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Asset allocation: comparison between the Markowitz approach and the Black-Litterman method

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Abstract

In 1952 Harry Markowitz published his best-known article, named *Portfolio Selection*, in the Journal of Finance. For the first time, the concept of mean-variance optimization was introduced, and this served as the foundation of modern portfolio theory and, later, for the Capital Asset Pricing Model (CAPM).

Developed as a solution to practical portfolio optimization problems faced at Goldman Sachs, the Black-Litterman method was firstly published in the Journal of Fixed Income in 1991. This approach overcomes the main limitations of the Markowitz model which tends to create concentrated and unstable portfolios that rely excessively on past performance, without comprising investors' views.

This thesis aims to compare Markowitz's portfolio allocation method with the one of Black and Litterman, from both a theoretical (chapter 3) and empirical (chapter 5) standpoint. With the aim to highlight the impact of the exogenous shock of Covid-19, this work examines a portfolio of 30 stocks diversified by geography, currency, and industry, over two different time horizons: 2015-2019 and 2020-2021. To capture the effects of the pandemic, certain industries were beforehand selected – airline and pharmaceutical companies as well as firms that exploit intangible economy, such as tech groups.

Under the hypothesis of normal distribution of the logarithmic returns, the mean-variance optimization problem for the two analysed methods has been solved. The resulting empirical analysis demonstrated that the dynamic Black-Litterman capital allocation leads to more balanced, diversified, and stable portfolios which comprise investors' views.

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1 Introduction

The goal of any individual willing to invest his money, be it a private citizen or a financial advisor, is to maximise the return on the capital invested by choosing the right instruments and the share of money to allocate to each of them.

When considering the most common financial assets, investors have a wide range of financial instruments to choose from, starting from the least risky government bonds, moving onto corporate bonds, then stocks, and financial derivatives. These instruments have different risk profiles and may be used to build a diversified portfolio that combines different asset classes at once, in order to achieve the risk level that best suits the goals of the individual. Stocks are generally the most talked-about asset class when considering financial assets, regardless of the investment objectives and the time horizon considered. Moreover, stocks are also the most volatile and risky asset class among the ones mentioned when excluding financial derivatives, which most people are not able to understand or cannot have access to.

At the beginning of the XX century, with the rise in popularity of the American stock market, several economists and researchers started to investigate the behaviour of financial instruments and study ways to price them in order to maximise returns on a single-stock basis. It was only later that researchers started looking into portfolio theory, where quantitative strategies were being designed to build better-performing portfolios. The breakthrough came in 1952, after the American economist Harry Markowitz published the essay titled *Portfolio Selection*, which gave birth to the so-called Modern Portfolio Theory (MPT).

The purpose of this work is to compare Markowitz and Black-Litterman asset allocation models, analysing both the upsides and downsides. To start with, Chapter 2 will provide a brief overview on the most relevant theories found in literature on the topic. The following section will then provide an in-depth display of the two theoretical frameworks that are going to be used for the analysis, along with their advantages and limitations. Subsequently, the fourth chapter will illustrate the data used for the empirical analysis, the methodology, key statistics, and findings. Chapter 5 will then focus on the empirical analysis, presenting outcomes and results prior to the final chapter where conclusions are drawn.

2 Literature Review and Research Question

This chapter aims at describing the main literature and empirical evidence concerning portfolio allocation strategies. A special focus will be given to the evolution of portfolio theory, starting from Markowitz's studies in the early 1950s up to the developments of Black-Litterman in the 1990s.

2.1 Portfolio asset allocation strategies

The article published in 1952 by Harry Markowitz introduced the concept of mean-variance optimization to assist investors in building more efficient investment portfolios. One of the starting points for Markowitz theory was *The theory of investment value* by John Burr Williams (1938), who stated that the present value of dividends provides a fair estimate of a security's value. Since future dividends tend to be unknown, Markowitz (1952) claimed that expected future returns could serve as a proxy for future dividend payments and, consequently, a useful indicator to determine a stock's value. In addition to a security's expected returns (also referred to as the "mean"), Markowitz also argued that risk (variance) is another aspect to consider when dealing with investments. Since portfolios are built using more than one asset, correlations between securities are also important in the process of risk assessment. The two main ideas behind the theories of Markowitz thus state that investors want to maximise their returns while also limiting their exposure to risk; to do so, it is necessary to build a portfolio of unrelated assets.

This approach was the first introducing the concept of diversification, serving as the foundation of modern portfolio theory and, later, for the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966). Indeed, testing whether a mean-variance portfolio of risky assets is efficient is the equivalent of testing the validity of the Capital Asset Pricing Model. Although the mean-variance approach and the optimal portfolios generated were based on a rigorous theory and robust demonstrations, acceptance among investors was limited. Richard Michaud (1989), for example, criticises the model stating that the estimates used for expected returns and variance are subject to estimation errors. This is because the model tends to overweight the assets that have larger expected returns, lower variance, and negative correlations. Therefore, an estimation error in one of these assets is more likely to have a large impact on the portfolio, negatively affecting performance and exposing investors to considerable risks. Also, Michaud criticises the use of historical data to produce these estimates, which contribute to the estimation errors described above. Other empirical research, namely from Gibbons (1981), Gibbons, Shanken, & Ross (1989), MacKinay & Richardson (1991) and Briére et al. (2013), highlighted the inefficiency of the market portfolio, finding that no mean-variance efficient portfolio can be found for American stocks.

Some authors have come up with solutions to tackle the flaws concerning error maximization in Markowitz's model. Professor Philippe Jorion (1986), for instance, suggests using the Bayesian method to determine the input variables used for asset allocation. His empirical findings using Bayes-Stein estimators eventually prove to be better, providing significant gains in portfolio selection.

Developed as a solution to practical portfolio optimization problems faced at Goldman Sachs, the Black and Litterman method (1992) applies the Bayesian method to combine economic priors based on the CAPM equilibrium with empirical data. By doing so, two major problems of the Markowitz mean variance approach are addressed, namely the difficulty in computing expected returns given the limited knowledge of an investor and the strong impact small changes in expected excess returns have on the optimal portfolio weights.

2.2 Research question

The purpose of this work is to compare Markowitz's portfolio allocation method with the Black-Litterman approach. The main research question this work aims at answering is the following: *What are the benefits of the Back-Litterman portfolio allocation model compared to the Markowitz approach?*

To answer this broader question, it is necessary to provide an answer to other sub-questions, which focus on different aspects of the two theories under scrutiny. What needs to be investigated is:

- How are portfolios constructed when using the Markowitz model and the Black-Litterman method?
- Why is the Black-Litterman method considered a dynamic approach to asset allocation?
- How do portfolio weights vary with the two approaches?

With the aim to highlight the impact of the exogenous shock of Covid-19, this work examines a portfolio of 30 stocks diversified by geography, currency, and industry, over two different time horizons: 2015-2019 and 2020-2021. To capture the effects of the pandemic, stocks belonging to specific industries were selected – airline and pharmaceutical companies as well as firms that exploit intangible economy, such as tech groups.

3 Theoretical Framework

This section aims at describing Markowitz modern portfolio theory and the Black-Litterman model in detail, presenting formulas, methodologies, and limitations for both.

3.1 Markowitz theory of portfolio optimization

In 1952 Harry Markowitz pioneered the field of modern portfolio theory by publishing his best-known article, named *Portfolio Selection*, on the Journal of Finance (1952). The publication represented a revolution for the financial world, since it was the first theory of portfolio optimization that emphasized the importance of risk management, analysed correlation between securities, and, providing the tools to quantitatively assess the riskiness of a portfolio, also gave instructions on how to implement diversification. Indeed, before Markowitz's essay, academic researchers in the field of finance mainly focused on the analysis of single securities, with the primary goal of valuing companies to achieve better returns without considering the impact of risk.

In his article, Markowitz argues that it is possible to create optimal portfolios where returns are maximised, while risks are reduced to the minimum. To do so, investors are required to hold more than one security, spreading their investment among them, not only considering individual returns, but also considering the correlation among them. This approach is often called *mean-variance approach* since both expected returns (mean) and volatility (variance) concur in choosing asset allocations. The underlying assumption is that investors with complete information make rational decisions and avoid unnecessary risk.

Markowitz's model introduces the notion of the efficient frontier, which is the curve where it is possible to find the different combinations of risk and return characterizing the efficient portfolios created with a set of securities. Therefore, each portfolio laying on the efficient frontier provides either the minimum volatility subject to an expected return or, likewise, the maximum expected return that can be obtained with a given volatility; because of these features, rational investors will want to hold efficient portfolios. However, not all portfolios are equal to the eye of the investor since the investor's utility function is unique and depends on subjective elements. Indeed, the author distinguishes two phases in the selection of a portfolio: first, the identification of the efficient frontier and of the efficient portfolios that are found on its boundary and, in the second phase, the selection of the portfolio that maximises the investor's utility.

3.1.1 Mean-variance approach

The portfolio selection process starts with a mean-variance analysis on the securities of interest. The model holds under several simplifying assumptions which involve both securities and investor behaviour. The assumptions are the following:

- Returns of financial assets follow a normal distribution and are independent and identically distributed
- The standard deviation of returns provides a proxy for measuring risk
- Markets are efficient and there are no *market frictions* (taxes, transaction costs, market segmentation, …)
- We are concerned with a single-period investment time horizon, meaning that money does not have to be reinvested in the following period

On the other hand, investors are rational and:

- Prefer high expected returns
- Dislike high return variances (they are risk-averse)
- When choosing among portfolios, they only consider expected returns and variance of returns over a defined period of time

Let C_0 be the capital that can be invested, t the time horizon, $i = 1, 2, \ldots, n$ the number of securities that are part of the portfolio of interest, R_i their return, σ_i^2 their variance and w_i the weight of each security in the portfolio at the beginning of t.

$$R_i = \frac{P_{i,t+1} - P_{i,t}}{P_{i,t}} = \frac{P_{i,t+1}}{P_{i,t}} - 1$$

$$\sigma_i^2 = \frac{\sum_{t=0}^{T}\left(R_{i,t} - E(R_i)\right)^2}{T-1}$$

$$w_i = \frac{P_i}{\sum_j^n P_j}$$

Being $\mu_i = E(R_i)$ the expected return of each security, the expected return of the portfolio is:

$$\mu_p = E(R_p) = E\left(\sum_{i=1}^{n} w_i R_i\right)$$

Using a compact matrix notation:

$$\mu_p = \Omega \cdot \theta^T$$

Where:

- $\Omega = \{w_1, w_2, \dots, w_n\}$ is the array containing the weight of securities
- $\theta = \{\mu_1, \mu_2, \dots, \mu_n\}$ is the array containing expected returns of securities and θ^T is the same array transposed

To calculate the variance of the portfolio, it is first necessary to define the variance-covariance matrix (Σ) from which it is possible to obtain the σ_{ij} terms used to find the portfolio's variance:

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}$$

Where:

$$\sigma_{ij}^2 = Cov(R_i, R_j) = E\left[(R_i - \mu_i)(R_j - \mu_j)\right]$$

And:

$$\sigma_{ii}^2 = Var(R_p) = \sigma_i^2$$

Thus:

$$\sigma_p^2 = Var(R_p) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j \sigma_{ij} = \sum_{i=1}^{n} w_j^2\, \sigma_j^2 + \sum_{i=1}^{n}\sum_{j=1 \neq i}^{n} w_i\, w_j \sigma_{ij}$$

Which in a compact notation can be written as:

$$\sigma_p^2 = \Omega \cdot \Sigma \cdot \theta^T$$

Considering that covariance between two securities is defined as:

$$Cov(R_i, R_j) = \Sigma_{i,j}\rho_{ij}\sigma_i\sigma_j$$

It is possible to see how correlation among securities increases or decreases the overall variance of the portfolio. Indeed, since the correlation term ρ_{ij} ranges between 1 and -1, there are 3 main situations, which in a portfolio of 2 securities can be summarizes as follows:

- Perfect **negative correlation** (ρ_{ij} = –1): portfolio's variance is equal to zero.
- **No correlation** (ρ_{ij} = 0): portfolio's variance is positive.
- Perfect **positive correlation** (ρ_{ij} = 1): two outcomes are possible.
- If short selling is allowed, portfolio's variance is equal to zero
- If short selling is not allowed, portfolio's variance remains positive

As Figure 3.1 shows, when the correlation among securities A and B decreases, the overall volatility of the portfolio follows it:

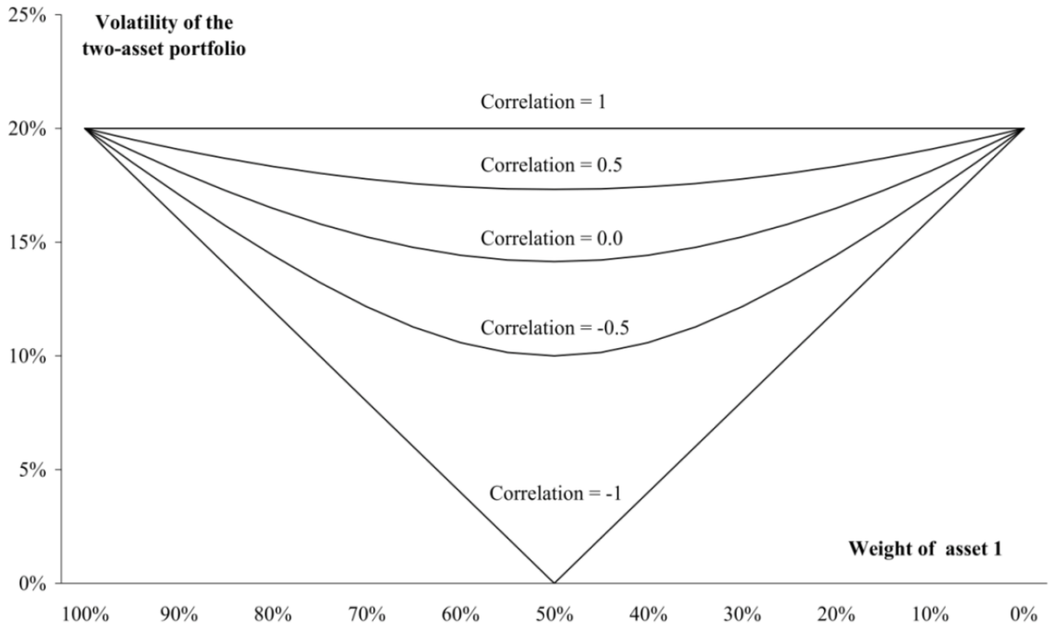

Figure 3.1. Relationship between the volatility of a two-asset portfolio and their different correlations.

Considering an equally weighted portfolio of *N* independent assets, in a realistic world where $\sigma_{ij} \neq 0$ for each $i \neq j$, we have a portfolio variance equal to:

$$\sigma_p^2 = \frac{\sum_{i=1}^{n} \frac{\sigma_i^2}{N}}{N} + \frac{N-1}{N} \sum_{i=1}^{n} \sum_{j=1 \neq i}^{n} \frac{\sigma_{ij}}{N(N-1)}$$

Thus:

$$\sigma_p^2 = \underbrace{\frac{1}{N}\,\bar{\sigma}^2}_{\substack{\text{Unsystematic risk,} \\ \text{diversifiable}}} + \underbrace{\left(1 - \frac{1}{N}\right)\bar{\sigma}_{ij}}_{\substack{\text{Systematic risk,} \\ \text{non-diversifiable}}}$$

Where $\bar{\sigma}^2$ and $\bar{\sigma}_{ij}$ are average values.

As N increases, the variance of an equally weighted portfolio becomes closer to the average covariance $\bar{\sigma}_{ij}$, so that the variance of individual assets no longer contributes to the total risk of the portfolio. What remains is known as systematic risk (or market risk) and it is non-diversifiable since it is not firm specific. An investor is subject to both systematic and unsystematic risk, with total risk depending on the allocation of funds and diversification.

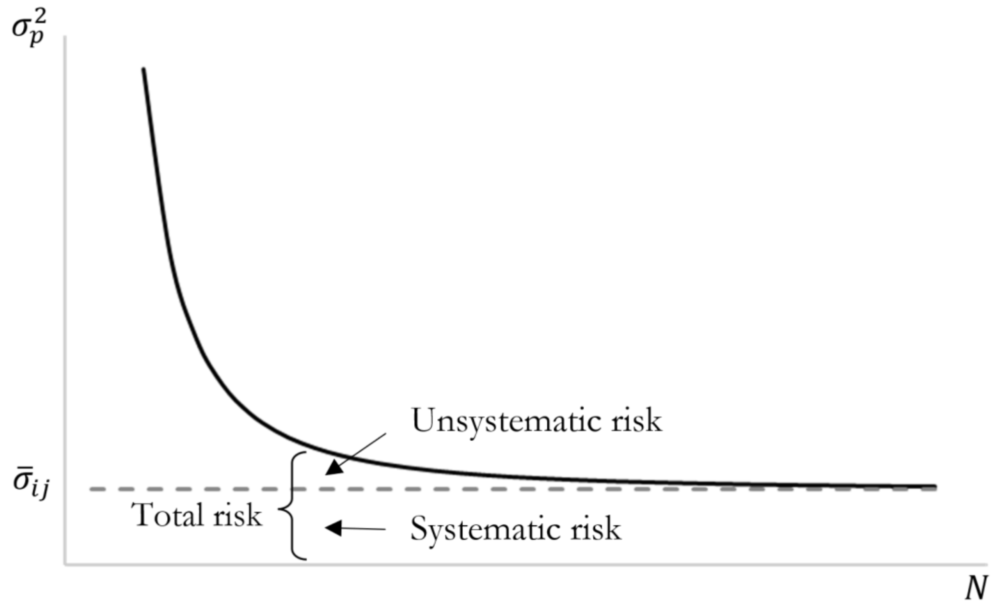

Figure 3.2. The effect of diversification on the risk of a portfolio.

3.1.2 The efficient frontier and the minimum variance portfolio

Once mean and variance coefficients have been calculated for the securities that are going to be part of the portfolio, it is possible to solve Markowitz's mean-variance optimization problem:

$$\max_{w} \sum_{i=1}^{n} w_i R_i$$

$$\text{subject to} \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{i,j}\, w_i w_j \leq V$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \ \geq 0, \forall i = 1, \dots, n$$

Where V indicates the risk threshold (variance) the investor does not want to exceed. By varying V, the problem's solution provides the investor with the efficient frontier of the portfolio, a curve representing the best trade-offs in terms of risk and expected returns for each risk level V. Figure 3.3. shows a graphic representation of the efficient frontier for a set of N securities.

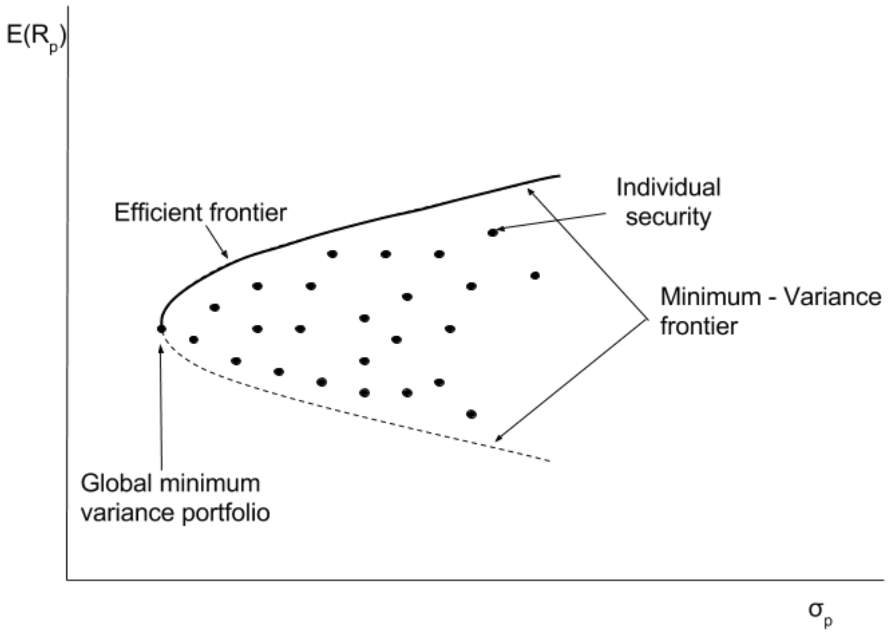

Figure 3.3. The efficient frontier for a set of N securities.

The image clearly shows the potential of diversification, since individual stocks lay below the efficient frontier, meaning that for a given level of risk, their returns are lower than the ones achievable in a diversified portfolio. It is also important to point out that the investor's risk appetite has to be considered, since not all portfolios possess the same characteristics. The most risk-averse investor will thus choose the *global minimum variance portfolio*, which is found on the leftmost point of the efficient frontier. Risk-taking investors, on the other hand, will prefer asset allocations leading to portfolios laying on the right of the frontier, more variance is associated with higher expected returns.

The same problem can also be solved using a different approach. Indeed, an investor could approach the problem by varying the desired return instead of the risk threshold. Hence, the optimization problem becomes:

$$\min_{w} \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{i,j}\, w_i w_j$$

$$\text{subject to } \sum_{i=1}^{n} w_i R_i = \mu$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0, \qquad \forall i = 1, \dots, n$$

Where μ indicates the expected return the investor wants to achieve by investing in the portfolio. The solution provides the investor with the weights of the securities and, by varying μ, it is possible to trace the efficient frontier just like in the previous optimization problem.

Let us now consider an example using a portfolio consisting of two uncorrelated stocks: S and C (with $\rho_{CS} = 0$). From the two optimization problems listed above, it is possible to generate the following diagram in Figure 3.4, representing the risk-return profiles for the different combinations of S and C. On the rightmost side of the curve, where point C is shown, the portfolio's composition is 100% on stock C and 0% on stock S. on the contrary, S indicates a portfolio's composition with a 100% ratio of stock S and 0% of C. It is worth noticing that C is a riskier stock compared to S, since portfolio C is characterised by a higher σ_p.

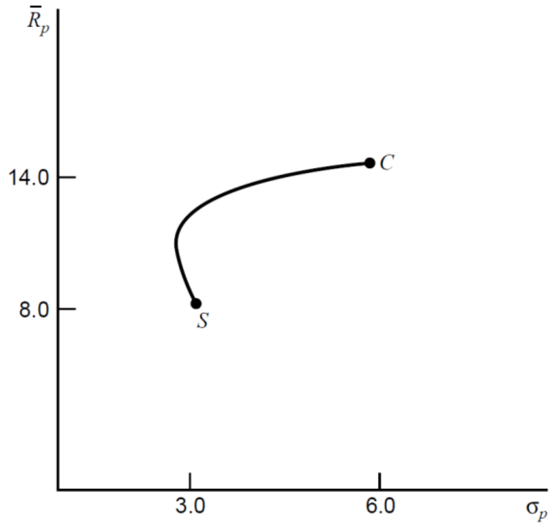

Figure 3.4. Expected return and standard deviation combinations for stocks S and C.

To find the efficient frontier, we must only consider the best risk-return combinations, meaning that for all those points where a risk profile σ_p has two possible return scenarios, it is necessary to discard the worst return profile. By doing so, we obtain the efficient frontier depicted in Figure 3.5, where portfolio S is discarded in favour of portfolio MV, which is the minimum variance portfolio that can be obtained with the two stocks in our example. Therefore, portfolio S cannot be considered an optimal portfolio since its risk profile is higher compared to MV's.

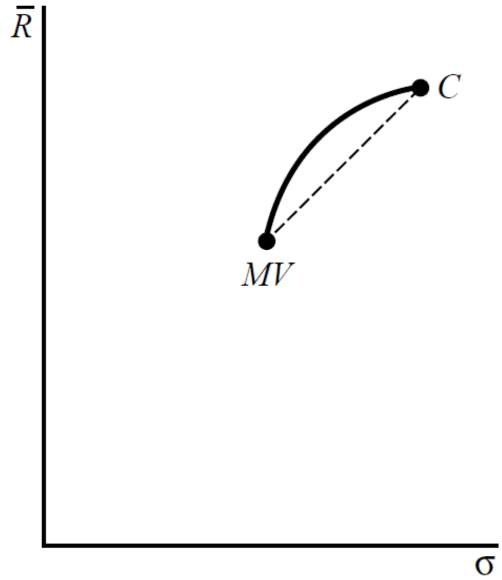

Figure 3.5. The efficient frontier for different combinations of S and C and the minimum variance portfolio (MV).

Going back to Figure 3.4, it is worth mentioning that the portfolio possibilities curve laying above the minimum variance portfolio is concave, while the portion below (the one to discard) is convex.

If, instead of having $\rho_{CS} = 0$, we assume different correlations, the shape of the efficient frontier changes accordingly. Figure 3.6 shows the most relevant combinations.

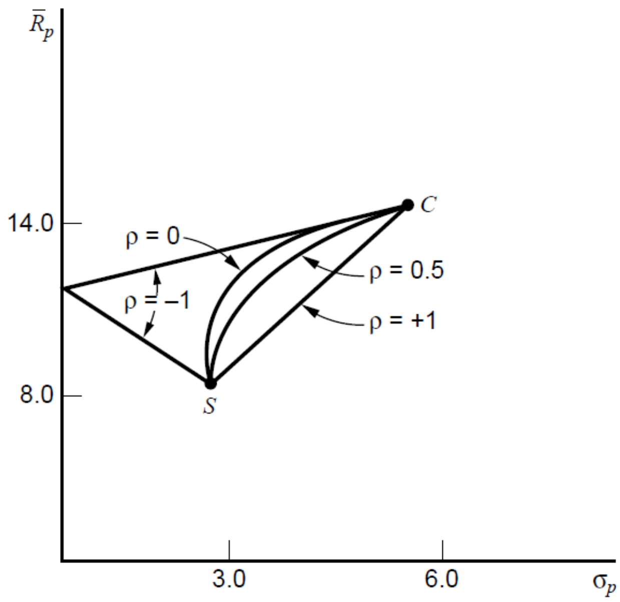

Figure 3.6. The effect of correlation on the shape of the efficient frontier.

The chart above shows how diversification leads to higher payoffs when correlation among securities is lower. On the other hand, higher correlations lead to riskier portfolios and, for any $\rho_{CS} > 0$, no combination of C and S leads to a σ_p lower than the minimum between σ_C and σ_S.

3.1.3 Short selling in an efficient portfolio

Thus far, short selling has not been contemplated in the analysis and we assumed that the investor possessed all the securities part of the portfolio. Introducing the possibility of short selling, investors can sell securities they do not own and profit from this action. Let us see an example of how this works.

Stock A currently trades at 100€ per share, but it is overvalued and investor *a* expects it to be worth 90€ at the end of period *T*. To profit from stock A, the investor (*a)*, instead of buying stock A at 100€ (and being on track for a potential loss of 10€ per share), sells the stock to another investor (*b*) at the current market price, 100€. This action is called short selling, and it is the equivalent of taking a short position on stock A. Since *a* does not own stock A, he will have to borrow it from someone else for a set period of time (*T*), with the promise of giving it back at the end of *T*. By taking a negative position, investor *a* will be given 100€ for the stock A that he sold to *b* and, at the end of period *T*, he will buy stock A on the market to settle the initial borrowing. If the forecast was right, the market price for stock A in *T* will be 90€. By doing so investor *a* will make a profit of 100€ – 90€ = 10€ by short selling.

Short sales clearly make sense when a security's expected returns are negative. However, since short sales allow investors to sell securities with low expected returns and use the proceeds to buy securities with higher expected returns, an investor may also hold a negative position in a stock with a positive outlook, as long as other stocks in the portfolio allow for higher returns. Moreover, an investor may be interested in short selling in order to take advantage of its impact on the stock's correlation with other securities. Indeed, if stocks A and B have a correlation $\rho_{AB} = 1$, once we short one of the two, be it A or B, their correlation changes to $\rho_{AB} = -1$.

To allow for short sales in Markowitz's portfolio, it is enough to remove the weights constraint $w_i \geq 0$. Optimal portfolios created through short selling will present negative weights for those securities that need to be sold short. Returning to the example described in section 3.1.2 with stocks S and C (and $\rho = 0.5$), and adapting the model to include short sales, it is possible to trace the following diagram:

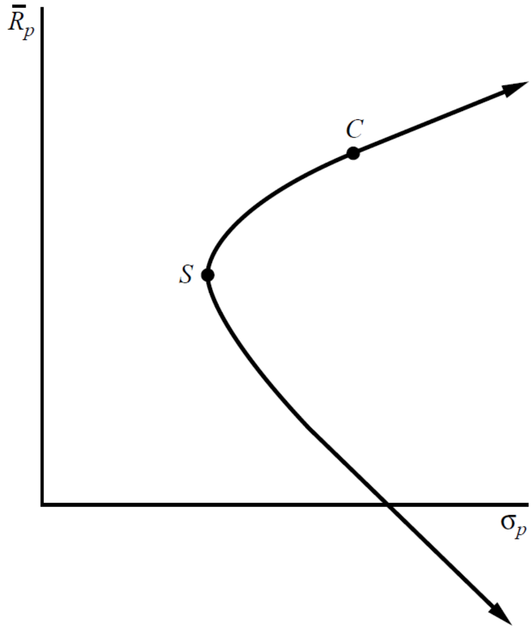

Figure 3.7. Expected return and standard deviation combinations of S and C when short selling is allowed.

The arrows in the diagram indicate that when short sales are allowed, expected returns are potentially unlimited. By only considering efficient portfolios, and indicating with B the point where short selling begins, the previous diagram becomes:

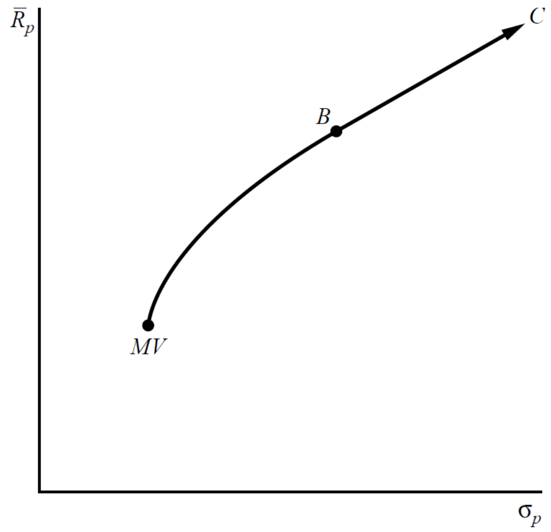

Figure 3.8. The efficient frontier when short selling is allowed.

Just like in Figure 3.5, we only consider the concave portion of the curve laying above MV.

3.1.4 The tangency portfolio

Let us now suppose that for an investor the goal is not to invest in the minimum variance portfolio, but rather to optimise his or her risk exposure. To do so, the investor needs to maximise the Sharpe Ratio[1] of the portfolio, which is also defined as a return-risk ratio, given its formula:

$$Sharpe\ Ratio = \frac{mean}{standard\ deviation}$$

The Sharpe Ratio represents the expected return per unit of risk; therefore, a risk-averse investor maximising SR will obtain the most risk-efficient portfolio, which is also called *tangency portfolio*. Graphically, the tangency portfolio is the point where a line through the origin is tangent to the efficient frontier (tg in Figure 3.9):

[1] From the economist William Sharpe.

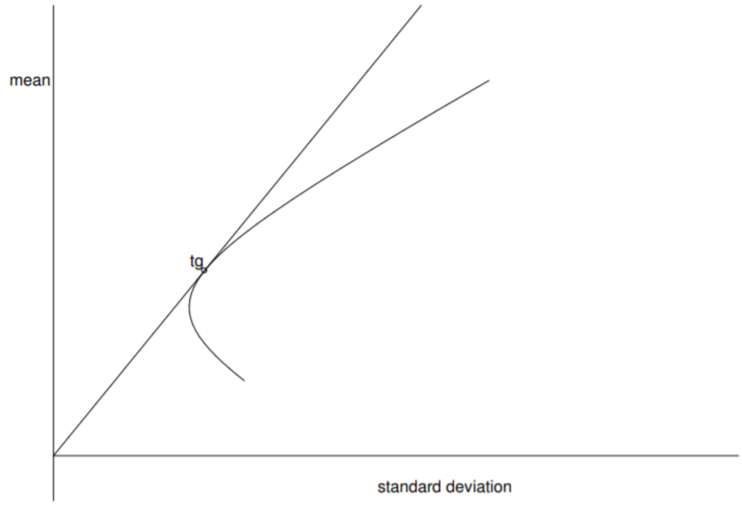

Figure 3.9. Graphical representation of the tangency portfolio.

3.1.5 Adding a risk-free security to the portfolio

After having considered portfolios consisting of risky assets only, and after having seen the effect of short selling on a portfolio's theoretical performance, it is possible to discuss the role of a risk-free asset (such as a government bond) in a portfolio.

To start with, riskless assets are characterised with a standard deviation equal to zero and with expected returns (which are in fact certain returns) that are equal to the risk-free rate R_F. For the sake of simplicity, let us assume that R_F is lower than the lowest expected return of the portfolio consisting of risky assets only (R_{MV}); the addition of a risk-free asset in a portfolio inevitably leads to a risk reduction as well as a reduction in the overall expected returns.

It is worth mentioning that buying a riskless asset such as a government bond can be seen as lending money at the R_F rate, while holding a negative position in such an asset equal to borrowing money at the risk-free rate. If an investor wishes to either lend or borrow money at a risk-free rate, while also holding a general portfolio A of risky assets, different combinations of risk and return can be achieved. A visual example is provided in Figure 3.10.

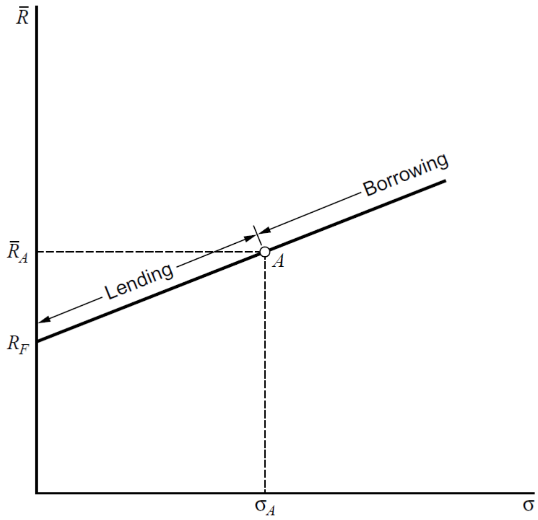

Figure 3.10. Expected return and standard deviation when portfolio A is combined with a risk-free asset.

Given that $w_F = 1 - w_A$, the average return the investor can expect from the combination is given from the following formula:

$$R_C = w_F R_F + w_A R_A$$

While the risk coefficient of the combination is:

$$\sigma_C = (w_F^2 \sigma_F^2 + w_A \sigma_A^2 + 2 w_F w_A \sigma_F \sigma_A \rho_{FA})^{\frac{1}{2}}$$

Since $\sigma_F = 0$, σ_C for the combination becomes:

$$\sigma_C = (w_A \sigma_A^2)^{\frac{1}{2}} = w_A \sigma_A$$

From which:

$$R_C = (1 - \frac{\sigma_C}{\sigma_A}) R_F + \frac{\sigma_C}{\sigma_A} R_A$$

Rearranging:

$$R_C = R_F + \left(\frac{R_A - R_F}{\sigma_A}\right) \sigma_C$$

This is the equation of the straight line with a slope of $\frac{R_A - R_F}{\sigma_A}$ and an intercept in (0;R_F) drawn in Figure 3.10 tangent to the efficient frontier.

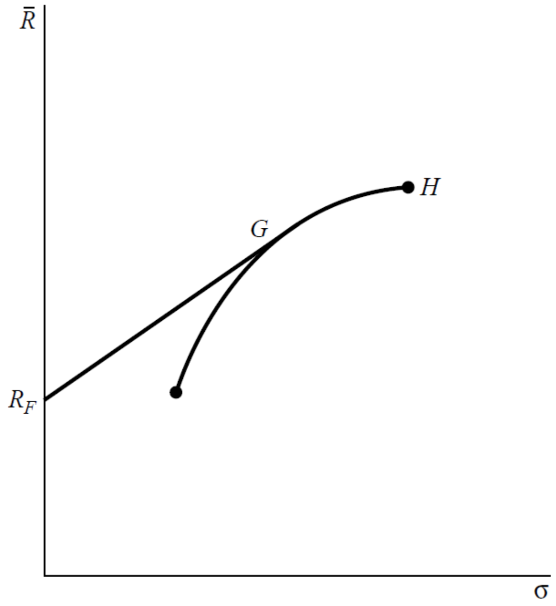

Figure 3.11. The efficient frontier when lending is allowed.

The efficient frontier with lending (but without borrowing) at the risk-free rate has now become the R_F-G-H curve. G is the point where the straight line is tangent to the efficient frontier found in Figure 3.8. If borrowing is possible, the curve defining the frontier changes shape, while still being found on the following R_F-G-H curve:

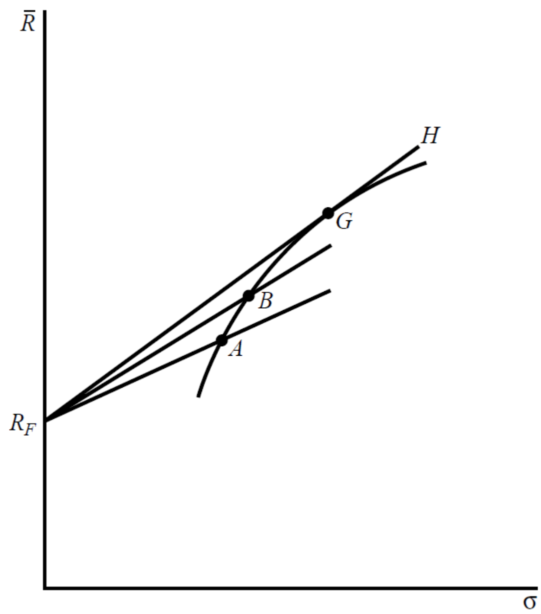

Figure 3.12. Efficient frontier R_F-G-H compared to the risk-return profiles of sub-optimal portfolios A and B.

A and *B* are other possible portfolios the investor could invest in, but they represent worse return-risk profiles, since it is possible to find better portfolios that offer a higher return for the same level of risk. Therefore, the efficient frontier is now represented from the straight-line connecting points R_F-*G*-*H*. This straight line is called *Capital Market Line* (CML), and the tangency portfolio found on the tangency point between the CML, and the efficient frontier (*G*) is called the *market portfolio*. It is worth remembering that investors with a higher risk aversion will prefer portfolios in the R_F-*G* portion of the CML, while investors who can tolerate a larger amount of risk will prefer portfolios on the *G*-*H* portion of the CML.

This finding is also known as the *One-Fund Separation Theorem*, which states that:

If the assets selected for investment includes a risk-free asset, then there exists a single fund F of risky assets such that every efficient portfolio can be constructed as a linear combination of the risk free asset and the fund F. These linear combinations constitute the capital market line, and no other efficient portfolios lie above the CML.

The CML has a slope of $SR = \frac{R_A - R_F}{\sigma_A}$, which is the Sharpe Ratio for portfolios with a risk-free asset. This ratio continues to represent the expected return per unit of risk. A risk-averse investor who wants to obtain the most risk-efficient portfolio must maximise this Sharpe Ratio, obtaining the best combination by adopting the market portfolio *G*, which is also the Tangency Portfolio. Clearly, the risk-free rate affects the slope of the CML and, analysing the shape of the efficient frontier when considering three different riskless securities combined with the same portfolio of risky assets, we obtain the following capital market lines (with market portfolios F, G and H):

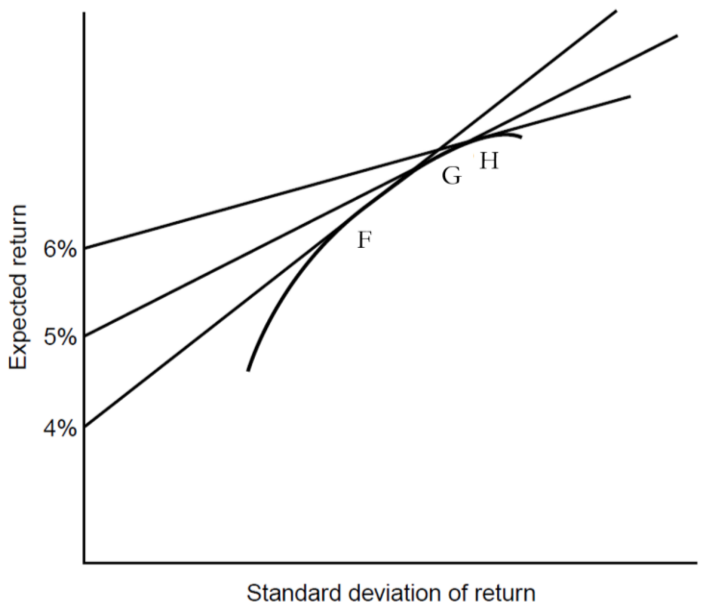

Figure 3.13. Tangency portfolios for different risk-free rates.

One last case affecting the shape of the efficient frontier is when the risk-free rate for borrowing (R_F) differs from the lending rate (R'_F), as shown in Figure 3.14.

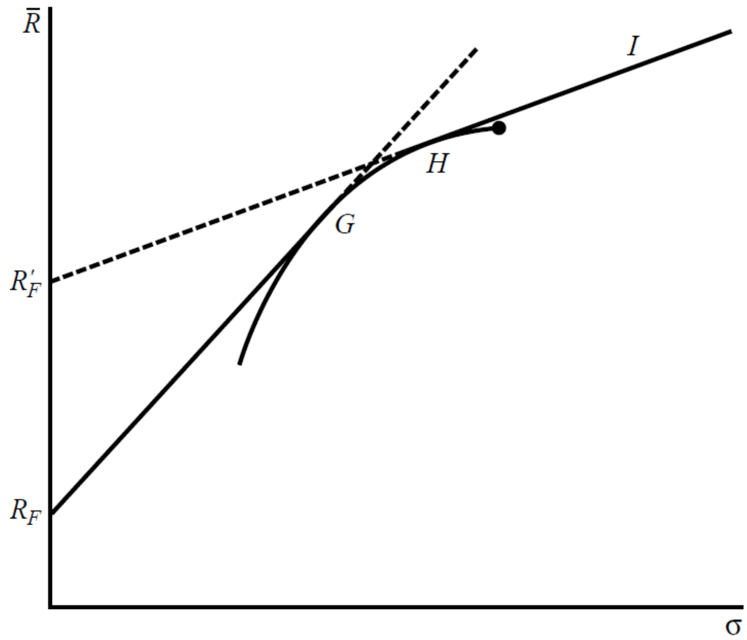

Figure 3.14. The efficient frontier with risk-free lending and borrowing at different rates.

In this case, the CML is not a straight line anymore and, for small differences between R_F and R'_F, the set of optimal portfolios for the investor is located between tangency points G and H.

3.1.6 The optimal portfolio for the investor

Once the quantitative phases of the portfolio selection process have been completed, investors must maximise their individual utility functions in order to find the optimal portfolio for themselves among the combinations available on the efficient frontier. The choice depends on the investor's appetite for risk.

Let us start by assuming that an investor's utility function in T is $u=u(T)$ and that Π is the set of efficient portfolios available to the investor (the opportunity set). The investor will want to maximise the expected utility obtained through his investment horizon. Briefly:

$$\max_{\pi} E[u(A_\pi)]$$

Given that efficient portfolios are found using the mean-variance approach, they solely depend on expected returns and variance (standard deviation). To include the utility function of the investor in the portfolio selection process, it is necessary to express it as a function of mean and variance, thus: *U=U(μ, σ)*. Expected utility, consequently, will increase with *μ* and decrease with *σ*. All portfolios belonging to the same utility curve (also called indifference curve) will have the same utility for the investor, balancing the effect of a higher exposure to risk with an increased expected return. Figure 3.15 shows a set of possible utility functions where it is easy to identify this relationship. Utility functions located in the upper portion of the chart are also associated with a higher utility, given that it is possible to achieve higher returns with the same amount of variance.

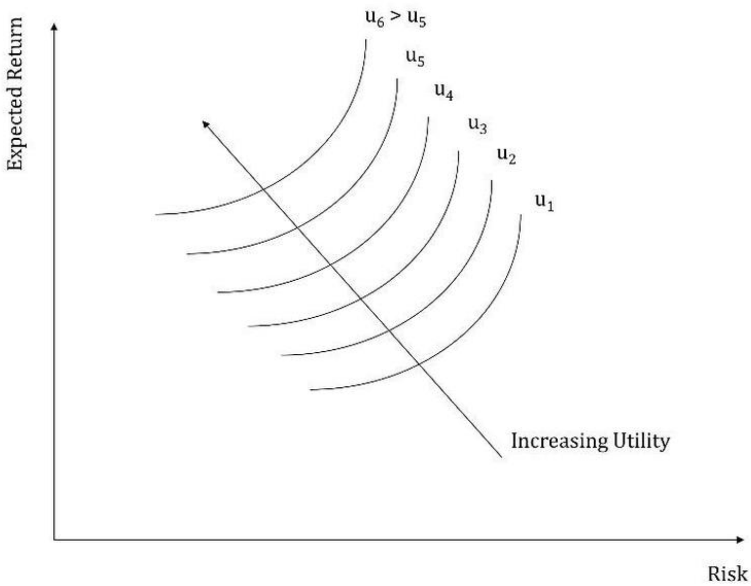

Figure 3.15. Risk and return relationship for different utility functions.

To find the optimal portfolio that suits the investor, it is necessary to overlap the utility functions chart with the diagram containing the efficient frontier and the capital market line. As Figure 3.16 shows, the optimal portfolio is represented by the tangency point between the efficient frontier and the indifference curve.

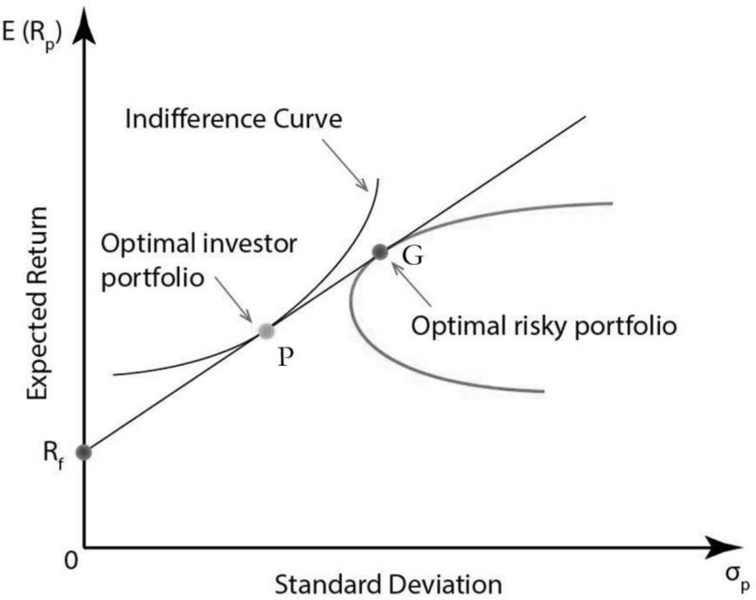

Figure 3.16. Risk and return profiles for the optimal portfolio P and the efficient portfolio G.

In the above diagram, G is the market portfolio previously found in section 3.1.5, while P is the optimal portfolio after considering the utility function of the investor. It is important to note that portfolio P has a lower expected return compared to portfolio G. The outcome shall not surprise since it is a consequence of the investors' low risk appetite.

3.1.7 The limits of the Markowitz model

Markowitz's model has several limits that make it hard to be efficiently used in the real world. The main hurdle in the model is due to errors deriving from the estimation of the three main parameters used to build the efficient frontier: expected returns, variance, and covariance. Since these measures can only be observed *ex-post*, it is necessary to estimate them.

By preferring stocks with higher expected returns, lower variability and negative covariances, the model tends to create portfolios concentrated on stocks with these characteristics, which represent a minority. As a consequence, these portfolios gravitate around a lower number of stocks, with the result of being less diversified and more subject to risks. To overcome this situation, it is necessary to include constraints regarding the number of securities in the portfolio or limits to their relative concentration (for example requiring each security to weigh less than 10% of the total portfolio).

Another critical issue in Markowitz's model is the reliance on past performance when analysing securities. Even a considerable amount of data regarding a security's historical performance is not enough to predict future behaviour. In fact, it is not possible to predict the future, and relying heavily on the past is often misleading when market conditions change. Also, a change in market conditions may require investors to adjust their portfolios accordingly, but the process is lengthy and requires new computations to find the new correlations, to re-define the efficient frontier, and the weights allocated to each security.

Moreover, the model does not include personal (for the investor) views on the future, be it the future of a stock, the future of the economy or any other kind of prediction. On the one hand, utility curves fix this issue, providing a tool to minimize risk for investors that wish to withhold their exposure in turbulent times. However, on the other hand, utility curves are the investor's very own, meaning that it is not possible to compare the portfolio of investor A to the one of investor B. This results in utility curves being *absolute* instead of *relative*, exposing the investor to a subjective view of the world which can turn out to be misleading and inefficient.

Because of these limitations academics and researchers have studied and developed methods to improve Markovitz's theory with the goal of improving the model and to better satisfy the needs of investors. The following section will describe one of these models, developed by Goldman Sachs analysts Fischer Black and Robert Litterman.

3.2 The Standard Capital Asset Pricing Model (CAPM)

The Standard CAPM is a general equilibrium model developed independently by Sharpe (1964), Lintner (1965) and Mossin (1966). Despite the stringent set of assumptions and its simplicity, the standard CAMP is an important milestone in the development of modern portfolio theory, well describing the relationship between market risk and expected return.

3.2.1 The Assumptions of the Standard CAPM

Equilibrium is an idealized state where forces are perfectly balanced, where supply equals demand. Even though this centre of gravity never really exists in financial markets, understanding its nature is of great importance, since it provides a framework to guide general principles of investing.

The Standard Capital Asset Pricing Model is based on the following assumptions:

1. There are no transaction costs (frictions) of buying and selling any asset
2. Every investor can sell and buy assets, regardless the size of their wealth
3. There are no income taxes
4. Individual investors cannot affect the price of a stock by buying or selling (perfect competition). All investors together determine prices by their actions

5. Investors make decisions only considering expected values and standard deviations of returns on their portfolios (mean variance investors)
6. Unlimited short sales are allowed, and investors can hold any fraction of an asset
7. The investor can borrow and lend any amount of money at one risk free rate
8. "Homogeneous belief 1". Investors care only about the mean and variance of returns on their portfolios and make decisions based on a single-period horizon defined in the same manner.
9. "Homogeneous belief 2". All investors have homogenous expectations regarding the inputs to the portfolio decision (e.g., risk free rate, expected returns, σ and ρ for the n risky assets)
10. All assets, including human capital, are marketable

3.2.2 Deriving the Standard CAPM

If all investors have homogenous expectations and are subject to the same lending and borrowing rates, they will hold the same risky portfolio which, in equilibrium, must be the market portfolio. Integrating what already outlined in section 3.1.5 and according to the *Mutual Fund Theorem*, all investors will invest in a combination of two portfolios, the Market Portfolio (M) and the risk-free asset, thus holding efficient portfolios that lie on the capital market line. Being $\bar{R}_E$ the expected return of the efficient portfolio and $\bar{R}_M$ the market expected return, the equation of the line connecting the risk-free asset and the market portfolio is:

$$\overline{R_E} = R_F + \left(\frac{\overline{R_M} - R_F}{\sigma_M}\right)\sigma_E$$

The first term on the right-hand side of the equation *(R_F)* measures the price of time. The second term is characterized by the Sharpe Ratio $SR = \frac{\overline{R_M} - R_F}{\sigma_M}$, representing the extra return per unit of risk, and σ_E, quantifying the amount of risk assumed. Therefore, we can rewrite the expected return of an efficient portfolio as:

$$Expected\ Return = Return\ of\ Time + Return\ of\ Risk$$

where $Return\ of\ Risk = Price\ of\ Risk * Amount\ of\ Risk$

As outlined in section 3.1.1, well-diversified portfolios are only characterized by systematic, non-diversifiable market risk. Beta is the relevant measure of risk for an individual security, quantifying how much of asset *i* 's return is driven by the market return, $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$.

Therefore, the $E(r_p) - \sigma$ space becomes the $E(r_p) - \beta$ space and the expected return of an individual asset (or any portfolio) can be rewritten as $\overline{R_\iota} = a + \beta_i * b$

In the case of the risk-free asset, $\overline{R_\iota} = \mathrm{R_F}$ with $\beta_F = 0$, we find that $a = R_F$

For the market portfolio, $\sigma_{MM} = \sigma_M^2$, $\beta_M = 1$, and we find that $b = \overline{R_\iota} - a = \overline{R_M} - \mathrm{R_F}$.

The following can then be written as:

$$\overline{R_\iota} = R_F + \beta_i * (\overline{R_M} - R_F)$$

This is the CAPM equation representing a straight line in The Expected return- Beta space, called SML – Security Market Line, as shown in Figure 3.17. The intercept occurs either when beta is zero or when there is no systematic risk (risk free asset). The slope represents the risk premium.

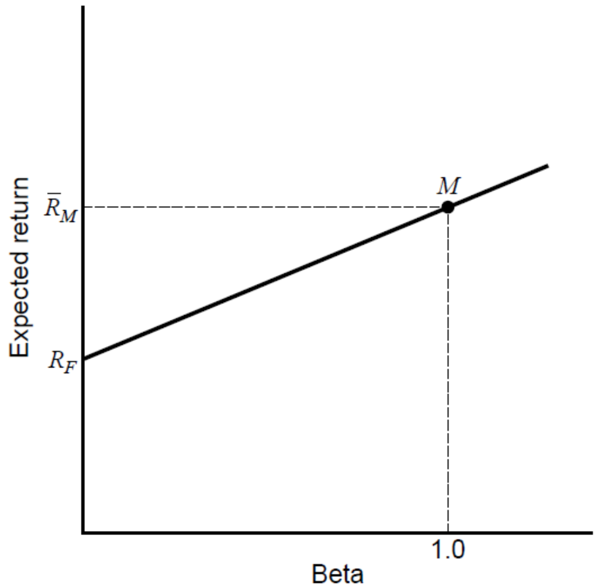

Figure 3.17. The security market line.

All investments and portfolios should lie on the SML. If this is not the case, a riskless arbitrage opportunity exists. However, the arbitrage would continue until equilibrium is established again. For example, if a stock plots above the SML, this means that the stock earns higher expected returns than the ones predicted by CAPM - therefore the stock is under-priced. Given the initial assumptions, all investors would recognize the arbitrage opportunity and they would buy. In turn, the stock price would rise, and the expected returns would fall until the equilibrium suggested by CAPM would be restored again. The opposite situation holds true too.

The CAPM equation holds great economic insight since it confirms that systematic risk is the one determining expected returns. Therefore, the investor is rewarded for bearing the market risk since the non-systematic risk can be diversified away.

Eventually, it is possible to classify stocks according to their value of beta:

- $\beta_i < 0$: stocks presenting an inverse correlation with the market
- $0 < \beta_i < 1$: defensive stocks with lower returns than the market
- $\beta_i > 1$: aggressive stocks with higher returns than the market
- $\beta_i = 1$: stocks that achieve the same returns as the market

3.2.3 The Limits of the Standard CAPM

If, on the one hand, the stringent set of initial assumptions that are endogenous to the model and that we have already discussed make it possible to develop a simple model for equilibrium, on the other hand they violate conditions that hold true in the real world. If the CAPM can explain equilibrium returns on the macro level, it is unable to model the behaviour of individual investors (micro level).

Since the CAPM assumptions leads to a simplification of the reality, several tests have been developed to establish the degree with which the model is able to describe the reality. Most of the early empirical studies used first pass and second pass (cross-sectional) regression to respectively estimate betas and test the hypothesis. However, this approach led to errors-in-variables problems that were apparently in contradiction with the CAPM. To overcome this, Black, Jensen and Scholes (1972) introduced the hypothesis that if investors can always lend at the risk-free rate, this is not the case when it comes to borrowing. Therefore, the so-called zero-beta portfolios exists, whose returns are uncorrelated with those of the market portfolio,

Another critique to CAPM and other equilibrium models was raised by Roll (1977), who argued that in the real world creating or observing a well-diversified portfolio is not possible. Therefore, valid empirical tests to CAPM cannot be performed. Eventually, computing an accurate beta coefficient of a security from historical data is difficult and many times proxies should be used. This, in turn, can affect the reliability of results.

To conclude, given the evident limitations of the CAPM model, economists have developed a new and different method to determine the prices of the assets: APT, Arbitrage Pricing Theory. Based on the Law of one price, APT is not based on the same restrictive assumptions of the CAPM, thus providing a more general description of equilibrium

3.3 The Black-Litterman model

The Black-Litterman global asset allocation model was first published by Fischer Black and Robert Litterman in the Journal of Fixed Income (1991). The following year, they published a more detailed analysis in the Financial Analyst Journal (1992). Developed as a solution to practical portfolio optimization problems faced at Goldman Sachs, the Black-Litterman method combines the CAPM and mean variance model. By doing so, it overcomes the main hurdles of Markowitz's asset allocation model.

According to the B&L model, market equilibrium and investors' expectations are the two main sources of information about future risk premiums. Given the uncertainty that characterizes both, they are expressed in terms of probability distribution. To allow the integration between the investor's personal views about asset returns with equilibrium excess returns, the B&L approach suggests an unconstrained portfolio reverse optimization technique as starting point. Taking a balanced market capitalization-weighted portfolio as a neutral point of reference enables the investor to express his feelings only on the assets he has a view about. As a consequence, portfolio weights will deviate from equilibrium weights according to the level of confidence of the investor regarding his views, their magnitude and τ, which specifies the weight of the view with respect to the market equilibrium. If the investor's views are aligned to those of the market, then the final optimal portfolio will present the same asset weights of the market portfolio, proportional to the market capitalization of each asset.

To sum up, the Black-Litterman approach is characterized by the following steps:

1. Computation of the implied excess equilibrium returns, by means of the reverse optimization technique
2. Integration between the implied excess returns and the views of the investors, by means of the Bayesian approach
3. Computation of the final optimal weights, using the excess expected returns obtained at point 2 as input to the mean variance optimization method

3.3.1 Reverse optimization

The mean variance method is extremely sensitive to the expected returns the investor provides as inputs. This results in portfolios that present large short and long positions. Given the unrealistic assumption that investors could accurately predict asset expected returns, Black and Litterman decided to rely on the equilibrium excess returns as neutral reference points for expected returns. In the ideal conditions where supply equals demand and all investors have homogenous expectations, the optimal portfolio coincides with the market portfolio, as stated in the CAPM. These assumptions are reasonable since, when expected returns deviate from CAPM, the imbalances tend to restore the equilibrium, as it has been outlined in section 3.2.2. Deriving the excess returns from the CAPM optimal portfolio, the *reverse optimization method* computes the implied excess returns.

The starting point of the reverse optimization technique is the quadratic utility function:

$$U = \Omega^T \theta - \frac{\lambda}{2} \Omega^T \Sigma \Omega$$

Where:

- U is the investors' utility
- Ω is the vector of the weights of each asset
- θ is the vector of equilibrium excess returns for each asset
- λ is the risk aversion parameter
- Σ is the covariance matrix of the assets

Since U is a concave function, it has a single global maximum, which can be found by computing the first order derivative of the utility function with respect to the weights and setting it equal to 0:

$$\frac{dU}{d\Omega} = \theta - \lambda \Sigma \Omega = 0$$

From this equation, it is possible to derive the unconstrained optimal portfolio weights, as in the standard mean variance approach:

$$\Omega = (\lambda \Sigma)^{-1} \theta$$

However, under the assumption that the optimal portfolio coincides with the market portfolio, the vector of the weights of each asset Ω can be calculated based on the market capitalization of all the assets in the portfolio, thus it can be rewritten as Ω_{mkt} . Indeed, instead of solving the above equation for Ω , we can reverse the problem and solve it for θ. The formula below is the closed form solution to the reverse optimization for deriving the vector of the implied excess returns, starting from an optimal mean variance portfolio - the market portfolio – without constraints:

$$\theta = \lambda \Sigma \Omega_{mkt}$$

The risk aversion coefficient λ determines the expected risk-return trade-off: it is the rate at which an investor is willing to accept less expected returns for less variance. It can be calculated by rearranging the above equation, multiplying both sides by Ω_{mkt}^T and replacing vector terms with scalar terms:

$$(E(R_M) - R_F) = \lambda \sigma_M^2$$

Thus obtaining:

$$\lambda = \frac{(E(R) - R_F)}{\sigma^2}$$

Where:

- $E(R)$ is the total return on the market portfolio
- R_F is the risk-free rate
- σ^2 is the variance of the market portfolio

The implied excess return vector obtained through the reverse optimization technique is the starting point to incorporate the views of the investors; in the case the investor has no view, he could simply buy a portfolio made accordingly the market capitalization of the assets. Therefore, the real value of the equilibrium concept consists in providing the investor a neutral framework to be adapted according to his views, goals, and constraints, without the requirement of expressing a complete set of expected excess returns of all the asset classes within the portfolio. In this way, the Black-Litterman approach generates portfolios that are less sensitive to changes in expected returns than the ones obtained through the Markowitz mean-variance model.

3.3.2 Investors' views

With the term *views*, Black and Litterman refer to the investors' subjective expectations about the returns of the assets. They can be expressed both in relative and absolute terms:

- *Absolute View:* if the investor thinks that a given asset is overvalued or undervalued

- *Relative View:* if the investor thinks that a given asset is going to outperform or underperform another asset

The investors' statements (*views*) about the expected returns are expressed with a certain degree of confidence, which ranges from 0% to 100%: the more the investor trusts his feeling, the higher the level of confidence, and vice versa. Moreover, the investor has a personal level of confidence in the equilibrium expected returns and this is measured by the so-called weight-on-views τ. The investor's views together with their level of confidence and the weight-on-views τ lead to deviations from the equilibrium weights, as will be described in detail in the following section 3.3.3.

3.3.3 Bayesian approach

The Black-Litterman model integrates the empirical source of information coming from the market with the subjective one of the investors by means of a Bayesian approach. By combining the implied excess returns with the investors' views, it is possible to derive the final expected returns (posterior distribution).

To provide the reader with a common vocabulary, the Bayes formula follows:

$$P(A|B) = \frac{P(B|A)\, P(A)}{P(B)}$$

Where:

- $P(A|B)$ is the conditional probability of A, given B – posterior distribution
- $P(B|A)$ is the conditional probability of B, given A – conditional distribution
- $P(A)$ is the probability of A – prior distribution
- $P(B)$ is the probability of B – normalizing constant.

Note that when solving for the posterior distribution, $P(B)$ will be neglected, since it is comprised within the constants of the integration. Core to the Black-Litterman (and Mean Variance) model is the assumption that returns are normally distributed. It follows that both prior and conditional distributions should be normally distributed. Eventually, the posterior distribution follows the normality assumption too.

Prior Distribution. According to the Bayesian Approach, $P(A) \sim N(x, \frac{S}{n})$, being x the mean, S the variance and n the sample size. Being τ the investor's confidence in the prior distribution, it is possible to state that the probability density function of the equilibrium returns follows a multivariate normal distribution:

$$P(A) \sim N(\theta, \tau\Sigma)$$

Conditional Distribution. According to the Bayesian Approach, $P(B|A) \sim N(\mu, \Sigma)$, being Σ the uncertainty in the estimate of μ. The starting point to calculate the conditional distribution in the Black Litterman approach is the mathematical expression of the investor's views. One of the biggest advantages of the Black-Litterman model consists in the possibility for the investor to express views only on a limited number of assets. Mathematically, if an investor has *k* views on *n* assets:

$$P \times E(R) = Q + \varepsilon$$

Where:

- P is a *k x n* matrix of the asset weights according to each view. Since the views are required to be fully invested, the sum of the views' weights should be 0 (relative views) or 1 (absolute views)
- Q is *k x 1* vector, expressing the expected excess returns for each *k* view
- ε is a normally distributed random variable that indicates the uncertainty of each view

$$P = \begin{bmatrix} P_{1,1} & \cdots & P_{1,n} \\ \vdots & \ddots & \vdots \\ P_{k,1} & \cdots & P_{k,n} \end{bmatrix} \qquad Q = \begin{pmatrix} Q_1 \\ \vdots \\ Q_k \end{pmatrix} \qquad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{pmatrix}$$

$\varepsilon \sim N(0, \Psi)$ and under the assumption that each view is unique and uncorrelated to the others, the covariance matrix Ψ is diagonal , with all off-diagonal entries equal to 0:

$$\Psi = \begin{bmatrix} \Psi_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \Psi_\mathrm{k} \end{bmatrix}$$

Therefore, in the view space $P(B|A) \sim N(Q, \Psi)$, while in the asset space it can be written $P(B|A) \sim N(P^{-1}Q, [P^T\Psi^{-1}P]^{-1})$.

Posterior Distribution. According to the Bayesian Approach $P(A|B) \sim N([\Sigma^{-1}\mu + nS^{-1}x]^T[\Sigma^{-1} + nS^{-1}]^{-1}, (\Sigma^{-1} + xs^{-1})^{-1})$. Substituting the prior and conditional distribution into this equation, the final posterior distribution can be computed:

$$P(A|B) \sim N([(\tau\Sigma)^{-1}\theta + P^T\Psi^{-1}Q][(\tau\Sigma)^{-1} + P^T\Psi^{-1}P]^{-1}, ((\tau\Sigma)^{-1} + P^T\Psi^{-1}P)^{-1})$$

This is often referred as the Black-Litterman master formula and can be rewritten as:

$$P(A|B) \sim N(\mu_{BL}, M)$$

Note that the posterior variance represents the variance of the posterior mean. To compute the variance of the returns that will then be used in the mean-variance optimization problem, further calculations should be made:

In the presence of views:

$$\Sigma_{BL} = \Sigma + M$$

In the absence of views:

$$\Sigma_{BL} = (1 + \tau) * \Sigma$$

3.3.4 Solving for the mean variance optimization problem

Once the Bayes theorem has been applied, all the asset returns, not only the ones on which the investor has expressed his views, have changed. This results from the correlation that exists among all the assets that leads to the spread of potential investor errors over all the assets, preventing weight concentrations.

The expected return vector and the covariance matrix are then used as input to the mean-variance optimization problem that will lead to the computation of the optimal portfolio, along with its assets' weights, mean and variance. At the end of the optimization process, the optimal portfolio will present asset weights that diverge from the market portfolio in an amount proportional to the magnitude and confidence of the views expressed by the investor.

By using the B&L approach, the investor can control the influence of his views, may they be relative or absolute, on the portfolio weights, thus implicitly determining at the same time his propensity to risk.

3.3.5 Critics and difficulties of the model

The Black-Litterman approach has often been criticized for the intrinsic subjectivity due to the introduction of the investor's views in the model. However, given that testing for all the possible variables is not feasible, every model will inevitably be subjective. Moreover, by taking the equilibrium excess returns computed through the CAPM as neutral reference points, the framework appears to be robust.

As in the case of Markowitz model, the initial limitative assumption about the normal distribution of returns is not always true in the real world. Another possible improvement that can be applied to the model concerns the construction of the P matrix. While He and Litterman (2003) assigns a percentage value to the assets and Satchell and Scowcroft (2000) prefer an equal weighting scheme, Idzorek (2004) proposes a market capitalization weighting scheme.

The two major problems of the B&L approach deal with the scalar parameter τ and the covariance matrix Ψ. In literature there is little guidance for setting τ. Black and Litterman (1992) and Lee (2000) suggest choosing τ close to zero, since the mean presents less uncertainty than the return. On the contrary, Satchell and Scowcroft (2000) set τ equal to 1. To conclude, interpreting $\tau\Sigma$ as the standard error of θ, Blamont and Firoozy (2003) propose $\tau = \frac{1}{n}$, where n is the number of observations.

As proposed by He and Litterman (2003), the easiest way to calibrate the model consists in making an assumption about the scalar value so that the ratio $\frac{\Psi}{\tau}$ is equal to the view variance $P\Sigma P^T$. It follows that a possible Ψ can be computed as $\Psi = diag(P(\tau\Sigma)P^T)$. When the covariance matrix is calculated this way, the value of τ becomes irrelevant since only the ratio $\frac{\Psi}{\tau}$ is included in the model.

4 Data and Methodology

This chapter presents the set of stocks chosen for the diversified portfolio that will be used for the empirical analysis. Specifically, statistical analyses of the historical stock prices and returns are carried out. This study is a fundamental prerequisite before proceeding to the empirical portfolio optimization modelling.

4.1 Data Selection

The choice of the financial instruments that will be used to create the portfolios using the Markowitz and the Black-Litterman models is of essential importance. This thesis considers a universe of 30 stocks, of which 10 US companies listed on the Nasdaq, NasdaqGS and New York Stock Exchange (NYSE), 15 European companies listed on the Euronext Paris, Milan Stock Exchange, Madrid Stock Exchange, Brussels Stock Exchange, and Frankfurt Stock Exchange and 5 UK companies listed on the London Stock Exchange.

With the aim to highlight the impact of the exogenous shock of Covid-19, the stocks are analysed over two different time horizons: 2015-2019 and 2020-2021. Moreover, to avoid excessive risk concentration, the selected assets belong to a well-diversified set of industries, e.g., from consumer electronics to beverages to auto and drug manufacturers, as shown in Table 4.1. However, to capture the effects of the pandemic, certain industries were before-hand selected – airline and pharmaceutical companies as well as firms that exploit intangible economy, such as tech groups.

From the website Yahoo Finance and for every stock, the historical daily adjusted close prices have been downloaded, over a time period ranging from 02/01/2015 to 30/12/2021. It must be pointed out that adjusted close is the closing price after all splits and dividend distributions. Given that the trading days differ among countries, and this could result in time series of asynchronous lengths, a filtering work has been carried out to cover a homogenous time period.

no.	Stock	Tick	Stock Exchange	Sector	Industry	Country	Currency
1	Apple Inc.	AAPL	NasdaqGS	Technology	Consumer Electronics	US	USD
2	Airbus SE	AIR.PA	Euronext Paris	Industrials	Aerospace & Defense	Netherlands	EUR
3	Amazon.com Inc.	AMZN	NasdaqGS	Consumer Cyclical	Internet Retail	US	USD
4	Air Products and Chemicals, Inc.	APD	NYSE - Nasdaq	Basic Materials	Specialty Chemicals	US	USD
5	AstraZeneca PLC	AZN.L	London Stock Exchange	Healthcare	Drug Manufacturers—General	UK	GBp
6	BASF SE	BAS.DE	Frankfurt Stock Exchange	Basic Materials	Chemicals	Germany	EUR
7	BP p.l.c.	BP.L	London Stock Exchange	Energy	Oil & Gas Integrated	UK	GBp
8	Davide Campari-Milano N.V.	CPR.MI	Milan Stock Exchange	Consumer Defensive	Beverages—Wineries & Distilleries	Italy	EUR
9	Enel SpA	ENEL.MI	Milan Stock Exchange	Utilities	Utilities - Diversified	Italy	EUR
10	easyJet plc	EZJ.L	London Stock Exchange	Industrials	Airlines	UK	GBp
11	Assicurazioni Generali	G.MI	Milan Stock Exchange	Financial Services	Insurance - Diversified	Italy	EUR
12	General Motors Company	GM	NYSE - Nasdaq	Consumer Cyclical	Auto Manufacturers	US	USD
13	Alphabet Inc.	GOOG	NasdaqGS	Communication Services	Internet Content & Information	US	USD
14	InterContinental Hotels Group PLC	IHG.L	London Stock Exchange	Consumer Cyclical	Lodging	UK	GBp
15	Interpump Group S.p.A.	IP.MI	Milan Stock Exchange	Industrials	Specialty Industrial Machinery	Italy	EUR
16	Deutsche Lufthansa AG	LHA.DE	Frankfurt Stock Exchange	Industrials	Airlines	Germany	EUR
17	Lockheed Martin Corporation	LMT	NYSE - Nasdaq	Industrials	Aerospace & Defense	US	USD
18	LVMH Moët Hennessy - Louis Vuitton, Société Européenne	MC.PA	Euronext Paris	Consumer Cyclical	Luxury Goods	France	EUR
19	Microsoft Corporation	MSFT	NasdaqGS	Technology	Software - Infrastructure	US	USD
20	L'Oréal S.A.	OR.PA	Euronext Paris	Consumer Defensive	Household & Personal Products	France	EUR
21	Pfizer Inc.	PFE	NYSE - Nasdaq	Healthcare	Drug Manufacturers—General	US	USD
22	The Procter & Gamble Company	PG	NYSE - Nasdaq	Consumer Defensive	Household & Personal Products	US	USD
23	Repsol S.A.	REP.MC	Madrid Stock Exchange	Energy	Oil & Gas Equipment	Spain	EUR
24	Banco Santander, S.A.	SAN.MC	Madrid Stock Exchange	Financial Services	Banks - Diversified	Spain	EUR
25	Sanofi	SAN.PA	Euronext Paris	Healthcare	Drug Manufacturers—General	France	EUR
26	SAP SE	SAP.DE	Frankfurt Stock Exchange	Technology	Software - Application	Germany	EUR
27	Solvay SA	SOLB.BR	Brussels Stock Exchange	Basic Materials	Chemicals	Belgium	EUR
28	Unilever PLC	ULVR.L	London Stock Exchange	Consumer Defensive	Household & Personal Products	UK	GBp
29	Volkswagen AG	VOW3.DE	Frankfurt Stock Exchange	Consumer Cyclical	Auto Manufacturers	Germany	EUR
30	Walmart Inc.	WMT	NYSE - Nasdaq	Consumer Defensive	Discount Stores	US	USD

Table 4.1. List of selected stocks.

4.2 Statistical Analysis of the data

Once all the relevant data were gathered, to capture the compounding effect, logarithmic returns were computed as follows:

$$r_t = ln\frac{P_t}{P_{t-1}}$$

Where:

- $\mathrm{r_t}$ is the logarithmic return at time t
- P_t is the adjusted close price at time t
- P_{t-1} is the adjusted close price at time $\mathrm{t} - 1$

Entering the logarithmic returns into Stata made it possible to compute the four fundamental moments of mean, variance, skewness, and kurtosis. While the mean indicates the average value around which central clustering occurs, the variance represents the dispersion around such value. As third momentum, skewness is a nondimensional measure of the asymmetry of a distribution about the mean. Therefore, a distribution is said to be *positive skewed* if the tail of the probability distribution is on the right, while it is *negatively skewed* if the tail is on the left. If there is no skewness, the normal distribution is the probability distribution. Eventually, kurtosis is the fourth nondimensional descriptive statistics that assesses the peakiness or flatness of a distribution in comparison to the normal one: by measuring the heaviness of the distribution tails, it indicates the probability of achieving extremely large or small returns.

The skewness and the kurtosis help testing the assumption of normality in the return distribution on which both the Mean-Variance Model developed by Markowitz and Black-Litterman approach are based.

Moreover, to test the normality distribution of the logarithmic returns, the Kolmogorov–Smirnov and the Shapiro–Wilk test have been performed using Stata. In both cases, the null hypothesis $\mathrm{H_0} =$ the random variable X follows a normal distribution can be rejected if the p-value is lower than the chosen significance level α. While the K-S test compares the cumulative empirical distribution function of the sample and the cumulative normal distribution, the S-W test compares two alternative estimators of the variance.

Even though for most of the stocks, the Kolmogorov–Smirnov and the Shapiro–Wilk tests rejected the null hypothesis about the normal distribution of the returns, we will assume it for the purpose of this thesis.

Eventually, a Q-Q plot has been drawn for each stock. It is a scatterplot where two sets of quantiles (observed and expected) are plotted one against the other. If the cumulative distribution of the observed variable follows a normal distribution, the points will lie on the diagonal line.

With the purpose of isolating the impact of Covid-19, the mean, variance, skewness, kurtosis, Sharpe ratio, Beta and R2 of each stock have been computed over three-time horizons, 2015-2021, 2015-2019 and 2020-2021, as respectively shown in Table 4.2, Table 4.3 and Table 4.4.

						2015-2021					
Stock	**Daily E[R]**	**Daily Variance**	**Daily Std. Dev.**	**Annual E[R]**	**Annual Variance**	**Annual Std. Dev.**	**Kurtosis**	**Skewness**	**Sharpe Ratio**	**Beta**	**Adj. R^2**
AAPL	0.12%	0.0003	1.85%	28.20%	0.0827	28.75%	6.33	-0.26	0.91	1.19	0.53
AIR.PA	0.07%	0.0006	2.45%	16.05%	0.1458	38.18%	16.49	-0.32	0.41	1.42	0.54
AMZN	0.14%	0.0004	1.92%	34.17%	0.0891	29.85%	6.32	0.50	1.08	0.97	0.33
APD	0.06%	0.0002	1.53%	13.94%	0.0569	23.86%	12.04	-0.28	0.50	0.97	0.51
AZN.L	0.05%	0.0002	1.57%	12.88%	0.0597	24.42%	10.80	-0.57	0.48	0.66	0.21
BAS.DE	0.01%	0.0003	1.67%	3.60%	0.0680	26.07%	6.95	-0.40	0.14	1.10	0.72
BP.L	0.01%	0.0004	2.03%	3.43%	0.0995	31.55%	13.37	0.02	0.07	1.41	0.57
CPR.MI	0.10%	0.0003	1.62%	24.08%	0.0636	25.21%	11.30	-0.51	0.89	0.57	0.29
ENEL.MI	0.06%	0.0003	1.65%	13.82%	0.0664	25.76%	26.04	-2.07	0.47	0.87	0.64
EZJ.L	-0.05%	0.0010	3.09%	-12.56%	0.2315	48.12%	13.87	-0.19	-0.28	1.28	0.20
G.MI	0.03%	0.0003	1.66%	7.46%	0.0665	25.78%	16.32	-1.19	0.22	0.90	0.69
GM	0.04%	0.0005	2.19%	10.50%	0.1167	34.16%	10.05	-0.10	0.25	1.14	0.35
GOOG	0.10%	0.0003	1.68%	24.56%	0.0687	26.22%	8.54	0.23	0.86	1.08	0.52
IHG.L	0.04%	0.0004	2.09%	10.25%	0.1059	32.54%	14.61	0.36	0.28	1.23	0.41
IP.MI	0.10%	0.0004	1.93%	24.96%	0.0901	30.01%	3.60	-0.44	0.78	0.70	0.31
LHA.DE	-0.04%	0.0007	2.65%	-10.63%	0.1703	41.27%	16.66	-0.96	-0.26	1.05	0.26
LMT	0.05%	0.0002	1.45%	11.33%	0.0510	22.59%	17.60	-0.87	0.42	0.78	0.36
MC.PA	0.11%	0.0003	1.74%	26.46%	0.0730	27.03%	3.00	-0.13	0.96	1.09	0.64
MSFT	0.12%	0.0003	1.70%	30.12%	0.0703	26.51%	11.00	-0.26	1.06	1.20	0.63
OR.PA	0.07%	0.0002	1.39%	17.80%	0.0468	21.63%	3.92	0.01	0.80	0.78	0.51
PFE	0.05%	0.0002	1.43%	13.34%	0.0499	22.34%	6.50	0.23	0.51	0.68	0.29
PG	0.05%	0.0001	1.20%	11.35%	0.0348	18.66%	13.96	0.20	0.50	0.62	0.34
REP.MC	0.00%	0.0005	2.17%	0.35%	0.1140	33.77%	8.62	0.16	-0.02	1.16	0.54
SAN.MC	-0.03%	0.0005	2.31%	-7.02%	0.1295	35.99%	11.96	-0.73	-0.23	1.49	0.78
SAN.PA	0.03%	0.0002	1.39%	6.20%	0.0467	21.60%	2.99	-0.15	0.27	0.68	0.39
SAP.DE	0.05%	0.0003	1.61%	12.61%	0.0629	25.08%	37.14	-2.17	0.50	0.90	0.53
SOLB.BR	0.01%	0.0003	1.83%	3.36%	0.0813	28.52%	9.97	-0.25	0.10	1.12	0.55
ULVR.L	0.04%	0.0002	1.32%	9.22%	0.0424	20.59%	9.29	0.45	0.40	0.64	0.27
VOW3.DE	0.01%	0.0006	2.37%	1.78%	0.1361	36.89%	12.89	-0.90	0.05	1.32	0.52
WMT	0.04%	0.0002	1.36%	9.55%	0.0448	21.17%	15.16	0.27	0.36	0.56	0.22

Table 4.2. Descriptive statistics over the time horizon 2015-2021.

2015-2019											
Stock	**Daily E[R]**	**Daily Variance**	**Daily Std. Dev.**	**Annual E[R]**	**Annual Variance**	**Annual Std. Dev.**	**Kurtosis**	**Skewness**	**Sharpe Ratio**	**Beta**	**Adj. R^2**
AAPL	0.09%	0.0003	1.58%	21.31%	0.0607	24.64%	4.05	-0.42	0.77	1.24	0.44
AIR.PA	0.10%	0.0003	1.71%	24.69%	0.0711	26.66%	1.58	0.04	0.90	1.16	0.55
AMZN	0.15%	0.0003	1.85%	35.79%	0.0832	28.84%	8.18	0.64	1.16	1.32	0.37
APD	0.06%	0.0001	1.20%	13.54%	0.0349	18.68%	2.88	-0.01	0.60	0.94	0.45
AZN.L	0.06%	0.0002	1.49%	14.42%	0.0535	23.13%	15.77	-0.88	0.57	0.84	0.25
BAS.DE	0.01%	0.0002	1.42%	3.08%	0.0490	22.13%	1.27	-0.17	0.13	1.08	0.74
BP.L	0.04%	0.0002	1.52%	9.52%	0.0559	23.63%	2.82	-0.19	0.35	1.22	0.51
CPR.MI	0.10%	0.0002	1.46%	24.30%	0.0521	22.81%	1.28	-0.08	0.98	0.52	0.25
ENEL.MI	0.07%	0.0002	1.45%	17.28%	0.0511	22.60%	3.88	-0.35	0.68	0.82	0.62
EZJ.L	0.00%	0.0005	2.25%	0.42%	0.1226	35.01%	15.83	-1.58	-0.03	0.76	0.09
G.MI	0.03%	0.0003	1.62%	7.71%	0.0639	25.28%	16.52	-1.09	0.23	0.93	0.65
GM	0.02%	0.0003	1.63%	5.11%	0.0645	25.40%	4.27	0.34	0.11	1.04	0.30
GOOG	0.08%	0.0002	1.53%	18.74%	0.0569	23.85%	11.06	0.68	0.69	1.21	0.45
IHG.L	0.06%	0.0002	1.56%	15.59%	0.0594	24.36%	26.65	-0.12	0.59	0.98	0.31
IP.MI	0.07%	0.0003	1.80%	18.15%	0.0790	28.11%	2.75	-0.22	0.58	0.67	0.27
LHA.DE	0.02%	0.0004	1.98%	4.65%	0.0955	30.91%	3.43	-0.48	0.14	0.85	0.23
LMT	0.07%	0.0001	1.12%	16.82%	0.0304	17.43%	5.00	-0.20	0.83	0.72	0.30
MC.PA	0.10%	0.0003	1.62%	25.05%	0.0639	25.28%	2.38	0.00	0.97	1.17	0.62
MSFT	0.11%	0.0002	1.47%	26.46%	0.0528	22.97%	6.66	0.03	1.05	1.31	0.57
OR.PA	0.06%	0.0002	1.28%	14.74%	0.0398	19.95%	2.80	0.11	0.71	0.85	0.53
PFE	0.03%	0.0001	1.15%	7.94%	0.0322	17.94%	3.69	-0.13	0.32	0.80	0.34
PG	0.04%	0.0001	1.00%	9.54%	0.0241	15.53%	6.29	0.24	0.47	0.56	0.23
REP.MC	0.02%	0.0003	1.76%	4.13%	0.0752	27.42%	4.37	-0.27	0.10	1.08	0.53
SAN.MC	-0.03%	0.0004	2.00%	-7.32%	0.0967	31.09%	15.52	-1.48	-0.28	1.53	0.82
SAN.PA	0.03%	0.0002	1.35%	7.34%	0.0444	21.07%	2.16	0.04	0.32	0.86	0.48
SAP.DE	0.07%	0.0002	1.38%	15.99%	0.0459	21.42%	7.98	0.39	0.73	0.91	0.56
SOLB.BR	0.01%	0.0002	1.57%	3.20%	0.0599	24.48%	2.52	-0.40	0.11	1.19	0.55
ULVR.L	0.06%	0.0002	1.25%	13.48%	0.0381	19.52%	10.54	0.54	0.62	0.78	0.31
VOW3.DE	0.00%	0.0005	2.13%	1.17%	0.1102	33.19%	17.10	-1.70	0.03	1.27	0.45
WMT	0.04%	0.0002	1.24%	9.09%	0.0374	19.34%	17.22	-0.14	0.35	0.61	0.18

Table 4.3. Descriptive statistics over the time horizon 2015-2019.

Stock	Daily E[R]	Daily Variance	Daily Std. Dev.	Annual E[R]	Annual Variance	Annual Std. Dev.	Kurtosis	Skewness	Sharpe Ratio	Beta	Adj. R^2
2020-2021											
AAPL	0.19%	0.0006	2.38%	45.44%	0.1376	37.09%	5.43	-0.17	1.19	1.15	0.62
AIR.PA	-0.02%	0.0014	3.70%	-5.54%	0.3327	57.68%	9.65	-0.28	-0.10	1.71	0.57
AMZN	0.12%	0.0004	2.07%	30.13%	0.1041	32.27%	3.12	0.24	0.90	0.73	0.33
APD	0.06%	0.0005	2.15%	14.94%	0.1122	33.49%	9.25	-0.35	0.41	0.98	0.55
AZN.L	0.04%	0.0003	1.76%	9.02%	0.0752	27.42%	3.75	-0.09	0.31	0.50	0.17
BAS.DE	0.02%	0.0005	2.18%	4.90%	0.1156	34.00%	7.21	-0.53	0.14	1.11	0.71
BP.L	-0.05%	0.0009	2.93%	-11.82%	0.2089	45.70%	9.08	0.14	-0.27	1.59	0.62
CPR.MI	0.10%	0.0004	1.95%	23.53%	0.0925	30.42%	17.43	-0.94	0.74	0.64	0.36
ENEL.MI	0.02%	0.0004	2.08%	5.17%	0.1047	32.36%	33.88	-3.35	0.13	0.94	0.68
EZJ.L	-0.19%	0.0021	4.56%	-45.00%	0.5041	71.00%	6.81	0.33	-0.64	1.78	0.32
G.MI	0.03%	0.0003	1.73%	6.85%	0.0729	27.00%	15.88	-1.40	0.22	0.84	0.79
GM	0.10%	0.0010	3.20%	23.97%	0.2476	49.75%	6.01	-0.27	0.46	1.21	0.38
GOOG	0.16%	0.0004	2.01%	39.11%	0.0984	31.37%	5.12	-0.32	1.21	0.99	0.64
IHG.L	-0.01%	0.0009	3.03%	-3.11%	0.2223	47.15%	5.80	0.50	-0.08	1.46	0.49
IP.MI	0.17%	0.0005	2.20%	42.01%	0.1176	34.30%	4.20	-0.76	1.20	0.74	0.37
LHA.DE	-0.20%	0.0015	3.84%	-48.85%	0.3569	59.74%	11.50	-0.86	-0.82	1.29	0.31
LMT	-0.01%	0.0004	2.06%	-2.39%	0.1027	32.05%	13.09	-0.94	-0.11	0.82	0.42
MC.PA	0.12%	0.0004	1.99%	30.01%	0.0960	30.99%	3.26	-0.31	0.97	1.01	0.70
MSFT	0.16%	0.0005	2.17%	39.31%	0.1142	33.79%	10.56	-0.49	1.13	1.12	0.71
OR.PA	0.10%	0.0003	1.63%	25.46%	0.0644	25.37%	4.34	-0.13	1.00	0.70	0.49
PFE	0.11%	0.0004	1.97%	26.86%	0.0943	30.71%	4.38	0.32	0.84	0.61	0.25
PG	0.07%	0.0003	1.60%	15.88%	0.0618	24.85%	12.49	0.13	0.59	0.66	0.46
REP.MC	-0.04%	0.0009	2.95%	-9.11%	0.2115	45.99%	6.61	0.39	-0.21	1.25	0.55
SAN.MC	-0.03%	0.0009	2.96%	-6.29%	0.2121	46.05%	7.02	-0.10	-0.14	1.44	0.73
SAN.PA	0.01%	0.0002	1.47%	3.33%	0.0524	22.89%	4.41	-0.51	0.15	0.48	0.28
SAP.DE	0.02%	0.0004	2.09%	4.17%	0.1056	32.49%	41.88	-3.75	0.13	0.90	0.50
SOLB.BR	0.02%	0.0006	2.36%	3.77%	0.1351	36.75%	10.87	-0.12	0.10	1.06	0.55
ULVR.L	-0.01%	0.0002	1.48%	-1.42%	0.0531	23.05%	7.07	0.32	-0.08	0.51	0.25
VOW3.DE	0.01%	0.0008	2.88%	3.33%	0.2013	44.87%	7.48	-0.03	0.07	1.38	0.62
WMT	0.04%	0.0003	1.62%	10.70%	0.0634	25.18%	11.28	0.73	0.38	0.53	0.28

Table 4.4. Descriptive statistics over the time horizon 2020-2021.

As outlined in section 3.1.4, the Sharpe ratio represents the expected return per unit of risk: therefore, it shows how much return an investor can achieve by taking additional risk. Denoting with $i = 1, \ldots, 30$ the number of stocks considered, for each stock the Sharpe ratio has been computed as:

$$SR = \frac{\overline{R_i} - R_F}{\sigma_i}$$

The national 10-year government bond yield is the risk-free rate considered for each stock, as shown in Table 4.5, Table 4.6 and Table 4.7. Note that if the annual yield was negative, it has been approximated with a 0% value.

Time Horizon 2015-2021	Belgium 10 Y Gov. Bond	France 10 Y Gov. Bond	Germany 10 Y Gov. Bond	Italian BTP 10 Y	Spain 10 Y Gov. Bond	UK Gilt 10 Y	US Treasury Yield 10 Y
Annual Yield	0.41%	0.41%	0.07%	1.67%	1.08%	1.09%	1.95%
Annual Variance	1.8E-05	1.9E-05	1.7E-05	4.9E-05	3.6E-05	2.5E-05	4.3E-05
Annual Std. Dev	0.43%	0.44%	0.41%	0.70%	0.60%	0.50%	0.66%

Table 4.5. 10-year government bond descriptive statistics over the time horizon 2015-2021.

Time Horizon 2015-2019	Belgium 10 Y Gov. Bond	France 10 Y Gov. Bond	Germany 10 Y Gov. Bond	Italian BTP 10 Y	Spain 10 Y Gov. Bond	UK Gilt 10 Y	US Treasury Yield 10 Y
Annual Yield	0.60%	0.61%	0.26%	1.96%	1.36%	1.31%	2.27%
Annual Variance	1.1E-05	1.2E-05	1.1E-05	3.5E-05	2.1E-05	1.5E-05	1.9E-05
Annual Std. Dev	0.33%	0.35%	0.33%	0.60%	0.46%	0.38%	0.44%

Table 4.6. 10-year government bond descriptive statistics over the time horizon 2015-2019.

Time Horizon 2020-2021	Belgium 10 Y Gov. Bond	France 10 Y Gov. Bond	Germany 10 Y Gov. Bond	Italian BTP 10 Y	Spain 10 Y Gov. Bond	UK Gilt 10 Y	US Treasury Yield 10 Y
Annual Yield	0.00%	0.00%	0.00%	0.96%	0.38%	0.53%	1.16%
Annual Variance	3.4E-06	2.9E-06	2.0E-06	1.3E-05	4.3E-06	8.0E-06	1.6E-05
Annual Std. Dev	0.18%	0.17%	0.14%	0.37%	0.21%	0.28%	0.40%

Table 4.7. 10-year government bond descriptive statistics over the time horizon 2020-2021.

Beta is the relevant measure of risk for an individual security, quantifying how much of asset i's return is driven by the market return. Under the single index model, beta can be estimated by using historical data as:

$$\overline{R_i} = \alpha_i + \beta_i \overline{R_m} + \varepsilon_i$$

Where:

- α_i is the part of return which is insensitive to the market return
- $\overline{R_m}$ is the market return
- $\varepsilon_i \sim N(0, \sigma_{\varepsilon_i})$

BEL 20 was taken as benchmark stock market index of Brussel Stock Exchange, CAC 40 of Euronext Paris, FTSE MIB of Milan Stock Exchange, FTSE 350 of London Stock Exchange, IBEX 35 of Madrid Stock Exchange, S&P 500 of NYSE and DAX 30 of Frankfurt Stock Exchange. By means of regression analysis, the respective betas were then computed.

4.2.1 Apple Inc.

While Figure 4.1 shows Apple Inc. historical prices performance in the last 7 years, Figure 4.2 exhibits the log return distribution over the same time horizon. Between 2015 and 2018 the stock price is quite stable. On 3rd January 2019 it experienced a major fall, after reporting lower than expected iPhones sales due to longer upgrade cycles and headwinds in China. In the first wave of the Covid-19 crisis, on 16th March 2020 Apple stock price dropped. However, in the following months, not only it regained all the ground lost, but it also climbed.

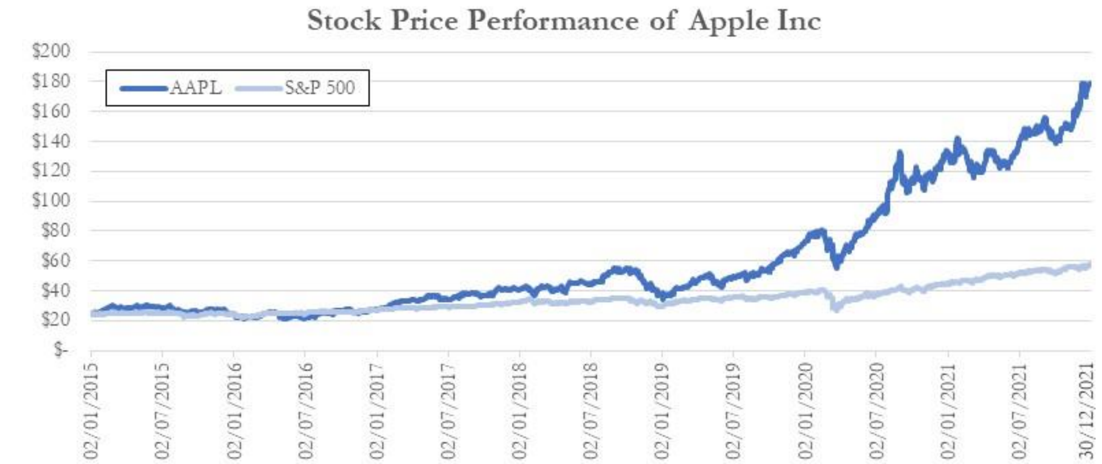

Figure 4.1. Stock price performance of Apple Inc.[2]

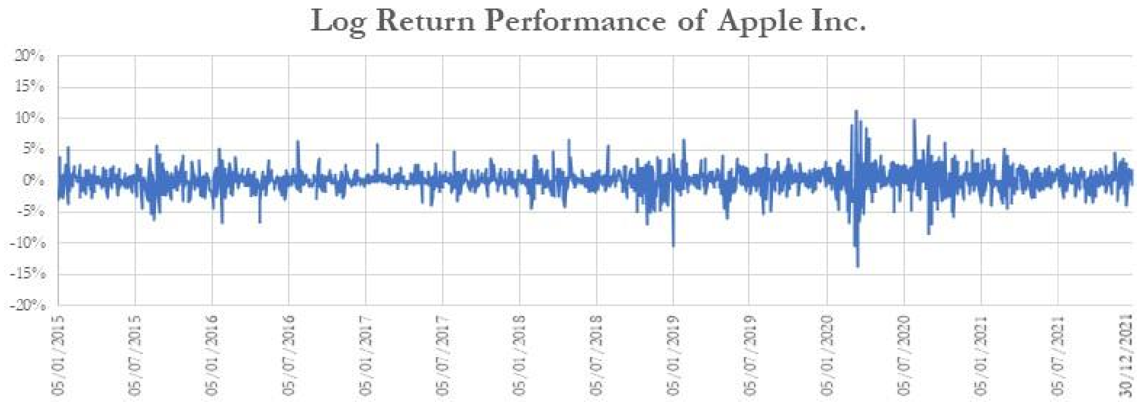

Figure 4.2. Log return performance Apple Inc.

The descriptive statistics for the daily logarithmic returns computed over 2015 to 2019 are reported in Table 4.8 and the distribution appears to be slightly left skewed, with thick tails as confirmed by Figure 4.3, which plots the log returns. Nevertheless, by analysing the Q-Q plot, an alignment between observed and theoretical quantiles can be observed, as shown in Figure 4.4.

[2] To make the data comparable, S&P 500 stock index price has been normalized using AAPL stock price on 02/01/2015 as base year

Apple Inc. log return descriptive statistics over 2015-2021	
Mean	0.12%
Standard Error	0.04%
Median	0.10%
Standard Deviation	1.85%
Sample Variance	0.0003
Kurtosis	6.3288
Skewness	-0.2586
Range	0.2509
Minimum	-13.77%
Maximum	11.32%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.8 Apple Inc. descriptive statistics over 2015-2021.

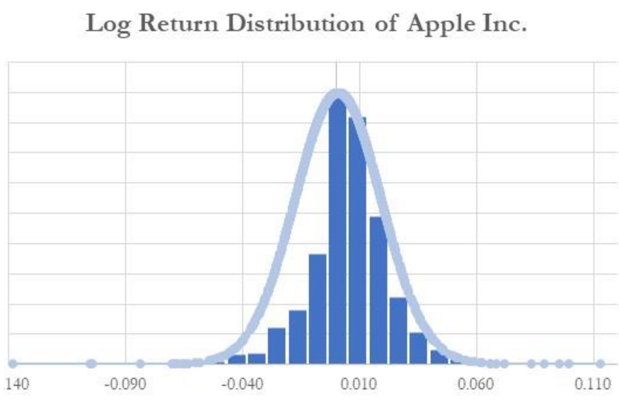

Figure 4.3. Log return distribution of Apple Inc. over 2015-2021.

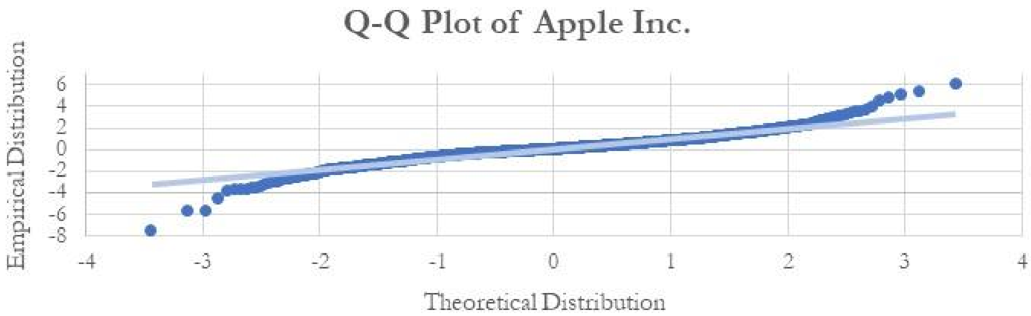

Figure 4.4. Q-Q Plot of Apple Inc. over 2015-2021.

To isolate the impact of Covid-19, the considered time period has been split into two subperiods, namely 2015-2019 and 2020-2021, and the respective descriptive statistics have been computed, as shown in Table 4.9 and Table 4.10. In 2020-2021 both the daily average return and the volatility more than doubled, while the kurtosis increased. Even though the Kolmogorov–Smirnov and the Shapiro–Wilk test both reject the null hypothesis about the normal distribution of the returns, we will assume it for the purpose of this thesis.

Descriptive statistics of Apple Inc. 2015-2019 log returns series	
Mean	0.09%
Standard Error	0.05%
Median	0.09%
Standard Deviation	1.58%
Sample Variance	0.0003
Kurtosis	4.0511
Skewness	-0.4152
Range	0.1711
Minimum	-10.49%
Maximum	6.62%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.9. Apple Inc. descriptive statistics over 2015-2019.

Descriptive statistics of Apple Inc. 2020-2021 log returns series	
Mean	0.19%
Standard Error	0.11%
Median	0.15%
Standard Deviation	2.38%
Sample Variance	0.0006
Kurtosis	5.4262
Skewness	-0.1682
Range	0.2509
Minimum	-13.77%
Maximum	11.32%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.002

Table 4.10. Apple Inc. descriptive statistics over 2020-2021.

4.2.2 Airbus SE

As shown in Figure 4.5, between 2015 and February 2020, Airbus stock price presents an overall upward trend. The only event to be pointed out in that timeframe consists in corruption allegations at the end of December 2018 that caused the stock price to drop. Covid-19 restrictions hit the airplane and aerospace sector hard: in particular, from 13th March to 4th April 2020, the stock price experienced a steep fall, heavily impacting the returns, as shown in Figure 4.6. Thanks to government aid and partial improvements of the crisis, the stock price started to rise again.

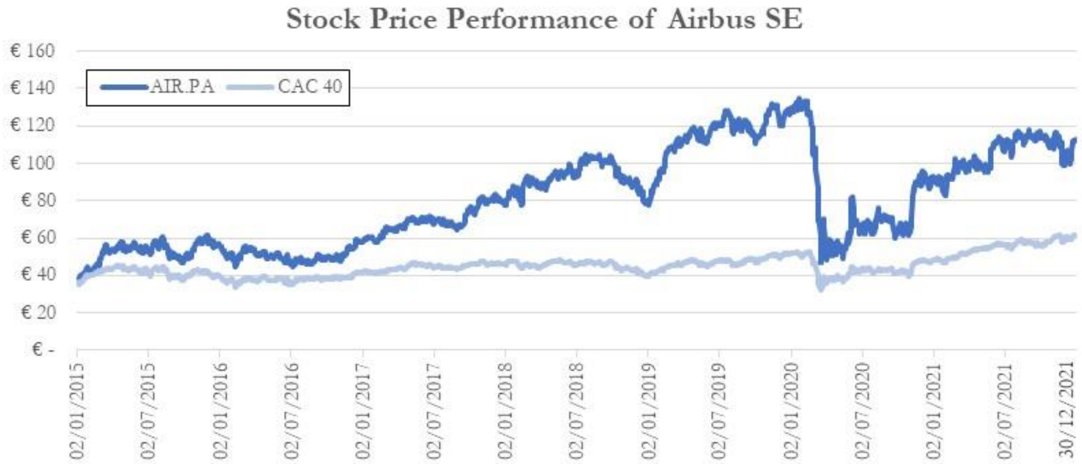

Figure 4.5. Stock price performance of Airbus SE.[3]

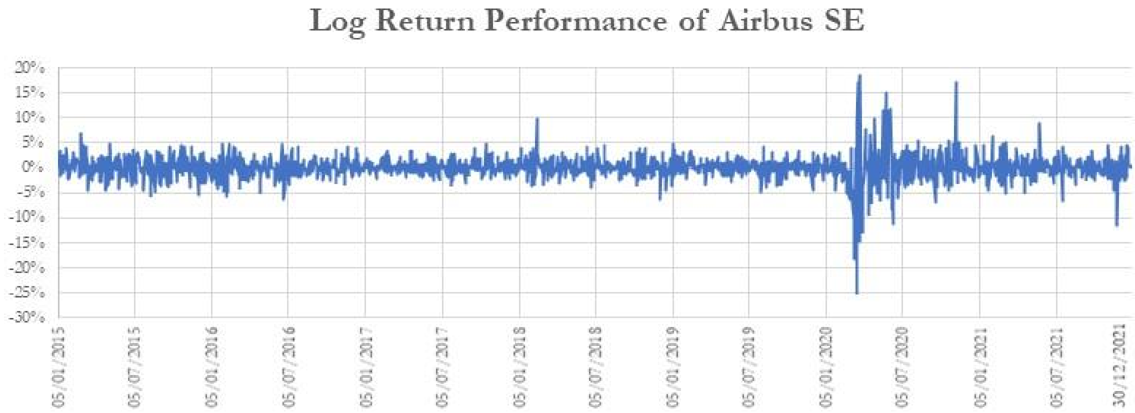

Figure 4.6. Log return performance Airbus SE.

Table 4.11 reports the descriptive statistics for the daily logarithmic returns computed over 2015 to 2019 and the distribution appears to be left skewed, with thick tails as confirmed by Figure 4.7, where the log return distribution has been plotted. Nevertheless, by analysing the Q-Q plot, an alignment between observed and theoretical quantiles can be observed, as shown in Figure 4.8.

[3] To make the data comparable, CAC 40 stock index price has been normalized using AIR.PA stock price on 02/01/2015 as base year

Airbus SE log return descriptive statistics over 2015-2021	
Mean	0.07%
Standard Error	0.06%
Median	0.06%
Standard Deviation	2.45%
Sample Variance	0.0006
Kurtosis	16.4905
Skewness	-0.3234
Range	0.4368
Minimum	-25.06%
Maximum	18.62%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.11. Airbus SE descriptive statistics over 2015-2021.

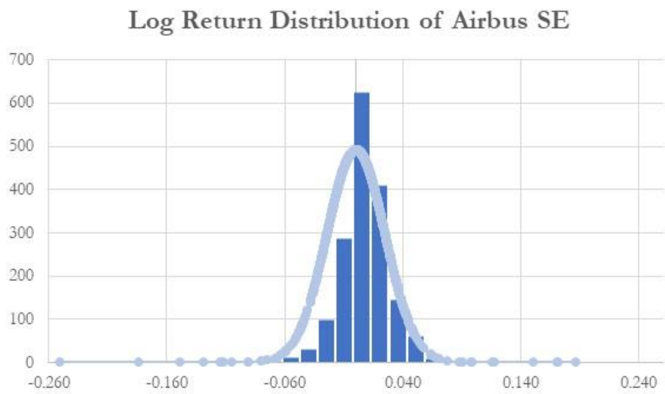

Figure 4.7. Log return distribution of Airbus SE over 2015-2021

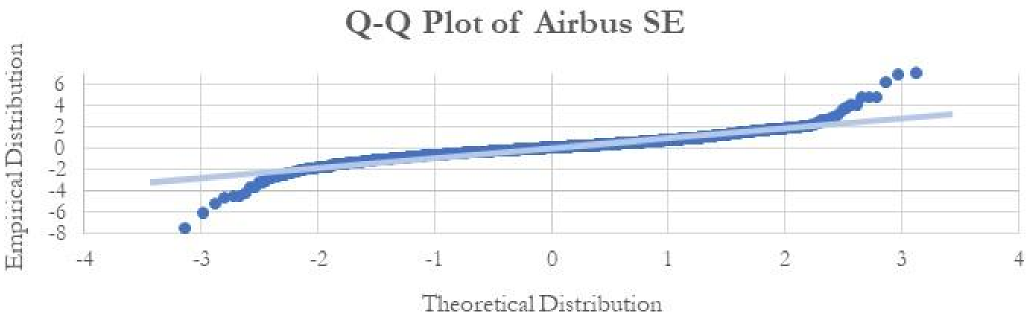

Figure 4.8. Q-Q Plot of Airbus SE. over 2015-2021.

Table 4.12 and Table 4.13 show the descriptive statistics computed respectively over 2015-2019 and 2020-2021. While in the first subperiod the daily distribution presents positive average return, with contained volatility, lower kurtosis, and positive skewness, during the pandemic the mean turned negative with increased standard deviation, higher kurtosis and negative skewness are found. If according to the Kolmogorov- Smirnov normality test, the logarithmic return distribution in the first sub period can be approximated with a normal one, the same cannot be said over the time horizon 2020-2021. Nonetheless, we will assume it for the purpose of this thesis.

Airbus SE log return descriptive statistics over 2015-2019	
Mean	0.10%
Standard Error	0.05%
Median	0.11%
Standard Deviation	1.71%
Sample Variance	0.0003
Kurtosis	1.5796
Skewness	0.0435
Range	0.1614
Minimum	-6.36%
Maximum	9.78%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.057

Table 4.12. Airbus SE descriptive statistics over 2015-2019.

Airbus SE log return descriptive statistics over 2020-2021	
Mean	-0.02%
Standard Error	0.17%
Median	-0.11%
Standard Deviation	3.70%
Sample Variance	0.0014
Kurtosis	9.6533
Skewness	-0.2836
Range	0.4368
Minimum	-25.06%
Maximum	18.62%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.13. Airbus SE descriptive statistics over 2020-2021

4.2.3 Amazon.com Inc.

In Figure 4.9 Amazon. Com Inc. historical price performance is shown over the last 7 years, while in Figure 4.10 the log return distribution over the same time period is reported. While the stock price shows an overall increasing trend, some events had a particular impact. For the first time, on 27th October 2017, Amazon share price traded above $1100. On 26th October 2018, the release of the firm's earnings report led to a negative stock return. Eventually, after an initial price drop due to the spread of the pandemic on 12th March 2020, the stock price shows an increasing trend, revealing Amazon as one of the biggest winners from Covid-19.

Figure 4.9. Stock price performance of Amazon.com Inc.[4]

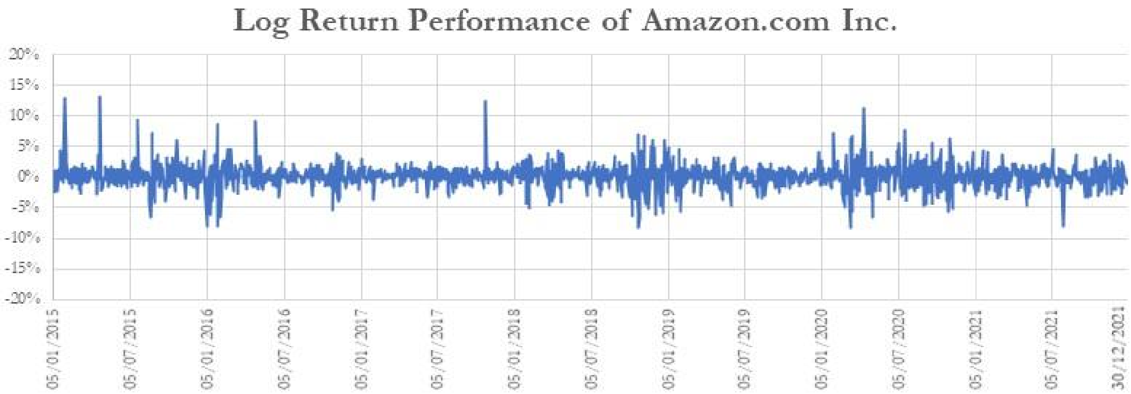

Figure 4.10. Log return performance of Amazon.com Inc.

The descriptive statistics for the daily logarithmic returns computed over 2015-2019 are reported in Table 4.14 and the distribution appears to be slightly right skewed, with thick tails as confirmed by Figure 4.11, where the log returns distribution has been plotted. Nevertheless, by analysing the Q-Q plot in Figure 4.12, an alignment between observed and theoretical quantiles exists.

[4] To make the data comparable, S&P 500 stock index price has been normalized using AMZN stock price on 02/01/2015 as base year

Amazon.com Inc. log return descriptive statistics over 2015-2021	
Mean	0.14%
Standard Error	0.05%
Median	0.14%
Standard Deviation	1.92%
Sample Variance	0.0004
Kurtosis	6.3191
Skewness	0.4984
Range	0.2147
Minimum	-8.25%
Maximum	13.22%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.14. Amazon.com Inc. descriptive statistics over 2015-2021.

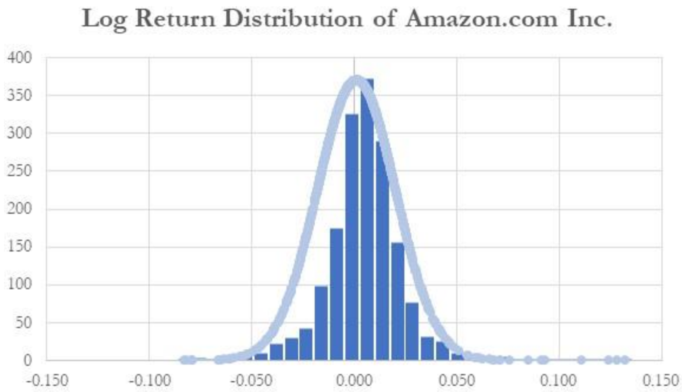

Figure 4.11. Log return distribution of Amazon.com Inc. over 2015-2021.

Figure 4.12. Q-Q Plot of Amazon.com Inc. over 2015-2021

As the descriptive statistics reported in Table 4.15 and Table 4.16 highlight, the daily distribution of returns over 2015-2019 presents a higher level of kurtosis and skewness than in 2020-2021. It has to be underlined the fact that in 2020-2021 the distribution is almost symmetrical, since the kurtosis value is almost 3. Even though the Kolmogorov–Smirnov and the Shapiro–Wilk test both reject the null hypothesis about the normal distribution of the returns, we will assume it for the purpose of this thesis.

Amazon.com Inc. log return descriptive statistics over 2015-2019	
Mean	0.15%
Standard Error	0.05%
Median	0.14%
Standard Deviation	1.85%
Sample Variance	0.0003
Kurtosis	8.1785
Skewness	0.6446
Range	0.2136
Minimum	-8.14%
Maximum	13.22%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.15. Amazon.com Inc. descriptive statistics over 2015-2019.

Amazon.com Inc. log return descriptive statistics over 2020-2021	
Mean	0.12%
Standard Error	0.09%
Median	0.15%
Standard Deviation	2.07%
Sample Variance	0.0004
Kurtosis	3.1222
Skewness	0.2371
Range	0.1939
Minimum	-8.25%
Maximum	11.13%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.021

Table 4.16. Amazon.com Inc. descriptive statistics over 2020-2021.

4.2.4 Air Products and Chemicals, Inc.

As shown in Figure 4.13, between 2015 and 2018, Air Products and Chemicals, Inc. stock price is quite stable. 2019 was a positive year for the company due to positive earnings report, adequate dividend policy and promising future expectations in the market. As the log return distribution in Figure 4.14 highlights, price falls occurred between the first wave of Covid-19 in February-March 2020 and the second one, in October-December 2020. Despite this, the stock performance recover, even exceeding the highest price reached in February 2020.

Figure 4.13. Stock price performance of Air Products and Chemicals, Inc.[5]

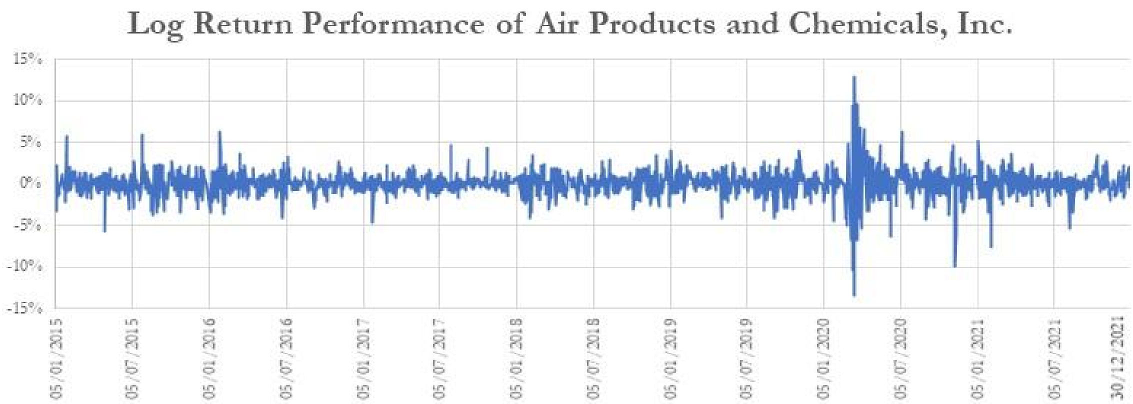

Figure 4.14. Log return performance of Air Products and Chemicals, Inc.

Table 4.17 shows the descriptive statistics for the daily logarithmic returns computed over 2015 to 2019 and the distribution appears to be slightly right skewed, with thick tails as Figure 4.15 confirms. Nevertheless, by analysing the Q-Q plot in Figure 4.16, an alignment between observed and theoretical quantiles exists.

[5] To make the data comparable, S&P 500 stock index price has been normalized using APD stock price on 02/01/2015 as base year

Air Products and Chemicals, Inc. log return descriptive statistics over 2015-2021	
Mean	0.06%
Standard Error	0.04%
Median	0.08%
Standard Deviation	1.53%
Sample Variance	0.0002
Kurtosis	12.0417
Skewness	-0.2793
Range	0.2633
Minimum	-13.47%
Maximum	12.86%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.17. Air Products and Chemicals, Inc. descriptive statistics over 2015-2021.

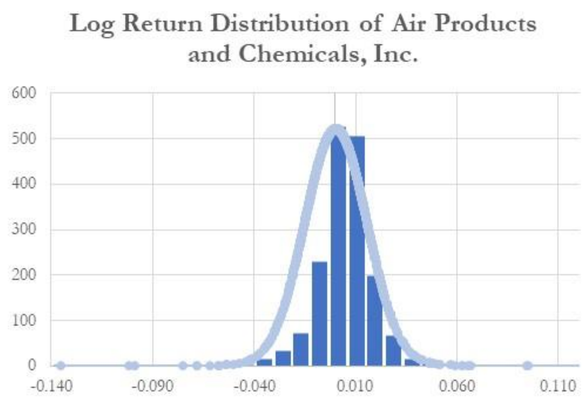

Figure 4.15. Log return performance of Air Products and Chemicals, Inc. over 2015-2021

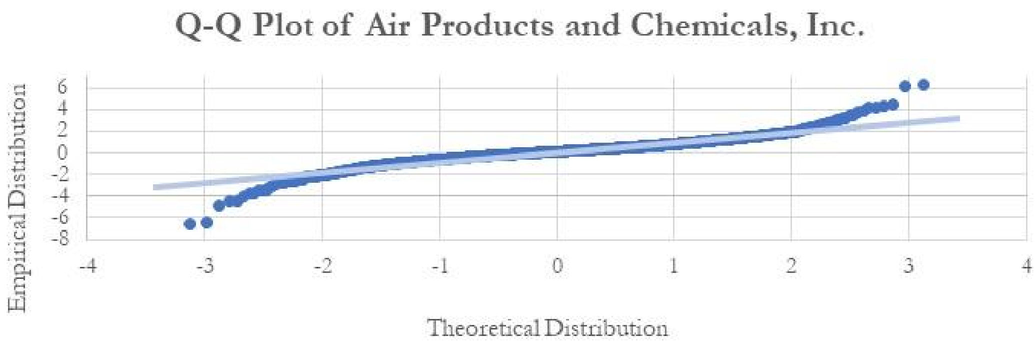

Figure 4.16. Q-Q Plot of Air Products and Chemicals, Inc.

Table 4.18 and Table 4.19 display the descriptive statistics of the daily log returns over two subperiods. 2020-2021 presents the same average daily return of 2015-2019, but with higher volatility. Moreover, the level of kurtosis is higher, and the curve appears left skewed. Even though the Kolmogorov–Smirnov and the Shapiro–Wilk test both reject the null hypothesis about the normal distribution of the returns, it will be assumed in the following part of this thesis.

Air Products and Chemicals, Inc. log return descriptive statistics over 2015-2019	
Mean	0.06%
Standard Error	0.03%
Median	0.06%
Standard Deviation	1.20%
Sample Variance	0.0001
Kurtosis	2.8787
Skewness	-0.0143
Range	0.1199
Minimum	-5.72%
Maximum	6.27%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.18. Air Products and Chemicals, Inc. descriptive statistics over 2015-2019.

Air Products and Chemicals, Inc. log return descriptive statistics over 2020-2021	
Mean	0.06%
Standard Error	0.10%
Median	0.18%
Standard Deviation	2.15%
Sample Variance	0.0005
Kurtosis	9.2472
Skewness	-0.3527
Range	0.2633
Minimum	-13.47%
Maximum	12.86%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.19. Air Products and Chemicals, Inc. descriptive statistics over 2020-2021.

4.2.5 AstraZeneca PLC

While Figure 4.17 shows AstraZeneca PLC historical prices performance in the last 7 years, Figure 4.18 presents the log return distribution over the same time horizon. From June 2016, the stock price presents an upward trend. On the 27th July 2017 the stock price dropped, after the failure of a key trial for lung cancer drug. If at the beginning of the pandemic the share price fell, immediately after it grew, due to the release of the Covid-19 vaccine.

Figure 4.17. Stock price performance of AstraZeneca PLC.[6]

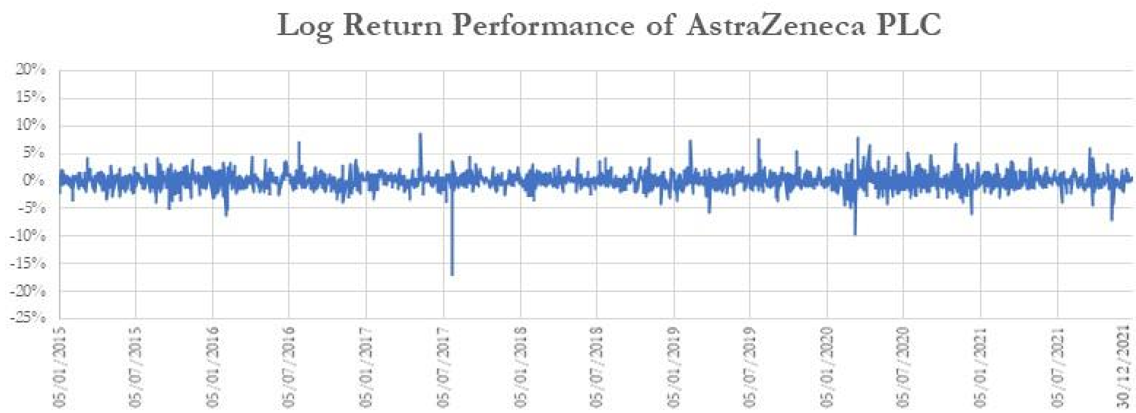

Figure 4.18. Log return performance of AstraZeneca PLC.

The descriptive statistics for the daily logarithmic returns computed over 2015-2019 are reported in Table 4.20 and the distribution appears to be left skewed, with thick tails as confirmed by Figure 4.19, which plots the log returns. Nevertheless, by analysing the Q-Q plot, an alignment between observed and theoretical quantiles can be observed, as shown in Figure 4.20.

[6] To make the data comparable, FTSE 350 stock index price has been normalized using AZN.L stock price on 02/01/2015 as base year

AstraZeneca PLC log return descriptive statistics over 2015-2021	
Mean	0.05%
Standard Error	0.04%
Median	0.05%
Standard Deviation	1.57%
Sample Variance	0.0002
Kurtosis	10.7977
Skewness	-0.5698
Range	0.2538
Minimum	-16.74%
Maximum	8.64%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.20. AstraZeneca PLC descriptive statistics over 2015-2021.

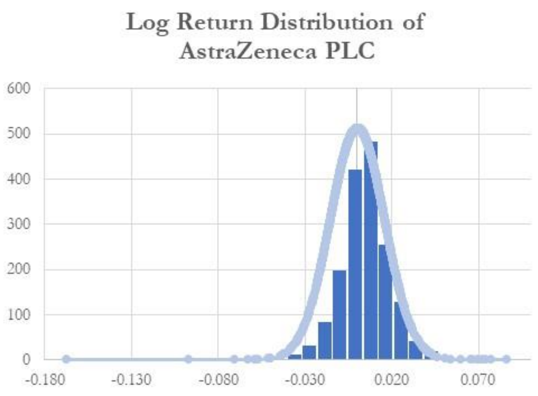

Figure 4.19 Log return performance of AstraZeneca PLC over 2015-2021

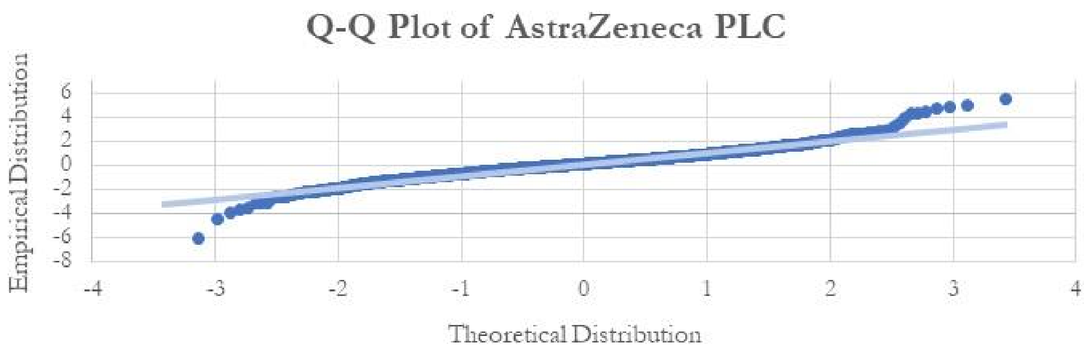

Figure 4.20. Q-Q Plot of AstraZeneca PLC over 2015-2021

Table 4.21 shows that the distribution of the daily logarithmic returns over 2015-2019 presents high level of kurtosis and negative skewness. As reported in Table 4.22, over 2020-2021, while the average daily return decreases, with higher volatility, skewness is almost negligible, with a kurtosis value similar to that one of a normal distribution. Under a level of significance of 5%, both the Kolmogorov–Smirnov and the Shapiro–Wilk test reject the null hypothesis about the normal distribution of the returns. Nevertheless, we will assume it for the purpose of this thesis.

AstraZeneca PLC log return descriptive statistics over 2015-2019	
Mean	0.06%
Standard Error	0.04%
Median	0.05%
Standard Deviation	1.49%
Sample Variance	0.0002
Kurtosis	15.7677
Skewness	-0.8768
Range	0.2538
Minimum	-16.74%
Maximum	8.64%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.21. AstraZeneca PLC descriptive statistics over 2015-2019.

AstraZeneca PLC log return descriptive statistics over 2020-2021	
Mean	0.04%
Standard Error	0.08%
Median	0.05%
Standard Deviation	1.76%
Sample Variance	0.0003
Kurtosis	3.7535
Skewness	-0.0871
Range	0.1740
Minimum	-9.67%
Maximum	7.73%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.027

Table 4.22. AstraZeneca PLC descriptive statistics over 2020-2021.

4.2.6 BASF SE

Figure 4.21 shows the performance of BASF SE share price over the last 7 years, highlighting the fact that the stock has gone through periods of rise and fall in price. As it can be seen in Figure 4.22, until the Covid-19 crisis there were no huge fluctuations in the stock price. However, it should be noted that, after an initial shock at the beginning of the pandemic, the stock price recovered.

Figure 4.21. Stock price performance of BASF SE.[7]

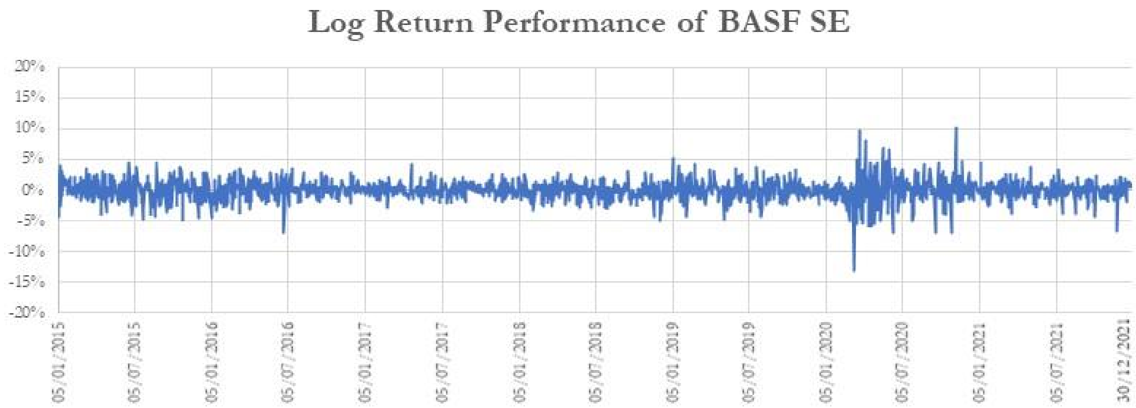

Figure 4.22. Log return performance of BASF SE.

Table 4.23 presents the descriptive statistics for the daily logarithmic returns computed over 2015 to 2019 and the distribution appears to be left skewed, with thick tails as confirmed by Figure 4.23, which plots the log return distribution. Nevertheless, in the Q-Q plot presented in Figure 4.24, an alignment between observed and theoretical quantiles can be observed.

[7] To make the data comparable, DAX 30 stock index price has been normalized using BAS.DE stock price on 02/01/2015 as base year

BASF SE log return descriptive statistics over 2015-2021	
Mean	0.01%
Standard Error	0.04%
Median	0.04%
Standard Deviation	1.67%
Sample Variance	0.0003
Kurtosis	6.9489
Skewness	-0.4023
Range	0.2328
Minimum	-13.08%
Maximum	10.19%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.23. BASF SE descriptive statistics over 2015-2021.

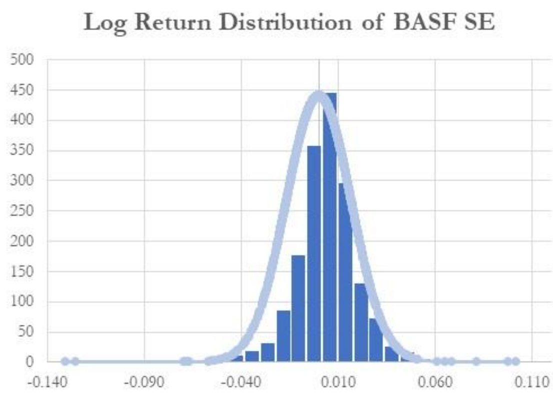

Figure 4.23. Log return distribution of BASF SE over 2015-2021.

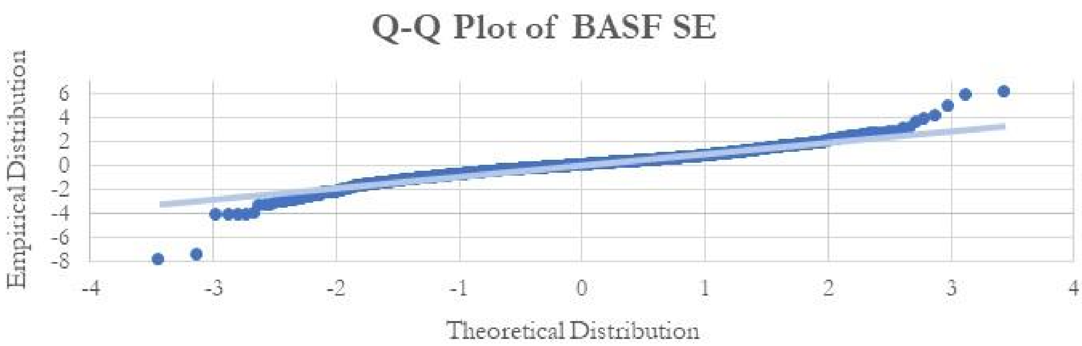

Figure 4.24. Q-Q Plot of BASF SE over 2015-2021.

Table 4.24 and Table 4.25 show the descriptive statistics computed respectively over 2015-2019 and 2020-2021. The skewness is always negative, while there are differences regarding the kurtosis: if in the first subperiod it is close to 1, in the latter it is more than 7. Even though the Shapiro-Wilk and Kolmogorov-Smirnov normality test reject the null hypothesis of normal distribution for both time periods, we will assume it for the purpose of this thesis.

BASF SE log return descriptive statistics over 2015-2019	
Mean	0.01%
Standard Error	0.04%
Median	0.03%
Standard Deviation	1.42%
Sample Variance	0.0002
Kurtosis	1.2670
Skewness	-0.1665
Range	0.1206
Minimum	-6.91%
Maximum	5.15%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.007

Table 4.24. BAS SE descriptive statistics over 2015-2019

BASF SE log return descriptive statistics over 2020-2021	
Mean	0.02%
Standard Error	0.10%
Median	0.05%
Standard Deviation	2.18%
Sample Variance	0.0005
Kurtosis	7.2148
Skewness	-0.5265
Range	0.2328
Minimum	-13.08%
Maximum	10.19%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.25. BAS SE descriptive statistics over 2020-2021

4.2.7 BP p.l.c.

As displayed in Figure 4.25 and Figure 4.26, from January 2015 to June 2018 the stock price was characterized by an overall increasing trend, followed by 6 months of stable returns. In the second quarter of 2019 the share price started to fall, impacted by the trade war between US and China. This downward trend was then amplified by the pandemic, that hit severely the oil & gas sector. However, BP p.l.c. price began recovering, after 28 October 2020, when it reached its lowest level in 7 years.

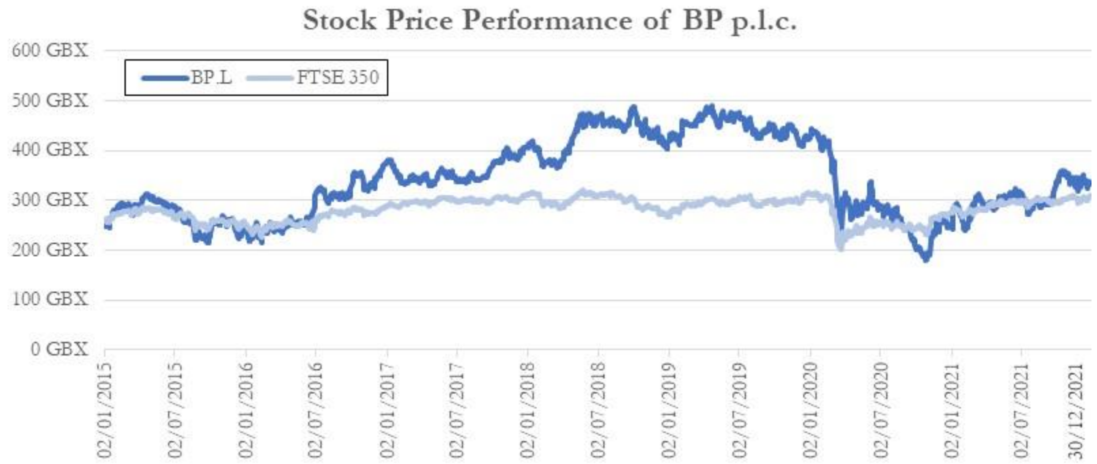

Figure 4.25. Stock price performance of BP p.l.c.[8]

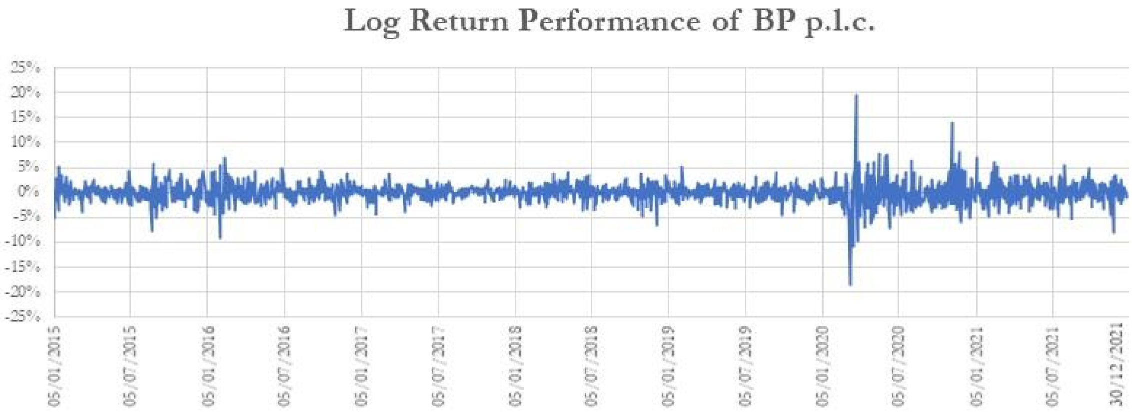

Figure 4.26. Log return performance of BP p.l.c.

Table 4.26 exhibits the descriptive statistics for the daily logarithmic returns computed over 2015 to 2019 and the distribution presents skewness close to zero and thick tails as Figure 4.27 displays. Nevertheless, the Q-Q plot in Figure 4.28 shows an alignment between observed and theoretical quantiles exists.

[8] To make the data comparable, FTSE 350 stock index price has been normalized using BP.L stock price on 02/01/2015 as base year

BP p.l.c. log return descriptive statistics over 2015-2021	
Mean	0.01%
Standard Error	0.05%
Median	0.00%
Standard Deviation	2.02%
Sample Variance	0.0004
Kurtosis	13.4397
Skewness	0.0255
Range	0.3791
Minimum	-18.36%
Maximum	19.54%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.26. BP p.l.c. descriptive statistics over 2015-2021.

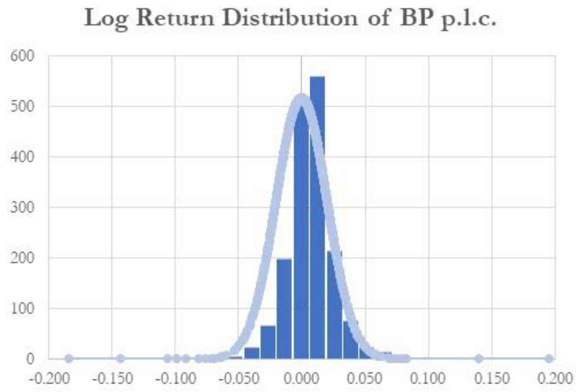

Figure 4.27. Log return distribution of BP p.l.c. over 2015-2021.

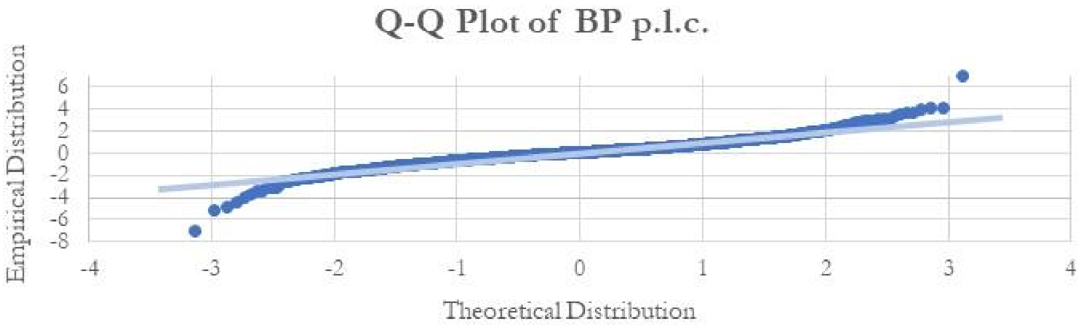

Figure 4.28. Q-Q Plot of BP p.l.c over 2015-2021.

Table 4.27 and Table 4.28 display the descriptive statistics of the daily log returns over two subperiods. While in 2015-2019 the kurtosis is less than 3 and the skewness is negative, 2020-2021 presents higher level of kurtosis and the curve appears right skewed. Moreover, the expected returns turned negative, while being subject to higher volatility. Even though the Shapiro-Wilk and Kolmogorov-Smirnov normality test reject the null hypothesis of normal distribution for both time periods, we will assume it for the purpose of this thesis.

BP p.l.c. log return descriptive statistics over 2015-2019	
Mean	0.04%
Standard Error	0.04%
Median	0.01%
Standard Deviation	1.52%
Sample Variance	0.0002
Kurtosis	2.8158
Skewness	-0.1885
Range	0.1601
Minimum	-9.08%
Maximum	6.93%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.001

Table 4.27. BP p.l.c. descriptive statistics over 2015-2019.

BP p.l.c. log return descriptive statistics over 2020-2021	
Mean	-0.05%
Standard Error	0.13%
Median	-0.08%
Standard Deviation	2.93%
Sample Variance	0.0009
Kurtosis	9.1729
Skewness	0.1419
Range	0.3791
Minimum	-18.36%
Maximum	19.54%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.008

Table 4.28. BP p.l.c. descriptive statistics over 2020-2021.

4.2.8 Davide Campari-Milano N.V.

As shown in Figure 4.29, the Davide Campari-Milano N.V. stock price was characterized by an upward trend, until the Covid-19 crisis. After an initial collapse occurred between February and March 2020, the share price started to rise again, showing a steeper growth than before. Figure 4.30 displays the log return performance of the stock.

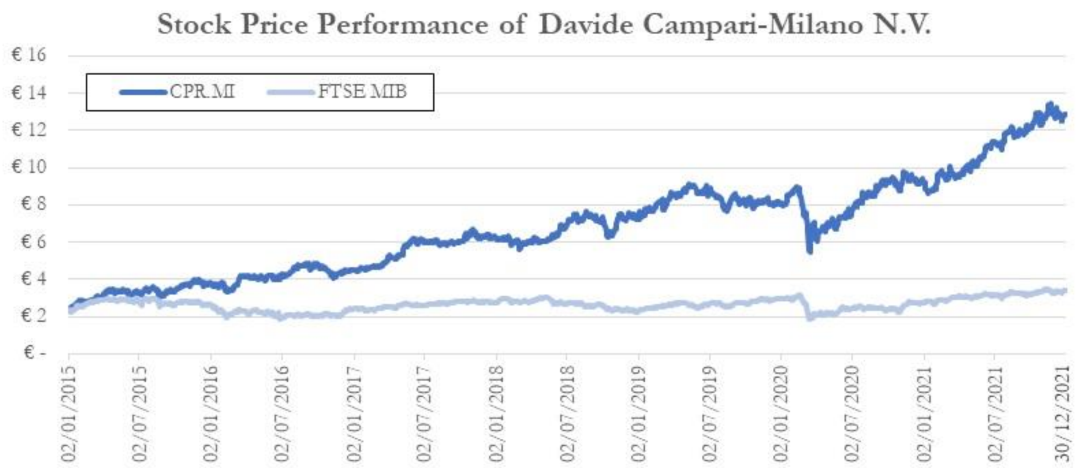

Figure 4.29. Stock price performance of Davide Campari-Milano N.V.[9]

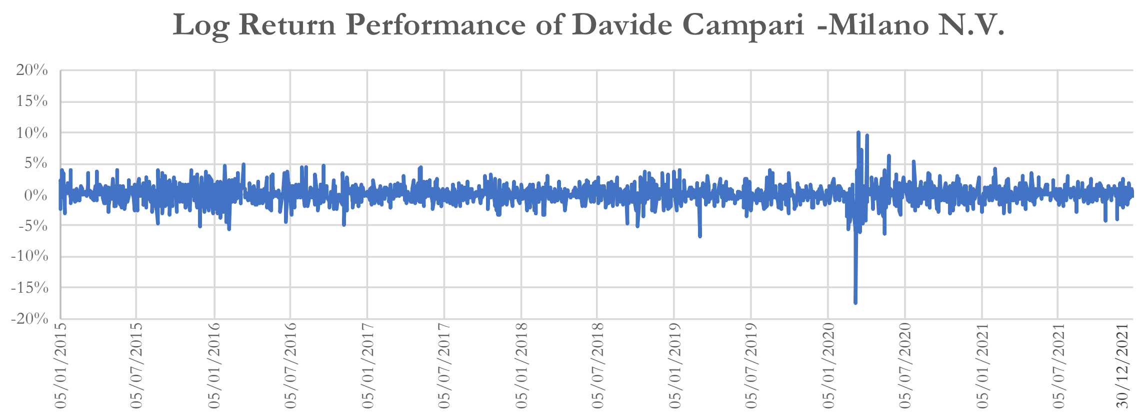

Figure 4.30. Log return performance of Davide Campari-Milano N.V.

As reported in Table 4.29, the daily logarithmic return distribution over 2015 to 2019 appears to be left skewed, with thick tails as confirmed by Figure 4.31., where the log return distribution has been plotted. Nevertheless, by analysing the Q-Q plot, an alignment between observed and theoretical quantiles can be observed, as shown in Figure 4.32.

[9] To make the data comparable, FTSE MIB stock index price has been normalized using CPR.MI stock price on 02/01/2015 as base year

Davide Campari-Milano N.V. log return descriptive statistics over 2015-2021	
Mean	0.10%
Standard Error	0.04%
Median	0.12%
Standard Deviation	1.62%
Sample Variance	0.0003
Kurtosis	11.3049
Skewness	-0.5100
Range	0.2770
Minimum	-17.57%
Maximum	10.13%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.29. Davide Campari-Milano N.V. descriptive statistics over 2015-2021.

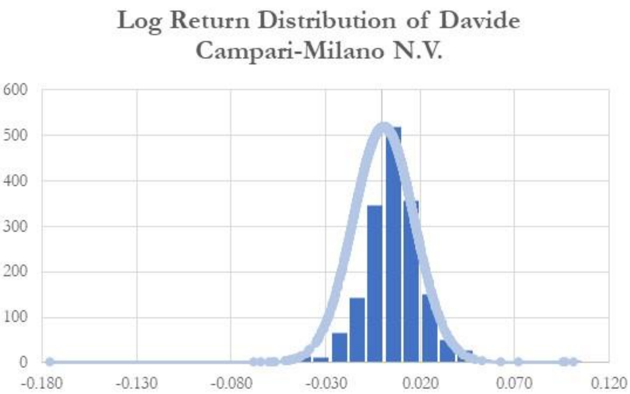

Figure 4.31. Log return performance of Davide Campari-Milano N.V. over 2015-2021.

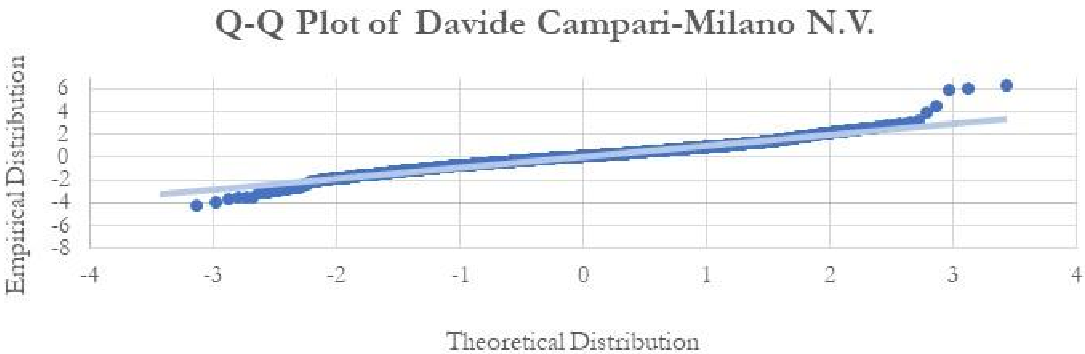

Figure 4.32. Q-Q Plot of Davide Campari-Milano N.V. over 2015-2021.

Table 4.30 and Table 4.31 display the descriptive statistics computed respectively over 2015-2019 and 2020-2021. While in the first subperiod the distribution presents kurtosis close to 1 and an almost negligible value of skewness, during the pandemic high kurtosis and negative skewness were found. If according to the Kolmogorov- Smirnov normality test, the logarithmic return distribution over 2015-2019 can be approximated with a normal one, the same cannot hold for 2020-2021. Nonetheless, we will assume it for the purpose of this thesis.

Davide Campari-Milano N.V. log return descriptive statistics over 2015-2019	
Mean	0.10%
Standard Error	0.04%
Median	0.12%
Standard Deviation	1.46%
Sample Variance	0.0002
Kurtosis	1.2799
Skewness	-0.0787
Range	0.1169
Minimum	-6.78%
Maximum	4.91%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.098

Table 4.30. Davide Campari-Milano N.V. descriptive statistics over 2015-2019.

Davide Campari-Milano N.V. log return descriptive statistics over 2020-2021	
Mean	0.10%
Standard Error	0.09%
Median	0.06%
Standard Deviation	1.95%
Sample Variance	0.0004
Kurtosis	17.4342
Skewness	-0.9352
Range	0.2770
Minimum	-17.57%
Maximum	10.13%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.31. Davide Campari-Milano N.V. descriptive statistics over 2020-2021.

4.2.9 Enel SpA

Figure 4.33 shows the performance of Enel SpA share price over the last 7 years. Between 2015 and the first quarter of 2017 the price was quite stable. Then, an upward trend followed which lasted until March 2020. As it can be clearly seen in Figure 3.2, on 12th March 2020 the stock price suffered a huge seatback. However, from May 2020 the stock price started to climb again, but with more volatility.

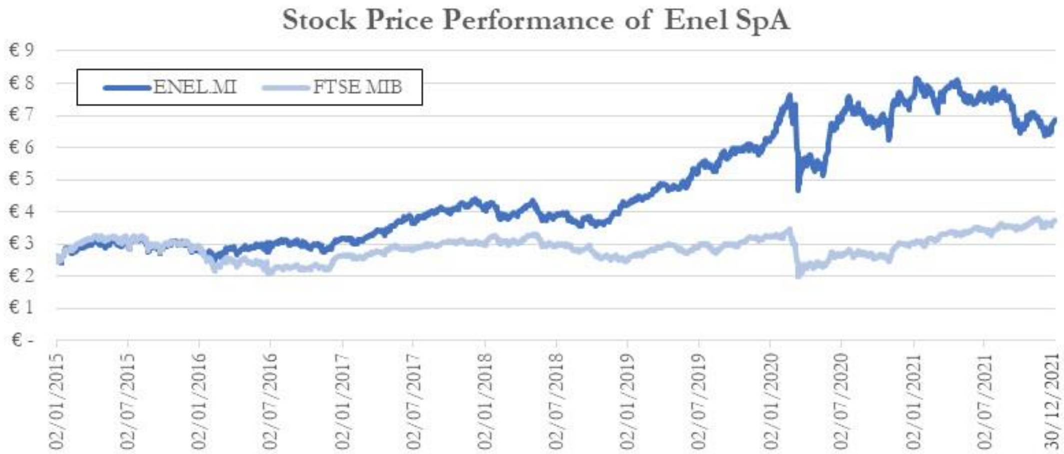

Figure 4.33. Stock price performance of Enel SpA.[10]

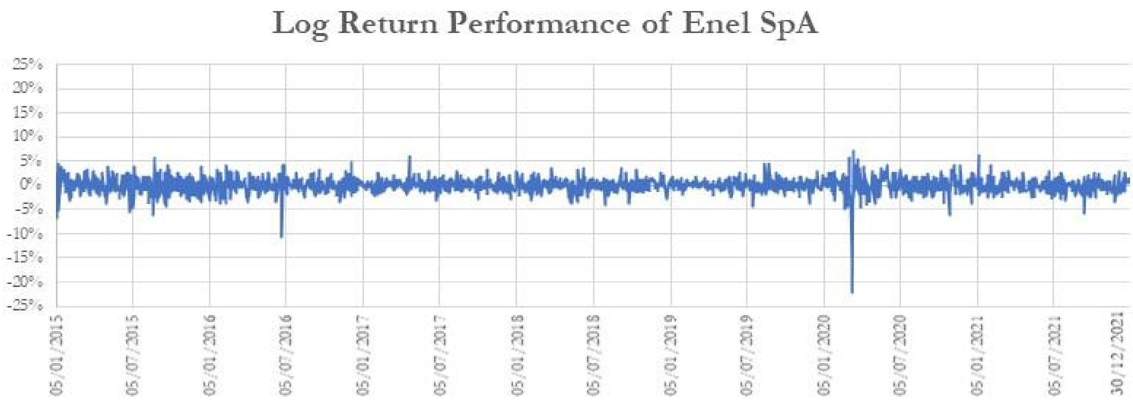

Figure 4.34. Log return performance of Enel SpA.

The descriptive statistics of the daily logarithmic returns computed over 2015-2021 are reported in Table 4.32 and the distribution appears to be left skewed, characterised by a huge value of kurtosis, as can be seen Figure 4.35, which plots the log returns. Nevertheless, by the Q-Q plot in Figure 4.36 reveals an approximated alignment between observed and theoretical quantiles can be observed.

[10] To make the data comparable, FTSE MIB stock index price has been normalized using ENEL.MI stock price on 02/01/2015 as base year

Enel SpA log return descriptive statistics over 2015-2021	
Mean	0.06%
Standard Error	0.04%
Median	0.05%
Standard Deviation	1.65%
Sample Variance	0.0003
Kurtosis	26.0419
Skewness	-2.0702
Range	0.2937
Minimum	-22.12%
Maximum	7.25%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.32. Enel SpA descriptive statistics over 2015-2021.

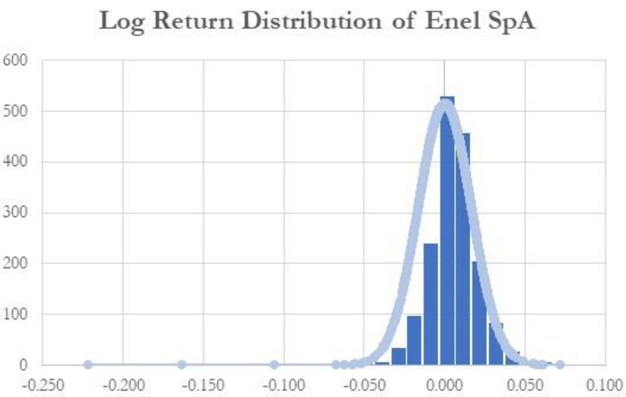

Figure 4.35. Log return distribution of Enel SpA over 2015-2021.

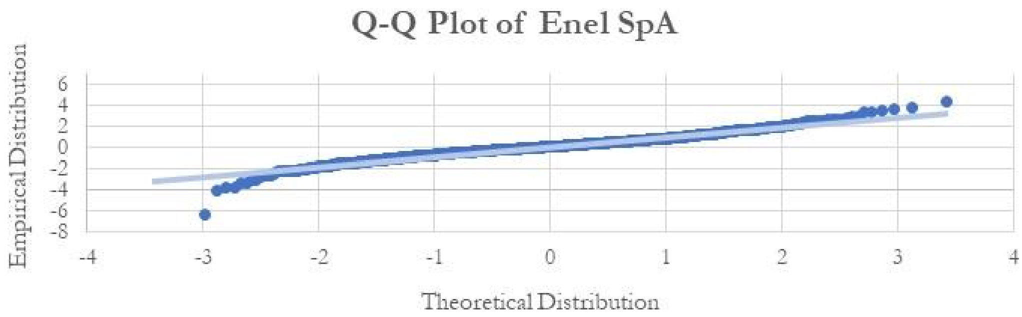

Figure 4.36. Q-Q Plot of Enel SpA. over 2015-2021.

By considering the values reported in Table 4.33 and Table 4.34, it is possible the impact of Covid-19 on Enel SpA daily stock return distribution. While in 2015-2019 it was characterized by both a low value of kurtosis and skewness, in 2020-2021 they skyrocketed. Moreover, the expected return decreased substantially, while the standard deviation increased. Even if, under the Kolmogorov–Smirnov and the Shapiro–Wilk test, the null hypothesis about the normal distribution of the returns should be rejected, we will assume it for the purpose of this thesis

Enel SpA log return descriptive statistics over 2015-2019	
Mean	0.07%
Standard Error	0.04%
Median	0.05%
Standard Deviation	1.45%
Sample Variance	0.0002
Kurtosis	3.8706
Skewness	-0.3499
Range	0.1665
Minimum	-10.59%
Maximum	6.07%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.001

Table 4.33. Enel SpA descriptive statistics over 2015-2019.

Enel SpA log return descriptive statistics over 2020-2021	
Mean	0.02%
Standard Error	0.09%
Median	0.03%
Standard Deviation	2.08%
Sample Variance	0.0004
Kurtosis	33.8830
Skewness	-3.3474
Range	0.2937
Minimum	-22.12%
Maximum	7.25%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.34. Enel SpA descriptive statistics over 2020-2021.

4.2.10 easyJet plc

By analysing both the easyJet stock price and log return performance, respectively displayed in Figure 4.37 and Figure 4.38, it is possible to see many fluctuations of various entity, which confirm the cyclical nature of airplane stock. On 27th June 2016 easyJet shares drop to three-year low after Brexit referendum. More recently, the Covid-19 crisis had a huge impact on the stock performance, characterized by a huge volatility.

Figure 4.37 Stock price performance of easyJet plc.[11]

Figure 4.38. Log return performance of easyJet plc.

In Table 4.35 the descriptive statistics for the daily logarithmic returns computed over 2015 to 2021 are reported, revealing a slightly left skewed distribution, with thick tails as confirmed by Figure 4.39. The Q-Q plot in Figure 4.40 analyses the alignment between observed and theoretical quantiles.

[11] To make the data comparable, FTSE 350 stock index price has been normalized using EZJ.L stock price on 02/01/2015 as base year

easyJet plc log return descriptive statistics over 2015-2021	
Mean	-0.05%
Standard Error	0.07%
Median	-0.02%
Standard Deviation	3.09%
Sample Variance	0.0010
Kurtosis	13.8652
Skewness	-0.1902
Range	0.5568
Minimum	-25.25%
Maximum	30.43%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.35. easyJet plc descriptive statistics over 2015-2021.

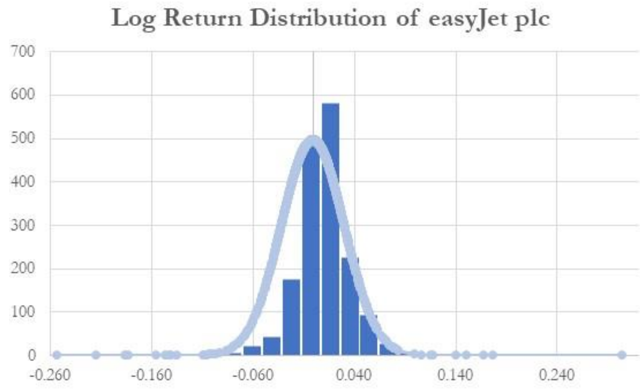

Figure 4.39. Log return distribution of easyJet plc over 2015-2021.

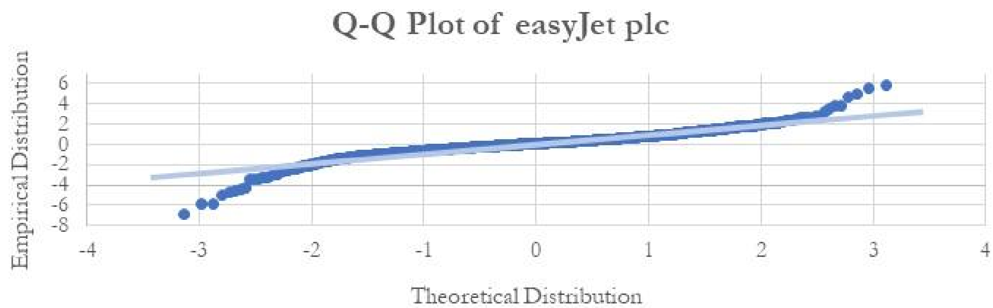

Figure 4.40. Q-Q Plot of easyJet plc over 2015-2021.

The comparison between the values in Table 4.36 and Table 4.37 help to understand the impact Covid-19 had on this business. If, on the one hand, the expected return in the period 2020-2021 became negative and characterized by greater volatility than in 2015-2019, on the other the kurtosis decreased, and the skewness became positive. Even though under the Kolmogorov–Smirnov and the Shapiro–Wilk test the null hypothesis about the normal distribution of the returns should be rejected, we will assume it for the purpose of this thesis.

easyJet plc log return descriptive statistics over 2015-2019	
Mean	0.00%
Standard Error	0.06%
Median	0.00%
Standard Deviation	2.25%
Sample Variance	0.0005
Kurtosis	15.8339
Skewness	-1.5756
Range	0.3330
Minimum	-25.25%
Maximum	8.05%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.36. easyJet plc descriptive statistics over 2015-2019.

easyJet plc log return descriptive statistics over 2020-2021	
Mean	-0.19%
Standard Error	0.21%
Median	-0.32%
Standard Deviation	4.56%
Sample Variance	0.0021
Kurtosis	6.8092
Skewness	0.3326
Range	0.5189
Minimum	-21.47%
Maximum	30.43%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.002

Table 4.37. easyJet plc descriptive statistics over 2020-2021.

4.2.11 Assicurazioni Generali

As depicted in Figure 4.42, the log return performance of Assicurazioni Generali over in 2015 and 2016 experienced relevant fluctuations, due to the overall financial market volatility that had an impact on the insurance sector. Starting from the end of 2016 until February 2020 the share price presents an upward slope, as can be interfered from Figure 4.41. The first wave of the pandemic hit the stock price, but it then recovered, being even able to outperform the highest value reached in February 2020.

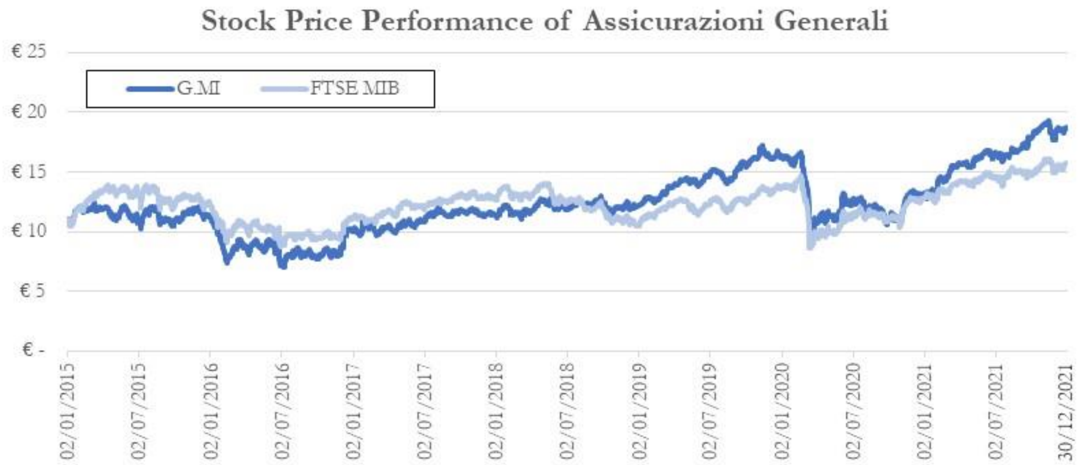

Figure 4.41. Stock price performance of Assicurazioni Generali.[12]

Figure 4.42. Log return performance of Assicurazioni Generali.

The descriptive statistics of Assicurazioni Generali over the three time horizon are reported in Table 4.38 (2015-2021), Table 4.39 (2015-2019) and Table 4.40 (2020-2021). Despite the

[12] To make the data comparable, FTSE MIB stock index price has been normalized using G.MI's stock price on 02/01/2015 as base year

pandemic, the four main moments are comparable, and the distribution of daily returns is left skewed, with big tails, as shown in Figure 4.43 for the timeframe 2015-2021. Figure 4.44 provides the Q-Q plot in the same time horizon.

Assicurazioni Generali log return descriptive statistics over 2015-2021	
Mean	0.03%
Standard Error	0.04%
Median	0.10%
Standard Deviation	1.66%
Sample Variance	0.0003
Kurtosis	16.3156
Skewness	-1.1944
Range	0.2884
Minimum	-18.35%
Maximum	10.49%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.38. Assicurazioni Generali descriptive statistics over 2015-2021.

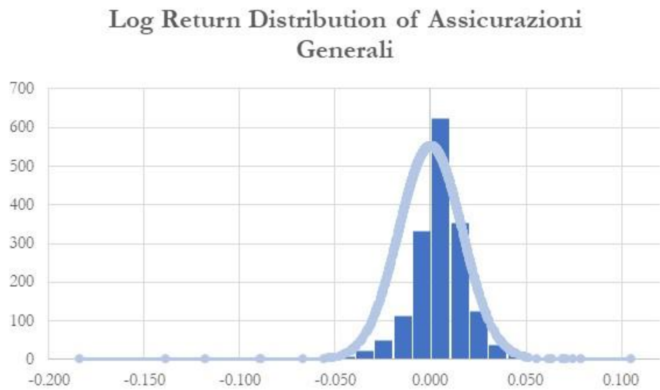

Figure 4.43. Log return distribution of Assicurazioni Generali over 2015-2021.

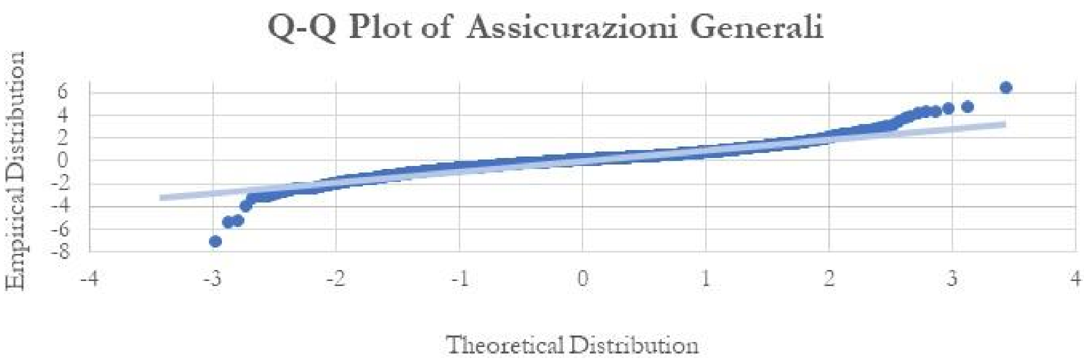

Figure 4.44. Q-Q Plot of Assicurazioni Generali. over 2015-2021.

Assicurazioni Generali log return descriptive statistics over 2015-2019	
Mean	0.03%
Standard Error	0.05%
Median	0.06%
Standard Deviation	1.62%
Sample Variance	0.0003
Kurtosis	16.5203
Skewness	-1.0938
Range	0.2625
Minimum	-18.35%
Maximum	7.89%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.39. Assicurazioni Generali descriptive statistics over 2015-2019.

Assicurazioni Generali log return descriptive statistics over 2020-2021	
Mean	0.03%
Standard Error	0.08%
Median	0.19%
Standard Deviation	1.73%
Sample Variance	0.0003
Kurtosis	15.8774
Skewness	-1.3996
Range	0.2436
Minimum	-13.87%
Maximum	10.49%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.40. Assicurazioni Generali descriptive statistics over 2020-2021.

4.2.12 General Motors Company

General Motors Company's stock is an example of cyclical stock, since its price is closely related to economic growth and business cycles, as shown in Figure 4.45. 2017 was a favourable year for the company, but from 2018 it experienced the consequences of the Trump's Trade War against China. As Figure 4.46 shows, at first Covid-19 significantly impacted the stock returns. Nevertheless, after this initial shock, the price started recovering, even exceeding the pre-pandemic levels.

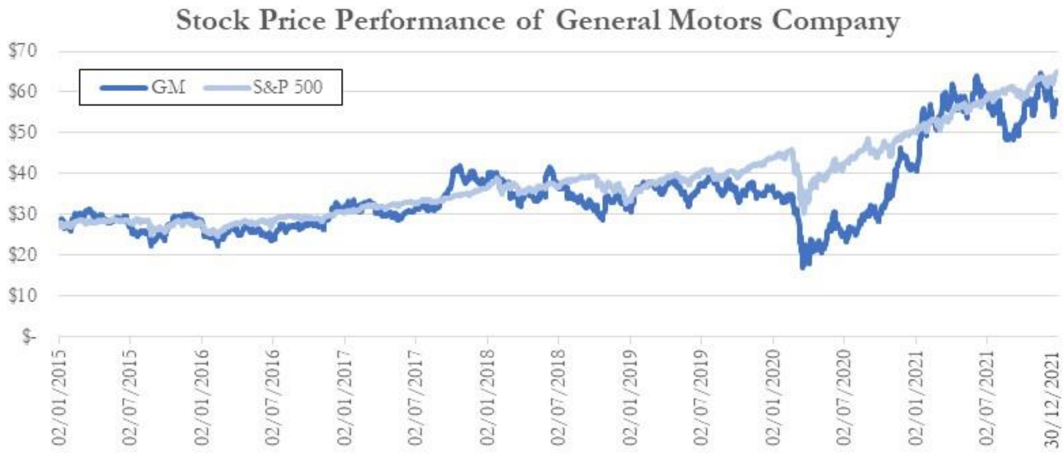

Figure 4.45. Stock price performance of General Motors Company.[13]

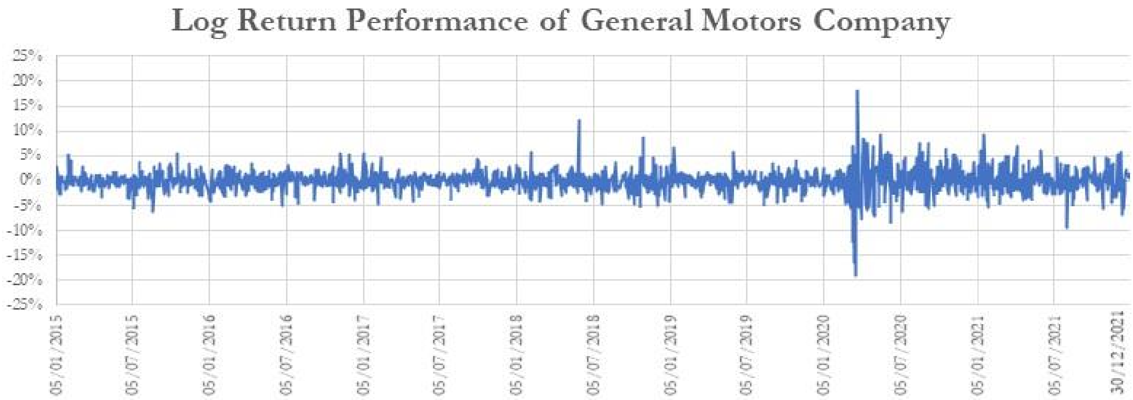

Figure 4.46. Log return performance of General Motors Company.

Table 4.41 describes the descriptive statistics of the logarithmic returns computed over 2015 to 2021. In particular, the distribution presents a value of skewness close to 0, with thick tails as confirmed by Figure 4.47. In the Q-Q plot in Figure 4.48 the alignment between observed and theoretical quantiles can be observed.

[13] To make the data comparable, S&P 500 stock index price has been normalized using GM stock price on 02/01/2015 as base year

General Motors Company log return descriptive statistics over 2015-2021	
Mean	0.04%
Standard Error	0.05%
Median	0.08%
Standard Deviation	2.19%
Sample Variance	0.0005
Kurtosis	10.0461
Skewness	-0.1004
Range	0.3721
Minimum	-19.02%
Maximum	18.18%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.41. General Motors Company descriptive statistics over 2015-2021.

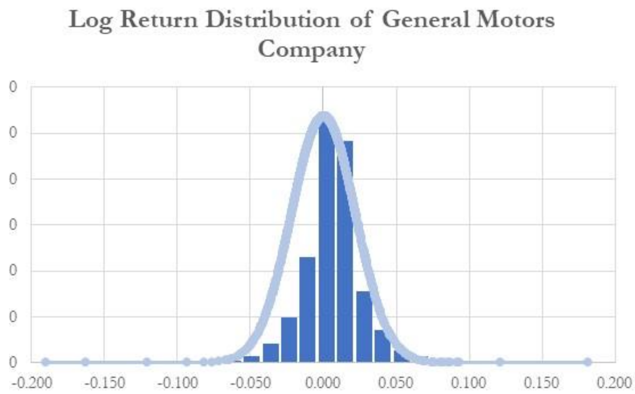

Figure 4.47. Log return distribution of General Motors Company over 2015-2021.

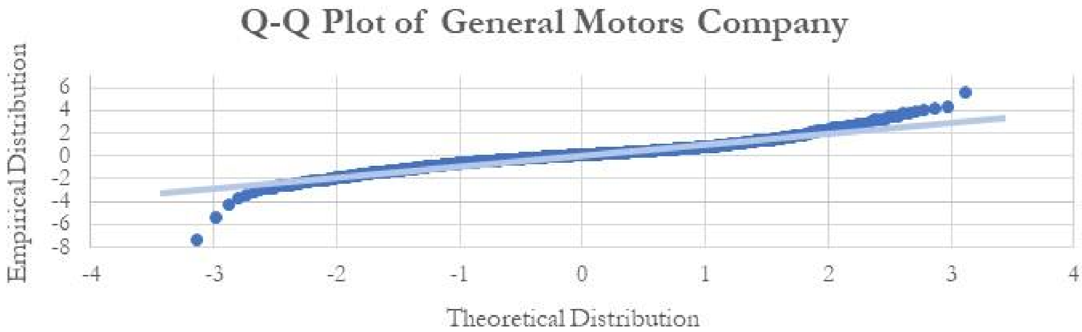

Figure 4.48. Q-Q Plot of General Motors Company over 2015-2021.

Comparing the values in Table 4.42 and Table 4.43 allows to understand the impact of the pandemic on this business. Surprisingly, the daily mean return computed over 2020-2021 is higher than in 2015-2019, at the cost of increased volatility though. Moreover, while the kurtosis remains higher than 3, the skewness reversed the sign, going from positive to negative.

General Motors Company log return descriptive statistics over 2015-2019	
Mean	0.02%
Standard Error	0.05%
Median	0.10%
Standard Deviation	1.63%
Sample Variance	0.0003
Kurtosis	4.2709
Skewness	0.3402
Range	0.1838
Minimum	-6.27%
Maximum	12.11%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.42. General Motors Company descriptive statistics over 2015-2019.

General Motors Company log return descriptive statistics over 2020-2021	
Mean	0.10%
Standard Error	0.15%
Median	-0.02%
Standard Deviation	3.20%
Sample Variance	0.0010
Kurtosis	6.0126
Skewness	-0.2665
Range	0.3721
Minimum	-19.02%
Maximum	18.18%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.031

Table 4.43. General Motors Company descriptive statistics over 2020-2021

4.2.13 Alphabet Inc.

As clearly shown in Figure 4.49 and Figure 4.50, Alphabet Inc. stock price has presented an increasing trend, with contained volatility, since the beginning of 2015. It was Covid-19 that significantly accelerated this tendency, making Google one of the most prospering companies in the pandemic.

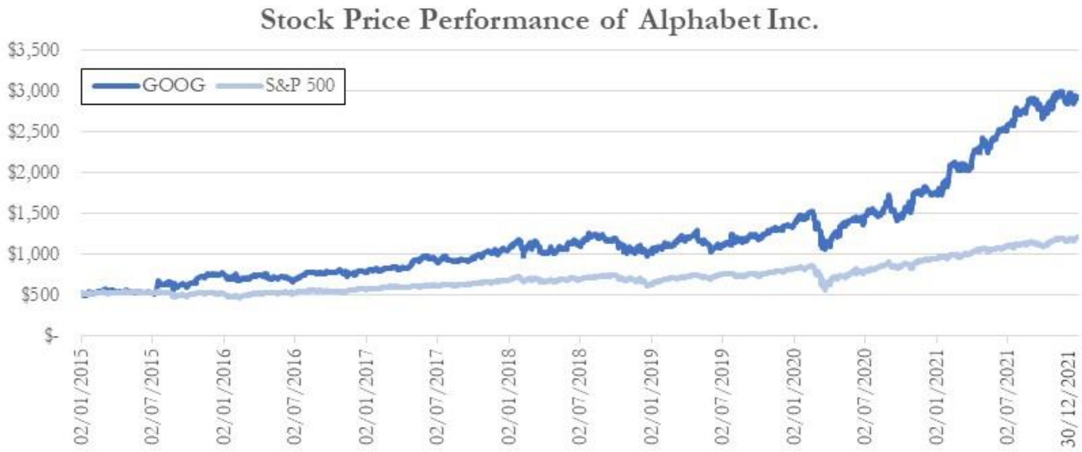

Figure 4.49. Stock price performance of Alphabet Inc.[14]

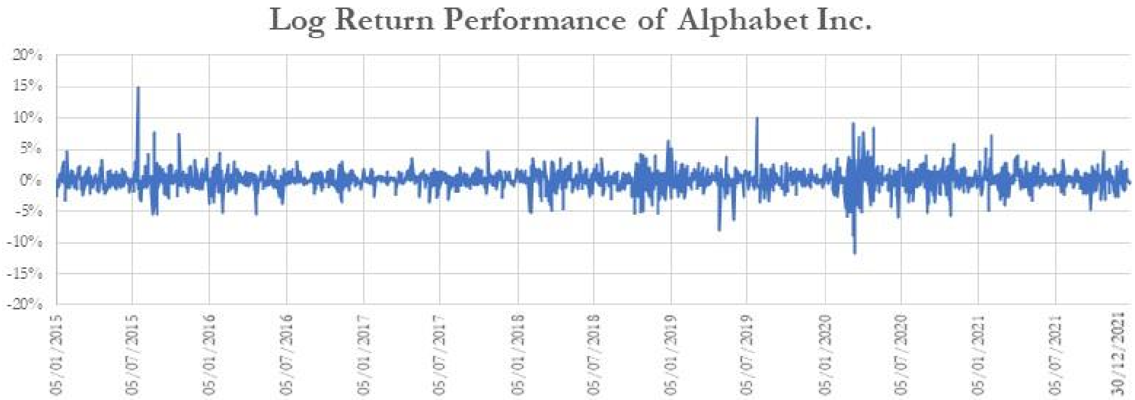

Figure 4.50. Log return performance of Alphabet Inc.

The descriptive statistics for the logarithmic returns computed over 2015 to 2021 are reported in Table 4.44 and the distribution appears to be right skewed, characterised by a huge value of kurtosis, as can be seen Figure 4.52, which plots the log returns. Nevertheless, the Q-Q plot in Figure 4.51 reveals an approximated alignment between observed and theoretical quantiles can be observed.

[14] To make the data comparable, S&P 500 stock index price has been normalized using GOOG stock price on 02/01/2015 as base year

Alphabet Inc. log return descriptive statistics over 2015-2021	
Mean	0.10%
Standard Error	0.04%
Median	0.12%
Standard Deviation	1.68%
Sample Variance	0.0003
Kurtosis	8.5435
Skewness	0.2264
Range	0.2665
Minimum	-11.77%
Maximum	14.89%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.44. Alphabet Inc. descriptive statistics over 2015-2021.

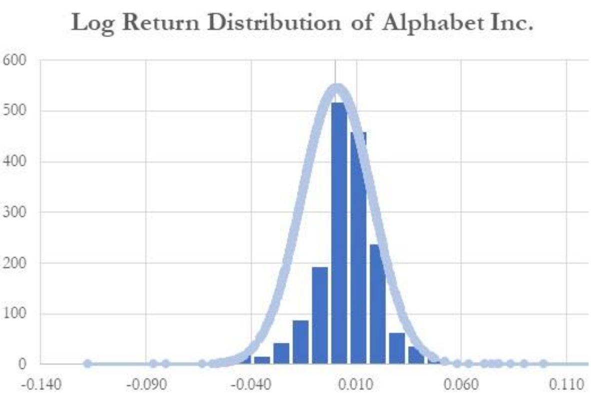

Figure 4.51. Log return distribution of Alphabet Inc. over 2015-2021.

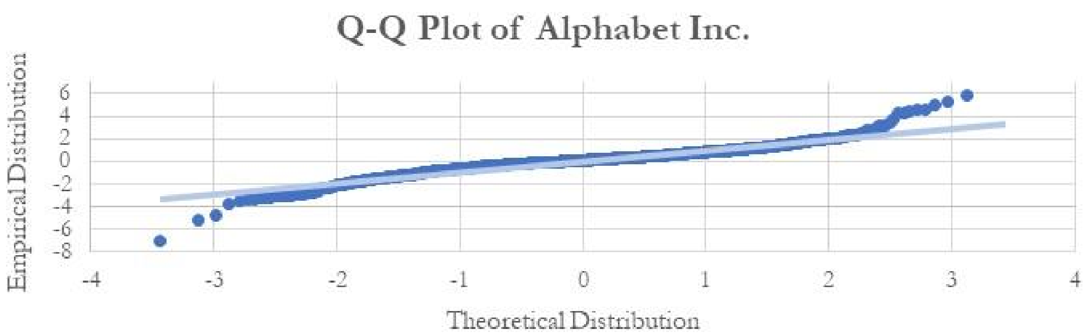

Figure 4.52. Q-Q Plot of Alphabet Inc. over 2015-2021.

By analysing the values in Table 4.45 and Table 4.46 it is possible to statistically understand why Alphabet Inc. is considered one on the main winner of the pandemic. The average daily return over 2020-2021 if compared to 2015-2019 doubled , followed by an increase in volatility. The kurtosis decreased and the skewness inverted sign.

Alphabet Inc. log return descriptive statistics over 2015-2019	
Mean	0.08%
Standard Error	0.04%
Median	0.07%
Standard Deviation	1.53%
Sample Variance	0.0002
Kurtosis	11.0597
Skewness	0.6795
Range	0.2290
Minimum	-8.01%
Maximum	14.89%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.45. Alphabet Inc. descriptive statistics over 2015-2019.

Alphabet Inc. log return descriptive statistics over 2020-2021	
Mean	0.16%
Standard Error	0.09%
Median	0.27%
Standard Deviation	2.01%
Sample Variance	0.0004
Kurtosis	5.1180
Skewness	-0.3221
Range	0.2075
Minimum	-11.77%
Maximum	8.99%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.46. Alphabet Inc. descriptive statistics over 2020-2021.

4.2.14 InterContinental Hotels Group PLC

InterContinental Hotels Group PLC is a cyclical company, and this nature can be seen in Figure 4.54. Figure 4.53 plots the stock price of the company, confirming that the hospitality sector has been one of the main victims of the pandemic.

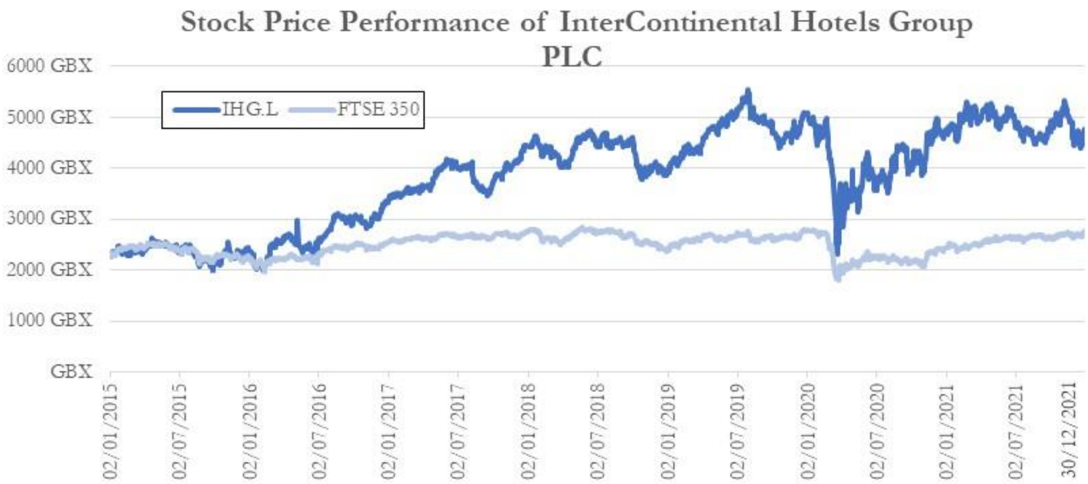

Figure 4.53. Stock price performance of InterContinental Hotels Group PLC.[15]

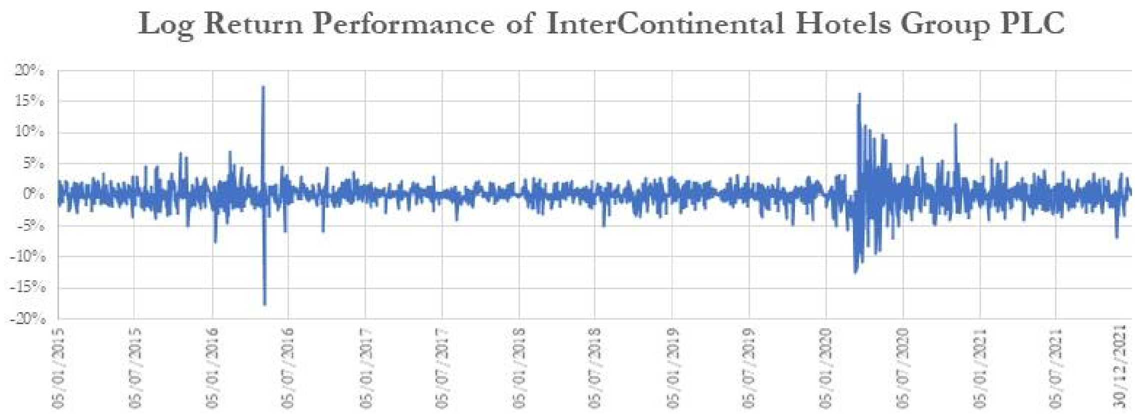

Figure 4.54. Log return performance of InterContinental Hotels Group PLC

The descriptive statistics of InterContinental Hotels Group PLC daily stock returns over the three time horizon are reported in Table 4.47 (2015-2021), Table 4.48 (2015-2019) and Table 4.49 (2020-2021). While the average mean over 2015-2019 is positive with a contained volatility, the time horizon 2020-2021 presents negative return and high volatility. In all cases, the null hypothesis of normality of distribution should be rejected. Nevertheless, the Q-Q plot in Figure 4.56 reveals

[15] To make the data comparable, FTSE 350 stock index price has been normalized using IHG.L stock price on 02/01/2015 as base year

an approximated alignment between observed and theoretical quantiles over 2015-2021 can be observed.

InterContinental Hotels Group PLC log return descriptive statistics over 2015-2021	
Mean	0.04%
Standard Error	0.05%
Median	0.06%
Standard Deviation	2.09%
Sample Variance	0.0004
Kurtosis	14.6071
Skewness	0.3609
Range	0.3504
Minimum	-17.56%
Maximum	17.47%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.47. InterContinental Hotels Group PLC descriptive statistics over 2015-2021.

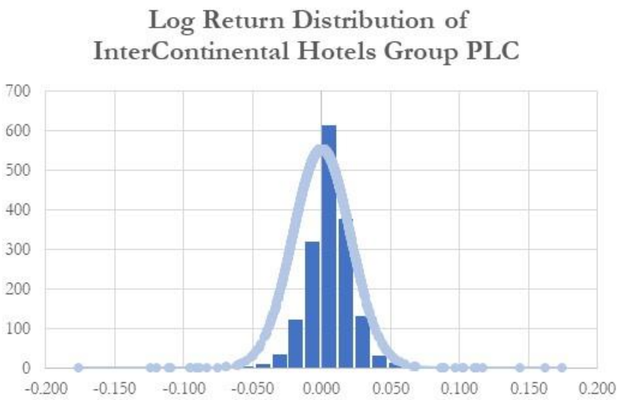

Figure 4.55. Log return distribution of InterContinental Hotels Group PLC over 2015-2021.

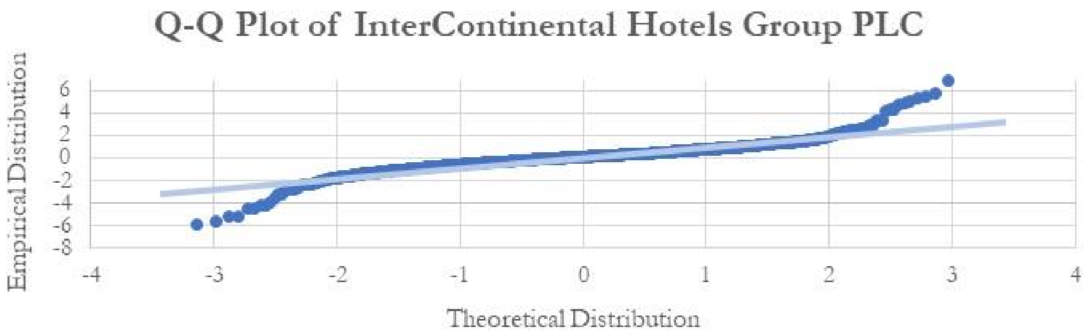

Figure 4.56. Q-Q Plot of InterContinental Hotels Group PLC over 2015-2021.

InterContinental Hotels Group PLC log return descriptive statistics over 2015-2019	
Mean	0.06%
Standard Error	0.04%
Median	0.10%
Standard Deviation	1.56%
Sample Variance	0.0002
Kurtosis	26.6491
Skewness	-0.1203
Range	0.3504
Minimum	-17.56%
Maximum	17.47%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.48. InterContinental Hotels Group PLC descriptive statistics over 2015-2019.

InterContinental Hotels Group PLC log return descriptive statistics over 2020-2021	
Mean	-0.01%
Standard Error	0.14%
Median	-0.11%
Standard Deviation	3.03%
Sample Variance	0.0009
Kurtosis	5.8010
Skewness	0.4986
Range	0.2865
Minimum	-12.38%
Maximum	16.27%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.49. InterContinental Hotels Group PLC descriptive statistics over 2020-2021.

4.2.15 Interpump Group S.p.A.

From the beginning of 2015, Interpump Group S.p.A stock price has been characterized by an increasing trend, that has steepened during the pandemic, as reported in Figure 4.57. Figure 4.58 plots the log return performance of the stock that presents oscillations at the beginning of the Covid-19 crisis.

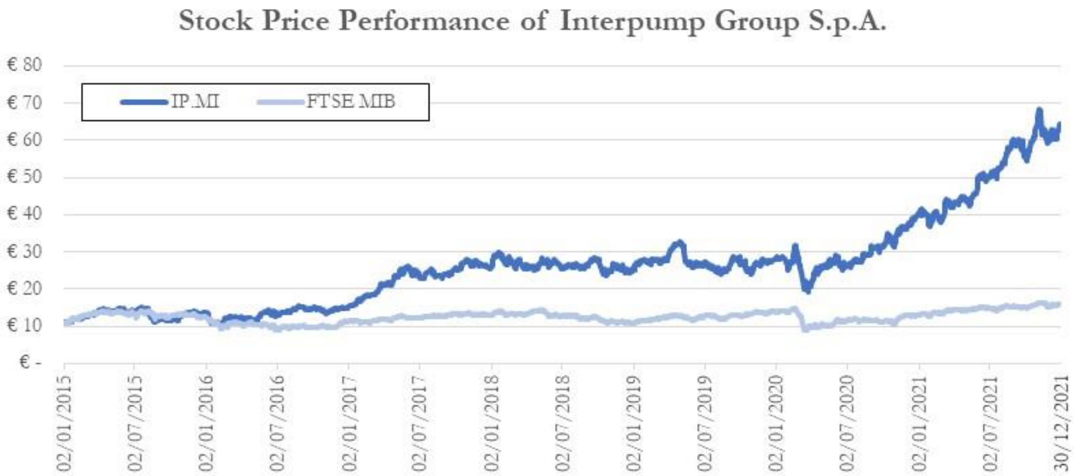

Figure 4.57. Stock price performance of Interpump Group S.p.A.[16]

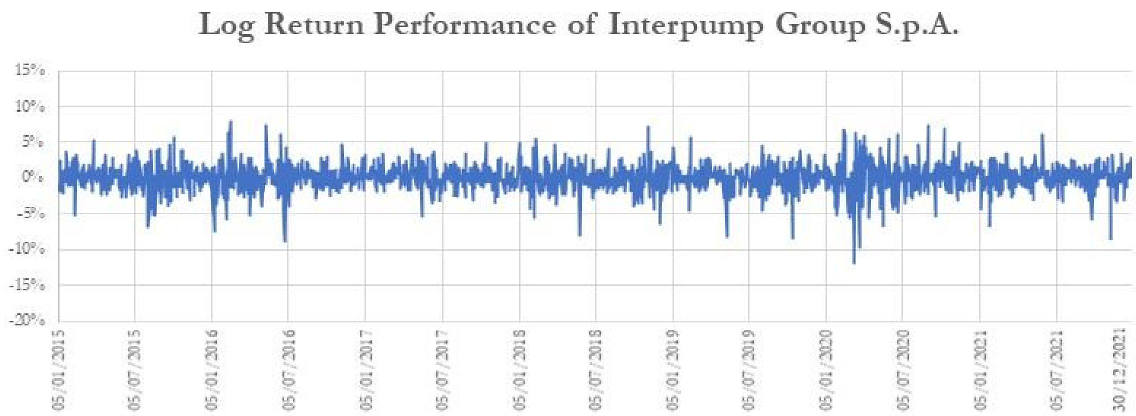

Figure 4.58. Log return performance of Interpump Group S.p.A.

In Table 4.50 the descriptive statistics for the daily logarithmic returns computed over 2015 to 2021 are reported, revealing a kurtosis close to 3 with a skewness close to 0 as shown in Figure 4.59. The Q-Q plot in Figure 4.60 displays the alignment between observed and theoretical quantiles.

[16] To make the data comparable, FTSE MIB stock index price has been normalized using IP.MI stock price on 02/01/2015 as base year

Interpump Group S.p.A. log return descriptive statistics over 2015-2021	
Mean	0.10%
Standard Error	0.05%
Median	0.08%
Standard Deviation	1.93%
Sample Variance	0.0004
Kurtosis	3.6023
Skewness	-0.4416
Range	0.1992
Minimum	-11.92%
Maximum	8.00%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.50. Interpump Group S.p.A. descriptive statistics over 2015-2021.

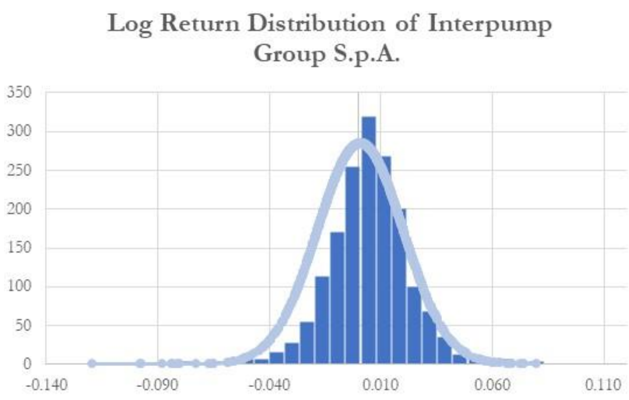

Figure 4.59. Log return distribution of Interpump Group S.p.A. over 2015-2021.

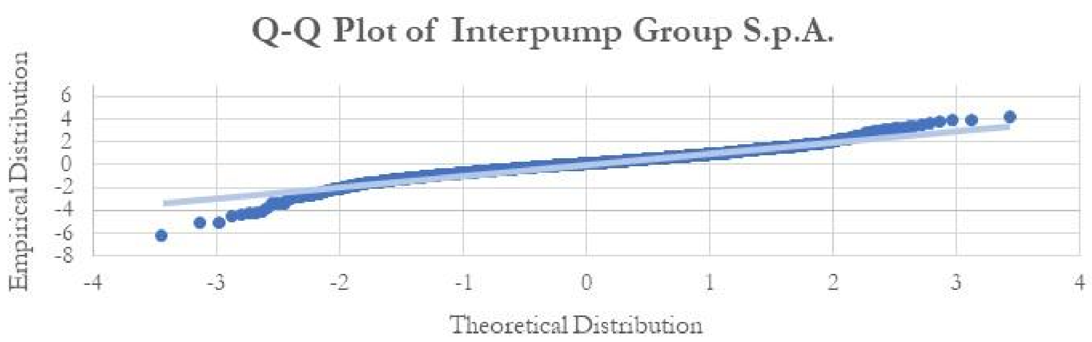

Figure 4.60. Q-Q Plot of Interpump Group S.p.A. over 2015-2021.

Table 4.51 and Table 4.52 helps to evaluate the impact of the pandemic over the stock price: the average daily return increased from 0.07% to 0.17%, while the volatility increased. At the same time both the kurtosis and the skewness increased.

Interpump Group S.p.A. log return descriptive statistics over 2015-2019	
Mean	0.07%
Standard Error	0.05%
Median	0.07%
Standard Deviation	1.80%
Sample Variance	0.0003
Kurtosis	2.7458
Skewness	-0.2218
Range	0.1681
Minimum	-8.81%
Maximum	8.00%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.002

Table 4.51. Interpump Group S.p.A. descriptive statistics over 2015-2019.

Interpump Group S.p.A. log return descriptive statistics over 2020-2021	
Mean	0.17%
Standard Error	0.10%
Median	0.23%
Standard Deviation	2.20%
Sample Variance	0.0005
Kurtosis	4.1980
Skewness	-0.7648
Range	0.1931
Minimum	-11.92%
Maximum	7.39%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.007

Table 4.52. Interpump Group S.p.A. descriptive statistics over 2020-2021.

4.2.16 Deutsche Lufthansa AG

As shown in Figure 4.60, Deutsche Lufthansa AG stock price overperformed the DAX30 in 2017-2019. However, the company was so impacted by the pandemic that in June 2020 was forced to leave the German index. Figure 4.61 reports the log return performance of the stock.

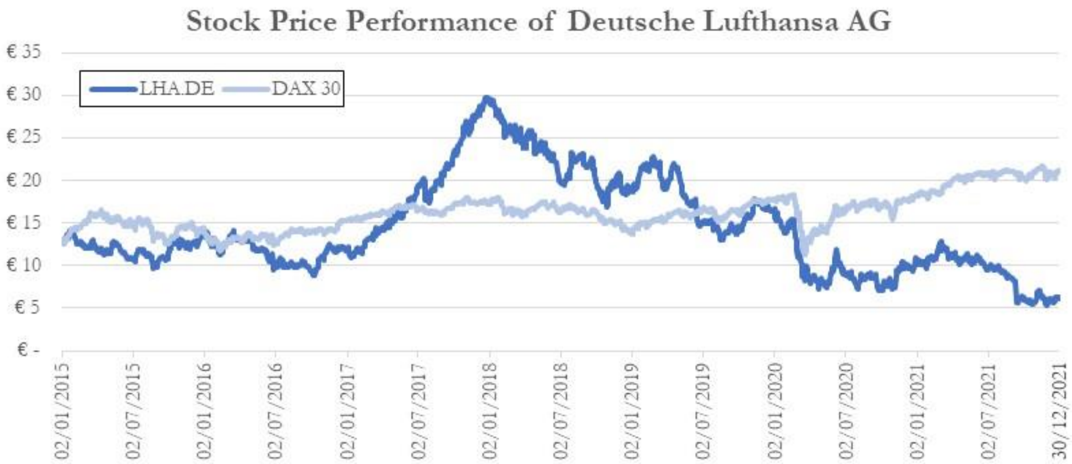

Figure 4.61. Stock price performance of Deutsche Lufthansa AG.[17]

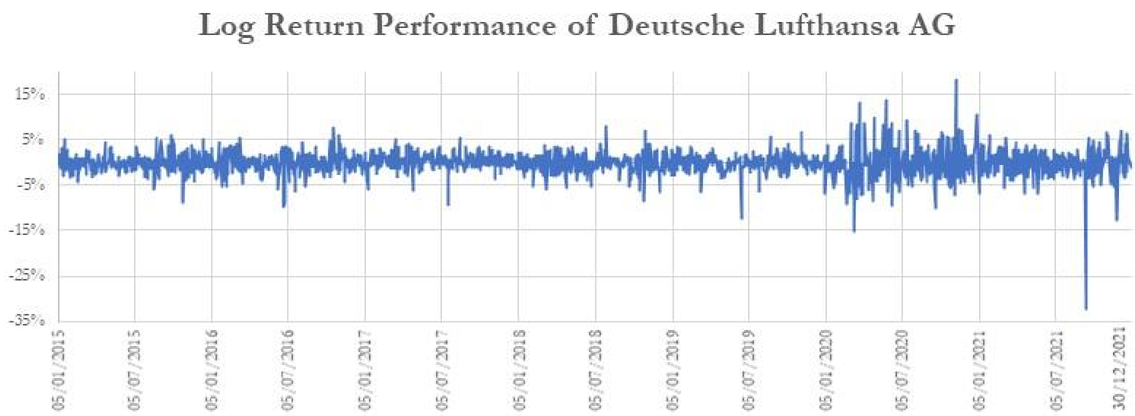

Figure 4.62. Log return performance of Deutsche Lufthansa AG.

The descriptive statistics of Deutsche Lufthansa AG stock returns over the three time horizon are reported in Table 4.53 (2015-2021), Table 4.54 (2015-2019), and Table 4.55 (2020-2021). While the average mean of the daily logarithmic returns over 2015-2019 is low, but positive with a contained volatility and a skewness close to 3, over the time horizon 2020-2021 the mean

[17] To make the data comparable, DAX 30 stock index price has been normalized using LHA.DE stock price on 02/01/2015 as base year

turns deeply negative characterized by a strong standard deviation. The Q-Q plot in Figure 4.64 shows the alignment between observed and theoretical quantiles over 2015-2021.

Deutsche Lufthansa AG log return descriptive statistics over 2015-2021	
Mean	-0.04%
Standard Error	0.06%
Median	0.00%
Standard Deviation	2.65%
Sample Variance	0.0007
Kurtosis	16.6632
Skewness	-0.9637
Range	0.5033
Minimum	-32.20%
Maximum	18.13%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.53. Deutsche Lufthansa AG descriptive statistics over 2015-2021.

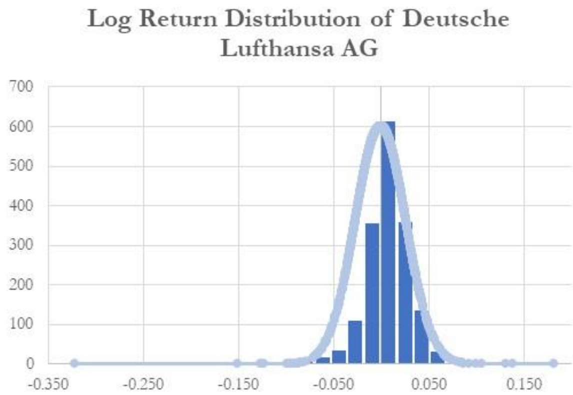

Figure 4.63. Log return distribution of Deutsche Lufthansa over 2015-2021

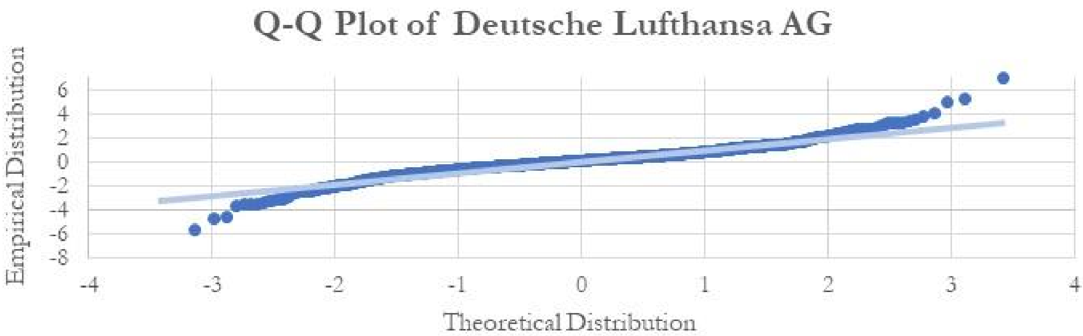

Figure 4.64. Q-Q Plot of Deutsche Lufthansa AG over 2015-2021.

Deutsche Lufthansa AG log return descriptive statistics over 2015-2019	
Mean	0.02%
Standard Error	0.06%
Median	0.08%
Standard Deviation	1.98%
Sample Variance	0.0004
Kurtosis	3.4336
Skewness	-0.4819
Range	0.2028
Minimum	-12.35%
Maximum	7.93%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.002

Table 4.54. Deutsche Lufthansa AG descriptive statistics over 2015-2019.

Deutsche Lufthansa AG log return descriptive statistics over 2020-2021	
Mean	-0.20%
Standard Error	0.17%
Median	-0.27%
Standard Deviation	3.84%
Sample Variance	0.0015
Kurtosis	11.5026
Skewness	-0.8583
Range	0.5033
Minimum	-32.20%
Maximum	18.13%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.006

Table 4.55. Deutsche Lufthansa AG descriptive statistics over 2020-2021.

4.2.17 Lockheed Martin Corporation

As shown in Figure 4.66, apart from random peak and falls, until the pandemic crisis the stock price returns of Lockheed Martin Corporation did not present relevant oscillations. Figure 4.65 displays how the performance of the stock always overperform the one of the underlying index, while following the underneath pattern.

Figure 4.65. Stock price performance of Lockheed Martin Corporation.[18]

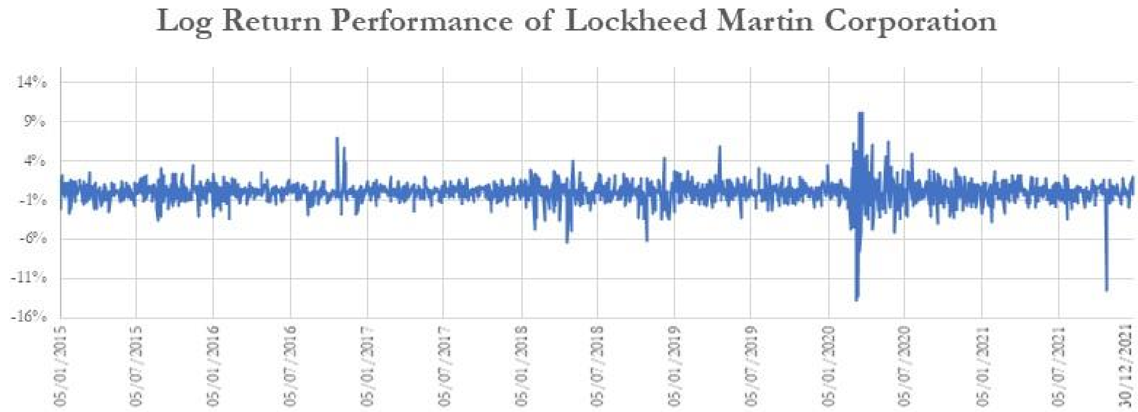

Figure 4.66. Log return performance of Lockheed Martin Corporation.

Table 4.56 exhibits the descriptive statistics of the logarithmic returns computed over 2015 to 2021 and the distribution appears to be left skewed, with tick tails as shown in Figure 4.67 displays. The Q-Q plot in Figure 4.68 plots the observed and theoretical quantiles.

[18] To make the data comparable, S&P 500 stock index price has been normalized using LMT stock price on 02/01/2015 as base year

Lockheed Martin Corporation log return descriptive statistics over 2015-2021	
Mean	0.05%
Standard Error	0.04%
Median	0.09%
Standard Deviation	1.45%
Sample Variance	0.0002
Kurtosis	17.5959
Skewness	-0.8747
Range	0.2384
Minimum	-13.65%
Maximum	10.19%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.56. Lockheed Martin Corporation descriptive statistics over 2015-2021.

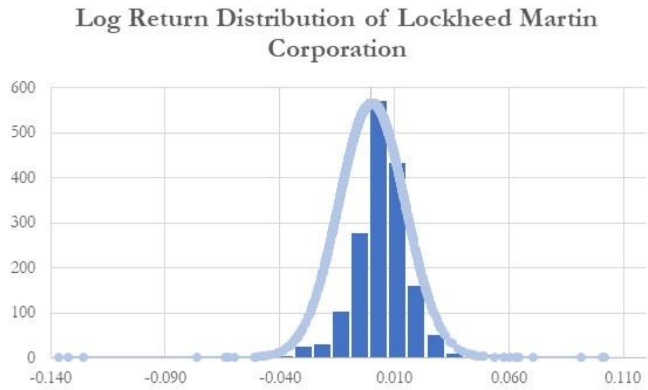

Figure 4.67. Log return distribution of Lockheed Martin Corporation over 2015-2021.

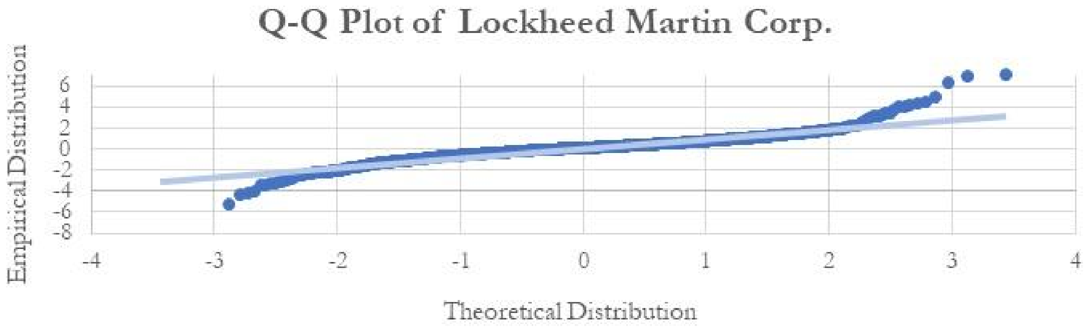

Figure 4.68. Q-Q Plot of Lockheed Martin Corporation.

To really appreciate the impact of Covid-19 over the daily stock return, Table 4.57 and Table 4.58 should be considered. In 2020-2021 the logarithmic return means turned negative with higher volatility. At the same time both kurtosis and skewness increased.

Lockheed Martin Corporation log return descriptive statistics over 2015-2019	
Mean	0.07%
Standard Error	0.03%
Median	0.09%
Standard Deviation	1.12%
Sample Variance	0.0001
Kurtosis	5.0017
Skewness	-0.1952
Range	0.1347
Minimum	-6.36%
Maximum	7.11%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.57. Lockheed Martin Corporation descriptive statistics over 2015-2019.

Lockheed Martin Corporation log return descriptive statistics over 2020-2021	
Mean	-0.01%
Standard Error	0.09%
Median	0.06%
Standard Deviation	2.06%
Sample Variance	0.0004
Kurtosis	13.0872
Skewness	-0.9397
Range	0.2384
Minimum	-13.65%
Maximum	10.19%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.58. Lockheed Martin Corporation descriptive statistics over 2020-2021.

4.2.18 LVMH Moët Hennessy Louis Vuitton, Société Européenne

As shown in Figure 4.69, LVMH stock price has increased over the years. Some recent events that impacted the price performance were the acquisition of Tiffany's and the pandemic: while the former was announced in November 2019 and completed in January 2021, the latter proved the resilience of the stock of the firm. Figure 4.70 plots the logarithmic return performance of the stock.

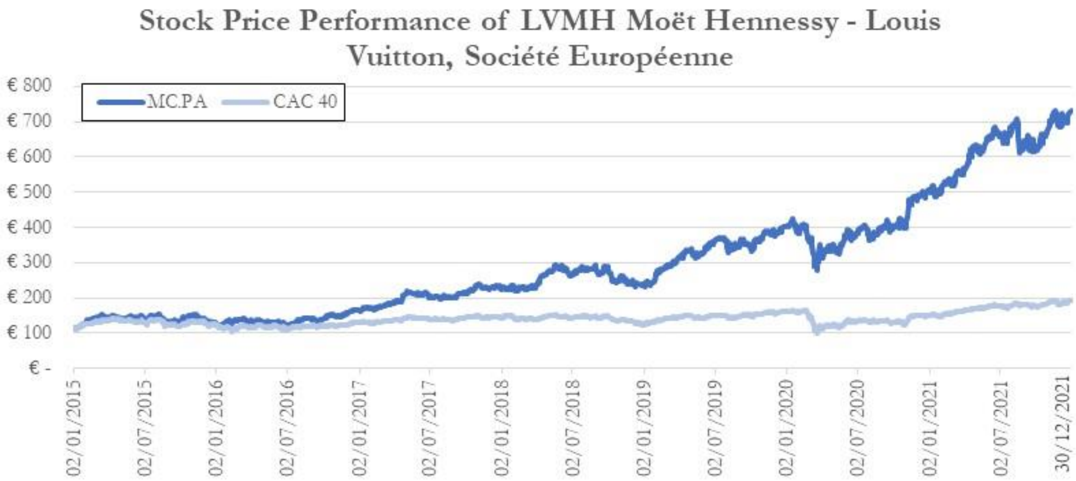

Figure 4.69. Stock price performance of LVMH.[19]

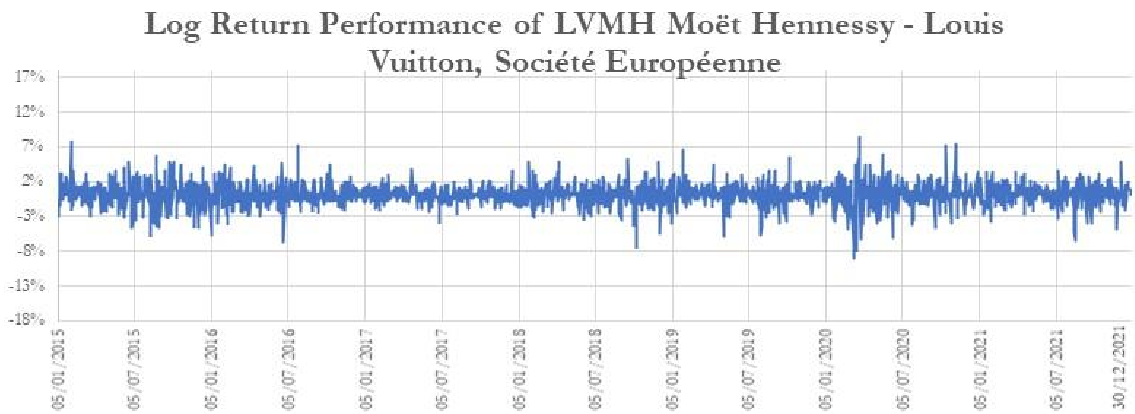

Figure 4.70. Log return performance of LVMH

Table 4.59 (2015-2021), Table 4.60 (2015-2019), and Table 4.61 (2020-2021) exhibit the descriptive statistics of LVMH stock returns over the three time horizon. The daily stock returns over 2020-2021 did not experience a significant increase in the average return, but the volatility increased. The kurtosis remains around 3, while the skewness close to 0, as Figure 4.71 shows. In

[19] To make the data comparable, CAC 40 stock index price has been normalized using MC.PA stock price on 02/01/2015 as base year

the Q-Q plot in Figure 4.72 the alignment between observed and theoretical quantiles over 2015-2021 is represented.

LVMH Moët Hennessy - Louis Vuitton, Société Européenne log return descriptive statistics over 2015-2021	
Mean	0.11%
Standard Error	0.04%
Median	0.11%
Standard Deviation	1.74%
Sample Variance	0.0003
Kurtosis	3.0018
Skewness	-0.1269
Range	0.1761
Minimum	-9.08%
Maximum	8.54%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.59. LVMH descriptive statistics over 2015-2021.

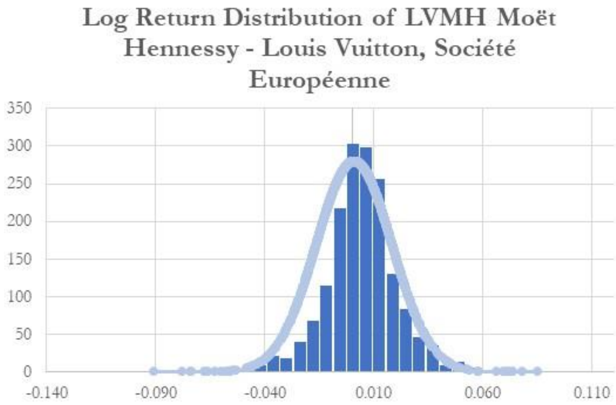

Figure 4.71. Log return distribution of LVMH over 2015-2021.

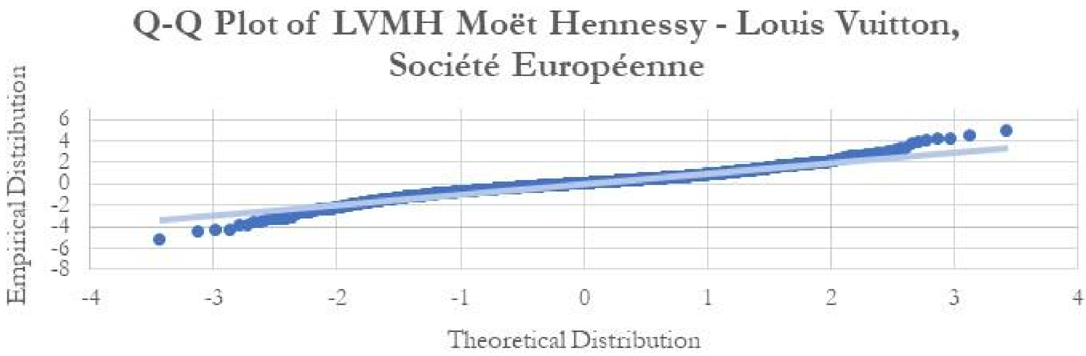

Figure 4.72. Q-Q Plot of LVMH over 2015-2021.

LVMH Moët Hennessy - Louis Vuitton, Société Européenne log return descriptive statistics over 2015-2019	
Mean	0.10%
Standard Error	0.05%
Median	0.10%
Standard Deviation	1.62%
Sample Variance	0.0003
Kurtosis	2.3806
Skewness	0.0012
Range	0.1523
Minimum	-7.41%
Maximum	7.82%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.001

Table 4.60. LVMH descriptive statistics over 2015-2019.

LVMH Moët Hennessy - Louis Vuitton, Société Européenne log return descriptive statistics over 2020-2021	
Mean	0.12%
Standard Error	0.09%
Median	0.18%
Standard Deviation	1.99%
Sample Variance	0.0004
Kurtosis	3.2601
Skewness	-0.3051
Range	0.1761
Minimum	-9.08%
Maximum	8.54%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.006

Table 4.61. LVMH descriptive statistics over 2020-2021.

4.2.19 Microsoft Corporation

As can be seen in Figure 4.74, Microsoft Corporation's stock returns did not present significant oscillations over the years, with the exception of the beginning of the pandemic, when the stock price increased, as Figure 4.73 displays.

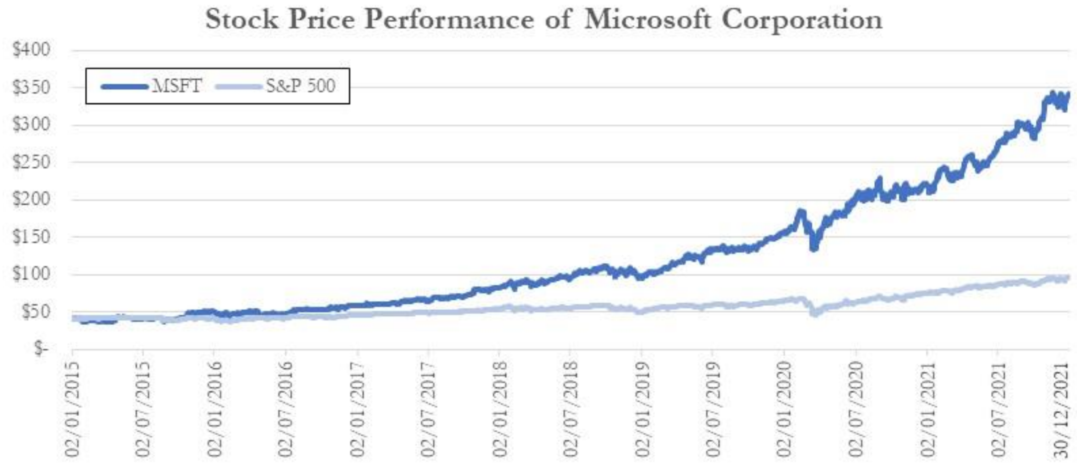

Figure 4.73. Stock price performance of Microsoft Corporation.[20]

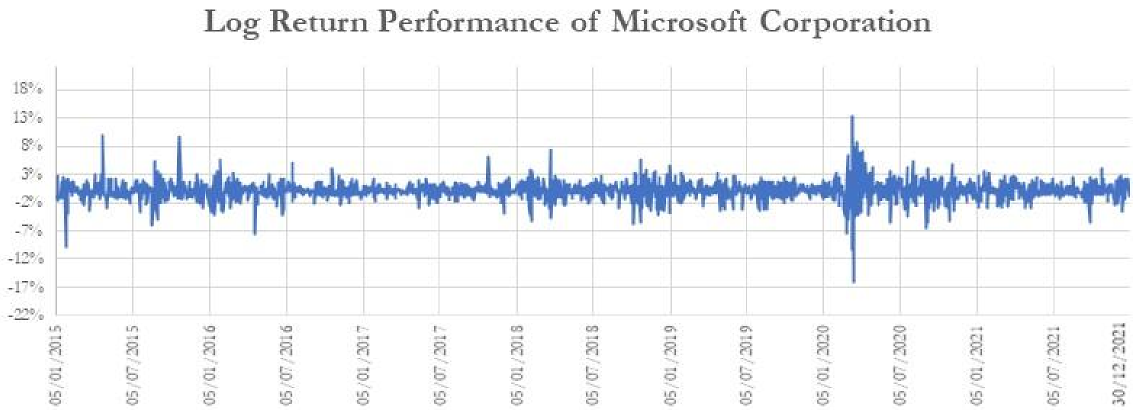

Figure 4.74. Log return performance of Microsoft Corporation

Table 4.62 exhibits the descriptive statistics of the daily logarithmic returns computed over 2015 to 2021. The distribution appears to be slightly left skewed, with a value of kurtosis close to 10, as shown in Figure 4.75, which plots the log returns. Nevertheless, the Q-Q plot in Figure 4.76 demonstrates an approximated alignment between observed and theoretical quantiles.

[20] To make the data comparable, S&P 500 stock index price has been normalized using MSFT stock price on 02/01/2015 as base year

Microsoft Corp log return descriptive statistics over 2015-2021	
Mean	0.12%
Standard Error	0.04%
Median	0.10%
Standard Deviation	1.70%
Sample Variance	0.0003
Kurtosis	10.9973
Skewness	-0.2589
Range	0.2924
Minimum	-15.95%
Maximum	13.29%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.62. Microsoft Corporation descriptive statistics over 2015-2021.

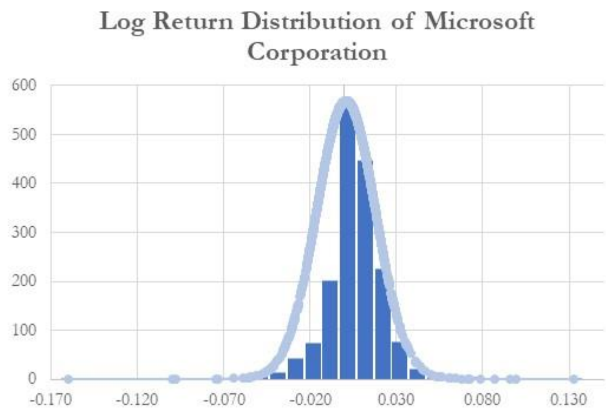

Figure 4.75. Log return distribution of Microsoft Corporation over 2015-2021.

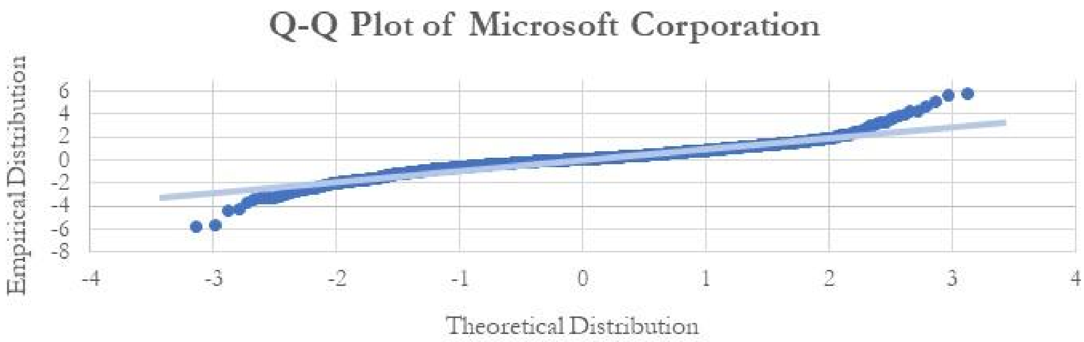

Figure 4.76. Q-Q Plot of Microsoft Corporation over 2015-2021.

Table 4.63 and Table 4.64 reports the descriptive statistics of daily log returns over respectively 2015-2019 and 2020-2021. The average returns in 2020-2021 are higher, characterized by increased volatility, higher level of kurtosis and the curve appears left skewed.

Microsoft Corp log return descriptive statistics over 2015-2019	
Mean	0.11%
Standard Error	0.04%
Median	0.10%
Standard Deviation	1.47%
Sample Variance	0.0002
Kurtosis	6.6580
Skewness	0.0270
Range	0.1965
Minimum	-9.71%
Maximum	9.94%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.63. Microsoft Corporation descriptive statistics over 2015-2019.

Microsoft Corp log return descriptive statistics over 2020-2021	
Mean	0.16%
Standard Error	0.10%
Median	0.15%
Standard Deviation	2.17%
Sample Variance	0.0005
Kurtosis	10.5600
Skewness	-0.4888
Range	0.2924
Minimum	-15.95%
Maximum	13.29%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.64. Microsoft Corporation descriptive statistics over 2020-2021.

4.2.20 L'Oréal S.A.

Benefitting from growing demand for its product, L'Oréal S.A.'s stock price has grown over the years, even more during the pandemic, as shown in Figure 4.77. Figure 4.78 displays the daily logarithmic returns.

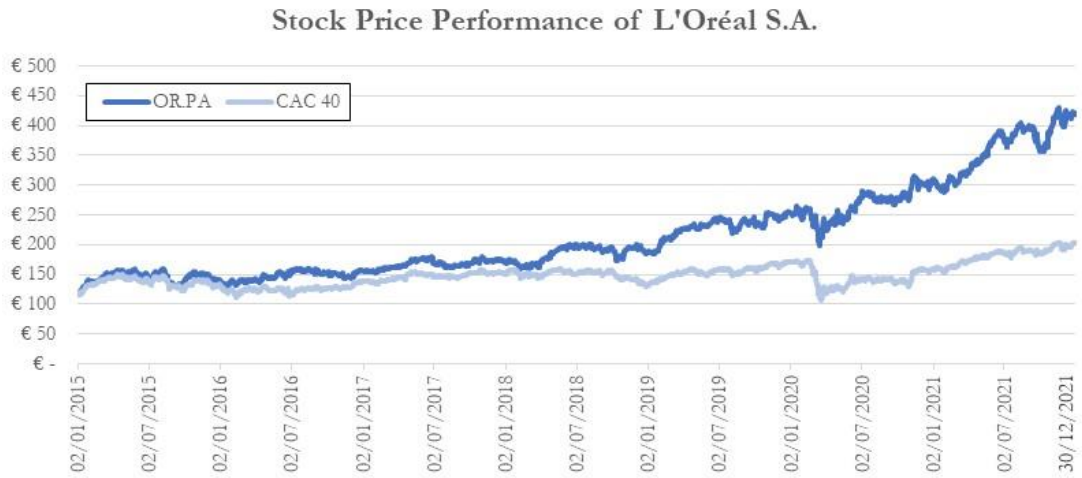

Figure 4.77. Stock price performance of L'Oréal S.A.[21]

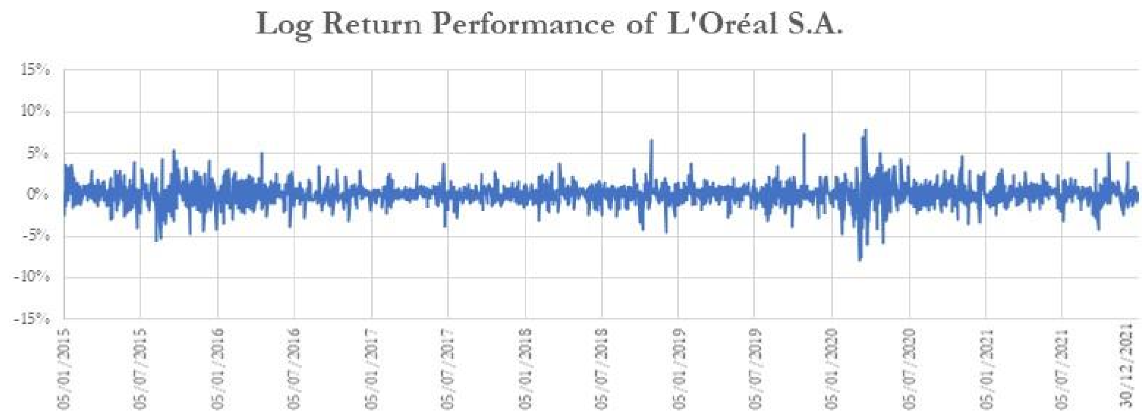

Figure 4.78. Log return performance of L'Oréal S.A.

The descriptive statistics of L'Oréal S.A.'s stock returns over the three time horizon are displayed in Table 4.65 (2015-2021), Table 4.66 (2015-2019), and Table 4.67 (2020-2021). The data are constant over the years, without experiencing significant changes. While the log return distribution over 2015-2021 is shown in Figure 4.79, the Q-Q plot is reported in Figure 4.80.

[21] To make the data comparable, CAC 40 stock index price has been normalized using OR.PA stock price on 02/01/2015 as base year

L'Oréal S.A. log return descriptive statistics over 2015-2021	
Mean	0.07%
Standard Error	0.03%
Median	0.10%
Standard Deviation	1.39%
Sample Variance	0.0002
Kurtosis	3.9195
Skewness	0.0134
Range	0.1563
Minimum	-7.91%
Maximum	7.72%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.65. L'Oréal S.A. descriptive statistics over 2015-2021.

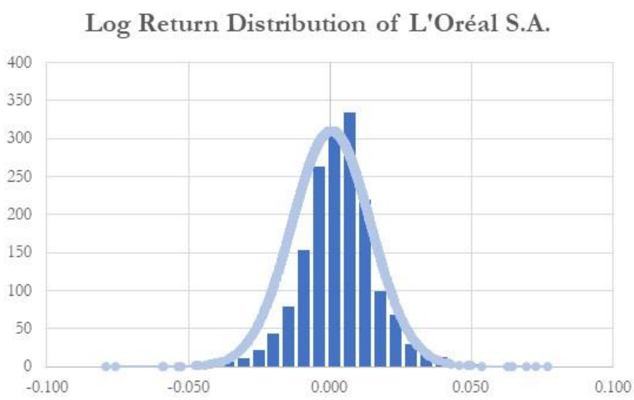

Figure 4.79. Log return distribution of L'Oréal S.A. over 2015-2021.

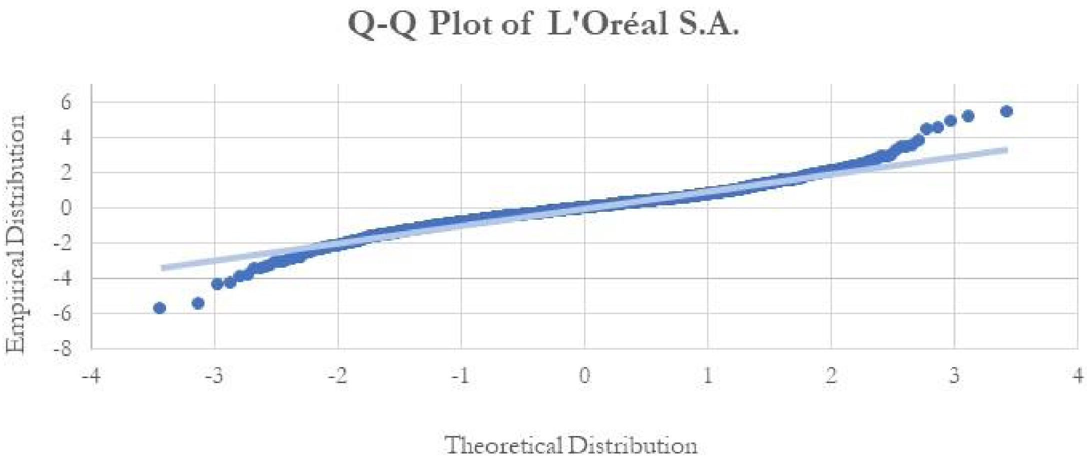

Figure 4.80. Q-Q Plot of L'Oréal S.A. over 2015-2021.

L'Oréal S.A. log return descriptive statistics over 2015-2019	
Mean	0.10%
Standard Error	0.07%
Median	0.14%
Standard Deviation	1.63%
Sample Variance	0.0003
Kurtosis	4.3401
Skewness	-0.1327
Range	0.1563
Minimum	-7.91%
Maximum	7.72%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.66. L'Oréal S.A. descriptive statistics over 2015-2019.

L'Oréal S.A. log return descriptive statistics over 2020-2021	
Mean	0.10%
Standard Error	0.07%
Median	0.14%
Standard Deviation	1.63%
Sample Variance	0.0003
Kurtosis	4.3401
Skewness	-0.1327
Range	0.1563
Minimum	-7.91%
Maximum	7.72%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.014

Table 4.67. L'Oréal S.A. descriptive statistics over 2020-2021.

4.2.21 Pfizer Inc.

As shown in Figure 4.81, Pfizer's stock price closely tracks the S&P 500 performance. Divergence appears during the pandemic, where the stock returns experienced huge fluctuations as displayed in Figure 4.82.

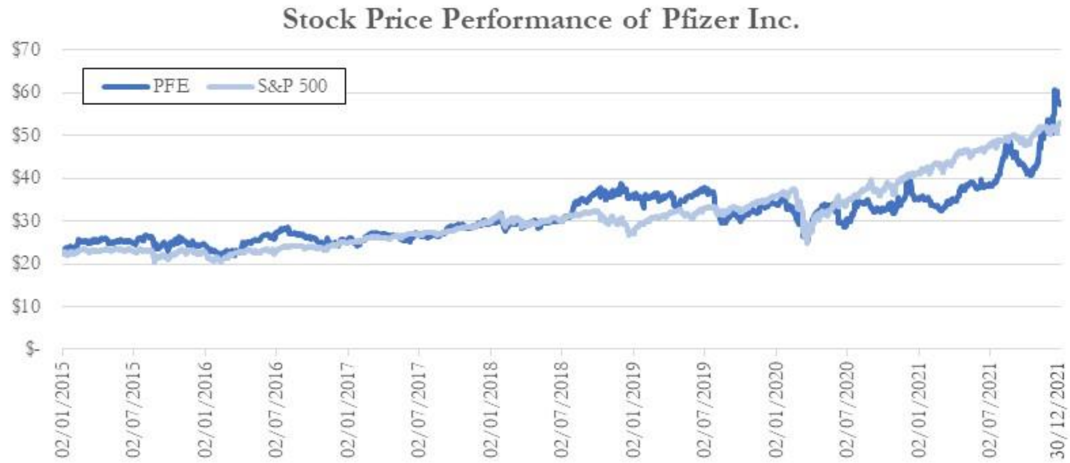

Figure 4.81. Stock price performance of Pfizer Inc.[22]

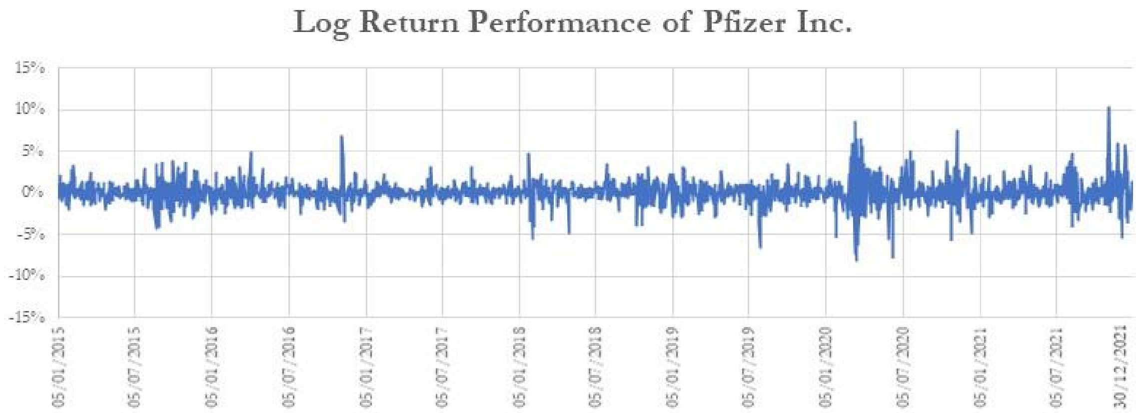

Figure 4.82. Log return performance of Pfizer Inc.

The descriptive statistics for the daily logarithmic returns computed over 2015-2021 are reported in Table 4.68 and the distribution appears to be right skewed, with thick tails as confirmed by Figure 4.83, which plots the log returns. Nevertheless, by analysing the Q-Q plot, an alignment between observed and theoretical quantiles can be observed, as shown in Figure 4.84

[22] To make the data comparable, S&P 500 stock index price has been normalized using PFE stock price on 02/01/2015 as base year

Pfizer Inc. log return descriptive statistics over 2015-2021	
Mean	0.05%
Standard Error	0.03%
Median	0.02%
Standard Deviation	1.43%
Sample Variance	0.0002
Kurtosis	6.5009
Skewness	0.2291
Range	0.1836
Minimum	-8.05%
Maximum	10.31%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.68. Pfizer Inc. descriptive statistics over 2015-2021.

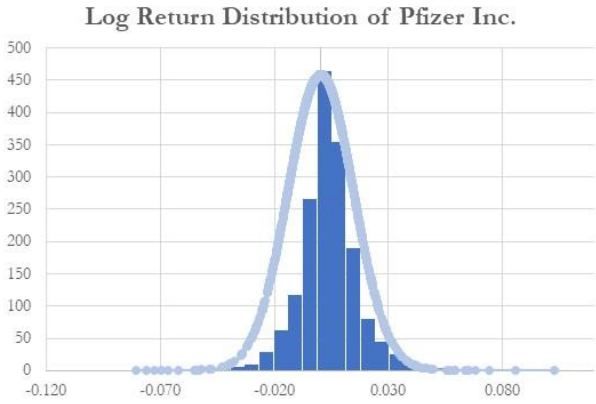

Figure 4.83. Log return distribution of Pfizer Inc. over 2015-2021.

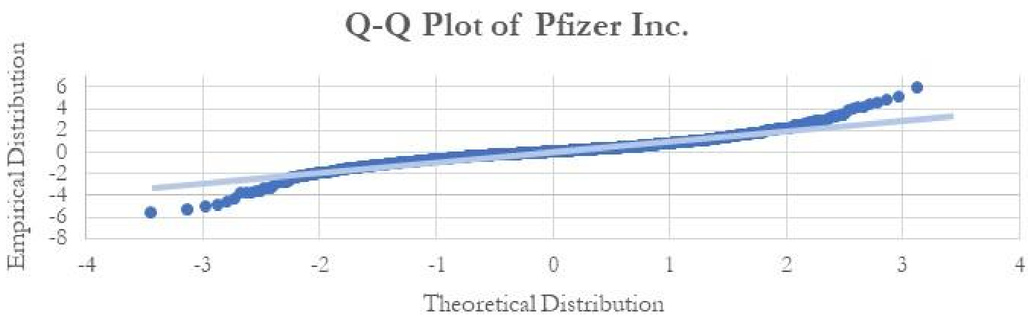

Figure 4.84. Q-Q Plot of Pfizer Inc. over 2015-2021.

To isolate the impact of Covid-19, the considered time period has been split into two subperiods, namely 2015-2019 and 2020-2021, and the respective descriptive statistics have been computed, as shown in Table 4.69 and Table 4.70. In 2020-2021 the daily average return peaked to 0.11%, with a slight increase in volatility. The kurtosis increased and the skewness turned negative. Even though the Kolmogorov–Smirnov and the Shapiro–Wilk test both reject the null hypothesis about the normal distribution of the returns, we will assume it for the purpose of this thesis.

Pfizer Inc. log return descriptive statistics over 2015-2019	
Mean	0.03%
Standard Error	0.03%
Median	0.03%
Standard Deviation	1.15%
Sample Variance	0.0001
Kurtosis	3.6934
Skewness	-0.1276
Range	0.1346
Minimum	-6.63%
Maximum	6.83%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.69. Pfizer Inc. descriptive statistics over 2015-2019.

Pfizer Inc. log return descriptive statistics over 2020-2021	
Mean	0.11%
Standard Error	0.09%
Median	-0.05%
Standard Deviation	1.97%
Sample Variance	0.0004
Kurtosis	4.3847
Skewness	0.3177
Range	0.1836
Minimum	-8.05%
Maximum	10.31%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.70. Pfizer Inc. descriptive statistics over 2020-2021.

4.2.22 The Procter & Gamble Company

The Procter & Gamble Company is a non-cyclical mature firm. stock. As a consequence, the log return performance does presents particular peaks only in exceptional times, like at the beginning of the Covid-19 crisis, as displayed in Figure 4.86. Figure 4.85 shows the stock price performance of the stock over the selected time horizon.

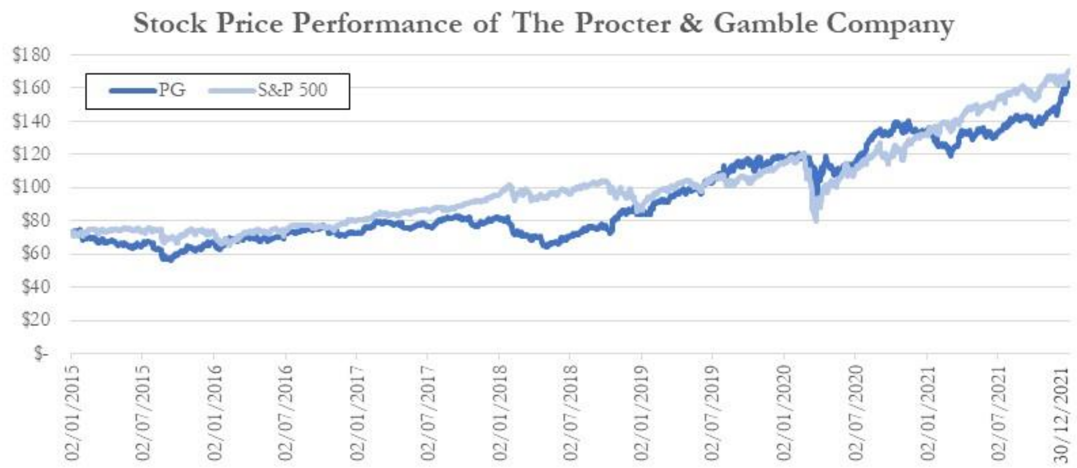

Figure 4.85. Stock price performance of The Procter & Gamble Company.[23]

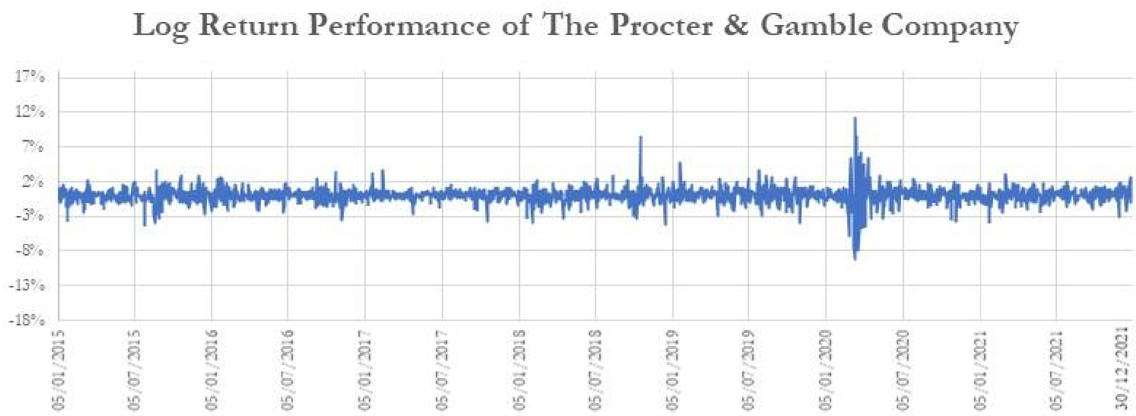

Figure 4.86. Log return performance of The Procter & Gamble Company.

The descriptive statistics of The Procter & Gamble Company are presented in Table 4.71 (2015-2021), Table 4.72 (2015-2019), and Table 4.73 (2020-2021). The daily stock returns over 2020-2021 did not experience a significant change in the average return, but the volatility increased. The kurtosis doubled, while the skewness decreased, as shown in Figure 4.87. In the Q-Q plot in Figure 4.88 the alignment between observed and theoretical quantiles over 2015-2021 is displayed.

[23] To make the data comparable, S&P 500 stock index price has been normalized using PG stock price on 02/01/2015 as base year

The Procter & Gamble Company log return descriptive statistics over 2015-2021	
Mean	0.05%
Standard Error	0.03%
Median	0.06%
Standard Deviation	1.20%
Sample Variance	0.0001
Kurtosis	13.9592
Skewness	0.2011
Range	0.2048
Minimum	-9.14%
Maximum	11.34%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.71. The Procter & Gamble Company descriptive statistics over 2015-2021.

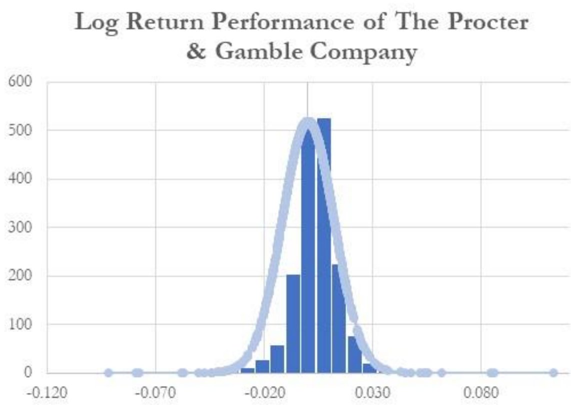

Figure 4.87. Log return distribution of The Procter & Gamble Company over 2015-2021.

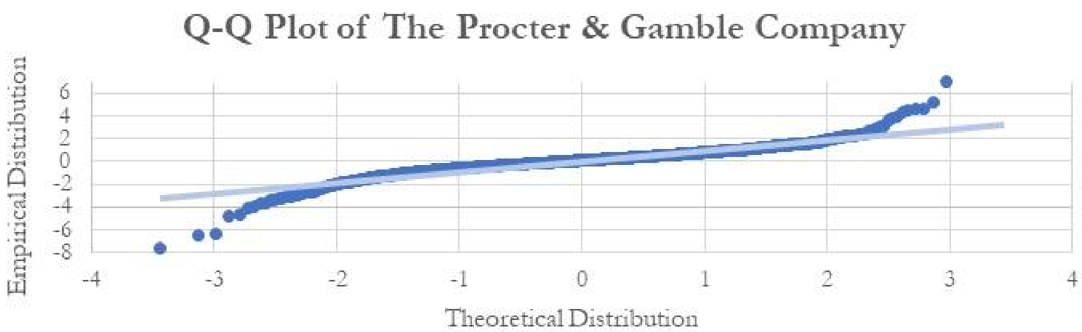

Figure 4.88. Q-Q Plot of The Procter & Gamble Company over 2015-2021.

The Procter & Gamble Company log return descriptive statistics over 2015-2019	
Mean	0.04%
Standard Error	0.03%
Median	0.04%
Standard Deviation	1.00%
Sample Variance	0.0001
Kurtosis	6.2928
Skewness	0.2404
Range	0.1252
Minimum	-4.09%
Maximum	8.43%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.72. The Procter & Gamble Company descriptive statistics over 2015-2019.

The Procter & Gamble Company log return descriptive statistics over 2020-2021	
Mean	0.07%
Standard Error	0.07%
Median	0.11%
Standard Deviation	1.60%
Sample Variance	0.0003
Kurtosis	12.4882
Skewness	0.1310
Range	0.2048
Minimum	-9.14%
Maximum	11.34%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.73. The Procter & Gamble Company descriptive statistics over 2020-2021.

4.2.23 Repsol S.A.

By looking at Figure 4.90, it is possible to argue that Repsol S.A.'s stock returns experienced oscillations in 2015-2016 and during the pandemic. Figure 4.89 shows the stock price performance of the company, compared with the one of the IBEX 35.

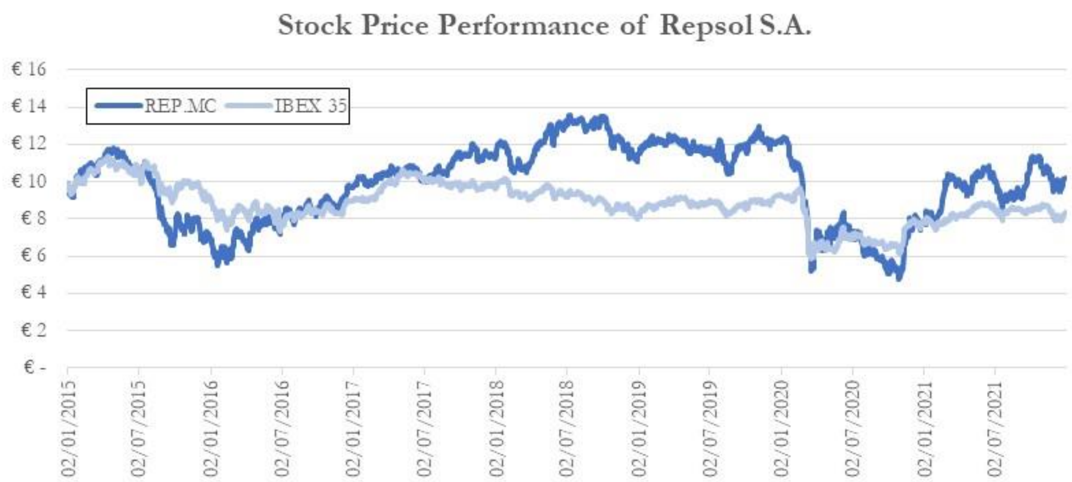

Figure 4.89. Stock price performance of Repsol S.A.[24]

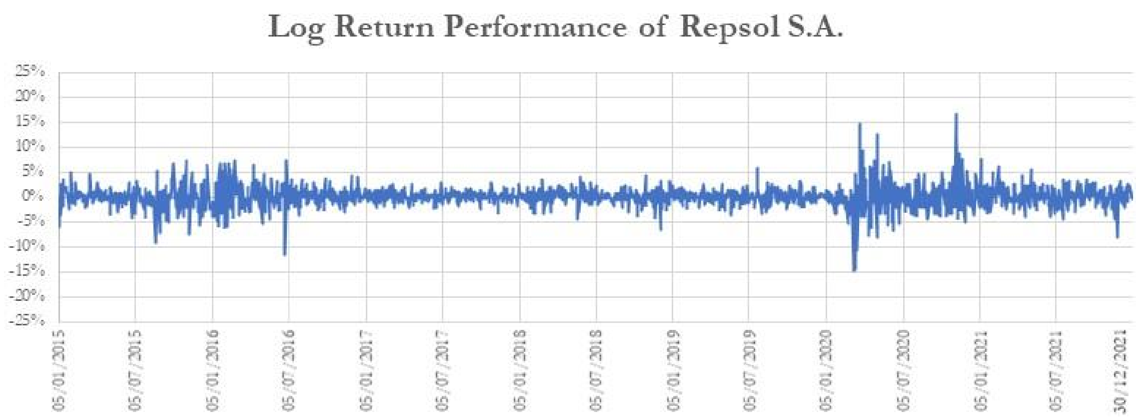

Figure 4.90. Log return performance of Repsol S.A.

As Table 4.76 (2020-2021) shows, the average daily stock returns over 2020-2021 turned negative, with increased volatility. The kurtosis was around 6, while the skewness turned positive. While Table 4.75 reports the descriptive statistics over 2015-2019, Table 4.74 refers to 2015-2021 Figure 4.91 shows the log return distribution. The Q-Q plot in Figure 4.92 shows the observed and theoretical quantiles over 2015-2021.

[24] To make the data comparable, IBEX 35 stock index price has been normalized using REP.MC stock price on 02/01/2015 as base year

Repsol S.A. log return descriptive statistics over 2015-2021	
Mean	0.00%
Standard Error	0.05%
Median	0.00%
Standard Deviation	2.17%
Sample Variance	0.0005
Kurtosis	8.6238
Skewness	0.1582
Range	0.3153
Minimum	-14.79%
Maximum	16.74%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.74. Repsol S.A. descriptive statistics over 2015-2021.

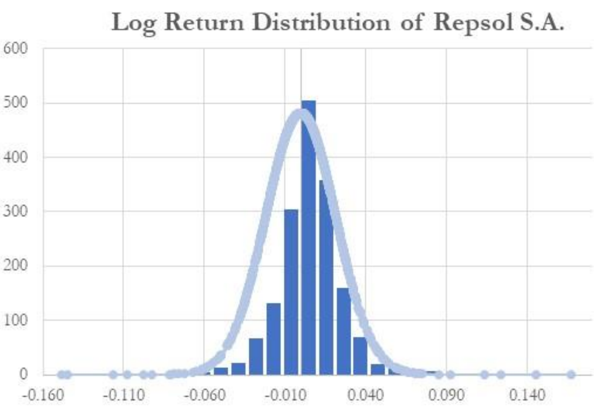

Figure 4.91. Log return distribution of Repsol S.A. over 2015-2021

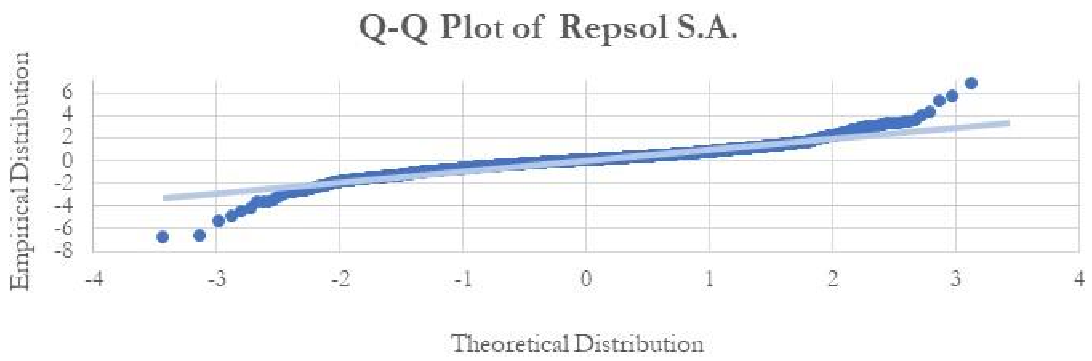

Figure 4.92. Q-Q Plot of Repsol S.A. over 2015-2021.

Repsol S.A. log return descriptive statistics over 2015-2019	
Mean	0.02%
Standard Error	0.05%
Median	0.03%
Standard Deviation	1.76%
Sample Variance	0.0003
Kurtosis	4.3672
Skewness	-0.2696
Range	0.1882
Minimum	-11.59%
Maximum	7.23%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.75. Repsol S.A. descriptive statistics over 2015-2019.

Repsol S.A. log return descriptive statistics over 2020-2021	
Mean	-0.04%
Standard Error	0.13%
Median	-0.16%
Standard Deviation	2.95%
Sample Variance	0.0009
Kurtosis	6.6102
Skewness	0.3893
Range	0.3153
Minimum	-14.79%
Maximum	16.74%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.005

Table 4.76. Repsol S.A. descriptive statistics over 2020-2021.

4.2.24 Banco Santander, S.A.

From the stock price performance showed in Figure 4.93, the cyclical nature of the financial services business emerges. By analysing Figure 4.94, it can be found that significant drops were on 09/01/2015 after fundraising and dividend cut, on 23/06/2016 after the Brexit announcement, and at the beginning of the pandemic.

Figure 4.93. Stock price performance of Banco Santander, S.A.[25]

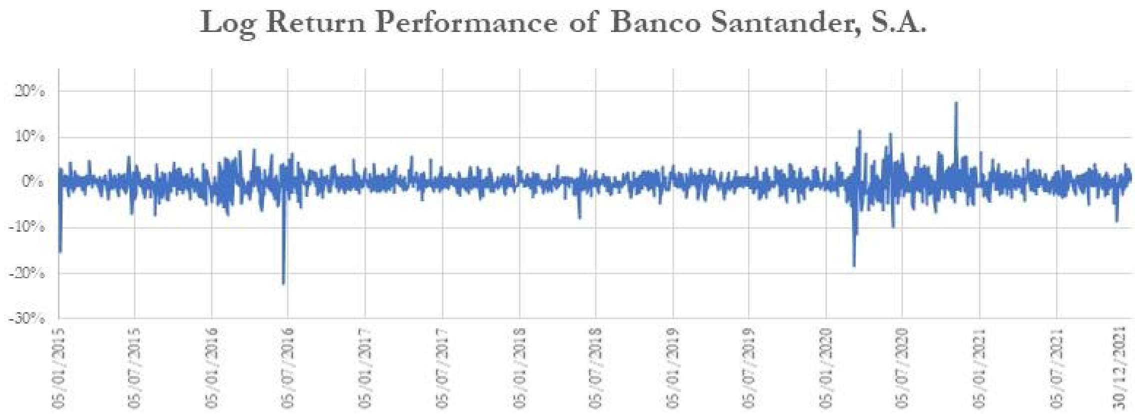

Figure 4.94. Log return performance of Banco Santander, S.A.

Table 4.77 exhibits the descriptive statistics of the logarithmic returns computed over 2015-2021 and the distribution appears to be left skewed, with tick tails as shown in Figure 4.95 displays. The Q-Q plot in Figure 4.96 plots the observed and theoretical quantiles.

[25] To make the data comparable, IBEX 35 stock index price has been normalized using SAN.MC stock price on 02/01/2015 as base year

Banco Santander, S.A. log return descriptive statistics over 2015-2021	
Mean	-0.03%
Standard Error	0.06%
Median	0.01%
Standard Deviation	2.31%
Sample Variance	0.0005
Kurtosis	11.9637
Skewness	-0.7349
Range	0.3975
Minimum	-22.17%
Maximum	17.58%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.77. Banco Santander, S.A. descriptive statistics over 2015-2021.

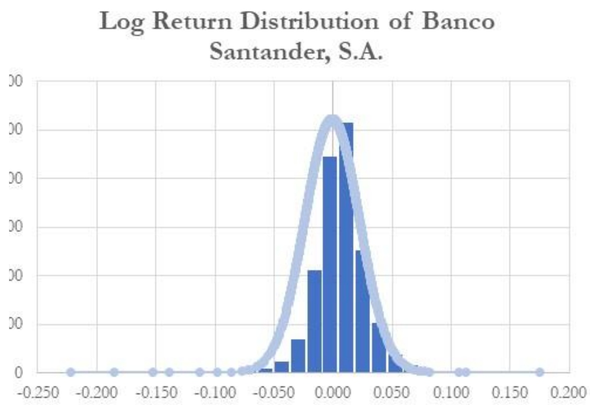

Figure 4.95. Log return distribution of Banco Santander, S.A. over 2015-2021.

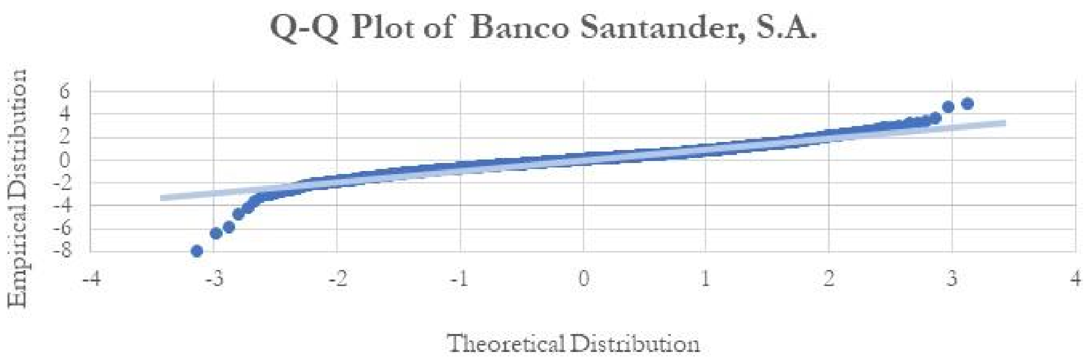

Figure 4.96. Q-Q Plot of Banco Santander, S.A. over 2015-2021.

Table 4.78 and Table 4.79 show the descriptive statistics computed respectively over 2015-2019 and 2020-2021. Both subperiods presents the same negative average returns, but over 2020-2021 the volatility increased. On the other hand, both the skewness and the kurtosis decreased.

Banco Santander, S.A. log return descriptive statistics over 2015-2019	
Mean	-0.03%
Standard Error	0.06%
Median	0.03%
Standard Deviation	2.00%
Sample Variance	0.0004
Kurtosis	15.5176
Skewness	-1.4752
Range	0.2948
Minimum	-22.17%
Maximum	7.31%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.78. Banco Santander, S.A. descriptive statistics over 2015-2019.

Banco Santander, S.A. log return descriptive statistics over 2020-2021	
Mean	-0.03%
Standard Error	0.13%
Median	-0.08%
Standard Deviation	2.96%
Sample Variance	0.0009
Kurtosis	7.0208
Skewness	-0.0959
Range	0.3604
Minimum	-18.46%
Maximum	17.58%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.015

Table 4.79. Banco Santander, S.A. descriptive statistics over 2020-2021.

4.2.25 Sanofi

Over the years, the stock price of Sanofi experienced several fluctuations, as shown in Figure 4.97, confirmed by Figure 4.98.

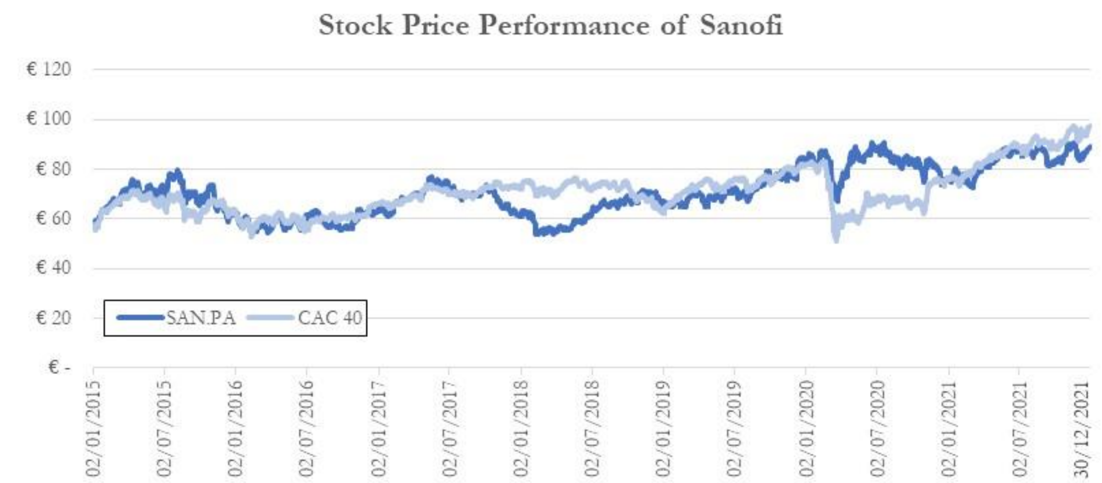

Figure 4.97. Stock price performance of Sanofi.[26]

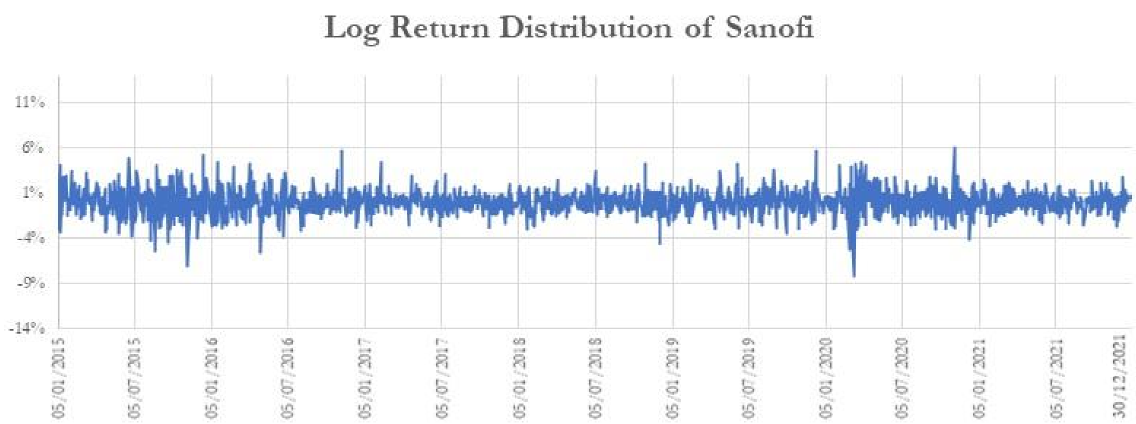

Figure 4.98. Log return performance of Sanofi.

Table 4.80 presents the descriptive statistics for the daily logarithmic returns computed over 2015 2021. The kurtosis is close to 3 with a slightly negative skewness, as shown in Figure 4.99. The Q-Q plot in Figure 4.100 displays the alignment between observed and theoretical quantiles.

[26] To make the data comparable, CAC 40 stock index price has been normalized using SAN.PA stock price on 02/01/2015 as base year

Sanofi log return descriptive statistics over 2015-2021	
Mean	0.03%
Standard Error	0.03%
Median	0.02%
Standard Deviation	1.39%
Sample Variance	0.0002
Kurtosis	2.9872
Skewness	-0.1457
Range	0.1431
Minimum	-8.19%
Maximum	6.12%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.80. Sanofi descriptive statistics over 2015-2021.

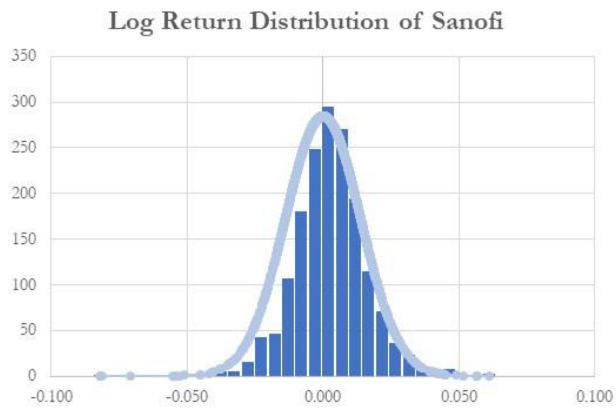

Figure 4.99. Log return distribution of Sanofi over 2015-2021.

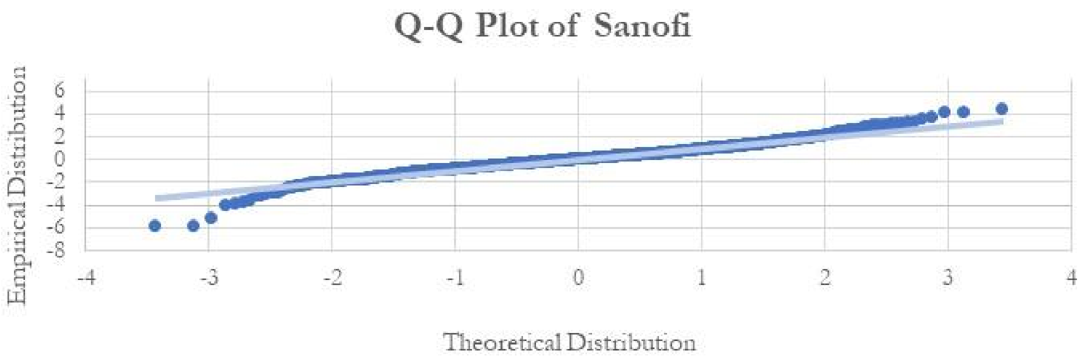

Figure 4.100. Q-Q Plot of Sanofi over 2015-2021.

Table 4.81 and Table 4.82 helps to evaluate the impact of the pandemic over the stock price: the average daily return decreased from 0.03% to 0.01%, while volatility slightly increased. Interestingly, according to the Kolmogorov–Smirnov test, the null hypothesis of normal distribution of stock returns can be accepted with a level of confidence of more than 95%.

Sanofi log return descriptive statistics over 2015-2019	
Mean	0.03%
Standard Error	0.04%
Median	0.02%
Standard Deviation	1.35%
Sample Variance	0.0002
Kurtosis	2.1564
Skewness	0.0404
Range	0.1278
Minimum	-7.08%
Maximum	5.70%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.007

Table 4.81. Sanofi descriptive statistics over 2015-2019.

Sanofi log return descriptive statistics over 2020-2021	
Mean	0.01%
Standard Error	0.07%
Median	0.03%
Standard Deviation	1.47%
Sample Variance	0.0002
Kurtosis	4.4121
Skewness	-0.5053
Range	0.1431
Minimum	-8.19%
Maximum	6.12%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.077

Table 4.82. Sanofi descriptive statistics over 2020-2021.

4.2.26 SAP SE

As displayed in Figure 4.101, the stock price performance of SAP SE presents a positive trend. Both the first and second wave of Covid-19 impacted the stock price, but in both cases it recovered. From Figure 4.102, it is possible to notice that the stock price experienced a significant drop of -25% on 26th October 2020, following the release of worse than imagined third-quarter results.

Figure 4.101. Stock price performance of SAP SE.[27]

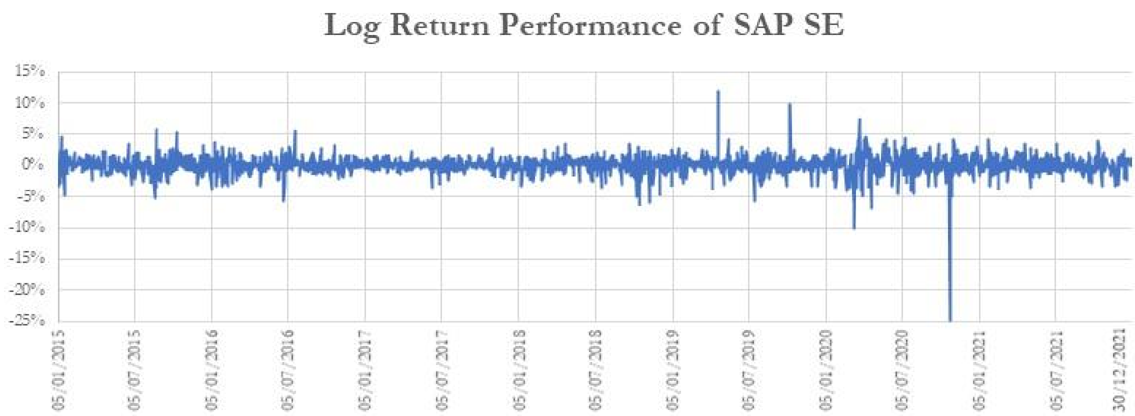

Figure 4.102. Log return performance of SAP SE.

The descriptive statistics of the daily logarithmic returns computed over 2015-2021 are reported in Table 4.83. The distribution appears to be left skewed, with a value of kurtosis close to 37, as shown in Figure 4.103, which plots the log returns. Figure 4.104 presents the Q-Q plot with the observed and theoretical quantiles.

[27] To make the data comparable, DAX 30 stock index price has been normalized using SAP.DE stock price on 02/01/2015 as base year

SAP SE log return descriptive statistics over 2015-2021	
Mean	0.05%
Standard Error	0.04%
Median	0.09%
Standard Deviation	1.61%
Sample Variance	0.0003
Kurtosis	37.1447
Skewness	-2.1654
Range	0.3659
Minimum	-24.77%
Maximum	11.82%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.83. SAP SE descriptive statistics over 2015-2021.

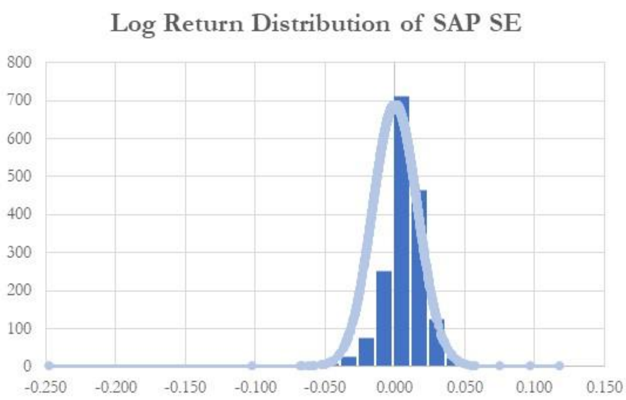

Figure 4.103. Log return distribution of SAP SE over 2015-2021.

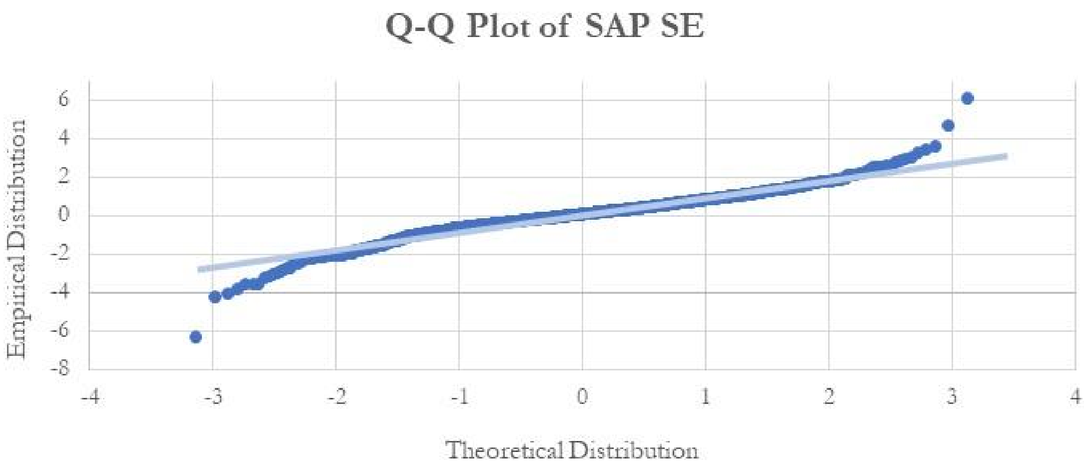

Figure 4.104. Q-Q Plot of SAP SE over 2015-2021.

Table 4.84 and Table 4.85 help to evaluate the impact of the pandemic over the stock price: the average return more than halved, while the volatility doubled. At the same time the kurtosis skyrocketed, and the skewness turned negative.

SAP SE log return descriptive statistics over 2015-2019	
Mean	0.07%
Standard Error	0.04%
Median	0.10%
Standard Deviation	1.38%
Sample Variance	0.0002
Kurtosis	7.9824
Skewness	0.3858
Range	0.1795
Minimum	-6.12%
Maximum	11.82%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.84. SAP SE descriptive statistics over 2015-2019.

SAP SE log return descriptive statistics over 2020-2021	
Mean	0.02%
Standard Error	0.09%
Median	0.04%
Standard Deviation	2.09%
Sample Variance	0.0004
Kurtosis	41.8766
Skewness	-3.7497
Range	0.3227
Minimum	-24.77%
Maximum	7.50%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.85. SAP SE descriptive statistics over 2020-2021.

4.2.27 Solvay SA

Over 2015-2021 Solvay SA's stock price oscillates in the range of € 52.3 (16/03/2020) and € 116.34 (13/08/2021), as shown in Figure 4.105. At first Covid-19 impacted the stock returns but they soon recovered, as Figure 4.106 shows.

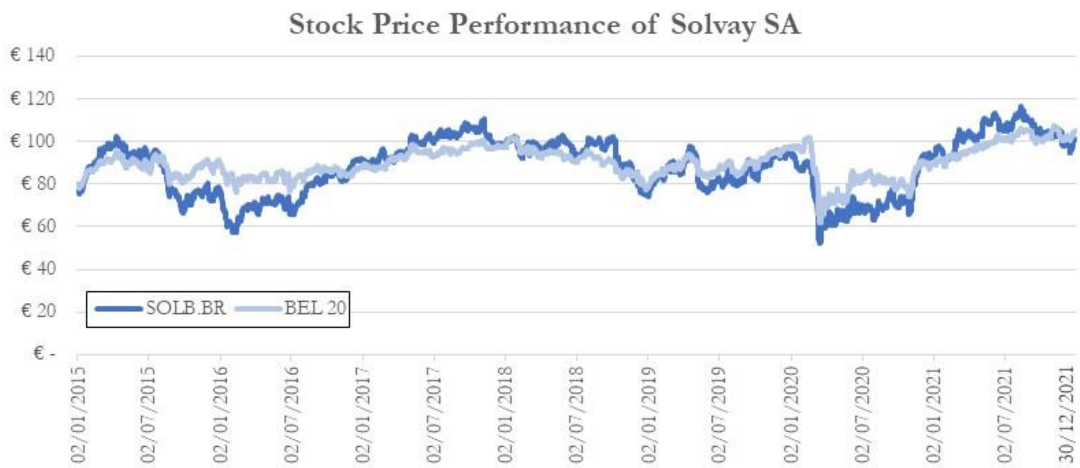

Figure 4.105. Stock price performance of Solvay SA.[28]

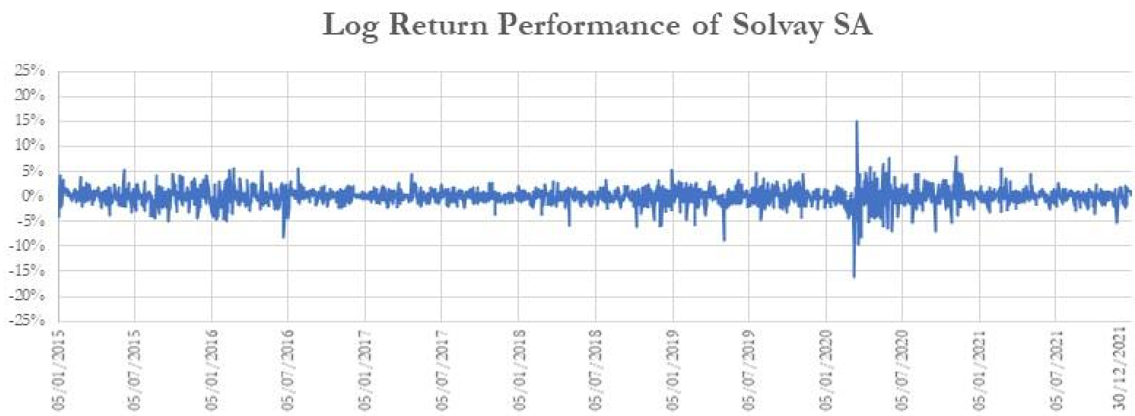

Figure 4.106. Log return performance of Solvay SA.

The descriptive statistics of Solvay SA are presented in Table 4.86 (2015-2021), Table 4.87 (2015-2019), and Table 4.88 (2020-2021). The daily stock returns over 2020-2021 did not experience a significant change in the average return, but the volatility increased. The kurtosis increased, while the skewness decreased. Figure 4.107 shows the log return distribution over 2015-2021 and the observed and theoretical quantiles are displayed in the Q-Q plot in Figure 4.108.

[28] To make the data comparable, BEL 20 stock index price has been normalized using SOLB.BR stock price on 02/01/2015 as base year

Solvay SA log return descriptive statistics over 2015-2021	
Mean	0.01%
Standard Error	0.04%
Median	0.04%
Standard Deviation	1.83%
Sample Variance	0.0003
Kurtosis	9.9743
Skewness	-0.2530
Range	0.3106
Minimum	-16.18%
Maximum	14.89%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.86. Solvay SA descriptive statistics over 2015-2021.

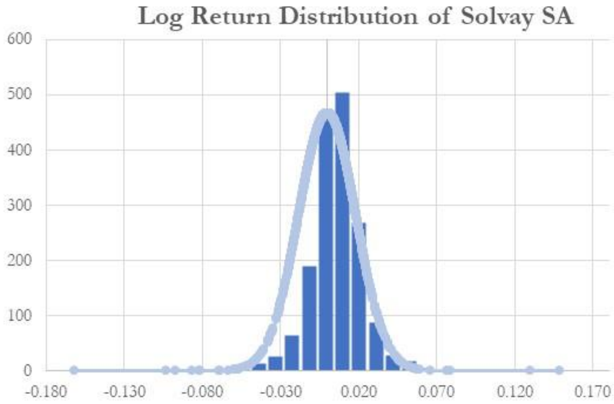

Figure 4.107. Log return distribution of Solvay SA over 2015-2021.

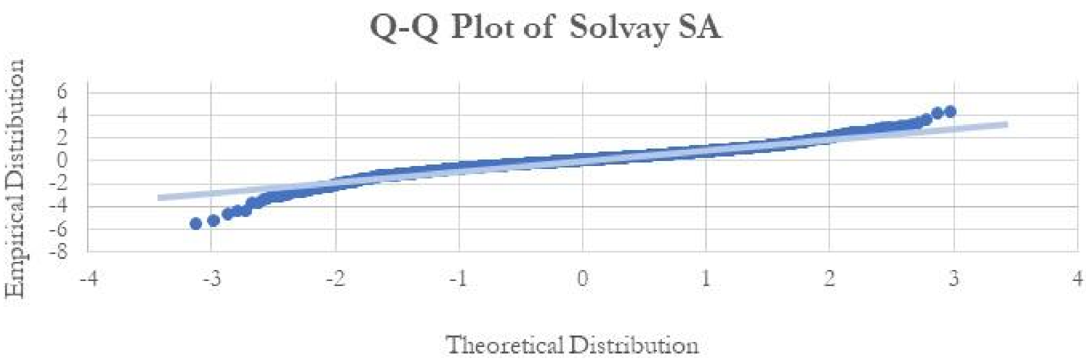

Figure 4.108. Q-Q Plot of Solvay SA over 2015-2021.

Solvay SA log return descriptive statistics over 2015-2019	
Mean	0.01%
Standard Error	0.05%
Median	0.04%
Standard Deviation	1.57%
Sample Variance	0.0002
Kurtosis	2.5239
Skewness	-0.3998
Range	0.1436
Minimum	-8.69%
Maximum	5.67%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.004

Table 4.87. Solvay SA descriptive statistics over 2015-2019.

Solvay SA log return descriptive statistics over 2020-2021	
Mean	0.02%
Standard Error	0.11%
Median	0.00%
Standard Deviation	2.36%
Sample Variance	0.0006
Kurtosis	10.8729
Skewness	-0.1203
Range	0.3106
Minimum	-16.18%
Maximum	14.89%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.88. Solvay SA descriptive statistics over 2020-2021.

4.2.28 Unilever PLC

Figure 4.109 shows the stock price performance of Unilever PLC. The highest price was reached on 5th September 2019, but then a decreasing trend followed. The pandemic hit the company, as the negative returns in Figure 4.110 reveals.

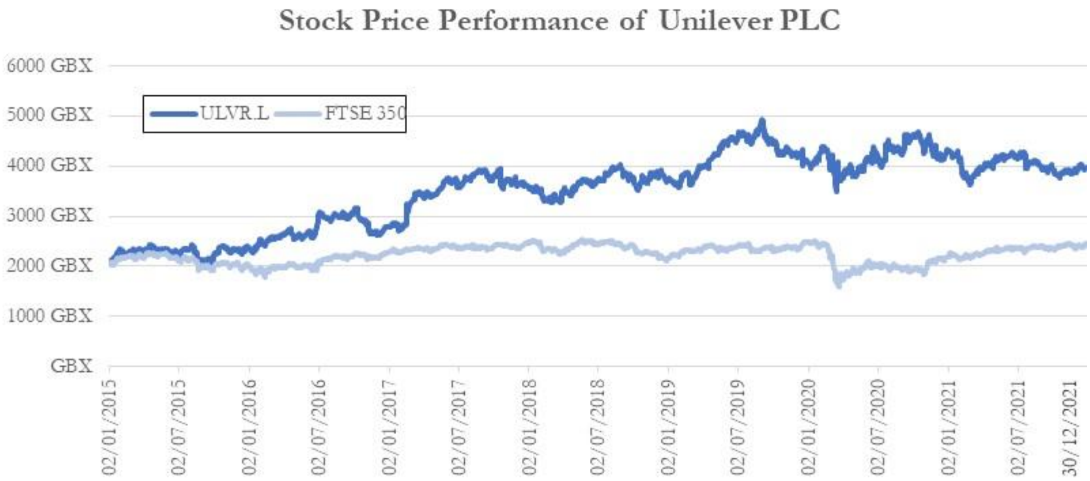

Figure 4.109. Stock price performance of Unilever PLC.[29]

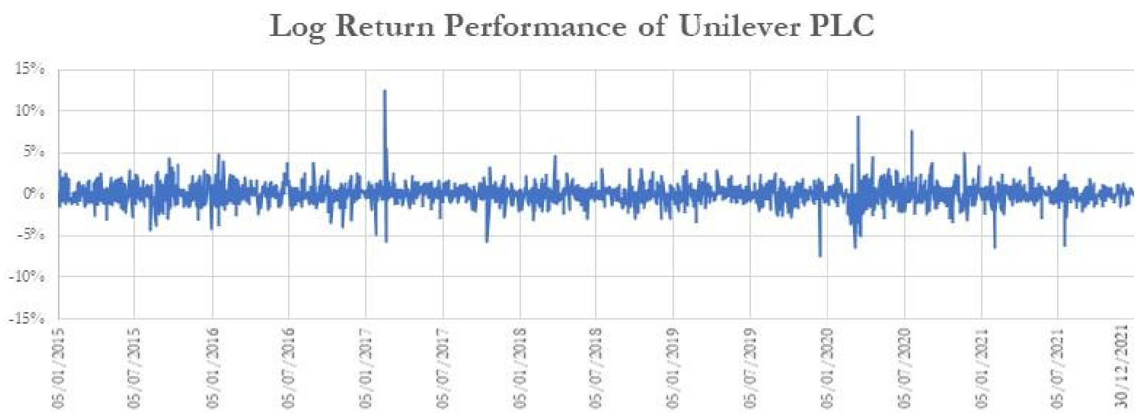

Figure 4.110. Log return performance of Unilever PLC.

Table 4.89 exhibits the descriptive statistics of the daily logarithmic returns computed over 2015-2021. The distribution appears to be slightly right skewed, with a value of kurtosis close to 9, as shown in Figure 4.111, which plots the log returns. Nevertheless, the Q-Q plot in Figure 4.112 demonstrates an approximated alignment between observed and theoretical quantiles.

[29] To make the data comparable, FTSE 350 stock index price has been normalized using ULVR.L's stock price on 02/01/2015 as base year

Unilever PLC log return descriptive statistics over 2015-2021	
Mean	0.04%
Standard Error	0.03%
Median	0.04%
Standard Deviation	1.32%
Sample Variance	0.0002
Kurtosis	9.2907
Skewness	0.4466
Range	0.2003
Minimum	-7.43%
Maximum	12.60%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.89. Unilever PLC descriptive statistics over 2015-2021.

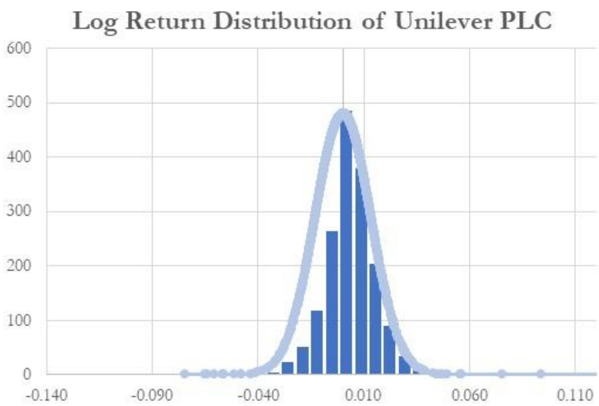

Figure 4.111. Log return distribution of Unilever PLC over 2015-2021.

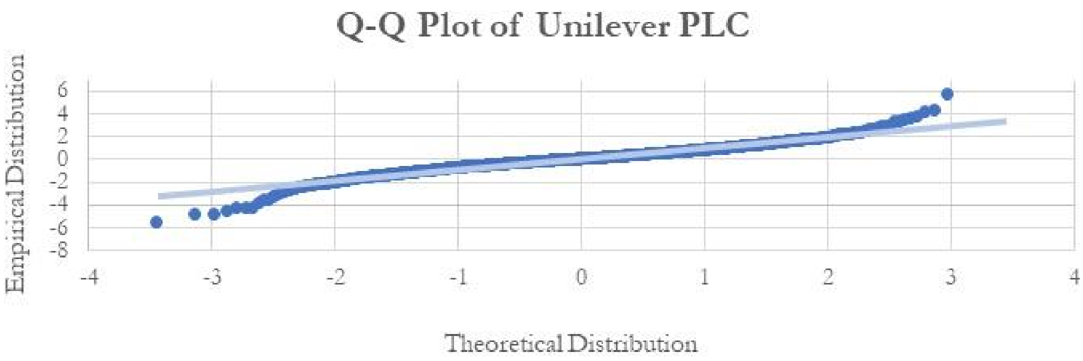

Figure 4.112. Q-Q Plot of Unilever PLC over 2015-2021.

To understand the impact of Covid-19, Table 4.90 and Table 4.91 reports the descriptive statistics over two subperiods, namely 2015-2019 and 2020-2021. In 2020-2021 the daily average return turned negative, characterized by a slight increase in volatility. Both the kurtosis and the skewness decreased. Even though the Kolmogorov–Smirnov and the Shapiro–Wilk test both reject the null hypothesis about the normal distribution of the returns, we will assume it for the purpose of this thesis.

Unilever PLC log return descriptive statistics over 2015-2019	
Mean	0.06%
Standard Error	0.04%
Median	0.05%
Standard Deviation	1.25%
Sample Variance	0.0002
Kurtosis	10.5430
Skewness	0.5379
Range	0.2003
Minimum	-7.43%
Maximum	12.60%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.90. Unilever PLC descriptive statistics over 2015-2019.

Unilever PLC log return descriptive statistics over 2020-2021	
Mean	-0.01%
Standard Error	0.07%
Median	0.00%
Standard Deviation	1.48%
Sample Variance	0.0002
Kurtosis	7.0731
Skewness	0.3242
Range	0.1588
Minimum	-6.50%
Maximum	9.38%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.002

Table 4.91. Unilever PLC descriptive statistics over 2020-2021.

4.2.29 Volkswagen AG

As Figure 4.114 reveals, the biggest drop in the log returns of the company occurred on 26th October 2020. Hit by the pandemic, the company's stock price started to rise again, as shown in Figure 4.113.

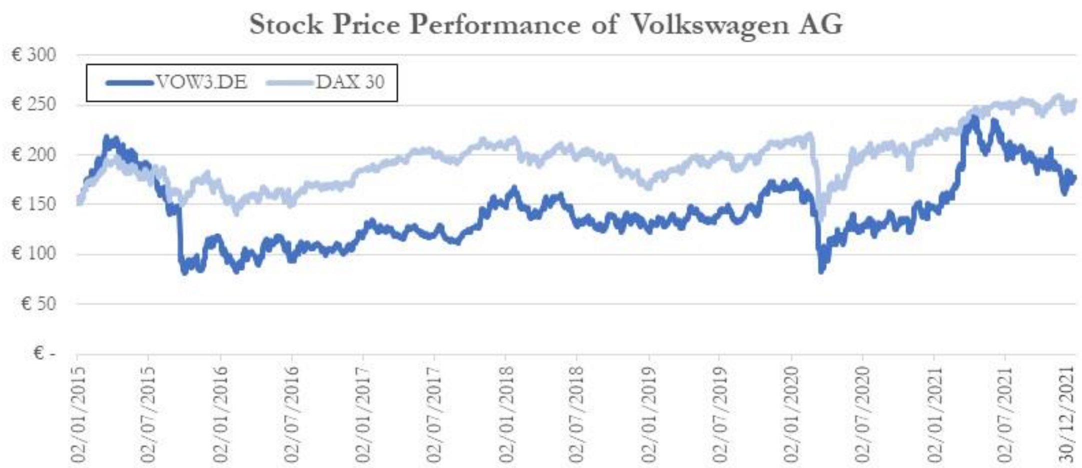

Figure 4.113. Stock price performance of Volkswagen AG.[30]

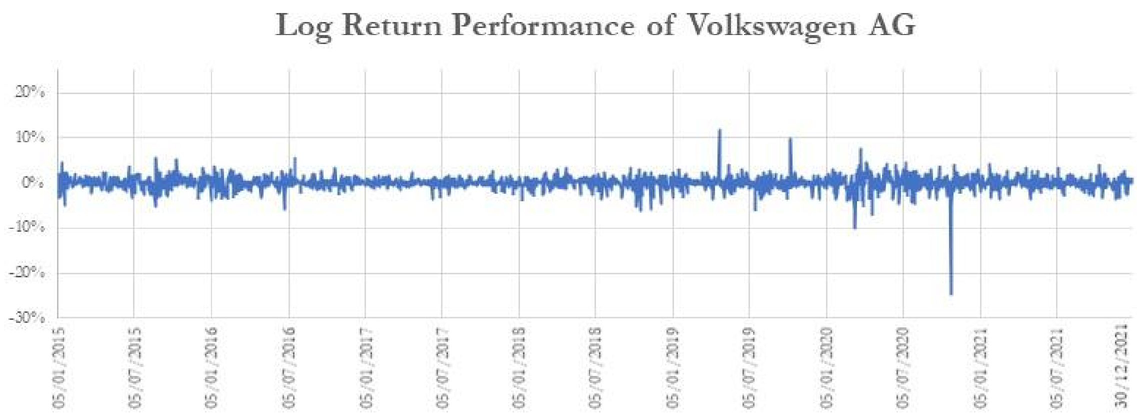

Figure 4.114. Log return performance of Volkswagen AG.

The descriptive statistics of Volkswagen AG are presented in Table 4.92 (2015-2021), Table 4.93 (2015-2019), and Table 4.94 (2020-2021). The daily stock returns over 2020-2021 did not experience a significant change in the average return, but the volatility increased. Both the kurtosis and the skewness decreased. Figure 4.115 plots the logarithmic return distribution over 2015-2021,

[30] To make the data comparable, DAX 30 stock index price has been normalized using VOW3.DE stock price on 02/01/2015 as base year

while in Figure 4.116 the Q-Q plot of the observed and theoretical quantiles over 2015-2021 can be observed.

Volkswagen AG log return descriptive statistics over 2015-2021	
Mean	0.01%
Standard Error	0.06%
Median	-0.03%
Standard Deviation	2.37%
Sample Variance	0.0006
Kurtosis	12.8904
Skewness	-0.8954
Range	0.3952
Minimum	-22.09%
Maximum	17.43%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.92. Volkswagen AG descriptive statistics over 2015-2021.

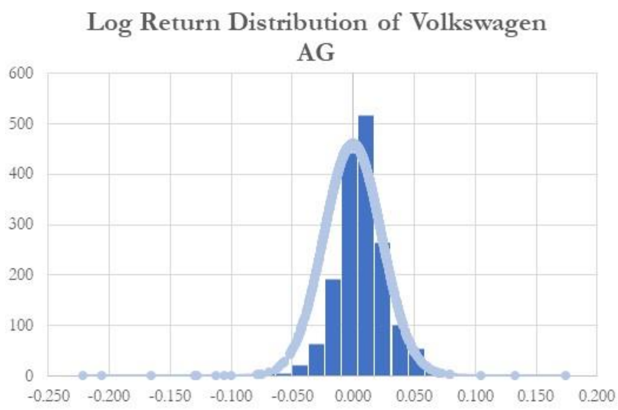

Figure 4.115. Log return distribution of Volkswagen AG over 2015-2021.

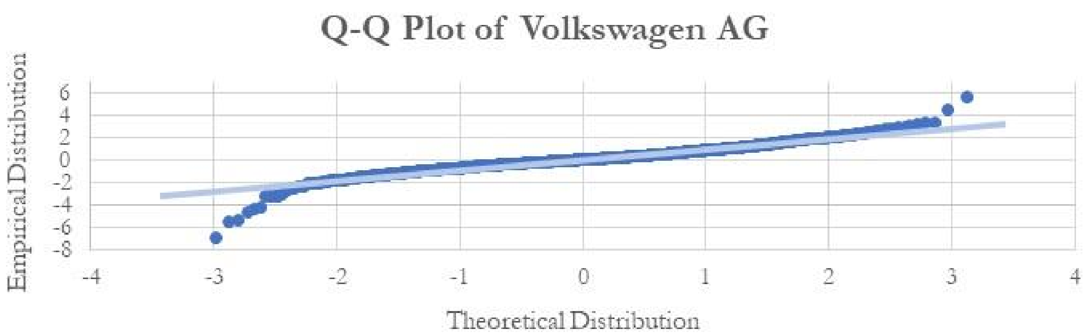

Figure 4.116. Q-Q Plot of Volkswagen AG over 2015-2021.

Volkswagen AG log return descriptive statistics over 2015-2019	
Mean	0.00%
Standard Error	0.06%
Median	-0.04%
Standard Deviation	2.13%
Sample Variance	0.0005
Kurtosis	17.1031
Skewness	-1.6962
Range	0.2896
Minimum	-22.09%
Maximum	6.88%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.93. Volkswagen AG descriptive statistics over 2015-2019.

Volkswagen AG log return descriptive statistics over 2020-2021	
Mean	0.01%
Standard Error	0.13%
Median	0.00%
Standard Deviation	2.88%
Sample Variance	0.0008
Kurtosis	7.4812
Skewness	-0.0301
Range	0.3394
Minimum	-16.50%
Maximum	17.43%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.94. Volkswagen AG descriptive statistics over 2020-2021.

4.2.30 Walmart Inc.

As shown in Figure 4.117, in 2015 the company's stock price experienced a steep decline, due to non-favourable sales forecast. While the title regained ground in the following periods, on 20th February 2020 the biggest one day drop since 1988 occurred, as displayed in Figure 4.118. The Covid-19 crisis did not hit severely the returns of the firm.

Figure 4.117. Stock price performance of Walmart Inc.[31]

Figure 4.118. Log return performance of Walmart Inc.

Figure 4.119 displays the log return distribution of Walmart Inc.'s stock returns over 2015-2021, while the Q-Q plot between observed and theoretical quantiles over 2015-2021 can be found in Figure 4.120.

[31] To make the data comparable, S&P 500 stock index price has been normalized using WMT's stock price on 02/01/2015 as base year

Table 4.95 (2015-2021), Table 4.96 (2015-2019), and Table 4.97 (2020-2021) present the descriptive statistics of Walmart Inc. The average daily stock return over 2020-2021 does not change from 2015-2019, but the volatility slightly increased. The kurtosis decreased, while the skewness turned positive.

Walmart Inc. log return descriptive statistics over 2015-2021	
Mean	0.04%
Standard Error	0.03%
Median	0.05%
Standard Deviation	1.36%
Sample Variance	0.0002
Kurtosis	15.1568
Skewness	0.2740
Range	0.2181
Minimum	-10.74%
Maximum	11.07%
Count	1698
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.95. Walmart Inc. descriptive statistics over 2015-2021.

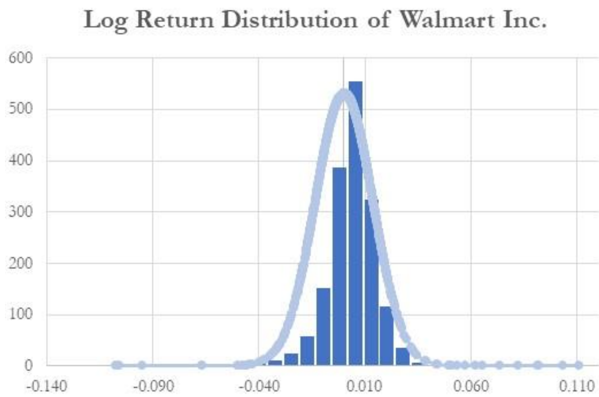

Figure 4.119. Log return distribution of Walmart Inc. over 2015-2021.

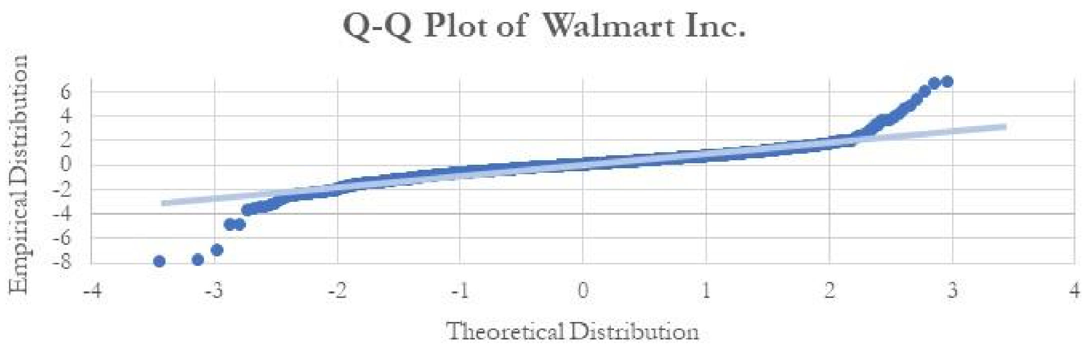

Figure 4.120. Q-Q Plot of Walmart Inc. over 2015-2021.

Walmart Inc. log return descriptive statistics over 2015-2019	
Mean	0.04%
Standard Error	0.04%
Median	0.07%
Standard Deviation	1.24%
Sample Variance	0.0002
Kurtosis	17.2178
Skewness	-0.1439
Range	0.2108
Minimum	-10.74%
Maximum	10.34%
Count	1213
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.96. Walmart Inc. descriptive statistics over 2015-2019.

Walmart Inc. log return descriptive statistics over 2020-2021	
Mean	0.04%
Standard Error	0.07%
Median	-0.01%
Standard Deviation	1.62%
Sample Variance	0.0003
Kurtosis	11.2839
Skewness	0.7318
Range	0.2058
Minimum	-9.51%
Maximum	11.07%
Count	485
Shapiro–Wilk p_value	0.000
Kolmogorov–Smirnov test p_value	0.000

Table 4.97. Walmart Inc. descriptive statistics over 2020-2021.

4.3 Correlation between returns

Once we have analysed the data, it is possible to compute the correlation between stocks' logarithmic returns. Pearson's correlation coefficient measures the strength of the relationship between two variables x and y:

$$\rho_{xy} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}}$$

Depending on the relationship between the daily log-returns, $-1 \leq \rho_{xy} \leq 1$. While $\rho_{xy} = -1$ indicates perfect negative correlation, $\rho_{xy} = 1$ shows perfect positive correlation, while $\rho_{xy} = 0$ suggests absence of relationship.

The correlation matrixes of the daily log-returns computed over the time period 2015-2021, 2015-2019 and 2020-2021 can respectively be found in Appendix 1, Appendix 2 and Appendix 3. Note that it is a symmetric matrix, having a value equal to 1 on the diagonal, given that each stock's return is perfectly correlated with itself. By means of conditional formatting, it is possible to visualize the intensity of the correlation, the higher the correlation, the greener the cell, while the lower the relationship the redder the cell.

The portfolio can be considered well diversified, given the low correlation the majority of the stocks show, turning even negative during Covid-19 crisis. In general, the relative intensity of the correlation among stocks remains congruent during all the three time periods. While the stock returns of the three Big Tech firms, Apple, Amazon and Google, are always highly correlated, even more during the pandemic, they show weak or almost no relationships with the other sectors. Note that in 2020-2021 Walmart Inc. is negatively correlated with Airbus SE, easyJet plc and InterContinental Hotels Group PLC.

Eventually, once the correlation matrix has been computed, the variance-covariance matrix can then be built. In fact, as outlined in section 3.1.1, given two stocks i and j, the correlation between two assets impacts the variance of the portfolio, since $Cov(R_i, R_j) = \Sigma_{i,j}\rho_{ij}\sigma_i\sigma_j$. In Appendix 4, Appendix 5 and Appendix 6 the annual variance-covariance matrixes for 2015-20, 2015-2019 and 2020-2021 are reported. Note that the daily variance has been multiplied for the average trading days per year for each time period considered.

5 Empirical Analysis

This chapter discusses the results of the empirical studies conducted on Excel, with the aim of creating a portfolio using the stocks presented in chapter 4. Section 5.1 applies Markowitz's method both in the case with and without short selling. In section 5.2 the B&L approach is applied only when short sales are allowed. To account for the impact of Covid-19, different sets of portfolios are created, using as inputs two distinct subsets of data, over 2015-2019 and 2020-2021.

5.1 Portfolio construction using the Markowitz model

Using the vector of expected returns in section 4.2 and the annualized variance-covariance matrixes in section 4.3 as inputs, the Markowitz's mean-variance optimization problem can be solved. With reference to section 3.1.2, when the Markowitz's mean-variance optimization problem is approached as a minimization problem of variance:

Without short selling:

$$\min_{w} \ \sigma_\pi^2$$
$$\text{subject to } E(R_\pi) = \tilde{R}$$
$$\sum_{i=1}^{n} w_i = 1$$
$$w_i \geq 0, \qquad \forall i = 1, \dots, n$$

With short selling:

$$\min_{w} \ \sigma_\pi^2$$
$$\text{subject to } E(R_\pi) = \tilde{R}$$
$$\sum_{i=1}^{n} w_i = 1$$

When the Markowitz's mean-variance optimization problem is approached as a maximization problem of expected returns:

Without short selling:

$$\max_{\mathrm{w}} \ E(R_\pi)$$
$$\text{subject to } \sigma_\pi^2 = \widetilde{\sigma^2}$$
$$\sum_{i=1}^{n} w_i = 1$$
$$w_i \geq 0, \forall i = 1, \dots, n$$

With short selling:

$$\max_{\mathrm{w}} \ E(R_\pi)$$
$$\text{subject to } \sigma_\pi^2 = \widetilde{\sigma^2}$$
$$\sum_{i=1}^{n} w_i = 1$$

The Excel Solver is a useful add-in instrument that allows to solve the above systems. By working on the decision variables, this tool complies with the constraints and returns the optimal result in the objective cell. As starting point, an equally weighted portfolio is computed, with its mean, variance, and standard deviation, as shown in Table 5.1 and Table 5.2.

Stock	Weight	Stock	Weight
AAPL	3.33%	MC.PA	3.33%
AIR.PA	3.33%	MSFT	3.33%
AMZN	3.33%	OR.PA	3.33%
APD	3.33%	PFE	3.33%
AZN.L	3.33%	PG	3.33%
BAS.DE	3.33%	REP.MC	3.33%
BP.L	3.33%	SAN.MC	3.33%
CPR.MI	3.33%	SAN.PA	3.33%
ENEL.MI	3.33%	SAP.DE	3.33%
EZJ.L	3.33%	SOLB.BR	3.33%
G.MI	3.33%	ULVR.L	3.33%
GM	3.33%	VOW3.DE	3.33%
GOOG	3.33%	WMT	3.33%
IHG.L	3.33%	Sum Weights	100.00%
IP.MI	3.33%	Expected Return	12.73%
LHA.DE	3.33%	Annual Variance	0.0188
LMT	3.33%	Annual Standard Deviation	13.69%

Table 5.1 Equally weighted portfolio composition over 2015-2019

Stock	Weight	Stock	Weight
AAPL	3.33%	MC.PA	3.33%
AIR.PA	3.33%	MSFT	3.33%
AMZN	3.33%	OR.PA	3.33%
APD	3.33%	PFE	3.33%
AZN.L	3.33%	PG	3.33%
BAS.DE	3.33%	REP.MC	3.33%
BP.L	3.33%	SAN.MC	3.33%
CPR.MI	3.33%	SAN.PA	3.33%
ENEL.MI	3.33%	SAP.DE	3.33%
EZJ.L	3.33%	SOLB.BR	3.33%
G.MI	3.33%	ULVR.L	3.33%
GM	3.33%	VOW3.DE	3.33%
GOOG	3.33%	WMT	3.33%
IHG.L	3.33%	Sum Weights	100.00%
IP.MI	3.33%	Expected Return	9.15%
LHA.DE	3.33%	Annual Variance	0.0084
LMT	3.33%	Annual Standard Deviation	23.31%

Table 5.2. Equally weighted portfolio composition over 2020-2021

Then, the efficient frontiers over the time horizons 2015-2019 and 2020-2021, with and without short selling, have been computed, by means of the Excel Solver. The limit constraints concern the expected return of the portfolio – which was set equal to an arbitrary target value -

and the assets full deployment – the sum of the weights should be equal to 1. Moreover, in the case of no short sales, an additional constraint on the non-negativity of the weights has been added.

For the purpose of comparison, for each time horizon 10 portfolios have been computed, by solving the variance minimization problem, with the same arbitrary values of target of expected returns. In the case of 2020-2021, 3 additional portfolios were added.

5.1.1 Case without short selling

Markowitz's efficient frontiers that have been built using the 2015-2019 and 2020-2021 logarithmic returns when short selling is not allowed are respectively shown in Figure 5.1 and Figure 5.2. The MVP obtained using 2015-2019 data presents $E(R_{MVP}) = 11.23\%$ with $\sigma_{MVP} = 10.58\%$, while the portfolio with the highest return is obtained when 100% of funds are allocated to AMZN, with $E(R_p) = 35.79\%$ and $\sigma_p = 28.84\%$. On the other hand, during 2020-2021, $E(R_{MVP}) = 9.63\%$ with $\sigma_{MVP} = 15.94\%$, while the maximum return is achieved by investing 100% of funds in AAPL, with $E(R_p) = 45.44\%$ and $\sigma_p = 37.09\%$.

Moreover, as Table 5.3 clearly shows, higher volatility characterizes 2020-2021 stock performance: it follows that in 2020-2021 the same expected returns of 2015-2019 are achieved with greater volatility, although, on the other hand, higher returns are possible.

As already discussed theoretically in section 3.1.7, one of the main limits of Markowitz's allocation model is the reliance on the historical performance of the stocks, without including the investor's personal views on the future. Therefore, to realize the same target return, the portfolio's stock weights change substantially depending on the time horizon considered. Referring to Figure 5.3 as an example, to achieve the same target return of 22% (Portfolio 6) without short selling, LMT (21.86%), AMAZN (19.86%), CPR.MI (17.94%), and PG (8.40%) are the four main stocks the investor should have invested his funds according to the 2015-2019 data. Relying on 2020-2021 returns, the weights of the four main stocks would have changed, resulting in WMT (15.81%), IP.MI (15.49%), PFE (14.07%) and AMAZN (13.31%). To achieve the optimal allocation, the investor should periodically review his portfolio from historical segment to historical segment, incurring high transaction costs that would erode profits.

Another critical issue of Markowitz's allocation model is its fragility with respect to diversification. As shown in Figure 5.4 and Figure 5.5, when the target expected returns increase, the model tends to concentrate on fewer stocks namely the ones that present higher expected returns, lower variance, and negative correlation. The high value of the variability indexes computed for each stock could be found in Appendix 7 and Appendix 8, together with the detailed portfolio weights.

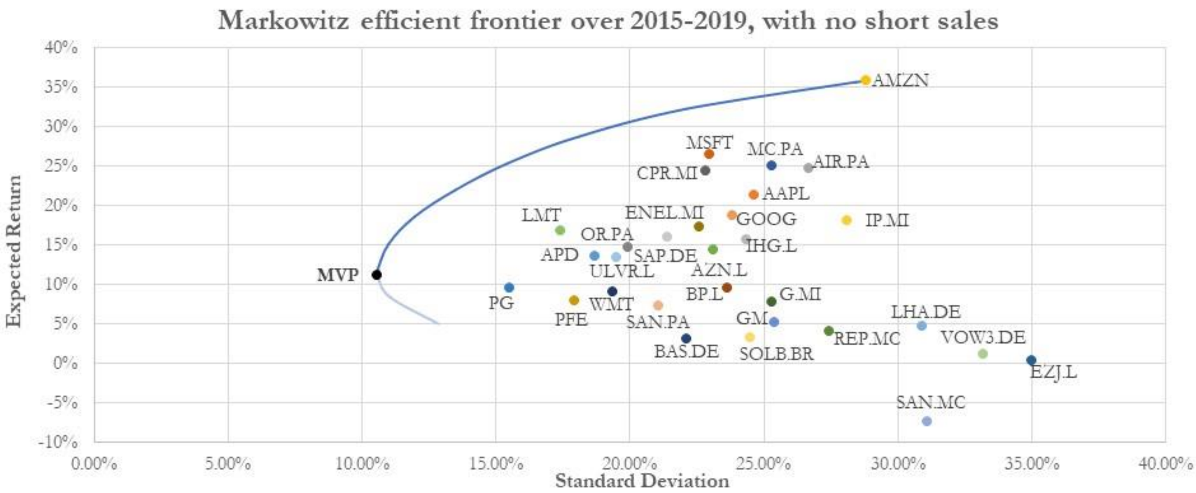

Figure 5.1. Markowitz efficient frontier over 2015-2019, with no short sales.

Portfolio/ Statistics	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6	Portfolio 7
E[R]	5.00%	8.40%	11.80%	15.20%	18.60%	22.00%	25.40%
Markowitz portfolio allocation 2015-2019, without short selling							
σ	12.84%	11.02%	10.59%	10.99%	11.94%	13.45%	15.42%
Markowitz portfolio allocation 2020-2021, without short selling							
σ	16.15%	15.95%	15.99%	16.23%	16.66%	17.26%	18.04%

Portfolio/ Statistics	Portfolio 8	Portfolio 9	Portfolio 10	Portfolio 11	Portfolio 12	Portfolio 13
E[R]	28.80%	32.20%	35.60%	39.00%	42.40%	44.00%
Markowitz portfolio allocation 2015-2019, without short selling						
σ	18.06%	22.10%	28.43%	N/A	N/A	N/A
Markowitz portfolio allocation 2020-2021, without short selling						
σ	19.03%	20.27%	21.81%	23.97%	26.89%	29.02%

Table 5.3. Comparison between Markowitz portfolio allocations without short sales - 2015-2019 vs 2020-2021.

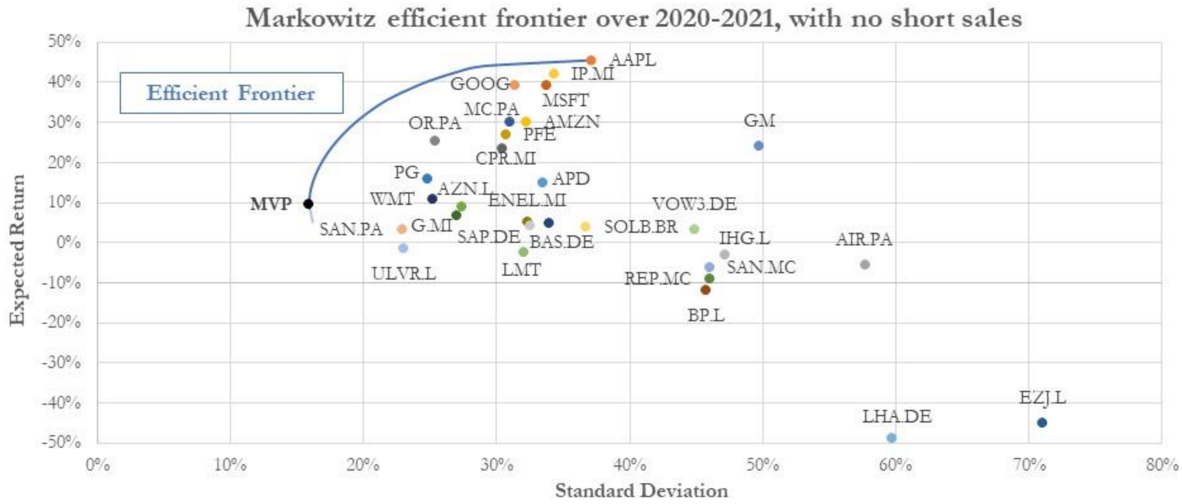

Figure 5.2. Markowitz efficient frontier over 2020-2021, with no short sales.

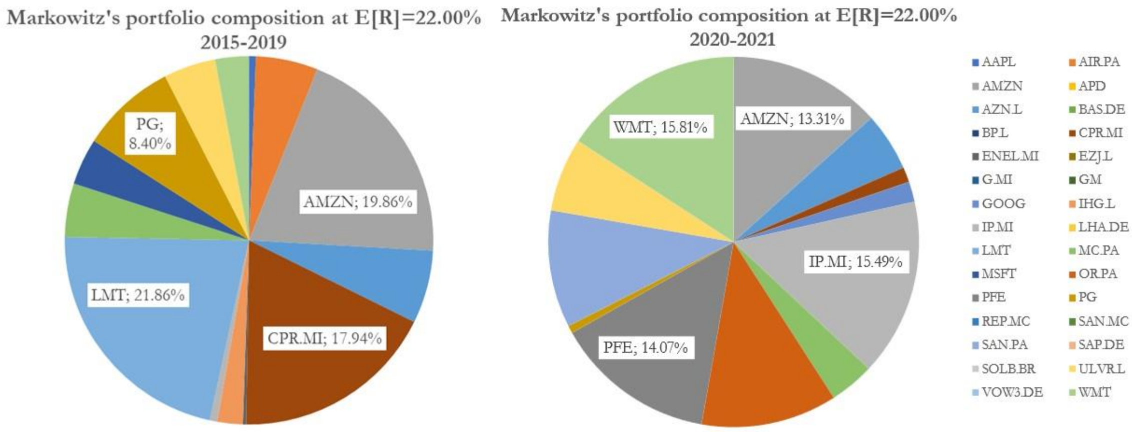

Figure 5.3. Comparison Markowitz portfolio allocation no. 6 without short sales - 2015-2019 vs 2020-2021.

Figure 5.4. Markowitz portfolio weights evolution without short selling over 2015-2019.

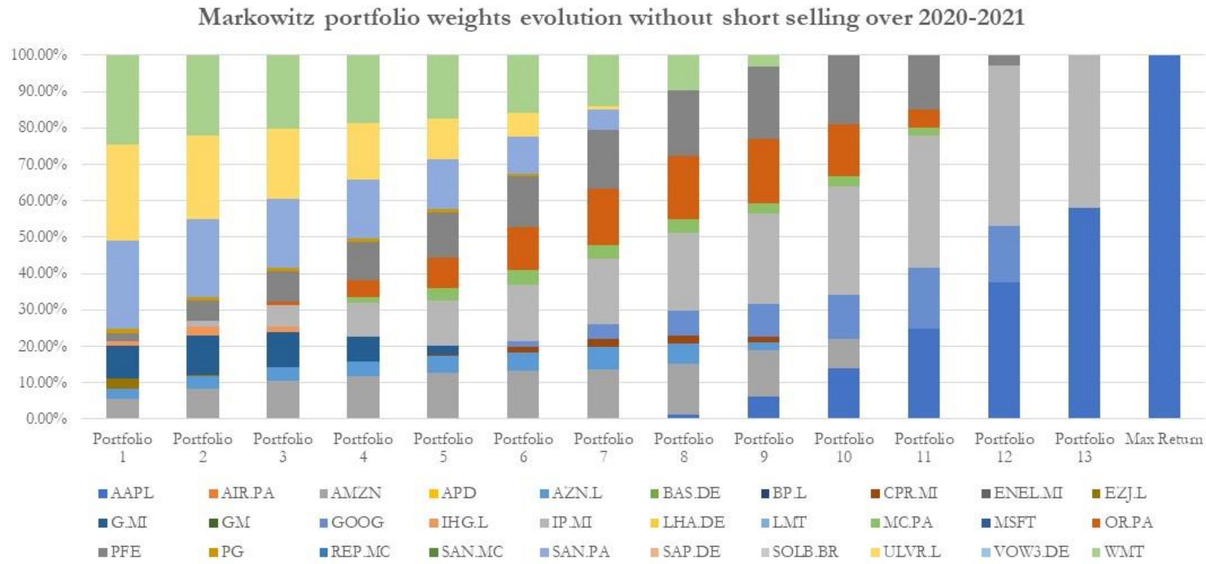

Figure 5.5. Markowitz portfolio weights evolution without short selling over 2020-2021.

5.1.2 Case with short selling

Markowitz's efficient frontiers that have been built using the 2015-2019 and 2020-2021 logarithmic returns when short selling is allowed are respectively shown in Figure 5.6 and Figure 5.7. While using 2015-2019 data, $E(R_{MVP}) = 10.36\%$ with $\sigma_{MVP} = 10.39\%$, during 2020-2021, $E(R_{MVP}) = 9.73\%$ with $\sigma_{MVP} = 14.44\%$.

As Table 5.4 exhibits, higher volatility characterizes 2020-2021 stock performance. Consequently, in 2020-2021 the same expected returns of 2015-2019 are achieved with greater volatility. The detailed portfolio weights can be found in Appendix 9 and Appendix 10.

As in the previous case when short selling is not allowed, to achieve higher target expected return, the portfolio allocation concentrates on stocks with higher expected returns, lower variance, and negative correlation. However, the portfolios built when short selling is allowed comprise more stocks, as Figure 5.8 shows. This diversification effect is even more pronounced over 2020-2021, displayed in Figure 5.9.

Portfolio/ Statistics	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6	Portfolio 7
E[R]	5.00%	8.40%	11.80%	15.20%	18.60%	22.00%	25.40%
Markowitz portfolio allocation 2015-2019, with short selling							
σ	10.62%	10.33%	10.31%	10.56%	11.05%	11.75%	12.64%
Markowitz portfolio allocation 2020-2021, with short selling							
σ	14.53%	14.45%	14.46%	14.56%	14.77%	15.07%	15.45%

Portfolio/ Statistics	Portfolio 8	Portfolio 9	Portfolio 10	Portfolio 11	Portfolio 12	Portfolio 13
E[R]	28.80%	32.20%	35.60%	39.00%	42.40%	44.00%
Markowitz portfolio allocation 2015-2019, with short selling						
σ	13.66%	14.81%	16.04%	N/A	N/A	N/A
Markowitz portfolio allocation 2020-2021, with short selling						
σ	15.91%	16.45%	17.05%	17.71%	18.43%	18.78%

Table 5.4. Comparison Markowitz portfolio allocations with short sales - 2015-2019 vs 2020-2021.

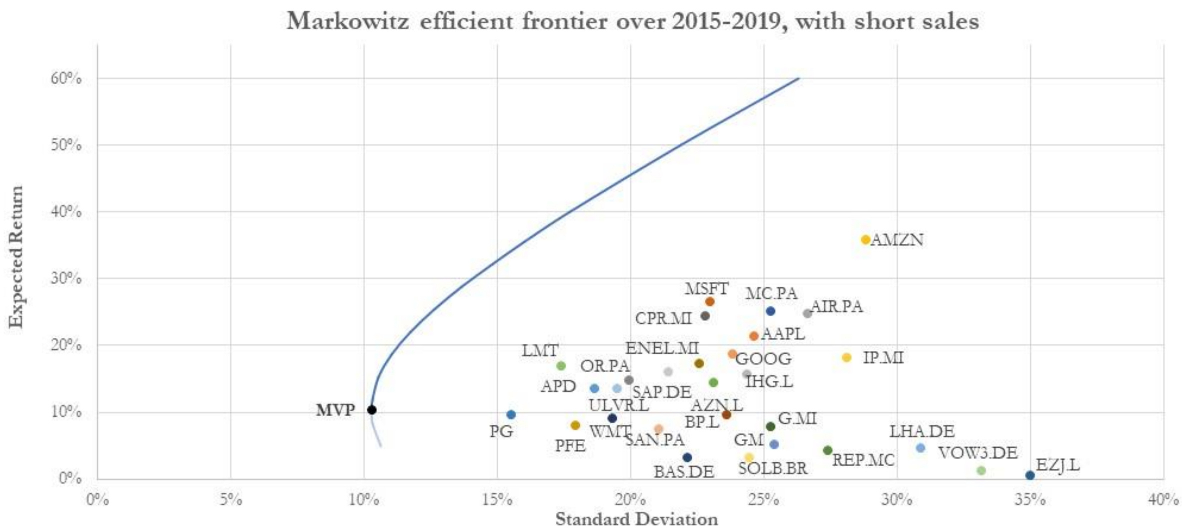

Figure 5.6. Markowitz efficient frontier over 2015-2019, with short sales.

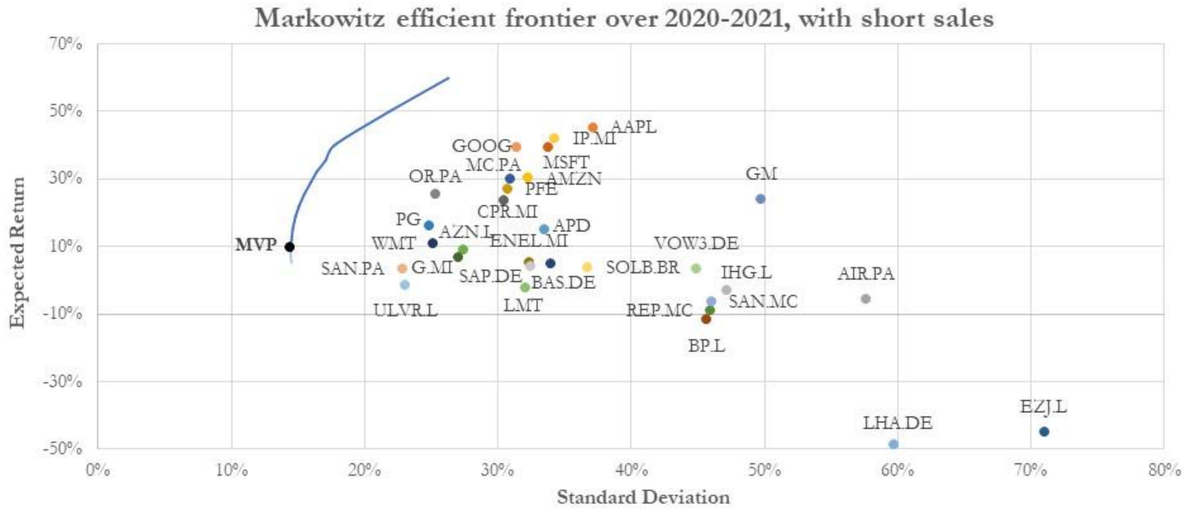

Figure 5.7. Markowitz efficient frontier over 2020-2021, with short sales.

Figure 5.8. Markowitz's portfolio weights evolution with short selling over 2015-2019.

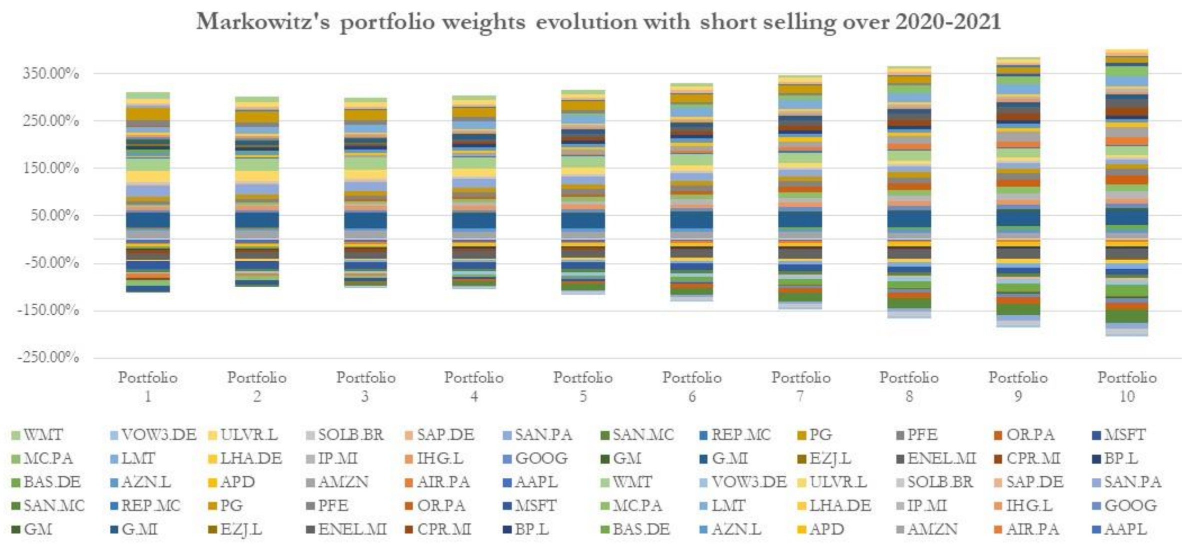

Figure 5.9. Markowitz's portfolio weights evolution with short selling over 2020-2021.

5.1.3 Comparison case with and without short selling

Figure 5.10 compares the efficient frontiers created using the historical data from 2015 to 2019 with and without short selling, while Figure 5.11 shows the same but using 2020-2021 returns. In both cases, the efficient frontiers plotted when short sales are allowed dominate the ones obtained when portfolio weights can only be positive. This constraint, in fact, leads to portfolios that for the same risk present lower expected return than those built with short selling. However, the intensity of the divergence in 2020-2021 is significantly smaller than in the case using 2015-2019 historical returns.

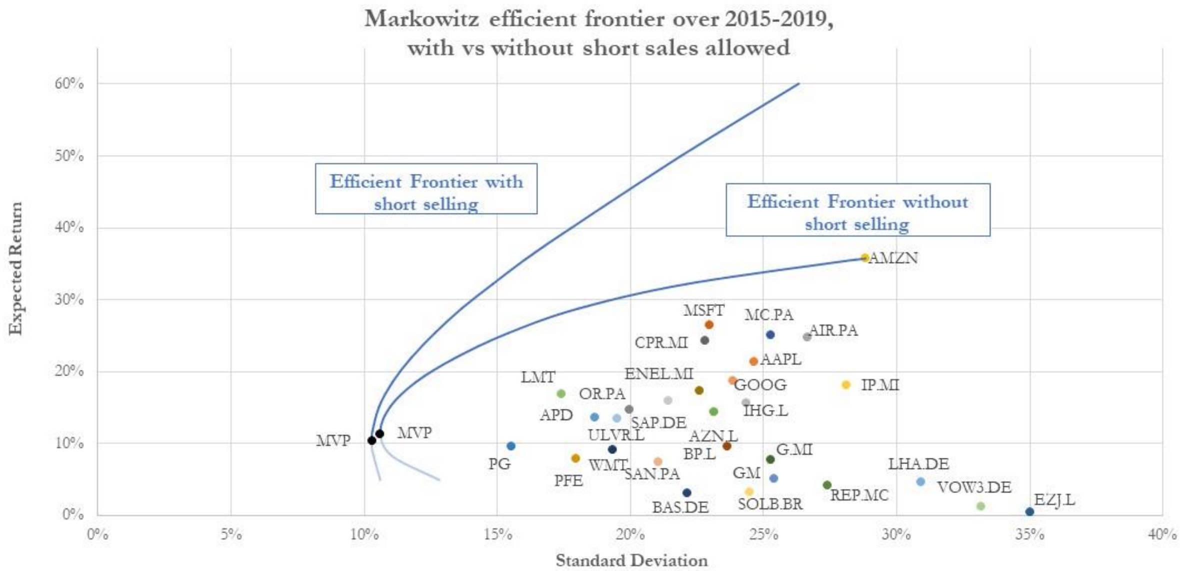

Figure 5.10. Markowitz efficient frontier over 2015-2019, with vs without short sales.

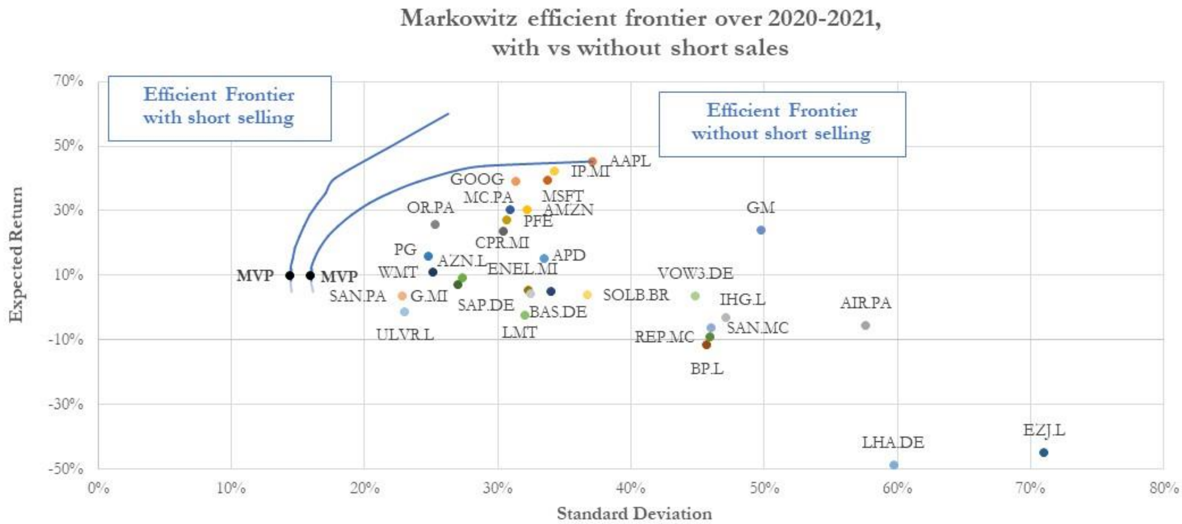

Figure 5.11. Markowitz efficient frontier over 2020-2021, with vs without short sales.

5.2 Portfolio construction using the Black-Litterman model - simplified formulation

As thoroughly theoretically presented in section 3.3., the Black-Litterman approach attempts to overcome the limitations of Markowitz's model. The following part of this section provides an empirical implementation of the Black-Litterman approach over the two selected time horizons 2015-2019 and 2020-2021. To simplify and compare with the results computed using the historical returns through the Markowitz model, the empirical work described in this section have been carried out using historical returns instead of excess returns, as it is instead done in the original Black-Litterman model.

5.2.1 Reverse Optimization

Since the B&L model takes the CAPM equilibrium returns as neutral reference points for the expected returns, the starting point of the empirical study should be the calculation of the implied returns by means of the *reverse optimization method*:

$$\theta = \lambda \Sigma \Omega_{mkt}$$

Where:

- θ is the vector of equilibrium returns for each asset
- λ is the risk aversion parameter
- Σ is the covariance matrix of the returns for each asset
- Ω_{mkt} is the vector of the weights of each asset computed based on the market capitalization of all the assets in the portfolio. While $\Omega_{mkt_{15/19}}$ can be found in Table 5.5, while $\Omega_{mkt_{20/21}}$ in Table 5.6.

Stock	Price at 30/12/2019	No. Shares at 14/04/2022	Market Cap in local currency at 30/12/2019	Market Cap in Euro at 30/12/2019	Ω_{mkt}
			2015-2019		
AAPL	71.72	16,320,000,000	$1,170,501,293,760	1,044,789,454,810 €	17.92%
AIR.PA	125.88	787,900,000	99,181,078,127 €	99,181,078,127 €	1.70%
AMZN	1,846.89	508,840,000	$939,771,515,233	838,840,054,497 €	14.39%
APD	224.33	221,720,000	$49,738,194,174	44,396,312,120 €	0.76%
AZN.L	71.24	1,550,000,000	£110,429,704,585	129,269,012,187 €	2.22%
BAS.DE	56.02	918,480,000	51,448,789,461 €	51,448,789,461 €	0.88%
BP.L	4.22	19,600,000,000	£82,668,293,568	96,771,504,451 €	1.66%
CPR.MI	8.03	1,130,000,000	9,075,148,650 €	9,075,148,650 €	0.16%
ENEL.MI	6.18	10,160,000,000	62,776,841,680 €	62,776,841,680 €	1.08%
EZJ.L	13.70	758,000,000	£10,381,605,748	12,152,707,689 €	0.21%
G.MI	16.24	1,570,000,000	25,504,471,020 €	25,504,471,020 €	0.44%
GM	36.00	1,450,000,000	$52,201,194,800	46,594,786,478 €	0.80%
GOOG	1,336.14	573,790,000	$766,663,779,207	684,324,089,320 €	11.74%
IHG.L	50.74	184,020,000	£9,336,899,848	10,929,774,962 €	0.19%
IP.MI	27.83	105,940,000	2,948,179,470 €	2,948,179,470 €	0.05%
LHA.DE	16.41	1,200,000,000	19,692,000,000 €	19,692,000,000 €	0.34%
LMT	370.86	266,530,000	$98,845,930,152	88,229,877,253 €	1.51%
MC.PA	400.67	502,750,000	201,435,146,222 €	201,435,146,222 €	3.46%
MSFT	154.64	7,500,000,000	$1,159,813,725,000	1,035,249,730,935 €	17.76%
OR.PA	252.32	535,410,000	135,095,072,032 €	135,095,072,032 €	2.32%
PFE	33.87	5,650,000,000	$191,391,552,150	170,836,099,449 €	2.93%
PG	117.85	2,400,000,000	$282,847,944,000	252,470,074,814 €	4.33%
REP.MC	12.18	1,470,000,000	17,909,393,670 €	17,909,393,670 €	0.31%
SAN.MC	3.33	17,050,000,000	56,849,303,500 €	56,849,303,500 €	0.98%
SAN.PA	83.39	1,260,000,000	105,071,601,600 €	105,071,601,600 €	1.80%
SAP.DE	114.92	1,180,000,000	135,605,292,020 €	135,605,292,020 €	2.33%
SOLB.BR	93.76	103,220,000	9,678,378,399 €	9,678,378,399 €	0.17%
ULVR.L	40.81	2,570,000,000	£104,882,754,111	122,775,751,963 €	2.11%
VOW3.DE	166.05	206,200,000	34,239,334,524 €	34,239,334,524 €	0.59%
WMT	115.60	2,770,000,000	$320,215,312,920	285,824,188,312 €	4.90%
Total				**5,829,963,449,617 €**	**100%**

Table 5.5. Weights of each asset based on market cap over 2015-2019.

2020-2021					
Stock	Price at 30/12/2021	No. Shares at 14/04/2022	Market Capitalization at 30/12/2021	Market Cap in Euro at 30/12/2021	Ω_{mkt}
AAPL	177.97	16,320,000,000	$2,904,523,456,320	2,565,275,116,622 €	25.03%
AIR.PA	112.68	787,900,000	88,780,572,000 €	88,780,572,000 €	0.87%
AMZN	3,372.89	508,840,000	$1,716,261,293,154	1,515,801,974,114 €	14.79%
APD	302.39	221,720,000	$67,045,914,126	59,214,951,356 €	0.58%
AZN.L	85.33	1,550,000,000	£1,322,631,045	1,538,484,432 €	0.02%
BAS.DE	61.78	918,480,000	56,743,693,482 €	56,743,693,482 €	0.55%
BP.L	3.33	19,600,000,000	£652,680,000	759,197,376 €	0.01%
CPR.MI	12.86	1,130,000,000	14,526,150,000 €	14,526,150,000 €	0.14%
ENEL.MI	6.85	10,160,000,000	69,617,539,200 €	69,617,539,200 €	0.68%
EZJ.L	5.57	758,000,000	£42,220,600	49,111,002 €	0.00%
G.MI	18.63	1,570,000,000	29,249,098,430 €	29,249,098,430 €	0.29%
GM	58.13	1,450,000,000	$84,288,501,450	74,443,604,481 €	0.73%
GOOG	2,920.05	573,790,000	$1,675,495,517,616	1,479,797,641,158 €	14.44%
IHG.L	47.68	184,020,000	£87,740,736	102,060,024 €	0.00%
IP.MI	64.45	105,940,000	6,827,832,682 €	6,827,832,682 €	0.07%
LHA.DE	6.18	1,200,000,000	7,416,000,000 €	7,416,000,000 €	0.07%
LMT	353.58	266,530,000	$94,239,673,935	83,232,480,019 €	0.81%
MC.PA	730.00	502,750,000	367,007,500,000 €	367,007,500,000 €	3.58%
MSFT	339.32	7,500,000,000	$2,544,900,052,500	2,247,655,726,368 €	21.93%
OR.PA	419.80	535,410,000	224,765,111,575 €	224,765,111,575 €	2.19%
PFE	57.96	5,650,000,000	$327,470,203,200	289,221,683,466 €	2.82%
PG	161.90	2,400,000,000	$388,550,097,600	343,167,446,200 €	3.35%
REP.MC	10.16	1,470,000,000	14,927,938,200 €	14,927,938,200 €	0.15%
SAN.MC	2.94	17,050,000,000	50,135,525,000 €	50,135,525,000 €	0.49%
SAN.PA	89.13	1,260,000,000	112,303,796,220 €	112,303,796,220 €	1.10%
SAP.DE	124.90	1,180,000,000	147,382,002,360 €	147,382,002,360 €	1.44%
SOLB.BR	101.11	103,220,000	10,436,334,110 €	10,436,334,110 €	0.10%
ULVR.L	39.67	2,570,000,000	£1,019,390,500	1,185,755,030 €	0.01%
VOW3.DE	177.48	206,200,000	36,596,375,175 €	36,596,375,175 €	0.36%
WMT	143.17	2,770,000,000	$396,580,894,460	350,260,245,987 €	3.42%
Total				**10,248,420,946,068.80**	**100.00%**

Table 5.6. Weights of each asset based on market cap over 2020-2021.

As discussed in section 3.3.1, the risk aversion parameter should be calculated as:

$$\lambda = \frac{(E(R) - R_F)}{\sigma^2}$$

Where:

- $E(R)$ is the total return on the market portfolio.
- R_F is the risk-free rate. In this case, R_F is the average return of the seven government bonds considered, as shown in Table 5.7 and Table 5.8
- σ^2 is the variance of the market portfolio

Time Horizon 2015-2019	Belgium 10 Y Gov. Bond	France 10 Y Gov. Bond	Germany 10 Y Gov. Bond	Italian BTP 10 Y	Spain 10 Y Gov. Bond	UK Gilt 10 Y	US Treasury Yield 10 Y	Average R_F
Annual Yield	0.60%	0.61%	0.26%	1.96%	1.36%	1.31%	2.27%	1.20%
Annual Variance	1.1E-05	1.2E-05	1.1E-05	3.5E-05	2.1E-05	1.5E-05	1.9E-05	1.8E-05
Annual Std. Dev	0.33%	0.35%	0.33%	0.60%	0.46%	0.38%	0.44%	0.41%

Table 5.7. R_F over 2015-2019.

Time Horizon 2020-2021	Belgium 10 Y Gov. Bond	France 10 Y Gov. Bond	Germany 10 Y Gov. Bond	Italian BTP 10 Y	Spain 10 Y Gov. Bond	UK Gilt 10 Y	US Treasury Yield 10 Y	Average R_F
Annual Yield	0.00%	0.00%	0.00%	0.96%	0.38%	0.53%	1.16%	0.43%
Annual Variance	3.4E-06	2.9E-06	2.0E-06	1.3E-05	4.3E-06	8.0E-06	1.6E-05	7.1E-06
Annual Std. Dev	0.18%	0.17%	0.14%	0.37%	0.21%	0.28%	0.40%	0.25%

Table 5.8. R_F over 2020-2021.

In first instance, the $\bar{R}$ and σ of the market portfolio have been computed as follows. Since the 30 selected stocks belong to 7 different market indexes, the annual expected return and standard deviation of each index has been multiplied for the % Market Capitalization of the stocks in the portfolio belonging to the index. Then, these results have then been added together to find the $\bar{R}$ and σ of the benchmark, as shown in Table 5.9 and Table 5.10. As it appears evident, the 30 selected stocks are a systematically distorted sample compared to the market portfolio. Therefore, the risk aversion parameter λ_1 that is obtained from these values and shown in Table 5.11 and Table 5.12 would lead to distorted equilibrium returns.

To solve this issue, an artificial market has been created and other two possible market's returns have been calculated. Specifically, λ_2 has been computed by applying the CAPM formula, using the weighted average of equilibrium returns and the betas of each stock. On the other hand, λ_3 has been determined using the weighted average of the stock returns as market benchmark. This latter lambda has eventually been selected as λ to be used as input for the computation of the implied returns.

What strikes the most is the lower risk aversion parameter for 2020-2021 than 2015-2019, while one should have expected the contrary. This difference should once more be attributed to the portfolio composition: with the aim to highlight the impact of the pandemic, most of the selected stocks proved to be resilient during the Covid-19 crisis. Only a minority of shares, e.g., EZJ.L or LHA.DE, suffered from the crisis.

			2015-2019			
Market Index	$\bar{R}$	σ	Market Cap stocks in the portfolio belonging to the index	% Market Cap stocks in the portfolio belonging to the index	Contribution to $\bar{R}$ of the Benchmark	Contribution to σ of the Benchmark
BEL20	3.79%	15.21%	9,678,378,399 €	0.17%	0.01%	0.03%
CAC 40	6.83%	17.03%	540,782,897,981 €	9.28%	0.63%	1.58%
DAX 30	6.10%	17.58%	240,985,416,005 €	4.13%	0.25%	0.73%
FTSE 350	3.50%	13.42%	371,898,751,252 €	6.38%	0.22%	0.86%
FTSE MIB	4.12%	21.77%	100,304,640,820 €	1.72%	0.07%	0.37%
IBEX 35	-1.48%	18.38%	74,758,697,170 €	1.28%	-0.02%	0.24%
S&P 500	8.96%	13.25%	4,491,554,667,989 €	77.04%	6.90%	10.21%
Total			**5,829,963,449,617 €**	**100.00%**		
				Total Benchmark	**8.07%**	**14.01%**

Table 5.9. $\bar{R}$ and σ of the benchmark over 2015-2019.

			2020-2021			
Market Index	$\bar{R}$	σ	Market Cap stocks in the portfolio belonging to the index	% Market Cap stocks in the portfolio belonging to the index	Contribution to $\bar{R}$ of the Benchmark	Contribution to σ of the Benchmark
BEL20	4.20%	25.96%	10,436,334,110 €	0.10%	0.00%	0.03%
CAC 40	9.08%	25.50%	792,856,979,795 €	7.74%	0.70%	1.97%
DAX 30	9.07%	25.71%	248,138,071,017 €	2.42%	0.22%	0.62%
FTSE 350	-0.39%	22.29%	3,634,607,863 €	0.04%	0.00%	0.01%
FTSE MIB	7.57%	28.45%	120,220,620,312 €	1.17%	0.09%	0.33%
IBEX 35	-4.91%	27.34%	65,063,463,200 €	0.63%	-0.03%	0.17%
S&P 500	19.72%	25.37%	9,008,070,869,771 €	87.90%	17.33%	22.30%
Total			**10,248,420,946,069 €**	**100.00%**		
				Total Benchmark	**18.32%**	**25.44%**

Table 5.10. $\bar{R}$ and σ of the benchmark over 2020-2021.

	2015-2019		
Statistical Measure	λ_1	λ_2	$\lambda_3 = \lambda$
E[R] of the Benchmark	8.07%	8.98%	20.54%
R_F	1.20%	1.20%	1.20%
σ of the Benchmark	14.01%	15.75%	15.75%
Lambda	**3.50**	**3.14**	**7.80**

Table 5.11. Lambda 2015-2019.

	2020-2021		
Statistical Measure	λ_1	λ_2	$\lambda_3 = \lambda$
E[R] of the Benchmark	18.32%	17.78%	33.75%
R_F	0.43%	0.43%	0.43%
σ of the Benchmark	25.44%	26.52%	26.52%
Lambda	**2.76**	**2.47**	**4.74**

Table 5.12. Lambda 2020-2021.

The implied equilibrium returns should follow a normal distribution $E(R)\sim N(\theta, \tau\Sigma)$, where, according to Black and Litterman (1992) and Lee (2000) , τ was set equal to 0.025. The $\tau\Sigma$ matrix in the absence of views over 2015-2019 and 2020-2021 can respectively be found in Appendix 11 and Appendix 12.

When the implied returns are used as posterior distribution in the absence of views, the variance of returns is computed as $(1+\tau)\Sigma$. Table 5.13 presents the implied equilibrium returns with their volatility.

Time horizon	2015-2019				2020-2021			
Stock	Historical Returns	σ of Historical Returns	Implied Equilibrium Returns	σ of Implied Equilibrium Returns	Historical Returns	σ of Historical Returns	Implied Equilibrium Returns	σ of Implied Equilibrium Returns
AAPL	21.31%	24.64%	23.43%	24.95%	45.44%	37.09%	42.60%	37.55%
AIR.PA	24.69%	26.66%	15.33%	27.00%	-5.54%	57.68%	21.12%	58.39%
AMZN	35.79%	28.84%	27.71%	29.20%	30.13%	32.27%	31.94%	32.67%
APD	13.54%	18.68%	12.40%	18.91%	14.94%	33.49%	26.24%	33.91%
AZN.L	14.42%	23.13%	7.99%	23.42%	9.02%	27.42%	9.61%	27.76%
BAS.DE	3.08%	22.13%	13.99%	22.41%	4.90%	34.00%	15.23%	34.42%
BP.L	9.52%	23.63%	9.77%	23.93%	-11.82%	45.70%	14.73%	46.27%
CPR.MI	24.30%	22.81%	10.16%	23.10%	23.53%	30.42%	15.35%	30.80%
ENEL.MI	17.28%	22.60%	10.52%	22.88%	5.17%	32.36%	18.90%	32.76%
EZJ.L	0.42%	35.01%	9.59%	35.44%	-45.00%	71.00%	18.97%	71.88%
G.MI	7.71%	25.28%	10.92%	25.60%	6.85%	27.00%	13.78%	27.34%
GM	5.11%	25.40%	13.31%	25.71%	23.97%	49.75%	25.89%	50.37%
GOOG	18.74%	23.85%	23.24%	24.14%	39.11%	31.37%	34.20%	31.76%
IHG.L	15.59%	24.36%	9.99%	24.67%	-3.11%	47.15%	18.71%	47.74%
IP.MI	18.15%	28.11%	12.74%	28.46%	42.01%	34.30%	15.28%	34.72%
LHA.DE	4.65%	30.91%	9.28%	31.29%	-48.85%	59.74%	16.79%	60.49%
LMT	16.82%	17.43%	9.81%	17.65%	-2.39%	32.05%	20.12%	32.45%
MC.PA	25.05%	25.28%	16.59%	25.60%	30.01%	30.99%	19.46%	31.37%
MSFT	26.46%	22.97%	24.00%	23.26%	39.31%	33.79%	39.66%	34.21%
OR.PA	14.74%	19.95%	11.29%	20.20%	25.46%	25.37%	15.11%	25.69%
PFE	7.94%	17.94%	10.24%	18.16%	26.86%	30.71%	16.35%	31.09%
PG	9.54%	15.53%	7.80%	15.72%	15.88%	24.85%	19.33%	25.16%
REP.MC	4.13%	27.42%	12.43%	27.76%	-9.11%	45.99%	16.76%	46.56%
SAN.MC	-7.32%	31.09%	15.52%	31.48%	-6.29%	46.05%	18.06%	46.62%
SAN.PA	7.34%	21.07%	10.39%	21.33%	3.33%	22.89%	8.47%	23.18%
SAP.DE	15.99%	21.42%	13.41%	21.69%	4.17%	32.49%	18.98%	32.90%
SOLB.BR	3.20%	24.48%	14.01%	24.78%	3.77%	36.75%	11.54%	37.21%
ULVR.L	13.48%	19.52%	7.35%	19.76%	-1.42%	23.05%	9.03%	23.34%
VOW3.DE	1.17%	33.19%	15.35%	33.61%	3.33%	44.87%	22.94%	45.43%
WMT	9.09%	19.34%	8.49%	19.58%	10.70%	25.18%	17.41%	25.50%

Table 5.13. Historical returns vs implied returns – simplified formulation.

5.2.2 Capital allocation without the investor's views

If the investor has no views on the future stocks' performance, he could build his portfolio relying on the implied equilibrium returns, whose vector can be found in Table 5.13. This neutral reference points together with the annual theoretical variance-covariance computed as $(1+\tau)\Sigma$ were the inputs to the mean variance optimization problem solved by means of the Excel Solver. Eventually, the stock allocation resulting from the B&L approach and the Markowitz method have been compared.

Case without short selling

The B&L efficient frontiers created using the implied returns when there are no views over 2015-2019 and 2020-2021 are respectively shown in Figure 5.12 and Figure 5.13. The MVP obtained using 2015-2019 data presents $E(R_{MVP}) = 9.17\%$ with $\sigma_{MVP} = 10.71\%$, while the portfolio with the highest return is obtained when 100% of funds are allocated to AMZN, with $E(R_p) = 27.71\%$ and $\sigma_p = 29.20\%$. On the other hand, during 2020-2021, $E(R_{MVP}) = 14.35\%$ with $\sigma_{MVP} = 16.13\%$, while the maximum return is achieved by investing 100% of funds in AAPL, with $E(R_p) = 42.60\%$ and $\sigma_p = 37.55\%$. The detailed portfolio weights can be found in Appendix 13 and Appendix 14.

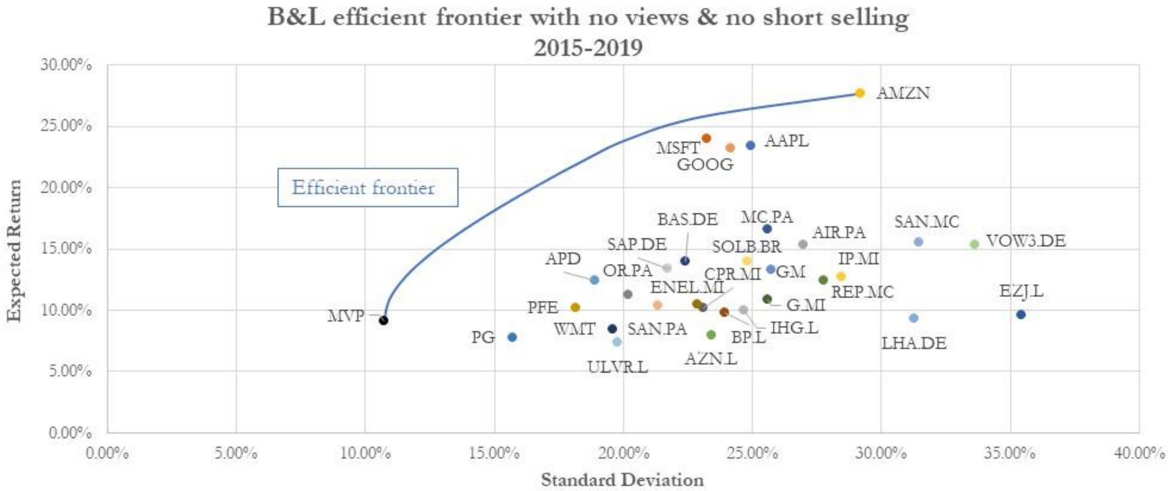

Figure 5.12. B&L efficient frontier with no views & no short selling over 2015-2019.

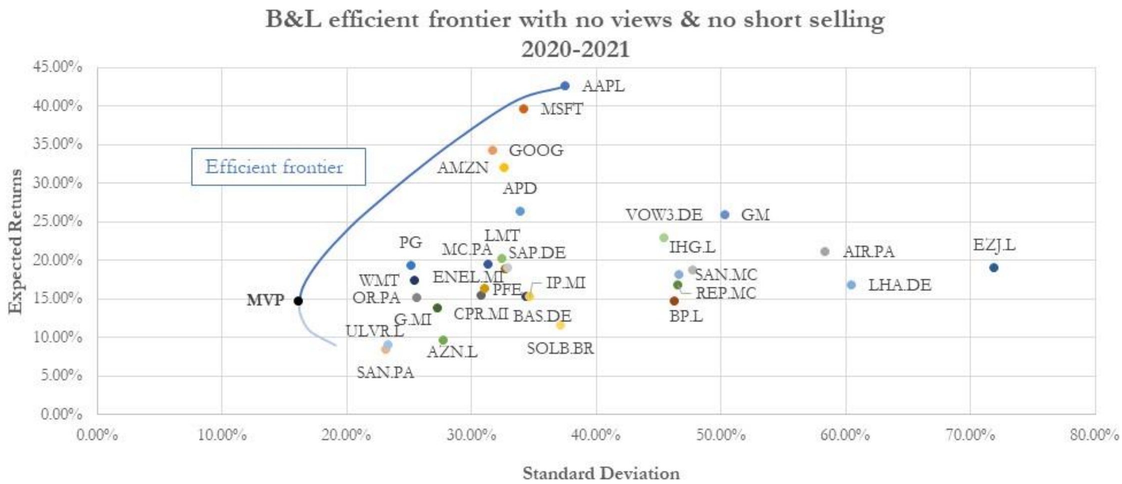

Figure 5.13. B&L efficient frontier with no views & no short selling over 2020-2021.

Case with short selling

The B&L efficient frontiers obtained using the implied returns over 2015-2019 and 2020-2021 when no views have been formulated are respectively shown in Figure 5.14 and Figure 5.15. When the implied returns over 2015-2019 are used , $E(R_{MVP}) = 8.25\%$ with $\sigma_{MVP} = 10.42\%$, while during 2020-2021, $E(R_{MVP}) = 9.88\%$ with $\sigma_{MVP} = 14.62\%$. The detailed portfolio weights can be found in Appendix 15 and Appendix 16.

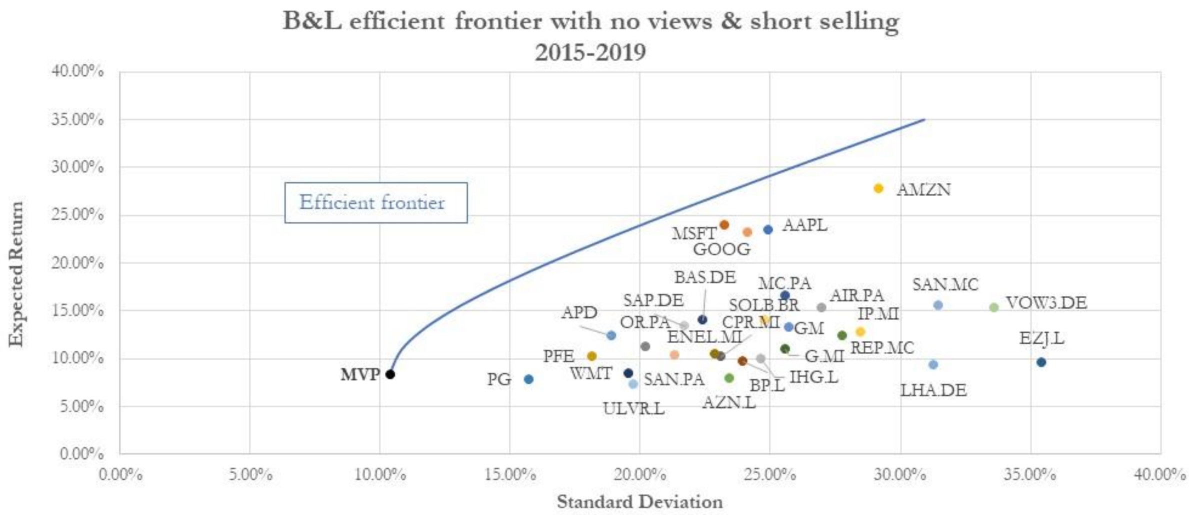

Figure 5.14. B&L efficient frontier with no views & short selling over 2015-2019.

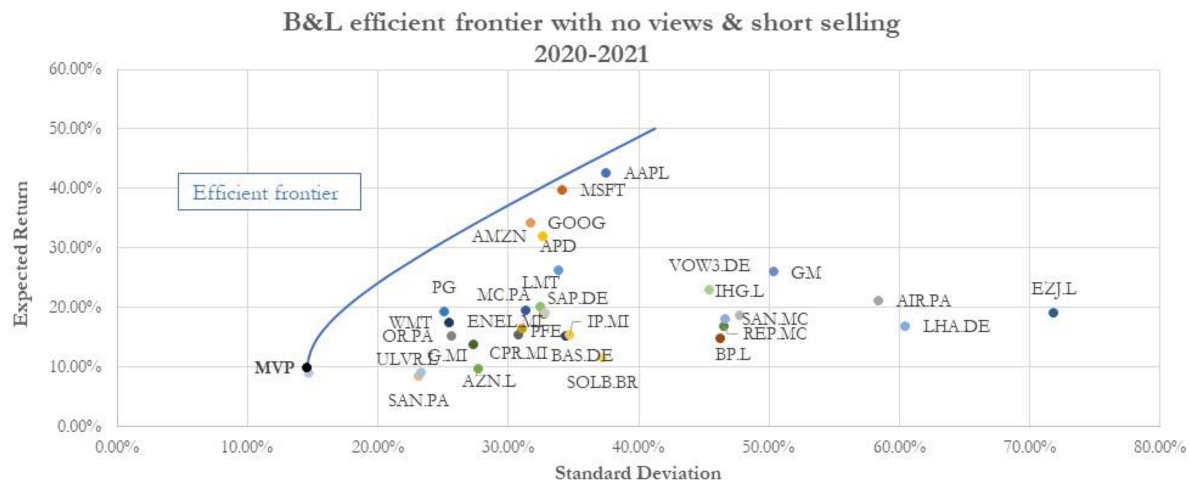

Figure 5.15. B&L efficient frontier with no views & short selling over 2020-2021.

Comparison with Markowitz stock allocation method

To develop their model, Black and Litterman addressed the allocation problem by working with the maximum degrees of freedom. Similarly, the comparison between the B&L method without views and the Markowitz approach has been made by allowing short selling. For this purpose, new Markowitz portfolios' weights have been computed, as shown in Appendix 17 and Appendix 18.

Portfolio concentration

The Herfindahl-Hirschman index is used as metrics of stock concentration which is in turn directly correlated with diversification: the lower the value, the less concentrated and more diversified the portfolio is.

As shown in Table 5.14, 80% of the portfolios obtained through the B&L approach during 2015-2019 are less concentrated, and this percentage increases to more than 90% during 2020-2021. Considering the narrower range of possible implied returns, that inevitably impacts the optimal allocation problem, it can be argued that during both the two time periods analysed, the portfolios obtained through the B&L approach are less concentrated, thus more diversified.

Figure 5.16 and Figure 5.17 visually display the inability of the Markowitz model to reduce the stock concentration in the portfolio.

Portfolio/ Statistics	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6	Portfolio 7	Portfolio 8	Portfolio 9	Portfolio 10	Portfolio 11	Portfolio 12	Portfolio 13
E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**30.84%**	**35.84%**	**40.84%**
Portfolio allocation 2015-2019													
B&L no views Herfindahl – Hirschman index	13.21%	9.95%	7.93%	7.16%	7.65%	9.38%	12.36%	16.60%	22.09%	28.82%	N/A	N/A	N/A
Markowitz Herfindahl – Hirschman index	15.59%	14.66%	14.21%	14.25%	14.78%	15.80%	17.30%	19.28%	21.76%	24.72%	N/A	N/A	N/A
Portfolio allocation 2020-2021													
B&L no views Herfindahl – Hirschman index	41.28%	35.20%	29.81%	25.11%	21.09%	17.76%	15.12%	13.17%	11.91%	11.33%	13.17%	19.92%	31.59%
Markowitz Herfindahl – Hirschman index	38.72%	37.81%	37.07%	36.50%	36.09%	35.85%	35.78%	35.87%	36.13%	36.55%	38.51%	41.66%	46.00%

Table 5.14. Comparison B&L implied equilibrium returns vs Markowitz historical returns portfolio allocation.

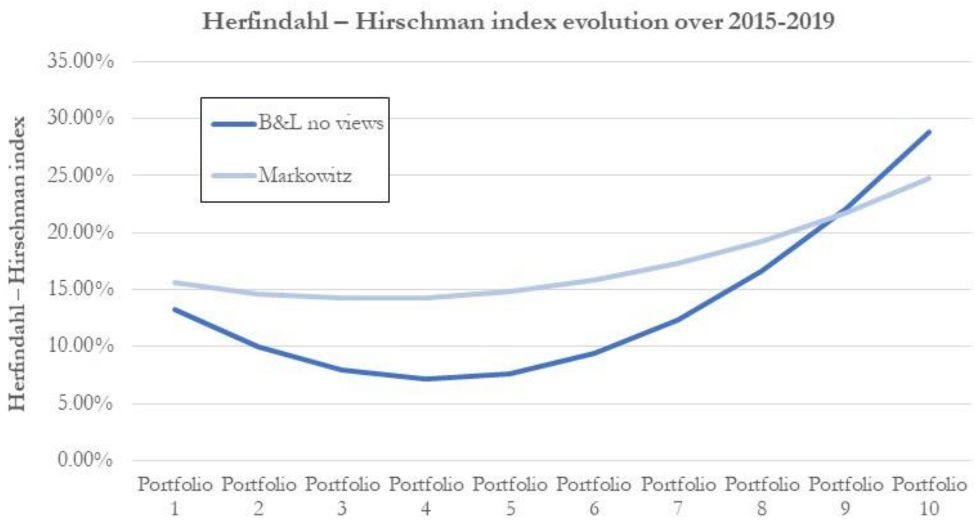

Figure 5.16. Herfindahl – Hirschman index evolution over 2015-2019.

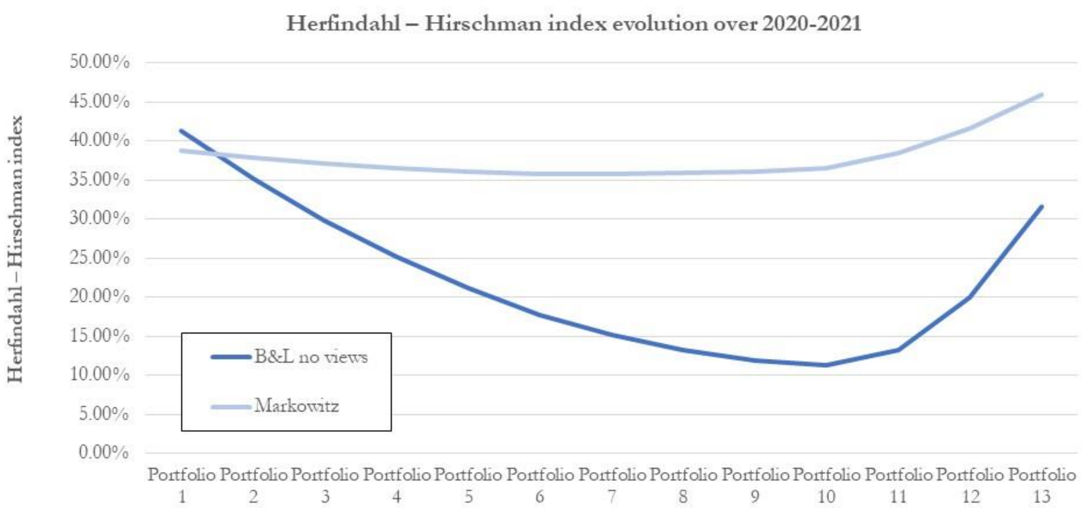

Figure 5.17. Herfindahl – Hirschman index evolution over 2020-2021.

Portfolio stability

One of the main drawbacks of Markowitz's approach consists in the strong impact that small changes in expected returns have on portfolio weights. By using implied equilibrium returns, the Black and Litterman approach creates more stable portfolios characterized by less abrupt variations in the portfolio weights as the correlation matrix changes.

The difference between the standard deviation of the weights of a given portfolio computed over one time period and the volatility of the same over another one can be considered a good indicator of portfolio stability - this metrics has been here indicated as "Portfolio Instability". In this case, since no investor's utility function has been specified nor a risk-free asset has been included in the portfolio, 18.35% (portfolio no. 6) has been set as target return and the portfolio instability index has been computed on that, as shown in Table 5.15. The higher the value, the more unstable the portfolio is. Since in this case Markowitz's portfolio instability is higher than Black-Litterman's, it can be stated that the latter produces more stable portfolios.

	2015-2019		2020-2021	
	Portfolio 6 Markowitz	Portfolio 6 B&L no views	Portfolio 6 Markowitz	Portfolio 6 B&L no views
Weights σ	**6.45%**	**4.49%**	**10.41%**	**6.94%**

Markowitz Instability	3.97%
B&L Instability	2.45%

Table 5.15. Markowitz & Black-Litterman portfolio instability indexes.

5.2.3 Bayesian approach

The implied returns over 2015-2019 and 2020-2021 shown in Table 5.13 constitute the prior distribution to the Bayes formula. They represent the starting point of the investor's capital allocation.

Views formulation

Views to be applied to the implied returns over 2015-2019

Context - It is end of April 2020, the Covid-19 pandemic has already hit the world and the investor formulates 2 relative views.

1) Relative View 1 – *Pfizer (PFE)* will outperform *easyJet (EZJ.L)* and *Lufthansa (LHA.DE)* by 35% in the next 12 months
2) Relative View 2 – *Apple (AAPL)* will outperform *Airbus (AIR.PA)* and *BP (BP.L)* by 40% in the next 12 months

Airplane Industry General Outlook - As of March 2020, Covid-19 has already disrupted the airline industry. To cite some relevant numbers, China, the first country which experienced restrictions, saw 71% drop in global flight capacity and Hong Kong 81%. In Italy, one of the first European countries hit by the virus, flights drop by 22%, increased by restrictions imposed in all the Schengen area. On March 11, the US declared 30-day travel ban, followed by the Canada that closed its borders on March 17. According to the Guardian (Jolly, 2020), on March 26, the European airline industry lost revenues are projected to be $76bn which represent almost 62% decrease from 2019's European airline turnover of $123bn (Statista, 2021).

easyJet (EZJ.L) – Already experienced losses for £205m, on March 30, easyJet grounded its entire fleet made of 344 Airbus aircraft for at least two months. The company forecasts that this event alone will lead to additional £1.2m losses, that would result in -20% revenues compared to 2019. If the shutdown will last 9 months, the company foresees costs of about £3m, -47% that would result in -20% revenues compared to 2019. To ensure liquidity, in April 2020 easyJet borrowed £600m loan from the Treasury and Bank of England. It also announced that other $500m will be borrowed from creditors. (Martin, 2020).

Lufthansa (LHA.DE) – Incurring €1.2m losses per hour, in Q1 2020 Lufthansa reported €1.2bn losses (Miller & Powley, 2020). At the end of April, the State intervened to bail out the company with $10bn of support (Peterson, Özgenc, & Moynihan, 2020).

Airbus (AIR.PA) – Covid-19 hit the aerospace industry severely. If in 2019 Airbus' annual revenues were $78.935bn (+4.91% from 2018), in Q1 2020 the company achieved $10.631bn (-15% from Q1 2019) (Sloan, 2020). While in March Airbus announced liquidity measures, in April 2020 it decided to reduce CapEx by $760.91m to $2.07bn and to suspend non-critical business activities. (Reuters, 2020).

Pfizer (PFE) – On April 28, Pfizer reported better than expected Q1 earnings ($0.80 /share). Even though the company's sales drop by 8% from the previous year to $12bn, investors are betting on Pfizer since it is working , together with BioNTech, on a vaccine against Covid-19 (Lovelace, 2020). To achieve the goal of getting the first vaccination by Q4 2020, the company has committed about $500m on R&D (Gibney, 2020).

BP (BP.L) – Because of the drop of oil demand and relative prices, on April 28, BP's revenues experienced a fall of 66%, with an increase in debt. Nevertheless, the dividends are kept at 10.5 cents. (Raval, 2020).

Apple (AAPL) – On April 30, Apple's publishes its revenues for the three months to March ($58.3 bn), showing 1% increase from the previous year, despite the pandemic. Even though iPhone sales in China decreased, the growth was mainly due to Apple's services, such as Apple Music, Apple TV+ and iCloud, which rose about 17% (Iyengar, 2020).

Views to be applied to the implied returns over 2020-2021

Context - It is March 2022, the current economic situation is impacted by increasing interest rates and the conflict in Ukraine and the investor formulates 2 relative views.

1) Relative View 1 – *Unilever (ULVR.L)* will outperform *BASF SE (BAS.DE)* by 25% in the next 12 months
2) Relative View 2 – *Amazon (AMZN)* will outperform *Volkswagen (VOW3.DE)* by 40% in the next 12 months

According to Berenberg (2022), the conflict in Ukraine determines high volatility and uncertainty. This, in turns, leads to higher inflation expectations and less growth. In this context of uncertainty, the Nasdaq suggests investing in the so-called recession proof stocks of firms operating in non-cyclical businesses (Samuel, 2022).

Unilever (ULVR.L) – As one of the more largest consumer staple companies, Unilever is considered one of the best examples of defensive stock. Over 2021, many insiders bought Unilever's shares. In particular, the biggest purchase was made in October 2021 by the CFO & Executive Director Graeme Pitkethly, who secured £750k worth of shares, at £40.86/share (Simply Wall St, 2022). According to the signalling theory, if an insider buys company's shares, this is perceived as a positive sign by the investors, who may think that the company will perform positively in the future, above the current stock price. What is more striking is the fact that they did not sell their stake in the company, despite the contained 4.5% underlying sales growth experienced by Unilever in 2021 (Unilever Investors Relations, 2022). Leveraging on a clear business strategy aimed at reducing the impact of material inflation, analysts expect 2022 Unilever's sales growth to be around 5.5%, beating the 3.5% forecast of the industry (Simply Wall St, 2022).

Amazon (AMZN) – Despite being considered a "consumer discretionary company", Amazon is generalized retailer, offering essential products at competitive prices. In this way, the company remains profitable, even when consumer spending is affected by rising inflation. To boost growth, the company is evolving into a full-service platform, expanding its core retail business with technology services such as Amazon Web Services and Amazon Prime Program. After the company published its strong earnings, Amazon stock was reaffirmed one of Goldman Sachs' 2022 top pick (Ribeiro, 2022) and Ivan Feinseth of Tigress Financial Partners raised its target price from $4,460 to $4,655, with a 50% upside from the current price (Ladenheim, 2022).

BASF SE (BAS.DE) – Even though less than 1.4% consolidated revenues come from Russia and Denmark, BASF has 73% stake in Wintershall Dea JV, with 50% operations in Russia. Since the beginning of February, the firm declared its intention to divest from this JV through an IPO (Tullo, 2022). Considering the uncertainty resulting from the Russian-Ukrainian conflict, the supply chain disruptions, the remaining Covid-19 effects and the rising energy prices, BASF's 2022 revenues forecast is down by more than 6%, from €78.6bn in 2021 to €74bn in 2022 (BASF SE Investors Relations, 2022).

Volkswagen (VOW3.DE) – With 199,000 vehicles sold, the Russian market represented 11.90% of Volkswagen Group's total revenues in 2021 and around 170,000 automobiles were produced at the company's plant in the country (Volkswagen AG, 2022). Not only the production facilities in Russia, but also some in Germany were closed, because of shortage of crucial parts. Forecasting a long-term disruption in the supply chain that could lead to increasing prices of raw materials and energy, Volkswagen declared to be considering expanding its operations outside EU, as to guarantee crucial supplies that it previously received from Russia. (Miller, 2022).

Bayesian Approach

Once the views have been formulated, the P matrix of the asset weights according to each view, and the Q vector, expressing the expected excess returns for each view, can be built, as shown in Table 5.16 and Table 5.17.

Following Satchell and Scowcroft (2000) an equal weighting scheme has been used for the construction of matrix P. Under this approach all the weights are proportional to the inverse of the number of assets which underperform or overperform:

$$weights \propto \frac{1}{no.\ of\ assets\ underperfoming/overperforming}$$

2015-2019

Link Matrix P/ Views (Q)		AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L
View 1	35.00%	0	0	0	0	0	0	0	0	0	-0.5
View 2	40.00%	1	-0.5	0	0	0	0	-0.5	0	0	0

Link Matrix P/ Views (Q)		G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA
View 1	35.00%	0	0	0	0	0	-0.5	0	0	0	0
View 2	40.00%	0	0	0	0	0	0	0	0	0	0

Link Matrix P/ Views (Q)		PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
View 1	35.00%	1	0	0	0	0	0	0	0	0	0
View 2	40.00%	0	0	0	0	0	0	0	0	0	0

Table 5.16. P matrix and Q vector over 2015-2019.

2020-2021

Link Matrix P/ Views (Q)		AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L
View 1	25.00%	0	0	0	0	0	-1	0	0	0	0
View 2	40.00%	0	0	1	0	0	0	0	0	0	0

Link Matrix P/ Views (Q)		G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA
View 1	25.00%	0	0	0	0	0	0	0	0	0	0
View 2	40.00%	0	0	0	0	0	0	0	0	0	0

Link Matrix P/ Views (Q)		PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
View 1	25.00%	0	0	0	0	0	0	0	1	0	0
View 2	40.00%	0	0	0	0	0	0	0	0	-1	0

Table 5.17. P matrix and Q vector over 2015-2019.

As next step, the covariance Ψ matrix of the error terms has been computed as $\Psi = diag(P(\tau\Sigma)P^T)$. The variance term is inversely proportional to the investor's level of confidence in his view: therefore, the closer the variance is to zero, the more certain the view is. As shown in Table 5.18, the variance of the error terms both over 2015-2019 and 2020-2021 are close to 0: therefore, it is possible to state that the views have been formulated with a high degree of certainty.

2015-2019

Ψ matrix	View 1	View 2
View 1	0.002637	0
View 2	0	0.001881

2020-2021

Ψ matrix	View 1	View 2
View 1	0.002843	0
View 2	0	0.00656

Table 5.18. Covariance matrix of the error term over 2015-2019 and 2020-2021.

At this point, it is possible to combine the specific personal investor's views with the equilibrium values through the so-called Black-Litterman master formula:

$$P(A|B) \sim N([(\tau\Sigma)^{-1}\theta + P^T\Psi^{-1}Q][(\tau\Sigma)^{-1} + P^T\Psi^{-1}P]^{-1}, ((\tau\Sigma)^{-1} + P^T\Psi^{-1}P)^{-1})$$

To compute the variance of the expected returns to be used in the mean variance optimizer and reported in , the following formula has been applied:

$$\Sigma_p = \Sigma + ((\tau\Sigma)^{-1} + P^T\Psi^{-1}P)^{-1}$$

Table 5.19 shows the B&L $E(R)|\theta$ and their σ over 2015-2019 and 2020-2021.

Time horizon	2015-2019		2020-2021	
Stock	**E(R)|θ**	**σ**	**E(R)|θ**	**σ**
AAPL	32.35%	24.87%	42.20%	37.54%
AIR.PA	7.36%	26.96%	3.75%	58.26%
AMZN	32.33%	29.18%	35.83%	32.62%
APD	13.00%	18.91%	21.23%	33.89%
AZN.L	6.59%	23.42%	10.31%	27.75%
BAS.DE	9.30%	22.39%	0.61%	34.26%
BP.L	5.04%	23.90%	0.16%	46.15%
CPR.MI	7.20%	23.09%	13.00%	30.79%
ENEL.MI	5.63%	22.87%	13.28%	32.73%
EZJ.L	-4.18%	35.32%	-1.37%	71.73%
G.MI	4.73%	25.58%	4.84%	27.26%
GM	13.56%	25.71%	11.76%	50.27%
GOOG	27.12%	24.13%	32.42%	31.76%
IHG.L	5.75%	24.66%	3.53%	47.61%
IP.MI	8.69%	28.46%	9.25%	34.70%
LHA.DE	-3.21%	31.19%	0.90%	60.38%
LMT	11.80%	17.64%	16.46%	32.44%
MC.PA	12.00%	25.59%	11.81%	31.32%
MSFT	27.77%	23.25%	39.56%	34.20%
OR.PA	8.01%	20.19%	13.18%	25.68%
PFE	15.06%	18.14%	14.78%	31.09%
PG	9.44%	15.72%	19.71%	25.16%
REP.MC	6.24%	27.74%	1.83%	46.43%
SAN.MC	7.75%	31.45%	1.85%	46.48%
SAN.PA	7.87%	21.32%	6.39%	23.17%
SAP.DE	9.94%	21.68%	13.63%	32.87%
SOLB.BR	9.00%	24.77%	0.88%	37.13%
ULVR.L	5.51%	19.76%	11.72%	23.31%
VOW3.DE	9.38%	33.59%	5.92%	45.22%
WMT	10.16%	19.58%	18.34%	25.49%

Table 5.19. B&L $E(R)|\theta$ and their σ over 2015-2019 and 2020-2021.

Comparing the implied equilibrium returns in with the B&L $E(R)|\theta$ helps to understand the impact of the views. Referring to the 2015-2019 case represented in Figure 5.18, when positive views are formulated, the stocks present higher expected returns, as in the case of AAPL and PFE. Conversely, when the views are negative, the returns drop, as for EZJ.L and LHA.DE. An analogous analysis could be done for the 2020-2021 time period, as shown in Figure 5.19.

Because of correlation between stocks shown in Appendix 19 and Appendix 20, not only the returns of the assets for which a view has been formulated, but also the returns of the other stocks for which no expectation has been made change.

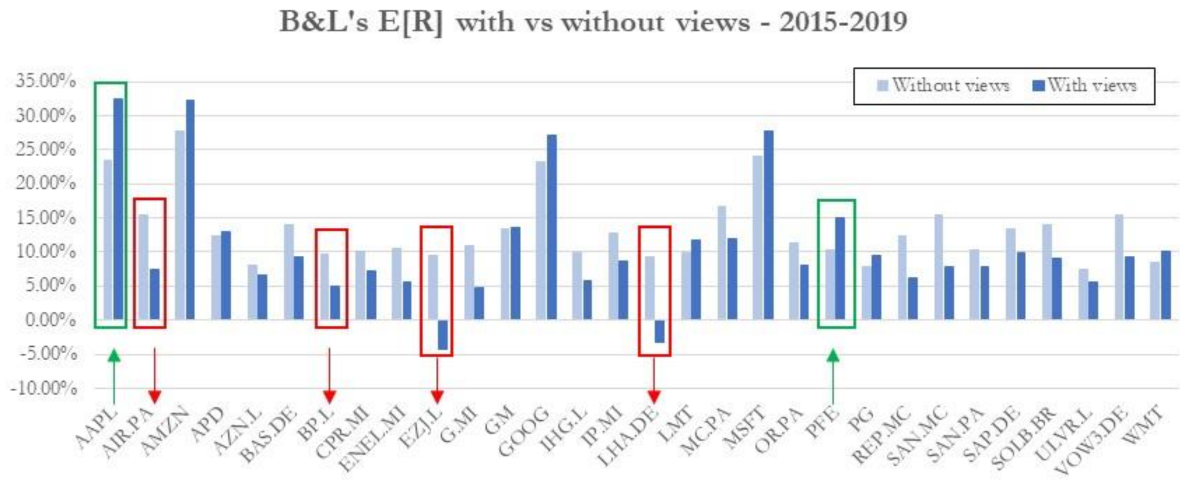

Figure 5.18. Implicit Equilibrium Returns vs B&L's $E(R)|\theta$ over 2015-2019.

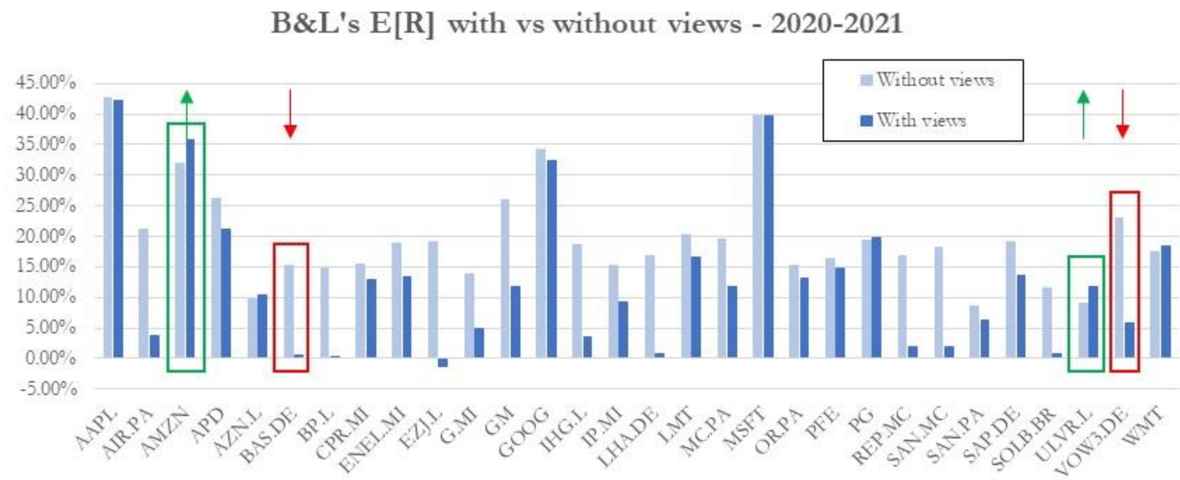

Figure 5.19. Implicit Equilibrium Returns vs B&L's $E(R)|\theta$ over 2020-2021.

5.2.4 Capital allocation with the investor's views

Comparison – Black-Litterman's capital allocation and Markowitz's

To be coherent with section 5.2.2, the comparison between the B&L method and the Markowitz approach has been made on portfolio no. 6 and by allowing short selling. The results are presented in Table 5.20. When the relative view about a stock is positive, the corresponding row has been highlighted in green, if negative in red. Depending on the degree of correlation between the stocks and the level of confidence of the view, not only the weights of the shares of the stocks directly impacted, but also those of the assets for which no expectation has been expressed, vary.

Time horizon	2015-2019			2020-2021		
Stocks	Portfolio 6 Markowitz	Portfolio 6 B&L no views	Portfolio 6 B&L with views	Portfolio 6 Markowitz	Portfolio 6 B&L no views	Portfolio 6 B&L with views
AAPL	2.41%	16.46%	18.72%	-4.09%	5.17%	3.35%
AIR.PA	1.84%	1.18%	-6.77%	-3.60%	-1.63%	-1.86%
AMZN	8.58%	13.31%	7.48%	16.69%	16.98%	19.72%
APD	5.88%	1.07%	2.79%	-6.05%	-2.67%	-2.98%
AZN.L	7.54%	2.65%	5.04%	6.26%	3.43%	3.74%
BAS.DE	-5.59%	0.96%	1.42%	0.20%	-2.20%	-10.32%
BP.L	7.20%	2.15%	-0.28%	-3.78%	-1.65%	-1.80%
CPR.MI	6.07%	0.28%	0.97%	-3.98%	-3.89%	-4.27%
ENEL.MI	6.86%	1.11%	1.35%	-16.93%	-9.29%	-10.20%
EZJ.L	1.00%	0.47%	-2.35%	0.55%	1.09%	1.19%
G.MI	7.78%	1.00%	4.04%	31.41%	20.32%	22.16%
GM	1.69%	1.01%	2.14%	1.79%	0.57%	0.56%
GOOG	-1.21%	10.94%	6.62%	6.52%	7.77%	7.16%
IHG.L	5.00%	0.60%	2.84%	8.92%	5.73%	6.26%
IP.MI	2.72%	0.16%	0.75%	8.51%	3.00%	3.27%
LHA.DE	4.06%	0.64%	-1.94%	-4.62%	-1.85%	-2.03%
LMT	16.84%	2.69%	9.07%	-5.13%	-0.92%	-1.07%
MC.PA	2.72%	2.58%	-2.15%	7.97%	4.09%	4.14%
MSFT	-3.98%	15.40%	2.50%	-13.66%	-1.78%	-3.96%
OR.PA	-6.25%	1.95%	-0.14%	6.65%	1.68%	1.63%
PFE	7.38%	3.61%	15.78%	9.99%	5.89%	6.17%
PG	19.08%	5.99%	15.02%	9.63%	7.22%	7.58%
REP.MC	1.86%	0.38%	0.72%	-0.86%	-0.68%	-0.75%
SAN.MC	-12.91%	0.33%	-3.15%	-5.33%	-2.65%	-2.94%
SAN.PA	-2.63%	1.78%	1.67%	17.04%	13.56%	14.70%
SAP.DE	5.37%	2.52%	3.58%	-1.23%	1.38%	1.38%
SOLB.BR	-3.98%	0.13%	-0.11%	4.78%	3.41%	3.72%
ULVR.L	8.09%	2.69%	5.91%	15.28%	12.96%	22.02%
VOW3.DE	-1.75%	0.51%	0.09%	-6.34%	-2.91%	-5.75%
WMT	8.30%	5.45%	8.39%	23.41%	17.85%	19.17%
Sum Weights	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Target Annual E[R]	18.35%	18.35%	18.35%	18.35%	18.35%	18.35%
Annual Variance	0.0121	0.0229	0.0165	0.0218	0.0280	0.0269
Annual Std. Dev.	11.00%	15.15%	12.83%	14.75%	16.73%	16.41%
Herfindahl – Hirschman index	15.80%	9.38%	12.72%	35.85%	17.76%	25.15%

Table 5.20. Portfolio no. 6 stock weights under different allocation methods.

Portfolio allocation using 2015-2019 data

If at the beginning of 2020 the investor relied on historical returns, the resulting portfolio would prefer stocks that offered high expected returns, low variance, and low correlation in the past. Solving the mean variance optimization problem for portfolio no. 6 using historical returns leads to stock concentration of PG (19.08%) and LMT (16.84%), as Figure 5.20 displays.

If the investor decided to rely on implied equilibrium returns without expressing his own expectations, he would obtain a portfolio characterized by a lower stock concentration, as Figure 5.21 shows. In this case, in fact, no short positions are needed, and the highest weighted stocks are AAPL with 16.46% and MSFT with 15.40%.

As the investor expresses his personal views, the Black-Litterman capital allocation becomes dynamic. Figure 5.22 shows that the stocks on which the investor has upward expectations are favoured, namely AAPL (18.72%) and PFE (15.78%). On the contrary, stocks characterized by downward views present negative weights – being respectively -6.77% AIR.PA, -0.28% BP.L, -2.35% EZJ.L and -1.94% LHA.DE.

To understand the impact of views, let us refer to Figure 5.23 and focus on AAPL stock. According to Markowitz's allocation approach, 2.41% of investor's funds should be allocated to AAPL. However, if the investor uses the Black-Litterman method he should hold a position of 18.72% on the stock.

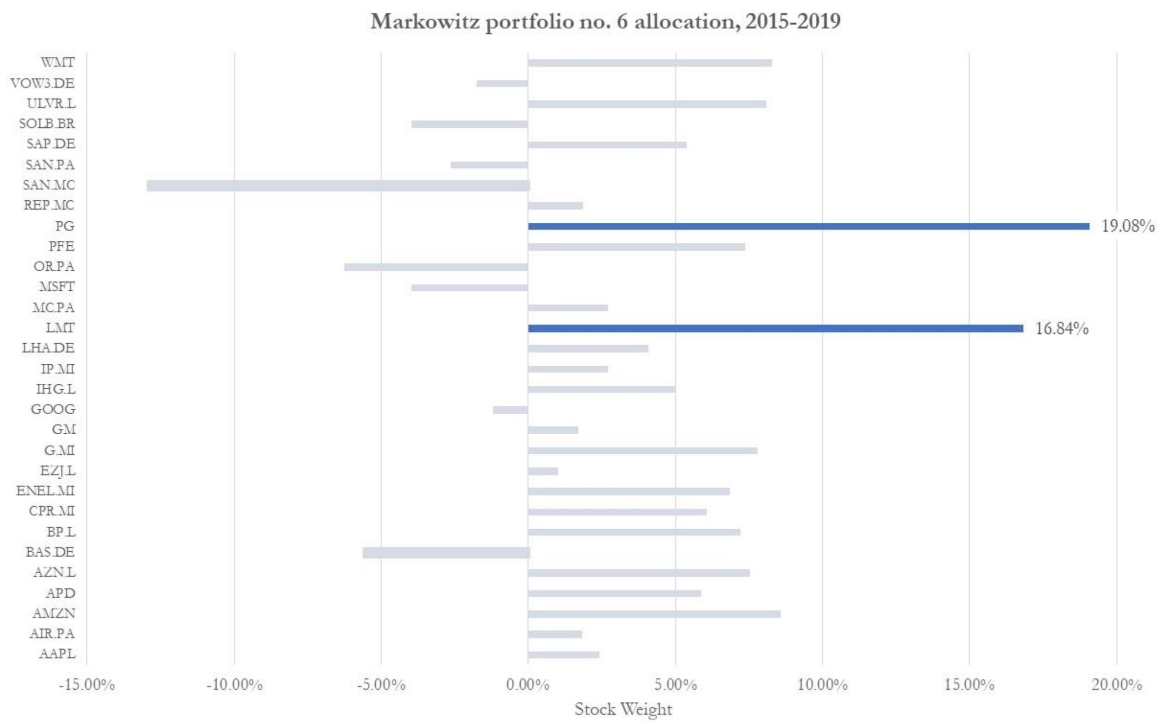

Figure 5.20. Markowitz portfolio no.6 allocation – 2015-2019.

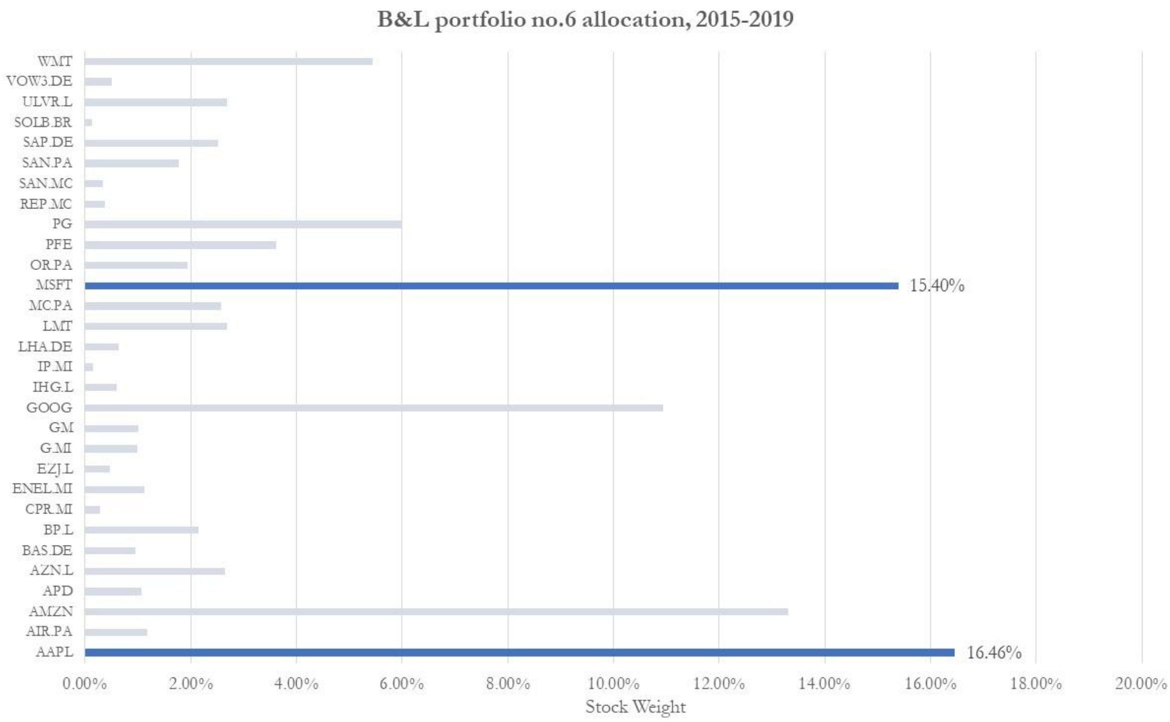

Figure 5.21. B&L portfolio no. 6 allocation without views, 2015-2019.

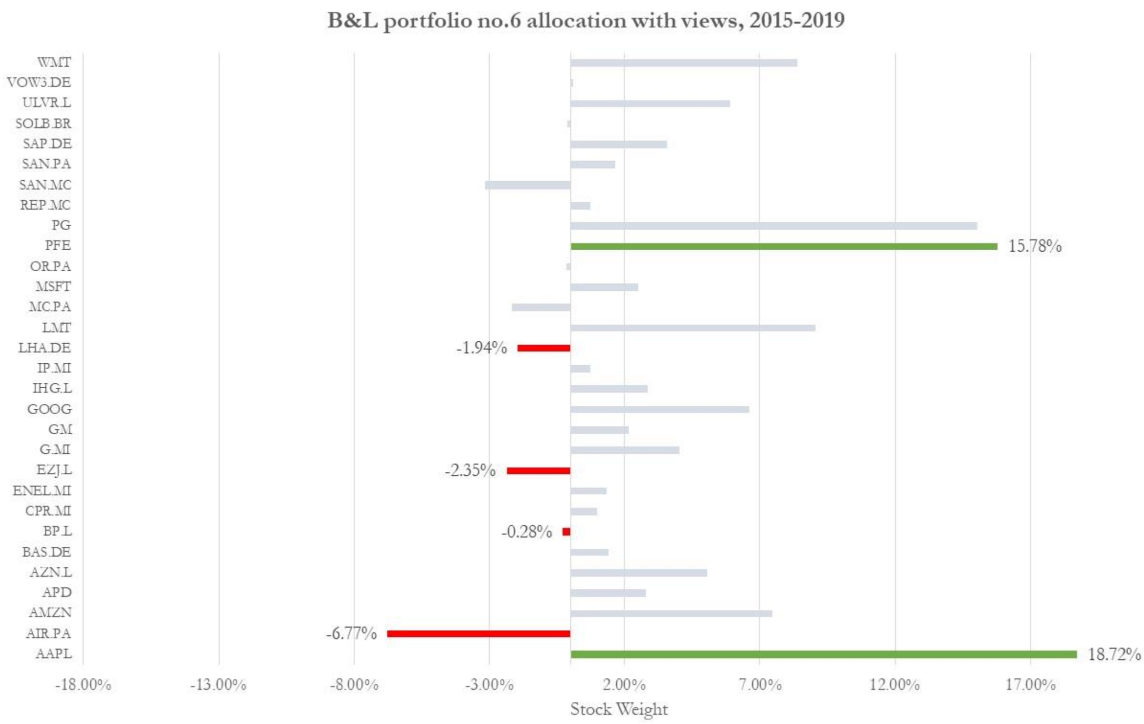

Figure 5.22. B&L portfolio no. 6 allocation with views, 2015-2019.

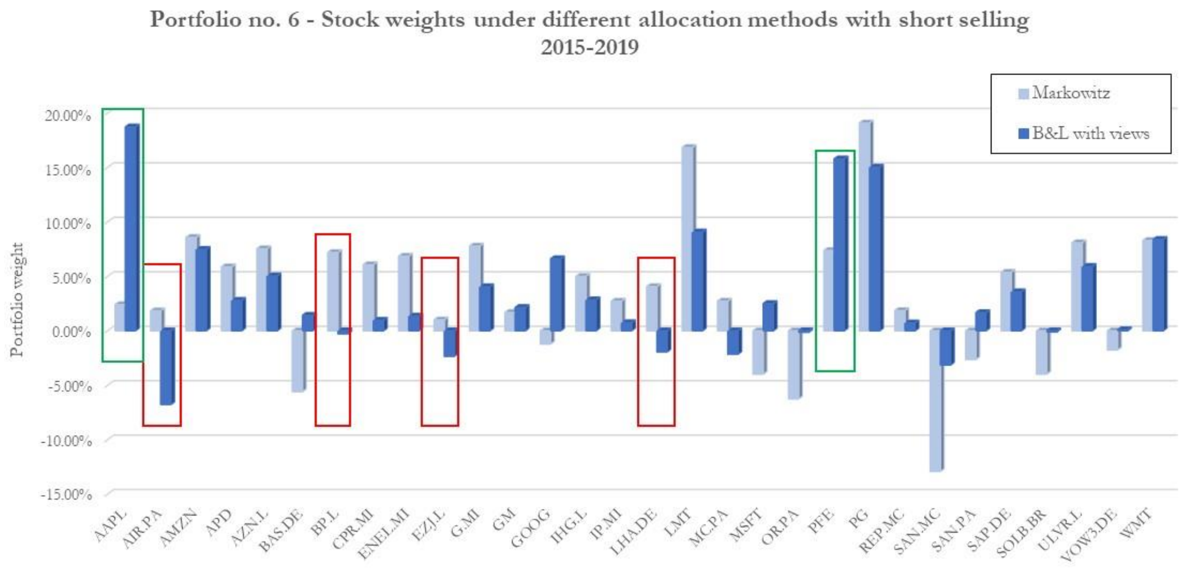

Figure 5.23. Portfolio no. 6 under different allocation methods, 2015-2019 data.

Portfolio allocation using 2020-2021 data

If at the beginning of 2022 the investor relied on historical returns to build his portfolio, the largest weights would be assigned to G.MI (31.41%) and WMT (23.41%), as displayed in Figure 5.24.

If portfolio no. 6 was created using B&L allocation method, this approach would still be preferable to Markowitz's in terms of stock concentration. As revealed by Figure 5.25. the two largest stock weights are reduced to 20.32% (G.MI) and 17.85% (WMT).

In line with the investor's views, Figure 5.26 shows that the dynamic asset allocation would favour ULVR.L (22.02%) and AMZN (19.72). It should be underlined that, although no positive views have been formulated on G.MI, it would nevertheless be the first stock per weight in the portfolio, followed by ULVR.L. The reason lies in the correlation that exists between stocks. On the other hand, the assets characterized by downward expectations present negative weights – being respectively -10.32% BAS.DE and -5.75% VOW3.DE.

To understand the impact of views, let us refer to Figure 5.27 and focus on ULVR.L stock. According to Markowitz's allocation approach, with 15.28%, ULVR.L would be the third stock per weight in the portfolio. Under the Black-Litterman method, 22.02% of the investor's funds should be allocated to ULVR.L, thus becoming the second most relevant stock in the portfolio.

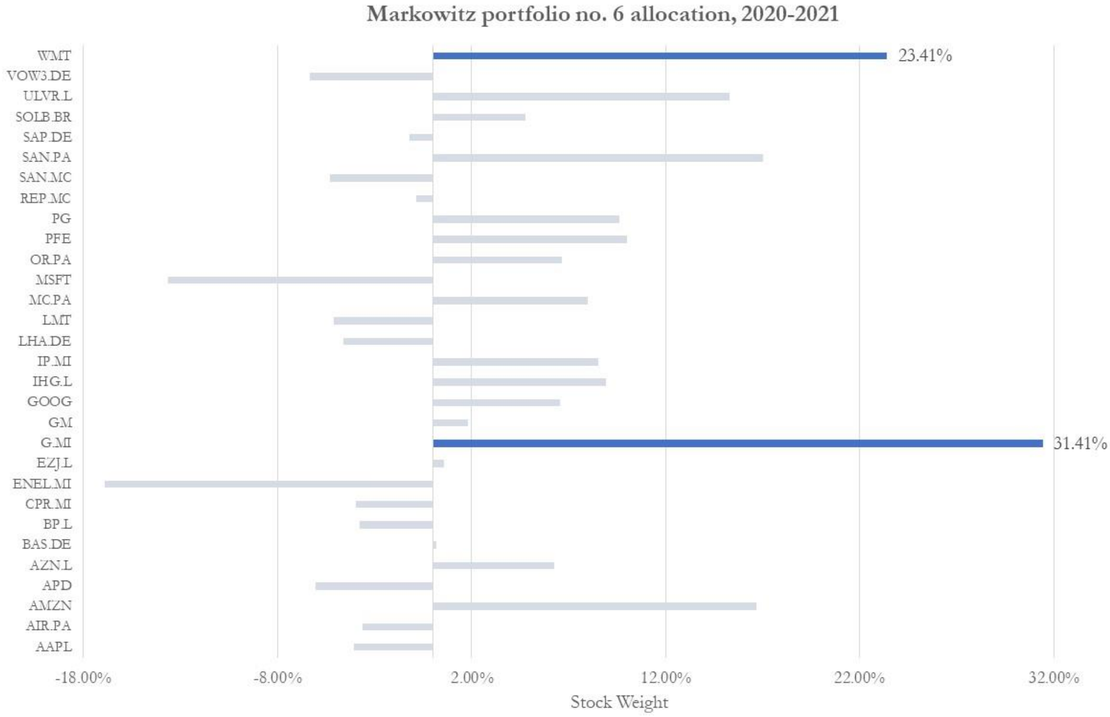

Figure 5.24. Markowitz portfolio no.6 allocation – 2020-2021.

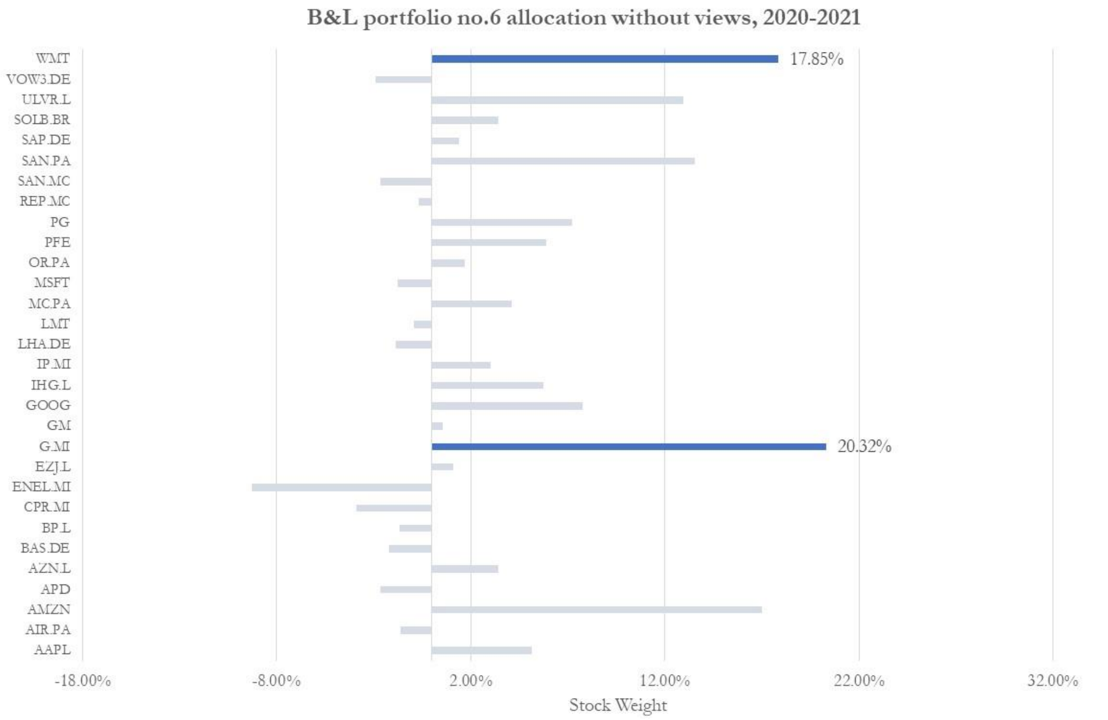

Figure 5.25. B&L portfolio no. 6 allocation without views, 2020-2021.

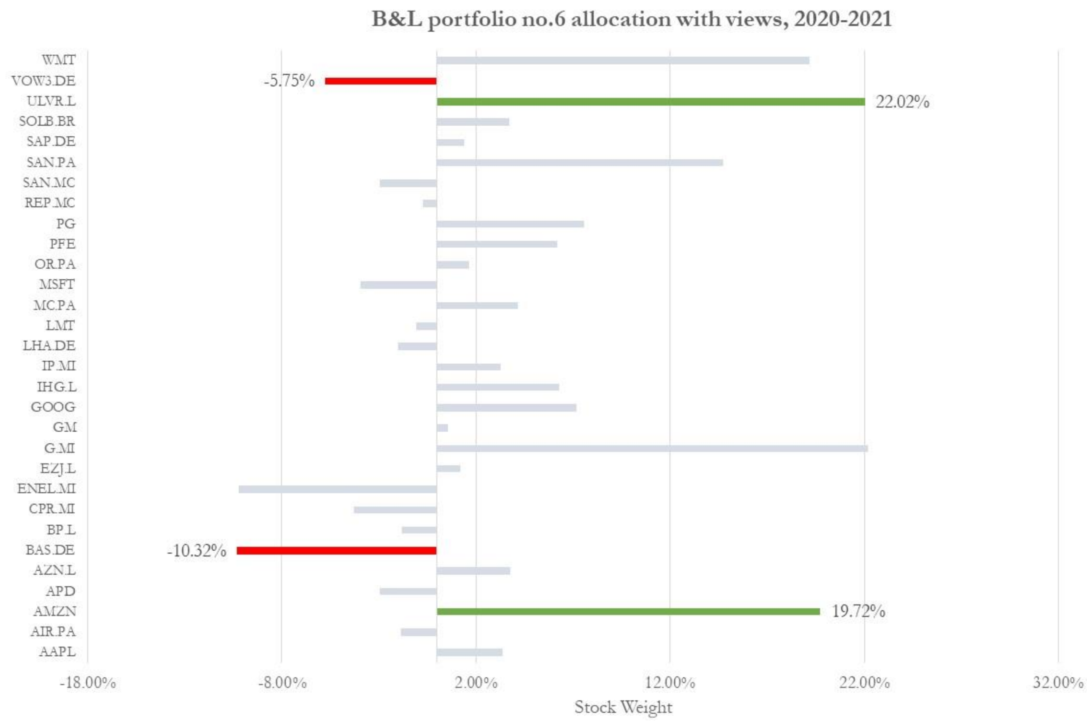

Figure 5.26. B&L portfolio no. 6 allocation with views, 2020-2021.

Figure 5.27. Portfolio no. 6 under different allocation methods without short selling, 2015-2019 data.

5.2.5 View sensitivity analysis

To assess the impact of each individual expectation, a view sensitivity analysis has been performed on portfolio no. 6., when short sales are allowed. Table 5.21 displays the changes in the portfolio weights, as different views are applied. While weight variations are highlighted in green if positive and in red if negative, different shades are employed to highlight the impact of the view: the more vivid the colour, the more relevant the change.

Time horizon	2015-2019				2020-2021			
View	Benchmark	All views 1	View 1 only	View 2 only	Benchmark	All views 1	View 1 only	View 2 only
Stocks	Portfolio 6 B&L no views	$\Delta_{1\&2}$	Δ_1	Δ_2	Portfolio 6 B&L no views	$\Delta_{1\&2}$	Δ_1	Δ_2
AAPL	16.46%	2.26%	-3.89%	5.94%	5.17%	-1.82%	-0.44%	-1.72%
AIR.PA	1.18%	-7.96%	-1.39%	-8.73%	-1.63%	-0.23%	-0.06%	-0.22%
AMZN	13.31%	-5.84%	-2.85%	-4.93%	16.98%	2.74%	0.05%	4.08%
APD	1.07%	1.72%	0.84%	1.45%	-2.67%	-0.30%	-0.07%	-0.29%
AZN.L	2.65%	2.39%	1.17%	2.01%	3.43%	0.31%	0.07%	0.30%
BAS.DE	0.96%	0.46%	0.23%	0.39%	-2.20%	-8.12%	-10.19%	-0.24%
BP.L	2.15%	-2.42%	1.32%	-4.05%	-1.65%	-0.16%	-0.04%	-0.15%
CPR.MI	0.28%	0.69%	0.34%	0.58%	-3.89%	-0.37%	-0.09%	-0.35%
ENEL.MI	1.11%	0.24%	0.11%	0.20%	-9.29%	-0.91%	-0.22%	-0.86%
EZJ.L	0.47%	-2.81%	-6.38%	1.19%	1.09%	0.10%	0.03%	0.10%
G.MI	1.00%	3.04%	1.49%	2.56%	20.32%	1.83%	0.44%	1.74%
GM	1.01%	1.14%	0.56%	0.96%	0.57%	-0.01%	0.00%	-0.01%
GOOG	10.94%	-4.32%	-2.11%	-3.65%	7.77%	-0.60%	-0.15%	-0.57%
IHG.L	0.60%	2.24%	1.10%	1.89%	5.73%	0.53%	0.12%	0.50%
IP.MI	0.16%	0.58%	0.29%	0.50%	3.00%	0.27%	0.06%	0.25%
LHA.DE	0.64%	-2.58%	-6.27%	1.39%	-1.85%	-0.18%	-0.04%	-0.17%
LMT	2.69%	6.38%	3.12%	5.38%	-0.92%	-0.16%	-0.03%	-0.15%
MC.PA	2.58%	-4.73%	-2.31%	-3.99%	4.09%	0.05%	0.01%	0.04%
MSFT	15.40%	-12.90%	-6.31%	-10.89%	-1.78%	-2.18%	-0.52%	-2.06%
OR.PA	1.95%	-2.09%	-1.03%	-1.77%	1.68%	-0.05%	-0.01%	-0.04%
PFE	3.61%	12.17%	15.96%	3.13%	5.89%	0.28%	0.07%	0.26%
PG	5.99%	9.03%	4.41%	7.62%	7.22%	0.36%	0.07%	0.33%
REP.MC	0.38%	0.35%	0.17%	0.28%	-0.68%	-0.07%	-0.02%	-0.07%
SAN.MC	0.33%	-3.48%	-1.70%	-2.94%	-2.65%	-0.29%	-0.07%	-0.27%
SAN.PA	1.78%	-0.11%	-0.05%	-0.09%	13.56%	1.14%	0.27%	1.07%
SAP.DE	2.52%	1.06%	0.52%	0.89%	1.38%	0.00%	0.00%	0.00%
SOLB.BR	0.13%	-0.24%	-0.12%	-0.20%	3.41%	0.31%	0.07%	0.29%
ULVR.L	2.69%	3.22%	1.58%	2.72%	12.96%	9.06%	10.43%	1.13%
VOW3.DE	0.51%	-0.42%	-0.20%	-0.35%	-2.91%	-2.84%	-0.07%	-4.18%
WMT	5.45%	2.94%	1.44%	2.48%	17.85%	1.32%	0.32%	1.26%

Table 5.21. The impact of views on portfolio no. 6.

View sensitivity analysis over 2015-2019

Figure 5.28 displays the impact of the individual views on portfolio no. 6 built using 2015-2019 implied returns. As expected, and in line with investor's expectations, when only view 1 is formulated, PFE experiences the largest positive weight change, while EZJ.L and LHA.DE are significantly negatively impacted. On the other hand, under view 2 only, AAPL weight undergoes the second largest positive change, while BP.L and AIR.PA weights are among the ones which decrease the most.

From the analysis it is also evident that view 1 dilutes the positive weight change that view 2 has on APPL and the contrary holds true regarding EZJ.L and LHA.DE.

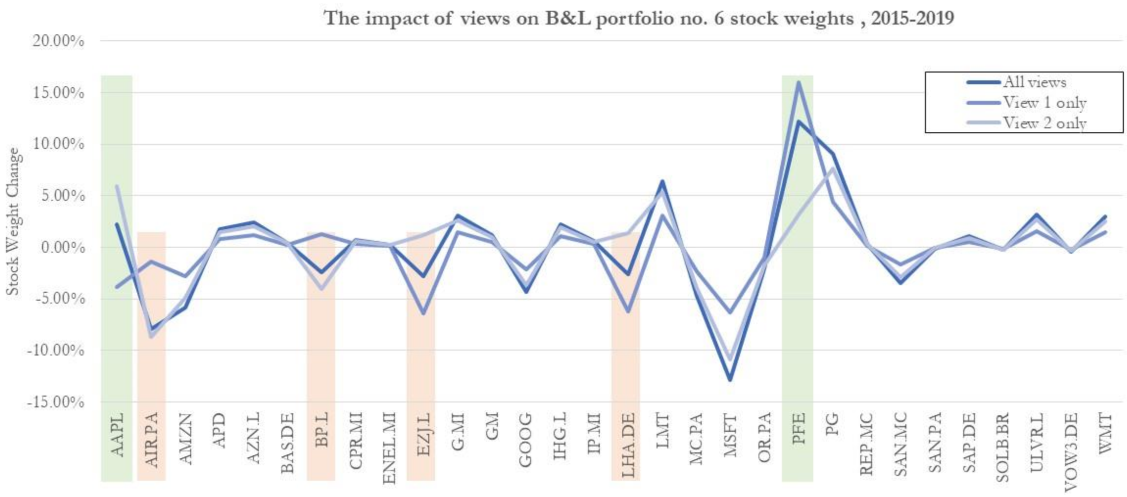

Figure 5.28. The impact of views on B&L portfolio no. 6 stock weights , 2015-2019.

View sensitivity analysis over 2020-2021

Figure 5.29 displays the impact of the individual views on portfolio no. 6 built using 2020-2021 implied returns. As expected, and in line with investor's expectations, when only view 1 is formulated, ULVR.L stock shows the largest positive weight change, while BAS.DE exhibits the most relevant negative variation. On the other hand, under view 2 only, AMZN weight increases the most, while VOW3.DE is the stock which is the most negatively affected by the view.

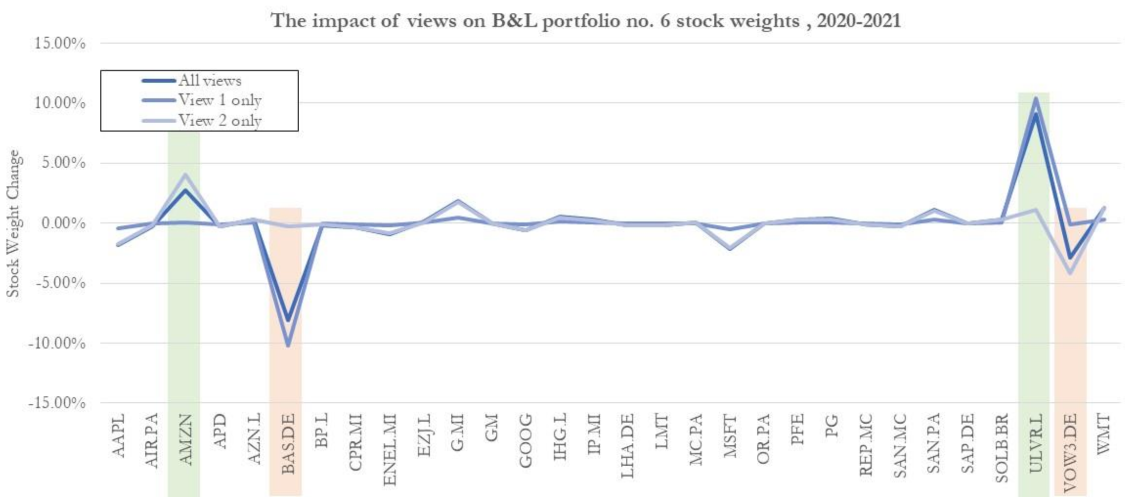

Figure 5.29. The impact of views on B&L portfolio no. 6 stock weights , 2020-2021.

Applying view 3 to portfolio no. 6, 2020-2021

Since the beginning of the conflict, many NATO countries and from the European Union started sending weapons to Ukraine (Erlanger, 2022).

Lockheed Martin (LMT) – According to BofA securities, the U.S. defense budget is projected to grow by 0.5%, from 3.5% to 4%. On March 7, the BofA senior equity analyst Ronald Epstein raised the target price of LMT from $410 to $485, representing an increase of +20% (Nigam, 2022). Over the past years, the firm was able to win profitable contracts, resulting in +5.5% consolidated operating profit in 2021 (from $ 8,644m in 2020 to $ 9,123m in 2021). For 2022, Lockheed Martin forecasts diluted earnings per share of $26.70, +17% from 2021. (Lockheed Martin Investors Relations, 2022). With a considerable cash balance, the group has publicly stated his commitment to invest $1 billion in manufacturing in Saudi Arabia, thus increasing the ROI (Reuters, 2022).

Given the above, let us suppose that the investor formulates the following view: *Lockheed Martin (LMT)* will outperform *BASF SE (BAS.DE)* by 45% in the next 12 months. As Figure 5.30 displays, coherent with the relative view, BAS.DE is the stock with the greatest negative position in the portfolio. On the contrary, if in line with the upward expectation, LMT weight increases from 2.69% in the case without views to 7.29%. However, the extent of this change is not as significant as one could have expected. The reason lies in the lower expected return, given the same risk, this stock shows compared to others.

The view on LMT shows how the return/risk structure of the portfolio impacts the effects of the investor's expectations. Hence, the need to explore the effects of the views one at a time (=sensitivity) arises.

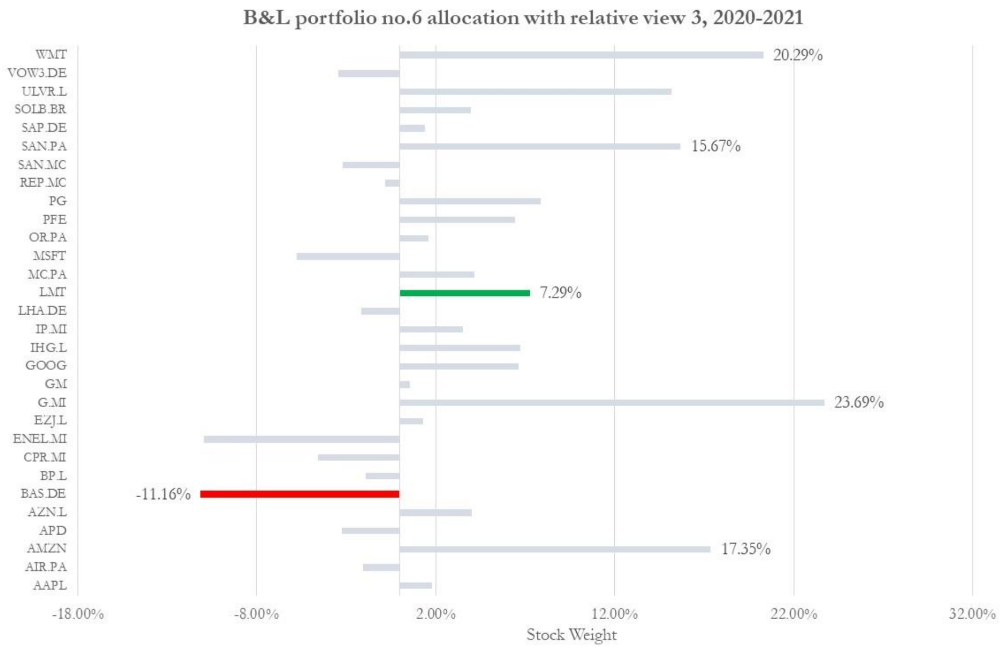

Figure 5.30. B&L portfolio no. 6 allocation with relative view on LMT, 2020-2021.

5.3 Portfolio construction using the Black-Litterman model – original formulation

In section 5.2 the Black-Litterman method has been empirically tested using returns instead of excess returns. To prove the validity of this assumption, the current section implements the B&L model in its original formulation, using excess returns.

The excess returns have been calculated by subtracting the daily average of the seven government bonds shown in Table 5.7 and Table 5.8 from the assets' logarithmic returns.

5.3.1 Reverse Optimization

Under the hypothesis that the CAPM equilibrium holds, the *reverse optimization method* computes the implied excess returns:

$$\theta = \lambda \Sigma \Omega_{mkt}$$

Where:

- θ is the vector of excess equilibrium returns for each asset
- λ is the risk aversion parameter already computed in section 5.2.1 and displayed in Table 5.9 and Table 5.10
- Σ is the covariance matrix of the excess returns for each asset, shown in Appendix 21 and Appendix 22.
- Ω_{mkt} is the same vector of the weights of each asset presented in section 5.2.1 in Table 5.5 and Table 5.6

The implied equilibrium excess returns should follow a normal distribution $E(R) \sim N(\theta, \tau\Sigma)$, where, according to Black and Litterman (1992) and Lee (2000) , τ was set equal to 0.025. The $\tau\Sigma$ matrix in the absence of views over 2015-2019 and 2020-2021 can be respectively found in Appendix 23 and Appendix 24.

When the implied excess returns are used as posterior distribution in the absence of views, the variance is computed as $(1+\tau)\Sigma$, as displayed in Table 5.22.

Table 5.24 displays the Δ between implied returns and implied excess returns with their respective volatility. The differences are always less than 0.4% and it is reasonable to expect that they will not have a strong influence on the subsequent Black-Litterman portfolio allocation. This minimal difference is attributable to the low mean value of the average risk-free rate and its neglectable volatility, as shown in Figure 5.31 and Table 5.23. The change is even less pronounced during 2020-2021 when the risk-free rate is close to 0.

Time horizon	2015-2019				2020-2021			
Stock	Historical Excess Returns	σ Historical Excess Returns	Implied Equilibrium Exc. Returns	σ Implied Equilibrium Exc. Returns	Historical Excess Returns	σ Historical Excess Returns	Implied Equilibrium Exc. Returns	σ Implied Equilibrium Exc. Returns
AAPL	21.27%	24.61%	23.31%	24.91%	45.09%	37.09%	42.60%	37.55%
AIR.PA	24.66%	26.64%	15.21%	26.97%	-5.90%	57.68%	21.12%	58.39%
AMZN	35.76%	28.81%	27.58%	29.17%	29.76%	32.27%	31.94%	32.67%
APD	13.50%	18.65%	12.29%	18.88%	14.57%	33.49%	26.24%	33.91%
AZN.L	14.39%	23.12%	7.90%	23.40%	8.66%	27.42%	9.61%	27.76%
BAS.DE	3.05%	22.10%	13.87%	22.37%	4.54%	34.00%	15.23%	34.42%
BP.L	9.49%	23.61%	9.67%	23.90%	-12.17%	45.70%	14.73%	46.27%
CPR.MI	24.26%	22.79%	10.07%	23.08%	23.16%	30.42%	15.34%	30.80%
ENEL.MI	17.24%	22.58%	10.42%	22.86%	4.81%	32.36%	18.90%	32.76%
EZJ.L	0.38%	34.99%	9.49%	35.43%	-45.34%	71.00%	18.97%	71.88%
G.MI	7.67%	25.26%	10.81%	25.57%	6.49%	27.00%	13.78%	27.34%
GM	5.08%	25.37%	13.20%	25.68%	23.60%	49.76%	25.89%	50.37%
GOOG	18.71%	23.81%	23.12%	24.11%	38.73%	31.37%	34.20%	31.76%
IHG.L	15.56%	24.34%	9.89%	24.65%	-3.47%	47.15%	18.71%	47.74%
IP.MI	18.12%	28.09%	12.64%	28.44%	41.63%	34.30%	15.28%	34.72%
LHA.DE	4.62%	30.89%	9.19%	31.27%	-49.19%	59.74%	16.79%	60.49%
LMT	16.79%	17.40%	9.72%	17.62%	-2.74%	32.05%	20.12%	32.45%
MC.PA	25.02%	25.25%	16.47%	25.57%	29.64%	30.99%	19.45%	31.37%
MSFT	26.42%	22.93%	23.88%	23.22%	38.93%	33.79%	39.66%	34.21%
OR.PA	14.71%	19.93%	11.19%	20.18%	25.10%	25.37%	15.11%	25.69%
PFE	7.90%	17.91%	10.15%	18.13%	26.50%	30.71%	16.35%	31.09%
PG	9.51%	15.50%	7.72%	15.69%	15.52%	24.85%	19.33%	25.16%
REP.MC	4.10%	27.39%	12.31%	27.73%	-9.46%	45.99%	16.76%	46.56%
SAN.MC	-7.35%	31.06%	15.40%	31.45%	-6.64%	46.05%	18.06%	46.62%
SAN.PA	7.31%	21.04%	10.29%	21.31%	2.97%	22.89%	8.47%	23.18%
SAP.DE	15.95%	21.40%	13.31%	21.66%	3.81%	32.49%	18.98%	32.90%
SOLB.BR	3.17%	24.45%	13.90%	24.75%	3.41%	36.75%	11.54%	37.21%
ULVR.L	13.45%	19.51%	7.27%	19.75%	-1.78%	23.05%	9.03%	23.33%
VOW3.DE	1.13%	33.17%	15.23%	33.58%	2.97%	44.87%	22.94%	45.43%
WMT	9.05%	19.32%	8.40%	19.56%	10.34%	25.18%	17.41%	25.50%

Table 5.22. Historical returns vs implied returns – original formulation.

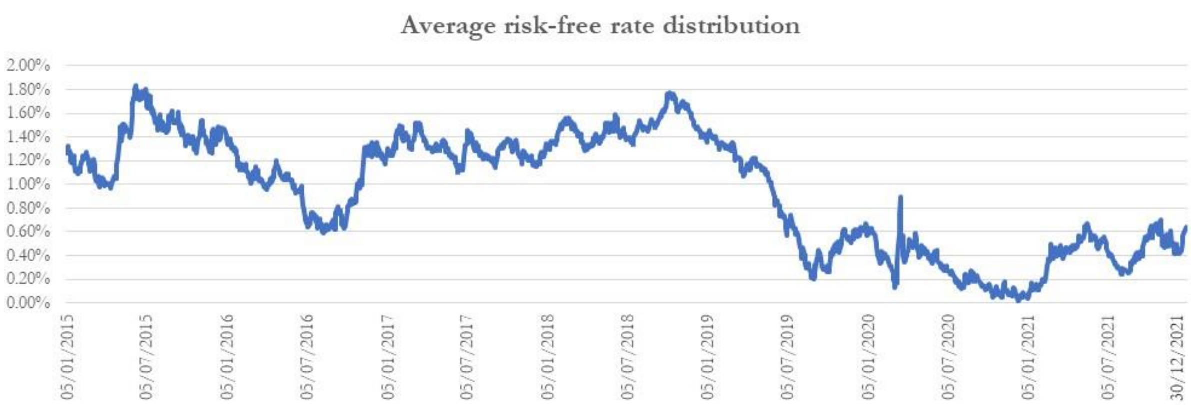

Figure 5.31. Average risk-free rate distribution.

Time Horizon	2015-2021	2015-2019	2020-2021
Annual Yield	0.96%	1.20%	0.36%
Annual Variance	2.4E-05	1.2E-05	3.1E-06
Annual Std. Dev	0.49%	0.34%	0.18%

Table 5.23. $\bar{R}$ and σ of the average risk-free rate.

Time horizon	2015-2019				2020-2021			
Stock	Δ Historical Returns	Δ σ of Historical Returns	Δ Implied Equilibrium Returns	Δ σ of Implied Equilibrium Returns	Δ Historical Returns	Δ σ of Historical Returns	Δ Implied Equilibrium Returns	Δ σ of Implied Equilibrium Returns
AAPL	0.033%	0.033%	0.119%	0.033%	0.357%	0.000%	0.001%	0.000%
AIR.PA	0.033%	0.028%	0.115%	0.029%	0.354%	0.000%	0.000%	0.000%
AMZN	0.033%	0.030%	0.123%	0.030%	0.369%	0.000%	0.001%	0.000%
APD	0.033%	0.035%	0.107%	0.036%	0.363%	0.001%	0.002%	0.001%
AZN.L	0.033%	0.015%	0.083%	0.015%	0.360%	0.001%	0.002%	0.001%
BAS.DE	0.033%	0.033%	0.112%	0.033%	0.359%	0.000%	0.000%	0.000%
BP.L	0.033%	0.024%	0.100%	0.024%	0.352%	0.001%	0.002%	0.001%
CPR.MI	0.033%	0.022%	0.095%	0.022%	0.366%	0.001%	0.002%	0.001%
ENEL.MI	0.033%	0.025%	0.101%	0.026%	0.359%	0.000%	0.001%	0.000%
EZJ.L	0.033%	0.015%	0.097%	0.015%	0.338%	0.000%	-0.001%	0.000%
G.MI	0.033%	0.026%	0.106%	0.026%	0.360%	0.000%	0.001%	0.000%
GM	0.033%	0.028%	0.112%	0.029%	0.367%	0.000%	0.000%	0.000%
GOOG	0.033%	0.033%	0.118%	0.034%	0.373%	0.000%	0.001%	0.000%
IHG.L	0.033%	0.020%	0.094%	0.020%	0.355%	0.000%	0.000%	0.000%
IP.MI	0.033%	0.023%	0.107%	0.024%	0.374%	0.000%	0.001%	0.000%
LHA.DE	0.033%	0.016%	0.096%	0.017%	0.337%	0.000%	0.000%	0.000%
LMT	0.033%	0.027%	0.093%	0.027%	0.356%	0.001%	0.002%	0.001%
MC.PA	0.033%	0.030%	0.115%	0.031%	0.369%	0.000%	0.001%	0.000%
MSFT	0.033%	0.038%	0.123%	0.038%	0.373%	0.000%	0.001%	0.000%
OR.PA	0.033%	0.026%	0.096%	0.026%	0.367%	0.001%	0.001%	0.001%
PFE	0.033%	0.029%	0.097%	0.029%	0.368%	0.000%	0.001%	0.000%
PG	0.033%	0.023%	0.084%	0.024%	0.363%	0.000%	0.001%	0.000%
REP.MC	0.033%	0.028%	0.115%	0.028%	0.353%	0.000%	0.000%	0.000%
SAN.MC	0.033%	0.030%	0.128%	0.030%	0.354%	0.000%	0.001%	0.000%
SAN.PA	0.033%	0.024%	0.096%	0.025%	0.358%	0.001%	0.002%	0.001%
SAP.DE	0.033%	0.028%	0.103%	0.029%	0.358%	0.001%	0.002%	0.001%
SOLB.BR	0.033%	0.030%	0.113%	0.030%	0.358%	0.000%	0.001%	0.001%
ULVR.L	0.033%	0.016%	0.080%	0.016%	0.356%	0.001%	0.001%	0.001%
VOW3.DE	0.033%	0.025%	0.119%	0.025%	0.358%	0.000%	0.001%	0.000%
WMT	0.033%	0.020%	0.087%	0.021%	0.361%	0.000%	0.001%	0.000%

Table 5.24. Δ between returns and excess returns with their respective volatility.

5.3.2 Capital allocation without the investor's views

If the investor has no views on the future stocks' performance, he could create his portfolio by relying on the implied equilibrium excess returns, whose vector can be found in Table 5.22. This neutral reference points together with the annual theoretical variance-covariance matrix computed as $(1+\tau)\Sigma$ were the inputs to the mean variance optimization problem solved by means of the Excel Solver. Eventually, the stock allocation resulting from the B&L approach and the Markowitz method have been compared to see if some differences from section 5.2.2 occur.

Comparison with Markowitz stock allocation method

New Markowitz and Black-Litterman portfolios' weights have been computed, by allowing short selling, as shown in Appendix 25, Appendix 26, Appendix 27 and Appendix 28.

Portfolio concentration

Regarding the Herfindahl-Hirschman index, identical results to the ones discussed for the simplified formulation have been obtained. Indeed, as shown in Figure 5.32 and Figure 5.33, 80% of the portfolios created by applying the B&L original method to 2015-2019 data are less concentrated, and this percentage increases to more than 90% during 2020-2021.

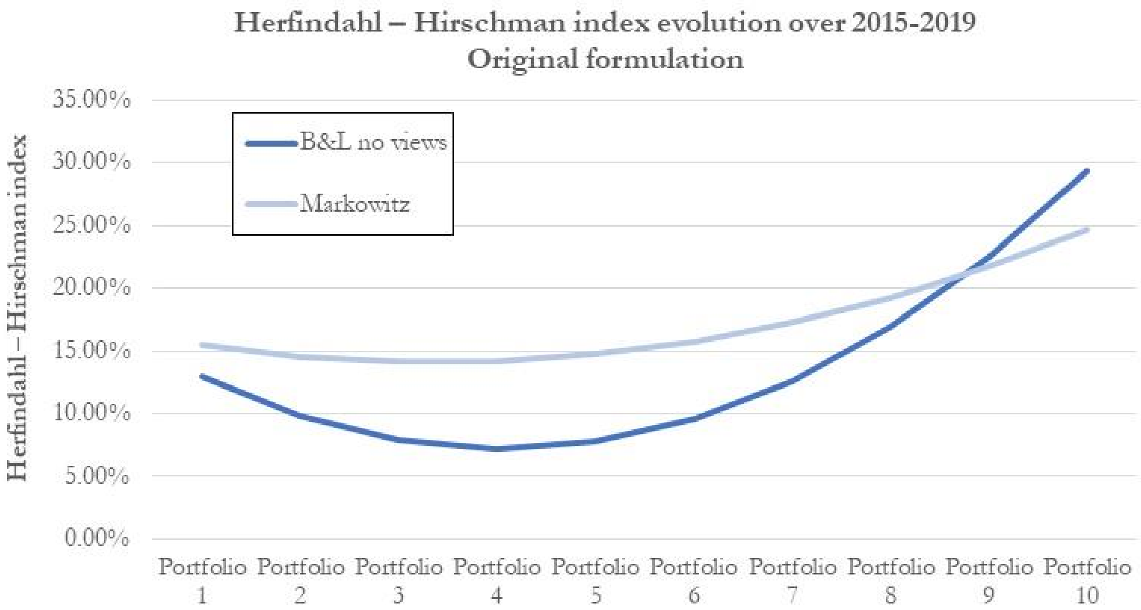

Figure 5.32. Herfindahl – Hirschman index evolution over 2015-2019 – original formulation.

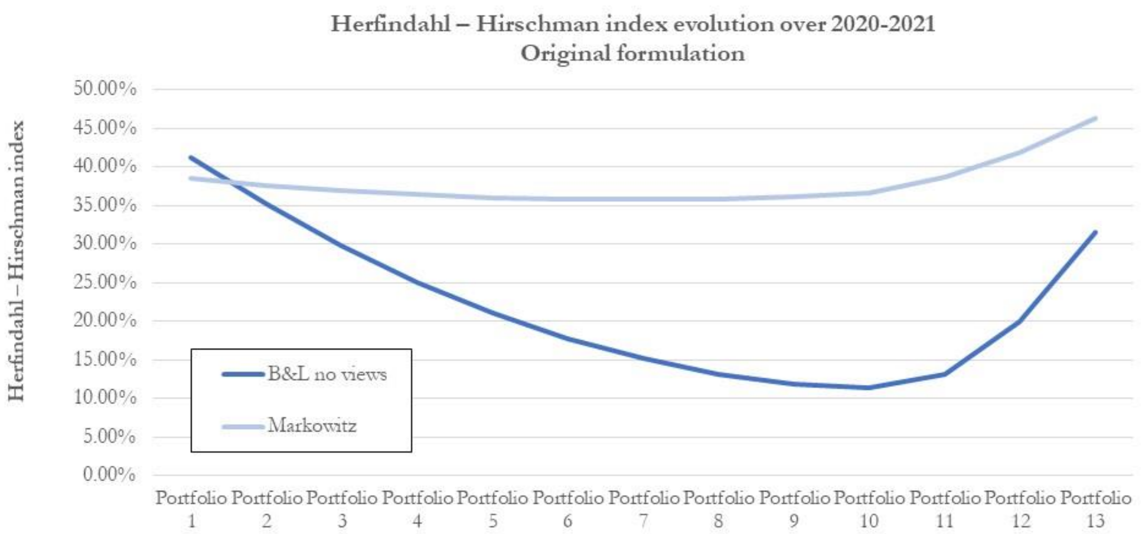

Figure 5.33. Herfindahl – Hirschman index evolution over 2020-2021 – original formulation.

Portfolio stability

As in the previous case, the metrics "Portfolio Instability" confirms the stability of the portfolios created by applying the original formulation of the Black-Litterman model. As Table 5.25 exhibits, portfolio no. 6 (with a target of 18.35% excess return) has proved to be more stable under the B&L's approach than the Markowitz's.

	2015-2019		2020-2021	
	Portfolio 6 Markowitz	Portfolio 6 B&L no views	Portfolio 6 Markowitz	Portfolio 6 B&L no views
Weights σ	**6.43%**	**4.54%**	**10.41%**	**6.93%**

Markowitz Instability	3.97%
B&L Instability	2.39%

Table 5.25. Markowitz & Black-Litterman portfolio instability indexes – original formulation.

5.3.3 Bayesian approach

The implied excess returns over 2015-2019 and 2020-2021 shown in Table 5.22 constitute the prior distribution to the Bayes formula. The same views formulated in section 5.2.3 and mentioned again below have been applied. They are the starting point of the investor's capital allocation.

Views to be applied to the implied excess returns over 2015-2019

Context - It is end of April 2020, the Covid-19 pandemic has already hit the world and the investor formulates 2 relative views.

3) Relative View 1 – *Pfizer (PFE)* will outperform *easyJet (EZJ.L)* and *Lufthansa (LHA.DE)* by 35% in the next 12 months
4) Relative View 2 – *Apple (AAPL)* will outperform *Airbus (AIR.PA)* and *BP (BP.L)* by 40% in the next 12 months

Views to be applied to the implied excess returns over 2020-2021

Context - It is March 2022, the current economic situation is impacted by increasing interest rates and the conflict in Ukraine and the investor formulates 2 relative views.

1) Relative View 1 – *Unilever (ULVR.L)* will outperform *BASF SE (BAS.DE)* by 25% in the next 12 months

2) Relative View 2 – *Amazon (AMZN)* will outperform *Volkswagen (VOW3.DE)* by 40% in the next 12 months

Bayesian Approach

Once the views have been formulated, the P matrix of the asset weights according to each view, and the Q vector, expressing the expected excess returns for each view, can be built. They are the same as the ones already displayed in in Table 5.16 and Table 5.17.

As next step, the covariance Ψ matrix of the error terms has been computed as $\Psi = diag(P(\tau\Sigma)P^T)$, as shown in Table 5.26. It should be underlined these values differ from the ones presented in Table 5.18, only from the 17th decimal number.

2015-2019		
Ψ matrix	**View 1**	**View 2**
View 1	0.002637	0
View 2	0	0.001881

2020-2021		
Ψ matrix	**View 1**	**View 2**
View 1	0.002843	0
View 2	0	0.00656

Table 5.26. Covariance matrix of the error term, 2015-2019 and 2020-2021 – original formulation.

At this point, it is possible to combine the specific personal investor's views with the equilibrium values through the Black-Litterman master formula, while computing the variance of the expected returns to be used in the mean variance optimizer as $\Sigma_p = \Sigma + ((\tau\Sigma)^{-1} + P^T\Psi^{-1}P)^{-1}$. Table 5.27 shows the B&L $E(R)|\theta$ and their σ over 2015-2019 and 2020-2021, while the Σ_p over the two time periods can be found in Appendix 29 and Appendix 30.

Time horizon	2015-2019		2020-2021	
Stock	E(R)|θ	σ	E(R)|θ	σ
AAPL	32.23%	24.84%	42.20%	37.54%
AIR.PA	7.24%	26.94%	3.75%	58.26%
AMZN	32.20%	29.15%	35.83%	32.62%
APD	12.89%	18.88%	21.23%	33.89%
AZN.L	6.50%	23.40%	10.31%	27.75%
BAS.DE	9.18%	22.36%	0.61%	34.26%
BP.L	4.93%	23.87%	0.16%	46.15%
CPR.MI	7.10%	23.07%	12.99%	30.79%
ENEL.MI	5.53%	22.84%	13.28%	32.73%
EZJ.L	-4.28%	35.30%	-1.37%	71.73%
G.MI	4.62%	25.55%	4.84%	27.26%
GM	13.44%	25.68%	11.76%	50.27%
GOOG	27.00%	24.10%	32.42%	31.76%
IHG.L	5.66%	24.64%	3.53%	47.61%
IP.MI	8.58%	28.43%	9.25%	34.70%
LHA.DE	-3.31%	31.18%	0.89%	60.38%
LMT	11.70%	17.62%	16.46%	32.44%
MC.PA	11.88%	25.56%	11.81%	31.32%
MSFT	27.64%	23.21%	39.56%	34.20%
OR.PA	7.91%	20.17%	13.17%	25.68%
PFE	14.96%	18.11%	14.78%	31.09%
PG	9.35%	15.69%	19.71%	25.16%
REP.MC	6.12%	27.71%	1.83%	46.43%
SAN.MC	7.61%	31.42%	1.85%	46.48%
SAN.PA	7.77%	21.30%	6.39%	23.17%
SAP.DE	9.83%	21.65%	13.63%	32.87%
SOLB.BR	8.88%	24.74%	0.87%	37.13%
ULVR.L	5.42%	19.74%	11.72%	23.31%
VOW3.DE	9.26%	33.57%	5.91%	45.22%
WMT	10.07%	19.56%	18.34%	25.49%

Table 5.27. B&L $E(R)|\theta$ and their σ, 2015-2019 and 2020-2021 – original formulation.

Table 5.28 displays the differences between the combined returns in Table 5.19 and the combined excess returns in Table 5.27. The Δ are always less than 0.14% and, regarding the 2020-2021 values, the variations can be appreciated only from the third decimals, due to the almost negligible influence of the risk-free rate that is close to zero. Given the small differences between combined returns and combined excess returns, it is reasonable to expect negligible changes in the subsequent mean-variance stock allocation, as it is analysed in the following section.

Time horizon	2015-2019		2020-2021	
Stock	Δ E(R)|θ	Δ σ	Δ E(R)|θ	Δ σ
AAPL	0.11837%	0.03336%	0.00094%	0.00008%
AIR.PA	0.11961%	0.02858%	0.00038%	-0.00032%
AMZN	0.12434%	0.03023%	0.00141%	0.00048%
APD	0.10969%	0.03557%	0.00202%	0.00071%
AZN.L	0.08624%	0.01500%	0.00221%	0.00116%
BAS.DE	0.11586%	0.03304%	0.00107%	-0.00007%
BP.L	0.10540%	0.02444%	0.00241%	0.00058%
CPR.MI	0.09834%	0.02216%	0.00235%	0.00108%
ENEL.MI	0.10488%	0.02584%	0.00167%	0.00049%
EZJ.L	0.09997%	0.01533%	0.00025%	-0.00033%
G.MI	0.11026%	0.02591%	0.00150%	0.00039%
GM	0.11471%	0.02872%	0.00033%	-0.00035%
GOOG	0.11937%	0.03375%	0.00097%	0.00010%
IHG.L	0.09721%	0.02003%	0.00071%	-0.00021%
IP.MI	0.11047%	0.02358%	0.00134%	0.00026%
LHA.DE	0.09886%	0.01671%	0.00078%	-0.00016%
LMT	0.09509%	0.02732%	0.00208%	0.00082%
MC.PA	0.11899%	0.03056%	0.00158%	0.00042%
MSFT	0.12497%	0.03811%	0.00138%	0.00038%
OR.PA	0.09991%	0.02619%	0.00187%	0.00091%
PFE	0.09910%	0.02937%	0.00141%	0.00043%
PG	0.08686%	0.02368%	0.00107%	0.00028%
REP.MC	0.11983%	0.02805%	0.00097%	-0.00010%
SAN.MC	0.13235%	0.03012%	0.00148%	0.00011%
SAN.PA	0.09990%	0.02481%	0.00203%	0.00115%
SAP.DE	0.10651%	0.02855%	0.00211%	0.00079%
SOLB.BR	0.11694%	0.03039%	0.00194%	0.00051%
ULVR.L	0.08389%	0.01624%	0.00161%	0.00090%
VOW3.DE	0.12321%	0.02484%	0.00131%	0.00006%
WMT	0.08907%	0.02051%	0.00095%	0.00018%

Table 5.28. Δ between combined returns and combined excess returns with their respective volatility.

5.3.4 Capital allocation with the investor's views

To prove the validity of the results discussed in section 5.2.4, portfolio no. 6 weights have been recomputed using excess returns as inputs. As expected, the values differ only for decimals and the variations are even less significant when the allocation considers 2020-2021 data.

Portfolio allocation using 2015-2019 data

If at the beginning of 2020 the investor relied on historical returns, the two highest weighted stocks in portfolio no. 6 would be PG (19.08%) and LMT (16.84%), as it has been displayed in Figure 5.20. From Figure 5.34, it is possible to notice that the same would happen if historical excess returns were employed, but the stock weights would be 19.02% (PG) and 16.82% (LMT).

If the investor has no views and decides to allocate his funds according to equilibrium values, he will obtain a portfolio characterized by a lower stock concentration than under Markowitz's. If implied returns are used as inputs to the means variance allocation, AAPL weight is 16.46% and MSFT's is 15.40% (Figure 5.21). Minimal variations occur if implied excess returns are employed, as shown in Figure 5.35.

As the investor expresses his personal views, the Black-Litterman capital allocation becomes dynamic and the stocks on which the investor has upward expectations are favoured. If the inputs are the combined returns, AAPL and PFE weights are 18.72% and 15.78%; when the combined excess returns are used, they become 18.88% and 15.83%. On the contrary, the B&L original formulation slightly increase the investor's short positions on those stocks that are characterized by downward expectations. As Figure 5.36 exhibits, this model leads to -6.79% AIR.PA, -0.35% BP.L, -2.40% EZJ.L and -2.00% LHA.DE, while the simplified version would allocate respectively -6.77%, -0.28%, -2.35% and -1.94%, as it has been displayed in Figure 5.22.

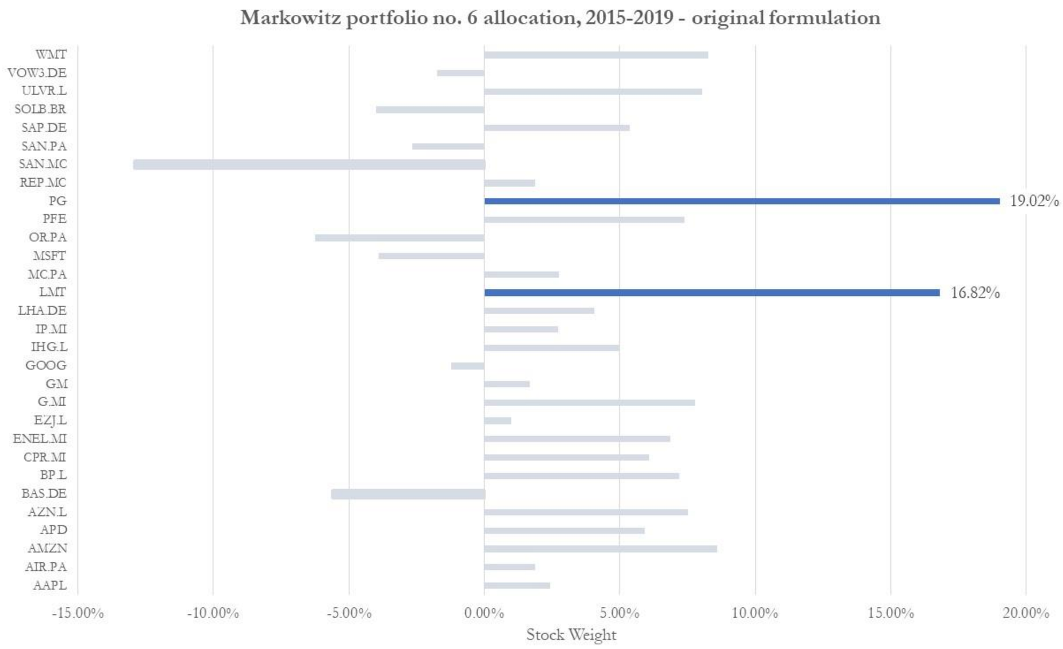

Figure 5.34. Markowitz portfolio no.6 allocation, 2015-2019 – original formulation.

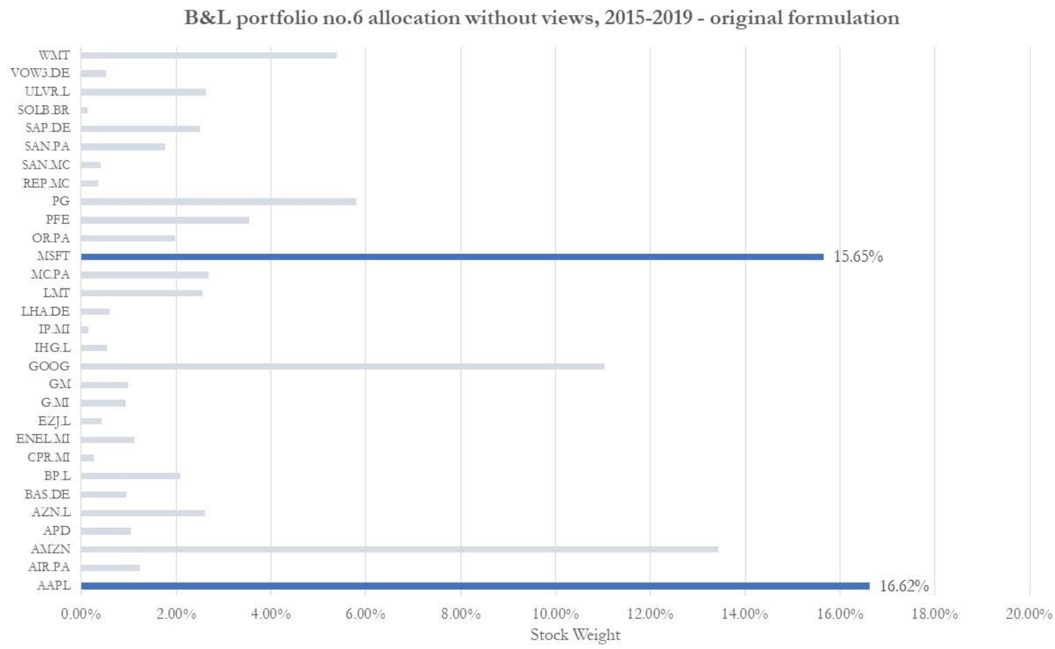

Figure 5.35. B&L portfolio no.6 allocation without views, 2015-2019 – original formulation.

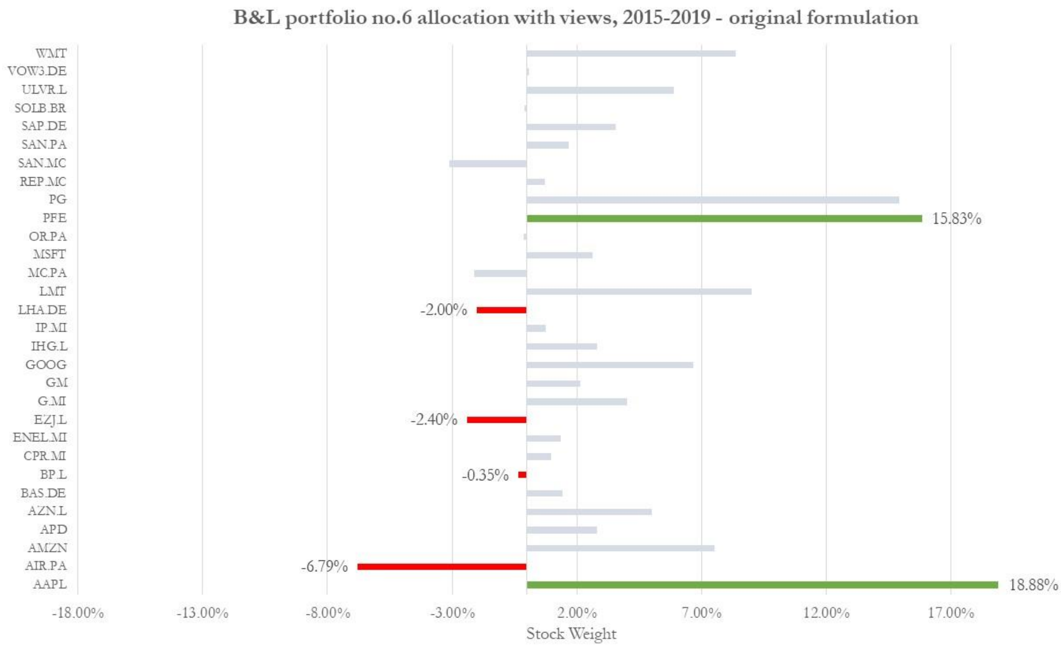

Figure 5.36. B&L portfolio no.6 allocation with views, 2015-2019 – original formulation.

Portfolio allocation using 2020-2021 data

If at the beginning of 2022 the investor relied on historical data, the largest weights would be assigned to G.MI (31.41% when historical returns are used, 31.40% when historical excess returns are the inputs) and WMT (23.41% and 23.30%), as displayed in Figure 5.24 and Figure 5.37.

Figure 5.25 and Figure 5.38 exhibit portfolio no. 6 stock allocation using respectively the simplified and the original B&L allocation method. In both cases, the two largest stock weights are 20.32% (G.MI) and 17.85% (WMT).

In line with the investor's views, the dynamic asset allocation that results from both formulations would favour ULVR.L (22.02% under the simplified version and 22.01% under the original formulation) and AMZN (19.72% in both), while penalise BAS.DE (-10.32% in both) and VOW3.DE (-5.75% and -5.76%). The visual representations are given in Figure 5.26 and Figure 5.39.

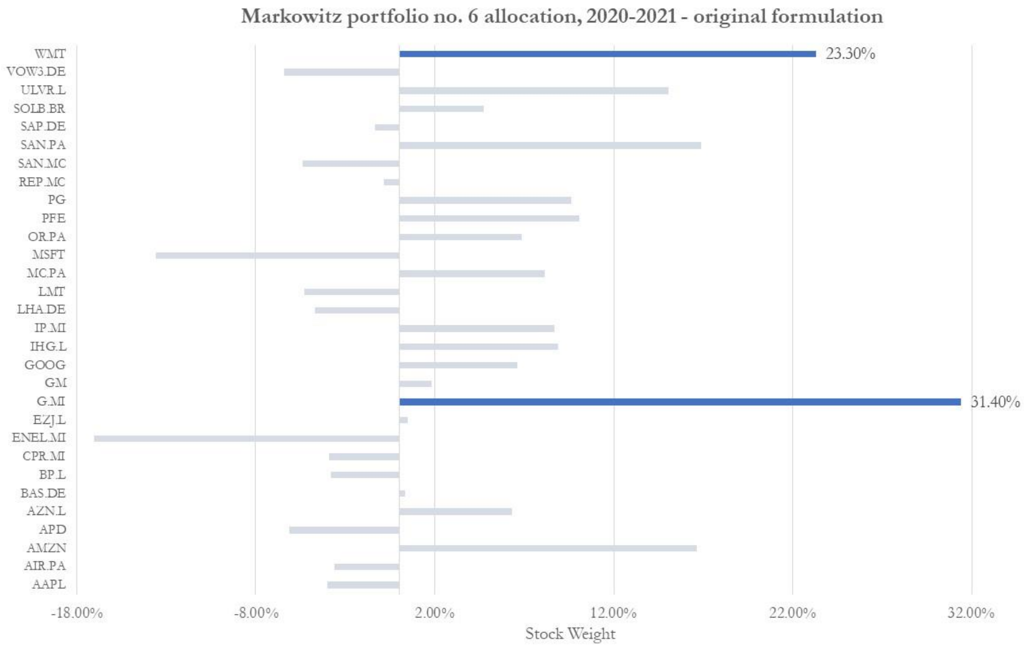

Figure 5.37. Markowitz portfolio no.6 allocation, 2020-2021 – original formulation.

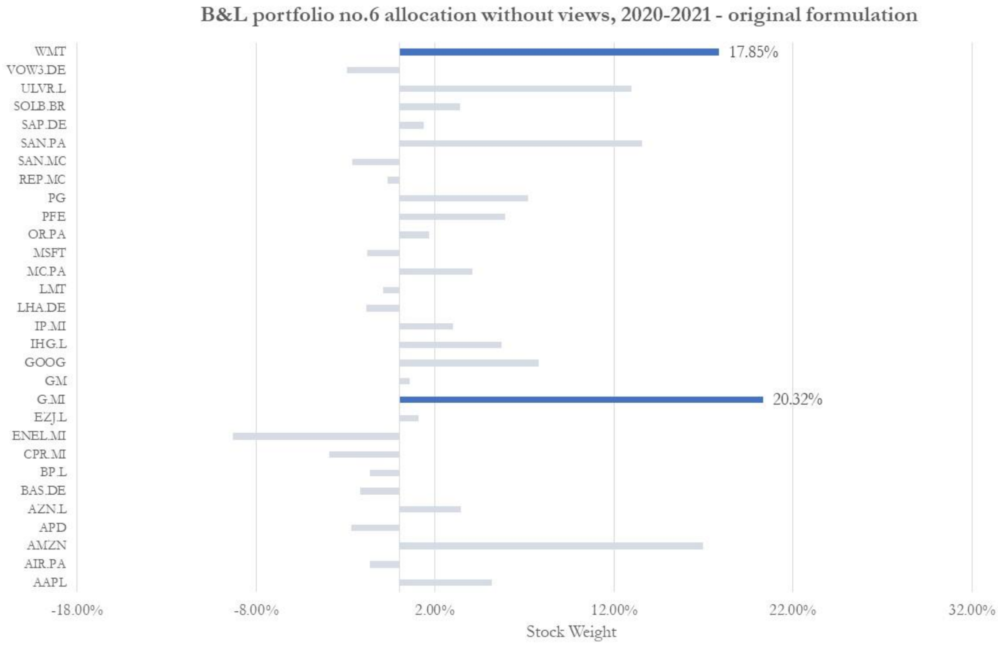

Figure 5.38. B&L portfolio no.6 allocation without views, 2020-2021 – original formulation.

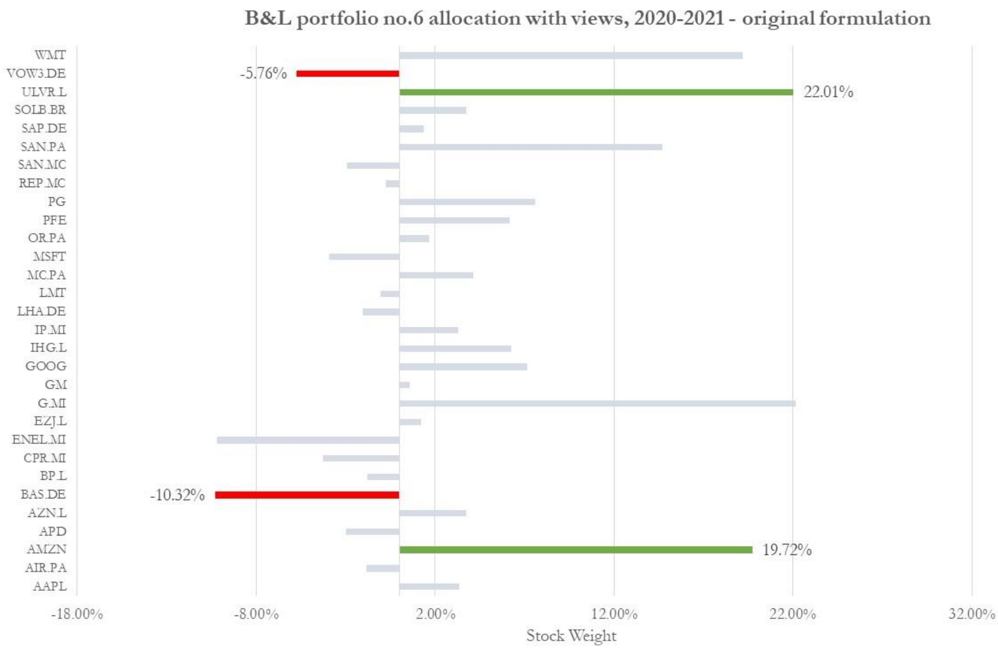

Figure 5.39. B&L portfolio no.6 allocation with views, 2020-2021 – original formulation.

6 Conclusion

By means of a diversified portfolio of 30 stocks and analysed over two different time horizons, 2015-2019 and 2020-2021, this work aims at answering the following main research question: *What are the benefits of the Back-Litterman portfolio allocation model compared to the Markowitz approach?*

The empirical analysis has highlighted the main limitations of the Markowitz model. By preferring securities that in the past exhibited higher expected returns with lower variability and lower or even negative covariances, the model has created portfolios concentrated on stocks with these characteristics, which represent a minority. Moreover, the Markowitz model has proved to be unstable, due to the strong impact that small changes in expected returns have on portfolio weights, since they strongly depend on past performance. Consequently, if the investor updated the historical returns series, this would lead to significant weight fluctuations. The fragility of Markowitz's model has also been observed when analysing the impact that changes in the covariance matrix have on weights. To conclude, the model does not include personal views on the future and the investor should periodically review his portfolio from historical segment to historical segment, incurring high transaction costs that would erode profits.

By applying the Bayes theorem, the Black-Litterman approach overcomes the above-mentioned limitations. While the original model combines implied equilibrium excess returns based on the CAPM with investors' views, this work employed equilibrium returns to favour the comparison. To prove the robustness of the results obtained under this simplification, the B&L model has been later implemented in its original formulation, thus confirming the validity of the findings. Moreover, the 30 selected stocks have proven to be a systematically distorted sample compared to the market portfolio, since many of them were resilient during the pandemic. Therefore, differently from the original approach, the risk aversion parameter λ has been determined using the weighted average of the stock returns as market benchmark.

As Black and Litterman addressed the allocation problem by working with the maximum degrees of freedom, so did this work. If the investor has no views on the future stocks' performances, he could create his portfolio relying on the implied equilibrium returns. Being neutral reference points, the tendency of the mean variance optimization to create aggregated portfolios is minimized. By using the Herfindahl-Hirschman index as metrics of stock concentration, this thesis has proved that 80% of the portfolios obtained through the B&L's approach during 2015-2019 are less concentrated than the ones obtained under Markowitz's, and this percentage increases to more than 90% during 2020-2021. Moreover, as the metrics "Portfolio Instability" shows, the use of the implied equilibrium returns leads to more stable portfolios characterized by less abrupt variations in the portfolio weights as the correlation matrix changes.

The implied returns over 2015-2019 and 2020-2021 represent the prior distribution to the Bayes formula and constitute the starting point to the investor's capital allocation. Two different sets of views, one for each time horizon, have been formulated. While the investor's expectations to be applied to 2015-2019 implied returns are affected by Covid-19, the other views consider the current economic situation impacted by increasing interest rates and the conflict in Ukraine. Since no investor's utility function has been specified nor a risk-free asset has been included in the portfolio, 18.35% has been set as target return. The impact of the views has then been evaluated on this portfolio: the stocks on which the investor has upward expectations are favoured; on the contrary, stocks characterized by downward views present negative weights.

Despite the evident advantages of the Black-Litterman model, however some limitations exist. The two major problems of this approach concern the scalar parameter τ and the covariance matrix Ψ. As in literature there is little guidance for setting them, many variants have been proposed over the years.

6.1 Further developments

This thesis thoroughly compares the Markowitz and Black-Litterman methods, but some future developments are still possible.

a) The 30 selected stock are listed on seven different Stock Exchanges and quoted in three currencies, namely Euro, Dollar and Pound. Including currency risk and return and partial hedging could enrich the analysis
b) A possible extension to the Black-Litterman model could be pursued by adding constraints to the allocation problem. By doing so, the investor can obtain different results from the ones derived from the implementation of the mean-variance model and the formulation of views. For example, constraints regarding the minimum or maximum weights of some stocks may be added, following a-priori choices of "political" nature.

c) Instead of assuming a normal distribution of the stock returns, a t-Student distribution could be employed. Although remaining symmetrical, however it considers leptokurtic assets' returns, leading to a better fit of the data

d) As alternative approaches to the mean-variance optimization, the mean – value at risk or the mean- expected shortfall models could be applied.

   VaR is defined as the worst loss that an investor can expect with a given probability for a particular confidence level. This statistical measure is preferred by portfolio managers since it captures within a single number all the risks associated to a portfolio. If the returns follow a normal distribution, then both the mean-variance and the mean-VaR optimization leads to the same results. However, if the distributions are not elliptical, then the outcome change.

   Expected Shortfall (ES), also known as Conditional-var (C-var) or Tail-var, is a risk measure that measures the amount of tail risk. It determines how large the losses can be if the return on the portfolio or asset falls below its "α-quantile". Unlike the VaR, the ES provides more information about the potential loss that the investor could sustain.

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Appendices

Correlation between stocks over 2015-2021

	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	1.000																													
AIR.PA	0.219	1.000																												
AMZN	0.560	0.142	1.000																											
APD	0.459	0.316	0.297	1.000																										
AZN.L	0.167	0.210	0.139	0.162	1.000																									
BAS.DE	0.288	0.576	0.181	0.463	0.236	1.000																								
BP.L	0.175	0.471	0.084	0.340	0.214	0.579	1.000																							
CPR.MI	0.235	0.318	0.206	0.289	0.295	0.391	0.268	1.000																						
ENEL.MI	0.232	0.400	0.205	0.396	0.301	0.534	0.417	0.501	1.000																					
EZJ.L	0.126	0.545	0.064	0.231	0.038	0.432	0.326	0.225	0.267	1.000																				
G.MI	0.215	0.512	0.141	0.344	0.178	0.615	0.477	0.413	0.611	0.420	1.000																			
GM	0.316	0.467	0.188	0.446	0.072	0.456	0.414	0.208	0.259	0.388	0.387	1.000																		
GOOG	0.598	0.277	0.647	0.452	0.164	0.314	0.195	0.263	0.288	0.196	0.269	0.363	1.000																	
IHG.L	0.209	0.596	0.117	0.356	0.160	0.510	0.480	0.296	0.341	0.536	0.422	0.435	0.255	1.000																
IP.MI	0.260	0.416	0.186	0.325	0.209	0.457	0.294	0.352	0.405	0.262	0.432	0.274	0.269	0.341	1.000															
LHA.DE	0.134	0.493	0.081	0.257	0.105	0.486	0.337	0.260	0.326	0.656	0.452	0.327	0.180	0.441	0.285	1.000														
LMT	0.368	0.269	0.251	0.467	0.173	0.325	0.258	0.307	0.290	0.182	0.249	0.295	0.363	0.263	0.202	0.206	1.000													
MC.PA	0.343	0.552	0.264	0.410	0.272	0.626	0.434	0.437	0.500	0.389	0.521	0.373	0.348	0.490	0.421	0.388	0.278	1.000												
MSFT	0.676	0.251	0.640	0.521	0.196	0.307	0.199	0.284	0.318	0.142	0.260	0.352	0.728	0.234	0.256	0.145	0.425	0.370	1.000											
OR.PA	0.272	0.407	0.241	0.375	0.383	0.505	0.369	0.528	0.562	0.233	0.448	0.234	0.288	0.368	0.366	0.278	0.309	0.660	0.332	1.000										
PFE	0.318	0.171	0.235	0.400	0.231	0.229	0.191	0.203	0.200	0.106	0.185	0.263	0.349	0.143	0.162	0.128	0.416	0.238	0.378	0.251	1.000									
PG	0.385	0.123	0.270	0.462	0.189	0.200	0.146	0.254	0.278	0.025	0.166	0.219	0.374	0.097	0.170	0.085	0.443	0.212	0.465	0.315	0.406	1.000								
REP.MC	0.197	0.511	0.111	0.374	0.135	0.624	0.788	0.288	0.467	0.394	0.578	0.426	0.238	0.474	0.342	0.403	0.269	0.462	0.235	0.359	0.205	0.160	1.000							
SAN.MC	0.209	0.544	0.139	0.362	0.133	0.675	0.561	0.323	0.545	0.461	0.706	0.434	0.268	0.439	0.387	0.481	0.265	0.523	0.248	0.409	0.198	0.165	0.689	1.000						
SAN.PA	0.201	0.338	0.147	0.246	0.495	0.446	0.329	0.409	0.494	0.149	0.411	0.201	0.218	0.254	0.317	0.229	0.251	0.455	0.240	0.540	0.279	0.216	0.341	0.377	1.000					
SAP.DE	0.309	0.457	0.285	0.368	0.285	0.539	0.329	0.414	0.484	0.301	0.463	0.273	0.343	0.412	0.411	0.321	0.252	0.553	0.381	0.544	0.211	0.211	0.363	0.423	0.450	1.000				
SOLB.BR	0.228	0.487	0.174	0.373	0.211	0.734	0.497	0.447	0.472	0.368	0.552	0.369	0.260	0.403	0.443	0.442	0.293	0.521	0.233	0.436	0.204	0.161	0.540	0.588	0.370	0.426	1.000			
ULVR.L	0.161	0.207	0.137	0.257	0.412	0.332	0.268	0.406	0.394	0.083	0.232	0.090	0.176	0.225	0.205	0.132	0.236	0.358	0.224	0.615	0.172	0.328	0.186	0.170	0.436	0.355	0.285	1.000		
VOW3.DE	0.269	0.501	0.170	0.366	0.197	0.628	0.462	0.289	0.420	0.403	0.533	0.471	0.297	0.464	0.381	0.404	0.246	0.539	0.273	0.386	0.181	0.138	0.523	0.573	0.325	0.452	0.514	0.192	1.000	
WMT	0.342	0.048	0.253	0.359	0.131	0.133	0.073	0.181	0.209	0.000	0.105	0.160	0.292	0.039	0.124	0.058	0.345	0.159	0.377	0.181	0.325	0.487	0.076	0.116	0.128	0.158	0.122	0.165	0.083	1.000

Appendix 1. Correlation matrix of the daily log-returns over 2015-2021.

Correlation between stocks over 2015-2019

	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	1.000																													
AIR.PA	0.263	1.000																												
AMZN	0.492	0.226	1.000																											
APD	0.397	0.331	0.276	1.000																										
AZN.L	0.143	0.310	0.108	0.154	1.000																									
BAS.DE	0.327	0.561	0.259	0.421	0.266	1.000																								
BP.L	0.187	0.347	0.155	0.283	0.274	0.457	1.000																							
CPR.MI	0.174	0.366	0.182	0.296	0.285	0.391	0.244	1.000																						
ENEL.MI	0.141	0.441	0.158	0.321	0.251	0.492	0.360	0.499	1.000																					
EZJ.L	0.140	0.295	0.097	0.201	0.072	0.322	0.019	0.199	0.247	1.000																				
G.MI	0.168	0.456	0.143	0.303	0.160	0.535	0.315	0.363	0.594	0.338	1.000																			
GM	0.321	0.281	0.240	0.414	0.049	0.332	0.236	0.156	0.176	0.198	0.286	1.000																		
GOOG	0.524	0.266	0.648	0.382	0.153	0.293	0.140	0.206	0.206	0.144	0.206	0.320	1.000																	
IHG.L	0.187	0.405	0.172	0.268	0.269	0.390	0.318	0.302	0.291	0.235	0.292	0.218	0.198	1.000																
IP.MI	0.250	0.445	0.191	0.299	0.211	0.463	0.280	0.329	0.369	0.275	0.388	0.228	0.219	0.343	1.000															
LHA.DE	0.133	0.339	0.107	0.203	0.103	0.404	0.104	0.240	0.297	0.567	0.398	0.213	0.145	0.264	0.268	1.000														
LMT	0.320	0.228	0.280	0.370	0.143	0.229	0.139	0.203	0.193	0.085	0.142	0.226	0.330	0.152	0.136	0.104	1.000													
MC.PA	0.329	0.584	0.263	0.386	0.287	0.638	0.361	0.431	0.483	0.317	0.467	0.279	0.302	0.399	0.426	0.345	0.227	1.000												
MSFT	0.566	0.315	0.618	0.466	0.158	0.344	0.227	0.255	0.263	0.136	0.235	0.348	0.664	0.230	0.242	0.147	0.409	0.342	1.000											
OR.PA	0.208	0.508	0.214	0.330	0.391	0.548	0.351	0.525	0.549	0.214	0.398	0.193	0.229	0.383	0.341	0.259	0.225	0.657	0.300	1.000										
PFE	0.301	0.227	0.283	0.354	0.197	0.216	0.172	0.163	0.159	0.088	0.165	0.319	0.342	0.129	0.184	0.115	0.372	0.214	0.382	0.222	1.000									
PG	0.256	0.155	0.203	0.336	0.128	0.164	0.142	0.222	0.215	0.048	0.114	0.200	0.262	0.087	0.113	0.049	0.288	0.202	0.348	0.280	0.301	1.000								
REP.MC	0.210	0.424	0.165	0.335	0.148	0.573	0.711	0.291	0.482	0.178	0.497	0.311	0.180	0.315	0.351	0.257	0.146	0.429	0.254	0.354	0.170	0.134	1.000							
SAN.MC	0.203	0.482	0.192	0.367	0.142	0.625	0.429	0.301	0.573	0.334	0.683	0.346	0.242	0.294	0.392	0.397	0.145	0.496	0.275	0.385	0.195	0.130	0.681	1.000						
SAN.PA	0.202	0.463	0.155	0.285	0.456	0.480	0.345	0.421	0.491	0.176	0.399	0.185	0.219	0.306	0.306	0.242	0.234	0.491	0.241	0.587	0.298	0.190	0.383	0.429	1.000					
SAP.DE	0.283	0.529	0.275	0.349	0.307	0.573	0.326	0.444	0.478	0.282	0.446	0.232	0.302	0.368	0.428	0.307	0.210	0.578	0.360	0.585	0.215	0.175	0.366	0.468	0.499	1.000				
SOLB.BR	0.296	0.532	0.246	0.363	0.229	0.742	0.437	0.365	0.442	0.312	0.508	0.297	0.269	0.392	0.443	0.391	0.182	0.577	0.301	0.450	0.230	0.137	0.533	0.584	0.416	0.509	1.000			
ULVR.L	0.125	0.290	0.110	0.201	0.397	0.323	0.298	0.411	0.391	0.096	0.200	0.052	0.131	0.321	0.200	0.118	0.170	0.383	0.189	0.654	0.141	0.310	0.186	0.163	0.443	0.384	0.263	1.000		
VOW3.DE	0.236	0.439	0.184	0.296	0.178	0.564	0.326	0.261	0.389	0.279	0.462	0.335	0.237	0.291	0.369	0.333	0.135	0.471	0.235	0.338	0.166	0.088	0.460	0.531	0.333	0.420	0.517	0.159	1.000	
WMT	0.229	0.163	0.182	0.259	0.063	0.122	0.104	0.126	0.157	0.067	0.097	0.194	0.209	0.104	0.117	0.076	0.287	0.161	0.268	0.157	0.295	0.364	0.090	0.118	0.109	0.159	0.131	0.117	0.075	1.000

Appendix 2. Correlation matrix of the daily log-returns over 2015-2019.

Correlation between stocks over 2020-2021

	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI
AAPL	1.000														
AIR.PA	0.193	1.000													
AMZN	0.675	0.070	1.000												
APD	0.520	0.308	0.341	1.000											
AZN.L	0.203	0.131	0.198	0.178	1.000										
BAS.DE	0.245	0.602	0.073	0.505	0.199	1.000									
BP.L	0.169	0.546	0.009	0.382	0.161	0.688	1.000								
CPR.MI	0.313	0.295	0.246	0.289	0.311	0.394	0.301	1.000							
ENEL.MI	0.339	0.382	0.281	0.474	0.378	0.583	0.477	0.505	1.000						
EZJ.L	0.119	0.687	0.033	0.252	0.005	0.529	0.522	0.259	0.291	1.000					
G.MI	0.296	0.640	0.139	0.425	0.214	0.764	0.720	0.508	0.656	0.563	1.000				
GM	0.316	0.579	0.145	0.469	0.102	0.565	0.532	0.265	0.339	0.508	0.548	1.000			
GOOG	0.694	0.303	0.651	0.537	0.183	0.342	0.258	0.344	0.399	0.256	0.384	0.418	1.000		
IHG.L	0.231	0.711	0.063	0.419	0.051	0.617	0.587	0.300	0.393	0.729	0.622	0.580	0.321	1.000	
IP.MI	0.277	0.423	0.178	0.367	0.207	0.457	0.327	0.388	0.460	0.268	0.520	0.336	0.347	0.358	1.000
LHA.DE	0.138	0.586	0.057	0.297	0.113	0.562	0.492	0.288	0.358	0.712	0.562	0.403	0.223	0.560	0.322
LMT	0.417	0.295	0.232	0.541	0.215	0.413	0.342	0.422	0.387	0.247	0.406	0.343	0.410	0.342	0.286
MC.PA	0.366	0.567	0.265	0.453	0.247	0.619	0.535	0.445	0.529	0.487	0.628	0.492	0.419	0.612	0.412
MSFT	0.800	0.211	0.689	0.577	0.253	0.266	0.179	0.321	0.383	0.152	0.309	0.362	0.813	0.243	0.278
OR.PA	0.357	0.344	0.289	0.435	0.369	0.454	0.404	0.532	0.582	0.265	0.544	0.286	0.377	0.371	0.405
PFE	0.335	0.136	0.185	0.437	0.282	0.243	0.208	0.251	0.245	0.121	0.225	0.221	0.362	0.154	0.140
PG	0.519	0.103	0.375	0.573	0.275	0.238	0.150	0.295	0.348	0.009	0.249	0.236	0.511	0.106	0.247
REP.MC	0.185	0.578	0.043	0.406	0.124	0.676	0.851	0.288	0.455	0.555	0.726	0.516	0.309	0.598	0.343
SAN.MC	0.216	0.609	0.064	0.362	0.123	0.730	0.685	0.351	0.512	0.578	0.767	0.518	0.302	0.572	0.384
SAN.PA	0.205	0.248	0.132	0.212	0.572	0.412	0.339	0.392	0.510	0.134	0.437	0.239	0.220	0.216	0.341
SAP.DE	0.339	0.415	0.309	0.388	0.260	0.503	0.336	0.378	0.492	0.321	0.506	0.312	0.397	0.454	0.396
SOLB.BR	0.151	0.467	0.071	0.386	0.190	0.726	0.555	0.551	0.508	0.422	0.639	0.436	0.249	0.418	0.448
ULVR.L	0.215	0.143	0.186	0.335	0.438	0.351	0.250	0.400	0.401	0.074	0.295	0.138	0.250	0.133	0.216
VOW3.DE	0.310	0.580	0.149	0.445	0.227	0.709	0.607	0.329	0.460	0.532	0.664	0.615	0.381	0.644	0.399
WMT	0.490	-0.050	0.376	0.476	0.240	0.149	0.045	0.260	0.280	-0.063	0.121	0.134	0.414	-0.023	0.134

	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL															
AIR.PA															
AMZN															
APD															
AZN.L															
BAS.DE															
BP.L															
CPR.MI															
ENEL.MI															
EZJ.L															
G.MI															
GM															
GOOG															
IHG.L															
IP.MI															
LHA.DE	1.000														
LMT	0.277	1.000													
MC.PA	0.455	0.348	1.000												
MSFT	0.148	0.446	0.411	1.000											
OR.PA	0.311	0.410	0.666	0.378	1.000										
PFE	0.141	0.453	0.274	0.376	0.290	1.000									
PG	0.116	0.578	0.229	0.590	0.363	0.503	1.000								
REP.MC	0.518	0.369	0.516	0.218	0.372	0.236	0.184	1.000							
SAN.MC	0.562	0.379	0.565	0.218	0.444	0.202	0.202	0.700	1.000						
SAN.PA	0.236	0.292	0.389	0.246	0.460	0.270	0.263	0.302	0.309	1.000					
SAP.DE	0.338	0.292	0.528	0.405	0.497	0.208	0.249	0.360	0.373	0.391	1.000				
SOLB.BR	0.494	0.396	0.451	0.156	0.420	0.179	0.187	0.548	0.592	0.311	0.335	1.000			
ULVR.L	0.155	0.325	0.315	0.277	0.552	0.218	0.361	0.192	0.183	0.422	0.321	0.322	1.000		
VOW3.DE	0.485	0.364	0.644	0.323	0.456	0.199	0.196	0.600	0.627	0.318	0.492	0.512	0.245	1.000	
WMT	0.042	0.417	0.155	0.522	0.216	0.365	0.640	0.063	0.113	0.162	0.157	0.111	0.243	0.095	1.000

Appendix 3. Correlation matrix of the daily log-returns over 2020-2021.

Annual variance-covariance matrix over 2015-2021																														
	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	0.083																													
AIR.PA	0.024	0.146																												
AMZN	0.048	0.016	0.089																											
APD	0.031	0.029	0.021	0.057																										
AZN.L	0.012	0.020	0.010	0.009	0.060																									
BAS.DE	0.022	0.057	0.014	0.029	0.015	0.068																								
BP.L	0.016	0.057	0.008	0.026	0.016	0.048	0.100																							
CPR.MI	0.017	0.031	0.015	0.017	0.018	0.026	0.021	0.064																						
ENEL.MI	0.017	0.039	0.016	0.024	0.019	0.036	0.034	0.033	0.066																					
EZJ.L	0.017	0.100	0.009	0.027	0.004	0.054	0.050	0.027	0.033	0.232																				
G.MI	0.016	0.050	0.011	0.021	0.011	0.041	0.039	0.027	0.041	0.052	0.066																			
GM	0.031	0.061	0.019	0.036	0.006	0.041	0.045	0.018	0.023	0.064	0.034	0.117																		
GOOG	0.045	0.028	0.051	0.028	0.011	0.021	0.016	0.017	0.019	0.025	0.018	0.032	0.069																	
IHG.L	0.020	0.074	0.011	0.028	0.013	0.043	0.049	0.024	0.029	0.084	0.035	0.048	0.022	0.106																
IP.MI	0.022	0.048	0.017	0.023	0.015	0.036	0.028	0.027	0.031	0.038	0.033	0.028	0.021	0.033	0.090															
LHA.DE	0.016	0.078	0.010	0.025	0.011	0.052	0.044	0.027	0.035	0.130	0.048	0.046	0.019	0.059	0.035	0.170														
LMT	0.024	0.023	0.017	0.025	0.010	0.019	0.018	0.018	0.017	0.020	0.014	0.023	0.022	0.019	0.014	0.019	0.051													
MC.PA	0.027	0.057	0.021	0.026	0.018	0.044	0.037	0.030	0.035	0.051	0.036	0.034	0.025	0.043	0.034	0.043	0.017	0.073												
MSFT	0.051	0.025	0.051	0.033	0.013	0.021	0.017	0.019	0.022	0.018	0.018	0.032	0.051	0.020	0.020	0.016	0.025	0.026	0.070											
OR.PA	0.017	0.034	0.016	0.019	0.020	0.028	0.025	0.029	0.031	0.024	0.025	0.017	0.016	0.026	0.024	0.025	0.015	0.039	0.019	0.047										
PFE	0.020	0.015	0.016	0.021	0.013	0.013	0.013	0.011	0.012	0.011	0.011	0.020	0.020	0.010	0.011	0.012	0.021	0.014	0.022	0.012	0.050									
PG	0.021	0.009	0.015	0.021	0.009	0.010	0.009	0.012	0.013	0.002	0.008	0.014	0.018	0.006	0.010	0.007	0.019	0.011	0.023	0.013	0.017	0.035								
REP.MC	0.019	0.066	0.011	0.030	0.011	0.055	0.084	0.025	0.041	0.064	0.050	0.049	0.021	0.052	0.035	0.056	0.021	0.042	0.021	0.026	0.015	0.010	0.114							
SAN.MC	0.022	0.075	0.015	0.031	0.012	0.063	0.064	0.029	0.051	0.080	0.066	0.053	0.025	0.051	0.042	0.071	0.022	0.051	0.024	0.032	0.016	0.011	0.084	0.130						
SAN.PA	0.012	0.028	0.010	0.013	0.026	0.025	0.022	0.022	0.027	0.016	0.023	0.015	0.012	0.018	0.021	0.020	0.012	0.027	0.014	0.025	0.013	0.009	0.025	0.029	0.047					
SAP.DE	0.022	0.044	0.021	0.022	0.017	0.035	0.026	0.026	0.031	0.036	0.030	0.023	0.023	0.034	0.031	0.033	0.014	0.038	0.025	0.030	0.012	0.010	0.031	0.038	0.024	0.063				
SOLB.BR	0.019	0.053	0.015	0.025	0.015	0.055	0.045	0.032	0.035	0.051	0.041	0.036	0.019	0.037	0.038	0.052	0.019	0.040	0.018	0.027	0.013	0.009	0.052	0.060	0.023	0.030	0.081			
ULVR.L	0.010	0.016	0.008	0.013	0.021	0.018	0.017	0.021	0.021	0.008	0.012	0.006	0.009	0.015	0.013	0.011	0.011	0.020	0.012	0.027	0.008	0.013	0.013	0.013	0.019	0.018	0.017	0.042		
VOW3.DE	0.029	0.071	0.019	0.032	0.018	0.060	0.054	0.027	0.040	0.072	0.051	0.059	0.029	0.056	0.042	0.061	0.021	0.054	0.027	0.031	0.015	0.009	0.065	0.076	0.026	0.042	0.054	0.015	0.136	
WMT	0.021	0.004	0.016	0.018	0.007	0.007	0.005	0.010	0.011	0.000	0.006	0.012	0.016	0.003	0.008	0.005	0.017	0.009	0.021	0.008	0.015	0.019	0.005	0.009	0.006	0.008	0.007	0.007	0.007	0.045

Appendix 4. Annual variance-covariance matrix over 2015-2021.

Annual variance-covariance matrix over 2015-2019																														
	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	0.061																													
AIR.PA	0.017	0.071																												
AMZN	0.035	0.017	0.083																											
APD	0.018	0.016	0.015	0.035																										
AZN.L	0.008	0.019	0.007	0.007	0.054																									
BAS.DE	0.018	0.033	0.016	0.017	0.014	0.049																								
BP.L	0.011	0.022	0.011	0.013	0.015	0.024	0.056																							
CPR.MI	0.010	0.022	0.012	0.013	0.015	0.020	0.013	0.052																						
ENEL.MI	0.008	0.027	0.010	0.014	0.013	0.025	0.019	0.026	0.051																					
EZJ.L	0.012	0.028	0.010	0.013	0.006	0.025	0.002	0.016	0.020	0.123																				
G.MI	0.010	0.031	0.010	0.014	0.009	0.030	0.019	0.021	0.034	0.030	0.064																			
GM	0.020	0.019	0.018	0.020	0.003	0.019	0.014	0.009	0.010	0.018	0.018	0.065																		
GOOG	0.031	0.017	0.045	0.017	0.008	0.015	0.008	0.011	0.011	0.012	0.012	0.019	0.057																	
IHG.L	0.011	0.026	0.012	0.012	0.015	0.021	0.018	0.017	0.016	0.020	0.018	0.013	0.011	0.059																
IP.MI	0.017	0.033	0.016	0.016	0.014	0.029	0.019	0.021	0.023	0.027	0.028	0.016	0.015	0.024	0.079															
LHA.DE	0.010	0.028	0.010	0.012	0.007	0.028	0.008	0.017	0.021	0.061	0.031	0.017	0.011	0.020	0.023	0.096														
LMT	0.014	0.011	0.014	0.012	0.006	0.009	0.006	0.008	0.008	0.005	0.006	0.010	0.014	0.006	0.007	0.006	0.030													
MC.PA	0.020	0.039	0.019	0.018	0.017	0.036	0.022	0.025	0.028	0.028	0.030	0.018	0.018	0.025	0.030	0.027	0.010	0.064												
MSFT	0.032	0.019	0.041	0.020	0.008	0.017	0.012	0.013	0.014	0.011	0.014	0.020	0.036	0.013	0.016	0.010	0.016	0.020	0.053											
OR.PA	0.010	0.027	0.012	0.012	0.018	0.024	0.017	0.024	0.025	0.015	0.020	0.010	0.011	0.019	0.019	0.016	0.008	0.033	0.014	0.040										
PFE	0.013	0.011	0.015	0.012	0.008	0.009	0.007	0.007	0.006	0.006	0.007	0.015	0.015	0.006	0.009	0.006	0.012	0.010	0.016	0.008	0.032									
PG	0.010	0.006	0.009	0.010	0.005	0.006	0.005	0.008	0.008	0.003	0.004	0.008	0.010	0.003	0.005	0.002	0.008	0.008	0.012	0.009	0.008	0.024								
REP.MC	0.014	0.031	0.013	0.017	0.009	0.035	0.046	0.018	0.030	0.017	0.034	0.022	0.012	0.021	0.027	0.022	0.007	0.030	0.016	0.019	0.008	0.006	0.075							
SAN.MC	0.016	0.040	0.017	0.021	0.010	0.043	0.032	0.021	0.040	0.036	0.054	0.027	0.018	0.022	0.034	0.038	0.008	0.039	0.020	0.024	0.011	0.006	0.058	0.097						
SAN.PA	0.011	0.026	0.009	0.011	0.022	0.022	0.017	0.020	0.023	0.013	0.021	0.010	0.011	0.016	0.018	0.016	0.009	0.026	0.012	0.025	0.011	0.006	0.022	0.028	0.044					
SAP.DE	0.015	0.030	0.017	0.014	0.015	0.027	0.016	0.022	0.023	0.021	0.024	0.013	0.015	0.019	0.026	0.020	0.008	0.031	0.018	0.025	0.008	0.006	0.022	0.031	0.023	0.046				
SOLB.BR	0.018	0.035	0.017	0.017	0.013	0.040	0.025	0.020	0.024	0.027	0.031	0.018	0.016	0.023	0.031	0.030	0.008	0.036	0.017	0.022	0.010	0.005	0.036	0.044	0.021	0.027	0.060			
ULVR.L	0.006	0.015	0.006	0.007	0.018	0.014	0.014	0.018	0.017	0.007	0.010	0.003	0.006	0.015	0.011	0.007	0.006	0.019	0.008	0.025	0.005	0.009	0.010	0.010	0.018	0.016	0.013	0.038		
VOW3.DE	0.019	0.039	0.018	0.018	0.014	0.041	0.026	0.020	0.029	0.032	0.039	0.028	0.019	0.024	0.034	0.034	0.008	0.040	0.018	0.022	0.010	0.005	0.042	0.055	0.023	0.030	0.042	0.010	0.110	
WMT	0.011	0.008	0.010	0.009	0.003	0.005	0.005	0.006	0.007	0.005	0.005	0.010	0.010	0.005	0.006	0.005	0.010	0.008	0.012	0.006	0.010	0.011	0.005	0.007	0.004	0.007	0.006	0.004	0.005	0.037

Appendix 5. Annual variance-covariance matrix over 2015-2019.

Annual variance-covariance matrix over 2020-2021

	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	0.138																													
AIR.PA	0.041	0.333																												
AMZN	0.081	0.013	0.104																											
APD	0.065	0.060	0.037	0.112																										
AZN.L	0.021	0.021	0.018	0.016	0.075																									
BAS.DE	0.031	0.118	0.008	0.058	0.019	0.116																								
BP.L	0.029	0.144	0.001	0.058	0.020	0.107	0.209																							
CPR.MI	0.035	0.052	0.024	0.029	0.026	0.041	0.042	0.093																						
ENEL.MI	0.041	0.071	0.029	0.051	0.034	0.064	0.071	0.050	0.105																					
EZJ.L	0.031	0.281	0.008	0.060	0.001	0.128	0.169	0.056	0.067	0.504																				
G.MI	0.030	0.100	0.012	0.038	0.016	0.070	0.089	0.042	0.057	0.108	0.073																			
GM	0.058	0.166	0.023	0.078	0.014	0.096	0.121	0.040	0.055	0.180	0.074	0.248																		
GOOG	0.081	0.055	0.066	0.056	0.016	0.036	0.037	0.033	0.041	0.057	0.033	0.065	0.098																	
IHG.L	0.040	0.193	0.010	0.066	0.007	0.099	0.127	0.043	0.060	0.244	0.079	0.136	0.048	0.222																
IP.MI	0.035	0.084	0.020	0.042	0.019	0.053	0.051	0.040	0.051	0.065	0.048	0.057	0.037	0.058	0.118															
LHA.DE	0.031	0.202	0.011	0.059	0.019	0.114	0.134	0.052	0.069	0.302	0.091	0.120	0.042	0.158	0.066	0.357														
LMT	0.050	0.055	0.024	0.058	0.019	0.045	0.050	0.041	0.040	0.056	0.035	0.055	0.041	0.052	0.031	0.053	0.103													
MC.PA	0.042	0.101	0.027	0.047	0.021	0.065	0.076	0.042	0.053	0.107	0.053	0.076	0.041	0.089	0.044	0.084	0.035	0.096												
MSFT	0.100	0.041	0.075	0.065	0.023	0.031	0.028	0.033	0.042	0.036	0.028	0.061	0.086	0.039	0.032	0.030	0.048	0.043	0.114											
OR.PA	0.034	0.050	0.024	0.037	0.026	0.039	0.047	0.041	0.048	0.048	0.037	0.036	0.030	0.044	0.035	0.047	0.033	0.052	0.032	0.064										
PFE	0.038	0.024	0.018	0.045	0.024	0.025	0.029	0.023	0.024	0.026	0.019	0.034	0.035	0.022	0.015	0.026	0.045	0.026	0.039	0.023	0.094									
PG	0.048	0.015	0.030	0.048	0.019	0.020	0.017	0.022	0.028	0.002	0.017	0.029	0.040	0.012	0.021	0.017	0.046	0.018	0.050	0.023	0.038	0.062								
REP.MC	0.031	0.153	0.006	0.063	0.016	0.106	0.179	0.040	0.068	0.181	0.090	0.118	0.045	0.130	0.054	0.142	0.054	0.073	0.034	0.043	0.033	0.021	0.211							
SAN.MC	0.037	0.162	0.009	0.056	0.016	0.114	0.144	0.049	0.076	0.189	0.095	0.119	0.044	0.124	0.061	0.155	0.056	0.081	0.034	0.052	0.029	0.023	0.148	0.212						
SAN.PA	0.017	0.033	0.010	0.016	0.036	0.032	0.036	0.027	0.038	0.022	0.027	0.027	0.016	0.023	0.027	0.032	0.021	0.028	0.019	0.027	0.019	0.015	0.032	0.033	0.052					
SAP.DE	0.041	0.078	0.032	0.042	0.023	0.056	0.050	0.037	0.052	0.074	0.044	0.050	0.040	0.070	0.044	0.066	0.030	0.053	0.045	0.041	0.021	0.020	0.054	0.056	0.029	0.106				
SOLB.BR	0.021	0.099	0.008	0.047	0.019	0.091	0.093	0.062	0.060	0.110	0.063	0.080	0.029	0.072	0.056	0.109	0.047	0.051	0.019	0.039	0.020	0.017	0.093	0.100	0.026	0.040	0.135			
ULVR.L	0.018	0.019	0.014	0.026	0.028	0.027	0.026	0.028	0.030	0.012	0.018	0.016	0.018	0.014	0.017	0.021	0.024	0.023	0.022	0.032	0.015	0.021	0.020	0.019	0.022	0.024	0.027	0.053		
VOW3.DE	0.052	0.150	0.022	0.067	0.028	0.108	0.124	0.045	0.067	0.170	0.080	0.137	0.054	0.136	0.061	0.130	0.052	0.090	0.049	0.052	0.027	0.022	0.124	0.130	0.033	0.072	0.084	0.025	0.201	
WMT	0.046	-0.007	0.031	0.040	0.017	0.013	0.005	0.020	0.023	-0.011	0.008	0.017	0.033	-0.003	0.012	0.006	0.034	0.012	0.044	0.014	0.028	0.040	0.007	0.013	0.009	0.013	0.010	0.014	0.011	0.063

Appendix 6. Annual variance-covariance matrix over 2020-2021.

Markowitz portfolios' weights without short selling over 2015-2019															
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**Max Return**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	0.00%	0.00%	0.20%	1.37%	1.87%	0.60%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.40%	0.68%	1.68
AIR.PA	0.00%	0.00%	0.00%	0.00%	2.77%	5.44%	7.72%	9.87%	5.98%	0.00%	0.00%	0.00%	3.18%	3.79%	1.19
AMZN	0.00%	0.00%	1.54%	7.39%	13.64%	19.86%	26.75%	40.45%	68.20%	98.35%	100.00%	0.45%	27.62%	32.81%	1.19
APD	0.00%	0.00%	2.33%	2.89%	1.76%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.15%	0.70%	1.16%	1.66
AZN.L	0.00%	4.99%	7.50%	7.68%	7.70%	6.41%	4.59%	0.00%	0.00%	0.00%	0.00%	7.47%	3.89%	3.50%	0.90
BAS.DE	8.13%	4.31%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.24%	2.77%	2.23
BP.L	0.26%	5.91%	5.37%	3.08%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	5.72%	1.47%	2.40%	1.64
CPR.MI	0.00%	0.00%	2.21%	7.37%	12.20%	17.94%	23.48%	27.68%	20.24%	1.65%	0.00%	1.28%	11.28%	10.48%	0.93
ENEL.MI	0.00%	0.00%	0.00%	2.01%	2.17%	0.29%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.45%	0.87%	1.95
EZJ.L	7.87%	5.32%	2.26%	0.11%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.60%	1.56%	2.81%	1.81
G.MI	0.00%	0.10%	2.45%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.74%	0.25%	0.77%	3.03
GM	3.91%	4.22%	1.69%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.09%	0.98%	1.71%	1.74
GOOG	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.23%	0.00%	0.00%	0.00
IHG.L	0.00%	0.00%	4.09%	4.92%	4.39%	2.23%	0.00%	0.00%	0.00%	0.00%	0.00%	3.94%	1.56%	2.13%	1.36
IP.MI	0.00%	0.00%	0.51%	1.94%	1.87%	0.66%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%	0.50%	0.78%	1.56
LHA.DE	0.00%	3.01%	2.86%	1.93%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.00%	0.78%	1.29%	1.65
LMT	0.00%	4.58%	14.99%	17.95%	20.96%	21.86%	21.09%	5.50%	0.00%	0.00%	0.00%	14.47%	10.69%	9.53%	0.89
MC.PA	0.00%	0.00%	0.00%	0.00%	1.44%	4.71%	7.50%	9.19%	5.58%	0.00%	0.00%	0.00%	2.84%	3.58%	1.26
MSFT	0.00%	0.00%	0.00%	0.00%	0.19%	4.08%	7.19%	7.30%	0.00%	0.00%	0.00%	0.00%	1.88%	3.10%	1.65
OR.PA	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
PFE	18.34%	15.17%	10.02%	5.60%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.73%	4.91%	7.12%	1.45
PG	27.72%	26.40%	22.53%	19.14%	15.52%	8.40%	0.00%	0.00%	0.00%	0.00%	0.00%	23.06%	11.97%	11.63%	0.97
REP.MC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
SAN.MC	15.83%	2.89%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.87%	4.99%	2.66
SAN.PA	4.58%	3.37%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.80%	1.70%	2.14
SAP.DE	0.00%	0.00%	0.14%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.11%	0.02%	0.04%	2.80
SOLB.BR	0.00%	0.06%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	3.16
ULVR.L	0.00%	6.38%	8.39%	7.37%	6.35%	4.57%	1.69%	0.00%	0.00%	0.00%	0.00%	8.53%	3.47%	3.48%	1.00
VOW3.DE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
WMT	13.36%	13.28%	10.91%	9.24%	7.15%	2.96%	0.00%	0.00%	0.00%	0.00%	0.00%	11.17%	5.69%	5.73%	1.01
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**5.00%**	**8.40%**	**11.80%**	**15.20%**	**18.60%**	**22.00%**	**25.40%**	**28.80%**	**32.20%**	**35.60%**	**35.79%**	**11.23%**			
Annual Variance	**0.0165**	**0.0122**	**0.0112**	**0.0121**	**0.0143**	**0.0181**	**0.0238**	**0.0326**	**0.0488**	**0.0809**	**0.0832**	**0.0112**			
Annual Std. Dev.	**12.84%**	**11.02%**	**10.59%**	**10.99%**	**11.94%**	**13.45%**	**15.42%**	**18.06%**	**22.10%**	**28.43%**	**28.84%**	**10.58%**			
Herfindahl–Hirschman index	**16.98%**	**13.19%**	**11.60%**	**10.83%**	**12.10%**	**14.10%**	**19.03%**	**26.68%**	**51.27%**	**96.76%**	**100.00%**	**11.92%**			

* The Variability Index of the weights has been computed using the first 10 portfolios.

Appendix 7. Markowitz portfolios’ weights without short selling over 2015-2019.

Markowitz portfolios' weights without short selling over 2020-2021																		
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**Portfolio 11**	**Portfolio 12**	**Portfolio 13**	**Max Return**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.14%	6.15%	13.86%	24.66%	37.41%	58.01%	100.00%	0.00%	10.86%	18.42%	1.70
AIR.PA	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
AMZN	5.54%	8.46%	10.57%	11.69%	12.84%	13.31%	13.74%	13.90%	12.85%	8.06%	0.00%	0.00%	0.00%	0.00%	9.24%	8.54%	5.46%	0.64
APD	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
AZN.L	2.86%	3.42%	3.81%	4.00%	4.37%	5.11%	6.23%	5.83%	2.14%	0.00%	0.00%	0.00%	0.00%	0.00%	3.56%	2.90%	2.29%	0.79
BAS.DE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
BP.L	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
CPR.MI	0.00%	0.00%	0.00%	0.00%	0.36%	1.32%	1.92%	2.07%	1.52%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.55%	0.83%	1.49
ENEL.MI	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
EZJ.L	2.87%	0.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%	0.79%	3.23
G.MI	8.74%	10.83%	9.53%	6.97%	2.47%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.59%	2.96%	4.33%	1.46
GM	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
GOOG	0.00%	0.00%	0.00%	0.00%	0.00%	1.76%	4.23%	6.85%	8.88%	12.29%	16.80%	15.75%	0.00%	0.00%	0.00%	5.12%	6.37%	1.24
IHG.L	1.52%	2.51%	1.44%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.34%	0.42%	0.84%	1.99
IP.MI	0.00%	1.44%	5.96%	9.28%	12.69%	15.49%	18.05%	21.41%	24.82%	29.80%	36.31%	43.96%	41.99%	0.00%	3.02%	20.09%	14.66%	0.73
LHA.DE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
LMT	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
MC.PA	0.00%	0.00%	0.00%	1.49%	3.25%	3.92%	3.67%	3.74%	3.02%	2.60%	2.24%	0.00%	0.00%	0.00%	0.00%	1.84%	1.64%	0.89
MSFT	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
OR.PA	0.00%	0.00%	0.87%	4.73%	8.37%	11.85%	15.59%	17.33%	17.72%	14.37%	5.13%	0.00%	0.00%	0.00%	0.00%	7.38%	7.16%	0.97
PFE	2.17%	5.59%	8.49%	10.45%	12.34%	14.07%	15.89%	17.98%	19.90%	19.02%	14.86%	2.88%	0.00%	0.00%	6.65%	11.05%	6.73%	0.61
PG	1.01%	0.97%	0.93%	0.97%	1.10%	0.67%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.91%	0.44%	0.50%	1.15
REP.MC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
SAN.MC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
SAN.PA	24.30%	21.53%	18.84%	16.31%	13.68%	10.24%	5.69%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	20.58%	8.51%	9.39%	1.10
SAP.DE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
SOLB.BR	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
ULVR.L	26.35%	22.75%	19.39%	15.39%	11.15%	6.44%	1.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	21.67%	7.88%	9.93%	1.26
VOW3.DE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
WMT	24.63%	22.16%	20.16%	18.72%	17.38%	15.81%	13.99%	9.76%	3.01%	0.00%	0.00%	0.00%	0.00%	0.00%	21.44%	11.20%	9.47%	0.85
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**5.00%**	**8.40%**	**11.80%**	**15.20%**	**18.60%**	**22.00%**	**25.40%**	**28.80%**	**32.20%**	**35.60%**	**39.00%**	**42.40%**	**44.00%**	**45.44%**	**9.63%**			
Annual Variance	**0.0261**	**0.0254**	**0.0256**	**0.0263**	**0.0277**	**0.0298**	**0.0326**	**0.0362**	**0.0411**	**0.0476**	**0.0574**	**0.0723**	**0.0842**	**0.1376**	**0.0254**			
Annual Std. Dev.	**16.15%**	**15.95%**	**15.99%**	**16.23%**	**16.66%**	**17.26%**	**18.04%**	**19.03%**	**20.27%**	**21.81%**	**23.97%**	**26.89%**	**29.02%**	**37.09%**	**15.94%**			
Herfindahl–Hirschman index	**20.23%**	**17.14%**	**14.66%**	**12.75%**	**11.99%**	**11.99%**	**13.13%**	**14.71%**	**16.33%**	**18.71%**	**24.61%**	**35.89%**	**51.28%**	**100.00%**	**16.23%**			

To achieve a better visual representation of the optimal portfolio allocation, 3 additional portfolios were added.

* The Variability Index of the weights has been computed using the first 13 portfolios.

Appendix 8. Markowitz portfolios' weights without short selling over 2020-2021.

Markowitz portfolios' weights with short selling over 2015-2019														
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	1.10%	1.43%	1.77%	2.10%	2.43%	2.77%	3.10%	3.44%	3.77%	4.10%	1.62%	2.60%	1.01%	0.39
AIR.PA	-8.13%	-5.59%	-3.05%	-0.51%	2.03%	4.57%	7.11%	9.65%	12.19%	14.73%	-4.13%	3.30%	7.69%	2.33
AMZN	-1.71%	0.91%	3.53%	6.15%	8.77%	11.39%	14.01%	16.63%	19.25%	21.87%	2.42%	10.08%	7.93%	0.79
APD	3.20%	3.88%	4.57%	5.25%	5.93%	6.61%	7.29%	7.98%	8.66%	9.34%	4.28%	6.27%	2.07%	0.33
AZN.L	6.81%	7.00%	7.18%	7.37%	7.55%	7.74%	7.92%	8.11%	8.29%	8.48%	7.10%	7.64%	0.56%	0.07
BAS.DE	6.75%	3.61%	0.47%	-2.67%	-5.81%	-8.94%	-12.08%	-15.21%	-18.36%	-21.50%	1.80%	-7.37%	9.50%	-1.29
BP.L	7.12%	7.14%	7.16%	7.18%	7.20%	7.22%	7.23%	7.25%	7.27%	7.29%	7.15%	7.21%	0.06%	0.01
CPR.MI	-1.45%	0.46%	2.38%	4.30%	6.21%	8.13%	10.05%	11.96%	13.88%	15.79%	1.57%	7.17%	5.80%	0.81
ENEL.MI	-2.02%	0.24%	2.50%	4.76%	7.02%	9.29%	11.55%	13.81%	16.07%	18.33%	1.54%	8.15%	6.85%	0.84
EZJ.L	4.51%	3.62%	2.72%	1.83%	0.94%	0.05%	-0.85%	-1.74%	-2.63%	-3.52%	3.10%	0.49%	2.70%	5.48
G.MI	5.94%	6.41%	6.88%	7.35%	7.82%	8.29%	8.75%	9.22%	9.69%	10.16%	6.68%	8.05%	1.42%	0.18
GM	4.08%	3.47%	2.86%	2.25%	1.64%	1.04%	0.43%	-0.18%	-0.79%	-1.39%	3.12%	1.34%	1.84%	1.37
GOOG	5.60%	3.87%	2.13%	0.40%	-1.34%	-3.07%	-4.81%	-6.54%	-8.28%	-10.01%	2.87%	-2.20%	5.25%	-2.38
IHG.L	4.65%	4.74%	4.83%	4.92%	5.01%	5.10%	5.19%	5.28%	5.37%	5.46%	4.79%	5.05%	0.27%	0.05
IP.MI	0.28%	0.90%	1.52%	2.15%	2.77%	3.39%	4.01%	4.63%	5.26%	5.88%	1.26%	3.08%	1.88%	0.61
LHA.DE	3.49%	3.64%	3.78%	3.93%	4.07%	4.22%	4.36%	4.51%	4.66%	4.80%	3.72%	4.15%	0.44%	0.11
LMT	13.11%	14.06%	15.01%	15.96%	16.91%	17.86%	18.81%	19.76%	20.71%	21.66%	14.60%	17.38%	2.88%	0.17
MC.PA	-12.27%	-8.45%	-4.63%	-0.82%	3.00%	6.82%	10.63%	14.46%	18.27%	22.09%	-6.26%	4.91%	11.56%	2.35
MSFT	-11.82%	-9.82%	-7.83%	-5.83%	-3.84%	-1.84%	0.15%	2.15%	4.14%	6.14%	-8.68%	-2.84%	6.04%	-2.13
OR.PA	0.90%	-0.92%	-2.74%	-4.56%	-6.38%	-8.20%	-10.02%	-11.85%	-13.67%	-15.49%	-1.96%	-7.29%	5.52%	-0.76
PFE	12.67%	11.32%	9.98%	8.63%	7.29%	5.95%	4.60%	3.26%	1.91%	0.57%	10.55%	6.62%	4.07%	0.62
PG	25.37%	23.77%	22.17%	20.57%	18.97%	17.37%	15.77%	14.17%	12.57%	10.97%	22.85%	18.17%	4.84%	0.27
REP.MC	0.46%	0.82%	1.17%	1.53%	1.89%	2.24%	2.60%	2.96%	3.32%	3.67%	1.02%	2.07%	1.08%	0.52
SAN.MC	-1.65%	-4.51%	-7.38%	-10.25%	-13.12%	-15.98%	-18.85%	-21.72%	-24.59%	-27.45%	-6.17%	-14.55%	8.68%	-0.60
SAN.PA	4.39%	2.60%	0.81%	-0.98%	-2.77%	-4.56%	-6.34%	-8.13%	-9.92%	-11.71%	1.57%	-3.66%	5.41%	-1.48
SAP.DE	3.91%	4.28%	4.65%	5.03%	5.40%	5.77%	6.14%	6.52%	6.89%	7.26%	4.50%	5.59%	1.13%	0.20
SOLB.BR	2.14%	0.58%	-0.97%	-2.53%	-4.09%	-5.65%	-7.21%	-8.77%	-10.33%	-11.89%	-0.31%	-4.87%	4.72%	-0.97
ULVR.L	9.15%	8.88%	8.61%	8.34%	8.07%	7.80%	7.54%	7.27%	7.00%	6.73%	8.72%	7.94%	0.81%	0.10
VOW3.DE	0.72%	0.09%	-0.54%	-1.17%	-1.80%	-2.43%	-3.06%	-3.69%	-4.32%	-4.95%	-0.27%	-2.11%	1.91%	-0.90
WMT	12.72%	11.59%	10.47%	9.34%	8.22%	7.09%	5.96%	4.84%	3.71%	2.59%	10.94%	7.65%	3.41%	0.45
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**5.00%**	**8.40%**	**11.80%**	**15.20%**	**18.60%**	**22.00%**	**25.40%**	**28.80%**	**32.20%**	**35.60%**	**10.36%**			
Annual Variance	**0.0113**	**0.0107**	**0.0106**	**0.0111**	**0.0122**	**0.0138**	**0.0160**	**0.0187**	**0.0219**	**0.0257**	**0.0106**			
Annual Std. Dev.	**10.62%**	**10.33%**	**10.31%**	**10.56%**	**11.05%**	**11.75%**	**12.64%**	**13.66%**	**14.81%**	**16.04%**	**10.29%**			
Herfindahl–Hirschman inc	**19.22%**	**15.99%**	**14.38%**	**14.37%**	**15.97%**	**19.17%**	**23.99%**	**30.41%**	**38.44%**	**48.07%**	**14.87%**			

* The Variability Index of the weights has been computed using the first 10 portfolios.

Appendix 9. Markowitz portfolios' weights with short selling over 2015-2019.

Markowitz portfolios' weights with short selling over 2020-2021																	
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**Portfolio 11**	**Portfolio 12**	**Portfolio 13**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	-7.18%	-6.39%	-5.60%	-4.82%	-4.03%	-3.24%	-2.45%	-1.66%	-0.88%	-0.09%	0.70%	1.48%	1.85%	-6.09%	-2.49%	3.02%	-1.21
AIR.PA	-2.75%	-2.96%	-3.18%	-3.40%	-3.62%	-3.84%	-4.05%	-4.27%	-4.49%	-4.71%	-4.92%	-5.14%	-5.25%	-3.05%	-4.04%	0.83%	-0.21
AMZN	19.06%	18.45%	17.85%	17.25%	16.64%	16.04%	15.44%	14.83%	14.23%	13.63%	13.03%	12.42%	12.14%	18.22%	15.46%	2.31%	0.15
APD	-3.68%	-4.28%	-4.89%	-5.49%	-6.09%	-6.69%	-7.30%	-7.90%	-8.50%	-9.10%	-9.71%	-10.30%	-10.59%	-4.52%	-7.27%	2.30%	-0.32
AZN.L	4.88%	5.23%	5.58%	5.93%	6.29%	6.64%	6.99%	7.35%	7.70%	8.05%	8.40%	8.76%	8.92%	5.37%	6.98%	1.35%	0.19
BAS.DE	-5.93%	-4.37%	-2.81%	-1.25%	0.31%	1.86%	3.42%	4.98%	6.54%	8.10%	9.67%	11.23%	11.96%	-3.76%	3.36%	5.97%	1.78
BP.L	-1.94%	-2.41%	-2.88%	-3.34%	-3.81%	-4.28%	-4.75%	-5.21%	-5.68%	-6.15%	-6.61%	-7.08%	-7.30%	-2.59%	-4.73%	1.79%	-0.38
CPR.MI	-7.39%	-6.52%	-5.65%	-4.78%	-3.92%	-3.05%	-2.18%	-1.31%	-0.44%	0.43%	1.31%	2.18%	2.59%	-6.18%	-2.21%	3.33%	-1.51
ENEL.MI	-13.84%	-14.63%	-15.41%	-16.20%	-16.98%	-17.77%	-18.56%	-19.34%	-20.13%	-20.92%	-21.71%	-22.49%	-22.86%	-14.93%	-18.53%	3.01%	-0.16
EZJ.L	2.35%	1.89%	1.43%	0.98%	0.52%	0.06%	-0.40%	-0.85%	-1.31%	-1.77%	-2.23%	-2.68%	-2.90%	1.71%	-0.38%	1.75%	-4.63
G.MI	31.82%	31.71%	31.61%	31.50%	31.40%	31.29%	31.19%	31.08%	30.98%	30.87%	30.77%	30.66%	30.61%	31.68%	31.19%	0.40%	0.01
GM	-0.23%	0.28%	0.80%	1.31%	1.82%	2.34%	2.85%	3.36%	3.87%	4.39%	4.90%	5.41%	5.65%	0.48%	2.83%	1.96%	0.69
GOOG	2.61%	3.60%	4.60%	5.60%	6.59%	7.59%	8.59%	9.58%	10.58%	11.58%	12.57%	13.57%	14.04%	3.99%	8.55%	3.82%	0.45
IHG.L	9.01%	8.99%	8.96%	8.94%	8.92%	8.89%	8.87%	8.84%	8.82%	8.79%	8.77%	8.74%	8.73%	8.98%	8.87%	0.09%	0.01
IP.MI	2.56%	4.07%	5.59%	7.10%	8.62%	10.13%	11.65%	13.17%	14.68%	16.20%	17.71%	19.23%	19.94%	4.66%	11.59%	5.80%	0.50
LHA.DE	-2.02%	-2.68%	-3.34%	-4.01%	-4.67%	-5.34%	-6.00%	-6.66%	-7.33%	-7.99%	-8.66%	-9.32%	-9.63%	-2.94%	-5.97%	2.54%	-0.43
LMT	-0.12%	-1.40%	-2.67%	-3.94%	-5.22%	-6.49%	-7.77%	-9.04%	-10.32%	-11.59%	-12.87%	-14.14%	-14.74%	-1.89%	-7.72%	4.88%	-0.63
MC.PA	2.42%	3.83%	5.25%	6.66%	8.08%	9.50%	10.91%	12.33%	13.74%	15.16%	16.57%	17.99%	18.65%	4.38%	10.85%	5.42%	0.50
MSFT	-16.07%	-15.46%	-14.84%	-14.23%	-13.61%	-13.00%	-12.38%	-11.77%	-11.15%	-10.54%	-9.93%	-9.31%	-9.02%	-15.22%	-12.41%	2.35%	-0.19
OR.PA	-1.50%	0.58%	2.65%	4.72%	6.80%	8.87%	10.95%	13.02%	15.10%	17.17%	19.25%	21.32%	22.30%	1.38%	10.86%	7.94%	0.73
PFE	6.33%	7.26%	8.19%	9.13%	10.06%	10.99%	11.92%	12.85%	13.78%	14.71%	15.64%	16.58%	17.02%	7.63%	11.88%	3.57%	0.30
PG	9.30%	9.38%	9.47%	9.55%	9.64%	9.72%	9.81%	9.89%	9.98%	10.06%	10.14%	10.23%	10.27%	9.41%	9.80%	0.33%	0.03
REP.MC	-1.29%	-1.18%	-1.07%	-0.96%	-0.85%	-0.74%	-0.63%	-0.52%	-0.42%	-0.31%	-0.20%	-0.10%	-0.05%	-1.13%	-0.64%	0.41%	-0.65
SAN.MC	-3.94%	-4.29%	-4.65%	-5.00%	-5.35%	-5.71%	-6.06%	-6.41%	-6.77%	-7.12%	-7.47%	-7.83%	-7.99%	-4.43%	-6.05%	1.35%	-0.22
SAN.PA	22.58%	21.17%	19.76%	18.35%	16.94%	15.52%	14.11%	12.70%	11.29%	9.87%	8.46%	7.05%	6.38%	20.62%	14.17%	5.41%	0.38
SAP.DE	2.77%	1.75%	0.73%	-0.29%	-1.30%	-2.32%	-3.34%	-4.36%	-5.38%	-6.40%	-7.41%	-8.43%	-8.91%	1.35%	-3.30%	3.90%	-1.18
SOLB.BR	5.57%	5.37%	5.17%	4.97%	4.77%	4.57%	4.37%	4.17%	3.97%	3.77%	3.56%	3.35%	3.26%	5.29%	4.38%	0.77%	0.18
ULVR.L	23.06%	21.08%	19.10%	17.11%	15.13%	13.15%	11.17%	9.19%	7.20%	5.22%	3.23%	1.24%	0.31%	20.31%	11.25%	7.59%	0.68
VOW3.DE	-3.91%	-4.53%	-5.15%	-5.77%	-6.39%	-7.01%	-7.63%	-8.25%	-8.87%	-9.49%	-10.10%	-10.72%	-11.01%	-4.77%	-7.60%	2.37%	-0.31
WMT	27.46%	26.43%	25.40%	24.37%	23.33%	22.30%	21.27%	20.23%	19.20%	18.17%	17.14%	16.11%	15.63%	26.03%	21.31%	3.95%	0.19
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**5.00%**	**8.40%**	**11.80%**	**15.20%**	**18.60%**	**22.00%**	**25.40%**	**28.80%**	**32.20%**	**35.60%**	**39.00%**	**42.40%**	**44.00%**	**9.73%**			
Annual Variance	**0.0211**	**0.0209**	**0.0209**	**0.0212**	**0.0218**	**0.0227**	**0.0239**	**0.0253**	**0.0271**	**0.0291**	**0.0314**	**0.0340**	**0.0353**	**0.0208**			
Annual Std. Dev.	**14.53%**	**14.45%**	**14.46%**	**14.56%**	**14.77%**	**15.07%**	**15.45%**	**15.91%**	**16.45%**	**17.05%**	**17.71%**	**18.43%**	**18.78%**	**14.44%**			
Herfindahl–Hirschman index	**41.22%**	**39.04%**	**37.42%**	**36.35%**	**35.83%**	**35.86%**	**36.44%**	**37.57%**	**39.25%**	**41.48%**	**44.27%**	**47.60%**	**49.35%**	**38.35%**			

To achieve a better visual representation of the optimal portfolio allocation, 3 additional portfolios were added.

* The Variability Index of the weights has been computed using the first 13 portfolios.

Appendix 10. Markowitz portfolios' weights with short selling over 2020-2021.

$\tau\Sigma$ matrix over 2015-2019

	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	0.0015																													
AIR.PA	0.0004	0.0018																												
AMZN	0.0009	0.0004	0.0021																											
APD	0.0005	0.0004	0.0004	0.0009																										
AZN.L	0.0002	0.0005	0.0002	0.0002	0.0013																									
BAS.DE	0.0004	0.0008	0.0004	0.0004	0.0003	0.0012																								
BP.L	0.0003	0.0005	0.0003	0.0003	0.0004	0.0006	0.0014																							
CPR.MI	0.0002	0.0006	0.0003	0.0003	0.0004	0.0005	0.0003	0.0013																						
ENEL.MI	0.0002	0.0007	0.0003	0.0003	0.0003	0.0006	0.0005	0.0006	0.0013																					
EZJ.L	0.0003	0.0007	0.0002	0.0003	0.0001	0.0006	0.0000	0.0004	0.0005	0.0031																				
G.MI	0.0003	0.0008	0.0003	0.0004	0.0002	0.0007	0.0005	0.0005	0.0008	0.0007	0.0016																			
GM	0.0005	0.0005	0.0004	0.0005	0.0001	0.0005	0.0004	0.0002	0.0003	0.0004	0.0005	0.0016																		
GOOG	0.0008	0.0004	0.0011	0.0004	0.0002	0.0004	0.0002	0.0003	0.0003	0.0003	0.0003	0.0005	0.0014																	
IHG.L	0.0003	0.0007	0.0003	0.0003	0.0004	0.0005	0.0005	0.0004	0.0004	0.0005	0.0004	0.0003	0.0003	0.0015																
IP.MI	0.0004	0.0008	0.0004	0.0004	0.0003	0.0007	0.0005	0.0005	0.0006	0.0007	0.0007	0.0004	0.0004	0.0006	0.0020															
LHA.DE	0.0003	0.0007	0.0002	0.0003	0.0002	0.0007	0.0002	0.0004	0.0005	0.0015	0.0008	0.0004	0.0003	0.0005	0.0006	0.0024														
LMT	0.0003	0.0003	0.0004	0.0003	0.0001	0.0002	0.0001	0.0002	0.0002	0.0001	0.0002	0.0003	0.0003	0.0002	0.0002	0.0001	0.0008													
MC.PA	0.0005	0.0010	0.0005	0.0005	0.0004	0.0009	0.0005	0.0006	0.0007	0.0007	0.0007	0.0004	0.0005	0.0006	0.0008	0.0007	0.0002	0.0016												
MSFT	0.0008	0.0005	0.0010	0.0005	0.0002	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0005	0.0009	0.0003	0.0004	0.0003	0.0004	0.0005	0.0013											
OR.PA	0.0003	0.0007	0.0003	0.0003	0.0005	0.0006	0.0004	0.0006	0.0006	0.0004	0.0005	0.0002	0.0003	0.0005	0.0005	0.0004	0.0002	0.0008	0.0003	0.0010										
PFE	0.0003	0.0003	0.0004	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0004	0.0004	0.0001	0.0002	0.0002	0.0003	0.0002	0.0004	0.0002	0.0322									
PG	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0002	0.0002	0.0006								
REP.MC	0.0004	0.0008	0.0003	0.0004	0.0002	0.0009	0.0012	0.0005	0.0007	0.0004	0.0009	0.0005	0.0003	0.0005	0.0007	0.0005	0.0002	0.0007	0.0004	0.0005	0.0002	0.0001	0.0019							
SAN.MC	0.0004	0.0010	0.0004	0.0005	0.0003	0.0011	0.0008	0.0005	0.0010	0.0009	0.0013	0.0007	0.0004	0.0006	0.0009	0.0010	0.0002	0.0010	0.0005	0.0006	0.0003	0.0002	0.0015	0.0024						
SAN.PA	0.0003	0.0006	0.0002	0.0003	0.0006	0.0006	0.0004	0.0005	0.0006	0.0003	0.0005	0.0002	0.0003	0.0004	0.0005	0.0004	0.0002	0.0007	0.0003	0.0006	0.0003	0.0002	0.0006	0.0007	0.0011					
SAP.DE	0.0004	0.0008	0.0004	0.0003	0.0004	0.0007	0.0004	0.0005	0.0006	0.0005	0.0006	0.0003	0.0004	0.0005	0.0006	0.0005	0.0002	0.0008	0.0004	0.0006	0.0002	0.0001	0.0005	0.0008	0.0006	0.0011				
SOLB.BR	0.0004	0.0009	0.0004	0.0004	0.0003	0.0010	0.0006	0.0005	0.0006	0.0007	0.0008	0.0005	0.0004	0.0006	0.0008	0.0007	0.0002	0.0009	0.0004	0.0005	0.0003	0.0001	0.0009	0.0011	0.0005	0.0007	0.0015			
ULVR.L	0.0002	0.0004	0.0002	0.0002	0.0004	0.0003	0.0003	0.0005	0.0004	0.0002	0.0002	0.0001	0.0002	0.0004	0.0003	0.0002	0.0001	0.0005	0.0002	0.0006	0.0001	0.0002	0.0002	0.0002	0.0005	0.0004	0.0003	0.0010		
VOW3.DE	0.0005	0.0010	0.0004	0.0005	0.0003	0.0010	0.0006	0.0005	0.0007	0.0008	0.0010	0.0007	0.0005	0.0006	0.0009	0.0009	0.0002	0.0010	0.0004	0.0006	0.0002	0.0001	0.0010	0.0014	0.0006	0.0007	0.0011	0.0003	0.0028	
WMT	0.0003	0.0002	0.0003	0.0002	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001	0.0001	0.0002	0.0002	0.0001	0.0002	0.0001	0.0002	0.0002	0.0003	0.0002	0.0003	0.0003	0.0001	0.0002	0.0001	0.0002	0.0002	0.0001	0.0001	0.0009

Appendix 11. $\tau\Sigma$ matrix over 2015-2019.

$\tau\Sigma$ matrix over 2020-2021																														
	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	0.0034																													
AIR.PA	0.0010	0.0083																												
AMZN	0.0020	0.0003	0.0026																											
APD	0.0016	0.0015	0.0009	0.0028																										
AZN.L	0.0005	0.0005	0.0004	0.0004	0.0019																									
BAS.DE	0.0008	0.0030	0.0002	0.0014	0.0005	0.0029																								
BP.L	0.0007	0.0036	0.0000	0.0015	0.0005	0.0027	0.0052																							
CPR.MI	0.0009	0.0013	0.0006	0.0007	0.0006	0.0010	0.0010	0.0023																						
ENEL.MI	0.0010	0.0018	0.0007	0.0013	0.0008	0.0016	0.0018	0.0012	0.0026																					
EZJ.L	0.0008	0.0070	0.0002	0.0015	0.0000	0.0032	0.0042	0.0014	0.0017	0.0126																				
G.MI	0.0007	0.0025	0.0003	0.0010	0.0004	0.0018	0.0022	0.0010	0.0014	0.0027	0.0018																			
GM	0.0015	0.0042	0.0006	0.0020	0.0003	0.0024	0.0030	0.0010	0.0014	0.0045	0.0018	0.0062																		
GOOG	0.0020	0.0014	0.0016	0.0014	0.0004	0.0009	0.0009	0.0008	0.0010	0.0014	0.0008	0.0016	0.0025																	
IHG.L	0.0010	0.0048	0.0002	0.0017	0.0002	0.0025	0.0032	0.0011	0.0015	0.0061	0.0020	0.0034	0.0012	0.0056																
IP.MI	0.0009	0.0021	0.0005	0.0011	0.0005	0.0013	0.0013	0.0010	0.0013	0.0016	0.0012	0.0014	0.0009	0.0014	0.0029															
LHA.DE	0.0008	0.0051	0.0003	0.0015	0.0005	0.0029	0.0034	0.0013	0.0017	0.0076	0.0023	0.0030	0.0010	0.0039	0.0016	0.0089														
LMT	0.0012	0.0014	0.0006	0.0015	0.0005	0.0011	0.0013	0.0010	0.0010	0.0014	0.0009	0.0014	0.0010	0.0013	0.0008	0.0013	0.0026													
MC.PA	0.0011	0.0025	0.0007	0.0012	0.0005	0.0016	0.0019	0.0010	0.0013	0.0027	0.0013	0.0019	0.0010	0.0022	0.0011	0.0021	0.0009	0.0024												
MSFT	0.0025	0.0010	0.0019	0.0016	0.0006	0.0008	0.0007	0.0008	0.0010	0.0009	0.0007	0.0015	0.0022	0.0010	0.0008	0.0007	0.0012	0.0011	0.0029											
OR.PA	0.0008	0.0013	0.0006	0.0009	0.0006	0.0010	0.0012	0.0010	0.0012	0.0012	0.0009	0.0009	0.0007	0.0011	0.0009	0.0012	0.0008	0.0013	0.0008	0.0016										
PFE	0.0010	0.0006	0.0005	0.0011	0.0006	0.0006	0.0007	0.0006	0.0006	0.0007	0.0005	0.0008	0.0009	0.0006	0.0004	0.0006	0.0011	0.0007	0.0010	0.0006	0.0024									
PG	0.0012	0.0004	0.0008	0.0012	0.0005	0.0005	0.0004	0.0006	0.0007	0.0000	0.0004	0.0007	0.0010	0.0003	0.0005	0.0004	0.0011	0.0004	0.0012	0.0006	0.0010	0.0015								
REP.MC	0.0008	0.0038	0.0002	0.0016	0.0004	0.0026	0.0045	0.0010	0.0017	0.0045	0.0023	0.0030	0.0011	0.0032	0.0014	0.0036	0.0014	0.0018	0.0008	0.0011	0.0008	0.0005	0.0053							
SAN.MC	0.0009	0.0040	0.0002	0.0014	0.0004	0.0029	0.0036	0.0012	0.0019	0.0047	0.0024	0.0030	0.0011	0.0031	0.0015	0.0039	0.0014	0.0020	0.0008	0.0013	0.0007	0.0006	0.0037	0.0053						
SAN.PA	0.0004	0.0008	0.0002	0.0004	0.0009	0.0008	0.0009	0.0007	0.0009	0.0005	0.0007	0.0007	0.0004	0.0006	0.0007	0.0008	0.0005	0.0007	0.0005	0.0007	0.0005	0.0004	0.0008	0.0008	0.0013					
SAP.DE	0.0010	0.0019	0.0008	0.0011	0.0006	0.0014	0.0012	0.0009	0.0013	0.0018	0.0011	0.0013	0.0010	0.0017	0.0011	0.0016	0.0008	0.0013	0.0011	0.0010	0.0005	0.0005	0.0013	0.0014	0.0007	0.0026				
SOLB.BR	0.0005	0.0025	0.0002	0.0012	0.0005	0.0023	0.0023	0.0015	0.0015	0.0028	0.0016	0.0020	0.0007	0.0018	0.0014	0.0027	0.0012	0.0013	0.0005	0.0010	0.0005	0.0004	0.0023	0.0025	0.0007	0.0010	0.0034			
ULVR.L	0.0005	0.0005	0.0003	0.0006	0.0007	0.0007	0.0007	0.0007	0.0007	0.0003	0.0005	0.0004	0.0005	0.0004	0.0004	0.0005	0.0006	0.0006	0.0005	0.0008	0.0004	0.0005	0.0005	0.0005	0.0006	0.0006	0.0007	0.0013		
VOW3.DE	0.0013	0.0038	0.0005	0.0017	0.0007	0.0027	0.0031	0.0011	0.0017	0.0042	0.0020	0.0034	0.0013	0.0034	0.0015	0.0032	0.0013	0.0022	0.0012	0.0013	0.0007	0.0005	0.0031	0.0032	0.0008	0.0018	0.0021	0.0006	0.0050	
WMT	0.0011	-0.0002	0.0008	0.0010	0.0004	0.0003	0.0001	0.0005	0.0006	-0.0003	0.0002	0.0004	0.0008	-0.0001	0.0003	0.0002	0.0008	0.0003	0.0011	0.0003	0.0007	0.0010	0.0002	0.0003	0.0002	0.0003	0.0003	0.0004	0.0003	0.0016

Appendix 12. $\tau\Sigma$ matrix over 2020-2021.

B&L portfolios' weights with no short selling & no views over 2015-2019

Stocks	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6	Portfolio 7	Portfolio 8	Portfolio 9	Portfolio 10	Max Return	MVP	Avg. Weight	Weight Std. Dev.	Variability Index*
AAPL	0.00%	3.89%	7.86%	10.91%	13.74%	16.46%	19.22%	22.00%	24.51%	20.20%	0.00%	0.00%	13.88%	8.11%	0.58
AIR.PA	0.00%	0.00%	0.00%	0.00%	0.16%	1.18%	2.13%	2.89%	0.83%	0.00%	0.00%	0.00%	0.72%	1.04%	1.45
AMZN	0.00%	3.72%	6.95%	9.17%	11.27%	13.32%	15.34%	17.64%	24.12%	52.62%	100.00%	0.46%	15.41%	14.80%	0.96
APD	1.63%	2.44%	2.56%	2.16%	1.63%	1.08%	0.46%	0.00%	0.00%	0.00%	0.00%	2.15%	1.20%	1.03%	0.86
AZN.L	7.63%	6.50%	5.39%	4.44%	3.53%	2.65%	1.81%	0.63%	0.00%	0.00%	0.00%	7.47%	3.26%	2.71%	0.83
BAS.DE	0.00%	0.00%	0.00%	0.44%	0.98%	0.96%	0.81%	0.11%	0.00%	0.00%	0.00%	0.00%	0.33%	0.43%	1.30
BP.L	5.80%	5.28%	4.72%	4.04%	3.18%	2.14%	1.21%	0.37%	0.00%	0.00%	0.00%	5.71%	2.68%	2.23%	0.83
CPR.MI	1.20%	1.30%	1.19%	0.98%	0.61%	0.28%	0.03%	0.00%	0.00%	0.00%	0.00%	1.28%	0.56%	0.56%	1.00
ENEL.MI	0.00%	0.00%	0.31%	0.80%	1.03%	1.12%	1.02%	0.15%	0.00%	0.00%	0.00%	0.00%	0.44%	0.49%	1.11
EZJ.L	2.68%	2.16%	1.71%	1.35%	0.93%	0.47%	0.00%	0.00%	0.00%	0.00%	0.00%	2.61%	0.93%	1.00%	1.08
G.MI	2.73%	2.78%	2.64%	2.30%	1.77%	1.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.74%	1.32%	1.25%	0.95
GM	1.87%	1.99%	1.88%	1.67%	1.36%	1.01%	0.59%	0.00%	0.00%	0.00%	0.00%	2.09%	1.04%	0.83%	0.80
GOOG	0.00%	3.32%	6.21%	7.83%	9.41%	10.94%	12.43%	13.82%	13.09%	0.00%	0.00%	0.23%	7.71%	5.19%	0.67
IHG.L	4.00%	3.35%	2.73%	2.13%	1.42%	0.60%	0.00%	0.00%	0.00%	0.00%	0.00%	3.94%	1.42%	1.54%	1.08
IP.MI	0.06%	0.31%	0.38%	0.45%	0.37%	0.16%	0.00%	0.00%	0.00%	0.00%	0.00%	0.25%	0.17%	0.18%	1.07
LHA.DE	3.11%	2.57%	2.11%	1.69%	1.19%	0.64%	0.03%	0.00%	0.00%	0.00%	0.00%	3.00%	1.13%	1.18%	1.04
LMT	14.61%	12.36%	9.95%	7.41%	5.00%	2.69%	0.47%	0.00%	0.00%	0.00%	0.00%	14.47%	5.25%	5.55%	1.06
MC.PA	0.00%	0.00%	0.00%	0.00%	0.90%	2.59%	4.21%	5.86%	6.38%	0.00%	0.00%	0.00%	1.99%	2.59%	1.30
MSFT	0.00%	0.00%	1.33%	6.19%	10.88%	15.39%	19.86%	24.46%	29.37%	27.18%	0.00%	0.00%	13.47%	11.43%	0.85
OR.PA	0.00%	0.00%	0.00%	0.02%	1.28%	1.92%	2.66%	3.26%	0.00%	0.00%	0.00%	0.00%	0.91%	1.28%	1.40
PFE	10.77%	9.40%	7.86%	6.31%	4.91%	3.62%	2.33%	0.04%	0.00%	0.00%	0.00%	10.73%	4.52%	4.00%	0.88
PG	23.70%	19.75%	16.12%	12.54%	9.16%	5.99%	2.87%	0.00%	0.00%	0.00%	0.00%	23.06%	9.01%	8.72%	0.97
REP.MC	0.00%	0.00%	0.00%	0.00%	0.12%	0.38%	0.24%	0.00%	0.00%	0.00%	0.00%	0.00%	0.07%	0.13%	1.81
SAN.MC	0.00%	0.00%	0.00%	0.00%	0.00%	0.33%	1.53%	2.47%	1.71%	0.00%	0.00%	0.00%	0.61%	0.93%	1.54
SAN.PA	0.00%	0.00%	0.57%	1.33%	1.66%	1.78%	1.83%	1.77%	0.00%	0.00%	0.00%	0.00%	0.89%	0.85%	0.95
SAP.DE	0.00%	1.34%	2.39%	2.90%	2.83%	2.52%	2.13%	1.39%	0.00%	0.00%	0.00%	0.11%	1.55%	1.19%	0.77
SOLB.BR	0.00%	0.00%	0.00%	0.00%	0.00%	0.12%	0.18%	0.00%	0.00%	0.00%	0.00%	0.00%	0.03%	0.07%	2.16
ULVR.L	8.82%	7.60%	6.48%	5.42%	3.91%	2.70%	1.54%	0.00%	0.00%	0.00%	0.00%	8.54%	3.65%	3.31%	0.91
VOW3.DE	0.00%	0.00%	0.00%	0.00%	0.31%	0.51%	0.65%	0.69%	0.00%	0.00%	0.00%	0.00%	0.22%	0.30%	1.37
WMT	11.39%	9.94%	8.64%	7.51%	6.47%	5.44%	4.42%	2.46%	0.00%	0.00%	0.00%	11.17%	5.63%	3.94%	0.70
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**27.71%**	**9.17%**			
Annual Variance	**0.0115**	**0.0119**	**0.0133**	**0.0157**	**0.0189**	**0.0229**	**0.0278**	**0.0336**	**0.0407**	**0.0534**	**0.0852**	**0.0115**			
Annual Std. Dev.	**10.72%**	**10.91%**	**11.54%**	**12.52%**	**13.74%**	**15.15%**	**16.68%**	**18.33%**	**20.16%**	**23.10%**	**29.20%**	**10.71%**			
Herfindahl–Hirschman index	**12.38%**	**9.41%**	**7.79%**	**7.11%**	**7.64%**	**9.38%**	**12.37%**	**16.56%**	**22.60%**	**39.16%**	**100.00%**	**11.92%**			

* The Variability Index of the weights has been computed using the first 10 portfolios.

Appendix 13. B&L portfolios' weights with no short selling & no views over 2015-2019.

B&L portfolios' weights with no short selling & no views over 2020-2021																		
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**Portfolio 11**	**Portfolio 12**	**Portfolio 13**	**Max Return**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	0.00%	0.00%	0.00%	0.00%	0.00%	1.56%	5.72%	8.75%	11.52%	14.28%	21.63%	29.29%	49.32%	100.00%	0.00%	10.93%	14.87%	1.36
AIR.PA	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.45%	0.76%	0.00%	0.00%	0.00%	0.09%	0.24%	2.53
AMZN	0.00%	0.00%	3.01%	10.20%	17.05%	18.48%	18.20%	17.67%	17.10%	16.57%	15.19%	13.96%	0.00%	0.00%	9.24%	11.34%	7.69%	0.68
APD	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.59%	0.00%	0.00%	0.00%	0.12%	0.44%	3.61
AZN.L	6.23%	5.49%	4.48%	3.42%	2.44%	2.16%	1.99%	1.74%	1.46%	1.20%	0.43%	0.00%	0.00%	0.00%	3.56%	2.39%	2.00%	0.84
BAS.DE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
BP.L	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
CPR.MI	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
ENEL.MI	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.43%	0.00%	0.00%	0.00%	0.11%	0.40%	3.61
EZJ.L	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.07%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%	3.61
G.MI	0.00%	8.98%	10.97%	10.52%	10.03%	9.05%	8.02%	7.23%	6.48%	5.64%	2.89%	0.00%	0.00%	0.00%	10.59%	6.14%	4.11%	0.67
GM	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.22%	0.72%	0.44%	0.00%	0.00%	0.00%	0.11%	0.23%	2.12
GOOG	0.00%	0.00%	0.00%	0.00%	0.00%	4.46%	7.21%	8.57%	9.64%	10.64%	13.27%	15.54%	5.02%	0.00%	0.00%	5.72%	5.55%	0.97
IHG.L	0.00%	0.00%	1.39%	2.50%	3.58%	3.60%	3.25%	2.92%	2.62%	2.25%	0.91%	0.00%	0.00%	0.00%	2.34%	1.77%	1.44%	0.82
IP.MI	0.00%	0.39%	2.86%	3.03%	3.08%	2.68%	2.26%	1.97%	1.69%	1.41%	0.55%	0.00%	0.00%	0.00%	3.02%	1.53%	1.22%	0.80
LHA.DE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.13%	0.00%	0.00%	0.00%	0.01%	0.04%	3.61
LMT	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.43%	0.94%	0.00%	0.00%	0.00%	0.11%	0.28%	2.63
MC.PA	0.00%	0.00%	0.00%	0.00%	0.00%	0.36%	1.29%	1.87%	2.29%	2.66%	3.54%	3.27%	0.00%	0.00%	0.00%	1.18%	1.38%	1.17
MSFT	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.70%	5.98%	9.21%	17.87%	27.09%	45.66%	0.00%	0.00%	8.35%	14.03%	1.68
OR.PA	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.05%	0.38%	0.75%	1.81%	0.00%	0.00%	0.00%	0.00%	0.23%	0.52%	2.27
PFE	0.00%	2.79%	6.01%	6.68%	6.91%	6.46%	5.93%	5.49%	5.06%	4.64%	3.43%	1.80%	0.00%	0.00%	6.65%	4.25%	2.43%	0.57
PG	0.00%	0.00%	0.00%	1.36%	4.60%	5.48%	5.76%	5.58%	5.31%	5.01%	4.07%	1.46%	0.00%	0.00%	0.91%	2.97%	2.49%	0.84
REP.MC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00
SAN.MC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.97%	0.00%	0.00%	0.00%	0.07%	0.27%	3.61
SAN.PA	47.18%	33.51%	23.93%	20.03%	16.18%	13.93%	12.17%	10.53%	8.91%	7.28%	3.05%	0.00%	0.00%	0.00%	20.57%	15.13%	13.55%	0.90
SAP.DE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.15%	0.39%	0.56%	0.78%	1.34%	0.65%	0.00%	0.00%	0.00%	0.30%	0.43%	1.43
SOLB.BR	2.60%	0.61%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.31%	0.00%	0.00%	0.00%	0.00%	0.27%	0.72%	2.67
ULVR.L	42.40%	32.41%	25.21%	21.07%	16.71%	14.03%	12.14%	10.44%	8.69%	6.95%	2.26%	0.00%	0.00%	0.00%	21.68%	14.79%	12.73%	0.86
VOW3.DE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.66%	0.00%	0.00%	0.00%	0.05%	0.18%	3.61
WMT	1.59%	15.82%	22.13%	21.19%	19.42%	17.76%	15.93%	14.11%	12.29%	10.50%	5.79%	0.00%	0.00%	0.00%	21.44%	12.04%	7.89%	0.66
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**30.84%**	**35.84%**	**40.84%**	**42.60%**	**14.35%**			
Annual Variance	**0.0367**	**0.0292**	**0.0266**	**0.0260**	**0.0270**	**0.0291**	**0.0320**	**0.0357**	**0.0400**	**0.0451**	**0.0619**	**0.0836**	**0.1134**	**0.1410**	**0.0260**			
Annual Std. Dev.	**19.15%**	**17.07%**	**16.30%**	**16.14%**	**16.42%**	**17.05%**	**17.89%**	**18.89%**	**20.01%**	**21.23%**	**24.88%**	**28.92%**	**33.67%**	**37.55%**	**16.13%**			
Herfindahl–Hirschman index	**40.71%**	**25.42%**	**18.94%**	**15.83%**	**14.07%**	**12.48%**	**11.19%**	**10.21%**	**9.73%**	**9.83%**	**12.99%**	**20.53%**	**45.43%**	**100.00%**	**16.23%**			

To achieve a better visual representation of the optimal portfolio allocation, 3 additional portfolios were added.

* The Variability Index of the weights has been computed using the first 13 portfolios.

Appendix 14. B&L portfolios' weights with no short selling & no views over 2020-2021.

B&L portfolios' weights with short selling & no views over 2015-2019														
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	2.72%	5.47%	8.22%	10.97%	13.71%	16.46%	19.21%	21.96%	24.71%	27.45%	1.62%	15.09%	8.32%	0.55
AIR.PA	-3.74%	-2.75%	-1.77%	-0.79%	0.20%	1.18%	2.16%	3.15%	4.13%	5.11%	-4.13%	0.69%	2.98%	4.33
AMZN	3.22%	5.24%	7.26%	9.28%	11.30%	13.31%	15.34%	17.35%	19.37%	21.39%	2.42%	12.31%	6.11%	0.50
APD	4.04%	3.44%	2.85%	2.26%	1.67%	1.07%	0.48%	-0.11%	-0.70%	-1.29%	4.28%	1.37%	1.79%	1.31
AZN.L	6.77%	5.95%	5.13%	4.30%	3.48%	2.65%	1.83%	1.01%	0.18%	-0.64%	7.10%	3.07%	2.49%	0.81
BAS.DE	1.74%	1.58%	1.43%	1.27%	1.12%	0.96%	0.81%	0.66%	0.50%	0.35%	1.80%	1.04%	0.47%	0.45
BP.L	6.78%	5.86%	4.93%	4.01%	3.08%	2.15%	1.22%	0.29%	-0.63%	-1.55%	7.15%	2.61%	2.81%	1.07
CPR.MI	1.47%	1.24%	1.00%	0.76%	0.52%	0.28%	0.04%	-0.19%	-0.43%	-0.67%	1.57%	0.40%	0.72%	1.80
ENEL.MI	1.51%	1.43%	1.35%	1.28%	1.20%	1.11%	1.04%	0.97%	0.88%	0.81%	1.54%	1.16%	0.24%	0.20
EZJ.L	2.91%	2.42%	1.93%	1.44%	0.96%	0.47%	-0.02%	-0.51%	-1.00%	-1.49%	3.10%	0.71%	1.48%	2.08
G.MI	6.26%	5.21%	4.15%	3.10%	2.05%	1.00%	-0.05%	-1.11%	-2.16%	-3.22%	6.68%	1.52%	3.19%	2.09
GM	2.96%	2.57%	2.18%	1.79%	1.40%	1.01%	0.61%	0.22%	-0.17%	-0.56%	3.12%	1.20%	1.19%	0.99
GOOG	3.46%	4.96%	6.45%	7.95%	9.45%	10.94%	12.44%	13.93%	15.43%	16.92%	2.87%	10.19%	4.53%	0.44
IHG.L	4.48%	3.70%	2.93%	2.15%	1.38%	0.60%	-0.18%	-0.95%	-1.73%	-2.50%	4.79%	0.99%	2.35%	2.38
IP.MI	1.18%	0.98%	0.77%	0.57%	0.36%	0.16%	-0.04%	-0.25%	-0.45%	-0.66%	1.26%	0.26%	0.62%	2.36
LHA.DE	3.49%	2.92%	2.35%	1.78%	1.21%	0.64%	0.07%	-0.50%	-1.07%	-1.64%	3.72%	0.93%	1.73%	1.86
LMT	13.72%	11.52%	9.31%	7.10%	4.89%	2.69%	0.48%	-1.73%	-3.94%	-6.14%	14.60%	3.79%	6.68%	1.76
MC.PA	-5.60%	-3.96%	-2.33%	-0.69%	0.95%	2.58%	4.22%	5.87%	7.50%	9.14%	-6.25%	1.77%	4.96%	2.81
MSFT	-6.90%	-2.44%	2.02%	6.47%	10.93%	15.40%	19.85%	24.31%	28.76%	33.22%	-8.68%	13.16%	13.50%	1.03
OR.PA	-1.68%	-0.95%	-0.23%	0.49%	1.21%	1.95%	2.66%	3.36%	4.10%	4.81%	-1.96%	1.57%	2.18%	1.39
PFE	10.03%	8.75%	7.46%	6.18%	4.90%	3.61%	2.33%	1.05%	-0.24%	-1.53%	10.55%	4.26%	3.89%	0.91
PG	21.60%	18.48%	15.36%	12.23%	9.11%	5.99%	2.86%	-0.26%	-3.38%	-6.50%	22.85%	7.55%	9.45%	1.25
REP.MC	0.97%	0.86%	0.73%	0.61%	0.49%	0.38%	0.26%	0.14%	0.01%	-0.11%	1.02%	0.43%	0.36%	0.84
SAN.MC	-5.69%	-4.48%	-3.27%	-2.07%	-0.87%	0.33%	1.54%	2.74%	3.95%	5.15%	-6.17%	-0.27%	3.65%	-13.65
SAN.PA	1.58%	1.62%	1.66%	1.70%	1.74%	1.78%	1.82%	1.86%	1.90%	1.94%	1.57%	1.76%	0.12%	0.07
SAP.DE	4.35%	3.98%	3.61%	3.25%	2.89%	2.52%	2.16%	1.79%	1.42%	1.06%	4.50%	2.70%	1.11%	0.41
SOLB.BR	-0.28%	-0.20%	-0.12%	-0.04%	0.04%	0.13%	0.21%	0.28%	0.36%	0.44%	-0.31%	0.08%	0.24%	2.92
ULVR.L	8.28%	7.16%	6.04%	4.93%	3.81%	2.69%	1.59%	0.48%	-0.65%	-1.76%	8.72%	3.26%	3.38%	1.04
VOW3.DE	-0.21%	-0.07%	0.07%	0.22%	0.37%	0.51%	0.65%	0.80%	0.94%	1.09%	-0.27%	0.44%	0.44%	1.00
WMT	10.54%	9.52%	8.50%	7.48%	6.46%	5.45%	4.43%	3.41%	2.39%	1.37%	10.94%	5.95%	3.08%	0.52
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**8.25%**			
Annual Variance	**0.011**	**0.012**	**0.013**	**0.016**	**0.019**	**0.023**	**0.028**	**0.034**	**0.040**	**0.047**	**0.011**			
Annual Std. Dev.	**10.45%**	**10.80%**	**11.51%**	**12.51%**	**13.74%**	**15.15%**	**16.68%**	**18.32%**	**20.03%**	**21.79%**	**10.42%**			
Herfindahl–Hirschman index	**13.21%**	**9.95%**	**7.93%**	**7.16%**	**7.65%**	**9.38%**	**12.36%**	**16.60%**	**22.09%**	**28.82%**	**14.87%**			

* The Variability Index of the weights has been computed using the first 10 portfolios.

Appendix 15. B&L portfolios' weights with short selling & no views over 2015-2019.

B&L portfolios' weights with short selling & no views over 2020-2021																	
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**Portfolio 11**	**Portfolio 12**	**Portfolio 13**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	-7.25%	-4.77%	-2.28%	0.20%	2.68%	5.17%	7.65%	10.13%	12.62%	15.10%	21.73%	28.38%	35.02%	-6.09%	9.57%	12.84%	1.34
AIR.PA	-3.20%	-2.89%	-2.57%	-2.26%	-1.94%	-1.63%	-1.32%	-1.01%	-0.69%	-0.38%	0.45%	1.29%	2.12%	-3.05%	-1.08%	1.62%	-1.50
AMZN	18.35%	18.07%	17.80%	17.53%	17.25%	16.98%	16.70%	16.43%	16.16%	15.89%	15.16%	14.42%	13.69%	18.22%	16.49%	1.41%	0.09
APD	-4.71%	-4.30%	-3.89%	-3.49%	-3.08%	-2.67%	-2.27%	-1.86%	-1.45%	-1.05%	0.04%	1.13%	2.22%	-4.52%	-1.95%	2.10%	-1.08
AZN.L	5.56%	5.14%	4.71%	4.28%	3.86%	3.43%	3.01%	2.58%	2.15%	1.72%	0.58%	-0.56%	-1.71%	5.37%	2.67%	2.21%	0.83
BAS.DE	-3.92%	-3.58%	-3.23%	-2.89%	-2.54%	-2.20%	-1.85%	-1.51%	-1.16%	-0.82%	0.09%	1.02%	1.93%	-3.76%	-1.59%	1.78%	-1.12
BP.L	-2.69%	-2.48%	-2.27%	-2.06%	-1.85%	-1.65%	-1.44%	-1.23%	-1.03%	-0.82%	-0.27%	0.28%	0.83%	-2.59%	-1.28%	1.07%	-0.83
CPR.MI	-6.42%	-5.91%	-5.41%	-4.90%	-4.40%	-3.89%	-3.39%	-2.88%	-2.38%	-1.87%	-0.53%	0.82%	2.17%	-6.18%	-3.00%	2.61%	-0.87
ENEL.MI	-15.52%	-14.27%	-13.03%	-11.78%	-10.53%	-9.29%	-8.04%	-6.79%	-5.55%	-4.30%	-0.97%	2.36%	5.70%	-14.93%	-7.08%	6.44%	-0.91
EZJ.L	1.78%	1.64%	1.50%	1.37%	1.23%	1.09%	0.96%	0.82%	0.68%	0.55%	0.18%	-0.19%	-0.55%	1.71%	0.85%	0.71%	0.83
G.MI	32.85%	30.35%	27.84%	25.34%	22.83%	20.32%	17.82%	15.31%	12.81%	10.30%	3.59%	-3.10%	-9.79%	31.68%	15.88%	12.95%	0.82
GM	0.47%	0.49%	0.51%	0.53%	0.55%	0.57%	0.59%	0.61%	0.63%	0.65%	0.70%	0.75%	0.80%	0.48%	0.61%	0.10%	0.16
GOOG	3.60%	4.43%	5.27%	6.10%	6.93%	7.77%	8.61%	9.44%	10.27%	11.10%	13.33%	15.57%	17.79%	3.99%	9.25%	4.31%	0.47
IHG.L	9.31%	8.60%	7.88%	7.16%	6.45%	5.73%	5.01%	4.30%	3.58%	2.87%	0.95%	-0.96%	-2.88%	8.98%	4.46%	3.70%	0.83
IP.MI	4.84%	4.47%	4.10%	3.74%	3.37%	3.00%	2.63%	2.27%	1.90%	1.53%	0.56%	-0.43%	-1.41%	4.66%	2.35%	1.90%	0.81
LHA.DE	-3.05%	-2.81%	-2.57%	-2.33%	-2.09%	-1.85%	-1.61%	-1.37%	-1.13%	-0.89%	-0.25%	0.40%	1.04%	-2.94%	-1.42%	1.24%	-0.87
LMT	-1.99%	-1.78%	-1.56%	-1.35%	-1.13%	-0.92%	-0.70%	-0.48%	-0.27%	-0.05%	0.52%	1.10%	1.68%	-1.89%	-0.53%	1.11%	-2.09
MC.PA	4.41%	4.35%	4.28%	4.22%	4.15%	4.09%	4.03%	3.96%	3.90%	3.83%	3.67%	3.49%	3.32%	4.38%	3.98%	0.33%	0.08
MSFT	-16.60%	-13.64%	-10.67%	-7.71%	-4.74%	-1.78%	1.19%	4.15%	7.12%	10.08%	18.01%	25.93%	33.87%	-15.22%	3.48%	15.32%	4.41
OR.PA	1.36%	1.42%	1.49%	1.55%	1.62%	1.68%	1.74%	1.81%	1.87%	1.94%	2.10%	2.27%	2.45%	1.38%	1.79%	0.33%	0.18
PFE	7.81%	7.42%	7.04%	6.66%	6.28%	5.89%	5.51%	5.12%	4.74%	4.36%	3.33%	2.30%	1.28%	7.63%	5.21%	1.98%	0.38
PG	9.64%	9.16%	8.67%	8.19%	7.70%	7.22%	6.73%	6.25%	5.76%	5.28%	4.00%	2.70%	1.40%	9.41%	6.36%	2.50%	0.39
REP.MC	-1.19%	-1.09%	-0.98%	-0.88%	-0.78%	-0.68%	-0.57%	-0.47%	-0.37%	-0.27%	0.01%	0.29%	0.56%	-1.14%	-0.49%	0.53%	-1.08
SAN.MC	-4.61%	-4.22%	-3.83%	-3.44%	-3.04%	-2.65%	-2.26%	-1.87%	-1.48%	-1.08%	-0.03%	1.02%	2.08%	-4.43%	-1.95%	2.03%	-1.04
SAN.PA	21.35%	19.79%	18.24%	16.68%	15.12%	13.56%	12.00%	10.44%	8.89%	7.33%	3.16%	-1.00%	-5.17%	20.62%	10.80%	8.05%	0.75
SAP.DE	1.35%	1.36%	1.36%	1.37%	1.37%	1.38%	1.39%	1.40%	1.40%	1.41%	1.43%	1.45%	1.46%	1.35%	1.40%	0.03%	0.02
SOLB.BR	5.49%	5.07%	4.66%	4.24%	3.83%	3.41%	3.00%	2.59%	2.17%	1.76%	0.65%	-0.46%	-1.57%	5.29%	2.68%	2.14%	0.80
ULVR.L	21.06%	19.44%	17.82%	16.20%	14.58%	12.96%	11.34%	9.72%	8.10%	6.48%	2.16%	-2.16%	-6.49%	20.31%	10.09%	8.37%	0.83
VOW3.DE	-4.96%	-4.55%	-4.14%	-3.73%	-3.32%	-2.91%	-2.50%	-2.09%	-1.68%	-1.28%	-0.19%	0.91%	2.00%	-4.77%	-2.19%	2.11%	-0.97
WMT	26.87%	25.07%	23.27%	21.46%	19.66%	17.85%	16.05%	14.24%	12.44%	10.63%	5.80%	0.98%	-3.85%	26.03%	14.65%	9.33%	0.64
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**30.84%**	**35.84%**	**40.84%**	**9.88%**			
Annual Variance	**0.0214**	**0.0215**	**0.0221**	**0.0234**	**0.0254**	**0.0280**	**0.0312**	**0.0351**	**0.0397**	**0.0449**	**0.0619**	**0.0836**	**0.1098**	**0.0214**			
Annual Std. Dev.	**14.64%**	**14.65%**	**14.87%**	**15.31%**	**15.94%**	**16.73%**	**17.68%**	**18.75%**	**19.92%**	**21.18%**	**24.88%**	**28.91%**	**33.14%**	**14.62%**			
Herfindahl–Hirschman index	**41.28%**	**35.20%**	**29.81%**	**25.11%**	**21.09%**	**17.76%**	**15.12%**	**13.17%**	**11.91%**	**11.33%**	**13.17%**	**19.92%**	**31.59%**	**38.35%**			

To achieve a better visual representation of the optimal portfolio allocation, 3 additional portfolios were added.

* The Variability Index of the weights has been computed using the first 13 portfolios.

Appendix 16. B&L portfolios' weights with short selling & no views over 2020-2021.

Markowitz portfolios' weights over 2015-2019 with short selling - to compare with B&L method														
Stocks	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6	Portfolio 7	Portfolio 8	Portfolio 9	Portfolio 10	MVP	Avg. Weight	Weight Std. Dev.	Variability Index*
AAPL	1.49%	1.68%	1.86%	2.04%	2.22%	2.41%	2.59%	2.78%	2.96%	3.14%	1.62%	2.32%	0.56%	0.24
AIR.PA	-5.14%	-3.75%	-2.35%	-0.95%	0.44%	1.84%	3.24%	4.64%	6.04%	7.43%	-4.13%	1.14%	4.23%	3.70
AMZN	1.37%	2.81%	4.25%	5.70%	7.14%	8.58%	10.02%	11.46%	12.90%	14.34%	2.42%	7.86%	4.36%	0.56
APD	4.01%	4.38%	4.75%	5.13%	5.51%	5.88%	6.26%	6.63%	7.01%	7.39%	4.28%	5.69%	1.14%	0.20
AZN.L	7.03%	7.13%	7.23%	7.33%	7.44%	7.54%	7.64%	7.74%	7.85%	7.95%	7.10%	7.49%	0.31%	0.04
BAS.DE	3.06%	1.33%	-0.40%	-2.13%	-3.86%	-5.59%	-7.31%	-9.04%	-10.77%	-12.49%	1.80%	-4.72%	5.23%	-1.11
BP.L	7.14%	7.15%	7.16%	7.18%	7.19%	7.20%	7.21%	7.22%	7.23%	7.24%	7.15%	7.19%	0.03%	0.00
CPR.MI	0.80%	1.86%	2.91%	3.96%	5.02%	6.07%	7.13%	8.18%	9.24%	10.29%	1.57%	5.55%	3.19%	0.58
ENEL.MI	0.64%	1.88%	3.12%	4.37%	5.61%	6.86%	8.10%	9.34%	10.59%	11.83%	1.54%	6.23%	3.77%	0.60
EZJ.L	3.46%	2.97%	2.48%	1.99%	1.50%	1.00%	0.51%	0.02%	-0.47%	-0.96%	3.10%	1.25%	1.49%	1.19
G.MI	6.50%	6.75%	7.01%	7.27%	7.53%	7.78%	8.04%	8.30%	8.56%	8.81%	6.68%	7.66%	0.78%	0.10
GM	3.36%	3.03%	2.69%	2.36%	2.02%	1.69%	1.36%	1.02%	0.69%	0.35%	3.12%	1.86%	1.01%	0.55
GOOG	3.56%	2.61%	1.65%	0.70%	-0.26%	-1.21%	-2.17%	-3.12%	-4.08%	-5.03%	2.87%	-0.73%	2.89%	-3.93
IHG.L	4.75%	4.80%	4.85%	4.90%	4.95%	5.00%	5.05%	5.10%	5.15%	5.20%	4.79%	4.97%	0.15%	0.03
IP.MI	1.01%	1.35%	1.70%	2.04%	2.38%	2.72%	3.07%	3.41%	3.75%	4.09%	1.26%	2.55%	1.04%	0.41
LHA.DE	3.66%	3.74%	3.82%	3.90%	3.98%	4.06%	4.14%	4.22%	4.30%	4.38%	3.72%	4.02%	0.24%	0.06
LMT	14.22%	14.75%	15.27%	15.79%	16.32%	16.84%	17.36%	17.89%	18.41%	18.93%	14.60%	16.58%	1.58%	0.10
MC.PA	-7.78%	-5.68%	-3.58%	-1.48%	0.62%	2.72%	4.82%	6.92%	9.02%	11.12%	-6.26%	1.67%	6.36%	3.80
MSFT	-9.47%	-8.38%	-7.28%	-6.18%	-5.08%	-3.98%	-2.88%	-1.79%	-0.69%	0.41%	-8.68%	-4.53%	3.33%	-0.73
OR.PA	-1.24%	-2.24%	-3.24%	-4.24%	-5.25%	-6.25%	-7.25%	-8.25%	-9.25%	-10.26%	-1.96%	-5.75%	3.03%	-0.53
PFE	11.09%	10.35%	9.60%	8.86%	8.12%	7.38%	6.64%	5.91%	5.16%	4.43%	10.55%	7.75%	2.24%	0.29
PG	23.48%	22.61%	21.73%	20.85%	19.96%	19.08%	18.20%	17.32%	16.44%	15.56%	22.85%	19.52%	2.67%	0.14
REP.MC	0.88%	1.08%	1.27%	1.47%	1.67%	1.86%	2.06%	2.26%	2.45%	2.65%	1.02%	1.76%	0.59%	0.34
SAN.MC	-5.02%	-6.60%	-8.18%	-9.75%	-11.33%	-12.91%	-14.49%	-16.06%	-17.64%	-19.22%	-6.17%	-12.12%	4.78%	-0.39
SAN.PA	2.28%	1.30%	0.31%	-0.67%	-1.65%	-2.63%	-3.62%	-4.60%	-5.59%	-6.57%	1.57%	-2.14%	2.98%	-1.39
SAP.DE	4.35%	4.55%	4.76%	4.96%	5.17%	5.37%	5.58%	5.78%	5.99%	6.19%	4.50%	5.27%	0.62%	0.12
SOLB.BR	0.31%	-0.55%	-1.40%	-2.26%	-3.12%	-3.98%	-4.83%	-5.69%	-6.55%	-7.41%	-0.31%	-3.55%	2.60%	-0.73
ULVR.L	8.83%	8.68%	8.53%	8.39%	8.24%	8.09%	7.94%	7.80%	7.65%	7.50%	8.72%	8.17%	0.45%	0.05
VOW3.DE	-0.02%	-0.36%	-0.71%	-1.06%	-1.40%	-1.75%	-2.10%	-2.45%	-2.79%	-3.14%	-0.27%	-1.58%	1.05%	-0.67
WMT	11.39%	10.77%	10.15%	9.54%	8.92%	8.30%	7.68%	7.06%	6.44%	5.82%	10.94%	8.61%	1.87%	0.22
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**11.23%**			
Annual Variance	**0.0106**	**0.0106**	**0.0107**	**0.0110**	**0.0115**	**0.0121**	**0.0129**	**0.0139**	**0.0150**	**0.0163**	**0.0112**			
Annual Std. Dev.	**10.31%**	**10.29%**	**10.35%**	**10.50%**	**10.71%**	**11.00%**	**11.36%**	**11.77%**	**12.24%**	**12.76%**	**10.58%**			
Herfindahl–Hirschman index	**15.59%**	**14.66%**	**14.21%**	**14.25%**	**14.78%**	**15.80%**	**17.30%**	**19.28%**	**21.76%**	**24.72%**	**14.87%**			

* The Variability Index of the weights has been computed using the first 10 portfolios.

Appendix 17. Markowitz portfolios' weights over 2015-2019 with short selling - to compare with B&L method.

Markowitz portfolios' weights over 2020-2021 with short selling - to compare with B&L method																	
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**Portfolio 11**	**Portfolio 12**	**Portfolio 13**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	-6.25%	-5.82%	-5.39%	-4.95%	-4.52%	-4.09%	-3.65%	-3.22%	-2.79%	-2.35%	-1.19%	-0.03%	1.13%	-6.09%	-3.32%	2.24%	-0.68
AIR.PA	-3.00%	-3.12%	-3.24%	-3.36%	-3.48%	-3.60%	-3.72%	-3.84%	-3.96%	-4.08%	-4.40%	-4.72%	-5.04%	-3.05%	-3.82%	0.62%	-0.16
AMZN	18.35%	18.01%	17.68%	17.35%	17.02%	16.69%	16.35%	16.02%	15.69%	15.36%	14.47%	13.58%	12.70%	18.22%	16.10%	1.72%	0.11
APD	-4.39%	-4.72%	-5.05%	-5.38%	-5.72%	-6.05%	-6.38%	-6.71%	-7.04%	-7.37%	-8.26%	-9.14%	-10.03%	-4.52%	-6.63%	1.71%	-0.26
AZN.L	5.29%	5.48%	5.68%	5.87%	6.07%	6.26%	6.46%	6.65%	6.84%	7.04%	7.56%	8.08%	8.60%	5.37%	6.61%	1.00%	0.15
BAS.DE	-4.09%	-3.23%	-2.38%	-1.52%	-0.66%	0.20%	1.05%	1.91%	2.77%	3.63%	5.92%	8.21%	10.51%	-3.76%	1.72%	4.43%	2.58
BP.L	-2.49%	-2.75%	-3.01%	-3.26%	-3.52%	-3.78%	-4.04%	-4.29%	-4.55%	-4.81%	-5.50%	-6.18%	-6.87%	-2.59%	-4.23%	1.33%	-0.31
CPR.MI	-6.37%	-5.89%	-5.41%	-4.93%	-4.46%	-3.98%	-3.50%	-3.02%	-2.54%	-2.06%	-0.79%	0.49%	1.77%	-6.18%	-3.13%	2.47%	-0.79
ENEL.MI	-14.77%	-15.20%	-15.63%	-16.06%	-16.50%	-16.93%	-17.36%	-17.80%	-18.23%	-18.66%	-19.82%	-20.97%	-22.13%	-14.93%	-17.70%	2.24%	-0.13
EZJ.L	1.81%	1.56%	1.31%	1.05%	0.80%	0.55%	0.30%	0.05%	-0.20%	-0.45%	-1.13%	-1.80%	-2.47%	1.71%	0.11%	1.30%	12.33
G.MI	31.70%	31.64%	31.58%	31.53%	31.47%	31.41%	31.35%	31.30%	31.24%	31.18%	31.02%	30.87%	30.71%	31.68%	31.31%	0.30%	0.01
GM	0.37%	0.66%	0.94%	1.22%	1.50%	1.79%	2.07%	2.35%	2.63%	2.91%	3.67%	4.42%	5.18%	0.48%	2.28%	1.46%	0.64
GOOG	3.78%	4.33%	4.88%	5.43%	5.97%	6.52%	7.07%	7.62%	8.17%	8.72%	10.18%	11.65%	13.11%	3.99%	7.49%	2.83%	0.38
IHG.L	8.98%	8.97%	8.96%	8.94%	8.93%	8.92%	8.90%	8.89%	8.88%	8.86%	8.83%	8.79%	8.76%	8.98%	8.89%	0.07%	0.01
IP.MI	4.34%	5.17%	6.01%	6.84%	7.68%	8.51%	9.34%	10.18%	11.01%	11.84%	14.07%	16.30%	18.53%	4.66%	9.99%	4.31%	0.43
LHA.DE	-2.80%	-3.16%	-3.53%	-3.89%	-4.26%	-4.62%	-4.99%	-5.36%	-5.72%	-6.09%	-7.06%	-8.04%	-9.02%	-2.94%	-5.27%	1.89%	-0.36
LMT	-1.62%	-2.32%	-3.02%	-3.72%	-4.42%	-5.13%	-5.83%	-6.53%	-7.23%	-7.93%	-9.81%	-11.68%	-13.56%	-1.89%	-6.37%	3.62%	-0.57
MC.PA	4.08%	4.86%	5.64%	6.42%	7.20%	7.97%	8.75%	9.53%	10.31%	11.09%	13.17%	15.25%	17.34%	4.38%	9.36%	4.03%	0.43
MSFT	-15.35%	-15.01%	-14.67%	-14.33%	-14.00%	-13.66%	-13.32%	-12.98%	-12.64%	-12.31%	-11.40%	-10.50%	-9.59%	-15.22%	-13.06%	1.75%	-0.13
OR.PA	0.94%	2.08%	3.22%	4.37%	5.51%	6.65%	7.79%	8.93%	10.07%	11.21%	14.27%	17.32%	20.37%	1.38%	8.67%	5.90%	0.68
PFE	7.43%	7.94%	8.45%	8.96%	9.48%	9.99%	10.50%	11.01%	11.53%	12.04%	13.41%	14.78%	16.15%	7.63%	10.90%	2.65%	0.24
PG	9.39%	9.44%	9.49%	9.53%	9.58%	9.63%	9.67%	9.72%	9.77%	9.82%	9.94%	10.07%	10.19%	9.41%	9.71%	0.24%	0.02
REP.MC	-1.16%	-1.10%	-1.04%	-0.98%	-0.92%	-0.86%	-0.80%	-0.74%	-0.68%	-0.62%	-0.46%	-0.30%	-0.14%	-1.13%	-0.75%	0.31%	-0.41
SAN.MC	-4.35%	-4.55%	-4.74%	-4.94%	-5.13%	-5.33%	-5.52%	-5.72%	-5.91%	-6.11%	-6.63%	-7.15%	-7.67%	-4.43%	-5.67%	1.01%	-0.18
SAN.PA	20.92%	20.15%	19.37%	18.59%	17.82%	17.04%	16.26%	15.48%	14.70%	13.93%	11.85%	9.78%	7.70%	20.62%	15.66%	4.02%	0.26
SAP.DE	1.57%	1.01%	0.45%	-0.11%	-0.67%	-1.23%	-1.79%	-2.35%	-2.91%	-3.47%	-4.97%	-6.47%	-7.96%	1.35%	-2.22%	2.90%	-1.30
SOLB.BR	5.34%	5.22%	5.11%	5.00%	4.89%	4.78%	4.67%	4.56%	4.45%	4.34%	4.05%	3.75%	3.45%	5.29%	4.59%	0.57%	0.12
ULVR.L	20.73%	19.64%	18.55%	17.46%	16.37%	15.28%	14.19%	13.09%	12.00%	10.91%	8.00%	5.08%	2.16%	20.31%	13.34%	5.64%	0.42
VOW3.DE	-4.64%	-4.98%	-5.32%	-5.66%	-6.00%	-6.34%	-6.68%	-7.02%	-7.37%	-7.70%	-8.62%	-9.53%	-10.44%	-4.77%	-6.94%	1.76%	-0.25
WMT	26.25%	25.68%	25.11%	24.54%	23.98%	23.41%	22.84%	22.27%	21.71%	21.13%	19.62%	18.10%	16.58%	26.03%	22.40%	2.94%	0.13
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**30.84%**	**35.84%**	**40.84%**	**9.63%**			
Annual Variance	**0.0209**	**0.0209**	**0.0210**	**0.0211**	**0.0214**	**0.0218**	**0.0222**	**0.0227**	**0.0233**	**0.0240**	**0.0263**	**0.0292**	**0.0327**	**0.0208**			
Annual Std. Dev.	**14.44%**	**14.44%**	**14.48%**	**14.54%**	**14.63%**	**14.75%**	**14.90%**	**15.07%**	**15.28%**	**15.50%**	**16.22%**	**17.09%**	**18.09%**	**14.44%**			
Herfindahl–Hirschman index	**38.72%**	**37.81%**	**37.07%**	**36.50%**	**36.09%**	**35.85%**	**35.78%**	**35.87%**	**36.13%**	**36.55%**	**38.51%**	**41.66%**	**46.00%**	**38.35%**			

To achieve a better visual representation of the optimal portfolio allocation, 3 additional portfolios were added.

* The Variability Index of the weights has been computed using the first 13 portfolios.

Appendix 18. Markowitz portfolios' weights over 2020-2021 with short selling - to compare with B&L method.

$B\&L's\ \Sigma_p$ matrix over 2015-2019																														
	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	0.0619																													
AIR.PA	0.0179	0.0727																												
AMZN	0.0357	0.0179	0.0852																											
APD	0.0187	0.0169	0.0152	0.0358																										
AZN.L	0.0084	0.0196	0.0074	0.0068	0.0548																									
BAS.DE	0.0184	0.0339	0.0170	0.0178	0.0139	0.0502																								
BP.L	0.0113	0.0223	0.0110	0.0128	0.0153	0.0245	0.0571																							
CPR.MI	0.0101	0.0228	0.0123	0.0129	0.0154	0.0202	0.0134	0.0533																						
ENEL.MI	0.0082	0.0271	0.0106	0.0139	0.0134	0.0252	0.0197	0.0263	0.0523																					
EZJ.L	0.0124	0.0281	0.0101	0.0135	0.0060	0.0254	0.0016	0.0162	0.0199	0.1247																				
G.MI	0.0108	0.0314	0.0107	0.0147	0.0096	0.0306	0.0192	0.0214	0.0347	0.0305	0.0654																			
GM	0.0206	0.0195	0.0180	0.0201	0.0030	0.0191	0.0145	0.0093	0.0104	0.0180	0.0188	0.0661																		
GOOG	0.0314	0.0174	0.0456	0.0174	0.0087	0.0159	0.0082	0.0115	0.0114	0.0123	0.0128	0.0198	0.0582																	
IHG.L	0.0116	0.0269	0.0124	0.0125	0.0155	0.0215	0.0187	0.0172	0.0164	0.0204	0.0184	0.0138	0.0118	0.0608																
IP.MI	0.0178	0.0341	0.0159	0.0161	0.0141	0.0295	0.0190	0.0216	0.0240	0.0276	0.0282	0.0167	0.0151	0.0241	0.0810															
LHA.DE	0.0105	0.0284	0.0099	0.0120	0.0075	0.0282	0.0078	0.0173	0.0212	0.0621	0.0317	0.0171	0.0110	0.0202	0.0237	0.0973														
LMT	0.0140	0.0109	0.0144	0.0124	0.0059	0.0091	0.0059	0.0083	0.0078	0.0054	0.0064	0.0103	0.0140	0.0066	0.0068	0.0058	0.0311													
MC.PA	0.0211	0.0403	0.0197	0.0187	0.0172	0.0365	0.0220	0.0255	0.0282	0.0286	0.0305	0.0184	0.0187	0.0251	0.0310	0.0275	0.0103	0.0655												
MSFT	0.0327	0.0198	0.0419	0.0205	0.0086	0.0180	0.0127	0.0137	0.0141	0.0113	0.0140	0.0208	0.0372	0.0132	0.0160	0.0107	0.0168	0.0204	0.0540											
OR.PA	0.0106	0.0277	0.0127	0.0126	0.0185	0.0248	0.0169	0.0245	0.0254	0.0152	0.0205	0.0101	0.0112	0.0190	0.0196	0.0163	0.0080	0.0339	0.0141	0.0408										
PFE	0.0136	0.0112	0.0150	0.0121	0.0084	0.0089	0.0075	0.0069	0.0067	0.0059	0.0078	0.0149	0.0150	0.0059	0.0095	0.0068	0.0119	0.0100	0.0161	0.0082	0.0329									
PG	0.0100	0.0066	0.0093	0.0100	0.0047	0.0058	0.0054	0.0081	0.0078	0.0027	0.0046	0.0081	0.0099	0.0034	0.0051	0.0025	0.0080	0.0082	0.0127	0.0089	0.0086	0.0247								
REP.MC	0.0148	0.0316	0.0135	0.0176	0.0096	0.0356	0.0471	0.0186	0.0305	0.0174	0.0352	0.0222	0.0121	0.0215	0.0277	0.0222	0.0072	0.0304	0.0165	0.0198	0.0086	0.0059	0.0770							
SAN.MC	0.0161	0.0408	0.0177	0.0218	0.0104	0.0440	0.0322	0.0218	0.0412	0.0370	0.0549	0.0280	0.0185	0.0228	0.0351	0.0388	0.0081	0.0398	0.0202	0.0244	0.0113	0.0065	0.0594	0.0989						
SAN.PA	0.0108	0.0266	0.0097	0.0115	0.0228	0.0229	0.0176	0.0207	0.0239	0.0133	0.0217	0.0101	0.0113	0.0161	0.0185	0.0161	0.0088	0.0268	0.0120	0.0253	0.0116	0.0064	0.0226	0.0287	0.0455					
SAP.DE	0.0154	0.0309	0.0174	0.0143	0.0156	0.0278	0.0169	0.0222	0.0237	0.0216	0.0247	0.0129	0.0159	0.0197	0.0264	0.0207	0.0080	0.0320	0.0182	0.0256	0.0085	0.0060	0.0220	0.0319	0.0231	0.0470				
SOLB.BR	0.0184	0.0355	0.0178	0.0170	0.0133	0.0411	0.0259	0.0209	0.0250	0.0273	0.0322	0.0189	0.0161	0.0239	0.0312	0.0301	0.0080	0.0366	0.0174	0.0225	0.0104	0.0054	0.0366	0.0455	0.0220	0.0273	0.0614			
ULVR.L	0.0062	0.0154	0.0064	0.0075	0.0184	0.0143	0.0141	0.0187	0.0177	0.0067	0.0101	0.0026	0.0063	0.0156	0.0112	0.0073	0.0059	0.0194	0.0087	0.0261	0.0051	0.0096	0.0102	0.0101	0.0187	0.0164	0.0128	0.0390		
VOW3.DE	0.0199	0.0398	0.0181	0.0188	0.0140	0.0424	0.0261	0.0202	0.0298	0.0330	0.0397	0.0290	0.0193	0.0241	0.0353	0.0348	0.0081	0.0404	0.0184	0.0229	0.0102	0.0047	0.0428	0.0560	0.0238	0.0306	0.0430	0.0105	0.1128	
WMT	0.0111	0.0086	0.0104	0.0096	0.0029	0.0053	0.0049	0.0057	0.0070	0.0047	0.0049	0.0098	0.0099	0.0050	0.0066	0.0047	0.0099	0.0081	0.0122	0.0062	0.0105	0.0112	0.0049	0.0073	0.0046	0.0068	0.0064	0.0045	0.0050	0.0383

Appendix 19. $B\&L\ \Sigma_p$ matrix over 2015-2019.

$B\&L's\ \Sigma_p$ matrix over 2020-2021																														
	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	0.1409																													
AIR.PA	0.0423	0.3394																												
AMZN	0.0827	0.0138	0.1064																											
APD	0.0662	0.0606	0.0379	0.1149																										
AZN.L	0.0212	0.0213	0.0180	0.0168	0.0770																									
BAS.DE	0.0317	0.1197	0.0085	0.0586	0.0191	0.1174																								
BP.L	0.0294	0.1461	0.0018	0.0596	0.0207	0.1085	0.2130																							
CPR.MI	0.0363	0.0527	0.0249	0.0301	0.0266	0.0416	0.0428	0.0948																						
ENEL.MI	0.0417	0.0726	0.0302	0.0525	0.0344	0.0653	0.0719	0.0509	0.1071																					
EZJ.L	0.0321	0.2866	0.0083	0.0610	0.0009	0.1293	0.1721	0.0570	0.0679	0.5146																				
G.MI	0.0304	0.1013	0.0126	0.0392	0.0163	0.0712	0.0904	0.0426	0.0585	0.1096	0.0743																			
GM	0.0599	0.1691	0.0242	0.0798	0.0142	0.0969	0.1228	0.0409	0.0555	0.1825	0.0749	0.2527																		
GOOG	0.0827	0.0561	0.0675	0.0578	0.0162	0.0373	0.0379	0.0336	0.0415	0.0583	0.0333	0.0669	0.1008																	
IHG.L	0.0414	0.1967	0.0103	0.0675	0.0068	0.1002	0.1286	0.0439	0.0611	0.2485	0.0804	0.1383	0.0487	0.2267																
IP.MI	0.0361	0.0853	0.0204	0.0431	0.0200	0.0542	0.0520	0.0414	0.0522	0.0662	0.0491	0.0584	0.0382	0.0588	0.1204															
LHA.DE	0.0314	0.2057	0.0117	0.0605	0.0191	0.1158	0.1365	0.0535	0.0704	0.3080	0.0922	0.1217	0.0428	0.1603	0.0671	0.3646														
LMT	0.0508	0.0556	0.0247	0.0594	0.0194	0.0459	0.0511	0.0421	0.0410	0.0571	0.0359	0.0558	0.0422	0.0527	0.0321	0.0540	0.1052													
MC.PA	0.0431	0.1031	0.0274	0.0480	0.0215	0.0663	0.0771	0.0429	0.0542	0.1090	0.0535	0.0772	0.0417	0.0910	0.0447	0.0856	0.0353	0.0981												
MSFT	0.1027	0.0422	0.0769	0.0670	0.0240	0.0314	0.0283	0.0339	0.0429	0.0374	0.0289	0.0624	0.0883	0.0397	0.0330	0.0306	0.0496	0.0442	0.1170											
OR.PA	0.0344	0.0514	0.0244	0.0378	0.0263	0.0400	0.0479	0.0420	0.0489	0.0488	0.0381	0.0369	0.0308	0.0453	0.0361	0.0481	0.0342	0.0536	0.0332	0.0659										
PFE	0.0391	0.0246	0.0188	0.0461	0.0243	0.0259	0.0298	0.0240	0.0249	0.0269	0.0190	0.0346	0.0357	0.0228	0.0151	0.0264	0.0456	0.0267	0.0400	0.0231	0.0967									
PG	0.0490	0.0152	0.0308	0.0489	0.0192	0.0206	0.0175	0.0228	0.0287	0.0016	0.0171	0.0299	0.0408	0.0128	0.0216	0.0177	0.0472	0.0181	0.0508	0.0235	0.0393	0.0633								
REP.MC	0.0323	0.1559	0.0070	0.0637	0.0160	0.1072	0.1822	0.0411	0.0690	0.1841	0.0917	0.1200	0.0456	0.1316	0.0549	0.1446	0.0555	0.0747	0.0348	0.0444	0.0341	0.0215	0.2156							
SAN.MC	0.0378	0.1644	0.0101	0.0568	0.0160	0.1159	0.1466	0.0502	0.0778	0.1921	0.0970	0.1205	0.0446	0.1261	0.0616	0.1573	0.0570	0.0820	0.0349	0.0529	0.0292	0.0237	0.1507	0.2160						
SAN.PA	0.0178	0.0333	0.0101	0.0166	0.0368	0.0327	0.0362	0.0280	0.0387	0.0221	0.0276	0.0277	0.0162	0.0237	0.0274	0.0329	0.0219	0.0282	0.0195	0.0273	0.0194	0.0154	0.0324	0.0332	0.0537					
SAP.DE	0.0418	0.0792	0.0334	0.0432	0.0238	0.0565	0.0508	0.0383	0.0529	0.0753	0.0453	0.0513	0.0415	0.0709	0.0451	0.0668	0.0311	0.0543	0.0456	0.0419	0.0213	0.0206	0.0547	0.0567	0.0297	0.1081				
SOLB.BR	0.0212	0.1005	0.0089	0.0484	0.0196	0.0922	0.0948	0.0630	0.0616	0.1117	0.0645	0.0810	0.0294	0.0734	0.0575	0.1104	0.0477	0.0522	0.0199	0.0401	0.0206	0.0175	0.0941	0.1018	0.0267	0.0407	0.1379			
ULVR.L	0.0189	0.0197	0.0143	0.0266	0.0283	0.0284	0.0271	0.0287	0.0307	0.0126	0.0189	0.0163	0.0186	0.0149	0.0176	0.0221	0.0246	0.0232	0.0222	0.0331	0.0158	0.0212	0.0210	0.0200	0.0228	0.0247	0.0281	0.0544		
VOW3.DE	0.0530	0.1522	0.0227	0.0681	0.0286	0.1096	0.1261	0.0458	0.0680	0.1719	0.0816	0.1394	0.0550	0.1381	0.0624	0.1317	0.0533	0.0910	0.0503	0.0529	0.0280	0.0225	0.1255	0.1314	0.0332	0.0730	0.0856	0.0261	0.2045	
WMT	0.0468	-0.0074	0.0313	0.0412	0.0170	0.0131	0.0054	0.0204	0.0234	-0.0114	0.0085	0.0172	0.0335	-0.0027	0.0119	0.0066	0.0345	0.0124	0.0455	0.0142	0.0290	0.0410	0.0076	0.0135	0.0096	0.0132	0.0106	0.0145	0.0112	0.0650

Appendix 20. $B\&L\ \Sigma_p$ matrix over 2020-2021.

Excess Returns - Annual variance-covariance matrix over 2015-2019																														
	AAPL	**AIR.PA**	**AMZN**	**APD**	**AZN.L**	**BAS.DE**	**BP.L**	**CPR.MI**	**ENEL.MI**	**EZJ.L**	**G.MI**	**GM**	**GOOG**	**IHG.L**	**IP.MI**	**LHA.DE**	**LMT**	**MC.PA**	**MSFT**	**OR.PA**	**PFE**	**PG**	**REP.MC**	**SAN.MC**	**SAN.PA**	**SAP.DE**	**SOLB.BR**	**ULVR.L**	**VOW3.DE**	**WMT**
AAPL	0.0605																													
AIR.PA	0.0171	0.0709																												
AMZN	0.0348	0.0172	0.0830																											
APD	0.0181	0.0163	0.0147	0.0348																										
AZN.L	0.0080	0.0190	0.0071	0.0066	0.0534																									
BAS.DE	0.0177	0.0330	0.0163	0.0173	0.0135	0.0488																								
BP.L	0.0107	0.0217	0.0105	0.0124	0.0149	0.0238	0.0557																							
CPR.MI	0.0096	0.0221	0.0119	0.0125	0.0150	0.0196	0.0130	0.0520																						
ENEL.MI	0.0077	0.0264	0.0102	0.0134	0.0130	0.0245	0.0191	0.0256	0.0510																					
EZJ.L	0.0120	0.0274	0.0097	0.0130	0.0058	0.0248	0.0014	0.0158	0.0194	0.1224																				
G.MI	0.0103	0.0306	0.0102	0.0142	0.0093	0.0298	0.0187	0.0208	0.0338	0.0298	0.0638																			
GM	0.0200	0.0189	0.0174	0.0195	0.0028	0.0185	0.0140	0.0089	0.0100	0.0175	0.0182	0.0644																		
GOOG	0.0306	0.0168	0.0444	0.0169	0.0084	0.0153	0.0078	0.0111	0.0110	0.0119	0.0123	0.0192	0.0567																	
IHG.L	0.0111	0.0262	0.0119	0.0121	0.0151	0.0209	0.0182	0.0167	0.0159	0.0199	0.0179	0.0133	0.0114	0.0593																
IP.MI	0.0171	0.0332	0.0154	0.0156	0.0136	0.0287	0.0185	0.0210	0.0233	0.0270	0.0275	0.0162	0.0145	0.0234	0.0789															
LHA.DE	0.0100	0.0278	0.0094	0.0116	0.0073	0.0275	0.0075	0.0168	0.0207	0.0612	0.0310	0.0166	0.0105	0.0198	0.0231	0.0954														
LMT	0.0136	0.0105	0.0140	0.0119	0.0057	0.0087	0.0056	0.0080	0.0075	0.0051	0.0061	0.0099	0.0136	0.0063	0.0065	0.0055	0.0303													
MC.PA	0.0203	0.0392	0.0190	0.0181	0.0167	0.0356	0.0214	0.0248	0.0275	0.0280	0.0297	0.0178	0.0181	0.0245	0.0301	0.0268	0.0099	0.0638												
MSFT	0.0319	0.0191	0.0408	0.0198	0.0083	0.0173	0.0122	0.0132	0.0135	0.0108	0.0135	0.0201	0.0362	0.0127	0.0155	0.0103	0.0163	0.0197	0.0526											
OR.PA	0.0101	0.0269	0.0122	0.0122	0.0180	0.0241	0.0164	0.0238	0.0247	0.0148	0.0200	0.0097	0.0108	0.0185	0.0190	0.0159	0.0077	0.0330	0.0136	0.0397										
PFE	0.0132	0.0107	0.0145	0.0117	0.0081	0.0085	0.0072	0.0066	0.0063	0.0054	0.0074	0.0144	0.0145	0.0056	0.0091	0.0063	0.0115	0.0096	0.0156	0.0078	0.0321									
PG	0.0097	0.0063	0.0090	0.0096	0.0045	0.0055	0.0051	0.0078	0.0075	0.0025	0.0044	0.0078	0.0096	0.0032	0.0048	0.0023	0.0077	0.0078	0.0123	0.0086	0.0083	0.0240								
REP.MC	0.0141	0.0308	0.0129	0.0170	0.0093	0.0346	0.0459	0.0181	0.0297	0.0170	0.0343	0.0215	0.0116	0.0209	0.0269	0.0216	0.0069	0.0296	0.0158	0.0193	0.0082	0.0056	0.0750							
SAN.MC	0.0154	0.0398	0.0170	0.0211	0.0101	0.0428	0.0314	0.0212	0.0401	0.0362	0.0535	0.0271	0.0178	0.0221	0.0341	0.0380	0.0077	0.0388	0.0194	0.0237	0.0107	0.0062	0.0579	0.0965						
SAN.PA	0.0104	0.0259	0.0093	0.0111	0.0221	0.0223	0.0171	0.0202	0.0233	0.0129	0.0211	0.0098	0.0109	0.0156	0.0180	0.0156	0.0085	0.0260	0.0115	0.0246	0.0112	0.0061	0.0220	0.0279	0.0443					
SAP.DE	0.0148	0.0301	0.0168	0.0138	0.0151	0.0270	0.0164	0.0216	0.0230	0.0211	0.0240	0.0125	0.0153	0.0191	0.0256	0.0202	0.0077	0.0312	0.0176	0.0249	0.0081	0.0057	0.0214	0.0310	0.0224	0.0458				
SOLB.BR	0.0177	0.0346	0.0172	0.0165	0.0129	0.0400	0.0252	0.0203	0.0243	0.0266	0.0313	0.0183	0.0155	0.0233	0.0304	0.0294	0.0076	0.0356	0.0168	0.0218	0.0100	0.0051	0.0356	0.0443	0.0213	0.0266	0.0598			
ULVR.L	0.0059	0.0150	0.0061	0.0072	0.0179	0.0138	0.0137	0.0182	0.0172	0.0065	0.0098	0.0025	0.0060	0.0152	0.0109	0.0070	0.0057	0.0188	0.0084	0.0254	0.0049	0.0093	0.0099	0.0098	0.0181	0.0159	0.0125	0.0380		
VOW3.DE	0.0192	0.0387	0.0174	0.0182	0.0135	0.0413	0.0254	0.0196	0.0290	0.0323	0.0387	0.0281	0.0186	0.0234	0.0343	0.0340	0.0077	0.0394	0.0177	0.0222	0.0098	0.0044	0.0417	0.0546	0.0231	0.0297	0.0419	0.0102	0.1100	
WMT	0.0108	0.0083	0.0100	0.0093	0.0028	0.0051	0.0047	0.0055	0.0067	0.0044	0.0046	0.0094	0.0095	0.0048	0.0063	0.0045	0.0096	0.0078	0.0118	0.0060	0.0101	0.0109	0.0046	0.0070	0.0044	0.0065	0.0061	0.0044	0.0047	0.0373

Appendix 21. Annual variance-covariance matrix over 2015-2019 – excess returns.

Excess Returns - Annual variance-covariance matrix over 2020-2021

	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L
AAPL	0.1376													
AIR.PA	0.0413	0.3327												
AMZN	0.0808	0.0130	0.1041											
APD	0.0646	0.0596	0.0369	0.1122										
AZN.L	0.0207	0.0207	0.0175	0.0163	0.0752									
BAS.DE	0.0309	0.1181	0.0080	0.0575	0.0186	0.1156								
BP.L	0.0286	0.1438	0.0013	0.0585	0.0202	0.1069	0.2089							
CPR.MI	0.0354	0.0517	0.0242	0.0294	0.0259	0.0407	0.0419	0.0925						
ENEL.MI	0.0407	0.0713	0.0293	0.0514	0.0335	0.0641	0.0706	0.0497	0.1047					
EZJ.L	0.0312	0.2815	0.0075	0.0600	0.0009	0.1276	0.1694	0.0558	0.0669	0.5041				
G.MI	0.0296	0.0996	0.0121	0.0384	0.0159	0.0701	0.0888	0.0417	0.0573	0.1079	0.0729			
GM	0.0584	0.1662	0.0233	0.0782	0.0139	0.0956	0.1209	0.0401	0.0546	0.1795	0.0737	0.2476		
GOOG	0.0807	0.0548	0.0659	0.0564	0.0157	0.0365	0.0370	0.0328	0.0405	0.0570	0.0326	0.0653	0.0984	
IHG.L	0.0404	0.1933	0.0096	0.0662	0.0066	0.0989	0.1266	0.0430	0.0600	0.2440	0.0791	0.1361	0.0475	0.2223
IP.MI	0.0352	0.0838	0.0197	0.0422	0.0194	0.0533	0.0512	0.0405	0.0511	0.0652	0.0482	0.0574	0.0373	0.0578
LHA.DE	0.0306	0.2021	0.0111	0.0594	0.0186	0.1141	0.1343	0.0524	0.0691	0.3022	0.0907	0.1199	0.0419	0.1577
LMT	0.0495	0.0546	0.0240	0.0580	0.0189	0.0450	0.0501	0.0411	0.0401	0.0561	0.0352	0.0547	0.0412	0.0517
MC.PA	0.0421	0.1013	0.0265	0.0470	0.0210	0.0653	0.0758	0.0420	0.0531	0.1071	0.0525	0.0759	0.0407	0.0894
MSFT	0.1002	0.0411	0.0752	0.0654	0.0234	0.0306	0.0276	0.0330	0.0419	0.0365	0.0282	0.0609	0.0862	0.0387
OR.PA	0.0336	0.0504	0.0237	0.0370	0.0257	0.0392	0.0469	0.0410	0.0477	0.0478	0.0373	0.0361	0.0300	0.0444
PFE	0.0381	0.0241	0.0183	0.0450	0.0237	0.0254	0.0292	0.0234	0.0243	0.0264	0.0186	0.0338	0.0349	0.0224
PG	0.0478	0.0148	0.0301	0.0477	0.0188	0.0201	0.0171	0.0223	0.0280	0.0015	0.0167	0.0292	0.0398	0.0125
REP.MC	0.0315	0.1534	0.0064	0.0626	0.0156	0.1056	0.1789	0.0403	0.0678	0.1812	0.0901	0.1181	0.0445	0.1296
SAN.MC	0.0369	0.1618	0.0095	0.0558	0.0156	0.1143	0.1442	0.0492	0.0763	0.1890	0.0954	0.1187	0.0436	0.1243
SAN.PA	0.0174	0.0327	0.0097	0.0163	0.0359	0.0321	0.0355	0.0273	0.0378	0.0218	0.0270	0.0272	0.0158	0.0233
SAP.DE	0.0408	0.0777	0.0324	0.0423	0.0232	0.0555	0.0499	0.0374	0.0517	0.0740	0.0444	0.0504	0.0405	0.0696
SOLB.BR	0.0206	0.0990	0.0085	0.0475	0.0191	0.0907	0.0932	0.0616	0.0604	0.1101	0.0634	0.0798	0.0288	0.0724
ULVR.L	0.0184	0.0190	0.0139	0.0259	0.0276	0.0275	0.0264	0.0280	0.0299	0.0121	0.0184	0.0158	0.0180	0.0144
VOW3.DE	0.0516	0.1501	0.0215	0.0668	0.0280	0.1081	0.1244	0.0450	0.0668	0.1696	0.0805	0.1373	0.0537	0.1362
WMT	0.0457	-0.0073	0.0306	0.0402	0.0166	0.0127	0.0052	0.0199	0.0228	-0.0113	0.0082	0.0167	0.0327	-0.0027

	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
IP.MI	0.1176															
LHA.DE	0.0659	0.3569														
LMT	0.0314	0.0530	0.1027													
MC.PA	0.0438	0.0841	0.0346	0.0960												
MSFT	0.0322	0.0298	0.0483	0.0431	0.1142											
OR.PA	0.0353	0.0471	0.0334	0.0524	0.0324	0.0644										
PFE	0.0148	0.0259	0.0446	0.0261	0.0390	0.0226	0.0943									
PG	0.0210	0.0172	0.0460	0.0176	0.0496	0.0229	0.0384	0.0618								
REP.MC	0.0540	0.1423	0.0544	0.0735	0.0339	0.0435	0.0334	0.0210	0.2115							
SAN.MC	0.0606	0.1547	0.0559	0.0807	0.0340	0.0518	0.0286	0.0231	0.1482	0.2121						
SAN.PA	0.0268	0.0322	0.0214	0.0276	0.0190	0.0267	0.0190	0.0150	0.0318	0.0326	0.0524					
SAP.DE	0.0441	0.0656	0.0304	0.0532	0.0445	0.0410	0.0208	0.0201	0.0537	0.0558	0.0291	0.1056				
SOLB.BR	0.0564	0.1085	0.0467	0.0513	0.0194	0.0392	0.0202	0.0170	0.0926	0.1002	0.0262	0.0400	0.1351			
ULVR.L	0.0171	0.0214	0.0240	0.0225	0.0216	0.0323	0.0154	0.0206	0.0203	0.0194	0.0222	0.0241	0.0273	0.0531		
VOW3.DE	0.0614	0.1300	0.0524	0.0895	0.0489	0.0519	0.0274	0.0219	0.1238	0.1296	0.0326	0.0717	0.0845	0.0254	0.2013	
WMT	0.0116	0.0063	0.0336	0.0121	0.0444	0.0138	0.0283	0.0400	0.0073	0.0131	0.0094	0.0128	0.0103	0.0141	0.0107	0.0634

Appendix 22. Annual variance-covariance matrix over 2020-2021 – excess returns.

$\tau\Sigma$ matrix over 2015-2019 - original formulation																														
	AAPL	**AIR.PA**	**AMZN**	**APD**	**AZN.L**	**BAS.DE**	**BP.L**	**CPR.MI**	**ENEL.MI**	**EZJ.L**	**G.MI**	**GM**	**GOOG**	**IHG.L**	**IP.MI**	**LHA.DE**	**LMT**	**MC.PA**	**MSFT**	**OR.PA**	**PFE**	**PG**	**REP.MC**	**SAN.MC**	**SAN.PA**	**SAP.DE**	**SOLB.BR**	**ULVR.L**	**VOW3.DE**	**WMT**
AAPL	0.0015																													
AIR.PA	0.0004	0.0018																												
AMZN	0.0009	0.0004	0.0021																											
APD	0.0005	0.0004	0.0004	0.0009																										
AZN.L	0.0002	0.0005	0.0002	0.0002	0.0013																									
BAS.DE	0.0004	0.0008	0.0004	0.0004	0.0003	0.0012																								
BP.L	0.0003	0.0005	0.0003	0.0003	0.0004	0.0006	0.0014																							
CPR.MI	0.0002	0.0006	0.0003	0.0003	0.0004	0.0005	0.0003	0.0013																						
ENEL.MI	0.0002	0.0007	0.0003	0.0003	0.0003	0.0006	0.0005	0.0006	0.0013																					
EZJ.L	0.0003	0.0007	0.0002	0.0003	0.0001	0.0006	0.0000	0.0004	0.0005	0.0031																				
G.MI	0.0003	0.0008	0.0003	0.0004	0.0002	0.0007	0.0005	0.0005	0.0008	0.0007	0.0016																			
GM	0.0005	0.0005	0.0004	0.0005	0.0001	0.0005	0.0004	0.0002	0.0002	0.0004	0.0005	0.0016																		
GOOG	0.0008	0.0004	0.0011	0.0004	0.0002	0.0004	0.0002	0.0003	0.0003	0.0003	0.0003	0.0005	0.0014																	
IHG.L	0.0003	0.0007	0.0003	0.0003	0.0004	0.0005	0.0005	0.0004	0.0004	0.0005	0.0004	0.0003	0.0003	0.0015																
IP.MI	0.0004	0.0008	0.0004	0.0004	0.0003	0.0007	0.0005	0.0005	0.0006	0.0007	0.0007	0.0004	0.0004	0.0006	0.0020															
LHA.DE	0.0003	0.0007	0.0002	0.0003	0.0002	0.0007	0.0002	0.0004	0.0005	0.0015	0.0008	0.0004	0.0003	0.0005	0.0006	0.0024														
LMT	0.0003	0.0003	0.0003	0.0003	0.0001	0.0002	0.0001	0.0002	0.0002	0.0001	0.0002	0.0002	0.0003	0.0002	0.0002	0.0001	0.0008													
MC.PA	0.0005	0.0010	0.0005	0.0005	0.0004	0.0009	0.0005	0.0006	0.0007	0.0007	0.0007	0.0004	0.0005	0.0006	0.0008	0.0007	0.0002	0.0016												
MSFT	0.0008	0.0005	0.0010	0.0005	0.0002	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0005	0.0009	0.0003	0.0004	0.0003	0.0004	0.0005	0.0013											
OR.PA	0.0003	0.0007	0.0003	0.0003	0.0004	0.0006	0.0004	0.0006	0.0006	0.0004	0.0005	0.0002	0.0003	0.0005	0.0005	0.0004	0.0002	0.0008	0.0003	0.0010										
PFE	0.0003	0.0003	0.0004	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0004	0.0004	0.0001	0.0002	0.0002	0.0003	0.0002	0.0004	0.0002	0.0321									
PG	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0002	0.0002	0.0006								
REP.MC	0.0004	0.0008	0.0003	0.0004	0.0002	0.0009	0.0011	0.0005	0.0007	0.0004	0.0009	0.0005	0.0003	0.0005	0.0007	0.0005	0.0002	0.0007	0.0004	0.0005	0.0002	0.0001	0.0019							
SAN.MC	0.0004	0.0010	0.0004	0.0005	0.0003	0.0011	0.0008	0.0005	0.0010	0.0009	0.0013	0.0007	0.0004	0.0006	0.0009	0.0009	0.0002	0.0010	0.0005	0.0006	0.0003	0.0002	0.0014	0.0024						
SAN.PA	0.0003	0.0006	0.0002	0.0003	0.0006	0.0006	0.0004	0.0005	0.0006	0.0003	0.0005	0.0002	0.0003	0.0004	0.0004	0.0004	0.0002	0.0007	0.0003	0.0006	0.0003	0.0002	0.0006	0.0007	0.0011					
SAP.DE	0.0004	0.0008	0.0004	0.0003	0.0004	0.0007	0.0004	0.0005	0.0006	0.0005	0.0006	0.0003	0.0004	0.0005	0.0006	0.0005	0.0002	0.0008	0.0004	0.0006	0.0002	0.0001	0.0005	0.0008	0.0006	0.0011				
SOLB.BR	0.0004	0.0009	0.0004	0.0004	0.0003	0.0010	0.0006	0.0005	0.0006	0.0007	0.0008	0.0005	0.0004	0.0006	0.0008	0.0007	0.0002	0.0009	0.0004	0.0005	0.0002	0.0001	0.0009	0.0011	0.0005	0.0007	0.0015			
ULVR.L	0.0001	0.0004	0.0002	0.0002	0.0004	0.0003	0.0003	0.0005	0.0004	0.0002	0.0002	0.0001	0.0002	0.0004	0.0003	0.0002	0.0001	0.0005	0.0002	0.0006	0.0001	0.0002	0.0002	0.0002	0.0005	0.0004	0.0003	0.0010		
VOW3.DE	0.0005	0.0010	0.0004	0.0005	0.0003	0.0010	0.0006	0.0005	0.0007	0.0008	0.0010	0.0007	0.0005	0.0006	0.0009	0.0008	0.0002	0.0010	0.0004	0.0006	0.0002	0.0001	0.0010	0.0014	0.0006	0.0007	0.0010	0.0003	0.0028	
WMT	0.0003	0.0002	0.0003	0.0002	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001	0.0001	0.0002	0.0002	0.0001	0.0002	0.0001	0.0002	0.0002	0.0003	0.0001	0.0003	0.0003	0.0001	0.0002	0.0001	0.0002	0.0002	0.0001	0.0001	0.0009

Appendix 23. $\tau\Sigma$ matrix over 2015-2019 – original formulation.

$\tau\Sigma$ matrix over 2020-2021 - original formulation

	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	0.0034																													
AIR.PA	0.0010	0.0083																												
AMZN	0.0020	0.0003	0.0026																											
APD	0.0016	0.0015	0.0009	0.0028																										
AZN.L	0.0005	0.0005	0.0004	0.0004	0.0019																									
BAS.DE	0.0008	0.0030	0.0002	0.0014	0.0005	0.0029																								
BP.L	0.0007	0.0036	0.0000	0.0015	0.0005	0.0027	0.0052																							
CPR.MI	0.0009	0.0013	0.0006	0.0007	0.0006	0.0010	0.0010	0.0023																						
ENEL.MI	0.0010	0.0018	0.0007	0.0013	0.0008	0.0016	0.0018	0.0012	0.0026																					
EZJ.L	0.0008	0.0070	0.0002	0.0015	0.0000	0.0032	0.0042	0.0014	0.0017	0.0126																				
G.MI	0.0007	0.0025	0.0003	0.0010	0.0004	0.0018	0.0022	0.0010	0.0014	0.0027	0.0018																			
GM	0.0015	0.0042	0.0006	0.0020	0.0003	0.0024	0.0030	0.0010	0.0014	0.0045	0.0018	0.0062																		
GOOG	0.0020	0.0014	0.0016	0.0014	0.0004	0.0009	0.0009	0.0008	0.0010	0.0014	0.0008	0.0016	0.0025																	
IHG.L	0.0010	0.0048	0.0002	0.0017	0.0002	0.0025	0.0032	0.0011	0.0015	0.0061	0.0020	0.0034	0.0012	0.0056																
IP.MI	0.0009	0.0021	0.0005	0.0011	0.0005	0.0013	0.0013	0.0010	0.0013	0.0016	0.0012	0.0014	0.0009	0.0014	0.0029															
LHA.DE	0.0008	0.0051	0.0003	0.0015	0.0005	0.0029	0.0034	0.0013	0.0017	0.0076	0.0023	0.0030	0.0010	0.0039	0.0016	0.0089														
LMT	0.0012	0.0014	0.0006	0.0015	0.0005	0.0011	0.0013	0.0010	0.0010	0.0014	0.0009	0.0014	0.0010	0.0013	0.0008	0.0013	0.0026													
MC.PA	0.0011	0.0025	0.0007	0.0012	0.0005	0.0016	0.0019	0.0010	0.0013	0.0027	0.0013	0.0019	0.0010	0.0022	0.0011	0.0021	0.0009	0.0024												
MSFT	0.0025	0.0010	0.0019	0.0016	0.0006	0.0008	0.0007	0.0008	0.0010	0.0009	0.0007	0.0015	0.0022	0.0010	0.0008	0.0007	0.0012	0.0011	0.0029											
OR.PA	0.0008	0.0013	0.0006	0.0009	0.0006	0.0010	0.0012	0.0010	0.0012	0.0012	0.0009	0.0009	0.0007	0.0011	0.0009	0.0012	0.0008	0.0013	0.0008	0.0016										
PFE	0.0010	0.0006	0.0005	0.0011	0.0006	0.0006	0.0007	0.0006	0.0006	0.0007	0.0005	0.0008	0.0009	0.0006	0.0004	0.0006	0.0011	0.0007	0.0010	0.0006	0.0024									
PG	0.0012	0.0004	0.0008	0.0012	0.0005	0.0005	0.0004	0.0006	0.0007	0.0000	0.0004	0.0007	0.0010	0.0003	0.0005	0.0004	0.0011	0.0004	0.0012	0.0006	0.0010	0.0015								
REP.MC	0.0008	0.0038	0.0002	0.0016	0.0004	0.0026	0.0045	0.0010	0.0017	0.0045	0.0023	0.0030	0.0011	0.0032	0.0014	0.0036	0.0014	0.0018	0.0008	0.0011	0.0008	0.0005	0.0053							
SAN.MC	0.0009	0.0040	0.0002	0.0014	0.0004	0.0029	0.0036	0.0012	0.0019	0.0047	0.0024	0.0030	0.0011	0.0031	0.0015	0.0039	0.0014	0.0020	0.0008	0.0013	0.0007	0.0006	0.0037	0.0053						
SAN.PA	0.0004	0.0008	0.0002	0.0004	0.0009	0.0008	0.0009	0.0007	0.0009	0.0005	0.0007	0.0007	0.0004	0.0006	0.0007	0.0008	0.0005	0.0007	0.0005	0.0007	0.0005	0.0004	0.0008	0.0008	0.0013					
SAP.DE	0.0010	0.0019	0.0008	0.0011	0.0006	0.0014	0.0012	0.0009	0.0013	0.0018	0.0011	0.0013	0.0010	0.0017	0.0011	0.0016	0.0008	0.0013	0.0011	0.0010	0.0005	0.0005	0.0013	0.0014	0.0007	0.0026				
SOLB.BR	0.0005	0.0025	0.0002	0.0012	0.0005	0.0023	0.0023	0.0015	0.0015	0.0028	0.0016	0.0020	0.0007	0.0018	0.0014	0.0027	0.0012	0.0013	0.0005	0.0010	0.0005	0.0004	0.0023	0.0025	0.0007	0.0010	0.0034			
ULVR.L	0.0005	0.0005	0.0003	0.0006	0.0007	0.0007	0.0007	0.0007	0.0007	0.0003	0.0005	0.0004	0.0005	0.0004	0.0004	0.0005	0.0006	0.0006	0.0005	0.0008	0.0004	0.0005	0.0005	0.0005	0.0006	0.0006	0.0007	0.0013		
VOW3.DE	0.0013	0.0038	0.0005	0.0017	0.0007	0.0027	0.0031	0.0011	0.0017	0.0042	0.0020	0.0034	0.0013	0.0034	0.0015	0.0032	0.0013	0.0022	0.0012	0.0013	0.0007	0.0005	0.0031	0.0032	0.0008	0.0018	0.0021	0.0006	0.0050	
WMT	0.0011	-0.0002	0.0008	0.0010	0.0004	0.0003	0.0001	0.0005	0.0006	-0.0003	0.0002	0.0004	0.0008	-0.0001	0.0003	0.0002	0.0008	0.0003	0.0011	0.0003	0.0007	0.0010	0.0002	0.0003	0.0002	0.0003	0.0003	0.0004	0.0003	0.0016

Appendix 24.$\tau\Sigma$ matrix over 2020-2021– original formulation.

Markowitz portfolios' weights over 2015-2019 with short selling - Excess Returns														
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	1.52%	1.70%	1.88%	2.07%	2.25%	2.44%	2.62%	2.80%	2.99%	3.17%	1.65%	2.34%	0.56%	0.24
AIR.PA	-5.11%	-3.71%	-2.31%	-0.92%	0.48%	1.88%	3.28%	4.68%	6.07%	7.47%	-4.11%	1.18%	4.23%	3.58
AMZN	1.40%	2.84%	4.28%	5.72%	7.16%	8.60%	10.04%	11.49%	12.93%	14.37%	2.43%	7.88%	4.36%	0.55
APD	4.03%	4.41%	4.78%	5.16%	5.53%	5.91%	6.28%	6.66%	7.04%	7.41%	4.30%	5.72%	1.14%	0.20
AZN.L	7.00%	7.11%	7.21%	7.31%	7.42%	7.51%	7.62%	7.72%	7.82%	7.93%	7.08%	7.47%	0.31%	0.04
BAS.DE	3.04%	1.32%	-0.41%	-2.14%	-3.87%	-5.59%	-7.31%	-9.05%	-10.78%	-12.51%	1.81%	-4.73%	5.23%	-1.11
BP.L	7.14%	7.15%	7.16%	7.17%	7.18%	7.19%	7.20%	7.21%	7.22%	7.23%	7.14%	7.18%	0.03%	0.00
CPR.MI	0.81%	1.87%	2.92%	3.98%	5.03%	6.09%	7.14%	8.19%	9.25%	10.30%	1.57%	5.56%	3.19%	0.57
ENEL.MI	0.65%	1.90%	3.14%	4.38%	5.63%	6.87%	8.12%	9.36%	10.61%	11.85%	1.54%	6.25%	3.77%	0.60
EZJ.L	3.45%	2.96%	2.47%	1.97%	1.48%	0.99%	0.50%	0.01%	-0.48%	-0.97%	3.10%	1.24%	1.49%	1.20
G.MI	6.48%	6.74%	7.00%	7.25%	7.51%	7.77%	8.03%	8.29%	8.54%	8.80%	6.66%	7.64%	0.78%	0.10
GM	3.37%	3.03%	2.70%	2.36%	2.03%	1.69%	1.36%	1.02%	0.69%	0.36%	3.13%	1.86%	1.01%	0.54
GOOG	3.56%	2.60%	1.65%	0.69%	-0.26%	-1.21%	-2.16%	-3.12%	-4.08%	-5.03%	2.88%	-0.74%	2.89%	-3.92
IHG.L	4.74%	4.79%	4.84%	4.88%	4.93%	4.98%	5.03%	5.08%	5.13%	5.18%	4.77%	4.96%	0.15%	0.03
IP.MI	1.02%	1.36%	1.70%	2.04%	2.39%	2.73%	3.07%	3.41%	3.76%	4.10%	1.26%	2.56%	1.04%	0.41
LHA.DE	3.65%	3.73%	3.81%	3.89%	3.97%	4.05%	4.13%	4.21%	4.29%	4.37%	3.71%	4.01%	0.24%	0.06
LMT	14.20%	14.73%	15.25%	15.77%	16.29%	16.82%	17.34%	17.86%	18.39%	18.91%	14.58%	16.56%	1.58%	0.10
MC.PA	-7.73%	-5.63%	-3.53%	-1.43%	0.67%	2.77%	4.87%	6.97%	9.07%	11.18%	-6.22%	1.72%	6.36%	3.69
MSFT	-9.40%	-8.30%	-7.20%	-6.10%	-5.00%	-3.91%	-2.81%	-1.71%	-0.61%	0.49%	-8.61%	-4.45%	3.32%	-0.75
OR.PA	-1.24%	-2.24%	-3.25%	-4.25%	-5.25%	-6.24%	-7.25%	-8.26%	-9.26%	-10.26%	-1.96%	-5.75%	3.03%	-0.53
PFE	11.08%	10.34%	9.60%	8.86%	8.12%	7.37%	6.64%	5.90%	5.16%	4.42%	10.55%	7.75%	2.24%	0.29
PG	23.42%	22.54%	21.66%	20.78%	19.90%	19.02%	18.14%	17.25%	16.37%	15.49%	22.79%	19.46%	2.67%	0.14
REP.MC	0.88%	1.08%	1.28%	1.47%	1.67%	1.87%	2.07%	2.26%	2.46%	2.66%	1.03%	1.77%	0.60%	0.34
SAN.MC	-5.00%	-6.58%	-8.15%	-9.73%	-11.31%	-12.89%	-14.47%	-16.04%	-17.62%	-19.20%	-6.13%	-12.10%	4.78%	-0.39
SAN.PA	2.27%	1.29%	0.30%	-0.68%	-1.66%	-2.65%	-3.63%	-4.62%	-5.60%	-6.58%	1.56%	-2.16%	2.98%	-1.38
SAP.DE	4.33%	4.54%	4.74%	4.95%	5.15%	5.36%	5.56%	5.77%	5.97%	6.18%	4.48%	5.26%	0.62%	0.12
SOLB.BR	0.31%	-0.55%	-1.40%	-2.26%	-3.12%	-3.98%	-4.84%	-5.69%	-6.55%	-7.40%	-0.31%	-3.55%	2.60%	-0.73
ULVR.L	8.79%	8.64%	8.49%	8.35%	8.20%	8.05%	7.90%	7.76%	7.61%	7.46%	8.69%	8.13%	0.45%	0.05
VOW3.DE	-0.02%	-0.37%	-0.71%	-1.06%	-1.41%	-1.75%	-2.10%	-2.45%	-2.79%	-3.14%	-0.27%	-1.58%	1.05%	-0.66
WMT	11.36%	10.74%	10.12%	9.50%	8.88%	8.27%	7.65%	7.03%	6.41%	5.79%	10.92%	8.57%	1.87%	0.22
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**11.23%**			
Annual Variance	**0.0105**	**0.0105**	**0.0106**	**0.0109**	**0.0114**	**0.0120**	**0.0128**	**0.0138**	**0.0149**	**0.0162**	**0.0112**			
Annual Std. Dev.	**10.27%**	**10.25%**	**10.32%**	**10.46%**	**10.68%**	**10.97%**	**11.33%**	**11.74%**	**12.21%**	**12.73%**	**10.58%**			
Herfindahl–Hirschman index	**15.50%**	**14.58%**	**14.14%**	**14.19%**	**14.72%**	**15.75%**	**17.25%**	**19.25%**	**21.74%**	**24.70%**	**14.79%**			

* The Variability Index of the weights has been computed using the first 10 portfolios.

Appendix 25. Markowitz portfolios' weights over 2015-2019 without short selling - excess returns.

Markowitz portfolios' weights over 2020 -2021 with short selling - Excess Returns																	
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**Portfolio 11**	**Portfolio 12**	**Portfolio 13**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	-6.18%	-5.74%	-5.31%	-4.87%	-4.44%	-4.00%	-3.56%	-3.13%	-2.70%	-2.27%	-1.10%	0.06%	1.22%	-6.09%	-3.23%	2.25%	-0.69
AIR.PA	-3.03%	-3.15%	-3.27%	-3.39%	-3.51%	-3.63%	-3.75%	-3.87%	-3.99%	-4.10%	-4.43%	-4.75%	-5.07%	-3.05%	-3.84%	0.62%	-0.16
AMZN	18.28%	17.95%	17.62%	17.29%	16.96%	16.62%	16.29%	15.96%	15.63%	15.30%	14.41%	13.52%	12.63%	18.22%	16.03%	1.72%	0.11
APD	-4.45%	-4.78%	-5.11%	-5.45%	-5.78%	-6.11%	-6.44%	-6.77%	-7.10%	-7.43%	-8.32%	-9.20%	-10.09%	-4.51%	-6.70%	1.71%	-0.26
AZN.L	5.33%	5.53%	5.72%	5.91%	6.11%	6.30%	6.50%	6.69%	6.89%	7.08%	7.60%	8.12%	8.64%	5.37%	6.65%	1.00%	0.15
BAS.DE	-3.93%	-3.07%	-2.21%	-1.36%	-0.50%	0.36%	1.21%	2.08%	2.94%	3.79%	6.09%	8.38%	10.67%	-3.76%	1.88%	4.43%	2.36
BP.L	-2.53%	-2.79%	-3.05%	-3.31%	-3.56%	-3.82%	-4.08%	-4.34%	-4.59%	-4.85%	-5.53%	-6.22%	-6.91%	-2.59%	-4.28%	1.33%	-0.31
CPR.MI	-6.27%	-5.80%	-5.32%	-4.84%	-4.36%	-3.88%	-3.40%	-2.92%	-2.44%	-1.97%	-0.69%	0.59%	1.88%	-6.18%	-3.03%	2.47%	-0.82
ENEL.MI	-14.85%	-15.28%	-15.72%	-16.15%	-16.58%	-17.02%	-17.45%	-17.88%	-18.31%	-18.74%	-19.90%	-21.06%	-22.21%	-14.94%	-17.78%	2.24%	-0.13
EZJ.L	1.76%	1.51%	1.26%	1.01%	0.75%	0.50%	0.25%	0.00%	-0.25%	-0.51%	-1.18%	-1.85%	-2.52%	1.71%	0.06%	1.30%	22.84
G.MI	31.69%	31.63%	31.58%	31.52%	31.46%	31.40%	31.34%	31.28%	31.23%	31.17%	31.01%	30.85%	30.70%	31.68%	31.30%	0.30%	0.01
GM	0.43%	0.71%	0.99%	1.28%	1.56%	1.84%	2.12%	2.40%	2.69%	2.97%	3.72%	4.47%	5.23%	0.48%	2.34%	1.46%	0.62
GOOG	3.89%	4.44%	4.98%	5.53%	6.08%	6.63%	7.18%	7.72%	8.27%	8.82%	10.29%	11.75%	13.22%	3.99%	7.60%	2.83%	0.37
IHG.L	8.98%	8.97%	8.95%	8.94%	8.93%	8.91%	8.90%	8.88%	8.87%	8.86%	8.82%	8.79%	8.75%	8.98%	8.89%	0.07%	0.01
IP.MI	4.50%	5.33%	6.17%	7.00%	7.84%	8.67%	9.51%	10.34%	11.18%	12.01%	14.24%	16.47%	18.70%	4.66%	10.15%	4.31%	0.42
LHA.DE	-2.87%	-3.23%	-3.60%	-3.96%	-4.33%	-4.70%	-5.06%	-5.43%	-5.79%	-6.16%	-7.13%	-8.11%	-9.09%	-2.94%	-5.34%	1.89%	-0.35
LMT	-1.75%	-2.45%	-3.15%	-3.86%	-4.56%	-5.26%	-5.96%	-6.66%	-7.37%	-8.07%	-9.94%	-11.82%	-13.69%	-1.89%	-6.50%	3.63%	-0.56
MC.PA	4.23%	5.01%	5.79%	6.57%	7.35%	8.13%	8.91%	9.68%	10.46%	11.24%	13.32%	15.41%	17.49%	4.38%	9.51%	4.03%	0.42
MSFT	-15.28%	-14.94%	-14.60%	-14.27%	-13.93%	-13.59%	-13.26%	-12.91%	-12.58%	-12.24%	-11.34%	-10.44%	-9.53%	-15.21%	-12.99%	1.75%	-0.13
OR.PA	1.17%	2.31%	3.45%	4.59%	5.73%	6.87%	8.01%	9.16%	10.30%	11.44%	14.49%	17.54%	20.59%	1.39%	8.90%	5.90%	0.66
PFE	7.53%	8.04%	8.55%	9.06%	9.57%	10.09%	10.60%	11.11%	11.62%	12.14%	13.51%	14.88%	16.25%	7.62%	11.00%	2.65%	0.24
PG	9.40%	9.44%	9.49%	9.54%	9.59%	9.63%	9.69%	9.73%	9.77%	9.82%	9.94%	10.07%	10.20%	9.41%	9.72%	0.24%	0.02
REP.MC	-1.16%	-1.10%	-1.04%	-0.97%	-0.91%	-0.85%	-0.79%	-0.74%	-0.68%	-0.62%	-0.46%	-0.30%	-0.14%	-1.14%	-0.75%	0.31%	-0.41
SAN.MC	-4.39%	-4.58%	-4.78%	-4.97%	-5.17%	-5.36%	-5.56%	-5.75%	-5.95%	-6.14%	-6.66%	-7.18%	-7.70%	-4.43%	-5.71%	1.01%	-0.18
SAN.PA	20.77%	19.99%	19.22%	18.44%	17.66%	16.89%	16.11%	15.33%	14.55%	13.77%	11.70%	9.62%	7.55%	20.62%	15.51%	4.02%	0.26
SAP.DE	1.47%	0.90%	0.34%	-0.22%	-0.78%	-1.34%	-1.89%	-2.46%	-3.02%	-3.58%	-5.07%	-6.57%	-8.07%	1.35%	-2.33%	2.89%	-1.24
SOLB.BR	5.31%	5.20%	5.09%	4.98%	4.87%	4.76%	4.65%	4.54%	4.43%	4.32%	4.02%	3.72%	3.43%	5.29%	4.56%	0.57%	0.13
ULVR.L	20.51%	19.42%	18.33%	17.24%	16.15%	15.06%	13.97%	12.88%	11.78%	10.69%	7.77%	4.86%	1.94%	20.31%	13.12%	5.64%	0.43
VOW3.DE	-4.70%	-5.04%	-5.38%	-5.72%	-6.07%	-6.41%	-6.75%	-7.09%	-7.43%	-7.77%	-8.68%	-9.59%	-10.51%	-4.77%	-7.01%	1.76%	-0.25
WMT	26.14%	25.57%	25.00%	24.43%	23.86%	23.30%	22.72%	22.16%	21.59%	21.02%	19.51%	17.99%	16.47%	26.02%	22.29%	2.94%	0.13
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**30.84%**	**35.84%**	**40.84%**	**9.36%**			
Annual Variance	**0.0208**	**0.0209**	**0.0210**	**0.0212**	**0.0215**	**0.0218**	**0.0223**	**0.0228**	**0.0235**	**0.0242**	**0.0265**	**0.0295**	**0.0330**	**0.0208**			
Annual Std. Dev.	**14.44%**	**14.45%**	**14.49%**	**14.55%**	**14.65%**	**14.78%**	**14.93%**	**15.11%**	**15.32%**	**15.55%**	**16.28%**	**17.16%**	**18.17%**	**14.44%**			
Herfindahl–Hirschman index	**38.52%**	**37.65%**	**36.94%**	**36.40%**	**36.03%**	**35.82%**	**35.78%**	**35.90%**	**36.19%**	**36.65%**	**38.70%**	**41.93%**	**46.36%**	**38.34%**			

To achieve a better visual representation of the optimal portfolio allocation, 3 additional portfolios were added.

* The Variability Index of the weights has been computed using the first 13 portfolios.

Appendix 26. Markowitz portfolios' weights over 2020-2021 without short selling - excess returns.

B&L portfolios' weights with short selling & no views over 2015-2019 - Original formulation														
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	2.84%	5.60%	8.35%	11.11%	13.87%	16.62%	19.38%	22.14%	24.89%	27.65%	1.65%	15.24%	8.35%	0.55
AIR.PA	-3.68%	-2.70%	-1.71%	-0.73%	0.26%	1.24%	2.22%	3.21%	4.20%	5.18%	-4.11%	0.75%	2.98%	3.99
AMZN	3.30%	5.33%	7.36%	9.38%	11.41%	13.43%	15.46%	17.49%	19.51%	21.54%	2.43%	12.42%	6.13%	0.49
APD	4.04%	3.44%	2.84%	2.24%	1.64%	1.05%	0.44%	-0.16%	-0.76%	-1.35%	4.29%	1.34%	1.81%	1.35
AZN.L	6.72%	5.90%	5.08%	4.25%	3.43%	2.61%	1.78%	0.96%	0.13%	-0.69%	7.08%	3.02%	2.49%	0.83
BAS.DE	1.74%	1.58%	1.42%	1.27%	1.11%	0.95%	0.80%	0.65%	0.49%	0.33%	1.81%	1.03%	0.47%	0.46
BP.L	6.74%	5.81%	4.88%	3.95%	3.02%	2.10%	1.17%	0.23%	-0.69%	-1.62%	7.14%	2.56%	2.81%	1.10
CPR.MI	1.47%	1.23%	0.99%	0.74%	0.51%	0.27%	0.03%	-0.21%	-0.45%	-0.69%	1.57%	0.39%	0.72%	1.87
ENEL.MI	1.51%	1.43%	1.35%	1.27%	1.19%	1.11%	1.04%	0.96%	0.88%	0.80%	1.55%	1.15%	0.24%	0.21
EZJ.L	2.88%	2.40%	1.91%	1.42%	0.93%	0.44%	-0.05%	-0.54%	-1.03%	-1.52%	3.10%	0.68%	1.48%	2.17
G.MI	6.21%	5.15%	4.10%	3.04%	1.99%	0.93%	-0.12%	-1.17%	-2.23%	-3.28%	6.67%	1.46%	3.19%	2.19
GM	2.96%	2.56%	2.17%	1.77%	1.38%	0.99%	0.59%	0.19%	-0.20%	-0.60%	3.13%	1.18%	1.19%	1.01
GOOG	3.52%	5.03%	6.53%	8.03%	9.53%	11.03%	12.54%	14.03%	15.53%	17.03%	2.87%	10.28%	4.54%	0.44
IHG.L	4.43%	3.66%	2.88%	2.11%	1.33%	0.55%	-0.22%	-1.00%	-1.78%	-2.55%	4.77%	0.94%	2.35%	2.50
IP.MI	1.17%	0.97%	0.76%	0.55%	0.35%	0.15%	-0.06%	-0.26%	-0.46%	-0.67%	1.26%	0.25%	0.62%	2.47
LHA.DE	3.46%	2.89%	2.32%	1.75%	1.18%	0.61%	0.04%	-0.54%	-1.10%	-1.67%	3.71%	0.89%	1.73%	1.93
LMT	13.62%	11.41%	9.19%	6.98%	4.77%	2.56%	0.34%	-1.87%	-4.08%	-6.29%	14.57%	3.66%	6.70%	1.83
MC.PA	-5.51%	-3.88%	-2.24%	-0.60%	1.04%	2.68%	4.32%	5.97%	7.61%	9.25%	-6.23%	1.86%	4.97%	2.66
MSFT	-6.68%	-2.21%	2.25%	6.72%	11.19%	15.65%	20.12%	24.59%	29.06%	33.52%	-8.61%	13.42%	13.52%	1.01
OR.PA	-1.65%	-0.92%	-0.19%	0.54%	1.27%	1.98%	2.70%	3.41%	4.13%	4.86%	-1.95%	1.61%	2.19%	1.36
PFE	9.99%	8.70%	7.41%	6.12%	4.83%	3.54%	2.25%	0.96%	-0.33%	-1.62%	10.55%	4.18%	3.91%	0.93
PG	21.43%	18.31%	15.18%	12.06%	8.93%	5.81%	2.68%	-0.45%	-3.57%	-6.70%	22.79%	7.37%	9.47%	1.28
REP.MC	0.97%	0.85%	0.73%	0.61%	0.49%	0.37%	0.24%	0.13%	0.00%	-0.12%	1.02%	0.43%	0.37%	0.86
SAN.MC	-5.61%	-4.40%	-3.20%	-1.99%	-0.79%	0.41%	1.61%	2.81%	4.01%	5.22%	-6.13%	-0.19%	3.64%	-18.87
SAN.PA	1.58%	1.62%	1.66%	1.70%	1.74%	1.78%	1.82%	1.87%	1.91%	1.95%	1.57%	1.76%	0.12%	0.07
SAP.DE	4.32%	3.96%	3.59%	3.22%	2.86%	2.50%	2.13%	1.77%	1.40%	1.04%	4.48%	2.68%	1.10%	0.41
SOLB.BR	-0.27%	-0.19%	-0.11%	-0.03%	0.05%	0.13%	0.20%	0.28%	0.36%	0.44%	-0.30%	0.09%	0.24%	2.73
ULVR.L	8.20%	7.09%	5.97%	4.85%	3.74%	2.63%	1.52%	0.41%	-0.71%	-1.82%	8.67%	3.19%	3.37%	1.06
VOW3.DE	-0.20%	-0.06%	0.08%	0.23%	0.37%	0.52%	0.66%	0.81%	0.95%	1.10%	-0.27%	0.45%	0.44%	0.98
WMT	10.48%	9.46%	8.44%	7.42%	6.40%	5.38%	4.36%	3.35%	2.33%	1.30%	10.92%	5.89%	3.09%	0.52
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**8.19%**			
Annual Variance	**0.011**	**0.012**	**0.013**	**0.016**	**0.019**	**0.023**	**0.028**	**0.034**	**0.040**	**0.048**	**0.011**			
Annual Std. Dev.	**10.41%**	**10.78%**	**11.50%**	**12.52%**	**13.77%**	**15.19%**	**16.74%**	**18.38%**	**20.10%**	**21.87%**	**10.38%**			
Herfindahl–Hirschman index	**13.01%**	**9.80%**	**7.85%**	**7.15%**	**7.71%**	**9.52%**	**12.59%**	**16.91%**	**22.49%**	**29.33%**	**14.79%**			

* The Variability Index of the weights has been computed using the first 10 portfolios.

Appendix 27. B&L portfolios' weights with short selling & no views over 2015-2019 - original formulation.

B&L portfolios' weights with short selling & no views over 2020-2021 - Original formulation																	
Stocks	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**	**Portfolio 7**	**Portfolio 8**	**Portfolio 9**	**Portfolio 10**	**Portfolio 11**	**Portfolio 12**	**Portfolio 13**	**MVP**	**Avg. Weight**	**Weight Std. Dev.**	**Variability Index***
AAPL	-7.25%	-4.77%	-2.29%	0.20%	2.68%	5.17%	7.65%	10.13%	12.62%	15.10%	21.74%	28.38%	35.02%	-6.09%	9.57%	12.84%	1.34
AIR.PA	-3.20%	-2.89%	-2.57%	-2.26%	-1.94%	-1.63%	-1.32%	-1.01%	-0.70%	-0.38%	0.45%	1.29%	2.12%	-3.05%	-1.08%	1.62%	-1.50
AMZN	18.35%	18.08%	17.80%	17.53%	17.25%	16.98%	16.71%	16.44%	16.16%	15.89%	15.16%	14.42%	13.69%	18.22%	16.50%	1.41%	0.09
APD	-4.70%	-4.30%	-3.89%	-3.48%	-3.07%	-2.67%	-2.27%	-1.86%	-1.45%	-1.04%	0.04%	1.13%	2.22%	-4.51%	-1.95%	2.10%	-1.08
AZN.L	5.57%	5.14%	4.71%	4.29%	3.86%	3.43%	3.00%	2.58%	2.15%	1.72%	0.58%	-0.56%	-1.71%	5.37%	2.67%	2.21%	0.83
BAS.DE	-3.92%	-3.58%	-3.23%	-2.89%	-2.55%	-2.20%	-1.85%	-1.51%	-1.16%	-0.82%	0.09%	1.02%	1.93%	-3.76%	-1.59%	1.78%	-1.12
BP.L	-2.68%	-2.47%	-2.26%	-2.06%	-1.85%	-1.65%	-1.44%	-1.23%	-1.02%	-0.82%	-0.27%	0.28%	0.83%	-2.59%	-1.28%	1.07%	-0.83
CPR.MI	-6.42%	-5.91%	-5.41%	-4.90%	-4.40%	-3.89%	-3.39%	-2.88%	-2.37%	-1.87%	-0.53%	0.82%	2.17%	-6.18%	-3.00%	2.61%	-0.87
ENEL.MI	-15.52%	-14.27%	-13.03%	-11.78%	-10.53%	-9.28%	-8.04%	-6.80%	-5.55%	-4.30%	-0.97%	2.36%	5.70%	-14.94%	-7.08%	6.44%	-0.91
EZJ.L	1.78%	1.64%	1.50%	1.37%	1.23%	1.09%	0.96%	0.82%	0.68%	0.55%	0.18%	-0.18%	-0.55%	1.71%	0.85%	0.71%	0.83
G.MI	32.85%	30.35%	27.84%	25.34%	22.83%	20.32%	17.81%	15.31%	12.81%	10.30%	3.59%	-3.09%	-9.79%	31.68%	15.88%	12.95%	0.82
GM	0.47%	0.49%	0.51%	0.53%	0.55%	0.57%	0.59%	0.61%	0.63%	0.65%	0.70%	0.75%	0.80%	0.48%	0.60%	0.10%	0.17
GOOG	3.60%	4.43%	5.26%	6.10%	6.93%	7.77%	8.61%	9.44%	10.27%	11.10%	13.33%	15.56%	17.79%	3.99%	9.25%	4.31%	0.47
IHG.L	9.31%	8.60%	7.88%	7.16%	6.44%	5.73%	5.01%	4.30%	3.58%	2.86%	0.95%	-0.96%	-2.88%	8.98%	4.46%	3.70%	0.83
IP.MI	4.84%	4.47%	4.10%	3.74%	3.37%	3.00%	2.63%	2.26%	1.90%	1.53%	0.56%	-0.43%	-1.41%	4.66%	2.35%	1.90%	0.81
LHA.DE	-3.05%	-2.81%	-2.57%	-2.33%	-2.09%	-1.85%	-1.61%	-1.37%	-1.13%	-0.89%	-0.25%	0.40%	1.04%	-2.94%	-1.42%	1.24%	-0.87
LMT	-1.99%	-1.78%	-1.56%	-1.34%	-1.13%	-0.91%	-0.70%	-0.49%	-0.27%	-0.05%	0.52%	1.10%	1.68%	-1.89%	-0.53%	1.11%	-2.09
MC.PA	4.41%	4.35%	4.28%	4.22%	4.15%	4.09%	4.03%	3.96%	3.89%	3.83%	3.67%	3.49%	3.32%	4.38%	3.98%	0.33%	0.08
MSFT	-16.60%	-13.63%	-10.67%	-7.70%	-4.74%	-1.78%	1.18%	4.16%	7.12%	10.08%	18.01%	25.93%	33.87%	-15.22%	3.48%	15.32%	4.40
OR.PA	1.36%	1.42%	1.49%	1.55%	1.61%	1.68%	1.75%	1.81%	1.88%	1.94%	2.10%	2.28%	2.45%	1.38%	1.79%	0.33%	0.18
PFE	7.80%	7.42%	7.04%	6.66%	6.27%	5.89%	5.51%	5.13%	4.74%	4.36%	3.33%	2.31%	1.28%	7.63%	5.21%	1.98%	0.38
PG	9.64%	9.15%	8.67%	8.18%	7.70%	7.22%	6.73%	6.24%	5.76%	5.28%	4.00%	2.69%	1.40%	9.41%	6.36%	2.50%	0.39
REP.MC	-1.19%	-1.09%	-0.99%	-0.89%	-0.78%	-0.67%	-0.57%	-0.47%	-0.37%	-0.27%	0.01%	0.28%	0.56%	-1.14%	-0.50%	0.53%	-1.08
SAN.MC	-4.61%	-4.22%	-3.83%	-3.43%	-3.04%	-2.65%	-2.26%	-1.87%	-1.48%	-1.08%	-0.03%	1.02%	2.08%	-4.43%	-1.95%	2.03%	-1.04
SAN.PA	21.35%	19.80%	18.24%	16.68%	15.12%	13.56%	12.00%	10.44%	8.88%	7.33%	3.16%	-1.01%	-5.17%	20.62%	10.80%	8.05%	0.75
SAP.DE	1.35%	1.36%	1.36%	1.37%	1.38%	1.38%	1.39%	1.40%	1.40%	1.41%	1.43%	1.45%	1.46%	1.36%	1.40%	0.03%	0.02
SOLB.BR	5.49%	5.07%	4.66%	4.24%	3.83%	3.41%	3.00%	2.59%	2.17%	1.76%	0.65%	-0.46%	-1.57%	5.29%	2.68%	2.14%	0.80
ULVR.L	21.06%	19.44%	17.81%	16.19%	14.57%	12.96%	11.34%	9.72%	8.10%	6.48%	2.16%	-2.17%	-6.49%	20.30%	10.09%	8.36%	0.83
VOW3.DE	-4.96%	-4.55%	-4.14%	-3.73%	-3.32%	-2.91%	-2.50%	-2.09%	-1.68%	-1.28%	-0.19%	0.91%	2.00%	-4.77%	-2.19%	2.11%	-0.97
WMT	26.87%	25.07%	23.26%	21.46%	19.65%	17.85%	16.04%	14.24%	12.44%	10.63%	5.80%	0.98%	-3.85%	26.02%	14.65%	9.33%	0.64
Sum Weights	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**			
Target Annual E[R]	**9.00%**	**10.87%**	**12.74%**	**14.61%**	**16.48%**	**18.35%**	**20.22%**	**22.09%**	**23.97%**	**25.84%**	**30.84%**	**35.84%**	**40.84%**	**9.88%**			
Annual Variance	**0.0214**	**0.0215**	**0.0221**	**0.0234**	**0.0254**	**0.0280**	**0.0312**	**0.0351**	**0.0397**	**0.0449**	**0.0619**	**0.0836**	**0.1098**	**0.0214**			
Annual Std. Dev.	**14.64%**	**14.65%**	**14.87%**	**15.31%**	**15.94%**	**16.73%**	**17.68%**	**18.75%**	**19.92%**	**21.18%**	**24.88%**	**28.91%**	**33.14%**	**14.62%**			
Herfindahl–Hirschman index	**41.28%**	**35.20%**	**29.80%**	**25.10%**	**21.09%**	**17.76%**	**15.12%**	**13.17%**	**11.91%**	**11.33%**	**13.17%**	**19.92%**	**31.59%**	**38.34%**			

To achieve a better visual representation of the optimal portfolio allocation, 3 additional portfolios were added.

* The Variability Index of the weights has been computed using the first 13 portfolios.

Appendix 28. B&L portfolios' weights with short selling & no views over 2020-2021 - original formulation.

$B\&L\ \Sigma_p$ matrix over 2015-2019 - original formulation

	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	0.0617																													
AIR.PA	0.0178	0.0726																												
AMZN	0.0355	0.0178	0.0850																											
APD	0.0185	0.0168	0.0151	0.0356																										
AZN.L	0.0083	0.0195	0.0073	0.0067	0.0548																									
BAS.DE	0.0182	0.0337	0.0168	0.0177	0.0138	0.0500																								
BP.L	0.0112	0.0222	0.0108	0.0127	0.0152	0.0243	0.0570																							
CPR.MI	0.0099	0.0226	0.0122	0.0128	0.0153	0.0201	0.0133	0.0532																						
ENEL.MI	0.0080	0.0270	0.0105	0.0138	0.0133	0.0250	0.0195	0.0262	0.0522																					
EZJ.L	0.0123	0.0279	0.0100	0.0133	0.0059	0.0252	0.0015	0.0161	0.0198	0.1246																				
G.MI	0.0107	0.0312	0.0106	0.0145	0.0095	0.0305	0.0191	0.0213	0.0346	0.0303	0.0653																			
GM	0.0204	0.0194	0.0178	0.0200	0.0029	0.0190	0.0144	0.0091	0.0102	0.0179	0.0187	0.0660																		
GOOG	0.0312	0.0173	0.0454	0.0173	0.0086	0.0157	0.0080	0.0114	0.0113	0.0122	0.0126	0.0197	0.0581																	
IHG.L	0.0115	0.0268	0.0123	0.0124	0.0154	0.0214	0.0186	0.0171	0.0163	0.0203	0.0183	0.0137	0.0117	0.0607																
IP.MI	0.0176	0.0340	0.0158	0.0160	0.0140	0.0293	0.0189	0.0215	0.0239	0.0275	0.0281	0.0166	0.0149	0.0239	0.0808															
LHA.DE	0.0103	0.0283	0.0097	0.0119	0.0074	0.0280	0.0076	0.0172	0.0211	0.0620	0.0316	0.0170	0.0108	0.0201	0.0236	0.0972														
LMT	0.0139	0.0108	0.0143	0.0122	0.0058	0.0089	0.0058	0.0082	0.0077	0.0053	0.0063	0.0101	0.0139	0.0065	0.0067	0.0057	0.0310													
MC.PA	0.0209	0.0401	0.0196	0.0185	0.0171	0.0364	0.0219	0.0253	0.0281	0.0285	0.0304	0.0182	0.0185	0.0250	0.0308	0.0274	0.0101	0.0653												
MSFT	0.0325	0.0197	0.0417	0.0203	0.0085	0.0178	0.0126	0.0136	0.0139	0.0111	0.0139	0.0206	0.0370	0.0131	0.0159	0.0106	0.0166	0.0202	0.0539											
OR.PA	0.0104	0.0275	0.0125	0.0125	0.0184	0.0247	0.0168	0.0244	0.0252	0.0151	0.0204	0.0099	0.0111	0.0189	0.0195	0.0162	0.0079	0.0338	0.0140	0.0407										
PFE	0.0135	0.0111	0.0148	0.0120	0.0083	0.0087	0.0074	0.0068	0.0065	0.0058	0.0076	0.0148	0.0148	0.0057	0.0094	0.0066	0.0118	0.0099	0.0160	0.0081	0.0328									
PG	0.0099	0.0065	0.0092	0.0099	0.0046	0.0057	0.0053	0.0080	0.0077	0.0026	0.0045	0.0080	0.0098	0.0033	0.0050	0.0024	0.0079	0.0080	0.0126	0.0088	0.0085	0.0246								
REP.MC	0.0146	0.0315	0.0133	0.0175	0.0095	0.0354	0.0470	0.0185	0.0304	0.0173	0.0351	0.0220	0.0120	0.0214	0.0275	0.0221	0.0071	0.0303	0.0163	0.0197	0.0085	0.0058	0.0768							
SAN.MC	0.0159	0.0406	0.0175	0.0217	0.0103	0.0438	0.0321	0.0217	0.0410	0.0369	0.0547	0.0278	0.0183	0.0226	0.0349	0.0387	0.0079	0.0397	0.0200	0.0243	0.0111	0.0063	0.0592	0.0987						
SAN.PA	0.0107	0.0265	0.0095	0.0114	0.0227	0.0228	0.0175	0.0206	0.0238	0.0132	0.0216	0.0100	0.0112	0.0159	0.0184	0.0160	0.0087	0.0267	0.0118	0.0252	0.0115	0.0063	0.0225	0.0286	0.0454					
SAP.DE	0.0152	0.0308	0.0173	0.0142	0.0155	0.0277	0.0168	0.0221	0.0235	0.0215	0.0246	0.0128	0.0157	0.0195	0.0262	0.0206	0.0079	0.0319	0.0181	0.0255	0.0084	0.0059	0.0219	0.0317	0.0229	0.0469				
SOLB.BR	0.0183	0.0353	0.0177	0.0169	0.0132	0.0410	0.0258	0.0207	0.0249	0.0271	0.0320	0.0188	0.0160	0.0238	0.0311	0.0300	0.0079	0.0364	0.0172	0.0224	0.0103	0.0053	0.0365	0.0453	0.0218	0.0272	0.0612			
ULVR.L	0.0061	0.0153	0.0063	0.0074	0.0183	0.0142	0.0140	0.0187	0.0176	0.0066	0.0100	0.0025	0.0062	0.0155	0.0111	0.0072	0.0059	0.0193	0.0086	0.0260	0.0050	0.0096	0.0101	0.0100	0.0186	0.0163	0.0127	0.0390		
VOW3.DE	0.0197	0.0396	0.0179	0.0187	0.0139	0.0422	0.0260	0.0201	0.0297	0.0329	0.0395	0.0288	0.0191	0.0239	0.0351	0.0346	0.0079	0.0403	0.0182	0.0228	0.0101	0.0046	0.0427	0.0559	0.0237	0.0304	0.0429	0.0104	0.1127	
WMT	0.0110	0.0085	0.0103	0.0095	0.0028	0.0052	0.0048	0.0056	0.0069	0.0046	0.0048	0.0097	0.0097	0.0049	0.0065	0.0046	0.0098	0.0080	0.0120	0.0061	0.0104	0.0111	0.0048	0.0072	0.0045	0.0067	0.0063	0.0045	0.0048	0.0382

Appendix 29. $B\&L\ \Sigma_p$ matrix over 2015-2019 – original formulation.

B&L Σ_p matrix over 2020-2021 - original formulation																														
	AAPL	AIR.PA	AMZN	APD	AZN.L	BAS.DE	BP.L	CPR.MI	ENEL.MI	EZJ.L	G.MI	GM	GOOG	IHG.L	IP.MI	LHA.DE	LMT	MC.PA	MSFT	OR.PA	PFE	PG	REP.MC	SAN.MC	SAN.PA	SAP.DE	SOLB.BR	ULVR.L	VOW3.DE	WMT
AAPL	0.1409																													
AIR.PA	0.0423	0.3394																												
AMZN	0.0827	0.0138	0.1064																											
APD	0.0662	0.0606	0.0379	0.1149																										
AZN.L	0.0212	0.0213	0.0180	0.0167	0.0770																									
BAS.DE	0.0317	0.1197	0.0085	0.0586	0.0191	0.1174																								
BP.L	0.0294	0.1461	0.0018	0.0596	0.0207	0.1085	0.2130																							
CPR.MI	0.0363	0.0527	0.0248	0.0301	0.0266	0.0416	0.0427	0.0948																						
ENEL.MI	0.0417	0.0726	0.0302	0.0525	0.0344	0.0653	0.0719	0.0509	0.1071																					
EZJ.L	0.0321	0.2867	0.0083	0.0610	0.0009	0.1293	0.1721	0.0570	0.0679	0.5146																				
G.MI	0.0304	0.1013	0.0126	0.0392	0.0163	0.0712	0.0904	0.0426	0.0585	0.1096	0.0743																			
GM	0.0599	0.1691	0.0242	0.0798	0.0142	0.0969	0.1228	0.0409	0.0555	0.1825	0.0749	0.2527																		
GOOG	0.0827	0.0561	0.0675	0.0578	0.0162	0.0373	0.0379	0.0336	0.0415	0.0583	0.0333	0.0669	0.1008																	
IHG.L	0.0414	0.1968	0.0103	0.0675	0.0068	0.1002	0.1286	0.0439	0.0611	0.2485	0.0804	0.1383	0.0487	0.2267																
IP.MI	0.0361	0.0853	0.0204	0.0431	0.0200	0.0542	0.0520	0.0414	0.0522	0.0662	0.0491	0.0584	0.0382	0.0588	0.1204															
LHA.DE	0.0314	0.2057	0.0117	0.0605	0.0191	0.1158	0.1365	0.0535	0.0704	0.3081	0.0922	0.1217	0.0428	0.1603	0.0671	0.3646														
LMT	0.0508	0.0556	0.0247	0.0594	0.0194	0.0459	0.0510	0.0421	0.0410	0.0571	0.0359	0.0558	0.0422	0.0527	0.0321	0.0540	0.1052													
MC.PA	0.0431	0.1031	0.0274	0.0480	0.0215	0.0663	0.0771	0.0429	0.0542	0.1090	0.0535	0.0772	0.0417	0.0910	0.0447	0.0856	0.0353	0.0981												
MSFT	0.1027	0.0422	0.0769	0.0670	0.0240	0.0314	0.0283	0.0338	0.0429	0.0374	0.0289	0.0624	0.0883	0.0397	0.0330	0.0306	0.0496	0.0442	0.1170											
OR.PA	0.0344	0.0514	0.0244	0.0378	0.0263	0.0400	0.0479	0.0420	0.0489	0.0488	0.0381	0.0369	0.0308	0.0453	0.0361	0.0481	0.0342	0.0536	0.0332	0.0659										
PFE	0.0391	0.0246	0.0188	0.0461	0.0243	0.0259	0.0298	0.0240	0.0249	0.0269	0.0190	0.0346	0.0357	0.0228	0.0151	0.0264	0.0456	0.0267	0.0400	0.0231	0.0967									
PG	0.0490	0.0152	0.0308	0.0489	0.0192	0.0206	0.0175	0.0228	0.0287	0.0016	0.0171	0.0299	0.0408	0.0128	0.0216	0.0177	0.0472	0.0181	0.0508	0.0235	0.0393	0.0633								
REP.MC	0.0323	0.1559	0.0070	0.0637	0.0160	0.1072	0.1822	0.0411	0.0690	0.1841	0.0917	0.1200	0.0456	0.1317	0.0549	0.1446	0.0555	0.0747	0.0348	0.0444	0.0341	0.0215	0.2156							
SAN.MC	0.0378	0.1644	0.0101	0.0568	0.0160	0.1159	0.1466	0.0502	0.0778	0.1921	0.0970	0.1205	0.0446	0.1261	0.0616	0.1573	0.0570	0.0820	0.0349	0.0529	0.0292	0.0237	0.1507	0.2160						
SAN.PA	0.0178	0.0333	0.0101	0.0166	0.0368	0.0327	0.0362	0.0280	0.0387	0.0221	0.0276	0.0277	0.0162	0.0237	0.0274	0.0329	0.0219	0.0282	0.0195	0.0273	0.0194	0.0154	0.0324	0.0332	0.0537					
SAP.DE	0.0418	0.0792	0.0334	0.0432	0.0238	0.0565	0.0508	0.0383	0.0529	0.0753	0.0453	0.0513	0.0415	0.0709	0.0451	0.0668	0.0310	0.0543	0.0456	0.0419	0.0213	0.0206	0.0547	0.0567	0.0297	0.1081				
SOLB.BR	0.0212	0.1005	0.0089	0.0484	0.0196	0.0922	0.0948	0.0630	0.0616	0.1117	0.0645	0.0810	0.0294	0.0734	0.0575	0.1103	0.0477	0.0522	0.0199	0.0401	0.0206	0.0175	0.0941	0.1018	0.0267	0.0407	0.1379			
ULVR.L	0.0189	0.0197	0.0143	0.0266	0.0283	0.0284	0.0271	0.0287	0.0307	0.0126	0.0189	0.0163	0.0186	0.0149	0.0176	0.0221	0.0246	0.0231	0.0222	0.0331	0.0158	0.0212	0.0210	0.0200	0.0228	0.0247	0.0281	0.0544		
VOW3.DE	0.0530	0.1522	0.0227	0.0681	0.0286	0.1096	0.1261	0.0458	0.0680	0.1719	0.0816	0.1394	0.0550	0.1381	0.0624	0.1317	0.0533	0.0910	0.0503	0.0529	0.0280	0.0225	0.1255	0.1314	0.0332	0.0730	0.0856	0.0261	0.2045	
WMT	0.0468	-0.0074	0.0313	0.0412	0.0170	0.0131	0.0054	0.0204	0.0234	-0.0114	0.0085	0.0172	0.0335	-0.0027	0.0119	0.0066	0.0345	0.0124	0.0455	0.0142	0.0290	0.0410	0.0076	0.0135	0.0096	0.0132	0.0106	0.0145	0.0112	0.0650

Appendix 30. *B&L* Σ_p matrix over 2020-2021 – original formulation.