Breakage of stay cables due to the impact of vehicles

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Abstract

Due to the dynamic environment in which cable-stayed bridges work, sudden failure of stay cables are likely to happen for diverse reasons. The more rapid is the failure the more severe will be the effect on the bridge, risking not just the breakage of one or more elements due to a zipper-type collapse, but of the whole system. The impact of a vehicle is the event that most likely is able to produce such a consequence and the regulations only consider a possible loss of just one element. For this reason in the following study the impact of a heavy vehicle is considered in order to see how a single cable would respond or how a system of cables would respond, if more than one is affected. a combination of an energy-based approach and a step by step solution is used to define the deformation and dynamic response and a sliding and no-sliding models are used for the interaction between vehicle and cable.
Keywords: stay cable, impact, vehicle, energy, deformation

1. Introduction

Cable-stayed bridges became popular due to the fact that they are perfectly suitable in situations that are in between the common suspension bridges, where the span to cover would not be long enough to be economically advantageous and cantilever bridges, where the span would be too long and the self-weight unsustainable. In this disposition there is a direct connection between the deck and the pylons with the stay cables, positioned in different possible configurations.
The most common materials used for these types of bridges are mostly two combined together, steel or concrete for pylons, post-tensioned concrete deck and steel cables, composed by several strands with high performance characteristics put together in helical shapes.

The origin of cable-stayed bridges goes back to 1595, where the Italian inventor Fausto Veranzio in his book *Machinae Novae* presented the first design attempt, in which stay cables were constructed by linked rods because there was not an industrial production yet of heavy structural cables. After this the first notable projects started in the 19th century, just to cite some of them the Albert Bridge (1872), Brooklyn Bridge (1883) and Bluff Dale bridge (Texas, 1890). The problem with these early bridges was that the static and dynamic concept was not well understood yet, and the impossibility to pre-tension the cables gave a lack in stiffness to the system, with consequent inadequate resistance to wind and vehicle induced vibrations.
The design improved in the 20th century where just after World War II, the Architect Elizabeth Mock presented a book where historical and aesthetic analysis were coupled together and all

the best-known bridges up to 1949 were described. The same year saw another publication that marked the future design of cable-stayed bridges with the Engineer Frank Dischinger, presenting a pioneering theory on how the cable theory could be used for design of cables. With his new theory new bridges started to improve both structurally and aesthetically, reaching spans of almost a 1000 m using new materials with high strength and new construction technologies, like the Skarnsund Bridge (Norway, 530m), the Pont de Normandie (France, 856m) and in the most recent days the Edong Yangtze River Bridge (China, 926m).

Due to their dimensions, complexity and importance from the safety point of view, each component is studied in terms of vulnerability and resistance to abrupt events like fire, blast, corrosion and especially impact from vehicles and the consequent dynamic response. One delicate issue is the case where one element is loss or absent momentarily due to these extreme events or maintenance and some regulations tried to manage these scenarios with some guidelines, considering for example a Dynamic Amplification Factor (DAF) to be applied to the intact components that should not have any repercussions deriving from the single failure. Some studies highlighted that the DAF of 2.0 generally taken by the regulations in some cases could not be enough to cover every possible scenario (Ruiz-Teran and Aparicio, 2006; Mozos & Aparicio, 2010). The main reason is that the DAF is strongly dependant on the position and type of the abrupt event, so it is not easy to generalise a single value to be always in a safe condition (Hoang, Kiyomiya & An, 2018).

This paper should be seen as an integration and improvement of the present studies on the abrupt loss of a stay cable due to a sudden impact of a vehicle, trying to have a better understanding of the response of a single or multiple cables to the impulsive force and the repercussions on the structure.

2. Objectives of the analysis

The main objectives of the following research are:

- To analyse the current studies and regulations to see what type of requirements exist;
- To analyse the response of a generic cable to the impact of a mass representing a vehicle, with variation of its geometrical and mechanical parameters;
- To analyse the influence of elastic and plastic response of the cable during the breakage event;
- To analyse how several cables could be affected;
- To define if one or more cables have to be considered as lost after the impact.

3. Research gap analysis

The five different regulations presenting guidelines for an impacting force on a bridge structure and its response (PTI, FIB, SETRA, EC1, EC3) consider just the effect of the loss of one cable or its temporary absence. So far there are no documented studies in which the reliability analysis of long-span cable-stayed bridges, subjected to sudden cables failure, has been conducted considering the dynamic excitations from wind and traffic and further an impact (Zhou & Chen, 2016).

4. Literature review

4.1. Regulations and guidelines for cable(s) loss

A cable(s) loss in cable-supported structures, mostly in cable-stayed bridges, is the results of extreme events like blast, fire or the impact of a vehicle. Different entities during the years have incorporated guidelines like the American Post-Tensioning Institute (PTI), Eurocode 1 and Eurocode 3, the French regulations Service d'Etudes Techniques des Routes et Autoroutes (SETRA) and the Fédération Internationale du Béton (FIB).

4.1.1. Post-Tensioning Institute (PTI) guidelines

The PTI D-45.1-12 2012 states that "*The impact dynamic force resulting from the sudden rupture of a cable shall be 2.0 times the static force in the cable, or the force as determined by non-linear dynamic analysis of a sudden cable rupture, but in no case less than 1.5 times the static force in the cable. This force shall be applied at both the top and bottom anchorage locations.*".

The guideline also considers a nominal yield strength equal to 90% of the ultimate strength.
The impact dynamic force stated above is calculated with the factor of 2 generally called Impact force factor (Kiviluoma et al,2015), it can be seen in Figure 1.It is defined as:

$$IF = \frac{\Delta N}{N_0} = \frac{|N_0 - N_{min}|}{N_0} \tag{1}$$

Where N_0 is the initial force in the cable, as soon as the cable breaks it reaches a minimum and begin to vibrate and at the end the tension goes to zero.
Note: *If a non-linear dynamic analysis is used, the dynamic model should be initialised with full permanent load and live load condition for the bridge.*

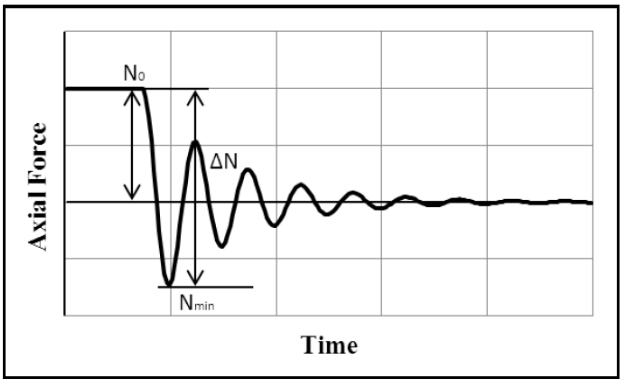

Figure 1- change in force after the breakage

4.1.2. Fédération Internationale du Béton (FIB) guidelines

The failure of one single stay cable should not lead to immediate failure of the entire cable-stayed structure. The Designer should take into account in his design accidental breakage of any one stay cable in the structure including the dynamic effects caused by the failure. Generally, redundant stay cable systems, i.e. systems consisting of multiple parallel tensile elements, are preferred to cables consisting of a single tensile element.

4.1.3. Service d'Etudes Techniques des Routes et Autoroutes (SETRA) guidelines

The SETRA entity gives its recommendations through the publication "Haubans - Recommandations de la commission interministérielle de la précontrainte".

Particular attention should be taken for the accidental action of the breakage of any stay cable, considering only one cable at the time. This rupture is represented by a force opposite to the tension of the shroud, exercised at its two anchors, and weighted by a dynamic amplification factor defined between 1.5 and 2.0. The dynamic amplification coefficient depends on the nature of the rupture (vehicle impact, corrosion of reinforcements, etc.) as well as the dynamic response of the structure. A coefficient value of 2.0 is a particularly severe enveloping value, corresponding to the unlikely case of a sudden break in the entire cable. For stay-cables with independent parallel frames, the simultaneous breaking of all the reinforcements being unlikely, the amplification factor can be reduced to 1.5. Local and global checks for the breakage of the shroud are carried out at the SLU, in combination of the effect of the breakdown, the working values are taken with their multiplicative coefficients.

4.1.4. Eurocode 1 - Actions on structures - Part 1-7: General actions - Accidental actions

This section defines accidental actions due to the following events:

- *impact from road vehicles (excluding collisions on lightweight structures);*
- *impact from forklift trucks;*
- *impact from trains (excluding collisions on lightweight structures);*
- *impact from ships;*
- *the hard landing of helicopters on roofs.*

For bridges, the actions due to impact and the mitigating measures provided should take into account, amongst other things, the type of traffic on and under the bridge and the consequences of the impact.

Actions due to impact should be determined by a dynamic analysis or represented by an equivalent static force.

NOTE 1: The forces at the interface of the impacting object and the structure depend on their interaction.

NOTE 2: The basic variables for impact analysis are the impact velocity of the impacting object and the mass distribution, deformation behaviour and damping characteristics of both the impacting object and the structure. Other factors such as the angle of impact, the construction of the impacting object and movement of the impacting object after collision may also be relevant.

For structural design the actions due to impact may be represented by an equivalent static force giving the equivalent action effects in the structure. This simplified model may be used for the verification of static equilibrium, for strength verifications and for the determination of deformations of the impacted structure.

For structures which are designed to absorb impact energy by elastic-plastic deformations of members (Le. soft impact), the equivalent static loads may be determined by taking into account both plastic strength and the deformation capacity of such members.

Impact is an interaction phenomenon between a moving object and a structure, in which the kinetic energy of the object is suddenly transformed into energy of deformation. To find the dynamic interaction forces, the mechanical properties of both the object and the structure should be determined. Static equivalent forces are commonly used in design.

Advanced design of structures to sustain actions due to impact may include explicitly one or several of the following aspects:

- *dynamic effects;*
- *non-linear material behaviour.*

Within the structure these forces may give rise to dynamic effects. An upper bound for these effects can be determined if the structure is assumed to respond elastically and the load is realised as a step function (i.e. a function that rises immediately to its final value and then stays constant at that value). In that case the dynamic amplification factor (i.e. the ratio between dynamic and static response) φ_{dyn} is 2.0. If the pulse nature of the load (i.e. its limited time of application) needs to be taken into account, calculations will lead to amplification factors φ_{dyn} ranging from below 1,0 up to 1,8 depending on the dynamic characteristics of the structure and the object. In general, it is recommended to use a direct dynamic analysis to determine φ_{dyn}.
In particular cases, when specific information is available, different design values may be chosen, depending on the target safety, the traffic intensity and the accident frequency.
In the absence of a dynamic analysis, the dynamic amplification factor for the elastic response may be assumed to be equal to 1,4.

4.1.5. Eurocode 3 - Design of steel structures - Part 1-11: Design of structures with tension components

The replacement of at least one tension component should be taken into account in the design as a transient design situation.
NOTE: The National Annex may define the transient loading conditions and partial factors for replacement.
Where required, a sudden loss of anyone tension component should be taken into account in the design as an accidental design situation.
NOTE 1: The National Annex may define where such an accidental design situation should apply and also give the protection requirements and loading conditions, e.g. for hangers of bridges.
NOTE 2: In the absence of a rigorous analysis the dynamic effect of a sudden removal may conservatively be allowed for by using the additional action effect E_d:

$$E_d = k \cdot (E_{d2} - E_{d1}) \tag{2}$$

Where:

- *k is 1,5*
- *E_{d1} represents the design effects with all cables intact;*
- *E_{d2} represents the design effects with the relevant cable removed.*

4.1.6. Eurocode 3 - Design of steel structures - Part 2: Steel Bridges

The design of the bridge should ensure that when the damage of a component due to accidental actions occurs, the remaining structure can sustain at least the accidental load combination with reasonable means.
NOTE: The National Annex may define components that are subjected to accidental design situations and also details for assessments. Examples of such components are hangers, cables, bearings.
The effects of corrosion or fatigue of components and material should be taken into account by appropriate detailing, see also EN 1993-1 -9 and EN 1993-1 -10.
NOTE 1: EN 1993-1-9, section 3 provides assessment methods using the principles of damage tolerance or safe life.
NOTE 2: The National Annex may be a choice of the design method to be used for fatigue assessment.

5. Breakage events

When talking about cables loss or failure in general of a cable-stayed bridge, different mechanisms are possible and normally they have in common the fact the there is a disproportion in size between the consequence and a triggering event. In case of impact, the progressive event is characterised by the fact that the collapse starts from one or few elements, then the repercussion of the loss is transmitted to the other parts that, if are not able to withstand to the redistributed forces or excessive deformations, collapse as well. Four classes and six different typologies of failure are possible (Starossek, 2007), but in case of a collision event the main one is the "zipper-type collapse". This type is mainly seen when a failure of one or several cable elements cause an impulsive overloading on the adjacent cables, that are not able to withstand the increased force and break.
When analysing the single cable, the failure can be classified in three different types: pure tensile, shearing and bending breakage (Hoang, kyiomiya & Tonxiang, 2015), in case of a collision the last one best describes the event, where there is a big force in a short period of time, but it is not intensive enough to suddenly break the cable without deforming it. The cable undergoes to a significant deformation, this increases the tension in the element that breaks if the ultimate strength is reached.

6. Cables

The cable is the main element of a cable-stayed bridge, ensuring its stability and resistance. The composition and characteristics are similar to the ones used for the prestressed concrete, this means high resistance and higher content of carbon respect to normal steel, that increases the strength but on the other side has the effect of reducing the ductility. The size of a single cable can reach a maximum of 0.034 m^2 normally (Gimsing & Georgakis, 2012), where the number of strands depends on the type of strand used (standard, super or compact).
The mechanical resistance of a cable derives from the properties of its basic components that are the wires. The wire is made by electro galvanized steel and it follows an elastic-plastic law with hardening, given in Figure 2.The value of the elastic modulus is normally a bit smaller than the normal structural steel, due to the increased quantity of Carbon as said before, and it can oscillate between values of 190-200 GPa (Gimsing & Georgakis, 2012), in this study it has been taken a mean value of 195 GPa, given by the regulations as well.

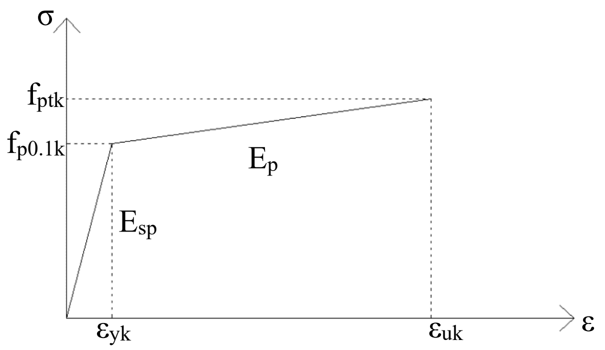

Figure 2-constitutive law of the steel

Where:
- $f_{ptk} = 1860\ MPa$
- $f_{p0.1k} = 1640\ MPa$
- $E_{sp} = 195\ GPa$

- $E_p = 8260\ MPa$
- $\varepsilon_{yk} = \frac{f_{p0,1k}}{E_{sp}} = 8.41‰$
- $\varepsilon_{uk} = 35‰$

In this study it has been used the standard strand, with the area of 139 mm^2 and a diameter of 0.62″ (equivalent to 15.7 mm) and for the weight is has been considered the normal density of the steel $\rho = 8000\ \text{Kg/m}^3$.

7. Spatial model

In consideration of a generic cable in the space, the two possible collisions are shown in Figure 3:

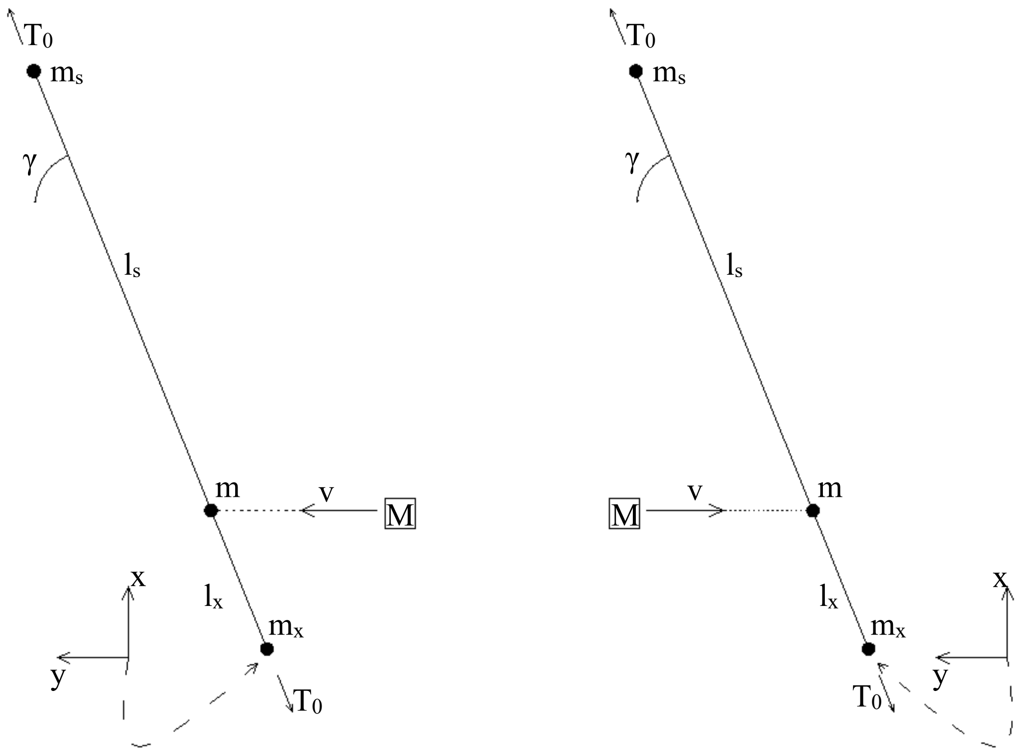

Figure 3-first type of impact on the left, second type of impact on the right

Where some of the parameters are defined as follow:

$$T_0 = p \cdot f_{ptk} \cdot A \tag{3}$$

$$\sigma_0 = p \cdot f_{ptk} \tag{4}$$

$$\varepsilon_0 = \frac{\sigma_0}{E_{sp}} \tag{5}$$

$$l_x + l_s = L \tag{6}$$

$$L_0 = \frac{L}{(1 + \varepsilon_0)} \tag{7}$$

$$\Delta L_0 = \frac{T_0 \cdot L_0}{E_{sp} \cdot A} \tag{8}$$

$$\Delta L_{0x} = \frac{\Delta L_0 \cdot l_x}{L} \tag{9}$$

$$\Delta L_{0s} = \frac{\Delta L_0 \cdot l_s}{L} \tag{10}$$

$$l_{0x} = \frac{l_x}{(1 + \varepsilon_0)} \tag{11}$$

$$l_{0s} = \frac{l_s}{(1 + \varepsilon_0)} \tag{12}$$

$$m = \frac{\rho \cdot L_0 \cdot A}{2} \tag{13}$$

$$m_x = \frac{\rho \cdot l_{0x} \cdot A}{2} \tag{14}$$

$$m_s = \frac{\rho \cdot l_{0s} \cdot A}{2} \tag{15}$$

When the collision starts, in the impacted point the mass is now m plus the mass of the vehicle M, as shown in Figure 4. The two masses keep moving in the horizontal direction until the cable breaks or, being able to withstand the impact, reaches the maximum displacement and it goes back.

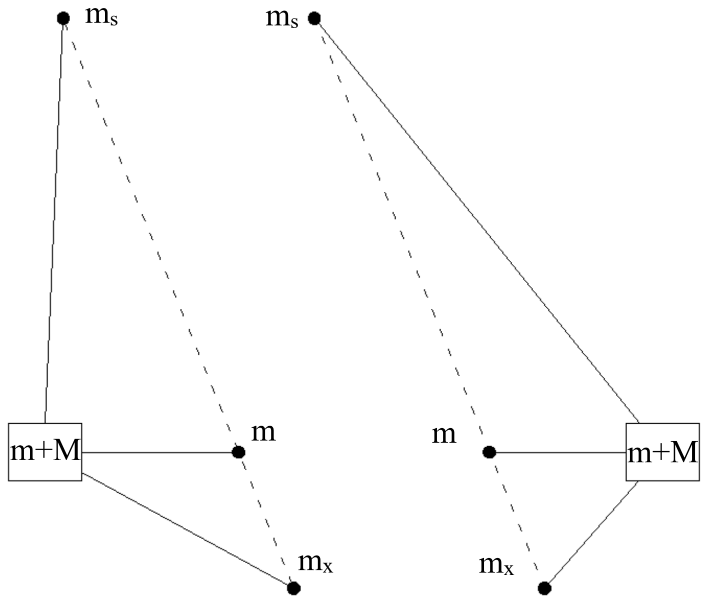

Figure 4-representation of the total mass during the impact,
first type of impact on the left, second type of impact on the right

In any moment of the displacement the geometry of the deformed cable can be defined by analysing the triangles shown in Figure 5. In specific, this will be done in relation to certain deformations of the cable, as it will be presented later in this paper.

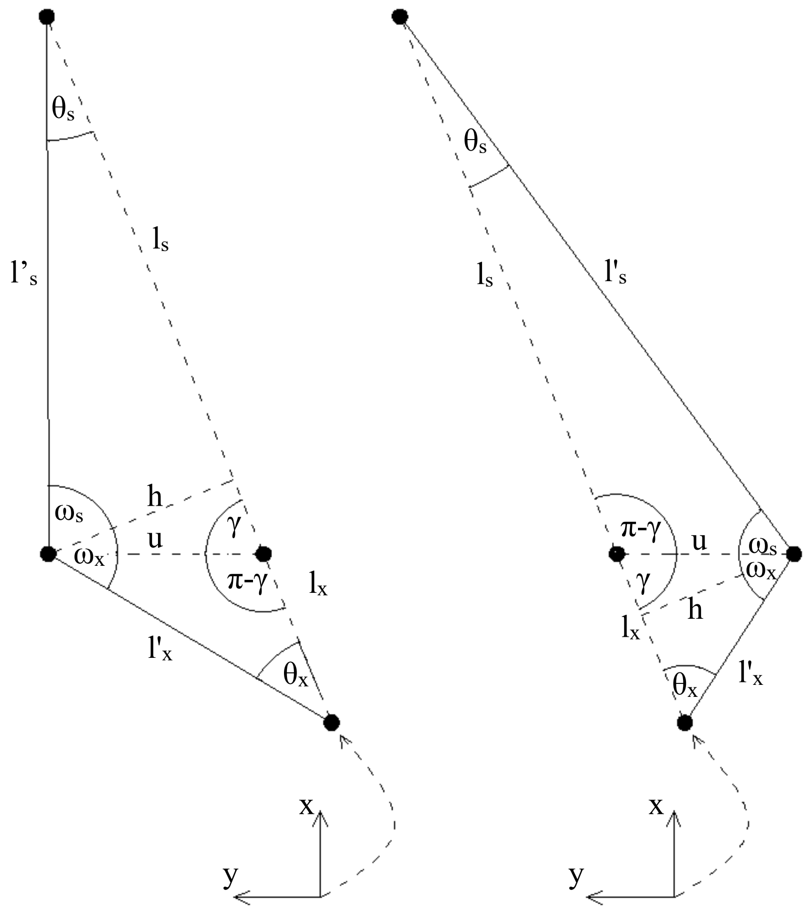

Figure 5-geometrical parameters of a cable during the displacement

8. Energetic approach

8.1. Vertical cable, middle point impact

The resistance of a cable is studied with an energetic approach, where before the impact there is a certain kinetic energy, given by the vehicle with a specific velocity, that is totally transferred to the cable in the form of elastic-plastic energy, if the cable is able to withstand the deformation required, or partly transferred if the elongation required is greater than the ultimate value and the cable breaks.

The study starts with a simple example of a vertical cable with the impact in the middle point, it is not realistic, but it will help to understand the phenomenon giving results for further implementation. In Figure 6 it is presented how the cable absorbs the energy in relation to which part of the constitutive law it is working in. Basically, the area under the graph is the elastic or elastic-plastic energy gained by the cable and the representation corresponds just to when the cable does not break, otherwise in case of breakage the whole area till $\varepsilon_{uk} \cdot L_0$ should be filled.

Considering now a generic area and length of the cable, the elongation required to absorb the energy is calculated assuming that the cable stays in its elastic range:

$$T_0 \cdot \Delta\varepsilon \cdot L_0 + \frac{1}{2} \cdot E_{sp} \cdot A \cdot \Delta\varepsilon^2 \cdot L_0 = \frac{1}{2} \cdot M \cdot v^2 \tag{16}$$

The strain $\Delta\varepsilon$ is calculated solving (16):

$$a = \frac{1}{2} \cdot E_{sp} \cdot A \cdot L_0 \tag{17}$$

$$b = T_0 \cdot L_0 \tag{18}$$

$$c = -\frac{1}{2} \cdot M \cdot v^2 \tag{19}$$

$$\Delta\varepsilon = \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \tag{20}$$

If $\Delta\varepsilon + \varepsilon_0$ is bigger than ε_{yk} then it goes into the plastic range and the equation is modified as follow:

$$T_0 \cdot \Delta\varepsilon \cdot L_0 + \frac{1}{2} \cdot E_{sp} \cdot A \cdot \left(\varepsilon_{yk} - \varepsilon_0\right)^2 \cdot L_0 + (f_{p01k} \cdot A - T0) \cdot (\Delta\varepsilon + \varepsilon_0 - \varepsilon_{yk}) \cdot L_0 + \frac{1}{2} \cdot E_p \cdot A \cdot \left(\Delta\varepsilon + \varepsilon_0 - \varepsilon_{yk}\right)^2 \cdot L_0 = \frac{1}{2} \cdot M \cdot v^2 \tag{21}$$

As before the strain $\Delta\varepsilon$ is calculated solving equation (21):

$$a = \frac{1}{2} \cdot E_p \cdot A \cdot L_0 \tag{22}$$

$$b = T_0 \cdot L_0 + (f_{p01k} \cdot A - T0) \cdot L_0 + E_p \cdot A \cdot (\varepsilon_0 - \varepsilon_{yk}) \cdot L_0 \tag{23}$$

$$c = \frac{1}{2} \cdot (E_{sp} + E_p) \cdot A \cdot L_0 \cdot (\varepsilon_0 - \varepsilon_{yk})^2 + (f_{p01k} \cdot A - T_0) \cdot (\varepsilon_0 - \varepsilon_{yk}) \cdot L_0 - \frac{1}{2} \cdot M \cdot v^2 \tag{24}$$

substituting equations (22)-(24) in equation (20).

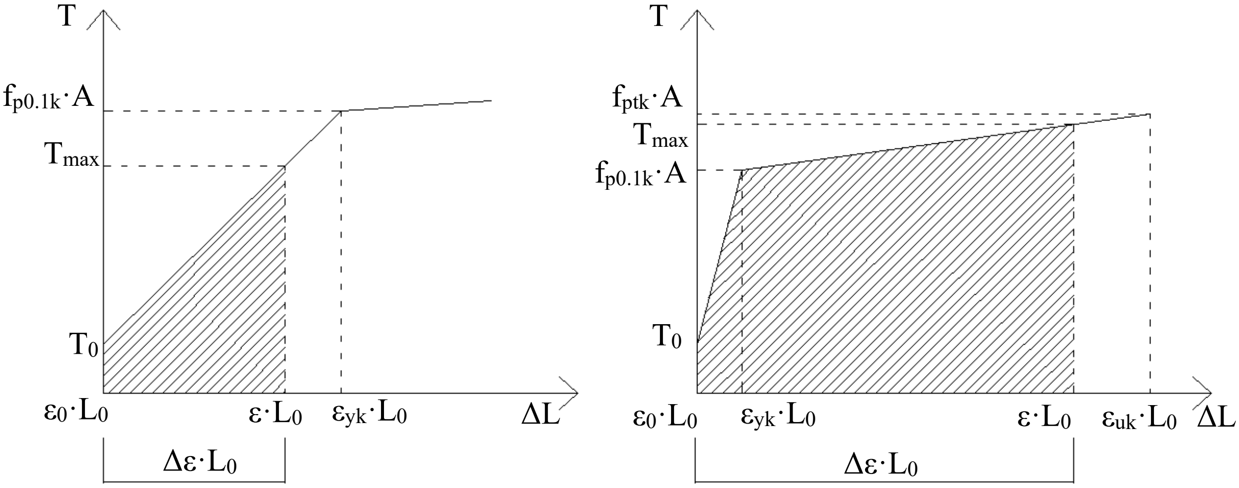

Figure 6-representation of the energy absorbed by the cable, just elastic range on the left, elastic-plastic range on the right

As it can be seen in equations (16), (21) and in Figure 7 there are two parameters that mainly affect the scale of the graph, the area A and the length L_0. If the area A is fixed, all the values on the vertical axis are fixed as well, so the only way to increment the area under the graph is to increase the length. in the same way if the length L_0 is fixed, the only way to increment the area is by increasing the area of the cable.

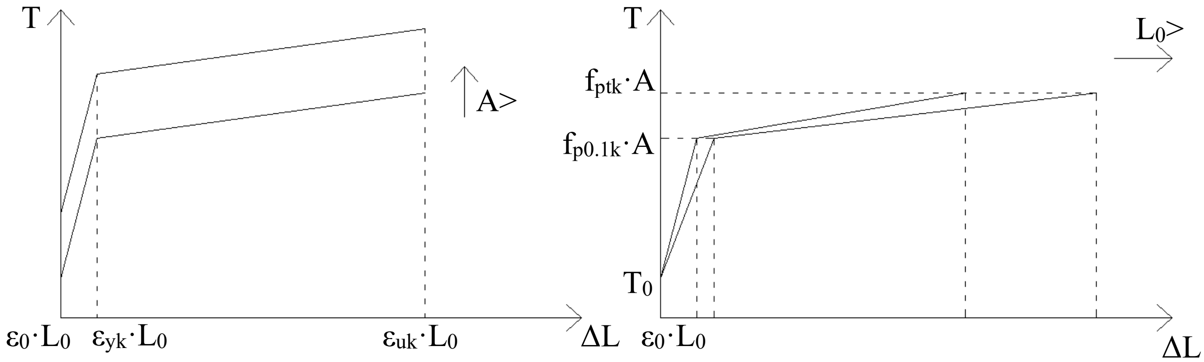

Figure 7-representation of the influence on the response of the area on the left and the length on the right

The same aspect can be presented more in specific by the analysis of equation (21) (as well as equation (16) in case of elastic range), obtaining a single equation in function of length and area. Starting with the substitution of equation (3) in equation (21) (or equation (16)), obtaining equation (25) (or (26)), then grouped in equation (27) (or (28)) with some simple manipulations, to have the single term "$A \cdot L_0$":

$$p \cdot f_{ptk} \cdot A \cdot \Delta\varepsilon \cdot L_0 + \frac{1}{2} \cdot E_{sp} \cdot A \cdot \Delta\varepsilon^2 \cdot L_0 = \frac{1}{2} \cdot M \cdot v^2 \tag{25}$$

$$\begin{aligned} p \cdot f_{ptk} \cdot A \cdot \Delta\varepsilon \cdot L_0 + \frac{1}{2} \cdot E_{sp} \cdot A \cdot \left(\varepsilon_{yk} - \varepsilon_0\right)^2 \cdot L_0 + (f_{p01k} \cdot A - p \cdot f_{ptk} \cdot A) \\ \cdot (\Delta\varepsilon + \varepsilon_0 - \varepsilon_{yk}) \cdot L_0 + \frac{1}{2} \cdot E_p \cdot A \cdot \left(\Delta\varepsilon + \varepsilon_0 - \varepsilon_{yk}\right)^2 \cdot L_0 \\ = \frac{1}{2} \cdot M \cdot v^2 \end{aligned} \tag{26}$$

$$\left(p \cdot f_{ptk} \cdot \Delta\varepsilon + \frac{1}{2} \cdot E_{sp} \cdot \Delta\varepsilon^2\right) \cdot A \cdot L_0 = \frac{1}{2} \cdot M \cdot v^2 \tag{27}$$

$$\begin{aligned} \Big(p \cdot f_{ptk} \cdot \left(\varepsilon_{yk} - \varepsilon_0\right) + \frac{1}{2} \cdot E_{sp} \cdot \left(\varepsilon_{yk} - \varepsilon_0\right)^2 + f_{p01k} \cdot \left(\Delta\varepsilon + \varepsilon_0 - \varepsilon_{yk}\right) + \frac{1}{2} \cdot E_p \\ \cdot \left(\Delta\varepsilon + \varepsilon_0 - \varepsilon_{yk}\right)^2\Big) \cdot A \cdot L_0 = \frac{1}{2} \cdot M \cdot v^2 \end{aligned} \tag{28}$$

The whole term in the parenthesis is now defined as deformative constant k(p) in equations (29) and (30) and it is function of the increment in strain $\Delta\varepsilon$ and the initial strain ε_0 as well as the percentage of initial tension p, but if a specific maximum elongation is set then k(p) is only influenced by the initial tension. Now equation (31) is obtained by the substitution of (29) (or (30)) in (27) (or (28)) and it can be seen how length and area are tied together by an hyperbolic function once the right hand side of the equation is defined:

$$k(p) = p \cdot f_{ptk} \cdot \Delta\varepsilon + \frac{1}{2} \cdot E_{sp} \cdot \Delta\varepsilon^2 \tag{29}$$

$$\begin{aligned} k(p) = \Big(p \cdot f_{ptk} \cdot \left(\varepsilon_{yk} - \varepsilon_0\right) + \frac{1}{2} \cdot E_{sp} \cdot \left(\varepsilon_{yk} - \varepsilon_0\right)^2 + f_{p01k} \cdot \left(\Delta\varepsilon + \varepsilon_0 - \varepsilon_{yk}\right) \\ + \frac{1}{2} \cdot E_p \cdot \left(\Delta\varepsilon + \varepsilon_0 - \varepsilon_{yk}\right)^2\Big) \end{aligned} \tag{30}$$

$$A \cdot L_0 = \frac{M \cdot v^2}{2 \cdot k(p)} \tag{31}$$

With equation (31) it is possible to define whether an existing cable breaks or not and set the minimum size to resist to the impact or to stay in a specific range in the constitutive law. In the following sections the values of k(p) are defined.

8.1.1. Breakage condition

If the cable breaks it means it has reached the maximum strain ε_{uk} and it is obviously in the plastic range. With this consideration, by substituting equation (32) in equation (30), equation (33) is derived where k(p) depends just on the initial strain now:

$$\Delta\varepsilon + \varepsilon_0 = \varepsilon_{uk} \tag{32}$$

$$k(p) = \left(p \cdot f_{ptk} \cdot (\varepsilon_{yk} - \varepsilon_0) + \frac{1}{2} \cdot E_{sp} \cdot \left(\varepsilon_{yk} - \varepsilon_0\right)^2 + f_{p01k} \cdot \left(\varepsilon_{uk} - \varepsilon_{yk}\right) + \frac{1}{2} \cdot E_p \cdot \left(\varepsilon_{uk} - \varepsilon_{yk}\right)^2\right) \tag{33}$$

The values of k(p), for a normal range of initial tension going from 30% to 60% of the ultimate strength, are presented in Table 1:

Table 1-values of the parameter k(p) for the breakage condition

p(%)	k(MPa)
30	52.63
40	52.00
50	51.21
60	50.23

It can be seen that even a variation in the initial tension has a small influence on the value of k(p), for this reason a conservative choice can be made selecting a unique value coming from p equal to 60% that will give a unique requirement for the breakage when substituted in equation (31).

8.1.2. Zero-tension condition

This is the condition where the cable has reached a certain plastic deformation without breaking, but when it goes back in place it has lost the whole initial tension T_0, keeping a residual deformation as shown in Figure 8.

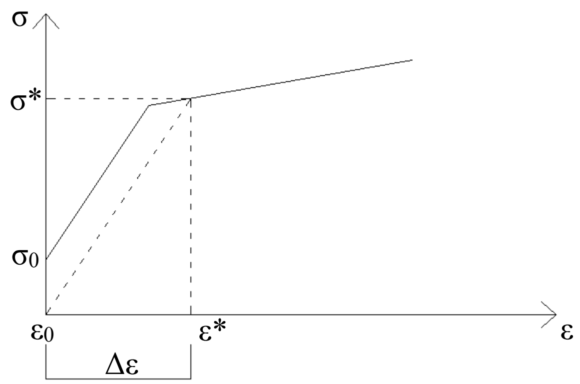

Figure 8-representation of the zero-tension condition

In order to get Δε the system (34) is solved:

$$\begin{cases} \sigma^* = E_{sp} \cdot \Delta\varepsilon \\ \sigma^* = E_{sp} \cdot \varepsilon_{yk} + E_p \cdot \left(\Delta\varepsilon + \varepsilon_0 - \varepsilon_{yk}\right) \end{cases} \tag{34}$$

Getting equation (35):

$$\Delta\varepsilon = \varepsilon_{yk} + \frac{E_p}{E_{sp} - E_p} \cdot \varepsilon_0 = \varepsilon_{yk} + \frac{E_p}{E_{sp} - E_p} \cdot \frac{p \cdot f_{ptk}}{E_{sp}} \tag{35}$$

Where:

$$\varepsilon^* = \Delta\varepsilon + \varepsilon_0 \tag{36}$$

Again the cable is moving over the plastic range, so by using directly equation (35) in equation (30) new values for k(p) can be calculated, given in Table 2:

Table 2-values of the parameter k(p)
for the zero-tension condition

p(%)	k(MPa)
30	11.04
40	12.08
50	12.95
60	13.65

8.1.3. Elastic condition

If the cable stays in this condition it never exits from the elastic range, so now the equation to consider is the (29), with the use of equation (37):

$$\Delta\varepsilon + \varepsilon_0 = \varepsilon_{yk} \tag{37}$$

$$k(p) = p \cdot f_{ptk} \cdot (\varepsilon_{yk} - \varepsilon_0) + \frac{1}{2} \cdot E_{sp} \cdot (\varepsilon_{yk} - \varepsilon_0)^2 \tag{38}$$

And Table 3 is given as a result of equation (38):

Table 3- values of the parameter k(p) for the elastic condition

p(%)	k(MPa)
30	6.10
40	5.48
50	4.68
60	3.70

8.2. Relation between displacement and size of the cable

Given the geometry of the cable, it can be seen how a combination of area and length influences the displacement by calculating first the elongation required to have a specific strain, then the displacement with the simple use of the Pythagoras's theorem with a focus on just one side due to the symmetry of this model (Figure 9).

$$\Delta L_x = \Delta\varepsilon \cdot \frac{L_0}{2} = (\varepsilon - \varepsilon_0) \cdot \frac{L_0}{2} \tag{39}$$

Using now equations (4), (5) and (7) in equation (39) a direct relation between the elongation and the length can be defined in equation (40), depending on the initial tension and the strain:

$$\Delta L_x = \frac{\left(\frac{\varepsilon}{2} - \frac{p \cdot f_{ptk}}{2 \cdot E_{sp}}\right)}{\left(1 + \frac{p \cdot f_{ptk}}{E_{sp}}\right)} \cdot L \tag{40}$$

and the displacement is calculated with equation (42) as shown in Figure 9:

$$u = \sqrt{(lx + \Delta Lx)^2 - lx^2} \tag{41}$$

$$u = \sqrt{\left(\frac{L}{2} + \frac{\left(\frac{\varepsilon}{2} - \frac{p \cdot f_{ptk}}{2 \cdot E_{sp}}\right)}{\left(1 + \frac{p \cdot f_{ptk}}{E_{sp}}\right)} \cdot L\right)^2 - \left(\frac{L}{2}\right)^2} = L * \sqrt{\left(\frac{1}{2} + \frac{\left(\frac{\varepsilon}{2} - \frac{p \cdot f_{ptk}}{2 \cdot E_{sp}}\right)}{\left(1 + \frac{p \cdot f_{ptk}}{E_{sp}}\right)}\right)^2 - \left(\frac{1}{2}\right)^2} \tag{42}$$

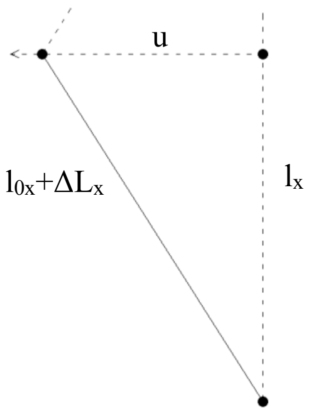

Figure 9-geometry of the displacement for a vertical cable

The relation between the maximum displacement and the length of the cable is linear once the initial tension and a strain are defined. In specific if the three different conditions presented previously are analysed the following procedure can be followed in order to get the displacement:

1. The range in which the cable will work is chosen: breakage, zero-tension or just elastic;
2. Given a specific initial tension, the tables give the value of k(p);
3. Given a specific length the cable has to cover, the area required to stay in the range chosen above is defined:

$$A = \frac{M \cdot v^2}{2 \cdot L \cdot k(p)} \tag{43}$$

4. With equations (40), (42) the increment in elongation and the displacement are calculated.

9. Inclined cable, generic point impact

In consideration now of a cable with an inclination and a shifted point of impact, as presented in Figure 3 and Figure 5, there is no symmetry anymore so the type of the impact has an influence on the response. One important aspect now is the interaction between the cable and the vehicle, these two can slide or stick in respect to each other during the impact, the real behaviour will not be either of them but something in between. For this reason, both approaches are going to be analysed in the following sections.

9.1. Sliding model

With this model the vehicle freely slides over the cable, this means that there are no differences between the inferior and superior part and the whole length L is responding in the same way. By comparing this situation with the vertical cable presented before, it can be seen then that there are no differences in terms of the way in which the cable transforms the kinetic energy in elastic-plastic energy and this means that the whole process performed before, from equation (16) to equation (38), is still valid giving at the end exactly the same results. The only aspect changing is the geometry, as a result the procedure to calculate the displacement will be different.

9.1.1. Relation between displacement and size of the cable

Given the non-symmetry the Carnot's theorem must be used to define the geometry. Equations (39) and (40) are still valid but in relation to the whole cable as shown in equations (44) and (45):

$$\Delta L = \Delta\varepsilon \cdot L_0 = (\varepsilon - \varepsilon_0) \cdot L_0 \tag{44}$$

$$\Delta L = \frac{\left(\varepsilon - \frac{p \cdot f_{ptk}}{E_{sp}}\right)}{\left(1 + \frac{p \cdot f_{ptk}}{E_{sp}}\right)} \cdot L \tag{45}$$

Due to the fact that an explicit formula for the displacement cannot be easily defined, the latter is calculated with an iterative procedure until the elongation that the displacement produces is exactly the one coming from equation (45):

1. The initial displacement is zero, u_{try}=0;
2. The initial elongation increment is zero as well because the displacement is zero, ΔL_{try}=0;
3. Using the Carnot's theorem the length of the two parts of the deformed cable is defined.
   a. In case of the first type of impact, presented on the left in Figure 5:

$$l'_x = \sqrt{{l_x}^2 + {u_{try}}^2 - 2 \cdot l_x \cdot u_{try} \cdot cos(\pi - \gamma)} \tag{46}$$

$$l'_s = \sqrt{{l_s}^2 + {u_{try}}^2 - 2 \cdot l_s \cdot u_{try} \cdot cos(\gamma)} \tag{47}$$

   b. In case of the second type of impact, presented on the right in Figure 5:

$$l'_x = \sqrt{{l_x}^2 + {u_{try}}^2 - 2 \cdot l_x \cdot u_{try} \cdot cos(\gamma)} \tag{48}$$

$$l'_s = \sqrt{{l_s}^2 + {u_{try}}^2 - 2 \cdot l_s \cdot u_{try} \cdot cos(\pi - \gamma)} \tag{49}$$

4. Given the deformed length the corresponding elongation is derived:

$$\Delta L_{try} = l'_x + l'_s - L \tag{50}$$

5. If $\Delta L_{try} - \Delta L \geq 0$ It means that the point where the elongation is the same as the one coming from the energy equation has been reached, the process can be stopped. The displacement corresponds to the last u_{try} calculated;

6. If $\Delta L_{try} - \Delta L < 0$ the displacement is increased ($u_{try} = u_{try} + 0.001$) and the procedure goes back to point 3.

The area is calculated in the same way with equation (43).
Now given the value of ε the tension and the stress can be defined:

1. If $\varepsilon \leq \varepsilon_{yk}$:

$$\sigma = E_{sp} \cdot \varepsilon \tag{51}$$

2. If $\varepsilon > \varepsilon_{yk}$:

$$\sigma = E_{sp} \cdot \varepsilon_{yk} + E_p \cdot (\varepsilon - \varepsilon_{yk}) \tag{52}$$

3. Tension:

$$T = \sigma \cdot A \tag{53}$$

9.2. No-sliding model

With this model now it is assumed that the friction forces between cable and vehicle are that high that there is no sliding. For this reason, the point of contact never changes during the impact, therefore the superior and inferior part of the cable now respond differently. The generic form of the energy equation is given by equation (54):

$$Energy(l_{0s}) + Energy(l_{0x}) = \frac{1}{2} \cdot M \cdot v^2 \tag{54}$$

With consideration of both types of impact, it cannot be said a priori which is the part reaching the breakage first or in which part of the constitutive law the two sides are working, then it is not possible to get an explicit equation in function of the size of the cable. For some range of displacement, depending on the type of impact, one side or the other undergoes a shortening and if this happens it means that it is losing the initial tension. The cable does not have resistance in compression so once the tension is lost it becomes just zero till the point it goes in traction again, if that happens. During this shortening range that side of the cable is not absorbing any energy because there is not any increment in elongation neither it is going in compression, as shown in Figure 10:

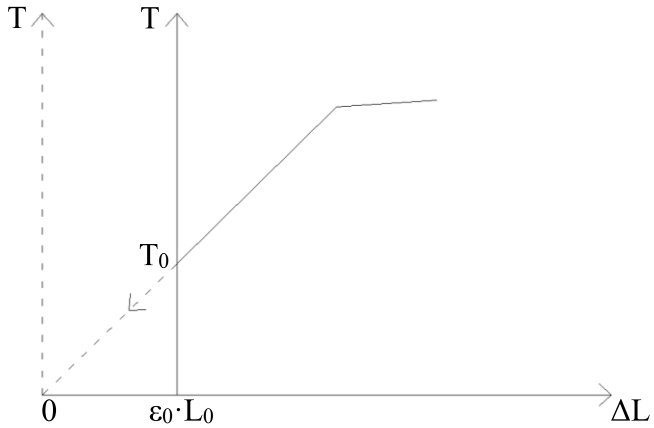

Figure 10-representation of the shortening in the energy graph

In general, an iterative procedure is required, but depending on the type of impact analysed some simplifications can be made as presented in the following sections. These simplifications derive from the fact that the part l_x is always shorter than l_s, because it depends on the height of the centre of mass of the vehicle that is generally small in relation to the length of the cable. Some numerical parameters will be presented later in the study.

9.2.1. First type of impact

This type allows to simplify the equation, because the part l_s always undergoes a shortening and this means the term Energy(l_s) is always zero, as shown in Figure 10. Theoretically, if the l_x part had enough resistance to allow l_s to reach the elongation range again, the latter would contribute to absorb the kinetic energy as well, but as said previously the two sides have a difference in length that makes l_x breaking before.

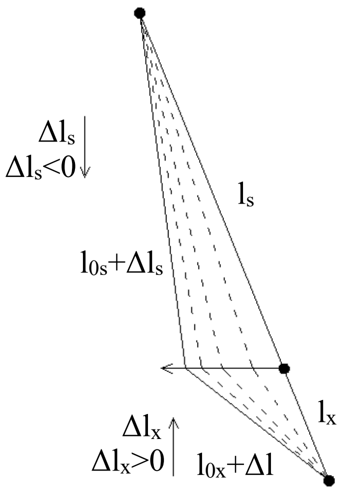

Figure 11- representation of the deformation of the two parts of the cable

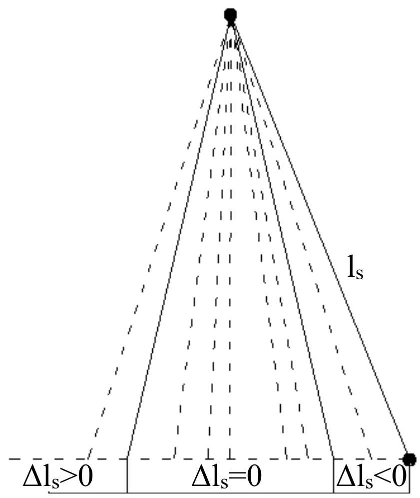

Figure 12- representation of the range of deformation of the part of the cable above impact

For the reason described above, it is possible to rewrite equations (16) and (21) with the use of the length l_{0x}:

$$T_0 \cdot \Delta\varepsilon \cdot l_{0x} + \frac{1}{2} \cdot E_{sp} \cdot A \cdot \Delta\varepsilon^2 \cdot l_{0x} = \frac{1}{2} \cdot M \cdot v^2 \tag{55}$$

$$\begin{aligned} T_0 \cdot \Delta\varepsilon \cdot l_{0x} + \frac{1}{2} \cdot E_{sp} \cdot A \cdot \left(\varepsilon_{yk} - \varepsilon_0\right)^2 \cdot l_{0x} + (f_{p01k} \cdot A - T0) \cdot (\Delta\varepsilon + \varepsilon_0 \\ - \varepsilon_{yk}) \cdot l_{0x} + \frac{1}{2} \cdot E_p \cdot A \cdot \left(\Delta\varepsilon + \varepsilon_0 - \varepsilon_{yk}\right)^2 \cdot l_{0x} = \frac{1}{2} \cdot M \cdot v^2 \end{aligned} \tag{56}$$

By following the same process, the k(p) values are the same for the three conditions presented previously. If now the centre of mass of the vehicle has a height of c, the length l_{0x} depends on the inclination of the cable as shown in equation (57):

$$l_{0x} = \frac{c}{(1 + \varepsilon_0) \cdot sin(\gamma)} \tag{57}$$

Equations (56)and (57) together define the final form:

$$A = \frac{M \cdot v^2}{2 \cdot c \cdot k(p)} \cdot (1 + \varepsilon_0) \cdot sin(\gamma) \tag{58}$$

Because of the centre of mass of the vehicle is fixed and the inclination of the cable can be fixed as well, the only missing parameter is the area, that now is a constant. There is no relation anymore with the length of the cable because the resisting part is the side below impact, that depends just on c and γ. If γ is not fixed, it can be seen how the area required grows with gamma, because l_{0x} decreases.

9.2.2. Relation between displacement and size of the cable

In terms of elongation, equation (40) is still valid, but instead of having half of the cable now the length is fixed to l_x:

$$\Delta L_x = \frac{\left(\varepsilon - \frac{p \cdot f_{ptk}}{E_{sp}}\right)}{\left(1 + \frac{p \cdot f_{ptk}}{E_{sp}}\right)} \cdot l_x \tag{59}$$

Now in consideration of Figure 5 (on the left) the maximum displacement can be calculated with equations (60)-(62):

$$\omega_x = sin^{-1}\left(\left(\frac{l_x}{l_x + \Delta L_x}\right) \cdot sin(\pi - \gamma)\right) \tag{60}$$

$$\theta_x = \pi - \omega_x - (\pi - \gamma) \tag{61}$$

$$u = \sqrt{{l_x}^2 + (l_x + \Delta L_x)^2 - 2 \cdot l_x \cdot (l_x + \Delta L_x) \cdot cos(\theta_x)} \tag{62}$$

The steps to follow are the same as presented before from 1. to 4., but equation (40), (42) and (43) are substituted with equations (59), (62) and (58).

9.2.3. Second type of impact

With this type of impact, no simplifications can be made a priori. Now the part that undergoes a shortening is l_x, but the difference from the previous situation is that the range in which it has negative or zero deformation is smaller, as shown in Figure 13 and Figure 14. As a result, in some situations the part breaking could be l_s, in others l_x and more it could be that if it is l_s to break, l_x could have reached the elongation range again having contributed to the absorption of the kinetic energy.

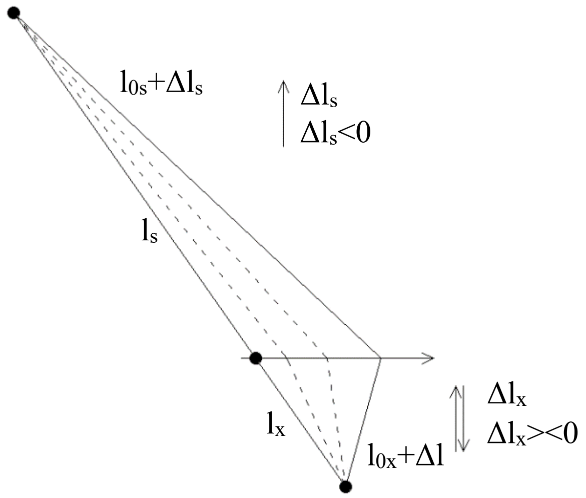

Figure 13- representation of the deformation of the two parts of the cable

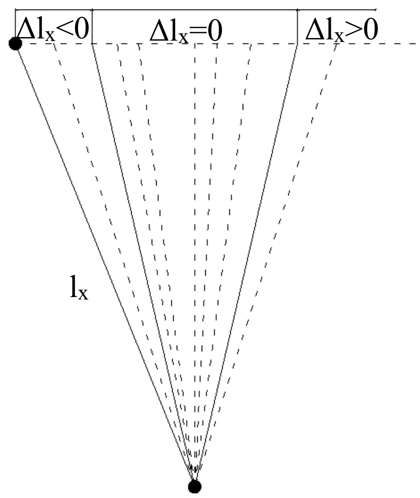

Figure 14-representation of the range of deformation of the part of the cable below impact

Because of the reasons just described it is not possible to get an explicit form like equations (31) and (58), the length and the inclination of the cable change the behaviour. As said before, now the iterative procedure is required:

1. Initial displacement u_{try}=0;
2. The elongation of the two sides is defined:

$$\Delta L_x = \sqrt{{l_x}^2 + {u_{try}}^2 - 2 * l_x \cdot u_{try} \cdot cos(\gamma)} - l_x \tag{63}$$

$$\Delta L_s = \sqrt{{l_s}^2 + {u_{try}}^2 - 2 \cdot l_s \cdot u_{try} \cdot cos(\pi - \gamma)} - l_s \tag{64}$$

$$\Delta\varepsilon_x = \frac{\Delta L_x}{l_{0x}} \tag{65}$$

$$\Delta\varepsilon_s = \frac{\Delta L_s}{l_{0s}} \tag{66}$$

3. Check of the maximum capacity:
   a. If $\Delta\varepsilon_x > \varepsilon_{uk}$;
   b. If $\Delta\varepsilon_s + \varepsilon_0 > \varepsilon_{uk}$;

   Then there has been the failure and the process is stopped. The parameters calculated are the ones at the breakage. If there is no breakage, the procedure proceeds to point 4.
4. The behaviour of l_s is defined.
   a. If $\Delta\varepsilon_s + \varepsilon_0 \leq \varepsilon_{yk}$ it is in the elastic range:

$$Energy(l_s) = T_0 \cdot \Delta\varepsilon_s \cdot l_{0s} + \frac{1}{2} \cdot E_{sp} \cdot A \cdot {\Delta\varepsilon_s}^2 \cdot l_{0s} \tag{67}$$

b. If $\Delta\varepsilon_s + \varepsilon_0 > \varepsilon_{yk}$ it is in the plastic range:

$$Energy(l_s) = T_0 \cdot \Delta\varepsilon_s \cdot l_{0s} + \frac{1}{2} \cdot E_{sp} \cdot A \cdot \left(\varepsilon_{yk} - \varepsilon_0\right)^2 \cdot l_{0s} + (f_{p01k} \cdot A - T_0) \cdot (\Delta\varepsilon_s + \varepsilon_0 - \varepsilon_{yk}) \cdot l_{0s} + \frac{1}{2} \cdot E_p \cdot A \cdot \left(\Delta\varepsilon_s + \varepsilon_0 - \varepsilon_{yk}\right)^2 \cdot l_{0s} \quad (68)$$

5. If $\Delta L_x < -\varepsilon_0 \cdot l_{0x} \rightarrow \Delta L_x = -\varepsilon_0 \cdot l_{0x}$.
6. The behaviour of l_x is defined.
   a. If $\Delta L_x \leq 0 \rightarrow Energy(l_x) = 0$;
   b. If $\Delta L_x > 0$:
      i. If $\Delta\varepsilon_x \leq \varepsilon_{yk}$ it is in the elastic range:

$$Energy(l_x) = \frac{1}{2} \cdot E_{sp} \cdot A \cdot \Delta\varepsilon_x^{\,2} \cdot l_{0x} \quad (69)$$

      ii. If $\Delta\varepsilon_x > \varepsilon_{yk}$ it is in the plastic range:

$$Energy(l_x) = \frac{1}{2} \cdot E_{sp} \cdot A \cdot \varepsilon_{yk}^{\,2} \cdot l_{0x} + f_{p01k} \cdot A \cdot (\Delta\varepsilon_x - \varepsilon_{yk}) \cdot l_{0x} + \frac{1}{2} \cdot E_p \cdot A \cdot \left(\Delta\varepsilon_x - \varepsilon_{yk}\right)^2 \cdot l_{0x} \quad (70)$$

7. If:

$$Energy(l_s) + Energy(l_x) = \frac{1}{2} \cdot M \cdot v^2 \quad (71)$$

The equilibrium configuration has been found and the procedure stops;

8. If:

$$Energy(l_s) + Energy(l_x) < \frac{1}{2} \cdot M \cdot v^2 \quad (72)$$

The displacement is updated and the process goes back to point 2:

$$u_{try} = u_{try} + 0.001 \quad (73)$$

When calculating the deformation of the part l_x the values related to the initial tension are not added because it has been considered that surely it loses the tension at the beginning, to go then again in elongation but now starting from zero. This aspect is not true in the case of a vertical cable, where the whole initial tension is kept or in case of a high inclination, where it is just

partly lost and the initial tension terms have to be added again. In reality these inclinations are not reached, so it is correct to assume this simplification.
The process just described is valid for every combination of γ and L, but under some circumstances the model can be simplified in the same way it has been done for the first type of impact. For every inclination there is a limit length for which when the cable reaches the breakage it is because the part l_s has reached the ultimate strain, but at the same time the part l_x has not reached the elongation range. This is the specular situation of the first type of impact and the final equation is going to be similar. The procedure that allows to define this limit length is presented below.

1. The limit displacement u* for which the part l_x does not go in elongation again is defined, as shown in Figure 15. When the displacement is u* then the length is the initial length l_{0x} again. The result is calculated starting from the equation (74) given by the Carnot's theorem:

$$l_{0x} = \sqrt{{l_x}^2 + u^{*2} - 2 \cdot l_x \cdot u^* \cdot cos(\gamma)} \tag{74}$$

$$u^{*2} - 2 \cdot l_x \cdot cos(\gamma) \cdot u^* + {l_x}^2 - {l_{0x}}^2 = 0 \tag{75}$$

The equation (75) can be simplified by neglecting the term ${l_x}^2 - {l_{0x}}^2$ because it is almost zero, as demonstrated in equation (76):

$${l_x}^2 - {l_{0x}}^2 = {l_x}^2 - \frac{{l_x}^2}{(1+\varepsilon_0)^2} \sim {l_x}^2 - {l_x}^2 = 0 \tag{76}$$

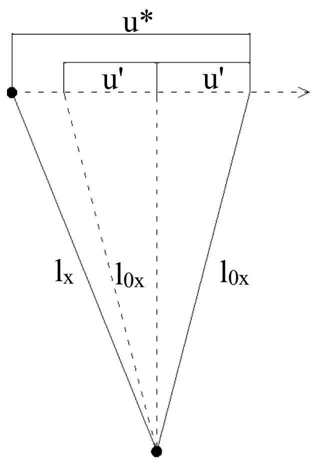

Figure 15-definition of the limit displacement

The remaining part is a second degree equation that gives the positive result in equation (77):

$$u^* = \frac{2 \cdot c}{tg(\gamma)} \tag{77}$$

2. The maximum length l'_s is the value that the second part does not have to exceed, depending on a set strain. This strain could be the ultimate resistance or one of the other conditions:

$$l'_s = (1+\varepsilon) \cdot l_{0s} = \frac{(1+\varepsilon)}{(1+\varepsilon_0)} \cdot l_s \tag{78}$$

3. With the use of the Carnot's theorem again the length l_s is defined using equation (80):

$$l'_s = {l_s}^2 + u^{*2} - 2 \cdot l_s \cdot u^* \cdot cos(\pi - \gamma) = {l_s}^2 + u^{*2} + 2 \cdot l_s \cdot u^* \cdot cos(\gamma) \tag{79}$$

$$\left(1 - \left(\frac{(1+\varepsilon)}{(1+\varepsilon_0)}\right)^2\right) \cdot {l_s}^2 + \frac{4 \cdot c}{tg(\gamma)} \cdot cos(\gamma) \cdot l_s + \left(\frac{2 \cdot c}{tg(\gamma)}\right)^2 = 0 \tag{80}$$

$$a = \left(1 - \left(\frac{(1+\varepsilon)}{(1+\varepsilon_0)}\right)^2\right) \tag{81}$$

$$b = \frac{4 \cdot c}{tg(\gamma)} \cdot cos(\gamma) \tag{82}$$

$$c = \left(\frac{2 \cdot c}{tg(\gamma)}\right)^2 \tag{83}$$

$$l_s = \frac{-b \; - \; \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \tag{84}$$

4. from l_s the total length L can be calculated:

$$L = l_s + \frac{c}{sin(\gamma)} \tag{85}$$

Given the result above, the whole iterative procedure from 1. to 8. can be substituted with the usual equation:

$$A \cdot l_{0s} = \frac{M \cdot v^2}{2 \cdot k(p)} \tag{86}$$

9.2.4. Relation between displacement and size of the cable

In a generic cable the displacement derives from the iterative procedure presented before, in the case where the simplification given in equation (86) can be followed the calculation is direct like it was for the first type of impact. In consideration of Figure 5 on the right:

$$\Delta L_s = \frac{\left(\varepsilon - \frac{p \cdot f_{ptk}}{E_{sp}}\right)}{\left(1 + \frac{p \cdot f_{ptk}}{E_{sp}}\right)} \cdot l_s \tag{87}$$

$$\omega_s = sin^{-1}\left(\left(\frac{l_s}{l_s + \Delta L_s}\right) \cdot sin(\pi - \gamma)\right) \tag{88}$$

$$\theta_s = \pi - \omega_s - (\pi - \gamma) \tag{89}$$

$$u = \sqrt{{l_s}^2 + (l_s + \Delta L_s)^2 - 2 \cdot l_s \cdot (l_s + \Delta L_s) \cdot cos(\theta_s)} \tag{90}$$

The steps to follow are the same as presented before from 1. to 4., but equation (40), (42) and (43) are substituted with equation (87), (90) and (86).

10. Step by step approach

The energy approach allows to define all the limit conditions for a single cable whether it breaks or not or under which range it is working during the impact. Linked with this aspect, there is the displacement that is important to understand if during the collision just one cable will be affected, or if it is bigger than the spacing, several of these. In case where more than one cable is touched the kinetic energy is absorbed by all of them, but when the second or further cables are hit the speed of the vehicle has decreased. This section is going to analyse this issue, to understand how at the end several cables respond. The procedure is the following one:

1. The system vehicle + cable has an initial velocity right at the starting moment of the impact, considering an inelastic collision:

$$v_0 = \frac{M}{(M + m)} \cdot v \tag{91}$$

2. The time step for the analysis has to be defined. Starting from the displacement u derived from the energy approach, imagining just for hypothesis a uniformly decelerated motion where the velocity at the end is zero in any case, the time is:

$$\Delta t = \frac{2 \cdot u}{v_0} \tag{92}$$

Given the fictitious total time, equation (92) is divided for 10^4 that gives the time step in equation (93):

$$ts = \frac{\Delta t}{10^4} \tag{93}$$

3. The initial condition of every point is defined.
   a. lumped mass:

$$m_{l_i} = \frac{\rho \cdot A \cdot (l_{0,i} + l_{0,i+1})}{2} \tag{94}$$

The point hit by the vehicle also has its mass:

$$m_{l_w} = \frac{\rho \cdot A \cdot (l_{0,i} + l_{0,i+1})}{2} + M \tag{95}$$

   b. velocity, the cable is considered still apart for the hit point that has the initial velocity v_0;
   c. initial shape, it can be simplified with a straight line. It has been performed a check with the catenary shape (Irvine, 1981; Russell & Lardner, 1981), that would be more precise, but due to the initial tension the difference between the two results does not lead to consistent errors:

$$x_i(0) = s \cdot sin(\gamma) \tag{96}$$

$$y_i(0) = s \cdot cos(\gamma) \tag{97}$$

   d. initial tension T_0, simplified as constant through the whole cable;
4. Additional displacement:

$$\Delta x_i(t) = v_{x,i}(t - ts) \cdot ts \tag{98}$$

$$\Delta y_i(t) = v_{y,i}(t - ts) \cdot ts \tag{99}$$

5. Nodes new position:

$$x_i(t) = x_i(t - ts) + \Delta x_i(t) \tag{100}$$

$$y_i(t) = y_i(t - ts) + \Delta y_i(t) \tag{101}$$

6. The node hit by the impact keeps moving horizontally with the vehicle:

$$x_w(t) = x_w(0) \tag{102}$$

7. If $y_w(t) \leq y_w(0)$ the impacted point reached the initial position without breaking and the process is stopped;
8. Strained length of every element in that moment in time:

$$l_i(t) = \sqrt{\big(y_i(t) - y_{i-1}(t)\big)^2 + \big(x_i(t) - x_{i-1}(t)\big)^2} \tag{103}$$

9. The strain is calculated.
   a. In case of the sliding model it is the total strain:

$$\Delta l_i(t) = \sum l_i(t) - L_0 \tag{104}$$

$$\varepsilon_i(t) = \frac{\Delta l_i(t)}{L_0} \tag{105}$$

   b. In case of the no-sliding model it is the strain of every element:

$$\Delta l_i(t) = l_i(t) - l_{0,i} \tag{106}$$

$$\varepsilon_i(t) = \frac{\Delta l_i(t)}{l_{0,i}} \tag{107}$$

10. The tension depends on which range of the constitutive law the element is working in, as shown in Figure 16.

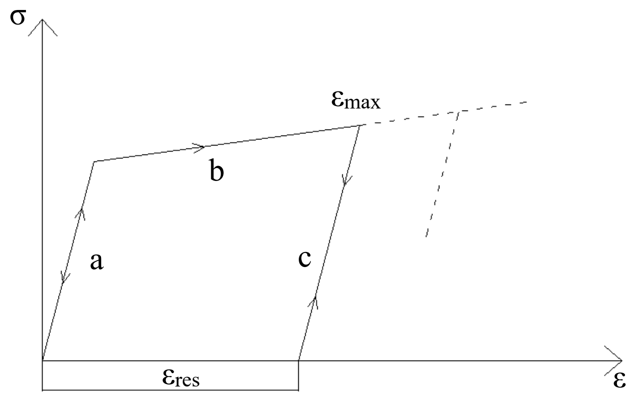

Figure 16-range identification for the element

a. The element is in the elastic range:

$$\sigma_i(t) = E_{sp} \cdot \varepsilon_i(t) \tag{108}$$

$$\varepsilon_{res,i} = 0 \tag{109}$$

b. The element is in the plastic range:

$$\sigma_i(t) = E_{sp} \cdot \varepsilon_{yk}(t) + E_p \cdot (\varepsilon_i(t) - \varepsilon_{yk}(t)) \tag{110}$$

$$\varepsilon_{max,i} = \varepsilon_i(t) \tag{111}$$

$$\varepsilon_{res,i} = \varepsilon_i(t) - \frac{\sigma_i(t)}{E_{sp}} \tag{112}$$

c. The element is in the elastic range again, but with a residual deformation that has been calculated at point b:

$$\sigma_i(t) = E_{sp} \cdot (\varepsilon_i(t) - \varepsilon_{res,i}) \tag{113}$$

And the tension at the end:

$$T_i(t) = \sigma_i(t) \cdot A \tag{114}$$

11. If $\varepsilon_i(t) > \varepsilon_{uk}$ the element broke and the procedure stops;
12. The tensions give the resultant of the forces in x and y direction at every node, knowing the positions:

$$F_{y,i}(t) = \frac{y_{i-1}(t) - y_i(t)}{l_i(t)} \cdot T_i(t) + \frac{y_i(t) - y_{i+1}(t)}{l_{i+1}(t)} \cdot T_{i+1}(t) \tag{115}$$

$$F_{x,i}(t) = \frac{x_{i-1}(t) - x_i(t)}{l_i(t)} \cdot T_i(t) + \frac{x_i(t) - x_{i+1}(t)}{l_{i+1}(t)} \cdot T_{i+1}(t) - 9.81 \cdot m_{li} \tag{116}$$

13. The acceleration is considered constant in the time step:

$$a_{x,i}(t) = \frac{F_{x,i}(t)}{m_{li}} \tag{117}$$

$$a_{y,i}(t) = \frac{F_{y,i}(t)}{m_{li}} \tag{118}$$

14. The velocity is considered constant in the time step:

$$v_{x,i}(t) = v_{x,i}(t - ts) + a_{x,i}(t) \cdot ts \tag{119}$$

$$v_{y,i}(t) = v_{y,i}(t - ts) + a_{y,i}(t) \cdot ts \tag{120}$$

15. The time is incremented by one time step and the procedure restarts from point 4.

At the end of the loop there are all the parameters to get the maximum tension and displacement, whether the cable breaks or not:

$$T_{max} = max\big(T_i(t)\big) \tag{121}$$

$$u = max\big(y_w(t) - y_w(0)\big) \tag{122}$$

The velocity of interest will be the value at the timestep in which the displacement is equal to the one defined at the beginning of the process, depending on the breakage of the cable or the reaching of the next element.

11. Unique solution

The procedures described so far allows to define requirements and results for a cable but with a separated analysis of the sliding and no-sliding models. These two implementations present the two extreme behaviours that the cable and the vehicle have while in contact, but the true interaction will not be either of them, but a combination and the result will be in between. The idea is that if the inclination is low the real behaviour will be closer to the sliding model, if the inclination is high the real behaviour will be closer to the no-sliding model. Following this reasoning the following statements are considered:

- If the cable is vertical the real response is the no-sliding model;
- If the cable is horizontal the real response is the sliding model;
- In between the real response is a weighted mean.

The weighted mean is calculated in relation of the angle γ, as presented in the following procedure. Consider the subscript "ns" for a value representing the no-sliding model and "s" for the sliding model:

1. A parameter is selected in both models, that could be the displacement as well as a minimum number of strands required or even other elements;
2. With equation (123) unique result is calculated:

$$z = z_{ns} \cdot \frac{2 \cdot \gamma}{\pi} + z_s \cdot \frac{\pi - 2 \cdot \gamma}{\pi} \tag{123}$$

12. Relation between the two approaches

A combination of the energy and the step by step approaches allows to define if during an impact one or more cables will be affected and how they will be affected. In consideration now of the simplification made before for specific situations, taking as an example the breakage condition the equations (31), (58) and (86) give the minimum requirements and two things can happen:

1. The cable breaks and the maximum displacement is smaller than the spacing. When this happens, right at the moment of the breakage the vehicle will have a new velocity with which the new kinetic energy is calculated and the same process described so far is repeated for a new cable;
2. The cable breaks or not but with a displacement that should be greater than the spacing. When this happens, it means that before reaching the maximum displacement one or more cables are affected as well and they contribute to the absorption of the kinetic energy. As in the previous point, every time that the following cable is reached a new kinetic energy is calculated depending on the velocity of the vehicle.

The second point is the one needing further implementation, so in relation to this the following process is followed in consideration of both the models:

1. Given a spacing b, the velocity that the impacted point has when $y_w(t)$-$y_w(0)$=b is calculated with the step by step approach. It is likely that for the no-sliding model the maximum displacement is smaller than the spacing, in this case it is considered the velocity at the breakage, assuming it does not decrease before the next cable;
2. At $y_w(t)$-$y_w(0)$=b the energy approach gives the tension. If the maximum displacement is smaller than the spacing, it is the ultimate resistance tension. The tension just calculated is going to be the new initial tension;
3. The starting hypothesis is that two cables are going to be affected by the impact, without reaching further ones. The energy absorbed at the maximum displacement of the first cable is calculated considering Figure 5. The energy absorbed now by the two cables has to be calculated with the following steps:

a. no-sliding model:
   i. assuming as said before that the maximum displacement is smaller than the spacing, the calculations are performed twice like two separated cables, where the only difference is the velocity that has decreased for the second element;
b. Sliding model:
   i. The new initial conditions of the first cable are:

$$\varepsilon_0 = \varepsilon(u = b) \tag{124}$$

$$p = \frac{\sigma(u = b)}{f_{ptk}} \tag{125}$$

   ii. when the first cable reaches the second, they work together and they could stop the vehicle without breaking the first one. The energy absorbed at the maximum displacement by both elements is calculated and it is checked if it is greater or smaller than the kinetic energy, using equation (123);
   iii. if it is greater it means that the displacement is too high, consequently the first cable will not break. The calculations are repeated to find the right displacement. if it is smaller it means the first cable breaks and the second cable keeps deforming;
   iv. if the first cable is to break, the second cable has now new initial conditions and it is analysed like an isolated cable.

4. if two cables cannot resist to the impact or further cables are reached, the calculations are performed for as many elements as necessary.

13. Numerical results

After having presented the procedure to define the generic requirements, in order to have the numerical limits some parameters have to be fixed. In general, for specific bridges there could be some limits in terms of maximum weight of the vehicles or maximum speed, consequently these cases can be studied with the use of these limits. The following study is presented as a generic situation where no limits are considered, hence the maximum size in terms of mass is taken as 40 tons, with the relative maximum velocity of 80 Km/h. A vehicle with this mass is a big truck, with a height of about 3m and a centre of mass that could be considered in the middle at 1.5m. The inclination of the cable is low, 20°. The following graphs present the results with the parameters stated above.

13.1. Sliding model, first type of impact

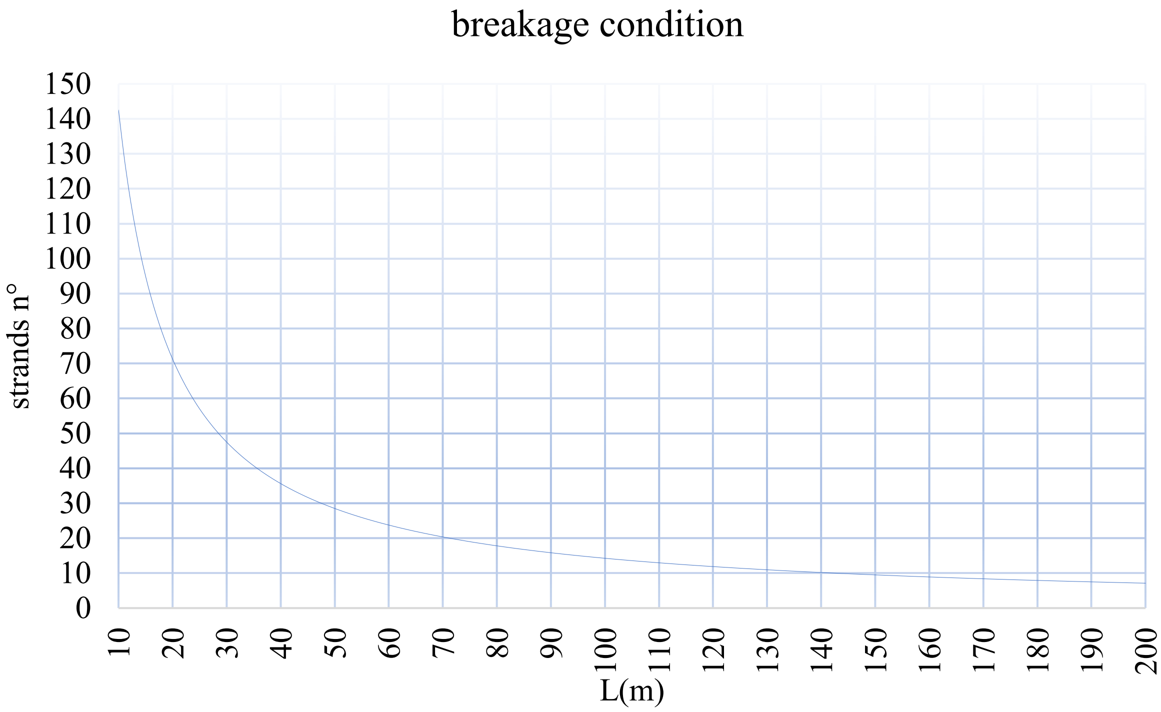

Figure 17-breakage condition for sliding model, first type of impact

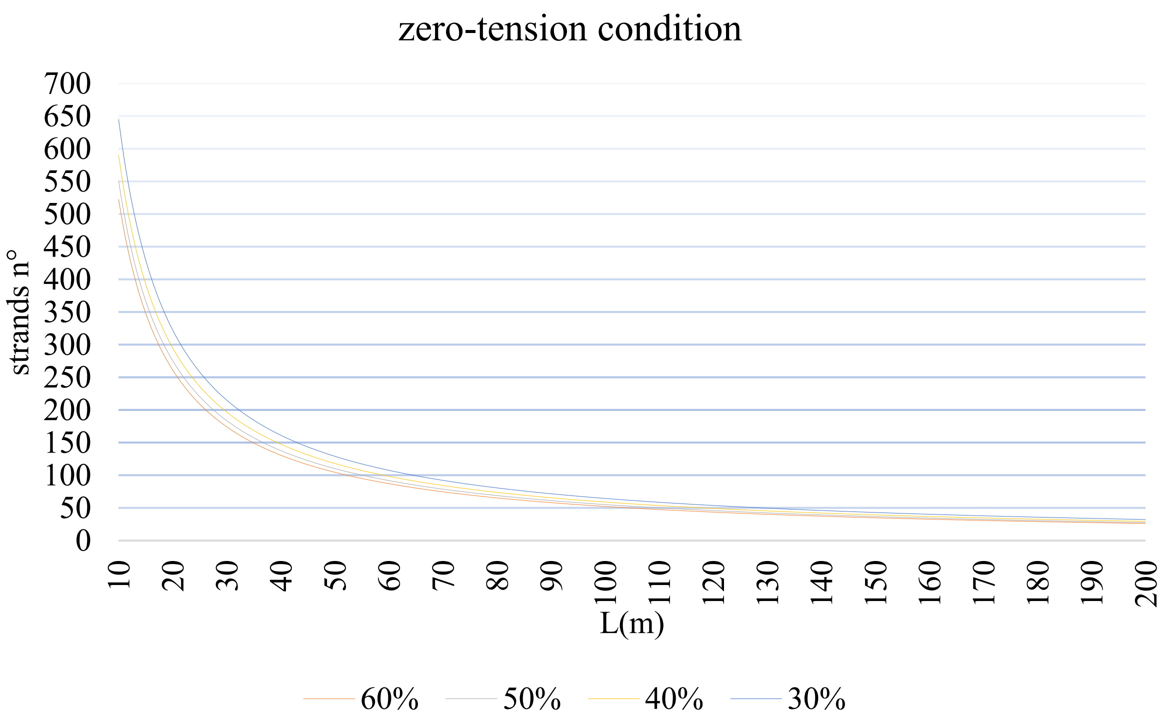

Figure 18-zero-tension condition for sliding model, first type of impact

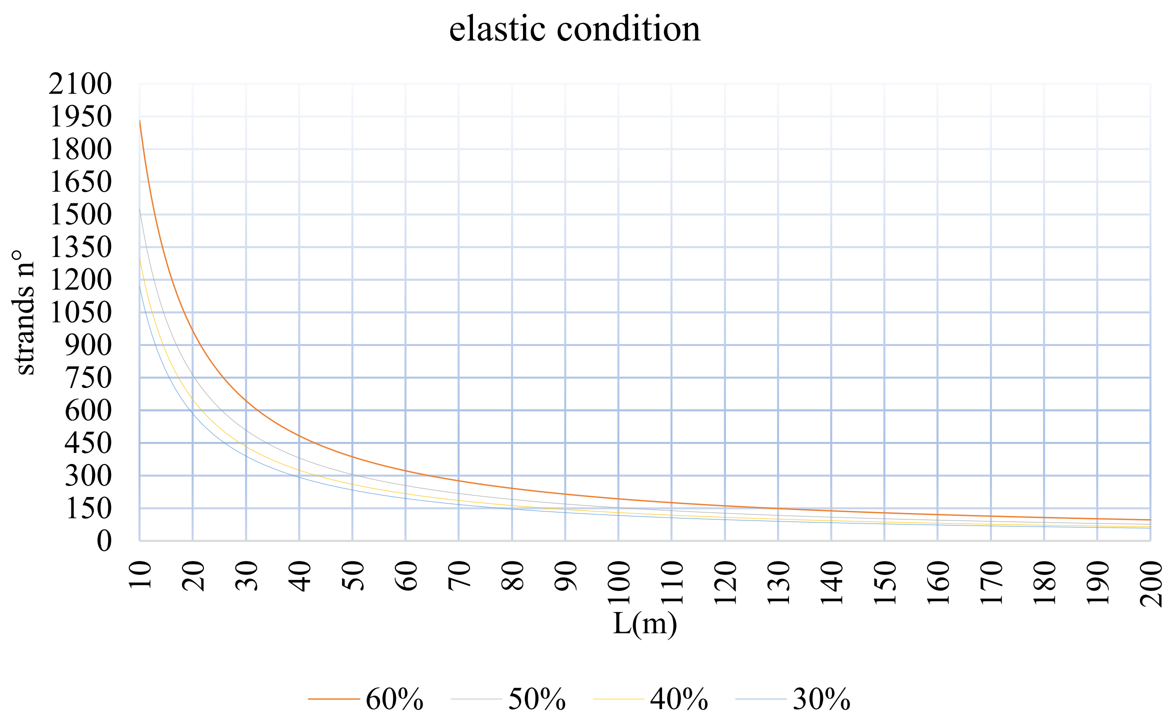

Figure 19-elastic condition for sliding model, first type of impact

Considering now 40% of initial tension as an example:

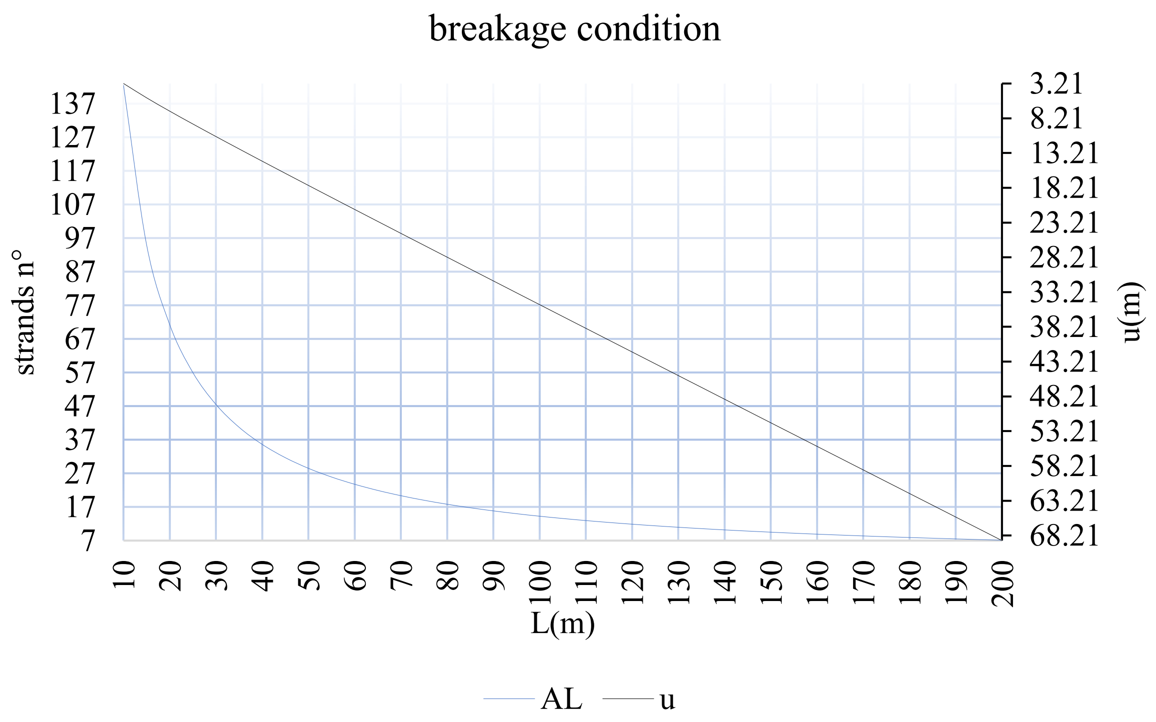

Figure 20-displacement in breakage condition for sliding model, first type of impact, initial tension at 40%

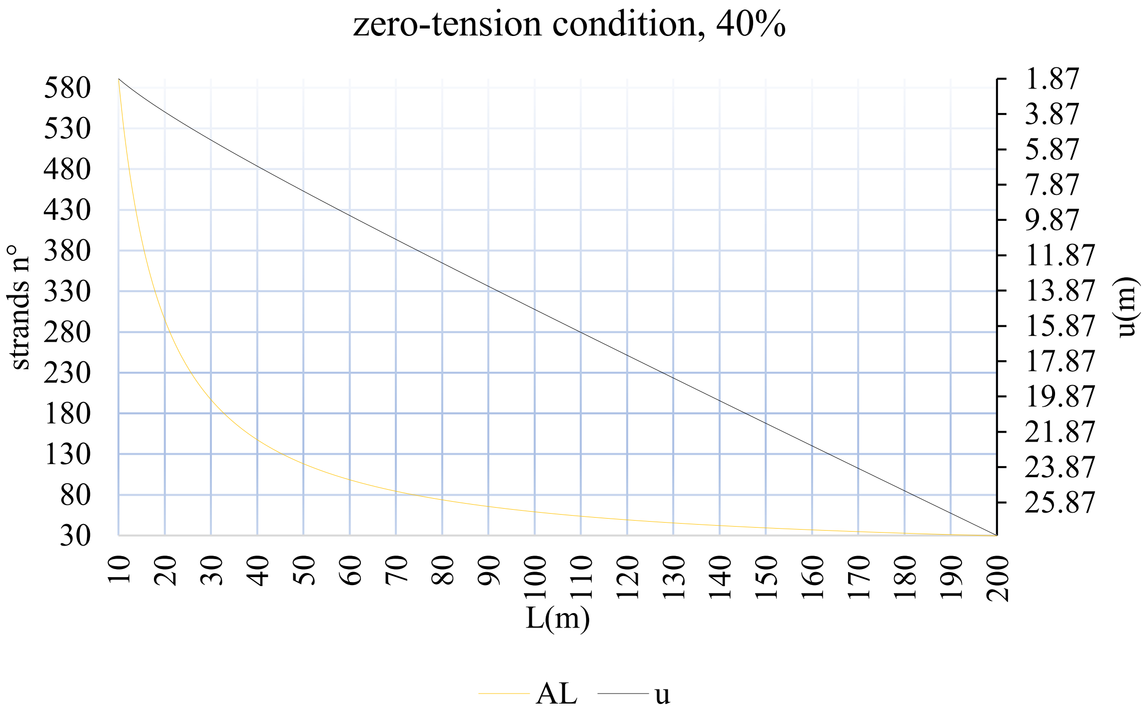

Figure 21-displacement in zero-tension condition for sliding model, first type of impact, initial tension at 40%

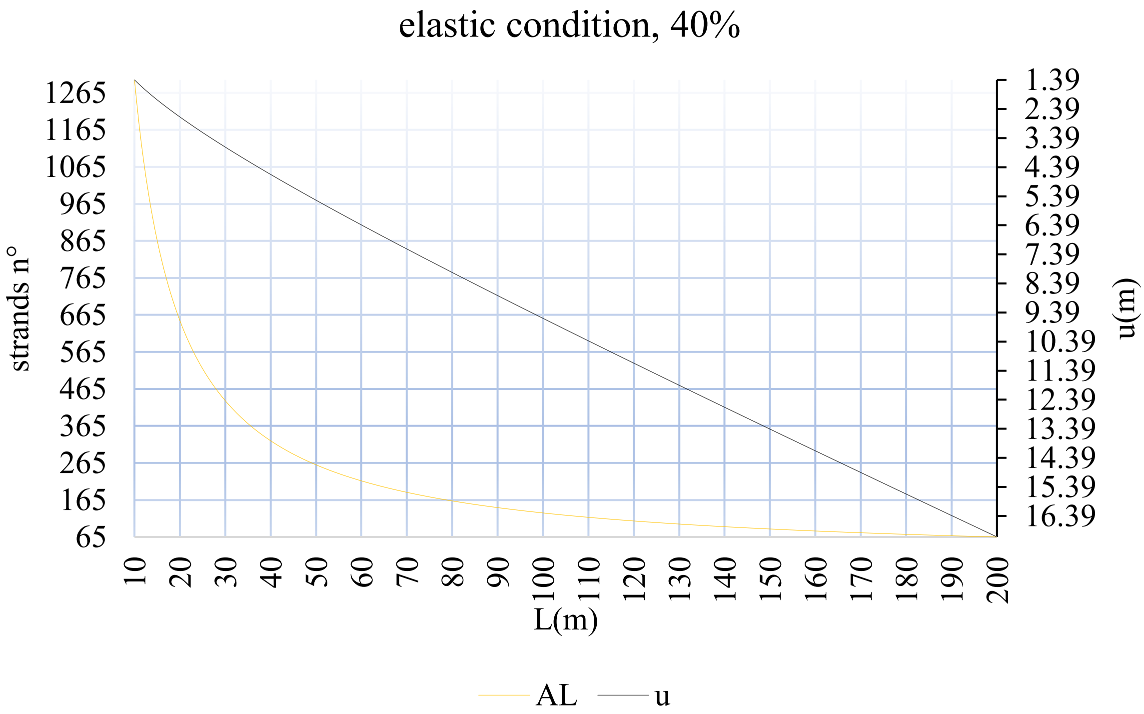

Figure 22-displacement in elastic condition for sliding model, first type of impact, initial tension at 40%

The plots can be merged in order to compare the different conditions and displacements, as shown in Figure 23 and Figure 24:

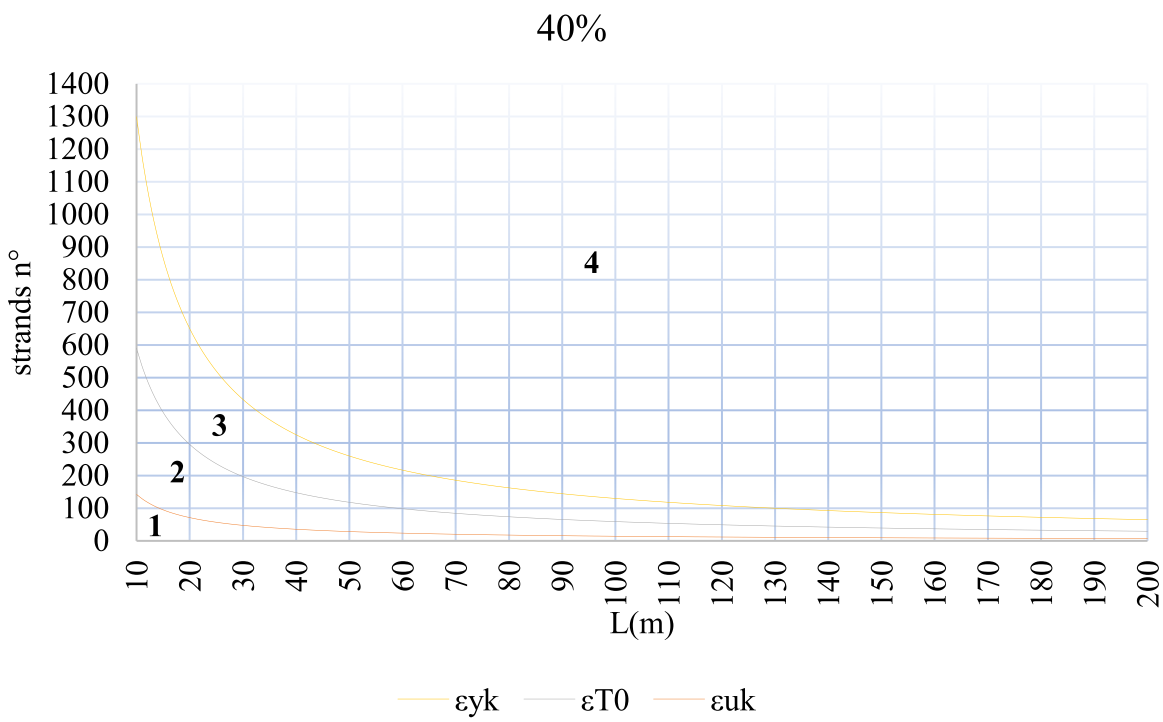

Figure 23-representations of the conditions for sliding model, first type of impact, initial tension at 40%

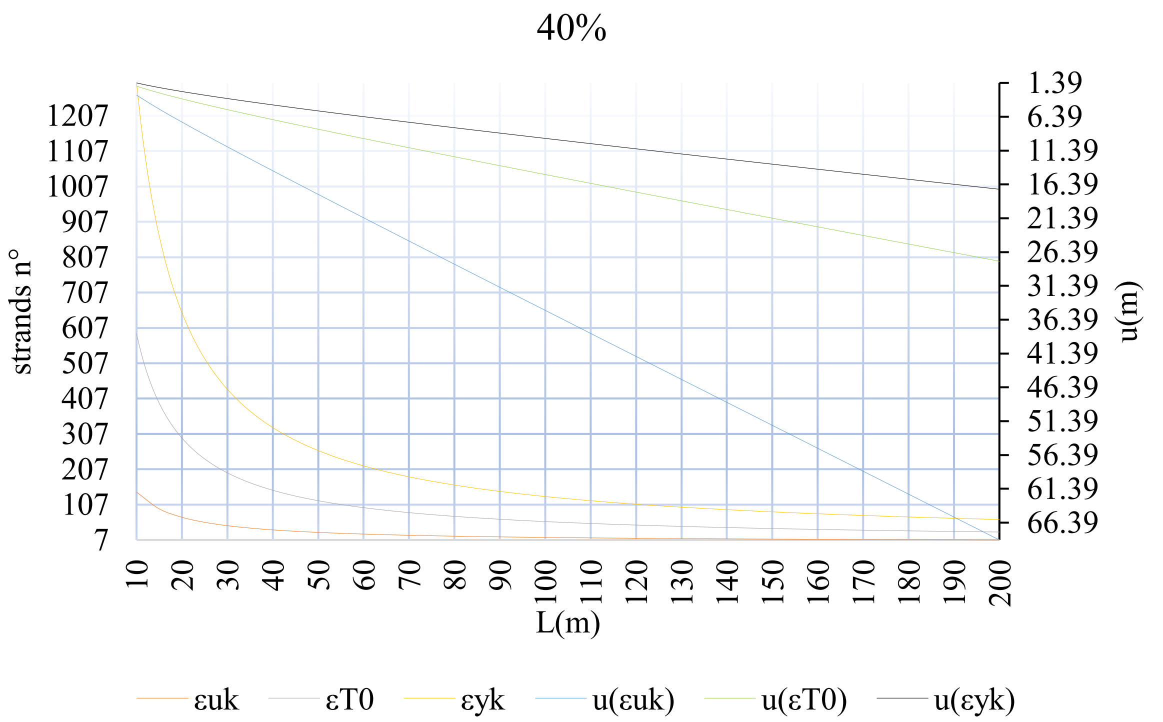

Figure 24-representation of the displacements for sliding model, first type of impact, initial tension at 40%

Analysing Figure 23, it can be seen how the curves divide the space into four different parts:

1. In this part the combination of area and length of the cable gives a value that is too small and the cable is not able to withstand the impact, therefore it breaks;
2. In this part the combination of area and length is big enough to not break, but when it goes back in place it keeps a residual deformation and the tension is totally lost;

3. In this part the combination of area and length is big enough to absorb all the energy, to not break and at the end when it goes back in place it still has some elastic tension and a small residual deformation;
4. In this part the combination of area and length of the cable is that big that allows the cable to stay in its elastic range.

13.2. No-sliding model, first type of impact

For this case the values are constant because all the parameters are fixed, the results are reported in Table 4:

Table 4-limits for no-sliding model, first type of impact

p	30%		40%		50%		60%	
condition	strands n°	u(m)	strands n°	u(m)	strands n°	u(m)	strands n°	u(m)
breakage	325	0.149	325	0.145	325	0.14	325	0.136
zero-tension	1471	0.0397	1346	0.0399	1257	0.04	1194	0.0402
elastic	2663	0.0258	2967	0.0214	3477	0.0169	4402	0.0125

These results are for an inclination of 20°, in Figure 25 and
Table 5 it can be seen how the numbers vary depending on γ for an initial tension at 40%:

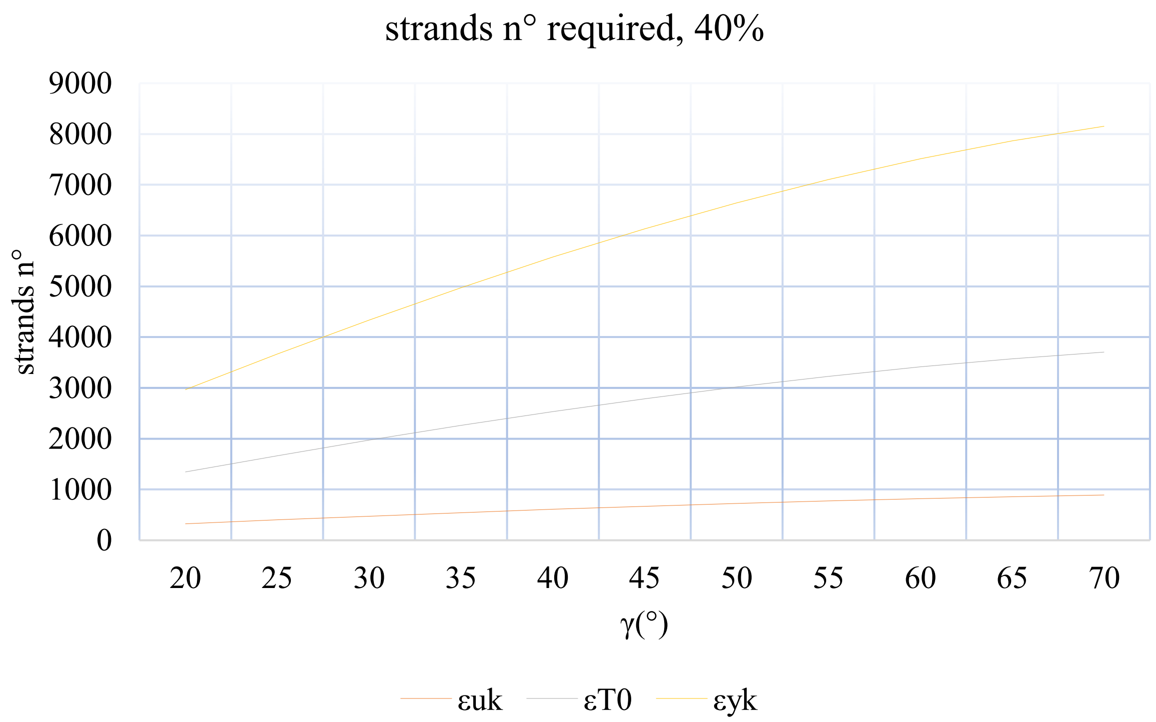

Figure 25-variation of the requirement depending on the inclination, for no-sliding model, first type of impact

Table 5-variation of the requirements depending on the inclination, for no-sliding model, first type of impact

	breakage	**zero-tension**	**elastic**
γ(°)	Strands n°	Strands n°	Strands n°
20	325	1346	2967
25	401	1666	3666
30	474	1971	4337
35	544	2262	4975
40	610	2534	5576
45	671	2788	6134
50	727	3020	6645
55	777	3230	7106
60	822	3415	7513
65	860	3574	7862
70	891	3706	8152

13.3. Sliding model, second type of impact

The graphs given in Figure 17, Figure 18 and Figure 19 are still valid, the only thing changing is the displacement due to the different geometry.

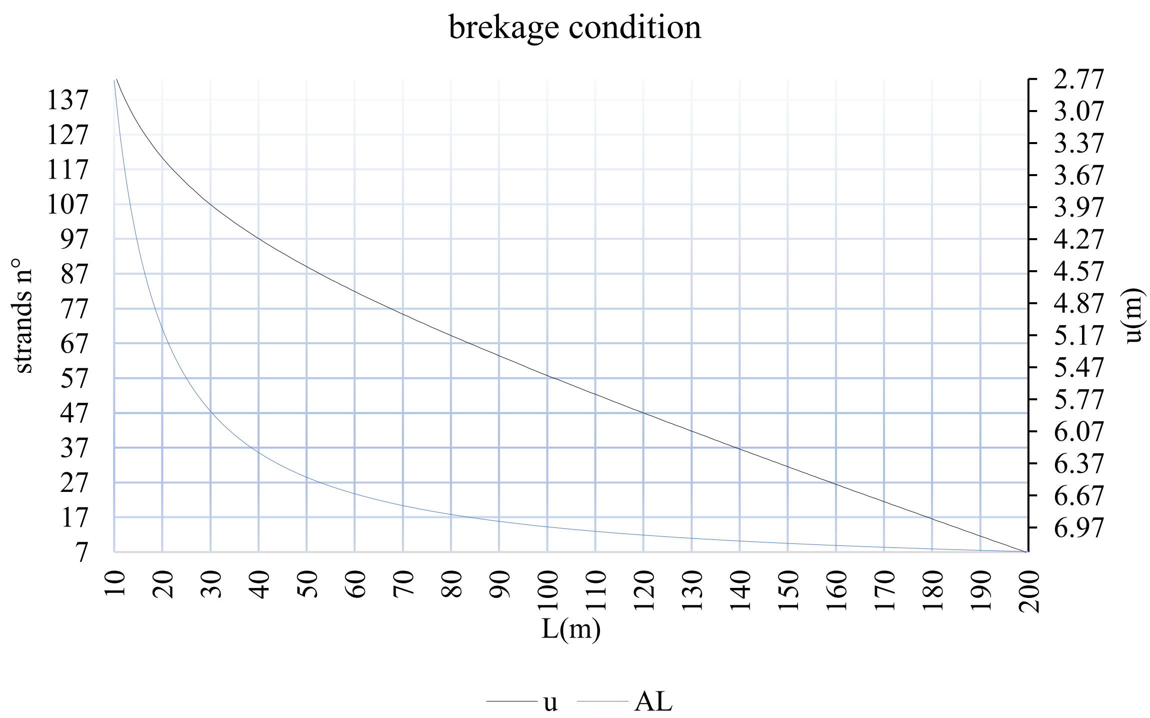

Figure 26-displacement in breakage condition for sliding model, second type of impact, initial tension at 40%

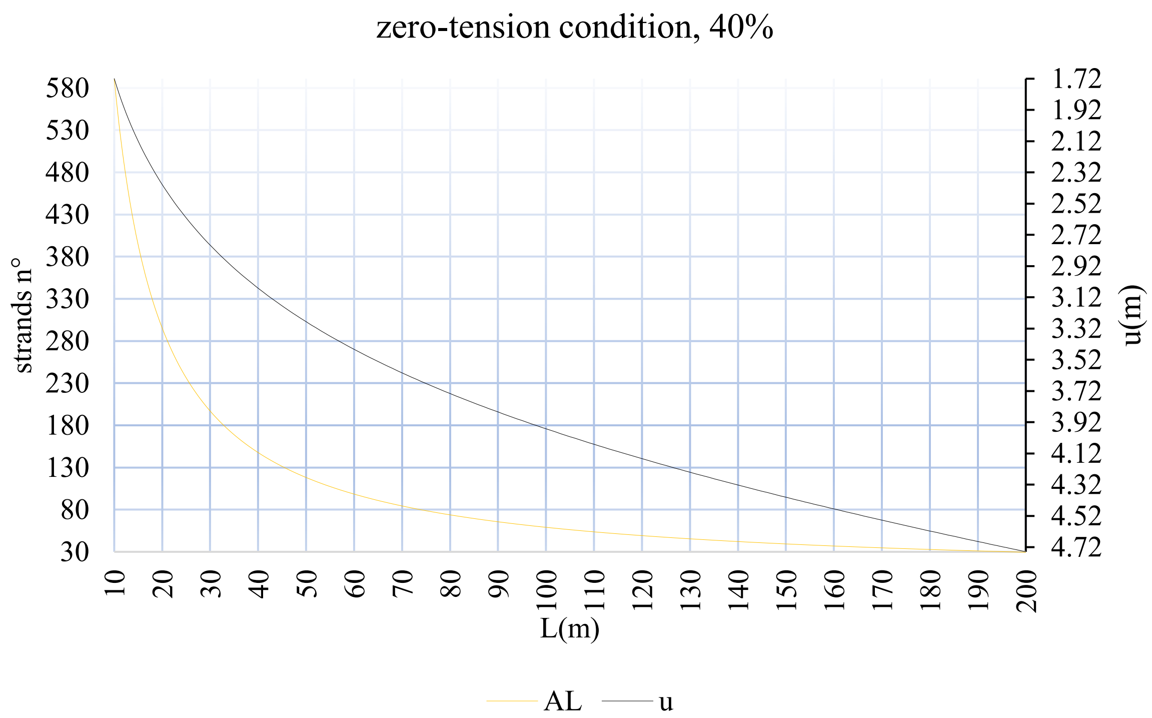

Figure 27-displacement in zero-tension condition for sliding model, second type of impact, initial tension at 40%

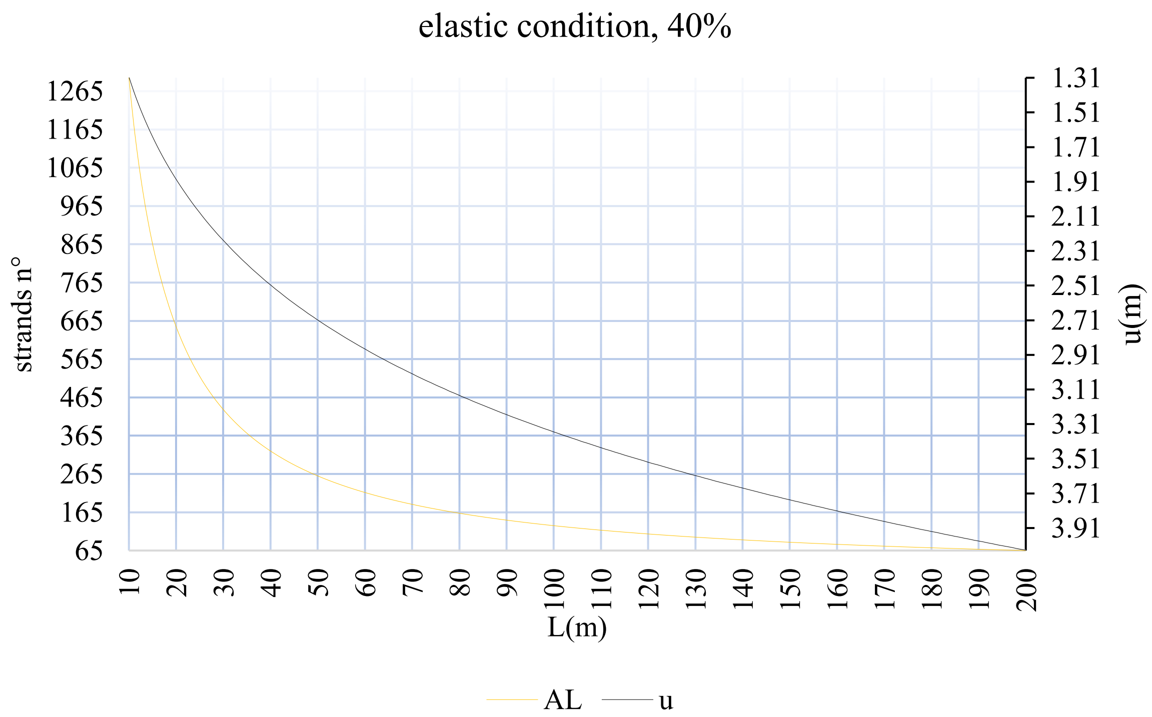

Figure 28-displacement in elastic condition for sliding model, second type of impact, initial tension at 40%

The plots can be merged in order to compare the different conditions and displacements, as shown in Figure 29:

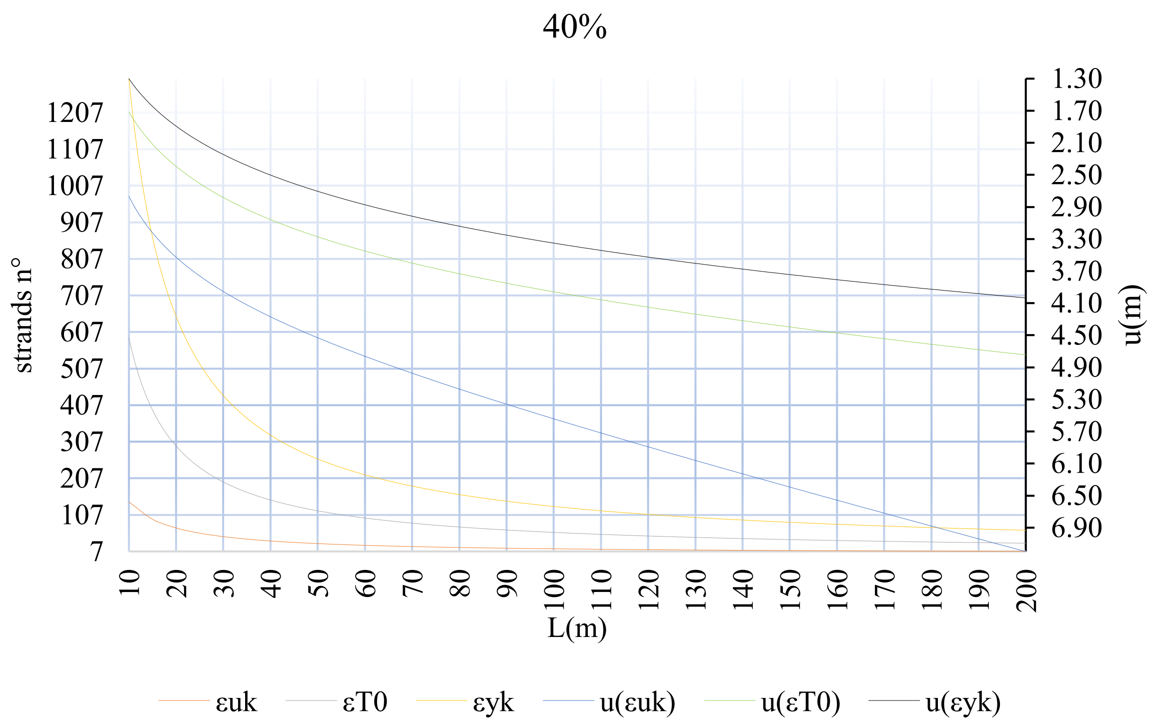

Figure 29-representation of the displacements for sliding model, second type of impact, initial tension at 40%

13.4. No-sliding model, second type of impact

As said previously in the paper, under specific circumstances some simplifications can be made and equation (86) can be used. The limit length given in equation (85) will allow to decide if the simplified equation is valid or not. Considering as an example an initial tension at 40%:

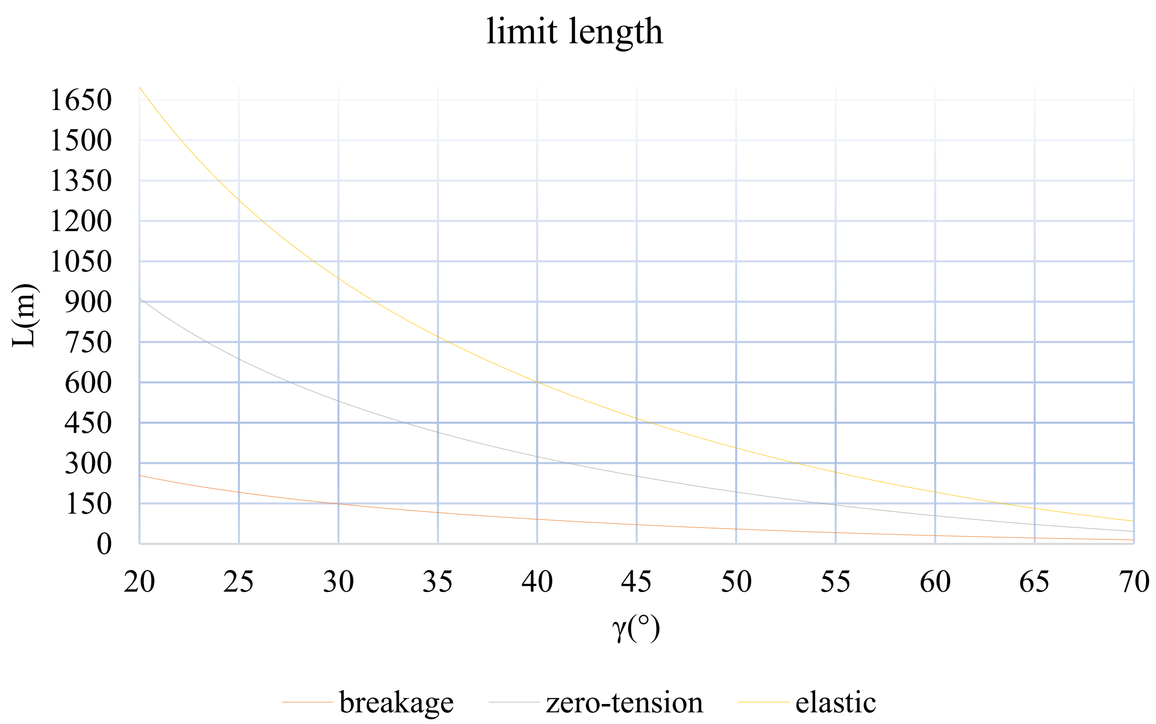

Figure 30-limit length for the three different conditions for no-sliding model, second type of impact

If the length is smaller than the length defined in Figure 30 then equation (86) is valid, otherwise the iterative procedure given in equations (63)-(73) has to be used. Given the inclination of 20° the simplified procedure can be followed for the lengths studied here.

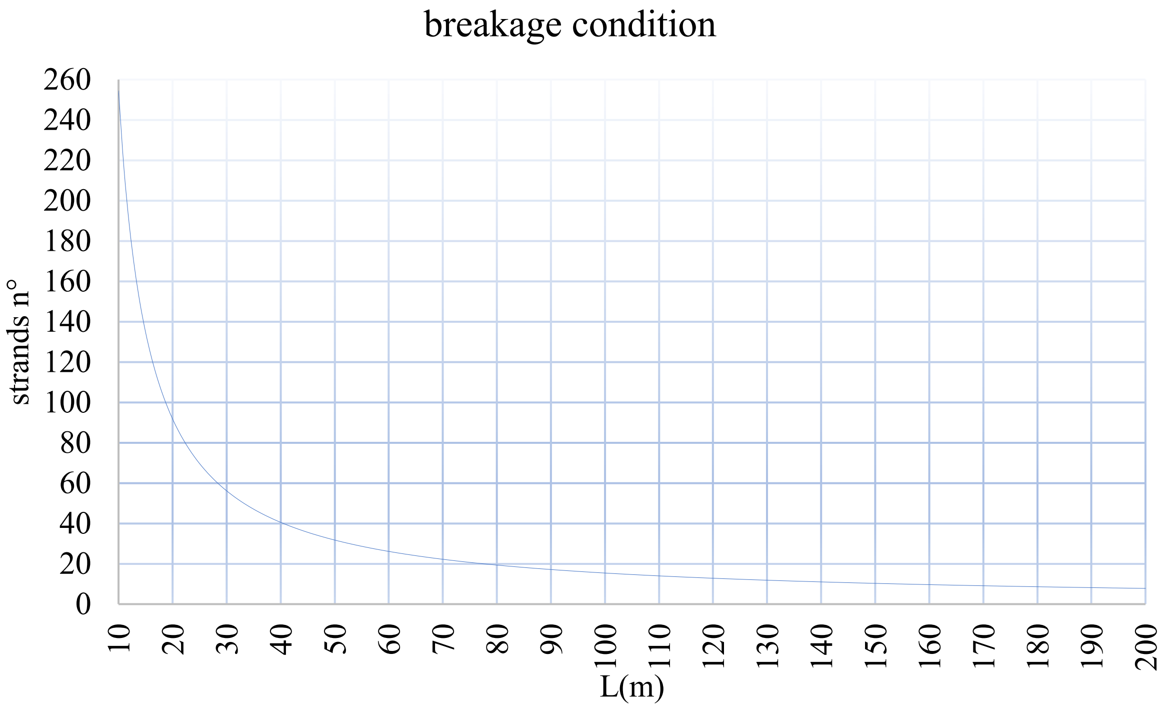

Figure 31-breakage condition for no-sliding model, second type of impact

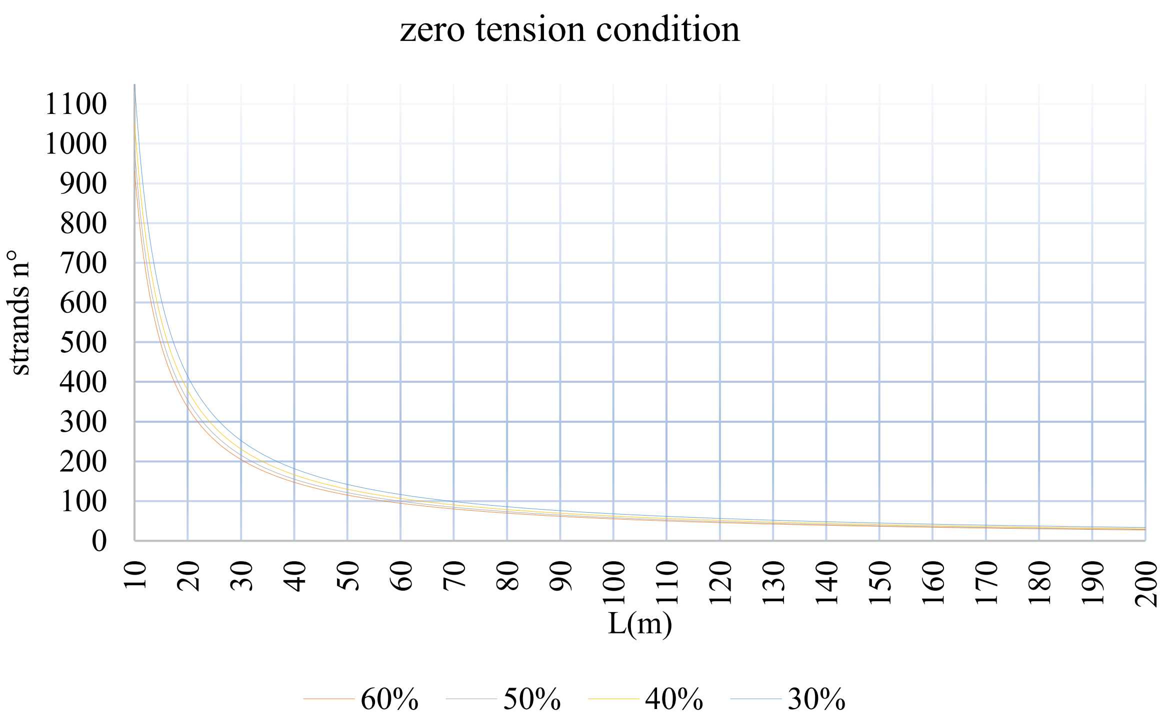

Figure 32-zero-tension condition for no-sliding model, second type of impact

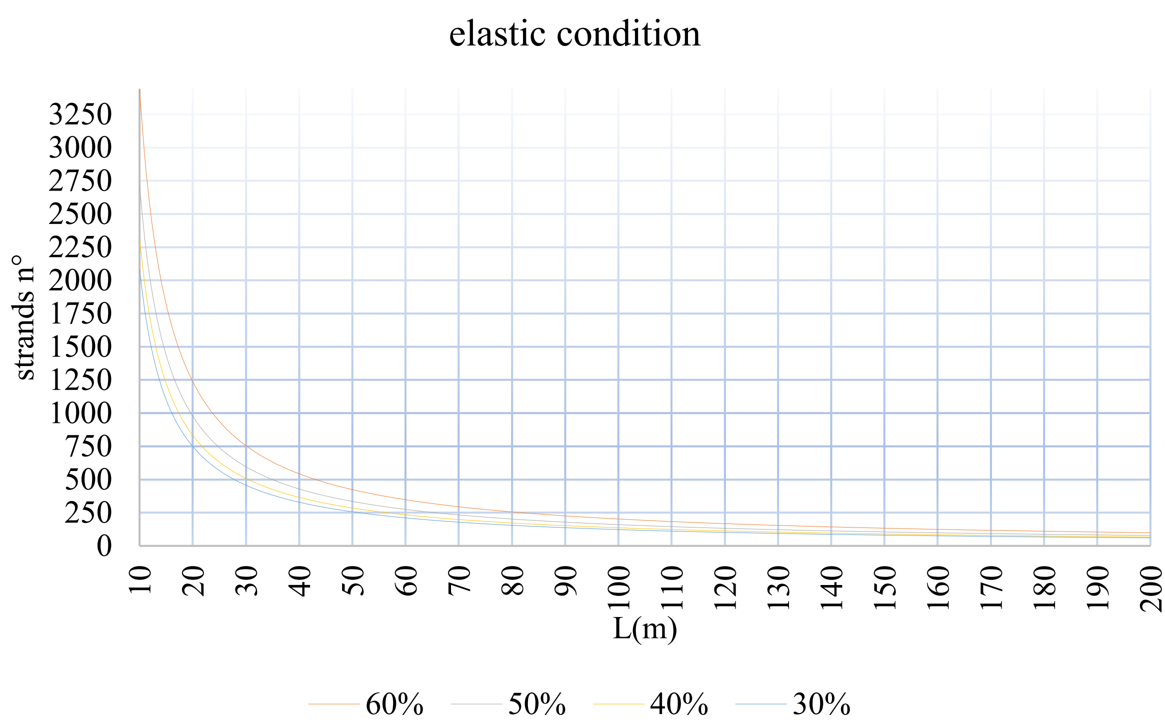

Figure 33-elastic condition for no-sliding model, second type of impact

Considering now 40% of initial tension as an example:

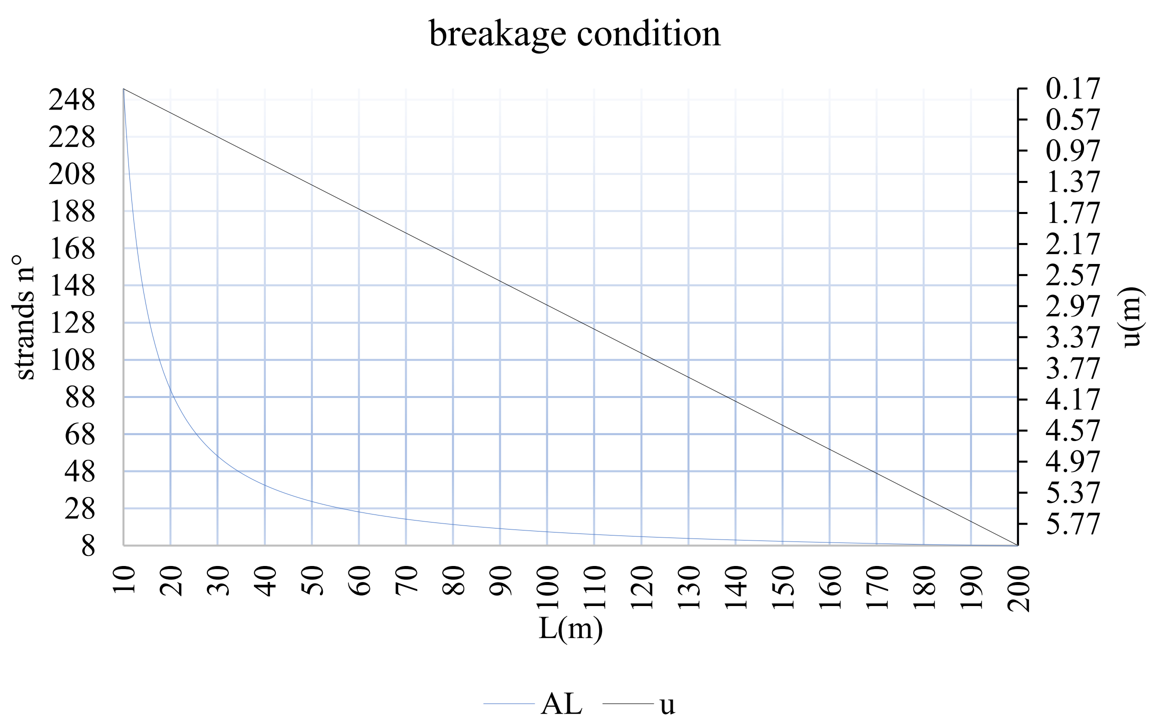

Figure 34-displacement in breakage condition for no-sliding model, second type of impact, initial tension at 40%

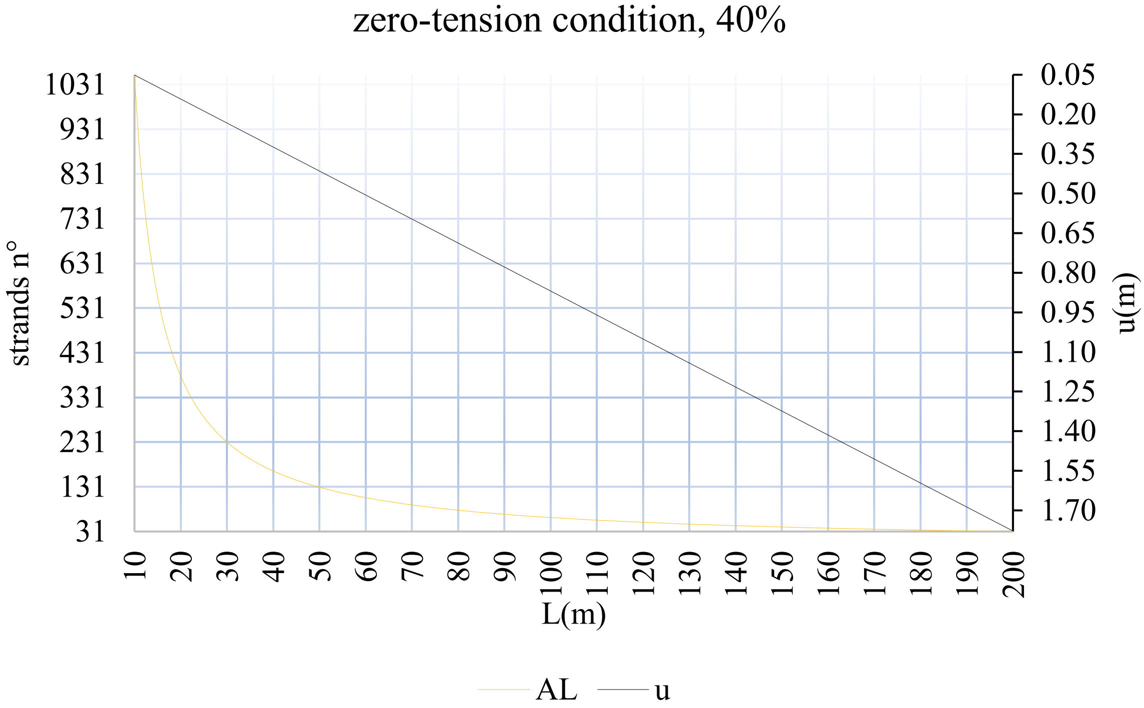

Figure 35-displacement in zero-tension condition for no-sliding model, second type of impact, initial tension at 40%

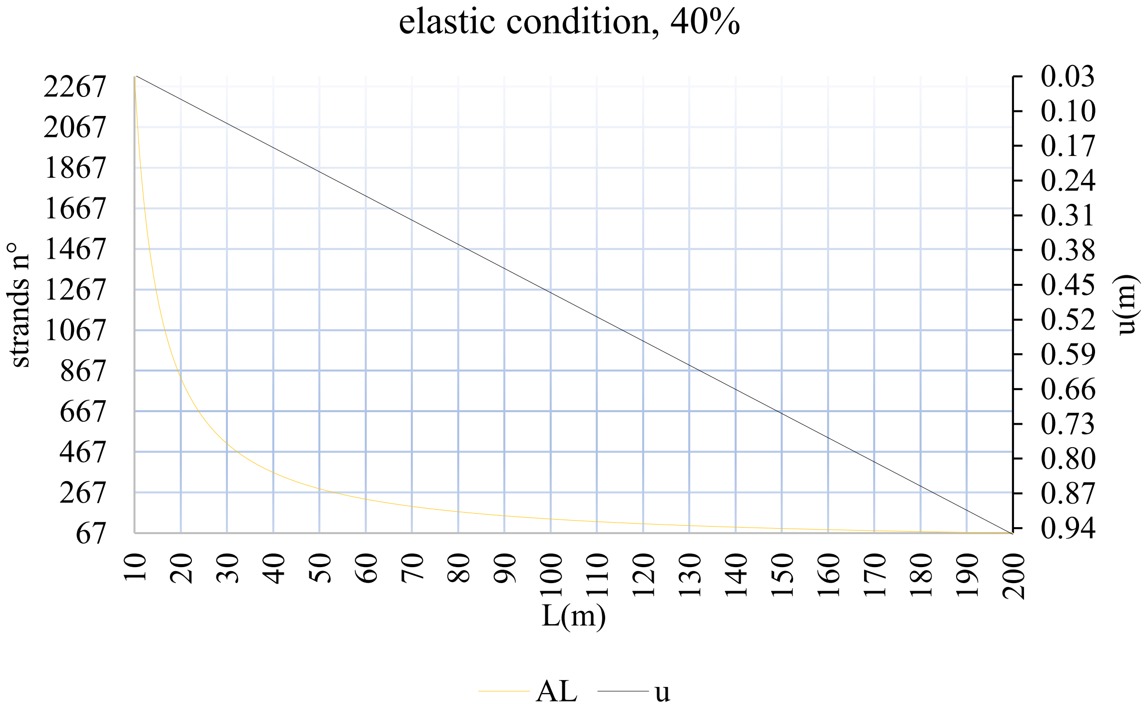

Figure 36-displacement in elastic condition for no-sliding model, second type of impact, initial tension at 40%

And again, the merged graphs in Figure 37 and Figure 38:

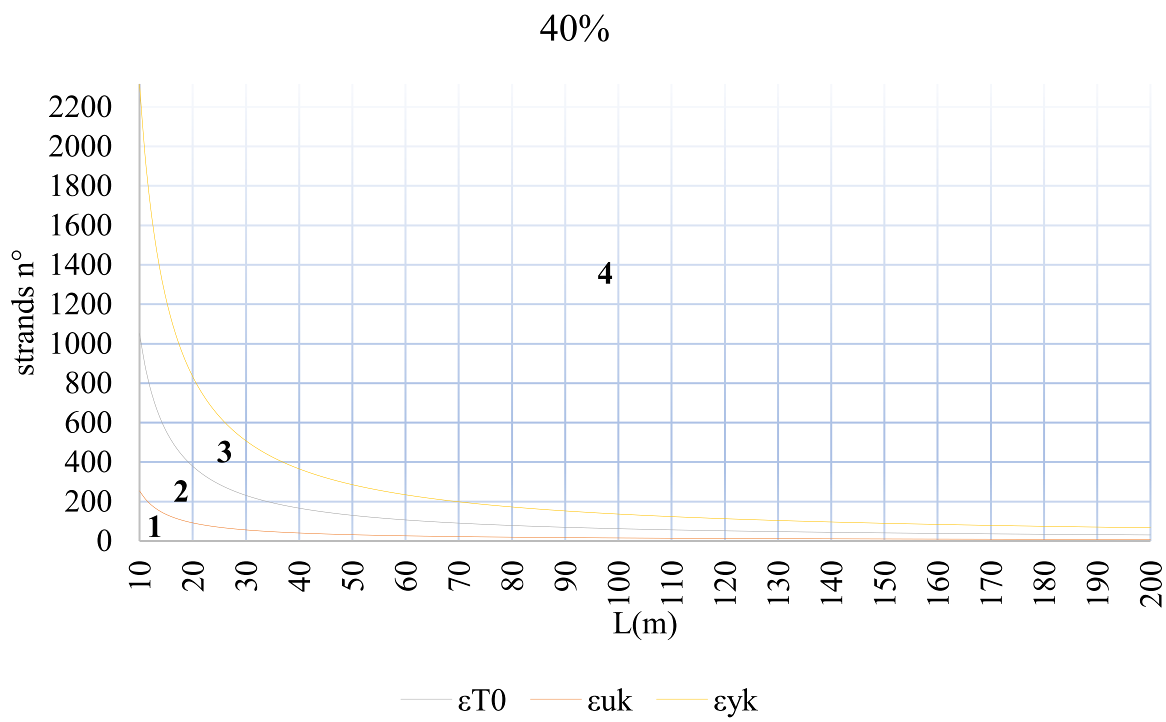

Figure 37-representations of the conditions for no-sliding model, second type of impact, initial tension at 40%

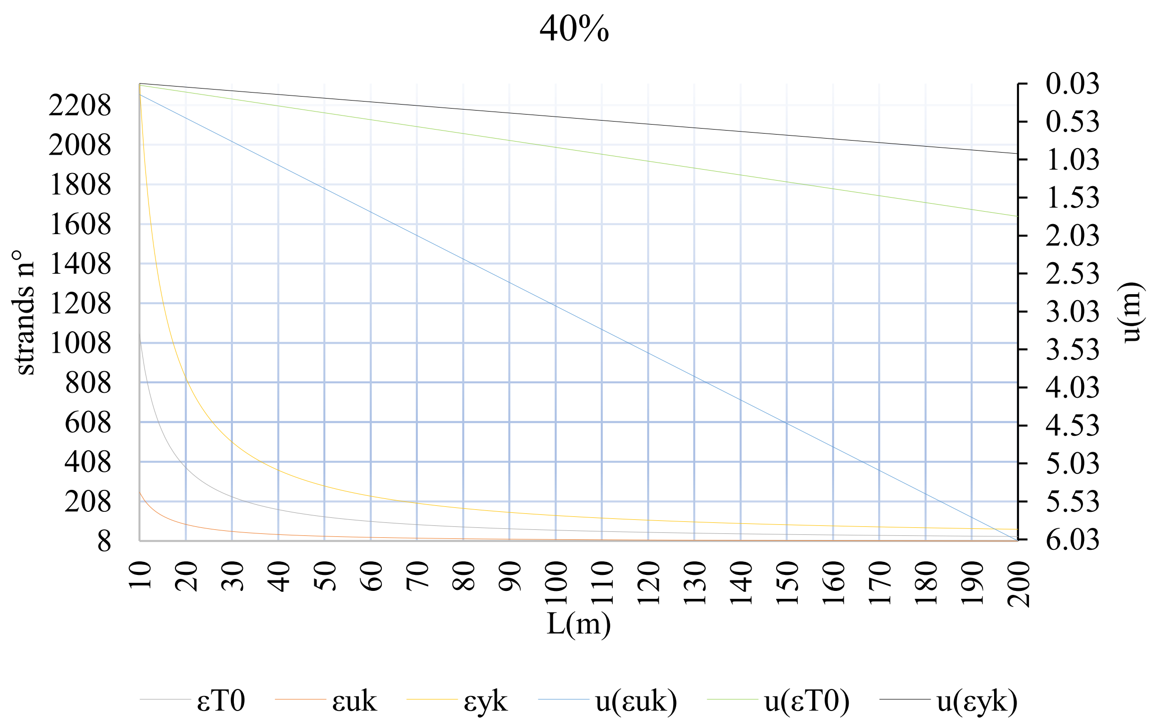

Figure 38-representation of the displacements for no-sliding model, second type of impact, initial tension at 40%

13.5. First type of impact, unique solution

Number of strands defined with the weighted mean:

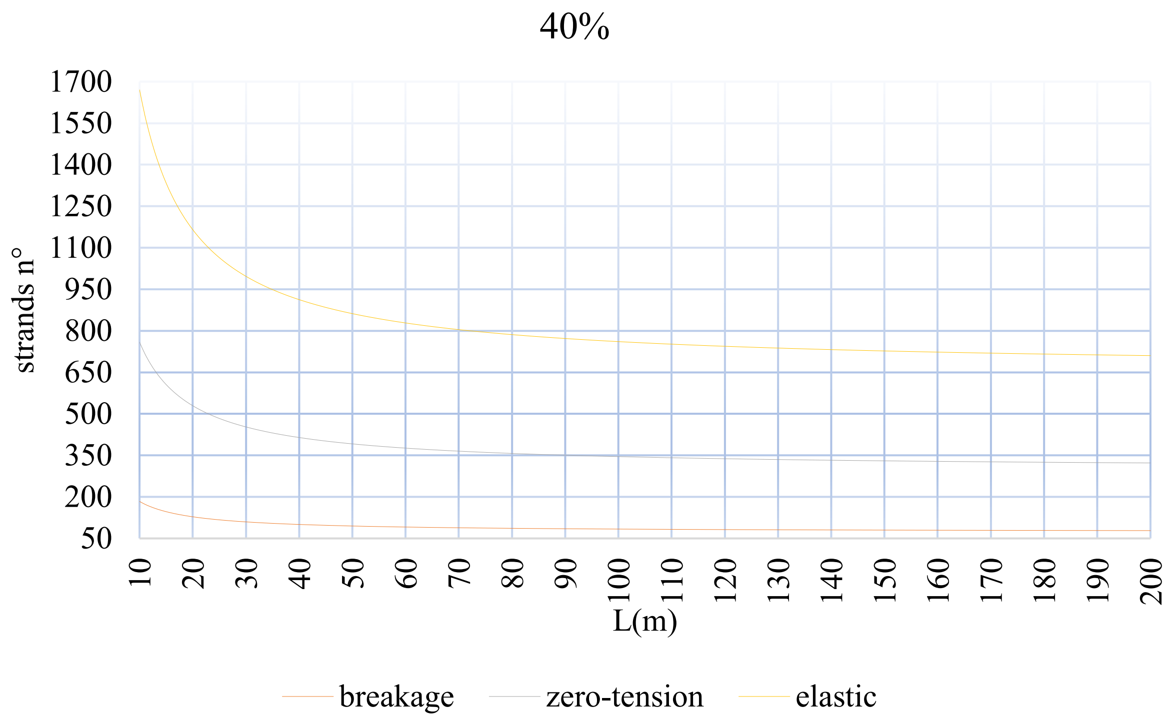

Figure 39-strands n° required for a specific length, first type of impact

13.6. First type of impact, unique solution

Number of strands defined with the weighted mean:

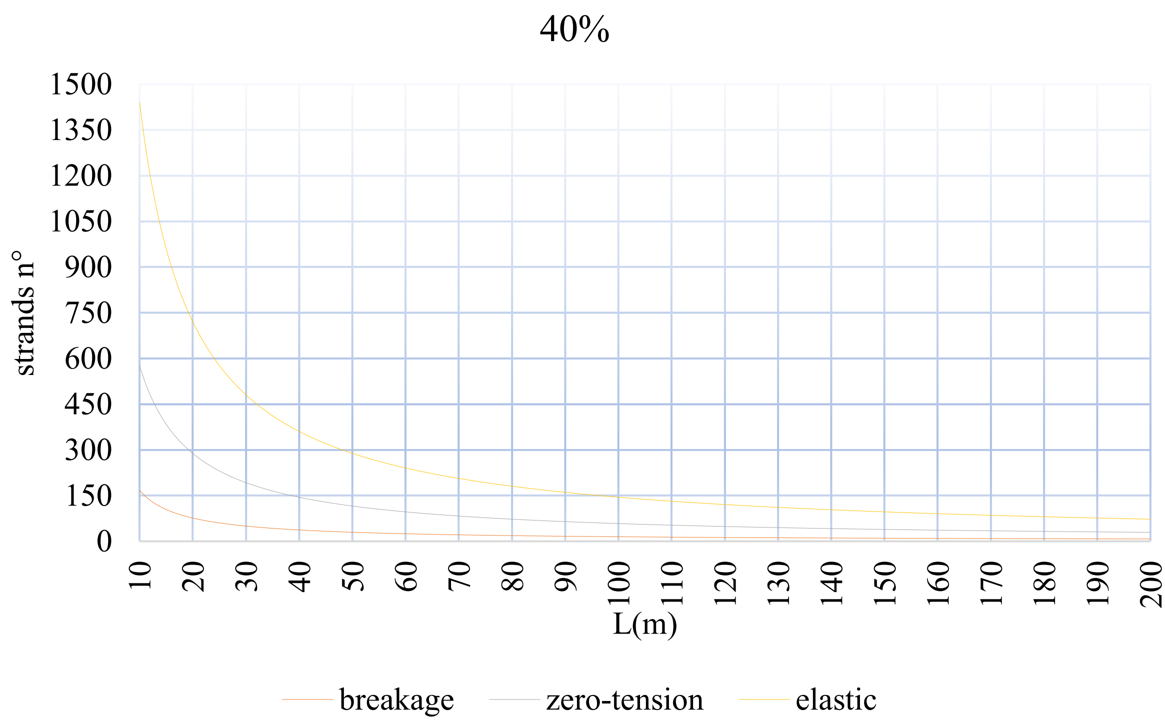

Figure 40- strands n° required for a specific length, second type of impact

14. Cables performance, example

Given the previous results, it is possible now to define what happens when the vehicle collides against a specific setup of cables. Consider the configuration given in Figure 41 and the first type of impact:

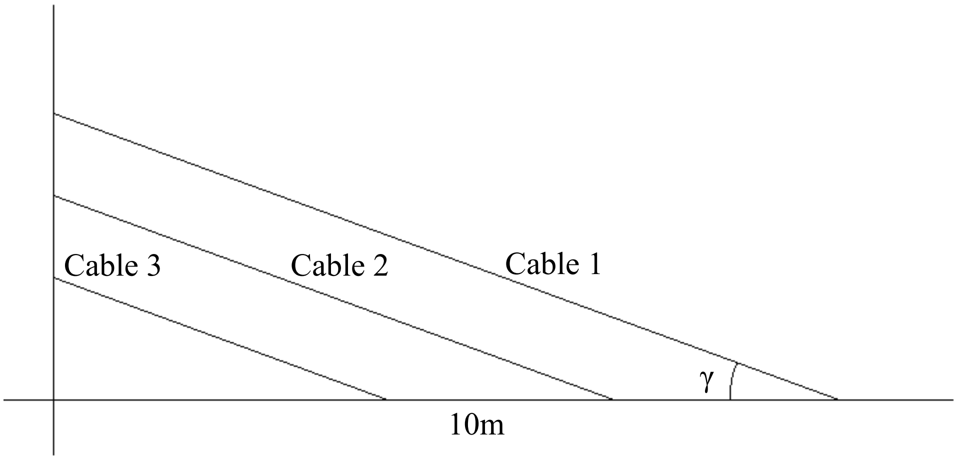

Figure 41-configuration of the set of cables

- Cable 1: 100m long, 50 strands;
- Cable 2: 89.36m long, 50 strands;
- Cable 3: 78.72m long, 50 strands.

The reported lengths are to be considered as strained lengths, the initial tension is at 40%.

$$\varepsilon_0 = 0.4 \cdot \frac{1860}{195000} = 3.82 \cdot 10^{-3} \tag{126}$$

Starting from cable 1, in considerations of both models, the total response is going to be defined. The minimum requirements for the length of 100m, starting from the breakage condition, are defined with the use of Figure 17 and Table 4 (result given by Figure 39):

- Sliding model, minimum number of strands 14;
- No-sliding model, minimum number of strands 325.

With the use of equation (123) the minimum value to not have breakage is:

$$strands\ n° = 325 \cdot \frac{20}{90} + 14 \cdot \frac{90-20}{90} \sim 84 \tag{127}$$

The result just found says that the cable alone will break, even if with the sliding model it would not. For this reason, the maximum displacement of the no-sliding model is the one at the breakage, while for the sliding model it is calculated depending on the strain required, using equation (20), then (44) and at the end the iterative procedure given by equations (46)-(50):

- Sliding model, maximum displacement 16.51 m;
- No-sliding model, maximum displacement 0.15 m.

$$u = 0.15 \cdot \frac{20}{90} + 16.51 \cdot \frac{90 - 20}{90} = 12.87\ m \tag{128}$$

It means that the cable before breaking has reached the second cable and they start to work together and using again the two models the procedure is repeated but now with two cables. The velocity at 10 m is defined with the step by step approach:

- Sliding model, 55.37 Km/h.
  The maximum displacement reached by the first cable is considered as a start for the second cable:

$$u_2 = 12.87 - 10 = 2.87\ m \tag{129}$$

$$l'_x = \sqrt{4.12^2 + 2.87^2 - 2 \cdot 4.12 \cdot 2.87 \cdot cos(\pi - \gamma)} = 6.89\ m \tag{130}$$

$$l'_s = \sqrt{85.24^2 + 2.87^2 - 2 \cdot 85.24 \cdot 2.87 \cdot cos(\gamma)} = 82.55\ m \tag{131}$$

$$\Delta L = 6.89 + 82.55 - 89.36 = 0.079\ m \tag{132}$$

$$\Delta\varepsilon = \frac{0.079}{89.36} \cdot (1 + 3.82 \cdot 10^{-3}) = 0.87 \cdot 10^{-3} \tag{133}$$

$$k_2(p) = 0.4 \cdot 1860 \cdot 0.87 \cdot 10^{-3} + \frac{1}{2} \cdot 195000 \cdot (0.87 \cdot 10^{-3})^2 = 0.73\ MPa \tag{134}$$

For the first cable, at 10 m:

$$l'_x = \sqrt{4.12^2 + 10^2 - 2 \cdot 4.12 \cdot 10 \cdot cos(\pi - \gamma)} = 13.94\ m \tag{135}$$

$$l'_s = \sqrt{95.88^2 + 10^2 - 2 \cdot 95.88 \cdot 10 \cdot cos(\gamma)} = 86.55\ m \tag{136}$$

$$\Delta L = 13.94 + 86.55 - 100 = 0.491\ m \tag{137}$$

$$\Delta\varepsilon = \frac{0.491}{100} \cdot (1 + 3.82 \cdot 10^{-3}) = 4.93 \cdot 10^{-3} \tag{138}$$

$$\varepsilon_0 = (3.82 + 4.93) \cdot 10^{-3} = 8.75 \cdot 10^{-3} \tag{139}$$

$$\sigma_0 = 195000 \cdot 8.41 \cdot 10^{-3} + 8260 \cdot (8.75 - 8.41) \cdot 10^{-3} = 1643\ MPa \quad (140)$$

$$p = \frac{1643}{1860} = 0.88 \quad (141)$$

For the first cable, at 12.87 m:

$$u_1 = 12.87m \quad (142)$$

$$l'_x = \sqrt{4.12^2 + 12.87^2 - 2 \cdot 4.12 \cdot 12.87 \cdot cos(\pi - \gamma)} = 16.80\ m \quad (143)$$

$$l'_s = \sqrt{95.88^2 + 12.87^2 - 2 \cdot 95.88 \cdot 12.87 \cdot cos(\gamma)} = 83.90\ m \quad (144)$$

$$\Delta L = 16.80 + 83.90 - 100 = 0.70m \quad (145)$$

$$\Delta\varepsilon = \frac{0.70}{100} \cdot (1 + 3.82 \cdot 10^{-3}) = 7.04 \cdot 10^{-3} \quad (146)$$

$$\varepsilon = (7.04 + 3.82) \cdot 10^{-3} = 10.86 \cdot 10^{-3} \quad (147)$$

$$k_1(p) = \Big(1643 \cdot (10.86 - 8.75) \cdot 10^{-3} + \frac{1}{2} \cdot 8260 \cdot \left((10.86 - 8.75) \cdot 10^{-3}\right)^2\Big) = 3.49\ MPa \quad (148)$$

- No-sliding model, 73.51 Km/h.
  With the assumption of the same displacement, the part of the cable below the impact breaks and the two cables are studied separately.

Now, the total energy absorbed and the velocity are calculated with equation (123):

$$v = 73.51 \cdot \frac{20}{90} + 55.37 \cdot \frac{70}{90} = 59.40\ Km/h \quad (149)$$

$$\frac{1}{2} \cdot 40000 \cdot \left(\frac{59.40}{3.6}\right)^2 = 5.45 \cdot 10^6\ Nm \quad (150)$$

$$\frac{139 \cdot 50}{1 + 3.82 \cdot 10^{-3}} \cdot \left((100 \cdot 3.49 + 89.36 \cdot 0.73) \cdot \frac{70}{90} + (4.37 \cdot 50.23) \cdot \frac{20}{90} \right) = 2.57 \cdot 10^{6}\ Nm \tag{151}$$

The kinetic energy is greater than the energy absorbed, it means that the elongation, hence the displacement, need to be increased. The consequence is that the first cable breaks and the study of the second cable is again performed like an isolated cable, with new initial conditions.

- Sliding model.
  The results for a displacement of 2.87 m have been calculated already in equations (129)-(134):

$$L = 6.89 + 82.55 = 89.44\ m \tag{152}$$

$$\varepsilon_0 = (3.82 + 0.87) \cdot 10^{-3} = 4.69 \cdot 10^{-3} \tag{153}$$

$$\sigma_0 = 195000 \cdot 4.69 \cdot 10^{-3} = 915\ MPa \tag{154}$$

$$p = \frac{915}{1860} = 0.49 \sim 0.50 \tag{155}$$

- No-sliding model.
  the initial stress is calculated as a weighted mean between sliding and no-sliding. In case of the no-sliding model at the displacement of 2.87 m the cable has already broken, so the maximum tension is taken.

$$\sigma_0 = 1860 \cdot \frac{20}{90} + 915 \cdot \frac{70}{90} = 1125\ MPa \tag{156}$$

$$\varepsilon_0 = \frac{1125}{195000} = 5.77 \cdot 10^{-3} \tag{157}$$

$$l_x = \frac{6.89}{1 + 5.77 \cdot 10^{-3}} = 6.85\ m \tag{158}$$

$$p = \frac{1125}{1860} = 0.60 \tag{159}$$

The last parameter to calculate is the new velocity. It is influenced by both cables between the displacement of 10 m and 12.87 m, for this reason the forces of both cables against the vehicle are considered in the step by step approach. The velocity to consider as a starting value is the new v_0 given by the impact with the second cable.

- Sliding model, 33.15 Km/h, with the starting velocity given in equation (160):

$$v_0 = \frac{40000 + 2770}{40000 + 2770 + 2475} \cdot 55.37 = 52.34\ Km/h \tag{160}$$

- No-sliding model, 63.86 Km/h, with the starting velocity given in equation (161):

$$v_0 = \frac{40000}{40000 + 2475} \cdot 73.51 = 69.23\ KM/h \tag{161}$$

At the end, the velocity is calculated again with equation (123):

$$v = 63.86 \cdot \frac{20}{90} + 33.15 \cdot \frac{90 - 20}{90} = 39.97\ Km/h \tag{162}$$

Now with the new velocity, the minimum requirements can be calculated with the same procedure as before. Starting with the breakage condition:

- Sliding model:

$$strands\ n° = \frac{40000 \cdot \left(\frac{39.97}{3.6}\right)^2}{2 \cdot 50.23 \cdot 89.44 \cdot 139} \cdot (1 + 4.69 \cdot 10^{-3}) \sim 4 \tag{163}$$

- No-sliding model:

$$strands\ n° = \frac{40000 \cdot \left(\frac{39.97}{3.6}\right)^2}{2 \cdot 50.23 \cdot 6.85 \cdot 139} \cdot (1 + 4.69 \cdot 10^{-3}) \sim 52 \tag{164}$$

With equation (123) calculate the minimum number of strands to not reach the breakage:

$$strands\ n° = 52 \cdot \frac{20}{90} + 4 \cdot \frac{90 - 20}{90} \sim 15 \tag{165}$$

Therefore, the second cable will not break, but the range in which it is going to work still has to be defined. Given the values of p in equations (155) and (159) and using Table 2, it is possible to first check the zero-tension limit.

- Sliding model:

$$strandsn° = \frac{40000 \cdot \left(\frac{39.97}{3.6}\right)^2}{2 \cdot 12.95 \cdot 89.44 \cdot 139} \cdot (1 + 4.69 \cdot 10^{-3}) \sim 16 \tag{166}$$

- No-sliding model:

$$strands\ n° = \frac{40000 \cdot \left(\frac{39.97}{3.6}\right)^2}{2 \cdot 13.65 \cdot 6.85 \cdot 139} \cdot (1 + 4.69 \cdot 10^{-3}) \sim 190 \tag{167}$$

$$strands\ n° = 190 \cdot \frac{20}{90} + 16 \cdot \frac{90 - 20}{90} \sim 55 \tag{168}$$

As a result, the cable will have reached the zero-tension limit and passed it and it will have totally lost the initial tension. The same steps can be repeated between the second and third cable, but at the end the last element will not be involved because the maximum displacement is smaller than the spacing.

15. Generic vehicle

In the whole process a big vehicle of 40 tons has been considered, the same reasoning could be done with a normal vehicle with a mass of about 2000 Kg, that now have a greater velocity, and see if this will give more restrictive conditions. Considering a velocity of 120 Km/h:

$$\frac{1}{2} \cdot M \cdot v^2 = \frac{1}{2} \cdot 2000 \cdot \left(\frac{120}{3.6}\right)^2 = 1.11 \cdot 10^6 Nm \tag{169}$$

In comparison with the previous result there is a difference of almost one order of magnitude:

$$\frac{1}{2} \cdot M \cdot v^2 = \frac{1}{2} \cdot 40000 \cdot \left(\frac{80}{3.6}\right)^2 = 9.88 \cdot 10^6\ Nm \tag{170}$$

In case of the sliding model when the usual equation (31) is analysed, the resultant value “AL” fixing the minimum requirement will be smaller and it will not present any worst situations.
In case of the no-sliding model there is a difference since now in consideration of a normal car, the centre of mass is lower in respect to the 1.5 m considered before. Considering now a height of 0.5 m and the first type of impact:

$$lx = \frac{0.5}{sin(\gamma)} \tag{171}$$

$$A = \frac{M \cdot v^2}{2 \cdot k(p)} \cdot \frac{sin(\gamma)}{0.5} \cdot (1 + \varepsilon_0) = \frac{M \cdot v^2}{k(p)} \cdot (1 + \varepsilon_0) \cdot sin(\gamma) \tag{172}$$

The terms k(p), sin(γ) and ε_0 are the same, by comparing the term with mass and velocity the result is:

$$\frac{M \cdot v^2}{3} = \frac{40000 \cdot \left(\frac{80}{3.6}\right)^2}{3} = 6.58 \cdot 10^6\ Nm \tag{173}$$

$$M \cdot v^2 = 2000 \cdot \left(\frac{120}{3.6}\right)^2 = 2.22 \cdot 10^6\ Nm \tag{174}$$

The value is almost 3 times smaller, so again this is not going to change any of the minimum requirements or present any more unfavourable situation.
In case of the impact in the opposite direction, in consideration of the simplified condition given by equation (85), the following equation is valid:

$$A \cdot \left(L - \frac{0.5}{(1 + \varepsilon_0) \cdot sin(\gamma)}\right) = \frac{M \cdot v^2}{2 \cdot k(p)} \tag{175}$$

The term in the parenthesis has increased while the term related to the mass and velocity has decreased, as stated above. Again, the requirements coming from this type of impact will be smaller than the previous one, giving a less severe outcome.
One aspect that could be further analysed when the mass starts to be relatively small is the fact that with the sliding model there is the possibility that the vehicle at one point loses contact with the road. This would happen in the cases of big cables, strong enough to resist to the impact and to push the vehicle upwards. As a result, the deformation in the cable will be even smaller presenting at the end conditions that could even be non-damaging for the system.

16. Conclusions

Cable-stayed bridges are imponent structures subjected to many aggressive events, such as the impact of a heavy vehicle. For this reason, they are constructed with a special attention to the redundancy of the cables to ensure that in case of a loss of an element, the rest of the structure will not suffer excessive damage or reach any ultimate state. This work has been developed to analyse the breakage of the cables using an energetic and step by step solution in order to confirm if the loss of one cable is a safe consideration or not and to analyse under which conditions an element could break. The full study on the response of a single isolated cable has been developed, presenting the limit conditions that lead to the failure, loss of the tension or just elastic behaviour. At the end a numerical example has been presented with a possible configuration of several elements, in order to see how many of them could be involved and how many could be considered as lost, if the breakage or total loss of tension has occurred. Finally, the main conclusions have been drawn and presented as follow:

- The size of the cable follows a hyperbolic function when a maximum strain is set, if the length increases the area could decrease and vice versa, without changing the response.
- When analysing short cables, the number of strands required starts to be really high. The breakage condition is more likely to be reached in respect to long cables, but in case of the zero-tension or elastic behaviour the minimum requirement is out of the normal construction range, as a result the loss has to be considered in any case.
- When analysing long cables, it is more likely that the cable will not break because the number of strands required starts to decrease, but still the elastic and zero-tension ranges are hard to guarantee due to the still high number of strands required.
- The maximum displacement reached is an important aspect both in case of breakage and not, because it allows to define weather the next cables will be touched and if for certain ranges more than one cable are going to work together against the vehicle. It can happen that one cable stops the vehicle, that several cables will be broken and they work as isolated cables, because the maximum displacement is always smaller than the spacing or that several cables will be touched by the impact due to the fact that the maximum displacement is way higher than the spacing. In the latter case it might be that there is no breakage, but the cables have passed the zero-tension condition and are close to the maximum strain.
- If the inclination grows the number of strands required increases, considered fixed the height of the centre of mass of the vehicle.
- The impact of a normal car is less severe than a heavy vehicle, even if it has a greater velocity.
- In the final numerical example, it has been proved that in case of the collision of a heavy vehicle at least two cables are going to be affected, where the first one will break and the second one will have lost totally the initial tension. The consequence is that in this and many other cases, considering the loss of just one cable could not be enough and the effect on the rest of the structure could be more severe than what expected.

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Appendix A – symbolic notations

IF	impact factor
N_0	initial tension in the cable
N_{min}	minimum tension in the cable
f_{ptk}	ultimate strength of steel
$f_{p0.1k}$	yielding strength of steel, stress at 0.1% of residual deformation
E_{sp}	elastic modulus
E_p	plastic modulus
ε_{yk}	yielding strain of steel
ε_{uk}	ultimate strain of steel
γ	inclination of the cable in respect of the horizontal direction
T_0	initial tension in the cable
σ_0	initial stress in the cable
L	strained length of the cable
l_x	strained length of the part of the cable below impact
l_s	strained length of the part of the cable above impact
ΔL_0, $\varepsilon_0 \cdot L_0$	initial elongation in the cable, due to the initial tension
ΔL_{0x}	initial elongation of the part of the cable below impact, due to the initial tension
ΔL_{0s}	initial elongation of the part of the cable above impact, due to the initial tension
L_0	unstrained length of the cable
ε_0	initial strain in the cable, due to the initial stress
l_{0x}	unstrained length of the part of the cable below impact
l_{0s}	unstrained length of the part of the cable above impact
M	mass of the vehicle
ρ	density of the steel
m	half of the mass of the cable concentrated in the point of the impact
m_x	mass concentrated in the bottom node
m_s	mass concentrated in the top node
v	velocity of the vehicle
l'_x	deformed length of the part of the cable below impact
l'_s	deformed length of the part of the cable above impact
ω_x, ω_s, θ_x, θ_s	angles formed by the cable in the deformed shape
h	height of the triangle formed by the cable in the deformed shape
u	maximum displacement of the cable in the point of the impact
T	tension in the cable after the impact
T_{max}	tension in the cable after the impact
$\Delta\varepsilon \cdot L_0$, ΔL	increment in elongation of the cable after the impact
$\varepsilon_{yk} \cdot L_0$	yielding elongation of the cable
$\varepsilon_{uk} \cdot L_0$	ultimate elongation of the cable
$\varepsilon \cdot L_0$	maximum elongation of the cable after the impact
$f_{po.1k} \cdot A$	yielding tension of the cable
$f_{ptk} \cdot A$	ultimate tension of the cable
k(p)	deformative constant
ΔL_x	increment in elongation of the part of the cable below the impact
ΔL_s	increment in elongation of the part of the cable above the impact
c	height of the centre of mass of the vehicle
v_0	initial velocity of the system vehicle + cable
Δt	fictitious duration of the impact
ts	time step for the step by step solution

s	Lagrangian coordinate along the cable
t	time
x	vertical coordinate
y	horizontal coordinate
Δx	displacement increment in the vertical direction
Δy	displacement increment in the horizontal direction
l_{0i}	strained length of the element
l_i	strained length of the element
v_x	velocity in the vertical direction
v_y	velocity in the horizontal direction
w	index used to define the point of impact
z	generic variable
$k_i(p)$	value of the constant for different cables if more than one has to be considered

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