2.1. Battery modelling

In the following, we derive the model for the electric battery, in which the power delivered by the battery (P_b) and the current state of charge of the battery (SOC) are the model input and output, respectively.

The SOC is defined as the ratio between the current battery charge Q_b and the nominal battery capacity Q_{nom}, i.e.,

$$SOC \equiv \xi \doteq \frac{Q_b}{Q_{nom}} \in [0,1]. \tag{1}$$

Differentiating both sides, we can relate ξ with the battery current I_b as follows

$$\dot{\xi} = \begin{cases} -\frac{1}{\eta_c}\frac{I_b}{Q_{nom}} & \text{if } I_b > 0 \text{ (discharge)} \\ -\eta_c \frac{I_b}{Q_{nom}} & \text{if } I_b < 0 \text{ (charge)}, \end{cases} \tag{2}$$

where $\eta_c \in (0,1)$ is named Coulombic efficiency and quantifies the fraction of current that is lost during charge/discharge [20].

Remark 1. Note that (2) has a step in $I_b = 0$, making it non-Lipschitz continuous (since the derivative is infinite). Therefore, we approximate the step with a sufficiently steep sigmoid function, so to let (2) be locally Lipschitz continuous in 0.

The battery is modelled as an ideal voltage source $V_{oc,b}$ with a series output resistance $R_{o,b}$. Hence, we have that $P_b = V_{oc,b}I_b - R_{o,b}I_b^2$, which can be solved for I_b, yielding

$$I_b = \frac{V_{oc,b} - \sqrt{V_{oc,b}^2 - 4R_{o,b}P_b}}{2R_{o,b}}. \tag{3}$$

Note that (3) expresses the battery current I_b (and, thus, $\dot{\xi}$) as function of the output power P_b.

The battery can be realized as a series of N_b smaller battery cells. In this case, $V_{oc,b}$ and $R_{o,b}$ can be expressed as $V_{oc,b} = N_b V_{oc,b,s}$ and $R_{o,b} = N_b R_{o,b,s}$, where $V_{oc,b,s}$ and $R_{o,b,s}$ are the open-circuit voltage and series output resistance of each single battery cell.

It is worth noticing that, in real batteries, $V_{oc,b}$ and $R_{o,b}$ are function of the battery SOC. In this paper, these relationships are obtained by means of piecewise polynomial fitting, using experimental data on $V_{oc,b,s}(\xi)$ and $R_{o,b,s}(\xi)$ taken from [20], and inserted in (3).

The model forces an upper limit on the battery power, i.e., $P_b \leq V_{oc,b}(\xi)^2/4R_{o,b}(\xi) \equiv \overline{P}_b^{\text{model}}(\xi)$. Real battery specifications provide as well upper-lower limits on P_b. Then, P_b has admissible values defined by

$$\underline{P}_b \equiv \underline{P}_b^{\text{spec.}} \leq P_b \leq \min\left(\overline{P}_b^{\text{model}}(\xi), \overline{P}_b^{\text{spec.}}\right) \equiv \overline{P}_b. \tag{4}$$

Battery	
N_b	200
Q_{nom}	60 Ah
$V_{oc,b,s}(\xi)$, $R_{o,b,s}(\xi)$	[20]
η_c	0.95
η_b	0.97
$[\underline{P}_b^{\text{spec.}}, \overline{P}_b^{\text{spec.}}]$	$[-289.24, 289.24]$ kW

[20] S. Onori, L. Serrao, G. Rizzoni, Hybrid Electric Vehicles: Energy Management Strategies, 1st Edition, Springer, 2016.