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Keywords: oval domes, history of engineering, history of construction, structural design

Research

Oval Domes: History, Geometry and Mechanics

Abstract. An oval dome may be defined as a dome whose plan or profile (or both) has an oval form. The word "oval" comes from the Latin *ovum*, egg. The present paper contains an outline of the origin and application of the oval in historical architecture; a discussion of the spatial geometry of oval domes, that is, the different methods employed to lay them out; a brief exposition of the mechanics of oval arches and domes; and a final discussion of the role of geometry in oval arch and dome design.

Introduction

An oval dome may be defined as a dome whose plan or profile (or both) has an oval form. The word "oval" comes from the Latin *ovum*, egg. Thus, an oval dome is egg-shaped. The first buildings with oval plans were built without a predetermined form, just trying to enclose a space in the most economical form. Eventually, the geometry was defined by using circular arcs with common tangents at the points of change of curvature. Later the oval acquired a more regular form with two axes of symmetry. Therefore, an "oval" may be defined as an egg-shaped form, doubly symmetric, constructed with circular arcs; an oval needs a minimum of four centres, but it is possible also to build ovals with multiple centres.

The preceding definition corresponds with the origin and the use of oval forms in building and may be applied without problem up to, say, the eighteenth century. From that point on, the study of conics in elementary courses of geometry taught the learned people to consider the oval as an approximation of the ellipse, an "imperfect ellipse": an oval was, then, a curve formed from circular arcs which approximates the ellipse of the same axes. As we shall see, the ellipse has very rarely been used in building.

Finally, in modern geometrical textbooks an oval is defined as a smooth closed convex curve, a more general definition which embraces the two previous, but which is of no particular use in the study of the employment of oval forms in building.

The present paper contains the following parts: 1) an outline of the origin and application of the oval in historical architecture; 2) a discussion of the spatial geometry of oval domes, i.e., the different methods employed to lay them out; 3) a brief exposition of the mechanics of oval arches and domes; and 4) a final discussion of the role of geometry in oval arch and dome design.

Historical outline of the origin and application of the oval in historical architecture

The first civilizations: Mesopotamia and Egypt

Rounded forms, many times not geometrically defined, were used in building from the most remote antiquity. These rounded forms may be called "oval". What the ancient builders were looking for was the most simple and economical way to enclose a space. As

1590-5896/07/020211-38 DOI 10.1007/s00004-007-0040-3

techniques were perfected, some of these plans were geometrically defined using cords and pegs to control their contours, i.e., employing circular arcs or combinations of them.

These enclosures were first covered by masonry in about 4000 B.C., by cantilevering the stones forming successive rings, until the space is closed at the top. This is what we call now a “false dome”. Fig. 1 shows the most ancient remains discovered so far in Asia Minor. Domes were used to form “stone huts” and the technique was developed, no doubt, in the context of permanent settlements associated with agriculture. It is a form of what we today call vernacular construction. The same technique of building has survived in some countries until the present day. (In Spain, for example, the vernacular buildings of *piedra seca*, dry stone, in Mallorca are similar to those first examples in Asia Minor [Rubió 1914].)

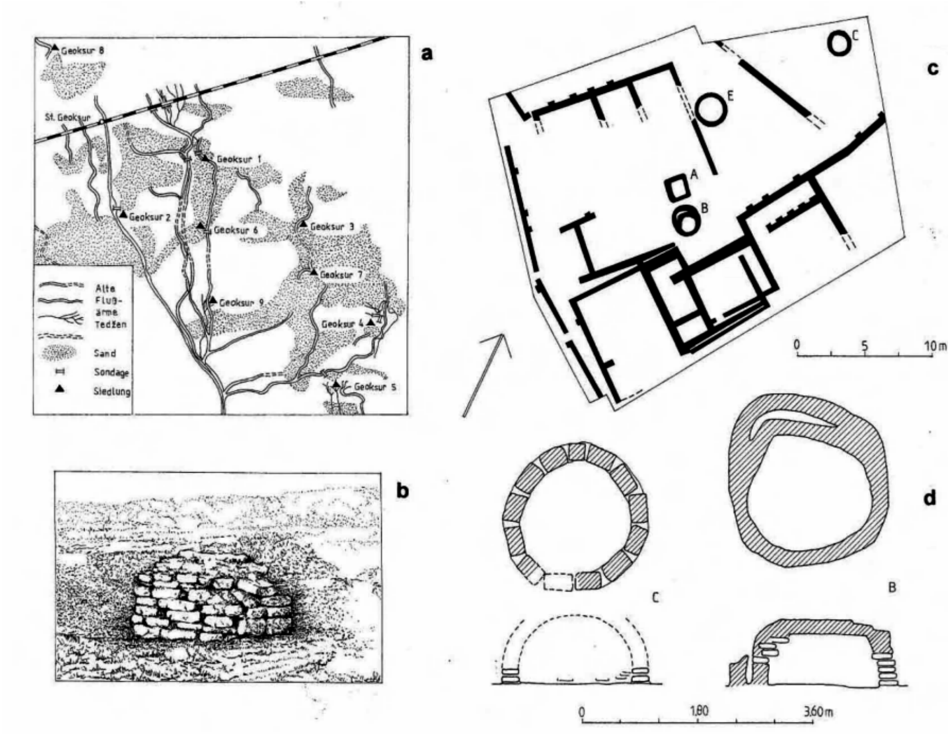


Fig. 1. The first domes covering a closed oval plan in Asia Minor ca. 4,000 B.C. [Baitimova 2002]

The invention of the arch apparently came later than that of the dome. The first arches were built in Mesopotamia or Egypt circa 3500 B.C. to construct the permanent covering of tombs. The books of Besenval [1984] and El-Naggar [1999] contain the most information on arch and vault building in those times.

The first arches were built with crude bricks. It was discovered that if the bricks are disposed in a certain manner in space they remain stable, their weight being transferred from one brick to the next until reaching the earth: the same force which tries to drag the bricks to the earth keeps them in position. It was an amazing invention and an enormous step forward from the more common custom of simply piling the bricks to form walls.

(The practice of brick wall building was itself an invention which evolved very slowly before the bricks and the different bondings were developed; see Sauvage [1998].)

In the first two millennia the builders experimented with several types of arches and vaults and there is no direct line of progress towards the voussoir arch with radial joints, which is our conceptual model. A perusal of the hundreds of surveys contained in the books of Besenval and El-Naggar makes evident a long period of "eclectic" experimentation, in which several forms and types of arches co-existed. Among them appeared the first oval vaults (fig. 2). Some of those vaults were built without centring, by building successive flat slices against a wall where the form of the arch was first drawn. The technique is still used in the north of Africa [Fathy 1976] (fig. 3).

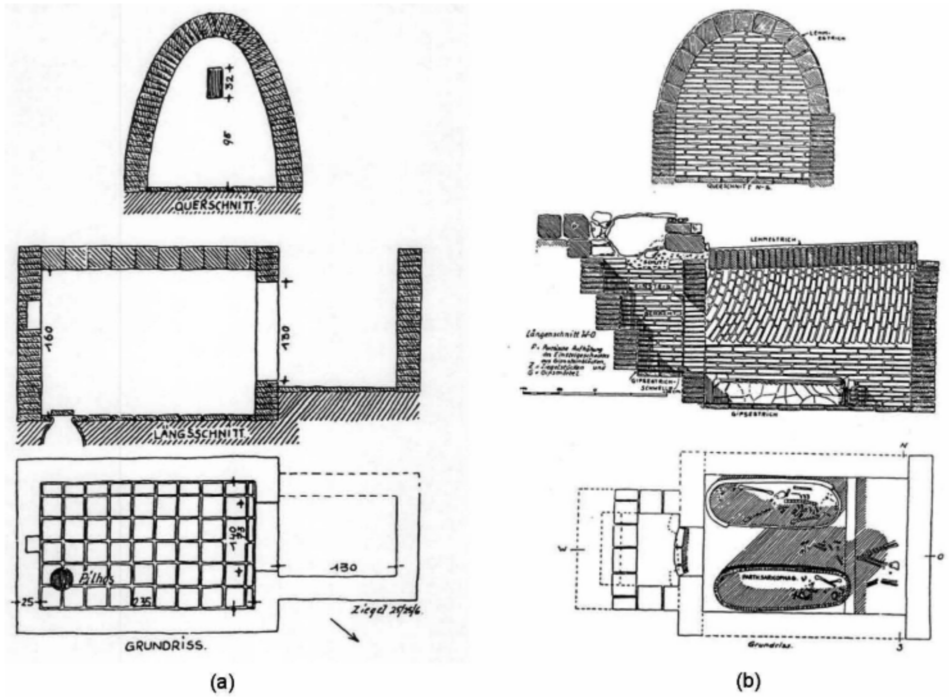


Fig. 2. Oval arches in Asur, Mesopotamia. a) With radial centres; b) Arches built without centring, by constructing successive slices leaned one against the next [Besenval 1984]

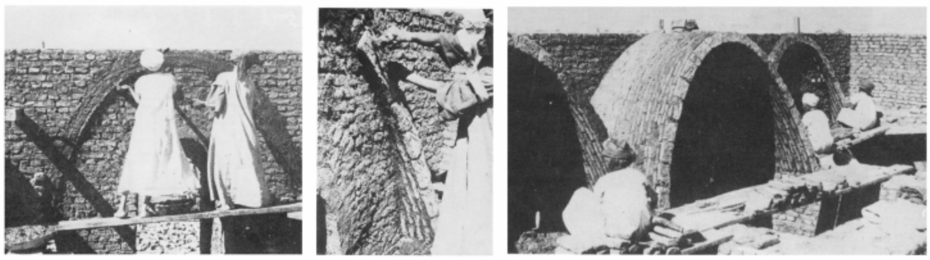


Fig. 3. Building oval barrel vaults in North Africa in the 1970's [Fathy 1976]

The first vaults were quite small, with spans of only about one or two meters, just enough to cover the tomb. This size favoured experimentation: the vault, if not of an adequate form, will distort and the observation of the movements gave the builders a “feeling” for the more adequate forms. Fig. 4a shows one of the plates of the book of El-Naggar [1999] which explains clearly the kind of forms adopted for the vaults. To an architect or engineer with some experience in masonry structures it will be evident that the vault at the bottom right side is the safest, adopting an oval form which will amply contain the trajectory of compressions (the line of thrust or inverted catenary) within the arch. Choisy [1904] was the first to point this fact as the origin of the oval arches (fig. 5a). (On the design of masonry arches see [Heyman 1995] and [Huerta 2006].)

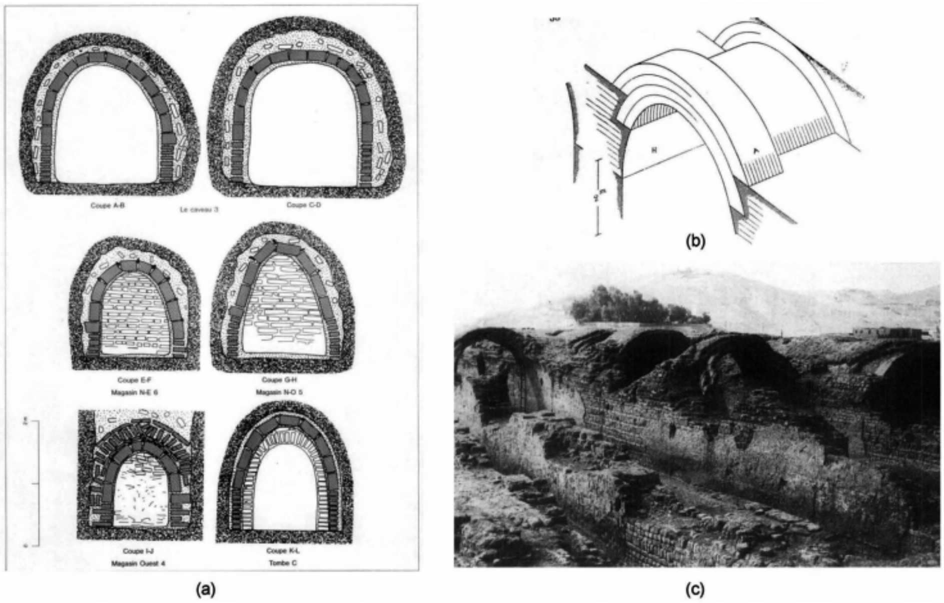


Fig. 4. a) Different Egyptian oval vaults, showing different degrees of distortion from the oval form [El-Naggar 1999]. The bottom-left vault, which shows no distortion, presents no danger of collapse; b) Construction of the vaults of the Ramesseum [Choisy 1904]; c) Barns of the Ramesseum [El-Naggar 1999]

When vaults grew bigger in the second millennium B.C. – for example, the vaults covering the barns of the Ramesseum (fig. 4b and c) have spans of almost five meters – a good regular building required that the form of the vault be fixed by some construction. The oval forms had to be defined geometrically. The Egyptians were experts in practical geometry using pegs and strings, and a form composed of circular arcs is the most logical. Choisy [1904], observing the form of many vaults, considering the practical geometry of the Egyptians (the 3-4-5 right triangle), and applying the logic of building, proposed a simple oval form and suggested how it might be constructed using a simple system of strings (fig. 5).

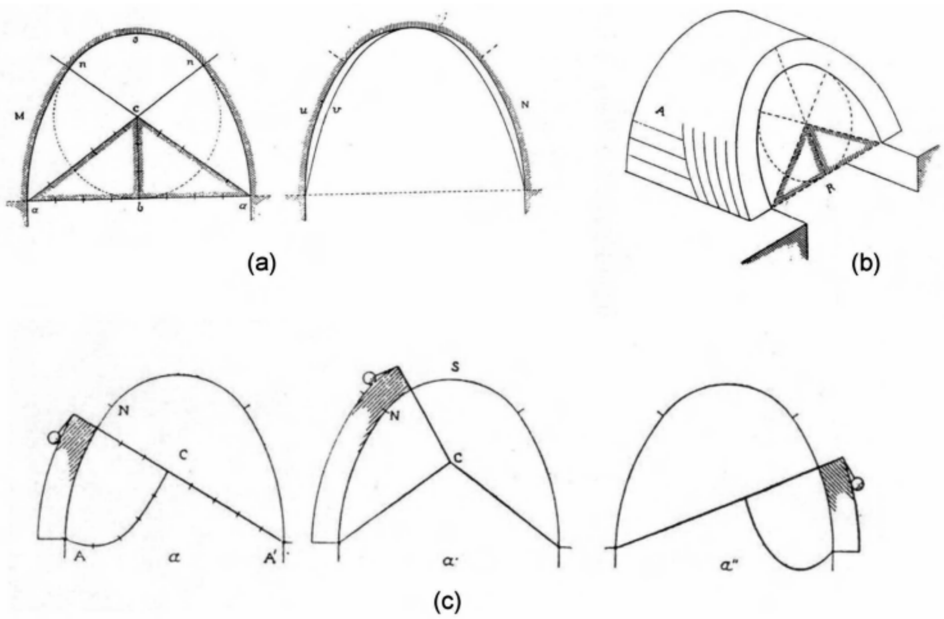


Fig. 5. The geometry and construction of Egyptian ovals, after Choisy [1904]. a) Egyptian oval with the 3-4-5 triangle and comparison with the catenary; b) Possible employment of the oval with leaning brick slice-arches; c) Use of a string method to draw the oval

A geometrical construction is not the only possibility. The mason may sketch the profile of the vault on a wall, perhaps making several corrections until he is satisfied with a certain form. Then, he may fix the form my drawing an horizontal line and measuring the vertical distances to it. Indeed, this was the method followed in a diagram from the Third Dynasty (3000-2700 B.C.) near the Step Pyramid of Saqqara [Gunn 1926] (fig. 6a and b). If the separation between the vertical lines is considered to be uniform, the profile does not correspond to the preceding "typical" oval or a circular arch, and this supports the previous hypothesis. However, Daressy [1927] demonstrated that if the last interval is presumed to be shorter than the others a circular arc may be somewhat adjusted to the curve (fig. 6c). The present author has adjusted an oval, following a simple geometrical construction (fig. 6d). Many other curves may be tried, but any interpretation should be made with caution, considering the historical context and the logic of practical building at that time.

Some scholars claim to have found ellipses and not ovals in the form of the Egyptian arches. In particular, the French archaeologist Daressy [1907] attributed an elliptical form to a drawing of the profile of an arch corresponding to the vaults of one of the tombs of Ramses VI (twelfth century B.C.). This hypothesis has been accepted as true by many scholars who have discussed the laying out of Egyptian arches. (See, for example, [Arnold 1991] and [Rossi 2004]). Heisel [1991] hinted at the possibility of employing a bent wooden strip to lay out the curve. ([Cejka 1978] has explored this method in his research on Islamic arches).

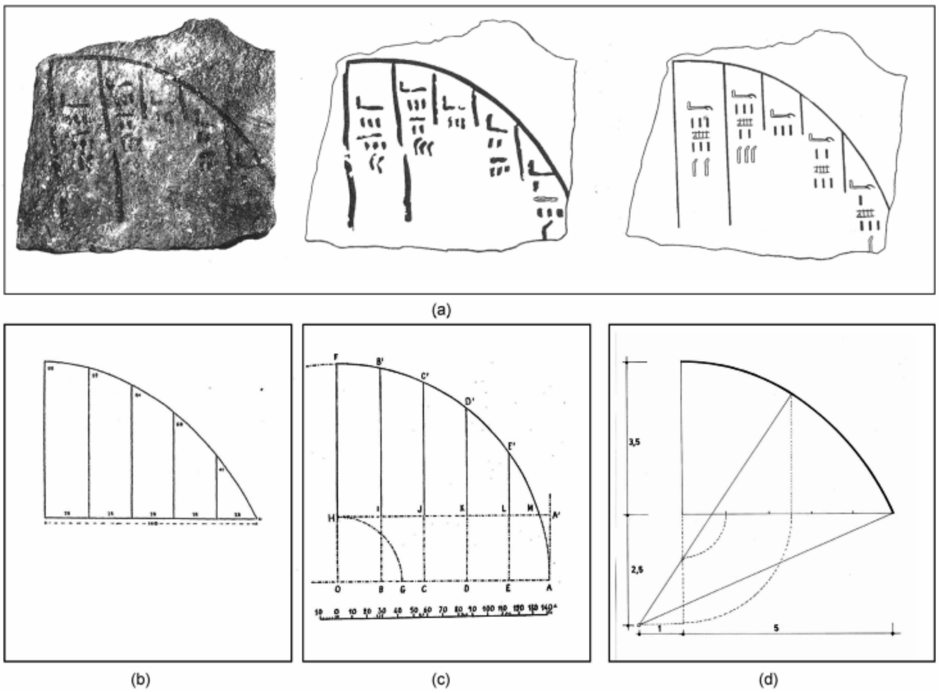


Fig. 6. a) The oldest diagram of an arch, Third Dynasty, 3000-2700 B.C. (photo and drawings after [Gunn 1926]; b) Drawing by Gunn [1926]; c) Geometrical interpretation as an arc of circle by Daressy [1927]; d) Oval which approximates to Gunn's drawing (author)

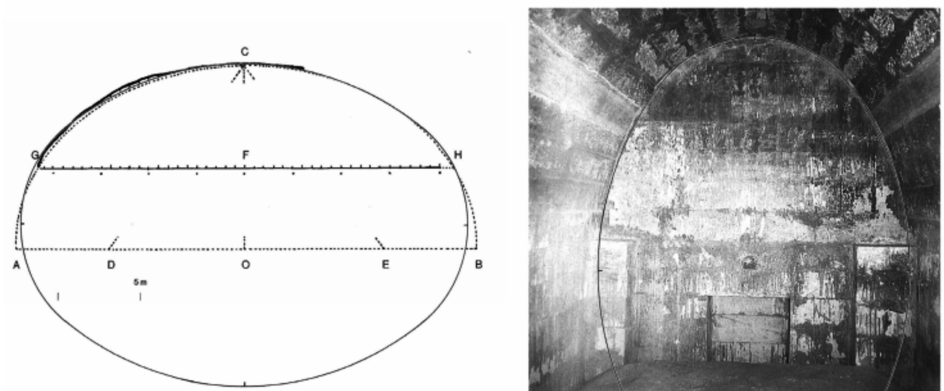


Fig. 7. a) Layout of an Egyptian oval dome. Comparative drawing of an Egyptian oval with an ellipse with foci in D and E ([El-Naggar 1999] after [Daressy 1907]). Choisy's oval has been superimposed by the author in a thin continuous line; b) Stone vault of an Egyptian tomb of oval profile [El-Naggar 1999]. Choisy's oval superimposed by the author

I disagree strongly with the hypothesis that the Egyptians used elliptical forms. The ellipse is a mathematical curve which was defined by Greek mathematicians the fourth century B.C. [Heath 1981]. The supposition that they have discovered by chance the string method for laying out the ellipse (the so-called "gardener's method") is also quite difficult to accept, given the long and painful birth of even the most simple and "obvious" inventions. In fact, Choisy's Egyptian oval adapts itself as well or better than the ellipse to the diagram, as fig. 8a makes evident, and if any geometrical construction was used, this appears a much more probable hypothesis. Finally, a drawing found in a wall of Luxor's Temple (figs. 8a and b) [Borchardt 1896] provides evidence of the employment of the oval by the Egyptians. In this case it is evident that the form corresponds to an oval, because of the great difference of the two radii. This is a very strong argument, to be added to the others, against the "ellipse hypothesis".

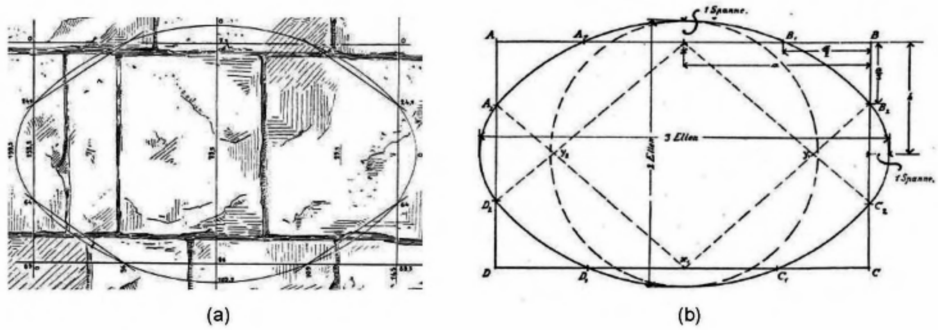


Fig. 8. Drawing of an oval discovered by Borchartd in 1896 in a wall of the Temple of Luxor. a) Borchardt's reproduction of the original drawing; b) One of his hypothesis for the geometrical generation of the oval [Borchartd 1896]

Greece and Rome

The Greeks knew the arch. Since stone was the usual building material, they employed the voussoir arch, almost without exception with a semicircular form. The arch was employed in secondary buildings, in sewers or for the gates of the city walls [Boyd 1978, Dornisch 1992]. Some cantilevered domes approached the oval form, but there was no systematic use of the ovals as in the brick architecture of Mesopotamia or Egypt.

The oval form appeared in Europe in Roman times for the design of amphitheatres (see Wilson Jones 1993]) (fig. 9a). Again, some scholars have tried to prove the use of ellipses instead of ovals. However, the oval form is the natural form for laying out the stands of the amphitheatre: it is impossible to construct parallel ellipses, and the only logical method is to use oval forms made of circular arcs. In any case, the differences between ellipses and ovals for the usual proportions are so small that, in fact, one see what one wants to see. Even the most precise mensuration does not serve to settle the matter [Rosin 2005]. It is not a matter of mensuration, but of the history of building traditions.

It appears that the Romans did not built oval domes: the central symmetry was considered a requisite and even in the experimentation of the domes in Hadrian's Villa all the forms present a centralized character [Rasch 1985]. Some exceptions may be found in the apses of thermae [Lotz 1955], and it is usually presumed that the octagon of the church of St. Gereon in Cologne rests on the oval foundations of a previous Roman building [Götz

1968] (fig. 9b) and [Krautheimer 1984] (fig. 9c). Choisy [1873] discovered that in the bridge of Narni, the central span of the inclined road was adapted by using an arch formed from two quarter-circles of different radii. Perhaps more examples can be found, but it appears that the Romans used the oval form for arches and domes only in exceptional cases.

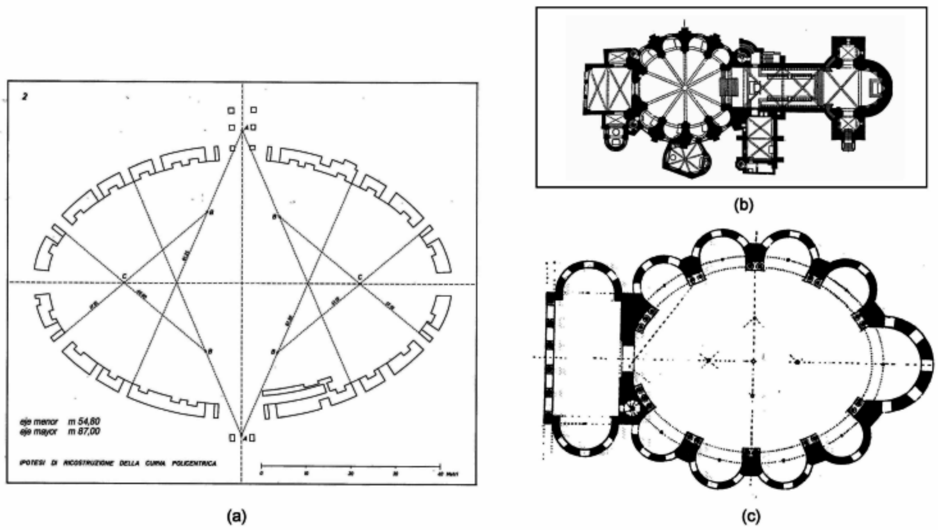


Fig. 9. a) One of the hypothesis concerning the oval geometry of the Roman Coliseum [1993]; b) Medieval octagon of the church of St. Gereon in Cologne; it is supposed to be built on the foundations of an oval Roman building [Götz 1968]; c) reconstruction by Krautheimer [1984]

The Middle Ages

Most medieval domes have a centralised form, probably due to the Roman influence. In Spain most Romanesque churches pertain to this type. However, there are also some exceptions, and the church of Santo Tomás de Olla, dated by Gómez Moreno [1919] to the tenth century, presents an octagonal dome on an oval plant, 6 x 5.5 m, surrounded by horseshoe arches (Fig, 10).

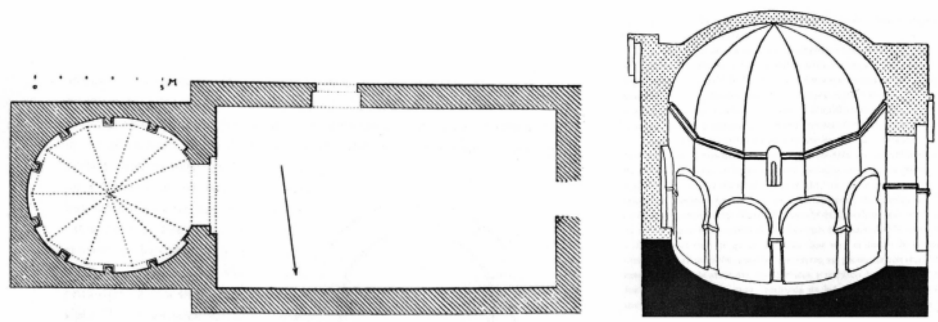


Fig. 10. Church of Santo Tomás de Olla, Spain, built in the tenth century [Gómez Moreno 1919]

In France, Chappuis [1976] has made what appears to be the only exhaustive study of the use of the oval form in the Middle Ages. He has studied some 400 Romanesque churches in the south of France which present some kind of oval arches or domes. Of these, 130 have domes with an oval plan. Fig. 11a shows Chappuis's classification of Romanesque ovals. Fig. 11b shows clearly the oval plan in two churches. Chappuis has found, then, a precedent in Europe for the Renaissance oval domes, which so far has not been noted by historians of architecture.

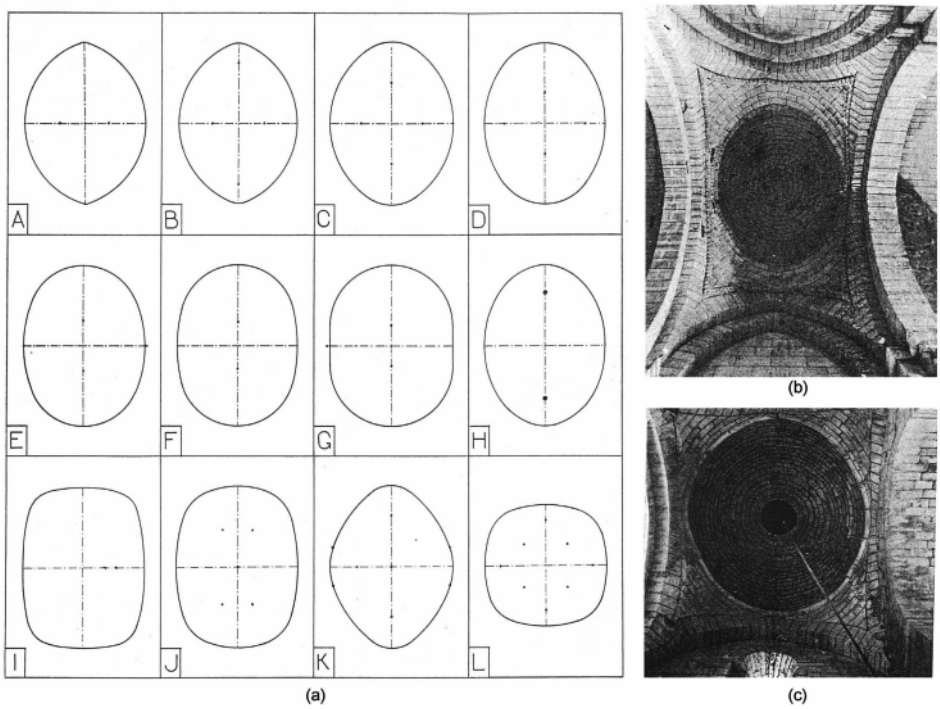


Fig. 11. The oval form in French Romanesque churches [Chappuis 1976]. a) Classification of ovals; b) Two examples of churches with oval plans: above, Saint-Martin de Gurçon, Dordogne, and below, Balzac, Charente

Another element in which oval arches can be found in Romanesque churches are the groined vaults, resulting from the intersection of two barrel vaults. It may that the medieval masons knew some method to lay out approximately the curve of the groin before construction, and the technique of the "lengthened arch" which will be discussed below, could have been used. But there are other techniques which make possible the construction without the physical building of the diagonal centring, as may be seen in the drawing by Mohrmann [Ungewitter 1890] (fig. 12). (The Romans may have used the technique in building their groined vaults.)

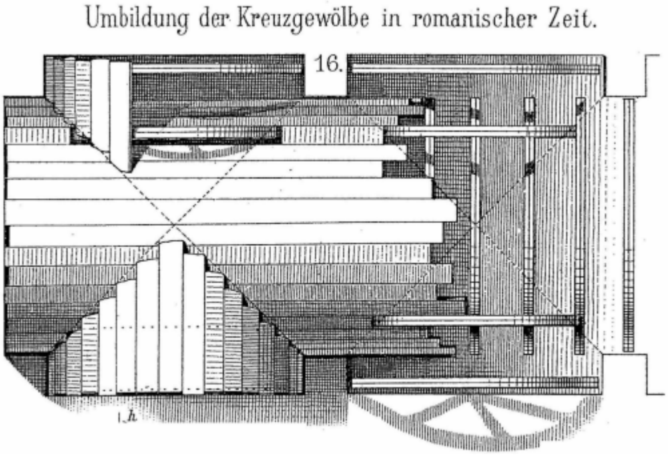


Fig. 12. Building of a Romanesque groined vault without knowing the form of the groin. The intersection is made "physically" with the wooden planks [Ungewitter 1890]

The Gothic is based on a great simplification of the building procedures. In the cross vaults the ribs, which are always composed of circular arcs, define the geometry of the vault, the masonry shell closing the space between ribs in the last step of the building (fig. 13). In the simple quadripartite vault, the cross ribs are semicircles and the transverse arches and the formerets (wall ribs) are adjusted to the desired height using pointed arches. (The best study on the geometry of the Gothic ribs is still that of Willis [1843].)

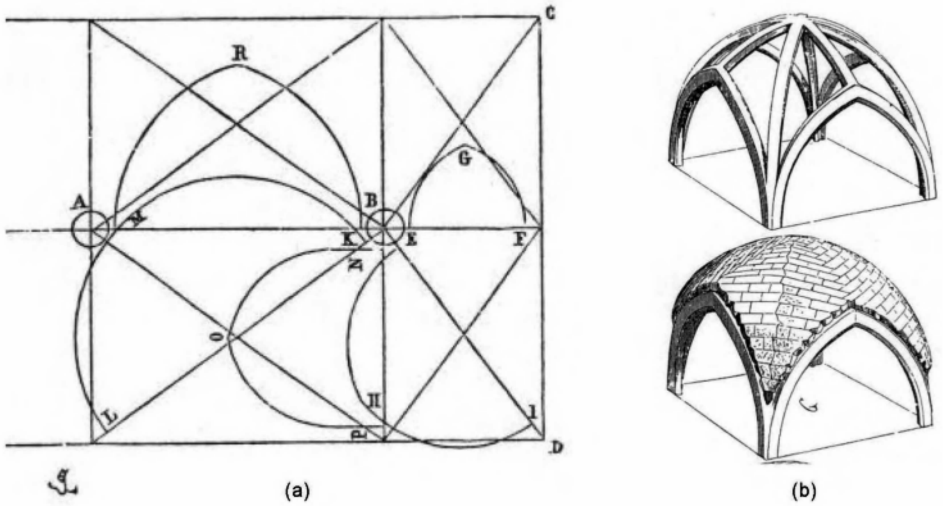


Fig. 13. The geometry of the Gothic cross vault is defined by that of the ribs, which are made of circular arcs. a) The technique permits correspondence with the forms of adjacent vaults; b) The ribs are built before and then the shell is closed [Viollet-le-Duc 1858]

The idea of defining beforehand the curve of intersection between two barrel vaults – difficult to lay out and construct – by a simple circular arc, seems to come from Byzantium; Choisy [1883] studied the matter in depth. The Byzantines did not use ribs to reinforce the groins, but this, in essence, is the only conceptual difference between the laying out of Byzantine and Gothic cross vaults (fig. 14).

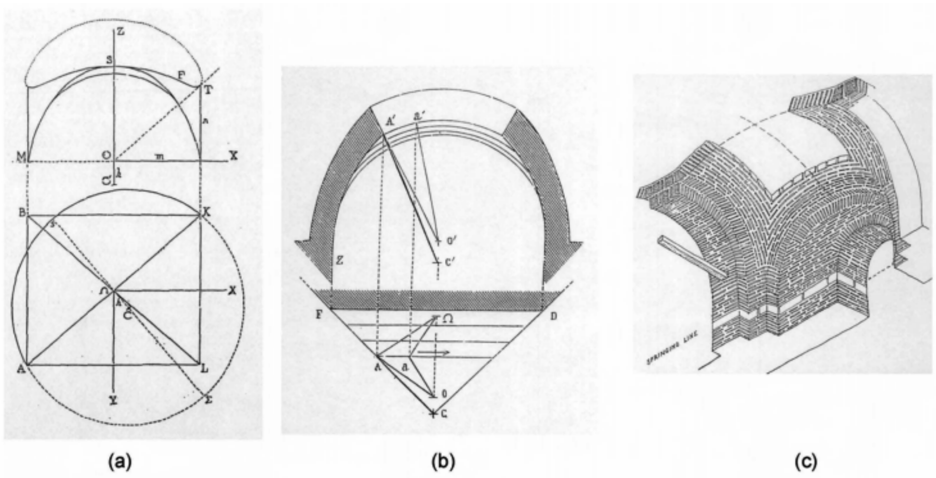


Fig. 14. Laying out of the groins in a Byzantine cross vault. a) The two groins and the four perimeter arches are circular segments, defined beforehand; b) The shell is "generated", hypothetically, by a system of strings rotating around an axis [Choisy 1883]; c) The different arch-slices are built without centring until the vault is closed [Ward-Perkins 1958]

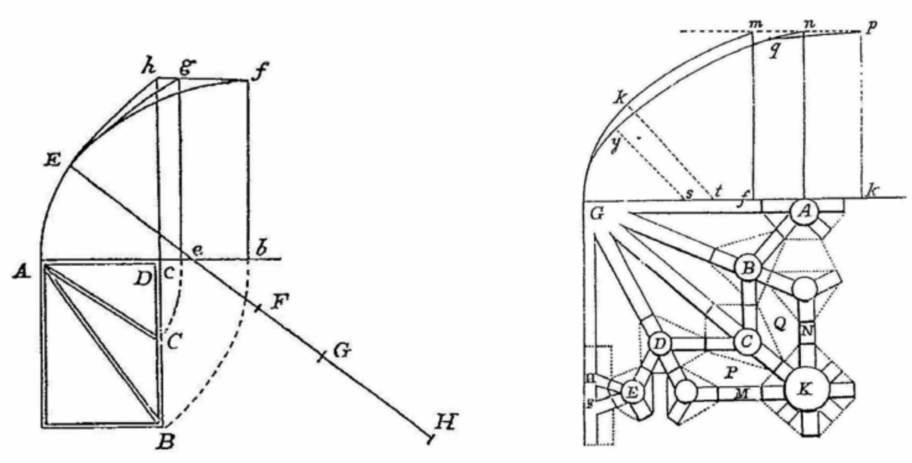


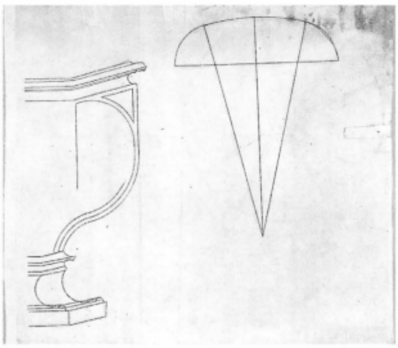
Fig. 15. Use of oval segmental ribs to define the geometry of late Gothic English vaults [Willis 1843]

Gothic geometry is coherent with the character of Gothic building: a requisite of economy, in the broadest sense of the term, permeates the whole Gothic building. The ingenuity of the master masons in solving easily, by simple methods, the most complicated problems still fascinate the historians of Gothic architecture today. In high Gothic, the ribs were always either semicircular or pointed arches of two centres. The flexibility of the system has been best explained in the drawings by Viollet-le-Duc (fig. 13a, above).

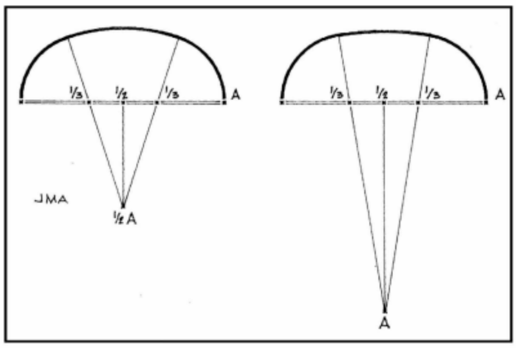
In late Gothic, however, in England, Germany and Spain, the Gothic masters began to employ oval forms, generated by the tangency of circles of different radii. The first to note the employment of this kind of "oval" ribs was Willis [1843] (fig. 15). Again, the tradition was well known in the Orient. In the comprehensive study of Cejka [1978] on the use of the arch in Islamic architecture, there are many examples of the same kind of geometries, which predate the European by several centuries.

The first documented evidence of the use of ovals is found in Germany. In the Wiener Sammlung, the richest collection of drawings of medieval buildings, two examples of ovals are found; one of them is represented in fig. 16a. Bucher [1968, 1972] has called attention to them and describes the method used by the medieval masons: the span of the arch is divided into a certain number of parts (three, four, ...); the two extreme points define the radius of the smaller circles; finally, on the axis of symmetry a third point is chosen to define the radius of the central arc (usually taking the span, or a fraction of it, from the springing line as in figs. 16a and b). The method is quite simple and may be adjusted by trial-and-error to any ratio of span to height. Fig. 16c illustrates some solutions to "exams" by the apprentice masons in Augsburg. The problem was to define the oval centrings for irregular vaults. It is evident that they were using a well-established technique. In the drawings only the final result is shown, but this must have been preceded by some experiments. Werner Müller, a German scholar who has contributed decisively to our knowledge of Gothic *Baugeometrie* and the history of stereotomy [Müller 1990, 2002] called attention to this important document [Müller 1972].

In Spain, the oval arch also appears as an element of Gothic architecture in the fifteenth and sixteenth centuries, used in the portals of churches and civil buildings, as well as in the ribs of the surbased vaults which support the choir at the foot of Spanish parish churches. Palacios [2003] has made the first research in this most interested aspect of Spanish building geometry. Two photographs of this kind of vault in the entrance to the church of San Juan de los Reyes in Toledo are shown in fig. 17.

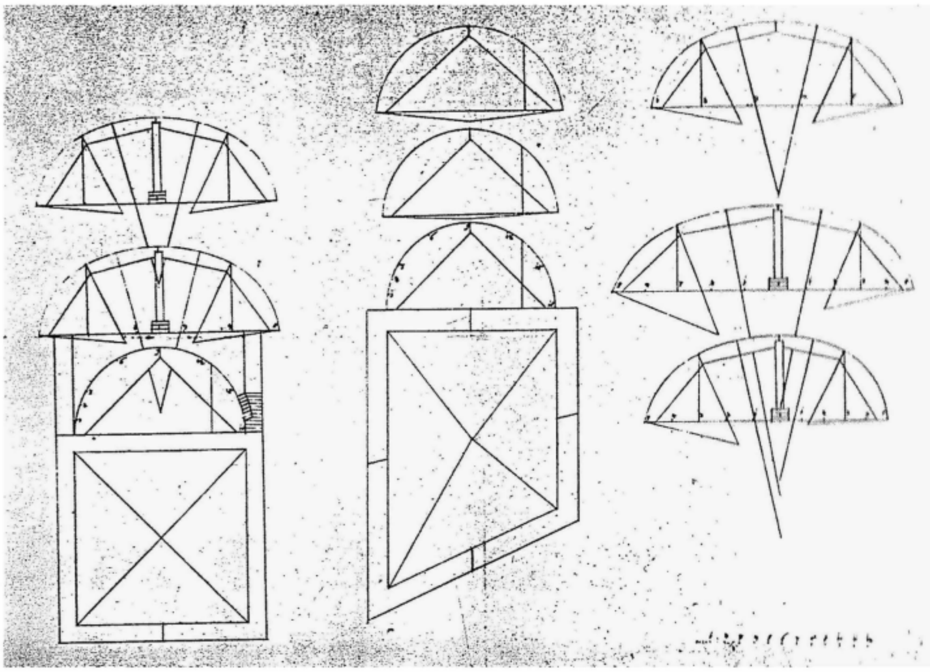


(a)



(b)

Fig. 16. Ovals in late Gothic German documents. a) Oval layout in the Wiener Sammlung [Koepf 1969]; b) Some types of Gothic ovals [Bucher 1972]



(c)

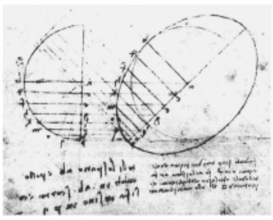
Fig. 16c. Exam of a mason apprentice in Augsburg [Müller 1972]

(a) (b)

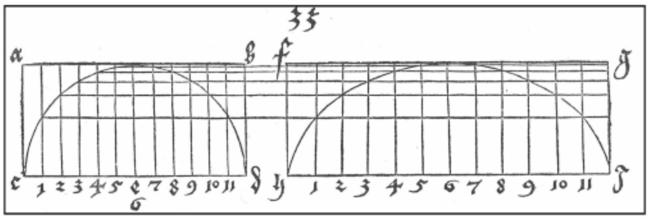
Fig. 17 . Gothic surbased vaults with oval ribs in the church of San Juan de los Reyes in Toledo, designed by Juan Guas (end of the fifteenth century). a) Vaults at the entrance (photo by G. López); b) Vault of the choir (photo by E. Rabasa)

The "lengthened arch"

At the beginning of the sixteenth century appears documentary evidence of another geometrical method for drawing oval arches or lines. It is first included in the *Codex Atlanticus* by Leonardo da Vinci (ca. 1510), as has been noted by Simona [2005] (fig. 18a). The method was first published in Dürer's *Unterweisung der Messung* in 1525 (fig. 18b): "The stonemasons will need to know how to stretch a half circle in length, maintaining all the others measures unchanged, and this is because the vaults need to close adequately" (my translation). The problem is to draw a surbased arch of a certain height and span by stretching the semicircle of the same height. The method consists in first inscribing the semicircle in a rectangle of ratio 1:2; next, the base of the rectangle (the diameter of the semicircle) is divided in twelve parts and vertical lines are drawn. Another rectangle of the same height and of the desired length is drawn and the base is divided in the same number of parts; again, vertical lines are drawn. The intersection of these lines with the horizontal lines from the intersection of the semicircle with the vertical lines in the first rectangle will give points of the desired arch. We know, of course, that the curve is an ellipse, but Dürer does not mention it; in fact, the mathematicians would discover this fact only one hundred years later. It was Guldin in 1640 who discovered the elliptical nature of the curve [Peiffer 1995]. Some authors have used the term "pseudo-ellipse" to refer to oval arches (for example, Bucher [1972] and Calvo [2002]), but this is misleading, as it suggests the desire to approximate a curve which was unknown to masons and architects before the beginning of the eighteenth century [West 1978].



(a)



(b)

Fig. 18. Drawing of an oval by stretching a circle. a) Leonardo da Vinci, *Codex Atlanticus* fol. 318 b-r, ca. 1510 [Simona 2005]; b) Albrecht Dürer, *Unterweisung der Messung* [1525]

Is this construction Leonardo's or Durër's invention, or are they describing a method pertaining to the tradition of the stonemasons? In the late Gothic drawings which have survived there is no trace of such a geometrical construction; as we have seen, they solved the problem by using ovals made of circular arcs. However, it is interesting to note that the same construction appears in other architectural and stone-cutting treatises written in the last half of the sixteenth century in Spain, namely those of Hernán Ruiz [ca. 1545], Ginés Martínez de Aranda [1590] and Alonso de Vandelvira [1580]. As procedures and building methods change very slowly, it is probable that the method was one of the geometrical tools of medieval master masons.

The oldest of these treatises is the so-called *Libro de arquitectura* of Hernán Ruiz el Joven, written between 1545 and 1562 [Navascués 1975]. It is a manuscript containing a translation of Book I of Vitruvius, followed by a collection of drawings of different problems of geometry, stone-cutting, buttress design, classical orders and designs of temples. Hernán Ruiz includes several drawings showing a method equivalent to that of Dürer but based on the parallel projection on an inclined plane. The method is applied three times: to draw the caissons of a *capilla por arista*, groined vault (fig. 19a); to draw a *capilla vaída*, sail vault (fig. 19b); and to derive a surbased arch from a semicircular arch (fig. 19c). The use in three different cases and the absence of any instructions suggest that the method was well known among Spanish stonemasons in the sixteenth century.

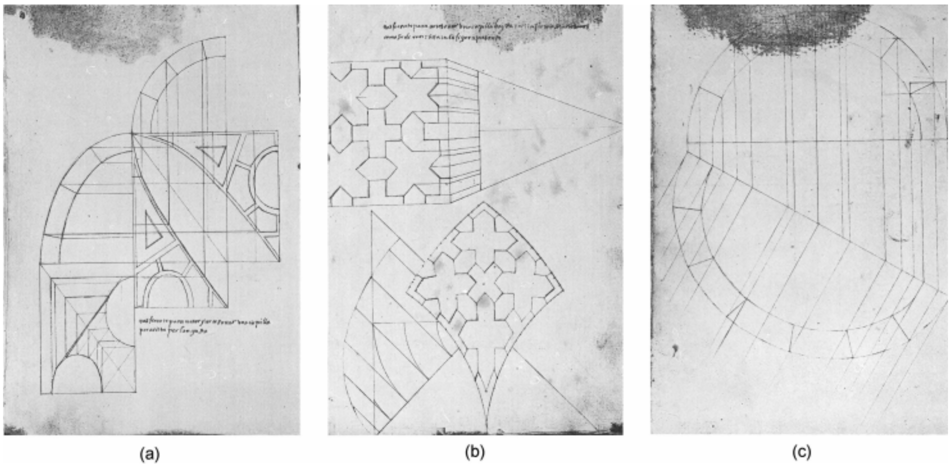


Fig. 19. The method of parallel projection in the *Libro de arquitectura* of Hernán Ruiz el Joven [ca. 1545]. a) Groined vault; b) Sail vault; c) arch construction. (Library of the E. T. S. de Arquitectura de Madrid)

Hernán Ruiz made extensive use of Serlio's treatise of architecture and he also copied Serlio's oval constructions (which will be discussed below). The method is not explicitly mentioned by Serlio, though he also uses parallel projection to obtain the intersection of the two barrels in his perspective drawing of a groined vault (fig. 20).

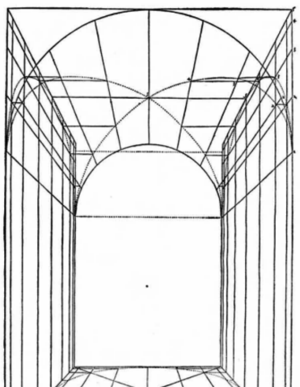


Fig. 20. Serlio's perspective drawing of a groined vault using parallel projection [1996, Book II]

Martínez de Aranda, a Spanish architect who flourished at the end of the sixteenth century, wrote a treatise on stonecutting in about 1590 which has survived in manuscript copy [Martínez de Aranda 1986]. He gives two methods to obtain oval arches: one by contraction or extension of coordinates (equivalent to that of Dürer) and the other by parallel projection (equivalent to that of Hernán Ruiz) (fig. 21a and b). However, it is the second method which is used extensively throughout the treatise. Both methods are succinctly explained in the first pages of the treatise under the title “Difinitiones”. In the stonecutting treatise of Vandelvira [1580] only the method of parallel projection is used, forming an essential tool in many drawings. However, he gives no explanation of it; the author either considers it evident or part of the common knowledge of a stonemason. The first occurrence of the method in the manuscript (fol. 13 r), where it is used to lay out a groined pendentive is shown in fig. 21c.

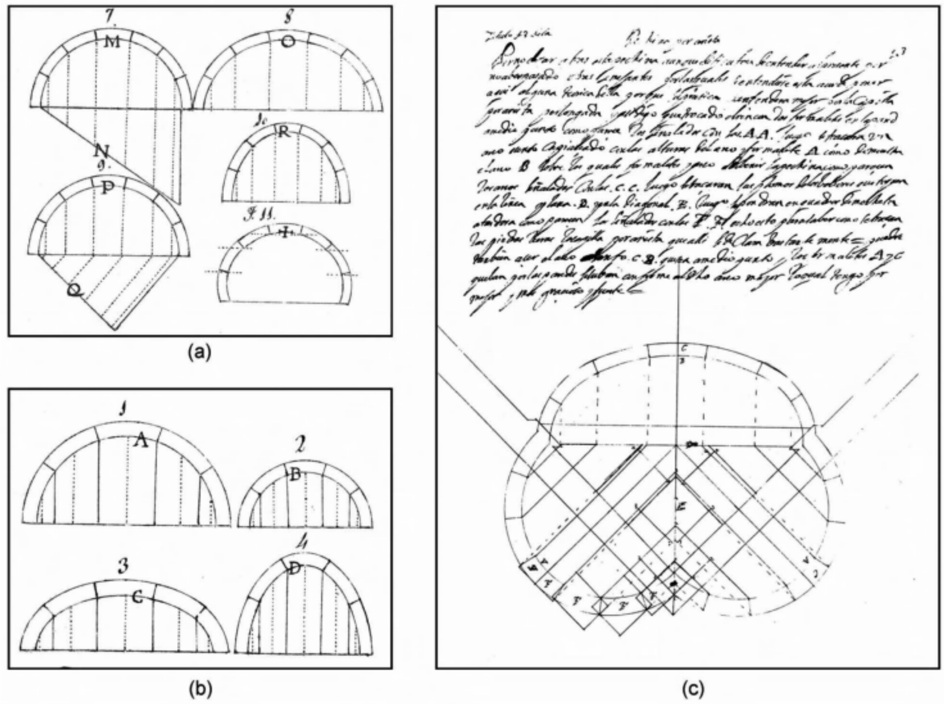


Fig. 21. The method of parallel projection for drawing oval arches: a and b) Martínez de Aranda [1590]; c) Alonso de Vandelvira [1580]

A long and detailed explanation of the two methods, a step-by-step exposition which can be followed by any apprentice, was given by Philibert De L’Orme in his book *Nouvelles inventions pour bien bastir* [1561]. A more succinct exposition is given in his book on architecture [1567]. De L’Orme calls the oval arch *Cherche rallongée*, lengthened arch, and again it seems that he is explaining a well known and established technique. De L’Orme gives two solutions which correspond approximately to the methods explained above (figs. 22a and b). He says explicitly that the curve cannot be draw with a compass and he shows the way to obtain the local curvature of the arch so that the templates for cutting the stones can be made.

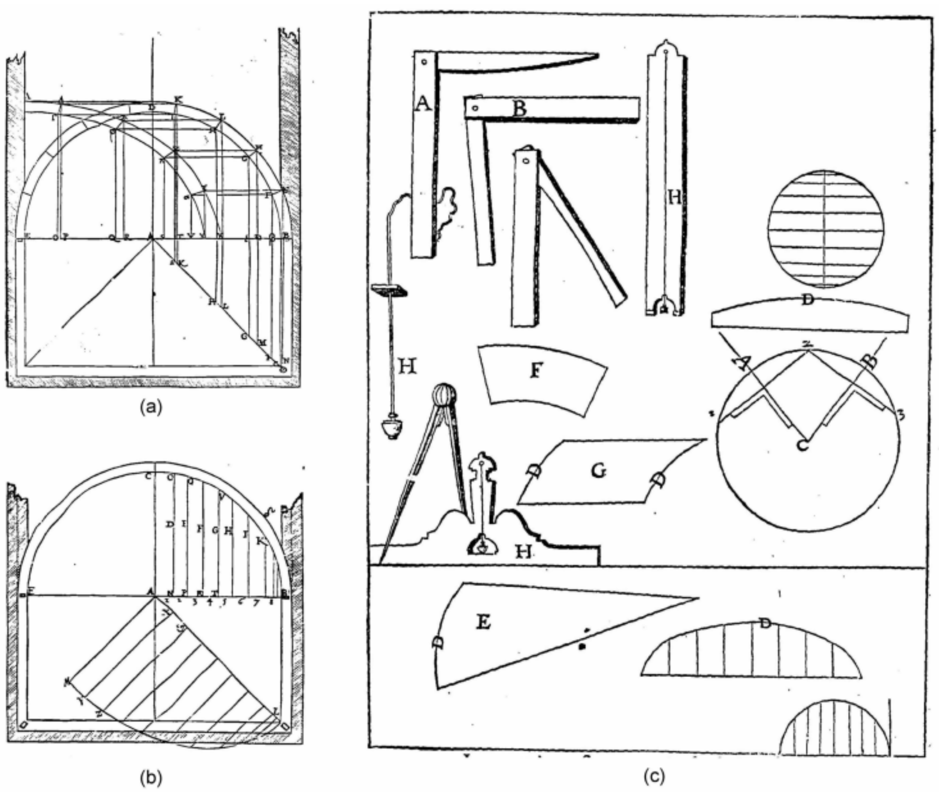


Fig. 22. Philibert De L'Orme's methods for constructing oval arches [1561, 1567]. a and b) two geometrical constructions; c) finding the centre of a circle when three points on the circumference are known

It is very simple: you take three successive points and draw two straight lines joining them. Then draw the perpendicular bisectors of each of these lines. The point of intersection of the two bisectors the centre of the circle which passes through the three points, as is shown in the sketch at the right of fig. 22c.

The same procedures were used by carpenters and were explained by Jousse in the first published handbook on carpentry printed in 1627 [1702] (fig. 23, next page). Of particular note is the drawing of median lines to obtain the radius of different parts of the arch (fig. 23c). It seems evident that a simplification to an oval of a few centres will be quite easy.

The oval in the Renaissance

We have seen that ovals were used in European late Gothic architecture. We have also seen that stonemasons developed a technique to lay out oval arches of any ratio of span to height by stretching or shortening circles or quarter-circles. The oval was not itself an invention of the Renaissance, but the oval form was used by some Renaissance architects to design a new concept of space for temples. Architectural historians have discussed the matter of the appearance of the oval in great detail; the fundamental contribution is still Lotz's monograph published in 1955. The literature is quite extensive; to name but a few fundamental contributions: Fasolo [1931], Zocca [1946], Müller [1967], Kitao [1974] and Nobile [1996]. On oval domes in Spain, see Gentil [1996] and Rodríguez de Ceballos [1983, 1990].

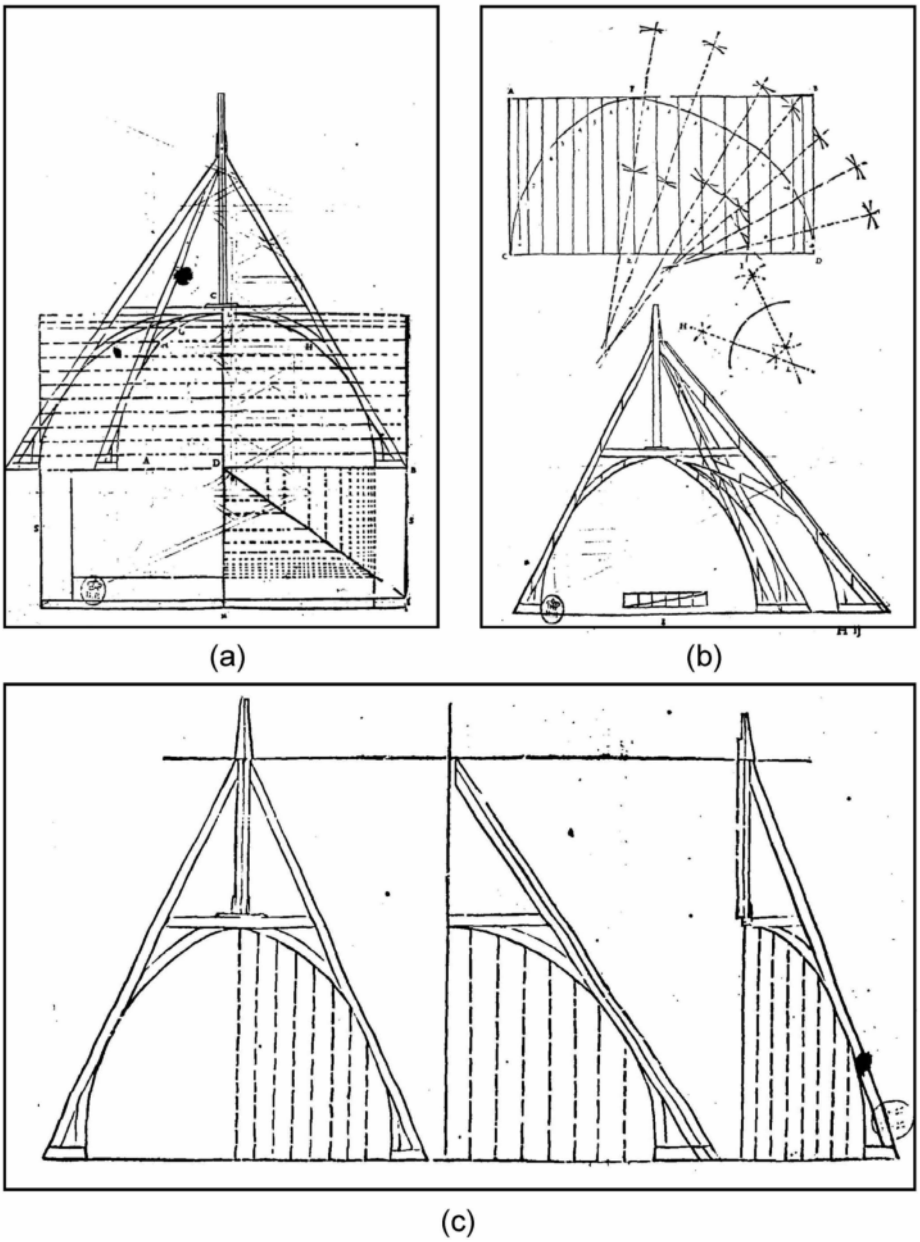


Fig. 23. The "lengthened arch" method in the manual of carpentry of Jousse [1627]

Apparently the idea of using the oval in different aspects of the arts was "in the air" at the beginning of the Cinquecento. Panofsky [1937] argued that Michelangelo's first project for the Tomb of Julius II already contained an interior oval space. According to Panofsky [1956], Correggio was the first painter to introduce an oval in a composition (*Madonna of St. Francis*, 1514, Dresden Gemäldegalerie), and Gian Maria Falconetto the first sculptor to employ one. In the book on perspective of Pélerin [1521] appear two ovals: the perspective drawing of an oval Renaissance arch and the oval frame of the illustration at the end of the treatise (fig. 24). It seems clear that the oval form exerted a new attraction to the artists at the beginning of the Cinquecento.

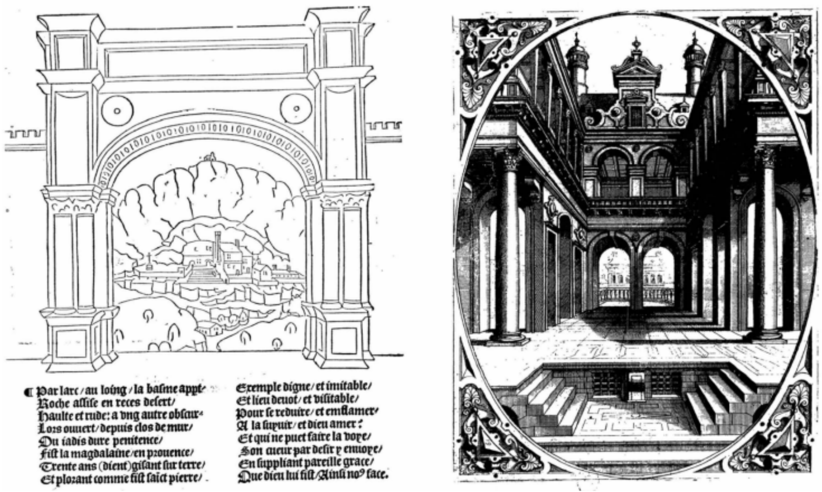


Fig. 24. Drawings with ovals in the treatise on perspective of Pélerin [1521]

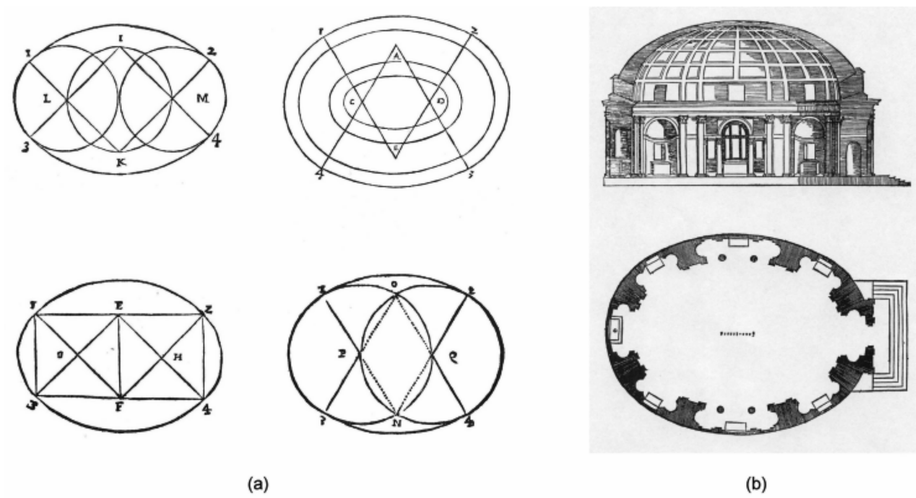


Fig. 25. a) Serlio's models for ovals in his Book I of 1545; b) Serlio's design of an oval temple in his Book V of 1547

The main architects in promoting the oval as a new form of defining the architectural space were Baldassare Peruzzi, Sebastiano Serlio and Giacomo Vignola. It was Peruzzi who first thought in taking advantage from the peculiarities of an oval space in church design, a compromise between the central space of the Quattrocento and the more linear character of traditional churches. However, his death left the diffusion of his ideas in the hands of his disciple Serlio. Indeed, Serlio's treatise [1996], one of the most popular architectural treatises ever published, was responsible of the spread of the oval form in the late Renaissance and Baroque in Europe. In his Book I on geometry, published in 1545, he includes a discussion on ovals. He says explicitly that it is possible to draw many different ovals and proposed four oval constructions (fig. 25a). These were copied again and again in later architectural manuals and were used many times in actual designs. However, architects and masons knew that for any two axes it is possible to construct many (in fact, infinite) different ovals and they departed from Serlio's models when desired.

Serlio includes another construction for ovals. He first alludes to the practice of masons of describing the ovals with strings: *molti muratore hanno una certa sua prattica, che col filo fanno simili volte*. The method alluded by Serlio has been considered by many scholars to be the so-called "gardeners-method" for drawing the ellipse with a string fixed in the two foci. Kitao [1974] provides another, simpler interpretation (fig. 26a). Further on, Serlio proposes another method, which he considers more precise: *Nondimeno se l'architetto vorrà procedere teoricamente, portato dalla ragione, potrà tener questa via*. He then gives a different method of projection. Two concentric circles are drawn and equally-spaced radii drawn to partition them. From the intersection of the radii with the larger circle are drawn vertical lines; from the intersection of the radii with the smaller circle are drawn horizontal lines. The intersections of these verticals and horizontals are points on the oval (fig. 26b). Of course, the curve is an ellipse, but Serlio did not name it so; more importantly, he did not know that it was an ellipse. This construction was also copied in subsequent treatises of the sixteenth and seventeenth centuries; after that, it gradually disappears from the manuals, although it continues to appear in the handbooks of practical geometry. (Hernán Ruiz, Vandelvira and Martínez de Aranda include it. However, in the usual stonecutting constructions they never used it; it only appears at the end of the manuscript by Vandelvira, in what looks as a theoretical exercise).

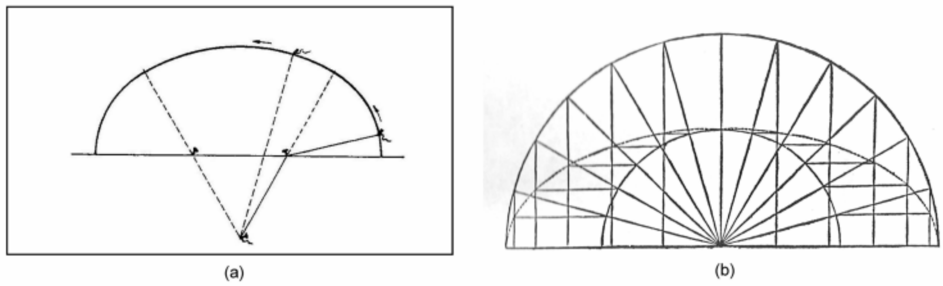
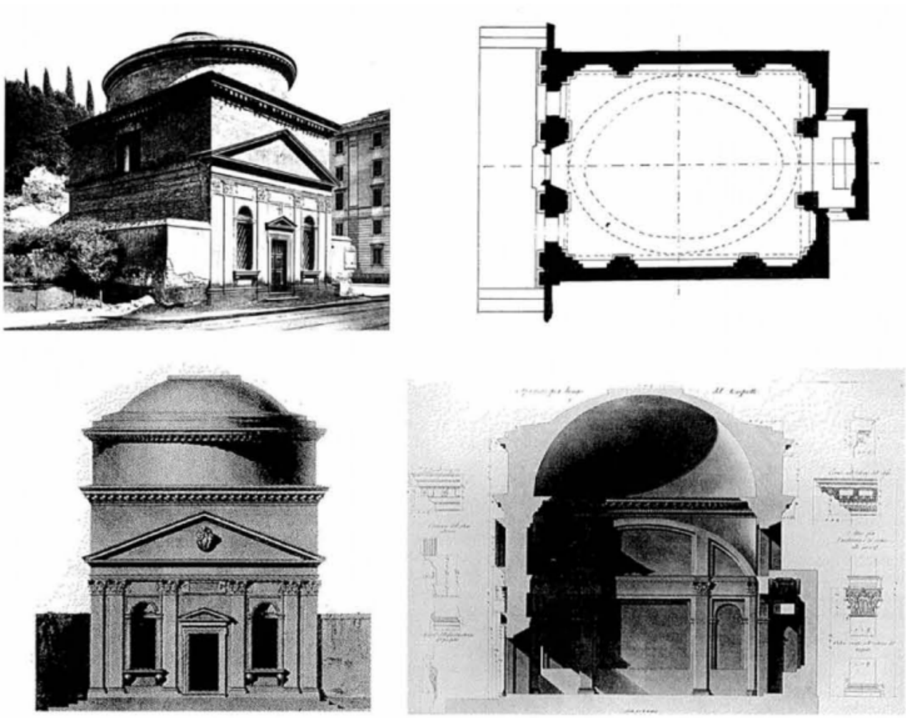
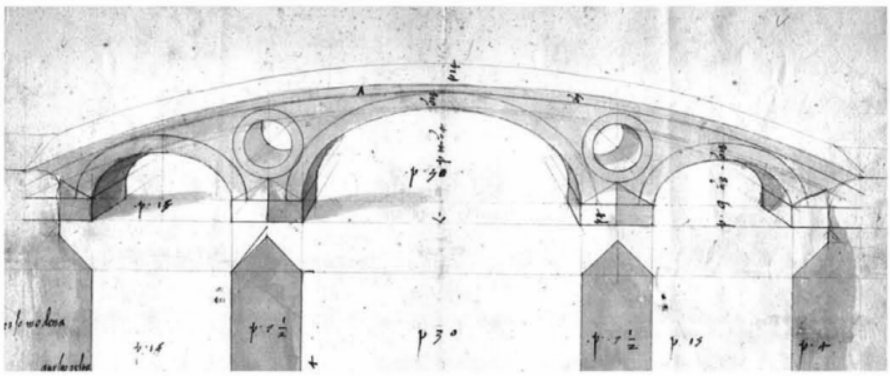


Fig. 26. a) Physical construction of an oval with a string and two pegs [Kitao 1974]; b) Serlio's construction of an oval *portato dalla ragione* [Serlio 1545]



(a)



(b)

Fig. 27. a) The first church built with an oval dome. S. Andrea in Via Flaminia by Vignola (1550-1554); b) First design of a bridge with oval arches by Vignola in 1547

When Serlio included the oval constructions in his Book I he was not thinking only of a geometrical problem; he had in mind the program of his master Peruzzi, the creation of a new type of temple. In his Book V on architecture, published two years later [1547], among a number of projects for temples, there appeared a design for an oval temple (fig. 25b), which exerted an enormous influence in the late Renaissance and Baroque. This is an enormous change from the previous uses of the oval: it became a fundamental argument of design and acquired an importance in defining architectural space which it had never had. Indeed, it surpassed the circle, which had been since classical antiquity the expression of geometrical perfection.

Peruzzi and Serlio prepared the theoretical way. It was Vignola who first put their ideas into practice building the first oval church, Sant'Andrea in Via Flaminia (1550-1554) (fig. 27a). Earlier, in 1547, Vignola had produced the first design of a bridge with surbased oval arches (fig. 27b). Vignola's design exerted no influence in the sixteenth and seventeenth centuries, but the use of oval arches in bridge design had become common by the beginning of the eighteenth century [Gautier 1716]. Polycentric oval arches were a central element in many of Perronet's bridges, in the second half of the eighteenth century [Perronet 1788].

After Serlio and Vignola, the oval dome spread quickly, not only in Italy, but in Spain [Gentil 1996], France [Châtelet-Lange 1978] and Central Europe [Fasolo 1931] as well. It is significant that Vandelvira's manuscript of stone cutting, ca. 1575, includes six different solutions of oval domes. There is no space in this short paper to enter into the subtleties of oval church design. The best architects of the Baroque exercised their ingenuity by solving the problems created by a non-central space. For example, Smyth-Pinney [1989] has studied in detail the design process followed by Bernini for the plan of S. Andrea al Quirinale and the subtle position of the axes of the perimeter chapels. Bernini's oval does not correspond to any of Serlio's models, and shows a delicate adjustment in the interior space (fig. 28). However, we may call Bernini's use of the oval "classical". As in Serlio's oval temple (fig. 25b, above), the section has the same form as the plan, and the references to the central churches of the Cinquecento are evident.

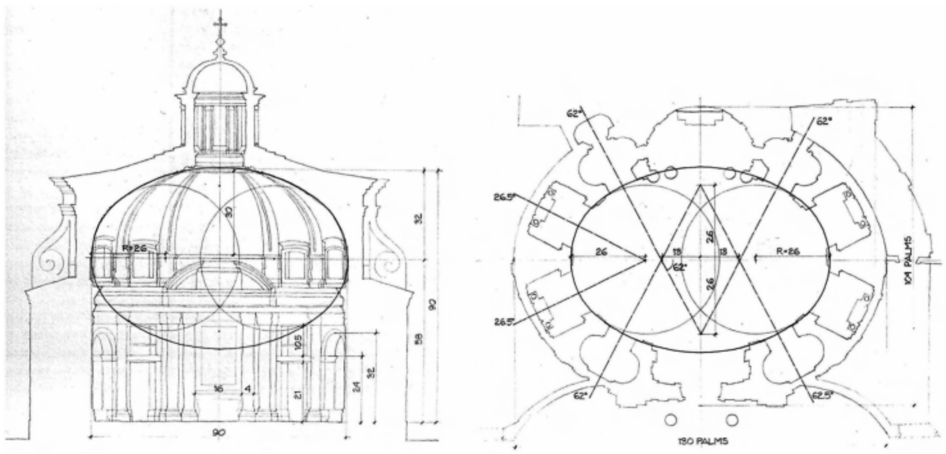


Fig. 28. The geometry of San Andrea al Quirinale [Smyth-Pinney 1989]

The project of Francesco Borromini for San Carlo alle Quattro Fontane (1663) presents a more complicated geometry, as the oval which generates the plan changes at the base of the dome (fig. 29). This last oval deviates very much from the usual form of ovals so far. No doubt, Borromini chose this form to provide "tension" in the space. Neither of the ovals corresponds with Serlio's models.

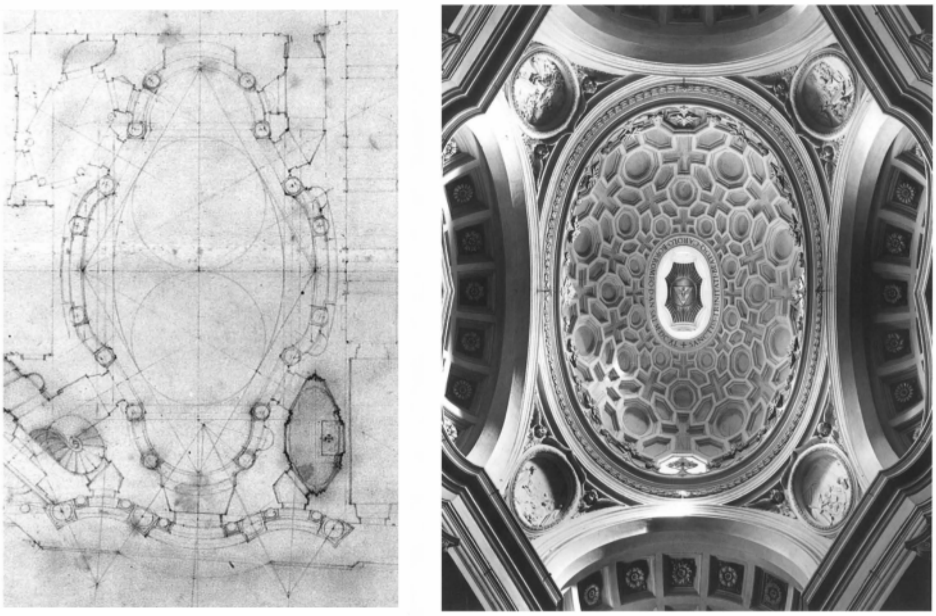


Fig. 29. San Carlo alle Quattro Fontane by Borromini. a) Design for the plan using a generating oval; b) Photograph of the dome showing the oval of impost at the base of the dome. Note the difference with the oval plan [Bellini 2004]

Geometry of oval domes

One of the main problems in the study of the geometry of oval arches and domes is the modern prejudice that an "oval" is an "approximation of the ellipse". In fact, as we have seen, as a geometrical figure the oval is much older than the ellipse, and there is a tradition of its use in building practice which can be traced back to the first arches in Mesopotamia and Egypt. As a mathematical concept, in geometrical terms, the ellipse is an intersection of a plane with a cone, or the locus of the points whose distance to two fixed points (foci) give a constant number. It is also an affine transformation of a circle. And we may define the curve mathematically in many other ways.

The study of the conics was a matter of "higher mathematics" up to the seventeenth century. The Greeks did not know a simple way to draw the ellipse. The string method was discovered by Anthemius of Tralles, a Byzantine architect and geometer of the sixth century. In the tenth century the Arab Alsigzï reported it, and in the late sixteenth and early seventeenth centuries it was known by Guidobaldo del Monte, Simon Stevin and Kepler [West 1978]. As a way to lay out arches, it was published in the treatise of fortification of Bachot [1598] (fig. 30a), and it began to appear in the building manuals at the beginning of the seventeenth century. In Spain, Fray Lorenzo de San Nicolás [1639], who wrote the most influential building manual in the Spanish language, after explaining the usual layout with circular arcs, cites the method and remarks that it is easy to lay out brick arches (fig. 30b and c). However, in the chapter on ovals he explains in detail the usual circular arc constructions, adding some new constructions to those of Serlio.

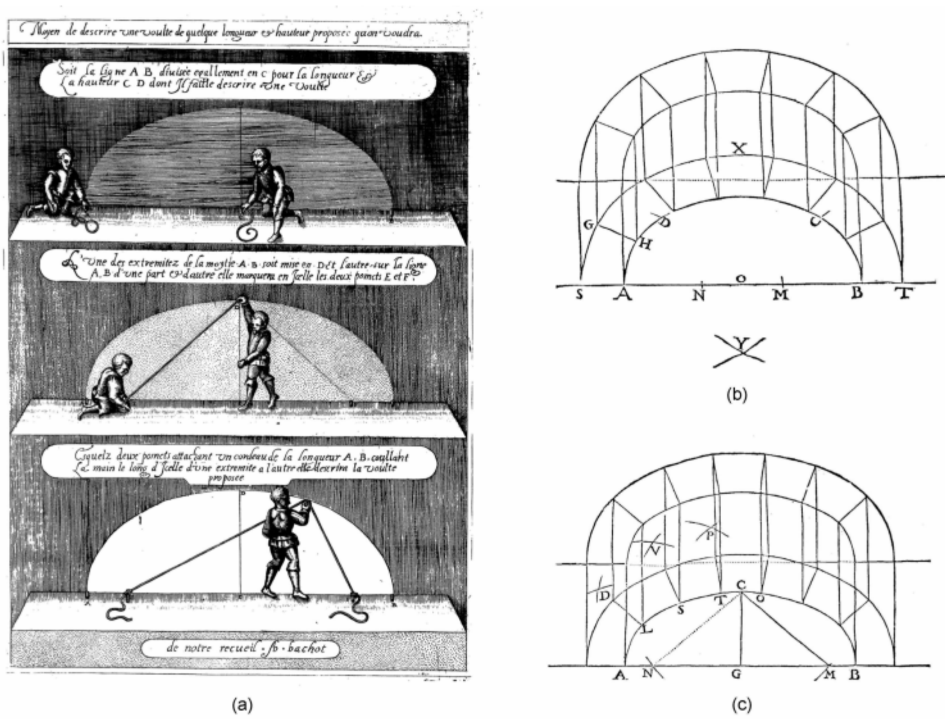


Fig. 30. a) Use of the string method to draw surbased arches in Bachot [1598]; b and c) Layout of surbased arches by employing circular arcs, and by the string method [San Nicolás 1639]

In the eighteenth century contempt for the oval on the part of learned engineers and architects began to grow. Frézier [1737] sharply criticized the use of ovals, and the title of Chapter IV of his Book II is illustrative: *De l'imitation des Courbes Régulieres par des compositions d'Arcs de Cercle*. He adds, *je ne conseille à personne d'avoir recours à cet artifice de l'ignorance*. However, he included a description of how to lay out different ovals. The treatise of geometry by Camus [1750] contains the most complete exposition of the different methods to lay out ovals for certain fixed conditions.

Thus it appears that in the mid-eighteenth century began a divorce between what is theoretically good and what was done in the practice. That a method is known does not necessarily imply that it must be used. The practical advantages of using circular arcs must have been taken into account. No doubt, Bernini and Borromini knew the methods of drawing ellipses, but they used ovals.

As was mentioned in the case of the Roman amphitheatres, some scholars believe that ascertaining whether the architect used an oval or an ellipse is a matter of mensuration. In at least one case, Rosin [2005] has demonstrated the practical impossibility of distinguishing an oval from an ellipse. Bendetti [1994] and Migliari [1995] arrived to the same conclusion after studying the geometry of the profile of the dome designed by Antonio de Sangallo for St. Peter's. Gentil [1996] claimed that the photogrammetrical survey of the oval dome of the Sala Capitular in the Cathedral of Seville demonstrated the use of ellipses, and other scholars agree with him [Rabasa 2000; Palacios 2003a]. The Sala has a ratio of length to width of 4:3, and we have seen in fig. 5a a geometrical construction for an oval of this proportion, the "Egyptian" oval. In fact, Vignola knew and used this constructions, as Gentil points out. For this geometry the difference between the ordinates of the oval and the corresponding ellipse amounts to less than 0.7% of the major span. It is not possible to reach this precision in building practice, and besides, the unavoidable movements suffered by masonry vaults and domes after decentring are also at least on the same order. We must arrive, then, at the same conclusion of the scholars cited above: determining the geometry of an actual oval of usual proportions is not a problem of mensuration. When Vigonla's oval is superimposed onto the reproduction of the photogrammetry in Gentil's article, the agreement is quite good (but of course, it is also with the ellipse) (fig. 31).

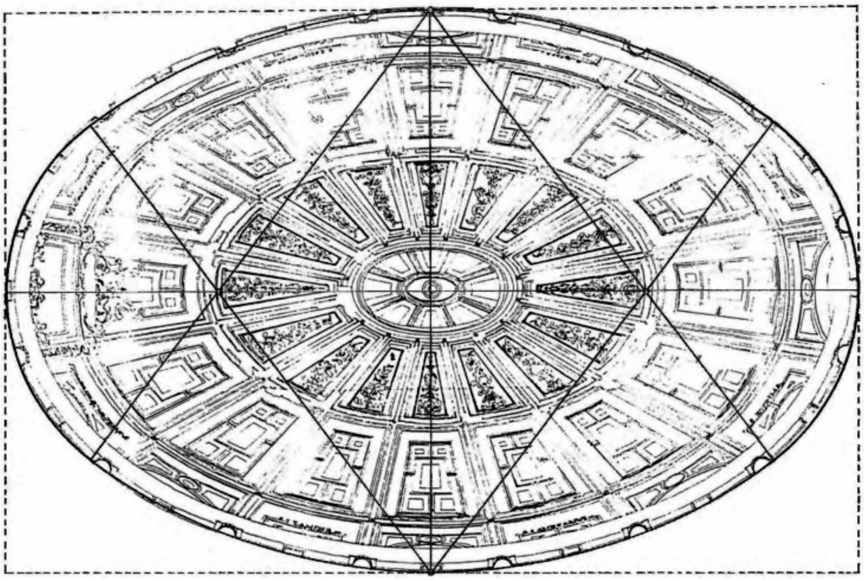


Fig. 31. The Sala Capitular of the Cathedral of Seville. Comparison of the photogrammetrical survey [Gentil 1996] with Vignola's 4:3 oval

To move forward towards solving the problem it is necessary to take into account the constructive tradition and the problems posed in practical building by the use of a curve such as the ellipse, which has a varying radius of curvature. In the case of a stone arch or dome, this will imply the use of different templates for each stone. We may also turn to documentary evidence. In the cases of S. Andrea and S. Carlo alle Quattro Fontane, the extant drawings show the employment of ovals. The same occurs in the case of the dome of Vicoforte [Aoki, et al. 2003]. In fact, this author does not know any plan of an oval dome of significant size (say a span of more than 10 m) that shows the use of the ellipse, although this does not mean that the ellipse may not have been used in exceptional cases. As an example, the Spanish architect Plo y Camin claimed in his treatise of architecture [1757] that he has laid out a brick oval dome with an ellipse compass formed by two wooden planks (no. 27 in fig. 32). This device can function only for domes of small size, as the bending of the planks will limit their size.

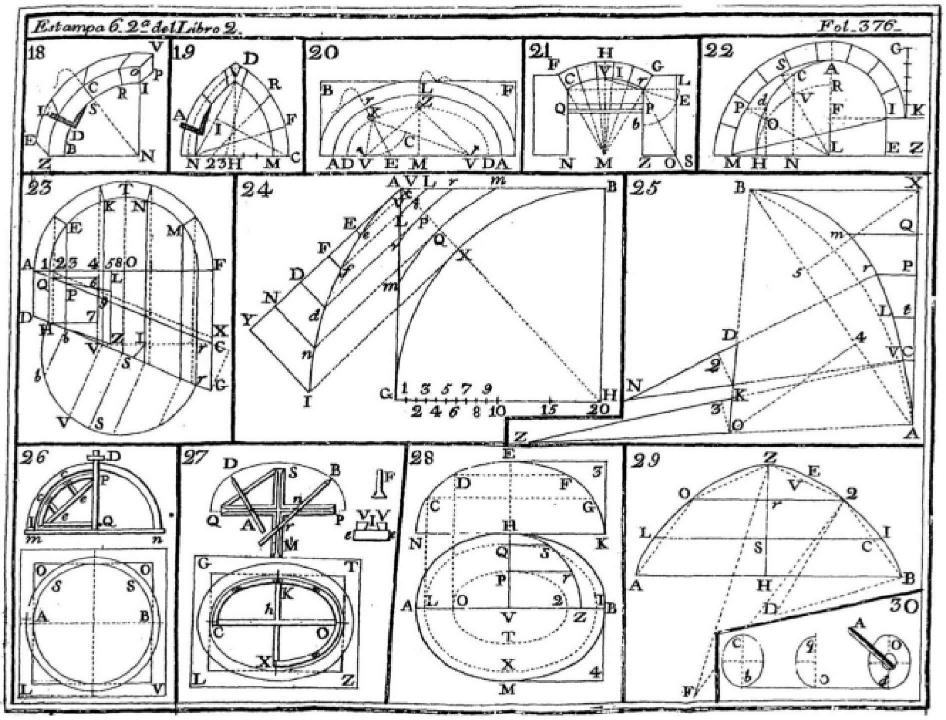


Fig. 32. Construction of oval domes in the treatise of Plo y Camín [1767]

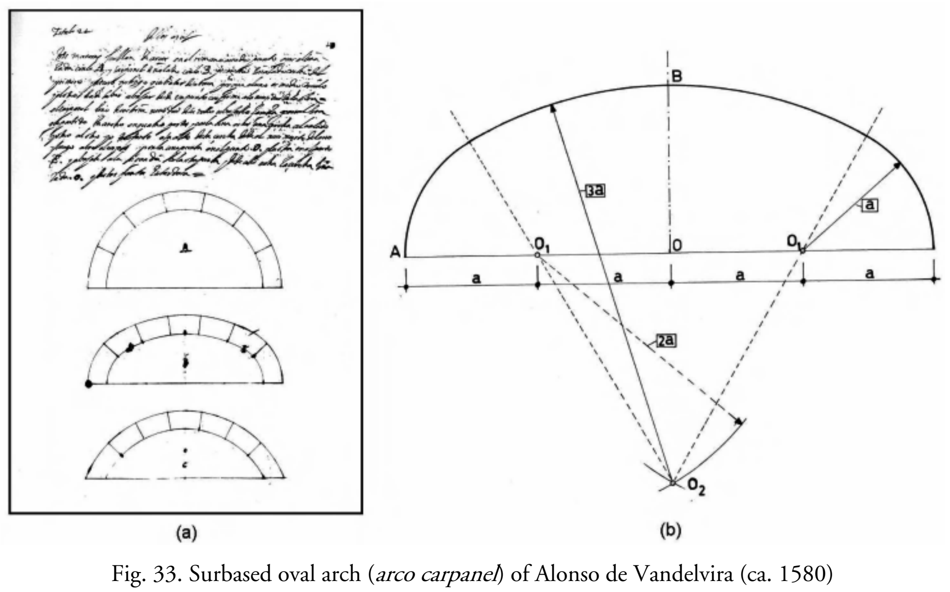


Fig. 33. Surbased oval arch (*arco carpanel*) of Alonso de Vandelvira (ca. 1580)

The oldest description of the geometry of oval domes is contained in the manuscript on stone cutting of Alonso de Vandelvira[1580]. There he explains six different types of *capillas ovales*, oval domes. All of them have the same plan – a *carpanel*, oval, of four centres – which Vandelvira uses througout his treatise (fig. 33). However, Vandelvira notes that other ovals can be chosen. The *capillas ovales tercera y cuarta*, third and fourth domes, provide a good opportunity for studying the geometry. Like the dome of the Sala Capitular (fig. 31), they are ribbed oval domes and the problem is to define the geometry of the "meridian" and "parallel" ribs.

Before discussing Vandelvira's construction some general comments on the building and geometry of oval domes should be made. The usual form of a dome has axial symmetry. In modern terms it may likened to a solid of revolution. For a master mason or architect, simply put, an arch defines the geometry of the dome. The advantages from the point of view of construction need not be explained. Besides, the dome may be erected without scaffolding, by closing successive rings of masonry. What is needed, then, are not centrings but "guides", a light scaffold which defines the geometry in space. Sometimes these arches were completed first and then the masonry shell between them was constructed. The advantage here is that the work can proceed faster, as the lime mortar of the arches will dry more quickly when four faces are exposed to the air as opposed to a single thick shell. Also, the placing of iron rings served to maintain the geometry while the mortar was setting. It was a complex process and there were several methods. It is necessary to keep in mind the remark of Rodrigo Gil de Hontañón when he described the closing of a cross vault: "... these things may be difficult to understand if one lacks experience and practice, or if one is not a stone mason, or has never been present at the closing of a rib vault" [García 1681]. The statement can be applied verbatim to the building of a masonry dome and reminds us of our basic ignorance about even the most simple operations of masonry construction.

The geometry of an oval dome is much more complex than that of the usual dome with a central vertical axis of symmetry. A modern architect or engineer will think of the usual ways of generating surface: rotation, translation, affine transformation, etc. The architect would have to think first in a general way of parameters which define the overall form of the intrados of the dome: the relationship between the two axes of the oval plan, the relationship between the height and the span, and the profile of the dome. All these parameters must have a certain relation with one another. Then comes the problem of how to build the dome, that is, how the surface of the intrados is going to be defined in the space, perhaps by a series of centrings. If the dome is to be made of stone, the geometrical definition must take into account the cutting of the stones. All the processes of building are subtended by a principle of economy.

A simple way of defining the geometry is to fix the profiles of the two sections following the major and minor axis of the oval plan. In Serlio's design of the oval temple (fig. 25, above), the curve which forms the section of the dome is the same oval as that of the plan. Because the height is the same as half the minor axis, the transverse cross section will be a circle, the simplest option. Any other transverse section is also a semicircle and by placing semicircular transverse centring the dome may easily built by successive rings, until it is closed. For a modern architect or engineer this is, of course, a surface of revolution around the major axis, but it is quite improbable that Serlio would have thought of it in such a way. Many Renaissance and Baroque oval domes have this kind of geometry. See, for example, San Andrea del Quirinale (fig. 28).

The same approach may applied considering that the transverse section has the form of the semi-oval and that the longitudinal section is a semicircle. This was the approach taken in Sant'Andrea in Via Flaminia (fig. 27a) and in the dome of the Cesarean Library in Vienna (fig. 34).

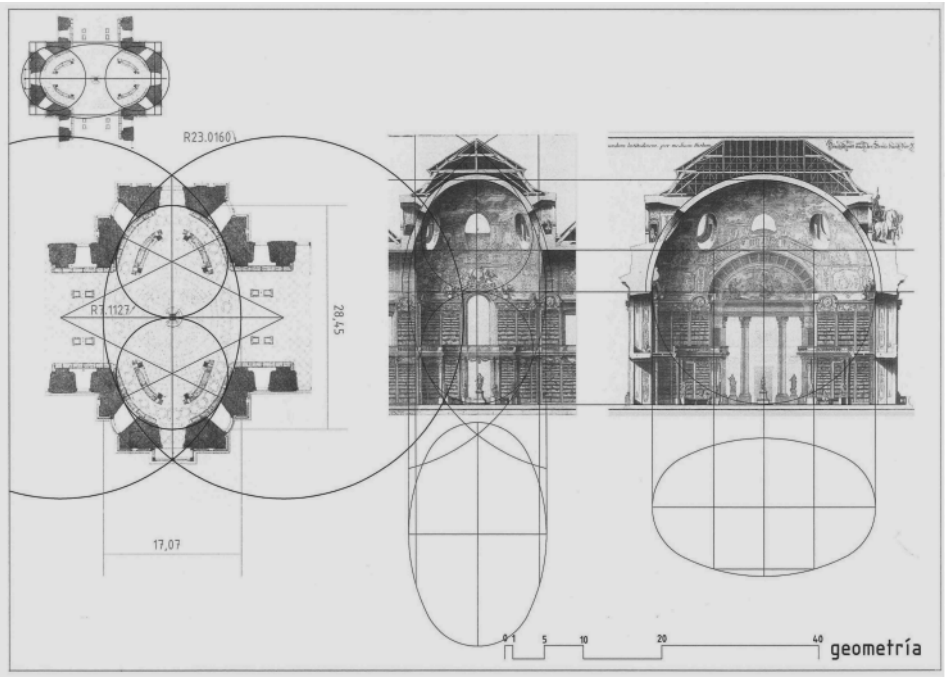


Fig. 34. Geometry of the oval dome of the Cesarean Library in Vienna (drawing by C. Pérez de los Ríos, after G. López Manzanares [2005])

Again, the logical placement of the successive centrings for the building of the dome will be the transverse sections, but what is the form of the shorter transverse sections? Now, it is not as simple as in the previous case, in which a semicircle can be supposed. It will be simple to think that these sections have the same form of the transverse minor section, that is, that they are similar ovals of diminishing size. To fulfil this condition the vertical ordinates of the circle and the oval must have a constant relationship, i.e., the oval is an ellipse. This is the mathematical point of view. Thus it follows that the oval plan must be elliptical to permit economical building. The point of view of the builder is that the oval fulfils the condition within the tolerances of building, something of the order of 1-3% of the span. Indeed, the deviation of the ordinates of the ovals used in building is less than this. Therefore the false supposition that transverse sections are all similar becomes "practically true". Building manuals are full of false "practical truths", to approximate values (for example, the square root of 2 is approximated to 7/5 with an error of 1% in the late Gothic German architectural manuals, and the practical value of π is 22/7, with less than 0.1% error [Huerta 2004]. The standards of surveying and levelling could not guarantee more precision and therefore the use of approximate numerical values and geometrical constructions is not only fully justified, but shows how clever the old master builders were.

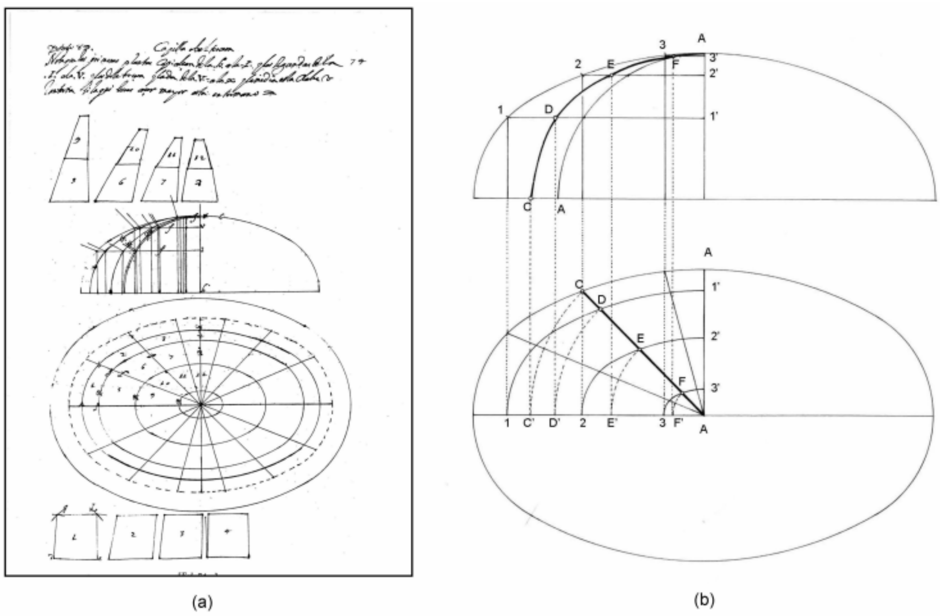


Fig. 35. a) Third oval dome (*capilla oval tercera*) of Vandelvira (Library of the E.T.S. de Arquitectura de Madrid); b) Method for drawing a "meridian rib"

Now, in order to discuss Vandelvira's solution for oval domes, we should keep in mind that he was not looking for the exact solution of the problem but rather for a practical solution that was easy to construct and draw. The process is clear with reference of the third oval dome (fig. 35a). He is considering an oval dome with the same longitudinal section as half the oval plan. Further, the cross section is a semicircle A-A. The longitudinal section is divided in seven parts by points 1, 2, 3, etc. (we work with only half section). Then horizontal lines are drawn and points 1′, 2′, 3′ are obtained on the semicircular cross section. Also, verticals from points 1, 2, 3, are projected onto the plan. Then, in the plan, we know that an horizontal section of the dome surface will give a line which must pass through points 1-1′, 2-2′, etc. What is this curve? If the surface is generated by the revolution of the oval plan around the long axis, we may draw this line point by point. Or, if the plan is supposed be an ellipse, then, it will be an ellipse. But the first procedure would be too long and Vandelvira did not know the second property. He then assumes that every horizontal section has a form similar to the oval of the plan. Of course, this is not true, as the proportion between the semi-axes A1-A1′, A2-A2′, ..., need not coincide with the proportion of the semi-axes of the oval plan. But the error is small in practice, and even the drawing can be "forced" by slightly opening or closing the compass, as the present author has done to make the drawing (a practice that any student of pre-CAD technical drawing of times knows very well; one of the problems of drawing by computer is that it is very difficult to "trick" a solution). Is Vandelvira ignorant? Frézier would have said this. But in fact, he is very clever: he is using a very simple graphical construction to solve a problem that, expressed in rigorous geometrical terms, would have been impossible. He is not a mathematician; he is a builder. In fact the approximation functions because of the small difference between ovals and ellipses alluded to. The construction will be exact for an ellipsoid of revolution, but he did not need to know this. Vandelvira uses the same approach in the solution of fourth oval dome (fig. 36) and, most probably, the Sala Capitular in Sevilla would have been laid out following the same method.

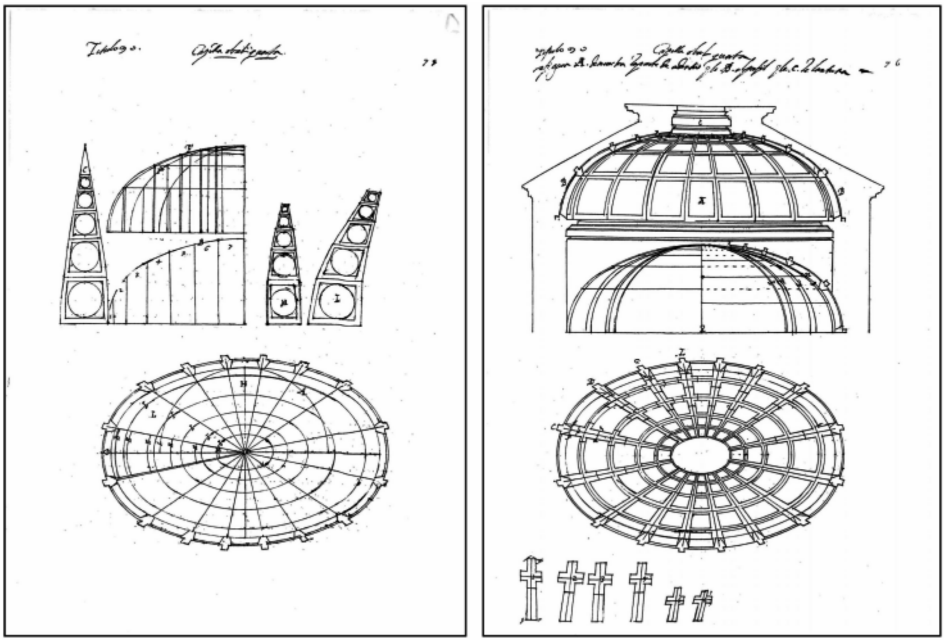


Fig. 36. Fourth oval dome (*capilla oval cuarta*) of Vandelvira (Library of the E.T.S. de Arquitectura de Madrid)

Mechanics of oval arches and domes

An “arch” is the natural way to span a void with a material in compression only, i.e., piling stones of a certain shape, so that the resulting geometry is stable: the stones,wanting to fall due to the force of gravity, remain in place due to their mutual interaction.

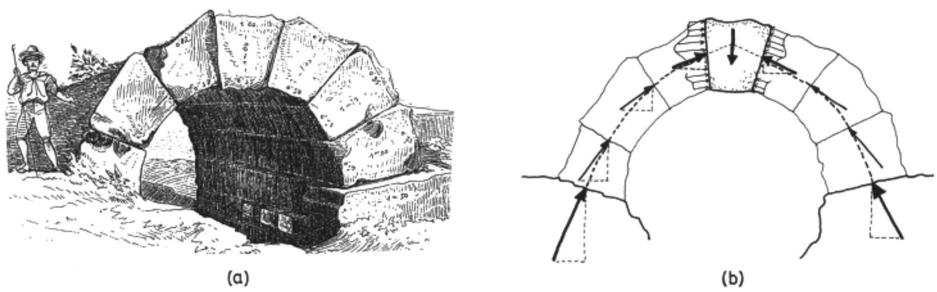


Fig. 37. Equilibrium of a voussoir masonry arch [Huerta 2004]

In fig. 37 we can see a massive Etruscan voussoir arch. Heavy stones have been cut in the form of wedges; then they have been placed on a formwork (a centring) beginning from the extremes. When the last stone in the middle (the keystone) was placed the centring was removed and the arch stood. (The best exposition of the mechanics of masonry structures is [Heyman 1995]).

Let us consider the free-body equilibrium of the keystone. In each joint (which we imagine more or less planar) there will exist a certain stress distribution. The stress resultant must be a compressive force, a "thrust"; the point of application is the "centre of thrust" and it must be contained within the plane of the joint. The two thrusts in the joints maintain the keystone in equilibrium. The same occurs with the other stones until we arrive at the springing of the arch. There the abutment must supply/resist a certain thrust. This is the "thrust of the arch", and the abutment must have adequate dimensions to resist it.

The masonry arch always pushes outwards; "the arch never sleeps", says an old proverb attributed to the Arabs. The locus of the centre of thrust forms a line, the "line of thrust". The form of this line depends, therefore, on the geometry of the arch. Of course, as masonry must work in compression, the line of thrust must be contained within the masonry arch.

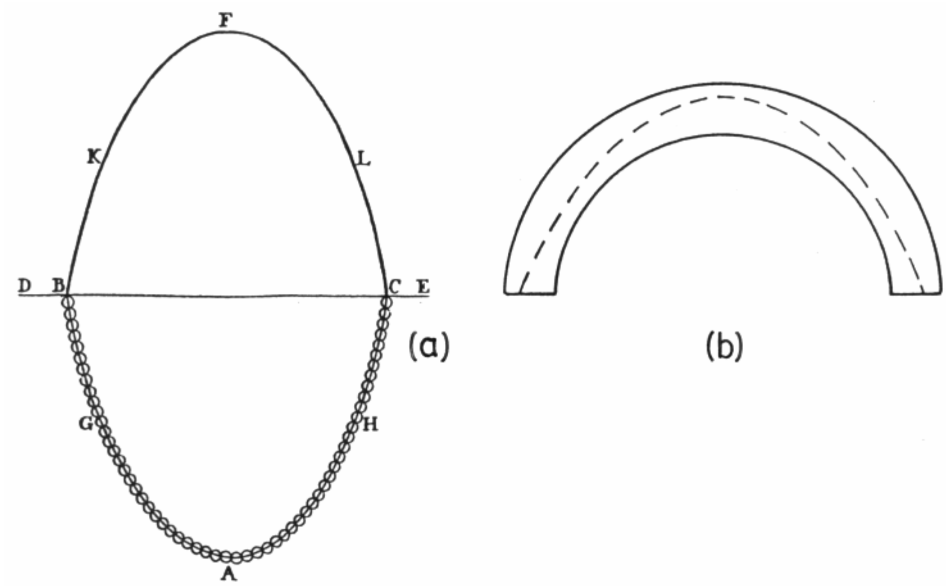
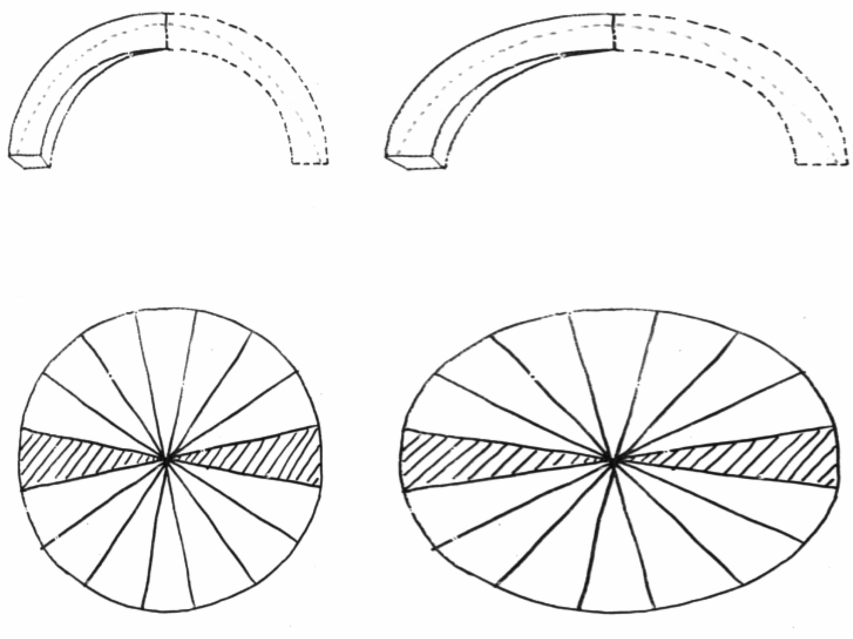


Fig. 38. Possible line of thrust of a semicircular arch [Heyman, 1995]

We may imagine one voussoir acting against the two voussoirs adjacent to it only through the centres of thrust. If we now invert the arch, what was a force of compression will be a force of tension: the voussoirs are hanging like a chain. The statics of arches and hanging cables is the same. (This was Hooke's brilliant idea, ca. 1670.) If all the voussoirs of the arch are the same size the line of thrust will have very nearly the form of an inverted catenary. But, it is not necessary that the arch have the form of catenary: it suffices that the catenary can be contained within the masonry. Therefore, in fig. 38 the arch is in equilibrium with an internal stress distribution represented by the inverted catenary. The drawing has no scale and it is evident that the safety is a matter of geometry, independent of size [Heyman 1995; Huerta 2006].

A dome can be imagined as composed by a series of arches obtained slicing the dome by meridian planes. Every two "orange slices" form an arch; if it is possible to draw a line of thrust within this arch, then we have found a possible equilibrium state in compression and the dome is safe, that is, it will not collapse (fig. 39a) [Heyman 1995]. The dome may have an oculus because a compression ring forms: the dome build a "keystone" when a ring is closed, and therefore masonry domes can be built without centring The safety condition is, then, a geometrical condition. Domes of similar forms and different sizes have the dame degree of safety: the drawing of fig. 39a has no scale.

The same "slicing technique" can be applied to oval domes [Huerta 2004]. Now the elementary arches are different, but equal the two opposed which can build a safe arch. If it is possible to find a line of thrust within each pair of elementary arches, the dome will be safe (fig. 39b). The only difference is that, since the arches are different, the compression ring has an oval form. As the oval form of the compressions need not coincide with the oval form of the oculus, the oculus must have sufficient thickness to accommodate the compression ring within the masonry. Again, the safety depends on geometry and not on the size of the dome.



(a) (b)

Fig. 39. Use of the "slicing technique" to study the safety of masonry domes. a) Domes of revolution; b) Oval domes

Scaling up and down is a particular case of an affine transformation. Rankine [1858] discovered that the stability of masonry structures subject only to dead load remained unaltered after an affine transformation: the line of thrust of the transformed arch is the affine transformation of the original line. The relative distances to the limits of masonry does not change, and, therefore, the safety is the same (fig. 40). (For a detailed discussion see [Huerta 2004]).

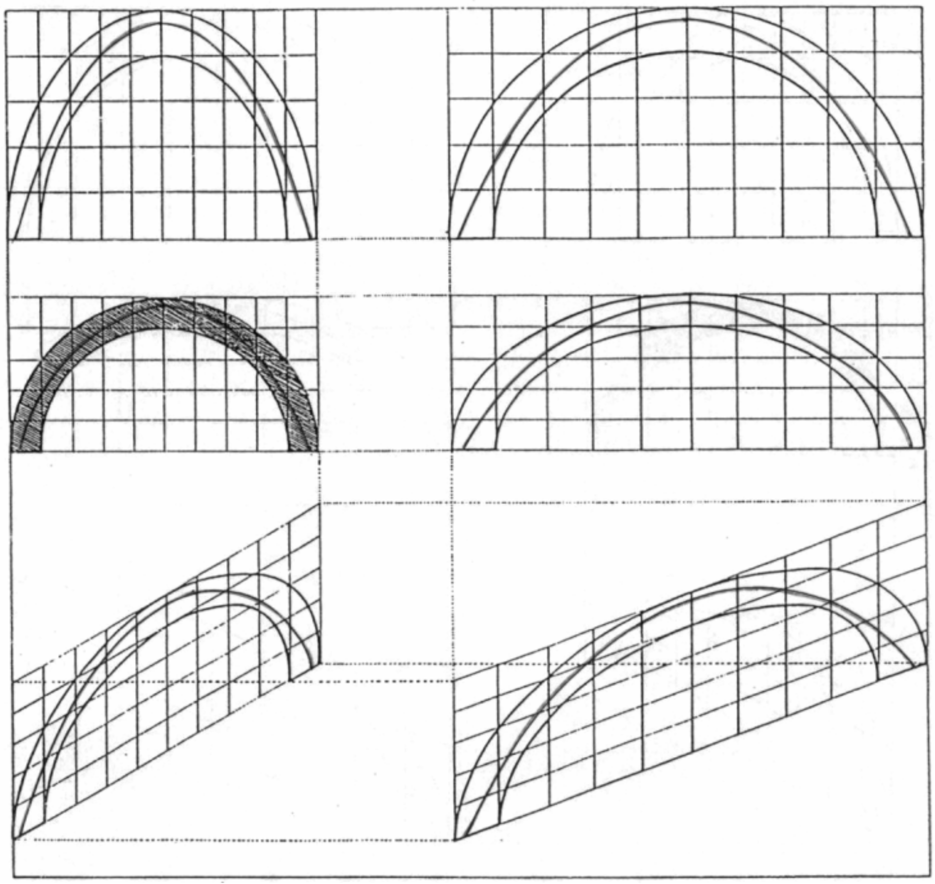


Fig. 40. Affine transformations of a masonry arch and its line of thrust: the safety remains the same [Huerta 2004]

The same occurs with domes. If we transform a stable dome by an affinity, the transformed dome has the same degree of safety and, again, the surface of thrust, representing the internal equilibrium, is the transformation of the original surface of thrust. Fig. 41a represents the statical analysis of the model of a simple dome proposed by Fontana [1694]. Fig. 41b represents another dome, obtained by contracting the height and width of the original dome. The line of thrust represents the equilibrium of the slice arch having the major axis. The stability remains unaltered. It may be that the confidence of the architects in the stability of oval domes originates in the intuition of this principle.

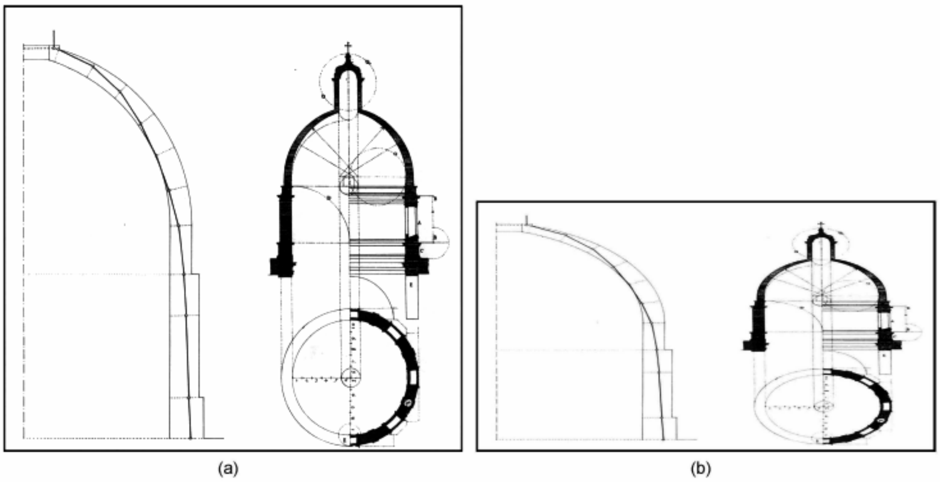


Fig. 41. a) Design of a simple dome by Fontana [1694] and analysis of its stability [Huerta 2004]; b) Safe oval dome obtained contracting the height and broad of the previous dome. The stability remains unaltered

Conclusion

Oval arches and domes form part of the tradition of building in masonry from the very beginning of the invention of the arch. The approximately egg-shaped forms were made regular through the use of practical geometry, perhaps about 2000 B.C., when the dimensions and the importance of the construction required it. The oval forms, generated by the tangency of circular arcs remained as an essential part of the architecture of Asia Minor and were incorporated in Islamic architecture, forming polycentric pointed arches. The Romans used the oval to design their amphitheatres. The oval arch reappeared in Europe in the Middle Ages as a device for solving practical problems: a dome over a rectangular bay, a surbased arch. In late Gothic, the master builders broadened the geometrical possibilities by employing oval ribs. They probably deduced the method of the "lengthened arch" that appears in the treatises of the sixteenth century. In the Cinquecento, Renaissance architects found in the oval the motif and the solution for a new architectural type: the church with an oval plan. This type was much appreciated during Baroque and late Baroque. The geometrical problems of laying out an oval surface were solved by the simplest methods, sufficiently close in approximation for practice. Though the ellipse was incorporated into building treatises in the seventeenth century, it was rarely used, and was probably never used at all in stone domes.

The mechanics of an oval dome is analogous to a dome of revolution, and the same techniques of analysis can be used. The dome will be stable if it is possible to find a thrust surface, in equilibrium with the loads, within the masonry. This is a matter of geometry. It turns out, that a "lengthened arch" or a "lengthened dome" has the same stability as the original, undistorted, arch or dome. Rankine demonstrated this property in the mid-nineteenth century, but it is obvious that the architects knew this property intuitively. This intuition was probably the basis for the confidence of architects who designed and constructed large oval domes from the beginning.

The study of the influence of mathematics in building cannot be made without considering the essential objective of a builder: to build. The complication of the geometry was always tempered by the common sense of the builder and the imperative necessity to erect a safe building in a certain time. It is true that architects enjoyed the play of geometry, but play presupposes some rules, and they found their liberty within the rules of construction.

References

AOKI, T., M.A. CHIORINO, and R. ROCATTI. 2003. Structural characteristics of the elliptical masonry dome of the Sanctuary of Vicoforte. Pp. 203-212 in *Proceedings of the First International Congress on Construction History*, S. Huerta, ed. Madrid: Instituto Juan de Herrera.

ARNOLD, Dieter. 1991. *Building in Egypt. Pharaonic stone masonry.* Oxford: Oxford University Press.

BACHOT, Ambroise. 1598. *Le Gouvernail d'Ambroise Bachot capitaine ingenieur du Roy, Lequel conduiraie curieux de Geometrie en perspective dedans l'architecture des fortifications, machines de guerre et plusieurs autres particularites et contenues.* Melun : Chez l'Auteur.

BAIMATOVA, Nasiba. 2002. Die Kunst des Wölbens in Mittelasien. Lehmziegelgewölbe (4.-3. Jh. v. Chr. - 8. Jh. n. Chr.). Dissertation: Institut für Vorderasiatische Altertumskunde, Freie Universität Berlin.

BELLINI, Federico. 2004. *Le cupole di Borromini: la "scientia" costruttiva in età barocca.* Milan: Electa.

BENEDETTI, Sandro. 1994. Oltre l'antico e il gotico. Il profilo della cupola vaticana di Antonio da Sangallo il Giovane. *Palladio* **14**: 157-166.

BESENVAL, Roland. 1984. *Technologie de la voûte dans l'Orient Ancien.* 2 vols. Paris: Editions Recherche sur les Civilisations.

BOYD, Thomas D. 1978. The arch and the vault in Greek architecture. *American Journal of Archaeology* **82**: 83-100.

BUCHER, François. 1968. Design in Gothic Architecture. A Preliminary Assessment. *Journal of the Society of Architectural Historians* **27**: 49-71.

BUCHER, François. 1972. Medieval Architectural Design Methods, 800-1560. *Gesta* **11**, 2: 37-51.

CALVO LÓPEZ, José. 2002. La semielipse peraltada. Arquitectura, geometría y mecánica en las últimas décadas del siglo XVI. Pp. 417-435 in *Actas del Simposium El Monasterio del Escorial y la Arquitectura* (San Lorenzo del Escorial, 8 al 11 de noviembre de 2002). San Lorenzo del Escorial.

CAMUS, M. 1750. *Elémens de géométrie théorique et pratique (Cours de mathématique, Seconde Partie).* Paris: Durand.

CARAZO, Eduardo and Juan Miguel OTXOTORENA. 1994. *Arquitecturas centralizadas. El espacio sacro de planta central: diez ejemplos en Castilla y León.* Valladolid: Servicio de Publicaciones de la Universidad de Valladolid.

CEJKA, Jan. 1978. Tonnengewölbe und Bogen islamischer Architektur. Wölbungstechnik und Form. Dissertation: München. Techn. Univ. Fachbereich Architektur.

CHAPPUIS, R. 1976. Utilisation du tracé ovale dans l'architecture des églises romanes. *Bulletin Monumental* **134**: 7-36.

CHOISY, Auguste. 1883. *L'art de bâtir chez les Byzantines.* Paris: Librairie de la Societé Anonyme de Publications Périodiques.

———.1904. *L'art de bâtir chez les égyptiens.* Paris: E. Rouveyre.

———. 1904. Note sur deux épures égyptiennes conservés à Edfou. *Journal of the Royal Institute of British Architects* **11**: 503-505.

DARESSY, Georges. 1907. Un tracé egyptienne d'une voûte elliptique. *Annales du Service des Antiquités de l'Egupte* **8**: 234-241.

DE L'ORME, Philibert. 1561. *Nouvelles inventions pour bien bastir et a petits fraiz.* Paris: Morel.

———. 1567. *Le premier tome de l'Architecture.* Paris: Morel.

DORNISCH, Klaus. 1992. *Die griechischen Bogentore. Zur Entstehung und Verbreitung des griechischen Keilsteingewölbes.* Frankfurt am Main: Peter Lang.
DÜRER, Albrecht. 1525. *Unterweisung der Messung.* Nürnberg. (facsimile edition, Nördligen: A. Uhl, 1983).
EL-NAGGAR, Salah. 1999. *Les voûtes dans l'architecture de l'Égypte ancienne.* Le Caire: Institut Français d'Archéologie Orientale.
FASOLO, Vincenzo. 1931. Sistemi ellittici nell'archittetura. *Architettura e Arti Decorative* 7: 309-324.
FATHY, Hassan. 1976. *Architecture for the Poor. An Experiment in Rural Egypt.* Chicago/London: University of Chicago Press.
FONTANA, Carlo. 1694. *Il tempio Vaticano e sua origine*. Rome : Nella Stamparia di Gio : Francesco Buagni.
FREZIER, Amédée-François. 1737-39. *La théorie et la pratique de la coupe de pierres et des bois pour la construction des voûtes et autres parties des bâtiments civils et militaires, ou traité de stéréotomie à l'usage de l'architecture.* Strasbourg/Paris: Charles-Antoine Jombert.
GAUTIER, Hubert. 1716. *Traité des Ponts.* Paris: Cailleau.
GENTIL BALDRICH, José María. 1996. La traza oval y la sala capitular de la catedral de Sevilla. Una aproximación geométrica. Pp. 77-147 in*Quatro edificios sevillanos.* J. A. Ruiz de la Rosa et al., eds. Seville: Colegio Oficial de Arquitectos de Andalucía, Demarc. Occidental.
GÓMEZ-Moreno, Manuel. 1919. *Iglesias Mozárabes. Arte español de los siglos IX al XI.* Madrid: Centro de Estudios Históricos.
GÖTZ, Wolfgang. 1968. *Zentralbau und Zentralbautendenz in der gotischen Architektur.* Berlin: Gebr. Mann Verlag.
GOYON, J.-C., J.-C. GOLVIN, C. SIMON-BOIDOT and G. MARTINET. 2004. *La construction Pharaonique, du Moyen Empire à l'époque gréco-romaine.* Paris: Picard.
HEATH, Thomas. 1981. *A History of Greek Mathematics.* 1921. Reprint, New York: Dover Publications.
HEISEL, Joachim P. 1993. *Antike Bauzeichnungen.* Darmstadt: Wissenschaftliche Buchgesellschaft.
HEYMAN, Jacques. 1995. *The Stone Skeleton. Structural Engineering of Masonry Architecture.* Cambridge: Cambridge University Press.
HUERTA, Santiago. 2004. *Arcos, bóvedas y cúpulas. Geometria y equilibrio en el cálculo tradicional de estructuras de fábrica.* Madrid: Instituto Juan de Herrara.
———. 2006. Galileo was wrong! The geometrical design of masonry arches. *Nexus Network Journal* 8: 25-52.
JOUSSE, Mathurin. 1702. *L'art de Charpenterie . . . corrigé et augmenté . . . par M. de La Hire*. 1627. Paris: Thomas Moette.
KITAO, Timothy K. 1974. *Circle and Oval in the Square of Saint Peter's. Bernini's Art of Planning.* New York. New York University Press.
KOEPF, Hans. 1969. *Die gotischen Planrisse der Wiener Sammlungen.* Vienna: Hermann Böhlaus Nachf.
KRAUTHEIMER, Richard. 1984. *Arquitectura paleocristiana e bizantina*. Madrid: Cátedra.
LÓPEZ MANZANARES, Gema. 2005. La contribución de R. G. Boscovich al desarrollo de la teoría de cúpulas: el informe sobre la Biblioteca Cesarea de Viena. Pp. 655-665 in *Actas del Cuarto Congreso Nacional de Historia de la Construcción* (Cádiz, 27-29 January 2005). S. Huerta, ed. Madrid: Instituto Juan de Herrera.
LOTZ, Wofgang. 1955. Die ovalen Kirchenräume des Cinquecento. *Römisches Jahrbuch für Kunstgeschichte* 7: 7-99.
LUCIANI, Roberto. 1993. *El Coliseo.* Madrid: Anaya.
MARTÍNEZ DE ARANDA, Ginés. [ca. 1590] *Cerramientos y trazas de montea.* Ms. Biblioteca de Ingenieros del Ejército de Madrid.
———. 1986. *Cerramientos y trazas de montea.* Madrid: Servicio Histórico Militar, CEHOPU.
MIGLIARI, Riccardo. 1995. Elissi e ovali: epilogo di un conflitto. *Palladio* 8, 16: 93-102.

MÜLLER, Johann Heinrich. 1967. *Das regulierte Oval. Zu den Ovalkonstruktionen im Primo Libro di Architettura des Sebastiano Serlio, ihrem architekturtheoretischen Hintergrund und ihrer Bedeutung für die Ovalbau-Praxis von ca. 1520 bis 1640.* Bremen.

MÜLLER, Werner. 1971. Der elliptische Korbbogen in der Architekturtheorie von Dürer bis Frézier. *Technikgeschichte* **38**: 93-106.

———. 1972. Die Lehrbogenkonstruktion in den Proberissen der Augsburger Mauermeister aus den Jahren 1553-1723 und die gleichzeitige französische Theorie. *Architectura* **2**: 17-33.

———. 1990. *Grundlagen gotischer Bautechnik.* München: Deutscher Kunstverlag.

———. 2002. *Von Guarino Guarini bis Balthasar Neumann. Zum Verständnis barocker Raumkunst*. Petersberg: Michael Imhof Verlag.

NAVASCUÉS PALACIO, Pedro. 1974. *El libro de arquitectura de Hernán Ruiz el Joven. Estudio y edición crítica.* Madrid: Escuela Técnica Superior de Arquitectura.

NOBILE. Marco Rosario. 1996. Chiese a pianta ovale tra Controriforma e Barocco: Il ruolo degli ordini religiosi. *Palladio* **9**, 17: 41-50.

PALACIOS GONZALO, José Carlos. 2003. *Trazas y cortes de cantería en el Renacimiento español.* Madrid: Munilla-Lería.

———. 2003. Spanish ribbed vaults in the 15th and 16th centuries. Pp. 1547-58 in *Proceedings of the First International Congress on Construction History*, S. Huerta, ed. Madrid: Instituto Juan de Herrera.

PANOFSKY, Erwin. 1937. The First Two Projects of Michelangelo's Tomb of Julius II. *Art Bulletin* **19**: 561-579.

———. 1956. Galileo as a Critic of the Arts: Aesthetic Attitude and Scientific Thought. *Isis* **47**: 3-15.

PELERIN, Jean. 1521. *De Artificiali Perspectiva.* Tulli.

PERRONET, Jean-Rodolphe. 1788. *Ses Oeuvres.* Paris: Didot.

PETRIE, W. M. Flinders. 1879. On Metrology and Geometry in Ancient Remains. *Journal of the Antropological Institute of Great Britain and Ireland*, Vol. 8, pp. 106-116.

PEIFFER, Jeanne. 1995. Dürer géométre. Pp. 15-131 in *Géométrie*, by A. Dürer. Paris: Éditions du Seuil.

PLO Y CAMIN, Antonio. 1767. *El Arquitecto Práctico, Civil, Militar, y Agrimensor, dividido en tres libros*. Madrid: Imprenta de Pantaleón Aznar.

RABASA DÍAZ, Enrique. 2000. *Forma y construcción en piedra. De la cantería medieval a la esteorotomía del siglo XIX.* Madrid: Akal.

RANKINE, W. J. M. 1858. *A Manual of Applied Mechanics.* London: Charles Griffin. (3rd ed. 1864.)

RASCH, Jurgen J. 1985. Die Kuppel in der römischen Architektur. *Architectura* **15**: 117-139

RODRÍGUEZ G. DE CEBALLOS, Alfonso. 1983. Entre el manierismo y el barroco, iglesias españolas de planta ovalada. *Goya* **177**: 98-107.

———. 1990. La planta elíptica: De El Escorial al clasicismo español. *Anuario del Departamento de Historia y Teoría del Arte* **2**: 151-172.

ROSIN, P. L. 2001. On Serlio's construction of ovals. *Mathematical Intelligencer* **23**: 58-69.

ROSIN, P. L. and E. TRUCCO. 2005. The amphitheatre construction problem. *Incontro Internazionale di Studi Rileggere L'Antico* (Rome, 13-15 December 2004).

ROSSI, Corina. 2004. *Architecture and Mathematics in Ancient Egypt.* Cambridge: Cambridge University Press.

RUBIÓ, Juan. 1914. Construccions de pedra en sec. *Anuario de la Asociación de Arquitectos de Cataluña* : 35-105.

RUIZ, Hernán el Jovan. ca. 1545. *Libro de arquitectura.* Ms. R.16, Biblioteca de la Escuela Técnica Superior de Arquitectura, Madrid.

SAN NICOLAS, Fray Lorenzo de. S. a. 1989. *Arte y uso de architectura. Primera parte*. 1639. Reprint Valencia: Albatros Ediciones.

SAUVAGE, Martin. 1998. *La brique et sa mise en oeuvre en Mésopotamie: des origines à l'époque Achéménide.* Paris: Centre de Recherche d'Archéologie Orientale.

SERLIO, Sabastiano. 1545. *Il Primo libro d'Architettura di Sebastiano Serlio. . .* Paris: 1545.

———. 1547. *Qvinto libro d'architettvra, nel quale se tratta de diuerse formede tempii . . .* Paris.
———. 1996. *Sebastiano Serlio on Architecture*. Vol. 1: Books I-V of *Tutte l'opere d'architettura et prospetiva* ; Vol 2. Books VI-VIII. V. Hart and P. Hicks, eds. New Haven: Yale University Press.
SIMONA, Michea. 2005. Ovals in Borromini's Geometry. Pp. 45-52 in *Mathematics and Culture II. Visual Perfection: Mathematics and Creativity.* M. Emmer, ed. Berlin: Springer.
SMYTH-PINNEY, Julia M. 1989. The Geometries of S. Andrea al Quirinale. *Journal of the Society of Architectural Historians* **48**: 53-65.
VANDELVIRA, Alonso de. 1580. *Exposición y declaración sobre el tratado de cortes de fábricas que escribió Alonso de Valdeelvira por el excelente e insigne architecto y maestro de architectura don Bartolomé de Sombigo y Salcedo, maestro mayor de la Santa Iglesia de Toledo.* Ms. R.10, Biblioteca de la Escuela Técnica Superior de Arquitectura, Madrid.
———. 1977. *Tratado de Arquitectura de Alonso de Vandelvira.* G. Barbé-Coquelin, ed. Albacete: Confederación Española de Cajas de Ahorros.
VIOLLET-LE-DUC, Eugene. 1858. Construction. Pp. 2-208 in vol. 4 of *Dictionnaire raisonnée de l'Architecture Française du XI au XVI siécle.* Paris: A. Morel.
WARD-PERKINS, John Bryan. 1958. Notes on the structure and building methods of early Byzantine Architecture. Pp. 52-104 in *The Great Palace of the Byzantine emperors. Second Report.* D. Talbot Rice, ed. Edinburgh: Walker Trust.
WEST, William Kyer. 1978. Problems in the cultural history of the ellipse. *Techonology and Culture* **19**: 709-712.
WILLIS, Robert. 1843. On the Construction of the Vaults of the Middle Ages. *Transactions of the Royal Institute of British Architects* **1**: 1-69.
WILSON JONES, Mark. 1993. Designing Amphitheatres. *Mitteilungen des deutschen archaologischen Instituts: Römische Abteilung* **100**: 391-442.
ZOCCA, Mario. 1946. *La cupola di S. Giacomo in Augusta e le cupole ellittiche di Roma. (Le cupole di Roma, 4).* Rome: Istituto di Studi Romani.

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