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Optimization of integrated energy systems: a mathematical model and solution approach

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Abstract

To address the growing challenges of efficient energy management, capacity planning and conflicting stakeholder objectives in hydrogen-based Integrated Energy Systems (IES), this thesis proposes a bi-level optimization framework applied to a multi-energy system integrating electricity, heat, and hydrogen. The model introduces an integrated energy bundle pricing strategy that reflects the interconversion of different energy sources.

The upper-level (leader) represents the system supplier, whose objective is to maximize overall profit by defining energy prices and offering bundle-based energy services to users. The lower level (follower) represents the consumers, who aim to minimize their energy costs by adapting their demand strategy based on offered prices and bundles.

We formulate the problem as a bi-level program and evaluate its validity and efficiency through computational experiments based on a real case study. The proposed model optimizes the planning and scheduling of each energy device within the integrated energy system over a defined planning timing. The optimization results show that the proposed model and method not only protects the interests of the operator and users but also demonstrates that incorporating a hydrogen fuel cell (HFC) and hydrogen storage tank significantly enhances the system's ability to meet user demand, ensures a positive profit for the leader, and contributes to substantial carbon emission reduction. This analysis highlights the value of hydrogen-focused solutions that support the transition toward a cleaner and more sustainable energy future.

Introduction

Integrated Energy Systems (IES) are becoming increasingly important in addressing the global energy challenge. As the world shifts away from fossil fuels, there is a growing need to adopt more sustainable and efficient energy solutions. Among these, hydrogen stands out as a clean and flexible energy carrier with strong potential in both industrial and domestic applications. IES plays a crucial role in combining various energy sources—such as electricity, heat, and hydrogen—while enabling the coordinated use of generation, conversion, and storage technologies[3].

The integration of these components requires advanced optimisation techniques to manage energy flows effectively, reduce costs, and ensure reliable supply. This is particularly relevant in systems where different stakeholders have conflicting objectives. In such cases, bi-level optimisation models are used to represent the interaction between decision-makers: the upper level (leader), usually the system operator or energy provider, aims to maximise profit, while the lower level (follower), representing consumers, aims to minimize energy costs[4].This leader–follower dynamic captures the real-world negotiation between supply and demand. One of the challenges in planning and managing IES is the complexity of decision-making over time. Long-term capacity planning must consider daily and seasonal variations in energy demand, resource availability, and market prices[4]. To address this, we apply a mathematical modelling approach that transforms real-time data into actionable insights. The model allows us to simulate different scenarios, test pricing strategies, and evaluate energy bundle offerings—combinations of electricity, heat, and hydrogen services—to improve cost efficiency and sustainability. The model is implemented in AIMMS and structured as a Mixed Integer Programming (MIP) problem to capture both continuous and binary decisions. By reformulating the bi-level problem into a single-level one using helper variable and logical constraints, we enable the model to be solved efficiently while retaining its strategic structure[1].

This thesis is structured in six chapters. It starts with the theories and ends with the conclusion; each chapter will be discussed in the following. The first chapter provides detailed information about the Integrated Energy Systems (IES), their optimization principles, and their framework and how each source integrated in system and their processing. The chapter ends with an explanation of the Bi-Level Optimization Framework for Integrated Energy Systems. The second chapter presents the proposed mathematical model used in this study. It begins by stating the modeling objectives, followed by data acquisition and the detailed formulation of the IES structure and constraints. The chapter includes the bi-level optimization model, defining both the leader's and follower's objectives. Various cost components are also introduced. Additionally, the chapter addresses energy demand balancing, satisfaction load modeling, and the bundle strategy, providing a comprehensive foundation for the optimization framework. The third chapter provides information about Optimization modeling in AIMMS, and the framework describes the implementation of the model in AIMMS, detailing sets, parameters, variables, and the overall optimization framework. The fourth chapter presents a literature review, identifying key research gaps and supporting the development of the proposed model. The fifth Chapter presents the results and analysis, comparing bundle strategies, evaluating convergence (UB/LB), and highlighting the role of hydrogen technologies in achieving profit, demand satisfaction, and emissions reduction. The last Chapter concludes with key findings, model implications, limitations, and future research directions.

1. Principle of Optimisation in Energy Integrated system

As energy stands at the forefront of global challenges, drawing significant attention. It is crucial to address this issue this time through the lens of Integrated Energy Systems (IES), with a special emphasis on green hydrogen. This research primarily focuses on capacity planning and aims to implement pricing policies within IES to enhance efficiency and cost savings. As hydrogen emerges as a flexible and eco-friendly energy carrier, attention has shifted toward managing IES based on hydrogen sources [7]. We apply a mathematical optimization approach to balance demand response and price strategies. Specifically, we develop and critically evaluate a bi-level optimization model that seeks to minimize consumer costs while maximizing supplier profits through optimal infrastructure capacity planning. Additionally, this study provides a structured analysis of each energy source within the IES framework. Furthermore, we assess the economic feasibility of bundled energy services, examining how strategic combinations of electricity, heat, and hydrogen can enhance cost efficiency, resource utilization, and market competitiveness. By integrating pricing strategies, infrastructure investment planning, and consumer behavior modeling, this research aims to provide a comprehensive solution for the evolving energy market, ensuring both sustainability and economic viability for future Integrated Energy Systems[2].

1.1 Introduction to Energy Integrated system

Integration Energy System (IES), in which it is described as "the coordinated planning and operation of the energy system 'as a whole' is concerned with interlinking different energy systems—like electricity, heating, and cooling—to work together in a synchronized way. It is concerned with interlinking energy sources (like PV panels, wind farms, or CHP) and energy demands (like houses, cars) through intelligent and efficient infrastructure[5].This is able to supply energy in an efficient, reliable, and environmentally friendly manner[3]. Renewable energy technologies have also been extensively documented for their green and low-carbon nature. However, it is also difficult for the traditional energy system to sustain large-scale consumption. Therefore, there is an urgent need to study how to maximise integrated energy structure and meet the demand[6]. During this period, innovation in renewable energy technologies is a global concern and led to the occurrence and emergence of different generations of novel technologies and methodologies to be a solution for this important problem. Different mechanisms of energy generation have different energy needs, particularly for energy system integration. IES is succeeded by the strategy that associates different modes of production with different energy requirements.

1.1.1 Framework

This interconnected system is aimed at improving energy management and to ensure that the energy is supplied with an ideal usability and to help to reduce resource redundancy. As can be seen from Figure 1, an Integrated Energy System (IES) consists of four types of energy needs: electricity, heat (cooling and heating), and hydrogen (fuel cell electric vehicles). The system is divided into several main components: energy generation, energy storage and conversion, and energy demand loads. Power generation comprises wind farms and solar panels generating electricity directly. A natural gas-fuelled cogenerator (CHP) is used to generate electricity indirectly. Unused energy is accumulated in energy storage. Batteries accumulate excess electrical energy, and excess heat is stored by thermal storage. Electricity is converted to heat by an electric boiler and a gas boiler through the combustion of natural gas. The gas boiler may also be operated using a mixture of hydrogen and natural gas as well, whose hydrogen ratio is constrained by operational conditions. A hydrogen electrolyser supplies hydrogen by means of electricity. In refrigeration, absorption chillers are put into the system, and the final energy provider is HFC, which makes use of hydrogen and produces heating and electricity. Demand for electricity, heat, and hydrogen is the final use of electricity, heat, and hydrogen by utilities[4].

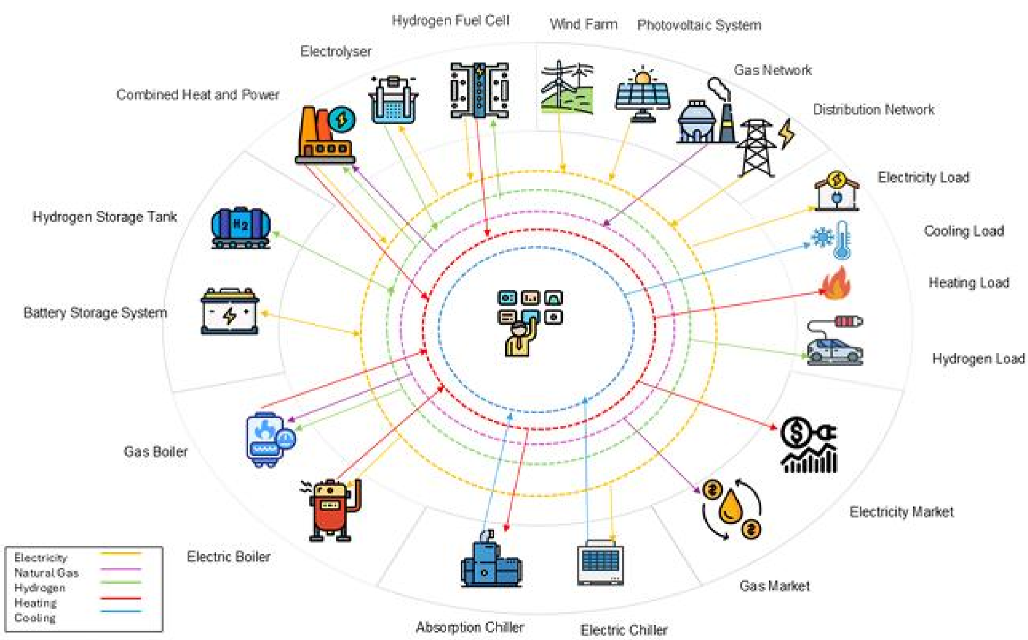


Figure 1: Structure of IES

Figure 1: Integrated Energy System Framework

1.2 Bi-Level Optimisation Framework for Integrated Energy Systems

IES sources are structured in a bi-level mathematical system which means for each source we define the constraints of processing and producing energy. The main purpose of optimising the Integrated Energy System (IES) is the goals of both decision-makers involved in the system. The upper level—typically representing the energy provider or system operator—aims to maximise overall profit by adjusting prices and managing resource allocation. On the other hand, the lower level—representing the users or consumers—focuses on minimising energy costs by selecting the most cost-effective bundles and adjusting their consumption strategies accordingly[2]. To effectively manage long-term capacity planning and daily operational decisions, IES stakeholders adopt extended planning, in terms of days and hours to better reflect the variation of sources, demand load, and energy price. In particular, the planning is expressed by sets of ($= \{1,\cdots,d,\cdots,D\}$) and ($= \{1,\cdots,d,\cdots,D\}$)and ($= \{1,\cdots,t,\cdots,T\}$)representing the planning days and time periods over each day along the long-term planning schedule[6]. In what follows, we model the operation of different energy production, and storage devices, followed by the description of the objective function. The upper level (leader) is the one who takes primary decisions; the lower level (follower) will respond to the problem by considering its own object (surely based on the decisions of the leader). This approach allows us to have the model where decisions are not taken from the air and practical outcome, but even by having in several iterations, we will have the great flexibility to check the leader and follower trade-off for enhancing the value of the solution. They are influenced by all the constraints as well. Its structure is quite fitting for instances that require coordinated balances between two decision-making entities for optimal outcomes. The combination of IES and a mathematical approach enables us to transform time-domain data into actionable insights, apply for more precise predictions of key cost indicators and ultimately enhance the efficiency and reliability of the energy system[2].

2. Proposed Mathematical Model

2.1 Objective

In this section, we provide an outline of the approach suggested in this thesis. As mentioned above, we leverage a bi-level mathematical optimization approach for deeping diving into details of optimizing the Integrated Energy Systems (IES). The model codified to ensure a reasonable result with employing real-world data such that there is a better evaluation of energy resource interactions within IES. The method is developed to use diverse energy sources, as explained in previous sections, such that the demand is met while holding specified system constraints and variables in different scenarios. System elements are modeled with a lot of care to its role in the larger energy infrastructure. This section describes the entire optimization process, such as the bundle strategy employed in this thesis. Our method is designed to take strategic steps in electricity, heat, and hydrogen energy to improve cost effectiveness as well as utilization of resources[8]. The optimization model introduced in the previous section has been solved using AIMMS, a mathematical programming software. In the final part of this section, a detailed explanation is provided on how AIMMS plays a critical role in structuring the decision-making problem by incorporating all parameters and efficiently searching for optimal solutions. In the following subsections, we provide a detailed explanation of each energy source within the system, discussing their role, drawbacks, and coupling within the IES framework.

2.2 Data Acquisition

In this section, we explain how data required in our optimisation model were collected and classified, and we mention logic needed behind forming the foundation of the optimisation model data set. Any sustainable energy system is only as good as the strong and sensible data it is based on. Since the aim of the current research is to provide a realist and feasible solution, data used must reflect the consumption patterns in real systems. The dataset includes household-level consumption data for electricity, heat, and hydrogen[12]. To ensure standardization and comparability, all energy demand values were converted to kilowatt-hours (kWh). This was particularly important for heat consumption, which is often expressed in cubic meters (m^3). A suitable conversion factor was applied to align it with electricity and hydrogen metrics. The data was pre-processed, where average hourly consumption values were computed across different months. Users were then classified into three typical household types—couples, small families, and larger families—based on statistical distributions. These profiles allowed us to model demand dynamically and include daily and seasonal variability. Additionally, a formula was implemented to simulate different consumption ranges (e.g., baseline, reduced usage, or increased usage), particularly to reflect behavioral responses to pricing strategies, environmental awareness, or external shocks like supply crises. This was especially relevant when analysing the European user base, where significant changes in electricity and gas prices have been recorded over the years. To this analysis, below, we incorporated multiple visualizations—such as time-series graphs of average electricity and gas prices across different sectors and continents which only we take into account residential in our model, and a pie chart showing the global distribution of electricity generation sources. These graphs helped validate trends in energy prices and consumption and highlighted the shift towards cleaner and more decentralized energy systems. The resulting dataset was therefore not only cleaned and unified but also enhanced to simulate realistic usage scenarios. This created a strong foundation for testing different optimisation strategies and evaluating the potential impact of pricing models, energy bundles, and consumption patterns within an Integrated Energy System (IES).

Continent	Households (quarterly change)	Households (annual change)	Business (quarterly change)	Business (annual change)
Africa	4.42%	0.67%	5.95%	4.72%
Asia	-4.44%	0.12%	-3.91%	-0.04%
Europe	-1.11%	-2.32%	-0.09%	-6.63%
North America	-2.28%	2.59%	-5.88%	-6.08%
Oceania	-4.31%	0.97%	-1.08%	23.96%
South America	0.78%	4.33%	2.86%	4.53%

Figure 2 :Percentage of change in electricity price by continent

Figure 3: Average Electricity prices in Europe

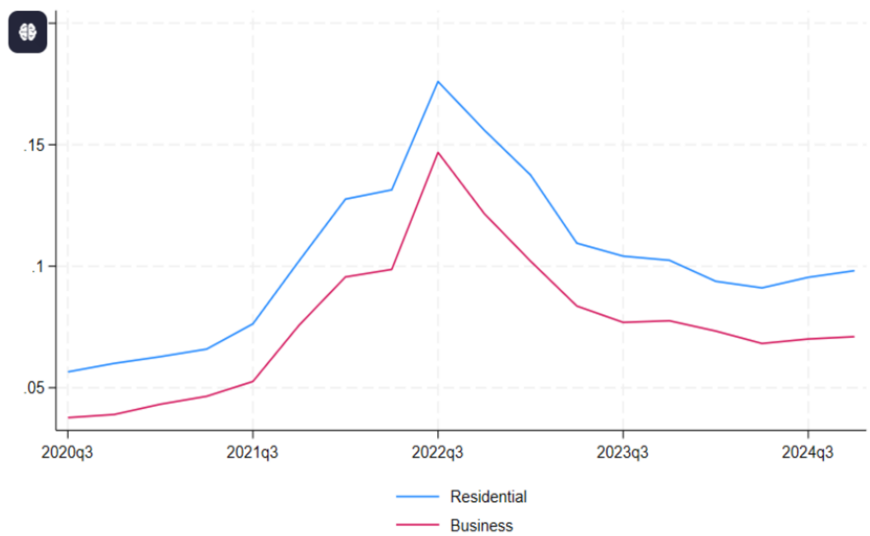


Figure 4: Average Gas prices in Europe

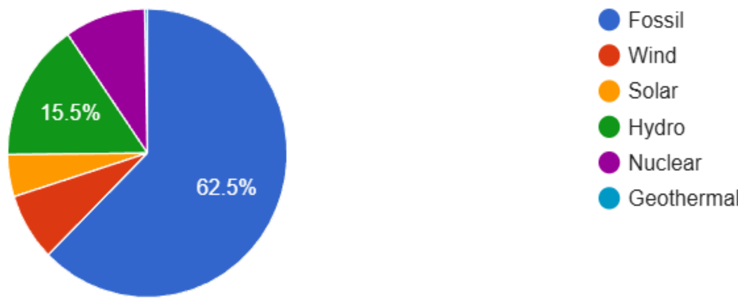


Figure 5 : World electricity generation from various sources, 2022

2.3 IES Mathematical Modeling (structure and constraints)

As we depicted an integrated energy system (IES), as we explained above. By defining our energy resources, we establish the objective function, decision variables, and constraints for each resource and how they integrate in this network from a mathematical point of view. To illustrate the idea behind energy system integration:

2.3.1 Photovoltaic System

Solar panels are systems that convert the sunlight into electric power. The dedicated area to the solar panels, Φ^{PV} (m2), is bounded by the parameter $A_{Available}$, which is the maximum available area. The electric power output of the PV system on day d at time instant, ${P_{mt}}^{Elect,PV}$ (kW), is[4][2] :

$$P_{mt}^{Elec,PV} = \eta_{PV} I_{mt} \Phi^{PV}$$
$$P_{mt}^{Elec,PV} \le P_{Max}^{Elec,PV}$$
$$\Phi^{PV} \le A_{Available}$$

Where:

- I_{mt} is the solar radiation intensity.
- Φ^{PV} is the panel area.
- ${P_{Max}}^{Elect,PV}$ is the system's maximum output.
- η^{PV} is efficiency of PV panel

Additionally, all available solar radiation is about $1kW/m2$, and since one of the significant constraints of this technology is the low energy density, which can only reach near 20%, we decide to set the efficiency of the photovoltaic module equal to 15% that is in line with an average monocrystalline module.

2.3.2 Combined Heat and Power Plant

The combined heat and power (CHP) convert the fuel into electric power. Thanks to the H2-NG technology, the fuel consumption in the CHP can be a mix of hydrogen and natural gas. This significantly reduces the methane consumption and the carbon footprint. The amount of hydrogen in the blended gas should be within a specific range and, in most cases, below 20% of the total volume[4].

The following equation describes the relation between fuel consumption and electric power output in the CHP:

$$P_{mt}^{H_2,CHP} + P_{mt}^{G,CHP} = \zeta_1 P_{mt}^{Elec,CHP} + \zeta_2 U_{mt}^{CHP}$$
$$P_{mt}^{H_2,CHP} \leq \kappa_{H_2}(P_{mt}^{H_2,CHP} + P_{mt}^{G,CHP})$$

Where:

- ${P_{mt}}^{H2,CHP}$, ${P_{mt}}^{G,CHP}$ are hydrogen and natural gas consumption.
- ${P_{mt}}^{Elect,CHP}$ is the electrical power output.
- ${U_{mt}}^{CHP}$ is the binary operation state (1 = On, 0 = Off)
- κ_{H2} is the hydrogen blending limit.

In the following, we mention the other important equations that need to be taken into consideration for having a more realistic optimisation solution: the output power of the CHP system should be the maximum capacity.

$$P_{mt}^{Elec,CHP} \leq MU_{mt}^{CHP}$$

$$P_{mt}^{Elec,CHP} \leq P_{cap}^{CHP}$$

$$P_{mt}^{Heat,CHP} = \frac{P_{mt}^{Elec,CHP}(1 - \eta_{CHP} - \eta_{Loss})}{\eta_{CHP}}$$

In the last equation we have the thermal output of the CHP, where η_{CHP} and η_{LOSS}, respectively, represent the power generation efficiency and heat dissipation loss coefficient.

2.3.3 Hydrogen Electrolyzer

The electrolyzer converts excess electricity into hydrogen using the following model:

$$M_{mt}^{H2,EL} = \frac{\eta^{ELec,EL} P_{mt}^{Elec,EL}}{LHV^{H2}}$$

Where:

- $M_{mt}{}^{H2,EL}$ is the hydrogen output.
- $P_{mt}{}^{Elect,EL}$ is the power input.
- LHV^{H2} is the lower heating value of hydrogen.
- Lower heating value (LHV) is set to 36.4 kW/kg.
- $\eta\ ^{Elec,EL}$is the power generation efficiency of the electrolyser.

The first equation shows the relationship between hydrogen production and consumption, where $P_{mt}{}^{H2,EL}$ represents the available hydrogen fuel in the system. Clearly, the second equation shows the maximum output of the electrolyser.

$$P_{mt}^{H_2,EL} = LHV^{H2} M_{mt}^{H2,EL}$$

$$P_{mt}^{Elec,EL} \leq P_{Cap}^{Elec,EL}$$

2.3.4 Gas Boiler (GB) and Electric Boiler (EB)

Gas and electric boilers generate heat using either gas or electricity.

$$P_{mt}^{Heat,GB} = \eta_{GB}(P_{mt}^{G,GB} + P_{mt}^{H_2,GB})$$
$$P_{mt}^{Heat,GB} \leq P_{cap}^{Heat,GB}$$

where variables:

- $P_{mt}{}^{G,GB}$ and $P_{mt}{}^{H2,GB}$ are the natural gas and hydrogen consumption in GB (in kW)
- $P_{mt}{}^{Heat,GB}$ (in kW) is the amount of thermal power that can be generated by GB on day d at time instant t,
- $P_{cap}{}^{Heat,GB}$ is the installation capacity of the GB.
- η_{GB} describes GB efficiency.

Similar to the CHP Plant, the volume of hydrogen blended with natural gas should be below a maximum value κ_{H2}.

$$P_{mt}^{H_2,GB} \leq \kappa_{H_2}(P_{mt}^{G,GB} + P_{mt}^{H_2,GB})$$

2.3.5 Electric Boiler

The Electric Boiler (EB) is another thermal device that takes electric power as input and produces heating power.

$$P_{mt}^{Heat,EB} = \eta^{EB} P_{mt}^{Elec,EB}$$
$$P_{mt}^{Elec,EB} \leq P_{cap}^{Elec,EB}$$

Where:

- $P_{mt}{}^{Heat,EB}$ and $P_{mt}{}^{Elect,EB}$ are the thermal output and electrical power input of the EB.
- η_{GB} denotes the EB efficiency
- $P_{mt}{}^{elect,EB}$ in represents the max power capacity of $P_{cap}{}^{elect,EB}$.

2.3.6 Hydrogen Fuel Cell

In this system, part of the hydrogen produced by EL is injected into the hydrogen fuel cell (HFC) to produce electric and thermal power.

$$P_{mt}{}^{Elect,HFC} = \eta^{\ Elect,HFC} * P_{mt}{}^{H2,HFC}$$

$$P_{mt}{}^{Heat,HFC} = \eta^{\ Heat,HFC} * P_{mt}{}^{H2,HFC}$$

$$P_{mt}{}^{H2,HFC} <= P_{cap}{}^{G,HFC}$$

Where:

- $P_{mt}{}^{H2,HFC}$ and $P_{mt}{}^{Elect,HFC}$ ($P_{mt}{}^{Heat,HFC}$) are the hydrogen input and power (thermal) output of HFC.
- $\eta^{\ Elect,HFC}$ and $\eta^{\ Heat,HFC}$ represent the electricity and thermal generation efficiency of the HFC
- $P_{cap}{}^{G,HFC}$ displays the HFC installed capacity.

2.3.7 Hydrogen Storage Tank

The hydrogen produced by EL can be stored by the HST to be used by other technologies that need hydrogen as input (GB, CHP, and HFC) or to satisfy the hydrogen load of the Fuel Cell Electric Vehicles:

$$Soc_{mt}^{H_2,HST} \leq Soc_{cap}^{H_2,HST}$$
$$P_{mt}^{H_2,Ch,HST} \leq Soc_{cap}^{H_2,HST} - Soc_{mt}^{H_2,HST}$$
$$P_{mt}^{H_2,Dis,HST} \leq Soc_{mt}^{H_2,HST}$$
$$Soc_{mt_1}^{H_2,HST} = Soc_{mT}^{H_2,HST}$$
$$Soc_{mt_1}^{H_2,HST} = Soc_{Initial}^{H_2,HST}$$
$$Soc_{mt}^{H_2,HST} = Soc_{m(t-1)}^{H_2,HST} + P_{mt}^{H_2,Ch,HST} - P_{mt}^{H_2,Dis,HST}$$

where:

- ${Soc_{cap}}^{H2,HST}$ is the installed capacity of the HST.
- ${Soc_{mt}}^{H2,HST}$ is the state of charge.
- ${P_{mt}}^{H2,Ch,HST}$ and ${P_{mt}}^{H2,Dis,HST}$ show the hydrogen charging and discharging in the HFC.
- The input parameter ${Soc_{Initial}}^{H2,HST}$ is the initial state of charge in HST.

2.3.8 Battery Storage System (BSS)

In the conversion part of the system, there is a battery where electricity can be stored when total power generation exceeds total load. This amount can be used during peak demand when power generation is insufficient. This is crucial due to the intermittency of renewable sources.

To model the operation of the Battery Storage System (BSS), we introduce non-negative variables ${Soc_{cap}}^{BSS}$, indicating the state of charge, power charging, power discharging, and installation capacity of the BSS on day *d* at time instant *t*:

$$Soc_{mt}^{Elec,BSS} = S_{BT}^{mt-1}(1-\delta^{BSS}) + (\eta^{BSS,Ch} P_{mt}^{Elec,Ch,BSS} - \frac{P_{mt}^{Elec,Dis,BSS}}{\eta^{BSS,Dis}})$$

$$Soc_{mt}^{Elec,BSS} \leq Soc_{cap}^{BSS}$$

$$P_{mt}^{Elec,Ch,BSS} \leq Soc_{cap}^{BSS} - Soc_{mt}^{Elec,BSS}$$

$$P_{mt}^{Elec,Dis,BSS} \leq Soc_{cap}^{BSS}$$

$$P_{mt}^{Elec,Ch,BSS} \leq M\chi_{mt}^{BSS}$$

where constraints the connection between the state of charge at two consecutive time instants, considering charging and discharging power. Parameters δ^{BSS} and $\eta^{BSS,Ch}$ ($\eta^{BSS,Dis}$) represent the battery self-discharge rate and charging/discharging efficiency, respectively. ensures that the state of charge does not exceed ${Soc_{cap}}^{BSS}$. The important point about this technology is that BSS does not allow simultaneous charging and discharging, which we manage by using a binary variable ${\chi_{mt}}^{BSS}$.

2.4 Bi level Mathematical Modelling

2.4.1 Objective Function of Leader and Costs of the Model

As we mentioned before, objective functions are defined for both the leader and the follower, along with the necessary constraints required to achieve an optimal solution. The leader's aim is to maximize profit (1), as represented where total revenue is discounted based on four main costs:

- Investment Expenses
- Energy Exchange Costs
- Environmental Costs
- Operating and Maintenance Costs

2.4.2 Objective Function of Leader

max: $Z^L = Revenue - (C_{INV}+C_{EPS}+C_{CO2}+C_{OM})$

Revenue = $\sum_{u=1}^{U} \sum_{M=1}^{M} \sum_{t=1}^{T} \sum_{k=1}^{K} ({r_{mt}}^{Elect,B} * {L_{u\,mt}}^{Elect} * Y_{u\,b} + {r_{mt}}^{Heat,B} * {L_{u\,mt}}^{Heat} * Y_{u\,b} + {r_{mt}}^{H2,B} * {L_{u\,mt}}^{H2} * Y_{u\,b})$

Where:

- ${L_{u\,mt}}^{Elect}$: Electricity demand load of user *u*
- ${L_{u\,mt}}^{Heat}$: Thermal demand load of user *u*
- ${L_{u\,mt}}^{H2}$: Hydrogen demand load of user *u*
- ${r_{mt}}^{Elect,B}$: Electricity price set for bundle *b* on day *m* at time t
- ${r_{mt}}^{Heat,B}$: Heat price set for bundle *b* on day *m* at time t
- ${r_{mt}}^{H2,B}$: Hydrogen price set for bundle *b* on day *m* at time t
- $Y_{u\,b}$: Binary parameter ,1=User u, buy this energy e in this bundle b with this price, 0 = Buy no energy

2.4.3 Investment Costs

Investment costs are the capital expenditure (CAPEX) required for acquiring and installing energy systems. The investment cost per unit of installed capacity is represented in different months M, and different times T by reflecting its operational. Here, K is the total number of energy devices in the Integrated Energy System (IES).

$$C_{INV} = \sum_{M=1}^{M} \sum_{t=1}^{T} \sum_{k=1}^{K} \Gamma^{k}{}_{cap} * (\mathrm{i}(i+1)^{y_k}) / ((i+1)^{y_k} - 1)$$

factors include:

- γ_k → is the device lifecycle,
- i→ The interest rate, which influences the financial viability of investments.
- $\Gamma^{k}{}_{cap}$ → represents the installed capacity of each device k.

Investment costs are amortised over the lifetime of the device, ensuring that the model accounts for the annual depreciation as a function of its useful lifespan.

2.4.4 Energy Exchange Costs

Energy exchange costs cover the expenses associated with purchasing electricity and gas from external suppliers when the system cannot meet energy demand internally. This includes:

$$C_{EPS} = \sum_{M=1}^{M} \sum_{t=1}^{T} \sum_{k=1}^{K} N_d * [(\lambda_{mt}{}^{Elect,Buy} * P_{mt}{}^{Elect,Buy} + \beta_{GAS} * (P_{mt}{}^{G,CHP}\mathrm{t} + P_{mt}{}^{G,GB})]$$

- β_{GAS} → The price of purchasing gas from external markets.
- $\lambda_{mt}{}^{Elect,Buy}$ → The price of purchasing electricity from the grid.
- $P_{mt}{}^{Elect,Buy}$ → The amount of electricity purchased

In our model, external energy procurement is necessary to balance supply and demand. The variables $P_{mt}{}^{G,CHP}$ and $P_{mt}{}^{G,GB}$ represent the amount of natural gas purchased from the gas network to supply:

- CHP plants (Combined Heat and Power systems)
- GB systems (Gas Boilers)

These purchases are made based on economic factors and availability constraints, ensuring that energy demand is met at the lowest possible cost.

2.4.5 Environmental Costs

Environmental costs reflect the financial penalties and regulatory charges associated with carbon emissions. This component is essential in evaluating the sustainability of different energy sources. The parameters include:

$$C_{CO2} = \sum_{M=1}^{M} N_d \sum_{t=1}^{T} \sum_{k=1}^{K} * [\ \phi^e\ (P_{mt}{}^{Elect,Buy} + P_{cap}{}^{,CHP}) + \phi^g\ (P_{mt}{}^{G,CHP}\text{t} + P_{mt}{}^{G,GB})\ ]$$

Where:

- r → The market carbon tax price, which imposes a financial penalty for emitting CO2.
- ϕ^e → The emission price of electricity generation, accounting for carbon intensity per unit of energy produced.
- ϕ^g → The emission price of gas consumption, reflecting the CO2 impact of natural gas use.

An important aspect of our model is the hydrogen fuel cell (HFC), which produces carbon-free electricity. Since HFCs do not emit CO2, no environmental compensation is applied to their electricity generation. notably The variables $P_{mt}{}^{Elect,Buy}$, $P_{mt}{}^{G,CHP}$ and $P_{mt}{}^{G,GB}$ represent the amount of electricity and natural gas purchased from the gas network to supply.

2.4.6 Operating and Maintenance (O&M) Costs

Operating and maintenance costs (OPEX) account for the recurring expenses required to keep the system functional. These include:

$$C_{OM} = \sum_{M=1}^{M} * \sum_{t=1}^{T} * \sum_{k=1}^{K} * \epsilon_k * \Gamma^k{}_{cap}$$

- Routine maintenance of energy infrastructure
- Operational expenses related to energy conversion and storage
- Wear and tear replacement costs for key components

The unit maintenance cost per device is represented by:

- ϵ_k → The O&M cost per unit of installed capacity for each device K in the system.
- $\Gamma^k{}_{cap}$ → represents the installed capacity of each device k.

Regular maintenance is crucial to ensure system reliability, efficiency, and longevity. High-maintenance energy assets, such as gas boilers and CHP plants, require greater O&M expenses compared to renewable energy sources, which generally have lower maintenance costs.

2.5 Objective Function of followers

The follower aims to minimize costs by determining whether it is more beneficial to purchase from us or competitors.

Min : $Z^F = \sum_{u=1}^{U} * \sum_{B=1}^{B} * \sum_{t=1}^{T} * \sum_{m=1}^{M} * ({r_{mt}}^{Elect,B} * {L_{u\,mt}}^{Elect} * Y_{u\,b} + {r_{mt}}^{Heat,B} * {L_{u\,mt}}^{Heat} * Y_{u\,b} + {r_{mt}}^{H2,B} * {L_{u\,mt}}^{H2} * Y_{u\,b}) + \sum_{u=1}^{U} * \sum_{t=1}^{T} * \sum_{m=1}^{M} * (cost^{Ext}\ {}_{P}{L_{u\,mt}}^{Elect} {Ext^{u}}_{p} + cost^{Ext}\ {}_{c}{L_{u\,mt}}^{Heat} {Ext^{u}}_{c} + cost^{Ext}\ {}_{H2}{L_{u\,mt}}^{H2} {Ext^{u}}_{H2}$

- u: Index for user
- Energy load = {P, C, H_2}, indexed by e
- E_b ⊂ Energy load: Set of energy loads included in bundle b (by definition)
- $Y_{u\,b}$: Binary variable that takes 1 if user u chooses bundle b, otherwise 0
- ${Ext^{u}}_{p}$: Binary variable that takes 1 if user u buys energy type P from a competitor

2.6 Energy Demand Balancing

As mentioned earlier, the IES is responsible for satisfying four types of demand; the Set of Constraints shows the system ensures balances energy demand across electricity, heat, and hydrogen, where to consider the energy type produced by different systems and on the other side of equation energy consumption in different energy sources and demand:

$$P_{mt}^{Elec,Buy} + P_{mt}^{Elec,PV} + P_{mt}^{Elec,WT} + P_{mt}^{Elec,CHP} + P_{mt}^{Elec,HFC} + P_{mt}^{BSS,dis} \geq \sum_{u=1}^{U} L_{u\,mt}^{Elec}(\sum_{b=1}^{B} y_{b\,u})$$

$$+ P_{mt}^{Elec,EB} + P_{mt}^{Elec,EL} + P_{mt}^{BT,ch}$$

$$P_{mt}^{Heat,CHP} + P_{mt}^{Heat,EB} + P_{mt}^{Heat,GB} + P_{mt}^{Heat,HFC} \geq \sum_{u=1}^{U} L_{u\,mt}^{Heat}(\sum_{b=1}^{B} y_{b\,u})$$

$$P_{mt}^{H_2,EL} + P_{mt}^{HST,dis} \geq \sum_{u=1}^{U} L_{u\,mt}^{H2}(\sum_{b=1}^{B} y_{b\,u}) + P_{mt}^{H_2,GB} + P_{mt}^{H_2,HFC} + P_{mt}^{H_2,HST,ch}$$

2.7 Load Satisfaction

By these three important equations, the model guarantees that the energy demand of all users is completely fulfilled. These equations ensure that the overall supply of electricity, heat, and hydrogen equals the demand of all the users. Through the implementation of these constraints, the model ensures also maximizing the use of resources.

Satisfy electricity demand of user u

$$\sum_{b=1}^{B} * \boldsymbol{a_b}^{\boldsymbol{Elect}} \boldsymbol{Y_{u\,b}} + \chi^{\boldsymbol{Elect}}{}_{\boldsymbol{u}} = \mathbf{1,\ u=1,....U\ ,\ b = 1,...,B}$$

Satisfy Heat demand of user u

$$\sum_{b=1}^{B} * \boldsymbol{a_b}^{\boldsymbol{Heat}} \boldsymbol{Y_{u\,b}} + \chi^{\boldsymbol{Heat}}{}_{\boldsymbol{u}} = \mathbf{1,\ u=1,....U\ ,\ b = 1,...,B}$$

Satisfy hydrogen demand of user u

$$\sum_{b=1}^{B} * \boldsymbol{a_b}^{\boldsymbol{H2}} \boldsymbol{Y_{u\,b}} + \chi^{\boldsymbol{H2}}{}_{\boldsymbol{u}} = \mathbf{1,\ u=1,....U\ ,\ b = 1,...,B}$$

Where:

- $a_b{}^{Elect}$: The parameter takes the value 1 if the bundle b includes the energy type electric.
- $a_b{}^{Heat}$: The parameter takes the value 1 if the bundle b includes the energy type heat.
- $a_b{}^{H2}$: The parameter takes the value 1 if the bundle b includes the energy type hydrogen.

- $\chi^{Elect}{}_{u}$binary variable takes 1if user u buys electricity from the competitor
- $\chi^{Heat}{}_{u}$binary variable takes 1if user u buys heat from the competitor
- $\chi^{H2}{}_{u}$ binary variable takes 1if user u buys hydrogen from the competitor
- $Y_{u\,b}$: Binary variable that takes 1 if user u chooses bundle b, otherwise 0

2.8 Bundles

In this part, we are going over one of the main strategies that we added to the system, with the aim of offering customers flexibility, savings, and efficient energy usage at a lower price based on the bundle they select. Our system offers five packages: electricity only, gas only, hydrogen only, electricity + gas, and gas + hydrogen, so customers have a range of alternatives optimally suited to their needs. Bundling not only achieves maximum efficiency for the system and minimizes wastage but also promotes utilization of cleaner forms of energy. Further, when bundled packages bring multiple sources of energy together into one package, customers get favorable prices, so bundled packages cost less than if one were to purchase each energy form separately. It is a demand peak management and grid stabilizing method.

2.8.1 Logical restrictions to define types of demands in every bundle

This section details the logical constraints that determine the structure of different types of energy requests (e.g., electricity, heat, hydrogen) within a bundle. These constraints define the bundles with the appropriate structure of energy services that are adequate for the demand of the consumers and the system's ability. The model gives cost-effective, feasible, and demand-based energy supply to the consumers through the imposition of these constraints.

$$(1- a_b{}^{Elect})\sum_{M=1}^{M}*\sum_{t=1}^{T}*\,P_{u\,mt}{}^{Elect} \leq M(1-Y_{u\,b}),\ \text{u=1,....U , b = 1,...,B}$$

$$(1- a_b{}^{Heat})\sum_{M=1}^{M}*\sum_{t=1}^{T}*\,P_{u\,mt}{}^{Heat} \leq M(1-Y_{u\,b}),\ \text{u=1,....U , b = 1,...,B}$$

$$(1- a_b{}^{H2})\sum_{M=1}^{M}*\sum_{t=1}^{T}*\,P_{u\,mt}{}^{H2} \leq M(1-Y_{u\,b}),\ \text{u=1,....U , b = 1,...,B}$$

Where:

- $a_b{}^{Elect}$- $a_b{}^{Heat}$- $a_b{}^{H2}$: The parameter takes the value 1 if the bundle b includes the energy type hydrogen.

In addition, there are complementary equations in bundle strategy as below:

- $a_b{}^{Elect} P_{u\,mt}{}^{Elect} \geq L_{u\,mt}{}^{Elect}\ Y_{u\,b}$, u=1,....U , b = 1,...,B
- $a_b{}^{Heat} P_{u\,mt}{}^{Heat} \geq L_{u\,mt}{}^{Heat} * Y_{u\,b}$, u=1,....U , b = 1,...,B
- $a_b{}^{H2} P_{u\,mt}{}^{H2} \geq L_{u\,mt}{}^{H2} * Y_{u\,b}$, u=1,....U , b = 1,...,B

2.8.2 Set Logical Prices Among Bundles

This keeps the prices of the bundles appropriate to their size and energy types. Larger bundles with more energy services should cost more than smaller ones. These pricing rules keep the bundles reasonable, competitive, and desirable to the users.

- $r_{mt}{}^{Elect,b1} \leq \alpha * r_{mt}{}^{Elect,b2}$, m = 1, ..., M , t = 1, ..., T, b1, b2 = 1, ..., B | Size(b1) > Size(b2)
- $r_{mt}{}^{Elect,b1} \geq 0$, m = 1, ..., M , t = 1, ..., T, b = 1, ..., B | $a^{Elect}{}_b$= 1
- $r_{mt}{}^{Heat,b1} \geq 0$, m = 1, ..., M , t = 1, ..., T, b = 1, ..., B | $a^{Heat}{}_b$= 1
- $r_{mt}{}^{H2,b1} \geq 0$, m = 1, ..., M , t = 1, ..., T, b = 1, ..., B | $a^{H2}{}_b$= 1

Where:

- $r_{mt}{}^{Elect,b1}$ -$r_{mt}{}^{Heat,b1}$ - $r_{mt}{}^{H2,b1}$: Energy prices for bundle b on day m at time

2.9 Transformation of Bi-Level Model into a Single-Level Problem

Our Bi-level optimisation problem is formulated in a way that represents how two different players make decisions like suppliers who set prices and consumers who react. Such models are, however, typically complicated and time-consuming to find because they represent two interconnected levels of decision-making. In order to get around the simplified model, we transformed the bi-level structure to an equivalent single-level problem. This was done by adding a special constraint called a cut that helps to transform the follower's (user's) decisions into the leader's (provider's) problem directly. This does not allow two problems to be solved separately. A second auxiliary variable, auxi (e, u, b, m, t), was added for the sake of carrying out this transformation. It combines bundle options, energy sources, and time slots into one optimization model. With that, we can now formulate the whole problem in AIMMS. This conversion allows the model to compute more complicated scenarios—like pricing, load balancing, and capacity planning—at a higher speed while still upholding the rules of how providers and consumers interact. This approach referred [1]which is explained in detail:

$$\sum_{u=1}^{U} * \sum_{B=1}^{B} * \sum_{t=1}^{T} * \sum_{m=1}^{M} * \sum_{e=1}^{E} * \ auxi_{e\ u\ b\ m\ t} \ * \ dem_{eumt} + \sum_{u=1}^{U} * \sum_{e=1}^{E} * \ {P_{eu}}^{Ext} * demand_{eu} \ * Ext - par_{e\ u\ it}) <= \sum_{u=1}^{U} * \sum_{B=1}^{B} * \sum_{t=1}^{T} * \sum_{e=1}^{E} * \sum_{m=1}^{M} * \ auxi_{e\ u\ b\ m\ t} \ * dem_{eumt} * \ Y - par_{e\ u\ it} \ ) \ + \sum_{u=1}^{U} * \sum_{e=1}^{E} * \ {P_{eu}}^{Ext} * \ demand_{eu} * Ext - par_{e\ u\ it})$$

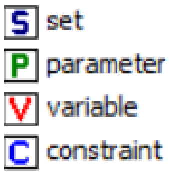
Where:

- $auxi_{eubmt}$: binary variable takes 1 o 0
- dem_{eumt} : The parameter of demand
- ${P_{eu}}^{Ext}$: The parameter price of competitor
- $Ext - par_{e\ u\ it}$: binary variable takes 1 o 0
- $Y - par_{e\ u\ it}$: binary variable takes 1

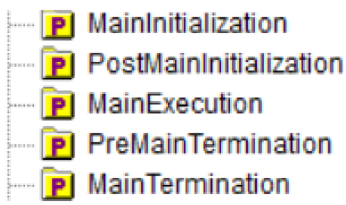
3. AIMMS

AIMMS stands for Advanced Interactive Multidimensional Modelling System. AIMMS is a modeling language applied for optimisation and decision-making based on mathematical approach. It's an environment in which the users can model and build mathematical equations. The relationships, variables, constraints, and objective functions can be defined by users. AIMMS supports a range of optimization algorithms, including linear programming and mixed integer programming. Users can choose the most suitable algorithm based on the nature of their optimization problem. We selected AIMMS to develop our model due to its strong capabilities perfectly suited to our problem because, with our bi-level model, we are required to have a wide level of flexibility and power to maintain all the contributing connection features and examine different solvers. AIMMS has numerous optimization techniques; thus, AIMMSs high-performance solvers such as CPLEX, GUROBI, and BARON to ensure that each challenging mathematical model are solved effectively and accurately. The second principal benefit is the various integration that it offers. We can import/export data and interact with external systems through libraries and reach excel file output easily. In the end, the interaction of all these advantages is great motivation to get use of this program for solving[11].

In our case, as mentioned above, in each mathematical model like ours where the objective function, decision variables, and constraints are defined and structured. These elements will be presented in, following format of AIMMS: Sets (S), Parameters (P), Variables (V), and Constraints (C): [11]



To achieve the optimal solution in our bi-level model, executing the main objective is a critical step. In AIMMS, this involves defining a Main Execution Module, where the primary objective function is solved by aligning decision variables and constraints which the main model that we will discuss later.[11]



3.1 Modeling in AIMMS

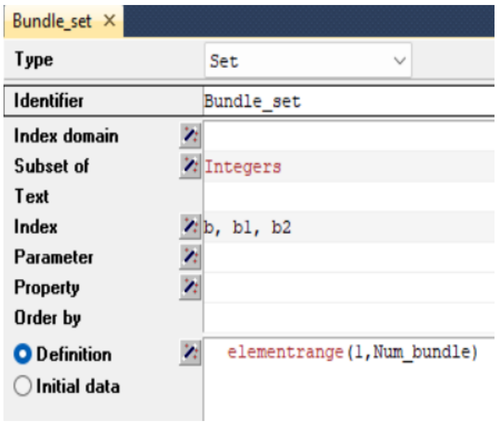
As described in Chapter 3, each equation in the model has been carefully defined in AIMMS using specific index domains. This allows the model to take advantage of logical connections between different components at every step of the optimisation process. In the following sections, additional parameters and values will be introduced to further enhance the model. These additions help improve the accuracy and completeness of the system, making the analysis reliable and the results more complete in finding reasonable and practical solutions[11].

3.1.1 Sets

A simple set in AIMMS is a finite collection of elements, either strings or integers. Sets use indices to reference elements in indexed expressions and statements [11].They may have optional attributes like Index Domain, Subset Of, Index, Parameter, Text, and Definition. AIMMS supports multi-dimensional sets, which are parameterized over other sets. For example, specifying an index domain (m,t,u,b) links each element to specific value combinations. Although sets and parameters may seem similar due to their multi-dimensional nature, sets contain elements, while parameters store numerical values. Additionally, a set's "parameter" attribute associates a parameter with each element as a specific property. The following sections provide AIMMS model sets with necessary explanations.

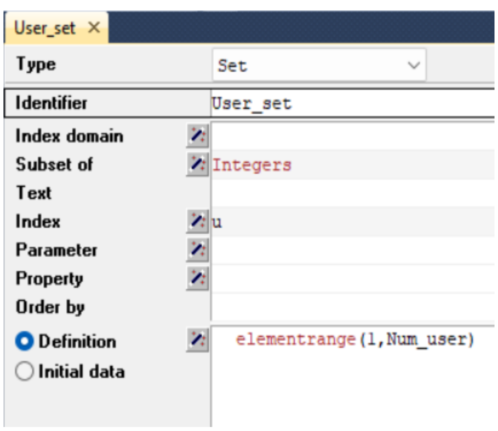
3.1.1.1 Bundle set

This set by identifier "bundle-set" contains the integer data format by considering the indexes b, b1, and "b2," which represent the number of bundles that are changeable from 1 to “Num_bundle" parameter. In the model, the system will reach the results based on this element range.



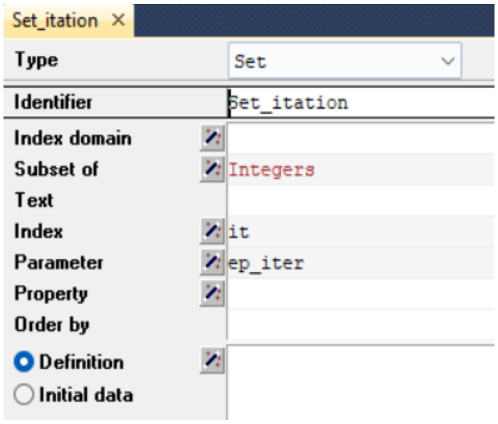
3.1.1.2 User set

Another important set to represent the number of users that will be considered in the model is the indexu,” like other indexes that make connections for the base to be specified in the whole model, the range set by the main parameter "Num-user."



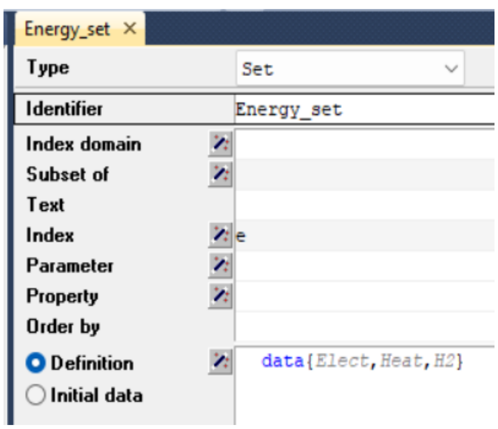
3.1.1.3 Set Iteration

The set of iterations plays a critical role in reaching the optimized solution, as each iteration balances the trade-off between the leaser and follower to improve overall output. Starting from the second iteration, this set reflects better real-life decision-making to select the best answer.



3.1.1.4 Energy set

Simply in this set, we identified the energy types that are involved in our model: Electricity, Heat and Hydrogen:

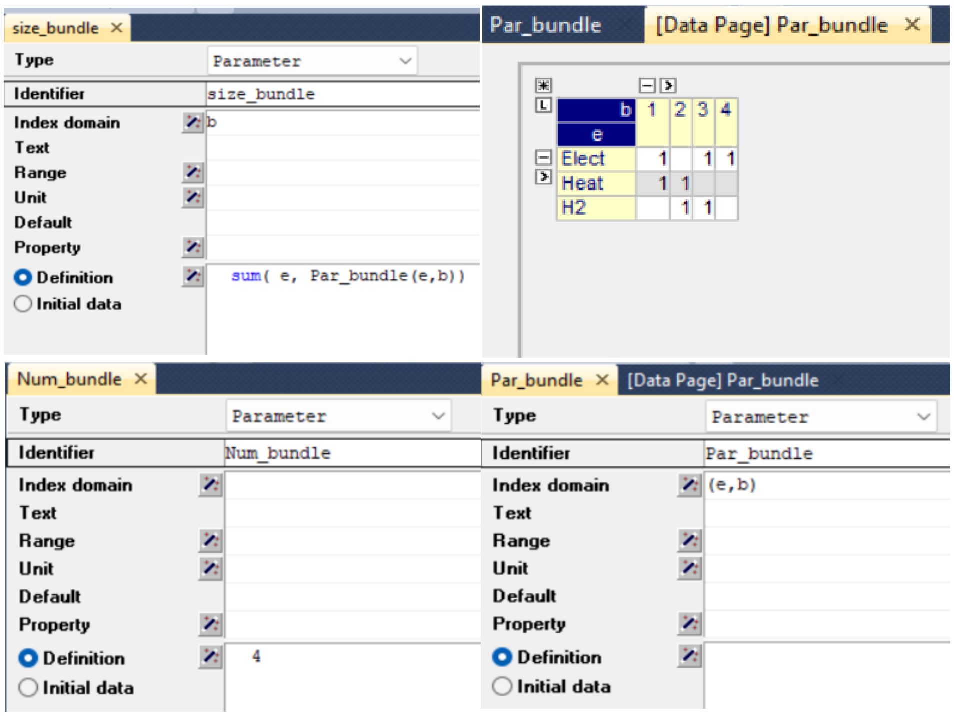


3.1.2 Parameter

In AIMMS, a parameter represents a known value, either numeric or string-based [11]. AIMMS distinguishes between parameters (known values) that can be manually adjusted, but from the other side, variables (unknowns in mathematical programs) that, after execution, In our case, we set two key parameters, like prices and demand (two important parameters), while variables will be computed based on inputs. The following sections provide explanations of AIMMS model parameters.

3.1.2.1 Bundle parameters

Num_bundle is a critical parameter that defines the total number of bundles available in the system. that can be adjusted by the user model to set different numbers of bundles to offer. In different scenarios, we inserted different numbers of bundles to check how the buyers reacted and how it had an impact on our profit. size_bundle This parameter calculates the size of each bundle by summing the values from Par_bundle (e, b) across all elements e. It helps determine how many energy types are included in each bundle. Par_bundle is another adjustable parameter by us. When can we design the energy type included in each bundle by putting "1" if we want the energy type “e” included in that specific bundle.



Y-par

This parameter is a binary parameter that defines whether each specific user chooses a particular bundle in a given iteration; its indexes are:

it: iteration

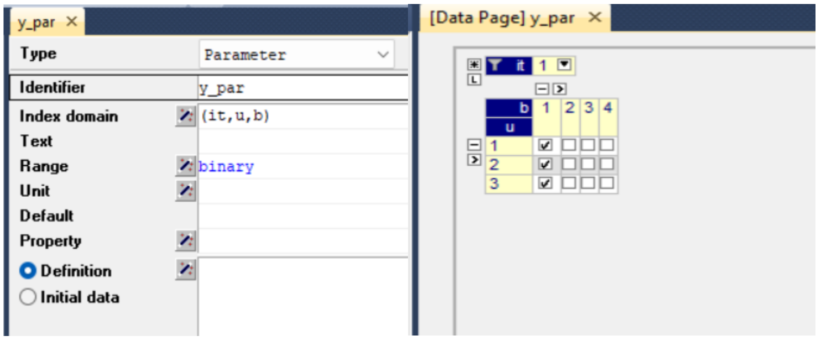
u: user

b: bundle

This binary parameter only gets 1 or 0:

- 1 indicates that the user selects the bundle b in iteration.
- 0 means that bundle b has not been selected by user u in the given iteration.

On the second page, we can see the table of data.



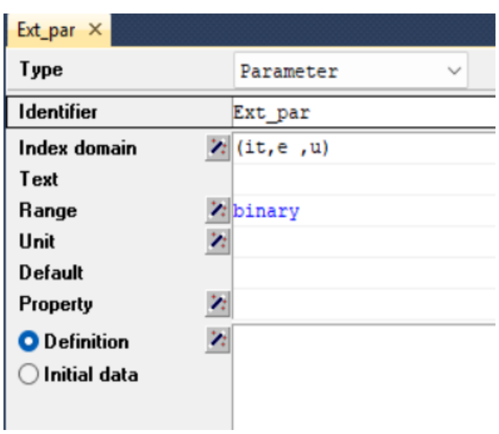
3.1.2.2 External parameters

In this section, key external parameters are defined with the aim of considering flexibility into the model by incorporating competition. With parameters, external competitors in our pricing strategies, giving users the freedom to choose between internal and external options. This approach improves realism and ensures a more optimized solution by considering competitors in the market.

Ext_par

This parameter indicates whether a user chooses an external competitor's energy source in a given iteration.

- If Ext_par(it, e, u) = 1, the user purchases energy from an external competitor.
- If Ext_par(it, e, u) = 0, the user chooses to buy from us.

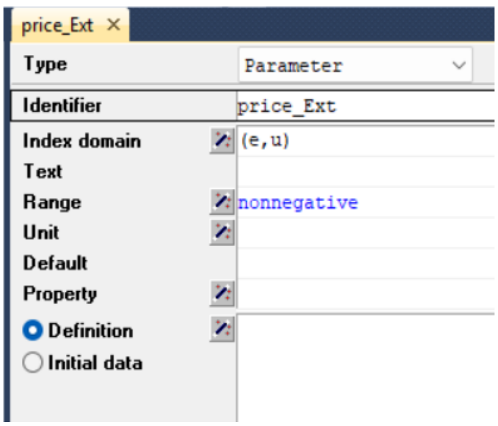


Price_Ext

is an external price that is associated with different indexes like “u” and "e.”

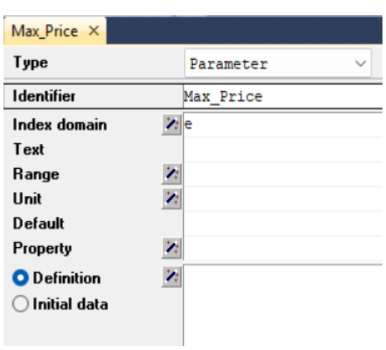
- e: energy
- u: user

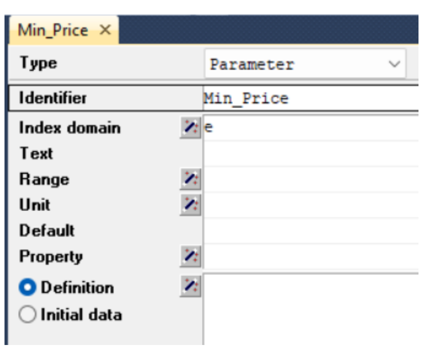
This parameter is nonnegative; it means it cannot get a negative value and ensure a valid price value.

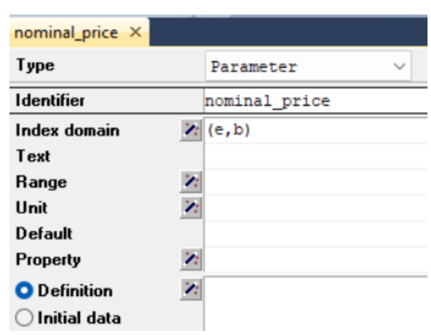


3.1.2.3 Price parameters

In this section, we define different price levels in our model. First, we set the maximum acceptable price of each type of energy, with the aim of having the upper limit in energy prices; secondly, we represent the minimum acceptable price to ensure that prices won't fall below the threshold. The last one will be defined by the model run, when the system is run and the optimal solution is reached.

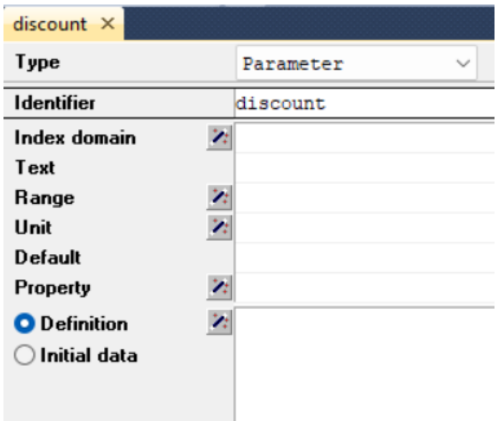






Discount

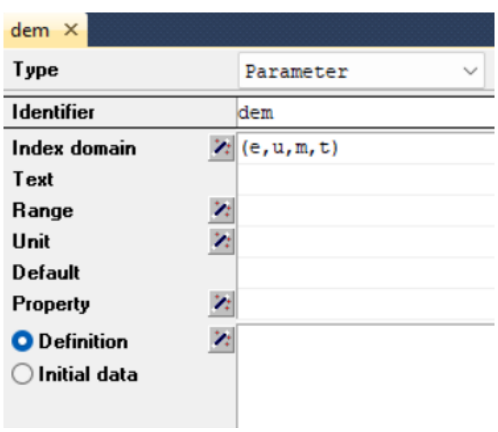
Represents discount values within the model. Unlike price_Ext, it does not have a predefined index domain, range, or unit, suggesting that it can be a general parameter for different scenarios. We basically define this parameter based on the prices.



3.1.2.4 Demand Parameter

This parameter defines how much energy each user type consumes based on their household composition. Its indexes are:

- e: energy
- u: user
- m: month
- t: hours



In addition to defining user demand, it is important to explain how users are allocated in the model. Family structure selection within this study comes from real statistics on how individuals live. It's tried to predict energy demand in a realistic way by assuming typical family structures. In CSIRO's Energize Insight, two-person families are the most common, followed by three- and four-person households. Energy usage increases as more people are in the household.

Three household types were employed in this study:

- User 1: A couple (two members)
- User 2: A couple with a child (three members)
- User 3: An extended family (four or five members)

Such users are real energy consumption. For example, families with children will consume more power than small families. Through this segmentation, the model can fit actual energy demand more accurately. This improves the accuracy of the system and aids in more accurate decision-making regarding planning and setting prices for energy [8].

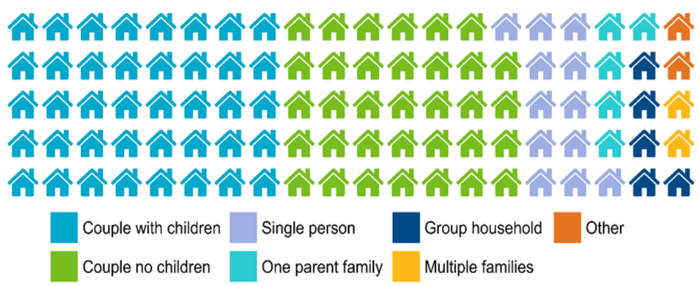


Figure 6: Household Allocation

3.1.3 Variable

In AIMMS, a variable represents an unknown value within a constraint. Variables receive value when a solver finds a solution. Once the model runs, AIMMS provides the optimal variable values as output. Variables have additional attributes beyond parameters, guiding the solver or storing solution-related information. Key attributes include IndexDomain, Text, Range, Unit, Property, Default, and Definition.

The following sections provide explanations of AIMMS model variables. These three AIMMS variables are the system's energy requirements for electricity, heat, and hydrogen (H2) and are indexed by (m, t), where:

- m: represents month.
- t: represents time,

3.1.3.1 Hydrogen Load

- Index Domain: (m, t)
- Range: Free

Description: This variable represents the hydrogen demand in the system over time and across different modules. Since it is free, it can take any value, meaning it may also represent excess production or deficits.

3.1.3.2 Heat Load

- Index Domain: (m, t)
- Range: Nonnegative (can only be zero or positive)

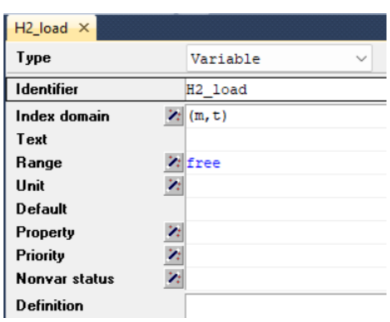
Description: This variable quantifies the thermal energy demand in the system, ensuring that heat consumption remains physically meaningful (no negative values). It is useful for optimising heating system operations.

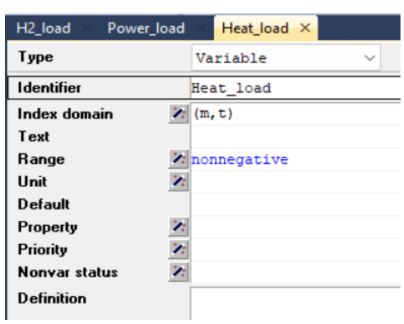
3.1.3.3 Electric Load

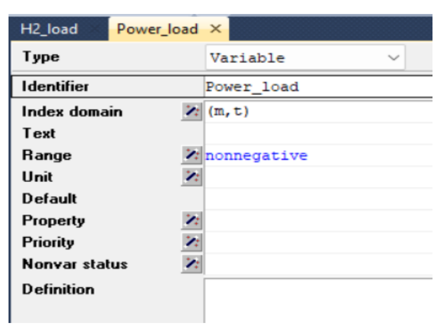
- Index Domain: (m, t)
- Range: Nonnegative (can only be zero or positive)

Description: This variable captures the electricity demand at different times and locations. Since it is nonnegative, it ensures that power consumption cannot be negative, aligning with real-world constraints.

All these three variables, which stand for the electric, heat and h2 load to the system :







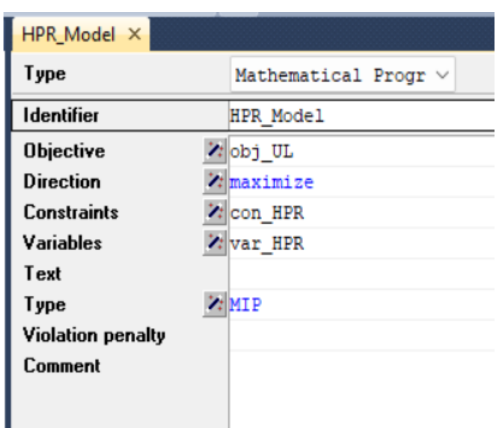
3.1.4 Constraints

In this part of the model, all the constraints defined for energy sources in Section 2 are declared and implemented with appropriate index sets. Constraints in AIMMS are mathematical expressions composed of variables, parameters, and constants. Constraints are used to limit variable values according to some conditions that should be met prior to producing an output. These constraints are the foundation of the optimisation model and are required in order to generate valid and realistic solutions. Below are further explanations on how these constraints are utilized in the AIMMS environment.

3.2 Mathematical program in AIMMS

3.2.1 HPR Model

We explain here first what the HPR_Model is, which is the core optimisation program developed in AIMMS which is critical to reach an optimisation solution. This model brings together all the necessary components—objective functions, constraints, and decision variables—used to represent both the leader (upper level) and follower (lower level) and both constraints(con_HPR) and its variables (var_HPR) for the systematic comprehension of this mathematical program's operation are explained below in details. To make the model solvable, we transformed the bi-level structure into a single-level problem, which is one of the key steps in building this system. The transformation ensures that all decisions are handled together under a unified optimisation framework. Type of MIP (Mixed Integer Programming) is the way to clarify in AIMMS, this setup provides a coherent optimisation structure that enables all the elements to function together with the aim of clarifying for system while considering all the constraints that we have which reflecting the trade-off between the two stakeholders.



3.2.2 HPR Constraint (con_HPR)

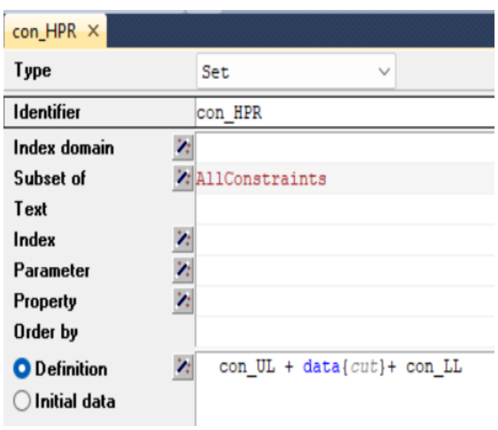
We define the constraints (con_HPR) that reflect the structure and viability of the HPR_Model. These constraints are gathered in a set called All Constraints, including:

- con_UL: Top-level constraints that define the system operator's strategic decisions, i.e., capacity planning and pricing.

- data(cut): Cut constraints that allow the bi-level structure to be reduced to a single-level model. This enables the problem to be transformed into a solvable one using regular optimisation solvers.

- con_LL: Lower-level constraints that reflect user-side decisions, such as energy bundle choices and price responses to consumption.

We declare this structure—con_UL + data(cut) + con_LL—because we wish AIMMS to take into account the logic and goals of both levels when solving the model. By integrating the provider's (upper-level) and the consumer's (lower-level) constraints into one formulation, AIMMS can examine all the interactions and trade-offs between the two decision-makers at once. Without the integrated constraint structure, the model could not produce a consistent solution that balances the objectives of both parties



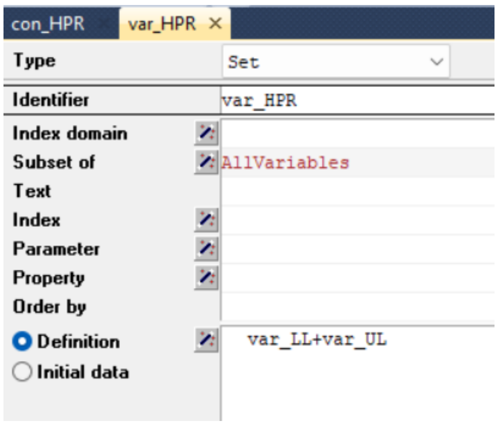
3.2.3 HPR Variables (var_HPR)

We declare the decision variables (var_HPR), they are classified under a set called AllVariables that includes:

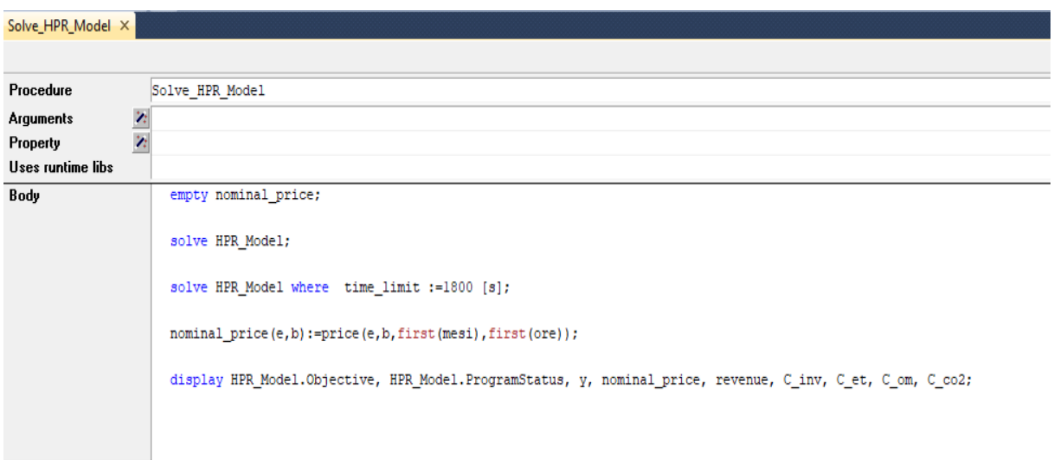
- var_UL: Upper-level decision variables system operator's strategic decisions, i.e., capacity planning and pricing.

- var_LL: Lower-level decision variables that manage operational details such as energy consumption, bundle selection, and user responses.

By including both upper- and lower-level variables, the model captures the full decision-making process. This allows the optimization to reflect strategic decisions made by the provider, along with detailed user responses, resulting in a solution that is balanced, realistic, and system wide.



All the solution codes were written with time limits in mind, to make sure the system works well within a realistic time frame and responds quickly during the optimization process.



4. Literature Review

This section discusses the process of gathering information and identifying the research gap for this thesis. We identify three of the most pertinent papers and analyze their shortcomings to identify the actual gap. Our final model attempts to bridge these gaps, presenting a superior solution and ensuring that we adopt the best possible research strategy. In filling these gaps, we contribute to developing an even more optimized and sustainable IES framework.

4.1 Article 1: Stackelberg-Based ICES Optimization

The analysis starts with the article *'Optimised Operation of Integrated Community Energy System Considering Integrated Energy Pricing Strategy: A Two-Layer Stackelberg Game Approach'*, exploring its methodology and leveraging the identified research gap.

Study No.	Article Title	Year	Authors	Aim	Methodology	Main Findings
1	Optimised operation of integrated community energy system considering integrated energy pricing strategy: A two-layer Stackelberg game approach	2024	Dongfeng Yang, Ziqian He, Yong Sun, Baoju Li, DeXin Li Xiaojun Liu, Chao Jiang	Optimize the operation of an integrated community energy system (ICES) by proposing an energy pricing strategy that considers energy source interconversion.	Two-layer Stackelberg game model: upper-layer operator maximizes revenue by setting energy prices; lower-layer users minimize energy costs by adjusting usage strategies.	The model achieves a unique Stackelberg equilibrium solution, balancing operator and user interests. It improves wind power absorption, reduces load fluctuation, and enhances energy system efficiency.

Table 1:Stackelberg-Based ICES Optimization Summary

Research Gap:

Both researchers formulate a mathematical approach like a two-level Stackelberg game for ICES pricing in this essay. The optimality outcomes of this paper indicate that investment in P2G, bus BSS, and V2G improves economic efficiency and wind power uptake, yet the model fails to handle user-specific energy bundles. The model that is offered in this thesis will establish a bi-level optimization where operators sell and offer value-added energy bundles (e.g., electricity + hydrogen), adding user engagement and maximizing consumer-supplier interaction. The model neither takes into consideration EV user satisfaction, battery wear compensation, nor revenue sharing. My work will include these and lead to a minimum level of carbon emissions to better enhance ICES economically and sustainably.

4.2 Article 2: ICES Optimization with Seasonal Thermal Storage

The second article, *'Optimisation of Integrated Energy Systems Considering Seasonal Thermal Energy Storage' (2023)*, is examined to identify key contributions and existing research gaps.

Study No.	Article Title	Year	Authors	Aim	Methodology	Main Findings
2	Optimization of integrated energy systems considering seasonal thermal energy storage	2023	Yixing Zhou, Chunhua Min, Kun Wang, Liyao Xie, Yuanhong Fan	Develop an optimization model for an integrated energy system (IES) with seasonal thermal energy storage to improve economic efficiency and reduce carbon emissions.	Mixed-integer linear programming (MILP) model optimizing IES capacity and operation over a one-year horizon. Includes ground source heat pump (GSHP), thermal storage, solar collectors, and waste heat recovery.	The IES with seasonal thermal storage reduces annual total cost by 9.1%, energy purchase costs by 23.4%, and carbon emissions by 12.6%. Seasonal thermal energy storage enhances system flexibility and efficiency.

Table 2:ICES Optimization with Seasonal Thermal Storage Summary

ResearchGap:

While this study optimises an integrated energy system (IES) with seasonal thermal energy storage, dynamic pricing schemes or energy bundling strategies are not considered. The current method is primarily concerned with capacity planning, whereas the model does include ground-source heat pumps and thermal storage but does not address operating efficiency with multi-energy interactions. Our research build on this by developing a bi-level optimization model that involves bundled energy services with dynamic pricing, and additionally, enhancing economic efficiency and renewable energy utilization efficiency.

4.3 Article 3: P2G Effectiveness in Integrated Energy Systems

The third article, *'Effectiveness of Power-to-Gas Technology for an Integrated Energy System'*, is reviewed to assess its findings and identify unresolved research challenges.

Study No.	Article Title	Year	Authors	Aim	Methodology	Main Findings
3	Effectiveness of Power-to-Gas Technology for an Integrated Energy System	2024	Sara Khodaparasti, Anna Pinnarelli, Antonio Cosma,Maria Elena Bruni	Develop a mathematical framework for modeling an integrated hybrid energy system incorporating Power-to-Gas technology and various renewable energy resources.	Formulated as a mixed-integer programming model, validated through computational experiments on a real case study. The model considers multiple energy types, conversion, and storage technologies.	Power-to-Gas enhances energy system efficiency, supports renewable integration, and reduces total costs. Incentivizing P2G investments lowers system costs by up to 6.6%, ensuring economic feasibility.

Table 3:P2G Effectiveness in Integrated Energy Systems Summary

ResearchGap:

While this study optimizes an integrated energy system with Power-to-Gas (P2G) technology, it prioritizes the minimization of investment and operation costs over neglecting demand-side flexibility or user-directed energy bundling strategy. The study prioritizes the economic benefits of the development of P2G and greater integration of renewables, but the model is not taking into account time-of-use prices for energy or dynamic price-based energy schemes. In addition, stakeholder-user interaction and active energy demand response are not yet taken into account. My research will introduce a bi-level optimization model that considers bundled energy services and adaptive pricing to improve system efficiency, economic viability, and user adaptability.

4.4 Article 4: Effectiveness of Stackelberg-Based Energy hybrid Integrated Energy Systems

The fourth article*, "Hybrid Energy Sharing Mechanism for Integrated Energy Systems Based on the Stackelberg Game"*, is reviewed to assess its findings and identify unresolved research challenges.

Study No.	Article Title	Year	Authors	Aim	Methodology	Main Findings
4	Hybrid Energy Sharing Mechanism for Integrated Energy Systems Based on the Stackelberg Game	2021	Peng Q., Wang X., Kuang Y., Wang Y., Zhao H., Wang Z., Lyu J.	Propose a hybrid energy sharing mechanism to coordinate energy flow in integrated systems using Stackelberg game theory.	Developed a bi-level optimization model where the energy operator acts as leader and prosumers as followers. Solved using a distributed algorithm; validated with a case study.	The model supports the shift from centralized to interactive energy markets. It builds a distributed sharing mechanism using an energy hubA privacy-preserving algorithm ensures convergence,

Table 4:Effectiveness of Stackelberg-Based Energy hybrid Integrated Energy Systems Summary

Research Gap:

While Peng et al. (2021) successfully apply a Stackelberg game to design a distributed energy sharing mechanism, their model mainly focuses on system-level coordination between operator and prosumers without explicitly modeling multi-energy bundling or hierarchical planning over time. The dynamic role of bundle combinations and time-sensitive decision layers—key to real-world IES—remains underexplored. Additionally, their transformation from bi-level to single-level lacks integration with advanced optimization tools like AIMMS for large-scale simulations. Therefore, there is a need for models that combine strategic interactions (e.g., Stackelberg structure), multi-energy bundling, and hierarchical time-based planning, while ensuring computational efficiency and scalability in practical applications.

Result

In this chapter, we look at the results of the model we built earlier to see how different energy sources work together to meet the needs of an Integrated Energy System (IES). Using AIMMS, we optimized the system to supply electricity, heat, and hydrogen in a way that meets user demand, keeps the leader's profit positive, and helps lower energy costs for consumers. The results give us helpful insights into how to use energy resources more effectively. The model uses real-world energy prices and includes different types of users to reflect realistic conditions. Technologies like the hydrogen tank and hydrogen fuel cell (HFC) play a key role, especially the HFC, which helps provide both heat and electricity. We also reduced the use of the gas boiler (GB), which relies on hydrogen and natural gas, to cut down on emissions. Another part of our analysis focuses on the bundle strategy and how it affects the leader and follower objectives during different iterations. Throughout the section, we use graphs to show how IES technologies contribute to energy supply over time and how different strategies impact system performance.

This graph shows how electricity is generated and purchased over time to meet the system's needs. HFC stands out as the main contributor, highlighting its key role in power production. EB and EL use electricity regularly to support heat and hydrogen production. When local sources aren't enough, electricity is purchased from outside to fill the gap. Other sources like solar panels (PV) and combined heat and power (CHP) make smaller. Overall, the system relies heavily on flexible options like HFC and grid electricity to meet demand efficiently, control costs, and keep the leader's remarkable positive profit.

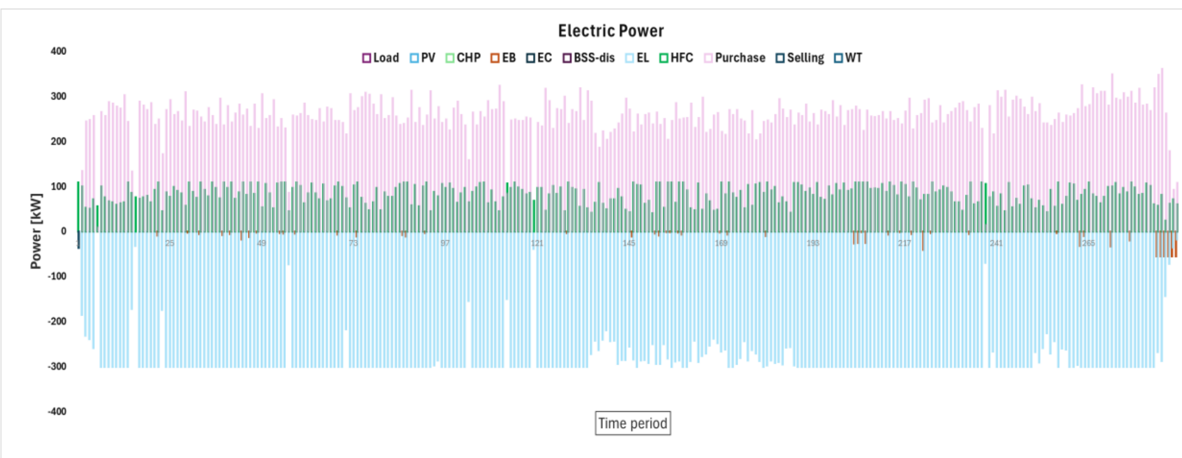


Figure 7: Energy production and procurement schedule: Electricity load

Figure 8 shows the heat power generation over time. Most of the heat demand is met by two main sources: the Electric Boiler (EB) and the Hydrogen Fuel Cell (HFC). HFC provides the majority of the heat throughout the entire period, showing it is the main source for thermal energy in the system. EB also contributes, mainly during certain peaks when HFC alone is not enough. Other sources like CHP (Combined Heat and Power) and GB (Gas Boiler) appear rarely and with small amounts, which suggests that the system prioritizes cleaner or more efficient options like EB and HFC. This setup reflects a strategy focused on using low-emission and flexible sources to meet heat demand while optimizing the system's performance.

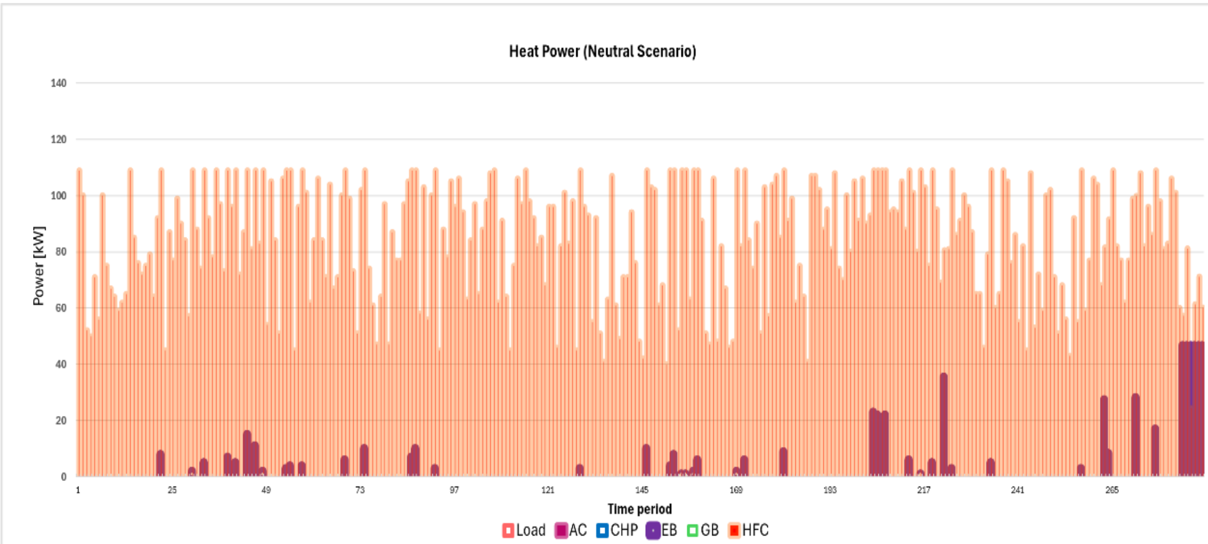


Figure 8 : Energy production and procurement schedule: Heat load

Figure 9 shows how hydrogen is produced, stored, and used over time in the system. One of the first things we notice is that the hydrogen tank plays an active role, it's frequently charged (HST-ch) and consistently discharged (HST-dis), which tells us that stored hydrogen is being used regularly to meet energy demand. On the production side, most of the hydrogen comes from the electrolyzer (H2-EL), which converts electricity into hydrogen. It operates steadily throughout the timeline and explains why it also appears as a big electricity consumer in the electricity graph. Hydrogen is then used in different ways: the hydrogen fuel cell (HFC) uses it to produce both electricity and heat and plays a key role in system performance, while the combined heat and power (CHP) unit uses smaller amounts. The storage level stays stable, within a range of about 132–228 kWh, which helps keep the system flexible and responsive. What's interesting is that the gas boiler (H2-GB) is never used, suggesting that hydrogen made from natural gas isn't part of the strategy—likely to keep the system cleaner and more sustainable. All in all, this part of the system clearly leans on clean hydrogen, smart storage, and efficient usage to meet energy needs and support the overall goal of maintaining profit while lowering emissions.

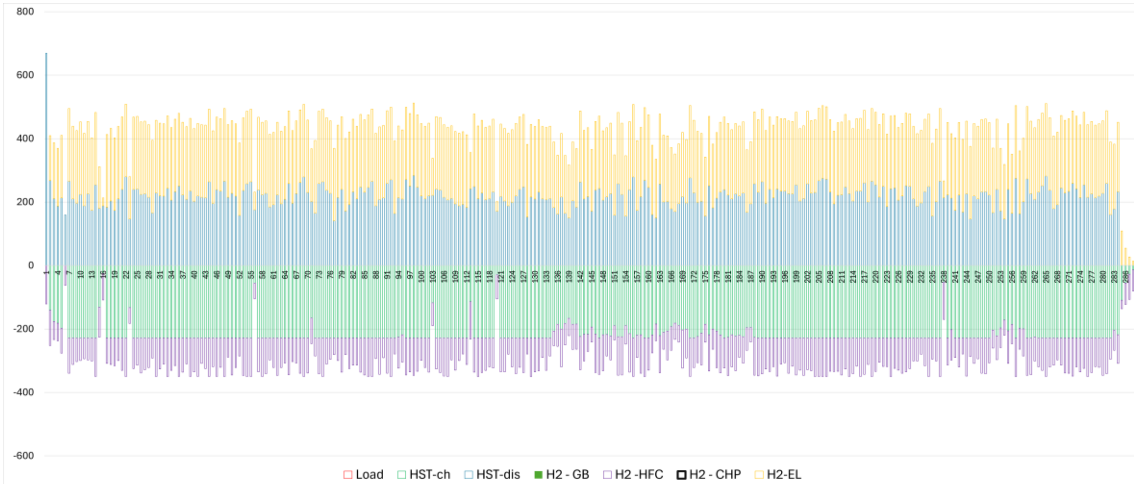


Figure 9: Energy production and procurement schedule: Hydrogen load

Figure 10 shows the behavior of the hydrogen storage tank over time, including its state of charge (HST-S), charging (HST-ch), and discharging (HST-dis). The green bars represent hydrogen being stored (HST-ch), and the red bars show hydrogen being discharged (HST-dis). The light pink area at the top illustrates the storage level at each time step. The graph reveals a consistent pattern of charging and discharging throughout the entire period, indicating that the system actively manages hydrogen flows to maintain flexibility. The storage level (HST-S) remains relatively stable, fluctuating between 200–600 kWh, which suggests that hydrogen storage never reaches critical low levels. The system avoids full depletion by continuously recharging the tank. This balance ensures that stored hydrogen is always available when needed, supporting the energy supply and confirming the importance of hydrogen storage as a strategic buffer in the IES.

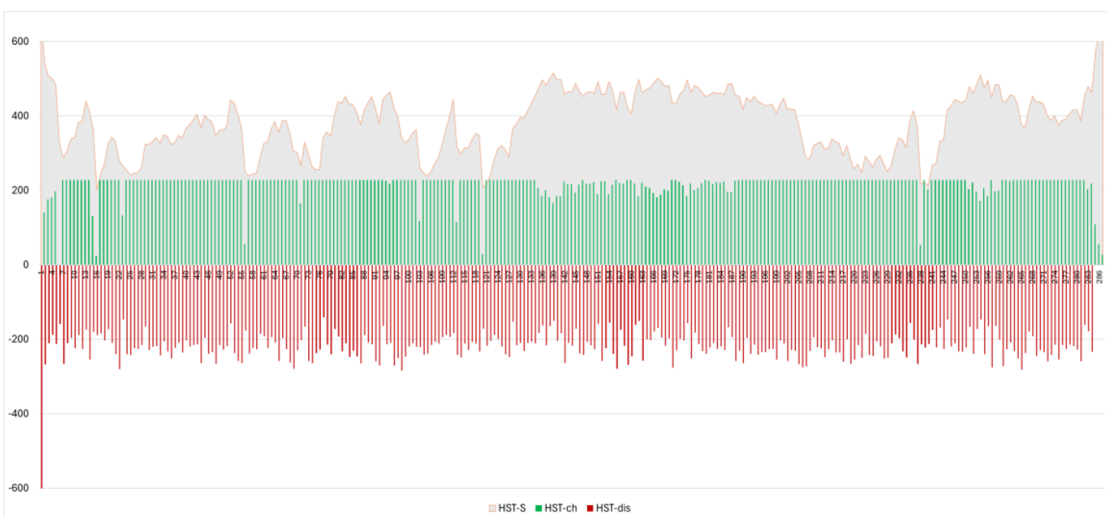


Figure 10 : Hydrogen Tank Storage Behaviour Over Time

Figure 11 illustrates the revenue performance across different bundle strategies: 3-bundle, 4-bundle, and 5-bundle configurations. As explained earlier in the thesis, bundles are designed to give consumers more flexibility in choosing energy types. In the 3-bundle setup, each bundle includes only one type of energy (electricity, heat, or hydrogen). The 4-bundle strategy keeps the first three single-energy bundles and adds a fourth bundle with both electricity and heat. The 5-bundle configuration introduces even more flexibility, including one bundle with electricity and heat, and another with heat and hydrogen. The graph shows that in the first iteration, all three bundle strategies start at the same revenue level. However, from the second iteration, the revenue drops slightly as the system begins to incorporate consumer preferences into the optimization. The 5-bundle case shows a recovery in revenue in the third and fourth iterations, suggesting that offering more flexible bundles may better match consumer demand. Despite a small decline after iteration 4, the 5-bundle setup maintains the highest overall revenue. This confirms that bundle-based strategies, especially with more mixed-energy options, can be beneficial for both the system and consumers by improving satisfaction and maintaining revenue performance.

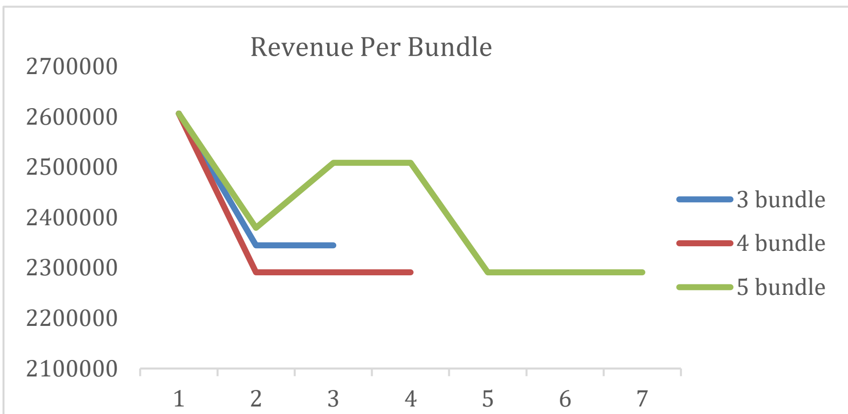


Figure 11: Revenue per Bundle

In this part of the results analysis, we dive deeper into the mathematical structure of the optimization process, focusing on how the model moves between the follower and leader objective goals to find the best solution. It represents the best value the leader could achieve, assuming follower response, while it follows to achieve reflects what is currently achievable with the follower's constraints. The smaller the gap between these two values, the more reliable and accurate the final result becomes. And here, the story begins—when we start changing the number of bundles. The simple case starts with the 3-bundle setup, where each bundle contains only one energy type (electricity, heat, or hydrogen). This result shows that the 3-bundle model reaches optimality very quickly—in just three iterations. At first, there's a large gap between their objectives, meaning the model is still searching for a good solution. But over time, the gap becomes smaller, and by the third step, are nearly equal, proving that the model has reached its best solution. As we move to more flexible bundle designs, such as 4 and 5 bundles, the trade-off between realism and complexity becomes a key factor in our optimization process.

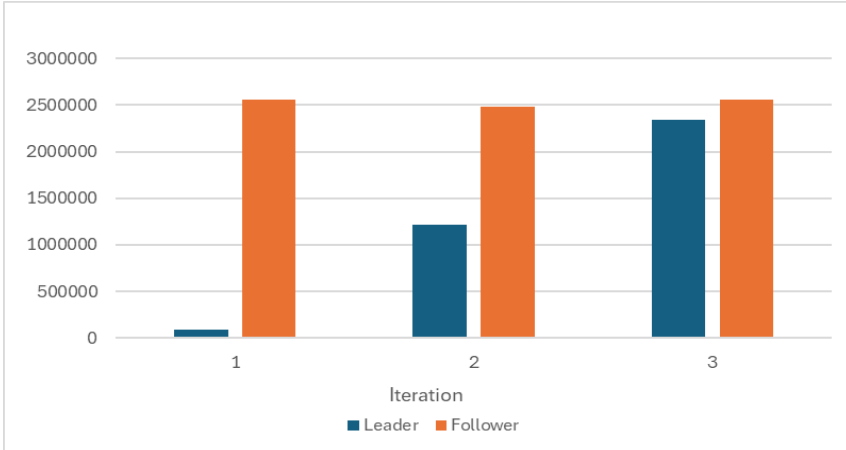


Figure 12: 3 bundle allocation per Leader and follower

In figure 13 continuing from the 3-bundle case, illustrates the optimization performance of the 4-bundle configuration over four iterations, which introduces one mixed-energy bundle (electricity + heat). Compared to the 3-bundle case, the model requires one additional iteration to reach optimality due to the added flexibility for consumers. The initial gap between objectives of follower and leader confirms the model is still exploring feasible solutions by considering both levels objectives. However, the second and fourth iterations improve significantly and approaches, indicating strong convergence. The system remains efficient while better capturing consumer diversity — an important step toward practical application of the model in real energy systems.

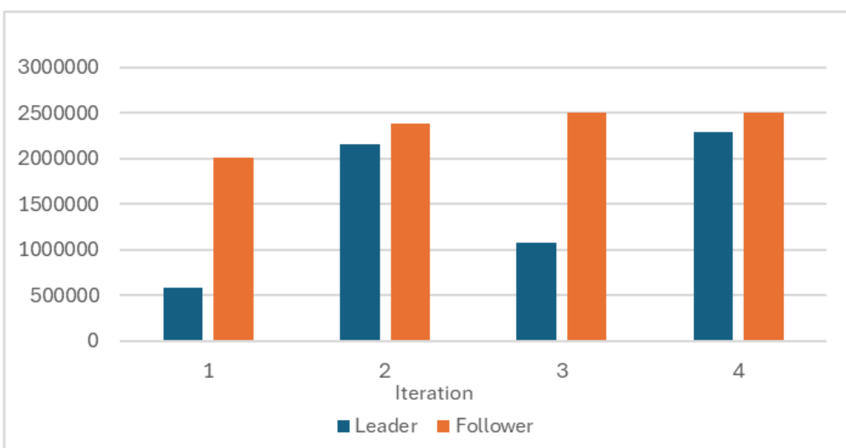


Figure 13: 4 bundle allocation per Leader and follower

Figure 14 displays the evolution of the Leader and Follower objective values across five iterations under a 5-bundle configuration. The bundle design includes three single-energy bundles (electricity, heat, hydrogen) and two mixed-energy bundles (electricity + heat, and heat + hydrogen), offering users high flexibility.

In the early iterations, the Follower's objective rises sharply, reaching its peak by iteration 3 and then stabilizing. The Leader's objective also increases, but at a slower rate, and remains consistently lower than the Follower's, reflecting the asymmetric benefits within the system. This confirms that users (followers) gain more from the added bundle flexibility, while the system operator (leader) faces limitations in optimizing their own outcome. From iteration 4 onward, both objectives plateau, indicating convergence. Overall, this result highlights the trade-off between user flexibility and system control: while the 5-bundle strategy boosts responsiveness and user satisfaction.

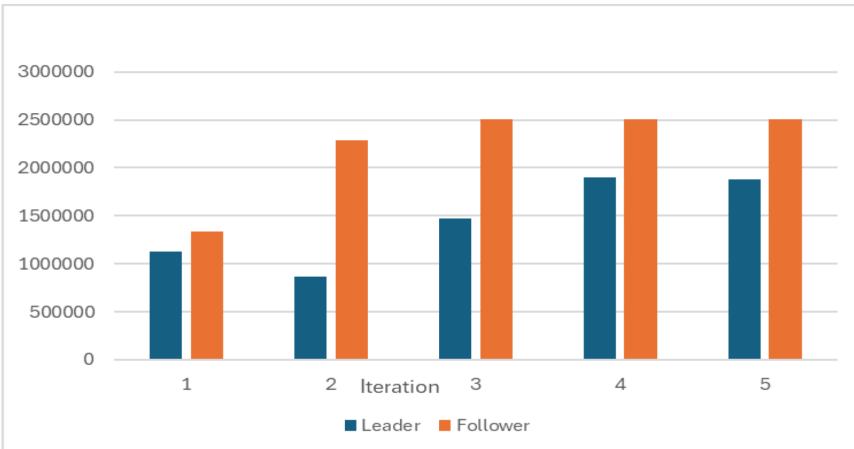


Figure 14: 5 bundle allocation per Leader and follower

This graph shows how the optimization model converges for each bundle strategy by comparing the objectives of leader and follower across iterations. The 3-bundle case reaches optimality quickly in just three steps, with the perfectly aligned by the third iteration. This reflects a simple problem structure where each bundle includes only one energy type, allowing fast and efficient convergence. The 4-bundle strategy, which introduces a mixed-energy bundle (electricity + heat), takes one additional iteration to converge. The leader gradually catches up with the follower, showing that the model still handles increased flexibility well, while capturing more realistic consumer behavior. This wider and slower convergence indicates that while the model becomes more realistic by offering consumers more choice, it also becomes computationally heavier and harder to solve. From an optimization perspective, a smaller gap means better coordination between leader and follower decisions in the bi-level structure. Therefore, this graph highlights the trade-off of computational effort: more bundles provide better representation of consumer behavior but require more time to reach reliable solutions. The model remains robust, but the strategy choice must balance realism and efficiency depending on the system's goals.

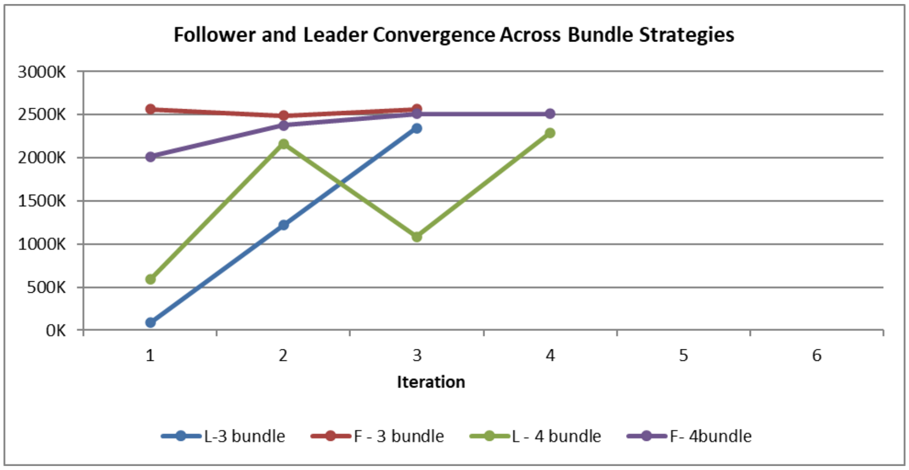


Figure 15: Bundle Strategies (3, 4 Bundles) Over Iterations

In the end, the results matched our main goals and showed that the model works well. Another important outcome is that the system didn't need to buy any gas, which is a big step given Europe's ongoing problems with gas price changes. We were able to cover all production costs while using our devices in an efficient and balanced way. This means the system stayed cost-effective and flexible. Additionally, our model proves to be reliable and realistic, especially because it treats competitors as active constraints within the system.

Conclusion

The main goal of this thesis was to optimize energy capacity planning using a mathematical approach. We developed a bi-level optimization model that captures the real-world interaction between two key actors in the system. By minimizing user costs, the model achieved a positive profit outcome. A central feature of the model was the bundle strategy, which allowed users to choose among different combinations of energy services—electricity, heat, and hydrogen. We tested various bundle configurations through multiple iterations to understand how flexibility impacts system performance. The results showed that more flexible options, such as the 5-bundle strategy, improve the system's ability to meet user demand. However, this added flexibility also increases model complexity and computational time. Throughout this process, hydrogen technologies have played a major role. The hydrogen fuel cell (HFC) and hydrogen storage tank (HST) played essential in keeping the system stable and efficient. Other important technologies include electric boilers (EB) and direct electricity purchase for supplying electricity, HFC and EB for heat supply, and electrolyzer (EL) that use electricity to produce hydrogen, which is then stored in the HST for later use.

Key Contributions

- Developed a bi-level capacity planning and pricing model that balances leader profit with user cost savings.
- Introduced a bundle-based pricing mechanism to simulate real-world energy service options and user choices.
- Modelling an electric-heat-hydrogen system involving EB, EL, CHP, HFC, GB, and HST under realistic pricing and demand conditions.
- Demonstrated the role of hydrogen technologies in reducing carbon emissions and increasing operational flexibility.
- Implemented the full model in AIMMS, enabling simulation of multiple scenarios and convergence analysis (UB/LB) the objective of leaders and follower.

Practical Implications

This model can support community energy operators in designing pricing strategies and energy plans that reflect both economic and sustainability goals. The bundle strategy enables customized service offerings for users with different energy needs, while the use of hydrogen-based components improves system environmental performance. The framework provides a foundation for smarter, cleaner energy management that aligns with future decarbonization targets.

Limitations

While the model effectively simulates multi-energy interactions, it does not include renewable generation sources such as wind turbines or photovoltaic (PV) panels, which are essential for assessing fully decarbonized systems. Their absence limits the system's ability to reflect variable renewable generation and zero-emission energy planning. Moreover, demand-side dynamics such as real-time user behavior or weather-driven variability were not included.

Future Work

Future studies can explore how this model performs when integrated with renewable generation sources like PV and wind, or how it can be extended to include real-time demand forecasting, or more detailed user satisfaction models. As the energy sector evolves, too can this framework—adapting to new technologies and policy mechanisms that support the transition toward sustainable community energy systems.

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