POLITECNICO DI TORINO

Collegio di Ingegneria Chimica e dei Materiali

Corso di Laurea Magistrale
in Ingegneria dei Materiali

Tesi di Laurea Magistrale

The influence of materials' choice in ski construction on subjective evaluation

Relatori
firma dei relatori
Prof.ssa Ada Ferri

....

Prof. Thomas Stöggl

....

Candidato
firma del candidato
David Cuppone

....

Marzo 2020

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Master of Science Course
in Materials Engineering

Master of Science Thesis

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Tutors
Signature of the Tutors
Prof.ssa Ada Ferri

....

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....

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....

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Sommario

1. Breve riassunto in lingua italiana

I. Introduzione

Lo studio presentato nella presente relazione prende spunto da numerose fonti bibliografiche reperibili in letteratura scientifica o in pubblicazioni di carattere più divulgativo concordi nel constatare il fatto che le vibrazioni dovute agli stimoli ricevuti dallo sci durante la discesa hanno un'influenza essenziale nel carattere della sciata e, di conseguenza, sulla sensazione che lo sciatore ha nel condurre il gesto tecnico. In molti dei documenti riportanti questa tematica, si fa espresso riferimento alle proprietà meccaniche degli sci impiegati e, in maniera particolare, si presta particolare attenzione alle caratteristiche dinamiche presentate dagli stessi.
Sulla base di queste considerazioni e delle esperienze affini a questo ambito d'interesse già maturate in campo scientifico, il gruppo di ricerca che ha condotto il lavoro qui esposto ha ideato e conseguentemente messo in pratica un metodo di studio per valutare la relazione che intercorre tra proprietà meccaniche degli sci, pattern vibrazionali che essi subiscono durante l'utilizzo e sensazione soggettiva che ne deriva allo sciatore.
Per sviluppare questa metodologia analitica, il team ha pensato di condurre dei test sulla neve utilizzando lo stesso tracciato con due paia di sci (gentilmente fornite dall'azienda produttrice Atomic®) aventi la stessa geometria ma con caratteristiche meccaniche diverse (attribuibili, dunque, unicamente alla diversa scelta dei materiali). Per valutare i livelli vibrazionali che intervengono durante la sciata, è stato deciso di usare come sistema di misura quattro accelerometri, con l'ausilio di due telecamere che riprendevano lo sciatore durante ogni fase della prova. Alla fine di ogni discesa, le sensazioni soggettive dello sciatore sono state raccolte mediante questionari di valutazione. I dati raccolti dai sensori di misura sono stati elaborati con l'impiego di fogli di calcolo per restituire grafici che mostrassero i pattern vibrazionali a cui lo sci è stato sottoposto nel tempo e confrontarli con i questionari del tester.
Infine, l'azienda produttrice Atomic® ha messo a disposizione del team un laboratorio per provvedere ai test di caratterizzazione delle proprietà meccaniche delle due paia di sci utilizzate. In questa maniera è stato possibile ricavare informazioni interessanti sulla rigidezza flessionale e sulla prima frequenza naturale di ogni paio di sci.

II. Materiali e schemi costruttivi impiegati nella realizzazione di sci

Lo strumento tecnologico che oggi si conosce con il nome di sci proviene da una lunghissima storia che risale fino agli albori dell'umanità. In un ambiente naturale in cui le temperature erano di gran lunga al di sotto delle medie registrate negli ultimi secoli, il problema di spostarsi agilmente e in sicurezza su manti nevosi è stata una difficoltà con cui alcuni gruppi umani si sono dovuti misurare quasi da subito. Le prime testimonianze di uomini che sfruttavano due assi di legno ai piedi per spostarsi con più facilità in una landa ricoperta di neve risalgono al terzo o secondo millennio a.C. e sono riconoscibili in pitture rupestri ritrovate nelle regioni nordiche dell'Europa o della Mongolia oppure sono state attestate da ritrovamenti di oggetti di risalenti a quell'epoca. Nonostante una storia così lunga, le innovazioni tecnologiche che più hanno rivoluzionato questo strumento e gli hanno dato il carattere che abbiamo in mente noi oggi appartengono quasi tutte al periodo storico che va dagli inizi del 1800 in poi. Una delle prime modifiche che ha cambiato radicalmente il profilo di questo oggetto e, conseguentemente, anche la tecnica d'utilizzo per governarlo è stata l'introduzione del camber ad opera di alcuni artigiani della regione di Telemark, in Norvegia. Il leggero profilo parabolico

dato allo strumento permetteva di distribuire i carichi più uniformemente e una trasmissione delle forze più efficace anche a livelli di deformazione contenuti.
Le prime pratiche che attestano un'attenzione più metodica alla scelta dei materiali risale alla fine dello stesso secolo, quando i produttori di sci cominciarono ad usare un tipo particolare di legno (il noce americano) in ragione di una combinazione di caratteristiche particolarmente vantaggiosa per questo tipo di applicazione.
Nei primi decenni del '900 cominciarono ad essere inserite le prime lamine di metallo sui fianchi degli sci per avere più presa e, dato il forte successo riscosso da questa innovazione, in breve tempo si arrivò all'introduzione della lamina d'acciaio continua e integrata. Quasi simultaneamente, costruttori e artigiani iniziarono a fare i primi tentativi nella direzione di sci laminati. I primi materiali da accoppiare alla tradizionale anima in legno furono fogli di alluminio e gli inserti in plastica per i fianchetti. Il primo esempio di sci assimilabile ai criteri costruttivi in uso ai giorni nostri risale al modello sviluppato da Howard Head nel 1949 che prevedeva una struttura a sandwich con un'anima in legno racchiusa tra due fogli di alluminio, fianchetti in plastica e una lamina d'acciaio continua su tutto il perimetro dello sci. Le varie parti erano saldamente tenute insieme da un cemento polimerico flessibile che permetteva anche un efficace smorzamento delle vibrazioni.
Con l'andare degli anni, viste anche le significative e incoraggianti scoperte in ambito scientifico e tecnologico riguardanti soprattutto i polimeri e la ricerca di nuovi materiali, è divenuto sempre più facile inserire questi componenti di nuova generazione all'interno degli sci. Alcune delle introduzioni più importanti e di ampio impiego sono state quelle di dotare gli sci di elementi rinforzanti in fibre (soprattutto vetro, ma anche carbonio o fibre polimeriche in alcuni casi) e la soletta in HDPE.
Una parte essenziale nella definizione del carattere e delle proprietà di uno sci è rivestita anche dallo schema costruttivo che si sceglie di adottare. Nella maggior parte dei casi, gli sci di moderna generazione presentano una struttura sandwich oppure cap, anche se non mancano esempi di configurazioni ibride. Uno schema esemplificativo di questi due modelli costruttivi può essere visualizzato in Figure 1. La prima è costituita dalla sovrapposizione all'anima centrale in legno di vari strati con diverse proprietà. I fianchi sono dritti e quasi in tutti i casi sono protetti da inserti in ABS, un materiale resistente ma elastico che consente una trasmissione dei carichi precisa ed efficace garantendo anche una buona rigidezza torsionale. In generale, questo schema costruttivo è utilizzato negli sci di fascia piuttosto alta (categoria race o race-carve), dal momento che uno strumento potenzialmente più performante spesso richiede anche più destrezza e abilità nell'utilizzo.
Per sci di fascia inferiore, solitamente si usa la costruzione cap, la quale prevede in alcuni casi l'accoppiamento di anima in legno e rivestimenti di altri materiali, mentre in altri viene adottata una struttura tubolare cava (spesso riempita con una schiuma) che dopo viene semplicemente ricoperta con uno strato superficiale riportante la serigrafia. In questo caso i fianchi sono completamente ricoperti da questo strato e generalmente la struttura presenta delle caratteristiche meno aggressive e performanti rispetto alla costruzione sandwich.

III. Metodi

Come già precedentemente esposto, la ricerca qui riportata si propone come principale obiettivo quello di riconoscere e comprendere la relazione tra le proprietà meccaniche di uno sci, i livelli vibrazionali a cui esso è sottoposto durante una normale discesa e la qualità della sensazione soggettiva che allo sciatore arriva.
Per condurre lo studio, si è deciso di usare come strumentazione di misura quattro accelerometri posti sulla superficie dello sci (tre nella parte frontale e uno in coda), accoppiati con una o due

telecamere per permettere un riferimento temporale esterno agli accelerometri. Questo è stato fatto in vista della fase di elaborazione dei dati dove sarebbe stato essenziale distinguere quali dati si riferivano a ciascuna curva della discesa.
Dal momento che non tutti i membri del gruppo avevano un buon livello di confidenza con gli accelerometri usati per questo studio, come prima esperienza è stata condotta una sessione di misura nel laboratorio del dipartimento dell'università. Oltre ad acquisire abilità nella gestione dei sensori, le prove erano volte anche a verificare i possibili funzionamenti indesiderati dei dispositivi, come ad esempio la presenza di rumore di fondo nelle misure e in che maniera influiva il limite di acquisizione massimo in ampiezza (± 16g). L'esperienza è stata condotta con due cicli di prova costituite da tre fasi differenti, ciascuna ripetuta per due volte. Nella prima fase, gli accelerometri accesi sono stati inseriti negli alloggiamenti di alluminio disposti sulla superficie dello sci ed esso è stato appoggiato su due supporti, uno in punta e uno in coda, senza che venisse applicata alcuna sollecitazione. Ciò è stato fatto per vedere se il segnale conseguente era affetto da rumore di fondo. La seconda fase ha previsto l'applicazione di una forza oscillante in corrispondenza dell'attacco dello sci ad opera dell'assistente che supervisionava ai test ed era volta a verificare che un normale ciclo di oscillazioni desse luogo a un segnale sempre interno alle due soglie massime di acquisizione e che non si verificassero eventi in input che eccedessero i limiti consentiti dalla strumentazione. La terza fase consisteva in un'applicazione di sollecitazioni cicliche come nella fase precedente ma con dei carichi di minore intensità. Ciò era volto all'avere un'idea approssimativa di come si presentano i dati registrati dagli accelerometri in un caso piuttosto vicino a quello di una normale sciata.
In seguito ai primi test svolti in laboratorio, una prima prova sulla neve è stata condotta per predisporre una procedura di raccolta e conseguente elaborazione dei dati con l'obiettivo di acquisire esperienza in merito alle misurazioni in pista e sviluppare un primo protocollo che consentisse di velocizzare e standardizzare certe operazioni in vista della sessione di test definitiva con le due paia di sci provviste dall'azienda produttrice. In questo caso si è proceduto all'esecuzione di cinque discese in campo libero secondo diversi stile e tecniche di sciata. Ogni discesa è stata filmata con una videocamera per permettere la distinzione dei dati riferiti ad ogni singola curva, anche se è sono stati riscontrati dei limiti importanti nell'utilizzare un solo strumento di ripresa. In molti casi, infatti, la convessità della pista non permetteva un contatto oculare diretto con i piedi dello sciatore e questo ha creato significative difficoltà nello stabilire il momento in cui finiva una curva ed iniziava la successiva. Per ovviare a questo inconveniente, si è deciso di procedere con due videocamere posizionate una all'inizio e una alla fine del tracciato per i test definitivi.
La seconda sessione di test è stata svolta in una località sciistica diversa dalla prima e ha coinvolto l'utilizzo delle due paia di sci appositamente costruite dall'azienda produttrice con stessa geometria e proprietà meccaniche differenti (un paio è stato progettato per essere più flessibile dell'altro), di un tracciato fisso dotato di porte da slalom e la partecipazione di un tester di professione per valutare la differenza tra un paio e l'altro.
Le due paia di sci sono state utilizzate in sequenza alternata per un numero complessivo di sei discese. Gli accelerometri sono stati posizionati sempre sullo sci destro del paio di sci utilizzato per la prova specifica e, alla fine di ugni discesa, un questionario vòlto a valutare la sensazione soggettiva è stato somministrato al tester.
I dati sono stati analizzati mediante funzioni di calcolo implementate nel pacchetto concernente l'analisi dei segnali di Matlab® che si basano su metodi piuttosto diffusi e affidabili come quello del periodogramma di Welch.
A completamento delle esperienze che il gruppo di ricerca si era proposto per questo studio, si è proceduto alla misurazione di alcune proprietà meccaniche fondamentali per la caratterizzazione degli sci che sono stati utilizzati nelle prove definitive. Questa fase è stata svolta nei laboratori gentilmente messi a disposizione dall'azienda produttrice Atomic® ed ha

permesso la valutazione sia delle proprietà meccaniche sia in campo statico (rigidezza flessionale) sia in campo dinamico (frequenze naturali). I dati ottenuti dalle misurazioni sono stati elaborati dal team di ricerca.

IV. Risultati

Purtroppo, non è stato possibile realizzare un'elaborazione definitiva e soddisfacente per quanto riguarda i dati acquisiti nella sessione di misura definitiva. Ciò è stato dovuto a difficoltà e ostacoli riscontrati nella fase preliminare di elaborazione dei dati (divisione curva per curva del set complessivo di dati). Il team ha giudicato comunque interessante e di rilevanza scientifica esporre il metodo pensato per questo tipo di studio prendendo in esame una singola curva ottenuta da una discesa della prima sessione di test. Per rappresentare la variazione del contenuto in frequenza nel tempo si è deciso di adottare la funzione spectrogram implementata da Matlab®. Si può prendere visione del grafico così ottenuto in Figure 22. La funzione di calcolo si basa sul metodo di Welch per la stima dello spettro di potenza di un segnale.
Gli algoritmi di calcolo di cui si è fatto uso per ottenere l'analisi in frequenza dei dati richiedono delle variabili di input (lunghezza e profilo della finestra di correlazione, sovrapposizione dei segmenti di analisi, numero di punti sul quale calcolare la DTFT, ecc.) che possono influenzare in maniera piuttosto significativa i risultati ottenuti. Per questo motivo, il team si è proposto di condurre una prova con i parametri che sembravano essere più ragionevoli, ma ha anche dovuto riconoscere che, per uno studio completo e affidabile dei dati acquisiti, è necessario l'ausilio e la supervisione di un esperto in analisi dei segnali.
Il gruppo di ricerca ha proceduto in seguito ai test necessari per caratterizzare le proprietà meccaniche statiche (rigidezza flessionale) e dinamiche (prima frequenza naturale) degli sci impiegati per i test.
Gli esiti ottenuti sono risultati essere in completo accordo con alcune considerazioni preliminari di carattere scientifico. Inoltre, è stato riscontrato come la differenza tra un paio e l'altro sia molto più evidente e facilmente riscontrabile per quanto riguarda la rigidezza flessionale piuttosto che per le frequenze naturali. Ad ogni modo, i risultati ottenuti hanno evidenziato che tra le due costruzioni esiste un divario in termini di caratteristiche meccaniche di una rilevanza sufficiente a permettere uno studio significativo degli argomenti che questo studio si è prefissato come obiettivi.
Infine, una valutazione generale dei questionari ottenuti per attestare la sensazione soggettiva del tester ha evidenziato che i giudizi numerici che sono stati raccolti hanno una varianza piuttosto alta e sono dispersi su un range piuttosto grande. Ciò ha fatto pensare ai ricercatori del team che possono essere considerati affidabili e rilevanti in termini scientifici perché una differenza consistente tra valutazione e valutazione implica che il tester sia stato in grado di riconoscere una netta differenza tra un caso e l'altro e questo risultato era proprio quello che ci si aspettava di misurare.

V. Conclusioni

I risultati ottenuti dall'esperienza condotta ha portato i membri del gruppo di ricerca a pensare che il metodo ideato per rilevare e distinguere i livelli vibrazionali agenti durante la sciata e correlarli in un secondo momento con le sensazioni soggettive espresse dal tester si è dimostrato adeguato e promettente. La scelta di procedere con due piai di sci che presentano differenze importanti in termini di proprietà meccaniche ma con la stessa geometria, è stata giudicata un'opzione vantaggiosa per poter distinguere in maniera più semplice e chiara le variazioni in termini di comportamento e di carattere che possono dipendere da una diversa scelta dei materiali.

Sebbene non sia stato possibile procedere ad un'elaborazione concisa e definitiva dei dati ottenuti dalla seconda sessione di test sulla neve, il team è concorde sul fatto che quella di procedere con l'analisi mediante le funzioni di calcolo supportate dal software Matlab® è una metodologia in grado di dare risultati significativi e soddisfacenti, per gli obiettivi che questo studio si era prefissato.
Gli esiti ottenuti mediante i questionari per valutare la sensazione soggettiva sono stati ritenuti validi e affidabili, sebbene questo possa dipendere dal fatto che il tester che ha condotto le prove vanta un'esperienza pluriennale in applicazioni e studi di questo tipo, motivo per cui potrebbero essere molto meno soddisfacenti se si decidesse di adottare la stessa metodologia con collaudatori meno esperti in procedure del genere.
In linea generale, il gruppo di ricerca è consapevole del fatto che molti tentativi e molte attività devono essere ancora svolte per dare un senso più completo e profondo al lavoro sin qui svolto, ma è concorde nel sostenere che il metodo con cui è stata condotta l'esperienza sia di adeguata rilevanza scientifica e che possa essere promettente per successivi sviluppi. Una delle proposte che sono state avanzate al fine di estendere il campo di questo studio, è quella di coinvolgere nei test sulla neve più collaudatori che possibilmente presentino caratteristiche diverse a livello di corporatura oppure livelli tecnici diversi. In questa maniera potrebbe essere esaminato come diversi sciatori utilizzano in maniera diversa lo stesso strumento e come questo si riflette sulla sensazione soggettiva finale.
Con maggiore attenzione dovrebbe essere valutata la differenza di proprietà meccaniche tra le due paia di sci utilizzate e, se giudicata non consistente, si potrebbe ripetere il test adottando diverse paia di sci pensati ciascuno per una disciplina diversa (slalom, discesa libera, free-style, ecc.).
Il metodo di analisi mediante accelerometri che poi ha fornito dati elaborabili mediante fogli di calcolo si è dimostrato molto versatile e promettente, soprattutto per il fatto che virtualmente si può dimostrare adatto per tipi di applicazioni molto diversi.
Essendo soddisfatto dei risultati fin qui ottenuti, il team di ricerca riconosce nel metodo di analisi qui esposto delle potenzialità che incoraggiano a uno studio più approfondito di questa materia e spera di poter conseguire importanti risultati in un futuro prossimo.

2. Introduction

The present work is an attempt in examining the relationship between skis' mechanical properties, vibrational patterns experienced on the snow and resulting subjective feeling of the skier. As materials chosen for manufacturing this kind of equipment have always played an essential role in determining the final character and properties of skis, this presentation shows how the concept of ski changed through the ages (Chapter 3) and how important innovations in materials' choice revolutionized the manufacturing methods, the possibilities of new products and the resulting technique in order to make a good use of them.
In Chapter 4 is reported a brief outline on the two principal layouts used for ski production, focusing on the properties enhanced by each of them and indicating which construction is more suitable and convenient for the different needs related to the performance sought by the skier and his technique level.
The work continues with a wide and complete overview on the materials used in modern ski manufacturing in Chapter 5. As the final properties of a material can depend heavily on the production process followed in order to obtain it, a particular attention is reserved to the aspects related to the industrial procedures and method possible to achieve the same material and how they can affect or modify some interesting features in the final equipment.
Chapter 6 provides a brief introduction to the works and experiences already performed in ski study related to the specific aspects and topics involved in this research. Moreover, the methods and the procedures adopted for every step of the project are described. After the first in-lab and on-snow tests made to have some practice with the measurement system and find a reliable protocol to follow for data collection, the presentation proceeds with the explanation of the final on-snow measurement session. In the last part, an introduction to frequency-domain data analysis and the specific methods used to extract the useful results for this study is given. Procedures and techniques followed for measuring the mechanical properties of the two pairs of skis used for the final session are described as well.
In the beginning of Chapter 7 is reported a short introduction to the Matlab® functions used for the codes of this study and particular attention is reserved to how the variance in input parameters implies modifications on the final outcomes. In addition, the results obtained from on-snow and in-lab (for skis' mechanical properties) tests are reported with some consideration regarding the meaningfulness of the achievements and their significance under a practical and scientific point of view.
Chapter 8 is dedicated to the conclusions arisen from the work and to assess the validity and effectiveness of the methodologies adopted for this project. Propositions and directions for future developments are suggested as well.

3. A brief historical overview

The history of skiing begins at the dawn of times. Evidences of this ancient practical can be found on pictographs and in ancient ski fragments discovered in some bogs mainly concentrated in the Northern European regions (around the Scandinavian peninsula) and in the Asian lands between China, Mongolia and Russia [1]. Where the skis took place for first is very hard to say, but, among the authors [2] [3], is recognized the idea that skis were a well-known means of transport by the 2000 BC at least.

For a long time, this device has been used mainly for practical reasons. At the very beginning of this history, skis were designed in order to allow an easy transportation of people and things. In addition, given the fact that populations living in the Northern regions had hunting as main subsistence resource, skis were extremely useful to follow reindeer and elk herds during cold seasons. After the XVII century, they were used also for military purposes. Around the XVIII century, a division of Swedish army trained and competed on snow skis. There are also historical sources showing that they were almost the only way, for some mountain communities gathered around mining sites in the USA, to avoid being isolated by the heavy winter snows [2]. In some places are still memorialized the mailmen covering these regions by skis with particular events occurring every year.

The slow process of changing skis from a survival practice to a recreational and touring activity took much time and it was scanned by technical innovations and new devices that made these objects easier to be handled and more reliable. One of the first most remarkable improvement in ski design was the cambered geometry, developed in Telemark (Norway) around 1840 [3]. Before that, the silhouette of a ski, looking in profile, was almost flat to the ground and this had the drawback of requiring for thick skis in order to provide a good flexural stiffness. The cambered geometry introduced a slight curvature on the ski to leave a slight gap between the ski sole and the flat surface of the ground, when it is not loaded. This would provide a more even pressure distribution on the snow, making possible to design thinner and lighter skis.

Before 1882, most of the skis were made of ash wood which has good mechanical properties and is easy to find [3]. At that time, some ski manufacturers started to produce hickory wood skis. This type of material presents a very appealing combination of stiffness, hardness and toughness, that turned into thinner and lighter skis which were more resistant to failures and to wear.

In 1926, the Austrian part-time mountaineer Rudolph Lettner [3] introduced the segmented steel edge, which allowed for a much better grip on hard snows letting the ski bending naturally. Nevertheless, there were still problems concerning edges' integrity: they needed to be screwed into the wood and, after long duty, they became loose. Most of all, it was not so uncommon that the edge piece could break in two, making the ski useless; that is why skiers had to carry with them spare edges, screws, screwdrivers and glues to make on-field reparations whether needed. Except for these drawbacks, the segmented steel edges have been widely used until the introduction of the continuous, integral steel edge in 1948.

In the previous years, some attempts were done to produce laminated skis in order to reduce the quantity of high-quality wood only to the lower parts and replace the upper with cheaper and lighter plywood or other kinds of soft woods, but the major problem was the lack of a waterproof resin which could guarantee the integrity of the object after many runs. When the chemical industry overcame this problem, the first laminated skis soon began to be produced. Around 1932 [3] the first three-layer laminated ski came to the market with an outstanding mix of properties such as lightness, liveliness and strength with respect to the previous ones. The appeal of this object, combined with the cheaper cost due to less use of noble materials, made possible for the sales to take off.

Among materials, also aluminum begun to be used in order to provide interesting mechanical properties. One of the first trials goes back to the 1945, when a group of three engineers from the Chance-Vought company took inspiration by a technical process, used for aircraft skins, to design an aluminum-laminated ski with a wood core. For commercial reasons, the production had to be stopped, but it was the first example of a mass-produced aluminum ski. Compared with the integral wood skis, these were easier to flex and much more resistant. In the years onwards, aluminum layers in the structure have been often used especially in downhill and GS (Giant Slalom) skis. In 1954, the pre-war champion Emile Allais, convinced the owner of the

Rossignol company to design the Metallais and Allais 60, which revolutionized the construction concept of high-speed skis [3].
In 1949, the engineer Howard Head launched the construction of a ski very similar to the modern equipment that can be used nowadays. It had a sandwich structure (composed by many overlapping layers of different materials) with a plywood core in the middle of two aluminum layers, presenting also continuous steel edges and plastic side-walls. All of these components came to be a single object after being loaded with pressure and heat inside a press. The breakthrough of this ski was the flexible contact cement allowing different parts to shear each other without weakening. It was one of the major commercial success in this sport.
Since 1950s onwards, many polymeric materials started to be used not only for their gluing properties. One of the first applications of plastic materials goes back to 1954 [3], when was introduced the first polyethylene sole. Soon, it got noticed that it provided good sliding properties on most of snow conditions, virtually eliminating the need for waxes. In addition, it was a cheap material and it was easy to be repaired. In later years, the research on the application of this material to skis brought to the sintered HDPE soles, which are the most diffused today.
In 1962 [3] Bob Lange invented the plastic-shell boots, which were designed to prevent mobility of the ankle joint during skiing. This radical upgrade in boot construction provided a much better transmission of the load from leg to the snow and thus a much better control. Even if this of Bob Lange was not a modification on the ski itself, it greatly influenced the way to conceive skis and the properties they are expected to possess, implying spillover effects also on the materials used to build them.
Another important change in ski design was the use of reinforcing materials. The first remarkable achievement came in 1959 [3] with the successful introduction of fiberglass. This innovation had a great impact especially on slalom skis, but research and trials in this direction has been conducted also in the following decades with the introduction of fibers of many kinds: polymeric (like Kevlar), ceramic (like carbon fibers) and many types of fiberglass. Now the research is still ongoing in this direction and, as many and many materials are discovered and come to be useful under an industrial point of view, the possibilities of further improving this old, but always new, object are still wide.

4. Ski constructions

The materials' layout used for ski construction plays a major role in the final properties and characteristics the ski will provide. Throughout the history [2] [3] [4], many attempts have been done to find new solutions to match desirable features and avoid displeasing drawbacks. The state of the art nowadays recognizes widely that most of the advantages come from a construction involving many different materials disposed in a certain way. Different arrangements between various parts, allow for different features of the final product, even if the single components are the same. Currently, the most used and well-known constructions are those called "sandwich" and "cap", even if there are also skis presenting a hybrid configuration. In Figure 1 are shown the cross-sections of these two constructions:

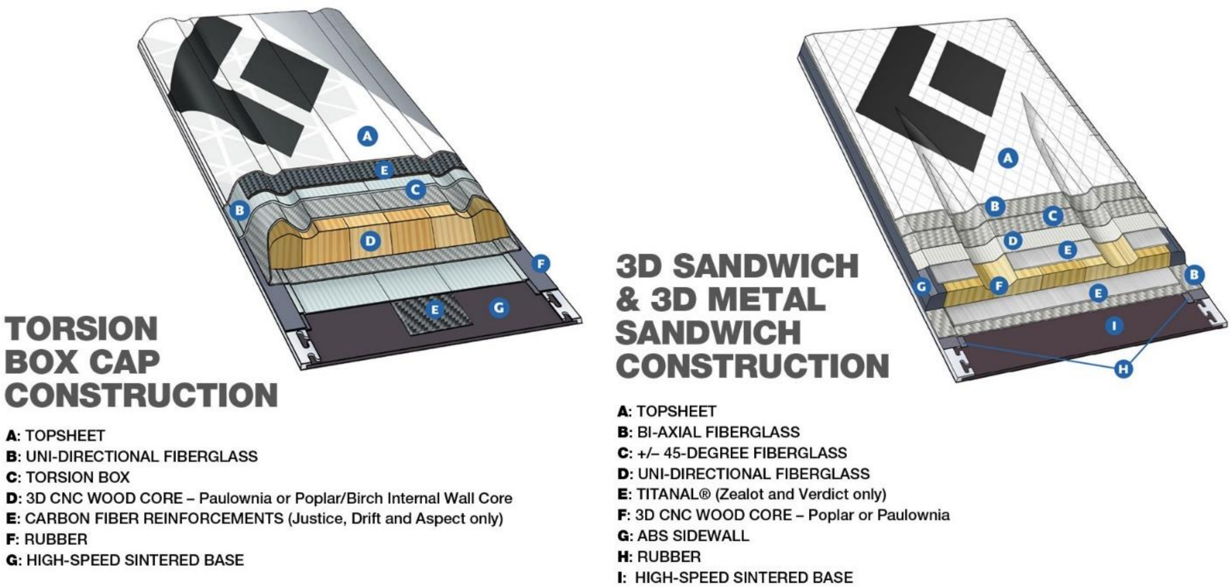

Figure 1: Cross-sections of typical cap and sandwich constructions. Pictures from [5].

At a first glance, the major difference appearing is the way the edges are realized. In the case of cap construction, the topsheet encloses the whole upper part of the ski until the edges, while for the sandwich construction the topsheet ends perpendicular to the flat lateral sides, which are protected by ABS slices. These plastic inserts are crucial to increase the stiffness of the ski so that the transmission of the load to the snow is more precise and effective. Furthermore, the extra-stiffness that the sidewalls provide, is also useful when high velocities occur and a more stable instrument is required in order to maintain a good control.

For this reason, most of times these two construction types are chosen by the manufacturers in order to match very different tastes and needs of the market. The cap construction is widely used to produce light and flexible skis with still a good torsional stiffness, hence providing a good turnability for beginners. This maneuverability is a good feature in the case of a skier at the first turns also because is more likely to forgive mistakes and incorrect use of the equipment while going downhill. On the other hand, an excessively soft ski could give bad feelings to an expert skier that would try to carve fast turns at high speeds. A low rigidity means big displacements and deformations, which could also lead to severe vibrations and inefficient load

distributions under the sole, jeopardizing the stability and the control of the skier. This is why most of times high-performance and race skis are designed with a sandwich construction, which can provide high levels of bending and torsion stiffness.
Although combining different materials in different arrangements can lead to a variety of properties which is virtually limitless, the concept of the ski lays on some ideas that are basically true for every one of them. First of all, every ski in its structure has the most performing material in the peripheric parts [2] [6], while the core has the role of a spacing element maintaining this gap between upper and lower layers. This is because the outermost areas of the cross-section are the ones affecting the most the mechanical properties of skis and so the stiffest and most resistant materials are placed here where they can act more effectively and be more useful. Conversely, since the core has no major role in terms of bending and torsion loads, it is frequently realized with light materials and often its construction is oriented towards vibration damping. This becomes a primary issue especially when the ski gets lighter, softer and shorter [2], as is the case for this equipment looking from 1860s afterwards [7].

5. Materials used in ski construction

As mentioned in paragraph 1, first skis were built by a single wooden slab. After the 1950s, many materials have been developed and gradually entered the market until becoming commonly used for industrial purposes. Many products obtained by a single material earlier came to be produced as a combination of different parts made of extremely diverse nature to get a mix of properties not achievable with a single material. Likewise, ski's manufacturing experienced radical changes in techniques and materials involved and also now development and research are still trying to get further.
As for many typical composite products, skis are made of many components all stack together and hold on by a polymeric resin. The purpose of the following treatise is to give a brief insight on the most common materials used in this field, provide some information about the production techniques of some of them that have a significant impact on final properties and try to give an indication on which materials are more suitable for which application. The direction chosen to illustrate the various components starts from the outermost parts and more familiar to every amateur of this sport and then proceeding towards the inner apparatuses, which in most of the cases are ignored by the mean consumer even if they are fundamental to define the ski's character and performance. Most of the dissertation here presented relies on the information exposed in [2] and [6].

5.1 Topsheet

The topsheet is the upper outermost layer of the ski and has the function of protecting the inner parts by wear, scratches and UV. Since it is also the main visible part of the object, is often decorated with drawings and graphics embedded underneath a polymeric film. An illustration of a typical topsheet is reported in Figure 2. Usually they are a couple of millimeters thick and the materials used for this purpose can be very diverse, such as nylon, wood, fiberglass, several kinds of plastics and composites.
The two principal techniques used to realize this component are encapsulation and sublimation. In the first one, graphics and decorations are disposed on a different support from the outer layer, which typically is a fabric, a paper sheet or something alike and then are covered with a transparent polymeric film. In this case, the choice of the inks and materials requires much attention and competence because, if the adhesion is not correct, delamination can occur and the superficial layer comes apart.
In sublimation technique, heat is used in order to enable the diffusion of the ink inside the polymeric sheet. In this case, it becomes directly the support for the ink with which it comes to be impregnated and this allows the decorations to be seen even if there are scratches and little cuts on the outer surface.

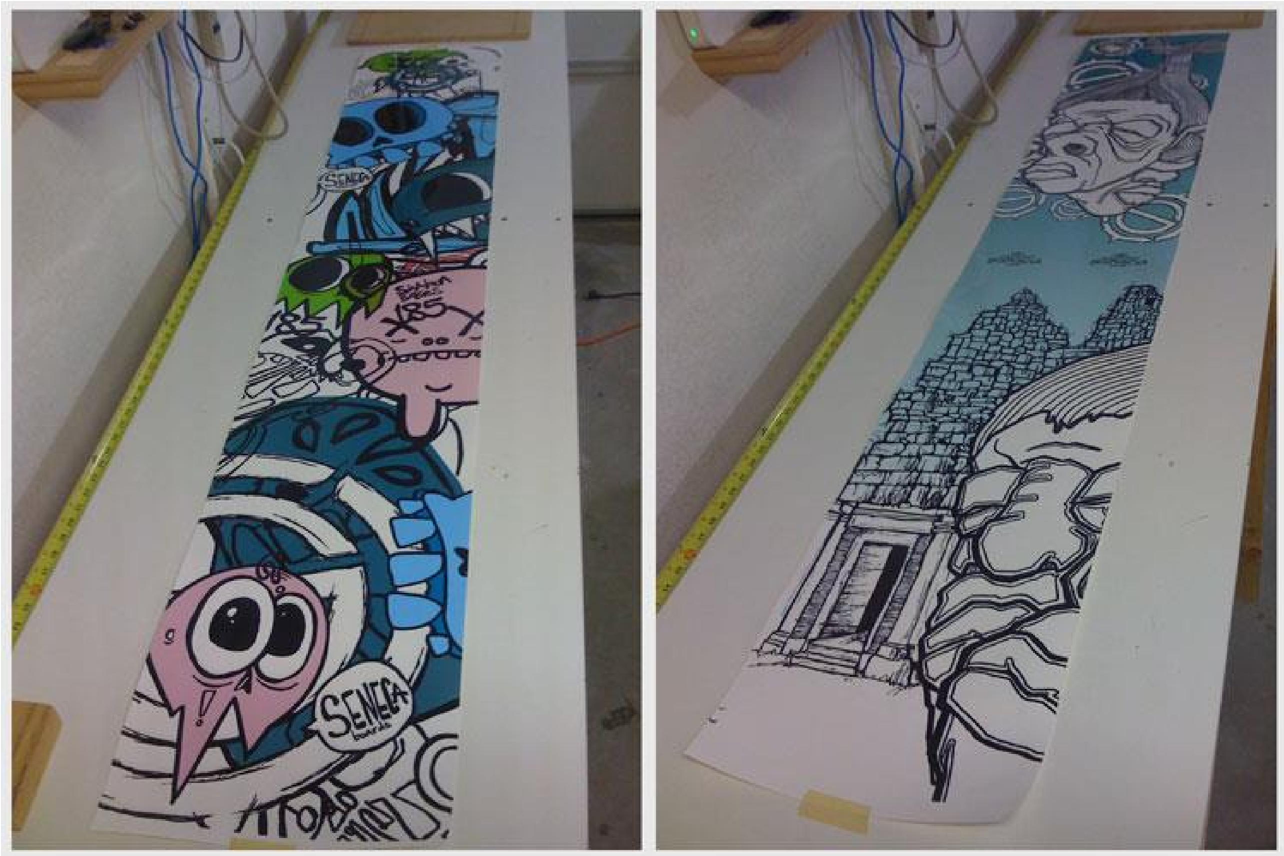

Figure 2: Example of topsheet used for ski manufacturing. Picture from [8].

5.2 Ski Base

The base plays a relevant role in ski performance and durability, since it is the part in direct contact with the snow during most of the time. Given that, at low temperatures, snow and ice grains can become very harsh and abrasive, a material capable of outstanding wear resistance is needed. On the other hand, it has to be tough enough to avoid cracks when hitting small rocks or other hard objects very likely to be on the slope. For this purpose, the almost only one material used is a particular form of polyethylene called UHMW-PE (the acronym stands for Ultra High Molecular Weight PolyEthylene), which can provide a good combination of these two main characteristics. In Figure 3 con be seen an example of a common polymeric layer used for ski construction. Moreover, its friction coefficient is very low and, as it comes from the polymerization of a non-polar molecule, it is highly hydrophobic and waterproof, which guarantees a good stability and constancy of the properties over long time and different conditions of the snow surface. Usually, it is labeled with a number which stands for the mean molecular weight of the material, normally between 3500 kg/mol and 7500 kg/mol. The higher it is, the better the properties in terms of toughness and wear resistance.

Figure 3: Layer of UHMW-PE ready to be shaped for ski construction. Picture from [9].

As well as for molecular weight, desirable attributes can also be influenced significantly by the crystallinity degree of the material. In general, the higher the crystallinity and the better are wear and impact resistance and, furthermore, the friction coefficient and water absorption gets lower. Crystallinity depends principally by the production technique used to obtain the final material. For UHMW-PE, the most used processes are extrusion and sintering.

In the first one, plastic is extruded and then cut into the desired shape. This technique is cheap and provides bases needing low maintenance, even if their durability is often low than others'. The sintering technique starts from the powder of UHMW-PE inside a mold reproducing the shape of the final component. Applying pressure and heat, the polymeric grains densifies until becoming a solid and compact slab of material. These two different processes give very different morphological structures to the final product. Extrusion is a technology that imposes a preferential orientation for the polymeric chains and this will turn into a higher degree of crystallinity and less porosity. On the contrary, the sintering technique gives numerous crystalline regions (called "spherulites" in specific terminology) with almost equiaxial shape and many amorphous spots between them. This turns into a higher porosity which will be unfavorable if the base surface is used without any treatment, but will be far easier to be

impregnated with wax and provides good retention. This is why most of the soles for race skis are sintered: waxes are always used in high-level competitions to modify in particular ways the gliding of the skis and a sintered base, with its higher porosity, will furnish a long-term effect of the treatment.
The UHMW-PE is almost never the only material the sole is made of. The common black color is normally given by the black carbon that is added to strengthen the material and provide a full-spectrum absorptivity. In this way, all the radiation in the visible spectrum coming to the ski sole, is transformed into heat and used to ease the formation of the thin melt-water film on which the skis glide [2]. Black carbon is not the only filler that is currently used to give particular features to sole's material. Many times, also graphite is embedded to provide anti-static properties, which will turn into a lower friction coefficient, and a better wax-retention. As an alternative, to lessen the friction coefficient also talc and other lubricants are mixed with the polymeric material [6].

5.3 Sidewalls

As previously mentioned, sidewalls are needed only for sandwich or semi-cap construction, and they are usually made in ABS (Acrylonitrile Butadiene Styrene). An example of typical ABS slices used for skis' sidewalls is shown in Figure 4.

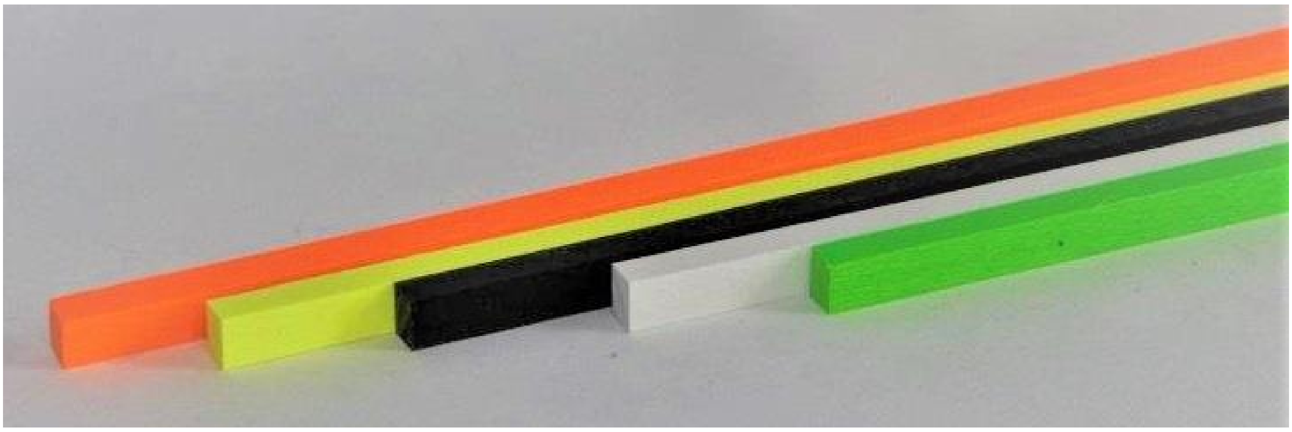

Figure 4: ABS slices used for skis' sidewalls. Picture from: [10].

This is one of the most valuable copolymers, since combines together the features coming from two thermoplastics (Styrene and Acrylonitrile) and one elastomer (Butadiene). As the thermal range in which it is normally used stands between $T_{g\text{-}PB}$ (usually <-90°C) and $T_{g\text{-}SAN}$ (around 100°C), the biphasic structure containing separate domains with very different mechanical behaviors, provides a material with outstanding properties in terms of hardness, stiffness, toughness, impact and scratch resistance. These are essential qualities for a part which is very likely to undergo cuts and shocks when the skis cross. What is much interesting, moreover, is the fact that the three components of the molecular structure can be mixed in different proportions in order to obtain specific characteristics from a wide spectrum of possibilities. This particular material is also chosen because sidewalls have to be stiff enough to guarantee a good load-transmission to the snow when the ski is set on the edge and a proper elasticity to let the ski flex in order to carry out a turn. Many times, to improve their impact resistance and give some vibrational damping, a thin strip of elastomer is inserted between sidewall and steel edge.

5.4 Edges

Edges are made of steel or stainless steel and have inserts with the characteristic T-shape to allow a better bond to the other parts of the skis. They are mounted between the last composite sheet and the sole of the ski. A typical ski edge is shown in Figure 5.

Generally, ski edges are assembled in two different configurations called full-wrap and partial wrap. For the full-wrap, the steel strip covers the whole perimeter of the ski except for the rear segment of the tail. This is the most resistant and solid layout and provides also a good protection to the tip which is the part more subjected to collisions and injuries but requires a complex procedure to be repaired, in case of failure.

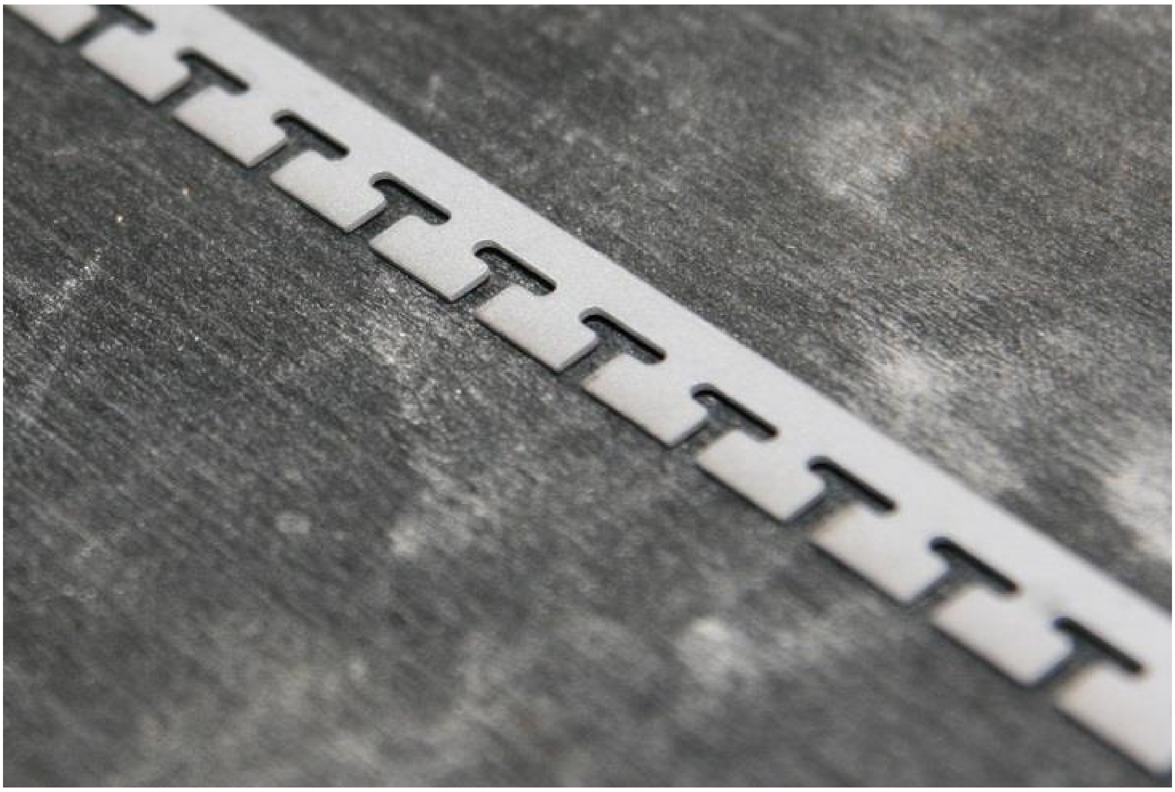

Figure 5: Particular of the T-shape inserts of a typical ski edge. Picture from [11].

For the partial wrap, the edge is laid only where the side of the ski is supposed to contact the snow. So, the two edges are two separate steel components mounted independently that leave the tip and the tail uncovered. This choice is motivated by the idea of giving a better turnability to the instrument by lightening the tip. In order to assure a good protection to the ski tip, some manufacturer has replaced the front part of the steel edge with a lighter alloy, while other ones go for hard-plastic inserts mounted with screws or rivets. In general, the fact of having a notch near the edge's end weakens resistance and toughness of the fixture and can carry to more frequent failures.

5.5 Composite layers

The part having a major role in ski behavior are the composite layers. These are inserted in the structure to provide the right stiffness to bending and twisting. In ski construction, many kinds of fibers are used in many configurations to give the appropriate character to the instrument and respond in different ways according to the loads that the ski undergoes. They can be disposed in two main ways called bi-axial wrap (when the fiber layers are shifted of 90° from one to the another) and tri-axial wrap (when they are disposed in the directions 45°, 0° and -45° on the plane). They have to be hold together by a polymeric resin and most of times they are taken by a pre-preg coil. Hereinafter is presented a wide overview of the materials used for composite layers.

Fiberglass is widely adopted in ski manufacturing because it is cheap and provides a good combination of specific strength and specific stiffness. This kind of reinforcement material is produced in several types, but the most used for ski production are type E (stands for electric, due to its high electrical resistivity) and type S (stands for structural, since is the one with the best mechanical properties).

For type E, the main constituents are SiO_2, CaO, Al_2O_3 and B_2O_3. They are the cheapest commonly produced and they still have good mechanical properties in terms of Young's modulus (about 80 GPa) and tensile strength (up to 3,5 GPa). Type S are more expensive fibers and can reach an elasticity modulus of 90 GPa and a tensile strength of 4,5 GPa. Their main constituents are SiO_2, Al_2O_3 and MgO.

Fiberglass is very sensitive to strength degradation and it can occur in many ways depending on different mechanisms. Fibers not properly protected or covered in a potential corrosive environment could heavily be affected by static fatigue, even if in the case of skis that would be very unlikely, since fibers are usually protected by the topsheet or the base. They have to be handled very carefully during processing because their high sensitiveness to abrasion can lessen remarkably their resistance if they scratch each other. Furthermore, they are very vulnerable to fatigue cracks. When this happens, the object loses its mechanical characteristic in terms of stiffness and dynamic behavior.

Carbon fibers are well known and widely used for mechanical designs, even though they began to be used in ski production not much time ago. They present a very interesting combination of stiffness and strength that could eventually also be by the production process. The main raw materials starting from to obtain them are PAN (PolyAcryloNitrile), rayon and petroleum pitch. Carbon fibers produced by PAN are treated by three main steps: stabilization, carbonization and graphitization. The whole process involves raising temperatures, inert atmosphere and mechanical stretching in order to form aromatic structures and progressively eliminate nitrogen atoms. Fibers undergoing different processing steps will turn into different mechanical properties. In this case, after carbonization, the fibers contain carbon layers not perfectly aligned, but rather rolled. These fibers are usually called "HS" (stands for "High Strength"), as their tensile strength reaches values around 3,5 GPa but a tensile modulus not higher than 250 GPa. Otherwise, if after carbonization there is also the graphitization step, the microstructure will have a more regular and overall orientation resulting in an elevate crystallinity. This affects remarkably the tensile modulus which will be much higher. That is the reason why they are called "HM" (stands for "High Modulus") and present values of tensile strength lower than 2,5 GPa and a modulus between 380 and 500 GPa.

Carbon fibers produced by petroleum pitch are obtained through a process involving three main steps (oxidation under stretch, carbonization and graphitization). Rising the temperature on the last step means to increase both tensile modulus and strength, even though the temperature required in order to obtain properties similar to fibers obtained from PAN are so high that the process is not economically convenient. A comparison between fibers obtained from PAN and pitch is given in Figure 6.

Aramidic fibers are a kind of organic fibers obtained by condensation of a dicarboxilic acid and a diamine. Their most interesting feature is the aromatic rings contained in the molecular structure, which provide them high thermal stability and high stiffness. In terms of Young's modulus and tensile strength, they have intermediate values between fiberglass and carbon fibers. Although they can be severely affected by UV degradation and water absorption which can cause a loss in mechanical properties until 50%, they are less sensitive to scratch damaging and more resistant in terms of bending fatigue with respect to fiberglass.

Kevlar® is the most common aramidic fiber used as reinforcing in composites. It is appealing for its high values of specific stiffness and resistance. Its density around 1,45 g/cm^3, which is perhaps the lowest among all the other fibers listed hereinbefore, makes it a very valid choice in order to increase mechanical properties without affecting significantly the weight. The mechanical behavior of the fibers is due mainly by both the rigid molecular structure and the several hydrogen bonds between the chains. The spinning process used to produce them, involving also a stretching stress, is aimed to align the macromolecules in the stretching direction, maximizing the interactions between chain and chain, giving rise to a huge increasement in mechanical properties. Nevertheless, the compression resistance of such molecules is quite poor. During compression, indeed, fibers tend to give rise to micro-bending phenomena which can cause the single molecular chains to move apart from each other, breaking the hydrogen bonds essentials for their resistance. In sport equipment, Kevlar® fibers are used also to damp vibrations, just like for sailing and tennis apparels.

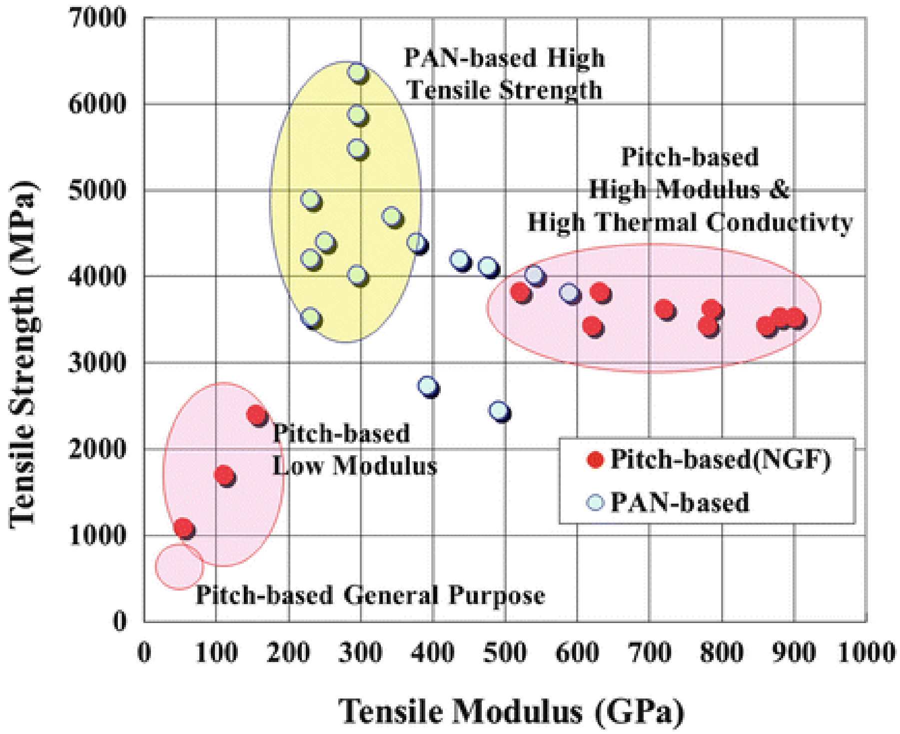

Figure 6: Typical mechanical properties of carbon fibers from different processes. Picture from: [12].

Basalt fibers are another kind of mineral fibers just as glass and carbon fibers, but they have higher resistance with respect to the first ones and are far cheaper than the second ones. They present slightly better mechanical characteristics than S type fiberglass, but have a cost between S type and E type fiberglass. Basaltic fibers are used for many purposes in many different fields, due to their high thermal and corrosion resistance and their insulation properties. Among the others, they have an important role in building and mechanical industry. In sport equipment, they are used as reinforcement in the production of skis and tennis rackets in place of fiberglass. Among the other reasons, manufacturers prefer to use this kind of fibers because, instead of fiberglass, they are a natural material and less toxic to be processed. A summary of the properties related to the main fibers used in ski construction is shown in Table 1.

Table 1: Overview of the main fibers for ski construction. Table from [6].

Material	Tensile Strength [GPa]	Young's Modulus [GPa]	Density [g/cm^3]	Relative cost
Basalt	4,15 – 4,8	100 – 110	2,65	Low
Carbon HM	2 – 2,4	380 – 400	1,95	High
Carbon HS	3 – 3,4	230 – 270	1,75	High
FG Type E	2 – 3,5	73 – 76	2,56	Very low
FG Type S	4 – 4,6	85	2,48	Low
Kevlar®	2,8 – 3	130		

Since the experience of many decades has indicated that the best characteristics for high-speed skis are achievable with metal sheets together with composite layers, most of times aluminum layers are embedded in ski structures. The most common alloy used is called Titanal®.
Titanal® is an aluminum alloy based on zinc, magnesium and copper. In Table 2 are reported the primary solutes of this alloy.

Table 2: Principal constituents of Titanal and two alloys of series 7000. Table from: [6].

Alloy	Cu %	Mg %	Zn %	Zr %
AMAG Titanal®	~ 1,7	~ 2,5	~ 7	~ 0,1
7075	~ 1,2 – 2,0	~ 2,1 – 2,9	~ 5,1 – 6,1	-
7050	~ 2,2	~ 2,2	~ 6,2	-

It was developed by the Austrian firm AMAG™ in order to produce a light alloy with interesting properties for winter sport's equipment, even if now it has many applications also in aeronautics. This specific alloy matches very well the requirements in ski construction of a lightweight, elastic, strong and tough material. As mentioned hereinbefore, the common construction used in order to produce high-performance skis and provide stability and reliability at high speeds is sandwich construction. In fact, racing skis have been found to perform better under high-speed
conditions if they have thin layers of high-strength aluminum in the lay-up, as they increase the torsional stiffness and provide useful vibrational damping [13]. Most of times, sheets of Titanal are inserted in this particular layout in order to couple anisotropic materials, just like the composite layers reinforced with fibers, with an isotropic material with the purpose of providing edge grip and solidity. The Titanal® layers addressed to ski construction undergo both a chemical treatment of anodizing with phosphoric acid and a thermal treatment T6 (quenching plus artificial ageing). The first one provides a rough surface where the polymeric matrix can easily penetrate and grab in order to strengthen the physical bond. The second one is intended to maximize the mechanical features of the alloy; in facts this can improve its properties until 20% more than the other commercial forms.

5.6 Epoxy resin

Epoxy resin is almost always used to manufacture skis. Thermosetting resins are preferred for this kind of production because they are more resistant to harsh environments, more reliable in terms of integrity and absorb less water. Some example can be found about the employment of polyester or phenolic resins, but the choice for epoxy is motivated by their higher mechanical properties, their excellent adhesion to fibers and other materials and good chemical resistance.

The most common way to polymerize an epoxy resin is to combine a prepolymer, consisting on monomers with epoxy rings, and a hardener, most of times it is a solution of amines.
The fundamental properties of the final resin depend on the cross-linking density. The larger it is, the higher will be the stiffness, the T_g and thermal and chemical resistance as well. On the other side, as the material becomes more rigid, it is more brittle and less tough too. The cross-linking density can heavily be affected by the chemical structure of the starting reagents, by the quantity and the kind of hardener used, as well as by the process conditions used (mainly temperature and pressure). In skis, thermosetting resins can be found both for impregnating the reinforcement fibers of composite layers, both as structural adhesive to hold together the several materials the skis are made of.

5.7 Core materials

In a ski structure, core has the main function of distancing the outer structural layers. Nevertheless, as it is a fundamental component in terms of volume, it has an essential role in shaping the mechanical behavior of the ski. Focusing on the static properties, its variation in thickness across the length affects strongly the stiffness profile and so the load distribution under the ski base. Furthermore, this is the component which has a major role in vibration damping. Since, in terms of volume, it is a very relevant part of a common ski, it is clear how lightness is a very appealing quality. On the other hand, a core material has to provide also a good compression resistance in order to maintain the distance between the skins also when it undergoes localized loads. Most common materials used for this purpose are wood, polymeric foams and aluminum or polymeric honeycombs.
Wood has been used for ski construction since the beginning of this practice and is still used for high-performance skis in sandwich constructions. This material presents a very particular morphology that could be called "natural composite", where the reinforcement component are cellulose fibers and the matrix is lignin, a substance based on cellulose as well. This layout gives to wood a natural elasticity and a liveliness which can't be provided by any of the synthetic materials and is steady over time.
Compared to polymeric materials, wood is capable of vibration damping more than all the other contestants and this guarantees a good feeling to the skier even at high speeds. The main drawback of wood is its weight which is surely higher than that of expanded structures like foams or honeycombs. This is why usually wood ski cores are manufactured by matching several slats of different kinds of wood, where one provides good stiffness and resistance, while the other lessens the overall weight. This could eventually give the possibility to tune minutely the mechanical properties combining several types of wood in different proportions or to design specific characters for different parts of the ski. The types of wood generally used are ash, birch, fir, poplar, beech. A traditional construction matches ash, for its specific strength and impact resistance, and fir, for its high elastic modulus and its lightness. In Table 3 are presented the main features of common woods involved in ski construction.

Table 3: Properties of woods used for ski construction. Table from: [6].

Wood type	Young's Modulus [GPa]	Resistance [MPa]	Density [g/cm^3]
Ash, black	11	88	0,526
Ash, white	12	108	0,638
Poplar	8	60	0,401
Birch	13	95	0,720
Silver fir	14	84	0,410

Polymeric foams have been used in common production processes since years 70s and experienced quite soon a very fast diffusion in many technological fields, like aircrafts and automotive. Their main vantage is in providing a light and soft material at low costs and high productivity rates. They have many applications also in aeronautics and building construction for thermal and acoustic insulation and, in ski production too, they have been widely used. Nevertheless, after a little while, it came out that these skis provided worse performance with respect to wood skis, especially in terms of stability and stiffness which are key features in high-speed disciplines. Now, polymeric foams are used only in the production of middle and low-performance skis. With this technological resource is feasible to manufacture cheap, light and maneuverable skis for a market target that, by a numerical point of view, represents the majority of the commercial skis produced.
Foams can be obtained by both thermoplastic and thermosetting polymers, even though only some of them are suitable for this purpose. In the first case, the cellular structure can be achieved by letting expand a gas inside the melt polymer and then cooling it down below the glass temperature; while, for thermosettings, the final morphology can be reached by curing. The mechanical properties of a foam can depend on the specific polymer used, the density and the particular morphology obtained after the expansion process. In general, thermosetting foams are a little bit stiffer and more brittle than thermoplastic. Open cells provide a lighter structure but less mechanical resistance (also in compression). With thermosetting resins can be obtained both open cells and closed cells, while for thermoplastic is easier to have closed cells.
Most of times, foams are manufactured by physical or chemical expansion, where some bubbling agents are mixed with the polymer at liquid state giving raise to the cellular structure that afterwards will have to be freezed, or by gas-aided injection molding, where a supercritical fluid is mixed with the polymer and, when it is injected into the cavity that has to be filled, it turns back to gas state creating the bubble structure inside the material.
The foam can be either injected or milled. In the first case, the outer shell of the ski is already formed and it has its mechanical strength, but has a cavity that has to be filled. The foam is injected from the tail of the ski and, expanding, pushes on the walls of the shell providing solidity and compactness to the skins of the item.
In the milling process, the foam core is obtained by material removal from a block of polymer previously treated to be as it has to be used. A mill carries the material to the final shape and then all the other components needed in the ski construction are overlaid to the core. The first method is the cheapest one while the second is more precise.
Polyurethanic foams are the most widely used in ski construction. They come from the polymerization of isocyanates and polyols. The final characteristics of the foam depend on the structure of the main reagents, on the catalysts and additives used to drive the reaction. Indeed, choosing accurately and mixing together in different proportions these compounds is possible to achieve several structures with very different characteristics of flexibility, toughness and weight. Usually for ski construction are used rigid or semi-rigid foams, depending on the character that the specific ski should possess.
Honeycombs are a kind of opened structures which reproduce a regular and periodic geometric pattern in two dimensions. The name derives from the fact that the first products developed in this brand had hexagonal cells, but nowadays many other shapes and profiles are used to provide specific and tailored properties. Some of them are shown in Figure 7.

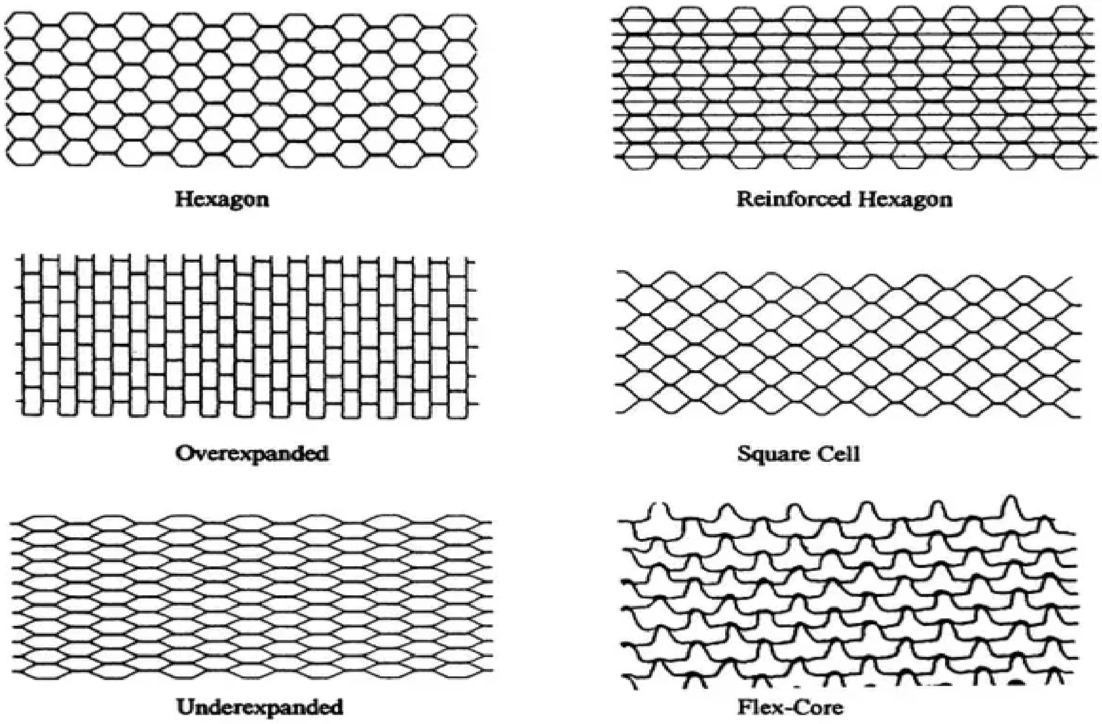

Figure 7: Common types of honeycombs. Picture from: [14].

Honeycombs are useful for several purposes and field of application and the hexagonal shape is still the most used for its good resistance-to-weight ratio. For particular application needing a higher flexibility and deformability is used the flex-core geometry.

Formerly, aluminum honeycombs have been used for their good stiffness and strength combined with a light weight. On the other hand, they are quite expensive and are not capable of an efficient reduction of vibration during the run. Now, most of the honeycombs for ski cores are in Nomex (aramidic material) which is particularly suitable for this application due to its mechanical properties with much saving in weight.

6. Methods

6.1 A brief overview on the previous experiences in ski study

Giving a general glance to the experiences that can be found in literature about ski testing and studying, can be easily noticed how many efforts and interest have been addressed in understanding the correlation and the connection between static mechanical properties and the skiing quality that they provide. Most of the experiences focus on the relationship between flexural and torsional stiffnesses and how different combinations of these can allow for diverse paces or be more indicated in particular conditions. To understand how they can influence the subjective sensation of the skier, some of them chose to rely on multi-item questionnaire [15] [16] [17]. In general, most of them agree about some fundamental considerations, like that a higher torsional rigidity provides a better control and easiness in carrying out a turn [16] [18], but it turns into a drawback on hard or icy snow [15] [19]. A high bending stiffness could mean a lower turnability at low speeds but guarantees high stability and good performance at high paces [20] [7]. Nevertheless, it always has to be taken into account how different snow conditions and skiers' expertise may require different properties [15] [20] [13].

As observed in literature [7], increasing attention and interest has been reserved to skis' dynamical properties and their influence since the early 80s onwards and many studies have been done to provide a good vibrational damping to the new products.

Having a ski with a good dynamic profile is essential in order to rely on it especially at high speeds and hard snows. The main concern is the fact that uncontrolled vibrations could lead to excessive displacements of the ski's edge bringing eventually to the loss of edge hold. Some authors pointed out how low-frequencies vibrations are the most dangerous in this case because they involve high-amplitude deformations [16]. Even if this topic has gathered rising attention by the scientific community, some studies revealed how some procedures settled to assess the main features of skis can be misleading in order to understand the important information about its dynamical behavior. For instance, it has been shown how the standard ISO 6267 is significantly uncomplete and insufficient to describe the character of a ski performing a real run [21] [22]. This is why some on-snow tests have been conducted to see the actual solicitations a common ski can undergo and recognize the vibration patterns that it experiences [17] [13]. As a consequence, many different systems have been developed with the purpose of making some trials on the phenomenon and give a particular damping profile to the instrument. Here can be recalled damping inertial mass [17], piezo-electric strips with a retroactive system [22] and polyamide sheets [23]. Although all of these experiences helped to understand and manage better with vibrational issues, the team involved in this project judged that there is still something missing in order to have a deep insight through this topic.

As the manufacturers well know and some authors already mentioned [13] [2] [6], the ski could give a bad feeling because the damping is too low, but also because it is too strong, which means to have an instrument seeming too inert and unreactive. This indicates how cannot be dealt with this problem just like there was more or less need for damping indiscriminately, but that the real demand is avoiding smartly and selectively some frequencies leaving the other ones unaltered. In order to do this, becomes crucial to distinguish between which vibrations are felt as bad and which ones as good and this is going to be the main issue of this work.

Moreover, it is clear that different skis have different mechanical configurations and show different behaviors and responses to the same vibration pattern. One of the principal interests in this study, then, is understanding what is the relationship between the skis' mechanical properties and the vibrations they undergo throughout a real run. Finally, since the subjective feeling is widely influenced by the technique of the skier and that the same ski could fit differently to diverse skiers due to their differences in body conformation, what is tried to do is also to have an idea about how much relevance has got the skier in influencing the skiing system and how this reflects in terms of overall impression.

6.2 Measurement system and in-lab tests

The measurement system chosen to carry out the study is composed by four accelerometers Physilog® 4 (version Gold and Silver) by Gait Up™ with dimensions 50 x 37 x 9.2 mm and an overall weight of 19 grams. All the features and characteristics available on the website of the producer [24]. The choice is motivated by the fact that this kind of devices are relatively simple to be handled, can work properly at the temperature and conditions normally present in a mountain environment and guarantees a high flexibility in terms of acquisition time and portability. Although this kind of sensors can collect simultaneously a wide variety of physical quantities, here the main interest is related to the acceleration in the z direction (that is the one perpendicular to the ski surface).

In order to assess and verify the effectiveness and the reliability of the measurement system, some in-lab tests have been executed.

The configuration used to place the sensors is similar to that one decided for the real on-snow tests and is composed of three accelerometers on the front part of the ski (Figure 8) and one on the back (Figure 9). They have been applied to the ski surface with a double-sided tape. The exact position of the aluminum cases is not referred as it was not interesting for the purpose of this practice.

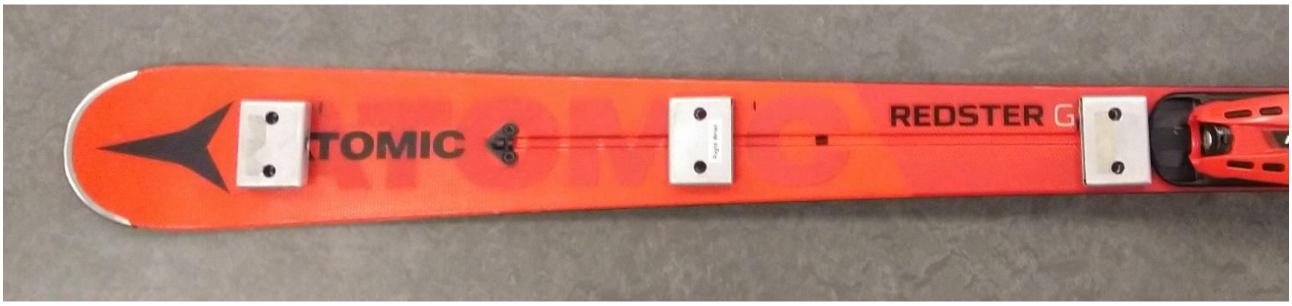

Figure 8: Aluminum cases placed on the front part of the ski to allocate the sensors.

Figure 9: Aluminum case placed on the rear part of the ski to allocate the sensor.

The sensors have been configured in order to work on the master/slave mode to let the synchronization be possible. This means that one of them have chosen to play the master role (that is to impose the timeline for all the others) while all the others worked on the slave mode (that is receiving the time reference by the signal of the Master). Once the accelerometers have been configured through the proper software, the Master has to be turned on for first and after that all the slaves. If they blink together at the same time, it means that they are synchronized.

To be sure that the data collected are meaningful, also a manual synchronization was performed. It was realized by stacking all the accelerometers in a pile already turned on and working, hold on by a tape and then applying a very strong up-and-down spike by the assistant. Since the acquisition range in terms of acceleration was set at ± 16 g, this very fast movement that they undergo all together should be a reliable and unmistakable signal in order to synchronize the time scales of the different sensors. This procedure was executed before and after every series of acquisitions.
The protocol adopted established three different types of solicitations and two times for each of them. An example of the timetable followed for the test, indicating only when every test finished, is shown in Table 4. From that, it can be easily seen how the time for each phase was a minute or even less and this is because the only interest was to have a qualitative feedback from the sensor to understand if they could fit well the needs of this study.

Table 4: Timetable of the in-lab tests

Trial	**Time (End Measurement) [hh:mm]**
Sync Event 1	11:12 , 11:57
Static 1	11:15 , 11:58
Static 2	11:16 , 11:58
High 1	11:13 , 11:59
High 2	11:14 ,11:59
Low 1	11:15 ,11:59
Low 2	11:15 ,12:00
Sync Event 2	11:17 , 12:01

Each name of the table refers to a different step of the trial and they are disposed in chronological sequence. As mentioned hereinbefore, the first action was the spike needed for manual synchronization (Sync Event 1). After that, the ski was undergone to no solicitation to detect the background noise of the sensors (Static 1 and 2) and then it was tested by the assistant providing the highest frequency vibration he could make manually (High 1 and 2) and finally it was tested with a low frequency sequence (Low 1 and 2). After the whole series, another manual synchronization was executed by the assistant (Sync Event 2). A typical graph of the accelerations detected on the tip of the ski is shown in Figure 10. Between the Sync Events and the nearest colored line can be seen some accelerations which are occurring when the devices were set in place.
In order to distinguish easily the various phases of the tests, on the Master sensor was set the synchro acquisition channel. By pushing on the power button of the Master, in the synchro channel nr.8 the device registers a square peak and this could be very useful to establish some references in time with the purpose of simplify the recognition of the time windows where the various phases are carried out. In this session of in-lab tests, this procedure was followed only in the second series of tests and this made possible to identify a major problem in this technique. When the sensors are all turned on and blink all together, the synchronization according to the Master signal is supposed to work and this means that the time scale should be the same for all of the sensors (example: the second 5 of the Master should correspond to the second 5 of all the

other devices). When this method was used, it had to be observed that the sensors maintain their own and individual time scale even if they blink simultaneously. This could bring some difficulties and undesired effects in data processing.
Being aware of this fact, the manual synchronization was recognized as mandatory in order to provide an efficient and reliable synchronization. Watching on Figure 10, it is easily seen how it represents an input easy to be identified and, since the sensors are piled up when they undergo the spike, it is sure that there is no significant delay among them. With this reference, the data can be trimmed to cut off the parts before and after the Sync Events and have data from all the sensors with all the same length and the same reference in time.

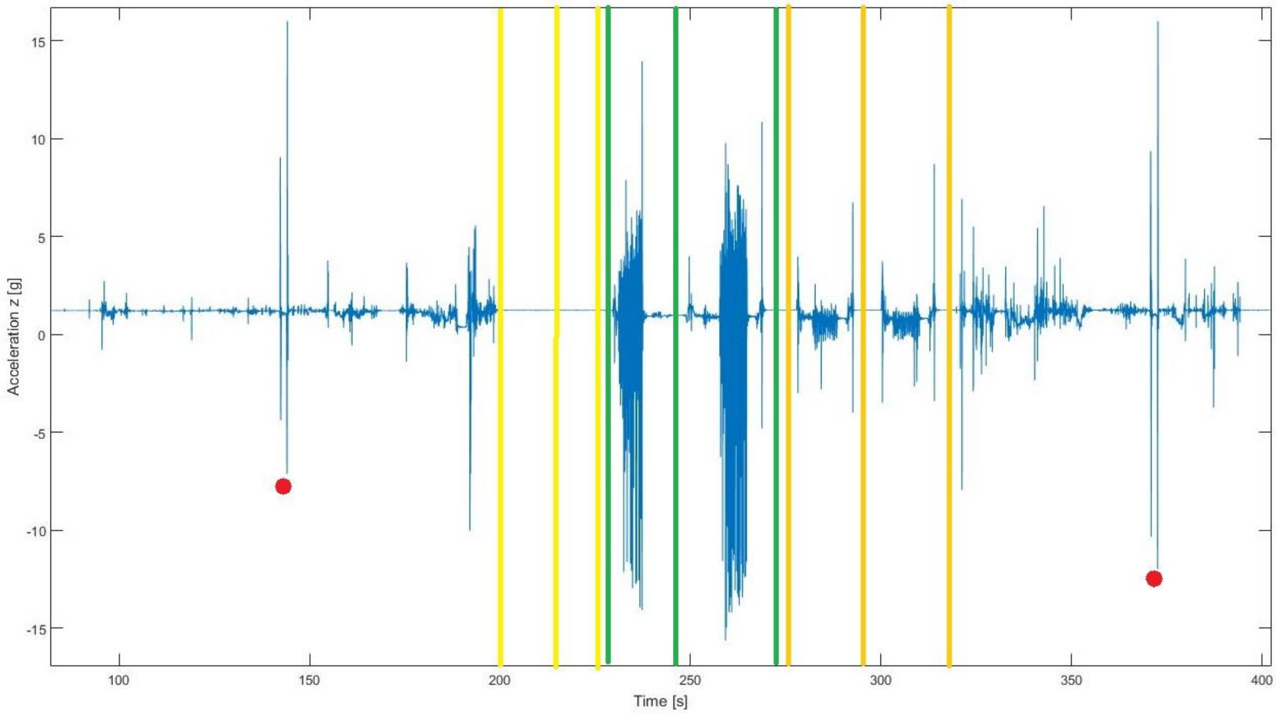

Figure 10: Acceleration detected on the ski tip. Here are pointed out the Sync Events 1 and 2 (Red Spots), Static 1 and 2 (between the yellow lines), High 1 and 2 (green line) and Low 1 and 2 (orange line).

As an additional time's reference, also spikes on the Master have been performed pushing on the power button three times before and after every series. This is not visible on the acceleration data, but is recognizable on a particular channel set on the Master for this purpose. This procedure is very useful because the spikes gives the reference of the start or the end of a series and, since the time scale is the same for every sensor, it allows to distinguish easily which of data correspond to which trial.
At the end of the experiment, the accelerometers could be extracted from the aluminum boxes and be turned off for downloading the collected data. To do this, is necessary to connect the sensors with a computer and open the files saved in the memory of the accelerometers with .BIN size. The software provided with the devices by the producer company allows to convert the original files to .csv dimension which can be easily opened in Microsoft Excel® or Matlab® in order to obtain graphs and make data processing.

6.3 On-snow tests – part 1

For the first session of tests, the location chosen was Kitzsteinhorn (maximum height 3203 m above sea level). The configuration of the accelerometers on the skis was the same for the previous in-lab tests and is shown in Figure 11. In order to avoid water infiltration, the sensors were protected with a waterproof wrap realized with a plastic film hold by an adhesive tape. The skis used in this experience have nothing with the ones supposed to support the following tests, as this session was performed only to make practice with the measurement system and the protocol for the on-snow trials.

Figure 11: Disposition of the accelerometers' measurement system (in foreground). On the right ski is mounted another measurement system involving piezoelectric sensors for other purposes and not relevant for this study.

For this application, the aluminum cases were placed on the ski with a thermosetting waterproof double tape provided by the firm 3M®. This operation required the surface of the ski to be cleaned from dust and dirt, so the best solution was to proceed with some paper wet with acetone. After that, the tape was put in place and the aluminum boxes were laid on that. As suggested by the instructions for the tape's use, the cases were loaded with 6 kg for about 30 seconds before heating up. To realize the curing of the tape, heat was provided with an electric hairdryer for about 20 minutes, taking care that all the cases were almost at the same temperature during this time, so that the process could occur in the same way for all the tape's pieces. The principal aim of this session of tests was to check how the measurement system behaves in a real-scenario run and get used to the common issues and the typical results that can occur in such an experience.
As none of the people performing these measurements were accustomed to this kind of procedure, it was taken the choice of flanking the accelerometers with a video camera. This would be useful because it states a very reliable time reference and shows at what time the skier is turning or is skiing straightly. In this case, the video camera was used to decide where a turn ends and starts the next one and the reference of this switching was chosen as the moment when the skis are flat to the ground. Since the acquisition frequency of the camera is 25 Hz and that

one of the sensors is 500 Hz, is clear that to every video frame correspond 20 points on the accelerometers and this makes the data processing much easier.
For the present purpose, it was judged to be enough to make five different runs of different length, different paces and on different slopes. A brief overview of the experience is summarized in Table 5.

Table 5: Outline of the runs performed.

Run Nr.	Pace	Slope	Number of turns	Video length (min)
1	very slow	not steep at all	8	1:30
2	very slow	same as Run 1	12	0:58
3	quite fast	steep	9	1:03
4	slow until the middle and then fast (quick turns)	same as Run 1	14	1:37
5	slow until the middle and then fast (quick turns)	steep (as Run 3)	17	1:50

At the very beginning of the experiment (i.e. before that the tester could fit the skis) and afterwards (i.e. after the last run), the accelerometers had undergone the reference spike while piled up all together. Before and after every run the tester performed a jump with the skis so that the accelerometers could have a major reference input simultaneously.
After the tests, data were collected on a hard disk and then plotted to assess the quality of the collection. The first and principal issue here observed is the fact that, in some points, the acceleration acquisition range suited by the sensors doesn't fit the input occurring during a real run. This conclusion was suggested by the fact that the plot of the data seemed to have unreasonable and unnatural cuts above +16 g and below -16 g for very large vibrations.
An example of this issue is given in Figure 12, where is shown the plot of data collected in the experience in Kitzsteinhorn. The red horizontal lines highlight the limits of the measurement range for the input acceleration (i.e. ±16 g), while the orange circles point out a part where is evident that the amplitude of the oscillations has been cut off by the measurement device.
Another issue related to the method used to collect data is related to the video camera. As the steep slope was not flat but rather convex, the feet of the skier were covered by the ground when he was far away and so the starting and ending point of a turn could be judged only taking some different reference (for instance, the posture of the knees, the position of the trunk, etc.) but clearly this cannot be as precise and reliable like watching when the skis are flat to the ground between a turn and another, as intended from the beginning. This problem is shown in Figure 13.
After a first comparison between data, the team realized that most of the times where signal amplitude exceeds the ±16 g range are due to a rapid break of the skier at the end of the run and is very rare that this case happens during a turn. Since the purpose of the present study was to detect the main frequencies occurring during the run and so this undesired effect should affect a part of data not relevant for analysis, it was judged to be negligible.
To solve the problem of the feet position, the team decided to employ two different cameras for the next test session and to locate one at the top and the other at the bottom of the slope to have always a direct eye contact at least with one of them.

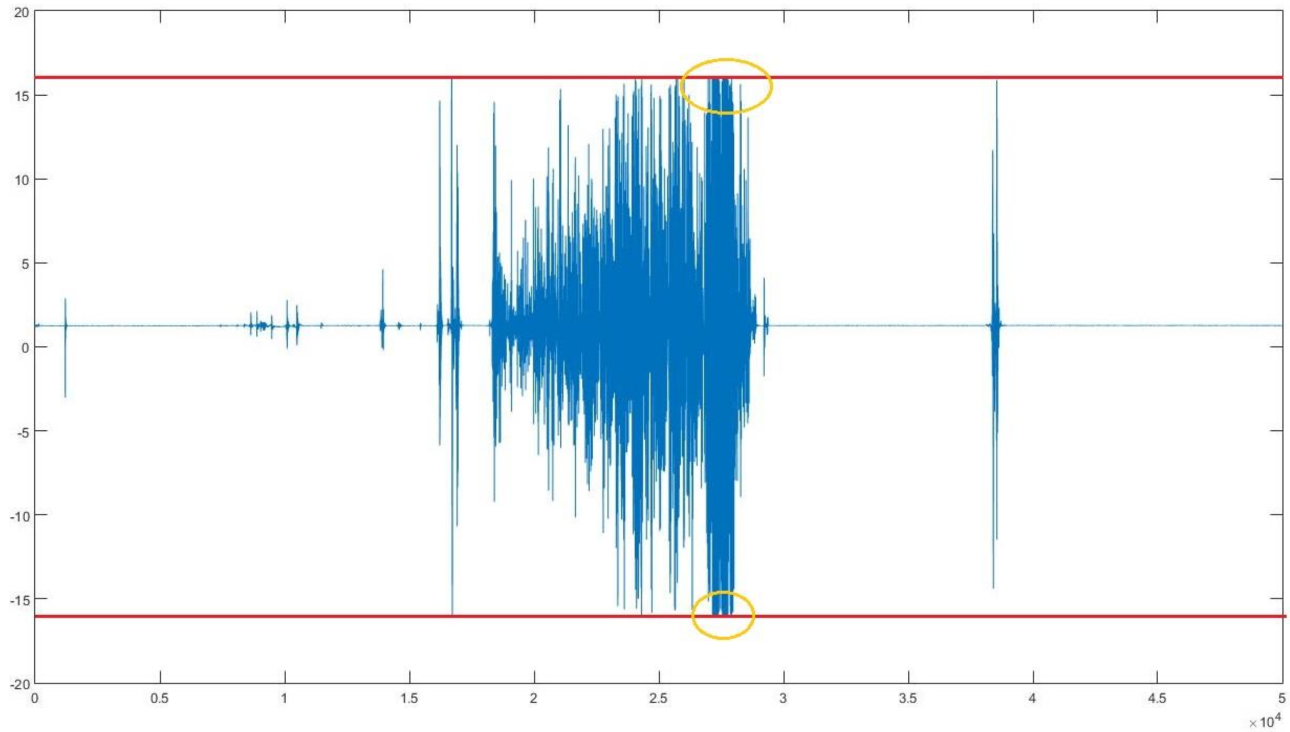

Figure 12: Plot of on-snow collected data. Red horizontal lines: limits of the acquisition range (± 16 g). Orange circles: points where is at most recognizable the peaks are cut off.

Figure 13: Tester performing the run. Notice how the convexity of the slope prevents the video camera from seeing the feet of the skier.

6.4 On-snow tests – part 2

The second and final stage of the on-snow tests was carried out in a location named Hochwurtzen (1850 m above sea level). For this session, two different pairs of skis were provided by the producer Atomic® with different mechanical properties in bending stiffness (one pair was labeled with a “W” standing for “weich”, which in German means “soft”). The firm reassured that the divergence depended on the fact that the wooden core of the softer ski was milled to be thinner than the other. As a consequence, also the plastic sidewalls were slightly narrower than for hard construction. One ski for each pair was chosen to mount the aluminum case to host the accelerometers and a similar procedure to the one followed for sticking the boxes to the surface of the ski in the first session was also here adopted. A sketch is shown in Figure 21 and the measures of the distances are given in Table 6.

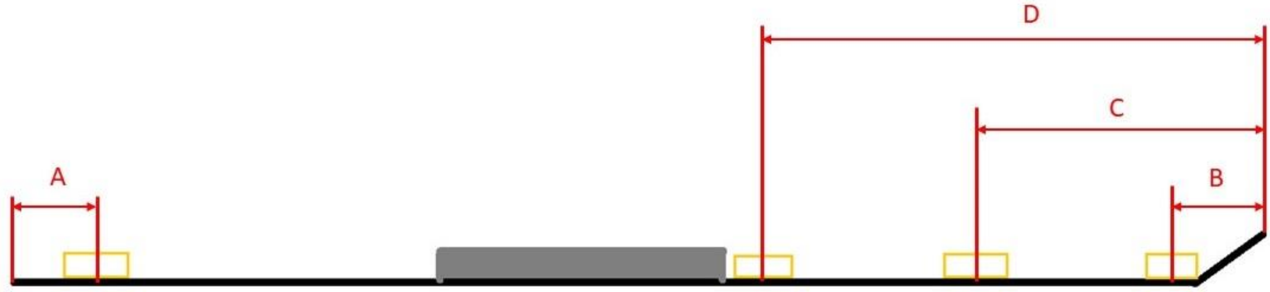

Figure 14: Disposition of the accelerometers on the skis.

Table 6: Accelerometers' positions on the skis.

	A	B	C	D
Distance (cm)	5	13	42	71

For each pair were carried out three runs in an alternative order (A, B, A, B, A, B). For all the measurements were used the same four sensors, so after every run they were taken from the old pair and placed on the new one to be used.
For this study, tests were performed on the same track realized by the men of the ski area according to the specifications given by the team conducting the study. The main features of the track are briefly summarized in Table 7.
The last parameter representing the quality of the snow (3 stands for poor, 2 for fair and 1 for good) is just an indicative information about how the snow consistence was felt to the touch, but is not a scientific measure. In this case is given only to assess that the snow was harder and more compact on the beginning of the track and softer from the middle to the end.
In this case as well, the sensors were turned on, piled up and then underwent all together a big up-and-down spike. Before every run, the tester was asked to perform a stomp with the ski mounting the measurement system, then positioning at the start of the slope, wait for the signal, perform the run and then make the same stomp with the same ski. This operation was chosen to insert some clear and unmistakable inputs defining where the run starts and where it finishes, so it would be easier to cut the single runs during data processing.
All the runs were performed by one single professional tester and, after each run, he was asked to fulfill a questionnaire about some crucial aspects involved in skiing. An overview of the questionnaire used for this purpose is shown in Appendix 1. Some items were chosen to assess general impressions (items 1, 2, 3 and 7), while others were intended to focus on the behavior of the ski in a particular phase of the turn (items 4, 5 and 6).
In order to prevent the issue related to the convexity of the ground that could eventually cover the skier’s feet, two different video cameras (one at the top and one at the bottom of the slope) were used to film every single run together.

Table 7: Main features of the track used for the tests.

Turn	Direction	Radius (m)	Feature	Snow Quality
1	R	?	First gate	1
2	L	24		1
3	R	24		1
4	L	24		1
5	R	23		2
6	L	24		3
7	R	26		2
8	L	27		3
9	R	27		2
10	L	26		2
11	R	27		2
12	L	26		2
13	R	25		2
14	L	-		2

6.5 Brief introduction to data analysis in frequency-domain

It is known [25] [26] how functions in time domain appearing hard to be interpreted and tricky to be manipulated can be converted into frequency domain by means of an operator usually called Fourier algorithms. The basic idea under this mathematical tool is that every periodic signal in time domain can be decomposed and written in frequency domain as a sum of sine and cosine functions. The general formulation of this idea is usually named Fourier series and is given in Equation 1:

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_T t + b_n \sin n\omega_T t) \tag{1}$$

Where:

$$\omega_T = \frac{2\pi}{T}$$

$$a_0 = \frac{2}{T}\int_0^T F(t)dt$$

$$a_n = \frac{2}{T}\int_0^T F(t)\cos(n\omega_T t)\,dt \quad (n = 1, 2, \dots)$$

$$b_n = \frac{2}{T}\int_0^T F(t)\sin(n\omega_T t)\,dt \quad (n = 1, 2, \dots)$$

The coefficients a_n and b_n represent the connection between time-domain and frequency-domain, as they apply an integration to a function that is the composition of the original signal F(t) and a periodic function having a frequency that is an integer multiple of the basic frequency of the signal ω_T. An example of this operation can be observed in Figure 15.

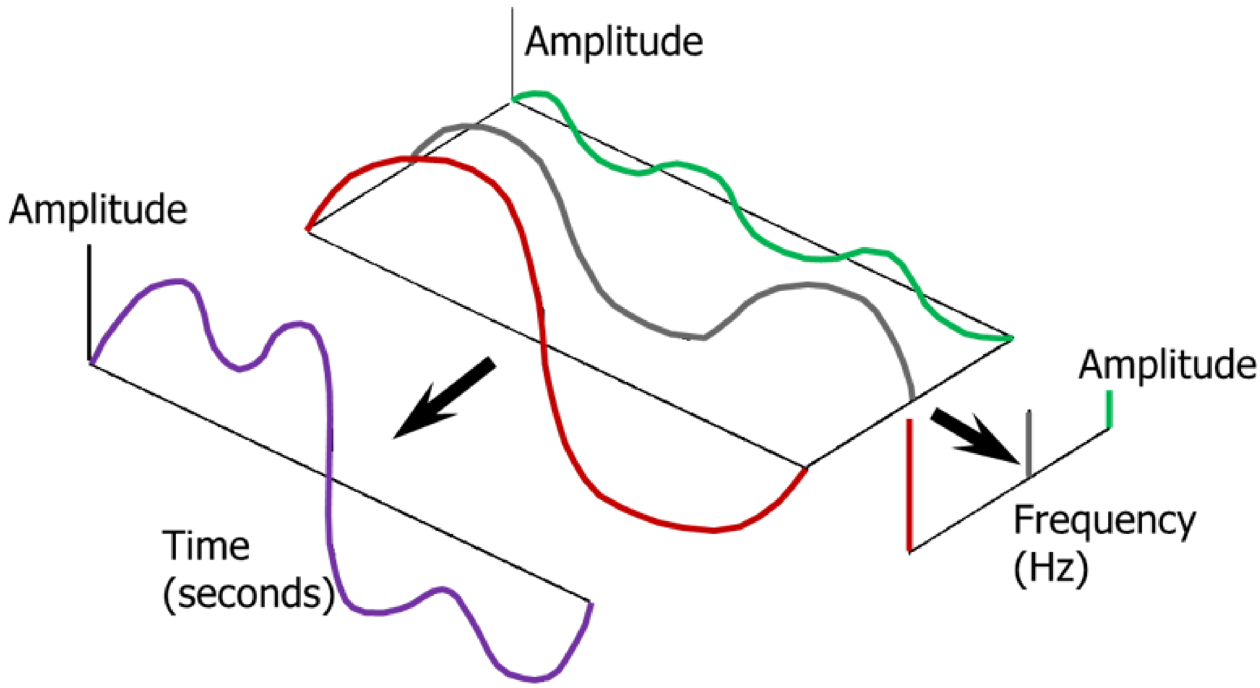

Figure 15: Application of Fourier series to decompose a time-domain signal in a frequency-domain spectrum. Picture from [27].

In this case can be seen how a periodic function in time domain maybe tricky to be understood or interpreted in its original shape, can be easily decomposed in a frequency spectrum (on the right of the picture) representing the fundamental frequencies involved in the creation of that signal with their respective amplitudes. There is also a more general form of the Fourier series called Fourier transform that extends the same concept of the periodic function to every possible function in time domain enlarging the period T from -∞ to +∞. In this way, the series becomes an integral and the result is a continuum spectrum in frequency domain.

6.6 Real data and real frequency-domain analysis

Fourier algorithms theoretically guarantee an infinite frequency resolution since their formulation rely on an endless time signal, but this is never the case for real sampling where the acquisition period must always be limited. Unfortunately, for practical applications, there is no way to achieve a continuous function in time domain, but what must deal with are discrete values collected by an analog-to-digital converter (usually called A/D converter). This implies that is never the case that there is the possibility to compute a continuous function x(t), but only a set of N values $\{x(t_k)\}$ equally spaced by the same interval Δt. In this case, N represents the number of acquisitions, k refers to the single acquisition of k-position (k= 1, 2, …, N) and $x(t_k)$ is the value of the acquisition at the time t_k. For this case, the form of the Fourier series turns into the expression shown in Equation 2 (most of times referred to as Discrete Time Fourier Transform, or DTFT):

$$x_k = x(t_k) = \frac{a_0}{2} + \sum_{i=1}^{N/2} \left(a_i \cos\frac{2\pi t_k}{T} + b_i \sin\frac{2\pi t_k}{T}\right) \quad (k = 1, 2, \dots, N) \tag{2}$$

Where the spectral coefficients are defined as:

$$a_0 = \frac{1}{N}\sum_{k=1}^{N} x_k$$

$$a_i = \frac{1}{N}\sum_{i=1}^{N} x_k \cos\frac{2\pi ik}{N}$$

$$b_i = \frac{1}{N}\sum_{i=1}^{N} x_k \sin\frac{2\pi ik}{N}$$

As the analysis has to be fulfilled on a finite set of data N-sized and they are taken with a finite resolution in time corresponding to Δt, some information is inevitably lost and is easy to incur in typical errors occurring with this kind of procedures usually called *aliasing* and *leakage*.
The first one consists in misunderstanding the frequency content of a signal by the fact that the acquisition frequency is too low and, for this reason, high frequencies present in the signal could appear as a low frequency content, even if they are not. An indicative sketch of this occurrence is shown in Figure 16. On the top of the picture it can be observed the case where many points are taken for each function period and thus the shape of the interpolating curve can fit well the profile of the original signal. In the picture below, instead, not enough points are taken for every cycle and this turns into an interpolating curve having a frequency far lower than the original signal and this will mislead the analysis of frequency content.

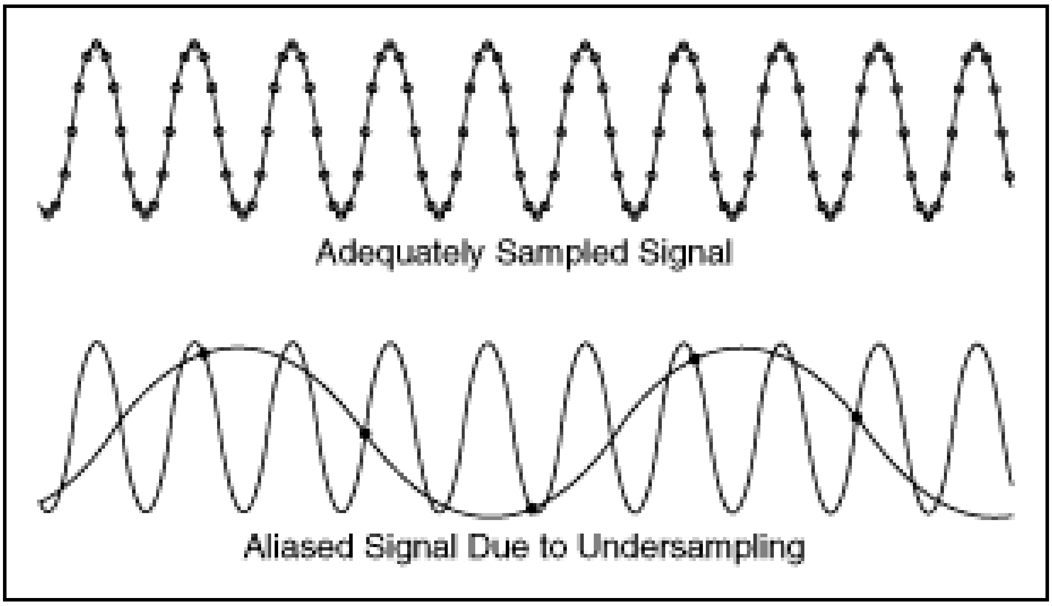

Figure 16: Comparison between an adequately sampled signal and another affected by aliasing. Picture from [28].

In order to avoid this kind of problem, the *Shannon's sampling theorem* states that the signal should be sampled at a frequency that is at least double with respect to the highest present in the signal. This frequency is also called *Nyquist frequency*, even if some studies showed that 2.5 samples per cycle is a better choice [29]. Obviously, this is not always possible and this because, in the most of the cases, such analysis is conducted for the very reason of knowing the frequency content of a signal, so is not possible to judge in advance what could be the highest present.
For the present study, the choice was to take the measurements at the highest frequency allowed by the accelerometers (500 Hz), knowing from literature [21] [13] [17] that there should be no important vibration components of the skis over 250 Hz.

Another common mistake occurring in data processing is the one usually called *leakage* and is related to the fact that the spectrum could contain some misleading frequencies. This is due to the fact that the computation of the discrete Fourier series implies that the acquisitions are intended as part of a periodic function. This is no problem in the case the signal is truncated at the exact end of its period, but if it is not, the shape of the curve will be modified and this will turn into the rising of some frequencies due exclusively to the incorrect sectioning of the signal. This occurrence is shown in Figure 17.
In order to avoid this inconvenience, most of times data are multiplied by a window that basically acts as a weighting function giving more importance to the central values than to the others located near the boundaries of the acquisition period. An example of this procedure is shown in Figure 18. Here it is clearly visible how the windowing function suppresses some frequencies next to the main one probably only due to the fact that the signal has been cut in a non-optimal way. The picture shows the application of an Hanning window to a data collection resulting in a depression of the boundary values with respect to the central, which lessen the noise and improves the quality of the final spectrum.

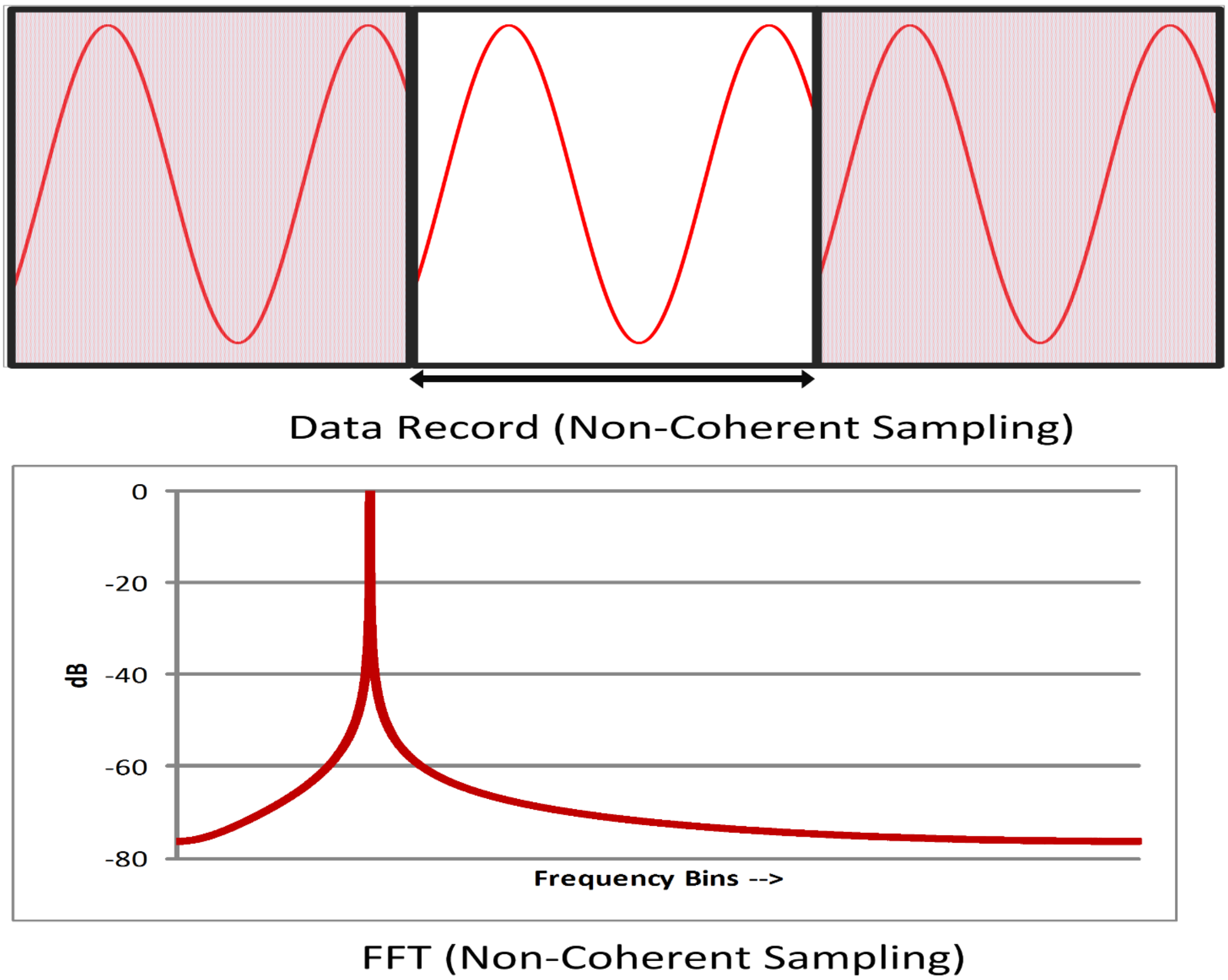

Figure 17: Broadening of spectrum around the main frequency due to leakage. Picture from: [30]

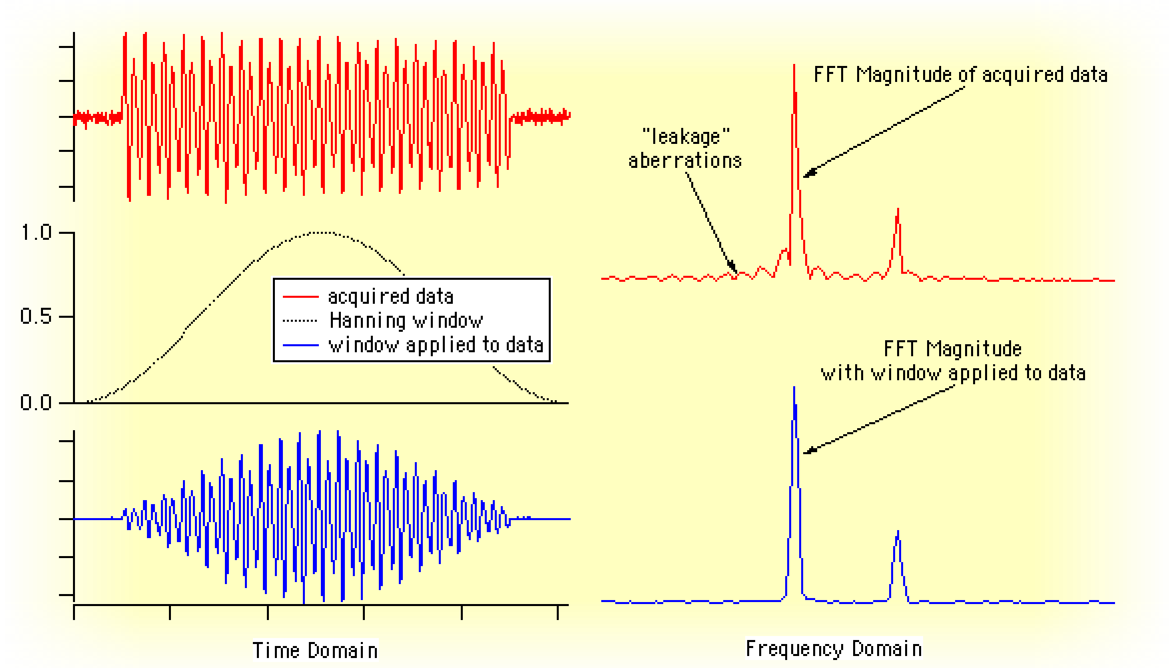

Figure 18: Windowing application on a data set. Picture from [31].

6.7 Power Spectral Density's theory and estimators

As for the purposes of this study the interest is focused on the correlation between the frequencies occurring during skiing and the subjective evaluation resulting from their action, it is essential to recognize which ones are the most important. For this reason, it was judged crucial to have an estimation of the Power Spectral Density (generally referred to as PSD) of the signal collected by means of acceleration data. Most of the concepts and formulations here explained were inspired by [32] [33] [34]. The PSD shows in terms of frequency-domain the same information that is expressed by the real data in time-domain, but the main difference is that PSD allows to recognize which frequencies acts predominantly with respect to others. The connection point between real data and PSD is power: the integral of PSD in frequency-domain must give the same total power resulting from the integral of real data in time-domain. To give a better idea of this feature, in Figure 19 is shown an example of the integration between two frequencies ω_a and ω_b is shown.

In this case, the area highlighted in red represents the power of the signal due to the frequencies between the two integration limits, called in the mathematic expression above P_{ab}. Notice how the integration needs a normalization constant of $1/2\pi$ to fit the original measurement units and give the same value of power as if it was obtained by the integration of the real data.

Let us give a correlation sequence $r_x[k]$ resulting by the expected value (that here can be approximated as the average) by the product of two functions composed by the same data set, but where one is shifted with respect to the other by k defined in Equation 3:

$$r_{xx}[k] = \frac{1}{L}\sum_{n=0}^{L-1} x[n]\, x[n-k] \qquad (3)$$

The PSD of this data set $S_{xx}(\omega)$ is given by the DTFT of the correlation sequence, as shown in Equation 4:

$$S_{xx}(\omega) = \sum_{-\infty}^{+\infty} r_{xx}[k]e^{-jk\omega} \tag{4}$$

Where j is the imaginary unit.

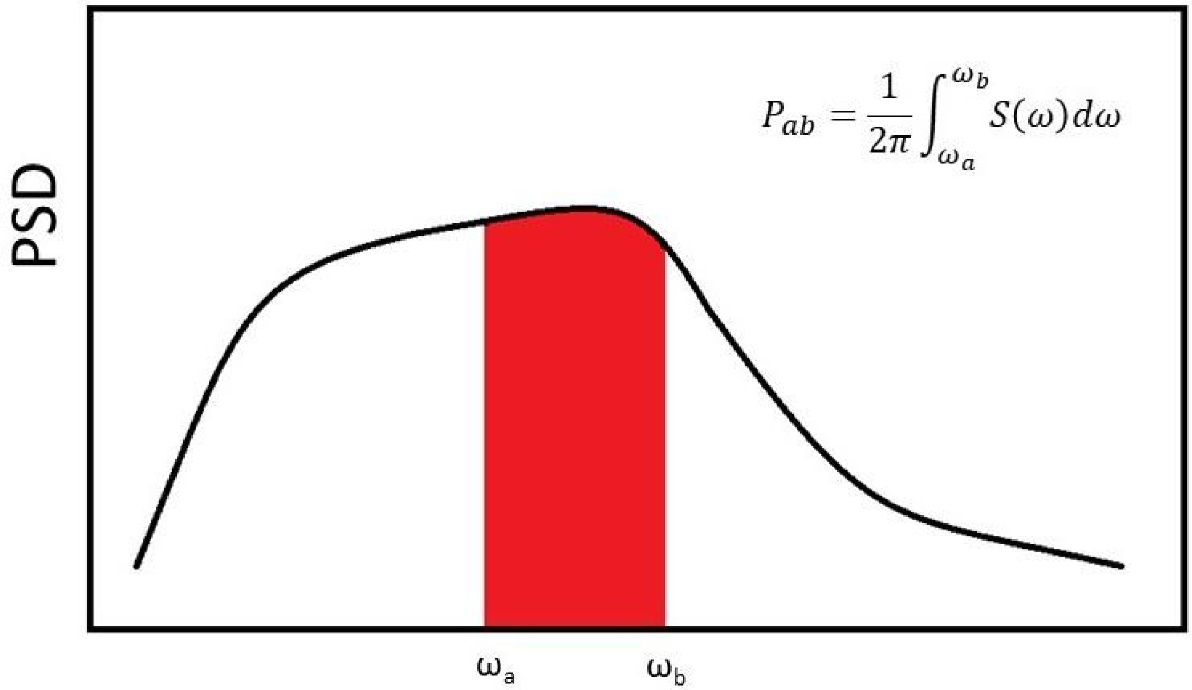

Figure 19: Integration of a typical PSD function between two frequencies.

By mathematical manipulation, if the correlation sequence is a real-valued sequence, the PSD must be an even function (means $S_{xx}(\omega) = S_{xx}(-\omega)$) and this is why in the graphs hereinafter showing only a one-sided representation will be adopted, since they represent the results of the analysis of real data that must always be real number.
There are many ways to obtain the PSD of a signal, but the method followed in this study relies on the theory of periodograms in reason of their versatility and their attitude to be implemented in processing codes. The periodogram allows to obtain an estimation of a signal's PSD basing on the average of the correlation sequence derived by multiplying the data set of that signal by a window function we can call here w[n]. If the window is rectangular, the result is called just "periodogram", while if the window has a different shape, it is called "modified periodogram". The introduction of this window function will affect also the normalization constant F that will have to take into account its module by a quantity thus defined in Equation 5:

$$F = \frac{1}{L}\sum_{n=0}^{L-1}(w[n])^2 \tag{5}$$

Recalling that the PSD defined as $S_{xx}(\omega)$ is the DTFT of the correlation sequence, by some mathematical steps, can be obtained this result exposed in Equation 6:

$$S_{xx}^{per}(\omega) = \frac{1}{LF}\left|X\left(e^{j\omega}\right)\right|^2 \tag{6}$$

Where $X(e^{j\omega})$ is the DTFT calculated for the frequency ω on the windowed function defined in Equation 7:

$$x_w = x[n]\, w[n] \tag{7}$$

This expression means that the value of power spectrum at frequency ω is proportional to the squared of the magnitude of the DTFT in that point, so the main idea of the spectrogram method is to achieve the PSD calculating the DTFT at various frequencies using the data set given as input.
To assess the validity and reliability of this method, some evaluations must be done in terms of expected value (i.e. mean value) that it can provide and the variance it is affected by with respect to the length of data set L. The procedure that allows to obtain the expected value is to make the convolution between the real power spectrum and the function C_{ww} representing the magnitude squared of the DTFT of the window function. The mathematical expression that follows is reported in Equation 8:

$$E\{S_{xx}^{per}(\omega)\} = \frac{1}{2\pi LF}\int_{-\pi}^{\pi} S_{xx}(\theta)\, C_{ww}\left(e^{(j\omega-\theta)}\right) d\theta \tag{8}$$

It can be shown how the spectrogram's expected value tends to the real value of PSD for L going to infinite. This means that, for a virtually infinite set of data, the value of periodogram in ω is centered in the value of the actual PSD and this means that it is a precise estimation method (virtually unbiased).
The mathematics behind calculation of variance is slightly more complex but the final result is that the variance of the method is proportional to the squared value of the real PSD. This means that, the more a particular frequency ω is present in a signal, the more periodogram will tend to give different final results and this fact does not depend on the number of samples collected. This suggests that periodogram is not a consistent estimator, as the variance of a consistent estimator should tend to zero when the number of samples goes towards infinity.
For this very reason, the aim of Welch's method relies on the fact that by averaging k independent functions the variance is reduced by a 1/k factor to lessen the variance of the spectrogram computation. Let us take a set of data as in Figure 20.
The data set (long Q acquisitions) is divided in k segments of length L with, eventually, some overlap between one another. The new convolution function between the data and the window function can be written as in Equation 9:

$$x_r[n] = x[rR + n]\, w[n] \tag{9}$$

Where n = 0, 1, …, L-1 and r = 1, 2, …, k.
Each of the segments thus obtained is used to compute a periodogram $I_r(\omega)$ and then all the periodograms are averaged together. Mathematically, this concept can be expressed with Equation 10 and Equation 11:

$$I_r(\omega) = \frac{1}{2\pi \sum_{n=0}^{L-1} w[n]^2}\left|\sum_{n=0}^{L-1} x_r[n] e^{-j\omega}\right|^2 \tag{10}$$

$$S_{xx}^{av}(\omega) = \frac{1}{k}\sum_{r=1}^{k} I_r(\omega) \tag{11}$$

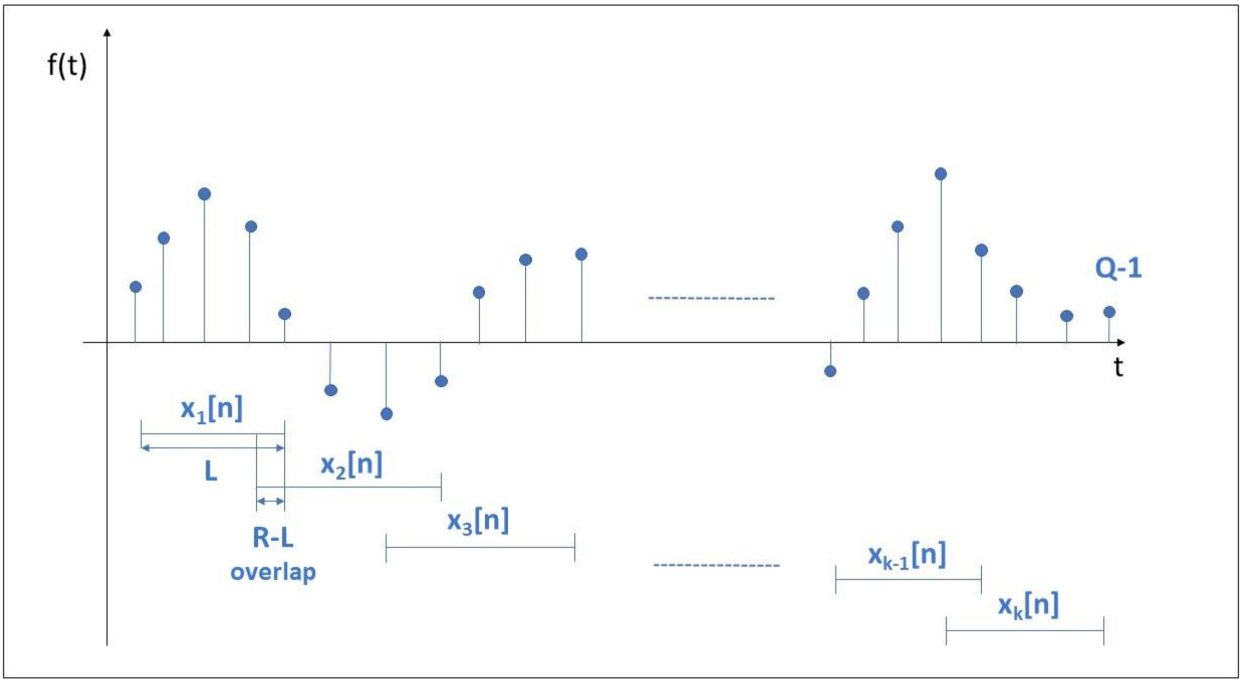

Figure 20: Application of Welch's method to a set of data.

From statistics it is known that the variation of this average must be contained between that one of the original periodogram and the one of the group of segments if they were completely independent (means without any overlap). This means:

$$\frac{1}{k}Var\{S_{xx}^{per}(\omega)\} \leq Var\{S_{xx}^{av}(\omega)\} \leq Var\{S_{xx}^{per}(\omega)\}$$

In data analysis, a major influence is played also by the profile of the window function used for convolution. A rectangular window (providing sharp edges at the limits of the segments, will turn into a better resolution but will affect the averaging procedure, since in the overlap the contiguous sections are far from being independent. On the other hand, choosing a window fainting towards the boundaries (like the previously mentioned Hanning window) allows to have a better result from averaging but worse outcomes in frequency resolution.
In general, given Q as a non-modifiable parameter, k and L can be varied according to the relationship in Equation 12:

$$kL \leq Q \tag{12}$$

Where, as a recall, k is the number of segments the total signal is divided into, L is the number of acquisitions that each segment contains and Q is the total number of acquisitions.
In particular, k has a major influence in reducing the variance, since it affects directly the constant 1/k expressing the factor of variance reduction at most, while L is involved in the expression of the expected value. Remember how hereinbefore was explained how the mean value of periodogram tends to the true value of PSD if L goes towards infinity. The fact that these two variables must be chosen accordingly to satisfy the previous relationship, means that there is a competition between which one rises turning into a decrease of the other one. Most of times, there is no suggestion or clue given by mathematics to make the right choice and what

makes the difference between a successful approach and a misleading attempt is the expertise of who analyses data.

6.8 Mechanical properties' tests

As the main goal of the project was to estimate how different mechanical properties of different skis can turn into different behavior in skiing and so different subjective feeling, an essential part of the work was to assess and define the mechanical characteristics of the skis used. All of the measurements were carried out in the laboratories of the ski company Atomic® and data collected were processed by the team of this study.

One of the most important and meaningful tests that usually are made on skis is the measurement of the bending stiffness along the ski length. In most of the cases, what is used is a three-point bending machine with a piston loading the ski on the binding against two supports, on the tip and tail. In this case too, a similar procedure was adopted and in Figure 21 can be seen a symbolic sketch of the layout involved in the measurement with the respective forces and distances outlined in

Table 8. Unfortunately, photos of the real machine were not allowed.

A	B	C	D	E	F
140	55	35	680	730	900

The measures of the unloaded and loaded profile were taken by an automatic laser device running all along the sole of the ski. The distance between two contiguous measurements could be set and, for this study, the choice was to take one every 10 mm, that guarantees a very accurate detection of the shape.

Table 8: Dimensions for the bending stiffness test (mm).

A	B	C	D	E	F
140	55	35	680	730	900

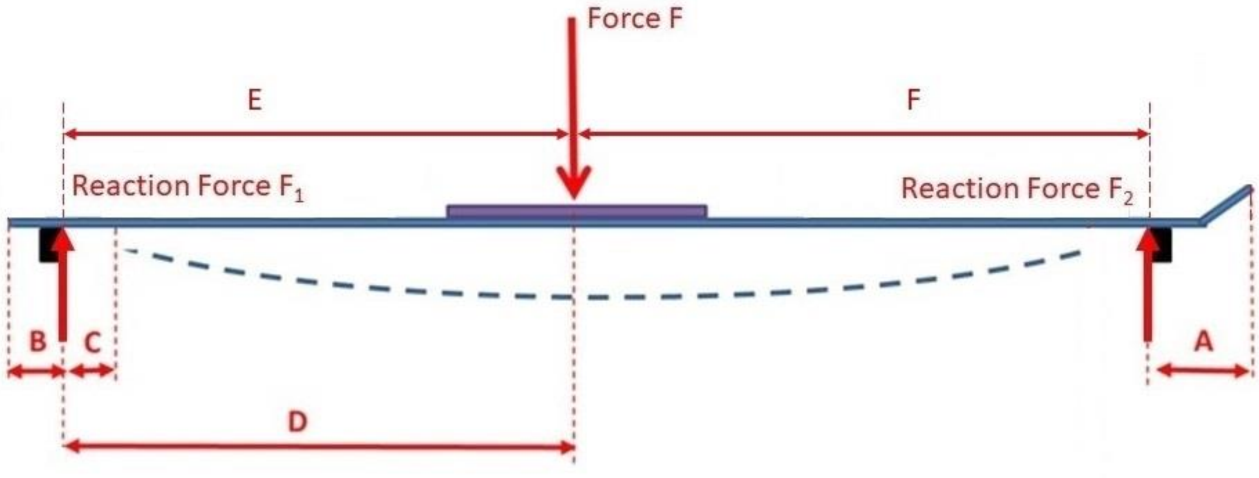

Figure 21: Symbolic representation of the scheme used for the bending stiffness test. Note: the length of the forces are not to be intended as proportional to their module.

Since the main goal of this study was about ski's vibration behavior, the team performed also some tests in order to assess the dynamic character of the skis. This was judged to be very important to have a whole and complete characterization of the skis because the static mechanical properties measured by means of bending machine are not necessary indicative for the dynamic behavior. Of course, the natural frequencies of a body depend on the stiffness

according to the well-known relationship $\omega_n = \sqrt{(k/m)}$ (where ω_n is the natural frequency, k is the stiffness of the body and m is the mass), but during vibration can have a major role other components (like damping effects of some materials) that cannot be correctly estimated from static tests alone. The machine employed for this purpose was a device usually dedicated to impact tests to assess the mechanical resistance and integrity of the equipment after a collision of the forebody (simulating landing after a jump). This means that the machine was not specifically designed for the aim the team used it for, but it allowed to clamp the skis at their binding plate, impose a displacement value and let the ski part oscillate freely (i.e. without contact with any other body during oscillation) until needed. The team judged it was good enough to suit the purpose of this test.
In order to detect the skis' natural frequencies, two different measurement systems were used: one was a one-axis accelerometer owned by the company and the other was one of the three-axis accelerometers already used by the team for the other tests. Unfortunately, the data collected by the first accelerometer were not accessible since they were stored in an informatic format not suited by the software used for data processing.
The aim of the measurements was to evaluate the first natural frequency of the skis used with the purpose of making a comparison between the two ski pairs and the vibration levels obtained by the on-snow tests. The choice was to measure both the tip and tail oscillations of both skis placing one single three-axis accelerometer at the most distant aluminum case from the binding plate. This is motivated by the fact that, not being interested in studying the mode shapes and frequencies of the higher modes, the team relied on the fact that every part of the ski oscillates at the same frequency but with a different displacement (parts far from the binding will vibrate largely since they are far from the clamping). This allows to obtain larger input values that will make the noise content less significant and hence help in data processing.
For every ski part, three different initial displacements were set and three different tests were performed for each displacement level, so that the data obtained could have a solid statistical basis. The tip of both skis was tested with displacement levels of 45, 37 and 30 mm, while the tails with 30, 20 and 10 mm. The device imposing the displacement was 285 mm and 200 mm, respectively for the cases of tip and tails, far from the end of the ski. Firstly, were performed the tests for the tip of both skis and subsequently for the tail.

7. Results

7.1 Working with Matlab®'s pwelch and spectrogram functions

Matlab®'s toolbox implements Welch's method with a function named pwelch and it allows for several syntaxes. For this study, the syntax chosen was one of the most complete and appears in this form:

```
[pxx,f] = pwelch(x,window,noverlap,nfft,fs)
```

Where, as input varibales, **x** represents the data set, **window** is the window vector indicating the length and the profile of the windows the data set must be divided into. If it is an integer (as in this case), the data set is divided into segments of that length. If no input value is provided, the software divides automatically the data set into eight segments. If not otherwise specified, the shape of the window will be automatically chosen as Hamming window.
The parameter **noverlap** stands for the number of points overlapping between two contiguous windows. In this case, the choice was to decide the percentage of overlap (50%) multiplying this constant for the number of points in a single window to return a number of points.
The value of **nfft** indicates the number of points taken to estimate PSD. The number **fs** represents the sampling frequency of the data set.
The output variables returned by pwelch function are the coefficients **pxx** of the PSD estimated and **f** are the respective frequencies going from 0 to the half of sampling frequency (in this case the anti-aliasing filter is provided by the software).
The Matlab®'s toolbox is suited with a very interesting plot function named "spectrogram" allowing to show, by means of chromatic scales, how the frequency content of a signal vary in time. The function's syntax used for the purposes of this study is pretty much the same of pwelch function already mentioned hereinbefore and the methods used by this function to obtain the PSD coefficients are similar to pwelch function.

7.2 Input variables in code functions for data processing

Data collected from a motion analysis test usually need some kind of pre-processing, like **filtering**. In this case, data were subjected to an informatic filtering process in order to delete some frequencies present in the signal that are assimilable to noise. In this case, we refer to some frequencies that are not interesting for the purposes of this study and that could have an amplitude comparable or even larger than the meaningful ones. Since by experience and literature [35] it can be verified that a common turn performed by an advanced or, eventually, race skier can last more or less one second, this means that at 1 Hz can be detected a major frequency content that is not due to ski vibration, but to the bending change from gate to gate. For this reason, the team chose a threshold of 2 Hz as the lower limit of the high-pass filter.
The procedure was performed with a fourth-order high-pass filter implemented in the Matlab®'s toolbox (function `butter`) and then applied by means of the function `filtfilt` in order to avoid the phase shift that this operation could imply.
Another issue examined by the team was the **length of data set**. The principal aim was to try if, increasing the size of data set, there was no influence on the final result. For a positive outcome, the idea would be to enlarge the data sets of all the other turns to the longest and then

using all the same values for the other input parameters. The sizing of data set was realized by a function developed by the department where this study was conducted (Department of Kinesiology and Sport Sciences, University of Salzburg) and basically relies on the reproduction of the same curve's shape on a data set with different length by means of a linear interpolation. Matlab® too implements such a function named `interp1` but the one provided by the department was faster and less demanding in terms of calculations for the system. The form used in this work presents the following syntax:

```
Data = ScaleTime(Data_old, Time_in, Time_end, DP_In)
```

Where Data is the new data set from the function, Data_old is the original data set, Time_in is the first value to take as input, Time_end is the last one and DP_In is the length of the new vector Data.
By increasing data length, two major effects are visible: the resolution in time tends to get better and the frequency content slides to lower frequencies. For the first effect, the improvement of frequency resolution is totally reasonable, as, for the PSD calculation, having a larger set of data is the same of having collected data on a larger time period (as a recall, frequency resolution Δf depends on sampling period according to the relationship $\Delta f = 1/T$, where T indicates the sampling period) and this turns into a refinement in frequency domain for the properties of the PSD spectrum.
The feature of the frequency content going down to lower frequency for rising data length is more intuitive than what could be expected and is related to the fact that if the same signal is spread on a larger time scale, the frequency contained in that signal seems lower that the original. This is a similar effect to the distortion of voices when some video-clip is reproduced in slow-motion. Sounds seem lower because the same frequency content is spread over a larger period.
The variation of the **window length** turned into a major modification in time and frequency resolution and they act as competitive variables (that is, if the first increases, the second decreases and vice versa).
By reducing the window size, is clear that the PSD evaluation is more accurate in time domain, since it tends to analyses tiny, small samples very well confined in time. On the other hand, the PSD algorithm has less and less points to evaluate the frequency content, so it could be hard to obtain large difference between the same small window and the sinusoid of several frequencies. This is why the power spectral coefficients would have pretty much the same value and the resolution will drop.
The variation in **noverlap** has implications in time resolution. The more noverlap increases, the more it gets better.
After some trials and debates, it is still not clear the relationship between this parameter and time resolution and the team didn't manage to find an acceptable reason to explain why time resolution experiences this variation by modifying this parameter. A possible hypothesis is related to the fact that, if on the same time period insist many windows, maybe it can be better estimated by several functions and this could improve time resolution, but this idea has to be verified.
Nevertheless, this effect could have a silver lining to allow for the identification of some small periods in time where most of the vibrations occurs, since the stripes are distinguished very clearly.
A drawback could be some vertical shades (they look like flames) comparing on the upper part of the graphs which are very unlikely due to real vibration patterns, while is very probable that they come from noise in computing the PSD coefficients. This is absolutely in agreement with the theory of periodogram and Welch's method saying that the gain in variance reduction is

lower when overlap is high. This is related to the fact that segments are less and less independent, so the averaging procedure is not that effective and some noise can occur, especially where the real value of PSD is large.
The variation in **nfft** gives a significant change in frequency resolution, even if time resolution too seems to be slightly influenced in a positive way.
As a recall, the team reminds how Matlab®'s help toolbox states that this number should always be a power of 2 and is not a wise choice to put a value smaller than window length.
The increasing frequency resolution occurring for larger values of nfft can be explained by the fact that they have, virtually, the same effect of widening the number of points used by the PSD algorithm to calculate spectrum coefficients. As stated hereinbefore, this gives the same outcome of sampling for a larger period of time and turns into a finer frequency resolution.
In general, this parameter should be chosen not too high because, otherwise, the zero padding segments exceeding the real window length will weight too much with respect to actual data and this could imply modifications in peaks' position and intensity in the spectrum.

7.3 Frequency analysis on real data

For this study, data processing came to be much more complex and demanding than what expected and no definitive results could be obtained. This was mainly due to the fact that a crucial step was to isolate every single turn and then making the average of the spectrum coefficients both for inner and outer ski for each run. The procedure required to cut turn by turn from the original data with the assistance of video-clips was not so efficient and reliable as imagined, since in some cases it was not clear where was the common time reference of the cameras and accelerometers. Moreover, the Matlab®'s function spectrogram was designed to return the PSD during a time period from a single set of data and not the average between many of them. Surely, a solution can be found for this code issue, but the team conducting the study was not able to find a satisfying way out during available time.
The results here shown have the only purpose of giving an idea of the method followed to make the comparisons between data collected in the on-snow test sessions and the questionnaires for subjective evaluation. Unfortunately, for the team there was no chance to proceed with the final data collected in the second session of on-snow tests, thus this presentation is intended to show the typical results obtained from the spectral analysis of a single turn using the code functions explained hereinbefore.
In Figure 22 is reported a plot of spectral analysis performed for a single turn taken from the first session of on-snow tests.
The input parameter for PSD calculation were chosen as follows:

- Data length: 512 points
- Cut-off frequency of high-pass filter: 2 Hz
- Window: Hamming (standard) window of 200 points
- Noverlap: 50% (standard)
- Nfft: 256 points

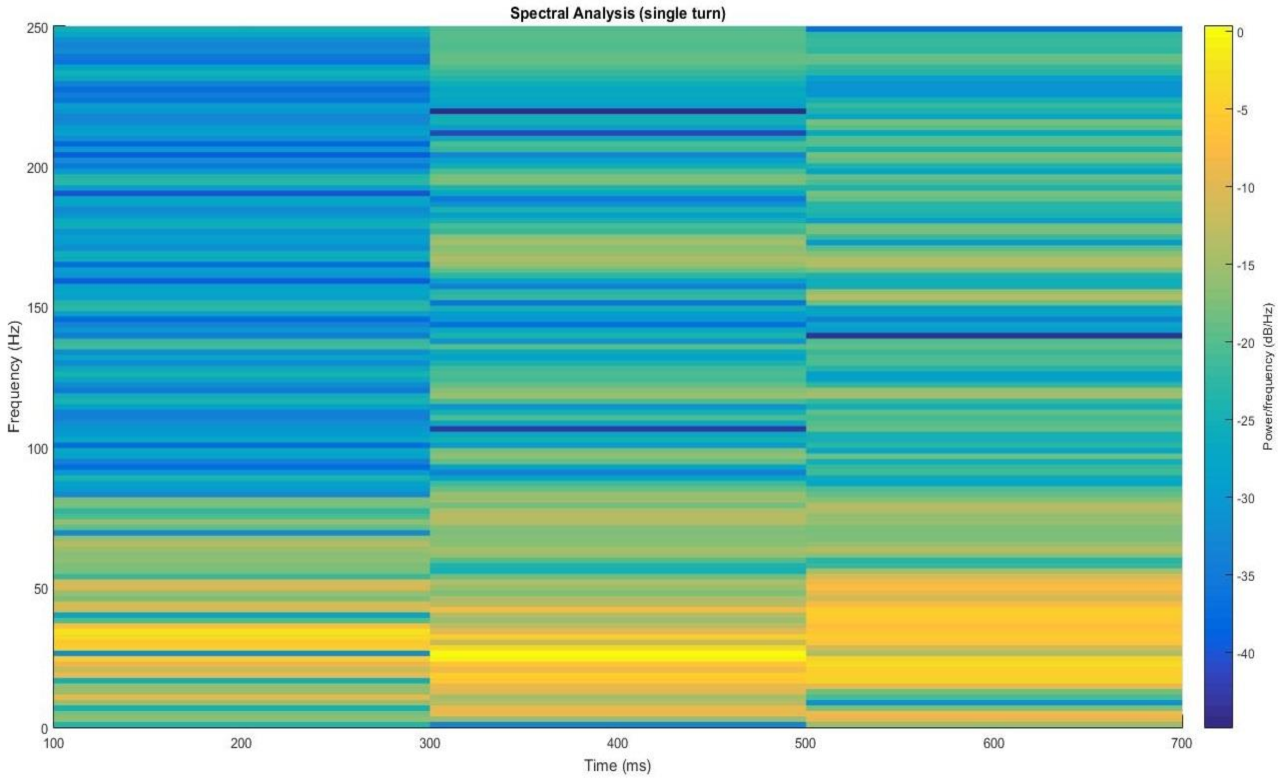

Figure 22: Spectral analysis for a single turn.

The first parameter (data length) was chosen to be the closest power of 2 to the original signal length (490 points). The 22 points added were all zeros according to the zero-padding method. This was mainly made to resize the data set to a power-of-two length allowing for fast calculations.
The cut-off frequency of the high-pass filter was set to 2 Hz according to literature reference for skiing vibration patterns [35] in order to have a spectrum representing only the frequencies of the signal due to skis' vibrations and not to the bending deflection the skis undergo from gate to gate.
The window length was chosen of 200 points because it was the shortest allowing for three-stripes representation in time domain (corresponding respectively to beginning, middle and final part of the turn). For higher values, a dispersion of the power was observed and hence the team supposed that 200 points was the best choice. The type of window used was Hamming window, according to Matlab®'s standard, since generally provides a good trade-off between leakage distortion and power dispersion.
Noverlap was set to 50% according to Matlab®'s standard.
The value of nfft was chosen to be the next power of 2 closest to window length.
As can be easily seen from Figure 22, the spectral analysis obtained is quite clear and some vibrational patterns can be distinguished without effort. For example, it can be observed how most of spectral power lays under an indicative threshold of 50 Hz for all the three parts of the turn. We can notice how in the beginning almost all the power gathers in a narrow stripe around 30 Hz, while in the middle and in the end, the plot seems to indicate that power is spread over a wider band of frequencies. In the middle part occurred the highest peak of the whole turn and it lays on a frequency range around 25 Hz.
Clearly, these results were obtained with the specific values of input parameters discussed hereinbefore and could change significantly with their variations. This is why this kind of analysis should be always supervised by an expert with deep expertise in signal processing. The

final considerations based on the graphs could be also slightly affected by the low frequency resolution provided by the representation, but it is unlikely that this could change the position and the intensity of the main peaks in a significant way. This means that this method could potentially give satisfying solutions in order to detect vibrational pattern for this kind of studies.

7.4 Skis' mechanical properties

The bending stiffness test was performed using the protocol explained hereinbefore. After every measurement session, the software returns a sketch of the shape of both loaded and unloaded ski. A comparison between the two cases for the hard and the soft constructions is shown in Figure 23.

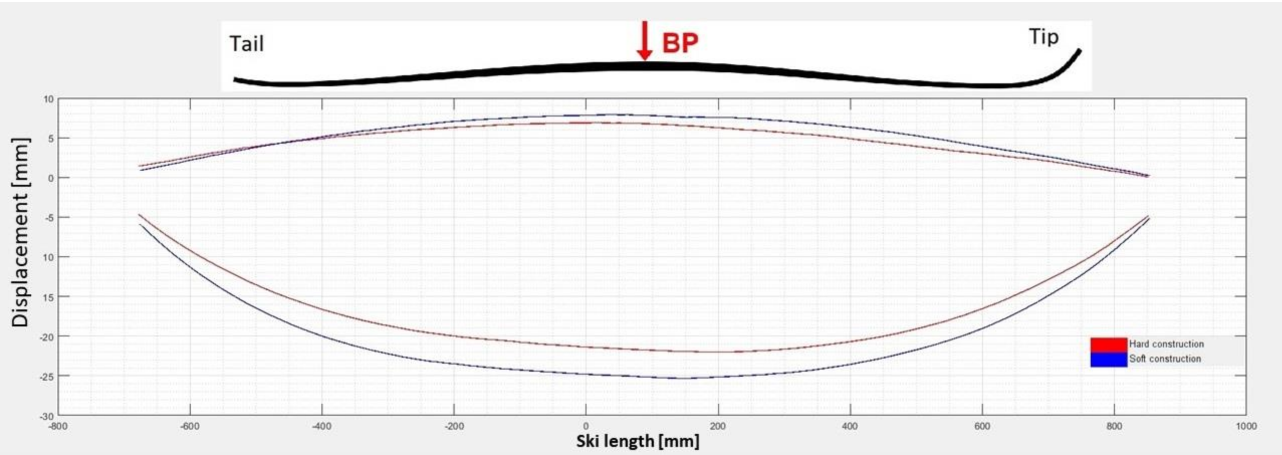

Figure 23: Comparison of bending deformation in hard construction (red curves) and soft construction (blue curves).

From this first schematic representation can be clearly seen how the soft construction bends more than hard construction during loading. This is totally in agreement with scientific common sense, as a softer body (i.e. with a lower elastic modulus) must undergo a larger deformation than a stiffer one when subjected to the same load. In this particular case, the soft ski experiences a maximum displacement of 25,3 mm at 869 mm from the tail, while the hard one 22 mm at 929 mm.

One more interesting feature to notice is the fact that from Figure 23 can be clearly seen how, even at unloaded configuration, the profiles of the skis are not on the same line. This, theoretically, should not happen, since the skis are supposed to have the same geometry and their difference in mechanical properties should be visible only when an external solicitation occurs. Actually, the same Atomic®'s operator assisting the team in data collection stated that is almost impossible to obtain the same geometry for two different skis by means of a common industrial process and that camber is probably one of the first features that is most likely to undergo some kind of slight variation. He also reassured, is very uncommon that the difference, even if detectable, is so large to affects significantly the ski's behavior on the snow. Indeed, most of times the skier doesn't realize anything and could not tell the difference.

The bending stiffness on the length of the skis was calculated with the expression found in [2] and reported in Equation 13:

$$M(x) = \frac{B}{R} = B(x)\frac{d^2y}{dx^2} \quad (13)$$

Where M(x) is the moment at x, the second derivative of the ordinate represents the profile's curvature of the neutral axis (which in this case we can approximate with the lower surface of the ski, since its thickness is far smaller than its length) and B(x) represents the bending stiffness at that point. Writing this formula, was assumed that the curvature of the profile was equal to 1/R, which would be the curvature radius, but this is true only if the curvature is constant point by point for all the length of the profile. Nevertheless, this is not important for the purposes of this study. Solving this equation with respect to B(x), we can obtain a simple relation to evaluate this parameter. As in this case the forces acting on the skis can be schematized as concentrated loads, the moment M(x) can be calculated with Equation 14 and 15:

$$M(x) = 55.2\, x \qquad 0 \le x \le 730\, mm \tag{14}$$

$$M(x) = 40.296 - 44{,}8\, x \quad 730 \le x \le 1630\, mm \tag{15}$$

A sketch of loads and moments applied to the skis is represented in Figure 24.
From the picture is well visible how the bending moment is basically a linear function going to a maximum in the first part and then decreasing after the application load point. As is suggested by common sense, the highest value is reached in correspondence of this coordinate. The distances were measured with a measuring tape and is reasonable that the loading force is located slightly nearer to the tail than the tip, since in almost every common ski is recognizable how the forebody is a little bit longer that the aftbody. This configuration helps the skier with load distribution and allows a better control.

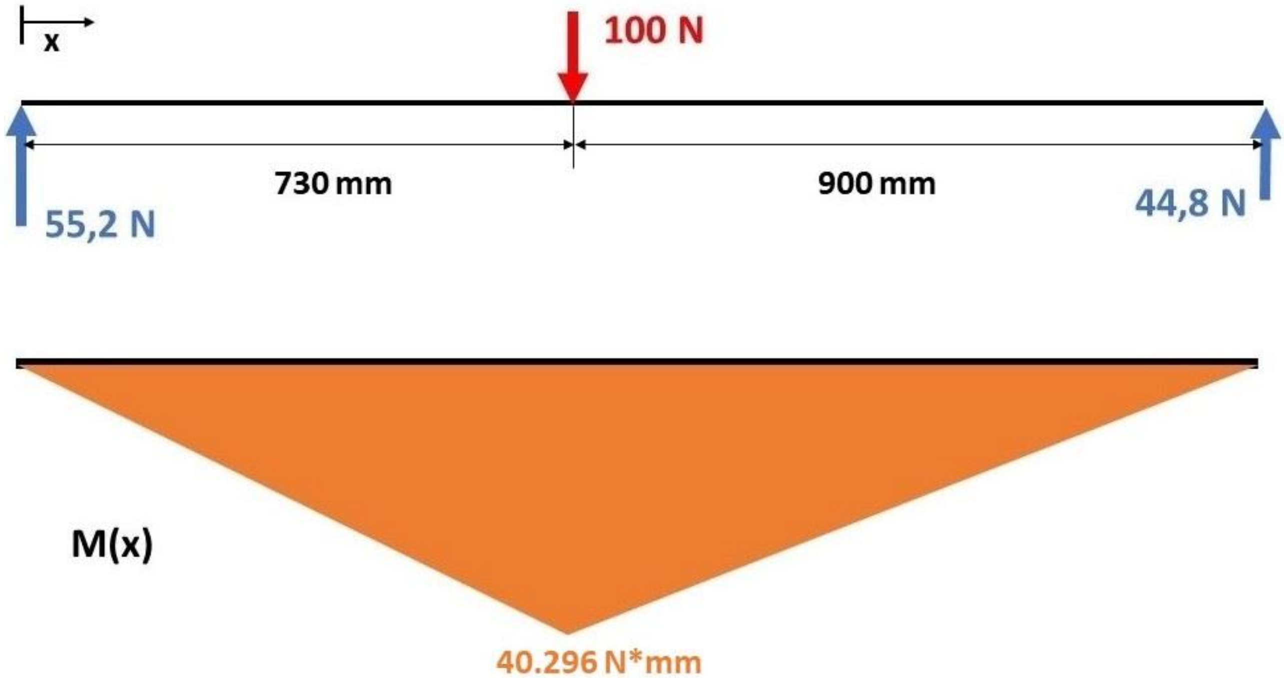

Figure 24: Sketch of the forces acting on the ski and resulting bending moment. The representation of the forces is just indicative and their length is not proportional to their module.

The calculations were executed by means of a Matlab® code reported in Appendix 2. In this case, to have an evaluation of the curvature along the length, the choice was to make a polynomial (7th order polyonomy) interpolation of the displacement points obtaining a function reproducing the profile of the bended shape and then making the second derivative of this function. The curves of the bending functions for both skis are shown in Figure 25.

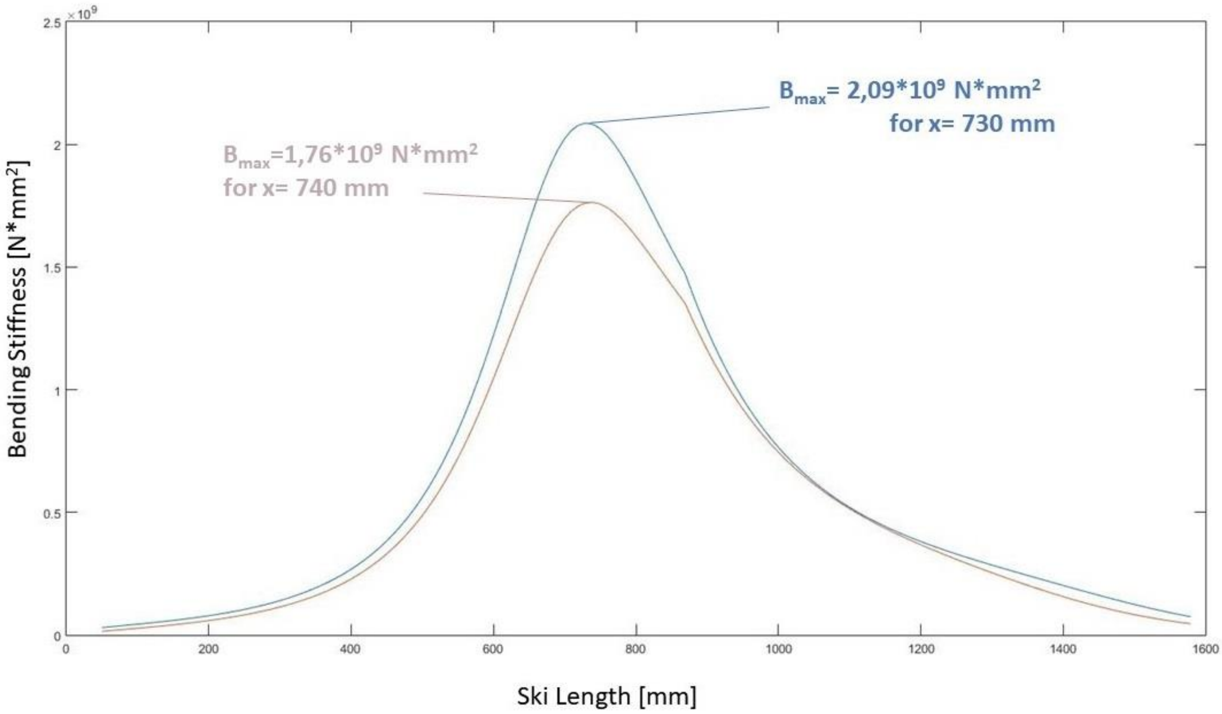

Figure 25: Bending stiffness curves for hard construction (blue) and soft construction (rose).

As visible in the picture, the hard construction has its maximum at about 730 mm (the resolution of this variable is related to the machine used for measurements that took a measure every 10 mm), while the soft construction reaches its highest value a little slightly later, at about 740 mm. The maximum of the hard construction exceeds the one of soft construction by 18,3%, which lead us to think that such a relevant difference could be reasonably detected by an expert skier by the different behavior showing during running.

The natural frequency of the first mode was calculated as the mean value of the first peak obtained by the Fast Fourier Transform of the time-domain signal. The team chose to proceed with the determination of only this frequency because it was the one detectable in the easiest and most reliable way from collected data and because, as seen from frequency analysis, most of power in a real turn is due to frequencies below 50 Hz. In order to do this, an automatic code reported in Appendix 3 was written by the team allowing to cut the significant data between the highest peak of oscillation (set as the initial value for frequency-domain analysis) and the next peak after a threshold corresponding to 1% of the highest peak value (set as final value). This because the team judged that after the decrement in amplitude exceeded 99%, there was no important information still present in data for the purpose here pursued.

In Figure 26 is reported a plot showing the three curves obtained by the three tests made for the hard construction's tail with initial displacement of 30 mm. Since it was the case with the largest variance between the results, it's easier to distinguish the curves of each test. From the picture is clearly visible how the peak due to the first natural frequency is very sharp and definite, while for the higher frequencies is not easy to distinguish a proper peak, even if some oscillation is revealed in correspondence of some ranges.

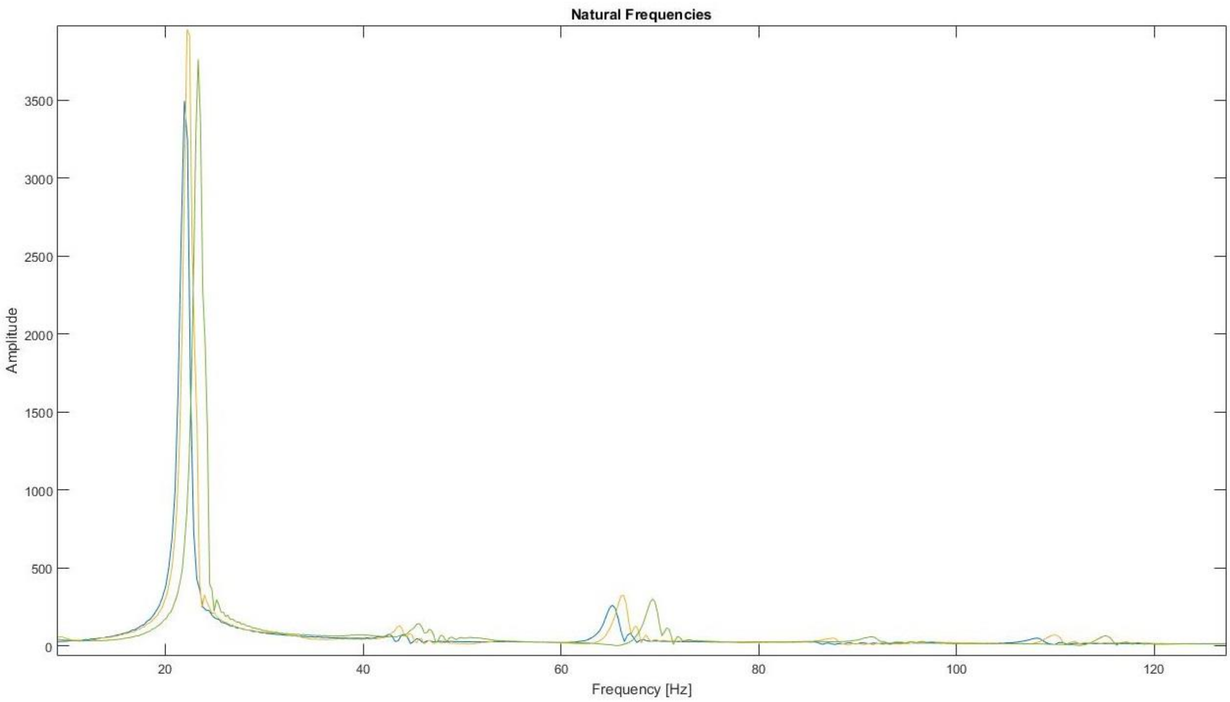

Figure 26: Plot of the FFT calculated on the three tests (blue curve: first trial, yellow second, green third) with maximum variance (hard construction's tail for a displacement of 30 mm). See also Appendix 4 for more information.

The complete screening of the results obtained from the calculations is reported in Appendix 4, while here in Table 9 are reported the most interesting and meaningful outcomes for this study. At a first glance, it can be immediately recognized how in both cases the values of natural frequency for the tails are larger than that resulting from the tips (in the case of hard ski of nearly 10 Hz). This can be related to the fact that, in modern alpine skis, tails are quite shorter than tips and this turns into a higher stiffness value, since the formula of the bending stiffness has got the beam length at denominator [36]. As previously recalled, the stiffness is at numerator in the relationship of the natural frequency of a body and this explains why larger stiffness turns into a higher natural frequency. The fact that the larger value of natural frequencies both for tips and tails come out from the hard construction derives from the same reason.

Table 9: Comparison of natural frequencies results for hard and soft construction.

hard ski				soft ski	
	tip	**tail**		**tip**	**tail**
mean [Hz]	12,135	22,327		11,181	19,825
Stand. Dev.	0,001	0,389		0,029	0,118
Percent [%]	0,011	1,741		0,260	0,596
	Diff. Tip [Hz]	**0,954**		**Diff. Tail [Hz]**	**2,501**
	Percent [%]	**8,532**		**Percent [%]**	**12,616**

The standard deviation of the second row of Table 9 was calculated on the mean values of the natural frequency provided by every set of tests with the same displacement level. The

percentual value below states the percentage between the standard deviation and the mean value.
In the two rows at the bottom of the table are reported the differences for the natural frequency values for tip and tail between hard and soft constructions. The percentual value below indicates the percentage between the difference and the lowest value of natural frequency detected for the respective part. We can see how the difference associated to the tails is quite larger than for the tips, but the team couldn't find a reasonable explanation for this fact.
Looking at the percentage of both parts, it is visible how the gap between the two constructions is slightly less evident than how revealed for bending stiffness, but it is still significant. However, the absolute difference is not that large and, for the purposes of this study, it would be essential to know if a variation of 2.5 Hz (the highest detected) can be clearly felt and distinguished by a tester and, subsequently, what is the impact it causes on the subjective evaluation.

7.5 Questionnaires for subjective evaluation

The results obtained from the questionnaires are reported in Table 10.

Table 10: Results of subjective evaluation questionnaires

Run	**1**	**2**	**3**	**4**	**5**	**6**	**Hard**	**Soft**
Directness	7	6	7	8	7	8	0	1,15
Controllability	8	6	6	4	5	7	1,53	1,53
Turnability	8	7	5	7	8	8	1,73	0,58
Innitiation	8	6	6	9	8	9	1,15	1,73
Stability	9	8	8	7	5	5	2,08	1,53
Completion	6	7	7	8	8	8	1,00	0,58
Overall	8	6	6	6	6	7	1,15	0,58

In the screening here presented, the columns from 1 to 6 represent the runs performed and, on every row, stands the rating obtained for that specific item. The last two columns show the standard deviation of each item for hard and soft construction respectively. This parameter is very important to assess the validity of the whole evaluation, since it is an indirect verification of the reliability of subjective estimation. As can be easily imagined, if the ratings were concentrated around a narrow interval of values, it could be also due to the uncertainty of the tester in judging the behavior of the skis. As stated in literature [16], the same pair of skis can be judged in a different manner by an advanced skier with respect to a pro or an athlete just because its limited dexterousness in using and understanding the device tends to gather his ratings around the mean values. While, in this case, the fact that the standard deviation is quite high confirms that the skier felt a difference from run to run and he could recognize it quite precisely.
Talking to the tester, he said that, after the first runs, he had the sensation there was a difference in behavior more between left and right ski of each pair rather than between pair and pair. This comment came a little bit unexpected, since the divergence in mechanical properties of the two pairs was pretty substantial and there was no plausible reason for explaining that two skis with the same construction could be felt more different than two with a diverse construction. The first supposition was that the measurement system could be responsible for this different feeling, since it was mounted only on the right ski of each pair, but the person of the team more

expert in using it said that it was very unlikely, since the weight of the accelerometers and the aluminum cases was negligible. Even if the questionnaires couldn't suit such an observation, this comment gathered the interest of the team and more research should be done in order to investigate where this difference in sensation could possibly come from.

8. Conclusions

The report presented hereinbefore was directed to the comprehension of the vibrational patterns usually occurring during a ski run, their dependence on the mechanical properties of the equipment used and their influence on the subjective evaluation of the skier performing the run. Referring to the methods followed for this kind of work, the team recognized as a valuable and reliable option to proceed with the employment of accelerometers, since they can provide accurate and meaningful data in the range of frequencies involved in this kind of application for all the time needed. The use of two different video cameras was essential in order to distinguish properly the feet position of the skier time by time and allowing to cut every turn at the right moment, even if some improvement should be done to this method to find a common reference time in order to simplify the pre-processing step.

The questionnaires for subjective evaluation provided good results in terms of accuracy and reliability, but this could be due to the fact that the skier employed for this task was a professional tester with a long expertise in this kind of procedures. Asking for the same assignment to advanced or even professional skiers not very accustomed to this type of methodology, probably the outcomes could be less significant.

In data processing, cutting the runs turn by turn required a major effort in terms of time and attention, since the feet position of the skier was not well recognizable at every time on the same tape and, to switch from one tape to the other, it was tricky to set a common time reference for both cameras. However, the study revealed how an external reference is mandatory to recognize the various skiing phases in the accelerometers' data. This suggested to the team that maybe a more versatile and simple way to find a time reference for the sensors could be looked for.

The employment of functions implemented by Matlab® for signal analysis returned good results, in team's opinion, but it was clear how necessary is to have an expert supervisor in dealing with such complex algorithms to obtain meaningful and reasonable outcomes. In general, this was judged to be a good tool to distinguish and evaluate the vibrational patterns intervening in a typical ski run.

The idea of harnessing two different ski pairs constructed with the same geometry but with two different materials' layout was considered appropriate for the aims of this study. Nevertheless, the mechanical tests performed on one ski of both pairs revealed that the gap in mechanical characteristics is more evident in terms of answer to static loads (bending stiffness) than for dynamical behavior (first natural frequencies). In particular, the main goal of the present study involved the evaluation of how diverse characters in the dynamical profile of a ski could affect the vibrational pattern detected when it performs on the snow. In order to assess this difference, the problem is that the software functions used to have an estimation of power spectra, present the results with a resolution in frequency near to 1 Hz (with some variations depending on the input variables chosen to carry out the calculations), so the doubt is if the methods used to represent the vibrational configuration in skiing is able to recognize a difference between the two pairs. Moreover, it is not clear to the members of the team if such a narrow gap between the first natural frequencies of the two pairs is reasonably distinguishable from the skier. More research and trials should be done in this sense to understand the impact of the sport equipment on the final performance. Maybe also other types of skis could be used for this purpose. In this case both pairs were constructed with a race profile, but it would be interesting to see if using

an all-mountain, a free-style or a women construction (a type of offer very diffused among ski companies nowadays) could turn into significantly different vibrational characteristics and patterns. Perhaps, the comparison between some GS and SL constructions would be useful too in order to see the divergence in dynamical behavior and response.
Another important issue related to the project here exposed is the fact that all the runs were performed by a single skier and this is kind of a limitation in assessing the subjective evaluation. It is easily comprehensible how the same ski could fit differently to different skiers according to the diversity in their physical characteristics and skiing technique. It would be very interesting to repeat the tests with multiple skiers in order to estimate how much this diversity can influence the outcomes in subjective evaluation.
The work here reported, unfortunately, was not able to reach definitive results (mainly due to issues and difficulties found in data processing), but the outcomes obtained from the various branches involved in this project suggested that there are possibilities to achieve significant insights for most of the activities undertaken and that there are good chances to make meaningful comparisons between the results from several experiences.
The team is persuaded that this method of study can lead to important accomplishments in the comprehension and practice of this sport and hopes to find more accurate and insightful conclusions in the near future.

9. Lista dei simboli

a_0 = first coefficient in Fourier Series
a_i = cosine coefficient or order i in DTFT
a_n = cosine coefficient of order n in Fourier Transform
B(x) = bending stiffness [N*mm^2]
b_i = sine coefficient of order i in DTFT
b_n = sine coefficient of order n in Fourier Transform
C_{ww} = magnitude squared of DTFT of window w
E{a} = expected value (mean value) of variable a
f = frequency [Hz]
F = normalization constant
F(t) = generic function of t
I_r = periodogram of segment long L samples
k = integer number, stiffness of a body
L = length of data set, length of segment
m = mass of a body [kg]
M(x) = bending moment [N*mm]
n = integer number
Q = length of data set
R = curvature radius [mm]
r = integer number
R = overlap percentage
r_{xx} = autocorrelation sequence of x
S_{xx} = power spectrum by autocorrelation of x
S_{xx}^{per} = averaged periodogram
T = period of a function [s]
t = time [a]
t_k = time of acquisition k [s]
Var{a} = variance of variable a
w = window function
x = distance along x axis [mm]
X = DTFT of a signal
x_n = function of discrete values
x_w = correlation function between x and window w
y = distance along y axis [mm]
Δf = frequency resolution [Hz]
θ = phase variable [rad]
ω = frequency [rad/s]

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Appendixes

Appendix 1. Questionnaire for subjective evaluation

28/11/2019

Ski Type ____ Run Nr. ____

Questionnaire for subjective evaluation

1. **Directness in force transmission**: How d**irectly** do the skis transmit the force to the snow?

(1= not direct at all / 10 = extremely direct)

	1	2	3	4	5	6	7	8	9	10
Steep	o	o	o	o	o	o	o	o	o	o
Flat	o	o	o	o	o	o	o	o	o	o
Overall	o	o	o	o	o	o	o	o	o	o

2. **Controllability**: How **easy** are the ski to control?

(1=very difficult / 10=very easy)

	1	2	3	4	5	6	7	8	9	10
Steep	o	o	o	o	o	o	o	o	o	o
Flat	o	o	o	o	o	o	o	o	o	o
Overall	o	o	o	o	o	o	o	o	o	o

3. **Turnability**: How much **effort** do the skis require to be turned?

(1= extreme effort / 10 = very little effort)

	1	2	3	4	5	6	7	8	9	10
Steep	o	o	o	o	o	o	o	o	o	o
Flat	o	o	o	o	o	o	o	o	o	o
Overall	o	o	o	o	o	o	o	o	o	o

4. **Ease of turn initiation**: How **eas**ily can the skis be set on edge?

(1=very difficult / 10=very easy)

	1	2	3	4	5	6	7	8	9	10
Steep	o	o	o	o	o	o	o	o	o	o
Flat	o	o	o	o	o	o	o	o	o	o
Overall	o	o	o	o	o	o	o	o	o	o

5. **Stability**: How stable were the skis during the turn?

(1= not at all / very well)

	1	2	3	4	5	6	7	8	9	10
Steep	o	o	o	o	o	o	o	o	o	o
Flat	o	o	o	o	o	o	o	o	o	o
Overall	o	o	o	o	o	o	o	o	o	o

6. **Easiness in turn completion**: How easily can the skis be released from one turn and be ready for the next one?

(1=very difficult / 10=very easy)

	1	2	3	4	5	6	7	8	9	10
Steep	o	o	o	o	o	o	o	o	o	o
Flat	o	o	o	o	o	o	o	o	o	o
Overall	o	o	o	o	o	o	o	o	o	o

7. **Overall impression**: How is the global feeling of the skis?

(1=terrible / 10 excellent)

	1	2	3	4	5	6	7	8	9	10
Steep	o	o	o	o	o	o	o	o	o	o
Flat	o	o	o	o	o	o	o	o	o	o
Overall	o	o	o	o	o	o	o	o	o	o

Appendix 2. Matlab® code for bending stiffness's calculation

```
% CALCULATING BENDING STIFFNESS

% import data from folder
soft_ski = importdata('Data Soft Load.xlsx');
leng = soft_ski(:,2) - soft_ski(1,2) + 50; % reference length for both data sets
hard_ski = importdata('Data Hard Load.xlsx');

%   saving the displacement collected for hard and soft ski
disp_hard = hard_ski(:,1);
disp_soft = soft_ski(:,1);

%   interpolating the data to obtain a polynom
ph = polyfit(leng,disp_hard,7); % for the hard ski
ps = polyfit(leng,disp_soft,7); % for the soft ski

%   creating a polynom from the coefficients and second derivative
poly_hard = poly2sym(ph);   % polynom hard ski
poly_soft = poly2sym(ps);   % polynom soft ski
curv_hard = diff(poly_hard,2);  % curvature hard ski's profile
curv_soft = diff(poly_soft,2);  % curvature soft ski's profile

%   converting the symbolic functions in real numbers
curv_real_hard = double(subs(curv_hard,'x',leng));
curv_real_soft = double(subs(curv_soft,'x',leng));

%   finding where is the maximum displacement of the ski and where the
%   moment funcion changes
[val,pos] = max(abs(disp_soft));
moment1 = 55.2*leng(1:pos);
moment2 = 55.2*leng(pos) - 44.8*(leng(pos:end) - leng(pos));

%   calculating the bending stiffness
% hard ski
BendStiff_hard(1:pos) = moment1./curv_real_hard(1:pos);
BendStiff_hard(pos:length(leng)) = moment2./curv_real_hard(pos:end);

% soft ski
BendStiff_soft(1:pos) = moment1./curv_real_soft(1:pos);
BendStiff_soft(pos:length(leng)) = moment2./curv_real_soft(pos:end);

%   showing the results
plot(leng,BendStiff_hard); hold on; plot(leng,BendStiff_soft);
clc
```

Appendix 3. Matlab® code for first natural frequency calculation

```
% CALCULATING FIRST NATURAL FREQUENCY
clc, clear all, close all

load('ski.mat');
fs = 500;
for t = 1:3
Data = ski(1).part(2).disp(3).try(t).x;
Data = Data - mean(Data);  % cutting off the mean value
[high,in] = max(abs(Data)); % finding the first peak
Data = Data(in:end);        % cutting data a the intial value
[pks,locs] = findpeaks(abs(Data));  % finding the last peak
a = high;
i = 1;
while a >= 0.01*high
    a = pks(i);
    i = i+1;
end
fin = locs(i);
Data = Data(1:fin);     % cutting data to the final value
nfft = length(Data);
spec = abs(fft(Data));  % FFT of the signal
spec = spec(1:floor(nfft/2));
xaxis = fs*(0:nfft/2-1)/nfft;      %scaling the x axis from bin to Hz
plot(xaxis,spec),
[ampl,pos] = max(spec);         % detecting where the first mode peak is
nfreq = xaxis(pos);
plot(xaxis,spec)                % plotting results for the same disp level
title('Natural Frequencies')
xlabel('Frequency [Hz]')
ylabel('Amplitude')
hold on
fprintf('Posizione frequenza naturale: %6.3f Hz \n',nfreq); % data presentation
end
```

Appendix 4. Complete screening of first natural frequency (values in Hz)

Errore. Il collegamento non è valido.

hard ski						
		tip			tail	
	max(45)	mid(37)	min(30)	max(30)	mid(20)	min(15)
a.	12,129	12,136	12,138	21,93	22,038	22,366
b.	12,127	12,137	12,138	22,2	22,3	22,375
c.	12,132	12,137	12,138	23,326	22,029	22,375
mean val	**12,129**	**12,137**	**12,138**	**22,485**	**22,122**	**22,372**
	0,003	**0,001**	**0,000**	**0,740**	**0,154**	**0,005**
	mean tip[1]	12,135		mean tail	22,327	
	St. Dev tip[2]	0,001		St.Dev. Tail	0,389	
	Percent[3]	0,011		Percent	1,741	

soft ski						
		tip			tail	
	max(45)	mid(37)	min(30)	max(30)	mid(20)	min(15)
a.	11,175	11,16	11,167	19,595	19,876	19,906
b.	11,233	11,164	11,164	19,668	19,874	19,905
c.	11,235	11,164	11,165	19,807	19,884	19,913
mean val	**11,214**	**11,163**	**11,165**	**19,690**	**19,878**	**19,908**
St. Dev	**0,034**	**0,002**	**0,002**	**0,108**	**0,005**	**0,004**
	mean tip	11,181		mean tail	19,825	
	St. Dev tip	0,029		St.Dev. Tail	0,118	
	Percent	0,260		Percent	0,596	

[1] Calculated as the mean of all the three mean values obtained from the three tests with different displacement levels.
[2] Calculated as the standard deviation between the three mean values obtained from the three tests with different displacement levels.
[3] Calculated as the percentual of the standard deviation with respect to the mean value of the respective ski's part.

Acknowledgements

I would like to thank very much all the people I worked with in the Sport Science and Kinesiology Department of Salzburg University. In particular, many thanks go to Prof. Josef Kröll for his suggestions and advices regarding the methodologies followed for this work and his continuous help and support during tests and data analysis, to Michael Buchecker for his essential and unique aid in data processing by means of the software, to Cory Snyder for the assistance and tutoring with measurement system and on-snow tests. My gratefulness goes also to Christoph Thorwarlt and Sebastian Fäber for their availability and cooperation for the mechanical properties' measurements. Of course, the team thanks very much the firm Atomic® for having allowed the possibility of conducting the tests with the machines of its laboratory.

A special thank is for Matteo Genitrini for his unique and valuable support in the organization of this thesis and in data processing.

Too long would be to recall every person that made something for me so that I could reach this final achievement but, since this could eventually be the end of my education program, I would like to give my thanks to the people that counted the most in my formation process. First of all, I would like to thank Prof. Gaetano Amato for his efforts and his patience in educating a lively and exuberant child, for having seen what was hidden yet and for having never lost the hope that, for a good person, to let his best part to come out is just a matter of time.

Many thanks go to Prof. Francesca Contarato, who imparted us a true and powerful passion for her subjects and taught us that there are some things that cannot be counted or measured and they are the ones that worth the most, since they are the very ones that make us humans.

I thank from the deep of my heart all my best friends (Nunzio Picchiotti, Norma De Marco, Lavinia Amenduni, Alessandro Castellani, Francesco Pancrazi and others) for having made me the man I am now, allowing me to write this work, and I thank my parents for having let me have the possibility to go along this road.