NEW EQUATION FOR THE RESILIENCE OF THE CLAMPED PART IN BOLTED JOINT

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SUMMARY

I N T R O D U C T I O N

The main objective of this thesis is finding a method to evaluate the stiffness of the clamped part analysing the result of simulations with finite elements. With the data provided by the samples, an equation is then extrapolated which describes the resilience of the clamped part.

To reach the final result, simulations have been executed on the program "Abaqus CAE", and the data obtained are analysed with the MatLab tool "Curve Fitting" .

In this thesis only metric bolt and threads with ISO standard dimensions are considered, at which a part in the first chapter is dedicated. Also in this chapter, the theory regarding the torque used to tighten the bolted connection and the calculation of the resilience of both the bolt and the clamped part is exposed. Always referencing to the German normative VDI.

The second chapter introduces the program ABAQUS/CAE in detail, and shows the procedure to construct the models samples of the bolted connections.

The last chapter explains the model used to calculate the overall stiffness of the clamped part and illustrates the integration of the equation on the data.

The study is made only in the field of elastic deformation of the material, this hypothesis is a big simplification but, aside of this, it covers the main part of cases of bolted connections since generally they are meant to be assembled and disassembled, with the presence of plastic deformation this would be not possible.

Chapter I

THEORY OF BOLTED JOINTS

1.1. Generality of threaded connections

A bolted joint is a detachable connection between two or more parts by means of one or more bolts. It is intended to transmit forces and moments between the joined parts in a clearly defined position relative to one another. Bolt generates the load that closes the parts together, the bolt should be designed to withstand the working loads. Transverse actions must be contrasted by friction, in many bolt connections and screw must not work in shear loading.

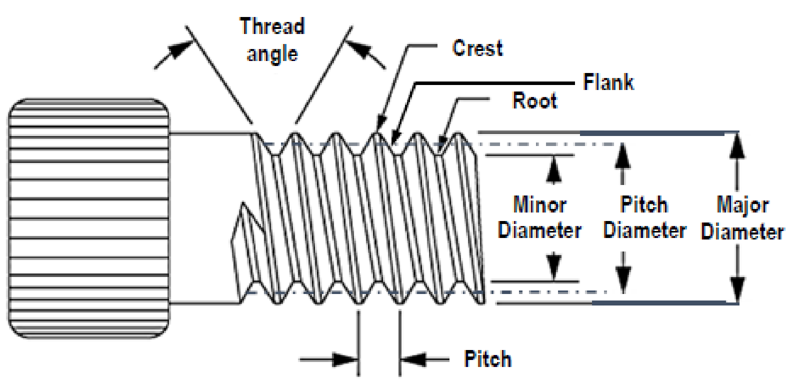


Fig. 1. *Bolt geometry*

A helicoidal thread used in screws or bolts is characterized by a helicoidal ridge wrapped around a cylinder. A sketch with the terminology of a screw threads is shown in Figure 1.

The pitch is the axial distance between two crest and the pitch diameter is the diameter at which each pitch is equally divided between the mating male and female threads Figure 2.

The inclination of the helix is the angle formed between a tangent to the helix with a perpendicular plane in respect to the axis of the bolt as shown in equation (1).

$$\alpha = \operatorname{atan}\frac{p}{\pi d} \qquad (1)$$

Generally this angle is lower than the friction angle of 6°-7°. The major diameter takes the name of nominal diameter; for every nominal diameter, exists only one gross thread and a small number of fine threads for special purpose.

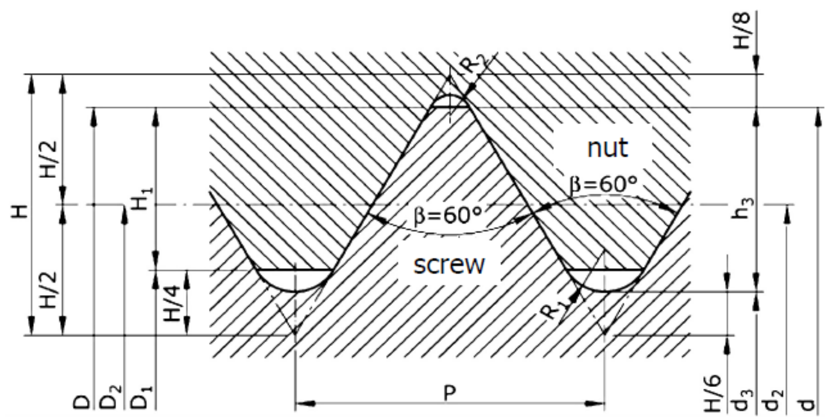


Fig. 2. *Threat parameters*

Figure 2 shows the thread coupling between screw and nut whose dimensional parameters are expressed in function of the pitch showed in the equation below (2).

$$\begin{gathered}
H = 0{,}86603 \cdot p \\
H_1 = \frac{5}{8} \cdot H = 0{,}54127 \cdot p \\
h_3 = \frac{17}{24} \cdot H = 0{,}61343 \cdot p \\
d_1 = D_1 = d - 2 \cdot H_1 = d - 1{,}08253 \cdot p \\
d_2 = D_2 = d - \frac{3}{4} \cdot H = d - 0{,}64952 \cdot p \\
d_3 = d - 2 \cdot h_3 = d - 1{,}22687 \cdot p \\
r = \frac{H}{6} = 0{,}14434 \cdot p \qquad (2)
\end{gathered}$$

1.2. Introduction of clamping

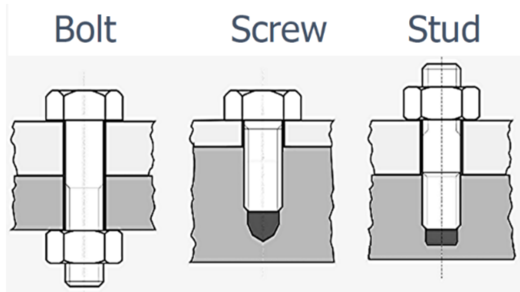


Fig. 3. *Types of connections*

The definition of a screw is an externally threaded headed fastener (the recessed drive socket in a setscrew is considered a head), which is tightened by applying torque to the head, causing it to be threaded into the material it will hold.

On the other hand, a bolt is an externally threaded headed fastener which is used in conjunction with a nut. Studs have a drive slot in one end to facilitate installing the stud, i.e. studs are not completely thread. In this study we focus on bolt threaded connection.

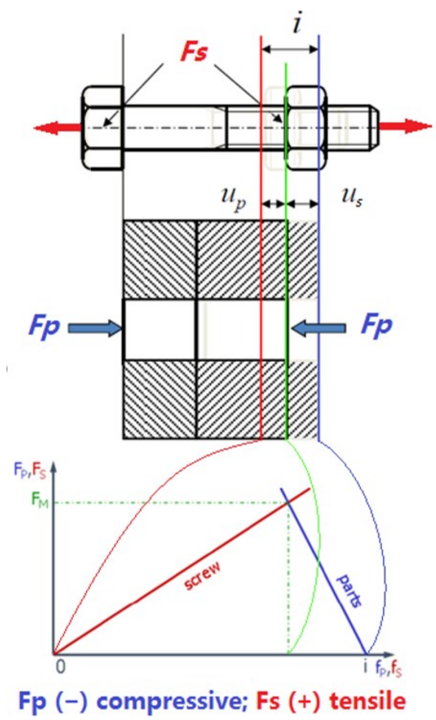


Fig. 4. *clamping of the bolt*

The clamping force produces tension in the bolt and that causes compression in the members that are shortened.

In figure 4, blue line is presented as the position of the bolt head and the nut first contacting with parts, at this moment bolt head and the nut just touched the two end surfaces of the clamped parts with zero load. Red line is presented as the position without load, if the parts to be clamped were not present, the inner head-to-nut distance would decrease of an amount "i" (interference). Green line is presented as the final equilibrium position reached by the nut, in this case the bolt head is obviously fixed, meanwhile, the clamped parts are compressed.

The bolt and the clamped part could be considered as a simple mechanical spring model, the sketch is shown as Figure 5.

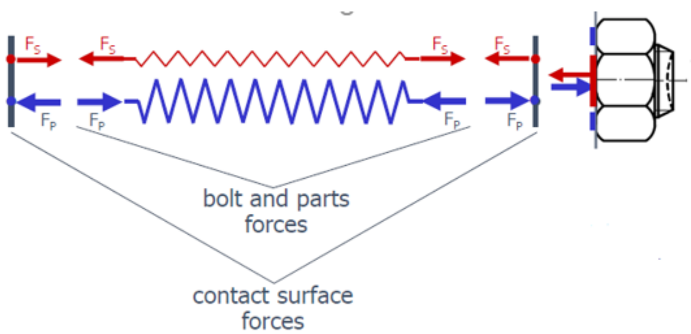


Fig. 5. *Spring model*

As shown in figure 6, bolt and part are displacing under load, *fs* and *fp* are measured from an initial position without clamping load; for the bolt this is the bolt head to nut face distance in the tightened configuration but with the parts removed; for the parts it is the position of the part with no load.
The resultant force on the bolt section, at the ends and through the length, is tensile Fs. The resultant force on the parts, at the ends and through the length, is a compress force Fp.
Due to Fs=Fp, at assembly they are called Fm.

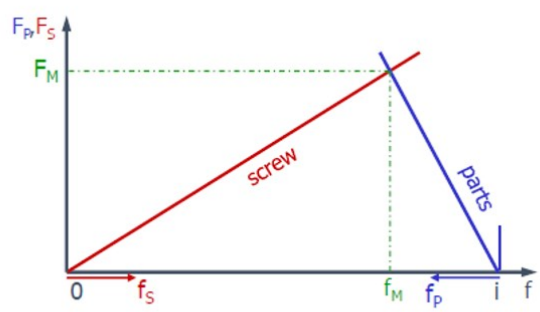


Fig. 6. *Clamping diagram*

1.3. Mechanical analysis of clamping

In order to compute the clamping torque, firstly, the bolted connection could be considered as a trapezoidal part sliding on edge of a wedged part, the trapezoid represents the part and the wedge the nut, showed in the figure 7.

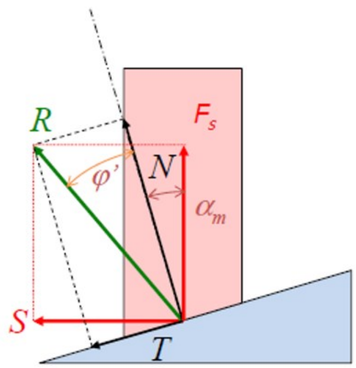


Fig. 7. *Trapezoidal model*

- F_s is the tensile assembly force in the bolt shaft
- (3) $tan\varphi' = \frac{tan\varphi}{cos\beta}$ is the fictious thread coefficient friction angle incremented taking into account the thread angle β
- α_m is the helix angle $\alpha_m = \text{atan}\frac{p}{\pi d}$ (4)
- S is the circumferential component $S = F_s * \tan(\varphi' + \alpha_m)$ (5)

The clamping torque M_t is composed of 2 contributes (6):

$$M_t = M_1 + M_2 \qquad (6)$$

Where torque due to the friction between the bolt's thread and nut's thread (7):

$$M_1 = \frac{F_s}{2} * d_m * \tan(\alpha_m + \varphi') \qquad (7)$$

Friction moment between the head of the bolt and the clamped part

$$M_2 = \frac{F_s}{2} * d_t * \tan \varphi_s \qquad (8)$$

Substituting (7) and (8) into the equation (6), (8) is obtained.

$$M_T = \frac{F_s}{2}\left[d_m \tan(\alpha_m + \varphi') + d_t \tan \varphi_s\right] \qquad (9)$$

We can approximate:

$$\tan(\alpha_m + \varphi') \approx \tan \alpha_m + \tan \varphi' \qquad (10)$$

Substituting the equation (10) in equation (9) we obtain the final equation (11):

$$M_T \approx \frac{F_s}{2}\left[d_m \tan \alpha_m + d_m \tan \varphi' + d_t \tan \varphi_s\right] \qquad (11)$$

In order to verify statically the resistance of the bolt, given clamping torque M_t the bolt body see an axial load F_s and a torque M_1 since M_2 regard only the zone under the head bolt and doesn't have effects in the shank.

To calculate the stresses we will refer to the minor section of the bolt with diameter d_3.

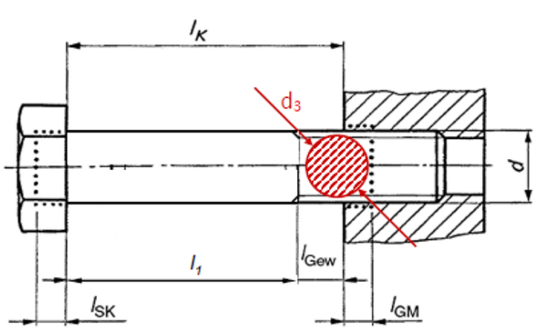


Fig. 8. *Bolt dimensions*

The resistant area is:

$$A_3 = \frac{\pi {d_3}^2}{4} \qquad (12)$$

The minimum polar resistant module is:

$$W_p = \frac{\pi {d_3}^2}{16} \qquad (13)$$

The axial stress will be so:

$$\sigma_a = \frac{4 \cdot F_s}{\pi \cdot {d_3}^2} \qquad (14)$$

While the torsion stress:

$$\tau = \frac{16 {M_T}^*}{\pi \cdot {d_3}^3} \qquad (15)$$

The Von Mises equivalent stress is then calculated in (16):

$$\sigma_{eq} = \sqrt{\sigma_a^2 + 3\tau^2} = \sigma_a \sqrt{1 + 3\left(\frac{\tau}{\sigma_a}\right)^2} \qquad (16)$$

Now the quotient $\frac{\tau}{\sigma_a}$ is called k:

$$\frac{\tau}{\sigma_a} = \frac{16 {M_T}^*}{\pi \cdot {d_3}^3} \cdot \frac{\pi \cdot d_3^2}{4F_s} = \frac{4}{d_3} \frac{{M_T}^*}{F_s} = \frac{4}{d_3} \frac{1}{2} \left(\frac{p}{\pi} + d_m \frac{\tan \varphi}{\cos \alpha} \right) = \frac{2}{d_3} \left(\frac{p}{\pi} + d_m \frac{\tan \varphi}{\cos \alpha} \right) \qquad (17)$$

Substituting the equation (13) into the equation (12) we obtain (18):

$$\sigma_{eq} = \sigma_a \sqrt{1 + 3\left[\frac{2}{d_3}\left(\frac{p}{\pi} + d_m \frac{\tan\varphi}{\cos\alpha}\right)\right]^2} = \sigma_a \sqrt{1 + 3 \cdot k^2} \qquad (18)$$

For the purpose of utilizing the bolt to the greatest possible extent, the nominal stress of the bolt at assembly is 90% of minimum guarantee $R_{p0.2}$

$$\sigma_{eq.\lim} = 0.9 \cdot R_{p0.2} \Rightarrow \sigma_{a,\lim} = \frac{0.9 \cdot R_{p0.2}}{\sqrt{1 + 3k^2}} \qquad (19)$$

1.4. Joint resilience

The bolt consists of a number of individual elements which can be considered cylindrical bodies of length l_i and cross section A_i , having resilience (20):

$$\delta_i = \frac{l_i}{EA_i} \qquad (20)$$

The cylindrical elements are arranged in series, the total elastic resilience δ_s is the sum of all resiliences within the clamp length

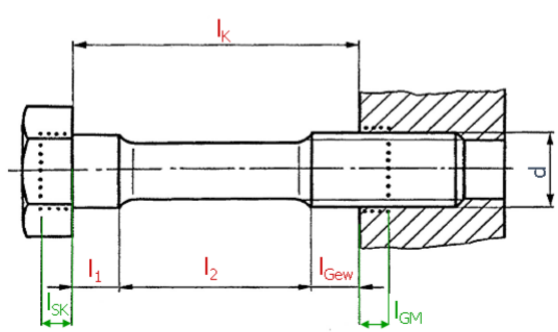


Fig. 9. *Subdivision of the bolt in elements*

Stresses in the clamped part affect a volume which widens moving away from the bolt head or nut. Calculation of part resilience is done on a "substitution" cone (Fig.10) having the same resilience, where:

- D_A is the part diameter;
- d_w is the contact bearing surface of the head of the bolt;
- l_k is the length of the clamped part;
- d_h is the hole diameter;
- $D_A{}'$ the maximum diameter of the cone of resilience;

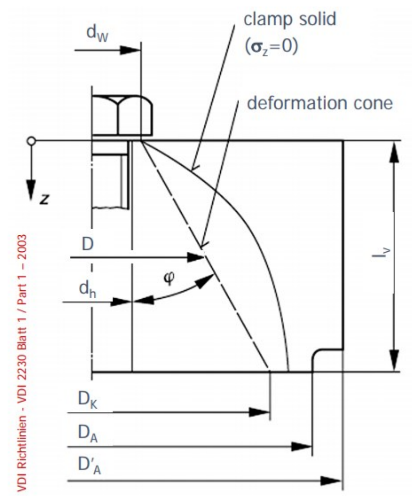
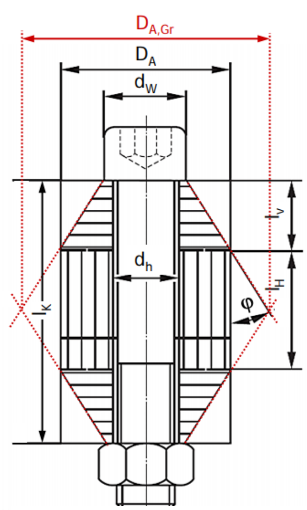


Fig. 10. *Substitution cone*

The equation of resilience vary depending on the type of connection. For a simple symmetric bolted joint with a nut like the one of (Fig. 11).



VDI ***Fig. 11****. Example of bolted joint*

If $D_A > D_{A,Gr}$:

$$\delta_p = \frac{1}{\pi \cdot E_p \cdot d_h \tan\varphi}\left[\ln\left(\frac{(d_w + d_h)\cdot(d_w + 2l_v \tan\varphi - d_h)}{(d_w - d_h)\cdot(d_w + 2l_v \tan\varphi + d_h)}\right)\right] \qquad (21)$$

If $dw < D_A < D_{A,Gr}$:

$$\delta_p = \frac{\frac{2}{d_h \tan\varphi}\ln\left(\frac{(d_w + d_h)\cdot(D_A - d_h)}{(d_w - d_h)\cdot(D_A + d_h)}\right) + \frac{4}{D_A^2 - d_h^2}\left[l_k - \frac{(D_A - d_w)}{\tan\varphi}\right]}{\pi \cdot E_p} \qquad (22)$$

In the case of tapped thread joint, to simplify the calculation of the plate resilience, the top cone and the bottom truncated cone are replaced by one substitution deformation cone of the same resilience, which can be followed by a sleeve.

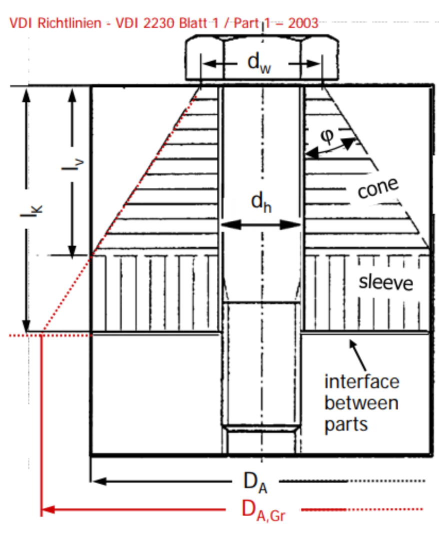


Fig. 12. *Example of taped threadt joint*

If $D_A > D_{A,Gr}$:

$$\delta_p = \frac{1}{\pi \cdot E_p \cdot d_h \tan\varphi}\left[\ln\left(\frac{(d_w + d_h)\cdot(d_w + 2l_v \tan\varphi - d_h)}{(d_w - d_h)\cdot(d_w + 2l_v \tan\varphi + d_h)}\right)\right] \tag{23}$$

If $dw < D_A < D_{A,Gr}$:

$$\delta_p = \frac{1}{\pi E_p} * \left\{\frac{1}{d_h tan\varphi}\ln(\frac{(d_w + d_h)(D_A - d_h)}{(d_w -)(D_A + d_h)} + \frac{4}{{D_A}^2 - {d_h}^2}[\, l_k - \frac{(D_A - d_w)}{2tan\varphi}]\right\} \tag{24}$$

C h a p t e r   I I

ABAQUS/CAE SIMULATION

2.1. Introduction of finite elements and Abaqus/CAE

The finite element analysis applies to each body that can be divided in a large number of elements of defined shape, every element then is considered as a field of integration of homogenic characteristics.

The principal characteristic of the finite elements method is to discretise creating a grid (called mesh) composed by elements of definite shape, on every elements the solution of the problem is assumed to be expressed by the linear combination of function called “base function” or “shape function”.

Nowadays there is a large variety of F.E.M analysis software, every software share the same 3 steps analysis:

- The pre-processing where the finite elements model is built
- The processing : resolution of the finite elements problem model
- The post-processing where the solution is presented

Some of the more common commercial model are: Femap, Ansys and Abaqus.

Abaqus can analyse complex structural mechanics system, in particular it is able to solve very large and complex problems and simulate highly nonlinear problems. We will utilise it for our analysis.

2.2. The model of bolted connection

In this thesis, we analyse a tapped thread bolted connection as the one in (Fig. 13).

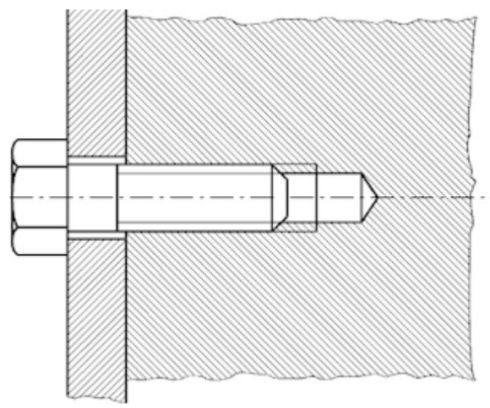


***Fig. 13**: example of tapped thread joint*

The profile of the bolted connection is assumed to be a symmetrical shape, therefore in this case we create a 2-dimensional axisymmetric model. In order to apply the load we separate the bolt into two parts: bolt head and shank.

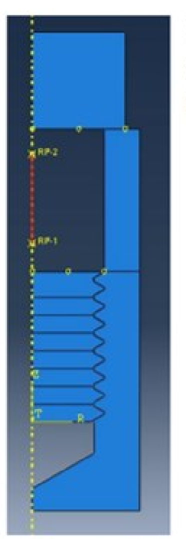


***Fig. 14**: assembly of the bolted joint*

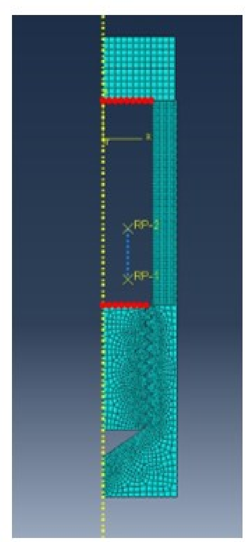


***Fig. 15**: assembly of the bolted joint in module mesh*

Selecting steel with class of resistance 8.8 as the material of the bolt we can insert elastic characteristic (Young modulus and coefficient of Poisson).

Reference point RP1 is constrained to the lower nodes of the head of the bolt while point RP2 is constrained to the upper nodes of the shank of the bolt (Fig. 15) , then we can tie RP1 to RP2 by using the command "Wire" in order to consider the deformability of the shank that we deleted.

The module "Constraints" allows to regard the shank between RP1 and RP2 as a spring with an elastic coefficient equal to the stiffness of the bolt shank (25):

$$k = \frac{EA}{l} \qquad (25)$$

Then a compressive load is applied on RP1 and RP2 to simulate the force due to the assembly preload

$$F_{s,\lim} = \sigma_{\lim} \cdot A_n = \frac{0.9 \cdot R_{p0.2}}{\sqrt{1+3 \cdot k^2}} \cdot \frac{\pi \cdot d_3^2}{4} \qquad (26)$$

We define the type of interaction between the surfaces, we select "tangential behaviour" and we define a friction coefficient in interaction properties of 0,12 as recommended on VDI in steel to steel contact in bolted joints. The zones that interacts are underlined in red.

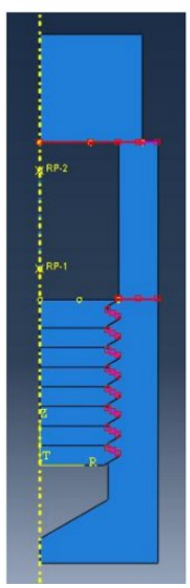


***Fig. 16**: interactions in the model*

Then the boundary conditions are defined as in figure 17:

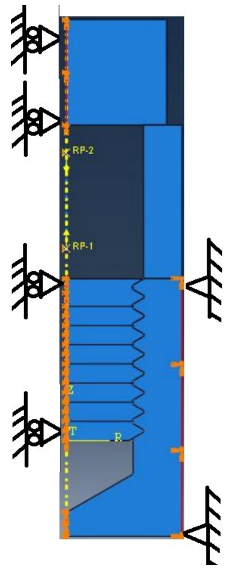


Fig. 17: *boundary conditions on the model*

We define the boundary condition "encastre" U1=U3=U2=UR1=UR2=0 on the right side of the nut, in order to represent the stiffness contribution of the material not displayed in the model, in which is created the threaded hole.

We select for the screw, on the revolution axis, U1=U3=UR2=0 which allows to the screw to translate only along the y axis.

ANALYSIS OF RESULTS

3.1. Equation of stiffness

Is not enough to take the ΔY between two nodes and the total force that go through the screw $F_{s,lim}$ and calculate the stiffness (27):

$$K = \frac{F_{s,lim}}{\Delta Y} \qquad (27)$$

This procedure is not scientific, because implicates to know already the stiffness, calculated with other theories or by experiments, and taking it as a reference to select the most representative couple of nodes, as made in precedent studies on this argument.

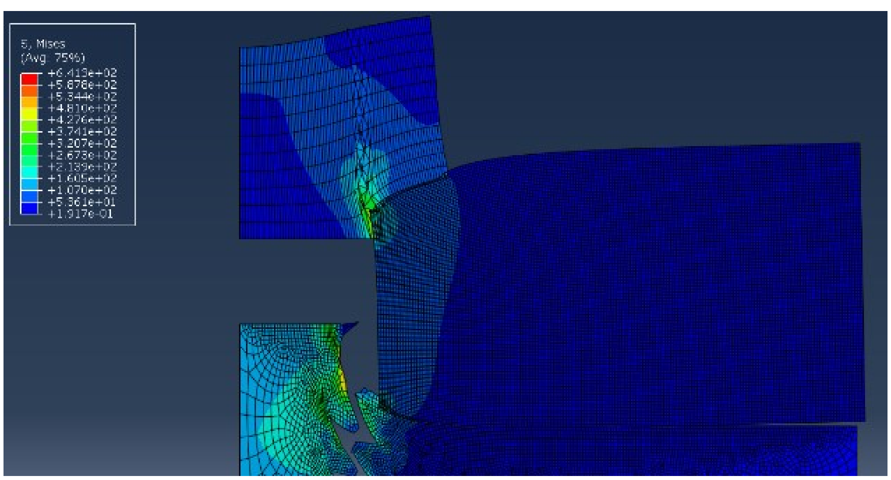


***Fig. 18**: f.e.m result for one of the models*

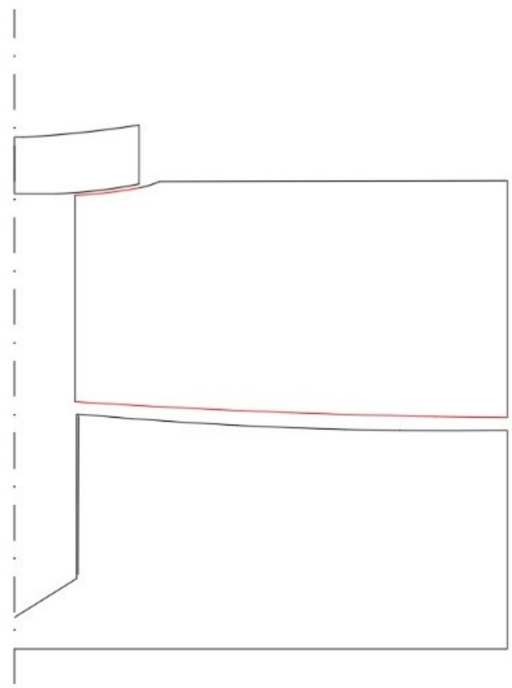


***Fig. 19**: surfaces in contact*

In (Fig. 19) it is possible to see in red the surfaces where the clamped part is in contact respectively with the nut and the bolt head.

The model adopted consist in taking these two red surfaces and delete all the material between them.

The two surfaces are considered as a thin sheet of metal at which are attached springs. From the upper surface we take only the part in contact with the head of the bolt because that is what effectively see the bolt and what interest to us.

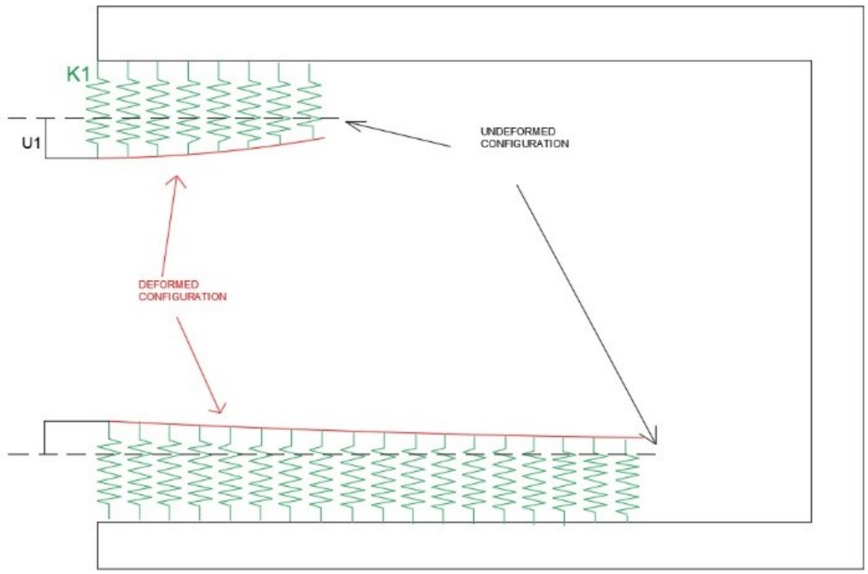


***Fig. 20**: spring model*

In this way it can be possible to simulate the support of the material that we have eliminated, but in a more coherent way with the end result of an equivalent spring. (Fig. 20)

There is one spring for every node of the mesh of the two surfaces.

The stiffness of every spring is calculated as the vertical force on the node i F_i divided by the vertical absolute displacement of the node U_i (28):

$$K_i = \frac{F_i}{U_{2i}} \qquad (28)$$

In Abaqus mode “visualization” we choose, with the command “select path”, the nodes that correspond to the two surfaces seen in Figure 18 and the vertical stress σ_{yi} and strain U_{yi} of the node are sent to an excel sheet.

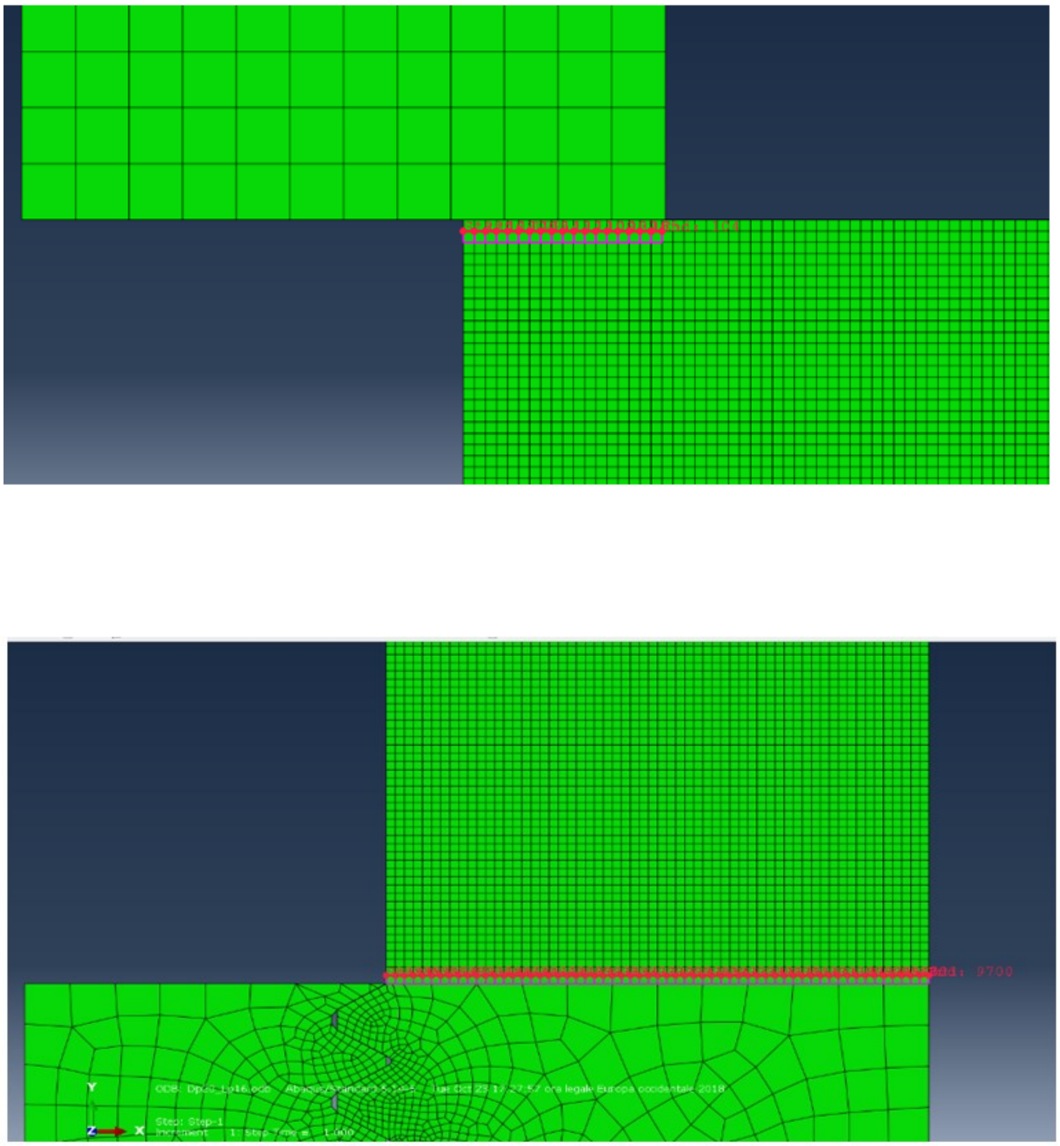


***Fig. 21**: nodes selected for the analysis*

The force on each single node is calculated (29):

$$F_i = \sigma_{yi} * A_i \qquad (29)$$

where A_i (figure 20) are the ring delimited by two nodes on the surfaces (30) (distance between nodes= 0.1 mm since that is the mesh increment)

$$A_i = 2\,\pi\,r\,0.1 \qquad (30)$$

These surfaces in Figure 21 are seen as a simple segment since we analyse the model as an axisymmetric 2D. Figure 22 is a saw from top plane.

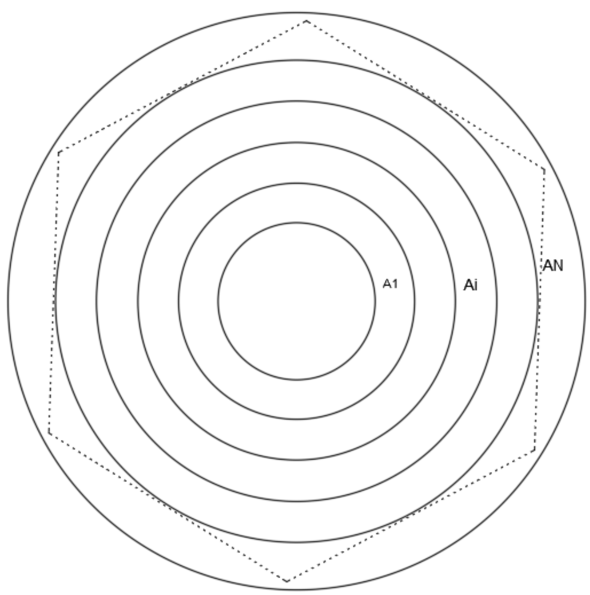


***Fig. 22**: Areas between the nodes upper surface of the part in contact with the hexagonal head of the bolt*

A $K_{eq\ sup}$ and a $K_{eq\ inf}$ are calculated simply summing respectively the upper and lower springs.

Keq sup
1630594

d	σi	Ui	r	Ai	Fi	Ki
0	-633,057	-0,01596	5	3,141593	-1988,81	124643
0,099856	-563,339	-0,01591	5,1	3,204425	-1805,18	113441
0,199463	-416,413	-0,01574	5,2	3,267256	-1360,53	86430,38
0,299096	-315,517	-0,0155	5,3	3,330088	-1050,7	67772,85
0,398696	-269,773	-0,01535	5,4	3,39292	-915,32	59624,43
0,498315	-235,025	-0,01515	5,5	3,455752	-812,189	53603,33
0,59792	-211,249	-0,01501	5,6	3,518584	-743,297	49504,08
0,697531	-193,561	-0,01484	5,7	3,581416	-693,224	46699,23
0,797143	-180,287	-0,01472	5,8	3,644247	-657,012	44630,74
0,896751	-169,643	-0,01457	5,9	3,707079	-628,879	43152,06
0,996364	-161,527	-0,01446	6	3,769911	-608,941	42113,4
1,095973	-154,741	-0,01433	6,1	3,832743	-593,082	41385,88
1,195586	-149,537	-0,01422	6,2	3,895575	-582,534	40956,6
1,295195	-145,183	-0,01411	6,3	3,958407	-574,693	40732,99
1,394808	-141,882	-0,01401	6,4	4,021239	-570,54	40733,81
1,494416	-139,215	-0,0139	6,5	4,08407	-568,564	40893,91
1,594029	-137,358	-0,0138	6,6	4,146902	-569,61	41261,18
1,693635	-136,017	-0,01371	6,7	4,209734	-572,597	41763,32
1,793248	-135,447	-0,01361	6,8	4,272566	-578,705	42506,08
1,892853	-135,361	-0,01353	6,9	4,335398	-586,845	43383,33
1,992465	-136,159	-0,01343	7	4,39823	-598,861	44585,23
2,092067	-137,637	-0,01335	7,1	4,461062	-614,008	45993,61
2,191678	-140,258	-0,01325	7,2	4,523893	-634,512	47879,19
2,291275	-144,452	-0,01318	7,3	4,586725	-662,563	50284,49
2,390885	-150,673	-0,01307	7,4	4,649557	-700,564	53598,41
2,490473	-160,739	-0,013	7,5	4,712389	-757,463	58257,46
2,590082	-180,367	-0,01288	7,6	4,775221	-861,29	66893,16
2,689653	-220,383	-0,01282	7,7	4,838053	-1066,23	83185,13
2,789286	-191,817	-0,01259	7,8	4,900885	-940,074	74686,13

***Fig. 23**: calculation of Keq sup for a model*

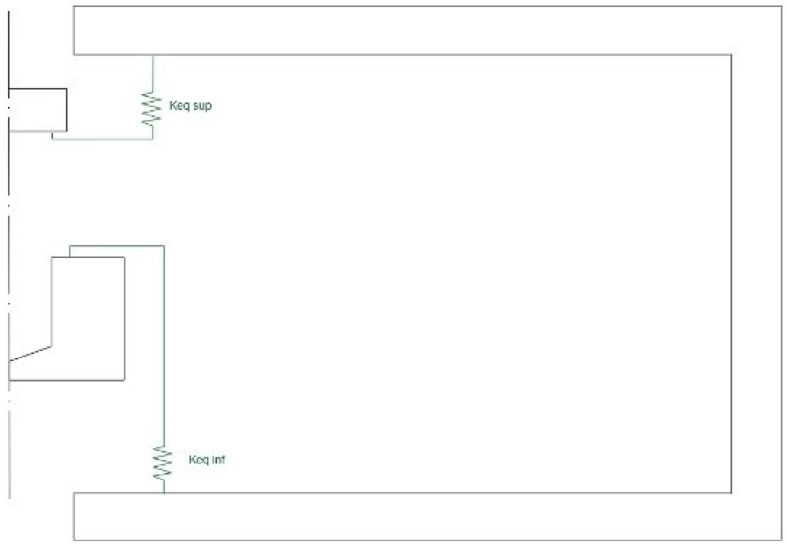


Fig. 24*: upper and lower equivalent springs*

The two springs in Figure 24 can be seen as in series since they see the same force on their ends maintaining the frame in equilibrium, the system can be simplified into the one of Figure 25.

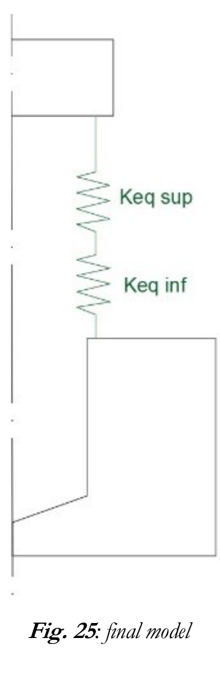


Fig. 25*: final model*

A K_{toteq} is calculated (31):

$$K_{toteq} = \frac{K_{eq\ sup} * K_{eq\ inf}}{K_{eq\ sup} + K_{eq\ inf}} \qquad (31)$$

The sum of all the vertical forces on the nodes $F_{tot} = \sum F_i$ will be less than the vertical force inside the screw $F_{s,lim}$ seen in equation 20, this is because of the horizontal components along X S11 which are equilibrated by tangential stress inside the clamped part (Figure 26).

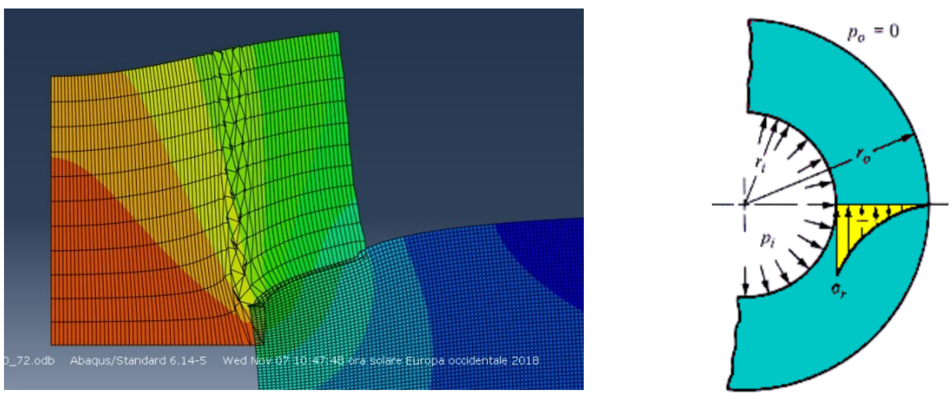


Fig. 26. *Radial and tangential stress inside the part*

In the calculation exposed until now we have only considered vertical stresses without taking into account the components along X and Z, the K_{toteq} needs to be incremented in order to utilise it with the $F_{s,lim}$ (32)

$$K_{tot\ corrected} = \frac{F_{s,lim}}{F_{tot}} K_{toteq} \qquad (32)$$

Then the resilience of the part is calculated (33):

$$\delta_p = \frac{1}{K_{tot\ corrected}} \qquad (33)$$

The theory says that the resilience depend on the quotients $\frac{L_k}{d_h}$ (where L_k is the thickness of the piece and d_h is the diameter of the hole in it equal to the class of the bolt) and $\frac{D_A}{d_w}$ (where D_A is the diameter of the piece and d_w the diameter of the bearing surface of the head of the bolt)

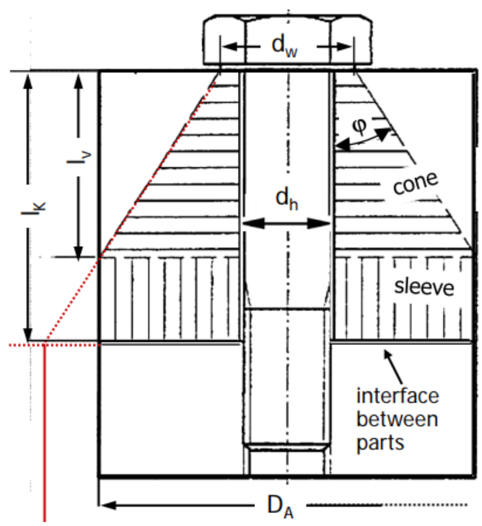


Fig. 27: *dimensions of the bolted joint*

The analysis has been done on 144 models of bolted joint with:

- quotients $\frac{L_k}{d_h}$ ranging from 1 to 12 and $\frac{D_A}{d_w}$ from 0,8 to 3,2
- bolts from M8 to M14

The results of the simulation on Abaqus are quite similar to the ones calculated utilising the German normative VDI Richtlinien .

δP simulation M8							
	0,88285	1,01868	1,69779	2,37691	2,64856	3,05603	DA/dw
1	9,65E-07	6,2E-07	3,2E-07	3,1E-07	3,1E-07	3E-07	
2	1,6E-06	1,2E-06	6,4E-07	4E-07	3,8E-07	3,6E-07	
4	3,8E-06	2,2E-06	7,5E-07	5,2E-07	4,8E-07	4,5E-07	
6	5,7E-06	3,1E-06	1E-06	6,5E-07	5,9E-07	5,2E-07	
8	7,6E-06	4,5E-06	1,3E-06	7,8E-07	7,7E-07	6E-07	
12	1,1E-05	6,7E-06	1,9E-06	1E-06	9E-07	7,6E-07	
Lk/dh							

***Fig. 28**: values of resilience of the simulation*

δP VDI Richtlinien M8							
	0,88285	1,01868	1,69779	2,37691	2,64856	3,05603	DA/dw
1	3,8E-07	3,7E-07	3,4E-07	3,2E-07	3,2E-07	3,1E-07	
2	5,3E-07	5,1E-07	4,5E-07	4,2E-07	4,1E-07	4E-07	
4	4,4E-06	2,4E-06	5,3E-07	4,9E-07	4,8E-07	4,6E-07	
6	6,6E-06	3,6E-06	1E-06	5,2E-07	5E-07	4,9E-07	
8	8,8E-06	4,9E-06	1,3E-06	8E-07	7,1E-07	6,2E-07	
12	1,3E-05	7,3E-06	1,9E-06	1,1E-06	9,1E-07	7,7E-07	
Lk/dh							

***Fig. 29**: values of resilience calculated utilising VDI Richtlinien*

Calculating the % difference we can see that the resilience calculated on Abaqus/CAE differ considerably from the VDI ones only for low values of $\frac{L_k}{d_h}$ and $\frac{D_A}{d_w}$:

%Difference VDI-simulation							
	0,88285	1,019	1,69779	2,37691	2,64856	3,05603	DA/dw
1	1,51	0,66	0,05	0,02	0,03	0,02	
2	2,08	1,32	0,42	0,06	0,06	0,09	
4	0,14	0,11	0,40	0,06	0,01	0,03	
6	0,14	0,14	0,02	0,26	0,16	0,08	
8	0,14	0,07	0,02	0,02	0,09	0,03	
12	0,14	0,08	0,03	0,02	0,02	0,02	
Lk/dh							

***Fig. 30**: % difference model-VDI*

Now, in order to obtain the equation of resilience of the piece the Matlab tool "curve fitting" is used for the simulation:

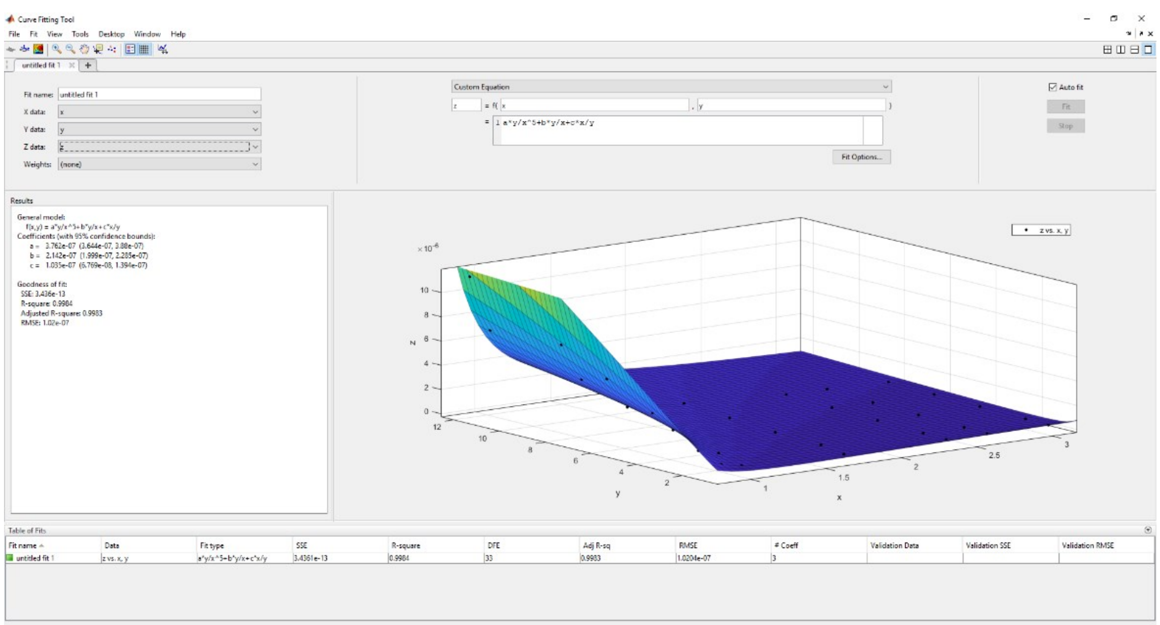


***Fig. 31**: Matlab curve fit interface*

We first fit the equation of resilience on the data of the M8 clamped connections(Figure 31).

We call X the values of the quotient $\frac{D_A}{d_w}$,Y the values of $\frac{L_k}{d_h}$ and Z the corresponding values of resilience.

Using the fitting mode "custom equation" we find, after many attempts, an equation that fit our data pretty well (34)

$$\delta_p = 10^{-8} * \left(37{,}62 * \frac{\frac{L_k}{d_h}}{(\frac{D_A}{d_w})^5} + 21{,}42 * \frac{\frac{L_k}{d_h}}{\frac{D_A}{d_w}} + 10{,}35 * \frac{\frac{D_A}{d_w}}{\frac{L_k}{d_h}} \right) \qquad (34)$$

This equation gives a good approximation of the data since its R^2=0,9984 (R^2 is a statistic that will give some information about the goodness of fit of a model. An R^2 of 1 indicates that the regression predictions perfectly fit the data) , while a polynomial approximation with X of grade 5 and Y of grade 5 gives only 0,9981.

The table below shows the %error between the results of the equation and the values calculated in Abaqus/CAE:

%Error equation-simulation							
	0,88285	1,018675	1,697792	2,37691	2,648556	3,056027	DA/dw
1	0,07	0,06	0,02	0,08	0,16	0,28	
2	0,19	0,02	0,39	0,21	0,21	0,17	
4	0,01	0,04	0,12	0,15	0,16	0,18	
6	0,00	0,07	0,08	0,06	0,06	0,08	
8	0,00	0,01	0,04	0,01	0,09	0,02	
12	0,00	0,01	0,00	0,12	0,15	0,17	
Lk/dh							

Fig. 32: *%Error between the values of resilience of the simulation and the ones of the equation*

Now we set the coefficient of $\frac{\frac{L_k}{d_h}}{(\frac{D_A}{d_w})^5}$ as a variable “a” and we fit the equation on the simulations with bolt diameter M10-M12-M14

- M10 : R^2=0,9758 ,a=2.193e-07
- M12: R^2=0,9911 ,a= 1.957e-07
- M14: R^2=0,985 ,a=1.875e-07

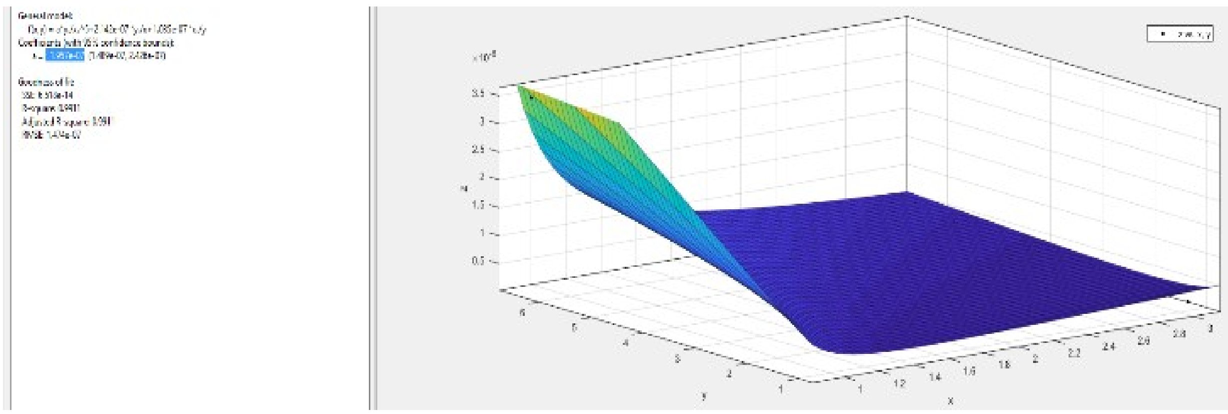


Fig. 33: *Equation fitted on the resiliences of M10*

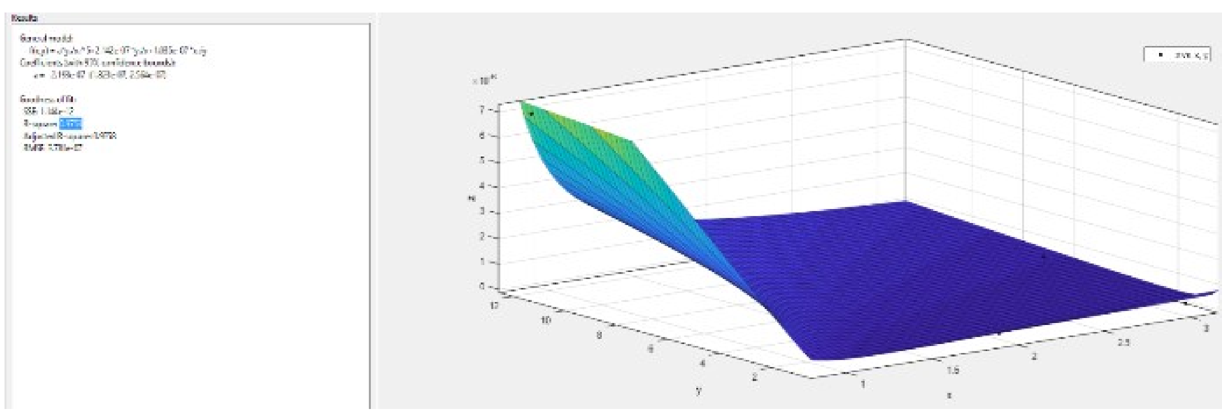


Fig. 34: *Equation fitted on the resiliences of M12*

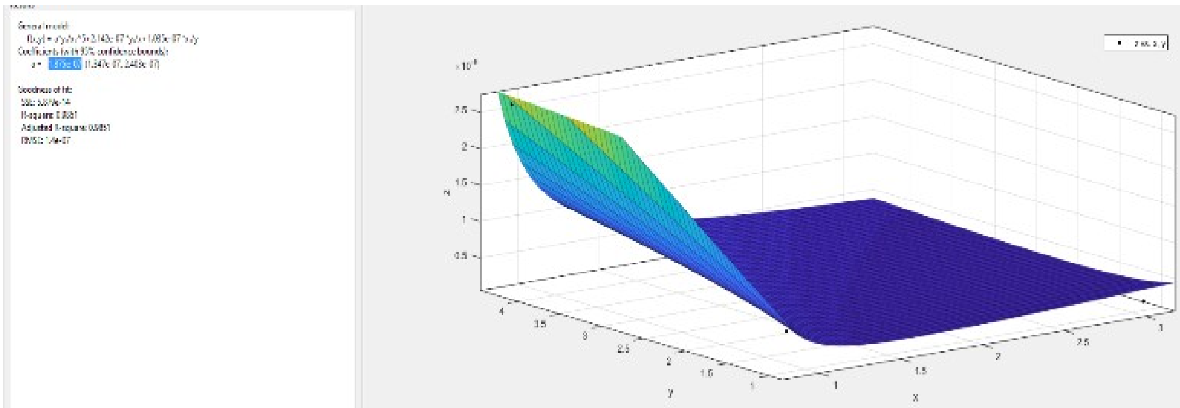


Fig. 35: *Equation fitted on the resiliences of M14*

Now it is possible to express the coefficient “a” as a function of the bearing surface diameter of the bolts d_w. In the toolbox “curve fit” we set coefficient “a” as Y and d_w as X (Fig. 36):

The dimensions of d_w for the bolts taken in consideration are:

- M8 : d_w=11.68mm
- M10 : d_w=15.6mm
- M12 : d_w=17.4mm
- M14 : d_w=20.5mm

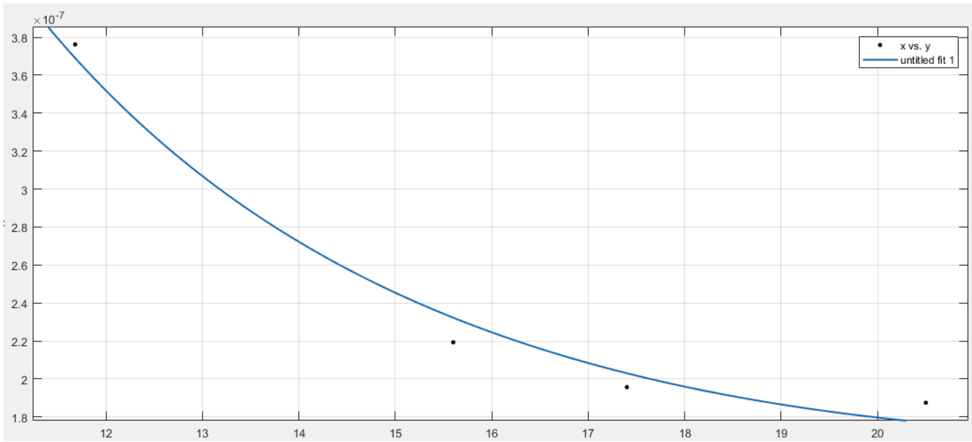


***Fig. 36**: curve fitting on values of "a"*

An handy equation that well fit the data is (35):

$$a = 10^{-5} * \left(\frac{475{,}1 * 10^3}{{d_w}^2} + 1{,}523 * {d_w}^2\right) \qquad (35)$$

Which has an R^2=0.9838 index of a good approximation of our data.

The final equation is:

$$\delta_p = 10^{-8} * \left(a * \frac{\frac{L_k}{d_h}}{(\frac{D_A}{d_w})^5} + 21{,}42 * \frac{\frac{L_k}{d_h}}{\frac{D_A}{d_w}} + 10{,}35 * \frac{\frac{D_A}{d_w}}{\frac{L_k}{d_h}} \right) \qquad (36)$$

With:

$$a = 10^{-5} * \left(\frac{475{,}1 * 10^3}{{d_w}^2} + 1{,}523 * {d_w}^2\right) \qquad (37)$$

Chapter IV

CONCLUSIONS AND FUTURE STUDIES

The major goal of this thesis has been the discover of a method capable of associate a 1D linear stiffness to an object that has complex deformation sum of bending, compression and radial strain.

The study has been focused only on tapered thread joints with a not so wide range of dimensions due to the time-consuming procedure of creating models on Abaqus/CAE.

A solution to this problem could be made automatizing the process of creating 3D models . Utilising an existing macro for Catia you can record the procedure to create a specified part like a spring or a bearing and then simply writing new dimensions in a VBA interface the part with new parameters is automatically created following the recording.Then the models created and meshed on Catia are simulated in Abaqus/CAE utilising the Abaqus-Catia Associative interface.

The procedure explained would reduce drastically the time utilized to create the models and would allow to have an equation that fit well every dimension adopted in practice.

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