Technische Universität München
Department of Electrical and Computer Engineering
Institute for Renewable and Sustainable Energy Systems

Politecnico di Torino
Dipartimento Energia "Galileo Ferraris"
Corso di Laurea Magistrale in Ingegneria Elettrica

Master’s Thesis

Financial Storage Rights for Integration of Battery Energy Storage Systems Using AC-OPF

written by

Hasan Gega

Advisors Prof. Maurizio Repetto,
Prof. Thomas Hamacher,
MSc. Andrej Trpovski, Dr. Arif Ahmed

Abstract

As renewable energy is gaining an increasing attention due to global warming, traditional grids are facing numerous challenges. A combination of different technologies is essential to ensure energy security and equity, while maintaining an acceptable level of environmental sustainability. Energy storage devices will play a key role at different levels of the electrical grids for maintaining a stable supply of energy, both in a short- and long-term context. As storage can provide a wide variety of services to the grid, an accurate regulatory framework is necessary for a detailed consideration of its benefits.
In this thesis, after introducing the basics of mathematical optimization and power system analysis, a discussion on the role and consideration of storage devices in electrical grids is presented. After defining an optimal power flow model in which storage is integrated as a "transmission asset", financial instruments called "Financial Storage Rights" are reviewed considering a nodal pricing scheme. Since manuscripts and reports concerning this topic have been considering Direct Current (DC) approximations of the optimal power flow equations, a full Alternating Current (AC) formulation is presented without neglecting losses, reactive power flows and voltage levels. Consequently, the mathematical computation of Financial Transmission Rights and Financial Storage Rights is performed using the same procedures as in the approximated model presented in [1]. The simulations have been carried out using the General Algebraic Modeling System (GAMS) software, in order to evaluate how approximations affect results quality, and to provide an overview on the potential use of Financial Storage Rights in electricity markets. Last, this thesis is concluded with a discussion of the potential future research scope.

Statement of Academic Integrity

I,

Last name: Gega
First name: Hasan
ID No.: 03739935

hereby confirm that the attached thesis,

Financial Storage Rights for Integration of Battery Energy Storage Systems Using AC-OPF

was written independently by me without the use of any sources or aids beyond those cited, and all passages and ideas taken from other sources are indicated in the text and given the corresponding citations appropriately. Tools provided by the institute and its staff, such as models or programs, are also listed.

I agree to the further use of my work and its results (including programs produced and methods used) for research and instructional purposes.

I have not previously submitted this thesis for academic credit.

Munich, June, 2021

Acknowledgements

I would like to express my gratitude to Andrej Trpovski and Arif Ahmed for their constant support during the preparation of my thesis. I would also like to thank Prof. Maurizio Repetto and Prof. Thomas Hamacher for their professional contribution and assistance throughout this work.
I would like to thank all the colleagues met in Turin and in Trondheim during my studies, it has been a great pleasure working with you. I am very grateful to my friends for their support and affection, I will never forget the moments we shared and how our friendship is important to me. My deepest appreciation goes to my beloved family, their patience and support during this journey. Last but not least, I would like to thank Selene for her trust, her love and these wonderful years together.
All of this would not have been possible without all of you.

Contents

Statement of Academic Integrity **3**

Contents **6**

List of Figures **8**

List of Tables **9**

1 Introduction **11**

2 Optimization in Electrical Engineering **15**
2.1 Primal and Dual Problem 18
2.2 Karush-Kuhn-Tucker Conditions 22

3 Power System Analysis **25**
3.1 The p.u. System 28
3.2 Conventional Power Flow 29
3.3 Optimal Power Flow 31
3.4 Applications 37
3.4.1 DC-OPF 37
3.4.2 Other Formulations 39
3.5 OPF and Dual Variables as Prices 39

4 Energy Storage **43**
4.1 Storage Characteristics 45
4.2 Technologies 47
4.2.1 Mechanical Energy Storage 47
4.2.2 Electrical and Magnetic Energy Storage 47
4.2.3 Thermal Energy Storage 48
4.2.4 Chemical Energy Storage 48
4.2.5 Electro-chemical Energy Storage 48
4.3 Storage Role and Advantages 50

5 Financial Rights **55**
5.1 Grid Congestions 55
5.2 Storage Congestions 58

6 Financial Storage Rights Calculation **63**
6.1 DC-OPF 63
6.2 AC-OPF 68

7 Simulations **77**
7.1 General Considerations 77
7.2 DC - AC Results Comparison 78
7.3 Storage Role 84
7.4 Financial Transmission Rights and Financial Storage Rights . 86

8 Conclusion **91**

9 Future Work **93**
9.1 Model Extensions 93
9.2 V2G 94

Bibliography **97**

List of Figures

1.1 Percentage regional electricity generation by fuel (2019) [2]. . . . 11
1.2 A Ragone plot for energy storage devices comparison. [5] 13

2.1 Classification of optimization problems. 18
2.2 Primal and Dual problem, linear case. 20
2.3 Example 1: LP Primal and Dual problem. 21
2.4 Example 1: LP Primal and Dual problem, matrix form. 21
2.5 Example 1: Duality between decision variables and Lagrangian variables (1) 21
2.6 Example 1: Duality between decision variables and Lagrangian variables (2) 22

3.1 A generic representation of a bus i, with multiple connection to the grid. 26
3.2 π-equivalent circuit for a generic branch. 27
3.3 Approximated voltage profiles in a transmission line with respect to the SIL. 36
3.4 Loadability curve for a transmission line [17]. 36
3.5 The analogy for DC-OPF 38
3.6 Two bus system example. 40
3.7 Two bus non-congested system example. 41
3.8 LMP in case of congestion in a two bus system 41

4.1 Technologies for energy storage. 44
4.2 Storage benefits according to the location in the supply chain [40] 45
4.3 Power-Energy view of storage benefits [59]. 51
4.4 Electrical Energy Storage (ESS) role for Peak shaving (a) and Load Levelling (b) [64]. 53

5.1 LMP in case of congestion in a two bus system. 57

7.1 5 bus system, single line diagram. 79
7.2 Relative errors on phase angles and net active powers, 10 time periods. 82
7.3 Relative errors on Locational Marginal Prices (LMPs). 83
7.4 Phase angle at bus 5. 85

7.5 Power generated from the slack bus and the most expensive generator . . . 86
7.6 LMPs [$\frac{k\$}{p.u.}$] in incrementing load simulation . . . 88
7.7 Financial Transmission Rights and congestion surplus match [$\frac{k\$}{p.u.}$]. 88
7.8 Financial Transmission Rights, congestion surplus and loss component in case of AC results [$\frac{k\$}{p.u.}$]. . . . 89
7.9 FSR computation, first approach [1]. . . . 90
7.10 FSR computation, second approach [71] . . . 90

9.1 New passenger car registrations by fuel type in the European Union: Q1 2021, [83] . . . 95
9.2 New passenger car registrations in the EU by alternative fuel type [83] . . . 96

List of Tables

7.1 Generators data [18]. . . . 79
7.2 Bus and Load data [18]. . . . 79
7.3 Lines data [18]. . . . 79
7.4 DC-OPF/AC-OPF results comparison: Voltages and phase angles. 80
7.5 DC-OPF/AC-OPF results comparison: Generated and net active and reactive powers. . . . 80
7.6 Relative error between DC-OPF/AC-OPF. . . . 81

Chapter 1

Introduction

Contemporary transmission and distribution grids are nowadays facing a challenging modernization process, in order to use energy in more effectively and efficiently: concepts such as Smart Grids, renewable generation and energy storage systems have increasingly been used in the last decades in order to better explain what modern electric networks are experiencing. A crucial aspect is the growing integration of solar and wind renewable sources, considering their intrinsic intermittency and need for an efficient way of storing the energy produced. Electrical consumption is going to increase, especially after a widespread uptake of Electric Vehicles (EVs). This will bring to higher energy demand, but also opportunities to use EVs batteries to improve the flexibility of the grid. Also, after a first period of use as on-board storage systems, second-life stationary applications may bring a huge benefit both in terms of capacity and battery recycling. Section 8.2 gives a view of the potential widespread diffusion of EV technologies, and second-life battery possibilities. The first step towards a lowering of

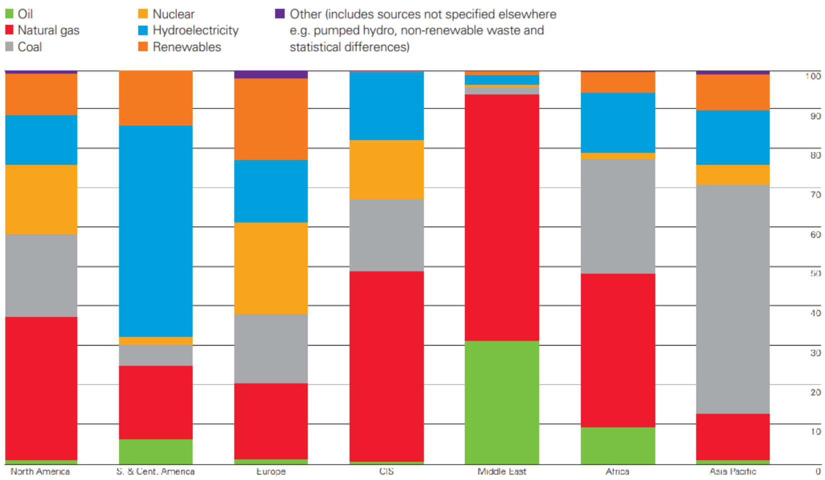


Figure 1.1 Percentage regional electricity generation by fuel (2019) [2].

the CO_2 footprint is the phasing-out of coal-fired power plants, that in Euro-

pean countries will occur in the next few years (2025 is the target for some European countries), while as a worldwide perspective might take a longer period. Figure 1.1 shows the different portfolios according to the locations.
As traditional plants will give way to local renewable sources, active power flows in high voltage lines could see a decrease in magnitude. While, on the one hand, it will represent an advantage in terms of savings in electricity bills for the final consumer and reduced CO_2 footprint, on the other hand this will test the network stability and resilience.
As discussed in chapter 3, events in which there are low load profiles and low power flowing in a transmission line could lead to a reactive power production at the transmission side. This is due to the capacitive effects of the lines, that could bring voltage levels to a higher level than their nominal values (this can be overcome via inductors in parallel). However, transmission grids will still play a crucial role to maintain a reliable energy supply and a stable network. Frequency is also another fundamental parameter to be taken into account, since in future grids there will be less conventional power plants and, therefore, less available synchronous generators for frequency support.
Also, renewable energy sources (RES) bring a high level of unpredictability and uncertainty of production. In this framework, storage is important for absorbing the surplus generated from RES when their output is higher than the actual demand of the grid, avoiding curtailment processes. The energy stored is then released when this aleatory generation drops, resulting in the possibility of active power control. Additionally, the presence of storage systems is essential if coupled with high power devices, in the framework of modern smart grids: e.g. in case of a widespread of fast charging technologies, the peak demand required could be unsustainable for the distribution systems.
Storage, along with other solutions, can be a flexibility resource also for the transmission side of the grid. Due to environmental issues and economic reasons, it might be difficult to build new lines. Therefore, it is common to see alternatives that match in other ways the new power requirements [3]:

– Storage devices that charge and discharge over a multi-period time frame, allowing a redistribution of high peak loads over the day;

– Demand response strategy aiming at utilizing electrical energy in a smart way by the modulation of the consumption, to avoid congestions in the grid and better redistribute the load in a larger timescale;

– An increase of energy efficiency brings advantages in terms of overall consumptions [3];

– Distributed generation sources helps alleviating transmission lines congestion because power production is closer to consumers, but the variability in production arises new challenges both in transmission and distribution grids.

Many storage technologies are currently available and in use, while others are still in development: some comprehensive overviews have been presented, see for example [4] and [5]. Each of these devices is more suitable in different circumstances, since their characteristics are extremely different. These dissimilarities can be efficiently outlined using a Ragone plot in term of specific energy (Wh/kg) and specific power (W/kg). The use of logarithmic axes shows how these values differ according to the technology used, considering also physical characteristics related to the volume or weight of the devices (Figure 1.2). See chapter 4 for a discussion about storage technologies for grid applications.

Considering their technological maturity and good compromise between energy density and power density, in this work we will focus on simulations using Battery Energy Storage Systems (BESS), their modelling in power systems, and their consideration within the financial framework of power system economics. Nevertheless, some of these considerations may be more broadly extended to the other storage technologies. Since the interface between BESS and the grid is based on converters, ancillary services such as reactive power and voltage/frequency control are also possible. This scenario is particularly significant in weak grids, wherein voltage and frequency excursions make BESS more valuable than in strong grids [6].

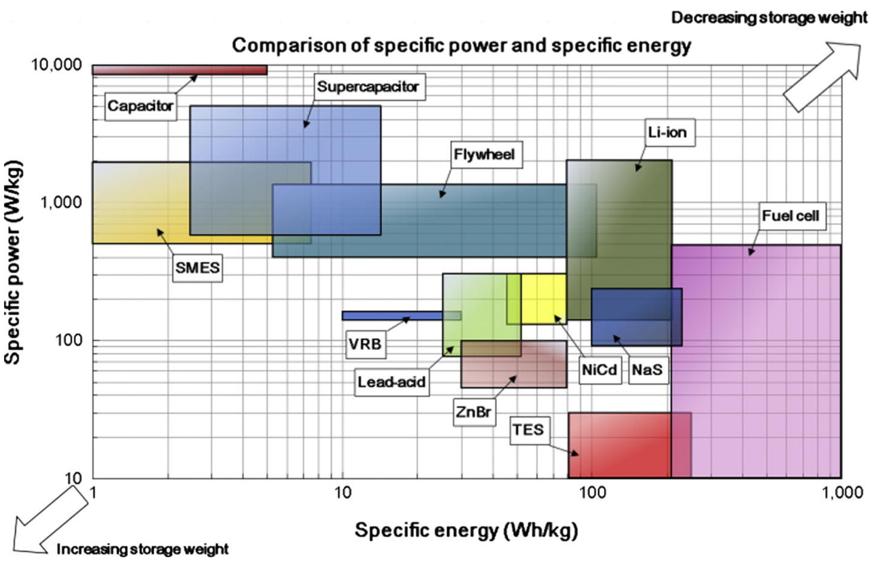


Figure 1.2 A Ragone plot for energy storage devices comparison. [5]

Future networks will see a widespread of decentralized producers, resulting in a less-predictable power flow. To get the most out of the existing network, a good autonomy in terms of energy storage will be essential. Storage helps also preventing congestions that would rise the prices in determinate areas, avoiding meanwhile high-cost infrastructure upgrades. BESS, as other storage technologies, can be considered as investments for the purpose of producing an income: for example, they can be charged when the electricity price is low and discharged when the cost rises. Whether they are used for maximizing energy penetration from renewable sources or for load power levelling, BESS can improve power system stability and efficiency.

A storage device is an attractive investment when profits exceed overall costs; for this reason, initial investment cost and operational expenses must be taken into account, also considering their durability and the cost of the power electronics conversion stage. Some of these ancillary services can be difficultly valued, [7] provides a description about non-technical issues related to BESS integration. One example is the inefficient pricing of some services provided by energy storage systems, that can result in unproductive amount of storage investments.

Furthermore, the regulatory treatment of storage brings to two opportunities: either providing regulated services or getting back their costs through the market. These and other methods such as a hybrid treatment or open access approach are discussed in [7].

System complexity of course increases if many storage devices are connected to the grid instead of building new transmission lines, but an appropriate treatment of storage could become a solid competitor to T&D upgrades (or at least could help with the deferral of new infrastructures).

Since storage can be categorized as a generation, transmission, or distribution asset due to its characteristics, every treatment can undervalue its potential revenue, because the services offered overlap among these sections [7]. Moreover, while they are usually classified as generation assets, transmission asset usage can be worth investigating. Thus, the battery does not buy or sell at the wholesale market, but gains just through rate payments [1]. This brings to considering storage as a passive and price insensitive device, aiming at maximizing its utility to the grid. For this reason, an efficient model is critical for not making storage operation uneconomic.

In this work, we will consider utility-scale storage systems as a transmission asset for different purposes. Financial storage rights, similar as financial transmission rights, are introduced to add a revenue source for the battery owner, while leaving the task of running the device to the system operator to maximize social welfare.

First, a general overview of mathematical optimization (chapter 2) and power systems operation (chapter 3) is presented for understanding the basics of optimal power flow with storage. Then, the role of storage is discussed in chapter 4, and the concept of financial rights is reviewed in chapter 5. The mathematical model is presented in chapter 6 and software simulations are summarized in chapter 7. Then, the final sections conclude the thesis.

Chapter 2

Optimization in Electrical Engineering

Mathematical Optimization is a branch of applied mathematics that studies methods and algorithms in order to find a solution to a maximization or minimization problem. Optimization is applied in every subfield of engineering, science and industry. Many methods can be used for solving an optimization problem, but some of them may be more suitable in certain cases, therefore, a basic knowledge and the classification of the methods must be clear when solving these problems.
When choosing the most appropriate method, the computational complexity is an important factor, even with modern computer performances. In some cases, it is useful to choose linear approximation of more complex problems, because computational time could be excessive. If an optimization problem has a complex structure, the user can also decide to "relax" some or all the entities that bring complexity to the problem. As an example, non-linear programming problems can be relaxed to apply linear programming algorithms, acting on the "non-linearities" via piecewise linear approximations.

In a general framework, an optimization problem consists of an objective function, to be minimized or maximized, given a domain for the variables subject to constraints:

$$min/max \quad f(x) \tag{2.0.1}$$
$$subject\ to \quad i_i(x) \leq 0, \ \ i = 1, ..., \tag{2.0.2}$$
$$e_j(x) = 0, \ j = 1, ..., \tag{2.0.3}$$
$$\tag{2.0.4}$$

Where:

- x is a vector containing all the primal decision variables (or degrees of freedom);
- $f(x)$ is the objective function to be minimized or maximized;

– $i_i(x)$ are the inequality constraints;

– $e_j(x)$ are the equality constraints.

When no constraints are considered in a problem, it is called unconstrained optimization. Since a general term can be fixed, some variables may be subject to an equality constraint. Almost every variable or combination of them is subject to inequality constraints because domain boundaries are important to maintain the solution physically feasible. The output of this problem is a vector containing the values to be assigned to the decision variables, in order to obtain the smallest objective function (largest in the case of maximization problems). As previously stated, when dealing with non-linear problems, the search of an optimal point can be hard because of computational reasons. Therefore, to avoid an excessive complexity, sometimes it is good to choose a "neighbourhood", that is part of the entire domain of the solution. It is possible that the optimal point found in the neighbourhood does not correspond to the global minimum of the solution domain, in this case it is called "local optimum". In most cases, the best option is a trade-off: choosing a neighbourhood wide enough to have a good quality solution, but not too large to avoid a problem extremely difficult to be computed [8].
With this clear, it is immediate to state that functions that have a unique minimum are simpler to be treated. Thus, convex optimization problems are easier to be solved rather than non-convex ones. A convex problem has an objective function and all the constraints that must be convex. The perception of convexity is straightforward in the case of a simple one real variable function: the function must be below any lines connecting two points of the function. For convex sets, any lines connecting two points of the set must rely entirely inside the set. Mathematically, this can be verified using the first order or the second order conditions [9].

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function whose first and second derivative (gradient ∇f and Hessian $\nabla^2 f$) exist in every point of the domain $dom(f)$. If f is convex then [9]:

$\longrightarrow$ $f(y) \geq f(x) + \nabla f(x)^T (y - x)$, for all $x, y \in dom(f)$;

$\longrightarrow$ $\nabla^2 f(x) \geq 0$, for all $x \in dom(f)$;

Again, relaxation methods are possible for simplifying studies of non-convex problems (convex relaxation).
Another important classification consists of Linear and Non-Linear Optimization problems (also called Linear Programming and Non-Linear Programming, LP and NLP). It is important to highlight that one single non-linearity in the objective function or one of the constraints, results in a Non-Linear Programming problem.
In case of minimization, in a general linear problem:

$$\begin{aligned} min \quad & f(x_i) = c_i x_i \\ s.t. \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

or, in matrix form:

$$\begin{aligned} min \quad & c^T x \\ s.t. \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

In this way, we defined the "Primal Problem". Another "Dual Problem" can be defined from these data, see subsection (2.1) for this important application.
In general, the simplex method introduced by Dantzig [10], is one of the most well-known methods for solving a Linear Programming problem. It consists of an algorithm that solves LP problems, by considering a feasible region defined by a geometric entity termed polytope. The polytope is drawn from the inequality constraints, and the optimal value is found evaluating the objective function in the feasibility region and will occur at the vertex of this area. Unlike the simplex method, that follows the path on the boundaries of the polytope, interior point methods cross the feasible region.
Countless other methods have been developed and applied for solving both LP and NLP problems. In figure (2.1) a not comprehensive list is presented.

The distinction between deterministic and stochastic methods is important because both of them are widely used in many subfields. Generally, if an algorithm has no random components, and seeks to find an exact solution is called deterministic. If somehow randomness is necessary because of the uncertain nature or challenging structure of the problem, randomized stochastic methods are used. In many applications deterministic methods can bring to a computationally expensive result, therefore, exact algorithms are impossible to be implemented in some cases. This results in an increasing success of meta-heuristics stochastic methods. Nevertheless, since the size of the grids considered in the simulations of this work is not too wide, deterministic methods have been preferred thanks to their specific procedures.
Hybrid approaches that use both exact and meta-heuristics methods are also possible, resulting in good quality of solutions with an acceptable time of computation. They can be classified into integrative combinations and collaborative combinations [11].
The solutions of the problems are different in case of integer variables: in

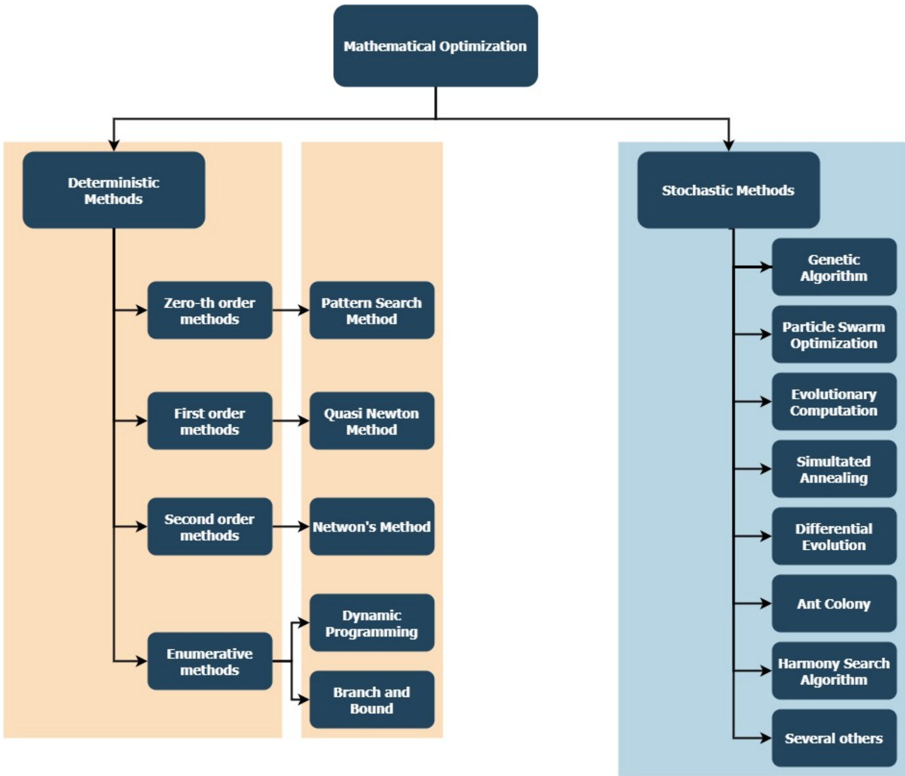


Figure 2.1 Classification of optimization problems.

this case we talk about Integer Programming (IP). If only some of the variables are integer, it is the case of Mixed Integer Programming (MIP). In conclusion, dealing with discrete variables rather than continuous variables brings to an even more complex structure of the problem. This chapter, and in particular the next sections, contains relevant mathematical background that will be useful when introducing financial storage rights.

2.1 Primal and Dual Problem

Given an optimization problem, that we call the “Primal Problem”, we can also define the “Dual Problem”, that is strictly related to the primal and in some cases might be very useful for finding the optimal solution of the primal. We can define the duality gap as the discrepancy between the solutions of these two problems. Only in some cases the duality gap is zero, i.e. the two problems have the same objective solution (e.g. in the case of a linear programming problem). Strong duality and weak duality can be defined, see [9] for this and other aspects of optimization. Given a problem in the form of eqs. (2.0), we can define the Lagrangian function associated:

$$\mathcal{L}(x,\lambda,\mu) = f(x) + \sum_i \lambda_i i_i(x) + \sum_j \mu_j e_j(x) \tag{2.1.1}$$

Where:

- $\lambda_i \geq 0$ is called "Lagrangian dual multiplier" or dual variable of the i-th inequality constraint
- μ_j is called "Lagrangian dual multiplier" or dual variable of the j-th equality constraint (note that in chapter 6 a Lagrangian function is defined, but the μ are replaced by λ also for equality constraints).

In this way, a weighted sum of the constraints increases the objective function. Then, the Lagrangian dual function can be defined:

$$\mathcal{L}_{dual}(\lambda,\mu) = \inf\left(\mathcal{L}(x,\lambda,\mu) = \inf\left(f(x) + \sum_i \lambda_i i_i(x) + \sum_j \mu_j e_j(x)\right)\right. \tag{2.1.2}$$

We can therefore state that:

- the dual function gives a lower bound to the optimal objective value;
- since we are looking for the best lower bound computable from the dual function, the dual problem is a maximization problem;
- the constraints for this maximization problem are the inequality constraints ensuring the non-negativity of the dual variables ($\lambda_i \geq 0$);
- the dual maximization problem is always a convex optimization problem, regardless of the nature of the primal problem.
- the number of constraints in the primal problem is equal to the number of variables in the dual problem, while the number of variables in the primal problem is equal to the number of constraints of the dual problem

The introduction of dual variables is extremely important. They are often called Lagrange multipliers or dual multipliers, but when they represent a cost, they are sometimes termed shadow prices, implicit prices or dual prices [12]. When using optimization modeling systems it sometimes take the acronym of marginal value. This will give relevant insights in chapters 5 and 6, where dual multipliers are considered as representing the cost of producing one extra MWh of energy after a supposed optimized re-dispatch in a specific location of the network [13]. From the definition of the Lagrangian function the value researched (minimum or maximum) can be found with the so-called Lagrange multipliers method. However, since this method only considers the presence of equality constraints, we are more interested on the Karush-Kuhn-Tucker approach that comprise also inequality constraints (see section 2.2 for more details). In case of linear programming problems, the dual problem can be easily found using the following notation:

$$\boxed{\begin{array}{c} min \quad [C]^T[x] \\ \\ s.t. \quad [A][x] \leq [B] \\ and \\ x_j \geq 0 \end{array}} \longleftrightarrow \boxed{\begin{array}{c} min \quad [B]^T[y] \\ \\ s.t. \quad [A]^T[y] \geq [C] \\ and \\ y_i \geq 0 \end{array}}$$

Figure 2.2 Primal and Dual problem, linear case.

The General Algebraic Modeling System (GAMS) language [by GAMS Development Corporation] is useful in this framework. After stating that there is a duality in the number of constraints and decision variables in the primal and dual problems, when solving an optimization problem it can be found that the values of the variables of the dual problem are equal to the values of the marginal costs associated to the inequality constraints of the primal problem (MARGINAL in GAMS). Vice versa, if solving the dual problem, the levels of the primal variables match with the marginals of the dual problem. The example in figure 2.3 (that is solved in [14] using dual simplex method) shows a simple linear programming problem and the results have been implemented using GAMS. Duality is even clearer in matrix form (figure 2.4). In this case, since in LP strong duality occurs, the objective functions of primal and dual problems take the same value.

The results from GAMS simulation is reported here. It can be seen that, apart from having found the same solution for the primal and dual problem, the dual variables value can be found under the column "MARGINAL" of the primal problem. This can be very useful in more complex problems.

$$\max \quad 120x_1 + 100x_2$$
$$s.t. \quad 2x_1 + 2x_2 \leq 8$$
$$5x_1 + 3x_2 \leq 15$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

$$\longleftrightarrow$$

$$\min \quad 8y_1 + 15y_2$$
$$s.t. \quad 2y_1 + 5y_2 \geq 120$$
$$2y_1 + 3y_2 \geq 100$$
$$y_1 \geq 0$$
$$y_2 \geq 0$$

Figure 2.3 Example 1: LP Primal and Dual problem.

$$\max \begin{bmatrix} 120 & 100 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$s.t. \begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 8 \\ 15 \end{bmatrix}$$
$$and$$
$$all \ x_j \geq 0$$

$$\longleftrightarrow$$

$$\min \begin{bmatrix} 8 & 15 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$
$$s.t. \begin{bmatrix} 2 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \geq \begin{bmatrix} 120 \\ 100 \end{bmatrix}$$
$$and$$
$$all \ y_i \geq 0$$

Figure 2.4 Example 1: LP Primal and Dual problem, matrix form.

```
                    LOWER      LEVEL      UPPER     MARGINAL

EQU Objective~        .          .          .        -1.0000
EQU ic1             -INF        8.0000     8.0000    35.0000
EQU ic2             -INF       15.0000    15.0000    10.0000

                    LOWER      LEVEL      UPPER     MARGINAL

VAR x1                .         1.5000     +INF        .
VAR x2                .         2.5000     +INF        .
VAR OF              -INF      430.0000     +INF        .
```

Figure 2.5 Example 1: Duality between decision variables and Lagrangian variables (1)

	LOWER	LEVEL	UPPER	MARGINAL
EQU Objective~	.	.	.	-1.0000
EQU ic1	-INF	120.0000	120.0000	1.5000
EQU ic2	-INF	100.0000	100.0000	2.5000

	LOWER	LEVEL	UPPER	MARGINAL
VAR y1	.	35.0000	+INF	.
VAR y2	.	10.0000	+INF	.
VAR OF	-INF	430.0000	+INF	.

Figure 2.6 Example 1: Duality between decision variables and Lagrangian variables (2)

2.2 Karush-Kuhn-Tucker Conditions

To prove if a solution is optimal, the Karush-Kuhn-Tucker conditions (KKT) can be computed [15]. They represent a more general approach with respect to the Lagrange multipliers method, since it also considers inequality constraints. After the definition of the dual problem, the Karush-Kuhn-Tucker conditions can be computed considering the gradient of the Lagrangian (the objective and constraints must be differentiable). The gradient (derivative with respect to the primal and dual variables) is set to zero, because this stationary condition brings to a maximum or minimum optimal point. The KKT conditions can be a way to solve simple optimization problems, but it is not realistically useful in the case of problems including several variables and constraints. Yet, these conditions are useful to approach the solutions, and in our cases to help finding a connection between the constraints through the use of the Lagrangian (see financial storage rights calculations in chapter 6).
Setting the gradient of the Lagrangian to zero, gives the **stationary condition**:

$$\nabla\mathcal{L}(x^*,\lambda,\mu) = \nabla f(x^*) + \sum_i \lambda_i \nabla i_i(x^*) + \sum_j \mu_j \nabla e_j(x^*) = 0 \qquad (2.2.1)$$

From the primal problem constraints we can define the **primal feasibility conditions**:

$$i_i(x^*) \leq 0,\ \ i = 1, ..., \qquad (2.2.2)$$

$$e_j(x^*) = 0,\ j = 1, ..., \qquad (2.2.3)$$

$$(2.2.4)$$

From the dual multipliers associated with the inequality constraints we can define the **dual feasibility conditions**:

$$\lambda_i \geq 0 \tag{2.2.5}$$

The KKT conditions stated above [16] do not give information about which inequality constraint is binding or not in the optimal solution found. Therefore, the following additional conditions labelled as **complementary slackness**, can be added to the model:

$$\lambda_i \, i_i(x^*) = 0 \tag{2.2.6}$$

A constraint is active (binding) if the corresponding dual multiplier is positive, while it is not binding if the corresponding multiplier is zero. We can also state that:

- Given a problem and its dual with strong duality, the solutions form a saddle point;
- The KKT optimality conditions are necessary, and also sufficient conditions in case of convex objective functions;
- Since several optimization problems related to power systems are nonlinear and non-convex it is relevant to remark that the KKT conditions are not sufficient in those cases.

Chapter 3

Power System Analysis

A generic electrical grid can be modelled as a graph consisting of buses and branches, representing every component of the power system (transformers, lines, physical points of connection, etc.). A system with n buses can be described by:

- 4n variables, comprising voltages, voltage angles, active and reactive net powers;

- 2n equations, representing the power flow equations for each bus.

Therefore, 2n variables are calculated through computation of the 2n power flow equations, whereas the remaining 2n variables must be specified according to the following criteria:

- PV nodes: active power and voltage magnitude in case of a generating unit connected to the bus;

- PQ nodes: active and reactive power in case of load buses, or transit buses;

- Slack bus ($V\delta$ node): voltage magnitude and phase angle for only one bus.

In some cases, during particular operating conditions, some nodes can change the specified variables. The active power cannot be assigned to every bus, because in quite large systems the total amount of power losses is non-negligible. Then, it is necessary to select one bus in which the active power is not assigned while phase angle and voltage magnitude are pre-specified, called as slack bus. Apart from the key-role played in the context of balancing the active power, the slack bus works also as a phase angle reference (usually set to 0°).

Computing voltages, power flows and other variables of interest is relevant for planning and operation of power systems. Therefore, an appropriate mathematical model is necessary to describe the system, ensuring that electrical energy is transferred from supply buses to loads. Generators, loads and the grid are represented using mathematical approximations. Each bus has a complex voltage $\overline{V_i}$ with respect to a reference, and the physical characteristics of branches and shunt elements can be described using the admittance matrix. A generic bus can be denoted with the notation in figure (3.1).

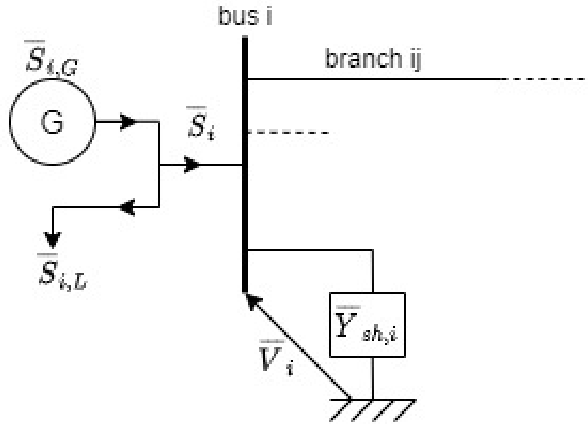


Figure 3.1 A generic representation of a bus i, with multiple connection to the grid.

Where $\overline{S} = P + jQ$ denotes the complex power and $\overline{Y}_{sh,i}$ is the shunt element for the bus in consideration. The currents and reactive powers can be represented using the same sign convention as the active power. We talk about **injected** power (or current), considering it positive if it is injected to the grid.
The grid is described using the admittance matrix Y_{bus}:

$$[\overline{Y}] = \begin{bmatrix} \overline{Y}_{11} & \dots & \overline{Y}_{1n} \\ \vdots & \ddots & \vdots \\ \overline{Y}_{n1} & \dots & \overline{Y}_{nn} \end{bmatrix} \tag{3.0.1}$$

The definition of the admittance matrix depends on the physical characteristics of the grid, in term of resistances and reactances.

A generic branch connecting two nodes can be described with its pi-equivalent circuit, considering a linear and passive behaviour of the lines:

$$\overline{Z}_L = (r + j\omega l)a \tag{3.0.2}$$

$$\overline{Y}_s = (g + j\omega c)\frac{a}{2} \tag{3.0.3}$$

in which the physical and electrical characteristics of the lines are described using the following coefficients:

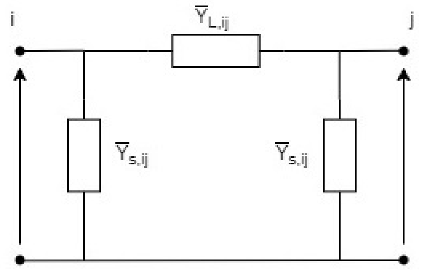


Figure 3.2 π-equivalent circuit for a generic branch.

– a is the length of the line [m];

– ra represents the resistance of the line [Ω];

– ωla is the reactance of the line [H];

– ga is the conductance of the line [S];

– ωca is the susceptance of the line [S].

It is relevant to highlight that:

* This is a lumped-element model that approximates the physical phenomena occurring along the lines;

* These approximations are valid for "not too long" lines (See [17] for a classification of line length and a discussion on when approximations can be considered valid);

* Typically, the power losses in the insulating material can be neglected, resulting in a capacitive shunt component (that can also be neglected for short overhead lines);

* Sometimes power losses can be neglected with respect to the active power flowing through the system (e.g in high voltage grids). Consequently, in some cases the resistances are neglected;

* The two shunt equivalent elements are equal for lines, considering a symmetrical behaviour, but they generally can be different (e.g. in transformers).

The bus admittance matrix can be constructed from the pi-equivalent circuit of the line, using the current nodal balance:

– The diagonal elements depend on all the admittances connected to a specific node. In case of shunt elements on the bus, they can be considered by adding its admittance value to the corresponding diagonal term (this is possible thanks to the use of admittances instead of impedances):

$$\overline{Y}_{ii} = \overline{Y}_{shunt,i} + \sum_{j}(\overline{Y}_{s,ij} + \overline{Y}_{L,ij}) \tag{3.0.4}$$

– Off-diagonal elements consist of the admittances connecting the two corresponding nodes (buses not connected result in a bus admittance element equal to 0):

$$\overline{Y}_{ij} = -\overline{Y}_{L,ij} \tag{3.0.5}$$

The main goal of these mathematical problems, as previously stated, is to find a solution to the system equations, using numerical methods to compute voltages, currents and powers in each point of the network. Voltages are considered as independent decision variables, while net currents are dependent variables that can be computed through the bus admittance matrix. Voltages cause current flows in proportion to the corresponding admittance:

$$\begin{bmatrix} \overline{I}_1 \\ \vdots \\ \overline{I}_n \end{bmatrix} = [\overline{Y}_{bus}] \begin{bmatrix} \overline{V}_1 \\ \vdots \\ \overline{V}_n \end{bmatrix} \tag{3.0.6}$$

The use of a bus impedance matrix Z_{bus} is also possible, selecting net injected currents as independent variables and voltages as dependent variables. This approach is useful for faults analysis, but unlike the bus admittance matrix, Z_{bus} is not a sparse matrix, and typically results in high computational demand.
The bus admittance matrix can be divided into conductance matrix G_{bus} and susceptance matrix B_{bus}. This approach can be useful when writing the optimization code using GAMS. In our cases, we will consider the values of G_{bus} and B_{bus} as constants, avoiding therefore a computationally demanding extended general approach comprising active transmission components such as PST (phase-shifting-transformers), tap-changing transformers or power electronics-based FACTS (flexible alternating current transmission systems). These components, as well as the status of switched reactors or capacitors bring also complexity to the nature of the variables, since some of them are discrete.

3.1 The p.u. System

When dealing with power systems, it is usually convenient the use of per unit dimensionless quantities (p.u) that represent a value of an electrical quantity with respect to a base quantity expressed with the SI:

$$X_{pu} = \frac{X}{X_{base}} \tag{3.1.1}$$

As base quantities it is common to use:

$$S_{base} = a\ unique\ three-phase\ power\ for\ the\ system$$
$$V_{base} = line\ to\ line\ voltages$$

The other base values can be found:

$$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}}$$
$$Z_{base} = \frac{V_{base}}{\sqrt{3}I_{base}} = \frac{V_{base}}{\sqrt{3}\frac{S_{base}}{\sqrt{3}V_{base}}} = \frac{V_{base}^2}{S_{base}}$$
$$Y_{base} = \frac{1}{Z_{base}}$$

Unless otherwise specified, all the quantities will be expressed in p.u.. Several advantages arise from this choice:

* 3 and $\sqrt{3}$ factors, typical in three-phase systems, are not present when using p.u. formulas;
* Most transformers have a voltage ratio equal to 1, because the voltages at the two ends are both expressed in p.u. (1:1);
* Magnitudes are quite close to 1 p.u., resulting in improved numerical stability [18].

3.2 Conventional Power Flow

In power system analysis the computation of active and reactive power flows is fundamental. It can be done through the definition of the complex power:

$$P + jQ = \overline{S} = \overline{V}\,\overline{I}^* = \overline{V}(\overline{Y}\,\overline{V})^* \tag{3.2.1}$$

where * denotes the complex conjugation. Then, we can write down the net current at a generic bus i as:

$$\overline{I}_i = \overline{Y}_{i1}\overline{V}_1 + ... + \overline{Y}_{ii}\overline{V}_i + ... + \overline{Y}_{in}\overline{V}_n = \sum_j \overline{Y}_{ij}\overline{V}_j \tag{3.2.2}$$

Considering rectangular coordinates for the admittance matrix entries and polar coordinates for complex bus voltages:

$$\overline{Y}_{ij} = G_{ij} + jB_{ij} \tag{3.2.3}$$

$$\overline{V}_i = |V_i|\angle\delta_i \tag{3.2.4}$$

From Euler's formula:

$$\overline{V}_i = |V_i|e^{j\delta_i} = |V_i|(cos\delta_i + jsin\delta_i) \tag{3.2.5}$$

Then, we can find the expression of active and reactive power flows, using the so-called 'bus injection model', from eqs. (3.2.1), (3.2.3), (3.2.4) and (3.2.5):

$$P_i = V_i \sum_j V_j \left[G_{ij}\, cos(\delta_i - \delta_j) + B_{ij}\, sin(\delta_i - \delta_j)\right] \tag{3.2.6}$$

$$Q_i = V_i \sum_j V_j \left[G_{ij}\, sin(\delta_i - \delta_j) - B_{ij}\, cos(\delta_i - \delta_j)\right] \tag{3.2.7}$$

Where the symbol $V_i = |\overline{V}_i|$ denotes the magnitude of voltage at bus i. It is important to note that these powers are the net values resulting from the dual operation of generating supply and loading demand: in a transit node without any loads or generation, the injected power is equal to zero.

$$\overline{S}_i = \overline{S}_{G,i} - \overline{S}_{L,i} \tag{3.2.8}$$

$$P_i = P_{G,i} - P_{L,i} \tag{3.2.9}$$

$$Q_i = Q_{G,i} - Q_{L,i} \tag{3.2.10}$$

An alternative notation of the power flow equations is characterized by the selection of different coordinates for voltages and admittances.
In the previous case (eqs. 3.2.6 and 3.2.7), voltages are expressed in polar coordinates ($\overline{V}_i = V_i\angle\delta_i$) while admittances in rectangular coordinates ($\overline{Y}_{ij} = G_{ij} + jB_{ij}$). It can also be useful to use polar coordinates for both voltages and admittances ($\overline{Y}_{ij} = Y_{ij}\angle\theta_{ij}$):

$$P_i = V_i \sum_j V_j \overline{Y}_{ij} cos(\delta_i - \delta_j - \theta_{ij}) \tag{3.2.11}$$

$$Q_i = V_i \sum_j V_j \overline{Y}_{ij} sin(\delta_i - \delta_j - \theta_{ij}) \tag{3.2.12}$$

The other representations are less useful in our context, and can be found in the literature (see [18]).
The main idea of the conventional power flow is to compute a deterministic solution without specifying any objective functions [18]. The loads are characterized using their powers, that can be considered as constant, even if most of the loads have voltage-dependent powers.

3.3 Optimal Power Flow

The OPF formulation is an extension of the conventional power flow. Traditionally, the first literature works are from Carpentier [19], Dommel and Tinney [20] from '60s. In the first paper, the power flow equations had been included for the first time in the Economic Dispatch classical formulation, in terms of equality constraints. According to Frank and Rebennack [18], planning of power systems operation have different time scales, from real-time simulations to a planning horizon of months or years. In this framework, the Optimal Power Flow formulation can be used for simulations in almost every time scale.
The main idea is to minimize an objective function that represents the costs associated to the production of electrical energy, seen as the summation of the costs in all generation nodes G:

$$min \sum_{i \in G} C_G \tag{3.3.1}$$

These costs are typically described using linear or convex functions (quadratic cost function). It is not unusual to choose different objective functions according to which quantity one may want to minimize. Some examples are given at the end of this chapter.
The constraints represent the power flow equations and the operational limits in terms of electrical quantities: powers, voltages, currents and phase angles.

$$P_i = P_G - P_L = V_i \sum_j V_j \left[G_{ij}\, cos(\delta_i - \delta_j) + [B_{ij}\, sin(\delta_i - \delta_j)\right] \tag{3.3.2}$$

$$Q_i = Q_G - Q_L = V_i \sum_j V_j \left[G_{ij}\, sin(\delta_i - \delta_j) - [B_{ij}\, cos(\delta_i - \delta_j)\right] \tag{3.3.3}$$

$$P_{G,i}^{min} \leq P_{G,i} \leq P_{G,i}^{max} \tag{3.3.4}$$

$$Q_{G,i}^{min} \leq Q_{G,i} \leq Q_{G,i}^{max} \tag{3.3.5}$$

$$V_i^{min} \leq V_i \leq V_i^{max} \tag{3.3.6}$$

$$\delta_i^{min} \le \delta_i \le \delta_i^{max} \tag{3.3.7}$$

Modern formulations include also line limits in terms of maximum current or maximum apparent power. The capability of the line has to be accurately assessed, in order to prevent damages on electrical systems components but also to avoid underestimation of the power that can flow, that may result in an inefficient operation of the grid. The capability of a line depends on the voltage level, on stability limits and thermal limits (see figure 3.4):

– The voltage level is typically fixed under an operating point of view. As a general rule, in order to increase the power transfer that can flow in long lines, voltage has to be increased. This is one of the reasons that brought to the growing development of HVDC technologies.

– Stability limits are extremely important, they are referred to voltage and angle stability. They are related to the generators and the dynamics of the loads [21]. When planning power systems, reactive power compensation methods can be designed in order to increase the maximum power that can flow in quite long lines.

– Thermal limits are related to the conductor loss of strength and especially the line sagging (they also avoid that lines protections activate) [17]. Wind velocity, external temperature and solar radiation are factors that affect the current limits of a line, that are consequently related to the thermal limits.

In terms of equations, the latter limit will be considered as a constant value of maximum power that can flow without compromising the thermal limits of the lines. The inequality constraint in the case of AC power flow studies, may be considered in terms of maximum current that can flow:

$$|\widetilde{V}_i - \widetilde{V}_j| \; y_{ij} \le I_{ij,MAX} \tag{3.3.8}$$

Or maximum apparent power:

$$P_{ij}^2 + Q_{ij}^2 \le S_{ij,MAX}^2 \tag{3.3.9}$$

Optimal power flow studies that comprise a constraint for line limits flow have an increased computational time (between 2 and 20 times according to [22], and an increase in the objective solution (up to 25% according to [22]) depending on the network size and characteristics. However, congestion is an important aspect to be considered when dealing with financial rights. The consideration of lines limits is therefore essential in our case.
For our calculations, we will consider the power limit (3.3.9). So, the expression for the power flowing in a transmission line is required, as well as for

the reactive power. For this purpose, we consider the complex power flowing through a line as:

$$P_{ij} + Q_{ij} = \overline{S}_{ij} = \overline{V}_i \overline{I}^*_{ij} \tag{3.3.10}$$

The current flowing through the line from bus i to bus j (and the shunt path) can be computed using Kirchhoff's Current Law in the pi-equivalent circuit (3.2):

$$\overline{I}_{ij} = \frac{y_{s,ij}}{2}\overline{V}_i + y_{L,ij}(\overline{V}_i - \overline{V}_j) \tag{3.3.11}$$

Computing the complex power:

$$\begin{aligned}\overline{S}_{ij} &= \overline{V}_i \overline{I}^*_{ij} = \overline{V}_i\left(\frac{\overline{Y}^*_{s,ij}}{2}\overline{V}^*_i + \overline{Y}^*_{L,ij}(\overline{V}^*_i - \overline{V}^*_j)\right) = \\ &= |\overline{V}|^2_i\left(\frac{\overline{Y}^*_{s,ij}}{2} + \overline{Y}^*_{L,ij}\right) - |\overline{V}_i||\overline{V}_j|\angle(\delta_i - \delta_j)\,\overline{Y}^*_{L,ij}\end{aligned}$$

Using Euler's Formula:

$$\overline{S}_{ij} = |\overline{V}_i|^2\left(\frac{\overline{Y}^*_{s,ij}}{2} + \overline{Y}^*_{L,ij}\right) - |\overline{V}_i||\overline{V}_j|\,(cos(\delta_i - \delta_j) + \, j\,sin(\delta_i - \delta_j)\,\overline{Y}^*_{L,ij} = \tag{3.3.12}$$

$$= |\overline{V}_i|^2\left(\frac{\overline{Y}^*_{s,ij}}{2} + \overline{Y}^*_{L,ij}\right) - [|\overline{V}_i||\overline{V}_j|\,cos(\delta_i - \delta_j) + |\overline{V}_i||\overline{V}_j|\,j\,sin(\delta_i - \delta_j)]\,\overline{Y}^*_{L,ij} \tag{3.3.13}$$

We introduce now the series and shunt admittances, and neglect the resistive behaviour of the insulating material (neglecting therefore the shunt conductance $G_{S,ij}$):

$$\begin{aligned}\overline{Y}_{L,ij} &= G_{ij} + jB_{ij}\\ \overline{Y}^*_{L,ij} &= G_{ij} - jB_{ij}\\ \frac{\overline{Y}_{S,ij}}{2} &= j\frac{B_{S,ij}}{2}\\ \frac{\overline{Y}^*_{S,ij}}{2} &= -j\frac{B_{S,ij}}{2}\end{aligned}$$

From (3.3.12) we can calculate the active and reactive power flowing through the line computing the real and imaginary parts of the complex power:

$$P_{ij} = \Re\{\overline{S}_{ij}\} \tag{3.3.14}$$

$$Q_{ij} = \Im\{\overline{S}_{ij}\} \tag{3.3.15}$$

Consequently:

$$\begin{aligned}\overline{S}_{ij} &= |\overline{V}_i|^2\left(-j\frac{B_{S,ij}}{2} + G_{ij} - jB_{ij}\right)\\ &\quad - \left[|\overline{V}_i||\overline{V}_j|\,cos(\delta_i - \delta_j) + |\overline{V}_i||\overline{V}_j|\,j\,sin(\delta_i - \delta_j)\right][G_{ij} - jB_{ij}]\end{aligned}$$

And then we can finally find the expression for the calculation of active and reactive powers through a generic transmission line:

$$P_{ij} = |\overline{V}_i|^2(G_{ij}) - G_{ij}|\overline{V}_i||\overline{V}_j|\,cos(\delta_i - \delta_j) - B_{ij}\,|\overline{V}_i||\overline{V}_j|\,sin(\delta_i - \delta_j)] \tag{3.3.16}$$

$$Q_{ij} = -|\overline{V}_i|^2\Big(\frac{B_{S,ij}}{2} + B_{ij}\Big) - G_{ij}\,|\overline{V}_i||\overline{V}_j|\,sin(\delta_i - \delta_j)] + B_{ij}|\overline{V}_i||\overline{V}_j|\,cos(\delta_i - \delta_j) \tag{3.3.17}$$

At this point, it can be relevant to calculate the power losses that occur through the line. This can be done simply via summation of the power flowing from node i to j with the power from j to i [23]:

$$\begin{aligned}P_{LOSS,ij} &= P_{ij} + P_{ji} =\\ &\qquad |\overline{V}_i|^2(G_{ij}) - G_{ij}|\overline{V}_i||\overline{V}_j|\,cos(\delta_i - \delta_j) - B_{ij}\,|\overline{V}_i||\overline{V}_j|\,sin(\delta_i - \delta_j)]\\ &\qquad + |\overline{V}_j|^2(G_{ji}) - G_{ji}|\overline{V}_j||\overline{V}_i|\,cos(\delta_j - \delta_i) - B_{ji}\,|\overline{V}_j||\overline{V}_i|\,sin(\delta_j - \delta_i)]\end{aligned}$$

Therefore, using trigonometric proprieties, we can consider $sin(-x) = -sin(x)$ and $cos(-x) = cos(x)$.

$$P_{LOSS,ij} = G_{ij}\,(|\overline{V}_i|^2 + |\overline{V}_j|^2) - 2\,G_{ij}\,|\overline{V}_i||\overline{V}_j|\,cos(\delta_i - \delta_j) \tag{3.3.18}$$

We will not consider how the maximum value of current is calculated, because as previously stated it depends on many factors such as wind speed and temperature, requiring solution of the non-linear conductor heat balance model. An approximated way of calculating the maximum apparent power flowing can be calculated considering the maximum current provided by the manufacturer in standard environment conditions, according to [23]:

$$S_{max} = 3\frac{V}{\sqrt{3}}I_{max} \tag{3.3.19}$$

Where V represents the phase to phase voltage. Formula (3.3.19) is not in p.u..
It is important to highlight that the maximum power we are considering here is related to **thermal limits**. For long lines the limits are set by voltage drop and stability limitations of the system [21]. Reactive power compensation is useful in this framework for increasing the stability limits, allowing longer lines to be built.
Also the reactive power flow in a generic line can be computed as well:

$$Q_{LINE,ij} = -\Big(\frac{B_{S,ij}}{2}+B_{ij}\Big)\,(|\overline{V}_i|^2+|\overline{V}_j|^2)+2\,B_{ij}\,|\overline{V}_i||\overline{V}_j|\,cos(\delta_i-\delta_j) \tag{3.3.20}$$

For the reactive power we can make a few comments:

– If the line needs reactive power for its operation, $Q_{LINE,ij} > 0$, while $Q_{LINE,ij} < 0$ in the opposite case.

– The reactive power in a transmission system depends on the active power flowing through the line with respect to the natural power (or SIL, Surge Impedance Loading).

– As a general rule, reactive power is generated by the line in "low load" conditions, while it is needed in "high load" conditions.

– This depends on the capacitive and inductive behaviours of the lines, if no reactive power is needed or produced, then these two effects compensate each other.

– Voltage profile through the line can be considered as constant - or "flat" - at the SIL (if we consider the same voltage at the two ends of the line), it increases at the middle of the line in "low power" conditions, while it decreases in "high power" conditions (see figure 3.3).

These aspects are important, along with the effective reactive power flow through the line. A transmission system can be considered approximately as a "constant voltage" system, not suitable for reactive power transfer [21]. The previous considerations are valid also when the voltage at the two ends of a generic line are not too far from their nominal values. This is one of the reasons that bring to simplifications of the OPF formulation to a DC-OPF.

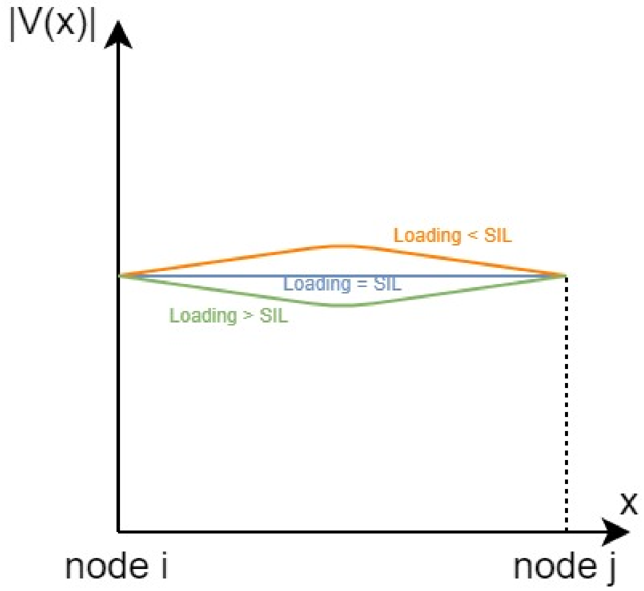


Figure 3.3 Approximated voltage profiles in a transmission line with respect to the SIL.

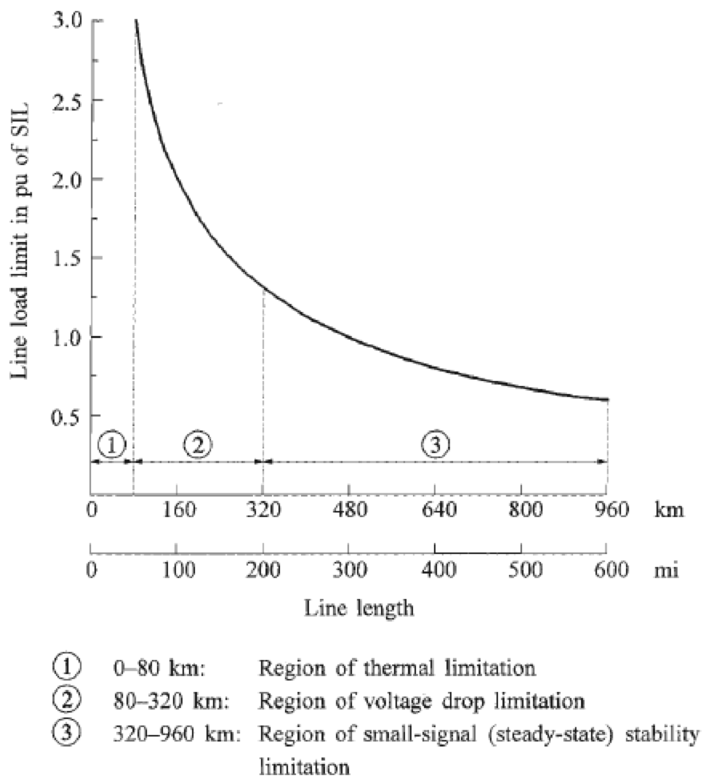


Figure 3.4 Loadability curve for a transmission line [17].

The inequality constraint (3.3.9) can be simplified, as well as the other constraints of this model, to an approximated model called DC-OPF, that will be discussed in subsection (3.4.1).
Finally, a sensitivity factor can be introduced if we want to avoid computing the voltage angles in order to calculate the power flowing through a line. It is called Power Transfer Distribution Factor (PTDF), and it is equal to the marginal increase of power in a line considering a marginal increase of power in a specified bus. See [24] for a detailed description.

3.4 Applications

In the following discussion, a short introduction about other models similar to the classical OPF formulation is listed:

- Economic Dispatch and Security-constrained ED: OPF was first introduced from the Economic Dispatch (ED) formulation, that did not consider any flows and transmission constraints. The objective function can be defined as in the OPF case, and we only have the inequality constraints for generation limits and an equality constraint that ensures the supply of the loads from the generating units $\sum_i P_{G,i} = P_L$. The security-constraint extensions comprises also the contingencies that may occur during the power system operation. Contingencies are usually related to N-1 security state, that considers the possibility that one component of the system (e.g. a line) can exhibit an outage;
- Unit Commitment and Security-constrained UC: this refers to a formulation that schedules the generators considering that they can also be turned off during a multi-period time scheme [18]. Start-up and shut down costs have to be considered in the objective function.
- Optimal Reactive Power Flow and Reactive Power Planning: the objective function minimizes the losses in the network, while considering in the second case also the insertion of new reactive power devices.

Finally, the DC-OPF approximation will be introduced in the next subsection. For a detailed discussion about those topics, see [18].

3.4.1 DC-OPF

One of the most well-known, and still widely used approximations of the full AC-OPF formulation, comes out from the following assumptions:

- Resistances and therefore power losses are neglected, based on the fact that transmission networks have low losses compared to the power flowing through the lines, and resistances are small if compared to the lines reactances;

– Voltages are considered as constants at their nominal value $V_i = 1\ p.u.$. In power systems they are typically bounded to their nominal value with good approximation, but this also means that reactive power generation is needed to keep the voltage stable [18].

– All reactive powers are neglected because small if compared to active power flows. In a voltage-constant transmission system the reactive power flow is related to the magnitude difference between the voltages, that in the first approximation were assumed to be equal for all the nodes.

– The voltage angles differences between connected buses is quite small, therefore: $cos\,(\delta_i - \delta_j) \approx 1$ and $sin\,(\delta_i - \delta_j) \approx (\delta_i - \delta_j)$.

It is called DC-OPF because the formulation is similar in form to a dc circuit (figure 3.5).

DC ⟷	AC
I	P
R	X
ΔV	$\Delta\delta$

Figure 3.5 The analogy for DC-OPF

These can be quite good approximations for power systems not excessively stressed. But, it is not uncommon to see voltages under their nominal values and voltage angle differences might be non-negligible [18]. Therefore, since in this work there is a focus on congestion management, stressed systems simulations might not be accurate using the DC-OPF.
In many papers, DC inaccuracies have been addressed using a power losses incorporation [25], [26]. Some of these approximations use a quadratic function to represent system losses, that can be relaxed using a piecewise-defined function to obtain a simpler problem; [27] and [28] give a comprehensive discussion about this topic, as well as convex relaxation methods for AC-OPF problems with storage integration.

3.4.2 Other Formulations

Optimal Storage Scheduling is another approach that can be used. In this case, described in [29] simulated with wind production, the profit of storage devices can be exploited through the maximization of the following function:

$$max\,(R - C) \tag{3.4.1}$$

Where R represents the total revenue of providing energy to the loads and C are the costs of energy from the transmission network. In this case, the network losses are also decreased.
The voltage profile enhancement is another example of optimization problem seeking to minimize the variation of the voltage from 1 p.u. at the load buses [30].
Multi-objective optimization is also a viable alternative when the objective functions to be optimized are more than one.

3.5 OPF and Dual Variables as Prices

The following section contains relevant links between power systems and electricity markets. An important distinction has to be made when dealing with marginal prices in power systems. Nodal pricing and zonal pricing are the most common choices that are used worldwide:

– Many countries (e.g. the European countries) use a **zonal pricing** scheme in which the concept of "zone" or area depends on two different approaches: a price zone might correspond to a country, or within the same country several price zones may be defined.

– Several countries (US, Singapore, New Zealand, Argentina and Chile [13]) adopt a **nodal pricing** scheme, or Locational Marginal Pricing (LMP) method.

The nodal marginal price is the cost for the supply of an extra unit of power (typically 1 MW) at a specific node. This price reflects the cheapest solution for re-dispatching this extra unit of power to that specific node, considering transmission constraints and capabilities [3]. Since this nodal price (or Locational Marginal Price) reflects also costs associated to the transmission system operation, it might be higher than the most expensive generator cost [31].
As a general rule, due to system losses and possible congestions, the LMP or nodal price is different in all the nodes. Only in the case of a lossless approximation without congestion ongoing we can consider the price for electrical energy to be equal in all the system buses.

In this work, we will deal with the nodal pricing scheme, but some considerations might be extended also to zonal pricing. This latter alternative considers the fact that only the interconnections between the "zones" can be congested. Sometimes a zone is a whole country that have such a highly meshed grid that congestion might hardly occur [32]. In a nodal pricing scheme costumers pay the energy according to the price of that specific location [33], [34], as said before accounting for losses and congestion surcharges:

$$LMP_i = LMP_E + LMP_L + LMP_C \tag{3.5.1}$$

Where the three terms are referred respectively to energy, losses and congestion marginal prices [31], [33].
We present now a simple example to show the price differences in a simple 2-bus system (figure 3.6). These considerations can be extended to more complex systems, for LMP and other relevant topics about power system economics see [3]. Note that the lines losses are neglected in order to focus on the energy economics of this problem.

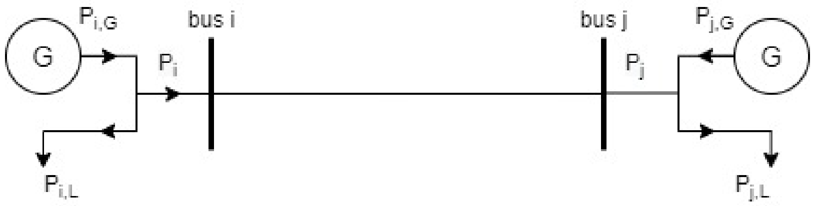


Figure 3.6 Two bus system example.

Let us suppose that generator i has a marginal price of $\pi_i = 25\frac{\$}{MWh}$ for supplying electrical energy, while generator j has a lower cost of $\pi_j = 20\frac{\$}{MWh}$. The transmission line limit is set to $100MW$, while the load demands are, at first, both equal to 60 MW. If we suppose that generator j has enough power output to supply both loads, then it will provide $120MW$ equally distributed to the two loads, and the line power flow would be $60MW$.

The nodal price in this first example is equal for both buses, and it corresponds to the marginal price of generator j, $20\frac{\$}{MWh}$. This is because the merit order brings on the market the generators with the least marginal costs [31], [35].

If, in a second case, we consider the load at the i bus to be $120MW$, generator j will provide a $100MW$ supply to load i, while maintaining the $60MW$ load at bus j, because line has a power limit of $100MW$.
This is a simple case that brings to a marginal price difference between

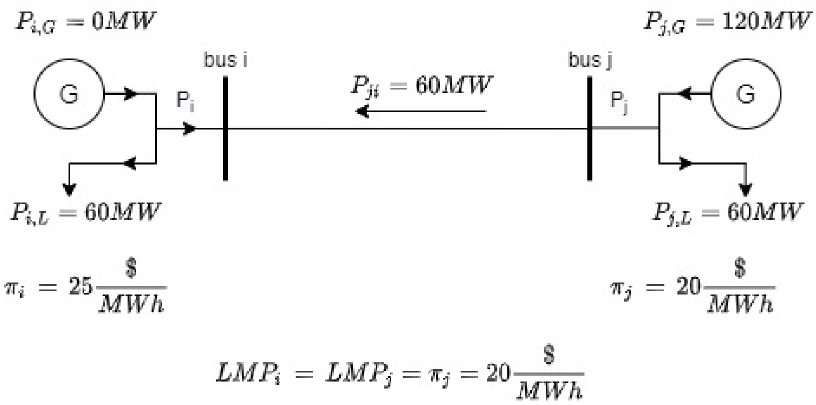


Figure 3.7 Two bus non-congested system example.

these two nodes, caused by transmission congestion. In this simple case the calculation of the nodal prices is straightforward, since an additional megawatt of load at bus i must come necessarily from generator i while at bus j from generator j:

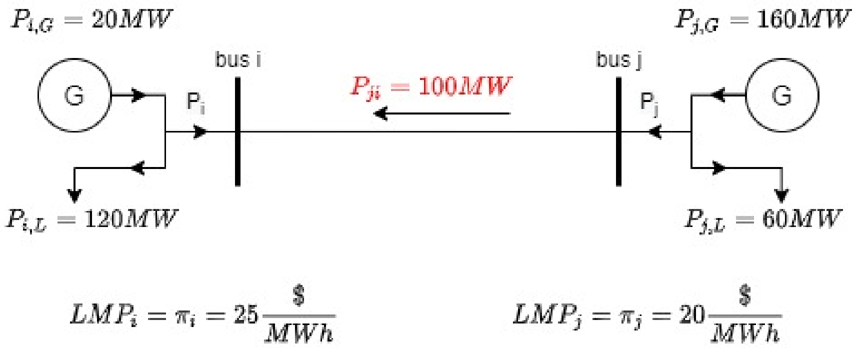


Figure 3.8 LMP in case of congestion in a two bus system

Considering a 3 bus system is a more complex problem, because the path of the power flow follows the physical rule of power systems according to lines admittances [31]. In this perspective we can see how can be useful to implement an OPF formulation with dual variables interpreted as prices, because in much more complex systems involving many buses and branches they reflect the marginal costs for supplying energy to that specific node.
Another crucial aspect that arises after a congestion event, is that a merchandising surplus (or congestion surplus in this case) results from the system operation. This is caused by the higher price paid by the customers with respect to the revenue gained by generators [3]. This surplus can be

calculated as:

$$Payments\,-\,Revenues\,=\,(LMP_i - LMP_j)\,P_{ji} = \\ =\,(25\frac{\$}{MWh} - 20\frac{\$}{MWh})100MW = 500\frac{\$}{h} \tag{3.5.2}$$

In chapter 5 we introduce a hedging financial instrument, named Financial Transmission Right (FTR), that redistributes the congestion surplus collected by the system operator when the lines are congested.
The DC-OPF formulation previously introduced, is popular when dealing with locational marginal pricing. However, since in markets and nodal prices schemes the accuracy of the result of simulations is important under the economical point of view, approximations may bring to results that do not reflect precisely the physical operation of the system [36]. AC-OPF is therefore getting more attention in the framework of energy markets [24]

Chapter 4

Energy Storage

The changes brought by the energy transition will have huge impacts on the whole electrical grid. Distributed energy sources will lead to a radical change in the flow direction in distribution grids, and the transmission level will also see relevant effects. Aleatory power production from PV and wind plants will also bring to an increased uncertainty in energy supply. Resilience and flexibility are some of the main features that we are looking for from power grids, qualities that traditional fuels-based plants can bring in efficiently in terms of voltage stability, frequency regulation and flexibility in production. In order to maintain the grid stable as it has been so far, we need to integrate new technologies and bring innovation.
In this framework, energy storage will be highly needed and will play a critical role. For this reason, many ways of storing energy have been tested, developed and deployed in distribution networks and also for utility-scale transmission grids. An important role will also be played by the residential-scale and microgrid storage, that will lead to an increasing level of self-consumption. However, in this work, we will deal with utility-scale energy storage systems, their role, consideration and integration in transmission systems.
The benefits brought by storage systems can comprise different services, from multi-period time shifting (also known as arbitrage) to short-term ancillary services. This multi-source income framework brings interesting insights in terms of return of investment. However, several regulatory difficulties arise when considering the benefits brought by storage devices to the electrical system [7].
The electrical grid is an efficient carrier for transporting energy, but when energy is produced asynchronously from the required demand, it has to be stored over a wide range of time: from milliseconds in case of supercapacitors, to days, months or seasonal energy balance in case of hydroelectric energy storage.
Straightforward ways to store electrical energy directly are not present in nature, apart from the energy in capacitors that is accumulated in an electric field. A conversion stage is therefore necessary for obtaining another form

of energy, and this brings to conversion losses and lack of flexibility.

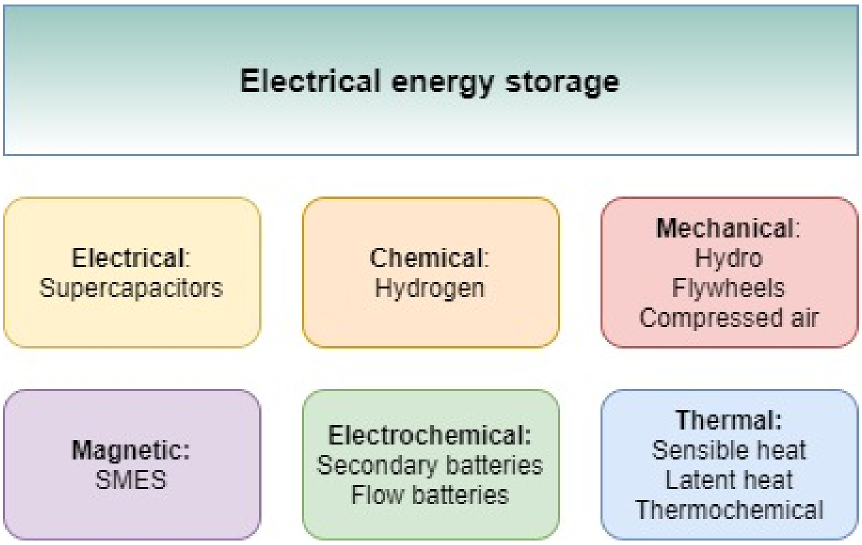


Figure 4.1 Technologies for energy storage.

The services delivered by storage devices are very different according to the type of technology considered. Large-scale pumped hydro storage systems mainly help in terms of energy time shifting, while electro-chemical storage can help with voltage support and frequency regulation but have a limited role in a "long-term" energy time shifting. As a general rule, multiple services can be provided by a single storage plant. In this work we will investigate some aspects of the integration of Battery Energy Storage Systems (BESS) in power systems, because of their potential benefit both in terms of power (in a short-term scale) and of energy (load shifting or peak shaving). Nevertheless, several features are also valid for the other types of technologies. A categorization of storage technologies is provided in figure (4.1).
As mentioned in the introduction, storage can be considered also under the grid planning point of view, especially as an alternative to new expensive lines (at least for their "deferral"). This aspect, along with the consideration of storage as a transmission asset, have a crucial role in the model introduced in the following sections. Transmission lines and storage devices have some similarities that can bring to a similar consideration in a market and regulatory framework according to [1]. In particular:

- Storage "moves" electric power ahead in time, while lines physically carry electric power.
- Both have high investment costs and low operating expenses.
- Both have constraints related to their power capabilities, storage has also energy constraints.

Storage benefits can be categorized according to the placement location (see figure 4.2), that is a critical aspect when deploying storage devices both in transmission (e.g. [37]) or distribution grids (e.g. [38], [39]). Along with services related to the resilience of the grid and the flexibility provided, it is also significant to highlight that it helps the decarbonization of the electricity sector and the reduction of emission when coupled with a growing penetration of green technologies. In addition, "non-technical" benefits can be listed as storage can provide market-related flexibility. We will further assess how a market participant can hedge against nodal price volatility using financial instruments called "Financial Storage Rights" in section 5.

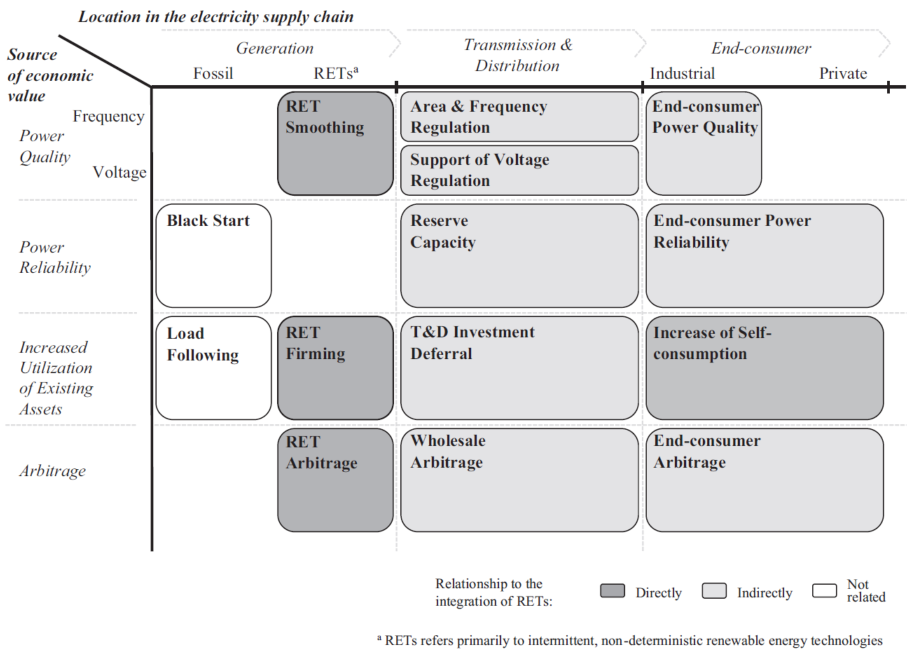


Figure 4.2 Storage benefits according to the location in the supply chain [40]

4.1 Storage Characteristics

Depending on the different devices or technology, storage can exhibit different physical operating parameters. Power and energy are key factors that can be compared using a Ragone Plot: see figure (1.2). Energy density and power density take into account also weight and volume aspects, that are crucial in some applications of storage. The efficiency has to be taken into account because both power losses and energy leakage coefficients can affect the quality of the benefits. Moreover, technology-related aspects such as the response time [41] have to be taken into account, especially for

applications in the electricity sector that might need great peaks of power in a very short time, or a huge amount of energy over time.
As we are extending our optimal power flow formulation to a system comprising storage technologies we are interested in the constraints related to its operation:

$$0 \leq \; p_{i,t}^{ch} \leq \; p_{i,t}^{ch,MAX} \tag{4.1.1}$$

$$p_{i,t}^{dch,MAX} \leq \; p_{i,t}^{dch} \leq 0 \tag{4.1.2}$$

$$E_{storage}^{MIN} \leq \; SOC_{i,t} \leq \; E_{storage}^{MAX} \tag{4.1.3}$$

$$SOC_{i,t} = SOC_{i,t-1} + \eta^{ch} p_{i,t}^{ch} \Delta\mathrm{t} + \frac{p_{i,t}^{dch}}{\eta^{dch}} \Delta\mathrm{t} \tag{4.1.4}$$

Where p_{ch} and p_{dch} denote the charging and discharging rates, while the $SOC_{i,t}$ denotes the State of Charge of the device at bus i, at the time step t. Expression (4.1.1) limits the charging power to its maximum value, and its minimum is set to zero because the discharging process is governed by another variable. The inequality constraint (4.1.2) is therefore needed to limit the power also when discharging. Inequality (4.1.3) represents the maximum and minimum energy that can be stored in the device. $E_{storage}^{min}$ can also be set to zero, but it is known from the literature that, especially for Li-ion-based technologies, the Depth of Discharge (that is, how much a storage device is discharged, $DOD = 100\% - SOC$) as well as the maximum State of Charge (SOC) affect the aging of the battery. It is therefore convenient to consider both energy limits. Equation (4.1.4) relates the multiperiod SOC value, as a function of the previous state of charge and the charging/discharging rates. Storage losses are modelled using efficiencies η^{ch} and η^{dch}, while the leakage coefficient of the batteries can be considered with an efficiency coefficient that multiplies $SOC_{i,t-1}$. The presence of two different efficiencies for charging and discharging powers justifies the utilization of two different variables representing the charging and discharging processes.
The state of charge can also be described by different equation, considering $SOC_{i,t+1}$, but our approach is more convenient when using the GAMS environment. Finally, sometimes it is also required to set the initial and final state of charge:

$$SOC_{i,0} = SOC_{i,END} \tag{4.1.5}$$

where END indicates the final state of charge at the end of the simulation.
Some storage integration approaches consider also the fact that batteries can provide reactive power to the grid. A quadratic constraint is necessary to consider the capability curve of the inverter that connects the device to

the grid [38]:

$$P_{i,t}^2 + Q_{i,t}^2 \leq S_{MAX,i,t}^2 \qquad (4.1.6)$$

In all those constraints the multi-period multi-bus approach has been adopted, considering i as the set of nodes and t the set of time steps.

4.2 Technologies

4.2.1 Mechanical Energy Storage

So far, most of the grid-scale storage is realized using pumped-storage hydro plants (in 2017 accounted about 97% of total storage [41]). It is a well-known technology that allows to store water in form of potential gravitational energy to an upper reservoir. It can provide energy and start a storage mode operation in a matter of seconds, resulting in a quite good response time [42]. Since the capital costs of such an investment are massive, building new pumped hydro-plants is a prohibitive task, considering also the environmental aspects of such huge plants. Only in Asia Pacific during the last years new hydroelectricity plants have been built, while the situation in the rest of the world is almost stable [2].
Flywheel devices store kinetic rotational energy that is exchanged using an electric motor as bidirectional converter. The use of magnetic bearings brings high advantages in terms of friction reduction, that is also lowered by the vacuum enclosure in which the rotating parts are placed. It is typically useful for frequency regulation, but there are also benefits in terms of active power shifting [43].
Compressed Air Storage Systems uses motors and compressors to store air in high-pressure chambers, that is released to produce electrical energy using an air turbine [44].

4.2.2 Electrical and Magnetic Energy Storage

The use of capacitors, and in particular ultra/super capacitors is based on direct storage of electrical energy in an electric field. It is relevant when high power requirements are necessary in very short periods. However, new capacitors allow to store much more energy than traditional technologies, but still small compared to the other grid-connected devices.
On the other hand, superconducting magnetic storage is based on the magnetic field energy produced by a DC (direct current) in a superconducting coil. Criogenically cooling the conductor brings the resistance to a very low value, resulting in high overall efficiency. Again, the power quality enhance-

ments are good, but the costs are in this case a limiting factor [45].

4.2.3 Thermal Energy Storage

Electrical energy can also be converted to thermal energy and, if not directly utilized for heating or cooling, can be reconverted back into electrical power within a large time-frame, from hours to months or years [46]. The implementation of heat pumps for different purposes such as ancillary services has received some attention [47]. There are several possibilities when dealing with thermal storage:

- Sensible heat: energy is stored increasing or decreasing the temperature of a material, without any phase changes;
- Latent heat: energy is stored or released when a material is changing phase at constant temperature;
- Thermochemical energy: in this case, reversible chemical reactions can require or emit heat and store energy changing the chemical bonds of the materials involved in the reaction [41].

4.2.4 Chemical Energy Storage

The conversion of electrical energy into chemical energy is possible thanks to the so-called water electrolysis process. The operation of an electrolyser brings to formation of hydrogen and oxygen from water. The hydrogen can be stored an then converted back to electricity using fuel cells, or converted to another type of gas or chemical (Power-to-Gas) [41].
Water electrolysis can be obtained using different technologies: polymer electrolyte membranes (PEM) alkaline electrolysis cells (AEC). Researches are also going towards solid oxide technology [48].
Several researcher projects suggest also that hydrogen will help the widespread uptake of renewable sources by seasonal storage of gas in the network [49] but it is still an emerging technology that, so far, may not be profitable due to the costs and the regulatory framework [48].
Nevertheless, hydrogen-based technologies can potentially bring many advantages in many fields, helping the society lowering the carbon emissions. See [50] for a comparison analysis between hydrogen and BESS.

4.2.5 Electro-chemical Energy Storage

Cost indices such as the Levelized Cost of Electricity (LCOE) dropped during the last years for Battery Energy Storage Systems (BESS), PV and wind installations [51]. Since the cost for lithium-ion batteries (LIB) has seen an

87% price drop in the last decade [52], and thanks to their maturity and good energy density as well as power density, BESS are receiving attentions for bringing benefits to transmission systems, distribution grids and also micro-grids.
A list of battery technologies according to the chemical components is listed:

– Lead-Acid Batteries: it is historically the most used and cheapest solution, therefore has seen a wide diffusion (e.g. in the automotive sector). Due to the fact that the lead is in solid state the material leakage is very limited. However, due to its heavy weight, has a low energy density, but with quite good power density. It has a limited role in electrical systems, but can be used when a high-power low amount of energy is required, e.g. for Uninterruptible Power Supply (UPS) applications [53];

– Nickel-based batteries: they present higher energy densities with respect to the lead-acid technology. NiCd (nickel cadmium) has not great relevance nowadays, but MiMeH (Nickel metal hydride) batteries have still some applications in the industry and in the automotive world [53];

– ZEBRA and Sodium-Sulphur batteries: they are molten-salt batteries that are characterized by very high temperature operating conditions. They show good characteristics in terms of energy density and have been utilized for both arbitrage, voltage regulation [54] and renewable energy sources integration [55];

– Lithium-ion batteries: they have the biggest market share and thanks to a reduction in prices it is the most used battery technology in many applications. They are common in portable applications and electric vehicles, but also for grid-scale applications such as time shifting or frequency regulation [41];

– Flow batteries: it is an early technology that is characterized by the fact that the reactants are outside of the battery and are stored in tanks. It is used for stationary applications because of the high volume required for all its components.

Together with power, energy, efficiency and costs, many other factors are practically important when designing a battery storage power station. The temperatures of operation, the cycle frequency, the minimum and maximum state of charge (SOC) are other aspects that affect the so-called aging of the battery pack, resulting in a natural degradation of the performance [53]. Due to the fact that batteries loose their original capacity as they age, it can be introduced the factor State of Health (SoH) of the battery: it refers to the number of Coulombs available at a given charging rate, compared to the original manufacturing conditions (It's about 100% when the battery is new, and decreases aging). Then, the charging power is important when

calculating aging consequences. It is also known that the SoH has different values if calculated with different charging rates: for this purpose it is necessary to specify at which C-rate [1] is the SoH referred to [53].
The SoH is an important factor when dealing with end-of-life applications of batteries that cannot provide an adequate performance in certain frameworks. The most relevant example is the reuse of old batteries from electric vehicles to stationary applications: there is an ongoing debate on how this storage devices can have a second life application and which difficulties may arise. In general, caused by the expected wide spread of an enormous amount of EVs, the battery could arise problems related to the recycling after their use (that depends also on the annual mileage of the vehicle [2] and how V2G applications will affect battery aging [57]). This problem can turn into an opportunity for stationary applications, but research on both reuse and recycle must bring efficient ideas on both these paradigms [56].

When dealing with Electric Vehicles the most relevant technology is the Li-ion Battery (LIB) thanks to its capability to provide good energy and power densities in quite small volumes, but when considering grid storage some of the above-mentioned alternatives may compete in terms of suitability. The article [58] gives an overview on the competitors to LIB-based technology when dealing with grid-scale storage. Weight and volume have less importance for stationary applications if compared to the automotive world. Technologies that can bring a high amount of maximum energy will be needed to increase the storage time-frame without increasing too much the expenses: e.g., flow batteries use quite cheap materials and if compared to LIB have a lower increasing cost when rising in volume [58].

4.3 Storage Role and Advantages

In the classification section it was clear that many technologies are available and consolidated, while some of them are undergoing a research process that will hopefully bring relevant solutions for the future issues of electrical grids. Operational limits are also key factors when choosing the most appropriate storage technologies: there is no storage device that can be suitable in every circumstance. Figure 4.3 shows how different services can be provided in different time scales.
Parameters such as frequency and voltage are important in power systems and must remain within their operating limits. Frequency is a crucial parameter, and is strictly related to the supply-demand balance. Moment by moment loads change their behaviour and power supply must follow these changes. It is, however, a change in the paradigm because so far the traditional power plants could provide an efficient balance by controlling their

[1]The C-rate factor describes the power required for a full-charge of the battery in one hour: e.g., if $C - rate = 2C$, the battery is charged in half an hour [53].

[2]A typical SoH for EVs at the end of their use is 0.7-0.8 [56].

input power according to the needs. In the framework of renewable energy sources, the grid must be reinforced with flexible devices that can help overcome these challenges.

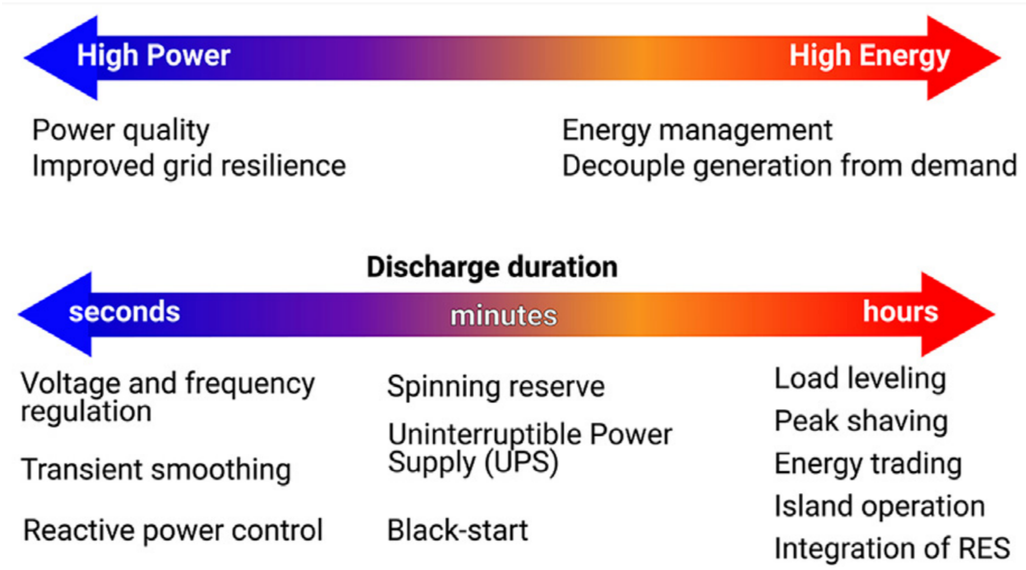


Figure 4.3 Power-Energy view of storage benefits [59].

If we analyze the different time scales:

– In the short term it is not uncommon to see voltage dips caused by outages or the insertion of high loads. The reactive power that, as stated in chapter 3, is strictly related to the voltage absolute value, has to be supplied by devices such as synchronous compensators in order to offset the inductive effect of most loads. Battery energy storage systems can also provide a reactive power regulation, since their connection to the grid is based on a DC/AC converter (inverter) that can operate in 4 quadrants of the P-Q characteristics. Frequency must also be kept under control, because it is strictly related to the rotating speed of generators. When a contingency, a loss of a large power plant, or a steep change in the load occurs, the electrical system have been traditionally providing the power required through the rotor inertia of the generators connected to the grid.

– Since storage does not require a start-up time such as traditional generators, they can provide an operating reserve very quickly (within some milliseconds [60]). Power systems have been traditionally providing spinning reserve thanks to the automatic speed controller of the synchronous generators connected to the grid. A decrease in the system frequency means also a decrease in the rotor speed. The speed controller (governor) then acts providing more torque to the generator, e.g. by supplying more fuel and the balance between power generated and loads is then re-established. This is called primary frequency regulation [61], [62].

– The secondary regulation then brings the frequency to its nominal value (e.g. 50 Hz in Europe) in a longer time-frame through an automatic centralized control.

In all these time periods BESS, and other storage devices can help the grid overcome temporal issues and transients. Other benefits comprise: lines congestions reduction, back-up power in case of outages, black start capabilities, RES curtailment avoidance, deferral of new lines. For our purposes, regardless on the typology of frequency regulation we can state that storage in general, and Battery Storage Systems in particular, can provide active power support that is strictly related to the frequency regulation (see [63] for a discussion on both primary and secondary regulation using BESS). We will focus on what is commonly referred as load shifting, peak shaving or arbitrage. That is, charging the battery (or the storage device in general) when electrical energy prices are cheap and discharge it when the price rise. We are reasonably considering high-load periods to be more expensive than low-load conditions.

Several problems arise when trying to evaluate the benefits brought by storage devices to the electrical grid. We discussed above that they comprise active and reactive power balance, but also voltage and frequency regulation. Not all these services are efficiently priced and, looking at storage devices as an investment, it might happen that they could bring to non-profitable results. In [7], a wide discussion about the non-technical issues and barriers for storage deployment is summarized. Starting from its consideration it is common to set storage as a generation asset that buy and sell energy according to nodal prices (arbitrage). In this work we will however assess which benefits could bring the consideration of storage as a transmission asset, that consists on leaving the operation task to the system operator that manages it according to its physical constraints in order to maximize social welfare. This consideration is facilitated by the fact that storage has some similarities with transmission lines, that have previously been outlined in this chapter.

However, it is relevant to point out that the differences are important. When lines are congested, we will consider the prices for energy different in the nodes of the system (using nodal pricing). Storage instead, arbitrages in the same node, but between two different time instants. This will bring to different results when dealing with financial rights in chapters 5-7.

Regarding ancillary services and grid services, it is important to provide a welcoming market for storage deployment and efficient consideration of its benefits that, in the past, were hardly valued or not valued at all [7].

The contribution of this thesis, is to try to help building a welcoming market framework under the arbitrage point of view (see figure 4.4). Several other contributions are essential to evaluate as much as possible the benefits brought by storage devices to electrical grids.

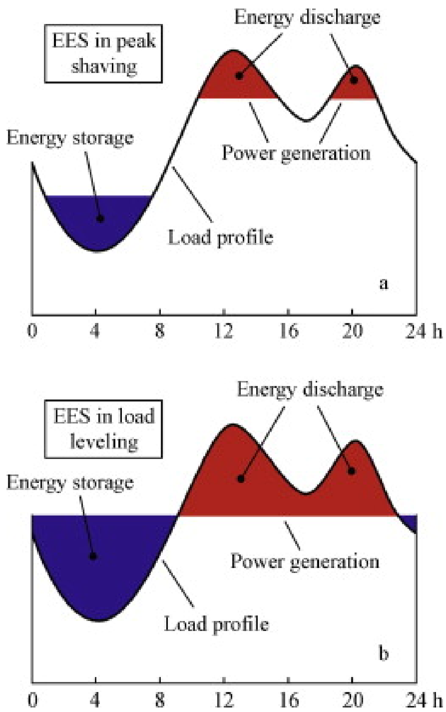


Figure 4.4 Electrical Energy Storage (ESS) role for Peak shaving (a) and Load Levelling (b) [64].

Chapter 5

Financial Rights

5.1 Grid Congestions

Nodal pricing scheme described in section (3.5) is strictly related to the concept of lines congestion. It has been explained how nodal prices can be found in simple 2 or 3 bus systems, but a mathematical model is needed when dealing with real systems comprising hundreds or thousands of buses (and an average of 3 lines connected to each bus).
One of the main results from such simulations is the cost of energy at each bus, also referred as LMP. Unpredictability in spot market can bring to very high costs, and may encourage the consumers to find a way to **hedge from price volatility**.
An approach presented in many works comprises the use of financial rights: what have already been adopted by some Independent System Operators (ISOs) is the concept of Financial Transmission Rights (e.g. PJM Interconnection LLC [65]). It is also possible when zonal pricing scheme is applied, considering only the lines interconnecting the different zones. An example is Europe's Forward Capacity Allocation (FCA) regulation that schedules the operation of cross-zonal interconnections and capacities in a forward market framework using long-term transmission rights (LTTRs) [66] [67]. However we will consider a nodal pricing scheme with different LMPs for each bus of the system.

A first approach can be the definition of Physical Transmission Rights (PTRs). They are bought by suppliers and consumers at auctions to have the right to physically use a transmission line for supplying or receiving power. This approach however has the problem that the flow follows the physical rules of the power system and not the intended path because of parallel flows [68]. Also, a market participant might want to buy physical rights and not use them only for increasing their profits. A "use them or lose them" condition can be binded to PTRs to avoid these circumstances [3].
A transmission right provides to its owner the property of a certain amount of

transmission capacity, giving the permission to use it. Furthermore, instead of considering this physical approach, **Financial Transmission Rights (FTRs)** can be introduced: their owners receive a profit that is proportional to the amount of FTR purchased and to the price for each FTR. This allows to their holder to gain a revenue in case of congestions. The value of these transmission rights is therefore zero if the system has no congestion events.
Financial Transmission Rights can be introduced using two different approaches:

- Point-to-Point Transmission Rights: they are defined between any two nodes of the grid, and the value of the right is the difference between the two nodal prices in $(e.g. \, in \, \frac{\$}{MWh})$ times the quantity of rights bought [69]. The rights are assumed to be simultaneously possible for the grid and their value must be less than the merchandising surplus of the system operator under congestion (also known as congestion surplus) [70];

- Flowgate or Flow-based Transmission Rights: they are defined using the definition of a "flowgate", that can be described as a line that can be congested. Since they are related to a specific line their feasibility is more related to the physical capacity of the line. Therefore, it can be calculated using the dual variable of the capacity constraint of the line [3].

Other approaches are compared in [68]. Also, simultaneous markets for both point-to-point and flowgate rights have been proposed [69].
Since we are more interested in the flowgate approach for the definition of financial storage rights, we will consider only this type of instruments, even though many system operators chose the other approach.
Then, we can extend the example introduced in subsection (3.5) for practically understanding the value of FTR.
Let us consider the congested case of fig. 3.8.
Here the price difference is quite low, but in some cases it can take high values.
The consumer $P_{i,L}$ may want to pay for hedging instruments together with energy prices. That is, paying for Financial Transmission Rights and gain a revenue that depends on the congesting conditions.
Assuming that the generators revenues and the consumers expenses are based on the nodal price, we decouple the production and consumption locations [3]. We can therefore find the two costs associated to the production at generators i and j:

$$ProductionCost_i = \pi_i \cdot P_{i,G} = 25 \frac{\$}{MWh} \cdot 20MW = 500 \frac{\$}{h} \tag{5.1.1}$$

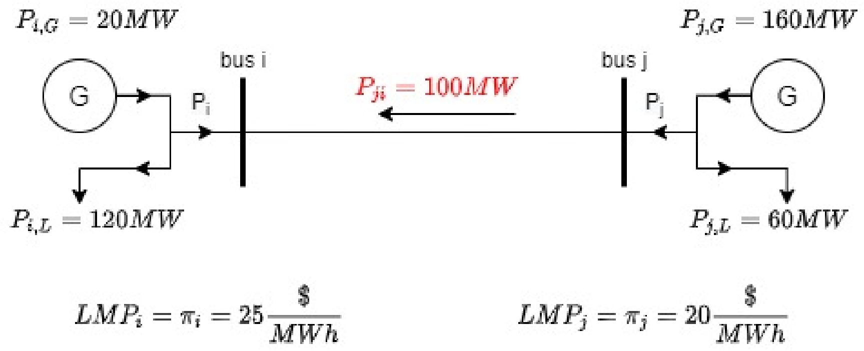


Figure 5.1 LMP in case of congestion in a two bus system.

$$ProductionCost_j = \pi_j \cdot P_{j,G} = 20\frac{\$}{MWh} \cdot 160MW = 3200\frac{\$}{h} \tag{5.1.2}$$

While the total cost for the consumers is:

$$ConsumtpionCost_i = \pi_i \cdot P_{i,G} = 25\frac{\$}{MWh} \cdot 120MW = 3000\frac{\$}{h} \tag{5.1.3}$$

$$ConsumptionCost_j = \pi_j \cdot P_{j,G} = 20\frac{\$}{MWh} \cdot 60MW = 1200\frac{\$}{h} \tag{5.1.4}$$

If we calculate the difference between the total costs for the consumers and the total revenues of the generators [3]:

$$\begin{gathered}(ConsumptionCost_i + ConsumptionCost_j) - \\ (ProductionCost_i + ProductionCost_j) = \\ = (\pi_i - \pi_j)P_{ij,MAX} = 4200 - 3700 = 500\frac{\$}{h}\end{gathered}$$

We obtained the value of the merchandising surplus (or congestion surplus). It can be profitable (or hedgeable) now, for consumer at node i, to recover their additional costs using Financial Transmission Rights and own the profits from the congestion between nodes j and i. The system operator can then decide to sell these financial instruments to market participants and reallocate the surplus coming from the system operation. Since the total surplus is $500\frac{\$}{h}$, it is not profitable for consumer at bus i to spend more than this amount. The maximum price that brings to a zero-hedge position is therefore:

$$FTR_{ji,MAX} = \pi_i - \pi_j = 5\frac{\$}{MWh} \tag{5.1.5}$$

It is important to highlight that:

– If consumer at bus i would be able to pay $0\frac{\$}{MWh}$ for the FTRs he would be perfectly hedged. It is equivalent to buying the energy according to the production location;

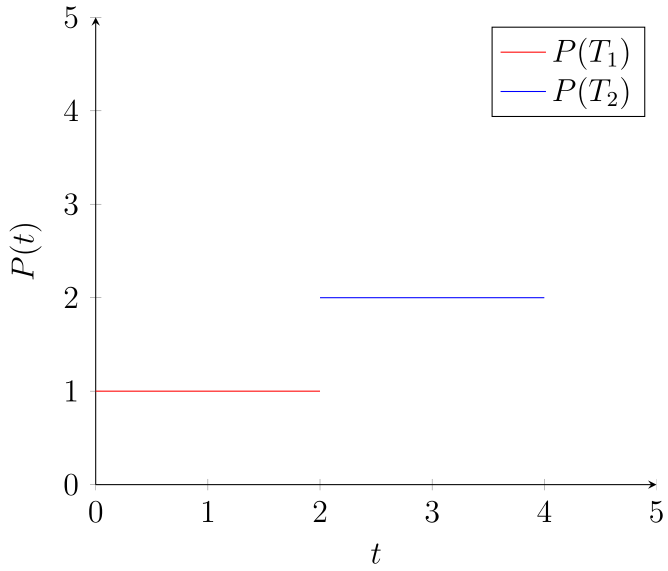
– What a market participant can purchase is a portion of the total amount of FTRs available;

– This two bus example can explain the basics around Flowgate Transmission Rights, while it does not comprehensively outline the whole framework also comprising the point-to-point approach;

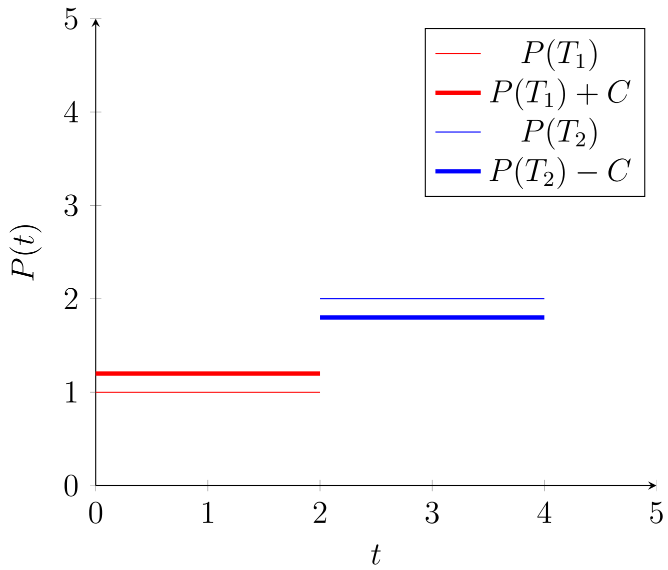
– Realistic situations are much more complex and a mathematical model is necessary to calculate the nodal prices and congestion surplus (this will be done in chapter 6 using dual variables, or shadow prices).

5.2 Storage Congestions

We already discussed about storage integration into electricity markets and the possibility of avoiding nodal price transactions, considering them as transmission assets [1]. Extending what we introduced in the previous section, storage can be used to protect against the unpredictability of nodal inter-temporal prices. In this case, we can consider or not the presence of transmission congestion, because storage provides a hedge against **temporal price fluctuations**.
There are two different approaches for Financial Storage Rights definition: [1] and [71]. Both of them propose the joint sale of both financial storage and transmission rights.
In the first model [1], similarly as with FTRs, financial storage rights lead to a gain only if they are used during a "storage congestion". Furthermore, storage rights cannot be negative while transmission rights can in some cases take negative value bringing to an obligation for its holder.
Understanding storage congestion might not be as intuitive as the transmission lines congestion is. Defining then this situation mathematically, we suppose that without the presence of storage the power demand has the following trend:



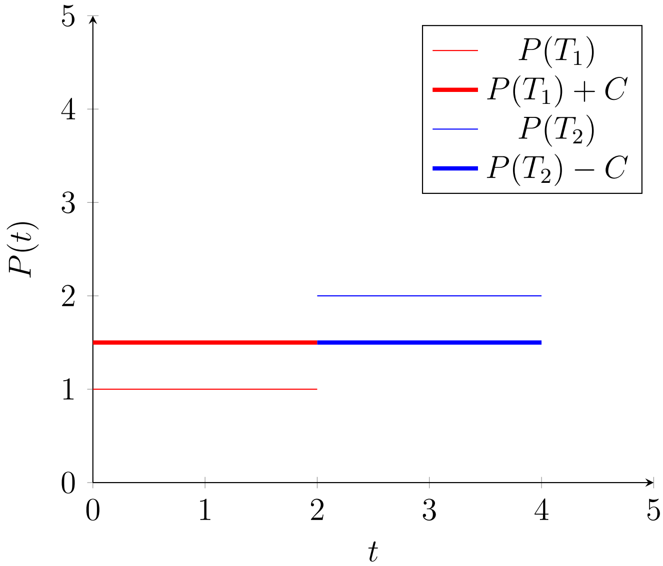
Generators and loads are perfectly balanced in terms of power. With the introduction of storage, let's suppose that the demand will increase by C during the first period (0,2) in order to charge the storage device. Then, it will be discharged during the second period (2,4) in order to supply part of the load and avoid for example congestion in transmission lines.



We can see that a storage congestion is ongoing, since the device is exploited to the fullest capacity. This situation occurs as soon as [1]:

$$C < \frac{P(T_2) - P(T_1)}{2} \tag{5.2.1}$$

In case the difference between the powers required in these periods divided by 2 equals the value of the power capacity, the storage unit would perfectly smooth the demand curve.



If the storage has a capacity higher than the above mentioned value, it should not be congested in our example, but it will use just a part of its total capacity. In case of congestion, the system operator would earn a revenue that is the exact amount that would be paid if the storage device acquired and sold power at nodal prices [1]. Merchandising surpluses are then redistributed to financial storage rights owners. Reasonably, this occurs if the price in the second period is higher than the first one. This situation might encourage consumers to protect against price unpredictability, and financial storage rights can hedge them in case of a congestion event. Since they are defined as financial rights, do not affect the physical operation of the system.
A multi-period Optimal Power Flow formulation is presented, in order to evaluate the effects of the storage presence. Moreover, a lossless DC power flow is an approximation widely used in power systems. However, DC PF does not consider voltage limits and stability, and can exhibit significant inaccuracy for heavily loaded systems [18]. Therefore, an AC Power Flow can be computed in order to overcome these issues and better evaluate storage effects within electric grids. Yet, this solution is difficult for the presence of non-convex constraints (voltages) and non-linear constraints (real and reactive powers) [72]. In order to incorporate energy storage technologies within markets, a multi-period OPF is necessary due to the time-coupled behaviour of power system dynamic components.

While considering storage as a generation asset would bring the owner a benefit related to arbitrage activities, the system operator could manage storage bringing to an increased overall benefit to the system. Reasonably, when seeking to maximize social welfare, the overall costs for producing and dispatching energy would be lower than in other cases. Also, if a detailed consideration of ancillary services would be available, storage would result in much more cost-effective investments. Under the system operator point of view, the approaches in the literature propose that an auction would be the best way to allocate financial rights to potential buyers. Also, both [71] and [73] state that this auction methodology is revenue adequate, that is, the budget surplus is greater than the overall value of financial rights. This conditions ensures that the system operator does not incur in a financial debt.

Having assessed the advantages for hedging consumers and the position of the system operator, we must evaluate if the storage owner's perspective is profitable. Instead of gaining from arbitrage, transactions at nodal prices bring to a revenue that is redistributed by the system operator to financial storage rights buyers, that have to pay for those financial instruments. If the FSR buyer partially hedges then a portion of the arbitrage revenue remains to the system operator. Then, the storage owner receives rate payment from the system operator from the sale of FSRs and thanks to the overall cost benefits both in terms of diminished objective value that for ancillary services provision to the grid. It is clear that an accurate benefits assessment is necessary for storage integration

As an alternative to this method, based on the definition of dual variables for storage congestion, [71] proposes an approach that calculates FSRs using nodal prices at the different time steps.

Chapter 6

Financial Storage Rights Calculation

6.1 DC-OPF

The definition of Financial Storage Rights (FSR), divided into Power Capacity Rights and Energy Capacity Rights, comes from the Lagrangian dual problem of a multi-period Optimal Power Flow linked by storage, with linearized approximation. This model had been introduced in [1], and will be extended to a full AC approach in the next section. The following mathematical model represents the DC Optimal Power Flow formulation in a generic transmission system:

$$min \qquad \sum_{i,t} f_{i,t}(p_{i,t}) \tag{6.1.1}$$

$$s.t. \quad p_{i,t} = p_{i,t}^{ch} + p_{i,t}^{dch} + \sum_{k} b_{ik} \left(\delta_i - \delta_k\right)_t \tag{6.1.2}$$

$$p_{i,t}^{MIN} \leq \; p_{i,t} \leq \; p_{i,t}^{MAX} \tag{6.1.3}$$

$$b_{ik} \left(\delta_i - \delta_k\right)_t \leq \; p_{ik}^{MAX} \tag{6.1.4}$$

$$0 \leq \; p_{i,t}^{ch} \leq \; p_{i,t}^{ch,MAX} \tag{6.1.5}$$

$$p_{i,t}^{dch,MAX} \leq \; p_{i,t}^{dch} \leq 0 \tag{6.1.6}$$

$$E_{storage}^{MIN} \leq \; SOC_{i,t} \leq \; E_{storage}^{MAX} \tag{6.1.7}$$

$$SOC_{i,t} = SOC_{i,t-1} + \eta^{ch} p_{i,t}^{ch} \Delta \mathrm{t} + \frac{p_{i,t}^{dch}}{\eta^{dch}} \Delta \mathrm{t} \tag{6.1.8}$$

Where i and t denote the set of nodes and time periods. From the OPF formulation, the Lagrangian dual problem can be expressed using dual multipliers λ_i for each constraint. Since some variables are limited with lower and upper bounds, two dual variables have to be assigned.

$$
\begin{aligned}
p_{i,t} = p_{i,t}^{ch} + p_{i,t}^{dch} + \sum_{k} b_{ik}\left(\delta_i - \delta_k\right)_t \quad &\longleftrightarrow \quad \lambda_{i,t}^{(1)} \\
p_{i,t}^{MIN} \leq\ p_{i,t} \leq\ p_{i,t}^{MAX} \quad &\longleftrightarrow \quad \lambda_{i,t}^{(2L)}, \lambda_{i,t}^{(2U)} \\
b_{ik}\left(\delta_i - \delta_k\right)_t \leq\ p_{ik}^{MAX} \quad &\longleftrightarrow \quad \lambda_{ik,t}^{(3)} \\
0 \leq\ p_{i,t}^{ch} \leq\ p_{i,t}^{ch,MAX} \quad &\longleftrightarrow \quad \lambda_{i,t}^{(4L)}, \lambda_{i,t}^{(4)} \\
p_{i,t}^{dch,MAX} \leq\ p_{i,t}^{dch} \leq 0 \quad &\longleftrightarrow \quad \lambda_{i,t}^{(5)}, \lambda_{i,t}^{(5U)} \\
E_{storage}^{MIN} \leq\ SOC_{i,t} \leq\ E_{storage}^{MAX} \quad &\longleftrightarrow \quad \lambda_{i,t}^{(6L)}, \lambda_{i,t}^{(6)} \\
SOC_{i,t} = SOC_{i,t-1} + \eta^{ch} p_{i,t}^{ch} \Delta\mathrm{t} + \frac{p_{i,t}^{dch}}{\eta^{dch}} \Delta\mathrm{t} \quad &\longleftrightarrow \quad \lambda_{i,t}^{(7)} \\
SOC_{i,0} = 0 \quad &\longleftrightarrow \quad \lambda_{i,0}^{(7)}
\end{aligned}
$$

The Lagrangian for this optimization problem can be constructed from the objective function and the constraints:

$$
\begin{aligned}
\mathcal{L}(p_i, \delta_i, \delta_k, p_i^{ch}, p_i^{dch}, SOC_i, \lambda_i)_t = &\sum_{i,t} (f_{i,t}(p_{i,t})) \\
&+ \lambda_{i,t}^{(1)}(p_{i,t}^{ch} + p_{i,t}^{dch} + \sum_{k} (b_{ik}(\delta_{i,t} - \delta_{k,t})) - p_{i,t}) \\
&+ \lambda_{i,t}^{(2L)}(p_{i,t}^{MIN} - p_{i,t}) + \lambda_{i,t}^{(2)}(p_{i,t} - p_{i,t}^{MAX}) \\
&+ \lambda_{ik,t}^{(3)}(b_{ik}(\delta_{i,t} - \delta_{k,t}) - p_{ik}^{MAX}) \\
&+ \lambda_{i,t}^{(4L)}(-p_{i,t}^{ch}) + \lambda_{i,t}^{(4)}(p_{i,t}^{ch} - p_{i,t}^{ch,MAX}) \\
&+ \lambda_{i,t}^{(5)}(p_{i,t}^{dch,MAX} - p_{i,t}^{dch}) + \lambda_{i,t}^{(5U)}(p_{i,t}^{dch}) \\
&+ \lambda_{i,t}^{(6L)}(E_i^{MIN} - SOC_{i,t}) + \lambda_{i,t}^{(6)}(SOC_{i,t} - E_i^{MAX}) \\
&+ \lambda_{i,t}^{(7)}(SOC_{i,t-1} + \eta^{ch} p_{i,t}^{ch} \Delta t + \frac{p_{i,t}^{dch}}{\eta^{dch}} \Delta t - SOC_{i,t})
\end{aligned}
\tag{6.1.9}
$$

Setting the derivatives of the Lagrangian with respect to the primal variables to zero, we get an optimal solution:

$$
\frac{\partial l}{\partial p_{i,t}} = \frac{\partial OF}{\partial p_{i,t}} - \lambda_{i,t}^{(1)} - \lambda_{i,t}^{(2L)} + \lambda_{i,t}^{(2U)} = 0 \tag{6.1.10}
$$

$$
\frac{\partial l}{\partial \delta_{i,t}} = \lambda_{i,t}^{(1)} \sum_{k} (b_{ik}) + \lambda_{ik,t}^{(3)} \sum_{k} (b_{ik}) = 0 \tag{6.1.11}
$$

$$
\frac{\partial l_k}{\partial \delta_{k,t}} = \lambda_{k,t}^{(1)} \sum_{k} (b_{ik}) + \lambda_{ki,t}^{(3)} \sum_{k} (b_{ik}) = 0 \tag{6.1.12}
$$

$$\frac{\partial l}{\partial p_{i,t}^{ch}} = \lambda_{i,t}^{(1)} - \lambda_{i,t}^{(4L)} + \lambda_{i,t}^{(4)} + \eta^{ch}\lambda_{i,t}^{(7)} = 0 \tag{6.1.13}$$

$$\frac{\partial l}{\partial p_{i,t}^{dch}} = \lambda_{i,t}^{(1)} - \lambda_{i,t}^{(5)} + \lambda_{i,t}^{(5U)} + \frac{\lambda_{i,t}^{(7)}}{\eta^{dch}} = 0 \tag{6.1.14}$$

$$\frac{\partial l}{\partial SOC_{i,t}} = -\lambda_{i,t}^{(6L)} + \lambda_{i,t}^{(6)} + \lambda_{i,t}^{(7)} - \lambda_{i,t-1}^{(7)} = 0 \tag{6.1.15}$$

The solution satisfies also the complementary slackness:

$$\lambda_{ik,t}^{(3)}(b_{ik}(\delta_{i,t} - \delta_{k,t}) - p_{ik}^{MAX}) = 0 \tag{6.1.16}$$

$$\lambda_{i,t}^{(4)}(p_{i,t}^{ch} - p_{i,t}^{ch,MAX}) = 0 \tag{6.1.17}$$

$$\lambda_{i,t}^{(5)}(p_{i,t}^{dch,MAX} - p_{i,t}^{dch}) = 0 \tag{6.1.18}$$

$$\lambda_{i,t}^{(6)}(SOC_{i,t} - E_{i}^{MAX}) = 0 \tag{6.1.19}$$

Note that only the conditions related to the lines limits and storage energy and power constraints have been written down. Assuming that each inequality constraint is binding, it behaves like an equality constraint, and the corresponding dual variable is positive and equal to the shadow price of that constraint. Considering all the dual variables as non-zero might be physically unfeasible, but our purpose is to find an expression between the dual variables. In case some of the inequality constraints will not be binding in our simulations, we will consider the corresponding dual variable as zero, and it will not affect our calculation.
Furthermore, multiplying $\lambda_{i}^{(1)}$ and (6.1.2) the merchandising surplus is obtained, because it is like computing the net surplus between "injections of payments and withdrawals":

$$\sum_{i,t} \lambda_{i,t}^{(1)} (\underbrace{p_{i,t}^{ch} + p_{i,t}^{dch}}_{\text{Storage term}} + \underbrace{\sum_{k} b_{ik}\,(\delta_i - \delta_k)_t}_{\text{Flowgate term}} - \overbrace{p_{i,t}}^{\text{Surplus term}}) = 0 \tag{6.1.20}$$

From (6.1.11) (and similarly for (6.1.12)) follows that:

$$\frac{\partial l}{\partial \delta_{i,t}} = (\lambda_{i,t}^{(1)} + \lambda_{ik,t}^{(3)}) \sum_{k} (b_{ik}) = 0 \tag{6.1.21}$$

Consequently, from (6.1.21) and the complementary slackness condition (6.1.16) we set the power flowing through a generic line at its maximum capacity p_{ik}^{MAX}, and then we can therefore replace the Flowgate term:

$$\sum_{i,t} \lambda_{i,t}^{(1)} \sum_{k} b_{ik}\,(\delta_i - \delta_k)_t \longrightarrow -\sum_{ik,t} \lambda_{ik,t}^{(3)} p_{ik}^{MAX} \tag{6.1.22}$$

Since we are considering the possibility that storage has a congestion, we can assume the corresponding inequality constraint as binding. For this reason:

$$p_{i,t}^{ch} = p_{i,t}^{ch,MAX} \tag{6.1.23}$$

$$p_{i,t}^{dch} = p_{i,t}^{dch,MAX} \tag{6.1.24}$$

$$SOC_{i,t} = E_i^{MAX} \tag{6.1.25}$$

For our purposes, in (6.1.13), (6.1.14) and (6.1.15) we can consider only the lambdas relevant to our constraints, since in a double inequality only one of the limits can be binding:

$$\lambda_{i,t}^{(1)} - \cancel{\lambda_{i,t}^{(4L)}} + \lambda_{i,t}^{(4)} + \eta^{ch}\lambda_{i,t}^{(7)} = 0 \tag{6.1.26}$$

$$\lambda_{i,t}^{(1)} - \lambda_{i,t}^{(5)} + \cancel{\lambda_{i,t}^{(5U)}} + \frac{\lambda_{i,t}^{(7)}}{\eta^{dch}} = 0 \tag{6.1.27}$$

$$\cancel{-\lambda_{i,t}^{(6L)}} + \lambda_{i,t}^{(6)} + \lambda_{i,t}^{(7)} - \lambda_{i,t-1}^{(7)} = 0 \tag{6.1.28}$$

We can therefore replace the the storage term in (6.1.20):

$$\sum_{i,t} \lambda_{i,t}^{(1)}(p_{i,t}^{ch} + p_{i,t}^{dch}) = \sum_{i,t} \lambda_{i,t}^{(1)}(p_{i,t}^{ch}) + \sum_{i,t} \lambda_{i,t}^{(1)}(p_{i,t}^{dch}) \tag{6.1.29}$$

From (6.1.26) and (6.1.27):

$$\sum_{i,t} \lambda_{i,t}^{(1)}(p_{i,t}^{ch}) = \sum_{i,t}(-\lambda_{i,t}^{(4)} - \eta_{ch}\lambda_{i,t}^{(7)})(p_{i,t}^{ch}) \tag{6.1.30}$$

$$\sum_{i,t} \lambda_{i,t}^{(1)}(p_{i,t}^{dch}) = \sum_{i,t}(\lambda_{i,t}^{(5)} - \frac{\lambda_{i,t}^{(7)}}{\eta_{dch}})(p_{i,t}^{dch}) \tag{6.1.31}$$

And therefore:

$$\begin{aligned}\sum_{i,t} \lambda_{i,t}^{(1)}(p_{i,t}^{ch} + p_{i,t}^{dch}) &= \sum_{i,t}(-\lambda_{i,t}^{(4)} - \eta_{ch}\lambda_{i,t}^{(7)})(p_{i,t}^{ch}) + \sum_{i,t}(\lambda_{i,t}^{(5)} - \frac{\lambda_{i,t}^{(7)}}{\eta_{dch}})(p_{i,t}^{dch}) \\ &= \sum_{i,t}(-\lambda_{i,t}^{(4)}(p_{i,t}^{ch}) + \lambda_{i,t}^{(5)}(p_{i,t}^{dch})) - \sum_{i,t}(\eta_{ch}\lambda_{i,t}^{(7)}(p_{i,t}^{ch}) - \frac{\lambda_{i,t}^{(7)}}{\eta_{dch}}(p_{i,t}^{dch}))\end{aligned} \tag{6.1.32}$$

This last term is equal to:

$$-\sum_{i,t} \lambda^{(7)}_{i,t}(\eta_{ch} p^{ch}_{i,t} - \frac{p^{dch}_{i,t}}{\eta_{dch}}) \tag{6.1.33}$$

That can be substituted using (6.1.25) and (6.1.8) with:

$$-\sum_{i,t} \lambda^{(7)}_{i,t}(E^{MAX}_i - SOC_{i,t-1}) \tag{6.1.34}$$

From (6.1.28):

$$-\sum_{i,t}(\lambda^{(6)}_{i,t} - \lambda^{(7)}_{i,t-1})(E^{MAX}_i - SOC_{i,t-1}) \tag{6.1.35}$$

Writing down the calculation:

$$-\sum_{i,t}(-\lambda^{(7)}_{i,t-1}E^{MAX}_i + \lambda^{(7)}_{i,t-1}SOC_{i,t-1} + \lambda^{(6)}_{i,t}E^{MAX}_i + \lambda^{(6)}_{i,t}SOC_{i,t-1}) \tag{6.1.36}$$

Using the following assumption:

$$\frac{\lambda^{(7)}_{i,t}}{\lambda^{(7)}_{i,t-1}} = \frac{SOC_{i,t}}{SOC_{i,t-1}} \tag{6.1.37}$$

According to (6.1.28):

$$-\sum_{i,t}(\cancel{-\lambda^{(7)}_{i,t-1}E^{MAX}_i} + \cancel{\lambda^{(7)}_{i,t-1}SOC_{i,t-1}} + \lambda^{(6)}_{i,t}E^{MAX}_i + \cancel{\lambda^{(6)}_{i,t}SOC_{i,t-1}}) \tag{6.1.38}$$

And therefore:

$$-\sum_{i,t} \lambda^{(7)}_{i,t}(\eta_{ch} p^{ch}_{i,t} - \frac{p^{dch}_{i,t}}{\eta_{dch}}) = \lambda^{(6)}_{i,t}E^{MAX}_i \tag{6.1.39}$$

Consequently, from (6.1.32):

$$\sum_{i,t} \lambda^{(1)}_{i,t}(p^{ch}_{i,t} + p^{dch}_{i,t}) = \sum_{i,t}(-\lambda^{(4)}_{i,t}(p^{ch}_{i,t}) + \lambda^{(5)}_{i,t}(p^{dch}_{i,t}) - \lambda^{(6)}_{i,t}E^{MAX}_i) \tag{6.1.40}$$

And finally, from (6.1.22) and (6.1.40) we can replace the terms in (6.1.20):

$$\overbrace{-\sum_{i,t}\lambda_{i,t}^{(1)}(p_{i,t})}^{\text{SO's Budget Surplus}} - \underbrace{\sum_{i,t}\lambda_{ik,t}^{(3)}p_{ik}^{MAX}}_{\text{FTR}} + \sum_{i,t}(\underbrace{-\lambda_{i,t}^{(4)}p_{i,t}^{ch} + \lambda_{i,t}^{(5)}p_{i,t}^{dch}}_{\text{PCR}} - \underbrace{\lambda_{i,t}^{(6)}E_i^{MAX}}_{\text{ECR}}) = 0 \tag{6.1.41}$$

Where:

- FTR = Flowgate Transmission Rights
- PCR = Power Capacity Rights
- ECR = Energy Capacity Rights
- The sum of PCR and ECR is the definition of Financial Storage Rights.

6.2 AC-OPF

A similar approach can be used starting from the following mathematical model, that represents the Optimal Power Flow formulation in a generic transmission system with a full AC representation:

$$min \qquad \sum_{i,t} f_{i,t}(p_{i,t}) \tag{6.2.1}$$

$$s.t. \quad p_{i,t} = p_{i,t}^{ch} + p_{i,t}^{dch} + V_{i,t}\sum_k V_{k,t}[g_{ik}cos\,(\delta_i - \delta_k)_t + b_{ik}sin\,(\delta_i - \delta_k)_t] \tag{6.2.2}$$

$$q_{i,t} = V_i\sum_k V_k[g_{ik}sin\,(\delta_i - \delta_k)_t - b_{ik}cos\,(\delta_i - \delta_k)_t] \tag{6.2.3}$$

$$p_{i,t}^{MIN} \le\ p_{i,t} \le\ p_{i,t}^{MAX} \tag{6.2.4}$$

$$q_{i,t}^{MIN} \le\ q_{i,t} \le\ q_{i,t}^{MAX} \tag{6.2.5}$$

$$P_{ik}^2\ +\ Q_{ik}^2\ \le\ S_{ik,MAX}^2 \tag{6.2.6}$$

$$V_{i,t}^{MIN} \le\ V_{i,t} \le\ V_{i,t}^{MAX} \tag{6.2.7}$$

$$\delta_{i,t}^{MIN} \le\ \delta_{i,t} \le\ \delta_{i,t}^{MAX} \tag{6.2.8}$$

$$0 \le\ p_{i,t}^{ch} \le\ p_{i,t}^{ch,MAX} \tag{6.2.9}$$

$$p_{i,t}^{dch,MAX} \le\ p_{i,t}^{dch} \le 0 \tag{6.2.10}$$

$$E_{storage}^{MIN} \le\ SOC_{i,t} \le\ E_{storage}^{MAX} \tag{6.2.11}$$

$$SOC_{i,t} = SOC_{i,t-1} + \eta^{ch}p_{i,t}^{ch}\Delta\mathrm{t} + \frac{p_{i,t}^{dch}}{\eta^{dch}}\Delta\mathrm{t} \tag{6.2.12}$$

Where:

$$P_{ik} = V_i^2(g_{ik}) - g_{ik}|V_i||V_k|\,cos(\delta_i - \delta_k) - b_{ik}\,|V_i||V_k|\,sin(\delta_i - \delta_k)] \tag{6.2.13}$$

$$Q_{ik} = -V_i^2\Big(\frac{b_{S,ik}}{2} + b_{ik}\Big) - g_{ik}\,|V_i||V_k|\,sin(\delta_i - \delta_k)] + b_{ik}|V_i||V_k|\,cos(\delta_i - \delta_k) \tag{6.2.14}$$

However, since the consideration of a quadratic constraint for the maximum power brings to a complex structure of the financial storage rights formulation, we will consider the maximum power as a maximum active power. A margin can be taken from that value in order to compensate possible deviations due to the reactive power flow through the line. Therefore:

$$g_{ik}V_{i,t}^2 - V_{i,t}V_{k,t}[g_{ik}cos\,(\delta_i - \delta_k)_t + b_{ik}sin\,(\delta_i - \delta_k)_t] \leq \; p_{ik}^{MAX} \tag{6.2.15}$$

From the OPF formulation, the Lagrangian dual problem can be expressed using dual multipliers λ_i for each constraint:

$$\begin{aligned}
p_{i,t} = p_{i,t}^{ch} + p_{i,t}^{dch} + V_{i,t}\sum_k V_{k,t}[g_{ik}cos\,(\delta_i - \delta_k)_t + b_{ik}sin\,(\delta_i - \delta_k)_t] &\quad\longleftrightarrow\quad \lambda_{i,t}^{(1)}\\
q_{i,t} = V_i\sum_k V_k[g_{ik}sin\,(\delta_i - \delta_k)_t - b_{ik}cos\,(\delta_i - \delta_k)_t] &\quad\longleftrightarrow\quad \lambda_{i,t}^{(8)}\\
p_{i,t}^{MIN} \leq\; p_{i,t} \leq\; p_{i,t}^{MAX} &\quad\longleftrightarrow\quad \lambda_{i,t}^{(2L)}, \lambda_{i,t}^{(2U)}\\
q_{i,t}^{MIN} \leq\; q_{i,t} \leq\; q_{i,t}^{MAX} &\quad\longleftrightarrow\quad \lambda_{i,t}^{(9L)}, \lambda_{i,t}^{(9U)}\\
V_{i,t}^{MIN} \leq\; V_{i,t} \leq\; V_{i,t}^{MAX} &\quad\longleftrightarrow\quad \lambda_{i,t}^{(10L)}, \lambda_{i,t}^{(10U)}\\
\delta_{i,t}^{MIN} \leq\; \delta_{i,t} \leq\; \delta_{i,t}^{MAX} &\quad\longleftrightarrow\quad \lambda_{i,t}^{(11L)}, \lambda_{i,t}^{(11U)}\\
g_{ik}V_{i,t}^2 - V_{i,t}V_{k,t}[g_{ik}cos\,(\delta_i - \delta_k)_t + b_{ik}sin\,(\delta_i - \delta_k)_t] \leq\; p_{ik}^{MAX} &\quad\longleftrightarrow\quad \lambda_{ik,t}^{(3)}\\
0 \leq\; p_{i,t}^{ch} \leq\; p_{i,t}^{ch,MAX} &\quad\longleftrightarrow\quad \lambda_{i,t}^{(4L)}, \lambda_{i,t}^{(4)}\\
p_{i,t}^{dch,MAX} \leq\; p_{i,t}^{dch} \leq 0 &\quad\longleftrightarrow\quad \lambda_{i,t}^{(5)}, \lambda_{i,t}^{(5U)}\\
E_{storage}^{MIN} \leq\; SOC_{i,t} \leq\; E_{storage}^{MAX} &\quad\longleftrightarrow\quad \lambda_{i,t}^{(6L)}, \lambda_{i,t}^{(6)}\\
SOC_{i,t} = SOC_{i,t-1} + \eta^{ch}p_{i,t}^{ch}\Delta\mathrm{t} + \frac{p_{i,t}^{dch}}{\eta^{dch}}\Delta\mathrm{t} &\quad\longleftrightarrow\quad \lambda_{i,t}^{(7)}\\
SOC_{i,0} = 0 &\quad\longleftrightarrow\quad \lambda_{i,0}^{(7)}
\end{aligned}$$

The Lagrangian for this optimization problem can be constructed from the objective function and the constraints:

$$
\begin{aligned}
l(p_i, \delta_i, \delta_k, p_i^{ch}, p_i^{dch}, SOC_i, q_i, V_i, \lambda_i)_t = \sum_{i,t}(f_{i,t}(p_{i,t})) \\
+ \lambda_{i,t}^{(1)}(p_{i,t}^{ch} + p_{i,t}^{dch} + V_{i,t}\sum_k V_{k,t}[g_{ik}cos\,(\delta_i - \delta_k)_t + b_{ik}sin\,(\delta_i - \delta_k)_t] - p_{i,t}) \\
+ \lambda_{i,t}^{(2L)}(p_{i,t}^{MIN} - p_{i,t}) + \lambda_{i,t}^{(2U)}(p_{i,t} - p_{i,t}^{MAX}) \\
+ \lambda_{ik,t}^{(3)}(g_{ik}V_{i,t}^2 - V_{i,t}V_{k,t}[g_{ik}cos\,(\delta_i - \delta_k)_t + b_{ik}sin\,(\delta_i - \delta_k)_t] - p_{ik}^{MAX}) \\
+ \lambda_{i,t}^{(4L)}(-p_{i,t}^{ch}) + \lambda_{i,t}^{(4)}(p_{i,t}^{ch} - p_{i,t}^{ch,MAX}) \\
+ \lambda_{i,t}^{(5)}(p_{i,t}^{dch,MAX} - p_{i,t}^{dch}) + \lambda_{i,t}^{(5U)}(p_{i,t}^{dch}) \\
+ \lambda_{i,t}^{(6L)}(E_i^{MIN} - SOC_{i,t}) + \lambda_{i,t}^{(6)}(SOC_{i,t} - E_i^{MAX}) \\
+ \lambda_{i,t}^{(7)}(SOC_{i,t} + \eta^{ch}p_{i,t}^{ch}\Delta t + \frac{p_{i,t}^{dch}}{\eta^{dch}}\Delta t - SOC_{i,t+1}) \\
+ \lambda_{i,t}^{(8)}(V_i\sum_k V_k[g_{ik}sin\,(\delta_i - \delta_k)_t - b_{ik}cos\,(\delta_i - \delta_k)_t] - q_{i,t}) \\
+ \lambda_{i,t}^{(9L)}(q_{i,t}^{MIN} - q_{i,t}) + \lambda_{i,t}^{(9U)}(q_{i,t} - q_{i,t}^{MAX}) \\
+ \lambda_{i,t}^{(10L)}(V_{i,t}^{MIN} - V_{i,t}) + \lambda_{i,t}^{(10U)}(V_{i,t} - V_{i,t}^{MAX}) \\
+ \lambda_{i,t}^{(11L)}(\delta_{i,t}^{MIN} - \delta_{i,t}) + \lambda_{i,t}^{(11U)}(\delta_{i,t} - \delta_{i,t}^{MAX})
\end{aligned}
\tag{6.2.16}
$$

Setting the derivatives of the Lagrangian with respect to the primal variables to zero, we get an optimal solution:

$$
\frac{\partial l}{\partial p_{i,t}} = \frac{\partial OF}{\partial p_{i,t}} - \lambda_{i,t}^{(1)} - \lambda_{i,t}^{(2L)} + \lambda_{i,t}^{(2U)} = 0 \tag{6.2.17}
$$

$$
\begin{aligned}
\frac{\partial l}{\partial \delta_{i,t}} = \lambda_{i,t}^{(1)}(V_{i,t}\sum_k V_{k,t}[-g_{ik}sin\,(\delta_i - \delta_k)_t + b_{ik}cos\,(\delta_i - \delta_k)_t]) \\
+ \lambda_{ik,t}^{(3)}(V_{i,t}V_{k,t}[g_{ik}sin\,(\delta_i - \delta_k)_t - b_{ik}cos\,(\delta_i - \delta_k)_t]) \\
+ \lambda_{ik,t}^{(8)}(V_{i,t}\sum_k V_{k,t}[g_{ik}cos\,(\delta_i - \delta_k)_t + b_{ik}sin\,(\delta_i - \delta_k)_t]) \\
- \lambda_{i,t}^{(11L)} + \lambda_{i,t}^{(11U)} = 0
\end{aligned}
\tag{6.2.18}
$$

$$\begin{aligned}\frac{\partial l}{\partial \delta_{k,t}} = \lambda_{k,t}^{(1)}(V_{k,t}\sum_i V_{i,t}[g_{ik}sin\left(\delta_k-\delta_i\right)_t - b_{ik}cos\left(\delta_k-\delta_i\right)_t]) \\ + \lambda_{ki,t}^{(3)}(V_{i,t}V_{k,t}[-g_{ik}sin\left(\delta_k-\delta_i\right)_t + b_{ik}cos\left(\delta_k-\delta_i\right)_t]) \\ + \lambda_{k,t}^{(8)}(V_{k,t}\sum_i V_{i,t}[-g_{ik}cos\left(\delta_k-\delta_i\right)_t - b_{ik}sin\left(\delta_k-\delta_i\right)_t]) \\ - \lambda_{k,t}^{(11L)} + \lambda_{k,t}^{(11U)} = 0\end{aligned} \quad (6.2.19)$$

$$\frac{\partial l}{\partial p_{i,t}^{ch}} = \lambda_{i,t}^{(1)} - \lambda_{i,t}^{(4L)} + \lambda_{i,t}^{(4)} + \eta^{ch}\lambda_{i,t}^{(7)} = 0 \quad (6.2.20)$$

$$\frac{\partial l}{\partial p_{i,t}^{dch}} = \lambda_{i,t}^{(1)} - \lambda_{i,t}^{(5)} + \lambda_{i,t}^{(5U)} + \frac{\lambda_{i,t}^{(7)}}{\eta^{dch}} = 0 \quad (6.2.21)$$

$$\frac{\partial l}{\partial SOC_{i,t}} = -\lambda_{i,t}^{(6L)} + \lambda_{i,t}^{(6)} + \lambda_{i,t}^{(7)} - \lambda_{i,t-1}^{(7)} = 0 \quad (6.2.22)$$

The solution satisfies also the complementary slackness conditions:

$$\lambda_{ik,t}^{(3)}(g_{ik}V_{i,t}^2 - V_{i,t}V_{k,t}[g_{ik}cos\left(\delta_i-\delta_k\right)_t + b_{ik}sin\left(\delta_i-\delta_k\right)_t] - p_{ik}^{MAX}) = 0 \quad (6.2.23)$$

$$\lambda_{i,t}^{(4)}(p_{i,t}^{ch} - p_{i,t}^{ch,MAX}) = 0 \quad (6.2.24)$$

$$\lambda_{i,t}^{(5)}(p_{i,t}^{dch,MAX} - p_{i,t}^{dch}) = 0 \quad (6.2.25)$$

$$\lambda_{i,t}^{(6)}(SOC_{i,t} - E_i^{MAX}) = 0 \quad (6.2.26)$$

Note that only the conditions related to the lines limits and storage energy and power constraints have been written down. Assuming that each inequality constraint is binding, it behaves like an equality constraint, and the corresponding dual variable is positive and equal to the shadow price of that constraint. Equations (6.2.20) to (6.2.22) and (6.2.24) to (6.2.26) did not change from the previous DC approximation, since the storage constraints are not influenced by the linearized approximation. Multiplying $\lambda_i^{(1)}$ and (6.2.2) gives the merchandising surplus:

$$\sum_{i,t}\lambda_{i,t}^{(1)}(\underbrace{p_{i,t}^{ch}+p_{i,t}^{dch}}_{\text{Storage term}} + \underbrace{V_{i,t}\sum_k V_{k,t}[g_{ik}cos\left(\delta_i-\delta_k\right)_t + b_{ik}sin\left(\delta_i-\delta_k\right)_t]}_{\text{Flowgate term}} - \overbrace{p_{i,t}}^{\text{Surplus term}}) = 0 \quad (6.2.27)$$

It is reasonable that the flowgate term has now a different value, since we are considering effects that have been neglected before. In particular, $\lambda_{i,t}^{(11L)}$ and $\lambda_{i,t}^{(11U)}$ are terms referred to voltage angles, that are limited due to stability reasons. However, because we are considering in our approach the

flowgate to be limited according to the thermal limits of the lines, we can find a similar expression for FTRs and FSRs considering the operation of the system far from these two stability limits.
Also, the following terms:

$$(V_{i,t}\sum_{k} V_{k,t}[g_{ik}cos\,(\delta_i-\delta_k)_t + b_{ik}sin\,(\delta_i-\delta_k)_t])(\lambda^{(8)}_{i,t}) \tag{6.2.28}$$

$$(V_{k,t}\sum_{i} V_{i,t}[g_{ik}cos\,(\delta_k-\delta_i)_t + b_{ik}sin\,(\delta_k-\delta_i)_t])(\lambda^{(8)}_{k,t}) \tag{6.2.29}$$

can be considered as negligible. Indeed, $\lambda^{(8)}_{i,t}$ is the term reflecting the dual variable of the net reactive power balance at each bus. The active power counterpart is described by $\lambda^{(1)}_{i,t}$ that reflects the costs of the active energy and re-dispatch (as already discussed in chapter 3). Simulations show results for $\lambda^{(8)}_{i,t}$ very close to zero. This term can be considered as reflecting the costs for additional reactive power requested to the system. However, we are not considering any reactive power costs because the transmission grid is not built for providing reactive power to the consumers. We will therefore neglect dual variables associated to the reactive power balance $\lambda^{(8)}$, as well as the stability-associated dual variables $\lambda^{(11L)}$ and $\lambda^{(11U)}$:

$$\begin{aligned}\frac{\partial l}{\partial \delta_{i,t}} = \lambda^{(1)}_{i,t}(V_{i,t}\sum_{k} V_{k,t}[-g_{ik}sin\,(\delta_i-\delta_k)_t + b_{ik}cos\,(\delta_i-\delta_k)_t]) \\ + \lambda^{(3)}_{ik,t}(V_{i,t}V_{k,t}[g_{ik}sin\,(\delta_i-\delta_k)_t - b_{ik}cos\,(\delta_i-\delta_k)_t]) = 0\end{aligned} \tag{6.2.30}$$

$$\begin{aligned}\frac{\partial l}{\partial \delta_{k,t}} = \lambda^{(1)}_{k,t}(V_{k,t}\sum_{i} V_{i,t}[g_{ik}sin\,(\delta_i-\delta_k)_t - b_{ik}cos\,(\delta_i-\delta_k)_t]) \\ + \lambda^{(3)}_{ki,t}(V_{i,t}V_{k,t}[-g_{ik}sin\,(\delta_i-\delta_k)_t + b_{ik}cos\,(\delta_i-\delta_k)_t]) = 0\end{aligned}$$

Consequently, from the complementary slackness condition (6.2.23) we set the power flowing through a generic line at its maximum capacity p^{MAX}_{ij}:

$$g_{ik}V^2_{i,t} - V_{i,t}V_{k,t}[g_{ik}cos\,(\delta_i-\delta_k)_t + b_{ik}sin\,(\delta_i-\delta_k)_t] = p^{MAX}_{ik} \tag{6.2.31}$$

and then we can therefore replace the Flowgate term using equation (6.2.30):

$$\sum_{i,t}\lambda^{(1)}_{i,t}\sum_{k}(V_{i,t}V_{k,t}[g_{ik}cos\,(\delta_i-\delta_k)_t + b_{ik}sin\,(\delta_i-\delta_k)_t]) \tag{6.2.32}$$

with:

$$\sum_{i,t} \lambda^{(3)}_{ik,t} p^{MAX}_{ij} - \sum_{i,t} \lambda^{(1)}_{i,t} g_{ik} V^2_{i,t} \tag{6.2.33}$$

It is clear how the solution is different from the DC case, since nodal prices have a losses-related component and cannot bring to the same results as in that case. Furthermore, the final expression will have a FTR component, but also a loss coefficient $\lambda^{(1)}_{i,t} g_{ik} V^2_{i,t}$.
Since we are considering the possibility that storage has a congestion, we can assume the corresponding inequality constraint as binding, for this reason:

$$p^{ch}_{i,t} = p^{ch,MAX}_{i,t} \tag{6.2.34}$$

$$p^{dch}_{i,t} = p^{dch,MAX}_{i,t} \tag{6.2.35}$$

$$SOC_{i,t} = E^{MAX}_{i} \tag{6.2.36}$$

For our purposes, in (6.2.20), (6.2.21) and (6.2.22) we can consider only the lambdas relevant to our constraints, since in a double inequality only one of the limits is binding:

$$\lambda^{(1)}_{i,t} - \cancel{\lambda^{(4L)}_{i,t}} + \lambda^{(4)}_{i,t} + \eta^{ch} \lambda^{(7)}_{i,t} = 0 \tag{6.2.37}$$

$$\lambda^{(1)}_{i,t} - \lambda^{(5)}_{i,t} + \cancel{\lambda^{(5U)}_{i,t}} + \frac{\lambda^{(7)}_{i,t}}{\eta^{dch}} = 0 \tag{6.2.38}$$

$$\cancel{-\lambda^{(6L)}_{i,t}} + \lambda^{(6)}_{i,t} + \lambda^{(7)}_{i,t} - \lambda^{(7)}_{i,t-1} = 0 \tag{6.2.39}$$

We can therefore replace the the storage term in (6.2.27):

$$\sum_{i,t} \lambda^{(1)}_{i,t} (p^{ch}_{i,t} + p^{dch}_{i,t}) = \sum_{i,t} \lambda^{(1)}_{i,t} (p^{ch}_{i,t}) + \sum_{i,t} \lambda^{(1)}_{i,t} (p^{dch}_{i,t}) \tag{6.2.40}$$

From (6.2.37) and (6.2.38):

$$\sum_{i,t} \lambda^{(1)}_{i,t} (p^{ch}_{i,t}) = \sum_{i,t} (-\lambda^{(4)}_{i,t} - \eta_{ch} \lambda^{(7)}_{i,t})(p^{ch}_{i,t}) \tag{6.2.41}$$

$$\sum_{i,t} \lambda^{(1)}_{i,t} (p^{dch}_{i,t}) = \sum_{i,t} (\lambda^{(5)}_{i,t} - \frac{\lambda^{(7)}_{i,t}}{\eta_{dch}})(p^{dch}_{i,t}) \tag{6.2.42}$$

And therefore:

$$\sum_{i,t} \lambda^{(1)}_{i,t}(p^{ch}_{i,t} + p^{dch}_{i,t}) = \sum_{i,t}(-\lambda^{(4)}_{i,t} - \eta_{ch}\lambda^{(7)}_{i,t})(p^{ch}_{i,t}) + \sum_{i,t}(\lambda^{(5)}_{i,t} - \frac{\lambda^{(7)}_{i,t}}{\eta_{dch}})(p^{dch}_{i,t})$$
$$= \sum_{i,t}(-\lambda^{(4)}_{i,t}(p^{ch}_{i,t}) + \lambda^{(5)}_{i,t}(p^{dch}_{i,t})) - \sum_{i,t}(\eta_{ch}\lambda^{(7)}_{i,t}(p^{ch}_{i,t}) - \frac{\lambda^{(7)}_{i,t}}{\eta_{dch}}(p^{dch}_{i,t})) \tag{6.2.43}$$

This last term is equal to:

$$-\sum_{i,t} \lambda^{(7)}_{i,t}(\eta_{ch}p^{ch}_{i,t} - \frac{p^{dch}_{i,t}}{\eta_{dch}}) \tag{6.2.44}$$

From (6.2.36) and (6.2.12):

$$-\sum_{i,t} \lambda^{(7)}_{i,t}(E^{MAX}_i - SOC_{i,t-1}) \tag{6.2.45}$$

From (6.2.39):

$$-\sum_{i,t}(\lambda^{(6)}_{i,t} - \lambda^{(7)}_{i,t-1})(E^{MAX}_i - SOC_{i,t-1}) \tag{6.2.46}$$

Writing down the calculation:

$$-\sum_{i,t}(-\lambda^{(7)}_{i,t-1}E^{MAX}_i + \lambda^{(7)}_{i,t-1}SOC_{i,t-1} + \lambda^{(6)}_{i,t}E^{MAX}_i + \lambda^{(6)}_{i,t}SOC_{i,t-1}) \tag{6.2.47}$$

Using the following assumption:

$$\frac{\lambda^{(7)}_{i,t}}{\lambda^{(7)}_{i,t-1}} = \frac{SOC_{i,t}}{SOC_{i,t-1}} \tag{6.2.48}$$

We can neglect the following terms, according to (6.2.39):

$$-\sum_{i,t}(\cancel{-\lambda^{(7)}_{i,t-1}E^{MAX}_i} + \cancel{\lambda^{(7)}_{i,t-1}SOC_{i,t-1}} + \lambda^{(6)}_{i,t}E^{MAX}_i + \cancel{\lambda^{(6)}_{i,t}SOC_{i,t-1}}) \tag{6.2.49}$$

Consequently:

$$-\sum_{i,t} \lambda^{(7)}_{i,t}(\eta_{ch}p^{ch}_{i,t} - \frac{p^{dch}_{i,t}}{\eta_{dch}}) = \lambda^{(6)}_{i,t}E^{MAX}_i \tag{6.2.50}$$

Then, from (6.2.43):

$$\sum_{i,t} \lambda_{i,t}^{(1)} (p_{i,t}^{ch} + p_{i,t}^{dch}) = \sum_{i,t} (-\lambda_{i,t}^{(4)} (p_{i,t}^{ch}) + \lambda_{i,t}^{(5)} (p_{i,t}^{dch}) - \lambda_{i,t}^{(6)} E_i^{MAX}) \qquad (6.2.51)$$

And finally, from (6.2.33) and (6.2.51) we can replace the terms in (6.2.27):

$$\overbrace{-\sum_{i,t} \lambda_{i,t}^{(1)} (p_{i,t})}^{\text{SO's Budget Surplus}} - \underbrace{\sum_{i,t} \lambda_{ik,t}^{(3)} p_{ij}^{MAX}}_{\text{FTR NEW}} + \sum_{i,t} \underbrace{(-\lambda_{i,t}^{(4)} p_{i,t}^{ch} + \lambda_{i,t}^{(5)} p_{i,t}^{dch}}_{\text{PCR}} - \underbrace{\lambda_{i,t}^{(6)} E_i^{MAX}}_{\text{ECR}}) + \overbrace{\sum_{i,t} \lambda_{i,t}^{(1)} g_{ik} V_{i,t}^2}^{\text{LOSS}} = 0 \qquad (6.2.52)$$

Where:

- FTR NEW = Flowgate Transmission Rights (neglecting $\lambda_{i,t}^{(11L)}$ and $\lambda_{i,t}^{(11U)}$ and assuming $\lambda_{i,t}^{(8)} = \lambda_{k,t}^{(8)}$)
- PCR = Power Capacity Rights
- ECR = Energy Capacity Rights
- The sum of PCR and ECR is the definition of Financial Storage Rights.
- **LOSS** is the component that brings to a LMP difference due to lines losses.

It is highly important to mention that the quantities p_{ik}^{MAX}, $p_{i,t}^{ch}$, $p_{i,t}^{ch}$ and E_i^{MAX} are the quantities that bring to the whole financial rights value. That value has to be redistributed to multiple participants, since they are allowed to buy a percentage of the financial right (transmission or storage).
In this model we made the assumption that voltage angles are within their limits. If we want to consider stability limits what is going to change in the previous expression is the flowgate term. Storage-related terms were introduced in the same way and bring to the same results.

Chapter 7

Simulations

7.1 General Considerations

For the purpose of testing and simulation, the software GAMS (General Algebraic Modeling System) have been used and different algorithms have been chosen from GAMS solvers database. The two different approaches chosen are relative to a DC approximation of the optimal power flow formulation, and the AC-OPF.
The main problem related to AC formulations is the computational complexity that, in real systems, can be a key factor because of the size of power systems. Therefore, DC approximations are widely used both in power system analysis and in market-related simulations.
For the implementation of the codes, it has been considered that:

- Grid data, as well as load and generation characteristics, are loaded into GAMS from an external Excel file. In this way, different grids can be studied without changing the code.
- The sets are defined for: nodes, branches, time periods, PV nodes and slack bus, nodes with batteries.
- Tap changing transformers and phase shifting transformers have not been considered in our model, because they can increase heavily the complexity of the system, see [18] for their integration in a OPF model.
- The decision variables and parameters depend on the model. In general, the unknown variables depend on the choice of each bus, as stated in chapter 3.
- Ramp rates have also been neglected, but they can be easily incorporated ensuring that, e.g., $P_{G,t+1} - P_{G,t} \leq P^{max}_{ramp_{up}}$.
- Network characteristics, load data and generator limits have been taken from a 5-bus example. Results for the AC-OPF are very close to the

original model presented in [18], except for very small numerical errors. Also, lines limits are introduced for giving a more realistic study of the grids, as well as a computation of financial transmission rights.

– the solvers used for the linear cases is CPLEX, and for NLP problems is CONOPT. Most of the simulations were non-linear even for the DC-approximation, because of the non-linear structure of the objective function.

The model starts with the definition of the physical characteristics of the grid in terms of resistances and reactances. It is also possible to consider parallel admittance both for lines features that in terms of shunt admittances. After building the bus susceptance matrix and in the case of AC-OPF also the bus conductance matrix, the bus admittance matrix can be found. The admittance angle matrix have not been computed, because of the choice of rectangular coordinates G_{ik} and B_{ik}. Given the resistance and reactance, the series conductance and susceptance can be found through:

$$G = \Re\{\overline{Y}\} = \frac{R}{R^2 + X^2} \tag{7.1.1}$$

$$B = \Im\{\overline{Y}\} = -\frac{X}{R^2 + X^2} \tag{7.1.2}$$

Then, the conductance and susceptance matrices are found using the rules explained in chapter 3 for diagonal and off-diagonal entries.
For having a specific overview of the storage advantages, the State of Charge at the beginning and at the end of the simulations are set to be equal, $SOC_{i,t_0} = SOC_{i,t_{final}}$.
All quantities, unless otherwise specified, are expressed in **per unit**.

7.2 DC - AC Results Comparison

In the following tables all the input data for the simple test case can be found, they are taken from an analogous example in [18]. For this simple case, congestion is not considered, but can simply be valued setting the capacity of some lines to 0.6 p.u. or less (or increasing the load).

Table (7.1) shows generating limits both for active and reactive power. 'Linear' and 'quadratic' are the coefficient that multiply the power and the squared power, considering a classical quadratic cost function. Table (7.2) shows the voltage limits for each bus, considering the slack bus as the connection to

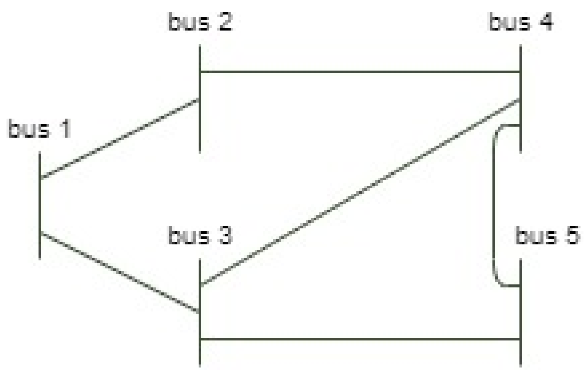


Figure 7.1 5 bus system, single line diagram.

	P_G,min	P_G,max	Q_G,min	Q_G,max	linear	quadratic
bus1	-20	20	-2	2	0.35	
bus2						
bus3	0.1	0.4	-0.2	0.3	0.2	0.4
bus4	0.05	0.4	-0.2	0.2	0.3	0.5
bus5						

Table 7.1 Generators data [18].

	V_min	V_max	g_shunt	b_shunt	P_Load	Q_Load
bus1	1	1				
bus2	0.95	1.05		0.3		
bus3	0.95	1.05	0.05			
bus4	0.95	1.05			0.9	0.4
bus5	0.95	1.05			0.239	0.129

Table 7.2 Bus and Load data [18].

		R	X	gS2	bS2	Limit
bus1	bus2		0.3			1
bus1	bus3	0.023	0.145		0.04	1
bus2	bus4	0.006	0.032		0.01	1
bus3	bus4	0.02	0.26			1
bus3	bus5		0.32			1
bus4	bus5		0.5			1

Table 7.3 Lines data [18].

the 'main grid'. The other columns represent the shunt elements and load data.
Table (7.3) lists the grid data in terms of resistances and reactances. Here the shunt elements represent the physical behaviour of the line. 'Limit' is the maximum power that can flow complying with the thermal limits of each line. The DC approximation neglects the reactive power flows, resistances and voltage limits. Therefore, the corresponding columns are not introduced in the model.
Results show a good approximation in terms of phase angles, and the voltage value is not too far between the two cases (about 5-6%). When computing active power, differences can be quite high because they are computed using voltage values, trigonometric functions and comprise the resistive part. Therefore, relative deviations on active power is more significant, while the reactive power has a maximum error, since it is neglected in the DC approximation, see Table (7.6).

	V [p.u.]		Angle [degree]	
	DC	AC	DC	AC
bus1	1	1	0	0
bus2	1	0.99	-8.077537	-8.435
bus3	1	0.9795	-3.585418	-3.803835
bus4	1	0.9776	-8.939141	-9.246847
bus5	1	0.95	-8.346616	-8.802888

Table 7.4 DC-OPF/AC-OPF results comparison: Voltages and phase angles.

	P [p.u.]		P_G [p.u.]		Q [p.u.]		Q_G [p.u.]	
	DC	AC	DC	AC	DC	AC	DC	AC
bus1	0.9015	0.94541	0.901	0.945	0	0.131914	0	0.132
bus2	0	0	0	0	0	0	0	0
bus3	0.1875	0.19421	0.187	0.194	0	0.02777	0	0.028
bus4	-0.85	-0.84309	0.05	0.057	0	-0.2	0	0.2
bus5	-0.239	-0.239	0	0	0	-0.129	0	0

Table 7.5 DC-OPF/AC-OPF results comparison: Generated and net active and reactive powers.

It can be relevant now to consider an increasing load framework, because we know that DC approximations suffer inaccuracy when the system is stressed and electrical quantities are closer to their operating limits. This can happen in real systems, and is really relevant when computing financial

	Relative Error [%]
Voltages	up to 5.26%
Voltage Angles	up to 5.74%
Active Power	up to 4.64%
Generated Active Power	up to 12.28%
Reactive Power	maximum
Generated Reactive Power	maximum

Table 7.6 Relative error between DC-OPF/AC-OPF.

transmission rights, since they take value only in the case of transmission congestion.
The same models have been investigated with an increase of load (+40%), and a multi-period increasing load (up to 100%). While inaccuracies relative to voltages and phase angles might be acceptable (up to 6-7%), generated and net active power errors increase more significantly. Having considered relative errors, it results that phase angles are even more accurate when the system is highly loaded (but still not congested).
Finally, when the load is increased too much, it has been seen that the DC-OPF is still feasible in this 5 bus example, while the AC-OPF is unfeasible using GAMS.

In figure (7.2), an incremental load has been applied in a 10 time steps framework. The reactive power has been chosen as decreasing, to accommodate the increase in real power. In the last time step, the power at bus 3 hits the operating limits of its generating unit, resulting in a zero error, while the power generated at bus 4 has a high error ($\sim 30\%$). From t7 onwards, congestions on the lines play a key role in the results accuracy.

It is more important for our purposes considering the errors in the nodal prices. The computation of marginal values in general has been done using the MARGINAL feature of GAMS. In order to find the nodal prices (or shadow prices), we must calculate the marginal of the nodal balance equation at each node. Figure (7.3) shows how in percentage these nodal prices differ when comparing the DC approximation with full AC-OPF.
LMPs, as expected, have a small deviation in low-load conditions. This is caused by the LMP factor due to losses, that is small if compared to the LMP factor due to congestions (see equation (3.5.1)). As the load increases leading to congestions in the grid, the LMP increases in value and lacks in accuracy. LMP at bus 1 has no error since the slack bus is the cheapest generating supply, and an increase in load would not lead to a re-dispatch calculation.
Locational marginal prices, as well as other marginal values are used for calculating financial transmission rights and financial storage rights (see equation (6.2.52)). Marginal values computation is often realized using DC-OPF

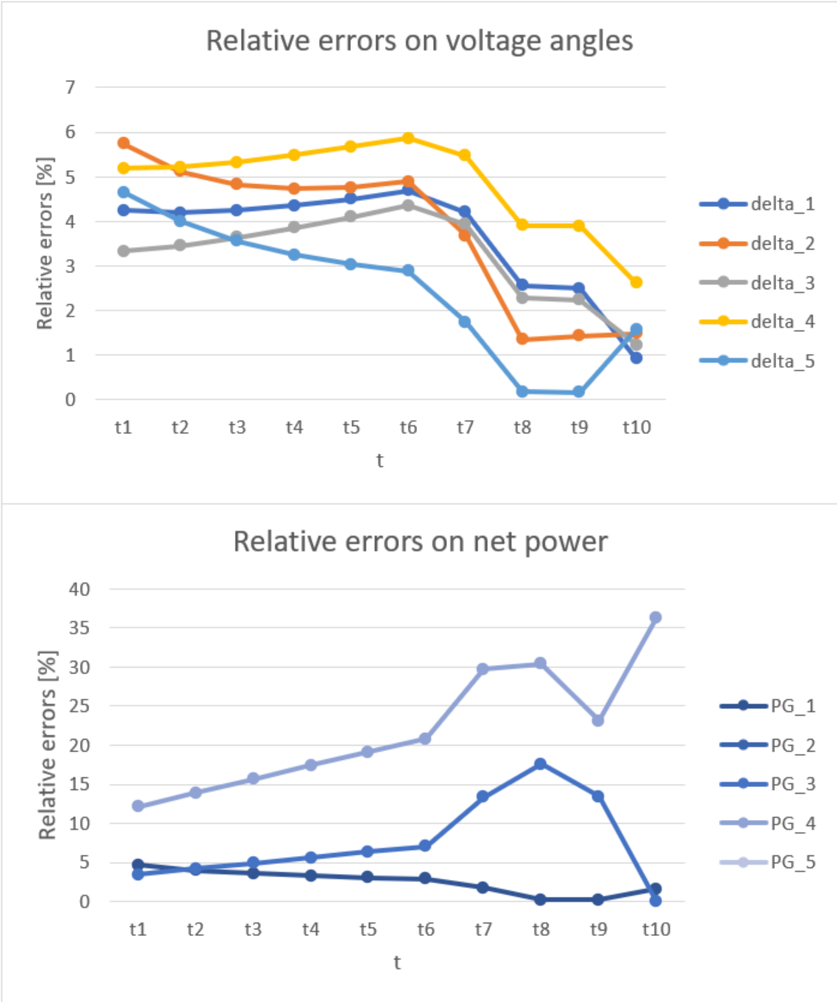


Figure 7.2 Relative errors on phase angles and net active powers, 10 time periods.

in real cases, and this might bring to non-negligible errors on LMPs calculation [74].
Computational complexity might still be a key factor when ignoring the use of AC-OPF for certain purposes. Therefore, it can be good to choose a trade-off between the two models to get closer in terms of results accuracy. This can be done by including real behaviours of variables into the DC-OPF or via relaxation methods of the AC-OPF.
Several ways have been introduced to improve the accuracy of the DC approximation. [75] proposes a loss compensation method and an α-matching DC-PF formulation. [27] assesses how line losses, the reactive power inclusion in the apparent power and phase angle consideration improves the accuracy, while [76] focuses on the network capabilities and introduces a reactive power flow integration into the DC-OPF.
One simple known way to integrate losses in the DC formulation is replac-

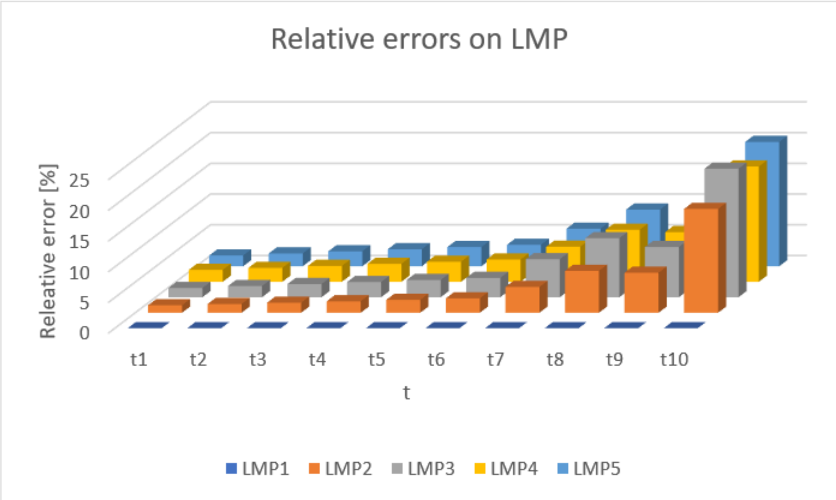


Figure 7.3 Relative errors on Locational Marginal Prices (LMPs).

ing the power flow balance equation [26] [77] with an equation considering a power losses approximation. The power flow in a line from nodes i to k can be computed as:

$$P_{ij} = V_i^2(G_{ij}) - G_{ij}|V_i||V_j|\,cos(\delta_i - \delta_j) - B_{ij}\,|V_i||V_j|\,sin(\delta_i - \delta_j)] \qquad (7.2.1)$$

And the losses are (see chapter 3 for the calculation):

$$P_{LOSS,ij} = P_{ij} + P_{ji} = \; G_{ij}\,(V_i^2 + V_j^2) - 2\,G_{ij}\,|V_i||V_j|\,cos(\delta_i - \delta_j) \qquad (7.2.2)$$

Neglecting the voltages level (that is set to 1 p.u.):

$$P_{LOSS,ij} \approx \; 2 \cdot G_{ij} - 2 \cdot G_{ij}\,cos(\delta_i - \delta_j) = \; 2G_{ij}(1 - cos(\delta_i - \delta_j)) \qquad (7.2.3)$$

Using small angle approximations:

$$cos(\delta_i - \delta_j) \sim 1 - \frac{(\delta_i - \delta_j)^2}{2} \qquad (7.2.4)$$

And therefore:

$$1 - cos(\delta_i - \delta_j) \sim \cancel{1} - \cancel{1} + \frac{(\delta_i - \delta_j)^2}{2} \qquad (7.2.5)$$

$$P_{LOSS,ij} \approx 2 \cdot G_{ij}\frac{(\delta_i - \delta_j)^2}{2} \qquad (7.2.6)$$

And finally:

$$P_{LOSS,ij} \approx G_{ij}(\delta_i - \delta_j)^2 \tag{7.2.7}$$

We can now replace the balance power flow equation in the DC approximation with:

$$P_G - P_L - \sum B_{ij}(\delta_i - \delta_j) = P_{LOSS,ij} \approx \frac{1}{2}G_{ij}(\delta_i - \delta_j)^2 \tag{7.2.8}$$

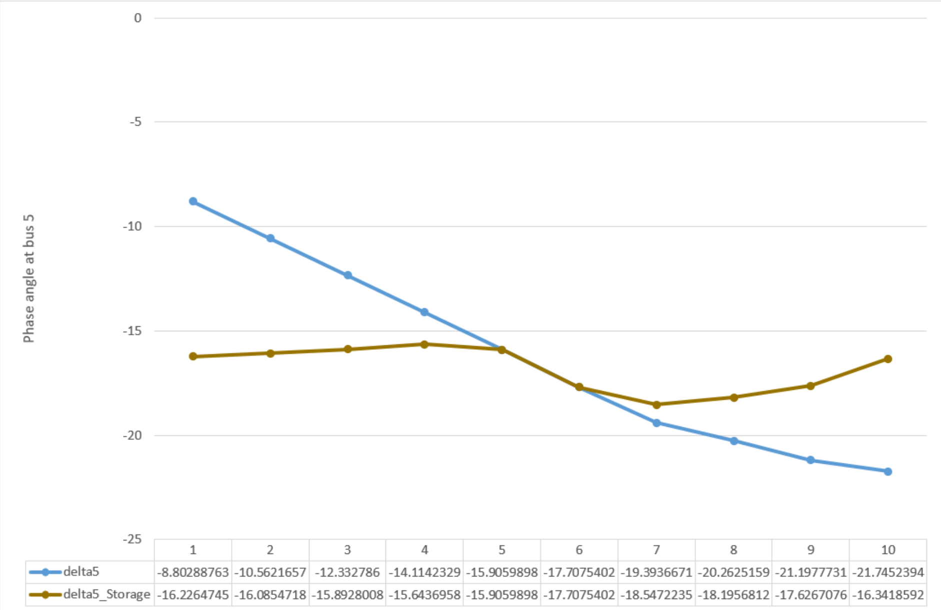
The $\frac{1}{2}$ factor is necessary for not including the losses of a single line in both the nodal balance at bus i and j. Furthermore [77] shows that this quadratic function can be approximated via piece-wise linearization, while [26] shows an alternative approximation as a conic quadratic problem.
Finally, [28] covers also the role of storage integration while computing power losses, and together with [78] state that DC-approximations play a critical role while assessing storage siting and sizing problem. This can lead to a *"sub-optimal energy storage systems integration decisions"* according to [28].

7.3 Storage Role

The storage integration brings several advantages that have been discussed in the previous chapters. As reactive power-related benefits are not evaluated in these simulations, storage devices are not allowed to exchange reactive power with the grid. This can be done, as with power balance equations, by adding charging and discharging terms to the reactive power balance equations. In that case, a quadratic inequality constraint has to limit the apparent power provided by the inverter/battery.

The similarities outlined in the previous chapters between lines and storage devices bring those devices to a positive consideration within a congestion framework. Transmission expansion planning are typically realized with the deployment of new lines, but as storage got cheaper and becoming well regarded, a deferral of new lines can be possible deploying large-scale storage systems.
Typically, energy prices are cheaper in low-load conditions and increase in peak periods. This increases the profit of storage devices and allows grids to redistribute the load to a larger time-frame. The following simulations, run under the increasing load conditions of the previous example, show how electrical quantities are affected by the presence of BESS.
A storage device is introduced at bus 5, since it is the only bus with a load without a generating unit. Several other choices are possible and would lead

to different results. However, it is not relevant for our purposes the placement choices of the storage device.



	1	2	3	4	5	6	7	8	9	10
delta5	-8.80288763	-10.5621657	-12.332786	-14.1142329	-15.9059898	-17.7075402	-19.3936671	-20.2625159	-21.1977731	-21.7452394
delta5_Storage	-16.2264745	-16.0854718	-15.8928008	-15.6436958	-15.9059898	-17.7075402	-18.5472235	-18.1956812	-17.6267076	-16.3418592

Figure 7.4 Phase angle at bus 5.

The phase angle at bus 5 follows a more stable trend when considering storage (brown line). The negativity of the angles means that power is going from the slack bus ($\delta_{1,t} = 0$) to the other nodes because of the cheaper cost for energy.
Regarding the generated powers, figure (7.5) refer to the generated power from the slack bus and the most expensive generator in the system. The storage system is charging in the first periods and discharging when load is higher (and then the costs are higher because of the quadratic objective function). More power is required from the (cheap) slack bus in the first time periods in order to charge the storage device, and less power is required from the (expensive) generator 4, that was asked to produce quite a lot of power at t_9 and t_{10} when storage was not yet introduced and congestions were increasing the costs of the system.
However, the single equations increased from 441 to 451, and the single variables from 441 to 471: 10 equations for the update of the SOC and 3 variables representing $P_{ch},\ P_{dch}\ and\ SOC$. This increase in variables and equations is linearly greater if considering more than one storage device.

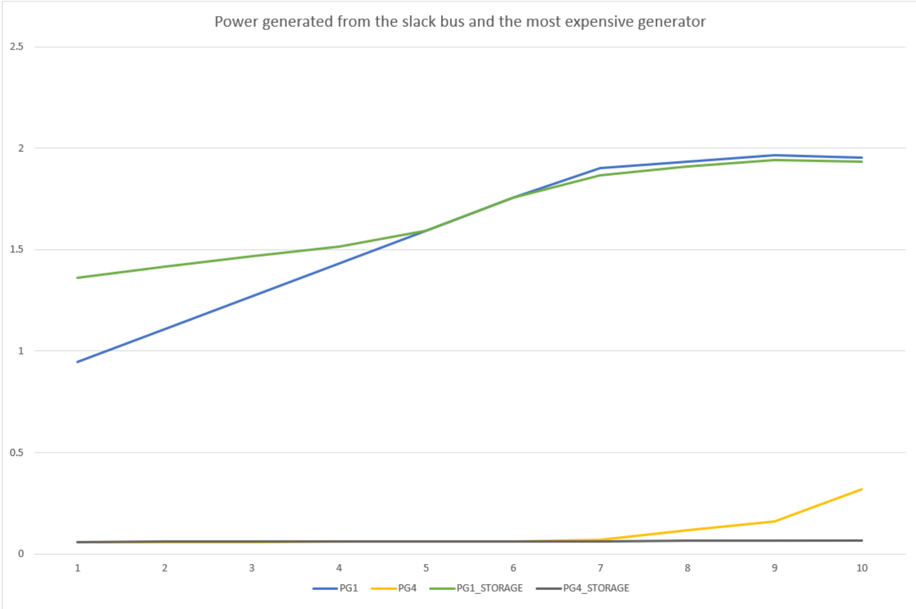


Figure 7.5 Power generated from the slack bus and the most expensive generator

7.4 Financial Transmission Rights and Financial Storage Rights

Storage characteristics have not been mentioned in the previous sections. The storage device introduced has a great storable energy and high power rate limits. In the 10-time-periods simulations it charged up to 1 p.u. and discharged back to the initial state of charge. Having introduced a multi-period monotonic power, it is reasonably charged a lot in the very first time steps and follows a dual behaviour discharging when the load and the costs are very high. However, the maximum charging and discharging powers are not reached and, according to eq. (6.2.52), only the Energy Capacity Right has value (for 3 time periods). If, for example, we can introduce a high-power, high-energy storage device in our system, we might be uncertain about the utility of Financial Storage Rights as well as storage arbitrage possibilities.
In other words, if the overall storage capability of the grid allows the system operator to perfectly balance the supply levelling the load (figure 4.4), then the investments in storage would be uneconomic because of the small price differences between the time periods investigated. According to [79], the price difference in a time framework, acts as "price signals" for investors in order to realize if investing in storage would be profitable or not. If, in the future, there will be a great amount of storage devices in the grid, the benefits would be less concrete and the investments would have a lower return.

Financial Storage Rights have a hedging role in electricity markets, and

allow system operator to redistribute the surplus resulting from storage congestion events. If the capacity of storage devices will be very high, the congestion will hardly occur and the rights value would be zero, just like the case in which a highly meshed grid will hardly have frequent congestion events due to multiple interconnections. In that case, the market participants would have less interests in hedging against price volatility that has been reduced by the presence of storage. On the other hand, the storage owner would not benefit from selling Financial Storage Rights and would only earn rate-based payments from the system operator. Therefore, the return of investment, would be exclusively related to these payments and the usefulness of storage for the grid.
For those reasons, in these section the storage device has been set to an energy capacity of 1 "p.u. per hour", charging and discharging rates at 0.2 p.u. and -0.2 p.u. respectively, while maintaining the efficiencies to the unity for simplicity.

The financial rights calculation comprise both flowgate and storage rights, but for the sake of simplicity we consider them in separate cases. We refer the reader to the papers previously cited for a comprehensive view of simultaneous rights allocations under "simultaneous feasibility" conditions. As previously discussed, it can be useful for a market participant to hedge against spatio-temporal price volatilities buying FTRs and FSRs [1].

We begin with a simple example in which we show how using DC approximation results in an exact match between merchandising surplus and financial transmission rights value. Summing up:

– In case of a DC approximation, nodal prices have no losses components and completely depend on the additional costs of energy and congestion re-dispatch costs.

– If no congestion occurs, the LMP is the same at every node and equals the linear costs of the slack bus ($0.35\frac{k\$}{p.u.}$), that is the cheapest generating unit (no quadratic components have been considered so far).

– Figure (7.6) shows that, increasing the load from t1 to t10, no congestions occur up to t7. Nodal prices difference at t8, t9, t10 is caused by congestion.

The line interested by the congestion event is branch 1-3. According to equation (6.2.52), we can calculate the congestion surplus (CS) or merchandising surplus as:

$$\sum_{i,t} \lambda_{i,t}^{(1)} p_{i,t} = MS \tag{7.4.1}$$

The total Financial Transmission Rights value (FTR) can be computed from the line shadow price (or dual variable):

$$\sum_{i,t} \lambda^{(3)}_{ij,t} p_{ij,MAX} = FTR \tag{7.4.2}$$

t1 - t7	t8	t9	t10
0.35	0.35	0.35	0.35
0.35	0.377493	0.417093	0.489618
0.35	0.398516	0.468398	0.596383
0.35	0.380425	0.424249	0.50451
0.35	0.391456	0.451169	0.56053

Figure 7.6 LMPs [$\frac{k\$}{p.u.}$] in incrementing load simulation .

	t1 - t7	t8	t9	t10
$\sum_{i,t} \lambda^{(1)}_{i,t} p_{i,t} = CS$	0	-0.062	-0.151	-0.314
$\lambda^{(3)}_{13,t} p_{13,MAX} = FTR$	0	0.062	0.151	0.314

Figure 7.7 Financial Transmission Rights and congestion surplus match [$\frac{k\$}{p.u.}$].

As expected, the FTR result in a zero value when there is no congestion, while they perfectly match with the congestion surplus when congestion occurs.

When computing the same simulation case with the AC approach, it is clear that the surplus cannot be equal to the FTR term due to a component that we called LOSS in the calculations in chapter 6. Trying to compute $\lambda^{(1)} g_{i,k} V_i^2$ gives an approximation on LMP component due to line losses. However, this approximation gave rough results in terms of accuracy. Several different computation possibilities have been presented for calculating the loss component of locational marginal prices, we refer the reader to [80], [81] and [82] for this topic. Typically, the loss component is found using penalty factors (or delivery factors) for each bus [82]. For the sake of simplicity, since our formulation is not directly referring to LMPs but relate shadow prices with power injections, using (6.2.52) we can directly compute the LOSS component from the congestion surplus and financial transmission rights value.

	t7	t8	t9	t10
$\sum_{i,t} \lambda^{(1)}_{i,t}\, p_{i,t} = CS$	-0.009934	-0.09449	-0.18225	-0.48728
$\lambda^{(3)}_{13,t}\, p_{13,MAX} = FTR$	0.0130094	0.09568	0.18008	0.478186
$LOSS$	-0.003075	-0.00119	0.002171	0.009094

Figure 7.8 Financial Transmission Rights, congestion surplus and loss component in case of AC results $[\frac{k\$}{p.u.}]$.

When comparing the two results tables, we show that the approximations bring to a non-negligible error.
As shown in figure (7.8), any loss component considered would not match perfectly the expression (6.2.52) in terms of congestion surplus and financial transmission rights. It has been demonstrated that, under non-convexity conditions, the revenue adequacy might not be obtained [80]. Thus, as a result, for our purposes the problem needs to be convex to ensure that the system operator does not incur in a financial deficit debt (even if quite low). Convex relaxation or losses incorporation methods are worth investigating in the framework of locational marginal prices and financial rights.

Having assessed the importance of precise mathematical models for the description of market-related FTRs and LMP calculations, we introduce now storage in our simulations and see how financial storage rights can be computed. As in the previous case, the marginal feature in GAMS can provide the computation of dual variables at each time period. We use here the same notation as in chapter 6, and calculate Energy Capacity Rights (ECRs) and Power Capacity Rights (PCRs). Charging and discharging rates are considered smaller than in section 7.3 for showing the role of PCRs. Meanwhile, a quadratic cost function is considered also for the slack bus.
As a first result, the Financial Storage Rights computation match with the nodal inter-temporal transaction scheme. In the previous chapter, we discussed about the possibility of implementing FSR either as a sequence of nodal price transaction over the time periods [71], or using a constraint-based approach [1]. In figure (7.10) we can see how Financial Storage Rights can be computed through the summation over all the time periods of the different components of eq. (6.2.52):

- CS is the congestion surplus table, calculated with nodal dual prices and power net balance.
- PCR_{ch}, PCR_{dch} and ECR are respectively the power capacity rights for charging, discharging and energy capacity rights;

CS	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10
bus1	0.417005	0.60557	0.685422	0.770079	1.016181	1.295489	1.608354	1.735862	1.868427	2.241152
bus3	0.284234	0.324692	0.339877	0.355123	0.396248	0.437801	0.479796	0.495516	0.511305	0.554063
bus4	-0.3568	-0.4891	-0.59767	-0.71413	-0.89641	-1.10046	-1.32669	-1.49559	-1.67282	-1.95239
bus5	-0.31292	-0.40719	-0.39291	-0.37576	-0.47992	-0.59728	-0.72809	-0.70333	-0.67565	-0.81701
										0.33008

PCR_ch	t1	t2	t4	t5	t6	t7	t8	t9	t10
bus5	-0.0279	-0.00758	Eps	Eps	Eps	Eps	Eps	0.007886	0.029365
									-0.03549

PCR_dch	t1	t2	t3	t9	t10
bus5	0.027901	0.007584	Eps	-0.00789	-0.02937
					-0.03725

ECR	t3	t4	t5	t6	t7	t10
bus5	-0.01906	-0.0516	-0.05216	-0.05275	-0.0196	0.62132
						-0.19517

Figure 7.9 FSR computation, first approach [1].

- The values in red boxes must not be considered when computing FSR, because they refer to dual variables not relevant in the congestion events (e.g. when the storage device has a $SOC = 0$ or $p_{ch} = 0$ or $p_{dch} = 0$);

Financial Storage Rights total value can be computed now as:

$$PCR_{ch} + PCR_{dch} + ECR = -0.03549 - 0.03725 - 0.19517 = -0.2679k\$ \tag{7.4.3}$$

Following now another approach, by calculating the arbitraging sequence of nodal transactions [71]:

$$\sum_{setC,t} \lambda^{1}_{setC,t} p_{setC,t} \tag{7.4.4}$$

Where setC denotes the set of nodes with batteries. The following figure shows how the same result is computed with this second approach.

	t1	t2	t3	t8	t9	t10
bus5	0.142561	0.162877	0.085231	-0.12426	-0.25641	-0.27789
						-0.2679

Figure 7.10 FSR computation, second approach [71]

The two results match for the value of Financial Storage Rights, but there is a remaining term, necessary to compensate the full congestion surplus, that corresponds to the loss component and to inaccuracies due to the simplification of the branch limit inequality constraint from (6.2.6) to (6.2.15).

Chapter 8

Conclusion

In this thesis, after introducing the main topics around the integration of storage devices in future grids, an AC-OPF model is presented. Financial Transmission Rights and Financial Storage Rights are calculated from the KKT conditions following a similar approach as in [1], but another method is also presented using nodal price transactions [71]. So far, to the best of our knowledge, all the papers dealing with FSRs focus on a DC approximation commonly used in many fields of power systems. However it is shown that, as expected, the approximations of DC models can yield imprecise results. Yet, when dealing with pricing and markets, results accuracy is an important factor.

It has been highlighted that dealing with an AC-OPF could result in problems if the structure of the problem is non-convex [80]. The most reasonable model for accurate financial rights calculation seems to be a convex AC-OPF or an enhanced DC-OPF.

This thesis is framed in the context of storage integration in transmission grids. Financial rights could help investors decouple the storage operation from any wholesale market mechanisms, leaving the system operator the task of utilizing it to maximize social welfare.

Since storage technologies will be fundamental to guarantee a good level of autonomy and flexibility, a detailed consideration of their benefits is essential for ensuring an appropriate return of investment.

Chapter 9

Future Work

9.1 Model Extensions

As we have seen, several problems may occur when choosing a DC approximation of Optimal Power Flow, or using a full-AC approach.
Also, coupled with the deployment of a huge amount of distributed generation devices, relatively small sized storage systems can help avoid congestion in grids where the configuration is radial or weakly meshed. Furthermore, several other services can be supplied by storage systems in general. Ancillary services can be provided by a variety of storage technologies. Adding such evaluations to the mathematical model would lead to a more accurate assessment of potential benefits. E.g., as discussed in chapter 3, optimal reactive power flow seeks to minimize power transfer losses, requesting reactive power from storage devices. If an increased penetration of renewable energy sources is expected, the merit order curve would lead to decreased overall prices, because PV and wind technologies have very low operating costs. In such framework, it may be reasonable to minimize network losses instead of production costs.
The incorporation of ancillary services can be a viable way. However, since AC-OPF is already computationally demanding, adding new complexities could lead to an infeasible problem. Probabilistic approaches can therefore see a variety of applications due to the nature of such uncertain problems.
Furthermore, due to AC-OPF complexity, it can be reasonable to get closer to its results without considering the full AC approach: relaxation methods for the AC model or losses integration for the DC approximation can lead to quite good results with reasonable computational complexity.
Conceptually, the model previously presented can be applied to meshed transmission grids. However, a similar approach can be used to extend the utilization of storage systems at the distribution level. However, pricing in distribution grids needs to follow the nodal scheme as in the transmission counterpart.
An accurate and detailed Financial Storage Rights framework would need

the incorporation of such instruments in auctions as shown in [73].
Finally, a more precise model with characteristics of storage devices that also depend on aging of the batteries would lead to more accurate results when dealing with return on investment calculations.

9.2 V2G

Road vehicles and automobiles are the most common means of transport in the world. Since future power grids will be highly dependent on renewable energy systems, EVs contain a battery and can provide an affordable high-power density supply for this grid support: this is named V2G (Vehicle-To-Grid) mode operation.
Only a few countries have nowadays a discrete number of plug-in electric vehicles, but in the near future there will be an exponential growth in sales that could bring to a potential widespread of V2G systems. Storage systems spread throughout the territory will lead to a direct instantaneous exchange of electric power with the grid, improving its stability, reliability and efficiency. As a potential forecast, millions of new EVs (Electric Vehicles) are expected to be manufactured in the upcoming years (figures (9.1) and (9.2)), leading to a non-negligible contribution brought by EVs connected to the grid, considering also that they are typically used just for a short time over the day.
Since the grid operates in AC, and the battery is charged using DC, a power electronic converter is required. High-power charging systems are decreasing the time required to a full charge of the battery, resulting in a high availability for V2G purposes. Since EVs have been considered as loads for the grid in the past, a new infrastructure is needed for the double purpose of charging the vehicle when needed (G2V operation mode) and discharging EVs batteries in V2G operation mode. This role is mainly covered by a bidirectional power electronic converter, and other devices in order to optimize the connection to the grid. If connected to the grid, EVs also offer other ancillary advantages: voltage and frequency regulation, reactive power compensation, active power regulation, current harmonic filtering, load balancing and peak load shaving (rearranging power required by the loads over a wide range of hours). V2G costs include batteries degradation, power electronics maintenance, additional devices for grid optimal connection and for charging/discharging the batteries according to the needs of the grid or the owner's specific claims.
Despite the huge negative impact brought by Covid-19 pandemic to the automobile industry, the share of hybrid and electric vehicles is growing, allowing the transition to an electrical energy-based transportation sector.

It is expected that the market share of EVs will keep growing and will be one of the key technologies for power grid autonomous operation. As we can

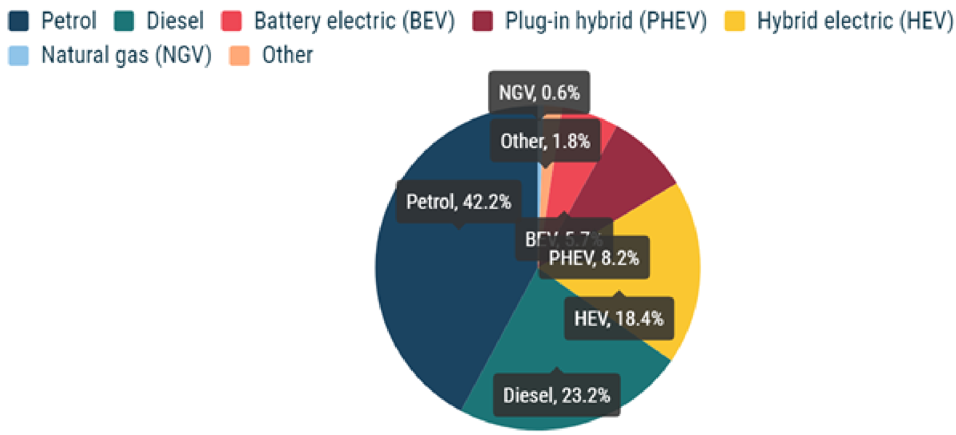


Figure 9.1 New passenger car registrations by fuel type in the European Union: Q1 2021, [83]

see from data publicly available on ACEA website [83], the market share is highly dependent on the country, and policies play a critical role. Concerning charging points, most of the stations are located in a few countries, but in the near future a widespread of both EVs and charging stations will be possible. Since power electronics is used for these applications, accurate control techniques are necessary for matching the requirements in terms of voltages and currents. Many other details and issues must be discussed when introducing V2G mode operation: power quality and connection to the distribution grid, SOC, SOH and EV's owner availability. It is essential that EVs widespread will be combined with a fewer fossil fuels-dependent society, for reduced overall emissions. There are many other issues related to EVs market (like batteries waste management, or bigger demand for new materials in automotive sector) but there are several advantages introducing V2G technology.

Therefore, some research projects have been established in order to better assess the benefits brought by V2G mode operation to the grid. One of the first pilot projects have been launched in September 2020 inside FCA's Mirafiori industrial area in Turin, Italy. The main idea of these projects is to 'aggregate' a great number of vehicles in the same point of connection, as flexibility resources that can provide services to the grid. Along with these pilot projects, it is important to fairly recognize the benefits brought by V2G [84]. Again, the regulatory framework has a great importance, because both the EV owner and the companies involved need to gain a revenue from V2G operation mode. The former expects an income because of the use of its battery, the latter ones need to recover the cost related to the use of innovative bidirectional converters and measurement devices, providing ancillary services [84].
Financial storage rights could be an interesting way to help integrate EVs

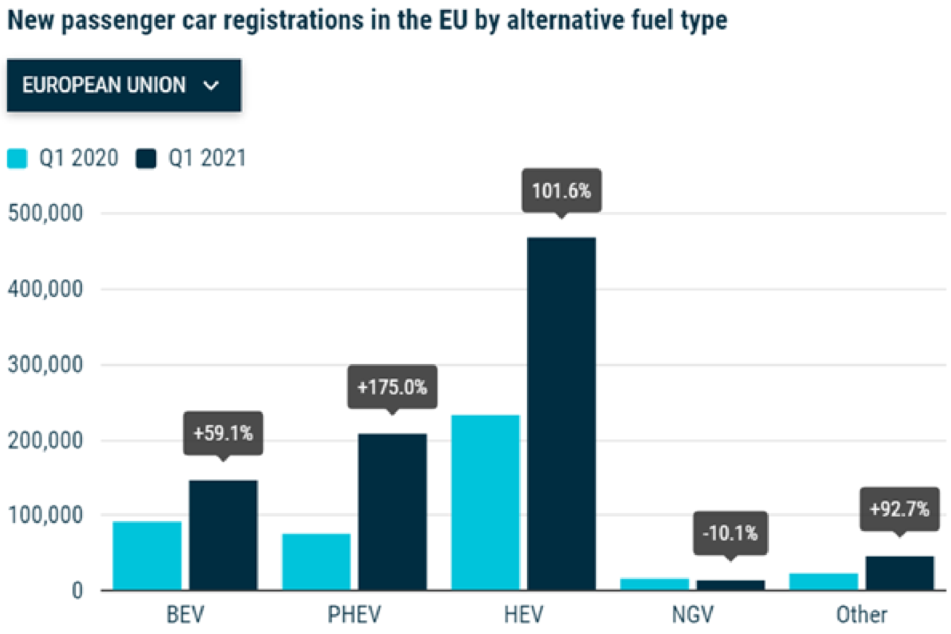


Figure 9.2 New passenger car registrations in the EU by alternative fuel type [83]

into the grid, in countries in which nodal pricing is applied. Several difficulties may arise from such an application, since research is still focused on bringing the right revenues to the owners of EVs when operating in V2G mode. However, large hubs connecting EVs in an aggregate way might be considered as a flexibility resource for the grid, similar to a utility-scale storage plant. In this framework, the consideration of these hubs as transmission assets could bring to an additional source of revenue for both the owner of the vehicle and the company providing the charging stations, because the overall social welfare would be enhanced.

The participation in wholesale markets is unfeasible for small-scale participants as well as distributed resources, because the market would be intractable with a huge number of participants, and because the regulations of the market is too complex for a single small contributor. The introduction of "aggregators" will help these distributed energy resources to manage the interaction with the wholesale markets [3].

All these considerations are placed in the framework of integrating new flexibility sources in the grid, as discussed in the previous chapters of this thesis.

Bibliography

[1] J. A. Taylor. “Financial Storage Rights”. In: *IEEE Transactions on Power Systems* 30.2 (2015), pp. 997–1005. DOI: 10.1109/TPWRS.2014.2339016 (cit. on pp. 1, 14, 44, 58–60, 63, 87, 89–91).

[2] bp. *Statistical Review of World Energy, 69th edition*. 2020. URL: https://www.bp.com/content/dam/bp/business-sites/en/global/corporate/pdfs/energy-economics/statistical-review/bp-stats-review-2020-full-report.pdf (cit. on pp. 11, 47).

[3] Daniel S. Kirschen and Goran Strbac. *Fundamentals on Power System Economics, second edition*. 2019 (cit. on pp. 12, 39–41, 55–57, 96).

[4] H. Ibrahim, A. Ilinca, and J. Perron. “Energy storage systems — Characteristics and comparisons”. In: *Renewable and Sustainable Energy Reviews* 12.5 (2008), pp. 1221 –1250. ISSN: 1364-0321. DOI: https://doi.org/10.1016/j.rser.2007.01.023. URL: http://www.sciencedirect.com/science/article/pii/S1364032107000238 (cit. on p. 13).

[5] Xing Luo et al. “Overview of current development in electrical energy storage technologies and the application potential in power system operation”. In: *Applied Energy* 137 (2015), pp. 511 –536. ISSN: 0306-2619. DOI: https://doi.org/10.1016/j.apenergy.2014.09.081. URL: http://www.sciencedirect.com/science/article/pii/S0306261914010290 (cit. on p. 13).

[6] L. Rouco and L. Sigrist. “Active and reactive power control of battery energy storage systems in weak grids”. In: *2013 IREP Symposium Bulk Power System Dynamics and Control - IX Optimization, Security and Control of the Emerging Power Grid*. 2013, pp. 1–7. DOI: 10.1109/IREP.2013.6629422 (cit. on p. 13).

[7] Ramteen Sioshansi, Paul Denholm, and Thomas Jenkin. “Market and Policy Barriers to Deployment of Energy Storage”. In: *Economics of Energy & Environmental Policy* 1.2 (2012), pp. 47–64. ISSN: 21605882, 21605890. URL: http://www.jstor.org/stable/26189491 (cit. on pp. 14, 43, 52).

[8] Elisa Pappalardo, Panos Pardalos, and Giovanni Stracquadanio. *Optimization Approaches for Solving String Selection Problems*. Jan. 2013, pp. 13–25. ISBN: 978-1-4614-9052-4. DOI: 10.1007/978-1-4614-9053-1 (cit. on p. 16).

[9] Stephen Boyd and Lieven Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004. DOI: 10.1017/CBO9780511804441 (cit. on pp. 16, 18).

[10] George Bernard Dantzig. *Linear Programming and Extensions*. Santa Monica, CA: RAND Corporation, 1963. DOI: 10.7249/R366 (cit. on p. 17).

[11] Jakob Puchinger and Günther R. Raidl. "Combining Metaheuristics and Exact Algorithms in Combinatorial Optimization: A Survey and Classification". In: *Artificial Intelligence and Knowledge Engineering Applications: A Bioinspired Approach*. Ed. by José Mira and José R. Álvarez. Berlin, Heidelberg: Springer Berlin Heidelberg, 2005, pp. 41–53 (cit. on p. 17).

[12] Joel Sobel. *Linear Programming Notes VI Duality and Complementary Slackness*. URL: https://econweb.ucsd.edu/~jsobel/172aw02/notes6.pdf (cit. on p. 19).

[13] URL: https://en.wikipedia.org/wiki/Electricity_market (cit. on pp. 19, 39).

[14] Bernard Kolman and Robert E. Beck. "3 - Further Topics in Linear Programming". In: *Elementary Linear Programming with Applications (Second Edition)*. Ed. by Bernard Kolman and Robert E. Beck. Second Edition. San Diego: Academic Press, 1995, pp. 155–247. ISBN: 978-0-12-417910-3. DOI: https://doi.org/10.1016/B978-012417910-3/50006-1. URL: https://www.sciencedirect.com/science/article/pii/B9780124179103500061 (cit. on p. 20).

[15] Harold W. Kuhn and Albert W. Tucker. "Nonlinear programming". In: *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*. Berkeley, CA, USA: University of California Press, 1951, pp. 481–492 (cit. on p. 22).

[16] URL: en.wikipedia.org/wiki/Karush\%E2\%80\%93Kuhn\%E2\%80\%93Tucker_conditions (cit. on p. 23).

[17] P. Kundur, N.J. Balu, and M.G. Lauby. *Power System Stability and Control*. EPRI power system engineering series. McGraw-Hill Education, 1994. ISBN: 9780070359581. URL: https://books.google.it/books?id=wOlSAAAAMAAJ (cit. on pp. 27, 32, 36).

[18] Stephen Frank and Steffen Rebennack. "An introduction to optimal power flow: Theory, formulation, and examples". In: *IIE Transactions* 48.12 (2016), pp. 1172–1197. DOI: 10.1080/0740817X.2016.1189626. eprint: https://doi.org/10.1080/0740817X.2016.1189626. URL: https://doi.org/10.1080/0740817X.2016.1189626 (cit. on pp. 29, 31, 37, 38, 60, 77–79).

[19] J. Carpentier. "Contribution a' l'etude du Dispatching Economique". In: 3 (1962), pp. 431–447 (cit. on p. 31).

[20] Hermann W. Dommel and William F. Tinney. "Optimal Power Flow Solutions". In: *IEEE Transactions on Power Apparatus and Systems* PAS-87.10 (1968), pp. 1866–1876. DOI: 10.1109/TPAS.1968.292150 (cit. on p. 31).

[21] E. Carpaneto. *Sistemi Elettrici di Potenza, Lecture Notes*. 2020 (cit. on pp. 32, 35).

[22] P. Lipka, R. O'Neill, and S. Oren. *Developing line current magnitude constraints for IEEE test problems, Optimal Power Flow Paper 7, Staff paper*. 2013. URL: https://www.ferc.gov/sites/default/files/2020-04/acopf-7-line-constraints.pdf (cit. on p. 32).

[23] S.A. Daza. *Electric Power System Fundamentals*. Artech House power engineering series. Artech House Publishers, 2016. ISBN: 9781630814328. URL: https://books.google.it/books?id=c2uwDgAAQBAJ (cit. on pp. 34, 35).

[24] Spyros Chatzivasileiadis. *Optimization in Modern Power Systems, DTU Course 31765, Lecture Notes*. Technical University of Denmark (DTU). 2018. URL: https://arxiv.org/pdf/1811.00943.pdf (cit. on pp. 37, 42).

[25] A.J. Conejo and J.A. Aguado. "Multi-area coordinated decentralized DC optimal power flow". In: *IEEE Transactions on Power Systems* 13.4 (1998), pp. 1272–1278. DOI: 10.1109/59.736264 (cit. on p. 38).

[26] R.A. Jabr. "Modeling network losses using quadratic cones". In: *IEEE Transactions on Power Systems* 20.1 (2005), pp. 505–506. DOI: 10.1109/TPWRS.2004.841157 (cit. on pp. 38, 83, 84).

[27] Carleton Coffrin, Pascal Van Hentenryck, and Russell Bent. "Approximating line losses and apparent power in AC power flow linearizations". In: *2012 IEEE Power and Energy Society General Meeting*. 2012, pp. 1–8. DOI: 10.1109/PESGM.2012.6345342 (cit. on pp. 38, 82).

[28] Anya Castillo and Dennice F. Gayme. "Evaluating the Effects of Real Power Losses in Optimal Power Flow-Based Storage Integration". In: *IEEE Transactions on Control of Network Systems* 5.3 (2018), pp. 1132–1145. DOI: 10.1109/TCNS.2017.2687819 (cit. on pp. 38, 84).

[29] Ke Qing et al. "Optimized operating strategy for a distribution network containing BESS and renewable energy". In: *2019 IEEE Innovative Smart Grid Technologies - Asia (ISGT Asia)*. 2019, pp. 1593–1597. DOI: 10.1109/ISGT-Asia.2019.8881304 (cit. on p. 39).

[30] Provas Kumar Roy and Susanta Dutta. IGI Global, 2019. Chap. Economic Load Dispatch: Optimal Power Flow and Optimal Reactive Power Dispatch Concept, pp. 46–64. URL: http://doi:10.4018/978-1-5225-6971-8.ch002 (cit. on p. 39).

[31] Santosh Raikar and Seabron Adamson. "8 - Renewable energy in power markets". In: *Renewable Energy Finance*. Ed. by Santosh Raikar and Seabron Adamson. Academic Press, 2020, pp. 115–129. ISBN: 978-0-12-816441-9. DOI: https://doi.org/10.1016/B978-0-12-816441-9.00008-8. URL: https://www.sciencedirect.com/science/article/pii/B9780128164419000088 (cit. on pp. 39–41).

[32] Ross Baldick. *Seams, ancillary services and congestion management: US versus EU Electricity Markets*. URL: https://medium.com/florence-school-of-regulation/seams-ancillary-services-and-congestion-management-us-versus-eu-electricity-markets-ab9ad53a16df (cit. on p. 40).

[33] Piotr F. Borowski. "Zonal and Nodal Models of Energy Market in European Union". In: *Energies* 13.16 (2020). ISSN: 1996-1073. DOI: 10.3390/en13164182. URL: https://www.mdpi.com/1996-1073/13/16/4182 (cit. on p. 40).

[34] Mette Bjørndal and Kurt Jørnsten. "Zonal Pricing in a Deregulated Electricity Market". In: *The Energy Journal* 22.1 (2001), pp. 51–73. ISSN: 01956574, 19449089. URL: http://www.jstor.org/stable/41322907 (cit. on p. 40).

[35] URL: https://en.wikipedia.org/wiki/Merit_order (cit. on p. 40).

[36] Brent Eldridge, Richard P. O'Neill, and Andrea R. Castillo. "Marginal Loss Calculations for the DCOPF". In: (Dec. 2016). DOI: 10.2172/1340633. URL: https://www.osti.gov/biblio/1340633 (cit. on p. 42).

[37] Nayeem Chowdhury, Giuditta Pisano, and Fabrizio Pilo. "Energy Storage Placement in the Transmission Network: A Robust Optimization Approach". In: *2019 AEIT International Annual Conference (AEIT)*. 2019, pp. 1–6. DOI: 10.23919/AEIT.2019.8893299 (cit. on p. 45).

[38] P. Lazzeroni and M. Repetto. "Optimal planning of battery systems for power losses reduction in distribution grids". In: *Electric Power Systems Research* 167 (2019), pp. 94–112. ISSN: 0378-7796. DOI: https://doi.org/10.1016/j.epsr.2018.10.027. URL: https://www.sciencedirect.com/science/article/pii/S0378779618303432 (cit. on pp. 45, 47).

[39] Hamidreza Mirtaheri et al. "Optimal Planning and Operation Scheduling of Battery Storage Units in Distribution Systems". In: *2019 IEEE Milan PowerTech*. 2019, pp. 1–6. DOI: 10.1109/PTC.2019.8810421 (cit. on p. 45).

[40] Benedikt Battke and Tobias S. Schmidt. "Cost-efficient demand-pull policies for multi-purpose technologies – The case of stationary electricity storage". In: *Applied Energy* 155 (2015), pp. 334–348. ISSN: 0306-2619. DOI: https://doi.org/10.1016/j.apenergy.2015.06.010. URL: https://www.sciencedirect.com/science/article/pii/S0306261915007680 (cit. on p. 45).

[41] European Commission. *Energy storage – the role of electricity*. URL: https://ec.europa.eu/energy/sites/ener/files/documents/swd2017_61_document_travail_service_part1_v6.pdf (cit. on pp. 45, 47–49).

[42] European Association for Storage of Energy (EASE). *Pumped Hydro Storage*. URL: https://ease-storage.eu/wp-content/uploads/2016/07/EASE_TD_Mechanical_PHS.pdf (cit. on p. 47).

[43] *Flywheel energy storage*. URL: https://en.wikipedia.org/wiki/Flywheel_energy_storage (cit. on p. 47).

[44] Paul Breeze. "Chapter 3 - Compressed Air Energy Storage". In: *Power System Energy Storage Technologies*. Ed. by Paul Breeze. Academic Press, 2018, pp. 23–31. ISBN: 978-0-12-812902-9. DOI: https://doi.org/10.1016/B978-0-12-812902-9.00003-1. URL: https://www.sciencedirect.com/science/article/pii/B9780128129029000031 (cit. on p. 47).

[45] Paul Breeze. "Chapter 5 - Superconducting Magnetic Energy Storage". In: *Power System Energy Storage Technologies*. Ed. by Paul Breeze. Academic Press, 2018, pp. 47–52. ISBN: 978-0-12-812902-9. DOI: https://doi.org/10.1016/B978-0-12-812902-9.00005-5. URL: https://www.sciencedirect.com/science/article/pii/B9780128129029000055 (cit. on p. 48).

[46] Bruno Pollet. *Energy Storage, Lecture Notes, Norwegian University of Science and Technology (NTNU)*. 2019 (cit. on p. 48).

[47] Z. E. Lee et al. "Providing Grid Services With Heat Pumps: A Review". In: *ASME Journal of Engineering for Sustainable Buildings and Cities* (2020). URL: https://doi.org/10.1115/1.4045819 (cit. on p. 48).

[48] Iain Staffell et al. "The role of hydrogen and fuel cells in the global energy system". In: *Energy Environ. Sci.* 12 (2 2019), pp. 463–491. DOI: 10.1039/C8EE01157E. URL: http://dx.doi.org/10.1039/C8EE01157E (cit. on p. 48).

[49] Stephen Clegg and Pierluigi Mancarella. "Storing renewables in the gas network: modelling of power-to-gas seasonal storage flexibility in low-carbon power systems". In: *IET Generation, Transmission & Distribution* 10.3 (2016), pp. 566–575. DOI: https://doi.org/10.1049/iet-gtd.2015.0439. eprint: https://ietresearch.onlinelibrary.wiley.com/doi/pdf/10.1049/iet-gtd.2015.0439. URL: https://ietresearch.onlinelibrary.wiley.com/doi/abs/10.1049/iet-gtd.2015.0439 (cit. on p. 48).

[50] Matthew A. Pellow et al. "Hydrogen or batteries for grid storage? A net energy analysis". In: *Energy Environ. Sci.* 8 (7 2015), pp. 1938–1952. DOI: 10.1039/C4EE04041D. URL: http://dx.doi.org/10.1039/C4EE04041D (cit. on p. 48).

[51] *BloombergNEF: 'Already cheaper to install new-build battery storage than peaking plants'*. URL: https://www.energy-storage.news/news/bloombergnef-lcoe-of-battery-storage-has-fallen-faster-than-solar-or-wind-i (cit. on p. 48).

[52] Andrej Trpovski et al. "A Hybrid Optimization Method for Distribution System Expansion Planning with Lithium-ion Battery Energy Storage Systems". In: *2020 IEEE Sustainable Power and Energy Conference (iSPEC)*. 2020, pp. 2015–2021. DOI: 10.1109/iSPEC50848.2020.9351208 (cit. on p. 49).

[53] Odne Stokke Burheim. "Chapter 7 - Secondary Batteries". In: *Engineering Energy Storage*. Ed. by Odne Stokke Burheim. Academic Press, 2017, pp. 111–145. ISBN: 978-0-12-814100-7. DOI: https://doi.org/10.1016/B978-0-12-814100-7.00007-9. URL: https://www.sciencedirect.com/science/article/pii/B9780128141007000079 (cit. on pp. 49, 50).

[54] Rahul Walawalkar, Jay Apt, and Rick Mancini. "Economics of electric energy storage for energy arbitrage and regulation in New York". In: *Energy Policy* 35.4 (2007), pp. 2558–2568. ISSN: 0301-4215. DOI: https://doi.org/10.1016/j.enpol.2006.09.005. URL: https://www.sciencedirect.com/science/article/pii/S0301421506003545 (cit. on p. 49).

[55] Terna Rete Elettrica Nazionale. *IL RUOLO DELLO STORAGE NELLA GESTIONE DELLE RETI*. URL: https://www.terna.it/it/media/news-eventi/ruolo-storage-gestione-reti (cit. on p. 49).

[56] Yash Kotak et al. "End of Electric Vehicle Batteries: Reuse vs. Recycle". In: *Energies* 14.8 (2021). ISSN: 1996-1073. DOI: 10.3390/en14082217. URL: https://www.mdpi.com/1996-1073/14/8/2217 (cit. on p. 50).

[57] Catherine Heymans et al. "Economic analysis of second use electric vehicle batteries for residential energy storage and load-levelling". In: *Energy Policy* 71 (2014), pp. 22–30. ISSN: 0301-4215. DOI: https://doi.org/10.1016/j.enpol.2014.04.016. URL: https://www.sciencedirect.com/science/article/pii/S0301421514002328 (cit. on p. 50).

[58] David Roberts. *Competitors to lithium-ion batteries in the grid storage market*. URL: https://www.volts.wtf/p/battery-week-competitors-to-lithium (cit. on p. 50).

[59] Maria C. Argyrou, Paul Christodoulides, and Soteris A. Kalogirou. "Energy storage for electricity generation and related processes: Technologies appraisal and grid scale applications". In: *Renewable and Sustainable Energy Reviews* 94 (2018), pp. 804–821. ISSN: 1364-0321. DOI: https://doi.org/10.1016/j.rser.2018.06.044. URL: https://www.sciencedirect.com/science/article/pii/S1364032118304817 (cit. on p. 51).

[60] C.D. Parker. "APPLICATIONS – STATIONARY | Energy Storage Systems: Batteries". In: *Encyclopedia of Electrochemical Power Sources*. Ed. by Jürgen Garche. Amsterdam: Elsevier, 2009, pp. 53–64. ISBN: 978-0-444-52745-5. DOI: https://doi.org/10.1016/B978-044452745-5.00382-8. URL: https://www.sciencedirect.com/science/article/pii/B9780444527455003828 (cit. on p. 51).

[61] Claudio Brivio, Stefano Mandelli, and Marco Merlo. "Battery energy storage system for primary control reserve and energy arbitrage". In: *Sustainable Energy, Grids and Networks* 6 (2016), pp. 152–165. ISSN: 2352-4677. DOI: https://doi.org/10.1016/j.segan.2016.03.004. URL: https://www.sciencedirect.com/science/article/pii/S2352467716300017 (cit. on p. 52).

[62] Philip C. Kjær and Rasmus Lærke. "Experience with primary reserve supplied from energy storage system". In: *2015 17th European Conference on Power Electronics and Applications (EPE'15 ECCE-Europe)*. 2015, pp. 1–6. DOI: 10.1109/EPE.2015.7311788 (cit. on p. 52).

[63] M. Benini et al. "Battery energy storage systems for the provision of primary and secondary frequency regulation in Italy". In: *2016 IEEE 16th International Conference on Environment and Electrical Engineering (EEEIC)*. 2016, pp. 1–6. DOI: 10.1109/EEEIC.2016.7555748 (cit. on p. 52).

[64] Haisheng Chen et al. "Progress in electrical energy storage system: A critical review". In: *Progress in Natural Science* 19.3 (2009), pp. 291–312. ISSN: 1002-0071. DOI: https://doi.org/10.1016/j.pnsc.2008.07.014. URL: https://www.sciencedirect.com/science/article/pii/S100200710800381X (cit. on p. 53).

[65] URL: https://learn.pjm.com/three-priorities/buying-and-selling-energy/ftr-faqs/what-are-ftrs.aspx#faq-box-text0 (cit. on p. 55).

[66] URL: https://acer.europa.eu/en/Electricity/MARKET-CODES/FORWARD-CAPACITY-ALLOCATION/Pages/default.aspx (cit. on p. 55).

[67] Petr Spodniak, Mari Makkonen, and Samuli Honkapuro. "Long-term transmission rights in the Nordic electricity markets: TSO perspectives". In: *2016 13th International Conference on the European Energy Market (EEM)*. 2016, pp. 1–5. DOI: 10.1109/EEM.2016.7521212 (cit. on p. 55).

[68] Hung po Chao et al. "Flow-Based Transmission Rights and Congestion Management". In: *The Electricity Journal* 13.8 (2000), pp. 38–58. ISSN: 1040-6190. DOI: https://doi.org/10.1016/S1040-6190(00)00146-9. URL: https://www.sciencedirect.com/science/article/pii/S1040619000001469 (cit. on pp. 55, 56).

[69] R. P. O'Neill et al. "A joint energy and transmission rights auction: proposal and properties". In: *IEEE Transactions on Power Systems* 17.4 (2002), pp. 1058–1067. DOI: 10.1109/TPWRS.2002.804978 (cit. on p. 56).

[70] William W. Hogan. "Contract networks for electric power transmission". In: *Journal of Regulatory Economics* 4 (1992), pp. 211–242. ISSN: 1573-0468. DOI: https://doi.org/10.1007/BF00133621 (cit. on p. 56).

[71] Daniel Munoz-Alvarez and Eilyan Bitar. *Financial Storage Rights in Electric Power Networks*. 2017. arXiv: 1410.7822 [math.OC] (cit. on pp. 58, 61, 89–91).

[72] J. Warrington et al. "A market mechanism for solving multi-period optimal power flow exactly on AC networks with mixed participants". In: *2012 American Control Conference (ACC)*. 2012, pp. 3101–3107. DOI: 10.1109/ACC.2012.6315477 (cit. on p. 60).

[73] Abu Alam and Joshua A. Taylor. "An auction for financial storage rights". In: *Mediterranean Conference on Power Generation, Transmission, Distribution and Energy Conversion (MEDPOWER 2018)*. 2018, pp. 1–5. DOI: 10.1049/cp.2018.1858 (cit. on pp. 61, 94).

[74] Kyri Baker. *Solutions of DC OPF are Never AC Feasible*. 2020. arXiv: 1912.00319 [math.OC] (cit. on p. 82).

[75] Yingying Qi, Di Shi, and Daniel Tylavsky. "Impact of assumptions on DC power flow model accuracy". In: *2012 North American Power Symposium (NAPS)*. 2012, pp. 1–6. DOI: 10.1109/NAPS.2012.6336395 (cit. on p. 82).

[76] S. Grijalva, P.W. Sauer, and J.D. Weber. "Enhancement of linear ATC calculations by the incorporation of reactive power flows". In: *IEEE Transactions on Power Systems* 18.2 (2003), pp. 619–624. DOI: 10.1109/TPWRS.2003.810902 (cit. on p. 82).

[77] A.L. Motto et al. "Network-constrained multiperiod auction for a pool-based electricity market". In: *IEEE Transactions on Power Systems* 17.3 (2002), pp. 646–653. DOI: 10.1109/TPWRS.2002.800909 (cit. on pp. 83, 84).

[78] Anya Castillo and Dennice F. Gayme. "Profit maximizing storage allocation in power grids". In: *52nd IEEE Conference on Decision and Control*. 2013, pp. 429–435. DOI: 10.1109/CDC.2013.6759919 (cit. on p. 84).

[79] Josh Taylor. *Financial Transmission and Storage Rights presented by Josh Taylor (webinar)*. URL: https://resourcecenter.smartgrid.ieee.org/education/webinar-videos/SGWEB0056.html (cit. on p. 86).

[80] Andy Philpott and Geoffrey Pritchard. "Financial transmission rights in convex pool markets". In: *Operations Research Letters* 32.2 (2004), pp. 109–113. ISSN: 0167-6377. DOI: https://doi.org/10.1016/j.orl.2003.06.002. URL: https://www.sciencedirect.com/science/article/pii/S0167637703001032 (cit. on pp. 88, 89, 91).

[81] James B. Bushnell and Steven E. Stoft. "Electric grid investment under a contract network regime". In: *Journal of Regulatory Economics* (1996). DOI: https://doi.org/10.1007/BF00133358 (cit. on p. 88).

[82] Fangxing Li, Jiuping Pan, and H. Chao. "Marginal loss calculation in competitive electrical energy markets". In: *2004 IEEE International Conference on Electric Utility Deregulation, Restructuring and Power Technologies. Proceedings*. Vol. 1. 2004, 205–209 Vol.1. DOI: 10.1109/DRPT.2004.1338494 (cit. on p. 88).

[83] ACEA. *NEW PASSENGER CAR REGISTRATIONS BY FUEL TYPE IN THE EUROPEAN UNION, Q1 2021*. URL: https://www.acea.auto/files/20210423_PRPC_fuel_Q1_2021_FINAL.pdf (cit. on pp. 95, 96).

[84] URL: https://www.terna.it/it/media/comunicati-stampa/dettaglio/Inaugurato-a-Mirafiori-il-progetto-pilota-Vehicle-to-Grid (cit. on p. 95).