POLITECNICO DI TORINO
Department of Control and Computer Engineering
Master's Degree in Mechatronic Engineering

Master Degree Thesis

Modeling of a Roller Test Bench for Car Homologation

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Sommario

Questa tesi è stata svolta in collaborazione con Control Sistem, azienda leader nello sviluppo di soluzioni ed apparecchiature nel settore automobilistico. Presso questa azienda è installato e viene utilizzato un banco prova a rulli per auto per l'omologazione. Al fine di migliorare le prestazioni di questa strumentazione, Control Sistem ha inteso l'utilità di disporre di un accurato modello matematico dinamico e di sviluppare un simulatore dinamico per il banco prova a rulli. La disponibilità di uno strumento di simulazione sia della dinamica del veicolo che di quella del banco prova è infatti di fondamentale importanza per analizzare le proprietà dinamiche più importanti del sistema auto/strada, ricavare informazioni utili dalla simulazione stradale e ottimizzare il lavoro di ricerca e sviluppo. Per la modellazione del sistema sono stati applicati i concetti fondamentali della meccanica classica sia al veicolo che al banco prova. Il modello dinamico risultante è stato sviluppato in Matlab/Simulink e validato con dati sperimentali misurati.

Summary

This thesis was carried out in collaboration with Control Sistem, a leading company in the development of solutions and equipment in the automotive sector. Up to now, a car roller test stand has been installed and used at this company for homologation. In order to improve the performance of this instrumentation, Control Sistem intended the utility to have an accurate dynamic mathematical model and to develop a dynamic simulator for the roller test bench. The availability of a simulation tool for both vehicle and bench dynamics is in fact of fundamental importance for analyzing the most important dynamic properties, obtaining useful information from road simulation and optimizing research and development work. For the modeling of the system, the fundamental concepts of classical mechanics were applied to both the vehicle and the bench. The resulting dynamic model was developed in Matlab/Simulink and validated with measured data.

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Chapter 1

Introduction

The key to this thesis work is to create a mathematical model of the roller test bench for homologation of vheicle, which would allow to anticipate the results of the system, in order to modify its design according to what is desired, and save time in setting the fundamental parameters of the Bench. For this purpose, it will also be necessary to use Matlab/Simulink to create the model.

A roller test room was born from the necessity of the vehicle manufacturers to test a car already complete and assembled. Initially the vehicles were driven on the road and tested directly on the field. Bringing the testing phase inside a hall allows to perform tests in repeatable working conditions not affected by external environmental factors (otherwise impossible to achieve in the field), and still perfectly simulate the conditions that the vehicle would encounter if the vehicle were to drive on an infinitely flat road, without wind and with constant temperature and constant humidity.

1.1 Composition of a roller test bench

This chapter looks at the bench's components, the software and hardware part, and explains the usefulness of these test benches.

1.1.1 Hardware components

A 4WD test bench for testing four-wheel driving cars must obviously have a roller system for each axle of the vehicle. Such a system is usually composed of a central engine twin-shaft, connected to two external drums, on top of which the wheels of the car are placed.
In some systems, there is not a single drum per wheel, but two rollers are used. The main one is called master roller and it is the one directly driven by the engine,

the second is referred to as the slave roller and is moved accordingly. accordingly. In this way, the wheel of the car will rest on both rollers. Having provided this information on the existence of benches equipped with a single roller or double roller it is necessary to underline the fact that in the following thesis project we have worked on a 2WD test bench with a single roller. A 2WD test room for the testing of two-wheel drive cars has a roller system only for the driving axle of the vehicle.

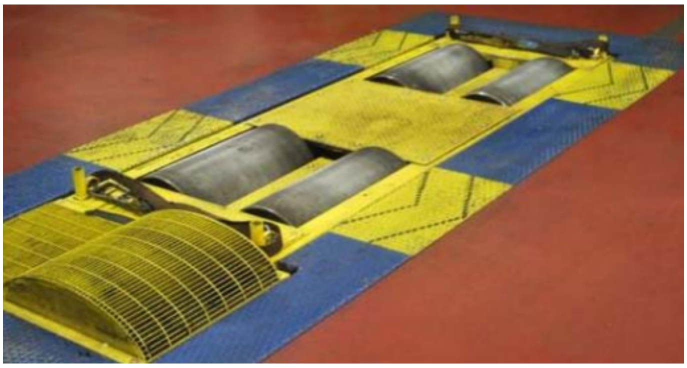

Figure 1.1: Bench equipped with master and slave rollers

The whole system rests on a ball bearing support to be totally released from the ground. In doing so, however, the engine would also be free to rotate together with the drums. To overcome this problem, the engine is anchored to the ground by means of a mechanical arm connected to a load cell. This brings a double advantage because it allows the reading of the torque that the motor generates. This type of motor is called a tilting casing motor.

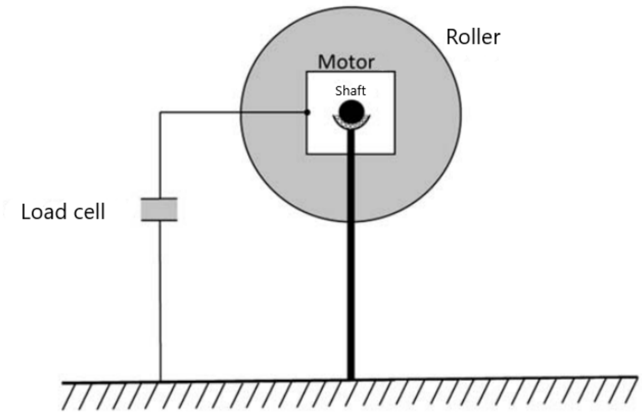

Figure 1.2: Front roller axis diagram

All of the above applies to both, front and rear axes. It is also good to know that the latter is shifted to adapt the distance from the first according to the wheelbase of the vehicle being tested. Therefore there is a mechanism that allows the movement of only the rear axle, in order to move it closer or further away from the front one.

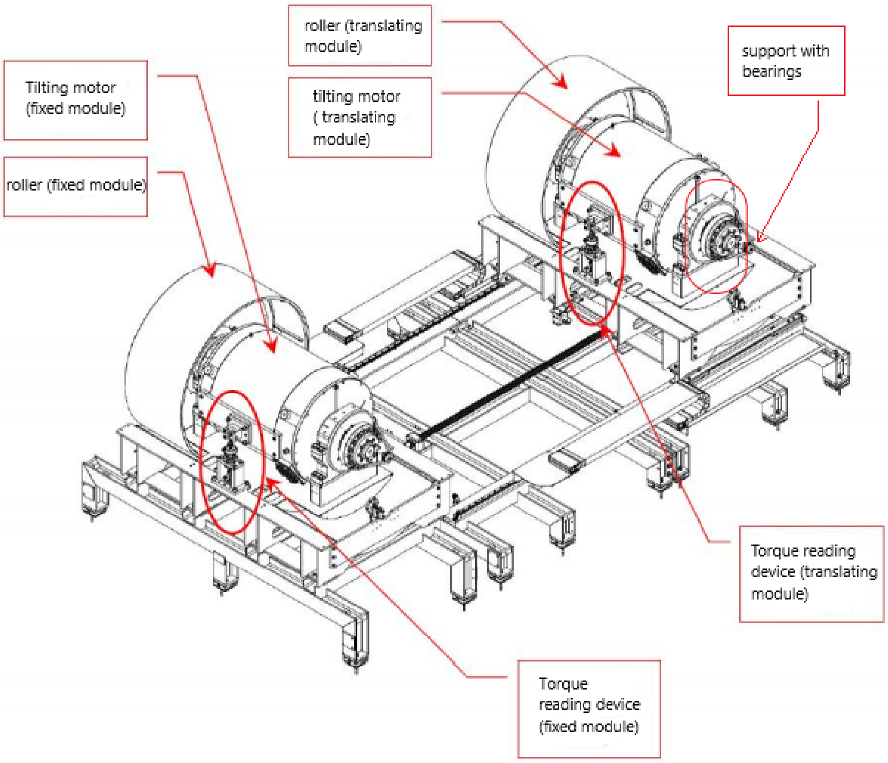

Figure 1.3: Translating module

1.1.2 Software components

The management architecture of a rollers testing room is composed of a control software called CSRolls which communicates with an application called DBSrolls. The latter manages and interfaces directly with a Wago PLC connected to the roller system and to the various sensors that make it up. The first is a room management software, which provides the user with the possibility of controlling the room to perform the various test cycles. Via Ethernet cable than it will drive the DBSrolls by sending work modes and commands. There is also a second Wago PLC directly connected to the CSRolls for the management of auxiliary signals for the control of the room. The blue arrows indicate an ethernet connection, the red arrow indicates a physical connection with the system components.

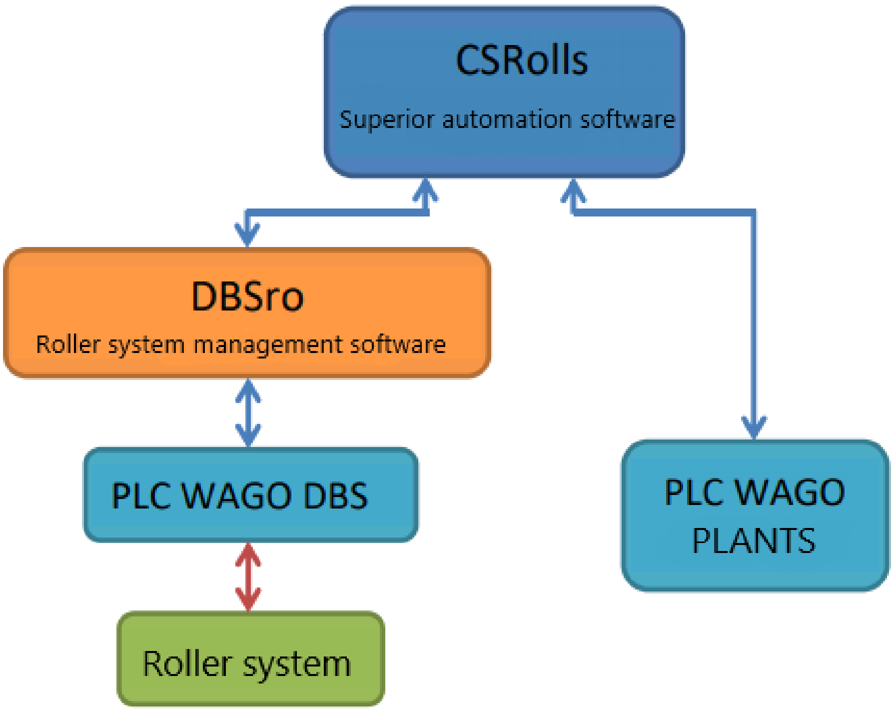

Figure 1.4: Communication network between layers

1.2 Fundamental quantities

The main measurement sensors of an automotive roller test bench are two:

- load cell;
- encoder.

The load cell is a force sensor connected at one end to the electric motor and at the other end to the ground. It is necessary to specify that this sensor is distanced from the electric motor by means of a mechanical arm which avoids that the motor rotates together with the shaft since it has a tilting casing that would rotate around the axis coinciding with that of the motor shaft. Therefore, the load cell measures the force F in Newton (N) necessary to prevent rotation and, since the arm is known and constant, the torque is univocally determined:

$$C_e = \tilde{F}b$$

This instrument can be said to be the most important measurement sensor present on the roller bench because it is the only one that measures all the force exchanged between the vehicle and roller, which, suitably processed, gives indications on the driving force supplied by the vehicle.

Figure 1.5: Load cell

The encoder is an angular velocity sensor, it detects the rotation speed of the rollers. It consists of a disc generally made of plastic material with windows of a known angle, a photodiode and a photoresistor. The photodiode sends the input signal through a light signal that will cross the windows of the disk. The photoresistor receives the light signal and will in turn send a logical output signal (1 if it receives light, 0 if it does not receive light). The type of encoder present on Control Sistem s.r.l.'s roller benches are relative (or incremental) encoders, which signal only the detectable increments (variations) with respect to another position taken as reference. These increments are independent from the direction of rotation which cannot be detected by this type of transducer.

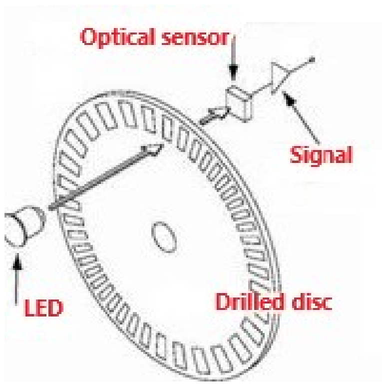

Figure 1.6: Encoder

Absolute encoders are not used because what is important to know are the impulses per revolution of the roller, while the absolute position of the roller is not useful data to detect.

1.3 The importance of simulating in a controlled environment

As previously mentioned, the most important reason for choosing to carry out tests in a controlled environment is the possibility of performing tests in perfectly repeatable working conditions. For example, if you want to test the car at various driving speeds and at a precise wind speed it will be almost impossible to have the repeatability of working conditions because it is not said that the wind speed will remain constant.

Possible tests conducted in controlled environment are tests of:

- Aerodynamics: car profile optimization activities;
- Reliability: tests at very low temperatures;
- Durability: synthesis tests on components;
- Performance: speed, traction force, braking force;
- Robustness of accessory equipment: air conditioning tests in roller rooms at high temperature and artificial sunlight;
- Refinement of calibrations and strategies for vehicle emissions;
- Vehicles homologation for a specific regulation.

The most important test conducted in an automotive roller test room is the RLS, also called road simulation. This cycle simulates the road course that is implemented on the car during its normal operation. In order to replicate the conditions that the vehicle would have on the road the rollers must rotate according to a very precise dynamic, they must be able to apply to the car the same resistant forces that it would encounter if it were really on the road.

Chapter 2

State of Art

The possibility to have available a mathematical model of the physical system of interest is very useful because it allows to predict and formally describe the behavior of that system. In an industrial environment this means not only relying on experience and common sense, but also to use an objective description able to provide predictions in terms of numerical values. In literature there are many articles or scientific books where physical systems are studied in order to create a mathematical model that represents them.
Most of these papers or books follow a certain ladder to arrive at the model. The various steps followed to create a mathematical model in the paper analyzed [16] are:

1. definition of a first simplified scheme of the system where by simplified it means formed only by the main components of major importance;

2. identification of the physical phenomena involved and creation of a free body diagram of the system;

3. extrapolation of the equation(s) of the system by means of equilibrium equations of forces and moments (Newton-Euler method) or Lagrange methods;

4. identification of the quantities of Input and Output necessary to the simulation system;

5. model validation.

The two methods mentioned in point 3. are two alterative methods to each other because they lead to the same dynamic equation and this is a procedure that

has been carried out in [1]. The Newtonian method is faster to use with less complex systems while the Lagrangian method is faster than the Newtonian method with more complex systems. In the literature some publications choose to use one method rather than the other for the reason just mentioned, however there are also cases where both were performed to be sure that they were performed properly.
Searching in the technical-scientific panorama mathematical-physical models that represent the entire operation of an automotive chassis dynamometer were not found results but were found ideas from which to start to get the desired model. Undoubtedly the starting point is the mathematical physical model of a car of which in the various sources have been found more alternatives, as [14] proposes. The most recurrent are three:

- double-track model;
- single-track model;
- longitudinal model.

2.1 Single-track model

Single-track model, this model (Riekert and Schunk 1940) allows the approximate, but physically plausible, description of the lateral dynamics of a vehicle. This model has the advantage of being simple, which allows for rapid integration. This simplicity, however, does not allow to consider phenomena which, according to the situation, can be not negligible, for example the phenomenon of pitching or rolling.

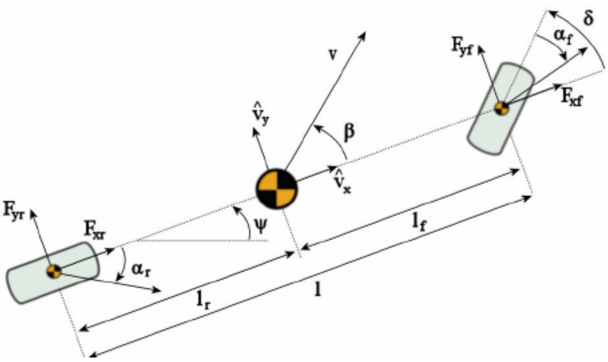

Figure 2.1: Single track model of a car on the road

2.2 Double-track model

Double-track model, this model assumes that the vehicle movements are planar movements and movements such as pitch, roll and vertical movements are neglected. At this model is arrived by adding longitudinal dynamics to the single-track model, so it will be able to perform both longitudinal and yaw movements. For this reason it is often used for estimating longitudinal and lateral states.

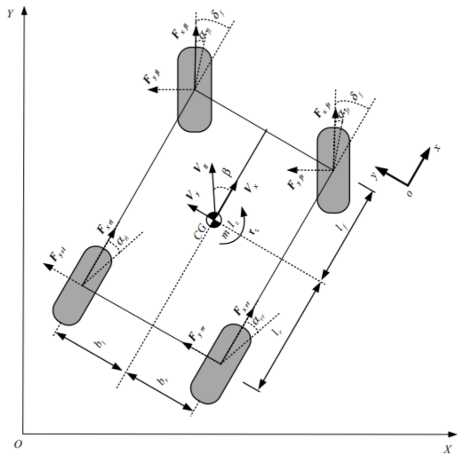

Figure 2.2: Double track model of a car on the road

2.3 Longitudinal model

Longitudinal model, the longitudinal model of the vehicle shown in the figure is applied to describe the longitudinal dynamics of the vehicle during braking and driving maneuvers. When only the longitudinal movement of the vehicle are considered at the slope angle of the road, the lateral movements of the vehicle and any other types are neglected. The left and right wheels of a vehicle can be combined into a wheel by ignoring the difference in movement between the left and right wheels.

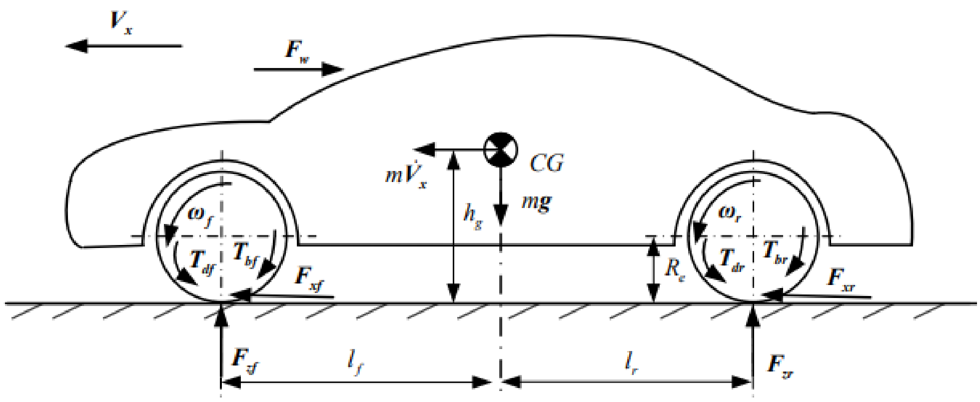

Figure 2.3: Longitudinal model of a car on the road

Since the aim of this thesis is to generate a model that simulates the behaviour of the chassis dynamometer, it was necessary to look at the industrial and university engineering scene in order to study the longitudinal dynamics of a car on the road. Research had to be carried out into the longitudinal dynamics of a vehicle on the road, since this knowledge would enable the test bench to test the car as if it were actually on the road. From the various papers analysed, the starting point is the longitudinal model, and in particular considering the longitudinal model of the research in [9] on a vehicle moving on a sloping road, the external longitudinal forces acting on the vehicle are the aerodynamic friction force, the weight force, the longitudinal forces at the wheels and the rolling resistance forces of the tyres.

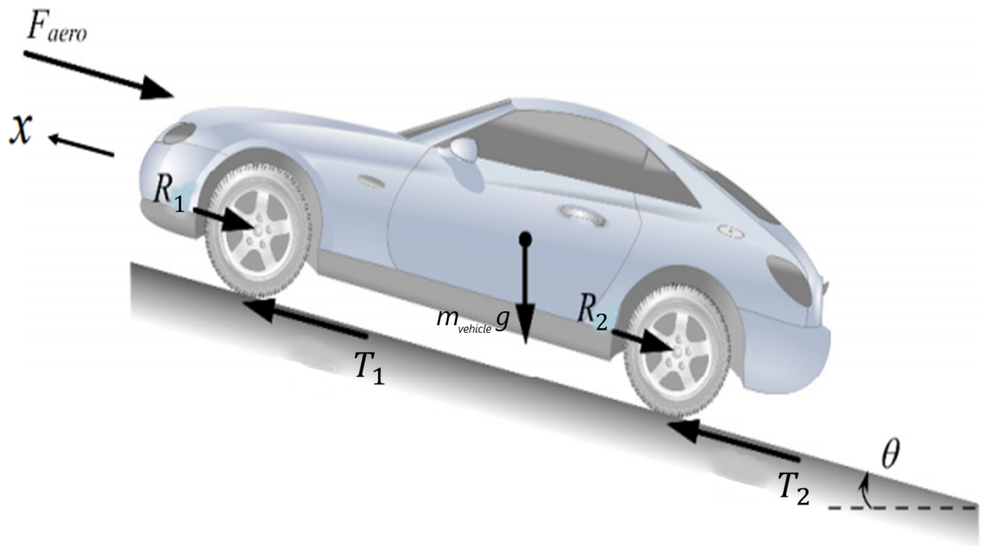

Figure 2.4: Longitudinal model of a car on a sloping road

The balance of forces along the x-axis obtained from this model is:

$$m_{vehicle}\ddot{x} = T_1 + T_2 - F_{aero} - R_1 - R_2 - m_{vehicle} g \sin\theta$$

where:

- T_1 is the longitudinal force at the front wheel;
- T_2 is the longitudinal force at the rear wheel;
- R_1 is the rolling resistance force at the front wheel;
- R_2 is the rolling resistance force at the rear wheel;
- F_{aero} is the aerodynamic friction force;
- $m_{vehicle}$ is the mass of the vehicle tested;
- g is the gravitational acceleration;
- θ is the slope of the road.

The following subtitles explain how these forces are usually calculated.

2.3.1 Aerodynamic friction force

The aerodynamic friction force is calculated as follows:

$$F_{aero} = \frac{1}{2}\rho C_d A_f (v + v_{wind})^2$$

where ρ is the air density, C_d is the coefficient of aerodynamic friction, A_f is the frontal area of the vehicle subjected to aerodynamic resistance, $v = \dot{x}$ is the longitudinal velocity of the vehicle and finally v_{wind} is the wind velocity.
As atmospheric conditions affect air density, they also affect drag. As a consequence of this relationship, the conditions to which all aerodynamic tests refer are a temperature of 15Ľ and an atmospheric pressure of 101.32 kPa. With this set, the resulting air density ρ is $1.225\frac{kg}{m^3}$. The frontal area A_f is usually calculated according to the studies carried out in [5], who calculates the area affected by drag as the area between $79 - 84\%$ of the area calculated with the vehicle width and height. However, this calculation is only accepted for vehicles with masses between 800 and 2000kg.

2.3.2 Longitudinal force at the wheels

The longitudinal wheel forces T_1 and T_2 are friction forces due to the wheel-road contact. In the literature in [7] from the study of Pacejka was defined a tyre model, which is now a standard in the field of vehicle dynamics simulation, is used to calculate the longitudinal forces at the wheel. The proposed Pacejka model is an empirical-mathematical model with the aim of reproducing the characteristic behaviour of the real component, based on mathematical formulae created ad hoc following experimental characterizations, independent of the physical reality that determining the behaviour acquired through measurements. The Pacejka tyre model, more commonly called, "Pacejka's Magic Formula", is therefore an empirical-mathematical model that attempts to summarise the experimental tyre performance through mathematical formulas. These have a precise structure in which quantified coefficients appear based on specific experimental tests.
From the experimental results obtained by Pacejka, the parameters on which the longitudinal force at the wheels depends were identified:

- s_L, longitudinal slip;
- F_Z, the weight force acting on the wheel;
- μ_L, the coefficient of longitudinal friction due to wheel-road contact.

Longitudinal slip is defined as the difference between the longitudinal speed in the direction of the wheel axis v and the rotational speed wr, the whole divided by v.

$$s_L = \frac{v - w_1 r}{v}$$

Once the longitudinal slip has been calculated and the coefficients related to the asphalt conditions (c_1, c_2, c_3, c_4 and c_5) appropriately chosen, it is possible to apply the "Pacejka's Magic Formula" to calculate the longitudinal friction coefficient due to the wheel-road contact.
The 'Magic Formula' is given below:

$$\mu(s_L) = (c_1(1 - e^{c_2 s_L}) - c_3 s_L)e^{-c_4 s_L v}(1 - c_5 F_Z^2)$$

The coefficient values for the asphalt conditions are values calculated by Pacejka derived from experimental characterisations following empirical tests.
Below are the ranges within which the empirical coefficients are chosen depending on the asphalt conditions:

- $0.05 \leq c_1 \leq 1.37$ from ice to cobblestone dry;
- $6.46 \leq c_2 \leq 306$ from cobblestone dry to ice;
- $0 \leq c_3 \leq 0.67$ from ice to cobblestone dry;

- $0.002\frac{s}{m} \leq c_4 \leq 0.004\frac{s}{m}$
- $c_5 \cong 0.00015kN^{-2}$

The figure below shows the graph μ_L-s_L where it is possible to appreciate the behaviour of the longitudinal friction coefficient as the longitudinal slip varies.

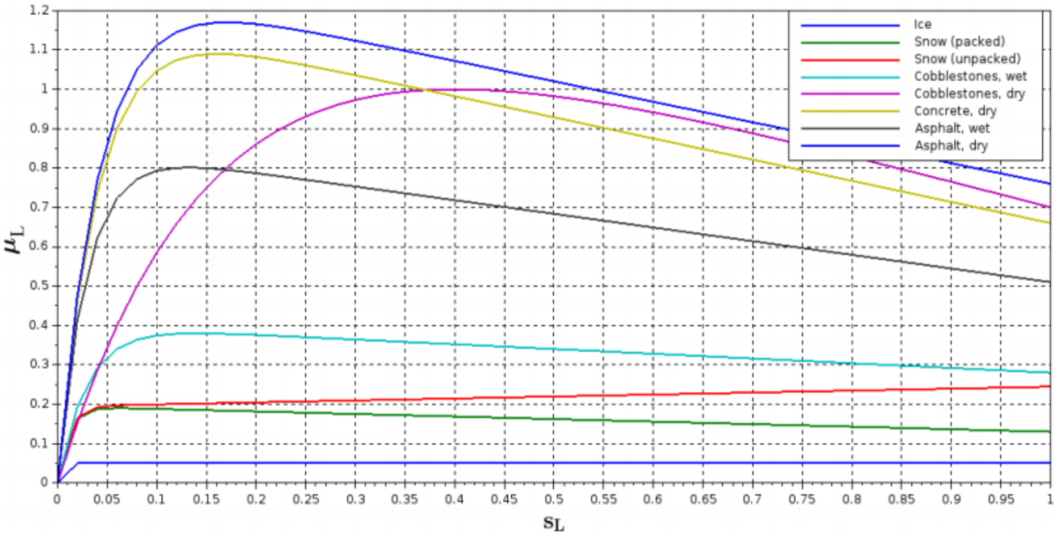

Figure 2.5: Graph μ_L-s_L

From the graph it is possible to see that the maximum values of the longitudinal coefficient of friction are in the range of 0.1-0.3 of longitudinal slip, which means that a minimum amount of longitudinal slip is necessary for good grip.
Finally, after applying the "Magic Formula" to calculate μ_L, from the definition of friction force and knowing the load normal to the wheel, it is possible to calculate the longitudinal force on the wheel:

$$T_{1,2} = \mu_L F_Z$$

2.3.3 Rolling resistance

In the end the last force applied to complete the model is the rolling resistance force. The rolling resistance acts while the tire rotates and it is a dissipative effect. Both the tire and the road are subject to deformation in the contact patch. The tire is elastic and new material from the tire continuously enter the contact patch as the tire rotates. Due to the normal load, this material is deflected vertically as it goes through the contact patch and then springs back to its original shape after it leaves the contact patch. Due to the internal damping of the tire material, the energy spent in deforming the tire material is not completely recovered when the

material returns to its original shape. These losses of energy can be represented by a force on the tires called rolling resistance that acts to oppose the motion of the vehicle. The loss of energy in tire deformation also results in a non-symmetric distribution of the normal tire load over the contact patch. When the tires are still, then the distribution of the normal load F_Z in the contact patch is symmetric with respect to the center of the contact patch.
Nevertheless, when the tires are rotating, the normal load distribution is non-symmetric, as shown in the figure below:

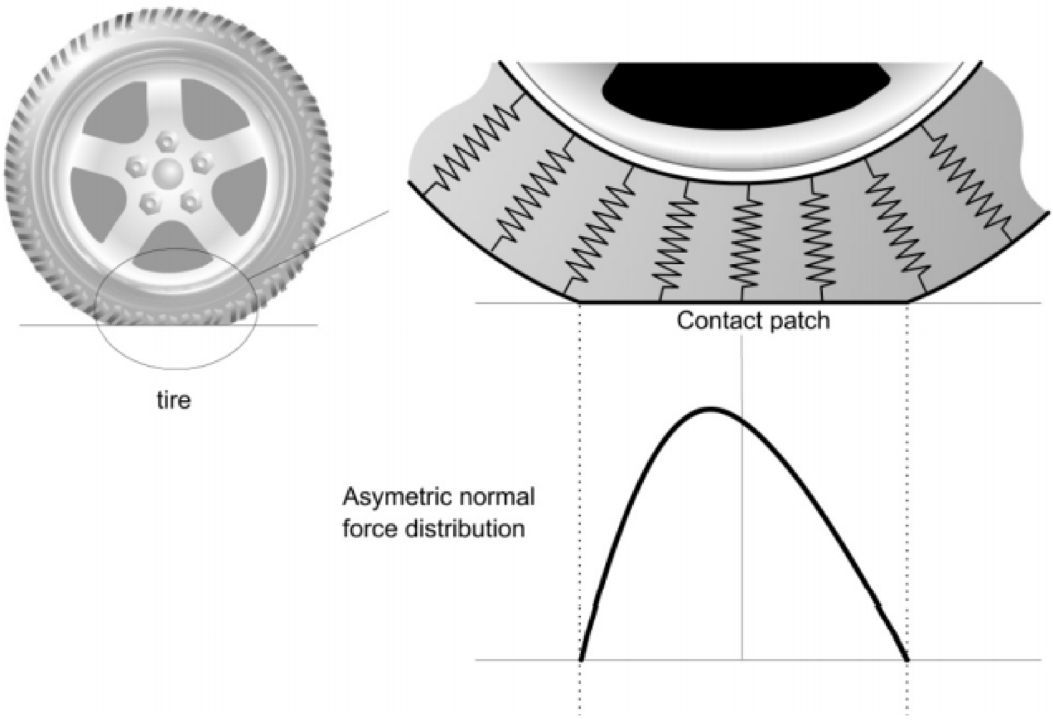

Figure 2.6: Contact patch zoom and normal force distribution to the wheel

The normal component is then displaced by a quantity u in the direction of motion and by balancing the rotation around the wheel centre we obtain the rolling resistance force described by the formula:

$$R_{1,2} = \left(\frac{u}{r}\right)F_Z = f_a F_Z$$

where $f_a = \frac{u}{r}$ is the rolling resistance coefficient.

This depth of research into the longitudinal model has been of great help in providing the basis for modelling a generic road car, which will be used to calculate the SET speed to be supplied to the rollers so that they can simulate the road behaviour of the vehicle under test.

Chapter 3

Modeling of Vehicle and Test Bench

In order to obtain a model that simulates the operation of a rollers test bench, it is necessary to first create a mathematical-physical model, first of a car on the road and then of a car on the bench. A mathematical-physical model is a set of equations and other mathematical relationships that represent physical phenomena, explaining hypotheses based on observation of reality. In general, a model is constructed from general laws and constitutive relations, of an experimental nature. In order to build mathematical models of cars on road and cars on rollers test bench and to obtain the related dynamic equations, two approaches have been followed: the Newtonian approach and the Lagrangian approach. If performed correctly they validate each other as they lead to the same dynamic equation. These two methods will be applied to both systems under analysis (car on road, car on test bench). In order to realize the mathematical models it has been necessary to represent the two systems according to a concentrated parameter schematization.

3.1 Model car on the road

The on-road car model was constructed based on the Longitudinal model from three assumptions:

- Absence of wind;
- Lack of rolling friction;
- Flat road.

In order to better understand the longitudinal dynamic behavior of a vehicle, it is necessary to identify the main forces acting on the car during its travel on the road, which are:

- Driving force;
- Resisting force;
- Inertial force.

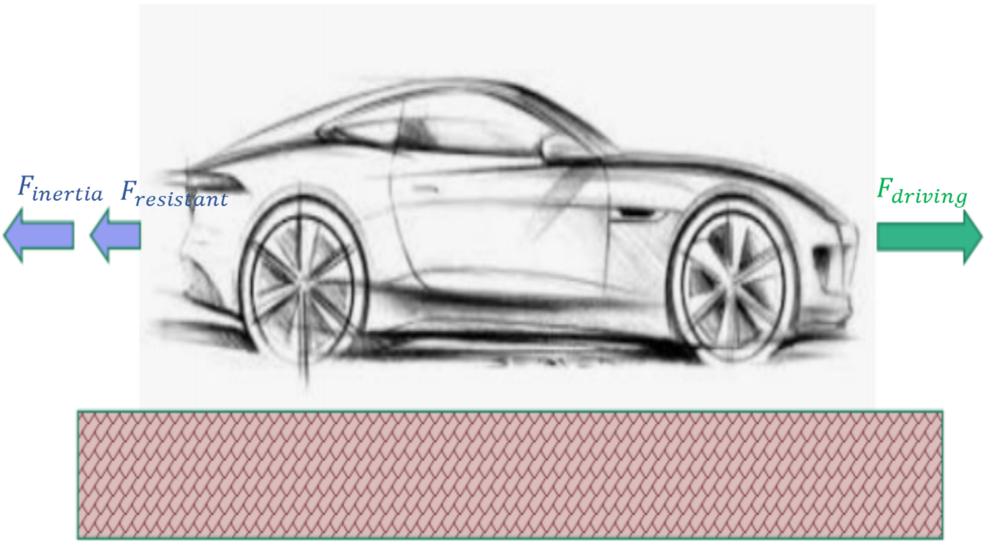

Figure 3.1: 2D car in longitudinal movement

The driving force is generated by a set of type processes:

- Chemical
  - combustion of endothermic engine
- Electric
  - electric or hybrid motors
- Mechanics
  - Transformation of torque at the drive shaft into torque at the wheels by means of gears, clutches and transmission
  - Transformation from torque to wheels to force to tire
  - Transformation from brake pad torque to tire force.

The resisting force is given by:

- Vehicle conformation
  - aerodynamics

- tire size and type
- road condition (uphill or downhill).

Inertial force is given by Newton's second principle of dynamics:

- $F = m_{vehicle}a$

3.1.1 Model car on the road: Newtonian approach

The Newtonian approach exploits the free body diagram of the system that you want to study by reporting the forces acting on the system itself. Once we have chosen a positive direction of the forces and moments we obtain the various equilibria around an arbitrary pole.

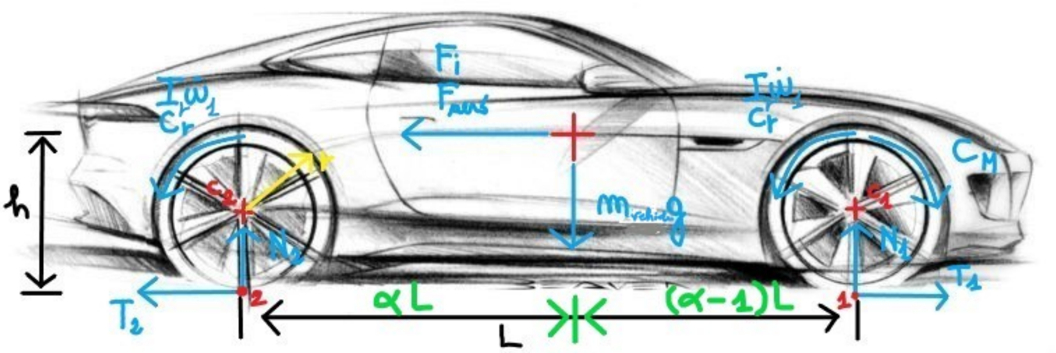

Figure 3.2: Diagram of the forces of a car on the road in Newton's method

The forces applied at the center of gravity of the vehicle are:

- F_i = inertial force;
- F_{aero} = aerodynamic friction force;
- $m_{vehicle}g$ = weight force.

The car under consideration is front wheel drive so the wheels of the front axle are driving wheels and the wheels of the rear axle are driven or dragged wheels. For this reason the two "wheel-systems" will be studied later in a different way in order to compose the final model.

Driving wheels

Figure 3.3: Force diagram for driving wheels

The forces and moments acting on the driving wheels are:

- N_1 = ground reaction force on the wheel;
- T_1 = dynamic friction force;
- C_M = driving force;
- $I_r\dot{w}_1$ = moment of inertia;
- C_r = resistant torque due to bearings and transmission;
- f_a = dynamic friction coefficient.

We apply the assumptions of adherence for a wheel that must not slip and must move of pure rolling motion so that the system has only one degree of freedom:

$T_1 \leq f_a N_1$
$\ddot{x} = r\dot{w}_1$

Driven or dragged wheels

Figure 3.4: Force diagram for driven wheels

The forces and moments acting on the driving wheels are:

- N_2 = ground reaction force on the wheel;
- T_2 = friction force;
- $I_r\dot{w}_1$ = moment of inertia;
- C_r = resistant torque due to bearings and transmission;
- f_a = dynamic friction coefficient.

We apply the assumptions of adherence for a wheel that must not slip and must move of pure rolling motion so that the system has only one degree of freedom:

$T_2 \le f_a N_2$
$\ddot{x} = r\dot{w}_1$

Calculation N_1 and N_2

In order to reach the dynamic equation it is necessary to obtain the equations of the reaction forces of the ground on the wheel and of the dynamic friction forces because they are terms not obtainable from any type of sensor present inside a generic car. The reactions of the ground on the wheel have been calculated in the following way:

Equilibrium at rotation around point 1:

$$C_M - 4C_r - 4I_r\dot{w}_1 + N_2L - F_ih - F_{aero}h - m_{vehicle}gL(1-\alpha) = 0$$

from the above equation we get :

$$N_2 = \frac{F_ih + F_{aero}h + m_{vehicle}gL(1-\alpha) - C_M + 4C_r + 4I_r\dot{w}_1}{L}$$

Equilibrium rotation around point 2:

$$C_M - 4C_r - 4I_r\dot{w}_1 - N_1L - F_ih - F_{aero}h + m_{vehicle}gL\alpha = 0$$

from the above equation we get:

$$N_1 = \frac{C_M - 4C_r - 4I_r\dot{w}_1 - F_ih - F_{aero}h + m_{vehicle}gL\alpha}{L}$$

Calculation T_1 and T_2

Considering instead the wheel subsystem, two equilibrium rotation are performed around the front wheel center and the rear wheel center to obtain the two dynamic friction forces. For the wheel subsystem it has been considered the entire axis and for this reason it will be found in the following equations a coefficient 2 that will multiply the resistant torque, the moment of inertia of the wheel and the moment generated by the friction force.

Equilibrium rotation around the point c_1:

$$C_M - 2C_r - 2I_r\dot{w}_1 - 2T_1 r = 0$$

from the above equation we get:

$$T_1 = \frac{\frac{C_M}{2} - C_r - I_r\dot{w}_1}{r}$$

Equilibrium rotation around the point c_2:

$$2T_2 r - 2C_r - 2I_r\dot{w}_1 = 0$$

from the above equation we get:

$$T_2 = \frac{C_r + I_r\dot{w}_1}{r}$$

Dynamic equation

Now by making an equilibrium to the horizontal translation knowing T_1 and T_2 we can obtain the dynamical equation of the car system.

Equilibrium at horizontal translation:

$$2T_1 - 2T_2 - F_{aero} - F_i = 0$$

Substituting the values of T_1 and T_2 we obtain:

$$\frac{C_M - 4C_r - 4I_r\dot{w}_1}{r} - F_{aero} - F_i = 0$$

noting that the driving torque and the resisting torque are divided by their own arm r, for clarity and convenience they are reported using the forces:

$$F_x - 4F_r - 4\frac{I_r\dot{w}_1}{r} - F_{aero} - F_i = 0$$

Knowing the relationship between angular acceleration and tangential acceleration

$$a = \dot{w}_1 r$$

it is possible to obtain an equation having as only unknown the tangential acceleration by replacing the angular acceleration with the tangential acceleration divided by the wheel radius:

$$\frac{I_r\dot{w}_1}{r} = \frac{I_r a}{r^2}$$

Substituting in the above equation gives the value of the tangential acceleration:

$$a = \frac{F_x - 4F_r - F_{aero}}{\left(m_{vehicle} + \frac{4I_r}{r^2}\right)}$$

F_r is a force due to friction, which is directly dependent on speed, so it is replaced with a coefficient that multiplies the speed:

$$F_r = k_1 v$$

F_{aero} is the aerodynamic friction force, which is directly dependent on the square of the velocity, so it can be substituted in this way:

$$F_{aero} = k_{aero} v^2$$

The force of inertia is given by Newton's second principle of dynamics, the recurring mass by acceleration:

$$F_i = m_{vehicle} a$$

Substituting the resistant force, dynamic force, and inertia force with the corresponding terms representing them, the equation for tangential acceleration becomes:

$$a = \frac{F_x - 4k_1 v - k_{aero} v^2}{\left(m_{vehicle} + \frac{4I_r}{r^2}\right)}$$

While the dynamic equation will take this form:

$$\ddot{x}\left(m_{vehicle} + \frac{4I_r}{r^2}\right) + \dot{x}^2 k_{aero} + \dot{x} 4k_1 = F_x$$

where $\ddot{x}$ corresponds to the tangential acceleration and $\dot{x}$ to the tangential velocity.

3.1.2 Model car on the road: Lagrangian approach

This is an analytical, energy-based approach. Once the generalized coordinate vector is chosen, one exploits the kinetic energy K, the potential energy of the system P, the knowledge of the external forces and of the dissipative forces to derive the dynamic equations of the system through derivatives with respect to the generalized coordinate vector and its derivative with respect to time, as can be seen in the equation below. This is the so-called Lagrangian equation:

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}_i}\right) - \frac{dL}{dq_i} = F_i - f_i^{fric}$$

where L is the Lagrange function which is defined as the difference between the total kinetic co-energy and the total potential energy of the system

$$L(q, \dot{q}) = K(q, \dot{q}) - P(q)$$

F_i represent the external forces acting on the system, while the f^{fric} represent the dissipative forces of the system.

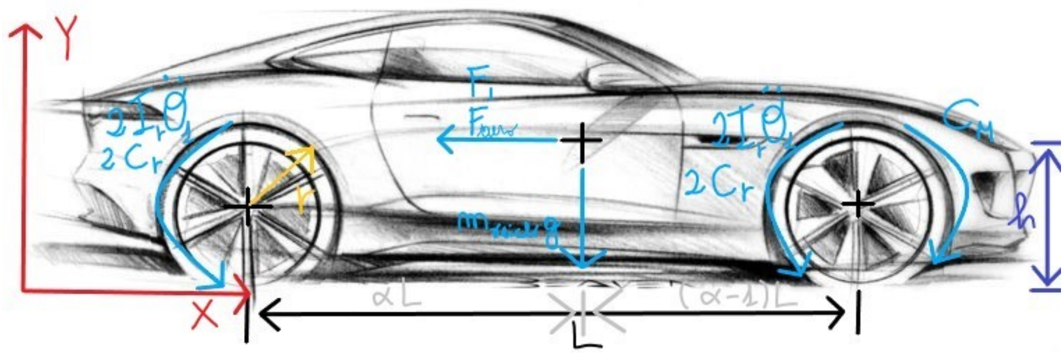

Figure 3.5: Diagram of the forces of a car on the road in Lagrange's method

The car system consists of five masses, where four masses represent the wheels m_r and the fifth represents the mass of the vehicle deprived of the wheels $m_{chassis}$. It is characterized by 2 generalized coordinates, defined as follows:

- θ_1 wheel rotation angle
- x absolute displacement of the vehicle

The relationship between the generalized coordinates is bound by their derivatives and holds

$$\dot{x} = \dot{\theta}_1 r$$

The center of gravity of the rear wheels is at the point of absolute coordinates

$$[x \quad r \quad 0]^T$$

The center of gravity of the front wheels is at the point of absolute coordinates

$$[x+L \quad r \quad 0]^T$$

The center of gravity of the chassis is at the point of absolute coordinates

$$[(x+L)\alpha \quad h \quad 0]^T$$

The squared norm of the rotation speed of the wheels is equal to ${\dot{\theta}_1}^2$.
The absolute velocity of the chassis is $v_{chassis} = (\dot{x} \quad 0 \quad 0)$ e la sua norma al quadrato vale $||v_{chassis}||^2 = \dot{x}^2$.
Now that it is known the norms of the velocities of the components of the system it is possible to calculate the total kinetic energy. The total kinetic energy is the sum of various components and precisely

$$K_{tot} = K_{chassis} + 4K_{wheel}$$

where

$$K_{chassis} = \frac{1}{2} m_{chassis} \dot{x}^2$$

$$K_{ruota} = \frac{1}{2} m_{wheel} \dot{x}^2 + \frac{1}{2} I_r {\dot{\theta}_1}^2$$

from which we get

$$K_{tot} = \frac{1}{2} m_{chassis} \dot{x}^2 + 2m_{wheel} \dot{x}^2 + 2I_r {\dot{\theta}_1}^2$$

The total potential energy is the sum of gravitational and elastic components and precisely

$$P_{tot} = 4m_{wheel}(-g)r + m_{chassis}(-g)h = m_{vehicle}(-g)h$$

Generalized forces are obtained by applying the principle of virtual work: there exists a point where the external force is applied and therefore the virtual displacement must be considered. This virtual work concerns the rotation of the wheels caused by the driving torque $\tau_M = F_x r$ and it holds

$$\delta W_M = [0 \quad 0 \quad \tau_M][0 \quad 0 \quad \delta\theta_1]^T = \tau_M \delta\theta_1$$

from the previous relations is obtained

$$\delta W_M = F_x \delta x$$

so it is easy to see that the generalized force corresponds to the driving force F_x. Regarding the dissipative forces, it is assumed that they are worth

$$f_i^{fric} = 4F_r + F_{aero}$$

Knowing the total kinetic energy and the total potential energy of the system, through their subtraction, it is calculated the Lagrangian function which is worth

$$L(q,\dot{q}) = \frac{1}{2} m_{chassis}\dot{x}^2 + 2m_{wheel}\dot{x}^2 + 2I_r\dot{\theta_1}^2$$

At this point we get to perform a series of derivatives, through which we can write the Lagrangian equation.

$$\frac{\delta L}{\delta q} = \frac{\delta L}{\delta x} = 0$$

$$\frac{\delta L}{\delta \dot{q}} = \frac{\delta L}{\delta \dot{x}} = m_{chassis}\dot{x} + 4m_{wheel}\dot{x} + 4I_r\frac{\dot{x}}{r^2}$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}}\right) = \frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right) = m_{chassis}\ddot{x} + 4m_{wheel}\ddot{x} + 4I_r\frac{\ddot{x}}{r^2}$$

The Lagrange's equation is:

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) - \frac{dL}{dx} = F_i - f_i^{fric}$$

$$m_{chassis}\ddot{x} + 4m_{wheel}\ddot{x} + 4I_r\frac{\ddot{x}}{r^2} = F_x - 4F_r - F_{aero}$$

The double derivative with respect to time of the absolute displacement of the vehicle $\ddot{x}$ corresponds to the longitudinal acceleration of the vehicle, so Lagrange's equation becomes

$$m_{chassis}a + 4m_{wheel}a + 4I_r\frac{a}{r^2} = F_x - 4F_r - F_{aero}$$

Now by collecting the acceleration as a first member, it is possible to combine the mass of the 4 wheels added to the mass of the chassis into a single term that will be called the total mass m_{tot}.

$$a\left(m_{chassis} + 4m_{wheel} + \frac{4I_r}{r^2}\right) = F_x - 4F_r - F_{aero}$$

$$4m_{wheel} + m_{chassis} = m_{tot}$$

$$a\left(m_{tot} + \frac{4I_r}{r^2}\right) = F_x - 4F_r - F_{aero}$$

At this point we isolate at first member the term that represents the tangential acceleration

$$a = \frac{F_x - 4F_r - F_{aero}}{m_{tot} + \frac{4I_r}{r^2}}$$

Regarding the dynamic equation, the equation that results is

$$\ddot{x}\left(m_{tot} + \frac{4I_r}{r^2}\right) = F_x - 4F_r - F_{aero}$$

which with the right substitutions of F_r and F_{aero} with their coefficients multiplied by their velocity dependence becomes

$$\ddot{x}\left(m_{tot} + \frac{4I_r}{r^2}\right) + \dot{x}^2 k_{aero} + \dot{x} 4k_1 = F_x$$

It can be seen, as expected, that the dynamic equations of the car system on the road obtained by performing two different approaches (Lagrange and Newton) are perfectly coincident. This leads to affirm that the two methods have been performed correctly.

3.2 Car on bench

Once the longitudinal dynamics of the vehicle has been studied, in order to arrive at the result of a roller bench model, the same study must be repeated considering the vehicle positioned inside a Roller Bench. Therefore, by studying this system, it is found that the resistant force will be divided into F_M due to the conformation of the vehicle and into F_p also called roller loss. This F_p is due to the frictional forces generated by the bearings that support the electric motor such that all the forces involved discharge onto the load cell.

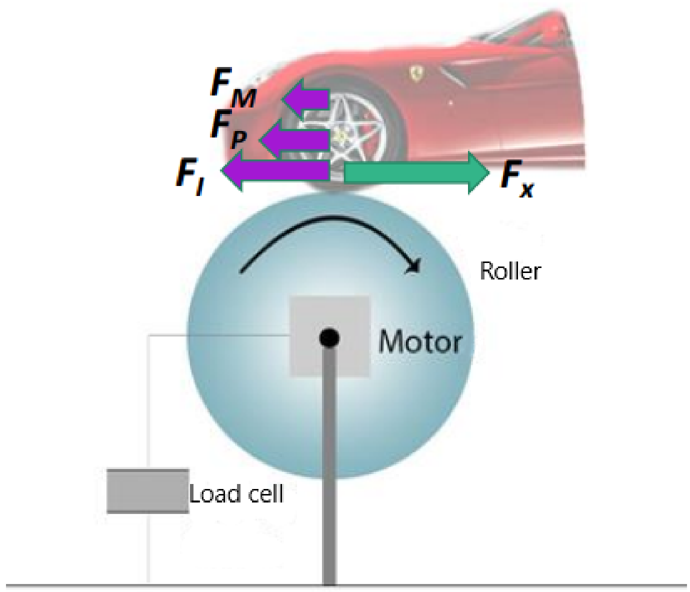

Figure 3.6: Car on bench diagram on front axle bench

- F_x: driving force of vehicle tangential to the roller;
- F_i: inertia force referred to the vehicle tangential to the roller;
- F_p: resistant force given by the presence of the roller;
- F_M: resistant force given by the aerodynamic and rolling resistance of the vehicle.

3.2.1 F_M and the road equation

In order to provide a more realistic simulation of the forces a vehicle is subjected to on the road, over the years a more accurate system than the simple inertia

simulation has been sought. Studies have therefore led to the formulation of an equation that takes the name of RLS, which is identified as F_M, which is a resistant force due to the conformation of the vehicle that is calculated through the use of the equation road (Road Load Simulation). The most widely used method to date is the RLS or Road Load Simulation that is a second-order equation that takes into account the inertia of the vehicle, the rolling friction between wheel and asphalt, and the aerodynamics of the vehicle.
The typical Road Load Simulation equation is as follows:

$$F_M = F_0 + F_1 v + F_2 v^2$$

$$[N] = [N] + [\frac{Ns}{m}] * [\frac{m}{s}] + [\frac{Ns^2}{m^2}] * [\frac{m^2}{s^2}]$$

$$Resistantforce = Inertia + Rollingfriction + Aerodynamics$$

F_0, F_1 and F_2 are the three parameters by which each car is characterized and affect the characteristic curve of the RLS. These parameters are provided by the vehicle manufacturer, who obtains these data from vehicle models or through real tests performed on the road, by performing a particular procedure from which to obtain these parameters, which is called coastdown (which will be explained later how it is performed on the bench), which on the road is performed in this way: the vehicle is brought to a constant speed of 135 km/h on a flat road in the absence of wind and then put in neutral gear is left to decelerate only because of friction. This deceleration is divided into intervals. In these intervals the deceleration and the elapsed time are observed and a and the elapsed time are observed and a resistant force is derived. These found forces are then averaged by interpolation with a least squares algorithm to find the parameters F_0, F_1 and F_2.

3.2.2 F_p, roller losses and calibration

F_p is the intrinsic resisting force of the roller and is due to the frictional forces generated by the bearings supporting the electric motor such that all the forces involved are discharged onto the load cell. Calculating this force correctly is a delicate operation, and is usually repeated several times, constantly. It depends on many factors, including temperature and rolling friction. rolling friction. To identify this force as precisely as possible, a calibration procedure is carried out. The calibration is used to characterize the mechanical system from the point of view of the intrinsic resistance to rotation by detecting the residual forces of the bench, it means the contribution that it must impose on the motor in order to keep the roller moving at the desired speed. Its result is to find three coefficients that express the quadratic braking curve of the bench. This curve is very important since each roller system is characterized by a univocal braking curve. This procedure therefore as a result allows to know the curve representing F_p as a function of the angular velocity of the rollers and can be performed in two different ways. The two calibration procedures are following:

- Mode 1 - based on constant resistant force:
  With unloaded rollers, it means without vehicle, I bring the system to an agreed speed, after which I impose a constant resistant force in order to decrease the speed of the rollers. I acquire the decelerations and compare them with the expected theoretical ones. From these data and from the difference between the measured and theoretical values, through a least squares algorithm, I extract the loss curve. It is usually a quadratic system.

- Mode 2 - based on stationary points:
  With unloaded rollers, the roller is driven at various preset speeds and the force values read by the load cell are saved, necessary to keep it at this speed. Again using a mathematical function, the points are interpolated to calculate the loss curve.

3.2.3 F_x, the driving force

During the roller test it is necessary to measure the force (or torque) generated by the vehicle, both to correctly control the speed of the rollers, and for possible testing requirements. To measure this force, one (or more than one in particular cases) load cell is used in the configuration shown in the figure. The motor is typically a twin-shaft with the two shafts connected to the two rollers (also called drums or flywheels). The shaft is decoupled from the ground since its support is typically equipped with ball bearings. The motor would therefore be free to turn on itself if it were not for the arm connecting it to the load cell that ties it to the ground. This causes all torque generated by the motor to be transmitted as a force to the load cell and measured.
Taking a moving car as an example, the force scheme is as follows:

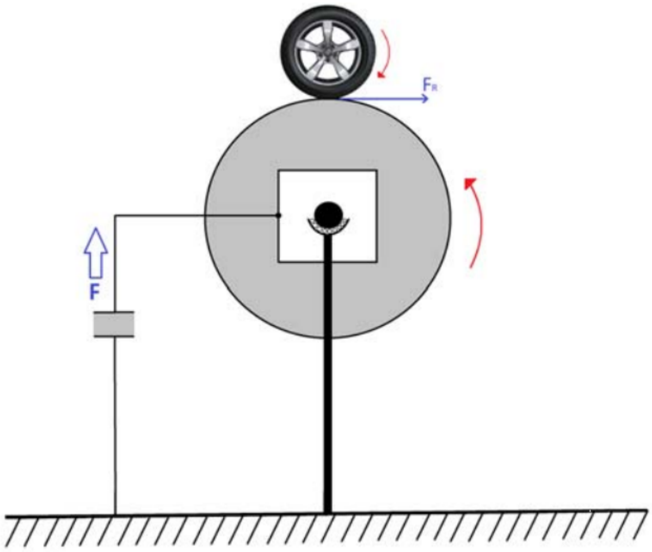

Figure 3.7: Force diagram of the traction axis

The reading of the load cell does not take into account only the driving force of the vehicle, but reads all the forces acting on the system and therefore the bench must be able to isolate the value of the driving force of the vehicle in order to make a correct road simulation. The formula used to derive the driving force is the following, where the inertia of the vehicle and the resistant forces due to the bench system are subtracted from the load cell reading:

$$F_x = -F_{cell} + F_r + m_{vehicle}a$$

The purpose of the bench is that one to simulate the road, therefore imagining not to be over a bench test rollers but to be on the road the components of the motive force will be

$$F_x = F_p + F_M + m_{vehicle}a$$

It is also necessary to add that following the request for tests on vehicles that can generate forces greater than the standard range (0-5kN), it is possible to insert a double load cell system. They have different scale and are inserted in series as shown in the figure.

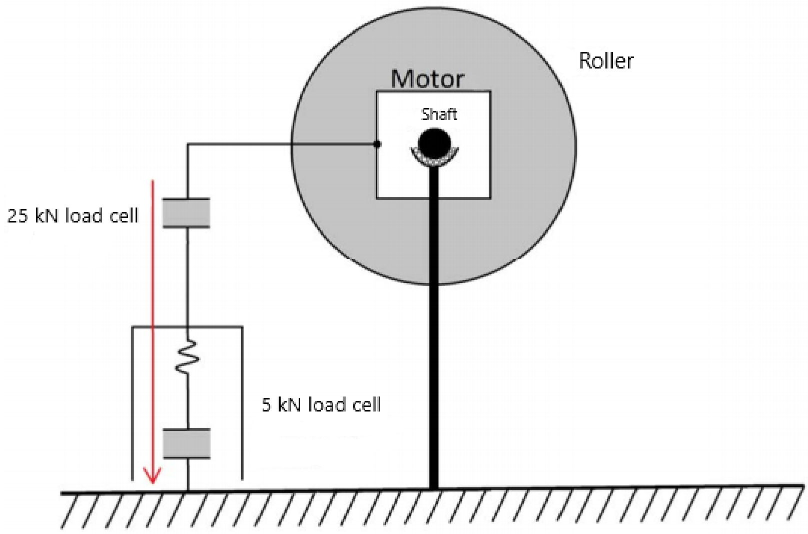

Figure 3.8: Double load cell reading scheme

The design is simplifying but the underlying concepts are easily highlighted. The spring present between the two load cells is designed to start compressing only for forces higher than 3500 N. The system is therefore predisposed to decouple the 5kN cell when the forces in play are higher, thanks precisely to the compression of the spring that will make the structure rest on the support plane. The forces will therefore pass through the 25kN load cell and will be then unloaded on the ground as shown by the red arrow in the figure. The actual structure is however more complex because the load cells work in both compression and elongation, but in both cases the smaller capacity cell is excluded when higher forces come into play.

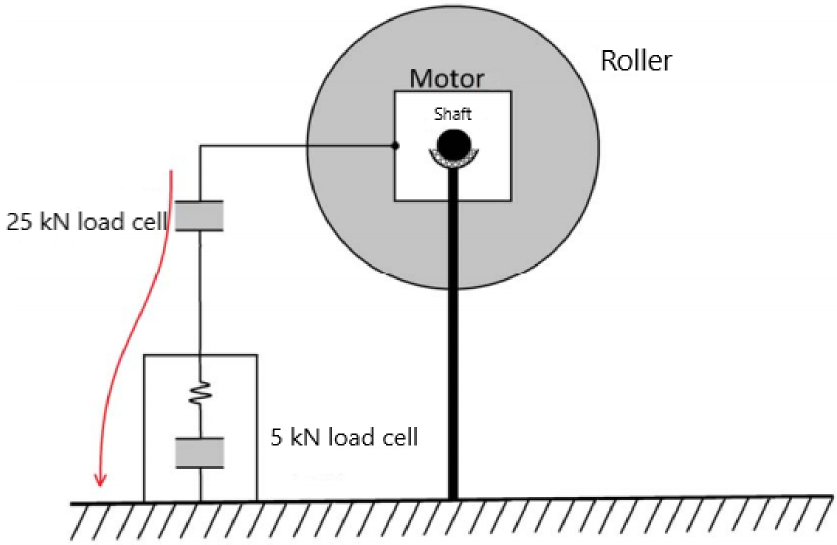

Figure 3.9: Single load cell reading scheme

The most consistent problem with this solution is the reading transition between the two cells. Since the system is real, and the two cells have different resolutions, it will be practically impossible to find an instant when both cells report exactly the same value. an instant in which both cells report exactly the same value. If you opt, for example for the direct passage of the feedback signal first of one and then of the other cell, two consecutive instants are created in which the read pair is not linear, but has a jump, and this could create stability problems to the system. The solution adopted is a weighted of the two load cell readings. For low force readings the reading of the load cell with a low full scale will have more weight (it means it will have more influence on the total reading), in case of larger readings more importance will be given to the reading of the 25kN load cell. The variation of these weights is linear with the variation of the detected force.

3.2.4 Data processing

Now that all the forces at play in the system have been identified, it is important to understand how they are to be used. Remembering that the main task of the roller test bench is to simulate the behavior that the vehicle would have on the road, it must be understood how to control the system to achieve the purpose. Therefore, since the electric motors are speed-controlled, it is necessary to extract the speed command to be given to the roller system, instant by instant. To do this, the acceleration that the vehicle would produce on itself if it were actually on the road must first be calculated.
From the parameters F0, F1 and F2 given by the manufacturer, the system is aware

of the resistant contribution acting on the vehicle (road surface resistance, aerodynamics, internal mechanics...). that acts on the vehicle (resistance road surface, aerodynamics, internal mechanics ...), from the calibration procedure also knows the intrinsic contribution of resistance of the system and through the coastdown is able to compensate the mechanical limit given by the rollers in order to correctly simulate the resistive contribution that would act on the car if it were on the road with the aim to correctly simulate the resistive contribution that would act on the car if it were on the road.
At this point, of the equation

$$F_x = F_p + F_M + m_{vehicle}a$$

is known everything except the acceleration. F_x is obtained instant by instant from the reading of the load cell, F_p and F_M both depend on the speed, which is read by the encoder and their coefficients are obtained from the two procedures already described and finally the mass of the vehicle is a known data provided by the manufacturer. Therefore it is now possible to obtain the instantaneous tangential acceleration:

$$a = \frac{F_x - F_p - F_M}{m_{vehicle}}$$

However, normally the rollers are driven in speed or torque, and not in acceleration. We therefore still have one more step to perform, which in the case of speed control is :

$$V = V_0 + at$$

Discretizing the time:

$$V_{t+1} = V_t + a\Delta t$$

Where V_{t+1} is the speed set point provided to the electric drive to bring the rollers to that speed, V_t is the speed read by the rollers in the current instant and Δt is the refresh time, it means the time that elapses between one reading and another.

3.2.5 Verification of RLS: The Coastdown

As it has been said many times before, the main task of a rollers test bench is to correctly simulate what the car would do on the road. It is therefore fundamental, before being able to use the bench for testing, to understand if it is actually able to correctly simulate what is required. To achieve this goal, the coastdown procedure is used. The coastdown procedure is used to define the parameters F0, F1 and F2 of the road equation and is conducted in this way: the vehicle positioned on the rollers is brought to a constant speed of 135 km/h and then put in neutral gear and allowed to decelerate only due to friction up to 15 km/h. This deceleration is divided into intervals. In these intervals the deceleration and the elapsed time are observed and a resistant force is derived.

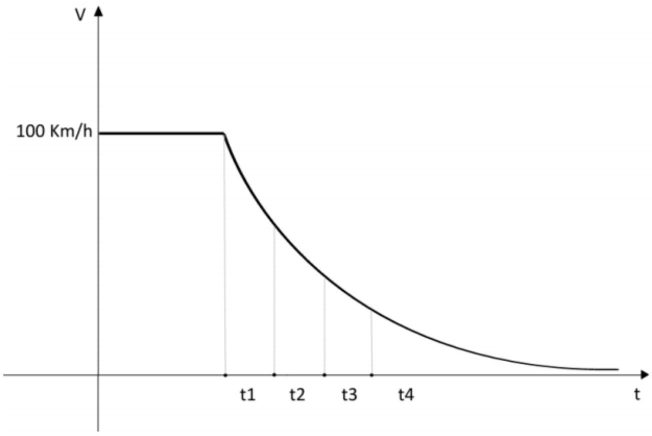

Figure 3.10: Speed-time graph of a coastdown test

$$\frac{\Delta v}{\Delta t} = a_{mean}$$

$$F = m_{vehicle} a_{mean}$$

These found forces are then averaged by interpolation with a least squares algorithm to find the parameters that make up F_M.
At the end of the coastdown procedure, the resulting force curve on the vehicle placed on the rollers shall be congruent with the above mentioned curve, committing a maximum error defined by standard. At the end of the cycle, by means of appropriate instruments, the difference between the forces is calculated in the different speed intervals and those obtained from the three road coefficients given by the customer. If the error is within tolerance, the coastdown is concluded since the bench correctly simulates the road behavior. If not, it is possible to recalculate three new coefficients of F0, F1 and F2 that will be used in place of those given by the customer. This recalculation can be repeated iteratively, as long as the errors obtained from the coastdown are in tolerance. It is important to understand that the results obtained from a recalculation must always be compared with the braking curve given by the original parameters F0, F1 and F2, because that is what you want to obtain. This recalculation is tolerated by vehicle manufacturers as the "Roller+Vehicle" combination slightly varies the previous forces that were identified with the calibration. It is then possible to recalculate the parameters and run a coastdown again. If the coastdown results in tolerance, it is possible to use the bench to test the vehicle, and one of the most used methods is precisely to perform driving cycles with preconfigured speed traces in order to analyze the pollutants produced by the vehicle.

3.2.6 Model car on bench: Newtonian approach

With the purpose of realizing a mathematical-physical model of a car on a chassis dynamometer, as already done in the previous sub-chapters for the car system on the road, the Newton-Euler method and the Lagrange method will be applied.
The chassis dynamometer considered in this study is a 2WD, which means that the cars tested on this dynamometer have only one drive axle, which in this case is the frontal axle.

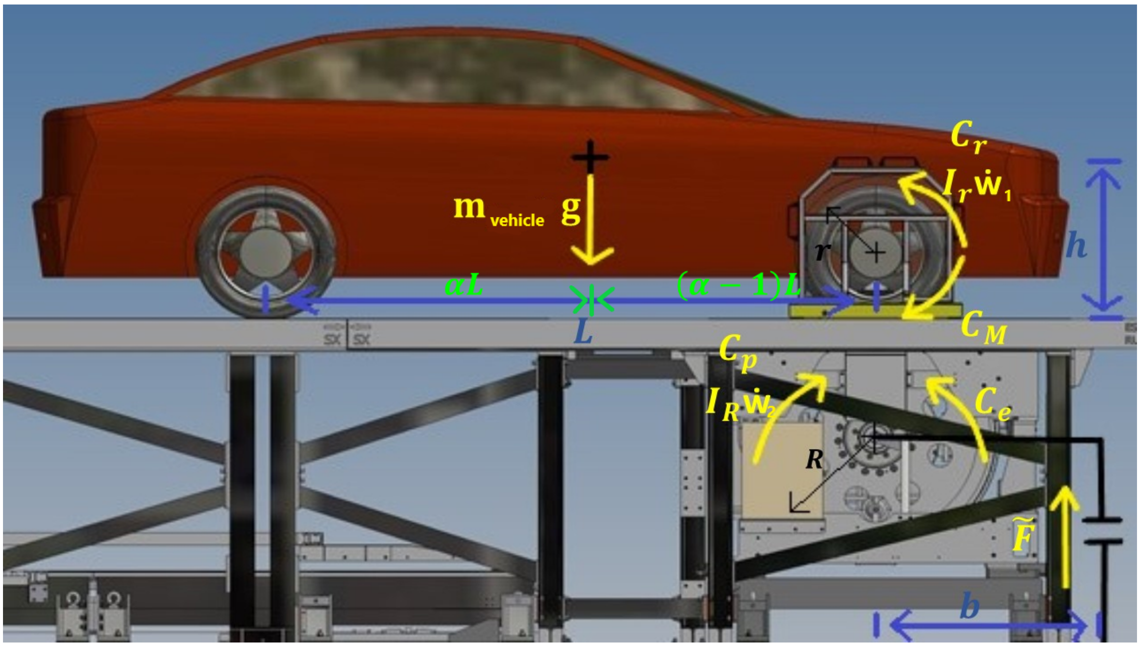

Figure 3.11: Diagram of the forces of a car on a bench in Newton's method

Also this model was built from the basis provided by the longitudinal car model. The forces and moments acting on the car+roller test bench system are:

- w_1 = angular velocity of the wheels;
- w_2 = angular speed of the rollers;
- C_p = Roller loss torque due to the resistance of the bearings that support the roller-motor axis so that all forces are loaded on the load cell;
- C_r = resistant torque due to the bearings and transmission of the car;
- C_M = driving torque of the car;
- C_e = driving torque of the electric motor of the rollers;
- $\tilde{F}$ = force reading [N] of the load cell;

- $m_{vehicle}g$ = car weight force;
- $I_r\dot{w}_1$ = moment of inertia due to the inertial mass of the wheel;
- $I_R\dot{w}_2$ = moment of inertia due to the inertial mass of the wheel;
- b = load cell arm, distance between roller center and load cell.

Load Cell Equation

Since we don't have an instrument to know the value of the car engine torque we will use the load cell reading to get it. As explained in the introduction, the load cell reads all the torque generated by the electric motor transmitted as a force.
Another consideration that needs to be made is regarding the speed at the point of contact between wheel and roller, which is considered equal for both rotating masses. Therefore, the relation that links the angular speed of the wheel and the angular speed of the roller is

$$v = w_1 r = w_2 R$$

By means of a rotation equilibrium around the center of the roller we derive the equation for the load cell reading. Before performing the rotation equilibrium it is necessary to carry all the torques acting on the wheels on the roller, and to do this we divide the torque of interest by the wheel radius r and multiply it by the roller radius R.

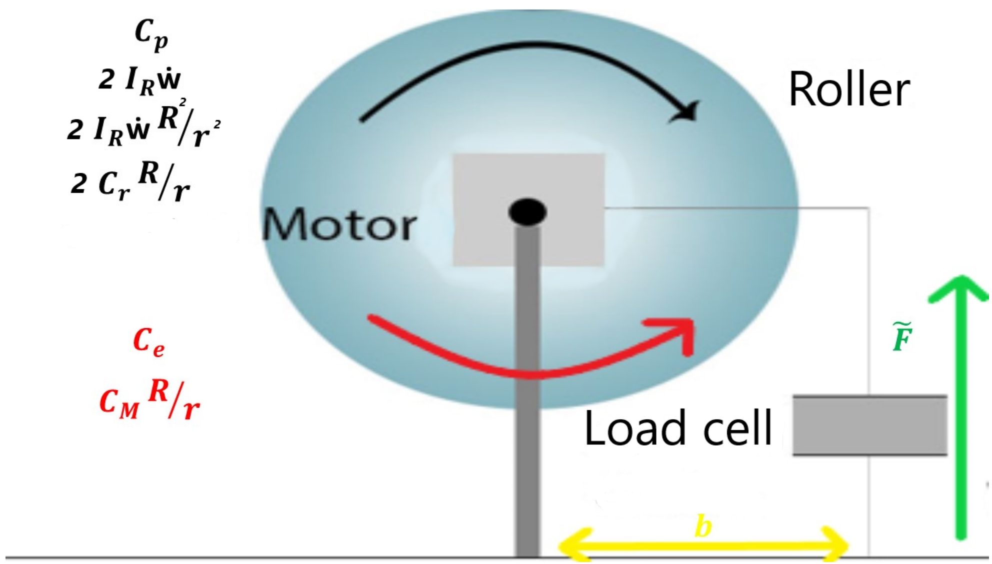

Figure 3.12: Force diagram for the front axle of the bench

Balance to the rotation around the center of the roller:

$$C_e + C_M \frac{R}{r} - C_p - 2I_R \dot{w}_2 - 2I_r \dot{w}_2 \frac{R^2}{r^2} - 2C_r \frac{R}{r} = 0$$

from the above equation we get:

$$C_M = 2I_r \dot{w}_2 \frac{R}{r} + 2C_r + (2I_R \dot{w}_2 + C_p - C_e)\frac{r}{R}$$

then dividing the C_M by the wheel radius:

$$F_x = \frac{C_M}{r}$$

$$F_x = 2I_r \dot{w}_2 \frac{R}{r^2} + 2F_r + (2I_R \dot{w}_2 + C_p - C_e)\frac{1}{R}$$

where C_e, motor torque of the electric motor corresponds to the load cell reading multiplied by the arm measured as the distance between the load cell and the center of the roller.

Dynamic equation

After having calculated the equation that uses the reading of the load cell to obtain the driving force of the vehicle under test, it is necessary to repeat the same calculations in order to extrapolate the dynamic equation of the system, but this time not taking into account the load cell. Therefore another equilibrium is performed on the rotation always around the center of the roller.

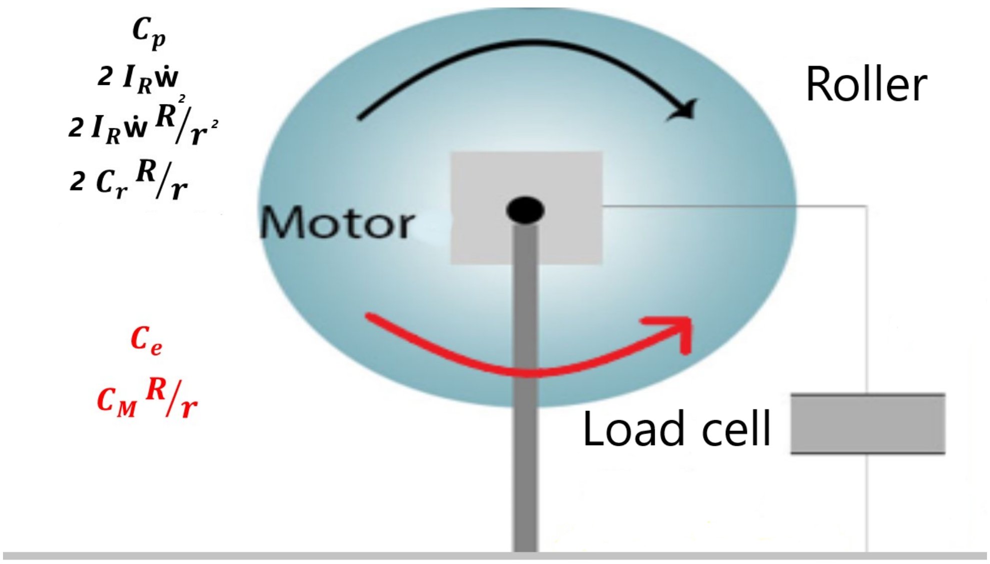

Figure 3.13: Force diagram by dynamic equation

Balance rotation around the center of the roller:

$$C_e + C_M\frac{R}{r} - C_p - 2I_R\dot{w}_2 - 2I_r\dot{w}_2\frac{R^2}{r^2} - 2C_r\frac{R}{r} = 0$$

by collecting it is got the dynamic equation of the system:

$$2\dot{w}_2(-I_R - I_r\frac{R^2}{r^2}) + C_e + (C_M - 2C_r)\frac{R}{r} - C_p = 0$$

From the dynamic equation it is possible to derive the angular acceleration:

$$\dot{w}_2 = \frac{C_e + (C_M - 2C_r)\frac{R}{r} - C_p}{2(I_R + I_r\frac{R^2}{r^2})}$$

are replaced the torques with the relative forces multiplied by their own arm:

$$\dot{w}_2 = \frac{F_e + F_x - 2F_r - F_p}{2R(\frac{I_R}{R^2} + \frac{I_r}{r^2})}$$

Multiplying the angular acceleration by the radius of the roller gives the tangential acceleration of the roller:

$$a = \frac{F_e + F_x - 2F_r - F_p}{2(\frac{I_R}{R^2} + \frac{I_r}{r^2})}$$

F_r and F_p are forces due to friction, which are directly dependent on velocity, so they are substituted like this:

$$F_r = k_1 v$$
$$F_p = k_2 v$$

Substituting in the equations for tangential acceleration and angular acceleration we get:

$$a = \frac{F_e + F_x - 2k_1 v - k_2 v}{2(\frac{I_R}{R^2} + \frac{I_r}{r^2})}$$
$$\dot{w}_2 = \frac{F_e + F_x - 2k_1 v - k_2 v}{2R(\frac{I_R}{R^2} + \frac{I_r}{r^2})}$$

Exploiting the relationship that binds the angular speed of the wheel to the angular speed of the roller it is possible to obtain also the equation that represents the angular acceleration of the wheel obtaining

$$\dot{w}_1 = \frac{F_e + F_x - 2k_1 v - k_2 v}{2r(\frac{I_R}{R^2} + \frac{I_r}{r^2})}$$

Regarding the dynamic equation of the system car+bench test rolls the equation that results from above equation is

$$2\left(\frac{I_R}{R^2} + \frac{I_r}{r^2}\right)\ddot{x} + (2k_1 + k_2)\dot{x} = F_e + F_x$$

3.2.7 Model car on bench: Lagrangian approach

The car+roller test bench system is composed of the masses of the four wheels m_r, the mass of the vehicle deprived of the wheels, and the masses of the two front rollers m_R.
It is characterized by 2 generalized coordinates, defined as follows:

- θ_1 wheel rotation angle
- θ_2 roller rotation angle

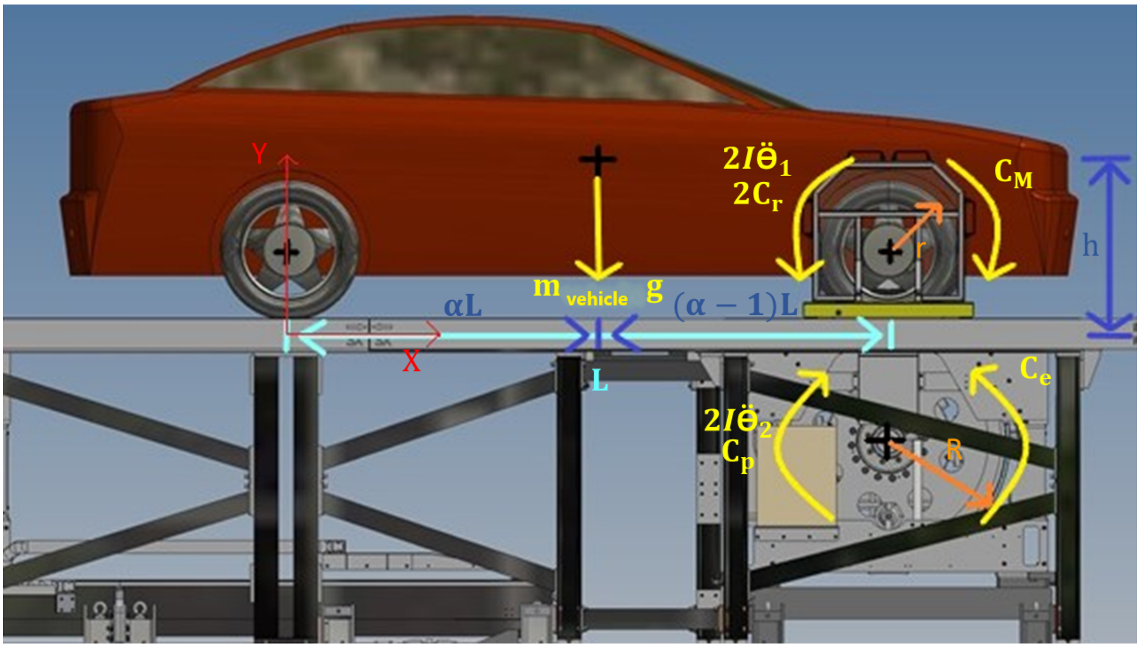

Figure 3.14: Diagram of the forces of a car on a bench in Lagrange's method

The relationship between the generalized coordinates is related by their tangential velocity, which is the same for both rollers and wheels, and is worth

$$v = \dot{\theta}_1 r = \dot{\theta}_2 R$$

$$\dot{\theta}_1 = \dot{\theta}_2 \frac{R}{r}$$

The center of gravity of the rear wheels turns out to be at the absolute coordinate point:

$$[0 \quad r \quad 0]$$

The center of gravity of the front wheels turns out to be at the absolute coordinate point:

$$[L \quad r \quad 0]$$

The center of gravity of the chassis is found to be at the absolute coordinate point:

$$[L\alpha \quad h \quad 0]$$

The center of gravity of the rollers is found to be at the absolute coordinate point:

$$[L\alpha \quad -R \quad 0]$$

The distances along the Z axis between right and left roller and right and left wheel have not been considered because they are negligible for this method.

The squared norm of the rotation speed of the wheels is equal to $\dot{\theta_1}^2$.
The squared norm of the rotation speed of the rollers is equal to $\dot{\theta_2}^2$.
Now that we know the square rotational velocities of the components of the system we can calculate the total kinetic energy. The kinetic energy developed in this system is exclusively due to rotational movements because the car cannot make any kind of translation movement when it is on a roller test bench. The total kinetic energy is the sum of various components and precisely

$$K_{tot} = 2K_{roller} + 2K_{wheel}$$

where

$$K_{roller} = \frac{1}{2} I_{roller} \dot{\theta_1}^2$$

$$K_{wheel} = \frac{1}{2} I_{wheel} \dot{\theta_2}^2$$

from which it is got

$$K_{tot} = I_{roller} \dot{\theta_1}^2 + I_{wheel} \dot{\theta_2}^2$$

and exploiting the relationship between the two generalized coordinates the equation representing the total kinetic energy becomes

$$K_{tot} = \dot{\theta_1}^2 \left(I_{roller} \frac{r^2}{R^2} + I_{wheel} \right)$$

The total potential energy is the sum of the gravitational and elastic components and precisely

$$P_{tot} = 4m_{wheel}(-g)r + m_{chassis}(-g)h + 2m_{roller}(-g)r = m_{vehicle}(-g)h + 2m_{roller}(-g)r$$

The generalized forces, in this case generalized torques, are obtained by applying the principle of virtual work and in this case the virtual work concerns the rotation of the wheels and the rotation of the rollers caused by the driving torque of the endothermic engine of the vehicle and by the driving torque generated by the electric motor of the bench, and it is worth

$$\delta W = C_M \delta\theta_1 + C_e \delta\theta_2$$

from the previous relations that link the two generalized coordinates it is obtained

$$\delta W = F_x r \delta\theta_1 + F_e R \frac{r}{R} \delta\theta_1$$

so it is easy to see that the generalized torque is

$$\tau_i = F_x r + F_e r$$

For what concern dissipative torques are concerned, it is assumed that they are worth

$$\tau_i^{fric} = 2C_r + C_p$$

$$\tau_i^{fric} = 2F_r r + F_p r$$

Knowing the total kinetic energy and the total potential energy of the system, through their subtraction, we calculate the Lagrangian function that is worth

$$L(q, \dot{q}) = \dot{\theta}_1{}^2 \left(I_{roller} \frac{r^2}{R^2} + I_{wheel} \right) - m_{vehicle}(-g)h - 2m_{roller}(-g)r$$

At this point we perform a series of derivations, through which it is derived the Lagrangian equation.

$$\frac{\delta L}{\delta q} = \frac{\delta L}{\delta \theta_1} = 0$$

$$\frac{\delta L}{\delta q} = \frac{\delta L}{\delta \dot{\theta}_1} = 2\dot{\theta}_1 \left(I_{roller} \frac{r^2}{R^2} + I_{wheel} \right)$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}}\right) = \frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}_1}\right) = 2\ddot{\theta}_1 \left(I_{roller} \frac{r^2}{R^2} + I_{wheel} \right)$$

The Lagrange equation is:

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) - \frac{dL}{dx} = \tau_i - \tau_i^{fric}$$

$$2\ddot{\theta}_1 \left(I_{roller} \frac{r^2}{R^2} + I_{wheel} \right) = F_x r + F_e r - 2F_r r - F_p r$$

The double derivative with respect to time of the generalized coordinate θ_1 corresponds to the angular acceleration of the wheels

$$\ddot{\theta}_1 = \dot{w}_1$$

so Lagrange's equation becomes

$$2\dot{w}_1\left(I_{roller}\frac{r^2}{R^2} + I_{wheel}\right) = (F_x + F_e - 2F_r - F_p)r$$

At this point it is isolated at first member the term that represents the angular acceleration of the wheels

$$\dot{w}_1 = \frac{(F_x + F_e - 2F_r - F_p)r}{2(I_{roller}\frac{r^2}{R^2} + I_{wheel})}$$

$$\dot{w}_1 = \frac{F_x + F_e - 2F_r - F_p}{2r(\frac{I_{roller}}{R^2} + \frac{I_{wheel}}{r^2})}$$

Exploiting the relation that binds the two generalized coordinates it is possible to obtain also the equation that represents the angular acceleration of the rollers

$$\dot{w}_2 = \frac{F_x + F_e - 2F_r - F_p}{2R(\frac{I_{roller}}{R^2} + \frac{I_{wheel}}{r^2})}$$

Finally, the resulting dynamic equation obtained from the previous equation, applying the relative substitutions seen above for F_r and F_p, is

$$2\left(\frac{I_R}{R^2} + \frac{I_r}{r^2}\right)\ddot{x} + (2k_1 + k_2)\dot{x} = F_e + F_x$$

In the final analysis, as observed for the road car model also for the car model on roller test bench it can be affirmed that the dynamic equations obtained with the two methods are correct having led to the same dynamic equation.

Chapter 4

Matlab-Simulink model

For the simulation of the bench it is employed the use of a model created in Matlab-Simulink environment so that comparisons can be made between the results obtained with the simulated tests and the results obtained with the empirical tests performed on real roller benches. The Matlab-Simulink model created was built by means of the union of the physical-mathematical model of car on the road with the mathematical-physical model of car on a roller test bench.

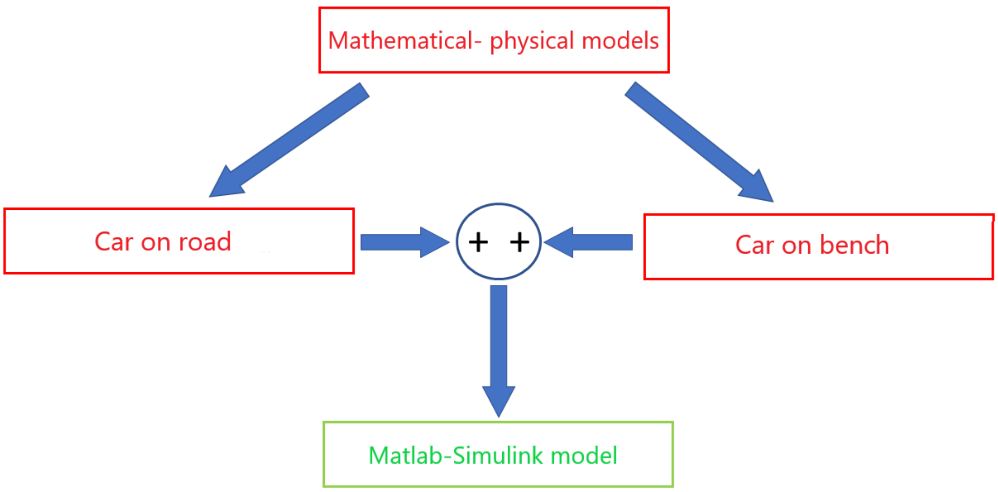

Figure 4.1: Matlab-Simulink model origin summary

From the mathematical-physical model of car on a roller test bench the equation of the load cell has been extrapolated, where in input is given the driving torque generated by the electric motor of the bench and in output is obtained the driving force generated by the car.

$$F_x = 2I_r\dot{w}_2\frac{R}{r^2} + 2F_r + (2I_R\dot{w}_2 + C_p - C_e)\frac{1}{R}$$

From the mathematical-physical model of a car on a roller test bench the road equation was used, where in input is given the driving force generated by the car on the bench and in output, solving the differential equation, is generated the tangential speed that the car would have had if it were really on the road.

$$a = \frac{F_x - 4k_1v - k_{aero}v^2}{(m_{vehicle} + \frac{4I}{r^2})}$$

The main model is as follows:

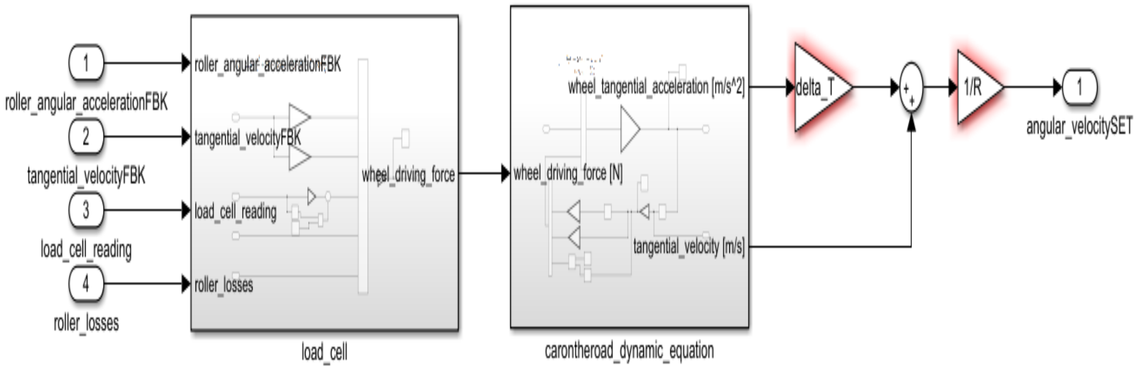

Figure 4.2: RLS subsystem of the main model

The load cell subsystem is the block containing the load cell equation, where the speed information, load cell readings and roller losses are provided as input. As input data for the various simulations we used real data from a test performed on a real Roller test Bench. In the following image you can see the content of the load cell subsystem representing the equation of the load cell in Simulink environment.

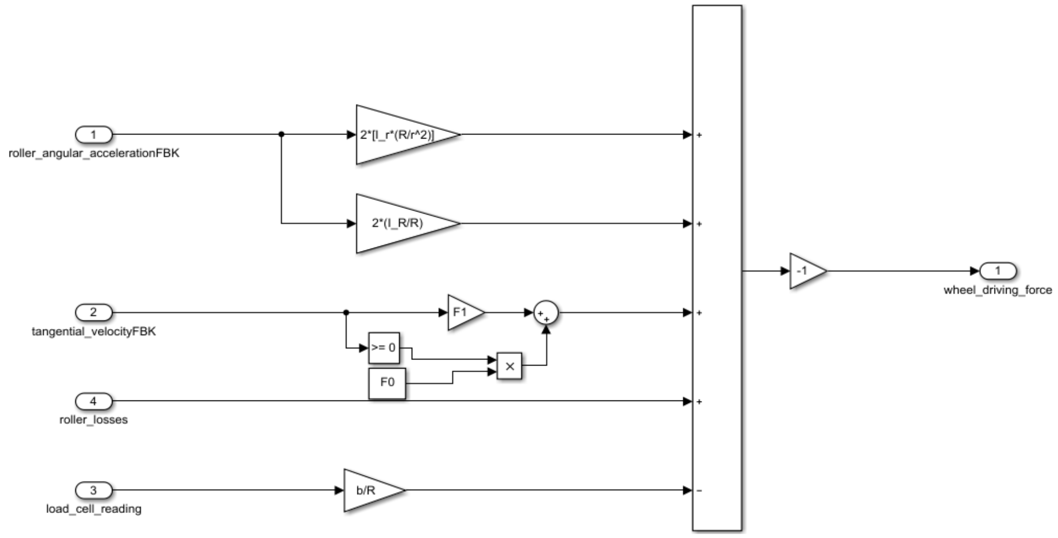

Figure 4.3: Load cell subsystem

The carontheroad dynamic equation subsystem contains the road equation and receives as input the driving force produced by the car from the load cell subsystem, while as output it generates the speed and the tangential acceleration at the wheels that the car would have had if it was really on the road. The image below shows the content of the subsystem carontheroad dynamic equation representing the road equation in Simulink language.

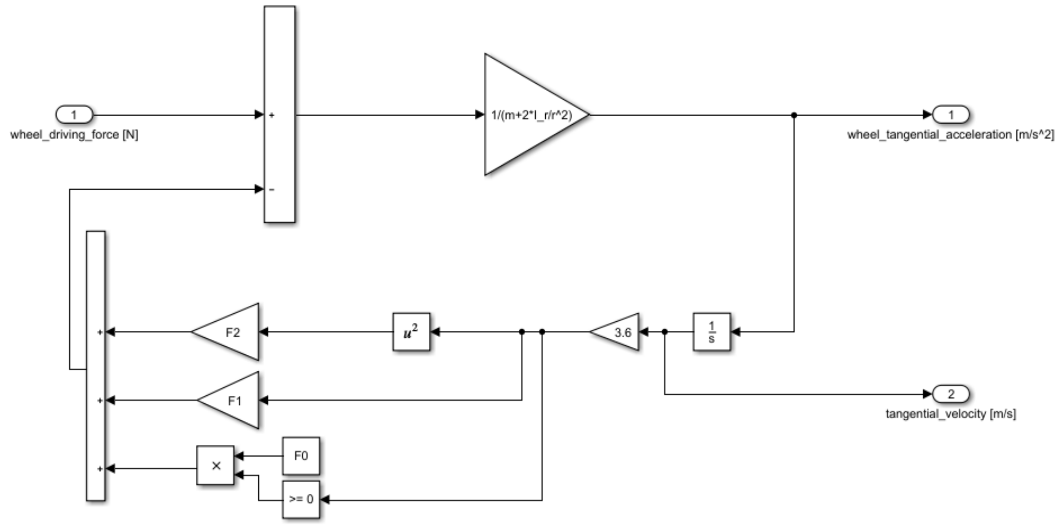

Figure 4.4: Carontheroad dynamic equation subsystem

It is necessary to emphasize the fact that the terms related to the rolling friction force, the resistant force due to the rotating parts present in the vehicle and the aerodynamic friction force acting on the vehicle present in both subsystems are replaced with the equation representing the braking curve F_M:

$$F_M = F_0 + F_1 v + F_2 v^2$$

The coefficients F_0, F_1 and F_2 are provided directly by the manufacturer of the vehicle under test.
Finally, in the next figure you can see the last part of the model, which generates the final output:

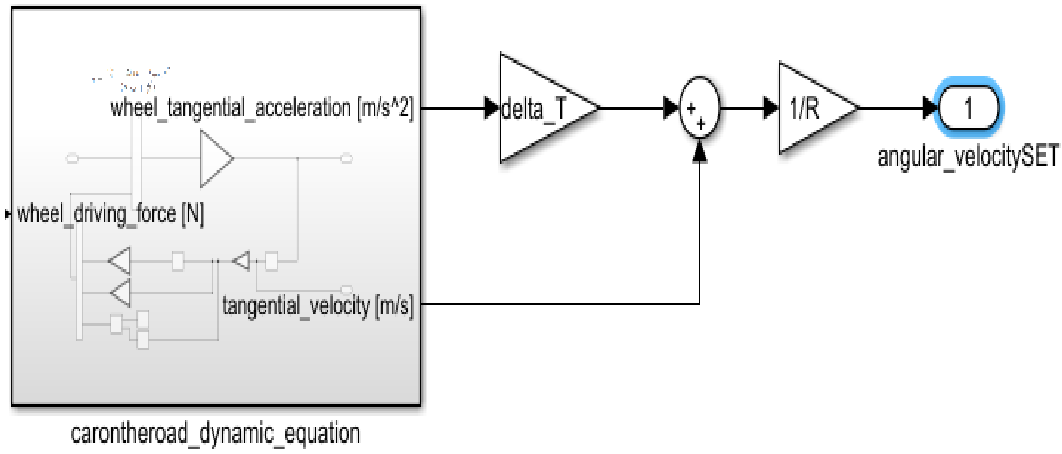

Figure 4.5: Angular velocity SET calculation

These blocks receive as input the tangential acceleration and tangential velocity at the wheels, and as output produce the SET angular velocity that must be supplied to the rollers so that they can simulate the road. What these blocks do mathematically is exploit the equation of uniformly accelerated motion discretized in time.

$$V_{t+1} = V_t + a\Delta t$$

Receiving from the carontheroad dynamic equation block the information of the acceleration and tangential velocity that the car would have if it were on the road, given the driving force F_x, at time instant t the tangential velocity SET at time instant t+1 is computed that must be supplied to the rollers in order to simulate the road.

4.1 Coast-down simulation

Using the Matlab-Simulink model, coast-down simulations were conducted. The purpose of the coast-down test is to verify if the roller test bench can perfectly reproduce the behavior of the car under test just as if it were really on the road, so this type of test has been chosen to validate the roller test bench model. As already explained in the previous chapters the coast down test is a cycle in which the vehicle is brought to a constant speed and then put in neutral and allowed to decelerate only because of friction. This deceleration is divided into intervals. In these intervals the deceleration and the elapsed time are observed and a resisting force is derived. The braking curve during the coast-down cycle is calculated by evaluating the deceleration between 125 km/h and 15 km/h.

In order to make comparisons between experimental results and simulated results, two variants of the main model have been created, which will be called the partially real model and the on-road car model. In the image below, the main model can be seen:

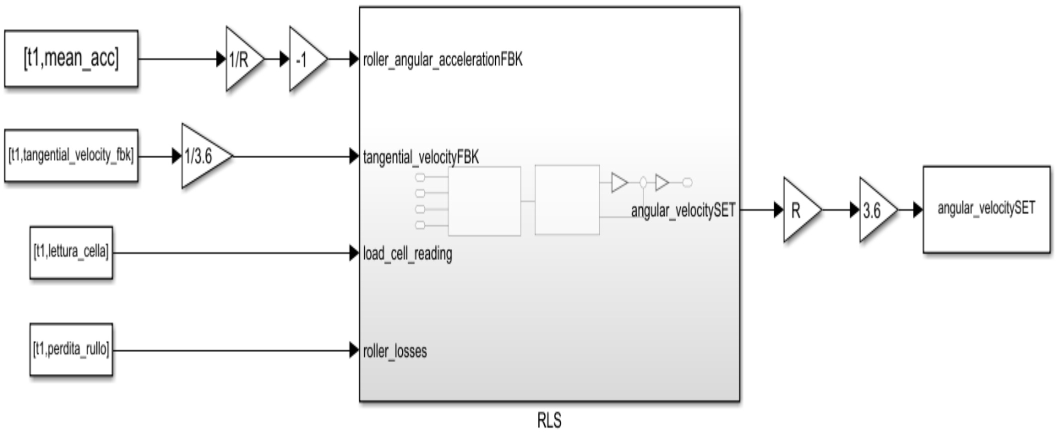

Figure 4.6: Main model

As it can be noticed all the inputs of the main model derive from the data taken from the empirical test carried out with a real rollers test bench.
In the following image instead we can see the partially real model:

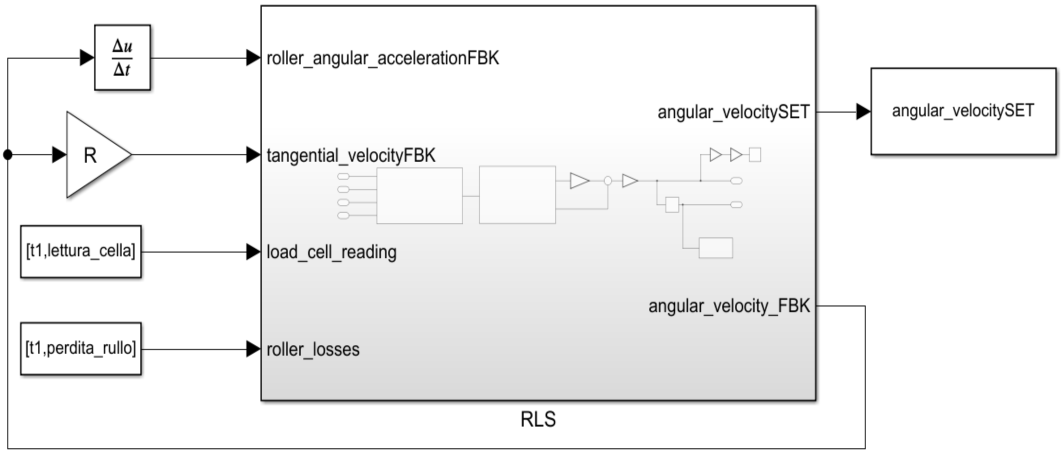

Figure 4.7: Partially real model

In this variant of the main model not all inputs come from data storage, only the information about load cell reading and roller losses. While the roller speed informations read from the encoder are calculated within the model.

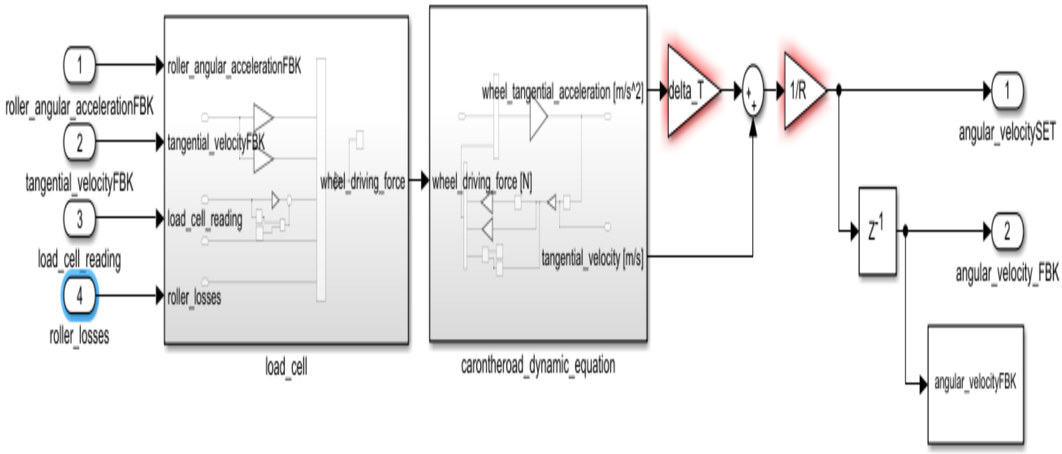

Figure 4.8: RLS subsystem of the partially real model

Browsing within the RLS subsystem, it is possible to see how the roller speed informations read by the encoder were calculated. The encoder reading was simulated by assuming that the speed read by the encoder at instant i is equal to the SET speed at instant i-1.

$$angularvelocityFBK(i) = angularvelocitySET(i-1)$$

This was enabled by the use of the Simulink delay block, which provides as output the input received in the previous integration step.
As a last step we go to analyze the vehicle model on the road:

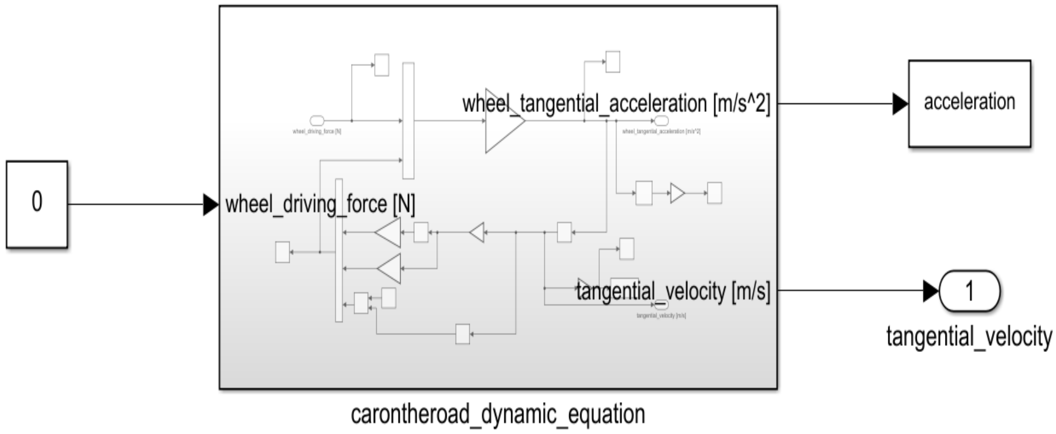

Figure 4.9: Model car on the road

In this case, as can be seen, the load cell subsystem containing the load cell equation is absent, this is because the model does not receive input information taken from the DBSrolls data store. The input model receives a constant driving force value of 0 N because what is simulated is a coast-down test and the car is in neutral gear during this test. Instead in output it produces the speed that the car would have if it was really on the road and was allowed to decelerate only because of friction from an initial speed of 125 km/h. The initial condition of 125 km/h for the tangential speed of the vehicle under test was entered into the integrator block present within the carontheroad dynamic equation subsystem.

4.1.1 Coast-down simulation: validation method

In order to validate the main model, data concerning speed information from the encoder, load cell readings and the bench's leakage curve were taken from a real coastdown test performed with a real roller test bench and all these data were used as input for the model. The data taken from the empirical test are data from a roller test bench that can perfectly simulate the behavior of the vehicle on the road.

The braking curve parameters provided by the manufacturer of the vehicle under test are:

- $F_0 = 99.2115$

- $F_1 = 0.7048$

- $F_2 = 0.1907$

Therefore based on the F_M equation of the braking curve

$$F_M = F_0 + F_1 v + F_2 v^2$$

$$F_M = 99.2112 + 0.7048v + 0.1907v^2$$

the braking curve that the model should try to follow is:

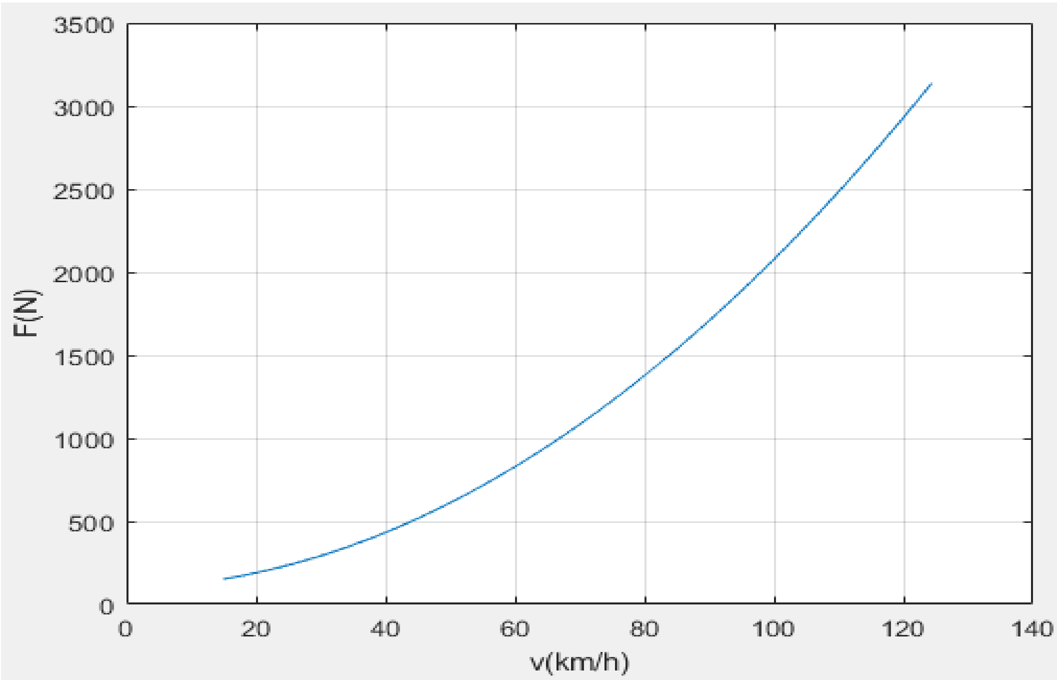

Figure 4.10: Theoretical braking curve of the bench

For what concern the braking curve simulated by the model it will be calculated using the output data of the SET acceleration of the rollers, which will be multiplied by the mass of the vehicle under test obtaining a resistant force.
The mass of the tested vehicle is 3300 kg.

$$F = a_{SET} m_{vehicle}$$

From these force values, the braking curve simulated by the model will be extrapolated using a second-order least squares interpolation.

In order to validate the model the NEDC (New European Driving Cycle) standard, requires to verify the percentage time error in the various speed intervals of 10 km/h, and compare it with a maximum value, which is 5% of the theoretical time value of that interval. Therefore, what you need to do is to calculate the time in the theoretical braking curve, obtained from the parameters provided by the vehicle manufacturer for each speed delta ranging from 125 km/h to 15 km/h at 10 km/h intervals. So 125 to 115, 115 to 105, and so on. Then calculate the time in the same intervals as the model curve. Then compare that the error made, for each interval, is within a range +/- 5%, except for speed ranges speed ranges below 55 km/h where the regulation is less stringent and the tolerance range increases by +/-10tolerance range increases to +/-10%. If this occurs, it means that the coastdown curve is accepted as correct.

4.1.2 Coast-down simulation: results

In the table below you can see the data from the coastdown test performed with a real roller test stand that will be used as a method of comparison with the simulation.

velocity range [km/h]	**Δt theoretical [s]**	**5% Δt**
124,4-115,4	2,97	0,1485
115,4-105,2	3,96	0,198
105,2-95,1	4,62	0,231
95,1-85,1	5,61	0,2805
85,1-75,2	7,26	0,363
75,2-65,2	8,91	0,4455
65,2-55,1	12,21	0,6105
55,1-45,1	16,5	0,825
45,1-35,1	23,43	1,1715
35,1-25,1	35,64	1,782
25,1-15,6	52,47	2,6235

Table 4.1: Theoretical coast-down test table

Three comparisons were made in order to validate the main model:

- comparison of theoretical braking curve and simulated braking curve using road car model;
- comparison of theoretical braking curve and simulated braking curve using the partially real model of car on bench;
- comparison of theoretical braking curve and simulated braking curve using the principal model of car on bench.

In the image below you can see the development of the simulated braking curve obtained using the car on road model compared with the theoretical braking curve.

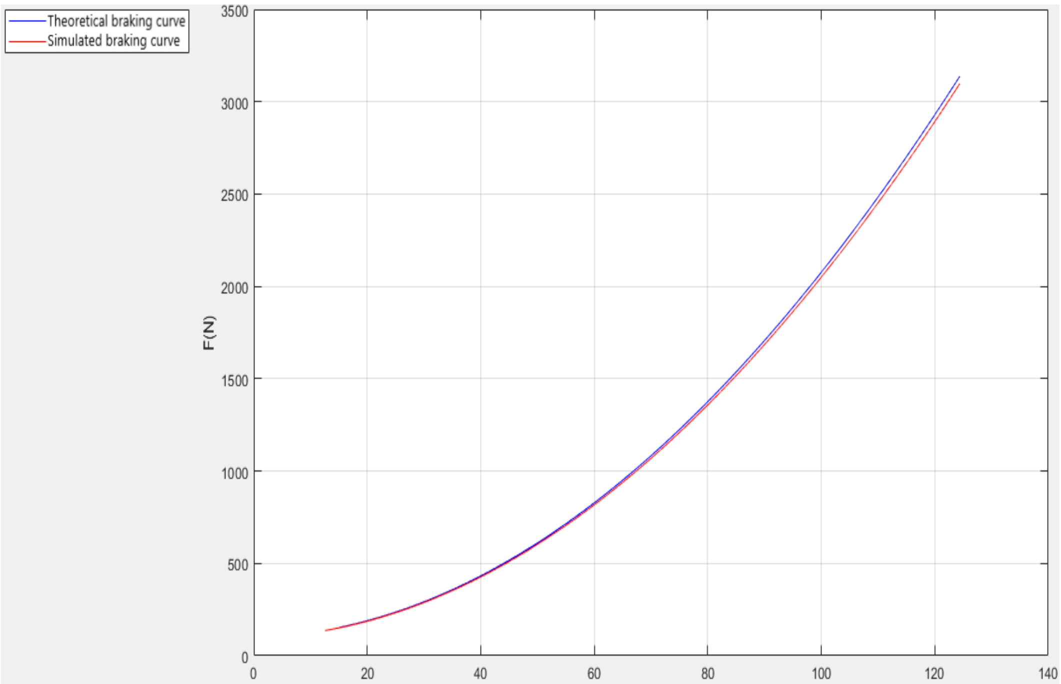

Figure 4.11: Comparison of theoretical and simulated braking curve of car on road model

As can be seen, the two curves are almost completely overlapping. This shows that the model is correctly simulating the road with respect to the three parameters F_0, F_1, and F_2 provided by the vehicle manufacturer. In order to better understand the comparison between the two curves and to attempt a validation using the competence standard, the NEDC (New European Driving Cycle), as explained above, a table was created in which the following data was entered:

- the velocity ranges of interest;
- Δt theoretical time in seconds, which indicates the time it should take the vehicle to move from one speed to another;
- the 5% of the theoretical Δt representing the tolerable error in simulation.;
- the Δt time in seconds of the simulation indicating the time elapsed to pass from one speed to another;
- the error in seconds committed by the simulation (difference between theoretical and simulated Δt);
- the percentage values of the absolute error of the simulation.

The table above can be seen below:

vel range [km/h]	**Δt theoretical [s]**	**5% Δt**	**Δt sim [s]**	**Δe**	**Δe %**
124,4-115,4	2,97	0,1485	2,706	0,264	8,888889
115,4-105,2	3,96	0,198	3,597	0,363	9,166667
105,2-95,1	4,62	0,231	4,323	0,297	6,428571
95,1-85,1	5,61	0,2805	5,181	0,429	7,647059
85,1-75,2	7,26	0,363	6,501	0,759	10,45455
75,2-65,2	8,91	0,4455	8,283	0,627	7,037037
65,2-55,1	12,21	0,6105	11,827	0,923	7,559378
55,1-45,1	16,5	0,825	15,312	1,188	7,2
45,1-35,1	23,43	1,1715	23,001	0,429	1,830986
35,1-25,1	35,64	1,782	36,069	-0,429	1,203704
25,1-15,6	52,47	2,6235	61,83	-9,36	17,83877

Table 4.2: Comparison table simulated coast-down test with car on road model

The table shows that all the speed intervals are within tolerance, except for the last interval between 25.1 km/h and 15.6 km/h. As far as the compliance with the regulations is concerned, simulating this model (car on road model), no problems have been found, also thanks to the simplicity of the constitutive blocks which do not receive input from external sources.

The next comparison to be made is that between the theoretical braking curve and the braking curve simulated using the car on bench model partially real. This comparison can be seen in the following image:

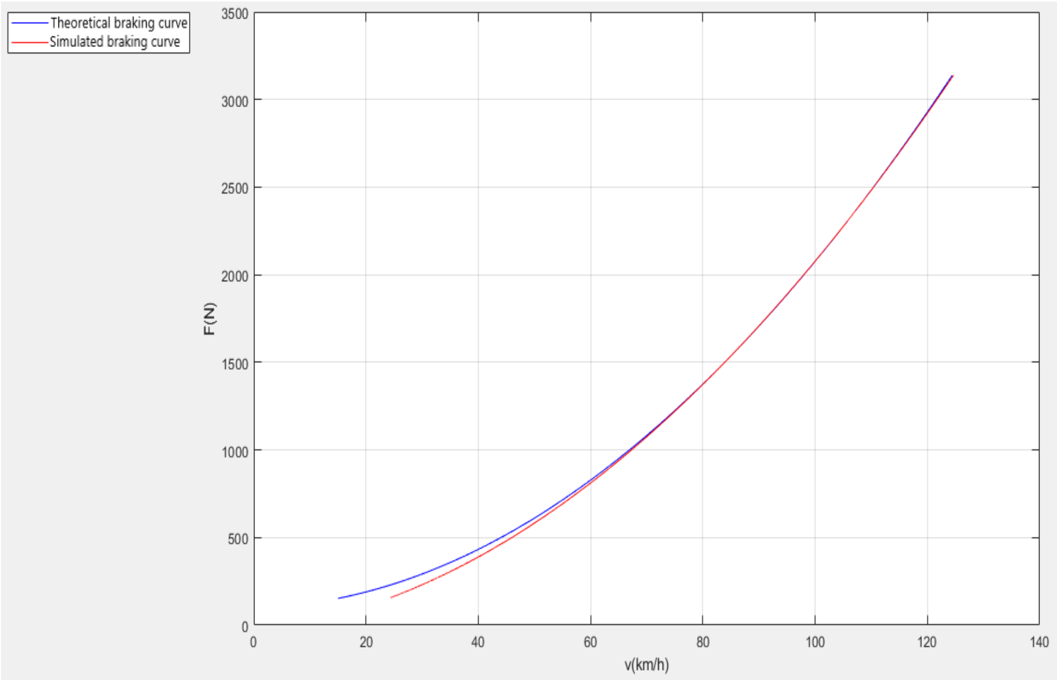

Figure 4.12: Comparison of theoretical and simulated braking curve of car on bench model partially real

The graph shows a significant deviation of the simulated braking curve from the theoretical one at low speeds. By analysing the table comparing the two curves, this error can be quantified.

vel range [km/h]	Δt theoretical [s]	5% Δt	Δt sim [s]	Δe	Δe %
124,4-115,4	2,97	0,1485	3,209	-0,239	8,047138
115,4-105,2	3,96	0,198	4,123	-0,163	4,116162
105,2-95,1	4,62	0,231	4,941	-0,321	6,948052
95,1-85,1	5,61	0,2805	5,181	0,429	7,647059
85,1-75,2	7,26	0,363	8,052	-0,792	10,90909
75,2-65,2	8,91	0,4455	9,783	-0,873	9,79798
65,2-55,1	12,21	0,6105	13,887	-1,677	13,73464
55,1-45,1	16,5	0,825	18,712	-2,212	13,40606
45,1-35,1	23,43	1,1715	27,001	-3,571	15,24114
35,1-25,1	35,64	1,782	43,069	-7,429	20,84456

Table 4.3: Comparison table of simulated coast-down test with car on bench model partially real

From the table it can be seen that from the speed range 55.1-45.1 km/h there is an increase in the percentage error which is outside the tolerance allowed by the regulations. This behaviour of the model is due to the fact that there is not a perfect synchronisation between the data collected with the empirical test carried out with a real test bench and the SET speed values calculated by the model. In the industrial environment, however, it is considered a good coast-down test taking this issue into account. Synchronisation was searched following a trial and error procedure by adding dummy inputs of roller loss and load cell readings equal to each other that correspond to the data taken during the empirical test at 124.4 km/h until the car had approximately reached the corresponding roller loss and load cell readings of the relative km/h.

Finally, the last comparison that was made was between the theoretical braking curve and the simulated braking curve using the pricipal model of car on bench. This is the most important comparison because the car model on the bench is the one that, together with the validation, would bring advantages to the company in terms of research and development and to save time in the testing phases. In fact, for the validation of this model, it was required to fully meet the requirements of the regulations. The comparison graph is shown below:

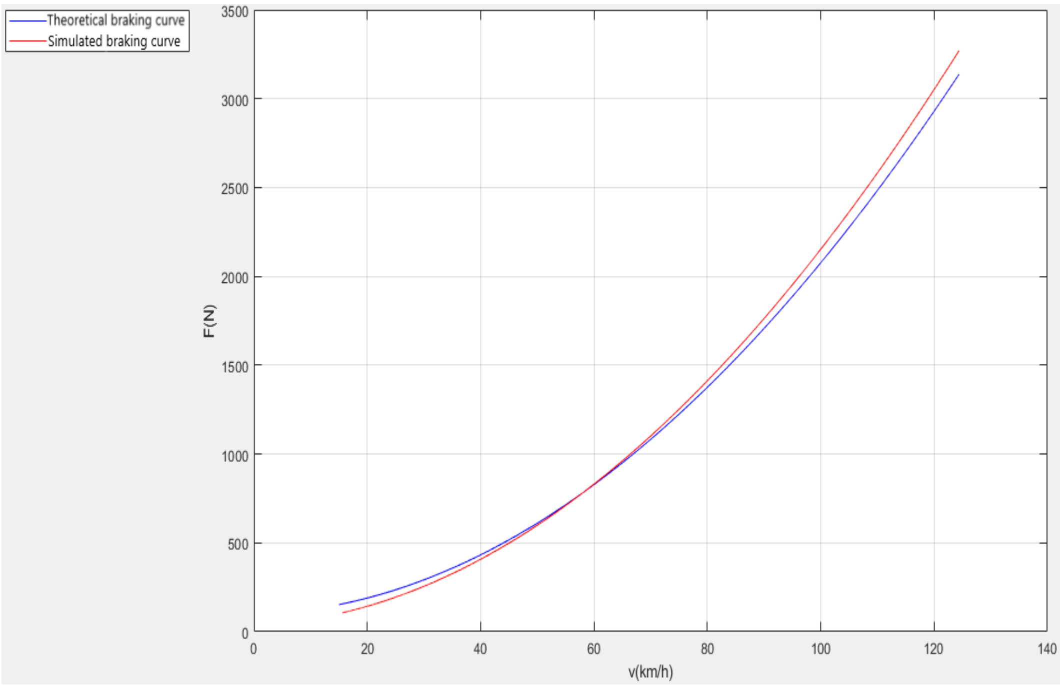

Figure 4.13: Comparison of theoretical and simulated braking curve of pricipal model of car on bench

The graph shows small deviations from the theoretical curve at the beginning and end of the simulation, but as can be seen in the table below, this does not affect

the fulfilment of the NEDC standard.
The results of the comparison with the simulation are shown below:

vel range [km/h]	**Δt theoretical [s]**	**5% Δt**	**Δt sim [s]**	**Δe**	**Δe %**
124,4-115,4	2,97	0,1485	2,86	0,11	3,703704
115,4-105,2	3,96	0,198	3,801	0,159	4,015152
105,2-95,1	4,62	0,231	4,386	0,234	5,064935
95,1-85,1	5,61	0,2805	5,441	0,169	3,012478
85,1-75,2	7,26	0,363	6,879	0,381	5,247934
75,2-65,2	8,91	0,4455	8,564	0,346	3,883277
65,2-55,1	12,21	0,6105	11,735	0,475	3,890254
55,1-45,1	16,5	0,825	15,537	0,963	5,836364
45,1-35,1	23,43	1,1715	22,555	0,875	3,734528
35,1-25,1	35,64	1,782	32,956	2,684	7,530864
25,1-15,6	52,47	2,6235	47,141	5,329	10,15628

Table 4.4: Comparison table of simulated coast-down test with principal model of car on bench

The table shows that the model behaves very well in simulation, as all ranges are within tolerance. These small errors of Δt that are recorded not only in this simulation but also in the previous one are due to the fact that the data supplied as input to the models, which were taken from a real coast-down test, are acquired with a delay mainly due to the processing that these data must undergo between the various communication layers, therefore the speed at which they are reported will be slightly different. In conclusion, it can be said that the principal model has been fully validated in accordance with the NEDC (New European Driving Cycle) standard.

Chapter 5

Conclusions and Future Developments

The following work is based on the study of a generic roller test bench for car homologation and in particular on the realization of a Matlab/Simulink model able to simulate the dynamic behaviour of the bench.

The first step was to study the bench in order to gain knowledge of its operating conditions. This knowledge was then applied using two methods, Newtonian and Lagrangian approach, to obtain dynamic equations describing the behaviour of cars on the road and cars on the bench. After that, the car on the bench model was implemented in Matlab/Simulink, which was called the pricipal model. This model, after passing validation, was made available to the company so that it could help them both to verify that the roller test bench behaves as expected and also to save time in finding the right trade off in setting the parameters F_0, F_1 and F_2.

Despite the results achieved, there are other possible implementations that could refine the model, for example, one possible analysis is to see if using an observer can improve some of the measurements calculated within the model, such as the driving force of the car and the angular velocity SET to be applied to the bench. Observers are useful in that they make it possible to know quantities which are not directly measurable, or whose measurement would require too expensive sensors. An observer makes it possible, by taking the inputs and outputs of the system, to know the values of other quantities.

In the case of estimating the driving force of the car one could resort to the use of observer sliding modes, which provide a non-linear input with the estimation error of the measurements, which is forced to cancel in a finite time, while the estimated state tends asymptotically to the true value of the state.

Regarding the estimation of the SET velocity to be applied to the bench, one could try to analyse the model with the addition of an EKF (extended Kalman filter), which is an observer used in the estimation of the parameters of a system which allows the Kalman filter theory to be applied and which could minimise the uncertainties arising from the system inputs.

Appendix A

Nomenclature

Symbol	Description
T_1	longitudinal force at front tires
T_2	longitudinal force at rear tires
R_1	rolling resistance of the front wheels
R_2	rolling resistance of the front wheels
F_{aero}	aerodynamic friction force
$m_{vehicle}$	total vehicle mass
m_{wheel}	wheel mass
$m_{chassis}$	chassis vehicle mass
v	longitudinal velocity of the vheicle
a	longitudinal acceleration of the vheicle
g	gravitational acceleration
θ	angle of road slope
C_d	coefficient of aerodynamic friction
A_f	frontal area of the vehicle subjected to aerodynamic friction
v_{wind}	wind velocity
ρ	air density
s_L	longitudinal slip
F_Z	weight force acting on the wheel
μ_L	longitudinal friction coefficient due to wheel-road contact
f_a	rolling resistance coefficient
F_i	inertia force
N_1	normal ground force on the front wheel
N_2	normal ground force on the rear wheel
C_M	driving torque of the vehicle
C_r	resistant torque due to bearings and transmission
L	wheelbase of the vehicle

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Symbol	Description
h	height of the vehicle's centre of gravity
c_1	center of front wheel
c_2	center of rear wheel
θ_1	angle of wheel rotation
θ_2	angle of rotation of the roller
r	wheel radius
R	roller radius
F_M	driving force of the vehicle
C_e	driving torque of the roller
F_e	driving force of the roller
b	arm of the load cell
$\tilde{F}$	reading the load cell
$\dot{w}_1$	angular acceleration of the wheel
$\dot{w}_2$	angular acceleration of the roller
$I_r\dot{w}_1$	moment of inertia of the wheel
$I_R\dot{w}_2$	moment of inertia of the roller
C_p	roller loss torque due to bearings

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