The simulation of dynamic behavior for gear-shaft-bearing system

Master Thesis in Automotive Engineering

Author:	MEIWEN LIAO (Student ID: S262575)
Supervisor:	Prof. Carlo Rosso
Co-Supervisor:	Ing. Claudio Marcellini
	Ing. Fabio Bruzzone
	Ing. Tommaso Maggi

Politecnico di Torino

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Abstract

As a very key part in automobile powertrain system, transmission's durability, stability, and NVH performance all have critical significance for vehicle's quality and performance. When the transmission is running, all of its transfer parts and support parts are suffering huge loads, which will make the transfer parts deform and deviate from the original position even be damaged. Meanwhile, manufacturing defects of parts and error of assembly also will influence the working precision and quality of the transmission. Therefore, analyzing and optimizing the loading status is an effective way to improve transmission's quality and performance.

This main structure of the research discusses different forms of gear-shaft-bearing systems in transmission, and analyzes the calculation of bearing reaction force and displacement under the gear meshing state through the stress balance condition. In the study, the different forms of systems were also modeled through Simulink to predict bearing reaction forces and displacements.

KEY WORDS: transmission, force analysis, dynamic model, Simulink

Content

1. Introduction

The development of the automobile industry has played an important role in the world's economic progress and the improvement of human quality of life. Modern cars mainly rely on the engine to generate driving force. But due to the complex and changing road conditions during the driving, the traction force and driving speed of the vehicle need to be able to change with the different driving conditions. However, because of the characteristics of the engines currently used in many cars, their output power and speed can only be varied in a relatively small range, which can cause a great waste of energy when the driving force does not match the actual state of use. Therefore, the car must transmit the engine output power to the drive wheels through the transmission, so as to effectively switch the power output state in the process of driving the car. So the car can achieve the balance of power performance, safety performance and economic performance according to the actual state of use. The main functions of the vehicle transmission are as follows.

(1) according to the actual driving conditions of the vehicle, change the actual torque and speed of the engine transmission to the vehicle drive wheels, and enable the vehicle to drive in a better fuel economy state.

(2) because the direction of rotation of the car engine in the working state cannot be changed, in order to enable the vehicle to achieve reverse driving, the transmission must have a reverse gear function.

(3) The transmission needs to have the function of interrupting the engine output power transmitting to the drive wheels, which is the neutral state, so that the engine can start under low load, idle speed, and gear switching.

The gear-shaft-bearing system is the most widely used power and motion transmission device in various mechanical equipment, which plays a key role in important equipment in metallurgy, aerospace, chemical industry, ships, petroleum, agricultural machinery, electric power systems, vehicles, mining equipment and machine building. According to statistics, 80% of the failures in transmission machinery are caused by bearings; 10% of the failures in rotating machinery are caused by gears . Therefore, the safe and reliable operation of these important equipment is related to the significant economic and social benefits. For example, if a large chemical company's centrifugal compressor unit (gear drive) fails, the economic loss of the company's production stoppage for one day can be as high as millions of dollars. In China's cement industry, the failure of widely used cement mill gearboxes causes a reduction in cement production of more than 2 million tons per year . Therefore, the dynamics of the gear system has been of great concern. Compared with foreign countries with advanced technology, the magnitude of vibration and noise of gearboxes in China is still relatively large. In China's mechanical systems with gear transmission, gear transmission generally produces large vibration and noise, and according to statistics, the vibration of the gear box itself and the vibration of the gears transmitted from the shaft system are the main source of radiation noise from the equipment. The strong mechanical vibration generated by the gearbox is transmitted to the outside world through the equipment casing, forming a powerful noise source, and the powerful noise source has a great impact on the quietness and concealment of the equipment. For some important equipment, pre-emptive action is the key to eliminating the other side to achieve victory, if the larger vibration and noise generated by the equipment itself is first detected by the enemy, thus exposing their position, will be devastated by the enemy's attack, the consequences are unimaginable. In addition, strong vibration and noise will also affect the normal work of the surrounding instruments, causing a huge impact on the physiological health of the staff.

2. Basic Structure and Working Principle of Automotive Gearbox

To study the automotive gearbox, we must first understand the function and classification of the automotive gearbox, know the basic mechanism of the automotive gearbox, and be familiar with the working principle of the automotive gearbox.

2.1 Classification and Functions of Automotive Gearboxes

Currently, the automotive market is expanding and the demand for gearboxes is also expanding further. The gearbox is an important transmission component in the chassis of a car, which can change the transmission ratio to meet the requirements of different driving conditions of the car, and can make the car reverse and cut off the transmission of power for a long time.

2.1.1 Classification of Automotive Gearboxes

Automotive gearboxes are classified according to the different modes of operation and are mainly divided into three types: mechanical, semi-automatic and automatic. In most cars, the driver directly manipulates the gear lever to change gears, which is the so-called mechanical transmission type, and is also the more common way we shift gears. There are two types of semi-automatic lever shift: one is automatic manipulation between a few fixed gears, but the other gears need to be manipulated by the driver; the other is pre-selected, where the driver pre-sets the gear to be used via an electronic device, and when the clutch pedal is depressed or the accelerator pedal is released while the car is moving, the electronic system will automatically perform the gear change. In the automatic transmission, the ratio selection is automatic and the driver only needs to control the accelerator pedal to control the speed of the car.

If classified according to the different gearing structures, they can be divided into two-shaft and three-shaft gearboxes, as shown in figures 1 and 2. Two-shaft gearboxes have an input shaft and an output shaft; three-shaft gearboxes have an additional intermediate shaft compared to two-shaft gearboxes, and the input and output shafts are in the same line. The two-shaft gearbox is efficient because the intermediate shaft is omitted and the power from the input shaft can be transmitted to the output shaft with only one pair of gears. Three-shaft gearboxes are more efficient than two-shaft gearboxes because they use multiple pairs of gears, so they can obtain a larger transmission ratio.

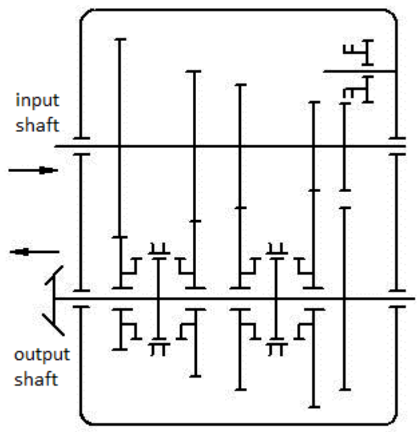

Figure 1. The transmission of two shaft gear

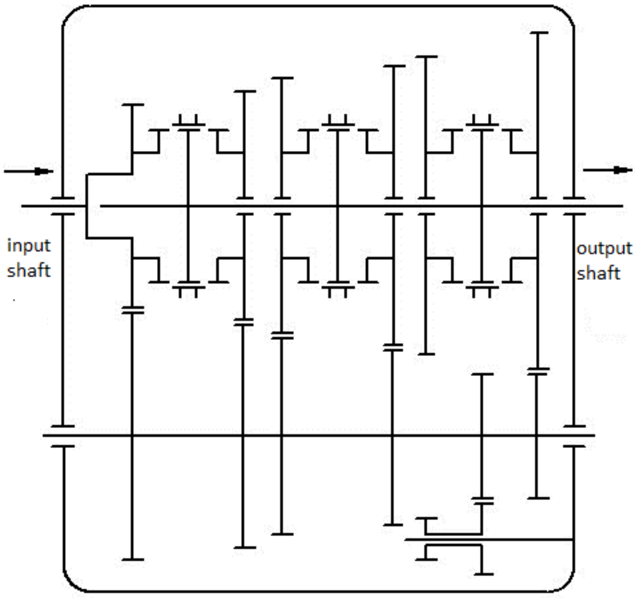

Figure 2. The transmission of three shaft gear

2.1.2 Functions of Automotive Gearboxes

The main function of the gearbox is to change the transmission ratio to achieve reverse driving, interrupt power transmission and achieve neutral gear.

Change the transmission ratio to meet the needs of different driving road conditions for traction force, so that the working performance of the engine can be maximized, in the mean while meeting the requirements of driving speed as much as possible. Due to the different driving conditions of the car, the driving torque and driving speed are required to change within a certain range; moreover, the car engine has the characteristic of small range of speed change, and its torque change range cannot meet the needs of the actual road conditions.

The gearbox allows the car to vary the torque and speed on the drive wheels over a wide range. For example, the speed could reach 100 km/h on the highway, while in the city the speed is often around 50 km/h.

To achieve reverse driving and meet the needs of the car backward driving. Because the engine crankshaft is generally fixed to rotate in one direction, and the car needs to realize reverse driving under certain working conditions. Therefore, the car needs to use the reverse gear function of the gearbox to realize reverse driving.

Interrupting power transmission to achieve neutral gear. When the engine starts, in idle condition, the car changes gears or needs to stop power output, gearbox realizes the function of interrupting power transmission to the driving wheels; when the clutch has been engaged, the gearbox can realize no power output. For example, the driver can release the clutch pedal and leave the driver's seat when the engine is not turned off.

2.2 Basic Structure of Automotive Gearbox

The gearbox is composed of two parts: the transmission mechanism and the transmission operating mechanism. The gear transmission mechanism is mainly composed of gears, shaft and bearing system, which transmits speed and torque; the gear manipulation mechanism is mainly composed of synchronizer and shift operation mechanism, which controls the transmission mechanism and realizes the change of transmission ratio.

2.2.1 Transmission Mechanism

The transmission mechanism is mainly composed of gears, shaft, bearing system and so on. Generally, the gearbox uses cylindrical gear transmission to change the speed of the transmission system to obtain a certain transmission ratio, and there are basically two transmission schemes: one is the two-shaft type with a pair of gears changing speed; the other is the three-shaft type with two pairs of gears changing speed. The simple arrangement of the gearing of these two schemes is shown in Figure 3 and Figure 4.

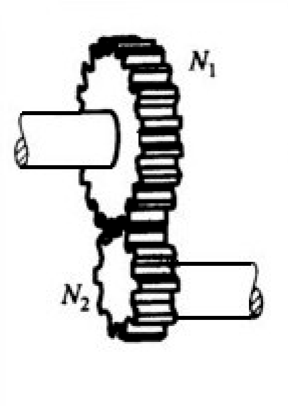

Figure 3. Twin-shaft

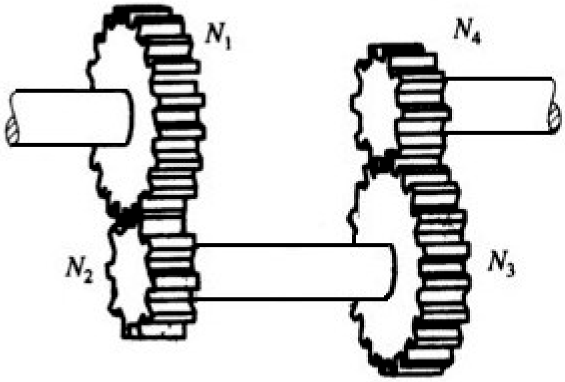

Figure 4. Three-shaft

2.2.2 Gearshift mechanism

The transmission operating mechanism is mainly composed of synchronizer, shift steering mechanism, etc., which controls the transmission mechanism and realizes the change of transmission ratio.

The synchronizer is mainly composed of synchronous locking ring, joint sleeve and slider, as shown in Figure 5. The currently used synchronizers are mainly inertia type and inertia force multiplier type. The way of operation is to achieve synchronization by the friction. There are locking angles on the joint sleeve, the ring of the gear to be jointed and the synchronizing locking ring. And the outer taper of the gear ring to be jointed is in contact with the inner taper of the synchronizing locking ring to produce friction. When designed, the locking angle and the taper surface have been properly selected so the gear sleeve which need to be engaged and the ring are quickly synchronized by the frictional action of the taper surface. Then their synchronization produces a locking effect that prevents the gears from meshing before synchronization.

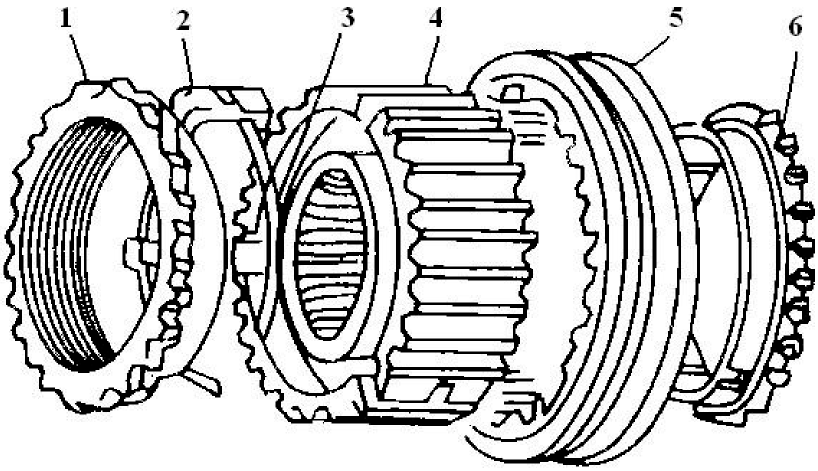

Figure 5. Inertial type of synchronizer

The shifting mechanism mainly includes shift fork shaft and fork. According to the different shifting positions, it can be divided into direct shifting type and Long distance-shift mechanism type, as shown in Fig. 6 and 7. The direct shifting mechanism means that the transmission is arranged near the driver's seat, and the shifting lever extends from the floor of the cab, then the driver can directly manipulate the shifting lever to toggle the shifting device inside the transmission cover for shifting. Long distance-shift mechanism means that the transmission is far away from the driver, so the shifting lever cannot be directly arranged on the transmission cover. For this reason, a set of transmission rod is installed between the shifting lever and the transmission, which constitutes a remote control method. Direct shift mechanism is compact, simple and easy to operate, suitable for most small cars and long-headed trucks, etc. The long-distance shifting mechanism should have enough rigidity, and the clearance between the connecting parts should be small otherwise the shifting feel is not obvious, which mainly used in sedans and light vehicles.

Shift manipulator must be accurate to put into gear, and need to avoid automatic off-gear, prevent the simultaneous hanging of two gears. Also it should be backed out of gear at any time to hang on any gear. In addition to it need to prevent the reverse gear by mistake.

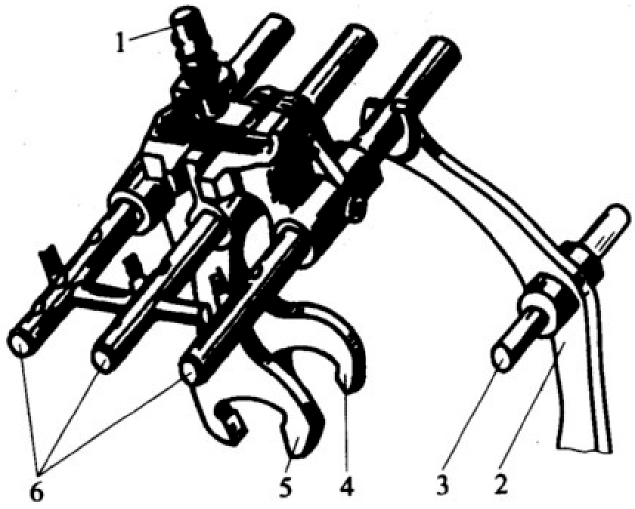

Figure 6. Direct-shift mechanism

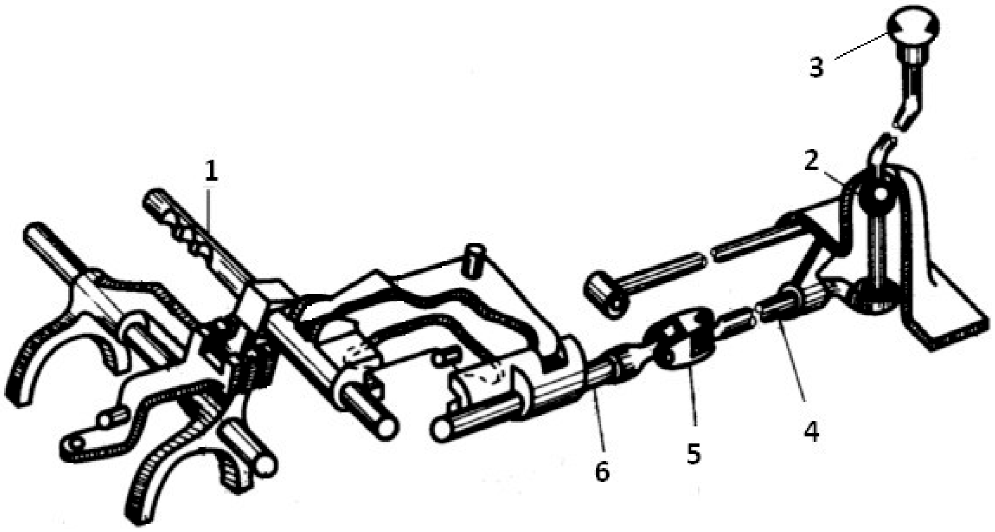

Figure 7. Long distance-shift mechanism

2.3 Operating Principle of Automotive Gearbox

Take a three-shaft five-speed gearbox as an example, and describe its working principle. The three-shaft five-gear gearbox is shown in Figure 8, has five forward gears and one reverse gear. It is composed of the housing, the first shaft, the intermediate shaft, the second shaft, the reverse shaft, the gears on each shaft and the operation mechanism. The input shaft is a gear shaft, and the intermediate shaft is assembled with six gears, which rotate as a whole. The first gear in front is in constant mesh with the input gear shaft gear, and the other five gears are in constant mesh with the gears on the output shaft and the gears on the reverse gear shaft. Three synchronizers are mounted on the output shaft. Through the synchronizer and the gear pair, the transmission ratio of each gear can be obtained, and furthermore, different speed and torque can be obtained. The transmission route of each gear is as follows:

First gear: Input shaft →gear pair 1 →Intermediate shaft →gear pair 4 →Synchronizer 2 →Output shaft

Second gear: Input shaft →gear pair 1 →Intermediate shaft →gear pair 3 →Synchronizer 2 →output shaft

Third gear: Input shaft →gear pair 1 →Intermediate shaft →gear pair 2 →Synchronizer 1 →output shaft

Fourth gear: Input shaft →synchronizer 1 →output shaft

Fifth gear: Input shaft →gear pair 1 →Intermediate shaft →gear pair 5 →Synchronizer 3 →Output shaft

Reverse gear: Input shaft →gear pair 1 →Intermediate shaft →gear pair 6 →Synchronizer 3→output shaft

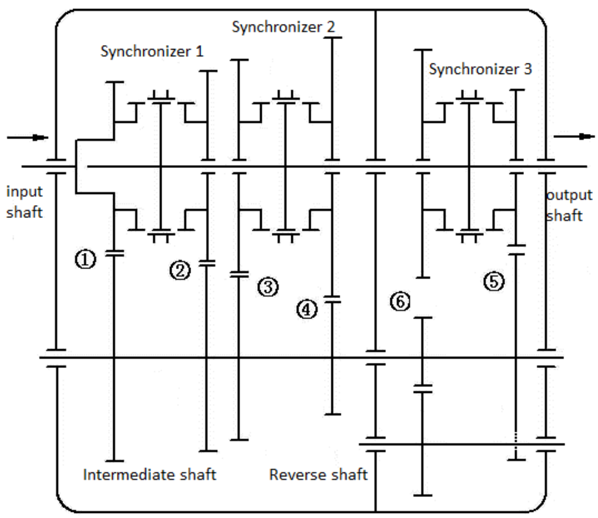

Figure 8. Three-axis five-speed gear transmission

3. Modeling of Gear Shaft Bearing System

In this section, the possible sources of vibration of the gearbox are analyzed, mathematical models of different system types are established, and dynamics simulation models of different system types are built by MATLAB/Simulink platform to obtain the effects of gear meshing forces on the gear shaft.

3.1 Overview of forces analysis of gearboxes

During the operation of an automotive transmission, the component parts are subjected to various forms of external forces, resulting in complex random vibrations. With the increasing demands of drivers on car driving performance, the design method of the vehicle transmission system is also tends to be high-speed, sophisticated, and needs to be able to withstand greater loads. As shown in Figure 9, the meshing force of the gear driving, the dynamic impact of the transmission power input and output, and the high-speed rotation of the bearings can all become sources of vibration in the transmission system. As the transmission gear train is closely connected to the transmission housing through the bearings, it results in various vibrations that will be transmitted to the transmission housing, which makes the housing also generate vibration. If the frequency of various vibration is similar or the same, it will produce a strong resonance, so that the transmission assembly issued a huge noise.

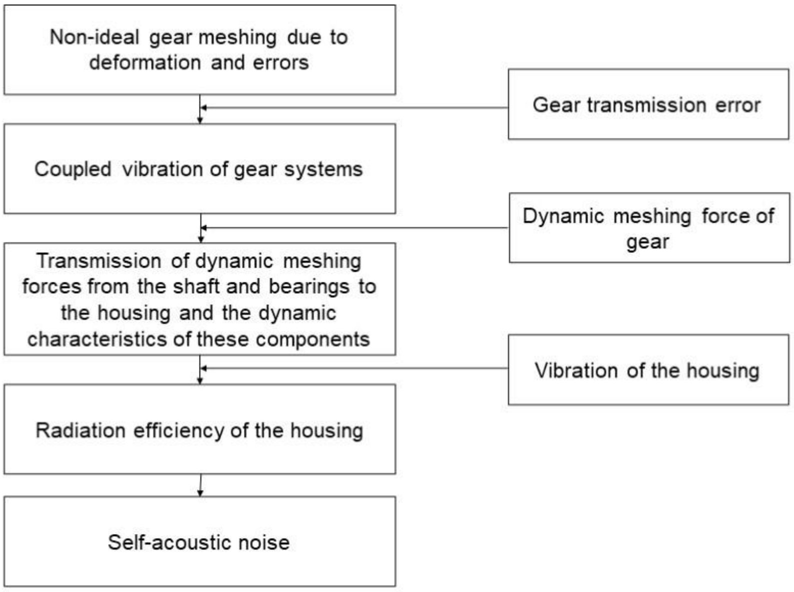

Figure 9. Cause of transmission noise

To evaluate the transmission gearing system design rationality, it mainly depends on its own composition of parts and the work quality of its load-bearing parts in the stressed state.

In automotive transmissions, gear, gear shaft and bearing components are the most important power transmission components in manual transmissions because of their high transmission efficiency and simple design layout. Gears are also the transmission parts with the highest machining accuracy requirements in automotive transmissions, and their meshing performance and service life will have a decisive impact on the accuracy, smoothness, and service life of the transmission, as well as the NVH performance of the transmission in use. For the transmission system in the case of loading, the gear set meshing normal can make the transmission operation stable, noise reduction and extended service life. If gear transmission system meshing performance is poor, it will causes that the transmission accuracy decline, gear surface is easy to wear, vibration and noise during using. Thus it undermines the comfort performance and safety of the vehicle. At the same time, the transmission housing plays a supporting role for the gear transmission system through the bearing, also plays a decisive role in the working accuracy of the transmission system.

In summary, in order to improve the overall performance of the transmission, this transmission force analysis research content will mainly focus on the gear support system.

3.2 Analysis of forces for different layout

The previous chapter has introduced the different types of transmissions, which can usually be divided into two-axis transmissions and three-axis transmissions. This paper focuses on the effect of different bearing and gear arrangements on the gear shaft. We first need to simplify the input, output, and intermediate shafts in the transmission by constraining the bearings at each end to 5 and 4 degrees of freedom, respectively, and then analyze the effect of the bearing reaction force on the gear shaft when the gear meshing force changes. The main simplified models are divided into three types, which explore the effect of a pair of gears meshing, and two pairs of gears meshing on the gear shaft respectively. For different models, different mechanical balance systems are established to calculate the bearing reaction forces. And the load on the bearing depends on the load carried by the supported shaft system components.

3.2.1 Forces analysis of layout 1

The first type is a gear mounted between two bearings and only one pair of gears meshes, which means that there is only one gear on this gear shaft, and the meshing force of this one gear is used as the input to this system. For type 1 system, the simplified model is shown below:

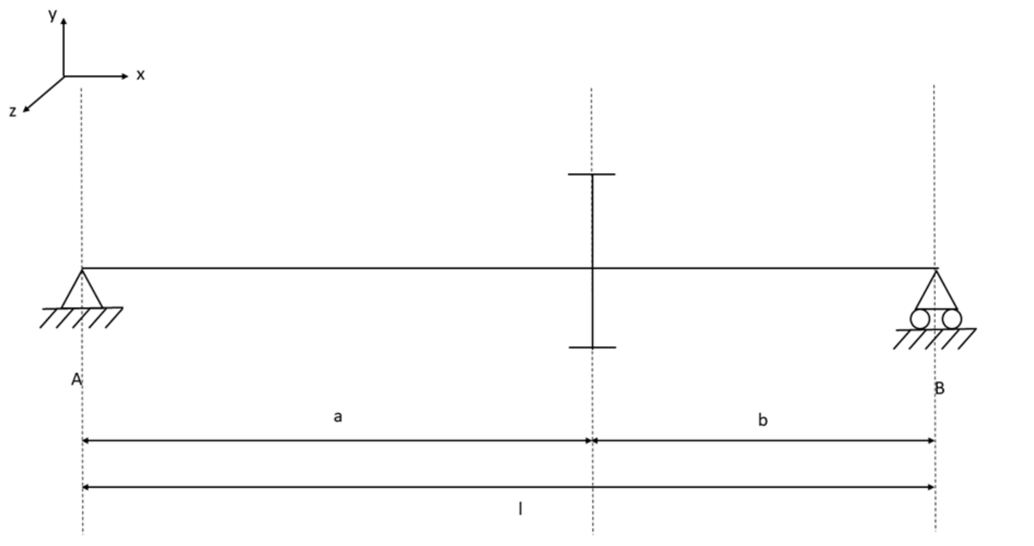

Figure 10. Type 1 system

In the yx plane, the force equation can be written as below:

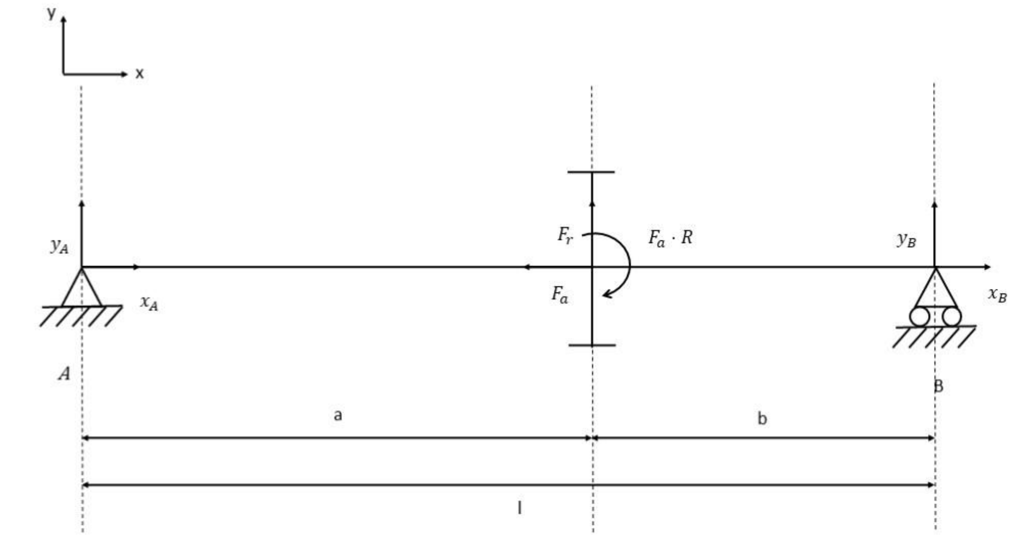

Figure 11. Force analysis of Type 1 system in yx plane

$$y_A + F_r + y_B = 0$$

$$x_A + x_B - F_a = 0$$

$$F_r \cdot a - F_a \cdot R + y_B(a + b) = 0$$

a and b are the distance from the bearing A and bearing B. l is the total distance of gear shaft.

y_A, y_B are the bearing reaction forces of A and B bearing along y direction.

x_A, x_B are the bearing reaction forces of A and B bearing along x direction.

F_a, F_r are the axial force and radial force respectively.

In the zx plane, the equation are:

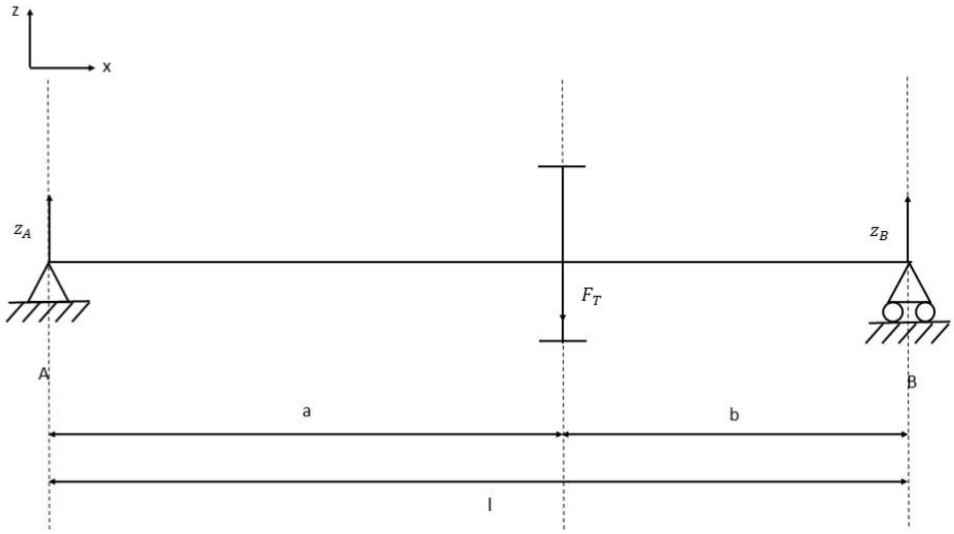

Figure 12. Force analysis of Type 1 system in zx plane

$$z_A + z_B - F_T = 0$$

$$z_B(a+b) - F_T a = 0$$

z_A, z_B are the bearing reaction forces of A and B bearing along z direction.

$F_T$are the tangent force of gear.

So the bearing reaction forces for 2 bearings A and B in different directions are:

$$y_A = \frac{-F_a R - F_r b}{a+b}$$

$$y_B = \frac{F_a R - F_r a}{a+b}$$

$$x_A = F_a$$

$$x_B = 0$$

$$Z_A = \frac{F_T b}{a + b}$$

$$Z_B = \frac{F_T a}{a + b}$$

3.2.2 Forces analysis of layout 2

For type 2 system, this system has two pairs of gear meshing forces, so there are two gears both mounted in the middle of the bearing. A simplified model is shown below.

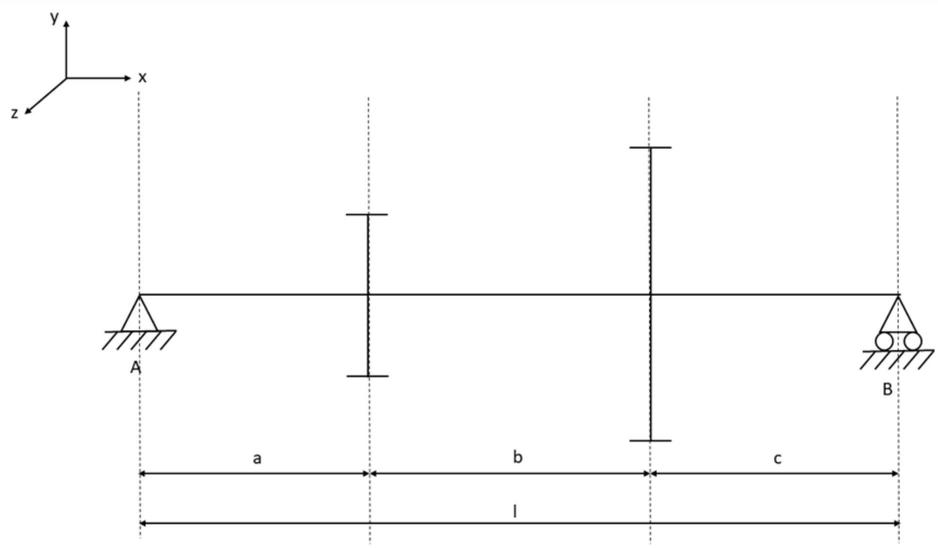

Figure 13. Type 2 system

In the yx plane, the force analysis is shown in the Figure 14.

y_A, y_B are the bearing reaction forces of A and B bearing along y direction.

x_A, x_B are the bearing reaction forces of A and B bearing along x direction.

F_{a_1}, F_{a_2} F_{R_1} F_{R_2} are the axial force are the radial force coming from gear 1 and gear 2 respectively.

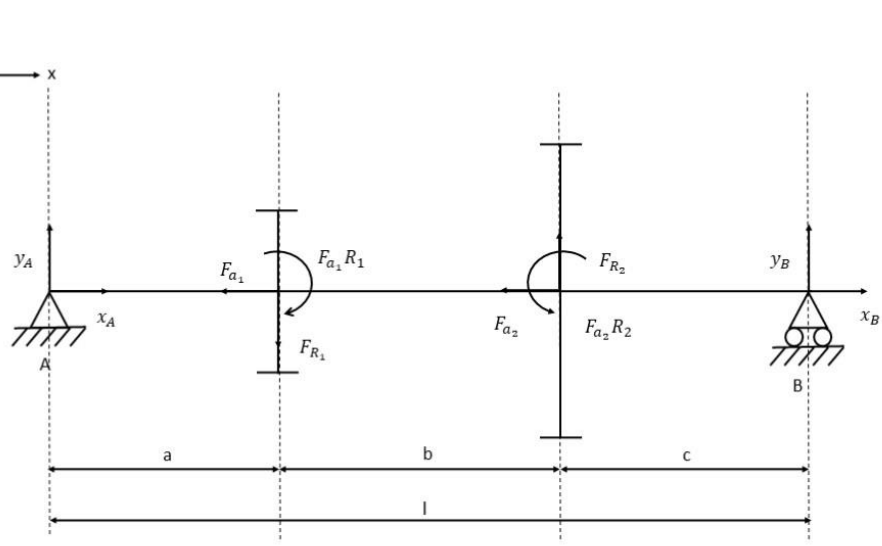

Figure 14. Force analysis of Type 2 system in yx plane

To compute this system's bearing reaction forces in different directions, it needs to split into 2 system, system Ⅰ and system Ⅱ.

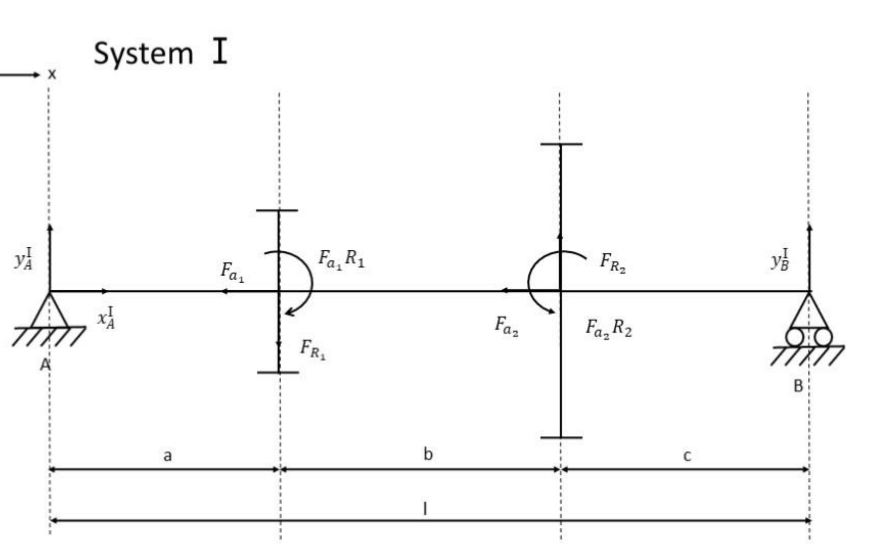

Figure 15. Force analysis of system Ⅰ for type 2 system

For system Ⅰ, on the point A, it has the forces along y and x directions and on the point B only has the force along y direction. So the force equilibrium equation for the y-direction and x-direction as well as the moment equilibrium equation at point A are:

$$y_A^{I} - F_{R_1} + F_{R_2} + y_B^{I} = 0$$

$$x_A^{I} - F_{a_1} - F_{a_2} = 0$$

$$-F_{a_1}R_1 - F_{R_1}a + F_{R_2}(a+b) + F_{a_2}R_2 + y_B^{I}(a+b+c) = 0$$

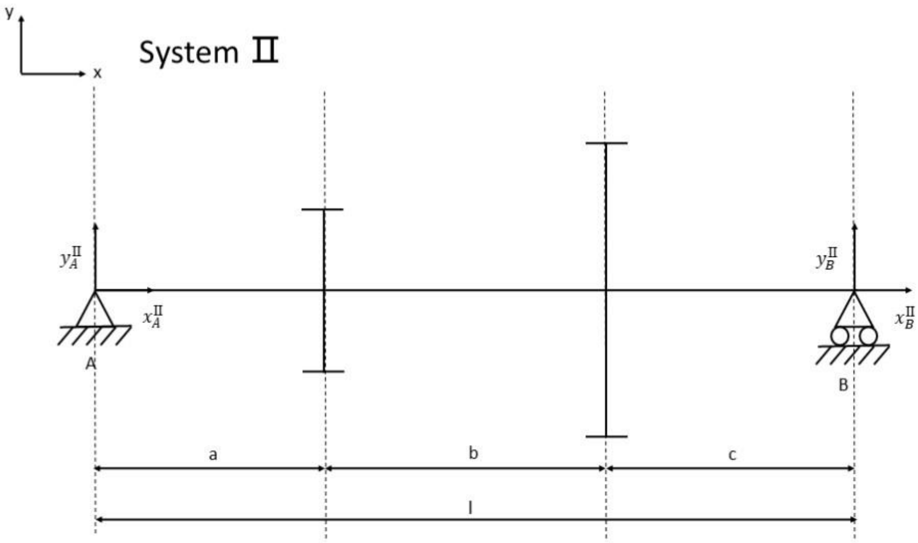

Figure 16. Force analysis of system Ⅱ for type 2 system

For system Ⅱ, this system is only concerned with the balance of the force on the bearings, so the equilibrium equation:

$$y_A^{II} + y_B^{II} = 0$$

$$x_A^{II} + x_B^{II} = 0$$

$$y_B^{II}(a+b+c) = 0$$

And in point B, we do not want any motion of x direction, so,

$$x_B = 0$$

$$\frac{x_B^{II} - x_A^{II}}{EA} + \frac{x_A^{I}}{EA} = 0$$

The results of system Ⅰ and Ⅱ in yx plane are:

$$y_A^{\mathrm{I}} = \frac{-F_{a_1}R_1 + F_{R_1}(b+c) + F_{a_2}R_2 - F_{R_2}C}{a+b+c}$$

$$y_B^{\mathrm{I}} = \frac{F_{a_1}R_1 + F_{R_1}a - F_{a_2}R_2 - F_{R_2}(a+b)}{a+b+c}$$

$$x_B^{\mathrm{I}} = 0$$

$$x_A^{\mathrm{I}} = F_{a_1} + F_{a_2}$$

$$y_A^{\mathrm{II}} = 0$$

$$y_B^{\mathrm{II}} = 0$$

$$x_A^{\mathrm{II}} = 0$$

$$x_B^{\mathrm{II}} = 0$$

This allows the bearing support force to be calculated in the yx plane and the results are shown below.

$$y_A = \frac{-F_{a_1}R_1 + F_{R_1}(b+c) + F_{a_2}R_2 - F_{R_2}C}{a+b+c}$$

$$y_B = \frac{F_{a_1}R_1 + F_{R_1}a - F_{a_2}R_2 - F_{R_2}(a+b)}{a+b+c}$$

$$x_A = F_{a_1} + F_{a_2}$$

$$x_B = 0$$

In the zx plane, only focus on the forces along z direction, so the equations can be written as:

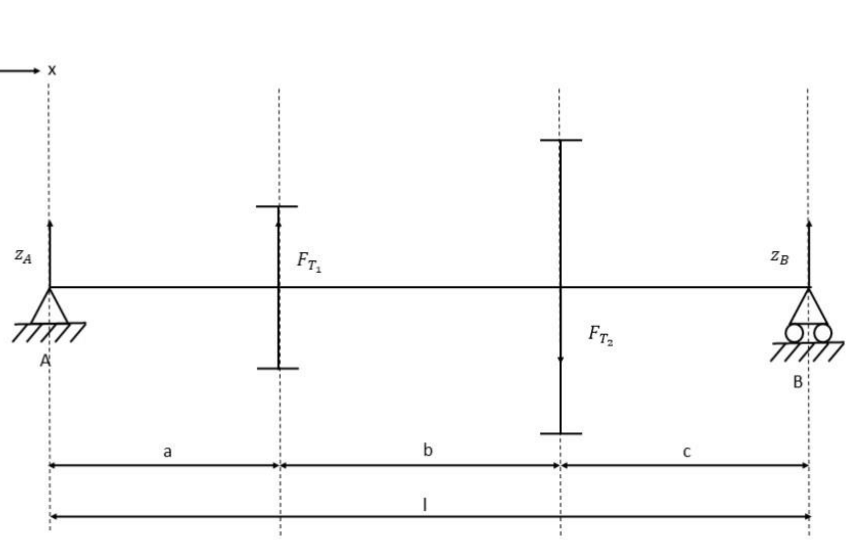

Figure 17. Force analysis of Type 2 system in zx plane

$$z_A + F_{T_1} - F_{T_2} + z_B = 0$$

$$F_{T_1} a - F_{T_2}(a + b) + z_B(a + b + c) = 0$$

Then the bearing reaction forces along z direction can be computed and results are shown below.

$$z_A = \frac{F_{T_2} c - F_{T_1}(b + c)}{a + b + c}$$

$$z_B = \frac{F_{T_2}(a + b) - F_{T_1} a}{a + b + c}$$

3.2.3 Forces analysis of layout 3

For type 3 system, this system also has two pairs of gears meshing, so in the simplified model, there are also two gears on the gear shaft. However, the two gears are not mounted in the middle of the bearings, but one gear is mounted between the bearings and the other gear is mounted on one end of the gear shaft. The diagram is shown below.

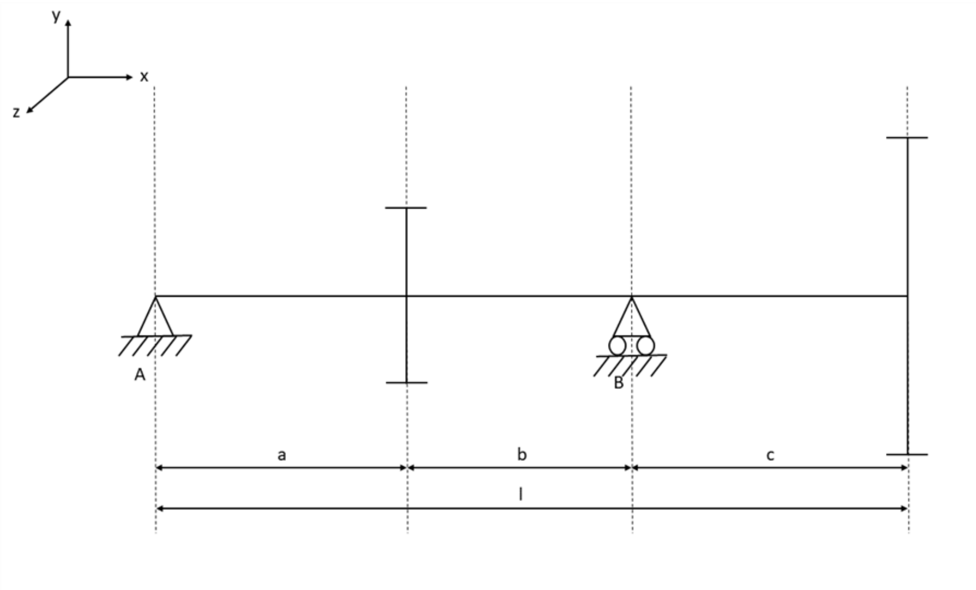

Figure 18. Type 3 system

In the yx plane, the force analysis is similar with type 2 system.

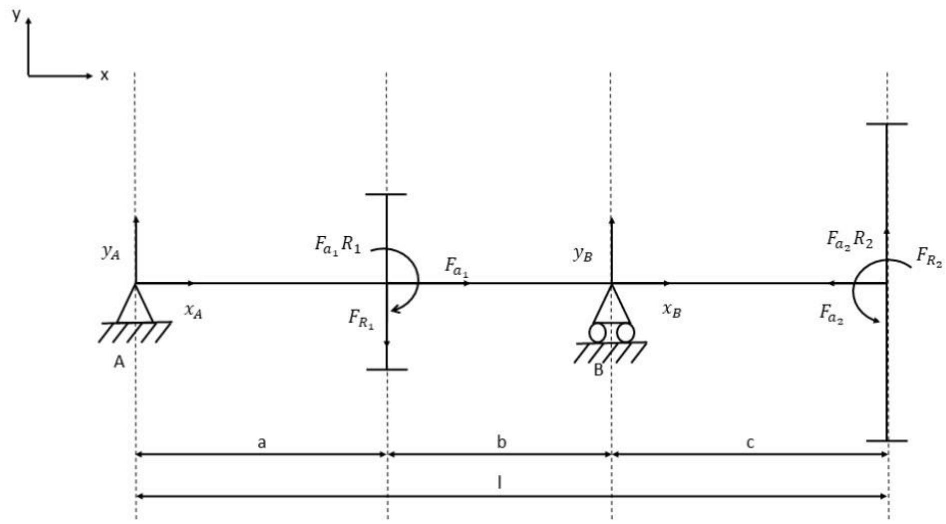

Figure 19. Force analysis of Type 3 system in yx plane

Also it needs to split into 2 system, system Ⅰ and system Ⅱ.

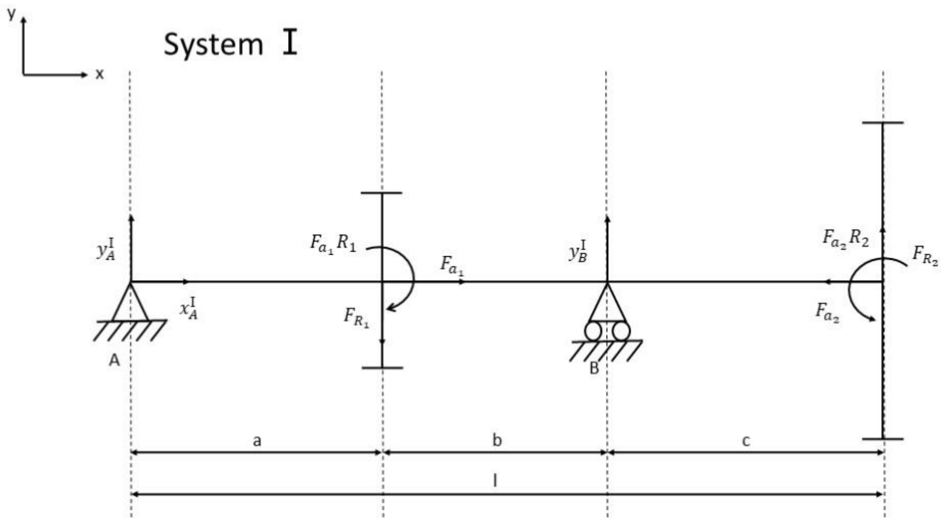

Figure 20. Force analysis of system Ⅰ for type 3 system

For system Ⅰ, by establishing the mechanical equilibrium equations for the y-direction and x-direction as well as the moment equilibrium equation at point A, we can obtain the equations shown below:

$$y_A^I - F_{R_1} + F_{R_2} + y_B^I = 0$$

$$x_A^I + F_{a_1} - F_{a_2} = 0$$

$$-F_{a_1}R_1 - F_{R_1}a + F_{R_2}(a+b+c) + F_{a_2}R_2 + y_B^I(a+b) = 0$$

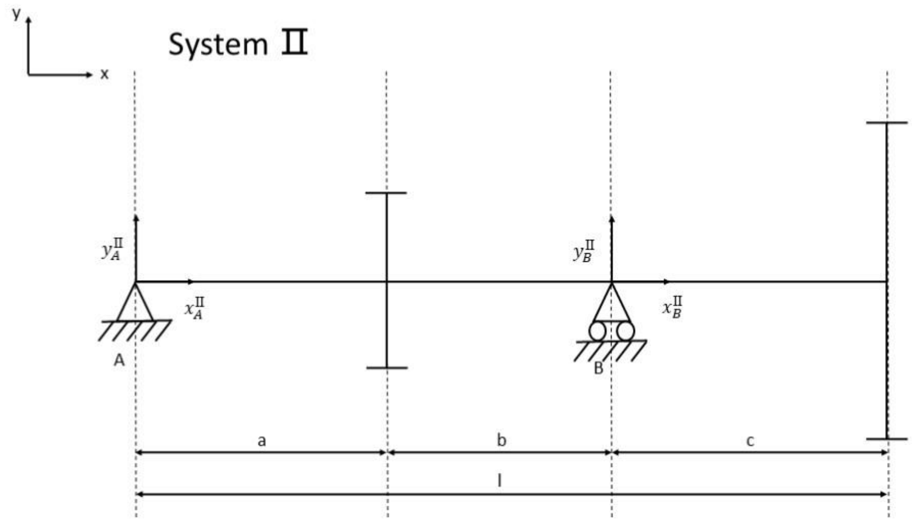

Figure 21. Force analysis of system Ⅱ for type 3 system

For system Ⅱ, the equations are:

$$y_A^{II} + y_B^{II} = 0$$

$$x_A^{II} + x_B^{II} = 0$$

$$y_B^{II} = 0$$

In point B, we also do not want any motion of x direction.

$$x_B = 0$$

$$\frac{x_B^{II} - x_A^{II}}{EA} + \frac{x_A^{I}}{EA} = 0$$

The bearing forces along y and x directions are:

$$y_A = \frac{-F_{a_1}R_1 + F_{R_1}b + F_{a_2}R_2 + F_{R_2}C}{a+b+c}$$

$$y_B = \frac{F_{a_1}R_1 + F_{R_1}a - F_{a_2}R_2 - F_{R_2}(a+b+c)}{a+b+c}$$

$x_A = F_{a_2} - F_{a_1}$

$x_B = 0$

In the zx plane, the analysis of the forces is much simpler and only requires the calculation of the forces in the z-direction.

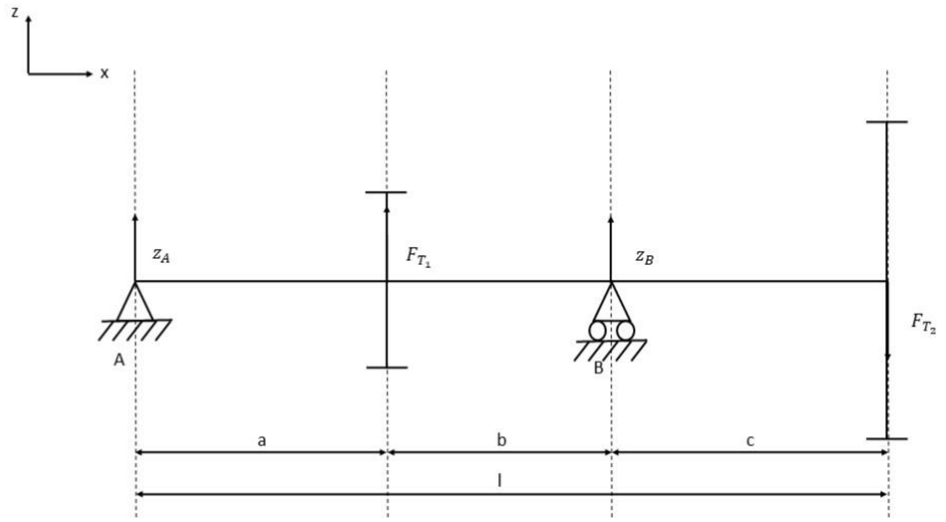

Figure 22. Force analysis of Type 3 system in zx plane

$$z_A + F_{T_1} - F_{T_2} + z_B = 0$$

$$F_{T_1}a - F_{T_2}(a+b+c) + z_B(a+b) = 0$$

The bearing forces along z direction are:

$$z_A = \frac{-F_{T_2}c - F_{T_1}b}{a+b+c}$$

$$z_B = \frac{F_{T_2}(a+b+c) - F_{T_1}a}{a+b+c}$$

3.3 Force analysis of helical gear

In the transmission drive system, because of the helix angle, the forces acting on the helical gears need to be decomposed to obtain the forces in the xyz plane.

During the helical gear transmission, the normal load acting on the tooth surface is still perpendicular to the tooth surface. As shown in the figure below, the F_n acting on the active wheel is located in the normal plane Pabc. F_n is decomposed into radial component force F_r and normal component force $F^{'}$, and then the normal component force $F^{'}$ is decomposed into circumferential force F_T and axial force F_a. The normal force F_n is then decomposed into three mutually perpendicular spatial components, and the magnitude of each component force is

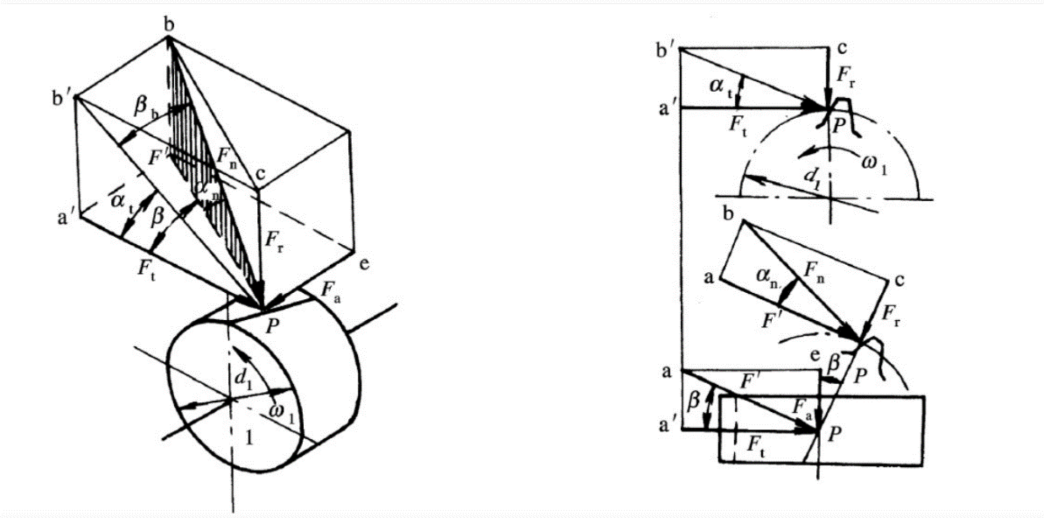

Figure 23. Force analysis of helical gear

$$F_T = F_n \cdot \cos\alpha \cdot \cos\beta$$

$$F_r = F_T \cdot \frac{\tan\alpha}{\cos\beta} = F_n \cdot \sin\alpha$$

$$F_a = F_T \cdot \tan\beta = F_n \cdot \cos\alpha \cdot \cos\beta \cdot \tan\beta = F_n \cdot \cos\alpha \cdot \sin\beta$$

3.4 Modeling of the system and dynamics equations

In order to see the vibration mechanism of the system more visually, the whole system can be viewed as a vibration system with the bearings as springs and dampers, the whole masses of the shaft and gears as equivalent masses, and the change in the meshing force of the gears or the bearing reaction forces as the starting force. The vibration system focuses on the displacement of the gear shaft in the shaft diameter direction and the change in force. As the gear meshing force and the change of the input torque or load torque of the transmission system cause the change of the bearing force on the gear shaft, it causes the gear shaft to vibrate to affect the service life of the gear shaft and generate noise pollution. The vibration dynamics model of the system is established, through which the vibration state of different gear-shaft-bearing systems can be visualized.

For a system in motion, the external force that provides the motion is the cause and the ability of the system to move is the effect. Figure 24 shows the system with two masses considering elasticity and damping. k is the linear elastic element, c_A and c_B are the damping coefficients.

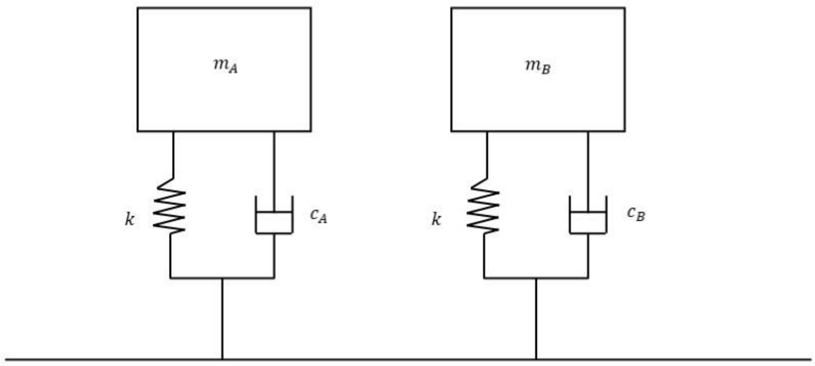

Figure 24. Modeling of system

The analytical model of the system can be deduced from the above figure as:

$$m_A\ddot{u}_1 + c_A\dot{u}_1 + ku_1 = y_A$$

$$m_A\ddot{v}_1 + c_A\dot{v}_1 + kv_1 = z_A$$

$$m_B\ddot{u}_2 + c_B\dot{u}_2 + ku_2 = y_B$$

$$m_B\ddot{v}_2 + c_B\dot{v}_2 + kv_2 = z_B$$

In the equations m_A and m_B are the equivalent masses of the gear and shaft at the bearing end points. u(i=1,2) and v(i=1,2) are the displacements of the gear shaft in the radial direction, that is, in the y and z directions. The above equation can be rewritten to obtain the model built in Simulink as below:

$$\ddot{u}_1 = \frac{y_A - c_A\dot{u}_1 - ku_1}{m_A}$$

$$\ddot{v}_1 = \frac{z_A - c_A\dot{v}_1 - kv_1}{m_A}$$

$$\ddot{u}_2 = \frac{y_B - c_B\dot{u}_2 - ku_2}{m_B}$$

$$\ddot{v}_2 = \frac{z_B - c_B\dot{v}_2 - kv_2}{m_B}$$

For different system types, the masses of the equivalent masses m_A and m_B are calculated slightly differently. The equivalent mass results for the 3 systems are given below.

For type 1 system:

$$m_B = \frac{m_{gear} \cdot a + m_{shaft} \cdot l_g}{l}$$

$$m_A = m_{gear} + m_{shaft} - m_B$$

For type 2 system:

$$m_B = \frac{m_{gear1} \cdot a + m_{gear2} \cdot (a + b) + m_{shaft} \cdot l_g}{l}$$

$$m_A = m_{gear1} + m_{gear2} + m_{shaft} - m_B$$

For type 3 system:

$$m_B = \frac{m_{gear1} \cdot a + m_{gear2} \cdot l + m_{shaft} \cdot l_g}{a + b}$$

$$m_A = m_{gear1} + m_{gear2} + m_{shaft} - m_B$$

4. Model in Simulink

According to the research purpose of this paper, the model of the gear-shaft-bearing system needs to be simulated and analyzed.

The mathematical model of the gear-shaft-bearing system has been determined in the previous chapter, and this chapter needs to convert the mathematical model into a simulation model by Simulink. The simulation results will be analyzed to determine the effect of dynamic gear meshing force on the gear shaft in different types of layouts.

4.1 Introduction to Simulink

MATLAB software is a computer software system that integrates numerical simulation calculation and graphics processing developed by MathWorks, Inc. in the United States, and is currently the most well-developed and widely used software system in the international arena. It is widely used in various industries such as mathematical computing, medical and health care, automobile manufacturing, finance. It has become the benchmark in the field of numerical simulation and computation by relying on its own simple and effective programming language and powerful computational drawing functions.

The system simulation tool chosen for this paper is the Simulink package in MATLAB. As one of the most important components of MATLAB, it can model, simulate and analyze dynamic systems, and is extremely widely used in the market. Compared with MATLAB, Simulink has the following advantages:

1) For very complex equations, it can present the mathematical model in a simple and clear graphical way. And with its own numerical solver, it reduces the complexity and error-prone nature of programming.

2) Since the structure of the simulation system is very complex, Simulink

can build and encapsulate subsystems to make the simulation module diagram clearer and more understandable.

4.2 Most used Simulink blocks

This section describes the modules in Simulink that will be used in building the gear-shaft-bearing system. Simulink contains many libraries of modules as follows:

1. Sources blocks, which provides various signal sources for the simulation.

2. Sinks blocks, which provides output device components for the simulation.

3. Continuous blocks, which provides continuous system for the simulation.

4. Discrete blocks, providing discrete components for simulation.

5. Math blocks, providing mathematical operation function components.

6. Function & Tables blocks, to custom functions and linear interpolation lookup table.

7. Nonlinear blocks, are non-continuous system components.

8. Signals & System blocks, provide for input, output and control of the relevant signals and related processing.

9. Subsystems blocks, which contains various subsystems.

In this Simulink system, the module libraries we used most include:

- Source blocks
- Output or sink blocks
- Math operations blocks
- Subsystems blocks
- Continuous blocks

The most used module in this system model from source block is the "From Workspace" module, where Fa, Fr, Ft from the decomposition of the gear mesh force along each direction is the main input module.

Figure 25. From Workspace

The From Workspace module reads data from the MATLAB workspace. The Data parameter of the module points to the workspace data, specified by a MATLAB computed expression containing a matrix or structure of signal values and time step tables. The format of its matrix or structure is the same as the data entered in the workspace. Its data type can be any type of real or complex signal.

Then We usually use "Constant" module in system modeling for gear radius, gear position data, etc. It generates a specified real or complex value that is independent of time and has the same data type as the module parameter constant value.

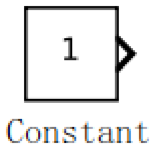

Figure 26. Constant

The input signal of the "Import" module is introduced from the outside of a system to the inside of the system, and creates input ports for subsystem or external inputs.

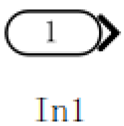

Figure 27. Import

The most used module in Sink module is the “Scope” module, which mainly displays a graph of the input about the simulation time, the module takes one input and can display a graph of multiple signals. Scope provides toolbar buttons to scale the displayed data, to display all the data, to save the settings of the axes in one simulation to the next, to limit the displayed data, and to save the data to the workspace.

Figure 28. Scope

The mathematical operations module is usually used to represent mathematical relationships to obtain the output quantities we need, such as bearing reaction forces and shaft displacements. The modules needed in the model include “Add”, “Divide” and “Gain” blocks. The “Gain” module is used to multiply the input by the built-in parameters to get the output. The gain value of the gain module can be a constant or a vector or matrix. Also the gain object can be one of the three.

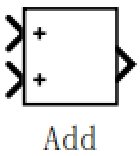

Figure 29. Add

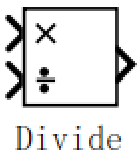

Figure 30. Divide

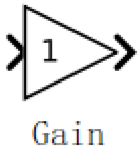

Figure 31. Gain

As the size and complexity of a system model increases, it needs to be simplified by combining modules into subsystems. A subsystem is a group of modules that are combined into a Subsystem module.

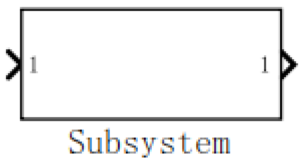

Figure 32. Subsystem

Simulink treats the “Integrator” module as a dynamic system with one state, and the output of the Integrator module is the integral value of the input signal with respect to time.

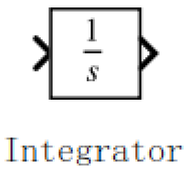

Figure 33. Integrator

4.3 Model presentation

The following three images show the total model of the previous three different types of gear-bearing-shaft systems. In the figures, the inputs are on the left, using the from workspace module, and the data are derived from the gear meshing forces, then the bearing reaction forces in different directions and the shaft displacements can be obtained by modeling the mathematical equations from the previous chapter 3. Figure 37 and 38 are the subsystems in the overall system model, and the main inputs are the bearing reaction forces to analyze the effect on the displacement of the gear shaft under modeling the bearing as a spring and a damper. Figure 39 and Figure 40 are the different gear meshing forces entered throughout the model, where figure 39 is Force_time.mat which is the constant torque and the change of speed. And figure 40 is DF_500.mat which is constant torque and constant speed.

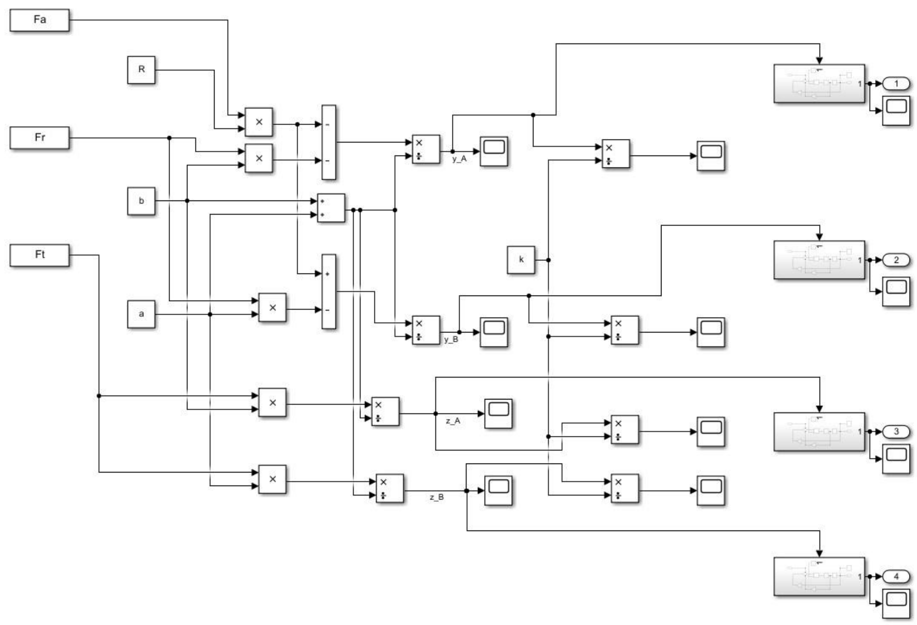

Figure 34. The overview of type 1 system model

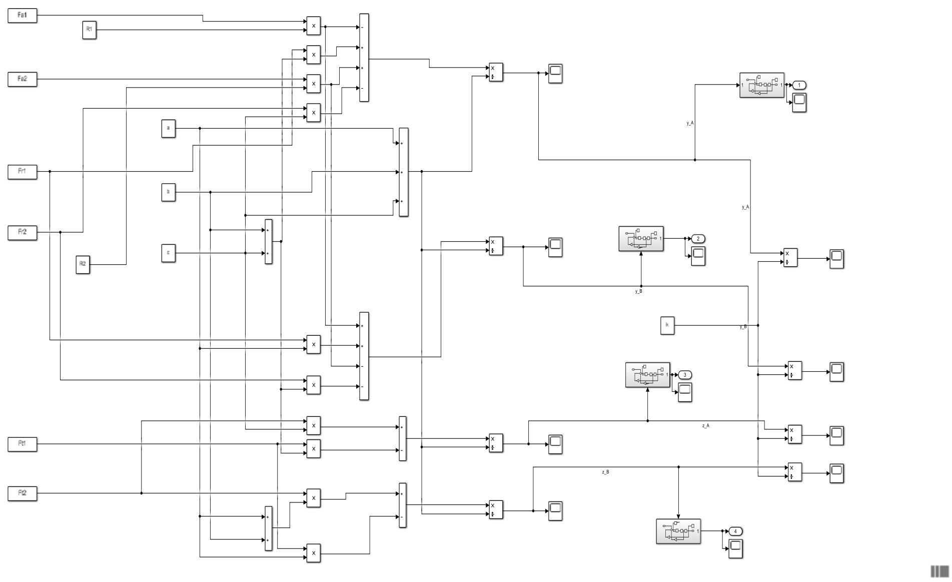

Figure 35. The overview of type 2 system model

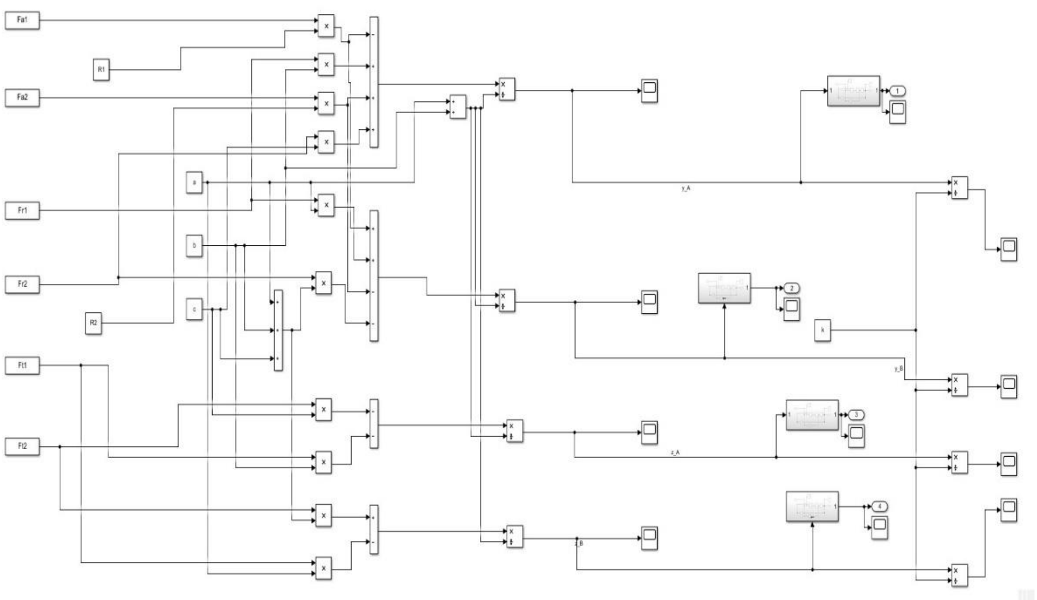

Figure 36. The overview of type 3 system model

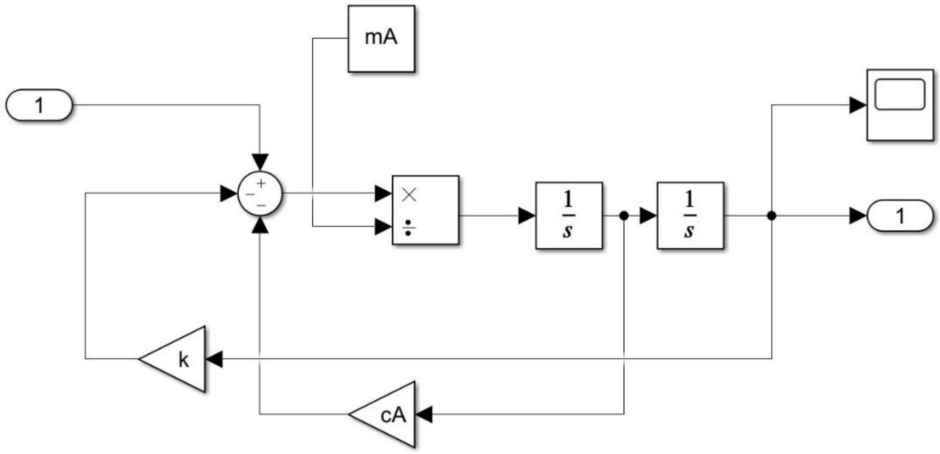

Figure 37. The subsystem on point A

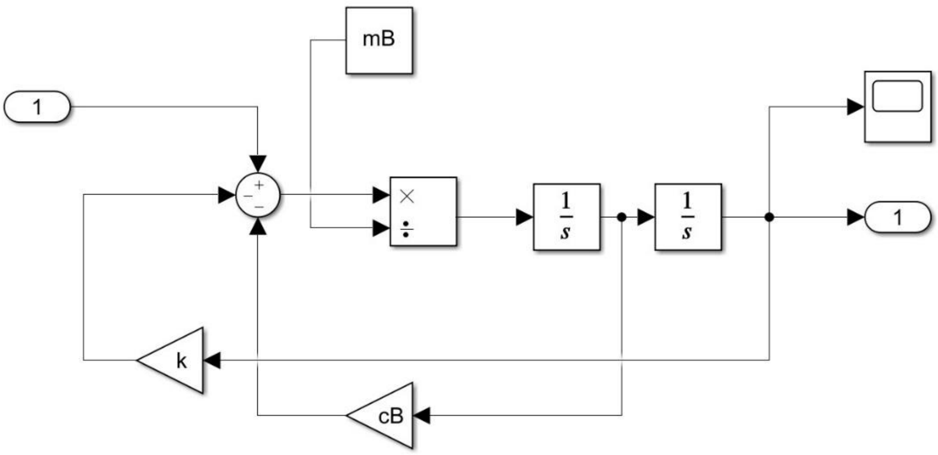

Figure 38. The subsystem on point B

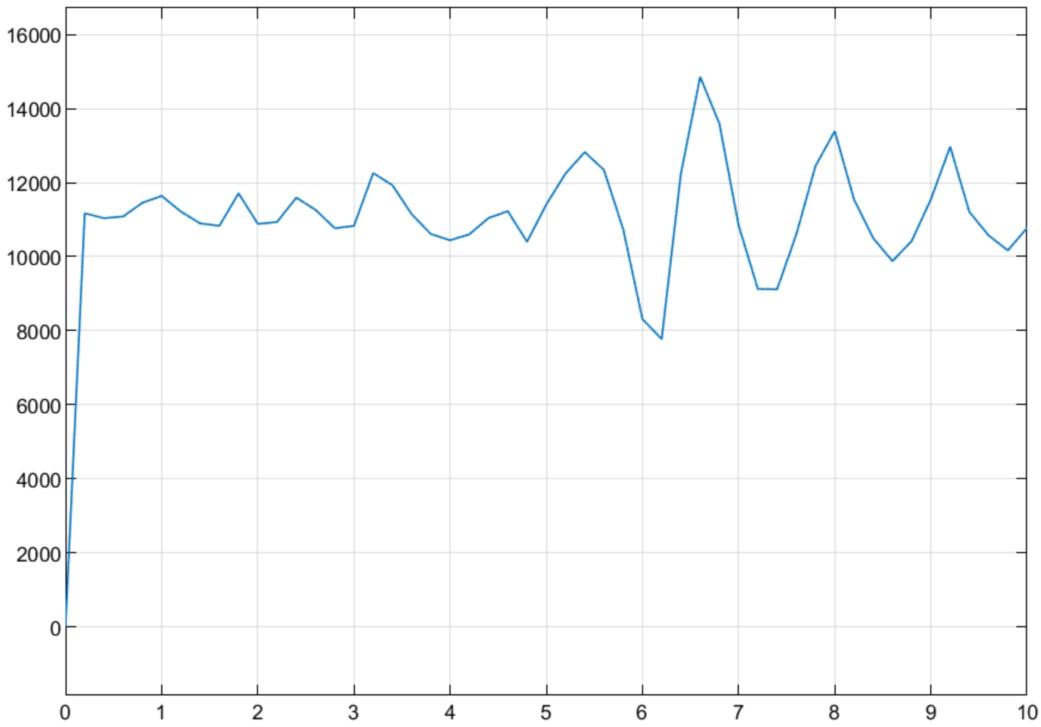

Figure 39. The meshing force of Force_time.mat

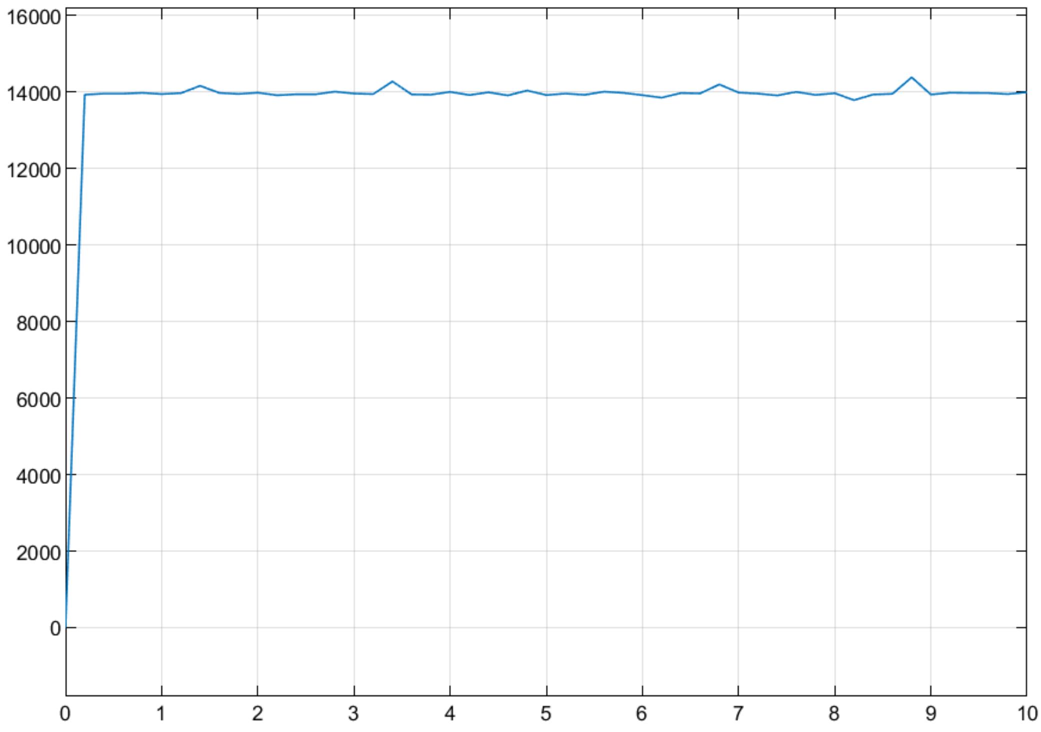

Figure 40. The meshing force of DF_500.mat

4.4 Analysis of results

Once all the data is set up and the model is built, I can run the whole system.

The following shows the results obtained for the three different types of systems, mainly the reaction forces at the two bearing ends and the displacement of the gear shaft in the radial direction that will be obtained after modeling the system. We have two different sources of input, one is constant torque and different speed, another one is constant torque and constant speed.

Load Force_time.Mat, which is constant torque and different speed, the results are shown below.

For the Type1 system:

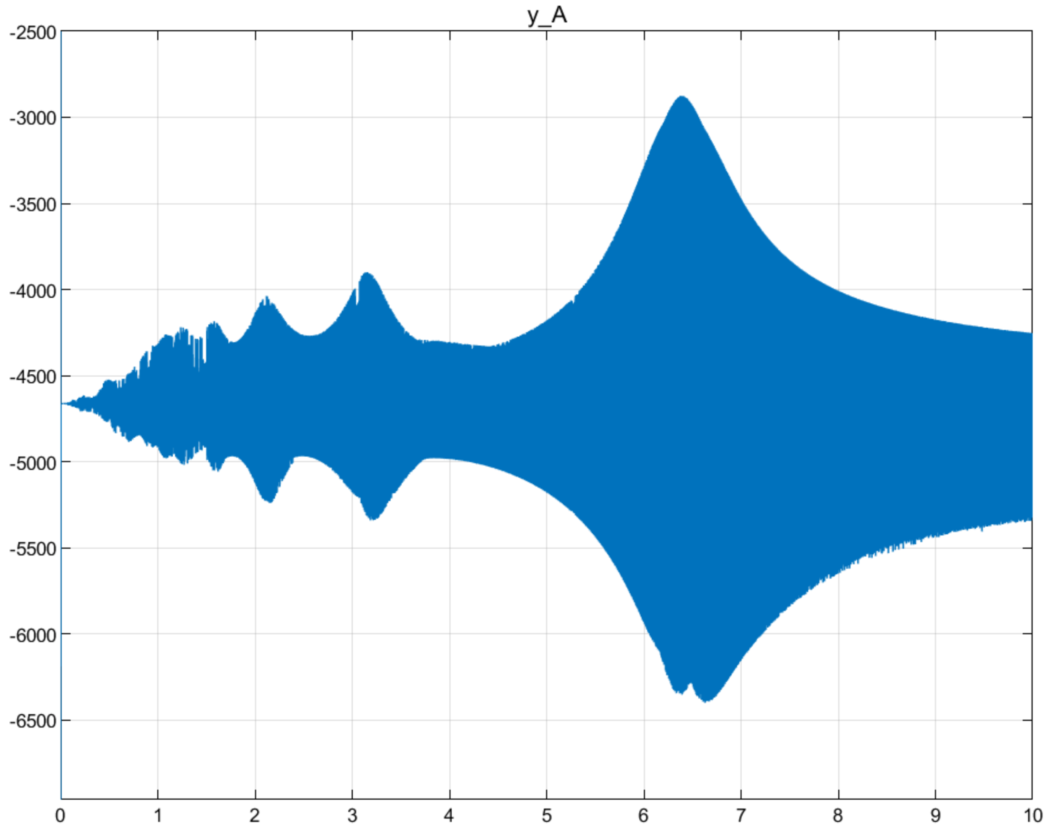

Figure 41. The bearing reaction force of bearing A in y direction

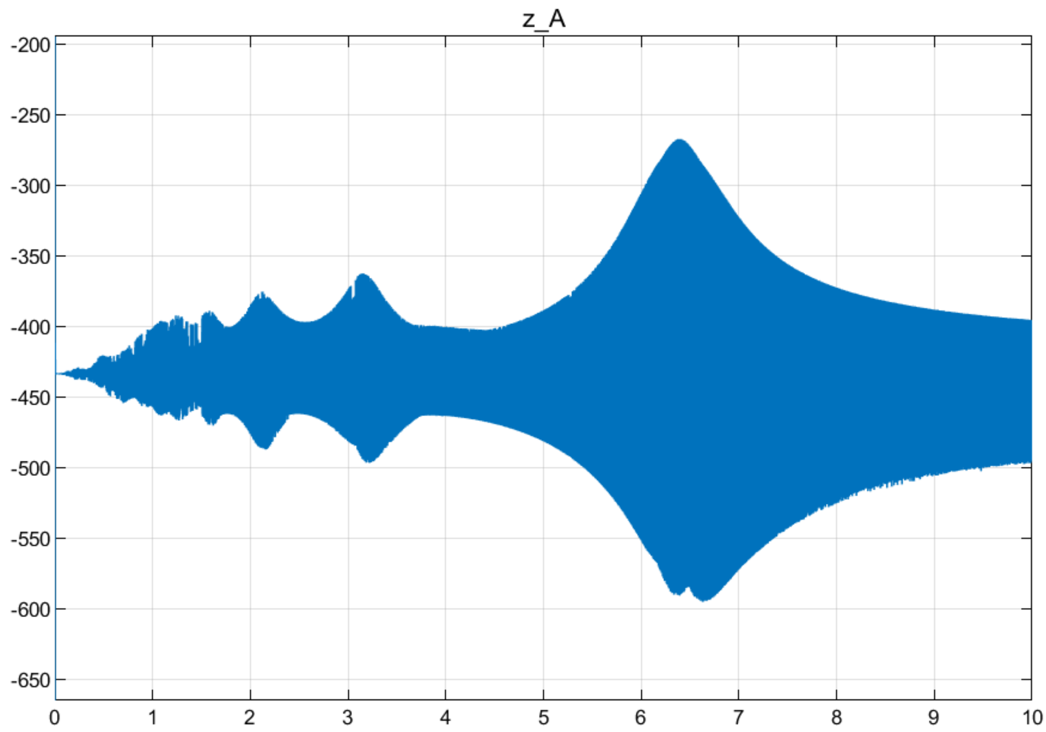

Figure 42. The bearing reaction force of bearing A in z direction

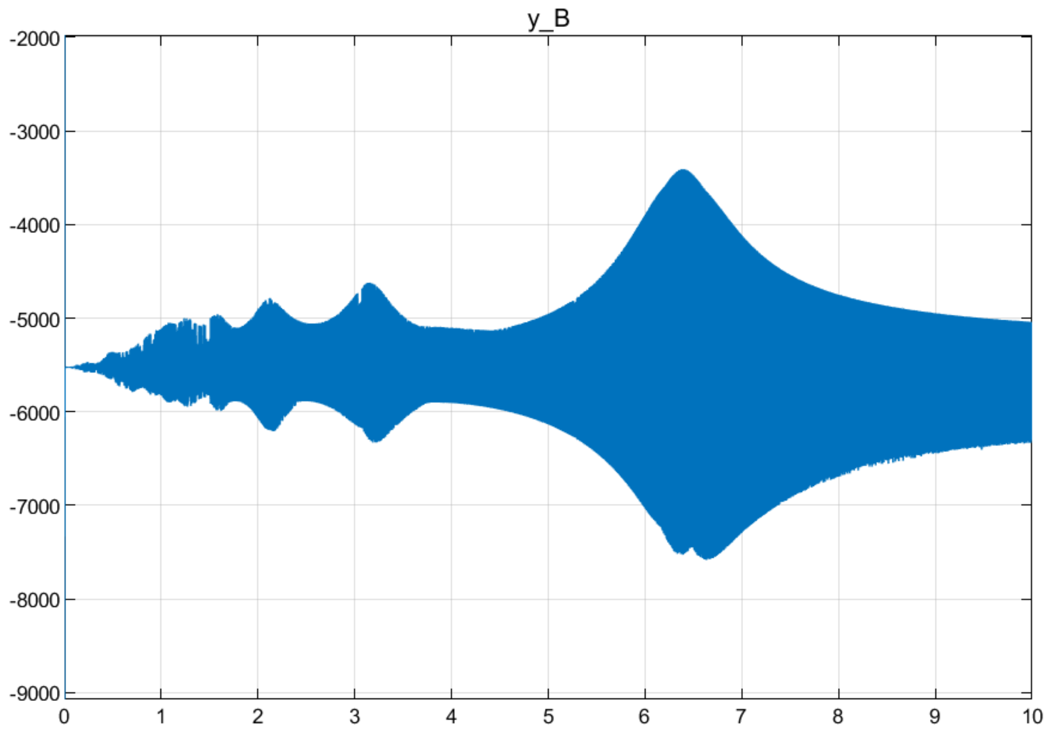

Figure 43. The bearing reaction force of bearing B in y direction

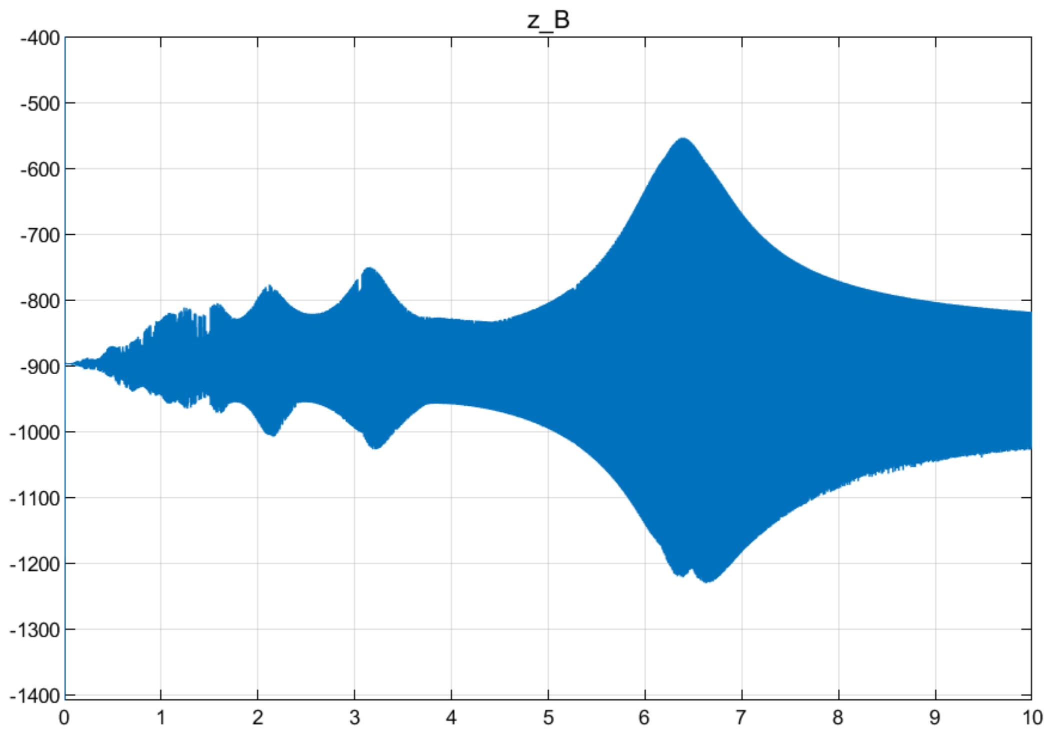

Figure 44. The bearing reaction force of bearing B in z direction

The above is the force acting on the two bearings obtained after the simulation of the gear-shaft-bearing system. It can be seen that for the type1 system, which is equivalent to the input shaft, the force acting in the y-direction is greater than the force in the z-direction for either bearing A or bearing B. The static force applied to bearing B is slightly greater than that of bearing A in either directions because the gear is mounted closer to bearing B on the shaft.

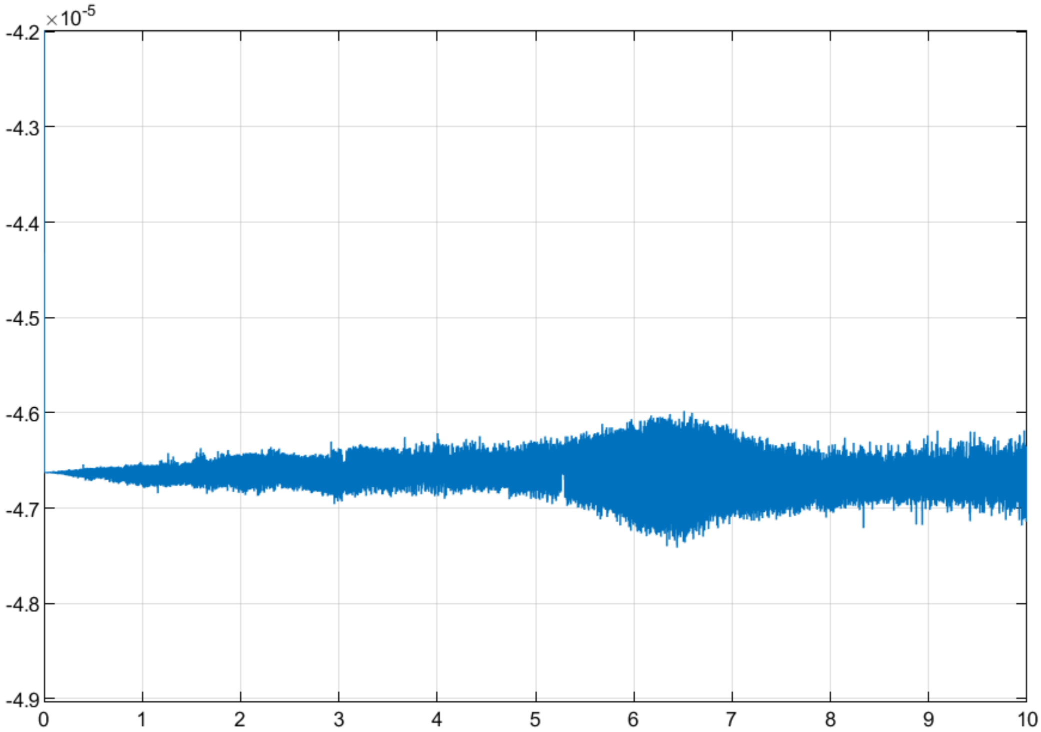

Figure 45. The displacement of bearing A in y direction

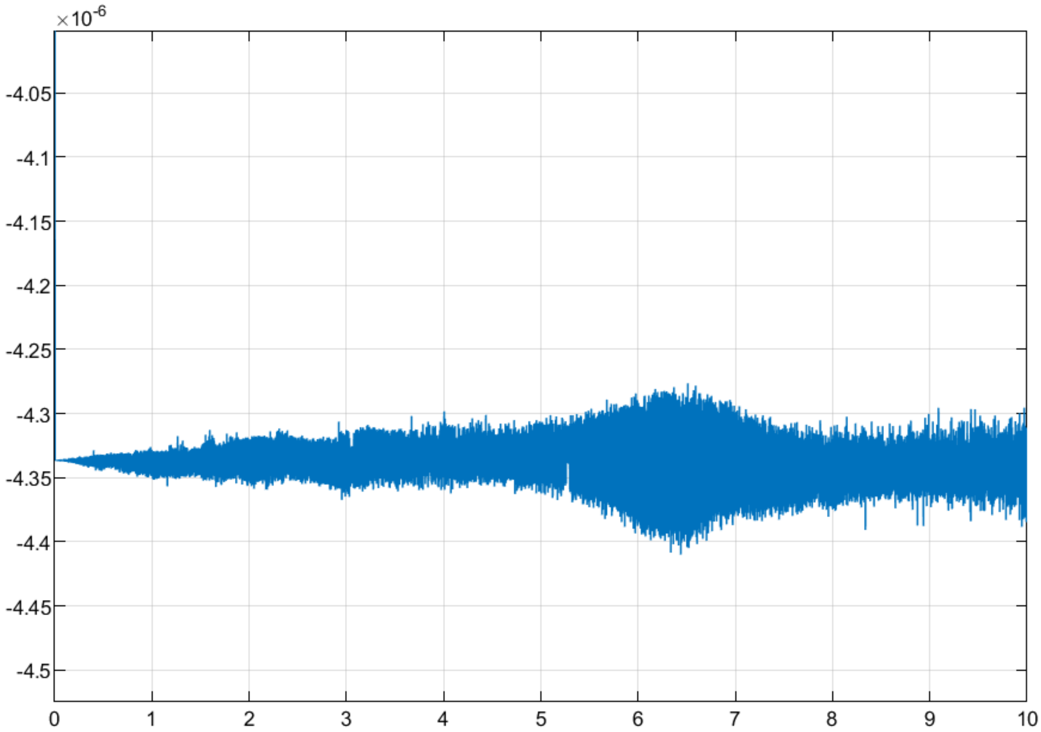

Figure 46. The displacement of bearing A in z direction

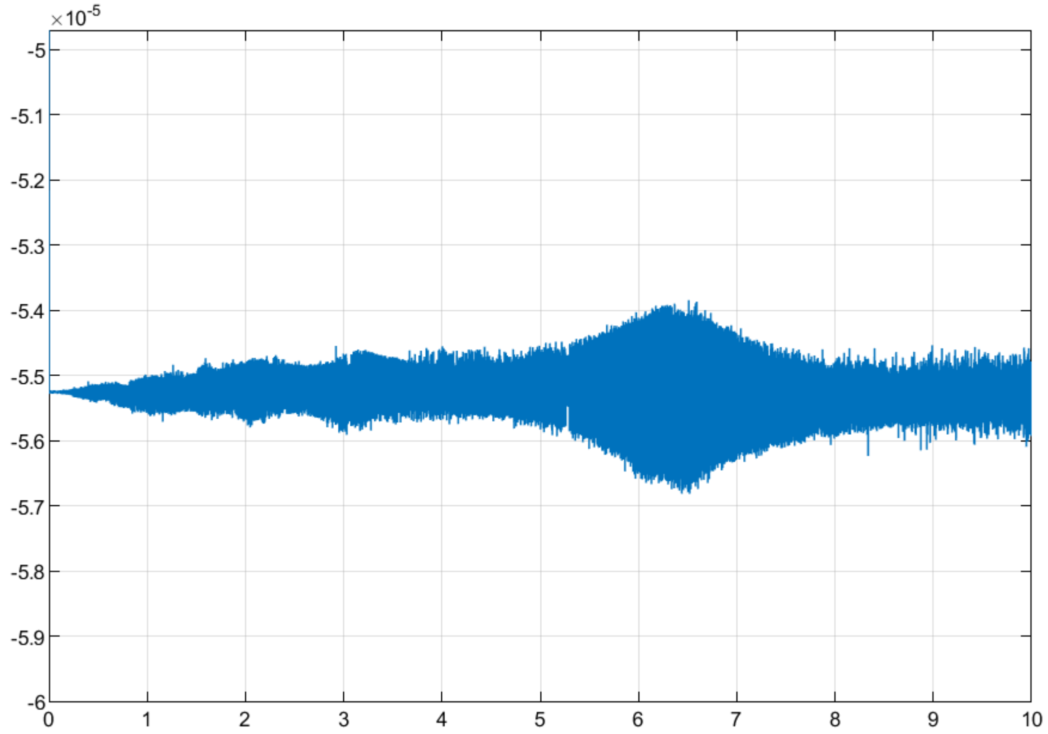

Figure 47. The displacement of bearing B in y direction

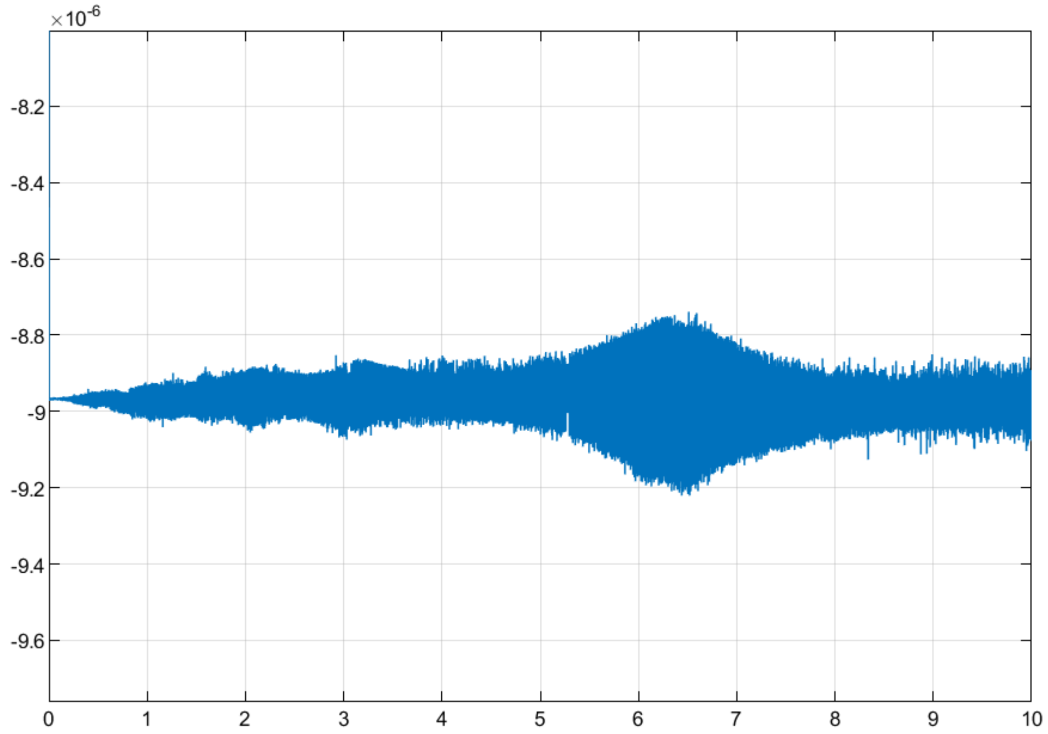

Figure 48. The displacement of bearing B in z direction

Through analysis of the displacement of the gear shaft in the shaft diameter direction, it reveals that the bearing reaction force is slightly larger on bearing B, resulting in a larger vibration amplitude at the B end of the gear shaft.

And if we observe Figure 39, we can see that around 6.5s, the bearing reaction force reaches a maximum value of about 6400, while at 0s, the bearing force is about 4600 in the static analysis. Meanwhile, observing Figure 43, it can be seen that around 6.5s, the maximum displacement is 4.73×10^{-5}, while after 0s the static displacement is 4.67×10^{-5},. Let y_{Amax} be the maximum bearing reaction force acting on bearing A along the y-direction, $y_{Astatic}$ be the static bearing reaction force around 0s, u_{Amax} be the maximum displacement, and $u_{Astatic}$ be the static displacement along the y-direction. Then a comparison can be made:

$$\frac{y_{Amax}}{y_{Astatic}} > \frac{u_{Amax}}{u_{Astatic}}$$

We also analyze the bearing reaction force z_A in the z-direction and the displacement v_A and find the same results:

$$\frac{z_{Amax}}{z_{Astatic}} > \frac{v_{Amax}}{v_{Astatic}}$$

This shows that there is no direct relationship between force and displacement, but rather a filter effect in both the y- and z-directions.

For bearing B, we can obtain the same conclusion in the y and z directions. The displacement does not show the huge fluctuations caused by the bearing reaction force, and its trend is more moderate compared to the bearing reaction force chart. So by comparing the ratio of the maximum dynamic bearing reaction force with the static bearing reaction force and the maximum gear shaft displacement with the displacement under static analysis, it can be more clearly to see that the maximum displacement of the gear shaft decreases. And this is because the bearing acts as a filter in the whole system, which reduces the magnitude of the displacement.

For the Type 2 system:

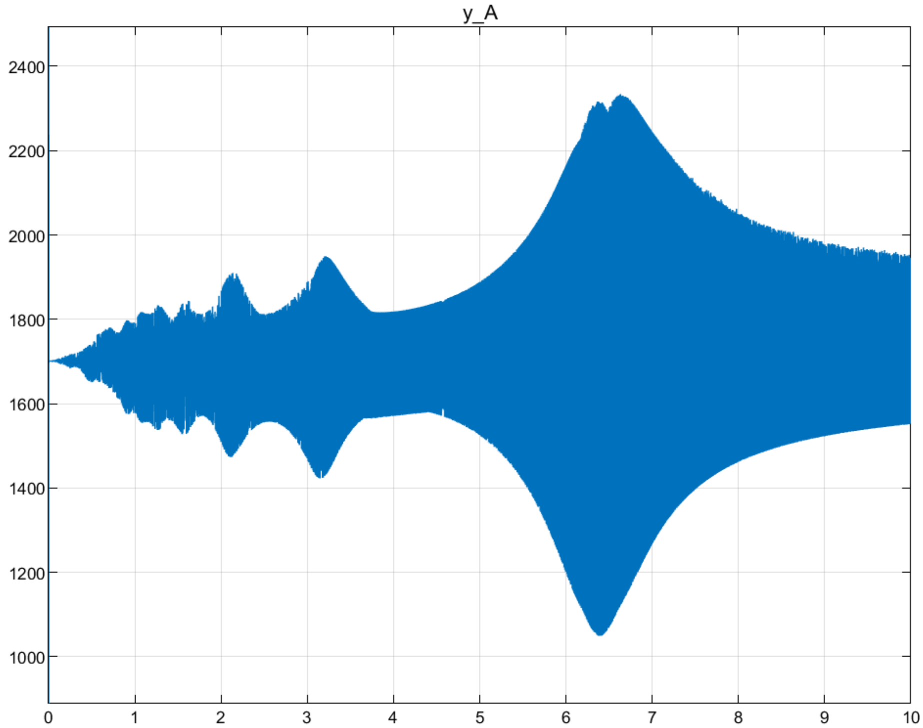

Figure 49. The bearing reaction force of bearing A in y direction

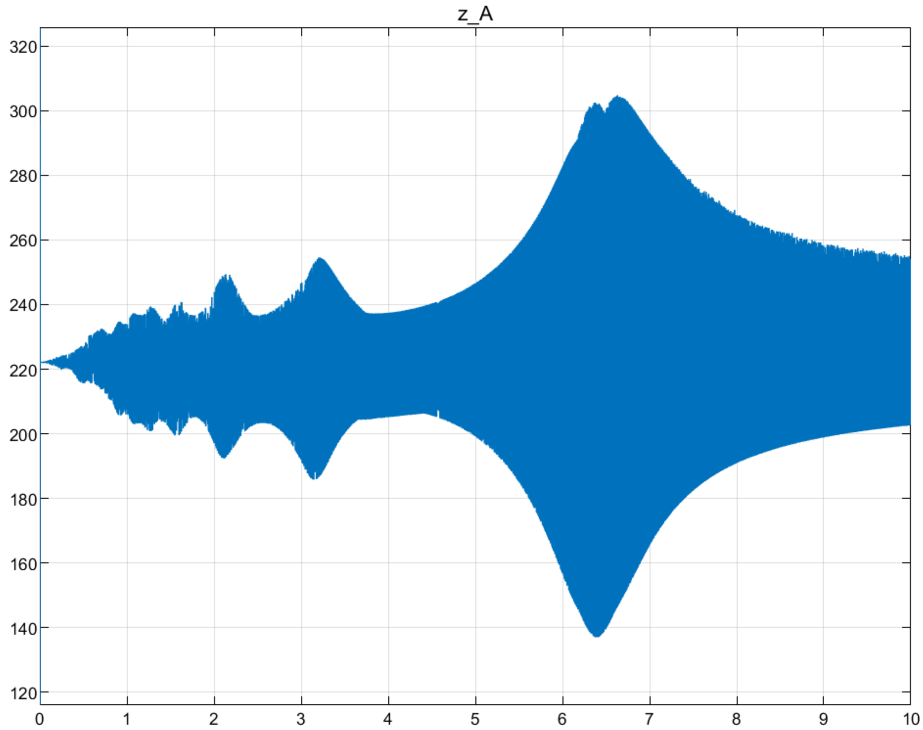

Figure 50. The bearing reaction force of bearing A in z direction

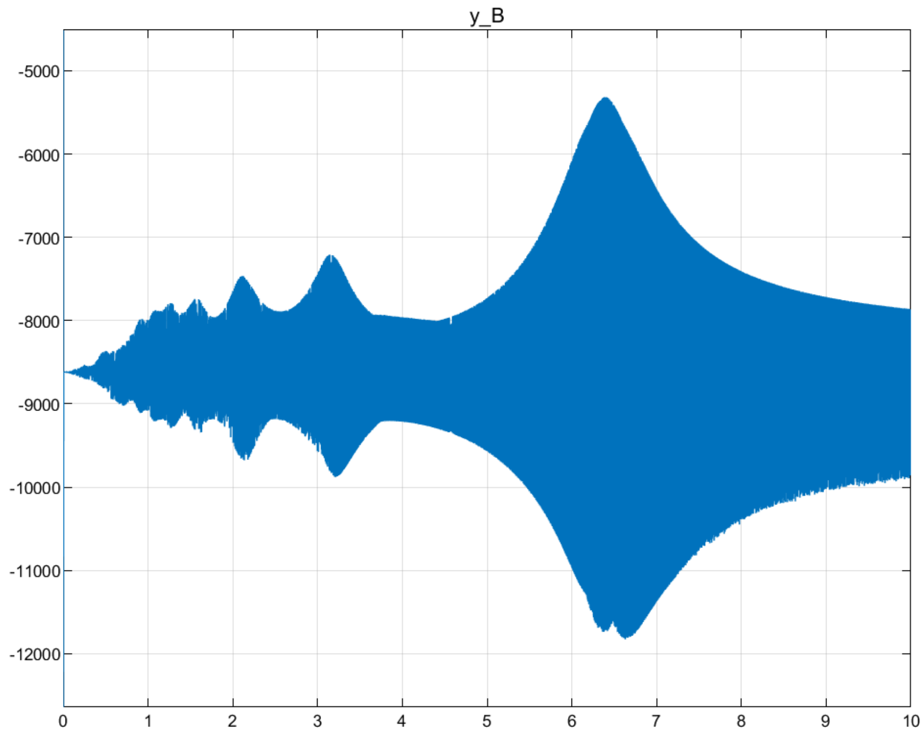

Figure 51. The bearing reaction force of bearing B in y direction

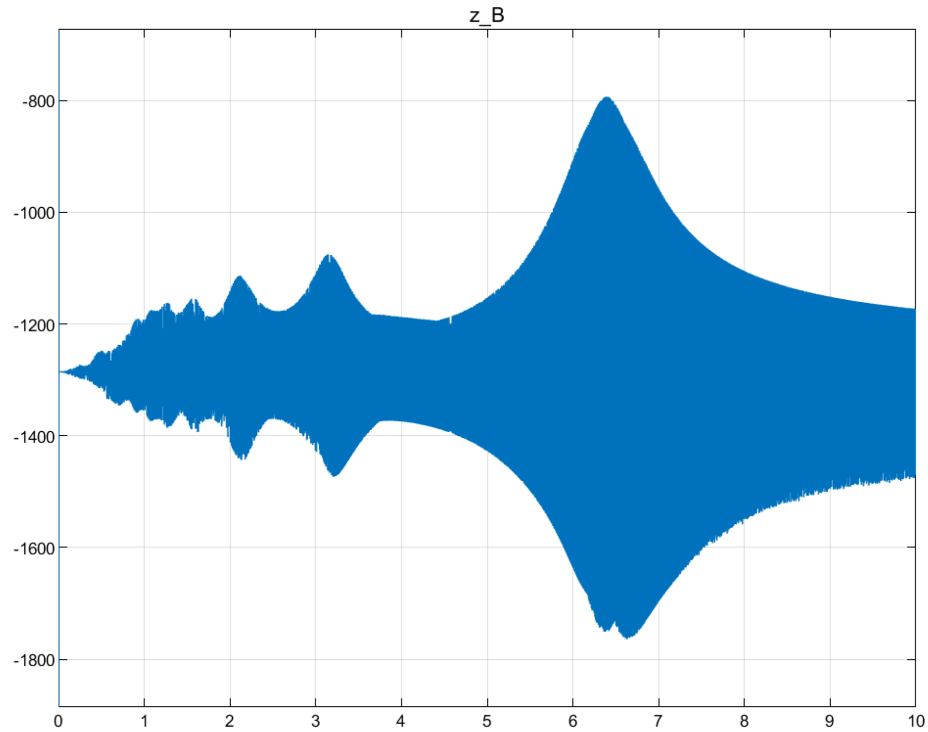

Figure 52. The bearing reaction force of bearing B in z direction

From the resulting figures, it appears that for the type2 system, the bearing reaction force in the y-direction is greater than that in the z-direction. The maximum force applied to the bearing B is greater than the maximum force applied to the bearing A.

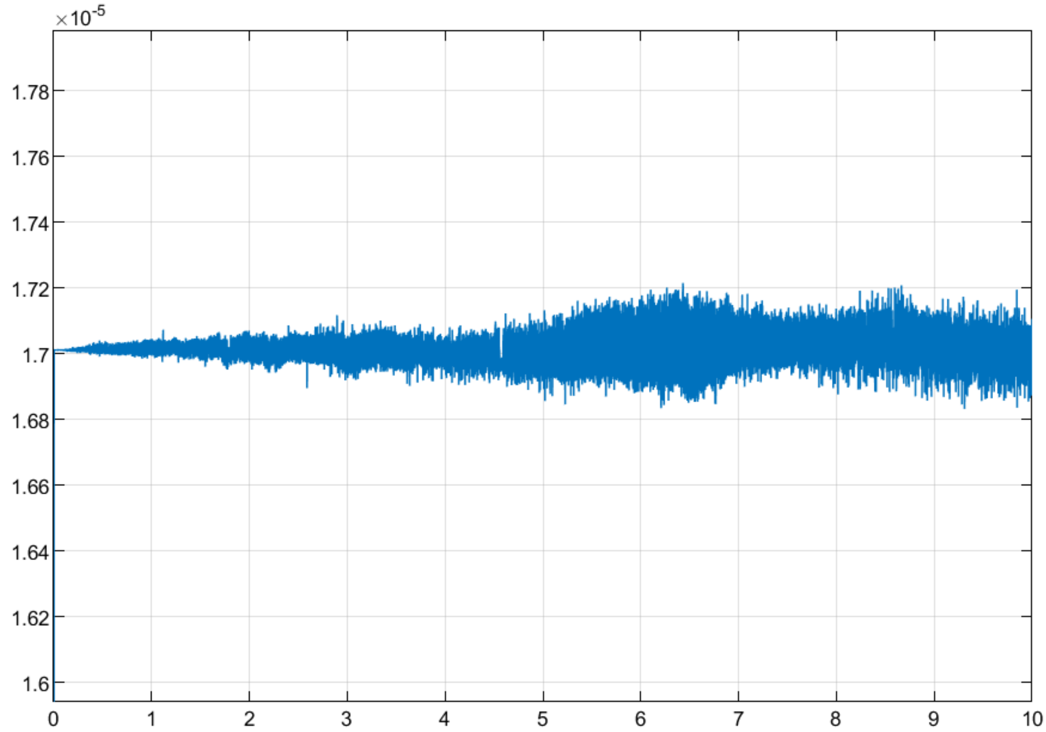

Figure 53. The displacement of bearing A in y direction

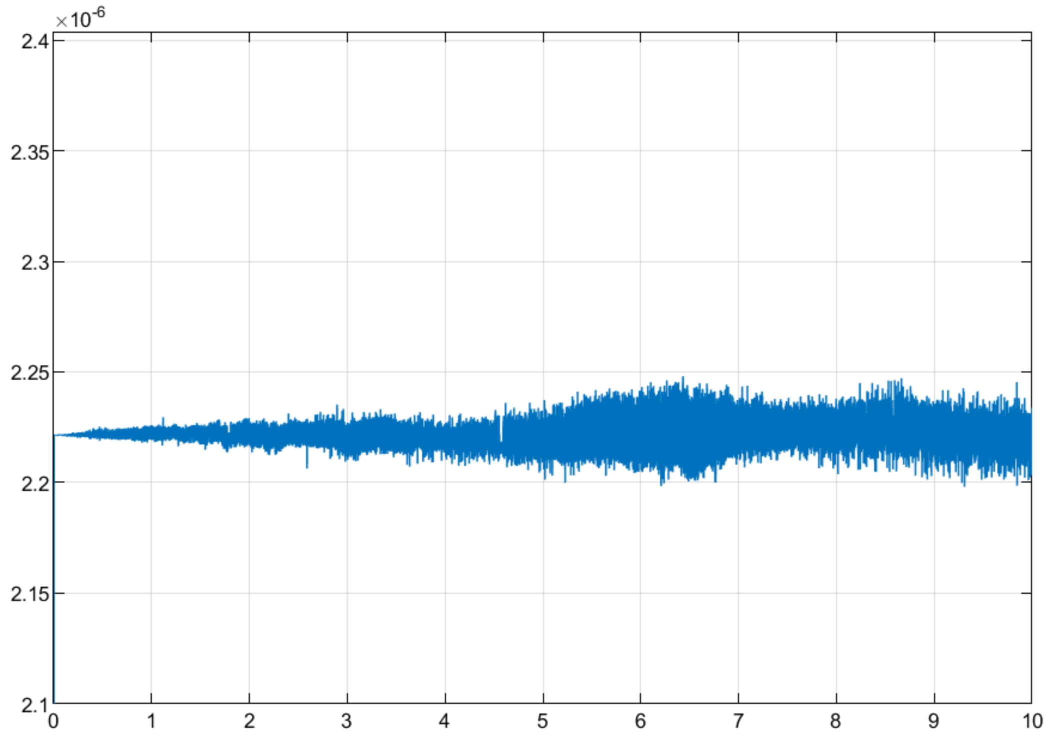

Figure 54. The displacement of bearing A in z direction

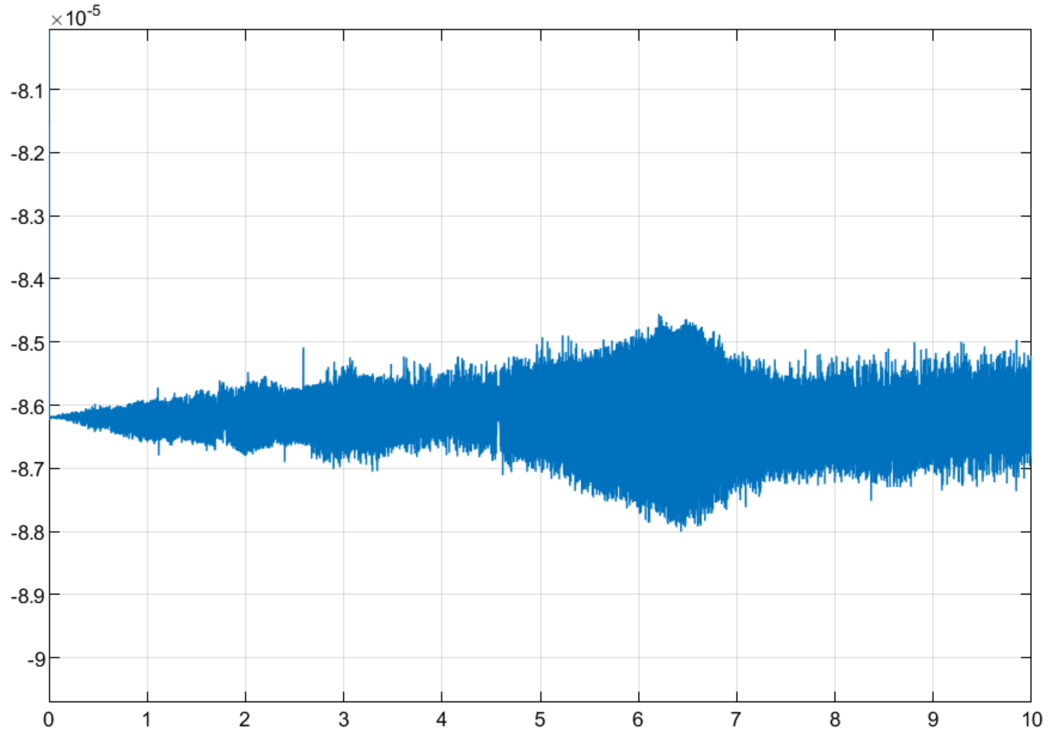

Figure 55. The displacement of bearing B in y direction

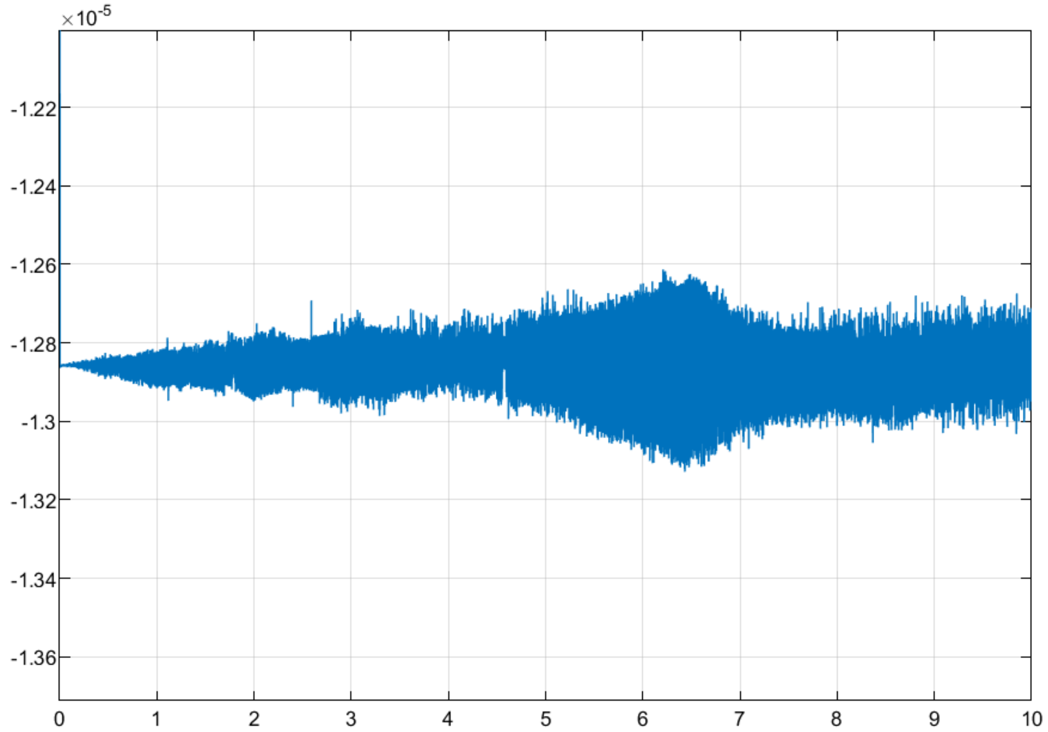

Figure 56. The displacement of bearing B in z direction

And if we observe Figure 47,48, 49, 50, we can see that around 6.5s, the bearing reaction forces reach a maximum value y_{max} and z_{max}, while at 0s, we can obtain the static forces y_{static} and z_{static}. Similarly observing the displacement of the two bearings in the y-direction and z-direction, the maximum displacement u_{max} and v_{max} at 6.5s and the static displacement u_{static} and v_{static} at 0s can be obtained. Comparing the ratio of these two, it can be found that:

$$\frac{y_{max}}{y_{static}} > \frac{u_{max}}{u_{static}}$$

$$\frac{z_{max}}{z_{static}} > \frac{v_{max}}{v_{static}}$$

So the analysis of the force and displacement figures above shows that in this system, the bearings act as a filtering effect. It makes the displacement change not as drastic as the bearing reaction force. So the displacement of the two bearing end points is more stable comparing with bearing reaction force. Moreover, compared with the type1 system, the displacement vibration at both ends of the gear shaft is smaller.

For the Type 3 system:

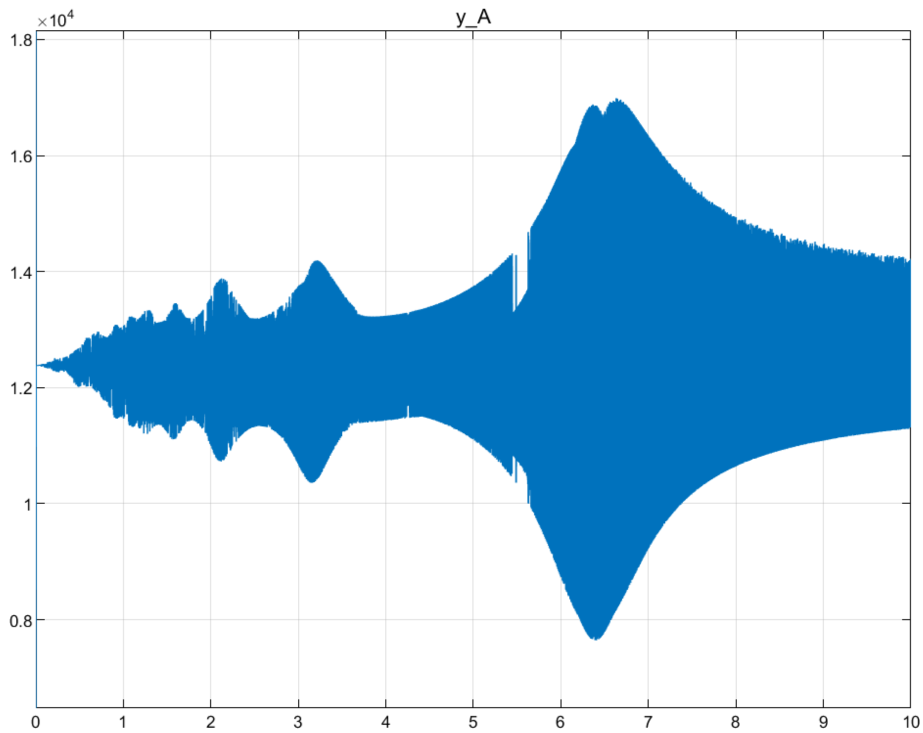

Figure 57. The bearing reaction force of bearing A in y direction

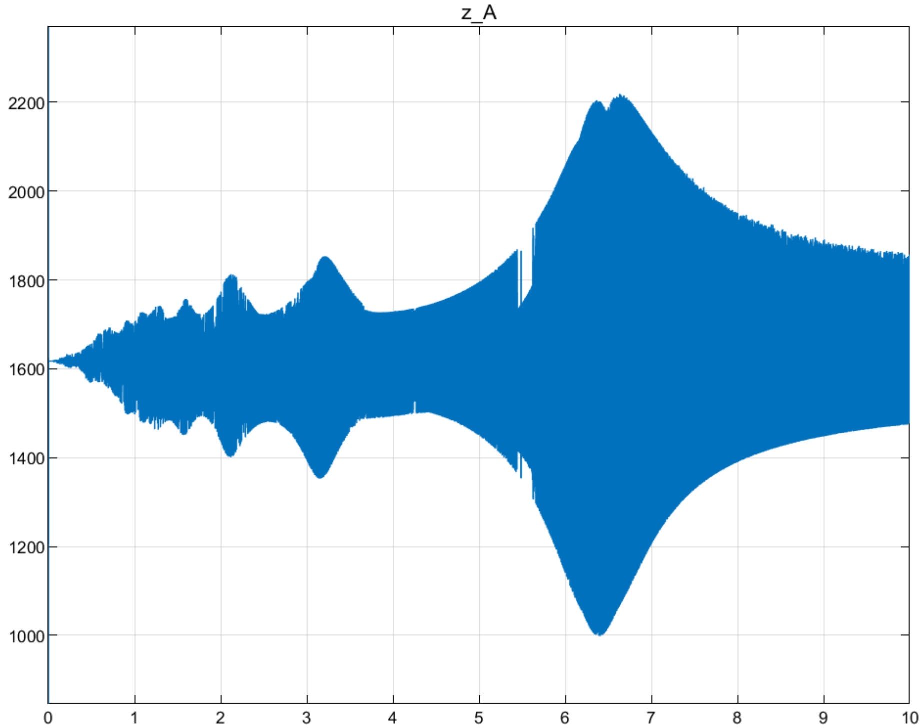

Figure 58. The bearing reaction force of bearing A in z direction

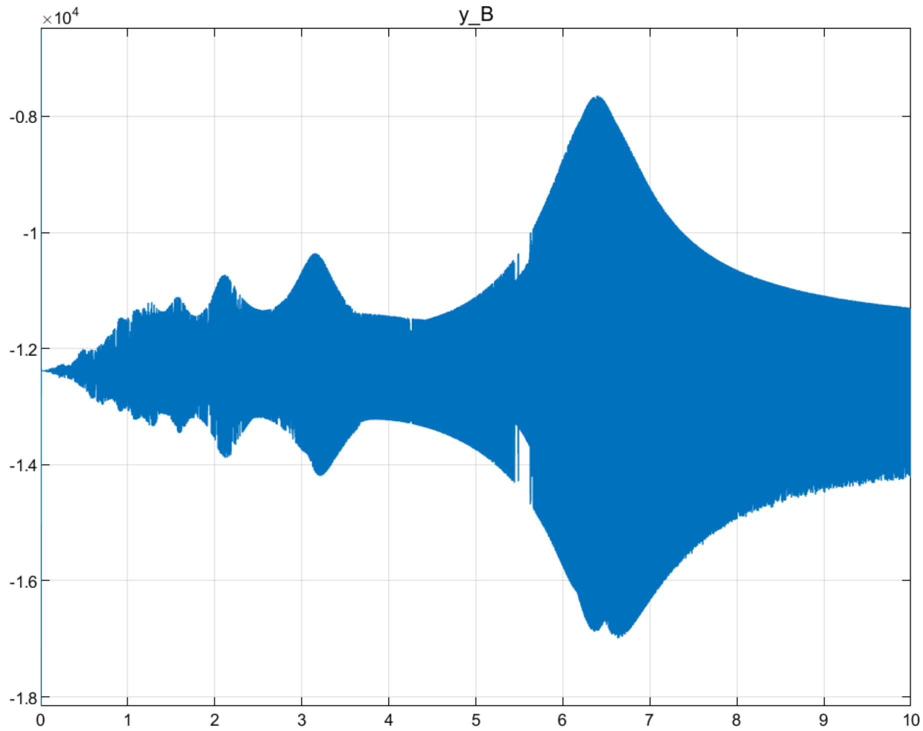

Figure 59. The bearing reaction force of bearing B in y direction

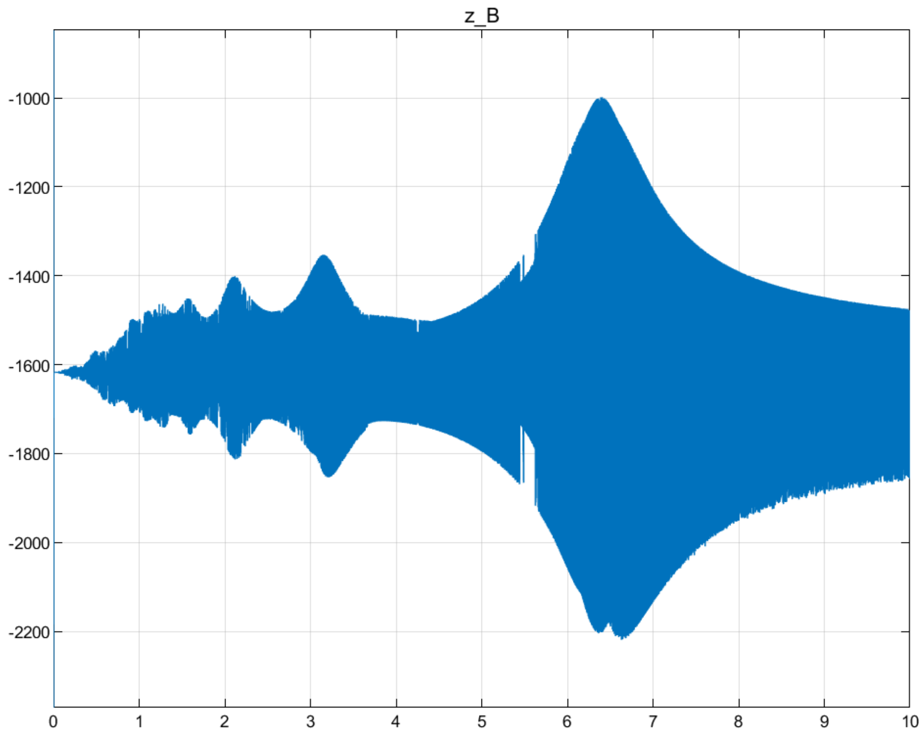

Figure 60. The bearing reaction force of bearing B in z direction

For type 3 system, we have chosen same diameter size gears to mount on the shaft. And the gears are mounted spaced apart from the bearings, which results in almost the same magnitude of force acting on the two bearings, but acting in opposite directions. And that can be observed by above figures.

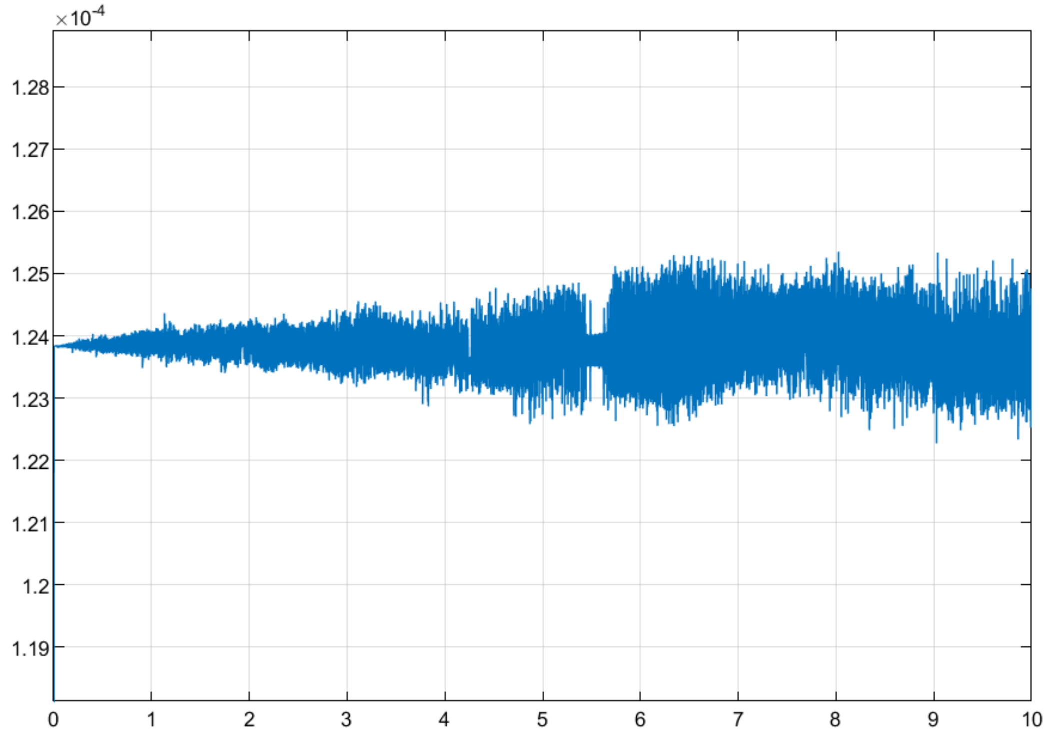

Figure 61. The displacement of bearing A in y direction

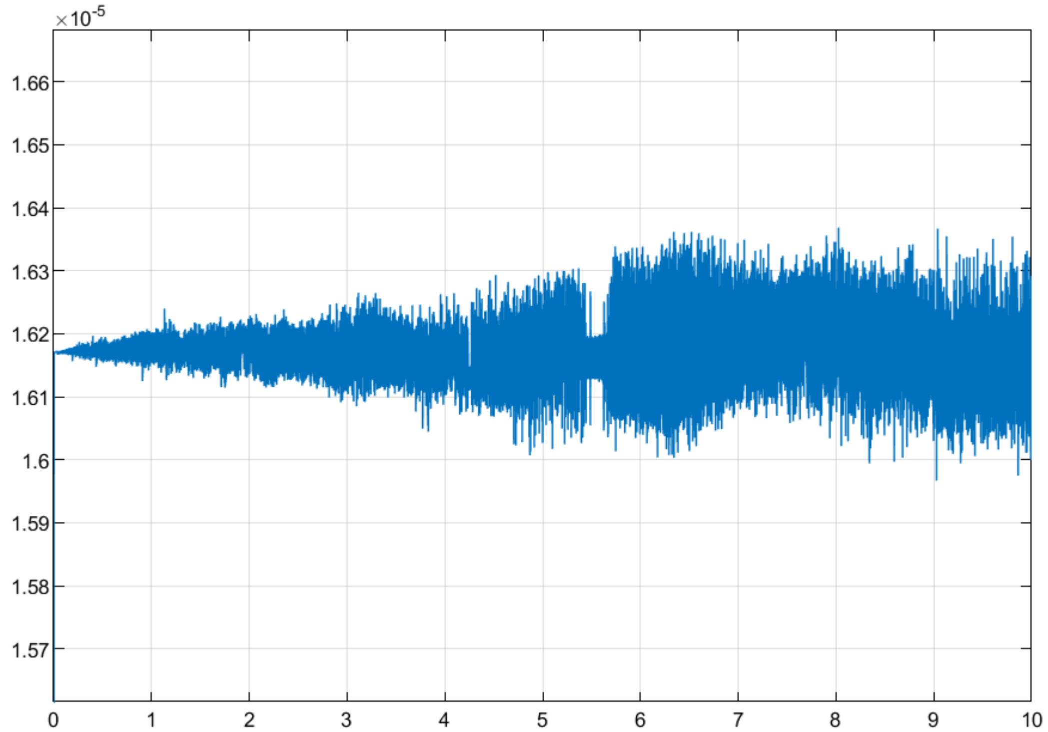

Figure 62. The displacement of bearing A in z direction

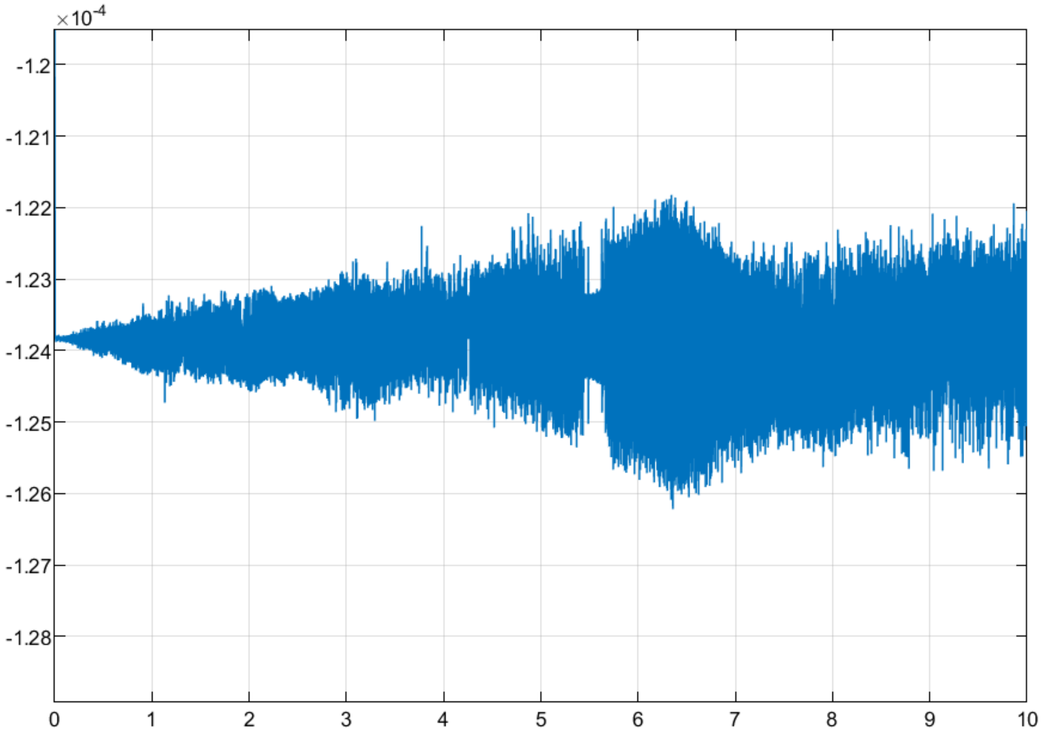

Figure 63. The displacement of bearing B in y direction

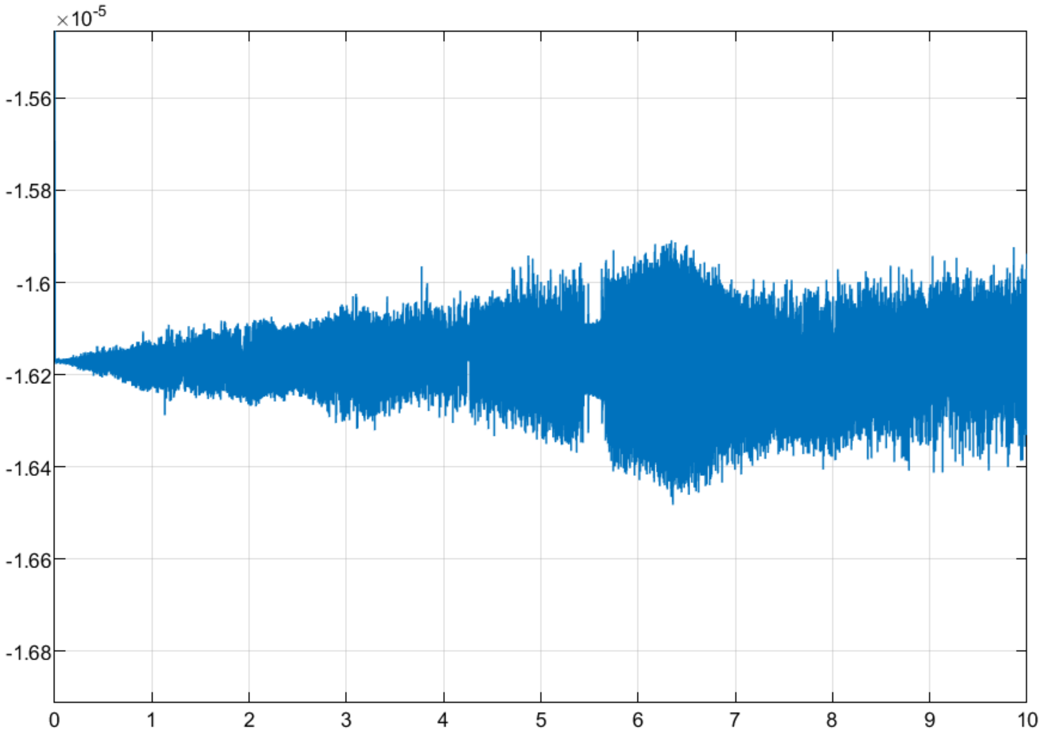

Figure 64. The displacement of bearing B in z direction

Analyzing the above force and displacement diagrams, we can draw the same conclusions with type 2 system.

$$\frac{y_{max}}{y_{static}} > \frac{u_{max}}{u_{static}}$$

$$\frac{z_{max}}{z_{static}} > \frac{v_{max}}{v_{static}}$$

So for type 3 system, both A and B bearings act equally as filters both along the y-direction and along the z-direction. It results the displacement of the gear shaft does not vary much under the effect of larger bearing reaction force, again due to the fact that the bearing filters out the displacement variation caused by the larger bearing reaction force in the whole system.

Then we load DF_500.mat which is constant torque and constant speed, the results are shown below.

For the Type1 system

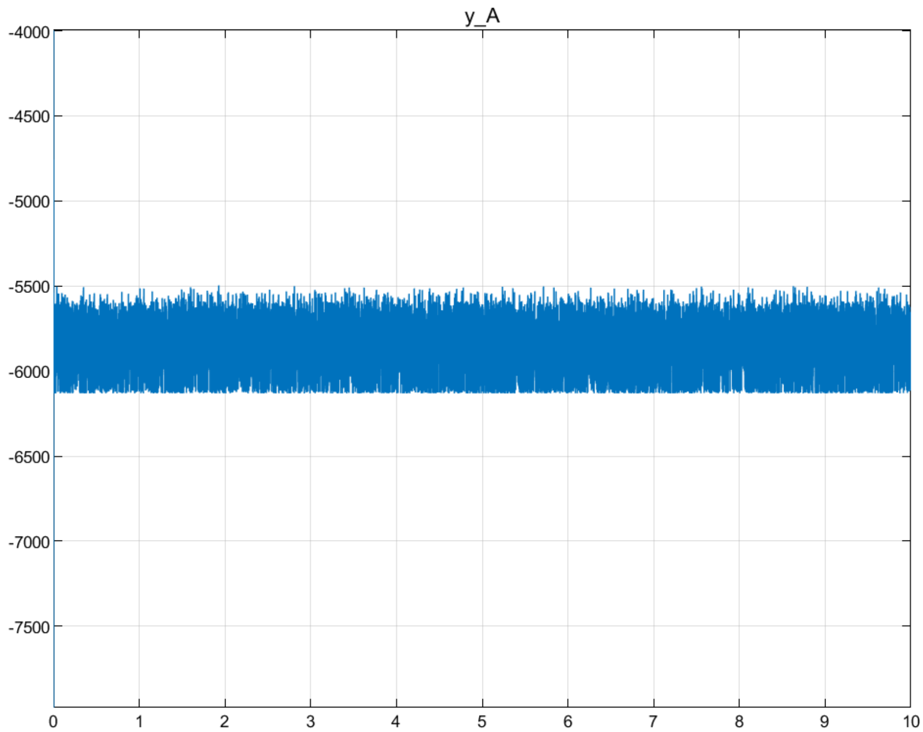

Figure 65. The bearing reaction force of bearing A in y direction

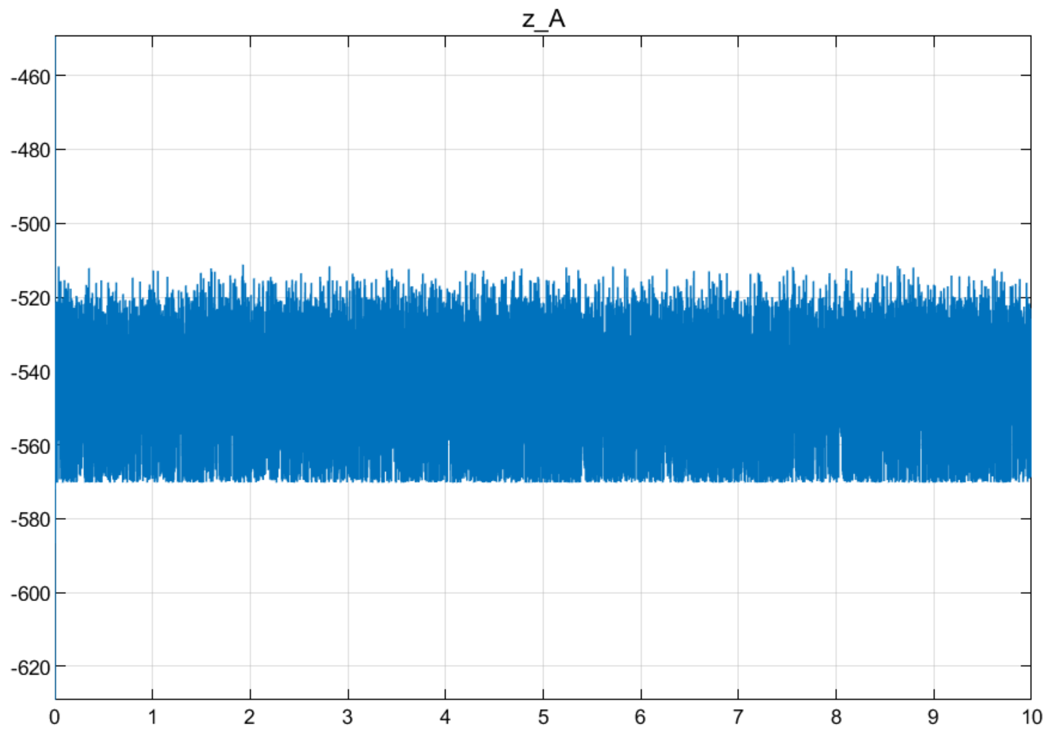

Figure 66. The bearing reaction force of bearing A in z direction

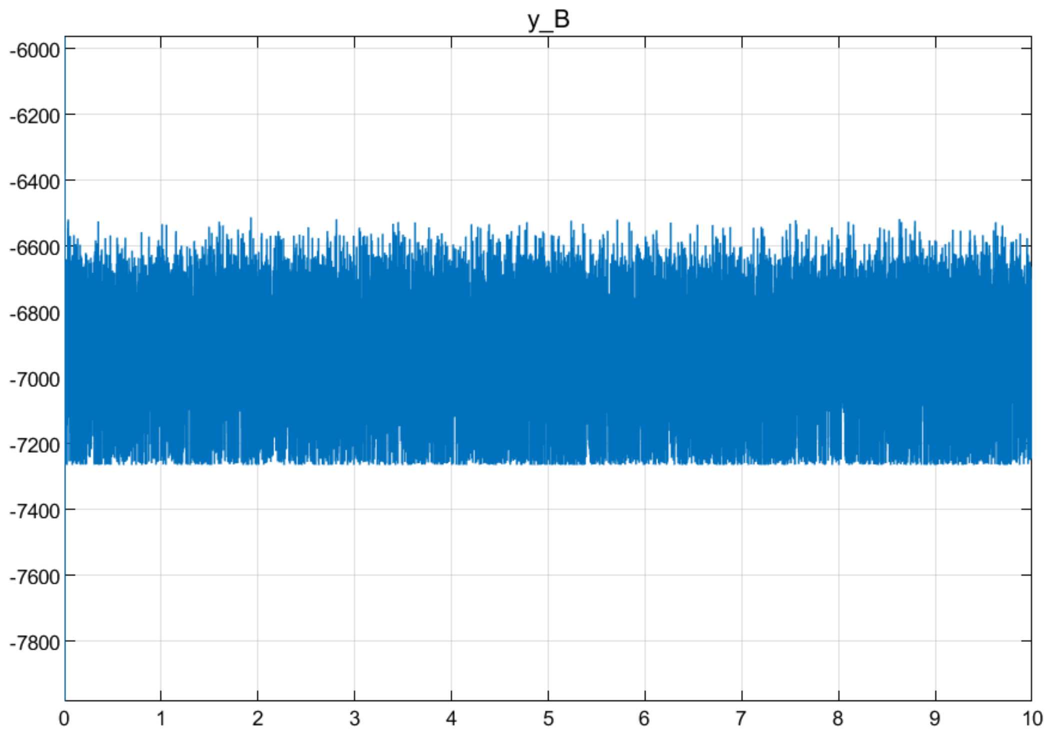

Figure 67. The bearing reaction force of bearing B in y direction

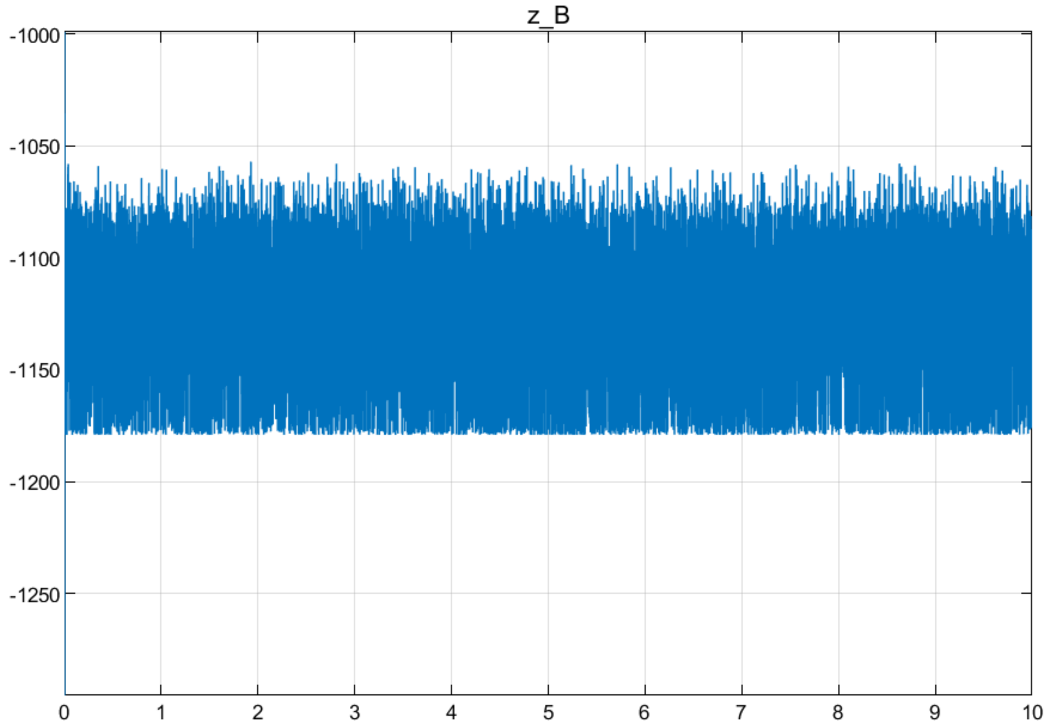

Figure 68. The bearing reaction force of bearing B in z direction

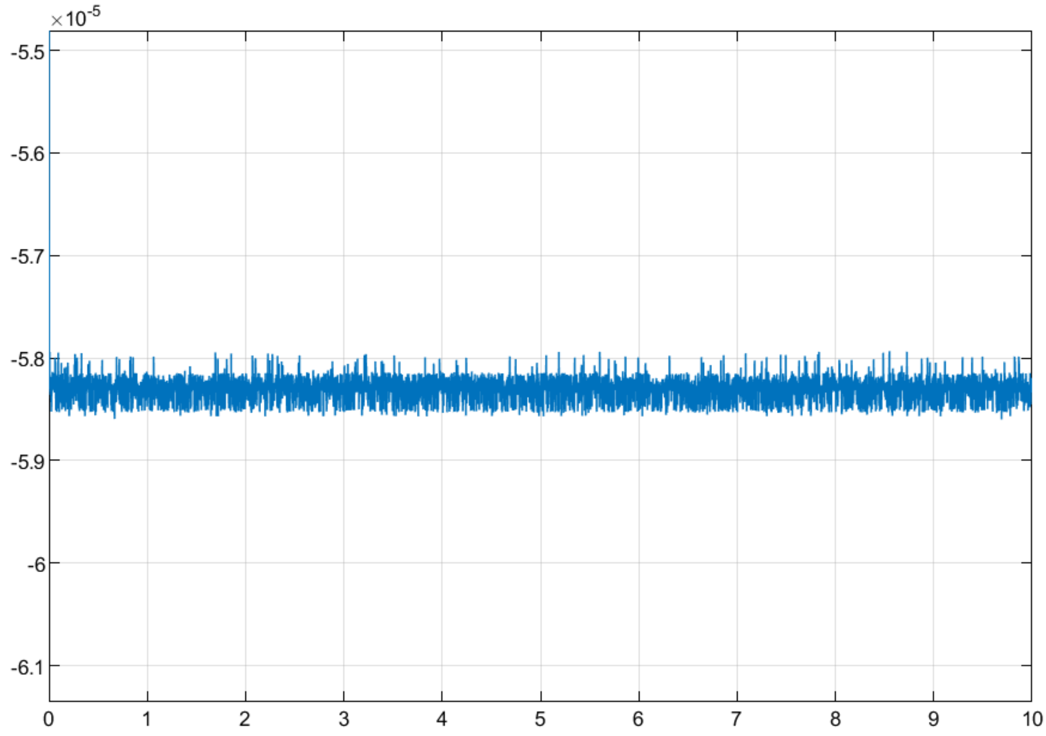

Figure 69. The displacement of bearing A in y direction

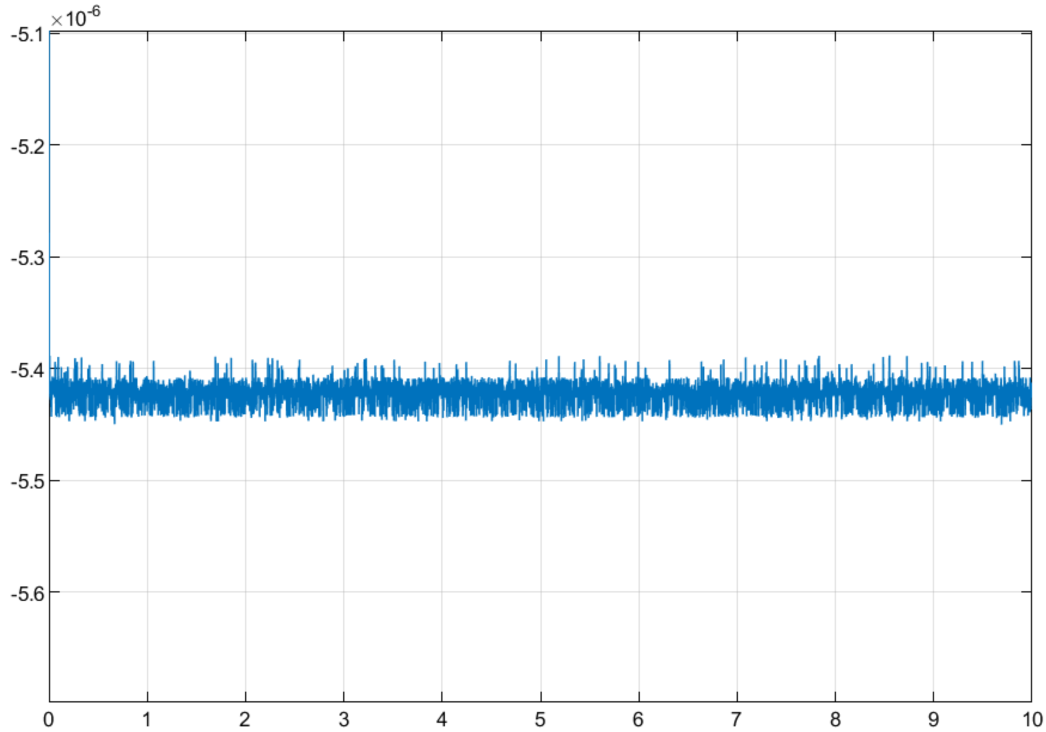

Figure 70. The displacement of bearing A in z direction

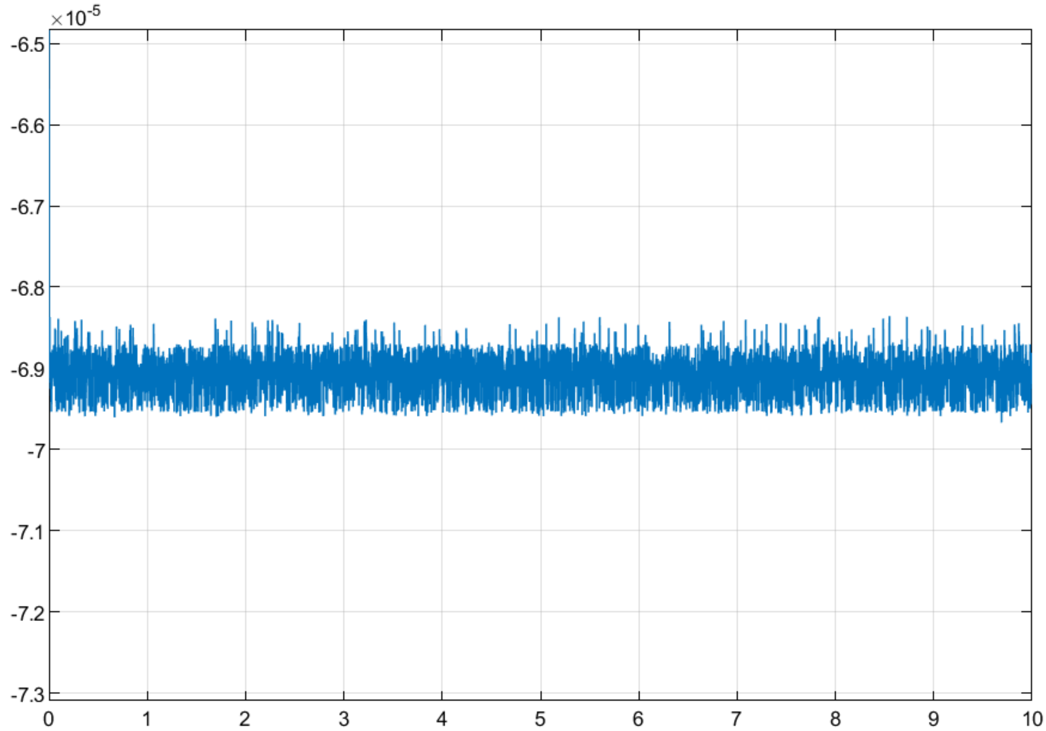

Figure 71. The displacement of bearing B in y direction

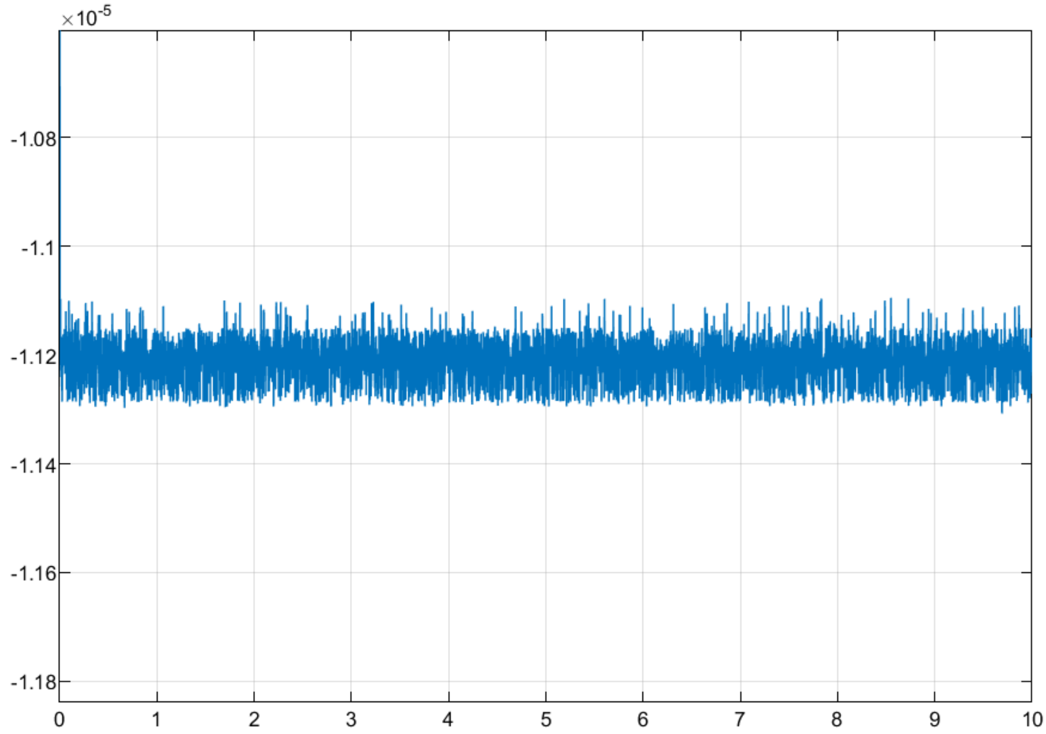

Figure 72. The displacement of bearing B in z direction

For the Type 2 system:

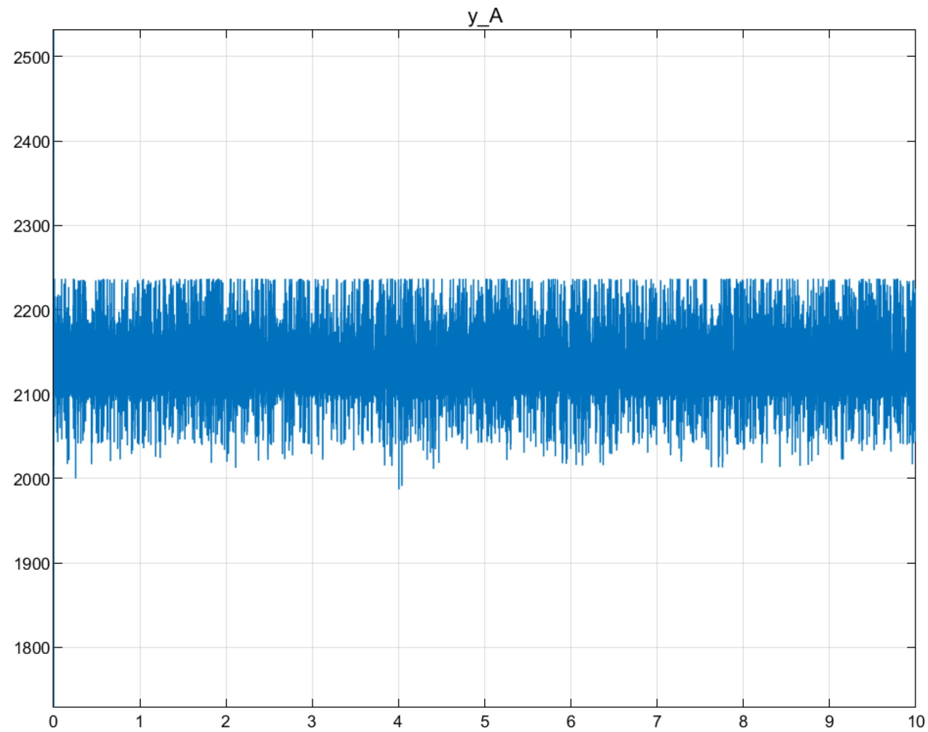

Figure 73. The bearing reaction force of bearing A in y direction

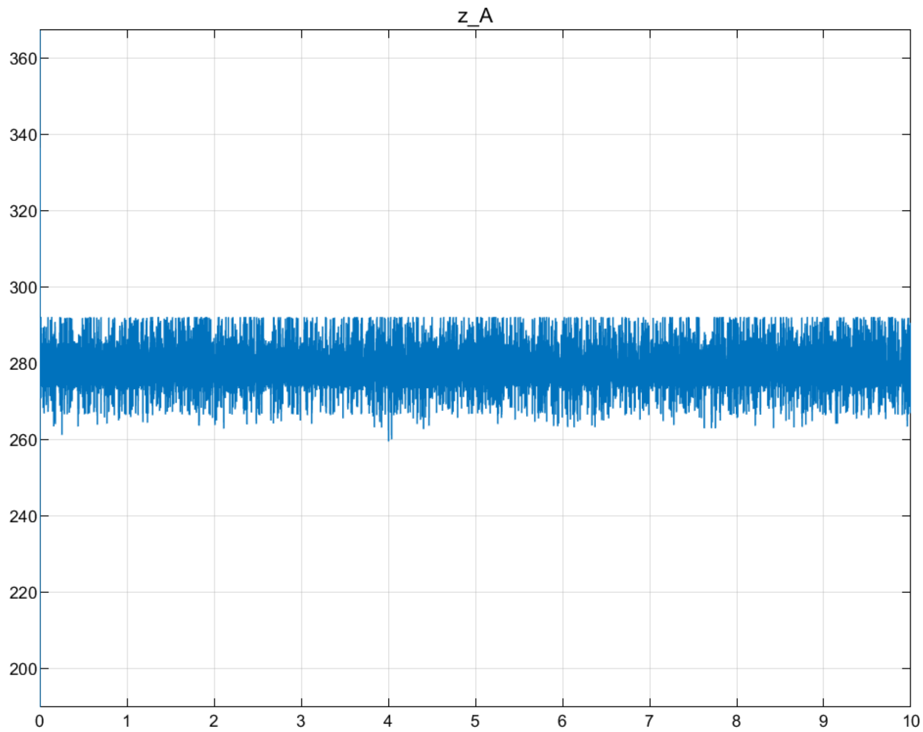

Figure 74. The bearing reaction force of bearing A in z direction

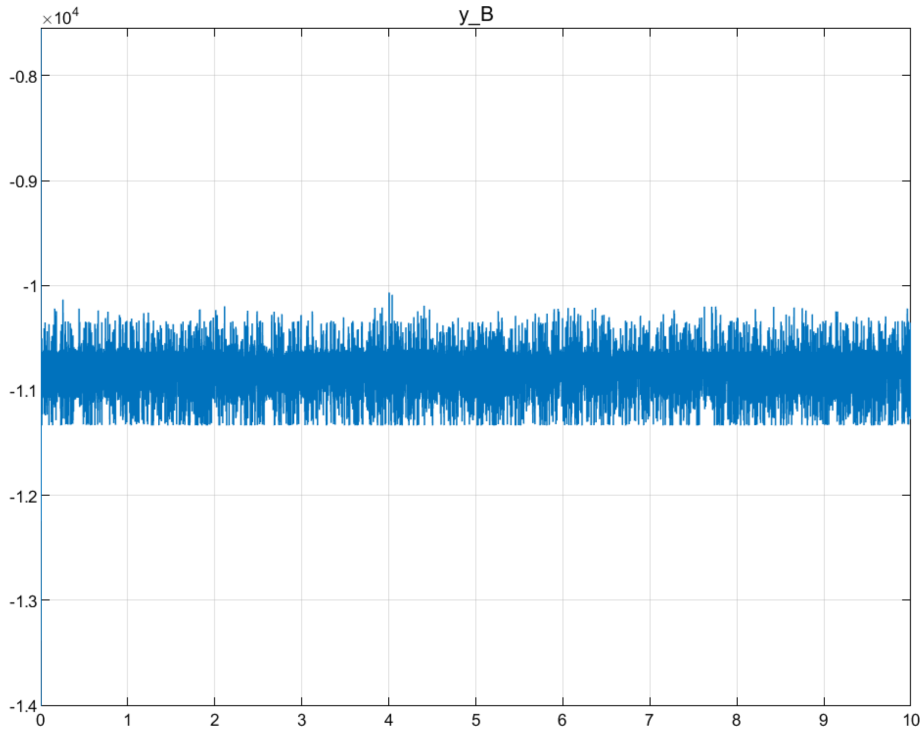

Figure 75. The bearing reaction force of bearing B in y direction

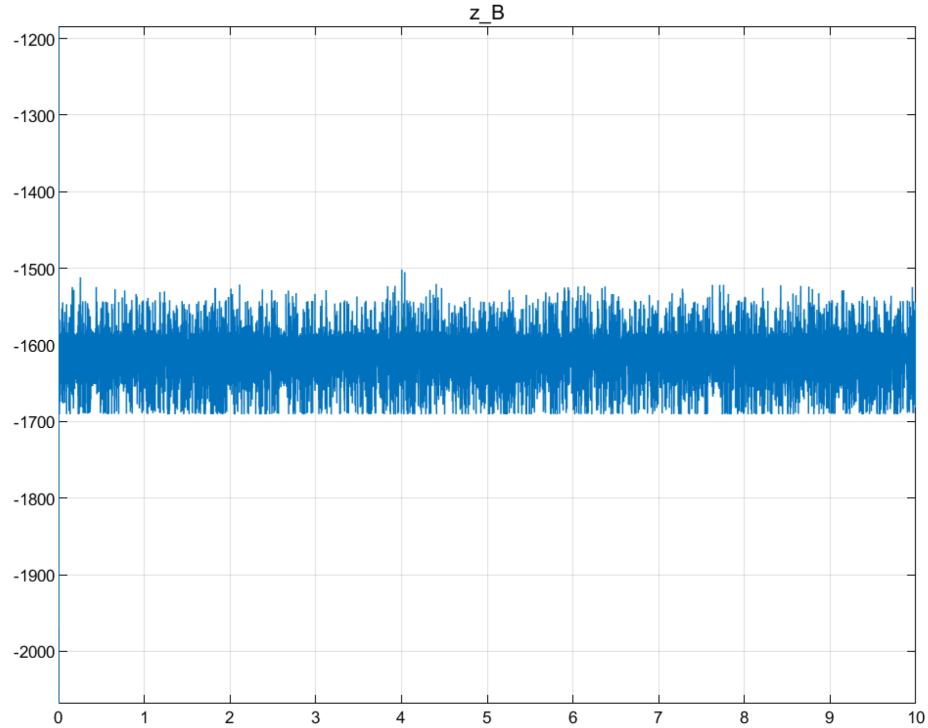

Figure 76. The bearing reaction force of bearing B in z direction

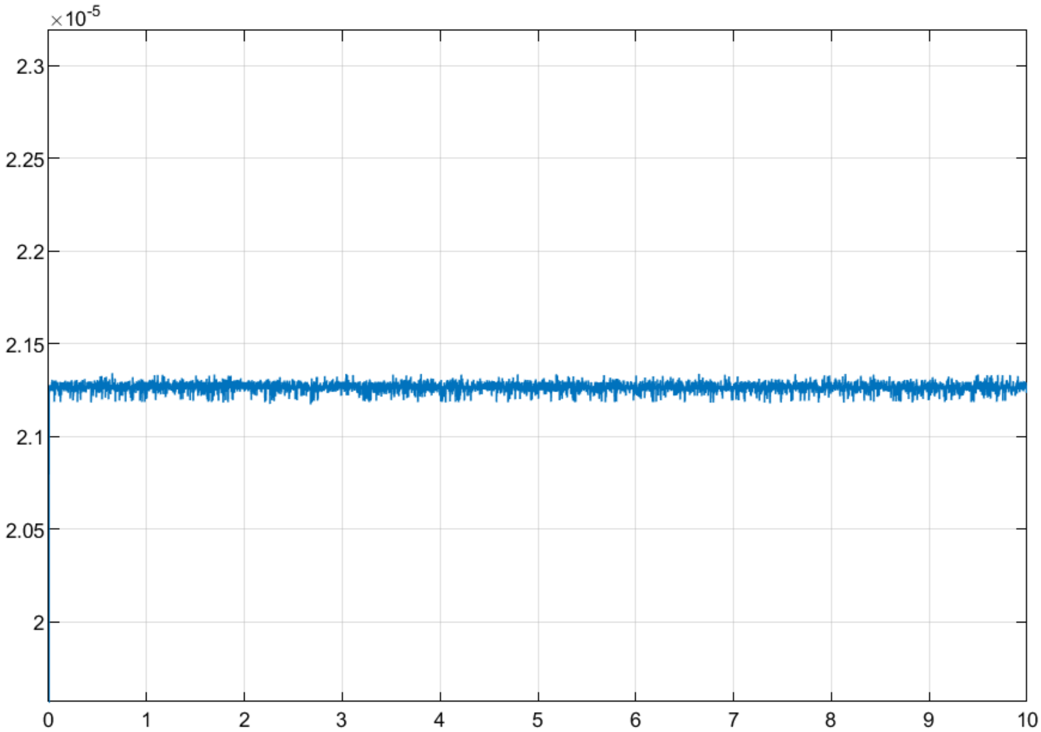

Figure 77. The displacement of bearing A in y direction

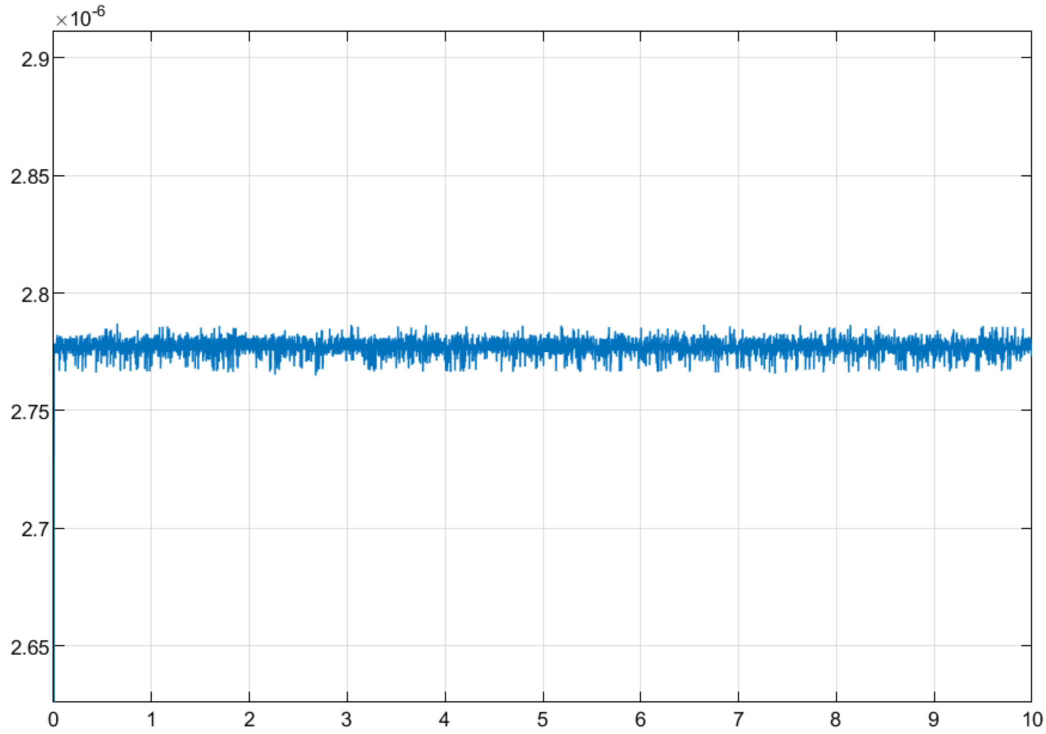

Figure 78.The displacement of bearing A in z direction

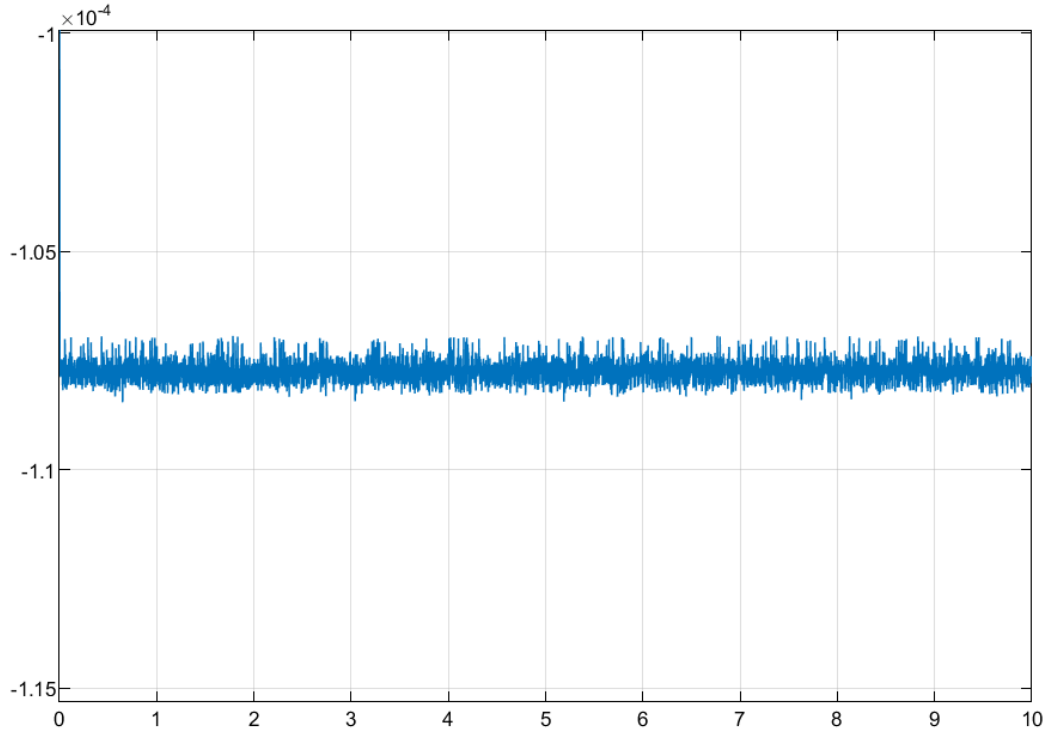

Figure 79. The displacement of bearing B in y direction

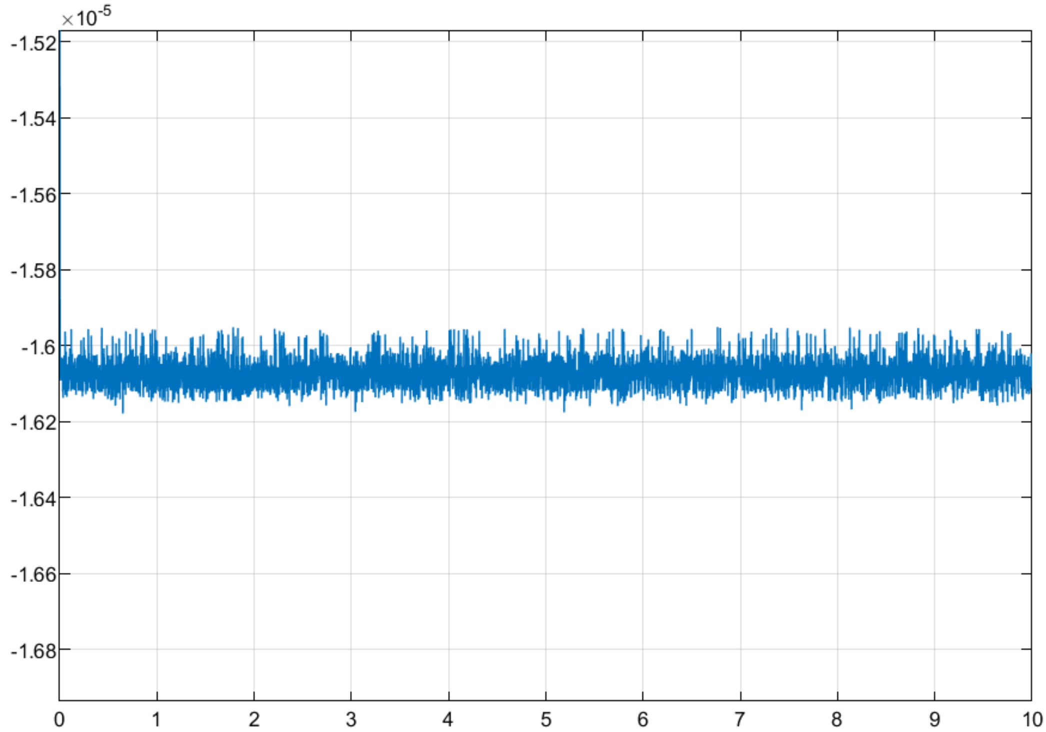

Figure 80. The displacement of bearing B in z direction

For the Type 3 system

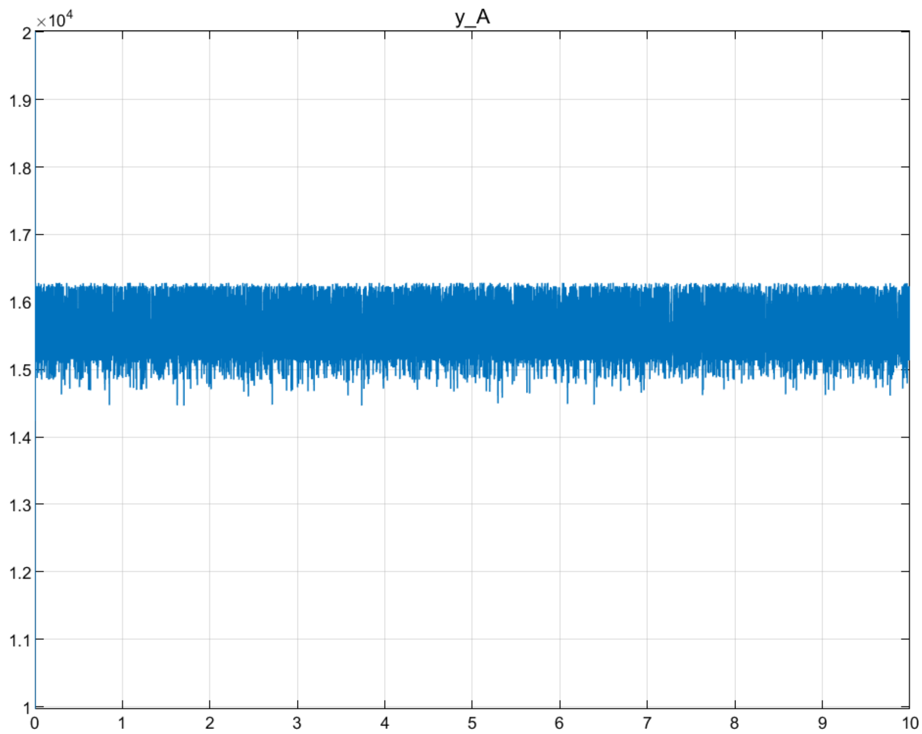

Figure 81. The bearing reaction force of bearing A in y direction

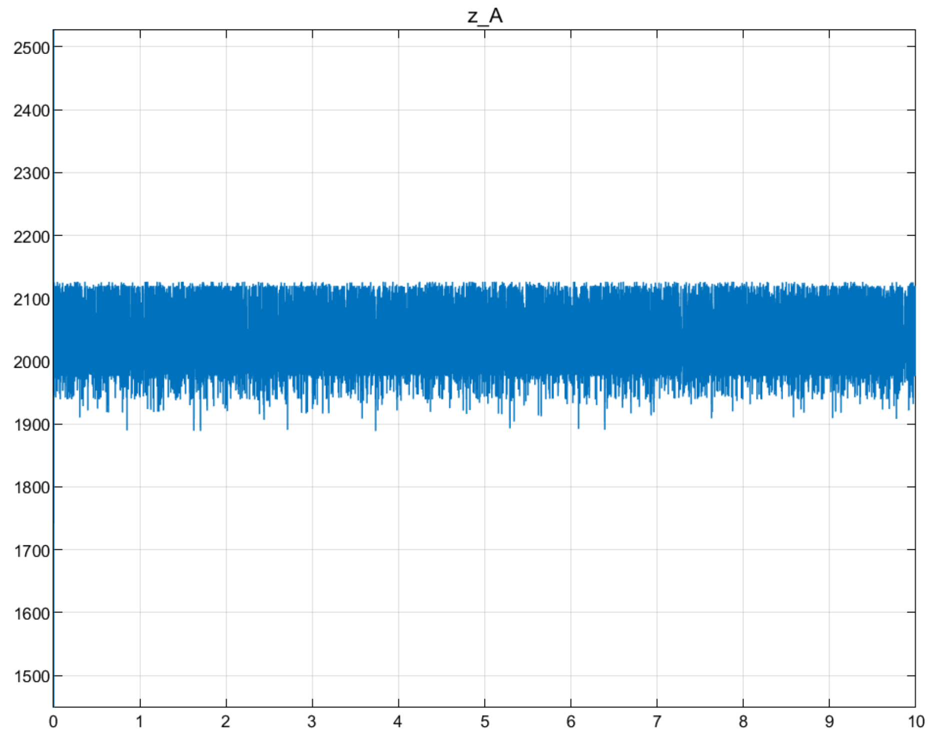

Figure 82. The bearing reaction force of bearing A in z direction

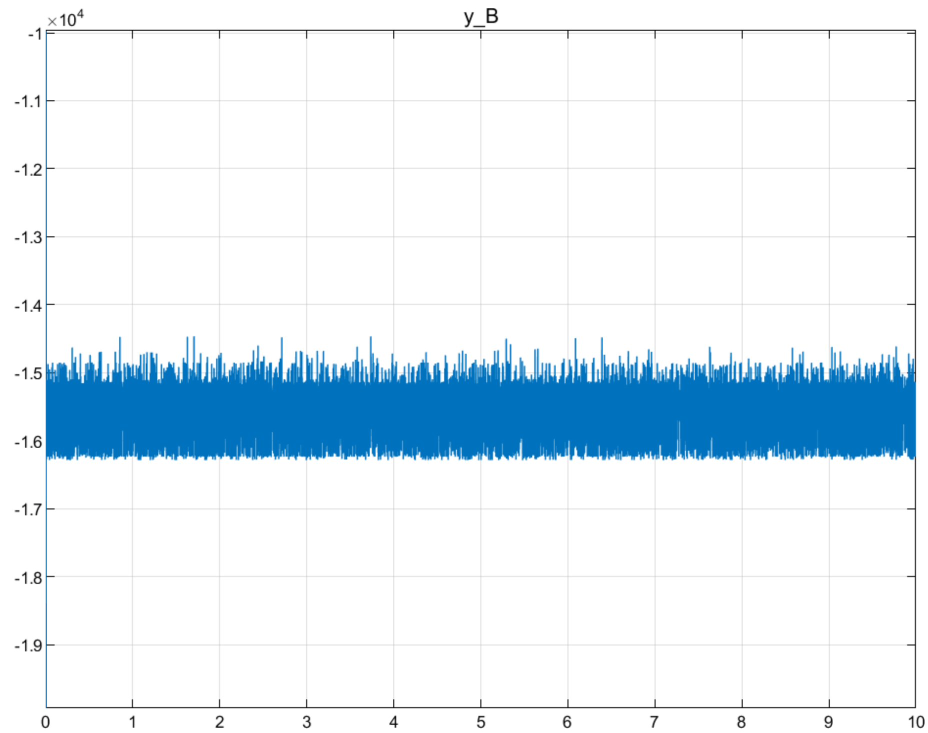

Figure 83. The bearing reaction force of bearing B in y direction

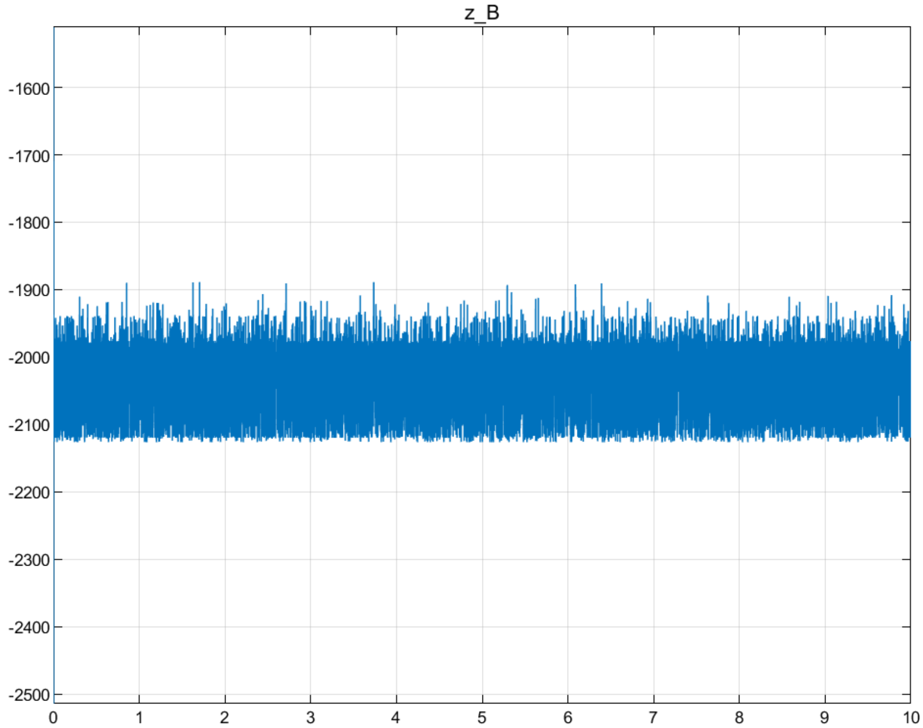

Figure 84. The bearing reaction force of bearing B in z direction

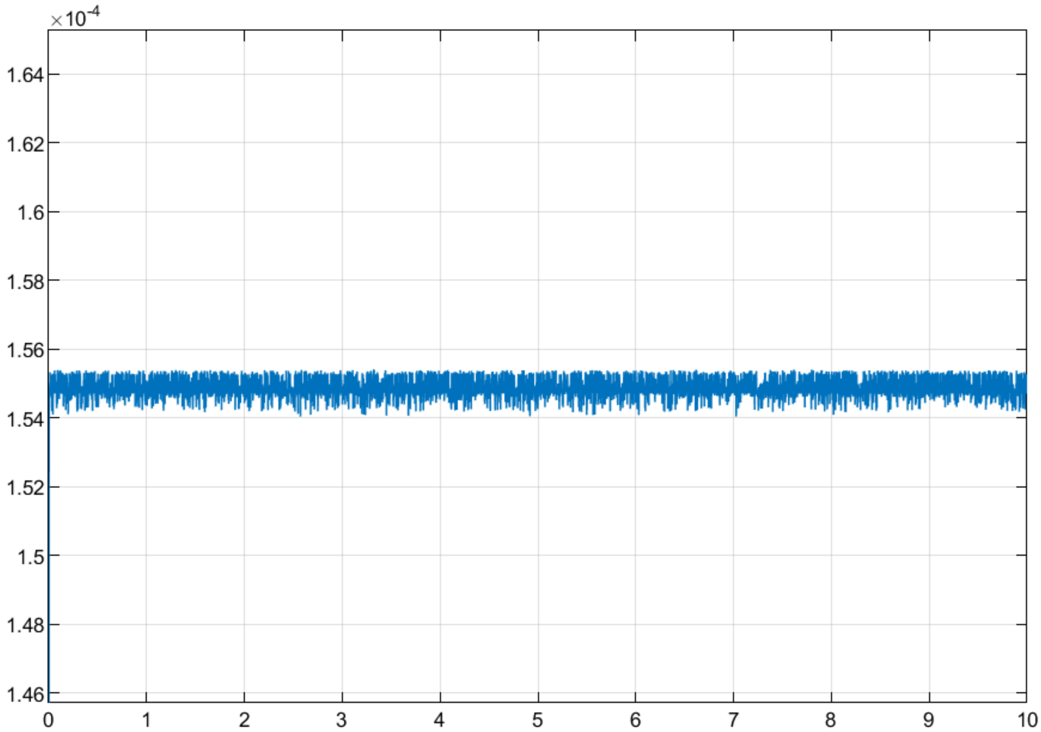

Figure 85. The displacement of bearing A in y direction

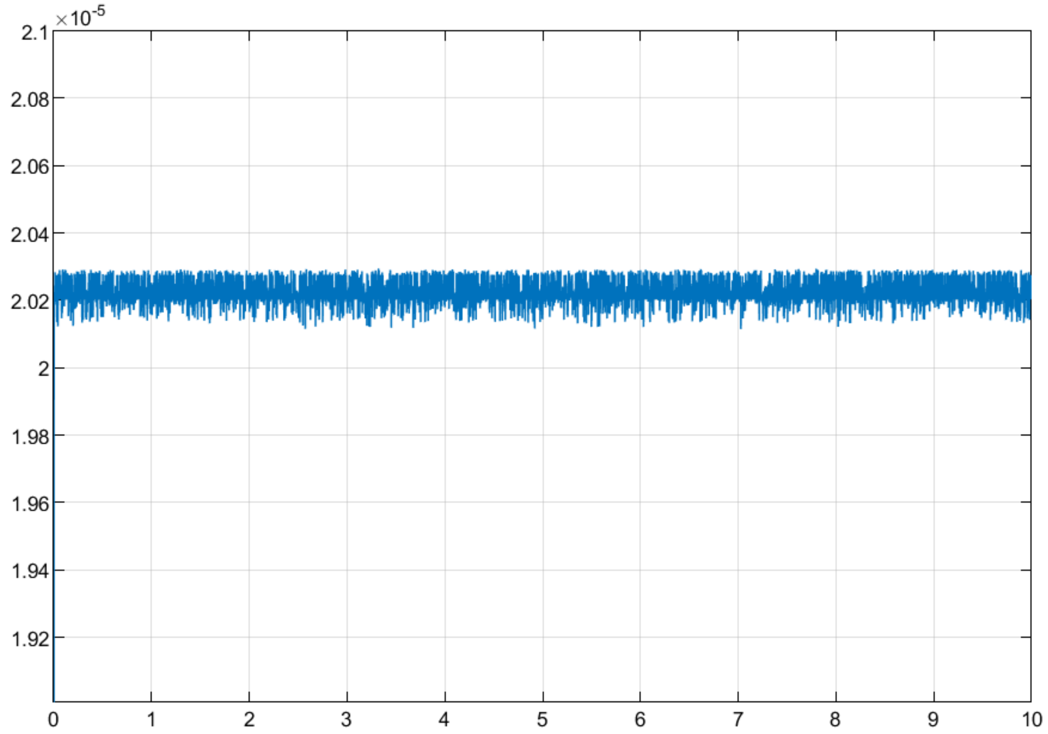

Figure 86. The displacement of bearing A in z direction

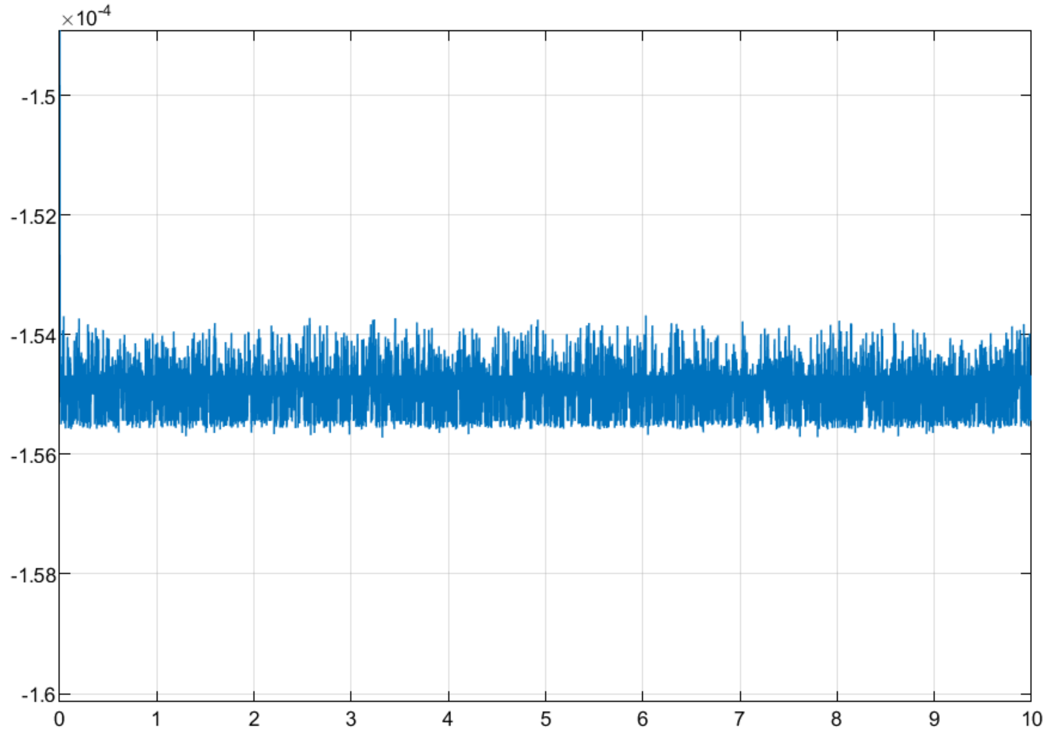

Figure 87. The displacement of bearing B in y direction

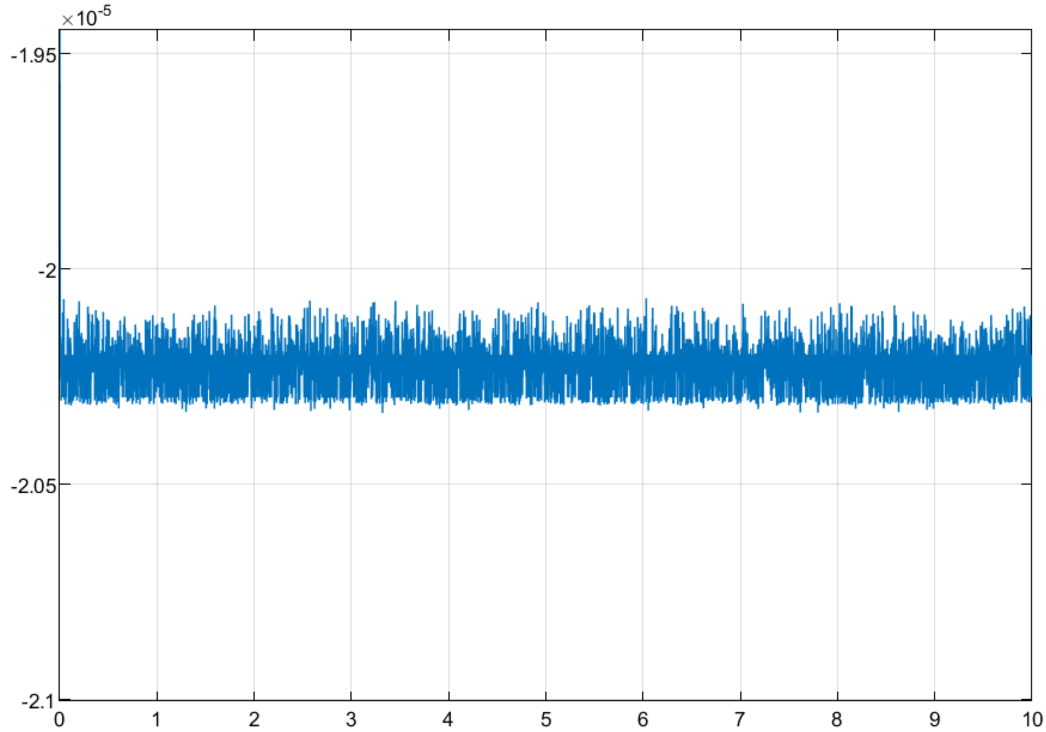

Figure 88. The displacement of bearing B in z direction

Since the input quantities of the engine are constant torque and constant speed, the amount of fluctuation of bearing reaction force and displacement around the bearing reaction force and displacement under static is not large and maintained in a certain range. But we can also carry out the analysis of these systems as in the previous load different speed file, it can be seen that whether type 1, type 2 or type 3 system, the ratio of its maximum bearing force to static bearing force is greater than the ratio of the maximum displacement to the amount of static displacement.

$$\frac{y_{max}}{y_{static}} > \frac{u_{max}}{u_{static}}$$

$$\frac{z_{max}}{z_{static}} > \frac{v_{max}}{v_{static}}$$

So even with a constant speed input, the bearings are equivalent to filters in the whole system.

5. Conclusion

The transmission is an indispensable accessory in the car, at the same time, the driver who seeks driving pleasure on the performance of the transmission and quality requirements are also increasing. In the transmission, the gear-shaft-bearing system, as a key characteristic that directly affects the transmission power transmission performance and vehicle NVH, reflects the core competence of transmission design and manufacturing technology. Based on the force analysis, the transmission quality of the gearbox in working condition is analyzed and studied. The preliminary assessment of the mechanical properties of the system was carried out by means of modeling in software, and the process and conclusions of this study are summarized as follows.

(1) A systematic description of the basic structure and working principle of the automotive gearbox was made. The basic structure of the gearbox, including the transmission mechanism and the gear control mechanism were analyzed, and the working principle of a three-shaft five-speed gearbox was illustrated as an example.

(2) The dynamics model of the gear transmission system was established. Based on the analysis of the basic structure of the automotive gearbox and a lot of literature reading, the internal forces of the gear-shaft-bearing system are analyzed, including the dynamic meshing forces of the gears, on which the dynamics model of gear-shaft-bearing system is established and the dynamics equations of each dynamics model are listed. This provides the basis for the analysis and dynamic response solution of the gear-shaft-bearing system.

(3) Using the combination of MATLAB software and Simulink model, the simulation of different systems under the dynamic meshing forces are implemented to analyze the mechanical properties of each load-bearing part in the working condition and the stress-strain state of the system.

(4) Whether the input is constant torque and constant speed or constant torque and different speed, the bearing acts as a filter in both the y and z directions, resulting in no direct relationship between the displacement change and the bearing reaction force. Therefore, the displacement amplitude is reduced and the vibration is also reduced.

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