POLITECNICO DI TORINO

Mechanical and Aerospace Engineering Department

Master’s Degree in Automotive Engineering

TORQUE VECTORING DIFFERENTIALS

COMPARISON AND EFFECT ON VEHICLE DYNAMICS

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SUMMARY

The thesis is about different torque vectoring technology used during the last 20 years on cars.

Many systems will be analysed using the approach of the applied mechanic and a different approach, normally used for gearboxes, named Velocity Diagram approach. The study is focus on the state of art of this technologies in order to show the pros & cons of the different solutions.

Then, one system is chosen, from the previous discussed, and it will be implemented in a car dynamic simulator to evidence the performances improvement fitting this kind of technology on a passenger car.

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a	Centre of gravity to front axle distance
b	Centre of gravity to rear axle distance
m	Vehicle total mass
I	Inertia moment about vehicle z axis
v	Longitudinal vehicle speed
r	Yaw rate
$\dot{r}$	Drag coefficient
m	Vehicle mass
L	Wheelbase
t	Track
a_y	Lateral acceleration
C_f	Front axle cornering stiffness
C_r	Rear axle cornering stiffness
β	Side slip angle
r_w	Wheels rolling radius
F_{aj}	Actuator force
T_j	Torque applied to element j
ω_j	Rotational speed of element j
$I_{j/k}$	Transmission ratio of j and k gear set
δ	Wheel steering angle

1 TORQUE VECTORING INTRODUCTION

We want to start our study focusing our attention to set some base-knowledge, for example starting from the definition of what does means “Torque vectoring functions” of a differential and where/when this term was born.

We talk about torque vectoring when the powertrain of a vehicle is able to change the dynamic behaviour of the vehicle itself according to current dynamic state. Practically, following some design parameter, during its working, a car is able to change its understeering/oversteering behaviour in order to improve performances, drivability and stability in lateral dynamic, modifying the torque applied to each driven wheel.

This way of working can be exploited on 2WD and 4WD vehicles both managed in mechanical or electronic way.

The first time in history toque vectoring appears was on lancer EVO IV GRS in 1996, with the system named Mitsubishi AYC (Active Yaw Control), the rear differential of the vehicle was able to split part of the torque coming from the engine on left and right wheel in order to create a yaw moment on vehicle to increase lateral dynamic performances.

During last years, the engineers are trying to improve, not only dynamic performances of the vehicle, also fuel consumption, comfort and safety of passenger vehicles with torque vectoring technique. The power of this technology is shown by some studies made by different research department, where was shown the possibility to use some torque-vectoring logic control instead of the ESP, with the result of better performances in many manoeuvre; so we’re talking about a system able to increase performances, not only on lateral dynamic, but also for safety and stability of vehicles.

2 OBJECTIVES

The objectives of this thesis are firstly to set the state of art of the torque vectoring technologies, nowadays used, in order to put in evidence the differences between the systems, finding out also the common points in order to find a future direction to develop those system to improve the performances and the safety of our cars.

Finally, the goal is starting from one of the technology investigated to try to develop a control strategy, using a car dynamic simulator as CarMaker, in combination with Matlab/Simulink used to tune and create our system. Our purpose is to maintain the same sensors fitted on our vehicles, in order to make the solution less invasive as possible. In order to do that, some new function will be implemented in the car ECU to correctly exploit our functions. Then the new functions will be used to control the system involved.

3 TORQUE VECTORING SYSTEMS AND DIFFERENTIALS CLASSIFICATION

It's necessary to speak about the differences of the different system in the market and to underline some important characteristic of all of them, to do that I'd like to introduce some families of system that are common for behaviour and working principle.

We know that torque vectoring strategies could be applied on ICE, EV, HV vehicles and are as suitable for 2WD vehicle as for 4WD vehicle; it's clear that, for our study, 4WD vehicle are more interesting.

- So we can make a classification on the basis of the technology used to change the transmission behaviour:
- Mechanically, using special type of joints or self-locking differentials.
- Mechanically Electro-hydraulic controlled, to change differential torque split between the wheels:
- ESP/ESC hardware, exploited by brake caliper.
- Electric motor or hydraulic actuator on some part of differential (better solution to avoid power loss).
- Full-electric, one electric motor for each wheel without any physical connection between them, this is the best solution in term of freedom, but it's the more complex for the software tuning and it's suitable only for EV and HV.

Set this concept, we can start analysing the system mounted on our cars, I'll give an explanation of one system for each category, going in crescent order of flexibility of the system.

I'll analyse the following torque vectoring systems to cover all the previous category:

Mitsubishi AYC, first road car in history to manage yaw moment to increase performances. The system was mounted on Lancer EVO IV in 1996. The car has 4WD with, mechanical open-diff as front differential, a viscous joint as centre differential, and a Torque Transferring Differential at rear axle (the one used to act yaw moment on the vehicle), that is a pneumatic controllable device, where the control is performed by a dedicated ECU.

Audi QUATTRO sport diff., permanent 4WD equipped with a crown-gear LSD as centre diff., for rear EDL differential and front axle an open differential, the torque vectoring control is operated by the clutches fitted in the rear differential case.

Brake-type torque vectoring using open differentials and the brake caliper adding a software in the ABS/ESP ECU that is able to generates yaw moment breaking the tyres of one side or one wheel of the vehicle.

Electronic Torque Vectoring Differential e-TVD are a family of differential, where we have an electric motor fitted, that is able, with its power, to control the amount and direction of the torque on each wheel (left and right in this case). At the moment we haven't a car sell on the market equipped with this kind of technology, but it's interesting to discuss more in deep this topic, because it represents the best solution of total freedom from engine driving torque.

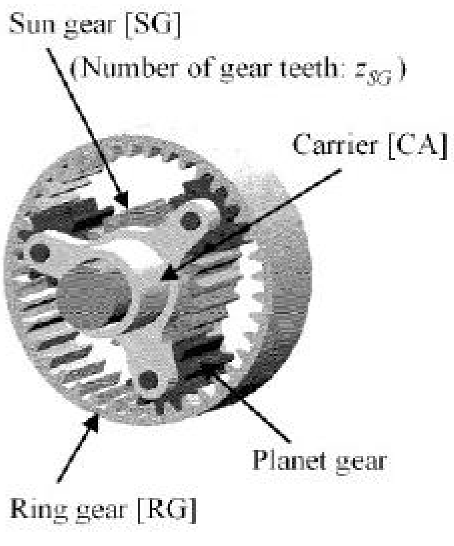
3.1 Example of analysis using velocity diagrams

We would like to make an example of analysis, we will evaluate the velocity diagram and some analytical parameter.

The velocity diagram Fig. 3.1.1, is a schematic representation useful to understand the behaviour of each element the a differential has. They are a simple 2D plot, where on y axis we have the revolving speed and on x axis the distances between each element is the inverse of the number of teeth of the gears, starting from one taken as reference (the carrier CA in the figure).

The diagram is drawn by the following procedure.

First, with reference to Fig. 3.1.1 (left), the rotating direction of the ring gear is confirmed as the reverse of the sun gear when the carrier is fixed. Then the point of the ring gear is placed on the horizontally opposite side of the sun gear from the point of the carrier in the velocity diagram, because of its reverse rotation. The horizontal distances of the ring gear and the sun gear from the carrier are proportional to the inverse of the number of teeth of each element. The three points drawn on the plot are joined together with the line named “Simple planetary gears” that represents the system analysed; the line can move up and down, on vertical direction, and can change its slope, this two movements are representing the two degrees of freedom of this system.



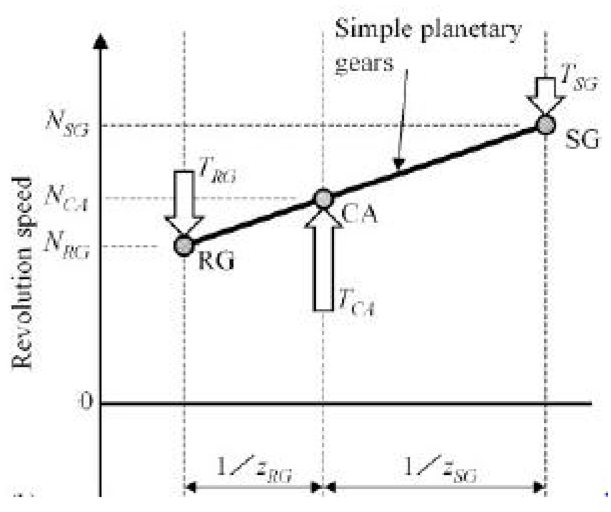


Figure 1 Velocity diagram of an ordinary differential

The torque acting on the system are the arrows and has to follow the lever rule, from which we can derive the following equations, useful to determine the torque on each component knowing only the input torque:

$$\begin{cases} T_{CA} = T_{SG} + T_{RG} \\ T_{SG}\left(\frac{1}{z_{SG}}\right) = T_{RG}\left(\frac{1}{z_{RG}}\right) \end{cases} \rightarrow \quad \begin{aligned} T_{SG} &= T_{CA}\frac{z_{SG}}{z_{SG}+z_{RG}} \\ T_{RG} &= T_{CA}\frac{z_{RG}}{z_{SG}+z_{RG}} \end{aligned} \tag{3.1.1}$$

$$\frac{N_{SG} - N_{CA}}{1/z_{SG}} = \frac{N_{CA} - N_{RG}}{1/z_{RG}} \tag{3.1.2}$$

3.2 Mitsubishi Active Yaw Control

The Mitsubishi AYC was the first case in history of yaw moment induction to improve vehicle performance. The driveline of this 4WD vehicle is a set of three differentials, but only the rear one is able to create a yaw moment on the car. The front and centre differentials are mounted on the gearbox Fig. 3.2.1 (left), under the hood, while the rear diff Fig. 3.2.1 (right) is on the rear axle; the first one is an open differential, than as centre diff we have a viscous joint, at rear we have torque-transferring differential.

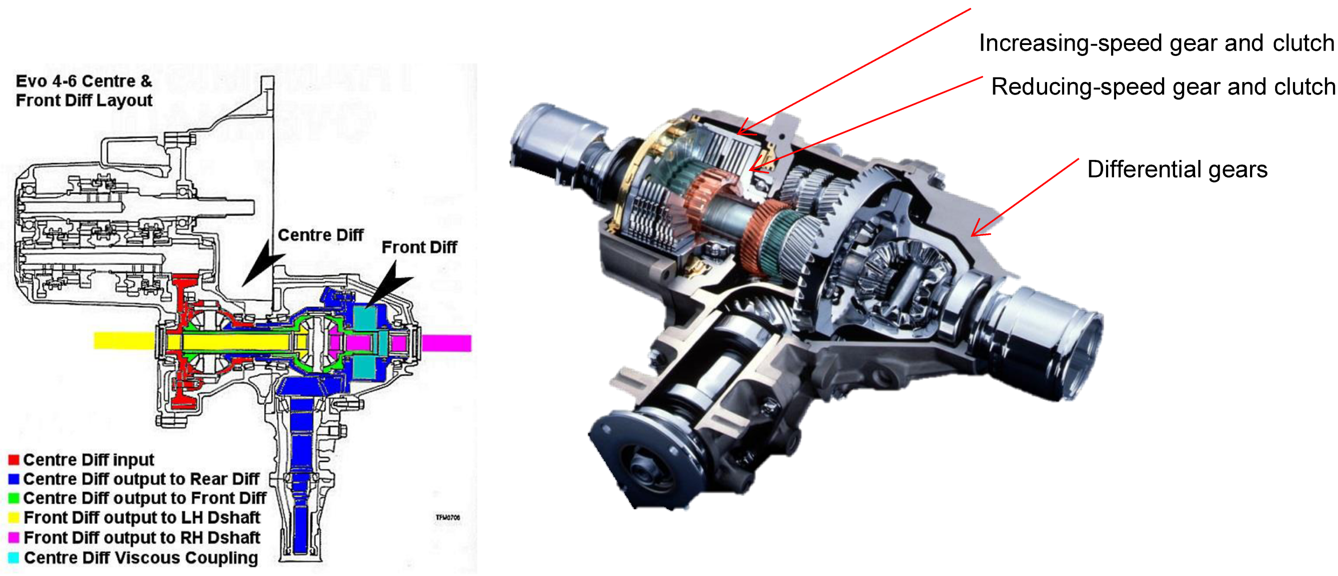


Figure 2 Front and centre diff (left), torque transferring differential (right)

The advantage of this kind of technology is an understeer reduction given by a yaw moment on centre of mass during corner and better distribution of forces acting on the vehicle, due to the fact that the most loaded wheel is the one subjected to the biggest traction force, this allows to cover corners faster and with lower steering angle.

The control is operated by an hydraulic system, cooperated by a dedicated ECU called ‘AYC-ECU’, that use the same set of sensors:

- Two mono-axial accelerometers, one for longitudinal acceleration, the other one for lateral acceleration. We have a couple of these sensors perpendicularly mounted, due to the fact in 1996 a yaw sensor for vehicles application was not yet invented, so Mitsubishi has to evaluate the entity of yaw moment combining the acceleration along vehicle’s x and y axis.
- Wheel speed sensors
- Steering wheel angle sensor
- Throttle valve position sensor

The hydraulic system can changes the dynamic behaviour of the car through some actuators that meters the hydraulic pressure of the actuator of the clutches in order to control the torque split between the wheels.

The torque transferring differential is composed by an open diff where, on the case are installed different sets of gear, one for each function; one for increase the left wheel torque and the other one to decrease it, each set of gear has its own clutch plates. The working principle is to increase/reduce the torque of the carrier, fixed the torque of the right sun gear.

3.2.1 Static and kinematic analysis

Now we start to explain, in terms of equation, the differential working using the classic approach of the applied mechanics, we start writing the equations referring to Fig 3.2.2. We define the convention of the transmission ratio i as the ratio between the output rotational speed and the input rotational speed. The input data of our analysis are the speeds of each wheel (ω_R, ω_L), the forces exerted by the linear actuators (F_{a1}, F_{a2}) and the input torque for the differential (TI).

The differential, starting from the left, is composed of an open differential modified so that its ring gear is fixed to the input gear of a two stage gear set. The two stage gear set is composed of a constant gear mesh (z_1 and z_4), a countershaft (carrying gears 4,5 and 6) and two output gears (2 and 3) each fixed to the inner disks of a different multi-disks clutch. The outer disks of the two multi-disks clutches are connected together through the same clutch drum which in turn is connected to the right differential output and to the right sun gear of the open differential.

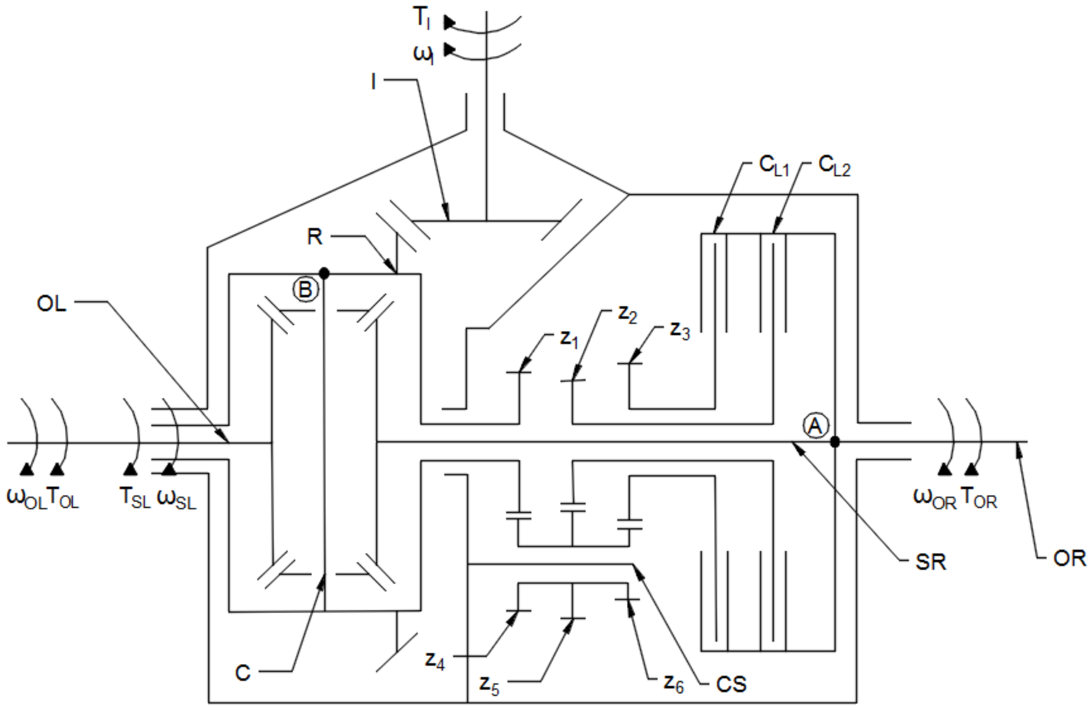


Figure 3 AYC differential scheme

With this data and knowing the number of teeth of each gear it's easy to evaluate ω_1, ω_2 and ω_3, i.e. the speed of the input and of the two outputs of the two stage transmission.

$$\begin{cases} \omega_1 = \omega_R = \omega_I i_{I/R} \\ \omega_2 = \omega_R i_{1/4} i_{5/2} = \omega_R \dfrac{z_1}{z_4}\dfrac{z_5}{z_2} = \omega_R i_{CL2} \\ \omega_3 = \omega_R i_{1/4} i_{6/3} = \omega_R \dfrac{z_1}{z_4}\dfrac{z_6}{z_3} = \omega_R i_{CL1} \end{cases} \tag{3.2.1}$$

Each clutch generates a torque that is proportional to the axial forces acting between the clutch disks and is equal to:

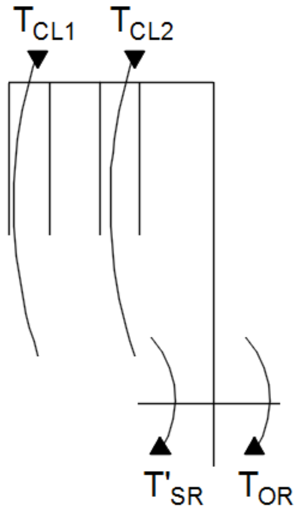
$$\begin{cases} T_{CL1} = F_{a1} r_{CL} f \cdot sgn(\Delta\omega_1) = F_{a1} r_{CL} f \cdot sgn(\omega_{SR} - \omega_3) \\ T_{CL2} = F_{a2} r_{CL} f \cdot sgn(\Delta\omega_2) = F_{a2} r_{CL} f \cdot sgn(\omega_{SR} - \omega_2) \end{cases} \quad (3.2.2)$$

Now we have to do the equilibrium of the mechanical node A. For a mechanical node we have the sum of the torque applied equal to zero and same rotational velocity.

$$\begin{cases} \sum_{i=1}^{4} T_i = 0 \\ \forall_{i,j} \omega_i = \omega_j \end{cases}$$

$$T'_{SR} + T_{OR} - T_{CL1} - T_{CL2} = 0$$

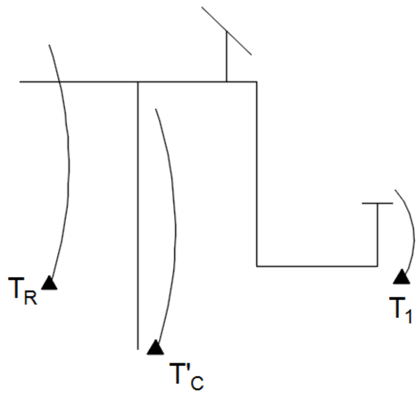
$$\rightarrow T_{OR} = T_{CL1} + T_{CL2} - T'_{SR} \quad (3.2.3)$$



From the equilibrium of the node B:

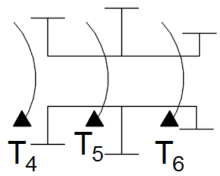
$$\sum_{i=1}^{4} T_i = 0$$

$$T_R + T'_C + T_1 = 0 \quad (3.2.4)$$



Torque balance of the countershaft:

$$T_4 + T_5 + T_6 = 0 \quad (3.2.5)$$



For the gear meshing of the countershaft:

$$\begin{cases} T_4 = T_1 \dfrac{z_4}{z_1} \\ T_5 = T_{CL2} \dfrac{z_5}{z_2} \\ T_6 = T_{CL1} \dfrac{z_6}{z_3} \end{cases} \tag{3.2.6}$$

Replacing the expressions of T4, T5 and T6 founded in the eq. (3.2.5) in the eq. (3.2.4) we will find T1.

$$T_1 \frac{z_4}{z_1} + T_{CL1} \frac{z_6}{z_3} + T_{CL2} \frac{z_5}{z_2} = 0$$
$$\rightarrow T_1 = -\frac{z_1}{z_4} (T_{CL1} \frac{z_6}{z_3} + T_{CL2} \frac{z_5}{z_2}) \tag{3.2.7}$$

Replacing the T_1 expression founded in eq. (3.2.6) in the eq. (3.2.3):

$$T_R + T_C' - \frac{z_1}{z_4} (T_{CL1} \frac{z_6}{z_3} + T_{CL2} \frac{z_5}{z_2}) = 0$$

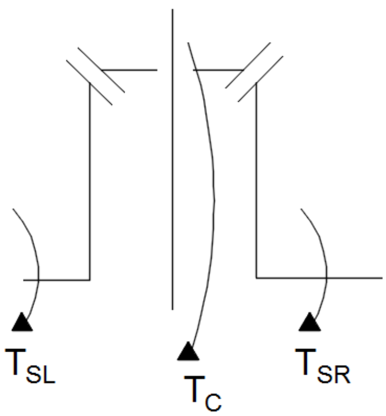
$$T_C' = -T_R + \frac{z_1}{z_4} (T_{CL1} \frac{z_6}{z_3} + T_{CL2} \frac{z_5}{z_2}) \tag{3.2.8}$$

For a normal open differential we have that:

$$T_{SR} = -\frac{T_{C'}}{2} = \frac{T_C}{2} \tag{3.2.9}$$

$$T_{SR} = -\frac{1}{2} \left[T_R - \frac{z_1}{z_4} (T_{CL1} \frac{z_6}{z_3} + T_{CL2} \frac{z_5}{z_2}) \right] \tag{3.2.10}$$

$$T_C + T_{SL} + T_{SR} = 0 \tag{3.2.11}$$



With the expression of T_{SR} founded in eq. (3.2.10), replacing it in the eq. (3.2.3) and for the 3rd Newton principle $T'_{SR} = - T_{SR}$.

$$T_{OR} = T_{CL1} + T_{CL2} - T'_{SR}$$

$$T_{OR} = T_{CL1} + T_{CL2} + \frac{1}{2}\left[T_R - \frac{z_1}{z_4}\left(T_{CL1}\frac{z_6}{z_3} + T_{CL2}\frac{z_5}{z_2}\right)\right] \quad (3.2.12)$$

We can evaluate T_{OL} from eq. (3.2.11), since we have found T_{SR} and T_C in eq. (3.2.10) and eq. (3.2.8):

$$T_{OR} = \frac{T_R}{2} + T_{CL1}\left(1 - \frac{z_1 z_6}{2 z_4 z_3}\right) + T_{CL2}\left(1 - \frac{z_1 z_5}{2 z_4 z_2}\right)$$

$$T_C + T_{SL} + T_{SR} = 0$$

$$T_{SL} = -T_C - T_{SR}$$

$$-T_{OL} = T_{SL} = -T_R + \frac{z_1}{z_4}\left(T_{CL1}\frac{z_6}{z_3} + T_{CL2}\frac{z_5}{z_2}\right) + \frac{1}{2}\left[T_R - \frac{z_1}{z_4}\left(T_{CL1}\frac{z_6}{z_3} + T_{CL2}\frac{z_5}{z_2}\right)\right]$$

$$T_{OL} = -\left[-\frac{T_R}{2} + \frac{T_{CL1}}{2}\frac{z_1 z_6}{z_4 z_3} + \frac{T_{CL2}}{2}\frac{z_1 z_5}{z_4 z_2}\right]$$

$$T_{OL} = \frac{T_R}{2} - \frac{T_{CL1}}{2}\frac{z_1 z_6}{z_4 z_3} - \frac{T_{CL2}}{2}\frac{z_1 z_5}{z_4 z_2} \quad (3.2.13)$$

So we can finally write down the equations on the torque acting on the wheels:

$$\begin{cases} T_{OR} = \frac{T_R}{2} + T_{CL1}\left(1 - 0.5\frac{z_1 z_6}{z_4 z_3}\right) + T_{CL2}\left(1 - 0.5\frac{z_1 z_5}{z_4 z_2}\right) = \frac{T_R}{2} + \frac{T_{CL1}}{2}(2 - i_{CL1}) + \frac{T_{CL2}}{2}(2 - i_{CL2}) \\ T_{OL} = \frac{T_R}{2} - 0.5 T_{CL1}\frac{z_1 z_6}{z_4 z_3} - 0.5 T_{CL2}\frac{z_1 z_5}{z_4 z_2} = \frac{T_R}{2} - \frac{T_{CL1}}{2} i_{CL1} - \frac{T_{CL2}}{2} i_{CL2} \end{cases}$$

$$\boxed{\begin{cases} T_{OR} = \frac{1}{2}\left[T_R + T_{CL1}(2 - i_{CL1}) + T_{CL2}(2 - i_{CL2})\right] \\ T_{OL} = \frac{1}{2}\left[T_R - T_{CL1} i_{CL1} - T_{CL2} i_{CL2}\right] \end{cases}} \quad (3.2.14)$$

The first thing we notice from the two equation is that we have a different torque split between the left and right wheel, then the entity of the two torque is different and that both the expression depends on the torque exerted by the clutch so we must do some considerations about the sign of these torque. The clutch torque sign depends only on $\mathrm{sgn}(\omega_{SR} - \omega_i)$ because the other terms are product of positive quantities so:

$$sgn(T_{CL1}) = sgn(\omega_{SR} - \omega_3) = sgn\left(\omega_{SR} - \omega_R \frac{z_1}{z_4}\frac{z_6}{z_3}\right) = sgn(\omega_{SR} - \omega_R i_{CL1}) \tag{3.2.15}$$

$$sgn(T_{CL2}) = sgn(\omega_{SR} - \omega_2) = sgn\left(\omega_{SR} - \omega_R \frac{z_1}{z_4}\frac{z_5}{z_2}\right) = sgn(\omega_{SR} - \omega_R i_{CL2}) \tag{3.2.16}$$

Since, for our study is important to know the sign of these torque, because they are fundamental to learn how this differential works, and since without having the real number of teeth of the gears of the components is impossible to accomplish our mission, the transmission ratios was deduced from the technical drawings available and are calculated as the ratio between the primitive radius of these gears. The results obtained are 0.761 and 1.228 respectively for i_{CL1} and i_{CL2}. At the moment to evaluate these sign we need to make the approximation of small speed differences between the wheels, with this condition we have:

$$sgn(T_{CL1}) = sgn(\omega_{SR} - \omega_R i_{CL1}) = sgn(\omega_{SR} - 0.761\omega_R) = +1 \rightarrow T_{CL1} > 0 \tag{3.2.17}$$

$$sgn(T_{CL2}) = sgn(\omega_{SR} - \omega_R i_{CL2}) = sgn(\omega_{SR} - 1.228\omega_R) = -1 \rightarrow T_{CL2} < 0 \tag{3.2.18}$$

We notice that the sign and the entity of the clutch torque is related only to the terms i_{CL1} and i_{CL2}, practically we have the clutch CL2 that operates as torque decreasing gears for the right wheel and the clutch CL1 increasing the torque acting of the right wheel. It is clear from eq. (3.2.14) that the best working conditions of the device are obtained when i_{CL1},i_{CL2}=1, because it's the only way to have the same torque split between the two side of the wheels, also to avoid of having a different behaviour during the two different clutches intervention, it must be underlined also that i_{CL1}, i_{CL2}>0 always because they are ratios between positive numbers.
Now we have to check what happens if the speed differences between the wheels aren't small, and if we are limited in a range of working condition. The condition that

ensure the working of the differential is the constant sign of the clutches torque. From eq. (3.2.17) and (3.2.18):

$$sgn(T_{CL1}) = +1 \; if \; (\omega_{SR} - \omega_R i_{CL1}) > 0 \tag{3.2.17}$$

$$\begin{cases} (\omega_{SR} - \omega_R i_{CL1}) > 0 \\ \omega_R = \dfrac{\omega_{SL} + \omega_{SR}}{2} \end{cases} \rightarrow \quad \omega_{SR} - \frac{i_{CL1}}{2}(\omega_{SR} + \omega_{SL}) > 0$$

$$\omega_{SR}\left(1 - \frac{i_{CL1}}{2}\right) > \frac{i_{CL1}}{2}\omega_{SL}$$

$$\omega_{SR} > \frac{i_{CL1}}{2 - i_{CL1}}\omega_{SL} \tag{3.2.19}$$

$$sgn(T_{CL2}) = -1 \; if \; (\omega_{SR} - \omega_R i_{CL2}) < 0 \tag{3.2.18}$$

$$\begin{cases} (\omega_{SR} - \omega_R i_{CL2}) < 0 \\ \omega_R = \dfrac{\omega_{SL} + \omega_{SR}}{2} \end{cases} \rightarrow \quad \omega_{SR} - \frac{i_{CL2}}{2}(\omega_{SR} + \omega_{SL}) < 0$$

$$\omega_{SR}\left(1 - \frac{i_{CL2}}{2}\right) < \frac{i_{CL2}}{2}\omega_{SL}$$

$$\omega_{SR} < \frac{i_{CL2}}{2 - i_{CL2}}\omega_{SL} \tag{3.2.20}$$

Using our value of i_{CL1} and i_{CL2}:

$$\omega_{SR} > \frac{i_{CL1}}{2 - i_{CL1}}\omega_{SL} \quad \rightarrow \quad \omega_{SR} > 0.614\omega_{SL} \quad \rightarrow \quad \frac{\omega_{SR}}{\omega_{SL}} > 0.614 \tag{3.2.21}$$

$$\omega_{SR} < \frac{i_{CL2}}{2 - i_{CL2}}\omega_{SL} \quad \rightarrow \quad \omega_{SR} < 1.590\omega_{SL} \quad \rightarrow \quad \frac{\omega_{SR}}{\omega_{SL}} < 1.591 \tag{3.2.22}$$

Until the above conditions are valid the differential will work in project condition, so we have to choose the right number of teeth of the gears in the two stage gear set in order to avoid to operate out of these conditions. The eq. (3.2.21) and (3.2.22) give us the possibility to remove the hypothesis of small difference of speed between the wheels and to discuss what happen in a real case.

The plot in Fig. 3.2.3 shown us the range of working of our system. When we are in the interval $0.614 < \frac{\omega_{SR}}{\omega_{SL}} < 1.591$, we have opposite sign from the two clutches, that means we're able to choose between both the working function increase/decrease torque on right wheel, but if we go out of the range, the only torque correction available creates an increase of torque on the faster wheel, that could be undesired.

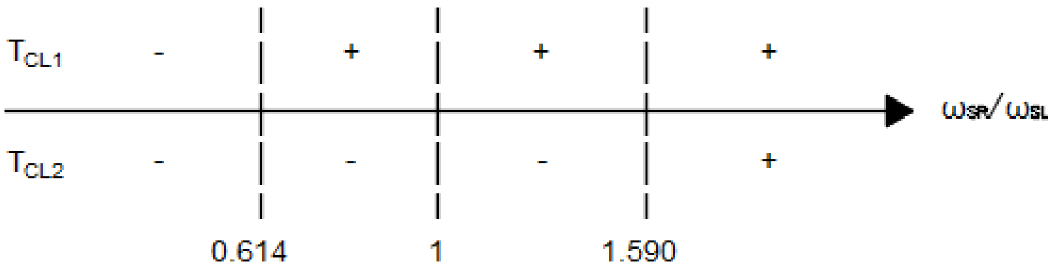


Figure 4 T_{CLi} sign plot in function of $\frac{\omega_{SR}}{\omega_{SL}}$

As we said before, the best working condition, as we know from eq. (3.2.14), is obtained with $i_{CL1} = i_{CL2} = 1$, because it's a wanted situation due to symmetric torque distribution between the wheels and same behaviour using the clutch 1 or 2, but the Fig. 3.2.4 evidence the reason why it's impossible to realize. If $i_{CL1} = i_{CL2} = 1$, the only torque correction avaible tends to increase the torque on the faster wheel.

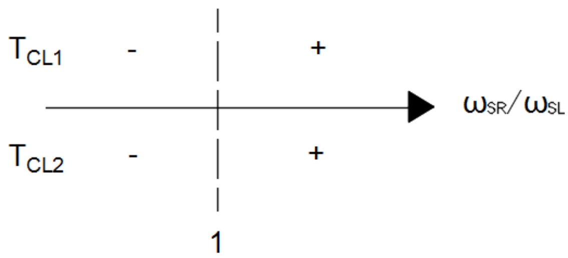


Figure 5 T_{CLi} sign plot in function of $\frac{\omega_{SR}}{\omega_{SL}}$ with $\mathbf{i_{CL1} = i_{CL2} = 1}$

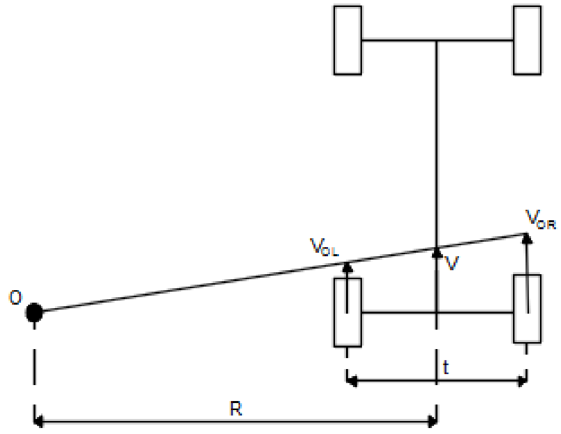


Figure 6 Rear axle of a vehicle with both wheels in pure rolling condition

Back to the studied situation, we have to ensure the range of working founded is wide enough to cover all the real conditions, to do that we try to calculate what could be the speed difference in a vehicle making the approximation of pure rolling of the tyres. In Fig. 3.2.5 we can see a scheme of what are the assumption of pure rolling condition of the rear tyres, if we make some calculation at constant curvature radius and variable speed we will obtain:

$$\frac{V}{R} = \frac{V_{OL}}{R - t/2} \quad \rightarrow \quad V_{OL} = \frac{V}{R}(R - t/2) \tag{3.2.23}$$

$$\frac{V}{R} = \frac{V_{OR}}{R + t/2} \quad \rightarrow \quad V_{OR} = \frac{V}{R}(R + t/2) \tag{3.2.24}$$

$$\frac{\omega_R}{\omega_L} = \frac{V_{OR}}{r_{tyre}} \frac{r_{tyre}}{V_{OL}} = \frac{\frac{V}{R}(R + t/2)}{\frac{V}{R}(R - t/2)} = \frac{(R + t/2)}{(R - t/2)} = 1.010 \tag{3.2.25}$$

The value in the eq. (3.2.25) is obtained with R=200m and t=1.90m, this value is in the range of working of the AYC differential. If we want to consider a variation of the radius of curvature, always in pure rolling condition, we can evaluate what happens if we keep constant the lateral acceleration, that in this case is fixed equal to 0.7g, a value that is near the limit handling condition of a vehicle. From the acceleration we can write the speed in function of radius.

$$a = \frac{V^2}{R} \quad \rightarrow \quad V = \sqrt{Ra} \tag{3.2.26}$$

$$\frac{V}{R} = \frac{V_{OL}}{R - t/2} \quad \rightarrow \quad V_{OL} = \frac{\sqrt{Ra}}{R}(R - t/2) \tag{3.2.27}$$

$$\frac{V}{R} = \frac{V_{OR}}{R - t/2} \quad \rightarrow \quad V_{OR} = \frac{\sqrt{Ra}}{R}(R - t/2) \tag{3.2.28}$$

$$\frac{\omega_R}{\omega_L} = \frac{V_{OR}}{r_{tyre}}\frac{r_{tyre}}{V_{OL}} = \frac{\frac{V}{R}(R + t/2)}{\frac{V}{R}(R - t/2)} = \frac{(R + t/2)}{(R - t/2)} \tag{3.2.29}$$

This time R is not a constant parameter, so we have to trace a plot of the quantity.

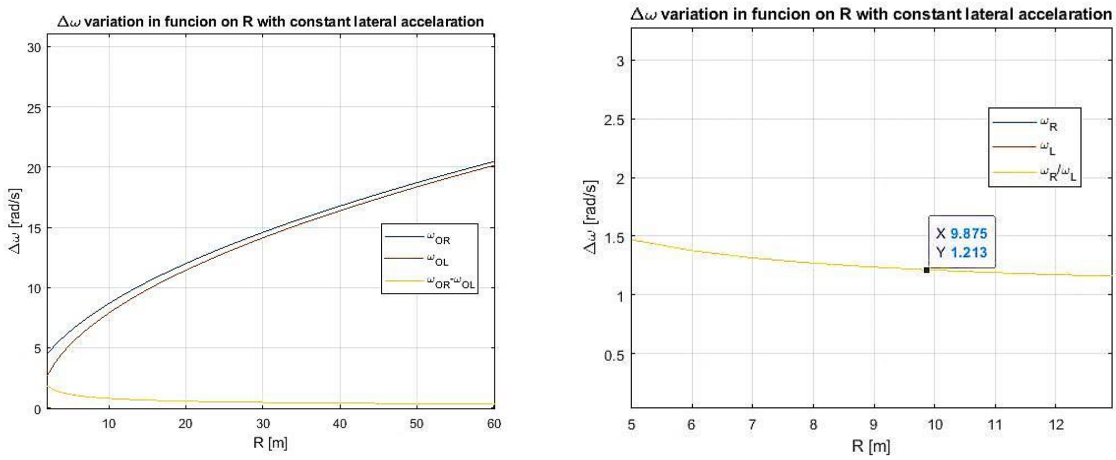


Figure 7 $\frac{\omega_{SR}}{\omega_{SL}}$ variation in function of the radius of the curvature a constant lateral acceleration

The plot in Fig. 3.2.6 evidence the fact we're in the right working range also in this case, a value near R=9m is shown because it's the minimum turning radius of a real car, we can also see that increasing the radius the ratio $\frac{\omega_{SR}}{\omega_{SL}}$ tends to '1'.

3.2.2 Static and Kinematic analysis using velocity diagram

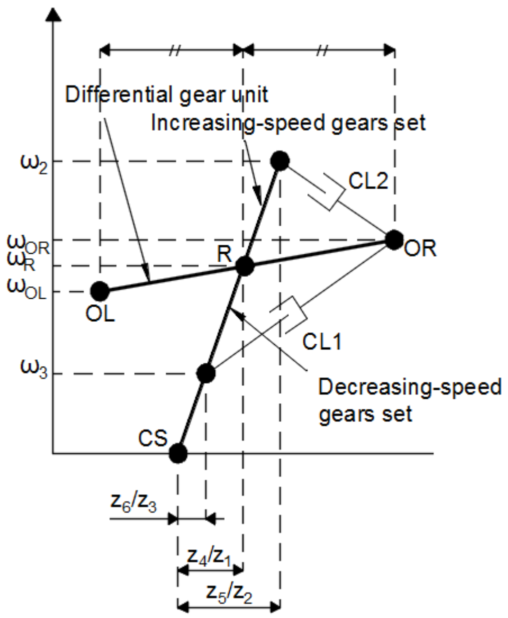


Figure 8 AYC velocity diagram

We have to evaluate again, the torque and the speed of the key elements of the differential, in order to compare the two different approach. The diagram in Fig. 3.2.7 is drawing using the rules explained above in the 3.1 section of this paper. The system is composed by an open differential, the lever OL-OR, with the ring gear R in the centre, the point OL and OR are opposite due to their velocity of rotation and same gear ratio; the two-stage gear set with the constant gear mesh are imagined as a differential with the ring gear fixed, the role of the ring gear is played by the countershaft CS. The position of CS is the first one to be set, is has zero revolution speed, so it has to be on horizontal axis. Starting from CS, the point 3, R and 2 are defined since they are all on the same segment, due to the fact they are part of the same epicyclic gear, all in the same part referred to CS because they revolute in the same direction. 2,R and 3 are then horizontally spaced by a quantity proportional to the transmission ratio of itself. R defined, we can put OL and OR equally spaced from R and in opposite position. In the scheme the two degrees of freedom of the differential are represented as the freedom of the point of vertically moving and by the lever of changing their slope, the situation represented so it's a time instant but not a static plot.

For a normal differential we have:

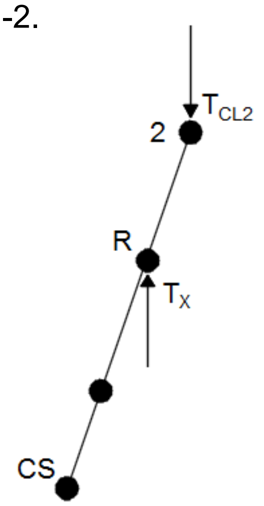
$$\omega_R = \frac{\omega_{OL} + \omega_{OR}}{2}$$

From the diagram:

$$\frac{\omega_2 z_2}{z_5} = \frac{\omega_R z_1}{z_4} \quad \rightarrow \quad \omega_2 = \omega_R \frac{z_1 z_5}{z_4 z_2} \tag{3.2.30}$$

$$\frac{\omega_3 z_3}{z_6} = \frac{\omega_R z_1}{z_4} \quad \rightarrow \quad \omega_3 = \omega_R \frac{z_1 z_6}{z_4 z_3} \tag{3.2.31}$$

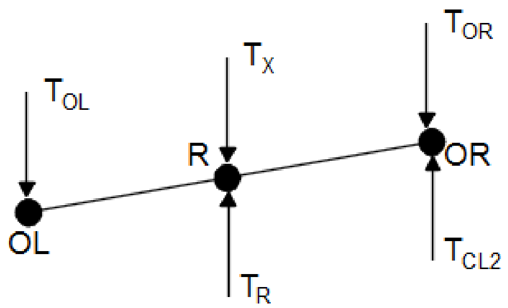
From the working principle we know that at least one clutch at times is working, so if is working the clutch CL2 we can write the equilibrium of the lever CS-2.



$$\frac{T_X z_4}{z_1} = \frac{T_{CL2} z_5}{z_2} \quad \rightarrow \quad T_X = T_{CL2} \frac{z_1 z_5}{z_4 z_2} \tag{3.2.32}$$

$$\begin{cases} T_X = T_{CL2} \dfrac{z_1 z_5}{z_4 z_2} \\ T_{OL} + T_{OR} + T_X = T_R + T_{CL2} \\ T_{OR} a - T_{CL2} a - T_{OL} a = 0 \end{cases} \tag{3.2.33}$$

$$\begin{cases} T_X = T_{CL2} \dfrac{z_1 z_5}{z_4 z_2} \\ T_{OL} + T_{OR} + T_X = T_R + T_{CL2} \\ T_{OR} = T_{CL2} + T_{OL} \end{cases}$$

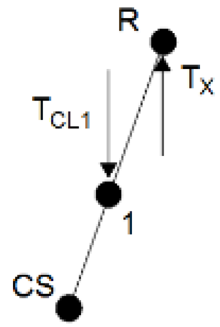


$$\rightarrow T_{OL} = T_R + T_{CL2} - (T_{CL2} + T_{OL}) - T_{CL2} \frac{z_1 z_5}{z_4 z_2} = \frac{1}{2}\left(T_R - T_{CL2} \frac{z_1 z_5}{z_4 z_2}\right) \tag{3.2.34}$$

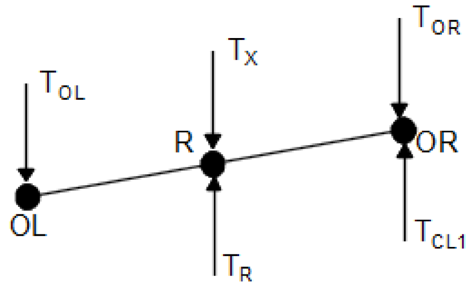
$$\rightarrow T_{OR} = T_{CL2} + T_{OL} = \frac{T_R}{2} + T_{CL2}\left(1 - \frac{z_1 z_5}{z_4 z_2}\right) \tag{3.2.35}$$

Now we write what happens if we make the clutch CL1 working:

$$\frac{T_X z_4}{z_1} = \frac{T_{CL1} z_6}{z_3} \quad \rightarrow \quad T_X = T_{CL1} \frac{z_1 z_6}{z_4 z_3} \qquad (3.2.36)$$



$$\begin{cases} T_X = T_{CL1} \frac{z_1 z_6}{z_4 z_3} \\ T_{OL} + T_{OR} + T_X = T_R + T_{CL1} \\ T_{OL} a + T_{CL1} a - T_{OR} a = 0 \end{cases} \qquad (3.2.37)$$



$$\begin{cases} T_X = T_{CL1} \frac{z_1 z_6}{z_4 z_3} \\ T_{OL} + T_{OR} + T_X = T_R + T_{CL1} \\ T_{OL} = T_{OR} - T_{CL1} \end{cases}$$

$$\rightarrow T_{OR} = T_R + T_{CL1} - T_{CL1} \frac{z_1 z_6}{z_4 z_3} - (T_{OR} + T_{CL1}) = \frac{T_R}{2} + T_{CL1} \left(1 - \frac{z_1 z_6}{2 z_4 z_3}\right) \qquad (3.2.34)$$

$$\rightarrow T_{OL} = T_{OR} - T_{CL1} = \frac{T_R}{2} - T_{CL1} \frac{z_1 z_6}{2 z_4 z_3} \qquad (3.2.35)$$

$$\begin{cases} T_{OR} = \frac{T_R}{2} + T_{CL1} \left(1 - \frac{z_1 z_6}{2 z_4 z_3}\right) + T_{CL2} \left(1 - \frac{z_1 z_5}{2 z_4 z_2}\right) \\ T_{OL} = \frac{T_R}{2} - T_{CL1} \frac{z_1 z_6}{2 z_4 z_3} - T_{CL2} \frac{z_1 z_5}{2 z_4 z_2} \end{cases} \qquad (3.2.38)$$

The results coming from the two method are the same are equal and all the consideration and calculation made for the sign of the torque and for the working range conditions are still valid, what is important to underline is the less calculation required by this approach and the simplicity added, so we can say that is better way of studying this complex system.

3.2.3 AYC working and control strategies

The control unit of system like this one in Fig. 3.2.8, has to evaluate the physical behaviour of the vehicle in order to choose the right working condition. The AYC-

ECU can set the right level of axial force applied on clutch plates to obtain the desired torque on the rear wheels.

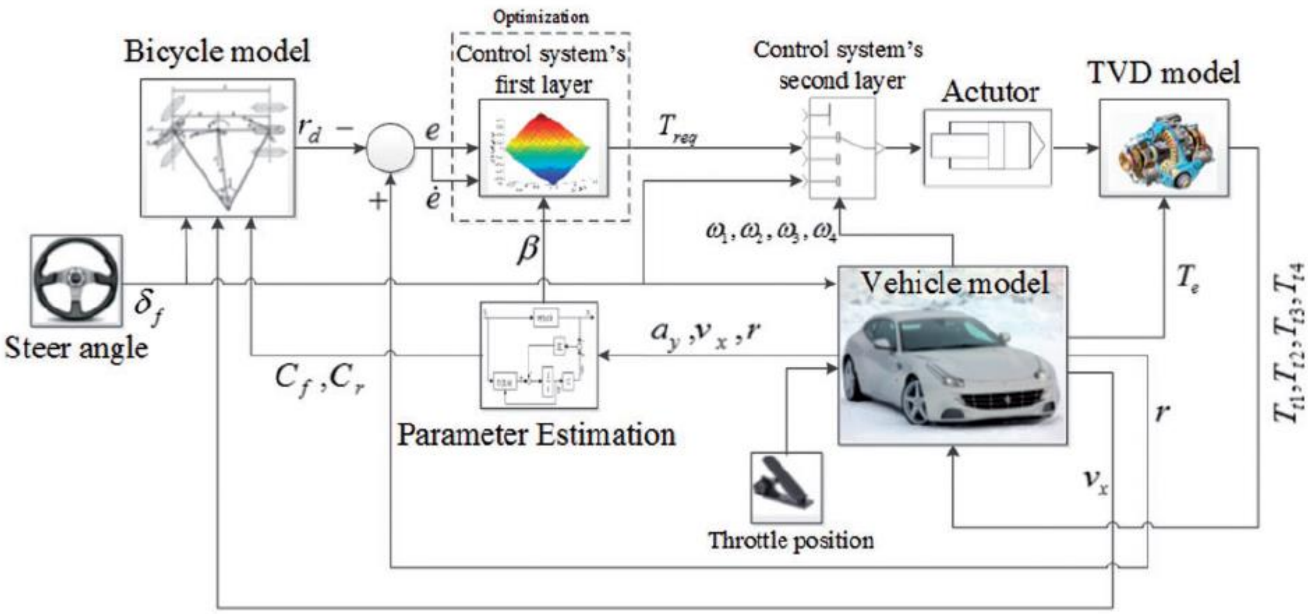


Figure 9 Schematic diagram of control system

According to the scheme on Fig. 3.2.8, the clutch actuators motion are determined by a logic implemented in the software that is based on the wheels speed, that is an input value coming from the wheel speed sensors, the steering angle, known from the dedicated sensor, and the T_{req}. The T_{req} is the torque required to obtain a precise handling correction. It’s evaluated from: the estimation of the side-slip angle β, that come from on a valuation based on the longitudinal speed v_x, yaw rate r and lateral acceleration a_y, all valuated accordin to 8 DOF vehicle model; the desired yaw rate r_d, extimated from steering angle, longitudinal speed and C_f, C_r cornering stiffness respectively for front and rear axle. To make a closed-loop evaluation the torque acting on each wheel are evaluated and inserted in the 8 DOF model. In Fig. 3.2.9 is represented the real working scheme fitted on the vehicle.

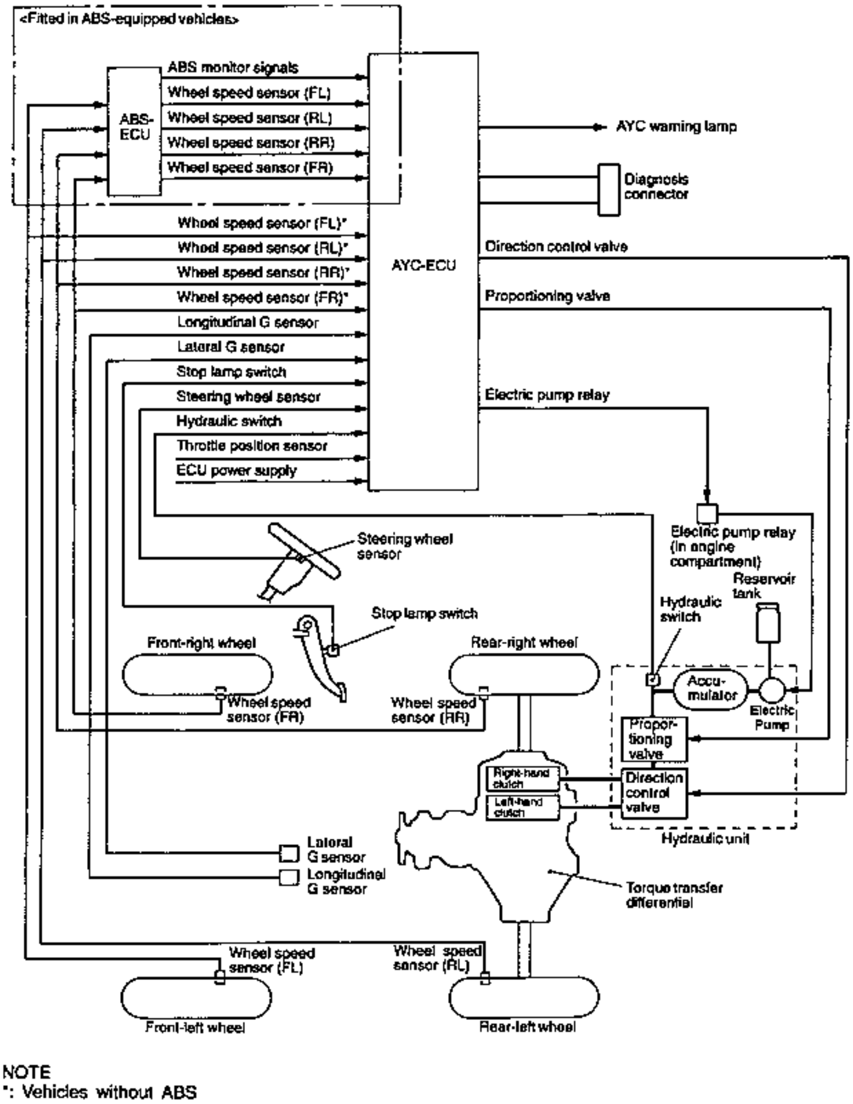


Figure 10 AYC control strategy scheme

3.3 Audi QUATTRO with sport differential

We will analyse the QUATTRO fitted on QUATTRO model that has the option to fit the sport differential as an example of Automatic Wheel Drive, with permanent four wheel driven, the differential that is able to exploit torque vectoring function is the rear one that is a mechanical differential electro-hydraulic actuated. The big deal with this technology is the possibility to counteract oversteer as understeer, according to driver intention and vehicle state and to balance the torque acting no only on front and rear differential but also on the rear left and right wheel. The transmission layout is composed by a centre crown-gear differential Fig. 3.3.1 (left) with two shaft exit, one for each axle of the car, front axle fit an open diff, while the rear differential is a torque-transfer type as the previous one analysed. The centre diff, during idle operation sends the 60% of the engine torque to the rear, but, according to instant conditions, the torque split between the two axes, can change from 85%-15% to 30%-70%, this give us a good level of freedom, compared to other system analysed. The rear differential shown Fig. 2.3.1, has two sets of gear, one for each sun gear, that is able, once a dedicated clutch is engaged, to directly transmit part of the torque directly from the carrier to the sun gear, with a fixed transmission ratio, but the entity of the transmitted torque could be metered, regulating the pressure of the oil pressing the clutch packs.

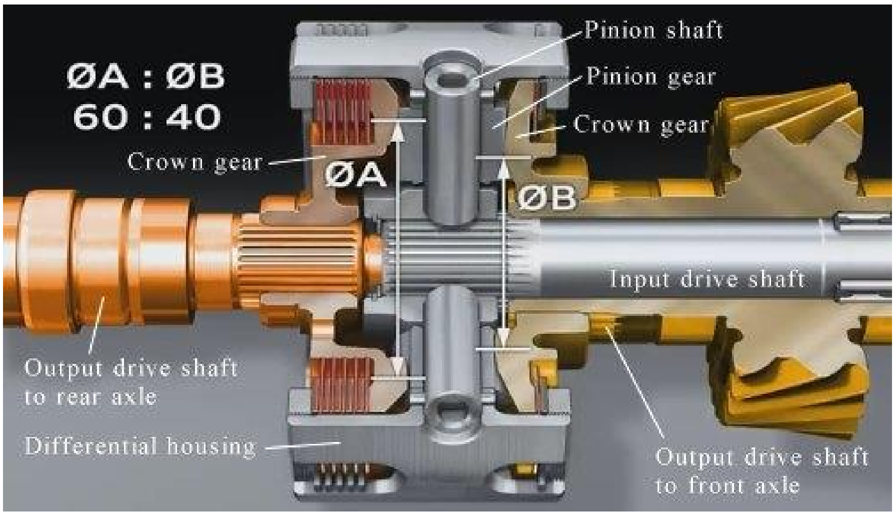


Figure 11 Crown-gear differential

The working principle it's similar to the previous one explained, we have a set of increasing-speed gears for each sun of the differential, and the clutches are solidly on one side to the case and at other at the gears set, this guarantees that the torque transmission will always be from the carrier to the gears set engaged.

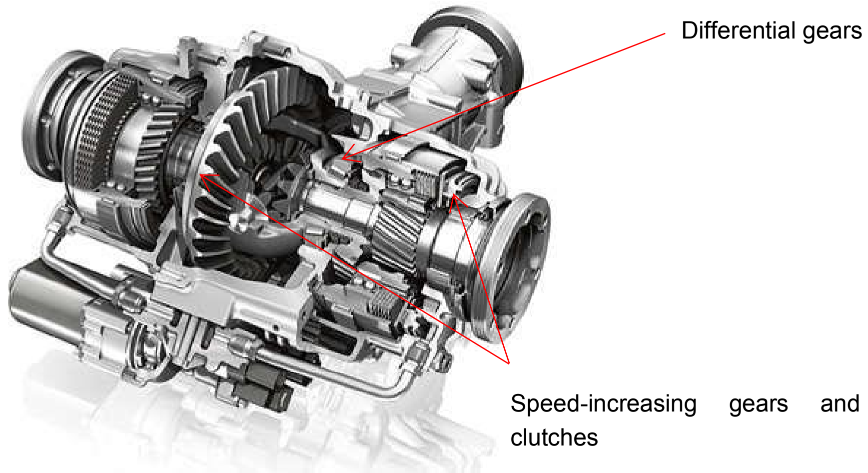


Figure 12 Audi torque transferring rear differential

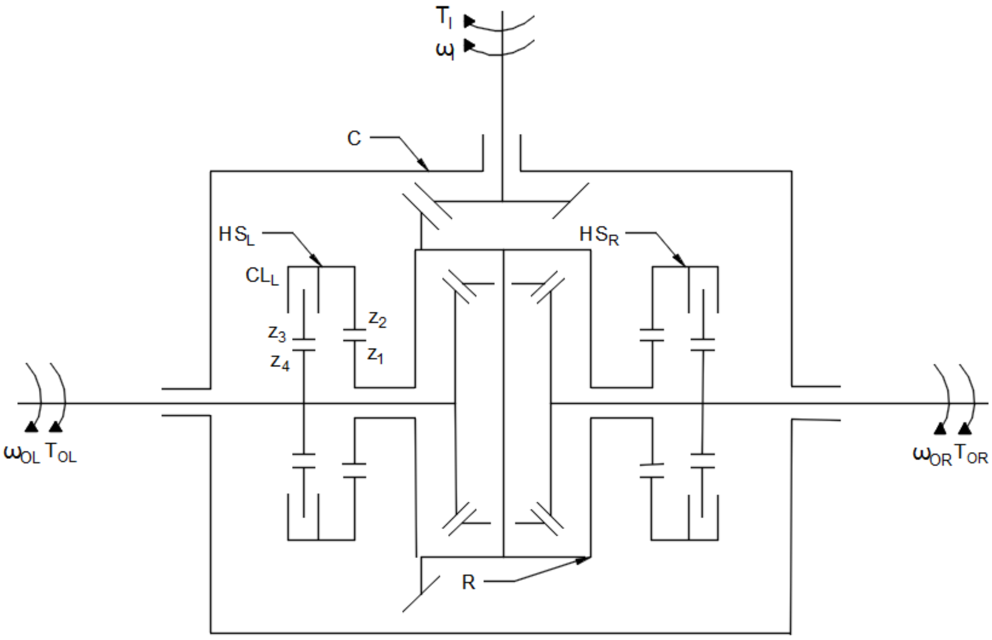


Figure 13 Audi torque transferring differential scheme

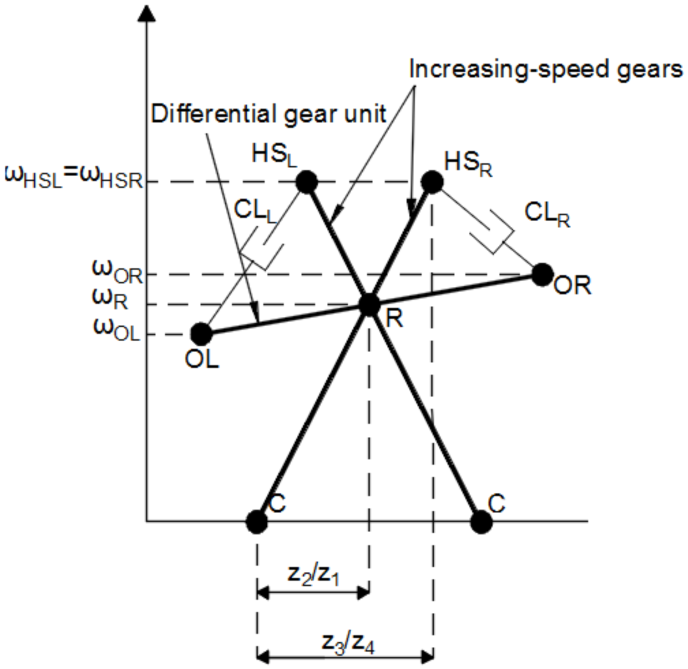
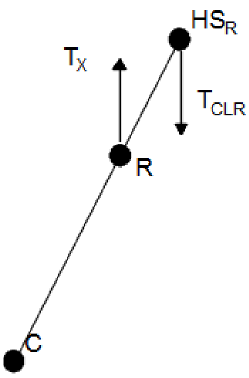


Figure 14 Audi torque transferring differential velocity diagram

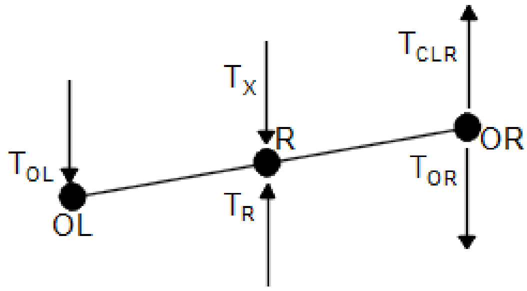
From velocity diagram in Fig. 3.3.4 and the equilibrium of the levers we can say:

$$\omega_{HSR} = \omega_{HSL} = \frac{\omega_R z_2}{z_1}$$

$$\frac{T_X z_3}{z_4} = \frac{T_{CLR} z_2}{z_1} \quad \rightarrow \quad T_X = T_{CL1}\frac{z_1 z_3}{z_2 z_4} \qquad (3.3.1)$$



$$\begin{cases} T_X = T_{CL1}\dfrac{z_1 z_3}{z_2 z_4} \\ T_{OL} - T_R + T_X + T_{OR} - T_{CLR} = 0 \\ T_{OL} - T_{OR} - T_{CLR} = 0 \end{cases} \qquad (3.3.2)$$



$$\begin{cases} T_X = T_{CL1}\dfrac{z_1 z_3}{z_2 z_4} \\ T_{OR} - T_{CLR} - T_R + T_{CL1}\dfrac{z_1 z_3}{z_2 z_4} + T_{OR} - T_{CLR} = 0 \\ T_{OL} = T_{OR} - T_{CLR} \end{cases}$$

$$\begin{cases} T_{OR} = \dfrac{T_R}{2} + T_{CLR}\left(1 - \dfrac{z_1 z_3}{2 z_2 z_4}\right) \\ T_{OL} = \dfrac{T_R}{2} - T_{CLR}\dfrac{z_1 z_3}{2 z_2 z_4} \end{cases} \tag{3.3.4}$$

This solution for electro-hydraulic type differentials is one the best interpretation, he can manage torque-split on the same axis, the entity of this torque slit is just limited by the transmission ratio of the secondary-gear elements. The big deal compared to previous one analysed is the possibility to have symmetric behaviour, independently from the clutch engaged, due to this symmetry the same calculation made for T_{CLR} are valid for the other clutch, just exchanging the terms depending on the clutch torque.

3.4 Brake-type torque vectoring

The brake-type torque vectoring it's the easiest way to obtain a yaw moment management without having big and complex differential. The organs involved in this function are normally equipped in a vehicle, since it requires only an open differential Fig. 3.4.1 and a brake system supported by ABS/ESP component that must be fitted on a nowadays car in order to satisfy the law requirements.

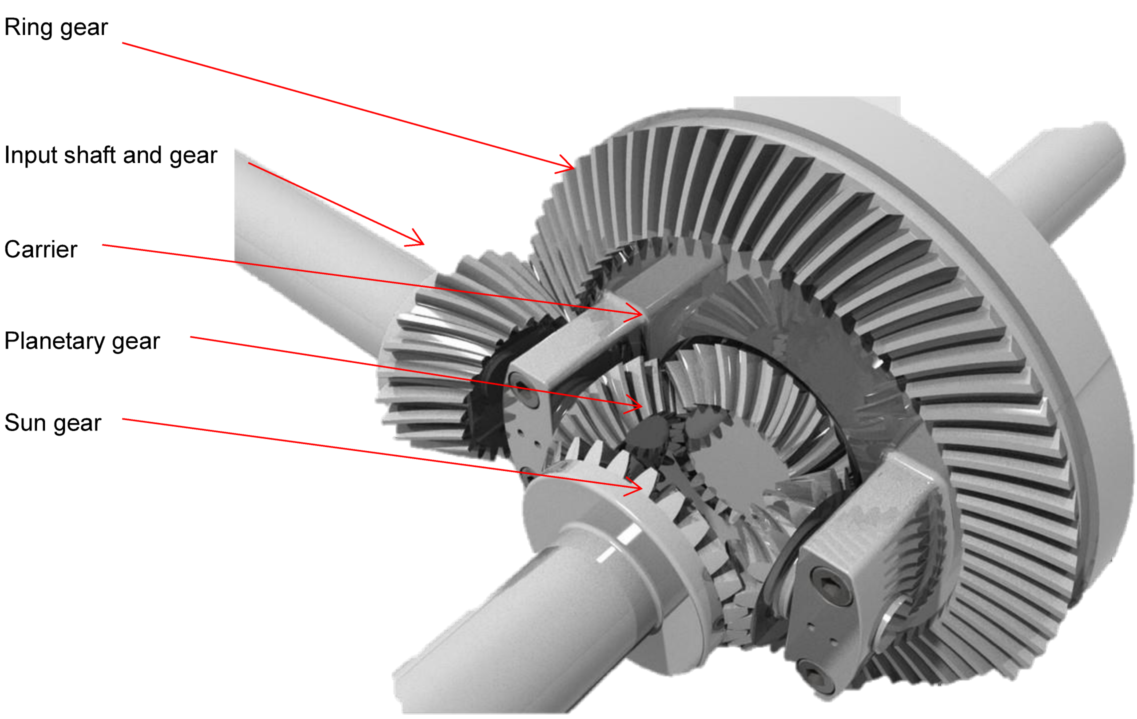


Figure 15 Open differential

The system, thanks to ESP set of sensors and ECU, is able to evaluate the driving condition and to recognize when is necessary to brake the inner wheels, this is necessary only at high speed or near to handling limit condition of the vehicle. The yaw moment is just created by a longitudinal force applied by the brakes that can modify the understeering coefficient of the car. The biggest advantage of this kind of solution is the possibility to maintain a standard open differential, that is a cheap and small device, moreover no other investment has to be done in order to add torque-vectoring features to a car, a part from an ESP software update; this advantage makes finally available to have TV characteristic on low segment car, not only on performance or high segment vehicle. The big drawbacks of this technology are mainly three:

- Impossibility of managing the amount of torque applied on single wheel drive tyre because with open differential the torque applied on each tyre will always be equal to half of the module of the input torque. Referring to Fig. 3.4.2, we evaluate only what happen in input and output of the differential, not inside; this means that we don't see τ_c, ω_c and τ_I, ω_I represent the torque and the speed applied to the carrier of the differential. So the torque expression of each wheel are evaluated equalizing one results of Willis equation with a torque equilibrium on the differential and an equation of power conservation assuming differential unitary efficiency:

$$\begin{cases} \omega_I = \dfrac{\omega_1 + \omega_2}{2} \\ \tau_1 + \tau_2 + \tau_I = 0 \\ \tau_1\omega_1 + \tau_2\omega_2 + \tau_I\omega_I = 0 \end{cases} \rightarrow \quad \tau_1 = \tau_2 = -\frac{\tau_I}{2}$$

- Increase of brake materials wear and in case of sport driving we risk to have overheated brake system when we have to brake
- Energy loss due to braking heat generation

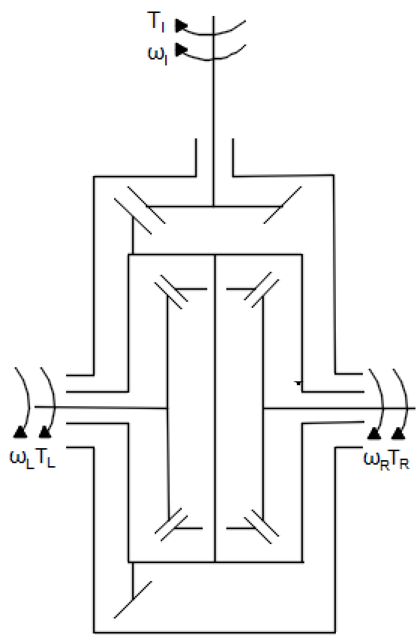


Figure 16 Open differential scheme

3.5 Torque vectoring with hybrid-electric differentials

Nowadays we haven't seen e-Differential fitted on ICE cars, but it's very interesting to analyse this system fitted on hybrid-electric vehicle because, for now, they are the only one method to control the module and sign of the torque of each wheel, independently from torque delivered by the engine. There are many possibilities to realize a solution like the one discussed above, but all of them starts from a torque transferring differential and a DC motor, fitted to control the increase/decrease speed gear set, as shown in Fig. 3.5.1 where 'M' indicates the position of the DC motor. This kind of differential, from now, are going to be called as E-TVD (Electronic-Torque Vectoring Differential).

E-TVD in this case are born to be fitted on 4WD vehicle, and they are mounted as differential for the rear axle, the big deal with this kind of technology is the possibility to avoid the heat power loss due to friction in brakes or clutch of differentials and the perfect manipulation of the torque between left and right side due to introduction of electric power that can be added or subtracted to engine power delivered. On Fig. 3.5.2 we have a velocity diagram of a E-TVD, on x axis we have the Speed Ratio, the ratio between the velocity of the component and the element 'I' taken as reference value; on y axis the revolution speed of each component considered, the segment represent the working of each gear unit. 'L' and 'R' letter stands for Left wheel and Right Wheel, the 'M' is the DC motor, 'I' represent the differential case, the sun gears of the first and second planetary gear units are connected to each other and are denoted by element 'C', the ring gear 'F' of the first planetary gear unit is fixed to the housing. The diagram consider an instant situation, where the vehicle is left cornering, this is evidenced by the fact that the right wheel is rolling faster than the left. All the possible way of function are obtained combining the three different segment, to control the yaw moment and lateral and longitudinal acceleration acting on the vehicle, in order to perform steady-state condition of driving close to the limit condition of the car. It's important to underline some fundamental principle of design of this technology:

- The electric motor is independent from the ICE engine and has only the function of torque driving but it doesn't drive the wheels.

- When the electric motor is not working the mechanism act as an open differential.
- The electric motor is always switched off when the vehicle is running straight.

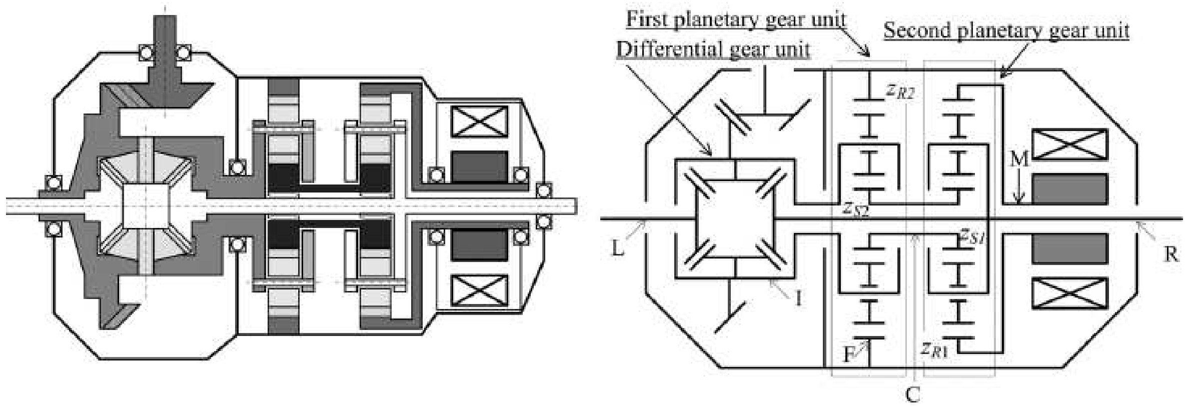


Figure 17 E-TVD gear scheme

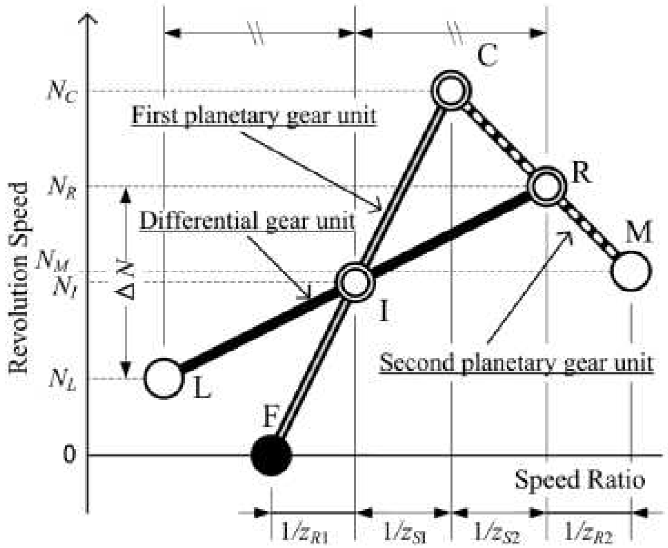


Figure 18 Velocity diagram of one E-TVD

Referring to Fig. 3.5.2, thanks to velocity diagram, we're able to writes down the equation that shows how the electric engine can influence the torque split between the two side of the vehicle. From Willis equation we have:

From basic geometric relations:

$$N_I = \frac{N_R + N_L}{2} \quad \rightarrow \quad \begin{cases} N_R = N_I + \dfrac{\Delta N}{2} \\ N_L = N_I - \dfrac{\Delta N}{2} \end{cases} \tag{3.4.1}$$

$$\frac{N_I}{1/z_{R1}} = \frac{N_C}{1/z_{R1} + 1/z_{S2}} \quad \rightarrow \quad N_C = \frac{z_{R1} + z_{S1}}{z_{S1}} N_I$$

$$\frac{N_C - N_R}{\frac{1}{z_{S2}}} = \frac{N_R - N_M}{\frac{1}{z_{R2}}} \quad \rightarrow \quad N_M = \frac{z_{R2} + z_{S2}}{z_{R2}} N_R - \frac{z_{S2}}{z_{R2}} N_C$$

$$N_M = \frac{z_{R2} z_{S1} - z_{R1} z_{S2}}{z_{S1} z_{R2}} N_I - \frac{z_{R2} + z_{S2}}{2 z_{R2}} \Delta N \qquad (3.4.2)$$

Now if I want that the electric engine is switched off when left and right wheels are running at the same speed it must be satisfied this relation:

$$\frac{z_{R2} z_{S1} - z_{R1} z_{S2}}{z_{S1} z_{R2}} = 0 \quad \rightarrow \quad \frac{z_{R1}}{z_{S1}} = \frac{z_{R2}}{z_{S2}} \qquad (3.4.3)$$

The last relation founded, tells us that we have to build two gear sets with the same transmission ratio in order to be able to follow the rule of electric motor switched off when the speed of the two wheels are the same. This means:

$$N_M = \frac{z_{R1} + z_{S1}}{2 z_{S1}} \Delta N \qquad (3.4.4)$$

Evaluated the influence of the electric motor on the carrier of the differential it's easy to have the expression of the torque on each wheel and how can be modified:

$$\begin{cases} T_R = \frac{T_I}{2} \pm \frac{z_{R1} + z_{S1}}{2 z_{S1}} T_M \\ T_L = \frac{T_I}{2} \mp \frac{z_{R1} + z_{S1}}{2 z_{S1}} T_M \end{cases} \qquad (3.4.5)$$

The $\pm$ sign indicates the possibility of choosing from the electric engine of behave like a motor or like a generator in order to apply torque to the carrier with different sign.

This solution show the biggest freedom degree because with a sufficient power delivered by the engine we can correct every torque coming from the engine.

The drawback is the high complexity required to set-up and design, the dimension and weight that can be limiting for vehicle design and the power required for the electric motor that requires a dedicated battery to supply.

3.6 Schematic compare between the analysed technologies

	Pros.	Cons.
Mitsubishi AYC rear diff.	• Handling performances improved • Management of torque distribution between the two side of the vehicle	• Heavy and complex differential to fit into the car and to design and set-up • Expensive solution, so can be fitted only on high segment car • The torque split between left & right requires different speed of the wheels and is limited by the gear ratio of the gear-sets
Audi QUATTRO sport diff	• More compact solution compared to AYC • The clutches and gear-sets use the same oil as differential • Same optimum performances as AYC system	• Expensive solution • The torque split between left & right requires different speed of the wheels and is limited by the gear ratio of the gear-sets • Heavy and complex differential to fit into the car and to design and set-up

Brake-type torque vectoring	• Cheap solution that makes it available also on mid/low-segment car • Less available space required to fit open differential	• Lowest performances compared to the other systems • Power loss due to braking action • Brake system heating and bigger wear of brake components
E-TVD	• Only system able to manage the torque split independently from engine torque • We can choose the torque acting on each wheel thanks to DC motor power introduction or taking • Best way to manage yaw moment, so best performances achieved	• Very complex solution to build and set-up • Very expensive solution • Big and heavy differential must be fitted on the vehicle • A dedicated electric power supply must be integrated to fulfil the energy request of the electric motor

4 TORQUE VECTORING CONTROL SYSTEM DESIGN

For our study, we have to start choosing a car to install our system, for that car we want:

- High segment vehicle, due to the fact that Torque Vectoring systems are expensive to fit.
- Rear wheel driven, for sake of simplicity.
- Good handling performances

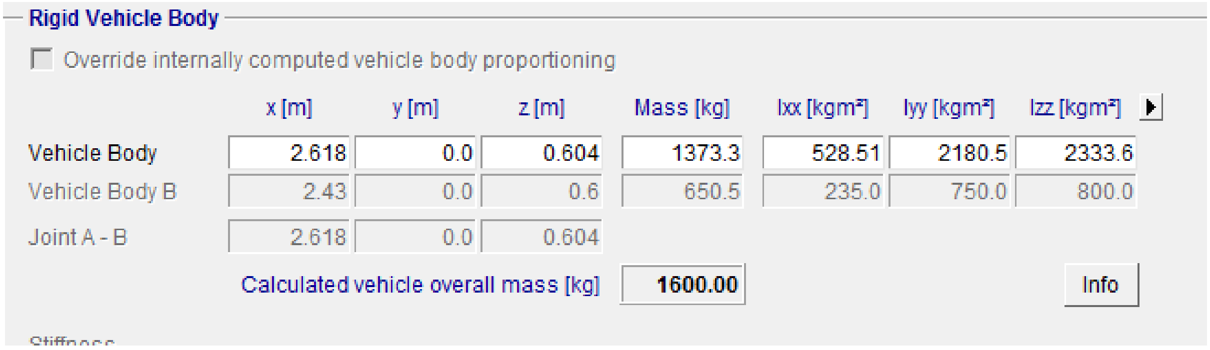
The manoeuvre set is an ISO step steer, for the repeatability of the results and the simplicity offered to tune the parameters.

The car will be virtually equipped with a torque transferring differential mounted of rear axle, that will allow it to absolve the torque vectoring function and our software will be added to the car ECU to simulate the working condition.

4.1 The car

The car is taken from the CarMaker library, considering the previous criteria, the BMW 5 well comply them. The above figures Fig. 4.1.1,2,3 shows the vehicle and the most interesting data for this analysis.

Figure 19 CarMaker car chosen



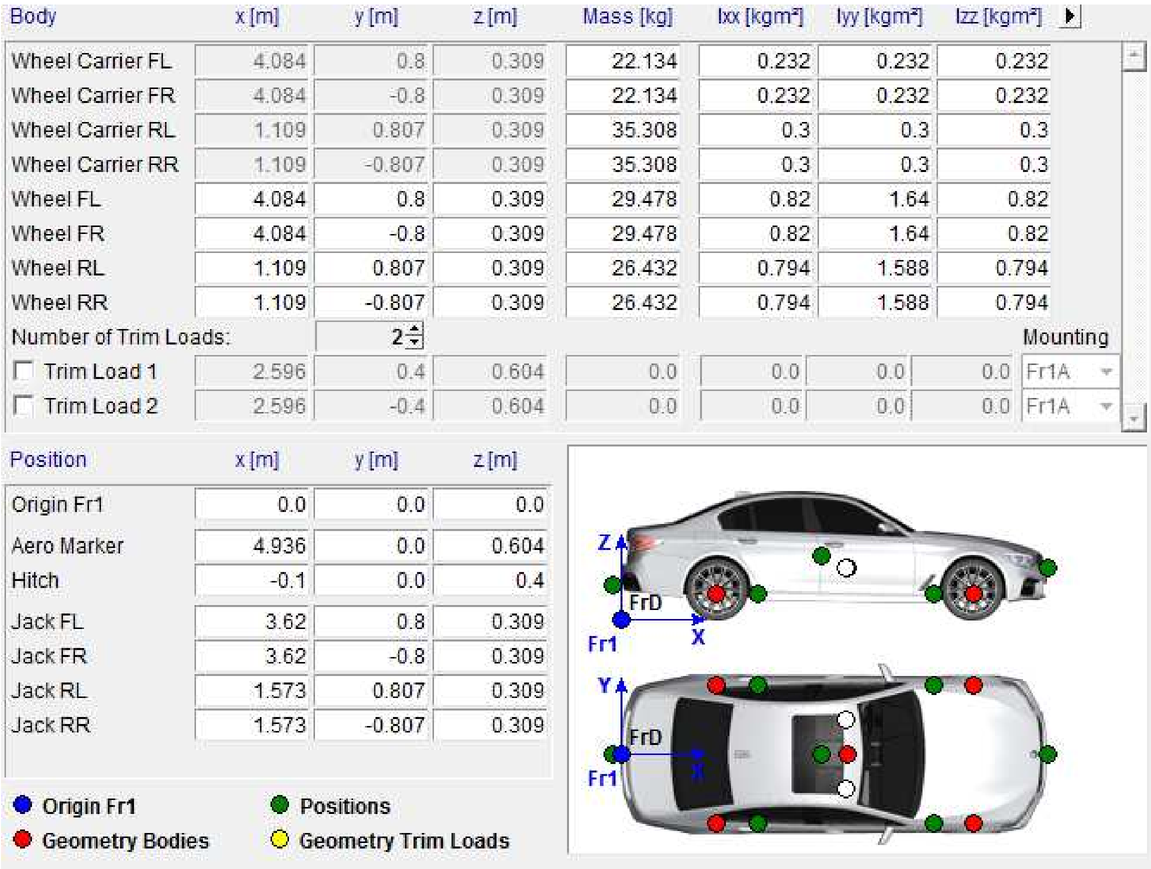
Rigid Vehicle Body

Override internally computed vehicle body proportioning

	x [m]	y [m]	z [m]	Mass [kg]	Ixx [kgm²]	Iyy [kgm²]	Izz [kgm²]
Vehicle Body	2.618	0.0	0.604	1373.3	528.51	2180.5	2333.6
Vehicle Body B	2.43	0.0	0.6	650.5	235.0	750.0	800.0
Joint A - B	2.618	0.0	0.604				

Calculated vehicle overall mass [kg] 1600.00 Info

Figure 20 Vehicle's mass m and inertia moment I_{zz}



Body	x [m]	y [m]	z [m]	Mass [kg]	Ixx [kgm²]	Iyy [kgm²]	Izz [kgm²]
Wheel Carrier FL	4.084	0.8	0.309	22.134	0.232	0.232	0.232
Wheel Carrier FR	4.084	-0.8	0.309	22.134	0.232	0.232	0.232
Wheel Carrier RL	1.109	0.807	0.309	35.308	0.3	0.3	0.3
Wheel Carrier RR	1.109	-0.807	0.309	35.308	0.3	0.3	0.3
Wheel FL	4.084	0.8	0.309	29.478	0.82	1.64	0.82
Wheel FR	4.084	-0.8	0.309	29.478	0.82	1.64	0.82
Wheel RL	1.109	0.807	0.309	26.432	0.794	1.588	0.794
Wheel RR	1.109	-0.807	0.309	26.432	0.794	1.588	0.794

Number of Trim Loads: 2

	x [m]	y [m]	z [m]	Mass [kg]	Ixx [kgm²]	Iyy [kgm²]	Izz [kgm²]	Mounting
Trim Load 1	2.596	0.4	0.604	0.0	0.0	0.0	0.0	Fr1A
Trim Load 2	2.596	-0.4	0.604	0.0	0.0	0.0	0.0	Fr1A

Position	x [m]	y [m]	z [m]
Origin Fr1	0.0	0.0	0.0
Aero Marker	4.936	0.0	0.604
Hitch	-0.1	0.0	0.4
Jack FL	3.62	0.8	0.309
Jack FR	3.62	-0.8	0.309
Jack RL	1.573	0.807	0.309
Jack RR	1.573	-0.807	0.309

Figure 21 Vehicle's data and reference frame position

From these tables we're able to evaluate:

- a=1.466 m
- b=1.509 m
- L=2.975 m
- m=1600 Kg
- I=2333.6 Kg/m^2
- t=1.6 m

4.2 Manoeuvre

Maneuver

No	Start	Dur	Long	Lat	Label/Description
					==== Global Settings / Preparation ====
0	0.0	30			Acceleration to 100 km/h
1	30.0	0.7	v=100	100	Steer step
2	30.7	30	v=100	±+0	Holding Steering Angle
3	60.7	==== END ====			

Figure 22 Manoeuvre sequences operated by the simulator

The decision of performing a step steer are two: performing an ISO manoeuvre we will have high repeatability of the results obtained and step steer manoeuvre is the simplest way to tune a control system of this kind.

The tests are done on a tarmac flat area, to avoid road condition and to collect data in the most objective way possible.

The car runs straight until reaches a speed of 100 Km/h, then, keeping the vehicle at constant speed, the driver perform a wheel steering rotation on 100° and hold this angle for 30s, after this time is passed the data recording ends.

This way of driving is particularly adapt to the tuning of our parameters, since we have a maintained input, so we can verify the time response of the system and its stability.

4.3 Torque vectoring control system design

The torque transferring differential is able to set a different torque split between left and right wheels. In order to perform the torque split it has, as seen before, two electro-hydraulic actuators, one for each clutch pack, that can control the forces to pull against each other the clutch disks.

Our target is to set-up a system able to decide the right level of forces that must be applied by the actuators to the clutches, to change properly the dynamic behaviour of our vehicle.

We will build a control strategy that is based on the reading of the normal sensors fitted of a present car:

- Wheel speed sensors
- Yaw sensor
- Steering wheel angle sensor
- Engine sensor to evaluate the torque input to differential

From these sensor we are able to evaluate all input variable for our system, a car normally using them is able to know:

- Longitudinal speed
- Lateral acceleration
- Steering angle
- Yaw velocity
- Yaw acceleration

All this variable will be used to our calculation.

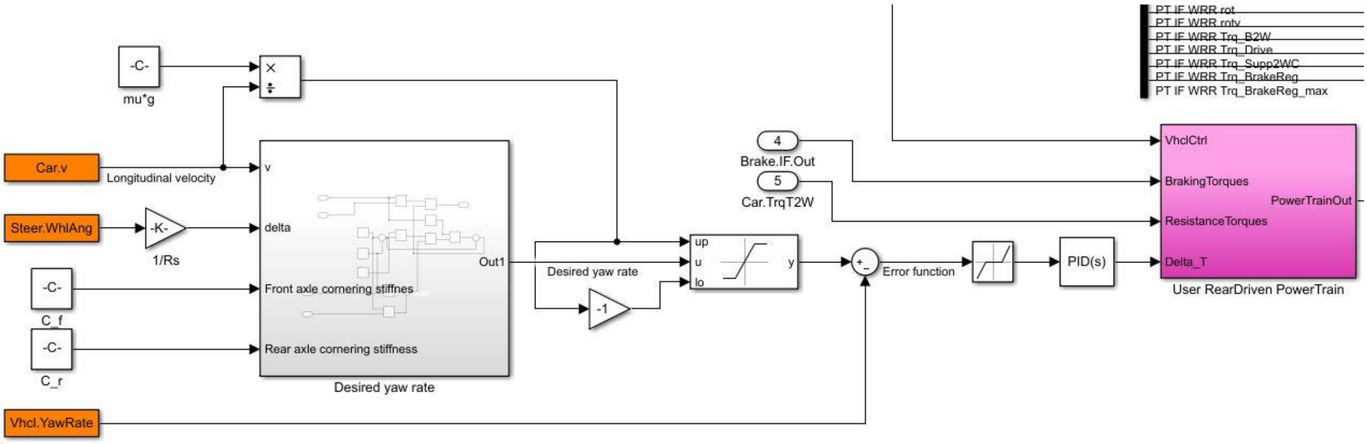


Figure 23 Torque vectoring control logic overview

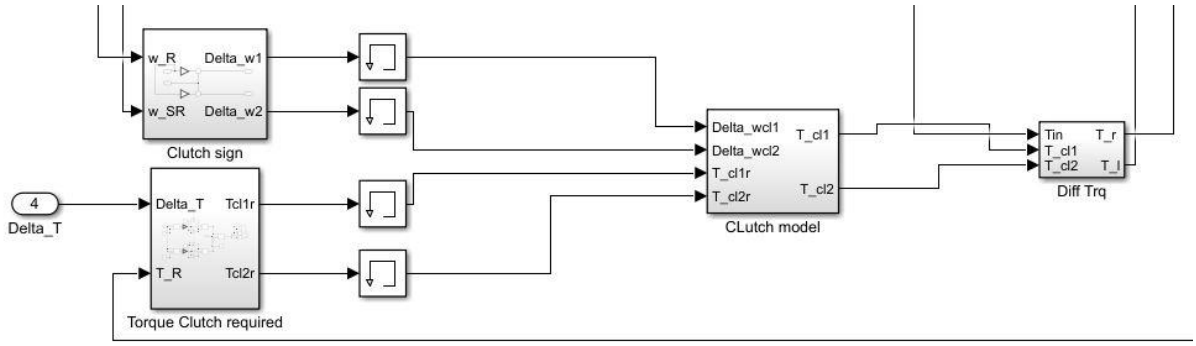


Figure 24 Mechanical part of the control system

The basic idea of all torque vectoring system is to create a controlled yaw moment on the vehicle to increase stability and performances. This moment is created applying different level of torque to the driven wheels. Since we have to create a precise yaw moment, we have to know the entity of this moment. In Fig. 4.7 its graphically represented how a different repartition of traction forces from left to right can manage the vehicle yaw rate, the above equation explain it mathematically.

If M is the opposite of the difference between the target yaw rate and the evaluated we have:

$$M = \frac{F_{xR}t}{2} - \frac{F_{xL}t}{2} = \frac{(F_{xR} - F_{xL})t}{2} = \frac{\Delta T_w R_w t}{2} \quad with\ \Delta T_w = (T_{OR} - T_{OL})\ , R_w\ wheel\ radius$$

$$M = PID(r_d - r) \quad \rightarrow PID(r_d - r) = \frac{(T_{OR} - T_{OL})R_w t}{2} \qquad (4.3.1)$$

With a correct tuning of the PID control we will be equal to substitute the sign of proportional in the eq. (4.3.1) with the equal sign.

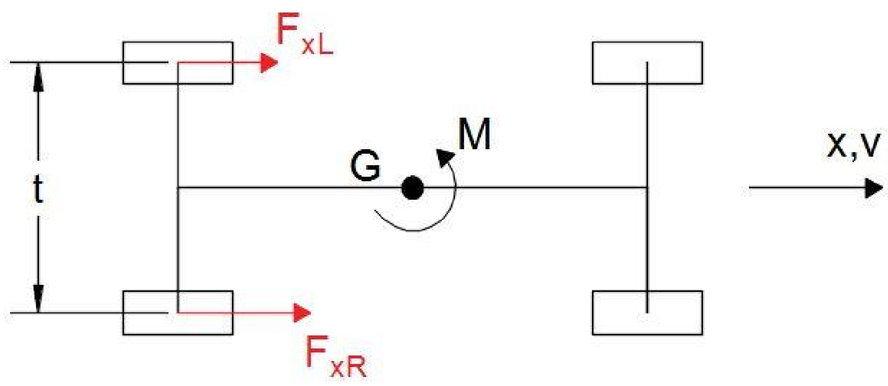


Figure 25 Vehicle top view with yaw moment created by traction forces

The model is mainly splitted in two part, a control part where the control logic is operated Fig. 4.5 and a mechanical part Fig. 4.6 where the decision of the control logic are transformed in mechanical action. The mechanical part is implemented inside the block of a rear wheel driven powertrain in our software, where it takes a PID controlled error function to decide which clutch must be activated and the entity of the clutch torque required.

We decide to take set the desired yaw rate, as the yaw rate of bicycle model of our vehicle running at constant speed at steady-state conditions. Once we know the desired yaw rate, we make a comparison with the real one evaluated by the vehicle sensor, the difference between this two values will be directly proportional to the forces that has to be exerted by the linear actuators.

We start explaining the control logic part of the system.

4.3.1 Desired yaw rate calculation

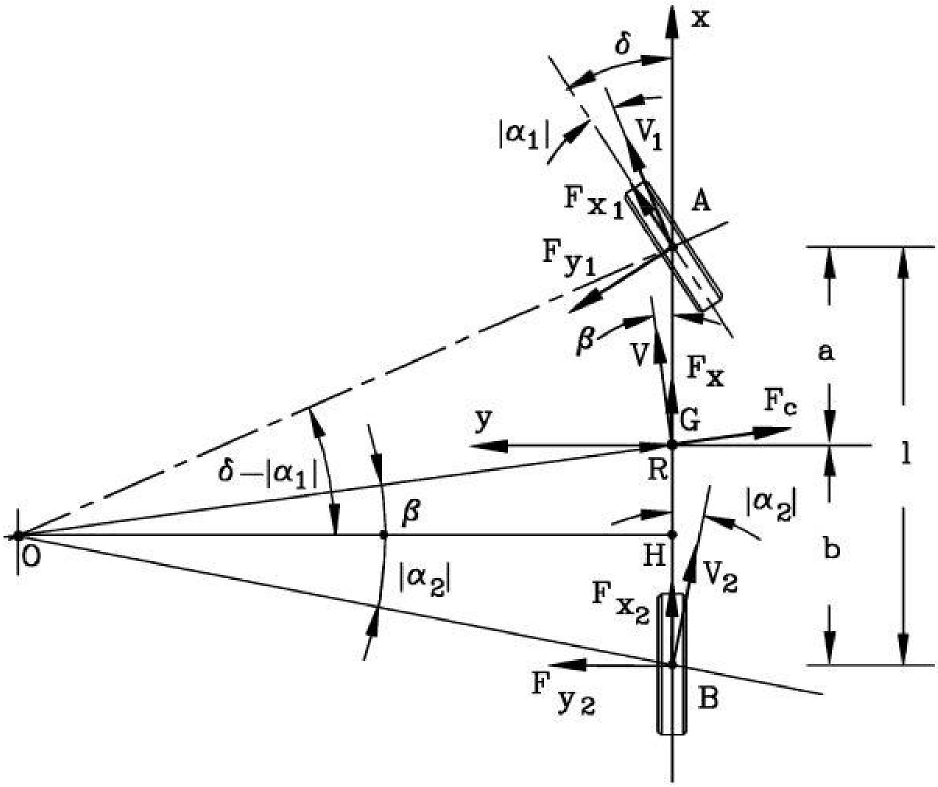


Figure 26 2 DOF model sketch and variables

The desired yaw rate is evaluated from the yaw rate gain for a 2 DOF model Fig. 4.5, linearized for steady-state condition, that means constant speed and curvature radius more bigger than wheelbase.

Starting from yaw rate gain eq.(4.3.1):

$$\frac{r}{\delta} = \frac{v}{L}\frac{1}{1+Kv^2} \quad K\ Stability\ coefficient \tag{4.3.2}$$

$$K = \frac{m}{l^2}\left(\frac{b}{C_f} - \frac{a}{C_r}\right) \quad C_f, C_r\ cornering\ stiffness\ of\ front\ and\ rear\ axle$$

$$\rightarrow \frac{r}{\delta} = \frac{v}{l + \frac{m}{l}\left(\frac{b}{C_f} - \frac{a}{C_r}\right)} \rightarrow r_d = \frac{v\delta}{l + \frac{m}{l}\left(\frac{b}{C_f} - \frac{a}{C_r}\right)} \tag{4.3.3}$$

The parameter C_f, C_r are evaluated from the tyres characteristic present in our vehicle model and it calculated as $\left(\frac{\partial F_y}{\partial \alpha}\right)_{\alpha=0}$ from Fig.4.8 and the results from the applied load are $C_f = 128916 \frac{N}{rad}$ $C_r = 171887 \frac{N}{rad}$.

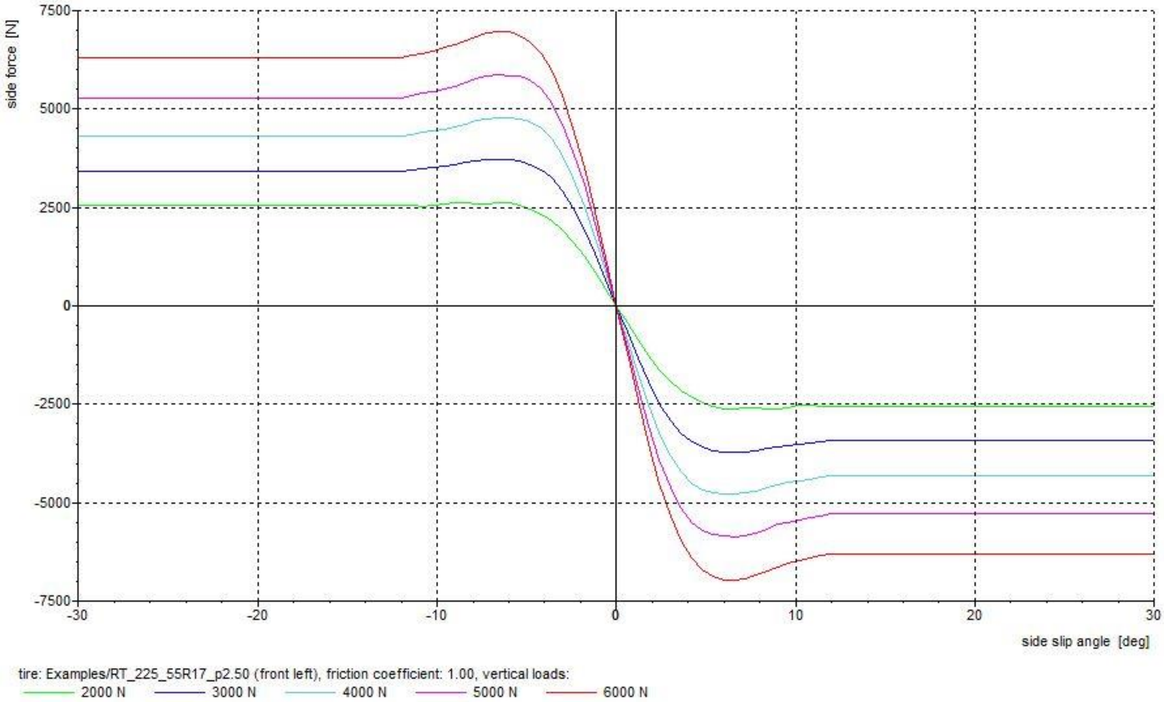


Figure 27 Tyre lateral characteristic in function of tyre side slip angle

We have to make attention during the evaluation of desired yaw moment, because it can reach high values that can make the vehicle unstable, so at the exit of the desired yaw moment block, it's necessary to add a saturation block to avoid instability, this block allows the system to act like the eq. (4.3.3). The maximum value for the yaw rate is multiplied by 0.8 for safety reasons, because a real car becomes instable earlier that a 2 DOF ideal model.

$$\begin{cases} a_{ymax} = \mu g \\ a_{ymax} = vr \end{cases} \rightarrow \quad |r_d| \leq \left|\frac{\mu g}{v}\right|$$

$$r_d = \min\left(\frac{v\delta}{l + \frac{m}{l}\left(\frac{b}{C_f} - \frac{a}{C_r}\right)}; 0.8\frac{\mu g}{v}\right) \tag{4.3.4}$$

It must be underlined that, a normal vehicle is not able to detect the friction coefficient, we have taken as known, because, for us, it's easy to evaluate from the simulator environment, but if we want to fit this control logic in a real vehicle, we have to insert an estimator of this parameter that must be sufficiently precise and rapid to avoid the instability condition during the transition from one surface to another. For calculation simplicity we act as we have this kind of estimator and we use the results.

In this block for calculation reasons is also implemented the reading of the wheel steering angle, that has its own gain due to transmission ratio of the steering system equal to 1:18.

4.3.2 System controller

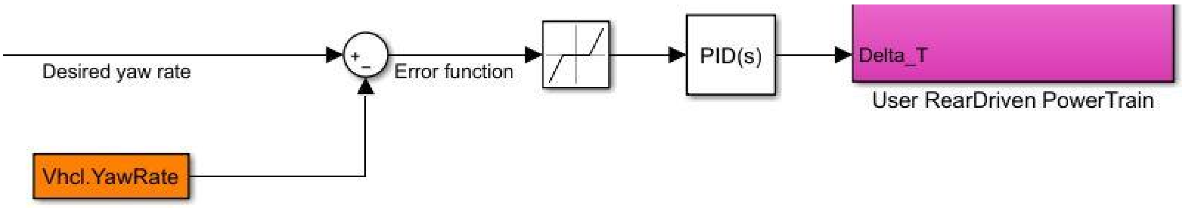


Figure 28 Controller scheme

The control of the system is exerted by a PID controller that can manage the error function built as the difference between the target yaw rate and the one evaluated from the vehicle sensors.

Before the PID, is inserted a Dead Zone block to avoid that the system start to act for very low differences between the quantities or for signal disturbances.

The PID gain are tuned to obtain a signal error that is directly proportional to the clutch torque required to correct the yaw rate error present during the vehicle operations. This tuning, was made during different experimental test with the vehicle dynamic simulator and gave the results of $K_P = 2000 \quad K_I = 3000 \quad K_D = 100$. Those values shows the best behaviour of the involved quantities during different manoeuvres done with different speeds.

Since we have finished the overview of the control logic part we can start talking about the mechanical part of the system, implement inside the control logic of the system.

4.3.3 Evaluation of the modulus of clutch torque

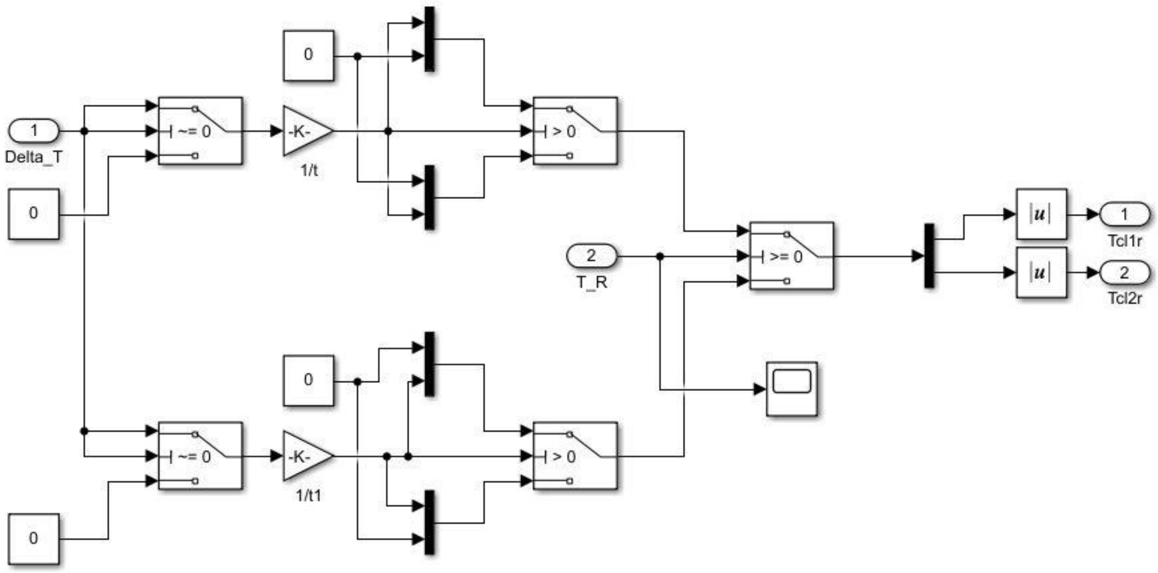


Figure 29 Overview of the 'Clutch torque required' block

The first block added, founded inside the powertrain model, is the calculator of the torque required, Fig. 4.10, by the clutches of the system to counteract a certain error situation.

The block has two input, 'Delta_T' that is the PID exit signal and 'T_R' torque acting on the right wheel. The system is symmetrical in the first part, this is due to the fact that when we have negative torque acting on the wheels the sign of clutch torque required must be swapped. In the first part are present two switch, the first one impose to the system to don't activate clutches when the error is close to '0', and the second part impose the values of each clutch torque to correct the situation (ex if T_R>0 & Delta_T>0 -> T_{cl1r}=Delta_T & T_{cl2r}=0). Before the exit of the block, two absolute value block are installed, to have a positive value of the variables because the clutch torque sign depends always from the speed difference applied in the discs pack. The sign of the clutch torque in decided in a separated block that has this function.

4.3.4 Clutch torque sign block

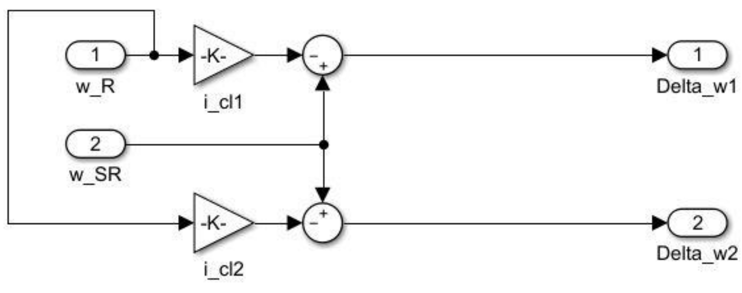


Figure 30 Clutch sign block overview

The clutch sign block is the Simulink implementation of the eq. (3.2.15-16).

$$sgn(T_{CL1}) = sgn(\omega_{SR} - \omega_R i_{CL1}) \quad (3.2.15)$$

$$sgn(T_{CL2}) = sgn(\omega_{SR} - \omega_R i_{CL2}) \quad (3.2.16)$$

The output of the system gives us the information about the sign of each clutch torque, the input variable are w_R, w_{SR}, rotational speed on the ring gear of the differential and rotational speed of the right wheels, both are known from the powertrain model.

4.3.5 Clutch model block

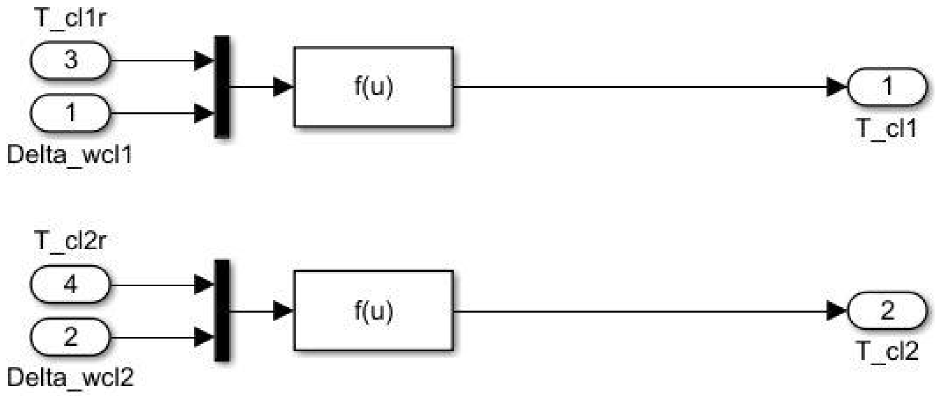


Figure 31 Clutch model block scheme

The ‘clutch model’ block merge together the two block previous analysed, ‘clutch torque sign’ and ‘torque clutch required’, indeed, the output signal of these blocks are becomes the input for this one here. A mux block merge together the signals referred to the same variable, then, a math function is operated to calculate the clutch torque in sign and modulus. The mathematical function multiply the modulus of the torque (ex. T_cl1r) by the hyperbolic tangent of Delta_wcl1 divider by 0.1 that is a constant value experimentally determined. The choice of using the hyperbolic tangent instead of sign is guided by the necessity of avoiding the discontinuity in ‘0’ value of the sign function.

The output of the system, as said, are the clutch torque that has to be inserted in the calculation of the wheel torque.

4.3.6 Clutch out from differential evaluation

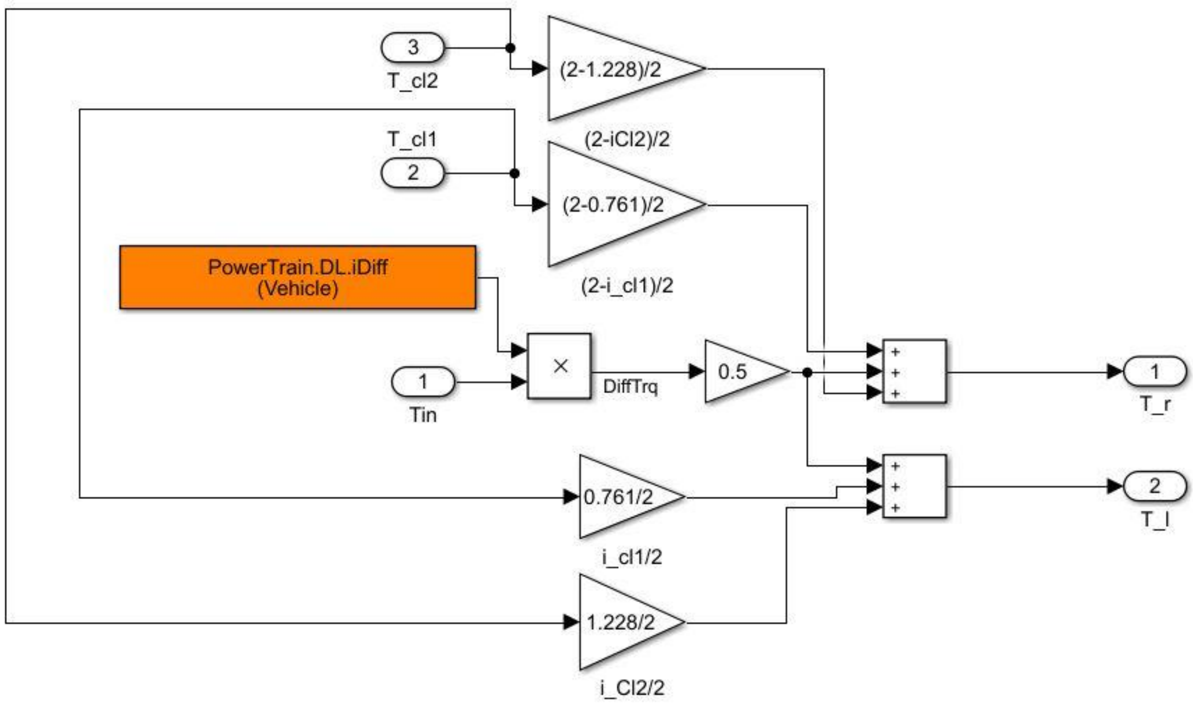


Figure 32 Calculator of left and right wheel torque

This subsystem is the implementation of eq. (3.2.14), the input are the clutch torque calculated in the last block analysed 'T_cl1','T_cl2' and the torque input to the differential 'Tin', the system out the torque delivered to each of the driven wheel and send it to the powertrain model to make system able to compute the dynamic calculation of the system.

$$\begin{cases} T_{OR} = \frac{1}{2}[T_R + T_{CL1}(2 - i_{CL1}) + T_{CL2}(2 - i_{CL2})] \\ T_{OL} = \frac{1}{2}[T_R - T_{CL1}i_{CL1} - T_{CL2}i_{CL2}] \end{cases} \quad (3.2.14)$$

5 RESULTS OF THE MANOEUVRE PERFORMED WITH THE DEVELOPED SYSTEM

Here will be presented the results vehicle selected equipped with the torque vectoring system explained performing the manoeuvre descripted, on the top of graph couple are shown the characteristic of the vehicle with torque vectoring and on the bottom the data of the same vehicle doing the same manoeuvre without torque vectoring. Only one plot one steer angle and velocity will be given because are equal for both vehicle.

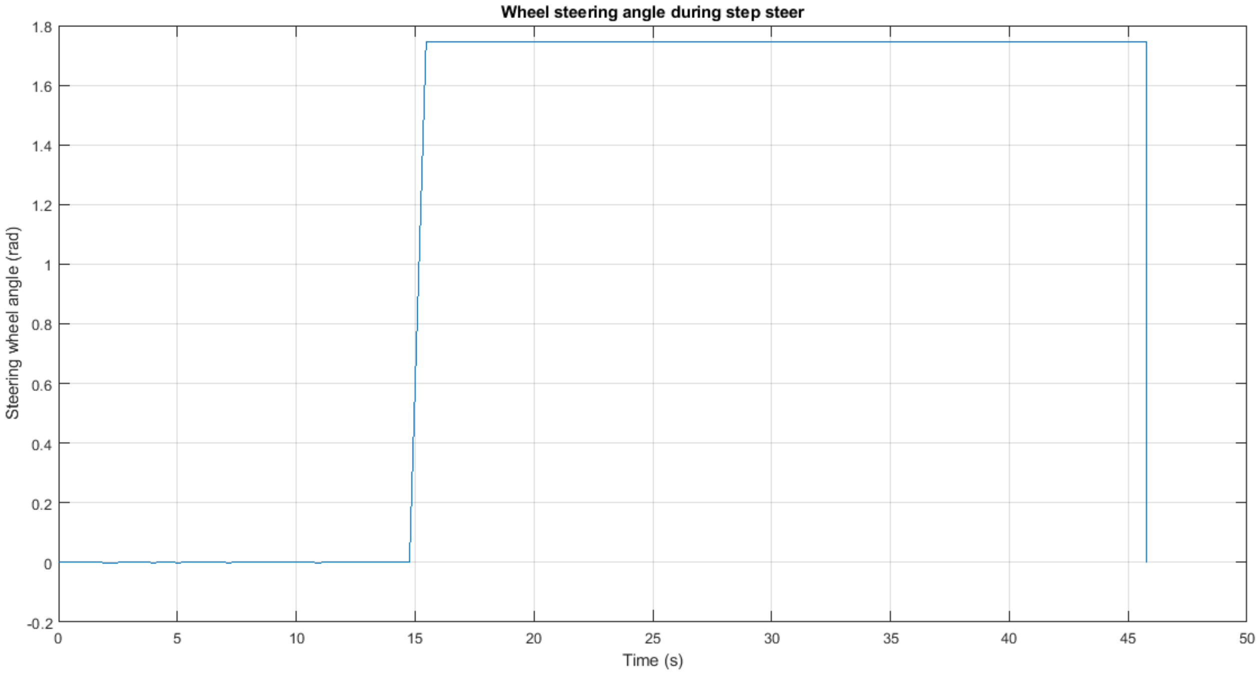
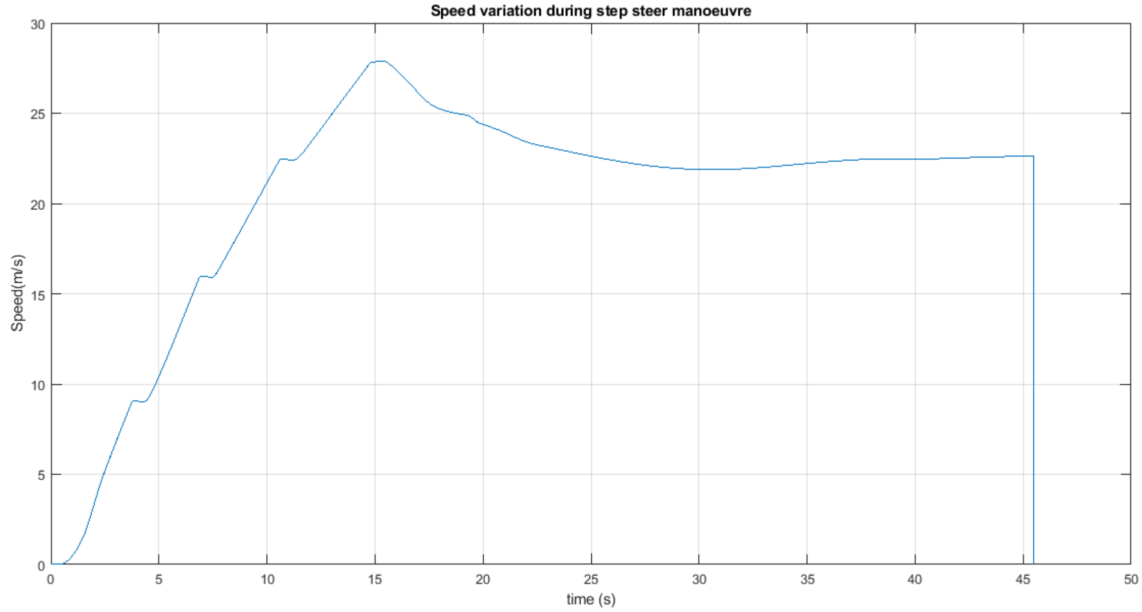


Figure 33 Manoeuvre time history



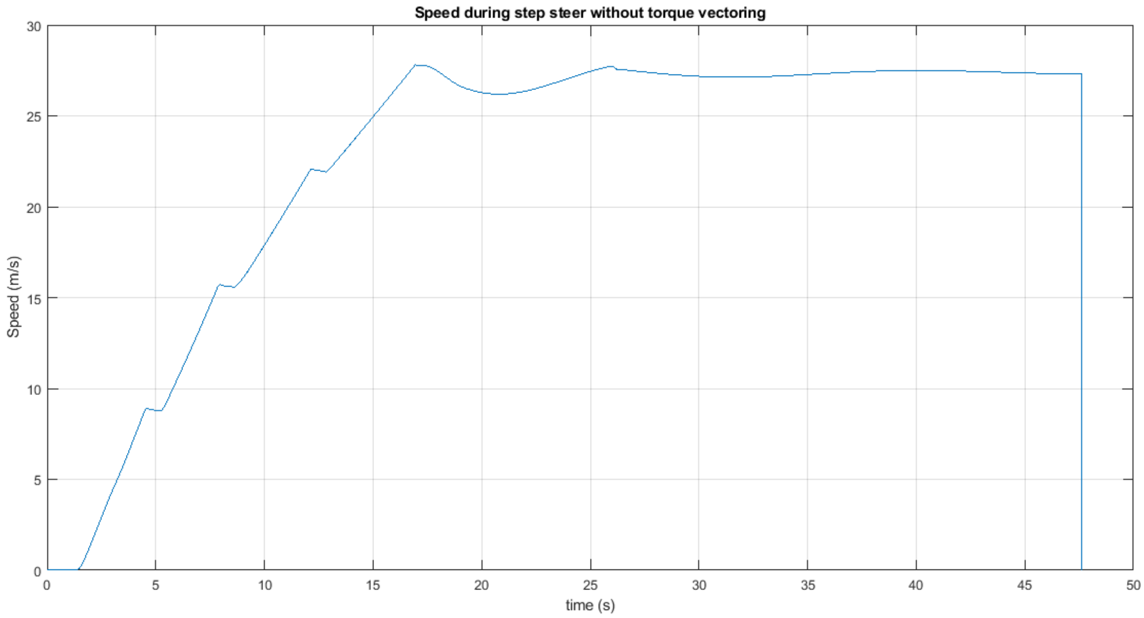
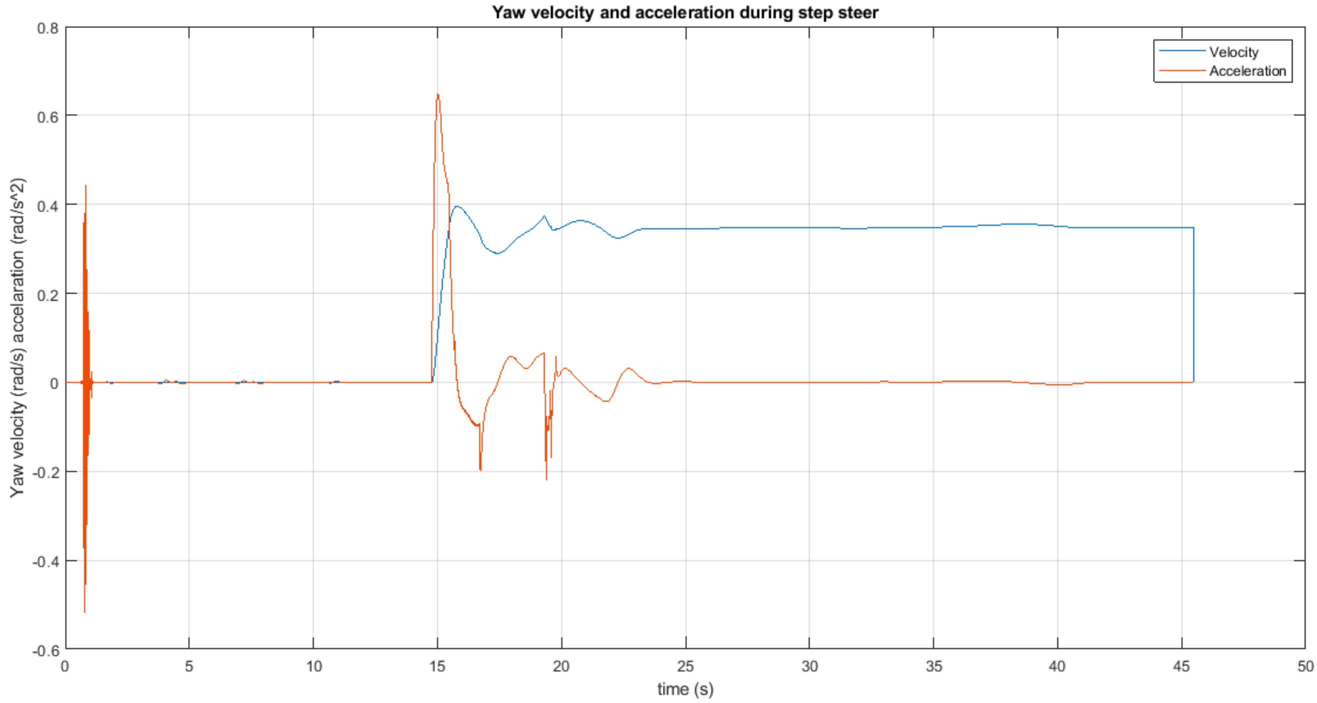


Figure 34 Vehicle speed during manoeuvre



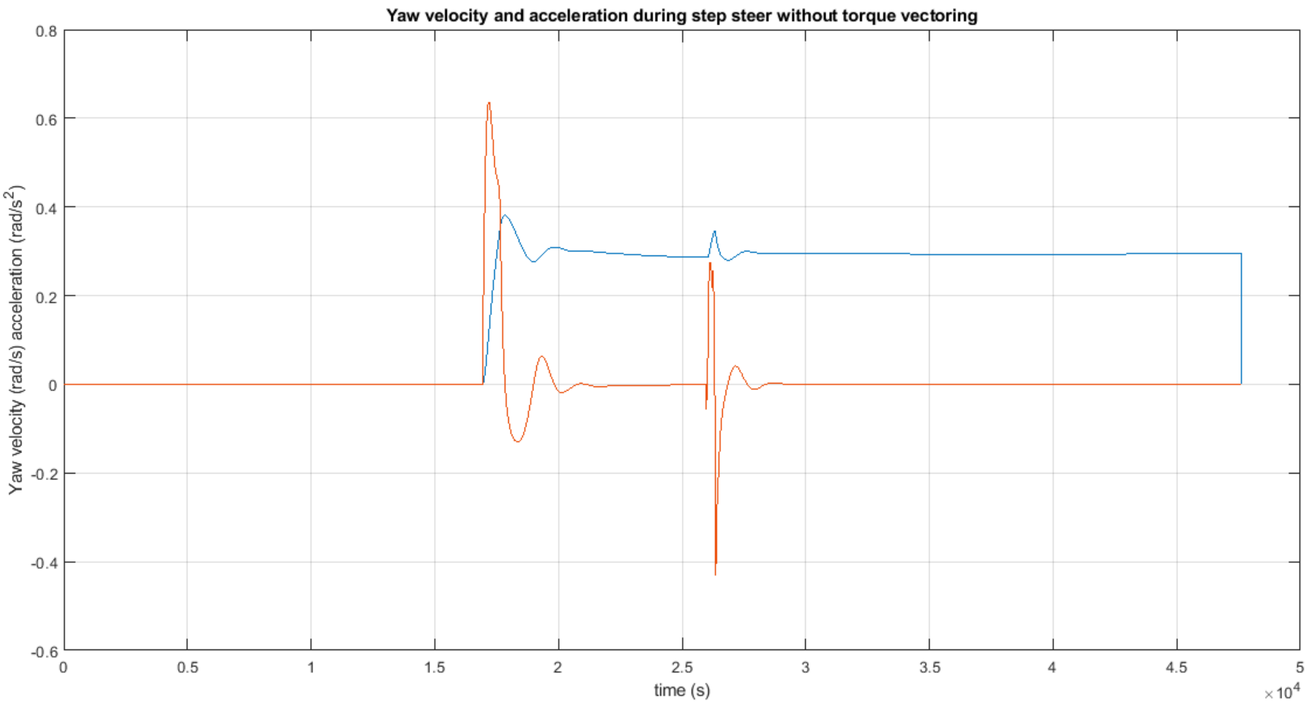


Figure 35 Comparison on yaw rate and yaw acceleration

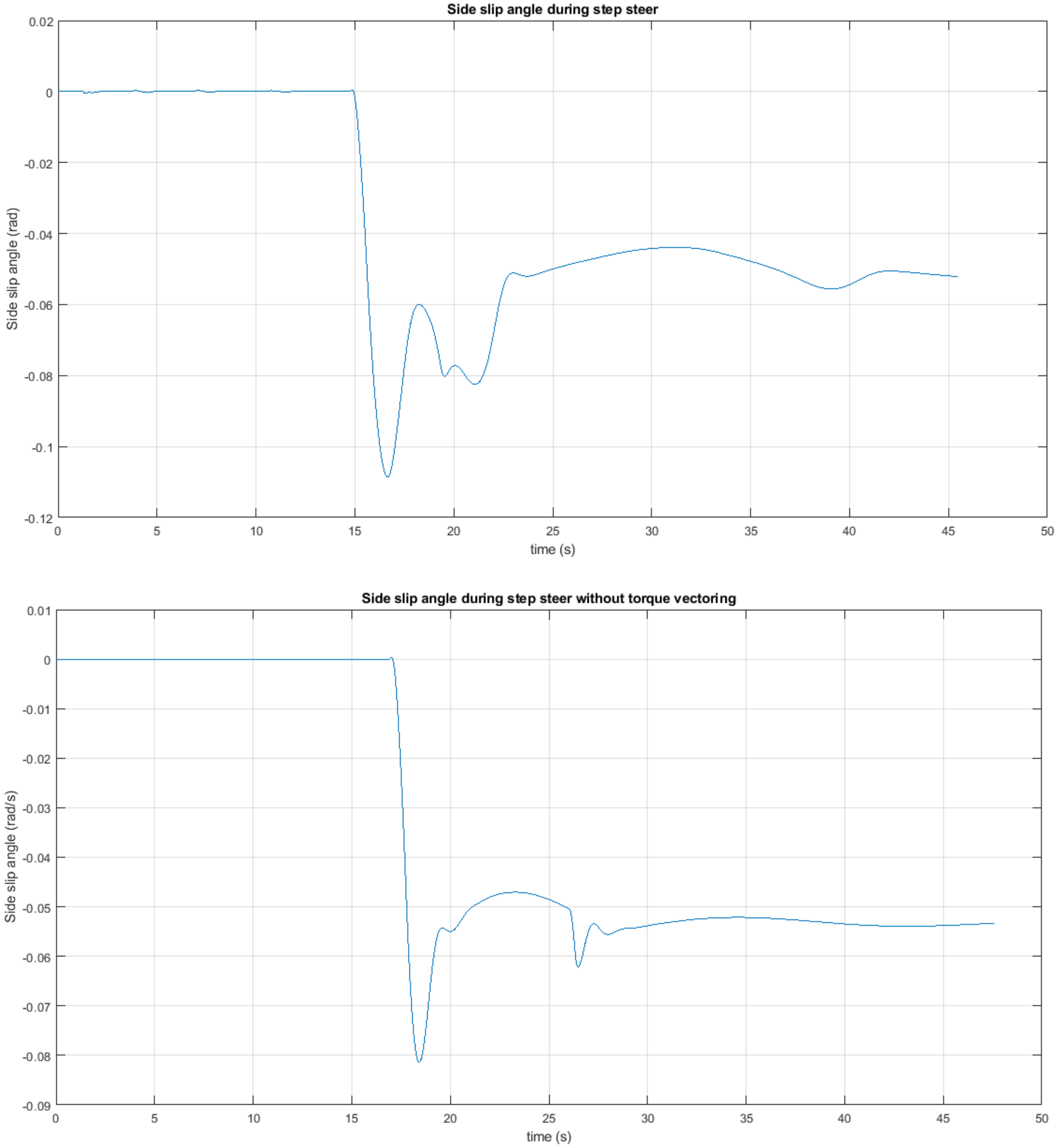


Figure 36 Side slip angle difference

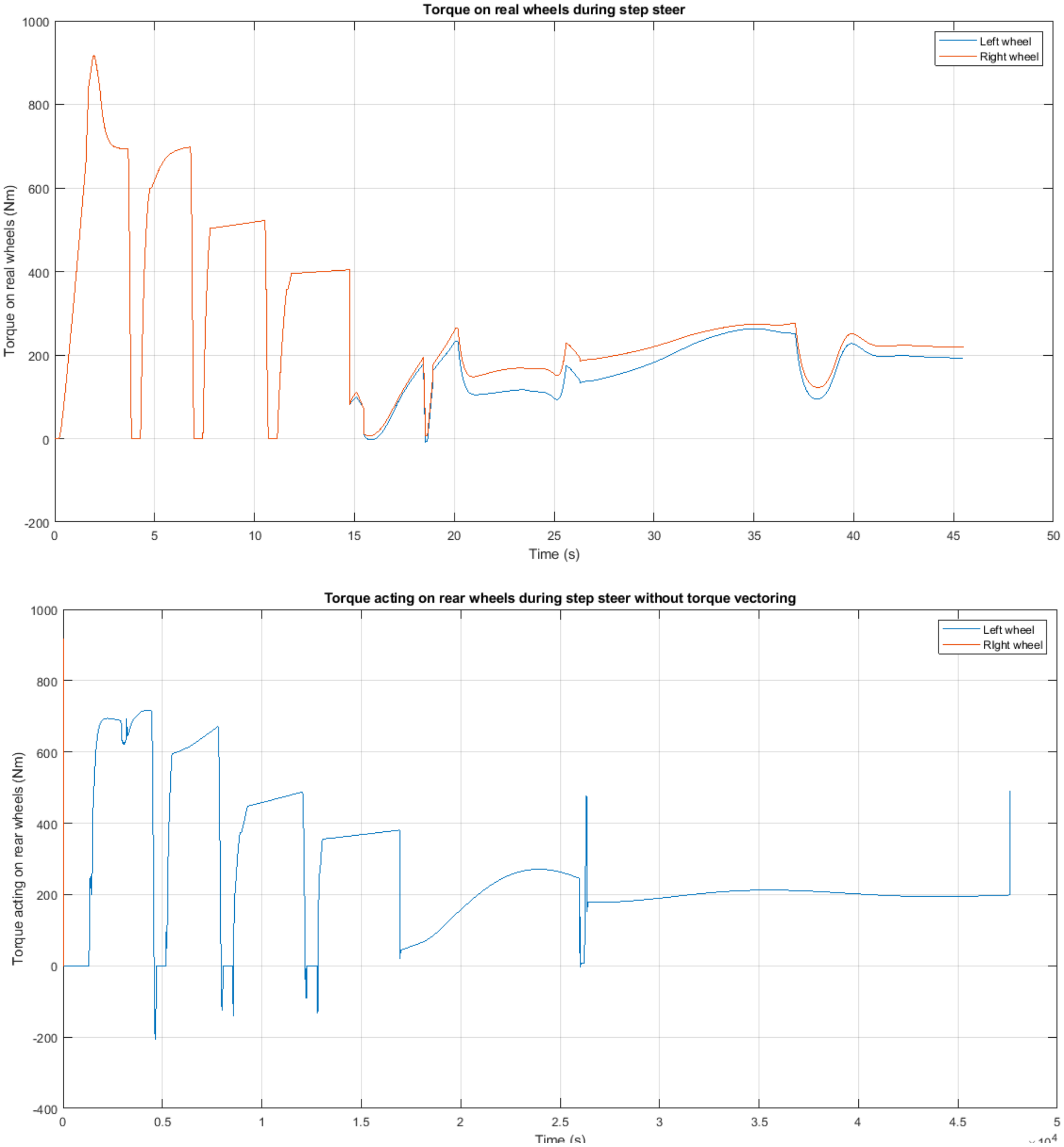


Figure 37 Traction Torque to rear wheels comparison

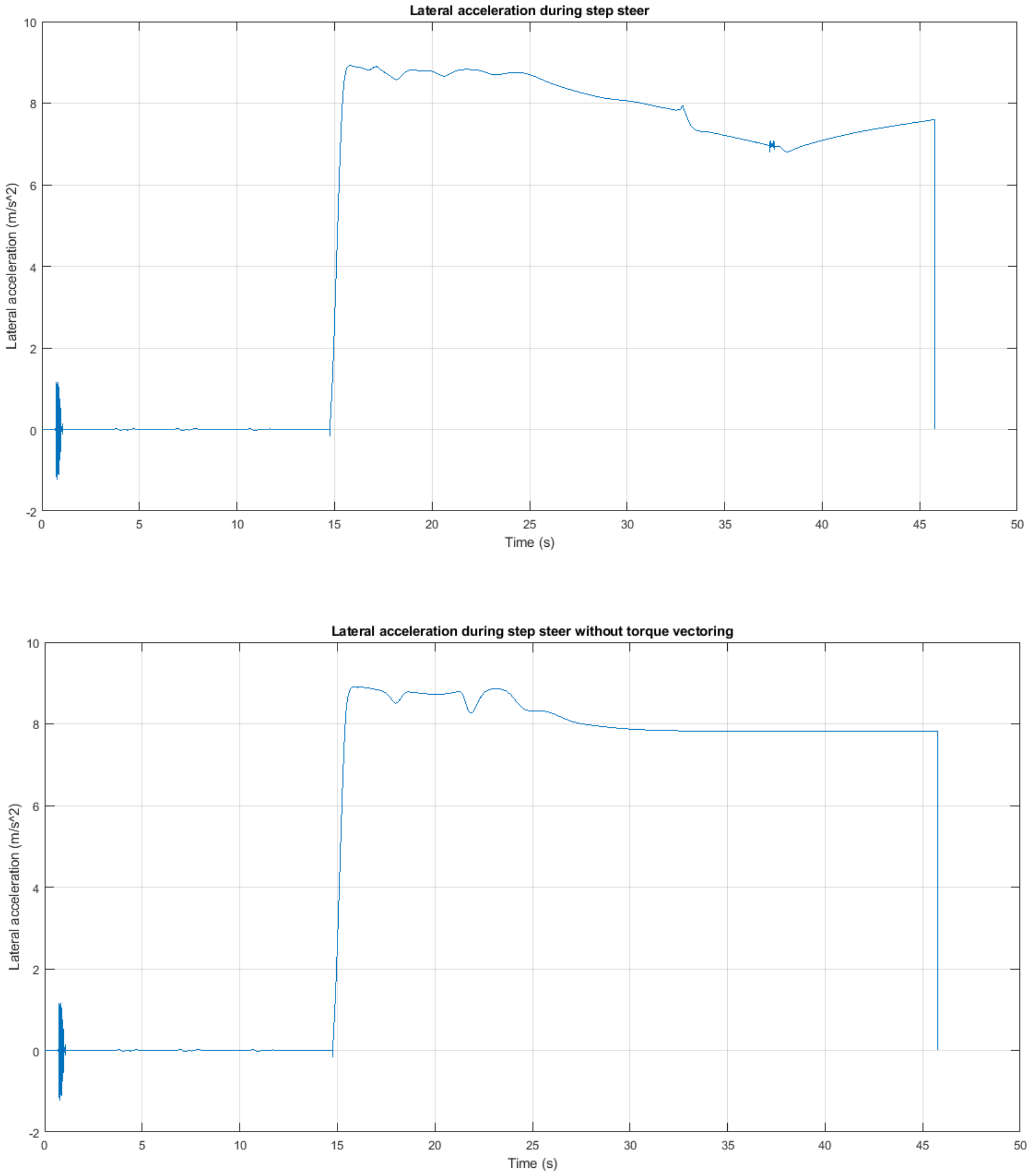


Figure 38 Lateral acceleration during step steer

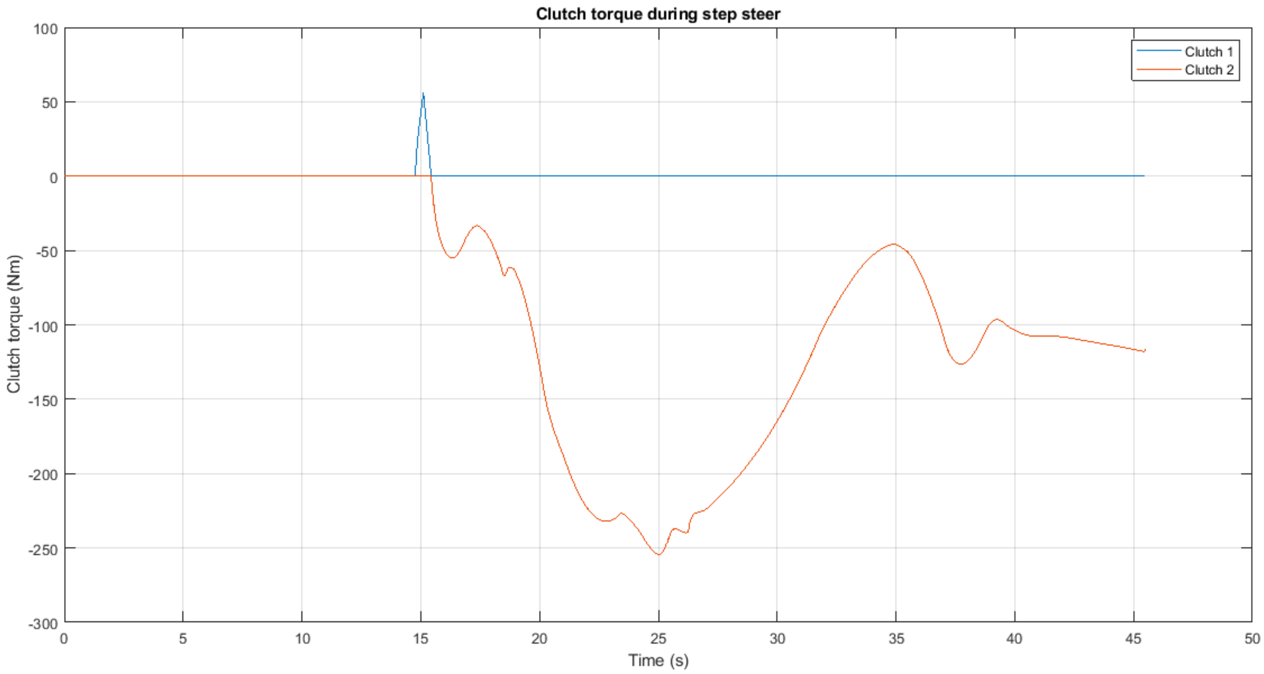


Figure 39 Clutch torque required

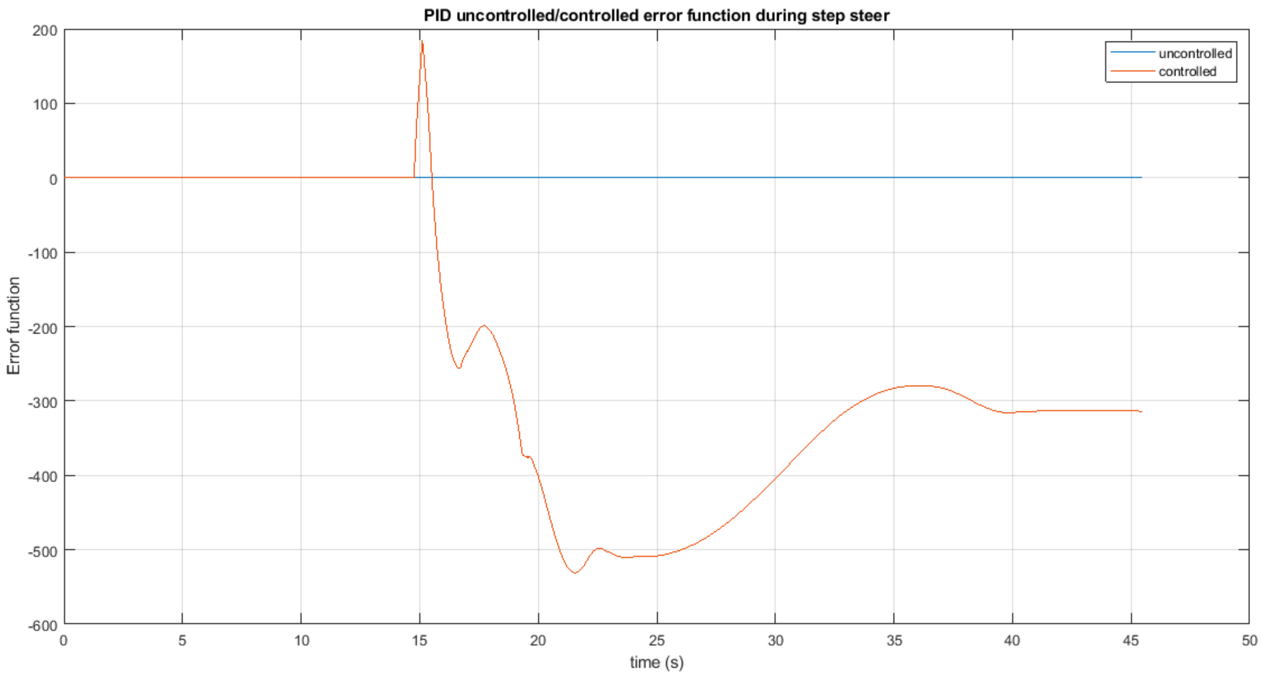


Figure 40 Error function uncontrolled and controlled by PID

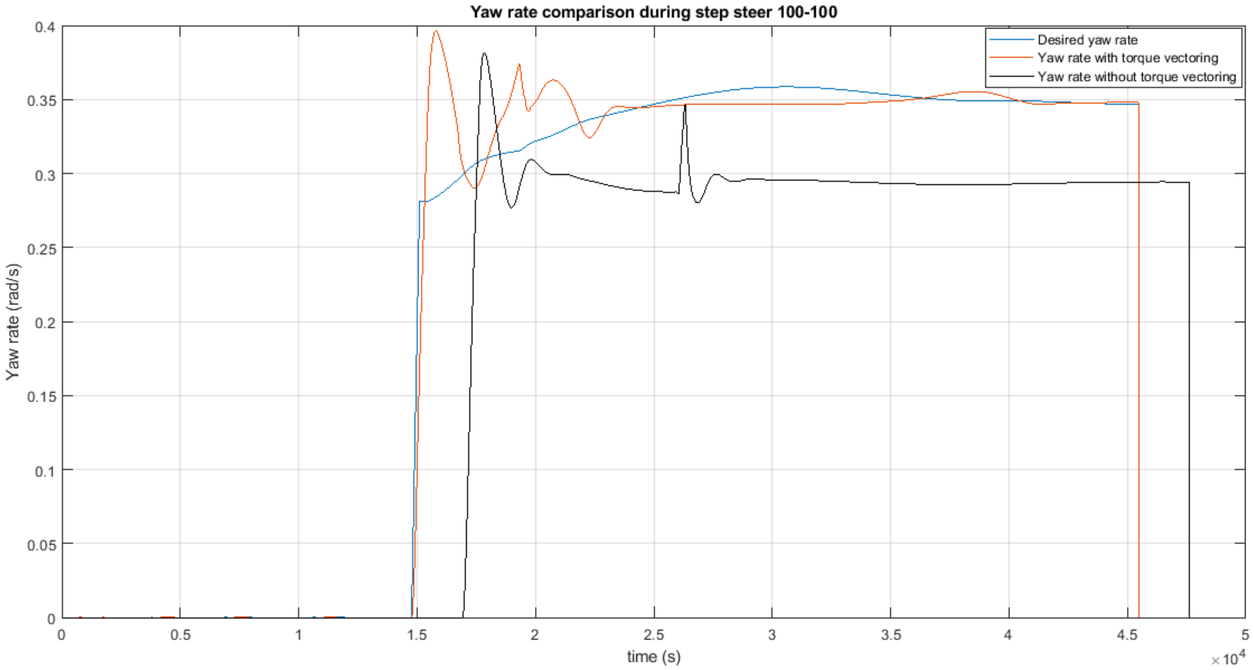


Figure 41 Comparison between desired yaw rate, and yaw rate with/out torque vectoring

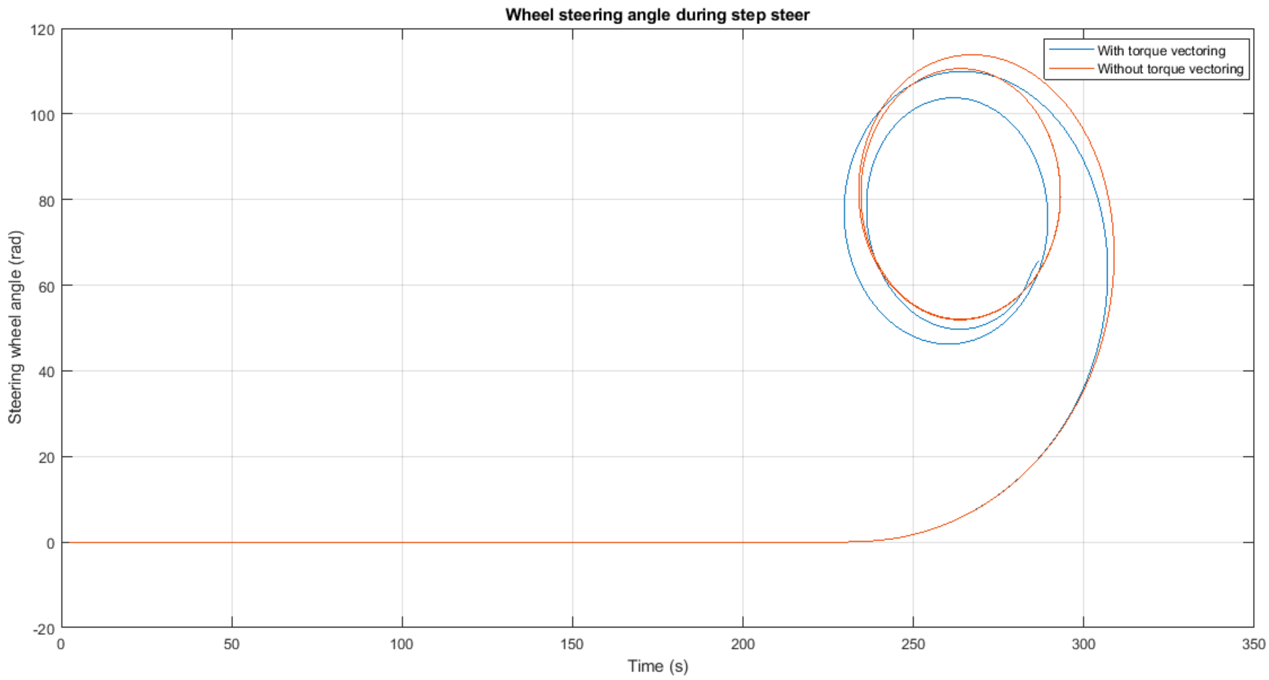
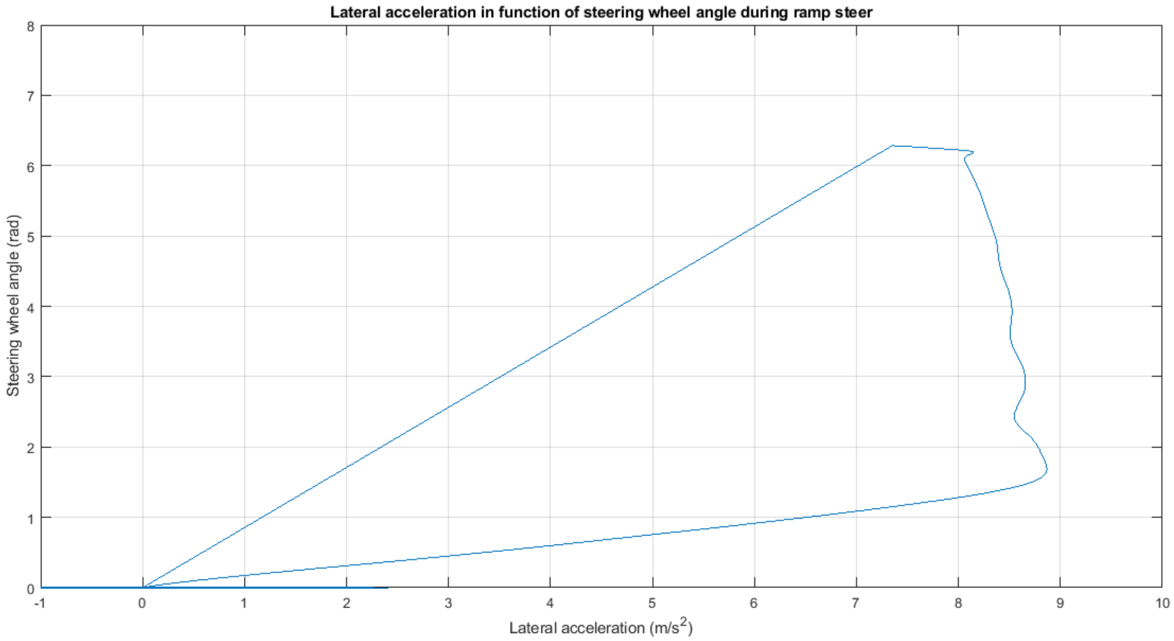


Figure 42 COG trajectory during step steer

The comparison plot shown us the big advantage of fitting this of system in a vehicle, we increase the stability ,and consequently the safety, because the yaw rate and its acceleration are more stable and also the torque acting on the tyres shows a more constant behaviour during the test. The biggest differences is made by the increasing of the dynamic performance, we can see that with same steer input and speed, the vehicle to cover the manoeuvre with lower curvature radius and better using the side slip angle that has more constant behaviour.

In order to evidence an important features, we decide to perform a ramp steer, a manoeuvre done with at constant speed of 50 Km/h slowly increasing the wheel steering angle 10°/s. The vehicle continue to run in circle until it reaches limit condition going out of the track. The condition of the track is high friction coefficient condition $\mu = 1$.

For this manoeuvre it’s drawn the variation of the lateral acceleration in function of the steering angle, this shows us the capability of the system to change the passive understeering coefficient of the vehicle, that is proportional to the derivative of the function, in fact in Figure 43 is shown how that the vehicle equipped with torque vectoring has a characteristic with a different slope from the other vehicle, this difference is directly proportional to the variation of the understeering coefficient (The difference between the two cases is $\sim 50\%$).



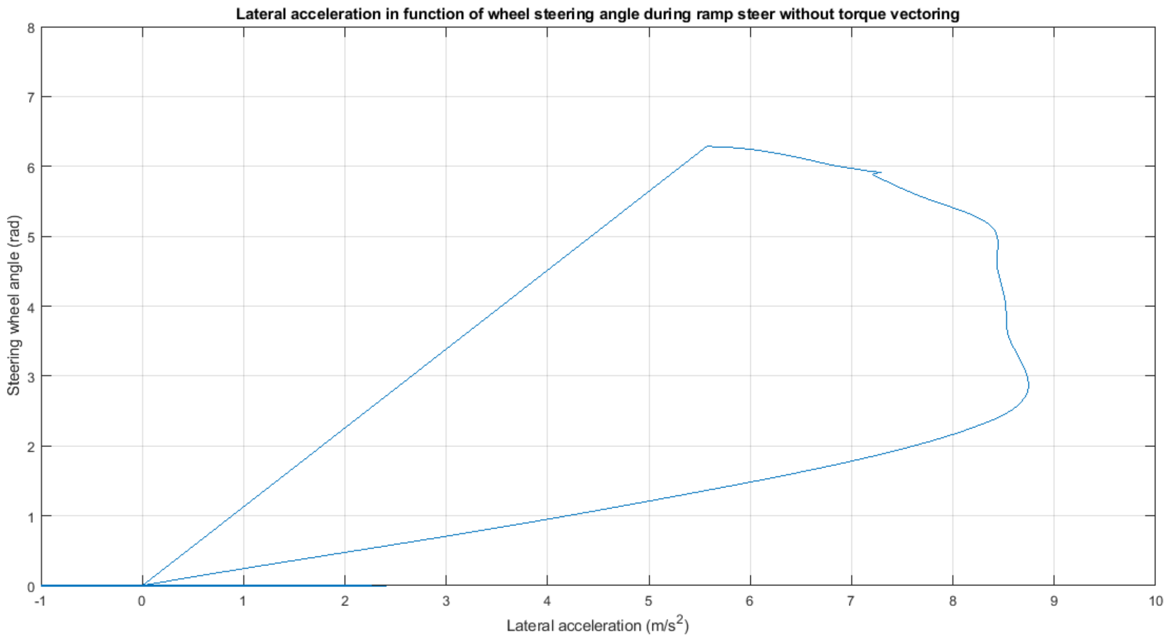


Figure 43 Lateral acceleration in function of steering angle

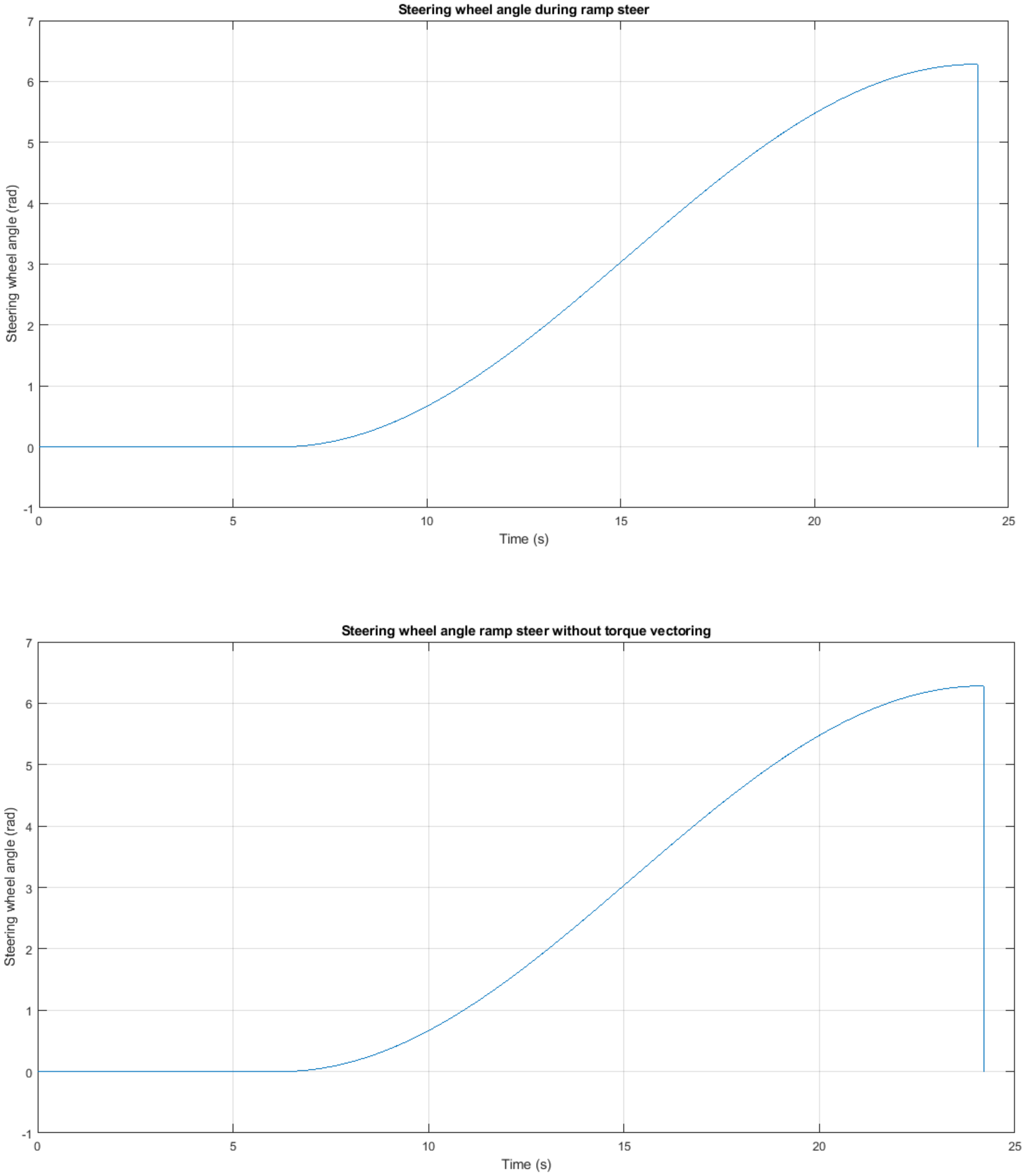
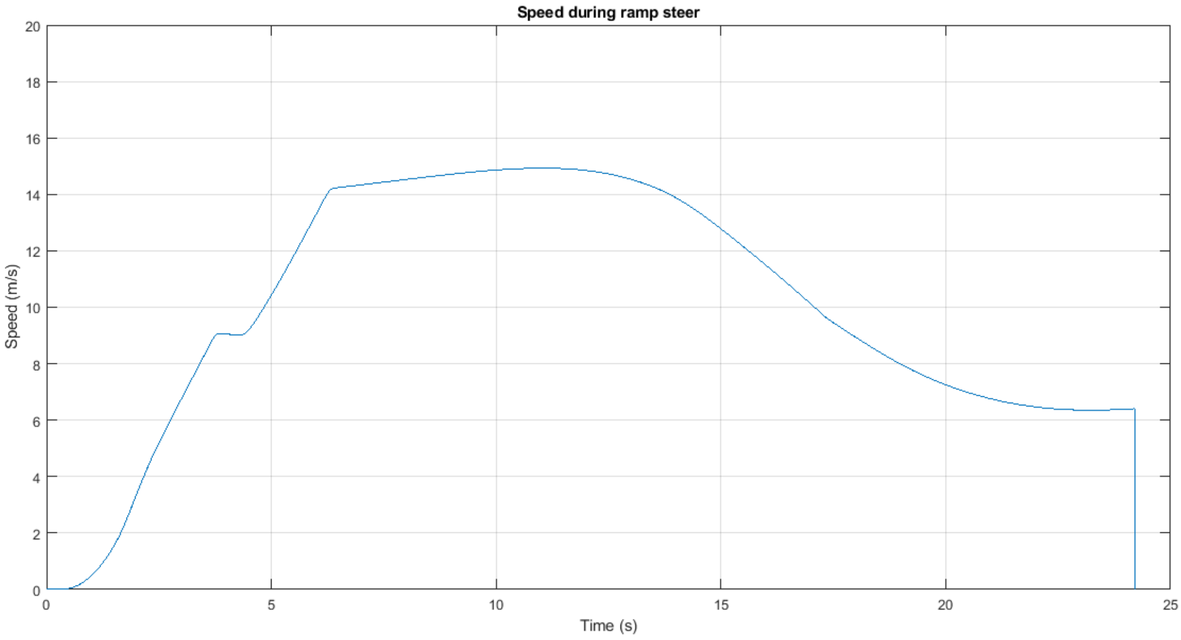


Figure 44 Steering wheel motion during ramp steer



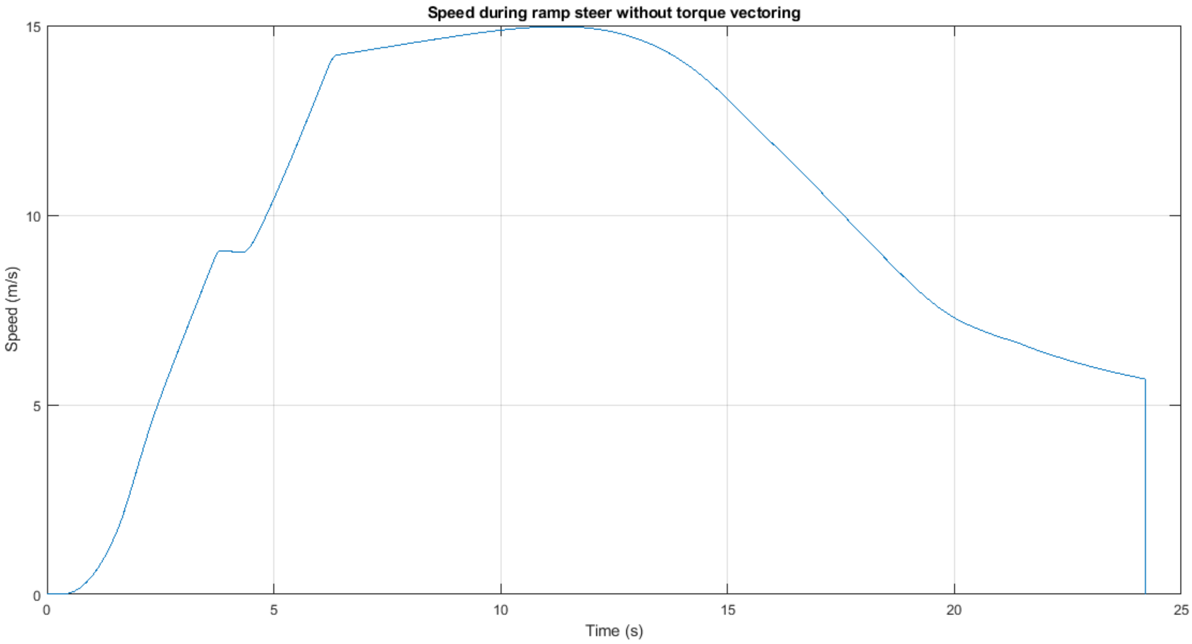
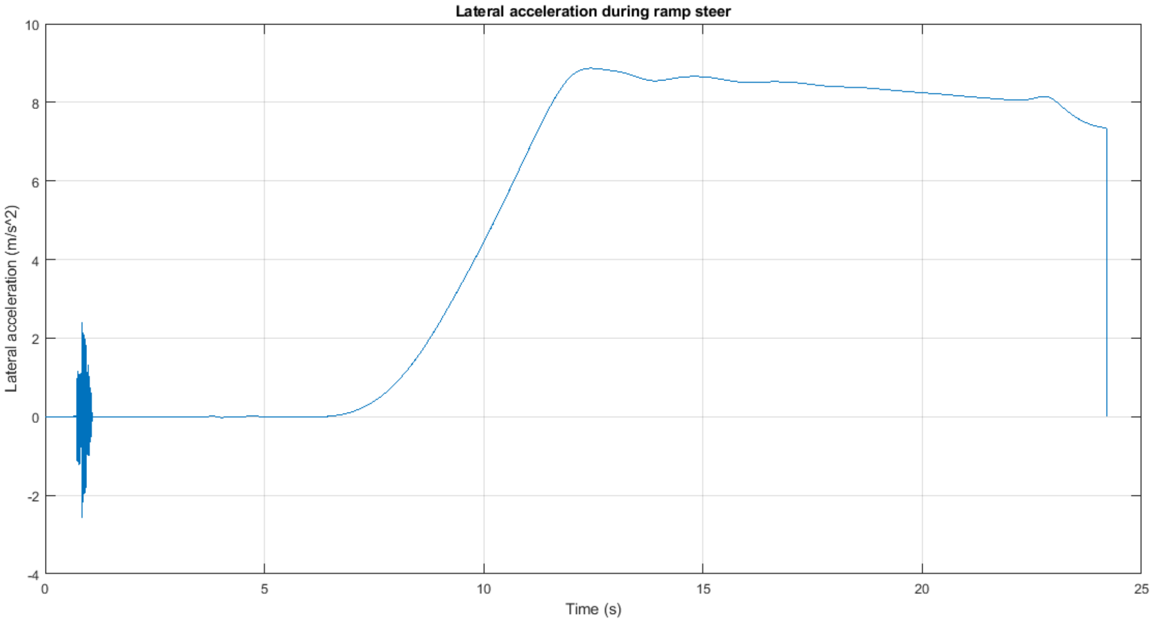


Figure 45 Speed variation during ramp steer



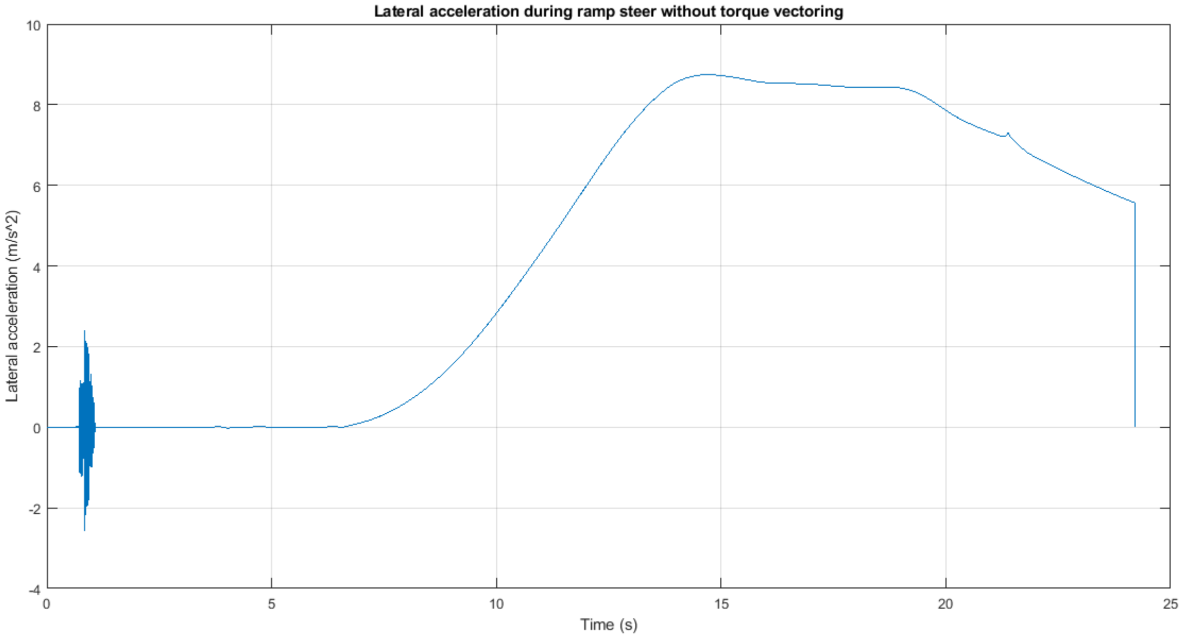
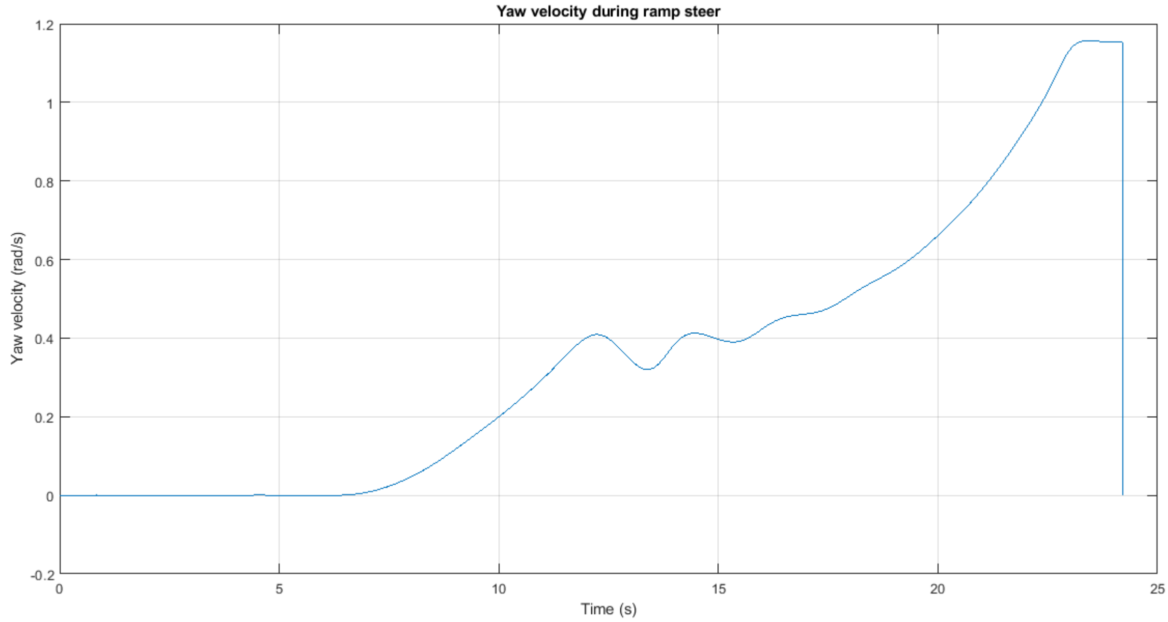


Figure 46 Lateral acceleration during ramp steer



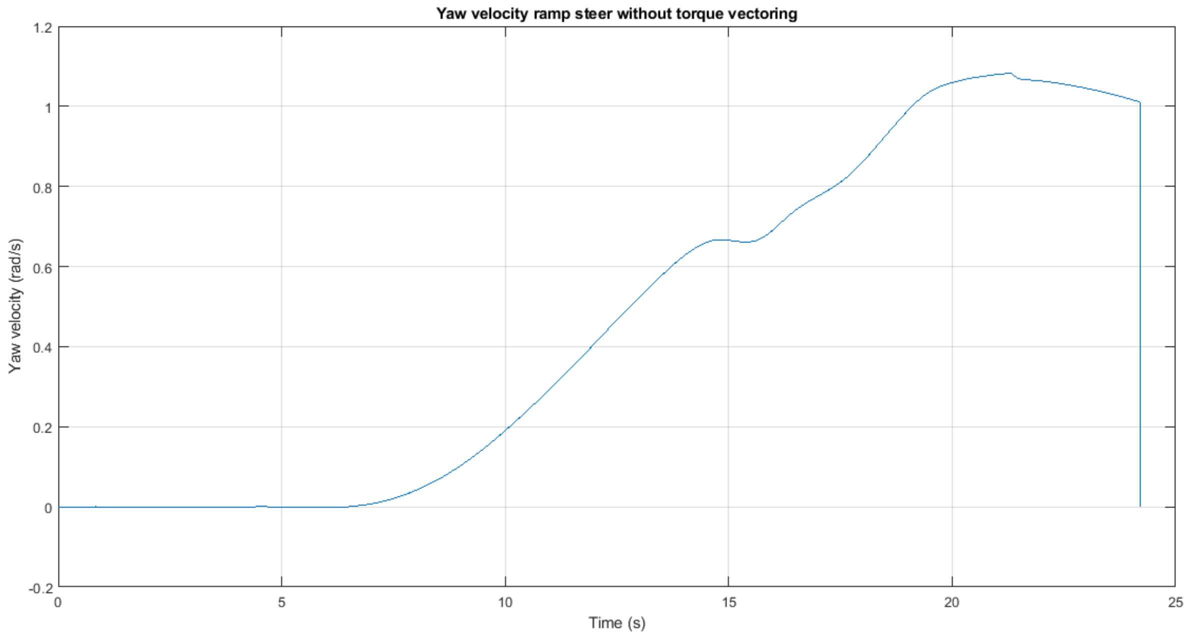
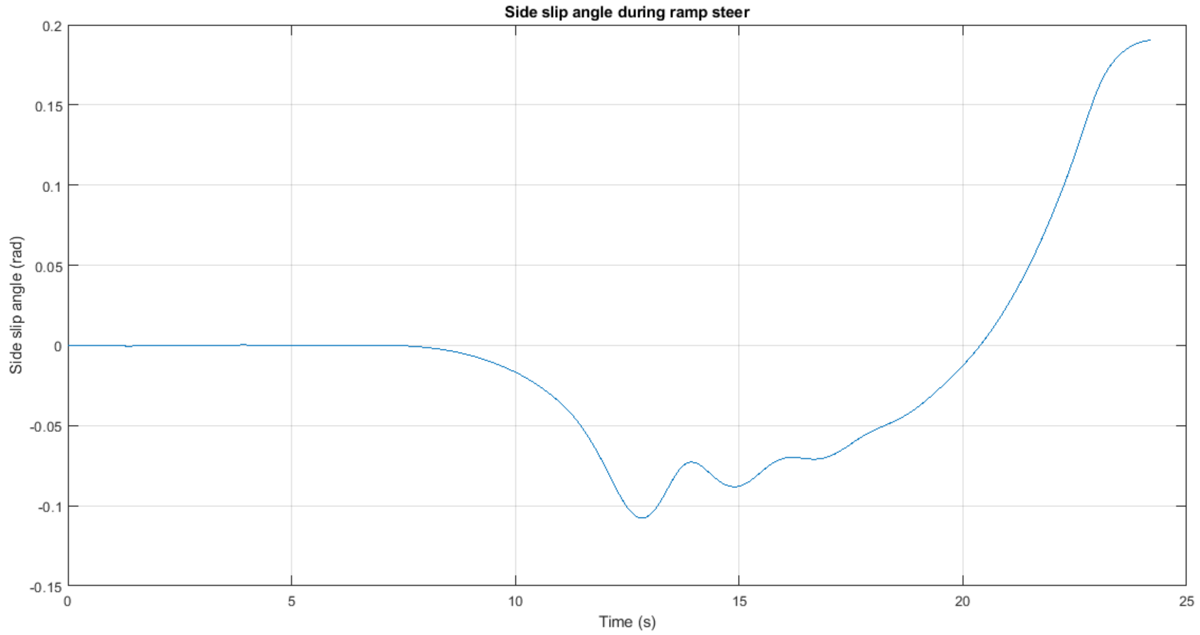


Figure 47 Yaw rate variation during ramp steer



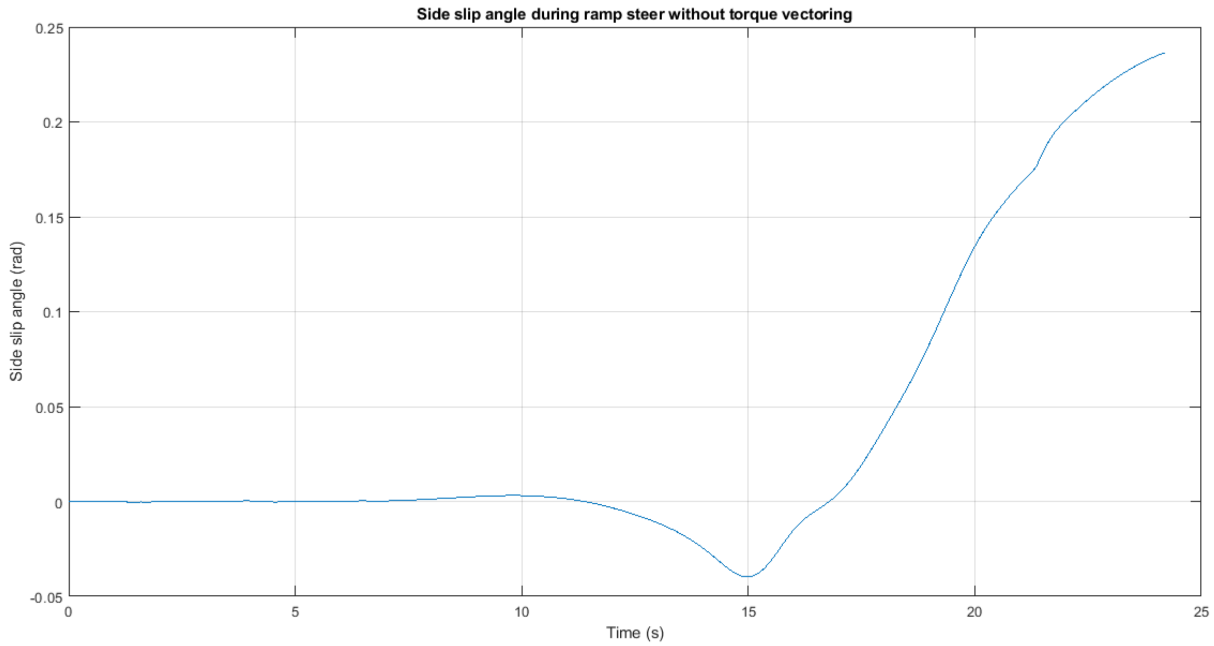
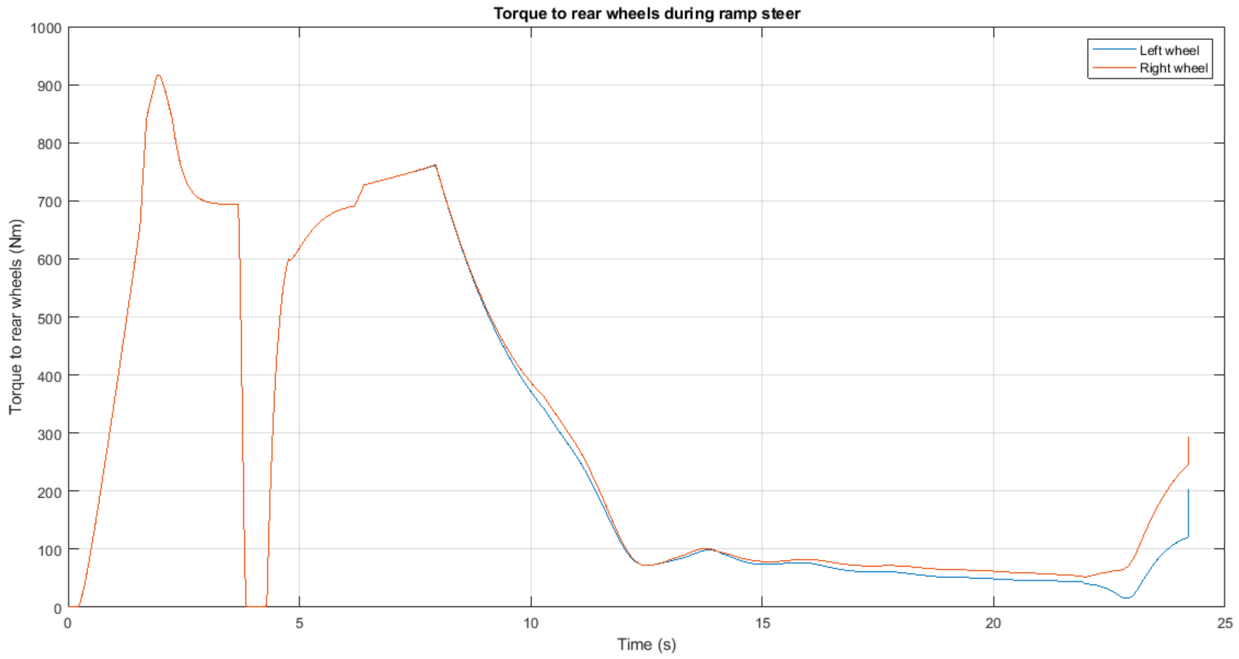


Figure 48 Side slip angle during ramp steer



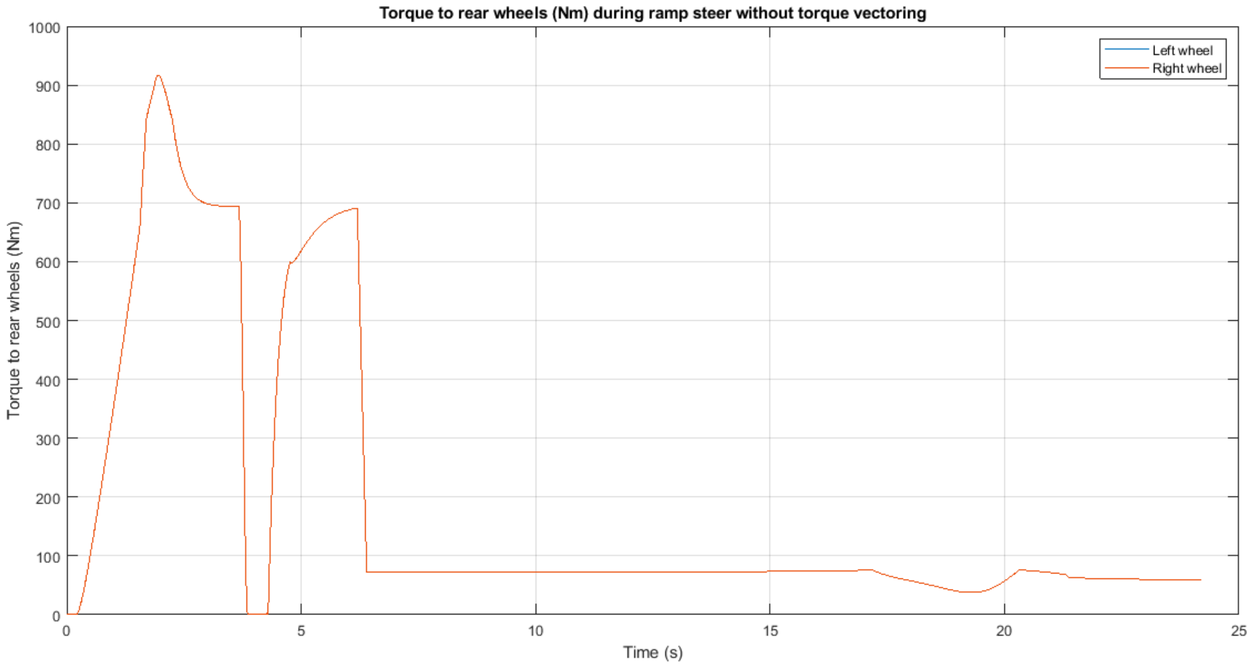


Figure 49 Torque to rear wheels during ramp steer

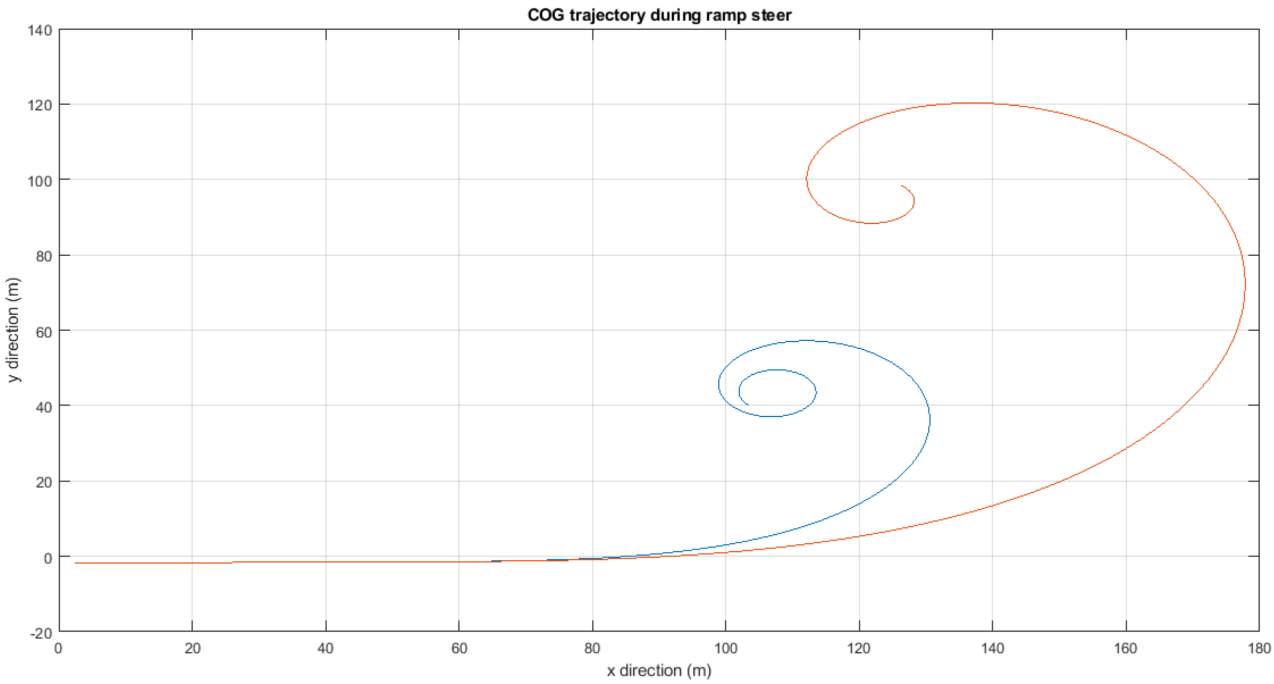


Figure 50 Centre of gravity position during ramp steer

6 CONCLUSION AND FUTURE PROSPECTS

Following a review of the graphs delivered it's clear the advantage of fitting kind of system in vehicle and many car manufacturer are developing car with analogous system, in particular to increase the performances of their vehicle.

Those system can be improved, with better and more complex differential, like the hybrid differential seen in this paper, that can are able to guarantee a better freedom degree in torque split, or improving the efficiency of the control system. There are some estimator for friction coefficient or cornering that are been developed and implemented in some car that could provide a more precise and rapid torque correction, there were made also some tests on real vehicle to replace ABS/ESP based safety system with special torque vectoring control, so we're sure that in the next year the technology progress will show us improvement in this direction.

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