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MASTER FINAL PROJECT
“Design and analysis of conformal base station antenna for
5G applications”

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ABSTRACT

Conformal array antenna is often installed on the surface of high speed aircraft or other mobile platform carriers. Conformal array antenna not only does not change the dynamic performance of the carrier, it is easy to install; Moreover, it can effectively reduce the radar scattering area (RCS) of the aircraft and improve the stealth performance [4] [11].

In practice, conformal array has occlusion effect. That is, some array elements cannot receive the incident signal from every direction. In this study, the uniform cylindrical conformal antenna array structure is studied for 5G applications, and good simulation results are obtained. The main work contents are as follows:

Firstly, the theory of the linear array antenna is introduced. Both planar array antennas and circular array antennas are discussed. The cylindrical conformal antenna can be viewed as a circular array stacked together. A comparison between planar, circular and cylindrical arrays are provided.

Then, the patch antenna conformal to the cylinder surface is designed using a commercial software, namely Microwave Studio by Computer Simulation Technology. The antenna elements are uniformly distributed on the surface of the cylindrical structure medium, and discrete ports are used to connect each patch antenna to a metal ground on the back of the substrate. The outer radius of the cylinder is 63.7mm and the inner radius of the cylinder is 62mm and the height is 150mm. The radiation pattern is generated by 1x16 rectangular identical patches fed with the same signal is numerically calculated and compared with the theoretical value.

Finally, in this thesis, the beam scanning performances of the antenna array is analyzed: multi-port excitation is performed and beam synthesis is carried out applying weighted feeding coefficients computed by a Matlab script. Comparison between ideal array, consisting of isotropic radiators, and directive radiators (patches) is presented and discussed.

Keywords: Linear Array, Planar Array, Circular Array, Cylindrical Conformal and Antenna Array

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Chapter 1 Introduction

1.1 Background and Significance of the Study

Antenna is widely used in radio systems such as communication, broadcasting, television, radar and navigation. It plays the role of broadcasting radio waves and is an essential device for effectively radiating and receiving radio waves [11].

Due to the rapid development of antenna technology, as well as many research directions of antennas in the world, the birth of new antennas is promoted. Array antennas are one of the research directions. They form arrays according to different parameters such as antenna feed current, spacing and electric length in a straight line or more complex form to obtain the best radiation directivity. The array antenna can adjust the directional performance of the radiation as needed. The result is smart antennas, such as those used in modern mobile communications. I believe that in the near future, these high-technology antennas will bring us the same high-quality communication environment [4] [11].

The total field of the array is determined by the vector addition of the field radiated by the individual elements. In an array of identical elements, there are at least five controls that can be used to shape the overall pattern of the antenna. These are [1] [10]:

1. the geometrical configuration of the whole array (linear, circular, rectangular, spherical, etc.)
2. the relative displacement between elements
3. the excitation amplitude of the individual elements
4. the excitation phase of the individual elements
5. the relative pattern of the individual elements

1.2 5G application

Achieving the goal of new 5G wireless will require active antenna arrays in new antenna designs to extend coverage, reduce interference and improve data carrying capacity. For 5G cellular networks are expected to work on millimeter wave(mmW) bands (EHF 30-300GHz) [2]. It is a very small wavelength, but possibility to place hundreds of miniaturized antennas, according to Recommendation ITU-R for International Mobile Telecommunications-2020 (IMT-2020) a single Transmission Reception Point (TRxP) will have up to 256 antennas at 30 GHz and up to 1024 antennas at 70 GHz [2].

For deploying the 5G network, it requires many antenna packages suitable for using indoor and outdoor, networking in a small cell with wide coverage, and various types of terminal equipment. Today, for the base station, the network resources of most cell towers are very crowded. For this purpose, building a compact 5G antenna integrating high and low frequency is the most cost-effective solution. In early 5G deployments, a 5G antenna will need to be placed next to an existing 4G LTE antenna, or the existing antenna will need to be replaced with a element that can be used as both a 4G LTE antenna and a low-frequency 5G antenna.

With the roll out of 5G at higher frequencies, m-MIMO arrays with many antenna elements will help reduce network congestion and increase base station capacity.

1.3 Organization of the thesis

The document is organized as follows: In Chapter 1, there is the introduction, a description of the background and the overview of the thesis. In Chapter 2, a description of the array antenna theory is presented, from the linear array to planar

array antenna to circular array antenna to cylindrical array antenna. In Chapter 3, a description of the work in CST: development of the Macro in CST and the figure of the structures and the radiation pattern plots. In Chapter 4, conclusion: the summary are presented.

Chapter 2 Design Basis and Theory of the Array Antenna

2.1 Introduction

Array antenna is a kind of special antenna which consists of two or more antenna elements arranged regularly or randomly and obtains predetermined radiation characteristics through appropriate excitation. Feeding is usually carried out by means of series or parallel feed.

Linear array antenna is the research basis of conformal antenna array, and on this basis, circular array antenna and planar array antenna are obtained. The cylindrical conformal antenna arrays are annular arrays superimposed together. As shown in the following Figures (a) [2], Figure (b) [2], Figure (c) [2]:



Figure 2.1(a) Uniform Linear Array ^[2]

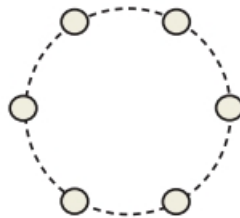


Figure 2.1(b) Uniform Circular Array ^[2]

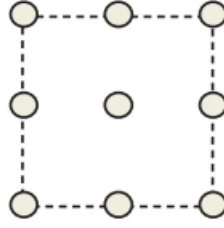


Figure 2.1(c) Uniform Planar Array ^[2]

2.2 Theory of Array Antenna

2.2.1 Linear Array Antenna

Linear array is the simplest array geometry and all elements are aligned along a straight line [22] [24]. To simplify the presentation and give a better physical interpretation of the techniques, a two-element array which is the minimum length linear array will first be considered. M-elements array analysis will then follow [27].

Two-element Array Antenna

Two-element antenna array consists of two antenna elements of the same type, same size, same phase and same amplitude. The two antenna element are represented by points, the spacing between them is $d=\lambda/2$.

Let us assume that the antenna under investigation is an array of two infinitesimal horizontal dipoles positioned along the one-axis [9]. The total field radiated by the two elements which assuming there is no coupling between the two elements is equal to the sum of the two antenna. As shown in Figure 2.2.1 (a) [9], two elements along z-axis, play in the y-z plane:

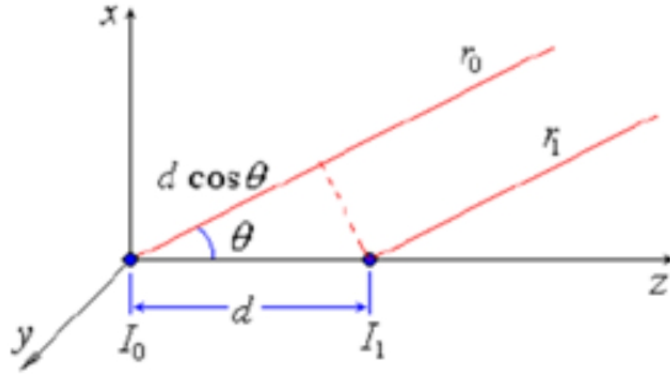


Figure 2.2.1 (a) 2-Elements Array Antenna ^[9]

The total radiation pattern equal to primary pattern which relates to the structure and position of the unit antenna multiply by array pattern. The formula is [10]:

$$f(\theta, \varphi) = f_1(\theta, \varphi) \times f_a(\theta, \varphi)$$

Where:

$f_1(\theta, \varphi)$ is Primary Pattern

$f_a(\theta, \varphi)$ is Array Pattern

M-elements Uniform Linear Array Antenna

Uniform linear antenna array is composed by multiple antenna elements which have the same type and same size, they are arranged in a straight line according to certain rules (equal spacing), as shown in Figure 2.2.1(b) [1].

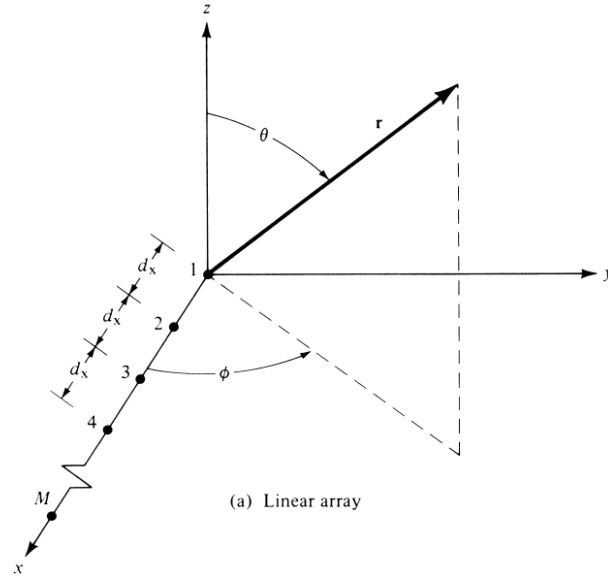


Figure 2.2.1 (b) M-Elements Array Antenna Geometry ^[1]

The M-elements Uniform Array Antenna, it is similar to the discussion of the two-elements antenna. The spacing between antennas is $d=\lambda/2$ and the Array Pattern of M-elements Uniform Linear Array(ULA) antenna is [10]:

$$f_a(\delta) = |1 + e^{j\psi(\delta)} + e^{j2\psi(\delta)} + \dots + e^{j(M-1)\psi(\delta)}|$$

$$f_a(\psi) = |\sin (M\psi / 2) / \sin (\psi / 2)|$$

Let M as the maximum, the function is normalized [10]:

$$f_a(\delta) = \frac{1}{M} |\sin (M\psi / 2) / \sin (\psi / 2)|$$

According to the product theorem of direction function, the Array Pattern multiply by direction function of the Array element is equal to the direction function of uniform linear Array. However, in practical application, the maximum radiation

direction of the array element should be consistent with the array factor as much as possible, and the direction of the array is controlled mainly by adjusting the array factor.

Steering Vector and Antenna Spacing

Assume that the array signal is a narrowband signal and the signal source is located in the far field of the array. It is believed that the signals incident in parallel and there is no amplitude difference between the signals received by each array element, only the phase difference caused by transmission delay exists.

For the narrowband signal model, the Array Pattern is [10]:

$$f_a(\delta) = |1 + e^{j\psi(\delta)} + e^{j2\psi(\delta)} + \dots + e^{j(M-1)\psi(\delta)}|$$

Where:

$$\psi(\delta) = 2\pi \frac{d}{\lambda} \sin \theta$$

Since the propagation delay for each received signal has the phase shift, the observations on each antenna are arranged as an $M \times 1$ vector. So, the steering vector can be get as [5]:

$$f_a(\theta) = \begin{bmatrix} 1 \\ e^{j2\pi \frac{d}{\lambda} \sin \theta} \\ \vdots \\ e^{j2\pi \frac{d}{\lambda} (M-1) \sin \theta} \end{bmatrix}$$

According to the Sampling theorem^[1] [5], the antenna spacing design is:

$$d \leq \frac{\lambda}{2}$$

2.2.2 Planar Array Antenna

Planar array antennas are the most common form of array antennas. The Uniform Planar Array is a Linear Array formed by the orderly arrangement of the elements along a straight line, and multiple elements are arranged in a rectangular grid form [22] [24]. In the Cartesian coordinate system, if the number of array elements arranged along the x axis is M, the number of array elements arranged along the y axis is N, and the spacing of array elements arranged along the x axis and y axis are dx and dy respectively, a two-dimensional rectangular uniform planar array of M x N elements is formed. As shown in Figure 2.2.2 [1]. Compared with the uniform linear array, the uniform planar array antenna add a factor that controls the shape of the antenna pattern. For this reason, the uniform planar array antenna has a more symmetrical pattern and lower side lobe.

[1] : Sampling theorem can be defined as the conversion of an analog signal into a discrete form by taking the sampling frequency as twice the input analog signal frequency [28].

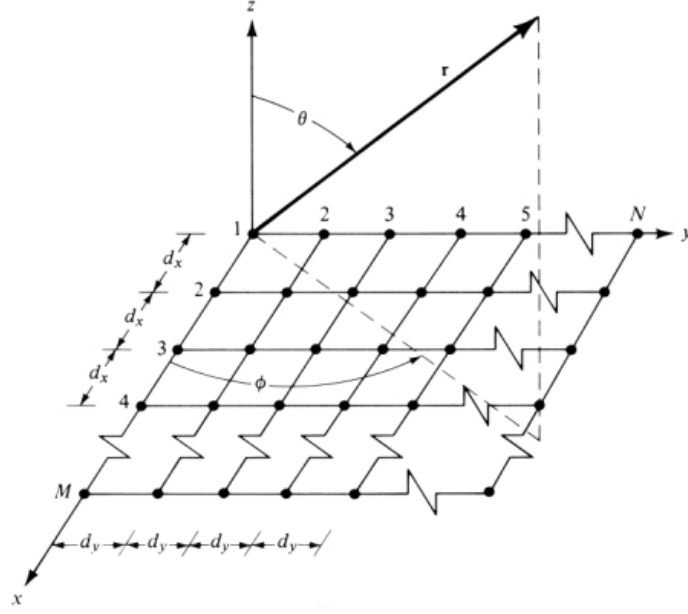


Figure 2.2.2 Uniform Planar Array Antenna geometry ^[1]

For the x axis, Array Pattern [10]:

$$f_{ax}(\theta, \varphi) = \left| \sum_{i=1}^{Nx} e^{j2\pi/\lambda(i-1)dx \cos \varphi \sin \theta} \right|$$

For the y axis, Array Pattern [10]:

$$f_{ay}(\theta, \varphi) = \left| \sum_{k=1}^{Ny} e^{j2\pi/\lambda(k-1)dy \cos \varphi \sin \theta} \right|$$

The total Array Pattern of the planar Array is multiply the x and y two parts [10] :

$$f_a(\theta, \varphi) = f_{ax}(\theta, \varphi) \times f_{ay}(\theta, \varphi)$$

$$f_a(\theta, \varphi) = \left| \sum_{i=1}^{Nx} \sum_{k=1}^{Ny} e^{j2\pi/\lambda[(i-1)dx \cos \varphi + (k-1)dy \sin \varphi \sin \theta]} \right|$$

Steering Vector

The angle pair for an observation of the planar is (θ, ϕ) . According to the calculation of the linear steering vector, the planar array steering vector can be obtained. Let that [10]:

$$\xi_x = 2\pi \frac{d_x}{\lambda} \cos \theta$$

$$\xi_y = 2\pi \frac{d_y}{\lambda} \sin \theta \sin \phi$$

The steering vectors of x-axis and y-axis are [5]:

$$a_x = \begin{bmatrix} 1 \\ e^{j\xi_x} \\ \vdots \\ e^{j\xi_x(M-1)} \end{bmatrix} \quad a_y = \begin{bmatrix} 1 \\ e^{j\xi_y} \\ \vdots \\ e^{j\xi_y(N-1)} \end{bmatrix}$$

By using the e Kronecker product ^[1], the total array pattern is written as the product of the array pattern for the x-axis and y-axis [5]:

$$a(\theta, \phi) = a_x \otimes a_y = \begin{bmatrix} 1 \\ e^{j(\xi_x + \xi_y)} \\ \vdots \\ e^{j[\xi_x(M-1) + \xi_y(N-1)]} \end{bmatrix}$$

[1]: In mathematics, the Kronecker product, sometimes denoted by \otimes , is an operation on two matrices of arbitrary size resulting in a block matrix. It is a generalization of the outer product (which is denoted by the same symbol) from vectors to matrices, and gives the matrix of the tensor product with respect to a standard choice of basis. The Kronecker product is to be distinguished from the usual matrix multiplication, which is an entirely different operation. The Kronecker product is also sometimes called matrix direct product [15].

2.2.3 Circular Array Antenna

All elements of the uniform circular array antennas are distributed on a ring. The uniform circular array antenna array is composed of N isotropic antenna elements

distributed with equal spacing on a ring located in the x - y plane and the radius of the ring is a . As shown in Figure 2.2.3 [1].

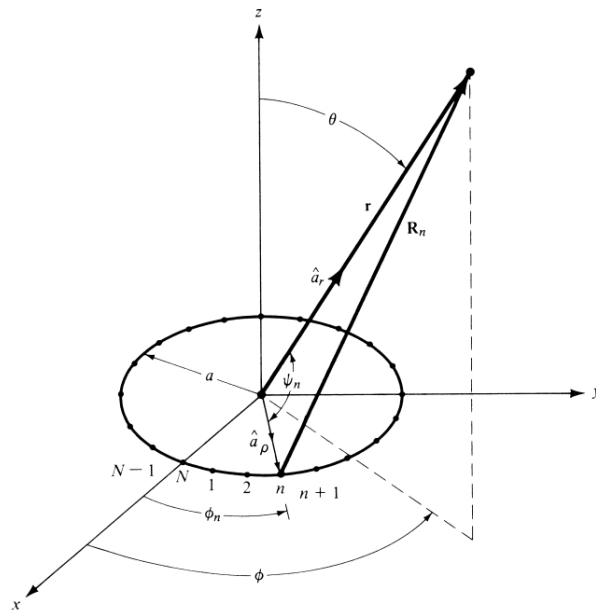


Figure 2.2.3 Geometry of an N-element Circular Array [1]

The total radiation field is [10]:

$$E(r, \theta, \phi) = \sum_{n=1}^N \tilde{I}_n R_n \quad [10]$$

Where:

\tilde{I}_n is the excitation current of n -th element

R_n is observation point distance of n -th element, and it can be expressed [10]:

$$R_n = (r^2 + a^2 - 2ar \cos \psi_n)^{1/2}$$

When $r \gg a$,

$$R_n = r - a \cos \psi_n = r - a \sin \theta \cos (\varphi - \varphi_n)$$

The expression of the total radiation field can be simplified as:

$$E(r, \theta, \varphi) = \frac{e^{-jkr}}{r} \sum_{n=1}^N \tilde{I}_n e^{jka \sin \theta \cos (\varphi - \varphi_n)}$$

Where:

$$\varphi_n = 2n\pi / N$$

In general, the excitation current of the array element can be expressed as

$$\tilde{I}_n = I_n e^{j\xi_n}$$

In this case, the radiation electric field expression is [10]:

$$E(r, \theta, \varphi) = \frac{e^{-jkr}}{r} \sum_{n=1}^N \tilde{I}_n e^{j[ka \sin \theta \cos (\varphi - \varphi_n) + \xi_n]}$$

Let $F(\theta, \varphi) = \sum_{n=1}^N \tilde{I}_n e^{j[ka \sin \theta \cos (\varphi - \varphi_n) + \xi_n]}$, which is an array function of circular array antenna with equally spaced distribution.

If the maximum point of the array function is: (θ_0, φ_0) , then the phase of the n-th array element current should be [10]:

$$\xi_n = -ka \sin \theta_0 \cos (\varphi_0 - \varphi_n)$$

Substitute in the above equation, we have [10]:

$$\begin{aligned} E(r, \theta, \varphi) &= \sum_{n=1}^N I_n e^{jka[\sin \theta \cos (\varphi - \varphi_n) - \sin \theta_0 \cos (\varphi_0 - \varphi_n)]} \\ &= \sum_{n=1}^N I_n e^{jka(\cos \psi - \cos \psi_0)} \end{aligned}$$

Where:

$$\cos \psi = \sin \theta \cos (\phi - \phi_n)$$

$$\cos \psi_0 = \sin \theta_0 \cos (\phi_0 - \phi_n)$$

In order to simplify the above equation, set [10]:

$$\rho_0 = a\sqrt{(\sin \theta \cos \phi - \sin \theta_0 \cos \phi_0)^2 + (\sin \theta \sin \phi - \sin \theta_0 \sin \phi_0)^2}$$

$$\gamma = \tan^{-1} \left[\frac{\sin \theta \cos \phi - \sin \theta_0 \cos \phi_0}{\sin \theta \sin \phi - \sin \theta_0 \sin \phi_0} \right]$$

The array function can be simplified as:

$$F(\theta, \varphi) = \sum_{n=1}^N I_n e^{jk\rho_0 \cos (\phi_n - \gamma)}$$

For a circular array with uniform current distribution, its array function can be expressed as [10]:

$$F(\theta, \varphi) = NI_0 \sum_{m=-\infty}^{+\infty} J_{mN}(k\rho_0) e^{jmN(\pi/2-\gamma)}$$

If N is large enough, the direction function of the circular array antenna can be approximated.

2.2.4 Uniform Cylindrical Array Antenna

The uniform Cylindrical Array (UCylA) can be regarded as a stack of circular array antennas [24]. Firstly, the coordinate diagram of cylindrical array antenna is given [6]. As shown in Figure 2.2.4 [6], the cylindrical with radius R is uniformly covered with M rows and N columns of antenna elements, and the row and the column spacing of array elements are d1 and d2. The global coordinate system (x,x,z) is established with the center of the cylinder bottom as the origin, and the local global coordinate system is established with the antenna element (i, j) as the origin [6].

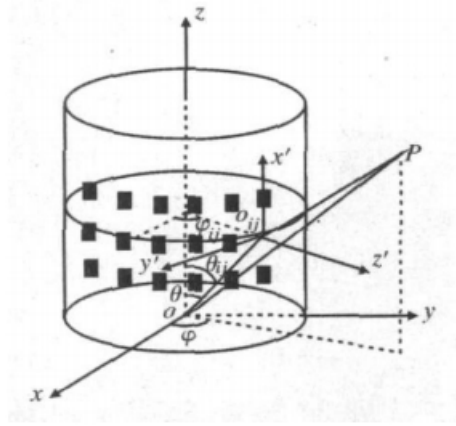


Figure 2.2.4 Coordinate Diagram of Cylindrical Array ^[6]

Any point of the spherical coordinates in the far field is P (θ, φ) in the global coordinate system. After coordinate transformation, the spherical coordinates is

expressed as $P(\theta', \varphi')$, the relationship is [7]:

$$\begin{cases} \theta' = \cos^{-1} (\sin \theta \cos (\varphi_{ij} - \varphi)) \\ \varphi' = \tan^{-1} \left(\frac{\sin \theta \sin (\varphi_{ij} - \varphi)}{\cos \theta} \right) \end{cases}$$

The pattern product theorem does not apply to cylindrical antenna arrays because the far-field radiation directions of array elements are different, which are related to their coordinate positions in the global coordinate system. Taking the origin O of the coordinate system as the reference center, each array element is regarded as a different independent item to superposition, and the total radiation field is expressed as [7]:

$$E(\theta, \varphi) = \sum_{i=1}^M \sum_{j=1}^N a_{ij} f_{ij}(\theta, \varphi) e^{jk(R \sin \theta \cos (\varphi - \varphi_{ij}) + z_{ij} \cos \theta)}$$

Where:

$$k = 2\pi / \lambda$$

$f_{ij}(\theta, \varphi)$ is the vector pattern function of array element in global coordinate system.

The theoretical calculation results of cylindrical conformal array antenna can be obtained by the above formulas.

2.2.5 Conformal Array Antenna

Conformal array antenna refers to the array antenna attached to the surface of the

carrier and attached to the carrier, that is, the array antenna needs to be conformally installed on a surface of a fixed shape, so as to form a conformal array antenna. In modern wireless communication systems, conformal array antennas is a hot spot research in the field of antennas because they can be conformal with the surface of carrier platforms running at high speed, such as aircraft, missiles and satellites, without damaging the shape structure and aerodynamic characteristics of the carrier.

The design process of conformal antenna is complex, and the shape of the conformal array antenna, the form of the array element and its distribution should be considered in the initial stage of design [4]. In addition, the influence of the conformal carrier and the mutual coupling effect between the elements on the resonant frequency, bandwidth and polarization of the array elements should be considered in the design [4] [12] [13].

Some type of the conformal Array Antenna, as shown in Figure 2.6 [4]:

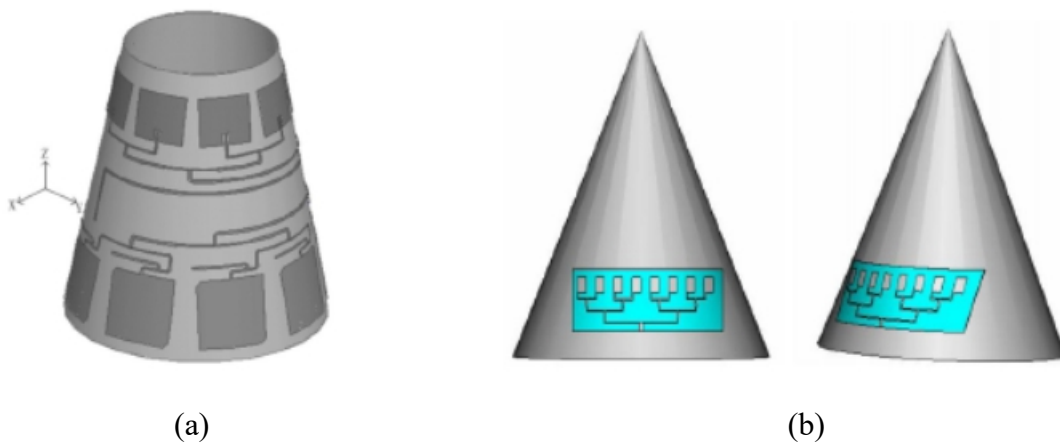




Figure 2.2.5 Conformal Array Antenna ^[4]

Advantages of Conformal Array

There has the advantages by using the conformal array antenna: The conformal antenna array has isotropic behavior; and the beam broadening does not cause pattern degradation; and do not have to divide the cells [4] [12].

2.2.6 Beamforming techniques

With the increasing prosperity of modern communication technology and the higher requirements for antenna in practical application. As a result, the beamforming array antenna is born [4].

Beamforming is a technology which adaptively adjusts the radiation pattern of antenna array according to the specific scene. It is a key technology of smart antenna system. It can distinguish between signals from different directions, and send signals to target users, and can improve the signal-to-noise ratio of the target users and the system energy utilization efficiency. At the same time, it can reduce the interference to other users and reduce the interference of the system and improve the edge user

throughput rate and the average throughput rate. In addition, the same time-frequency resources can be used to transmit signals to users in different locations to achieve space division multiplexing, which makes full use of space resources to improve spectrum utilization and system capacity [16].

The traditional communication mode of single antenna is the electromagnetic wave propagation from single antenna to single antenna between base station and mobile phone. In the absence of physical adjustment, the antenna radiation direction is fixed, which results in the limited number of users serving the same frequency at the same time. In the beamforming technology, the base station has multiple antennas, which can automatically adjust the phase of the signal emitted by each antenna, so that it can form an effective superposition of electromagnetic waves at the receiving point of the mobile phone, and generate stronger signal gain to overcome the loss, so as to achieve the purpose of improving the received signal intensity [17].

Basic principles of beamforming

The most basic method to realize beamforming is to add the signals of each antenna array element after proper delay, so that the signals in the target direction are superimposed and enhanced, while the other directions are weakened to varying degrees. This method is usually used for analog signals. The digital signals can be added by multiplying the signals of each antenna array element by the complex weighting coefficient, as shown in the Figure 2.2.6-1 [5]. The weighted coefficient can not only adjust the signal phase, but also the signal amplitude. For the narrowband signal, the signal in the target direction can be enhanced, and the interference and noise in other directions can be suppressed by setting the zero point reasonably.

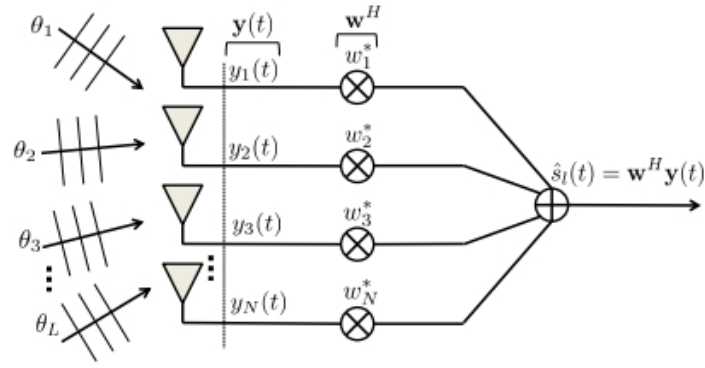


Figure 2.2.6-1 Beamforming Basic Schematic Diagram [5]

3D Beamforming Simulation

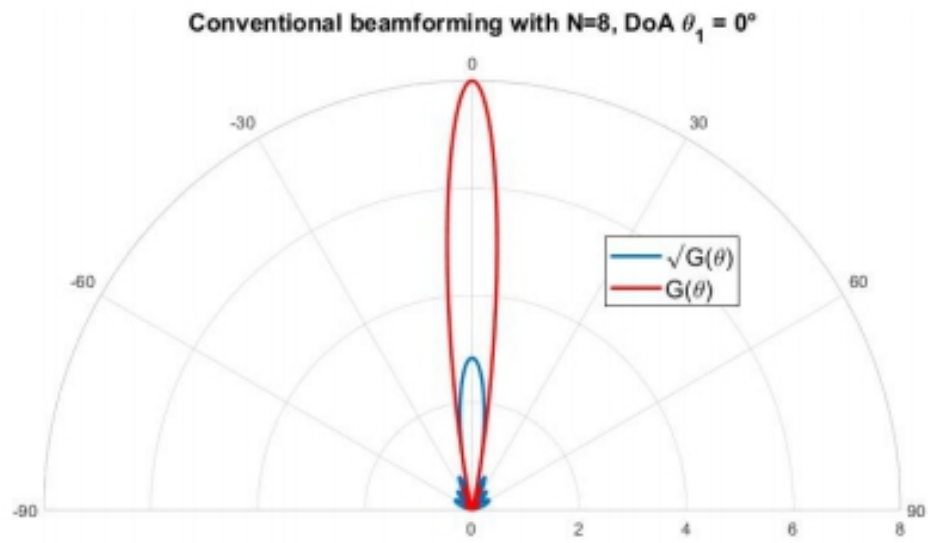
After entering the 5G era, as the antenna array expands from one dimension to two dimension, the beamforming has developed into a three-dimensional multi-faceted industry, which can control the horizontal and vertical shape of the antenna pattern at the same time and evolve into 3D beamforming. 3D beamforming enables the base station to direct the signal to the target user more accurately according to the different spatial distribution of the user.

One-dimensional uniform linear arrays cannot realize three-dimensional beamforming in real sense, so two-dimensional antenna arrays must be used to realize three-dimensional beamforming.

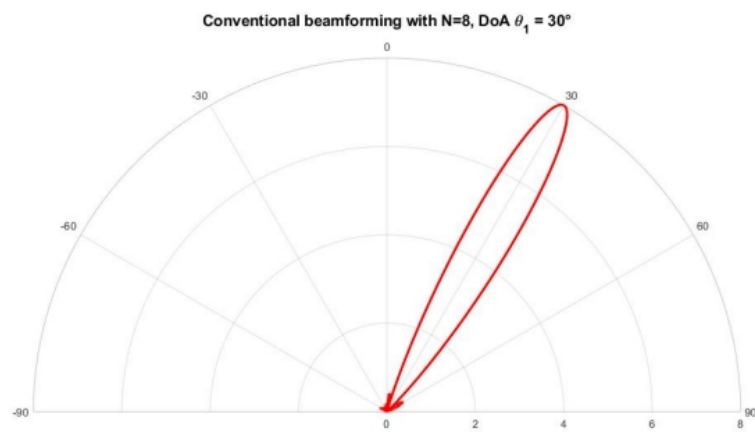
For uniform linear arrays, changing the DOA (Direction of Arrival) of the received signal (Impinging wavefront) can change the beam width of the 2D radiation pattern.

For example, as shown in the Figure 2.2.6-2 [5], the uniform linear arrays set along the X-axis: $M=8$ that keeps the number of array elements and the horizontal Angle unchanged. After changing the vertical angle and increasing the vertical angle, the bar-shaped beam formed by the uniform linear array becomes wider in the vertical direction, indicating that the beam width gradually increases with the increase of DOA. The resolution of the vertical angle of the uniform linear array is reduced,

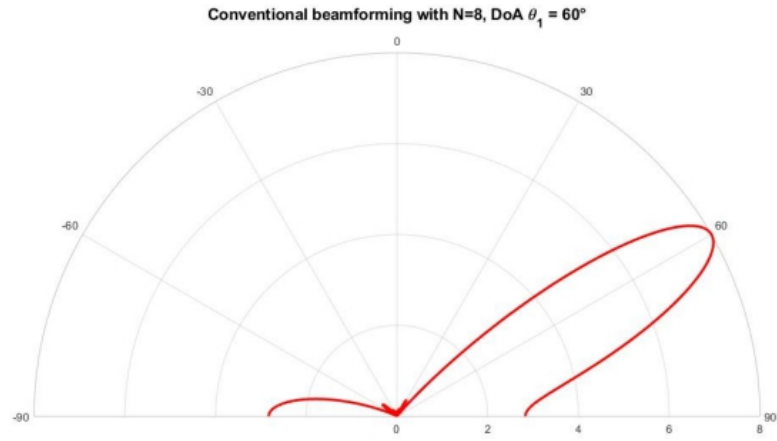
which results in the poor shape of the vertical beam.



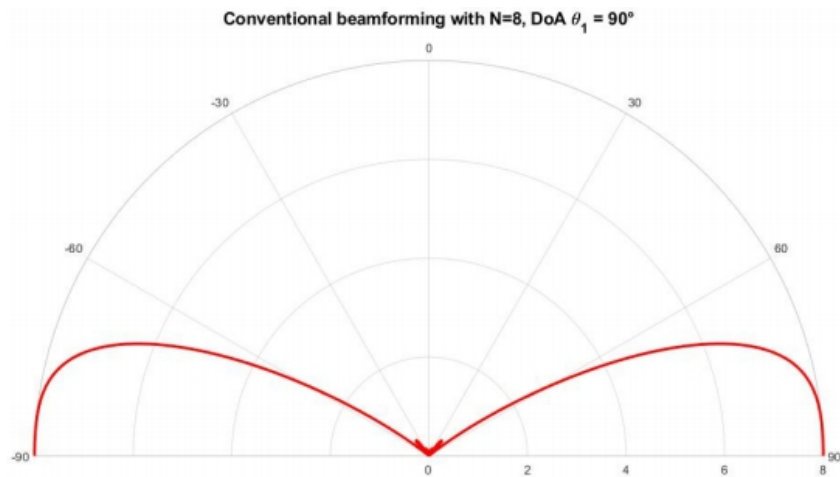
(a)



(b)



(c)

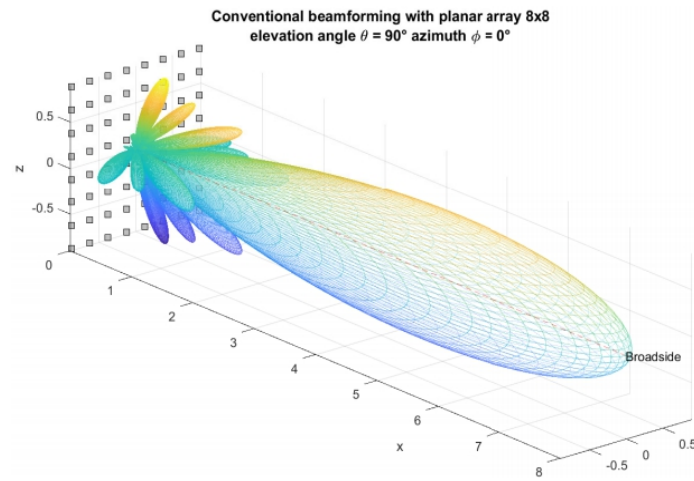


(d)

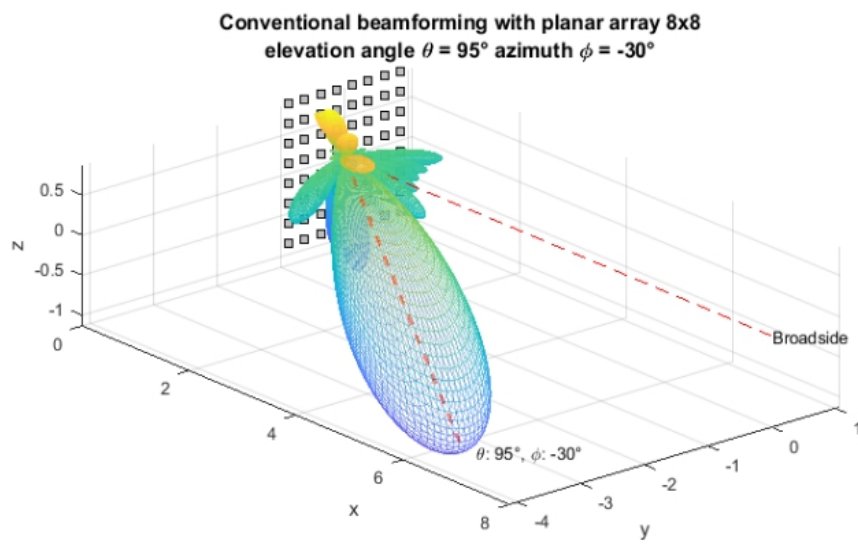
Figure 2.2.6-2 2D Radiation Pattern of a Uniform Linear Array ^[5]

For the uniform planar array antennas. For example, as shown in the Figure 2.2.6-3 [5], a uniform planar array of x-axis and y-axis elements $M=8$, $N=8$, with spacing $dx=dy=\lambda/2$ is used to transmit a narrowband signal in different directions. The 3D beamforming effect picture is obtained. The vertical resolution of the uniform

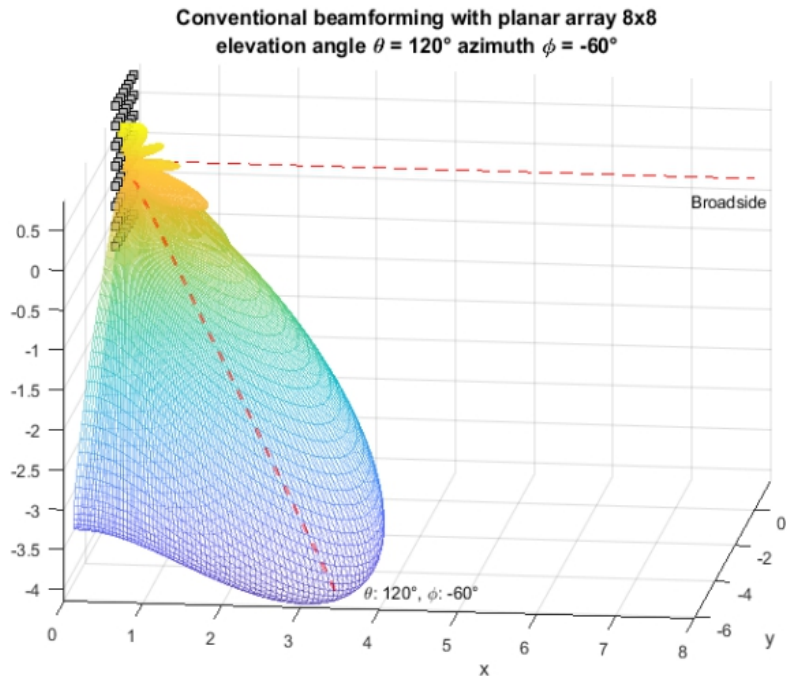
planar array is not affected by the change of horizontal angle, but decreases significantly with the increase of pitch angle.



(a)



(b)

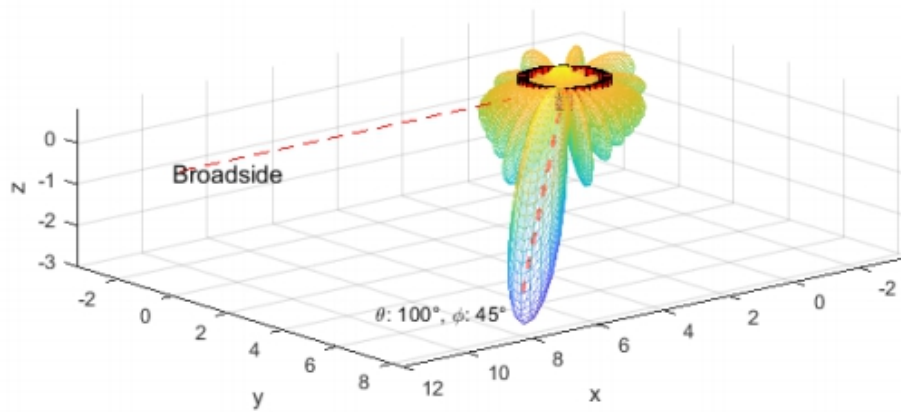


(c)

Figure 2.2.6-3 3D Radiation Pattern of a Uniform Planar Array [5]

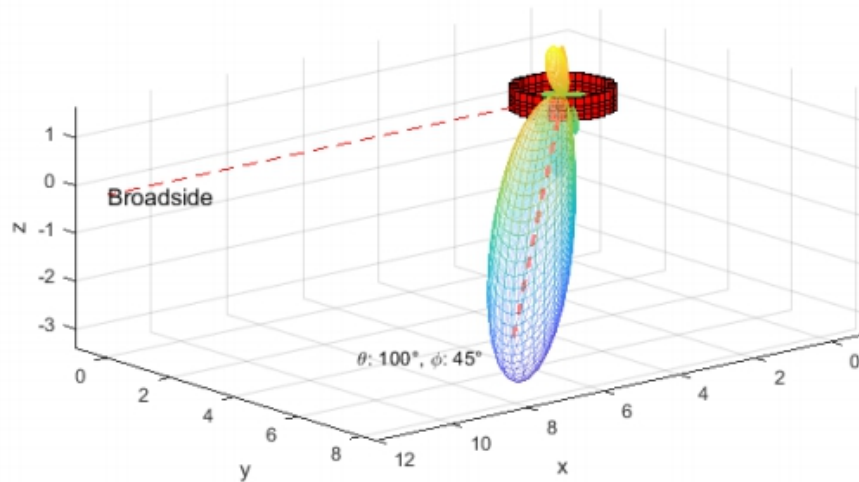
The beamforming resolution of the uniform cylindrical array is better than uniform planar array. For example, four uniform circular arrays along the Z axis are superimposed to form a uniform cylindrical array. As shown in the Figure 2.2.6-4 [5], it has better beamforming performance by using directive elements than using isotropic elements. For directive elements, side lobes are suppressed.

Conventional BF with cylindrical array of 36 isotropic elements per circle and 4 rings
elevation angle $\theta = 100^\circ$ azimuth $\phi = 45^\circ$



(a)

Conventional BF with cylindrical array of 36 directive elements per circle and 4 rings
elevation angle $\theta = 100^\circ$ azimuth $\phi = 45^\circ$



(b)

Figure 2.2.6-4 3D Radiation Pattern of a Uniform Cylindrical Array ^[5]

Chapter 3 Design of Cylindrical Conformal Antenna

3.1 Introduction

The patch conformal antenna has the characteristics of small size, light weight, easy installation, easy conformal and so on. It often widely used in satellite communication, navigation, telemetry and remote control, weapon fuze, medical devices, etc [4] [6] [26]. For the high-speed moving carriers, such as grenades, etc., it will rotate at a high speed in the process of movement. Therefore, it is required that the antenna equipped must be able to radiate omnidirectional without affecting the dynamic characteristics of the cylinder.

Printed conformal antenna structure is generally composed of a dielectric substrate, a radiator and a ground [25]. The thickness of the dielectric substrate is far less than the wavelength, the metal layer at the bottom of the substrate is connected with the ground, and the metal thin layer with a specific shape on the front is used as the radiator [26]. The formation of the radiator can be changed as required.

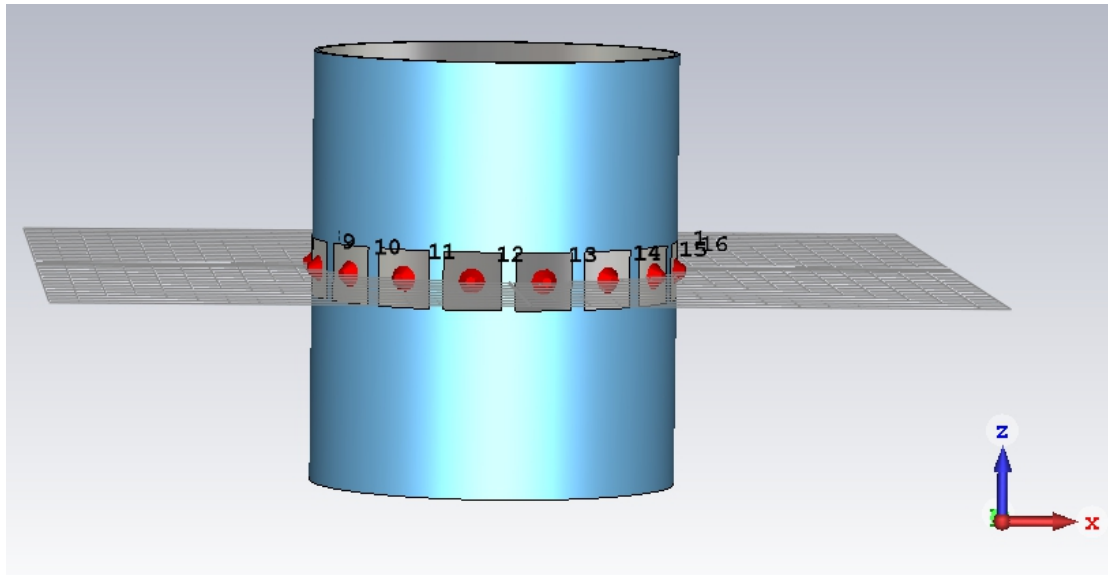
In this chapter, a cylindrical conformal antenna array is designed. The antenna is composed of 16 single-layer rectangular patch antennas with a central frequency of 13.99 GHz which can achieve omnidirectional radiation.

3.2 Single Layer Cylindrical Conformal Antenna

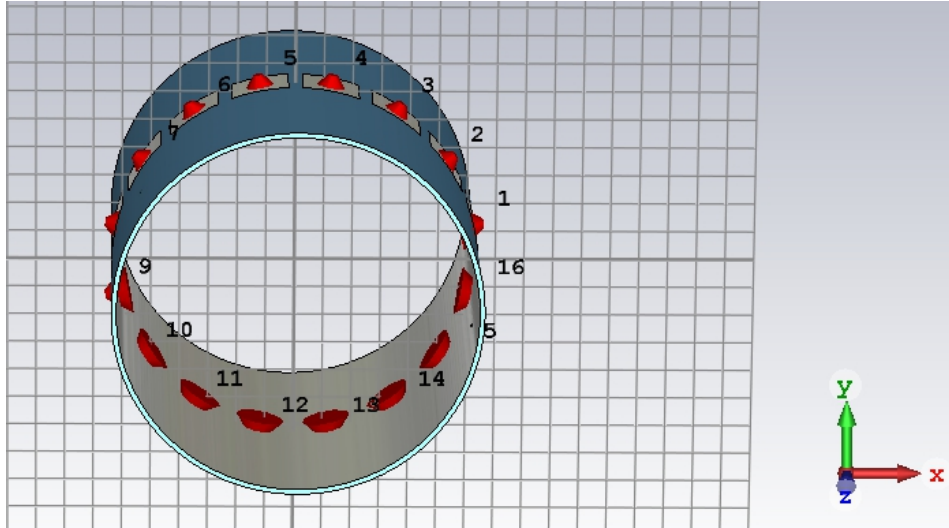
3.2.1 Antenna Array Structure

In this section, the structure of the cylindrical conformal antenna array is

designed, as shown in Figure 3.2.1-1. The antenna is composed of three parts: the inner surface of which name is the ground, the middle is a dielectric substrate with a thickness of 1.7 mm, and the outer surface is the rectangular metal patches array. As shown in Figure 3.2.1-2, it is a diagram of the patch plane array. The array antenna is fed through the patch antenna and the patch is conformal on the surface of the cylinder. A single layer of 16 patches with equal distance and equal height surround the surface of the cylinder. The data of the structure of conformal cylindrical array antenna is shown in following Table 3.2.1-3.



(a)



(b)

Figure 3.2.1-1 Cylindrical Conformal Array Antenna Structure

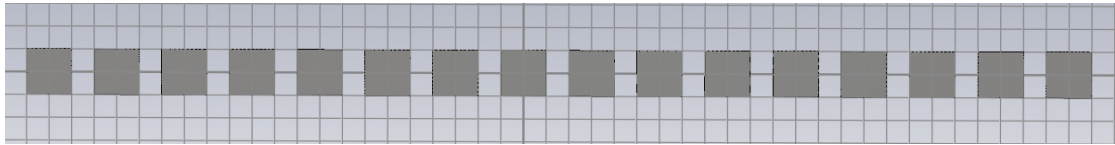


Figure 3.2.1-2 Patch Planar Array

Table 3.2.1-3 Data of the Cylindrical Structure

	materials	length[mm]	radius[mm]	width[mm]	height[mm]
ground	PEC	/	62	/	150
dielectric substrate	Vacuum	/	63.7(outer radius) 62 (inner radius)	/	150
patch(radiator)	PEC	10	/	10	/

3.2.2 Development of Macro In CST

Simulation Setting

The frequency unit set as [GHz], the length unit as [mm].

The frequency range is from 1 GHz to 40 GHz; and the material type of the background is set to: Normal.

For the boundary condition, the Open (Add Space) boundary was selected. It simulates the infinite half of the free space and increases the whole area of the simulation space by 1/8 wavelength, because the open boundary has the best performance when it is 1/8 wavelength away from the source.

Design of the Cylinder

In the “Modeling”, select "Cylinder" as the medium substrate. The height of the cylinder is 150 mm, the outer radius is 63.7 mm, the inner radius is 62 mm, and the thickness is 1.7 mm. The direction along the Z axis, the coordinate set to : (outer radius, inner radius, Xcenter, Ycenter, Zmin,Zmax) = (63.7, 62, 0, 0, -150/2, 150/2), and the material is: "Vacuum".

In the “Modeling”, select "Cylinder" as the ground. The height of the cylindrical layer is 150 mm, the outer radius is 62 mm, and the inner radius is 62 mm. The direction is along Z axis, and the coordinates are set as : (outer radius, inner radius, Xcenter, Ycenter, Zmin,Zmax) = (62, 62, 0, 0, -150/2, 150/2), and the material is: "Copper (pure)".

Array element patch design

The "Brick" in the Modeling is selected as the array element, and the length and width of the array element are respectively 10 mm and 10 mm. The coordinates are

set as $(Xmin, Xmax, Ymin, Ymax, Zmin, Zmax) = (63.7, 63.7, -10/2, 10/2, -10/2, 10/2)$, and the material is "PEC".

The patch antenna is bent and conformal to the surface of the cylinder. Then in "Transform", 15 patch array elements are copied with the center of Z-axis and the rotation angle of $\alpha=360/16$ degree.

Discrete Port Setting

The Discrete Port is defined by two points, the center of the each patch as the starting point and the corresponding ground on the ground floor as the end point, and is connected through the Discrete Port with the Port source located in the middle of the wire. The Port excitation type is: S-parameter. The global coordinates of the starting and ending points of the discrete ports are set to: $(X1, Y1, Z1)=(63.7+0.05, 0.0, 0.0)$ and $(X2, Y2, Z2) = (62-0.05, 0.0, 0.0)$, respectively.

The wire between the starting point and the end point of the discrete element must be along the grid edge, which cannot be seen in the model window. Only in the Mesh View can see the actual distribution of lines for the Mesh.

In "Transform", create discrete ports for each patch element centered on the Z axis with $\alpha=360/16$ degree rotation angle.

3.2.3 Antenna Performance

Setting up "Field Monitors" does time domain solver at frequency: $f=13.9$ GHz. All ports are excited for time domain solver which is shown in Figure 3.2.3-1.

As shown in Figure 3.2.3-2, the plot of S-Parameters shows that the resonant frequency point of the patch antenna is located at 13.9 GHz which is good matching frequency and the center frequency is very low. The thin medium substrate is the main reason for the width of the impedance band.

Time Domain Solver Parameters

Solver settings

Mesh type:

Hexahedral

Accuracy:

-40

dB

☐ Store result data in cache

Stimulation settings

Source type:

All Ports

☐ Inhomogeneous port accuracy enhancement

Mode:

All

☐ Calculate port modes only

☐ Superimpose plane wave excitation

S-parameter settings

☐ Normalize to fixed impedance

50

Ohm

☐ S-parameter symmetries

S-Parameter List...

Adaptive mesh refinement

☐ Adaptive mesh refinement

Adaptive Properties...

Sensitivity analysis

☐ Use sensitivity analysis

Properties...

Start

Close

Apply

Optimizer...

Par. Sweep...

Acceleration...

Specials...

Simplify Model...

Help

Figure 3.2.3-1 Time Domain Simulation Setup

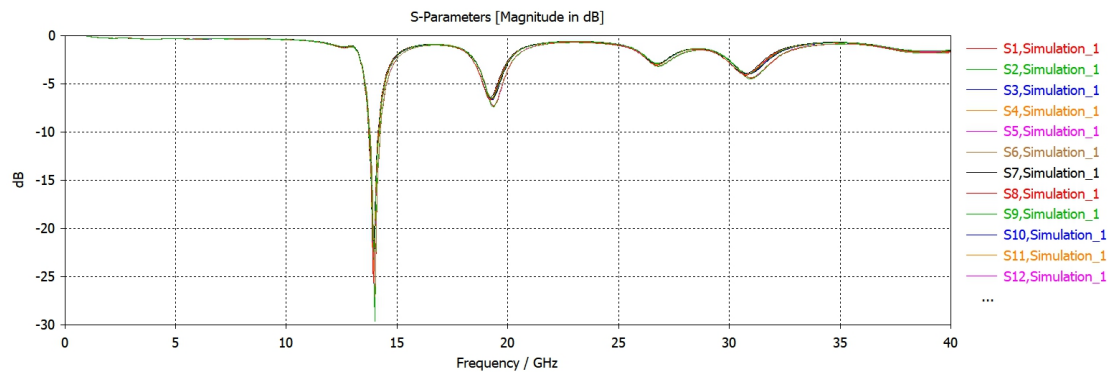


Figure 3.2.3-2 S-Parameters

3.3 Radiation Pattern

Antenna pattern is also called radiation pattern, or far-field pattern [18]. It is an important pattern to measure the performance of the antenna. The antenna parameters can be observed from the radiation pattern [19]. It is synthesized by the some antenna element radiation pattern under the given amplitude and phase excitation [20].

For the cylindrical conformal antenna array simulation structure: 1x16, each element has an independent feed port. In this way, during simulation, CST will perform feed simulation for each port in turn and the other ports will automatically match absorption. At the end of the simulation, the active pattern of excitation for each feed port (i.e., each patch element) is obtained. After the active pattern of all 16 patches is obtained, the “Combine Results” in the post-processing module can be used for pattern synthesis. All ports simulation setup is shown in Table 3.3-1.

Table 3.3-1 Ports Simulation Setup for Equal Amplitude and Phase

	port1	port2	port3	port4	port5	port6	port7	port8
amplitude [V]	1	1	1	1	1	1	1	1
phase [degree]	0	0	0	0	0	0	0	0

	port9	port10	port11	port12	port13	port14	port15	port16
amplitude [V]	1	1	1	1	1	1	1	1
phase [degree]	0	0	0	0	0	0	0	0

The 2D radiation pattern are shown in Figure 3.3-2, Figure 3.3-3, and 3D radiation pattern is shown in Figure 3.3-4.

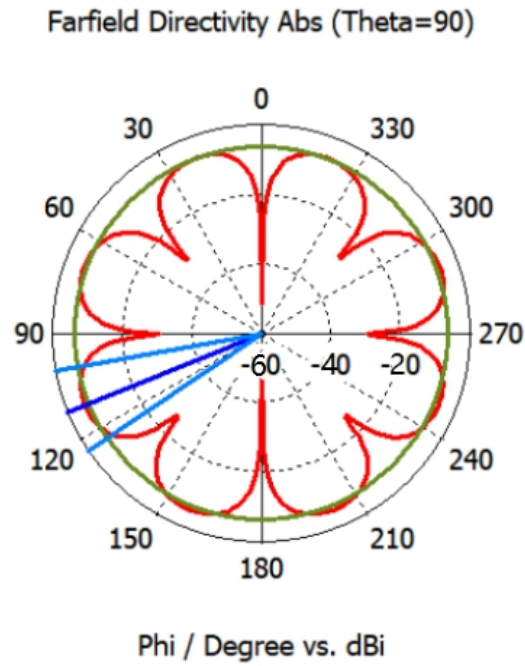


Figure 3.3-2 2D radiation Pattern for Equal Amplitude and Phase in the x0y Plane (Polar Representation)

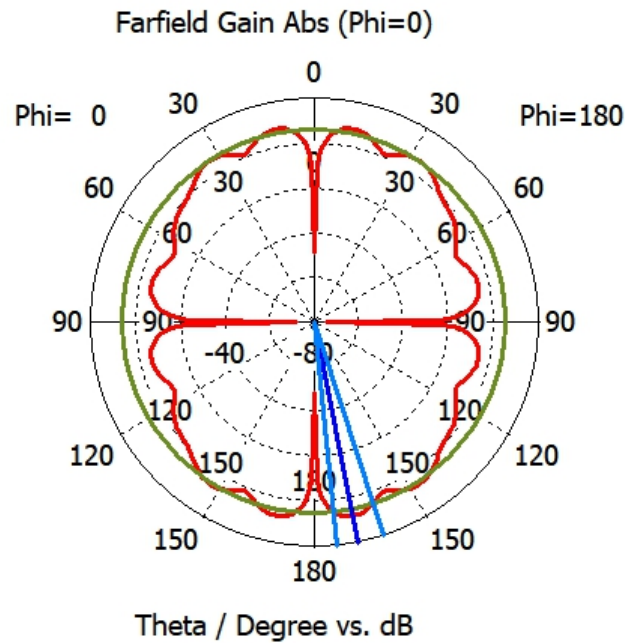


Figure 3.3-3 2D Radiation Pattern for Equal Amplitude and Phase in the x0z Plane (Polar Representation)

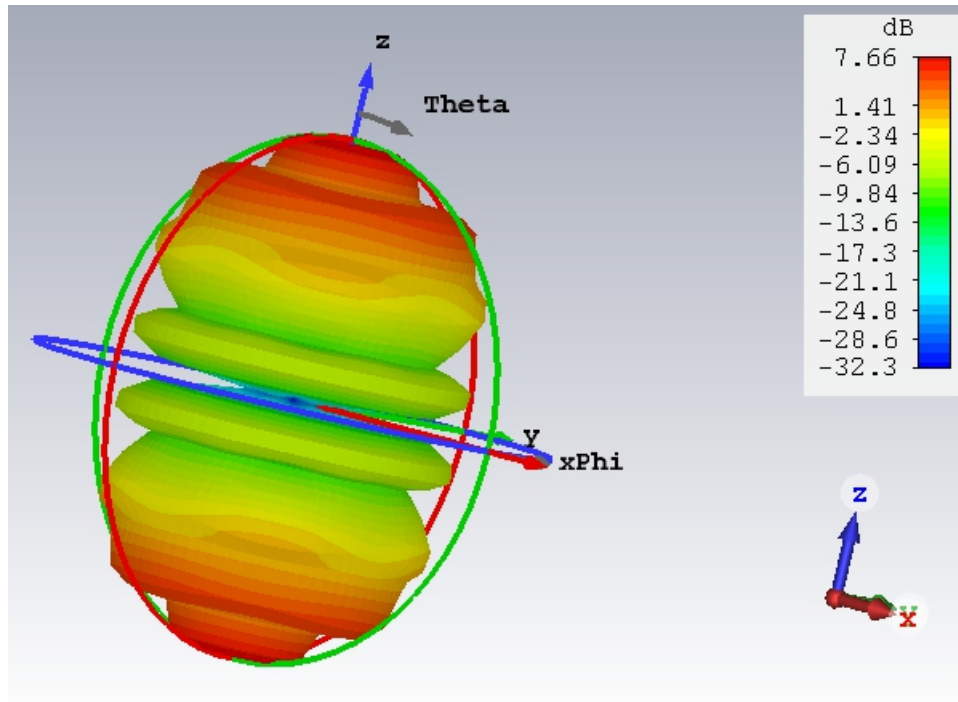


Figure 3.3-4 3D Radiation Pattern for Equal Amplitude and Phase

In order to compare the performance of the ideal array beamforming obtained by MATLAB and the actual radiation pattern simulated by CST. The excitation signals are obtained by using MATLAB which get the amplitudes and phases that correspond to feed each port in CST.

The existence of ground plane is considered in the simulation of CST antenna array. Therefore, in MATLAB, it is necessary to use two concentric circular arrays to generate excitation signals. Two groups of circular array excitation coefficients are obtained. But in the CST, only consider the weights of the external array. The image (internal array) will be considered by the numerical model as image of the external one due to the presence of the ground plane.

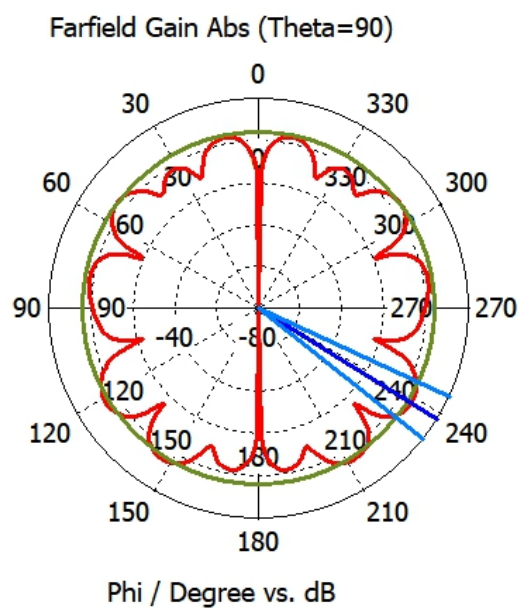
For CST, the phase of each port is center of symmetry which fed by MATLAB ideal values, as shown in Table 3.3-5. The 2D radiation pattern are shown in Figure 3.3-6, Figure 3.3-7, and 3D radiation pattern is shown in Figure 3.3-8.

Table 3.3-5 Ports Simulation Setup for Symmetric Phase

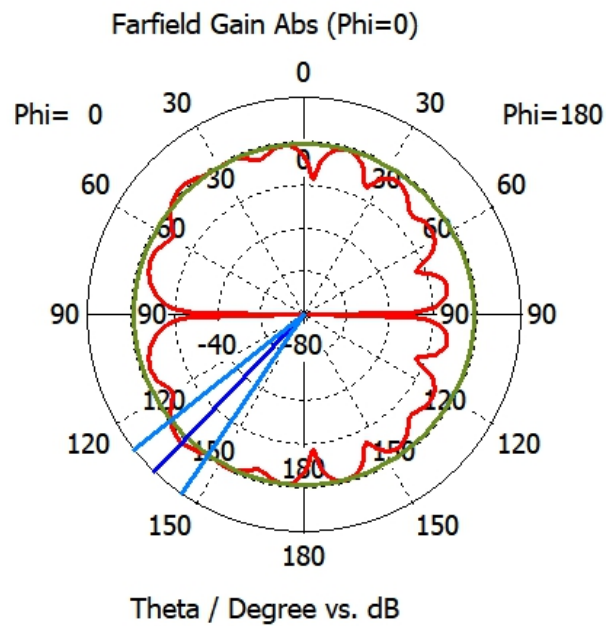
	port1	port2	port3	port4	port5	port6
amplitude [V]	1	1	1	1	1	1
phase [degree]	-118.5769	170.7119	0.0000	-170.7119	118.5769	-170.7119

	port7	port8	port9	Port10	port11
amplitude [V]	1	1	1	1	1
phase [degree]	-0.0000	170.7119	170.7119	-0.0000	-170.7119

	port12	port13	port14	port15	port16
amplitude [V]	1	1	1	1	1
phase [degree]	118.5769	-170.7119	0.0000	170.7119	-118.5769



**Figure 3.3-6 2D Radiation Pattern for Symmetric Phase in the x0y Plane
(Polar Representation)**



**Figure 3.3-7 2D Radiation Pattern for Symmetric Phase in the x0z Plane
(Polar representation)**

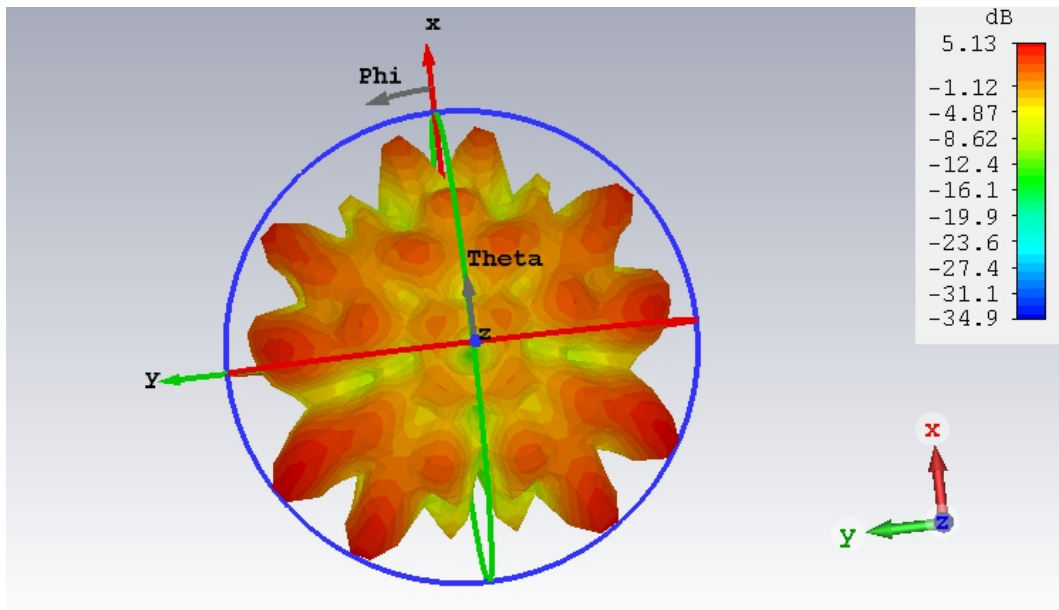


Figure 3.3-8 3D Radiation Pattern for Symmetric Phase

In order to set the main lobe direction in this direction ($\theta=90^\circ$, $\phi=0^\circ$), the port feed amplitude is optimized with a symmetric amplitude. The value of each port feed amplitude and phase is shown in the Table 3.3-9. The radiation pattern is optimized by using “Parameter Sweep”, which are shown in Figure 3.3-10.

Table 3.3-9 Ports Simulation Setup for Symmetric Amplitude

	port1	port2	port3	port4	port5	port6	port7	port8
amplitude [V]	0.1	1	1	1	0.45	0.2	0.1	0.1
phase [degree]	0	90	-180	0	45	135	45	0

	port9	port10	port11	port12	port13	port14	port15	port16
amplitude [V]	0.1	0.1	0.2	0.45	1	1	1	0.1
phase [degree]	-180	-135	-45	-135	-180	0	-90	-180

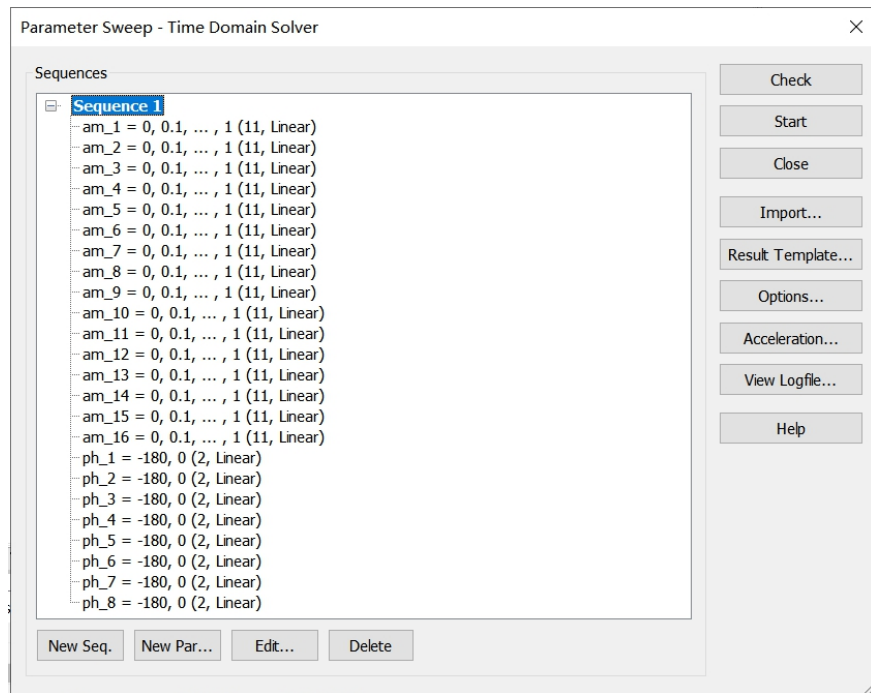


Figure 3.3-10 Parameter Sweep Setup

3.4 Results

3.4.1 Simulation in CST

At 13.9 GHz, compare the beamforming of the ideal array simulate in MATLAB which structure has two uniform concentric circulars with 8 elements per ring and in CST which structure is 1x16 patches uniformly conformal the cylinder, consisting of isotropic radiators, and directive radiators (patches) is presented and discussed.

In CST, the antenna array radiation pattern is shown in Figure 3.4.1. It shows the main lobe direction in ($\theta=90^\circ$, $\phi=0^\circ$), and the main lobe magnitude is 9.48 dB. It also shows many side lobes.

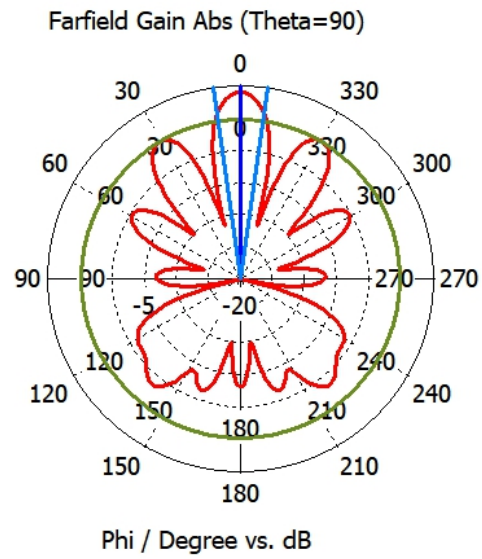


Figure 3.4.1 (a) 2D Radiation Pattern in the x0y Plane (Polar representation)

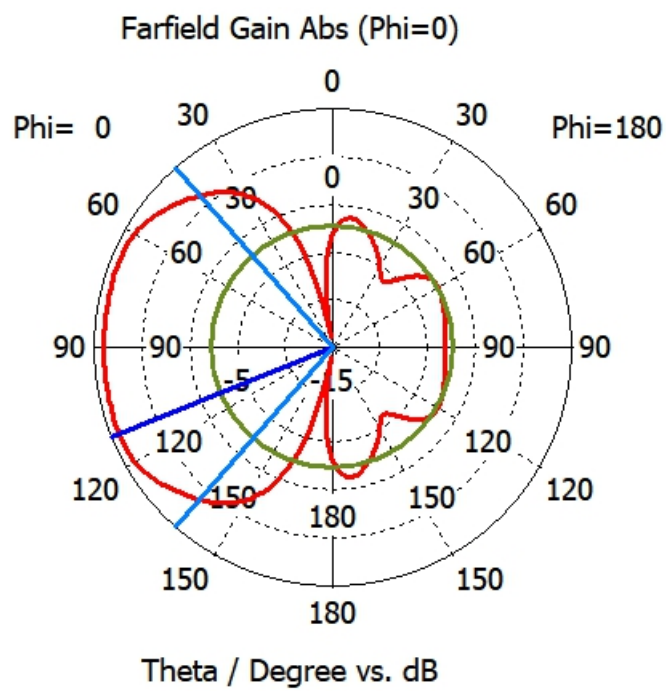


Figure 3.4.1 (b) 2D Radiation Pattern in the x0z Plane (Polar Representation)

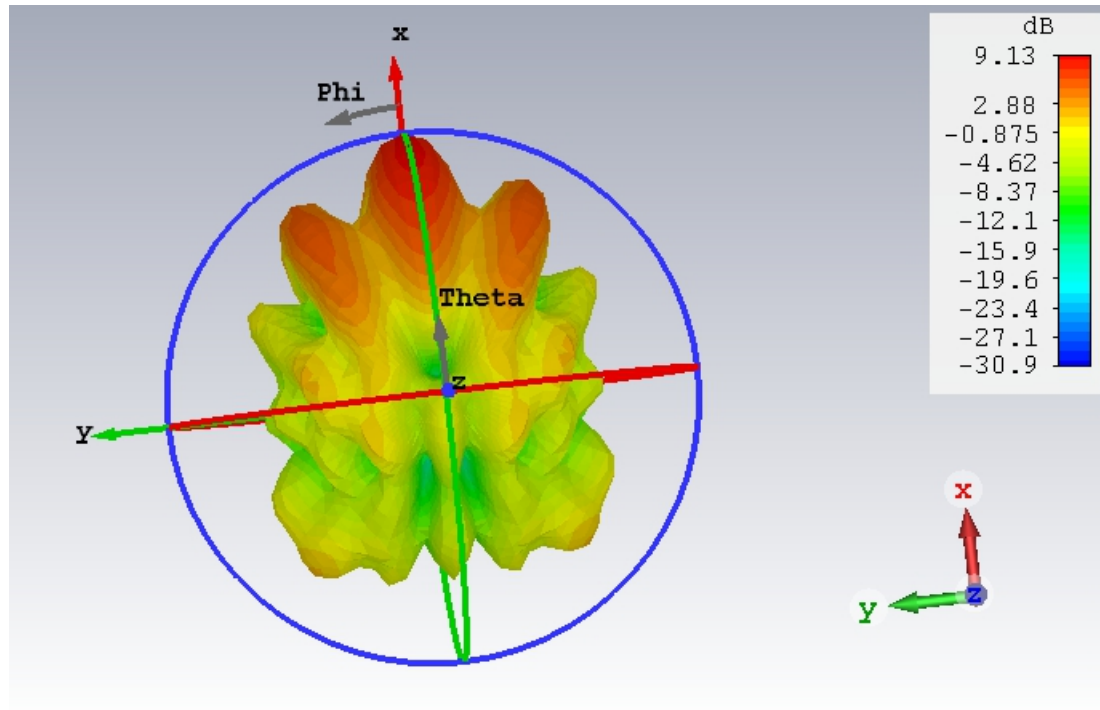


Figure 3.4.1 (c) 3D Radiation Pattern

Figure 3.4.1 Antenna Array Radiation Pattern

3.4.2 Simulation in MATLAB

Isotropic Antenna Element in MATLAB

In MATLAB, the antenna array radiation pattern of isotropic antenna element is shown in Figure 3.4.2. The figures show that the direction of antenna element pattern is omnidirectional and the main lobe direction of antenna array in ($\theta=90^\circ$, $\phi=0^\circ$), and the main lobe magnitude in linear is 4. It also shows some side lobes.

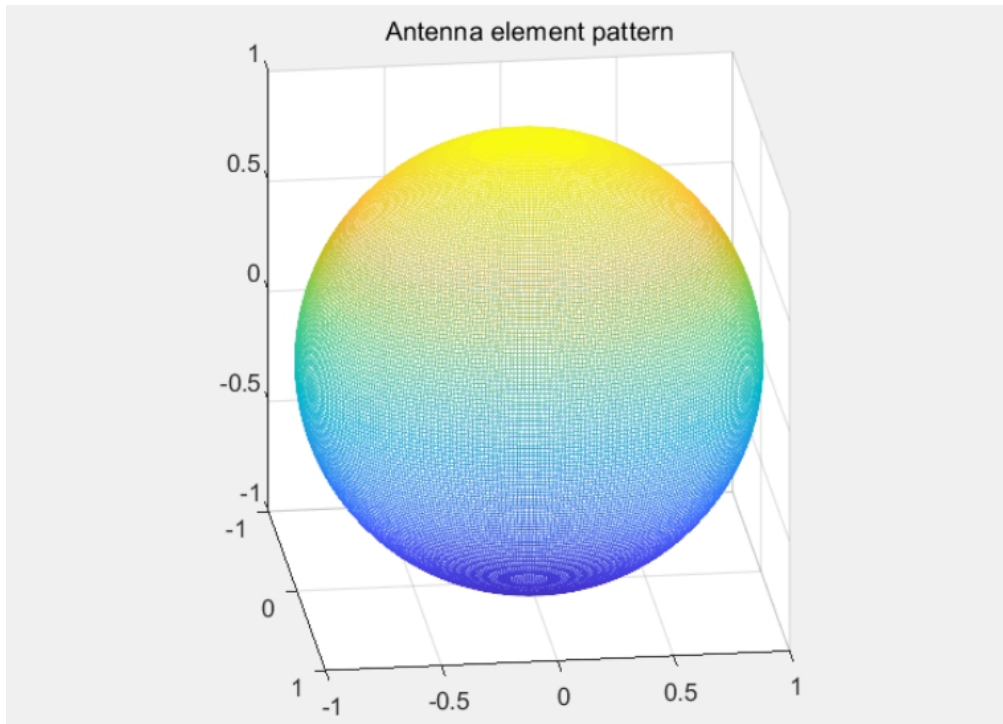


Figure 3.4.2-1 (a) Isotropic Antenna Element 3D Radiation Pattern

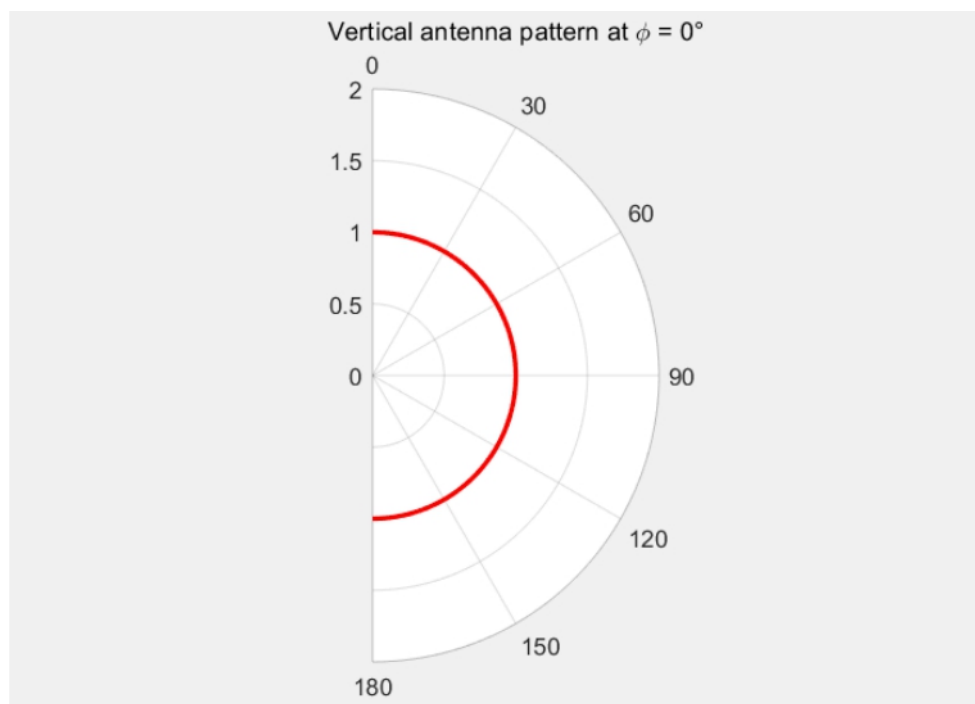


Figure 3.4.2-1 (b) Vertical Isotropic Antenna Element 2D Polar Plot

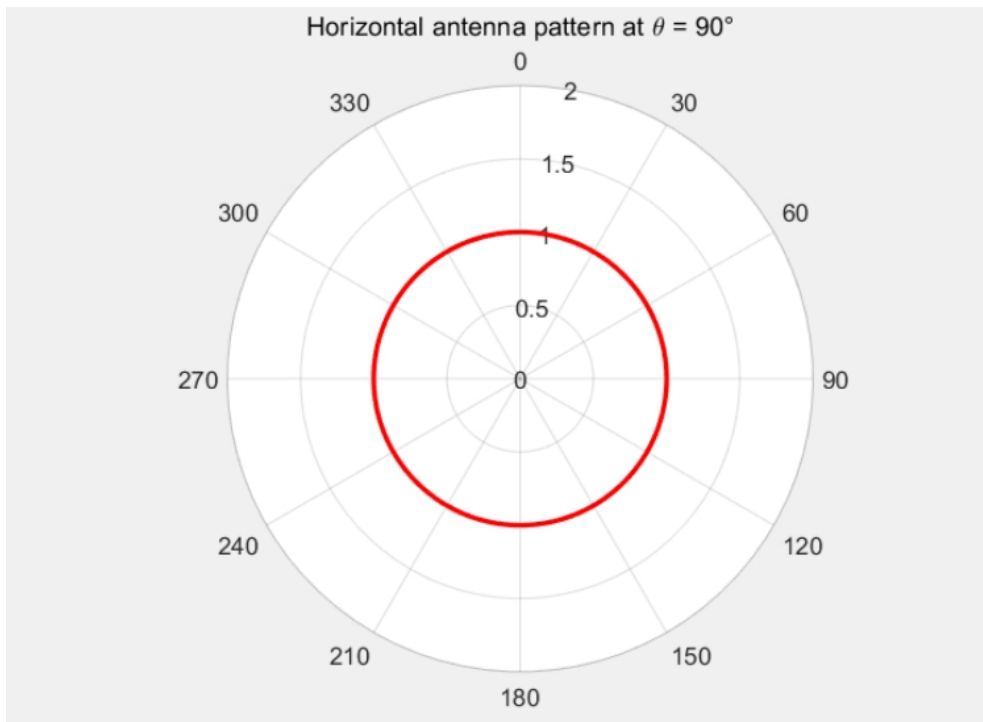


Figure 3.4.2-1 (c) Horizontal Isotropic Antenna Element 2D Polar Plot

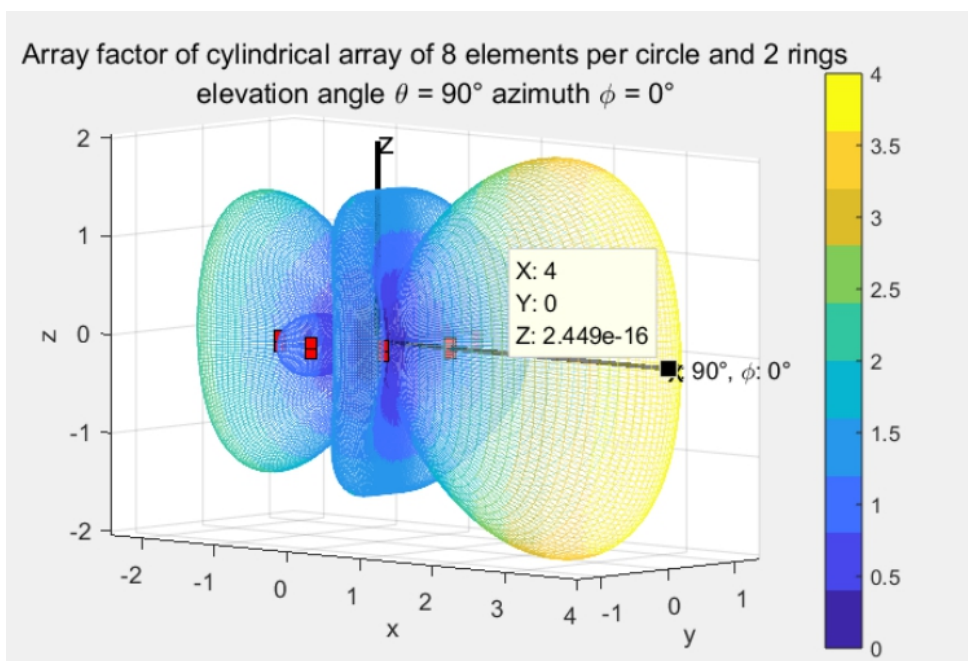


Figure3.4.2-1 (d) Isotropic Antenna Array 3D Radiation Pattern

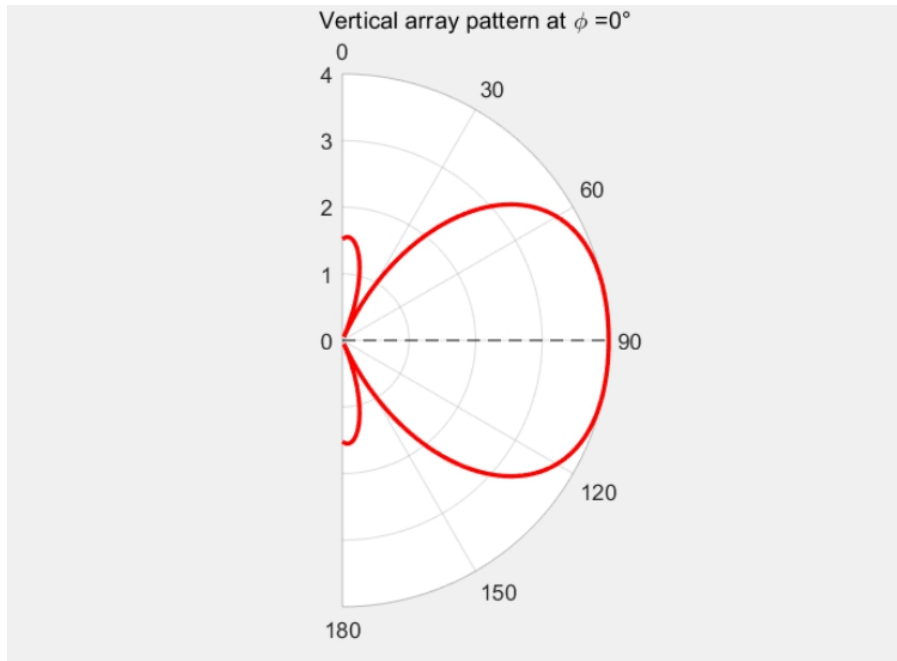


Figure 3.4.2-1 (e) Vertical Isotropic Antenna Array 2D Polar Plot

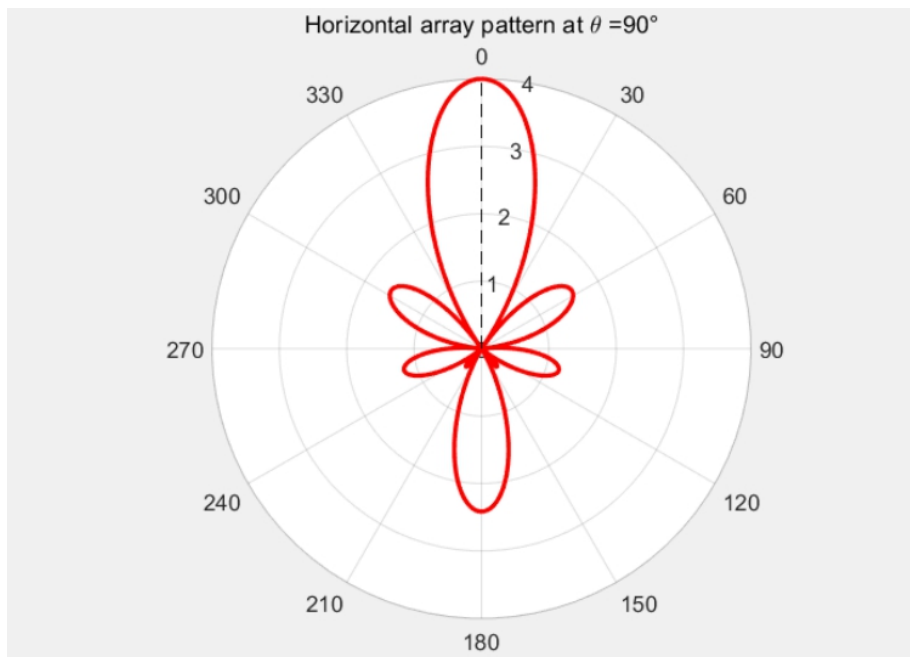


Figure 3.4.2-1 (f) Horizontal Isotropic Antenna Array 2D Polar Plot

Figure 3.4.2-1 Isotropic Antenna Array Radiation Pattern

Directive Antenna Element in MATLAB

In MATLAB, the antenna array radiation pattern of directive antenna element is shown in Figure 3.4.3. The figures show the direction of antenna element pattern in ($\theta=90^\circ$, $\phi=0^\circ$) and the antenna array main lobe direction also in ($\theta=90^\circ$, $\phi=0^\circ$), and the main lobe magnitude in linear is 1.773. It also shows very low side lobes.

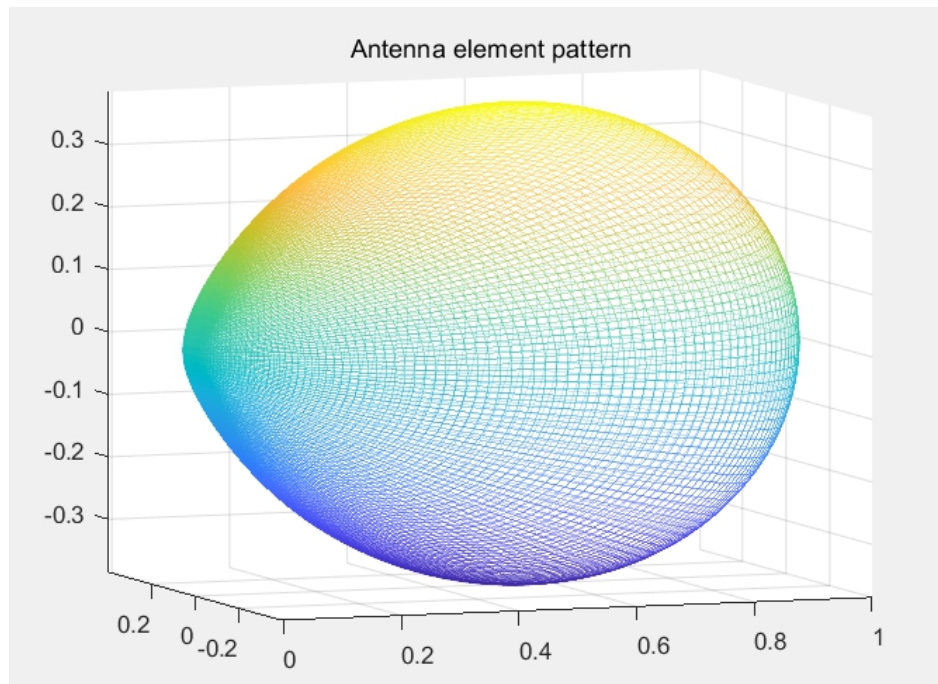


Figure 3.4.2-2 (a) Directive Antenna Element 3D Radiation Pattern

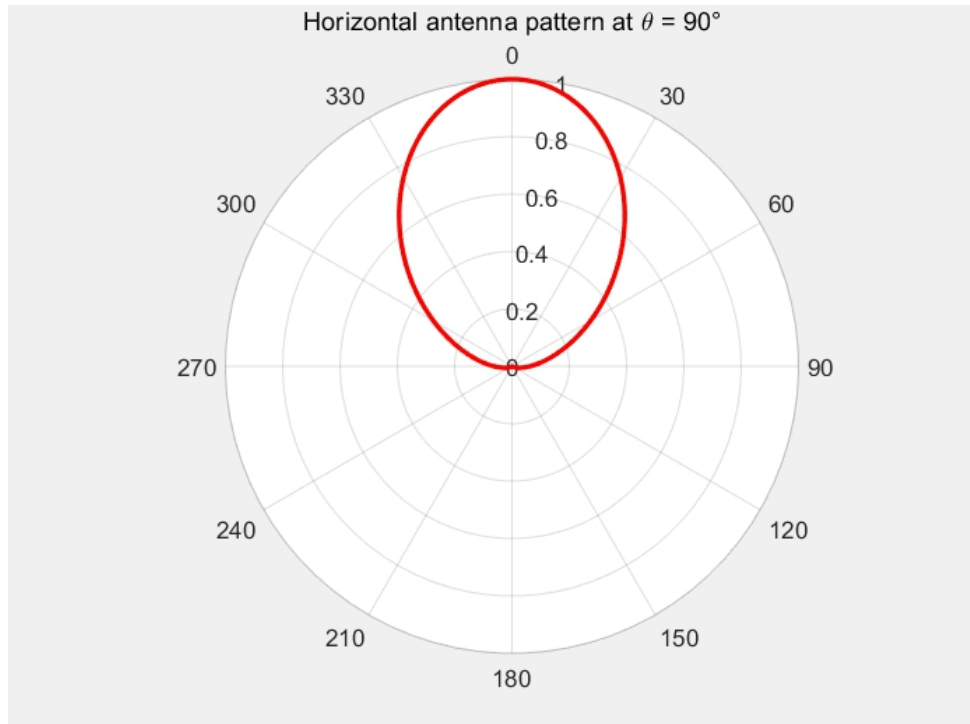


Figure 3.4.2-2 (b) Horizontal Directive Antenna Element 2D Polar Plot

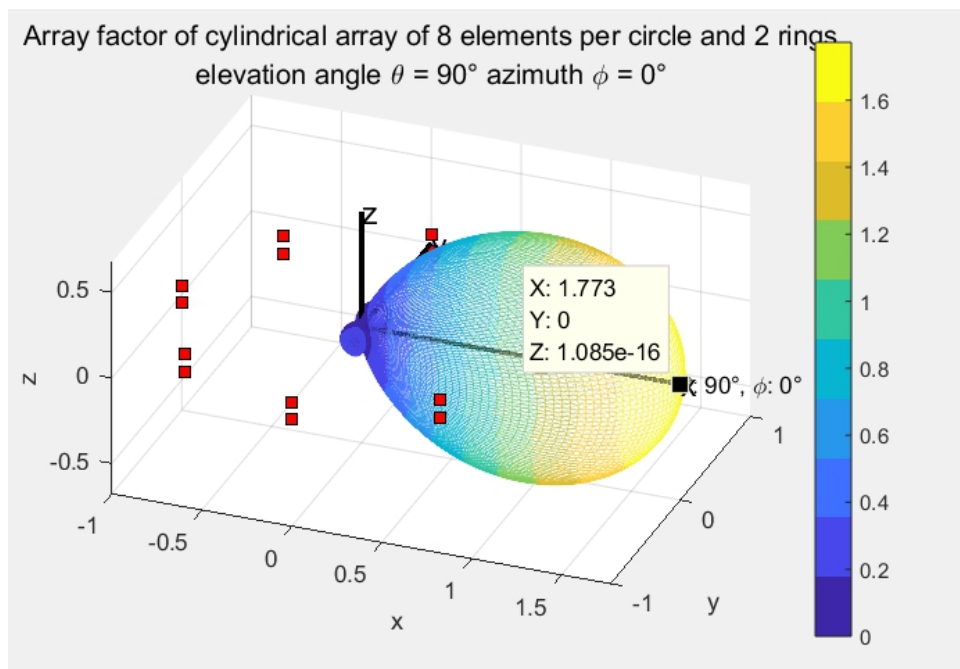


Figure3.4.2-2 (c) Directive Antenna Array 3D Radiation Pattern

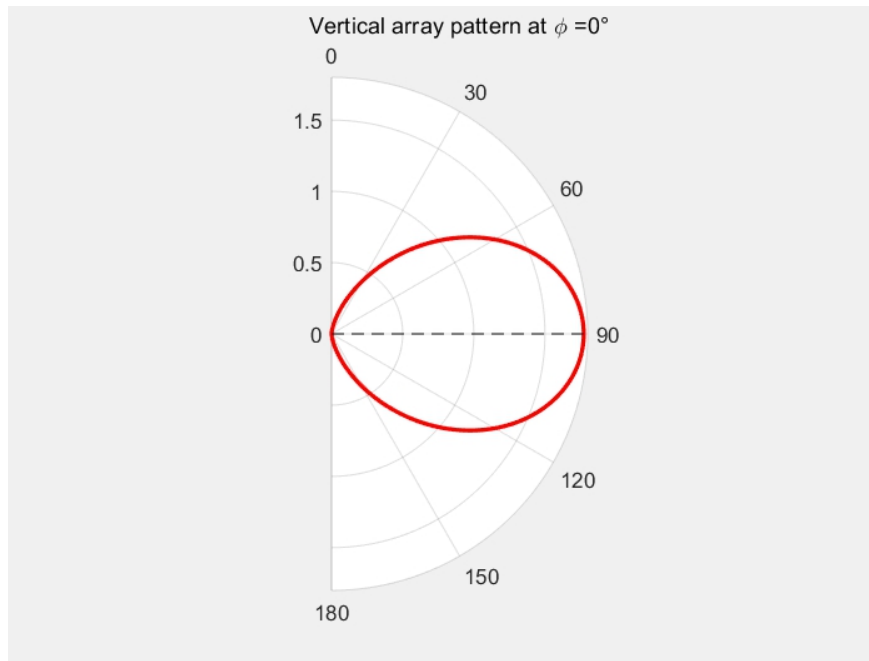


Figure 3.4.2-2 (d) Vertical Directive Antenna Array 2D Polar Plot

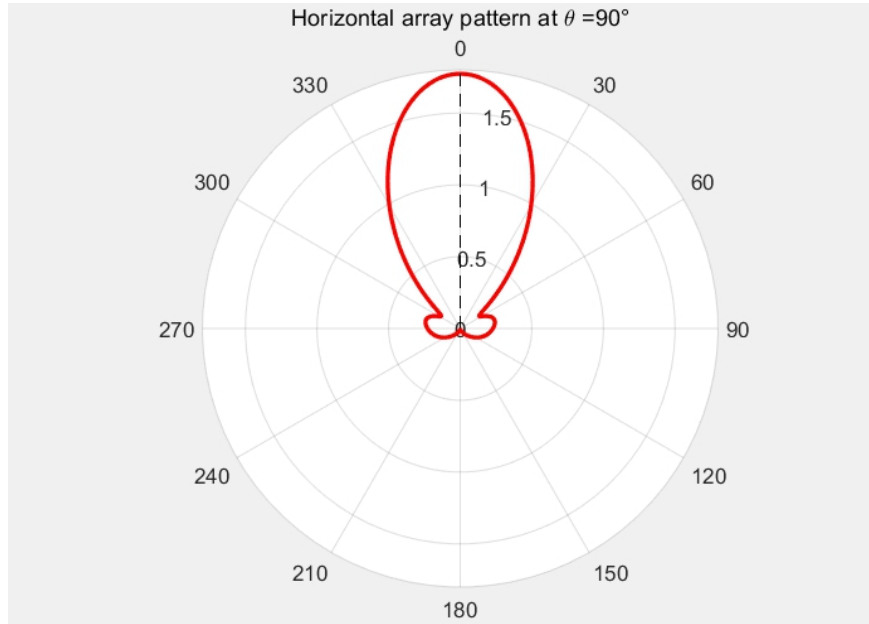


Figure 3.4.2-2 (e) Horizontal Directive Antenna Array 2D Polar Plot

Figure 3.4.2-2 Directive Antenna Array Radiation Pattern

Chapter 4 Conclusion

With the development of military, the performance requirements of the array antenna system carried by various aircraft such as airborne, space-borne and missile-borne are getting higher and higher, and the conformal array antenna is getting more and more attention due to its own advantages. In order to realize the precise guidance of millimeter wave, the research of millimeter wave conformal antenna array is of great significance [8].

In this thesis, the conformal array antenna on the surface of the cylinder is displayed. In the design process, time-domain solver, optimization and other related principles are used. And the beam scanning performances of the antenna array is analyzed: multi-port excitation is performed and beam synthesis is carried out applying weighted feeding coefficients computed by a Matlab script. Comparison between ideal array, consisting of isotropic radiators, and directive radiators (patches) is presented and discussed.

In CST, The synthesis algorithm of beam shaping is studied. Firstly, the radiation pattern of each antenna element is obtained by solving each antenna element respectively. In order to improve the directivity of far-field radiation pattern of antenna array, through the data post-processing, the phase and amplitude of each feed port are optimized with the symmetry of the first eight ports and the last eight ports, so that the maximum direction of far-field pattern is in the same direction as the simulation results of MATLAB. In order to improve the degree of optimization, bind the amplitude of each feed port remains unchanged, the phase of each feed port is optimized to make the maximum direction of the far field plot in the same direction as the simulation results of MATLAB and enhanced gain characteristic. Due to the mutual coupling between each antenna element, there are many side lobes.

Antennas have different radiating or receiving capacities in different directions of space. In Matlab, the beamforming which is synthesized by directive antenna

elements radiation pattern has directivity, few side lobes. The gain is relatively high and the anti-jamming ability is relatively strong, so it has strong antenna practicality for long distance point - to -point communication [21]. The beamforming that is synthesized by isotropic antenna elements performance has non-directional. It is typically used as a hub for point-to-point communications [21].

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