

Master of Science in Communication Engineering

Master Degree Thesis

Performance studies of entanglement swapping in quantum networks using NetSquid simulator.

Professor: PhD. Roberto Proietti

Candidate: Ten. ing. Christian Lavezzari

Academic year 2023/2024

Summary

Quantum networks are essential for advancing quantum computing and secure communication, enabling quantum information to be shared across multiple nodes. These networks can support applications like distributed quantum computing, quantum cryptography, and enhanced sensing. Such technologies will pave the way for the future quantum internet. However, scaling such networks presents significant challenges, including maintaining qubit coherence over long distances and efficiently distributing and routing entanglement among numerous nodes. In this scenario, optical communications offer a promising solution to distribute qubits, due to its large bandwidth and low loss characteristics even over long distances. The work in this thesis leverages Netsquid, a quantum hardware and network simulator, to simulate a network in which multiple quantum nodes are interconnected. The goal is to implement such a quantum network and investigate the performances of entanglement distribution and swapping operations in the presence of noise and losses. Thus, the distributed entangled resources are used to perform teleportation operations. By simulating entanglement distribution, entanglement swapping, and quantum teleportation, we assess how these operations are affected by realistic conditions like channel noise and quantum hardware imperfections. Using NetSquid, we implemented a 4 x 4 nodes network with quantum and classical channels (the last ones needed for running the swapping and teleportation protocols and exchanging other network control plane data). It is worth to mention that the network can be extended to larger configurations if needed. In the simulated network, quantum nodes can both send and receive qubits, enabling scenarios of distributed quantum computing. We observed how noise, decoherence and loss can influence the fidelity of single-hop and multi-hop entanglement-swapping-based distribution and the success rates of quantum operations.

These findings contribute to a better understanding of the design rules for robust quantum networks, laying the groundwork for future research towards the realization of a quantum internet.

Acknowledgements

This thesis marks the achievement of a long, challenging and growth period of study that has led me to reach many personal goals, but also to an extended professional development. Firstly, I would like to express my deepest gratitude to my advisor, Professor Roberto Proietti, for his expertise and immense support throughout this "quantum" journey. His guidance has been invaluable, and his availability unmatched during the entire process. I am also profoundly thankful to Jake Smith for his significant support and fundamental assistance with the explanations and implementation of the NetSquid code. His insights greatly contributed to the advancement of my work.

I wish to extend my heartfelt thanks to my family and parents, Tiziano, Patrizia and Greta for their unwavering support during these years of hard work. They have always believed in me, helping and encouraging me even during the most difficult times, making the toughest moments easier and remembering me the importance to have, build and maintain a strong and unite family. Their constant presence, despite any physical distance, has been a source of great comfort for me and i will be always thanksful for this. My gratitude also goes to my grandparents Flora, Giuseppe, Antonietta whose support has been fundamental and whose i am grateful everyday to have grown with. They have always believed in me, aiding and sustaining me even in the darkest moments. A thought and a deep thanks also goes to an angel, my Grandfather Giuseppe Lavezzari, who always reminded me to study and to give my best. This thesis is also for you, i know you are proud of me even from the sky. Last but not least a special thanks goes to my special and unique girlfriend Camilla, who has always stood by me, supporting me even in moments of discouragement and challenge.

Her presence has been a pillar of strength and a constant light on the horizon throughout this hard journey. Finally, I would like to say a huge thanks to my aunts, uncles, cousins, and all those people who have cared about me, my progress and my carrier development over the years. Their interest and affection have been a constant source of motivation.

Contents

List of Figures

Chapter 1

Introduction

1.1 Quantum revolutions and applications in IT

Modern technologies, which boosted the growth of the actual information society and megacities, are more dependent than ever from telecommunications, computer science and availability of connectivity services. Such a huge amount of information is shared mostly thanks to optical fiber interconnections all around the globe. Such propagation medium is the most used one to distribute information thanks to its low losses and high bandwidth, enabling all the current internet applications (emails, web browsing, file sharing, video streaming, online banking, social networking, video conferences, generative AI, etc.) distributed computing, and future secure quantum applications. Indeed, fiber-based networks, together with free-space satellite based communications for very long distance scenarios, represent the most viable solution to implement reliable quantum communication channels for emerging quantum communication and network technologies in support of the future quantum internet. Each aspect of the everyday life develops through the previously mentioned technologies, from social media, to more crucial and critical sectors like financial, economic, transportation, homeland security and defense. Even if few are aware of this, such technologies were born following the **first quantum revolution**, which toke place around the mid part of 20-th century, laying the foundational principles of quantum mechanics. It is in these years that some key enabling technologies, like semiconductors, lasers, optical fibers, magnetic resonance imaging, digital cameras and nuclear power become parts of the word revolutionizing our way to see it.

While semiconductors have a paramount importance for the modern electronics, lasers, optical fibers and networks, play a big role in modern communications which are the backbone of the global internet connections and de-facto standard for fixed metropolitan network access solution, but also to interconnect cell towers. To show the paramount importance of intercontinental fiber optics communications around the globe, a 2024 map, taken from this [reference](https://www.submarinecablemap.com/) , is showed in figure [1.1.](#page-11-0)

Figure 1.1: 2024 map of global submarines cables. [\[1\]](#page-102-0)

All these, and many others, are the basic building blocks that allows to handle the huge amount of information, in terms of bits, that need to be send, stored, and kept safe 24 hours a day, all the days. Practically, this is possible thanks to some big rooms or building hosting data centers which enable access to a shared pull of computational resources as well as documents, file, social media profiles, medical and banking services. The bandwidth demands duplicates each year and the latency required by the real time applications, like autonomous driving, boosted the development of 5G NR (New Radio) in parallel with multi access edge computing [solutions,](https://newslab.iith.ac.in/research/mec.html) as can be seen in the scheme reported below [1.2.](#page-12-0) Passing onto the **second quantum revolution**, which nowadays is still ongoing, it is characterized by the ability to manipulate and control individual quantum systems, such as atoms, ions, electrons, photons, molecules and it is based on quantum physics.

Figure 1.2: Multi Access Edge Computing scheme. [\[2\]](#page-102-1)

This revolution is unlocking new technological possibilities that extend beyond the capabilities of classical physics. Innovations from this era include quantum computing, quantum cryptography, and quantum communication and networking.

As mentioned above, optical fibers are the best solution to carry and distribute quantum information due to their capability to keep losses low and achieve high speed even over long distances. Quantum computers promise exponential speedups for certain computations, potentially breaking current encryption methods, while quantum cryptography offers unbreakable encryption based on quantum mechanics laws such as non cloning principle but not only. Quantum sensors, by means of quantum entanglement and superposition, are set to revolutionize fields such as metrology, radar detection and navigation with unprecedented precision and accuracy.

In 2001, IBM, a leader in the computing industry, created a 7-qubit quantum computer. This was a pivotal moment that marked the beginning of rapid advancements in the field. By 2019, simulations of quantum computers with 56 qubits were conducted, and the period from 2019 to 2022 saw exponential growth in the number of qubits used, leading to the creation of the first cloud-based quantum computer. In late 2019 and early 2020, IBM launched a 53-qubit quantum supercomputer and introduced the "Q System One," the first integrated quantum computing system accessible via the cloud. This system can integrate with existing service providers worldwide. IBM continued its advancements with the release of the Eagle processors, which support 127 qubits and have paved the way for the Q System Two, capable of operating with quantum CPUs of up to 1,121 qubits in the coming years. The race for quantum supremacy has also seen significant contributions from Google.

Between 2018 and 2019, Google developed the Bristlecone and Sycamore quantum processors, featuring 72 and 53 qubits, respectively. Google's strategy not only focuses on increasing computational power but also on developing quantum algorithms tailored to optimize the performance of quantum hardware. This includes the creation of the first open-source quantum framework for NISQ (Noise Intermediate Scale Quantum) computers, which have between 50 and 100 qubits. Microsoft has also joined the game, but the Canadian company D-Wave Systems has made notable steps forward by providing a 2,000-qubit quantum computer to Lockheed Martin in 2011 for research and development. The latest frontier in quantum computing is represented by the D-Wave 2000Q, although the most significant progress is being made in collaboration with Google at the Quantum Artificial Intelligence Lab, where research is conducted on a 512-qubit computer in partnership with NASA. The continuous evolution of quantum computers necessitates a rethinking of fields such as computing, cryptography, and cybersecurity as will be clear in the next lines. In order to solve vulnerabilities in existing cryptosystems against actual and future quantum attacks, the field of post-quantum cryptography has emerged. This is also driven by the "harvest now, decrypt later" scenario, where attackers can store intercepted encrypted data for future decryption using powerful quantum computers. This is particularly concerning for the secure exchange of symmetric keys, which must remain secure for decades, as well as for military and government institutions. Post-quantum algorithms often require larger public parameters and ciphertexts, and may involve longer execution times compared to classical cryptography so necessitates a careful balance to avoid bottlenecks in memory and processing. The NIST's standardization process, started in 2016, has played a critical role in the development of post-quantum cryptographic algorithms. In 2022, the first four standards were selected, including CRYSTALS-Kyber and CRYSTALS-Dilithium, but there were not the only ones. Ongoing research aims to identify more robust algorithms, as evidenced by the fourth round of evaluations and new proposals. To facilitate the transition from classical to post-quantum algorithms, hybrid cryptosystems have been introduced, combining classical and post-quantum security measures in order to take advantages from the strong points of each technology. These systems use techniques like Concatenate KDF and Cascade KDF for key generation, achieving dual-layered security to enhance resilience during the transitional period. Quantum cryptography, particularly Quantum Key Distribution (QKD), has been significantly influenced by principles of quantum mechanics.

This ensure secure communication by encoding data as polarized photons, where any interception alters the quantum states, alerting the legitimate parties, even if some practical implementation challenge remains like authentication and Secret Sharing, distance Limitations and photon Polarization [\[16\]](#page-103-5). Passing onto quantum networks, these are on front line to guarantee secure information transfer across various sectors, including finance, healthcare, and critical infrastructure. This is due to the evolution of cyber threats that expose traditional cryptographic methods and requires more robust security frameworks development [?] [\[17\]](#page-103-6). One of the examples of quantum networks is the deployment of quantum communication infrastructure in China. The Chinese Quantum Communication Network spans over 2000 kilometers, providing secure communication between cities like Beijing and Shanghai. This network utilizes QKD to protect sensitive communications and has successfully integrated quantum satellite technology to extend secure communications beyond terrestrial limits. A recent study highlights the network's ability to transmit quantum keys over long distances, achieving notable success in maintaining key security in various applications. [\[18\]](#page-103-7) [\[19\]](#page-103-8) Moving further, recent advancements in quantum network technology have led to innovative applications, such as the development of quantum repeater technologies and the integration of machine learning (ML) algorithms to optimize key distribution processes. For instance, researchers have demonstrated that machine learning can enhance the efficiency of QKD protocols, allowing for real-time adaptation to varying network conditions. [\[20\]](#page-103-9) [\[21\]](#page-103-10) These abilities are not only essential for safeguarding sensitive data but also enhances the potential for advanced applications such as distributed quantum computing, secure data sharing, and more sophisticated cryptographic protocols that can withstand future potential quantum attacks, also against cloud critical infrastructures. Moreover, it is worth to mention that the implementation of quantum networks can significantly booster cybersecurity measures, providing a proactive approach to emerging threats in an increasingly interconnected and challenging world. [\[22\]](#page-103-11) [\[23\]](#page-103-12) [\[24\]](#page-103-13) All considered, the work in this thesis focus on simulations studies that aim to investigate the effect of noise, decoherence and losses impact on quantum operations in a multiple node quantum network that can form a basic infrastructure supporting a variety of future different quantum applications . The thesis is organized as follow: firstly, an introduction about quantum technologies and how they can be applied to IT sector is provided. Moving to the second section, background information about qubits, quantum operations and quantum hardware components are provided. Finally, a chapter presenting system level architecture and simulation results can be found, as well as conclusions and future works.

1.2 Military applications: quantum warfare scenario

In the last 70 years, technological advancements have profoundly influenced every aspect of global, regional and proxy conflicts. The development of improved electronic devices, including radar, sonar, radio, and other tools designed to detect or transmit electromagnetic waves, has shifted military focus towards a newly recognized Operational Environment (OE): the electromagnetic spectrum (EMS).

Although initial efforts in this domain were observed as early as the First World War, it was during the Second World War that tactics such as distortion and interference, specifically targeting command and control (C2), navigation, and communication systems, became prevalent. A review of post-World War II conflicts highlights that the ability to execute precise operations within the electromagnetic environment (EME) has become critical to achieving success in warfare. Such operations have played key roles in the Vietnam War, the Gulf War, Iraq, Afghanistan to not mention the russo-ucrain war in which drone and counter drone warfare is currently widley used by all the fighting forces.

According to NATO doctrine, the operating environment (OE) encompasses the "set of conditions and circumstances that influence the employment of military capabilities, including the decisions of the commander." With this definition in mind, it becomes clear that electronic warfare (EW)— which involves the use of electromagnetic energy to conduct offensive or defensive operations within the EME— is an integral aspect of modern military strategy and it has paramount importance both in force protection, to detect and disrupt enemy drones and communications, but also in offensive operations with the use of suicides drone or loitering munitions that requires a "data link" connection to the user if no AI or autonomous applications are considered [\[25\]](#page-103-14).

From the analysis of Figure [1.3,](#page-16-0) it is clear that the use of electronic, optoelectronic, radio, satellites, and computer systems by land, air, and naval forces has made them more united, interconnected, and dependent on each other than ever before, without mentioning the pervasive informative and psychological warfare that also heavily rely on communication and social media to carry out their effects.

Figure 1.3: electronic and informative aspects of multi domain operations.

Furthermore, there are two additional domains that complement the three traditional ones and require doctrinal, training, and methodological adaptations, as can be seen in Figure [1.4.](#page-16-1) The Cyber and Space domains have rapidly emerged and pervade all operational scenarios, creating networks, communication lines, or areas with limited/denied access $(A2/AD$ bubbles) [\[26\]](#page-104-0).

Figure 1.4: Cross domain operations and maneuvers.[\[3\]](#page-102-2)

Initially "invisible", operations in these sectors are only noticed when they produce destructive or compromising effects on physical infrastructure or critical services. This is becoming always more true if counter drone operations are considered: despite multiple defence strategies have been presented, the EM countermeasure are a clear example of what previously stated. A more cyber related example are the cyber operations carried out by the US Cyber Command against Iranian intelligence in response to the damage to several oil tankers in the Gulf of Oman or the famous Stuxnet virus developed to slow down the iranian nuclear program [\[27\]](#page-104-1).

Another technological step that has involved weaponry and weapon systems is their interconnection into networks, both military and civilian, and the implementation of features that make the operation of the weapon or the life of the operator dependent on the software in execution. These are known as cyber-physical systems and they are critical because new kind of cyberattacks can potentially lead to the disruption of the whole system. It is in this highly interconnected and complex scenario that present and future quantum technologies they make their entrance on the scene, leading to a new advanced branch often defined as [quantum technology military applications](https://www.researchgate.net/publication/350341569_Quantum_Warfare_Definitions_Overview_and_Challenges/link/605aae9b92851cd8ce61bac9/download?_tp=eyJjb250ZXh0Ijp7ImZpcnN0UGFnZSI6InB1YmxpY2F0aW9uIiwicGFnZSI6InB1YmxpY2F0aW9uIn19) [\[4\]](#page-102-3): these have the potential to disrupt and significantly impact numerous aspects of human military activity so quantum advancements are of particular interest to the defense and security industries, as well as government entities.

In the following lines a summary of the various potential military applications of quantum technology is presented, considering the ability to introduce new capabilities, enhance effectiveness, and improve precision of currenlty available weapons and weapons systems. Next years scenarios, necessitates the development of updated and innovative military strategies, doctrines and policies. Specific military applications sectors are: cyber, space, electronic, and underwater, as well as intelligence, surveillance, target acquisition, and reconnaissance (ISTAR) and even more.

Starting a comprehensive view can be obtained looking at the scheme below: this is a rendering of the possible quantum military applications and their simultaneous use on the future battlefield.

Figure 1.5: Quantum warfare rendering. [\[4\]](#page-102-3)

Most of the presented technologies are still not ready to be deployed in the field due to theoretical, technical or size, weight and power requirements. Despite this, some quantum applications are now at TRL 6 or 7 and so it is reasonable to image that will be implemented on military platforms in next years. The first application, that is also present among the civil ones, is the simulation of new chemical and biological molecules that play a fundamental role in the CBRN (chemical, biological, radiological and nuclear) defence. In this field the use of quantum computing or hybrid classical and quantum hardware can lead both to the simulation of new substances but also of new antidotes or drugs that can be potentially useful even for the civilian population. Similarly, the development of new materials can take advatanges of quantum techologies: examples are room temperature superconducting for SQUID (superconducting quantum interference device) magnetometers, stealth or high temperature tolerance materials.

Applications of quantum technologies to the cyber domain, and especially in cybersecurity, is set to transform digital security by introducing both powerful new attack vectors and resilient defenses. The implementation of quantum crypto-agility is critical, particularly to counter threats posed by quantum algorithms like Shor's, which can break traditional encryption methods based on integer factorization and elliptic-curve cryptography (ECC).

As quantum-safe encryption becomes necessary, organizations should consider adopting Quantum Key Distribution (QKD), despite its vulnerabilities at system endpoints. Quantum computing presents significant opportunities in cyber warfare, enabling highly effective attacks on current encryption systems while also driving the development of quantum-resilient algorithms. The urgency for deploying post-quantum cryptography is high, especially as adversaries may already be stockpiling encrypted data, anticipating future quantum decryption capabilities. Fully Homomorphic Encryption (FHE) offers a novel approach by allowing data processing without decryption, which is crucial for sensitive information in military and intelligence contexts.

Quantum computing's impact extends beyond encryption, offering enhancements in optimization, machine learning (ML), and artificial intelligence (AI). These technologies will likely be integrated into hybrid quantum-classical systems, improving everything from military logistics to decision-making processes. Although challenges remain, such as the need for cryogenic cooling in current qubit designs, ongoing advancements in quantum computing promise to revolutionize defense strategies and capabilities, making quantum technology a vital area of focus for future cybersecurity and military operations. Another vital area of applications for future quantum technologies are the quantum communication networks: beyond the already mentioned QKD capabilities, whose space applications will be revised later in this work, it is worth to focus on timing, positioning and position, navigation and timing (PNT) applications. Quantum timing promise to improve the clock synchronization across various systems by utilizing quantum entanglement. This advancement enables more precise coordination between atomic clocks, such as those used in GPS, and local digital clocks. The high precision of quantum clocks is crucial for the effective operation of C4ISR (Command, Control, Communications, Computers, Intelligence, Surveillance, and Reconnaissance) systems. Accurate synchronization of data and operations across radar systems, electronic warfare platforms, command centers, and weapon systems is essential for modern military applications, and quantum timing technologies offer a substantial improvement over current methods which will be exploited also for unmanned autonous vehicle in all domain and cross domain operations.

Passing onto the PNT applications, there are three main sectors of interest:

• Quantum inertial navigation: systems represent a major leap in navigational accuracy, offering up to 100 times greater precision compared to traditional inertial navigation systems.

These systems are particularly valuable in GPS-denied or challenging operational environments, such as underwater, underground, or regions subjected to GPS jamming.

Unlike classical inertial navigation, which suffers from cumulative errors over time, quantum inertial navigation maintains high levels of accuracy over extended periods. This makes it especially useful for applications like submarine navigation, where quantum inertial systems can reduce errors to mere hundreds of meters per month, compared to miles with classical systems. Given its advanced development stage, quantum inertial navigation is one of the most mature quantum technologies and is expected to reach a technology readiness level (TRL) of 6 in the near future. As the technology continues to miniaturize, it could be deployed in smaller platforms, including unmanned autonomous vehicles and missiles.

- Enhanced GNSS: The integration of quantum clocks, with their superior precision, into GNSS (Global Navigation Satellite Systems) promises to significantly improve the accuracy of positioning and navigation. Future GNSS systems may incorporate quantum internet connectivity to further enhance timing distribution and clock synchronization, providing robust defenses against GNSS spoofing and deception. This would be particularly beneficial in maintaining the integrity and reliability of navigational data in critical military operations.
- Non-GNSS Navigation: An alternative approach to traditional satellite-based navigation involves the use of quantum magnetometers in conjunction with maps of Earth's magnetic anomalies. This quantum-based navigation system provides a viable solution in environments where GNSS signals are either unavailable or unreliable. By leveraging the natural magnetic anomalies of the Earth, this technology offers a resilient and accurate method for navigation, independent of external satellite infrastructure, making it ideal for operations in GPS-deprived or contested environments.

Resuming the discussion on satellite QKD applications, utilizing a satellite as a quantum trusted relay is an exceptionally promising solution, as it effectively bypasses the issue of key generation speed. The speed of key generation is inversely proportional to the distance covered by the fiber optic link; however, in the atmosphere, attenuation is virtually negligible compared to traditional telecommunications channels.

This makes satellite applications potential game changers for quantum key distribution (QKD) over extremely long distances. The theoretical limit of this technology pertains to the lifespan, and thus the distance traveled, of individual photons once they enter the Earth's atmosphere. Numerous tests and research conducted between the late 1990s and the early 2000s demonstrated that the polarization states of individual photons can pass through the Earth's atmosphere unscathed and correctly reach the ground station. The first real test was conducted in 2007 through the collaboration between the Chinese Academy of Sciences (CAS) and the Austrian Academy of Sciences (AAS), leading to the creation of the QUESS (Quantum Experiment at a Space Scale) project. The goal of this endeavor was to demonstrate the potential use of a satellite equipped with a photon source as a quantum relay to facilitate communication between two ground stations in Europe and China. The experiment was successful, implementing a free-space QKD link with Decoy State over a distance of 143 km in space, while maintaining a good key generation rate and, importantly, a low attenuation of approximately 35 dB. Following the establishment of the feasibility of this type of link, subsequent tests focused on their resilience to the environmental conditions and motion dynamics characteristic of satellites or aircraft, as compared to a terrestrial base station. A significant experiment was conducted between a rapidly moving aircraft, at approximately 290 km/h, and a ground station, successfully maintaining low noise levels and a contained Quantum Bit Error Rate (QBER), which positions these applications as truly viable technologies.

A crucial concept frequently referenced in this discussion is the Low Earth Orbit (LEO) satellite. To maintain distances compatible with experimental data, the relay must be positioned in Low Earth Orbit, which corresponds to altitudes ranging between 300 and 1000 km. Operating within this range, at an altitude of 500 km above the Earth's surface, is the world's first satellite equipped with a QKD payload: the Chinese satellite "Micius," launched in 2016. This launch marked a significant advancement in the integration of space and quantum telecommunications sectors.

Initially, Micius was used to implement Decoy State QKD, achieving a key rate of 1 kbps over a distance exceeding 1000 km. Repeated tests indicated that, given good atmospheric conditions and accepting a coverage window of approximately 4.5 minutes per day, the efficiency of the satellite link is 20 orders of magnitude greater than that of a fiber optic link. The following image [1.6](#page-22-0) illustrates the vast distances covered by Micius, the excellent QBER results, and the sizes of the final secret keys generated.

Figure 1.6: Positions, distances and results obtained from quantum key generation with the Micius satellite.[\[5\]](#page-102-4)

Following its initial missions, the Micius satellite functioned as a reliable relay, enabling a secure QKD OTP (One-Time Pad) communication link between a Chinese and an Austrian university. This link facilitated the exchange of images in a completely secure manner. The locations involved in this exchange are shown in Figure [1.6,](#page-22-0) and during the satellite's passes over these locations, encrypted data could be downloaded for approximately 5 minutes. The satellite link demonstrated impressive performance, achieving a key generation rate of 3 Kb/s over a distance of roughly 1000 km, marking this as the first instance of intercontinental quantum communication.

In a recent declassified document from the Defence Intelligence Agency (DIA), part of the US DoD (Department of Defence), [quantum applications for space communications,](https://www.dia.mil/FOIA/FOIA-Electronic-Reading-Room/FileId/170047/) are explored and in particular the possibility to control, from the earth, a rover on mars. Also the intelligence gathering, information acquisition and the surveillance in general, will be affected by the quantum technologies: these hold the potential to revolutionize ISTAR (Intelligence, Surveillance, Target Acquisition, and Reconnaissance), a critical capability for modern military operations that require precise situational awareness across multi-domain battlefields. Quantum sensing technologies, when deployed on land, sea, aerial vehicles, and low-Earth orbit (LEO) satellites, could lead to significant advancements. Quantum gravimeters and gravitational gradiometers, for example, promise highly accurate measurements that could transform applications in geophysics, seismology, archaeology, mineral detection, and precise georeferencing, including topographical mapping of the seabed for underwater navigation.

In particular, this last point will gain always more weight in the future scenarios, due to the constant developement of USV (Unmanned Surface Veichles) for ISR (Intelligence Survelliance and Reconnaises) operations. An example is the DARPA (Defense Advanced Research Projects Agency) [\[6\]](#page-102-5) [Manta Rey,](https://www.darpa.mil/news-events/2024-05-01) [1.7,](#page-23-0) that in future could host quantum communications devices for precise underwater operations along with nuclear submarines.

Figure 1.7: Darpa Manta Rey USV.[\[6\]](#page-102-5)

Going a little bit deeper on this topic, submarines, especially larger ones, are expected to be among the first to adopt quantum inertial navigation and communications systems due to their ability to accommodate the larger quantum devices and cryogenic cooling systems required. Quantum inertial navigation can provide precise guidance without relying on external signals, which is crucial for stealth operations. Additionally, quantum gravimeters and magnetometers can map the underwater environment, such as sea floors or icebergs, without the need for sonar, which is easily detectable from enemy forces. Quantum magnetometers could become a key tool in anti-submarine warfare. Advanced devices like SQUID magnetometers may detect submarines from distances of up to 6 kilometers, a significant improvement over classical magnetic anomaly detectors, which have a range of only a few hundred meters. Deploying an array of quantum magnetometers along coastlines could cover vast areas, effectively creating exclusion zones for submarines. Moreover quantum magnetometers could also be used for detecting underwater mines, potentially using unmanned underwater vehicles (UUVs) for safer and more efficient operations.

To conclude this long paragraph, it is worth to mention the Electronic Warfare (EW) and space warfare improvements due to the use of quantum laws. These two sectors, as demonstrated by the recent conflict between Russia and Ukraine, are fundamental due to their ability to disrupt enemy communications capabilites, as well as drone guidance systems. Quantum antennas, particularly those based on Rydberg atoms, offer a significant advantage in EW by providing small, versatile devices capable of intercepting a wide range of frequencies with high precision. These antennas can measure signals ranging from AM and FM to weak and strong electromagnetic fields, all while maintaining a compact size. The future may see arrays of these quantum antennas that can dynamically adjust to different bandwidths and determine the angle of arrival of signals. However, the need for cryogenic cooling of Rydberg atoms remains a challenge. Quantum computing also offers improvements to RF spectrum analysis in EW, allowing for more effective signal processing and analysis through quantum optimization and machine learning techniques. Additionally, quantum timing could enhance the precision of signals intelligence (SIGINT) and counter-radar jamming operations, improving the overall effectiveness of EW systems. Focusing on quantum channel, various attacks could be developed, though their practicality remains uncertain. For example, a man-in-the-middle attack could exploit vulnerabilities in early quantum networks, such as authentication issues. Other sophisticated attacks, like photon number splitting or Trojan-horse attacks, target the quantum physics level but may be difficult to execute, especially in space environments. More likely, QEW attacks might focus on disrupting quantum communication through denial-of-service tactics, such as jamming receivers to generate excessive noise or using directed energy weapons to damage or destroy sensors. These methods aim to interrupt or degrade the quantum channel's functionality rather than directly intercepting the quantum data.

Finally, satellite-based quantum communication has already been demonstrated to play a pivotal role in establishing an integrated quantum network over long distances. Current quantum communication satellites, functioning as trusted repeaters, face challenges similar to those of optical fiber-based systems. These challenges include vulnerabilities to cyber attacks, which are being addressed through advanced security protocols such as Measurement-Device-Independent Quantum Key Distribution (MDI-QKD), designed to safeguard against potential threats.

The deployment of quantum technologies in space also underscores the need for enhanced capabilities to detect and track other satellites, space-borne objects, and space debris.

Traditional radar systems, like the [Space Fence project within the US Space Surveillance](https://www.lockheedmartin.com/en-us/products/space-fence.html) [Network,](https://www.lockheedmartin.com/en-us/products/space-fence.html) have limitations in tracking small objects and managing the vast quantities of space debris. Quantum radar and lidar, particularly those operating in the optical regime, offer a promising alternative. Simulations indicate that space-based quantum radar could provide significantly improved detection and tracking sensitivity compared to existing ground-based systems, such as [GEODSS \(Ground-based Electro-Optical Deep Space](https://www.spaceforce.mil/About-Us/Fact-Sheets/Article/2197760/ground-based-electro-optical-deep-space-surveillance/) [Surveillance\)](https://www.spaceforce.mil/About-Us/Fact-Sheets/Article/2197760/ground-based-electro-optical-deep-space-surveillance/) [\[7\]](#page-102-6) which ground bases are shown in the next figure [1.8.](#page-25-0)

Figure 1.8: US space survelliance network. [\[7\]](#page-102-6)

As quantum sensing and communication technologies become more integrated into space operations, the field of quantum electronic warfare is expected to gain increasing relevance. This evolving landscape will drive advancements in both offensive and defensive strategies in space, shaping the future of military and space exploration endeavors.

Chapter 2

Background information

2.1 Qubits and quantum networks

The development and advancement of quantum technology are set to revolutionize secure communication and quantum computing. Central to this revolution are qubits and quantum networks, which form the backbone of quantum information processing and transmission. Qubits are unique in their ability to exist in multiple states simultaneously, a property stemming from the principles of quantum mechanics. This superposition, along with entanglement, that is a phenomenon where qubits become interlinked and the state of one instantly influences the state of another, enables quantum computers to process vast amounts of data at unprecedented speeds. Moreover, it is worth to mention that qubits can be realized using different technologies, like trapped ions, superconducting circuits, and photons. Each technology has its advantages and disadvantages. For instance, trapped ions offer high fidelity and long coherence times, superconducting circuits provide scalability, and photonic qubits are ideal for long-distance quantum communication due to their minimal interaction with the environment and compatibility with existing fiber optic infrastructure. A qubit, or quantum bit, is the fundamental unit of quantum information, analogous to a bit in classical computing. The term was coined by Benjamin Schumacher and unlike classical bits, which can be either 0 or 1, a qubit can exist simultaneously in a superposition of both states. This property is a direct consequence of the principles of quantum mechanics and is what gives quantum computers their immense potential.

Mathematically, a qubit is represented as a linear combination of its basis states, denoted as $|0\rangle$ and $|1\rangle$.

This can be expressed as:

$$
|\Psi\rangle = \alpha |0\rangle + \beta |1\rangle
$$

Where α and β are the complex numbers that represent the probability amplitudes of the qubit being in state $|0\rangle$ and $|1\rangle$, respectively. The probabilities of the qubit being measured in either state are given by $|\alpha|^2$ and $|\beta|^2$ and this need to satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1.$

The qubit can be visualized using the *Bloch sphere representation* [2.2,](#page-28-1) which provides an intuitive geometric interpretation. On the Bloch sphere, any qubit state can be represented as a point on the surface of a unit sphere. The north and south poles of the sphere correspond to the classical states $|0\rangle$ and $|1\rangle$, while any point on the surface represents a superposition of these states. This visualization is useful for understanding the continuous nature of quantum states and the effects of quantum operations.

The Bloch sphere is defined by the angles θ and φ , which correspond to the qubit's position on the sphere.

These angles are related to the probability amplitudes α and β as follows:

$$
\alpha = \cos(\frac{\theta}{2})
$$

$$
\beta = e^{i\phi}\sin(\frac{\theta}{2})
$$

where θ is the polar angle and ϕ is the azimuth angle.

Moreover, remaining on the Bloch sphere representation, can be stated that superposition and pure states lie on the surface of the sphere, while mixed states (expressed by classical probabilities) lie inside the sphere.

Moving on to the natural application of qubits we come at quantum computing. Qubits are the building blocks of quantum computation promising a huge increasing in computation and speed thank to their ability of theoretically represent an infinite amount of information. Such goal can be achieved thanks to the the ability of manipulate qubits through quantum gates and entanglement.

Quantum gates has a paramount importance because allow to apply operations to change the qubit state, that is to say, to rotate the previously mentioned Bloch spehere. In the following an overview of the most meaningful quantum gates and related operations is presented:

Figure 2.1: 3D Bloch sphere qubit representation. [\[8\]](#page-102-7)

Figure 2.2: Different qubit states on 3D Bloch sphere.[\[9\]](#page-102-8)

- Pauli gates:
	- $-$ X gate (NOT gate): inverts the state of a qubit, so $|0\rangle$ \leftrightarrow $|1\rangle.$
	- $-$ Y gate: Combines X and Z gates with a phase flip, so $|0\rangle\rightarrow i|1\rangle$ and $|1\rangle\rightarrow-i|0\rangle.$
	- $-$ Z gate (Phase flip): swap the qubit phase, so $|0\rangle$ \rightarrow $|0\rangle$ and $|1\rangle$ \rightarrow $-|1\rangle$
- Hadamard gate (H gate): creates states superposition, so:

$$
- |0\rangle = \rightarrow \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)
$$

$$
- |1\rangle = \rightarrow \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)
$$

- CNOT gate: invert the state of the target qubit if the state of the control one is $|1\rangle$.
	- $|00\rangle \rightarrow |00\rangle$ $|01\rangle \rightarrow |01\rangle$
	- $|10\rangle \rightarrow |11\rangle$
	- $|11\rangle \rightarrow |10\rangle$
- CZ gate: apply a Z gate to the target qubit if the state of the control is $|1\rangle$
	- $|00\rangle \rightarrow |00\rangle$ $|01\rangle \rightarrow |01\rangle$ $|10\rangle \rightarrow |10\rangle$ $|11\rangle \rightarrow -|11\rangle$

In systems composed of multiple qubits, another quantum mechanical property called quantum entanglement comes into play. Entanglement causes the states of different qubits to influence each other, further enhancing the computational speed offered by quantum computers. Despite the advantages, there are significant practical challenges associated with the use of qubits:

- Delicacy of Qubits: Qubits are extremely sensitive to environmental changes such as temperature fluctuations, vibrations, or electromagnetic waves, which can quickly disrupt their quantum state, leading to information loss.
- Temperature Requirements: Quantum computers require cryogenic temperatures near absolute zero (-273.15°C) to function properly. Achieving and maintaining such low temperatures is costly and complicated, often involving rare gases like helium, nitrogen, or argon, although magnetic cooling techniques are being developed as an alternative. On this side some promising studies have shown that in the future the temperature requirements could be not valid anymore.

Passing onto the quantum networks they are responsible for the transmission of quantum information over distances, enabling applications such as Quantum Key Distribution (QKD) and distributed quantum computing.

Different types of quantum networks leverage specific quantum phenomena to achieve secure and efficient communications, as can be observed from the list below:

- *Entanglement based networks*: Using entanglement to create secure communication links, these networks allow quantum information to be teleported between nodes. Any eavesdropping attempt disturbs the entangled state, ensuring security and allowing the parties to restart the transmission.
- Quantum Key Distribution: QKD uses quantum mechanics to generate and distribute encryption keys securely. Protocols like BB84 utilize the polarization states of photons to establish a shared secret key between parties, making eavesdropping detectable.
- Hybrid quantum networks: These networks combine different quantum communication methods to optimize performance and extend communication range. For example, integrating entanglement-based protocols with QKD enhances security and robustness over long distances.

Creating scalable and reliable quantum networks faces several challenges, such as the loss of qubits over long distances. Quantum repeaters extend the range of quantum communication by creating entanglement between intermediate nodes and performing entanglement swapping to establish end-to-end entanglement. This last mentioned feature has a paramount importance due to the fact that teleportation consumes entanglement and it is also related to the delicacy of quantum information.

Satellite-based quantum communication has emerged as a promising solution to overcome distance limitations, as shown by the successful launch of the Micius satellite. This demonstrated the feasibility of satellite-based QKD and entanglement distribution, showcasing a significant step towards a global quantum internet but also highlighting the possible implementation of these technologies in the space defence sector to improve security of intercontinental and overseas units.

Finally, it is important to remember that the path towards a full global quantum internet is still long so integrating quantum networks with classical infrastructure is essential for practical implementation. This requires robust network interfaces capable of handling hybrid classical-quantum data frames and sophisticated synchronization mechanisms to manage the timing of quantum operations across the network.

2.2 Bell state and BSM

With the term *Bell states* we refers to particular quantum states of two qubits that are maximally entangled. Those play a fundamental role in some key quantum applications like teleportation, quantum cryptography and networking.

The four Bell states are displayed below:

- \bullet $|\Phi^+\rangle = \frac{1}{4}$ $\frac{1}{2}$ ($|00\rangle + |11\rangle$)
- \bullet $|\Phi^-\rangle = \frac{1}{\sqrt{2}}$ $\frac{1}{2}$ ($\ket{00}$ - $\ket{11})$
- \bullet $|\Psi^+\rangle = \frac{1}{\sqrt{2}}$ $\frac{1}{2}$ ($|01\rangle + |10\rangle$)
- $\bullet\;\ket{\Psi^-}=\frac{1}{\sqrt{2}}$ $\frac{1}{2}$ ($|01\rangle$ - $|10\rangle$)

To retrieve one of this Bell states is required to start from a qubit in one basis state and apply a some specific operations in the correct sequence.

To obtain the $|\Phi^+\rangle$ state is required to firstly apply the H gate and after the CNOT as can be appreciated in the following lines.

- Hp: the starting point are two qubits with state $|00\rangle$
- Apply Hadamard (H) gate to the first qubit to transform the |0〉 state in a superposition of both $|0\rangle$ and $|1\rangle$ that is to say:
	- $H |0\rangle = \frac{1}{\sqrt{2}}$ $\frac{1}{2}$ ($|0\rangle + |1\rangle$)
	- The state of the 2 qubits system becomes: $H |0\rangle |0\rangle = \frac{1}{\sqrt{2}}$ $\frac{1}{2}$ $(|0\rangle+|1\rangle)|0\rangle=\frac{1}{\sqrt{2}}$ \overline{c} $(|00\rangle + |10\rangle)$
- Apply CNOT gate using the first qubit as control and the second one as target.

$$
-\text{ CNOT} \left(\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \right) = \frac{1}{\sqrt{2}}(\text{CNOT}|00\rangle + \text{CNOT}|10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = |\Phi^+\rangle
$$

To obtain the $|\Phi^{-}\rangle$ state is enough to use the previously described $|\Phi^{+}\rangle$ state as starting line and apply a Z gate to the first qubit:

• $|\Phi^-\rangle = (Z \otimes I) |\Phi^+\rangle = \frac{1}{\sqrt{2}}$ $\frac{1}{2}$ $(\ket{00}$ - $\ket{11})$ It is worth to mention that the ⊗ is the tensorial, or Kroneker, product and it is used to combine two vector spaces or operators in a bigger space. For instance, if one operates on two vectors that have two dimensions each, the resulting vector will have four dimensions. Passing onto the $|\Psi^+\rangle$ and assuming always starting from two qubits in the $|00\rangle$ state, the required steps are the following:

• Apply Hadamard (H) gate to the first qubit:

$$
- H |0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)
$$

- The state of the 2 qubits system becomes: $H |0\rangle |0\rangle = \frac{1}{\sqrt{2}}$ $\frac{1}{2}$ ($|00\rangle + |10\rangle$)
- Apply X gate to second qubit:
	- $(I \otimes X)|00\rangle = |01\rangle$
	- The state of the 2 qubits system becomes: $\frac{1}{\sqrt{2}}$ $\frac{1}{2}$ $(|01\rangle + |11\rangle)$
- Apply CNOT gate using the first qubit as control and the second one as target.

$$
-\text{ CNOT}\left(\frac{1}{\sqrt{2}}(|01\rangle+|11\rangle)\right)=\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)=|\Phi^-\rangle
$$

Lastly, to retrieve the $|\Psi^-\rangle$, it is possible to start from $|\Psi^+\rangle$ and apply a Z gate on the second qubit.

 $\bullet \vert \Psi^- \rangle = (I \otimes Z) \vert \Psi^+ \rangle = \frac{1}{\sqrt{2}}$ $\frac{1}{2}$ (|01\-|10\)

To analyze the Bell State Measurement, often referred to as BSM, is valuable to previously introduce the expression of a two-qubit generic quantum state:

• $|\Psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

The numerical part of this expression shows the four basis states of two qubits:

- $|00\rangle$: both qubits are in $|0\rangle$ state
- $|01\rangle$: first qubit is in $|0\rangle$ state, the second qubit is in $|1\rangle$ state
- $|10\rangle$: first qubit is in $|1\rangle$ state, the second qubit is in $|0\rangle$ state
- $|11\rangle$: both qubits are in $|1\rangle$ state

Focusing on the coefficients $(\alpha, \beta, \gamma, \delta)$, this is made by complex numbers whose show the probabilities related to each state, that is to say the probability that the system is in one of these states. It is important to remember that in quantum mechanics a normalization condition exist so the following equality holds:

•
$$
|\Psi\rangle = |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1
$$

It is also fundamental to underline that, until a measurement is performed, the system is not in a defined state, but it is in a combination of all these states.

All considered, it is possible to come to the measurement part, that play the role of determine in which of the four states the two qubits are. Starting from the above mentioned generic quantum state, $|\Psi\rangle=\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle,$ a CNOT and a Hadamard (H) port are applyed. Finally the two qubits are measured in the computational basis with the classical results (2 bits) that show which was the initial Bell state.

2.3 Entanglement distribution and swapping

Quantum entanglement is a fundamental resource for quantum communication and quantum networks. The process of distributing and managing entanglement over long distances is crucial for the development of a quantum internet, which promises secure communication, enhanced computational capabilities, and new applications in quantum sensing and metrology.

Focusing on the networking applications of quantum entanglement there are two main general cases: one node, e.g. Alice, wants to entangle herself with another node, e.g. Bob, or multiple Alices want to establish entanglement towards one Bob. In the next lines both cases will be analyzed but the staring point it is always the same: on Alice side we have a Bell pair, that in practical implementation are represented by two qubits.

- One to One entanglement distribution: One of the Bell states, qubits, it is send to Bob so now he and Alice are entangled with the meaning that was previously explained.
- Multiple entanglement distribution: For simplicity we explain the two Alices and one Bob case, that is to say the simplest multiple entanglement case, but the concept can clearly be extended as one wish. Node Alice1 has a Bell pair and send one qubit of this to Bob, also referred to as "intermediate node". On the other side, also Alice2 has her Bell pair and perform the same operation of Alice1. It is worth to mention that now Bob has entanglement with both Alices because he has a Bell pair from both locally at his node.
	- $-$ At this moment Bob can perform *Bell State Measurement* (BSM) to swap the entanglement and entangle Alice1 with Alice2. This explanation matches with the definition of entanglement swapping that ensure two qubits that have never interacted before become entangled.

For the sake of completeness, a general view of what previously described is can be found in the image below [2.3.](#page-35-0)

Figure 2.3: Entanglement distribution and swapping process scheme. [\[10\]](#page-102-9)

In this context it is important to mention the fidelity that is a measure of how the resulting state of the two qubits, that have never interacted before, is closer to an ideal one if no noise or losses are considered.
2.4 Teleportation

Quantum teleportation is a groundbreaking protocol that enables the transfer of quantum information between two distant locations without physically transmitting the qubits and was first proposed by Charles Bennett and his colleagues in [1993](https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.70.1895) [\[28\]](#page-104-0).

The process relies on a shared entangled state between the sender (Alice) and the receiver (Bob) and in particular works as follow. Alice (the sender) and a Bob (receiver) share a pair of qubits in an entangled state, aslo referred to as Bell pair. This shared entanglement is used as a resource necessary for teleportation. To teleport a quantum state from Alice to Bob, Alice performs a joint measurement, that is to say a Bell-state measurement (BSM), on the qubit to be teleported and her portion of the entangled pair. Doing so, the two qubits are projected into one of the four maximally entangled Bell states, collapsing their quantum state and producing two classical bits of information. Afterwords, these two bits are used to encode the result of the measurement and need to be transmitted to Bob through a classical communication channel.

It is absolutely fundamental to note that quantum teleportation requires the coexistence of quantum and classical channels. The quantum channel facilitates the initial entanglement distribution, while the classical channel carries the two classical bits generated during the Bell-state measurement.

Once Bob has received the classical information, use it to apply one of four possible unitary operations (identity, X gate, Y gate, or Z gate) to his part of the entangled pair. A detail that can not be underestimated is that the quantum information (state) do not traverse the link because the process is enabled by the classical results and the entanglement connection as can be appreciated in the next scheme [2.4.](#page-37-0)

Quantum teleportation has profound implications for quantum networks, as it enables the transfer of quantum information without the need for a direct quantum link between the sender and receiver. Of course this is not magic and indeed a swapping operation is previously requied to create the entanglement connections between the nodes involved in the teleportation.

Another really important aspect is the possibility to use the teleportation operation to enable quantum repeater and extend the range of such a component. This can pave the way towards network capable of distributing quantum information over extremely long distances.

Figure 2.4: Teleportation process scheme. [\[10\]](#page-102-0)

Moving on, teleportation has also some really strong consequencies in quantum computing world. In this field, the advantage is that qubits can be transferred between distant nodes at the same time without the need of a quantum channel.

Such a capabiltity acts a backbone of the protocols that aims to integrate multiple quantum processors in to a larger quantum system. Recently, teleportation experiments shows the practical applicability of this quantum operation in real world, changing forever the telecommunication [field](https://arxiv.org/pdf/1707.00934) [\[29\]](#page-104-1) and opening the way to future quantum internet.

2.5 Fidelity computation and density matrix

In quantum information science, fidelity is a key metric to evaluate the similarity of the obtained quantum output with respect to the correspondent ideal state. Simulating quantum systems, especially if noisy qubits are involved, havily rely on fidelity to have a quantitative measure of accuracy. Considering what just stated, also NetSquid takes care of computing the fidelity of qubits that undergone to various network operations like teleportation and entanglement. Moreover, fidelity is a key allied to fully understand the impact of quantum operations on the delicate information carried by qubits. To describe the state of quantum system density matrix are often used. This is particularly true if in the simulation some source of noise are involved. In general is good to discriminate between:

- Pure states: can be described using state vectors which contains the probaility amplitudes of the quantum state
- Mixed states: can be defined as "ensemble of pure states with a probablity associated to each one" [\[30\]](#page-104-2).

Thus, density matrix is a valuable way to represent both pure states and mixed states in a quantum system.

Considering a pure state, the density matrix ρ is given by:

$$
\rho = |\psi \rangle |\psi|
$$

where, assuming the quantum state being $|\Phi\rangle = \frac{1}{\sqrt{2}}$ $\frac{1}{2}$ (|00\ $+i|11\rangle$)|−\ the correspondent state vector, as reported by a Netsquid [documentation example,](https://docs.netsquid.org/latest-release/api_qubits/netsquid.qubits.qformalism.html) is [\[31\]](#page-104-3):

$$
\begin{pmatrix}\n\frac{1}{2} \\
\frac{-1}{2} \\
0 \\
0 \\
0 \\
0 \\
\frac{1}{2}i \\
\frac{-1}{2}i\n\end{pmatrix}
$$

However, in practice, quantum systems contains qubits that are often in mixed states, thus the density matrix assumes the following general structure:

 $\rho = \sum_{i=1}^m p_i |\psi_i\rangle\!\langle\psi_i|$

where ψ_1, \ldots, ψ_m are the probability that the system is a pure state. Apply this general formulation to the previous example state, one find:

$$
\rho = \begin{pmatrix}\n\frac{1}{4} & -1 & 0 & 0 & 0 & 0 & -1i & \frac{1}{4}i \\
-\frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & \frac{1}{4}i & -\frac{1}{4}i \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4}i & -\frac{1}{4}i & 0 & 0 & 0 & 0 & \frac{1}{4}i & -\frac{1}{4}i \\
-\frac{1}{4}i & \frac{1}{4}i & 0 & 0 & 0 & 0 & -\frac{1}{4}i & \frac{1}{4}\n\end{pmatrix}
$$

Focusing on the fidelity, it is worth to mention that the fidelity of two pure states, ψ and ϕ , is computed as:

$$
F(\psi, \phi) = |\langle \psi | \phi | \rangle|^2
$$

The actual interpretation is quite straight forwards since this gives the measure on how much the two state vectors are overlapped. If the result is one, means that the fidelity is maximum and the two state are equal. On the contrary, if is near to zero or zero, means that them are near orthogonal or orthogonal.

Passing to mixed states, density matrices are involved in the fidelity computation process. If one consider two mixed states the fidelity is computed as:

$$
F(\rho, \sigma) = \left| Tr(\sqrt{\sqrt{\rho} \sigma \sqrt{\rho}}) \right|^2
$$

As already explained, mixed states accounts for situation in which quantum state have different probabilistic mixture of different pure states. This resemble very well communications systems scenarios in which qubits need often to be simulated and transmitted over a noisy environment.

In practice, as reported from this paper [\[32\]](#page-104-4), if the complexity of a quantum system grows also the fidelity computatation complexity increases. In particular dense matrix are involved thus finding a way to reduce such a issue would be a great plus.

A scalable solution is offered by the Density Matrix Renormalization Group (DMRG) algorithm, that compresses quantum states, to reduce the computational weight without sacrificing accuracy. This important result can be really fundamental in large scale quantum network development which rely on multi qubits systems.

2.6 Enabling technologies for quantum networks

One of the critical components in quantum network, a part from the already cited optical fibers, nodes and systems are the sources of single entangled photons. They enable the generation of photon pairs whose quantum states are intrinsically linked, without issues related to distances. As one can imagine, such a characteristic is fundamental for applications like quantum teleportation, distributed quantum computing and quantum key distribution (QKD).

In the following lines an overview of how entangled photon pairs are produced is presented. One of the most used techniques is the spontaneous parametric down-conversion (SPDC) in nonlinear optical crystals. In this kind of process a pump photon interacts with a nonlinear medium, leading to the emission of two lower energy photons which are commonly referred to as the "signal" and "idler" photons. This light particles can be entangled in a wide range of ways polarization, time, or energy. Passing onto the spontaneous four-wave mixing (SFWM) this take place in optical fibers or waveguides and take advantage of the third-order nonlinearity of the medium to generate entangled photon pairs. Even if its name sound complex it has the big advantage to be compatible with the already existing optical fiber infrastructure and infrastructure [\[33\]](#page-104-5). Unfortunately, both SPDC and SFWM sources produce photons probabilistically, leading to issues with multiphoton events that can compromise the security and efficiency of quantum communication protocols. Trying to avoid this problem researchers are focusing on deterministic single-photon sources. These promises to be able to emit one and only one photon on demand. One of the most suitable technologies to be used for that purpose are quantum dots embedded in photonic crystal cavities. In particular they are semiconductor nanostructures that can be electrically or optically excited to be forced to emit a single photons with high purity [\[34\]](#page-104-6). Thanks the research field, another suitable approach is using atom-like defects in solid-state materials like, for instance, nitrogen-vacancy centers in diamond or silicon vacancies in silicon carbide. One of the top rated characteristic of these systems is their capability to behave as stable (single) photon emitters at room temperature [\[35\]](#page-104-7). Transmission distances and key generation rates are two of the most important parameters that can take advantage of the upper mentioned technologies. All considered, entangled photon single sources are one of the key technologies responsable for the developement of quantum communication and networking and will be even more critical in the future when, with further improvements, will contribute to realize a global quantum internet [\[36\]](#page-104-8).

2.7 Quantum memories and interfaces

One of the most critical component in quantum world is quantum memory. Such hardware play a fundamental role in the development of large-scale quantum networks due to the fact that enables the storage and retrieval of qubits from nodes or future quantum computers and networks. Store qubits is a crucial capability to handle:

- Stationary qubits: that are in quantum processors or memories.
- Flying qubits: that are photons transmitted through optical channels.

In this field, the most challenging and delicate sector is the one that accounts for the electro - optical conversion from flying qubits (photons) to stationary ones and viceversa. Electro optical transduction bridges the gap between photonic qubits used for communication and matter-based qubits used for storage and processing. On one side, Photons are excellent to carry quantum information over long distances due to their weak interaction with the environment, thus decoherence is minimized. Unfortunately such a characteristic makes them unsuitable for tasks requiring qubit manipulation or memory storage. On the other hand, qubits that are stationary, like those in atoms, ions, or solid-state systems, can be easily manipulated and stored but are not really suitable for long-distance transmission. From the following ideal representation [2.5,](#page-42-0) one can have some graphical hints of the topic.

Figure 2.5: Graphical representation of flying - stationary qubit interface. [\[10\]](#page-102-0)

The aim of an effective transduction interface is to enable coherent transfer of quantum states between these two types of qubits without degrading the quantum information. This was the object of the following technologies:

- Atomic Ensembles and Quantum Memories: Atomic ensembles have shown the capability to absorb and emit photons, making them suitable for storing photonic qubits. Techniques like electromagnetically induced transparency (EIT) enable the reversible mapping of photonic states into collective atomic excitations, effectively storing the quantum information in a stationary medium. [\[9\]](#page-102-1)
- Opto-mechanical Systems: This kind of systems take advantage of mechanical vibrations to mediate interactions between optical and microwave photons. Coupling optical cavities to mechanical resonators, they are able to transfer quantum states between photons and mechanical modes, which can then interact with other quantum systems. [\[37\]](#page-104-9)
- Superconducting Circuits: In this case, qubits operating at microwave frequencies can be interfaced with optical photons through nonlinear optical materials or by using microwave to optical photons converters. Thus, the advantages of the superconducting qubits are combined with photons for communication.[\[38\]](#page-104-10)
- Rare-Earth-Doped Crystals: Similar kind of structures can behave as quantum memories by absorbing photons and storing their quantum state in the electronic states of the dopant ions. The advantages are longer coherence times and the ability to interface with photons at telecommunication wavelengths.[\[39\]](#page-104-11)

Theoretically, the electro optical conversion is pretty easy and straightforward. Unfortunately, when those concepts are applied to the real word, challenges arises. Some of them are efficiency and bandwidth, noise, decoherence and scalability. The firsts are important due to the fact that unmatched frequency ranges of optical photons and stationary qubits require careful engineering to ensure that the transduction process is resonant and efficient. On the other hand, scalability remains also critical because such interfaces need to account for a large number of inputs and be compatible with large scale communications systems. Recently, nano-photonic structure development [\[40\]](#page-104-12) and hybrid systems, leads to some improvements in this field promising to be a fundamental components for the future quantum internet realization.

Passing onto quantum memories, also them face significant challenges related to coherence time. This is defined as the time duration for which a quantum memory can hold the quantum state of a qubit without significant loss of information. Noise sources from the surrounding environment and decoherence events degrade the stored quantum information over time. Systems like atomic ensembles, solid-state memories, and ion doped crystals enable to reach coherence times ranging from milliseconds to seconds. In any case, these times are still insufficient for long-distance quantum communications and advanced quantum computing applications. The actual research aims to develop quantum memories capable of storing qubits for longer periods, as minutes or even hours with minimal degradation of the store quantum state. Such a goal would be a boost in the development of quantum networks, but also in the funds assigned to those infrastructures, due to the increased reliability and scalability that would be obtained.

Some components that will gain serious advantages from such a goal are quantum repeaters. Those rely on quantum memories to buffer qubits at intermediate nodes playing a critical role in extending communication distances beyond the current limits of optical fibers. It is worth to remember that for quantum communications EDFAs or general optical amplifiers can not be used because of the no cloning theorem. [\[41\]](#page-104-13)

One of the most promising sector for quantum communications is space. In such a scenario, the world race to develop reliable and secure space-based quantum communications is fully open. These systems are in charge of push the limit over ground-based quantum repeaters, enabling long-distance communication that spans thousands of kilometers. Deploying quantum memories on satellites offers a solution to the loss limitations that are present in terrestrial optical fibers. In terrestrial quantum networks, the distance is limited to hundreds of kilometers without the use of quantum repeaters. The use of quantum memories in space, facilitates ultra long distance communication, leading to the opportunity to reach distances between Earth and the Moon or even Mars. This approach extends the communication range and enables the transmission of entangled photons over large distances, overcoming atmospheric interference. One of the most promising project is the NASA Deep Space Quantum Link (DSQL) that is currently involved in investigate the potential for quantum teleportation between Earth and the Moon. Such an experiment requires quantum memories capable of maintaining quantum coherence for up to 1.3 seconds to account for the time required by the signals to travel between these two locations. Quantum memories in this context will play a critical role in experiments related to quantum gravity, in which entanglement is tested under varying gravitational forces.

It is worth to notice that, even if great strides forward have been achieved, quantum memories for space applications are still facing significant engineering challenges. The harsh space domain conditions, including temperature fluctuations, radiation, and vacuum, are threats to the stability of quantum states. Satellite are moving always towards miniaturization leading to the spread of hundreads of nano satellites. Integrate quantum memories into such platforms is a key challenge that can be addressed using warm vapor cells and cold atomic gases. These are among the candidates for quantum memories in space due to their potential to fit very small dimensions robustness to environmental factors. In particular, warm vapor cells, such as Rubidium based systems, demonstrate promising storage times of up to seconds in laboratory conditions. These technologies are going toward space ready versions, with minimal cooling requirements. This makes them suitable for satellite deployment in some years from now. [\[42\]](#page-105-0) [\[43\]](#page-105-1)

On the other hand, cryogenic systems like Rare Earth Ion Doped crystals (REIDs) offer longer storage times and higher fidelity but require complicate cooling systems, which complicates their integration into satellites. One need also to consider that improved miniaturized cryogenic systems are making REIDs a more viable option for future space missions. To conclude this paragraph, we can say that quantum memories are likely to be applied on board of satellites even if challenges still remains on decoherence time, stability, cooling systems and on electro - optical conversion.

2.8 Optical communication channel (in support of quantum communication)

The aim of this paragraph is to outline the core components of an optical communication channel, their functions, and the techniques used to maximize the performances. Optical communication channels are used to transmit information exploiting light as the carrier and revests a paramount importance in quantum communications and network because are used to distribute classical information like the BSM results and signalling messages. This is typically done sending light through optical fibers, enabling short reach or long reach communications. Channels like that are essential in modern telecommunications, because offers high-speed, long-distance data transmission with minimal signal loss or extremely fast and wide band ones, like in the data center applications. One of the most known and critical use of the fibers is in the long reach transoceanic cables that manage to distribute 90% of global internet traffic.

An optical communication channel requires, at least, a laser source, a modulator, one or more optical fibers and a photodiode at the receiver side. In the following lines some key aspects of each component are addressed.

• Laser source: This can be seen as the "core" of any optical communication system, serving as the fundamental source of light used to transmit data over optical fibers. In high-performance optical networks, Distributed Feedback lasers (DFB) are commonly employed due to their high output power, narrow linewidth, and stability. Such characteristics make them particularly suitable for long-distance communication and advanced modulation techniques. As one can imagine, there are plenty of laser but here we stick to DFB ones that are often used for long-distance, high-speed applications. These are semiconductor-based and have a grating embedded in the active region, which provides the feedback necessary to produce coherent light with a stable wavelength. DFB lasers are single-frequency lasers, meaning they produce light at a very specific wavelength with minimal unwanted fluctuations. This narrow linewidth is crucial for reducing interference and dispersion over long distances, enabling an easier work at the receiver side and making possible to extract information from the signal. In optical communication, linewidth refers to the spectral width of the laser light.

A narrow linewidth is essential because it reduces the broadening of light signals as they travel through the fiber, thus maintaining the clarity and integrity of the transmitted data. Moving on, another advantage of DFB lasers is their temperature stability. In many optical systems, environmental factors can cause the laser's wavelength to drift and this need to be avoided to not deviate from the system design conditions. Such lasers are engineered to maintain a stable output, even in fluctuating conditions. This feature is critical in dense data environments like Dense Wavelength Division Multiplexing (DWDM) systems. Since today this kind of system play a fundamental role is it worth to dedicate some lines on them. In DWDM systems, the capacity of a single optical fiber is maximized by transmitting multiple channels at different wavelengths, also known as "optical frequencies", simultaneously. Each wavelength can carry a separate data stream, thus increase the bandwidth of the fiber. As one can get from from the previous lines, DFB lasers are perfectly suitable for DWDM because they can generate highly precise wavelengths, minimizing interference between closely spaced channels.

DWDM systems typically operate within specific optical windows where fiber attenuation is minimized. These windows are centered around 1550 nm, a wavelength at which standard optical fibers have the low possible losses and can thus transmit data over long distances without significant signal degradation. This range is part of the C-band, the most used in telecommunications field, as can be seen from the following scheme from Polito optcom group. In particular this is retrieved from an optical course of full professor [Roberto Guadino.](https://www.optcom.polito.it/people/full-professor/roberto-gaudino)

Figure 2.6: Attenuation versus wavelength graph showing: S band, O band and C band. [\[11\]](#page-103-0)

Furthermore, it is worth to mention that DFB lasers are tunable, so they can be adjusted to different wavelengths within the 1550 nm window. This feature is essential in DWDM networks, where each channel needs to operate at a distinct, but really nearby, wavelength to avoid crosstalk. Accurate DFB lasers ensures that each channel is well separated, maximizing the capacity of the optical fiber and so optimizing also the cost of the system. Another noise source that can be faced using precise tuning of DFB lasers is the chromatic dispersion that, if not properly addressed, can cause distorsions on the signal.

• **Modulator:** In optical communication, the process where data is encoded onto a light beam is called modulation. This plays a key role in determining the performance and efficiency of the system. There are two main types of modulations, each one with its own advantages and disadvantages:

– Direct modulation: Direct modulation refers to varying the laser's drive current to encode data directly onto the light source. The advantages are that it is simpler and cost-effective due to the fact that it eliminates the need for external components, making it a suitable choice for low and medium speed optical communication systems. However, among the drawbacks of direct modulation one can find the frequency chirp. Usually this is described as a shift in wavelength during operations, which can degrade signal quality, especially over long distances. Frequency chirp increases dispersion in the fiber, reducing the system's capacity to transmit high data rates and thus poses a threat to the overall system capacity. Additionally, since the laser's output power and wavelength are directly modulated, maintaining laser stability becomes a challenge, further limiting its applicability in high speed or long distance communications, like transoceanic cables.

For the sake of completeness, a direct modulation scheme is attached below making clear the use of the laser to drive the modulation.

Figure 2.7: Direct modulation scheme.[\[12\]](#page-103-1)

– External modulation: For high performance systems, external modulation is preferred because it avoids the stability issues inherent in direct modulation. In this technique, the laser generates a continuous, unmodulated light beam, which is then modulated externally using devices such as Mach-Zehnder Modulators (MZM). The MZM works by creating interference between two light beams, allowing for precise tuning of both the amplitude and phase of the light signal. A general scheme of an external modulation is attached below.

Figure 2.8: External modulation scheme. [\[12\]](#page-103-1)

Both the previously reported scheme are not created by the author but are taken from this [source.](https://www.rfwireless-world.com/Terminology/Direct-modulation-vs-External-modulation.html) It is worth to mention that external modulators support advanced modulation formats, including Quadrature Amplitude Modulation (QAM). QAM modulates both the amplitude and phase of the carrier light, allowing multiple bits to be transmitted per symbol. For example, 16-QAM can transmit 4 bits per symbol, while 64-QAM can transmit 6 bits per symbol as can be appreciated from the attached graph [\[13\]](#page-103-2).

Figure 2.9: Variuous modulation graphs. [\[13\]](#page-103-2)

These formats are really important in high-speed systems because they significantly improve spectral efficiency, allowing for the transmission of more data over the same optical bandwidth. This kind of devices also allow for the implementation of coherent detection techniques, where both the amplitude and phase of the incoming light are recovered with high precision. Coherent modulation and detection schemes, such as those using Quadrature Phase-Shift Keying (QPSK) or higher-order QAM, are essential for long-haul optical systems and DWDM networks.

• Optical fibers and EDFAs: Optical fibers are basically super thin cylinders of extremely pure glass. Two primary types of fibers are used: single-mode (SMF) and multi-mode (MMF).

In long-haul communications, like transoceanic cables, single-mode fibers are favored due to their ability to minimize modal dispersion. This is a form of distortion of the signal that arises due to the light taking different paths through the fiber. Generally, single mode fibers are characterized by a really small core, around 8-10 µm, which allow the light to travel in a single path, thus reducing signal degradation and ensuring high quality transmission over distances of thousands of kilometers. On the other hand, Multi Mode Fibers requires a bigger core ranging from 50 to 62.5 µm as explained [here.](https://learn.aflglobal.com/enterprise/single-mode-vs-multimode) The scheme below graphically shows what is explained in this section, highlighting the different dimensions of the core and cladding.

Figure 2.10: SMF versus MMF internal structure. $[14]$

It is worth to mention that optical fibers can not handle all the work alone and it is natural that after long distances the signal power reach low levels so countermeasures need to be taken. The solution comes from the Erbium-Doped Fiber Amplifiers (EDFAs) are often deployed along the transmission path to amplify the optical signal directly, without converting it to an electrical signal. Such a plus is possible thanks to the use of stimulated emission from erbium ions in the fiber. This technique is widely adopeted today, particularly for wavelengths in the c band, where optical fiber exhibits minimal attenuation (0.2 db/km).

Nowadays the demand for bandwidth is constantly increasing boosting the use of Dense Wavelength Division Multiplexing (DWDM) systems as well as the study of more efficient types of modulations. In DWDM, as already explained, multiple signals are transmitted simultaneously over the same fiber at different wavelengths.

Thus, EDFAs acquire even a more important role in keeping the signal at the correct power level to avoid SNR degradation.

• Photodiode: Up to now the optical communication system description focused on the transmitting side and on the physical channel mean. It is now required to consider the receiver side, in which a crucial component like the photodiode is present. Its works is to convert the light signal, taken from the fiber optic, in an electric signal to be further processed.

In optical communication field two main types of photodiodes are used: PIN and Avalanche (APDs) ones. PIN photodiodes are the simplest and most common one due to their easy structure with a wide Intrinsic layer sandwiched between P-type and N-type semiconductors, as the name suggest. Even if this is not a comprehensive explanation of this electronic devices, can be sad that they absorb photons and generate electron-hole pairs, which are then converted into an electrical signal. For completeness, it is worth to say that PIN photodiodes are often used in short reach applications.

On the other hand, considering high speed and long distance systems, APDs photodiodes are preferred due to their higher sensitivity. Sensitivity is an important parameters in optical field telling which is the minimum power, at the receiver side, for which the system is still able to retrieve information from the signal. So higher sensitivity means that the receiver is able to process even really low power incoming signal, adding a great plus to the overall system. Of course this advantage is not for free and indeed the drawback is the high cost of these components.

APDs exploit the principle of avalanche multiplication,in which a single photon can generate multiple electron hole pairs, amplifying the weak optical signal received over long distances.

To summarize this background sections two optical systems examples are reported:

• Short reach system: These kind of systems can be usually find in data centers or local networks. In such scenario, a Distributed Feedback (DFB) laser is used to generate continuous light, which is modulated by a Mach-Zehnder Modulator (MZM) to encode data. The optical signal is then transmitted through a single mode fiber (SMF) over distances from a few hundred meters to a few kilometers.

At the receiver side, a PIN photodiode takes care of convert the light from the optical to the electrical domain. This operation allow to further process the signal to retrieve the data.

• Long reach system: Considering a long reach point to point scenario, like the submarines cables for global communications, the biggest difference is the presence of optical amplifiers to account for SNR degradation. We can imagine that a DFB laser, modulated by an MZM, transmits light through a SMF that span hundreds or even thousands of kilometers.

During the path, EDFAs can ben founded to a interval that depends from the system architecture. At the receiving side, an APD photodiode detects the weak signal, while advanced Digital Signal Processing (DSP) techniques, are applied to correct for impairments such as chromatic dispersion and polarization mode dispersion.

Finally, it is important to consider how and why the previously presented optical components are important for quantum networks. A concept that was already presented in section [2.7](#page-42-1) but that is important to recall, is that Erbium Doped Fiber Amplifiers (EDFAs) can not be used in quantum network due to the no cloning theorem. Moving on, can be sad that quantum networks share a lot of similarities with classical optical communication system. On the other hand they also face some unique challenges regarding quantum characteristics like entanglement, fidelity, coherence and so on. In this scenario, quantum networks need to handle a more delicate kind of information with respect to the classical optical one, reason for which noise, losses and imperfections impact is multiplied. In the following point the various use of what explained above are reported:

• Laser Source for Quantum Networks: As mentioned, the imperfections and the goodness of the optical components becomes of a paramount importance dealing with quantum information. For instance, using lasers in quantum network, requires to have a really good laser coherence to be able to maintain entangled photon pairs or preparing single-photon states. Again, DFB lasers can be used to gain an accurate control over the photon phase. This is crucial in some quantum application like quantum communications and Quantum Key Distribution (QKD). Lastly, as already mentioned in section [2.6,](#page-41-0) spontaneous parametric down-conversion (SPDC) sources can be used to generate photon entanglement.

- Modulators in Quantum Networks: The use of this components is also important in quantum networks due to the fact that they allow precise modulation of the phase and amplitude of single-photon states. One of the tasks assigned to them is to apply transformations to quantum states to encode quantum informations based on the used quantum protocol.
- Optical Fibers in Quantum Networks: Single mode fibers (SMF) are extensible used to carry flying photons (qubits) over quantum networks. One of the most important parameters to be minimized is modal dispersion. This is crucial to maintain the coherence between quantum states.
- Photodiodes in Quantum Networks: Sensitivity is another key parameter. To reach high sensitivities values, Avalanche Photodiodes (APDs) come into play thanks to their ability to detect single photons. Such a characteristic is possible thanks to the fact that a a single photon can generate multiple electron-hole pairs, amplyfing the weak quantum signal at the receiver side.[\[44\]](#page-105-2)

However, APDs presents some intrisic noise sources that brings to the use of technologies like Superconducting Nanowire Single-Photon Detectors (SNSPDs) which guarantee higher detection efficiency and lower noise [\[45\]](#page-105-3) [\[36\]](#page-104-8).

2.9 Quantum channel noise model

In quantum networks, accurately modeling noise is essential for the reliable transmission of quantum information. Quantum channels are susceptible to various types of errors due to interactions with the environment, leading to decoherence, losses and dephasing issues. Being able to understand quantum channel noise models is crucial to develop some strategies to tackle the problem, such as develop error correction techniques, so codes, and design robust quantum communication protocols. One fundamental noise model is the depolarizing channel, which represents a scenario where a qubit, with a certain probability p, loses its original state and becomes a completely mixed state. Mathematically, the depolarizing channel acting on a qubit state ρ is described as:

$$
\mathcal{E}_{\text{depol}}(\rho) = (1 - p)\rho + p\frac{I}{2}
$$

where I is the identity operator. This model captures well random errors that can occur uniformly in all directions on the Bloch sphere, making it a useful approximation for various types of noise. In the following plot, the effect of this kind of noise model on the Block sphere can be appreciated.

Figure 2.11: Effect of the depolarizing channel on the block sphere for $p = 0.5$. [\[15\]](#page-103-4)

Another important model is the *amplitude damping channel*, which describes the process of energy dissipation, such as spontaneous emission in optical systems. In this case, γ is the probability that a qubit in the excited state ket 1 decays to the ground state ket 0.

The transformation is described by means of Kraus operators that have the following meaning: E_0 corresponds to the case in which there is no errors and quantum state remains unaffected except for a scaling factor related to the probability of no error (indeed p is the probability of an error to occur). On the contrary, E_1 is referred to the case in which an error occurs, indeed the square root of γ is considered.

$$
E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}, \quad E_1 = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix}
$$

The final state, after the application of the amplitude damping channel, is given by:

$$
\mathcal{E}_{\mathrm{amp}}(\rho)=E_0\rho E_0^\dagger+E_1\rho E_1^\dagger
$$

The effect of the amplitude damping channel on the Block sphere is reported in the next plot. As can be noticed, the sphere shrinks and move in the north pole (ket 0 state) direction.

Figure 2.12: Effect of the amplitude damping channel on the block sphere for $p = 0.8$. [\[15\]](#page-103-4)

Passing onto the *phase damping channel*, also known as the *dephasing channel*, it models the loss of quantum coherence with no energy dissipation. It captures the effects of random fluctuations in the environment that disturb the relative phase between quantum states. The Kraus operators for this channel are:

$$
E_0 = \sqrt{1 - \lambda} I, \quad E_1 = \sqrt{\lambda} \sigma_z,
$$

where λ is the probability of a phase error and σ_z is the Pauli Z operator.

The final state, after the application of the phase damping channel, is given by:

$$
\mathcal{E}_{\text{phase}}(\rho) = (1 - \lambda)\rho + \lambda \sigma_z \rho \sigma_z,
$$

Moving on, one can find channels that models specific errors related to qubit as:

- Bit-flip Channel: Represents errors where a qubit flips from ket 0 to ket 1 or vice versa with probability p.
- **Phase-flip Channel**: Describes errors introducing a phase shift of π in the qubit state.
- Bit-phase-flip Channel: Combines bit-flip and phase-flip errors.

Before going deeper into these three representation is worth to mention that $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices representing each operation as reported below:

$$
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
$$

Starting from the **Bit-flip Channel**, the Kraus operational elements are the following one:

$$
E_0 = \sqrt{1-p} I, \quad E_1 = \sqrt{p} \,\sigma_x,
$$

In the next graph the effect of this model can be appreciated.

Figure 2.13: Effect of the bit-flip channel on the block sphere for $p = 0.3$. [\[15\]](#page-103-4)

Considering the **Phase-flip Channel**, we have:

$$
E_0 = \sqrt{1 - p} I, \quad E_1 = \sqrt{p} \,\sigma_z,
$$

and the corresponding sphere representation is:

Figure 2.14: Effect of the phase-flip channel on the block sphere for $p = 0.3$. [\[15\]](#page-103-4)

Lastly, passing to the **Bit-phase-flip Channel**, one obtain:

$$
E_0 = \sqrt{1 - p} I, \quad E_1 = \sqrt{p} \,\sigma_y,
$$

The related and combined effect on the block sphere is reported below:

Figure 2.15: Effect of the phase-flip channel on the block sphere for $p = 0.3$. [\[15\]](#page-103-4)

Chapter 3

System level architecture and results

At the beginning of this chapter we provide an overview of NetSquid and how this has been used in our work. NetSquid is a python framework used to simulate quantum hardware and is capable of handle quantum operations, links and networks. It is worth to mention that this simulator is the core of the pratical implementation of this master thesis, and it was used in parallel with a theoretical understanding of the real world processes.

Moving on, one can find an explanation on how the different operations are implemented in the code, how the various information are sent between nodes as well as a description of the hybrid classical quantum network topology.

Moreover, types of noises and decoherence penalties will be presented, as well as considerations about qubit encoding. Finally, simulations results will be addresses using also graphs to compare different scenarios and multiple operative conditions.

3.1 Simulation software description: Netsquid

NetSquid is a sophisticated python framework specifically developed and tailored to simulate quantum hardware and networks operations and protocols. As quantum technologies continue to evolve, the need for robust tools to model, analyze, and visualize quantum communication processes has become increasingly important and will be more and more in the future. Offering a modular architecture that simplifies the implementation of complex quantum systems, NetSquid rises to this challenge and present itself as a promising tool for present and future quantum research.

Before moving on, an overview of the "NetSquid package" is provided in next figure taken from the official NetSquid website [\[46\]](#page-105-4):

Figure 3.1: General view of Netsquid software structure.

The core Components of NetSquid are crafted to fulfill specific roles within the quantum networking environment and, practically, make the researchers able to construct realistic simulations and conduct experiments efficiently in a low cost way. These components are presented in the following list:

• Base Component Class: At the base of NetSquid there is the base component class that can be founded in netsquid.components.component.

This provides the fundamental attributes and methods necessary for creating and managing various quantum components, ensuring consistency and coherence across the framework.

- Channels for Quantum Communication: Since in quantum networks, communication heavily relies on channels, NetSquid offers several types of them answering to a wide range of needs:
	- Quantum Channel: As the name suggest, this is tailored to transmit qubits (quantum states). Noise models as well as loss one are available and included, allowing for realistic simulations that account for the imperfections inherent in quantum communication. Again this is a pro that encourage many researches to use NetSquid.
	- Classical Channel: In a lot of cases, and this work is no exception, quantum protocols require to transfer classical information. The ClassicalChannel component handles this classical communication effectively, ensuring that the framework can simulate hybrid quantum-classical operations seamlessly.
	- CQ Channel: To handle scenarios that necessitate simultaneous transmission of both quantum and classical information, the CQChannel acts as a hybrid channel, integrating the functionalities of both quantum and classical channels even if this is not mandatory to be used.

One of the hot topic in quantum research activities are quantum memories. Actually this is one of the hardest challenge to be solved and limits the practical applications of quantum information theory to the field of computing and communication.

As already explained in chapter 2, at section [2.7,](#page-42-1) the problem is how to keep qubits stored avoiding that the delicate quantum information is lost. Practically speaking this is related to the decoherence time and NetSquid is the perfect platform to simulate quantum memories behavior since allow to this, and other, parameters. More precisely, QuantumMemory component supports a variety of operations, such as preparing, measuring, and retrieving qubits, which facilitate the manipulation of quantum information. This capability is essential for tasks like quantum teleportation and entanglement distribution that are the core of this thesis. As for the classical computer science and communication world, instructions and program are needed as well as processors. Of course if one brings this in a quantum fashion will come up to:

- Instructions: The instruction set in NetSquid allows users to define a range of operations that can be applied to qubits and other components. This includes single-qubit gates, like Z, X etc, measurements, and more complex operations essential for executing quantum protocols.
- Quantum Programs: Summarizing, quantum program is a sequence of instructions specifying how qubits should be manipulated. This abstraction makes it easier for users to construct intricate quantum operations and protocols without getting bogged down in the details of each step and also helps in handling and calling functions or requests.
- Quantum Processor: This is at the core of a quantum node even if it is not mandatory to implement each node using it. Manages the execution of quantum programs, allocates qubits, and ensures that the various operations are performed in a coordinated manner.

Furthermore, a really fundamental concept is how the qubit are generated and how one can simulate this. The Quantum Source component is responsible for creating single qubits or entangled pairs, which are necessary for initializing quantum communication protocols. In practice, this capability is needed in quantum key generation and, as in our case, entanglement distribution.

It is worth to mention that no results can be obtained without a proper measure of the quantum information. Such a goal can be obtained in a more ideal way with noiseless quantum gates or with Quantum Detector. In the second case, this is component that is designed for measuring the states of qubits and can handle a lot of parameters to simulate real hardware. As previously described, [2.2,](#page-31-0) retrieve these state is mandatory to extract classical information from quantum systems. This measurement capability enables the evaluation of quantum protocols, and pave the way a more deep integration between network and physical level analysis.

Each network requires, at least, two nodes and it is clear that these play a prime order role in NetSquid. Each node can host multiple components, including quantum memories, processors, and channels, allowing for flexible configurations that can be tailored to specific research needs. A comfortable characteristic of NetSquid is that it is equipped with pre-defined protocols for various quantum communication tasks, such as quantum key distribution, teleportation, and entanglement swapping.

These protocols can be easily customized or extended, providing researchers with a solid foundation upon which to build more complex experiments.

Another key point is the possibility, to include noise models in the simulations to reproduce realistic quantum environments by fine tuning the noise levels inside the quantum system or network. Models account for different types of noise, loss, and other factors that affect quantum communication. In NetSquid, as can be appreciated from the official documentation, the formalism has a paramount importance in how the quantum states are represented and thus also in which situation one need to be preferred. In particular, NetSquid provides five formalisms to represents qubits and the applied operations:

- DM (density matrices) stands for DenseDMRepr representation.
- SPARSEDM (sparse density matrices) stands for SparseDMRepr representation.
- KET (Ket state vectors) stands for KetRepr representation.
- STAB (stabilizer tableau) stands for StabRepr representation.
- GSLC (graph states with local cliffors) stands for GSLCRepr representation.

It is worth to mention that KET, DM and SPARSEDM are used when one deals with universal quantum computing. In such cases density matrices allow the user to simulate mixed states to account for realistic situations in which the exact state is unknown On the other hand, STAB e GSLC formalism can not be used to simulate universal quantum computing. Moreover, unless the state is highly entangled, GSLC is faster than the STAB. What causes the GSLC being slower in highly entangled situation is the high number of edge in the correspondent GSLC graph.

Thus, when noise need to be considered or tracked with high accuracy, DM or SPARSEDM are the best choice. On the contrary, if noise is not important in the simulation and can be aproximated to zero, KET need to be preferred.

Taking everything in to account, can be sad that this framework really helps in research and development or experimental activities. Simulation is gaining always much importance due to its capability to create a plenty of different conditions with low costs with respect to real experiments, when they are possible. NetSquid express modular design that comprehends a rich array of components and functionalities, playing a critical role in advancing the field of quantum information science keeping the costs low. For any further understanding of NetSquid simulator, please check the official documentation available [here.](https://docs.netsquid.org/latest-release/api_reference.html)

3.2 Code description and implementation

This section aims to explain how the code is written and the main features of the implementation. An overview of the network is presented as well, showing the importance of understand network topology and implementation in quantum networks.

A central point of our work was to keep the code as simple as possible. This allow to concentrate more attention on quantum operations itself without getting too much in trouble in tricky coding enabling an easier error debugging if needed. In the first lines some global variables related to noise and loss can be founded, making the initial code part a "control suite" for most of the simulator. These variables address for some physical impairments in the communication and are referred to node distance, link noise rate, depolarization rate etc. Moreover, an enabling variable for losses, loss enabled, is present as well and can be set to True or False following the user needs. Similarly, some flags to handle various plotting and printing of various parameters are insert, giving completeness to our work. As already anticipated, a brief network overview is required to understand why some functions are needed and is presented in the next lines. As already anticipated, to simulate our quantum topology two overlapping networks are required:

- A quantum network: used to distribute quantum information (qubits) and to create entanglement connections.
- A classical network: used to distribute classical information as success signals required to handle various operations and subprotocols.

In particular the four node at the left (A nodes) can simultaneously distribute entanglement towards four right nodes (B nodes) to create a "baseline network" meaning that each A node has an entanglement connection with each B node. Practically speaking, bell pairs are created and qubits are sent from Alice nodes to Bob nodes to create entanglement connections. Moving on, we implemented also a swapping capability thanks to which two A nodes can obtain a connection between themselves by means of an intermediate B node will be clear in the next lines and sections. The swapping operation is crucial to implement more complex process like teleportation. Consider, for example, a three nodes case: Alice1, Alice2 and Bob1. Alice1 and Alice2 hosts two bell pair, one for each node, and send respectively one qubit to bob1 to create two entanglement connections. Doing so their are linked with Bob1 but no each other.

To overcome this issue swapping operation can be carried out, performing a BSM (Bell State Measurement) on the qubit pair at Bob1 side. Based on the BSM outcome, different quantum gates are applied to the qubits.

Doing so Alice1 and Alice2 now share a new entanglement connection among the halves of the bell pairs that has been kept on their side.

As for the other quantum operations the fidelity need to be computed to check the goodness of the process and, eventually, the noise impact.

Finally teleportation can occurs both from Alice to Bob nodes and between different Alice nodes if them have previously swapped the entangled, so if they share an entanglement connection.

It is worth to consider that the most meaningful case is the second one, teleport from Alice node to another Alice node, because this highlight the role of the intermediate Bob node and underline the advantages of quantum network operations versus classical one. Also in this case a specific fidelity computation is present to check the quality of the process. Moreover, entanglement connection restoration is implemented to recreated consumed connections by teleportation operations.

The simulation of these three quantum operations directly reflects on the nodes structures and requires different internal organizations for A and B nodes. In particular A nodes need to hosts 9 qubits divided as follows:

• 4 qubit pairs: used to reproduce bell states to create entanglement connections between each A and B node. They occupy memory positions from 0 to 7.

• 1 qubit to be teleported: inserted in the last position of each node, so the eighth, is the one involved in the teleportation operation. The precise measurement process to achieve this important result is described further below.

The following scheme, [3.2,](#page-67-0) reports graphically what explained above:

Figure 3.2: A node structure scheme.

On the other hand, B nodes, have 5 memory positions divided as follow:

- 4 memory positions: required to host the qubits send by A side to create entanglement connections.
- 1 memory position: used to insert the qubit responsible for the restoration of the entanglement connection consumed when a teleportation operation is performed from Alice to Bob. This is the last position.

The following scheme, [3.3,](#page-68-0) reports graphically what explained above:

Figure 3.3: B node structure scheme.

Once that the different node structures are clear, one can easily comprehends the overall "baseline network" structure that is showed in the figures below.

The first one, [3.4,](#page-69-0) concentrates on the memory positions involved in the entanglement distribution and clearly shows which memory locations are used as extremities of each entanglement connection.

In this scenario, qubits - that is to say Bell pairs - are inserted in memory positions of A nodes and after only the correct one are send, thanks to quantum connections, to the right node B memory postions.

In particular, qubits in position 0, 2, 4, 6 are send, while qubits in position 1, 3, 5, 7 are kept on node A side. For instance, considering nodes A1, B1 and B2, the qubit in pos[1] of A1 will be entangled with the qubit in pos[0] of B1 and the qubit in pos[3] of A1 will be entangled with the one in pos[0] of B2.

Figure 3.4: Network scheme showing the starting and ending memory position of each entanglement connection.

The second one, [3.5,](#page-70-0) aims to show the actual memory content after the entanglement distribution operation, so it is more focused on presenting the memory status.

Figure 3.5: Network scheme to show the final memory state after entanglement distribution.

The previous figure [3.5,](#page-70-0) has the following meaning: red memory positions are the one that remain empty after the entanglement distribution because were used to store the qubits that was sent towards B nodes. On the other hand, green memory positions are the one that contains a qubit. On A side, this means that in those positions are contained the qubits representing half of the orginal Bell pair. On B side, this shows that the qubits are correctly received and inserted in memory to create entanglement connections. It is worth to mention that, at this stage, qubits in "mem_pos_8" of all A nodes, are not involved in the operations because will be used after for teleportation. The empty memory positions inside the B nodes, are reserved for qubits to be inserted only when a re establishment of an entanglement after teleportation from Alice to Bob is required. In the following lines a deeper analysis of the various quantum operations is presented following the actual coding implementation:

- Entanglement distribution: We start implementing a reduced version of the whole four by four topology, so a two by two is considered initially. In such a scenario four nodes in total are considered, that is to say A1, A2 and B1, B2. Our network is capable of distribute entanglement, that is to say entangled qubits states as qubits. Before considering to distribute entanglement it is required to create and insert qubits (bell pairs) in to nodes A quantum memories. If the reduced network is considered, two bell pairs are created so four total qubits need to handled and inserted in nodes A memory positions from [0] to [3]. In such a scenario the first qubit (pos[0]) of A1 is send to the first position (pos[0]) of B1 and the third qubit (pos[2]) of A1 is send to first position (pos[0]) of B2. Similarly, the first qubit (pos[0]) of A2 is send to the second position (pos[1]) of B1 and the third qubit (pos[2]) of A2 is send to second position (pos[1]) of B2. The described flow has as result to create entanglement connections between A1, A2 and B1, B2 with these specific end points:
	- Node A1 and B1: pos[1] of A1 pos[0] of B1
	- Node A1 and B2: pos[3] of A1 pos[0] of B2
	- Node A2 and B1: pos[1] of A2 pos[1] of B1
	- Node A2 and B2: pos[3] of A2 pos[1] of B2

Considering the full 8 nodes network topology this considerations can be extended and created the following entangled connections:

Furthermore, an entanglement fidelity computation is carried on considering the qubits pairs that are previously described. As an example one can consider this pairs but this can be extended to any node pair.

- First pair: qubit in position zero of B1 (so the one that was in pos [0] of A1) and the qubit that is in the third position of A1 (pos[0] $B1 - pos[1]$ A1).
- Second pair: qubit received by B2 and inserted in pos [0] and the fourth one $(pos[3])$ in A1 node memory $(pos[0]$ B2 - $pos[3]$ A1).
- Entanglement swapping: As it is known, the entanglement swapping operation allow two nodes that are not entangled to be connected by means of a new entanglement connection. Practically speaking, this is really useful due to the fact that paves the way for teleportation between nodes that were not originally connected. In this thesis work such a capability is used to enable the teleportation from A nodes to other A nodes. Moreover the capability to link two network entities, performing operations on an intermediate node, should not be under estimate because can have a strong impact on networking. In the following points a detailed analysis on how the swapping operations are implemented is provided for both network versions:
- Four nodes network:. The swapping implementation is based on the previously explained entanglement distribution. For the sake of clarity we do the following example among nodes A1, B1 and A2. The end state is to swap entanglement between A1 and A2 using B1 as intermediate node. On A1 memory we have $q_0_0, q_0_1, q_0_2, q_0_2, q_0_3,$ up to q_0_7 . On A2 memory we have q_2_0, q_2_1, q_2_2, q_2_3, up to q_2_7. After the entanglement has been distributed the firsts memory positions on A1 and A2 are empty, that is to say $pos[0]$ of A1 and $pos[0]$ of A2. Thus entanglement swapping use qubits in pos[0] and [1] of B1 and the qubits in pos[1] of A1 and A2. Next, bell state measurement (BSM) is carried out on B1 qubits and conditional operations are applied on A nodes qubits based on measurements outcomes. In particular:
	- ∗ if the result corresponds to the |01> bell state, X operator is applied.
	- ∗ if the result corresponds to the |10> bell state, Z operator is applied.
	- ∗ if the result corresponds to the |11> bell state, X and Z operators are applied.

In the following graph what explained can be graphically appreciated:

Figure 3.6: Sample scheme of quantum swapping operation between Alice1, Alice2 and Bob1.

One can also extend this example considering A1 and A2 swapping entanglement using B2 as intermediate node.

- Eight nodes network: Considering a full network of four Alices and four Bobs, it is simply required to extend the previous concept to more qubits and node entanglement connections, but nothing else changes.
- Teleportation: It is crucial to remember that teleportation operation requires to have an entanglement connections between two nodes. For this reason, if the two nodes can be directly connected, it is enough to rely on an simple entanglement distribution, otherwise, if the nodes can not be directly connected, a swapping operation is required.

After have done this assumption, in the next lines the project implementation for teleportation is discussed. Firstly, one need to remember that the qubit to be teleported is the last one in Alice nodes memory, pos [7], so "mem_pos_8". Considering the most meaningful teleportation case, the one between two nodes that can not be directly connected, the following steps are done: on A side a BSM is carried out over the qubit to be teleported and the qubit that is the extremity of the entanglement connection. After this, the result of the BSM is sent to the other A node, that based on the classical result, choose which quantum operation need to be applied to the target qubit to convert it and in particular:

- If the BSM result shows a qubit in 00 state: no correction is required.
- If the BSM result shows a qubit in 01 state: apply X gate.
- If the BSM result shows a qubit in 10 state: apply Z gate.
- If the BSM result shows a qubit in 11 state: apply X and Z gate.

For an extended view of the quantum gate operations, one can check this section [2.2.](#page-31-0) Concluding, all this steps bring to the conversion of the target qubit in to the one to be teleported that was on alice side.

In the scheme below a scheme to graphically visualize the teleportation operation is provided, considering A1 and A2 using B1 as intermediate node.

Figure 3.7: Sample scheme of quantum teleportation operation between Alice1 and Alice2

• Entanglement restoration after teleportation: As it is known, the teleportation process consumes the entanglement distribution between the two involved nodes. This can be noticed also in the previous figure, in which - at the end - there is no more the green line representing the swapped entanglement connection. For this reason, in order to properly simulate the overall teleportation operation, a re-establishment of the entanglement is required. In particular, it is important to consider the positions of the qubits that are the extremities of the entanglement connections. For instance, looking at the four node network, the entanglement need to be re-established between position 1 of A1 and A2 if B1 is used as intermediate node or between position 3 of A1 and A2 if the intermediate node is B2.

After each teleportation operation between A and B nodes, the entanglement is re-established and the qubit postions are computed based on the involved nodes. On the other hand, considering teleportation from A to A the re-establishment process is included in the previously done swapping operation. This means that before each teleportation from A to A node a swapping operation is required to create, or re-create, the required entanglement connection.

Figure 3.8: swapping to re-establish entanglement for teleportation

• Retransmission mechanism: Considering losses in our network leads to need of retransmissions. This is because qubits - photons - are lost during their trip over the various links.

In our project the retransmission mechanism has been developed to work on the four nodes network and further details are showed in the next lines. An explanatory scheme is also provided in figure [3.9](#page-80-0) and shows that each node has 4 ports: 2 quantum one and 2 classical one. The quantum ports are required to send or receive qubits over the link, while the classical one are used handle the classical signalling required. The following list hallow to better link the classical and quantum port names to the proper node.

- Node A1:
	- ∗ Quantum ports: qout_A_11, qout_A_12
	- ∗ Classical ports: cin_A_11, cin_A_12
- Node B1:
	- ∗ Quantum ports: qin_B_11, qin_B_12
	- ∗ Classical ports: cout_B_11, cout_B_12
- Node A2:
	- ∗ Quantum ports: quantum ports: qout_A_21, qout_A_22
	- ∗ Classical ports: cin_A_21, cin_A_22
- Node B2:
	- ∗ Quantum ports: quantum ports: qin_B_21, qin_B_22
	- ∗ Classical ports: cout_B_21, cout_B_22

The mechanism works as follow: if a qubit is lost in one of the links and so it is not inserted in node B memory, a classical retransmission request is send from the B node towards the A one that was responsable for the original sending. The A nodes is waiting for some requests to come on its classical ports. If one arrives, the node re-create a bell pair, insert it in memory, keep one qubit for itself and send the other to the correct bob using the proper quantum port. On Bob side, it is also waiting for the retransmitted qubit. Thus, when this arrives, the node insert it in the proper memory position based on which A perform the re-transmission.

Such a mechanism seems easy but involves some really delicate code and conceptual aspects. For instance, timing is critical. Checking if a retrasmission message has been send, send it, or check if a qubit has been received and inserted in memory are all unseful operations if done at the wrong time. For this reasons we need to account also for messages and qubits propagation time over the network, so the following two times are considered. It is worth to mention that the processing time factor is required even if in our nodes there are no additional delays for the execution of the operations. Parameters and units of measurement:

- Propagation time and Timeout window: [ns]
- Node distance: [km]
- Speed of light $=c = .0003$ [km/ns]
- $-$ Processing time $= 1$ ns
- Propagation time: Theoretically this should account only for the time required by qubits - photons - to travel the node distance. Practically, to be able to appreciate the fact that a qubit is received and inserted in node B memory, it is required to add the upper mentioned processing time. For the sake of simplicity in this work we define only one "propagation time" but, if one wants, two can defined: a theoretical one - in which only node distance and speed of light are considered, and a practical one - in which also the processing time is taken in to consideration.
- Timeout window: This is the total time that a B node need to wait from the moment in which it send the classical retransmission message towards A, until the moment in which the retransmitted qubit is received and inserted in node B memory. If the retransmitted qubit it is not received after this time, the B node ask for another retranmission.
- Propagation time $= ((node distance)/c) + processing time$
	- ∗ Example: considering a node distance of 1 m, the Propagation time with the processing - is 6 ns.
- Timeout window or total wait time: $= 2 * ((node distance)/c) + processing time$
	- ∗ Example: considering the upper values, the Timeout window is 11 ns.

Figure 3.9: Nodes, ports and network scheme used for the retransmission.

The propagation time is used when a nodes needs to wait for the propagation of a quantum or classical message over the network. On the ohther hand, the timeout window or total wait time, is used when a node need to send a classical retransmission message, wait that the other node process the message and send back the qubit as it is also mentioned before.

It is worth to notice that for each of the three basic quantum network operations, we compute fidelity. Such a parameter has a paramount importance in determine the goodness of the quantum information at the receiver side. Furthermore, can be used to understand how and how much noise and distance affects qubits.

Before passing onto the analysis of the three different fidelity, it is worth to mention that we used the squared Netsquid parameters equal to False. Thus, the mininum fidelity for entanglement and swapping is 0.25 and for teleportation is 0.5.

• Entanglement fidelity: This is the fidelity between the two qubit of the bell pair used to create entanglement among different nodes. It is important to address this value to understand the goodness of the entanglement connections.

- Swapping fidelity: This is the fidelity between two extremities of the entanglement connections created between two nodes that can not be directly connected, for example A1 and A2.
- Teleportation fidelity: This is the fidelity between the target state and the one of the qubit to be teleported on A node side.

3.2.1 Hybrid classical and quantum network topology

As already presented in the previous paragraphs, our work aims to investigate the impact of noise, decoherence and distances on quantum operations like entanglement, swapping and teleportation and to investigate the related network performances. To do so, two overlapped networks are required. The most easily understandable, is the quantum one that is used to distribute entanglement between four A nodes and four B nodes, so to create a baseline network. On the other hand, the classical network has a paramount importance because is responsible to transport the classical BSM results used in the teleportation operation and for the classical messages used in the retransmission operations.

3.3 Types of noise and decoherence penalties

As one can catch from the upper paragraphs, NetSquid is a powerful tool for simulating quantum systems, focusing on the realistic modeling of noise, errors, and delays that impact quantum networks and devices. The models provided by NetSquid capture real-world imperfections in quantum operations, ensuring that the simulations reflect reality as closely as possible.

These models are essential for studying how noise, errors, and operational delays affect the performance of quantum communication and computing systems, leading NetSquid to be a really important tool in the quantum simulation field. Noise can not be avoided in quantum systems, and NetSquid offers several detailed noise models to simulate these effects. One of the most common models is the *DepolarNoiseModel*, which simulates depolarizing noise. This type of noise causes qubits to lose their coherence, turning them into a maximally mixed state with a certain probability. Depolarizing noise is especially important in quantum communication systems, as qubits are likely to experience decoherence over long distances, such as through optical fibers. This plays an important role due to the challenges poses by actual fiber optic networks worldwide. This model is the one used in this thesis work, both for the optical fiber links and for the quantum memories.

Another key noise model in NetSquid is the *DephaseNoiseModel*, which simulates dephasing noise. Dephasing occurs when qubits lose their phase coherence while keeping their energy states unchanged. This is a common source of errors in quantum computing, especially in quantum memory and during gate operations. NetSquid allows users to define specific noise rates, making it possible to simulate environments with different levels of decoherence. Moreover it is possible to apply different dephasing noises to specific qubits in predefined quantum memory positions, boosting the capability to reproduce with high fidelity the real quantum hardware. These noise models can be applied also to channels and gates so this gives users the flexibility to simulate noise in complex networks.

Models referred to errors in NetSquid simulate the imperfections that occur in quantum devices, in this part we focus on the *GateErrorModel* simulates errors during quantum gate operations. In reality, quantum gates, which manipulate qubits according to quantum logic, are not perfect, and small imperfections can lead to errors in quantum circuits. NetSquid's error models allow users to simulate both systematic errors, which happen consistently, and random errors, which occur probabilistically.

This is important because allow to handle both situation in which errors are created by noise sources that evolves in time, so follows the laws of probability, and someothers in which the errors repated sistematically in time because are due to some HW imperfections. Furthermore, NetSquid provides error models for quantum memory and channels. Quantum memory can be affected by noise or errors during the storage and retrieval of qubits. The MemoryErrorModel simulates these errors, which are especially important in long-term quantum communication where qubits need to be stored for a while before being measured or used in further operations. These models play a fundamental role to understand and test the reliability of quantum operations and the feasibility of large-scale quantum networks. In addition to the previously mentioned aspects, delays are another important factor to be considered. Considering that qubits take time to travel through channels like optical fibers, or when quantum operations take time to complete. NetSquid includes delay models such as the FibreDelayModel, which simulates the time delay caused by qubits traveling through fiber optic cables. This model is important for quantum communication networks, where timing mismatches and desynchronization issues can affect the efficiency of protocols. Not only, FixedDelayModel, which simulates constant delays associated with quantum operations or processes, is provided. By using delay models for gates, measurement devices, and channels, users can analyze how timing constraints produce effects on quantum algorithms and protocols. Delays are especially important in distributed quantum networks, where they can affect the success rate of operations like entanglement swapping, quantum teleportation and quantum key distribution.

As one can clearly notice, the real added value of a simulation framework like NetSquid, is the capability to integrate noise models, errors and delays models. This is the core of a simulation activity that aims to mimic the real world phenomena because they really take place all together. On the other side, it is also important to be able to keep them separated to be able to study some specific component of noise behavior. For instance, in quantum networks, qubits may experience depolarizing noise while traveling through quantum channels, encounter gate errors during operations, and be affected by transmission delays. One of the critical hardware component of quantum networks, that is still under development, is quantum memory. In this field combining noise and error models is critical for studying the reliability of qubit storage. As the time passes the qubits that are stored can undergone a degradation process due to memory errors, and a even more worst one when qubits are retrieved.

3.4 Simulations results

In this part our simulations results are presented as results of quantum network operations versus different parameters and code settings. Firstly the impact of distance, in km, and Link Noise Rate [Hz] on quantum operation fidelity without any losses is analyzed. Moving on, the same analysis is carried out considering losses in the fiber connections and, consequently, using a custom re transmission mechanism implemented by us to overcome qubits (photons) losses. About the retransmission, that are described in this section [3.1,](#page-61-0) results about the number of retransmissions versus distances are presented. Lastly, memory idle noise impact on quantum operations fidelity is addressed. Next section, [3.5,](#page-99-0) accounts for the conclusions and possible future related works.

3.4.1 Quantum network operations fidelity vs distance and LNR (L.E. $= False$)

In this section the results obtained investigating the various fidelity outcomes versus link noise rate (LNR) and distances are reported. Link noise rate is expressed in Hz and distances in km, as can be appreciated from the graph axes and legends. The LNR values on the x axes are plotted using a logarithmic scale to be able to fit the wide range of numbers that is required. As the name of the sections suggests, here losses are not considered so the impairments are the distance and the link noise rate. It is worth to mention that we investigate the fidelity values for each quantum operation. This is reflected in the graphs presented in the next lines:

- Entanglement fidelity vs Link Noise Rate vs Distance [3.10](#page-86-0)
- Swapping fidelity vs Link Noise Rate vs Distance [3.11](#page-86-1)
- Swapping fidelity vs Link Noise Rate vs Distance with repeater [3.12](#page-87-0)
- Swapping fidelity vs Link Noise Rate vs Distance with repeater values comparison [3.14](#page-88-0)
- Teleport fidelity vs Link Noise Rate vs Distance [3.15](#page-90-0)

All the numerical results used to plot the previously mentioned results are shown in the table inserted after the graphs.

In the first graph, [3.10,](#page-86-0) one can appreciate how the entanglement fidelity decreases for increasing values of distances and link noise rate (LNR). For instance, fixing a value of entanglement fidelity of 0.7 the LNR for 1 meter is 5e7 Hz, while for 100 km is 5e2 Hz. These two opposite situations highlight how spread and different can be the values in our analysis and how the distance influences the link capability to face noise. Similar considerations can be done for the second graph, [3.11.](#page-86-1)

Figure 3.10: Entanglement fidelity vs distance vs LNR without losses.

Swapping fidelity vs Link Noise Rate

Figure 3.11: Swapping fidelity vs distance vs LNR without losses.

The next plot is obtained considering half of the link distance of the previous one. Such a choice has been done to underline the advantage of eventually using quantum repeater to interconnect the nodes of this kind on topology. The advantage is that this optical component allows to halve the link length and so the distance that the signal need to travel.

Figure 3.12: Swapping fidelity vs distance vs LNR with no losses, with repeater.

Observing the next two plots, one can appreciate what previously explained. For instance, fixing a value of swapping fidelity of 0.6 the LNR is about 7e2 Hz for a link length of 50 km, as can be noticed here [3.14.](#page-88-0) On the other hand, peeking a values of fidelity of 0.6 but with a 25 km fiber length, the link noise rate is about 3e3 Hz, as showed in this graph [3.13.](#page-88-1) From this one can conclude that, as expected, using a component that allow to practically consider half link length is a great plus for the network, enabling the connection to reach the same fidelity value for higher LNR values.

In the next two subplots, one can graphically see the values used to do the comparison written above.

Figure 3.13: LNR values comparison for the same fidelity, 50 km case (with no repeater).

Swapping fidelity vs Link Noise Rate - with repeater

Figure 3.14: LNR values comparison for the same fidelity, 25 km case (with repeater).

In the next table the fidelity values for the different swapping scenarios are reported versus various LNR [Hz].

Lastly, Teleportation is considered. Even in this case the same analysis is done and the obtained results are similar to the entanglement and swapping case. This proof the goodness of our work since entanglement and swapping are the two preliminary operations with respect to the teleportation. In other words, without have properly done that two is impossible to obtain a well done teleportation with reliable fidelity values.

Figure 3.15: Teleportation fidelity vs distance vs LNR without losses.

45e7 45e6 45e5 45e4 45e3 45e3 45e2 45e2 0.258 0.250 0.500

50e7 | 50e6 | 50e5 | 50e4 | 50e3 | 50e3 | 50e2 | 50e2 | 0.255 | 0.250 | 0.500

Finally, a table with the numerical results of entanglement, swapping, and teleportation that were used to plot the previous graphs is provided:

3.4.2 Quantum network operations fidelity vs distance and LNR (L.E. $=$ True)

In this section the entanglement, swapping and teleport fidelity results versus distance and LNR are reported. Losses are consider, indeed the Loss Enabled (L.E.) parameter is equal to True, assuming a standard single mode fiber with alpha equal to 0.25 [dB/Km]. As already mentioned, this graphs and tables are obtained using a qubit - photons retransmission mechanism to overcome the losses. The considered distances are: 10m , 100m, 1km, 10km and 100km.

It is worth to mention that, due to the retrasnmissions, the fidelities values obtained in this graphs are practically the same of the previous one, underlining the importance of being able to send a qubit - photon - when others are lost due to link losses.

Figure 3.16: Entanglement fidelity vs distance vs LNR with losses.

Figure 3.17: Swapping fidelity vs distance vs LNR with losses.

Figure 3.18: Teleportation fidelity vs distance vs LNR with losses.

For the sake of completeness a table reporting all the numerical values used to plot the graphs is inserted.

3.4.2.1 Average retransmissions number versus distance

In this section the number of retransmissions experienced by the network are presented. It is worth to read carefully the section part at which this link brings [3.9](#page-80-0) in which the retransmission mechanism is explained along side a scheme that explain graphically connections and ports.

In the following table the results are reported and clearly shows that for longer distances the network experience more retransmissions. This is obtained, considering a SMF with and alpha of 0.25 and enabling losses. Furthermore, the values in the table are the result of an averaging process over 10 different iterations for each distance, changing each time the random seed used for the simulation. A bar plot is also inserted thanks to which one can graphically appreciate the concept.

Number of Retransmissions vs Distance

Figure 3.19: Bar plot showing retranmissions number versus distance for nodes B1 and B2.

These results highlight the importance of implementing a quantum retransmission mechanism in quantum networks to overcome losses and noisy links. Without this kind of solutions the qubit - flying photons - transmission can not be done.

3.4.3 Fidelity vs Memory idle noise

In this section entanglement fidelity and swapping fidelity versus memory idle noise results are presented. Before starting to investigate this precise impact, it is worth to mention that in our implementation the memory idle noise use the same noise model of the link noise rate (LNR), that is to say the Depolar noise model, as mentioned in the dedicated section. As one can imagine, considering noisy memories leads to a decreasing goodness of the quantum operations that are analyzed. Figure [3.20](#page-97-0) show exactly this trend.

Entanglement Fidelity vs Memory Idle Noise

Figure 3.20: Swapping and entanglement Fidelity vs memory idle model.

The following table reports the obtained numerical values used to plot the upper-mentioned graphs.

3.5 Conclusions and future works

Taking everything in to account, this thesis work contributes to the development of quantum network studies and simulation. In the next years these fields will gain more and more importance in the science community thanks also to the development and improvements of quantum hardware components like quantum repeaters, single-photon sources, and high-precision quantum detectors. Such a goal will pave the way to a wide spread implementation of quantum networks all around the world.

Focusing on this thesis work, we simulated a network capable to distribute entanglement and swapping and able to perform teleportation, even in presence of losses and noise. Coding various protocols, noise models, and decoherence effects into the communication links and memories, we analyzed the impact of these factors on network performance also thanks to a quantum retransmission mechanism which allows to handle qubits lost over the links. Our findings underline the relevant influence of noise and decoherence on qubit fidelity and the success rates of quantum operations such as entanglement swapping and quantum teleportation. The simulations quantified the level of entanglement fidelity degradation as function of noise in the quantum channel, which can severely affect the reliability of quantum communication and teleportation. Such a result underlines the importance of developing robust error correction and mitigation techniques to preserve quantum information over long distances, enabling the word spread implementation of quantum networks.

Our simulator can be extended to networks with more nodes, integrated with advanced quantum protocols, allowing for more comprehensive studies of network behaviors under various conditions. The software can also be modified to account for more realistic and noisy quantum gates and memories. For instance, one can investigate how decoherence affect qubits that need to be stored in memories for longer time intervals due to retransmission processes. Future works can also explore the possibility to scale such kind of network for a bigger number of users, or can be more focused on increase the level of realism of the quantum hardware of the node memories. Addressing these areas, future research can significantly advance the field of quantum networking, paving the way for the widespread implementation of quantum networks around the world, thus leading to more likely implementation of a global quantum internet. Such an achievement would revolutionize fields like secure communication, distributed quantum computing, and high-precision sensing, and being of interest for many state and non - state actors, like government and agencies.

In summary, the work in thesis provides a valuable insights on the behavior of quantum networks both in ideal and realistic conditions and highlights the critical factors that influence their performance, like decoherence and losses.

Bibliography

- [1] https://www.submarinecablemap.com/. submarinecablemap, 2024.
- [2] Shashwat Kumar (Ph.D). https://newslab.iith.ac.in/research/mec.html. NeWS lab, 2020.
- [3] https://imrmedia.in/multi-domain operations/. https://imrmedia.in/multi-domain-operations/. https://imrmedia.in/multi-domain-operations/, 2022.
- [4] Michal Krelina. Quantum warfare definitions overview challenges. Faculty of Nuclear Sciences and Physical Engineering, pages –, 2021.
- [5] Rafi Letzter. China quantum key network the largest ever is officially online. Physical Review Letters, 2018.
- [6] outreach darpa. Manta ray uuv prototype completes in water testing. darpa web site, 2024.
- [7] US space force wikipedia. Ground based electro optical deep space surveillance. US space force site - wikipedia site, 2020.
- [8] Xinglei Dou Lei Liu. A new qubits mapping mechanism for multi-programming quantum computing. ResearchGate, 2020.
- [9] M. Hajdušek and R. Van Meter. Quantum communications online lecture notes. MITPress NewMath, 2023.
- [10] Angela Sara Cacciapuoti, Francesco Saverio Cataliotti Marcello Caleffi, and Giuseppe Bianchi Stefano Gherardini, Francesco Tafuri. Quantum internet: Networking challenges in distributed quantum computing. quantum physics, 2018.
- [11] gaudino roberto. https www optcom polito it people full professor roberto gaudino. polito web site - slide materials, 2023.
- [12] www.rfwireless world.com. External versus internal modulation. www.rfwireless-world.com, 2019.
- [13] Liang Shu Kun Xu Zhenming Yu, Zhiquan Wan. Loss weight adaptive multi-task learning based optical performance monitor for multiple parameters estimation. ReserachGate, 2019.
- [14] Patrick Dobbins. Single-mode vs. multimode fiber. Applications Engineering and Field Engineering for AFL.
- [15] Michael A. Nielsen Isaac L. Chuang. Quantum computation and quantum information. cambridge university press, 2010.
- [16] Agenzia per la cybersicurezza nazionale. Acn crittografia quantistica quantum safe. Agenzia per la cybersicurezza nazionale, 2024.
- [17] Bechmann-Pasquinucci H. Briegel Scarani, V. The security of practical quantum key distribution. $Reviews of Modern physics, 2009$.
- [18] et al. Yin, J. Satellite-based quantum key distribution. Nature, pages 343–348, 2017.
- [19] et al. Zhang, L. Review on quantum key distribution and its applications in secure communication. IEEE, 2020.
- [20] et al. Pan, J.-W. Experimental demonstration of high-speed quantum key distribution over a 2000 km optical fiber network. Nature Photonics, pages 218–223, 2021.
- [21] et al. Zhang, C. Machine learning-enhanced quantum key distribution: Theory and experiment. Nature Communications, 2022.
- [22] X. et al. Wang. Quantum key distribution and its future development. Journal of Physics, 2018.
- [23] et al. Ladd, T. D. Quantum computers. Nature, pages 45–43, 2010.
- [24] et al. Van Meter, R. Quantum networks. Nature Reviews Physics, pages 366–380, 2018.
- [25] Brennan Deveraux. Loitering munitions in ukraine and beyond. Texas national security review, pages –, 2022.
- [26] Italian MOD. defence approach to mdos. DEFENCE GENERAL STAFF, pages 22,44, 2022.
- [27] Gianluca Zanella. Usa israel cyber attack. inside over, 2024.
- [28] Brassard G. Crépeau C. Jozsa R. Peres A. Wootters W. K. Bennett, C. H. Teleporting an unknown quantum state .. NPhysical Review Letters, 1993.
- [29] Ji-Gang Ren et al. Ground to satellite quantum teleportation. Quantum Physics, 2017.
- [30] Emil Marinov. pure-mixed-and-entangled-quantum-states.
- [31] netsquid documentation. docs.netsquid.org.
- [32] Yiqing Zhou Cyprien Lambert E. Miles Stoudenmire Xavier Waintal Thomas Ayral, Thibaud Louvet. Density-matrix renormalization group algorithm for simulating quantum circuits with a finite fidelity. PRX QUANTUM, 2023.
- [33] J. Fan and A. Migdall. Generation of cross-polarized photon pairs in a microstructure fiber with frequency-conjugate laser pump pulses. NIST, 2005.
- [34] Fattal D. Vučković J. Solomon G. S. Yamamoto Santori, C. Indistinguishable photons from a single photon device. nature, 2002.
- [35] Dirk Englund3 Igor Aharonovich1, 2* and 2 Milos Toth1. Solid state single photon emitters. nature photonics, 2016.
- [36] H. J. Kimble. The quantum internet. *quantum physics*, 2008.
- [37] NIKITA VOSTROSABLIN. Pulsed quantum optomechanical interfaces. -, 2020.
- [38] J. I. Cirac and ohters. quantum state transfer and entanglement distribution among distant nodesin a quantum networ. physical review letters, 1997.
- [39] Mikael Afzelius Christoph Clausen, F´elix Bussi'eres and Nicolas Gisin. Quantum storage of polarization qubits in birefringent and anisotropically absorbing materials. quantum physics, 2012.
- [40] Darrick E. Chang1. Quantum nonlinear optics photon by photon. nature photonics, 2014.
- [41] shi-hai wei and ohter. $T_{\text{owards}_real_world_quantum_networks_{ar}eview.lpr -jurnal.org, 2022.}$
- [42] Mustafa Gundogan et al. A case for quantum memories in space. Decadal Survey on Biological and Physical Sciences Research in Space 2023-2032, 2021.
- [43] Matthias Meister ID David Edward Bruschi Andreas Wolfgang Schell ID Janik Wolters ID Jan-Michael Mol ID, Luisa Esguerra and Lisa Wörner ID 1. Quantum memories for fundamental science in space. quantum-ph, 2023.
- [44] 3 Kriti Tyagi 1 2 3 * Sangita Tripathy, 1 and Pratiksha Pratap. A comprehensive study of various superconductors for superconducting nanowire single photon detectors applications. iScience, 2024.
- [45] Nicolas Gisin and Rob Thew. Quantum communication. *quantum physics*, 2008.
- [46] NS SW producer. Netsquid_package. Netsquidwebsite, 2017.