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DEPARTMENT OF CONTROL AND COMPUTER ENGINEERING

MASTER OF SCIENCE IN MECHATRONIC ENGINEERING

Simulink Control Model Of An Active Pneumatic Suspension System In Passenger Cars

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Abstract

Starting from 1950, greater interest has been gained on active suspensions with respect to semi-active and passive systems.
In a vehicle, the suspension system aims at satisfying the contrasting requirements of both comfort and handling, by insulating passengers from vehicle vibrations and providing constant tyre-road contact, respectively. One of the main advantages of active suspension is to provide the possibility of managing the trade off between vertical acceleration reduction and suspension travel usage through the choice of a suitable control strategy.
The aim of this work is to set up a basic control model of an automotive active pneumatic system, to meet the essential performances of control and comfort.
The active pneumatic suspension system is based on a set of equations for the quarter car model, the pneumatic valve and the air spring.
The quarter car model is derived from the Second Newton's Law and accounts for the dynamics of a quarter of the vehicle mass (sprung mass) and of the suspension and tyre mass (unsprung mass).
The proportional valve is modeled using the orifice equation for the air mass flow rate, exploiting a proportional relationship between the valve active area and the control input u. It is responsible for the air flow regulation to the actuator.
The air spring, i.e. the actuator, is represented through the pressure variation equation inside a variable volume chamber. This variation is responsible for the active force production.
To generate the right control signal u and to deal with the system nonlinearities, mainly arising from the complex air behavior, a nonlinear control algorithm based on backstepping technique is adopted.
As last step, simulations of the developed model are run in MATLAB-Simulink environment under different driving scenarios to test the effectiveness of the adopted solution with respect to a passive system.

Dedicated to my family

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Chapter 1

Introduction

Nowadays, in order to satisfy the requirements of the market, also low-cost car are being equipped with devices which in the past were a prerogative of high end cars. The aim of this trend is to provide also less expensive cars with technological features that can increase the comfort performances. An example of these features are active suspension systems. The suspensions play a fundamental role in a car, as they can provide insulation from road vibrations, making the travel enjoyable for passengers, along with ensuring safety.
In particular, active suspensions represent a great step forward if compared to the traditional solutions. By exploiting data coming from sensors and vehicle instrumentation, they can satisfy the driver needs, which in a travel may continuously change between comfort and handling.
Following this idea, to exploit an active suspension system in a city car can lead to a great enhancement of passengers comfort as it can cope with some driving situations typical of the urban scenario. To implement this active system an environment friendly and cheap solution is represented by pneumatic actuators, of which air springs are an example.

The aim of this work is then to set up a basic control model of an automotive active pneumatic system of air springs, to meet an optimal compromise between control and comfort. The developed model is conceived for a city car in cornering and bump crossing. In particular the designed active suspension shall compensate for rolling, caused by lateral load transfer, and for road disturbances.

The structure of this work is organized as follows: in Chapter 2 the main concepts and requirements of a suspension system are presented, along with a review of some available control strategies for active suspensions.
In Chapter 3 the modeling of each component of the active suspension is addressed. In particular the physical equations that have been integrated in the MATLAB

Simulink environment are shown.
Chapter 4 is dedicated to the illustration of the control law and its related features.
Finally, in Chapter 5 the comparison between the active system and a passive one is performed by means of simulation and the results are discussed.

Chapter 2

Background And State Of Art

2.1 Vehicle Suspension Requirements

The suspension system in a vehicle aims at fulfilling the requirements of both comfort, for passengers, and road handling, for the driver. These two aspects, however, characterize in a general way what the suspension system should provide to the driver and the vehicle. Hence, they must be translated into physical quantities that can be observed to evaluate the suspension performances.
In this section, the concepts of comfort and road handling will be defined in detail and the trade-off between them will be discussed.

2.1.1 Ride Comfort

In the automotive field, ride comfort can be referred to as the ability of the vehicle suspension system of insulating passengers and payloads from vibrations caused by the road profile roughness [44]. This capability is very relevant because, besides affecting the quality of a road trip, it has been proven that excessive vibrations have an impact also on the human health [48]. The result of this studies led to the identification, in standard ISO 2631, of the limits for what concerns the vibrations that human beings can undergo in time. An example is shown in figure (2.1).
As mentioned above, however, this requirement needs to be translated into a physical quantity which can be measured or observed, so that the system performances can be evaluated. This quantity should also be controllable, so that suitable control strategies if an active solution is adopted.
More specifically, the car body vertical acceleration is the selected magnitude related to this aspect [18][49], being the effective factor experienced by driver and passengers inside the car.
To provide better comfort, vertical acceleration must be minimized and oscillations must be highly attenuated: this means to allow the suspension to greatly

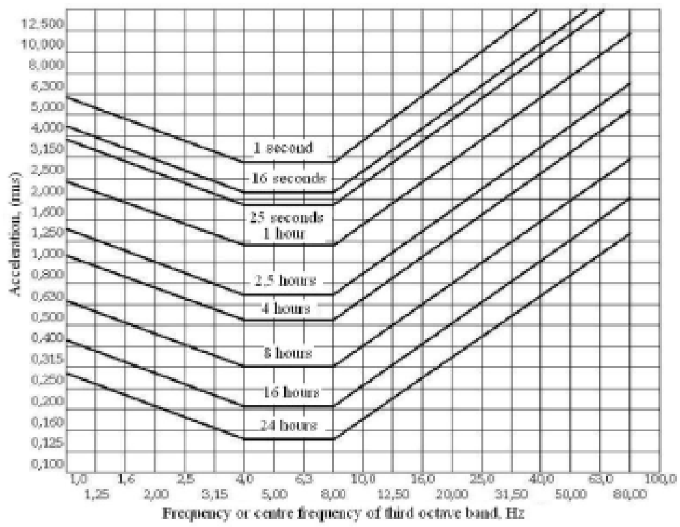


Figure 2.1: Vertical vibration exposure criteria (ISO 2631) [48]

compress and elongate, exploiting a big amount of the suspension travel, which represents the relative displacement between the sprung and the unsprung mass. This situation, however, is dangerous because with great values of the suspension travel there is the possibility of reaching its upper or lower limit, thus causing excessive wear of components and possible damages.[30]
As a consequence, to increase comfort, the suspension system shall show a more soft and flexible behavior, which can be also interpreted as a high damping performance requirement in order to reduce the vertical acceleration values [7]. A stiffer behavior will lead to the opposite situation.

2.1.2 Vehicle Handling

In [13] a definition of handling is given as "the interaction between three main factor, namely driver, vehicle and environment, during vehicle transportation". An other effective definition of vehicle handling is given in [37], where it is defined as "the responsiveness of the vehicle to the driver inputs, or more in general, as the ease of control".
Hence, it can be interpreted as the capability of the vehicle to correctly respond to the behavior that the driver is trying to impose, also under critical driving conditions.
To reach the goal of defining a physical quantity that links handling to the suspension system, it must be considered that the mean through which a vehicle interacts with the road, allowing actions like steering and acceleration, is the wheel.

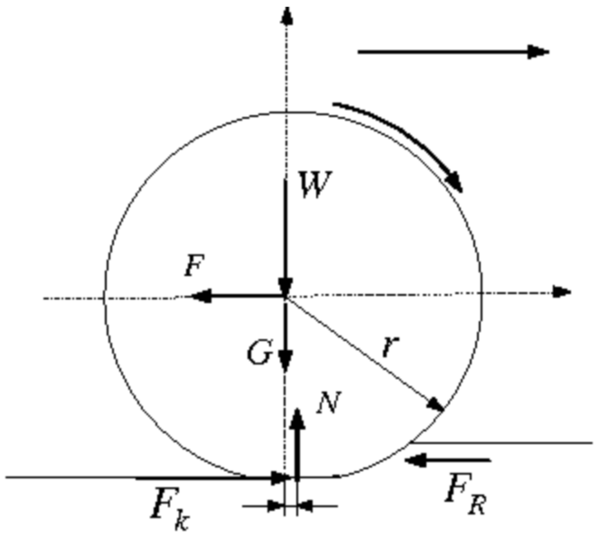


Figure 2.2: Wheel ground interaction [39]

In particular, the driver acts on the car actuators and as a consequence the forces exchanged in correspondence to the tyre contact patch are modified, allowing the regulation of the vehicle behavior during driving [25]. In this perspective, the quantity that is selected to account for handling is the suspension travel[49]. Thanks to its deflection, the suspension system is capable of guaranteeing continuous wheel-ground contact, required to control the vehicle.
Minimizing the displacement between sprung and unsprung mass, thus imposing a more rigid behavior for the suspension system, will prevent tyre-road contact loss and will provide increased road handling, improving safety. On the other hand, imposing a low damped behavior will lead to a greater vehicle vertical acceleration and consequently to discomfort for the passengers.

2.1.3 Suspension Requirements Trade-Off

As shown in the two previous sub-sections, it is evident that the two main objectives for a suspension system are contrasting: from one side comfort is required to avoid side effects on passengers health as well as to make the travel enjoyable. On the other side, handling is a mandatory requirement to guarantee safety and stability for both vehicle and passengers.
The former needs suspension to show a soft behavior with high damping performances, while the latter requires the system to be rigid and low damped, to reduce the displacement between sprung and unsprung mass.
This conflict can be solved through suitable regulation of the parameters related

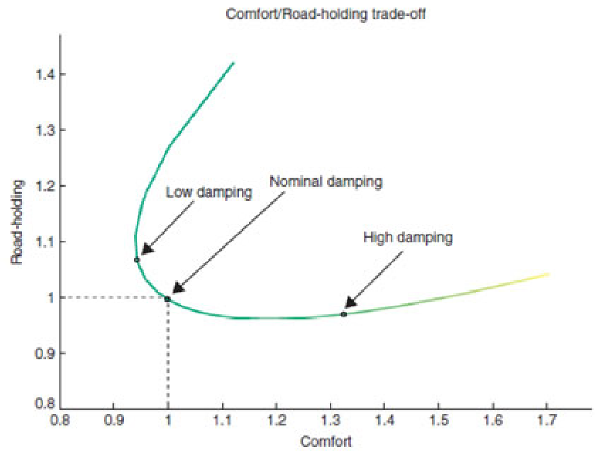


Figure 2.3: Suspension Trade-Off [1]

to the mechanical components (i.e. stiffness and damping ratios) during design in passive systems, while in semi-active and active systems a control algorithm is used to manage the trade off and (active) and to modulate the damper action (semi-active).

2.2 Suspension Types

Suspension systems can be classified according to the way in which they are able to interact with the vehicle in order to provide isolation from vibration and handling. In particular, they can be distinguished basing on the possibility or not of producing an active force, i.e. to generate and introduce energy into the system [47], and also on the presence of electronic control for their actuation.
According to this classification, suspensions can be defined as passive, semi-active and active. In the following, this three concepts will be illustrated.

2.2.1 Passive Systems

Passive suspension systems are, in general, made up of purely mechanical elements such as springs and dampers [49].
The mechanical spring can compress or extend, exerting an elastic force. This allows the suspension to contrast the disturbance forces acting on the system. The shock absorber works against the spring displacement, in order to damp the oscillations.
Applying force balancing to the system of figure (2.4) under the hypothesis of a

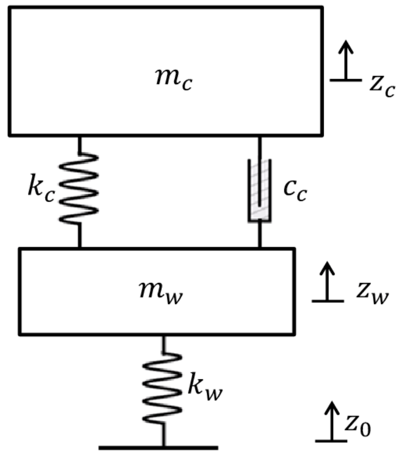


Figure 2.4: Passive Suspension Quarter Car Model [55]

linear relationship between force and displacement for the spring, and force and velocity for the damper, it is obtained:

$$m_c \ddot{z}_c = -k_c(z_c - z_w) - c_c(\dot{z}_c - \dot{z}_{us}) \tag{2.1}$$

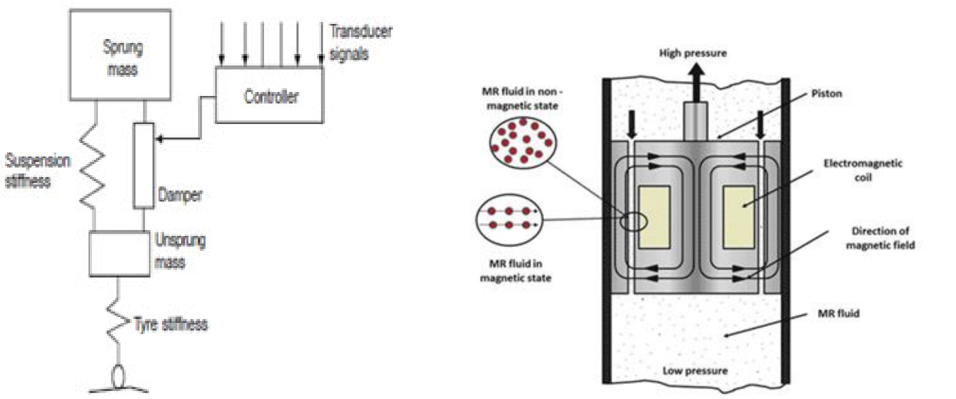
$$m_w \ddot{z}_w = k_c(z_c - z_w) + c_c(\dot{z}_c - \dot{z}_w) - k_w(z_w - z_0) \tag{2.2}$$

As can be seen from equations (2.1) and (2.2), this kind of systems present a drawback: since only the elastic and the damping force terms are present, the energy which comes from the external disturbances is transformed in potential elastic energy, because of the spring, and dissipated by means of the damper. However, no energy is directly produced by the suspension to be added to the system [47], as no additional force input is present. This means that the system is only able to react passively to external disturbances in accordance with the components capability.
To cope with this aspect, damping coefficient and stiffness are designed in order to show a balanced behavior. The aim of this procedure is to ensure to the suspension the capability to face the widest set of external perturbation still providing good performances [34]. However, once these parameters have been selected they are fixed and the only changes that can happen are due to aging and wear, which causes a decrease of the performances.
Therefore, analyzing this kind of system from a general point of view, passive suspensions represent a good compromise, thanks to their simple structure, but they show a limited effectiveness with respect to the road condition variation [34].

2.2.2 Semi-Active Systems

Semi-active suspensions represent a step forward with respect to passive ones. This kind of systems are not constituted by only purely passive mechanical components, but they present in addition a variable actuator. The working principle of this systems consists in adjusting the actuator damping coefficient to modify the damping force contribution. This allows to regulate the quickness of energy dissipation in response to the system conditions [26] [24]. The actuator that are usually employed can belong to different domains, spacing from dampers based on fluid that can be influenced by means of electric or magnetic fields to pneumatic isolators with variable stiffness characteristics. In order to provide a better understanding, electrorheological or magnetorheological dampers can be briefly analysed, being them widespread in this kind of applications. The main idea behind this components is to make the hydraulic fluid contained in the chambers of the cylinder sensitive to the presence of magnetic or electrical fields [58]. As can be seen from figure (2.5b), providing the piston with a proper structure and regulating the intensity of the applied field, the fluid properties, and more specifically the viscosity, will vary. As a result, the actuator behavior will depend on the field strength, showing a low or high damped characteristic.
If force balance is computed for the system of figure (2.5a), the same equations as (2.1) and (2.2) are obtained. It must be remarked that, in this case, the damping coefficient can be varied by means of an electronic control. Hence, as in the previous case the energy balance depends only on the spring and the damper forces. This means that also semi-active systems are not capable of introducing active force term, as the control aims only at adjusting the introduced damping.[47]

The control system to manage the whole system, which includes the ECU and



(a) (b)

Figure 2.5: (a) Semi-Active Configuration [1], (b) Tesla MR Damper [16]

the required sensors, provides greater flexibility thanks to the different control algorithms available, at the expense of an increased complexity. As a reference, in [32] some control strategies are shown.
In conclusion, by adding some complexity in terms of electronic components, it is possible to make the system response compliant, up to a limited extent, with respect to the road surface roughness. This allows to reach better performances.

2.2.3 Active Systems

Active suspension systems differentiates from the other suspension types for the presence of an electronically controlled actuator. This element is used to apply an active force to counteract to the disturbances coming from the road [49]. The actuators that are usually employed in this kind of application are of different types [43].
In figure (2.6) a possible configuration for an active system is shown. However, other system layouts can be defined, where either the elastic force, the damping force contribution, or both can be included in the active force generated by the actuator so that the corresponding spring or damper can be eliminated [49]. This means that the active element could cover multiple roles in the suspension, as it can also take the place of the classical components that constitutes the suspension [45].
Referring to figure (2.6) and computing the dynamics for both m_b and m_w, it is obtained:

$$m_b\ddot{x}_b = -k_s(x_b - x_w) - b_s(\dot{x}_b - \dot{x}_w) + f_s \tag{2.3}$$

$$m_w\ddot{x}_w = k_s(x_b - x_w) + b_s(\dot{x}_b - \dot{x}_w) - k_t(x_w - r) - f_s \tag{2.4}$$

Comparing equations (2.3) and (2.4) with (2.1) and (2.2), it can be seen that an additional term f_s is present, which is the active force. This term represents the main difference between an active systems and semi-active and passive ones. The former is capable of modifying the force balance, introducing energy into the system by means of f_s, on the contrary of the latter ones where this term is missing [47].
The other main advantage offered by active system is given by the possibility of choosing a control strategy which allows to adjust the system behavior in order to manage at best the trade-off between comfort and handling. This is a strong point for this kind of suspension since, thanks to the different possibilities offered by the control theory, the vibration isolation problem can be tackled from different perspectives. As also semi-active suspension are controlled, it must be noted that in active suspension the aim of control is to drive the actuator in order to produce the force f_s, while the aim of semi-active system is to manage the damping

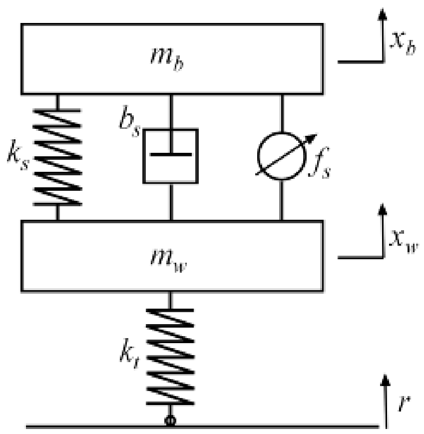


Figure 2.6: Quarter car model active configuration [35]

coefficient, as it will provide different grade of energy dissipation by means of its variation.
Despite the great performances that can be achieved and the high level of flexibility provided by these systems, it must be taken into account that they have a more complex structure, due to hardware and software requirements. This translates also in higher power consumption and costs.

2.3 Active Pneumatic Suspensions

As explained in the previous section, since their introduction, active suspensions provided a great innovation in the way in which a suspension system is conceived. In particular, thanks to the advance in the development of sensors, a larger amount of information can be gathered about the road conditions as well as regarding the instantaneous state of the active system. This data, combined with a suitable control strategy can realize a solution capable of providing great performances.
As this work is centered on the development of a basic control model for an active pneumatic system based on air springs, this kind of actuator is addressed in this section.
In the first part air spring are considered from a general point of view. The main characteristics of this actuators are pointed out, such as the main advantages and disadvantages, along with the main design shapes that are available in market. In the last part some application of air spring in automotive vibration isolation are presented, along with a real suspension system example.

2.3.1 Air Spring

Air springs represent a valid alternative to other types of actuator commonly employed in automotive active solutions, like hydraulic or electromechanical ones. In particular, air spring have a reduced cost and are also cleaner, e.g. with respect to hydraulic actuators, as they exploit compressed air as mean of actuation. Among the other qualities associated to this type of solution, as shown in [40] it must be remarked:

- the ability of maintaining a constant vehicle body height with respect to possible changes in the load applied to the spring. If a variation happens, it is sufficient to correct the internal pressure according to the new load condition.[34]
- the capability of providing a stiffness characteristic which is variable without affecting the natural frequency of the overall active system.

As a drawback, on the other hand, it must be taken into account all the additional hardware which is required to manage the pneumatic system, which should fit in a limited space [34]. In fact, a pneumatic active system equipped with air springs, in general, comprehends: connection pipes to link the air spring with the pressure supply and to the exhaust, as well as to connect the system components; a compressor, required to act as the high pressure source; pneumatic valves, to regulate the air flow in the pneumatic circuit and the electronic circuitry to drive the overall active system (fig.(2.7)).
Furthermore, air springs can be used along with an air tank which cover the role of additional air reservoir. This is useful to increase the spring volume and to introduce the possibility of modulating the spring stiffness.
A schematic example of a pneumatic circuit involving air springs is shown in figure (2.11), where the main blocks representing the supply (and exhaust) system and the valve block are evidenced.

Figure 2.7: Components of an active system with air spring [9]

These actuators are available in the market under different form. In particular, as stated in [40], it is possible to differentiate this actuators basing on their shape, as this characteristic will influence also the properties of the spring:

- Convoluted bellows: as can be seen from figure (2.8), this kind of spring possess a structure which is divided into convolutions. They allow small relative displacement and are characterized by high stiffness [34]

Figure 2.8: Double convoluted bellow [52]

- Rolling lobes: this kind of springs are provided with a cylindrical structure as the one shown in figure 2.9. As stated in [34], they are characterized by reduced dimensions, which implies also the presence of a smaller spring effective area.

Figure 2.9: Rolling lobe [4]

- Tapered sleeves: in terms of structure this components are very similar to the rolling lobe, however the shape is different. An example of this kind of actuators is shown in figure 2.10.

Figure 2.10: Tapered sleeve cross section [6]

2.3.2 Example of application

Among the possible use of air spring inside active suspension systems, one possible application is represented by the height leveling control.
In this implementation, the air spring is controlled in order to modify the height of the car according to desired conditions. As shown in [27], where a control law is developed for this purpose, the overall system is composed by the aforementioned component and a schematic representation is given in figure (2.11). The main operations related to the suspension can be divided into two main groups, namely the increasing and the decreasing of the vertical position of the sprung mass. During the former phase the air flows from from the air compressor to the actuator, while the flow follows the inverse path during the latter one. The aim of this system, in general, is to facilitate the driver and passengers operation during, for example, the entering and exiting of the car [27].
An other possible use of air spring is given in [14], where the spring is used as an actuator to generate an active force in order to counteract the road disturbances. It is evident that, in this case, the air spring is not only responsible for the leveling of the sprung mass height but is used to counteract the perturbations coming from the road.
If practical solution implemented on vehicles are considered, the AIRmatic suspension system can be considered. [34]

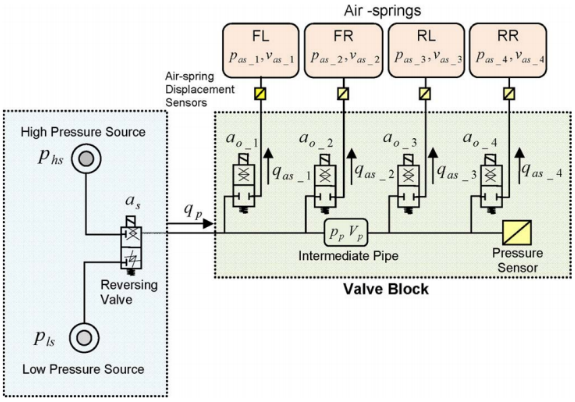


Figure 2.11: Height leveling control scheme [27]

This active suspension, developed by Mercedes-Benz, is a combined system where the main elements are represented by air springs and hydraulic actuators [5]. As can be seen from the schematic in figure (2.12), the suspension features a pneumatic system, whose aim is to regulate the air flow in the air spring actuators. This operation lead to the modification of the spring stiffness. The hydraulic damper, on the other hand, has a variable damping coefficient.
Hence, exploiting the combination of the air spring and of the damper it is possible to regulate the suspension performances according to the to the road surface and to driver needs [34][5].

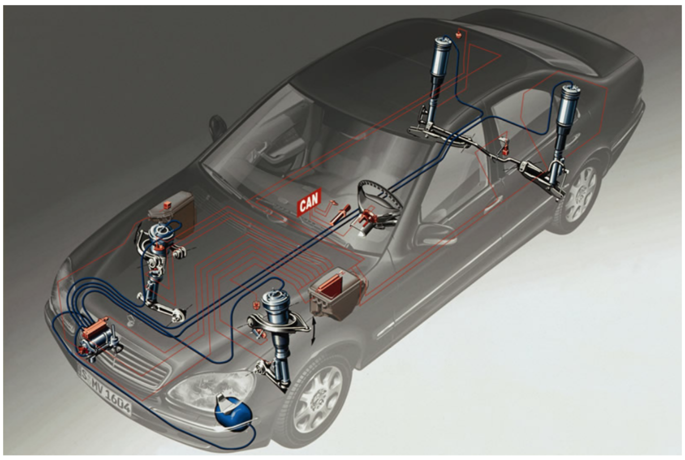


Figure 2.12: AIRmatic suspension schematic [5]

2.4 Literature Review On Active System Control Strategies

Since the idea of an active suspension was brought forward, it raised an increasing interest in the automotive field because of the advantages it can introduce and that has been presented in the previous section. In parallel to the active suspension technology, several strategies have been developed in order to properly control the system and meet the requirements in terms of safety, comfort and handling along with a low energy consumption. As can be found in literature, different models for the suspension system and the vehicle can be developed, either linear or nonlinear. As a first step, strategies for linear systems are considered. This kind of models are based on approximations that can be obtained through mathematical linearization process from nonlinear models. Among the most used and widespread solutions for controlling these systems, optimal control techniques, like LQR, must be mentioned.

In [53] researchers adopted LQR regulator to control a half car model. They developed two control laws based on two different performance indexes. The analysis of the results proved that a performance index accounting also for passengers accelerations provides better results with respect to a traditional one.

An other simple but effective solution is represented by PID controllers. Authors

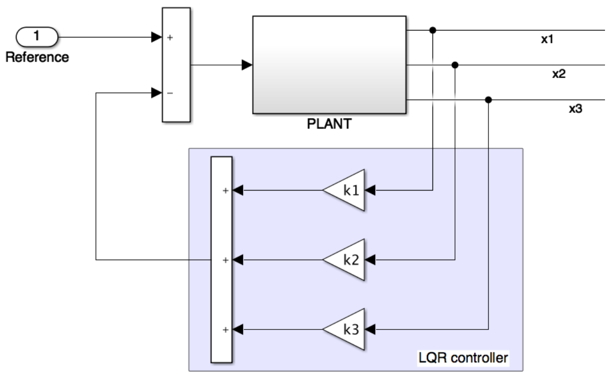


Figure 2.13: LQR control scheme

in [29] developed a control structure based on PID controller for a linear quarter car model. After tuning the three gains associated to the controller, they proved its effectiveness for both step inputs and for ISO road profiles.

One of the main issues related this kind control strategies, dealing with suspension control, is that they rely on linear models for the vehicle and the suspension components, while the actual system behavior is in general more complex and nonlinear [57]. This can lead to the overlooking of some non negligible nonlinearities [10]and thus to degradation of the controller performances. Hence, nonlinear control strategies for active suspension systems are considered, starting from sliding mode control to more complex strategies where linear and nonlinear control technique are mixed.

In [10] the authors proved the superiority of a nonlinearly controlled electro-

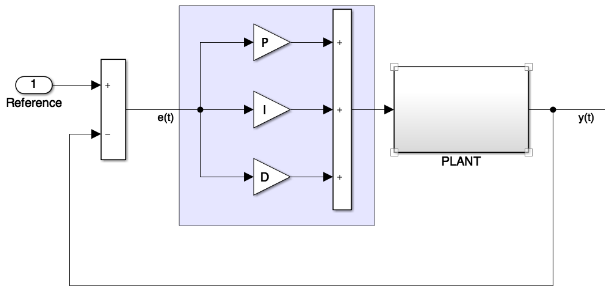


Figure 2.14: PID control scheme

hydraulic suspension with respect to both a passive system and a linearly controlled one, exploiting sliding mode control.
In [30] a controller based on backstepping strategy is realized. A nonlinear term, which is function of the suspension travel, is used to manage the comfort-handling trade off to improve the system response.
The same authors then, in [31], improved their previous work ([30])by making their algorithm capable of adapting to the road conditions by changing in real time some algorithm internal parameters.
A fuzzy control strategy for an active hydraulic suspension is presented in [17]. The authors developed a controller which embeds an inner loop for force tracking, while the outer one realizes the fuzzy logic.
Researchers in [56] developed a sliding mode controller for a suspension system equipped with an air spring and an actuator. This nonlinear controller, whose output operates the actuator, tracks a force reference coming from a quarter car model controlled through an LQR based strategy.
In [33] is realized a control strategy for an active suspension system by combining LQR optimal control, belonging to linear control theory, and backstepping, belonging to nonlinear control theory. The LQR controller produces the reference input $\widetilde{u}$ that is tracked by the backstepping based controller which also compensate for the system nonlinearities.
In [11] the authors presented a controller based on sliding control mode and fuzzy logic to control a suspension model which exploits an air spring. The aim of the fuzzy adaptive system is to deal with nonlinearity and uncertainty coming from the air spring, while the sliding mode controller aims at tracking the reference coming from a linear active suspension model.

Chapter 3

System Modeling

In this section, the system model that has been used to study the active control strategy is presented. Firstly an overview of the overall model is given, then each component is addressed in order to give a more detailed insight, along with the equations that constitute their mathematical description.

3.1 System Overview

The model that will be presented has been developed in order to simulate the behavior of a pneumatic active suspension system in presence of some different driving situations.
The system is composed by four main blocks: the controller, the valve, the actuator model and the car model, as shown in figure 3.1. The main idea behind this structure, is to obtain the correct air mass flow rate through the pneumatic valve so that pressure inside the actuator, i.e. the air spring, is modified and a suitable

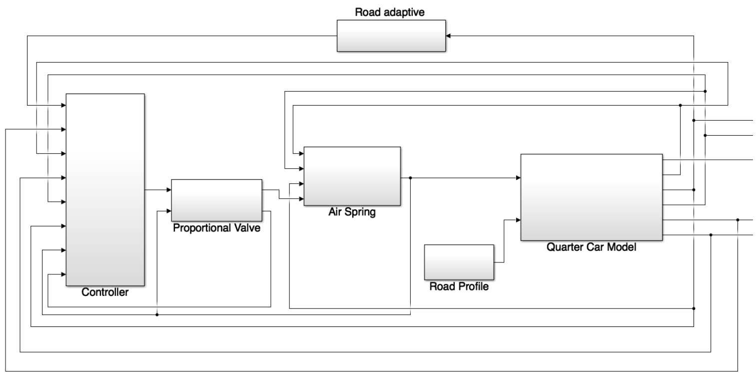


Figure 3.1: Simulink model of the active suspension

force is produced to counteract the disturbance coming from the road.
To obtain the desired system behavior a controller, responsible for the production of the correct input voltage for operating the valve is required, and it will be discussed in the next chapter.

3.2 Car Model

To test the performance of the control strategy when applied to the active suspension, it is required to model the vehicle on which the system is supposed to be mounted on.
The vehicle model that will be used should show a good insight of the vertical dynamics, with particular attention to the suspension travel and the vertical acceleration, which are representative of the handling capability and comfort, respectively.
Among all the possible representations of the car dynamics which can be found in literature, the 2 DOF quarter car model is selected since it shows low complexity combined with a suitable representation of the vertical dynamics [54].
This model, as the name says, represents only one quarter of the vehicle body, focusing the attention on the wheel-suspension assembly part of the car. It is constituted by two masses, namely the sprung and the unsprung mass, which are connected by the components representing the suspension system. The sprung mass, denoted with symbol m_s, incorporates a part of the total vehicle mass, and accounts also for eventual passengers and payloads. The unsprung mass, denoted with symbol m_{us}, refers to the mass of tyre, brake and part of the suspension weight.
As for the suspension components, it is assumed that the relationships existing between the elastic force $F_{elastic}$ and the displacement and the one existing between

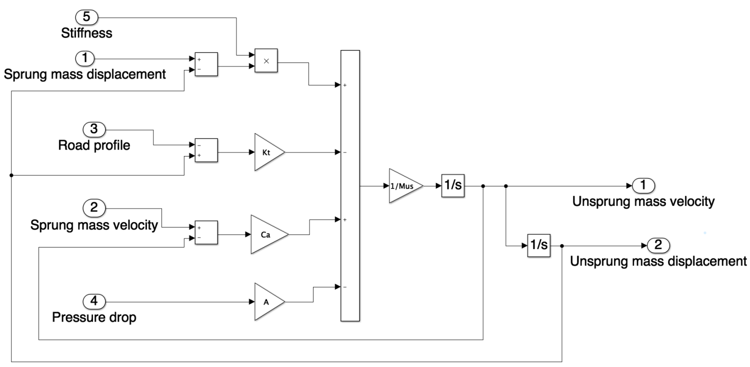


Figure 3.2: Sprung mass Simulink model

the damping force $F_{damping}$ and the velocity, are of linear type. The active element, i.e. the actuator, is represented by an air spring. It also substitutes the mechanical spring. Following [21], in fact, the total force produced by the air spring can be defined as the sum of different contribution, among which the active force F_{active}, due to the internal pressure and air mass variation, and the elastic force $F_{elastic}$ given by the varying stiffness of the spring. It follows that in this case, even if the active suspension is constituted only by a damper and the active element, it can be modeled as the classical parallel of a damper, a spring and an actuator, where both the stiffness and the active force come from the air spring.
Finally, to model the tyre, a spring with stiffness K_t is introduced.
By combining all these considerations, it is obtained a system as the one of figure (2.6). Applying the second Newton's Law to the model of figure (2.6), it is possible to obtain the equations of motion for the sprung and unsprung mass. Referring to equations (2.3) and (2.4), $\ddot{x}_b, \dot{x}_b$ and x_b are the sprung mass acceleration, velocity and displacement. The same notation holds for the unsprung mass magnitudes, indicated with the subscript "w".
Equations (2.3) and (2.4) are then represented in the MATLAB-Simulink software are shown in figure (3.2) and (3.3).

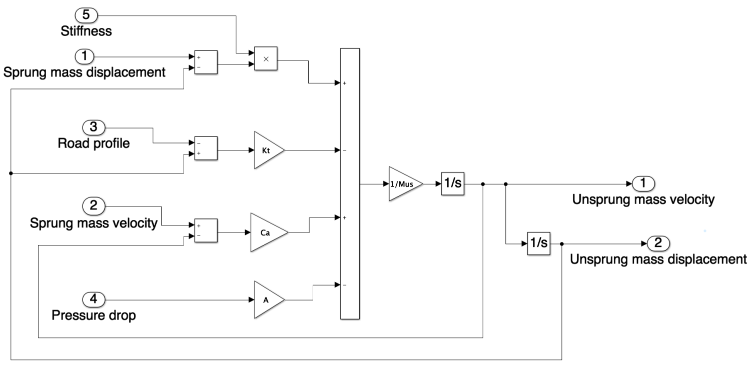


Figure 3.3: Unsprung mass Simulink model

3.3 Actuator Model

In an active suspension system, the actuator represents the element which exerts the active force in order to compensate the road disturbances.
In this specific case, the active element is represented by an air spring. It can be described as a variable volume bag delimited by a rubber wall, filled with compressed air [40]. This kind of actuator are available in commerce under different models, which differ basically in the form and in the performances they can pro-

vide. For this application, a double convolution type air spring has been used as a reference.
Following [50], assuming an ideal gas behavior for the air, adiabatic working conditions and homogeneous temperature in the whole system, it can be obtained:

$$P_{as} = \frac{kP_{as}}{m_{as}}\dot{m_{as}} - \frac{kP_{as}}{V_{as}}\dot{V_{as}} \tag{3.1}$$

where P_{as} is the air spring internal pressure, $\dot{P_{as}}$ is the air spring pressure gradient, V_{as} is the air spring volume, $\dot{V_s}$ is the volume variation, k is the adiabatic exponent for air (k=1.4), m_{as} and $\dot{m_{as}}$ are the air mass and the air mass variation in the spring, respectively.
According to the ideal gas law, it can be written:

$$P_{as}V_{as} = m_{as}RT \tag{3.2}$$

where R is the ideal gas constant ($\frac{J}{KgK}$) and T is the temperature.
From equation (3.2) it is obtained that:

$$\frac{P_{as}}{m_{as}} = \frac{RT}{V_{as}} \tag{3.3}$$

which, if substituted in equation (3.1) gives:

$$P_{as} = \frac{kRT}{V_{as}}\dot{m_{as}} - \frac{k}{V_{as}}P_{as}\dot{V_{as}} \tag{3.4}$$

Equation (3.4) represents the governing law for the change of pressure inside the air spring, and as can be seen it depends on both the mass flow rate variation and the spring volume variation with respect to time. While the expression for the former term will be derived in the next section, as it depends on the valve behavior, the volume is strictly related to the actuator characteristics.
This parameter varies nonlinearly with the air spring relative displacement [40], as the spring extension or compression will cause a change in the actual volume. In particular, the function that characterizes its variation can be determined either using mathematical relationship or fitting data coming from experiments on a test bench [40],[15].
In analytical form, following [27], the volume variation can be expressed as:

$$V_{as} = V_0 + A_{eff}(x_{sprung} - x_{unsprung}) \tag{3.5}$$

where V_0 is the initial volume due to the static height of the spring and A_{eff} is the air spring effective area.

Figure 3.4: Air spring actuator [2]

The A_{eff} is an other key parameter for modeling the air spring behavior. It can be described as "a non-constant imaginary area over which the spring's relative internal pressure is assumed to act" [40]. As it happens for the volume, its associated characteristic can be determined through experiments.
In order to explain the considerations in the following, a datasheet of a double convoluted air spring (Continental, ContiTech Modelseries C) is also considered as a reference.
As stated in [15], double convoluted bellows presents a value of the effective area parameter that results to be non constant with respect to the spring relative displacement. In this case, however, as can be seen from the reference of figure (3.5), the volume change is almost linear with respect to the height variation. It is therefore assumed that the area of the active spring is constant. Hence, basing on this hypothesis, expression (3.5) can be derived with respect to time, obtaining:

$$\dot{V}_{as} = A_{eff}(\dot{x}_{sprung} - \dot{x}_{unsprung}) \tag{3.6}$$

The complete expression for the air spring pressure variation can be thus obtained by substituting eq.(3.6) into (3.4):

$$P_{as} = \frac{kRT}{V_{as}}\dot{m}_{as} - \frac{kA_{eff}}{V_{as}}P_{as}(\dot{x}_{sprung} - \dot{x}_{unsprung}) \tag{3.7}$$

As a last step, the active force produced by the air spring must be addressed. In the previous section, it has been assumed to split this term into the elastic force and the active force following [21]. The elastic contribution depends on the spring stiffness, whose dynamic behavior is computed as [15]:

$$C = \frac{\mathrm{d}F}{\mathrm{d}z} \tag{3.8}$$

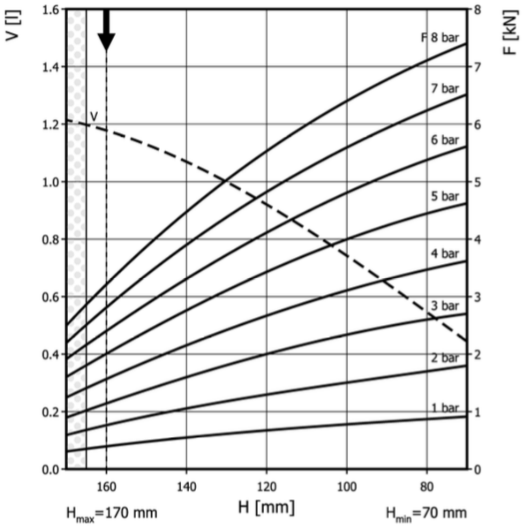


Figure 3.5: Air spring characteristic [3]

By referring to figure (3.5), however, it can be noted that for each static pressure level, the change in the slope of each force-deflection characteristic is very small as the curves are almost parallel. In addition, a positive or negative variation of the internal pressure produces a correspondent increase or decrease of the force, as the characteristic will be shifted upwards or downwards in the plot. Since the slope of these curves represents the stiffness (see eq.(3.8)), it is then assumed that the spring has also a constant stiffness coefficient. According to this further consideration, the elastic force provided by the air spring will be proportional to its relative displacement by means of stiffness, whereas the increase or decrease of the total force produced is due to the pressure variation.
Hence, the stiffness can be computed in the static equilibrium point (p_0, h_0) of the spring following [40]:

$$K = \frac{kP_{eq}\mu A_{eq}}{V_{eq}} - (P_{eq} - P_{atm})\alpha \tag{3.9}$$

where P_{eq} is the spring internal pressure at the equilibrium point, μ is the volume derivative with respect to the spring height evaluated at the equilibrium point, V_{eq} is the value of the volume at the equilibrium point and α is the value of the effective area derivative with respect to height computed at the equilibrium point.

3.4 Pneumatic Valve

The pneumatic valve represents the physical link between the controller and the actuator, since it regulates the air flow rate either to or from the air spring on the basis of the voltage control output u. The aim of the valve, in particular, is to connect the air spring to the supply source $\mathrm{P_s}$ during the charging phase, and to the ambient $\mathrm{P_a}$ during the discharging.
The model presented in this section correspond to a 3/3 pneumatic proportional spool valve.
This kind of flow regulators features an electromechanical stage which control the

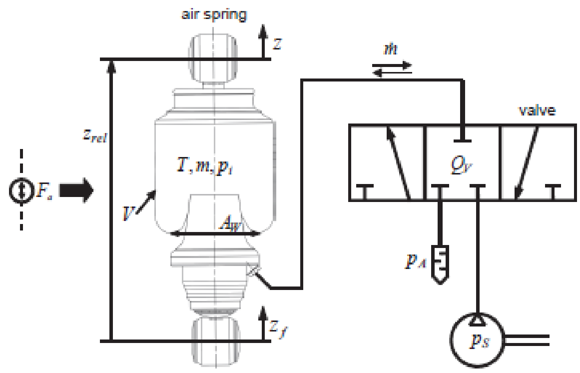


Figure 3.6: Valve-air spring scheme [20]

internal spool displacement. This control is achieved by unbalancing the forces that keep the spool in the equilibrium position by means of a force proportional to the applied voltage. As a consequence of the spool movement, either a fraction or the whole valve area will be available for letting the air flow [38].
The dynamics of this stage, which consist of mass-spring-damper second order differential equation, is illustrated in [42]. However, it has been neglected during the modeling phase since it has been assumed to be faster than the overall system dynamics.
The other key point of the valve is represented by the flow stage, which models the flow of air through the air path.
According to [12] and [42], an expression for the mass flow rate is given by:

$$\dot{m}_{\mathrm{v}} = \begin{cases} C_f A_v C_1 \dfrac{P_u}{\sqrt{T}} & if \quad \dfrac{P_d}{P_u} \leq P_{cr} \\[2ex] C_f A_v C_2 \dfrac{P_u}{\sqrt{T}} \left(\dfrac{P_d}{P_u}\right)^{\frac{1}{k}} \sqrt{1 - \left(\dfrac{P_d}{P_u}\right)^{\frac{(k-1)}{k}}} & if \quad \dfrac{P_d}{P_u} \geq P_{cr} \end{cases} \tag{3.10}$$

Besides the valve related coefficients C_{f}, C_1 and C_2, it can be noted that expression (3.10) depends on the the critical pressure ratio P_{cr} and on the valve opened area A_{v}.
The P_{cr} expression, defined as:

$$P_{cr} = \left(\frac{2}{R\,(k-1)} \right)^{\frac{k}{k-1}} \tag{3.11}$$

is not involved in the modeling process as it depends only on the constant value k, which for air is selected equal to k=1.4 in adiabatic conditions.
Hence, only Av must be shaped to correctly represent the valve behavior. This parameter, as stated above, is defined on the basis of the spool displacement, being the valve of proportional type [38]. However, since the spool dynamics has been assumed to be very fast and thus neglected, in this case it has been established a proportional relationship directly between A_{v} and the controller input u.
In particular, following [8], the active area of the valve orifice can be expressed as:

$$A_v = \frac{A_{max}}{u_{max}} u \tag{3.12}$$

where $\mathrm{A_{max}}$ is the maximum value of the active area, corresponding to the fully opened position of the valve, and $\mathrm{u_{max}}$ is the maximum voltage that can be applied to the valve solenoid. This leads to a linear characteristic between A_v and u (figure 3.7) As can be seen from equation (3.12), every value of u different from zero will produce a non-null value for the active area $\mathrm{A_v}$. This means that the valve is not showing any deadzone, i.e. it is assumed to be of zero lapped type [59].

Always following [8], the mass flow rate equation can thus be rearranged in extended form as:

$$\dot{m}_{\mathrm{v}} = \begin{cases} \dfrac{A_{max}}{u_{max}} P_{sup} \sqrt{\dfrac{2k}{RT\,(k-1)}}\, \phi\left(\dfrac{P_{sup}}{P_{as}}\right) u & if \quad u > 0 \\ \dfrac{A_{max}}{u_{max}} P_{as} \sqrt{\dfrac{2k}{RT\,(k-1)}}\, \phi\left(\dfrac{P_{as}}{P_{atm}}\right) u & if \quad u < 0 \end{cases} \tag{3.13}$$

where $\mathrm{P_{sup}}$ is the supply pressure, $\mathrm{P_{spring}}$ is the internal spring pressure, $\mathrm{P_{amb}}$ is the ambient pressure, R is the perfect gas constant ($\frac{J}{KgK}$) and T is the working temperature.
In order to make the expression more compact, it is defined:

$$\Omega = \begin{cases} P_{sup} \quad \phi\left(\dfrac{P_{sup}}{P_{as}}\right) u & if \quad u > 0 \\ P_{as} \quad \phi\left(\dfrac{P_{as}}{P_{atm}}\right) u & if \quad u < 0 \end{cases}$$

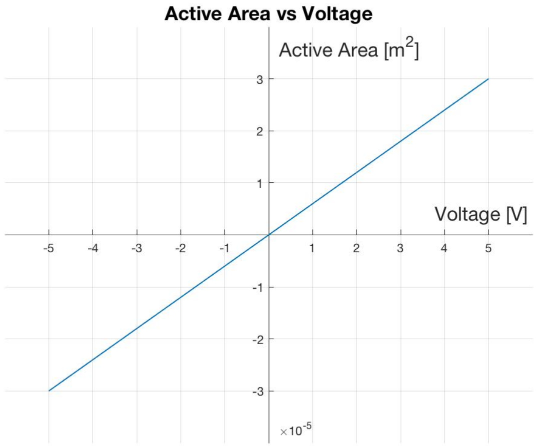


Figure 3.7: Orifice Area VS Voltage characteristic

Thus eq.(3.14) becomes:

$$\dot{m}_{\mathrm{v}} = \frac{A_{max}}{u_{max}} \Omega u \tag{3.14}$$

The $\phi(\sigma)$ function introduced above is chosen as:

$$\phi(\sigma) = \begin{cases} \sqrt{\sigma^{\frac{k+1}{k}} \left(\sigma^{\frac{1-k}{k}} - 1 \right)} & for \quad 0.528 \leq \sigma \leq 1 \\ 0.58 & for \quad 0 < \sigma < 0.528 \end{cases} \tag{3.15}$$

and σ is defined as:

$$\sigma = \frac{P_{downstream}}{P_{upstream}} \tag{3.16}$$

Equation (3.15) reflects behavior of air when σ (eq.3.16) is above or below the critical pressure ratio of eq.(3.11), introducing chocked flow condition in equation (3.13).

From equation (3.13) it can be noted that the inflation or deflation operations are defined on the base of the control input sign. More specifically, it is assumed that the mass flow rate can be either positive or negative according to the sign of u. If $\dot{m}_v > 0$, it means that air is flowing from the supply source to the actuator, while $\dot{m}_v < 0$ represent the discharging of air into the ambient.

Chapter 4

Control Algorithm

This chapter is devoted to describe the strategy used to control the suspension model discussed in the previous chapter. It has been adapted to the pneumatic active model, starting from the original work of [30] and [31] where it was defined for an hydraulic system. The nonlinear control strategy exploits the properties of backstepping to regulate the suspension behavior according to the road conditions. Firstly an overview of the backstepping technique theory is given. Then, the main features that the algorithm can realize are presented, along with the computation required to obtain the desired controller output u.

4.1 Fundamentals of Backstepping Control

Backstepping is a recursive procedure which belongs to the field of the nonlinear control strategy. Exploiting this procedure allows to design a feedback control law for the plant to be stabilized, in parallel with a Lyapunov function [19]. Basing on the constructed function, according to the Lyapunov Criterion [46], the stability of the control law can be proved.
This method can be applied to a specific set of systems that are represented in the so called "strict feedback form" [28], which can be expressed as:

$$\begin{aligned}
\dot{x} &= f_0(x) + g_0(x)z1, \\
\dot{z_1} &= f_1(x, z_1) + g_1(x, z_1)z_2, \\
&\;. \\
&\;. \\
\dot{z_n} &= f_n(x, z_1, ..., z_n) + g_n(x, z_1, z_2, ..., z_n)u,
\end{aligned} \tag{4.1}$$

As can be seen from equations (4.1), due to its particular form, the whole system can be interpreted as constituted by smaller blocks, each of which is represented

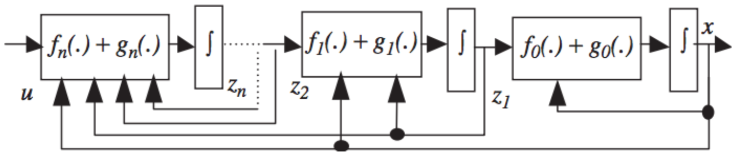


Figure 4.1: Strict Feedback System [51]

by a dynamic equation. Taking also figure (4.1) as a reference, it can be seen that the subsystem associated to x depends on the z_1 variable, which acts as an input to it. It follows that, in order to provide stabilization to the x subsystem, it is possible to exploit z_1 [41]. In particular z_1 is used to define a "virtual control input" [19]. The objective becomes then the stabilization of z_1 which, similarly to the previous step, can be achieved by means of z_2. This procedure is repeated until an explicit expression of the controller output u is reached. [36]
It is then clear that the main idea of this control strategy is to proceed backwards to define an expression of the input u that, if applied to the system to be controlled, progressively provides stability to each subsystem.[36]
The control laws that must be defined in the intermediate steps can be exploited to compensate the nonlinearities that may appear in the subsystems expressions. This kind of approach provides backstepping a great advantage with respect to other control techniques, as it allows to deal with the system nonlinearities in a smarter way. There exist in fact nonlinear terms which improves the achievement of stability for a subsystem by superimposing their behavior over other unuseful nonlinear terms, thus eliminating their negative effect [19]. It follows that they can be exploited into intermediate control laws in order to compensate unwanted nonlinearities while keeping the useful ones.[19] This allow to avoid the exact mathematical cancellation which is more problematic.[41]

4.2 Working Principle

As stated in Chapter 2, the suspension system imposes a trade off between passenger comfort and road handling.
By carefully analyzing these two divergent requirements, it can be noted that there exist common element between them, represented by the suspension travel. This quantity, in fact, should be allowed to assume greater or smaller values according to which of the two aspects should be prioritized.
In the literature, a backstepping controller implementation to cope with this situation is given in [30].

As shown by the authors, a possible way to improve the suspension performances is to favor the passengers comfort when the relative displacement between the sprung and unsprung mass is far enough from the limits of the suspension stroke. On the other hand, the controller should limit the suspension travel to guarantee handling when it overcomes a safe threshold.
In order to control the pneumatic active suspension, the control strategy showed in [30] is adapted to the developed model.

4.2.1 Trade-Off Managing

Following the work of [30], a variable nonlinear term (eq. 4.4) is used to cope with the trade-off. The aim of this term is to act as a weighting function on the controlled variable (eq.(4.3)). When the weight is low, more importance is given to the minimization of the sprung mass displacement. This means that the controller will enhance comfort. As the output weight increases, the controller will target the reduction of the suspension stroke usage.This happens when the road disturbances become significant.
The first step to obtain this behavior from the controller, is to introduce a new unsprung mass expression:

$$\dot{\bar{x}}_{unsprung} = -(\varepsilon_0 + \kappa_1 \varphi(\zeta))(\bar{x}_{unsprung} - x_{unsprung}) \tag{4.2}$$

and to define the controlled variable as:

$$y = x_{sprung} - \bar{x}_{unsprung} \tag{4.3}$$

As can be seen from equation (4.2), the nonlinear expression is given by:

$$(\varepsilon_0 + \kappa_1 \varphi(\zeta)) \tag{4.4}$$

where ε_0 and κ_1 are positive constant and $\varphi(\zeta)$ is a function of the suspension travel ζ.
More specifically, φ is a piecewise function which depends on two variables m_1 and m_2:

$$\varphi(\zeta) = \begin{cases} \left(\dfrac{\zeta - m_1}{m_2}\right)^4 & \zeta > m_1 \\ 0 & |\zeta| \leq m_1 \\ \left(\dfrac{\zeta + m_1}{m_2}\right)^4 & \zeta < -m_1 \end{cases} \tag{4.5}$$

By observing expressions (4.5) and figure (4.2a), it can be noted that when the suspension travel is within the [-m_1,m_1] interval, the output of this function is zero.

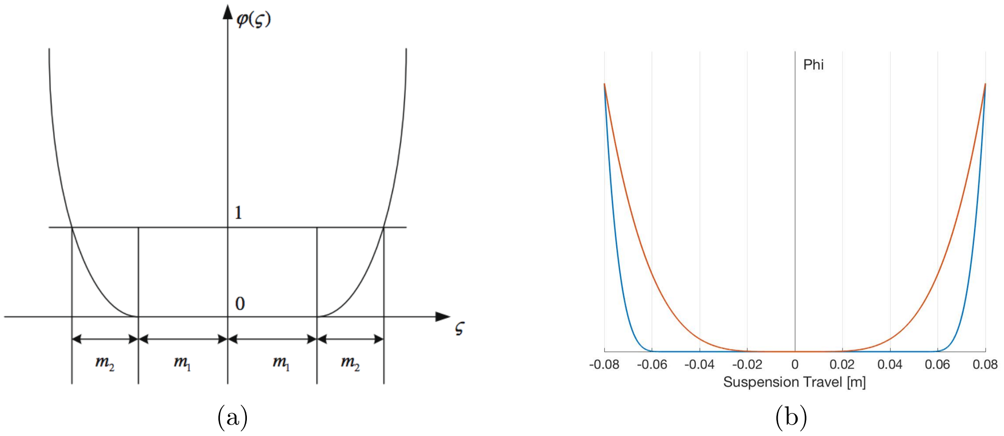


Figure 4.2: (a) Phi parametrization [23] (b) Phi function examples

This range correspond to suspension travel values that are considered acceptable, and the controller can focus on comfort. By introducing $\varphi(\zeta) = 0$ in eq.(4.3), it will lead to a low weight generated by eq.(4.4) which gives:

$$y \approx x_{sprung} \tag{4.6}$$

This means that the algorithm is aiming at minimizing the vertical displacement of the sprung mass. As a consequence, the vertical acceleration will be minimized too, providing the required comfort. If the suspension travel values become larger and exceed the m_1 limit either in compression or extension, instead, the weight of the output of function (4.4) increases the importance of $\dot{x}_{unsprung}$ in equation (4.3). Hence, in this situation the controller will change its objective, giving priority to the suspension travel reduction:

$$y = x_{sprung} - x_{unsprung} \tag{4.7}$$

According to the considerations made above, it is pointed out that equation (4.4) influences the output value of equation (4.2). In turn, equation (4.2) affects the definition of the controlled variable y (eq.(4.3)). In particular, depending on the importance that is given to $\dot{x}_{unsprung}$ in eq.(4.3), the control effort will be centered either on comfort or on handling.
Hence, the additional function (4.2) allows the controller to decide in which way the system should respond to the road perturbations, , i.e. by softening or stiffening.

4.2.2 Adaptation To Road Conditions

As stated in [30],[31] and as can be seen from figure (4.2a), the φ function (4.5) is parametrized in terms of m_1 and m_2. It follows that these two parameters

influence the output of function (4.4) and thus the controller objective (eq.(4.3)). More specifically, the m_1 parameter defines a range (see eq.(4.6)) for which the output of the function φ is zero. This means that, when the relative displacement between $\mathrm{x_{sprung}}$ and $\mathrm{x_{unsprung}}$ is in this safe interval, the relevance of the $\dot{x}_{unsprung}$ term will be small in eq.(4.3) and the controller will target comfort.
The m_2 parameter on the other hand, as shown in figure (4.2), defines the rapidity with which the output of function (4.5) changes when the suspension travel is outside the comfort interval. It follows that this value affects the quickness with which the controller changes its aim from comfort to suspension travel limitation. Following the original work of the authors in [31], the controller is provided with the possibility of selecting these parameters according to the road condition in order to improve the suspension system behavior.
Firstly, the road conditions are divided into two main groups, namely rough and smooth. Then, two different sets for the (m_1,m_2) couples are defined, one for each road condition, in order to obtain the best behavior from the suspension.
In particular, the m_1 parameter should be quite large when road conditions are good and small when the road is rough. This choice is reasonable: the controller aim when the road is smooth should be mainly comfort. When the road conditions are worse, on the other hand, the controller should focus more on the suspension travel reduction.
As for m_2, its value should be able to balance the effect of m_1. Therefore, it is selected as small for smooth roads, as the controller should promptly react to bumps. A larger value is necessary when the road unevenness are relevant, so that the controller can have an effective but smoother reaction to the road unevenness.
As last step, the controller should be able to assess the conditions of the road on which the vehicle is traveling to select the best parameters couple. To achieve this goal, a possible solution is represented by the choice of two values of the suspension travel that, if overcome, will define the transition from a smooth road to a rough one and vice versa. It is then defined the t_{SR} parameter for the passage from a smooth to a rough profile, and the t_{RS} for the opposite situation. Hence, with a simple comparison, the controller can decide which are the road conditions and consequently which is the best couple to be used.
The implementation of this feature in Simulink Stateflow is shown in figure (4.3). Every rectangle represents a specific state of the road, which in turn produces as output a given couple of the (m_1,m_2) parameters. Each arrow represents a transition from one state to the other, and the strings among the square brackets are the conditions that must be met in order to traverse the edge that links two contiguous states. As can be seen, in the initial condition the road is assumed to be smooth (State 1) and the suspension travel value is compared with the t_{SR} threshold. When this limit is overcome (transition from State 1 to State 2), and

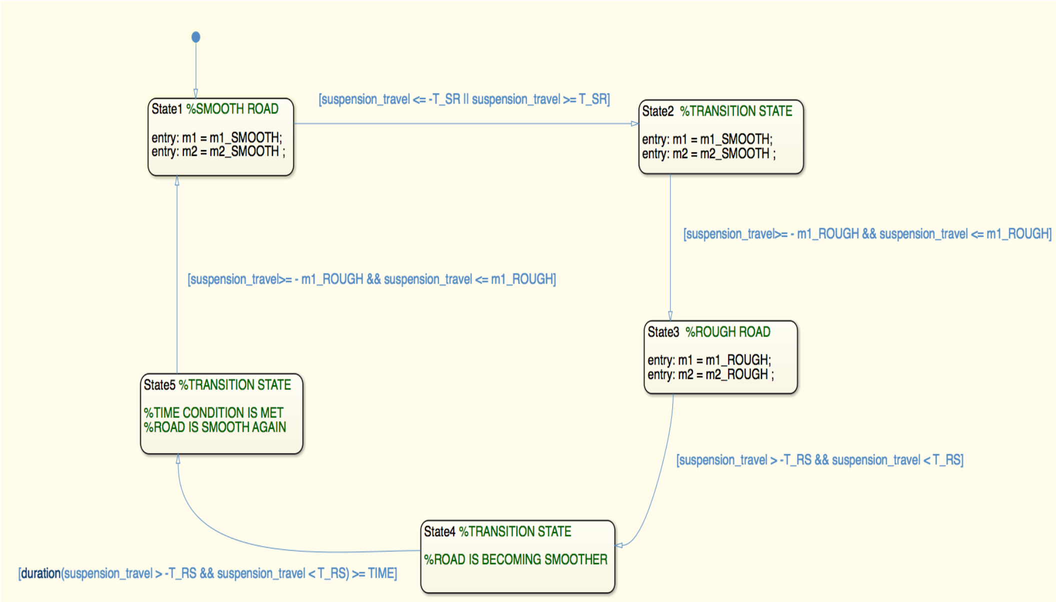


Figure 4.3: Simulink Stateflow scheme of road adaptation capability

as soon as the nonlinearity of equation (4.5) is not active (transition from State 2 to State 3), the (m_1,m_2) parameters are changed as the condition of the road surface have worsened (State 3). At this point, either the road continues to be rough (permanence in State 3) or it starts to become smoother (transition from State 3 to State 4), i.e. the suspension travel falls below the t_{RS} limit. To state if it is possible to return in the starting state (State 1), namely if the control can focus again on comfort, it is also checked if the relative displacement between x_{sprung}-x_{unsprung} is maintained below the t_{RS} limit for a give time interval (transition from State 4 to State 5). If these two conditions are met then the coefficients are changed again.

4.3 Control Law

In order to introduce the computations that leads to the definition of the controller output u, the state variables notation is introduced.
The state vector is defined as

$$x = \begin{bmatrix} x_{sprung} \\ \dot{x}_{sprung} \\ x_{unsprung} \\ \dot{x}_{unsprung} \\ P_{as} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \tag{4.8}$$

Starting from the equations obtained for each component in Chapter 3, the full system can be rewritten in terms of state equations as:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} \dot{x}_2 \\ -\frac{1}{M_b}[K_a(x_1 - x_3) + C_a(x_2 - x_4) - A_{eff}x_5] \\ x_4 \\ \frac{1}{M_{us}}[K_a(x_1 - x_3) + C_a(x_2 - x_4) - K_t(x_3 - r) - A_{eff}x_5] \\ \frac{kRT}{V(z)}[\frac{A_{max}}{u_{max}}\Omega u - \frac{A_{eff}}{RT}x_5(x_2 - x_4)] \end{bmatrix} \tag{4.9}$$

In the design it is also included equation (4.2):

$$\dot{\bar{x}}_3 = -(\varepsilon_0 + \kappa_1 \varphi(\zeta))(\bar{x}_3 - x_3)$$

The design of the controller is performed always following [30]. It is then defined:

$$e_1 = x_1 - \bar{x}_3 \tag{4.10}$$

Following the backstepping procedure, the e_1 error is derived with respect to time:

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{\bar{x}}_3 \\ &= x_2 + (\varepsilon_0 + \kappa_1 \varphi(\zeta))(\zeta - e_1) \end{aligned} \tag{4.11}$$

where ζ is the suspension travel. Now the aim is to stabilize the e_1 subsystem. As can be seen, equation (4.11) depends on the state variable x_2, which can thus be used as a control input. Hence:

$$e_2 = x_2 - \alpha_1 \tag{4.12}$$

$$\alpha_1 = -c_1 e_1 - (\varepsilon_0 + \kappa_1 \varphi(\zeta))\zeta \tag{4.13}$$

Now, similarly to the previous step, the aim becomes the stabilization of e_2. Since x_5 appears in $\dot{e}_2$ expression, it will be used to provide the stability.
Consequently it is defined $e_3 = x_5 - \alpha_2$ and the time derivative of (4.12) is computed:

$$\begin{aligned} \dot{e}_2 &= \dot{x}_2 - \dot{\alpha}_1 \\ &= -\frac{1}{M_b}\left[K_a(x_1 - x_3) + C_a(x_2 - x_4) - A\underbrace{(e_3 + \alpha_2)}_{\mathrm{x}_5}\right] + g_2 \end{aligned} \tag{4.14}$$

where $g_2 = -\dot{\alpha}_1$.
The α_2 is then used to cancell the unusefull terms as well as to provide stability:

$$\alpha_2 = \frac{M_b}{A_{eff}}\left[-c_2 e_2 - e_1 + \frac{1}{M_b}[k_a(x_1 - x_3) + C_a(x_2 - x_4)\right] - g_2 \tag{4.15}$$

In order to stabilize the e_3 subsystem, it is computed its time derivative:

$$\begin{aligned}
\dot{e}_3 &= \dot{x}_5 - \dot{\alpha}_2 \\
&= \frac{kRT}{V(z)} \left[\frac{A_{max}}{u_{max}} \Omega u - \frac{A_{eff}}{RT} x_5 (x_2 - x_4) \right] - \dot{\alpha}_2 \\
&= \frac{kRT}{V(z)} \frac{A_{max}}{u_{max}} \Omega u - \frac{A_{eff}}{V(z)} x_5 (x_2 - x_4) + \underbrace{g_3 + (d_3 + n_3 h_3)\, r}_{\text{-}\dot{\alpha}_2}
\end{aligned} \tag{4.16}$$

where:

$$n_3 = \frac{M_b K_t}{A_{eff} M_{us}} \tag{4.17}$$

$$d_3 = n_3 \left(\frac{C_a}{M_b} - \varepsilon_0 \right) \tag{4.18}$$

$$\begin{aligned}
g_3 = & \frac{M_b}{A_{eff}} \left[(c_2 + c_1) \left(-c_2 z_2 - z_1 + \frac{A}{M_b} z_3 \right) \right] + c_1 \kappa_1 \frac{\mathrm{d}\varphi}{\mathrm{d}\zeta} z_1 \\
& + \left[c_1^2 - 1 + c_1 (\varepsilon_0 + \kappa_1 \varphi(\zeta)) \right] \left[-c_1 z_1 - (\varepsilon_0 + \kappa_1 \varphi(\zeta)) z_1 + z_2 \right] \\
& + \frac{1}{M_b} \left[K_a (x_2 - x_4) + C_a w_1 \right] - (\varepsilon_0 + \kappa_1 \varphi(\zeta)) w_1 \\
& - 2\kappa_1 \frac{\mathrm{d}\varphi}{\mathrm{d}\zeta}^2 - k_1 \frac{\mathrm{d}^2\varphi}{\mathrm{d}\zeta^2}^2 \zeta - \kappa_1 \frac{\mathrm{d}\varphi}{\mathrm{d}\zeta} w_1 \zeta
\end{aligned} \tag{4.19}$$

In equation (4.16) it has been highlighted the derivative of α_2. The $\dot{\alpha}_2$ term contains the derivative of the unsprung mass velocity, namely $\dot{x}_4$, which as can be seen from equation (4.9) contains the road disturbance term r. Since r can not be predicted a priori, in order to compensate for the nonlinearities introduced by $\dot{\alpha}_2$, all the terms related to the road disturbance has been isolated and brought out from the derivative expression (see eq.(4.17), eq.(4.18), eq.(4.19)) because they can not be cancelled.
Finally, from equation (4.16) the controller output u appears, the procedure can be terminated by defining α_3:

$$\alpha_3 = \left[-c_3 e_3 - \frac{A_{eff}}{M_b} e_2 - b_3 h_3^2 e_3 + \frac{k A_{eff}}{V(z)} (x_2 - x_4) x_5 - g_3 \right] \tag{4.20}$$

and finally obtaining the explicit expression for u as

$$u = \frac{V(z)}{kRT} \frac{u_{max}}{A_{max}} \frac{1}{\Omega} \alpha_3 \tag{4.21}$$

It must be noted that $\Omega \neq 0$ as the pressure will always be limited between the ambient pressure P_{atm} and the supply pressure P_{sup}

Chapter 5

Simulations And Results

In this chapter are presented the results of the simulation that have been run in order to test the performances of the control model. As a term of comparison it has been used a quarter car model equipped with a passive suspension system.
The magnitudes of the two models that will be observed are represented by the vertical body acceleration, the suspension travel and the roll angle ϕ.
In the following, 3 section will be presented, each corresponding to a different testing condition that will be explained in detail along with the results.

5.1 Test 1: Car Body Roll Behavior

In this section is analyzed the capability of the system to cope with the roll behavior of the vehicle due to lateral load transfer during cornering. Firstly a brief description of the load transfer phenomenon is given, then the setup for the test and the result achieved are discussed.

5.1.1 Roll behavior description

Besides considering the rejection of the disturbances coming from the road, there are other driving situations in which it is required the intervention of the active suspension system. One of this case is represented by cornering, where the vehicle lateral dynamics is involved. In particular, when a car undertakes a curve, the load shifting phenomenon takes place. This means that an additional weight term will act on the wheel which is traveling on the outer part of the curve, as it is shown in figure (5.1), where Δz represents the lateral load transfer term.
Due to this phenomenon the vehicle will bend laterally, around the roll axis, forcing the suspension to compress in correspondence of one wheel of the axle and to

extend on the other one. The active suspension should be able to compensate for this situation by means of the actuator force.

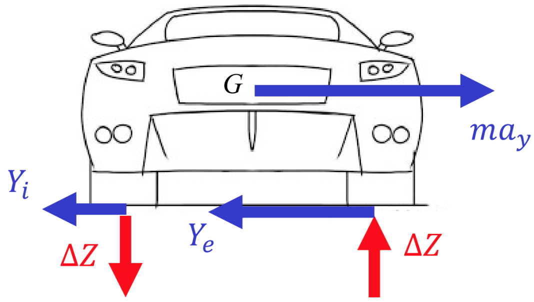


Figure 5.1: Lateral load transfer in cornering

5.1.2 Test Setup

To simulate this condition, it is possible to exploit the quarter car model instead of the half car model. In particular, following [22] the load transfer and the consequent roll behavior can be represented in the quarter car model as a disturbance force which acts on the sprung mass. The profile of the load which is used to model the disturbance derives from [22], where the measured data during a real test on a circuit are exploited. The realization in the Simulink environment of the measured force ([22]) is shown in figure 5.2. As can be seen, it is represented as a force acting for 3s with a value of 1920 N

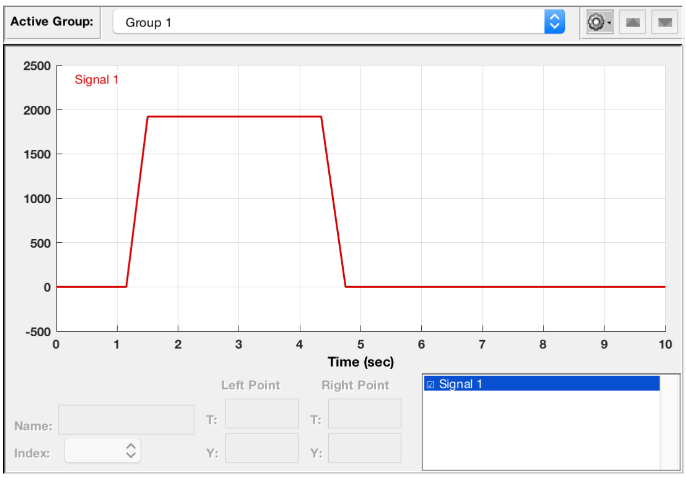


Figure 5.2: Simulink force profile representing roll behavior

In order to perform the test, the overall system model is modified with respect to the original scheme. In particular the road disturbance is set to zero, as only the behavior during cornering must be observed. Furthermore, in the sprung mass force balance is added the force term representing the load behavior (figure (5.3)). Always according to [22], the test conditions lead to a negligible displacement for

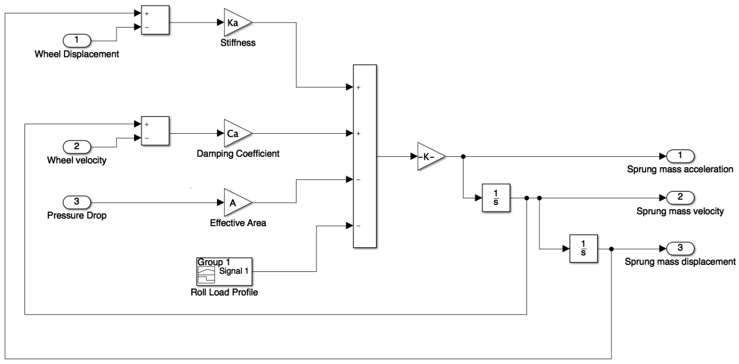


Figure 5.3: Simulink sprung mass scheme for active roll compensation

the unsprung mass. As a consequence, its influence is not considered in the definition of the rolling behavior of the car, which will be evaluated only basing on the sprung mass dynamics.
Taking into account this last consideration, the observed quantity is defined as the the roll angle, which can be expressed starting from the sprung mass displacement as [22]:

$$\phi = \arctan\left(\frac{2x_s}{T}\right) \tag{5.1}$$

where x_s is the sprung mass displacement and T is the track width.

5.1.3 Results And Comparison

The simulation is then performed according to the described setup and the results are shown in figure 5.4.
It can be seen from the plot that there is a significant difference between the behaviors of the passive and of the active suspension.
In particular, the active suspension is able to provide a great reduction in the roll angle value, as well as eliminating the oscillation that are present in the response of the passive system. As a further proof, the performance achieved by the controller are also compared with the original result presented in [22], where the same analysis was performed with an electromagnetic active suspension.
In order to establish the comparison, it must be accounted that the electromagnetic

damper is assumed to have a nonlinear characteristic, as well as the other quarter car parameters like sprung mass, unsprung mass and stiffness constants present different values with respect to the ones used to simulate the controller.

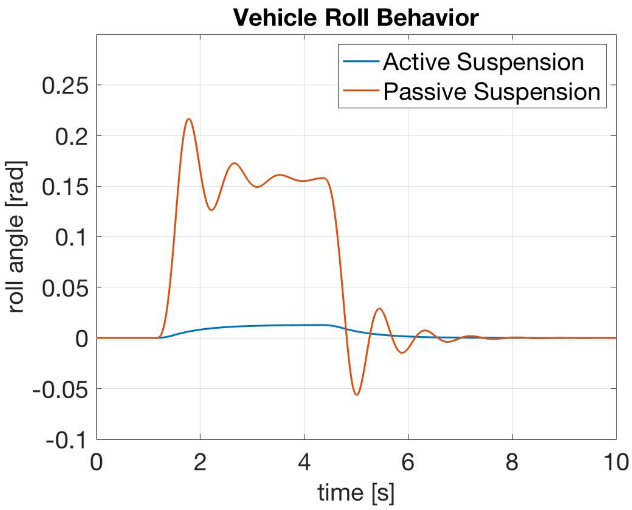
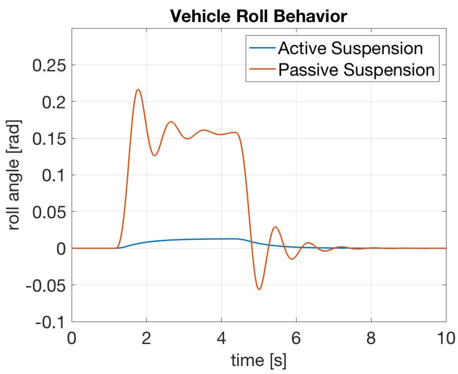
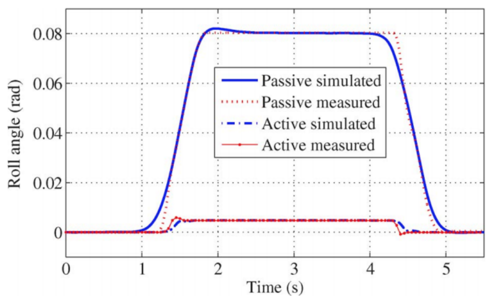


Figure 5.4: Vehicle roll behavior comparison

The comparison between the active pneumatic solution and the electromagnetic one are shown in figure (5.5).



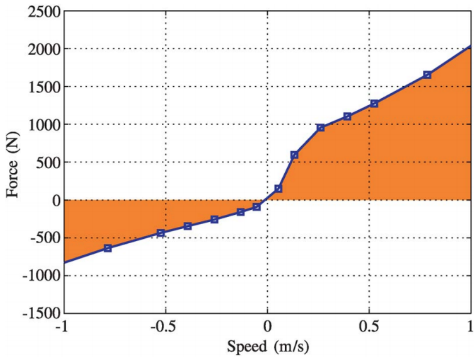
(a)



(b)

Figure 5.5: (a) Pneumatic system roll, (b) Electromagnetic system roll

As can be seen from figure (5.5a), the results achieved by implemented controller are similar to those reached by the active damper of [22], as both presents a dramatic reduction with respect to the roll angle values coming from the passive system. This testify the consistence of the results obtained by the implemented controller.



Parameter	Value
Sprung mass	387 Kg
Passive system stiffness	29.1 $^{kN}/_{m}$
Active system stiffness	31.2 $^{kN}/_{m}$

Figure 5.6: Parameters for setup of [22]

5.2 Test 2: Single Bump Road Profile

As a second test, a road profile constituted by a single sinusoidal bump is used. The aim of this simulation is to represent the classical scenario when the car hits a bump, in order to evaluate the vertical acceleration and the suspension travel.

5.2.1 Test Setup

The road shape is given, in analytical form, by the expression [33]:

$$r = \begin{cases} a\left(1 - cos\left(2\pi f t\right)\right) & t_2 \leq t \leq t_1 \\ 0 & \text{otherwise} \end{cases} \tag{5.2}$$

where the a parameter correspond to half of the bump height expressed in meter, f is the frequency of the road disturbance, t represents the time and t_1 and t_2 are two frequency dependent thresholds, required to isolate a single cosine oscillation. The first simulation is run with a frequency of f=4Hz and an amplitude of 0.05m, corresponding to a value of a=0.025:

$$r = \begin{cases} 0.025\left(1 - cos\left(8\pi t\right)\right) & 0.5s \leq t \leq 0.75s \\ 0 & \text{otherwise} \end{cases} \tag{5.3}$$

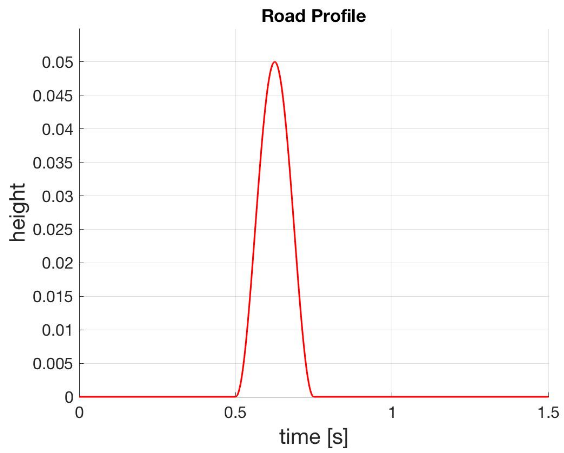


Figure 5.7: 0.5 cm bump

5.2.2 Test Results

The result of the test, showing the comparison between passive and active accelerations and suspension travel values, are shown in figure (5.9) and figure (5.8) respectively.
In response to the road disturbance, the suspension system compresses and then start to oscillate. In particular, as soon as the bump is hit, the system undergo a negative deflection, which results to be smaller for the passive suspension, as can be seen in figure (5.8).
This behavior, however, is perfectly coherent with the adopted control strategy.

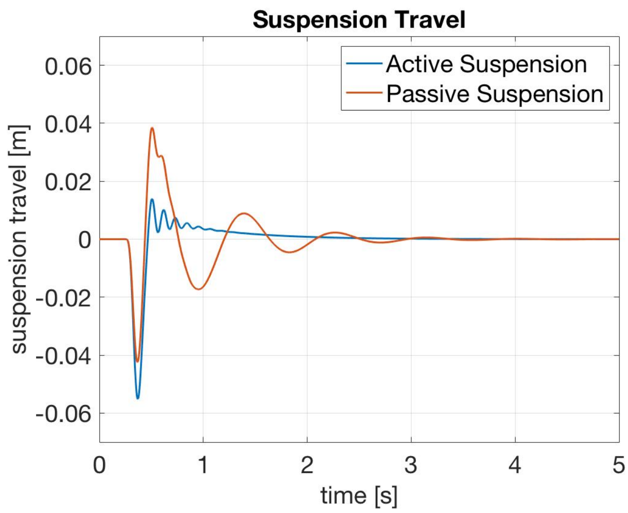


Figure 5.8: Suspension travel comparison for 5cm bump

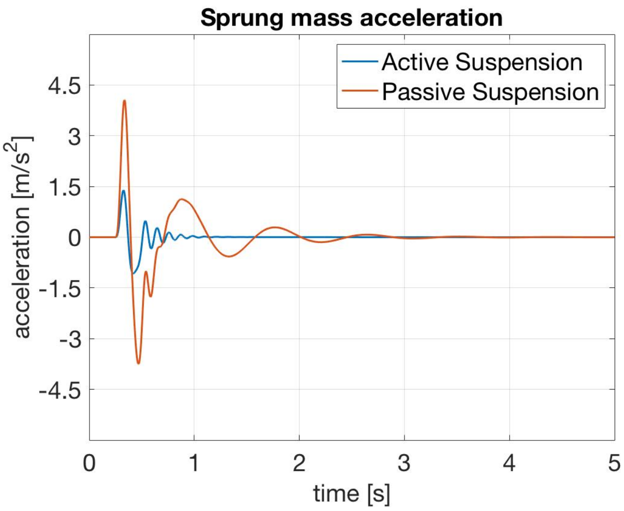
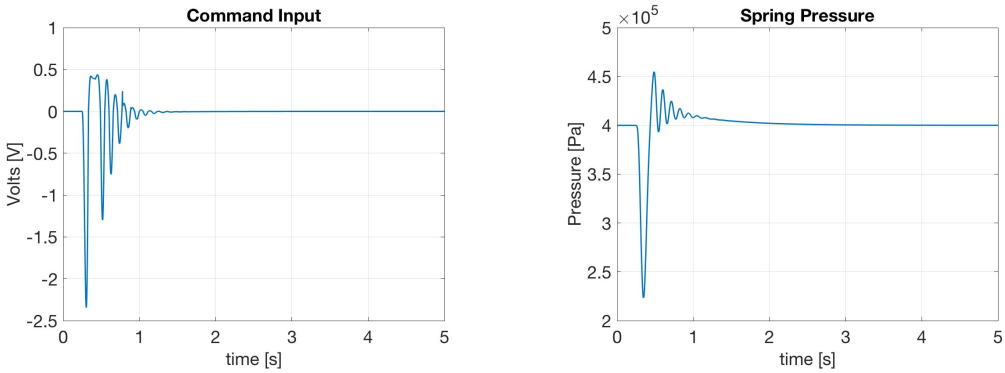


Figure 5.9: Sprung mass vertical acceleration for 5cm bump

Since the bump height does not cause the suspension travel to overcome the limits for which the controller should then focus on reducing the suspension deflection, the aim of the active control remains the reduction of the vertical acceleration. This is achieved when the suspension is allowed to exploit a larger amount of its available stroke, which is what happens in the plot of figure 5.8.
It can also be seen from figure (5.9) that the values of the vertical acceleration for the active system are greatly reduced with respect to the passive ones. This eventually translates into a greater comfort level.
In order to establish the effectiveness of the controller,also the values of the command u produced by the controller and the internal pressure variation for the spring must be observed.



(a) (b)

Figure 5.10: (a) Command input u, (b) Air spring pressure variation

As can be seen from figure 5.10, the limits for both the variables that are shown are respected. More specifically, the input voltage is within the [-5V,5V] range and the air spring internal pressure does not overcome the minimum and maximum pressure level shown in the datasheet, defined as 1Bar and 8Bar respectively. As a consequence, it can be stated that the controller is working properly.
In particular, it is capable of imposing a soft behavior to the pneumatic suspension in response to the road bump, thus increasing the level of comfort for the passengers, since the values of the suspension travel are far from the maximum limit.
In table 5.2 is shown a summary of the performances that have been reached with the active suspension with respect to the passive system.
The numerical data confirm the performed analysis

Parameter	**Passive System**	**Active System**	**Improvement**
Max. Vertical Acceleration	4.05 m/s^2	1.39 m/s^2	65.8%
Max. Suspension Deflection	0.0386 m	0.0139 m	63.8%

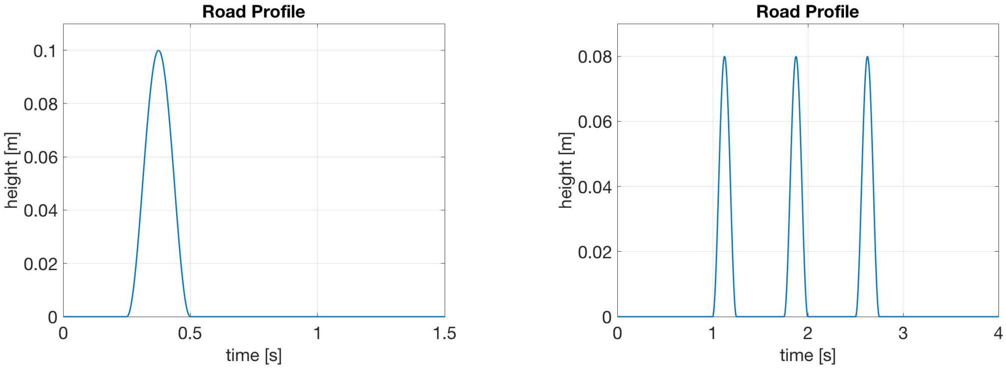
Table 5.2: Single bump results comparison

5.3 Test 3: Road Adaptability And Higher Bumps

So far, the model capabilities have been tested for the primary aim of this active suspension, namely the compensation of the rolling behavior, and also for a bump of medium amplitude. In order to check if the system is able to cope with higher bumps, as well as if the road adaptive feature produces an effective increase of the performances also for a pneumatic system, a rougher road profile is considered.

5.3.1 Test Setup

The road profile for this test is shaped as in test 2, exploiting equation (5.2). The height of the bump is firstly set to 10cm, which according to [30] is a sufficient height to bring the suspension near its stroke limits. Successively, the road roughness is increased and a new profile is defined following [31], in order to analyze the behavior of the system when the condition of the road are worsening.
In this test, besides the values of the suspension travel and the vertical acceleration, it will also be observed the variation of the parameters (m_1,m_2) (see eq.(4.6) and figure (4.2b)).



(a) (b)

Figure 5.11: (a) 10 cm bump, (b) Rough road pavement profile

5.3.2 Test Results And Discussion

The simulations are run but with both the road profiles the execution stops and it can not be completed by Simulink. In order to understand the reasons of the failure, it is firstly considered the active force produced by the air spring. As stated in the previous chapter, it is obtained as the product between the pressure variation and the air spring active area. Because of the increment of the height of the bump, also the force required to compensate the disturbance is increased. As the area parameter has been considered constant, in order to produce an increase of the force to cope with the bump, the pressure variation should assume greater values in terms of magnitude. As a consequence, the physical pressure limits imposed by the spring are overcome when the bump is hit. In order to verify this hypothesis, the parameter of the actuator are modified. In particular, it is considered an air spring with with a bigger area (tab.5.3), but with a force-displacement characteristic similar to the original one, i.e. almost linear, so that the consideration on the actuator force made in chapter 3 are still valid. The other parameters, included stiffness, are maintained fixed. With the new air spring parameter, the road profiles (figures (5.11a) and (5.11b)) are tested again. The attempts are successful, and the simulations are completed.
The first test performed is the single bump. As can be clearly seen from figure (5.12), due to the increased height of the bump, both the active and the passive suspensions greatly compress when the bump is reached. In this case, however, the deflection of the active system overcomes the lower boundary related to comfort, defined in equation (4.6) by parameter -m_1. As a consequence, the controller will aim at preserving handling, thus limiting the suspension travel. Hence, the active suspension shows reduced value, during both compression and extension, with respect to the passive system.

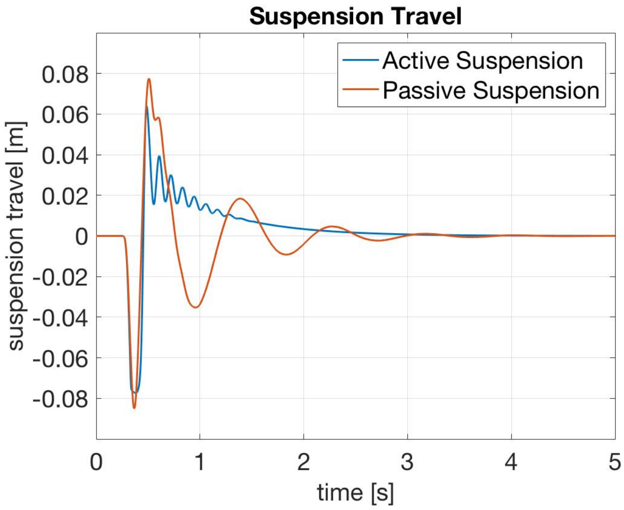


Figure 5.12: Suspension travel comparison for 10cm bump

The side effect of the change in the control objective, on the other hand, is the increase of the vertical acceleration, as can be seen from figure (5.13). The vertical acceleration for the active suspension shows a great positive peak of more than 30 m/s^2 whereas the maximum value for the passive system is around 10 m/s^2.

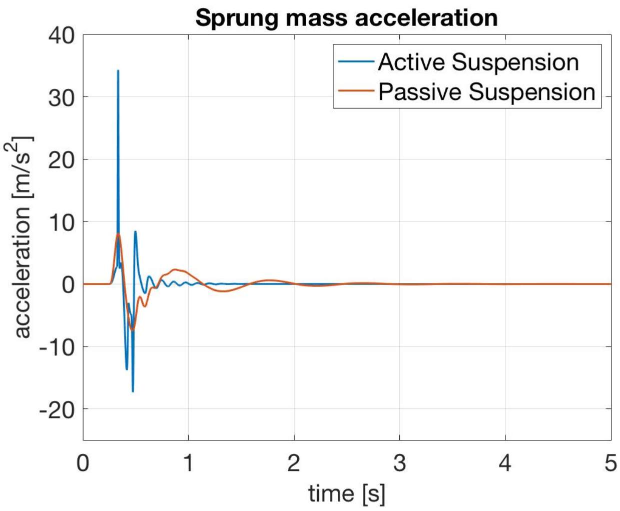


Figure 5.13: Vertical acceleration comparison for 10cm bump

This is a consequence of the trade-off managing. The road surface requires a more rigid behavior from the active suspension, which implies a decrease of the comfort. It must be considered, however, that peaks values are experienced for a very short period of time and the oscillations, both in the acceleration and in the suspension

travel, are damped quickly for the active system.
The second test performed concerns a rough road profile. It is simulated as a series of bumps of a given height, in this case 8cm, in a row.
Plot (5.14) shows the suspension travel values along with the m_1 parameter, which should vary when the the condition of the road pavement becomes worse.

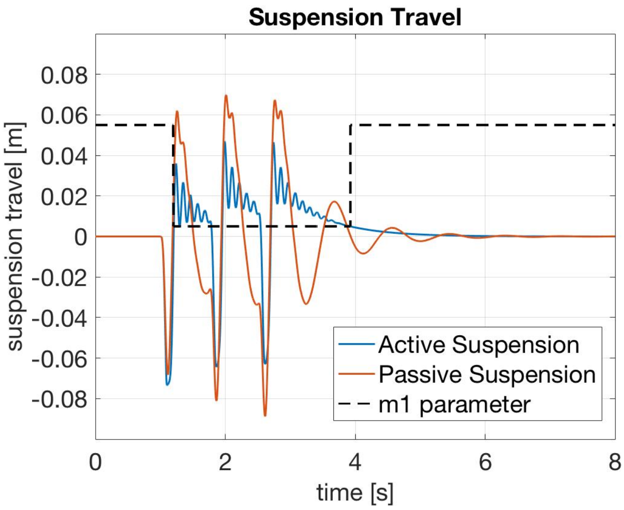


Figure 5.14: Rough road suspension travel

As can be clearly seen, the active system performs better with respect to the passive suspension. An important consideration, however, is related to the m_1 parameter. As it is evident from the dark dashed line in the graph, the controller is able to recognize the change in the road conditions.According to the behavior explained in section 4.2.2, when the required conditions are met, the parameter is changed from the value suitable for smooth roads ($m_{1\mathrm{smooth}}$) to the one conceived for roughs roads ($m_{1\mathrm{rough}}$). This condition correspond to the first step variation in the plot. This value then is maintained until the road surface conditions improves, when the second step variation happens. The variation of the correspondent m_2 parameter, not shown in plot (5.14), is complementary to the one of m_1.
The plot for the vertical acceleration is shown in figure (5.15). It can be seen that, also in this case, this magnitude presents slightly greater values with respect to the passive system, as a consequence of the suspension trade-off managing. In particular, the highest peak appears when the (m_1,m_2) couple is the one for smooth roads, whereas after the adaptation the peaks are significantly reduced.
From the performed analysis for both the single bump and the the rough road profile, it can be stated that the controller is performing according to the expected behaviors. In the former case it is able to limit the suspension deflection. In the latter, besides guaranteeing the reduction of the suspension relative displacement,

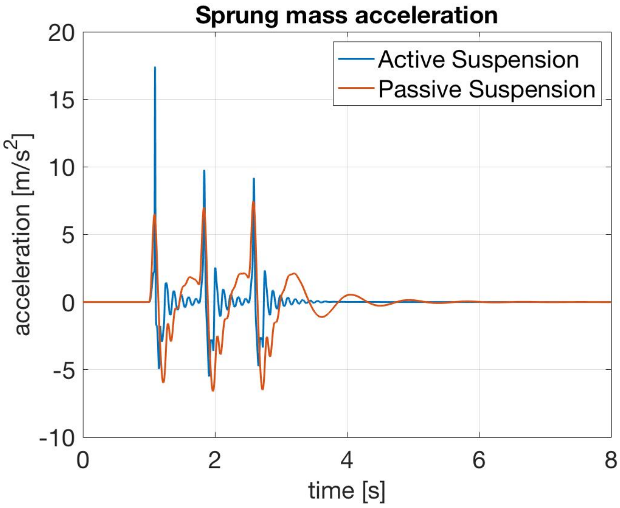


Figure 5.15: Rough road vertical acceleration

the controller changes the m parameters from "smooth" to "rough" and vice versa, according to the road conditions in order to improve the system response.
However, in order to establish the goodness of the achieved performances, the values of the controller output u must be considered. This parameter is strictly related to the maximum and minimum voltages applicable to the pneumatic valve as well as to the system energy consumption. Hence, this verification is necessary because, besides providing the required comfort or handling, the voltage values and the energy consumption must be in an acceptable range.
Before performing the analysis, it is reminded that the limits for u in order to control the pneumatic valve, are represented by the operating range [-5V,5V].

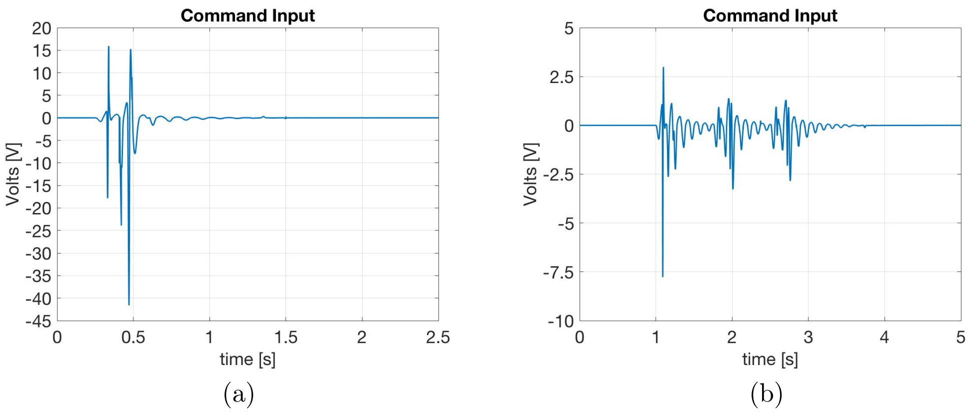


Figure 5.16: (a) Voltage values for 10 cm bump, (b)Voltage values for rough profile

The values of the control output for the single 10 cm bump disturbance are reported in figure (5.16a).
As can be seen from the plot, the controller output voltage exceeds the maximum and the minimum values which are allowed for the valve control.
Considering the series of 3 bumps of 8cm height, the controller output is shown in figure (5.16b). In this case it can be noted a peak value that overcomes the maximum voltage that can be applied to the valve. It is reached when the first bump is faced, then there is a reduction and the values return within the safe range. As a result of this further analysis, it can be stated that the controller shows the expected behavior when the suspension faces bumps of greater size. In particular, the trade off is correctly handled and the algorithm parameters are adjusted according to the variation of the road profile condition. However, the voltage limits are exceeded. As a consequence, also the energy consumption imposed by the system when dealing with higher bumps or with a rough road profile is higher.

Parameter	Value
Initial Air Spring Area	0.0075 m^2
New Air Spring Area	0,021 m^2

Table 5.3: New area parameter

Chapter 6

Conclusions And Future Work

This work started with a research in the literature of the main concepts about suspension systems. The following step consisted in the investigation of air springs, as they have been selected as actuator for the pneumatic suspension model. In particular, air springs present many advantages if compared to other solutions. First of all, they have a reduced weight and cost, which are important factors in the design of a car. They are also more environment-friendly than hydraulic systems, as they exploit compressed air. This means that possible leakage will not affect the environment, as air would be released in the atmosphere. Furthermore, they increase the isolation from vibration of the vehicle body and reduce the wear of tyres. The main results of this phase, was to point out that active systems can handle the trade-off between passengers comfort and handling imposed by the suspension.

The next step consisted in the modeling of the pneumatic suspension. Firstly, the required components have been identified. The fundamental elements, besides the controller and the car model, are the pneumatic valve and the air spring. The former is necessary to regulate the air flow during charging and discharging phases, while the latter is responsible for the elastic force contribution and the active force production. The 3/3 proportional valve is characterized only through its flow stage, as its dynamic is assumed to be faster than the one of the system. The air flow rate is determined according to a proportional relationship between the opening of the valve orifice and the applied voltage. The actuator model has been characterized basing both on the data provided by the manufacturer and on the physical phenomena that occurs in the air spring. For this purpose, an air spring with almost linear characteristc between force and displacement and volume and displacement has been used. This allowed to assume the spring stiffness and the effective area as constant values. To model the vehicle, the quarter car model has been exploited, as it gives a proper representation of the vehicle vertical dynamics. This aspect is fundamental to evaluate the performances of the system. To conclude the modeling phase, the set of equation which represents the suspension has been integrated in MATLAB Simulink .

The last step required the implementation of a suitable strategy to control the suspension. During the literature research of the first phase, many solutions have been analyzed. Since nonlinear control offered the possibility to deal properly with the suspension trade-off and the pneumatic valve nonlinearities, a backstepping control alghoritm has been selected. In particular, it has been adapted to the pneumatic system, as it was originally conceived for an hydraulic system. The main feature of this strategy is the possibility to move the controller effort from comfort to handling and vice versa. Exploiting the suspension travel measurements, the controller can regulate the response of the suspension, making it stiffer or compliant. In addition, the controller can infer the road condition from the suspension relative displacement. In this way, it can adjust two internal parameters to further enhance the suspension response .

After completing the system modeling and the controller implementation, tests have been executed by means of Simulink software. The results obtained have been compared with those of a passive system, equipped with a mechanical spring and a damper. The performances are expressed in terms of vertical acceleration for comfort, suspension travel for handling and roll angle for cornering.
The first test aimed at verifying the response of the active suspension when the car bends around the roll axis due to load transfer. The results showed a great reduction of the roll angle for the active system. As a further validation, the achieved enhancement proved to be comparable with the performances of an active electromagnetic suspension under the same test situation.
The second test is concerned with the response of the active suspension when a bump of medium size is crossed. The controlled acted as expected, as it improved the comfort by imposing a low damped behavior to the suspension.
The driving scenario of the third test represented worse road conditions. Firstly a single bump of greater height is simulated. Then, a rough road profile constituted by three bumps in a row is used. In both cases the controller handled the trade-off as expected. In particular, with the selected bumps heights, the suspension should reach the compression limit of its stroke. To avoid this situation, the controller changed its objective in order to reduce the suspension travel. The control voltage generated by the controller to achieve these results, on the other hand, are unsuitable. As the aim of this work was to realize a basic control model for a pneumatic suspension capable of dealing with common driving situation, the objective can be considered fulfilled. The response of the pneumatic active system controlled through backstepping are improved both in cornering and in

medium height bump crossing. However, when the road conditions become worse, the performances are maintained at the expense of a higher controller output.

6.1 Future Work

The developed model performs well when operating in a given range of working conditions. The implementation of the dynamic characteristic for the spring stiffness and the introduction of the valve dynamics would be beneficial to obtain a system behavior closer to the real case. Furthermore, the response of the system with bumps of higher size must be investigated in order to reduce the generated command. This will allow to expand the capabilities of this model.

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Appendices

Appendix A

Model Parameters

In table A.1 are shown the simulation parameters for the air spring, the valve and the quarter car model.

Parameter	**Value**
Sprung mass M_b	290 Kg
Unsprung mass M_{us}	59 Kg
Air Spring Stiffness K_a	20000 $^N/_m$
Passive Spring Stiffness K_{pas}	16812 $^N/_m$
Damping Coefficient C_a	1000 Nm
Tire Stiffness K_t	190000 $^N/_m$
Max. Valve Opened Area	$3 \cdot 10^{-5} m^2$
Max. Valve Supply Voltage	5 V
Critical Pressure Ratio b	0.528
Polytropic Coefficient k	1.4
Ideal Gas Constant R	287 $^J/_{KgK}$
Wokring Temeprature T	290 K
Atmospheric Pressure P_{atm}	1 Bar
Supply Pressure P_s	10 Bar
Spring Area A	0.0075 m^2
Spring Static Volume V_0	0.0011 m^3

Table A.1: Suspension simulation parameters

Appendix B

Algorithm Parameters

In table B.1 are shown the values used for the controller parameter from [30].

Parameter	**Value**
c1	500
c2	500
c3	500
b3	0.01
e0	1.5
$m_{1smooth}$	0.055
$m_{2smooth}$	0.005
m_{1rough}	0.055
m_{2rough}	0.055
k_1	0.0125

Table B.1: Algorithm Parameters