

# RobustOptFourthTrial

June 23, 2024

## 0.0.1 Robust Pricing Optimization - Two Products - Price Chosen within a Discrete Set of Values

### Packages Import

```
[1]: import numpy as np
import pandas as pd
import gurobipy as grb
import matplotlib.pyplot as plt
from sklearn.metrics import mean_absolute_error
import statsmodels.api as sm
from scipy import stats as st
```

Generate both samples from linear demand functions but with different parameters, they represent sold quantities of dissimilar products made by the same company (production costs are not equal indeed).

```
[2]: # PRODUCT 1
np.random.seed(42)
n = 20
var = 1
prod_cost = 10
# demand function's real slope and intercept
alpha_real_1 = 200
beta_real_1 = -5
# lower and upper bounds
lower_p1 = prod_cost
upper_p1 = -alpha_real_1 / beta_real_1 # upper bound to maintain the demand
↳ greater or equal to zero, below this value it will be negative

eps= np.random.normal(0, var, size=(n, 2))
price_1 = np.random.uniform(lower_p1, upper_p1, n)
demand_1 = alpha_real_1 + beta_real_1 * price_1 + eps[:, 0]
```

```
[3]: # PRODUCT 2
np.random.seed(42)
prod_cost_2 = 15
# demand function's real slope and intercept
alpha_real_2 = 350
```

```

beta_real_2 = -7
# lower and upper bounds
lower_p2 = prod_cost_2
upper_p2 = -alpha_real_2 / beta_real_2 # upper bound to maintain the demand
↳greater or equal to zero, below this value it will be negative

price_2 = np.random.uniform(lower_p2, upper_p2, n)
demand_2 = alpha_real_2 + beta_real_2 * price_2 + eps[:, 1]

```

**Robust solution nominal problem** The max-min optimization is actually the simple max because there is no uncertainty in the nominal case, all the coefficients are known. The optimal objective value obtained will be the benchmark to which compare the other results since it would be the best revenue possible. Since this time the price has to be selected within a discrete set of values, we set parameter vtype equal to “I” which stands for Integer.

```

[4]: try:
# create the model
    model = grb.Model()
# create decision variables
    p = model.addVars(2, ub=[upper_p1, upper_p2], lb=[lower_p1, lower_p2],
↳vtype='I', name='p')
# set objective function
    expr = (alpha_real_1 + beta_real_1*p[0])*p[0]+(alpha_real_2 +
↳beta_real_2*p[1])*p[1]
    model.setObjective(expr, sense=grb.GRB.MAXIMIZE)
# solve the problem
    model.optimize()
    for v in model.getVars():
        print('%s %g' % (v.varName, v.x))
    print('Obj: %g' % model.objVal)
except grb.GurobiError as e:
    print('Error code' + str(e))
except AttributeError:
    print('Encountered an attribute error')
obj_nominal = model.objVal

```

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Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0x7ec0ea4a  
Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 4e+02]

QObjective range [1e+01, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5175.0000000

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 6375 5175

Optimal solution found (tolerance 1.00e-04)

Best objective 6.375000000000e+03, best bound 6.375000000000e+03, gap 0.0000%

p[0] 20

p[1] 25

Obj: 6375

```
[5]: # DATAFRAME FOR LINEAR REGRESSION 1
data_1 = pd.DataFrame({'price': price_1, 'demand': demand_1})
#print(data_1)
```

```
[6]: # DATAFRAME FOR LINEAR REGRESSION 2
data_2 = pd.DataFrame({'price': price_2, 'demand': demand_2})
#print(data_2)
```

```
[7]: conf_level = 0.05 # significance level
X_1 = pd.DataFrame({'price': data_1['price'].copy()})
y_1 = pd.DataFrame({'demand': data_1['demand'].copy()})
X_1['constant'] = 1
# LINEAR REGRESSION 1
np.random.seed(42)
returns_1 = np.linalg.lstsq(X_1, y_1, rcond=None)
params_1 = returns_1[0]
alpha_1 = params_1[1]
beta_1 = params_1[0]
sum_sq_residuals_1 = returns_1[1] # sum of squared residuals
dof_1 = X_1.shape[0] - len(params_1) # degrees of freedom
mse_1 = sum_sq_residuals_1 / dof_1 # mean squared error
cov_1 = mse_1 * np.diagonal(np.linalg.inv(X_1.T @ X_1)) # covariance matrix
se_1 = np.sqrt(cov_1) # standard errors
t_1 = st.t.ppf(1 - (conf_level / 2), dof_1) # t-distribution quantile function
```

```

delta_1 = t_1 * se_1 # margin of error
# confidence interval 95%
alpha_up_1 = alpha_1 + delta_1[0]
alpha_low_1 = alpha_1 - delta_1[0]
beta_up_1 = beta_1 + delta_1[1]
beta_low_1 = beta_1 - delta_1[1]

```

```

[8]: X_2 = pd.DataFrame({'price': data_2['price'].copy()})
y_2 = pd.DataFrame({'demand': data_2['demand'].copy()})
X_2['constant'] = 1
# LINEAR REGRESSION 2
np.random.seed(42)
returns_2 = np.linalg.lstsq(X_2, y_2, rcond=None)
params_2 = returns_2[0]
alpha_2 = params_2[1]
beta_2 = params_2[0]
sum_sq_residuals_2 = returns_2[1] # sum of squared residuals
dof_2 = X_2.shape[0] - len(params_2) # degrees of freedom
mse_2 = sum_sq_residuals_2 / dof_2 # mean squared error
cov_2 = mse_2 * np.diagonal(np.linalg.inv(X_2.T @ X_2)) # covariance matrix
se_2 = np.sqrt(cov_2) # standard errors
t_2 = st.t.ppf(1 - (conf_level / 2), dof_2) # t-distribution quantile function
delta_2 = t_2 * se_2 # margin of error
# confidence interval 95%
alpha_up_2 = alpha_2 + delta_2[0]
alpha_low_2 = alpha_2 - delta_2[0]
beta_up_2 = beta_2 + delta_2[1]
beta_low_2 = beta_2 - delta_2[1]

```

Uncertainty set construction through uniform sampling, we extract  $n$  realizations of uncertain parameters  $\alpha$  and  $\beta$  from their interval estimates for each product type.

```

[9]: np.random.seed(42)
n_samples = 50
alpha_sim_1 = np.random.uniform(alpha_low_1, alpha_up_1, size=n_samples)
alpha_sim_2 = np.random.uniform(alpha_low_2, alpha_up_2, size=n_samples)
beta_sim_1 = np.random.uniform(beta_low_1, beta_up_1, size=n_samples)
beta_sim_2 = np.random.uniform(beta_low_2, beta_up_2, size=n_samples)

```

Now we define some attributes for the uncertainty set: mean, standard deviation, maximum and minimum of scenarios

```

[10]: alpha_min_1 = min(alpha_sim_1)
alpha_max_1 = max(alpha_sim_1)
beta_min_1 = min(beta_sim_1)
beta_max_1 = max(beta_sim_1)
alpha_min_2 = min(alpha_sim_2)
alpha_max_2 = max(alpha_sim_2)

```

```

beta_min_2 = min(beta_sim_2)
beta_max_2 = max(beta_sim_2)
alpha_mean_1 = sum(alpha_sim_1)/n_samples
alpha_mean_2 = sum(alpha_sim_2)/n_samples
beta_mean_1 = sum(beta_sim_1)/n_samples
beta_mean_2 = sum(beta_sim_2)/n_samples
alpha_std_1 = np.sqrt(sum((alpha_sim_1-alpha_mean_1)**2)/(n_samples-1))
alpha_std_2 = np.sqrt(sum((alpha_sim_2-alpha_mean_2)**2)/(n_samples-1))
beta_std_1 = np.sqrt(sum((beta_sim_1-beta_mean_1)**2)/(n_samples-1))
beta_std_2 = np.sqrt(sum((beta_sim_2-beta_mean_2)**2)/(n_samples-1))

```

Budget of uncertainty indicates how great will be the conservatism of the optimization problem's solution. We create an array of 11 values of delta to be tested

```

[11]: delta_min = 0
delta_max_alpha = max(max(np.abs(alpha_min_1-alpha_mean_1)/alpha_std_1, np.
↳abs(alpha_max_1-alpha_mean_1)/alpha_std_1),
                       max(np.abs(alpha_min_2-alpha_mean_2)/alpha_std_2, np.
↳abs(alpha_max_2-alpha_mean_2)/alpha_std_2))
delta_max_beta = max(max(np.abs(beta_min_1-beta_mean_1)/beta_std_1, np.
↳abs(beta_max_1-beta_mean_1)/beta_std_1),
                      max(np.abs(beta_min_2-beta_mean_2)/beta_std_2, np.
↳abs(beta_max_2-beta_mean_2)/beta_std_2))
delta_max = max(delta_max_alpha, delta_max_beta)
delta_test = np.linspace(delta_min, delta_max, num=10)

```

```

[12]: print(delta_test)

```

```

[0.          0.20153702 0.40307405 0.60461107 0.8061481  1.00768512
 1.20922215 1.41075917 1.6122962  1.81383322]

```

To understand how uncertainty affects the objective function value, we solve the optimization problem many times extracting each iteration with uniform distribution the coefficients estimations.

```

[13]: # Monte Carlo simulations
np.random.seed(42)
n_scenarios = 50
alpha_scen_1 = np.random.uniform(alpha_min_1, alpha_max_1, size=n_scenarios)
beta_scen_1 = np.random.uniform(beta_min_1, beta_max_1, size=n_scenarios)
alpha_scen_2 = np.random.uniform(alpha_min_2, alpha_max_2, size=n_scenarios)
beta_scen_2 = np.random.uniform(beta_min_2, beta_max_2, size=n_scenarios)
obj_val = []
for i in range(n_scenarios):
    model = grb.Model()
    p = model.addVars(2, ub=[upper_p1, upper_p2], lb=[lower_p1, lower_p2], u
↳vtype='I', name='p')
    expr = (alpha_scen_1[i] + beta_scen_1[i]*p[0])*p[0] + (alpha_scen_2[i] + u
↳beta_scen_2[i]*p[1])*p[1]

```

```
model.setObjective(expr, sense=grb.GRB.MAXIMIZE)
model.optimize()
obj_val.append(model.objVal)
```

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xad7ae762

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [8e+00, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5546.4292269

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 7813.52 5546.43

Optimal solution found (tolerance 1.00e-04)

Best objective 7.813515625651e+03, best bound 7.813515625651e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x49619aa6

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [9e+00, 2e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]  
Found heuristic solution: objective 5049.0735431  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6187.97 5049.07

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.187968305674e+03, best bound 6.187968305674e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0x9395df66  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:  
Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [8e+00, 2e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]  
Found heuristic solution: objective 5020.5219141  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6253.54 5020.52

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.253542564063e+03, best bound 6.253542564063e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set

[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x71045f83

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [8e+00, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5245.4189829

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 6757.5 5245.42

Optimal solution found (tolerance 1.00e-04)

Best objective 6.757503211328e+03, best bound 6.757503211328e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0

(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set

[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x1102f284

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [1e+01, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5517.5417705

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.05 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 7607.82 5517.54

Optimal solution found (tolerance 1.00e-04)

Best objective 7.607819633127e+03, best bound 7.607819633127e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x1724634c

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [8e+00, 2e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5082.2251903

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 6370.11 5082.23

Optimal solution found (tolerance 1.00e-04)

Best objective 6.370113018573e+03, best bound 6.370113018573e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xbb40926c

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]  
 QObjective range [1e+01, 1e+01]  
 Bounds range [1e+01, 5e+01]  
 RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5189.9733888  
 Presolve removed 0 rows and 2 columns  
 Presolve time: 0.00s  
 Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
 Thread count was 1 (of 8 available processors)

Solution count 2: 6399.22 5189.97

Optimal solution found (tolerance 1.00e-04)  
 Best objective 6.399223796788e+03, best bound 6.399223796788e+03, gap 0.0000%  
 Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
 [SSE2|AVX|AVX2|AVX512]  
 Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
 Model fingerprint: 0xf5b8a199  
 Model has 2 quadratic objective terms  
 Variable types: 0 continuous, 2 integer (0 binary)  
 Coefficient statistics:

Matrix range [0e+00, 0e+00]  
 Objective range [2e+02, 3e+02]  
 QObjective range [1e+01, 1e+01]  
 Bounds range [1e+01, 5e+01]  
 RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5275.9458623  
 Presolve removed 0 rows and 2 columns  
 Presolve time: 0.00s  
 Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
 Thread count was 1 (of 8 available processors)

Solution count 2: 6676.74 5275.95

Optimal solution found (tolerance 1.00e-04)  
 Best objective 6.676740411584e+03, best bound 6.676740411584e+03, gap 0.0000%  
 Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0x7b96264c  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 2e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 4884.6121098  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 5573.07 4884.61

Optimal solution found (tolerance 1.00e-04)  
Best objective 5.573069593336e+03, best bound 5.573069593336e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0x5d994d4d  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5280.8792803  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6688.65 5280.88

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.688653247921e+03, best bound 6.688653247921e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0x68c56827  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5049.2414236  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6023.1 5049.24

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.023095172439e+03, best bound 6.023095172439e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0xaf143704  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
 Objective range [2e+02, 3e+02]  
 QObjective range [1e+01, 1e+01]  
 Bounds range [1e+01, 5e+01]  
 RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5203.8821180  
 Presolve removed 0 rows and 2 columns  
 Presolve time: 0.00s  
 Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
 Thread count was 1 (of 8 available processors)

Solution count 2: 6435.83 5203.88

Optimal solution found (tolerance 1.00e-04)  
 Best objective 6.435828119797e+03, best bound 6.435828119797e+03, gap 0.0000%  
 Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
 [SSE2|AVX|AVX2|AVX512]  
 Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
 Model fingerprint: 0x4bd37488  
 Model has 2 quadratic objective terms  
 Variable types: 0 continuous, 2 integer (0 binary)  
 Coefficient statistics:

Matrix range [0e+00, 0e+00]  
 Objective range [2e+02, 3e+02]  
 QObjective range [9e+00, 1e+01]  
 Bounds range [1e+01, 5e+01]  
 RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5329.6252069  
 Presolve removed 0 rows and 2 columns  
 Presolve time: 0.00s  
 Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
 Thread count was 1 (of 8 available processors)

Solution count 2: 6962.36 5329.63

Optimal solution found (tolerance 1.00e-04)  
 Best objective 6.962356637783e+03, best bound 6.962356637783e+03, gap 0.0000%  
 Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0xf4c05115  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:  
Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5157.4350208  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6304.06 5157.44

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.304062122663e+03, best bound 6.304062122663e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0x02f9be68  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:  
Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 2e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 4836.9059376  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 5520.43 4836.91

Optimal solution found (tolerance 1.00e-04)  
Best objective 5.520429362526e+03, best bound 5.520429362526e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0xc252ebbf  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5402.8423086  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 7134.95 5402.84

Optimal solution found (tolerance 1.00e-04)  
Best objective 7.134946652168e+03, best bound 7.134946652168e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0xc9123ec1  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 2e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 4963.0869438

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 5767.67 4963.09

Optimal solution found (tolerance 1.00e-04)

Best objective 5.767666593152e+03, best bound 5.767666593152e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xfdc36883

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [9e+00, 2e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5017.9955138

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 6140.39 5018

Optimal solution found (tolerance 1.00e-04)

Best objective 6.140387509840e+03, best bound 6.140387509840e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0

(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x6ee05dfa

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [1e+01, 2e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 4757.8553336

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 5315.49 4757.86

Optimal solution found (tolerance 1.00e-04)

Best objective 5.315488615087e+03, best bound 5.315488615087e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0

(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x0c90b3c4

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [8e+00, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5334.7744060

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 7080.85 5334.77

Optimal solution found (tolerance 1.00e-04)  
Best objective 7.080848654600e+03, best bound 7.080848654600e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0xf897a719  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[2e+02, 3e+02]
QObjective range	[9e+00, 1e+01]
Bounds range	[1e+01, 5e+01]
RHS range	[0e+00, 0e+00]

Found heuristic solution: objective 5346.7065768

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.05 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6992.36 5346.71

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.992361063280e+03, best bound 6.992361063280e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0xeaec9fa5  
Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 2e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 4768.7638639

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 5363.27 4768.76

Optimal solution found (tolerance 1.00e-04)

Best objective 5.363268740072e+03, best bound 5.363268740072e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0

(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xbc01390a

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5062.5826630

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 6019.37 5062.58

Optimal solution found (tolerance 1.00e-04)

Best objective 6.019370389683e+03, best bound 6.019370389683e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x2d45c4fb

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [9e+00, 2e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5047.9383436

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 6214.11 5047.94

Optimal solution found (tolerance 1.00e-04)

Best objective 6.214110664078e+03, best bound 6.214110664078e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xa46e02b2

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [9e+00, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5309.1690651

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6847.44 5309.17

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.847442871987e+03, best bound 6.847442871987e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0xb67fa205  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [9e+00, 2e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 4993.9897668  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6040.44 4993.99

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.040443268807e+03, best bound 6.040443268807e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0x3326cf85

Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [9e+00, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5355.2849514  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 7020 5355.28

Optimal solution found (tolerance 1.00e-04)  
Best objective 7.019997603390e+03, best bound 7.019997603390e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0xc7ece898  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 4993.3821569  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 5835.53 4993.38

Optimal solution found (tolerance 1.00e-04)

Best objective 5.835528307815e+03, best bound 5.835528307815e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xac77784c

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[2e+02, 3e+02]
QObjective range	[1e+01, 1e+01]
Bounds range	[1e+01, 5e+01]
RHS range	[0e+00, 0e+00]

Found heuristic solution: objective 5430.0665389

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 7254.17 5430.07

Optimal solution found (tolerance 1.00e-04)

Best objective 7.254171959320e+03, best bound 7.254171959320e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xfcd7934d

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[2e+02, 3e+02]
QObjective range	[1e+01, 2e+01]
Bounds range	[1e+01, 5e+01]
RHS range	[0e+00, 0e+00]

Found heuristic solution: objective 4832.0895088

Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 5471.62 4832.09

Optimal solution found (tolerance 1.00e-04)  
Best objective 5.471621995953e+03, best bound 5.471621995953e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0xf4f320f8  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [8e+00, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5137.1843968  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6454.34 5137.18

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.454341742043e+03, best bound 6.454341742043e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xe3bc3e39  
 Model has 2 quadratic objective terms  
 Variable types: 0 continuous, 2 integer (0 binary)  
 Coefficient statistics:  
   Matrix range       [0e+00, 0e+00]  
   Objective range    [2e+02, 3e+02]  
   QObjective range   [9e+00, 2e+01]  
   Bounds range       [1e+01, 5e+01]  
   RHS range          [0e+00, 0e+00]  
 Found heuristic solution: objective 4928.2362546  
 Presolve removed 0 rows and 2 columns  
 Presolve time: 0.00s  
 Presolve: All rows and columns removed  
  
 Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
 Thread count was 1 (of 8 available processors)  
  
 Solution count 2: 5841.56 4928.24  
  
 Optimal solution found (tolerance 1.00e-04)  
 Best objective 5.841558687815e+03, best bound 5.841558687815e+03, gap 0.0000%  
 Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
 (22631.2))  
  
 CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
 [SSE2|AVX|AVX2|AVX512]  
 Thread count: 4 physical cores, 8 logical processors, using up to 8 threads  
  
 Optimize a model with 0 rows, 2 columns and 0 nonzeros  
 Model fingerprint: 0xc95fd1  
 Model has 2 quadratic objective terms  
 Variable types: 0 continuous, 2 integer (0 binary)  
 Coefficient statistics:  
   Matrix range       [0e+00, 0e+00]  
   Objective range    [2e+02, 3e+02]  
   QObjective range   [1e+01, 1e+01]  
   Bounds range       [1e+01, 5e+01]  
   RHS range          [0e+00, 0e+00]  
 Found heuristic solution: objective 5415.4782771  
 Presolve removed 0 rows and 2 columns  
 Presolve time: 0.00s  
 Presolve: All rows and columns removed  
  
 Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
 Thread count was 1 (of 8 available processors)  
  
 Solution count 2: 7196.56 5415.48

Optimal solution found (tolerance 1.00e-04)  
Best objective 7.196561477718e+03, best bound 7.196561477718e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: Oxa8cc9675

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[2e+02, 3e+02]
QObjective range	[1e+01, 1e+01]
Bounds range	[1e+01, 5e+01]
RHS range	[0e+00, 0e+00]

Found heuristic solution: objective 5324.7908769

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 6885.11 5324.79

Optimal solution found (tolerance 1.00e-04)

Best objective 6.885106626064e+03, best bound 6.885106626064e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: Ox764f575b

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[2e+02, 3e+02]
QObjective range	[1e+01, 2e+01]
Bounds range	[1e+01, 5e+01]
RHS range	[0e+00, 0e+00]

Found heuristic solution: objective 4958.8257491  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 5786.99 4958.83

Optimal solution found (tolerance 1.00e-04)  
Best objective 5.786991232868e+03, best bound 5.786991232868e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x4d881e85

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [1e+01, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5234.8020269

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6535.87 5234.8

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.535866099774e+03, best bound 6.535866099774e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0x3fb392a9  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:  
Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [9e+00, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]  
Found heuristic solution: objective 5431.6908117  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 7275.46 5431.69

Optimal solution found (tolerance 1.00e-04)  
Best objective 7.275457726274e+03, best bound 7.275457726274e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0x775759ba  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:  
Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [9e+00, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]  
Found heuristic solution: objective 5232.8656414  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6589.89 5232.87

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.589886784880e+03, best bound 6.589886784880e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0xa21ed837  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [8e+00, 1e+01]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5270.9673654  
Presolve removed 0 rows and 2 columns  
Presolve time: 0.00s  
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)  
Thread count was 1 (of 8 available processors)

Solution count 2: 6823.62 5270.97

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.823621940311e+03, best bound 6.823621940311e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros  
Model fingerprint: 0x05a7b666  
Model has 2 quadratic objective terms  
Variable types: 0 continuous, 2 integer (0 binary)  
Coefficient statistics:

Matrix range [0e+00, 0e+00]  
Objective range [2e+02, 3e+02]  
QObjective range [1e+01, 2e+01]  
Bounds range [1e+01, 5e+01]

```

    RHS range      [0e+00, 0e+00]
Found heuristic solution: objective 4983.3419753
Presolve removed 0 rows and 2 columns
Presolve time: 0.00s
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)
Thread count was 1 (of 8 available processors)

Solution count 2: 5890.46 4983.34

Optimal solution found (tolerance 1.00e-04)
Best objective 5.890463292226e+03, best bound 5.890463292226e+03, gap 0.0000%
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros
Model fingerprint: 0x0a249efa
Model has 2 quadratic objective terms
Variable types: 0 continuous, 2 integer (0 binary)
Coefficient statistics:
  Matrix range      [0e+00, 0e+00]
  Objective range   [2e+02, 3e+02]
  QObjective range  [1e+01, 2e+01]
  Bounds range      [1e+01, 5e+01]
  RHS range         [0e+00, 0e+00]
Found heuristic solution: objective 4803.3608772
Presolve removed 0 rows and 2 columns
Presolve time: 0.00s
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)
Thread count was 1 (of 8 available processors)

Solution count 2: 5414 4803.36

Optimal solution found (tolerance 1.00e-04)
Best objective 5.414003321250e+03, best bound 5.414003321250e+03, gap 0.0000%
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

```

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xca6bb572

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [9e+00, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5482.3095481

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 7468.95 5482.31

Optimal solution found (tolerance 1.00e-04)

Best objective 7.468951576957e+03, best bound 7.468951576957e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x9e98d54e

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [9e+00, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5495.0396242

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 7528.86 5495.04

Optimal solution found (tolerance 1.00e-04)

Best objective 7.528863388946e+03, best bound 7.528863388946e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xf0a4661b

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [1e+01, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5269.0397827

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 6675.39 5269.04

Optimal solution found (tolerance 1.00e-04)

Best objective 6.675394967272e+03, best bound 6.675394967272e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x13a13e35

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [9e+00, 1e+01]

```

    Bounds range      [1e+01, 5e+01]
    RHS range        [0e+00, 0e+00]
Found heuristic solution: objective 5114.8466008
Presolve removed 0 rows and 2 columns
Presolve time: 0.00s
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)
Thread count was 1 (of 8 available processors)

Solution count 2: 6339.75 5114.85

Optimal solution found (tolerance 1.00e-04)
Best objective 6.339751247535e+03, best bound 6.339751247535e+03, gap 0.0000%
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros
Model fingerprint: 0xe623e89c
Model has 2 quadratic objective terms
Variable types: 0 continuous, 2 integer (0 binary)
Coefficient statistics:
    Matrix range      [0e+00, 0e+00]
    Objective range   [2e+02, 3e+02]
    QObjective range  [1e+01, 1e+01]
    Bounds range      [1e+01, 5e+01]
    RHS range         [0e+00, 0e+00]
Found heuristic solution: objective 5060.1212180
Presolve removed 0 rows and 2 columns
Presolve time: 0.00s
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)
Thread count was 1 (of 8 available processors)

Solution count 2: 6079.68 5060.12

Optimal solution found (tolerance 1.00e-04)
Best objective 6.079680546558e+03, best bound 6.079680546558e+03, gap 0.0000%
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

```

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xbcc2b1b2

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [1e+01, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5323.6662395

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.03 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 6848.39 5323.67

Optimal solution found (tolerance 1.00e-04)

Best objective 6.848393855797e+03, best bound 6.848393855797e+03, gap 0.0000%

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0

(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x5a332ee7

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [1e+01, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5419.0812088

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 7199.43 5419.08

Optimal solution found (tolerance 1.00e-04)  
Best objective 7.199425677799e+03, best bound 7.199425677799e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0xc46b5506

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

QObjective range [1e+01, 1e+01]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Found heuristic solution: objective 5322.2269820

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)

Thread count was 1 (of 8 available processors)

Solution count 2: 6886.47 5322.23

Optimal solution found (tolerance 1.00e-04)  
Best objective 6.886470945282e+03, best bound 6.886470945282e+03, gap 0.0000%  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 2 columns and 0 nonzeros

Model fingerprint: 0x8b34a46b

Model has 2 quadratic objective terms

Variable types: 0 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [2e+02, 3e+02]

```

    QObjective range [1e+01, 1e+01]
    Bounds range     [1e+01, 5e+01]
    RHS range        [0e+00, 0e+00]
Found heuristic solution: objective 5267.5798237
Presolve removed 0 rows and 2 columns
Presolve time: 0.00s
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.02 seconds (0.00 work units)
Thread count was 1 (of 8 available processors)

Solution count 2: 6660.77 5267.58

Optimal solution found (tolerance 1.00e-04)
Best objective 6.660773234659e+03, best bound 6.660773234659e+03, gap 0.0000%

The value of simulation is then computed as the average of the optimal objective values for each
scenario under the deterministic threshold

```

```
[14]: obj_val = np.array(obj_val)
      sum(obj_val[obj_val < obj_nominal])/sum(obj_val < obj_nominal)
```

```
[14]: 5897.846116000538
```

```
[15]: fig, ax = plt.subplots(figsize=(8,5))
      ax.scatter(np.arange(0, sum(obj_val < obj_nominal)), obj_val[obj_val <
      ↪obj_nominal], color='red', label='MonteCarloObjectives')
      ax.hlines(xmin=0, xmax=sum(obj_val < obj_nominal), y=obj_nominal, color='black',
      ↪linestyle='--', label='DeterministicObjective')
      plt.grid()
      plt.legend()
      plt.savefig('MonteCarlosimulations_2.png')
      plt.show()
```

Now we see the difference with the robust approach, setting different budget of uncertainty. Modifying those quantities we will change the solution level of conservatism, depending on what are our needs and risk aversion.

```
[16]: # WORST CASE SCENARIO - REFORMULATION WITH STRONG DUALITY
      rob_obj_val = []
      for i in range(len(delta_test)):
          #create the robust model
          robust_model = grb.Model()
          #create decision variables
          z_r = robust_model.addVar(lb=0, vtype='C', name='z_r')
```

```

price_r = robust_model.addVars(2, lb=[lower_p1, lower_p2], ub=[upper_p1,
↪upper_p2], vtype='I', name='price_r')
mu_ra = robust_model.addVars(4, lb=0, vtype='C', name='mu_ra')
mu_rb = robust_model.addVars(4, lb=0, vtype='C', name='mu_rb')
gamma_ra = robust_model.addVars(4, lb=0, vtype='C', name='gamma_ra')
gamma_rb = robust_model.addVars(4, lb=0, vtype='C', name='gamma_rb')
#set objective function
robust_model.setObjective(z_r, sense=grb.GRB.MAXIMIZE)
#set constraints
robust_model.addConstr(mu_ra[0]*alpha_min_1 - mu_ra[1]*alpha_max_1 +
↪mu_ra[2]*alpha_min_2 - mu_ra[3]*alpha_max_2 +
mu_rb[0]*beta_min_1 - mu_rb[1]*beta_max_1 +
↪mu_rb[2]*beta_min_2 - mu_rb[3]*beta_max_2 -
gamma_ra[0]*(alpha_mean_1 +
↪delta_test[i]*alpha_std_1) + gamma_ra[1]*(alpha_mean_1 -
↪delta_test[i]*alpha_std_1) -
gamma_ra[2]*(alpha_mean_2 +
↪delta_test[i]*alpha_std_2) + gamma_ra[3]*(alpha_mean_2 -
↪delta_test[i]*alpha_std_2) -
gamma_rb[0]*(beta_mean_1 + delta_test[i]*beta_std_1)
↪+ gamma_rb[1]*(beta_mean_1 - delta_test[i]*beta_std_1) -
gamma_rb[2]*(beta_mean_2 + delta_test[i]*beta_std_2)
↪+ gamma_rb[3]*(beta_mean_2 - delta_test[i]*beta_std_2) >= z_r,name='c1')
robust_model.addConstr(price_r[0] - mu_ra[0] + mu_ra[1] + gamma_ra[0] -
↪gamma_ra[1] == 0, name='c2')
robust_model.addConstr(price_r[1] - mu_ra[2] + mu_ra[3] + gamma_ra[2] -
↪gamma_ra[3] == 0, name='c3')
robust_model.addConstr(price_r[0]**2 - mu_rb[0] + mu_rb[1] + gamma_rb[0] -
↪gamma_rb[1] == 0, name='c4')
robust_model.addConstr(price_r[1]**2 - mu_rb[2] + mu_rb[3] + gamma_rb[2] -
↪gamma_rb[3] == 0, name='c5')
#solve the problem
robust_model.params.NonConvex = 2
robust_model.optimize()
rob_obj_val.append(robust_model.objVal)

```

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 3 rows, 19 columns and 27 nonzeros

Model fingerprint: 0xdee46802

Model has 2 quadratic constraints

Variable types: 17 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [1e+00, 3e+02]  
QMatrix range [1e+00, 1e+00]  
QLMatrix range [1e+00, 1e+00]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 2 columns

Presolve time: 0.00s

Presolved: 9 rows, 17 columns, 53 nonzeros

Presolved model has 2 bilinear constraint(s)

Solving non-convex MIQCP

Variable types: 15 continuous, 2 integer (0 binary)

Root relaxation: objective 9.128134e+03, 10 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	9128.13445	0	3	- 9128.13445	-	-	-	0s
0	0	7150.83761	0	4	- 7150.83761	-	-	-	0s
0	0	6509.89714	0	4	- 6509.89714	-	-	-	0s
0	0	6501.60923	0	2	- 6501.60923	-	-	-	0s
0	0	6422.97016	0	4	- 6422.97016	-	-	-	0s
0	0	6411.69332	0	2	- 6411.69332	-	-	-	0s
0	0	6392.05431	0	4	- 6392.05431	-	-	-	0s
0	0	6382.04991	0	4	- 6382.04991	-	-	-	0s
0	0	6378.12712	0	3	- 6378.12712	-	-	-	0s
0	0	6377.10115	0	3	- 6377.10115	-	-	-	0s
H	0	0			6376.6242817	6376.96860	0.01%	-	0s

Cutting planes:

MIR: 1

Explored 1 nodes (27 simplex iterations) in 0.09 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 6376.62

Optimal solution found (tolerance 1.00e-04)

Best objective 6.376624281668e+03, best bound 6.376968600391e+03, gap 0.0054%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]  
 Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 3 rows, 19 columns and 27 nonzeros  
 Model fingerprint: 0xab1462b0  
 Model has 2 quadratic constraints  
 Variable types: 17 continuous, 2 integer (0 binary)

Coefficient statistics:  
 Matrix range [1e+00, 3e+02]  
 QMatrix range [1e+00, 1e+00]  
 QLMatrix range [1e+00, 1e+00]  
 Objective range [1e+00, 1e+00]  
 Bounds range [1e+01, 5e+01]  
 RHS range [0e+00, 0e+00]

Presolve time: 0.00s  
 Presolved: 9 rows, 19 columns, 57 nonzeros  
 Presolved model has 2 bilinear constraint(s)

Solving non-convex MIQCP

Variable types: 17 continuous, 2 integer (0 binary)

Root relaxation: objective 8.935482e+03, 12 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	8935.48160	0	3	- 8935.48160	-	-	-	0s
0	0	7026.97366	0	4	- 7026.97366	-	-	-	0s
0	0	6308.10839	0	4	- 6308.10839	-	-	-	0s
0	0	6302.11672	0	2	- 6302.11672	-	-	-	0s
0	0	6249.26121	0	4	- 6249.26121	-	-	-	0s
0	0	6239.15722	0	2	- 6239.15722	-	-	-	0s
0	0	6220.28539	0	4	- 6220.28539	-	-	-	0s
0	0	6214.09044	0	3	- 6214.09044	-	-	-	0s
0	0	6213.07526	0	3	- 6213.07526	-	-	-	0s
0	0	6209.92837	0	3	- 6209.92837	-	-	-	0s
H	0	0			6209.1583400	6209.38565	0.00%	-	0s

Cutting planes:  
 MIR: 1

Explored 1 nodes (29 simplex iterations) in 0.09 seconds (0.00 work units)  
 Thread count was 8 (of 8 available processors)

Solution count 1: 6209.16

Optimal solution found (tolerance 1.00e-04)

Best objective 6.209158340033e+03, best bound 6.209385653055e+03, gap 0.0037%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 3 rows, 19 columns and 27 nonzeros

Model fingerprint: 0xc1a59f00

Model has 2 quadratic constraints

Variable types: 17 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range	[1e+00, 3e+02]
QMatrix range	[1e+00, 1e+00]
QLMatrix range	[1e+00, 1e+00]
Objective range	[1e+00, 1e+00]
Bounds range	[1e+01, 5e+01]
RHS range	[0e+00, 0e+00]

Presolve time: 0.00s

Presolved: 9 rows, 19 columns, 57 nonzeros

Presolved model has 2 bilinear constraint(s)

Solving non-convex MIQCP

Variable types: 17 continuous, 2 integer (0 binary)

Root relaxation: objective 8.742829e+03, 12 iterations, 0.00 seconds (0.00 work  
units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	8742.82876	0	3	- 8742.82876	-	-	-	0s
0	0	6903.10971	0	4	- 6903.10971	-	-	-	0s
0	0	6106.31963	0	4	- 6106.31963	-	-	-	0s
0	0	6102.62421	0	2	- 6102.62421	-	-	-	0s
0	0	6075.55227	0	4	- 6075.55227	-	-	-	0s
0	0	6068.17812	0	2	- 6068.17812	-	-	-	0s
0	0	6060.16859	0	4	- 6060.16859	-	-	-	0s
0	0	6055.26575	0	3	- 6055.26575	-	-	-	0s
0	0	6054.54097	0	2	- 6054.54097	-	-	-	0s
0	0	6054.16752	0	3	- 6054.16752	-	-	-	0s
H	0	0			6053.3376317	6053.79459	0.01%	-	0s

Cutting planes:

MIR: 2

Explored 1 nodes (28 simplex iterations) in 0.08 seconds (0.00 work units)  
Thread count was 8 (of 8 available processors)

Solution count 1: 6053.34

Optimal solution found (tolerance 1.00e-04)

Best objective 6.053337631652e+03, best bound 6.053794586845e+03, gap 0.0075%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 3 rows, 19 columns and 27 nonzeros

Model fingerprint: 0x0554d9c7

Model has 2 quadratic constraints

Variable types: 17 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [1e+00, 3e+02]

QMatrix range [1e+00, 1e+00]

QLMatrix range [1e+00, 1e+00]

Objective range [1e+00, 1e+00]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Presolve time: 0.00s

Presolved: 9 rows, 19 columns, 57 nonzeros

Presolved model has 2 bilinear constraint(s)

Solving non-convex MIQCP

Variable types: 17 continuous, 2 integer (0 binary)

Root relaxation: objective 8.550176e+03, 12 iterations, 0.00 seconds (0.00 work  
units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	8550.17591	0	3	-	8550.17591	-	-	0s
0	0	6779.24575	0	4	-	6779.24575	-	-	0s
0	0	5959.17479	0	4	-	5959.17479	-	-	0s
0	0	5929.16528	0	4	-	5929.16528	-	-	0s

	0	0	5923.16338	0	2	-	5923.16338	-	-	0s	
	0	0	5914.16053	0	4	-	5914.16053	-	-	0s	
	0	0	5906.93063	0	3	-	5906.93063	-	-	0s	
	0	0	5906.65815	0	4	-	5906.65815	-	-	0s	
	0	0	5904.24718	0	3	-	5904.24718	-	-	0s	
	0	0	5903.90197	0	4	-	5903.90197	-	-	0s	
H	0	0					5902.6222973	5903.16793	0.01%	-	0s

Cutting planes:

MIR: 1

Explored 1 nodes (28 simplex iterations) in 0.08 seconds (0.00 work units)  
Thread count was 8 (of 8 available processors)

Solution count 1: 5902.62

Optimal solution found (tolerance 1.00e-04)

Best objective 5.902622297308e+03, best bound 5.903167925166e+03, gap 0.0092%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 3 rows, 19 columns and 27 nonzeros

Model fingerprint: 0x786d5237

Model has 2 quadratic constraints

Variable types: 17 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [1e+00, 3e+02]

QMatrix range [1e+00, 1e+00]

QLMatrix range [1e+00, 1e+00]

Objective range [1e+00, 1e+00]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Presolve time: 0.00s

Presolved: 9 rows, 19 columns, 57 nonzeros

Presolved model has 2 bilinear constraint(s)

Solving non-convex MIQCP

Variable types: 17 continuous, 2 integer (0 binary)

Root relaxation: objective 8.357523e+03, 12 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	8357.52306	0	3	- 8357.52306	-	-	-	0s
0	0	6655.38180	0	4	- 6655.38180	-	-	-	0s
0	0	5833.39175	0	4	- 5833.39175	-	-	-	0s
0	0	5793.45920	0	4	- 5793.45920	-	-	-	0s
0	0	5785.47269	0	2	- 5785.47269	-	-	-	0s
0	0	5773.49292	0	4	- 5773.49292	-	-	-	0s
0	0	5763.63095	0	3	- 5763.63095	-	-	-	0s
0	0	5763.50978	0	4	- 5763.50978	-	-	-	0s
0	0	5762.22289	0	3	- 5762.22289	-	-	-	0s
0	0	5762.02553	0	4	- 5762.02553	-	-	-	0s
H	0	0			5760.9496951	5761.56604	0.01%	-	0s

Cutting planes:

MIR: 1

Explored 1 nodes (29 simplex iterations) in 0.12 seconds (0.00 work units)  
Thread count was 8 (of 8 available processors)

Solution count 1: 5760.95

Optimal solution found (tolerance 1.00e-04)

Best objective 5.760949695058e+03, best bound 5.760949695058e+03, gap 0.0000%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0  
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set  
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 3 rows, 19 columns and 27 nonzeros

Model fingerprint: 0xf9566620

Model has 2 quadratic constraints

Variable types: 17 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [1e+00, 3e+02]

QMatrix range [1e+00, 1e+00]

QLMatrix range [1e+00, 1e+00]

Objective range [1e+00, 1e+00]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Presolve time: 0.00s

Presolved: 9 rows, 19 columns, 57 nonzeros

Presolved model has 2 bilinear constraint(s)

Solving non-convex MIQCP

Variable types: 17 continuous, 2 integer (0 binary)

Root relaxation: objective 8.164870e+03, 12 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work		
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time	
0	0	8164.87021	0	3	- 8164.87021	-	-	-	0s	
0	0	6531.51785	0	4	- 6531.51785	-	-	-	0s	
0	0	5714.84167	0	4	- 5714.84167	-	-	-	0s	
0	0	5661.36959	0	4	- 5661.36959	-	-	-	0s	
0	0	5650.67518	0	2	- 5650.67518	-	-	-	0s	
0	0	5634.63355	0	4	- 5634.63355	-	-	-	0s	
0	0	5628.32307	0	3	- 5628.32307	-	-	-	0s	
0	0	5627.11522	0	3	- 5627.11522	-	-	-	0s	
0	0	5626.56891	0	4	- 5626.56891	-	-	-	0s	
0	0	5626.35349	0	4	- 5626.35349	-	-	-	0s	
H	0	0			5624.2104331	5625.62143	0.03%	-	0s	
	0	2	5625.62143	0	4	5624.21043	5625.62143	0.03%	-	0s

Cutting planes:

MIR: 2

Explored 3 nodes (33 simplex iterations) in 0.09 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 5624.21

Optimal solution found (tolerance 1.00e-04)

Best objective 5.624210433124e+03, best bound 5.624676596064e+03, gap 0.0083%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 3 rows, 19 columns and 27 nonzeros

Model fingerprint: 0xd52dec9f

Model has 2 quadratic constraints

Variable types: 17 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [1e+00, 3e+02]

QMatrix range [1e+00, 1e+00]

QLMatrix range [1e+00, 1e+00]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Presolve time: 0.00s  
Presolved: 9 rows, 19 columns, 57 nonzeros  
Presolved model has 2 bilinear constraint(s)

Solving non-convex MIQCP

Variable types: 17 continuous, 2 integer (0 binary)

Root relaxation: objective 7.972217e+03, 12 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	7972.21736	0	3	- 7972.21736	-	-	-	0s
0	0	6407.65390	0	4	- 6407.65390	-	-	-	0s
0	0	5625.37216	0	4	- 5625.37216	-	-	-	0s
0	0	5543.82027	0	4	- 5543.82027	-	-	-	0s
0	0	5527.50990	0	2	- 5527.50990	-	-	-	0s
0	0	5503.04433	0	4	- 5503.04433	-	-	-	0s
0	0	5498.92867	0	3	- 5498.92867	-	-	-	0s
0	0	5498.43214	0	3	- 5498.43214	-	-	-	0s
0	0	5496.58741	0	4	- 5496.58741	-	-	-	0s
H	0	0			5495.6091363	5495.67824	0.00%	-	0s

Explored 1 nodes (28 simplex iterations) in 0.08 seconds (0.00 work units)  
Thread count was 8 (of 8 available processors)

Solution count 1: 5495.61

Optimal solution found (tolerance 1.00e-04)  
Best objective 5.495609136339e+03, best bound 5.495678243866e+03, gap 0.0013%  
Set parameter NonConvex to value 2  
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]  
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 3 rows, 19 columns and 27 nonzeros  
Model fingerprint: 0xb8d8204a  
Model has 2 quadratic constraints  
Variable types: 17 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [1e+00, 3e+02]  
QMatrix range [1e+00, 1e+00]  
QLMatrix range [1e+00, 1e+00]  
Objective range [1e+00, 1e+00]  
Bounds range [1e+01, 5e+01]  
RHS range [0e+00, 0e+00]

Presolve time: 0.00s

Presolved: 9 rows, 19 columns, 57 nonzeros

Presolved model has 2 bilinear constraint(s)

Solving non-convex MIQCP

Variable types: 17 continuous, 2 integer (0 binary)

Root relaxation: objective 7.779565e+03, 12 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
	0	7779.56452	0	3	- 7779.56452	-	-	-	0s
	0	6283.78994	0	4	- 6283.78994	-	-	-	0s
	0	5535.90266	0	4	- 5535.90266	-	-	-	0s
	0	5426.27096	0	4	- 5426.27096	-	-	-	0s
	0	5404.85260	0	2	- 5404.85260	-	-	-	0s
	0	5382.40582	0	4	- 5382.40582	-	-	-	0s
	0	5375.00964	0	3	- 5375.00964	-	-	-	0s
	0	5374.67689	0	4	- 5374.67689	-	-	-	0s
	0	5373.16756	0	4	- 5373.16756	-	-	-	0s
H	0				5370.9857524	5372.01418	0.02%	-	0s
	0	2 5372.01217	0	2	5370.98575	5372.01217	0.02%	-	0s

Explored 3 nodes (35 simplex iterations) in 0.08 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 5370.99

Optimal solution found (tolerance 1.00e-04)

Best objective 5.370985752364e+03, best bound 5.370985752364e+03, gap 0.0000%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 3 rows, 19 columns and 27 nonzeros

Model fingerprint: 0x81a6e83d

Model has 2 quadratic constraints

Variable types: 17 continuous, 2 integer (0 binary)

Coefficient statistics:

Matrix range [1e+00, 3e+02]

QMatrix range [1e+00, 1e+00]

QLMatrix range [1e+00, 1e+00]

Objective range [1e+00, 1e+00]

Bounds range [1e+01, 5e+01]

RHS range [0e+00, 0e+00]

Presolve time: 0.00s

Presolved: 9 rows, 19 columns, 57 nonzeros

Presolved model has 2 bilinear constraint(s)

Solving non-convex MIQCP

Variable types: 17 continuous, 2 integer (0 binary)

Root relaxation: objective 7.594734e+03, 9 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	7594.73366	0	3	- 7594.73366	-	-	-	0s
0	0	6164.82299	0	4	- 6164.82299	-	-	-	0s
0	0	5449.86765	0	4	- 5449.86765	-	-	-	0s
0	0	5313.43015	0	4	- 5313.43015	-	-	-	0s
0	0	5294.43699	0	2	- 5294.43699	-	-	-	0s
0	0	5273.41339	0	4	- 5273.41339	-	-	-	0s
0	0	5261.78991	0	3	- 5261.78991	-	-	-	0s
0	0	5259.98502	0	4	- 5259.98502	-	-	-	0s
H	0	0			5258.4708061	5259.03206	0.01%	-	0s

Cutting planes:

MIR: 1

Explored 1 nodes (23 simplex iterations) in 0.08 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 5258.47

Optimal solution found (tolerance 1.00e-04)

Best objective 5.258470806060e+03, best bound 5.258470806060e+03, gap 0.0000%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]  
 Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 3 rows, 19 columns and 27 nonzeros  
 Model fingerprint: 0x8b9303a8  
 Model has 2 quadratic constraints  
 Variable types: 17 continuous, 2 integer (0 binary)

Coefficient statistics:  
 Matrix range [1e+00, 3e+02]  
 QMatrix range [1e+00, 1e+00]  
 QLMatrix range [1e+00, 1e+00]  
 Objective range [1e+00, 1e+00]  
 Bounds range [1e+01, 5e+01]  
 RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns  
 Presolve time: 0.00s  
 Presolved: 9 rows, 18 columns, 55 nonzeros  
 Presolved model has 2 bilinear constraint(s)

Solving non-convex MIQCP

Variable types: 16 continuous, 2 integer (0 binary)

Root relaxation: objective 7.518064e+03, 9 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work		
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time	
0	0	7518.06398	0	3	- 7518.06398	-	-	-	0s	
0	0	6114.98769	0	4	- 6114.98769	-	-	-	0s	
0	0	5413.44955	0	4	- 5413.44955	-	-	-	0s	
0	0	5266.39011	0	4	- 5266.39011	-	-	-	0s	
0	0	5249.62516	0	2	- 5249.62516	-	-	-	0s	
0	0	5230.70598	0	4	- 5230.70598	-	-	-	0s	
0	0	5216.67160	0	3	- 5216.67160	-	-	-	0s	
0	0	5216.01102	0	4	- 5216.01102	-	-	-	0s	
H	0				5213.3890309	5215.03522	0.03%	-	0s	
	0	2	5215.03522	0	4	5213.38903	5215.03522	0.03%	-	0s

Cutting planes:  
 MIR: 1

Explored 3 nodes (28 simplex iterations) in 0.08 seconds (0.00 work units)  
 Thread count was 8 (of 8 available processors)

Solution count 1: 5213.39

Optimal solution found (tolerance 1.00e-04)

Best objective 5.213389030920e+03, best bound 5.213389030920e+03, gap 0.0000%

```
[17]: print(rob_obj_val)
```

```
[6376.6242816682025, 6209.158340032898, 6053.337631651906, 5902.622297308244,  
5760.949695057683, 5624.210433124288, 5495.609136339142, 5370.985752364289,  
5258.470806059773, 5213.389030920211]
```

Having a great budget of uncertainty is not always useful because if the linear regression performs well we will prefer to stay close to parameters estimations, hence we have to apply a tradeoff between the conservatism of solution and the revenue we are eager to lose.

See minimum, maximum and average percentage variation between robust optimal solution and Monte Carlo simulation

```
[18]: rob_obj_val = np.array(rob_obj_val)  
perc_var = (sum(obj_val[obj_val < obj_nominal])/sum(obj_val <=  
->obj_nominal)-rob_obj_val)/(rob_obj_val)
```

```
[19]: print(perc_var, sum(perc_var)/len(perc_var))
```

```
[-0.07508333 -0.05013759 -0.02568691 -0.00080916  0.02376282  0.04865317  
 0.07319243  0.09809379  0.12158959  0.13128832] 0.03448631419702809
```

```
[ ]:
```