POLITECNICO DI TORINO

Corso di Laurea Magistrale
in Ingegneria Civile

Tesi di Laurea Magistrale

Robustness assessment in reliability terms
of reinforced concrete structures in seismic zone

Relatore: Prof. Paolo Castaldo

Correlatore: Ing. Diego Gino

Candidata:

Elena Miceli

Marzo 2020

Summary

In the past decades structural robustness has become an important issue among civil engineering studies, in the light of what has happened in terms of accidental events like the World Trade Center terrorist attack (New York, 11th September 2001).

Normally, buildings are designed to withstand to normal actions, whose types and intensities are defined according to the building's importance. Nevertheless, when an action is unlike to occur, or when it occurs whit a larger intensity than in normal scenarios, the building cannot resist to this event and a collapse is likely to initiate. When the facility is able to avoid disproportionate collapse, if a local damage occurs after an accidental event, the structure is judged robust from a structural point of view.

In this work of thesis, a probabilistic approach is applied to evaluate the robustness of a reinforced concrete building, since a reliability evaluation of the problem can be useful in the contest of robustness analysis, in order to take into account the uncertainties affecting structural performance.

The frame under analysis is located in a highly seismic area of Italy, in L'Aquila city. Previous works of thesis have demonstrated that the structure, as it has been designed according to code rules, is not adequate, from a robustness point of view, to resist to an accidental event, like an explosion or an impact, which causes the removal of the central column. This means that different approaches with respect to the ones prescribed by code rules should be followed to enhance structural robustness.

The starting point of this thesis is the same frame previously described, with the structural details that have been judged the most appropriate from a robustness point of view. A probabilistic approach has been applied, by sampling materials and loads parameters with a Latin Hypercube Sampling (LHS), one-hundred times.

A FEM software, called ATENA 2D, has been used. The main analyses that have been performed, for all the one-hundred scenarios, are of two types: a *pushdown analysis* and a *reliability analysis*. The former was necessary to compute the dynamic amplification coefficients, to be applied to the loads in the second type of analysis, after the column is removed. The latter is needed for the reliability evaluation of the structural robustness.

These dynamic amplification coefficients, different for all the one-hundred scenarios, have been evaluated using an approach proposed by Izzuddin et al. [1], which consists on performing a pushdown analysis, obtaining the capacity curve and computing the dynamic displacement. This corresponds to the intersection point between the internal energy and external work curves. If the equilibrium is not found (i.e. if the curves do not intersect), the structure reaches collapse, which has happened for almost the 50% of the cases.

After that, the reliability analysis was performed for all the one-hundred scenarios, so that it was possible to verify the method proposed by Izzuddin; all the scenarios where the equilibrium was not found did not effectively reach convergence criteria in the probabilistic analysis, which confirmed the initial results.

In the end, the strains at different points of the frame have been monitored, both for the concrete and the reinforcement parts, so to evaluate the local probability of failure. This has resulted in a global reliability evaluation.

Ringraziamenti

A conclusione di un percorso di vita così pregnante, ringraziare le persone che vi hanno partecipato è solo una parte di quanto meritano.

Ringrazio la mia Famiglia, Mamma Maria che mi ha insegnato a insistere sempre, ed è grazie a lei che raggiungo i miei successi, Papà Nunzio che mi ha temprata e resa forte, e i miei fratelli, che sono il mio punto di riferimento: la piccola Miriam, che sempre e sarà la mia gioia, anche da lontano e Marcello, anche e soprattutto quando mi fa disperare. Grazie allo Zio Emanuele, con cui è sempre bello scherzare.

Grazie a Fil, perché mi aiuta a superare i miei limiti e a combattere le mie incertezze ed è sempre il mio porto sicuro. Ringrazio Luisella, il cui affetto non finisce mai di emozionarmi e Muin, con i suoi modi particolari, perché mi hanno accolto a braccia aperte nelle loro famiglie e mi fanno sempre sentire a casa.

Grazie a Bartolomeo e Gabriele, i miei due soci nei lavori di gruppo, senza di voi questo percorso avrebbe avuto tutto un altro colore. In particolare grazie a Bartolomeo, con il quale ho condiviso ogni successo degli ultimi anni e con il quale è nata una sincera amicizia.

Grazie agli altri due compagni di cantina, Giulio e Mario, che hanno reso quella stanza un ritrovo di siculi, più che un semplice laboratorio, e mi hanno fatta sentire vicino alla mia terra nonostante i chilometri di distanza. Ringrazio anche i ragazzi della magistrale del primo anno, perché mi hanno fatta tornare ai tempi del liceo.

Infine, grazie alle persone che hanno reso possibile questo lavoro di tesi. Grazie al Professore Paolo Castaldo, per la fiducia che ha sempre riposto nei miei confronti e che apprezzo molto e grazie a Diego, perché il suo aiuto e le sue intuizioni sono sempre stati preziosi.

In conclusione, anche se non è un vero e proprio ringraziamento, vorrei ricordare, in modo che resti indelebile, il periodo storico in cui questa tesi verrà discussa, quello del COVID-19, perché mi sta insegnando a capire ciò che è davvero importante.

Questa tesi è dedicata a mio Nonno Turi, il mio angelo custode

Contents

Preface……………………………………………………………………..1

1 Structural Robustness 4

1.1 Definition of Robustness 4

1.1.1 Code rules 4

1.2 Modelling and Intensity of accidental actions 6

1.2.1 Classification of the accidental actions 7

1.2.2 Modelling of the accidental actions 8

1.3 Risk of disproportionate collapse 12

1.3.1 The concept of risk 12

1.3.2 Probabilistic risk analysis 13

1.3.3 Risk analysis based on scenarios 14

1.4 Risk reduction strategies 14

1.4.1 Classification of design approaches 14

1.4.2 Summary of the possible strategies 17

1.5 Design for robustness 17

1.5.1 Conceptual design 17

1.5.2 Structural modelling 19

1.5.3 Types of analysis 20

1.5.4 Design for reinforced concrete structures cast in place 20

1.6 Probabilistic and semi-probabilistic quantification of the robustness……………………………………………………………..26

2 Structural Reliability 29

2.1 Definition of reliability 29

2.2 Basic principles 30

2.2.1 Limit state design 31
2.2.2 Levels-of-Approximation approach 32
2.2.3 Uncertainties classification..33
2.3 Problem formulation for structural reliability................................37
2.3.1 Probability of failure..37
2.3.2 Reliability index 39
2.4 Classification of reliability methods 39
2.4.1 Methods of Level III...40
2.4.2 Methods of Level II 47
2.4.3 Methods of Level I 51
2.4.4 Methods of Level 0...52
2.5 Target reliability 52
2.6 Safety format..54
2.6.1 Probabilistic safety format..55
2.6.2 Partial safety factor format 55
2.6.3 Global resistance format...57
3 RC building: design characteristics...59
3.1 Introduction...59
3.2 Geometrical characteristics..60
3.3 Material characteristics 62
3.3.1 Concrete..62
3.3.2 Steel 62
3.4 Durability..63
3.4.1 Exposure class 63
3.4.2 Structural class..64
3.4.3 Concrete cover..64

3.5 Actions ... 66
3.5.1 Permanent actions ... 68
3.5.2 Variable actions ... 70
3.5.3 Seismic action ... 72
3.6 Modal analysis ... 74
3.7 Structural verification and dimensioning ... 75
3.7.1 Beam design: bending at ULS ... 76
3.7.2 Beam design: shear at ULS ... 78
3.7.3 Beam design: SLS ... 80
3.7.4 Column design: bending and compression at SLU ... 83
3.7.5 Column design: shear at SLU ... 83
3.7.6 Joints design ... 85
3.7.7 Summary of the design solution according to capacity design 87
3.8 Design characteristics of the frame under analysis ... 87
3.8.1 Summary of previous robustness analysis ... 88
3.8.2 Summary of the design solution, according to robustness analysis ... 89
4 ATENA 2D ... 90
4.1 Software generalities ... 90
4.2 Pre-processing ... 91
4.2.1 Graphical interface ... 91
4.2.2 Materials ... 92
4.2.3 Topology ... 99
4.2.4 Loads and supports ... 104
4.2.5 Run ... 109
4.3 Post processing ... 112

4.3.1 Cracks 113

4.3.2 Bar reinforcement 114

4.3.3 Scalars 115

4.3.4 Output document 115

5 FEM Model and Reliability analysis 116

5.1 Introduction 116

5.2 Basic variables sampling 117

5.2.1 Resistance basic variables 120

5.2.2 Action basic variables 127

5.3 Material's constitutive law 133

5.3.1 Concrete constitutive law 135

5.3.2 Steel reinforcement constitutive law 138

5.4 Geometry and mesh for analyses 1,2 and 3 139

5.5 Analysis 1 - Model with column 141

5.5.1 Loads cases 141

5.5.2 Analysis steps 142

5.6 Analysis 2 - Pushdown analysis 143

5.6.1 Load cases 144

5.6.2 Analysis steps 145

5.6.3 Dynamic amplification coefficient 148

5.6.4 Dynamic linear analysis simulation 152

5.7 Analysis 3 - Reliability Analysis 154

5.7.1 Load cases 155

5.7.2 Analysis steps 156

5.7.3 Output analysis 158

5.7.4 Calculation of local probability of failure $Pfmax$ 159

Conclusions 163

List of Figures

Figure 1.1: Temporal behaviour of the front wave (CNR-DT 214/2018) 11

Figure 1.2: Formation of catenary action in a damaged building (http://www-personal.umich.edu/~eltawil/catenary-action.html) 16

Figure 1.3: Tie forces (DoD 2016 [16]) 16

Figure 1.4: Methods for designing collapse-resistant structures (Haberland and Starossek 2009 [17]) 17

Figure 1.5: Frame structure with transfer columns (Kokot and Solomos, 2012 [18]) 18

Figure 1.6: Membrane stresses (CNR-DT 214 2018) 21

Figure 1.7: Load-displacement curve (CNR-DT 214 2018) 22

Figure 1.8: Two-dimensional reinforced concrete frame (Lew et al. [19]) 23

Figure 1.9: Diagram of imposed displacement-reaction (CNR-DT 214 2018) 23

Figure 1.10: Diagram of imposed displacement-horizontal displacement (CNR-DT 214 2018) 23

Figure 1.11: Symbols and conventions moments and rotations in the plastic hinges (CNR-DT 214 2018) 25

Figure 2.1: Accuracy on the estimate of the actual behaviour versus time devoted to the analysis for various LoA (fib MC 2010) 32

Figure 2.2: Limit state domain with 2 random variables $X1$ and $X2$ 38

Figure 2.3: Harbitz’s Importance Sampling Method 45

Figure 2.4: Reliability index according to Cornell 1969 48

Figure 2.5: Hasofer-Lind reliability index, with linear limit state function 49

Figure 2.6: Target value identification: differences for the design of new structures versus upgrading of existing structures (fib Bulletin 80). 53

Figure 3.1: Front view of the building 61

Figure 3.2: Plan view of the building 61

Figure 3.3: Slab scheme (dimensions in cm) 68

Figure 3.4: Scheme of the internal walls (dimensions in cm) 69

Figure 3.5: Response spectrum at ULS 74

Figure 3.6: Response spectrum at SLS 74

Figure 4.1: Main tools of the software (www.cervenka.cz) 90

Figure 4.2: Graphical interface 92

Figure 4.3: Basic tab for *SBeta Material* 93

Figure 4.4: Tensile tab (type of tension softening) for *SBeta Material* 94

Figure 4.5: Tensile tab (crack model) for *SBeta Material* 94

Figure 4.6: Compressive tab for *SBeta Material* 95

Figure 4.7: Shear tab (retention factor) for *SBeta Material* 95

Figure 4.8: Shear tab (tension-compression interaction) for *SBeta Material* 96

Figure 4.9: Miscellaneous tab for *SBeta Material* 96

Figure 4.10: Basic tab for *Reinforcement* 97

Figure 4.11: Miscellaneous tab for *Reinforcement* 97

Figure 4.12: Basic tab for *Plane Stress Elastic Isotropic* 98

Figure 4.13: Miscellaneous tab for *Plane Stress Elastic Isotropic* 98

Figure 4.14: CCT command description for Materials 1,2,3 and 4 99

Figure 4.15: CCT command description for Materials 5,6 and 7 99

Figure 4.16: Joint definition through *Joint* tab 100

Figure 4.17: Joint definition through CCT command.. 100

Figure 4.18: Line definition through *Line* tab... 101

Figure 4.19: Line definition through CCT command 101

Figure 4.20: Macro-elements definition through *Macro-elements* tab 102

Figure 4.21: Macro-elements definition (left) and mesh characteristics (right) through CCT command 102

Figure 4.22: Reinforcement definition through *Reinforcement* tab (topology section) 103

Figure 4.23: Reinforcement definition through *Reinforcement* tab (property section) 103

Figure 4.24: Reinforcement definition through CCT command.......................... 103

Figure 4.25: Definition of fixed support load case through load case tab 104

Figure 4.26: Application of *Supports load case* to the lines of the columns' basis 105

Figure 4.27: Definition of *Supports load case* through CCT command............. 105

Figure 4.28: Representation of the 2D frame with fixed supports....................... 105

Figure 4.29: Definition of *Body force load case* through load case tab 106

Figure 4.30: Definition of *Body force load case* through CCT commands......... 106

Figure 4.31: Definition of *Forces load case* through load case tab..................... 107

Figure 4.32: Application of *Forces load case* to the lines of the beams 107

Figure 4.33: Definition of *Forces load case* through CCT commands 107

Figure 4.34: Representation of the 2D frame with distributed line loads on the beams 108

Figure 4.35: Definition of *Prescribed deformation load case* through load case tab 108

Figure 4.36: Application of *Prescribed deformation load case* to the joint on top of the central column... 109

Figure 4.37: Definition of *Prescribed deformation load case* through CCT command... 109

Figure 4.38: Definition of Analysis steps through *Analysis steps tab* 110

Figure 4.39: Definition of Analysis steps through CCT commands....................110

Figure 4.40: Definition of monitoring points through *Monitoring point tab*.......111

Figure 4.41: Definition of monitoring points through CCT commands.............111

Figure 4.42: New *solution parameters* (General section).................................112

Figure 4.43: New *solution parameters* (Conditional Break Criteria section)......112

Figure 4.44: View of the post processing mode (User's Manual for ATENA2D [34]) 113

Figure 4.45: Example of cracks pull-down menu..114

Figure 4.46: Example of *bar reinforcement* pull-down menu............................114

Figure 4.47: Example of *Scalars* pull-down menu..115

Figure 4.48: Example of *text printout*..115

Figure 5.1: Concrete compressive strength - Lognormal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot....................121

Figure 5.2: Steel yield strength - Lognormal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot................................122

Figure 5.3: Steel ultimate strength - Lognormal distribution: a) Probability density function; b) Histogram and Distribution fit c) Scatter plot.................................123

Figure 5.4: Steel ultimate strain - Lognormal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot................................125

Figure 5.5: Steel Elastic Modulus - Lognormal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot................................126

Figure 5.6: Correlation between reinforcement basic variables: a) correlation between fy and fu; b) correlation between fy and εu; c) correlation between fu and εu; d) correlation between fy and Es 127

Figure 5.7: Reinforced concrete specific-weight - Normal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot.128

Figure 5.8: Scatterplots for specific weight: a) reinforced concrete (independent sampled variable); b) concrete cover (dependent variable)................................128

Figure 5.9: Permanent structural load - Normal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot................................129

Figure 5.10: Permanent non-structural load - Normal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot....................130

Figure 5.11: Floor variable load - Gumbel distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot................................131

Figure 5.12: Roofing variable load - Gumbel distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot................................132

Figure 5.13: Scheme for Material 1 (Beam D), Material 2 (Beam ND), Material 3 (Column) and Material 4 (Joints) and Material 5 (NC Concrete)........................134

Figure 5.14: Stress-strain Relationship, unconfined vs confined concrete (Saatcioglu and Razvi [32]) 135

Figure 5.15: Stress-strain Relationship, unconfined vs confined concrete (Saatcioglu and Razvi [32]): a) beam dissipative area; b) beam non dissipative area; c) column; d) joints 136

Figure 5.16: Joints representation of the 2D frame: in the left model with column, in the right, model without the column..139

Figure 5.17: Lines representation of the 2D frame: in the left model with column, in the right, model without the column..140

Figure 5.18: Representation of the 2D frame with joints, lines and macro-elements: in the left model with column, in the right, model without the column 140

Figure 5.19: Representation of the 2D frame with longitudinal and transversal reinforcement: in the left model with column, in the right, model without the column 141

Figure 5.20: Scheme of the pushdown analysis..145

Figure 5.21: Location of the monitoring points on the 2D frame for the Analysis 2 - Pushdown Analysis 146

Figure 5.22: Results of pushdown analysis on the N simulations, showing the collapse branch: a) curve displacement-load; b) capacity curve non-dimensional with respect to the peak values 146

Figure 5.23: Results of pushdown analysis on the N simulations, stopped at the ultimate value: a) curve displacement-load; b) capacity curve non-dimensional with respect to the peak values 147

Figure 5.24: Pushdown analysis results for a generic simulation: a) vertical displacement of the central column vs horizontal displacement of the lateral column; b) vertical displacement of the central column vs load 148

Figure 5.25: Energy balance approach (Izzuddin [1]) 149

Figure 5.26: Energy curves: a) case where the equilibrium is reached b) case where the equilibrium is not reached.. 150

Figure 5.27: Dynamic Amplification factors for the N simulations: a) DAFs for the 53 simulations where equilibrium is reached; b) DAF*s for the 47 simulations where equilibrium is not reached... 151

Figure 5.28: Frame modelled in ADINA Structures: on the left hand side model with the column; on the right hand side model without the column and reaction at the centre of the first floor 153

Figure 5.29: Output of the Dynamic Linear analysis on ADINA: displacement versus time 154

Figure 5.30: Scheme for the strains evaluation: a) location of the sections; b) location of the nodes per each section 158

Figure 5.31: Scheme of the sections: a) directly affected members; b) indirectly affected members.. 159

Figure 5.32: Convolution integral for the maximum Pf of directly affected sub-sections: a) beam confined concrete strain; b) beam reinforcement strain.......... 160

Figure 5.33: Scheme of the failure probabilities at each sub-section 161

Figure 5.34: Failure probabilities and reliability indices of the nodes located at a distance (x,y) from the point where the column is removed: a) 3D plot of Pf; b) 3D plot of β; c) Contour plot of Pf; d) Contour plot of β 162

List of tables

Table 2.1: Relationship between the failure probability Pf and the reliability index β 39

Table 2.2: Suggested range of target reliability from *fib Model Code 2010* for *new* and *existing* structures 54

Table 3.1: Minimum values for the Design Working Life V_N 60

Table 3.2: Use coefficient CU 60

Table 3.3: Characteristics of concrete 62

Table 3.4: Characteristics of steel 62

Table 3.5: Exposure class related to corrosion induced by carbonation 63

Table 3.6: Recommended limiting values for composition and properties of concrete 64

Table 3.7: Recommended structural classification depending on the exposure class XC 64

Table 3.8: Minimum cover requirement with regard to bond 65

Table 3.9: Minimum cover requirements with regard to durability for 65

Table 3.10: Combination coefficients 68

Table 3.11: Permanent structural load of the slab 69

Table 3.12: Permanent non-structural load of the slab 69

Table 3.13: Internal walls' weight 70

Table 3.14: Permanent non-structural load for internal walls 70

Table 3.15: Exposure coefficient ce , as function of the height 71

Table 3.16: Wind pressure, as function of the height 72

Table 3.17: Modal analysis for the first 12 vibration modes 75

Table 3.18: Beam's geometry 76

Table 5.1 correlation coefficients between the 10 basic variables [-] 120

Table 5.2: Mechanical properties of tested rebars (monotoning tensile tests). Data refer to average value of three tests 124

Table 5.3: Correlation coefficients between reinforcement basic variables [-] ... 126

Table 5.4: Summary of the sampled basic variables 133

Table 5.5: Basic inputs for SBeta Material (Materials 1,2,3,4 and 5) 138

Table 5.6: Tensile, Compressive, Shear and Miscellaneous inputs for SBeta Material (Materials 1,2,3,4 and 5) 138

Table 5.7: Basic and Miscellaneous inputs for Reinforcement (Material 6) 138

Table 5.8: Load cases for Analysis 1 - Model with column 142

Table 5.9: Load cases for Analysis 2 - Pushdown analysis 144

Table 5.10: Load cases for Analysis 3: Reliability analysis 155

Preface

Recent events have demonstrated how it is unsafe to assume that structures designed for normal conditions can withstand abnormal or accidental load conditions. During the past decades, especially because of social and political factors, it has been observed an increase of the events that might initiate failures. In this context it seems necessary to study the robustness of the structures, in terms of capability to find alternative stress paths in order to redistribute the applied loads and to avoid disproportionate collapse.

A situation which deserves deeper analyses is the particular case of the sudden column loss, as a consequence of abnormal events or excessive loads. Indeed, this phenomenon can initiate a chain reaction of structural element failures, eventually resulting in partial or full collapse.

In the context of the evaluation of the robustness of a structure, reliability analysis can represent a very good tool. The reliability is defined as the capability of a structure to fulfil the specified requirements, during the designed working life. To do so, a probabilistic analysis can be conducted, in order to perform an evaluation of the local probability of failure that can be, in further analyses, transferred to the global scale.

For the scope of this thesis, a probabilistic analysis has been conducted on a reinforced concrete building designed according to seismic criteria. Previous studies on the same structure have demonstrated that capacity design is not able to guarantee enough robustness, thus different choices on the structural design have been suggested, especially for what concern the longitudinal reinforcement. The frame, as it has been suggested to be designed according to these studies, is the starting point of this work of thesis.

The Chapter 1 concerns the basic notions related to the structural robustness, according to the *CNR*, titled as “Istruzioni per la valutazione della robustezza delle costruzioni”. In particular, the definition of the term structural robustness according to many code rules are presented. Then, the concepts of accidental events and risk scenarios are emphasized, and attention is paid on the strategies of risk reduction. Finally, robustness design and probabilistic evaluation, only for reinforced concrete structure, concludes the chapter.

The Chapter 2 deals with other basic notions, this time about structural reliability, with references on the fib Model Code 2010[2] and JCSS Probabilistic Model Code[3]. Again, the starting point is the definition of the term reliability and its importance on the structural engineering field. Then, the discussion continues with the limit states analysis, sources of uncertainties and their classifications and level of approximation methods (LoA methods). Methods of reliability analysis are analysed, with particular emphasis on the reliability target and, finally, safety format considerations conclude the dissertation.

The Chapter 3 is about the design procedure for the building under analysis. In particular, capacity design prescriptions are followed, since the building is located in a high seismicity area, i.e. L'Aquila city in Italy. After a discussion about material characteristics and durability aspects (i.e. exposure class, structural class and concrete cover), the actions definition and quantification follow. Then, structural verification and dimensioning according to SLS and ULS are presented, with particular description of reinforcement detailing, both longitudinal and transversal. Finally, the actual detailing used for the case study, according to robustness design, are summed up.

The Chapter 4 is about the software description, i.e. ATENA 2D, which allows to perform Static Non-linear Finite Element Analysis (i.e. Static NLFEA). The description is divided into two phases: pre-processing and post-processing. In the former, basics about material, geometry, load and support modelling are given; additionally, the types of solution parameters for the non-linear analysis are described. In the latter, the output and graph tools are defined, with particular attention on the crack and bars yielding visualization.

The Chapter 5 deals with the core of this work of thesis: i.e. reliability analysis to evaluate the robustness of the structure under study. At first, the sampling description of the basic material and load variables is given, according to the Latin Hypercube Sampling (LHS). Then, the basics of the Finite Element Model are defined in terms of material, geometry and mesh characteristics. Then, the three main analyses will be presented: Model with column (i.e. Analysis 1), Pushdown Analysis (i.e. Analysis 2) and Reliability analysis (i.e. Analysis 3), where each of them is a Static NLFEA, with the difference that the second and third ones are conducted on the model without the central column (i.e. the column that is suddenly removed according to the robustness analysis). Finally, the local and global probability of failure are evaluated, by elaborating the outputs of the Analysis 3 in terms of principal total strains. This has allowed to compute the reliability of the structure.

1 Structural Robustness

Before introducing the analysis done for this work of thesis, it is important to define in an exhaustive way the concept of robustness. This is the purpose of the following chapter, where first a definition of robustness is introduced, then accidental actions and risk scenarios are analysed; risk reduction strategies follow, together with the concepts of not proportional collapse and progressive collapse and finally robustness design is treated. All the information presented in this chapter are based on what is written in the DT 214 of 2018[4], titled as "Istruzioni per la valutazione della robustezza delle costruzioni", which is a document drawn up by the CNR (National Council of the Researches).

1.1 Definition of Robustness

When a structure is designed, certain types of actions and combinations are used according to what is prescribed by the code rules. These actions are the permanent, the variable and the seismic loads. The aim is always to reach a certain level of safety which depends on the importance of the structure and is function of the consequences that a certain damage can cause. Moreover, code rules prescribe that structures should have enough *robustness*. It is in this contest that the robustness of a structure with respect to an exceptional action is defined as the capability of avoiding disproportionate damage, that should not be excessive with respect to the exceptional action that has caused this damage. The term exceptional means that that action has not been considered in the design or, if considered, it is with a lower intensity.

1.1.1 Code rules

In this section, the main code rules that define robustness are presented.

The § 2.1 of EN 1990: Eurocode - Basis of structural design [5], is about the fundamental requirements that a design should guarantee: a structure shall be designed and executed in such a way that it will not be damaged by events such as explosion, impact and consequences of human errors to an extent disproportionate to the original cause. in order to avoid potential damage, it shall be appropriately chosen one or more of the following prescriptions:

- avoid, eliminate or reduce the hazard to which a structure can be exposed
- select a structural form that has low sensitivity to the hazard
- select a structural design that can resist to an accidental removal of an element
- avoid structural schemes that can suddenly collapse
- provide the structural members with appropriate tying elements

The EN 1991-1-7 Eurocode – Actions on structures: accidental actions from impact and explosions [6] at § 3.2, defines the way to consider the accidental actions depending on some criteria such as: measures taken for preventing or reducing the severity of the action, probability of occurrence, consequences of failure, public perception and level of acceptable risk. Moreover, it suggests measures to be taken to mitigate the risk related to accidental actions, recognizing one of them in ensuring enough *robustness* by § 3.2(3c):

- designing some “key components” to increase the probability of survive of the structure after the occurrence of an accidental action
- designing structural details using material and elements with enough levels of ductility
- realizing sufficient redundancy meaning that actions caused by the accidental event should be transferred to alternative load paths.

EN 1992-2: 2005 Eurocode 2: Design of concrete structures – Part2: Concrete bridges - Design and detailing rules [7] at § 5.10.1(106) and 6.1(109) suggests design rules for increasing the robustness in case of corrosion of precompression cables or stranded wires.

In Italy the NTC 2008 [8], at § 2.1, gave a definition of the robustness against accidental actions as an additional performance requirement that a structure must guarantee, in addition to the safety with respect to ultimate limit states and serviceability limit states. Moreover NTC 2008 indicated that the *robustness* of a structure could be verified by a global analysis adding to the usual actions, nominal ones oriented by two horizontal and orthogonal directions, having intensity equal to 1% of the gravitational loads.

The new NTC of the 2018 [9], at § 2.2.5, adds to the previous different design strategies to guarantee enough robustness, as function of the use of the structure, such as:

- use load combinations adding accidental loads to the other design actions
- prevent the effects of the accidental actions or reduce their intensity
- use a structural form and type as less sensitive as possible to the accidental actions
- use a structural form and type that can resist to the local damage caused by the accidental action
- adopt redundant, robust and ductile structures as much as possible
- use monitoring systems, active or passive

Finally, fib Model Code 2010 [2],at § 3.3.1.3, states that *robustness* plays a key role for the capability of a structure to maintain its functionality during situations in which accidental actions are present or in consequence of human errors. It emphasizes the importance of ensuring robustness for the preservation of human life, protection of human properties and environment, preservation of operations.

1.2 Modelling and Intensity of accidental actions

When dealing with the structural robustness evaluation, risk scenarios can be due to a single action, caused by a natural accidental event or by human being, or to a combination of actions. In any case, these actions are different to the ones taken into account in the limit states or because they are of the same type but with higher intensity (being associated to a very low occurrence probability) or because they are types of actions not defined by code rules.

The main problem remains the way of defining these actions and identifying the way of resisting to them. A starting point is for sure to classify accidental actions, as the CNR suggests.

1.2.1 Classification of the accidental actions

For an engineer it is important to identify and then quantify the types of accidental actions to which a structure can be subjected. These actions can have natural or anthropic origin, so the possible hazards connected to them can be classified in:

- Category 1: related to hazards caused by natural phenomena (for example earthquakes, meteorological events or landslides) or hazards generated by unintentional human actions such as explosion or fire, when not intentional
- Category 2: constituted by intentional human actions like acts of vandalism or terrorist attacks
- Category 3: connected to hazards caused by conceptual/design/execution errors related to the structure

In order to model these types of actions, taking into account the possible interaction between the event and the structure, we can have:

- <u>distributed loads of accidental entities</u>, such as overpressure caused by explosions, pressures due to fluid motion (like air motion when tornado come or water when talking about flooding or snow in case of avalanche)
- <u>impact loads,</u> for example vehicle crushes, vessels, aircraft
- <u>accelerations</u> imposed to the structure, like seismic actions
- <u>imposed deformations/ induced displacements</u> like the ones caused by foundation failure due to a landslide, reduction of the mechanical strength of materials caused by a fire or displacements caused by an earthquake
- <u>conceptual/design/execution errors</u>

Another way to classify accidental actions is by looking at their temporal distribution. Thus, one can distinguish between:

- static load
- dynamic load
- impulsive load

To apply one or another model, it is important to know that the same type of action can be static or dynamic as function of the main vibration periods which characterizes the structure itself.

Modelling of accidental actions

1.2.2 Modelling of the accidental actions

Once that the actions have been classified, it is then necessary to identify the way to model these loads, so to describe their intensity, their effects of the structure, quantify the risk and the mitigation measures to be taken. These models depend on the considered category.

For what concerns the Category 1 (hazards having natural origin or involuntary human origin), to have a realistic scenario and so to better evaluate the consequences on a structure it is necessary to:

1. define a model which describes the occurrence frequency of the event. For those events caused by human actions, it is not always so easy to define a model of the occurrences
2. identify a model that describes the future effects
3. define a model which describe the intensity of the action and its model of application (load pressure, impulsive load, ...)
4. suggest a model which describes the effects of the eventual mitigation procedures and hazard reduction

Regarding hazards being part of Category 2 (vandalism or terroristic acts) it is not possible to define an occurrence model because the past events are not significative from a probabilistic point of view. Moreover, it is not easy to identify the intensity of the actions because, being dependent on human behaviours, they can be always different and various. The only exception is the case of vandalism acts by means of explosive bombs, because in this case it is possible to model the wave pressure equivalent to the quantity of TNT that has been used.

Finally, hazards of Category 3 (conceptual/design/execution errors) cannot be treated from a statistical point of view but it is possible to prevent them with quality control processes both during design and construction phases.

1.2.2.1 Phenomena caused by seismic action

Earthquake: the type of seismic action taken into account by code rules are the *far field* earthquakes, when the distance between the epicentre and the structure is long enough to consider attenuation laws to model the earth acceleration induced by the earthquake. When the earthquake is *near fault,* the response spectrum model suggested by code rules cannot be applied anymore because, being an action characterized by long period impulses, the structure does not oscillate with its vibration period. Moreover, *near fault* actions can also involve vertical acceleration which are not considered when adopting the usual design requirements.

Tsunami: this phenomenon is caused by underwater motions of the crust or landslides which involve material motion over the sea. These phenomena involve the propagation of very high waves that can reach coasts and cause flooding. The highest tsunami wave that has been registered in Italy is of 5 m for the south Italy and Sicily and 1,5 m for the Sardinia island. These types of actions can be modelled as: impact pressures caused by the fluid, impact loads caused by rocks and buoyancy loads.

1.2.2.2 Natural gravitational phenomena

Landslide with transportation of loose material: it is caused by the loss of stability and/or cohesion of a soil or fractured rock mass. This phenomenon starts because of the presence of water. The impact load of a landslide like this if function of the density of the loose mass involved ρ and of the velocity of the mass in correspondence of the construction v with the following law (Bugnion et al., 2012[10]):

$$p = C\rho v^2 \tag{1.1}$$

where C is a coefficient which considers that the flow is deviated by the construction and it is evaluated using this formula:

$$C = 9v^{-1.3}(gh)^{0.65} \tag{1.2}$$

where g is the gravity acceleration and h is the depth of the moving mass.

Debris flow: similar to the previous because involves the motion of loose material due to the water action. These phenomena involve material having very low plasticity index (PI<5%) and high velocities (0.05-20 m/s). This action can be modelled in two ways: or by means of a hydrostatic pressure with a triangular distribution function of the elevation or using a hydrodynamic approach. The latter is described by the following law (Suda et al.,2009[11]):

$$p = 4.5\rho v^{0.8}(gh)^{0.6} \tag{1.3}$$

where ρ and v are the density and velocity of the loose mass involved, g is the gravity acceleration and h is the depth of the flow.

Other types of phenomena that involve gravitational motion are *rock avalanches* (impact loads whose intensities are evaluated by equating the potential energy and the kinetic one, usually it is assumed an impact energy of 500 kJ), *snow avalanches* (distinguished into sliding, powdery and mist avalanches and whose intensity is evaluated with the (1.1) and *volcanic eruptions* (modelled with a lateral uniform pressure when effusive phenomena or impact load when explosive eruption).

1.2.2.3 Foundation settlements

Subsidence: it can involve partial or total displacement of the soil where the foundation of a structure is inserted. It can cause cracks that can be more or less severe or problems to the facilities.

Modification of the water level: it can be caused by the nature (because of seasonal effects) or by human actions. When the level arises, water overpressures are present and so reduction of effective stresses can be induced on the foundations. When the level decreases, the foundation can settle down and cause a damage into the structure. As function of the rigid rotation induced to the structure, the level of damage can be described.

Flooding: caused by the nature (very intense precipitation) or by human actions (hydraulic constructions) and modelled with hydrostatic or hydrodynamic pressures, function of the elevation of the submerged area and the velocity of the flow. Sometimes, impact forces can be applied when the flooding transports bodies of huge masses.

1.2.2.4 Meteorological phenomena

Tornado and storms: the main problem related to these phenomena are not only the very high pressures caused by the wind but also the motion of objects that can become potential impact forces against the construction.

1.2.2.5 Detonations

Detonation on an open space: they cause the release of energy and glowing gases, with the creation of a spherical front wave, having a surface that is called shock wave. When this shock wave beats the surface of the structure, located at a certain distance, there is a quasi-instantaneous increase of pressure called P_{so}. Then, the pressure decreases in an exponential way becoming, at a certain instant, lower than the atmospheric pressure. This instant defines two phases: before it there is the positive phase, when the majority of the damages occur, after this time the negative phase occurs.

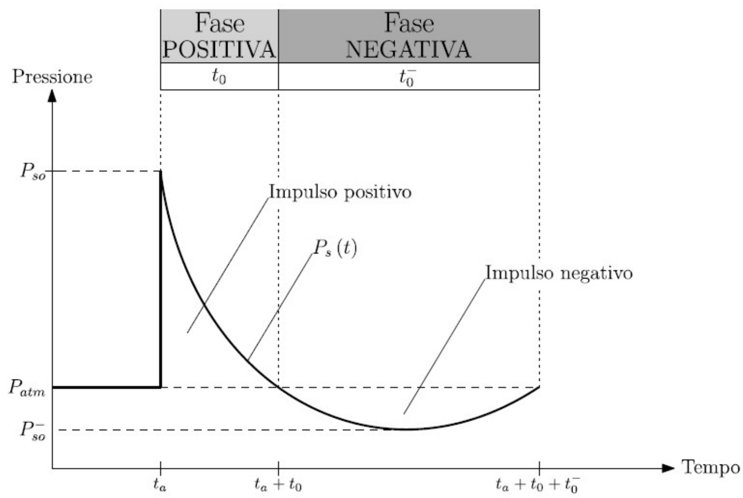


Figure 1.1: Temporal behaviour of the front wave (CNR-DT 214/2018)

1.2.2.6 Impacts

There are two ways to analyse these actions: with a dynamic or with a static equivalent approach. These loads depend on the stiffness of the elements involved (impacting and impacted bodies).

Motor vehicles: they can involve both buildings and infrastructures. There are tables (Eurocode 1 EN 1991-1-7 CEN 2006[12]) that indicate the value of the impact load to be considered as function of the traffic category and the position of the structure with respect to the road.

Vessels: it regards structures located close to channels or harbours. Tables exist that show the impact force (lateral and frontal) as function of the class of the vessel and they depend on the type of environment involved (if maritime or fluvial area).

Aircrafts: these analyses depend on the height of the building, that should be in general larger than the mean height of the area. There are some specific guidelines to consider this phenomenon, for example the USNRC 2018 (Nuclear Regulation Commission)[13] or the EN 1991-1-7 CEN 2006[12].

1.3 Risk of disproportionate collapse

The definition "disproportionate collapse" is referred to a collapse whose characteristics are extreme, in terms of extent, with respect to the event that has caused the damage. This definition is often confused with the "progressive collapse", that happens when a local damage involves progressively the majority or the whole structure (like a domino effect). While the former collapse looks mainly at the extent of the damage, the latter is a specific definition of the way of collapsing.

When dealing with robustness analysis, it is of paramount importance to identify the level of acceptance of the risk connected to a certain event, stated that a zeroing of the risk is not possible to be achieved.

1.3.1 The concept of risk

When a structure is subjected to a certain accidental action, it becomes immediate to evaluate the risk associated to that action, being related to the possibility that that hazard could become harmful. The risk is defined as:

$$R = P(*)V(*)E \tag{1.4}$$

where P is the hazard (probability that a certain event, with a certain intensity, happens in a certain period of time and area) , V is vulnerability (inclination of a system to be subjected to the consequences of a certain event) and E is the exposure (measure of the value of people and properties related to the system).

From this definition, it is easy to understand the difficulties to quantify the risk and the related level of acceptance, because its perception can be different depending on the actors involved. For example, community believes perceives the risk associated to an air crush larger than a car accident, even if the latter are more frequent and so, from a statistical viewpoint, are riskiest.

Being the collapse of a structure usually a very rare event, it is adopted a level of risk defined *de minimis*, behind which regulatory requirements are not necessary. This value is assumed around 10^{-7} $per\ year$ (Pate-Cornell, 1994[14]).

1.3.2 Probabilistic risk analysis

A rational approach to evaluate the risk is the PRA (Probabilistic Risk Analysis), which is based on the quantification and it leads to adopt mitigation measures for reducing the damages.

Referring to the particular case of disproportionate collapse, the probability of collapse $P[C]$ is defined by the following equation:

$$P[C] = P[C|SL] \cdot P[SL|H] \cdot P[H] \tag{1.5}$$

where:

- $P[C]$ is the annual probability of collapse, due to the event H;
- $P[H]$ is the occurrence probability of the event H, assumed equal to the mean annual occurrence λ_H;
- $P[SL|H]$ represents the conditional probability of local damage (SL) on the structure, given H;
- $P[C|SL]$ represents the conditional probability of disproportionate collapse (C), given the local damage on the structure (SL);

In order to prevent the phenomenon of disproportionate collapse, it is possible to:

- prevent the occurrence of accidental events, so reducing $P[H]$ or λ_H;
- prevent local damages that can trigger the collapse, so reducing $P[SL|H]$;
- prevent the structural collapse working on the structural design, so reducing $P[C|LS]$.

The first option (reduction of $P[H]$), is the only independent from the structural design strategies and can be reduced by looking at the possible accidental events. The second option (reduction of $P[SL|H]$) is dependent on the design strategy adopted for the local resistance and can be often not the cheapest one. The third option is the most consolidate in the field of mitigation robustness measures to be taken because it is the only term on which the engineers can work on. Its evaluation will be discussed in the following chapters.

1.3.3 Risk analysis based on scenarios

When evaluating the annual occurrence probability of an event ($P[H]$), a sufficient quantity of data is necessary, so to have a reliable measure. When special events occur, such as terroristic attacks, the occurrence frequency is not easy to be computed, so it is necessary to use a different approach, a deterministic one, defined as risk analysis based on a scenario S. The equation becomes:

$$P[C|S] = P[C|SL] \cdot P[SL|H] \tag{1.6}$$

1.4 Risk reduction strategies

To reduce the risk connected to a certain scenario many actions should be taken: first, define the risk scenario, then stabilize the performance requirements that the structure have to maintain in the occurrence of that specific scenario, thus calculate the failure probability and finally evaluate the consequences of the failure (in terms of non-satisfaction of the requirements).

There are two types of performance requirements: the general ones, valid for any kind of structure and connected to human life losses and environmental damages and the specific ones, related to the use class of the structures and dependent on the economic losses.

1.4.1 Classification of design approaches

There are three kinds of classification, elaborated in the ASCE/SAI 7-05 standards [15]:

- General approach used: prescriptive design approach or performance-based approach
- Method used for the design of the structural system: direct or indirect method
- Risk scenario: specific or generic.

Usually, indirect design methods are used in the prescriptive approaches because they guide more or less all the choices in terms of elements of the structural systems such as columns and beams layout, minimum resistance of the connections or minimum dimensions of the elements. Of course, the advantage is that they are very simple to be applied, but on the contrary the engineer is not free to choose different solutions. On the other hand, the direct method allows the engineer to personally select the strategy in order to avoid local damages or, eventually, the occurrence of a disproportionate collapse in their presence.

In the following, the second classification will be discussed in more detail.

1.4.1.1 Direct design methods

This approach consists on directly evaluate the resistance capacity of the structure and adopt measures in order to avoid the disproportionate collapse. There are two kinds of methods:

- *Local resistance method*, which consists on increase the resistance of key elements (whose collapse might cause a disproportionate collapse against accidental actions)
- *Alternative load path method,* that means that the structure is able to transfer the loads even if there is a local damage. This approach is usually done by removing a structural element (e.g. a supporting column on a building or a cable in a guyed bridge) and evaluate the behaviour of the structure after the removal.

1.4.1.2 Indirect design method

The goal is to reach an alternative equilibrium configuration through catenary action, which means the development of large deformation so that loads are mainly taken by vertical components of axial forces that develop in the beams, these components are indeed the catenary forces. Of course, to be this phenomenon efficient, the structural element should have enough ductility. This aspect is clearly explained in the Figure 1.2:

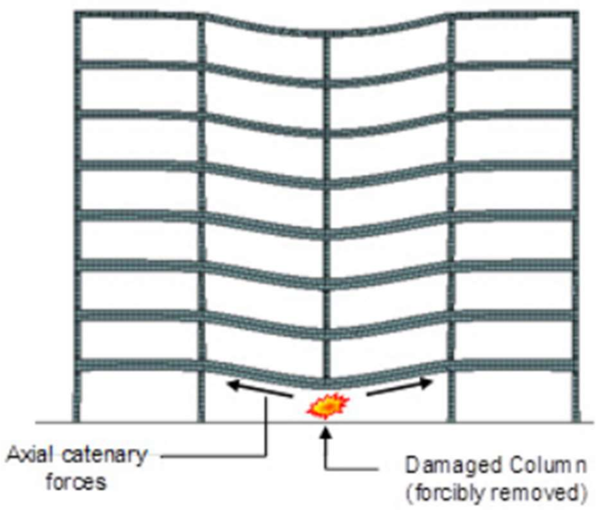


Figure 1.2: Formation of catenary action in a damaged building (http://www-personal.umich.edu/~eltawil/catenary-action.html)

In order to reach this goal, the engineer should design three-dimensional tie systems such as: corner, peripheral and internal ties in the two orthogonal slab directions, horizontal ties between columns or walls and vertical ties. These mechanisms are shown in Figure 1.3:

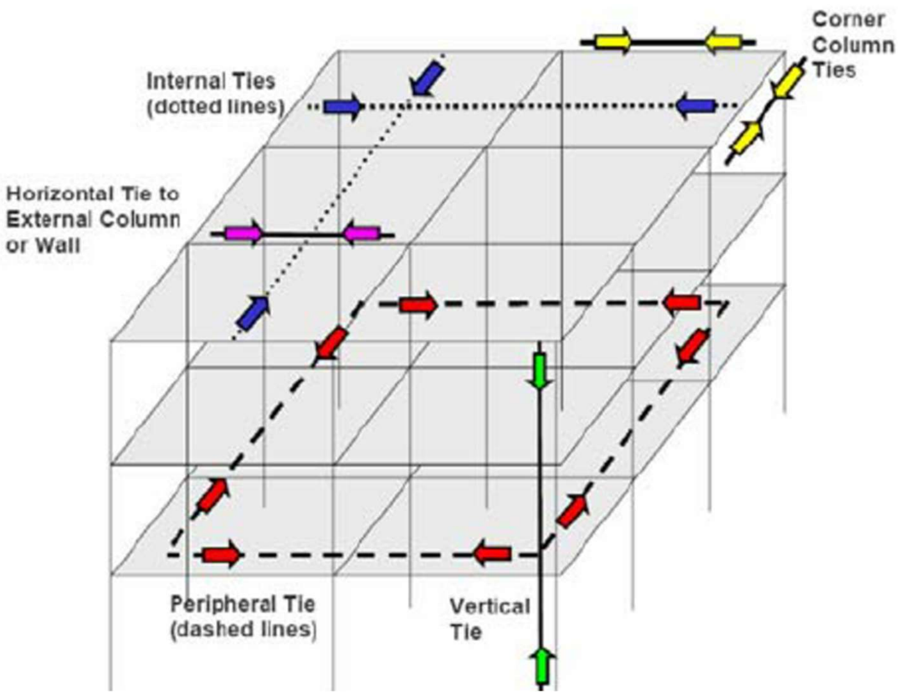


Figure 1.3: Tie forces (DoD 2016 [16])

1.4.2 Summary of the possible strategies

To reduce the risk connected to the accidental event, it is possible to adopt non-structural measures for the control of the accidental action or structural measures to evaluate the local damage and its evolution. The Figure 1.4 shows the procedure to be adopted in order to reduce the risk, using the approaches described in this sub-chapter:

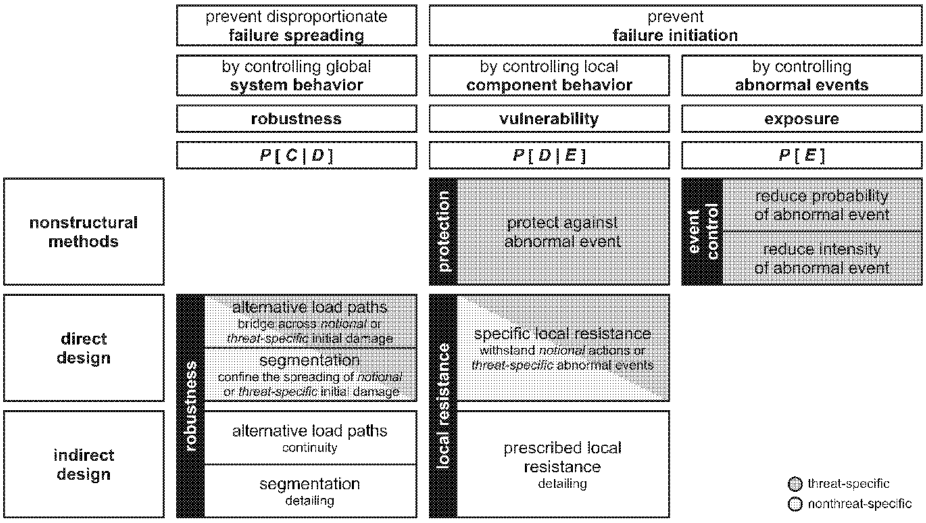


Figure 1.4: Methods for designing collapse-resistant structures (Haberland and Starossek 2009 [17])

1.5 Design for robustness

1.5.1 Conceptual design

There are many expedients that an engineer can use to reduce the risk associated to a disproportionate collapse: redundancy, ties, ductility, uniform distribution of structural elements, enough resistance against shearing actions and capability of resisting to changes in direction of the actions.

There are three methods to be adopted in conceptual design to increase structural robustness: local resistance method, alternative load path and compartmentalization.

1.5.1.1 Local resistance method

It consists on avoiding the failure of key elements (those which can cause a disproportionate collapse if subjected to a local damage), which means to reduce the probability $P[SL|H]$ of equation (1.5).

This method is usually adopted with those structures where an alternative path of the applied load is difficult to be achieved, thus where there is lack of redundancy. This happens for buildings with transfer columns, where the loss of a continuous column is probable to cause a disproportionate collapse (see Figure 1.5).

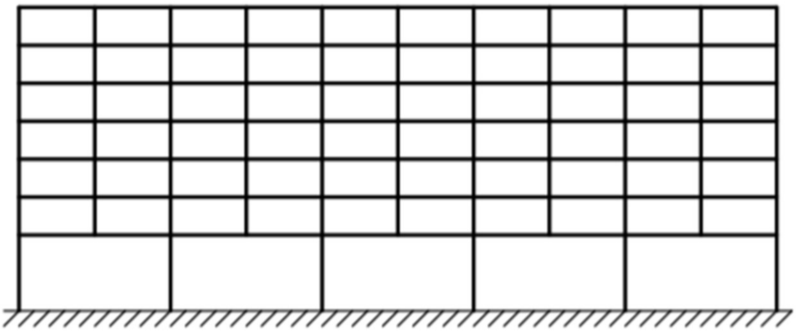


Figure 1.5: Frame structure with transfer columns (Kokot and Solomos, 2012 [18])

To apply the local resistance method, one can design individually the only key element or consider other structural elements to increase the resistance of the key element. The latter is the usually adopted when working with existing structures.

Moreover, the method is often based on the preliminary identification and quantification of the accidental action (in this case the analysis is non-linear and dynamic) but can also be applied even if the accidental action is generic (in this case the method is non-linear and static). Even if in both cases the method is considered a *direct* one, when the local resistance is reached with prescriptive construction details and not with explicit calculations, the method is *indirect*.

1.5.1.2 Alternative load path

The idea is to avoid a disproportionate collapse when a local damage occurs, which means to redistribute the loads in alternative paths. This method holds even if the accidental events are not preliminary identified but using generic scenarios.

The approach consists on removing a structural element and verify the robustness adopting static or dynamic non-linear analysis. This is the case of a *direct* approach.

When an *indirect* approach is used, this method is called catenary effect, and it consists on using three-dimensional ties so to increase continuity, ductility and alternative load paths.

1.5.1.3 Compartmentalization

The approach aims at avoiding the domino effect in case of a local damage by isolating the collapsed part of the structure. The idea is to create a compartment where the edges are "strong elements" that avoid the extension of the collapse of the "weak elements" or create a compartment where the weak elements are disconnected from the rest of the structure when collapse.

1.5.2 Structural modelling

The evaluation of the structural robustness is a very complex procedure due to the number of variables involved and the uncertainties connected to them. From this point of view, an important role is played by the modelling assumed to evaluate the materials' behaviour:

Constitutive linear-elastic models: they are the simplest ones, so the easiest to be applied, especially in a preliminary phase, when a first evaluation of the critical points is needed. Inadequate for complex phenomena like disproportionate collapse, because, due to the high strain values, non-linearities are involved.

Constitutive non-linear models, dependent/independent from the application speed of the load: these models can be used when dealing with disproportionate collapse, because they can describe the non-elastic behaviour that occurs when high strains are involved, and plasticization cannot be neglected. In addition, with these models it is possible to consider the increase of the resistance due to the instantaneous application of the load (e.g. for explosions or vehicle impacts), by taking into account the application speed of the force.

Local/global models: in order to have a better control of the outputs, both global and local models are used. The formers are considered to evaluate general behaviours like stress analysis or displacements in the whole structure. The seconds are necessary when studying critical points like connections, application points of the load, discontinuities etc.

1.5.3 Types of analysis

Dynamics effects are involved when there is an instantaneous transition between the original and the deformed configuration. These effects can be considered in different ways, depending on the type of analysis:

Static linear analysis: this approach can be used even when dynamic effects are involved, by amplified the effects with a dynamic amplification coefficient. Of course, this approach is a simplified one, and cannot be used when dealing with geometrical and/or materials non-linearities, catenary effect, redistribution of stresses.

Static non-linear analysis: the catenary effects and geometrical non-linearities can be considered with this approach. In any case, it is important to accurately evaluate the modelling of materials behaviour and the non-linearities associated to the connections. In the end, as previously mentioned, dynamic effects can be considered by using dynamic amplification coefficients.

Dynamic linear analysis: this approach allows to evaluate the dynamic effects connected to the occurrence of a local/global damage but does not lead to a good evaluation of the non-linearities.

Dynamic non-linear analysis: this is the most accurate analysis because it adopts three-dimensional, non-linear models and considers high deformations and transitional effects. On the other hand, not all the calculation programs can implement this approach. Moreover, only expert engineers can use and control this analysis, because of the complexity involved.

1.5.4 Design for reinforced concrete structures cast in place

Reinforced concrete structures show many advantages against accidental actions: they are easy to be designed with structural redundancy, sections can show high ductility under bending actions, instability is controlled thank to the large columns cross-section, the large masses of the structure can resist against particular events such as explosions. On the other hand, the large masses do not help when the engineer wants to adopt an alternative load path approach because high forces are involved. Moreover, fragile mechanisms (e.g. shear, torsion, anchoring) should be avoided because they can prevent ductile behaviours, thus *capacity design* like the one adopted in earthquake engineering should be considered.

1.5.4.1 Membrane effects in RC structures

Whit the concept of membrane effect it is intended the born of axial stresses in beam elements, or radial and tangential stresses in plates, when large deformations occur, with the consequence of an increase in resistance. This benefit can be obtained not only in presence of an accidental action, like a column removal (Figure 1.6b), but also when loads are applied, having larger modulus with respect to those planned during the design phase (Figure 1.6a).

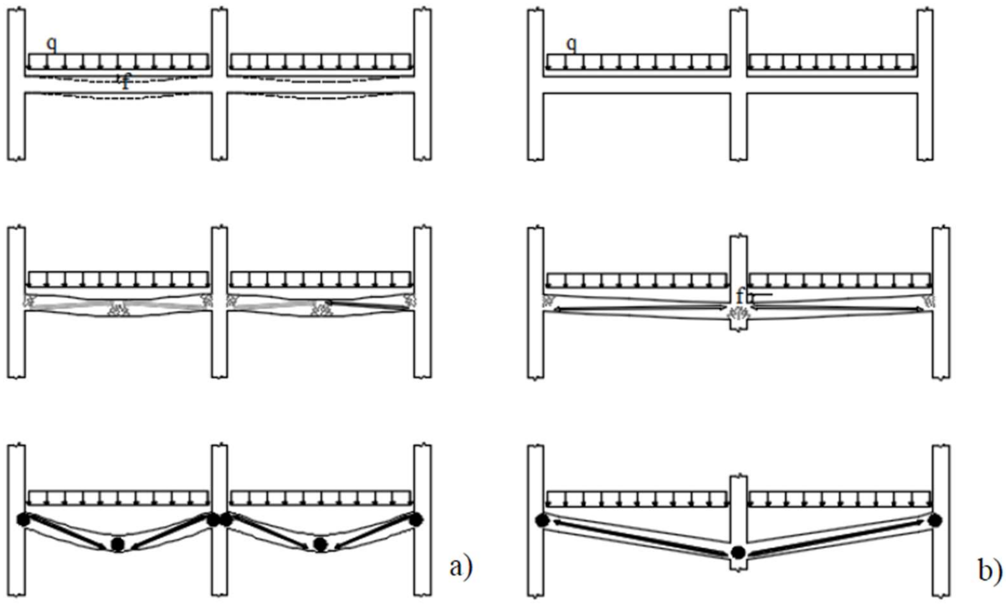


Figure 1.6: Membrane stresses (CNR-DT 214 2018)

The Figure 1.7 shows the dependency of the applied load (uniform distributed load q) with respect to the displacement f, for both cases. The response of the structure, when no membrane effects neither non-linearities are evaluated is very different with respect to the curve when these effects are considered. This demonstrates that membrane effects and geometrical non-linearities accomplish a growth in the structural resistance.

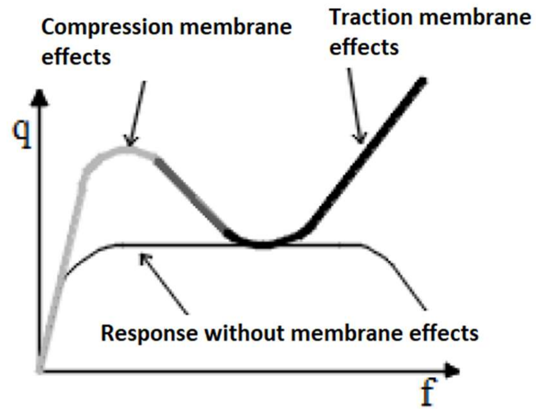


Figure 1.7: Load-displacement curve (CNR-DT 214 2018)

In reinforced concrete structures, compressive membrane effects born when fissures occur, while traction membrane stresses are related to the plasticization phenomena.

When span length is not too long, compressive membrane stresses are relevant even for low strain values, while for large spans the contribution of the previous is negligible and traction membrane effects become significant in the evaluation of the resistance.

In addition, while traction membrane effects do not depend on the concrete strains but on the reinforcement ultimate strains, the compressive effects are function of the initial conditions of the concrete (i.e. creep, shrinkage, previous deformations).

1.5.4.2 Structural behaviour in case of column removal

The structural behaviour of a reinforced concrete structure subjected to a column removal, can be described by means of an experimental evidence on a two-dimensional frame (Figure 1.8).

A vertical displacement is imposed on the point P1, located where the column has been removed, and it can be monitored as function of the reaction on the point P1 (Figure 1.9) and the horizontal displacement in point P2 (Figure 1.10).

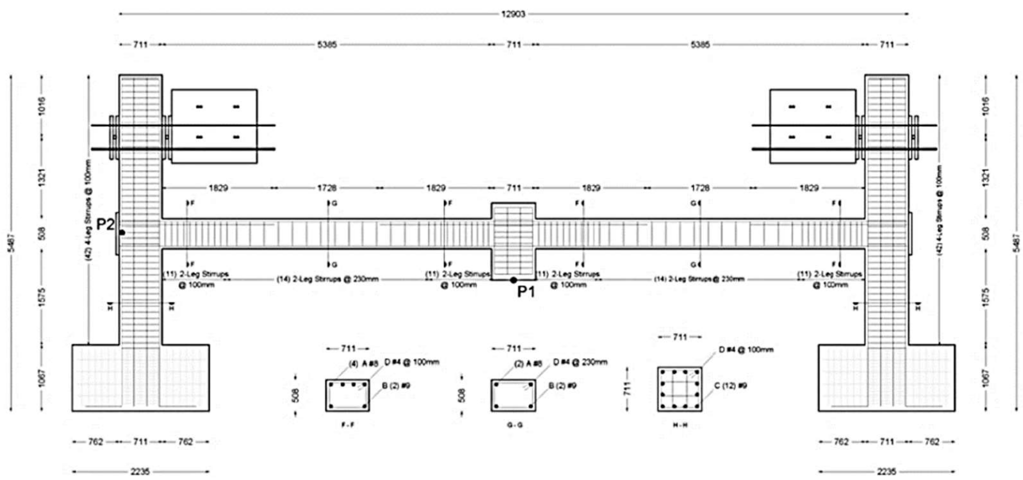


Figure 1.8: Two-dimensional reinforced concrete frame (Lew et al. [19])

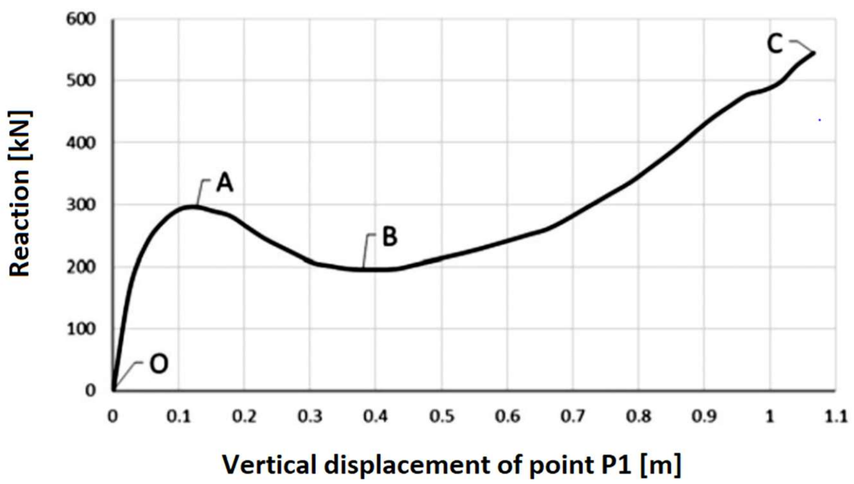


Figure 1.9: Diagram of imposed displacement-reaction (CNR-DT 214 2018)

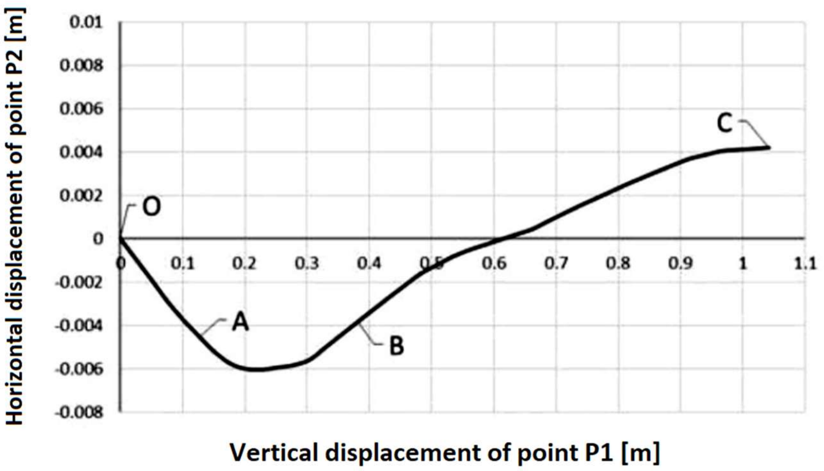


Figure 1.10: Diagram of imposed displacement-horizontal displacement (CNR-DT 214 2018)

The structural behaviour can be explained by looking at three different phases:

- Line OA (*flexural behaviour*): it depends on the bending behaviour of the beam and lasts with the formation of plastic hinges at the connection points between beam and column (negative plastic moment in correspondence of the lateral columns and positive plastic moment at the central column). The point P2 is displaced towards the external part (negative value) because of the fissurization and consequent elongation of the beam. In addition, the beam is compressed because of the stiffness of the columns that opposes the beam elongation.
- Line AB (*softening*): characterized by a softening phase and so reduction of the reaction of point P1. The horizontal displacement starts being less negative. Also, the compression action is reduced.
- Line BC (*catenary effect*): the reaction increases again with the increasing of the vertical displacement in point P1. The horizontal displacement becomes positive (so the external columns move towards the inner part) and the beam is in tension. This is due to a combination of the tensional membrane effects and the catenary effect due to the reinforcement bars.

It is important to underline that if the reinforcement does not continue over the external columns, the softening is not possible and only flexural behaviour of the beam takes the load. This implies that the process is interrupted in point A, with the collapse of the structure.

1.5.4.3 Design in case of column removal

By means of a simplified approach, it is possible to compute the maximum resistant load in the flexural phase $P_{MAX,FL}$ (point A) and the one corresponding to the catenary behaviour $P_{MAX,CAT}$ (point C), when a column is removed (all the symbols used in the formulas can be understood by looking at Figure 1.11).

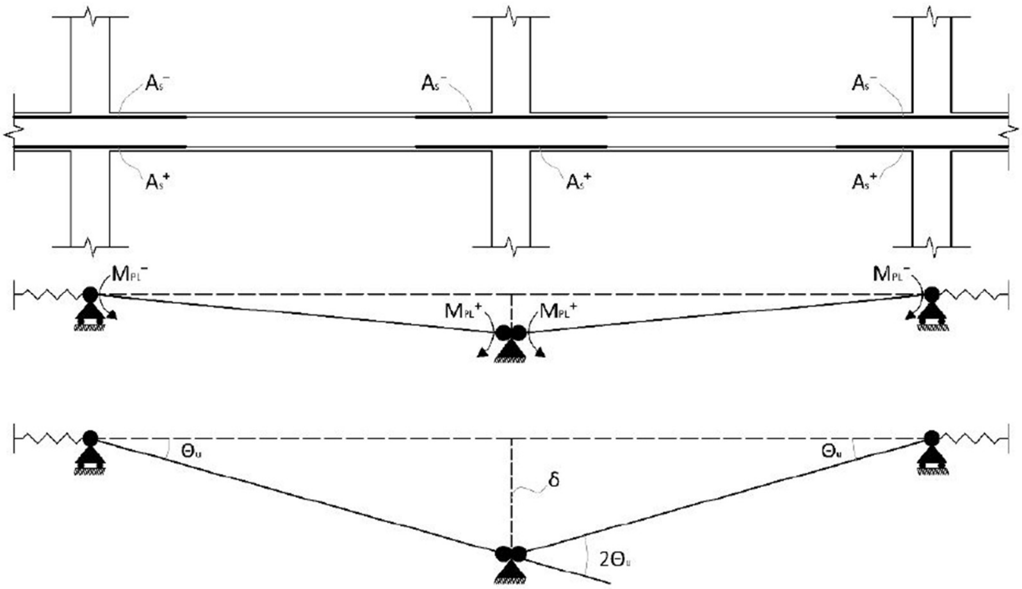


Figure 1.11: Symbols and conventions moments and rotations in the plastic hinges (CNR-DT 214 2018)

The maximum resistant load in the flexural phase can be evaluated as following:

$$P_{MAX,FL} = \frac{2(M_{PL}^{+} + M_{PL}^{-})}{L} \tag{1.7}$$

where M_{PL}^{+} and M_{PL}^{-} are the plastic moment of the beam in correspondence of the connections with the column and they can be approximately computed as:

$$M_{PL}^{+} = 0.9 A_s^{+} f_y d \tag{1.8}$$

$$M_{PL}^{+} = 0.9 A_s^{+} f_y d \tag{1.9}$$

where A_s^{+} and A_s^{-} are the reinforcements areas of the beam in correspondence of the joint beam-column, respectively for positive and negative moment, d is the effective height of the beam, f_y is the yielding strength of the reinforcement, computed according to the coefficient suggested by code rules in case of accidental actions.

For what regards the catenary effect shown in point A of Figure 1.9, we do have:

$$P_{MAX,CAT} = 2\frac{\delta}{L}A_{s,cont}f_t = 2\theta_u A_{s,cont}f_t \tag{1.10}$$

where δ and θ_u are respectively the displacement and rotational capacities of the point where the column is removed, $A_{s,cont}$ is the reinforcement, continuous over the length $2L$ of the beam and f_t is the ultimate strength of the reinforcement, computed according to the coefficient suggested by code rules in case of accidental actions. For the evaluation of the rotational capacity and the displacement caused by the removal of the column, it is necessary to refer to experimental data.

It is important to underline that the catenary effect is beneficial only if the load that develops can be retained by the portion of the structure that is not directly interested by the damage. So, attention should be payed to the critical columns like the corner or perimetral ones. Finally, the catenary effect is beneficial only if it determines an effective growth of the resistant load, so if $P_{MAX,CAT} \geq P_{MAX,FL}$.

1.6 Probabilistic and semi-probabilistic quantification of the robustness

Considering the state of art, code rules and standards do not consider an enough amount of actions or events, that can represent a threat for the safety and performance of a structural system. This has posed the need to consider risks that in the past was not considered or evaluated using a deemed-to-satisfy approach, i.e. a sort of checklist of the event rather than real structural computation design.

A key consideration is that many events can be more or less harmful as function of the target risk that is considered; to make an example, an explosion can have a low hazard and so a low effect on the risk evaluation if the target probability of collapse $P(C)$ is of 10^{-5} per year, but it can have a larger influence if the target is 10^{-7} per year.

Moreover, the risk evaluation based on a time scale of 1 year can have different consequences on the exposures if the time scale is of 50 years or 100 years. This has posed the need to evaluate in the design procedures only the events that cause a probability of collapse which overcome a certain target level, defined *de minimis*.

The evaluation of the conditional probabilities presented in equation (1.5), can be evaluated through a Probabilistic Risk Analysis (PRA), which, in case of structural systems, can be seen as a Structural Reliability Analysis. According to it, the failure is reached when the demand S (intended as the effects generated by the actions) exceeds the resistance *R*. The failure probability can be thus be expressed by:

$$P_f = \int F_R(x) \cdot f_S(x)\, dx \tag{1.11}$$

where $F_R(x)$ is the cumulative distribution function of the resistance and $f_S(x)$ is the probability density function of the demand. More details on the computation of the failure probability follow on the next chapter.

Referring to a generic structure, in order to perform a performance-based design or assessment, it is necessary to establish a-priori the target risk that can be accepted. When dealing with the specific event of a collapse whose consequences can determine lives losses, it is possible to define the following target:

$$P(C) \leq p_{th} \tag{1.12}$$

where p_{th} is the de minimis risk, usually comprised between 10^{-5} and 10^{-7}, according to Pate-Cornell [14].

For the specific case of alternative load path design method, the probability of collapse as defined in equation (1.5) is reduced to the quantification of $P(C|SL)$ and can be evaluated according to the convolution integral of Equation (1.11).

Thus, according the expression (1.12), the probability of collapse, conditioned to a local damage, can be expressed as:

$$P(C|SL) \leq p_{th}/\lambda_H \tag{1.13}$$

Assuming the hazard probability λ_H equal to $10^{-6}/yr$ and $10^{-7}/yr$, the performance requirement expressed in (1.13) for the probability of collapse, given a local damage, is of the order of $10^{-2}/yr - 10^{-1}/yr$. This means that the reliability index β - more details will follow about its computation – is of the order of 1.5. This value is significantly lower with respect to the one assumed for the ultimate state of new structures, normal use conditions, that is equal to 3.8, corresponding to a probability of failure of $10^{-4}/year$.

To conclude, referring to a *de minimis* risk, which regards an event whose occurrence probability per year is lower than $10^{-7}/yr$, this probability become of the order of $5 \cdot 10^{-6}/50yr$, considering a building with nominal life of 50 years. Thus, since the reliability index for a building of class 2, whit a nominal life of 50 years, is equal to 3.8, and corresponds to a $P(C)$ of $7.3 \cdot 10^{-5}/50yr$, the $P(C|SL)$, according to the *de minimis* target, should be lower than $6.9 \cdot 10^{-2}/50yr$.

2 Structural Reliability

This chapter is an attempt to define the second key aspect of this work of thesis, which is the structural reliability. Probabilistic approach has been used to evaluate the reliability of the structure in terms of β index.

In the following, a brief introduction on the basics is given, in terms of definitions, limit state analysis, uncertainties and their classification and level of approximations. Then, methods of reliability analysis are discussed, and target reliability indices are followingly presented. Finally, safety format considerations conclude this chapter.

Many of the information given in this chapter are elaborated from Implementation of Eurocodes – Handbook 1 and 2 [20][21], Advances in reliability methods for reinforced concrete structures (D. Gino, 2019) [22], Probability, Reliability and Statistical Methods in Engineering Design (A. Haldar, S. Mahadevan, 2000) [23]. The reference code rules are JCSS Probabilistic Model Code [3] and ib Model Code 2010 [2].

2.1 Definition of reliability

The importance of structural reliability can be understood in the light of the concept that uncertainties, regarding structural performance, can never be entirely eliminated and have to be considered in any structural design. In general, all the parameters adopted in engineering have some degree of uncertainty and should be considered as random variables. For example, by loading until failure a certain number of "identical" specimens of a steel bar, the load capacity of the bar is computed as the mean of the failure loadings, thus it is a random variable. In view of this context, satisfactory performance cannot be absolutely guaranteed, while probabilistic assurance of performance, referred to as reliability, can be given [23].

Many are the definitions of the term "reliability":

- in ISO 2394 [24], it is described as "the ability of a structure or structural member to fulfil the specified requirements, during the working life, for which it has been designed"
- EN 1990 [5], Section 2, it is recommended to design and build a structure with appropriate degree of reliability and in economic way, such that, during its intended life, it will: sustain all actions likely to occur and respect serviceability conditions

It is important to notice that behind the previous definitions, other relevant concepts are hidden such as:

- performance requirements, related to the structural failure
- time period, connected to the service life
- reliability level, mutual with respect to the probability of failure P_f
- conditions of use, limiting input uncertainties

It is important to underline that the term reliability cannot be interpreted in an absolute way, as if the structure has to be judged or reliable or not reliable. People can believe that if a structure is reliable, a failure is not possible to occur. Contrary, a reliable structure may collapse, meaning that there is always a failure probability, even small or negligible. Thus, failures are always considered to occur, being part of the reality; it is the occurrence of the phenomena to be quantified and limited, through economic, human safety and structural considerations.

To verify all the aspect of structural reliability, as described in section 2 of EC2, in terms of basic requirements, it is first necessary to define limit states design and uncertainties related to it.

2.2 Basic principles

Basic topics are introduced in the following, in order to lay the foundations for reliability analysis procedure. These are: limit state design, level-of-approximation approaches and uncertainties.

2.2.1 Limit state design

As already mentioned, according to EN1990 [5], structural reliability is connected to the ability of a structure to maintain certain *performance requirements* during its *service life*. Performance requirements are mainly related to structural resistance, ductility, durability and robustness. The service life is intended as the design working life if talking about new structures and residual service life if it is the case of existing ones.

Related to these concepts, limit states are defined, according to EN1990 [5], as "states beyond which the performance requirements are no longer satisfied". They can be related to persistent situations during the working life of the facility or to transient because regarding the execution of the structure or because of unintended uses or extreme events.

Two are the basic limit states:

- ultimate limit states (ULS) are states associated to collapse or close to structural failure in general. These states concern the safety of people and/or the safety of the structure. They include: loss of equilibrium of the structure or any part of it, considered as a rigid body (i.e. EQU in EN 1990); failure by excessive deformation, transformation into a mechanism, rupture, loss of stability of the structure or any part of it, including supports and foundations (i.e. STR or GEO in EN 1990); failure by fatigue or time-dependent effects.
- serviceability limit states (SLS) are states beyond which specified criteria such as structure's use or function are no longer valid. They include: deformation or deflection which impede the appearance of the structure, the comfort of people or the functionality of the structure or that cause unacceptable damage to finishes or non-structural elements; vibrations which affect the comfort of users or limit the functional performance of the structure; damage that hinder the appearance, durability and functioning of the structure.

2.2.2 Levels-of-Approximation approach

When designing structural members, approximations of reality are necessarily made. The level of accuracy of these approximations defines the level-of-approximation (LoA). The LoA approach consists on "a design strategy where the accuracy on the estimate of a structural member (behaviour or strength) can be, if necessary, progressively refined through a better estimate of the physical parameters involved in the design equations" (fib Model Code 2010 [2]).

The main idea behind this approach is that the behaviour or strength of a structural member is defined by a set of parameters, correlated through a certain number of design equations. These parameters can be physical properties (e.g. crack width), mechanical properties (e.g. steel yielding strength) or geometrical (e.g. height of a beam cross-section). The accuracy involved to estimate these parameters is refined in each new LoA adopted, while the time devoted to the analysis increases more and more. This concept is explained in the following figure (Figure 2.1), taken from fib MC 2010.

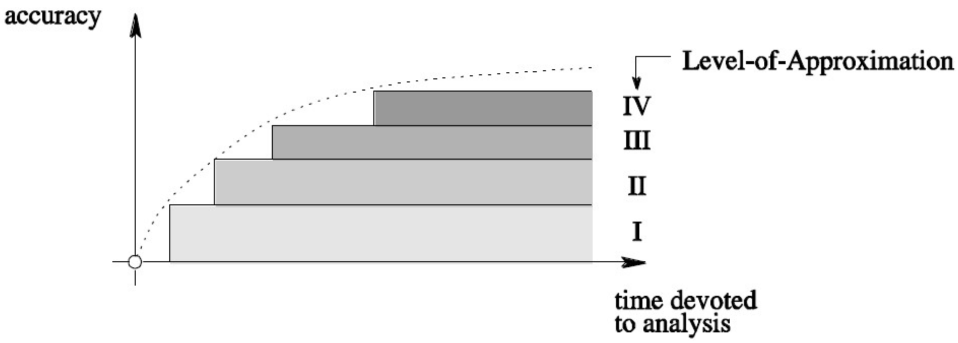


Figure 2.1: Accuracy on the estimate of the actual behaviour versus time devoted to the analysis for various LoA (fib MC 2010)

In general, four LoA are defined and the choice of one of them depends on the type of analysis performed, on the context (i.e. preliminary of detailed analysis) and on the degree of advantages that can be reached or not if a larger LoA is selected. The differences between levels are followingly described:

- LoA I: it is usually adopted for preliminary design because it is simple and low time consuming. It consists on simple and safe hypothesis about physical parameters adopted in the design equations. When using the first

LoA, any other higher levels are not needed if the structure shows enough strength under the assumptions of LoA I.

- LoA II and LoA III: they represent the evolution of LoA I, reached by means of analytical procedures. The internal forces and other geometrical and mechanical parameters are evaluated according to simplified analytical formulas. These levels can be adopted for design of new structures as well as for assessment of existing ones.
- LoA IV: it allows to have the best estimates of the physical parameters and/or design equations thank to numerical procedures. The disadvantage of this approach is that it is very time consuming, thus it is suggested to use it only for specific needs. For example, for executive design of new structure when it is a very complex one or for assessment of existing structures when criticisms are present. In fact, when significant savings are needed to avoid or limit strengthening of the structures, the LoA are very powerful approaches.

2.2.3 Uncertainties classification

In all stages of execution and use, construction works are made of a series of systems that suffer from a variety of uncertainties. These differ depending on many aspects such as the nature of the structure itself, environmental conditions and applied actions. In general, it is possible to distinguish between two sources of uncertainties: *noncognitive sources* (qualitative) and *cognitive sources* (quantitative).

Noncognitive sources can be divided into three types:

- inherent uncertainty, defined as the intrinsic randomness in all physical observations (e.g. actions, material properties and geometric data); meaning that if a measure is repeated many times for the same physical quantity it does not lead to the same value, due to many fluctuations in the environment, procedure, human errors, instrument and so on. To overcome this uncertainty, it is possible to collect a huge number of samples in order to increase the reliance of the quantity. However, no infinite measures can be taken, thus this aspect is unavoidable

- statistical uncertainty, meaning that it is not possible to have precise information about the variability of the physical quantity due to a limited size of data. Therefore, quantitative measures of confidence are added to reliability evaluation
- modelling uncertainty intended as the approximation made when trying to represent system's behaviour with models. An example is represented by finite element models, where the attempt is to use idealized mathematical relationships and numerical procedures to represent structural behaviours. Unfortunately, important differences can occur between the structural model and the real behaviour. Statistical description of modelling error can be included as an additional variable in the reliability analysis

An example can be used to better explain these three types of uncertainty: if wind load has to be estimated, an inherent uncertainty is given by the wind speed, since it is possible to record different wind speed data, but these are inherently random. Moreover, the statistical uncertainty can be evaluated by looking at past observations. However, it is necessary to convert the statistical information about wind speed, in wind pressure, using Bernoulli's theorem, thus adding another uncertainty (i.e. modelling uncertainty).

Cognitive sources are related to the vagueness of the problem, due to the attempt of abstracting the reality. These may come from:

- definitions of certain parameters such as: structural performance, quality, deterioration, skill and experience of workers and engineers, environmental impact of projects, conditions of existing structures
- definition of interrelationships among parameters of the problems

As already mentioned, while inherent variability cannot be eliminated, being an intrinsic property of parameters, the other sources of uncertainty may be reduced, by testing, measurements, procedures and predictive models. This difference leads to the distinction between *aleatory* uncertainties, regarding the randomness of the variables that is an intrinsic property, and *epistemic* uncertainties, related to the lack of knowledge in the definition of the structural model. In general terms, the inherent variability of material properties and actions are considered as *aleatory* sources, while measurement and human errors, statistical uncertainty and model uncertainty are *epistemic* ones.

2.2.3.1 Aleatory uncertainties for resistance models

Properties of materials represent an important class of variables which influence the structural reliability. In design calculations, materials properties are represented by characteristic values, which are nothing but fractiles of appropriate distributions.

In the following, a probabilistic modelling of resistance models in reinforced concrete structure is given by JCSS Probabilistic Model Code [3].

Concrete

The reference property of concrete is the compressive strength f_{co} of standard test specimens (cylinder of 300 mm height and 150 mm diameters), tested according to standard conditions and at age of 28 days. This variable is assumed to be distributed as a lognormal distribution, with:

- expected value equal to the mean value f_{cm}, obtained by testing results or by code prescription
- coefficient of variation V_c equal to 0.15

The other variables such as concrete tensile strength f_{ct}, Modulus of Elasticity E_c, fracture energy G_f, peak strain at concrete compressive strength and ultimate strain are can be evaluated indirectly, as function of the concrete compressive strength. Expressions given by code rules EC2 [7] are available, otherwise it is possible to use other probabilistic models given by JCSS [3].

Reinforcement

The reference properties of structural steel are the yield strength f_y, the ultimate tensile strength f_u, the modulus of elasticity E_s and the ultimate strain ε_u. For sake of simplicity, only the distribution concerning the yield strength will be given. This variable is assumed to be distributed as a lognormal distribution, with:

- expected value equal to the mean value f_{ym}, obtained by testing results or by code prescription

- coefficient of variation V_y equal to 0.05

2.2.3.2 Epistemic uncertainties for resistance models

It is important to take into account not only the aleatory uncertainties of material models but also the epistemic ones. In fact, when trying to represent a material behaviour by means of a model, could it be physical, semi-empirical or empirical, there is always an uncertainty due to simplified assumptions in their definitions or disregarding of some parameters that could have been important.

To quantify epistemic model uncertainties related to resistance, the JCSS Probabilistic Model Code [3] has proposed a procedure that is explained in the following. First of all, it is necessary to consider that:

- the database of experimental observations has to give all the parameters for the realization of the tests and the computation of the resistance
- the range of parameters included in the set of experimental results gives the limits of the resistance model
- statistical inference for the observed sample has to be computed in order to test the goodness-of-fit (i.e. most likely probabilistic distribution and its parameters)

It is possible to represent model uncertainties through the following expression:

$$R(X,Y) \approx \vartheta \cdot R_{Model}(X) \tag{2.1}$$

where:

- $R(X,Y)$ is the actual response of a structure in general
- ϑ is the model uncertainty random variable, due to factors affecting test
- $R_{Model}(X)$ is the response (or the resistance) estimated by the model
- X is a vector of basic variables considered in the resistance model
- Y is a vector of variables that are not considered in the model but may affect the resistance mechanism (e.g. variables for which their influence is still not completely clear or widely assessed).

It is important to notice that Y variables are considered in the right-hand side part of equation by means of ϑ variable. In particular, the model uncertainty random variable ϑ can be estimated by computing a vector of experimental observation and another vector of model response, performing the following ratio:

$$\vartheta_j = R_j(X,Y)/ R_{Model}(X) \tag{2.2}$$

By means of (2.2), ϑ_j represents the j-th value of the model uncertainty random variable of the selected experimental database.

In the end, by means of statistical inference, it is possible to define the probabilistic distribution and related parameters to represent the model uncertainty.

According to the JCSS Probabilistic Model Code [3], the model uncertainty random variable ϑ is distributed as a lognormal distribution.

2.3 Problem formulation for structural reliability

In quantitative sense reliability may be seen as the complement of the probability failure P_f. An exhaustive determination of failure significance, read in terms of performance requirements (i.e. limit states), is thus necessary to be discussed. After that, the reliability index can be presented, as function of the failure probability.

2.3.1 Probability of failure

In the following, failure is intended in a general sense, indicating "any undesired state of a structure (e.g. collapse or excessive deformation) which is unambiguously given by structural conditions" [21].

Structural behaviour can be described by a set of variables $X_i = [X_1, X_2, \dots, X_i, \dots, X_N]$. These variables can represent actions, mechanical properties, geometrical data and model uncertainties.

Another quantity to be defined is the limit state function of a structure (could be ultimate, serviceability, durability or fatigue), also called performance function, usually expressed via an implicit form like:

$$Z(X) = 0 \tag{2.3}$$

Being the limit state function the value at which $Z(X)$ equals zero, it has to be defined in such a way that for positive values (i.e. for $Z(X) \geq 0$), the state is favourable or safe, while for negative values (i.e. for $Z(X) < 0$), the state is identified unfavourable or failure state Figure 2.2.

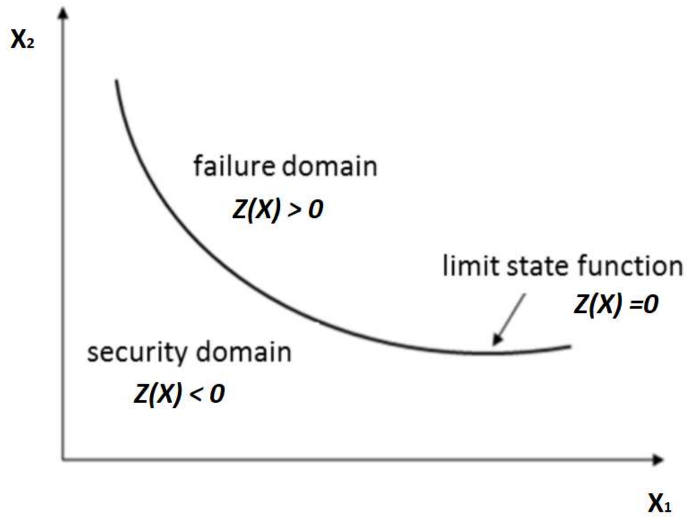


Figure 2.2: Limit state domain with 2 random variables X_1 and X_2

According to this, the probability of failure associated to the aforementioned limit states, can be expressed as:

$$P_f = P\{Z(X) < 0\} \tag{2.4}$$

Depending on the time-dependency of the basic variable X_i the expression of the probability of failure can be more complicated. In particular, if the basic variables $X_i = [X_1, X_2, \ldots, X_i, \ldots, X_N]$ are described by time independent joint probability density function $f_x(x)$, the probability function is reduced to an integral:

$$P_f = \int_{Z<0} f_x(x)dx \tag{2.5}$$

More complicated procedures have to be used if the basic variables are time-dependent, but this is out of the scope of this dissertation.

Conversely, the probability of survival P_s, can be defined as:

$$P_s = 1 - P_f \tag{2.6}$$

Another aspect to be mentioned is that the probability of failure P_f is usually estimated considering a specific reference period that commonly is the design or residual service life, depending if dealing with respectively new or existing structures.

2.3.2 Reliability index

Reliability index is defined as the negative value of a standardized normal value corresponding to the probability of failure P_f. The mathematical expression is thus the following:

$$\beta = -\Phi^{-1}(P_f) \tag{2.7}$$

where the quantity Φ^{-1} represents the inverse standardized normal distribution function.

This expression stresses the fact that P_f and β are mutual representation of the same aspect, that is the structural reliability.

The following table, that is table 1 of [21], represents the relationship between the two quantities:

Table 2.1: Relationship between failure probability P_f and reliability index β

P_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1.3	2.3	3.1	3.7	4.2	4.7	5.2

As it is possible to notice, the larger is the reliability index β, the more reliable is the structure, thus the lower is the failure probability P_f and the less probable is to exceed a specified limit state.

2.4 Classification of reliability methods

The evaluation of structural reliability requires a failure probability computation; this can be done through advanced methods or using a simplified approach. Of course, the latter is used when one wants to avoid too much computational efforts. There are different levels of reliability analysis, whit the term level identifying the extent of information about the analysed problem. Four basic levels exist:

- Level III (probabilistic)

- Level II (probabilistic)
- Level I (semi-probabilistic)
- Level 0 (deterministic)

Starting from the level III, the degree of probability theory implementation decreases, as also the computational efforts.

2.4.1 Methods of Level III

This method allows to exactly compute the probability of failure, by solving the integral of Equation (2.5). These methods are appropriate for structures that are of major economic importance. Different strategies can be used:

- analytical solutions
- numerical integration
- Monte Carlo's simulation

The analytical solutions can hold for few and very simple cases. This is because the solution depends on the variables vector, all must be independent and normally distributed, while on the limit state region, it must be defined by hyper-planes. Thus, it cannot be considered as a general solution.

The numerical integration can be considered an exact solution in the sense that it is possible to reach the desired precision. The simple trapezoidal rule of integration gives very good results if there are not too many variables (4 or 5), but the complexity of integration increases exponentially with the number of variables. Thus, it is useful only when a small number of variables are involved.

In the end, for complex system, the most suitable method is the Monte Carlo's simulation, based on a random sampling of the variables and carrying a long number of artificial experiments. To avoid those difficulties explained before (e.g. not-independent variables, big number of variables involved), variance reduction and importance methods are used.

In the following, two examples will be shown:

- the first, illustrating an application of analytical solutions to a problem with two independent variables

- the second, presenting the Monte Carlo's method, together with reduced sampling techniques (i.e. importance sampling and Latin Hypercube sampling).

2.4.1.1 Reliability analysis with two independent variables: analytical method

In the following, a problem composed by two independent random variables having any distribution is presented: the action effect E, having density function $f_E(e)$ and the resistance R, having density function $f_R(r)$. The limit state function can be expressed as:

$$Z = g(R, E) = R - E \tag{2.8}$$

Thus, the probability of failure, according to Eq. (2.5), can be analytically solved as:

$$P_f = \int_{Z<0} f_{R,E}(r, e) dr de = \int_{Z<0} f_R(r) \cdot f_E(e) dr de \tag{2.9}$$

The solution of Eq. (2.9), can be obtained in two ways:

$$\begin{aligned} P_f &= \int_{-\infty}^{+\infty} P[(R < e) \cap (e \leq E \leq e + de)\ ]\, de \\ &= \int_{-\infty}^{+\infty} F_R(e) \cdot f_E(e) de \end{aligned} \tag{2.10}$$

$$\begin{aligned} P_f &= \int_{-\infty}^{+\infty} P[(r \leq R \leq r + dr) \cap (E > r)\ ]\, dr \\ &= \int_{-\infty}^{+\infty} f_R(r) \cdot [1 - F_E(r)] dr \end{aligned} \tag{2.11}$$

If the basic random functions R and E are function of other random variables, such that $R = g_R(R_1, R_2, \dots, R_N)$ and $E = g_E(E_1, E_2, \dots, E_M)$, the expression become multiple integrals, difficult to be solved analytically. Thus, Monte Carlo's method or numerical integration are the only possible solutions.

However, if the random variables R and E are both normally or lognormally distributed, the analytical expression becomes much simpler, as followingly explained.

Supposing to have R and E normally distributed, whit mean values μ_R, μ_E and variances σ_R^2, σ_E^2, respectively, then the limit state function $Z = R - E$, expressed in Eq. (2.8), is normally distributed as well, whit mean value $\mu_Z = \mu_R - \mu_E$ and variance $\sigma_Z^2 = \sigma_R^2 + \sigma_E^2$.

Thus, the failure probability P_f is expressed as:

$$P_f = P[Z < 0] = \Phi\left[-\frac{\mu_Z}{\sigma_Z}\right] = \Phi[-\beta] \tag{2.12}$$

where, as already mentioned, Φ is the cumulative standard normal distribution and β is the reliability index.

If R and E are both lognormally distributed, the solution can be obtained as well, by applying the logarithm to the variables and following the same procedure as before.

2.4.1.2 Monte Carlo's method and sampling techniques

The name Monte Carlo itself has no meaning, except that it was used first by von Neumann during the II World War as a code word for nuclear weapons work, in a laboratory of New Mexico. Most commonly, the name Monte Carlo is associated with a place where gamblers take risks. Nowadays, this method is applied to evaluate the risk or reliability associated to complicated engineering systems (Haldar and Mahadevan [23]).

The Monte Carlo simulation technique is composed by the following procedure, based on six essential elements:

1) Define all the random variables involved
2) Define the probabilistic characteristics of all the random variables in terms of probabilistic distribution and corresponding parameters (i.e. perform the statistical inference)
3) Generate the values of these random variables
4) Perform numerical experimentation, which means to evaluate the problem deterministically for each set of realizations of all the random variables
5) Extract probabilistic information from these N realizations
6) Determine the accuracy and efficiency of the simulation

In the following, this method will be applied in the specific case of computing the failure probability P_f. Firs of all, according to Eq. (2.5), the probability of failure can be expressed as:

$$P_f = \int_{g(X_i)<0} f_{X_i}(x_i)dx_i = \int_{-\infty}^{+\infty} I[g(X_i)]f_{X_i}(x_i)dx_i \quad i = 1,2, \dots N \qquad (2.13)$$

where $I[g(X_i)]$ is the indicator function defined as:

$$I[g(X_i)]\begin{cases} 0 & if\ g(X_i) \geq 0 \\ 1 & if\ g(X_i) < 0 \end{cases} \qquad i = 1,2,\dots,N \qquad (2.14)$$

As already mentioned, in order to evaluate if the single realization of the random variables X_i belongs to the safe or to the failure region, it is necessary to define the limit state function. The probability of failure can thus be estimated by the number of samples that gives the structural failure (i.e. for which $g(X_i) < 0$).

Thus, the estimated failure probability P_f of a problem whit n samples can be written as:

$$P_f \approx P_f^n = \frac{1}{n}\sum_{j=1}^{n} I[g(X_i)] \qquad i = 1,2,\dots,N;\ j = 1,2,\dots,n \qquad (2.15)$$

Finally, to evaluate the accuracy of the simulation as in point (6) of the procedure, it is possible to compute the coefficient of variation of the solution as function of the number of samples n. In particular, it is assumed that each simulation cycle constitutes a Bernoulli trial, and the number of failures in N trials is assumed to follow a binomial distribution. Then, the coefficient of variation of the P_f at the jth sample is:

$$V_{P_f}^j = \frac{\sqrt{\frac{(1-P_f^j)P_f^j}{j}}}{P_f^j} \qquad j = 1,2,\dots,n \qquad (2.16)$$

It is important to notice that as the number of samples approaches to infinite, then the coefficient of variation goes to zero, meaning that the evaluation is accurate.

In general, the level of accuracy depends on the unknown probability of failure. In many engineering problems, the probability of failure is smaller than 10^{-5}, meaning that, on average, only 1 out of 100000 trials would fail. So, at least 100000 simulations are needed to estimate that probability, but, for a reliable estimate, at least 10 times this minimum (i.e. 1 million simulations) is recommended.

In the light of the effort that is needed to perform such a simulation, to reduce the computational effort (i.e. to decrease the number of simulations), several sampling techniques are used. Some of the commonly used sampling techniques are:

- systematic sampling
- importance sampling
- stratified sampling
- Latin hypercube sampling
- adaptive sampling
- randomization sampling
- conditional expectation

In the following, two of them will be described: importance sampling method and Latin hypercube sampling method (LHS).

Importance sampling method

The main concept behind this method is to concentrate the distribution of sampling points in the region that mainly contributes to the failure probability, defined the region of most importance. This is to avoid the spreading of these points evenly among the whole range of possible values of the basic variables. One way to achieve this is given by Harbitz [25], illustrated in Figure 2.3, where the simulation is given only outside the β-sphere, because no failure occurs inside that sphere.

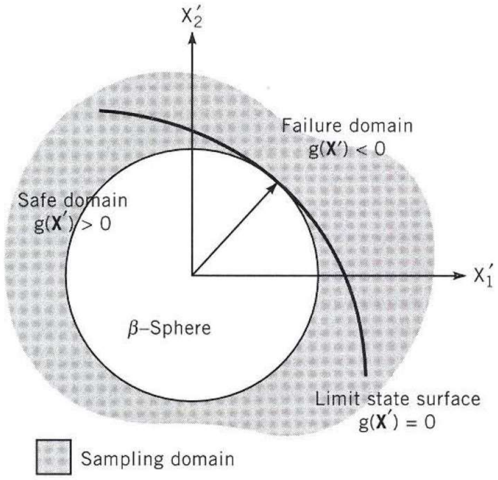


Figure 2.3: Harbitz's Importance Sampling Method

Called U the standard normal space, where there is the largest contribution to the failure probability, the P_f can be evaluated as:

$$P_f = \int_{-\infty}^{+\infty} I[g(U_i)] f_{U_i}(u_i) dU_i \tag{2.17}$$
$$= \int_{-\infty}^{+\infty} I[g(U_i)] \frac{f_{U_i}(u_i)}{f_{Z_i}(u_i)} f_{Z_i}(u_i) dU_i$$

where $f_{U_i}(u_i)$ is the jointed probability density function expressed in the standard normal space and $f_{Z_i}(u_i)$ is the sampling density function. The best choice is to adopt a sampling density function $f_{Z_i}(u_i)$ that is proportional to the jointed probability density function $f_{Z_i}(u_i)$, that is $f_{Zi}(ui)$ α $|f_{Ui}(ui)|$.

The procedure to be adopted it thus the following:

1) generate a vector z_i' of random variables having standard normal distribution
2) define the design point u_i^*, which is the point on the limit surface of the standard normal space having the lower distance from the origin. In formula it is $u_i = u_i^* + \Sigma_{ij}$, where Σ_{ij} is generally represented by the unit matrix
3) calculate the $f_{Ui}(ui)$ and $f_{Zi}(ui)$ as:

$$f_{U_i}(u_i) = \frac{1}{(2\pi)^{\frac{n}{2}}} exp\left[-\frac{1}{2} u_i^T \cdot u_i\right] \qquad i = 1,2,\ldots,N \tag{2.18}$$

$$f_{Z_i}(u_i) = \frac{1}{(2\pi)^{\frac{n}{2}} \cdot det\, \Sigma_{ij}} exp\left[-\frac{1}{2}(u_i - u_i^*)^T \cdot \Sigma_{ij}^{-1} \cdot (u_i - u_i^* \qquad (2.19)$$

4) Transform in the original space of basic variables and evaluate the limit state function for the correspondent realization x_i in order to determine the value of $I[g(u_i)]$.

In the end, the estimation of the probability of failure P_f with n samples is:

$$P_f \approx P_f^n = \frac{1}{n}\sum_{j=1}^{n} I[g(U_i)]\frac{f_{U_i}(u_i)}{f_{Z_i}(u_i)} \quad i = 1,2,\dots,N;\ j = 1,2,\dots,n \qquad (2.20)$$

Latin hypercube method (LHS)

The LHS method is very efficient when performing a reliability analysis by means of non-linear finite element method, as the case of this work of thesis. In fact, this method allows to reduce the number of numerical simulations required in order to characterize the probabilistic distribution of the structural resistance. In fact, it can be demonstrated that when the coefficient of variation of basic variables is lower or equal to 0.2, a number of 30-40 samples are sufficient in order to determine mean, variance and probabilistic distribution of the investigated structural resistance (Gino, 2019 [22]).

The basic concept behind LHS method is that the variables are sampled by their probabilistic distribution and, successively, randomly combined. The sampling algorithm ensures that each distribution function is sampled uniformly between the interval of probabilities $(0,1)$.

The procedure to adopt is the following:

1) for each variable X_i the probability interval (0,1) is subdivided in n non-overlapping equiprobable sub-intervals (h_{inf}, h_{sup})
2) in each one of the n sub-intervals, one value between (h_{inf}, h_{sup}) is sampled randomly from a uniform distribution and the corresponding value of the basic variable X_i is evaluated
3) a random permutation between the n values sampled for each variable X_i is performed in order to randomly combine the outcomes. In this way, the n input sets of basic variables to perform the simulations is defined.

2.4.2 Methods of Level II

With these methods the reliability is evaluated using the information on first and second moments of the random variables only. These are the first-order second moment (FOSM) and advanced first-order second moment (AFOSM) methods.

First-order second moment method (FOSM)

This method is also called *mean value first-order second-moment* (MVFOSM) method in literature. The name is derived from the fact that the evaluation of the performance function (i.e. limit state function) is based on a first-order Taylor series approximation, linearized at the mean values of the random variables, and also because it uses only means and covariances (i.e. second-moment statistics) of the random variables.

The original formulation by Cornell (1969) is based on the same problem with two variables discussed on section 2.2.3.1. Considering R and S to be statistically independent normally distributed random variables, also Z is normally distributed. The probability of failure depends on the ration between mean value of Z and its standard deviation, as follows:

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{2.21}$$

Thus, the reliability index can be seen as the distance between the mean value μ_Z from the failure condition, that is $Z = 0$, intended as the numbers of standard deviation of the limit state function to reach the zero value. This concept is graphically explained in Figure 2.4.

Significant errors can occur by using this method if the limit state function is non-linear, because of the neglection of higher order terms can be a problem. Moreover, the index β seems not to be constant value under different but mechanically equivalent formulations of the same performance function. The advanced method tries to overcome the latter problem.

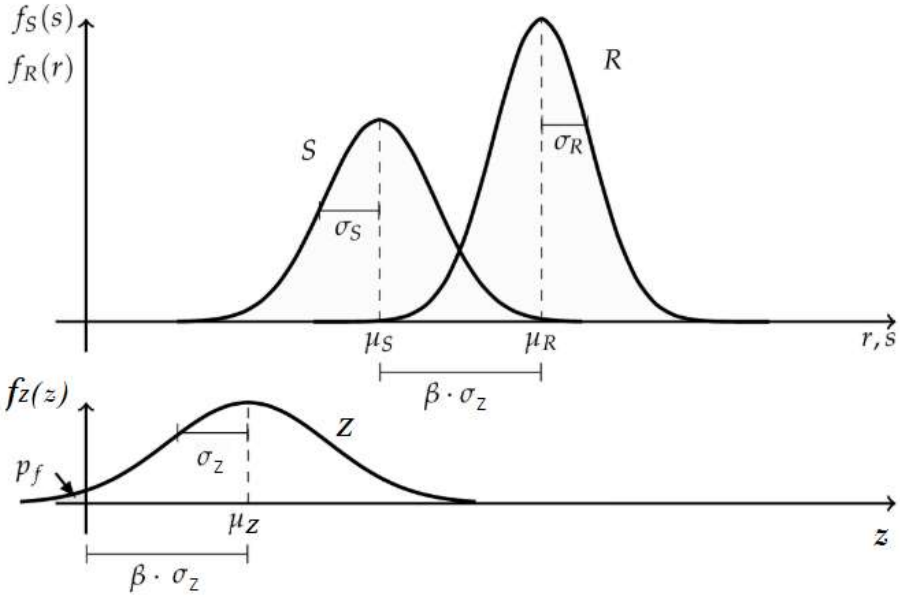


Figure 2.4: Reliability index according to Cornell 1969

Advanced first-order second moment method (FOSM)

This method is also called Hasofer-Lind (H-L) method, taking its name from the homonym researches who have proposed the methodology. It is applicable to normal random variables.

The safety index β_{HL} is defined as the minimum distance from the origin of axes in the reduced coordinate system $\boldsymbol{X'}$ to the limit state surface:

$$\beta_{HL} = \sqrt{(x'^{*})'(x'^{*})} \tag{2.22}$$

The minimum distance point is denoted as x'^{*} in the reduced coordinate system and x^{*} in the original one, and it is called design point or checking point.

The Figure 2.5, taken from Haldar and Mahadevan [23], explains the concept.

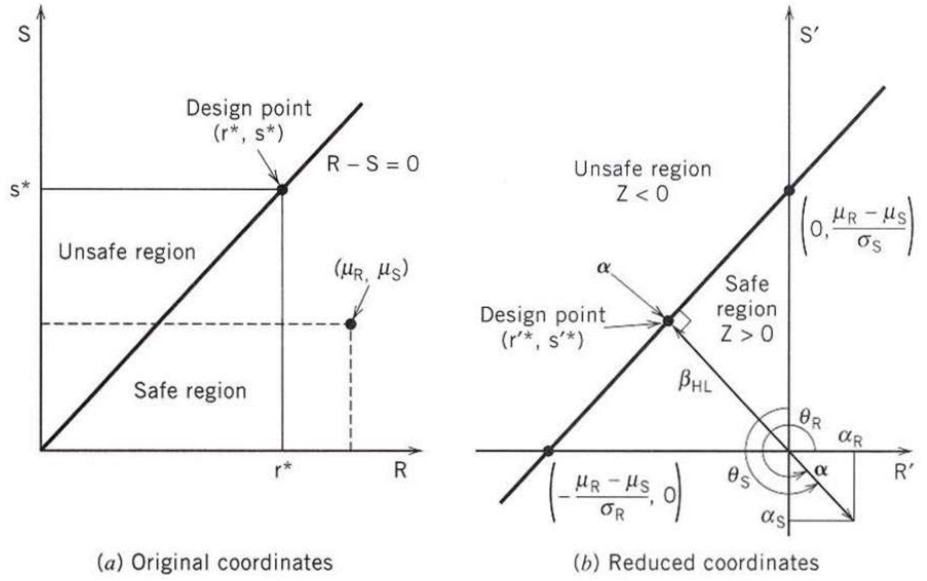


Figure 2.5: Hasofer-Lind reliability index, with linear limit state function

The best option for the design point, in terms of consistency and invariance of the solution for a different formulation of the limit state function, is the point $(R_d\,, E_d)$ closest to the mean $\mu_E\,, \mu_R$ indicated in the previous figure. Thus, the design point coordinates can be written as:

$$R_d = \mu_R - \alpha_R \beta \sigma_R \tag{2.23}$$

$$E_d = \mu_E - \alpha_E \beta \sigma_E \tag{2.24}$$

The coefficients α_E and α_R are called sensitivity factors of variables E and R. They can be seen as the direction cosines of the failure boundary, thus:

$$\alpha_E = -\sigma_E \Big/ \sqrt{\sigma_E^2 + \sigma_R^2} \tag{2.25}$$

$$\alpha_R = \sigma_R \Big/ \sqrt{\sigma_E^2 + \sigma_R^2} \tag{2.26}$$

and $\sqrt{\alpha_E^2 + \alpha_R^2} = 1$.

Eurocodes suggest recommended values for sensitivity factors as follows:

$$\alpha_E = -\sigma_E \Big/ \sqrt{\sigma_E^2 + \sigma_R^2} = -0{,}7 \tag{2.27}$$

$$\alpha_R = \sigma_R \Big/ \sqrt{\sigma_E^2 + \sigma_R^2} = 0.8 \tag{2.28}$$

The validity of the previous assumption is in a limitation on the ratio between the two standard deviations as follows: $0.16 < \sigma_E/\sigma_R < 7.6$. When this requirement is not satisfied, since the previous assumptions (i.e. $\alpha_E = -0.7$ and $\alpha_R = 0.8$) are extremely on the safe side, it is possible to assume $\alpha_E = -1$ and $\alpha_R = 1$.

The design values E_d and R_d are thus defined as fractiles of normal distribution:

$$P(E > E_d) = \Phi(+\alpha_E\beta) = \Phi(-0.7\beta)\,; \quad E_d = \mu_E + 0.7\beta\sigma_E \tag{2.29}$$

$$P(R \leq R_d) = \Phi(-\alpha_R\beta) = \Phi(-0.8\beta)\,; \quad R_d = \mu_R - 0.8\beta\sigma_R \tag{2.30}$$

where, as already mentioned, the symbol Φ stands for a standardized normal distribution. For $\beta = 3.8$, then the fractiles e_d and r_d correspond respectively to the probabilities of 0.999 and 0.001.

Moreover, the previous values on the coefficients of sensitivity are valid for dominant (or leading) random variables. When dealing with non-dominant (or accompanying) random variables, the value of sensitivity factors is reduced by pre-multiplying for 0.4. In this case, the following holds:

$$P(E > E_d) = \Phi(+0.4\alpha_E\beta) = \Phi(-0.28\beta) \tag{2.31}$$

$$P(R \leq R_d) = \Phi(-0.4\alpha_R\beta) = \Phi(-0.32\beta) \tag{2.32}$$

Again, for $\beta = 3.8$, then the fractiles of the accompanying (or non-dominant) variables correspond respectively to the probabilities of 0.9 and 0.1.

Finally, if the two R and E variables are lognormal distributed, the above equations become:

$$P(lnE > lnE_d) = \Phi(\alpha_E\beta) = \Phi(-0.7\beta)\,; E_d = \mu_E \exp(0.7\beta\ \sigma_E/\mu_E) \tag{2.33}$$

$$P(lnR \leq lnR_d) = \Phi(\alpha_R\beta) = \Phi(-0.8\beta); R_d = \mu_R \exp(-0.8\beta\sigma_R/\mu_R) \tag{2.34}$$

In conclusion, design values are the upper fractiles (for actions) or lower fractiles (for resistance), which correspond to a certain probability of being exceeded (actions) or not reached (resistance).

The methods of level II are the basis for calibration of methods of level I.

2.4.3 Methods of Level I

This approach consists on representing the basic variables with their characteristic value, which corresponds to a low quantile in case of resistance distributions and to a high quantile in case of actions distributions.

The method consists on verifying whether the limit state is not exceeded when all basic variables in the limit state equations are replaced by the so-called design valued (identified with a letter d as a subscript). In the simple case of a limit state function as the one of section 2.2.3.1, it is necessary to verify that:

$$R_d \geq E_d \tag{2.35}$$

with E_d and R_d defined respectively as:

$$E_d = E(F_{d,1}, F_{d,2}, \dots; a_{d,1}, a_{d,2} \dots; \vartheta_{d,1}, \vartheta_{d,1}) \tag{2.36}$$

$$R_d = R\left(X_{d,1}, X_{d,1}, \dots; a_{d,1}, a_{d,2}, \dots; \vartheta_{d,1}, \vartheta_{d,1}\right) \tag{2.37}$$

where F is an external action, X is a material property, a is a geometrical property, and ϑ, as already mentioned, is the model uncertainty.

Partial safety factors for material properties (i.e. γ_m) and actions (i.e. γ_f), in general, are now introduced, as function of their characteristic values, as:

$$\gamma_m = R_k/R_d \text{ for resistances} \tag{2.38}$$

$$\gamma_f = E_d/E_k \text{ for load effects} \tag{2.39}$$

The design values can be evaluated according to the previous method (i.e Level II), in particular with Eq. (2.29) and (2.30), if normally distributed or Eq. (2.33) and (2.34) if lognormally distributed. For what concerns the characteristic values, they can be obtained in general as the 5% quantile of the probabilistic distribution of the resistances, the 50% quantile of the probabilistic distribution of permanent actions and the 95-98% quantile of the probabilistic distribution of variable actions.

The semi-probabilistic limit state approach according to EN1990 [5] is based on this methodology, and it is used in practise for design and assessment of structures.

2.4.4 Methods of Level 0

These methods are pure deterministic ones. The basic verification is performed based on the following equation:

$$R_{nom} \geq \gamma E_{nom} \tag{2.40}$$

The nominal values are in general used accounting for one global safety factor γ that is determined empirically.

This method does not allow to quantify the level of reliability within assessment or design and may leads to underestimation of the structural safety without any control about it.

The probability-based models previously described are definitely more reliable than these types of approach, thus these are no longer adopted neither implemented.

2.5 Target reliability

The target reliability level is defined in fib Model Code 2010 [2] as "an acceptable failure probability corresponding to a specified reference period, which is required to assure the performance of a structure or structural component for which it has been designed".

Referring to the limit state approach, the maximum acceptable failure probability is function of the limit state, meaning ultimate or serviceability, the consequences of structural failure, the relative costs for safety measures and the reference period.

In general, the following aspect need to be evaluated: human safety and economical implication.

For what concerns the former, the total costs C_{tot} of a structure during its working life can be evaluated as:

$$C_{tot} = C_i + P_f D \tag{2.41}$$

where C_i are the initial costs to build the new structures C_{build} or to renovate the existing one $C_{upgrade}$ and $P_f\ D$ is the expected failure costs related to the working life (i.e. design service life for new structures and residual service life for existing ones).

In order to reach the optimum target level, it should be identified a value, corresponding to the minimization of total costs C_{tot} and that should be lower to the minimum requirements for human safety.

This is different if considering new or existing structures, as the Figure 2.6 shows:

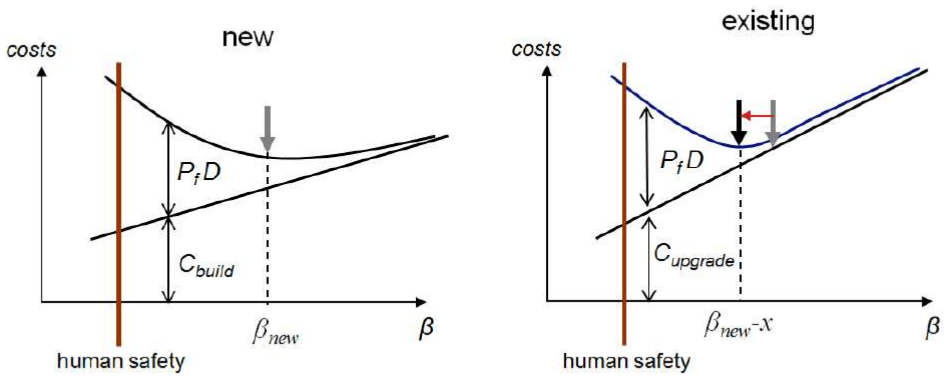


Figure 2.6: Target value identification: differences for the design of new structures versus upgrading of existing structures (fib Bulletin 80).

As it is possible to observe, the cost for upgrading of the existing structure are higher than the costs to build the new ones. Thus, optimum target reliability indexes for existing structures are lower if compared to the ones for new structures.

In Table 2.2, taken from fib MC 2010 [2], suggested values of target reliability are presented, different between new and existing structures.

If the structure is in presence of multiple equally important failure, according to fib MC 2010 [2], it should be designed for a larger level of reliability. Moreover, the target levels presented in the table are intended for structures for which failure is preceded by a certain level of warning (e.g. ductile failure modes). This way, it is possible to take preventive measures so to limit the eventual consequences of structural failure, in particular preventing human lives losses.

Brittle failure modes occurrences, meaning failure modes that are not warned by the structure, should be avoided by means of design procedures and correct detailing. Nevertheless, if a structural component is designed considering brittle failure mode, the target reliability level should be larger.

Table 2.2: Suggested range of target reliability from *fib Model Code 2010* for *new* and *existing* structures

Limit states	**Target reliability index β**	**Reference period**
New structures		
Serviceability (SLE)		
reversible	0.0	Service life
irreversible	1.5	50 years
irreversible	3.0	1 year
Ultimate (SLU)		
Low consequences of failure	3.1	50 years
	4.1	1 year
Medium consequences of failure	3.8	50 years
	4.7	1 year
High consequences of failure	4.3	50 years
	5.1	1 year
Existing structures		
Serviceability (SLE)	1.5	Residual service life
Ultimate (SLU)		
	3.1 – 3.8*	50 years
	3.4 – 4.1*	15 years
	4.1 – 4.7*	1 year

*depending from costs for safety measures and upgrading of the structure; more detailed information can be derived from *fib Bulletin 80*.

2.6 Safety format

Safety format is the procedure that allow to perform the verification of a structure with respect to a particular limit state. This verification consists on a probability-based method and fib MC 2010 [2] proposes the following safety formats:

- *Probabilistic safety format*, also called *fully probabilistic design method*, is a method that permits to quantify the reliability requirements in terms of reliability index β and reference period. This method is more suited for the assessment of existing structures, in particular for the computation of residual service life
- *Partial safety factor format*, is a simplified verification concept, based on past experiences and calibrated in such a way that general reliability requirements are satisfied. Usually adopted for verifying structural design
- *Global resistance format,* as the name foretells, it consists on a global resistance verification with partial safety factors. It is suited for design based on non-linear analysis, where numerical simulations are adopted to perform the verification of limit states

- *Deemed-to-satisfy approach,* consists on a set of rules for dimensioning, material and product selection and execution procedures, given in standard, aiming at maintaining the target reliability lower than the relevant limit state. Used for verifying service life design of new structures
- *Design by avoidance,* applicable both for verification of traditional structural design and design for service life, consists on avoiding or reducing harmful effects (e.g. protecting the structure from certain loads such as wind, wave loads, impacts and so on)

In the following, more details are given for the first three safety formats.

2.6.1 Probabilistic safety format

The probabilistic safety format consists on a probabilistic assessment of the safety of the structure, by means of an estimation of the failure probability P_f, or, mutually, of the reliability index β.

The procedure is the same discussed in the previous section, where to estimate the failure probability, the verification is:

$$P_f = P[g(X_i) \leq 0] \leq P_{f,T} \qquad i = 1,2,\dots,N \tag{2.42}$$

where $g(X_i)$ is the performance function (or limit state function), the $g(X_i) \leq 0$ represents the failure or unsafe condition and $P_{f,T}$ are the target probability of failure indexes, here reported in section 2.5, according to Table 2.2.

The relation between the reliability index β and the failure probability P_f is reported in this chapter, in sub-section 2.3.2. Moreover, for the methodology to be adopted for the evaluation of the basic variables X_i as well as the failure probability P_f it is suggested to look at the previous sections, in particular section 2.4.

2.6.2 Partial safety factor format

The idea behind this format is to separate the treatment of uncertainties from various cases by means of design values assigned to variables. According to fib MC 2010, this consists on selecting the representative values of variables and the partial safety factors so to meet reliability requirements in terms of β index.

Distinction should be made between basic and other variables. The formers are actions (F), material or product properties (X), some geometrical quantities (a), variables which account for the model uncertainties (ϑ). For these, design values include reliability margins. For the other variables, whose dispersion may be neglected or is covered by a set of partial factors, their most likely values are assumed.

The requirement consists on the following expression:

$$g(F_d, X_d, a_d, \vartheta_d, C) \geq 0 \tag{2.43}$$

with C representing serviceability constraints.

2.6.2.1 Design values of basic variables

Design variables of basic values are expressed as following:

- Design values of *actions*:

$$F_d = \gamma_F F_{rep} \tag{2.44}$$

  where F_{rep} is the representative value of actions and γ_F is a partial safety factor

- Design values of *material or product property*:

$$f_d = f_k / \gamma_m \tag{2.45}$$

  or if uncertainty in the design model is considered:

$$f_d = f_k / \gamma_M = f_k / (\gamma_m * \gamma_{Rd}) \tag{2.46}$$

  where f_k is the characteristic value of the resistance, γ_m is a partial safety factor for a material property, γ_{Rd} is a partial safety factor related to uncertainty of resistance model plus geometric deviations if they are included in the model, $\gamma_M = \gamma_m * \gamma_{Rd}$ is a partial safety factor for a material property accounting for the model uncertainties

- Design values of *geometrical property* are usually taken equal to their design values a_d

2.6.2.2 Determination of partial safety factors

Partial safety factors for basic variables are expressed as following:

- *Materials*:

$$\gamma_M = \gamma_{Rd}\,\gamma_m \tag{2.47}$$

$$\gamma_{Rd} = \gamma_{Rd1}\,\gamma_{Rd2} \tag{2.48}$$

  where γ_{Rd} is a partial safety factor accounting for model uncertainty set equal to 1.05 and γ_{Rd2} is a partial safety factor accounting for geometrical uncertainty equal to 1.05. Assuming normal distribution for material uncertainties the value of γ_M is equal to 1.5 for concrete cylinder compressive strength with a coefficient of variation equal to 0.15, while for bar reinforcements γ_M is equal to 1.15 with a coefficient of variation equal to 0.05. Finally, the related target of reliability is defined by $\beta = 3.8$ according to Table 2.2.
- *Permanent actions (G) and variable loads (Q)*:

$$\gamma_G = \gamma_{Sd}\,\gamma_g \text{ and } \gamma_Q = \gamma_{Sd}\,\gamma_q \tag{2.49}$$

  where γ_{Sd} is a partial safety factor accounting for model uncertainty and set equal to 1.05 while γ_G and γ_Q are partial safety factors for permanent and variable actions respectively, described in Section 2.4.3, Eq. (2.39).

2.6.3 Global resistance format

The uncertainties of the structural behaviour are integrated in a global design resistance and can also be expressed by a global safety factor. Again, these values should be selected in order to meet the requirements for the reliability index β.

The representative variable for the global resistance is the structural resistance R. The uncertainty is expressed by the following values of resistance:

- R_m mean value of resistance
- R_k characteristic value of resistance (corresponding to a probability of failure of 5%)
- R_d design value of resistance

The value of action F is considered in the same way as in the partial safety factor method, sub-section 2.6.2.

The safety condition is met when:

$$F_d \leq R_d,\ R_d = R_m/\gamma_R^* \gamma_{Rd} \tag{2.50}$$

where F_d is the design external action defined according to the partial factor format; γ_R^* is denoted as the global resistance safety factor, which accounts for material aleatory uncertainties; γ_{Rd} represents the resistance model uncertainty safety factor, which accounts for the resistance model uncertainty.

The value of the model uncertainty factor depends on the quality of formulation of resistance model, recommended values are:

- $\gamma_{Rd} = 1.0$ for no uncertainties
- $\gamma_{Rd} = 1.06$ for low uncertainties
- $\gamma_{Rd} = 1.1$ for high uncertainties

It is important to underline the differences between global and partial safety factors. The former refer to the global structural response evaluated by means of mean values of material properties, instead, partial safety factors refer just to each material property (i.e. concrete compressive strength, reinforcement yielding strength) evaluated with its characteristic value for local verification of structural members [22].

3 RC building: design characteristics

This work of thesis is about the evaluation of reliability for robustness of a real building designed in seismic area. Previous works of thesis had already investigated the robustness, but not in probabilistic way, of the same building and they are the following: “Robustezza Strutturale di edifici intelaiati in calcestruzzo armato: analisi parametrica e nuove proposte progettuali” by Fortunato Mauro [26] and “Robustezza Strutturale di costruzioni multipiano in calcestruzzo armato: analisi parametrica di telai 2D per mezzo di modelli globali e locali” by Luca Capri [27].

In the following, the building is described in terms of design characteristics according to code rules (i.e. capacity design) and in the end, the frame, object of this work of thesis, is presented, as a result of the conclusions made in previous studies.

3.1 Introduction

The building under analysis is a reinforced concrete structure, with residential use, located in L’Aquila city, in Abruzzo region of Italy. The elevation of the city is of 714 meters above sea level and the zone is judged of class II from a seismic point of view.

The references in terms of code rules are the following:

- D.M. 17 Gennaio 2018: Aggiornamento delle “Norme tecniche per le costruzioni” [9]
- EN1990 Eurocode 0: Basis of structural design [5]
- EN1992 Eurocode 2: Design of concrete structures [28]
- EN1998 Eurocode 8: Design of structures for earthquake resistance [29]
- EN206-1 Concrete - Part1: Specification, performance, production and conformity [30]

The building is a new one and the type of construction is “Costruzione con livelli di prestazione ordinari”, which means construction with ordinary performance levels.

Table 3.1: Minimum values for the Design Working Life V_N

	TIPI DI COSTRUZIONI	Valori minimi di V_N (anni)
1	Costruzioni temporanee e provvisorie	10
2	Costruzioni con livelli di prestazioni ordinari	50
3	Costruzioni con livelli di prestazioni elevati	100

Thus, referring to the table 2.4.I of the DM2018 [9], here reported in Table 3.1, the design working life of the building is $V_N = 50\ yr$, because it is associated to a construction type 2.

Moreover, the building is considered of Class of Use number II, defined at §2.4.2 of DM2018 [9] meaning that it is a building whose use is associated to normal crowding, not being dangerous for the environment and without essential public and social functions.

For the Class of Use number II, the use coefficient C_U is equal to 1.0, according to table 2.4.II of the Italian code rules, here reported in Table 3.2:

Table 3.2: Use coefficient C_U

CLASSE D'USO	I	II	III	IV
COEFFICIENTE C_U	0,7	1,0	1,5	2,0

Referring to § 2.4.3 of DM2018 [9], the design period for the seismic action is:

$$V_R = V_N \cdot C_U = 50\ yr \tag{3.1}$$

3.2 Geometrical characteristics

The building is composed by four floors plus the roofing, characterized by an inter-floor height of 3 meters and a span length of 5 meters; also, the influence depth in the transversal direction is of 5 meters. Having the columns of the entire building a cross-section of 60x60 centimetres, the effective span length is equal to 4.4meters.

For what concerns the beams, their cross-section is 40x50 centimetres, so that the effective inter-floor height is equal to 2.5 meters. To better visualize these geometrical aspects, a front view (Figure 3.1) and a plan view (Figure 3.2) are shown.

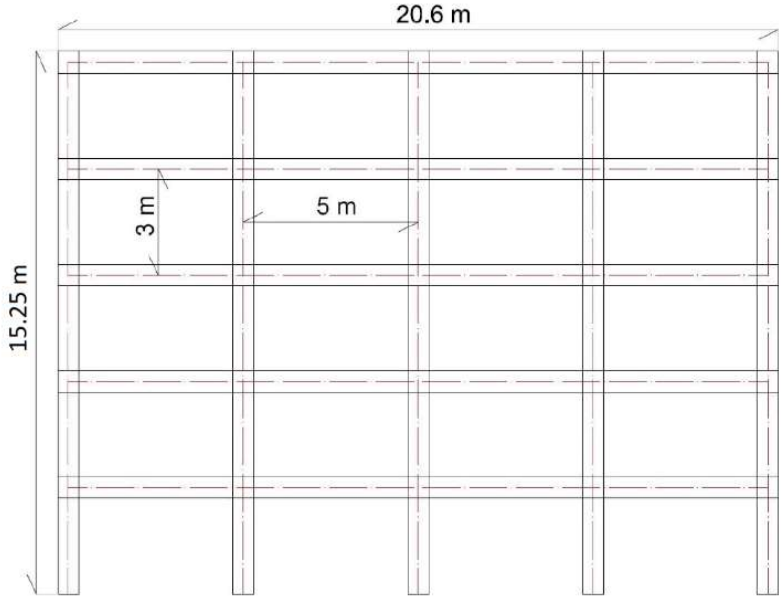


Figure 3.1: Front view of the building

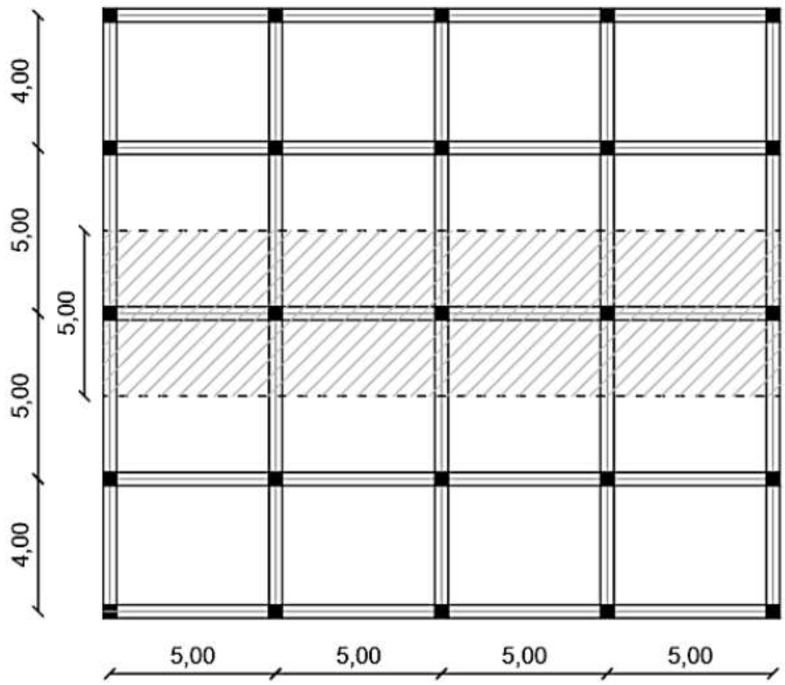


Figure 3.2: Plan view of the building

3.3 Material characteristics

3.3.1 Concrete

The concrete used to realize columns and beams has the following features:

Table 3.3: Characteristics of concrete

Concrete resistance class	C25/30
Cube strength R_{ck}	30 N/mm^2
Cylinder strength f_{ck}	25 N/mm^2
Partial safety coefficient γ_C	1.5 [-]
Long duration load coefficient α_{cc}	0.85 [-]
Design value of compressive strength f_{cd}	14.17 N/mm^2
Mean value of axial tensile strength f_{ctm}	2.56 N/mm^2
Characteristic axial tensile strength f_{ctk}	1.8 N/mm^2
Design value of tensile strength f_{ctd}	1.2 N/mm^2
Ultimate strain at ULS ε_{cu}	3.5 ‰
Specific weight γ	25 kN/m^3
Elastic modulus E_{cm}	31476 N/mm^2
Poisson's coefficient υ	0.2 [-]

3.3.2 Steel

The steel used for the bars and the stirrups has the following characteristics:

Table 3.4: Characteristics of steel

Steel class	B450/C
Characteristic tensile strength f_{tk}	540 N/mm^2
Characteristic yield strength f_{yk}	450 N/mm^2
Partial safety coefficient γ_S	1.15 [-]
Design yield strength f_{yd}	391 [-]
Ratio between characteristic tensile and yield strength	1.15 - 1.35 [-]
Characteristic strain at maximum load ε_{uk}	75 ‰
Design yielding strain ε_{syd}	1.96 ‰
Ultimate strain at ULS ε_{ud}	$0.9\varepsilon_{uk} = 63‰$
Elastic modulus E_s	200000 N/mm^2
Poisson's coefficient υ	0.3 [-]

3.4 Durability

The reference code rules are defined in EN1992 Eurocode 2: Design of concrete structures [28], at section 4: Durability and cover to reinforcement. The durability regards the stability and strength of the structure, that shall be maintained during all the design working life. Its value depends on the environmental conditions, intended as the chemical and physical conditions to which the structure is exposed in addition to the mechanical actions.

3.4.1 Exposure class

By looking at the table 4.1-section2 of EC2 [28], it is possible to define the exposure class of the building, depending on the corrosion induced by carbonation (Table 3.5):

Table 3.5: Exposure class related to corrosion induced by carbonation

XC1	Dry or permanently wet	Concrete inside buildings with low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure class XC2

The environmental conditions to which the case study is exposed are “wet, rarely dry”, so the exposure class is XC2.

According to the EN 206-1 [30], at table F.1, recommended limiting values for composition and properties of concrete are presented, depending on the class previously defined (Table 3.6):

Table 3.6: Recommended limiting values for composition and properties of concrete

	Exposure classes																	
	No risk of corrosion or attack	Carbonation-induced corrosion				Chloride-induced corrosion						Freeze/thaw attack				Aggressive chemical environments		
						Sea water			Chloride other than from sea water									
	X0	XC 1	XC 2	XC 3	XC 4	XS 1	XS 2	XS 3	XD 1	XD 2	XD 3	XF 1	XF 2	XF 3	XF 4	XA 1	XA 2	XA 3
Maximum w/c	---	0,65	0,60	0,55	0,50	0,50	0,45	0,45	0,55	0,55	0,45	0,55	0,55	0,50	0,45	0,55	0,50	0,45
Minimum strength class	C12/15	C20/25	C25/30	C30/37	C30/37	C30/37	C35/45	C35/45	C30/37	C30/37	C35/45	C30/37	C25/30	C30/37	C30/37	C30/37	C30/37	C35/45
Minimum cement content (kg/m^3)	---	260	280	280	300	300	320	340	300	300	320	300	300	320	340	300	320	360
Minimum air content (%)	---	---	---	---	---	---	---	---	---	---	---	---	4,0[a]	4,0[a]	4,0[a]	---	---	---
Other requirements												Aggregate in accordance with prEN 12620:2000 with sufficient freeze/thaw resistance					Sulfate-resisting cement[b]	

[a] Where the concrete is not air entrained, the performance of concrete should be tested according to an appropriate test method in comparison with a concrete for which freeze/thaw resistance for the relevant exposure class is proven.

[b] When SO_4^{2-} leads to exposure classes XA2 and XA3, it is essential to use sulfate-resisting cement. Where cement is classified with respect to sulfate resistance, moderate or high sulfate-resisting cement should be used in exposure class XA2 (and in exposure class XA1 when applicable) and high sulfate-resisting cement should be used in exposure class XA3.

3.4.2 Structural class

The Eurocode 2 [28], at suggests at §4.4.1.2(5) suggests to start from a structural class S4 when the design working life is of 50 years, as for the case study. In the table 4.3N of the aforementioned normative, there are recommended values for structural classification, depending on the exposure class (Table 3.7):

Table 3.7: Recommended structural classification depending on the exposure class XC

Structural Class							
Criterion	Exposure Class according to Table 4.1						
	X0	XC1	XC2 / XC3	XC4	XD1	XD2 / XS1	XD3 / XS2 / XS3
Design Working Life of 100 years	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2	increase class by 2
Strength Class [1) 2)]	≥ C30/37 reduce class by 1	≥ C30/37 reduce class by 1	≥ C35/45 reduce class by 1	≥ C40/50 reduce class by 1	≥ C40/50 reduce class by 1	≥ C40/50 reduce class by 1	≥ C45/55 reduce class by 1
Member with slab geometry (position of reinforcement not affected by construction process)	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1
Special Quality Control of the concrete production ensured	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1	reduce class by 1

Having the case study an exposure class XC2 and a concrete class <C35/45, the structural class remains equal to the initial suggested value, thus S4.

3.4.3 Concrete cover

Again, the reference code rule is the Eurocode 2 [28], in particular §4.4.1 titled Concrete cover. First of all, concrete cover is defined as the minimum possible distance between the surface of the reinforcement and the nearest concrete surface. The nominal concrete cover is computed as the sum of a minimum part c_{min} and an allowance in design for deviation Δc_{dev}:

$$c_{nom} = c_{min} + \Delta c_{dev} \tag{3.2}$$

The minimum concrete cover is necessary to ensure a safe transmission of bond forces, to protect the steel against corrosion (durability purposes) and to guarantee enough fire resistance. Its value is computed as follows:

$$c_{min} = \max\{c_{min,b}\ ;\ c_{min,dur} + \Delta c_{dur,\gamma} - \Delta c_{dur,st} - \Delta c_{dur,add}\ ; 10\} \tag{3.3}$$

where:

- $c_{min,b}$ is the minimum cover for bond requirement, according to table 4.2 of [28] (here Table 3.8):

Table 3.8: Minimum cover requirement with regard to bond

Bond Requirement	
Arrangement of bars	Minimum cover $c_{min,b}$*
Separated	Diameter of bar
Bundled	Equivalent diameter (ϕ_n)(see 8.9.1)
*: If the nominal maximum aggregate size is greater than 32 mm, $c_{min,b}$ should be increased by 5 mm.	

- $c_{min,dur}$ is the minimum cover due to environmental conditions, according to table 4.4N of [28] (here Table 3.9):

Table 3.9: Minimum cover requirements with regard to durability for

Environmental Requirement for $c_{min,dur}$ (mm)							
Structural Class	**Exposure Class according to Table 4.1**						
	X0	XC1	XC2 / XC3	XC4	XD1 / XS1	XD2 / XS2	XD3 / XS3
S1	10	10	10	15	20	25	30
S2	10	10	15	20	25	30	35
S3	10	10	20	25	30	35	40
S4	10	15	25	30	35	40	45
S5	15	20	30	35	40	45	50
S6	20	25	35	40	45	50	55

- $\Delta c_{dur,\gamma}$ is the additive safety element, whose recommended value according to Eurocode 2 [28], at §4.4.1.2(6), is $0\ mm$;
- $\Delta c_{dur,st}$ is the reduction of minimum cover for use of stainless steel, whose recommended value according to Eurocode 2 [28], at § 4.4.1.2(7), is 0 mm;
- $\Delta c_{dur,add}$ is the reduction of minimum cover for use of additional protection, whose recommended value according to Eurocode 2 [28], at §4.4.1.2(8), is 0 mm;

Taking into account all the previous suggestions, the assumed values are:

- $c_{min,b} = 18\text{ mm}$ for the beams and $c_{min,b} = 20\text{ mm}$ for the columns, according to the design bars' diameter (i.e. $\phi 18$ for the beams and $\phi 20$ for the columns);
- $c_{min,dur} = 25\ mm$, being the structural class S4 and the exposure coefficient XC2;
- $\Delta c_{dur,\gamma} = \Delta c_{dur,st} = \Delta c_{dur,st} = 0\ mm$

Thus, according to (3.3), the minimum concrete cover is $c_{min} = \max\{20;\ 25; 10\} = 25\text{mm}$.

In conclusion, considering that the recommended value of Δc_{dev} is $10mm$ according to §4.4.1.2 of Eurocode 2 [28], referring to equation (3.2), $c_{nom} = c_{min} + \Delta c_{dev} = 25 + 10 = 35\ mm$, used both for columns and beams.

3.5 Actions

The reference normative are the DM2018 [9] and, particularly for the seismic actions, the Eurocode 8 [29]. From a general point of view, an action is defined as a consequence, or a group of consequences, that can determine the realization of a limit state in a structure. In § 2.5.1 of DM2018 [9], actions can be classified as function of their intensity variation over the time, as follows (definitions are taken from Eurocode 0 [5]):

- Permanent loads (G): "action that is likely to act throughout a given reference period and for which the variation in magnitude with time is negligible, or for which the variation is always in the same direction (monotonic) until the action attains a certain limit value"
- Variable loads (Q): "action for which the variation in magnitude with time is neither negligible nor monotonic"
- Accidental (exceptional) loads (A): "action, usually of short duration but of significant magnitude, that is unlikely to occur on a given structure during the design working life"
- Seismic actions (E): "action that arises due to earthquake ground motions"

Depending on the limit states to be considered, actions can be combined as follows:

- Fundamental combination, usually assumed for Ultimate Limit State (ULS):

$$\gamma_{G1}G_1 + \gamma_{G2}G_2 + \gamma_P P + \gamma_{Q1}Q_{k1} + \gamma_{Q2}\Psi_{02}Q_{k2} + \gamma_{Q3}\Psi_{03}Q_{k3} + \ldots \quad (3.4)$$

- Rare combination, usually assumed for irreversible Serviceability Limit State (SLS):

$$G_1 + G_2 + P + Q_{k1} + \Psi_{02}Q_{k2} + \Psi_{03}Q_{k3} + .. \quad (3.5)$$

- Frequent combination, usually assumed for reversable Serviceability Limit State (SLS):

$$G_1 + G_2 + P + \Psi_{11}Q_{k1} + \Psi_{22}Q_{k2} + \Psi_{23}Q_{k3} + .. \quad (3.6)$$

- Quasi-permanent combination (SLS), usually assumed for reversable long term effects:

$$G_1 + G_2 + P + \Psi_{21}Q_{k1} + \Psi_{22}Q_{k2} + \Psi_{23}Q_{k3} + .. \quad (3.7)$$

- Accidental combination, usually assumed for Serviceability Limit State (SLS) and long-term effects:

$$G_1 + G_2 + P + \mathrm{A_d} + \Psi_{21}Q_{k1} + \Psi_{22}Q_{k2} + \Psi_{23}Q_{k3} + .. \quad (3.8)$$

- Seismic combination, usually assumed for both SLS and ULS, when considering seismic actions:

$$G_1 + G_2 + P + \Psi_{21}Q_{k1} + \Psi_{22}Q_{k2} + .. \quad (3.9)$$

where γ_{G1}, γ_{G2} and γ_P are the partial coefficient of respectively permanent structural loads, permanent non-structural loads and live loads, while Ψ_{ij} are the combination coefficients related to the jth variable action. The latter depend on the type of action, on the category of the structure and on the design situation, as defined in table A1.1 of Eurocode 0 [5], here reported in Table 3.10:

Table 3.10: Combination coefficients

Action	ψ_0	ψ_1	ψ_2
Imposed loads in buildings, category (see EN 1991-1-1)			
Category A : domestic, residential areas	0,7	0,5	0,3
Category B : office areas	0,7	0,5	0,3
Category C : congregation areas	0,7	0,7	0,6
Category D : shopping areas	0,7	0,7	0,6
Category E : storage areas	1,0	0,9	0,8
Category F : traffic area, vehicle weight ≤ 30kN	0,7	0,7	0,6
Category G : traffic area, 30kN < vehicle weight ≤ 160kN	0,7	0,5	0,3
Category H : roofs	0	0	0
Snow loads on buildings (see EN 1991-1-3)*			
Finland, Iceland, Norway, Sweden	0,70	0,50	0,20
Remainder of CEN Member States, for sites located at altitude H > 1000 m a.s.l.	0,70	0,50	0,20
Remainder of CEN Member States, for sites located at altitude H ≤ 1000 m a.s.l.	0,50	0,20	0
Wind loads on buildings (see EN 1991-1-4)	0,6	0,2	0
Temperature (non-fire) in buildings (see EN 1991-1-5)	0,6	0,5	0
NOTE The ψ values may be set by the National annex. * For countries not mentioned below, see relevant local conditions.			

3.5.1 Permanent actions

3.5.1.1 Permanent structural loads (G_1)

The permanent structural loads (G_1) are defined by the self-weight of the beams and the columns, and the self-weight of the reinforced concrete and hollow tiles mixed floor slab. The former are directly computed by the FEM software, function of the specific weight ($25\ kN/m^3$ for the reinforced concrete and $24\ kN/m^3$ for the concrete only) and the cross-section of the beams and the columns, while the latter is computed taking into account the slab scheme (Figure 3.3).

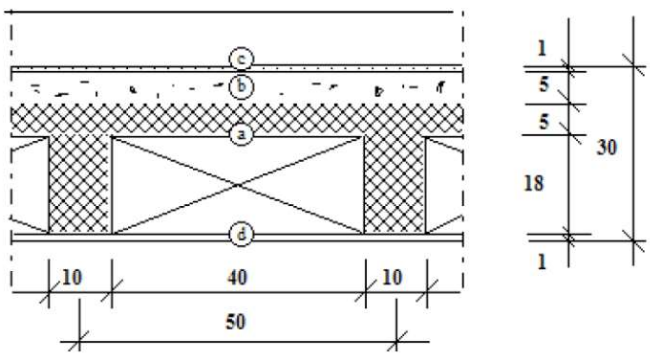


Figure 3.3: Slab scheme (dimensions in cm)

Considering all the contributions (Table 3.11), the permanent structural load of the slab is equal to $3.20\ kN/m^2$.

Table 3.11: Permanent structural load of the slab

	$Width\ [m]$	$Thickness[m]$	$Weight\ [kN/m^3]$	$g_1\ [kN/m^2]$
Slab	1.00	0.05	25	1.25
Rib	2x0.10	0.18	25	0.90
Brick	2x0.40	0.18	7.3	1.05
				3.20

3.5.1.2 Permanent non-structural load (G_2)

The permanent non-structural load (G_2) depends on the non-structural parts of the slabs (screed, floor and plaster) and the inner walls. The former is equal to $1.40\ kN/m^2$, as Table 3.12 shows:

Table 3.12: Permanent non-structural load of the slab

	$Width\ [m]$	$Thickness[m]$	$Weight\ [kN/m^3]$	$g_2\ [kN/m^2]$
Screed	1.00	0.05	16.0	0.8
Floor	-	-	-	0.20
Plaster	1.00	0.02	20.0	0.40
				1.40

The other part of G_2, related to the internal walls, can be computed considering the Figure 3.4:

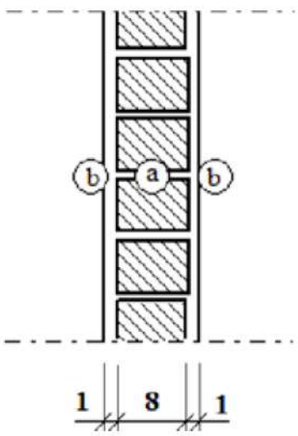


Figure 3.4: Scheme of the internal walls (dimensions in cm)

Considering the specific weight of the elements composing the internal walls, it is possible to obtain the permanent non-structural load as follows (Table 3.13):

Table 3.13: Internal walls' weight

	$\boldsymbol{Width}$ $[\boldsymbol{m}]$	$\boldsymbol{Weight}$ $[\boldsymbol{kN/m^3}]$	$\boldsymbol{g_2}$ $[\boldsymbol{kN/m^2}]$
Brick	0.08	6.0	0.48
Plaster	0.01	20.0	0.40
			0.88

To calculate the G_2 value associated to the internal walls, code rules suggest loads per square meters to refer, starting from the load per meter. This value can be easily computed for the specific case by multiplying the previous load for the height of the floor, as follows:

$$G_2 = 2.65 \cdot 0.88 = 2.33kN/m \tag{3.10}$$

Thus, considering the DM 2018 [9] at § 3.1.3, the g_2 value can be assumed equal to 1.20 kN/m^2:

Table 3.14: Permanent non-structural load for internal walls

per elementi divisori con		$G_2 \leq 1{,}00$ kN/m : $g_2 = 0{,}40$ kN/m²;
per elementi divisori con	1,00 <	$G_2 \leq 2{,}00$ kN/m : $g_2 = 0{,}80$ kN/m²;
per elementi divisori con	2,00 <	$G_2 \leq 3{,}00$ kN/m : $g_2 = 1{,}20$ kN/m²;
per elementi divisori con	3,00 <	$G_2 \leq 4{,}00$ kN/m : $g_2 = 1{,}60$ kN/m²;
per elementi divisori con	4,00 <	$G_2 \leq 5{,}00$ kN/m : $g_2 = 2{,}00$ kN/m².

The total permanent non-structural load is then equal to $\boldsymbol{g_2 = 2.60\ kN/m^2}$.

3.5.2 Variable actions

3.5.2.1 Live loads

This is the overload due to the use of the building, and, referring to table 3.1.II of DM2018 [9], it is equal to 2.00 kN/m^2 for the floors and 0.50 kN/m^2 for the roofing.

3.5.2.2 Wind load

This action is defined in § 3.3 of DM2018 [9] as follows:

$$p = q_r \cdot c_e \cdot c_p \cdot c_d \tag{3.11}$$

where:

- q_r is kinetic wind pressure

  The kinetic wind pressure q_r is computed using the following formula:

$$q_r = \frac{1}{2}\rho v_r^2 \tag{3.12}$$

  Considering an air density $\rho = 1{,}25\ kg/m^3$ and a reference wind velocity $v_r = 31.3\ m/s$ for L'Aquila city. Thus, by plug in these values on (3.12), it results $q_r = 612{,}3\ N/m^2$.

- c_e is the exposure factor

  The exposure coefficient c_e is given by the following formulae:

$$c_e(z) = k_r^2 c_t \ln(z/z_0)\,[7 + c_t \ln(z/z_0)] \quad \text{for } z \geq z_{min} \tag{3.13}$$

$$c_e(z) = c_e(z_{min}) \qquad \text{for } z < z_{min} \tag{3.14}$$

  where:

  - k_r, z_0, z_{min} are given in table 3.3.II of DM2018 [9] depending on the exposure category of the site. Since the category is V, it results: $k_r = 0.23,\ z_0 = 0{,}70m\ , z_{min} = 12m$
  - c_t is the topography coefficient and the normative suggests $c_t = 1$

  Thus, it is possible to define the exposure coefficient as function of the height z, expressed in meters, by substituting the coefficients in (3.12) and (3.13):

Table 3.15: Exposure coefficient c_e, as function of the height

$z\ [m]$	0	3	6	9	12	15
$c_e\ [-]$	1.48	1.48	1.48	1.48	1.48	1.63

- c_p is the shape coefficient (or aerodynamic) and it is assumed equal to 0.80 for the upwind surface and -0.45 for the downwind surface
- c_d is the dynamic factor, the normative suggests to assume $c_d = 1$

In conclusion, by plug in all these coefficients in (3.11) and taking into account an influence area of 5 meters, it is possible to obtain the distribution of the wind pressure along the height:

Table 3.16: Wind pressure, as function of the height

z $[m]$	p_{up} $[kN/m]$	p_{down} $[kN/m]$
0	3.60	-1.80
3	3.60	-1.80
6	3.60	-1.80
9	3.60	-1.80
12	3.60	-1.80
15	4.00	-2.00

3.5.2.3 Snow load

This action is defined in § 3.4 of DM2018 [9] as follows:

$$q_s = \mu_i * q_{sk} * C_E * C_t \quad (3.15)$$

where:

- q_{sk} is the characteristic ground snow load and it depends on the location of the city (L'Aquila is in zone III) and on the elevation of the site (714 meters over the sea level for L'Aquila). Thus, referring to § 3.4.2, it results $q_{sk} = 2.72 kN/m^2$
- μ_i is the shape coefficient and depends on the inclination angle of the roofing. For the specific case, the roof is planar, thus referring to table 3.4.II of DM2018 [9], $\mu_i = 0.8$
- C_E is the exposure coefficient, suggested unitary in § 3.4.4
- C_t is the thermal coefficient, suggested unitary in § 3.4.5

In conclusion, the snow load is $q_s = 2.17\ kN/m^2$ for the case study.

3.5.3 Seismic action

This action is described in § 3.2 of DM2018 [9], where the design seismic action is defined as function of the seismic hazard of the site, associated to the morphological and stratigraphic characteristics of the ground where the structure is located.

Seismic hazard is correlated to $S_e(T)$, that is the elastic horizontal ground acceleration response spectrum also called "elastic response spectrum”. This spectrum corresponds to a ground acceleration equal to the design ground acceleration on ground type A multiplied by the soil factor S, defined as:

$$S_e(T) = f(a_g, F_0, T_c^*) \tag{3.16}$$

where:

- a_g is the design ground acceleration on ground type A
- F_0 is the maximum horizontal amplification factor
- T_c^* is the corner period at the upper limit of the constant acceleration region of the elastic spectrum

To determine these values, the DM2018 [9] recommends the use of an Excel sheet, made available by the Ministry of Infrastructure and Transport. The procedure is divided into 3 phases.

The first phase (“fase 1” of the Excel Macro) regards the identification of the dangerousness of the site. Thus, it is needed to identify the region and city in which the building is placed. Then the geographical coordinates are automatically obtained.

The second phase (“fase 2” of the Macro) regards the choice of the design strategy. The input parameters are:

- Design working life (in year). The building is class 2 (buildings with ordinary level of performance) so $V_N = 50\ yr$
- Use coefficient $c_U = 1{,}0$

In the last phase (“fase 3” of the Macro), the information are:

- Limit state considered: both ULS and SLS are considered;
- Ground type: B “deposits of very dense sand, gravel, or very stiff clay, at least several tens of m in thickness, characterised by a gradual increase of mechanical properties with depth” (Table 3.1 of EC8 [29])
- Topographic class: T_3
- Behaviour factor $q_0 = 4.5 \cdot \alpha_u/\alpha_1 = 4.5 \cdot 1.3 = 5.85$, having chosen a ductility class A and $\alpha_u/\alpha_1 = 1.3$ for multi-storey frames

- Structural factor $q = K_R \cdot q_0 = 1 \cdot 5.85 = 5.85$, having chosen a reduction factor $K_R = 1$ as the normative suggests for regular structures in elevation
- Damping factor: $\xi = 5\%$ as conventionally assumed for reinforced concrete structures

In the following, the response spectrum at ultimate limit state (Figure 3.5) and at the serviceability limit state (Figure 3.6) are represented.

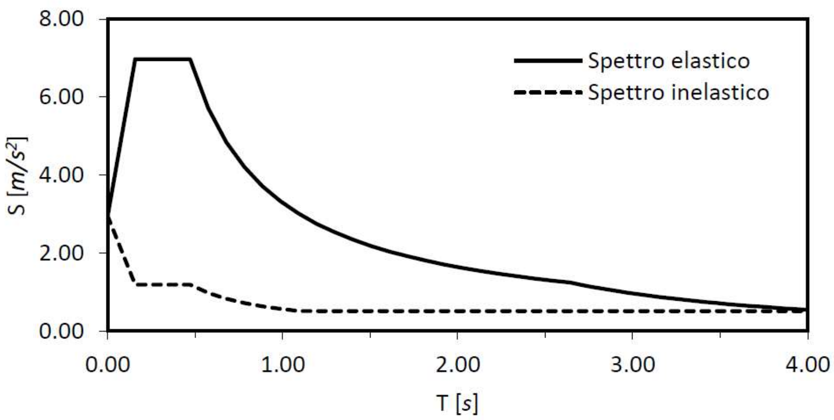


Figure 3.5: Response spectrum at ULS

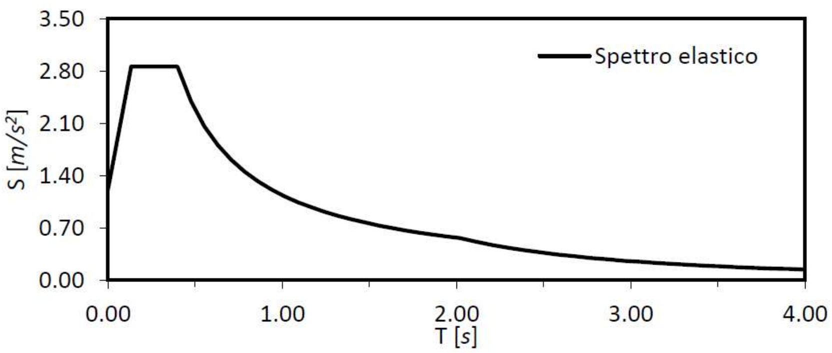


Figure 3.6: Response spectrum at SLS

3.6 Modal analysis

Considering the DM2018, it is requested to consider all the eigenvalues and eigenvectors that can contribute significatively to the response of a structure, from a global point of view. In particular, it is necessary to verify that:

- The sum of the modal masses of the considered vibration modes should represent the 85% of the total mass of the structure

- The considered modes should have a modal larger than the 5% of the total mass of the structure

In the specific case, the building is characterized by regularity both in plan and in elevation, thus it is possible to conduct the modal analysis following a linear approach.

The periods and the modal masses involved are presented in the following table:

Table 3.17: Modal analysis for the first 12 vibration modes

Mode	**Period**	**Mass mobilized along X**	
		For each mode	Sum
$[-]$	$[s]$	[%]	[%]
1	0.45	82.223	82.223
2	0.14	10.792	93.015
3	0.08	4.320	97.335
4	0.05	0.000	97.335
5	0.05	2.041	99.376
6	0.05	1E-04	99.376
7	0.05	4E-18	99.376
8	0.04	0.617	99.994
9	0.04	3E-18	99.994
10	0.04	2E-03	99.996
11	0.03	2E-18	99.996
12	0.03	5E-17	99.996

It is possible to notice that the 85% is already reached with the first two modes and also that the 5% is overcame for them. So, the indication suggested by code rules are respected.

Another consideration is that the modal mass has been computed by applying the following formula, again suggested by code rules:

$$G_1 + G_2 + \sum \Psi_{2j} Q_{kj} \tag{3.17}$$

3.7 Structural verification and dimensioning

The building design takes into account the criticalities connected to the seismicity of the area; thus, the capacity design is the main criterion. According to this, the structural elements should be designed in a certain order, consider the importance that they assume, in order to favour ductile mechanisms (bending) rather than fragile ones (shearing). Since beams are ductile elements, these are the elements that firstly have to reach plasticization, such that they can absorb and dissipate the energy in case of earthquake. This means that it is first designed the longitudinal reinforcement of the beams and, as function of it, the shear reinforcement of beams and both the bars and stirrups of columns are designed. In the following, the dimensioning and verification at ultimate limit state and serviceability limit state are analysed.

3.7.1 Beam design: bending at ULS

The geometry of the beams is followingly resumed in Table 3.18:

Table 3.18: Beam's geometry

$\boldsymbol{B}$ $[mm]$	$\boldsymbol{H}$ $[mm]$	$\boldsymbol{c}$ $[mm]$	$\boldsymbol{d'}$ $[mm]$	$\boldsymbol{d}$ $[mm]$
400	500	35	52	448

The normative suggests to compare the active bending moment M_{Ed} with the resistant one M_{Rd} so that:

- if $M_{Ed} < M_{Rd,lim}$ the reinforcement is simply in tension
- if $M_{Ed} > M_{Rd,lim}$ also the compressive reinforcement is needed

where the limiting value of the resistant bending moment is obtained by

$$M_{Rd,lim} = 0.2961\ B\ d^2 f_{cd} = 503.9\ kNm \tag{3.18}$$

Even if the active bending moment is always less than the resistant one, also the reinforcement in compression will be designed so that robustness will be increased.

Knowing the material characteristics, and the coefficient $\beta_1 = 0.8095$ and $\beta_2 = 0.4160$, a first trial of x_U (neutral axis position) is possible, thanks to the binomial formula. The minimum amount of reinforcement in the tensile zone is then:

$$A_{s,min}[mm^2] = \frac{\beta_1 B f_{cd} x_U}{f_{yd}} \tag{3.19}$$

Having chosen the bar diameters, it is necessary to respect what it is recommended by code rules (DM2018 § 7.4.6.2.1):

- both in the upper and lower part of the cross section, and along the entire length of the beam, the reinforcement diameters should be at least 14 mm
- in correspondence of the critical zones (i.e. at both extremities of the beam), the reinforcement in compression should be not less than half of the one in tension, such that: $\rho_s' \geq 0.5\rho_s$. In the other sections, it should be guaranteed that $\rho_s' \geq 0.25\rho_s$, where:

$$\rho_s' = \frac{A_s'}{BH} \tag{3.20}$$

$$\rho_s = \frac{A_s}{BH} \tag{3.21}$$

- in each section, the limits of the geometrical reinforcement ρ_s are:

$$\frac{1.4}{f_{yk}} \leq \rho_s \leq \rho_s' + \frac{3.5}{f_{yk}} \tag{3.22}$$

After having defined reinforcement in compression and in tension, it is necessary to verify if these values are sufficient to guarantee enough resistance. Firstly, the depth of the neutral axis position is computed as follows:

$$x_U = \frac{f_{yd}(A_s - A_s')}{\beta_1 f_{cd} B} \tag{3.23}$$

In order to verify the initial assumption on the strengths of the material (field 3, where $\varepsilon_{cd} = 0.35\%$ and $\varepsilon_s > 0.196\%$), an x_U value is computed, solving the equation:

$$C + S' - S = 0 \tag{3.24}$$

where $C = \beta_1 f_{cd} B x_U$, $S = f_{yd} A_s$ and $S' = f_{yd} A_s' = \varepsilon_s' E_s A_s'$

Once x_U is computed, the verifications are:

- Ductility check: $x_U < 0.45d$
- Bending moment check: $M_{Ed} < M_{Rd}$ where the resistant bending moment is computed as:

$$M_{Rd} = \beta_1 f_{cd} B x_U (d - \beta_2 x_U) + A_s' f_{yd} (d - d') \tag{3.25}$$

For the sake of simplicity, only bars having diameter of 18 mm have been chosen, and they are located as follows:

- 3 continuous bars in the lower part and along the entire length of the beam, enough to retain the applied positive bending moment
- 2 bars in the upper part, such that on the nodes beam-column the total amount of bars is 4
- from 1 to 3 extra bars are added on the upper part in correspondence of the nodes, so to retain the peaks of negative applied bending moment

3.7.2 Beam design: shear at ULS

The shear action is determined as function of the resistant bending moment, as the capacity design prescribes. Also the gravitational loads are involved in the computation, as follows:

$$V_{Ed} = \gamma_{Rd} \frac{M_{Rb,1} + M_{Rb,2}}{l_c} + \frac{1}{2}(G + \psi_2 Q) l_c \tag{3.26}$$

where:

- l_c is the length of the simply supported beam
- $M_{Rb,1}$ is the resistant bending moment of the first support
- $M_{Rb,2}$ is the resistant bending moment of the second support
- γ_{Rd} overstrength factor, taken equal to 1.0 for $q \leq 3$, or to 1.2 otherwise
- $G + \psi_2 Q$ gravitational load

The computation of the transversal reinforcement is based on the Ritter-Morsch model, also called strut-and-tie model, based on a truss system with parallel chords (i.e. longitudinal reinforcement) connected by means of pin joints, where the concrete compressive struts are inclined with an angle ϑ and the shear reinforcement represents the tensile web members, where the inclination angle is $\alpha = 90°$ [31].

The design resistant shear, referred to the concrete struts, is computed as follows:

$$V_{Rcd} = 0.9 d b_w \alpha_c \upsilon f_{cd} \frac{cotg\alpha + cotg\vartheta}{1 + cotg^2\vartheta} \tag{3.27}$$

On the other hand, the design resistant shear, referred to the steel stirrups, is:

$$V_{Rsd} = 0.9 d \frac{A_{sw}}{s} f_{yd} (cotg\alpha + cotg\vartheta) sin\alpha \tag{3.28}$$

where:

- d is the effective height of the cross-section
- b_w is the width of the cross-section
- α_c is a coefficient that considers the tension state of the compressive chord, for this case equal to 1
- υf_{cd} design value of compressive strength, reduced with a quantity $\upsilon = 0.5$
- α stirrups inclination with respect to the longitudinal reinforcement direction (i.e. horizontal)
- ϑ strut inclination, value that should respect the limits: $1 \leq cotg\vartheta \leq 2.5$ and $cotg\vartheta = 1$ for CD "A"
- A_{sw} is the area of the transversal reinforcement
- s is the distance between consecutive transversal stirrups

The verifications consist of guaranteeing that:

- $V'_{Ed} < V_{Rcd}$ where V'_{Ed} is the shear value in correspondence of the joint with the column
- $V''_{Ed} < V_{Rs}$ where $V''_{Ed}$is the shear value at a distance d from the joint with the column
- $V_{Rsd} > V_{Rcd}$ for ductility reasons

In order to define the stirrups design, two zones are distinguished: the dissipative and the non-dissipative parts of the beam. The former has a length of 1.5 (for CD "A") and 1.0 (for CD "B") times the height of the beam, that is the area where the plasticization occurs. The non-dissipative zones regard the remaining cross-sections of the beam.

There are limitations regarding both the dissipative and non-dissipative areas, defined at DM2018 § 4.1.6.1.1 and § 7.4.6.2.1. In particular:

In the dissipative area the stirrups should have a step s that is the minimum among:

- a quarter of the effective height d of the cross-section
- 175 mm and 225 mm respectively for CD "A" and CD "B"
- 6 times and 8 times the minimum diameter of the longitudinal bars respectively for CD "A" and CD "B"
- 24 times the diameter of the transversal reinforcement

In the non-dissipative zones, the stirrups should respect the following limitations:

- The cross-section of the stirrups should be at least $A_{st} = 1.5b\ mm^2/m$ where b is the minimum width of the web
- At least there should be 3 stirrups per meter
- The step should be not larger than 0.8 times the effective height d of the cross-section

In the end, the final configuration consists of stirrups having all the same diameters equal to 8 mm and the step between two consecutive stirrups is:

- 7.5 cm in the dissipative zones
- 15 cm in the non-dissipative zones

3.7.3 Beam design: SLS

3.7.3.1 Stress limitation

These limitations are defined at § 7.2 of EC2 [28], where it suggested to limit the compressive stress in the concrete in order to avoid longitudinal cracks, micro-cracks or high levels of creep.

Limitations are the following:

- $\sigma_c < 0.6\ f_{ck}$ for the characteristic combination
- $\sigma_c < 0.45\ f_{ck}$ for the quasi-permanent combination
- $\sigma_s < 0.8\ f_{yk}$ for the characteristic combination

Stresses in concrete and steel are computed using Navier formula:

$$\sigma_c = \frac{M}{I_{om,x}} y \tag{3.29}$$

$$\sigma_s = n \frac{M}{I_{om,x}} y \tag{3.30}$$

where x is the position of the neutral axis, n is the homogenization coefficient and y is the distance from the neutral axis (i.e. y is taken as the distance of the longitudinal bars for the steel stresses calculation while for the concrete stresses computation is the extremity of the cross-section because it is the most stressed part).

All the cross-sections of the beams result verified.

3.7.3.2 Crack control

These limitations are defined at § 7.3 of EC2 [28], where it is defined a maximum crack width w_{max}, to be limited to an extent that will not prevent the functionality or durability of the structure. These values are defined in table 7.1N of the normative, as function of the load combination and the exposure class.

To compute the crack width w_k, to be compared to the limit value w_{max} (equal to 0.3 mm for the specific case), the formula is:

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \tag{3.31}$$

where:

- $s_{r,max} = k_3 c + k_1 k_2 k_4 \Phi / \rho_{eff}$ is the maximum crack distance
- c is the concrete cover
- k_1 is a coefficient that depends on the reinforcement bond (0.8 for bars having strongest bond and 1.6 for smooth bars)
- k_2 is a coefficient that depends on the strain distribution (0.5 for bending and 1.0 for pure traction)
- $k_3 = 0.4$
- $k_4 = 0.425$
- ε_{sm} reinforcement mean strain
- ε_{cm} concrete mean strain

The difference between the two strains in formula (3.31) is computed as:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_{s,max} - \frac{k_t f_{ctm}}{\rho}\left(1 + \alpha_e \rho_{eff}\right)}{E_s} \tag{3.32}$$

where

- k_t is a factor that is function of load duration (0.6 for short duration and 0.4 for long duration)
- $\alpha_e = E_s/E_c$
- $\rho_{eff} = A_s/B\ h_{c,eff}$

Moreover, the difference at
(3.32) should be larger or equal to 0.6 $\sigma_{s,max}/E_s$.
All the cross-sections of the beams result verified.

3.7.3.3 Deflection control

These controls are defined at § 7.4 of EC2 [28], where it is mentioned that "the deformation of a member or structure shall not be such that it adversely affects its proper functioning or appearance". In particular:

- the deflection of a beam, slab or cantilever under quasi-permanent loads should not exceed the ratio span/250 in order to not impair the appearance and general utility of the structure
- deflections that could damage adjacent parts of the structure should be limited. Thus, for the deflection after construction, span/500 is normally an appropriate limit for quasi-permanent loads.

According to DM2018 [9], for beams having a span larger than 10 m, it is possible to omit other verifications, whether the slenderness ratio $\lambda = 1/h$ respects the following limitation:

$$\lambda \leq K\left(\frac{11 + 0.0015 f_{ck}}{\rho + \rho'}\right)\left(\frac{500 A_{s,eff}}{f_{yk} A_{s,calc}}\right) \tag{3.33}$$

where $A_{s,eff}$ and $A_{s,calc}$ are respectively the tension reinforcement effectively present on the most stressed cross-section and the corresponding reinforcement that has been calculated, while K is a correction coefficient which depends on the structural scheme. All the beams result verified.

3.7.4 Column design: bending and compression at SLU

The reference code rules are the DM2018 [9] at § 4.1.6.1.2 and 7.4.6.2.2, where it is prescribed a limitation on the longitudinal reinforcement, such that:

- longitudinal bars should have diameter larger or equal than 12 mm and can not have distance larger than 300 mm
- the reinforcement area should be at least equal to $0.10N_{Ed}/f_{yd}$ and not less than $0.003A_c$
- the ratio ρ between the area of the longitudinal reinforcement and the gross concrete cross-section should be within the limits: $1\% \leq \rho \leq 4\%$.

For the case study, all the columns have a cross-section of $600x600 = 3600mm^2$; 12 bars of $20\ mm$ diameter each have been chosen, in particular 4 are located at the sides and 8 are in the intermediate location, for a total are of $3770\ mm^2$.

Moreover, for the capacity design, plasticization of the beam should occur before the one of the columns. This condition is reached if the following occurs (§ 7.4.4.2.1 DM2018 [9]):

$$\sum M_{c,Rd} \geq \gamma_{Rd} \sum M_{b,Rd} \tag{3.34}$$

where:

- γ_{Rd} overstrength factor, taken equal to 1.3
- $M_{c,Rd}$ resistant moment of the column, computed as function of the axial stresses at the node, caused by seismic combinations
- $M_{b,Rd}$ resistant moment of the beam at the node

The reinforcement that has been adopted results verified.

3.7.5 Column design: shear at SLU

The shear action is computed as function of the resistant bending moments $M^s_{c,Rd}$ and $M^i_{c,Rd}$that act respectively at the superior and inferior ends of the column, with the following expression (taken from § 7.4.4.2.1 DM2018 [9]):

$$V_{Ed} = \gamma_{Rd} \frac{M_{c,Rd}^{s} + M_{c,Rd}^{i}}{l_p} \tag{3.35}$$

where:

- γ_{Rd} overstrength factor, taken equal to 1.3 for CD “A” and 1.1 for CD “B”
- l_p length of the pillar

The resistant shear is computed according to Ritter-Morsch, where the formulas and procedure are exactly the same of sub-chapter 3.7.2, equations (3.27) and (3.28).

Moreover, at § 7.4.6.1.2 it is prescribed that the length of the dissipative zone should be the maximum between:

- the height of the cross section
- 1/6 of the effective height of the column
- 45 cm
- the free height of the column, if it is less than 3 times the height of the cross-section

In addition, according to § 7.4.6.2.2, in the dissipative areas the following conditions should be guaranteed:

- the bars at the corners of the cross-section have to be retained by the stirrups
- at least one bar over two, among the ones located at the sides, should be retained by the stirrups
- the non-retained bars should be located at less than 20 cm from the adjacent retained one for CD “A” and 15 cm for CD “B”

The diameter of the containing stirrups should be at least of 6 mm and the step should be larger than the minimum between:

- 1/3 or 1/2 of the minimum side of the column cross-section for CD “A” and CD “B” respectively
- 175 mm for CD “B” or 125 mm CD “A”
- 6 or 8 times the diameter of the longitudinal bars for CD “A” and CD “B” respectively

In all the columns and along the entire length, 2 stirrups having 2 arms each have been adopted, for a total of 4 stirrups. The diameter is equal to 8 mm for all the stirrups and the step is of 10 cm.

With this value of reinforcement, all the columns result verified.

3.7.6 Joints design

A joint is the area of the column where the latter intersects the beam. For the capacity design, it is important that the collapse is not reached by the joint before it is reached by the adjacent beams and columns. The verifications prescribe that the maximum compression and tension at the joint do not overcome the strength of the concrete.

According to § 7.4.4.3.1 of DM2018 [9], the horizontal shear acting on a joint, for each seismic direction, can be computed as:

- For internal joints

$$V_{jbd} = \gamma_{Rd}(A_{s1} + A_{s2})f_{yd} - V_c \tag{3.36}$$

- For external joints

$$V_{jbd} = \gamma_{Rd}A_{s1}f_{yd} - V_c \tag{3.37}$$

where:

- A_{s1} is the beam reinforcement in the upper chord
- A_{s2} is the beam reinforcement in the lower chord
- V_c is the shear that acts on top of the joint, at the level of the column
- γ_{Rd} overstrength factor, taken equal to 1.2 for CD "A" and 1.1 for CD "B"

In order to verify that the concrete strength is not reached, the verification is:

$$V_{jbd} \leq \eta\, f_{cd}\, b_j\, h_{jc}\sqrt{1 - \nu_d/\eta} \tag{3.38}$$

where:

- $\eta = \alpha_j(1 - f_{ck}/250)$ and α_j equal to 0.48 for external joints and 0.60 for internal

- $\nu_d = N_{Ed}/(A_c f_{cd})$ normalized compressive stress, acting in the column above the joint
- h_{jc} distance from the more external bars of the column
- b_j effective width of the joint, equal to the minimum between: the maximum among the sides of column and beam cross-sections and the minimum between the sides of column and beam cross-section, both increased to half of the column's cross-section height

In order to avoid the diagonal cracking of the joint, thanks to the presence of transversal reinforcement, it is necessary to verify that:

- For internal joints

$$A_{sh} f_{ywd} \geq \gamma_{Rd} (A_{s1} + A_{s2}) f_{yd} (1 - 0.8\nu_d) \tag{3.39}$$

- For external joints

$$A_{sh} f_{ywd} \geq \gamma_{Rd} A_{s2} f_{yd} (1 - 0.8\nu_d) \tag{3.40}$$

In all the joints 2 stirrups having 2 arms each have been adopted, for a total of 4 stirrups. The diameter is equal to 8 mm for all the stirrups and the step is of 5 cm.

With this value of reinforcement, all the columns result verified.

3.7.7 Summary of the design solution according to capacity design

The principal characteristics of the frame, as it has been designed according to Eurocodes and DM2018, are summarized in the following list:

- Columns’ cross section: 60 x 60 cm
- Beams cross section: 40x50 cm
- Columns reinforcement:

  transversal: stirrups with 4 arms, diameter 8 mm, steps 10 cm

  longitudinal: 12 bars with diameter 20 mm
- Beams reinforcement:

  transversal (dissipative zone): stirrups with 2 arms, diameter 8 mm, steps 7.5 cm

  transversal (non-dissipative zone): stirrups with 2 arms, diameter 8 mm, steps 15 cm

  longitudinal (dissipative zone): 3 bars with diameter 18 mm in the lower chord, 5 bars with diameter 18 mm in the upper chord

  longitudinal (non-dissipative zone): 3 bars with diameter 18 mm in the lower chord, 2 bars with diameter 18 mm in the upper chord
- Joints transversal reinforcement: stirrups with 4 arms, diameter 8 mm, steps 5 cm
- Concrete cover: 35 cm

3.8 Design characteristics of the frame under analysis

As already mentioned, the frame under analysis has not the same characteristics, in terms of structural properties, of the real structure, as it has been discussed until now. This is because previous works of thesis have tried different design strategies in order to enhance the robustness of the same structure.

In the following, the different strategies are summed up and the structural characteristics of the frame under analysis are listed.

3.8.1 Summary of previous robustness analysis

First of all, different analysis has shown that the behaviour of the frame as it has been designed, according to seismic capacity design, is not enough to avoid disproportionate collapse in case of removal of a column. In particular, the discontinuity between the dissipative and non-dissipative interfaces, in terms of reinforcement and concrete confinement, causes the formation of plastic hinges where they should not appear ideally and rupture of the longitudinal reinforcement, which is the key element in order to guarantee robustness.

Three are the fundamental detailing that has improved significantly the behaviour of the frame, against column removal:

- **Continuity** of the longitudinal bars along the beams, which increases the ultimate displacement
- **Symmetry** of the longitudinal bars of the beams between upper and lower chord, which increases the flexural and membrane strength
- **Equality** of longitudinal reinforcement among floors, which increases the flexural and membrane strength

The three conditions imply the introduction of 5Φ18, both in the upper and lower chord, continuous along the beams, and equal in all the floors. Thus, an increase of 10% of longitudinal reinforcement determines an increase of the resistance against column removal of 30%. Moreover, this has solved the problem related to the formation of plastic hinges in parts of the frame where it should not occur.

After this, another trial has concerned the increase of the number of bars up to 7Φ18, again respecting the three conditions. This has resulted in an increase of the bending peak, but a reduction of the ultimate displacement, which means a more fragile behaviour.

Another attempt has regarded the centralization of the bars, by reducing the effective height and increasing the concrete cover. This has implied a decrease of resistance immediately after the bending peak but a delay of resistance-fall in correspondence of the maximum displacement.

3.8.2 Summary of the design solution, according to robustness analysis

Starting from the initial design characteristic and considering the conclusion of the study made on the same structure, in the following the design solution adopted for this work of thesis will be listed:

- Columns cross section: 60 x 60 cm
- Beams cross section: 40x50 cm
- Column reinforcement:

  transversal: stirrups with 4 arms, diameter 8 mm, steps 10 cm

  longitudinal: 12 bars with diameter 20 mm
- Beams reinforcement:

  transversal (dissipative): stirrups with 2 arms, diameter 8mm, steps 7.5cm

  transversal (non-dissipative): stirrups with 2 arms, diameter 8mm, steps 15cm

  longitudinal: 5 bars with diameter 18 mm both in the upper and lower chord
- Joints transversal reinforcement: stirrups with 4 arms, diameter 8 mm, steps 5cm
- Concrete cover: 53 cm

Moreover, the loads characteristics do not change with respect to the initial design, since the geometrical and material characteristics, as well as the use and of the structure, remains invariant. The loads values, distributed along beams, are followingly resumed:

- Permanent structural load G_1 for the beams and columns: function of material specific-weight ($\rho = 24\ kN/m^3$ for concrete and $\rho = 25\ kN/m^3$ for reinforced concrete)
- Permanent structural load G_1 for the slabs: 16 kN/m
- Permanent non-structural load G_2 13 kN/m
- Variable loads Q_p for the floors: 10 kN/m
- Variable loads Q_c for the roofing: 2.5 kN/m

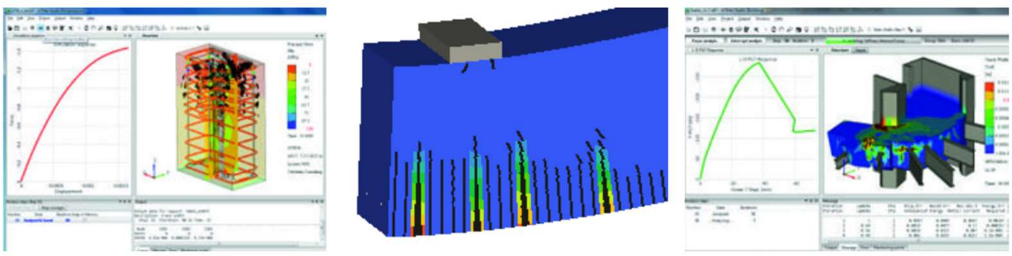
4 ATENA 2D

The ATENA software has been used in three phases: first, to model the frame with the presence of the column in order to compute the reaction at the base of the central column, then for a pushdown analysis to evaluate the dynamic effect involved in the structure in the accidental event of a sudden column loss and finally to perform a reliability analysis. In the following, the software is described in terms of generalities, input parameters (materials, topology, loads and supports), meshing and post-processing.

4.1 Software generalities

ATENA is a fully windows program for non-linear analysis of reinforced concrete structures. The name stands for Advanced Tool for Engineering Nonlinear Analysis. Many are the outcomes using this software (www.cervenka.cz):

- Modelling for concrete, reinforcement (both with discrete or smeared bars), steel, rock, soil and masonry (Figure 4.1)
- Crack propagation visualization (Figure 4.1)
- Real-time visualisation of results (Figure 4.1)
- Analysis of modern fiber reinforced concrete materials: SHCC, ECC, HPRFC, UHPFRC
- Dynamics, statics, creep, thermal and moisture analyses
- Modelling of high temperature and fire loading on concrete structures
- Modelling of structural durability and reinforcement corrosion.



Modeling Of Reinforcement | Realtime Visualisation Of Results | Crack Visualisation

Figure 4.1: Main tools of the software (www.cervenka.cz)

ATENA also allows to perform a three-dimensional analysis, that will not be used for the scope of this work of thesis, because a two-dimensional frame is considered.

A positive aspect of ATENA 2D, is the possibility of managing the software not via graphical interface but by writing the CCT file that can be imported in the software. This file, that can be opened through a text viewer, contains alphanumeric commands which describe all the input to be given to the software in order to run the analysis. This way of proceeding is very useful for this work of thesis because dealing with a sampling of 100 different mechanical characteristic combinations, it is not so easy when you have to insert by graphical interface all the different 100 combinations.

In the following, both the procedures will be explained: via graphical interface and via CCT file.

4.2 Pre-processing

The item is active only in the pre-processing mode that can be made from the menu item *Calculations*. The pre-processing phase is necessary for the definition of all input data, that can be accessed by the graphical interface and are structured as follows:

1. General data
2. Materials
3. Topology
4. Loads and Supports
5. Run

4.2.1 Graphical interface

As already mentioned, with ATENA 2D the designer can work in a user-friendly graphical environment (Figure 4.2). The icons placed horizontally on the higher part of the interface, allow you to save, create the mesh, run the analysis, zoom in or out, select points, lines or macro elements and so on. All the commands are available on a sliding bar on the left-hand side, that allows you to define in an ordered way all the input to be given to the program. The icon shaped like an eye, leads to switch on and off the elements of the design, such as the points, the nodes, the lines, their labels etc.

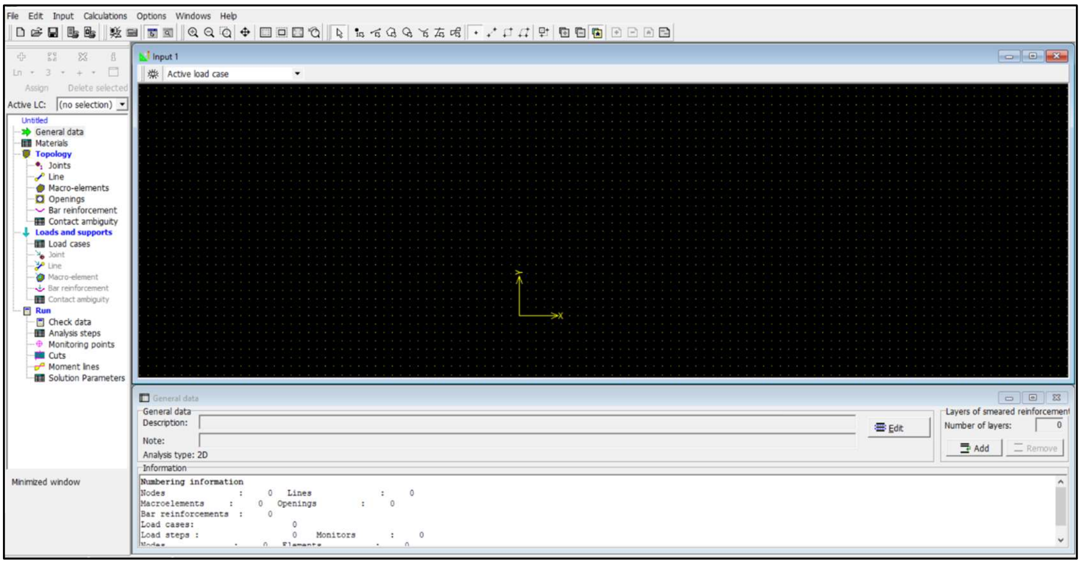


Figure 4.2: Graphical interface

Finally, the general data command allows to give the name to the project, insert comments, indicate the decimal points to be used and also lets to have information about the numbers of nodes, lines, bar reinforcements, load cases etc., under the window called numbering information (on the lower part of the interface).

4.2.2 Materials

Being a powerful software, ATENA 2D allows you to model a huge variety of materials: *Plane Stress/Strain elastic isotropic, 3D Non Linear Cementitious 2, 3D Variable Non Linear Cementitious, SBeta Material, Microplane4 Material, 3D BiLinear Steel Von Mises, 2D Interface, Reinforcement, Cycling Reinforcement, Smeared Reinforcement, Spring, Bond for Reinforcement, 3D Ducker-Prager Plasticity, Material With Random Fields*. Each of them is suitable for specific types of material behaviours, such as rock like materials, fibre reinforced concrete, plastic materials like steel, truss elements modelling a spring and so on.

For the purpose of this work, three types of materials have been used and followingly explained: *SBeta Material* for the concrete, *Reinforcement* for the steel rebars and *Plane stress elastic isotropic* for modelling the infinitely rigid plates.

4.2.2.1 SBeta Material

The name SBeta is the abbreviation to indicate the German definition for StahlBETonAnalyse which means “analysis of reinforced concrete”. This material model includes the following effects:

- Non-linear behaviour in compression including hardening and softening
- Fracture of concrete in tension
- Biaxial strength failure criterion
- Reduction of compressive strength and shear stiffness after cracking
- Tension stiffening effect
- Two crack models: fixed crack direction and rotated crack direction

Basic

In the Basic tab it is necessary to insert information about Elastic modulus, Poisson’s ratio, Tensile strength and Compressive strength. They are needed to describe the uniaxial stress-strain law (with non-linear behaviour in compression) and the biaxial failure law.

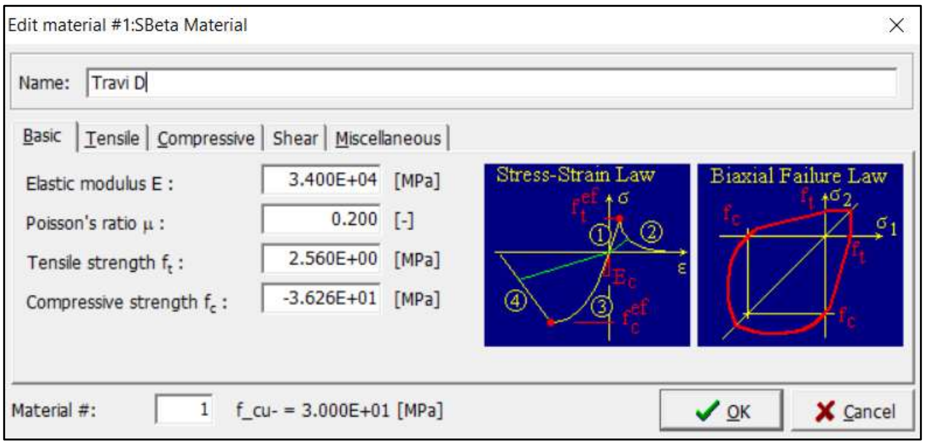


Figure 4.3: Basic tab for *SBeta Material*

Tensile

In this tab, it is necessary to select the type of tension softening and crack model.

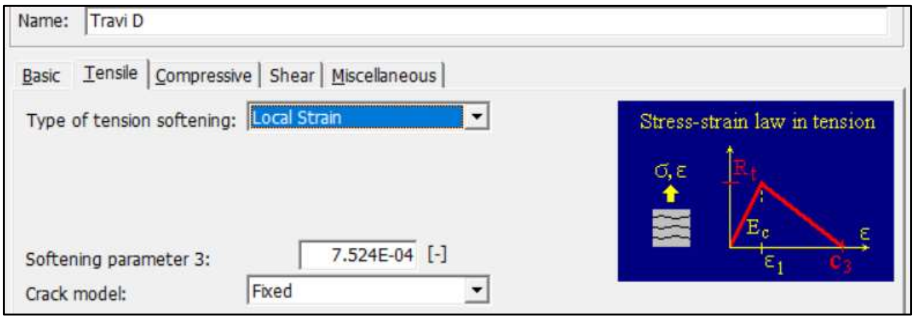


Figure 4.4: Tensile tab (type of tension softening) for *SBeta Material*

There are five different options to describe the softening model: *Exponential crack opening law*, *Linear crack opening law*, *Linear softening based on local strain* (that is the one selected for this case), *SFRC Based on Fracture Energy* and *SFRC Based on Strain*.

For the *Local Strain model*, the descending branch of the stress-strain diagram is defined by the end strain C_3, corresponding to the complete release of stress. This parameter, called in the figure *Softening Parameter 3*, computed as ten times the strain corresponding to the axial tensile strength of concrete.

Moreover, it is necessary to select the Crack Model among two options: *fixed* and *rotated.* In the first one, the crack direction is given by the principal stress direction at the moment of the crack initiation and remains fixed during further loading, while in the second if the principal strain axes rotate during the loading, the direction of the cracks rotate too.

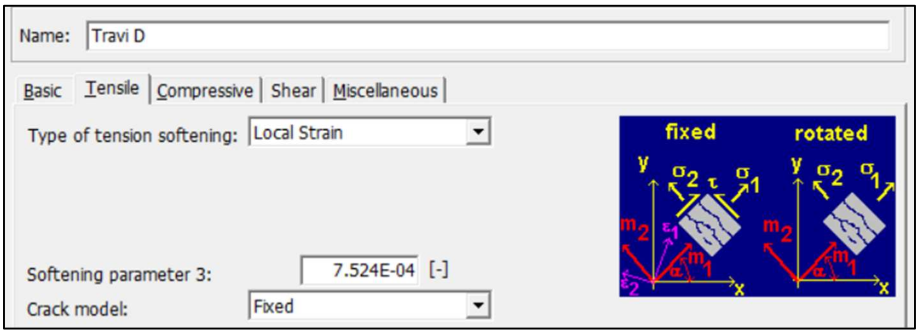


Figure 4.5: Tensile tab (crack model) for *SBeta Material*

Compressive

In this tab, it is necessary to insert the information related to the compressive behaviour of concrete and in particular the compressive strain computed using the

Saatcioglu & Razvi model [32], the RCS parameter (Reduction of compressive strength due to cracks), equal to 0.8 as the ATENA Manual suggests [33]. This parameter represents the maximum strength reduction under the large transverse strain. Moreover, it is necessary to select the type of compression softening among the *Softening modulus* and the *Crush band.* In the first it is defined a slope of the softening law by means of the softening modulus E_d, computed as the product between the compression softening parameter c_d and the concrete elastic modulus E_c.

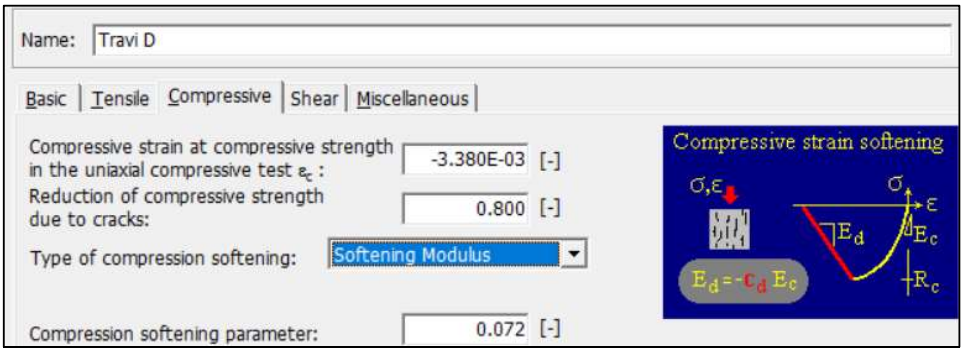


Figure 4.6: Compressive tab for *SBeta Material*

Shear

In this tab, the assumptions regard the *Shear retention factor* and the *Tension-compression interaction*. For the former, it is possible to select between *variable* and *fixed;* variable means that it considers a reduction of shear stiffness after cracking, otherwise it is fixed. For the latter, it is possible to select among *Linear*, *Hyperbola A* and *Hyperbola B*.

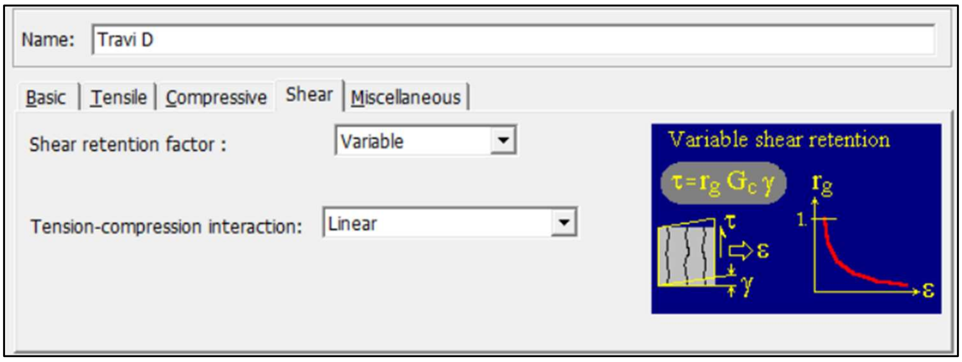


Figure 4.7: Shear tab (retention factor) for *SBeta Material*

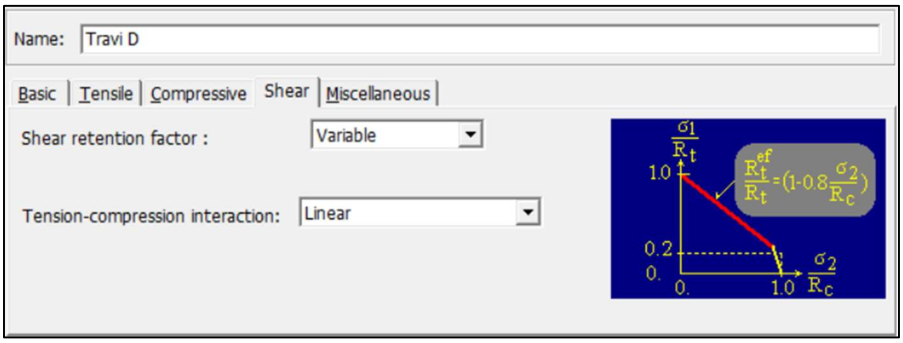


Figure 4.8: Shear tab (tension-compression interaction) for *SBeta Material*

Miscellaneous

In this last tab, the generalities about the material are defined in terms of specific weight ρ in MN/m^3 and coefficient of thermal expansion α in $1/K$.

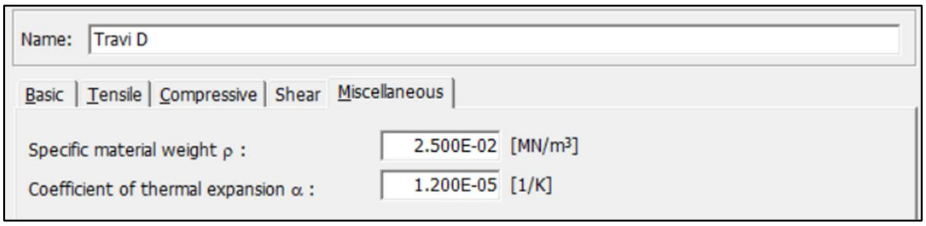


Figure 4.9: Miscellaneous tab for *SBeta Material*

4.2.2.2 Reinforcement

This type is assigned to the longitudinal and transversal lines of the model that represent the rebars and the stirrups. Also in this case, many assumption should be considered to define this material model.

Basic

All reinforcement material models in ATENA exhibit the same behaviour in tension as well as in compression. In the basic, it is necessary to define the type of stress-strain law, among different models: *Linear* (in which only the elastic modulus should be inserted), *Bilinear* (where also the yielding strength should be added to the elastic modulus), *Multilinear* (with an arbitrary graphical representation) and *Bilinear with Hardening* (where also the limit stress and strength should be considered in addition to the elastic modulus and the yielding strength). The *Bilinear with Hardening* model is the one selected for the case study.

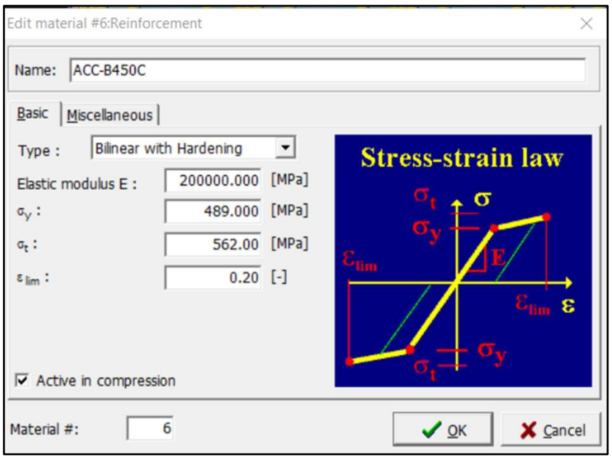


Figure 4.10: Basic tab for *Reinforcement*

Miscellaneous

In this last tab, the generalities about the reinforcement are defined in terms of steel weight Rho in MN/m^3 and coefficient of thermal expansion $ALPHA$ in $1/K$.

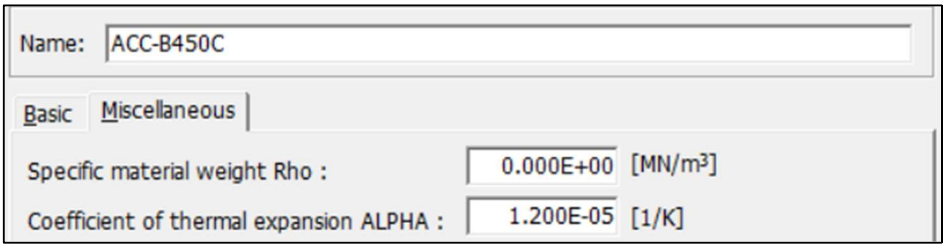


Figure 4.11: Miscellaneous tab for *Reinforcement*

4.2.2.3 Plane stress elastic isotropic

This last type of material has been used to model the plates elements located in the nodes beam-column (only on the external parts) and at the base of the columns. The need of this elements is due to the necessity of monitoring the displacements during the loading phases and to apply the imposed displacements for the pushdown analysis.

Basic

In this tab no assumption should be made about the constitutive law, being already defined by the material type (i.e. *Plane stress elastic isotropic*). Only the value corresponding to the elastic modulus and the Poisson's ratio should be inserted. It is important to underline the very high value corresponding to the elastic modulus is not realistic but necessary to avoid incoherent deformations during the pushdown analysis.

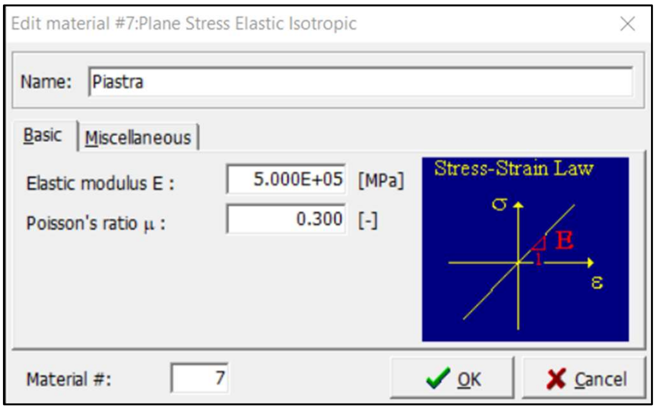


Figure 4.12: Basic tab for *Plane Stress Elastic Isotropic*

As in the previous cases, this tab contains the information of the specific weight and the coefficient of thermal expansion.

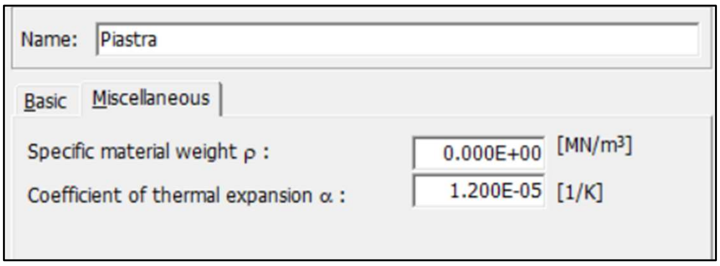


Figure 4.13: Miscellaneous tab for *Plane Stress Elastic Isotropic*

4.2.2.4 CCT material description

As already explained, another possibility is to write a CCT file, that can be opened with a text viewer, which contains all the necessary information to define the material characteristics and behaviour. In the following the CCT for the 7 materials that have been inserted in the model are presented. More details about their description is given in the next chapter, i.e. Chapter 5.

```
BEG_MAT 1
  VER_MAT 0
  NAME "Travi D"
  TYPE "CCSBETAMaterial"
  E  3.400E+04
  Mu       0.200
  Ft  2.560E+00
  Fc -3.626E+01
  ISOFT 3.0
  C3  7.524E-04
  Eps_C -3.380E-03
  CompRed       0.800
  CSOFT 2.0
  Cd       0.072
  Shear Variable
  CS 0.6
  Rho  2.500E-02
  Alpha  1.200E-05
END_MAT 1

BEG_MAT 2
  VER_MAT 0
  NAME "Travi ND"
  TYPE "CCSBETAMaterial"
  E  3.364E+04
  Mu       0.200
  Ft  2.560E+00
  Fc -3.501E+01
  ISOFT 3.0
  C3  7.604E-04
  Eps_C -2.990E-03
  CompRed       0.800
  CSOFT 2.0
  Cd       0.089
  Shear Variable
  CS 0.6
  Rho  2.500E-02
  Alpha  1.200E-05
END_MAT 2

BEG_MAT 3
  VER_MAT 0
  NAME "Pilastro"
  TYPE "CCSBETAMaterial"
  E  3.527E+04
  Mu       0.200
  Ft  2.560E+00
  Fc -4.099E+01
  ISOFT 3.0
  C3  7.253E-04
  Eps_C -4.860E-03
  CompRed       0.800
  CSOFT 2.0
  Cd       0.050
  Shear Variable
  CS 0.6
  Rho  2.500E-02
  Alpha  1.200E-05
END_MAT 3

BEG_MAT 4
  VER_MAT 0
  NAME "Nodi"
  TYPE "CCSBETAMaterial"
  E  3.700E+04
  Mu       0.200
  Ft  2.560E+00
  Fc -4.809E+01
  ISOFT 3.0
  C3  6.913E-04
  Eps_C -7.090E-03
  CompRed       0.800
  CSOFT 2.0
  Cd       0.018
  Shear Variable
  CS 0.6
  Rho  2.500E-02
  Alpha  1.200E-05
END_MAT 4
```

Figure 4.14: CCT command description for Materials 1,2,3 and 4

```
BEG_MAT 5
  VER_MAT 0
  NAME "CLS-NC"
  TYPE "CCSBETAMaterial"
  E  3.271E+04
  Mu       0.200
  Ft  2.560E+00
  Fc -3.187E+01
  ISOFT 3.0
  C3  7.822E-04
  Eps_C -2.000E-03
  CompRed       0.800
  CSOFT 2.0
  Cd       0.097
  Shear Variable
  CS 0.6
  Rho  2.400E-02
  Alpha  1.200E-05
END_MAT 5

BEG_MAT 6
  VER_MAT 0
  Func_Type 4
  xvalues 0.000E+00 2.445E-03 2.000E-01 2.020E-01
  yvalues 0.000E+00 4.890E+02 5.620E+02 0.000E+00
  NAME "ACC-B450C"
  TYPE "CCReinforcement"
  RHO  0.000E+00
  ALPHA  1.200E-05
  COMPRESSION 1
END_MAT 6

BEG_MAT 7
  VER_MAT 0
  NAME "Piastra"
  TYPE "CCPlaneStressElastIsotropic"
  E  5.000E+05
  Mu       0.300
  Rho  0.000E+00
  Alpha  1.200E-05
END_MAT 7
```

Figure 4.15: CCT command description for Materials 5,6 and 7

4.2.3 Topology

As it is possible to notice in the left hand side tab of the graphical interface (Figure 4.2), after materials it is necessary to define the topology, intended as the geometrical characteristics of the fem model. In this section it is necessary to define *Joints*, *Line*, *Macro-elements*, *Openings* (not for the case study), *Bar Reinforcement*, *Contact Ambiguity* (not for the case study).

4.2.3.1 Joints

The points that define the fem model can be inserted by clicking on the joint tab of the graphical interface. There, it is required to select the x and y coordinate and, if necessary, the refinement type (Figure 4.16).

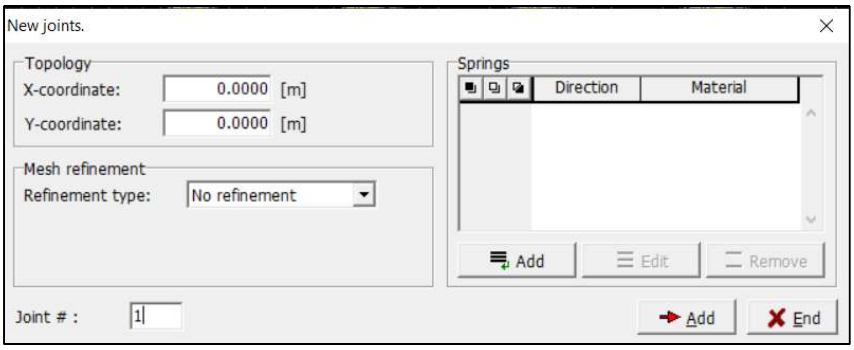


Figure 4.16: Joint definition through *Joint* tab

A much more easy way to insert all the joints, instead of clicking every time on the joints tab, is to work on the CCT file, as shown in Figure 4.17.

```
BEG_JOINTS
COUNT 20
    1      0.0000      0.0000
    2      0.0530      0.0000
    3      0.3000      0.0000
    4      0.5470      0.0000
    5      0.6000      0.0000
    6      0.0000      0.6000
    7      0.0530      0.6000
    8      0.3000      0.6000
    9      0.5470      0.6000
   10      0.6000      0.6000
   11      0.0000      2.1000
   12      0.0530      2.1000
   13      0.3000      2.1000
   14      0.5470      2.1000
   15      0.6000      2.1000
   16      0.0000      2.7500
   17      0.0530      2.7500
   18      0.3000      2.7500
   19      0.5470      2.7500
   20      0.6000      2.7500

END_JOINTS
```

Figure 4.17: Joint definition through CCT command

4.2.3.2 Line

After having defined the joints, the next procedure is to define the lines connecting them. Again, it is possible to use the *Line* tab, on the left-hand side of the graphical interface, or by means of a CCT file. The latter is the most suggested, since it is possible to create an Excel file that can write “automatically” the CCT commands.

In Figure 4.18, it shown how to use the Line tab; it is first necessary to select the line type among *Line*, *Arc* or *Circle.* For our case, only the *Line* type has been used, and in this case, it is requested only to define the marker of the joint which represents the origin and the one for the end.

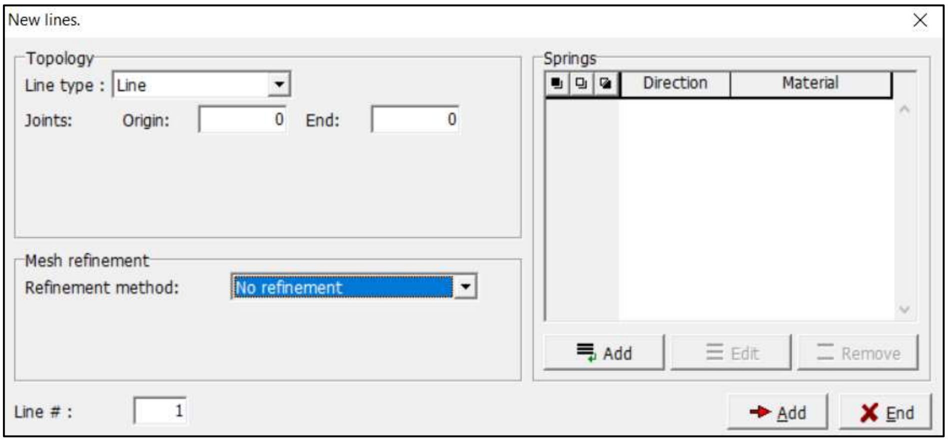


Figure 4.18: Line definition through *Line* tab

The Figure 4.19 represents the other possibility, that is to work with the CCT file.

```
BEG_LINES
COUNT 10
    1 LI     1     2
    2 LI     2     3
    3 LI     3     4
    4 LI     4     5
    5 LI     1     6
    6 LI     2     7
    7 LI     3     8
    8 LI     4     9
    9 LI     5    10
   10 LI     6     7
END_LINES
```

Figure 4.19: Line definition through CCT command

4.2.3.3 Macro-elements

These elements are defined by identifying the lines that surround their perimeter, so they follow the *Lines* definition. By typing on the Macro-elements tab (Figure 4.20), it is necessary to define the *Boundary list*, which contains the markers of the four lines that surround the element; the *Mesh type*, among *Triangles*, *Quadrilaterals* and *Mixed*. Then, it is necessary to insert the *Element size* and the *Material* among the previous defined. Moreover, it should be defined the *Thickness* and the Quadrilateral elements type, among *CCIsoQuad*, *CCQ10* and *CCQ10SBeta*.

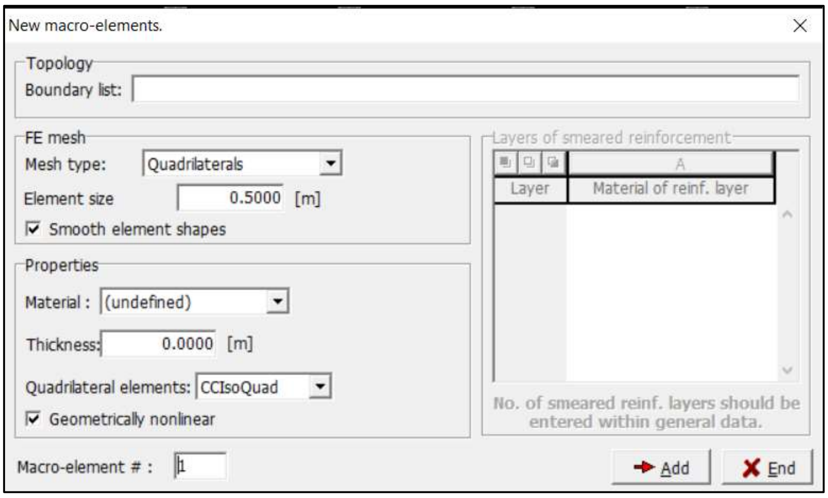


Figure 4.20: Macro-elements definition through *Macro-elements* tab

Again, an example of a CCT format is presented (Figure 4.21). On the left and side, the macro-element definition is presented: the first number is the marker of the macro-element, the second is the marker of the material, the third is the thickness, the other four numbers are the *Boundary List* lines and finally NON LINEAR stands for the geometrically non-linearity. On the right-hand side, the mesh characteristics are defined: the first number is the mark of the macro-element, while 0,100 stands for the mesh size.

```
BEG_MACROELEMENTS
COUNT 10
    1     5      0.6000 1,5-6,10 NONLINEAR
    2     3      0.6000 2,6-7,11 NONLINEAR
    3     3      0.6000 3,7-8,12 NONLINEAR
    4     5      0.6000 4,8-9,13 NONLINEAR
    5     5      0.6000 10,14-15,19 NONLINEAR
    6     3      0.6000 11,15-16,20 NONLINEAR
    7     3      0.6000 12,16-17,21 NONLINEAR
    8     5      0.6000 13,17-18,22 NONLINEAR
    9     5      0.6000 19,23-24,28 NONLINEAR
   10     3      0.6000 20,24-25,29 NONLINEAR
END_MACROELEMENTS
```

```
BEG_MACROELEMENTS
COUNT 10
    1 QUAD      0.1000 REFINE ISOQUAD
    2 QUAD      0.1000 REFINE ISOQUAD
    3 QUAD      0.1000 REFINE ISOQUAD
    4 QUAD      0.1000 REFINE ISOQUAD
    5 QUAD      0.1000 REFINE ISOQUAD
    6 QUAD      0.1000 REFINE ISOQUAD
    7 QUAD      0.1000 REFINE ISOQUAD
    8 QUAD      0.1000 REFINE ISOQUAD
    9 QUAD      0.1000 REFINE ISOQUAD
   10 QUAD      0.1000 REFINE ISOQUAD
END_MACROELEMENTS
```

Figure 4.21: Macro-elements definition (left) and mesh characteristics (right) through CCT command

4.2.3.4 Bar Reinforcement

The last element necessary to complete the topology of the model is the *Bar Reinforcement* element. This tab contains two sections: *Topology* and *Properties*. In the former (Figure 4.22), it necessary to select the *Segment type* among *Polyline of straight segment* and *Circle*. Moreover, by clicking on Add, an *Add segment* window appears where it should be specified the *Origin* and the *End* coordinates of the segment. In the latter (Figure 4.23), the previously defined material should be specified (it should be a *Reinforcement* type material) and the area of the rebar. A *perfect connection* bond between concrete and steel is specified.

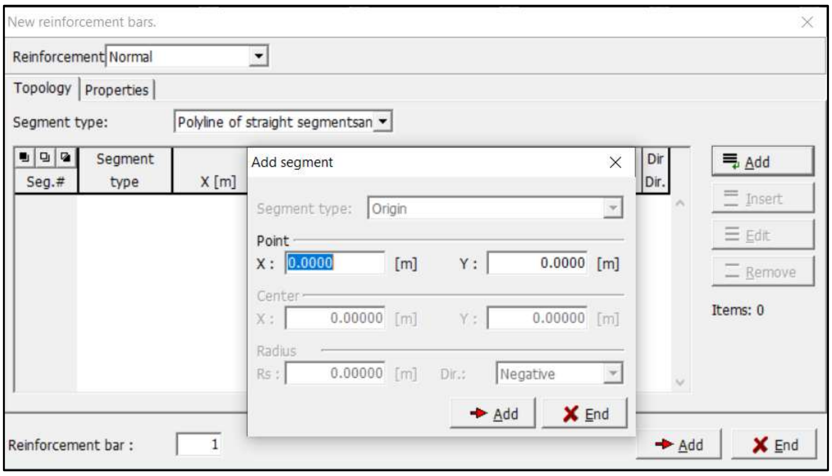


Figure 4.22: Reinforcement definition through *Reinforcement* tab (topology section)

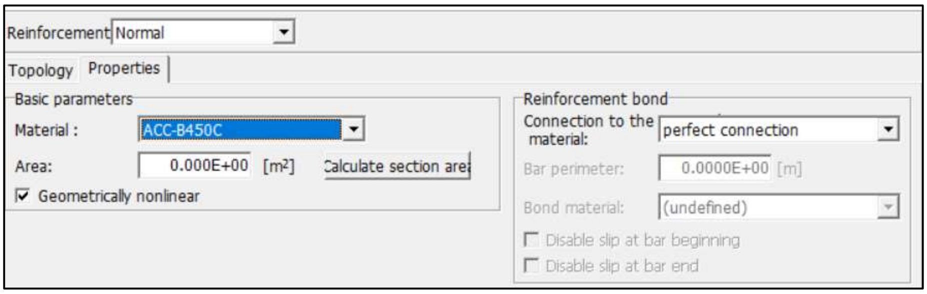


Figure 4.23: Reinforcement definition through *Reinforcement* tab (property section)

Regarding the CCT command, an example is shown (Figure 4.24). The number 6 stands for the mark of the reinforcement material previously defined, the next number is the area of the rebar, while BEG are the coordinate of the origin and LIN are the coordinate of the end point.

```
BEG_REINFORCEMENTS
COUNT  2
   1      6  1.272E-03 NORM      2 NONLINEAR
         BEG      0.0500       2.8020
         LIN     20.5500       2.8020
   2      6  1.272E-03 NORM      2 NONLINEAR
         BEG      0.0500       3.1980
         LIN     20.5500       3.1980
END_REINFORCEMENTS
```

Figure 4.24: Reinforcement definition through CCT command

4.2.4 Loads and supports

In this section it is possible to define a new load case, and to assign it to all the elements that regard the topology: Joints, Line, Macro-element, Bar reinforcement and Contact ambiguity.

In the Atena 2D software, it is possible to add seven different types of load cases: body force, forces, supports, prescribed deformation, temperature, shrinkage and pre-stressing.

In the following, only the general procedure to define the load cases is described, according to the only load cases that are used for the case study.

4.2.4.1 Supports load case

To fix the base of the columns, it is necessary to introduce a new load case called *Supports* (Figure 4.25). The required information are the LC name (load case name), the LC code (support in this case) and the dead load direction (-Y for the specific case).

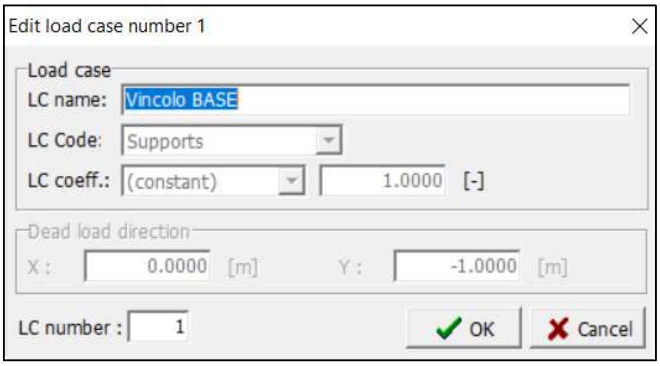


Figure 4.25: Definition of fixed support load case through load case tab

After that, it is necessary to assign it to the lines that represent the base of the columns, so it is necessary to click on the *set active* button of the load case and then click on *Line,* that is a tab present in the *Loads and supports* sub-section of the main interface of the software. Then, it is necessary to select all the lines where the supports load case should be applied and a window appears so to apply the load case to the lines (Figure 4.26). In this window, it is necessary to define the directions to be fixed (both X and Y for the specific case of a fixed support).

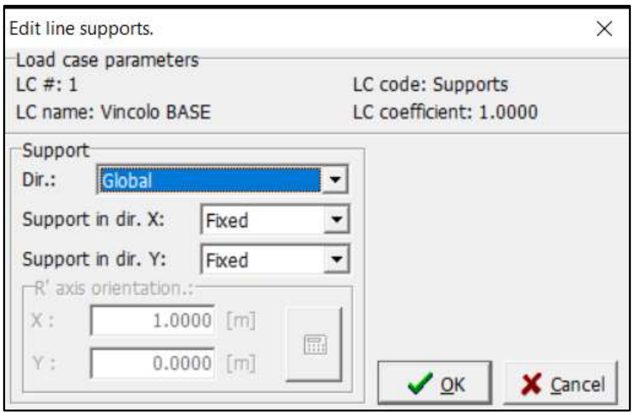


Figure 4.26: Application of *Supports load case* to the lines of the columns' basis

The same procedure can be applied by writing on the CCT file (Figure 4.27). It is necessary to define in particular the mark of the line (1736 for the specific case), and the direction of the fixities (both X and Y).

```
BEG_LOADCASE
  NO 1
  NAME "Vincolo BASE"
  CODE SU
  COEFTYPE CONST
  COEFFICIENT      1.0000
  BEG_JOINTLOAD
  COUNT 0
  END_JOINTLOAD
  BEG_LINELOAD
  COUNT 1
     1736    -1 GLOB FIXD FIXD
  END_LINELOAD
END_LOADCASE
```

Figure 4.27: Definition of *Supports load case* through CCT command

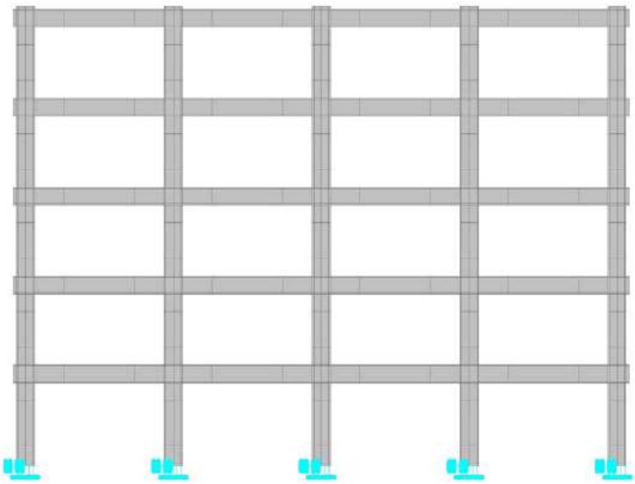


Figure 4.28: Representation of the 2D frame with fixed supports

4.2.4.2 Body force load case

This load case is essential to define the self-weight of the entire structure. The only information to be given are again the LC name, the LC Code and the Dead Load direction Figure 4.29. Differently to the previous case, it is not necessary to apply this load case to elements of the structure such joints, lines or macro-elements, because its application to the whole structure is already recognized by the software.

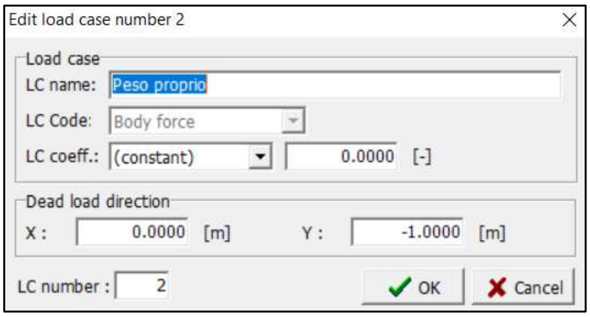


Figure 4.29: Definition of *Body force load case* through load case tab

```
BEG_LOADCASE
  NO 2
  NAME "Peso proprio"
  CODE SW
  COEFTYPE CONST
  COEFFICIENT      0.0000
  DIRECTION      0.0000     -1.0000
END_LOADCASE
```

Figure 4.30: Definition of *Body force load case* through CCT commands

4.2.4.3 Forces load case

This load case is the one used to define distributed loads such as: permanent structural and non-structural loads, live loads and reaction of the removed column before its removal. The procedure to define the load case is similar to the ones described previously.

To assign the load case to the lines, it is again necessary to press the *set active* button of the load case, then select the lines where to apply it, and complete the information of the *Edit line loading* window (Figure 4.32). It is necessary to select the type of line forces, among *Continuous full length*,*Point load*, *Partial and Quadrilateral*, *Quadrilateral*. Then, select the direction of the load, (global Y along line) and the value of the load with the correct sign

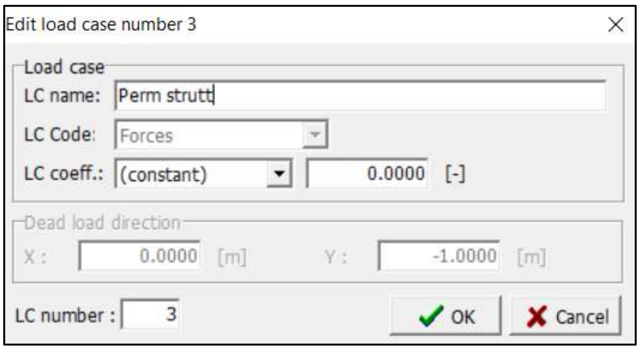


Figure 4.31: Definition of *Forces load case* through load case tab

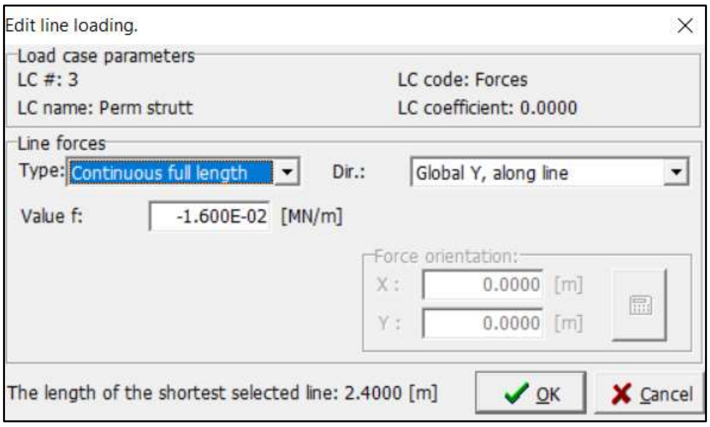


Figure 4.32: Application of *Forces load case* to the lines of the beams

The other approach is to write the load on the CCT file (Figure 4.33).

```
BEG_LOADCASE
  NO 3
  NAME "Perm strutt"
  CODE FO
  COEFTYPE CONST
  COEFFICIENT      0.0000
  BEG_JOINTLOAD
  COUNT 0
  END_JOINTLOAD
  BEG_LINELOAD
  COUNT 120
      278    -1 COFL GLYA -1.600E-02
  END_LINELOAD
END_LOADCASE
```

Figure 4.33: Definition of *Forces load case* through CCT commands

It is again possible to show the selected load case, as in the following figure (Figure 4.34):

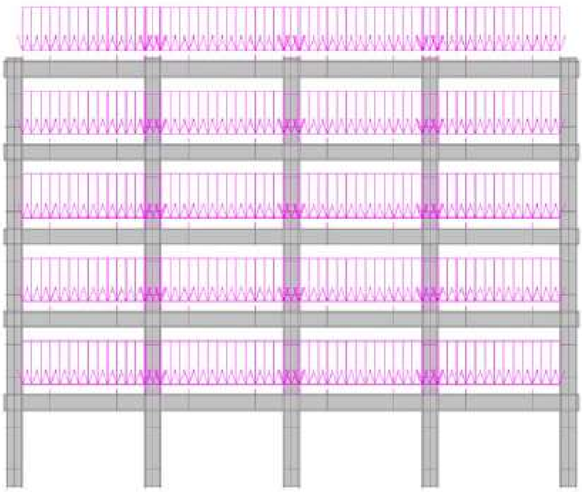


Figure 4.34: Representation of the 2D frame with distributed line loads on the beams

4.2.4.4 Prescribed deformation load case

The last load case that has been used for the purposes of this work of thesis is the prescribed deformation, in order to perform the pushover analysis. Again, it is necessary to edit a new load case, by specifying the LC name, the LC Code and the Dead load direction.

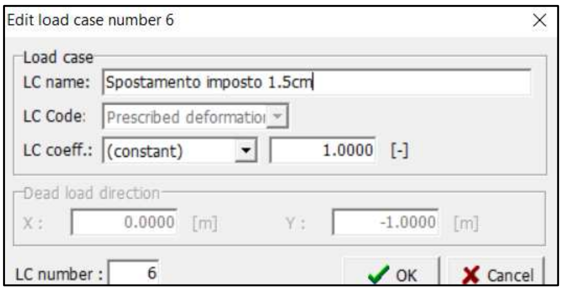


Figure 4.35: Definition of *Prescribed deformation load case* through load case tab

Then, to assign it, it is necessary to press the *set active* button of the load case and select the joint where to apply this prescribed deformation. The entity of the prescribed deformation is defined in the *Edit prescribed displacements* window (Figure 4.36).

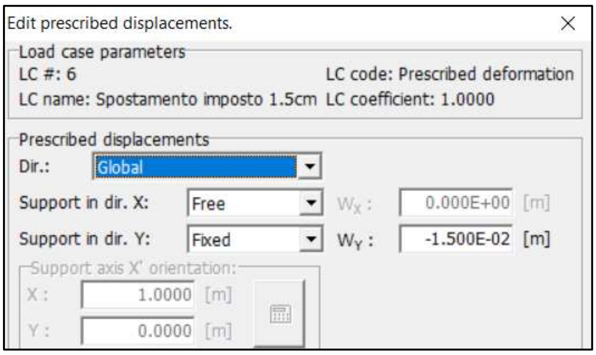


Figure 4.36: Application of *Prescribed deformation load case* to the joint on top of the central column

Finally, the command to be used in the CCT file, are presented in the Figure 4.37:

```
BEG_LOADCASE
  NO 6
  NAME "Spostamento imposto 1.5cm"
  CODE DF
  COEFTYPE CONST
  COEFFICIENT      1.0000
  BEG_JOINTLOAD
  COUNT 1
      963    -1 GLOB FREE FIXD -1.500E-02
  END_JOINTLOAD
  BEG_LINELOAD
  COUNT 0
  END_LINELOAD
END_LOADCASE
```

Figure 4.37: Definition of *Prescribed deformation load case* through CCT command

4.2.5 Run

In the *Run* sub-section, it is possible to select different options such as *Check data*, *Analysis steps*, *Monitoring points*, *Cuts*, *Moment lines*, *Solution parameters*. This information are the final inputs to be given to the software in the pre-processing phase and regard the analysis itself. The most relevant are the *Check data, Analysis steps*, *Monitoring points* and *Solution parameters*.

4.2.5.1 Check data

This tab is an instrument that allows to perform a rapid check of the model, in order to evaluate if errors are present in the model, as defined in the other sub-sections (materials, topology, load and supports and load steps). By clicking on the *Check data* tab, if everything is correctly defined, the following writing appears: *Data O.K.*.

4.2.5.2 Analysis steps

This tab contains the core of the analysis, in terms of types of loading to be applied at each step.

Figure 4.38: Definition of Analysis steps through *Analysis steps tab*

The first information is the load cases to be applied at that step; to make an example if LC1 is the supports load case and LC2 is the self-weight, it is necessary to insert in the load cases space the numbers 1,2. This load step can be applied with a unitary multiplier, and in this case the whole LC will be applied in that load step, or with a different multiplier (e.g. 0,1) and in that case only the 10% of the LC will be applied. The *Solution Parameters* section, allows the user to define the way of solving the model among two standard types that are: *Standard Newton-Raphson* and *Standard Arc-Length*.

The Figure 4.39 is an example of how a CCT can be written to define the load steps. In that example, LC 1 and LC2 are applied in 5 load steps with a multiplier of 0.2, such that at the fifth load step the entire entity of both load cases will be applied. The first number is the load step marker, the second is the multiplier, the third is the type of soulution parameter and the last two numbers are used to identify the markers of the load cases.

```
BEG_CALCSTEPS
COUNT 5
    1 0.2000 1 F T 1,2
    2 0.2000 1 F T 1,2
    3 0.2000 1 F T 1,2
    4 0.2000 1 F T 1,2
    5 0.2000 1 F T 1,2
END_CALCSTEPS
```

Figure 4.39: Definition of Analysis steps through CCT commands

4.2.5.3 Monitoring points

The monitoring points, as the words clarify, are points of the model used to save solutions parameters at each step, such as displacements or reaction forces. to insert a monitoring point, it is necessary to click on the Monitoring points tab, under the run sub-section and click add, such that the New monitors window appears. There, it is necessary to give a name to the monitoring point, assign a location in terms of X and Y coordinates, specify the location (if *Nodes* or *Integration points*) and define the value to monitor (*Displacements, External_Forces* and *Reactions* if the location is nodes, *Total_Elem_Init_Strain* and *Total_Elem_Init_Stress* if the location is integration points). Then, the item identifies the component of the value; to make an example, for displacement the notation is: 1-x, 2-y, 3-rotz.

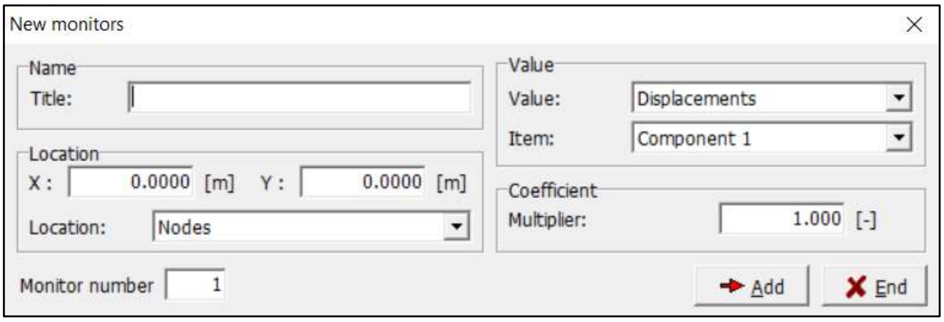


Figure 4.40: Definition of monitoring points through *Monitoring point tab*

The same, can be done writing the commands on a CCT file (Figure 4.41).

```
BEG_MONITORS
COUNT 2
    1 "Reazione" 1.000 JO 10.3000 15.4000 "Reactions" "Component 2"
    2 "Spostamento" 1.000 JO 10.3000 15.3000 "Displacements" "Component 2"
END_MONITORS
```

Figure 4.41: Definition of monitoring points through CCT commands

4.2.5.4 Solution parameters

This last important option represents basically the techniques and parameters to the iterative nonlinear solution of the equilibrium equations at each of load step.

Figure 4.42 shows the window that appears when adding a new solution parameter. Apart from giving the title, two basic solution methods can be chosen: *Newton-Raphson* and *Arc Length*. The latter should be used for force loading up to near peak load or in post peak, while the other is recommended in all the other cases. Line search icon can be selected in combination with both of them to accelerate a convergence rate. The stiffness update can be of two type: *Each iteration*, meaning that the structural stiffness matrix is calculated and assembled each iteration and this should be accomplished with the *Stiffness type Tangent*, because it may change each iteration, or *Each step*, meaning that it is updated only at the first iteration of each step, so the *Elastic Stiffness type* is preferred. The last four rows of the General tab are the error tolerances, thus the limits for various criteria. If these limits are respected the iteration stops and the calculation passes to the following step.

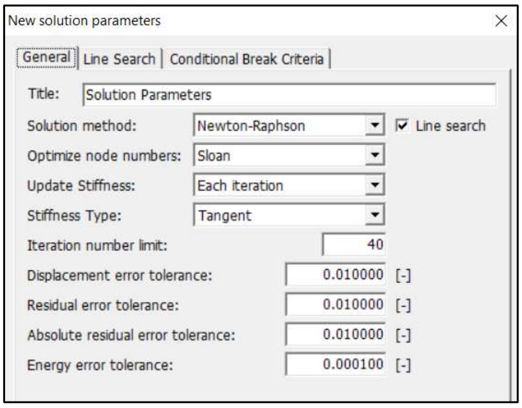


Figure 4.42: New *solution parameters* (General section)

The conditional break criteria can be set for both the methods to stop the computation if an error exceed the prescribed tolerance, as set in Figure 4.42, multiplied by the prescribed factor presented in Figure 4.43.

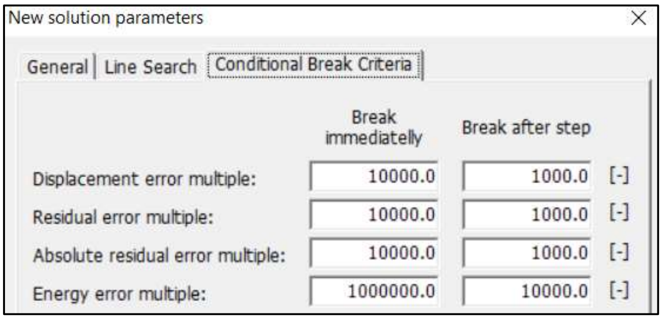


Figure 4.43: New *solution parameters* (Conditional Break Criteria section)

4.3 Post processing

After having given to the software all the inputs necessary to perform the fem analysis, the following passages are mesh generation and finite elements analysis.

These have to be done, obviously in that order, by clicking on the respectively icons of the Menu Calculations. When the analysis starts, the post-processing is active and only tools for graphical post processing are active. Moreover, ATENA 2D allows to visualize, in graphical form, the solutions not only at the end of the analysis but in real time at the end of each load step.

The Figure 4.44, taken from User’s Manual for ATENA 2D [34], represents all the available tools in the post-processing phase. It is possible to select a specific load step and visualize graphically all the possible results in terms of *Springs, Forces MNQ, Cracks, Bar reinf., Interfaces, Scalars, Vectors* and *Tensors*. In the following, attention will be paid on *Cracks, Bar reinf* and *Scalars*.

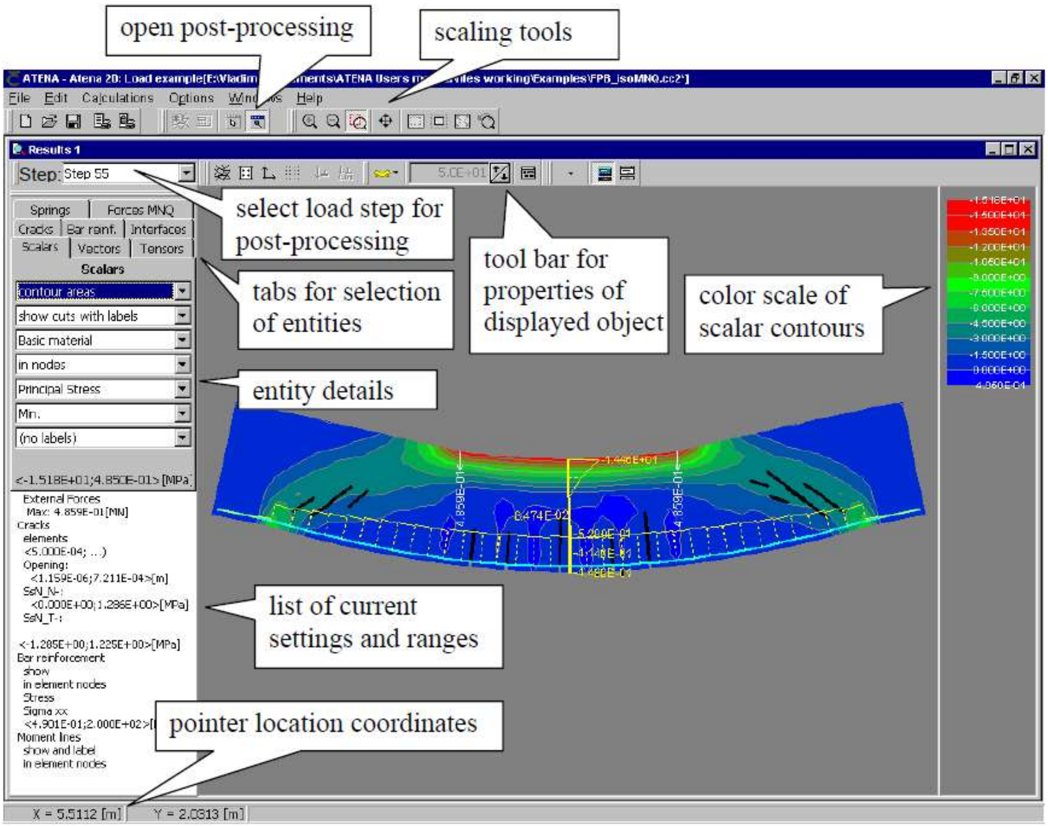


Figure 4.44: View of the post processing mode (User’s Manual for ATENA2D [34])

4.3.1 Cracks

Cracks can be displayed by choosing the tab Cracks shown in Figure 4.44 (on the left-hand side). Details of the display should be selected from the pull-down menus (Figure 4.45).

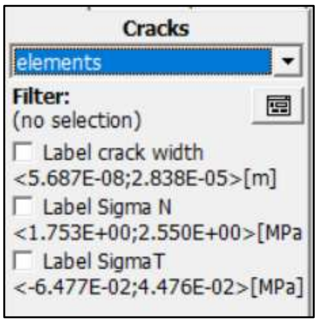


Figure 4.45: Example of cracks pull-down menu

The display type can be *no graphics* (cracks are not shown), *elements* (one crack per element is shown) and *integration points* (one crack per integration point is shown). Crack entities can be of three types: *Label crack width* (which shows the numerical value of the crack width), *Label sigma N* (normal stress on the crack face is shown) and *Label sigma T* (Numerical value of the shear stress on the crack face is shown).

4.3.2 Bar reinforcement

This tab allows to display stress and strains in bars reinforcement (Figure 4.46)

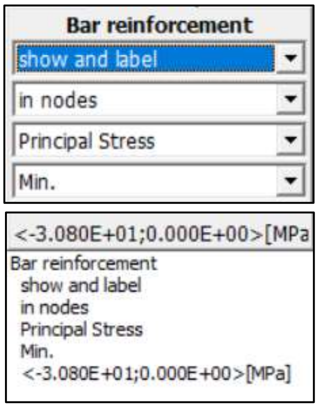


Figure 4.46: Example of *bar reinforcement* pull-down menu

The display type can be *no graphics, show* and *show and labels,* the averaging method can be in nodes or in element nodes while the entity can be chosen among *Engineering Strain, Principal Engineering Strain, Stress, Principal Stress, Plastic Strain* and *Principal Plastic Strain*. Finally, the component depends on the entity type, for example if the *Principal Stress* is selected, it is possible to choose among *Max., Min., Vmax x, Vmax y, Vmin x, Vmax y*.

4.3.3 Scalars

The display type can be chosen between *no graphics, rendering, contour areas* and *contour lines.* Again, the averaging can be done *in nodes* or in *element nodes* and the entity depends on the material model chosen. For the basic material the possibilities are *Displacements, Elem Total Temperature, Strain, Principal Strain, Stress, Principal Stress, Von Mises Stress, Total Strain, Principal Total Strain, Sbeta State Variables, Performance Index, Crack Width.* In the end, *Component* and *Label* can be selected, depending on the type of entity chosen.

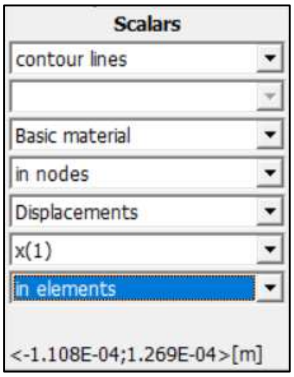


Figure 4.47: Example of *Scalars* pull-down menu

4.3.4 Output document

By clicking on the Text printout icon, a data tree structure is displayed, such that many output data can be generated. Another possibility is to export a CCO file which contains the results of all the steps of the monitoring points that have been defined in the pre-processing phase.

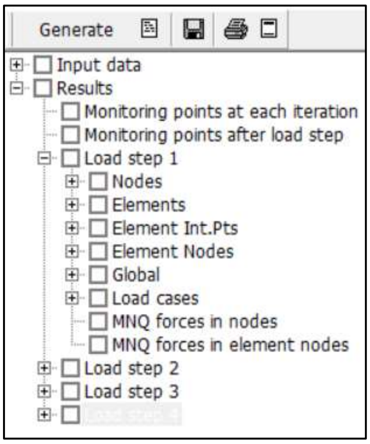


Figure 4.48: Example of *text printout*

5 FEM Model and Reliability analysis

In this chapter the FEM characteristics of the static non-linear finite element model adopted to perform the reliability analysis are evaluated. During the introduction, the three steps needed to perform the reliability analysis are presented. Then, a discussion about the sampling of basic variables, i.e. material and geometrical characteristics, according to Latin Hypercube Sampling method, follows. After that, the FEM material constitutive law assumptions, geometry and mesh characteristics are evaluated. In the end, the three aforementioned analyses are discussed, in terms of results and the aim of this thesis, i.e. local and global reliability factor conclude the chapter.

5.1 Introduction

To perform the analysis, that is to simulate the removal of the central column because of an accidental event, earlier non-linear analyses are needed; indeed, ATENA 2D does not allow to remove geometrical elements from an analysis step to the following. Thus, the simulation of the column removal is made by creating a model without the central column and, during the initial analysis-steps, applying a force in the nodes of the beam adjacent to the absent column. This force is equal to the reaction at the base of the column (minus the weight of the column itself) so to simulate the presence of the column. Then the column is removed in the next analysis steps (i.e. the reaction in the aforementioned nodes is posed equal to zero) and the loads of the central spans are amplified so to simulate the dynamicity of the event. To do so, three consecutive analyses should be performed:

- **Analysis 1 - Model with column** (i.e. with the presence of the central column), where all the design actions are applied, needed to calculate the reaction at the base of the column to be removed. The software used is ATENA 2D and the analysis is a Static NLFEA

- **Analysis 2 - Pushdown analysis** (i.e. performed in a model without the central column), needed to calculate the amplification coefficients according to Izzuddin et al. [1]. The software used is ATENA 2D and the analysis is a Static NLFEA
- **Analysis 3 -Reliability analysis**, as explained above (i.e. simulation of the removal of the column and amplification of the loads in the central span). The software used is ATENA 2D and the analysis is a Static NLFEA

These analyses are equal in terms of geometrical characteristics (apart from the analysis 1 where the only difference is the presence of the central column), mesh sizes, material and mechanical properties. The only change appears in the analysis-steps, i.e. in the way the analysis is performed itself.

Moreover, to perform a reliability analysis according to what has been explained on the Chapter 2 of this work of thesis (i.e. Method of Level III for reliability analysis), a probabilistic sampling of the basic variables has to be performed. Thus, the aforementioned analyses have been repeated N times, where N is the dimension of the samples for all the basic variables, posed equal to 100.

An additional analysis, between the 2nd and the 3rd has been needed in order to validate the assumption of amplifying the loads only on the central spans of the frame and not elsewhere. This validation has consisted on a **dynamic linear analysis**, performed on a software called ADINA. For sake of simplicity, this analysis has been conducted only once, taking the mean values of material and loads, and not N times.

5.2 Basic variables sampling

The reference code rules needed to perform the sampling of the basic variables are the JCSS – Probabilistic Model Code [3] and the fib Model Code 2010 [2]. In particular, the *Probabilistic Method*, according to fib MC 2010 [8] has been used, which consists on running several non-linear finite element analyses (NLFEAs), adopting a sampling technique such as Monte Carlo’s simulation or Latin Hypercube Sampling. For the purpose of this thesis, a Latin Hypercube Sampling has been selected (explained in sub-section 2.4.1.2 of this thesis). In short, this technique consists on dividing the cumulative density function of a standard distribution into N equal partitions and then choosing a random data point in each partition.

The resistance basic variables that has been sampled are:

- Concrete compressive strength f_c
- Reinforcement yield strength f_y
- Reinforcement ultimate strength f_u
- Reinforcement ultimate strain ε_{su}
- Reinforcement elastic modulus E_s

The action basic variables that has been sampled are:

- Reinforced concrete specific-weight ρ
- Permanent structural load G_1
- Permanent non-structural load G_2
- Floor variable loads Q_p
- Roofing variable loads Q_c

The LHS has been performed for all the $n = 10$ variables, with the MATLAB command called "*X_LHS=lhsnorm(MU,C,N)*". The input to be given to this function are:

- MU is a vector (1 x n) which contains the mean values (or the logarithmic mean or 0) for each variable. More details will be followingly explained.
- $[C]_{nxn} = [D * Ro * D]_{nxn}$ is a matrix (n x n), where $[D]_{nxn}$ is a diagonal matrix (n x n) which contains on the diagonal the variances (or the logarithm of the variances or 1) for each basic variable and $[Ro]_{nxn}$ is the covariance matrix. More details are followingly explained.
- N is the number of samples for each basic variable, chosen equal to 100.
- $[X_{LHS}]_{Nxn}$ is a (Nxn) matrix which contains in each column the $\left[X_{LHS,i}\right]_{Nx1}$vector coming as an output from the sampling and $\mathrm{i} = 1,2,3, \dots ,10$, since ten are the basic variables.

Depending on the basic variable under analysis, three types of distributions have been used:

- *Normal distribution:* in this case the mean and variance values to insert in the MU vector and D matrix respectively are the same of the distribution since the LHS method works on a Normal CDF. Thus, the output of the generic basic variable $\left[X_{LHS,i}\right]_{Nx1}$ coincides with the vector of samples $\left[X_{sam,i}\right]_{Nx1}$ of the generic basic variable X_i

- *Lognormal distribution:* in this case it is necessary to perform the logarithm of the mean and variance of the generic basic variable X_i, according to the following relation, where $X_{m,i}$ is the mean of the generic basic variable and V_i is the coefficient of variation:

$$MU_{\mathrm{i}} = \ln\left(X_{m,i}\right) - \log\left(V_i^2 + 1\right) \tag{5.1}$$

$$D_{\mathrm{ii}} = \sqrt{\log\left(V_i^2 + 1\right)} \tag{5.2}$$

Then, after the $\left[X_{LHS,i}\right]_{Nx1}$vector is generated from the LHS sampling, for the generic basic variable X_i, it is necessary to perform the exponential function in order to relate the output of the LHS that are normally distributed to the basic variable that is lognormally distributed

- *Gumbel distribution:* in this case the mean and variance values to be given as inputs to the MU vector and D matrix, are posed equal to 0 and 1 respectively, as if the distribution is a standardized normal distribution. After that, to come back to a Gumbel distribution, the vector which contains the samplings of the generic basic variable X_i is computed from the output of the LHS sampling as follows:

$$X_{sam,i} = -\ln\left(-\,ln\left(\Phi^{-1}\left(X_{LHS,i}\right) * \vartheta_{2,i} + \vartheta_{1,i}\right)\right) \tag{5.3}$$

where Φ^{-1} is the cumulative density function of the standard normal distribution and $\vartheta_{1,i}$ and $\vartheta_{2,i}$ are the two parameters of the Gumbel distribution computed as:

$$\vartheta_{2,i} = \sigma_i * \sqrt{6}/\pi \tag{5.4}$$

$$\vartheta_{1,i} = X_{m,i} - 0.5772 * \sigma_i * \sqrt{6}/\pi \tag{5.5}$$

with $X_{m,i}$ the mean value of the generic basic variable and σ_i the variance of the generic basic variable.

Finally, the covariance matrix C contains all the correlation coefficient between the sampled variables. When the coefficient is 0, it indicates no correlation, if 1 the correlation is the largest possible (i.e. between the same variable). The correlation between variables are shown in the following table:

Table 5.1: Correlation coefficients between the 10 basic variables [-]

	f_c	f_y	f_u	ε_{su}	E_s	ρ	G_1	G_2	Q_p	Q_c
f_c	1	0	0	0	0	0	0	0	0	0
f_y	0	1	0.75	-0.45	0	0	0	0	0	0
f_u	0	0.75	1	-0.6	0	0	0	0	0	0
ε_{su}	0	-0.45	-0.6	1	0	0	0	0	0	0
E_s	0	0	0	0	1	0	0	0	0	0
ρ	0	0	0	0	0	1	0	0	0	0
G_1	0	0	0	0	0	0	1	0	0	0
G_2	0	0	0	0	0	0	0	1	0	0
Q_p	0	0	0	0	0	0	0	0	1	0
Q_c	0	0	0	0	0	0	0	0	0	1

It is interesting to notice that, according to fib Model Code 2010 [8], there is a positive correlation between the steel yielding and ultimate strength, while there is a negative correlation between the ultimate strain and the latters. On the other hand, when there is no correlation, meaning when dealing with independent variables, the coefficient is zero.

In the following, details for each basic variable about distribution, mean value and variance are given.

5.2.1 Resistance basic variables

5.2.1.1 Concrete compressive strength f_c

According to JCSS – Probabilistic Model Code [3], the concrete compressive strength is a lognormal distribution with the following characteristics:

$$\mathrm{f_c} \sim \mathrm{LN}(\mathrm{f_{cm}}, \mathrm{V_c}) \tag{5.6}$$

where $V_c = \sigma_c / f_{cm} = 0.15$ is the coefficient of variation and f_{cm} is the mean value for the lognormal distribution of concrete cylinder compressive strength computed as suggested by EC2 [28]:

$$f_{cm} = f_{ck} \exp(1.645\, V_c) \tag{5.7}$$

with f_{ck} the characteristic compressive cylinder strength of concrete at 28 days, expressed as $f_{ck} = 0.83 * R_{ck}$ and R_{ck} is the characteristic compressive cube strength, equal to $30\ MPa$ for C25/30. Thus, it results that $f_{ck} = 24.9\ MPa$ and $f_{cm} = 31.87\ MPa$.

For what concerns all the other concrete variables such as the elastic modulus E_c, the tensile concrete strength f_{ct}, the compressive strain at the peak stress ε_{co}, the ultimate compressive strain ε_{cu}, etc., those represent aleatory dependent variables. Thus, in this preliminary stage, these variables do not need to be sampled, since their value depend on the only independent basic variable f_c.

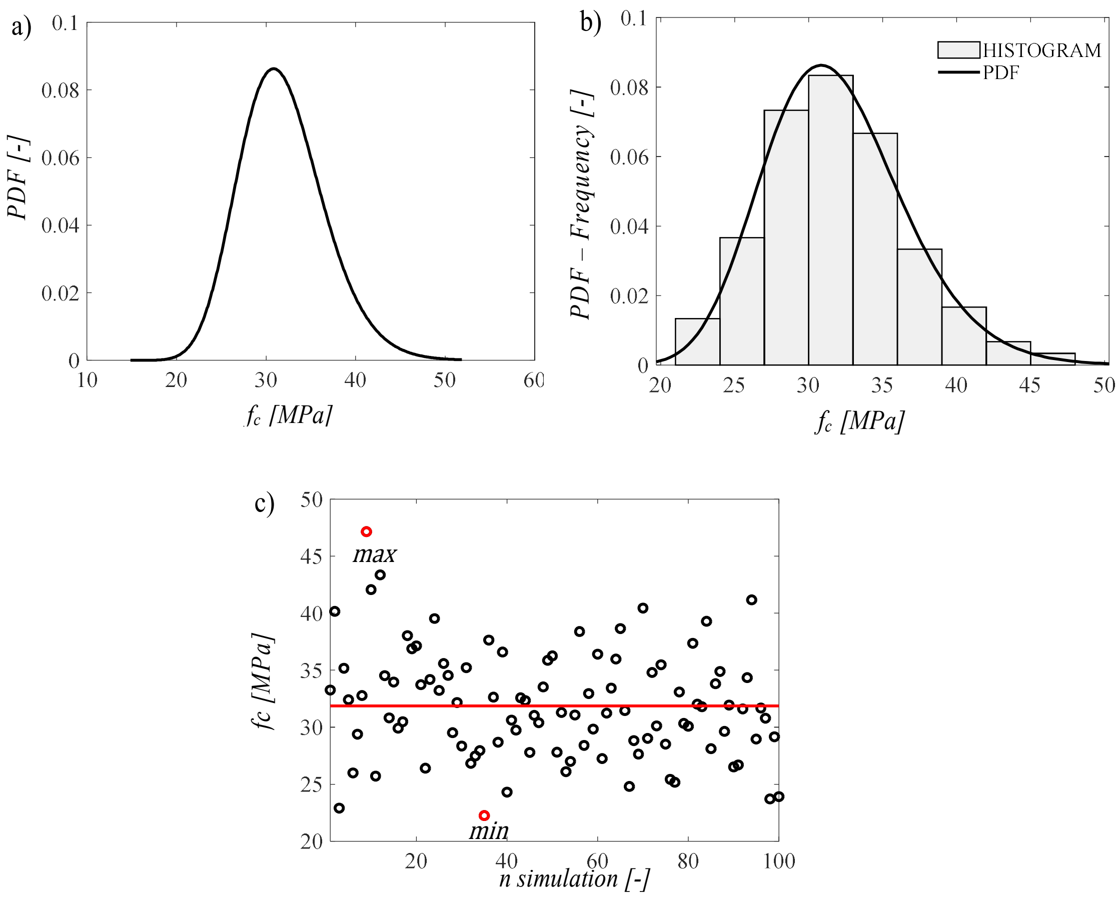


Figure 5.1: Concrete compressive strength - Lognormal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot

5.2.1.2 Reinforcement yield strength $\mathrm{f_y}$

According to JCSS – Probabilistic Model Code [3], the reinforcement yielding strength is a lognormal distribution with the following characteristics:

$$\mathrm{f_y} \sim \mathrm{LN}\left(\mathrm{f_{ym}}\,, \mathrm{V_{sy}}\right) \tag{5.8}$$

where $V_{sy} = \sigma_{sy}/f_{ym} = 0.05$ is the coefficient of variation and f_{ym} is the mean value of the lognormal distribution for the reinforcement yield strength computed as suggested by EC2 [28]:

$$f_{ym} = f_{yk}\, exp\left(1.645\, V_{sy}\right) \tag{5.9}$$

with f_{yk} the characteristic yield strength of reinforcement, equal to $450\ MPa$ for steel B450C. Thus, it results that $f_{ym} = 488.58\ MPa$.

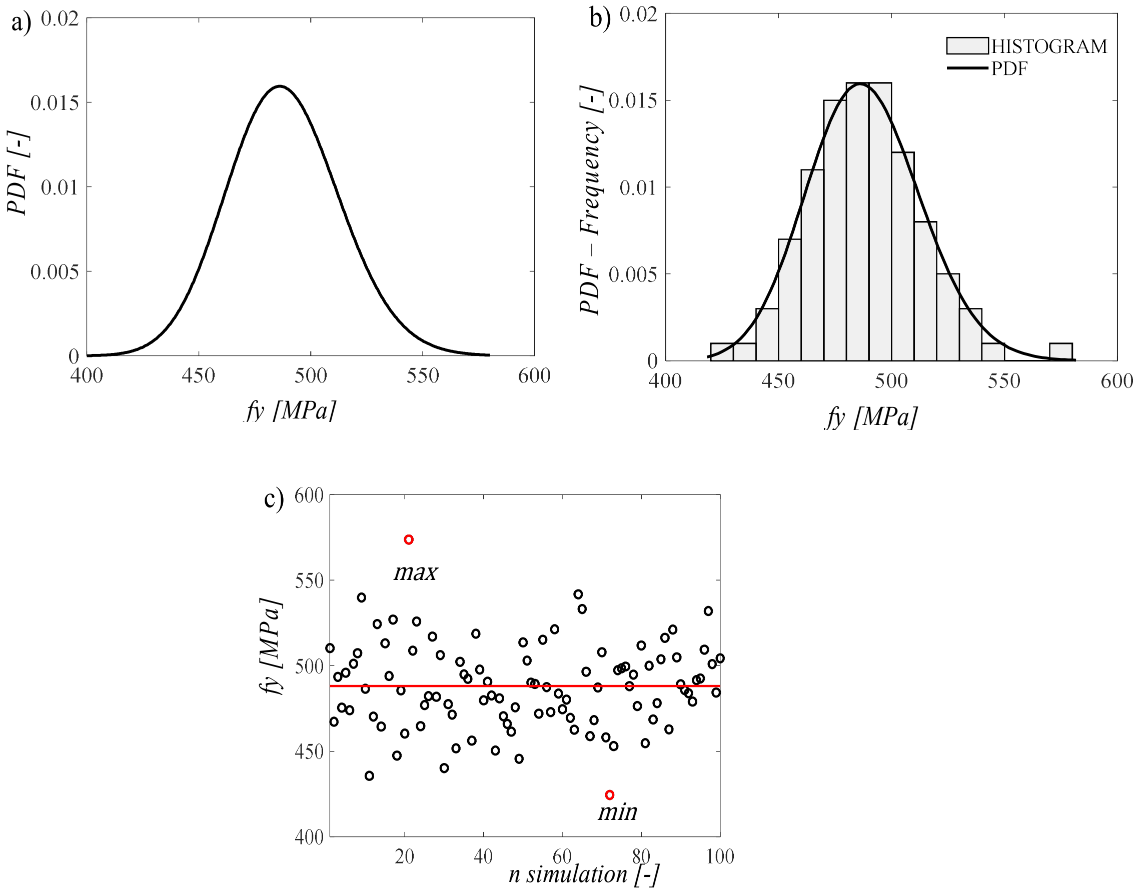


Figure 5.2: Steel yield strength - Lognormal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot

5.2.1.3 Reinforcement ultimate strength f_u

According to JCSS – Probabilistic Model Code [3], the reinforcement ultimate strength is a lognormal distribution with the following characteristics:

$$f_u \sim LN(f_{um}, V_{su}) \tag{5.10}$$

where $V_{su} = \sigma_{su}/f_{um} = 0.05$ is the coefficient of variation f_{um} is the mean value of the lognormal distribution for the ultimate strength and it is obtained as function of the yield strength as follows:

$$f_{um} = f_{ym}(1 + k) \tag{5.11}$$

with f_{ym} the mean value for the yield strength of reinforcement and $k = 0.15$ is a coefficient that determines the relation between the yield and the ultimate strength of concrete. Thus, it results that $f_{um} = 561.86\ MPa$.

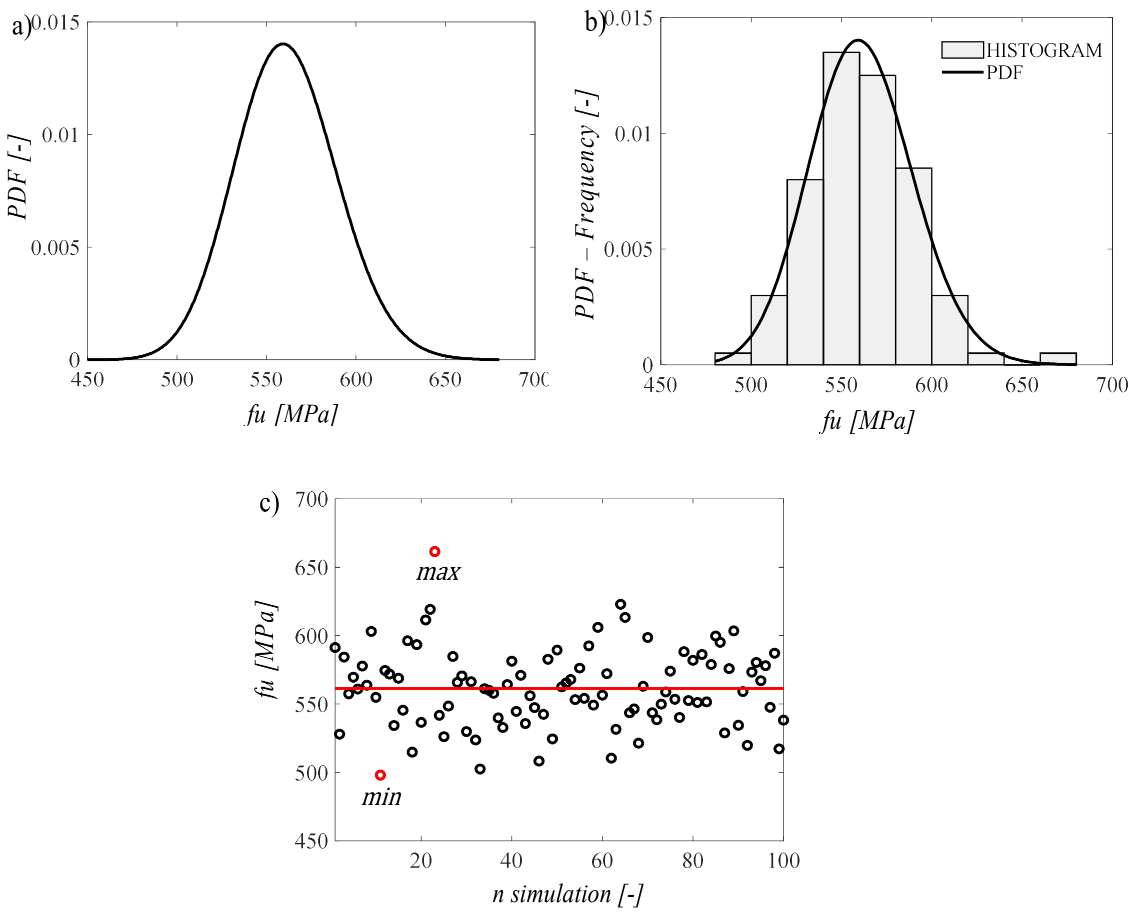


Figure 5.3: Steel ultimate strength - Lognormal distribution: a) Probability density function; b) Histogram and Distribution fit c) Scatter plot

5.2.1.4 Reinforcement ultimate strain ε_{su}

According to JCSS – Probabilistic Model Code [3], the ultimate strain for reinforcement is a lognormal distribution with the following characteristics:

$$\varepsilon_{su} \sim LN(\varepsilon_{sum}, V_{su}) \tag{5.12}$$

The coefficient of variation $V_{su} = \sigma_{su}/\varepsilon_{sum} = 0.09$ is taken according to different articles found in literature ([35], [36], [37], [38] and [39]), where experiments have been conducted on steel samples. Moreover, the ultimate strain is taken as $\varepsilon_{sum} = 0.14$, contrary to the suggested value in Eurocodes (i.e. 7.5%). This is due to the fact that the structure has been designed according to robustness criteria, so a larger, more realistic value, should be selected.

The validity of that 14 % is found in Caprili and Salvatore [40], where the paper presents data coming from a wide experimental test campaign executed on different typologies of steel reinforcing bars. In the following, table 4 of the aforementioned paper is presented, where the ultimate strain experimental values for B450C ϕ16 are reported (the symbol used is A_{gt} [%]):

Table 5.2: Mechanical properties of tested rebars (monotonic tensile tests). Data refer to average value of three tests

Steel grade/diameter/process/producer	R_m [MPa]	σ_{Rm}	R_e [MPa]	σ_{Re}	R_m/R_e	$\sigma_{Rm/Re}$	A [%]	σ_A	A_{gt} [%]	σ_{Agt}
B450C-16-TEMP-R Prod.1(1)	640.5	32.2	537.3	27.1	1.19	0.01	23.9	1.3	8.9	1.10
B450C-16-TEMP-R Prod.1(2)	542.7	3.10	446.7	1.90	1.21	0.00	30.3	2.4	15.4	1.70
B450C-16-TEMP-R Prod.1(3)	615.4	2.50	517.8	5.60	1.19	0.01	25.4	0.2	13.8	1.50
B450C-16-TEMP-R Prod.2	601.1	4.80	479.3	14.6	1.25	0.03	28.8	0.5	17.5	1.62

According to Table 5.2, the mean value can be taken as:

$$\varepsilon_{sum} = \frac{\sum A_{gt}}{4} = \frac{8.9+15.4+13.8+17.5}{4} = 13.9\% \cong 14\% \tag{5.13}$$

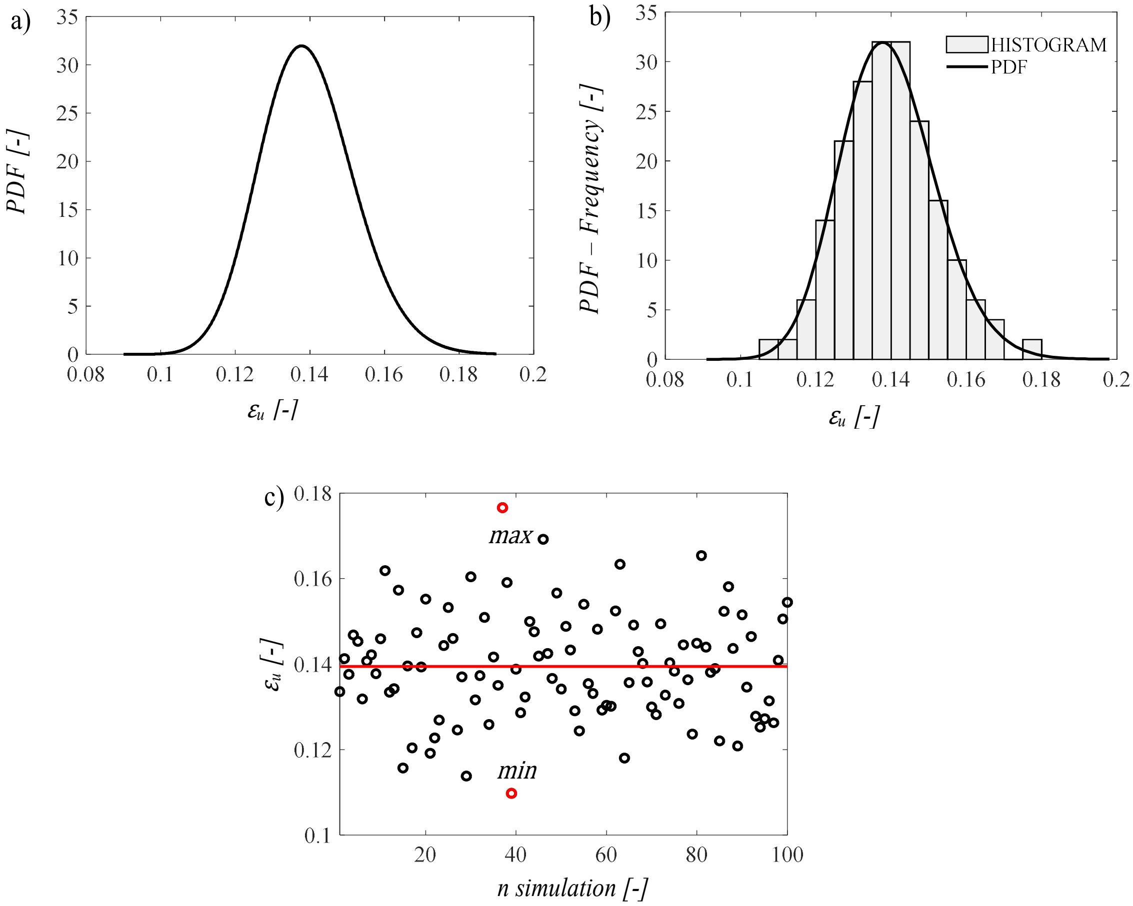


Figure 5.4: Steel ultimate strain - Lognormal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot

5.2.1.5 Reinforcement Elastic Modulus E_s

According to JCSS – Probabilistic Model Code [3], the Elastic Modulus for reinforcement is a lognormal distribution with the following characteristics:

$$E_s \sim LN(E_{sm}, V_{E_s}) \tag{5.14}$$

where $E_{sm} = 210000\ MPa$ is the mean value of the lognormal distribution for the Elastic Modulus, according to EC2, while $V_{E_s} = \sigma_{E_s}/E_{sm} = 0.03$ is the coefficient of variation.

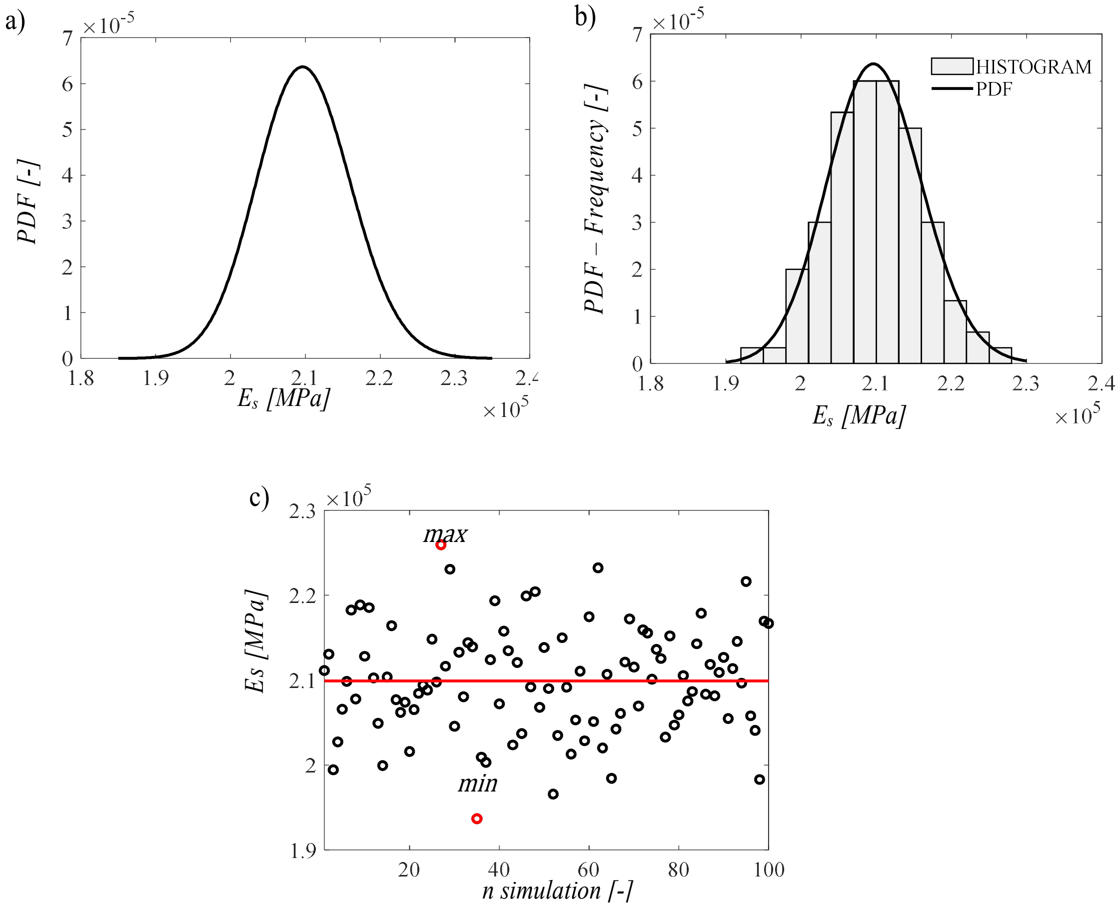


Figure 5.5: Steel Elastic Modulus - Lognormal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot

5.2.1.6 Correlation between reinforcement basic variables

In the following, the correlation between the 4 basic variables of reinforcement are discussed. Indeed, according to fib Model Code 2010, these are the only variables that, for the specific case, show a correlation different from zero. In the following table, the correlation coefficients are reported:

Table 5.3: Correlation coefficients between reinforcement basic variables [-]

	f_y	f_u	ε_{su}	E_s
f_y	1	0.75	-0.45	0
f_u	0.75	1	-0.6	0
ε_{su}	-0.45	-0.6	1	0
E_s	0	0	0	1

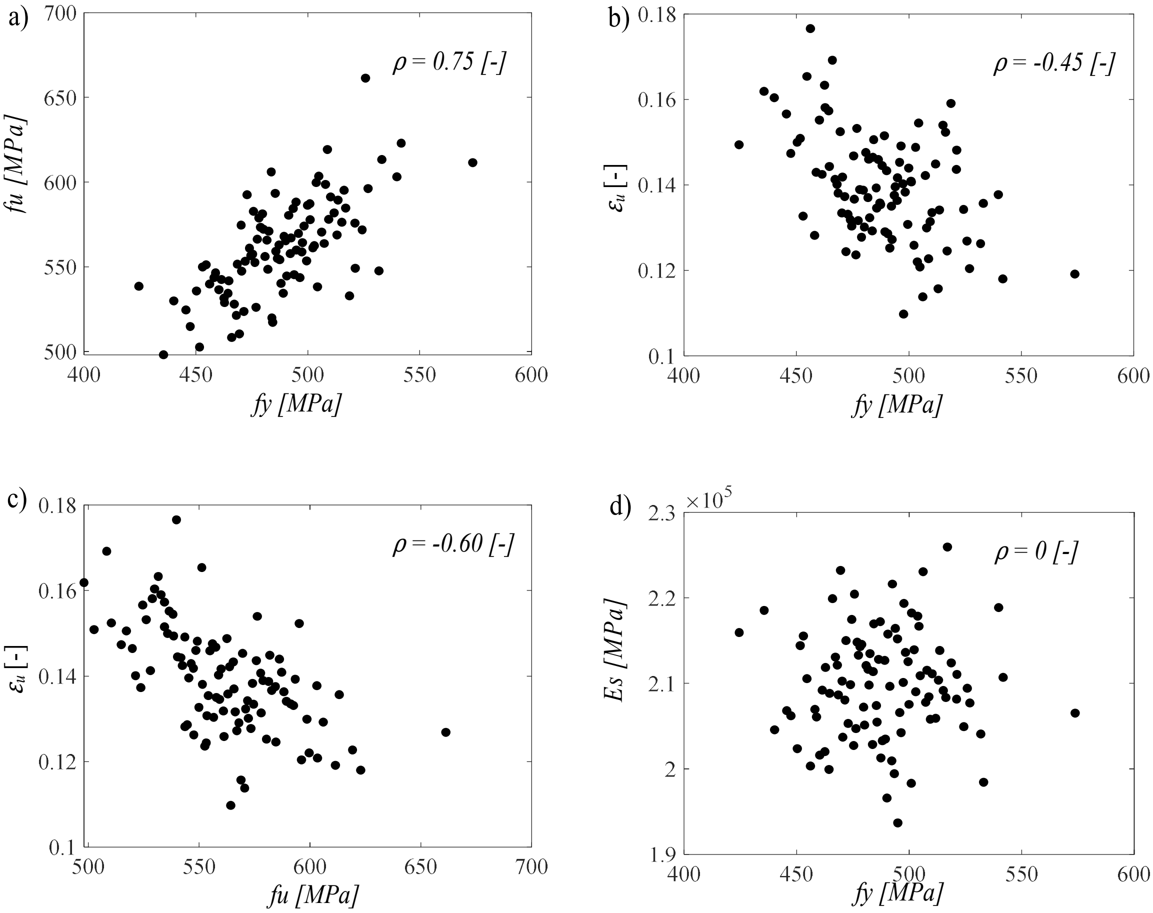


Figure 5.6: Correlation between reinforcement basic variables: a) correlation between f_y and f_u; b) correlation between f_y and ε_u; c) correlation between f_u and ε_u; d) correlation between f_y and E_s

5.2.2 Action basic variables

5.2.2.1 Reinforced-concrete specific-weight ρ

According to JCSS – Probabilistic Model Code [3], the reinforced-concrete specific-weight is a normal distribution with the following characteristics:

$$\rho \sim \mathrm{N}(\rho_m, V_\rho) \tag{5.15}$$

where $\rho_m = 25\ kN/m^3$ is the mean value of the normal distribution for reinforced-concrete specific-weight, according to EC2, while $V_\rho = \sigma_\rho/\rho_m = 0.05$ is the coefficient of variation.

The concrete specific-weight, that is the part occupying the concrete cover, is taken as the value assumed by the sampled reinforced concrete specific-weight minus one, since it is a dependent variable.

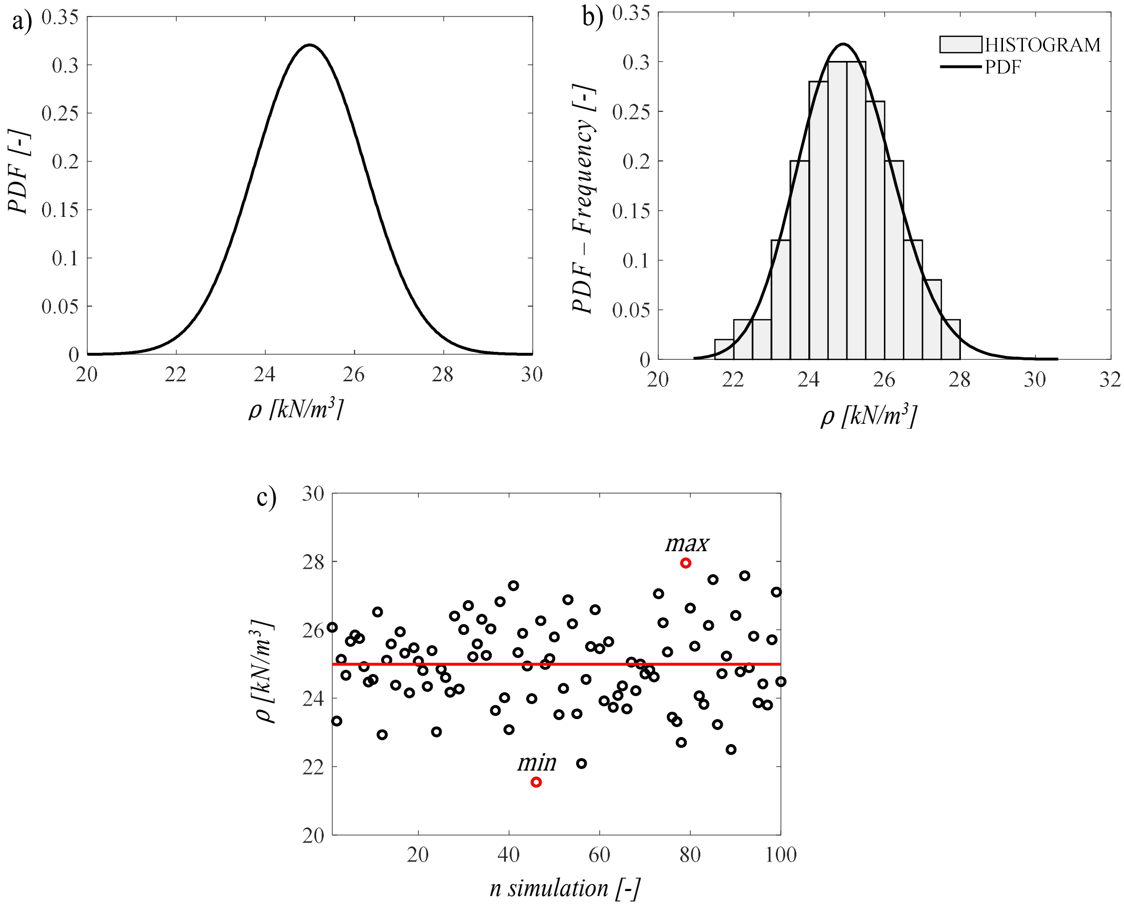


Figure 5.7: Reinforced concrete specific-weight - Normal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot

The following figure show the dependency between the reinforced concrete self-weight (i.e. basic independent variable) and the concrete self-weight (dependent variable).

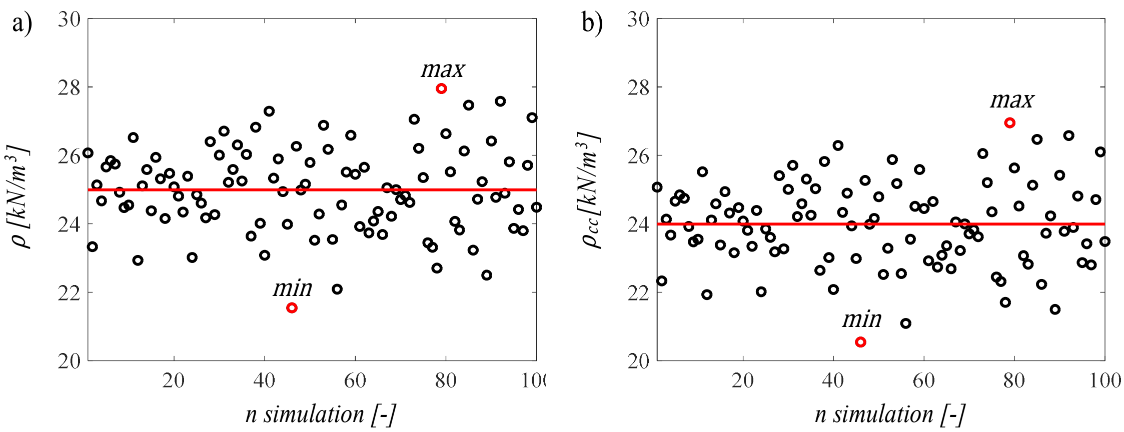


Figure 5.8: Scatterplots for specific weight: a) reinforced concrete (independent sampled variable); b) concrete cover (dependent variable)

5.2.2.2 Permanent structural load of the slab G_1

According to JCSS – Probabilistic Model Code [3], the permanent structural load is a normal distribution with the following characteristics:

$$G_1 \sim N\left(G_{1_m}, V_{G_1}\right) \tag{5.16}$$

where $G_{1_m} = 16\ kN/m$ is the mean value of the normal distribution for the permanent structural load of the slab, according to the design value (sub-section 3.5.1.1), while $V_{G_1} = \sigma_{G_1}/G_{1_m} = 0.05$ is the coefficient of variation.

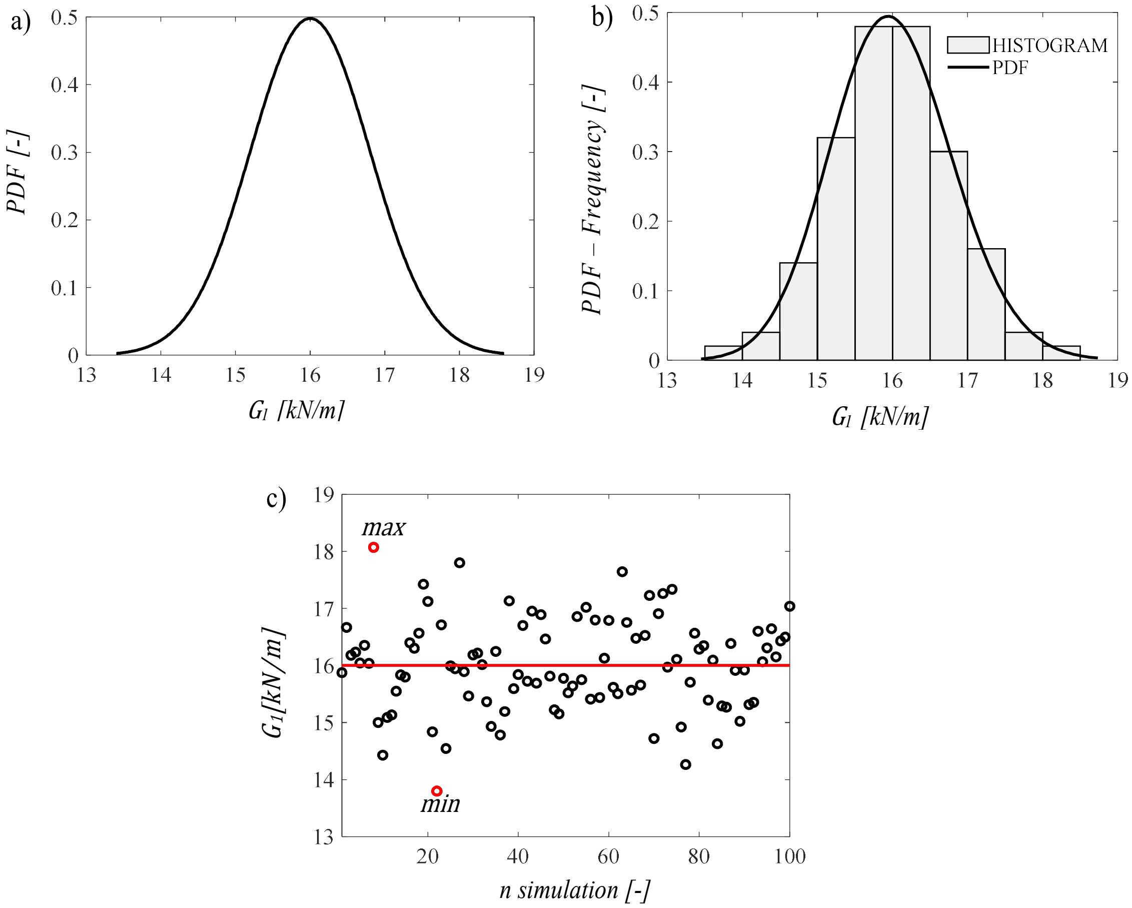


Figure 5.9: Permanent structural load - Normal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot

5.2.2.3 Permanent non-structural load G_2

According to JCSS – Probabilistic Model Code [3], the permanent non-structural load is a normal distribution with the following characteristics:

$$G_2 \sim N\left(G_{2_m}, V_{G_2}\right) \tag{5.17}$$

where $G_{2_m} = 13\, kN/m$ is the mean value of the normal distribution for the permanent non-structural load, according to the design value (sub-section 3.5.1.2), while $V_{G_2} = \sigma_{G_2}/G_{2_m} = 0.05$ is the coefficient of variation.

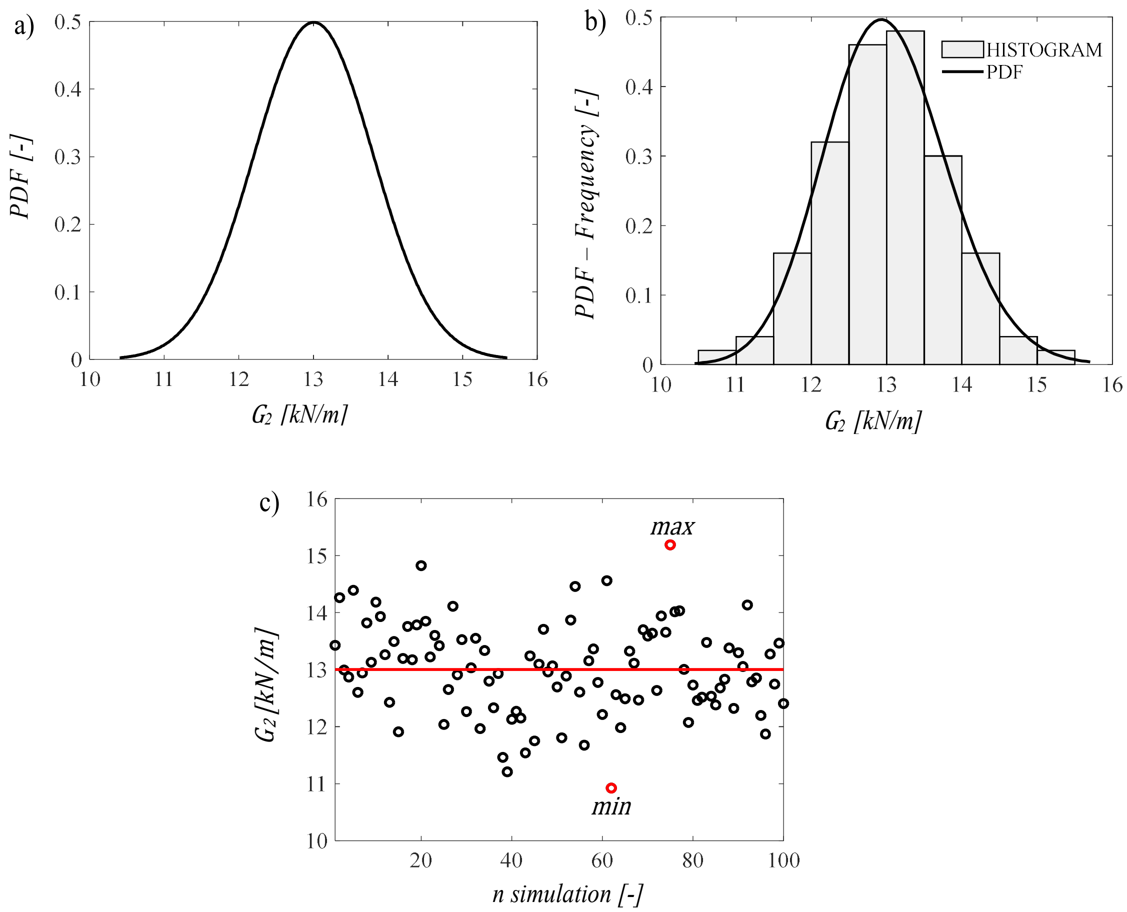


Figure 5.10: Permanent non-structural load - Normal distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot

5.2.2.4 Floor variable loads Q_p

According to JCSS – Probabilistic Model Code [3], the variable loads of the floor is a Gumbel distribution with the following characteristics:

$$Q_p \sim \text{Gumbel}\left(Q_{p_m}, V_{Q_p}, \vartheta_{1_{Q_p}}, \vartheta_{2_{Q_p}}\right) \tag{5.18}$$

where $Q_{p_m} = 6.5\ kN/m$ is the mean value of the Gumbel distribution for the variable loads of the floors. In particular, Q_{p_m} corresponds to the characteristic value (i.e. fractile 98%) of the design value equal to $10\ kN/m$ (sub-section 3.5.2.1). Furthermore, $V_{Q_p} = \sigma_{Q_p}/Q_{p_m} = 0.20$ is the coefficient of variation, while $\vartheta_{2_{Q_p}} = 1.0136$ and $\vartheta_{1_{Q_p}} = 5.91$ are the Gumbel distribution parameters computed with Eq. (5.4) and Eq. (5.5) respectively.

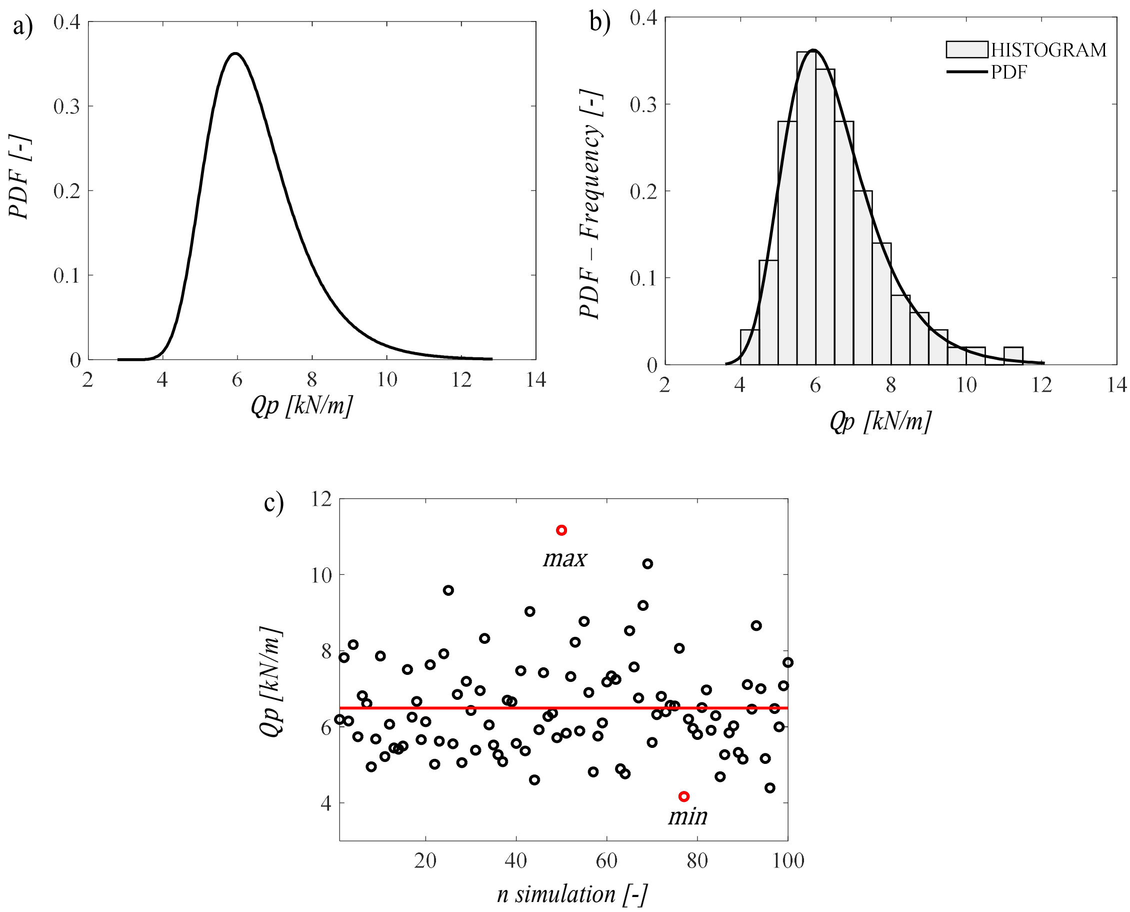


Figure 5.11: Floor variable load - Gumbel distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot

5.2.2.5 Roofing variable loads Q_c

According to JCSS – Probabilistic Model Code [3], the variable loads of the floor is a Gumbel distribution with the following characteristics:

$$Q_c \sim \text{Gumbel}\left(Q_{c_m}, V_{Q_c}, \vartheta_{1_{Q_c}}, \vartheta_{2_{Q_c}}\right) \tag{5.19}$$

where $Q_{c_m} = 1.6\ kN/m$ is the mean value of the Gumbel distribution for the variable loads of the roofing. In particular, Q_{c_m} corresponds to the characteristic value (i.e. fractile 98%) of the design value equal to $2.5\ kN/m$ (sub-section 3.5.2.1). Furthermore, $V_c = \sigma_{Q_c}/Q_{c_m} = 0.20$ is the coefficient of variation, while $\vartheta_{2_{Q_c}} = 0.249$ and $\vartheta_{1_{Q_c}} = 1.46$ are the Gumbel distribution parameters computed with Eq. (5.4) and Eq. (5.5) respectively.

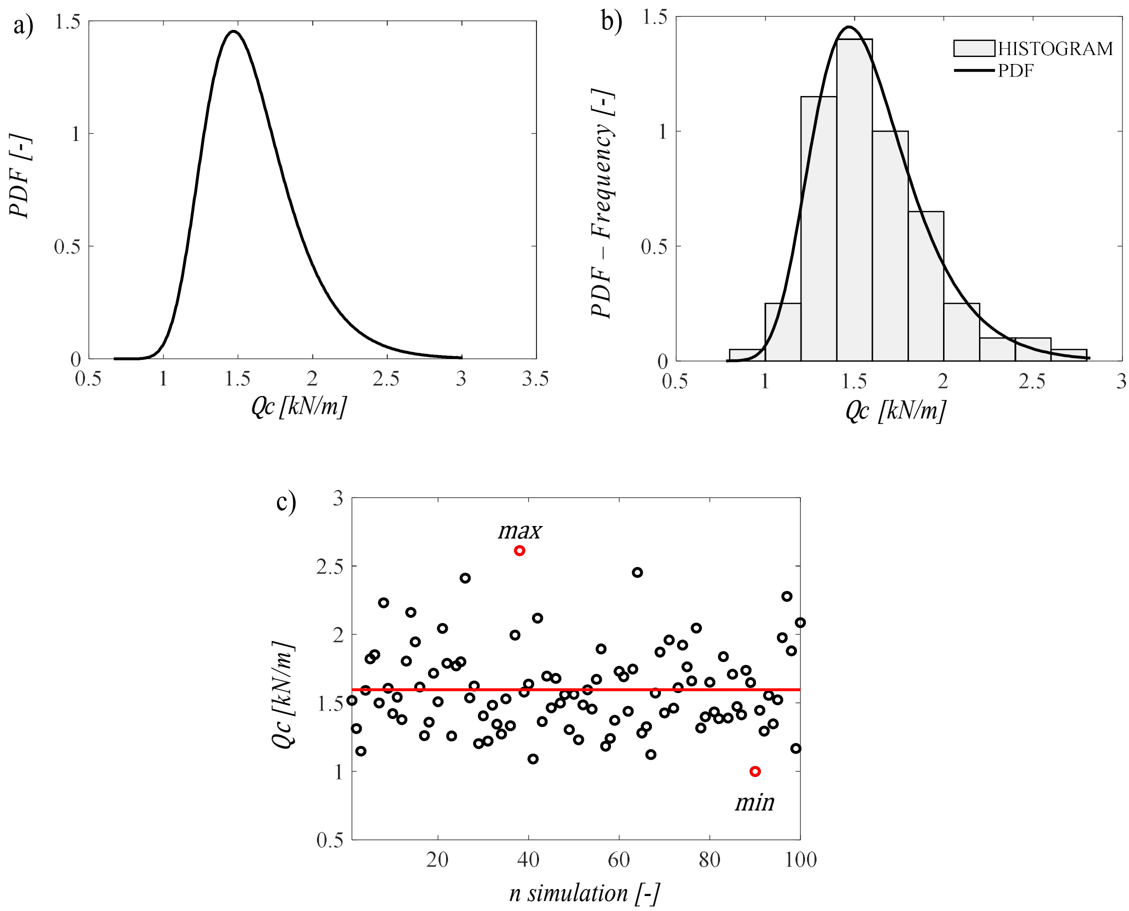


Figure 5.12: Roofing variable load - Gumbel distribution: a) Probability density function; b) Histogram and Distribution fit; c) Scatter plot

In the following, all the assumption made about distribution type, mean value and coefficient of variation, are summed up:

Table 5.4: Summary of the sampled basic variables

	Distribution type	Mean value	Coefficient of Variation
Concrete compressive strength f_c	LN	31.87 MPa	0.15 [-]
Reinforcement ultimate strength f_u	LN	488.58 MPa	0.05 [-]
Reinforcement yield strength f_y	LN	561.86 MPa	0.05 [-]
Reinforcement ultimate strain ε_{su}	LN	0.14 [-]	0.09 [-]
Reinforcement elastic modulus E_s	LN	210000 MPa	0.03 [-]
Reinforced concrete specific-weight ρ	N	25 kN/m^3	0.05 [-]
Permanent structural load G_1	N	16 kN/m	0.05 [-]
Permanent non-structural load G_2	N	13 kN/m	0.05 [-]
Floor variable loads Q_p	GUMBEL	6.5 kN/m	0.20 [-]
Roofing variable loads Q_c	GUMBEL	1.6 kN/m	0.20 [-]

5.3 Material's constitutive law

Two types of materials compose the building: concrete and steel reinforcement. Depending on the location (i.e. beam, column or joint or dissipative, non-dissipative zone), the design of the stirrups' reinforcement changes, thus the confinement action that the reinforcement creates on the concrete is different. According to this, as already mentioned in the previous chapter, seven types of material have been modelled in ATENA 2D:

- Material 1: Beam D. It is an *SBeta Material* which refers to the confined concrete of the beam in the dissipative zone (D stands for dissipative), i.e. the area of the beam located at a distance from the joint lower than 90 cm; the stirrups are made of $\phi 8$ step 7.5 cm while the longitudinal has 5 bars with $\phi 18$ both in upper and lower chord
- Material 2: Beam ND. It is a *Sbeta Material* that refers to confined concrete of the beam of the non-dissipative zone (ND); the transversal reinforcement is $\phi 8$ step 15 cm while the longitudinal is composed by 5 bars with $\phi 18$ both in upper and lower chord
- Material 3: Column. It refers to the confined concrete column and it is again a *Sbeta Material*; the transversal reinforcement is $\phi 8$ step 10 cm while the longitudinal is composed by 12 bars with $\phi 20$

- Material 4: Joints. It is a *Sbeta Material* and refers to the confined concrete located in the joints between columns and beams; the transversal reinforcement is $\phi 8$ step 5 cm while the longitudinal is composed by 12 bars with $\phi 20$
- Material 5: NC Concrete. It is a *Sbeta Material* is used to define the concrete area not-confined by the reinforcing bars (NC), thus the concrete cover.
- Material 6: Steel B450C: it refers to the steel, category B450C, used for the longitudinal reinforcement and transversal reinforcement, so it is defined with a *Reinforcement* type of material.
- Material 7: Plates. This material is a *Plane stress elastic isotropic* type and it has been used to model steel plates of 10 cm, having infinite stiffness, located at the external parts of the column-beam joints and at the bases of each column, where fixities are applied. They are needed for monitoring, to apply the imposed displacement in the pushdown analysis and to insert the constraints at the base of the columns. In BASIC it is assumed E=500000 MPa and $\upsilon = 0.3$, while in MISCELLANEOUS $\rho = 0$

In the following, a scheme of the majority of the Materials that have been used is presented. The horizontal and vertical green lines define the longitudinal and transversal reinforcement respectively, i.e. Material 6.

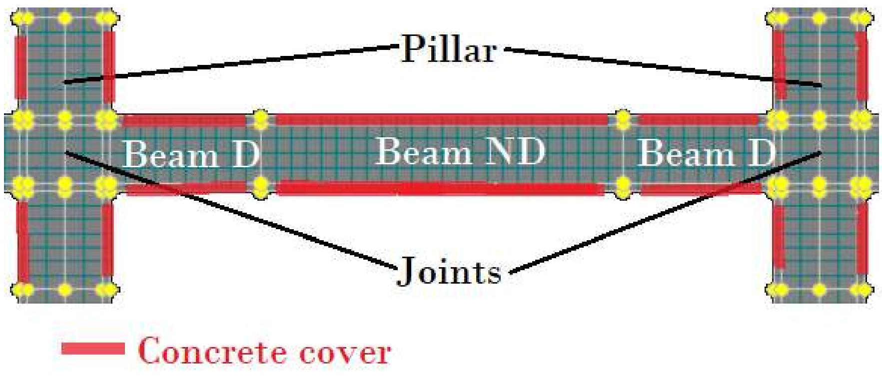


Figure 5.13: Scheme for Material 1 (Beam D), Material 2 (Beam ND), Material 3 (Column) and Material 4 (Joints) and Material 5 (NC Concrete)

5.3.1 Concrete constitutive law

The constitutive law of the concrete is based on a model proposed by Saatcioglu and Razvi (1992) [32]. The non-linear behaviour of confined concrete under transversal reinforcement action is a multiaxial state of stress, since passive lateral pressure develops as it expands under the influence of axial compression. Transverse strains caused by lateral pressure counteract the tendency of material to expand laterally, and result in increased strength.

The model of the unconfined concrete shows a peak stress f'_{cc}, corresponding to a strain equal to ε_1 and a softening behaviour after having reached the 20% of the peak stress, in correspondence of a strain ε_{20}.

The model allows also to implement the unconfined concrete behaviour, according to the f'_{co} value (peak stress of unconfined concrete), assumed equal to the f_c sampled value in subsection 5.2.1.1, $\varepsilon_{01} = 0.002$ (strain corresponding to the peak stress of unconfined concrete) and $\varepsilon_{085} = 0.085$ (strain at 85% strength level beyond the peak stress of unconfined concrete).

The two behaviours can be seen in the following figure, taken from [32]:

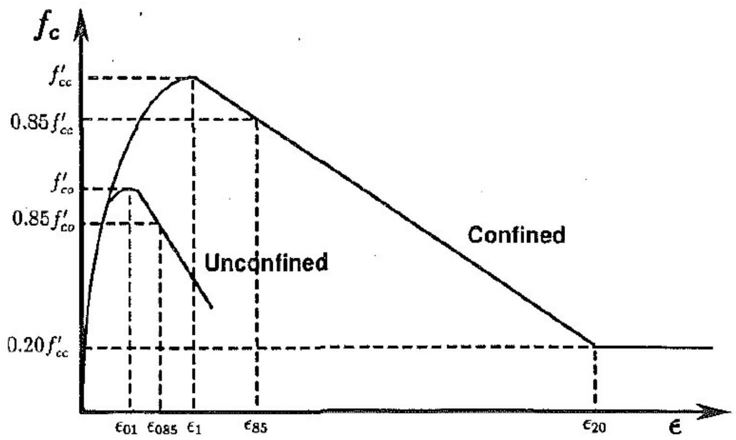


Figure 5.14: Stress-strain Relationship, unconfined vs confined concrete (Saatcioglu and Razvi [32])

The model has been implemented thanks to an Excel Sheet, where the input arguments are the mean concrete compressive strength, the section geometry, the concrete cover, the reinforcement yield strength f_y, the diameter of the longitudinal reinforcement (enclosed by the stirrups) and finally the diameter, step and arms of the transversal reinforcement.

As Figure 5.15 shows, the confined concrete shows a larger strength with respect to the unconfined one, in all the four types of materials, due to the triaxial state of confined concrete. Moreover, having the column and joints a transverse reinforcement with 4 arms, the strengthening is larger with respect to the one of the beams (both for the D and ND areas). In addition, the joints parts of the column show a larger strengthening than in other parts of the column itself, due to a larger step of the stirrups. For the same reason, in the dissipative part of the beam the peak strength is larger than in the non-dissipative parts.

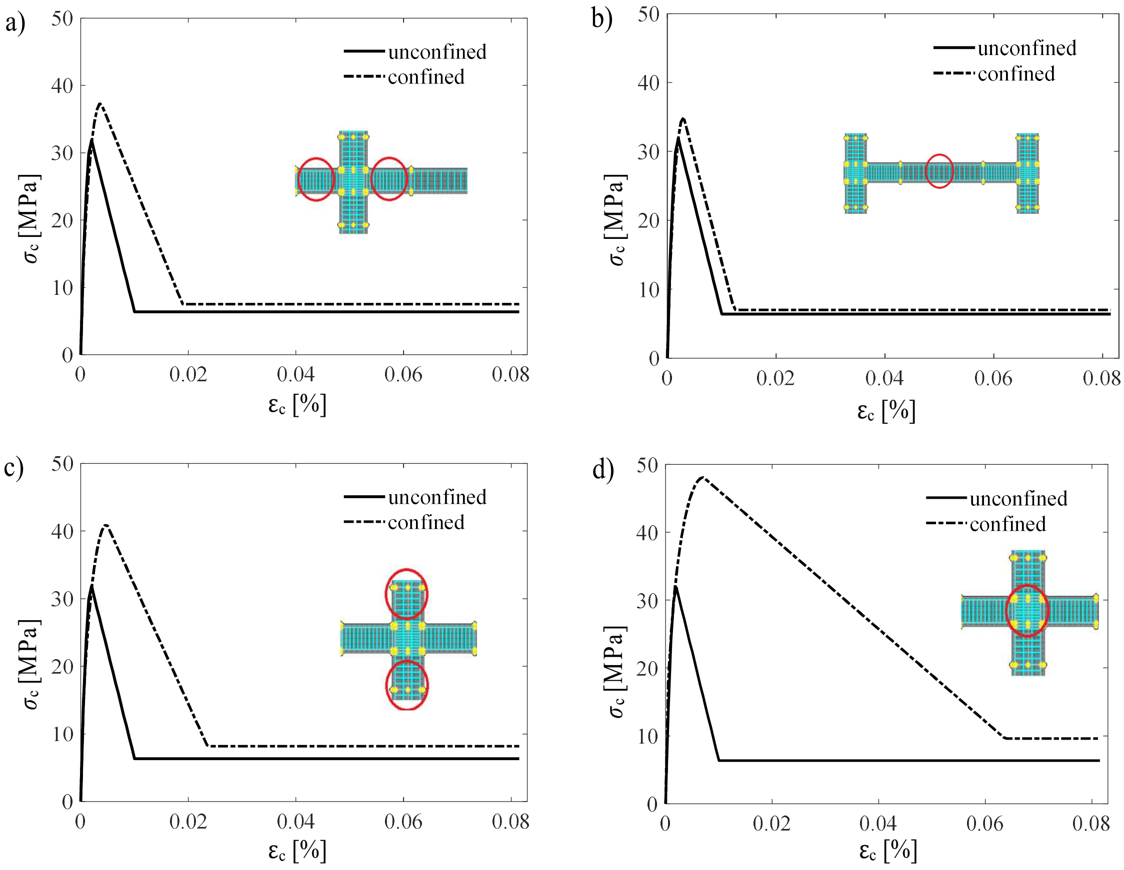


Figure 5.15: Stress-strain Relationship, unconfined vs confined concrete (Saatcioglu and Razvi [32]): a) beam dissipative area; b) beam non dissipative area; c) column; d) joints

It is important to notice that the majority of the input arguments have been sampled according to the probabilistic approach or depends on these sampled values. This means that also the constitutive law changes for all the *N* simulations.

According to the previous chapter, sub-section 4.2.2, the following assumption have been made in ATENA 2D for *Sbeta Material*:

- Tensile: the type of tension softening *Local Strain*, according to which the descending branch of the stress-strain diagram is defined by the end strain C_3, called *Softening parameter 3*, computed as follows (5.20):

$$C_3 = 10 * \frac{f_{ctm}}{E_{cm,TG}} \tag{5.20}$$

  where (all variables defined in Eurocode 2 [28] at §3.1):
  - $f_{ctm} = 0.3 * (f_{ck})^{2/3}$ is the mean value of axial tensile strength of concrete (assumed the same both for confined and unconfined concrete)
  - $E_{cm,TG} = E_{cm} * 1.05$ is the tangent elastic modulus
  - $E_{cm} = 22000 * \left(\frac{f_{cc}}{10}\right)^{0.3}$ is the secant modulus of elasticity of concrete
  - f_{cc} is the f'_{co} or f'_{cc} peak value of Saatcioglu and Razvi model, depending if unconfined or confined concrete

  Moreover, the selected crack model is *Fixed*

- Compressive: the type of compression softening is *Softening Modulus* where it is defined a slope of the softening law by means of the softening modulus E_d, computed as the product between the *compression softening parameter* c_d and the concrete elastic modulus E_c. Other input parameters are ε_1 or $\varepsilon_{01} = 0.002$, depending if confined or unconfined concrete respectively and the *RCS parameter* (*Reduction of Compressive Strength due to cracks*), equal to 0.8 as the ATENA Manual suggests [33]

- Shear: the *shear retention factor* is considered *Variable*, so a reduction of shear stiffness after cracking is assumed and the tension-compression interaction is *Linear,* where the only parameter is the *FSF (Fixed Shear retention Factor),* suggested values are 0.1-0.2 given in ATENA Manual [33]

The following tables represent a summary of all the selected input parameters:

Table 5.5: Basic inputs for SBeta Material (Materials 1,2,3,4 and 5)

	BASIC			
	f_{cc} $[MPa]$	f_{ctm} $[MPa]$	$E_{cm,TG}$ $[MPa]$	υ $[-]$
1) Beam D	37.45	2.56	34328	0.2
2) Beam ND	35.01	2.56	33641	0.2
3) Column	41.00	2.56	35272	0.2
4) Joint	48.09	2.56	37003	0.2
5) NC Concr	31.87	2.56	32706	0.2

Table 5.6: Tensile, Compressive, Shear and Miscellaneous inputs for SBeta Material (Materials 1,2,3,4 and 5)

	TENSILE	COMPRESSIVE			SHEAR	MISCELLANEOUS	
	C_3 $[-]$	ε_{cc} $[-]$	$R.C.S.$ $[-]$	$C.S.P.$ $[-]$	$F.S.F.$ $[-]$	ρ $[kN/m^3]$	α $[1/K]$
1) Beam D	0.000745	0.00375	0.8	0.0575	0.2	25	0.000012
2) Beam ND	0.000760	0.00298	0.8	0.0893	0.2	25	0.000012
3) Column	0.000725	0.00486	0.8	0.0497	0.2	25	0.000012
4) Joint	0.000691	0.00709	0.8	0.0184	0.2	25	0.000012
5) NC Concr	0.000782	0.002	0.8	0.0974	0.2	24	0.000012

It is important to underline that these tables refer to the values coming from the mean value of the sampled basic variables. Thus, apart from the constant parameters (i.e. RCS, CSP, FSF, α), these tables change N times, according to the sampled basic variables.

5.3.2 Steel reinforcement constitutive law

The steel reinforcement has been modelled in Material 6: Steel B450C. The assumption on its behaviour is *Bilinear with Hardening*. The input values are summed up in the following table

Table 5.7: Basic and Miscellaneous inputs for Reinforcement (Material 6)

BASIC				MISCELLANEOUS	
σ_y $[MPa]$	σ_t $[MPa]$	E $[MPa]$	ε_{lim} $[MPa]$	ρ $[kN/m^3]$	$ALPHA$ $[1/K]$
488.58	561.86	210000	0.14	0	0.000012

where:

- σ_y is the yield strength of steel, assumed equal to the sampled value f_y, according to Eq. (5.9)

- σ_t is the ultimate strength, assumed equal to the sampled value f_u, according to Eq. (5.11)

Again, all the input given in the BASIC, change according to the sampled basic variables, thus N times.

5.4 Geometry and mesh for analyses 1,2 and 3

According to the previous chapter, sub-section 4.2.3, the geometry of the FEM model built in ATENA 2D, defined in the section topology, requires the definition of joints, lines, macro-elements and reinforcement.

Using an Excel sheet, that has allowed to automatize the procedure, all the points delimiting the confined and non-confined concrete areas, the longitudinal and transversal reinforcement and the steel plates of 10 cm have been inserted. In total, 1025 and 1005 are the joints regarding respectively the model with and without the central column. At the end, from the graphical interface it is possible to represent them (Figure 5.16):

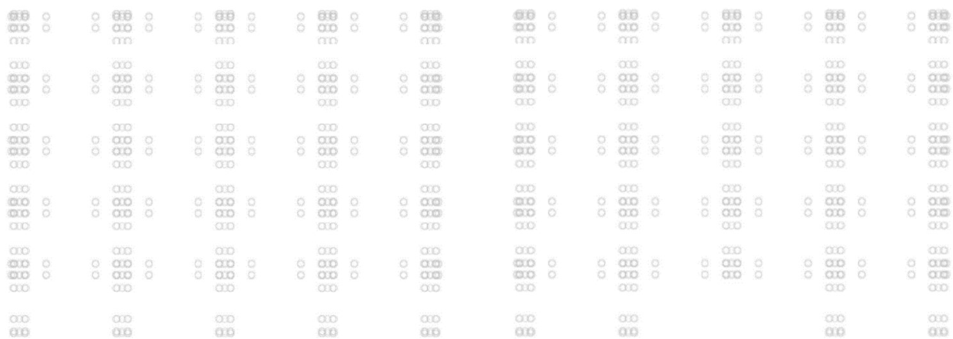


Figure 5.16: Joints representation of the 2D frame: in the left model with column, in the right, model without the column

Figure 5.17 is about the representation of the lines of the final model. In total, 1890 and 1854 lines have been used respectively for the model with and without the central column.

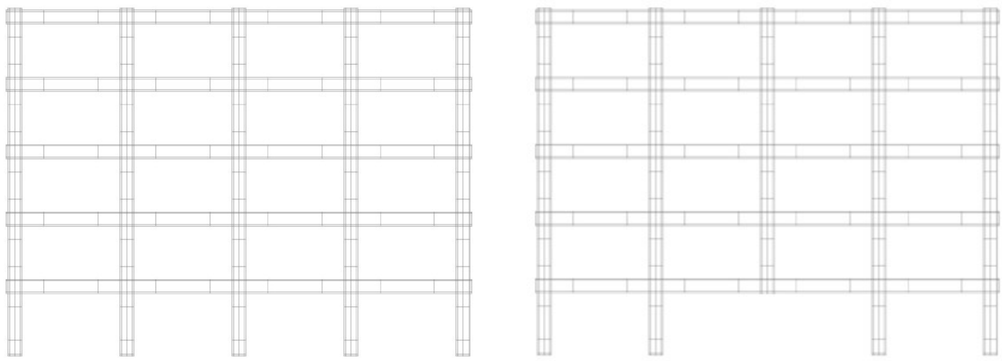


Figure 5.17: Lines representation of the 2D frame: in the left model with column, in the right, model without the column

In total, 850 and 834 are the number of macro-elements for the model with and without the central column respectively. At the end of this process, the model appears like in Figure 5.18:

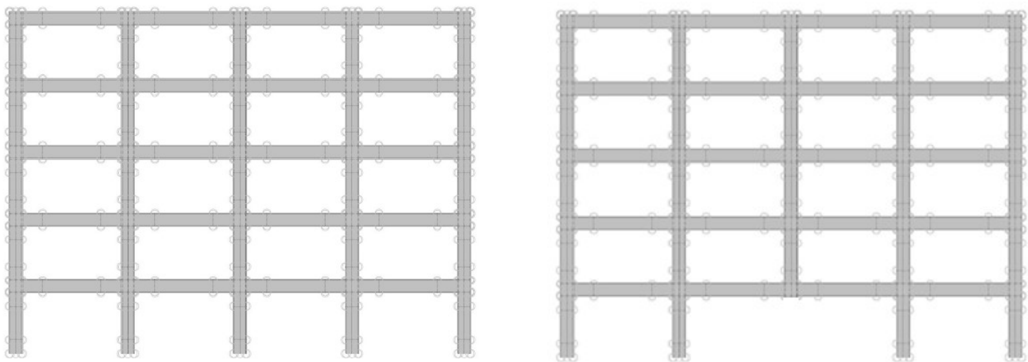


Figure 5.18: Representation of the 2D frame with joints, lines and macro-elements: in the left model with column, in the right, model without the column

In total, 1825 and 1796 are the *Reinforcements type* lines inserted in the model with and without the central column respectively. In particular, 4 vertical continuous lines represent the longitudinal reinforcement for the columns and 2 horizontal continuous lines for the beams. These lines are located in such a way that the concrete cover has a thickness of 53 cm both for columns and beams. The transverse reinforcement has a step of 5 cm for the joints, 10 cm for the column, 7.5 cm for the dissipative parts of the beam and 15 cm for the non-dissipative parts.

When all the topology elements have been considered, the model appears like in Figure 5.19: Representation of the 2D frame with longitudinal and transversal reinforcement: in the left model with column, in the right, model without the column

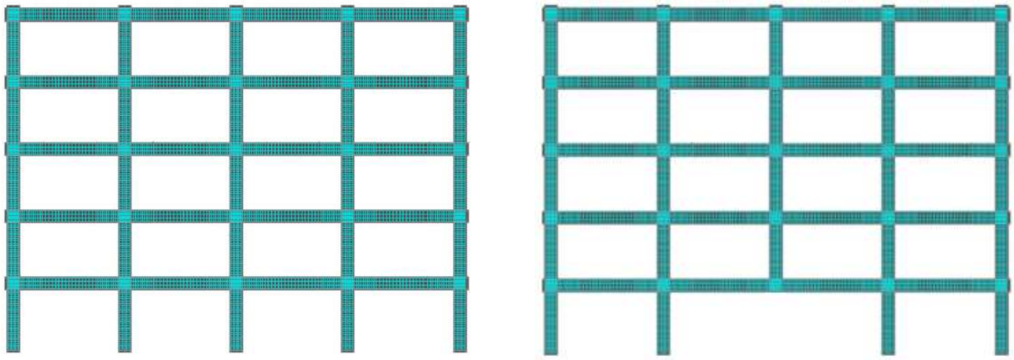


Figure 5.19: Representation of the 2D frame with longitudinal and transversal reinforcement: in the left model with column, in the right, model without the column

For what concerns the mesh, the type selected is *Quadrilateral*, with quadrilateral elements of type *CCIsoQuad*. The thickness of 60 cm for the columns and 40 cm for the beams. The *Element size* is 0.1 m.

5.5 Analysis 1 - Model with column

This model has been created in order to calculate the reaction at the base of the central column. The reaction is needed for two purposes:

- To compute the gravity loading concentrated force P_o at the top of the removed column. This is necessary to apply a method proposed by Izzuddin [1] in order to compute the dynamic amplification factors for all the N simulations
- To apply the load case "Reaction" at the top of the column removed, in order to simulate the removal of the column in the reliability analysis (i.e. analysis 3)

To do so, the software ATENA 2D has been adopted. For what concerns materials, geometry and load characteristics, they have all been explained in the previous sub-section. In the following, the load cases and the analysis steps inserted in ATENA 2D are summed up.

5.5.1 Loads cases

The following table shows the load cases that have been considered for this analysis:

Table 5.8: Load cases for Analysis 1 - Model with column

Load case number	Description
1	Base support
2	Self weight
3	Permanent structural loads
4	Permanent non-structural loads
5	Variable loads

The base support load case is created according to sub-section 4.2.4.1 of the previous chapter and it is applied at the lines composing the basis of the five pillars. The self-weight is applied on the whole structure and it is directly computed by the software with the body force command, as function of geometry and density, according to sub-section 4.2.4.2. The permanent loads, both structural and non-structural, are applied along all the beams lines. The variable loads are defined in such a way that two different values are applied if the beam is of the floor or of the roofing. These last load cases are defined in ATENA 2D according to what has been explained in sub-section 4.2.4.3.

5.5.2 Analysis steps

For the purpose of this analysis, it is needed to apply all the actions simultaneously, no combination coefficients are used. Indeed, since this analysis is functional to the reliability analysis (i.e. analysis 3), the approach is probabilistic, thus no combinations of loads, like in the semi-probabilistic approach adopted by code rules, is needed.

The analysis steps adopted are the following:

- Analysis step 1: load cases 1,2,3 and 5 with a coefficient of 0.5
- Analysis step 2: load cases 1,2,3 and 5 with a coefficient of 0.5
- Analysis step 3: load cases 1 and 4 with a coefficient of 0.5
- Analysis step 4: load cases 1 and 4 with a coefficient of 0.5

According to this, in the first two steps all the permanent loads and self-weight are applied and in the last two the variable loads are applied as well. All the analysis steps have been performed according to the *Standard Newton-Raphson* defined in sub-section 4.2.5.4, with a maximum of 40 iterations each step.

No monitoring points are needed for the purpose of this analysis, because the only output is the reaction computed at the five nodes composing the base of the central column. Thus, after running the analysis, the *TextPrintout* command allows to obtain the *Reactions* at the *Nodes*. Of course, these reactions should be taken correspondingly to the last analysis step (i.e. the fourth), at which all the gravity loads have been applied.

The reactions that are obtained as such, for the five nodes of the line, should be summed in order to compute the total reaction at the base of the central column. After that, it is necessary to subtract the reaction just computed the weight of the column in order to obtain the actual reaction at the top of the column accidentally dismissed. It is important to underline that the weight of the removed column is not constant, since the specific-weight of reinforced concrete ρ is a variable that has been sampled N times. At the same time, N simulations should be performed, according to the sampled basic variables and the dependent ones.

At the end of this analysis, a vector called P_0 of dimension 1xN is obtained, containing all the 100 reactions at the top of the removed column, for all the 100 simulations.

5.6 Analysis 2 - Pushdown analysis

This analysis is necessary to obtain the capacity curves, needed for the evaluation of the dynamic amplification coefficients, according to Izzuddin [1].

Three types of pushdown analyses can be conducted [41]:

- Load Controlled Pushdown Analysis (LC-PD) this is the proposed one by the General Services Administration (GSA) Guidelines, according to which the load is increased step by step until a specific level is reached, under a given column removal scenario. It is an equivalent static approach where the dynamic enhancement is simulated by increasing the gravity load step by step
- Displacement Controlled Pushdown Analysis (DC-PD) it consists on apply an imposed displacement in the location of the removed column, leaving the loading pattern unchanged (i.e. gravity load). At each step the equivalent load corresponding to the increased displacement is monitored

- Staged Construction Pushdown Analysis (SC-PD) where the gravity loads are applied on the undamaged structure and then the column is suddenly removed, while the gravity loads remain unchanged

Since the original loading pattern is equivalent in both LC-PD and DC-PD, the results remain the same in both cases, until the ultimate loads are reached. For the purpose of this analysis, the second approach has been adopted (i.e. Displacement Control Pushdown Analysis).

According to the prescription of the CNR "Istruzioni per la valutazione della robustezza delle costruzioni" [4], here reported in Chapter 1 of this thesis, the pushdown method is firstly a *direct design method*, because it aims at explicitly evaluate the capacity of the structure. Among the possible direct methods, this can be seen as an *alternative load path method*, since it aims at avoiding the collapse thanks to a load redistribution made by the sustaining elements. Moreover, a *non-linear analysis* is adopted, such that it takes into account the non-linear behaviour of both concrete and steel, according to which the dissipation of energy and redistribution of loads can be modelled. Finally, it is a *static analysi*s, thus it can emphasize the geometrical non-linearities and catenary effects.

For the purpose of this analysis, the software ATENA 2D has been used. Materials, geometry and load characteristics, have all been explained in the previous sub-sections. The remaining considerations regard the load cases and the analysis steps. The frame is the one without the central column (right part of the Figure 5.19), since the pushdown analysis should be performed on the damaged structure, as already mentioned.

5.6.1 Load cases

The following table shows the load cases that have been considered for this analysis:

Table 5.9: Load cases for Analysis 2 - Pushdown analysis

Load case number	Description
1	Base support
6	Imposed displacement of 1.5 cm
7	Imposed displacement of 3.0 cm

The base support is a fixity, created according to sub-section 4.2.4.1 of the previous chapter and applied at the lines composing the basis of the four columns. The imposed displacements are created according to sub-section 4.2.4.4, defined as *Prescribed Deformation* in ATENA 2D. They have been assigned at the node of the centreline of the central column, in correspondence of the top of the frame (Figure 5.20: Scheme of the pushdown analysis).

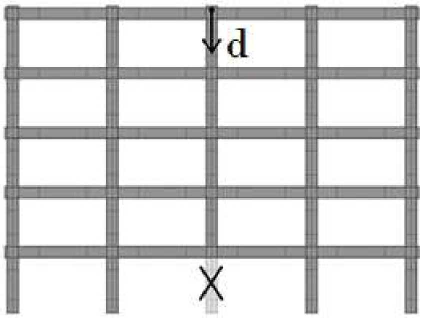


Figure 5.20: Scheme of the pushdown analysis

5.6.2 Analysis steps

The analysis steps adopted are the following:

- Analysis steps from 1 to 10: load cases 1 and 6 with a multiplier of 1.0
- Analysis steps from 11 to 45: load cases 1 and 7 with a multiplier of 1.0

According to this, at the end of the step 10, the imposed displacement is equal to 15 cm, while at the end of the 45th step, additional 1.05 m have been imposed, such that the total amount of imposed displacement in the last step is equal to 1.2m. The choice of imposing the displacement in two different way, lower for the first 10 steps and larger on the lasts, is due to numerical issues and to better capture the change in slope of the capacity curves. All the analysis steps have been performed according to the *Standard Newton-Raphson* defined in sub-section 4.2.5.4, with a maximum of 40 iterations.

The monitoring points have been inserted according to what is drawn in Figure 5.21: Location of the monitoring points on the 2D frame for the Analysis 2 - Pushdown Analysis. In particular, two monitoring points at the top of the central column are needed to monitor the reaction and the displacement during the pushdown analysis. The lateral monitoring points are necessary to control the lateral displacements of the structure during the analysis.

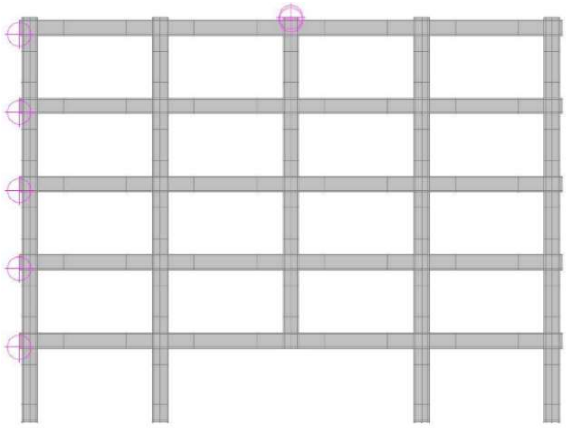


Figure 5.21: Location of the monitoring points on the 2D frame for the Analysis 2 - Pushdown Analysis

After having run the analysis, the output to be elaborated is the capacity curve, i.e. a displacement-reaction curve. To do so, the CCO file can be exported from ATENA 2D, which contains the results of the monitoring points, in the specific case the two points on top of the central column where displacements and reactions are monitored at each step.

For the purpose of this analysis, *N* pushdown analyses, thus *N* capacity curves have been elaborated, where every time material characteristics change. The following figure shows the capacity curves on the left (i.e. curve displacement-load) while, on the right, the non-dimensional capacity curves are reported, where displacement and loads have been divided for the correspondent peak values.

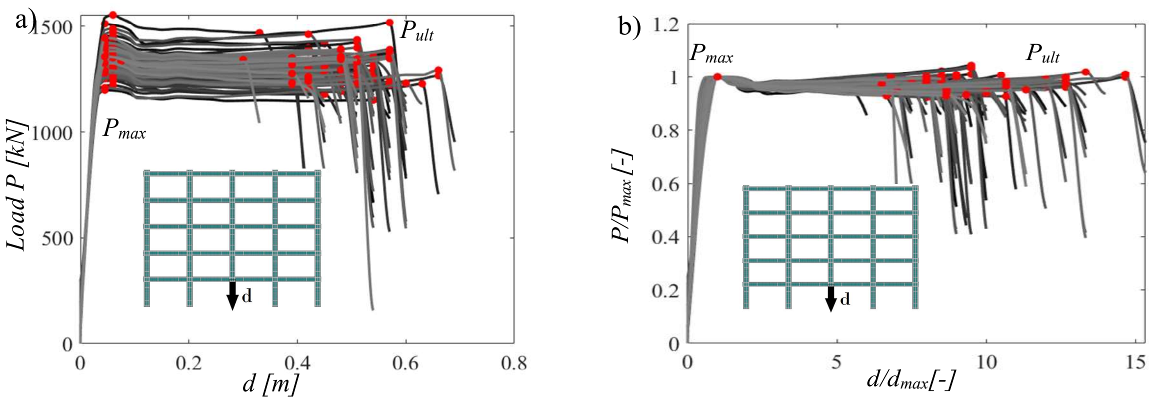


Figure 5.22: Results of pushdown analysis on the *N* simulations, showing the collapse branch: a) curve displacement-load; b) capacity curve non-dimensional with respect to the peak values

All the curves show a first peak in the resistance, corresponding to the flexural behaviour, then a softening phase followed by a constant-resistance phase occur. After having reached the ultimate capacity, failure happens, thus the curves descend drastically.

The curves of Figure 5.22 has been stopped at a step after the ultimate value P_{ult} is reached; even if what happens after the ultimate value has no meaning, from a physical point of view, it is interesting to notice how the ultimate capacity descends when the failure is reached.

The following curves (Figure 5.23) are the same of the previous, but shown without the descending branch, that, in practise, has no physical meaning because when the ultimate value is reached, the structure collapses.

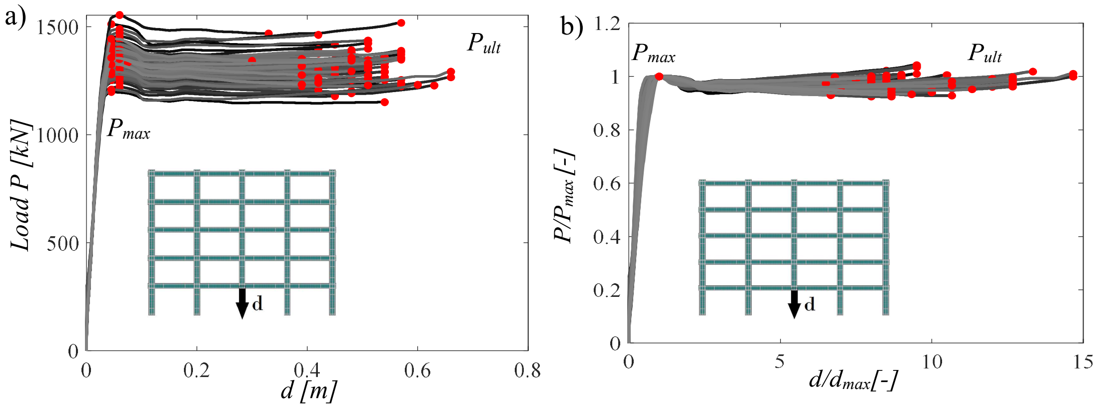


Figure 5.23: Results of pushdown analysis on the N simulations, stopped at the ultimate value: a) curve displacement-load; b) capacity curve non-dimensional with respect to the peak values

From the Figure 5.23b it is easier to observe the capacity recovery, in the catenary response, if present, i.e. when the P_{ult} is larger than the P_{max}. It is interesting to notice that in the majority of the cases (89 %), the ultimate resistance is lower than the peak one, meaning that the catenary branch is governed by a ductility capacity of the reinforcement bars rather than a catenary effect. This aspect can be observed by looking at the horizontal displacements of the external column, that is always outward, both when the maximum and ultimate load values are reached meaning that the beam are always in their compressive phase (Figure 5.24a).

In the following, the two significant graphs presented in chapter 1, to evaluate the membrane effects involved in the accidental event of a sudden column loss, are reported for a generic simulation among the N possible. The Figure 5.24a, shows the horizontal negative (outward) displacement of the external column versus the increasing vertical imposed displacement of the central one. At the ultimate state, when the displacement is about half a meter, the horizontal displacement is not able to change its sign (becoming positive, thus inward), due to the absence of the catenary effect.

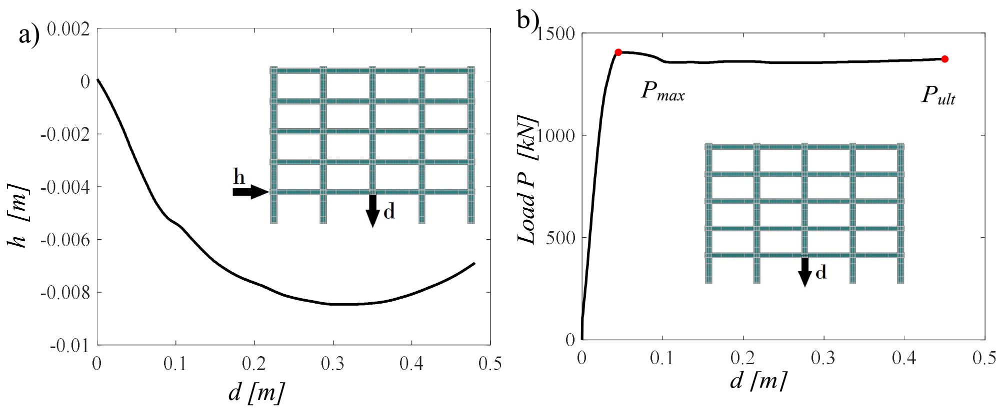


Figure 5.24: Pushdown analysis results for a generic simulation: a) vertical displacement of the central column vs horizontal displacement of the lateral column; b) vertical displacement of the central column vs load

5.6.3 Dynamic amplification coefficient

The accidental event of a sudden column loss as regards the structural response is a dynamic event, accomplished by strong non-linearities in terms of geometry and material behaviour. Although DoD provisions[16] suggest to perform a dynamic non-linear analysis on the damaged structure, this is not the best solution since this type of analysis is not always easy, especially for the computational demand and the difficulties for practical application in structural design. Thus, a static non-linear analysis, considering an equivalent dynamic approach can be a better solution. This is allowed by the DoD and GSA guides [16][42], where a static assessment is used, based on a constant dynamic amplification factor (DAF) $\lambda_d = 2$, for gravity loading above the lost column. However, a different approach can be used in order to compute numerically the DAF, since the aforementioned value has been judged too much conservative. In particular, Izzuddin [1] has proposed a method, applied on a steel multi-storey building, where a different range of λ_d has been computed (i.e. between 1.3 and 1.5).

The idea behind this method is that the phenomenon of sudden column loss is similar to a sudden application of gravity load on the affected sub-structure; at the beginning, at the instance of column loss, the gravity load is larger than the static structural resistance, due to the dynamicity of the phenomenon, thus the incremental of deformations is transformed into additional kinetic energy, causing an increase in the velocities. Since deformations increase more and more, the static structural resistance increases as well, causing a reduction in the kinetic energy, i.e. in the velocities. The maximum dynamic displacement is reached when its derivative is zero, thus, according to the definition of kinetic energy, when the kinetic energy is reduced back to zero. This coincides with the point at which the work done by the gravity loads becomes equal to the energy absorbed by the structure.

The expressions of the internal energy U_n and external work W_n are followingly reported:

$$U_n = \int_0^{u_{d,n}} P \, du_s \tag{5.21}$$

$$W_n = \lambda_n P_o u_{d,n} \tag{5.22}$$

Where the internal energy U_n corresponds to the area under the capacity curve (i.e. curve displacement u_s – load P) up to $u_{d,n}$ while the external work W_n is the product between the level of sudden applied gravity loading ($P_n = \lambda_n P_o$) and the corresponding maximum dynamic displacement ($u_{d,n}$). This can be graphically seen by the following figure:

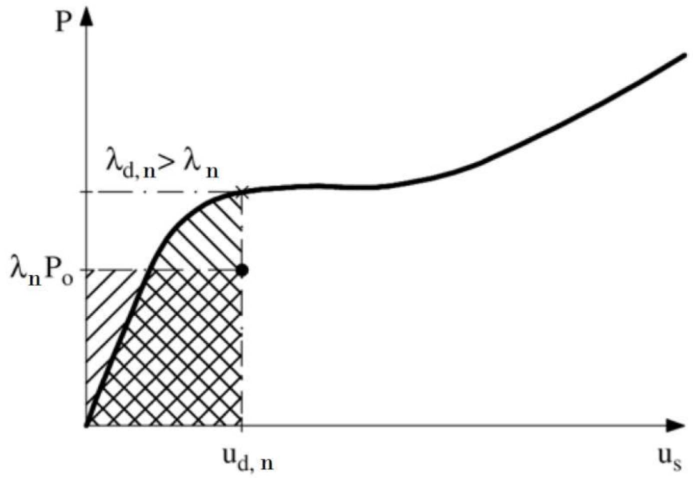


Figure 5.25: Energy balance approach (Izzuddin [1])

According to the previous considerations, by equating (5.21) and (5.22), the dynamic amplification factor can be evaluated:

$$\lambda_{d,n} = P_d/P_o \tag{5.23}$$

According to this, P_d is the dynamic load, i.e. the level of amplified loading that correspond to a dynamic displacement, computing as the point of the capacity curve corresponding to $u_{d,n}$.

To evaluate the level of sudden applied gravity loading P_o, this can be seen as equal and opposite to the reaction applied at the point of the removed column, according to the procedure that has been followed in sub-section 5.5. Two situations can occur: if the two curves (i.e. internal energy curve and external work curve) find an intersection, the structure is able to sustain the removal of the column, otherwise, the equilibrium is not reached under that loading-material conditions.

The procedure of equating the energies has been conducted for all the N simulations, and the result is that the 53% of the curves find a state of equilibrium, thus the two curves intersect. In the remaining 47% there is not any intersection, meaning that the structure, with that materials and loads combination, is not able to sustain the event of a sudden column loss.

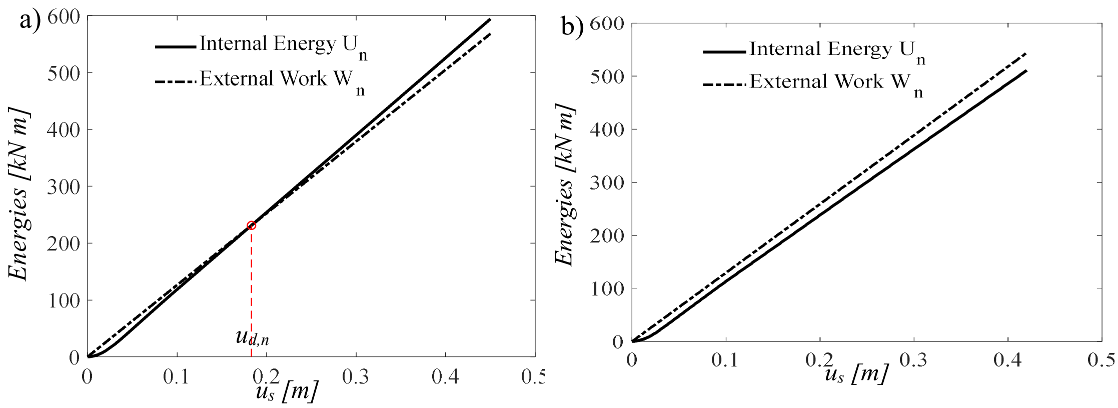


Figure 5.26: Energy curves: a) case where the equilibrium is reached b) case where the equilibrium is not reached

For all the cases where the equilibrium is reached, the dynamic displacement is found, correspondingly to the displacement at the intersection point ($u_{d,n}$ in Figure 5.26a). After that, the corresponding load on the capacity curve is found and saved as P_d and then the DAF is computed according to formula (5.23).

For the other cases, where the intersection is not present, the DAF is computed as the maximum possible DAF, i.e. computed as:

$$\lambda_{d,n}^{*} = P_{max}/P_{o} \tag{5.24}$$

where P_{max} corresponds to the first peak of the capacity curve, above which the structure is no more able to sustain other gravity loads, under sudden column loss.

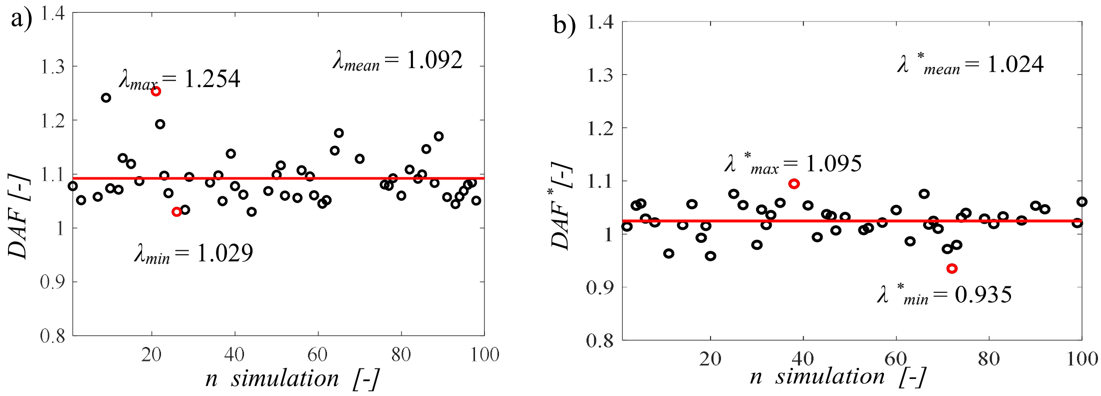


Figure 5.27: Dynamic Amplification factors for the N simulations: a) DAFs for the 53 simulations where equilibrium is reached; b) DAF*s for the 47 simulations where equilibrium is not reached

Figure 5.27 shows the values of the DAF and DAF* as function of the number of simulations (from 1 to 100). On the left-hand side, the DAF regards the cases where the equilibrium is reached, even in the event of the column loss. The maximum amplification coefficient is $\lambda_{max} = 1.254$, the minimum is $\lambda_{min} = 1.029$ and the mean value is $\lambda_{mean} = 1.092$. For what concerns the right hand side of the figure, the DAF*s are reported, regarding the cases where the structure collapses because of the accidental event. The coefficient values are characterized by $\lambda_{max}^{*} = 1.095$, the minimum is $\lambda_{min}^{*} = 0.935$ and the mean value is $\lambda_{mean}^{*} = 1.024$. It is interesting to notice that there are nine cases, among the 47 where the equilibrium is not reached, where the DAF* is lower than one, meaning that the maximum value of the capacity curve is lower than the dynamic gravity loading. From a physical point of view, this means that the structure is not able to sustain the gravity loading itself, no matter the event of the column loss.

5.6.4 Dynamic linear analysis simulation

At this point of the analysis, it is wondered if to apply the dynamic amplification factor only on the central spans of the frame and disregarding what could happen elsewhere, could be a limiting assumption in the context of the aim of this work.

To validate this assumption, it was first modelled the frame in ADINA, by means of a dynamic linear analysis. ADINA is the acronym for Automatic Dynamic Incremental Nonlinear Analysis. It is a commercial engineering simulation software program used worldwide in industry and academia to solve structural, fluid, heat transfer and electromagnetic software. Since this model was needed only for validation purposes, only one model has been created having as input the mean values of the sampled material and load variables.

For the scope of this thesis, the *ADINA Structures* tool has been used, and the *Dynamic-Implicit* linear analysis has been conducted. Again, two models are needed: the first, with the central column, in order to compute the reaction at the top of column itself and the second, without the column, so to simulate the sudden column loss.

A unique material has been modelled with as an Isotropic Linear Elastic Material, with the following characteristic:

- Elastic modulus $E = 31500000\ kN/m^2$
- Poisson's ratio $\upsilon = 0.3$
- Density $\rho = 25\ kN/m^3$

The geometry is created according to the dimensions already discussed while for the mesh it has been used a *Shell* type mesh, with thickness of 0.4 m for the beams and 0.6 m for the columns, and mesh size of 0.1 m. The only constraints are the fixed supports at the base of the columns.

As already mentioned, the loadings are defined in terms of mean values, thus:

- G_1 self-weight of beams and columns computed by the software as function of $\rho = 25\ kN/m^3$ geometrical dimensions
- $G_1 = 16\ kN/m$ permanent structural load of the slab

- $G_2 = 13\ kN/m$ permanent non-structural load
- $Q_p = 6.5\ kN/m$ variable load on the floors
- $Q_c = 1.6\ kN/m$ variable load on the roofing

For the model with the column, the time steps have been selected such that:

- From 0 to 5 the loadings increase linearly up to their maximum value
- From 0 to 29.5 the loadings are maintained constant

At the end, the reaction at the base of the central column has been computed and the weight of the removed column has been subtracted.

For the model without the column, the time steps have been created such that:

- From 0 to 5 the loadings and the reaction increase linearly up to their maximum value
- From 5 to 9 the loadings and the reaction remain constant
- From 9 to 9.2 the loadings remain constant and the reaction goes linearly to zero (so to simulate the removal of the column)
- From 9.2 to 29.2 the loadings remain constant and the reaction is maintained equal to zero

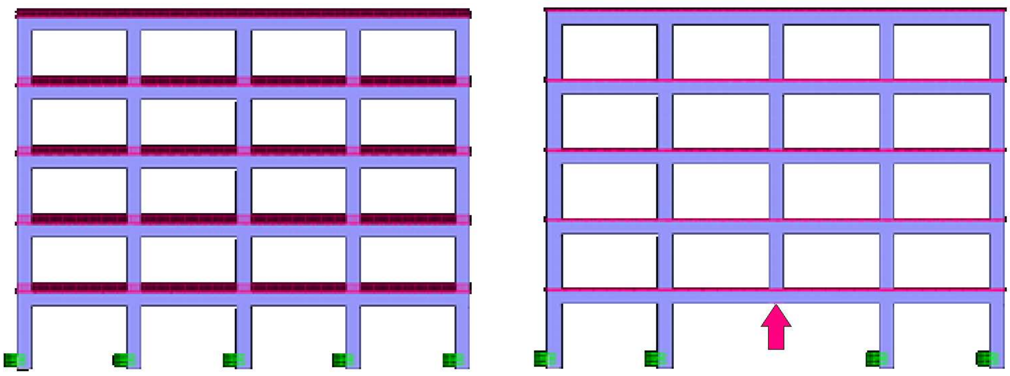


Figure 5.28: Frame modelled in ADINA Structures: on the left hand side model with the column; on the right hand side model without the column and reaction at the centre of the first floor

The following picture shows the output of the simulation, in particular it is shown the displacement of the point at the centre of the first floor, where the column is removed, as function of time.

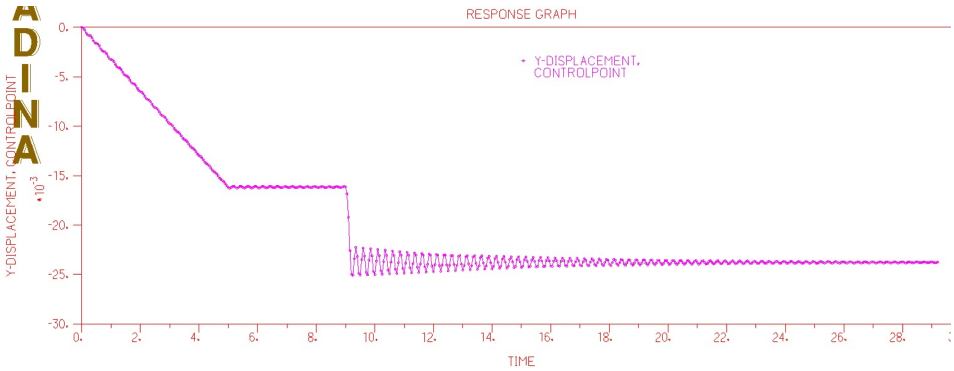


Figure 5.29: Output of the Dynamic Linear analysis on ADINA: displacement versus time

The ratio between the dynamic displacement (i.e. the peak value of the displacement after the sudden removal of the column) and the static one (i.e. when the displacement reaches the steady-state response) has been computed. This can be seen as a measure of the DAF. This result has been compared with the same ratio, but computed at different points of the external spans of the structure. It has been observed that the ratio is not comparable with the former, meaning that the DAFs should be applied only on the central spans of the structure.

To conclude, this assumption is commonly used in literature: [1] on evaluating the progressive collapse of steel-framed buildings, [43] for the vulnerability of three steel frames to disproportionate actions, [41] to study steel moment resisting frames designed according to current seismic codes and [44] for a numerical study on the structural response of a 5-story steel frame building under the sudden loss of columns.

5.7 Analysis 3 - Reliability Analysis

This is the last step that should be performed in order to reach the goal of this thesis, i.e. evaluation of the reliability of this structure, designed according to robustness and seismic criteria. To do so, the information given by the previous two analyses, in terms of reaction at the point where the column is removed (from the analysis 1) and the dynamic amplification factors (from the analysis 2) is used, for all the N simulations. After that, the strains at relevant nodes of the frame are extrapolated and a local and then global reliability evaluation is conducted.

The frame under analysis, in terms of geometrical and mesh characteristics, is the one without the central column, as described in sub-section 5.4 (right part of the

Figure 5.19). In the following, more details are given about the load cases and analysis steps, as modelled in the NLFEA software ATENA 2D.

5.7.1 Load cases

The following table shows the load cases that have been considered for this analysis:

Table 5.10: Load cases for Analysis 3 - Reliability analysis

Load case number	Description
1	Base support
2	Self-weight
3	Permanent structural loads
4	Permanent non-structural loads
5	Variable loads
6	Amplified columns weight
7	Amplified beams weight
8	Amplified permanent structural loads
9	Amplified variable loads
10	Amplified permanent non-structural loads
11	Temporary reaction

The base support is a fixity, created according to sub-section 4.2.4.1 of the previous chapter and applied at the lines composing the basis of the four pillars. The self-weight is applied on the whole structure, as function of geometry and specific weight ρ and computed according to sub-section 4.2.4.2. The other gravity loadings, defined in load cases 3,4 and 5, are applied as loads per meter on the upper horizontal lines of all the beams (apart from the variable loads where distinction is made between floors and roofing). The other load cases, i.e. from 6 to 10, regard only the lines of the beams of the central spans, since these are the only gravity loadings that should be amplified according to the DAFs and DAF*s previously computed. Finally, the temporary reaction is a load per meter applied oppositely to the gravity loadings, used to simulate the presence of the column. Thus, it is equal to the concentrated reaction P_0, divided by 0.6 m to obtain the load per meter, and applied at the line of the top of the column removed. These last load cases have been created according to the procedure for *Force* load cases, as described in 4.2.4.3.

5.7.2 Analysis steps

Different assumptions have regarded the analysis steps, depending on the simulation that has to be performed, i.e. if the equilibrium has been found or not according to the energy-balance calculations, and on the value of λ or λ^*:

If the simulation reaches an equilibrium, and the $1.1 \leq \lambda < 1.2$:

- Steps from 1 to 10: load cases 1, 2, 3, 4, 5 and 11 with multiplier 0.1
- Steps from 11 to 20: load cases 1 and 11 with a multiplier of -0.1
- Step 21: load cases 1 and 6, 7, 8, 9, 10 with a multiplier equal to 0.1
- Step 22: load cases 1 and 6, 7, 8, 9, 10 with a multiplier equal to $\lambda - 1.1$

If the simulation reaches an equilibrium, and the $\lambda \geq 1.2$:

- Steps from 1 to 10: load cases 1, 2, 3, 4, 5 and 11 with a multiplier of 0.1
- Steps from 11 to 20: load cases 1 and 11 with a multiplier of -0.1
- Step 21: load cases 1 and 6, 7, 8, 9, 10 with a multiplier equal to 0.1
- Step 22: load cases 1 and 6, 7, 8, 9, 10 with a multiplier equal to 0.1
- Steps 23: load cases 1 and 6, 7, 8, 9, 10 with a multiplier equal to $\lambda - 1.2$

If the simulation does not reach an equilibrium, and the $\lambda^* < 1.0$:

- Steps from 1 to 10: load cases 1, 2, 3, 4, 5 and 11 with a multiplier of 0.1
- Steps from 11 to 20: load cases 1 and 11 with a multiplier of -0.1

In the other cases, i.e. for $\lambda < 1.1$ or $\lambda^* > 1.0$:

- Steps from 1 to 10: load cases 1, 2, 3, 4, 5 and 11 with a multiplier of 0.1
- Steps from 11 to 20: load cases 1 and 11 with a multiplier of -0.1
- Step 21: load cases 1 and 6, 7, 8, 9, 10 with a multiplier equal to $\lambda - 1.0$ or $\lambda^* - 1.0$

The first 20 analysis steps are equal in all the N simulations. In particular, according to the first ten, the frame is completely loaded with gravity loadings distributed on the entire frame and contemporarily there is the temporary reaction, i.e. load case 11, which simulates the presence of the column. Then, on the subsequent ten steps, thus until the twentieth, the reaction is completely removed so to simulate the column loss, while the gravity loadings remain loaded. The reason why 10 steps with a multiplier of 0.1 have been adopted, is to avoid numerical problems due to excessive loadings, especially for the phase when the column is removed.

The analysis step 21 is the one starting from which different assumptions have been considered depending on the simulations; in particular, this is the step where an additional percentage of gravity loadings, only on the central spans of the frame is added, according to the DAF or DAF*. If its value is lower than 1.0, as has happened in some cases where the equilibrium has not been reached, the structure is expected to fail at the twentieth analysis step, because not able to reach an equilibrium even under the static gravity loadings (i.e. without any accidental event). If the DAF is larger than 1.1, the amplification is made in two or three consecutive steps, i.e. in the 21st and 22nd, or in the 21st, 22nd and 23rd, in order to avoid numerical problems due to a large multiplier. Finally, if the DAF is lower than 1.1, as it happens in the majority of the cases, the gravity loadings are amplified at the 21st step and all the simulations, where the equilibrium is not reached, are expected to fail at that step.

To verify the results of the energy-balance model, i.e. to verify that some of the simulations fail under a specific combination of materials and loads while others reach an equilibrium, additional twenty analysis steps, i.e. up to the 40th (or 41st or 42nd or 43rd depending on the case) whit a multiplier of 0.005, have been considered. The results are that:

- for the simulations where the equilibrium is expected to be reached, even with an additional amplification of the gravity loadings of more than 7%, the structure resists.
- for the simulations where equilibrium is not expected to be reached, but the DAF* is larger than 1, the structure collapses at the 21st step, or around that, meaning with an additional amplification of 2%
- for the simulations where equilibrium is not expected to be reached, but the DAF* is lower than 1, the structure collapses at the 20st step

Moreover, it has been observed that for all the simulations where the equilibrium was reached according to the energy-balance model, if the DAF is large enough, i.e. larger than 1.1, the structure has always resisted to an additional amplification of the 10%. This represents a validation of the physical meaning of the Dynamic Amplification Factor; the larger it is, the more gravity loading the structure is able to sustain under the event of sudden column loss.

For what concerns the Solution Parameters adopted, for the first ten analysis steps the *Standard Newton-Raphson* defined in sub-section 4.2.5.4, has been used. For the remaining steps, a new *Newton-Raphson* solution method has been selected, with less restrictive criteria. In particular, all the error tolerances have been doubled with respect to the ones of the standard method, i.e. displacement error tolerance 0.02, residual error tolerance 0.02, absolute residual error tolerance 0.02 and energy error tolerance 0.0004. Finally, the number of iterations has been increased up to 200.

5.7.3 Output analysis

After having run the analysis, for all the N simulations, the output to be elaborated are the strains at the nodes. This can be obtained thank to the *TextPrintout* command that allows to export the *Principal Total Strains* of the step where the gravity loadings of the central spans are amplified, i.e. step 20, 21, 22 or 23 depending on the case, on all the nodes. After that, a MATLAB code has been written in order to manipulate these data. In particular, the strains have been considered in specific position of the structures, according to the following scheme:

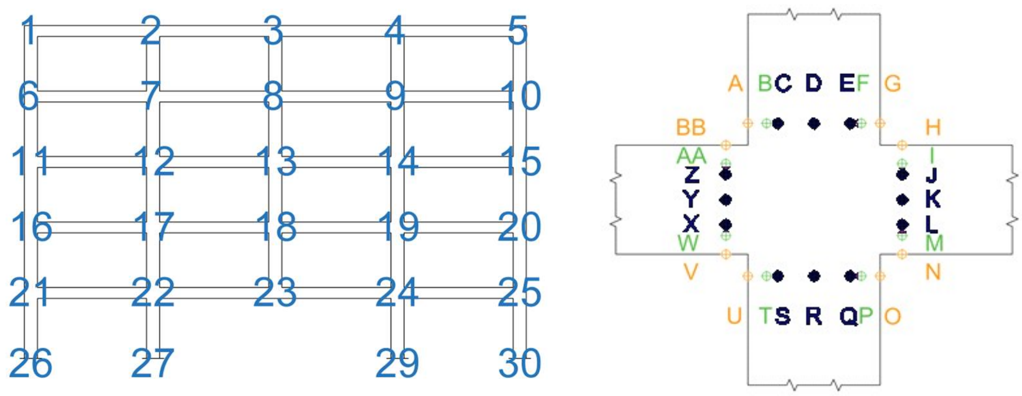


Figure 5.30: Scheme for the strains evaluation: a) location of the sections; b) location of the nodes per each section

The inner nodes (in black) defined by the letters J, K, L, X, Y, Z are the nodes of the confined concrete, part of the dissipative zone of the beams, thus having constitutive law according to Figure 5.15a. The other inner nodes (in black) defined by the letters C, D, E, Q, R, S are nodes of the confined concrete as well, but the constitutive law, according to Figure 5.15c, is different since they are parts of the column. For what concerns the external nodes (in orange), i.e. A, G, H, N, O, U, V, BB, they define the concrete cover, so non-confined concrete. Finally, the intermediate nodes (in green) defined by B, F, I, M, P, T, W, AA represent the reinforcing longitudinal bars.

Moreover, distinction is made between the sections of the central spans and the sections of the external ones:

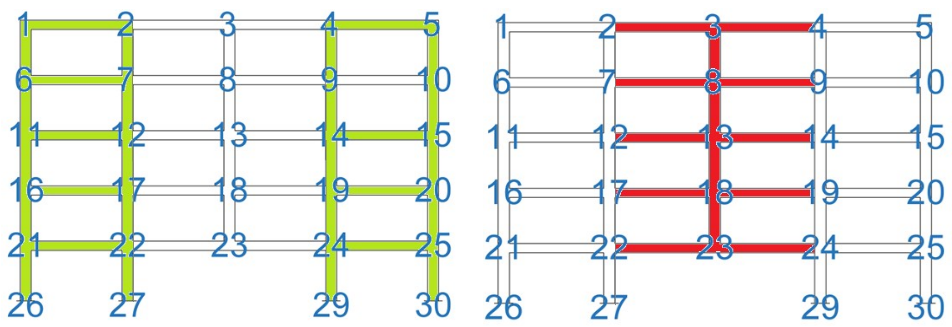


Figure 5.31: Scheme of the sections: a) directly affected members; b) indirectly affected members

5.7.4 Calculation of local probability of failure P_f^{max}

For each section, and for each node of the sections, the strains given as an output have been compared with the ultimate strains, both of the reinforcement and of the concrete. For the concrete the ultimate strain is intended as the one corresponding to the peak. The procedure that has been followed is above explained:

1) Nodes where the concrete is in tension have been removed from the data, since only the failure of concrete in compression has to be evaluated, for these points the probability of failure is the minimum realistic $P_f = 10^{-7}$

2) Among the values of concrete in compression, the nodes of the concrete cover have been removed, since it has been decided to evaluate only the failure of the confined concrete
3) The probability of failure P_f is computed for all the nodes, according to a convolution integral calculation
4) The local probability of failure for each sub-section is computed as the maximum between the probabilities (i.e. P_f^{max}). The sub-section is intended as one of the 4 sides composing the section (e.g. the ensemble of nodes ABCDEFG of a generic section)

The evaluation of the failure probability has been already explained in the previous chapters (i.e. 1.6 and 2.3.1). For the specific case a convolution integral has been used, according to the following expression:

$$P_f = P(S > R) = \int_{r=-\infty}^{\infty} \int_{s=-\infty}^{r} f_{S,R}(s,r)dsdr \qquad (5.25)$$

where $f_R(x)$ is the probability density function of the resistance R (ultimate strains) and $f_S(x)$ is the probability density function of the demand S (strains at the nodes). The two PDFs are obtained respectively from the vector of the ultimate strain values, sampled N times, and the strains taken as outputs from the N FEM simulations. According to this, the convolution integral represents the probability that the strains at the nodes have reached or overcome the ultimate strains.

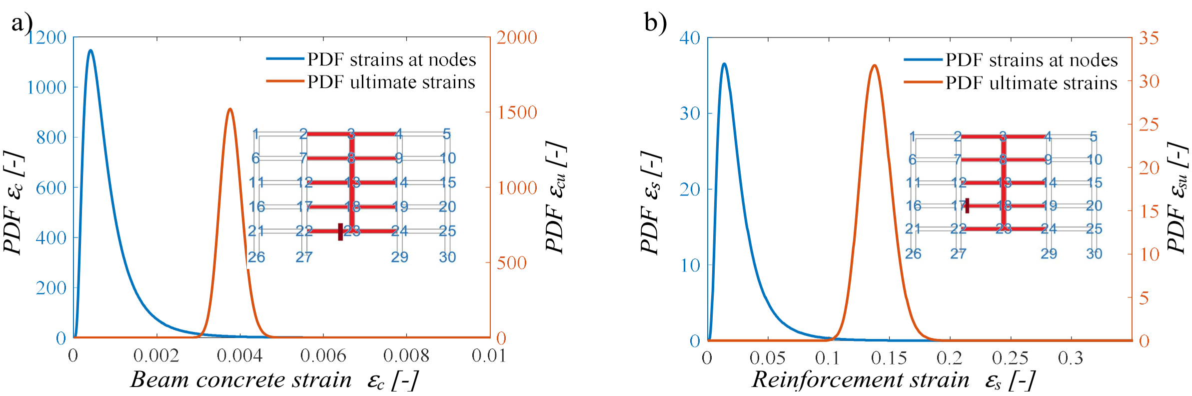


Figure 5.32: Convolution integral for the maximum P_f of directly affected sub-sections: a) beam confined concrete strain; b) beam reinforcement strain

Two situations have been observed (Figure 5.33: Scheme of the failure probabilities at each sub-section):

- For the sub-sections indirectly affected by the column loss (Figure 5.31a), i.e. located outside the central spans, the P_f^{max} is lower than the limiting value everywhere
- For the sub-sections directly affected by the column loss (Figure 5.31b), i.e. located inside the central spans, the P_f^{max} is always different than zero and larger than 10^{-4} for the longitudinal reinforcement of the beams. While it is lower than the limiting value on the columns' nodes.

This can be observed in the following scheme:

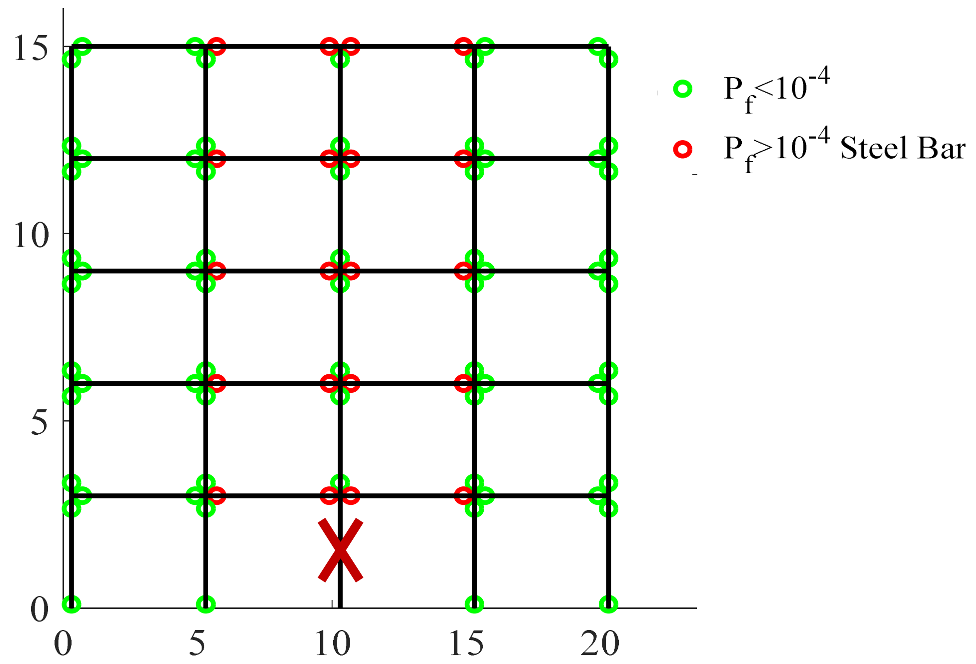


Figure 5.33: Scheme of the failure probabilities at each sub-section

Moreover, the results in Figure 5.34 show that, among the nodes of the central spans, there is a larger local probability of failure at the bars close to the joints beam column, and in particular in correspondence of the lateral pillars, this is due to the fact that plasticization occur in order to allow for the displacement of the nodes where the column is removed. These points are indeed the more stressed. Furthermore, there are slightly larger probabilities in the nodes of the middle floors, i.e. the second and the third floor.

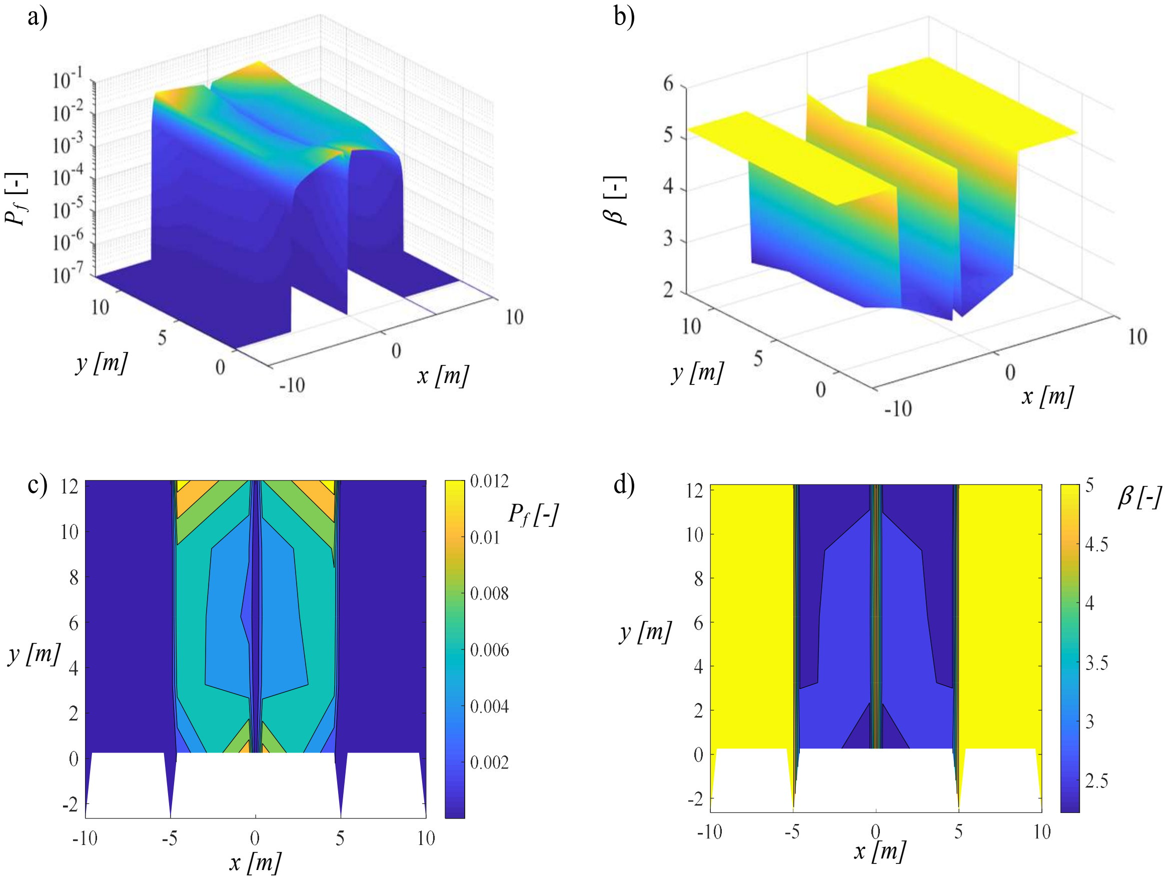


Figure 5.34: Failure probabilities and reliability indices of the nodes located at a distance (x,y) from the point where the column is removed: a) 3D plot of P_f; b) 3D plot of β; c) Contour plot of P_f; d) Contour plot of β

Conclusions

This work of thesis has consisted on a method to evaluate the reliability of a structure, designed according to robustness criteria, located in a high seismicity area, i.e. L'Aquila city.

At first, the structural detailing based on capacity design and code rules has been calculated, referring to a design working life of 50 years. Since the structure, according to that detailing, has been judged not adequate from a robustness point of view, in particular not able to create alternative load paths under the phenomena of sudden column loss, improvement have been needed. In particular, the robustness has been enhanced by creating continuity of the bars along each beam, symmetry of the bars between the upper and lower chord and equality of the bars between the floors.

A probabilistic method has been adopted to evaluate the reliability of the structure, designed according to the improvements previously defined. In particular, 10 basic variables have been sampled in line to what is prescribed by code rules (fib Model Code 2010 and Probabilistic Model Code), in terms of probabilistic distribution, mean value and coefficient of variation. For the *Resistances*, the basic variables are concrete compressive strength, reinforcement yield strength, reinforcement ultimate strength, reinforcement ultimate strain, reinforcement elastic modulus. For the *Actions,* the basic variables are the reinforced concrete specific weight, the permanent structural loads, the permanent non-structural loads, the variable loads of the floors and the variable loads of the roofing. A sample of 100 elements for each variable has been judged enough to perform the analysis, since the minimum should be of 30-40.

A Static Non-Linear Analysis has been evaluated the best method to perform the reliability analysis, by simulating the removal of the central column and the ATENA 2D software has been used for this purpose. Two preliminary analyses have been needed for this scope, in particular for the computation of the Dynamic Amplification Factors (DAFs): one with the presence of the central column to compute the reaction at the top of the removed column and another, called pushdown analysis, to compute the capacity curves, both performed for all the 100 simulations.

The model used for the computation of the DAFs is based on an energy-balance method and it has resulted in a distinction between three possible situations: a case where the specific combination of basic variables has caused the collapse of the structure under the event of central column loss, a case where the collapse happens before the accidental event, because too unfavourable combinations of variables and the last case where the structure is able to sustain the accidental event.

By performing the reliability analysis, i.e. removing the central column and amplifying the loads only on the central spans of the frame, the results found with the energy-based model have been confirmed: all the cases where the structure should have collapsed, have not effectively reached the convergences criteria of the FEM reliability analysis, thus have failed.

Moreover, the results of the reliability analysis have conducted on a probability of failure computation, by comparing the ultimate strains with the strains at the nodes, for all the sections at the joints beam-column of the frame. This has confirmed that the structure can be judged reliable, since only the central spans of the frame reach a too large probability of failure and in particular on the points where plastic hinges form, leading to a stresses' redistribution.

Future works can follow from this work of thesis, in particular it could be interesting to apply this methodology, i.e. reliability evaluation with a static non-linear model and dynamic-equivalency in the amplification of the loads, on a frame designed according to capacity design criteria. Other possibilities could be to adopt a Dynamic Non-Linear approach to reach the same goal of this thesis.

References

[1] B. A. Izzuddin, A. G. Vlassis, A. Y. Elghazouli, and D. A. Nethercot, “Progressive collapse of multi-storey buildings due to sudden column loss - Part I: Simplified assessment framework”, *Eng. Struct.*, vol. 30, no. 5, pp. 1308–1318, May 2008.

[2] fib, "fib Model Code for Concrete Structures", 2010 Weinheim, Germany: Wiley-VCH Verlag GmbH & Co. KGaA, 2013.

[3] JCSS, “JCSS - Probabilistic Model Code”, 2001.

[4] CNR (Consiglio Nazionale delle Ricerche), "Istruzioni per la valutazione della robustezza nelle costruzioni", 2018.

[5] European Committee for Standardization, “Eurocode: Basis of structural design", 1990.

[6] European Committee for Standardization, “Eurocode 1: Action on structures - Part 1-7: General actions - Accidental actions”, 1991.

[7] European Committee for Standardization, “Eurocode 2: Design of concrete structures - Part 2: Concrete Bridges - Design and detailing rules”, 2005.

[8] Ministero delle Infrastrutture e dei Trasporti, “Norme tecniche delle costruzioni", 2008.

[9] Ministero delle Infrastrutture e dei Trasporti, “Decreto 17 gennaio 2018: Aggiornamento delle "Norme tecniche per le costruzioni", 2018.

[10] J. K. Mitchell and R. J. Blong, “Volcanic Hazards: A Sourcebook on the Effects of Eruptions”, *Geogr. Rev.*, vol. 76, no. 1, p. 113, Jan. 1986.

[11] J. Suda, A. Strauss, F. Rudolf-Miklau, and J. Hübl, “Safety assessment of barrier structures”, *Struct. Infrastruct. Eng.*, vol. 5, no. 4, pp. 311–324, 2009.

[12] European Committee for Standardization, “Eurocode 1: Actions on

structures - Part 1-7: General actions - Accidental actions”, 2006.

[13] U.S. Nuclear Regulatory Commission, "Regulatory Guide 50.150 Aircraft impact assessment", Washington D.C., 2018.

[14] M. E. Paté-Cornell, “Quantitative safety goals for risk management of industrial facilities”, *Struct. Saf.*, vol. 13, no. 3, pp. 145–157, Mar. 1994.

[15] ASCE/SEI 7-05, "Minimum Design Loads for Buildings and Other Structures", Reston, VA: American Society of Civil Engineers, 2005.

[16] D. Stevens *et al.*, “DoD Research and Criteria for the Design of Buildings to Resist Progressive Collapse”, *J. Struct. Eng.*, vol. 137, no. 9, pp. 870–880, Sep. 2011.

[17] U. Starossek and M. Haberland, “Disproportionate Collapse: Terminology and Procedures”, *J. Perform. Constr. Facil.*, vol. 24, no. 6, pp. 519–528, Dec. 2010.

[18] S. Kokot and G. Solomos, “Progressive collapse risk analysis: literature survey, relevant construction standards and guidelines”, Publications Office of the European Union, 2012.

[19] H. S. Lew, Y. Bao, F. Sadek, J. A. Main, S. Pujol, and M. A. Sozen, “An Experimental and Computational Study of Reinforced Concrete Assemblies under a Column Removal Scenario”, Gaithersburg, MD, Oct. 2011.

[20] M. Holicky, A. Materna, G. Sedlacek, and L. Sanpaolesi, "Implementation of Eurocodes Handbook 1 - Basis of structural design. Leonardo Da Vinci Project", Garston, Watford, UK, 2004.

[21] M. Holicky *et al.*, "Implementation of Eurocodes Handbook 2 - Reliability backgrounds. Leonardo Da Vinci Project", Prague, 2005.

[22] D. Gino, “Advances in reliability methods for reinforced concrete structures”, Politecnico di Torino, 2019.

[23] A. Haldar and S. Mahadevan, “Probability, Reliability and Statistical Methods in Engineering Design”, *Bautechnik*, vol. 77, no. 5, May 2000.

[24] International Standard Organization, “ISO 2394 - General principles on reliability for structures”, 1998.

[25] A. Harbitz, “An efficient sampling method for probability of failure calculation”, *Struct. Saf.*, vol. 3, no. 2, pp. 109–115, Jan. 1986.

[26] F. Mauro, “Robustezza strutturale di edifici intelaiati in calcestruzzo armato: analisi parametrica e nuove proposte progettuali”, Politecnico di Torino, 2019.

[27] L. Capri, “Robustezza Strutturale di costruzioni multipiano in calcestruzzo armato: analisi parametrica di telai 2D per mezzo di modelli globali e locali”, Politecnico di Torino, 2019.

[28] European Committee for Standardization, “Eurocode 2: Design of concrete structures - Part 1-1 : General rules and rules for buildings”, 2004.

[29] European Committee for Standardization, “Eurocode 8: Design of structures for earthquake resistance - Part 1: General rules, seismic actions and rules for buildings”, 2003.

[30] EN European Standard, “EN 206-1 Concrete - Part 1: Specification, performance, production and conformity”, 2000.

[31] D. Grandić, “Shear resistance of reinforced concrete beams in dependence on concrete strength in compressive struts” , *Teh. Vjesn. Gaz.*, vol. 22, no. 4, pp. 925–934, Aug. 2015.

[32] M. Saatcioglu and S. R. Razvi, “Strength and ductility of confined concrete”, *J. Struct. Eng. (United States)*, vol. 119, no. 10, pp. 3109–3110, 1993.

[33] V. Červenka, L. Jendele, and J. Červenka, “ATENA Program Documentation Part 1 Theory”, 2013.

[34] V. Červenka and J. Červenka, “ATENA Program Documentation Part 2-1 User’s Manual for ATENA 2D”, 2013.

[35] A. J. Kappos, M. K. Chryssanthopoulos, and C. Dymiotis, “Uncertainty analysis of strength and ductility of confined reinforced concrete members”, *Eng. Struct.*, vol. 21, no. 3, pp. 195–208, 1999.

[36] Y. Lu and X. Gu, “Probability analysis of RC member deformation limits for different performance levels and reliability of their deterministic calculations”,

Struct. Saf., vol. 26, no. 4, pp. 367–389, 2004.

[37] A. Slobbe, Á. Rózsás, D. L. Allaix, and A. Bigaj-van Vliet, "On the value of a reliability-based nonlinear finite element analysis approach in the assessment of concrete structures", *Struct. Concr.*, no. December 2018, pp. 1–16, 2019.

[38] D. F. Wiśniewski, P. J. S. Cruz, A. A. R. Henriques, and R. A. D. Simões, "Probabilistic models for mechanical properties of concrete, reinforcing steel and pre-stressing steel", *Struct. Infrastruct. Eng.*, vol. 8, no. 2, pp. 111–123, 2012.

[39] Shamim, A. Sheikh, Dharmendra, V. Shah and S. K. Shafik, "Confinement of High-Strength Concrete Columns", 1994.

[40] S. Caprili and W. Salvatore, "Mechanical performance of steel reinforcing bars in uncorroded and corroded conditions", *Data Br.*, vol. 18, pp. 1677–1695, Jun. 2018.

[41] M. Ferraioli, A. Maria Avossa, and A. Mandara, "Assessment of Progressive Collapse Capacity of Earthquake-Resistant Steel Moment Frames Using Pushdown Analysis", *Open Constr. Build. Technol. J.*, vol. 8, no. 1, pp. 324–336, Dec. 2014.

[42] General Services Administration, "Progressive collapse analysis and design guidelines for new federal office buildings and major modernization projects", Washington (DC, USA), 2003.

[43] G. Xu and B. R. Ellingwood, "An energy-based partial pushdown analysis procedure for assessment of disproportionate collapse potential", *J. Constr. Steel Res.*, vol. 67, no. 3, pp. 547–555, Mar. 2011.

[44] H. R. Tavakoli, "Numerical Study of Progressive Collapse in Framed Structures: A New Approach for Dynamic Column Removal", *Int. J. Eng.*, vol. 26, no. 7 (A), Jul. 2013.