

# Analysis of branching zero variance Monte Carlo games for radiation shielding problems

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November 26th 2024



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# Outline

- 1 Introduction
- 2 Games under investigation
- 3 Derivation of the zero variance schemes
- 4 Model problem
- 5 Conclusion

- Solution of the Boltzmann transport equation using **Monte Carlo methods**.
- Monte Carlo is widely used in **radiation shielding problems**, but techniques to reduce the variance are needed → **importance sampling**.
- **"Zero variance games"**:
  - the exact solution can be achieved;
  - need the solution of the importance equation.

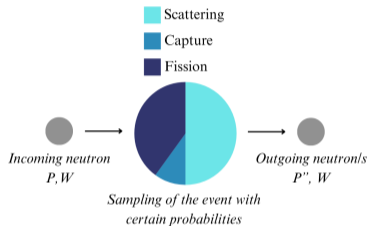
What happens to the zero variance games for neutron transport problems where **neutron multiplication is allowed** (e.g., ex-core flux monitoring during reactor start-up)?  
**How would they perform?**

# Different kind of games

- Monte Carlo Game are defined by:
  - **stochastic rules**  $\rightarrow$  Source  $Q$ , Flights  $T$ , Collisions  $C$ ;
  - **suitable estimator**.
- Types of Games:
  - **natural (analog)**;
  - **non-analog**  $\rightarrow$  weight generation rules to preserve average tally for target response function (population control techniques often applied).
- Regarding the multiplicative phenomenon:
  - **branching games**  $\rightarrow$  allow creation of branching histories;
  - **branchless games**  $\rightarrow$  "1-in-1-out" approach.

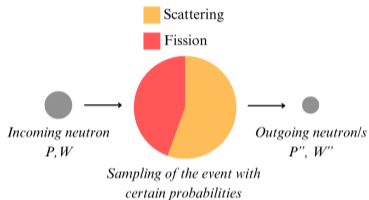
# Comparison of games

## Branching Analog



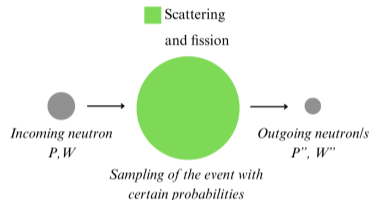
- All the reaction channels are available for the particle.
- Particle weight is not modified.

## Branching Implicit Capture



- Capture reaction channel is suppressed.
- Remaining reaction channels probability and particle weight are rescaled.

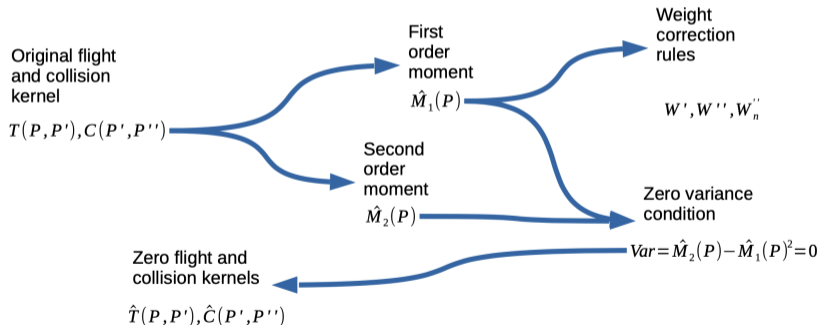
## Branching Forced Fission



- A similar scheme is implemented in the TRIPOLI-4® code.
- The events of scattering and fission are not mutually exclusives.

# Zero variance

- Sampling strategies such that **every particle carries the expected contribution to the detector** → **even with 1 particle the exact solution can be achieved.**
- Original "rules" modified by the zero variance condition.



# Rules of the zero variance games

- Zero variance game's rules are derived by weighting:
  - flight kernel  $\rightarrow T$
  - collision kernel  $\rightarrow C$
  - source  $\rightarrow Q$

**based on the expected contribution to score from sampled state.**

- Given the new game rules the weight corrections are:

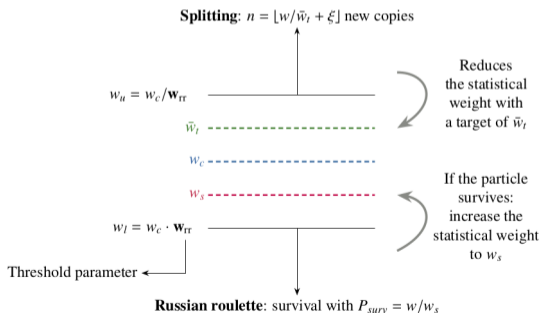
$$W' = \frac{T(P, P')}{\hat{T}(P, P')}$$

$$W''_n = W' \frac{c_n(P') C_n(P', P'')}{\hat{c}_n(P') \hat{C}_n(P', P'')}$$

$$W_Q = \frac{Q(P)}{\hat{Q}(P)}$$

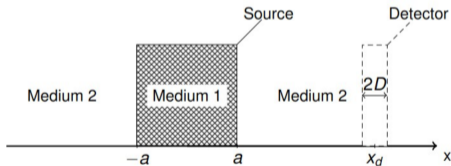
# Population control

- Problem of **"immortal" particles** → population control methods used to stop the simulation.
  - **Russian Roulette** (R.R.): a particle history is terminated if the weight of the particle is too low to contribute significantly to the final score.
- Zero variance can be achieved only if population control is progressively suppressed:  
**weight cutoff (wct) = 0 → vanishing R.R. limit**

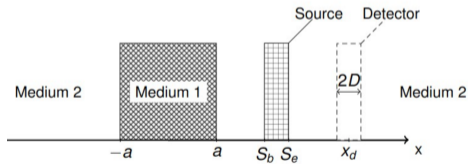


# Verification of the proposed schemes

- Benchmark configuration → **1D infinite rod, 2- directions, mono-energetic problem with two media.**
- With the **Chenille Monte Carlo transport code (MGMC)**:
  - the detector response ( $R_{avg}$ ) and the error ( $R_{err}$ ) are estimated;
  - in the limit of **vanishing population control** the zero variance schemes are verified.



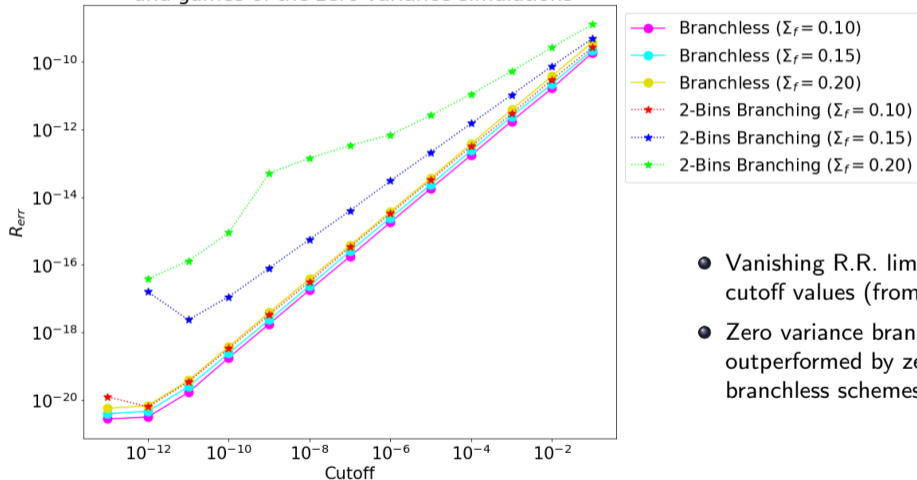
Case A geometry



Case B geometry

# Case A: $R_{err}$ evolution

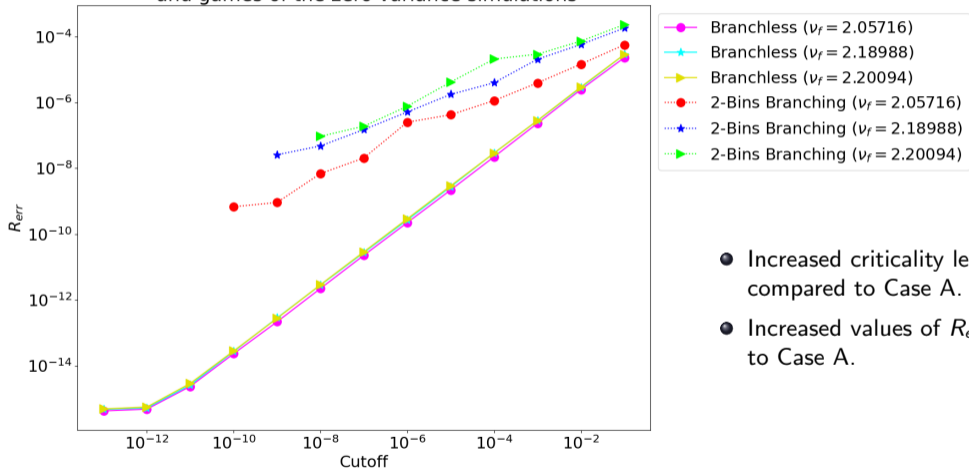
Error trend for different cutoffs, fission cross sections and games of the zero variance simulations



- Vanishing R.R. limit  $\rightarrow$  decreasing cutoff values (from right to left).
- Zero variance branching schemes outperformed by zero variance branchless schemes.

# Case B: $R_{err}$ evolution

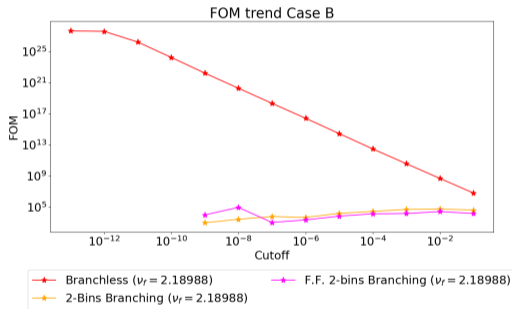
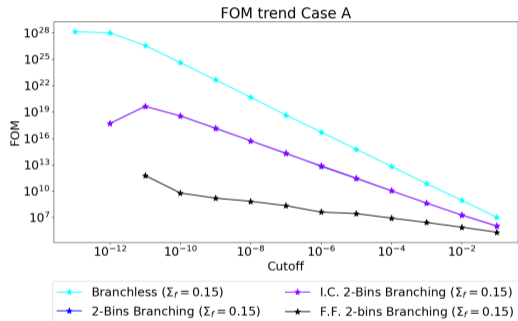
Error trend for different cutoffs, fission cross sections and games of the zero variance simulations



- Increased criticality levels compared to Case A.
- Increased values of  $R_{err}$  compared to Case A.

# General FOM comparison

- Chosen Figure Of Merit  $\rightarrow FOM = \frac{1}{(R_{analytical}/R_{err})^2 \Delta t}$



- In terms of FOM: Branchless  $>$  Branching analog  $\approx$  Branching implicit capture  $>$  Branching forced fission

- Objectives achieved:
  - new zero-variance games derived **from implicit capture/forced fission games**;
  - implemented in Chenille (MGMC) and **compared with analog and branchless zero-variance games**;
  - tested on problems with:
    - different criticality levels,
    - various importance functions,
    - different fission child probability densities.
- Results:
  - branching zero-variance: suitable for multiplicative media, unbiased, but **feasibility decreases with branching histories**;
  - branchless zero-variance: better overall **FOM performance**;
  - still relevant for production Monte Carlo codes.

**Thank you!**  
Questions?