Politecnico di Torino

Master's Degree in
Mechanical Engineering

Master's Degree Thesis

Automatic Generation of Gearbox Layouts

1859

Advisors:

Prof. Carlo Rosso

Candidate:

Amin Amiripour

A.Y. 2023/2024

Abstract

This thesis presents a comprehensive approach to the automatic generation of a gearbox layout using advanced computational methods. The primary objective is to develop an automated system capable of designing a gearbox based on user-defined inputs such as input torque, desired output torque, material characteristics, and the working conditions of the gearbox. The system performs calculations for both bevel and cylindrical gears, determining their dimensions and the forces they exert. Subsequently, a finite element analysis (FEM) is conducted in MATLAB to accurately dimension the shafts by assessing the static loads. Based on the FEM results, suitable bearings are selected from the SKF catalogue, ensuring optimal performance and reliability.

The entire process is automated through MATLAB scripts, which save the calculated data into Excel files for further use. These parameters are then imported into NX Siemens, where a parametric design of the gears, shafts, and bearings is created, resulting in a complete CAD model of the gearbox. This integration of computational design, finite element analysis, and parametric CAD modeling streamlines the gearbox design process, enhancing efficiency, accuracy, and customization capabilities.

The automated system developed in this thesis offers significant improvements over traditional manual design methods, reducing the time and expertise required to produce high-quality gearbox layouts. The results demonstrate the potential of this approach to revolutionize gearbox design in mechanical engineering, providing a robust foundation for future enhancements and applications in the field.

Contents

List of Figures

List of Tables

Introduction

The design of gearboxes is a fundamental task in mechanical engineering, with applications ranging from automotive transmissions to industrial machinery. Traditionally, the design process has been manual and labor-intensive, requiring extensive calculations, precise material selection, and meticulous drafting of components. This approach, while effective, is time-consuming and prone to human error, limiting the efficiency and accuracy of the design process.

The advent of computational tools and automation technologies offers a promising solution to these challenges. Automated design systems can significantly enhance the speed and precision of gearbox development, allowing for rapid prototyping and customization to meet specific operational requirements. This thesis explores the development of an automated system for generating gearbox layouts, leveraging advanced computational methods and software integration to streamline the design process.

The primary objective of this research is to create a system that accepts user-defined inputs—such as input torque, desired output torque, material characteristics, and the working conditions of the gearbox—and automatically generates a complete gearbox layout. This involves calculating the dimensions and forces for both bevel and cylindrical gears, performing finite element analysis (FEM) to accurately dimension the shafts, and selecting appropriate bearings from the SKF catalogue based on the calculated forces.

The methodology is implemented in MATLAB, with scripts designed to handle each stage of the design process. The calculated data is saved in Excel files, which are then used to create a parametric design in NX Siemens. This integration ensures that the final output is not only precise but also easily modifiable, allowing for quick adjustments and iterations.

The significance of this work lies in its potential to transform traditional gearbox design. By automating the process, the system reduces the reliance on manual calculations and expert knowledge, making advanced gearbox design accessible to a broader range of engineers and designers. Moreover, the system's ability to generate detailed CAD models facilitates seamless transition from design to manufacturing, enhancing the overall efficiency of product development.

In summary, this thesis aims to demonstrate the feasibility and benefits of automated gearbox design through the development and implementation of a comprehensive design system. The following sections will detail the methodologies used, the implementation process, and the results obtained, highlighting the advantages and potential applications of this innovative approach.

Literature Review

1. Introduction to Gearbox Design

Gearboxes are critical components in many mechanical systems, responsible for transmitting power and adjusting torque and speed between connected machinery. The design of gearboxes involves selecting appropriate gear types, calculating dimensions, and ensuring that all components can withstand the operational loads and conditions.

2. Traditional Gearbox Design Methods

Historically, gearbox design has relied heavily on manual calculations and empirical methods. Engineers use fundamental principles of mechanics and material science to determine the dimensions and specifications of gears, shafts, and bearings. While effective, these methods are time-consuming and require a high level of expertise.

3. Computational Methods in Gearbox Design

Recent advancements in computational tools have revolutionized gearbox design. Software such as MATLAB, SolidWorks, and NX Siemens provide powerful capabilities for automating complex calculations and simulations. These tools allow for more accurate and efficient design processes, reducing the likelihood of errors and enabling more sophisticated designs.

4. Finite Element Analysis (FEM) in Mechanical Design

Finite Element Analysis (FEM) is a numerical method used to predict how a product reacts to real-world forces, vibration, heat, and other physical effects. FEM is particularly useful in mechanical design for evaluating the structural integrity of components under various loading conditions. The application of FEM in shaft dimensioning, as explored in this thesis, allows for precise and optimized designs that ensure reliability and performance.

5. Bearing Selection Criteria and Tools

Bearings are essential for reducing friction between moving parts and supporting loads. The selection of bearings must consider factors such as load capacity, speed, lubrication, and environmental conditions. Tools like the SKF catalogue provide comprehensive data and guidelines for choosing the appropriate bearings, ensuring that they meet the specific requirements of the gearbox design.

6. Integration of CAD Tools in Automated Design

Computer-Aided Design (CAD) tools, such as NX Siemens, play a crucial role in translating computational design outputs into detailed, manufacturable models. Parametric design capabilities enable the creation of flexible models that can be easily adjusted based on varying input parameters. The integration of MATLAB and NX Siemens, as implemented in this thesis, exemplifies the synergy between computational calculations and CAD modeling.

1. Cylindrical Gears

Definition and Functionality

Cylindrical gears, also known as spur gears or helical gears depending on the tooth design, are gears with teeth parallel to the axis of rotation. They are used to transmit motion and power between parallel shafts. These gears are fundamental in many mechanical systems due to their simplicity and efficiency in power transmission.

Types of Cylindrical Gears

1. **Spur Gears:**
   - **Description:** Have straight teeth that are parallel to the gear's axis. They are the simplest type of cylindrical gears.
   - **Applications:** Commonly used in gearboxes, conveyors, and general machinery where high-speed and high-torque transmission are required.
2. **Helical Gears:**
   - **Description:** Have teeth that are cut at an angle to the face of the gear. This design allows for gradual engagement of teeth, resulting in smoother and quieter operation compared to spur gears.
   - **Applications:** Widely used in automotive transmissions, industrial machinery, and robotics due to their efficiency and reduced noise.

Applications of Cylindrical Gears

Cylindrical gears are used in various industries due to their versatility and effectiveness in power transmission. Common applications include:

- **Automotive Transmissions:** Helical gears are prevalent in car gearboxes, ensuring smooth shifting and power transfer.
- **Industrial Machinery:** Used in a wide range of machines, including conveyors, mixers, and pumps.
- **Robotics:** Provide precise and reliable motion control in robotic arms and automated systems.
- **Aerospace:** Employed in aircraft engines and auxiliary power units for efficient power transmission.

Design Considerations

When designing cylindrical gears, several critical factors must be considered to ensure optimal performance and reliability:

- **Gear Ratio:** Determines the speed and torque relationship between the input and output shafts.
- **Tooth Profile:** The shape and size of the gear teeth, which affect the gear’s strength and operational smoothness.
- **Material Selection:** Choosing materials with appropriate properties (e.g., strength, hardness, wear resistance) based on the application requirements.
- **Load Capacity:** Ensuring the gear can withstand the expected loads without failure.
- **Lubrication:** Proper lubrication is essential to reduce friction and wear, extending the gear's lifespan.

Advantages of Cylindrical Gears

1. **High Efficiency:** Spur and helical gears provide efficient power transmission with minimal energy loss.
2. **Simplicity:** Spur gears are easy to manufacture and install, making them cost-effective.
3. **Smooth Operation:** Helical gears offer smooth and quiet operation due to the gradual engagement of teeth.
4. **Versatility:** Applicable in a wide range of industries and mechanical systems.
5. **High Load Capacity:** Double helical gears can handle high torque loads without generating significant axial forces.

Disadvantages of Cylindrical Gears

1. **Noise:** Spur gears can be noisy at high speeds due to abrupt tooth engagement.
2. **Axial Thrust:** Helical gears generate axial thrust forces that require additional bearings or supports to counteract.
3. **Complex Manufacturing:** Helical gears are more complex and expensive to manufacture compared to spur gears.
4. **Alignment Sensitivity:** Cylindrical gears require precise alignment to avoid excessive wear and noise.
5. **Lubrication Requirements:** Proper lubrication is crucial for maintaining performance and longevity, requiring regular maintenance.

Spur Gear

Spur gears are the most basic type of gears, making them ideal for demonstrating the fundamentals of gear meshing. They are extensively used in a wide range of applications. The terminology of spur gears is shown in the figure 1.1.

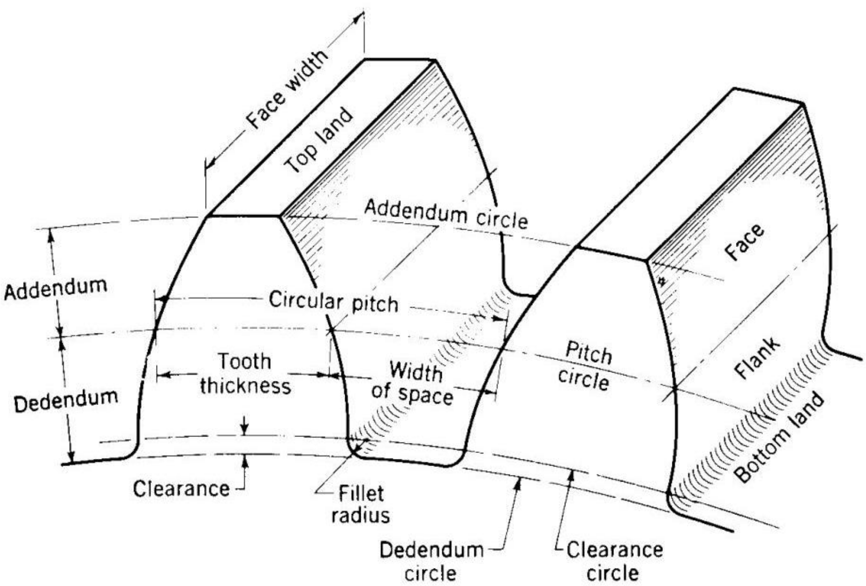

Figure 1.1 Spur Gear Terminology

Helical Gears

Helical gears are a type of cylindrical gear with teeth cut at an angle to the axis of rotation, forming a helix shape. This angled teeth design allows for gradual engagement between meshing gears, resulting in smoother and quieter operation compared to spur gears. You can see the terminology of helical gears in the figure 1.2:

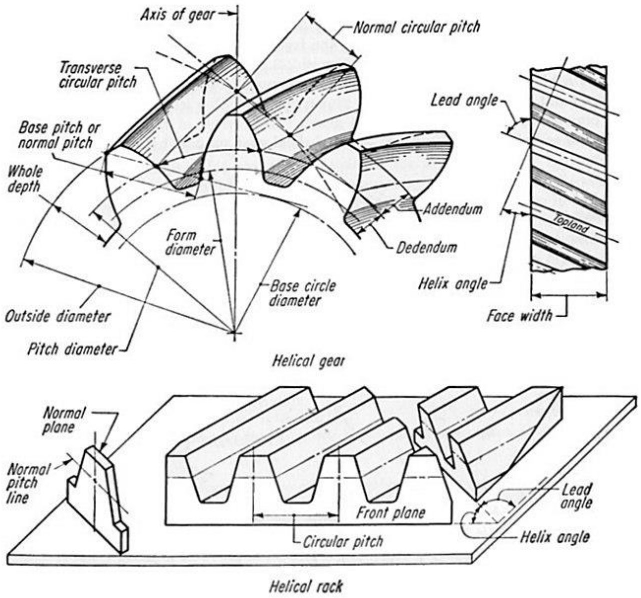

Figure 1.2 Helical Gear Terminology

Definition of Cylindrical Gear Parameters

Before delving into the design of cylindrical gears, it is important to understand some key definitions:

Planes in Helical gears: In helical gear design, there are two important planes: the tangent plane, which is perpendicular to the gear axis and tangent to the pitch cylinder, and the normal plane, which is perpendicular to the tooth surface and tangent to the base cylinder of the gear. Gear parameters are first calculated in normal plane, then they are converted to the tangent plane using the helix angle. Subscript n refers to normal plane and t refers to tangential one.

Helix Angle ($\boldsymbol{\beta}$): The helix angle determines the angle at which the teeth of helical gears are inclined relative to the gear axis. It ranges between 0 to 45 degrees.

Pressure Angle ($\boldsymbol{\alpha}$): The pressure angle defines the angle between the line of force transmission and the tangent to the pitch circle of gear teeth. It influences tooth strength, contact smoothness, and manufacturing considerations in gear design. It is a standardized value and the standard values which are mostly used in cylindrical gears are:

- **Standard Normal Pressure Angles:** [14.5, 20, 22.5, 25]

The relationship between tangential and normal pressure angle is as below:

$$\tan(\alpha_t) = \frac{\tan(\alpha_n)}{\cos(\beta)}$$

Module: Module is a standardized measure of the size of a gear tooth, defining the pitch and ensuring compatibility and proper meshing between gears in gear design. It can be measured both in the normal plane (normal module) and the tangent plane (tangential module), providing flexibility in gear tooth dimensioning:

- **Standard normal modules**: [1,1.25,1.5,2,2.5,3,4,5,6,8,10,12,16,20,25,32,40,50]

Transverse module which is measured in the transverse plane of the gear can be calculated using this formula:

$$m_t = \frac{d}{z}$$

$$m_n = \frac{m_t}{\cos(\beta)}$$

Where m_t is the module measured in transverse plane, m_n is the module measured in the normal plane, d is the diameter of the primitive circle, and z is the number of teeth.

Pitch or Primitive Circle: The pitch circle in gears is an imaginary circle that defines the theoretical line of contact between two meshing gears, essential for determining the gear ratio and tooth dimensions:

$$d = m_t * z$$

Base Circle: The base circle is a theoretical circle from which the involute tooth profile is generated in gears, crucial for calculating gear tooth dimensions and ensuring proper meshing:

$$d_b = d * \cos(\alpha_t)$$

Addendum Circle: The addendum circle is located outside the pitch circle and defines the outermost point of the gear tooth, determining the height of the tooth above the pitch circle:

$$d_a = d + 2 * h_a$$

Dedendum Circle: The dedendum circle is located inside the pitch circle and defines the depth of the tooth space, ensuring proper clearance between meshing gears for smooth operation.

$$d_f = d - 2.5 * h_f$$

Where h_a and h_f in standard racks are equal to m_n and $1.25 * m_n$ respectively.

Pitch: Pitch in gears refers to the distance between corresponding points on adjacent teeth along the pitch circle. It determines the size and spacing of gear teeth, crucial for maintaining accurate gear ratios and ensuring smooth meshing between gears:

$$p_t = \frac{\pi * d}{z}$$

$$p_n = p_t * \cos(\beta)$$

Axial pitch in gears refers to the distance along the gear axis between corresponding points on adjacent teeth. It is important in helical gears to determine the spacing and engagement of teeth, influencing the overall design and functionality of the gear system:

$$p_x = p_t * \cot(\beta)$$

Gear Teeth: Cylindrical gear teeth are designed using involute curves. This design ensures a constant pressure angle and smooth motion transmission. You can see this method which is implemented in this thesis in the figure 1.3:

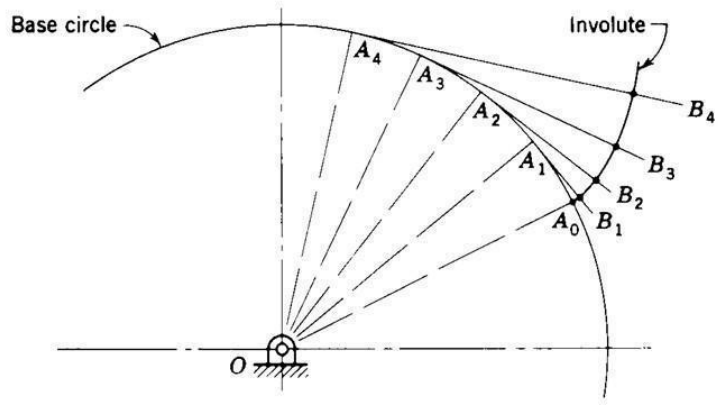

Figure 1.3 Involute Curves

Cylindrical Gear Load Capacity

After completing the gear design, it is essential to calculate the gear's load capacity and establish appropriate safety margins to ensure compliance with required safety standards. The load capacity of cylindrical gears is assessed using ISO 6336, a standard guideline for gear design. It is crucial to evaluate safety factors independently for each type of potential damage such as pitting, tooth root breakage, tooth flank fracture, scuffing, micropitting, etc. In each case, the adequacy of the gear's load capacity is determined by ensuring that the computed safety factor S meets or exceeds its respective minimum safety factor S_{min}.

During the calculation of load capacity, several influence factors are computed according to the loading and geometry of the gears such as:

- Application Factor K_α
- Internal Dynamic Factor K_v
- Zone factor Z_H
- Elasticity Factor Z_E
- Stress Correction Factor Y_S
- Etc.

After deriving all the influence factors, 2 safety factors are computed for both pinion and wheel:

1. Safety factor for bending stress (Safety against tooth breakage): S_f
2. Safety factor for surface durability (Safety against pitting): S_H

If the calculated safety factor is below the required level, an iterative process is initiated. This involves increasing parameters such as gear width and module until the safety factors meet the minimum required standards.

2. Bevel Gear

Bevel gears are a type of gear that transmits motion and power between intersecting shafts at an angle. Unlike parallel shaft gears, bevel gears feature intersecting shafts, allowing power to be transmitted between non-parallel axes. This feature makes them ideal for applications where shafts must reverse direction, such as automotive differentials, hand drills, and marine propulsion systems.

Bevel gears are widely used in various industries and applications, including:

- Automotive**:** Differential gears, steering systems.
- Aerospace**:** Aircraft engines, helicopter rotors.
- Industrial Machinery**:** Machine tools, printing presses.
- Marine**:** Ship propulsion systems.
- Mining and Construction**:** Heavy machinery.

Advantages of Bevel Gears

1. **Efficient Power Transmission**:
   - Bevel gears efficiently transmit power between intersecting shafts at various angles, allowing for versatile mechanical designs.
2. **Compact Design:**
   - Compared to other types of gears, bevel gears can achieve high gear ratios and torque transmission in a relatively compact space.
3. **Versatility in Applications:**
   - Bevel gears are used across diverse industries including automotive, aerospace, marine, and industrial machinery, showcasing their versatility.
4. **Smooth and Quiet Operation:**
   - Modern designs like spiral and Zerol bevel gears offer smoother and quieter operation compared to straight bevel gears, reducing noise levels.
5. **Customizable Configurations:**
   - Different types such as straight, spiral, hypoid, and Zerol bevel gears offer varying advantages in terms of load capacity, speed, and noise reduction, allowing for tailored solutions to specific application requirements.

Disadvantages of Bevel Gears

1. **Complex Manufacturing Process:**
   - Especially true for spiral and hypoid bevel gears, manufacturing involves complex machining processes and precise alignment requirements, leading to higher production costs.
2. **Higher Costs:**
   - Bevel gears can be more expensive to manufacture compared to simpler gear types due to their complexity and the materials required for high strength and durability.
3. **Maintenance and Alignment Challenges:**
   - Proper alignment and maintenance are critical for ensuring smooth operation and longevity of bevel gears, requiring skilled technicians and periodic inspections.
4. **Limited Load Capacity at Extreme Angles:**

- Bevel gears may experience reduced load capacity and efficiency at extreme operating angles, necessitating careful design considerations for optimal performance.

Types of Bevel Gears

1. Straight Bevel Gears: Straight bevel gears have straight teeth and are the simplest form of bevel gears. They are cost-effective and easy to manufacture but tend to produce more noise and vibration compared to other types. Straight bevel gears are commonly used in applications where noise is not a critical factor.

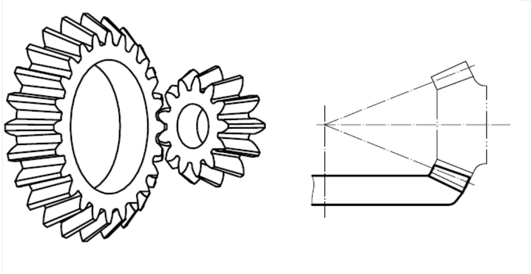

Figure 2.1 Straight Bevel Gear

2. Spiral Bevel Gears: Spiral bevel gears have curved teeth, which spiral around the gear axis. This design improves tooth contact and reduces noise and vibration compared to straight bevel gears. Spiral bevel gears offer higher load capacity and smoother operation, making them suitable for high-performance applications.

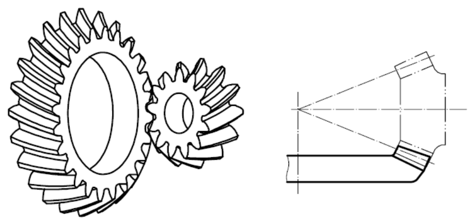

Figure 2.2 Spiral Bevel Gear

3. Zerol Bevel Gears: Combining the benefits of spiral and straight bevel gears, Zerol bevel gears are a unique kind of bevel gear. With a spiral angle of 0 degrees, their curved teeth resemble those of spiral bevel gears. In comparison to straight bevel gears, Zerol bevel gears operate more smoothly and quietly, and they are simpler to produce than spiral bevel gears.

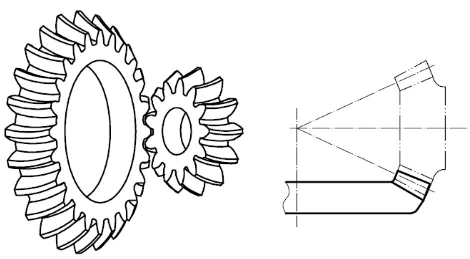

Figure 2.3 Zerol Bevel Gear

4. Hypoid Bevel Gears: These gears do not have teeth on the pitch cones; instead, their teeth are offset from the gear axis. Because of their special geometry, hypoid gears may transfer motion between non-intersecting and non-parallel shafts, which makes them appropriate for situations where there are spatial restrictions or shaft misalignment.

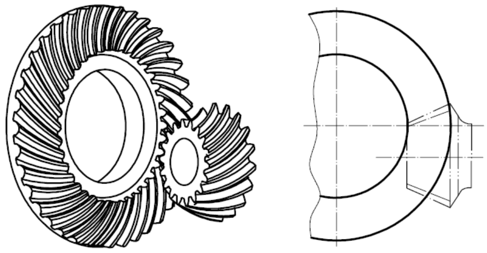

Figure 2.4 Hypoid Bevel Gear

Bevel Gear Terminology

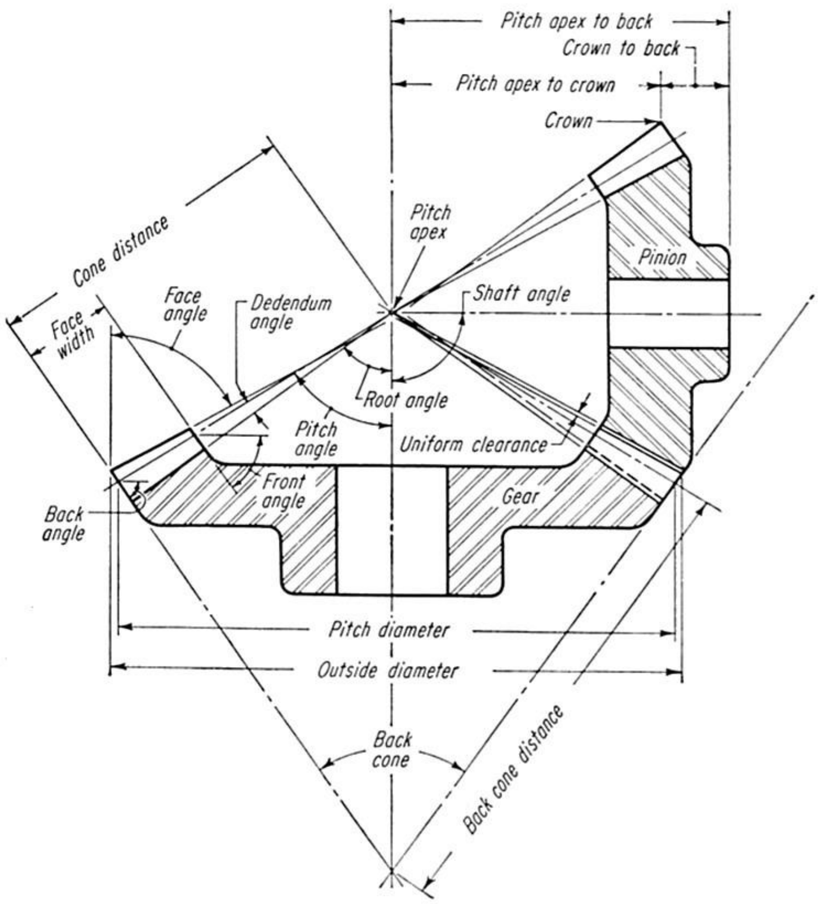

Figure 2.5 Bevel Gear Terminology

Geometric design of the bevel gears consists of two part:

1. Determining pitch cone parameters
2. Designing the teeth

Pitch Cone Parameters:

Shaft Angle (Σ): It is the angle between the intersecting shafts. In our case it is 90°.

Outer Pitch Diameter (d_e): The diameter of the pitch circle, which is an imaginary circle that passes through the points where the teeth of two meshing gears theoretically contact each other.

$$d_e = m_{et} * z$$

Where m_{et} is the module measured in the transverse plane of the back cone.

Pitch Angle (δ): The angle between the pitch cone and the axis of the gear. It is crucial in determining the orientation of the gear teeth.

$$\delta_1 = tan^{-1}\left(\frac{\sin(\Sigma)}{\cos(\Sigma) + u}\right)$$

$$\delta_2 = \Sigma - \delta_1$$

Where u is the gear ratio and δ_1 and δ_2 are the pitch angles for the pinion and gear respectively.

Outer Cone Distance: The distance from the apex of the pitch cone to the pitch circle. It is essential for the correct positioning and meshing of bevel gears.

$$R_{e1,2} = \frac{d_{e2}}{2 * \sin(\delta_2)}$$

Root Angle($\boldsymbol{\delta_f}$): The root angle of a bevel gear is the angle between the root cone and the gear axis. The root cone extends from the bottom of the gear teeth to the gear axis.

Face Angle ($\boldsymbol{\delta_a}$): The face angle of a bevel gear is the angle between the face cone and the gear axis. The face cone extends from the top of the gear teeth to the gear axis.

There are 3 different ways to determine the root and face angles according to the configurations we use for the tooth depth of bevel gears:

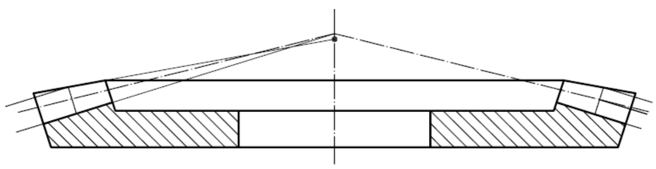

Figure 2.6 Standard Depth Taper

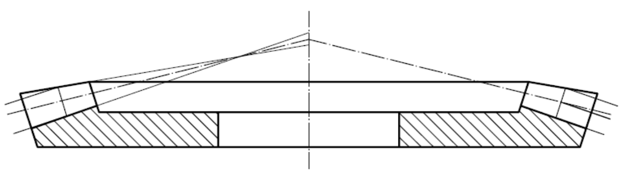

Figure 2.7 Constant and Modified Slot Width

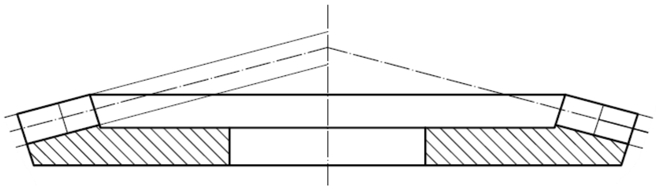

Figure 2.8 Uniform Depth

The configuration which is used in this thesis is standard depth taper. According to ISO, for standard depth taper the formulas used for determining the face and root angles are:

- Sum of dedendum angles: $\theta_{fs} = tan^{-1}\left(\frac{h_{fm1}}{R_{m2}}\right) + tan^{-1}\left(\frac{h_{fm2}}{R_{m2}}\right)$
- Addendum angle of gear: $\theta_{a2} = tan^{-1}\left(\frac{h_{fm1}}{R_{m2}}\right)$
- Dedendum angle of gear: $\theta_{f2} = \theta_{fs} - \theta_{a2}$
- Face angle of gear: $\delta_{a2} = \delta_2 + \theta_{a2}$
- Root angle of gear: $\delta_{f2} = \delta_2 - \theta_{f2}$
- Face angle of pinion: $\delta_{a1} = sin^{-1}(sin\Sigma * \sin(\delta_{a2}) + \cos\Sigma * \cos(\delta_{f2}))$
- Root angle of pinion: $\delta_{f1} = sin^{-1}(sin\Sigma * \sin(\delta_{a2}) + \cos\Sigma * \cos(\delta_{a2}))$

h_{am}, h_{fm}: Mean addendum and dedendum
R_m: Mean cone distance

Pressure angle: The most common pressure angle used for bevel gears is 20°.

Teeth design and virtual cylindrical gears:

Virtual cylindrical gears are hypothetical cylindrical gears that correspond to the actual bevel gears. They are used as an aid in designing and analyzing bevel gears. The pitch cone of the bevel gear is imagined to be unwrapped into a flat plane, forming a cylindrical shape. These virtual gears simplify the understanding of bevel gear geometry by allowing the application of cylindrical gear equations to bevel gears. This helps in calculating parameters like tooth dimensions, pitch, and pressure angles. Virtual cylindrical gears enable easier computation and visualization of the complex interactions within bevel gears, facilitating accurate design and ensuring efficient performance.

The parameters of virtual cylindrical gears are computed as below:

- Pitch diameter: $d_{v1,2} = \frac{d_{m1,2}}{\cos(\delta_{1,2})}$
- Base diameter: $d_{vb1,2} = d_{v1,2} * \cos(\alpha)$
- Tip diameter: $d_{va1,2} = d_{v1,2} + 2 * h_{am1,2}$
- Root diameter: $d_{vf1,2} = d_{v1,2} - 2 * h_{fm1,2}$
- Number of virtual teeth: $z_{v1} = z_1 * \frac{\sqrt{u^2+1}}{u}$, $z_{v2} = z_2 * \sqrt{u^2+1}$

It should be mentioned that the number of the virtual teeth in virtual cylindrical gears does not have to be an integer unlike real cylindrical gears.

Geometric design of the bevel gears can be done using abovementioned formulas. When the geometric design is done, it is the time to calculate the load capacity of bevel gears.

Bevel gear load capacity

The load capacity of the bevel gears has been determined according to ISO 10300 standards, which define criteria for assessing gear endurance and strength across different operating conditions. This approach ensures that the gears meet industry standards for reliability and performance.

The procedure for determining the load capacity of bevel gears is very similar to the procedure used for cylindrical gears. Several influence factors are computed according to the geometry and loading condition of the gears and at the end, the safety factors are calculated:

1. Safety factor for bending stress (Safety against tooth breakage): S_f
2. Safety factor for surface durability (Safety against pitting): S_H

If the computed safety factor falls short of the required level, a step-by-step process begins. This includes adjusting parameters like gear module until the safety factors reach the specified minimum standards.

3. Shaft

In the context of mechanical engineering, accurately analyzing the structural integrity and performance of shafts is crucial for ensuring the reliability and efficiency of machinery. This section details the methodology and findings of static analyzing the shaft using the Finite Element Method (FEM) using Timoshenko beam theory in MATLAB. These methods allow for a comprehensive assessment of the shaft's static behavior under different loading conditions, taking into account shear deformation and rotational bending effects.

Timoshenko Beam Theory

Timoshenko beam theory is a refinement of the classical Euler-Bernoulli beam theory. It includes shear deformation and rotational bending effects, making it more accurate for short beams or beams subjected to high loads. The key differences are:

1. **Shear Deformation**: Unlike Euler-Bernoulli theory, Timoshenko beam theory considers the shear deformation, which is significant in short beams.
2. **Rotational Bending Effects**: The theory accounts for the rotations of the cross-sections that are perpendicular to the axis of the beam.

Finite Element Method (FEM)

The Finite Element Method is a powerful computational technique used to solve complex structural problems. It divides a large system into smaller, simpler parts called finite elements. The behavior of each element is described by equations, which are then assembled into a larger system of equations that models the entire problem.

In this analysis, the shaft is discretized into finite elements, and the Timoshenko beam theory is applied to each element. The steps involved are:

1. **Discretization**: Dividing the shaft into a finite number of elements. In our case study, we have 3 different shafts. The methodology implemented for discretizing the shafts is different for each type:

- Input and output shafts: These shafts are divided into 4 parts with 4 diameters and to create shoulders suitable for holding 1 gear and 2 bearings. Each individual part is then divided into 5 equal-length parts. So in total, these shafts are divided to 20 elements with 21 nodes.
- Middle shafts: These shafts consist of 5 parts with 5 different diameters in order to make appropriate shoulders for 2 gears and 2 bearings. Each part then is divided into 5 equal-length parts so that in total, there are 25 elements.

2. **Element Stiffness Matrix and Stress-Strain Matrix**: The stiffness matrix for a Timoshenko beam element is derived by considering both bending and shear effects. This matrix is larger and more complex than that of the Euler-Bernoulli beam theory due to the additional degrees of freedom. Each node has 6 degrees of freedom leading to a total 12 degrees of freedom for each element. The degrees of freedom consist in 3 translational displacements (along x, y and z) and 3 rotational displacements (around x, y and z). Below you can see the elemental stiffness matrix of Timoshenko beam:

$$[K] =$$

$$E * \begin{bmatrix} \frac{A}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{A}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12I_z}{(1+\phi_y)l^3} & 0 & 0 & 0 & \frac{6I_z}{(1+\phi_y)l^2} & 0 & -\frac{12I_z}{(1+\phi_y)l^3} & 0 & 0 & 0 & \frac{6I_z}{(1+\phi_y)l^2} \\ 0 & 0 & \frac{12I_y}{(1+\phi_z)l^3} & 0 & -\frac{6I_z}{(1+\phi_z)l^2} & 0 & 0 & 0 & -\frac{12I_y}{(1+\phi_z)l^3} & 0 & -\frac{6I_y}{(1+\phi_z)l^2} & 0 \\ 0 & 0 & 0 & \frac{GI_x}{lE} & 0 & 0 & 0 & 0 & 0 & -\frac{GI_x}{lE} & 0 & 0 \\ 0 & 0 & -\frac{6I_y}{(1+\phi_z)l^2} & 0 & \frac{(4+\phi_z)I_y}{(1+\phi_z)l} & 0 & 0 & 0 & \frac{6I_y}{(1+\phi_z)l^2} & 0 & \frac{(2-\phi_z)I_y}{(1+\phi_z)l} & 0 \\ 0 & \frac{6I_z}{(1+\phi_y)l^2} & 0 & 0 & 0 & \frac{(4+\phi_y)I_z}{(1+\phi_y)l} & 0 & -\frac{6I_z}{(1+\phi_y)l^2} & 0 & 0 & 0 & \frac{(2-\phi_y)I_z}{(1+\phi_y)l} \\ -\frac{A}{l} & 0 & 0 & 0 & 0 & 0 & \frac{A}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12I_z}{(1+\phi_y)l^3} & 0 & 0 & 0 & -\frac{6I_z}{(1+\phi_y)l^2} & 0 & \frac{12I_z}{(1+\phi_y)l^3} & 0 & 0 & 0 & -\frac{6I_z}{(1+\phi_y)l^2} \\ 0 & 0 & -\frac{12I_y}{(1+\phi_z)l^3} & 0 & \frac{6I_z}{(1+\phi_z)l^2} & 0 & 0 & 0 & \frac{12I_y}{(1+\phi_z)l^3} & 0 & \frac{6I_y}{(1+\phi_z)l^2} & 0 \\ 0 & 0 & 0 & -\frac{GI_x}{lE} & 0 & 0 & 0 & 0 & 0 & \frac{GI_x}{lE} & 0 & 0 \\ 0 & 0 & -\frac{6I_y}{(1+\phi_z)l^2} & 0 & \frac{(2-\phi_z)I_y}{(1+\phi_z)l} & 0 & 0 & 0 & \frac{6I_y}{(1+\phi_z)l^2} & 0 & \frac{(4+\phi_z)I_y}{(1+\phi_z)l} & 0 \\ 0 & \frac{6I_z}{(1+\phi_y)l^2} & 0 & 0 & 0 & \frac{(2-\phi_y)I_z}{(1+\phi_y)l} & 0 & -\frac{6I_z}{(1+\phi_y)l^2} & 0 & 0 & 0 & \frac{(4+\phi_y)I_z}{(1+\phi_y)l} \end{bmatrix}$$

Where the description of parameters used in matrix is:

- Beam cross section (m^2): $A = \frac{\pi * d^2}{4}$ where d is the diameter of the shaft.
- Young modulus (pa): E
- Moments of Inertia: $I_x = I_p = \frac{\pi * d^4}{32}$, $I_y = I_z = \frac{\pi * d^4}{64}$
- Shear modulus: G
- The ratio between the shear and the flexural flexibility of the beam: $\phi_y = \frac{12EI_z}{GAl^2}$, $\phi_z = \frac{12EI_y}{GAl^2}$

The stress-strain matrix [D], or constitutive matrix, applies Hooke's Law to link stress and strain in materials, crucial for linear elastic behavior in 2D and 3D stress scenarios. In finite element analysis, [D] is vital for determining element stiffness and predicting structural responses accurately.

$$[D] = \frac{E(1-v)}{(1+v)(1-2v)} * \begin{bmatrix} 1 & \frac{v}{1-v} & \frac{v}{1-v} & 0 & 0 & 0 \\ \frac{v}{1-v} & 1 & \frac{v}{1-v} & 0 & 0 & 0 \\ \frac{v}{1-v} & \frac{v}{1-v} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2v}{2(1-v)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2v}{2(1-v)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2v}{2(1-v)} \end{bmatrix}$$

3. **Assembly**: The assembly of stiffness matrix is a fundamental step in the Finite Element Method analysis. It involves combining the matrices of individual elements into a global matrix that represents the entire structure. To assemble the global stiffness matrix, the stiffness matrices of individual elements must be transformed from local coordinate systems to the global coordinate system. This transformation accounts for the orientation and position of each element within the overall structure. The global stiffness matrix $[K]_{tot}$ is constructed by superimposing the matrices of all elements. This involves summing the contributions of each element at the corresponding degrees of freedom (DOFs).

4. **Boundary Conditions**: Boundary conditions are critical in the Finite Element Method (FEM) as they define how the model interacts with its environment. They ensure that the numerical solution of a physical problem adheres to the physical constraints and conditions that the actual system would experience. There are two main types of boundary conditions:

5. **Solution**: After applying the boundary conditions, the modified system of equations is solved to obtain the nodal displacements or other primary variables. This solution provides the foundational data needed to understand the system's behavior. In the post-processing stage, these nodal values are used to calculate derived quantities such as strains, stresses, and reaction forces.

Implementation in MATLAB

The process begins with defining the geometric dimensions of each part of the shafts:

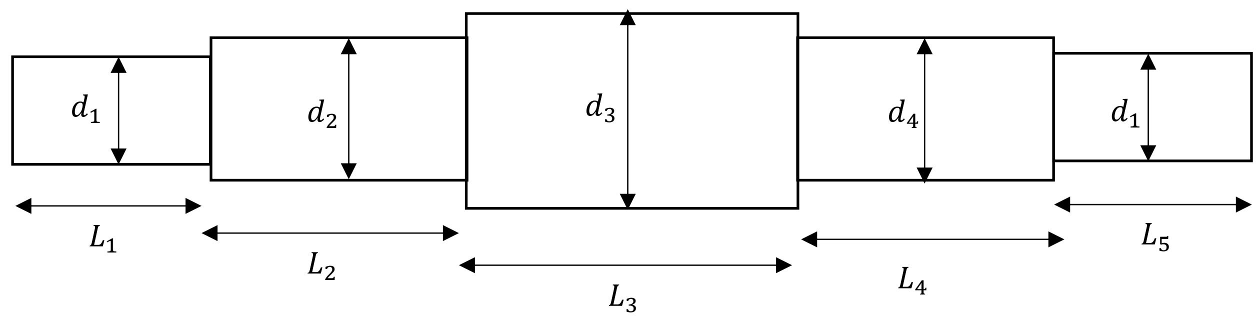

Figure 3.1 Middle Shaft

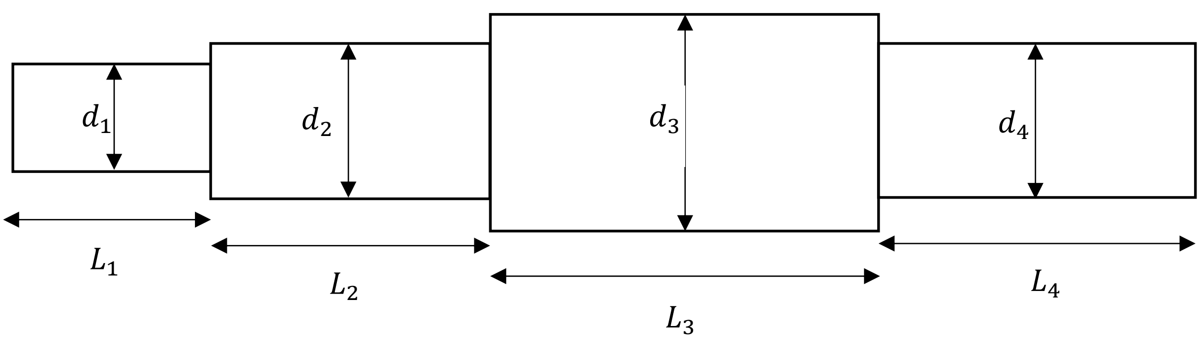

Figure 3.2 Input / Output Shaft

Length of shaft:

Middle shafts: The gears are mounted on the second and fourth part of these shafts. So the length of these parts of the shaft is equal to the width of the gear plus a clearance which is needed for the gears to operate properly. On the other hand, bearings are mounted on the first and fifth part of the shaft. So in a same way, their length is considered equal to the width of the bearings plus a clearance. Third part of the shaft has a constant value in a way that it can provide enough space between the 2 operating gears and at the same time, it can minimize the length of the shaft.

Input and output Shafts: There is only one gear on these shafts. So these shafts need one less part. The gear is mounted on the second part of these shafts and bearings are mounted on the first and fourth parts. Length of first part of the input shaft is dimensioned in a same way as before, except the fact that the value of the exceedance of the shaft must be added to it since the input shaft has an extra length which is outside of the casing of the gearbox to mate with the driving machine. Length of the second, third, and fourth parts of this shaft is determined exactly like before. The length of the output shaft is determined in a same way. The difference is that according to the configuration of the gearbox (same-sided or opposite-sided) and the number of the stages, the shaft exceedance is either added to the first part or the last part. In order to understand better, 2 two-stage gearbox layouts are presented below with different configurations:

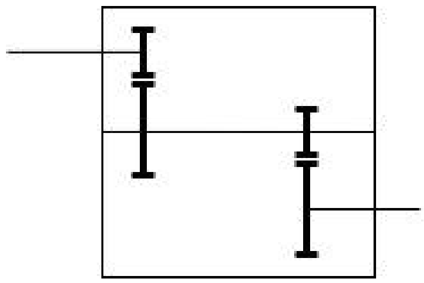

Figure 3.3 Opposite Sided Configuration

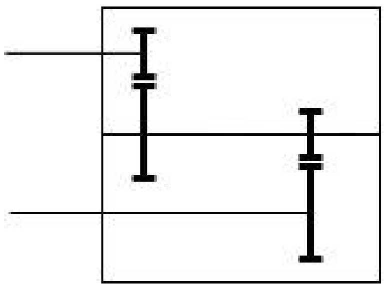

Figure 3.4 Same Sided Configuration

As you can see from the figures 3.3 and 3.4, in the opposite-sided configuration, the shaft exceedance is added to the fourth part of the shaft, but in the parallel-sided configuration, the shaft exceedance is added to first part. Also the second part of the shaft must be modified so that it can compensate the whole length of the gearbox.

Diameter of shaft:

After the length is defined, it is time for choosing the diameters. Choosing the diameters is an iteration process. Diameter of each part is chosen from a diameter vector which is compatible with the bore diameters of the SKF bearings. After the analysis is done, if the safety factor met the required safety factor, chosen diameters are finalized. If not, a larger diameter is chosen and this process is continued until proper diameters are found.

Stiffness Matrix:

The next step after determining the geometry of shaft, is to write the stiffness matrix for each node. In our case study, each part of the shafts is divided into 5 smaller pieces, leading to a total number of 25 elements for the middle shafts and 20 elements for the input and output shafts. The stiffness matrix ($[K]$) is computed for each node, then using MATLAB codes and algorithms, all the matrices for each node are combined together forming a global stiffness matrix ($[K]_{tot}$).

Force Vector: Each gear exerts 5 different force and moments on the shaft. So in general, all the elements of the force vector of the shaft are 0, except the ones which refer to the nodes related to gears. The forces are along x, y, and z axes (F_x, F_y, F_z) and as a result of these forces, we have 2 moments around x and z axes (M_x, M_z). Since we have 2 gears on the middle shafts, we have 10 non-zero elements in the force vector and for the input and output shafts, there are 5 non-zero elements which have to be put in the force vector in accordance with their nodes. Forces exerted by gears can be seen in the figure 3.5:

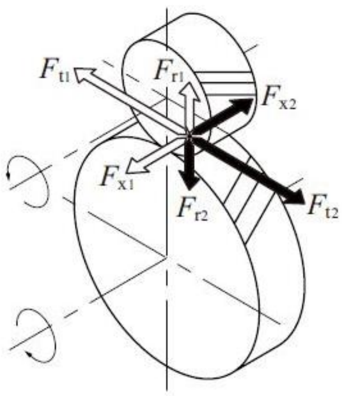

Figure 3.5 Gear Forces

The above-mentioned forces can be calculated as below:

- Tangential load: $F_t = F_z = \frac{2000*T}{d}$ where T is the input torque of the stage and d is the diameter of pitch circle in millimeters.
- Axial load: $F_a = F_x = F_t * \tan(\beta)$
- Radial load: $F_r = F_y = F_t * \cos(\beta) * \tan(\alpha_t)$
- Moment around x axis: $M_x = F_t * \frac{d}{2000}$
- Moment around z axis: $M_z = F_a * \frac{d}{2000}$

Boundary Conditions: To proceed, the boundary conditions must be applied. Bearings and gears constrain some degrees of freedom of the shaft. The constrained degrees of freedom related to bearings are:

$$\{u, v, w\}$$

And the gears constrain only one degree of freedom:

$$\{\phi_x\}$$

After the constrained DOFs are specified, the corresponding row and column in stiffness matrix and the corresponding row in force vector must be deleted.

Finding displacements: The displacements will be derived using the below formula:

$$[X] = [K]^{-1}[F]$$

After deriving the $[X]$ matrix, we need to put 0 for the displacements of the constrained nodes that we removed before.

Total Force Vector: By deriving the vector for all the displacements, now we can find the final force vector containing all the forces and reaction forces of the shaft using the below formula:

$$[F] = [K][X]$$

Deflections: Deflection in shafts refers to the bending or displacement of a shaft under applied loads, such as torque or forces. It is influenced by factors like material properties, shaft geometry, and the type of load applied. Controlling deflection is crucial to maintaining alignment and ensuring the proper functioning of mechanical systems. Once the displacements are known, deflection of the shaft in each plane can be determined. The Allowable deflection is determined by the user. If the maximum deflection of the shaft is more that the allowable deflection, the iteration process continues with larger diameters in order to control the deflection.

Strains: Having the $[X]$ vector, displacements and rotations along each axis are subdivided into different vectors:

$$\{u\} = \{u_1 \quad u_2 \quad u_3 \quad \dots \quad u_{N-1} \quad u_N \quad u_{N+1}\}^T$$

$$\{v\} = \{v_1 \quad v_2 \quad v_3 \quad \dots \quad v_{N-1} \quad v_N \quad v_{N+1}\}^T$$

$$\{w\} = \{w_1 \quad w_2 \quad w_3 \quad \dots \quad w_{N-1} \quad w_N \quad w_{N+1}\}^T$$

$$\{\phi_x\} = \{\phi_{x,1} \quad \phi_{x,2} \quad \phi_{x,3} \quad \dots \quad \phi_{x,N-1} \quad \phi_{x,N} \quad \phi_{x,N+1}\}^T$$

$$\{\phi_y\} = \{\phi_{y,1} \quad \phi_{y,2} \quad \phi_{y,3} \quad \dots \quad \phi_{y,N-1} \quad \phi_{y,N} \quad \phi_{y,N+1}\}^T$$

$$\{\phi_z\} = \{\phi_{z,1} \quad \phi_{z,2} \quad \phi_{z,3} \quad \dots \quad \phi_{z,N-1} \quad \phi_{z,N} \quad \phi_{z,N+1}\}^T$$

Using these vectors, the strains ε_{xx}, ε_{yy}, ε_{zz}, γ_{xy}, γ_{yz}, γ_{xz} can be calculated. To do so, we need a matrix called B matrix(strain-displacement matrix), which is the derivative of shape functions, for calculating strains of each element:

- ε_{xx}:

$$B = \left[-\frac{1}{l}, \frac{1}{l}\right]$$

$$\varepsilon_{xx}(i) = B * [u(i); u(i+1)]$$

- ε_{yy}:

$$B = \begin{bmatrix} \dfrac{\frac{-6}{l} + 12 * \frac{X}{l^2}}{l(1+\varphi_y)} & \dfrac{\frac{-4}{l} + 6\frac{X}{l^2} - \frac{\varphi}{l}}{1+\varphi_y} & \dfrac{\frac{6}{l} - 12\frac{X}{l^2}}{l(1+\varphi_y)} & \dfrac{\frac{-2}{l} + 6\frac{X}{l^2} + \frac{\varphi}{l}}{1+\varphi_y} \end{bmatrix}$$

$$\varepsilon_{yy}(i) = B * [v(i); \phi_z(i); v(i+1), \phi_z(i+1)]$$

- ε_{zz}:

$$B = \begin{bmatrix} \dfrac{\frac{6}{l} - 12\frac{X}{l^2}}{l(1+\varphi_z)} & \dfrac{\frac{-4}{l} + 6\frac{X}{l^2} - \frac{\varphi}{l}}{1+\varphi_z} & \dfrac{\frac{-6}{l} + 12\frac{X}{l^2}}{l(1+\varphi_z)} & \dfrac{\frac{-2}{l} + 6\frac{X}{l^2} + \frac{\varphi}{l}}{1+\varphi_z} \end{bmatrix}$$

$$\varepsilon_{zz}(i) = B * \left[w(i); \phi_y(i); w(i+1), \phi_y(i+1)\right]$$

- γ_{xy}:

$$B = [B_{11} \quad B_{12} \quad B_{13} \quad B_{14}]$$

$$B11 = \frac{\frac{-\varphi}{l} - 6\frac{X}{l^2} + 6\frac{X^2}{l^3}}{1+\varphi} - \frac{-6\frac{X}{l} + 6\frac{X^2}{l^2}}{l(1+\varphi)}$$

$$B12 = \frac{l * (\frac{1}{l} + \frac{\varphi}{2l} - \frac{\varphi X}{l^2} - 4\frac{x}{l^2} + 3\frac{X^2}{l^3})}{1+\varphi} - \frac{1 - 4\frac{X}{l} + 3\frac{X^2}{l^2} + \varphi(1 - \frac{X}{l})}{1+\varphi}$$

$$B13 = \frac{\frac{\varphi}{l} + 6\frac{X}{l^2} - 6\frac{X^2}{l^3}}{1+\varphi} - \frac{6\frac{X}{l} - 6\frac{X^2}{l^2}}{l(1+\varphi)}$$

$$B14 = \frac{l * (\frac{-\varphi}{2l} + \frac{\varphi X}{l^2} - 2\frac{x}{l^2} + 3\frac{X^2}{l^3})}{1+\varphi} - \frac{-2\frac{X}{l} + 3\frac{X^2}{l^2} + \varphi\frac{X}{l})}{1+\varphi}$$

$$\gamma_{xy} = B * [V(i); \phi_z(i); v(i+1); \phi_z(i+1)]$$

- γ_{xz}:

$$B = [B_{11} \quad B_{12} \quad B_{13} \quad B_{14}]$$

$$B11 = \frac{\frac{-\varphi}{l} - 6\frac{X}{l^2} + 6\frac{X^2}{l^3}}{1+\varphi} + \frac{-6\frac{X}{l} + 6\frac{X^2}{l^2}}{l(1+\varphi)}$$

$$B12 = -\frac{l * (\frac{1}{l} + \frac{\varphi}{2l} - \frac{\varphi X}{l^2} - 4\frac{x}{l^2} + 3\frac{X^2}{l^3})}{1+\varphi} - \frac{1 - 4\frac{X}{l} + 3\frac{X^2}{l^2} + \varphi(1 - \frac{X}{l})}{1+\varphi}$$

$$B13 = \frac{\frac{\varphi}{l} + 6\frac{X}{l^2} - 6\frac{X^2}{l^3}}{1+\varphi} + \frac{6\frac{X}{l} - 6\frac{X^2}{l^2}}{l(1+\varphi)}$$

$$B14 = -\frac{l * (\frac{-\varphi}{2l} + \frac{\varphi X}{l^2} - 2\frac{x}{l^2} + 3\frac{X^2}{l^3})}{1+\varphi} - \frac{-2\frac{X}{l} + 3\frac{X^2}{l^2} + \varphi\frac{X}{l})}{1+\varphi}$$

$$\gamma_{xz} = B * [w(i); \phi_y(i); w(i+1); \phi_y(i+1)]$$

- γ_{yz}

$$B = \left[-\frac{1}{l}, \frac{1}{l}\right]$$

- $\gamma_{yz}(i) = B * [\phi_x(i); \phi_x(i+1)]$

- X is the distance from the starting point of the element. In this case study, each element is divided to 10 small elements and strains are calculated at each point. The maximum strain is then used for calculating the stress.

All the calculated strains are then combined in a universal vector containing all of them:

$$\{\varepsilon\} = \{\ \varepsilon_{xx,1} \quad \varepsilon_{yy,1} \quad \varepsilon_{zz,1} \quad \gamma_{xy,1} \quad \gamma_{yz,1} \quad \gamma_{xz,1} \quad \cdots \quad \varepsilon_{xx,N+1} \quad \varepsilon_{yy,N+1} \quad \varepsilon_{zz,N+1} \quad \gamma_{xy,N+1} \quad \gamma_{yz,N+1} \quad \gamma_{xz,N+1}\}^T$$

Then the vector of stresses can be found:

$$\{\sigma\} = [D]\{\varepsilon\}$$

The stress tensor for each node is defined as:

$$[\sigma]_i = \begin{bmatrix} \sigma_{xx,i} & \tau_{xy,i} & \tau_{xz,i} \\ \tau_{xy,i} & \sigma_{yy,i} & \tau_{yz,i} \\ \tau_{xz,i} & \tau_{yz,i} & \sigma_{zz,i} \end{bmatrix}$$

In order to transform the tensor of stresses to principal stresses, calculation of Eigen problem Is necessary as below:

$$det \begin{bmatrix} \sigma_{xx,i} - \sigma & \tau_{xy,i} & \tau_{xz,i} \\ \tau_{xy,i} & \sigma_{yy,i} - \sigma & \tau_{yz,i} \\ \tau_{xz,i} & \tau_{yz,i} & \sigma_{zz,i} - \sigma \end{bmatrix} = 0$$

The eigenvalues σ that solve the self-problem of the principal's tensions in each node are $\sigma_{1,i}$, $\sigma_{2,i}$, and $\sigma_{3,i}$.

The equivalent stress is then must be calculated for each node using Von-Mises methodology:

$$\sigma_{eq,1} = \frac{1}{\sqrt{2}} \sqrt{\left(\sigma_{1,i} - \sigma_{2,i}\right)^2 + \left(\sigma_{2,i} - \sigma_{3,i}\right)^2 + (\sigma_{1,i} - \sigma_{3,i})^2}$$

The vector of equivalent tension for each node is then will be:

$$\left\{\sigma_{eq}\right\} = \{\sigma_{eq,1} \quad \sigma_{eq,2} \quad \sigma_{eq,3} \quad \cdots \quad \sigma_{eq,N-1} \quad \sigma_{eq,N} \quad \sigma_{eq,N+1}\}^T$$

The maximum of $\left\{\sigma_{eq}\right\}$ vector is then used to calculate the safety factor:

$$SF = \frac{R_{p02}}{\sigma_{eq,max}}$$

If the safety factor was below the required safety factor of the user, the iteration process continues with larger diameters until this condition is satisfied:

$$SF \geq SF_{User}$$

4. Bearings

Bearings are mechanical components designed to reduce friction between moving parts while supporting radial and axial loads. They are essential in a wide range of machinery, ensuring smooth and efficient operation. By minimizing friction, bearings enhance performance, extend the lifespan of equipment, and reduce energy consumption.

Rolling-element bearings can be classified into several types based on their design and application:

Ball Bearings

Ball bearings are a type of rolling-element bearing that use spherical balls to maintain the separation between the bearing races. They are designed to reduce rotational friction and support both radial and axial loads. Ball bearings are commonly used in a wide range of applications, including household appliances, electric motors, and automotive components, due to their efficiency and versatility. Their design allows for smooth and low-friction motion, making them ideal for high-speed operations.

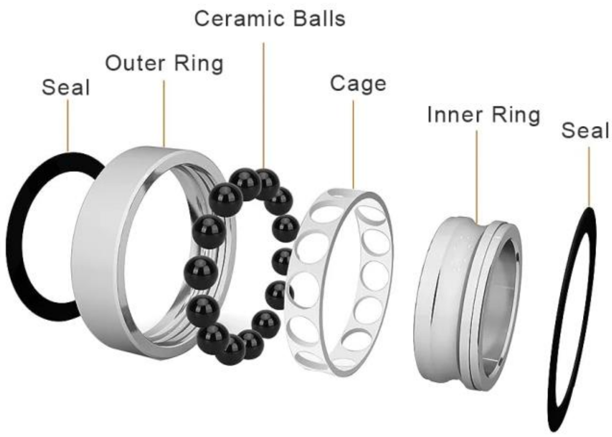

Figure 4.1 Ball Bearing Elements

Deep Groove Ball Bearings: Deep groove ball bearings are a subtype of ball bearings known for their ability to support both radial and axial loads. Their design features deep, uninterrupted raceway grooves which allow for higher load capacity and efficient operation. These bearings are highly versatile, making them suitable for a wide range of applications. Their construction ensures low friction, high-speed capability, and minimal maintenance requirements.

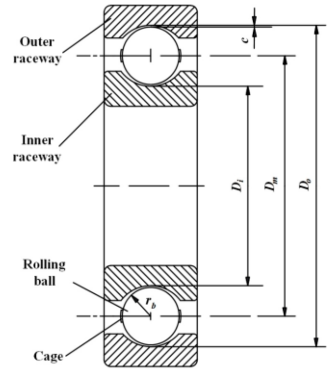

Figure 4.2 Deep Groove Ball Bearing

Angular Contact Ball Bearings: Angular contact ball bearings are designed to handle both radial and axial loads, but unlike deep groove ball bearings, they can support significant axial loads in one direction due to the angle at which the balls contact the races. This angular contact allows for higher axial load capacity and improved load distribution. These bearings are commonly used in applications requiring precise alignment and high-speed operation, such as in machine tool spindles, automotive transmissions, and high-performance pumps. Their design allows for the combination of multiple bearings to handle complex load conditions.

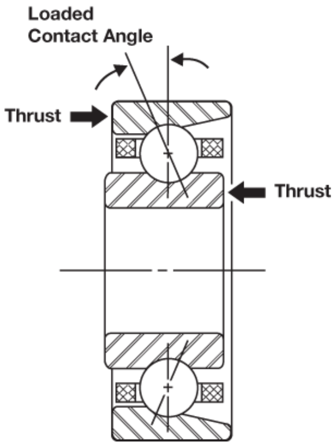

Figure 4.3 Angular Contact Ball Bearing

Roller Bearings

Roller bearings are a type of bearing that use cylindrical or tapered rollers to maintain the separation between moving parts of a machine. Unlike ball bearings, which use spherical balls, roller bearings are designed to carry heavy loads and reduce friction between rotating parts. They are commonly used in applications where high radial loads are present, such as in conveyor belt rollers, gearboxes, and heavy machinery. The rollers provide a larger contact area with the races, distributing loads more evenly and increasing the bearing's load-carrying capacity. Roller bearings come in various designs, including cylindrical, needle, tapered, and spherical, each suited to specific applications and load requirements.

Cylindrical Roller Bearings: Cylindrical roller bearings are a type of roller bearing that use cylindrical rollers to maintain the separation between moving parts. They are particularly effective at handling high radial loads and offer low friction, making them suitable for high-speed applications. These bearings provide excellent performance in conditions where heavy radial loads and moderate axial loads are present. They are commonly used in industrial gearboxes, electric motors, and large machinery.

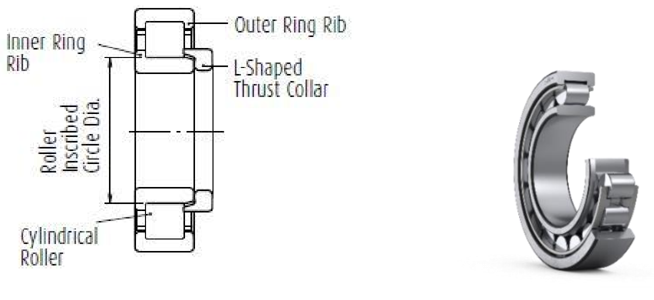

Figure 4.4 Cylindrical Roller Bearing

Needle Bearings: Needle bearings are a type of roller bearing that use long, thin cylindrical rollers resembling needles to reduce friction between moving parts. They have a high load-carrying capacity and are particularly effective in applications with limited radial space. Due to their small cross-section, needle bearings are ideal for compact designs and can handle significant radial loads despite their size. They are commonly used in automotive components, such as transmissions and universal joints, as well as in industrial machinery and aerospace applications. Needle bearings come in various designs, including radial, thrust, and combined types, to suit different load and space requirements.

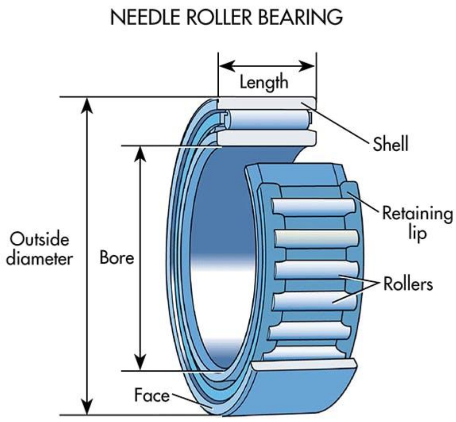

Figure 4.5 Needle Bearing

Bearing Selection Process:

- **Initiation**: The process of selecting bearings for the gearbox starts immediately after the completion of static shaft analysis.

- **Catalog Compilation**: Bearings in the SKF catalog are organized into an Excel spreadsheet, including comprehensive details about their specifications and characteristics for thorough assessment.

- **MATLAB Integration**: The Excel file is then integrated into MATLAB, allowing MATLAB to have all the details about the bearings.

- **Force Analysis and Bore Diameter Determination**: Using Finite Element Method (FEM) of the shafts within MATLAB, the forces acting on the bearings are derived and the necessary bore diameter required for optimal performance under operational conditions are specified.

- **Selection Criteria Consideration**: Beyond force analysis, critical factors such as environmental conditions (e.g., temperature, contamination levels), specific user-defined preferences (e.g., noise level, reliability), and gearbox-specific operational demands (e.g., rotational speeds) are carefully evaluated. These considerations ensure that the chosen bearings not only meet but exceed performance and operational standards, guaranteeing optimal functionality and longevity in varied operating environments.

- **Final Step:** After the bearing selection process is done, all the chosen bearings with their characteristics are saved in an Excel file for further steps.

5. Gearbox Design

Now that the elements of the gearbox are explained, it's time to go deeply through the process of the dimensioning the gearbox. The flowchart in the figure 5.1 explains the whole process for creating the gearbox. Each step will be then explained with details:

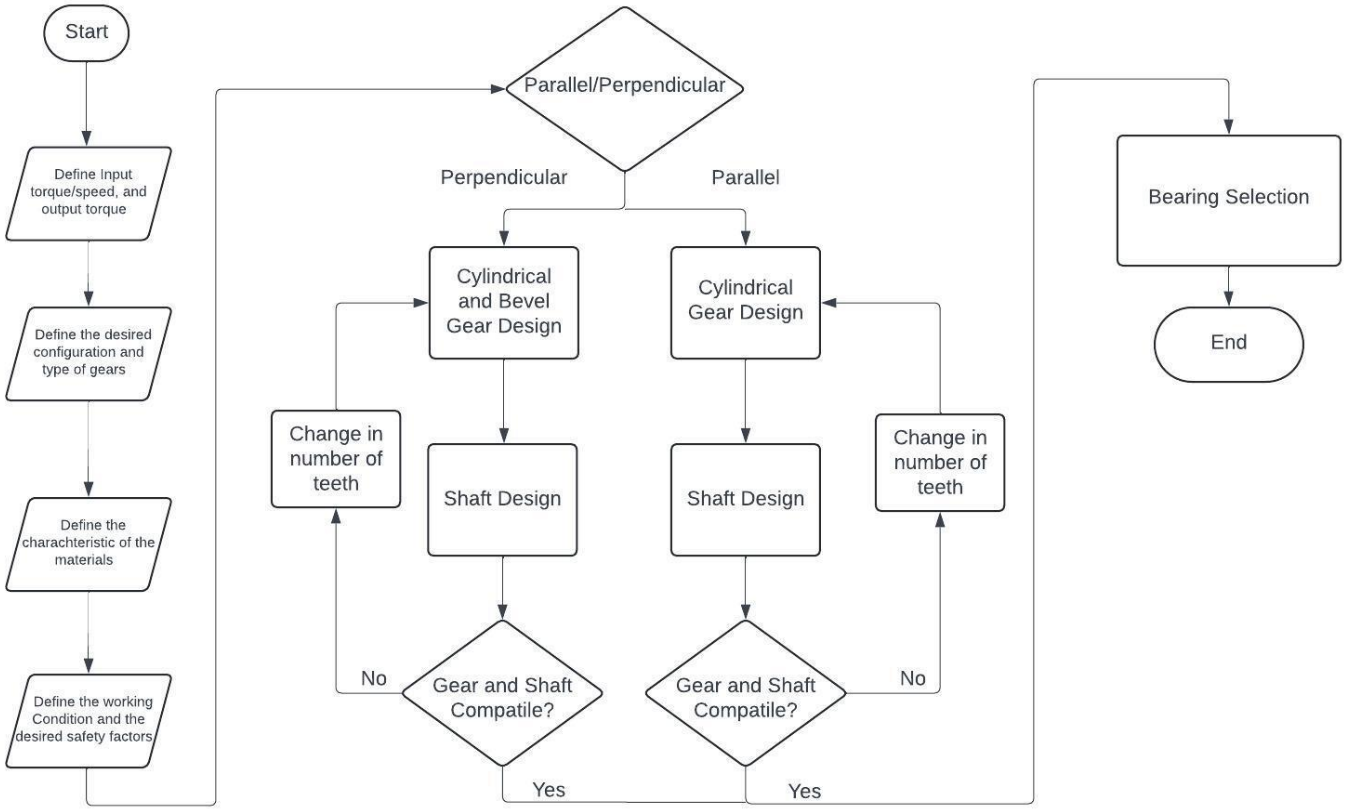

Figure 5. 1 Flowchart of the Gearbox Design

Initiation:

The process starts with the code asking the user to input the data of the input and output of the gearbox. The user, according to his/her desired application, must input the value for the input torque and rotational speed that is going through gearbox (by means of an external source like an electric motor) and the desired output torque.

Configuration:

In this step, user is required to choose a configuration between the 4 available configurations. The configurations available for this case study are:

- Parallel configuration (Same-Sided): In this configuration, the input and output shaft have parallel axes and are on the same side of the gearbox.

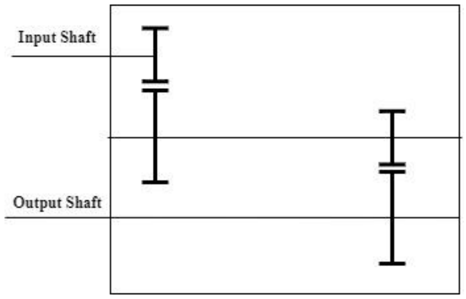

Figure 5. 2 Parallel Same-Sided Configuration

- Parallel Configuration (Opposite-Sided): In this configuration, the input and output shaft have parallel axes, but they are not on the same side of the gearbox

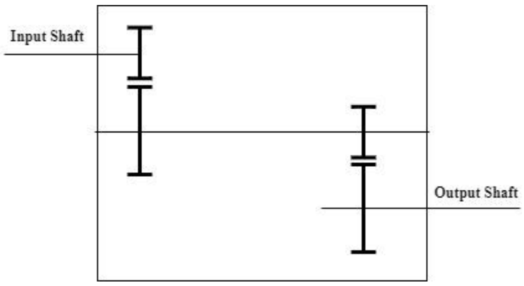

Figure 5. 3 Parallel Opposite-Sided Configuration

- Perpendicular Configuration: In this configuration, the axes of the input and output shaft are perpendicular:

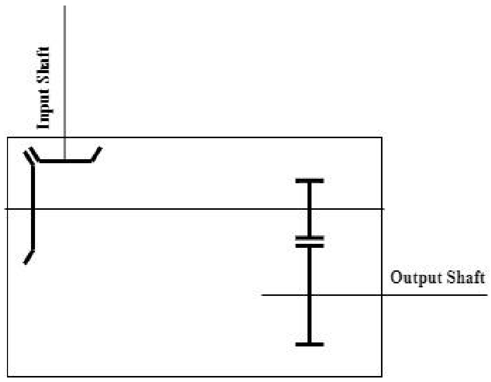

Figure 5. 4 Perpendicular Configuration

- Compound Reverted Gear Train: This configuration is a form of parallel configurations, but in this case, the input and output shafts have inline axis. It is only possible to use this configuration when the number of stages are equal to 2.

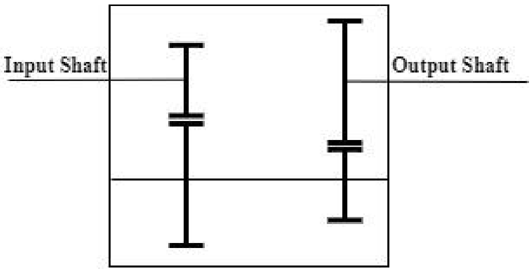

Figure 5. 5 Compound Reverted Configuration

Type of gears:

In this case study, we have used only the straight type of bevel gears. But for the cylindrical gears, the user has the opportunity to choose between helical or spur gears.

Characteristics of materials:

One of the most important steps for designing a gearbox in to choose proper materials for your elements. The user will be asked with some questions about the material he/she wants to use for the gears and also for the shafts. This helps the user to have a gearbox with the material that are available. At first, user needs to define the mechanical properties of the material of the gear. These properties are:

- Type of the material (Cast iron, through hardened steel, Nitrided wrought steel, etc.)
- Allowable stress number for contact ($\sigma_{H\ lim}$)
- Nominal stress number for bending ($\sigma_{F\ lim}$)
- Young modulus (E)
- Poisson ratio(υ)
- Brinell Hardness (HB)

In the next step, the material properties of the shaft must be defined by user. These properties are:

- Yield strength (R_{p02})
- Young modulus (E)
- Poisson ratio(υ)

Working Condition:

In order to be able to design a gearbox properly, it is needed to know the conditions that the gearbox in going to work in. These conditions are:

1. Shock level: The driving machine of the gearbox, and also the machine that is going to be driven by the gearbox have a level of shock. These shock levels are:
    - Uniform
    - Light shocks
    - Medium shocks
    - Hard shocks

    It is necessary to define these shock levels to be able to calculate application factor (K_A) for gears
2. Duty cycle: It is defined as the working hours of the gearbox in a day. After the application factor is computed using shock levels, it will be corrected according to the duty cycle of the gearbox.
3. Input and output shaft exceedances: According to the working conditions, and the need of the user, user can define how much he/she wants the input and output shafts of the gearbox to be outside of the gearbox. This value will be added to the total lengths of the input and output shafts before applying the FEM analysis.

Safety Factor:

According to the application of the gearbox, user can define the proper safety factors for shafts and gears.

Gearbox initiative parameters:

After user has put all the necessary information, the next step is defining initiative parameters of the gearbox. This consist of determining:

- Number of stages: This is a crucial step for starting to design the gearbox. According to the input and output torque of the gearbox, the overall gear ratio can be computed:

$$U_{all} = \frac{T_{out}}{T_{in}}$$

  According to the overall ratio of the gearbox, the number of stages can be defined. The case study supports gearboxes with at most 4 number of stages and the maximum value of the gear ratio for each stage is considered to be 5. So the maximum overall gear ratio is 1:625. So according to the overall gear ratio, the total number of stages is defined by the code.
- Ratio per stage: The gear ratio for each stage is computed in the following way:

$$a = \left(\frac{U_{all}}{R^{(n-1)*\frac{n}{2}}}\right)^{\frac{1}{n}}$$

$$U_i = a * R^{i-1}$$

  Where n is the number of stages and U_i is the ratio per each stage. The value R is a constant value than can be equal to 1, for equal division of the ratios on each stage, it can be less than 1 where the ratios will be decreasing as we go for the last stages, and it can be greater than one to have an increasing order for the gear ratios.

Element Design:

1. **Gears:** After the calculation of the initiative parameters, depending on the choice of the user, whether the configuration is perpendicular or parallel, the process of designing the elements of the gearbox starts. If the chosen configuration is perpendicular, the process starts with designing the bevel gears and if the number of stages exceeds one, then the cylindrical gears are designed. On the other hand, if the chosen configuration is parallel (same-sided, opposite-sided, or compound reverted gear train), the code directly starts designing the cylindrical gears. Once the process for designing the gears are done, their data will be saved in an Excel file for the further use.
2. **Shafts:** After designing each pair of gear, the forces exerted by each one of them are extracted from the code and are implemented to the shaft code. These data, along with the material data said by the user will help the code to run an iterative process in order to find the best set of diameters for the shafts. The data related to shaft dimensions will then be saved in an Excel file for future use.
    - **Compatibility check:** Once the gears and shafts are dimensioned, a safety check is run. The reason to that is to compare the diameters of the shaft and gears. The approach is to have gears with the dedendum diameters which are at least 20% larger than the diameter of the shaft. While in the first try, this condition might not satisfy and we can encounter a shaft which is larger than the gear. So a safety check is done and if the condition does not stand, the number of teeth on pinion will be increased and the gear will be dimensioned again and so will the shaft. This process continues until the gear and the shaft can pass the safety check.
3. **Bearings:** After the finite element analysis of the shaft is done, the reaction forces of the bearing can be extracted. Using those forces and general criteria of the bearings (e.g. environment, noise level, etc.) the bearings will be then chosen from the SKF catalog and the information related to them, will be saved in an Excel file.

6. CAD Design:

Computer-Aided Design (CAD) involves using software to create, modify, analyze, or optimize a design. CAD design improves productivity by allowing for quick iterations and modifications, enhances accuracy with precise measurements and geometries, and facilitates better visualization through 3D modeling. It also enables easy sharing and collaboration among team members and speeds up the manufacturing process by providing detailed and precise digital blueprints. Overall, CAD is a powerful tool that streamlines the design process and enhances the quality of the final product.

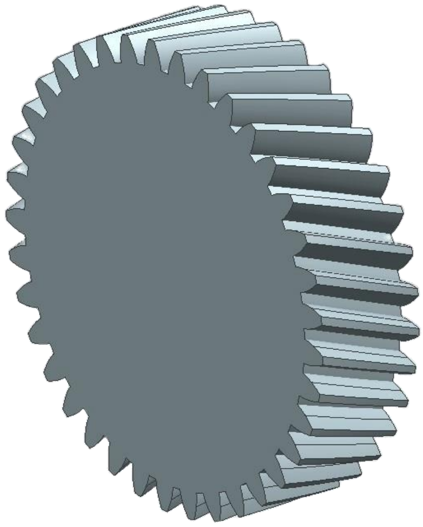

Figure 6. 1 An Example of CAD Design

Parametric CAD design:

Parametric CAD design uses parameters and constraints to define and control the relationships and dimensions within a design, enabling easy and consistent updates. This approach enhances efficiency by allowing quick adjustments without redrawing the entire model. Benefits include increased accuracy, streamlined design modifications, and improved consistency across parts and assemblies. It also facilitates automation and optimization, making it ideal for complex projects and iterative design processes. Parametric design in Siemens NX involves using parameters and constraints to define and control relationships within a model, ensuring automatic updates and consistency. The steps are:

1. **Create a New Sketch**: Start by sketching the basic geometry on a chosen plane.
2. **Define Parameters**: Set dimensions and other parameters to control the sketch.
3. **Add Constraints**: Apply geometric constraints to manage relationships between sketch elements. In order to have a proper parametric design that can handle every situation, you must make sure that the sketch you have drawn is fully constrained.
4. **Create Features**: Transform the sketch into 3D features like extrusions or revolves.
5. **Apply Expressions**: Use mathematical expressions to link parameters, enabling complex interdependencies.
6. **Assemble Components**: Combine parts into assemblies, applying constraints to define their interactions.

7. **Update and Modify**: Adjust parameters or constraints as needed; the model updates automatically.

Initiation:

When the process of designing elements is done, MATLAB saves all the data required for CAD design of those elements into separate Excel files. This file will be imported to Siemens NX, a leading software in this field, in order to design the dimensioned elements in an effective and accurate way. The process of parametric design will be introduced below:

Importing parameters to Siemens NX:

In Siemens NX, there is a section called expressions. Expressions allow you to define and manipulate design parameters using equations and relationships. There are some commands that can be used in the Expression part in order to read data from Excel. These commands are:

- *ug_cell_read("directory of the excel file","Row and column of the parameter in excel")*
  This command allows Siemens NX to import a parameter from a certain cell in a certain Excel file.
- *ug_read_list("directory of the excel file","Cell",True/False)*
  You can define a vector in expression parts, and using this command, you will be able to read a row or a column of the Excel file. The "*Cell*" is the cell you want to be the starting point of your row or column. If you put *True*, NX will read the row starting from the chosen cell and if you put *False*, NX will read the column. After the list is ready, you can assign each parameter of the vector to the parameters you want using this command

$$nth\ (number\ of\ the\ element\ in\ the\ vector, vector\ name)$$

You can find a picture of this process done in the thesis in the figure 6.2:

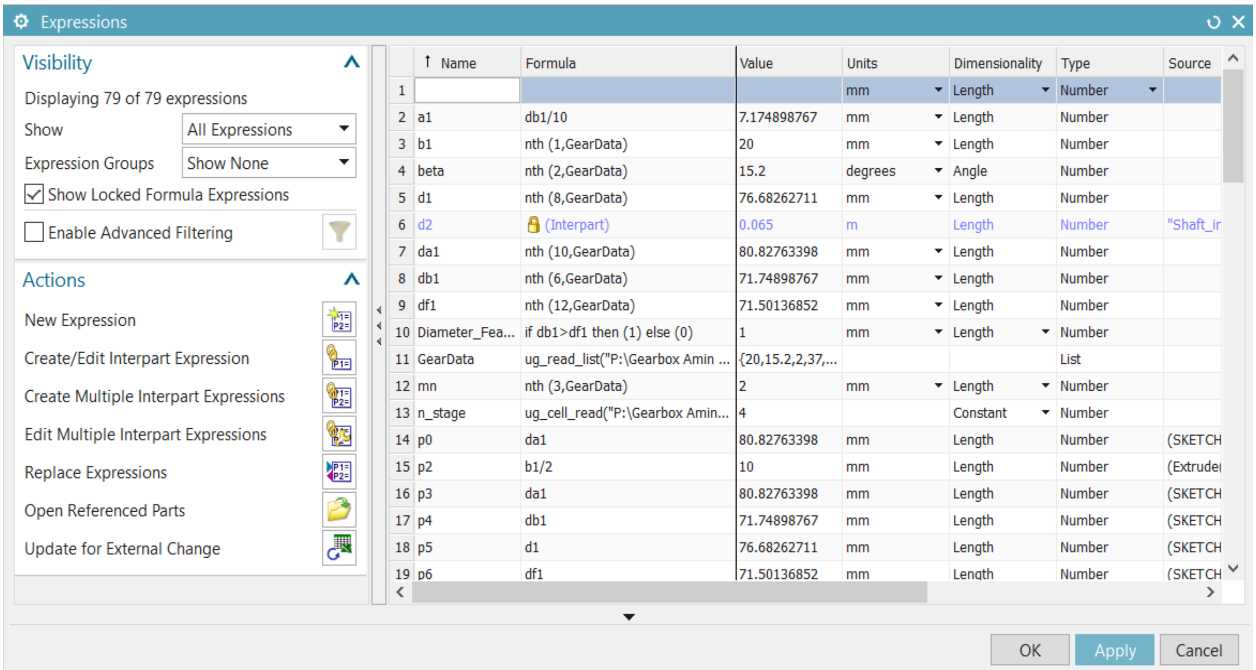

Figure 6. 2 Expression in NX Siemens

Cylindrical Gears:

There are 2 ways for creating the teeth of the gears. The teeth can either be added or can be subtracted. In order to make it more similar to what happens in reality, the approach chosen for generating the teeth of the gear is to subtract the space between the teeth from the gear blank design. So as a result, the addendum circle is drawn and will be extruded. Then the 4 main circles of the gear (Primitive, base, addendum, and dedendum circles) are sketched on the face of the blank gear. To generate the teeth of the gear, the method which is used is the involute of circles:

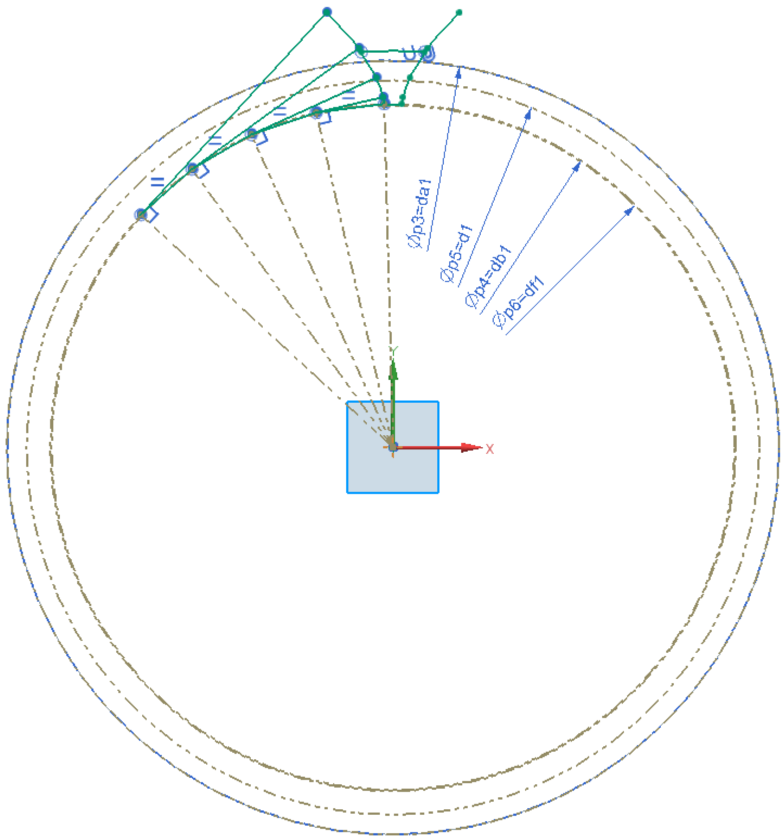

Figure 6. 3 Involute of Circles

In order to draw the involute of circles like shown in the figure 6.3, first the base circle is divided to a number of equal parts. Then the radial lines will be drawn from the center of the circles and they are connected to each other by lines which have a constant and equal value (a). Beginning from the second point, perpendicular lines to the radial line are constructed. The length of these lines are equal to a, $2a$, $3a$, and so on. For the final step, we connect the endpoints of these lines using Spline command in NX and the involute is ready. The produced involute must be mirrored about the vertical axis so that it can fully construct the space between the teeth.

After the involute is fully developed, it should be cut through the gears blank body. To do so, we need to define the Lead of the gear:

$$Lead = \frac{\pi * d_p}{\tan (\beta)}$$

This value is used for constructing a helix on the gear body so that we can use it as a guideline in order to sweep our sketch through the body of the gear in a way that the teeth with proper helix angle be constructed. In order to produce the helix, we use pitch diameter as the diameter of the helix. The pitch of helix is set equal to the lead of the gear and the length of the pitch is set equal to the gear width. When the guideline is ready, we can use it to create the teeth:

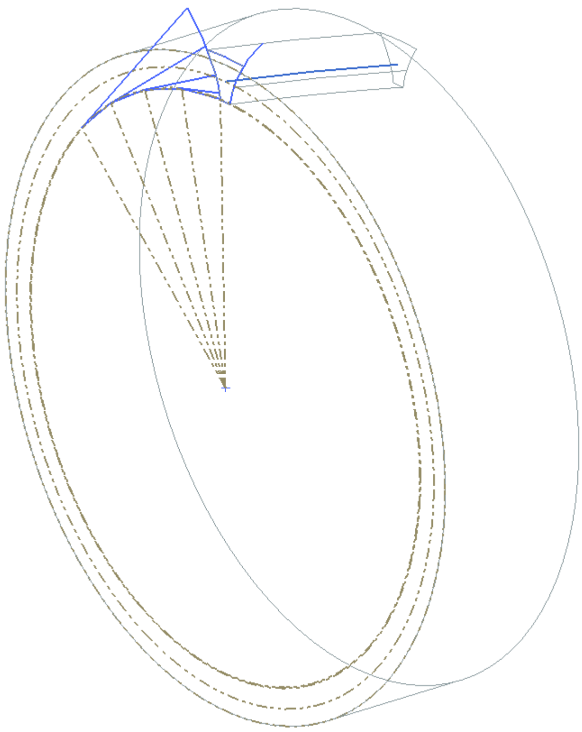

Figure 6. 4 Generating the Teeth of Gear

After this step, the swept part will be subtracted from the gear blank body and then it will be patterned around the body of the gear to create the teeth of the gear:

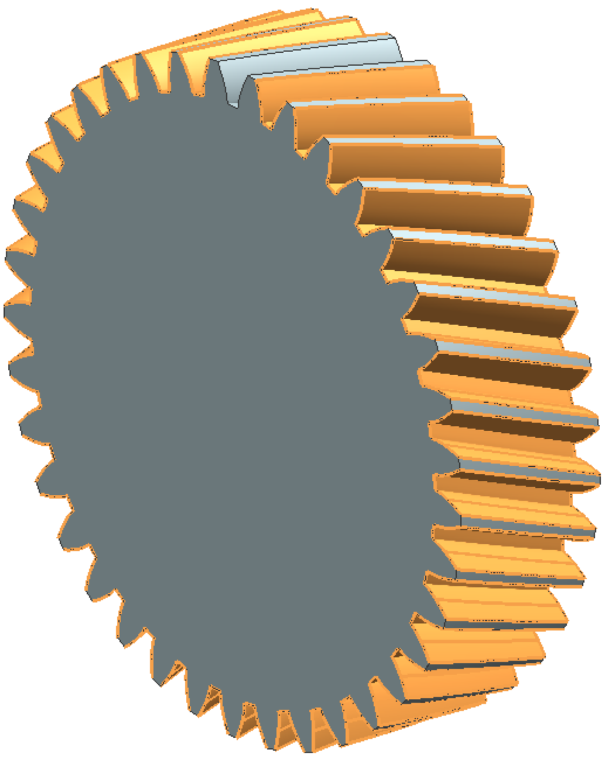

Figure 6. 5 Completed Gear

Bevel Gear:

Designing bevel gears is a more complex procedure than designing cylindrical gears. At first step, using data that are computed in MATLAB and are stored in the Excel file, the sketch of the cone can be designed:

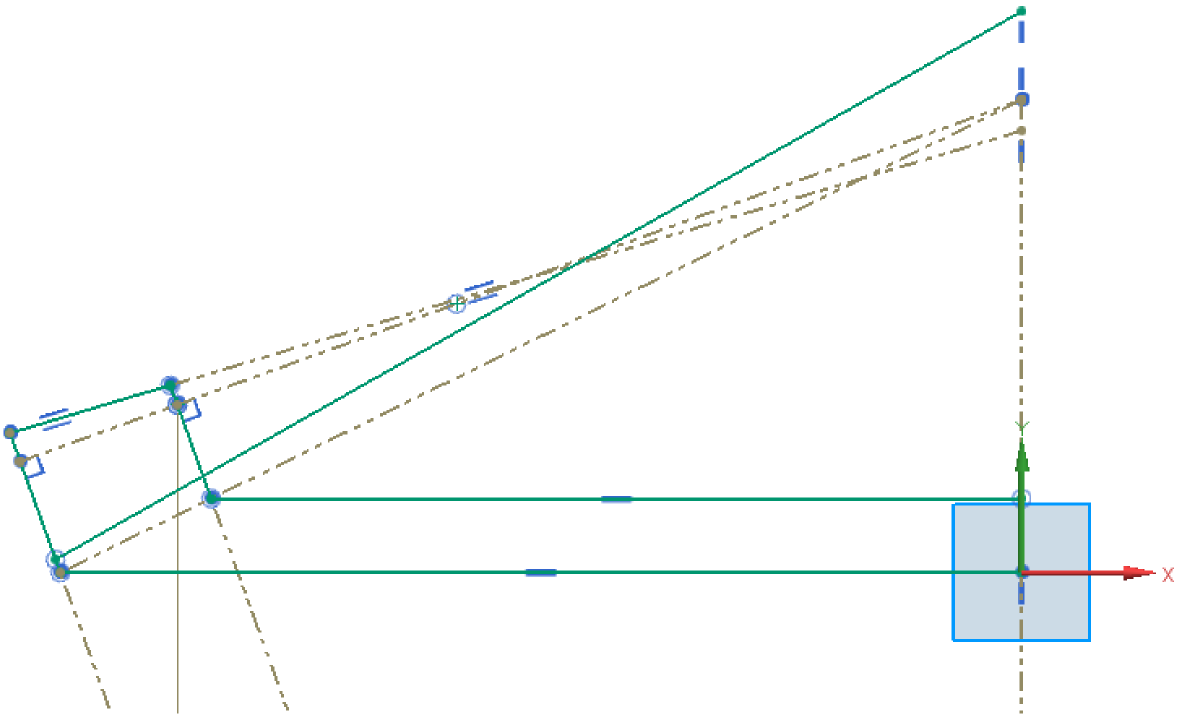

Figure 6. 6 Bevel Gear Cone Sketch

After sketching the cone, the cone must be revolved around the Y axis. The resulting figure will be as below:

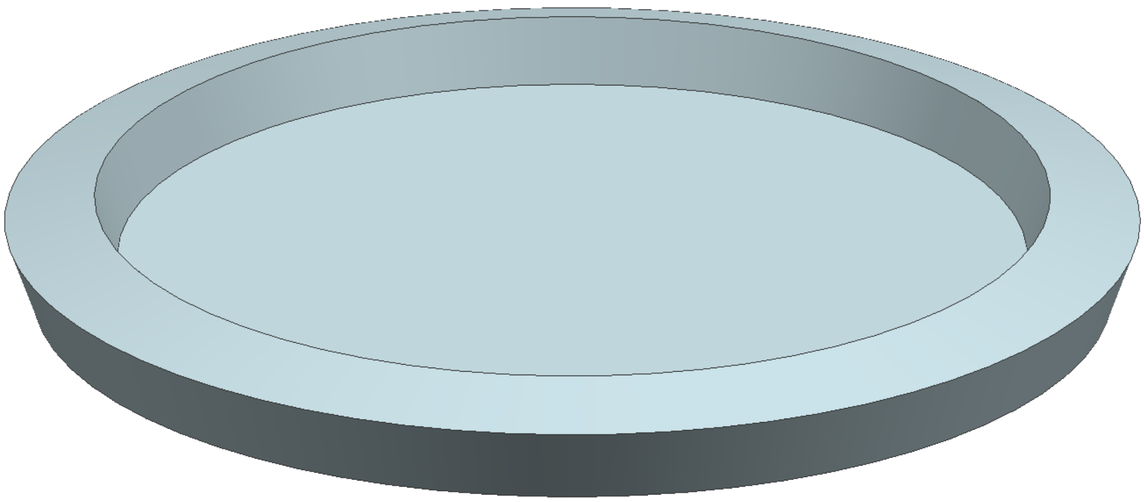

Figure 6. 7 Bevel Gear Cone

Once the cone is revolved, it is time to design the teeth. The procedure to design the teeth in bevel gears is similar to the one that is used for cylindrical gear. The main difference is that the teeth, have to be drawn on a plane, which is tangent to the back of the cone. Virtual circles calculated before are drawn in that plane and using the previous steps, the involute of circles are drawn:

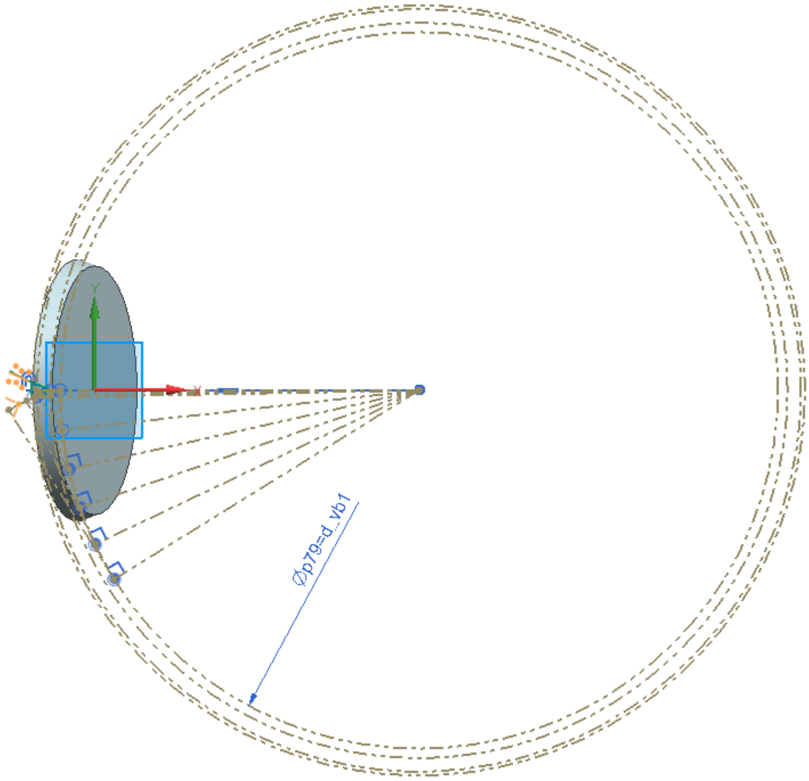

Figure 6. 8 Involute of Circles in Bevel Gears

Here is a detailed view of the drawn tooth:

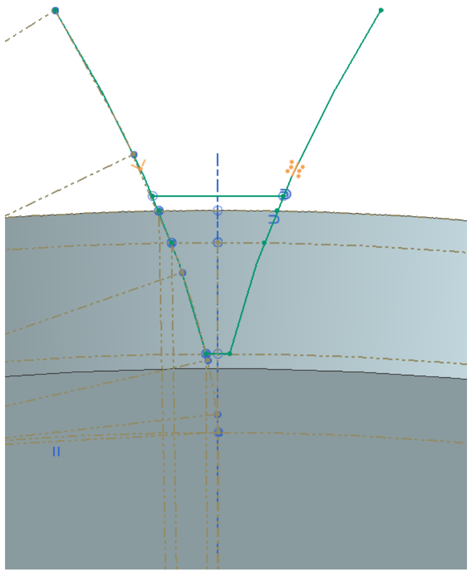

Figure 6. 9 Detailed View of Involute Curves

Once the space between the teeth is drawn, it should be swept along the line that connects the root of the teeth to the center of the cone. Then, the swept part must be subtracted and patterned around the whole bevel gear blank body as shown:

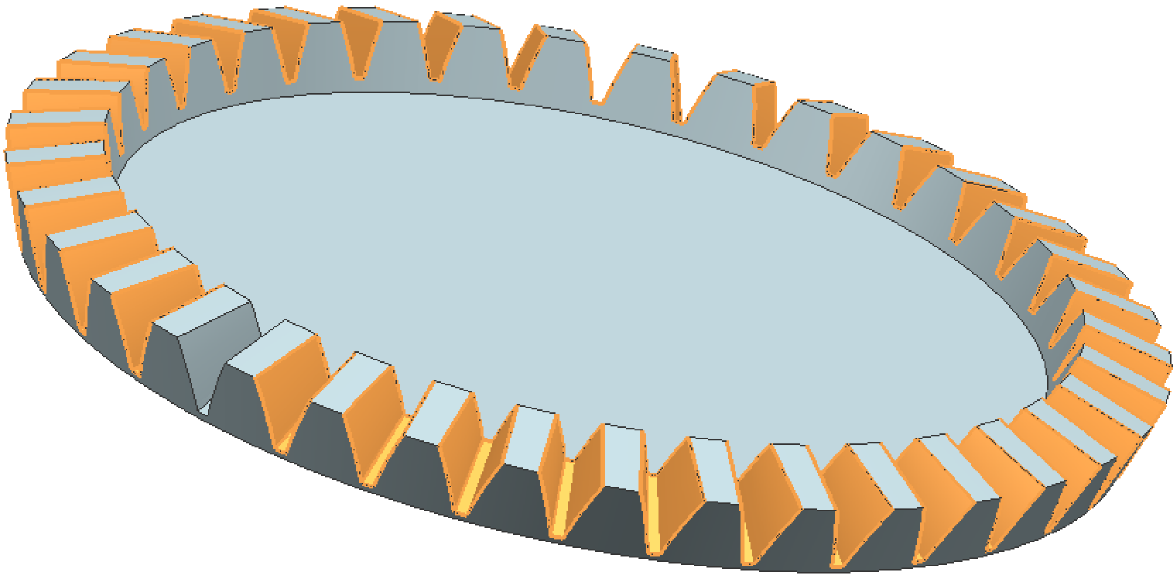

Figure 6. 10 Bevel Gear Completed

Shaft:

Designing the shaft is simple. The dimensions of the shaft are fed to the expressions of the NX by the Excel file. For input and output shafts there are 4 lengths and 4 diameters and for the middle shaft, there are 5. For each part of the shaft, a circle is drawn with the diameter equal to the diameter of that part of the shaft. This circle will then be extruded with the length of the corresponding part of the shaft. Examples of middle shaft and input shaft can be seen in figures 6.11 and 6.12:

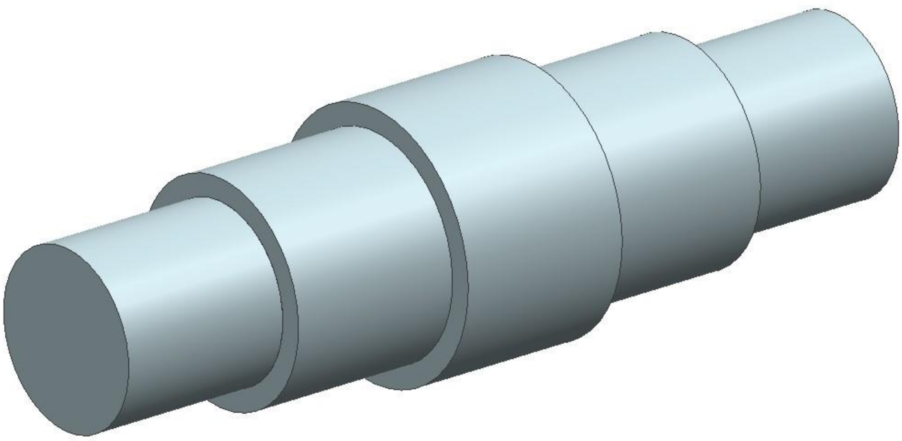

Figure 6. 11 Middle Shaft

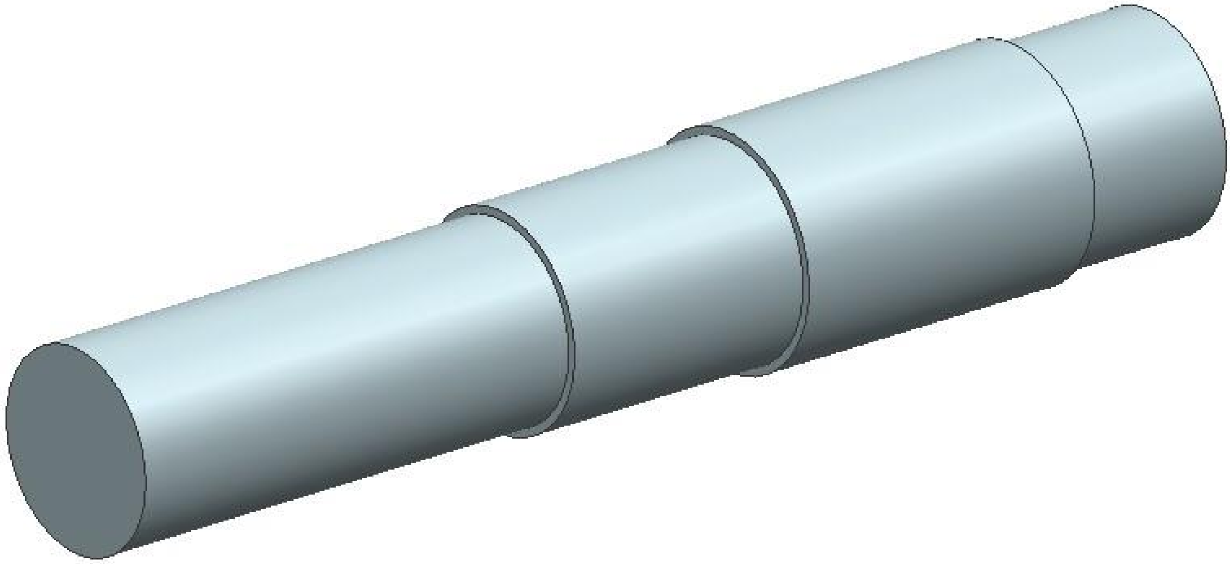

Figure 6. 12 Input Shaft

Bearings:

Bearings also like shafts, have a simple procedure for drawing. Since the bearings are chosen from the SKF catalogue, only a simplified version of the bearings will be drawn for the purpose of showing in the final assembly. The code chooses the bearing and it saves the data in the Excel file. This data, such as bore diameter, outside diameter, and bearing width, will be used in order to draw a simplified version of the bearings in the NX:

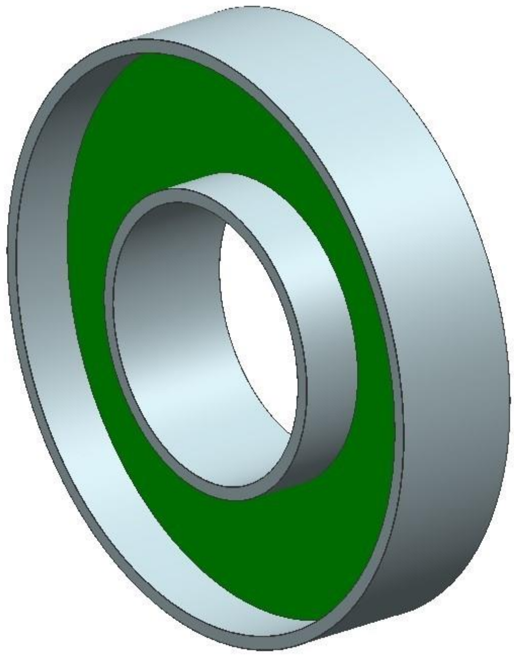

Figure 6. 13 Bearing

Interpart Expression:

Another essential element of assembly in parametric modeling is the interpart expression. Defining the relationships between the parameters of each individual component is necessary to establish parametric modeling. These inputs should be specified within the assembly and linked through the interpart expression. This ensures that any changes to the assembly's input are reflected in the individual components.

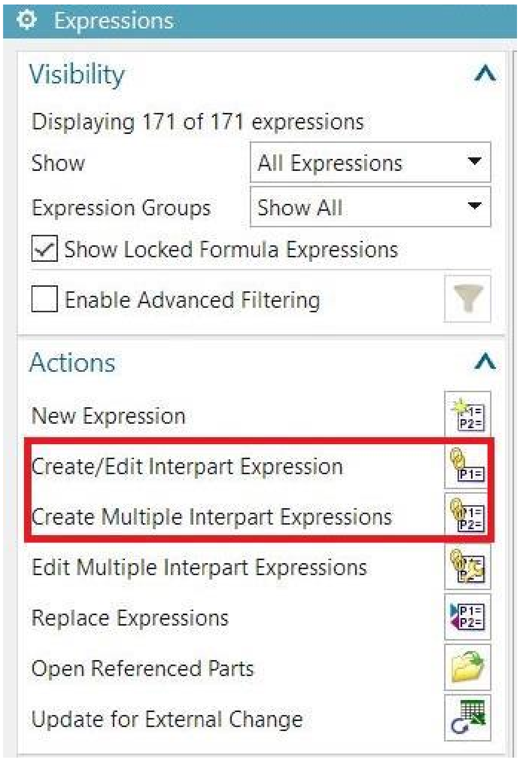

Figure 6. 14 Interpart Expression

Interpart expression can be determined in the expression window as shown in figure 6.14. As an example, the diameter of the second part of the shaft is chosen as an interpart expression for the bore diameter of the gear.

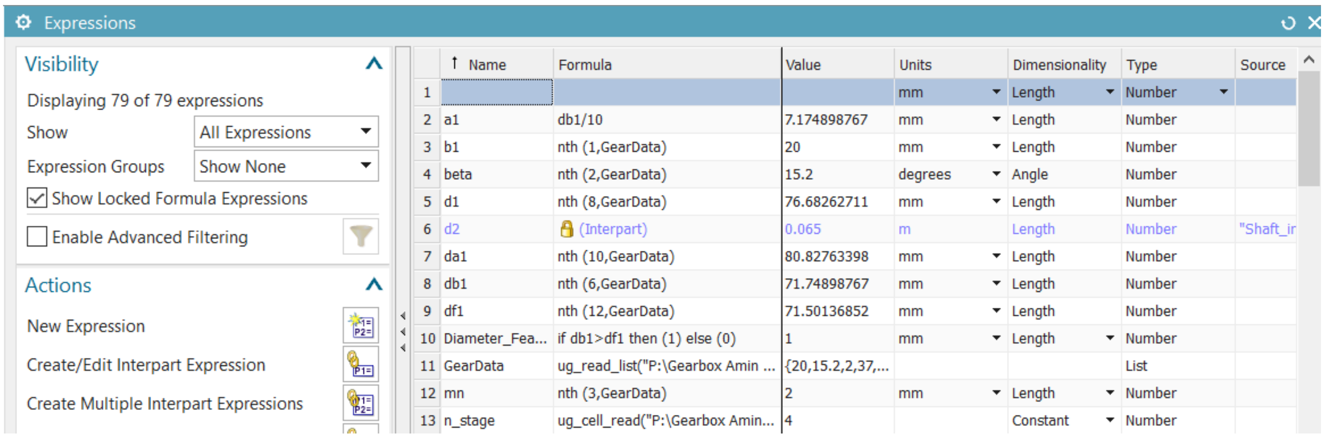

Figure 6. 15 Interpart Expression in Expressions

As you can see, in the picture above, the interpart expression is shown with the yellow lock in the formula section.

Assembly:

Now that each part is designed individually, the assembly process starts. In order to properly assemble the parts in NX, there exist some constraints that must be implemented in NX correctly. The available constraints in NX assembly are:

Touch/Align: It is used to position and orient components relative to each other by aligning their faces, edges, or points. This constraint ensures that the selected elements either touch or align with each other, maintaining the specified spatial relationship during the assembly process.

Concentric: The concentric constraint ensures that the selected cylindrical or spherical faces of different components share the same centerline. This alignment allows the parts to rotate around the common axis while maintaining their concentric positioning.

Distance: The distance constraint sets a specific distance between selected faces, edges, or points of different components. This constraint maintains the defined separation, ensuring precise spatial relationships within the assembly.

Fix: The fix constraint locks a component in its current position, preventing any movement or rotation. This constraint ensures that the component remains stationary within the assembly, serving as a stable reference point for other parts.

Parallel: The parallel constraint ensures that selected faces, edges, or axes of different components remain parallel to each other.

Perpendicular: The perpendicular constraint ensures that selected faces, edges, or axes of different components are oriented at a 90-degree angle to each other.

Align/Lock: The align/lock constraint allows selected faces, edges, or points of different components to be positioned precisely relative to each other. This constraint ensures that the specified elements maintain their alignment and relative position, effectively locking their spatial relationship during assembly.

Fit: The fit constraint, constraints two objective with equal radii, such as circular or elliptical edges, or cylindrical or spherical faces.

Bond: The bond constraint is used to rigidly connect or join selected components together. This constraint simulates a permanent attachment, ensuring that the parts behave as a single unit without relative movement between them.

Center: The center constraint, centers one or two objects between a pair of objects, or centers a pair of objects along another object.

Angle: The angle constraint allows you to specify a desired angle between selected components or features. This ensures that the parts are positioned at the specified angular relationship relative to each other, maintaining precise alignment within the assembly.

Assembly procedure:

At the first step, the input shaft is inserted in the assembly part. It then will be fixed to be used as a reference for the proceeding steps. Next part to be inserted is the gear. First gear will be added in the assembly environment. Using the concentric constraint, the hole in gear body, which is designed using interpart expression using the diameter of the shaft, will be put on the shaft on its place. The next step will be to add bearings. 2 bearings must be added, as there will be 2 bearings mounted on the input shaft, and using concentric constraint, they will be put on the shaft. The result is shown in figure 6.16:

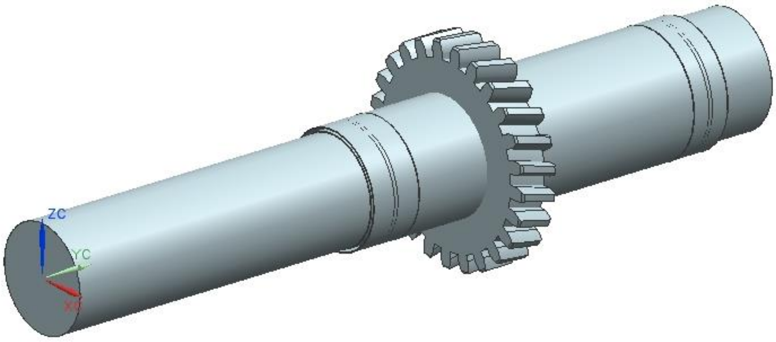

Figure 6. 16 Assembly of Input Shaft

Once the input shaft is assembled, we need to proceed to implementing other shafts. Depending on the number of stages, the next shaft can be output shaft or middle shaft. Let's assume that we have a gearbox with number of stages equal to 2. So the next shaft would be a middle shaft. The middle shaft will be inserted into assembly environment. Same procedure will take place for this shaft. The difference is that 2 gears will be mounted on this shaft. Once all the gears and bearings are mounted on the shaft, 2 interpart expressions must be added to our expressions. These 2 expressions are diameters of the primitive circles of the first 2 gears. These 2 values are used to set a distance between the shafts. This distance is called center distance of the gears and is equal to:

$$a = \frac{d_1 + d_2}{2}$$

Using align constraint, we need to align the faces of the gear 1 and gear 2 so that they can mesh. The resulting assembly by far is shown in figure 6.17:

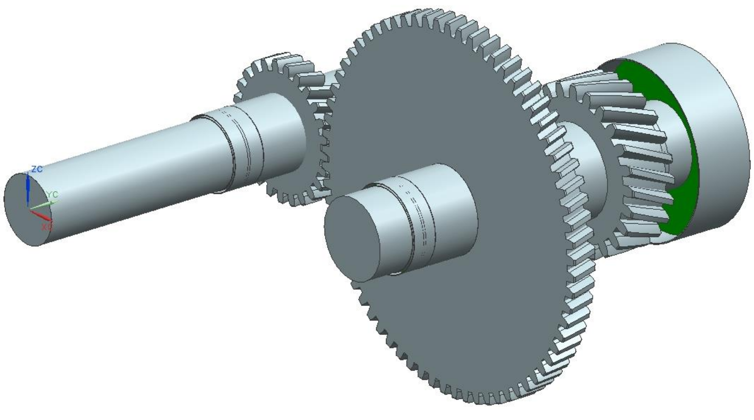

Figure 6. 17 Assembly of Input and Middle Shaft

The last part is to add the output shaft, and mount the gears and bearing on it. Then using the diameters of the third and fourth gear, which are implemented to assembly environment using interpart expression, we can determine the center distance of the gears. Same as before, we should align the faces of gear 3 and 4 so that they can mesh properly. The final assembly of a 2 stage helical gear opposite sided gearbox is shown in figure 6.18:

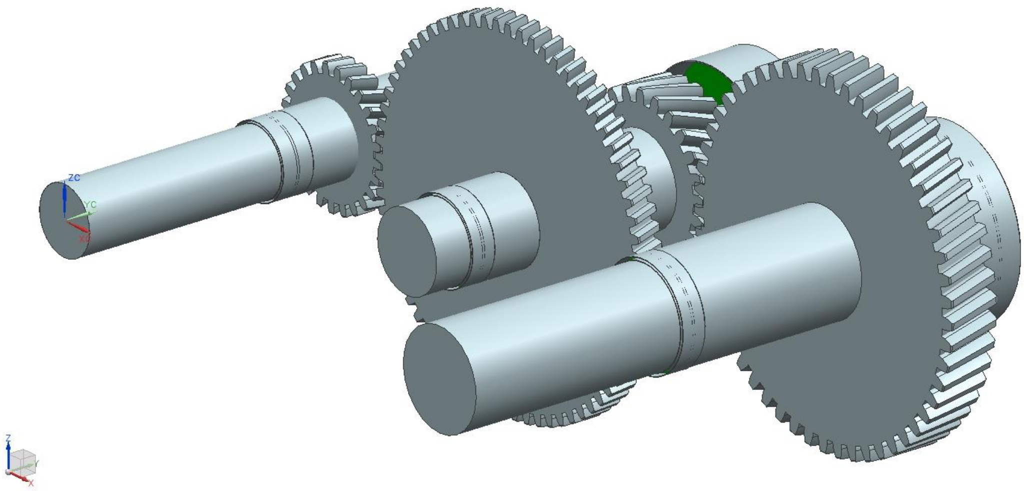

Figure 6. 18 Completed Assembly

The finished gearbox from the front view will be like this:

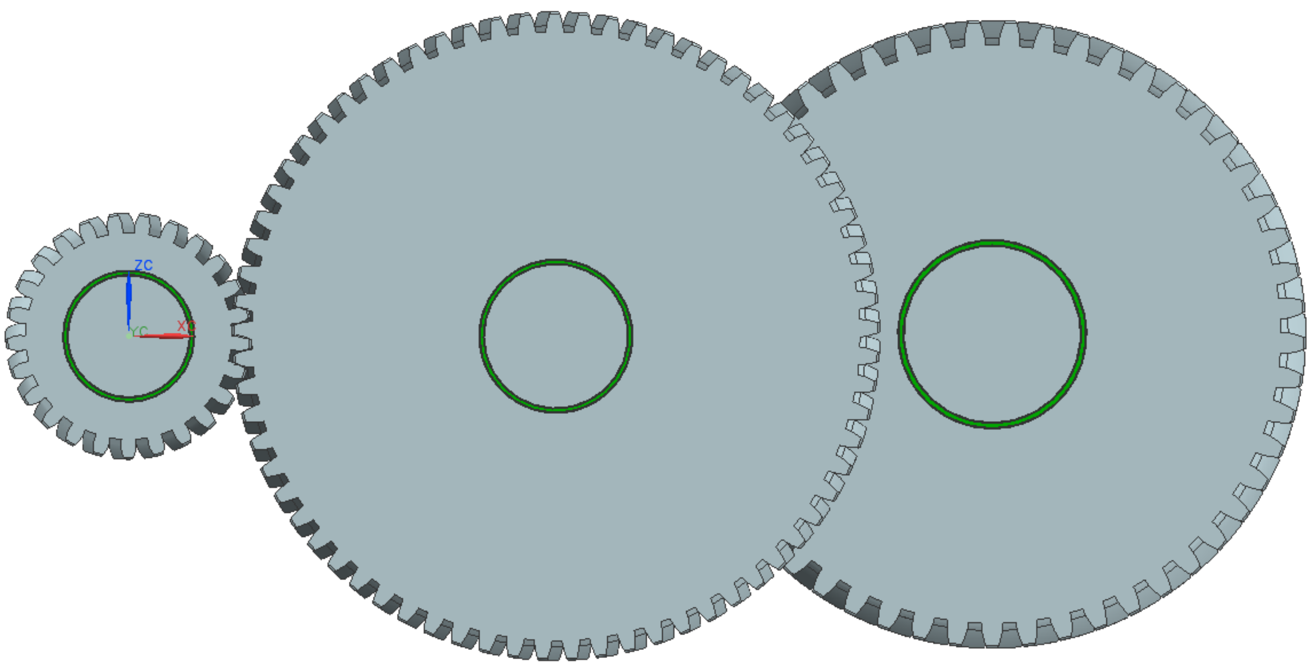

Figure 6. 19 Front View of the Assembly

7. Results

The code implemented in this thesis, can obtain infinite number of gearboxes and configurations. To show the results of the code, some examples of the code are shown below:

Example 1:

Input Data:

Table 7. 1 Gearbox Information

Input Torque	$300\ (Nm)$
Input Speed	$1200\ (rpm)$
Output Torque	$37800\ (Nm)$
Shock Level : Driving Machine	Medium Shocks
Shock Level : Driven Machine	Hard Shocks
Duty Cycle	$10\ hrs./day$
Configuration	Parallel - Same Sided input and output shafts
Gear Type	Cylindrical – Helical gear

Table 7. 2 Gear Material Information

Material type	Case-Hardened Wrought Steel
Brinell Hardness(HB)	600
Allowable stress number for contact (σ_{Hlim})	$1500 * 10^6\ \ (N/m^2)$
Nominal Stress number for bending (σ_{Flim})	$480 * 10^6\ \ (N/m^2)$
Young's Modulus	$2.1 * 10^{11}\ (Pa)$
Poisson Ratio	0.33

Table 7. 3 Shaft

Shaft Data	
Input Shaft Exceedance	$150\ (mm)$
Output Shaft Exceedance	$150\ (mm)$
Minimum Safety Factor	1.5
Shaft Material Data	
Young Modulus	$2.1 * 10^{11}\ (Pa)$
Poisson Ratio	0.3
Yield Strength ($R_{p0.2}$)	$700 * 10^6\ (Pa)$

Output Data:

Table 7. 4 General Output Data

Number of Stages	4
Output speed	8.9418 (rpm)

Table 7. 5 Cylindrical Gear Data

	First Pair	Second Pair	Third Pair	Fourth Pair
Gear Ratio	4.8148	3.8076	3	2.44
Input Torque (Nm)	300	1444.44	5500	16500
Input Speed	1200	249.23	65.4545	21.8181
Module (mm)	3	2	10	16
Number of teeth - Pinion	27	26	26	25
Number of teeth - Gear	130	99	78	61
Width (mm)	25	45	45	55
Pitch Diameter – Pinion (mm)	83.9364	134.7127	269.4254	414.50
Pitch Diameter – Gear (mm)	404.13	512.9446	808.2763	1011.4
Pressure Angle	14.5	14.5	14.5	14.5
Helix Angle	15.2	15.2	15.2	15.2

Table 7. 6 Shaft Dimensions

	Input Shaft	Middle Shaft 1	Middle Shaft 2	Middle Shaft 3	Output Shaft
Diameter 1	55	75	75	140	170
Diameter 2	60	80	80	150	180
Diameter 3	65	85	85	160	190
Diameter 4	60	80	80	150	180
Diameter 5	-	75	75	140	-
Length 1	198	52	52	65	226
Length 2	65	65	55	55	140
Length 3	100	100	135	70	115
Length 4	48	65	65	130	76
Length 5	-	52	52	65	-

Table 7. 7 Bearing Data - Part 1

	Left Bearing on Input Shaft	Right Bearing on Input Shaft	Left Bearing on First Middle Shaft	Right Bearing of First Middle Shaft
Type of Bearing	A rullini e gabbia	A rullini e gabbia	Obliqui a due corone di sfere	Orientabili a rulli
d (mm)	55	60	75	75
D (mm)	60	65	130	160
B (mm)	27	30	41.3	37
Dynamic Load Rating (kN)	35.8	41.3	95.6	1.5
Static Load Rating (kN)	96.5	116	88	285
Fatigue Load Limit (kN)	12	14.3	3.75	34.5
Designation	K55x60x27	K60x65x30	3215A	*21315E

Table 7. 8 Bearing Data - Part 2

	Left Bearing on Second Middle Shaft	Right Bearing on Second Middle Shaft	Left Bearing on Third Middle Shaft	Right Bearing of Third Middle Shaft
Type of Bearing	Orientabili a rulli	A rullini e gabbia	Radiale a sfere	Radiale a sfere
d (mm)	75	75	140	140
D (mm)	160	81	250	250
B (mm)	37	30	42	42
Dynamic Load Rating (kN)	285	50.1	199	199
Static Load Rating (kN)	325	143	212	212
Fatigue Load Limit (kN)	34.5	18	6.4	6.4
Designation	*21315E	K75x81x30	7228BCBM	7228BCBM

Table 7. 9 Bearing Data - Part 3

	Left Bearing on Output Shaft	Right Bearing on Output Shaft
Type of Bearing	Radiale a sfere	Obliqui ad una corona di sfere
d (mm)	170	180
D (mm)	260	280
B (mm)	42	46
Dynamic Load Rating (kN)	172	195
Static Load Rating (kN)	204	240
Fatigue Load Limit (kN)	5.85	6.7
Designation	7034BGM	7036BGM

The final CAD design of the gearbox is shown in figure 7.1 and 7.2:

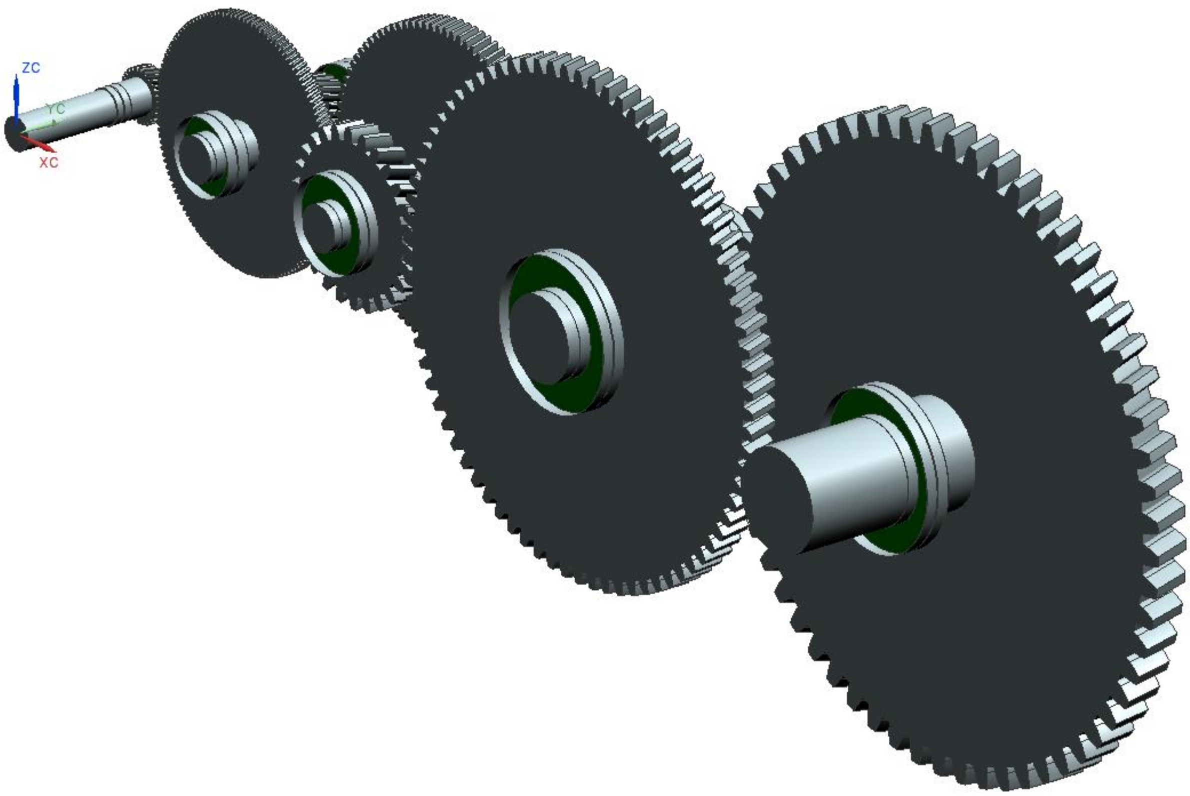

Figure 7. 1 Final CAD Design of Gearbox – Example 1

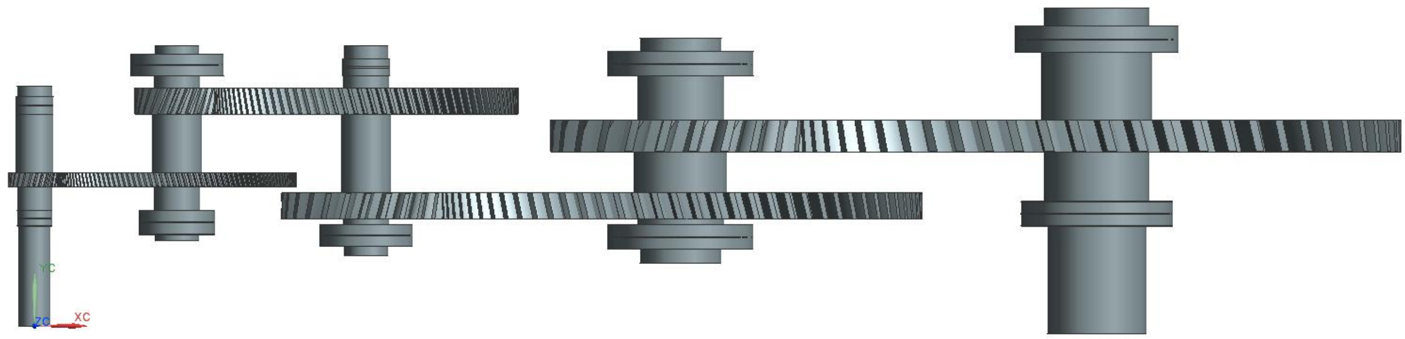

Figure 7. 2 Top View of the Final Design – Example 1

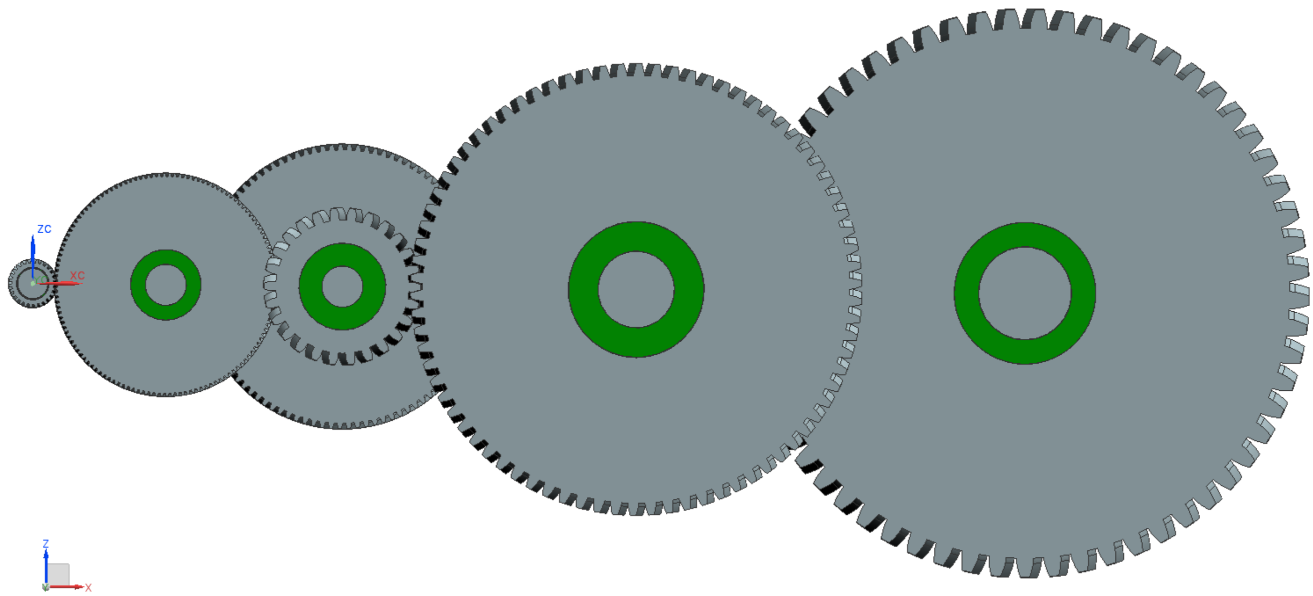

Figure 7. 3 Front View of the Final Design – Example 1

Example 2:

Input Data:

Table 7. 10 Gearbox Information

Input Torque	300 (Nm)
Input Speed	1200 (rpm)
Output Torque	7800 (Nm)
Shock Level : Driving Machine	Medium Shocks
Shock Level : Driven Machine	Hard Shocks
Duty Cycle	10 $hrs./day$
Configuration	Perpendicular
Gear Type	Bevel Gear / Cylindrical – Helical gear

Table 7. 11 Gear Material Information

Material type	Case-Hardened Wrought Steel
Brinell Hardness(HB)	600
Allowable stress number for contact (σ_{Hlim})	$1500 * 10^6\ (N/m^2)$
Nominal Stress number for bending (σ_{Flim})	$480 * 10^6\ (N/m^2)$
Young's Modulus	$2.1 * 10^{11}\ (Pa)$
Poisson Ratio	0.33

Table 7. 12 Shaft

Shaft Data	
Input Shaft Exceedance	150 (mm)
Output Shaft Exceedance	150 (mm)
Minimum Safety Factor	1.5
Shaft Material Data	
Young Modulus	$2.1 * 10^{11}\ (Pa)$
Poisson Ratio	0.3
Yield Strength $(R_{p0.2})$	$700 * 10^6\ (Pa)$

Output Data:

General Data

Number of Stages	3
Output speed	50.4 (rpm)

Table 7. 13 Bevel Gear Data

	Pinion	Gear
Gear Ratio	2.20	
Input Torque (Nm)	300	
Input Speed	1200	
Outer Transverse Module (mm)	8	
Number of Teeth	13	48
Face Width	24	24
Outer Reference Diameter	104	384
Pitch Angle	15.16	74.84
Face Angle	19.26	76.48
Root Angle	13.51	70.73
Pressure Angle	20	20

Table 7. 14 Cylindrical Gear Data

	First Pair	Second Pair
Gear Ratio	2.92	2.20
Input Torque (Nm)	1107.69	3234.46
Input Speed	325	111.3
Module (mm)	5	8
Number of teeth - Pinion	25	24
Number of teeth - Gear	73	53
Width (mm)	45	55
Pitch Diameter – Pinion (mm)	129.53	198.96
Pitch Diameter – Gear (mm)	378.23	439.37
Pressure Angle	14.5	14.5
Helix Angle	15.2	15.2

Table 7. 15 Shaft Dimensions

	Input Shaft	Middle Shaft 1	Middle Shaft 2	Output Shaft
Diameter 1	55	75	65	110
Diameter 2	60	80	70	120
Diameter 3	65	85	75	130
Diameter 4	60	80	70	120
Diameter 5	-	75	65	-
Length 1	198	52	50	208
Length 2	39	75	20	75
Length 3	20	100	155	20
Length 4	75	75	75	58
Length 5	-	52	50	-

Table 7. 16 Bearing Data – Part 1

	Left Bearing on Input Shaft	Right Bearing on Input Shaft	Left Bearing on First Middle Shaft	Right Bearing of First Middle Shaft
Type of Bearing	A rullini e gabbia	A rullini e gabbia	A rullini e gabbia	Obliqui ad una corona di sfere
d (mm)	55	60	75	75
D (mm)	60	65	81	160
B (mm)	27	20	20	37
Dynamic Load Rating (kN)	35.8	28.1	35.8	133
Static Load Rating (kN)	96.5	72	93	106
Fatigue Load Limit (kN)	12	8.8	11.6	4.15
Designation	K55x60x27	K60x65x20	K75x81x20	7315BECBY

Table 7. 17 Bearing Data – Part 2

	Left Bearing on Second Middle Shaft	Right Bearing on Second Middle Shaft	Left Bearing on Output Shaft	Right Bearing of Output
Type of Bearing	A sfere a quattro punti di contatto	A rullini e gabbia	A rullini e gabbia	Obliqui ad una corona di sfere
d (mm)	65	65	110	120
D (mm)	140	73	117	215
B (mm)	33	30	24	40
Dynamic Load Rating (kN)	176	5.9	53.9	165
Static Load Rating (kN)	156	125	160	163
Fatigue Load Limit (kN)	6.55	15.6	18.6	5.3
Designation	*QJ313MA	K65x73x30	K110x117x24	7224BCBM

The final CAD design of the gearbox is shown in figures 7.3 and 7.4:

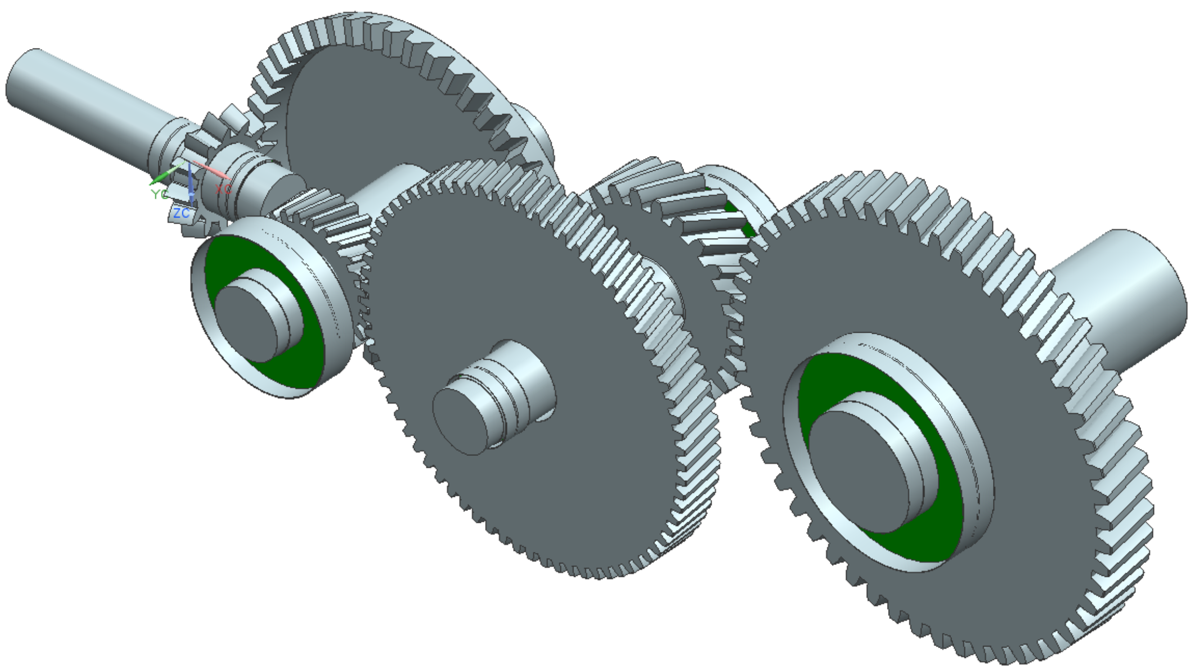

Figure 7. 4 Final CAD Design of the Gearbox - Example 2

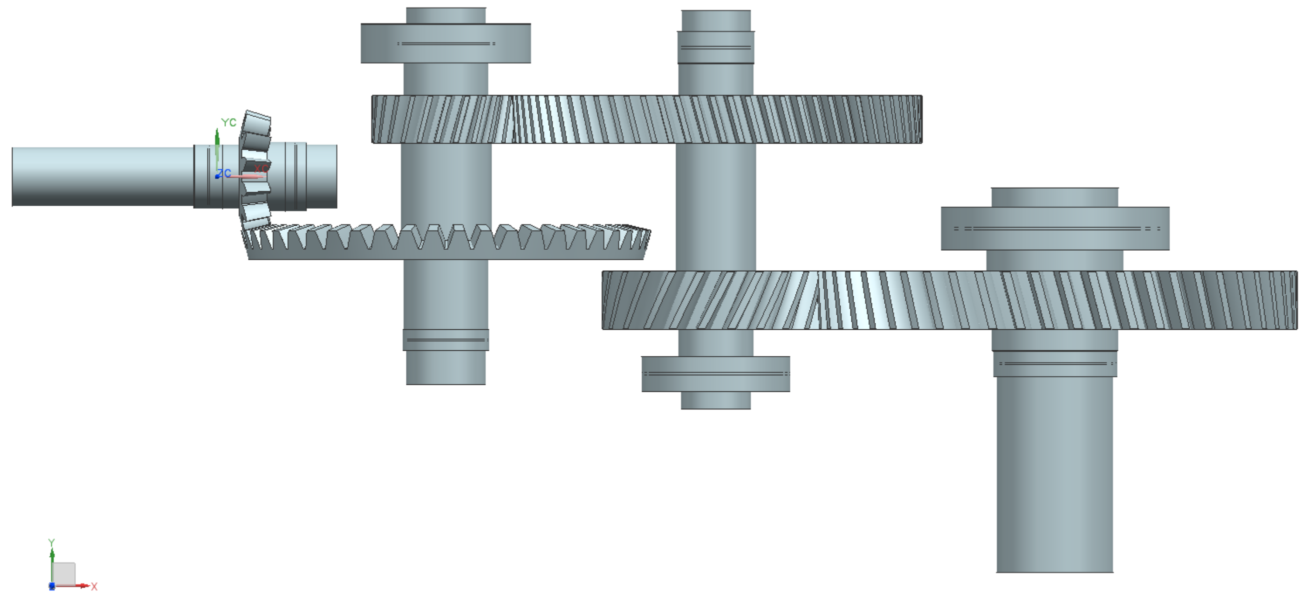

Figure 7. 5 Top View of the Final Design - Example 2

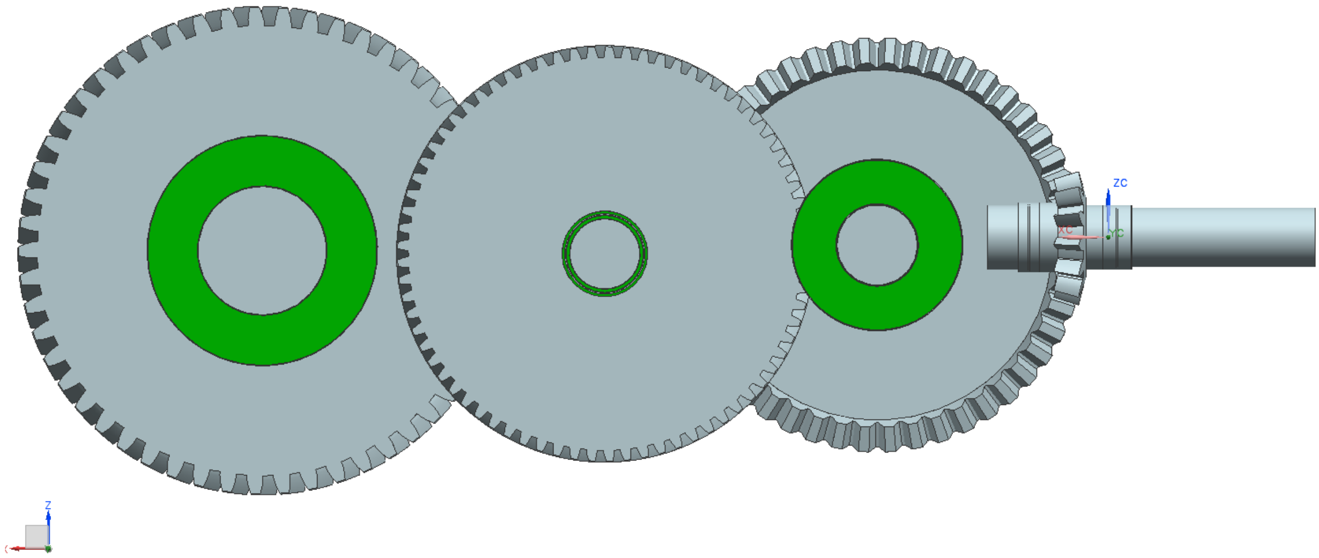

Figure 7. 6 Front View of the Final Design – Example 2

Example 3:

Input Data:

Table 7. 18 Gearbox Information

Input Torque	$300\ (Nm)$
Input Speed	$1200\ (rpm)$
Output Torque	$1800\ (Nm)$
Shock Level : Driving Machine	Uniform Shocks
Shock Level : Driven Machine	Uniform Shocks
Duty Cycle	$10\ hrs./day$
Configuration	Compound Reversed Gear Train
Gear Type	Cylindrical – Helical gear

Table 7. 19 Gear Material Information

Material type	Case-Hardened Wrought Steel
Brinell Hardness(HB)	600
Allowable stress number for contact (σ_{Hlim})	$1500 * 10^6\ (N/m^2)$
Nominal Stress number for bending (σ_{Flim})	$480 * 10^6\ (N/m^2)$
Young's Modulus	$2.1 * 10^{11}\ (Pa)$
Poisson Ratio	0.33

Table 7. 20 Shaft

Shaft Data	
Input Shaft Exceedance	$100\ (mm)$
Output Shaft Exceedance	$100\ (mm)$
Minimum Safety Factor	1.5
Shaft Material Data	
Young Modulus	$2.1 * 10^{11}\ (Pa)$
Poisson Ratio	0.3
Yield Strength ($R_{p0.2}$)	$700 * 10^6\ (Pa)$

Output Data:

Table 7. 21 General Data

Number of Stages	2
Output speed	200 (rpm)

Table 7. 22 Cylindrical Gear Data

	First Pair	Second Pair
Gear Ratio	2.44	2.44
Input Torque (Nm)	300	732
Input Speed	1200	491.8
Module (mm)	4	4
Number of teeth - Pinion	25	25
Number of teeth - Gear	61	61
Width (mm)	25	35
Pitch Diameter – Pinion (mm)	103.62	103.62
Pitch Diameter – Gear (mm)	252.84	252.84
Pressure Angle	25	25
Helix Angle	15.2	15.2

Table 7. 23 Shaft Dimensions (mm)

	Input Shaft	Middle Shaft	Output Shaft
Diameter 1	55	65	80
Diameter 2	60	70	85
Diameter 3	65	75	90
Diameter 4	60	70	85
Diameter 5	-	65	-
Length 1	148	50	42
Length 2	55	55	55
Length 3	20	200	20
Length 4	38	55	152
Length 5	-	50	-

Table 7. 24 Bearing Data – Part 1

	Left Bearing on Input Shaft	Right Bearing on Input Shaft	Left Bearing on Middle Shaft	Right Bearing of Middle Shaft
Type of Bearing	A rullini e gabbia	A rullini e gabbia	A rullini e gabbia	Radiali a due corone di sfere
d (mm)	55	60	65	65
D (mm)	60	65	70	140
B (mm)	20	30	20	48
Dynamic Load Rating (kN)	27	41.3	29.2	121
Static Load Rating (kN)	67	116	76.5	106
Fatigue Load Limit (kN)	8.15	14.3	9.3	4.5
Designation	K55x60x20	K60x65x30	K65x70x20	4313ATN9

Table 7. 25 Bearing Data - Part 2

	Left Bearing on Output Shaft	Right Bearing on Output Shaft
Type of Bearing	A rullini e gabbia	Radiali a due corone di sfere
d (mm)	80	85
D (mm)	88	150
B (mm)	30	36
Dynamic Load Rating (kN)	68.2	85
Static Load Rating (kN)	176	150
Fatigue Load Limit (kN)	22	36
Designation	K80x88x30	4217ATN9

The final CAD design of the gearbox is shown in figures 7.5 and 7.6:

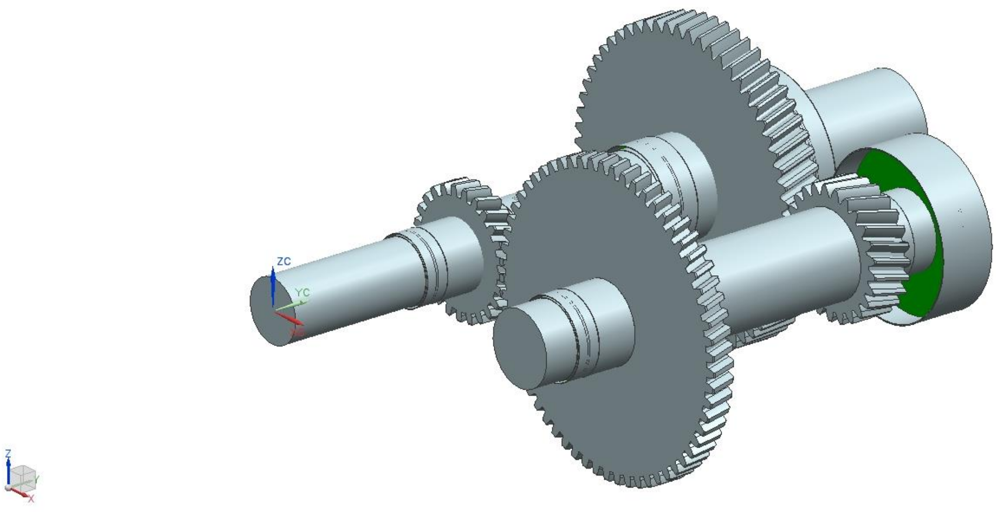

Figure 7. 7 Final Design of the Gearbox - Example 3

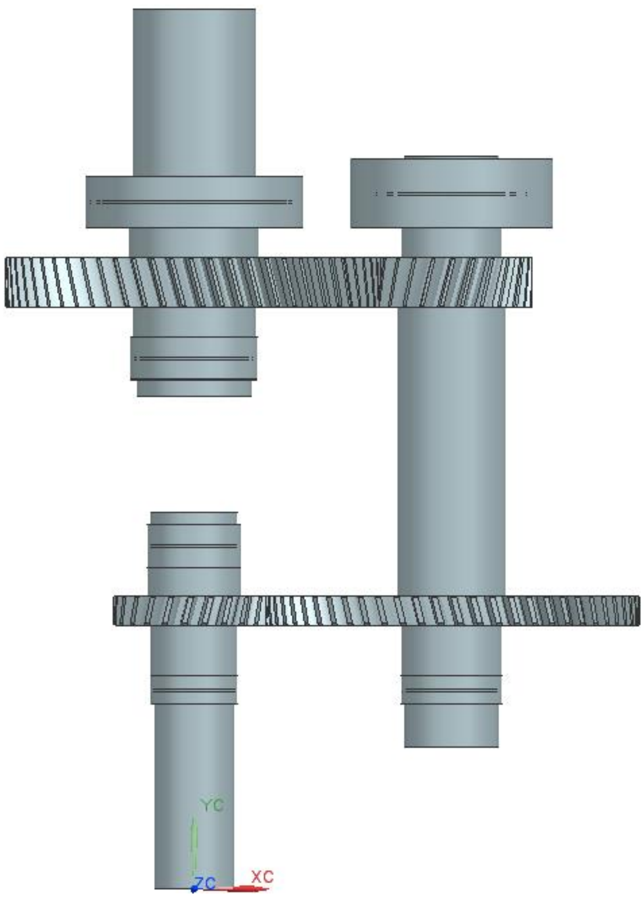

Figure 7. 8 Top View of the Final Design - Example 3

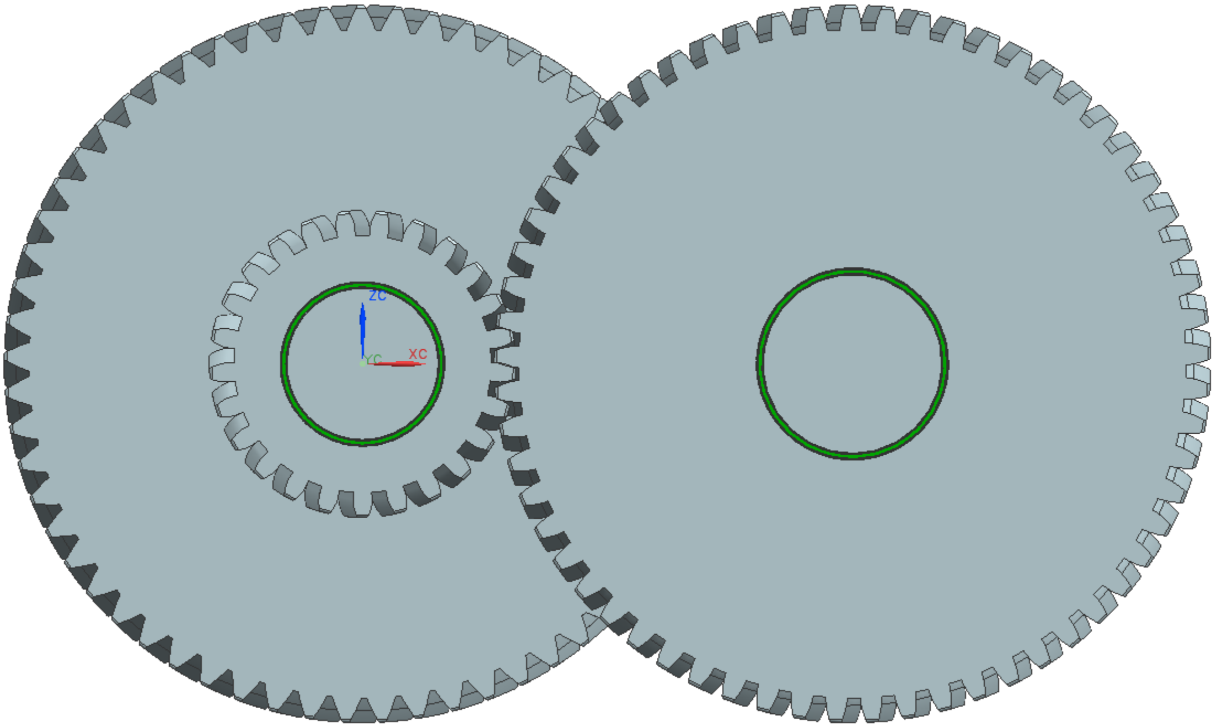

Figure 7. 9 Front View of the Final Design - Example 3

8. Conclusion and Future Developments

This project focused on the comprehensive design and analysis of an industrial gearbox, integrating various components and methodologies to ensure a robust and functional design. The key achievements and contributions of this project are summarized as follows:

1. **Design of Bevel Gears in MATLAB and NX Siemens:**
    - Complete design of bevel gears was carried out using MATLAB for computational modeling and NX Siemens for parametric design. This dual approach ensured precision and facilitated easy modifications.
2. **Modification of Cylindrical Gears:**
    - The design of cylindrical gears was pre-existing; however, modifications were made to enhance their performance and integration with the overall gearbox design.
3. **Static Analysis of the Shaft Using FEM:**
    - A static analysis of the shaft was performed using the Finite Element Method (FEM). The primary improvement involved correcting the strain calculation by incorporating all B matrices (derivatives of shape functions) for all six degrees of freedom. This correction significantly improved the accuracy of the analysis.
4. **Fixing the Bearing Code:**
    - The bearing part of the project involved code that was initially non-functional. Significant effort was invested in debugging and fixing this code, ensuring that the bearings operated correctly within the gearbox design.
5. **Integration and Synchronization of Components:**
    - The most crucial aspect of this project was the synchronization of all elements—gears, shaft, and bearings—into a cohesive system. A comprehensive code was developed to integrate these components, enabling the design of a complete industrial gearbox.
6. **Introduction of Multiple Gearbox Configurations:**
    - Four different types of gearbox configurations were introduced, allowing the code to design a specific configuration based on user needs. This flexibility is a significant advancement, catering to diverse industrial requirements.

Future Developments

While the project has successfully achieved its primary objectives, several areas for future development have been identified to further enhance the gearbox design and its applications. These include:

1. **Connection Mechanisms for Gears and Shaft:**
    - Future work can focus on designing robust connection mechanisms such as keys and splines to securely attach gears to the shaft. This will ensure reliable power transmission and operational stability.

2. **Dynamic Analysis of the Shaft:**
    - Conducting a dynamic analysis of the shaft is crucial to determine its fatigue strength under varying operational conditions. This analysis will help in predicting the lifespan of the shaft and identifying potential failure points.
3. **Introduction and Evaluation of Shaft Notches:**
    - Notches on the shaft can be introduced to evaluate their impact on stress concentration and overall performance. This will involve detailed stress analysis and testing to ensure that the notches do not compromise the shaft's integrity.
4. **Enhanced User Interface for Configuration Selection:**
    - Developing a more user-friendly interface for selecting gearbox configurations. This will make the design tool more accessible to users with varying levels of expertise.
5. **Automation and Optimization:**
    - Implementing automation and optimization algorithms to streamline the design process. This can include genetic algorithms or other optimization techniques to find the most efficient and effective gearbox designs based on predefined criteria.

Bibliography

1. SKF Group. (2016). Rolling Bearing Catalogue
2. ISO. (2007). International Standard 21771 – Cylindrical involute gears and gear pairs concept and geometry
3. ISO. (2005). International Standard 6336 – Calculation of load capacity of spur and helical Gears
4. ISO. (2007). International Standard 23509 – Bevel and hypoid gear geometry
5. ISO. (20014). International Standard 10300 – Calculation of load capacity of bevel gears
6. Richard G. Budynas, J. Keith Nisbett, Shigley's Mechanical Engineering Design