POLITECNICO DI TORINO

Master's Degree in AUTOMOTIVE ENGINEERING

Master's Degree Thesis

Adaptive integrate control for traction control

Supervisors

Prof. Aldo SORNIOTTI

Phd. Davide LAZZARINI

Candidate

CHEN YANG

A.Y. 2024/2025

Abstract

In this thesis, three different automotive control systems which are used to optimize the vehicle stability are compared and analyzed, including a passive system and two adaptive control systems which are using Nonlinear Model Predictive Control (NMPC) architecture. Due to the effect of the Road irregularities, vehicles generate vertical and longitudinal acceleration oscillations. It affects the ride commfort and driving safety. Therefore, the vehicle traction control system need to respond the road profile to optimize the driving comfort and safety.

In the absence of electrification in conventional internal combustion engine vehicles, the passive system is the main solution. However, with the development of electric vehicles, NMPC is an important and mature control solution strategy to solve linear and nonlinear control problems and predict future states while satisfying constraints. The objective of this study to develop an NMPC architecture with preview function for adaptive traction control system. The NMPC architecture with preview can receive road surface information previously and adjust the torque distribution compared to the NMPC architecture without preview. In comparison, the NMPC controller without preview function can only make adjustments when the road profile changes.

This case uses a four-wheel vehicle model to simulate the vehicle under four different working conditions based on the vehicle dynamic model. These road profiles include tip-in, variable friction coefficient, step and complex road condition. The performance of the three different suspension systems was evaluated, it consists the images evaluation and KPI data evaluation. In the part of image simulation. It can visually show the comparison of acceleration, tire angular velocity difference and slip ratio. In the part of KPI data evaluation, it evaluates the driving comfort and overall control performance.

The final results show that the NMPC control can effectively control the stabilization performance of the vehicle and improve vehicle safety and driving comfort. But compared with the NMPC structure without preview function, the NMPC structure with preview function can be better control the stability of the vehicle and reduce the slip ratio of the vehicle. Especially in complex working conditions, it can adaptively balance the stability and comfort of the vehicle of the vehicle to achieve the best driving state.

ACKNOWLEDGMENTS

Thanks to professors for their guidance and support during my thesis work.
Thanks to all the professors and tutors who helped me throughout my studies. I truly respect and appreciate their efforts.
Special thanks to my parents for their constant love and support.

Table of Contents

1 Introduction **1**

1.1 Motivation 1

1.2 Thesis Outline 2

2 Theoretical Framework **3**

2.1 Optimal control problem (OCP) 3

2.2 Nonlinear Model Predictive Control (NMPC) 3

3 System architecture and model design **6**

3.1 Structure environment 6

3.2 Full vehicle model 7

3.3 Vehicle dynamic 8

3.3.1 Vertical dynamic 9

3.3.2 Longitudinal dynamic 11

3.4 Enveloping model and Road preview 14

3.5 Nonlinear Model Predictive Control 16

3.5.1 Cost function 16

3.5.2 State and Control Definitions 17

3.5.3 Online Parameters 18

3.5.4 Constraints 18

3.6 Simulink 19

4 Results **20**

4.1 CASE-1: Step profile with friction coefficient drop and tip-in . . 21

4.1.1 Plot 21

4.1.2 KPI 23

4.2 CASE-2: Speed bump with friction coefficient drop and tip-in . 26

4.2.1 Plot 26

4.2.2 KPI 27

4.3 CASE-3: Step profile with variable friction coefficient drop and tip-in 30

4.3.1 Plot 30

4.3.2 KPI 32
4.4 CASE-4: B-road with friction coefficient drop and tip-in 35
4.4.1 Plot 36
4.4.2 KPI 37

5 Conclusion **41**

Bibliography **43**

List of Figures

2.1 Prediction horizon [1] 4
2.2 NMPC structure 4

3.1 Structure environment 6
3.2 Car model 9
3.3 Tire structure model 10
3.4 Enveloping model 14
3.5 Road preview 15
3.6 Structure of simulink 19

4.1 Step profile with friction coefficient drop and tip-in 22
4.2 Step profile with friction coefficient drop and tip-in 23
4.3 Step profile with friction coefficient drop and tip-in 24
4.4 Step profile with friction coefficient drop and tip-in 25
4.5 Step profile with friction coefficient drop and tip-in 25
4.6 Speed bump with friction coefficient drop and tip-in 26
4.7 Speed bump with friction coefficient drop and tip-in 28
4.8 Speed bump with friction coefficient drop and tip-in 29
4.9 Speed bump with friction coefficient drop and tip-in 29
4.10 Speed bump with friction coefficient drop and tip-in 30
4.11 Step profile with variable friction coefficient drop and tip-in . . 31
4.12 Step profile with variable friction coefficient drop and tip-in . . 33
4.13 Step profile with variable friction coefficient drop and tip-in . . 33
4.14 Step profile with variable friction coefficient drop and tip-in . . 34
4.15 Step profile with variable friction coefficient drop and tip-in . . 35
4.16 B-road with friction coefficient drop and tip-in 36
4.17 B-road with friction coefficient drop and tip-in 38
4.18 B-road with friction coefficient drop and tip-in 39
4.19 B-road with friction coefficient drop and tip-in 39
4.20 B-road with friction coefficient drop and tip-in 40

List of Tables

3.1 Main parameters in-wheels motors 8
3.2 Main parameters suspension 8
3.3 Main parameters Tire 8

4.1 CASE-1 KPI Test 23
4.2 CASE-2 KPI Test 28
4.3 CASE-3 KPI Test 32
4.4 CASE-4 KPI Test 38

Chapter 1

Introduction

1.1 Motivation

In the recent years. With the development if electric vehicles (EV) and hybrid electric vehicles (HEV). The electrification and intelligence of the automotive industry had accelerated significantly. Electrification is not only reflected in powertrain innovations, but also extends to the entire vehicle control architecture and chassis control. In the area of chassis control, since the battery system provides efficient energy support for active control,active electromechanical suspension system, braking energy recovery system, torque distribution system have been developed.These technology imposes higher demands on the vehicle's dynamic performance and meets consumer expectations for control accuracy, driving stability and comfort. This transformed the traditional chassis control method based on mechanical structure into electronic control as core, with real-time feedback capabilities of intelligent control system.This technology imposes higher demands on the vehicle's dynamic performance and meets consumer expectations for control accuracy, driving stability and comfort.
Passive suspension cannot dynamically adjust the damping and stiffness, it is difficult to achieve the multi-task optimization under complex road conditions. For example, when the vehicle passes over bumps or low-adhesion roads, the passive suspension cannot actively reduce the vehicle vibration or adjust wheel contact force. Compared with the passive suspension system, the active suspension system detect the real-time state of vehicle by using V2X sensors,and response quickly based the electronic actuator to achieve the active adjustment of vehicle performance. The development of active suspension offers a good balance between comfort and performance.
However, the jerk is a challenge in the design of active suspension. Jerk is the time derivation of acceleration, it is the key characteristic of the dynamic shock and drive comfort measurements. Some high jerk often occurs in the following

situations, Tip-in,step,and variable friction coefficient. Now days, it is often solved by the MPC and reinforcement learning (RL). About the MPC, it can deal with many variables under constraints, but it doesn't handle with the complex work condition. About the RL, it can solve optimization problems adaptively, but it needs high cost and many times to train it. So, this paper offers a preview NMPC to solve the optimization problem. This preview NMPC can obtain the anti-jerk and multi-task collaborative optimization.

1.2 Thesis Outline

The structure of this thesis is following:

- Chapter 2 introduce the theoretical framework, it includes the optimal control problem and nonlinear model predictive control.
- Chapter 3 is the system architecture and model design. It introduces the structure environment and the full vehicle model. It consists the formulations of the vehicle dynamic and the nonlinear model predictive control.
- Chapter 4 shows the results and discusses the performance of controller in different scenarios.
- Chapter 5 is the last chapter, it consists the conclusion and what the future will do.

Chapter 2

Theoretical Framework

2.1 Optimal control problem (OCP)

The OCP (optimal control problem) is the core of modern control theory and widely used for trajectory optimization in dynamic system control. The objective is to determine a control strategy to achieve optimal performance while following the constraints and dynamics of the system. For example, OCP can be used to optimize the vehicle's wheel slip ration to minimize the consumption of energy, improve the stability, and ensure the driving safety.

Cost function is needed to be calculated for the optimization performance of the controller.It's mathematical form is usually expressed as:

$$\begin{aligned} &\text{minimize} \quad J = \int_{t_0}^{t_f} L(\mathbf{x}(t), \mathbf{u}(t), t)\, dt + \Phi(\mathbf{x}(t_f)), \\ &\text{subject to} \quad \dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t), t), \end{aligned} \tag{2.1}$$

In the cost function, J is the cost function, L is the state cost, Φ is the terminal cost, u represents the control action vector, and x is the state vector. By improving the cost function, it can define a global optimal solution.

2.2 Nonlinear Model Predictive Control (NMPC)

The NMPC (Nonlinear Model Predictive Contro) is a MIMO nonlinear system. It uses the non-linear system model to predict the future state of the system and calculate the optimal control input under constraints. NMPC uses receding-horizon optimization and feedback corrections to achieve closed-loop control.In detail, NMPC operates the following steps in each control cycle:

- **Prediction**: To create the state trajectory of the time interval $[t_k, t_k + T_p]$. It presents in the $Figure$2.1.

- **Optimization**: To achieve the optimal control input $[u(t_k), u(t_k+1), \ldots]$ by solving the OCP in the time interval.
- **Execution and update**: To implement the last control input $u(t_k)$, then repeat optimization in the next cycle.

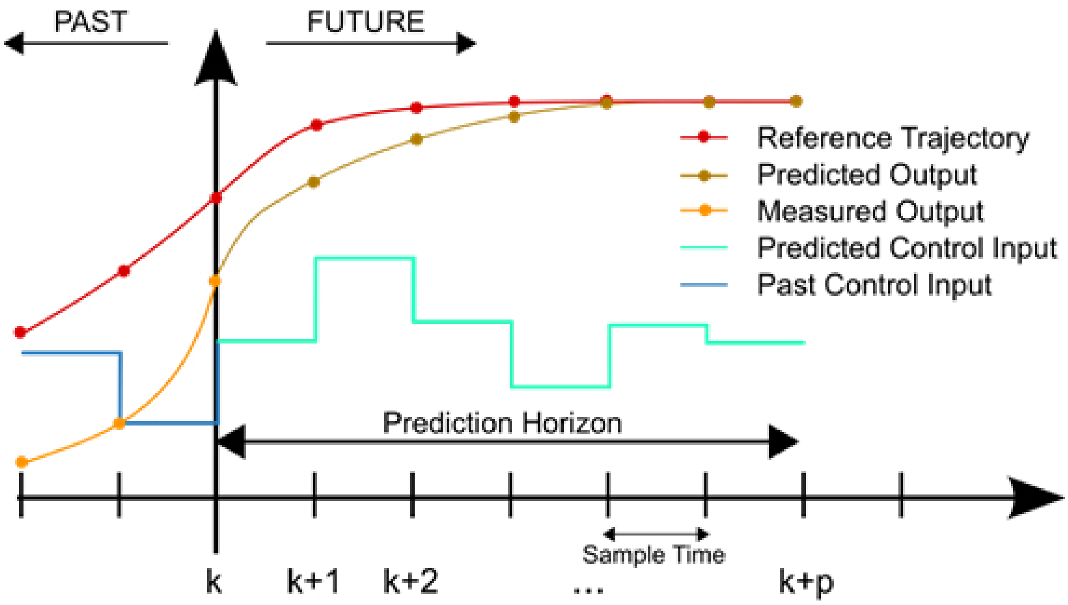

Figure 2.1: Prediction horizon [1]

The NMPC structure use a feedback loop to optimize the control of the system in real time. The controller predicts the dynamic behavior of the system in the future time according to the current state and reference trajectory, then it defines the optimal control inputs. Then it transfers the signal to the plant block, and the last state of system will be measured and fed back to the NMPC controller for update. It is a closed-loop control. The NMPC can be solved in the MATLAB. The structure is created in the Simulink. The acado toolkit can be solved the NMPC control problem to calculate the performance of the NMPC. The simple structure represents in the $Figure$2.2.

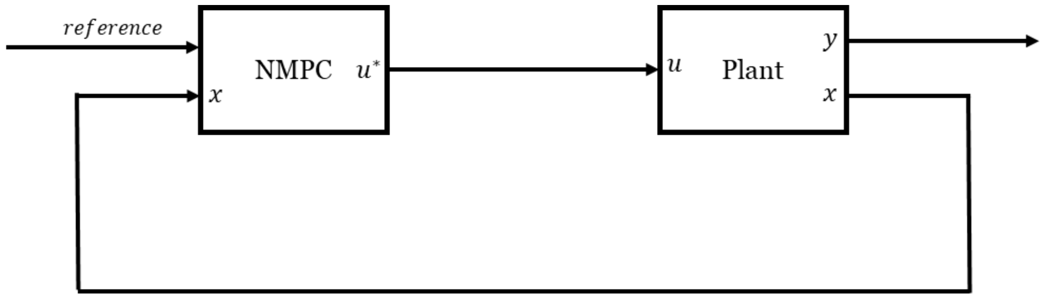

Figure 2.2: NMPC structure

Toolbox

In order to solve the NMPC problem. It needs ACADO toolkit to calculate. ACADO toolkit is a software environment and collection of algorithms for automatic control and dynamic optimization. It offers a general framework to optimize control directly by using varieties of algorithms. It includes model predictive control, state and parameter estimation and robust optimization. ACADO toolkit is implemented based on C++ code and access into MATLAB interface.The object-oriented design allows to easily couple existing optimization packages and extend them with user-written optimizations.

Chapter 3

System architecture and model design

3.1 Structure environment

The structure environment is consists of the following block. They are reported in the Fig.3.1.

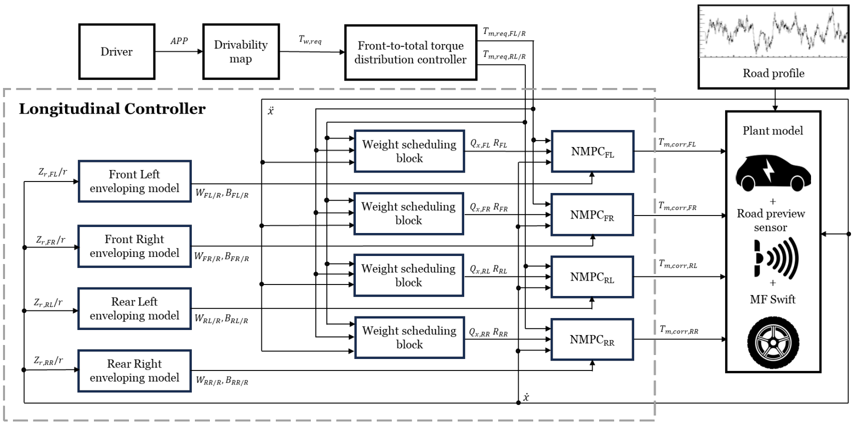

Figure 3.1: Structure environment

- The drive block, the driver define the accelerator pedal position (APP) according to the road situation.

- The drivability map converts the APP signal into the desired driving force request $T_{w,req}$ smoothly.

- The front-to-total torque distribution controller defines the distributions of the total torque demand to the four wheels. It distributes the front and rear axle torque ratio according to the vehicle dynamic requirements. The outputs are the torque for each wheel $T_{m,req,FL/FR/RL/RR}$.

- Each wheel corresponds to an enveloping model, they are used to estimate the contact characteristics between the wheel and the road. The inputs are the each tire vertical load. And the outputs is the maximum available tire adhesion. It represents the maximum traction or braking force of the wheel under the current load.

- The weight scheduling block adjust the cost function weight according to the real-time road characteristics and the information of the vehicle dynamics. The inputs are the state of system $\ddot{x}$ and the outputs of the enveloping model. The outputs of the weight scheduling block is the cost function weight $Q_{x,FL/FR/RL/RR}$, $R_{x,FL/FR/RL/RR}$. They are used to optimal the cost function for solutions.

- The NMPC calculates the optimal torque inputs for each tire. The inputs are the state vector $\dot{x}$, the desired driving force request $T_{w,req}$, cost function weight $Q_{x,FL/FR/RL/RR}$, $R_{x,FL/FR/RL/RR}$. The NMPC predicts the slip ratio in the future and minimize the cost function. Then it inserts the outputs $T_{x,FL/FR/RL/RR}$ control into the plant.

- The Road preview block uses the sensor to evaluate the road information. The outputs of road preview can evaluate the road surface change. It can help the NMPC to adjust the strategy during the prediction horizon.

- The plant model block simulates the real dynamic of vehicle and interacts with the external environment to generate state feedback for the close-loop control. It creates three outputs, the angle velocity difference, the longitudinal velocity and the slip ratio. In this block, it uses the MF-swift tire model to simulate the nonlinear characteristics of tires. Then it creates the wheel longitudinal force.

3.2 Full vehicle model

It uses a four-wheel independent drive electric vehicle to simulate. This vehicle adopts a front dual motor and rear dual motor layout and equips with an active suspension system. This vehicle supports torque vector distribution and road preview functions. The key parameters about vehicle are shown in **Table 3.1**. The **Table 3.2** and **Table 3.3** show the main parameters about the suspension and tire.

Parameter	Description	Value	Unit
m_b	Sprung mass	2789	$[kg]$
$I_{b,y}$	Vehicle body inertia	2200	$[kg \cdot m^2]$
a	Front semi-wheelbase	1.4727	$[m]$
b	Rear semi-wheelbase	1.4553	$[m]$
h_g	COG height	0.631	$[m]$
$m_{u,F}$	Front unsprung mass	30	$[kg]$
$m_{u,R}$	Rear unsprung mass	30	$[kg]$
$I_{y,F}$	Wheel inertia	1.3890	$[kg \cdot m^2]$
$I_{y,R}$	Wheel inertia	1.3890	$[kg \cdot m^2]$
R_F	Front wheel radius	0.3725	$[m]$
R_R	Rear wheel radius	0.3725	$[m]$

Table 3.1: Main parameters in-wheels motors

Parameter	Description	Value	Unit
AP_F	Anti properties percentages (5%)	0.05	$[-]$
AP_R	Anti properties percentages (5%)	0.05	$[-]$
p_inst	Front to Total distribution installation	0.5	$[-]$
$C_{x,F}$	Front damper parameter x_{axis}	1800	$[\frac{N \cdot s}{m}]$
$K_{x,F}$	Front stiffness parameter x_{axis}	600000	$[\frac{N}{m}]$
$C_{x,R}$	Rear damper parameter x_{axis}	1800	$[\frac{N \cdot s}{m}]$
$K_{x,R}$	Read stiffness parameter x_{axis}	600000	$[\frac{N}{m}]$
$K_{z,F}$	Front stiffness parameter z_{axis}	33000	$[\frac{N}{m}]$
$K_{z,R}$	Rear stiffness parameter z_{axis}	33000	$[\frac{N}{m}]$

Table 3.2: Main parameters suspension

Parameter	Description	Value	Unit
$C_{t,F}$	Tangential Front Tyre damper	50	$[\frac{N \cdot s}{m}]$
$K_{t,F}$	Tangential Front Tyre Stiffness	317634	$[\frac{N}{m}]$
$C_{t,R}$	Tangential Rear Tyre damper	50	$[\frac{N \cdot s}{m}]$
$K_{t,R}$	Tangential Rear Tyre Stiffness	317634	$[\frac{N}{m}]$
$K_{r,F}$	Radial Front Tyre Stiffness	317634	$[\frac{N}{m}]$
$K_{r,R}$	Radial Rear Tyre Stiffness	317634	$[\frac{N}{m}]$
$f_{0,F}$	Rolling resistance coefficient	0.01075	[-]
$f_{0,R}$	Rolling resistance coefficient	0.01075	[-]
$f_{2,F}$	Rolling resistance coefficient	0	[-]
$f_{2,F}$	Rolling resistance coefficient	0	[-]

Table 3.3: Main parameters Tire

3.3 Vehicle dynamic

In this section, The **Figure 3.2** shows the schematic for the front half of the vehicle. It demonstrates the dynamic of vehicle. It includes the rotational

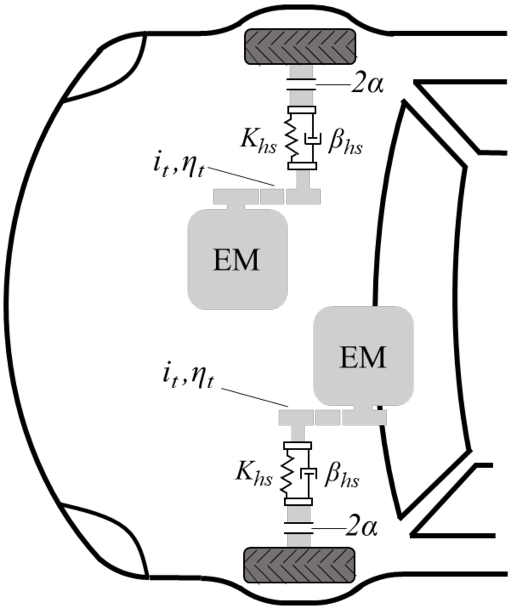

Figure 3.2: Car model

dynamic, lateral dynamic and the longitudinal dynamic. The following are the descriptions of the dynamic about the prediction models for the front half of the vehicle.

3.3.1 Vertical dynamic

It is first to define the relative displacement and speed. The ϵ_z and $\dot{\epsilon}_z$ are vertical relative displacement and vertical relative speed.

$$\begin{aligned} \epsilon_z &= z_b - z_u \\ \dot{\epsilon}_z &= \dot{z}_b - \dot{z}_u \end{aligned} \tag{3.1}$$

The following are the vertical balance of the sprung and unsprung force.

- Vertical force balance of the sprung mass

$$\begin{aligned} \ddot{z}_{b,F,L} &= (-F_{k,z,F,L} - F_{c,z,F,L}) \cdot \frac{2}{m_b} \cdot \frac{l}{b} \\ \ddot{z}_{b,F,R} &= (-F_{k,z,F,R} - F_{c,z,F,R}) \cdot \frac{2}{m_b} \cdot \frac{l}{b} \\ \ddot{z}_{b,R,L} &= (-F_{k,z,R,L} - F_{c,z,R,L}) \cdot \frac{2}{m_b} \cdot \frac{l}{b} \\ \ddot{z}_{b,R,R} &= (-F_{k,z,R,R} - F_{c,z,R,R}) \cdot \frac{2}{m_b} \cdot \frac{l}{b} \end{aligned} \tag{3.2}$$

Where the m_b is the total sprung mass. And the $F_{k,z}$ is the vertical

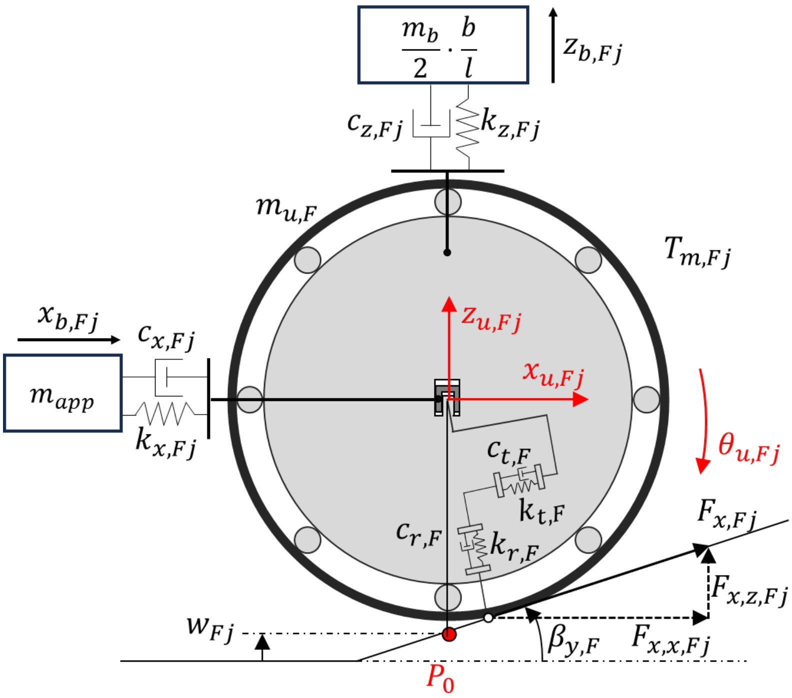

Figure 3.3: Tire structure model

stiffness force which is applied on the z-axis.

$$
\begin{aligned}
F_{k,z,F,L} &= K_{z,F} \cdot \epsilon_{z,F,L} = K_{z,F} \cdot (z_b - z_{u,F,L}) \\
F_{k,z,F,R} &= K_{z,F} \cdot \epsilon_{z,F,R} = K_{z,F} \cdot (z_b - z_{u,F,R}) \\
F_{k,z,R,L} &= K_{z,R} \cdot \epsilon_{z,R,L} = K_{z,R} \cdot (z_b - z_{u,R,L}) \\
F_{k,z,R,R} &= K_{z,R} \cdot \epsilon_{z,R,R} = K_{z,R} \cdot (z_b - z_{u,R,R})
\end{aligned}
\tag{3.3}
$$

The $F_{c,z}$ is the vertical damper force which is applied on z-axis. It is a continuous nonlinear function to approximate. a_1,b_1,b_2,c_1,c_2,d_1 and d_2 are constant coefficients.

$$
\begin{aligned}
F_{c,z,F,L} &= a_1 + b_1 \cdot atan(c_1 \cdot \dot{\epsilon}_{F,L} + d_1) + b_2 \cdot atan(c_2 \cdot \dot{\epsilon}_{F,L} + d_2) \\
F_{c,z,F,R} &= a_1 + b_1 \cdot atan(c_1 \cdot \dot{\epsilon}_{F,R} + d_1) + b_2 \cdot atan(c_2 \cdot \dot{\epsilon}_{F,R} + d_2) \\
F_{c,z,R,L} &= a_1 + b_1 \cdot atan(c_1 \cdot \dot{\epsilon}_{R,L} + d_1) + b_2 \cdot atan(c_2 \cdot \dot{\epsilon}_{R,L} + d_2) \\
F_{c,z,R,R} &= a_1 + b_1 \cdot atan(c_1 \cdot \dot{\epsilon}_{R,R} + d_1) + b_2 \cdot atan(c_2 \cdot \dot{\epsilon}_{R,R} + d_2)
\end{aligned}
\tag{3.4}
$$

- Vertical force balance of the unsprung mass

$$
\begin{aligned}
\ddot{z}_{u,F,L} &= (F_{k,z,F,L} + F_{c,z,F,L} + F_{r,z,F} - F_{t,z,F} + F_{x,z,F,L}) \cdot \frac{1}{m_{u,F}} \\
\ddot{z}_{u,F,R} &= (F_{k,z,F,R} + F_{c,z,F,R} + F_{r,z,F} - F_{t,z,F} + F_{x,z,F,R}) \cdot \frac{1}{m_{u,F}} \\
\ddot{z}_{u,R,L} &= (F_{k,z,R,L} + F_{c,z,R,L} + F_{r,z,R} - F_{t,z,R} + F_{x,z,R,L}) \cdot \frac{1}{m_{u,R}} \\
\ddot{z}_{u,R,R} &= (F_{k,z,R,R} + F_{c,z,R,R} + F_{r,z,R} - F_{t,z,R} + F_{x,z,R,R}) \cdot \frac{1}{m_{u,R}}
\end{aligned}
\tag{3.5}
$$

Where the $m_{u,F}$ is the front unsprung mass, the $m_{u,R}$ is the rear unsprung mass. Among them, the $F_{k,z}$ and $F_{c,z}$ are represented in 3.3 and 3.4. The $F_{r,z}$ is the radial force is applied on the z-axis. It depends the grounding pressure and maximum friction of the tire. $F_{t,z}$ is the tangential force is applied on the z-axis. It is caused by the unevenness of road.

$$
\begin{aligned}
F_{r,z,F} &= (w - z_u) \cdot cos(\beta_y) \cdot K_{r,F} \cdot cos(\beta_y) \\
F_{r,z,R} &= (w - z_u) \cdot cos(\beta_y) \cdot K_{r,R} \cdot cos(\beta_y) \\
F_{t,z,F} &= (w - z_u) \cdot sin(\beta_y) \cdot K_{t,F} \cdot sin(\beta_y) \\
F_{t,z,R} &= (w - z_u) \cdot sin(\beta_y) \cdot K_{t,R} \cdot sin(\beta_y)
\end{aligned}
\tag{3.6}
$$

$F_{x,z}$ is the F_x total longitudinal forces which is applied on the x axis. It is calculated by the Pacejka model. It represents that:

$$
F_x = D \cdot sin(C \cdot atan(B \cdot \sigma_x - E \cdot (B \cdot \sigma_x - atan(B \cdot \sigma_x))) + s_{vx}; F_{x,z} = F_x \cdot sin(\beta_y) \tag{3.7}
$$

3.3.2 Longitudinal dynamic

It is first to define the relative displacement and speed. The ϵ_x and $\dot{\epsilon}_x$ are longitudinal relative displacement and longitudinal relative speed.

$$
\begin{aligned}
\epsilon_x &= x_b - x_u \\
\dot{\epsilon}_x &= \dot{x}_b - \dot{x}_u
\end{aligned}
\tag{3.8}
$$

The following are the longitudinal balance of the sprung and unsprung force.

- Longitudinal force balance of the sprung mass

$$
\begin{aligned}
\ddot{x}_{b.F,L} &= \frac{1}{m_{app,F}}\Bigg[-F_{k,x,F,L} - F_{c,x,F,L} + \frac{T_{m,req,F,R}}{R_F} + \frac{T_{m,req,R,L}}{R_R} + \\
&\qquad \frac{T_{m,req,R,R}}{R_R} - F_{drag} - 0.5F_{roll,F} - F_{roll,R}\Bigg] \\
\ddot{x}_{b.F,R} &= \frac{1}{m_{app,F}}\Bigg[-F_{k,x,F,R} - F_{c,x,F,R} + \frac{T_{m,req,F,L}}{R_F} + \frac{T_{m,req,R,L}}{R_R} + \\
&\qquad \frac{T_{m,req,R,R}}{R_R} - F_{drag} - 0.5F_{roll,F} - F_{roll,R}\Bigg] \\
\ddot{x}_{b.R,L} &= \frac{1}{m_{app,R}}\Bigg[-F_{k,x,R,L} - F_{c,x,R,L} + \frac{T_{m,req,R,R}}{R_R} + \frac{T_{m,req,F,R}}{R_F} + \\
&\qquad \frac{T_{m,req,F,L}}{R_F} - F_{drag} - 0.5F_{roll,R} - F_{roll,F}\Bigg] \\
\ddot{x}_{b.R,R} &= \frac{1}{m_{app,R}}\Bigg[-F_{k,x,R,R} - F_{c,x,R,R} + \frac{T_{m,req,R,L}}{R_R} + \frac{T_{m,req,F,R}}{R_F} + \\
&\qquad \frac{T_{m,req,F,L}}{R_F} - F_{drag} - 0.5F_{roll,R} - F_{roll,F}\Bigg]
\end{aligned}
\tag{3.9}
$$

Among them, the m_{app} is the apparent sprung mass for the longitudinal force balance. It considers the unsprung masses and mass moments of inertia for rotating.

$$
\begin{aligned}
m_{app,F} &= m_b + 2 \cdot m_{u,R} + m_{u,F} + \frac{2 \cdot I_{y,R}}{R_R^2} + \frac{I_{y,F}}{R_F^2} \\
m_{app,R} &= m_b + 2 \cdot m_{u,F} + m_{u,R} + \frac{2 \cdot I_{y,F}}{R_F^2} + \frac{I_{y,R}}{R_R^2}
\end{aligned}
\tag{3.10}
$$

The $F_{k,x}$ is the longitudinal stiffness force which is applied on the x-axis.

$$
\begin{aligned}
F_{k,x,F,L} &= K_{x,F} \cdot \varepsilon_{x,F,L} = K_{x,F} \cdot (x_b - x_{u,F,L}) \\
F_{k,x,F,R} &= K_{x,F} \cdot \varepsilon_{x,F,R} = K_{x,F} \cdot (x_b - x_{u,F,R}) \\
F_{k,x,R,L} &= K_{x,R} \cdot \varepsilon_{x,R,L} = K_{x,R} \cdot (x_b - x_{u,R,L}) \\
F_{k,x,R,R} &= K_{x,R} \cdot \varepsilon_{x,R,R} = K_{x,R} \cdot (x_b - x_{u,R,R})
\end{aligned}
\tag{3.11}
$$

The $F_{c,x}$ is the longitudinal damper force which is applied on the x-axis.

$$
\begin{aligned}
F_{c,x,F,L} &= C_{x,F} \cdot \epsilon_{x,F,L} = C_{x,F} \cdot (\dot{x}_b - \dot{x}_{u,F,L}) \\
F_{c,x,F,R} &= C_{x,F} \cdot \epsilon_{x,F,R} = C_{x,F} \cdot (\dot{x}_b - \dot{x}_{u,F,R}) \\
F_{c,x,R,L} &= C_{x,R} \cdot \epsilon_{x,R,L} = C_{x,R} \cdot (\dot{x}_b - \dot{x}_{u,R,L}) \\
F_{c,x,R,R} &= C_{x,R} \cdot \epsilon_{x,R,R} = C_{x,R} \cdot (\dot{x}_b - \dot{x}_{u,R,R})
\end{aligned}
\tag{3.12}
$$

And R_F and R_R are the radius of the front and the rear wheel. The

$T_{m,req,ij}$ is the requested torque at each wheel. F_{drag} is the aerodynamic drag force, where where ρ_{air} is the air density, C_d is the aerodynamic drag coefficient, and A_{car} is the frontal area of the vehicle.

$$F_{drag} = \frac{1}{2}\rho_{\text{air}} C_d A_{\text{car}} \dot{x}_b^2 \tag{3.13}$$

The $F_{roll,F}$ and $F_{roll,R}$ is the The front and rear rolling resistance forces. The $f_{roll,F}$ and $f_{roll,R}$ are the rolling resistance coefficient. f_0 and f_2 are the constant coefficients.

$$\begin{aligned} f_{roll,F} &= f_{0,F} + f_{2,F} \cdot \dot{x}_b^2 \\ f_{roll,R} &= f_{0,R} + f_{2,R} \cdot \dot{x}_b^2 \\ F_{roll,F} &= (f_{0,F} + f_{2,F}\dot{x}_b^2) \cdot m_{\text{total}} \cdot \frac{b}{L} \cdot g \\ F_{roll,R} &= (f_{0,R} + f_{2,R}\dot{x}_b^2) \cdot m_{\text{total}} \cdot \frac{a}{L} \cdot g \end{aligned} \tag{3.14}$$

- Longitudinal force balance of the unsprung mass

$$\begin{aligned} \ddot{x}_{u,F,L} &= \frac{1}{m_{u,F}} \cdot (F_{k,x,F,L} + F_{c,x,F,L} - F_{r,x,F} - F_{t,x,F} + F_{x,x,F,L}) \\ \ddot{x}_{u,F,R} &= \frac{1}{m_{u,F}} \cdot (F_{k,x,F,R} + F_{c,x,F,R} - F_{r,x,F} - F_{t,x,F} + F_{x,x,F,R}) \\ \ddot{x}_{u,R,L} &= \frac{1}{m_{u,R}} \cdot (F_{k,x,R,L} + F_{c,x,R,L} - F_{r,x,R} - F_{t,x,R} + F_{x,x,R,L}) \\ \ddot{x}_{u,R,R} &= \frac{1}{m_{u,R}} \cdot (F_{k,x,R,R} + F_{c,x,R,R} - F_{r,x,R} - F_{t,x,R} + F_{x,x,R,R}) \end{aligned} \tag{3.15}$$

Where the $m_{u,F}$ is the front unsprung mass, the $m_{u,R}$ is the rear unsprung mass. Among them, the $F_{k,x}$ and $F_{c,x}$ are represented in 3.10 and 3.11. The $F_{r,x}$ is the radial force is applied on the x-axis. $F_{t,z}$ is the tangential force is applied on the z-axis.

$$\begin{aligned} F_{r,x,F} &= (w - z_u) \cdot cos(\beta_y) \cdot K_{r,F} \cdot sin(\beta_y) \\ F_{r,x,R} &= (w - z_u) \cdot cos(\beta_y) \cdot K_{r,R} \cdot sin(\beta_y) \\ F_{t,x,F} &= (w - z_u) \cdot sin(\beta_y) \cdot K_{t,F} \cdot cos(\beta_y) \\ F_{t,x,R} &= (w - z_u) \cdot sin(\beta_y) \cdot K_{t,R} \cdot cos(\beta_y) \end{aligned} \tag{3.16}$$

The $F_{x,x}$ is the longitudinal component of F_x.

$$F_{x,x} = F_x \cdot cos(\beta_y) \tag{3.17}$$

3.4 Enveloping model and Road preview

The Enveloping Model is a geometric filtering model used to process raw pavement elevation data to generate an equivalent effective road profile for tire-pavement contact. It use Schmeitz's tandem enveloping model to implement.[2]. It is to solve the limitations of raw road surface data. It can not get the detailed data by using the pacejka model and MF-swift to calculate. Because the tire contact is not smooth due to elastic deformation. And the NMPC require the accurate information on the longitudinal forces, which is not directly available from raw data.

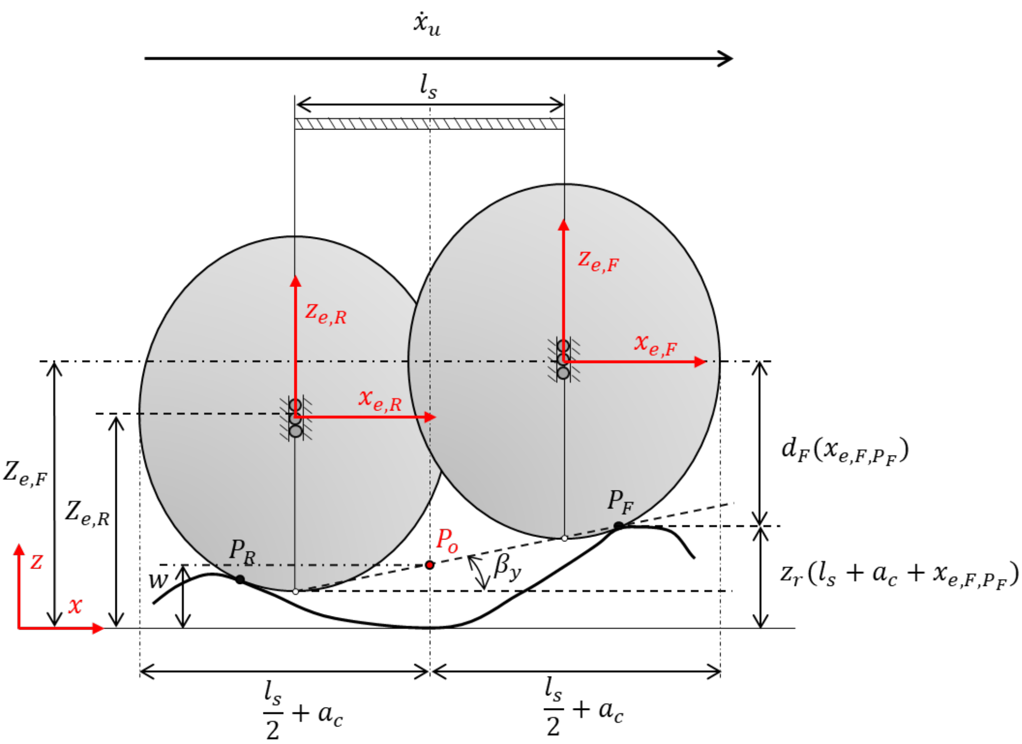

Figure 3.4: Enveloping model

In the $Figure$3.3, This is the tire-road contact model.It is consists of two elliptical cams of equal rigidity with a fixed lateral spacing of l_s.l_s is a fixed horizontal distance. The ellipsoid moves longitudinally with the wheels. It independently moves according to the road surface evaluation. In the control system, the input is the raw pavement elevation, the output w and β_y are provided as external inputs to the NMPC controller. The w is the equivalent pavement height at the point P_0 which is under the middle of tire. The β_y is the equivalent longitudinal slope in the tire contact zone.

$$\begin{aligned} u(x_u) &= \frac{Z_{e,F} + Z_{e,R}}{2} - b_c \\ tan\beta_y(x_u) &= \frac{Z_{e,F} + Z_{e,R}}{l_s} \end{aligned} \tag{3.18}$$

where b_c is the vertical semi-axis of the ellipses. And a_c is the horizontal ellipse's semi-axis, c is the ellipse shape parameter,$x_{e,F}$ and $z_{e,F}$ are the front ellipse's local axis system.

$$(\frac{x_{e,F}}{a_c})^2 + (\frac{z_{e,F}}{b_c})^2 = 1 \tag{3.19}$$

The bottom profile of the front ellipse, d_F, it can expressed as a function of: $x_e, f_c \in [-a_c, a_c]$:

$$d_F(\mathbf{x}_{e,F}) = b_c \left\{ 1 - \left[\frac{\mathbf{x}_{e,F}}{a_c} \right]^c \right\}^{\frac{1}{c}} \tag{3.20}$$

Then it uses the similar methods to calculated the corresponding distance for the rear ellipse d_r. $Z_{e,F}$ and $Z_{e,R}$ are obtained as the highest values of the combination of the road height z_r. The distances d_{fc} and d_{rc} across the possible range for $x_{e,fc}$ and $x_{e,rc}$:

$$\begin{aligned} &Z_{e,fc} = \max\left(z_r\left(x_u, x_{e,fc}\right) + d_{fc}\left(x_{e,fc}\right)\right) \\ &x_{e,fc} \in [-a_c, a_c] \\ &Z_{e,rc} = \max\left(z_r\left(x_u, x_{e,rc}\right) + d_{rc}\left(x_{e,rc}\right)\right) \\ &x_{e,rc} \in [-a_c, a_c] \end{aligned} \tag{3.21}$$

where z_r is the position of the longitudinal wheel, and it is considered as the longitudinal position along the local axis of the respective ellipse.

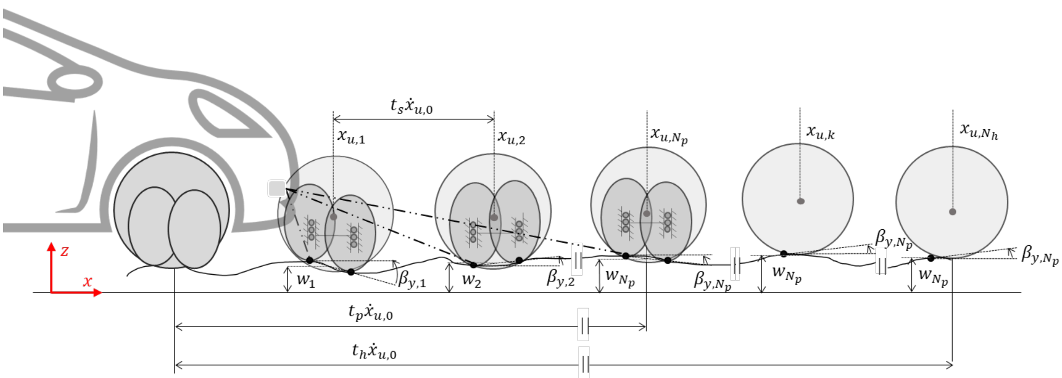

Figure 3.5: Road preview

In the *Figure*3.4, this is a simplified model of the road preview. The enveloping model offers the road height and slope information into the nmpc prediction model. β_y and w are along the preview time $t_p = N_p \cdot t_s$, where N_p is the number of preview points, and t_s is the sample time steps. In this case, the t_h is very short, it assumes that it doesn't have speed variation. Then, the vehicle speed which is along the prediction steps can be calculated as:

$$x_{u,k} = x_{u,0} + \dot{x}_{u,0} t_s k \tag{3.22}$$

where the $x_{u,0}$ is the initial position, and $\dot{x}_{u,0}$ is the initial longitudinal speed. The k is the time step. If $t_h > t_p$, the effective road data keep constant and equal to the final measured value of the prediction section.

$$\begin{aligned} W_{F,L} &= [w_0, w_1, ..., w_{N_p-1}, w_{N_p}, ..., w_{N_p}] \\ B_y &= [\beta_{y,0}, \beta_{y,1}, ..., \beta_{y,N_p-1}, \beta_{y,N_p}, ..., \beta_{y,N_p}] \end{aligned} \tag{3.23}$$

3.5 Nonlinear Model Predictive Control

This part is the explanation of each part of the NMPC. The design of NMPC consists of the state vector, the control actions, the cost function and the constraints. All the parts are designed in the Simulink.

3.5.1 Cost function

At each time step j_c, the NMPC algorithm determines the optimal sequence of control inputs U that minimizes a given cost function. The system constraints are also considered to predict the system dynamics within a predefined time horizon t_h. The discretized form of the optimal control problem formulation is:

$$\begin{aligned} \underset{u}{\operatorname{argmin}}\, J &= J_{\text{terminal}} + J_{\text{stage}} \\ &= \frac{1}{2} \left\| z_{N_h} - Z_{\text{ref},N_h} \right\|_{Q_x}^2 + \frac{1}{2} \sum_{k=j_c}^{j_c+N_h-1} \left[\left\| z_k - Z_{\text{ref},k} \right\|_{Q_x}^2 + \left\| U_k \right\|_R^2 \right] \end{aligned} \tag{3.24}$$

Subject to:

$$x_0 = x_{\text{int}} \tag{3.25}$$

$$x_{k+1} = f_d\left(x_k, u_k, p_k\right) \tag{3.26}$$

$$z_k = g_d(x_k, u_k, p_k) \tag{3.27}$$

$$U_{\min} \leq U_k \leq U_{\max} \tag{3.28}$$

where J consists of a terminal cost J_{terminal} and a stage cost J_{stage}. The terminal cost J_{terminal} aims to minimize the response error at the end of t_h, while the stage cost J_{stage} optimizes the response along t_h. k represents one step along the prediction horizon t_h, N_h is the number of steps, i.e., $t_h = N_h * t_s$, where t_s is the sampling time used to discretize OCP. z is vector of predicted system output, it consists the longitudinal acceleration of the vehicle body. It is the main variable which is related to the longitudinal vehicle comfort:

$$z_k = [\ddot{x}_{b,k}] \tag{3.29}$$

and its corresponding reference vector is z_{ref}:

$$z_{ref,k} = [\ddot{x}_{b,ref,k}] \tag{3.30}$$

where $\ddot{x}_{b,ref,k}$ is the reference value of the longitudinal acceleration at each time step. It is calculated by the vehicle model formulation:

$$\begin{aligned}\ddot{x}_{b,ref} = [&\frac{T_{m,req,FL} + T_{m,req,FR}}{R_{lad,F}} + \frac{T_{m,req,RL} + T_{m,req,RR}}{R_{lad,R}} - F_{roll} - F_{drag}] \\ &\cdot \frac{1}{m_{tot} + 2\frac{J_{u,y,F}}{R_F^2} + 2\frac{J_{u,y,R}}{R_R^2}}\end{aligned} \tag{3.31}$$

It neglects the effect pf road irregularities on the reference longitudinal acceleration profile. It is the required acceleration value within the consideration of range of controller. The weighting matrices Q_x and R are positive diagonal matrices used to penalize deviations in state and control deviations. x is the state vector, U is the control input vector;

$$U = [u_{j_c}, u_{j_c+1}, \dots, u_{j_c+N_h-1}]^T$$

is the decision variable vector. p_k represents online parameters or disturbances. x_{int} is the initial value of the state vector. f_d is a vector field describing the discrete versions of the prediction models arranged in a nonlinear state space; g_d is is a function representing the output of the system; $U_{\min}/U_{\max}$ are the limits of the control actions U.

3.5.2 State and Control Definitions

State vector:

$$x_{ij} = \left[\dot{z}_{b,ij},\ z_{b,ij},\ \dot{z}_{u,ij},\ z_{u,ij},\ \dot{x}_{b,ij},\ x_{b,ij},\ \dot{x}_{u,ij},\ x_{u,ij},\ \dot{\theta}_{\omega,ij},\ \theta_{\omega,ij}, \dot{\theta}_{s,ij},\ \theta_{s,ij},\ T_{m,ij}\right]$$

Control action:

$$u = [\Delta T_{m,FL},\ \Delta T_{m,FR},\ \Delta T_{m,RL},\ \Delta T_{m,RR},\ \varepsilon_{\sigma_x,FL},\ \varepsilon_{\sigma_x,FR},\ \varepsilon_{\sigma_x,RL},\ \varepsilon_{\sigma_x,RR}]$$

$\Delta T_{m,ij}$ represent the adjustments in motor torque for each wheel. There are the control actions which are computed by the NMPC algorithm to optimize vehicle performance. And $\varepsilon_{\sigma_x,ij}$is the slack variable of the longitudinal tire slip ratio. This allows to implement a soft constraint on the slip ratio error.

3.5.3 Online Parameters

The parameter vector P_{ij} includes real-time system conditions:

$$P_{ij} = [w_{FL},\ \beta_{FL},\ T_{m,\text{req},FL},\ T_{m,\text{req},FR},\ T_{m,\text{req},RL},\ T_{m,\text{req},RR},\ \mu_{\text{actual}},\ \sigma_{\text{ref}}]$$

It includes eight parameters that provide real-time system conditions and external disturbance. It allows the NMPC controller to adapt dynamically to real-time road and driving conditions.

3.5.4 Constraints

Hard constraint (motor torque):

$$\text{Lb}_{T_m} \leq T_{m,\text{req},ij} + T_m \leq \text{Ub}_{T_m}$$

Where the Lb_{T_m} and Ub_{T_m} are the lower and upper motor torque limit. Hard constraints ensure safety and physical feasibility. $T_{m,\text{corr},ij} = T_{m,\text{req},ij} + T_m$ is the motor torque after correction. It must be under the hard constraint.
Soft constraint (slip ratio):

$$\sigma_{x,ij} - \sigma_{x,\text{thr},ij} + \varepsilon_{\sigma_x,ij} \geq 0 \tag{3.32}$$

$$\varepsilon_{\sigma_x,ij} \geq 0 \tag{3.33}$$

The first equation means that σ_x, ij cannot be lower than $\sigma_{x,thr}, ij$. But it can allow the slack of slip ratio. And under hard constraints, if some variables cannot satisfy the constraints, the optimization problem may have no solutions. So $\varepsilon_{\sigma_x,ij}$ allow small violations of constrains so that the optimization problem is still solvable.

3.6 Simulink

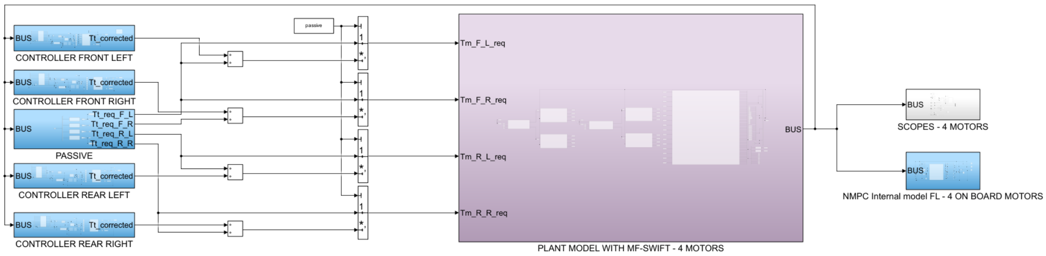

Figure 3.6: Structure of simulink

$Figure3.5$ is the structure of simulink. It shows the vehicle model system structure based on the 4-wheel drive. It uses four individual wheel control block to calculate the drive torque requirement for each wheel $T_{t,corrected}$. And the system has a passive block, it controlled by the switch. It used to simulate the results of passive system. Then it compared the performance with NMPC system. After they calculate the toruqe, it will transfer to the plant, it is the model based on the MF-swift. The output connect to the scopes and internal model. The scope block is used to show the different simulation result. This model structure achieves a closed-loop path from controller output, switch and vehicle dynamics response to feedback evaluation.

Chapter 4

Results

In the results section, it is made up of two parts. The first part is a plot which is generated by MATLAB. It is used to visualize the test simulation results. The second part is composed of data. It use some KPI performance characteristics to evaluate the different strategies.
In the part of plot. This is the figure result generated by MATLAB after running the simulation. In this figure, there are six subplots, the longitudinal acceleration, the angular velocity difference, the slip ratio, the torque, the road profile and the friction coefficient. It demonstrates the different strategies (passive system, base NMPC system, preview NMPC system) simulated under different working condition. The black line represents the reference, the green line represents the passive system. The red line represents the NMPC base system and the blue line represents the NMPC system with preview. Among them, the dashed line is mainly used to distinguish the relevant variables or reference signals of the rear wheel.
In the part of KPI. The KPI script shows the results of 4 tests and saves in the table. The test 1 evaluates the basic longitudinal vibration comfort.

$$\mathrm{RMS}(\ddot{x}_b) = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} (\ddot{x}_b(t) - \ddot{x}_{b,\mathrm{ref}}(t))^2 \, dt} \tag{4.1}$$

The **RMS**$\ddot{x}_b$ is the root mean square of body acceleration error.It is used to evaluate the tracking accuracy between body acceleration ($\ddot{x}_b$) and the reference acceleration(a_{ref}). If the value is low, it means the control strategy is more accurate to track the reference signals.

$$\mathrm{VDV}(\ddot{x}_b) = \sqrt[4]{\int_{t_1}^{t_2} (\ddot{x}_b(t) - \ddot{x}_{b,\mathrm{ref}}(t))^4 \, dt} \tag{4.2}$$

The **VDV**$\ddot{x}_b$ is the vibration dose value of body acceleration. It is used to evaluate the impact of transient shock of acceleration error on passenger comfort.

If the value is low, it means that the control strategy can optimize the transient vibration. The passenger's comfort will be better.
The test 2 evaluates the dynamic of wheel under cut-off frequency. It chooses 8Hz as the cut-off frequency.

$$\Delta V = v(t_2) - v(t_1) \quad \text{(m/s)} \tag{4.3}$$

ΔV is the velocity difference. It used to calculate the difference of velocity at the final simulation.

$$\Delta\omega = \max_{t\in[t_1,t_2]} \left|\dot{\theta}(t) - \dot{\theta}_{\text{ref}}(t)\right| \quad \text{(rad/s)} \tag{4.4}$$

And the $\Delta\omega$ is the angular velocity difference between suspension and wheel. It represents the dynamic vibration between suspension and wheel.
The test 3 is used to evaluate the slip ratio control of wheel.

$$\lambda = \frac{\omega r - v}{\max(\omega r, v)} \times 100\% \quad (\%) \tag{4.5}$$

The $\lambda_{\textbf{err-max}}$ is the maximum slip ratio error. If the value is low, it represents the control strategy can control the slip ratio and keep the traction control.
The test 4 is used to evaluate the comprehensive performance under all working conditions. It demonstrates and verifies its robustness covering a a variety of complex conditions.

4.1 CASE-1: Step profile with friction coefficient drop and tip-in

4.1.1 Plot

The **Figure 4.1** is the plot of the case 1, it demonstrates the vehicle passes a step profile with friction coefficient drop and tip-in. The following is the detailed analysis.

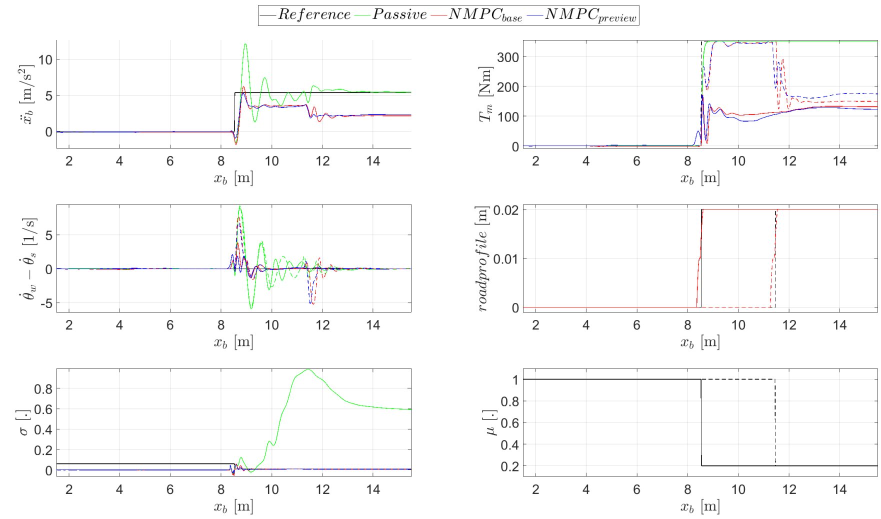

Figure 4.1: Step profile with friction coefficient drop and tip-in

- In the figure of longitudinal acceleration. As we can see, the step caused the shock. The passive system produces the maximum longitudinal acceleration. It repeated shocks within 8-12m, then it is close to the reference. The NMPC system can control the longitudinal acceleration significantly. The NMPC model decrease the shock at the step and decrease the longitudinal acceleration after 12m. Compared with the base and preview. The preview system has lower longitudinal acceleration than the base system. And it quickly stabilizes around 12m to reduce fluctuations. The preview system can perceive the road information in advance and respond quickly. Then the NMPC with preview system has the best performance.

- In the figure of angular velocity difference. As we can see, In the front wheels, the NMPC system significantly decrease the shock when the step happened. The NMPC with preview system has the minimum angular velocity difference. It represents this system can keep the vehicle stable. But when the friction coefficient drops. The NMPC system responds quickly to keep stable, then dashed line represents the rear wheels produce larger angular velocity difference. We also found that the preview system has the shock before the base system.

- In the figure of slip ratio. Obviously, the NMPC system controls the slip ratio during the whole simulation. It produces the fluctuations at the step, but it stabilizes quickly, then it is close to the reference. Among them, the preview system has the lower slip ratio than the base system. In the other side, the passive system cannot control the slip ratio adaptively.

- In the figure of torque. As we can see, the passive system applied the torque when the step happened. It doesn't optimize adaptively. But the NMPC system adaptively applies the torque to keep the vehicle stable. And the torque is lower than the torque which is applied by the passive system. It means it can decrease the power consumption to optimize.

4.1.2 KPI

The **Table 4.1** is the KPI performance of the case 1, it demonstrates the vehicle passes a step profile with friction coefficient drop and tip-in. The following is the detailed analysis.

Test	**System**	**RMS**($\ddot{x}_b$)	**VDV**($\ddot{x}_b$)	ΔV	$\Delta\omega_f$	$\Delta\omega_r$	$\lambda_{\mathbf{max}}$
Test-1	Passive	1.7899	2.9426	-	-	-	-
	Base	1.6208	2.5987	-	-	-	-
	Preview	1.5515	2.2926	-	-	-	-
Test-2	Passive	2.3395	3.7505	0.0000	8.9789	9.2522	-
	Base	1.2312	1.8387	0.5637	3.8323	7.7118	-
	Preview	1.1234	1.5130	0.5480	2.1142	6.7902	-
Test-3	Passive	2.3395	3.7505	0.0000	8.9789	-	0.9750
	Base	1.2312	1.8387	0.5637	3.8323	-	0.0473
	Preview	1.1234	1.5130	0.5480	2.1142	-	0.0416
Test-4	Passive	1.6061	3.1038	0.0000	8.9789	-	0.9750
	Base	0.8459	1.5218	0.5637	3.8323	-	0.0473
	Preview	0.7719	1.2522	0.5480	2.1142	-	0.0416

Table 4.1: CASE-1 KPI Test

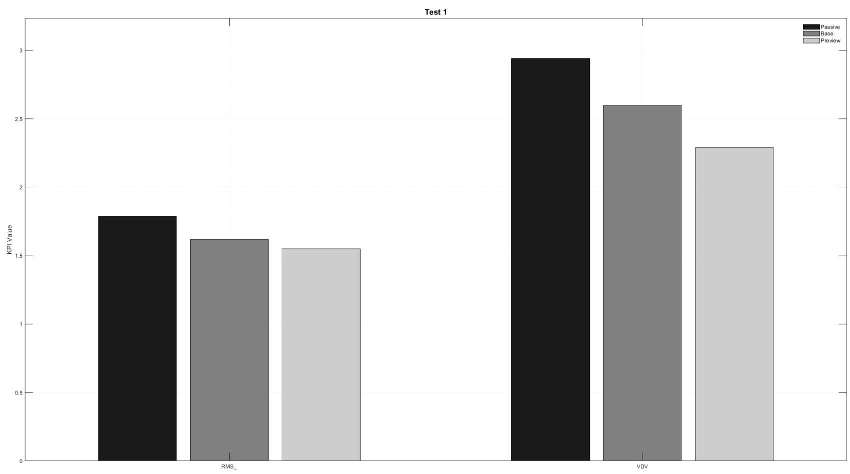

Figure 4.2: Step profile with friction coefficient drop and tip-in

Test 1. The **Figure 4.2** shows the comparisons between them. Compared with the dates. The value of NMPC system is lower than the passive system. It means the NMPC system has effect in decrease the body vibration on step profiles. And between base NMPC and preview NMPC, the preview strategy reduced RMS$\ddot{x}_b$ 13.3% and reduced VDV$\ddot{x}_b$ 22.1%. It is better than the base strategy. The preview strategy improves the driving comfort.

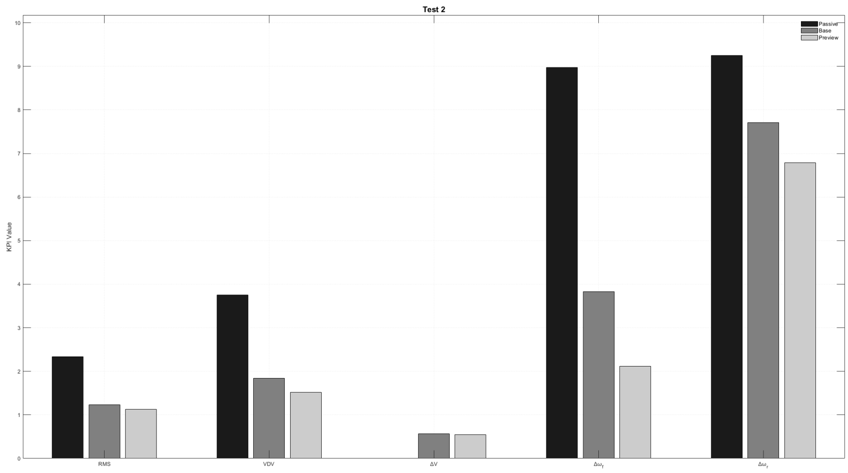

Figure 4.3: Step profile with friction coefficient drop and tip-in

Test 2. The **Figure 4.3** shows the comparisons between them. In the part of high frequency vibration. The preview NMPC system reduced the RMS$\ddot{x}_b$ 52% and the VDV$\ddot{x}_b$ 59.7%. It is better than base NMPC system and passive system. In the part of wheel stability. The preview strategy decreased 76.4% than passive system on the front wheel angular. In all, the preview strategy improves the comfort and stability under high frequency disturbances obviously.

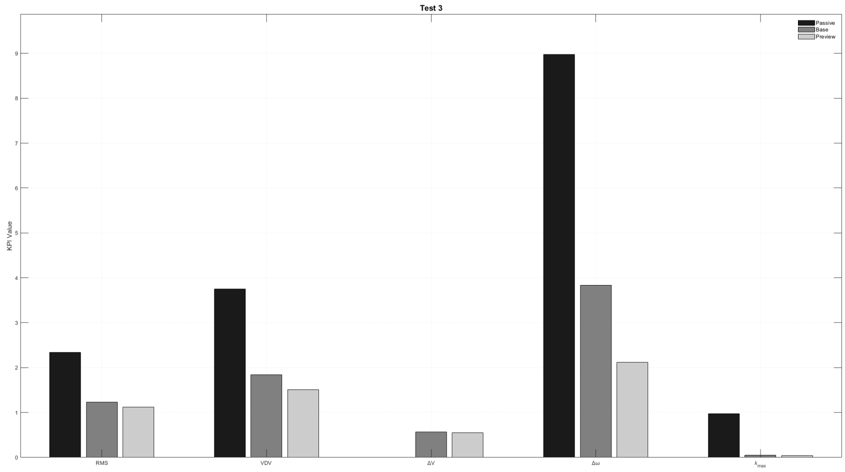

Figure 4.4: Step profile with friction coefficient drop and tip-in

Test 3. The **Figure 4.4** shows the comparisons between them. In the part of slip control. The NMPC reduced the slip ratio significantly. The preview strategy reduced 95.7% on slip ratio. The preview NMPC system can keep traction control stable on low-friction.

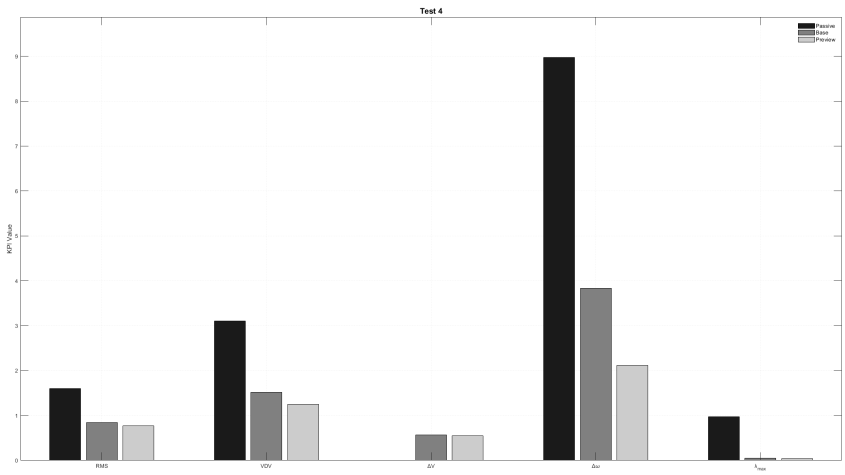

Figure 4.5: Step profile with friction coefficient drop and tip-in

Test 4. The **Figure 4.5** shows the comparisons between them. By evaluating covering the full road profile. The preview strategy reduced the 52% on RMS$\ddot{x}_b$ and 59.7% on VDV$\ddot{x}_b$. And it can keep the lowest slip ratio. It verifies the robustness and adaptability under complex scenarios.

4.2 CASE-2: Speed bump with friction coefficient drop and tip-in

4.2.1 Plot

The **Figure 4.6** is the plot of the case 2, it demonstrates the vehicle passes a speed bump with friction coefficient drop and tip-in. The following is the detailed analysis.

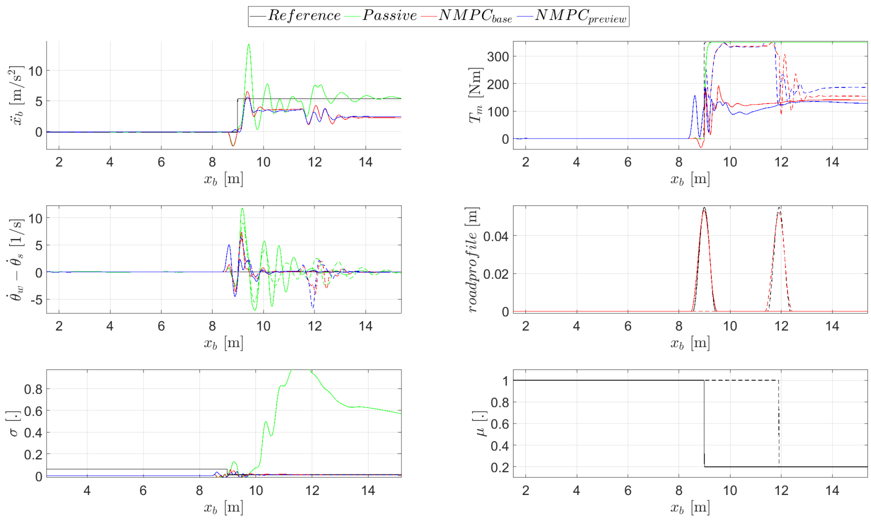

Figure 4.6: Speed bump with friction coefficient drop and tip-in

- In the figure of longitudinal acceleration. As we can see, the passive has a negative acceleration before the bump point. And it caused the maximum shock after the friction coefficient drops. After a period of fluctuation, the acceleration approaches the reference value. The base NMPC system also has a negative acceleration before the bump point. Because, they don't have the preview ability. But it doesn't produce a large shock when the friction coefficient drops. It quickly stabilizes fluctuations in acceleration. Finally, its acceleration value is lower than the reference value. The preview NMPC system has a positive value before the bump point. Because it anticipates in advance and adapts actively to handle the bump. Compared with the base NMPC system. The shock is most closed to the reference value. It can stabilize more quickly and less vibration for the rear wheel. Finally, the acceleration reaches to the base closely.

- In the figure of angular velocity difference. In the figure of angular velocity difference. The passive system has a small angular velocity difference at the bump point, then produces the maximum shock when the friction coefficient drops. It fluctuates up and down and finally gradually converges to 0. The NMPC system decreases the shock when the friction coefficient drops, but it has a larger angular velocity difference than passive system. The preview strategy produces the maximum difference. It demonstrates that increases the velocity to control the following slip ratio.

- In the figure of slip ratio. As we can see, the passive system doesn't be applied any control. It produces the largest slip ratio, especially around 12m. Otherwise, the NMPC system controls the slip ratio better. It controls the slip ratio to reach the reference values around 12m. Before the friction coefficient drops, the preview strategy responds previously based on previewed road information. Then it keeps stable more quick than base strategy.

- In the figure of torque. As we can see, the passive system can not control the applied torque adaptively. It directly applied the torque at bump point. The NMPC can control the torque adaptively. At 12m, it actively adapts the applied torque with the road information and keep the vehicle stable quickly. Finally, it keeps a smaller torque than passive system. It demonstrates the ability to reduce the energy consumption of suspension system. Between base strategy and preview strategy, the preview strategy actively responses before the bump point and have smaller energy consumption than base strategy in the whole simulation.

4.2.2 KPI

The **Table 4.2** is the KPI performance of the case 2, it demonstrates the vehicle passes a speed bump with friction coefficient drop and tip-in. The following is the detailed analysis.

Test	System	RMS($\ddot{x}_b$)	VDV($\ddot{x}_b$)	ΔV	$\Delta\omega_f$	$\Delta\omega_r$	$\lambda_{\mathbf{max}}$
	Passive	1.9517	3.2590	-	-	-	-
Test-1	Base	1.3920	2.0425	-	-	-	-
	Preview	1.4433	2.2027	-	-	-	-
	Passive	2.6440	4.5276	0.0000	11.763	9.0489	-
Test-2	Base	1.2809	2.0513	0.4391	7.4551	6.6271	-
	Preview	1.0178	1.6124	0.4296	5.2922	6.7321	-
	Passive	2.6440	4.5276	0.0000	11.763	-	1.0167
Test-3	Base	1.2809	2.0513	0.4391	7.4551	-	0.0483
	Preview	1.0178	1.6124	0.4296	5.2922	-	0.0260
	Passive	1.8123	3.7470	0.0000	11.763	-	1.0167
Test-4	Base	0.8784	1.6977	0.4391	7.4551	-	0.0483
	Preview	0.6988	1.3344	0.4296	5.2922	-	0.0260

Table 4.2: CASE-2 KPI Test

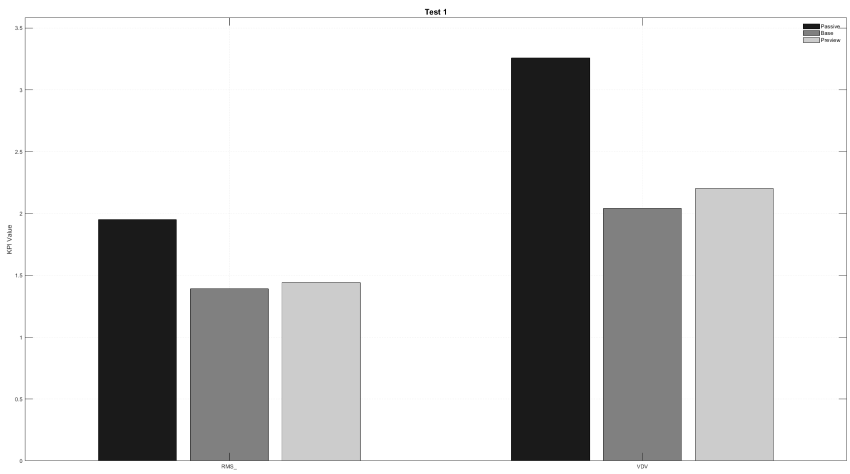

Figure 4.7: Speed bump with friction coefficient drop and tip-in

Test 1. The **Figure 4.7** shows the comparisons between them. The performance of NMPC system is better than passive system. Because the NMPC system optimize the distribution of torque to reduce the vehicle vibration. The base strategy reduced 28.7% on RMS$\ddot{x}_b$ and 37.3% on VDV$\ddot{x}_b$. But the value of preview strategy is a little higher than base strategy. Due to preview ability, it may consider other complex control to cause that the performance of base strategy is better.

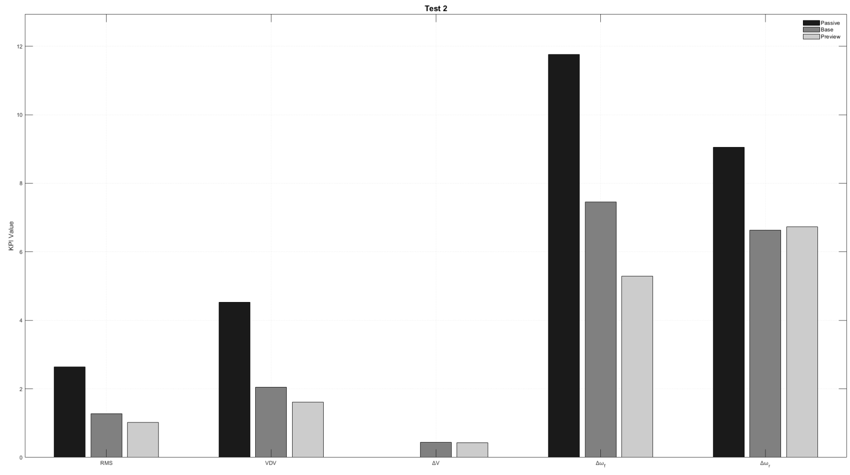

Figure 4.8: Speed bump with friction coefficient drop and tip-in

Test 2. The **Figure 4.8** shows the comparisons between them. In the part of driving comfort testing. Compared with the passive system, the preview strategy reduced 61.5% on RMS$\ddot{x}_b$ and 64.4% on VDV$\ddot{x}_b$. Compared with base strategy, the preview strategy reduced 20.5% on RMS$\ddot{x}_b$, 21.4% on VDV$\ddot{x}_b$, it means the preview strategy reduced the high frequency vibration obviously. In the part of wheel stability, the preview reduced 55.0% on front wheel angular difference. Although the rear wheel angular difference is a little higher than the base strategy. It demonstrates improvements of traction stabilization. About the velocity difference, the base and preview strategy both control the distribution to keep the velocity. But the preview strategy is better than base strategy.

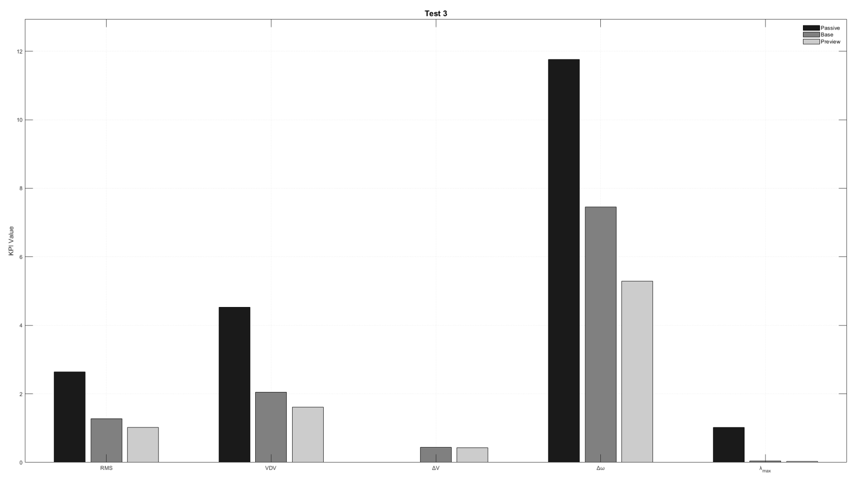

Figure 4.9: Speed bump with friction coefficient drop and tip-in

Test 3. The **Figure 4.9** shows the comparisons between them. In the part of slip ratio testing. The performance of preview strategy has been significantly improved. It reduced 97.4% on RMS$\ddot{x}_b$ compared with passive system. It demonstrated the distribution of torque to decrease the slip ratio. Compared with base strategy, it reduced 46.2% on the slip ratio. It demonstrates the preview ability to get the previous road information, then it responses actively to reduce the slip ratio.

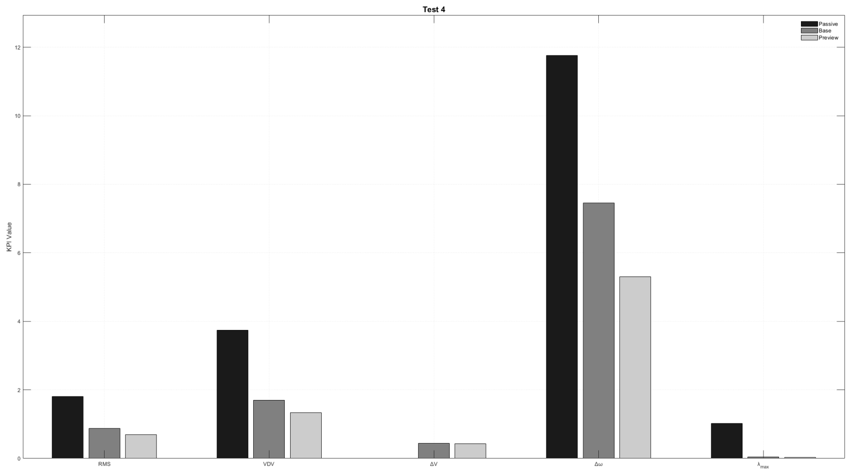

Figure 4.10: Speed bump with friction coefficient drop and tip-in

Test 4. The **Figure 4.10** shows the comparisons between them. In the whole simulation of covering the full road profile. The preview reduced 61.5% on RMS$\ddot{x}_b$ and 64.4% on VDV$\ddot{x}_b$. And it also keeps the lowest value of slip ratio error and angular velocity difference. It demonstrates that it can minimize the transient shock and response in low friction coefficient area quickly. The preview strategy avoids power loss and risk of loss of control during the whole road profile simulation. It verifies its robustness and adaptability.

4.3 CASE-3: Step profile with variable friction coefficient drop and tip-in

4.3.1 Plot

The **Figure 4.11** is the plot of the case 3, it demonstrates the vehicle passes a step profile with variable friction coefficient drop and tip-in. The following is the detailed analysis.

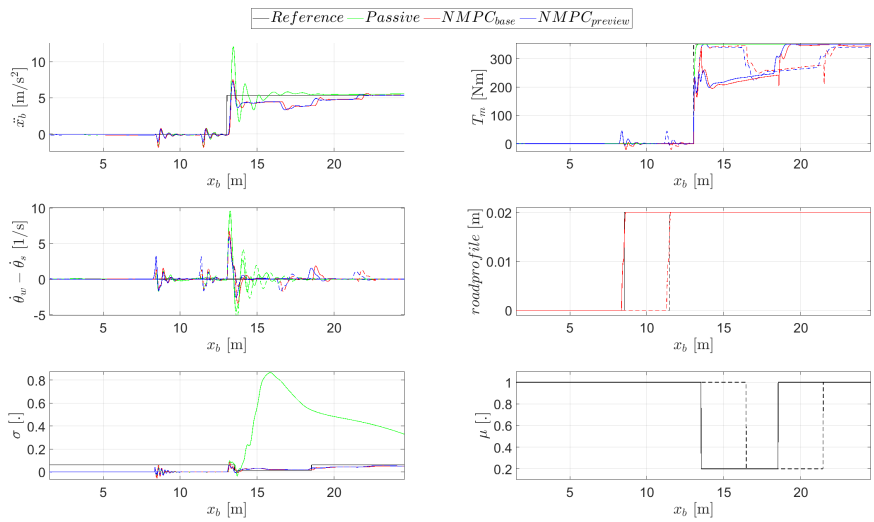

Figure 4.11: Step profile with variable friction coefficient drop and tip-in

- In the figure of longitudinal acceleration. As we can see, the passive system produces shocks at front wheel step, rear wheel step and drop of friction coefficient. Especially, there is the maximum shock when the friction coefficient drops. After a period of fluctuation, the acceleration approaches the reference value. Then NMPC base system also have the shock at the step points. But it decreases the shock at the friction coefficient point. Then it keeps the acceleration stable quickly. Finally, the acceleration increases when the friction coefficient rise and approaches to the reference value. About the preview strategy, it doesn't produce a negative shock at the step points due to preview ability. And when the friction coefficient drops or rises, the system controls the acceleration previous than base strategy. It demonstrates that the preview strategy can response previously according to the road profile.

- In the figure of angular velocity difference. As we can see, the passive system produces smaller fluctuations at the step points but it produces maximum fluctuation when the friction coefficient drops. It means that the passive system can not handle variable friction coefficient actively. The NMPC base system decreases the shock when the friction coefficient drops. But it increases the shock when the steps happened. And it produces the shocks when the friction coefficient rises. It means that the NMPC base system considers other complex requirements for keep the vehicle stable. About the preview strategy. It produces a larger shock than base strategy, but it decreases the shock when the friction coefficient rises. It demonstrates the preview ability to adjust the distribution of torque according to different road profile.

- In the figure of slip ratio. The passive system doesn't control actively, then the slip ratio is out of control. It reaches the highest after 15m, then it decreases. About the NMPC system, it can control the vehicle stable based on the distribution of torque. The system controls slip ratio to approach the reference value when the step or variables happened. Between base and preview strategy, the preview strategy has a lower slip ratio value. The preview strategy has a better performance.

- In the figure of torque. In the figure of torque. The passive system applies a 350Nm torque until the end of simulation. The NMPC base system applies a negative force to response step. It demonstrates the base strategy can adjust the distribution of torque according to the real-time road profile. Then the torque increases to keep the vehicle stable when the friction coefficient rises. About the NMPC preview system, it adjusts the distribution of torque before the steps and variable friction coefficient happen. It demonstrates the preview ability to response the previewed road profiles previously.

4.3.2 KPI

The **Table 4.3** is the KPI performance of the case 3, it demonstrates the vehicle passes a step profile with variable friction coefficient drop and tip-in. The following is the detailed analysis.

Test	System	RMS($\ddot{x}_b$)	VDV($\ddot{x}_b$)	ΔV	$\Delta\omega_f$	$\Delta\omega_r$	$\lambda_{\mathbf{max}}$
Test-1	Passive	0.2287	0.4694	-	-	-	-
	Base	0.2414	0.5248	-	-	-	-
	Preview	0.0905	0.1897	-	-	-	-
Test-2	Passive	0.2709	0.5228	0.0000	9.5652	9.3457	-
	Base	0.2826	0.5804	0.0057	6.2139	6.1464	-
	Preview	0.0106	0.2154	-0.0152	6.1413	6.1050	-
Test-3	Passive	0.2709	0.5228	0.0000	9.5652	-	0.8548
	Base	0.2826	0.5804	0.0057	6.2139	-	0.0168
	Preview	0.0106	0.2154	-0.0152	6.1413	-	0.0229
Test-4	Passive	0.1865	0.4330	0.0000	9.5652	-	0.8548
	Base	0.1944	0.4804	0.0057	6.2139	-	0.0168
	Preview	0.0749	0.1783	-0.0152	6.1413	-	0.0229

Table 4.3: CASE-3 KPI Test

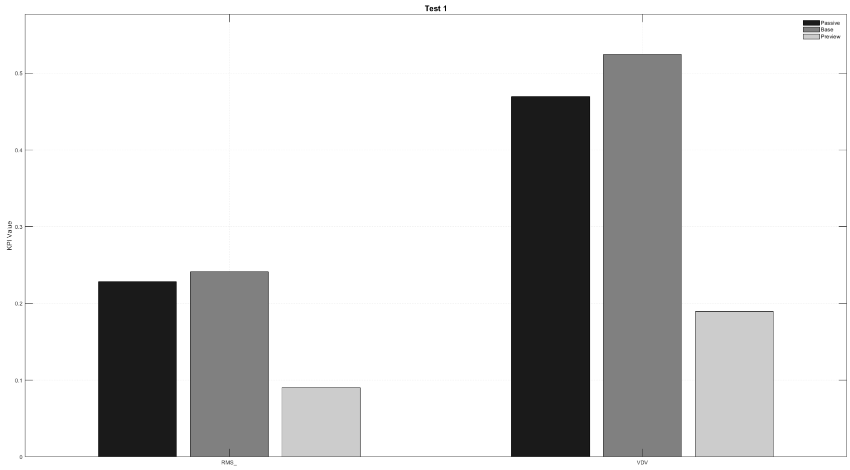

Figure 4.12: Step profile with variable friction coefficient drop and tip-in

Test 1. The **Figure 4.13** shows the comparisons between them. The performance of NMPC preview system is the best. Compared with passive system, it reduces 60.4% on RMS$\ddot{x}_b$ and reduces 59.6% on VDV$\ddot{x}_b$, it demonstrates to optimize driving comfort by preview ability under regular road profile. But the value of base strategy is higher than passive system. It means that it may consider other constraints to lead to overly aggressive control response.

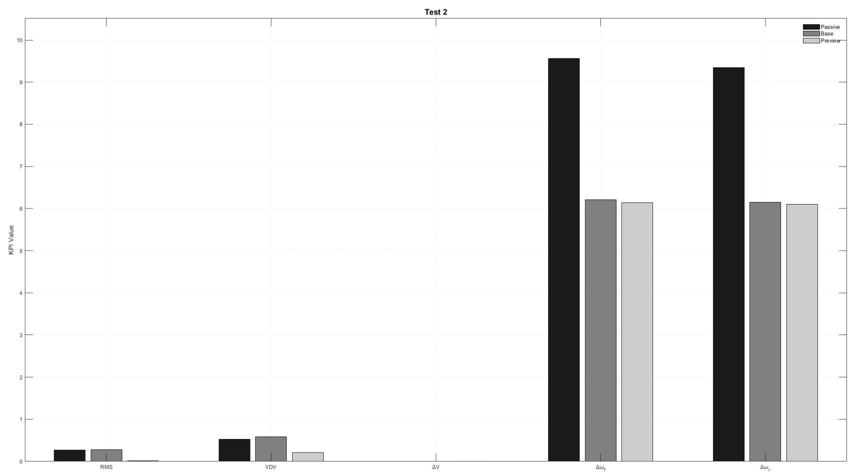

Figure 4.13: Step profile with variable friction coefficient drop and tip-in

Test 2. The **Figure 4.14** shows the comparisons between them. In the part of comfort testing. Compared with passive system, the NMPC preview system reduces 96.1% on RMS$\ddot{x}_b$ and 58.8% on VDV$\ddot{x}_b$. It verifies the control ability under high frequency disturbance. In the part of wheel stabilization testing. The performance of NMPC system is better than passive system. The preview is better between preview strategy and base strategy. Compared with passive system, it decreases 35.8% in the front wheel angular difference. It demonstrates that the preview strategy is more stable for traction system. The DeltaV of preview strategy, it demonstrates that it decreases actively to avoid slip and keep vehicle stable and efficient under low friction coefficient area.

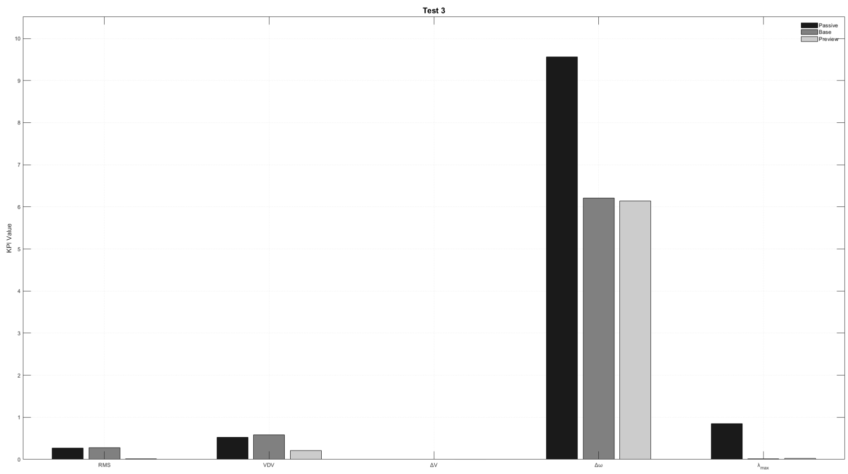

Figure 4.14: Step profile with variable friction coefficient drop and tip-in

Test 3. The **Figure 4.15** shows the comparisons between them. About the slip ratio test. The performance of NMPC system is obviously better than passive system. Between base strategy and preview strategy, the base strategy controls the slip ratio is better than preview strategy. It demonstrates that base strategy may limit torque more conservatively to reduce slip ratio. Otherwise, the preview strategy allows a slight error to improve driving comfort by keeping balance between vibration and slip.

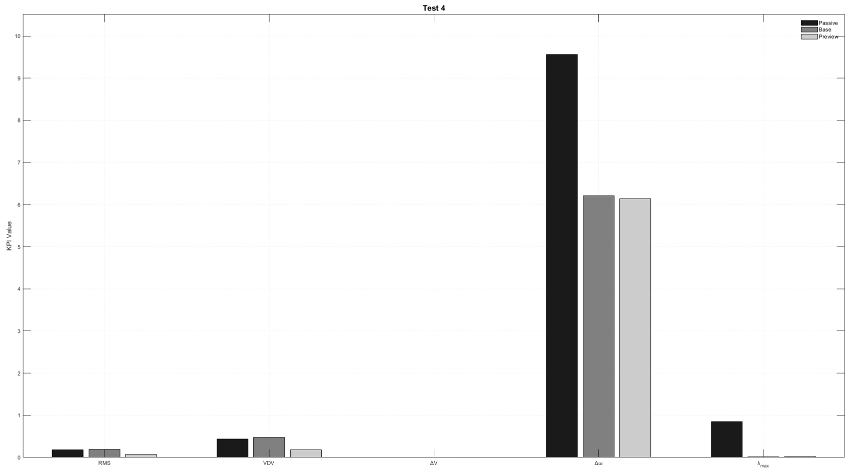

Figure 4.15: Step profile with variable friction coefficient drop and tip-in

Test 4. The **Figure 4.16** shows the comparisons between them. During the whole simulation under completed road profile. The NMPC preview system reduces 59.8% RMS$\ddot{x}_b$ lower than passive, 58.9% VDV$\ddot{x}_b$ lower than passive. Although, the slip ratio is littler higher than base strategy, but it has better performance in the vibration control and dynamic stabilization.

4.4 CASE-4: B-road with friction coefficient drop and tip-in

The **Figure 4.16** is the plot of the case 4, it demonstrates the vehicle passes a step profile with variable friction coefficient drop and tip-in. The following is the detailed analysis.

4.4.1 Plot

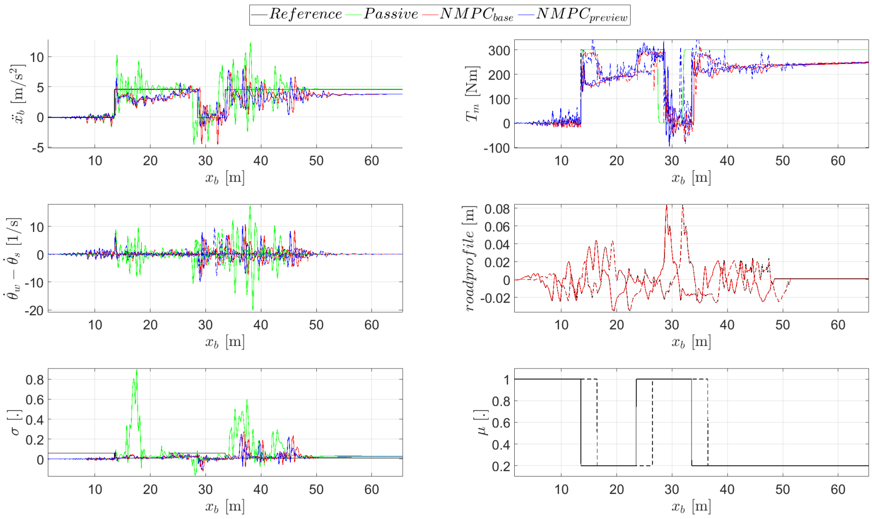

Figure 4.16: B-road with friction coefficient drop and tip-in

- In the figure of longitudinal acceleration. As we can see, around 10m, the passive system changes to adjust the vehicle acceleration as the road profile changes. But it produces large shocks when the torque is tipped in. Between 30m to 40m, as the tip-in and friction coefficient drops, the passive system is unable to control actively control the distribution of torque thus leading to large shocks. After 50m, the acceleration gradually stabilizes and approaches as the working conditions gradually stabilize. About the NMPC base strategy, it also responds to the real-time road profile, but it can optime the distribution of torque, then it can reduce the shock compared with passive system. But when the friction coefficient drops of rises up, the NMPC system may take to increase the wheel angular velocity to reduce the slip ratio. As we can see the NMPC system produces greater angular velocities when the friction is changing. About the preview strategy, it can respond in advance when the road profile changes, as we can see, around 10m, the preview strategy produces positive accelerations. It demonstrates to adjust actively to handle the road profile. Between 40m to 50m, it responds quickly in advanced than base strategy. In conclusion, the preview has the best performance.

- In the figure of angular velocity difference. The passive system produces larger wheel angular wheel difference before 20m. And it doesn't handle complex work situation, like around 40m. It demonstrates that the passive system doesn't control actively, there is a risk of losing control of the vehicle. The NMPC base system can reduced the angular velocity difference compared with passive system. It demonstrates that the NMPC base system can control the distribution of torque to control the wheel stabilization. About the NMPC preview, the preview strategy may consider other complex constraints. As we can see, around 10m, the fluctuations which produced by preview strategy is larger than base strategy. Around 45m, it may increase the wheel velocity to reduce the slip ratio. In conclusion, the NMPC preview system has the best performance. Although it produces some larger fluctuations than passive system when the friction coefficient drops or rises, it keeps the wheel stable covering the whole work situation testing.

- In the figure of slip ratio. As we can see, the passive system is obviously unable to control the slip ratio when the coefficient drops or rises. Especially when the friction coefficient drops, the slip ratio exceeds 0.8 and the vehicle is in the risk of slipping. About the NMPC base system, it decreases the slip ratio when the coefficient drops or rises. Compared with passive system when the friction coefficient drops, the NMPC system controls the slip ratio to approach to the reference. It avoids the vehicle slipping. About the preview strategy, it not only reduces the fluctuations even more, but it also adjusts before the friction coefficient changes compared with base strategy. In conclusion, the NMPC system is better than passive system, between base strategy and preview strategy, the preview strategy performance is better.

- In the figure of torque. It is obvious that the passive system is applied a certain torque to handle covering the whole work situation. The NMPC base system changes the torque according to work situation. It demonstrates that the NMPC base system can optimize the distribution of torque to keep the balance between vehicle stabilization and energy consumption. Compared with base strategy, the preview strategy can respond in advanced, it can handle better under bump or step work situation.

4.4.2 KPI

The **Table 4.4** is the KPI performance of the case 4, it demonstrates the vehicle passes a step profile with variable friction coefficient drop and tip-in. The following is the detailed analysis.

Test	System	RMS($\ddot{x}_b$)	VDV($\ddot{x}_b$)	ΔV	$\Delta \omega_f$	$\Delta \omega_r$	$\lambda_{\mathbf{max}}$
Test-1	Passive	0.2689	0.3964	-	-	-	-
	Base	0.2807	0.3996	-	-	-	-
	Preview	0.1802	0.2295	-	-	-	-
Test-2	Passive	0.3132	0.4359	0.0000	20.2360	9.1261	-
	Base	0.3219	0.4377	0.0163	10.8040	7.3096	-
	Preview	0.2104	0.2789	-0.0347	10.1780	9.7677	-
Test-3	Passive	0.3132	0.4359	0.0000	20.2360	-	0.8899
	Base	0.3219	0.4377	0.0163	10.8040	-	0.2657
	Preview	0.2104	0.2789	-0.0347	10.1780	-	0.2381
Test-4	Passive	0.2224	0.3580	0.0000	20.2360	-	0.8899
	Base	0.2284	0.3600	0.0163	10.8040	-	0.2657
	Preview	0.1538	0.2321	-0.0347	10.1780	-	0.2381

Table 4.4: CASE-4 KPI Test

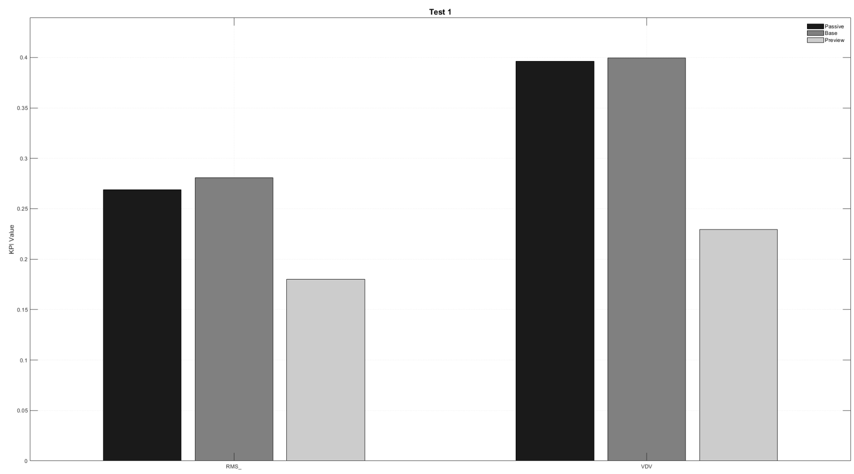

Figure 4.17: B-road with friction coefficient drop and tip-in

Test 1. The **Figure 4.17** shows the comparisons between them. The values of NMPC base system are higher than passive system. It demonstrates that it cannot optimize good under some complex work condition to produces the larger fluctuation of vehicle acceleration. But the preview strategy adjusts the distribution of torque by the preview road profile. It is able to reduced the shock and vibration. It reduces 33.0% on RMS($\ddot{x}_b$), and 42.1% on VDV($\ddot{x}_b$). In conclusion, the preview strategy has the best performance.

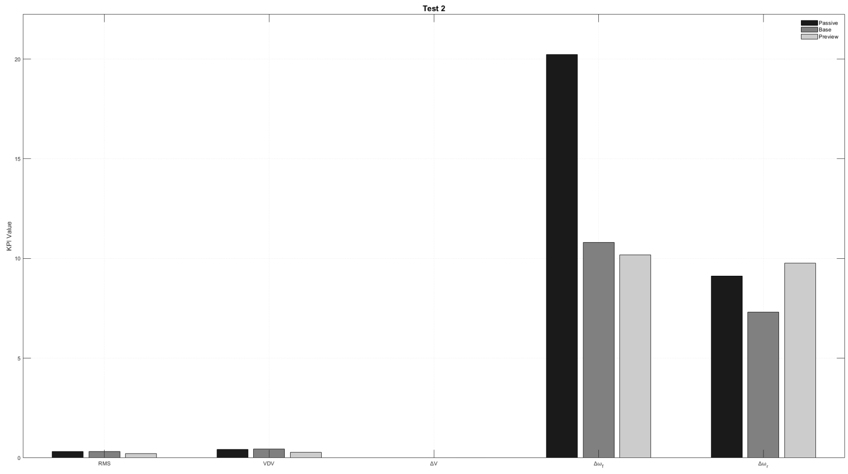

Figure 4.18: B-road with friction coefficient drop and tip-in

Test 2. The **Figure 4.18** shows the comparisons between them. The preview reduces 49.7% on the front wheel angular velocity difference. It demonstrates that it is useful to optimize distribution of torque to keep wheel stable. But it produces a negative $\Delta\omega_r$, because it decreases the torque to avoid the wheels slipping actively under low friction coefficient, then it causes the loss of velocity. It is the reasonable optimization for improve vehicle stabilization. The base may take the increase of vehicle velocity to reduce the angular velocity difference. But it sacrifices the slip ratio control.

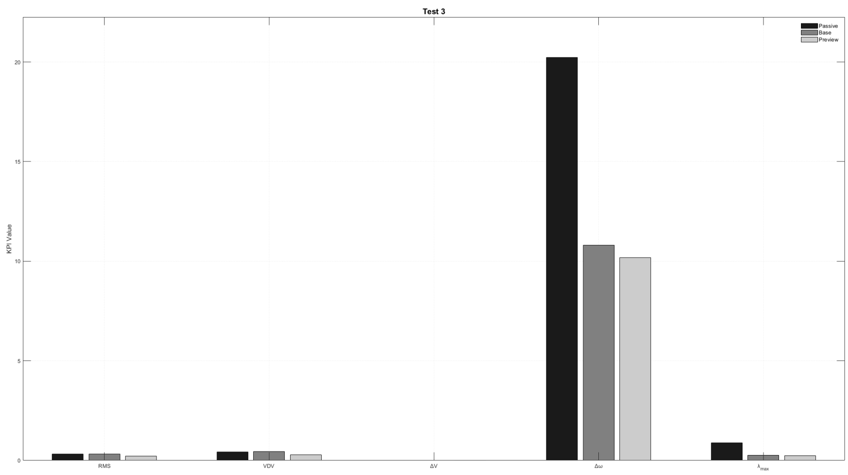

Figure 4.19: B-road with friction coefficient drop and tip-in

Test 3. The **Figure 4.19** shows the comparisons between them. Compared with test 2. it mainly focuses on the slip ratio control. As we can find, the

NMPC preview strategy reduced 73.3% slip ratio compared with passive system. It sacrifices a little velocity to keep the traction control to avoid the wheel slipping. Otherwise, the base strategy fails to adjust the distribution of torque in advance due to lack of previewed capability, it caused the slip ratio is 11.6% higher than preview strategy.

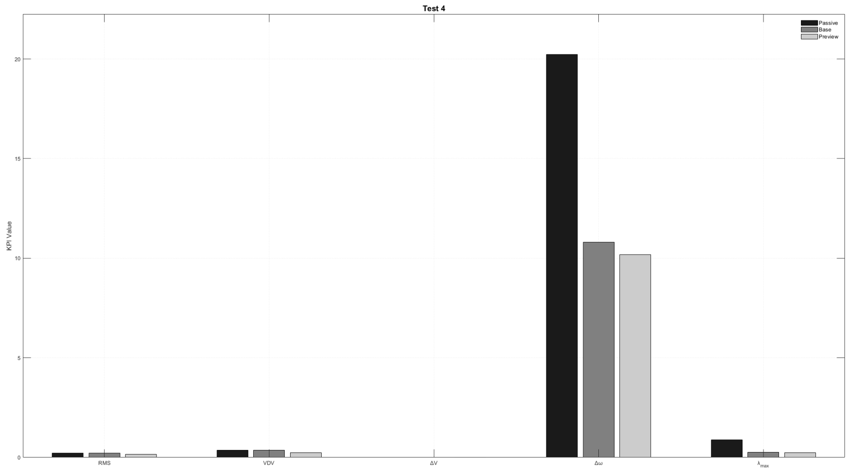

Figure 4.20: B-road with friction coefficient drop and tip-in

Test 4. The **Figure 4.20** shows the comparisons between them. During the whole period simulation testing, the preview strategy reduces 30.9% on RMS($\ddot{x}_b$) and 35.2% on VDV($\ddot{x}_b$) compared with passive system, and keeps the lowest slip ratio error. It demonstrates that the preview strategy optimizes comfort, stabilization and traction efficiency covering the whole testing. In addition to ensure stability through a little velocity reduction, but it keeps the safety of vehicle at the same time.

Chapter 5

Conclusion

This study presents an adaptive integrated control framework with road preview based on nonlinear model predictive model (NMPC). It is applied to electric vehicles which are equipped with active shaft and torque distribution systems. The goal is to optimize multiple objectives, including ride comfort, vehicle stability by solving the problems of vehicle jerks, vibrations and traction control under complex driving conditions, such as stepped roads, variable friction surfaces and tip-in.

- It develops a new NMPC architecture which is integrates real-time road preview information function. It can control actively to adjust torque distribution and shaft dynamics within the prediction range by the road preview information. Compared with the passive system, it responds more quickly.

- The four test cases validate the framework's advantages over the passive system and the base NMPC system.Under case1 (step profile with friction coefficient drop and tip-in). In the test of whole working condition, the preview NMPC reduced 95.7% on slip ratio, 52% on RMS$\ddot{x}_b$ and 59.7% on VDV$\ddot{x}_b$. Under case2 (speed bump with friction coefficient drop and tip-in). In the test of whole working condition, the preview NMPC reduced 46.2% on slip ratio, 61.5% on RMS$\ddot{x}_b$ and 64.4% on VDV$\ddot{x}_b$. But in case 3(step profile with variable friction coefficient drop and tip-in) and case 4(step profile with friction coefficient drop and tip-in), the slip ratio's KPI are a little higher than the base NMPC, but the preview NMPC effectively control the RMS$\ddot{x}_b$ and VDV$\ddot{x}_b$. In the all, the preview NMPC demonstrates the advantages of optimization performance, it can adjust sacrifice the slip ratio to keep the vehicle stable.

This study shows the preview NMPC structure improves the balance between comfort and stability of electrical vehicle through adaptive traction control.

In this structure, although different parameters can be adjusted according to different road surfaces, the adjustment parameters can only target specific working conditions, and the calculation of different road surfaces is complicated, which makes it difficult to realize full scene coverage. Afterwards, we will continue to introduce artificial intelligence algorithms to realize full active adjustment and full-length near coverage, which can better improve the driving and passenger experience.

Bibliography

[1] S. G. Zielinski, T. G. Baehr, and A. T. Zell. *Path Following for an Omnidirectional Robot Using a Non-Linear Model Predictive Controller for Intelligent Warehouses*. ResearchGate. May 2023 (cit. on p. 4).

[2] A. J. C. Schmeitz and Willem Versteden. *Structure and Parameterization of MF-Swift, a Magic Formula-based Rigid Ring Tire Model*. ResearchGate. Sept. 2009 (cit. on p. 14).