POLITECNICO DI TORINO

MASTER'S DEGREE IN ARCHITECTURE FOR HERITAGE

MASTER'S THESIS

THE MAIN DOME OF SAN LORENZO CHURCH

HISTORICAL RESEARCH AND GEOMATICAL ANALYSIS OF SAN LORENZO'S MAIN DOME



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ABSTRACT

This thesis investigates the geometry of the ribbed dome of San Lorenzo Church in Torino, a masterpiece by Guarino Guarini. The church is renowned for its structural innovation, its proportions, and use of light to manipulate vision, making the dome appear higher through its repeated pattern. The study begins with a general historical overview of Guarini's background, detailing his architectural projects and treatises, and placing his work within the broader context of Baroque architecture. Following this, the thesis delves into the history and architecture of San Lorenzo Church, with a specific focus on the main ribbed dome. It incorporates debates and discussions from eminent researchers such as Professor Susan Klaiber, Henry Millon, Elwin Clark Robison, and other scholars, whose work significantly influenced this study.

The thesis is divided into two main sections: historical research and geometric analysis. The historical section contextualizes Guarini and his architecture. The geometric section details the survey methods employed, including 3D photogrammetry and laser scanning, to document the church and the dome. This documentation facilitated the identification of the dome 's geometry and its potential deformations. The research focused on the ribs of the dome, and it involved the identification and selection of what seemed to be the most intact rib for detailed geometric analysis, aiming to understand Guarini's design intentions.

By comparing Guarini's theoretical approach from his treatise "Architettura Civile" with the acquired survey data, the study proposes a 3D model which aims to represent Guarini's original design for the dome. The analysis reveals a strong correlation between Guarini's methods for creating ovals and ellipses and the current geometry of the dome's ribs. Despite minor discrepancies likely due to structural settlements, the study provides insights into Guarini's design process. The geometric analysis further opens to considering the structural dynamics of the dome, and factors such as load distribution and material properties.

Although a definitive conclusion on Guarini's original design for the dome remains elusive, the thesis discusses the most plausible geometric configurations for the San Lorenzo Church dome. The research underscores the importance of combining historical knowledge with modern technological methods to uncover the intricacies of historical architectural masterpieces.

Keywords:

Guarino Guarini, San Lorenzo Church, Ribbed Dome, Architectural Geometry, Structural Innovation, Historical Research ,3D Photogrammetry, Laser Scanning, Architectural Treatises, Perspective Manipulation, Oval and Ellipse Geometry, Structural Deformation Analysis

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METHODOLOGY

This thesis employs a mixed-methods approach, combining historical research with an experimental geometrical survey to thoroughly investigate the San Lorenzo Church, with a specific focus on its ribbed dome. The methodology is divided into two primary sections: a historiographical analysis and a geometrical survey. **Historiographical Analysis**:

1. Conceptual Framework and Historical Context:

- The research begins with a detailed historiography of the original concept, geometry, and architectural characteristics of the San Lorenzo Church, with an emphasis on the ribbed dome. This section delves into the intrinsic features of the building, supported by images and drawings presented in the thesis.

- A comprehensive background of Guarino Guarini, the esteemed architect of San Lorenzo Church, is provided. This includes an overview of his contributions to architecture, his role within the Savoy dynasty, and a review of his architectural treatises. Guarini's theoretical perspectives are crucial for understanding the design principles applied in the ribbed dome.

2. Historical Documentation and Analysis:

- Collection of historical documents, including architectural drawings, written descriptions, and previous research studies. These documents offer insights into the design and construction techniques used by Guarini.

- Comparative analysis of historical records to identify the evolution of the architectural design and any documented modifications to the structure over time.

Experimental Geometrical Survey

1. Survey Methodology:

- Laser Scanning and Photogrammetry: Advanced technologies such as laser scanning and photogrammetry drones were employed to conduct a detailed geometrical survey of the ribbed dome. These methods allow for high-precision measurements.

- Data Collection: The survey focused on capturing the exact dimensions, shapes, and alignments of the dome's elements. This data was essential for identifying any deviations or deformations in the dome's geometry.

2. Data Analysis:

- Quantitative Analysis: The collected measurements were analyzed to distinguish the differences from a quantitative perspective. This involved comparing the current geometrical data with historical records and the ideal geometrical model proposed in previous studies.

- Qualitative Analysis: A qualitative assessment was conducted to interpret the structural conditions and any observed irregularities.

3. 3D Model Creation and Hypothesis Testing:

- An ideal 3D model of the ribbed dome was created using the extracted measurements. This model served as a reference to hypothesize about the original geometric intentions of Guarini and to identify potential deformations and irregularities in the current structure.

- Hypothesis Formation: Based on the 3D model and the gathered geometrical data, several hypotheses regarding the dome's geometry were formulated. These hypotheses aimed to explain any deviations

observed in the current structure, considering potential deformation over time.

Final Interpretation:

The thesis concludes with an analytical interpretation of the combined historical and geometrical data. This interpretation draws parallels between the theoretical concepts proposed by Guarini and the practical realities observed in the current structure, offering insights into the architectural and geometrical intricacies of the San Lorenzo Church's ribbed dome.

By employing this mixed-methods approach, the thesis aims to provide a detailed and multifaceted understanding of the San Lorenzo Church's ribbed dome, combining historical context with precise geometrical analysis.

PART I:

CHAPTER 1



FIGURE 1 - PORTRAIT OF GUARINO GUARINI

GUARINO GUARINI AND HIS PIONEER ARCHITECTURE?

Exploring the architectural works of Guarino Guarini, one is immediately struck by the seamless integration of mathematics in his visionary designs. Susan Elizabeth Klaiber, in her influential Ph. D thesis "Guarino Guarini's Theatine Architecture," emphasizes Guarini's brilliance in not only mastering form and structure but also in understanding the deep connection between scholarly pursuits and architectural innovation. For Guarini, architecture was more than a profession; it was a sacred calling that merged philosophy, mathematics, and theology into a harmonious creative expression. His profound appreciation for mathematical principles is evident in all his works, setting him apart from his contemporaries. As we delve into Guarini's architectural legacy, we are drawn into a complex web of his mathematical insights, with each structure reflecting his imaginative and scholarly precision.

Guarino Guarini was born on January 17, 1624, to Raimondo Guarini and Eugenia Marescotti. ¹ Little is known about his early life until he joined the Theatine order on November 27, 1639. ² He completed his novitiate at S. Silvestro al Quirinale in Rome, where he pursued theological studies before moving to Venice. In Venice, Guarini's architectural journey began to take shape, evidenced by his engagement in theological courses and his ordination in early 1648 upon returning to Modena.³

Guarini's arrival in Turin marked a significant turning point, as he gained substantial patronage that propelled his architectural career. Turin became the epicenter of his achievements, where he dedicated himself to architectural projects. His architectural writings, such as "Modo di misurare le fabriche" (1674) and "Trattato di fortificazione" (1676), were intended for a broader audience, reflecting his desire to spread architectural knowledge beyond academic circles.⁴

Guarini's treatises highlight the deep connection between mathematics and architecture, emphasizing the practical application of mathematical principles in design and construction. "Modo di misurare," for instance, underscores the importance of precise measurement in construction, offering practical methods rooted in mathematical concepts. Guarini viewed architecture as an extension of mathematical principles: "Thaumaturga Mathematicorum miraculorum insigni, vereque Regali architectura coruscat," meaning "The magic of wondrous mathematicians shines brightly in the marvellous and truly regal architecture."

Guarini's architectural philosophy also emphasized the sensory pleasure in design, elevating it beyond mere mathematical calculation to artistic expression. His belief in the interplay between mathematics and aesthetic experience informed his practice, highlighting the balance between rationality and sensory perception in architectural creation.

In his first architectural publication, "Modo di misurare," Guarini lays the foundation for the necessity of precise measurement, responding to Turin's rapid expansion at the time. Beyond the introductory remarks, he provides a concise overview of arithmetic, delving into techniques for calculating the area and volume of various geometric shapes, with only occasional direct ties to architectural applications. ⁵

^{1 -} J. Southorn, Power and Display in Seventeenth-Century Italy. Cambridge, 1988, p. 56.

^{2 -} Modena capitoli. f. 52r (it could be part of a register or manuscript that contains chapters or provisions regarding Modena.), with the required three announcements of Guarini's intention to join the order on 30 September 1639, 14 October 1639 and 27 November 1639. Cf. ASMo, ECA 364, Libro delle cose piil memoribile della casa di S. Vincenzo, f. 89r: "A di 27 di 9bre entra in Relig.e Guarino del Sig. Rinaldo Guarini e di Sig.a Eugenia Marescotta d'eta d'anni 15. E q[ue]sta mattina parte p.[er] Roma...a fare il novitiato in S. Silvestro."

^{3 -} The profession required preliminary statements on 11 and 14 April, see Sandonnini, p. 488 and Guarini's autograph vows in AGT, MS 134, f. 8lr.

^{4 -} AGT, MS 26 (Gregorio Carafa), p. 209; This substantially revises traditional versions of Guarini's biography, which suggest he remained in Rome until late 1647.

^{5 -} Tricomi, F. "Guarini matematico." In *GGIB*, II: 551-557. Robison has presented a revised assessment of Guarini's mathematical writings: Robison, E. C. *Guarino Guarini's Church of San Lorenzo in Turin*. Ph.D. diss., Cornell University, 1985, 154-166. For a sample discussion of 17th-century geometry books

In the "Trattato di fortificazione," the initial focus similarly lies on practical applications.⁶ Guarini's legacy endures as a testament to the integration of mathematics and aesthetics in architectural theory and practice, inspiring future generations of architects and scholars to explore the dynamic interplay between science and art in architecture.

In architectural discourse, few figures have sparked as diverse critical responses as Guarino Guarini. H.A. Meek, in "Guarino Guarini and His Architecture," argues that Guarini's work defies easy categorization, presenting a unique and fantastical language that challenges conventional classification. ⁷ Guarini's architectural vision is not merely a product of his environment but an influence that shaped the discourse on architectural innovation and expression.

Guarini's appreciation for contemporary French architectural trends and his incorporation of Islamic forms into his designs demonstrate his embrace of diverse influences. His use of unconventional solutions, such as openwork domes, represents a significant departure from traditional architectural norms. Rather than imitation, Guarini's approach involved thematic assimilation, akin to the harmonic development in musical composition.⁸

One of Guarini's most notable contributions was his pioneering use of openwork stone domes, which departed from conventional structural practices. These visually striking domes obscured the true statical foundations of the structures. Guarini's architectural innovations, particularly his manipulation of domes, warrant detailed analysis within the broader context of art history. Exploring influences from Islamic and Gothic sources, as well as insights from Guarini's own writings in "Architettura Civile," provides valuable understanding of his creative process and philosophical foundations.

In summary, H.A. Meek's examination of Guarino Guarini's architectural legacy highlights the complexity and innovation in his work. Guarini's architectural language transcends imitation, embracing diverse influences and challenging norms to create a distinctive aesthetic that continues to captivate scholars and enthusiasts.

considered in the context of 17th-century mathematics, see the Maieril, L. "Il quinto postulate euclideo da C. Clavio [1589] a G. Saccheri [1733]." *Archive for History of Exact Sciences* 27 (1982): 297-334, esp. p. 305 on Guarini.

^{6 -} G. Guarini, Euclides adauctus. Turin, 1671, unpaginated dedication; first quoted by R. Wittkower, "Guarini the Man," in his Studies in the Italian Baroque. London, 1975, p. 186. Guarini goes on to praise the duke's building projects in Turin and at Venaria Reale.

^{7 -} Guarini himself wrote in Architettura Civile: 'we often see that people change their fashions, and what was once admired as beautiful becomes a bhorred as deformed, and what one nation likes displeases another'

^{8 -} Argan, G.C.'La tecnica del Guarini '. GGIB.I.p.42

CHAPTER 2

GUARINI'S CAREER LIFE AND PROJECTS

Guarino Guarini (1624-1683), a Theatine priest, theologian, and architect by profession, was also recognized as a polymath, delving into themes of astronomy and philosophy. His intellectual prowess extended to prolific publications across various fields of study. ⁹ Teaching philosophy and mathematics in different venues, serving as a confessor, and offering architectural consultations and site supervision, Guarini emerged as one of the most original European architects of the latter half of the seventeenth century. Born in Modena, within the Duchy of Este, and passing away in Milan, Guarini left architectural footprints across Modena, Messina, Paris, Vicenza, Verona, and Turin. While five of his seven churches still stand today¹⁰, along with a palace, two castles, and an altar ¹¹, his ambitious projects for Theatine churches in Lisbon, Nice, Prague, and Vicenza, as well as a grand palace in Paris and churches for other orders in Messina, Turin, Europe, and Casale Monferrato, stand as testaments to his architectural vision. Notably, his innovative vaulted wooden structure for a courtyard, later transformed into a hall at Palazzo Madama, showcased his ingenuity. Although engravings of most of his projects were published in 1686, additional works attributed to him, including a villa near Turin, further enrich his legacy. ¹²

Though Guarini's exact role in the history of geometry remains somewhat unclear¹³, there's little doubt that he stood out as a unique architect, among many inspired by Francesco Borromini, in creating original and captivating works comparable to Borromini's. His projects also demonstrate familiarity with the works of Gian Lorenzo Bernini, who, along with Borromini and Pietro da Cortona, was active in Rome during Guarini's

^{9 -} Richard Pommer is the first to suggest that for Guarini_l'architettura fu una forma di conoscenza (see l'architettura del settore in Piemonte. Le strutture aparte di Juvarra, Alferi and Vitume, edited by G. Dardanello, Turin 2003 [ed. 1967), p. 10). Guarini's Puritan colleagues respected erudition, were literary trained and appreciated the promise of science as a way to increase knowledge.

^{10 -} The chapel of the Shroud, San Lorenzo, the Consolata and the Immaculate Conception are still in existence in Turin, and Santa Maria d'Araceli in Vicenza. For these five churches, plus the Theatine house in Modena (modified), Sainte Anne-la-Royale in Paris (destroyed), the Santissima Annunziata and the Theatine house in Messina (destroyed) and San Gaetano in Vicenza (not built) see the essays that make up the third part of this volume entitled Projects and architecture.

^{11 -} For castello di Govone see M. PASSANTI, Nel mondo magico di Guarino Guarini, Torino 1963, pp. 6-8; A. LANGE, Designs and documents of Guarino Guarini, in Guarino Guarini e l'internazionalità del Barocco, a cura di V. Viale, proceedings of the international conference promoted by the Academy of Sciences of Turin (Turin, 30 September-5 October 1968), Turin 1970, vol. I, pp. 91-344, especially pp. 222-227; E. BORRA, Govone e il suo castello. Nel solco della storia del Piemonte, Borgo San Dalmazzo 1986; H. A. MEEK, Guarino Guarini, Milano 1991 (ed. or. 1988), p. 176; L. MO- RO, Il castello di Govone, Torino 1997; G. DARDANELLO, Guarino Guarini, in Storia dell'architettura italiana. Il Seicento, a cura di A. Scotti Tosini, Milano 2003, vol. II, pp. 588-613, in particolare p. 612, nota 53.

^{12 -} For Santa Maria Della Divina in Lisbona see PASSANTI 1963, pp. 67-73; POMMER 2003, pp. 62, 69, nota 14 a p. 70; R. WITTEKOWER, Arte e architettura in Italia 1600-1750, Torino 1972 (ed. or 1958), pp. 355-336, nota 13 a p. 377; M. PASSANTI, Disegni integrativi di lastre del trattato della Architettura Civiles, in Guarino Guarini 1970, vol. I, pp. 425-448, especially pp. 437-440; MEEK 1991, pp. 21/24; S. KLAIBER, Guarino Guarini's Theatine Architecture, Ph.D. dissertation, Columbia University, New York 1993, cap. VI; G. SCHNEIDER, GuarinoGua rini. Ungebaute Bauten, Wiesbaden 1997, pp. 13-17; A. MORROGH, Guarini e la ricerca dell'originalità. La Chiesa per Lisbona and related projects , in «Journal of the Society of Architectural Historians», vol. LVII,n. 1, marzo 1998, pp. 6-29.

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For San Gaetano a Nizza see. P. PORTOGHESI, Guarino Guarino, Milano n. 1956, p. 14; PASSANTI 1963, pp. 101-1 63-89; J. MORGANSTERN, Guarino Guarini: La Chiesa dei Padri Somaschi per Messina, master's thesis , New York University, New York 1964, pp. 43-44, note 24-28 di pp. 55-57; MELE 2003, pag. 12; LANGE 1970, pag. 120-124; E. C. ROBISON, Guarino Guarini's church of San Lorenzo in Turin , Ph.D. Thesis, Cornell University, Ithaca 1985, p. 234-242; MITEK 1991, pp. 101-1 175, 168, fig. 161-162; KLAIBER 1993, pag. 427-435, note 2-19 alle pp. 427-435. 444-447; SCHNEIDER 1997, pp. 19-24.

For San Gaetano a Vicenza see P. PORTOCHESI, Guarini a Vicenza. La chiesa di S. Maria d'Araceli, in «Critica d'arte», n.s., 1957, n.20, PP. 108-128; n. 21, PP. 214/229; PASSANTI 1963, PP. 92/97; MORGANSTERN 1964, PP. 36-40; MEEK 1991, PP. 166, 173, 175; KLAIBER 1993, cap. VII; A. ROCA DE AMI, CIS, II progetto di Guarini.

For the chiesa di S. Gaetano a Vicenza, in «Palladio», n.s., VI, n. 12, luglio dicembre 1993, PP. 109/114; S. KLAIBER, A New Drawing for Guarini's San Gaetano, Vicenza, in «The Burlington Magazine», XXXVI, n. 1097, agosto 1994, PP. 501-505; MORROGH 1998, pp. 17/19; M. BARAUSSE, F. BARBIERI, G. DARDANELLO, S.KLAIBER e A. ROCA DE AMICIS, Guarini a Vicenza.

For the Chiesa dei Somaschi a Messina cfr. C. CESCHI, Progetti del Guarini e del Vittone per la chiesa di San Gaetano a Nizza, in «Palladio», V, 1941, pp. 171/177; PORTOGHESI 1956a, pp. 8, 13; PASSANTI 1963, pp. 108.113; MORGANSTERN 1964, PP. 30/33, note 9/11 .LANGE 1970, PP. 75-90; KLAIBER 1993, PP. 400-403, note 87 91 a pp. 423/425;

For San Filippo Neri in Turin see G. CHEVALLEY, Vicende costruttive della chiesa di San Filippo Neri di Torino, in «Bollettino del Centro di Stur di Archeologici ed Artistici del Piemonte», II, 1942, PP. 6399; POR, TOGHESI 1956a, p. 13; PASSANTI 1963, PP. 58-61 POMMER 2003, PP. 61-72; L. TAMBURINI, Le Chiese di Torino dal Rinascimento al Barocco, Torino 1968, cap. XXVIIIXIX; KLAIBER 1993, PP. 336, 343; SCHNEIDER 1997, PP. 25-30.

¹³⁻ See H. A. MILLON, Guarino Guarini and the Palazzo Carignano in Turin, Ph.D. dissertation, Harvard University, Cambridge 1964, pp. 121/123, note 44-46; W. MÜLLER, The Autenticity of Guarini's Stereotomy in his Architettura Civile, in «Journal of the Society of Architectural Hi storians», vol. XXVII, n. 3, settembre 1968, PP. 202/208; F.G. TRICOMI, Guarini matematico, in Guarino Guarini 1970, vol. II, pp. 551-557;

theological studies (1639-1647) at the house of San Silvestro al Quirinale.¹⁴ It's possible that Guarini also spent time at the Theatine residence in Venice, where he may have had the chance to admire the works of Baldassare Longhena.¹⁵

Guarini's exploration of geometry likely fueled his fascination with Borromini's architectural works, where the underlying geometric principles of the layout are seldom apparent. Conversely, in Guarini's constructions, the geometric framework is not only acknowledged but also accentuated, with recurring geometric patterns. Despite his buildings openly reflecting geometric concepts, in his predominantly mathematical treatise on architecture, he cautions against the misuse of mathematics and architectural theory: "Architecture, while reliant on mathematics to a significant degree, is also an art that seeks to captivate rather than repel the sense of reason. Thus, although many of its principles are rooted in mathematical principles, if adherence to these principles compromises visual aesthetics, they should be altered, discarded, or even contradicted." For Guarini, geometry serves as a valuable tool in theoretical architectural development, but in practical application, it should yield to visual evaluations.

Guarini's architecture, being somewhat more approachable than Borromini's, has notably influenced subsequent generations of architects, extending its reach beyond Piedmont. This influence was particularly pronounced in Germany and Austria, surpassing that in France and Italy. Knowledge of Guarini's architecture, facilitated by publications from 1686 and 1737, as well as through the structures erected in Piedmont, is discernible in the works of prominent German and Austrian architects of the 18th century, including Lucas von Hildebrandt, the Dientzenhofer family, Johann Bernhard Fischer von Erlach, Johann Michael Fischer, and Balthasar Neumann.

The Turin State Archives hold approximately ninety percent of the architect's known drawings and a significant portion of the documents pertaining to Guarini's projects in the Savoy capital and its surrounding areas.

HIS YEARS IN MODENA

Guarini, along with three of his five brothers, joined the Theatine order in 1639 at the age of fifteen.¹⁶ He became a novice and relocated to Rome (1639-1647), possibly spending some time in Venice for training. Returning to Modena to take his vows in 1648, he entered the Theatine house annexed to San Vincenzo. There, he quickly became involved in the construction of the church and was appointed as a professor of philosophy in 1650. In 1653, his alternative project for one of the churches of San Vincenzo was approved. However, due to his young age of sixteen, Guarini was only able to make occasional visits to his family in the duchy. Apart from brief stays in Parma and Guastalla in 1656, and a reference in the chapter registers of Modena in the summer of 1657, little is known about Guarini's whereabouts until 1660 when he is documented in the Theatine house in Messina. In Messina, he taught philosophy and mathematics and constructed his first known building, the facade of the Santissima Annunziata and the house of the order. Both structures were damaged in the 1783 earthquake and destroyed in 1908, but their appearance has been preserved through engravings and photographs taken before the earthquake.

^{14 -} Ibid., p. 17; A. ROCA DE AMICIS, Notizie su Guarino Guarini nell Archivio Generale dei Teatini, in «Regnum Dei», L, n. 120, January December 1994, PP. 69/103; ID., Il primo Guarini e Borromini: nuove considerazio-ni, in Francesco Borromini, a cura di C. L. Frommel e E. Sladek, atti del convegno internazionale (Roma, 13-15 gennaio 2000), Milano 2000, PP. 451/457.

For Guarino Guarini and Rome see S. BENEDETTI, Guarini ed il Barocco romano, in Guarino Guarini 1970, vol. I, pp. 705/750

^{15 -} hypothesized by Susan Klaiber (1993, pp. 399-400, note 87-89); also see DARDANELLO 2003, p. 590; Richard Pommer (2003, nota 51, p.18 16- KLAIBER 1993, p. 16. The introductory essay proposed here is a revised and updated version of a sheet published in MacMillan Encyclo-pedia of Architects, a cura di A. K. Placzeck, New York-Londra 1982, vol. II, pP 265-279.



FIGURE 2- FACADE OF THE CHURCH OF S. VINCENZO OF MODENA, BOTH OLD AND NEW - GUARINO GUARINI, 1662, ASMO, MUNICIPAL ASSISTANCE AGENCY

HIS YEARS IN PARIS

In the summer of 1662, Guarini returned to Modena, where he worked on projects for the Teatina house of San Vincenzo. He may have returned from Messina earlier that year following the death of his mother. In September or October of 1662, he traveled to Paris, where he was actively involved in the construction of the new Theatine church, Sainte-Anne-la-Royale. The foundation stone was laid by the Prince of Conti on 28 November 1662. Despite using existing foundations, Guarini made significant changes to the original design by Maurizio Valperga. ¹⁷ Construction continued until 1666 when it halted due to financial constraints. Disputes over construction costs led Guarini to leave Paris for Italy in late 1666. He arrived in Turin on 4 November and was welcomed into the local Teatina house.

^{17 -} For an exhaustive discussion of Valperga's project for Sainte-Anne-Ia- Royale and other works by the architect Efr. DARDANELLO 2003, pp. 52-54

Upon Guarini's departure from Paris, only the pillars and walls of the cross and transept of Sainte-Anne-la-Royale had been erected. Later on, alterations were made to the church's axis, ¹⁸ and although the spaces were vaulted, this was only a temporary measure completed between 1714 and 1720 (the building was essentially demolished in the early 1820s). ¹⁹

The project is extensively documented through surveys conducted in Paris by various architects, engravings illustrating the plan, section, and elevation in Architettura Civile, and a preparatory drawing for the section table, bearing Guarini's autograph. As Guarini's first project widely documented, Sainte-Anne deserves the attention it has garnered.²⁰



FIGURE 3- GUARINO GUARINI – ARCHITETTURA CIVILE - ELEVATION OF SAINTE ANNE LA ROYALE IN PARIS

FIGURE 4- GUARINO GUARINI-ARCHITETTURA CIVILE - SECTION OF SAINTE ANNE LA ROYALE IN PARIS

The architectural layout is quite distinctive, featuring a Greek cross with a choir extension and a central dome spanning two levels. What sets it apart is the fact that the elongated transverse octagonal chapels are wider than the domed cross, and the tambour is wider than the area below it. These octagonal chapels boast vaults

^{18 -} When work began again, the orientation of the church was rotated. The transmo spans, already completed, became the initial spans of the nave and presbytery, while a low closed dome was built on the cross vault. In the other direction, incongruent semicircular apses with half-dimensional arches were added to define a central space. The entire building was covered by a French-style terrace

^{19 -} See the recent contribution of Susan Klaiber (2013, pp. 16-36) and Giuseppe Dardanello (2003, pp. 331-193, note 15-21 on p. 610)

with twin ribs, resembling those found in Borromini's design for the Propaganda Fide chapel. However, in the Parisian church, they support unusually large octagonal lanterns. The chapels, serving as larger spatial units both in plan and section, are clearly delineated from the transept yet firmly connected to the central space through diagonally positioned main pylons. The desired spatial continuity is highlighted by the uncommon arches with a V-shaped section, which start from the corners of the pylons and connect the cross and the chapels.

Considering Guarini's later interests, Valperga's plan for Sainte-Anne-la-Royale presents an opportunity. With the foundations already prepared for the four immense pylons of the transept, Guarini could reduce their size by building on top of them. The new pylons, three-dimensional and smaller than those originally envisioned by Valperga, opened towards the nave, the cross, and the transept, rather than enclosing the area of the drum and the dome above. At Sainte-Anne-la-Royale, Guarini recognized the effectiveness of multidirectional diagonal pylons, which later became a distinctive feature in the design of his longitudinal churches.



FIGURE 5- GUARINO GUARINI – ARCHITETTURA CIVILE - PLAN OF SAINTE ANNE LA ROYALE IN PARIS

Guarini introduced a sense of tension between the components in the design of the drum and the dome. The tamburo is conceived as two concentric structures, with the smaller inner one providing support for the double-level dome. Consequently, visitors would have had the opportunity to gaze upwards through the impost frame, past the twin columns of the internal drum, towards the sixteen windows of the external drum,

and further upward towards the small domes with lanterns nestled within the afnnular vault between the two drums.²¹

Above the drum, the inner surface of the first dome's cap, pierced in its lower half by eight windows (matching the arches of the internal drum), featured bands of paired ribs. Each pair of ribs formed an arch above the corresponding windows, originating from a single low pedestal. Intersecting in this manner, the ribs created a substantial opening for the second dome, which had a diameter exceeding sixty percent of the lower dome's. Atop the second dome rested an octagonal lantern with a spiral finial.

During an inspection of the construction site on June 14, 1665, Bernini expressed his confidence in the success of the church. Chronicles also mention that in 1664, the Bolognese priest Sebastiano Locatelli predicted that the church of Sainte-Anne-la-Royale would surpass Val-de-Grâce in beauty.

HIS YEARS IN TURIN

CHAPEL OF THE SHROUD (S. SINDONE)

Upon his arrival in Turin, Guarini received a commission to complete a chapel to house the main relic of the Savoy dynasty: The Holy Shroud, believed to be the shroud of Christ's body. In April 1667, a wooden model of his project was constructed, possibly depicted in the plan and section published in 1682 in the Theatrum Sabaudiae. In June 1668, Guarini was appointed as the ducal engineer of the chapel, a position he held until his death.

The Chapel of the Shroud boasts a rich history. ²² Its origins date back to 1611 when construction commenced on an oval chapel, intended to stand between the main apse of the cathedral and the forthcoming western wing of the ducal palace. Ascanio Vitozzi and Carlo di Castellamonte were responsible for the chapel's initial design. In 1655-1656, the chapel was elevated to align with the main floor of the ducal palace, positioning it between the cathedral's apse and the newly constructed western wing. To facilitate access, side stairs were planned on both sides of the apse, connecting the cathedral to the chapel, which was intended to remain open and visible from the nave .²³

In terms of its plan, the dome consists of six hexagons stacked on top of each other. Above the sixth level, at the dome's summit, there is an oculus partly concealed by a twelve-pointed star, curved to resemble a slender and open dome. The ellipsoidal top of the lantern displays the dove representing the Holy Spirit in glory, illuminated from below by twelve oval windows hidden at the dome's base. Similar to the structural approach seen in Sainte-Anne-la-Royale and San Lorenzo, in the Shroud, both the actual load-bearing framework and the initiated construction were present. In 1664, Amedeo di Castellamonte proposed a new project, which had made little progress by the time Guarini arrived, due to a dispute with an oratory occupying a part of the site. It appears that Guarini's new plan for a centrally located church gained approval from the order and the duke before 1668. Thus, Guarini had the opportunity to develop a structure entirely of his own, and construction commenced in January 1670.²⁴

^{21 -} The eight large elliptical arches give shape to a drum and a dome which are progressively more and more permeable to light as you go up. 22 - For the initial history of the Shroud see G. DARDANELLO, Progetti per le prime cappelle della Sindone a Torino, in Politica e cultura nell'età di Carlo Emanuele I. Torino, Parigi, Madrid, edited by M. Ma soero, S. Mamino e C. Rosso

J. B. SCOTT, Architecture for the Shroud. Relic and Ritual in Turin, Chicago Londra ,2003, parte I, cap. II, pp. 9,85.

^{23 -} In 1657 a completely new project by Bernardino Quadri and Amedeo di Canellamonte was approved, and Quadri was appointed architect. The previous structure was demolished and the new and tall chapel, circular instead of oval, began in the same year

^{24 -} For the first church of San Lorenzo see; KLAIBER 1993; EAD., The First Ducal Chapel of San Lorenzo: Turin and the Escorial, in Politica e cultura 1999, PP.329/343;



FIGURE 6- GUARINO GUARINI – ARCHITETTURA CIVILE – PLAN OF THE CHAPEL OF THE SHROUD (S. SINDONE)



FIGURE 7- GUARINO GUARINI – ARCHITETTURA CIVILE – SECTION OF THE CHAPEL OF THE SHROUD (S. SINDONE)

Twelve years after Guarini's arrival in Turin, in 1678, the dome was finished, and by the end of 1679, a cross was installed atop the lantern. Although the main altar was yet to be completed, the church was inaugurated on 12 May 1680, and Guarini celebrated the first mass there. The altar, started in September 1680, remained unfinished at Guarini's death and was consecrated only in 1696. ²⁵ While part of the opulent polychrome marble covering of the church's lower level was completed during Guarini's lifetime, the decoration of the church and chapels continued well into the eighteenth century.

SAN LORENZO CHURCH

Since this thesis focuses on the Chiesa di San Lorenzo, it will be thoroughly discussed in the following chapters. However, to provide an overall review of this remarkable church, we should note that the engraved plan for San Lorenzo comprises a narthex enhanced with banded ribs reminiscent of those seen in Sainte-Anne-la-Royale. It also includes a central space crowned by a dome, convex chapels situated at the four corners, and shallower convex chapels along the lateral axes. The choir, oval and domed, along with a vaulted annular retrochoir, complete the architectural ensemble.

^{25 -} For the main altar of San Lorenzo see. G. DARDANELLO, Cantiere di corte e imprese decorative a Torino, in Figure del Barocco in Piemonte, edited by G. Romano, Turin 1988, pp. 201-202. A precedent for the main altar of San Lorenzo, and perhaps Guarini's most important altar project, was the altar of San Nicolò in Verona, completed according to Portoghesi in 1683, the year of the death of the Teatino (see 1956a, pp. 16-20). The altar, on three levels and concave in plan, practically occupies the entrance to the choir. Numerous concentric rows of fine marble columns (smooth on the first and third levels, spiral-shaped on the second) frame the projecting group and more decidedly concave of the lower level.



FIGURE 8- GUARINO GUARINI – ARCHITETTURA CIVILE – PLAN OF CHIESA DI SAN LORENZO

The trapezoidal spandrels rest upon the convex corners of the chapels and ascend towards an impost frame, which is interrupted by eight horizontal oval openings. This discontinuous impost frame provides support for the ribbed drum-dome and the lantern. The drum-dome is additionally punctuated by large oval vertical openings on its lower level. Above, at the points where pairs of arches intersect, the sails have been removed, allowing the intersections of the arches to form the octagonal base ring, encircled by light, for the lantern. At an even higher point, on the intrados of the lower cap of the lantern, pairs of arched ribs, strikingly similar to those applied to the lower level of the dome of Sainte Anne-la-Royale, commence in opposite directions, intersecting to create an opening for the terminal dome of the lantern, featuring eight oval vertical windows at its base.



FIGURE 9 - (RIGHT) - GUARINO GUARINI – ARCHITETTURA CIVILE – SECTION OF CHIESA DI SAN LORENZO FIGURE 10- (LEFT) - GUARINO GUARINI – ARCHITETTURA CIVILE – ELEVATION OF CHIESA DI SAN LORENZO

A drum and dome cannot be supported effectively by such a disjointed impost frame, where spandrels and arches offer only minimal reinforcement. In reality, four sizable arches, concealed behind the walls of the corner chapels, ascend behind the oval windows of the impost frame to uphold the dome's drum.²⁶ Four smaller arches extend diagonally from the four larger ones, resting on their backs and collectively forming an octagonal foundation for the arches of the drum-dome.

SANT'ANDREA CHURCH

In early 1672, Guarini received a commission, likely from Carlo Emanuele II, to design a church for the Missionaries of San Vincenzo De Paoli, who had recently arrived in Turin in 1655. With an initial donation from the duke, construction of the Immaculate Conception commenced in June 1673, although actual work began in 1675. However, by 1677, two years after the duke's passing, funds were depleted, uncertain progress at the main frame level. It wasn't until 1694, when several bequests were made, that construction resumed. Three years later, in 1697, the church was finally consecrated. The arrangement of the Immaculate Conception, one of Guarini's three longitudinal churches, alongside San Filippo Neri in Turin and Santa Maria Ettinga in Prague, is documented. It comprises two circular bays with ribbed domes, separated by a narrowed transept. This

^{26 -} At Sainte-Anne-la-Royale, Guarini does not reveal any support the weight of the external drum. The hidden arches above the level of the chapel vault also supported four diagonal pillars. In Turin his desire to amaze, confuse and seduce has grown to the point of involving the entire structure.

transept is bordered by diagonal pylons and features a continuous vault with diagonal ribs converging at its center. $^{\rm 27}$



FIGURE 11- (UP) - GUARINO GUARINI – ARCHITETTURA CIVILE - SAN FILIPPO NERI – TORINO – INTERIOR SECTION



FIGURE 12- (DOWN RIGHT) - GUARINO GUARINI – ARCHITETTURA CIVILE - SAN FILIPPO NERI – TORINO – ELEVATION FIGURE 13- (DOWN LEFT) - GUARINO GUARINI – ARCHITETTURA CIVILE - SAN FILIPPO NERI – TORINO – PLAN

^{27 -} The circular bays contain side chapels, the transept, chapels the hexagonal ones; a small retrocoto with dome was placed behind the large altar; none of the three spaces is dominant. The facade, with a central curve that reverses to the sides, reflects the first circular span. The plan of the facade belongs to the group of his churches which also includes Sainte Anne-la-Royale in Paris, Santa Maria della Divina Provvidenza in Lisbon, Santa Maria Eninga in Prague.

When Guarini undertook the design of the church and sanctuary of the Consolata in Turin in 1678, he faced the challenge of working with an existing structure. It seems that shortly after 1675, the abbot commissioned a new church to replace a deteriorating Romanesque building with a longitudinal layout. However, this initial plan was deemed insufficient, and by the time Guarini was enlisted to revise the design, at least four of the main nave pillars had already been erected. An engraving of Guarini's proposed plan and section for the church of Sant'Andrea and the adjacent hexagonal sanctuary of the Consolata reveals the incorporation of elements from the pre-existing structure. By 1701, much of the construction was completed, except for the western hemicycle, which still housed the main altar and the crypt of the original Romanesque church. The dome over the hexagonal sanctuary was finished by 1703. In 1729, Filippo Juvarra added a new sanctuary and altar to the complex. Later, in 1899-1904, Carlo Ceppi enhanced the sanctuary by adding oval and hexagonal chapels on three sides, imbuing the space with his neo-Baroque aesthetic. Guarini's design transforms the original longitudinal layout of the church of Sant'Andrea, which was already partially constructed, into an oval shape. He replaces the entrance and apse with two hemicycles, each featuring three radial chapels, positioned on either side of a rectangular central bay. The entrance is relocated from the eastern side to the southern side, leading into a rectangular space between the two hemicycles. This space aligns with the new hexagonal sanctuary, situated on the opposite (north) side of the hall. The sanctuary was intended to have a low hexagonal star vault as its roof, supported by four rhomboidal pillars and elevated above an ambulatory. This ambulatory, in turn, is covered with alternating circular and elliptical domes.²⁸

RACCONIGI CASTLE

While the sole documented payment from Emanuele Filiberto Amedeo di Savoia Carignano to Guarini for designing the Racconigi castle's prison dates back to 1677, it's plausible that the project predates site preparation and construction commenced in 1676. The transformation of the original medieval castle with its four square corner towers and an open courtyard remained incomplete at the time of Guarini's passing. Progress might have slowed after 1679, as available funds were redirected toward constructing Palazzo Carignano in Turin, which commenced in the same year. By 1712, as depicted in a print by Bartolomeo Giuseppe Tasnière, only the northern wing of the castle and the tall central pavilion erected to replace the former courtyard had been completed, offering a glimpse from a southern elevated perspective.

^{28 -} The lighting in the two structures was different. In Sant'Andrea there were lunettes above each chapel, windows at the base of the semi-domes with ribs of the hemicycles, as well as in the barrel vaults of the central span; in the chapel of the sanctuary, however, there was a third level of nearly square windows below the two upper levels.



FIGURE 14- G. GUARINI. PLAN AND FACADE OF THE PALLACIO OF THE HOLY PRINCE FILIBERTO OF SAVOY IN RACCONIGI FOR THE REYDIFICATION PROJECT

Guarini's envisioned project for a staircase featuring sinuous ramps on the north façade, leading towards the garden, seemingly never came to fruition. Instead, a pair of symmetrical stairs, possibly from the early 18th century and attributed to an unidentified designer, were in place by the central platform, based on a drawing dating approximately between 1710 and 1730. The current staircase, which descends towards the garden, was designed by Giambattista Borra and replaced the previous structure in 1755.

The northern aspect of the castle, aside from the stairs, largely retains Guarini's original design: a central tower segment, boasting four floors and seven bays, features a terrace bordered by slightly projecting single bays. This is complemented by corner bodies projecting outward, each comprising three bays and adorned with hipped roofs crowned by small elements with arched roofs. Positioned centrally within the edifice, the elevated pavilion that illuminates the hall (three bays on the facade, five on the sides) is topped with a bell-shaped roof. The southern side, along with the eastern and western wings, the hall, and the chapel, underwent renovations and reconstruction by Borra between 1756 and 1760, showcasing early neoclassical features with hints of Anglo-Saxon influences. Atop the straight flight of stairs, enclosed by two retaining walls, a pediment atop a tetrastyle portico signals the entrance to the main hall. In the 1830s and 1840s, two-story wings designed by Ernesto Melano were appended to the east and west sides.



FIGURE 15- G. GUARINI. COMPLETE PROJECT FOR THE CASTLE, COURTYARD OF HONOR, STABLES AND COMPLETE VIEWS TO THE SIDES OF THE CASTLE. PLAN



FIGURE 16- COLLABORATOR OF G. GUARINI. PROJECT WITH DECORATIONS FOR THE SOUTHERN ACCESS STAIRCASE



FIGURE 17- G. GUARINI. GOOSENECK STAIRCASE. PLAN STEPS AND RETAINING WALLS, AND DECORATION ELEVATION

PALAZZO CARIGNANO

Emanuele Filiberto Amedeo di Savoia-Carignano (1628-1709), the commissioner of Palazzo Carignano, the most significant palace by Guarini, was intimately acquainted with Teatine architecture due to his involvement in projects in Racconigi, San Lorenzo, and the chapel of the Shroud, which was nearly completed in 1679. It seems Emanuele Filiberto had a longstanding interest in architecture, as chronicles suggest he engaged in lively discussions with Guarini as early as 1674, demonstrating a sophisticated understanding of architecture and fortifications in various aspects. On 9th June 1680, Emanuele Filiberto appointed Guarini as the theologian of Casa Carignano. Despite being deaf and mute since birth, the prince was proficient in lip-reading and speech. At his mother's insistence, Maria di Borbone Soissons, he married to secure the family lineage, as he was the potential heir of the dynasty until the birth of Charles Emmanuel III in 1699, son of the dukes of Savoy, Victor Amadeus II, and Anne of Orléans. When he commissioned Guarini to design Palazzo Carignano in 1679, Emanuele Filiberto probably didn't anticipate the role it would play in his mother's dynastic ambitions. He might have simply thought that the palace would pass to the Soissons branch or to the duke himself upon his death. Perhaps Guarini envisioned a palace with a majestic appearance, drawing inspiration from Bernini's three projects for the Louvre, without his client's awareness. Emanuele Filiberto's heirs would eventually reside in the palace and succeed to the leadership of the duchy. The residence was built on land owned by Emanuele Filiberto's father, Tommaso di Savoia-Carignano, since the 1740s, outside the eastern walls of Turin, towards the Po. Guarini was compensated for the palace project on 6th August 1679. Excavation for the foundations began in January 1680, and by October of the same year, the entrance vault was completed. Between the end of 1681 and the beginning of 1682, the rustic walls and roof were finished. The atrium and the large hall were vaulted between July 1682 and the end of 1683. However, plans to complete the eastern half of the palace and close the courtyard were ultimately abandoned. Internal decoration began in December 1683 and was sufficiently advanced by the summer of 1693 to bless the rooms, but it was only completed in the early years of the following century. The palace's garage and stables were finished in 1697-1698, while some other works were completed in 1686. Palazzo Carignano stands out as the most significant palace project in Italy during the late seventeenth century, boasting a distinct regal and representative character.

Following Guarini's passing, his architectural legacy was carried on by Michelangelo Garove (Racconigi Castle, San Filippo Neri, Palazzo Carignano), Antonio Bertola (Chapel of the Shroud, San Filippo Neri, Palazzo Carignano), and Giovanni Francesco Baroncelli (Palazzo Carignano, Racconigi Castle). Other structures designed and constructed by these architects demonstrate a deep understanding of Guarini's work. The publication of some of Guarini's drawings in the "Dissegni di Architettura Civile et Ecclesiastica" of 1686 likely sparked increased interest in his work. With the passing of Baroncelli (1694), Garove (1713), and Bertola (1719), Guarini's direct influence gradually waned. The prominent Piedmontese architects of the subsequent generation, such as Francesco Gallo and Gian Giacomo Plantery, while occasionally showing familiarity with Guarini's buildings, tended to adhere to the restrained style introduced to Piedmont by Amedeo di Castella Monte, following the tradition of Bernini and Carlo Fontana.



FIGURE 18- ARCHIVIO DI STATO TORINO WEBSITE - G. GUARINI. PROJECT OF THE ENTIRE CARIGNANO PALACE WITH TWO ATRIUMS TO THE EAST AND WEST, RECTANGULAR WITH PILLARS AND COLUMNS



FIGURE 19- ARCHIVIO DI STATO TORINO WEBSITE - G. GUARINI. STUDY FOR THE DOUBLE STAIRCASE ON A BROKEN LINE, BETWEEN THE OVAL ATRIUM AND THE FACADE



FIGURE 20- ARCHIVIO DI STATO TORINO WEBSITE - G. GUARINI. STUDY FOR THE PROJECT OF THE OVAL ATRIUM, FACADE AND STAIRWELLS.

FIGURE 21- ARCHIVIO DI STATO TORINO WEBSITE - G. GUARINI. STUDY FOR THE PLANNED FAÇADE (BOTTOM) AND THE OVAL AND RECTANGULAR ATRIUMS



FIGURE 22- ARCHIVIO DI STATO TORINO WEBSITE - G. GUARINI AND COLLABORATOR Z. STUDY FOR THE ATRIUMS



FIGURE 23- ARCHIVIO DI STATO TORINO WEBSITE - G. GUARINI. STUDY FOR THE PROJECT OF THE OVAL ATRIUM



FIGURE 24- ARCHIVIO DI STATO TORINO WEBSITE - G. GUARINI. STUDY FOR THE VAULT, WITH LUNETTES OF THE ATRIUM, PERHAPS FOR ITS ARMATURE



FIGURE 25- ARCHIVIO DI STATO TORINO WEBSITE - G. GUARINI. « FIRST SHELF AND SIDE OF THE HALL». LONGITUDINAL SECTION OF THE OVAL AND RECTANGULAR ATRIUMS, TOWARDS THE STAIRCASE



FIGURE 26- ARCHIVIO DI STATO TORINO WEBSITE - G. GUARINI. « PROFILE OF THE LARGE STAIRCASE ». SECTION OF ONE OF THE STAIRCASES

NICE, CASALE, VICENZO

Guarino Guarini's architectural legacy includes several unrealized projects, including the church of San Gaietani, which he designed for the Theatines of Vicenza in 1675. Although the project remained unbuilt, Guarini was later invited to submit a design proposal for the rebuilding of the church of San Stefano in Nizza (today nice, France) Monferrato. Due to time constraints and ongoing projects in Turin²⁹, Guarini's design and site inspection were submitted in absentia. The Savoy Dynasty, which held possession of Nizza at the time, planned to finance the project fully, but the dynasty's military conflicts with France hindered the plan, and Guarini's design was ultimately rejected. It was not until the eighteenth century ³⁰that the project was given to Bernardo Antonio Vittone, who built the church on a different site in Nizza. ³¹

Guarini's unrealized church designs are notable for their innovative approach to sacred architecture. An examples is San Filippo Neri in Casale Monferrato, which was later constructed by Sebastiano Guala without adhering to Guarini's original plans. The design represents variation of the central quincunx church concept, characterized by a modified Greek cross plan with adjunct spaces at the four corners.

The design of San Gaietani, possibly conceived in the late 1660s, features a central plan based on a pentagon inscribed within a circle. Five semi-circular chapels, including one designated for the altar with a columnar screen akin to San Lorenzo, open onto the space of the inscribed pentagon. The apse chapels contribute to an undulating facade, and the entrance is situated within a narrow span between the side chapels. In the drum, intersecting arcs form a second inscribed pentagon, and the dome supports a pentagonal lantern with a five-pointed star opening at its center. Although the church was ultimately constructed on a different site and according to a new design by Vittone, Guarini's design remains an important example of his innovative approach to architecture.

^{29 -} Nino Carboneri, "Introduzione," xiii. "L'arrivo a Torino si riteneva conseguente ad una visita a Nizza per i progetti della Chiesa di San Gaetano; più probabilmente il sopralluogo fu compiuto già da Torino, ove venne chiarmato direttamente da Parigi per la fabbrica della Chiesa di San Lorenzo (pure dei Teatini), che si trascinava da tempo."

³⁰ Oeschlin, "Tra due fuochi," 285. "Per Wittkower Vittone è un 'genio ossessionato.' Una definizione che ricorda molto le considerazioni, già di per sé eccessivamente mistificate, degli storici dell'arte su Borromini e Guarini, divenute topoi ripetuti circa la figura dell'artista in epoca barocca. Infine Wittkower chiude le sue considerazioni su un arco di trecento anni di evoluzione storica dell'architettura con un rinvio all'interesse di Vittone per le armonie musicali: 'il principio e la ine si incontrano.' La logica della storia dell'arte e dello sviluppo stilistico, assieme a una concezione limitativa dell'epoca barocca come fenomeno in sé concluso, hanno il sopravvento su una visione non preconcetta di un'architettura in rapido e continuo sviluppo nel corso del XVIII secolo." 31 - Guarini, Guarino Meek, 139-40.





FIGURE 27- (LEFT) - GUARINO GUARINI – ARCHITETTURA CIVILE – S. GAIETANI– NICE – ELEVATION AND PLAN FIGURE 28- (RIGHT) - GUARINO GUARINI – ARCHITETTURA CIVILE – S. GAIETANI- NICE–SECTION AND PLAN

LONGITUDINAL CHURCHES

Guarini's projects for longitudinal churches, namely San Filippo Neri in Turin, Santa Maria della Divina Province in Lisbon, and Santa Maria Ettinga in Prague, are cataloged in this sequence in his Architettura Civile. San Filippo, designed in 1679, comprises three octagonal bays with side chapels—reminiscent of Sainte Anne-la-Royale—formed by diagonally oriented pillars. At both ends, there's a main chapel with an apse and an entrance of identical shape, defining a double-ended plan. The nave, entrance bay, and apse feature ribbed vaults, while transverse arches delineate the individual bays, all reaching the same height without a dominant spatial center.

In the design of the Prague church (1679), also featuring a double termination, an oval entrance chapel and a bay preceding the choir flank a larger octagonal central space. Lanterns adorn the center of the vault in these three areas, positioned at the intersection of the ribs. Diagonal pylons delineate the separation of each section, simultaneously emphasizing the central span while maintaining its dominance.



FIGURE 29- GUARINO GUARINI – ARCHITETTURA CIVILE - SANTA MARIA DELLA DIVINA PROVINCE – LISBON – SECTION



FIGURE 30- GUARINO GUARINI – ARCHITETTURA CIVILE - SANTA MARIA DELLA DIVINA PROVINCE – LISBON – PLAN

In the design of the church in Lisbon, configured in the shape of a Latin cross, the nave comprises two nearly circular bays with oval chapels. The transept is composed of oval chapels, the apse takes a semicircular form, and spatially, the cross appears as an eroded square. Each section of the nave, the chapels, the sanctuary, the transepts, and the cross is adorned with lanterns. The vaulted system displays a more fluid and undulating structure, characterized by a more pronounced rhythm compared to San Filippo and Santa Maria Ettinga. This design likely represents the most recent addition to the group, possibly conceived between 1678 and 1680.
CHAPTER 3

GUARINI'S TREATISES

Guarino Guarini was an influential architect and mathematician who authored a variety of treatises covering subjects such as architecture, fortifications, geometry, philosophy, theology, and theatrical works. His writings demonstrate his deep understanding of both practical and theoretical aspects of these fields, often incorporating mathematical principles into his discussions.

His seminal works include:

- The **Placita Philosophica** (A System of Philosophy, 1665), which establishes his philosophical system and influences his architectural approach.
- The Architettura Civile (Civil Architecture, 1737), a practical manual on civil architecture published posthumously, highlighting his contributions to architectural design.
- the Euclides Adauctus (The Advancement of Euclid, 1671), which applies projective and descriptive geometry to stereotomy, marking an early and influential work in this area.
- the Modo di Misurare le Fabbriche (Methods of Measurement for Construction, 1674), focusing on methods of measuring buildings and essential for understanding construction techniques.
- the Trattato di Fortificazione (Treatise on fortifications, 1676), a treatise on military fortifications that applies mathematical principles to defensive architecture.
- the Leges Temporum et Planetarum (The Laws of Time and the Planets, 1683), which discusses the laws of time and planetary movements, reflecting his interest in astronomy.
- The Compendio della Sfera Celeste (Compendium of the Celestial Sphere, 1683), covering the celestial sphere and its relation to architectural principles.
- The Cælestis Mathematicae (Celestial Mathematics, 1683), which synthesizes his earlier work on celestial mathematics.
- Guarino Guarini is the author of ten published works two more works that are not mentioned as his significant works are **Dissegni d'archittetura civile, et ecclesiastica** (Designs for Civil and Ecclesiastical Architecture, 1683) and La Pietà Trionfante (The Triumph of Mercy, 1660)

PLACITA PHILOSOPHICA

Guarino Guarini's architectural theory and practice were deeply rooted in his philosophical system, primarily articulated in his early work "Placita Philosophica".³² The Placita, a comprehensive 868page treatise published in 1665, established Guarini's complex philosophical framework that he developed throughout his life. This philosophical foundation is integral to understanding his culminating practical theory of civil architecture, as seen in his later publication "Architettura Civile".³³

Guarini's philosophy, which encompassed mathematics, geometry, and celestial themes, was interpreted within a didactic, pragmatic, and structural framework in his architectural works, such as Saint-Anne-la-Royale and San Lorenzo, commissioned in 1668 shortly after the Placita's publication.³⁴ His dedication to notable figures like Torres in the Placita and Ferrari in the "Modo di Misurare" reflects Guarini's inclination to interweave philosophical, political, and power dynamics in his writings and designs. This synthesis of Guarini's philosophical thought profoundly influenced his architectural craft, particularly during his Parisian period in the 1660s, which built upon his earlier experiences in Messina, Modena, and Paris.³⁵



FIGURE 31- GUARINO GUARINI, PLACITA PHILOSOPHICA, (PARIS: APUD DIONYSIUM THIERRY, 1665).

ARCHITETTURA CIVILE

Gianfrancesco Mairesse's posthumous publication in Turin in 1737 ³⁶, The Architettura Civile, embodies Guarino Guarini's comprehensive pragmatic philosophy of civil architecture. Unlike his previous works in Latin, this treatise is the only one by Guarini written in the Italian vernacular. This book functions as a practical architecture handbook, offering concepts and operational instructions for building design and construction. As a climax of his architectural career, Guarini's approach in the Architettura Civile combines academic understanding with real-world applications. Guarini's philosophical and intellectual depth are also evident in the work, despite its practical orientation. It highlights Guarini's conviction that architecture is a synthesis of art and science and provides guidelines for balancing practicality and aesthetic beauty in architectural design. Guarini's avant-garde treatment of architectural form and space, addressing concerns like proportion, symmetry, and spatial order, is shown in the Architettura Civile. The Architettura Civile, which was published after Guarini's death, has a significant number of errors and erroneous references, indicating that Guarini's final editorial control may not have had a positive impact on it. Although Bernardo Vittone, the editor, could have added something, there is indication that he did not completely understand Guarini's geometric reasoning. This is especially important since comprehending Guarini's architectural principles requires a grasp of his geometric ideas, which are expounded in his other writings, such as Euclides Adauctus.

Guarini explores projective and descriptive geometry in Euclides Adauctus, which was published in 1671. This work demonstrates his methodical approach to architectural design. His contributions to the development of orthogonal projections are highlighted by his theories on geometric projections, which may have originated from his previous work Ortografia Gettata. This essay demonstrates Guarini's conviction that geometry is an essential instrument for philosophical and architectural research, bridging geometric accuracy with metaphysical

36 - For a detailed description of Guarini's designs, and the engravings produced for the Dissegni d'architettura Civile, see: Aldo Bertini, "Il Disegno del Guarini e le Incisioni del Trattato di 'Architettura Civile'" in Guarino Guarini e l'Internazionalità del Barocco (Torino: Accademia delle Scienze, 1970), 597-610.

^{32 -} Mitrović, Branko. "Guarino Guarini's Architectural Theory and Counterreformation Aristotelianism: Visuality and Aesthetics in Architectura Civile and Placita Philosophica.", 2020, 375-396.

^{33 -} Flynn, Elisabeth L. "A Baroque Architectural Text: The Architettura Civile of Guarino Guarini". Arris. Volume 1. Chapel Hill: The University of North Carolina Press, 1989, pp. 18-28.

^{34 -} Conversano, Elisa. "La cultura architettonica nelle missioni teatine in Oriente e l'architettura di Guarino Guarini". Tesi di dottorato, Scuola dottorale Cultura e trasformazione della città e del territorio, 2011/2012. Tutor: Prof. Mario Panizza.

^{35 -} The division of Guarini's life into three periods is proposed by Wittkower in the speech at the "Guarino Guarini e l'internazionalità del Barocco" conference held in Turin in 1968

comprehension. In conclusion, Guarini had a significant impact on Baroque architecture and mathematical philosophy, as shown by the Architettura Civile and Euclides Adauctus taken together. Together, the former, a useful manual for architects, and the latter, a seminal work on geometric theory, demonstrate Guarini's lasting influence as a Renaissance architect and thinker.

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FIGURE 32- GUARINO GUARINI, ARCHITETTURA CIVILE... (TURIN: APPRESSO GIANFRANCESCO MAIRESSE ALL'INSEGNA DI SANTA TERESA DI GESÙ, 1737)

EUCLIDES ADAUCTUS

Euclides Adauctus, a ground-breaking book by Guarino Guarini published in 1671, greatly enhanced the use of projective and descriptive geometry, especially in stereotomy. Guarini is recognized as a significant figure among the early contributors to geometric theory because of this work, which is important to the history of the Science of Representation. In addition to outlining mathematical techniques for practical architectural applications, Euclides Adauctus also aims to uncover a logical philosophical basis for existence. Guarini's investigation reveals his profound interest in metaphysics as well as physics, highlighting his conviction that the cosmos had innate harmony and order, which he thought could be understood mathematically. Guarini maintained that one may learn about the divine order ruling the cosmos by revealing the geometric and mathematical foundations underlying physical forms. Euclides Adauctus, published six years after his seminal Placita Philosophica, is Guarini's second significant book. Its place in Guarini's publishing chronology and the purposeful use of Latin in its language, which highlights its scientific rigor and intended audience among the era's intellectuals and practitioners, both emphasize the book's significance. In the context of Orthogonal Projections, an area where Guarini's efforts helped lay the way for later advancements in architectural representation, Euclides Adauctus assumes particular significance.



FIGURE 33-GUARINO GUARINI, EUCLIDES ADAUCTUS ET METHODICUS MATHEMATICAQ; UNIVERSALIS ... (TURIN: TYPIS BARTHOLOMAEI ZAPATAE BIBLIOPOLAE, 1671)

MODO DI MISURARE LE FABBRICHE

The 1674 publication of Guarino Guarini's Modo di Misurare le Fabbriche is dedicated to Giovanni Andrea Ferrari, painter, minister of finance under Amedeo di Castellamonte, and count of Bagnolo. Although Guarini's book has a limited impact, its subject matter is extensive and has the potential to increase the duke's influence. (Effeto è il libro delle mie deboli forze, la materia di lui è una vastità proportionata all' ampie sue prerogative).³⁷ Designed to improve the technical skills of builders and architects in the House of Savoy, this treatise embodies Guarini's pragmatic approach to architectural philosophy. According to Guarini, the treatise's goal is to make the House of Savoy into a machine that is precisely in sync with celestial sphere movement (so bene moderna questa machina, che non vi e movimento di sfera sì perfettamente aggiustato), which will enable Ferrari to perform his duties as Vittorio Amedeo I, the Duke of Savoy's minister of finance, more effectively. ³⁸ One may find a detailed inventory of all the instruments needed for building surveys and solid surface measurements in the Modo di Misurare le Fabbriche. Guarini's meticulous dedication to detail highlights the importance of measuring methods in architectural design, especially when creating intricate structural components like domes and vaults. Through the description of these tools and methods of measurement, Guarini aims to standardize and enhance building procedures, guaranteeing accuracy and effectiveness in carrying out architectural projects supported by the Savoy dynasty. Beyond its technical features, the work combines mathematical ideas with practical architecture, reflecting Guarini's larger intellectual aspirations. His focus on empirical observation and geometric accuracy highlights his conviction that scientific knowledge has the transforming ability to shape architectural forms. Thus, Guarini's dedication to expanding theoretical knowledge and practical application in the area of architecture throughout the Baroque period is demonstrated by the Modo di Misurare le Fabbriche.



FIGURE 34- GUARINO GUARINI, MODO DI MISURARE LE FABRICHE (TURIN: PER GL'HEREDI GIANELLI, 1674).

^{37 -} Guarino Guarini, Compendio della Sfera Celeste (Turin: Giorgio Colonna, 1675).

^{38 -} Guarino Guarini, Modo di Misurare le Fabbriche (Turin: Per gl'Heredi Gianelli, 1674), 3-4.

TRATTATO DI FORTIFICAZIONE

A 1676 publication, the Trattato di Fortificazione, demonstrates Guarini's fascination in military construction. This work offers guidelines for building fortresses to defend towns and covers defensive fortification construction procedures, despite the author's never holding a steady job as a military architect. Guarini emphasized the importance of exact military strategy using mathematics and fortification systems in his dedication to Prince Ludovico Giulio di Carignano.³⁹ The dissertation gives detailed instructions on how to use mathematical concepts to construct defensive fortifications in Piedmont and Turin. "Every part of the fortress has the ability to defend the city not only by direct offense from the front but also by parallel and oblique defense," says Guarini, introducing the first fortification construction theory.⁴⁰



FIGURE 35- GUARINO GUARINI, TRATTATO DI FORTIFICATIONE, CHE HORA SI USA IN FIANDRA, FRANCIA & ITALIA (TURIN: APPRESSO GL'HEREDI DI CARLO GIANELLI, 1676).

LEGES TEMPORUM ET PLANETARUM

Leges Temporum et Planetarum by Guarini was published after his death. His understanding of universalism in astronomy and mathematics is demonstrated by this work, the Compendio della Sfera Celeste, and the Caelestis Mathematicae. The Compendio della Sfera Celeste (1675) offered notions that were expanded upon in the Leges Temporum et Planetarum (1678). These pieces increase the Duke of Savoy's impact by examining the heavenly sphere and its relationship to politics and power.



FIGURE 36-GUARINO GUARINI, LEGES TEMPORUM (TURIN: EX TYPOGRAPHIA HAEREDUM CAROLI IANELLI, 1678).

^{39 -} Guarini, Fortificatione, "Che saggio di virtù militare non pompeggio nel Conte de Soisons.; James McQuillan, "The Treatise on Fortification by Guarino Guarini" in the Nexus Network Journal 16 (2014)

^{40 -} Gommes, Portogallo, 37 - "Principio I: Ogni parte della fortezza dove potersi diffendere da cittadini non solo con offesa diretta, e per fronte, ma anche con difessa paraella, & obliqua."

COMPENDIO DELLA SFERA CELESTE

The celestial sphere is discussed in terms of politics and astronomy in the Compendio della Sfera Celeste, which was published in 1675. With its extensive topic matter, this dissertation, devoted to Truchi, seeks to increase the Duke of Savoy's influence. The Leges Temporum et Planetarum and the Caelestis Mathematicae, which expand on similar subjects, were influenced by this book.



Appreflo Giorgio Colonna,

FIGURE 37- GUARINO GUARINI, COMPENDIO DELLA SFERA CELESTE (TURIN: APPRESSO GIORGIO COLONNA, 1675)

CÆLESTIS MATHEMATICAE

The Compendio della Sfera Celeste and the Leges Temporum et Planetarum, two earlier works, were developed into the posthumously released Caelestis Mathematicae in 1683. Guarini's deep interest in mathematics and astronomy is evident in this dissertation, which links the aspirations of the Savoy dynasty to the motions of the celestial sphere. Soon after Guarini's passing, the Cælestis Mathematicae was released, concluding his contributions to the science of celestial mechanics.



FIGURE 38- GUARINO GUARINI, CAELESTIS MATHEMATICAE PARS PRIMA [-SECUNDA] (MILAN: EX TYPOGRAPHIA LUDOVICI MONTIAE, 1683)

CHAPTER 4

EARLY HISTORY OF SAN LORENZO, TURIN

The history of San Lorenzo in turin, crucial to understanding Guarino Guarini's remarkable project there, has often been obscured in literature. Exploring its evolution from its origins in 1557 sheds light on guarini's achievement and the church's dual role as both theatine and ducal. According to Klaiber's research, the church's history is intertwined with duke Emanuele Filiberto's vow to build churches dedicated to st. Lawrence after a significant victory in 1557 ⁴¹.

Contrary to the common belief that the church stood on its present site since the 1560s, evidence suggests that Emanuele Filiberto's initial church of S. Lorenzo was situated elsewhere. ⁴²

Later, in 1608-09, further renovations expanded and remodeled the church.⁴³ When the theatines were granted the current site in 1634, it marked a pivotal move from a private chapel to a public church.

This shift in ownership coincided with duke Vittorio Amedeo I's urbanistic considerations, aiming to restructure the area around the Palazzo di S. Giovanni and Piazza Castello.⁴⁴ His plans for a new palace and SS. Sindone chapel redirected attention and purpose away from the original S. Lorenzo site.

The transition of S. Lorenzo to the theatines in 1634 was a strategic move to alleviate crowding and provide a prominent location for a new church. However, this change may have switched the church from a centralized to a latin cross plan with its facade on the side street. Slowed down the project.⁴⁵

Klaiber hints at possible inspiration for S. Lorenzo's design from Philip II's escorial, which was constructed on a centralized plan with a large dome.⁴⁶ This inspiration possibly influenced the "tempio di forma sferica" of Vittorio Amedeo I, Guarini's patron.

Guarini's San Lorenzo, as the only remaining theatine church, poses intriguing questions and opportunities for historians' despite being well-documented. Recent research has significantly contributed to understanding the church, including discoveries about early 17th-century work on its facade.

^{41 -} On these vows see L. Cibrario, Storia di Torino. 2 vols., Turin, 1846, ii, p. 469; and AGT, Torino.

^{42 -} In 1563, he chose to renovate the small romanesque church of S. Maria del Presepe, dedicating it to S. Lorenzo. (Cibrario, II, p. 469. Cibrario's placement of the church next to the city wall is confirmed by a 1583 description of the area. See AST, Corte, Arcivescovadi e vescovadi, Torino, mazzo) This restoration commenced in 1563 and saw substantial alterations, including consecration of the high altar in 1564 (Tamburini, p. 201, n. 6, derived from Crepaldi, p. 19)-Crepaldi, p. 11. Crepaldi traces the church back to 1177. But the church of Santa Maria della Neve (Crepaldi, p. 13) with which he equates Santa Maria del Presepe, was probably another name for Santa Maria de Dompno or de Dompuo. One of the three churches on the site of the current Cathedral of S. Giovanni in Turin (F. Rondolino, II Duomo di Torino. Turin, 1898, pp. 46, 193). Robison, S. Lorenzo, pp. 67-68, 84, is rightly skeptical of Crepaldi's identification of the current vestibule as the former romanesque church, but he does not suspect that S. Maria del presepe was never located on the current site of S. Lorenzo.

^{43 -} See A. Barghini, "il Palazzo Ducale a Torino (1562-1606)," Atti e rasseqna tecnica della societa deal' Inqeqneri e architetti in Torino, n.s. 42, n. 7-8, July-August 1988, pp. 127-134. Barghini, p. 130, notes the existence of a chapel of S. Lorenzo in the palace compound in the 1570s.

^{44 -} A copy of Giovanni Caracha's plan is found in BRT, Incisioni, Ill.15. The plan by Gerolamo Righettino plan is found in AST, corte, Museo Storico. Both are reproduced in A. Cavallari Murat, ed., forma urbana e architettura nella Torino Barocca. Turin, 1968, II, pp. 316-317, 319-320.

^{45 -} One clear account of the church's location next to the city wall is given in AST, riunite, art. 207, reg. 15 [ducal building accounts 1615-1620]

^{46 -} Pollak, Turin, p. 48 and her fig. 35, publishes a sketch by Ascanio Vitozzi for a stairway apparently located at the abutment of the new palace and the gallery leading to the Castello. She dates the sketch CA. 1584, i.e. Part of the mid-1580s palace construction campaign.

GUARINI AT SAN LORENZO, TURIN

The historical background of S. Lorenzo in Turin spanned over a century, yet when Guarini arrived in the city in 1666, the site on Piazza Castello lacked a significant architectural presence.⁴⁷ His initial recorded architectural engagement in Turin was related to the SS. Sindone: in April 1667 ⁴⁸

The development of Guarini's design for S. Lorenzo seems to have occurred in cycle with his involvement in the SS. Sindone. Records suggest that by August 1668, Guarini was among the priests consulted, indicating the potential adoption of his design by the duke and theatines by that time. It's proposed that Guarini presented the design as early as 1667. Construction for S. Lorenzo commenced in early 1670, signifying the duke's approval of the new design.⁴⁹

The reasons behind the modifications to the original design are up for debate. The church's current design was already the work of renowned court architect Amedeo di Castellamonte at the time. It seems unlikely, given the level of architectural skill at the court, that the duke asked Guarini to make revisions to the plans.⁵⁰ Furthermore, there is no evidence that Guarini was specifically tasked with designing S. Lorenzo when he was called upon by the duke or the Padre generale in Rome.

CONSTRUCTION OF GUARINI'S CHURCH

The observations regarding Guarini's construction process at San Lorenzo church in Turin, outlined in Susan Elizabeth Klaiber's analysis, suggest a relative freedom from prior design constraints despite speculation regarding the utilization of Castellamonte's foundations.⁵¹ Klaiber references the 'Libro della Fabrica della chiesa nuova di San Lorenzo,' archived in AST, corte, which meticulously details the church's construction and has been thoughtfully summarized by Robison.⁵²

Construction activities commenced in january 1670 and included demolitions along Strada Nuova in April and may of the same year. While bricks were delivered by June 1670, interpretations of tie rods delivered in September 1670 differ, with some suggesting potential reinforcement for a temporary chapel under the entranceway.⁵³ Foundation works initiated in June 1670 and continued until at least July 1671, parallel to the preparation of bases and capitals for columns, ensuring their readiness for subsequent stages.⁵⁴

^{47 -} A. Lange, "Disegni e documenti di Guarino Guarini ", p. 233; schede vesme. II, p. 551

^{48 -} There were payments noted in the ducal accounts for a model representing the "New design" for the Sindone. Later, in may 1668, Carlo Emanuele II formally appointed Guarini as the "Ingegniero" for the SS. Sindone, entitling him to an annual stipend of 1,000 lire [the carpenter Giovanni Rosso was paid for the model on 30 April 1667 (AST, Riunite, art. 179/8, 1667 in 1669, n. 8, noted in Carboneri, "Vicenda," p. 106, n. 38). On the patent, see schede vesme. Ii, p. 551]

^{49 -} Crepaldi, p. 46: "nel 1667 il Guarini presento alia corte il progetto che, come vedemmo, venne accolto entusiasticamente, e sotto la sua direzione, nel 1668, vennero intrapresi i lavori..." all evidence from the S. Lorenzo Libro della Fabrica. However, indicates work began on Guarini's project in 1670.
50 - Pommer comments on the relative lack of architectural sophistication at the Savoy court when discussing Bettino's 1675 plan for S. Filippo Neri in turin: "That such a design could have been chosen under royal AUSPICES for the largest church in Turin suggests that architectural tastes in Piedmont were completely fluid, that a man like Bettir.o was considered almost on a par with Guarini." (R. Pommer, eighteenth century architecture in Piedmont: the open structures of Juvarra. Alfieri and Vittone. New york, 1967, p. 80.)

^{51 -} On Castellamonte's foundations see for instance Schede Vesme. II, p. 551. Contrary to this and to Robison's conjecture (pp. 118-123), there is no evidence of preexisting foundations on the site prior to the 1670 campaign. See Dardanello, figure, p. 19 6, who publishes a letter of 14 June 1670 from the confraternity to the duke: the site description is found in AGT, Turin; its accompanying drawing has been lost. The site survey stemmed from the Turin Senate's investigation of January 1670 into the complaints of the confraternita degli schiavi as recorded in the Sommario. P. 9 and ACTA, pp. 426-432. 52 - this description lists the following features: A porta grande leading from the strada nuova to a portico and thence to the oratory; three shops owned by the theatines; a chapel known as the scala Santa: the church officiated by the theatines in 1670 at the extreme east of the site adjacent to Piazza Castello; the exact position of these features on the lot is often difficult to determine, but we can attempt a tentative reconstruction.

^{53 -} tie rods could hardly be placed in the chapels of the new church in late 1670 since the columns for the corner chapels were not put in place until 1673, see below. While no tie rods are visible in the chapels in the church today, tie rods still reinforce the atrium-portico of S. Lorenzo.

^{54 -} See Libro della Fabrica. July 1671 "al Sig.R Ferrari nodaro per la scrittura di capitulatione delli capitelli con li mastri Scarpellini."

Between 1672 and 1676, subsidiary structures like the entranceway, corridor, sacristy, oratory, and a new kitchen were established.⁵⁵ Notably, construction at the southwest corner of the church commenced following the demolition of the old oratory in July 1673.

Substantial progress was evident by 1675, with columns being set, cornices installed, and payments made for the dome's wooden covering. Rapid advancements continued throughout 1676, focusing particularly on the upper levels and the crucial pendentive zone supporting the cupola.⁵⁶

Financial expenditures peaked notably in 1676 and 1678, signifying intensified construction efforts, particularly towards the cupola's support system.⁵⁷ However, a temporary pause in progress occurred in July 1677, as payments dwindled. Klaiber's analysis indicates Guarini's plan anticipated completion up to the dome's base by 1678, requiring continuous work and allowing time for masonry to cure.

The completion of the entire dome required two more construction seasons after 1677. The commencement of dome work in March 1678, subsequent payments for the cornice, and. Some sort of iron reinforcement is highlighted in Klaiber's observatio was placed on the "Arconi fondamentali della coppela" in July 1678, probably the large hidden arches supporting the dome or perhaps the intersecting ribs of the dome itself, which have iron ties just above the large lobed windows.⁵⁸ by September 1678, the exterior cornice on top of the drum had been installed. The dome's construction culminated on October 27, 1679, marked by the placement of the cross atop the lantern, an occasion celebrated by the city with fireworks.⁵⁹

According to Elwin C. Robison's work in "Optics and mathematics in the domed churches of Guarino Guarini," the plans for S. Lorenzo were finalized by late 1669, marking the commencement of construction in January 1670. Robison highlights Guarini's evolving ideas on optics and design during the two years between conceptualizing the SS. Sindone and envisioning S. Lorenzo. Despite minor adjustments made to the proportions of S. Lorenzo during construction, Guarini's incorporation of conic sections in crafting the dome was integral to the initial design.⁶⁰

^{55 -} Libro della Fabrica. May 1672, "al fontana piccapietre per una pietra di lavello alia cucina nuova mentre s'e lasciata la vecchia per 1'oratorio."

^{56 -} the theatines spent 12,596.15.10 lire Piemontesi on construction in 1676 and 11,145.1.8 in 1678 (Libro della Fabrica). One added cost in 1676 involved repairs to a vault damaged by rain. Payment for these repairs appears in the Libro della Fabrica in October 1676; The Madama Reale had donated 500 lire for this purpose on 27 August 1676 (AST, riunite, patenti controllo finanze, reg. 159, f. 69, noted in BRT, ms 161, L. Cibrario, 511.) 57- E. C. Robison, "Optics and mathematics in the domed churches of Guarino Guarini," JSAH. 50, 1991, pp. 396-397.

^{58 -} See Libro della Fabrica. October 1679; Tamburini, p. 203; and Schede Vesme. Ii, p. 551.

^{59 -} Amedeo Romagnano di Virle summarized the progress of construction at the end of the 1678 season in a letter of 18 January 1679 to the Madama Reale: "Il ne reste plus a faire de Cette Eglise, que le dome, qui est la plus belle, et la plus difficile partie, et parce qu'on ne la Peut commencer, que 1'on aye de quoy 1'achever toutte en mesme temps..." it is unclear what Romagnano refers to here: certainly some work must have been completed on the main dome by this point, yet it seems unlikely that the theatines would devote an entire construction season simply to the upper dome/lantern.

^{60 -} The different versions of the design were first noticed by H. Millon in a review of G. Crepaldi, La real chiesa di San Lorenzo in Torino (Turin, 1963), in ART bulletin, XIVII, 1965, 531-532. For a discussion of the modification of the design of S. Lorenzo, see "Guarino Guarini's church of San Lorenzo in Turin," Ph.D. Diss., Cornell university, 1985, 255-261.

CHAPTER 5

DISTINCTIVE FEATURE OF S. LORENZO'S GENERAL DESIGN

Guarini's entrance in Turin in 1666 was a watershed point in the city's architectural history, notably his designs for the Real Chiesa di San Lorenzo.

Exploring Guarini's Real Chiesa di San Lorenzo reveals a plethora of structural innovation that challenges traditional architectural assumptions. Leoncini and Quarello address this in their article "La Doppia Struttura della Chiesa di S. Lorenzo del Guarini, Esempio di Architettura nella Torino del Seicento." This understanding of the church's <u>proportions</u>, <u>structural system</u>, and <u>use of light</u> as a construction material demonstrates Guarini's inventive spirit and imaginative approach to building.

Upon entering the church, visitors experience wonder as they find themselves in an area where the distinctions between form and structure become one. Guarini defied convention in architecture, as seen by the dome, which appears to be hanging over the drum and is held up by thin red marble columns. Viewers are encouraged to explore further into the secrets of his design via the contemplation and discovery that this complicated interplay of parts evokes.

LIGHT PENETRATION

San Lorenzo Church by Guarino Guarini is a unique structure that is best appreciated in person. Its real grandeur is seen when one enters and looks up at the high dome, which has a distinctly geometric shape and appears to be almost magical when lit by sunshine. S. Lorenzo's genuine splendor is best appreciated by witnessing it directly, even after extensive description. The dome is high above the nave, so as you go in, you have to look up to see it.

Guarini created a striking sense of height in his center domed cathedrals that significantly outstripped the real size of the buildings. He used shape, light, color, and shade in concert to create interior spaces that seemed to go on forever. To create this appearance, Guarini experimented with a number of methods, such as manipulating repetitive shapes and using light and concealed structures strategically.

According to Guarini's observations, "The objects that are white appear larger than those of dark color, or black.... The more illuminated place or object seems greater than that which is dark." ⁶¹ This way of thinking emphasized his dedication to designing environments that went beyond geographical bounds and had a profound effect on those who entered them.

Furthermore, the enormous oculus that opens into the upper cupolino efficiently illuminates the vault's crown, which is usually a difficult place to light. This increases the amount of light in this area. This focus on lighting is also visible from the outside, where a large number of windows force light into the perforated dome (Figure 39). Guarini also used repeating geometries to give the impression of greater height. ⁶²

Guarini's task during plan conception was to maximize light penetration into the church's lower levels—a feat that traditional dome systems would block with enormous piers. Guarini's creative design called for the church's primary vessel to be a large bridge with arches that were supported by squinches and concealed piers, evoking the style of Gothic cathedrals. This framework structurally, concealed beneath a sheathing. ⁶³

^{61 -} Guarini, Architettura civile, tratt. III, cap. XXI, oss. 6, "Gli oggettiche sono bianchi paiono più grandi, che di colore oscuro, o nero"; "Il luogo ovvero oggetto piu illuminato sembra maggiore di quello che sia l'oscuro."

^{62 -} Guarini, Architettura civile, tratt. III, cap. XXII, oss. 11. " ... I volti paiono sempre meno svelti di quello sono, e massime le cupole di mezzo tondo, le quali dal terzo in su paiono piane, occupando una luce men chiara il loro fondo, e nascondendo la loro curvita, che in quel sito è poca. Però chi vorrà far volte svelte bisognerà non servirsi del semi-circolo, ma farle come insegneremo abbasso." As a geometer, Guarini understood that the curve of a semicircular dome does not change. The viewer's perception of the curve of a hemispherical dome, however, can vary.

^{63 -} The sheathing, an undulating screen of eight serlianas, visually supports the pendentives and dome while defining chapels and otherwise shaping the space of the nave.



FIGURE 39-PICTURE TAKEN BY THE AUTHOR – SAN LORENZO DOME – TORINO

INNOVATIVE ARCHITECTURAL APPROACH

Guarini's architectural legacy, which is most visible in his domed churches such as the chapel of the SS. Sindone in Turin, the church of S. Lorenzo in Turin, and the project for the church of S. Gaetano in Vicenza, is distinguished by complicated geometries, linked spaces, and buried structures. While architecture in lateseventeenth-century Rome inclined toward formalism, Guarini revitalized current architecture in Turin, paving the way for architectural changes that lasted long into the eighteenth century. His stay as a Theatine novice in Rome exposed him to the seminal works of Bernini, Borromini, and Cortona, which probably impacted his architectural style. Carlo Randoni's attempts to fortify the structure in the nineteenth century demonstrate Guarini's architectural durability. Randoni's careful effort, guided by Guarini's design ideas, addressed structural challenges without the use of contemporary techniques. The preservation of Randoni's alterations demonstrates Guarini's vision for the Real Chiesa di San Lorenzo goes beyond ordinary architecture, highlighting how light, shadow, and space interact. Inspired by Arab and Byzantine styles, Guarini created a design that broke away from tradition and matched the complexity of Moorish Spain's structures.

STRUCTURAL AND GEOMETRICAL ASPECTS

Regarding Robison's point of view in Optics and mathematics in domed churches of Guarini, Guarino Guarini's innovative approach to architectural design, particularly in his domed churches, wasn't driven solely by mathematical or structural principles but was deeply influenced by considerations of optics and aesthetics. During the seventeenth century, while Guarini was in Paris, ⁶⁵ debates on aesthetics and optics were thriving, shaping his ideas and techniques. Guarini's unconventional architectural concepts were undoubtedly shaped by his unique background and experiences. For instance, in his Parisian church of Sainte-Anne-la-Royale, Guarini introduced a vertical stacking of distinct zones, a design motif he would further develop in his subsequent projects. ⁶⁶ The interlacing bands in the dome established a visual rhythm that characterized many of his later works. Robison says this magical feeling comes from how Guarini played with how we see things in his dome designs. He used the shape of the dome, special lighting, repeated patterns, and hidden framework to make the space look bigger and the dome seem taller. ⁶⁷ The church's shape reflects Guarini's careful attention to detail, seen in how he arranged the interior space and manipulated structural elements. The drum, pendentives, and arches give the impression of solidity, hiding the structure that supports them. Additional features like iron tie-rods and keys add to the overall stability, enhancing the church's strength.

The clear separation between the upper and lower parts of the church makes the difference in design even more noticeable. Today, after renovation, the dome's sharp, geometric shape, lit up by sunlight, stands out against the warm, decorated lower level. It feels like the dome is far away and it is almost magical. The upper zones of the church are noticeably brighter than the lower areas, in line with Guarini's theory on optics, which suggests that brighter objects appear larger. However, due to our perspective from the main dome's oculus, the upper zones cannot physically seem closer to us. Instead, the interlacing ribs of the main dome serve as a reference point for assessing the size of the cupolino. This visual contradiction between the fixed reference of the main dome and the perceived larger size of the cupolino disconnects the upper part of the church from the observer, enhancing the impression of dome height.⁶⁸

67 - E. C. Robison, "Optics and Mathematics in the Domed Churches of Guarino Guarini," JSAH. 50, 1991, pp. 394-395.

^{64 -} Cristina Leoncini, Ugo Quarello, "La doppia struttura della chiesa di S. Lorenzo del Guarini, esempio di architettura nella Torino del seicento", thesis advisor Prof. Arch. Mario Dalla Costa, Politecnico di Torino, faculty of architecture, academic year 1994/95. p.369-392

^{65 -} Guarini's travels enhanced his academic education by bringing him into contact with ideas of the leading intellectuals of the day, especially during his stay in Paris from 1662 to 1666.

^{66 -} This complex vertical stacking of distinct zones was an idea that Guarini had developed before or immediately upon his arrival in Paris. Guarini designed Sainte-Anne-Ia-Royale in a very short time. The first mention of his name in the documents of the Theatine order in Paris was 26 Oct, then-completed transept was capped by a low dome and utilized as a nave until its demolition in the late nineteenth century. See A. Lange, "Disegni e documenti di Guarino Guarini," in Guarino Guarini e l'internazionalità del barocco, I, 110; and D. Coffin, "Padre Guarino Guarini in Paris," JSAH, XV, 1956, 3-11, esp. 5.

^{68 -} Guarini, Architettura civile, tratt. III, cap. XXII, oss. 11. " ... I volti paiono sempre meno svelti di quello sono, e massime le cupole di mezzo tondo, le quali dal terzo in su paiono piane, occupando una luce men chiara il loro fondo, e nascondendo la loro curvita, che in quel sito è poca. Però chi vorrà far volte svelte bisognerà non servirsi del semi-circolo, ma farle come insegneremo abbasso." As a geometer, Guarini understood that the curve of a semicircular dome does not change. The viewer's perception of the curve of a hemispherical dome, however, can vary.

But when you take a closer look, you see the dome's support system, which seems almost like a miracle. There are windows in the cornice ring, holes in the top of the serlianas, and thin columns holding up the pendentives and more. These details show Guarini's love for unique designs and his fascination with Gothic architecture, which he thought seemed like it shouldn't stand up but did. Even when it was being built or just after it was finished, people were amazed by S. Lorenzo's unusual design. Donato Rossetti, describing Guarini's plans in 1674, called them "...che qui chiamano cose belle capricciose, ed io con un solo epiteto direi cose strane." ('beautifully strange). In 1680, Emanuele Filiberto of Savoy, who supported Guarini, praised S. Lorenzo for its clever and unusual design. During the siege of 1706 in Turin, there were stories about rockets going through the dome of S. Lorenzo without hurting it.⁶⁹



FIGURE 40- PICTURE TAKEN BY THE AUTHOR FROM THE WINDOW ON TOP OF SERLIAN ZONE - SAN LORENZO - TORINO

^{69 -} D. Rossetti, letter of 5 September 1674 to Prince Leopoldo de' Medici, in A. Fabroni, Lettere inedite di uomini illustri. II, Florence, 1775, p. 248; patent in AST, Riunite, Registri dei conti della casa dei principi di Carignano, Emanuele Filiberto Amedeo di Savoia, XXI, f. 49r, published in Schede Vesme. II, p. 555 and in S. Cordero di Pamparato, "II padre Guarino Guarini teologo del principe di Carignano," II duomo di Torino. II 4, 1928, pp. 22-24.

PLAN

Guarini did not choose to the usual Latin cross layout for S. Lorenzo. Instead, he came up with a centralized plan. The planning process has been the object of several different interpretations. Passanti for instance explained how Guarini's plan for S. Lorenzo started as a Greek cross but got changed with some elastic deformations along the way.⁷⁰



FIGURE 41- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI

According to Passanti's Nel Mondo Magico di Giarino Guarini as you can see in illustration (fig 1) When we increase the total dimension L, the Bacino diameter of the crossing grows while the arms' depth decreases, expanding the central space. However, the central compartment and four arms remain distinct. Conversely, if we halt L and barrel width, and smooth the bacino edges of the crossing, the arms' depth decreases, central space control increases, but part identification weakens due to merging caused by bevels. In San Lorenzo, the central environment and lantern expand greatly, with the drum height small but already indicating space fusion.

^{70 -} Passanti (1963), p.128-9

Brinckmann, had published a copy of Guarini's plan around 1870(Figure 42) on which he had superimposed a number of circles of equal radius, passing through the major points of articulation, with another one and a half times larger; these, he claimed, underlay the whole geometry of the church.⁷¹

Brinckmann's scheme was corrected around 1920 by the Belgian Scholars J. Vanderperren and J. Kennes, in a scheme (Figure 43) that sees the major circles as key points from which undulatory movements emanate, on the basis of 'wave lengths' located outside the building itself.⁷²



FIGURE 42- TURIN, S. LORENZO. GEOMETRICAL BASES OF THE PLAN. ACCORDING TO BRINCKENMANN



FIGURE 43- TURIN, S. LORENZO. GEOMETRICAL BASES OF THE PLAN. ACCORDING TO VANDERPERREN AND KENNES

^{71 -} Brinckenmann (1931), p.231

^{72 -} J. Vanderperren and J. Kennes, 'De systema-tische ruimtelike wereld van Guarino Guarini' in at, No. 31 (September 1976), fig. 2, p. 74, and p. 89.

The centralized plan of S. Lorenzo reflects both practical necessities and symbolic aspirations, echoing Vittorio Amedeo's vision of a "tempio di forma sferica" for the palatine chapel relocated to Piazza Castello.⁷³ Guarini likely devised the plan using interlocking circles or arcs, a technique akin to Brinckmann's proposal. Without extant preparatory drawings, the precise evolution of Guarini's design remains mysterious. This methodology allows for multiple interpretations of the church's fundamental geometry, ranging from a square to an octagon, or even a circle, each emphasized on different horizontal levels.(Figure 44-Figure 45)⁷⁴



FIGURE 44- GUARINO GUARINI – ARCHITETTURA CIVILE – SAN LORENZO'S PLAN



FIGURE 45- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – PLAN AND HORIZONTAL SECTION FROM THE DRUM LEVEL FIGURE 46- GUARINO GUARINI – ARCHITETTURA CIVILE – SAN LORENZO'S PLAN

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^{73 -} For a church "di forma sferica" the architect had to erect a dome over this plan, bringing light into a church surrounded by buildings on three sides. S. Lorenzo would function as a court chapel with its ducal tribunes, but it would also have to be a Theatine church.

^{74 -} Passanti, for instance, sees the basic form of the plan as a Greek cross with shallow arms (M. Passanti, Nel mondo maqico del Guarino Guarini. Turin, 1963, p. 128), while Wittkower (Pelican, p. 410) and Muller (p. 49) read a regular octagon as the basis.



FIGURE 47- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – PLAN OF THE GROUND FLOOR AND THE PROJECTION OF THE DOME OUTLINE WITH DIMENTIONS

PENDENTIVE ZONE AND HIDDEN STRUCTURE

The interior of S. Lorenzo can be understood as having two main structural systems: one that's deeply hidden and another that's more visible. The primary load-bearing work happens at the deeper level. Essentially, there are four significant brick arches starting high up in the corners of the main structure, known as the 'chamfered' diagonal serlianas. These arches extend over circular openings in the cornice at the four main points, not diagonals, and their highest points touch the tops of these openings. (Figure 50-a) Additionally, at each corner where pairs of these arches extend, four smaller squinch arches are added (Figure 50-b)(Figure 48), connected to the major arches about a third of the way along their curves. This creates a connection between diagonal and normal arches eight times, corresponding inside the church to the starting points of the interlaced dome arches and outside to the angles of the drum's eight faces. Each dome loading point essentially rests on about a third of the major arch below it, requiring the base system to be reinforced with metal tie rods to prevent deformations. Guarini's 'key' also reveals a unique support system of large timbers placed under each major arch, directed to the intrados at third points along the arch. These triangular prisms at each corner help manage the vertical loads, minimizing the horizontal thrust from the interlaced dome arches and supporting the weight of the lantern above.





FIGURE 49- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – THE ILLUSTRATION OF HIDDEN STRUCTURE WHICH IS INTEGRATED IN THE BUILDING WITH THE RELATED HORIZONTAL SECTION



FIGURE 48- THE HIDDEN STRUCTURE IN 3D FORMAT FROM A RESEARCH PAPER "UNFOLDING SAN LORENZO" BY NTOVROS VASILEIOS

FIGURE 50- TURIN, S. LORENZO. AXONOMETRIC VIEW DEMONSTRATES THE HIDDEN STRUCTURE FROM 1920

The large pendentives in the Real Chiesa di San Lorenzo are adorned with unique irregular hexagons that frame paintings of the four evangelists, a feature that was apparently unique in Italy (Figure 51).⁷⁵ This distinctive design serves as a bold reminder of Guarini's years in Paris, showcasing his innovative and distinctive architectural style. The pendentives visually support a cornice ring, which forms the base for the entire dome. This ring is composed of an architrave supporting cherub-head brackets and is pierced with eight kidney shape windows, adding to the architectural complexity of the structure. The placement of these windows precisely at the points where the cornice would seem to require the most reinforcement. While an unbroken annular shape might seem structurally sound, Guarini introduces eight horizontal oval openings within the cornice ring, creating a sense of unease. Yet, this unease pales in comparison to the revelation that each of the four pendentives appears to transfer its considerable load, via the chamfers of the diagonal convexities, onto the slender columns and responds of a serliana at ground level. (Figure 51).

^{75 -} seem to derive directly from one of Mansart's drawings for the Bourbon chapel at St.-Denis - Klaiber, Susan Elizabeth. "Guarino Guarini's Theatine Architecture." – p266



FIGURE 51- PLACEMENT OF THE OPENINGS IN LOCATION OF CRONICE RING AND PLACES THAT SEEMTO NEED THA MAXIMUM REINFORCEMENT - SAN LORENZO – TORINO

Following the anchoring of the church's corners with hidden structures, Guarini defined a square with rounded corners, embellishing its perimeter with carved niches and undulating walls. Convex arcs and mandorla-shaped corner chapels contribute to the perception of the lowest level as an octagon with curved sides. The vertical development of S. Lorenzo defies predictability, as its interior comprises non-structural sheathing within a concealed structural cage. ⁷⁶

At this stage, the area where the congregation gathers is surrounded by a set of curved surfaces. These curved shapes, which stand out prominently at the main points of the compass, have smooth, rounded edges, while the diagonal ones are made up of connected parts, including a central slope bordered by two curved sections. You can notice them most clearly at the level just below the cornice, where they provide support for the walls and structures around the altars. (Figure 52)

^{76 -} Iconographic interpretations of S. Lorenzo range from general readings such as the cupola as dome of heaven to the obscure, in which the cupola represents astrological charts, St. Lawrence's gridiron, or planetary orbits. Compare the varied approaches of Millon, "Guarini," Macmillan Encyclopedia. pp. 267-268; Robison, S. Lorenzo, pp. 271-285; Meek, p. 60; E. Battisti, "Schemata nel Guarini," GGIB. II, pp. 114-116;



FIGURE 52- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – PLAN AND FORM OF THE SERLIANE AND ANDULATIG WALLS

The undulating borders of the pendentives in the Real Chiesa di San Lorenzo continue to exert dynamic forces, despite their penetration into physical space. As we observe the development of the entrance-level convexities, we witness their transformation from convex to concave shapes as their chamfers evolve into pendentives (Figure 54). As we discussed earlier in the text at the lowest level of the pendentives, the intricate ground floor plan transitions into a Greek cross with short arms, providing a support for the ribbed dome and lantern. (Figure 53)⁷⁷

^{77 - &}quot;The massive wall knows no rest until it reaches the round base of the dome; here the massive swelling material comes to a point of repose, the terminal point of the first "instalment" - Vanderperren and Kennes, p. 89.



FIGURE 53- (LEFT), TURIN, S.LORENZO. COMPLEX FLOOR PLAN TRANSFORMED INTO A GREEK CROSS AT THE LOWEST LEVEL FIGURE 54- (RIGHT)- TURIN, S. LORENZO. PLAN AND TE CONVEXITIES AND CONCAVITIES OF THE PENDENTIVE ZONE

The mind which devised the virtually independent actual structure hiding behind S. Lorenzo's improbable apparent structure certainly was capable of inventing the cupola of intersecting arches. Millon first published the rare engraved plan and section in AST, Riunite which illustrates an early design for the system of hidden arches supporting the cupola; the executed system is more complex (Figure 55).⁷⁸ Four huge brick arches rest on the corners of the church behind the corner chapels, with massive squinches built out behind the hollow pendentives and above the pierced chapel vaults providing the actual springing. The engraving shows buttress infill between the lower zone of the arches and the side walls of the church. The engraving also indicates brick corbels underneath each arch supporting a wooden truss which seems far too fragile to lend any real support to the structure; this is perhaps why they were eliminated in the executed church.⁷⁹

The engraving of the "Chiave della cupola" in the Real Chiesa di San Lorenzo differs from the original plan and the executed church. (Figure 55) The engravings show the chapels or niches on the middle of each side pushed toward the center of the plan, and all four of the small niches on each side aligned. As built, the lateral altar niches are shallower than those in the engravings and the corner chapel altar niches deeper and more curved.

The four arches each rise along one wall of the church, approximately concentric with and in the same plane as the short barrel vaulted arms at pendentive level. The arches peak just above the windows cut into the cornice ring above these barrel vault segments. Squinches span the corners between the large arches up to this level, bracing the structure and creating a continuous base for the dome just above the pierced cornice. The eight

^{78 -} The engraving is found in AST, Riunite, Tipi e disegni, n. 17; See Millon, "Crepaldi," p. 531, where he dates the engraving before 1679; see also Lange, p. 234, cat. no. 4, who dates the engraving 1677.

^{79 -} Further discrepancies exist between various engravings and the executed church: The Chiave della cupola engraving and the Disseani section of the church both show the large arches as hollow (or encasing timber ties as Meek, p. 57, suggests). The proportions of the arch sections, however, differ in the two engravings: The Disseani section illustrates more massive arches, taller and thinner in section, yet with a smaller hollow area than shown in the Chiave della cupola. This change corresponds roughly to thechanges in execution, therefore the chiave engraving predates the Disseani engraving.

squinch and arch abutments behind this attic zone form the springing points for the ribs of the dome. Iron ties further reinforce the hidden structural system, and ties also run directly above the large windows of the dome.



FIGURE 55– GUARINO GUARINI - HIDDEN SUPPORTING ARCHES OT SAN LORENZO FROM ARCHITETTURA CIVILE

DOME

As a worshiper steps into S. Lorenzo, they embark on a journey of discovery, gradually unraveling the church's elaborate intricacies as the mass unfolds. The interplay of shifting light illuminates the warm hues of rose, white, and pale grey tones, enhancing the richness of the lower zone's marble, stone, and stucco adornments. Yet, the apex of exposure lies in the splendid dome, with its crisp geometry bathed in radiant light. How did Guarini conceptualize and execute such a remarkable space within the confines of the established program for S. Lorenzo? (Figure 56)



FIGURE 56- PICTURE BY THE AUTHOR – THE NAVE AND ALTAR OF THE SAN LORENZO CHURCH – TORINO

The impulse initiated at the entrance level persists upward as ellips shape ribs rise, creating an openwork dome allowing light to penetrate progressively (Figure 57). Atop the interlaced arches forming an octagon, Guarini places a lantern through which daylight streams, avoiding conventional Baroque celestial scenes for an open skylight, evoking an illusion of infinite space where the vertical dimension dissipates into the boundless area.

Elizabeth Klaiber highlights Guarini's skillful manipulation of light and architectural structure in S. Lorenzo to enhance the sense of perspective within the space. The focal point of the church is undoubtedly the expansive dome, characterized by its interlacing ribs that stretch across the 15-meter diameter, forming a captivating octagonal arrangement (in plan forms two square grids rotated forty-five degrees with respect to one another). ⁸⁰ Guarini finalized the plans for S. Lorenzo by late 1669, marking the commencement of construction in January 1670. During the intervening two years, between the design phases of SS. Sindone and S. Lorenzo,

^{80 -} The geometry of the dome has been compared to that of Islamic domes at Iznik (Paolo Verzone, "Struttura delle cupole del Guarini," in Guarino Guarini e l'internazionalita del barocco, I, 401-413), domes in the mosque in Cordoba, or such later medieval domes as the one at Torres del Rio (Adolfo Florensa, "Guarini ed il mondo islamico," in Guarino Guarini e l'internazionalitd del barocco, I, 637-665).

It should be noted that Leonar- do's sketchbooks contain several diagrams for centrally planned churches that trace out a geometry like that of Guarini's ribs. For example, see R. Mainstone, "Guarini and Leonardo," Architectural Review, CXLVII, 1970, 454; and Oechslin, "Guarini e Lobkowitz," 586. I am not suggesting that Guarini came upon his geometric idea from Leonardo's treatise (although it is certainly a possibility);

Guarini's understanding of optics and architectural design evolved significantly. While minor adjustments were made to the proportions of S. Lorenzo during construction.⁸¹



FIGURE 57- QUALITATIVE AND QUANTITATIVE ANALYSIS OF NATURAL LIGHT IN THE DOME OF SAN LORENZO, TURIN, SIX OPENNINGS AND THE ONE UNDER THE DRUM FROM INTERIOR AND EXTERIOR OF THE DOME

Once we move past the level of the cornice ring, the complex surface structure that hides the actual support mechanisms disappears. What lies beyond, like the openwork dome and the lantern it holds, represents the genuine structure - remarkable enough on its own without any concealment. This creates a stark contrast between the pressures and deceptive nature of the lower zone, and the clear honesty of the higher, more heavenly areas, which could carry theological or stylistic significance.⁸²

Through Klaiber's dissertation, it becomes evident that up to the dome's level, S. Lorenzo may appear unconventional but not entirely groundbreaking. However, Guarini's departure from Italian Renaissance-

^{81 -} The different versions of the design were first noticed by H. Millon in a review of G. Crepaldi, La real Chiesa di San Lorenzo in Torino (Turin, 1963), in Art Bulletin, XLVII, 1965, 531-532. For a discussion of the modification of the design of S. Lorenzo, see "Guarino Guarini's Church of San Lorenzo in Turin," Ph.D. diss., Cornell University, 1985, 255-261.

H. A. Meek, Guarino Guarini and His Architecture, New Haven, 1988, 48, each call this a parabolic vault. The curve of an ellipse is similar to that of a parabola. The ribs of S. Lorenzo, however, are tangent to the vertical at the point of their springing. A parabola with a vertical axis (as would necessarily be the case with a masonry dome) can never become tangent to a vertical plane because it asymptotically approaches the limits of the generating cone.

^{82 -} To G. L. Marini, an architectural writer who has little patience for either concettismo or semiotics, the church of S. Lorenzo as a whole nevertheless 'truly ... corresponds to a religious concept'; it makes him think of the description of the eternal light given by Dante at the end of the Divine Comedy: 'Within its depths I saw ingathered, bound by love in one volume, the scattered leaves of all the universe' (Paradiso, 33, 86-7) – Marini (1963), p. 92. For a sharp critique of Marini's view of Guarini, see H. A. Millon, Review of Marini (1963) [q.v.), Art Bulletin XLVII (1965), pp. 532-3.

Baroque conventions becomes strikingly apparent in S. Lorenzo's dome. Here, Guarini ventures into uncharted territory, crafting a remarkably original architectural form that seems to draw inspiration from Gothic or Islamic architectural traditions.⁸³ The dome, stands out for its exposed network of intersecting structural ribs, visible from the interior, with openings punctuated in the vault infill between the ribs. Such innovative elements in the S. Lorenzo cupola hint at Guarini's visionary approach to architecture and can be traced back to his earlier works. The intricate details of the dome structure in S. Lorenzo from Susan Klaiber's view is explored. Observing the dome, one notices how the eight ribs emerge from sixteen pedestals situated in the attic story just above the cornice ring. Each rib traverses three of the dome's eight large windows, intersecting with four other ribs before reaching its designated pedestal. What's striking is how the ribs on either side of each window arch above it, creating a curved triangular space below.

From Meek's perspective, Guarini's placement of the windows exactly where the cornice appears to need the most reinforcement. The pendentives support the fractured cornice ring of the dome, punctuated by large oval vertical windows and springing vaulting ribs (Figure 58-Figure 59). These ribs, rather than converging at a central point, form an eight-pointed star with a regular octagon in the center. The interplay of parallel arches creates an endless trajectory, launching the spectator from one springer to the other without conclusion.⁸⁴



FIGURE 58- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – PASSANTI'S FREEHAND DRAWING FROM THE MAIN DOME RIBS STRUCTURE AND THE OPENINGS

FIGURE 59- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – PLAN AT CRONICE RING OF THE DRUM, SHOWING PIERS FROM WHICH ELLIPS SHAPE RIBS SPRING

^{83 -} On the S. Lorenzo dome, see A. Terzaghi, "Origini e sviluppo della cupola ad arconi intrecciati nell'architettura barocca del Piemonte," Atti del X Conaresso di storia dell'architettura. Rome, 1959, pp. 369- 379; A. Blunt, "Guarini and Leonardo," Architectural Review. 147, 1970, pp. 164-166; and R. Mainstone, ibid. p. 454;

P. Verzone, "Struttura delle cupole del Guarini," GGIB. I, pp. 401-413; W. Oechslin, "Bemerkungen zu Guarino Guarini und Juan Caramuel de Lobkowitz," Raggi, vol. 9, no. 3, 1969, pp. 91-109. (See also W. Oechslin, "Osservazioni su Guarini e Juan Caramuel de Lobkowitz," GGIB. I, Turin, 1970, pp. 573-595, an Italian version of above article.)

^{84 -} H. A. Meek, Guarino Guarini and his architecture, new haven, 1988, 48, each call this a parabolic vault. The curve of an ellipse is similar to that of a parabola. The ribs of S. Lorenzo, however, are tangent to the vertical at the point of their springing. A parabola with a vertical axis (as would necessarily be the case with a masonry dome) can never become tangent to a vertical plane because it asymptotically approaches the limits of the generating cone.

The support system underlying the dome likely influenced its distinctive eight-ribbed structure. Robison suggests that the intersections of the eight squinches inspired the formation of the dome's ribs, with windows strategically positioned above the weaker points at the arches' peaks. In the absence of detailed engineering studies, we can only speculate that the eight ribs, along with their curved, thin-shelled, perforated infill, contribute to a lighter dome design suited to the point-supports of the arch system, rather than the more continuous support typical of conventional pier, pendentive, and cornice solutions. At the apex of the dome, a conventional octagonal lantern rests, its eight piers positioned atop the vertices of the octagonal opening. From these piers spring eight ribs, forming the base of the lantern's dome. However, unlike the dome below, each rib intersects only two others, resulting in a simpler vault pattern reminiscent of the rib-bands observed in the Ste.-Anne cupola.⁸⁵

Many scholars are tempted to interpret this exposed structure as symbolic of divine revelation, contrasting with the concealed structure below, which mirrors deceptive earthly appearances. ⁸⁶ This interpretation adds another layer of complexity to Guarini's architectural vision, inviting further exploration into the relationship between form, function, and symbolism in his designs.



FIGURE 60- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – PLAN AT CRONICE RING OF THE DRUM, SHOWING PIERS FROM WHICH ELLIPS SHAPE RIBS SPRING

^{85 -} Millon, "Guarini," Macmillan Encyclopedia, p. 267; Robison, S. Lorenzo, pp. 55-56.

^{86 -} For various nuances in the interpretation of the S. Lorenzo dome as dome of heaven or as revelation of the hidden structure below, see Meek, p. 60; Muller, Unendlichkeit. p. 49; Robison, S. Lorenzo, pp. 275-285.



FIGURE 61- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – SECTION OF THE CHURCH WITH INDICATION OF HORIZONTAL SECTION LOCATIONS



FIGURE 62- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – HORIZONTAL SECTION D LOOKING DOWNWARDS WITH THE RIBS PROJECTION



FIGURE 63- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – SITE PLAN



FIGURE 64-PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – HORIZONTAL SECTION FROM RIBS SPRINGING LOCATION LOOKING UPWARD UP TO THE FIRST INTERSECTION OF THE RIBS IN THE TRIANGULAR PERFORATIONS



FIGURE 65- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – HORIZONTAL SECTION FROM THE FIRST INTERSECTION OF THE RIBS IN THE TRIANGULAR PERFORATIONS LOOKING UPWARDS UP TO THE OCTAGONAL OPENING ON THE TOP PART OF THE RIBS OF THE MAIN DOME



FIGURE 66- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – SECTION C LOOKING UPWARD

One of the most intriguing aspects of the church is the presence of inverted pentagonal openings carved into the spandrels above the windows. These openings contribute to the church's enigmatic character. ⁸⁷ Moving upwards from these initial intersections, the ribs enclose an octagonal opening at the dome's crown. Perforations strategically placed between the ribs and within the vault webbing encircle the perimeter of this octagon. (Figure 67)

These sixteen perforations alternate between triangles along the octagon's sides and scallops positioned between the ribs at each vertex. The presence of triangles, pentagons, and scallops underscores the ribs' structural role as a network of supporting elements independent of the vault infill. (Figure 67) Such meticulous structural details not only enhance the architectural complexity of S. Lorenzo but also reflect Guarini's innovative design principles, inviting further exploration and analysis.



FIGURE 67- PASSANTI – NEL MONDO MAGICO DI GUARINO GUARINI - TURIN, S. LORENZO – PLAN OF THE GROUND FLOOR LOOKING UPWARDS WITH ALL THE 16 PERFORATIONS

^{87 -} Pentagons also appear in the dome at SS. Sindone, as recessed panels in the fields under each window.

The dome underwent its first structural reinforcements and repairs shortly thereafter. Additional reinforcements were introduced in the 1820s, accompanied by the closure of the pentagonal openings in the dome. Around 1830, the painter Pietro Fea embarked on a decorative campaign, adorning the chapel vaults, arch intrados above the serliana windows, pendentives, and the entire dome with his paintings. During World War II in 1943, S. Lorenzo endured minor damages, resulting in leaks across the cupola. These damages were addressed in 1956-1957, when the roof underwent replacement, and the 19th-century paintings within the dome were replaced with a neutral color scheme that remains present to this day.⁸⁸

If we look into Guarini's profound understanding of both the optical and structural aspects of his dome designs regarding Robison's paper. Guarini, while primarily concerned with the optical properties of his domes, demonstrated a comprehensive grasp of the structural behavior of the interlacing ribs. He acknowledged the considerable horizontal thrusts produced by hemispherical and segmental vaults, particularly emphasizing the influence of heavy lanterns on these thrusts. In his writings, Guarini emphasized that hemispherical or segmental vaults should not bear the weight of a lantern due to their rounded shape, which inherently limits their load-bearing capacity. Guarini proudly elaborates on how his vaulting shapes, inspired by conic sections, addressed the structural weaknesses inherent in hemispherical domes. Despite the significant structural improvement achieved by selecting an elliptical shape for the dome over a hemispherical profile, Guarini's meticulous examination of the dome suggests that his choice of the elliptical form was primarily driven by optical considerations rather than structural optimization.⁸⁹



FIGURE 68- METHOD OF FORMING AN ELLIPSE USING TWO CIRCLES FROM ARCHITETTURA CIVILE- TAV. VI, FIG. 9.

In traditional masonry construction, achieving an optimal design

involves ensuring that no areas of tension arise within the masonry. Given masonry's inherent strength in compression and limited strength in tension, ⁹⁰ especially considering variations in mortar quality, the optimal shape for a simple masonry dome with uniform thickness is a catenary curve. Guarini understood the structural superiority of this shape, resembling the natural curve of a rope suspended between two points. ⁹¹ While a catenary curve closely approximates a parabola, Guarini's decision not to adopt a parabolic shape, despite recognizing the structural drawbacks of the ellipse, underscores his emphasis on other factors. Guarini prioritized the dome's ability to capture and reflect light effectively, along with its resemblance to traditional hemispherical dome shapes, as decisive criteria in selecting the dome curvature.

Robison's comprehensive examination of Guarini's dome, he explores dome's geometric, structural, and optical intricacies. Robison points out that the dome's elliptical section, as constructed, differs slightly from the depiction in the Dissegni engraving, which portrays a more elongated version. He attributes Guarini's choice of an elliptical section to his aim of "counteract the flat appearance of hemispherical domes."⁹² This examination will be discussed later on in the geomatic chapter.

^{88 -} On the restorations, see Tamburini, pp. 215-216, and Robison, S. Lorenzo, who also summarizes the church's fortunes in the Napoleonic era (when the Theatines lost the church in 1801) and afterwards, pp. 141-151.

^{89 -} Guarini, Architettura civile, tratt. III, cap. XXVI, oss. 8, "... circa le volte semisferiche, o semisferoidali, o lenticolari, s'ha da avvertire, che non si caricheranno col lanternino, come si fa alle cupole, perchte quando sono tonde, ovvero meno del tondo non lo possono portare."

^{90 -} Paul Hutchinson and Robert Mark, in their study of the Pantheon ("On the Structure of the Roman Pantheon," Art Bulletin, LXVIII, 1986, 24-34), suggest that any area of a dome in tension, even those within the limits of the tensile strength of the mortar, will crack in time, due to secondary stress from curing, thermal gradients, differential settlement, and wind loading.

^{91 -} This principle was first written down by Robert Hooke in his diary in 1670 and again in 1675: "... as hangs the flexible line, so but inverted will stand the rigid arch." See E. Benvenuto, An Introduction to the History of Structural Mechanics, New York, 1991, 328-329. When a dome is loaded with a very large lantern, as in the case of S. Lorenzo, its optimal design shape is altered from a simple catenary curve, where the hypothetical "rope" has an equal weight along its length, to a curve in the shape of a rope with additional weight hanging from its center point.

^{92 -} On the geometry of the dome, see Robison, S. Lorenzo, pp. 243-249; 255-259; and now E. C. Robison, "Optics and Mathematics in the Domed Churches of Guarino Guarini," JSAH. 50, 1991, pp. 384-401. Robison identifies the geometric basis of the dome as an ellipsoid of revolution which counteracted the "flat appearance of hemispherical domes", thus creating the "illusion of greater height" ("Optics and Mathematics," p. 394). Robison also notes AC, III.xxvi.8, where Guarini discourages placing lanterns on vaults of hemispherical section (or shallower vaults).



FIGURE 69- (LEFT)-ROBISON - OPTICS AND MATHEMATICS IN THE DOMED CHURCHES OF GUARINO GUARINI - S. LORENZO. SECTION, WITH GEOMETRIC CONSTRUCTION OF ELLIPTICALPROFILE OF DOME SUPERIMPOSED

FIGURE 70-(RIGHT)- ROBISON - OPTICS AND MATHEMATICS IN THE DOMED CHURCHES OF GUARINO GUARINI - S. LORENZO. HORIZONTAL SECTION CUT BELOW THE SPRINGING OF THE DOME (AT AN ELEVATION OF 21.5 M.), WITH THE CIRCLE CORRESPONDING TO THE MAJOR AXIS OF THE DOME ELLIPSE AND THE CI

Based on Robison's conduction, a simplified finite element analysis of the dome, representing the ribs as isotropic elements, to illustrate the fundamental deflection pattern of the rib system. ⁹³ The cupolino's load exerts pressure on the vault's crown, causing it to descend, and pushes the lower segments of the vault ribs outward. In Figure 19, a sectional axonometric of the dome and cupolino is depicted, with the roof structure removed to reveal the rib arrangement. For clarity, the ribs on the extrados of the vault are portrayed as they would be observed from within the building; however, in reality, the upper portions of the elliptical ribs of the main dome have stepped masonry masses constructed on their extrados, supporting the beams of the tile roof.

Guarini's engineering skills and instinctive understanding of masonry vault behavior are evident in the placement of his iron tension ring, which, despite not being a conventional circular ring, comprises an octagonal belt of iron tie rods, and in his treatment of the lower rib sections. As demonstrated by the computer analysis, the maximum outward deflection of the ribs, along with the most significant bending moment leading to potentially damaging tensile stresses, occurs in the lower section of the elliptical ribs within the zone of the large windows, a region challenging to reinforce.

Guarini adopted two key strategies to counter prevailing architectural trends: firstly, he positioned the iron tension ring just above the windows, ⁹⁴ and secondly, he crafted the superstructure around the dome with a

^{93 -} The analysis was a linear analysis using three-dimensional beam elements acting in both tension and compression, which is a simplifi- cation of the actual nonlinear response of such a dome. Nevertheless, the qualitative deflection information provides a good understanding of the basic dome behavior and helps in interpreting Guarini's understand- ing of the dome behavior.

^{94 -} Guarini's iron reinforcement is distinguished from later iron tie rods and cramps (formed from machine-rolled bar stock) by its rounded, hand-hammered profile. Guarini's iron ring about the dome is located approximately 30 cm. below the larger, nineteenth-century, iron hoop. Guarini's tension rods are relatively small in cross section, and they probably would offer limited resistance to significant movement in the dome. It is Guarini's qualitative

scalloped, concave profile, enhancing the depth of the ribs in the haunches. Traditionally, hemispherical domes generated significant horizontal thrust at the haunches, often mitigated by iron tension rings. Despite his confidence in the elliptical shape of his dome, Guarini acknowledged the need for reinforcement, evident in the deep section of the ribs and the placement of iron reinforcement above the windows. This underscores that his choice of dome shape was not solely driven by structural concerns but also by aesthetic and optical considerations.



FIGURE 71- ROBISON - OPTICS AND MATHEMATICS IN THE DOMED CHURCHES OF GUARINO GUARINI - S. LORENZO. SECTIONAL AXONOMETRIC VIEW OF DOME, SHOWING LOCATION OF IRON HOOP REINFORCEMENT. SUCCESSIVE LAYERS OF THE DOME HAVE BEEN ELIMINATED TO SHOW ITS CONSTRUCTION

Islamic, Romanesque or Gothic sources may account for the dome's intersecting rib pattern, but Guarini's perforations within it are entirely unprecedented in the history of architecture. The perforations along with the dome's hidden support system make possible sources such as the much smaller dome of the mosque in Cordoba seem insignificant in the face of Guarini's powers of invention. ⁹⁵ However, as I'll elaborate on in subsequent chapters, it's highly improbable that Guarini ever traveled to Spain. Therefore, we can only speculate, along with others, about the origins of the cupola. It could be attributed to Guarini's exploration of Gothic architecture, his familiarity with Borromini's Propaganda Fide, or perhaps his exposure to Islamic architectural influences in Sicily.

According to observations made by Oechslin and Blunt, Leonardo da Vinci suggested dome designs resembling those of S. Lorenzo, featuring intersecting ribs and even openings between them in one sketch. Mainstone further notes that Leonardo attributed these ideas to "del tedesco in domo," indicating a Gothic influence from Northern Italy. Pommer also identifies S. Lorenzo's similarity to Gothic architecture, describing "...the vault is truly a tower rather than a dome". A study of the church's section supports this interpretation, given its considerable height relative to the main vessel's area.⁹⁶

understanding, however, which is significant to this discussion, as the ability to quantify structural behavior correctly for even simple beams was not possible until Navier's developments of flexural theory in the early nineteenth century.

^{95 -} Sigfried Giedion first popularized the comparison between Cordoba and S. Lorenzo in Space. Time and Architecture. Cambridge, MA, 1st ed, 1941, pp. 58-9, although others had noted the similarity before; see the Lisbon chapter below. For alternative sources for the domes see Oechslin, "Bemerkungen", pp. 104-7 and P. Portoghesi, Guarino Guarini. Milan, 1956, p. [6], Admittedly, the Cordoba dome displays a far more stunning similarity with the S. Lorenzo dome than any of these other examples.

^{96 -} Pommer, p. 9. Pommer's comment recalls Francesco Paciotto's critique of the Escorial dome, in which he likened the exceptionally high dome to a Gothic tower:

CONNECTION BETWEEN GUARINI'S ARCHITECTURE IN SAN LORENZO DOME WITH SPAIN AND PORTUGAL AND DIFFERENT DEBATES ON THIS QUESTION

In the discourse surrounding Guarini's Lisbon church, there exists a compelling conjecture that ties its genesis to a possible journey undertaken by the architect to Spain and Portugal during the late 1650s. Opinions among scholars diverge regarding the church's chronology, with some attributing its creation to the early stages of Guarini's career and others positing it as a culmination thereof. The ambiguity extends to whether the church was ever erected, its potential existence rendered uncertain by the Lisbon earthquake of 1755, which likely consumed it if indeed it stood. Our insights into Guarini's involvement with this endeavor are primarily from two engravings preserved in the Architettura civile. The engravings portraying his S. Maria della Divina Providence were dedicated to Ardizzone. Despite efforts to glean a definitive timeline from this dedication, it remains elusive.

In scholarly discourse, opinions regarding the dating of Guarini's church exhibit notable differences. Two main factions emerge: one asserts the church's design to have originated in the "blank years" of the late 1650s, while other scholars advocate for a later date around 1680. These proposed dates correspond to significant events in the history of the Lisbon church, situated at opposite ends of Guarini's career. The evaluations put forth by scholars, drawing from historical and stylistic evidence, pose relevant inquiries regarding methodological approaches, particularly concerning the stylistic benchmarks utilized to establish the chronology of the Architettura civile engravings.

Proponents of the early dating group tend to rely on circumstantial indications suggesting Guarini's presence in Spain and Portugal during the late 1650s. Conversely, advocates of the later date reference pivotal moments such as the 1681 building license granted to the Lisbon Theatines or the year 1698. ⁹⁷ Few contend that the church predates Guarini's architectural endeavors in Messina or Paris. Passanti, for instance, places the church's construction around 1680, highlighting its heightened complexity compared to Guarini's preceding longitudinal church designs. ⁹⁸ Millon wonders that the church "well may be the latest of the group," potentially conceived in 1681, yet provides limited elaboration on his rationale. ⁹⁹ Portoghesi similarly regards S. Maria della Divina Providenza as the "most mature" within Guarini's series of longitudinal churches.

Robison, credited with identifying the 1680 copy of the plan in the Archivio di Stato in Turin, approximates the church's origin to circa 1680. He notes the distinct architectural features of the church, suggesting that elements such as the shell-like piers with their substantial voids align more closely with later designs than with the conservative structure of Ste.-Anne-la-Royale.(Figure 4)¹⁰⁰ Given the diverse array of opinions, it becomes evident that further historical and stylistic insights are necessary to arrive at a conclusive determination regarding the church's chronology.

Regarding Klaiber's writtings the influences behind Guarini's design for S. Maria della Divina Provvindenza by delving into the court environments of Turin and Paris. However, the historical context of Guarini's Lisbon church has often been linked to a speculative journey to the Iberian Peninsula between mid-1657 and early 1660. As outlined by Meek, which revolves around Guarini's architectural influences from Islamic Spain evident in his works across France and Italy, as well as his proposals for the Lisbon church. "what Guarini brought back from Islamic Spain to put into his own architecture in France and Italy, and what he actually proposed for his Lisbon church." ¹⁰¹ Additionally, details from Guarini's initial publications contribute to this discourse. Evidence suggesting a Spanish influence on Guarini's architecture includes his references to the Gothic churches of Salamanca and Seville in his renowned chapter on Gothic architecture in Architettura civile. Guarini's exposure to Islamic architecture in Spain is also highlighted, particularly the resemblance between

[&]quot;...it is more like a belfry than a dome, and truly a ridiculous thing, of a measure never seen among the ancients nor among moderns, unless perhaps among Germans or Goths, of whose architecture I do not speak, for knowing its rule or measure, being entirely contrary to my practice." (G. Kubler, Building the Escorial. Princeton, 1982, p. 49.)

^{97 -} when the Theatines laid a cornerstone for a new church

^{98 -} M. Passanti, Nel mondo magico di Guarino Guarini.

Turin, 1963, pp. xv, 68-70.

^{99 -} Millon, "Guarini," Macmillan Encyclopedia, p. 273.

^{100 -} E. C. Robison, Guarino Guarini's Church of San Lorenzo in Turin. Ph.D. diss., Cornell, 1985, pp. 307-8. The comparison with the Ste.-Anne plan should be modified with our new understanding of Valperga's contribution to that design. For the copy of the plan, see AST, Corte, Cat. 95, mazzo 1, n. 3. 101 - Meek, p. 12.

the rib patterns at S. Lorenzo and those found in Cordoba's mosque, as illustrated by Sigfried Giedion in Space, Time and Architecture. ¹⁰²

Since Guarini employed the order precisely for the Lisbon church, Ramirez argues that he went to Spain and Portugal during the "blank years" of mid-1657-1660 to design Santa Maria della Divina Providenza and there applied ideas he absorbed while he was in Lisbon.

The most recent argument for Guarini's travels in the Iberian Peninsula is among the most serious and credible. In 1981 Juan Antonio Ramirez introduced a new element into the case: the Spanish Benedictine Fray Juan Ricci, who wrote treatises on painting and architecture in the late 1650s and early 1660s.¹⁰³ Ricci's painting treatise had perhaps not yet been written during Guarini's "blank years" and remained unpublished until the twentieth century, so if Guarini knew of the treatise it must have been through personal contact. Ricci left Spain for Italy in 1662, and he completed his painting treatise by that date; the architectural treatise dates from 1663. Ramirez speculates that Guarini met Ricci or his followers in Madrid or Medina del Campo and learned of his ideas.

Had Guarini learned of the complete undulating order from Ricci in Spain, or did he develop the order himself? Ramirez asks this question as well, "... could it not be an example of one of those rare coincidences where two ingenious minds, with a similar cultural equipment, arrive at the same conclusions independently of each other?"¹⁰⁴ We must judge ourselves which coincidence is more plausible, a meeting between Ricci and Guarini in Spain, or simultaneous independent development of the ideas.

Despite similarities of form and language, however, substantial differences exist between the two men's ideas. Ramirez points out that while the bases, pedestals and capitals of Ricci's order undulate, Guarini remains silent about the appearence of these members in his order.¹⁰⁵ Ramirez's argument deserves careful consideration; at the very least it points out an admittedly remarkable coincidence. But the Ricci connection is not sufficient in itself to prove Guarini traveled through Spain.

All discussions of Guarini's possible Iberian travels neglect some historical details. Spain and Portugal were engaged in conflicts from the Braganza restoration of 1640 until the Treaty of Lisbon, in which Spain recognized an independent Portugal in 1668.¹⁰⁶ Thus, travel to one country by no means implied travel to the other. Further, virtually all travel to Portugal from Italy in these years was by sea, not overland. A stop in Madrid or Cordoba thus appears improbable. The only direct connection between Guarini and Portugal is the plan and section of S. Maria della Divina Providenza in Lisbon, presented as plates 17 and 18 in the Architettura civile.

Bearing all this in mind, one may, in the absence of documentary evidence, be tempted to look instead for circumstantial evidence in support of the hypothesis that Guarini visited Prague and Lisbon in the blank years - blank to us, that is - between 1657 and 1660, For Prague there is little to show, for Lisbon a great deal. The circumstantial evidence of a visit by Guarini to the Iberian peninsula is of two main kinds: what Guarini brought back from Islamic Spain to put into his own architecture in France and Italy, and what he actually proposed for his Lisbon church.

At this point, when no buildings have as yet been described or analyzed, we may summarize the position in this way: one of the most striking of all Guarini's architectural devices is the openwork dome, made up of interlaced ellips shape arches that sweep over the centre of the space they roof leaving an octagonal opening in the middle, above which further structures are superimposed. The closest parallel to this type of construction is to be seen in two of the domes at the Great Mosque of Cardoba completed before AD.976, where the superficial similarity between the two designs is very striking.

^{102 -} S. Giedion, Space. Time and Architecture. Cambridge,

MA, 1st ed., 1941, pp. 58-9. However, Otto Schubert first made the point in his Geschichte des Barock in Spanien. Esslingen, 1908. See Terzaghi, p. 377, n. 1, and De Bernardi Ferrero, 'Disegni'. pp. 23-8 for a survey of the early literature on this subject.

^{103 -} E. Lafuente Ferrari, ed., La vida y obra de Fray Juan Ricci. Fray Juan Ricci, escritor de arte v pintor de la escuela de Madrid por Elias Tormo v Monzo. Biografia del pintor D. Juan Andres Ricci, monie de Monserrat. por el P. Celestino Gusi. Textos v iuicios criticos sobre Fray Juan Ricci con el Tratado de la Pintura Sabia del P. Ricci. Madrid, 1930; Brebe tratado de Arcruitectura acerca del Orden Salomonico Entero. 1663, still MS [?], Montecassino. 104 - "But why should we suppose an influence of Ricci in Guarini, and not the reverse?" (Ramirez, p. 178).

^{105 -} Ramirez, p. 180, suggests Guarini refers to Ricci when he discusses Salomonic columns, stating: "Pero alcuni hanno giudicato, che cio sia un'ordine speziale..." (AC. III.viii.3). But the statement is taken out of context, and Guarini continues, "...ma perche tutte le Colonne, benche Doriche, o Joniche possono esser a vite, o torte; quindi e, che non essendo accompagnate da alcuna propria Cornice non si puo chiamar ordine." Thus, Guarini refers to those who have considered the twisted column alone a separate order.

^{106 -} Spain held the Portuguese throne from 1580-1640. After the 1640 restoration, civil war between Spanish loyalists and the supporters of Joao IV was a threat until the king's death in 1656 (Kubler, Portuguese Plain Architecture, p. 146). The Spanish then invaded Portugal in 1663-65.

The other Guarinian feature that seems to have Arabo-Spanish and Mudejar origins is the 'telescopic disposition of vertical space; that is, the development of the section as a series of independent regions of decreasing horizontal dimension as the building rises. This type of volumetric design is characteristic of Mudejar architecture.

Islamic prototypes for the S. Lorenzo nest shape arches were noted as far back as 1908 in O. Schubert's book on the history of the Baroque in Spain,¹⁰⁷ and have been the subject of several studies since. Guarini may be presumed to have seen the 'prototype' of the S. Lorenzo arches in Spain," notably in the Great Mosque in Cordoba, now th cathedral, completed under Caliph Al-Hakim II. It was Siegfried Giedion's publication of the cupola of S. Lorenzo and one of the domes in the Great Mosque at Cordoba in his famous book Space, Time and Architecture. that first popularized Guarini's achievements - and sources - with a wider public. Giedion reproduced his illustrations of Guarini's cupola and the Cordoban dome on facing pages, in photographs of identical size, while readily admitting that the dimensions of these Moorish domes are humble in comparison with Guarini's daring masterpiece. Guarini does not mention the Great Mosque of Cordoba in his Treatise, but he does refer to the great church of Seville in Andalusia and the cathedral of Salamanca in Castile.¹⁰⁸ but it is certain that Guarini's stay in Messina would have allowed him to become familiar with Arab monuments of which far more were standing in the seventeenth century than today.¹⁰⁹ But at all events, as Paolo Verone remarks, 'in reality Islamic art has had a catalytic effect on Guarmis fervent and gifted mind, since it was also permeated with geometrical stylizations'¹¹⁰ Thus, to take the most obvious example, none of the Islamic domes suppresses the web - the infill between the ribs - in the way that Guarini does, with the consequent diffusion of high-level light that converts his dome from one with a single continuous mural boundary into an aerial cage, opening onto an outer zone.



FIGURE 72-A 'NEW MAQSURAH' DOME IN THE MOSQUE OF AL-HAKIM, CORDOBA, CORDOBA, COMPARED WITH THE DOME OF S. LORENZO, TURIN (PHOTOGRAPHED BEFORE DAMAGE SUFFERED IN WORLD WAR II)

109 - See on this point Eugenio Galdieri in the foreword to Florensa, pp. 637-8.

^{107 -} O. Schubert, Geschichte des Barock in Spanien (Esslingen 1908), p. 176.

^{108 -} AC Ill.xiii.1. Guarini's bête noire, the Spanish architect-bishop Juan Caramuel de Lobkowitz, also refers to these buildings in his book Architectura civil recta y obliqua (Vigevano 1678), p. 74, a book which Guarini read with the closest attention. This does not, of course, exclude the possibility of Guarini having seen them himself.

^{110 -} Verzone, p. 404.
ANNEX

GUARINO GUARINIS'S LIFE IN A GLANCE

This chapter of my thesis provides a concise overview of Guarini's life, presented in the form of a timeline. It is important to acknowledge that the papers and website of Professor Susan Klaiber have played a crucial role in the development of my master's thesis. Specifically, this chapter draws extensively from Susan Klaiber's WordPress website, constituting a direct citation. Due to the significance of this summary, I have adapted the timeline to better align with the criteria discussed in my thesis, presenting it in a more selective and graphically illustrative manner.

The information used in this timeline is based on Professor Klaiber's website and according to her claim on the website is derived from three primary sources:

"- Guarini's location as attested to by letters, Theatine chapter minutes, and other contemporary documents.

- The construction dates of his major architectural projects, when known, and of the few unexecuted projects with clearly fixed dates.

- The dates and places of publication of his books."¹¹¹

The main sources of information for this timeline are as follows:

- Sandonnini, Tommaso. "Il Padre Guarino Guarini modenese." "Atti e memorie delle RR. Deputazioni di storia patria per le provincie modenesi e parmensi", series III, volume V, part II (1890): 483-534.

- Baudi di Vesme, Alessandro. "Guarini, Guarino." In "Schede Vesme. L'arte in Piemonte dal XVI al XVIII secolo", vol. 2. Turin: Società piemontese di arti belli e archaeologia, 1966: 550-558.

- Lange, Augusta. "Disegni e documenti di Guarino Guarini." In V. Viale, editor, "Guarino Guarini e l'internazionalità del Barocco", vol. 1. Turin: Accademia delle Scienze, 1970: 91-344.

¹¹¹ Klaiber, Susan, Guarino Guarini's Timeline, https://guarinoguarini.blogspot.com/, June,2024

TIMELINE OF GUARINO GUARINI (ANNEX)

January 17, 1624 -

Born in Modena

Baptized on 22 January 1624 "Addì 22 Gennaio 1624 - Guarino figlio del S.r Rinaldo Guarini et della Signora Eugenia Marescotti sua moglie fu battezzato."



June 6, 1634 📫

Cornerstone laid for San Lorenzo, Turin

Plans for San Lorenzo, Turin, originated in a vow by Duke Emanuele Filiberto on 10 August 1557. In June 1634 the church was moved to its present location and construction begun according to plans by Carlo di Castellamonte. Guarini took over the project in the late 1660s.



Image: Google Books / Bibliothèque municipale de Lyon The cornerstone insription of San Lorenzo, Turin. From Giuseppe Silos, Historiarum clericorum regularium, vol. 2 (Rome: Heredum Corbelletti, 1655): 444

November 27, 1639 -

Enters Theatine order as novice

Guarini spent his novitiate at San Silvestro al Quirinale, the first Theatine church in Rome.



Image: www.info.roma.it Giuseppe Vasi, San Silvestro al Quirinale, 1761

- 73 -



Vows to Theatine order, Rome

To make his profession to the Theatine order, Guarini signed documents on 11 and 14 April 1641 ending his novitiate us Juaninius Digeens Matinensis In

Image: Archivio Generale dei Teatini, Rome / Susan Klaiber (CC BY-NC-ND 3.0) Guarini's autograph vows, 14 April 1641

December 29, 1647 📑 — July 2, 1657

Modena

After his novitiate and Theatine education, Guarini is first mentioned again in Modena at the Theatine casa of San Vincenzo in late 1647. He remains based there until

at least July 1657.



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Image: Biblioteca Estense / urbanistica.comune.modena.it Perspective plan of Modena, first half 17th century

June 4, 1653

Project for dome, Modena

Guarini's first documented architectural project is an unexecuted dome for the Theatine church San Vincenzo, Modena

"...cupola della nostra chiesa di legno coperta di piombo, sul modello fatto di cartone dal P. Don Guarino"

Rome

November 20, 1655 — December 6, 1655

In his capacity as elected preposito of the Modena Theatines, Guarini attends a Theatine chapter general meeting at the order's mother church San Silvestro al Quirinale, Rome.



Image: Wikimedia Commons / public domain

The church, casa, and gardens of San Silvestro al Quirinale (detail from Giambattista Nolli, map of Rome, 1748)

Messina

Guarini is in Messina "for many years" (at least June 1659-early 1662), where he teaches at the Archiepiscopal Seminary and builds the facade of the Theatine church Santissima Annunziata

Image: Wikimedia Commons / public domain Guarino Guarini, facade of Santissima Annunziata, Messina, 1660, destroyed 1908.

1660

La pietà trionfante

Published in Messina



LA PIETA Trionfante RAGICOMEDIA MORALE

November 8, 1661

Image: Biblioteca Braidense / opac.braidense.it/vufind/Recor...

Cornerstone laid for Sainte-Anne-la-Royale, Paris

Construction of the Theatine church in Paris was funded with a bequest of Cardinal Mazarin, and begun under the supervision of Maurizio Valperga. Guarini took over a year later.

Image: Wikimedia Commons / public domain The Theatines' property and future building site for Sainte-Anne-la-Royale on the Quai Malaguais, across from the Grande Galerie of the Louvre (Gomboust 1652)

Rome

October 26, 1662 -September 27, 1666

During this four-year period, Guarini is regularly documented in Paris. He takes over construction of Sainte-Anne-la-Royale and substantially alters the initial design, but the

- church remains unfinished.
- Image: archive.org / public domain

Guarino Guarini, elevation of Sainte-Anne-la-Royale, Paris, from Dissegni d'architettura, 1686, plate 10.











Image: © Google Street View

Guarino Guarini, Santa Maria d'Araceli, Vicenza (today: Santa Maria in Araceli), 1675-1680.



1676

Palazzo Carignano, Turin

Work begins on 11 May 1679, with a first payment to Guarini recorded on 6 August 1679. Most construction completed by 1684, interior work continued until 1693.



Image: Marco Plassio, Wikimedia Commons (CC BY-SA 3.0) Guarino Guarini, Palazzo Carignano, Turin, 1679-1693.

1683

Caelestis Mathematicae

Two volumes, published in Milan, in press at the time of Guarini's death.

MATHEMATICAE PARS PRIMA. IN QUA LEGES ANTIQUÆ ET NOVÆ TEMPORYM. AC PLANETARYM DISRAVYVA A V C T O R E O GVARINO GVARINO

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March 6, 1683

Dies in Milan

"[la] morte del Padre D. Guarino Guerini...seguì la notte del sabato" - the Savoy resident in Milan in a letter to Duke Vittorio Amedeo II in Turin, Tuesday, 9 March 1683



Image: archive.org / public domain Detail from the Guarini portrait frontispiece of the Dissegni d'architettura (Turin: 1686)

1737 -

Architettura civile

Published in Turin, edited by Bernardo Antonio Vittone



Image: archive.org / public domain

<u>part II</u>

CHAPTER 6

GEOMATIC ANALYSIS OF SAN LORENZO'S RIBBED DOME

MATERIAL AND METHOD OF THE SURVEY

The conservation of built heritage is regarded as a fundamental aspect of modern society. A artistic, cultural, and intrinsic values join together making built heritage extremely important. Complementary to this, the need to analyze the buildings' own structural values, the characteristic behavior of their masonry, as well as the complex interaction between components, and the lack of documentation make the analysis of such constructions remarkably difficult. Currently, numerous regulations propose the integration of different approaches, ¹¹² including: (i) the study of the construction's history; (ii) inspection; (iii) monitoring; and (iv) structural analysis.

In the field of built constructions, remote sensors, especially photogrammetric and laser scanner systems, have proven valuable for analysis. These sensors provide accurate and dense point clouds with geometric and radiometric values, which can be used to assess buildings through non-intrusive means. Despite this, the data provided (in the form of dense and accurate point clouds) is largely untapped, as it is often used normally for construction of simplified CAD models. ¹¹³ A geomatic survey was conducted in this thesis to generate a 3D model describing the geometry of the San Lorenzo dome.

DATA COLLECTION

Geomatic tools such as laser scanners, drones, and photogrammetry are employed to gather geometric and radiometric data. Laser scanners capture precise 3D measurements of the building's exterior and interior. Drones equipped with cameras or LiDAR (Light Detection and Ranging) sensors collect aerial imagery and topographic information. Photogrammetry involves taking overlapping photographs from various angles to create 3D models. In 2024 A geomatic survey was conducted to generate a 3D model accurately describing an essential part of the structure of the San Lorenzo Dome, integrating both LiDAR and drone photogrammetry techniques.

The LiDAR scan was executed from nine strategic points: four were symmetrically positioned directly below the dome, with the other locations visible in the accompanying figures (Fig-73). However, due to the substantial distance between the LiDAR setup locations and the central dome, the point density was reduced, particularly regarding the dome's height. Additionally, since the laser scans were conducted from ground level, certain sections of the dome, such as the upper regions of the drum beneath the oval windows, were not captured accurately. To address this, a drone equipped with photogrammetry techniques was utilized to capture these challenging-to-reach areas.

^{112 -} Saloustros, S.; Pelà, L.; Roca, P.; Portal, J. Numerical analysis of structural damage in the church of the poblet monastery. Eng. 2015, 48, 41–61. 113 - Villarino, A.; Riveiro, B.; Gonzalez-Aguilera, D.; Sánchez-Aparicio, L. The integration of geotechnologies in the evaluation of a wine cellar structure through the finite element method. Remote Sens. 2014, 6, 11107–11126.



FIGURE 73 - LOCATION OF THE LASER SCANNER ON THE POINT CLOUD PLAN OF S.LORENZO FROM GROUND FLOOR AND SERLIANE ZONE

SPECIFICATIONS OF THE LASER SCANNER

The geometry of the dome was acquired using a combination of LiDAR laser scanning and drone photogrammetry. Specifically, it was necessary to perform scans at four laser stations. Each scan provided a horizontal coverage of 360°, acquiring a point cloud with an average resolution of 6mm at 10m, using four single shots per point, with a level of detail (LOD) of 1-15. This method resulted in a total of approximately eight million raw points in XYZ coordinates for the geometric definition of the San Lorenzo Dome.

Laser scanner	Faro Focus ^{3D} X 330
Distance measurements	Phase shift
Wavelenght	1550 nm
Distance Range	0.6-330 m
Horizontal and Vertical Range	305/360 °
Distance accuracy	± 2 mm @ 10 m
Acquistion speed	up to 976.000 pt/s
Camera	RGB integrated
Weight	5.20 kg

Resolution	Quality	Scan time in min	No. of points in million	Point distance in mm/10m distance
1/32	4x	2:03	0,7	49,087
1/20	4x	2:14	1,7	30,680
1/16	4x	2:23	2,7	24,544
1/10	4x	3:05	7,0	15,340
1/8	4x	3:44	10,9	12,272
1/5	4x	6:31	28,0	7,670
1/4	4x	9:06	43,7	6,136

FIGURE 75-SPECIFICATIONS OF THE LASER SCANNER

FIGURE 74-RESOLUTION AND QUALITY OF THE LASER SCANNER (FARO FOCUS X330)

	tati scansione	Tensioni dei	punti di scansione				
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ScanFit				3.90	San_Lorenzo_Scan_002		
ScanFit				3.58	San_Lorenzo_Scan_008		
ScanFit				3.50	San_Lorenzo_Scan_004		
ScanFit				3.28	San_Lorenzo_Scan_001		
ScanFit				3.14	San_Lorenzo_Scan_007		
ScanFit				3.09	San_Lorenzo_Scan_005		
ScanFit				2.91	San_Lorenzo_Scan_006		
ScanFit				2.72	San_Lorenzo_Scan_003		
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Vista c Preleva Applica	a		Statistic	ppia visualizzazione o che complessive Medio: 3.4456 < 4 mm: 58.2	OK Ann OK [mm]		

OBSERVED IN THE CONDUCTED SURVEY. AS EVIDENT FROM THE DATA, APPROXIMATELY 60% OF THE COLLECTED DATA EXHIBITS AN AVERAGE ERROR OF LESS THAN 3.5 MM. THE FIGURE ABOVE ILLUSTRATES THE AVERAGE ERROR AMOUNT FOR EACH SCAN INDIVIDUALLY. NOTABLY, THE OVERLAP OF LASER SCAN POINTS SIGNIFICANTLY CONTRIBUTES TO A DECREASE IN THE AVERAGE ERROR. CONSEQUENTLY, THE CONDUCTED SURVEY ACHIEVES MAXIMUM ACCURACY AND MINIMUM DISCREPANCY.

OBSTACLES AND LIMITATIONS

In this section of the survey, several challenges were encountered that significantly impacted the accuracy of the final product due to the absence of a dense and well-distributed network of ground control points (GCPs). Given the limited density of the executed laser scan, it is important to note that the height from the ground floor to the top of the main dome measures approximately 33.5 meters. As a result, positioning the laser scan equipment on either the ground floor or the serlian level (that is, at a height of approximately 15.6 m from the ground floor) did not yield a sufficiently dense point cloud for the higher levels, such as the top of the main dome and the upper portion of the ribs. Furthermore, the laser could not reach the lower part of the oval windows on top of the cornice ring, making this area unavailable in the extracted point cloud. To capture the missing portion of the point cloud, an alternative method was implemented using a drone to ascend to the top of the main dome and complete the data acquisition. However, this stage also faced challenges due to the lack of a well-distributed network. Both the LiDAR equipment and the drone require network connectivity to obtain the coordinate system. Weak network connectivity hindered data transmission and the drone's ability to synchronize its coordinate system. This network limitation resulted in disorganized points in the extracted photogrammetry at locations affected by network inadequacies. Consequently, it became necessary to subsequently adjust and synchronize these two point clouds from a coordinate perspective.

What are Ground Control Points (GCPs)?

Ground Control Points (GCPs) are physical markers strategically placed in the survey area with known,

accurately surveyed coordinates. These points act as reference markers for the drone's images and assist in georeferencing the collected data. They provide a known spatial reference that ties drone-captured imagery to real-world coordinates.

In photogrammetry with drones, GCPs are crucial for ensuring the accuracy and reliability of the final output, especially when creating detailed 3D models or maps of structures. In the absence of a dense and well-distributed network of GCPs, several issues can arise:

- The model may lack accurate georeferencing, leading to misalignment of the drone-captured images and inaccuracies in the 3D model.
- Spatial distortions, such as scaling issues or misshapen elements, may occur due to the software's inability to properly align and orient the images.
- Measurements and dimensions within the model may not accurately represent the real-world scale.

Importance of GCPs in Photogrammetry:

- Georeferencing and Accuracy: Drones capture images from various angles, which are then used to create a 3D model of the San Lorenzo Dome. GCPs provide known reference points in these images, allowing software to accurately position and orient the images relative to each other and the realworld coordinates. Without GCPs, the accuracy of the model might be compromised, as the software would lack precise reference points to align and scale the imagery correctly.
- Error Correction: GCPs help correct various errors that can occur during image capture and processing, such as lens distortion, camera sensor inaccuracies, or inaccuracies in drone positioning. They act as control points to minimize these errors and ensure the model's accuracy.
- 3. Scale and Orientation: GCPs assist in establishing the correct scale and orientation of the 3D model. They help in accurately placing the model in its real-world context, ensuring that measurements and spatial relationships within the model align with the actual dimensions.

WORKFLOW OF MATCHING COORDINATE SYSTEM OF PHOTOGRAMMETRY

WITH THE LIDAR POINT CLOUD AND GENERATING THE POINT CLOUD FROM

THE PHOTOGRAMMETRY

In this process, two primary software tools play a vital role: MetaShape and Cloud Compare. The workflow begins by importing all photos extracted from drone videos into MetaShape.
 Since the photos lack initial alignment based on location and rotation, the first step is to eliminate any existing alignment. Subsequently, the photos are aligned using the "Align" function under the workflow tab in MetaShape. It is preferred to set the accuracy to high, ensuring each photo undergoes thorough review.

MetaShape then utilizes points extracted from the drone photos to generate a 3D model. To ensure consistency in orientation with the coordinate system of the LIDAR point cloud, metric values are introduced. Aligning the drone's extracted points involves synchronizing them with the orientation and scaling of the LIDAR data.

Achieving alignment requires consideration of three critical pieces of information:

- A. Interior Orientation: Parameters related to the camera's internal characteristics, such as focal length and lens distortion.
- B. Relative Orientation: Spatial relationship between images, typically involving camera positions and orientations relative to each other.
- C. Absolute Orientation: Specifically, the alignment along the z-axis derived from the LIDAR scan's coordinate system.

These elements are essential for integrating drone-captured imagery with LIDAR data, ensuring accurate spatial alignment and scaling in the final 3D model.



FIGURE 77- IMPORTING THE DRONE PHOTOS INSIDE THE METASHAPE AND ALIGNING THE PHOTOS

In Metashape, integrating precise location data from the LIDAR point cloud is essential. This involves
opening the LIDAR file in Cloud Compare to accurately incorporate LIDAR data into the Metashape
workflow. Opening the Lidar file in Cloud Compare is crucial, particularly for identifying points visible
in both the drone photos and the LIDAR point cloud. This step proved challenging in the case of San
Lorenzo due to the dome's fractal geometry, initially complicating the exact pinpointing of locations.
To address this challenge, we cross-referenced details from the spherical pendentives in the drone
video with the point cloud to identify the selected points.

Maintaining a uniformly distributed set of points is critical because Metashape treats some as Ground Control Points (GCPs) for absolute orientation and scaling, and others as Control Points (CPs) for accuracy checks. Capturing screenshots and documenting coordinates in a clear format (e.g., point20;5.286;59.863;234.191) ensures that Metashape can interpret the data accurately. While a minimum of 15 points is recommended, approximately 26 points were selected for the San Lorenzo project: 15 from intersections of ribs, 3 from corners of the serlian windows, and 8 from within the oval windows inside the dome.

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							V P point2	14.777000	í	62.013000	62.013000 238.846000	62.013000 238.846000 0.005000	62.013000 238.846000 0.005000 0.003026	62.013000 238.846000 0.005000 0.003026 8
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							🗸 🏲 point8	16.202000	54.511000	24	0.174000	0.174000 0.005000	0.174000 0.005000 0.018707	0.174000 0.005000 0.018707 10
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port CSV	/					×	🗸 🏲 point10	15.773000	60.064000	239.9030	000	0.005000	000 0.005000 0.005114	000 0.005000 0.005114 6
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FIGURE 78- INPUTTING THE PRECISE LOCATION OF THE POINTS BY THEIR COORDINATES OBTAINED FROM THE LIDAR POINT CLOUD AND INSERTING THE COORDINATES INTO METASHAPE

In MetaShape, establishing a local coordinate system is a detailed process that involves several
precise steps. First, you navigate to the reference tab and import a prepared note file that serves as a
reference for coordinating the alignment process. Once imported, activate the local coordinate
system within MetaShape to ensure all subsequent positioning and alignment are based on this
designated framework.

The next crucial step is to precisely position markers in all relevant photos based on their original locations in the Lidar point cloud. This meticulous approach guarantees accurate alignment of photos within MetaShape, ensuring each marker corresponds correctly to its spatial coordinates as defined by the Lidar data. This methodical workflow ensures that the project maintains consistency and accuracy throughout the alignment process.

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FIGURE 80- PRECISELY POSITION MARKERS IN ALL RELEVANT PHOTOS BASED ON THEIR ORIGINAL LOCATIONS IN THE LIDAR POINT CLOUD

E	178		* ¢		0 II.	L.	头				
Mar	kers				X err (m)		Y err (m)		Z err (m)	
√	•	point10			-0.0042	224		-0.002489		-0.001457	
√	۲	point11		0.000336			-0.002800	-0.000185			
√	•	point12			-0.012226			-0.003858	-0.003076		
√	Þ	point13			-0.0357	715		0.028706		-0.020986	
√	۲	point15			0.0100	39		-0.023524		-0.001674	
√	P	point20			0.0069	33		0.003801		0.011655	
√	Þ	point21			0.0097	49		0.015387		-0.028820	
√	▶	point22			0.0294	48		-0.006873		-0.017135	
√	Þ	point23			-0.0019	940		0.000236		-0.005660	
√	۲	point24			0.0154	62		-0.001478		0.007559	
✓	۲	point25		(0.0011	03		0.007799		0.019281	
√	►	point26			-0.0078	347		0.006511		0.015823	
√	۲	point27			0.0051	41		-0.010017		-0.022303	
✓	۲	point28			-0.005	153		-0.005915	0.001327		
√	۲	point29		(0.0053	09		-0.009547	0.027449		
√	۲	point30			0.0059	24		-0.006598	0.023911		
✓	۲	point31			0.0243	397		-0.004211		0.012040	
√	Þ	point32			-0.003	731		-0.002362		0.000590	
✓	۲	point33		(0.0039	96		0.017134		0.006073	
√	۲	point34			-0.006	114		0.031299		-0.009612	
√	۲	point35			-0.000	958		0.003281		0.002291	
✓	۲	point100		(0.0246	53		0.016876		-0.004294	
✓	۲	point200			0.0084	74		-0.012214		0.004370	
√	۲	point300			0.0132	87		-0.011463		0.012717	
ota	l Eri	ror									
Co	ontro	ol points			0.0128	01		0.013708		0.012404	
Cł	neck	points									

FIGURE **79-** THE DISCREPANCY BETWEEN THE LOCAL COORDINATE SYSTEM AND THE ACTUAL LOCATION INDICATED FOR EACH POINT BASED ON ITS POSITION IN THE DRONE IMAGES.

- In the final step, generate the point cloud from the photogrammetry data in MetaShape by navigating
 to the workflow tab and selecting "Build point cloud." Once this process is complete, it is crucial to
 compare the resulting point clouds generated from both LIDAR and photogrammetry using Cloud
 Compare. This comparison involves thorough examination of different sections both vertically and
 horizontally to ensure their precise alignment.
- In Cloud Compare software, for a detailed assessment, select both the LIDAR and photogrammetry
 point clouds and utilize the "distances/cloud-cloud distance" tool found in the tools tab. This tool
 enables a comprehensive comparison, where you designate the LIDAR point cloud as the "reference"
 and the photogrammetry point cloud as the "compared," facilitating meticulous evaluation of their
 alignment.

To visually assess accuracy, focus on the photogrammetry point cloud and adjust color settings by selecting "scalar field" and setting the color scale from blue to green to yellow to red. This creates a color-coded point cloud visualization where different colors indicate varying degrees of adjustment: blue for areas needing the most attention, progressing through green, yellow, and red. This method helps pinpoint areas requiring further refinement and aids in identifying potential alignment issues within the data.





FIGURE 81- THE "DISTANCES/CLOUD-CLOUD DISTANCE" TOOL IN CLOUD COMPARE SOFTWARE WHICH INDICATES COLOR-CODED VISUALIZATION AIDS IN IDENTIFYING REGIONS WITH POTENTIAL ALIGNMENT ISSUES

POINT CAB AND EXTRACTING SECTIONS AND PLAN

Having achieved proper scaling between the photogrammetry and the point cloud data, the subsequent stages focus on fulfilling the primary objective of this thesis: examining the geometry of the ribbed dome. The initial task involves refining the ribs using Cloud Compare. Here, a methodical approach is employed to define the borders of main arches or each rib precisely, ensuring the exclusion of extraneous elements like the dome's inner shell, surface perforations, or any unwanted components. The aim is to isolate the eight ribs with accuracy.

Subsequently, the software POINT CAB is utilized to derive precise rib plans and section lines for each individual rib. During this extraction process, a notable challenge arises. The intrados of the ribs exhibits a curved form reminiscent of a segment of an ellipse rotated 360 degrees around an axis. This curved structure implies that the interior of each rib does not conform to a flat plane but rather conforms to a continuously curved surface.

Visualizing these ribs, which have experienced minor settlements, tensions, and displacements since the 17th century, underscores their complex structural history. Although these deviations may not be immediately perceptible, they are meticulously documented through the point cloud data. Analysis of the point cloud reveals bends, tensions, and deviations at the intersections of the ribs, influencing the compression of certain rib sections. Significant attention is paid to identifying bends and tilts observed in the rib plans. The process of drawing section lines from the centerline of the ribs encounters challenges due to their curved nature. This curvature results in irregularities in projecting the rib's shape, leading to unconventional orientations of the section lines. This irregularity within the rib structure poses a substantial challenge, necessitating careful examination of the sections to identify those that accurately depict a section line ideally passing through the rib's centerline.





SECTION LINES ON THE POINT CAB PLAN

The figure below illustrates the section lines on Point Cab and the horizontal section of the dome from the cornice part. Each section line is designated by a specific number and differentiated by a color code corresponding to its respective rib. These section lines are intended to follow the centerline of the ribs from their starting to ending points. However, it is evident that these section lines cannot remain precisely centered on the rib throughout its entire length. The reasons for this will be further elaborated in subsequent sections.





7m





CONCLUSION

9 m

Based on the analysis of the point cloud sections, numerous deformations and bends are observed along the paths of the ribs. Ribs numbered 1, 2, 4, and 7 notably deviate significantly from their intended centerline due to bending, indicating substantial displacement from their original positions. Ribs 3, 5, and 8 also exhibit deviations, albeit less pronounced, from their centerlines as observed in Figure 1, where the section lines show irregular deformations along their paths. These deviations are likely caused by historical stresses exerted over time for various reasons.

Interestingly, the intersections of the ribs, serving as anchor points in the nest-shaped structure to maintain stability, do not seem to exhibit the same level of deformation. Consequently, the stresses applied do not uniformly affect the entire length of the ribs, resulting in varying degrees of deformation among them. Rib number 3 shows some minor deviations, indicating better structural integrity compared to others. In contrast, rib number 6 emerges as the most stable and reliable in terms of maintaining its position and structural integrity over time, exhibiting minimal to no significant deformations. This assessment underscores the dynamic nature of historical structures and highlights the importance of detailed analysis to understand their structural conditions accurately.

PROCESS OF CREATING THE IDEAL 3D MODEL OF S. LORENZO:

The Geometry of the Plan

1. As previously stated, the initial step in understanding the geometry of S. Lorenzo's plan relies on the extracted point cloud obtained during the laser scanning process, which is considered the most precise and reliable source for such analyses. As we have mentioned before This process starts with cleaning up the ribs and removing any unnecessary points using Cloud Compare. Following this, the refined point cloud is imported into PointCab software to obtain a precise plan.





2. The center of the dome is determined by connecting the intersections of the ribs, specifically where triangular perforations occur on the dome's surface to allow light penetration (referred to as point O).



3. At the approximate center of the dome's plan, we import the centerline of the rib from the PointCab plan and place it on a perpendicular reference plane positioned at the location of rib 6 on the plan. On this reference plane, a circle (circle A) is drawn to intersect the springing points (P and Q) of the rib. Additionally, a corresponding circle (circle B) is drawn on the plan to encompass all starting and ending points of the other ribs visible in the PointCab plan.



4. A line is extended from the dome's center on the plan to the top vertex of the triangular perforation formed by the intersections of the ribs (OC). Another line perpendicular to OC is drawn with the same

approach (OD). Where these two lines intersect circle B, two tangent lines to circle B, perpendicular to OC and OD, are drawn. The point where these tangent lines intersect circle B is labeled as point F. Using a similar method, points G, H, and I are determined.

- 5. Subsequently, points F, G, H, and I are connected to form a square. When this square is rotated 45 degrees around the same center point, the resulting shape forms an octagon, delineating the starting locations of the ribs.
- 6. To delineate the ribs, the process starts from one vertex of the octagon (vertex J). This vertex is connected to the third vertex on both sides (points K and M), leaving two remaining vertices on the sides. This method is repeated to establish the centerlines of the remaining ribs.



- 7. Comparing the PointCab plan with our own, we delineate the outer shell of the dome using a hypothetical circle (Circle N).
- 8. Based on measurements from the point cloud indicating a thickness and depth of approximately 50x70 cm for the ribs, another circle (Circle R) is drawn with a 70 cm offset from circle A to represent the inner shell of the dome and its thickness. This inner shell encompasses the nest-shaped web formed by intersecting ribs. Each rib's centerline in the plan is offset by 25 cm to outline the ribs. The smaller octagon within the intersected ribs serves as the opening at the dome's top.
- 9. An additional concentric circle (Circle S) is used to depict the surface of the intrados of the ribs, offset accordingly. These concentric circles collectively define the elliptical geometry of the dome and its ribs.
- 10. To incorporate windows and allow light penetration, existing openings at the top and triangular shapes are integrated into the dome's surface. Additionally, a pentagon and circular sector shape





need to be included for this purpose.

- 11. Between the ribs on circle B, oval-shaped windows are positioned as perforations, forming an arcshaped opening that penetrates through the dome's thickness. Placing this arc within the space between the ribs prepares for detailing the undulated edge at the dome's base.
- 12. The center of circle B is positioned on the plan along the line passing through the dome's center (O) and the vertex of the square (F). Ensuring this circle is tangent to circle B at two points corresponding to the intersections of the ribs (points T and U).
- 13. The developed plan represents the integrated geometry and serves as a foundational step for proceeding with the 3D model, requiring careful interpretation for accurate depiction.



COMPARING IDEAL 3D SECTIONS WITH POINT CLOUD SECTION LINES

From the ideal 3D model, sections of the ribs are extracted and compared with the section lines obtained from the point cloud using PointCab software. This comparison aims to ascertain the alignment of the 3D model with the centerline of the ribs. The rib section line that closely matches the 3D model is selected to advance to subsequent phases of analysis. Based on our assessment, rib number 6 is identified as the best match with the ideal 3D section lines due to its minimal deviations.



Ð

SECTION 8

SECTIONA

SECTION









COMPARING IDEAL 3D PLAN WITH THE POINT CLOUD HORIZONTAL SECTION

PLANES

The comparison between the horizontal section of the Point cloud and the Ideal plan is illustrated below across three distinct zones: cornice ring, pentagonal perforations, and top octagon void. The current placement of the rib springing locations directly above the cornice ring aligns perfectly with the ideal plan. This alignment is similarly observed in the top hexagon upon superimposition. However, in the area of the pentagonal perforation situated above the oval windows, some discrepancies are noted. Such variations are common, as traditional construction practices often deviated from initial designs, with adjustments frequently made on-site. Moreover, the dome has been subject to structural settlements and deformations, affecting its original plan.



FIGURE 84-ELEVATION OF S.LOREMZO'S MAIN DOME WITH THE RELATIVE HIGHT CODES FROM GROUND FLOOR





In the previous stage, section lines extracted from the ribs in PointCab were compared with the ideal 3D sections to determine the most accurate representation of Guarini's intended shape for the S. Lorenzo church. Moving forward, rib 6 is selected for further analysis. The traditional method involves extracting coordinates (X, Y, Z) from multiple points along the rib's center and inputting them into 3Dmax software to generate a polyline that defines the precise centerline section of the rib.



FIGURE 88- POINT CLOUD (LOOKING UPWARD FROM THE BOTTOM VIEW)



FIGURE 89- COORDINATS OF RIB 6'S CENTERLINE (CLOUD COMPARE)





FIGURE 90- COORDINATS OF RIB 6'S CENTERLINE (CLOUD COMPARE)



FIGURE 92- INSERTING THE COORDINATES OF THE POINTS INTO 3DMAX SOFTWARE TO GENERATE THE ACCURATE CENTERLINE OF THE RIB 6

EXPLAINING THE PROCESS

As previously detailed, this method stands as the most precise approach to delineate the centerline section of rib 6, achieved through meticulous point-by-point drawing. The process commences by selecting approximately 77 points along the rib's center, ensuring each point is precisely positioned on the rib's exact center. Each coordinate extracted must maintain a precision level of 0.001 meters. Using software such as 3D Max, these coordinates are inputted to create a polyline that connects all selected points, thereby defining the centerline section of the rib.

The emphasis on the rib's centerline in this research is rooted in its integral role as part of the overarching dome structure, characterized by an elliptical or oval section revolving around a perpendicular axis aligned with the dome's center on the plan. This double curvature results in an intrados within the ribs. The centerline represents the most reliable area in terms of structural integrity and point cloud density. Comparing ribs based on their centerlines is deemed more reliable than using their borders. Therefore, pinpointing the accurate centerline of rib 6 is critical for subsequent phases aimed at elucidating the dome's geometric profile, particularly whether it conforms to an oval or elliptical shape.

In the figure below (fig 93), the centerline of rib 6 is showed. The red dashed line represents the polyline drawn using coordinates, while the black line denotes the section line extracted from the Point cloud via PointCab software. Although they exhibit notable similarity, the slight discrepancies between them, discernible in this comparison, hold significance for guiding our forthcoming analyses.



AN EARLIER INTERPRETATION OF S. LORENZO GEOMETRY:

The geometric foundation of San Lorenzo church is not explicitly depicted in any original drawings or sketches. Instead, the available sources include engravings of the plan, section, and elevation published in Architettura Civile, as well as the completed building itself. Despite the lack of primary source material on Guarini's geometric ordering scheme for this church, several important geometric relationships between the building's elements can be established, providing insight into Guarini's use of geometry in the design.

A fundamental aspect of the San Lorenzo design is the dome of interlacing ribs, whose geometric form logically interacts with the geometric ordering scheme of the church. Therefore, understanding the geometry of the dome is essential. The design of San Lorenzo grows outward from the main dome, emphasizing the importance of the main vault in the overall design of the church. Guarini himself emphasized the significance of vaults, stating that they are the principal part of the building. This emphasis is reflected in his writings, where he criticized previous treatise writers for treating vaulting superficially.

Given these general assumptions, we will follow Elwin Robsin's interpretation of S. Lorenzo's dome geometry. Robison suggested in his ph. D dissertation in 1985 that Guarini's design parallels the emphasis given to vaulting in his treatise, as the geometry of the main vault has a close relationship to the geometry of the entire church. Guarini, in Architettura Civile creates a parametric set of equations that describes the formation of ellipses caused by the binary movement of the two axis points (focos). The geometric function described by the ellipse results from the interaction between concentric or parallel rings that make contact with the sphere and are inclined at varying degrees according to their angles. The elliptical forms created by the interaction between the sphere and focal points are applied in the building of the dome above the sanctuary space.



FIGURE 94- METHOD OF FORMING AN ELLIPSE USING TWO CIRCLES FROM ARCHITETTURA CIVILE- TAV. VI, FIG. 9.

As Robison specified in his PHD dissertation"The exact form the the San Lorenzo dome is that of an ellipsoid of revolution, with the interlacing ribs placed along the surface of the ellipsoid.¹¹⁴ An ellipsoid of revolution is a solid of revolution, formed by rotating an ellipse about its central axis, creating an egg-shaped, or rugby ball shaped volume. Since this figure is applied to a vaulting problem, only one half of the solid is used by Guarini.Each rib of the vault Is a cut section of the ellipsoid of revolution, and Is itself an ellipse, although smaller in Size.The exact size of the generating ellipse is found most easily by taking the known points of its outline and algebraically determining its major and minor axes.¹¹⁵ In this case the minor axis of the generating ellipse is equal to the diameter of the plan of the dome. However, the major axis of the ellipse cannot be directly measured, for it passes through the open oculus, and there is no architectural element which defines its height. This major axis can be found by using the general equation of an ellipse. Since the intersection of the ribs lies on the surface of the ellipsoid of revolution, the horizontal and vertical distances of this point from the center of the dome can be entered as x and y coordinates into the general equation of an ellipse, and the major axis is easily found."¹¹⁶

In subsequent stages, Robinson's interpretation of the dome's geometric structure will undergo verification against our 2024 survey data and a meticulously crafted ideal 3D model. This comparative analysis aims to better undestand Robison's claim, and to check it against our precise survey data so the reliability of this claim will be discovered.



FIGURE 95- OPTICS AND MATHEMATICS IN THE DOMED CHURCHES OF GUARINO GUARINIA S. LORENZO – ROBISON-HORIZONTAL SECTION CUT BELOW THE SPRINGING OF THE DOME (AT AN ELEVATION OF 21.5 M.), WITH THE CIRCLE CORRESPONDING TO THE MAJOR AXIS OF THE DOME ELLIPSE AND THE CIRCUMSCRIBED SQUARE SUPERIMPOSED BY ROBINSON



FIGURE 96- OPTICS AND MATHEMATICS IN THE DOMED CHURCHES OF GUARINO GUARINIA S. LORENZO – ROBISON- S. LORENZO. SECTION, ELLIPSE CONSTRUCTION ON SECTION OF SAN

114- Morganstern and Pommer both call this vault a parabolic Vault, and in fact the ellipse form does appear similar to a parabola, However, the ribs are tangent to a. vertical plane at the point of their springing, a parabola with a vertical axis (as would necessarily be the case with a masonry dome) can never become tangent to a vertical plane because it must always line within the confines of the generating cone. 115 - This procedure which would have been impossible for Guarini to accomplish algebraically, but It should be remembered that we are following Guarini's

path backwards, and reconstructing his procedure from its result. Guarini started from given diameters, and so the two circle method is all that he would have required.

116 - The general equation for an ellipse is x2 / a2 + y 2 / b2 = 1
CHECKING THE RIB 6 SECTION LINE ACQUIRED FROM THE POINT CLOUD WITH

GUARINI'S METHOD FOR ELLIPSE CREATION

The process of creating an ellipse according to Guarini's method proceeds as follows:

1. Begin with two concentric circles representing rib 6 in San Lorenzo's structure: the smaller circle denotes the intrados of the ribs, while the larger circle marks the intersections of the ribs' centerlines.

2. Establish a horizontal line passing through the centers of these concentric circles.

Draw a diagonal line, which serves as the diameter of the circles, intersecting at their centers.
 At the intersection point of the diagonal line with the larger circle, draw a parallel horizontal line to the initial horizontal line.

5. From the point where the diagonal line intersects the smaller circle, draw a vertical line perpendicular to the horizontal lines, intersecting the horizontal line from the intersection of the diagonal line and the larger circle.

6. Repeat this process with varying angles of the diagonal line to determine additional intersection points.

7. Connect these points sequentially to form an ellipse, representing the potential geometry of rib 6's centerline.





FIGURE 97- SUPERIMPOSING GUARINI'S GEOMETRICAL METHOD FOR CREATING ELLIPSE ACCORDING TO SAN LORENZO'S PLAN AND THE SECTION LINE OF RIB 6 THAT IS OBTAINED BY EXPORTING THE POINTS COORDINATION WHICH ARE LOCATED ON RIB 6 CENTERLINE

CONCLUSION OF THE METHOD

After applying Guarini's method to create an ellipse and comparing it with the precise centerline of rib 6 obtained through coordinates, the final result depicted in the figure above shows a perfect match in the rib's springing zone. This alignment was expected since the smaller circle was aligned with the rib's springing zone on its intrados. However, when examining the entire curve divided into five equal sectors, the greatest deviation in curvature appears in the fourth sector. This asymmetrical deviation suggests that it is likely influenced by historical deformations rather than an inherent flaw in the method itself. If the method were fundamentally incompatible with the rib's geometry, similar deviations would appear symmetrically across other sectors. Despite these deviations, the elliptical geometry does not perfectly conform to rib 6 beyond the springing zone, suggesting that another geometric approach may better represent the ribs of S. Lorenzo. In the subsequent phase, an oval geometry will be scrutinized to assess whether it provides a more suitable fit for the actual state of the ribbed dome's geometry.

GUARINI'S OVAL CREATION METHOD ACCORDING TO AECHITETTURA CIVILE

Guarini's theoretical treatment of the oval with regards to his architectural theory can be found in Architettura civile, Trattato II, Cap. VI, Oss. 7, p. 59 (Lastra 3, fig. 7). Here is a Brief translation of Guarinis writing in Architectura civile about the theory of creating an oval ("Ovato"):

"SEVENTH OBSERVATION.

On the ovate made with multiple portions of the circle.

Let there be two circles, either contiguous, or cut apart, or at a distance afar, or equal, or unequal. Draw a line that passes through their centers AF, ending in C, and I, points of their circumferences, and from there take two equal parts CG, and IO, which are longer than half the drawn line CI, and from centers of the circles A, and F, and with the interval AO, and GF draw two arcs MGH, and MOH, and from the points, where M, and H are cut, draw the two lines MV for the centers A, and MT for F, and the other two HR, and HS, and with the center in H, describe with the interval HS an arc, which will end in R, and with the center M another arc with the interval MT, which will end in V, and hence an Ovate will be made; and if the circles are equal, it will be as acute towards C as towards I, but if they are unequal, the Ovate will be sharper on that side where the circle is smaller. I try this operation in our Euclid in Tract. 18. prop. 6. on page 289."

that is all he has to say about the oval. As you can see, he calls it "ovate" (ovato). what he means (which is obvious from the discription and the figure 96) is what we today would call an oval. It is likely, however, that he was not being dismissive abou the matter but rather that the references to the "Ovato" were different from those of other curves, at the time when he wrote Architettura Civile.



FIGURE 98- METHOD OF FORMING AN OVAL USING TWO CIRCLES FROM ARCHITETTURA CIVILE, TRATTATO II, CAP. VI, OSS. 7, P. 59 (LASTRA 3, FIG. 7)

CHECKING RIB 6 SECTION LINE AQUIRED FROM THE POINT CLOUD WITH GUARINI'S SUGGESTED

METHOD ABOUT OVAL CREATION



1. The process begins with the centerline of rib 6. A circle (circle 1) is inscribed within rib 6, positioned to touch the edges at the rib's springing location. Two additional circles (circle 2 and circle 3), sharing identical diameters, are positioned along the extended horizontal diameter of circle 1. The centers of circles 2 and 3 lie on the circumference of circle 1. At the intersections of circles 1, 2, and 3, four points are identified. Points A and B are located closest to the centerline of rib 6.

2. Lines are drawn from points A and B towards the center of circle 1 (point O), extending until they intersect with rib 6, yielding points C and D on rib 6. Subsequently, a circle is drawn through points C and D that maintains the most tangent contact with rib 6 (circle 4). Connections are made from points C and D to the center of this newly drawn circle (O'), extending these lines to intersect circle 4 at points E and F.

3. The starting points of rib 6's centerline are connected with a narrow line. The entire previous tangent circle (circle 4), along with its diameters, is duplicated and moved downwards (circle 5). Circle 5's diameters intersect with those of circle 4 at two points (G and H), positioned along the narrow line connecting the starting and ending points of rib 6's centerline springing location.

4. Circles are created from points G and H with radii GC and HD (circles 6 and 7). This results in an oval shape formed by two arcs and a portion of the tangent circle.





FIGURE 99- SUPERIMPOSING GUARINI'S GEOMETRICAL METHOD FOR CREATING ELLIPS ACCORDING TO SAN LORENZO'S PLAN AND THE SECTION LINE OF RIB 6 THAT IS OBTAINED BY EXPORTING THE POINTS COORDINATION WHICH ARE LOCATED ON RIB 6 CENTERLINE

CONCLUSION OF THE METHOD

After following Guarini's process for creating an oval and superimposing it with the precise centerline of rib 6, the final result shown in the figure above reveals a perfect match at the point of maximum curvature, with a slight deviation observed at the springing location. Compared to the ellipse method, the oval form better aligns with the path of rib 6's centerline. The deviation at the springing location may stem from the dome's degradation and deformations over time. In this method, aligning the circle with the rib's sagging zone typically results in minimal or zero deviation, whereas the primary misalignment occurs in the springing zone—contrasting with the ellipse method, where the springing zone was aligned with the ellipse creation process. Therefore, certainty about which one of these two geometric method. Guarini intended for this dome is not absolute, as changes over time complicate the precise reconstruction of Guarini's original design intent. Subsequently, exploration will focus on potential deformations are more likely to occur in domes to determine which geometry most faithfully represents the dome's original design.

CHECKING HEIGHT DEVIATIONS AT THE INTERSECTION POINT WHICH ARE

INTENDED TO BE ON A SAME HORIZONTAL PLANS



CHECKING HEIGHT DEVIATIONS AT THE INTERSECTION POINT WHICH ARE

INTENDED TO BE ON A SAME HORIZONTAL PLANS

In this step, the goal is to export the coordinates of the intersection points of the dome's ribs to verify if they align on a flat horizontal surface at a specific height. Ideally, these points should coincide on the same horizontal plane at their intersected locations. This hypothesis will undergo testing in this section. It's important to note that the X and Y coordinates are referenced in a local coordinate system, while the Z coordinate denotes the relative height (Z = 207.11m). This distinction is crucial as it enables analysis not only of the points' spatial positioning but also of their elevation relative to the flat horizontal surface. Subsequently, we will measure the maximum and minimum deviations in height among these points situated on the same horizontal plane.



FIGURE 100- THE GROUND FLOOR POINT CLOUD PLAN AND THE COORDINATES OF A POINT ON THE FLAT SURFACE OF THE GROUND FLOOR ARE UTILIZED TO ACQUIRE THE RELATIVE HEIGHT QUANTITY WITH RESPECT TO THESE SPECIFIC COORDINATES. THE COORDINATE OF Z IS 207.11 METERS.

Number	X Coordinate	Y Coordinate	Z Coordinate	Relative height	
1	10.45	59.26	240.40	33.29 m	
2	12.91	60.51	240.12	33.01 m	
3	15.39	59.63	240.14	33.03 m	
4	16.59	57.28	240.37	33.26 m	
5	15.78	54.88	240.36	33.25 m	
6	13.45	53.73	240.32	33.21 m	Max Height : 33.29 m
7	10.87	54.59	240.34	33.23 m	Min Height : 33.01 m
8	9.70	56.95	240.39	33.28 m	Deviation amount : 0.28 m
1	10.67	59.91	240.26	33.15 m	
2	12.25	60.68	240.04	32.93 m	
3	13.47	60.81	239.96	32.85 m	
4	15.07	60.25	239.97	32.86 m	
5	16.05	59.45	240.04	32.93 m	
6	16.80	57.93	240.19	33.08 m	
7	16.87	56.64	240.32	33.21 m	
8	16.39	55.19	240.29	33.18 m	
9	15.58	54.23	240.18	33.07 m	
10	14.11	53.49	240.19	33.08 m	
11	12.85	53.43	240.16	33.05 m	
12	11.18	53.95	240.14	33.03 m	
13	10.32	54.75	240.22	33.11 m	
14	9.50	56.34	240.27	33.16 m	Max Height : 33.21 m
15	9.45	57.49	240.32	33.21 m	Min Height : 32.85 m
16	9.92	58.98	240.32	33.21 m	Deviation amount: 0.36 m
17	11.07	61.10	239.67	32.56 m	
18	14.54	61.33	239.45	32.34 m	
19	17.21	59.10	239.58	32.47 m	
20	17.41	55.64	239.84	32.73 m	
21	15.23	53.10	239.67	32.56 m	May Llaight (22.91 m
22	11.73	52.84	239.60	32.49 m	IVIAX Height : 32.81 m
23	9.10	55.11	239.71	32.60 m	Min Height : 32.34 m
24	8.91	58.52	239.92	32.81 m	Deviation amount: 0.47 m

	X coordinate	Y Coordinate	Z Coordinate	Relative height	
1	10.11	59.63	240.24	33.13 m	
2	12.87	61.02	239.92	32.81 m	
3	15.76	60.05	239.91	32.80 m	
4	17.11	57.29	240.22	33.11 m	
5	16.20	54.53	240.17	33.06 m	Max Haight , 22 12 m
6	13.50	53.18	240.07	32.96 m	Wax Height : 55.15 m
7	10.62	54.10	240.08	32.97 m	Min Height : 32.80m
8	9.24	56.85	240.22	33.11 m	Deviation amount: 0.33 m
9	8.65	58.97	239.66	32.55 m	
10	10.63	61.28	239.45	32.34 m	
11	11.24	61.60	239.42	32.31 m	
12	14.31	61.78	239.14	32.03 m	
13	15.00	61.57	239.15	32.04 m	
14	17.38	59.55	239.26	32.15 m	
15	17.72	58.98	239.34	32.23 m	
16	17.84	55.86	239.58	32.47 m	
17	17.64	55.22	239.59	32.48 m	
18	15.67	52.90	239.44	32.33 m	
19	15.04	52.63	239.39	32.28 m	
20	11.94	52.39	239.32	32.21 m	
21	11.29	52.65	239.36	32.25 m	
22	8.92	54.63	239.42	32.31 m	Max Height : 32.64 m
23	8.67	55.23	239.50	32.39 m	Min Height : 32.03 m
24	8.44	58.33	239.75	32.64 m	Deviation amount: 0.61 m
1	10.79	61 77	239 21	32.10 m	
2	14 77	62.01	238.85	31 74 m	
3	17.88	59.43	239.05	31.94 m	
4	18.12	55.44	239.42	32.31 m	
5	15.52	52.43	239.19	32.08 m	
6	11.52	52.16	239.07	31.96m	Max Height : 32.38 m
7	8.50	54.74	239.22	32.11 m	Min Height · 31 74 m
8	8.18	58.76	239.49	32.38 m	



CONCLUSION

From the points' height, it is evident that in the green and yellow zones, the maximum and minimum relative heights are being observed. Consequently, it is obvious that we have a slight tilt in all the horizontal surfaces that include specific intersection points. The deviation between the points that are supposed to be on a flat surface is between 0.28 and 0.64 meters. The maximum deviation can be recognized in the zones with lower absolute height, located in the first intersecting location between the ribs, in the green and yellow highlighted zones, in the first intersecting location of the rings, nearly 60 centimeters of deviation between the height of the points has been reached. To understand the causes of these deformations, further research is necessary, which extends beyond the scope of this thesis. This comprehensive subject requires detailed analysis of tilts and settlements from multiple perspectives to identify affected zones accurately. In-depth research into structural analysis, including analyzing construction methods, masonry arrangements, and potential degradations typical of this type of dome, and many other factors are essential. Additionally, investigating historical construction records and previous restoration efforts could provide insights into the structural behavior observed today.

While this falls outside the focus of this thesis, it is valuable to consider Heyman's claims in his book regarding possible deformations in domes. His analysis of the structural mechanics and stress distributions in historical domes could offer a theoretical framework to understand these deformations. This consideration could help determine which geometries Guarini likely intended when designing the San Lorenzo dome.

POSSIBLE DEFORMATION IN DOMES ACCORDING TO HEYMAN

EQUILIBRIUM OF MASONRY ARCH AND DOME

As is well known, all components of a masonry structure must maintain three-dimensional equilibrium to ensure stability and longevity. In a true arch, the stone on brick voussoirs work solely under compression. Materials such as natural stone, mortar, and brick excel in compression but are weak in tension. In contrast to timber, which has comparable strength in both tension and compression but limited durability, masonry materials exhibit approximately ten times greater compressive strength than tensile strength (Cowan, 1977). Three critical assumptions regarding the behavior of masonry arches, as outlined by J. Heyman (1982), include: 1. Sliding failure is prevented by sufficient friction between voussoirs or effective interlocking.

2. Masonry lacks tensile strength.

3. Masonry possesses infinite compressive strength.

The equilibrium of a masonry arch is conceptualized through a line of thrust, tracing the path of resultant compressive forces through the structure. If the line of thrust extends outside the arch section, the material would need to resist tension, which masonry cannot withstand. Determining the line of thrust involves methods such as physical hanging models or graphic statics. While numerous compressive thrust lines may exist in a typical masonry arch, the arch's geometry dictates the range of horizontal forces it can accommodate.

Managing thrust is the primary challenge in vaulted masonry structures, ensuring it is safely directed to the ground within the masonry fabric (Ochsendorf and Block, 2009). The line of thrust must remain within the middle third of the arch section to maintain stability and provide a significant safety margin. Deviation from this middle third may lead to cracking and increased compression in specific areas but is unlikely to cause collapse (Heyman, 1995). Collapse occurs only when the thrust line becomes tangent to a point on the arch section, highlighting the critical link between collapse mechanisms and arch geometry.

To prevent reliance on tensile forces under any loading condition, unreinforced masonry structures must be designed with correct forms and considerations for the elastic properties of materials. Mortar plays a crucial role in distributing internal forces across voussoirs, contributing to the overall stability and durability of the structure (Ochsendorf and Block, 2009).



FIGURE 101- FOR AN ARCH IN ORDER TO BE STABLE, IT IS NECESSARY THAT THE LINE OF THRUST SHOULD LIE WITHIN THE MASONRY. (A) MINIMUM AND MAXIMUM HORIZONTAL THRUSTS IN A SEMICIRCULAR MASONRY ARCH: COLLAPSE UNDER THE MIN AND MAX FORCE (SOURCE:

THE DOME AS A STRUCTURAL ELEMENT

A dome serves as a rounded vault designed to span expansive areas, illustrated by iconic structures such as Hagia Sophia (AD 532, span: 31m), St. Peter's in Rome (1560, span: 42.5m), and the Pantheon (AD 120, span: 43m). It comprises multiple semi-arches that converge at the crown, defined by meridians and parallel circles as shell elements. Since the early 19th century, forces acting on a dome have been categorized into two main types (Heyman, 1995):

1. Vertical 'arching' stresses or meridional forces.

2. Horizontal circles or hoop forces.

Arching stresses remain compressive throughout the dome, while hoop stresses are compressive in the upper part and tensile in the lower part. In a hemispherical dome, this transition typically occurs at a circle inclined at a 52° angle from the crown and a 37° angle from the horizontal (Cowan, 1977).



FIGURE 103- HYBRID DOUBLE DOME - A. TAVAKOLI DINANI - VERTICAL 'ARCHING STRESSES OR MERIDIONAL FORCES AND HORIZONTAL 'HOOP' STRESSES IN A THIN HEMISPHERICAL DOME.

Simplifying the behavior of domes, especially those with cracks, into a series of arches has been documented since the 17th century. This approach was extensively outlined by Poleni (1748) in his analysis of the Vatican's dome. Poleni conducted experiments where he determined the catenary shape by suspending weights between two pins that represented the span. He then superimposed this catenary shape onto the dome's cross-section, concluding that the dome was structurally sound as long as the catenary remained entirely within it, with sufficient hoop forces at the base to prevent spreading of the sections. However, this solution proved insufficient, prompting Luigi Vanvitelli to add five additional tie chains to the dome in 1744. The characteristic 'orange slice' crack pattern highlights meridional cracks, segmenting the dome into distinct sections





FIGURE 102- THE STONE SKELETON - JACQUES HAYMAN -HOOP STRESS RESULTANTS NECESSARY FOR THE EQUILIBRIUM OF A HEMISPHERICAL SHELL.







FIGURE 106- THE STONE SKELETON - JACQUES HAYMAN - HOOK HANGING CHAIN

CONCLUSION, REGARDING TO THE POSSIBLE DEFORMATIONS AND WHICH

GEOMETRY IS MORE PROBABLE TO BE GUARINI'S INTENTION

In studying the San Lorenzo dome geometry, particularly whether it is oval or elliptical, we observed a slight deviation in the springing zone using the oval method. By analyzing the possible deformations, the dome may have experienced over the years, we concluded that the dome likely exhibits a structural behavior similar to thr "orange slice" deformation described by Heyman. This would be consistent with the small deviation in its springing zone, making it more probable that Guarini intended the dome to have an oval shape rather than an elliptical one. Since domes are more prone to this type of deformation and given the presence of supportive tie rods within the dome structure, it is almost unlikely for deformation to occur where these tie rods are located. Therefore, although based on our analysis there is no absolute answer to Guarini's geometric intention in constructing the San Lorenzo dome, an oval geometry appears to more closely reflect the dome's actual shape than the elliptical shape.

FUTURE DEVELOPMENT

GUARINI'S ARCHITECTURE AND PERSIAN ARCHITECTURE

While working on this thesis, the intrinsic features of Islamic architecture and the similarities between Guarini's designs in the S. Lorenzo dome and certain Islamic and Persian mosques, such as the Cordoba mosque, the khagi mosque in Isfahan, Iran, and the Amir Chakhmagh mosque in Yazd, Iran, were explored. The investigation of the geometry of the S. Lorenzo dome focused on the intersection of circles, which is crucial in the creation of oval geometry. The proportion and location of the circles play a vital role in this process, drawing upon knowledge of Persian mosque architecture and proportional equations to inform the analysis.

The study of traditional design methods, as outlined in Heyman's Stone Skeleton, highlighted that the rules of proportion and geometry remained the foundation of both aesthetic and structural principles in western architecture until the 18th and late 19th centuries. These rules provide a precise understanding of the design and behavior of masonry. Persian proportional values were applied to the geometric survey of San Lorenzo to see if they would hold true. Surprisingly, the results fit perfectly, piquing further interest in exploring whether Guarini was inspired by Persian architecture and whether he integrated Islamic proportional values into the geometry of the dome, separate from the overall resemblance to Islamic architecture.

Persia boasts an impressive collection of brick structures that showcase effective techniques for creating complex forms, often striving for infinite permanence. The French-American art historian Oleg Grabar, in his 1986 study on Islamic architecture, highlights the dome as a "true glory" and a source of innovative technical developments in Iranian architecture. This architectural beauty, which spread throughout the Islamic region, is evident in the numerous mosques, mausoleums, and madrasas built in Iran and Central Asia during the 15th to 18th centuries. The system became more complex by doubling shells and hybridizing them with wooden components.

The establishment of the Safavid dynasty in the 16th century sparked a new foreign interest in the country, particularly under the charismatic leadership of Shah Isma 'il I (1501–1524). However, news of these developments primarily reached Western Europe through second-hand reports written by Venetian representatives stationed in Istanbul. For nearly a century, only a handful of Westerners, mostly Portuguese and Italian travelers, visited Iran, leaving behind fragmented descriptions of life and politics in the Safavid realm. The rise of Shah 'Abbas I (1587–1629) and his outward-looking agenda, marked by an energetic foreign policy, created a political and economic environment that attracted more European interest in Iran as a land of religious, commercial, and strategic opportunity.

During this period, Catholic missionaries entered the country and set up convents in various locations, while European diplomats frequented Isfahan in hopes of persuading Shah 'Abbas to join Europe's rulers in their

struggle against the Ottoman Empire. The list of notable visitors between 1600 and 1722 includes prominent figures such as Don García de Silva y Figueroa, Pietro Della Valle, Adam Olearius, Cornelis Speelman, Jean-Baptiste Tavernier, Raphaël du Mans, Jean de Thévenot, John Fryer, Jean Chardin, Engelbert Kaempfer, François Sanson, Cornelis de Bruyn, and Artemii Volynskii.

Their travelogues, which vastly enhanced and improved the volume and quality of knowledge in Europe about Iran, transformed the country from a mythical realm to a place explored and described from an experiential, empiricist perspective. Many travelers, however, were eager to publish their accounts for fame and financial gain. Returning home, they faced a competitive book market that bred both opportunities and anxieties.

Pascal Coste, a French architect, prepared famous drawings from Iranian monuments, but his account lacks precise geometrical information (in 1867) for analytical purposes. For example, his drawings of the Shah Mosque (fig-106) and Chahar Bagh madrasah show Western structural techniques, such as connecting two shells of a double dome like St. Mark's Cathedral in Venice, which is far from the intrinsic facets of Persian masonry double domes. (fig-107) Another example of misperception is given by Wolfgang Born.

Due to the Mongol invasion in the 13th century, only a few documents survived that describe the geometry of arches and domes until the 15th century, such as Suhayl al-Qui of the 10th century. The main reference for researchers since the 15th century is the work of Ghiayth Al-Din Kashani, who made valuable contributions to the field of building construction and vaulted structures (fig-110) (Oghlu, 2000). He offers tables for defining geometrical properties of arches, such as the length of intrados, the area of the façade, the rise of intrados, the height of convexity, and the empty area underneath the arch. These tables play a crucial role in building construction and estimating the required building materials and scaffoldings (Kashani, 1967; Memarian, 2014). Contemporary literature developed by Prinia (1981, 1991) and Memarian (2014) has also followed Al Kashani's studies. For exploring the inner dome's geometry, selected curvatures are studied and compared. Kashani introduced three types of arches, and three other arches of Panj o Haft, Sebakhshi Tond, and Sebakhshi Kond are presented. (fig-108)



FIGURE 107- ESFAHAN SHAH MOSQUE, P.COSTE (1867), NEW YORK PUBLIC LIBRARY, DIGITAL COLLECTION



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In 1655, Guarini entered a ten-year period of scarcely documented travels and experiences. As a studious young architect, we might suppose he had the opportunity to travel to Spain and Portugal, he gained knowledge about the three great religions of the West, which he brought to a culminating point in the design of San Lorenzo.

While it can not be proved it is plausible at least that Guarini's exposure to Persian architecture during his time in France, coupled with the availability of travelogues and architectural information from that era, influenced his work. Given Guarini's passion for reading and investigating the writings and drawings of other architects, it is likely that he had access to some resources regarding Persian architecture, which led him to incorporate elements of Persian construction methods and architecture into his own projects. This hypothesis, while currently speculative, warrants further investigation and is a potential topic for my future academic research.

Therefore, I propose to conclude this thesis with an open-ended question that sets the stage for my next project: "The Effect of Persian and Islamic Architecture as Inspiration to European Architects." This inquiry will delve deeper into the potential influences of Persian and Islamic architecture on European architectural design, exploring the extent to which these cultural and stylistic elements were incorporated into European architectural projects during the period in question.

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FIGURE 92- INSERTING THE COORDINATES OF THE POINTS INTO 3DMAX SOFTWARE TO GENERATE TH	E ACCURATE CENTERLINE OF THE RIB
FIGURE 93-RED DASHED LINE SHOWS PRECISE POLYLINE FROM 3D MAX COORDINATES, WHILE BLACK	LINE IS POINTCAB-EXTRACTED
SECTION. MINOR DIFFERENCES ARE CRUCIAL FOR FURTHER ANALYSIS	ERROR! BOOKMARK NOT DEFINED.
FIGURE 94- METHOD OF FORMING AN ELLIPSE USING TWO CIRCLES FROM ARCHITETTURA CIVILE- TAV NOT DEFINED.	/. VI, FIG. 9ERROR! BOOKMARK
FIGURE 95- OPTICS AND MATHEMATICS IN THE DOMED CHURCHES OF GUARINO GUARINIA S. LORENZ	ZO – ROBISON- HORIZONTAL
SECTION CUT BELOW THE SPRINGING OF THE DOME (AT AN ELEVATION OF 21.5 M.), WITH THE	CIRCLE CORRESPONDING TO THE
MAJOR AXIS OF THE DOME ELLIPSE AND THE CIRCUMSCRIBED SQUARE SUPERIMPOSED BY ROL DEFINED.	BINSONERROR! BOOKMARK NOT
FIGURE 96- OPTICS AND MATHEMATICS IN THE DOMED CHURCHES OF GUARINO GUARINIA S. LORENZ	20 – ROBISON- S. LORENZO.
SECTION, ELLIPSE CONSTRUCTION ON SECTION OF SAN.	ERROR! BOOKMARK NOT DEFINED.
SECTION LINE OF RIB 6 THAT IS OBTAINED BY EXPORTING THE POINTS COORDINATION WHICH	ARE LOCATED ON RIB 6 CENTERLINE FRROR! BOOKMARK NOT DEFINED.
FIGURE 98- METHOD OF FORMING AN OVAL USING TWO CIRCLES FROM ARCHITETTURA CIVILE, TRAT	ΓΑΤΟ ΙΙ, CAP. VI, OSS. 7, P. 59
(LASTRA 3, FIG. 7)	ERROR! BOOKMARK NOT DEFINED.
FIGURE 99- SUPERIMPOSING GUARINI'S GEOMETRICAL METHOD FOR CREATING ELLIPS ACCORDING T	O SAN LORENZO'S PLAN AND THE
SECTION LINE OF RIB 6 THAT IS OBTAINED BY EXPORTING THE POINTS COORDINATION WHICH	ARE LOCATED ON RIB 6 CENTERLINE
FIGURE 100- THE GROUND FLOOR POINT CLOUD PLAN AND THE COORDINATES OF A POINT ON THE FL	AT SURFACE OF THE GROUND
FLOOR ARE UTILIZED TO ACQUIRE THE RELATIVE HEIGHT QUANTITY WITH RESPECT TO THESE S	PECIFIC COORDINATES. THE
COORDINATE OF Z IS 207.11 METERS	ERROR! BOOKMARK NOT DEFINED.
FIGURE 101- FOR AN ARCH IN ORDER TO BE STABLE, IT IS NECESSARY THAT THE LINE OF THRUST SHOL	JLD LIE WITHIN THE MASONRY. (A)
MINIMUM AND MAXIMUM HORIZONTAL THRUSTS IN A SEMICIRCULAR MASONRY ARCH: COLL	APSE UNDER THE MIN AND MAX
FORCE (SOURCE: HEYMAN, 1982: OCHSENDORF	ERROR! BOOKMARK NOT DEFINED.
FIGURE 102- THE STONE SKELETON - JACQUES HAYMAN -HOOP STRESS RESULTANTS NECESSARY FOR	
'HOOP' STRESSES IN A THIN HEMISPHERICAL DOME	FRROR! BOOKMARK NOT DEFINED.
FIGURE 104- THE STONE SKELETON - JACQUES HAYMAN - GREATLY EXAGGERATED SCHEMATIC ILLUST	RATION OF CRACKING OF A DOME
DUE TO INCREASE OF SPAN	ERROR! BOOKMARK NOT DEFINED.
FIGURE 105- POLENI -1748	ERROR! BOOKMARK NOT DEFINED.
FIGURE 106- THE STONE SKELETON - JACQUES HAYMAN - HOOK HANGING CHAIN	ERROR! BOOKMARK NOT DEFINED.
FIGURE 107- ESFAHAN SHAH MOSQUE, P.COSTE (1867) , NEW YORK PUBLIC LIBRARY , DIGITAL COLLEC DEFINED.	TIONERROR! BOOKMARK NOT
FIGURE 108- GEOMETRICAL STUDIES OF PERSIAN ARCHES, IN THE KEY TO ARITHMETIC (MIFTAH AL-HI	SAB) ,1427, NATIONAL LIBRARY
PERSIAN AND ARABIANS BY PASCAL COSTE, (1867)	BIZANTINE, KUIVIAN, SASANIED, ERROR! BOOKMARK NOT DEFINED.
FIGURE 110- DIFFERENT ARCH TYPES AND MATRIX CALCULATION FROM THE ORIGINAL MANUSCRIPT	OF AL-KASHI'S RESALEH TAGH VA
AJAZ WRITTEN BY JAZBI IN 1987	ERROR! BOOKMARK NOT DEFINED.

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