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THESIS

Design of External Gears for Hydraulic Pumps

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Abstract

External gear pumps are widely utilized in various applications due to their simplicity, robustness, and versatility. This thesis presents the development of a MATLAB application designed to optimize the dimensions of external gear pump gears under specific dimensional constraints, with a primary focus on achieving the required pump displacement. The application efficiently calculates essential gear parameters, such as module, number of teeth, and center distance. It also evaluates the flow ripple, a critical factor influencing pump behavior and noise levels.

The optimization process incorporates user-defined constraints, ensuring that the resulting gear design meets the operational requirements, in case the input parameters cannot achieve a solution within the constraints, it offers the closest possible solution. Additionally, the application can be used to automatically generate drawings of the optimized gears. These drawings can be visualized on the app or be exported in formats compatible with leading CAD software, facilitating seamless integration into the design and manufacturing workflow. Also, they have great utility for simulation in CFD software.

The proposed MATLAB application permits rapid iteration in simulation environments instead of experimentally obtaining the pump characteristics. This will be an extremely useful tool for research and rapid prototyping work, saving time and resources.

Chapter 1

Introduction

1. Introduction

Fluid power technology, encompassing both hydraulics and pneumatics, plays a crucial role in modern industrial and mobile applications due to its unmatched power density, precise control, and reliability. Hydraulic systems, in particular, offer significant advantages in terms of power transmission, especially in applications requiring high force and torque in a small package. From construction machinery to aerospace systems, fluid power is integral to the operation of a wide range of equipment. As industries continue to evolve, the demand for more efficient, compact, and quiet hydraulic systems is growing, pushing the boundaries of fluid power technology.

Often wrongly believed to be a dirty technology, fluid power will be an integral part of future technology despite the advance of electrification. While this latter technology offers significant advantages in low power applications, it will not replace hydraulic power in most applications for the foreseeable future. These technologies will work in combination with each other, increasing the energy efficiency and reducing emissions and waste.

In the context of hydraulic systems, external gear pumps are a fundamental component, converting shaft power into hydraulic power, valued for their simplicity, durability, and ability to operate under a wide range of conditions. Despite their widespread use, the design of external gear pumps presents several challenges, particularly in optimizing the gear dimensions to meet specific operational and dimensional requirements while minimizing undesired effects such as flow ripple. This phenomenon, associated with pressure fluctuations in the hydraulic system, can lead to increased noise, vibration, and fatigue, thereby affecting the overall performance and longevity of the hydraulic system.

The future of fluid power technology lies in the continuous improvement of components like external gear pumps, where optimization and automation play key roles. Advances in computational tools have opened new avenues for enhancing the design process, allowing engineers to achieve higher levels of precision and efficiency. This thesis addresses the need for an advanced tool that can optimize the design of external gear pump gears.

The work is based on a previous application built on the AMESim environment, while this was a groundbreaking tool in its time, the application resulted highly challenging to update and debug. Therefore it was decided to implement a similar tool in MATLAB, which offers several advantages. Among them, the possibility to easily upgrade and troubleshoot the software as well as the ability to run the program as a standalone application without having additional software beyond the Windows operating system. Additional features that were not present in the preceding app were developed and incorporated into this version.

A MATLAB application has been developed to automate the optimization of gear dimensions for external gear pumps. The application is composed of three primary components.

The first component calculates the gear parameters based on user-defined constraints, such as the

maximum width of the pump and the maximum diameter of the gears, while targeting a specific pump displacement. Additional constraints, including fixed inter-axial distance, fixed external diameter of the gears, or both simultaneously, can also be applied. This process results in a set of gear parameters that can be further utilized within the app. Since both gears in an external gear pump are identical, only one set of parameters needs to be calculated.

The second component evaluates the kinematic flow rate of the external gear pump. This process includes calculating the angular derivatives of the delivery, suction, and trapped volumes of the pump, which are essential for simulating the unit using a lumped parameters approach. The method relies on knowing the lengths of the vector rays between the centers of the gears and the contact points. From the geometric parameters the pump displacement and the kinematic flow ripple index are derived and plotted.

Finally, the third module utilizes the gear parameters calculated in the first step, or any user-provided parameters, to generate the gear designs. It checks for impossible geometries and teeth interference. Additionally, it exports the gear designs as a set of coordinates compatible with leading CAD software. The gears can also be visualized in 2D directly within the application, eliminating the need for external software.

With recent advancements in the strength and precision of parts produced through additive manufacturing techniques like 3D printing and sintering, it is now feasible to create gears without the traditional need for a rack cutter. These modern processes allow for simpler geometry at the root radius of the gears, as clearance for the rack is no longer necessary. This application supports the use of gears produced by both traditional rack cutting and alternative methods such as additive manufacturing across all modules.

As the demand for more efficient and quieter hydraulic systems continues to rise, tools like the one presented in this thesis will become increasingly important. By automating the design process and ensuring optimal gear dimensions, this MATLAB application contributes to the advancement of fluid power technology, positioning it to meet the challenges and opportunities of the future.

2. Geometry of the external gears

2.1 Gear description

External gear pumps consist of two inter meshing straight cut gears housed within a casing. Both gears are identical and rotate in opposite directions. One gear, known as the driving gear, is powered by an external prime mover, while the other, called the driven gear, rotates in response to the driving gear. As the gears rotate, fluid is trapped between the teeth and the casing, creating a vacuum at the pump inlet. This vacuum draws fluid into the pump, where it is then carried around the perimeter of the gears to the outlet. The meshing action of the gear teeth forces the fluid out of the outlet under pressure.

2.2 Nomenclature and input parameters

The following basic parameters of the gear set are the minimum necessary to unequivocally define the dimensions of each gear and their center distance.

- N_d : number of teeth of each gear
- m_0 : module
- θ_0 : nominal pressure angle
- l_k : operating center distance
- h_{sn} : normal play between the teeth
- b : axial thickness of the gears
- R_e : external radii
- h_f : tip chamfer height
- A : tool addendum coefficient
- D : tool dedendum coefficient
- h_r : tool fillet height coefficient

2.3 Additional derived geometric parameters

From the aforementioned parameters, it is possible to derive additional characteristics and parameters useful to draw and characterize the gear set.

- Pitch Radius:

$$R_{p0} = \frac{N_d m_0}{2} \quad (1.1)$$

- Nominal Center Distance:

$$I = 2R_{p0} \quad (1.2)$$

- Working Pressure Angle:

$$\theta_k = \arccos \left(\frac{I}{I_k} \cos \theta_0 \right) \quad (1.3)$$

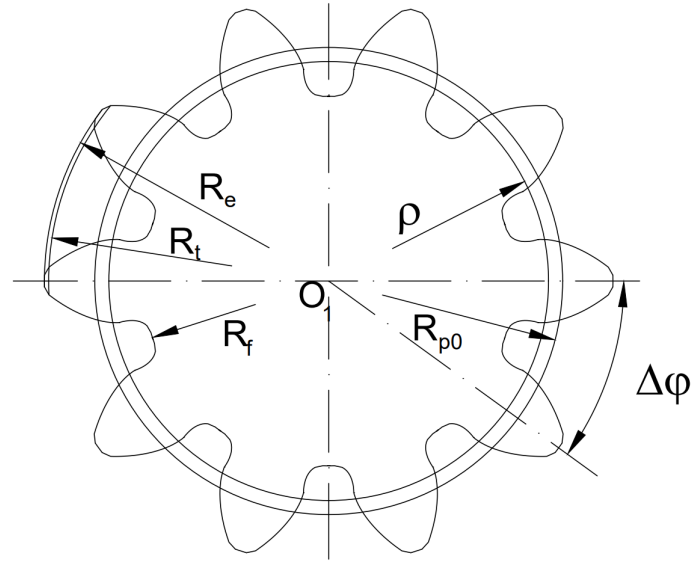


Figure 1.1: Spur gear geometry and main parameters

- Working Module:

$$m = m_0 \frac{\cos \theta_0}{\cos \theta_k} \quad (1.4)$$

- Working Pitch Radius:

$$R_{pl} = R_{p0} \frac{\cos \theta_0}{\cos \theta_k} \quad (1.5)$$

- Base Radius:

$$\rho = R_{p0} \cos \theta_0 \quad (1.6)$$

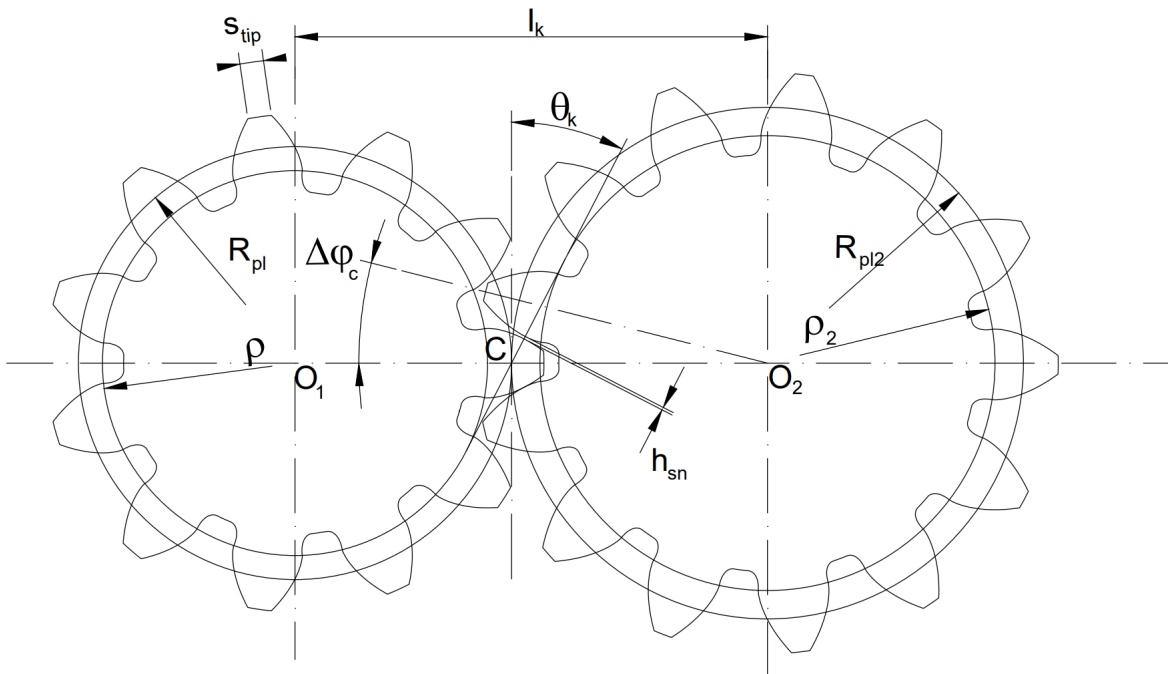


Figure 1.2: Engagement between two gears: geometry and main parameters

- Root Radii:

$$R_f = R_{p0} - Am_0 + x_g m_0 \quad (1.7)$$

Where x_g is the correction factor with play which will be defined and calculated later

- Length of the Contact Segment:

$$TT' = 2\rho \tan \theta_k \quad (1.8)$$

- Circular Pitch:

$$p = \pi m_0 \frac{\cos \theta_0}{\cos \theta_k} \quad (1.9)$$

- Tooth Thickness at the Pitch Circle:

$$s_{R_{p0}} = \frac{\pi m_0}{2} + 2x_g m_0 \tan \theta_0 \quad (1.10)$$

- Angular Pitch:

$$\Delta\varphi = \frac{2\pi}{N_d} \quad (1.11)$$

- Play at the Working Pitch:

$$h_s = \frac{h_{sn}}{\cos \theta_k} \quad (1.12)$$

- Angular Phasing of the Driven Gear with Respect to the Driving Gear:

$$\Delta\varphi_d = \frac{\Delta\varphi}{2} - \frac{h_s}{2R_{p1}} \quad (1.13)$$

3. Typical Dimensions of Involute Profile Gears

3.1 Gear Profile

The gears found in external gear pumps are involute gears. This means the teeth are shaped according to an involute of a circle, specifically, the base circle of radius ρ . This is the most common gear profile used in modern mechanical systems because of its ability to maintain smooth transmission of power and torque between meshing gears.

Involute gears can handle slight variations in center distance between gears without significantly affecting performance. The contact between two teeth occurs along a straight line, inclined to the gear center line. The angle of this contact line is called the pressure angle, which is typically 20° or 25° .

$$\text{inv}(\theta) = \tan(\theta) - \theta \quad (1.14)$$

It is possible to calculate the tooth thickness at any point of the involute profile using the following relation:

$$\frac{s^*}{R^*} = \frac{s_0}{R_{p0}} + 2\zeta \quad (1.15)$$

where the quantity ζ is defined by the equation:

$$\zeta = \varphi_0 - \varphi = \text{inv}(\theta_0) - \text{inv}(\theta^*) \quad (1.16)$$

The angle θ_0 is the nominal pressure angle ; the angle θ^* , based on simple geometric considerations, can be derived as a function of the radius for which the thickness is required:

$$\theta^* = \arccos\left(\frac{\rho}{R^*}\right) \quad (1.17)$$

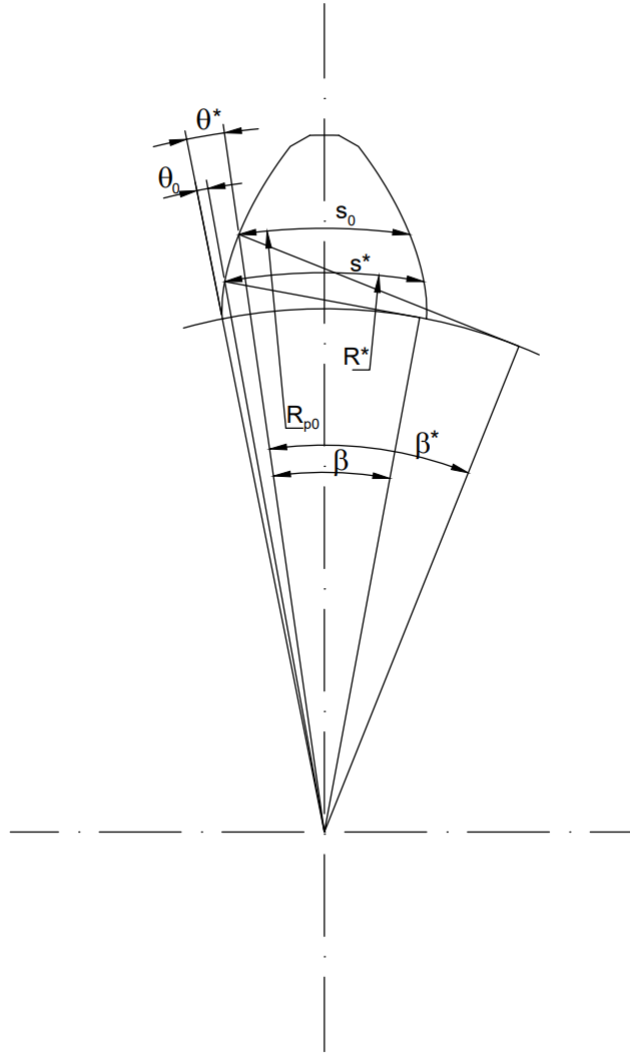


Figure 1.3: Tooth thickness along its length

From the previous relations, the final expression for the calculation of tooth thickness is obtained:

$$s^* = R^* \left[\frac{s_0}{R_{p0}} + 2(\text{inv}(\theta_0) - \text{inv}(\theta^*)) \right] \quad (1.18)$$

3.2 Calculation of Profile Correction Factor

Profile shifting in gears refers to the intentional displacement of the gear tooth profile along the radial direction, either outward or inward, relative to the pitch circle. The correction factor x_g must be calculated.

When the correction factor is greater than 0, we have positive profile shifting. In this case, the gear teeth are cut larger than normal, moving the tool radially outward. This increases the strength of the teeth and reduces the risk of interference.

Conversely, when negative profile shifting ($x_g < 0$), the tool is moved inwards. The teeth became smaller than normal and interference is more likely to happen.

To calculate x_g , first we define a condition of no play within the teeth. Meaning the pitch of the gear is equal to double the tooth thickness.

$$s_1 + s_2 = p \quad (1.19)$$

where s and p are respectively the circular thickness and pitch of the two inter meshing gears. From equation eq.(1.15), the tooth thicknesses can be substituted at a generic radius:

$$s_1 + s_2 = R_1 \left(\frac{s_0}{R_{p0}} + 2\zeta_1 \right) + R_2 \left(\frac{s_0}{R_{p0}} + 2\zeta_2 \right) = p \quad (1.20)$$

Since both gears are identical and because we are on the working pitch radius of the gear, equation eq.(1.20) is modified as follows:

$$s_1 + s_2 = \frac{N_d p}{\pi} \left(\frac{s_0}{R_{p0}} + 2\zeta \right) = p \quad (1.21)$$

The angle θ used to calculate the quantity ζ is the working pressure angle θ_k ; therefore it results that:

$$\zeta = \text{inv}(\theta_k) - \text{inv}(\theta_0) \quad (1.22)$$

knowing that:

$$s_{R_{p0}} = \frac{p_0}{2} + 2xm_0 \tan \theta_0 \quad (1.23)$$

with the appropriate substitutions and simplifying:

$$x_g = \frac{N_d}{2 \tan \theta_0} [\text{inv}(\theta_k) - \text{inv}(\theta_0)] \quad (1.24)$$

The previous equation started from assuming teeth engagement without play, if we introduce a play h_s between the teeth we obtain:

$$s_1 + s_2 + h_s = p \quad (1.25)$$

The final result is:

$$x_g = \frac{N_d}{2 \tan \theta_0} \left[\text{inv}(\theta_k) - \text{inv}(\theta_0) - \frac{h_{sn}}{2m_0 N_d \cos \theta_0} \right] \quad (1.26)$$

3.3 Calculation of the Length of the Engagement Segment

The involute of the base circle does not constitute the entire profile of the gear. It has a radius of starting of the involute profile a radius at the end. There are R_i and R_t respectively.

The meaning and calculation of this radii will be discussed in the next chapter.

To calculate the length of the engagement segment, it is necessary to know the unused parts of the contact segment; these can be functions of the radius of the start of the involute of the gear or of the external radius.

Based on fig. 1.4, the following relations are derived:

$$\delta = \max \left(TT - \sqrt{R_t^2 - \rho^2}, \sqrt{R_i^2 - \rho^2} \right) \quad (1.27)$$

Thus, the length of the engagement segment is defined by:

$$l = TT - 2\delta \quad (1.28)$$

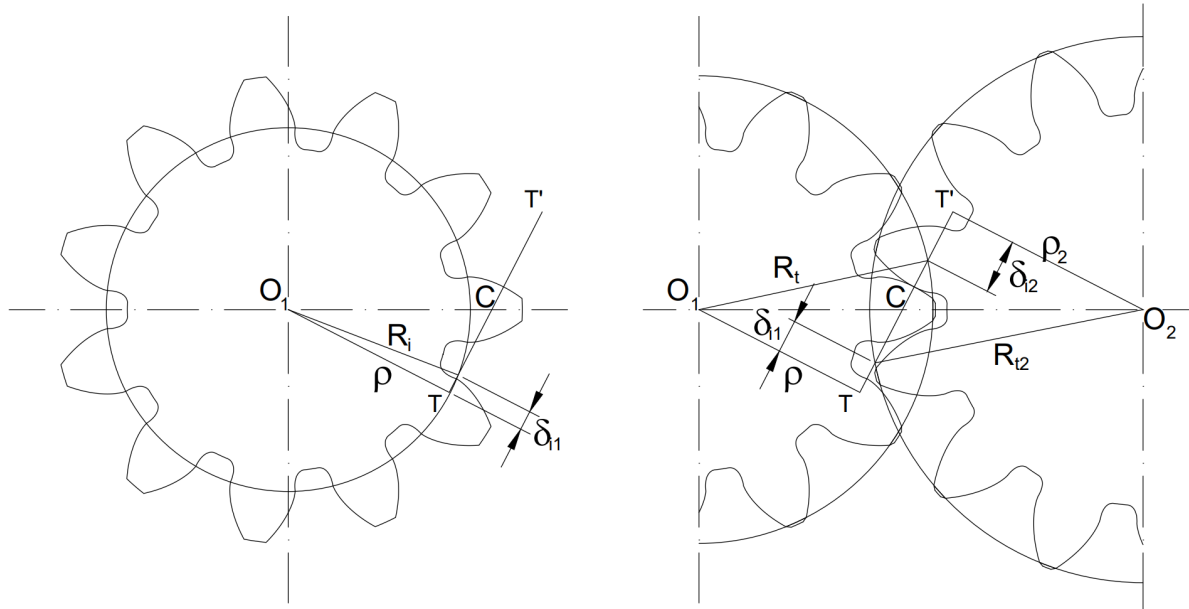


Figure 1.4: Engagement and contact segments

3.4 Contact ratio

The contact ratio is a dimensionless parameter that expresses the number of teeth simultaneously in engagement during one rotation of the gear equal to one angular pitch.

Thus, using the length of the engagement segment, we obtain:

$$\varepsilon = \frac{l}{\rho \Delta \varphi} \quad (1.29)$$

To ensure a smooth operation, ε must be greater than one, although it is optimal that $\varepsilon > 1.1$.

Chapter 2

Automatic Design of Profiles for External Gears

1. Manufacturing Processes of Gears

The gears used for this type of pumps can be produced in various ways; for example, using cutting tools or by additive manufacturing. A key objective of this thesis is to automatically generate two types of gear profiles:

- In this method, a tool rack is employed for cutting the gear. The root circle is connected to the involute segment by a curve, which is shaped directly by the cutting tool.
- Additive Manufacturing (e.g., Sintering): In this approach, the involute section begins at the base radius, and a straight radial segment connects it to the root circle. Additionally, an optional fillet may be introduced at the junction between the root and the involute to enhance the transition.

The graphical comparison between the two profiles is shown in fig. 2.1.

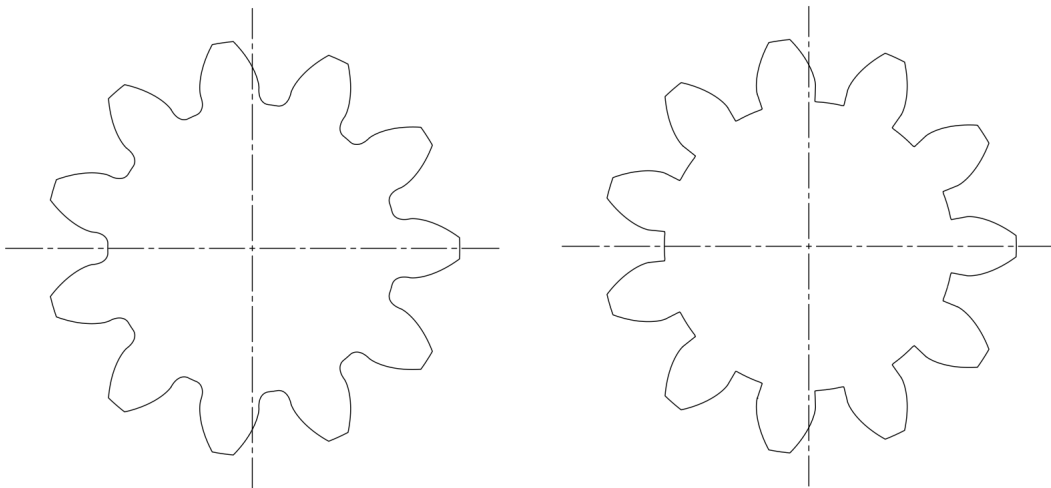


Figure 2.1: Comparison of profiles

For automatic profile generation, a text file (.txt) must be created containing two column vectors that represent the sequence of the desired profile coordinates in a Cartesian reference system. The resulting gear profile is then constructed by connecting successive points with straight line segments.

The length of these segments, and consequently the accuracy of the profile, can be controlled by the user. Specifically, the user defines the minimum number of points (N_p) that should be calculated

over the angular pitch of the gear as input data, which implicitly determines the precision of the profile.

In the following paragraphs, a variable φ will always appear; it represents the revolution of the gear during the cutting process and is used to calculate the profile coordinates in the various sections. The angular increment $d\varphi$ is therefore imposed by the equation:

$$d\varphi = \frac{\Delta\varphi}{N_p} \quad (2.1)$$

2. Profile Obtained by Cutting

This type of profile can be selected during the data entry phase by choosing the "Rack cutter" mode from the drop-down menu corresponding to the "Manufacturing Process" parameter.

2.1 Tool Geometry

The reference tool chosen to determine the profile geometries is a rack: during the assumed cutting, the motions considered are:

- rotation with angular velocity ω of the gear to be cut
- translation of the tool with velocity v

The ratio between the velocity of the tool and that of the gear is precisely the pitch radius R_{p0} :

$$\frac{v}{\omega} = R_{p0} = \frac{N_d m_0}{2} \quad (2.2)$$

Therefore, the bases of the motion are the circumference of radius R_{p0} and a line parallel to the tool reference line but shifted by a quantity equal to $x_g m_0$.

From fig. 2.2, it can be seen that the rack is composed of two straight segments: one inclined at angle θ_0 and the other perpendicular to the tooth axis. These two segments are connected by a radius:

$$q = \frac{h_{rac}}{1 - \sin \theta_0} \quad (2.3)$$

The straight segments are responsible for generating the involute and the root circle of the gear, while the connection between them generates the fillet.

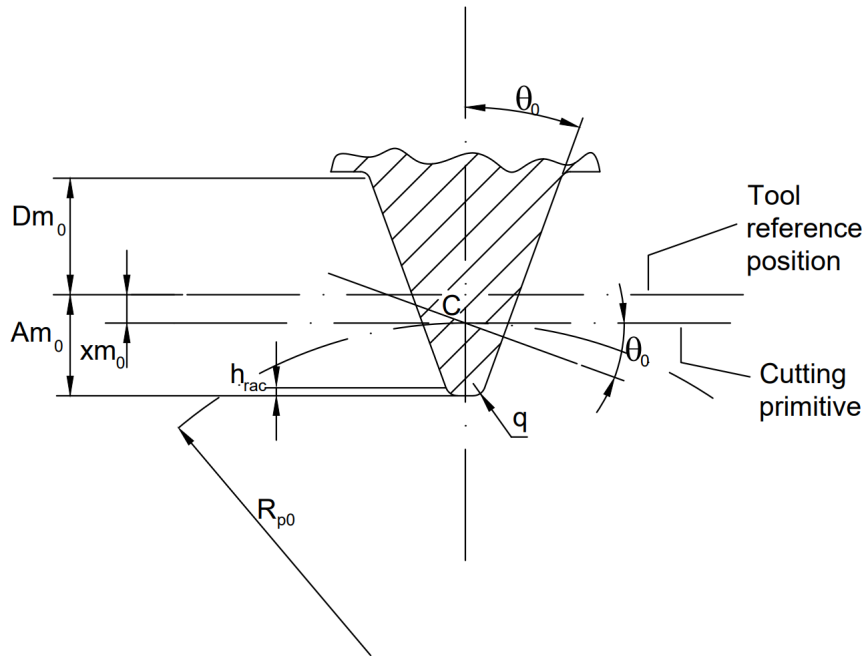


Figure 2.2: Geometry of the rack cutter

The straight segment dedicated to the creation of the involute is the one inclined at angle θ_0 , meaning it defines the nominal pressure angle.

The geometry of the tool is identified by coefficients that correspond to the ratio between the dimension they refer to and the module; the user will define the proportions of the rack using these coefficients.

$$a = Am_0 \quad ; \quad d = Dm_0 \quad ; \quad h_{rac} = h_r m_0 \quad (2.4)$$

2.2 Reference Systems and Rotation Matrices

The profile drawing is developed only on a plane as only straight cut gears are of concern in this case, but for the general case, spatial coordinates will be needed; moreover, there will be three reference systems:

- fixed reference system f
- reference system fixed to the gear 2
- reference system fixed to the cutting tool l

To find the coordinates of the points that describe the profile, two different approaches can be taken:

- represent the locus of contact points between gear and gear or between tool and gear in the fixed reference system and then rotate, with a coordinate transformation, the points obtained in the gear reference system.
- represent the profile of the tool in its own reference system and rotate the coordinates obtained in the gear reference system.

Using the second approach, the coordinates of a generic point must be expressed with a column vector of four elements; the last of which will be taken as unity:

$$r_m = [x_m \quad y_m \quad z_m \quad 1]^T \quad (2.5)$$

From fig. 2.3, it can be seen that a rotation φ of the gear corresponds to a translations of the tool; the two quantities are related by the following relation:

$$s = R_{p0}\varphi \quad (2.6)$$

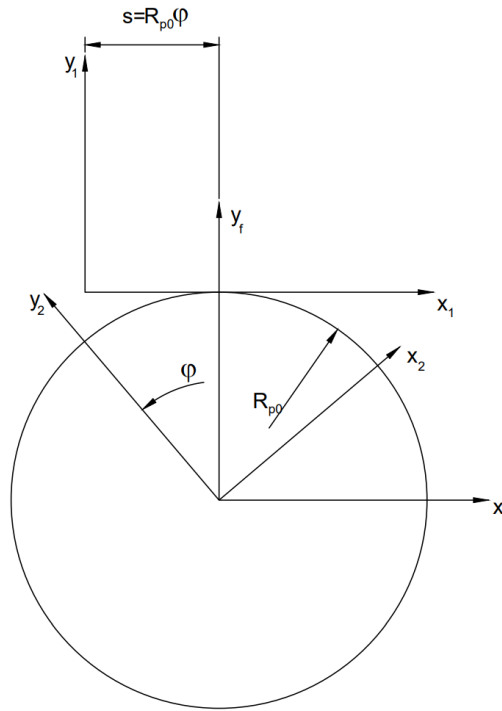


Figure 2.3: Reference system

To switch from one reference system to another, a matrix rotation approach is used; the notation adopted for these matrices is of the type M_{f1} , which indicates the coordinate transformation matrix from the reference system l to the fixed reference system f .

Below are the matrices for a reference change from system 2 to system 1; to obtain the opposite reference change, it will be sufficient to invert the matrices.

- Rotation matrix from 2 to f :

$$M_{f2} = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 & 0 \\ \sin \varphi & \cos \varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.7)$$

- Translation matrix from f to l :

$$M_{lf} = \begin{bmatrix} 1 & 0 & 0 & \varphi\rho \\ 0 & 1 & 0 & -\rho \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.8)$$

- Rotation-translation matrix from 2 to 1:

$$M_{12} = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 & \varphi \rho \\ \sin \varphi & \cos \varphi & 0 & -\rho \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.9)$$

3. Gear Profile Equations

For simplicity of calculation and computational time, it was chosen to draw only half of an angular pitch of the gear, mirror it with respect to the tooth axis, and repeat all the found points, rotating them, for a number of times equal to the number of teeth of the gear.

A tooth, produced by a cutting tool, consists of four sections:

- an arc of a circle of radius R_e
- an involute section of a circle from radius R_e to radius R_i
- a connecting section between the involute and the root circle generated by the fillet or tool's corner
- an arc of a circle of radius R_f

3.1 Tip Circle Equations

Using the center of the gear as a reference point and the Cartesian coordinate system it is possible to determine the location of each point of each section of the profile. Once the external radius R_e is known, the equations for the tip section of the tooth are:

$$x = R_e \sin \varphi \quad ; \quad y = R_e \cos \varphi \quad (2.10)$$

3.2 Involute Section Equations

With reference to fig. 2.4, for a parametric representation of the involute section, the following equations are used:

$$\alpha = \tan \beta - \beta \quad ; \quad R = \frac{\rho}{\cos \beta} \quad (2.11)$$

For a representation using implicit functions, however, it is necessary to use a function or a system capable of defining the tool profile coordinates. These points must be related to the rotation φ of the gear being generated, that is, with the angle by which the reference system 2 rotates during the cutting process.

To find the desired system of equations, some kinematic considerations are necessary; knowing that the profiles, in the involute section, are conjugated:

- The tangents to the two profiles (gear-gear or gear-tool) at the contact point are coincident.
- The normals to the profiles at the contact point are coincident and pass through the center of instantaneous rotation.

Given any point in the plane belonging to a generic curve, it is possible to derive the equation of the tangent and the normal to the curve at that point.

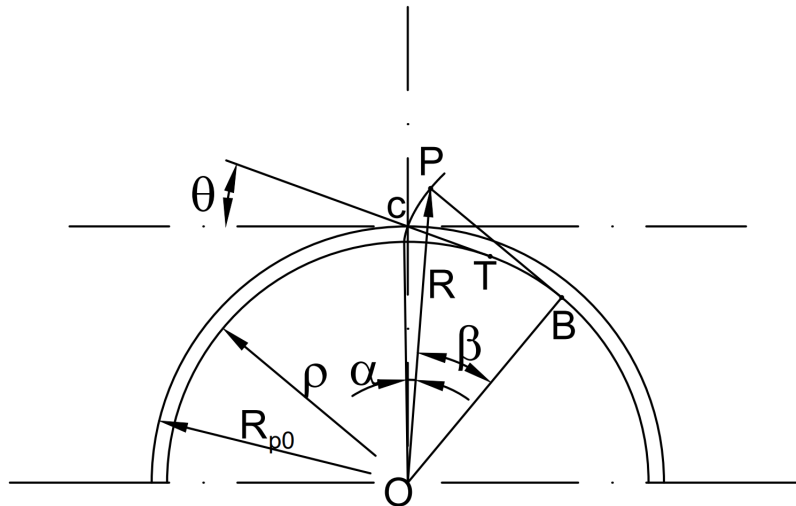


Figure 2.4: Involute of a circle

For the tangent, we can write:

$$\frac{X - x}{T_x} = \frac{Y - y}{T_y} \quad (2.12)$$

where:

- x, y are the coordinates of the point in question
- X, Y are the coordinates of point D
- T_x, T_y are the components of the unit vectors i and j that identify the tangent in the plane reference system.

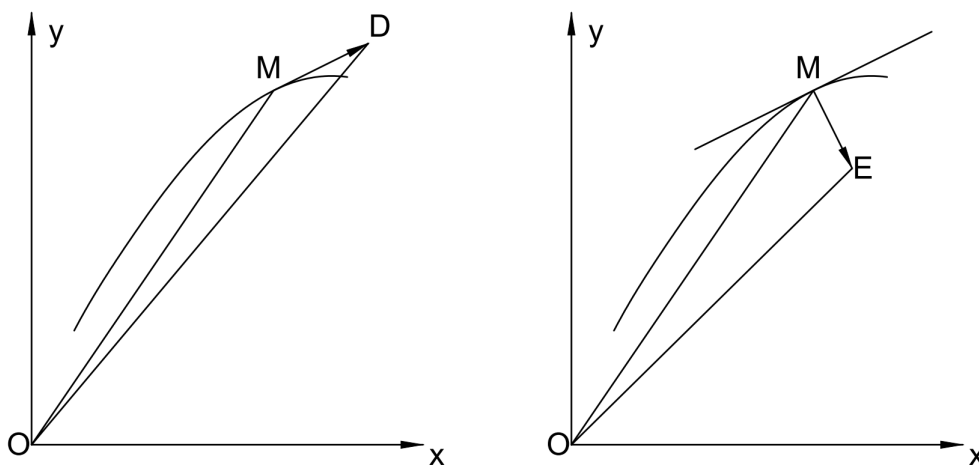


Figure 2.5: Normal and tangent versors of a curve

For the normal, we can write:

$$\frac{X - x}{N_x} = \frac{Y - y}{N_y} \quad (2.13)$$

where:

- x, y are the coordinates of the point in question
- X, Y are the coordinates of point E
- N_x, N_y are the components of the unit vectors i and j that identify the normal in the plane reference system.

By imposing that the normal to the two profiles at the contact point passes through the center of instantaneous rotation, a meshing equation is obtained, i.e., an expression capable of relating the coordinates of the contact point of the tool and gear as a function of the parameter φ .

Therefore, in our case, equation eq.(2.13) will be used, where:

- x, y are the coordinates of the contact point in the tool reference system
- X, Y are the coordinates of the center of instantaneous rotation in the tool reference system
- N_x, N_y are the components of the unit vectors i and j that identify the normal to the two profiles in the plane reference system.

Based on fig. 2.6, the components of the unit vectors i and j in the tool reference system for the tangent and the normal to the tool profile can be derived:

$$T_x = \sin \theta_0 \quad ; \quad T_y = \cos \theta_0 \quad (2.14)$$

Therefore, the components referred to the normal are:

$$N_x = \cos \theta_0 \quad ; \quad N_y = -\sin \theta_0 \quad (2.15)$$

From fig. 2.7, it can be seen how the contact point between the gear and the tool moves along the flank of the latter within two limit positions M^* and M , which correspond to the distances $-u_1$ and u_2 relative to the origin.

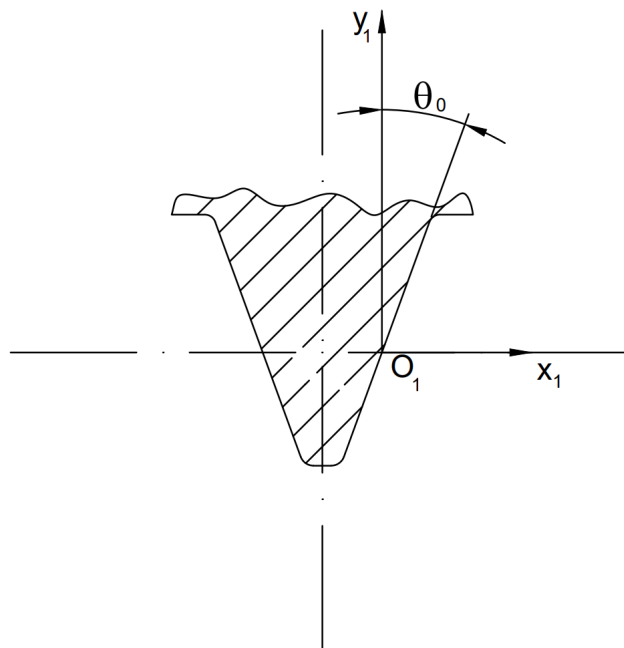


Figure 2.6: Components of a normal and tangent versors

By introducing an auxiliary coordinate u , which varies from $-u_1$ to u_2 , it is possible to express the coordinates of the contact point in the tool's own reference system:

$$x_1 = u \sin \theta_0 \quad ; \quad y_1 = u \cos \theta_0 \quad (2.16)$$

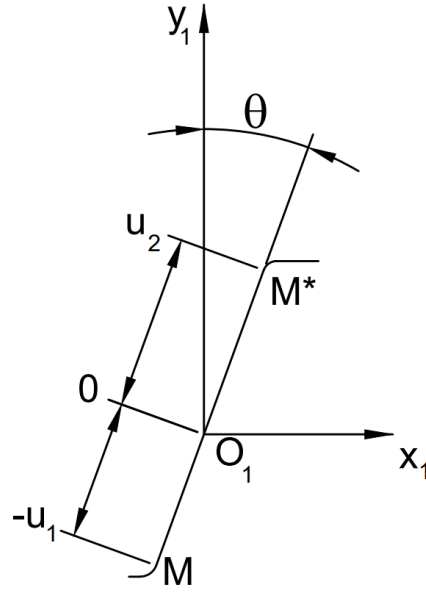


Figure 2.7: Contact points of the involute profile

From fig. 2.3, it is possible to derive the coordinates of the center of instantaneous rotation in the tool reference system:

$$X = R_{p0} \varphi \quad ; \quad Y = 0 \quad (2.17)$$

Substituting the expressions from eq.(2.15) to eq.(2.17) into eq.(2.13), the desired meshing equation is obtained:

$$f(u, \varphi) = \frac{R_{p0} \varphi - u \sin \theta_0}{\cos \theta_0} - \frac{u \cos \theta_0}{\sin \theta_0} = 0 \quad (2.18)$$

which simplifies to:

$$f(u, \varphi) = u - R_{p0} \varphi \sin \theta_0 = 0 \quad (2.19)$$

At this point, it is necessary to find the tooth shape equations: it is sufficient to transfer the coordinates of the contact point, already found for the tool, to the gear reference system; to do this, the coordinate transformation matrices derived in the previous paragraph can be used. Therefore, the expression is:

$$r_1 = M_{2f} M_{f1} r_2 = M_{21} r_2 \quad (2.20)$$

The rotation-translation matrix M_{12} is invertible, and its inverse M_{21} is:

$$M_{21} = \begin{bmatrix} \cos \varphi & \sin \varphi & R_{p0}(\sin \varphi - \varphi \cos \varphi) \\ -\sin \varphi & \cos \varphi & R_{p0}(\cos \varphi + \varphi \sin \varphi) \\ 0 & 0 & 1 \end{bmatrix} \quad (2.21)$$

Based on what has been previously stated, the coordinate transformation matrix should be a 4x4, but since the problem is planar, the third row and third column, which refer to the calculation of the z coordinate, have been preemptively eliminated.

Thus, two equations are obtained that, solved with respect to x_2 and y_2 , are able to provide us with the involute profile referred to the gear's base circle:

$$x_2 = u \sin(\varphi + \alpha) + R_{p0}(\sin \varphi - \varphi \cos \varphi) \quad (2.22)$$

$$y_2 = u \cos(\varphi + \alpha) + R_{p0}(\cos \varphi + \varphi \sin \varphi) \quad (2.23)$$

The two derived expressions are still functions of both the parameter φ and the parameter u ; to eliminate this double dependency, we use the meshing equation, eq.(2.32). The equations then transform into:

$$x_2 = R_{p0} \sin \varphi - R_{p0} \varphi \cos \varphi \cos(\alpha + \varphi) \quad (2.24)$$

$$y_2 = R_{p0} \cos \varphi - R_{p0} \varphi \cos \alpha \sin(\alpha + \varphi) \quad (2.25)$$

Fillet Equations

The fillet of the gear is generated by the corner of the rack tool or by the connection between the two straight segments. Below, only the treatment referring to the second case is proposed since the tool with a sharp corner generates a fillet that can be derived by simplifying the more complex case.

Conceptually, the procedure does not differ much from that used for the involute section; this time, however, the contact point does not move along the straight flank of the tool but along the fillet from point M_1 to point M_2 . The auxiliary variable u is thus replaced by an auxiliary variable α which, once the fillet radius is known, identifies the coordinates of the contact point in the tool's own reference system.

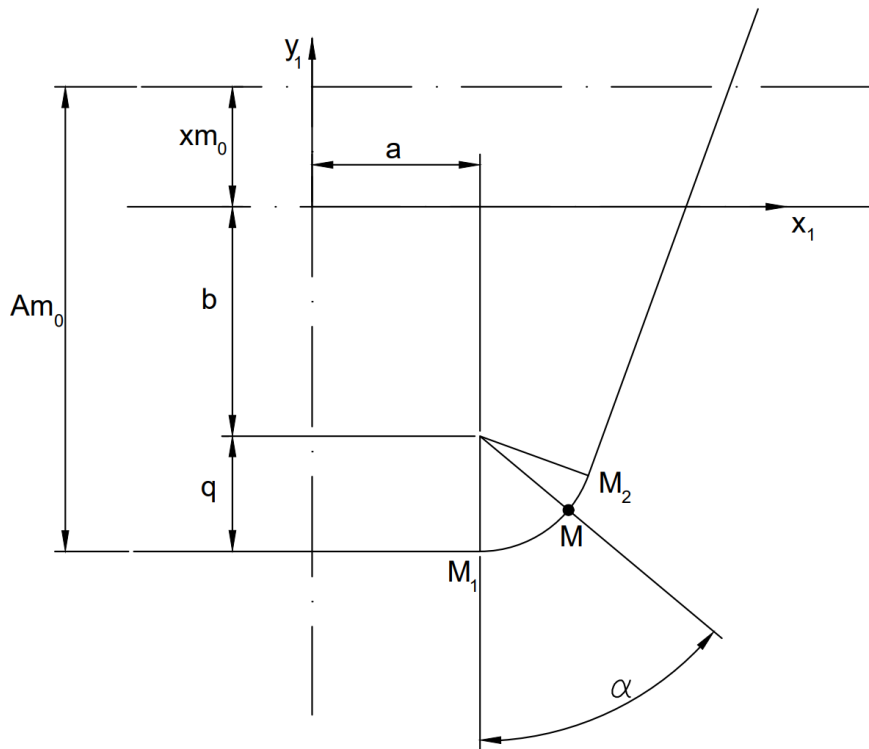


Figure 2.8: Contact points of the fillet

the relationships expressing these coordinates can be derived:

$$x_1 = a + q \sin \alpha \quad ; \quad y_1 = -b - q \cos \alpha \quad (2.26)$$

where a and b are two constants that depend on the geometry of the tool and are given by:

$$b = Am_0 - q - x_g m_0 \quad (2.27)$$

$$a = \frac{\pi}{4} m_0 - \frac{q}{\cos \theta_0} - (b + x_g m_0) \tan \theta_0 \quad (2.28)$$

As for the calculation of the involute section, the coordinates of the center of instantaneous rotation are:

$$X = R_{p0} \varphi \quad ; \quad Y = 0 \quad (2.29)$$

The components of the unit vectors referred to the profile normal are:

$$N_x = q \sin \alpha \quad ; \quad N_y = -q \cos \alpha \quad (2.30)$$

In this case, the meshing equation is:

$$\frac{R_{p0} \varphi - a - q \sin \alpha}{q \sin \alpha} + \frac{b + q \cos \alpha}{q \cos \alpha} = 0 \quad (2.31)$$

which simplifies to:

$$R_{p0} \varphi - a + b \tan \alpha = 0 \quad (2.32)$$

Thanks to a coordinate transformation very similar to that for the involute section, the equations of the fillet in the gear reference system are obtained:

$$x_2 = q \sin(\alpha - \varphi) + a \cos \varphi - b \sin \varphi + R_{p0}(\sin \varphi - \varphi \cos \varphi) \quad (2.33)$$

$$y_2 = -q \cos(\alpha - \varphi) - a \sin \varphi - b \cos \varphi + R_{p0}(\cos \varphi + \varphi \sin \varphi) \quad (2.34)$$

This time, however, the meshing equation does not allow an easy substitution of the auxiliary variable α , so the value of the latter must be calculated as a function of the variable φ .

The resulting system from the previous considerations is:

$$x_2 = q \sin(\alpha - \varphi) + a \cos \varphi - b \sin \varphi + R_{p0}(\sin \varphi - \varphi \cos \varphi) \quad (2.35)$$

$$y_2 = -q \cos(\alpha - \varphi) - a \sin \varphi - b \cos \varphi + R_{p0}(\cos \varphi + \varphi \sin \varphi) \quad (2.36)$$

$$R_{p0} \varphi - a + b \tan(\alpha) = 0 \quad (2.37)$$

3.3 Root Circle Equations

For the calculation of the coordinates, it is necessary to know in advance the value of the root radius; this, based on the geometric characteristics of the tool and the operating conditions of the two gears, is calculated using:

$$R_f = R_{p0} - Am_0 + x_g m_0 \quad (2.38)$$

The coordinates are calculated in a manner entirely analogous to the external or tip circle section:

$$x = R_f \sin \varphi \quad ; \quad y = R_f \cos \varphi \quad (2.39)$$

4. Phasing and Existence Field of the Equations of the Various Sections

The profile equations obtained in the previous section have defined existence fields and, moreover, the coordinates need to be rotated to correctly compose the various sections of the tooth.

In this section, relationships are sought that allow defining the rotation angles necessary to correctly phase the various sections and the existence limits imposed on the parameter φ .

4.1 Rotation of Various Sections

Using a rotation matrix very similar to M_{f2} , the equation used to rotate the coordinates of various sections is:

The final relationship to be applied is:

$$\begin{bmatrix} X_r \\ Y_r \end{bmatrix} = \begin{bmatrix} \cos \chi & -\sin \chi \\ \sin \chi & \cos \chi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (2.40)$$

where:

- x, y are the coordinates of the profile still to be rotated
- X_r, Y_r are the already rotated coordinates
- χ is the angle by which the various sections need to be rotated and will be calculated from time to time in the following paragraphs.

4.2 Tip Circle

The drawn profile may or may not have a chamfer on the tooth tip: in the case of zero-height chamfer, the tip thickness of the tooth is calculated and once the outer radius of the gear is known, it is easy to calculate the existence limits of this section:

$$s_{Re} = R_e \left[\frac{s_{prim}}{R_{p0}} + 2(\text{inv}(\theta_0) - \text{inv}(\theta_{Re})) \right] \quad (2.41)$$

where:

$$\theta_{Re} = \cos^{-1} \left(\frac{\rho}{R_e} \right) \quad (2.42)$$

$$s_{prim} = \frac{\pi m_0}{2} - 2x_g m_0 \tan(\theta_0) \quad (2.43)$$

The angle for which the equations eq.(2.10) are valid results is:

$$\beta = \frac{s_{Re}}{2R_e} \quad (2.44)$$

and therefore the parameter φ must vary in the range:

$$-\frac{\beta}{2} < \varphi < 0 \quad (2.45)$$

If, on the other hand, there is a chamfer on the tooth tip, the existence field of the equations eq.(2.10) is:

$$-\alpha'_1 < \varphi < 0 \quad (2.46)$$

This section does not require any phasing.

4.3 Involute

To determine the existence limits on the parameter φ , it is enough to impose that the involute coordinates belong to a circumference of radius R_e or radius R_i depending on whether the upper or lower limit of the involute section is desired.

It is therefore necessary to solve the following equation with respect to the parameter φ :

$$X^2 + Y^2 = W^2 \quad (2.47)$$

where X and Y are the coordinates of the involute already derived earlier eq.(2.24) and eq.(2.25), while W is replaced with R_e or R_i .

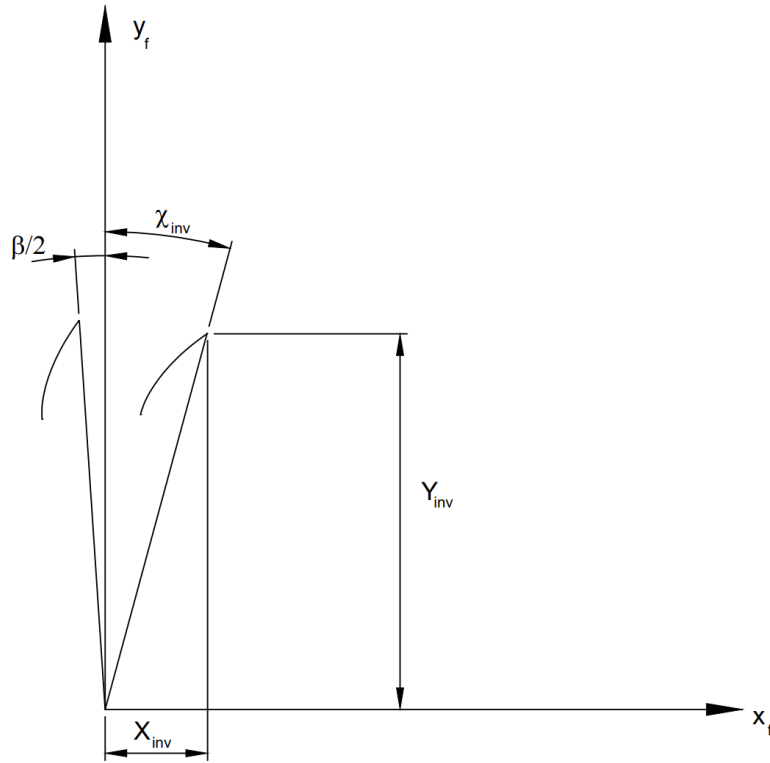


Figure 2.9: Phasing of the involute segment

Solving, we find:

$$\varphi = \frac{1}{2R_{p0}((\sin \alpha)^2 - 1)} [2R_{p0} \cos \alpha \sin \alpha - 2\sqrt{(R_{p0} \cos \alpha \sin \alpha)^2 + (R_{p0} \sin \alpha)^2 - (W \sin \alpha)^2 - R_{p0}^2 + W^2}] \quad (2.48)$$

At this point, it is necessary to find an angle χ_{inv} , which is obtained by substituting the value of φ found with eq.(2.48) (for radius R_e) in equations eq.(2.24) and eq.(2.25) and thus:

$$\chi_{inv} = \arctan \left(\frac{X_{inv}(\varphi_{R_i})}{Y_{inv}(\varphi_{R_i})} \right) \quad (2.49)$$

By rotating by this angle, summed with $\frac{\beta}{2}$, the coordinates of the involute derived earlier are correctly phased.

4.4 Root Circle

This section, like the tip circle, does not require any phasing but only existence limits; being the last section, the lower limit is represented by half the angular pitch of the gear. The upper limit χ_{root} is found, instead, considering that the root circle is generated only by the straight section of the tool parallel to the direction of motion.

$$-\frac{\Delta\varphi}{2} < \varphi < \chi_{root} \quad (2.50)$$

where:

$$\chi_{root} = \frac{a}{R_{p0}} \quad (2.51)$$

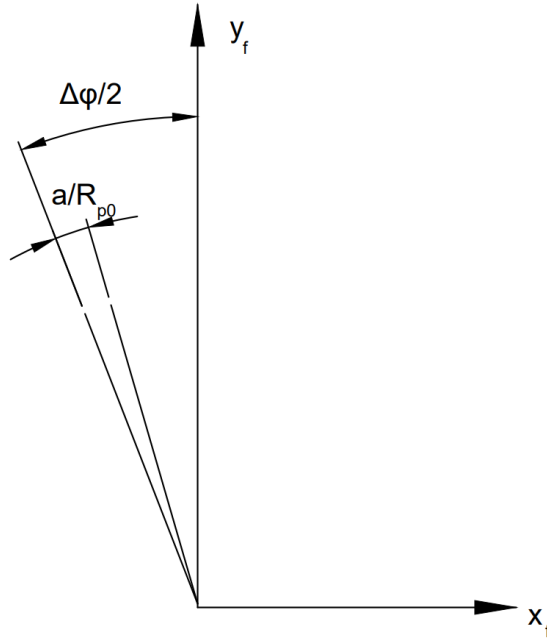


Figure 2.10: Phasing of the root segment

4.5 Fillet

Regarding the existence limits on the parameter φ , the approach is different; in fact, not having been able to eliminate a parameter thanks to the meshing equation, a treatment similar to that used for the involute section is not feasible.

It is known from previous paragraphs that the section in question is generated by the fillet between the two straight flanks of the tool; as already mentioned, the contact point between the tool and the gear moves between points M_1 and M_2 fig. 2.8, consequently, it is possible to deduce the existence field of the parameter φ , necessary for the calculation of the fillet, deriving the existence limits of the auxiliary variable α .

Therefore, knowing that:

$$0 < \alpha < \frac{\pi}{2} - \theta_0 \quad (2.52)$$

from the meshing equation eq.(2.32), we get:

$$\frac{a}{R_{p0}} < \varphi < \frac{b - a \tan\left(\frac{\pi}{2} - \theta_0\right)}{R_{p0}} \quad (2.53)$$

To find the correct phasing, the coordinates found with equations eq.(2.35), eq.(2.36) and eq.(2.37) must be rotated by an angle equal to χ_{root} . This latter is obtained by substituting the lower limit value of φ in the aforementioned equations to then derive the corresponding angle, in a manner completely analogous to what has been done for the involute; considering that the root circle is generated by the straight tool section parallel to the reference line, the sought rotation angle φ corresponds to a tool translation equal to the quantity a (equation eq.(2.28)).

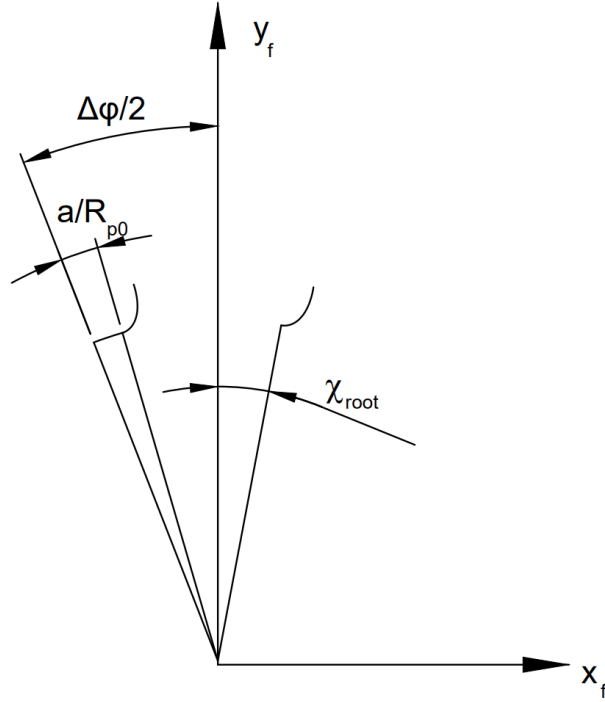


Figure 2.11: Phasing of the fillet segment

$$\varphi^* = \frac{a}{R_{p0}} \quad (2.54)$$

And by substituting, we find:

$$\chi_{base} = \frac{X_{base}(\varphi^*)}{Y_{base}(\varphi^*)} \quad (2.55)$$

So the angle by which the coordinates eq.(2.35) and eq.(2.36) need to be rotated to obtain the correct phasing is:

$$\chi_{tot} = \chi_{base} + \frac{\Delta\varphi}{2} - \frac{a}{R_{p0}} \quad (2.56)$$

The upper limit for calculating the fillet can only be obtained if the radius at which the involute section and fillet intersect is known; this is called the conjugation radius or also the initial involute radius (R_i) and is calculated in the next subsection.

4.6 Calculation of the Starting of Involute Radius

Due to cutting, the tooth profile may be hollowed at the base with consequent structural weakening; if this occurs, it is in a condition of undercutting.

To avoid the problem and strengthen the profile, instead, one must fall into a condition where the tooth thickness is always increasing from the tip radius to the root radius (condition of non-undercutting).

To obtain information on the type of cut being addressed, it is necessary to find a discriminating condition between the two cases.

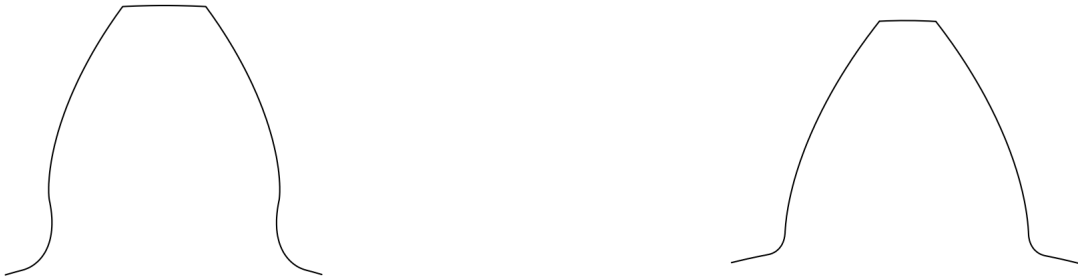


Figure 2.12: Undercutting of teeth

Conditions of Non-Undercutting

Knowing that the involute section is generated by the straight section belonging to the flank of the tool, the intersection of the flank itself with the contact line in cutting conditions is imposed; the tool, therefore, must translate until the last point of its straight flank meets the contact segment. The circumference for which pure rolling occurs is that of radius R_{p0} , but the tool reference line must be translated by an amount equal to $x_m m_0$.

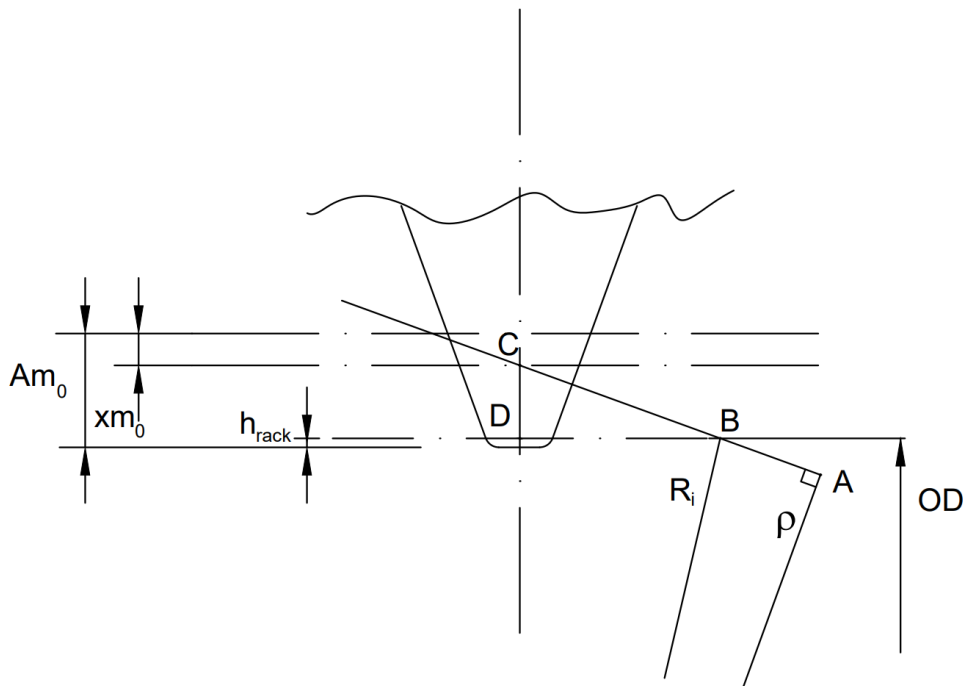


Figure 2.13: Calculation of the start of involute radius

Based on simple geometric considerations, the following relationships are obtained:

$$h_{rac} = h_r m_0 \quad (2.57)$$

$$q = \frac{h_{rac}}{(1 - \sin \theta_0)} \quad (2.58)$$

$$\tan \alpha_G = \frac{GC - CL}{r_b} = \frac{GC - CL}{r_b} \quad (2.59)$$

and therefore:

$$\tan \alpha_G = \tan \theta_0 - \frac{4(Am_0 - x_g m_0 - h_{rac})}{Nm_0 \sin 2\theta_0} \quad (2.60)$$

The non-undercutting condition is obtained when $\alpha_G \geq 0$; if this occurs, the radius R_i can be easily found with:

$$R_i = \frac{\rho}{\cos \alpha_G} = \frac{Nm_0 \cos \theta_0}{2 \cos \alpha_G} \quad (2.61)$$

Iterative procedure

The automatic drawing of profiles allows obtaining the radius R_i as the intersection between the involute section and the root fillet of the gear.

Operationally, the two sections are drawn (using the previously calculated equations) in a restricted area of the profile, to increase calculation precision and maintain a decent execution speed of the program.

The following allowed deviation offer a compromise between calculation speed and precision:

- Angular step for equation calculation:

$$\Delta\varphi = \frac{\Delta\varphi}{5000}$$

- Maximum deviation between root fillet and involute:

$$|X_{root} - X_{inv}| < 10^{-7} \text{ m} \quad |Y_{root} - Y_{inv}| < 10^{-7} \text{ m}$$

Once the previous conditions are met, it is possible to obtain the upper limit of the existence field of the root fillet and calculate the radius R_i with the Pythagorean theorem:

$$R_i = \sqrt{X_{root}^2 + Y_{root}^2} \quad (2.62)$$

5. Profile Obtained by Sintering

To comply with the transmission conditions typical of involute gears, the profile part that comes into contact with the other gear can be designed as in the case of a profile obtained by cutting. The same consideration is valid for the chamfer of the tip and for the root circle.

The only section of the profile that differs from the previous case is the root fillet; this time, in fact, the gear is not created by enveloping all the positions occupied by the tool's rack during meshing and therefore it can enjoy a bit more freedom.

Consequently, the aim is to maximize the useful engagement section for meshing by setting the starting radius of the involute coinciding with the base radius; moreover, the involute section is connected to the root circle through a radial segment.

Thus, all the relationships derived before remain valid, except for the existence limits of the bottom circle which become:

$$-\frac{\Delta\varphi}{2} < \varphi < \chi_{bottom} \quad (2.63)$$

where this time χ_{bottom} is derived through the tooth thickness at the base radius:

$$s_p = \frac{s_{pitch}}{R_{p0}} + 2inv(\theta_0) \quad (2.64)$$

$$\chi_{bottom} = \frac{s_p}{2\rho} \quad (2.65)$$

The final user of the program can select this type of profile during data entry by setting the "Other" mode.

For this type of gears, it is possible to insert a chamfer on the tooth tip. Also, a chamfer in the junction of the involute and the bottom circle can be introduced.

5.1 Chamfer Equations

If a chamfer is applied to the tooth tip as shown in fig. 2.14, angles α_1'' and α_1' must be calculated, which respectively delimit the section of the circumference of radius R_t and the involute profile. Additionally, the radius R_t must be calculated.

The angle α_1'' can be calculated similarly to what was done with eq.(2.41):

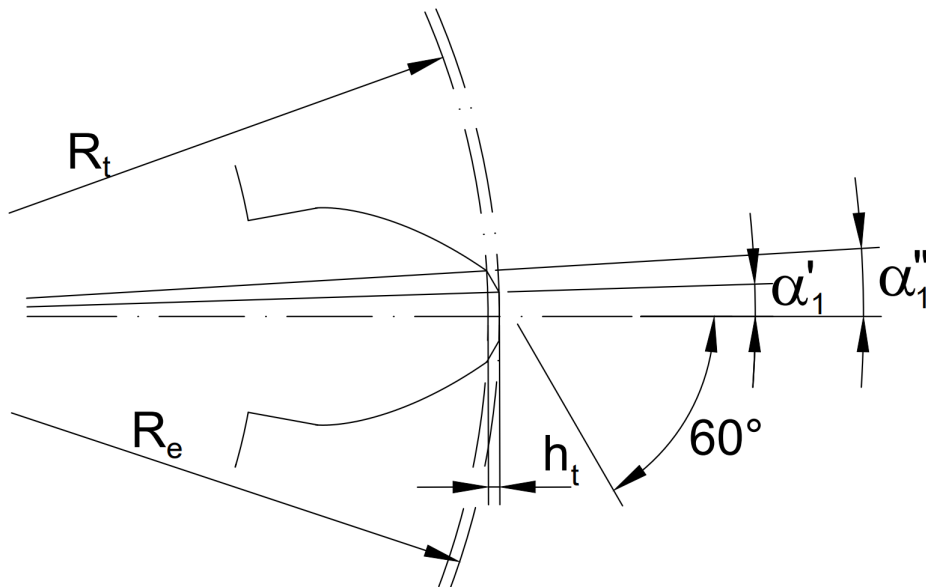


Figure 2.14: Tooth tip chamfer

$$\alpha_1'' = \frac{s_{Rt}}{2R_t} \quad (2.66)$$

where:

$$s_{R_t} = R_t \left[\frac{s_{R_{p0}}}{R_{p0}} + 2 \left(\text{inv}(\theta_0) - \text{inv} \left(\cos^{-1} \left(\frac{\rho}{R_t} \right) \right) \right) \right] \quad (2.67)$$

and therefore:

$$\alpha_1'' = \frac{s_{R_{p0}}}{2R_{p0}} + 2 \left(\text{inv}(\theta_0) - \text{inv} \left(\cos^{-1} \left(\frac{\rho}{R_t} \right) \right) \right) \quad (2.68)$$

The length of segment DK is:

$$DK = OH + HK - R_t = R_t \cos \alpha_1'' + \frac{R_t \sin \alpha_1''}{\tan(60^\circ)} - R_e \quad (2.69)$$

where O_1 is the center of the gear.

With some approximation, we obtain α_1' :

$$\alpha_1' = \frac{DB}{R_e} \approx \frac{DK \tan(60^\circ)}{R_e} \quad (2.70)$$

The height of the chamfer h_t is:

$$h_t = R_e \cos \alpha_1' - R_t \cos \alpha_1'' \quad (2.71)$$

Operationally, an initial value for R_t is imposed ($R_t = R_t - dR$ with dR very small) and α_1' and α_1'' are calculated using eq.(2.71) to find h_t . If the value found is less than the imposed value of h_t , then dR is increased, and the process is repeated until the calculated h_t is close to the imposed value.

The chamfer on the tip does not create particular problems for the design of the gears; however, it is necessary to replace the radius R_e with the radius R_t in equation eq.(2.48); additionally, it is not necessary to add the angle $\frac{\beta}{2}$ to the variable χ_{inv} ; it will be sufficient to add the angle α_1'' just calculated.

5.2 Bottom fillet Equations

Optionally, it is possible to include a fillet radius in the junction between the internal radial segment and the fillet radius. The fillet is tangent to both segments and the user chooses the value of this radius.

When this fillet is included the involute and tip segments remain identical, only the bottom circle's existence χ_{bottom} is reduced in phasing angle.

$$\chi_{bottom(fillet)} = \chi_{bottom} - \sin\left(\frac{r}{r+R_f}\right) = \chi_{bottom} - \sin(\beta) \quad (2.72)$$

Where r is the radius of the fillet and R_f is the radius of the bottom circle.

The location of the center of the fillet radius is found by imposing the tangency condition to both segments. It can be proven the location, in the gear reference system is:

$$X_{0_{fillet}} = r \quad ; \quad Y_{0_{fillet}} = \frac{r}{\tan(\beta)} \quad (2.73)$$

An auxiliary variable γ is introduced to define the points that belong to the fillet. The field of existence of this new variable is:

$$0 \leq \gamma \leq \frac{\pi}{2} - \beta \quad (2.74)$$

Finally, in the gear reference system the equations that define the entire fillet are:

$$X_{fillet} = r - r\sin(\gamma) \quad ; \quad Y_{fillet} = \frac{r}{\tan(\beta) - r\sin(\gamma)} \quad (2.75)$$

Chapter 3

Automatic Drawing of Gear Profiles: Software Implementation

1. Introduction

The first module of the software consists of the implementation of the automatic design equations described in the first two chapters. The software is based on the MATLAB app designer.

This chapter details the structure and functionality of the MATLAB application, its user interface, and the underlying algorithms used to design the gears. In addition, it describes how the generated profiles can be exported as CAD files.

2. Input parameters

The "External Gears Drawing" tab is the main user interface for this module. On the left the input parameters are introduced by the user, these are:

- Normal Pressure Angle [deg]
- Number of Teeth
- Tooth Fillet Coefficient
- Module [mm]
- External Radius [mm]
- Addendum Coefficient
- Tooth Tip Chamfer [mm]
- Dedendum Coefficient
- Normal Clearance Between Teeth [mm]
- Manufacturing Process: 'Rack Cutting' or 'Other' can be selected
- Center Distance [mm]
- Number of Points N_p to be drawn per half of each teeth
- Gear Thickness [mm]
- Initial Angle [deg] for displaying the gears in a rotated position

- Internal Radius [mm] in case of sintering
- Root Fillet Radius [mm] optional, in case of sintering
- Include Root Fillet Radius (checkbox)

The checkboxes do not allow non numeric values to be entered and make sure that the values are within realistic boundaries.

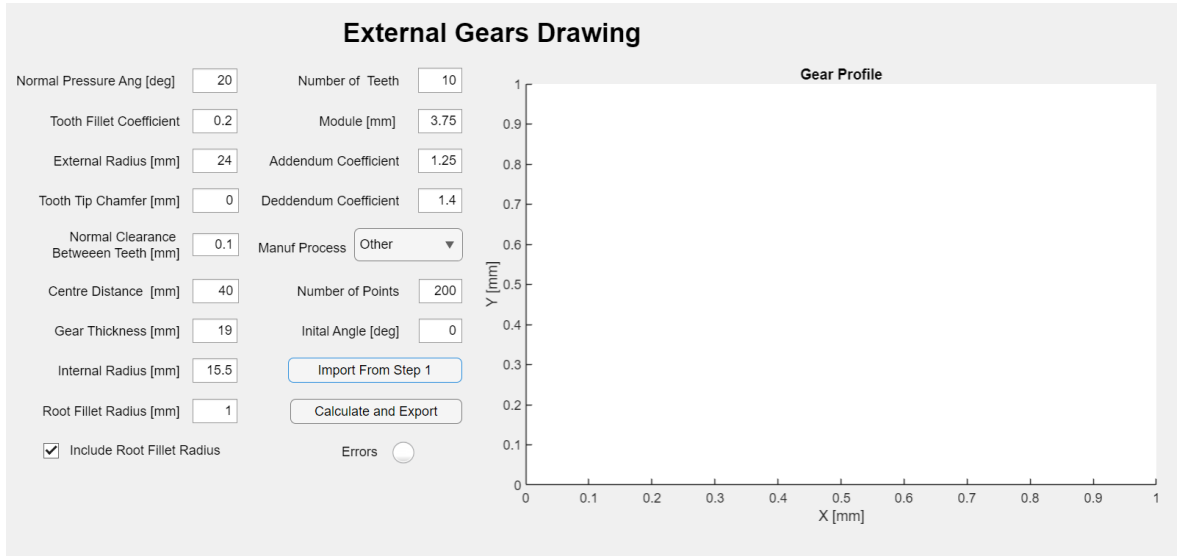


Figure 3.1: User interface of the MATLAB App

When 'Rack Cutting' is selected in the 'Manufacturing Process' drop down, the internal radius, root fillet radius and its check box are disabled because these parameters are not relevant to the creation of a gear with a rack cutter. Also, the tooth tip chamfer box is disabled and its value is set to 0 mm. Additionally, when a solution created by the optimizer module is available, the 'Import From Step 1' button is enabled. It allows the direct import of a complete set of parameters.

3. Calculation

3.1 Input parameters processing

When the 'Calculate' button is pressed, the script starts running. First it imports all the input parameters as variables that will be fed to the algorithm.

All measures are converted to SI units, millimeters are converted to meters and the normal pressure angle, which is entered in degrees, is converted to radians.

Then, the following additional parameters are calculated:

- Operating Pressure Angle θ_k
- Base Pitch t_0
- Pitch Radius R_{p0}
- Normal Centre Distance I
- Working Pitch Radius R_{pl}

- Base Radius R_o
- Angular Pitch $\Delta\varphi$
- Play Between Teeth hs
- Contact Line Length TT
- Max Cutting Module $m0max$
- Min Cutting Module $m0min$

Then, the operating pressure angle is verified to be a real parameter, in case it is not, the calculation is stopped and an error message is displayed.

The correction factor x_g is evaluated with the introduction of the play hs . With the knowledge of this parameter, the thickness of the teeth at the nominal pitch radius and at the working pitch radius are calculated.

In case there is no tooth tip chamfer h_t , the tip radius R_t is equal to the external radius. Contrarily, when a sintering manufacturing process is chosen and h_t is not zero, the process described in sub-section 5.1 of Chapter 2 is used to calculate the tip radius R_t and the phasing angle α'_1 . Subsequently, the thickness of the tooth tip is computed and compared to a minimum accepted value of 1mm.

The minimum number of teeth in the case of rack cutting is verified and it must comply with:

$$Z_{min} = 2 \frac{\sin(\theta)^2}{A - x_g} \quad (3.1)$$

In case of additive manufacturing there is no inferior boundary for the number of teeth.

The bottom radius is computed differently for each manufacturing process. If sintering is used its value is defined by the user, otherwise it is computed as:

$$R_f = R_{p0} - Am_0 + x_g m_0; \quad (3.2)$$

Several parameter value compatibility issues and possible errors are verified. They will receive a thorough explanation in a dedicated chapter.

3.2 Tooth Segment Coordinates Calculation

First, the radius for the start of the involute R_i must be found in case of rack cutting, this is achieved by the process described in section 4.6 of Chapter 2.

Thereafter, the angular increment of φ by which the tooth will be divided in order to save its coordinates in a vector is computed as:

$$I_\varphi = \frac{\Delta\varphi}{Np} \quad (3.3)$$

Then, with the equations presented in Chapter 2, all the fields of existence for every segment of the teeth profile are computed.

Subsequently, two vectors, each with the coordinates in the x and y axes respectively for one half of one tooth, are computed as well.

The points of each segment are stored in the vectors in the following order.

1. Tip or external circle
2. Tip chamfer (in case it exists)
3. Involute profile
4. Root fillet radius (in case it exists)
5. Root circumference

In case an error is detected, the lamp will illuminate with a red color and the description of all the error found can be seen in the "Errors/Warnings" tab.

3.3 Plotting

Once the vector is completed, the entire gear profile is defined. The next step is to mirror the vector to generate the other half of the tooth and then replicate it according to the total number of teeth on the gear.

It is desirable to plot both gears in the same window in order to see them in the working position and visualize their engagement. Since there is a constraint of engagement between them, the driven gear has to be plotted with a rotation angle $\Delta\varphi_c$ respect to the driver gear and an axial distance I_k .

If the number of teeth is even:

$$\Delta\varphi_c = \frac{\Delta\varphi}{2} - \frac{hs}{2R_{pl}} \quad (3.4)$$

Otherwise:

$$\Delta\varphi_c = -\frac{hs}{2R_{pl}} \quad (3.5)$$

Then, each gear is saved into a table containing all its points in the two dimensional plane. In the user plot area of the app, the profile of the gears can be visualized.

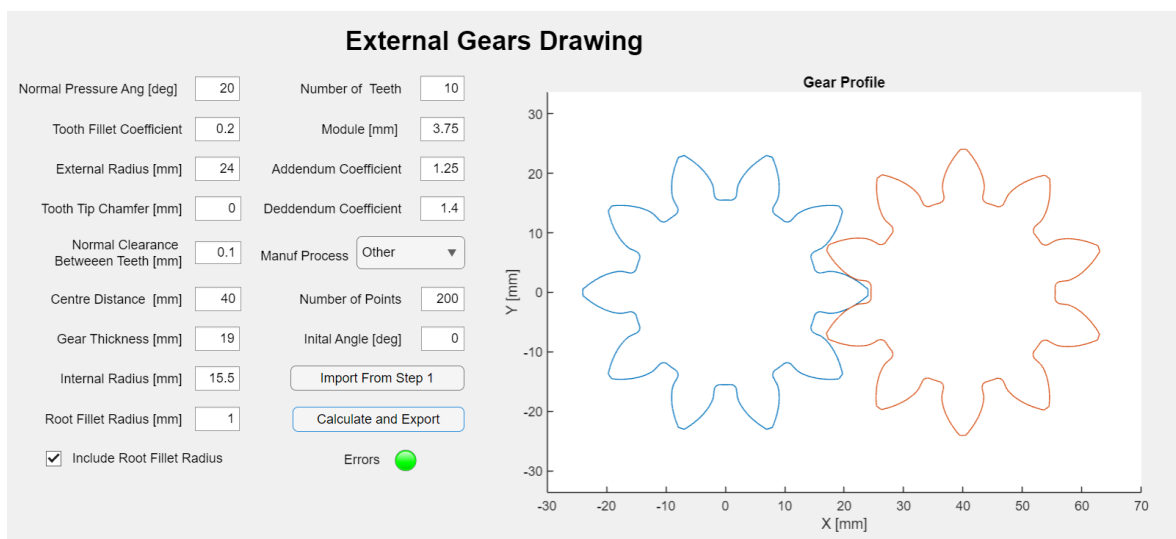


Figure 3.2: Gear profile after drawing without errors

3.4 Exporting of the profiles for CAD software

The table containing the complete profile is saved into a .txt file complying with the format required to import it into AutoCAD. The user only needs to input the gear axial thickness into the CAD software in order to define the three dimensional shape of the gear.

Chapter 4

Automatic Design of Gears

1. Introduction

The first module of the app consists of the automatic design of gears given user imputed parameters and constraint. The fundamental value entered is the target displacement to be achieved. The script analyses all possible combinations of parameters within the constraints and chooses the pair of gears with the maximum pumping area, i.e., the maximum ratio between displacement and axial thickness. When possible, more than one solution is presented, with varying number of teeth.

The program not only checks for compliance of the dimensional constraints, it also excludes flawed solutions that can have interference, low contact ratio, etc. If there is no possible solution complying with all constraints, the closest solution in terms of displacement is displayed.

This script works for gears made by cutting with a tool rack and also gears made by additive manufacturing.

2. Input Parameters

The parameters that need to be defined in order to search the most adequate pair of gears are:

- Target **displacement** V_{tar} desired by the user
- Maximum **axial thickness** B of the gears
- Maximum **overall length** L of the coupled gears
- Maximum **external diameter** R_e of each gear
- Maximum **ratio** $\lambda = \frac{2R_e}{B}$ between the external diameter and axial thickness of the gears
- Minimum **bottom diameter** $2R_f$ of the gears
- Minimum **contact ratio** ε
- Minimum **thickness at the tooth tip** S_t
- **Side clearance** h_{sn} and **tip clearance** h_{tt} of the teeth equal to a value set by the user

Depending on the calculation mode used, it may be possible to also impose:

- a **center distance** int equal to a value defined by the user
- an exact **external diameter**

- the **manufacturing method**, rack cutting or other

Figure 4.1: Optimizer input parameters

3. Calculation Modes

There are three fundamentally different ways in which the program can search for the solutions:

- **Fixed center distance and fixed external diameter:** this mode is useful when modifying an existing pump without changing the casing, where these two values cannot be changed. Only the number of teeth and the module can be changed.
- **Fixed center distance and variable external diameter:** this is useful then the center distance is defined by the casing or another external component that may require it. In this mode also the external diameter is subject to optimization.
- **Variable center distance and variable external diameter:** is usually the best for designing a new pump. Usually provides the largest number of solutions since it has the most variables to combine.

4. Additional Input Parameters and Advanced Options

This module contains an "Advanced Settings" tab in which parameters that are not strictly necessary for the definition of the gear can be found. However, these are crucial for the functioning of the optimization script. These variables are:

- **Step For Module Increment [mm]:** the value by which the module is incremented in each loop iteration.
- **Step For External Radius Increment [mm]:** the value by which the external radius is incremented in each loop iteration.

External Gears Design: Step 1

Manuf Process: Other

Calculation Mode: int:fix ; Re:fix

Pump Displacement [cc/rev]:

Centre Distance [mm]:

External Radius [mm]:

Max Axial Thickness [mm]:

Minimum Int Diam [mm]:

Max diam/thickness Ratio:

Normal Pressure Angle [deg]:

Max Speed [rpm]:

Clearance Between Teeth [mm]:

Clearance at Tooth Tip [mm]:

Max Ext Diameter [mm]:

Max Overall Width [mm]:

Errors ●

Solutions Analysed: 564
3 profile(s) found:
9 teeth
10 teeth
11 teeth

External Gears Design: Step 1
Gears Generation Mode: Without Rack Cutter
Calculation mode: fixed centre distance - fixed external radius

Input parameters:
Target displacement [cc/rev] = 20.000
Centre distance (int) [mm] = 40.000
External radius (Re) [mm] = 24.000
Max axial thickness (Bmax) [mm] = 20.000
Max ratio diameter/thickness [-] = 5.000
Normal pressure angle (teta0) [deg] = 20.000
Clearance between teeth (hsn) [mm] = 0.100
Max speed [rpm] = 6000.000

Profile(s) found:

Nd	m0	int	Re	Ri	B	L	V	Vmax
[-]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[cc/rev]	[cc/rev]
9	4.130	40.000	24.000	15.500	19.500	88.000	20.046	20.560
10	3.750	40.000	24.000	15.500	19.000	88.000	19.792	20.833
11	3.530	40.000	24.000	15.500	19.000	88.000	19.931	20.980

Nd	P.A.	dq	vsmax	eps.	st	xg	tetak
[-]	[cm2/rev]	[%]	[m/s]	[-]	[mm]	[-]	[deg]
9	10.280	22.714	12.248	1.102	1.700	0.405	29.166
10	10.417	18.481	11.892	1.234	1.026	0.383	28.241
11	10.490	16.262	10.298	1.420	1.006	0.162	24.188

Legend:
P.A. = pumping area
dq = flow ripple index
vsmax = max. profiles sliding velocities
eps. = contact ratio
st = tooth tip width
xg = correction factor

Choose Solution To Export for Drawing 9 Teeth

Figure 4.2: Calculation mode: fixed center distance - fixed external diameter

- Step For Centre Distance Increment [mm]: the value by which the center distance is incremented in each loop iteration.
- Rounding For Axial Gear Thickness: analogue to the aforementioned, but for the axial thickness of the gear.
- Addendum Coefficient
- Minimum Number of Teeth
- Maximum Number of Teeth
- Tool Fillet Coefficient
- Tooth Tip Chamfer [mm]
- Minimum Width of the Tooth Tip [mm]
- Minimum Contact Ratio [mm]

External Gears Design: Step 1 Advanced Options

Step For Module Increment [mm]	<input type="text" value="0.01"/>
Step For External Radius Increment [mm]	<input type="text" value="0.1"/>
Step For Centre Distance Inc [mm]	<input type="text" value="0.1"/>
Rounding For Axial Gear Thickness	<input type="text" value="0.5"/>
Addendum Coefficient	<input type="text" value="1.25"/>
Min Number of Teeth	<input type="text" value="7"/>
Max Number of Teeth	<input type="text" value="15"/>
Tool Fillet Coefficient	<input type="text" value="0.2"/>
Tooth Tip Chamfer [mm]	<input type="text" value="0"/>
Minimum Width of the Tooth Tip [mm]	<input type="text" value="1"/>
Minimum Contact Ratio	<input type="text" value="1.1"/>

Figure 4.3: Optimizer advanced options

5. Logic and working principle of the script

The program starts by making the appropriate unit conversions and checks the input parameters for conflict or inconsistencies. Only after that the iteration starts.

There are four nested loops, from outermost to innermost, the variable to optimize for is:

- Number of teeth: a solution is searched for each number of teeth from the minimum specified to the maximum.
- Center distance: only for the third calculation mode, the center distance is change according to a user defined step. The search interval varies depending on previous constraints.
- Module: the module's value varies incrementally according to the user defined step.
- External Radius: similarly to the previous two loops, the radius varies according to a user defined step

Before leaving the outermost loop, if a solution verifies all requirements, it is saved in a vector. For a given number of teeth, if a solution with greater pumping area is found, it replaces the previous saved solution.

Finally, the best solution for each number of teeth is displayed.

5.1 Solutions found

The desired outcome of the optimization is that several possible solutions are found. They may not exactly match the required displacement because a non negligible step for the dimensions is used, which produces a discrete number of solutions.

5.2 No Possible Solution With Given Target Displacement

When a viable gear profile is found but to achieve the target displacement the required axial thickness B is greater than the maximum specified, the closest solution is displayed. This can also be exported to the drawing module.

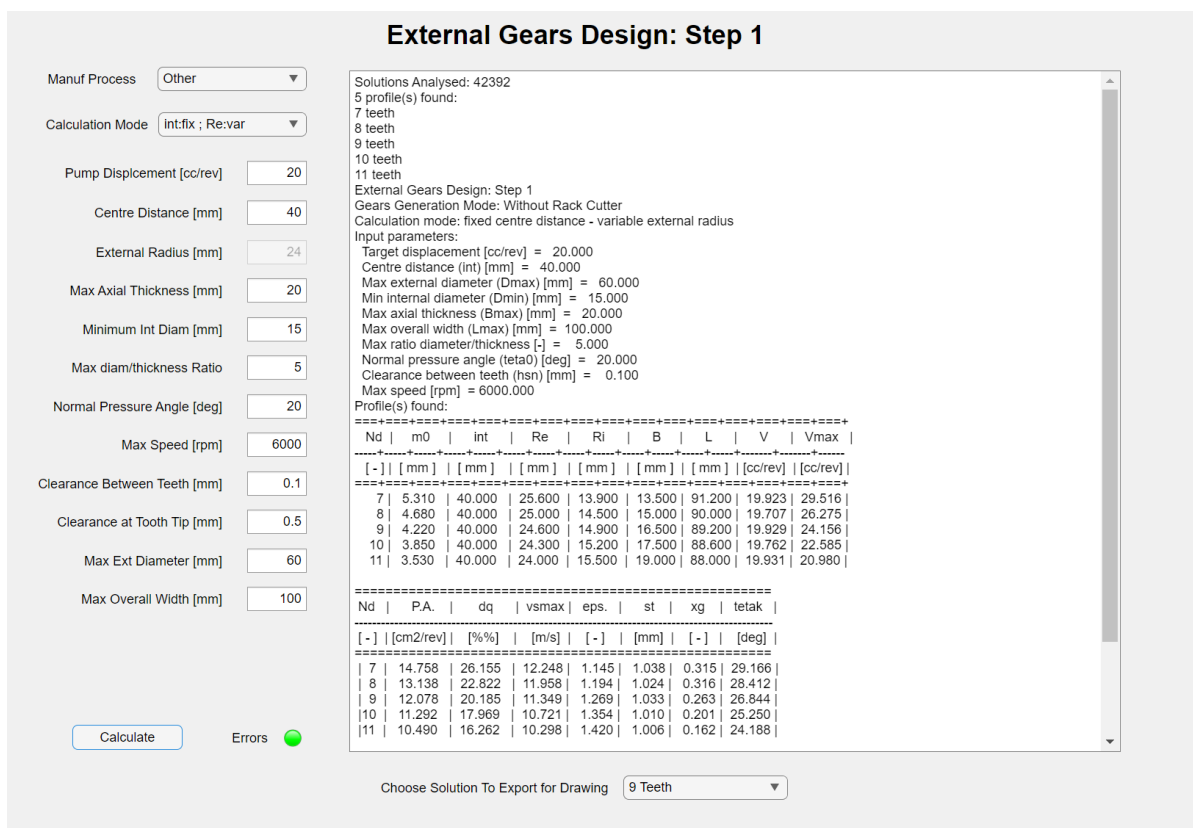


Figure 4.4: Calculation mode: fixed center distance - variable external diameter

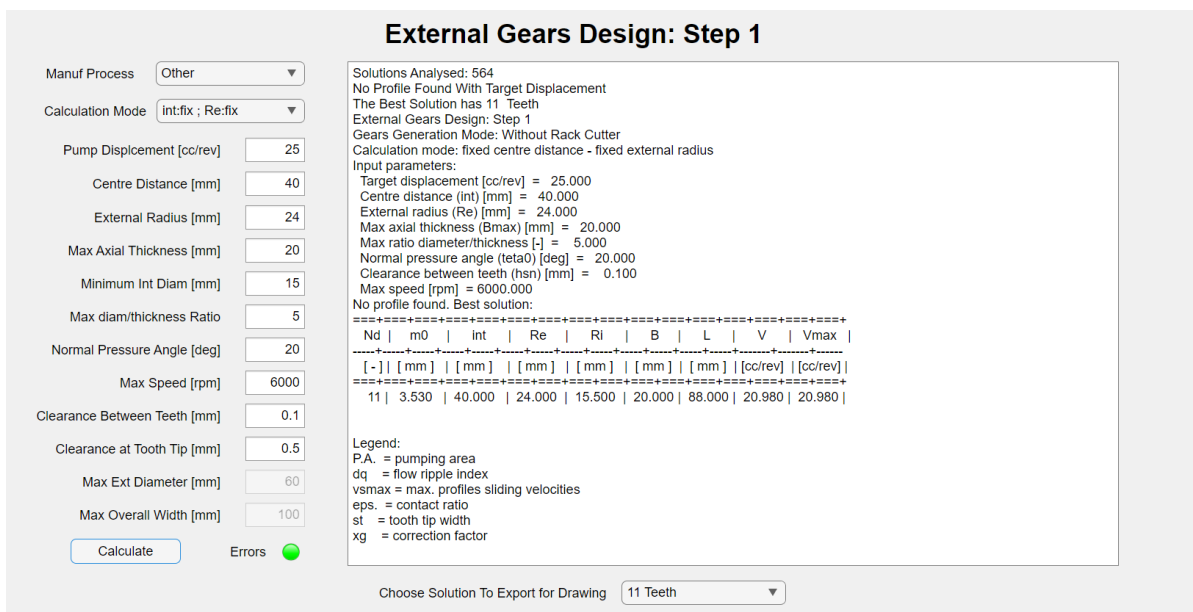


Figure 4.5: No Possible Solution With Given Target Displacement: Closest Solution

5.3 No Solution Found

In some cases a set of parameters does not provide a viable set of external gears, not even one that has a smaller displacement than desired. This usually happens when the target displacement is too small and the maximum diameter/thickness ratio is not compatible with said displacement.

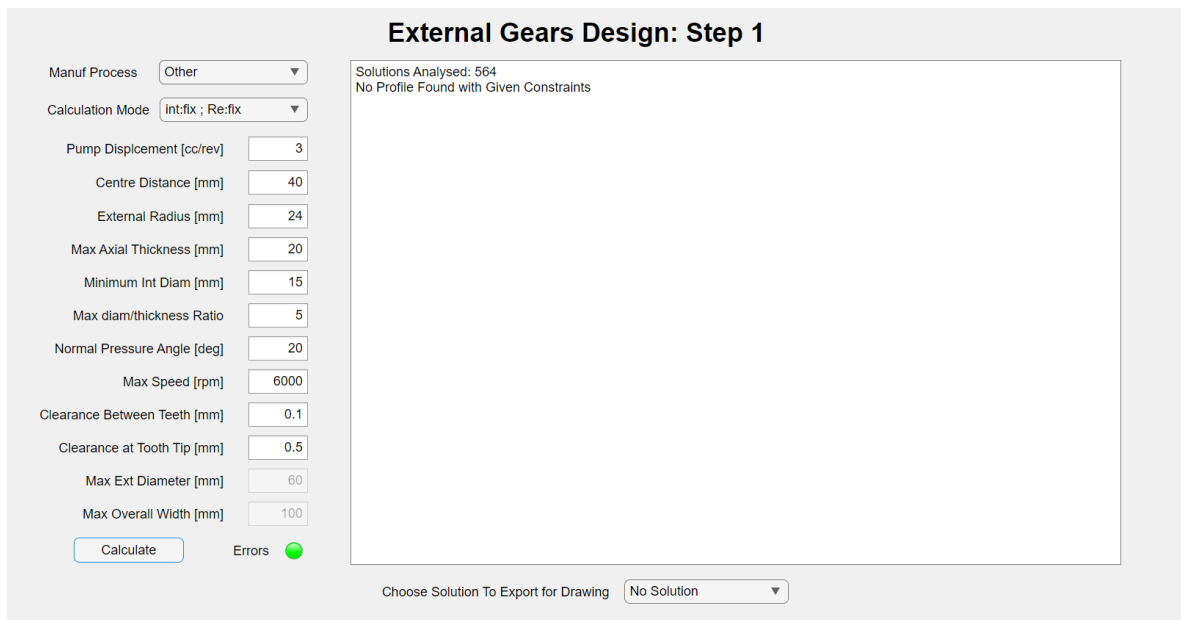


Figure 4.6: No solution found within the desired constraints

6. Results Display

When the program has finished going through all the possible solutions and no errors are found, the text area is used to display the results. First, the number of solutions analyzed and the most important input parameters are displayed. If solutions are found, even if they do not reach the required displacement, they are displayed in a table, where each line corresponds to one number of teeth.

Additionally, for each solution, the following information is displayed:

- **Pumping area** $P.A.$ [cm²/rev]
- Flow **ripple index** dq [%]
- Maximum **profile sliding velocity** vs_{max} [m/s]
- **Contact ratio** eps
- **Tooth tip width** s_t [mm]
- Profile **correction factor** x_g
- Working **pressure angle** θ_k [deg]

This information is also optionally saved into a text file with identical content to what is displayed into the text part. Additionally, it is possible to export the parameters to the gear drawing part of the app.

External Gears Design: Step 1

Manuf Process: Other

Calculation Mode: int:var ; Re:var

Pump Displacement [cc/rev]:

Centre Distance [mm]:

External Radius [mm]:

Max Axial Thickness [mm]:

Minimum Int Diam [mm]:

Max diam/thickness Ratio:

Normal Pressure Angle [deg]:

Max Speed [rpm]:

Clearance Between Teeth [mm]:

Clearance at Tooth Tip [mm]:

Max Ext Diameter [mm]:

Max Overall Width [mm]:

● Errors

Solutions Analysed: 246242
8 profile(s) found:

7 teeth
8 teeth
9 teeth
10 teeth
11 teeth
12 teeth
13 teeth
14 teeth

External Gears Design: Step 1
Gears Generation Mode: Without Rack Cutter
Calculation mode: variable centre distance - variable external radius

Input parameters:
 Target displacement [cc/rev] = 20.000
 Max external diameter (Dmax) [mm] = 60.000
 Min internal diameter (Dmin) [mm] = 15.000
 Max axial thickness (Bmax) [mm] = 20.000
 Max overall width (Lmax) [mm] = 100.000
 Max ratio diameter/thickness [-] = 5.000
 Normal pressure angle (teta0) [deg] = 20.000
 Clearance between teeth (hsn) [mm] = 0.100
 Max speed [rpm] = 6000.000

Profile(s) found:

Nd	m0	int	Re	Ri	B	L	V	Vmax
[-]	[mm]	[mm]	[mm]	[mm]	[mm]	[mm]	[cc/rev]	[cc/rev]
7	5.790	43.700	28.000	15.200	11.500	99.700	20.393	35.466
8	5.200	44.300	27.800	16.000	12.000	99.900	19.798	32.997
9	4.730	44.700	27.600	16.600	13.000	99.900	20.093	30.912
10	4.330	45.000	27.400	17.100	14.000	99.800	20.310	29.015
11	3.980	45.300	27.200	17.600	15.000	99.700	20.293	27.057
12	3.700	45.600	27.100	18.000	15.500	99.800	19.929	25.714
13	3.460	45.900	27.000	18.400	16.500	99.900	20.071	24.329
14	3.200	46.300	26.800	19.000	18.000	99.900	19.779	21.976

Nd	P.A.	dq	vsmax	eps.	st	xg	tetak
[-]	[cm2/rev]	[%%]	[m/s]	[-]	[mm]	[-]	[deg]
7	17.733	25.880	13.464	1.148	1.018	0.326	29.363
8	16.498	22.437	13.095	1.218	1.004	0.298	28.064
9	15.456	19.816	12.533	1.296	1.033	0.246	26.502
10	14.507	17.692	12.077	1.368	1.028	0.205	25.285
11	13.529	16.029	11.915	1.415	1.016	0.195	24.747
12	12.857	14.576	11.562	1.483	1.000	0.158	23.799
13	12.164	13.473	11.245	1.538	1.035	0.121	22.948
14	10.988	12.758	12.109	1.472	1.010	0.238	24.588

Choose Solution To Export for Drawing: 9 Teeth

Figure 4.7: Calculation mode: variable center distance - variable external diameter

Chapter 5

Flow Ripple Calculation

1. Introduction

The final module of the program is dedicated to the flow ripple calculation. In hydraulic systems, flow ripple refers to the fluctuations in flow rate caused by the mechanical operation of the gears during the pumping cycle. These fluctuations can lead to noise, vibrations, and inefficiencies, negatively affecting the system's overall performance.

Additionally, it evaluates the pumping area, which is defined by the relationship between the pump's displacement and the axial thickness of its gears. By multiplying it by the gear thickness the pump displacement is obtained. Also, the flow ripple index, a dimensionless parameter that quantifies the severity of the flow ripple is calculated.

The modeling of the flow ripple uses a lumped parameters approach, which requires the knowledge of the length of vector rays between the centers of the gears and the contact points. This method is much easier to implement and requires less computational time than a CFD approach, while having more than enough precision for the task.

2. Input Parameters

The flow ripple calculation starts after the gear profile has been automatically drawn in the previous section. The user has to first define the gear parameters and calculate the profiles, those same parameters will be the ones used for this calculation.

After the aforementioned step, the user must switch to the **Flow Ripple Calculation** tab. There, additional parameters must be imputed.

1. *Shaft speed* [rpm]: the rotational speed of the gears for the calculation
2. *Number of points*: the amount of points at which the flow rate will be evaluated in the rotation, 6000 by default.
3. *Start Angle* [deg]: the driving gear angle at which the evaluation will start, 0 by default.
4. *End Angle* [deg]: the driving gear angle at which the evaluation will end, 360 by default, this means evaluating a full rotation.

Finally, the user can choose to export the flow rate and the driven gear angle in two columns in a .txt file.

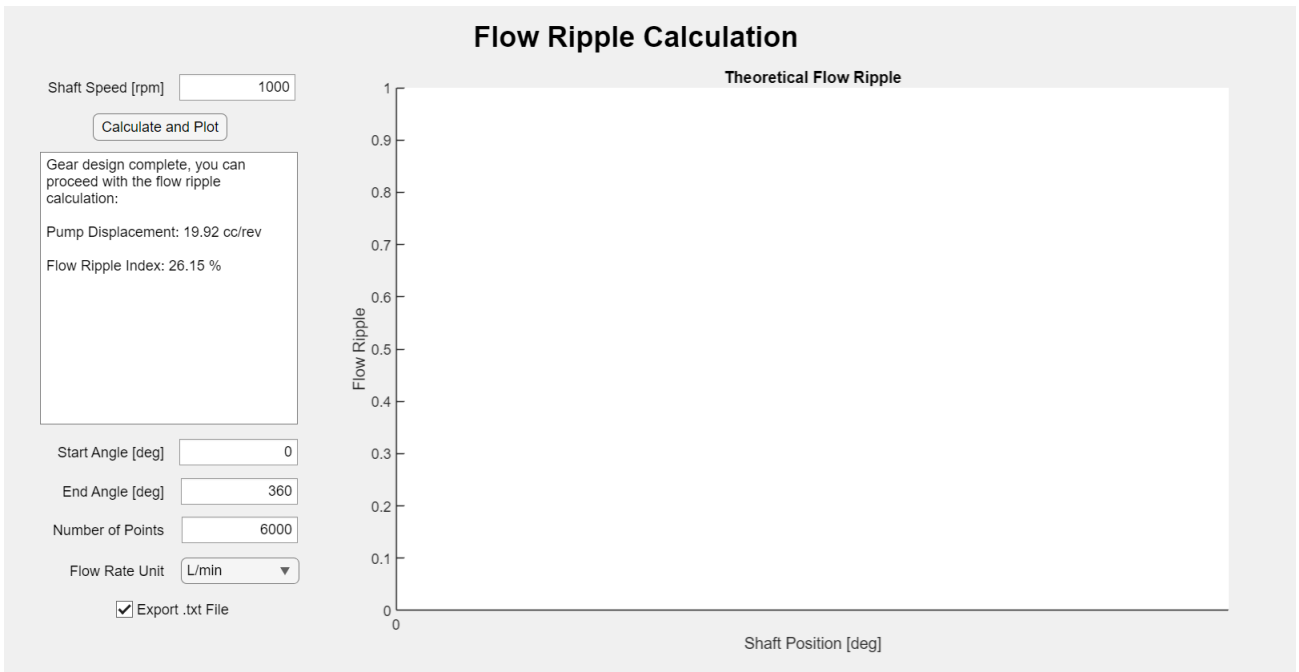


Figure 5.1: Input parameters for the flow ripple calculation

2.1 Calculation of displacement and flow ripple index

The base pitch t_0 is:

$$t_0 = \pi \cdot m_0 \cdot \cos(\theta_0) \quad (5.1)$$

The pumping area of the external gear pump is:

$$Pa = R_e^2 - \frac{R_{p0} \cdot I_k^2}{2 \cdot I} - \frac{t_0^2}{12} \quad (5.2)$$

Where R_e is the external radius, R_{p0} is the pitch radius, I_k is the working centre distance, and I is the nominal center distance.

The displacement of the pump is:

$$V = Pa \cdot b \quad (5.3)$$

Where b is the thickness of the gear.

The flow ripple index δ_Q is:

$$\delta_Q = \frac{1}{\frac{4R_e^2 - I^2}{t_0^2} - \frac{1}{3}} \quad (5.4)$$

The displacement and the flow ripple index are displayed at the text area.

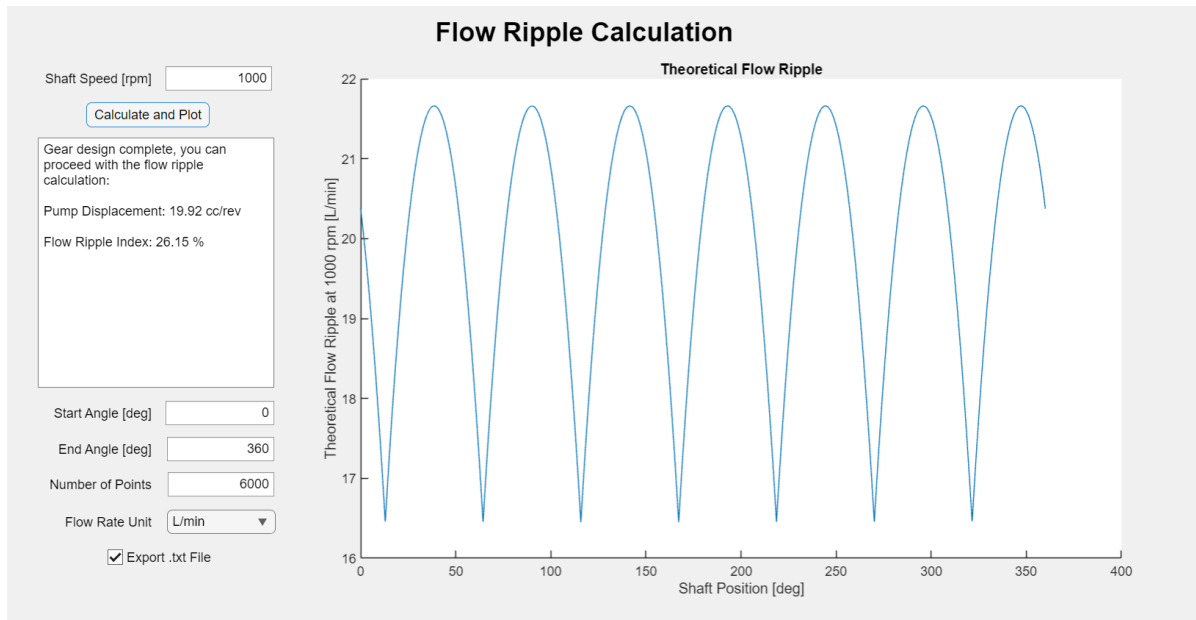


Figure 5.2: Tab for calculating the flow ripple

3. Calculation of the flow ripple

The mathematical model for the calculation depends only on the geometry of the gears. No compressibility of the fluid is considered.

An auxiliary variable for the delivery volume v_d is presented and it is a function of the rotation angle φ of the driver gear. It has the following properties:

- It varies from 0 to $\Delta\varphi$ when φ varies from 0 to $+\infty$,
- It equals zero when a new contact begins between a tooth of the driver gear and one of the driven gear, i.e., when the trapped volume is created.

v_d can be obtained as:

$$v_d = \varphi + \varphi_s - \text{fix}\left(\frac{\varphi + \varphi_s}{\Delta\varphi}\right) \cdot \Delta\varphi \quad (5.5)$$

where $\text{fix}()$ is a function that rounds the argument to the nearest integer towards zero; φ_s is the value that v_d assumes when the gears are in the reference condition; and $\varphi = 0$ represents the rotation that must be imposed on the driver gear so that from the moment when a new contact point P_1^0 is generated, the reference condition is reached with the contact point at P_0 (fig. 5.3). Its value can be obtained as:

$$\varphi_s = \tan(\theta_k) - \varphi_1' - v_0 \quad (5.6)$$

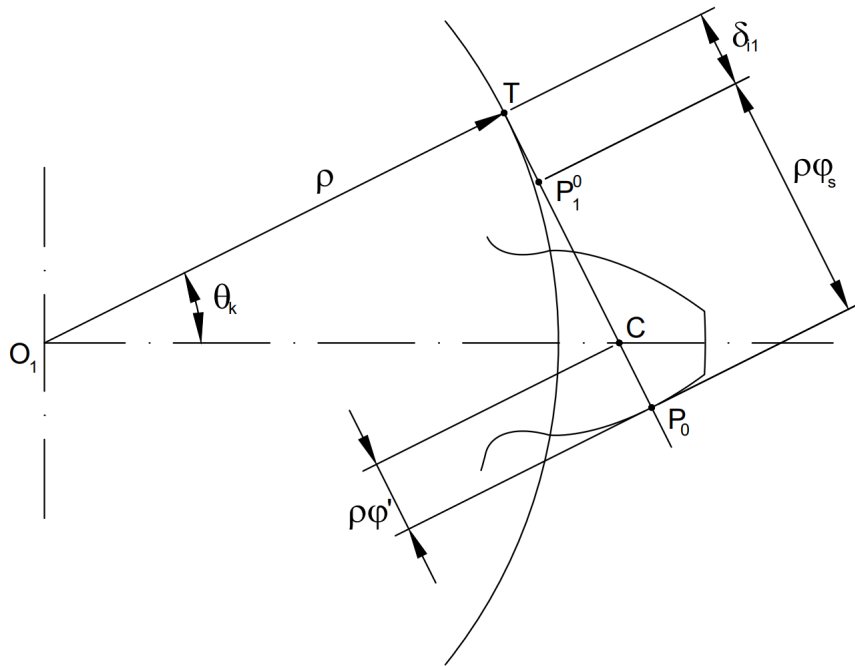


Figure 5.3: Graphic representation of φ_s

where v_0 is the angle corresponding to the unused section of the contact segment on the driving gear side, and φ'_1 corresponds to the rotation of the driver gear necessary to bring the instantaneous contact point C to point P_0 where the gear is in the reference condition fig. 5.4.

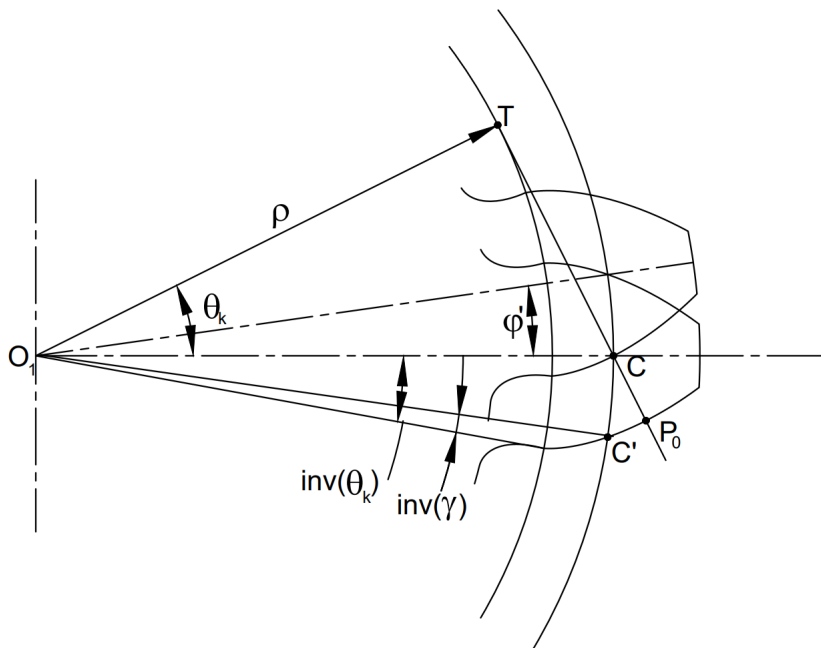


Figure 5.4: Graphic representation of φ'_1

From eq.(1.27) δ is the unused part of the contact segment on ether gear's side.

$$v_0 = \frac{\delta}{\rho} \quad (5.7)$$

Next φ'_1 is:

$$\varphi'_1 = \frac{s_{R_{p0}}}{2 \cdot R_{p0}} \quad (5.8)$$

Where $s_{R_{p0}}$ is the thickness of the tooth at the pitch circle, obtained from eq.(1.18). With this information it is possible to find v_d as a function of the angle φ .

An analogous auxiliary variable, v_s , for the suction volume is determined as a function of the rotation angle φ , with the following properties:

- It varies from 0 to $\Delta\varphi$ when φ varies from 0 to $+\infty$,
- It equals zero when two teeth separate (i.e., when the trapped volume disappears).

The equation for its formulation is:

$$v_s = \varphi + \varphi'_s - \text{fix} \left(\frac{\varphi + \varphi'_s}{\Delta\varphi} \right) \Delta\varphi \quad (5.9)$$

Where, similarly, φ'_s is the angle that v_s assumes when the gears are in the reference condition, and $\Delta\varphi - \varphi'_s$ represents the rotation that must be imposed on the wheel so that from the reference position, the contact point P_0 reaches the end of the engagement segment in P_2^0 fig. 5.5.

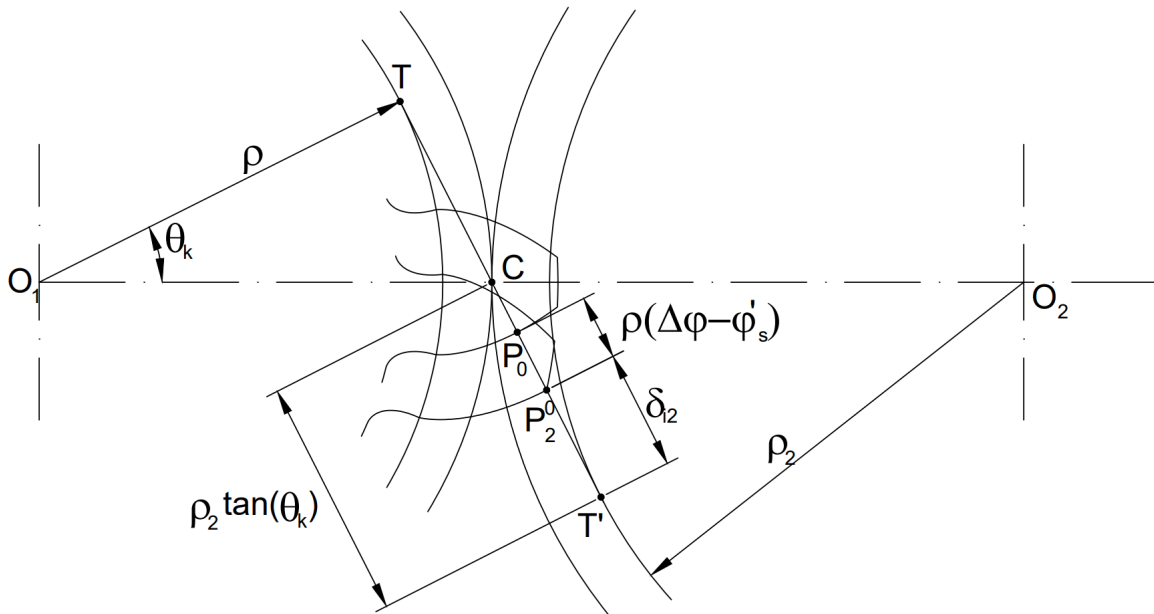


Figure 5.5: Graphic representation of φ'

In this case, the relations are derived for the driver gear, and the results must consider the transmission ratio ($\tau = -1$)

$$\varphi'_s = \Delta\varphi - \tan(\theta_k) + v_0 + \varphi'_1 \quad (5.10)$$

Timing Angles

The timing angle φ_{bt} by which the driver gear must rotate so that the trapped volume is generated, is defined as:

$$\varphi_{bt} = \Delta\varphi - \varphi_s \quad (5.11)$$

The angle φ_{bt} by which the driver gear must rotate so that the trapped volume is disappears, is defined as:

$$\varphi_{et} = \Delta\varphi - \varphi'_s \quad (5.12)$$

The script analyzes the delivery volume and derivatives at a discrete number of points, set by default to 6000 points in a full rotation of the driver gear. The angle φ represents the angular position of the driver gear.

First, the auxiliary variable v_d is calculated for each value of φ with eq.(5.5)

The length of the segment belonging to the contact line, which goes from the tangent point with the base circle to the contact point between the teeth of two gears is:

$$\delta_d = \rho(v_0 + v_d) \quad (5.13)$$

Based on the definition of v_d , the range of existence of δ_d is:

$$\rho v^0 \leq \delta_m < \rho(v^0 + \Delta\varphi) \quad (5.14)$$

For each value of v_d , the length of vector for the driver gear referred to the first contact point is:

$$R_d^2 = \rho^2 + \delta_d^2 \quad (5.15)$$

And substituting, we obtain:

$$R_d^2 = \rho^2 [1 + (v_d + v_0)^2] \quad (5.16)$$

And for the second contact point:

$$R_{d2}^2 = \rho^2 [1 + (v_d + v_d + \Delta\varphi)^2] \quad (5.17)$$

For the driven gear, the analogous expressions are:

$$R_c^2 = \rho^2 [TT - 1 + (v_d + v_0)^2] \quad (5.18)$$

And for the second contact point:

$$R_{c2}^2 = \rho^2 [TT - 1 + (v_d + v_d + \Delta\varphi)^2] \quad (5.19)$$

Finally, the derivative of the delivery volume is:

$$dV_d = \frac{b}{2} \cdot (R_d^2 + R_c^2 - 2Re^2) \quad (5.20)$$

Where Re is the tooth tip radius of the gear.

Now, the variation of the trapped volume is required. First the auxiliary variables v_{dbt} and v_{det} , which correspond to the angle at which the trapped volume begins and ceases to exist respectively, are defined as:

$$v_{dbt} = \varphi_{bt} + \varphi_s - \text{fix}\left(\frac{\varphi_{bt} + \varphi_s}{\Delta\varphi}\right) \cdot \Delta\varphi \quad (5.21)$$

$$v_{det} = \varphi_{et} + \varphi_s - \text{fix}\left(\frac{\varphi_{et} + \varphi_s}{\Delta\varphi}\right) \cdot \Delta\varphi \quad (5.22)$$

Then, only if v_d is within v_{dbt} and v_{det} the volume is trapped and the variation of the trapped volume is:

$$dV_t = \frac{b}{2} \cdot (R_{d2}^2 + R_{c2}^2 - R_d^2 - R_c^2) \quad (5.23)$$

Otherwise:

$$dV_t = 0 \quad (5.24)$$

At last, the theoretical delivery flow rate is:

$$Q = -\omega(dV_d + \min(0, dV_t)) \quad (5.25)$$

Where ω is the angular speed of the driving gear in rad/s and $\min()$ is a function that outputs the minimum of the two arguments.

Since Q depends on the angular position φ , the calculation is repeated for every value of φ . When the end of the angular interval is reached. Q and φ are plotted in the axes y and x respectively.

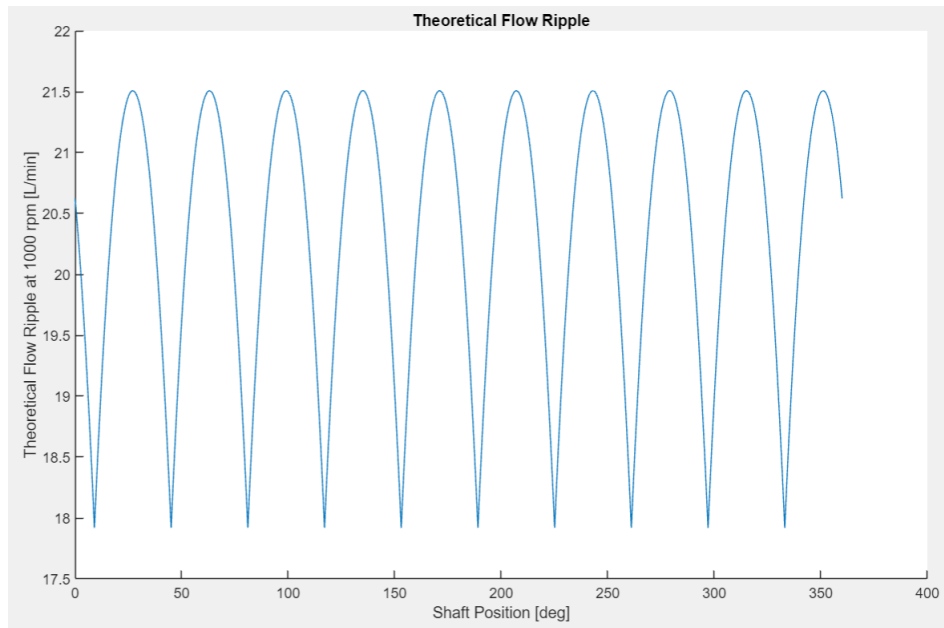


Figure 5.6: Plotting of the delivery flow rate versus angular position of the driver gear for a 10 teeth pump

Chapter 6

Error Handling

1. Introduction

This final chapter addresses the handling of errors that may occur during the execution of the script across various sections. These errors can arise from invalid or incompatible parameter inputs, or from parameters falling outside the expected boundaries, which may only become evident after calculations have begun. The second category of errors can include issues such as interference, invalid geometry, excessively thin tooth tip thickness, and other geometrical inconsistencies.

The purpose of this section is to provide a comprehensive list of all errors considered and to describe how each is managed by the software. Notably, errors do not interrupt the program's execution, as it can be beneficial to visually assess the nature of the issue. A status indicator, positioned at the bottom of both the optimizer and profile drawing sections, turns red when an error is detected. Detailed information about the error is then displayed in the "Errors/Warnings" tab for further review.

2. External Gears Drawing

This section discusses the error handling in the External Gears Drawing Module.

2.1 Verification of the Module

The module must be within:

$$m_{0_{max}} = \frac{I_k}{N_d \cos \theta_0} \quad (6.1)$$

And:

$$m_{0_{min}} = 0.65m_0 \quad (6.2)$$

This limitation is derived from the expression used to calculate the working pressure angle, which was already presented in previous chapters:

$$I \cos \theta_0 = I_k \cos \theta_k \quad (6.3)$$

With the appropriate simplifications and by imposing that the cosine of θ_k is equal to the maximum value the upper boundarie is derived.

The error message displayed when the module exceeds the imposed limits is:

”The module must be between: $m_{0_{min}}$ and $m_{0_{max}}$ ”

2.2 Verification of the Working Pressure Angle

The error message has a theoretical justification quite similar to that presented for the module. In fact, to calculate the angle θ_k , the following expression must be solved:

$$\theta_k = \arccos \left(\frac{N_d m_0 \cos \theta_0}{I_k} \right) \quad (6.4)$$

Thus, it is necessary to verify the following inequality to ensure the existence of the angle:

$$\frac{N_d m_0 \cos \theta_0}{I_k} < 1 \quad (6.5)$$

The error message displayed if this inequality is not satisfied is:

”You must reduce the number of teeth or the module.”

2.3 Correct Setting of External Radius

The user has the option to set the values of the external radii. These radii must be consistent with the meshing conditions presented by the gear. Therefore, the following relationship must be satisfied:

$$R_t > R_{pl} \quad (6.6)$$

In this case, the error message that appears is:

”The tip radius must be greater than: R_{pl} ”

2.4 Constraint on the External Radius in ”Rack Cutter” Mode

If the profiles are generated by a cutting tool, it is necessary to impose limitations on the maximum radius of the initial tendon. Therefore, the following relationship must be satisfied:

$$R_t < R_{p0} + x_g m_0 + d m_0 \quad (6.7)$$

The displayed message is:

”The tip radius must be less than: $R_{p0} + x_g + d m_0$ ”

2.5 Calculation of the Minimum Number of Teeth

It is important to know the minimum number of teeth for a gear generated by cutting; the most unfavorable condition for meshing is that given by the coupling with the rack cutter.

If this condition is not met, the tooth of the gear will be undercut at the base, which could cause serious problems and compromise the integrity of the gear or the cutter. Once the program calculates the minimum number of teeth, it compares the number of teeth of the two gears and imposes this limitation:

$$z_{\min} = \frac{2}{(\sin \theta_0)^2} (a - x_g) \quad (6.8)$$

If the number of teeth of the gear is less than z_{\min} , the following message will be displayed:

”The gear does not have enough teeth to be cut by a rack”

2.6 Non-Interference Conditions

The non-interference conditions coincide, in practice, with the non-undercutting conditions already discussed in previous chapters. The difference, in this case, is that the meshing occurs with the mating gear, not with the rack cutter.

To avoid interference between the two gears, the contact between them must occur within, and not beyond, segment TT' . The maximum limit for R_t can then be derived with the following inequality:

$$R_t \leq \sqrt{(\rho_2 \sin \theta_k)^2 + (l_k - \rho_2 \cos \theta_k)^2} \quad (6.9)$$

which simplifies to:

$$R_t \leq \sqrt{l_k^2 + \rho_2^2 - 2l_k \rho_2 \cos \theta_k} \quad (6.10)$$

If the condition $R_t < R_{t_{\max}}$ is not met, the program will display the following message:

”Interference detected: Adjust the design parameters.”

2.7 Further Verification of Interference

Until now, limits have been set on the external radii that derive from the meshing between the gear and the tool. It is also necessary to impose limits on the meshing between the gear and the counter-gear:

In this situation, the following two inequalities must be respected, otherwise, the error presented in Figure 6 occurs:

$$R_e + R_f < l_k \quad (6.11)$$

The displayed message is:

”The gears are interfering; reduce the tip radius or increase the tool addendum.”

2.8 Invalid Tool Geometry

It may happen that the set of input data generates an invalid tool rack geometry or one that is inconsistent with the initial assumptions used for the profile design. Below are the two types of errors that may arise and their causes:

$$a < \frac{sR_{p0}}{2} \quad (6.12)$$

$$b < Am_0 - x_g m_0 \quad (6.13)$$

These two inequalities may not be satisfied due to the imposed center distance value. The error message that is displayed in this case is:

”The imposed center distance leads to an invalid tool geometry; reduce the gap with the nominal center distance or increase the shear module.”

$$a > 0 \quad (6.14)$$

This other limitation derives from the assumption that the tool has a straight segment with a near-zero limit designed to create the root circle of the gear. If this were not the case, there would be a cusp at the tip of the tool’s tooth, and the generated gear would not even have a root circumference. Furthermore, the center of the fillet radius would not be positioned at point O_1 . This problem can be solved by reducing the fillet radius between the two straight segments of the tool. The displayed message is:

”The tool fillet radius height leads to an invalid geometry.”

2.9 Calculation of Tooth Tip Thickness

The calculation model is capable of informing the user when the tooth tip thickness is too thin. Specifically, a warning message is triggered if the thickness is less than 1 mm. The message that will appear is:

”Tooth tip thickness less than 1 mm.”

3. Automatic Design of Gears Optimizer

This section is dedicated to the possible errors that appear in the second module of the program.

3.1 Centre Distance and Maximum Overall Width

The first error checks whether the input center distance I_k exceeds half of the maximum overall width L when using the variable external diameter mode. The dimensions must comply with:

$$I_k \leq \frac{L}{2} \quad (6.15)$$

If this condition is false, the program will display the following error message:

”Centre distance must be less than half the maximum overall length”

3.2 Maximum External Diameter Exceeds Centre Distance

This error checks whether the center distance I_k is greater than or equal to the maximum external diameter D_{max} when using the fixed center distance modes. The following relationship must be true:

$$I_k \leq D_{max} \quad (6.16)$$

Otherwise the corresponding error message is:

”Centre distance must be less than maximum external diameter”

3.3 External Radius and Centre Distance Condition

This error condition checks if the external radius R_t is less than or equal to half of the center distance I_k when using the fixed external radius mode:

$$R_t \geq \frac{I_k}{2} \quad (6.17)$$

If this condition is not met, the following error message is displayed:

”External radius must be greater than half the center distance.”

3.4 Minimum External Diameter Exceeds Maximum Internal Diameter

This error condition checks whether the minimum internal diameter D_{min} is greater than or equal to the maximum external diameter D_{max} when using any of the variable center distance modes:

$$D_{min} \geq D_{max} \quad (6.18)$$

If the inequality holds, the following error message is displayed:

”Maximum external diameter dmax must be greater than minimum internal diameter dmin.”

3.5 External Diameter and Minimum External Diameter Condition

This error checks if the minimum external diameter D_{min} is greater than or equal to twice the root diameter R_t when using the fixed external diameter mode:

$$D_{min} \geq 2R_t \quad (6.19)$$

If this is true, the error message displayed is:

”External diameter must be greater than half the minimum external diameter”

3.6 Diameter/Axial Thickness Ratio Exceeds Maximum Limit

This error condition is triggered if the diameter/axial thickness ratio λ exceeds $\frac{2R_t}{b}$ when using the fixed center distance mode:

$$\lambda \geq \frac{2R_t}{b} \quad (6.20)$$

The corresponding error message is:

”Max lambda must be greater than two times the external diameter / max axial thickness”

3.7 Minimum Diameter Exceeds Required Condition

The final error checks whether the minimum internal diameter D_{min} exceeds $2I_k R_t H_{tt}$ when using the fixed external diameter mode:

$$D_{min} \geq 2I_k R_t H_{tt} \quad (6.21)$$

If this condition is met, the program will display the error message:

”Minimum diameter must be less than 2(int-re-htt)”

Chapter 7

Conclusions

This thesis presents the development of a MATLAB-based application designed to optimize the design of external gear pump gears. The software successfully meets the defined objectives by providing an automated process for gear profile generation, gear sizing and optimization and study of the kinematic flow ripple. Through the integration of analytical and computational methods, it ensures that the generated gears adhere to the required performance specifications, while significantly reducing design time cost.

A key improvement over previous versions is the flexibility that MATLAB uses and the ability to easily implement changes and fixes in the software. Additionally, the software features an improved user interface, allowing for efficient input of design requirements and effortlessly copying the parameters from one section to the next.

The ability to rapidly prototype external gear pumps is of great importance, particularly in today's competitive engineering landscape where reducing time-to-market is critical. This application facilitates early-stage design exploration, reducing reliance on physical prototypes and iterative experimental processes. Overall, this work provides a valuable tool for researchers and industry professionals, enhancing the efficiency and accessibility of external gear pump design.

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