



**DETECTION OF PIPELINE UPHEAVAL BUCKLING  
BY MEANS OF FIBRE-OPTIC SENSORS**

A THESIS WRITTEN BY

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# ACKNOWLEDGEMENTS

At the end of an arduous journey, travellers stop to recover strength and, before setting off again for new destinations, direct grateful thoughts to those who have guided, encouraged or merely accompanied them along the way. I am much obliged to my

- supervisor Prof. Godio, whose human qualities, passion and scientific competence have been profoundly inspiring;
- family (Father, Mother, Aunt Maria and the late Grandmother Antonia), which has taught me (in the words of Natalia Ginzburg) *not the little virtues but the great ones: not thrift but generosity and indifference to money, not prudence but bravery and contempt for danger, not astuteness but frankness and love of truth, not diplomacy but altruism and abnegation, not ambition but the desire to be and to know*. This will forever be my polestar.



# PREFACE

One day a giant challenged Thor to raise his cat from the floor: the god of thunder grabbed it by the belly and began to lift; but the more he pulled, the more the feline arched its back. This mythological image (found in Snorri Sturluson's *Edda*) aptly summarises the salient features of upheaval buckling, a mechanism whereby a segment of buried pipeline (comparable to the extraordinarily stretchable creature of Norse fable) bows upwards out of the trench under excessive axial compression (analogous to the mighty effort expended by the hammer-wielding deity). Amongst the numerous factors determining the size of the involved force (e.g. mechanical properties of materials, impact of constraints, interaction with soil, loading history), elevated operating pressure (beyond 10 N/mm<sup>2</sup>) and temperature (up to 100 °C above the external one) emerge as the most influential. The flowering of on- and (to a greater extent) offshore oil and gas fields worldwide has brought transport infrastructure into prominence: the possibility that overstress may culminate in rupture (with the attendant risks to man, nature and the economy) strongly calls for the adoption of continuous efficient reliable robust monitoring systems.

Today fibre-optic sensors offer an attractive alternative to their conventional electrical counterpart because they

- are light, versatile, cost-effective, safe to handle, of small diameter, resistant to corrosion and fatigue, immune to electromagnetic disturbance, unlikely to initiate fires or explosions, dielectric (ergo suitable for concrete and FRP);
- streamline the sensing architecture thanks to their dual role of gauges and pathways for signals;
- can be organised in serial and parallel multiplex arrays, which enable truly distributed measurements.

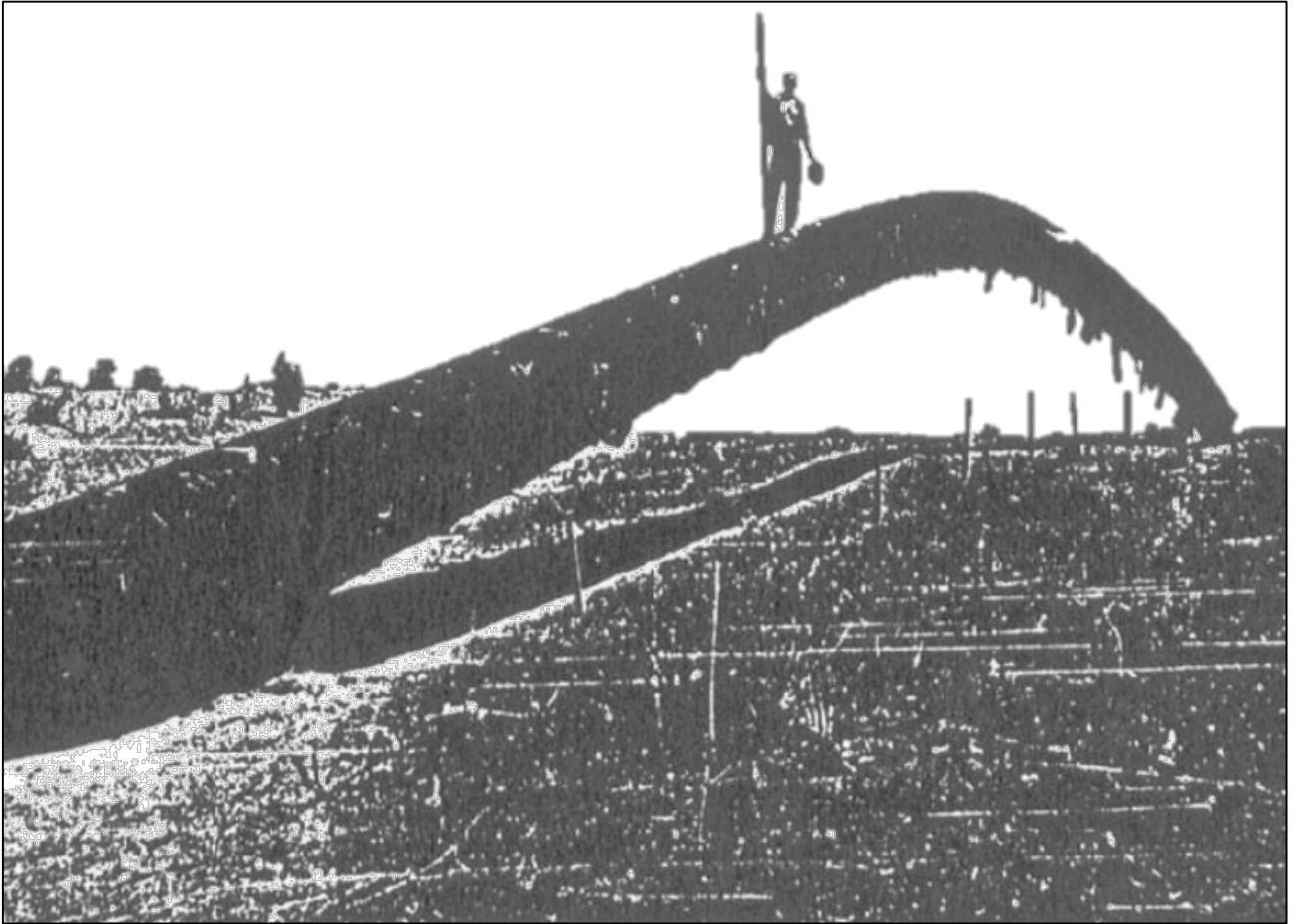
The present thesis aims at assessing the ability of such instruments to detect the upheaval buckling of pipelines. It is divided into three sections:

- the first examines the classical theory of buckling,
- the second discusses the key aspects of fibre-optic technology; starting from these conceptual premises,
- the third proceeds to analyse the results of a specially designed laboratory simulation.

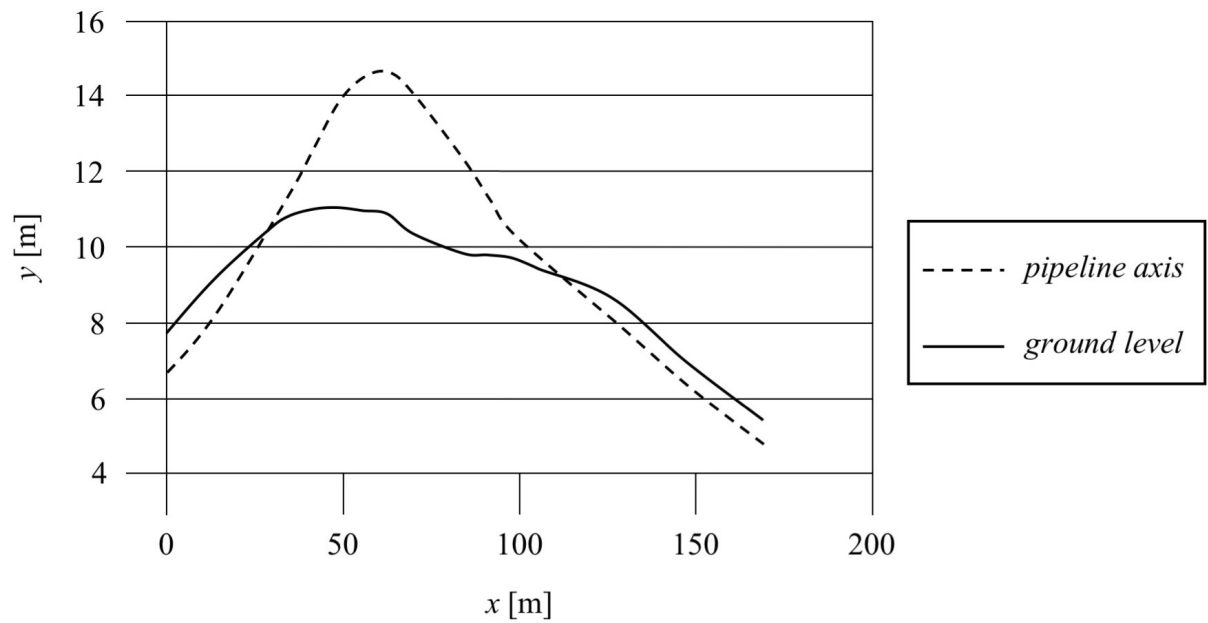
A commitment is made to keeping the writing style concise and to the point, in pursuance of the maxim that *frustra fit per plura quod potest fieri per pauciora* ("it is vain to do with more what can be done with less"). Lastly, perusal of the appendices (amplifying the information imparted in the text) ought not to be omitted.



FIG. 1 Thor and the cat (linocut by Virginie Le Gall).



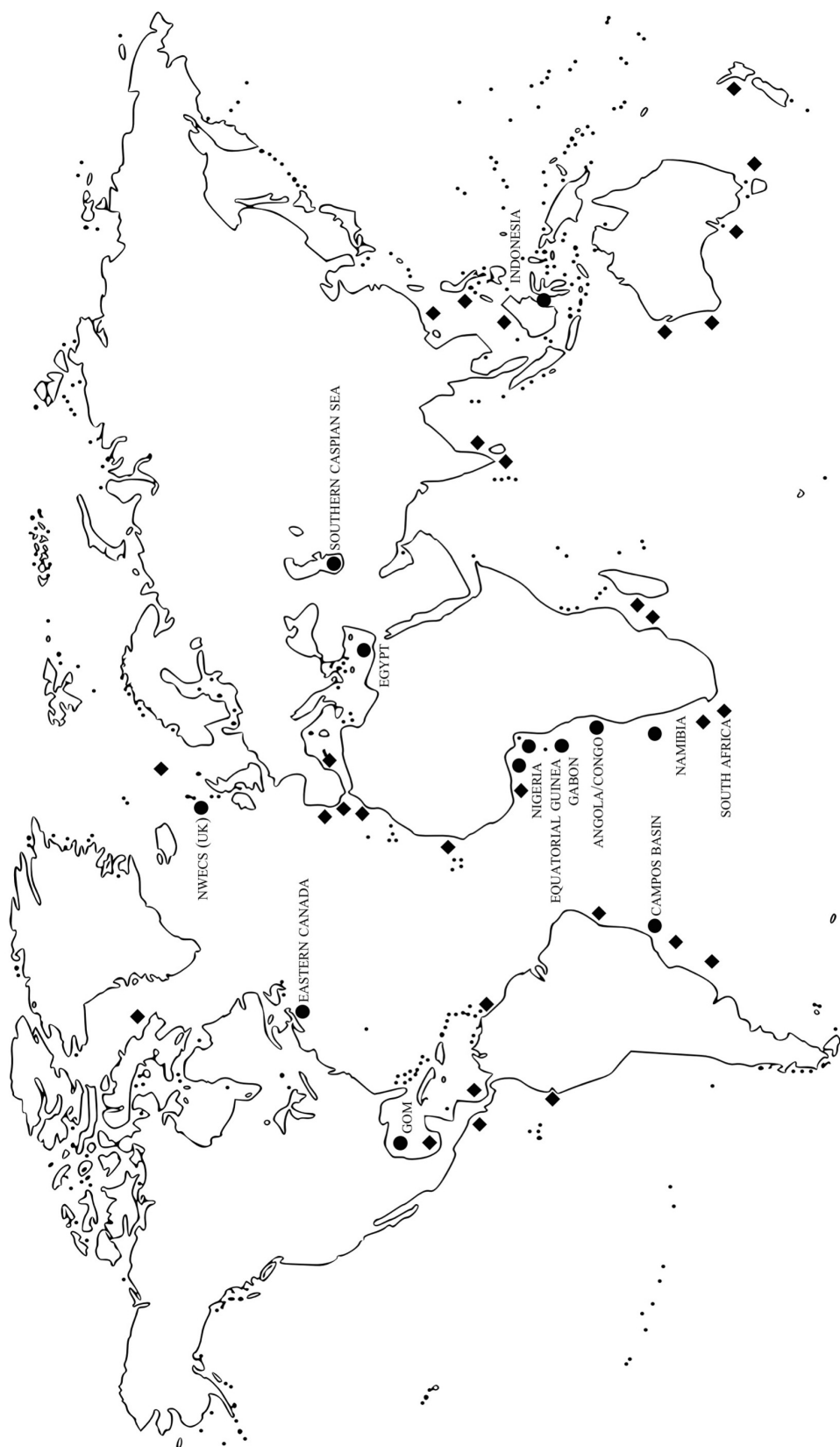
(a)



(b)

**FIG. 2** (a) Photograph and (b) profile of a buckled 1020-mm (40-in.) onshore gas pipeline near Tashkent in Uzbekistan (adapted from Ref. 9, pp. 447-448).





**FIG. 3** Current (●) and future (◆) deep-water areas with proven hydrocarbon reserves (adapted from EDMUNDO CORONA, STELIOS KYRIAKIDES – *Buckling and collapse*, in *Mechanics of offshore pipelines*, Elsevier, 2007, vol. 1, p. 5).

TABLE 1

Examples of major on- and offshore pipelines (adapted from ibid., pp. 2-3)

<i>Trans-Alaska (USA)</i>		<i>Trans-Mediterranean</i>		<i>Blue Stream</i>		<i>Medgaz</i>
Type	Onshore	On-/offshore	On-/offshore	On-/offshore	On-/offshore	
<i>Product</i>	Oil	Gas	Gas	Gas	Gas	
<i>From-to</i>	Prudhoe Bay – Valdez	1. Hassi R'Mel (DZ) – Cape Bon (DZ) 2. Cape Bon – Mazara del Vallo (IT) 3. Mazara del Vallo – Mortelle (IT) 4. Mortelle – Favazzina (IT) 5. Favazzina – Bologna (IT)	1. Stavropol (RUS) – Dzhubga (RUS) 2. Dzhubga – Samsun (TR) 3. Samsun – Ankara (TR)	1. Hassi R'Mel (DZ) – Beni Saf (DZ) 2. Beni Saf – Almeria (E) 3. Almeria – Albacete (E)		
<i>Length [km]</i>	1287	1. 920 2. 160 3. 352 4. 44 5. 1051	1. 370 2. 774 3. 501	1. 550 2. 200 3. 300		
<i>Capacity</i>	800 Mstb/d	3 MMMScf/d	1.5 MMMScf/d	1.5 MMMScf/d		
<i>p [bar]</i>	80	–	1. 75-100 2. 250 3. 75	–		
<i>Commissioned in</i>	1977	1983	2003	2009		
<i>D × t [in.]</i>	48×0.462-0.562	1. 48 2. 20 3. 48 4. 10 + 20	1. 56 + 48 2. 24×1.254 3. 48×0.563	2. 24×1.170		
<i>Steel grade</i>	X60, X65, X70	2. X65	2. X65	2. X70		
<i>Max. water depth [m]</i>	n/a	2. 600 4. 360	2. 2150	2. 2160		
<i>Cost [\$bn]</i>	10	3	2.5	2. 0.75		

TABLE 1 (CONT.)

<i>Baku-Ceyhan</i>		<i>Mardi Gras (USA)</i>		<i>Independence Trail (USA)</i>		<i>Jansz &amp; Gorgon Projects (AUS)</i>	
<i>Type</i>	Onshore	On-/offshore	On-/offshore	On-/offshore	On-/offshore	On-/offshore	On-/offshore
<i>Product</i>	Oil	Oil/gas	Gas	Gas	Gas	Gas	Gas
<i>From-to</i>	Baku (AZ) – Ceyhan (TR)	1. NaKika – MP 260 2. Thunder Horse – SP 89E 3. SP 89E – LOOP 4. Holstein/Mad Dog/Atlantis – Beaumont (via SS 332) 5. Holstein/Mad Dog/Atlantis – Neptune (via SS 332)	Independence Hub – Tennessee Gas Pipeline	1. Jansz Field – Barrow Island 2. Gorgon Field – Barrow Island			
<i>Length [km]</i>	1760	1. 160 (Okeanos, gas) 2. 115 (Proteus, oil) 3. 145 (Endymion, oil) 4. 180 (Caesar, oil) 5. 180 (Cleopatra, gas)	220		1. 180 2. 70		
<i>Capacity</i>	160 Mstb/d	1. 500 MMscf/d 2. 580 Mstb/d 3. 750 Mstb/d 4. 450 Mstb/d 5. 500 MMscf/d	850 MMscf/d		825 MMscf/d		
<i>p [bar]</i>	–	–	–	–	–	–	–
<i>Commissioned in</i>	2006	2006	2007	2010			
<i>D × t [in.]</i>	42 + 36	1. 24 + 20×0.941-1.000 2. 28×1.088-1.210 + 24×1.095-1.200 3. 30 4. 28×1.045-1.148 + 20×0.971-1.227 5. 20×0.750-0.880 + 16×0.665-0.898	24		1. 24×1.236 + 28×1.173 2. 30×1.327		
<i>Steel grade</i>	–	X65	–	–	X65		
<i>Max. water depth [m]</i>	n/a	1310-2225	2450		1350		
<i>Cost [\$bn]</i>	3.6	1	0.28		–		



# 1 UPHEAVAL BUCKLING OF PIPELINES

## 1.1 ANALYTICAL MODEL

The problem (according to Hobbs's formulation, reviewed by Calladine and Maltby) essentially concerns a long straight uniform heavy elastic pipe (assimilated to an Eulerian strut), laid on a flat rigid base at ambient temperature in stress-free conditions and constrained to lie in the vertical plane, regardless of the loads thereon exerted by its surroundings.

First, let  $T$  increase by a certain amount: the pipe would elongate with a strain

$$\varepsilon_1 = \alpha \Delta T \quad (1.1)$$

( $\alpha$  being the coefficient of linear thermal expansion) were it not assumed to be fastened to the base at either extremity; as a consequence, a compressive (positive) force is generated, equal to

$$P_1 = AE\alpha\Delta T, \quad (1.2)$$

in which  $A$  denotes the cross-sectional area and  $E$  the Young's modulus. By the same token, the axial strain due to the pressure  $p$  inside the pipe (characterised by an inner radius  $r$ , a wall thickness  $t$  and a Poisson's ratio  $\nu$ )

$$\varepsilon_2 = \frac{1}{E} \left( \frac{pr}{2t} - \nu \frac{pr}{t} \right) \quad (1.3)$$

(here expressed in terms of the longitudinal and circumferential or hoop stresses in thin-walled cylindrical shells) is fully restrained, giving rise to

$$P_2 = \frac{Apr}{t} (0.5 - \nu). \quad (1.4)$$

In total,

$$P_0 = AE\alpha\Delta T + \frac{Apr}{t} (0.5 - \nu). \quad (1.5)$$

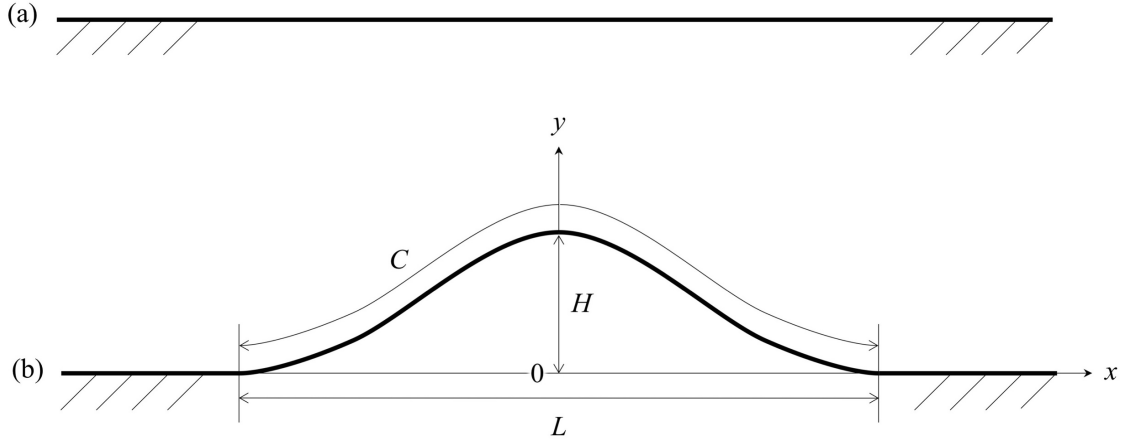
Next, following King and Palmer's approach, suppose that a single symmetrical buckle forms, having a height  $y$ , function of the horizontal distance  $x$  (it is convenient to locate the origin of the Cartesian coordinate system on the base, immediately below the apex); consider an infinitesimal element thereof, in translational and rotational equilibrium under the action of the normal force  $P$ , shear force  $S$ , bending moment  $M$  and weight per unit length  $w$ , wherefore

$$-S + wdx + (S + dS) = 0 \Rightarrow \quad (1.6)$$

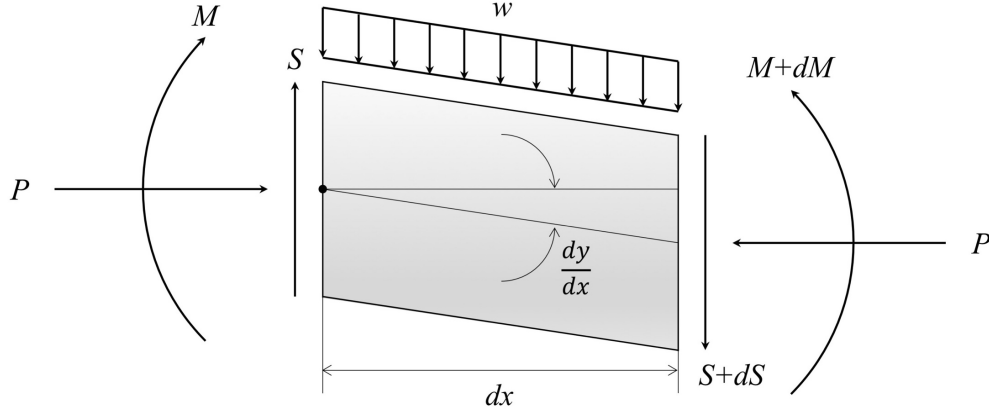
$$\Rightarrow \frac{dS}{dx} + w = 0, \quad (1.7)$$

$$-M - wdx \frac{dx}{2} + P \frac{dy}{dx} dx - (S + dS)dx + (M + dM) = 0 \Rightarrow \quad (1.8)$$

$$\Rightarrow \frac{dM}{dx} + P \frac{dy}{dx} - S = 0. \quad (1.9)$$



**FIG. 1.1** (a) Straight pipe. (b) An upheaval buckle, with the coordinate system and principal dimensions. Note that the vertical scale is exaggerated.



**FIG. 1.2** Forces and moments acting on an element of the buckle.

Differentiation of (1.9) with respect to  $x$  and addition of (1.7) thereto lead to the governing equation

$$\frac{d^2 M}{dx^2} + P \frac{d^2 y}{dx^2} + w = 0. \quad (1.10)$$

Provided that  $|dy/dx| \leq 0.1$  (hypothesis of small deflections and slopes), a proportionality subsists between  $M$  and curvature through flexural rigidity or stiffness (product of  $E$  and the second moment of area  $I$ ):

$$M = EI \frac{d^2 y}{dx^2}. \quad (1.11)$$

Finally, (1.10) can be rewritten as

$$EI \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} + w = 0. \quad (1.12)$$

Now, naming the maximum height of the buckle  $H$  and the distance which separates the “peel” points  $L$ , the hereunder listed boundary conditions obtain:

$$y(0) = H, \quad (1.13)$$

$$y\left(\pm \frac{L}{2}\right) = 0, \quad (1.14)$$

$$\frac{dy}{dx}\left(\pm \frac{L}{2}\right) = 0, \quad (1.15)$$

$$\frac{d^2y}{dx^2}\left(\pm \frac{L}{2}\right) = 0. \quad (1.16)$$

(1.12-15) are fulfilled by

$$y(x) = \frac{wEI}{P^2} \left[ 1 + \frac{m^2 L^2}{8} - \frac{m^2 x^2}{2} - \frac{\cos(mx)}{\cos(mL/2)} \right] \quad (1.17)$$

(derived step by step in Appendix 1), which incorporates a wavenumber satisfying the relation

$$m^2 = \frac{P}{EI}, \quad (1.18)$$

whereas (1.16) requires that

$$\tan \frac{mL}{2} = \frac{mL}{2} \quad (1.19)$$

or, as lowest root,

$$mL = 8.99. \quad (1.20)$$

From (1.18) and (1.20),

$$P = 80.8 \frac{EI}{L^2}. \quad (1.21)$$

Obviously, the contour of the pipe in the uplifted configuration  $C$  is greater than the original  $L$ , whence it ensues that  $P$  in the buckle must be less than  $P_0$  away therefrom. This “geometric shortening” is computed from

$$C - L = \int_{-L/2}^{L/2} 0.5 \left( \frac{dy}{dx} \right)^2 dx = \quad (1.22)$$

$$= 75.6 \frac{w^2 (EI)^{1.5}}{P^{3.5}}. \quad (1.23)$$

On the assumption (unrealistic in the case of actual pipelines, yet directly relevant to such short pin-ended tubes as those used in small-scale experiments) that slip along the base in the tracts flanking the buckle is completely prevented,

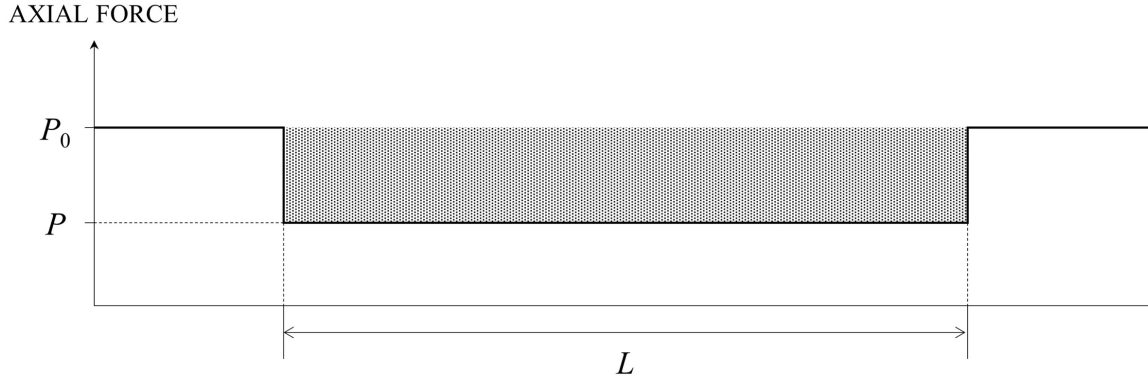
$$C - L = \frac{(P_0 - P)L}{AE}. \quad (1.24)$$

By equating (1.24) to (1.23),

$$P_0 = P + 16.0 \times 10^{-6} \frac{w^2 A E L^6}{(EI)^2}, \quad (1.25)$$

with a minimum at

$$L_{min} = \left[ 1.69 \times 10^6 \frac{(EI)^3}{w^2 AE} \right]^{0.125}. \quad (1.26)$$



**FIG. 1.3** Distribution of axial force in the buckled pipe. The stippled area is proportional to geometric shortening.

## 1.2 NUMERICAL EXAMPLE

Fig. 1.4 depicts (1.25) on a graph of  $P_0$  against  $L$ , produced by a MATLAB script (v. Appendix 2) from data belonging to the experimental apparatus (v. Table 1.1).  $P_0$  represents the control parameter of the phenomenon inasmuch as it is linked with  $\Delta T$  and  $p$  by (1.5). Its curve falls sharply for small lengths (when the first summand on the right-hand side dominates) and, after crossing a minimum, climbs steeply as  $L$  grows (once the second takes over). Such a shape implies that

- a buckle cannot appear until  $P_0 > P_{0,min}$ ;
- the equation admits two distinct solutions, associated to a shorter (always mechanically unstable) and a longer span respectively.

Plotting (1.21) makes it clear that  $P_0 \gg P$  for  $L$  big enough and their difference is proportional to geometric shortening in agreement with (1.24).

The transition from the unstable to the stable branch (“dynamic snap”,  $1 \rightarrow 2$ ) happens at lower forces and less dramatically in the presence of an initial crookedness, to be eventually replaced by a progressive enlargement of the imperfection (v. Fig. 1.5).

**TABLE 1.1**  
Characteristics of LDPE PN-4 pipes (mfd. by Picenum Plast)

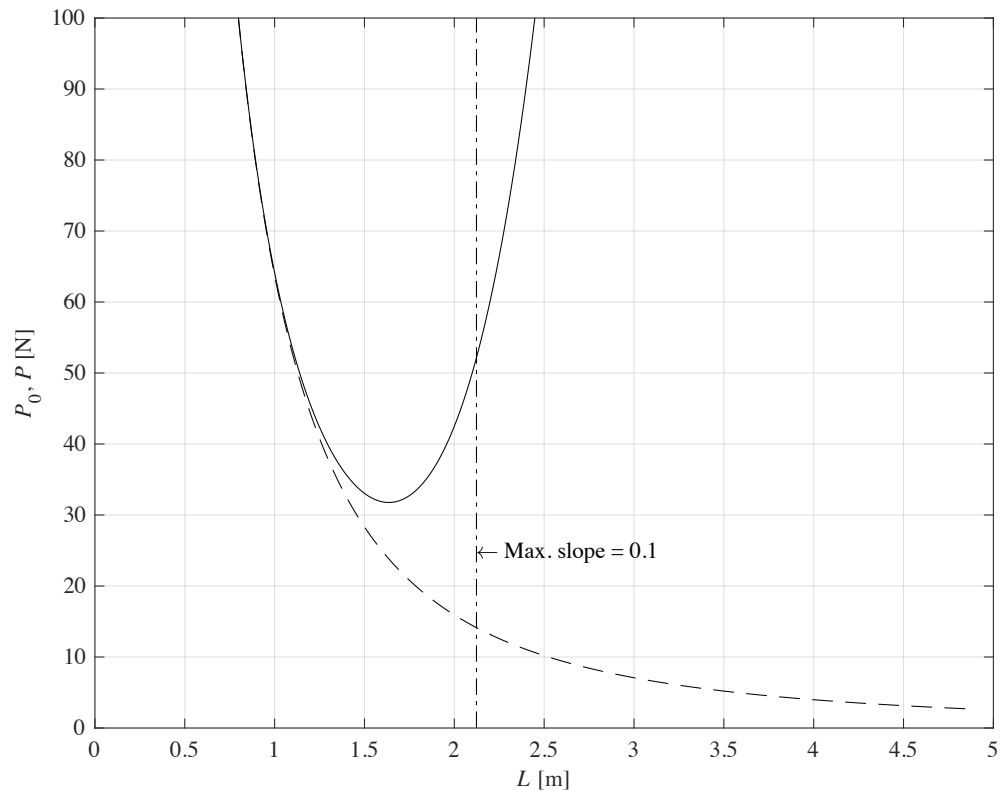
<i>Parameter</i>	<i>Symbol</i>	<i>Value</i>	<i>Unit</i>
Outer diameter	$D$	20*	mm
Wall thickness	$t$	1.6*	mm
Weight per unit length	$w$	0.000934*	N/mm
Yield strength	$\sigma_y$	14*	N/mm <sup>2</sup>
Young’s modulus	$E$	200*	N/mm <sup>2</sup>
Thermal coefficient	$\alpha$	200*	$\mu\epsilon/^\circ\text{C}$
Vicat softening temperature	$VST$	76-109**	$^\circ\text{C}$
Poisson’s ratio	$\nu$	0.45***	—

\* Italian Institute of Plastics.

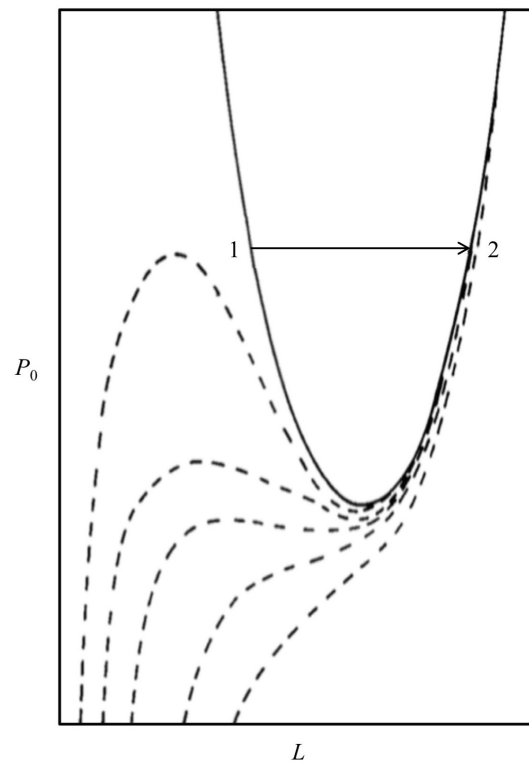
\*\* GEORGE WYPYCH – *Handbook of polymers*, ChemTech Publishing, 2016, p. 179.

\*\*\* ROY J. CRAWFORD – *Plastics engineering*, Butterworth-Heinemann, 1998, p. 59.





**FIG. 1.4**  $P_0$  (—) and  $P$  (---) vs  $L$ .



**FIG. 1.5** Imperfection sensitivity of equilibrium paths (adapted from Ref. 8, p. 8).

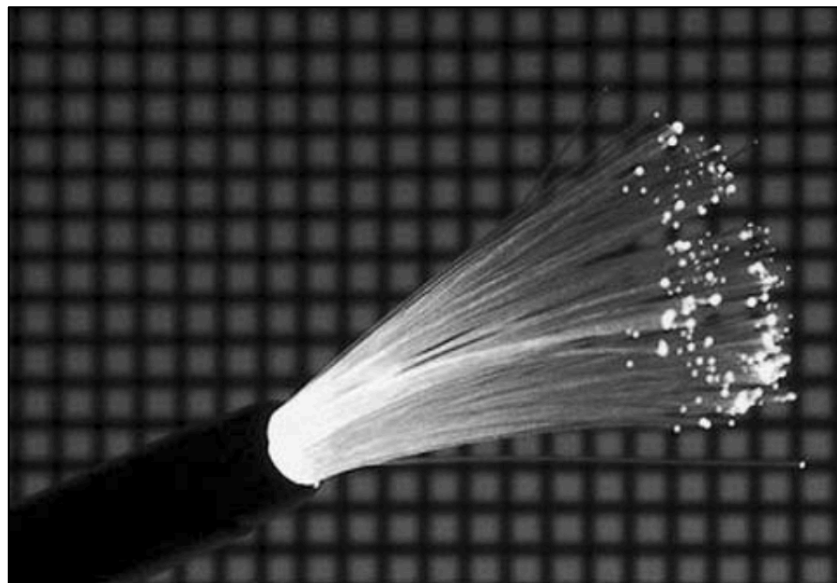


## 2 FIBRE-OPTIC SENSING TECHNOLOGY

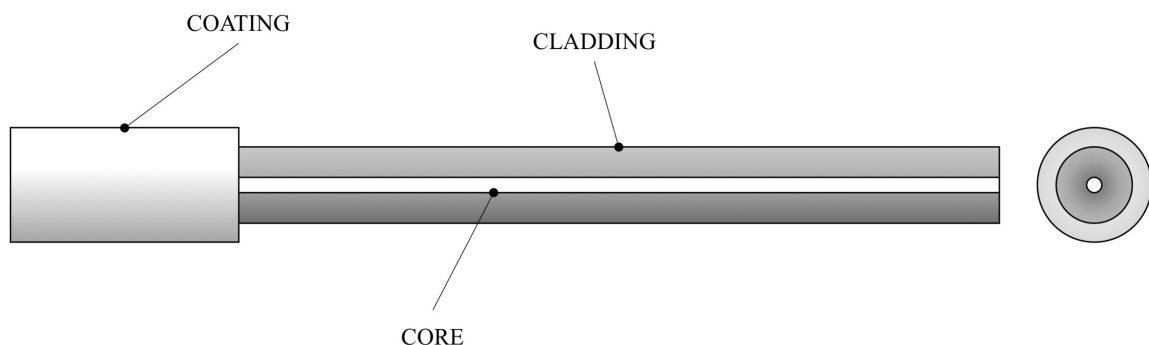
### 2.1 OPTICAL FIBRES

In his treatise (which constitutes the chief source of this section), Measures describes them as thread-like objects (approximately as fine as a strand of hair), drawn from highly pure fused silica ( $\text{SiO}_2$ ) and capable of carrying light over vast distances with negligible loss. They consist of a core (within which rays propagate), surrounded and sustained by a cladding of lower refractive index  $n$  (minimising dissipation); moreover, an acrylate (or polyimide, when embedded in a composite material) coating safeguards against rapid disintegration by the agency of moisture. Employment in harsh environments may necessitate a tough protective jacket.

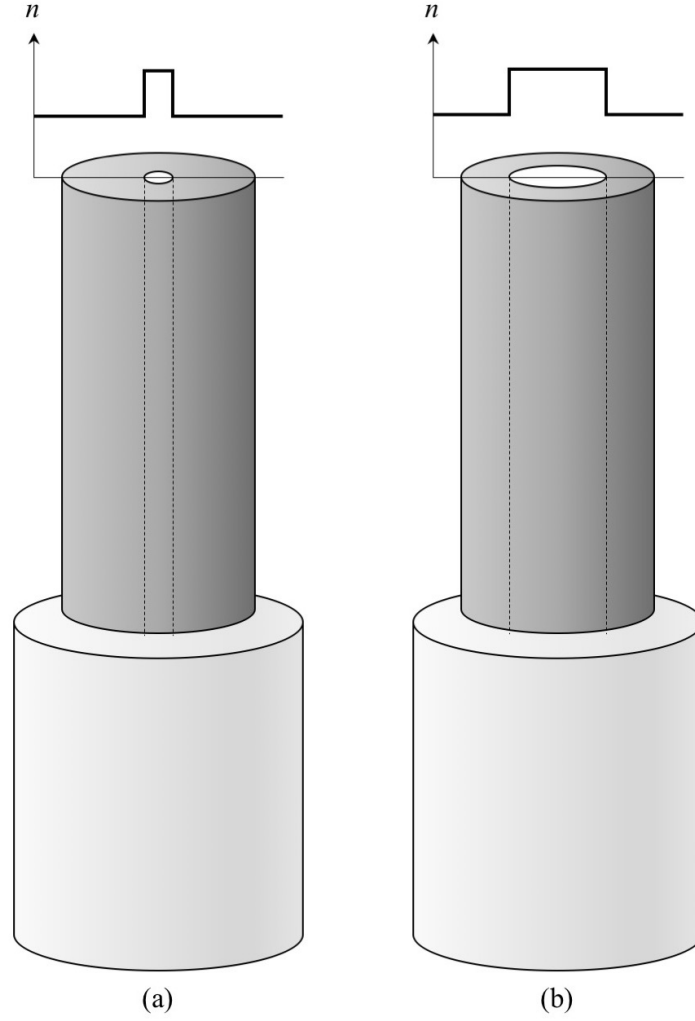
A common classification loosely distinguishes between single- and multimode fibres. In the former (whereon all interferometric sensors are based, including Bragg gratings – v. § 2.2), light travels as a plane wave (i.e. wavefronts are parallel planes perpendicular to the direction of propagation) inside a tiny core with a diameter of  $5\text{--}10\text{ }\mu\text{m}$ , thus limiting the temporal spread of short pulses and widening the transmission bandwidth; whilst the latter can convey significantly greater power since they have a core diameter of up to  $100\text{ }\mu\text{m}$  (*ceteris paribus*).



**FIG. 2.1** Bundle of fibres (from Ref. 11, p. 161).



**FIG. 2.2** Main parts of a fibre.



**FIG. 2.3** Radial profiles of  $n$  for (a) single- and (b) multimode fibres.

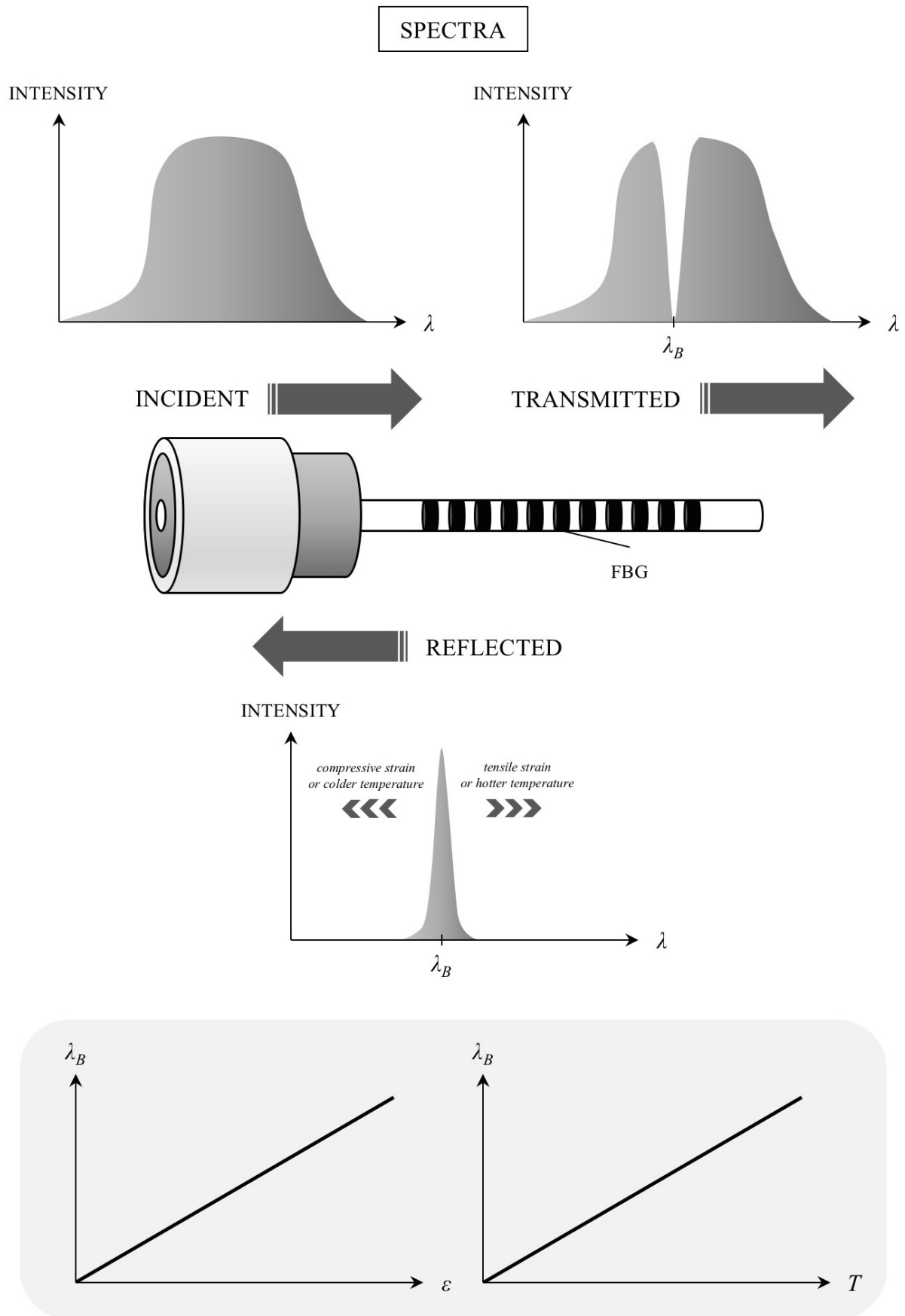
## 2.2 FIBRE BRAGG GRATINGS

An FBG is a longitudinal sinusoidal variation in the  $n$  of a core, permanently induced by exposure to an intense interference pattern of ultraviolet light (an effect christened photosensitivity by its discoverer Hill in 1978). It serves as an excellent band-stop filter in that virtually no backscatter occurs from each consecutive peak in the aforesaid variation, save for wavelengths in the region of

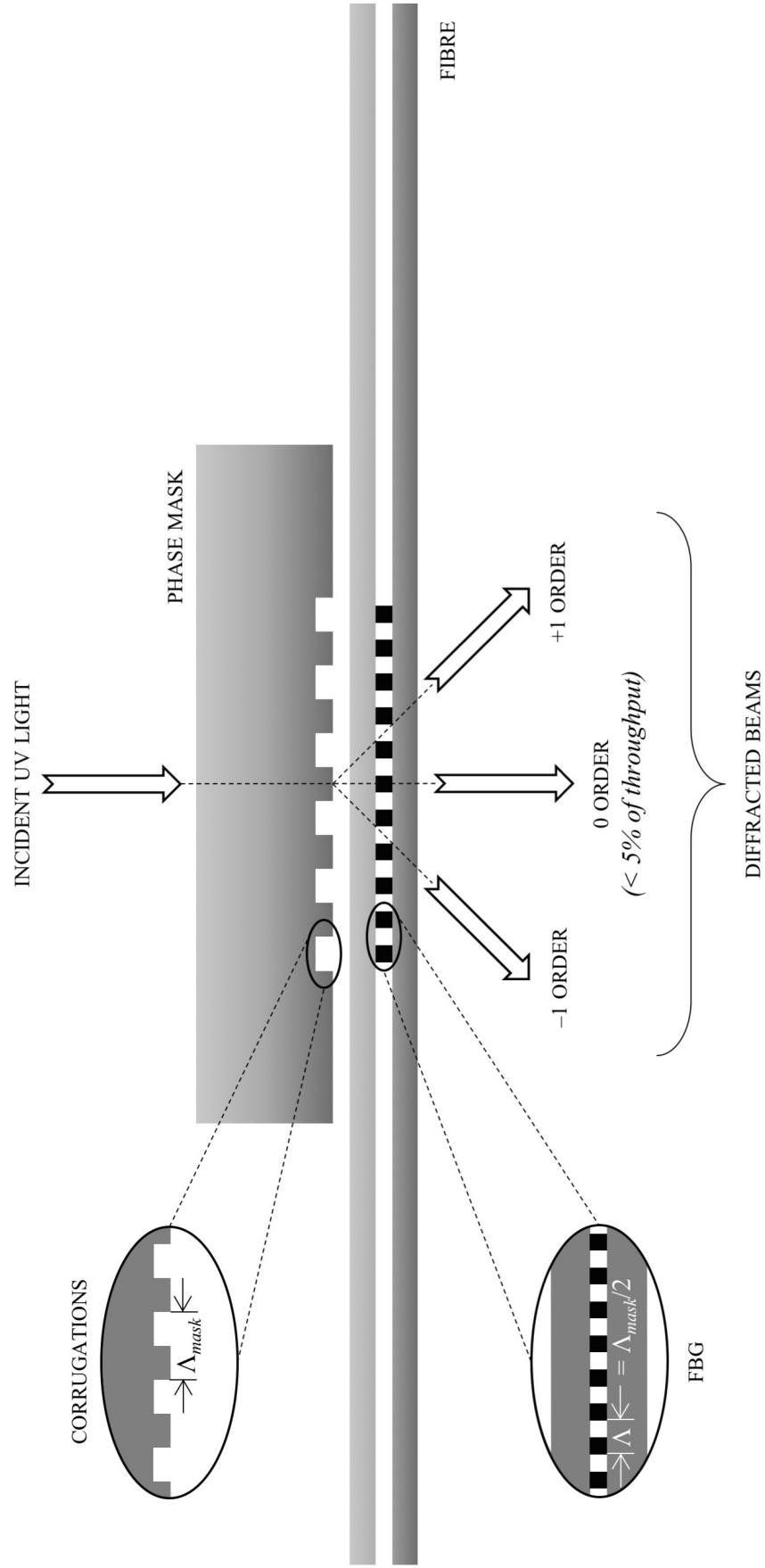
$$\lambda_B = 2n\Lambda, \quad (2.1)$$

wherein  $\Lambda$  designates the grating period. Local changes in strain or temperature affect both parameters and hence linearly shift  $\lambda_B$ , thereby rendering the grating an intrinsic sensor (v. § 2.3).

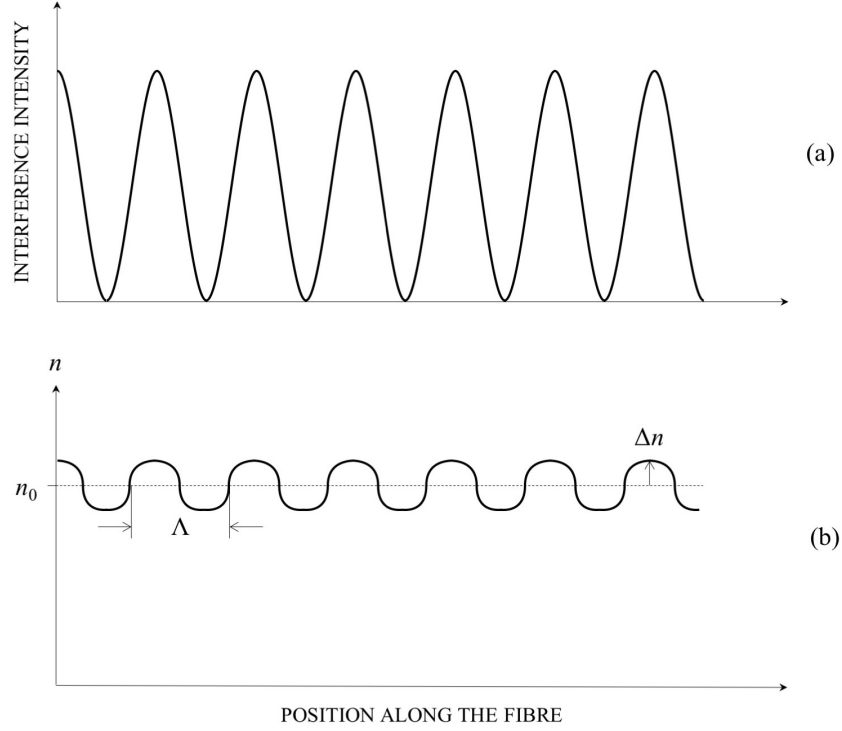
The established procedure for fabricating FBGs entails photolithographically etching a 1D square-wave-profiled surface-relief structure on one of the faces of a silica-glass slab; such phase mask, being transparent to the incident UV radiation, diffracts it into the  $\pm 1$  orders (the 0 one is suppressed by properly adjusting the depth of corrugations), which interfere to create a periodic pattern, that imprints a grating (with  $\Lambda = \Lambda_{mask}/2$ , independent of the inscribing-beam  $\lambda$ ) in the core.



**FIG. 2.4** Working principle of FBGs.



**FIG. 2.5** Fabrication of FBGs.



**FIG. 2.6** (a) Interference pattern and (b) resultant variation in the  $n$  of a core.

### 2.3 OPTICAL-FIBRE STRAIN AND TEMPERATURE SENSITIVITY

The fundamental parameter of FBG sensors is the path length

$$\zeta_L = n\Lambda, \quad (2.2)$$

function of strain (or stress, in accordance with Hooke's law) and temperature as stated in § 2.2:

$$\zeta_L = \zeta_L(\sigma, T). \quad (2.3)$$

A 1<sup>st</sup>-order-truncated Taylor-series expansion of (2.3) gives

$$\Delta\zeta_L = \left(\frac{\partial\zeta_L}{\partial\sigma}\right)_T \Delta\sigma + \left(\frac{\partial\zeta_L}{\partial T}\right)_\sigma \Delta T; \quad (2.4)$$

here  $(\partial\zeta_L/\partial\sigma)_T$  and  $(\partial\zeta_L/\partial T)_\sigma$  symbolise the derivatives of  $\zeta_L$  with respect to  $\sigma$  and  $T$  at a reference state  $(\sigma_0, T_0)$ . In the light of (2.2), (2.4) can be further expanded as

$$\Delta\zeta_L = \left[n\left(\frac{\partial\Lambda}{\partial\sigma}\right)_T + \Lambda\left(\frac{\partial n}{\partial\sigma}\right)_T\right]\Delta\sigma + \left[n\left(\frac{\partial\Lambda}{\partial T}\right)_\sigma + \Lambda\left(\frac{\partial n}{\partial T}\right)_\sigma\right]\Delta T \Rightarrow \quad (2.5)$$

$$\Rightarrow \Delta\zeta_L = n\Lambda \left\{ \left[ \left(\frac{\partial\epsilon}{\partial\sigma}\right)_T + \frac{1}{n}\left(\frac{\partial n}{\partial\epsilon}\right)_T \left(\frac{\partial\epsilon}{\partial\sigma}\right)_T \right] \Delta\sigma + \left[ \left(\frac{\partial\epsilon}{\partial T}\right)_\sigma + \frac{1}{n}\left(\frac{\partial n}{\partial T}\right)_\sigma \right] \Delta T \right\}. \quad (2.6)$$

If  $E$  and  $\alpha$  of the fibre are introduced, (2.6) becomes

$$\Delta\zeta_L = n\Lambda \left\{ \left[ 1 + \frac{1}{n}\left(\frac{\partial n}{\partial\epsilon}\right)_T \right] \frac{\Delta\sigma}{E_F} + \left[ \alpha_F + \frac{1}{n}\left(\frac{\partial n}{\partial T}\right)_\sigma \right] \Delta T \right\}, \quad (2.7)$$

the second addend in the first/second square bracket corresponding to the strain/thermo-optic effect. By defining the sensitivity to strain

$$S_\varepsilon \equiv 1 + \frac{1}{n} \left( \frac{\partial n}{\partial \varepsilon} \right)_T \quad (2.8)$$

and that to temperature

$$S_T \equiv \alpha_F + \frac{1}{n} \left( \frac{\partial n}{\partial T} \right)_\sigma, \quad (2.9)$$

(2.7) reduces to

$$\frac{\Delta \zeta_L}{\zeta_L} = S_\varepsilon \Delta \varepsilon + S_T \Delta T. \quad (2.10)$$

The normalised  $\Delta \zeta_L$  and  $\Delta \lambda_B$  are proportional, so that

$$\frac{\Delta \zeta_L}{\zeta_L} = \frac{\Delta \lambda_B}{\lambda_B}. \quad (2.11)$$

In conclusion, the basic structural-sensing equation (2.10) takes the form

$$\frac{\Delta \lambda_B}{\lambda_B} = S_\varepsilon \Delta \varepsilon + S_T \Delta T. \quad (2.12)$$

(2.12) suggests that any  $\Delta \lambda_B$  caused by a deviation in temperature is indistinguishable from one ascribable to the application of an exclusively mechanical force; therefore, some sort of compensation is needed. Imagine that a fibre is installed on a host structure not subjected to external loads but experiencing only a  $\Delta T$ ; then an axial stress will develop therein, owing to the discrepancy in  $\alpha$ :

$$\Delta \sigma = E_F (\alpha_H - \alpha_F) \Delta T. \quad (2.13)$$

Substitution of (2.13) into (2.12) yields

$$\frac{\Delta \lambda_B}{\lambda_B} = [S_\varepsilon (\alpha_H - \alpha_F) + S_T] \Delta T \quad (2.14)$$

and, allowing for stress-related strain,

$$\frac{\Delta \lambda_B}{\lambda_B} = S_\varepsilon [\Delta \varepsilon_m + (\alpha_H - \alpha_F) \Delta T] + S_T \Delta T. \quad (2.15)$$

Rearrangement of (2.15) delivers

$$\Delta \varepsilon_m = \frac{1}{S_\varepsilon} \frac{\Delta \lambda_B}{\lambda_B} - \Delta \varepsilon_T, \quad (2.16)$$

in which

$$\Delta \varepsilon_T \equiv \left( \alpha_H - \alpha_F + \frac{S_T}{S_\varepsilon} \right) \Delta T \quad (2.17)$$

stands for the apparent thermal strain. This can fatally vitiate measurements. By way of illustration, it was estimated that, given a  $\Delta T = 96^\circ \text{C}$  sufficient to trigger buckling in the Trans-Alaska Pipeline (v. Table 1), the  $\Delta \varepsilon_T$  suffered by gauges of the kind under study herein (v. Fig. 2.7 and Table 2.1)



would far exceed  $\Delta\epsilon_m$ : 1678 vs 393  $\mu\epsilon$  (v. Appendix 3 for calculations). Even if  $\alpha_H$  and  $\alpha_F$  coincided, the fact that  $S_T$  is about an order of magnitude greater than  $S_\epsilon$  could still pose serious difficulties.

Farrell et al. devised an interrogation technique which promises to overcome this drawback. In sketchy outline, the signal is sent from a broadband light source to an edge filter (FBG 1), split into a reference and a sensing arm (FBG 2), received by photodiodes (detectors). It is reiterated that a  $\Delta T$  translates into a  $\Delta\lambda_B$ ; by contrast, the shape (integral) of the reflected spectrum  $G(\lambda)$  remains unaltered (v. Fig. 2.4). That being so, the ratio of the system

$$Q = 10 \log_{10} \left[ \frac{\int G_1(\lambda_{B1,0} + \Delta\lambda_{B1,T}) G_2(\lambda_{B2,0} + \Delta\lambda_{B2,T} + \Delta\lambda_{B2,\epsilon}) d\lambda}{\int G_1(\lambda_{B1,0} + \Delta\lambda_{B1,T}) d\lambda} \right] \quad (2.18)$$

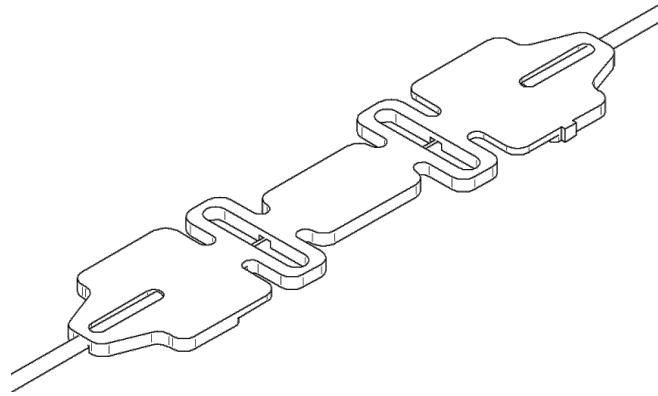
simplifies to

$$Q = 10 \log_{10} \left[ \frac{\int G_1(\lambda_{B1,0}) G_2(\lambda_{B2,0} + \Delta\lambda_{B2,\epsilon}) d\lambda}{\int G_1(\lambda_{B1,0}) d\lambda} \right]. \quad (2.19)$$

None of the quantities in (2.19) pertains to  $T$ , so the technique is *prima facie* temperature-insensitive. Indeed, tests (v. Fig. 2.9) demonstrated that by increasing

- strain from 0 to 1100  $\mu\epsilon$  at room temperature,  $Q$  decreases monotonically at a rate (slope) going successively up and down;
- $T$  from 15 to 50 °C at a fixed strain of 320  $\mu\epsilon$ ,  $\Delta Q = \pm 0.15$  dB, meaning that  $\Delta\epsilon_T = \pm 2.2$   $\mu\epsilon$  or  $S_T = \pm 0.063$   $\mu\epsilon/^\circ\text{C}$  (two orders of magnitude less than the value in Table 2.1).

For the sake of thoroughness, it is worth mentioning that, in similar fashion, the mismatch between the  $\alpha$ 's of the substrate and the measuring grid, combined with the dependence of the grid resistance on  $T$ , contributes to the “thermal output” exhibited by electrical sensors; however, as shown in Fig. 2.11, the temperature drift of a resistive gauge is substantially less severe than that of an optical one mounted on various supports.

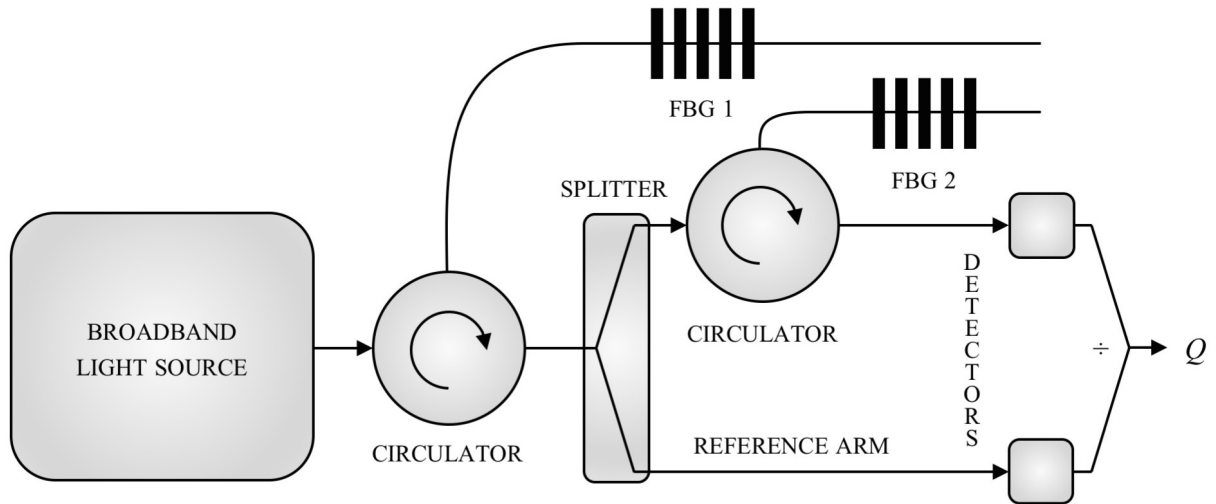


**FIG. 2.7** FBG strain gauge (mfd. by Micron Optics) and...

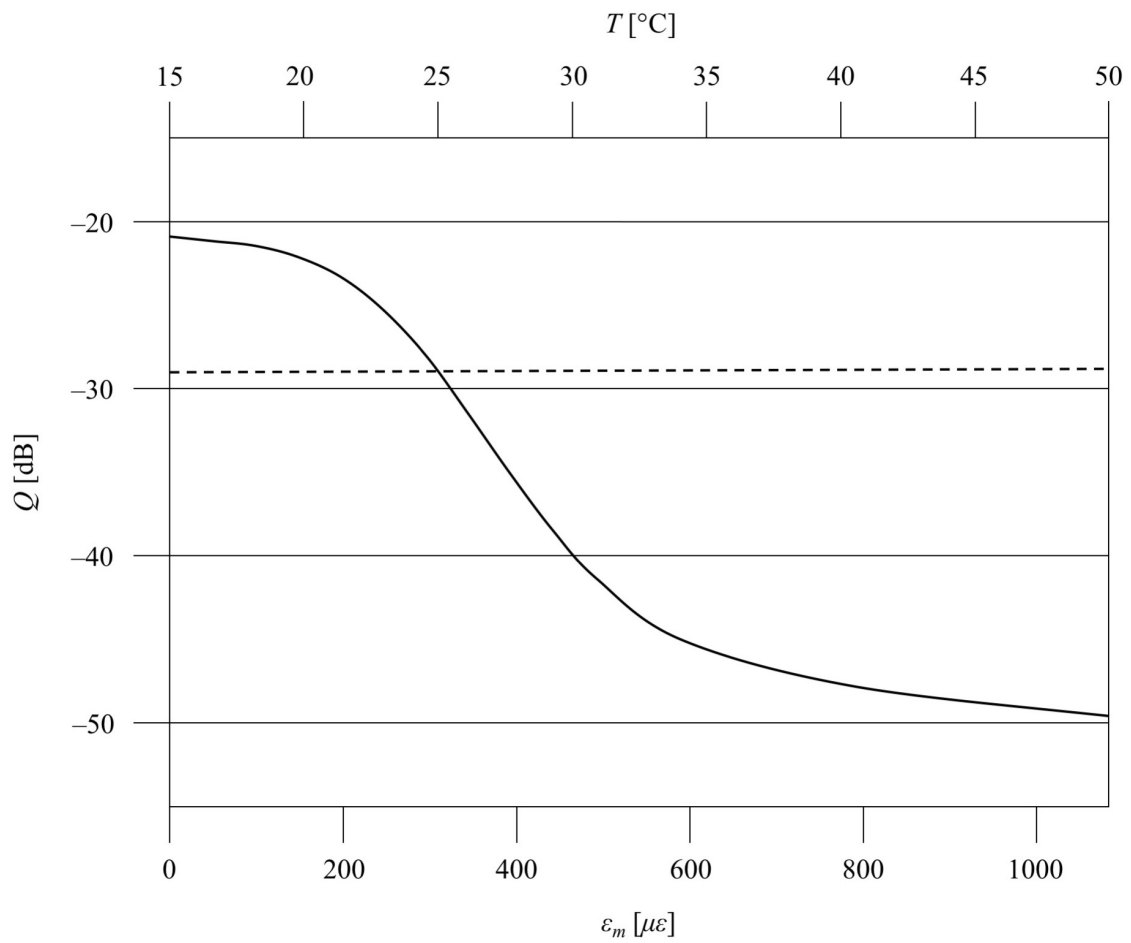
**TABLE 2.1**  
...its characteristics

<i>Parameter</i>	<i>Symbol</i>	<i>Value</i>	<i>Unit</i>
Bragg wavelength	$\lambda_B$	1532-1552*	nm
Strain sensitivity	$S_\epsilon$	$10^{-6}$ *	/ $\mu\epsilon$
Temperature sensitivity	$S_T$	$6.156 \times 10^{-6}$ *	/ $^\circ\text{C}$
Thermal coefficient	$\alpha_F$	0.7	$\mu\epsilon/^\circ\text{C}$

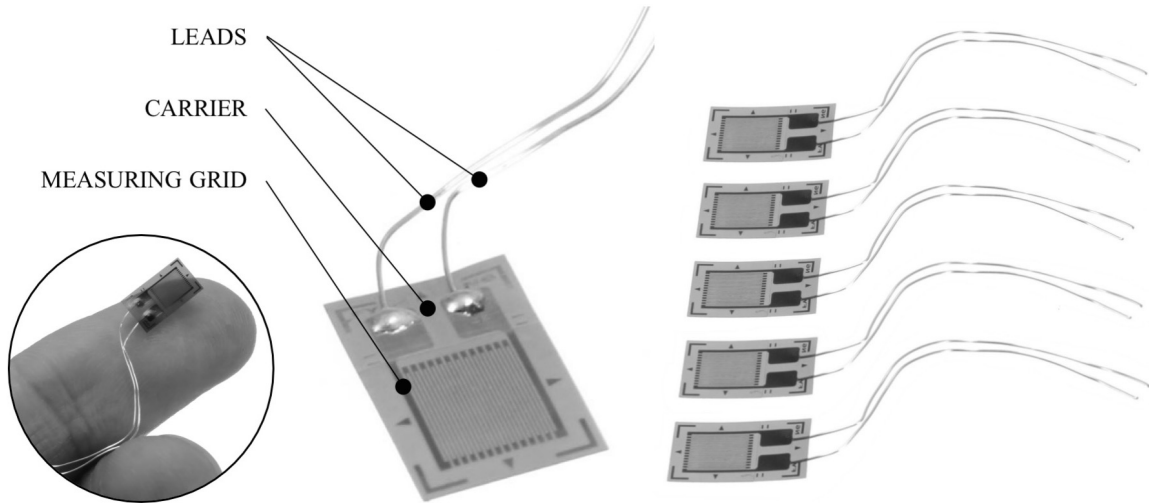
\* At 22 °C.



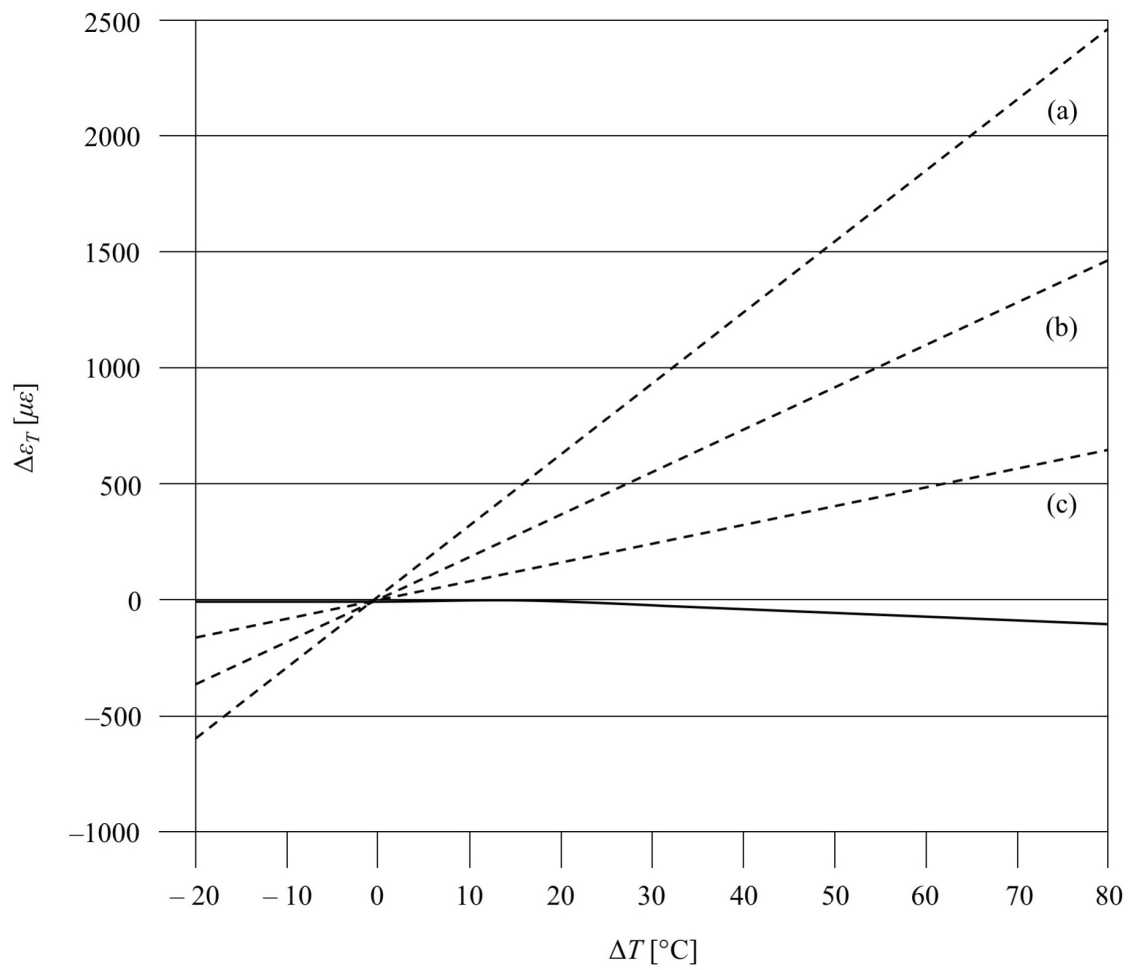
**FIG. 2.8** Diagram of a temperature-insensitive interrogation technique for FBG sensors (adapted from Ref. 6, p. 654).



**FIG. 2.9**  $Q$  vs  $\epsilon_m$  at room temperature (—) and  $T$  at a fixed strain (---) (adapted from *ibid.*, p. 655).



**FIG. 2.10** Electrical strain gauges.



**FIG. 2.11**  $\Delta\epsilon_T$  vs  $\Delta T$  for an electrical (—) and an optical gauge (---) attached to (a) aluminium / (b) steel / (c) glass (adapted from Ref. 10, p. 227).



# 3 LABORATORY SIMULATION

## 3.1 MODUS OPERANDI

The longitudinal strain at an arbitrary point on the exterior of a buckled pipe of outer radius  $R$ , distant  $h$  from the neutral axis and making with it an angle  $\theta$  comprises an axial and a bending component:

$$\varepsilon_l = \varepsilon_a + \varepsilon_b \quad (3.1)$$

or, equivalently,

$$\varepsilon_l = \varepsilon_a + \varepsilon_{b,max} \sin \theta, \quad (3.2)$$

wherein

$$\varepsilon_a = \frac{P_0 - P}{AE}, \quad (3.3)$$

$$\varepsilon_b = \frac{Mh}{EI}, \quad (3.4)$$

$$\varepsilon_{b,max} = \varepsilon_b|_{\theta=\pi/2} = \frac{MR}{EI}. \quad (3.5)$$

(3.2) is insoluble for it contains three unknowns ( $\varepsilon_a$ ,  $\varepsilon_{b,max}$ ,  $\theta$ ), unless as many measurements of  $\varepsilon_l$  per cross section are performed. Ansari et al. proposed that sensors should be placed at intervals of  $120^\circ$  around the circumference (that is to say, at 12, 4 and 8 o'clock):

$$\varepsilon_{l,1} = \varepsilon_a + \varepsilon_{b,max} \sin \theta, \quad (3.6)$$

$$\varepsilon_{l,2} = \varepsilon_a + \varepsilon_{b,max} \sin \left( \theta + \frac{2\pi}{3} \right), \quad (3.7)$$

$$\varepsilon_{l,3} = \varepsilon_a + \varepsilon_{b,max} \sin \left( \theta + \frac{4\pi}{3} \right). \quad (3.8)$$

After some manipulation,

$$\varepsilon_a = \frac{\varepsilon_{l,1} + \varepsilon_{l,2} + \varepsilon_{l,3}}{3}, \quad (3.9)$$

$$\theta = \tan^{-1} \left[ \frac{2\varepsilon_{l,1} - \varepsilon_{l,2} - \varepsilon_{l,3}}{\sqrt{3}(\varepsilon_{l,2} - \varepsilon_{l,3})} \right], \quad (3.10)$$

$$\varepsilon_{b,max} = \frac{\varepsilon_{l,2} - \varepsilon_{l,3}}{\sqrt{3}} \quad (3.11)$$

if  $\theta = 0, \pi$  or else

$$\varepsilon_{b,max} = \frac{2\varepsilon_{l,1} - \varepsilon_{l,2} - \varepsilon_{l,3}}{3 \sin \theta}. \quad (3.12)$$

Two such triads of FBG gauges were clamped (the material, chosen by reason of its relatively high  $\alpha$ , repels most commercial adhesives) to a 1-m-long polyethylene tube (connected to a supply of hot water and equipped with brass terminal fittings as well as frictionless alignment gates, all bolted or

screwed to a spruce board): the first close to the inlet; the second roughly at the centre of a pre-existing bend, exploited to “catalyse” buckling (v. § 1.2). A  $p$  of 1 bar (as read on the dial of the manometers) was attained by partially opening the outlet valve; the average  $T$  regularly measured with an infrared thermometer;  $\varepsilon_l$  acquired by an interrogator, transferred to a computer, saved in a text file (excerpted in Appendix 4) and processed in MATLAB (v. Appendix 5). For details, consult Tables 1.1, 2.1 and Figs. 3.2-4 (made in AutoCAD).

### 3.2 RESULTS

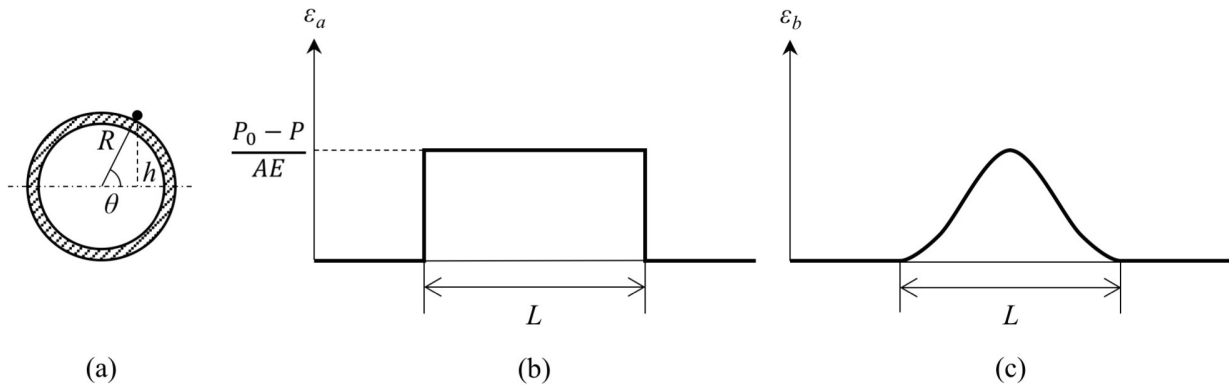
As evidenced by Fig. 3.6, Triad 1 (sensors 1-3) registered an increment in tensile strain (positive); instead, Triad 2 recorded concurrent expansion in the upper portion (sensor 4) and contraction in the lower one (sensors 5-6). The traces ultimately transform into uninterpretable parallel lines on account of temperature compensation (v. § 2.3).

The bending behaviour is not necessarily indicative of buckling, being observable also in the phases preceding or succeeding this event, which can nonetheless be unequivocally identified by distinctive trends of  $\varepsilon_a$  and  $\varepsilon_b$ , extracted from  $\varepsilon_l$  (v. Fig. 3.1b-c). Fig. 3.7 reveals that the tension encountered in the inceptive stage (attributable to the instantaneous contact with the heating fluid) was gradually counterbalanced by the reaction of the constraint at the outlet, perceived first by Triad 2, undergoing sudden compression and subsequently releasing it (a circumstance compatible with geometric shortening – v. § 1.1). Fig. 3.8 “inserts the missing piece in the jigsaw” by evincing a surge in  $\varepsilon_b$  within the bend.

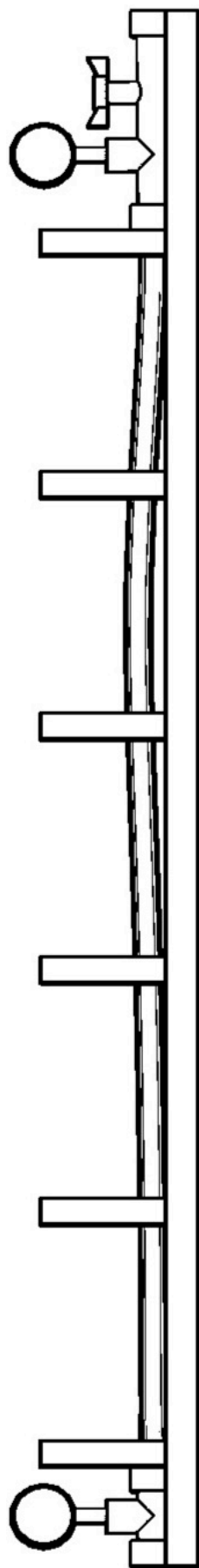
### 3.3 CLOSING REMARKS

To recapitulate,

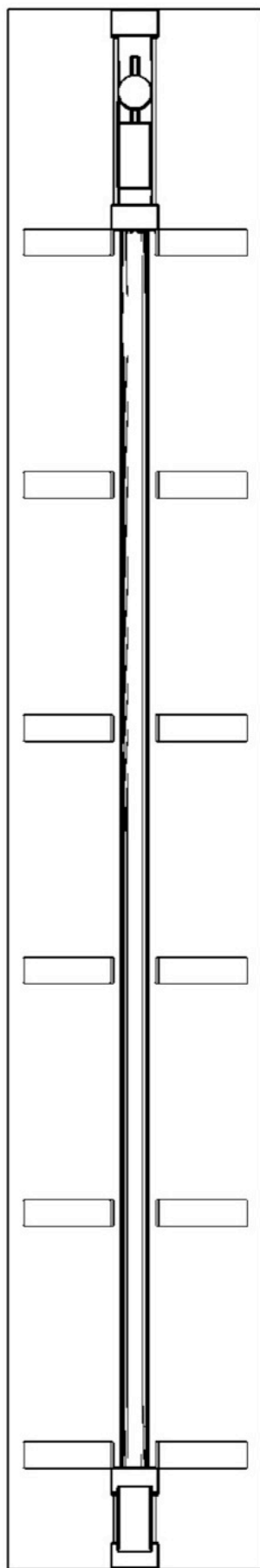
- *upheaval buckling is a mechanism whereby a segment of buried pipeline bows upwards out of the trench under excessive axial compression* (Preface);
- *the possibility that overstress may culminate in rupture strongly calls for the adoption of successful monitoring systems* (ibid.);
- *today fibre-optic sensors offer an attractive alternative to their conventional electrical counterpart* (ibid.);
- *unfortunately, the apparent thermal strain can fatally vitiate measurements* (§ 2.3);
- *this disadvantage notwithstanding, our experiment verified the potential of FBG gauges to “diagnose” buckling in a correct and prompt manner.*



**FIG. 3.1** (a) Cross-section of a buckled pipe. Distributions of (b)  $\varepsilon_a$  and (c)  $\varepsilon_b$ .

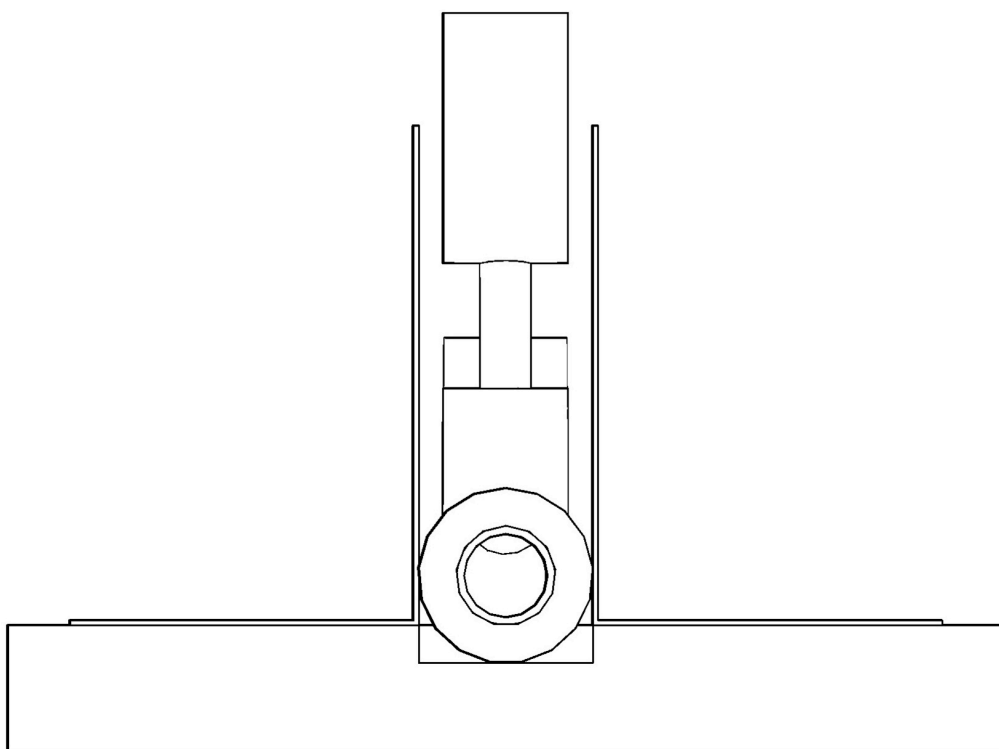


(a)

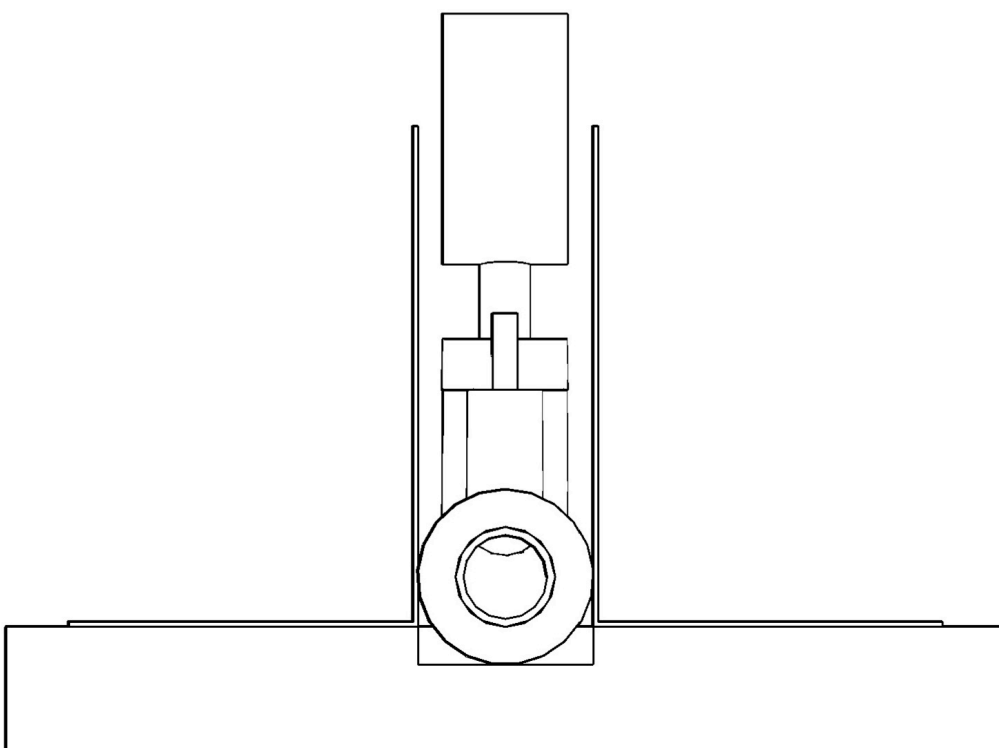


(b)

FIG. 3.2 (a) Front and (b) top views of the apparatus (scale 1:5).



(a)



(b)

**FIG. 3.3** (a) Left and (b) right views of the apparatus (scale 1:1.5).



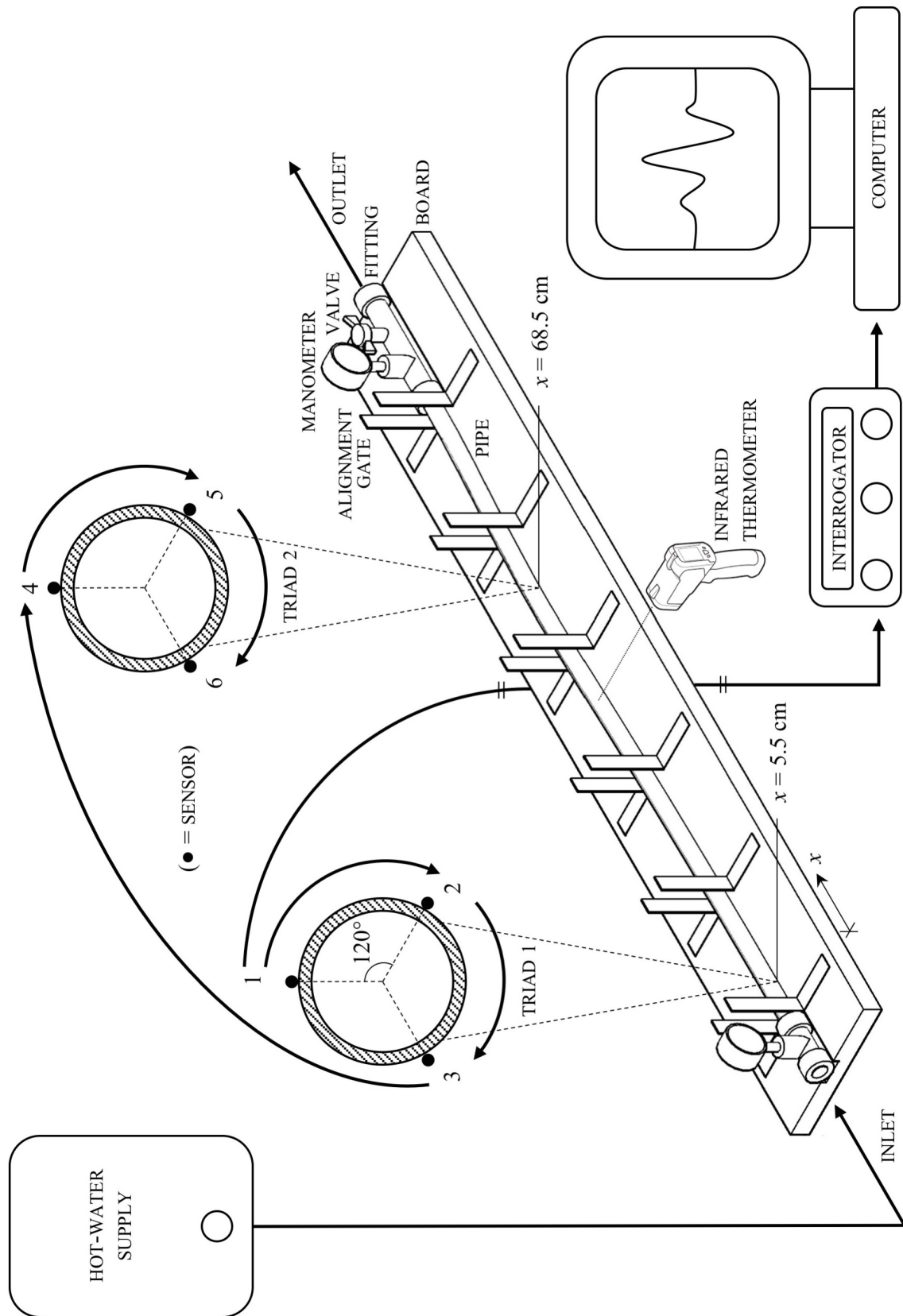
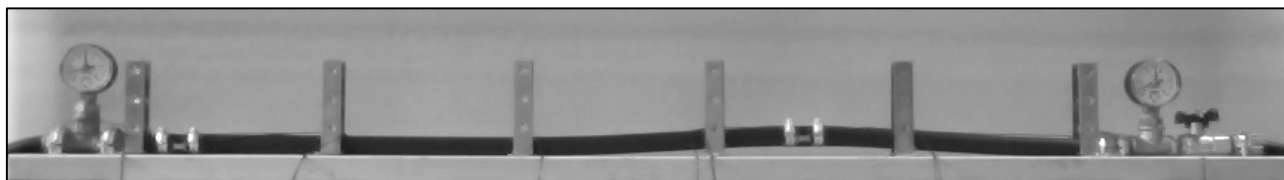


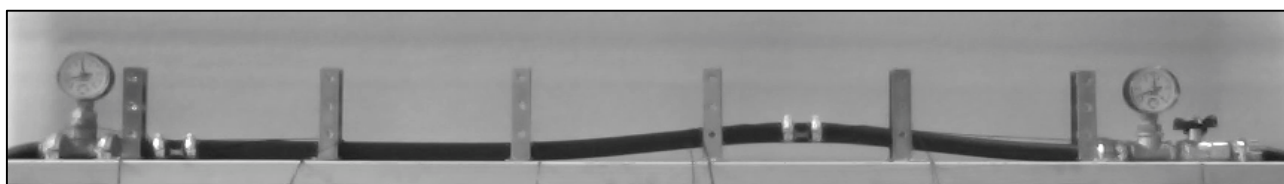
FIG. 3.4 Diagram of the experiment.



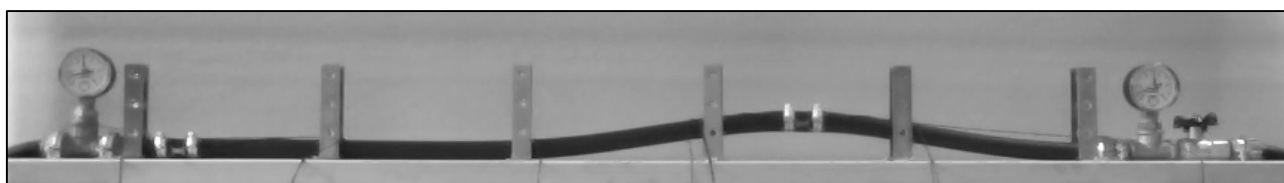
(Time = 0 s)



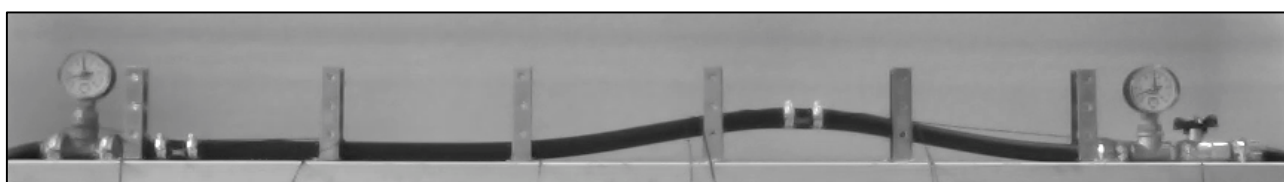
(Time = 5 s)



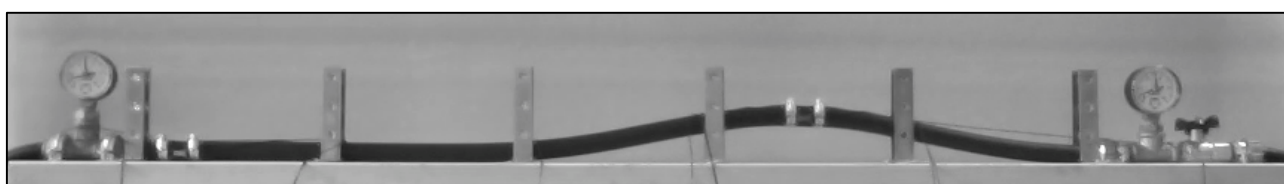
(Time = 10 s)



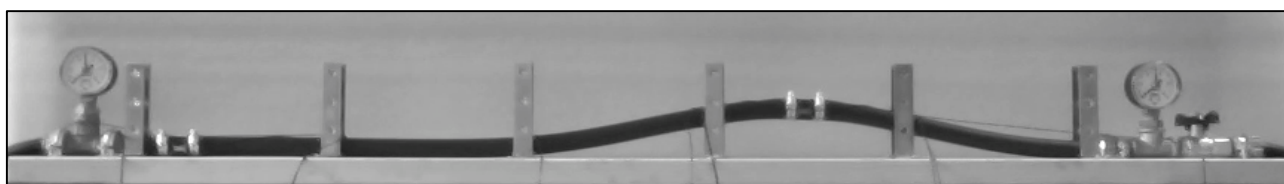
(Time = 15 s)



(Time = 20 s)

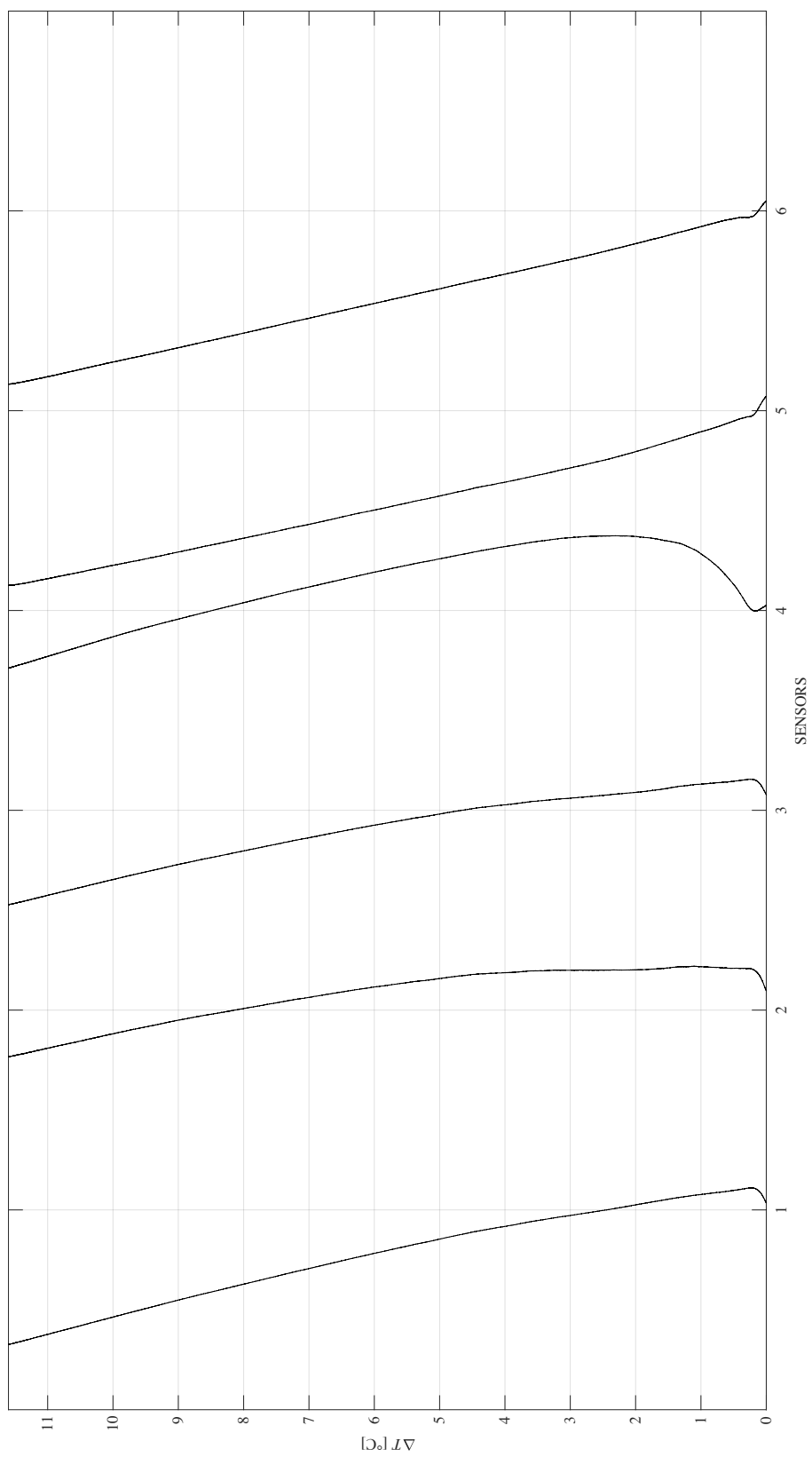


(Time = 25 s)

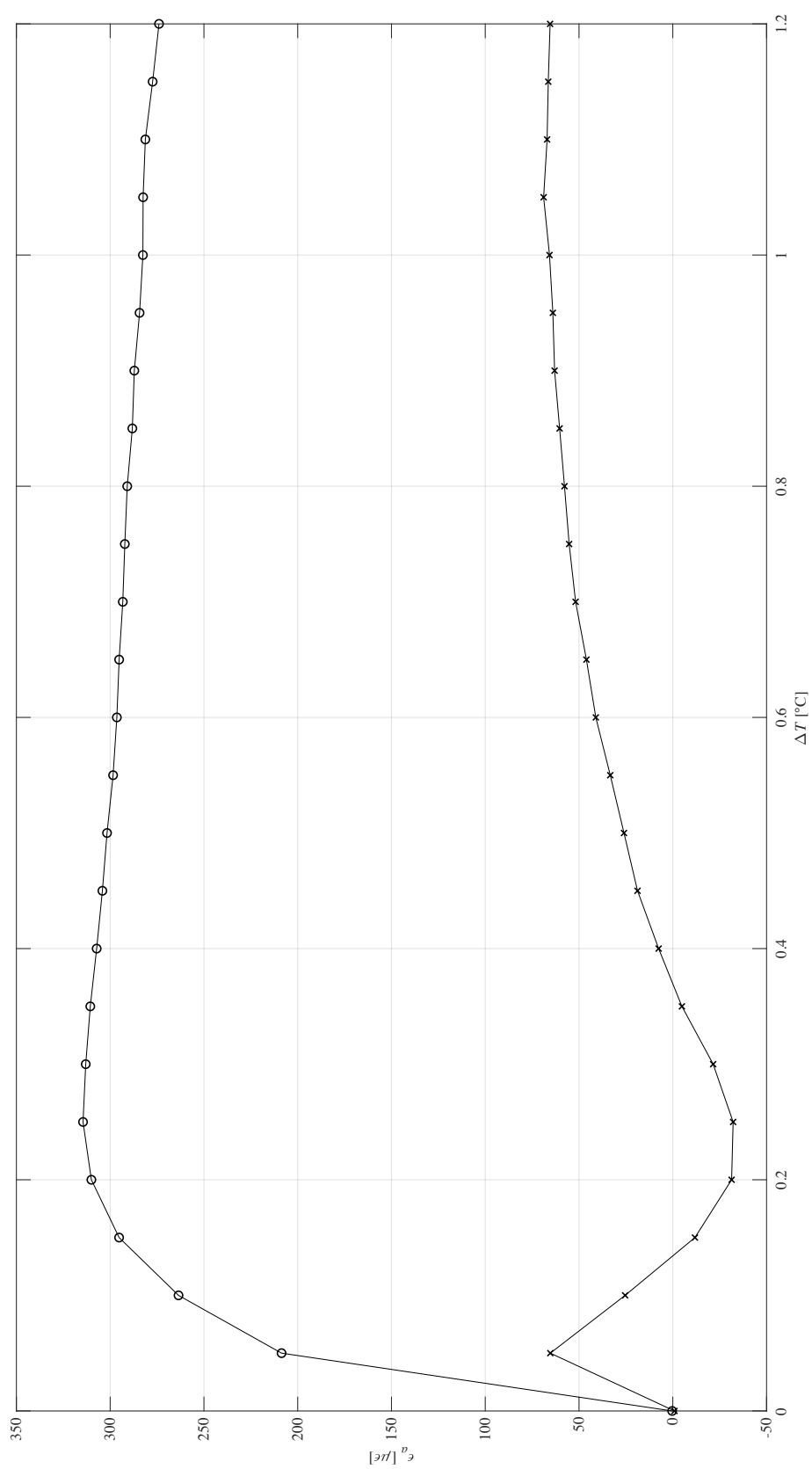


(Time = 30 s)

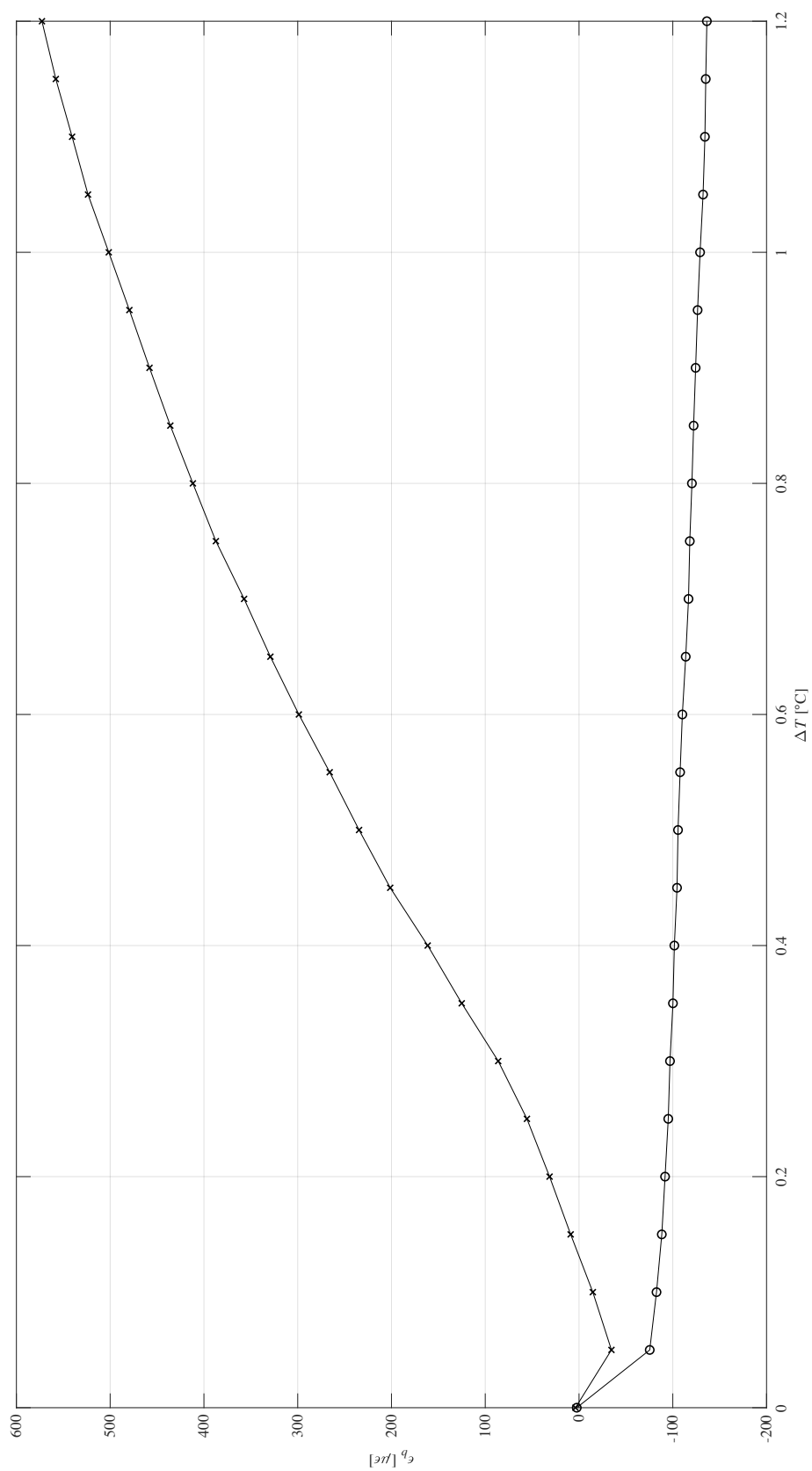
**FIG. 3.5** Selected frames from the video of the experiment.



**FIG. 3.6**  $\Delta T$  vs temperature-compensated  $\varepsilon_i$  for Triads 1 (sensors 1-3) and 2 (sensors 4-6).



**FIG. 3.7**  $\varepsilon_a$  vs  $\Delta T$  for Triads 1 (o) and 2 (x).



**FIG. 3.8**  $\epsilon_b$  vs  $\Delta T$  for Triads 1 (o) and 2 (x).



# APPENDIX 1

$$EI \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} + w = 0 \Rightarrow \dots\dots\dots (1.12)$$

$$\Rightarrow EI y'''' + P y'' = -w \dots\dots\dots (I)$$

This is a 4<sup>th</sup>-order linear inhomogeneous ODE with constant coefficients.

## COMPLEMENTARY FUNCTION

$$EI \rho^4 + P \rho^2 = 0$$

$$\rho^2 (EI \rho^2 + P) = 0$$

$$\rho^2 = 0 \Rightarrow$$

$$\Rightarrow \rho_1 = 0$$

$$EI \rho^2 + P = 0 \Rightarrow$$

$$\Rightarrow \rho_{2,3} = \pm i \sqrt{\frac{P}{EI}} =$$

$$= 0 \pm im =$$

$$= \tau \pm i\omega$$

$$y_0(x) = k_1 e^{\rho_1 x} + e^{\tau x} [k_2 \cos(\omega x) + k_3 \sin(\omega x)] =$$

$$= k_1 + k_2 \cos(mx) + k_3 \sin(mx)$$

## PARTICULAR INTEGRAL

$$y_p(x) = \beta x^2 + \gamma x + \delta$$

$$y_p' = 2\beta x + \gamma$$

$$y_p'' = 2\beta$$

$$y_p''' = 0$$

$$y_p'''' = 0$$

$$P 2\beta = -w \Rightarrow$$

$$\Rightarrow \beta = -\frac{w}{2P} \dots\dots\dots \text{from (I)}$$

$$y_p(x) = -\frac{wx^2}{2P} + \gamma x + \delta$$

## GENERAL INTEGRAL

$$\begin{aligned}
 y(x) &= y_0(x) + y_p(x) = \\
 &= k_1 + k_2 \cos(mx) + k_3 \sin(mx) - \frac{wx^2}{2P} + \gamma x + \delta = \\
 &= k_1^* + k_2 \cos(mx) + k_3 \sin(mx) - \frac{wx^2}{2P} + \gamma x
 \end{aligned}$$

## BOUNDARY CONDITIONS

$$y\left(\frac{L}{2}\right) = k_1^* + k_2 \cos\left(\frac{mL}{2}\right) + k_3 \sin\left(\frac{mL}{2}\right) - \frac{wL^2}{8P} + \frac{\gamma L}{2} = 0 \dots\dots\dots \text{(II), from (1.14)}$$

$$\begin{aligned}
 y\left(-\frac{L}{2}\right) &= k_1^* + k_2 \cos\left(-\frac{mL}{2}\right) + k_3 \sin\left(-\frac{mL}{2}\right) - \frac{wL^2}{8P} - \frac{\gamma L}{2} = \\
 &= k_1^* + k_2 \cos\left(\frac{mL}{2}\right) - k_3 \sin\left(\frac{mL}{2}\right) - \frac{wL^2}{8P} - \frac{\gamma L}{2} = 0 \dots\dots\dots \text{(III), from (1.14)}
 \end{aligned}$$

$$y'\left(\frac{L}{2}\right) = -k_2 \sin\left(\frac{mL}{2}\right)m + k_3 \cos\left(\frac{mL}{2}\right)m - \frac{wL}{2P} + \gamma = 0 \dots\dots\dots \text{(IV), from (1.15)}$$

$$\begin{aligned}
 y'\left(-\frac{L}{2}\right) &= -k_2 \sin\left(-\frac{mL}{2}\right)m + k_3 \cos\left(-\frac{mL}{2}\right)m + \frac{wL}{2P} + \gamma = \\
 &= k_2 \sin\left(\frac{mL}{2}\right)m + k_3 \cos\left(\frac{mL}{2}\right)m + \frac{wL}{2P} + \gamma = 0 \dots\dots\dots \text{(V), from (1.15)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(II)} + \text{(III)} &= 2k_1^* + 2k_2 \cos\left(\frac{mL}{2}\right) - \frac{wL^2}{4P} = 0 \Rightarrow \\
 \Rightarrow k_1^* &= -k_2 \cos\left(\frac{mL}{2}\right) + \frac{wL^2}{8P} \dots\dots\dots \text{(VI)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(IV)} + \text{(V)} &= 2k_3 \cos\left(\frac{mL}{2}\right)m + 2\gamma = 0 \Rightarrow \\
 \Rightarrow \gamma &= -k_3 \cos\left(\frac{mL}{2}\right)m \dots\dots\dots \text{(VII)}
 \end{aligned}$$

$$\begin{aligned}
 -k_2 \cos\left(\frac{mL}{2}\right) + \frac{wL^2}{8P} + k_2 \cos\left(\frac{mL}{2}\right) + k_3 \sin\left(\frac{mL}{2}\right) - \frac{wL^2}{8P} - k_3 \cos\left(\frac{mL}{2}\right)\frac{mL}{2} &= \\
 = k_3 \left[ \sin\left(\frac{mL}{2}\right) - \cos\left(\frac{mL}{2}\right)\frac{mL}{2} \right] &= 0 \Rightarrow \\
 \Rightarrow k_3 = 0 \Rightarrow \dots\dots\dots &\text{from (II)} \\
 \Rightarrow \gamma = 0 \dots\dots\dots &\text{from (VII)}
 \end{aligned}$$

$$k_2 = -\left(k_1^* - \frac{wL^2}{8P}\right) \frac{1}{\cos(mL/2)} \dots\dots\dots \text{(VIII), from (VI)}$$

$$\begin{aligned}
 \left(k_1^* - \frac{wL^2}{8P}\right) \frac{\sin(mL/2)}{\cos(mL/2)} m - \frac{wL}{2P} &= \\
 = \left(k_1^* - \frac{wL^2}{8P}\right) \tan\left(\frac{mL}{2}\right) m - \frac{wL}{2P} &= \\
 = \left(k_1^* - \frac{wL^2}{8P}\right) \frac{m^2 L}{2} - \frac{wL}{2P} = 0 \Rightarrow \\
 \Rightarrow k_1^* = \frac{wL^2}{8P} + \frac{w}{m^2 P} \Rightarrow \dots\dots\dots &\text{from (IV)}
 \end{aligned}$$



$$\Rightarrow k_2 = -\frac{w}{m^2 P \cos(mL/2)} \dots\dots\dots \text{from (VIII)}$$

### SOLUTION

$$\begin{aligned} y(x) &= \frac{wL^2}{8P} + \frac{w}{m^2 P} - \frac{w \cos(mx)}{m^2 P \cos(mL/2)} - \frac{wx^2}{2P} = \\ &= \frac{wEI}{P^2} \left[ 1 + \frac{m^2 L^2}{8} - \frac{m^2 x^2}{2} - \frac{\cos(mx)}{\cos(mL/2)} \right] \dots\dots\dots (1.17) \end{aligned}$$



# APPENDIX 2

```
clear all
close all

%% INPUT (from Table 1.1)

D = 20; %[mm]
t = 1.6; %[mm]
w = 0.000934; %[N/mm]
E = 200; %[N/mm^2]

%% CALCULATIONS

d = D-2*t;
A = (pi/4)*(D^2-d^2);
I = (pi/64)*(D^4-d^4);
Lmin = (1.69*10^6*((E*I)^3/(w^2*A*E)))^0.125; %(1.26)
L = 0.1*Lmin:1:3*Lmin;
P = 80.8*((E*I)./L.^2); %(1.21)
P0 = P+16.0*10^-6*((w^2*A*E.*L.^6)/(E*I)^2); %(1.25)
Lmaxslope = (11.6*((E*I)/w))^0.333; %From eq. 12 in Ref. 8

%% OUTPUT

fig = figure(); %Fig. 1.4
plot(L./10^3,P0,'-k',L./10^3,P,'--k');
xlim([0 5]);
ylim([0 100]);
line([Lmaxslope/10^3 Lmaxslope/10^3],[0 100],'Color','k','LineStyle','-');
xlabel('\itL [m]');
ylabel('\itP_0, \itP [N]');
txt = text(Lmaxslope/10^3,25,'\leftarrow Max. slope = 0.1');
set(findall(fig,'-property','FontName'),'FontName','Times New Roman','FontSize',11);
grid on;
```



# APPENDIX 3

```
clear all
```

```
close all
```

---

```
%% DATA
```

```
p = 80; %[bar] (*)
```

```
D = 48; %[in.] (*)
```

```
t = 0.462; %[in.] (*)
```

```
wpipe = 235; %[lbf/ft] (**)
```

```
Swoil = 300.5; %[lbf/bbl] (**)
```

```
SWfill = 1500; %[kgf/m^3] (***)
```

```
hfill = 8; %[ft] (**)
```

```
E = 200; %[GPa] (****)
```

```
aH = 12*10^-6; %[/°C] (****)
```

```
nu = 0.3; %(****)
```

```
aF = 0.7; %[microstrain/°C] %(****)
```

```
Se = 10^-6; %[/microstrain] %(****)
```

```
ST = 6.156*10^-6; %[/°C] %(****)
```

```
%(*) Table 1 / (**) 2013 TAPS Fact Book / Typical values for (***) gravel, (****) steel / (****) Table 2.1
```

---

```
%% CONVERSIONS
```

```
p = p*10^5; %[Pa]
```

```
D = D*0.0254; %[m]
```

```
t = t*0.0254; %[m]
```

```
wpipe = wpipe*1.35582; %[N/m]
```

```
Swoil = Swoil*2.85; %[kgf/m^3]
```

```
hfill = hfill*0.3048; %[m]
```

```
E = E*10^9; %[Pa]
```

---

```
%% CALCULATIONS
```

```
d = D-2*t;
```

```
A = (pi/4)*(D^2-d^2);
```

```
I = (pi/64)*(D^4-d^4);
```

```
woil = Swoil*((pi/4)*d^2);
```

```
wfill = SWfill*hfill*D;
```

```
w = wpipe+woil+wfill;
```

```
Lmin = (1.69*10^6*((E*I)^3/(w^2*A*E)))^0.125; %(1.26)
```

```
P = 80.8*((E*I)/Lmin^2); %(1.21)
```

```
P0 = P+16.0*10^-6*((w^2*A*E*Lmin^6)/(E*I)^2); %(1.25)
```

```
dT = (1/(A*E*aH))*(P0-(((0.5-nu)*A*p*(D/2))/t)) %(1.5)
```

```
strain = ((P0-P)/(A*E))*10^6 %(1.24), (3.3)
```

```
tempcomp = ((aH/10^-6)-aF+(ST/Se))*dT %(2.17)
```



# APPENDIX 4

TIME		SENSOR_1	SENSOR_2	SENSOR_3	SENSOR_4	SENSOR_5	SENSOR_6
07/06/2019	16:09:29.667	2.737	-1.124	-0.625	2.884	-1.762	-3.784
07/06/2019	16:09:29.668	3.857	-0.500	0.624	2.007	-1.762	-2.397
07/06/2019	16:09:29.669	3.235	-1.124	-0.251	3.386	-1.636	-4.792
07/06/2019	16:09:29.670	3.733	0.624	0.125	4.514	-1.133	-3.531
07/06/2019	16:09:29.671	3.359	-0.873	-0.251	3.134	-1.384	-4.792
07/06/2019	16:09:29.672	3.982	-0.873	0.000	3.261	-1.888	-2.648
07/06/2019	16:09:29.673	2.365	0.000	0.875	2.884	-2.013	-3.909
07/06/2019	16:09:29.674	3.857	-1.748	0.624	3.134	-0.882	-4.792
07/06/2019	16:09:29.675	3.733	-0.873	-0.625	3.386	-1.888	-4.287
07/06/2019	16:09:29.676	2.861	0.000	0.000	2.884	-3.146	-3.909
07/06/2019	16:09:29.677	2.489	-0.624	0.375	2.884	-2.139	-4.035
07/06/2019	16:09:29.678	2.861	-0.873	0.375	3.511	-2.139	-4.161
07/06/2019	16:09:29.679	3.111	-0.873	-0.625	3.762	-1.007	-2.900
07/06/2019	16:09:29.680	3.111	-0.500	0.250	3.887	-1.636	-5.171
07/06/2019	16:09:29.681	3.484	-0.500	1.000	3.636	-2.895	-3.784
07/06/2019	16:09:29.682	3.608	-1.124	-0.376	3.009	-0.882	-3.026
07/06/2019	16:09:29.683	3.733	-1.248	0.875	2.507	-1.888	-3.909
07/06/2019	16:09:29.684	3.111	-0.624	-0.876	2.382	-2.391	-3.909
07/06/2019	16:09:29.685	2.613	-1.124	0.125	3.386	-2.139	-3.909
07/06/2019	16:09:29.686	3.733	-1.124	-0.501	2.759	-1.636	-4.035
07/06/2019	16:09:29.687	3.359	-0.873	0.000	3.009	-1.636	-3.531
07/06/2019	16:09:29.688	2.489	-0.624	0.875	2.884	-2.139	-4.792
07/06/2019	16:09:29.689	2.489	-1.748	-0.251	3.386	-1.384	-3.784
07/06/2019	16:09:29.690	3.111	-1.124	0.250	3.261	-1.384	-4.161
07/06/2019	16:09:29.691	2.986	-0.873	0.624	2.382	-2.517	-3.784
07/06/2019	16:09:29.692	3.982	-1.124	0.375	2.884	-1.888	-3.153
07/06/2019	16:09:29.693	2.986	-1.498	0.125	2.507	-2.139	-4.161
07/06/2019	16:09:29.694	3.484	-1.124	-1.502	4.013	-1.384	-3.909
07/06/2019	16:09:29.695	3.608	0.375	-1.126	2.507	-2.139	-4.792
07/06/2019	16:09:29.696	2.861	-0.873	-0.126	3.134	-1.007	-3.531
07/06/2019	16:09:29.697	2.613	-1.124	-0.625	3.134	-2.895	-5.171
07/06/2019	16:09:29.698	3.608	-1.124	0.750	2.634	-1.636	-2.900
07/06/2019	16:09:29.699	2.986	-0.873	0.250	2.759	-1.636	-3.279
07/06/2019	16:09:29.700	2.861	-0.624	0.624	3.261	-2.266	-4.161
07/06/2019	16:09:29.701	2.613	-0.624	-0.876	3.134	-1.133	-4.918
07/06/2019	16:09:29.702	2.737	0.125	-0.376	2.257	-2.517	-4.161
07/06/2019	16:09:29.703	1.867	-0.624	-0.501	2.759	-1.762	-3.909
07/06/2019	16:09:29.704	2.861	-1.124	-0.251	3.009	-1.384	-4.792
07/06/2019	16:09:29.705	2.861	-1.498	0.375	3.009	-1.762	-4.161
07/06/2019	16:09:29.706	3.733	-1.622	0.000	3.009	-1.258	-2.018
07/06/2019	16:09:29.707	2.613	-1.748	-0.126	2.507	-2.642	-2.648
07/06/2019	16:09:29.708	2.613	-1.124	-0.501	3.261	-2.517	-4.792
07/06/2019	16:09:29.709	2.613	-1.748	0.375	4.138	-2.391	-4.161
07/06/2019	16:09:29.710	3.359	-1.124	-0.126	1.880	-1.258	-4.413
07/06/2019	16:09:29.711	3.111	-1.124	0.499	3.009	-2.139	-3.784
07/06/2019	16:09:29.712	2.861	-1.748	-0.501	3.887	-1.762	-4.161
07/06/2019	16:09:29.713	3.608	0.000	-0.501	3.134	-1.636	-3.909
07/06/2019	16:09:29.714	2.365	0.125	0.000	2.382	-2.642	-3.656
07/06/2019	16:09:29.715	2.365	-1.748	-0.625	3.511	-1.636	-3.909
07/06/2019	16:09:29.716	3.484	-0.500	0.125	3.134	-1.007	-3.656
07/06/2019	16:09:29.717	4.106	-0.873	-0.251	3.636	-2.266	-2.900
07/06/2019	16:09:29.718	2.489	-0.873	0.624	3.636	-1.384	-3.531
07/06/2019	16:09:29.719	3.235	0.375	-0.376	2.884	-2.642	-3.026
07/06/2019	16:09:29.720	2.861	-1.498	-0.501	3.762	-1.762	-5.171
07/06/2019	16:09:29.721	2.489	-0.624	-0.501	2.759	-0.755	-3.784
07/06/2019	16:09:29.722	3.608	-0.873	0.375	3.511	-1.762	-4.792
07/06/2019	16:09:29.723	2.986	-0.500	0.375	2.884	-0.882	-3.026
07/06/2019	16:09:29.724	2.365	-1.248	-0.126	3.134	-1.888	-3.279
07/06/2019	16:09:29.725	2.986	-0.249	0.375	3.386	-1.762	-3.909
07/06/2019	16:09:29.726	3.235	-0.873	-0.251	2.507	-1.762	-3.531
07/06/2019	16:09:29.727	2.861	-1.498	0.375	3.386	-1.888	-3.531
07/06/2019	16:09:29.728	3.733	-1.124	-0.376	2.507	-1.762	-3.784
07/06/2019	16:09:29.729	2.489	-0.249	-0.625	2.759	-2.642	-2.018
07/06/2019	16:09:29.730	2.986	-0.249	1.000	4.138	-1.384	-3.026
07/06/2019	16:09:29.731	2.861	-1.248	0.624	2.884	-1.762	-4.540
07/06/2019	16:09:29.732	3.359	-0.624	0.624	3.511	-1.258	-4.161
07/06/2019	16:09:29.733	3.235	0.125	0.375	3.134	-1.133	-2.522
07/06/2019	16:09:29.734	3.733	-0.500	-0.501	3.386	-2.391	-3.531
07/06/2019	16:09:29.735	3.359	-0.500	-0.126	2.634	-1.636	-3.279
07/06/2019	16:09:29.736	3.111	-1.124	0.750	3.762	-1.133	-3.909
07/06/2019	16:09:29.737	3.111	0.749	-0.501	4.013	-1.384	-3.909
07/06/2019	16:09:29.738	2.861	-0.624	0.875	2.759	-1.384	-3.279
07/06/2019	16:09:29.739	4.355	-0.624	0.499	3.134	-1.007	-4.540
07/06/2019	16:09:29.740	2.861	-0.249	0.875	3.386	-2.013	-3.909
07/06/2019	16:09:29.741	3.733	-0.249	0.000	3.134	-1.133	-2.522
07/06/2019	16:09:29.742	3.111	0.000	2.376	2.884	-0.882	-4.035
07/06/2019	16:09:29.743	3.982	0.375	1.625	3.386	-1.258	-4.918
07/06/2019	16:09:29.744	4.355	0.000	0.499	3.386	-0.882	-4.792

[...]





# APPENDIX 5

```
function FBG_Strain_Gauge_Data_Analysis_Tool()
```

---

```
clear all
```

```
close all
```

---

```
%% INPUT
```

```
aH = 200; %[microstrain/°C] (*)
```

```
aF = 0.7; %[microstrain/°C] (**)
```

```
Se = 10^-6; %[/microstrain] (**)
```

```
ST = 6.156*10^-6; %[/°C] (**)
```

```
time1 = 10; %[s]
```

```
time2 = 20; %[s]
```

```
time3 = 30; %[s]
```

```
T0 = 23.8; %[°C] (***)
```

```
T1 = 25.6; %[°C] (***)
```

```
T2 = 31.4; %[°C] (***)
```

```
T3 = 35.4; %[°C] (***)
```

```
%(*) Table 1.1 / (**) Table 2.1 / (***) Measured
```

---

```
%% IMPORT DATA FROM TEXT FILE
```

```
[filename,~] = uigetfile({'*.txt'}, 'Please, choose a file:');
```

```
delimiter = '\t';
```

```
if nargin <= 2
```

```
    startRow = 2;
```

```
    endRow = inf;
```

```
end
```

```
formatSpec = '%*q%f%f%f%f%f%f%[\n\r]';
```

```
fileID = fopen(filename, 'r');
```

```
textscan(fileID, '%[\n\r]', startRow-1, 'WhiteSpace', '', 'ReturnOnError', false);
```

```
dataArray = textscan(fileID, formatSpec, endRow-startRow+1, 'Delimiter', delimiter, 'TextType', 'string', ...  
    ... 'EmptyValue', NaN, 'ReturnOnError', false, 'EndOfLine', '\r\n');
```

```
fclose(fileID);
```

```
strain = [dataArray{1:end-1}];
```

```
clearvars filename delimiter startRow endRow formatSpec fileID dataArray ans;
```

```
[a,~] = size(strain);
```

```
time = [0, time1, time2, time3];
```

```
T_meas = [T0, T1, T2, T3];
```

```
regr = polyfit(time, T_meas, 3);
```

```
for i = 1:6
```

```
    for ii = 1:1:a
```

```
        T_int(ii,i) = regr(1)*((ii-1)*10^-3)^3+regr(2)*((ii-1)*10^-3)^2+regr(3)*((ii-1)*10^-3)+T0;
```

```
    end
```

```
end
```

```

dT = T_int-T0;
strain_c = strain-((aH-aF+ST/Se).*dT); %(2.16-17)

```

---

```

%% CALCULATIONS AND OUTPUT

fig1 = figure(); %Fig. 3.6
dT_T = transpose(dT(:,1));
x = (1:6);
wigggle(x,dT_T,strain_c,'wigggle',2000);
axis([0 max(x)+1 0 max(dT_T)]);
xticks(1:max(x));
yticks(0:max(dT_T));
xlabel('SENSORS');
ylabel('\Delta\itT [°C]');
grid on;
%(3.9-12):
iii = 1;
for iv = 0:0.05:1.2
    k = find(dT_T >= iv,1);
    ea1(iii) = (strain_c(k,1)+strain_c(k,2)+strain_c(k,3))/3;
    theta1 = atan((2*strain_c(k,1)-strain_c(k,2)-strain_c(k,3))/(sqrt(3)*(strain_c(k,2)-strain_c(k,3))));
    if (theta1 == 0) || (theta1 == pi)
        eb1_max = (strain_c(k,2)-strain_c(k,3))/sqrt(3);
    else
        eb1_max = (2*strain_c(k,1)-strain_c(k,2)-strain_c(k,3))/(3*sin(theta1));
    end
    eb1(iii) = eb1_max*sin(theta1);
    ea2(iii) = (strain_c(k,4)+strain_c(k,5)+strain_c(k,6))/3;
    theta2 = atan((2*strain_c(k,4)-strain_c(k,5)-strain_c(k,6))/(sqrt(3)*(strain_c(k,5)-strain_c(k,6))));
    if (theta2 == 0) || (theta2 == pi)
        eb2_max = (strain_c(k,5)-strain_c(k,6))/sqrt(3);
    else
        eb2_max = (2*strain_c(k,4)-strain_c(k,5)-strain_c(k,6))/(3*sin(theta2));
    end
    eb2(iii) = eb2_max*sin(theta2);
    iii = iii+1;
end

fig2 = figure(); %Fig. 3.7
plot(0:0.05:1.2,ea1,'-ok',0:0.05:1.2,ea2,'-xk');
xlabel('\Delta\itT [°C]');
ylabel('\it\epsilon_{\ita} [\it\mu\epsilon]');
grid on;

fig3 = figure(); %Fig. 3.8
plot(0:0.05:1.2,eb1,'-ok',0:0.05:1.2,eb2,'-xk');
xlabel('\Delta\itT [°C]');

```

```

ylabel('{\it\epsilon}_{\itb} [\it\mu\epsilon]');
grid on;
set(findall(fig1,'-property','FontName'),'FontName','Times New Roman','FontSize',11);
set(findall(fig2,'-property','FontName'),'FontName','Times New Roman','FontSize',11);
set(findall(fig3,'-property','FontName'),'FontName','Times New Roman','FontSize',11);

```

---

```

function wiggle(x,t,Data,style,dmax,showmax,plImage,imageax,~) %(C) 2004 Thomas Mejer Hansen

```

---

```

if (nargin == 9)
    np=3;
    subplot(np,np,1);
    wiggle(Data);
    subplot(np,np,2);
    wiggle(Data,dmax);
    subplot(np,np,3);
    wiggle(x,t,Data);
    subplot(np,np,4);
    wiggle(x,t,Data,style,dmax);
    subplot(np,np,5);
    wiggle(x,t,Data,style,dmax,showmax);
    subplot(np,np,6);
    wiggle(x,t,Data,style,dmax,showmax,plImage);
    if isempty(dmax)
        dmax = nanmax(abs(Data(:)));
    end
    subplot(np,np,7);
    wiggle(x,t,Data,style,dmax,showmax,plImage,dmax./10);
    return
end

showmax_def = 100;
style_def = 'wiggle';
if (nargin == 1)
    Data = x;
    t = [1:1:size(Data,1)];
    x = [1:1:size(Data,2)];
    dmax = max(Data(:));
    style = style_def;
    showmax = showmax_def;
end

if (nargin == 2)
    Data = x;
    dmax = t;
    t = [1:1:size(Data,1)];

```

```

    x = [1:1:size(Data,2)];
    style = style_def;
    showmax = showmax_def;
end
if (nargin == 3)
    style = style_def;
    dmax = nanmax(abs(Data(:)));
    showmax = showmax_def;
end
if (nargin == 4)
    dmax = nanmax(abs(Data(:)));
    showmax = showmax_def;
end
if (nargin == 5)
    showmax = showmax_def;
end
if (nargin < 7)
    plImage = 0;
end
if isempty(dmax)
    dmax = nanmax(abs(Data(:)));
end
if isempty(showmax)
    showmax=100;
end
if (nargin == 7)
    imageax = [-1 1].*dmax;
end
if (plImage == 1)
    imagesc(x,t,Data);
    if (length(imageax) == 1)
        imageax = [-1 1].*abs(imageax);
    end
    caxis(imageax);
    hold on;
end
if (showmax > 0)
    if (length(x) > 1)
        dx=x(2)-x(1);
    end
    ntraces = length(x);
    d = ntraces/showmax;
    if (d <= 1);

```

```

        d = 1;
    end
    d = round(d);
    dmax = dmax/d;
    for i=1:d:length(x)
        xt = dx*Data(:,i)'./dmax;
        plot(xt+x(i),t,'k-');
        if (i == 1)
            hold on;
        end
    end
    end
    axis([x(1) x(i)+1 min(t) max(t)])
end
hold off;

```



# NOMENCLATURE

## ENGLISH

$A$	pipe cross-sectional area	$P_1$	$P_0$ temperature component
$C$	buckle contour	$P_2$	$P_0$ pressure component
$D$	pipe outer diameter	$p$	pipe internal pressure
$E$	pipe Young's modulus	$Q$	interrogation-system ratio
$E_F$	optical-fibre Young's modulus	$R$	pipe outer radius
$G$	reflected spectrum	$r$	pipe inner radius
$H$	buckle maximum height	$S$	buckle shear force
$h$	vertical distance from pipe neutral axis	$S_T$	optical-fibre temperature sensitivity
$I$	pipe cross-section second moment of area	$S_\epsilon$	optical-fibre strain sensitivity
$L$	buckle length	$T$	temperature
$M$	buckle bending moment	$t$	pipe wall thickness
$m$	$= \sqrt{P/EI}$ , wavenumber	$VST$	pipe Vicat softening temperature
$n$	optical-fibre refractive index	$w$	pipe weight per unit length
$P$	buckle normal (axial) force	$x$	horizontal coordinate (// pipe axis)
$P_0$	pre-buckle normal (axial) force	$y$	vertical coordinate ( $\perp$ pipe axis)

## GREEK

$\alpha$	coefficient of linear thermal expansion (CLTE)	$\epsilon_T$	optical-fibre apparent thermal strain
$\alpha_F$	optical-fibre CLTE	$\zeta_L$	optical-fibre path length
$\alpha_H$	host-structure (pipe) CLTE	$\theta$	angle with respect to pipe neutral axis
$\epsilon$	strain	$\Lambda$	optical-fibre grating period
$\epsilon_1$	$P_1$ -related strain	$\Lambda_{mask}$	phase-mask grating period
$\epsilon_2$	$P_2$ -related strain	$\lambda$	optical-fibre wavelength
$\epsilon_a$	$\epsilon_l$ axial component	$\lambda_B$	optical-fibre Bragg wavelength
$\epsilon_b$	$\epsilon_l$ bending component	$\nu$	pipe Poisson's ratio
$\epsilon_l$	buckled-pipe longitudinal strain	$\sigma$	stress
$\epsilon_m$	optical-fibre mechanical strain	$\sigma_y$	pipe yield strength





# UNIT CONVERSIONS

---

$$\begin{array}{lcl} \text{bar} & \times & 1.0^* \quad \text{E-01} = \text{N/mm}^2 \\ & ( \times & 1.0^* \quad \text{E+05} = \text{N/m}^2 \text{ or Pa} \quad ) \end{array}$$

$$\text{stb} \quad \times \quad 1.589873 \quad \text{E-01} = \text{sm}^{3**}$$

$$\begin{array}{lcl} \text{in.} & \times & 2.54^* \quad \text{E+01} = \text{mm} \\ & ( \times & 2.54^* \quad \text{E-02} = \text{m} \quad ) \end{array}$$

$$\text{scf} \quad \times \quad 2.831685 \quad \text{E-02} = \text{sm}^{3**}$$

$$\mu\epsilon \quad \times \quad 1.0^* \quad \text{E+00} = \mu\text{m/m}$$

---

\* Exact factor.

\*\* At the stock-tank (for oil) or standard (for gas) conditions of 15 °C and 101325 Pa (atmospheric pressure).



# BIBLIOGRAPHY

1. ANSARI, FARHAD; FENG, XIN; MENG, DEWEI; WU, WENJING; ZHOU, JING – *Distributed monitoring method for upheaval buckling in subsea pipelines with BOTDA sensors*, in *Advances in structural engineering*, SAGE, 2017, vol. 20, no. 2, pp. 180-190.
2. ARREGUI, FRANCISCO; CLAUS, RICHARD O.; MATIAS, IGNACIO – *Optical-fibre sensors*, in *Handbook of optics*, McGraw-Hill, 2001, vol. 4, ch. 15.
3. BROWN, TOM G. – *Optical fibres and fibre-optic communications*, in *Handbook of optics*, op. cit., ch. 1.
4. CALLADINE, CHRISTOPHER R.; MALTBY, TIMOTHY C. – *An investigation into upheaval buckling of buried pipelines (I). Experimental apparatus and some observations*, in *International journal of mechanical sciences*, Elsevier, 1995, vol. 37, no. 9, pp. 943-963.
5. ID. – *An investigation into upheaval buckling of buried pipelines (II). Theory and analysis of experimental observations*, in *International journal of mechanical sciences*, op. cit., pp. 965-983.
6. FARRELL, GERALD; SEMENOVA, YULIYA; SUN, AN; WANG, PENGFEI; WU, QIANG – *High-resolution temperature-insensitive interrogation technique for FBG sensors*, in *Optics and laser technology*, Elsevier, 2009, vol. 42, no. 4, pp. 653-656.
7. HILL, KENNETH O. – *FBGs*, in *Handbook of optics*, op. cit., ch. 9.
8. HOBBS, ROGER E. – *Pipeline buckling caused by axial loads*, in *Journal of constructional-steel research*, Elsevier, 1981, vol. 1, no. 2, pp. 2-10 (republished as *In-service buckling of heated pipelines*, in *Journal of transportation engineering*, ASCE, 1984, vol. 110, no. 2, pp. 175-189).
9. KING, ROGER A.; PALMER, ANDREW C. – *Subsea pipeline engineering*, PennWell, 2008, ch. 14.
10. KLECKERS, THOMAS – *Fibre Bragg sensors compared with electrical strain gauges for use in force measurement. Prospects and potentials*, in *Proceedings of the 19<sup>th</sup> IMEKO world congress on fundamental and applied metrology (Lisbon, 6-11 September 2009)*, pp. 226-229.
11. MEASURES, RAYMOND M. – *Structural monitoring with fibre-optic technology*, Academic Press, 2001, chs. 5-7.

