction of the ASMEJournal of Basic Engineering, pp. 35-45 (March 1960).
[5]Gelb, A. 1974. Applied Optimal Estimation, MIT Press, Cambridge, MA
[6] Brown, R. G. and P. Y. C. Hwang. 1992. Introduction to Random Signals and Applied Kalman Filtering, Second Edition, John Wiley and Sons, Inc.
[7]Jacobs93 Jacobs, O. L. R. 1993. Introduction to Control Theory, 2nd Edition. Oxford University Press.
[8Julier, Simon and Jeffrey Uhlman. "A General Method of Approximating Nonlinear Transformations of Probability Distributions," Robotics Research Group, Department of Engineering Science, University of Oxford
Chapter4
[1]://people.uwec.edu
[2]:(Mirsa, 1996, p.60)
[3]://e-education.psu.edu
Chapter5
[1]://cars245.net
[2]://www.satsleuth.com
[3]://en.wikipedia.org/wiki/Gyroscope
Chapter6
[1]:A 3D State Space Formulation of a Navigation Kalman Filter for Autonomous Vehicles,Alonzo Kelly,1994.
Chapter7
[1]://www.ros.org
[2]://www.clearpathrobotics.com