Autonomous driving car model and its control

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Abstract

This thesis work focused on the research of the autonomous driving car model and of the steering and speed control of autonomous vehicles.

The different level of vehicle dynamic model are considered such as kinematic model and bicycle model with the linear or nonlinear tire model.

The controller is split in 2 parts, the first one is longitudinal controller corresponding to change of speed, and the other is lateral one which is mainly related to the steering angle. The lateral controller is based on the MPC theory, and the longitudinal one use the PID theory. The entire controller, which is developed in the matlab and simulink, should work well simultaneously to follow the desired path. To test it, the different scenario is generated by the autonomous driving toolbox in matlab.

To get more reliable data of the vehicle state, we are using the Carsim software co-simulating with matlab, making our consideration more comprehensive.

It’s important to see how the controller parameters (step horizon, cost function) have the effect on the results.

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Chapter 1 Introduction:

1.1 General idea of autonomous driving car

Autonomous driving car is the car that fulfills the transportation task without the human intervention, using the computer inside the car to simulate the behavior of the driver and make decision.

Getting the environment data from the variety of sensor, based on the information of road, localization of vehicle, and the obstacle the controller will give the command.

Here the autonomous driving system can be divided by the 4 parts which is system management, environment sensing, path planning, and vehicle control.

(1) environment sensing

The same as the human driving car, autonomous driving car need the information of driving environment in real time. There are 2 ways to get the data, the one is by the help of all the sensor inside the vehicle , combined with the environment to have sensor fusion, trying to let vehicle ‘understand ’ the environment, the other is through the internet to supply the data of the outside area. For example by the vehicular networking vehicle can easily get the road data in front of it also the driving trend of the car which is around the autonomous driving car.

Environment sensing system uses these data correlating with the model inside the computer to understand and recognize the situation.

Environment sensing system is key parts to have reliable driving behavior which means it is also the most difficult part to fulfill. As we all know to drive in the city at least autonomous driving car need recognize the road the traffic light and all the traffic sigh related to traffic rules.

Getting the data from environment sensing system transfer it to path planning system.

(2)Path planning system

Path planning means inside the environment which has obstacle, based on a certain standard such as minimal the length of path or minimal the energy to be used, finding a path from starting point to the final destination without any interruption.

At present path planting system mainly used theory coming from research about robotics. In general there are 2 parts: global path planning and local path planning.

Global path planning means under the condition of knowing the total map of environment, using the local environment information such as the location of obstacle and the boundary of the road, autonomous driving car can confirm the optimist path to follow.

But when the condition is changing such as the other car interrupt the path that autonomous driving car followed, it is important to use the local path planning to re-plan the path. Local path planning, which according to sensor data representing local environment information, generate the path need to be followed. It is under the guide of global path planning; In the process of planning consider not only the minimal energy to be used, safety issues but also problem of dynamic environment constraints.

In the process of local path planning another issue should pay attention to is the motion planning, which means that local path should consider the constraint of vehicle dynamic.

Using model predictive control theory into autonomous driving path planning process, the important issue is that how to solve the vehicle dynamic constraints. We will discuss it in the following chapter.

(3) vehicle control system

Vehicle control system, in which include longitudinal and lateral direction control, generate the command to follow the path coming from path planning system. The same as the robotic control there is difference between the path and the trajectory. Trajectory which considers space and time simultaneously is one kind of path. The essence of path

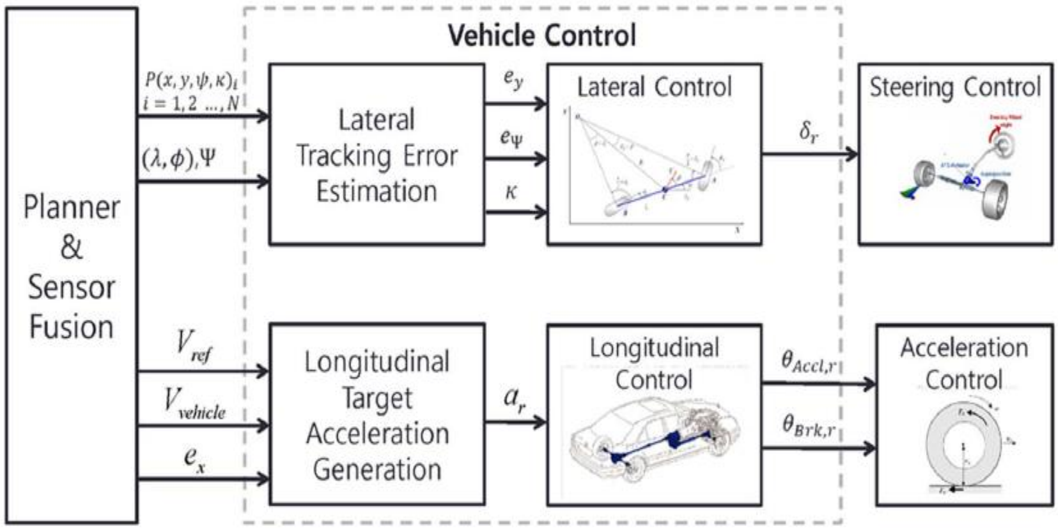


figure 1 vehicle control

planning is trying to minimal the error between the vehicle location and the reference trajectory through control of vehicle motion. If we talk about the trajectory it also includes error within time.

Vehicle control system is important part of autonomous driving car. Environment sensing, path planning must integrate with vehicle control system. Under all the condition of driving, especially with very high speed, vehicle dynamic has much more effect on the results. All the parameter such as side slip of the vehicle, the friction coefficient between the tire and the road, aerodynamic need to consider compared with low speed condition.

(4)system management

Autonomous driving car first is a system work, in which there are a lot of sensors, actuators, and electronic control unit need to work together, to fulfill variety of functions such as car following, lane keeping, emergency stopping, and obstacle avoidance. Different function need to cooperate with different sensor and actuators to make sure the safety and comfortable regulation.

When the new condition is facing or the autonomous driving car receive the new requirement from the passage the system management should assign different function to actuators based on data coming from different sensor.

At the same time system management should supervise the whole autonomous driving car to check what kind of situation of the parts inside the car.

1.2 MPC for path planning and tracking application.

Vehicle dynamic model play an important role in solving the problem of autonomous driving car path planning and tracking. Introducing the dynamic model in path planning process could increase the real-time capability by reducing the computation quantity.

Felipe KUHNE purpose a control scheme based on vehicle kinematic model for wheeled mobile robot. Using MPC to have better constraints condition, by the help of Quadratic programming (QP) to handle linearization problem of wheeled mobile robot model.[1]

Razvan.C.Rafaila consider autonomous driving car model with dynamic model but take in to account the nonlinear tire force. The nonlinear model predictive control model use optimization algorithm to have the minimal value of cost function.[2]

Matthew Brown put the local path planning and path tracking in same control system using model predictive control. All the results must be generated before calculation which takes into account the desired path positioned in two safe envelopes.

The one envelope represents stability problem optimization the other gives indicator of obstacle avoidance.[3]

Jonathan Y.Goh generates insight from professional drivers. They give an approach to control autonomous drifting to have better performance of path tracking problem.

The reference path is generated point by point from all the equilibrium of drifting points considering the side slip angle of tire is no more very small meaning non linear of tire force.[4]

Jiechao LIU investigates the effect of different degree of freedom on performance of model predictive controller. The indicators such as time to reach the target point and the deviation from desired path are used to judge the performance.[5]

Steven C peters design path tracking controller consider differential flatness to explore the limits of tire force with better performance in trajectory tracking problem.[6]

Path following problem is formed to geometry way to design the control algorithm [7-11], it is important to consider the closed loop controller including both longitudinal and lateral direction.[12],and the driver model detailed information in paper[13 14].

1.3 thesis problem statement

This thesis tries to investigate the application of model predictive control (**MPC**) in autonomous driving car. Starting from the vehicle kinematic and dynamic model building within linear or nonlinear tire model, following the discussion of concepts about the model predictive control such as optimization, cost function, quadratic programming and constraints.

A nonlinear mathematical model for a vehicle with dynamic constrains is developed based on the vehicle data coming from Carsim. Test in the simulink with various kinds of scenarios to see the final performance of the controller. Carsim vehicle model is embedded as simulink s-function and controller is designed by the help of matlab mpc toolbox.

An important issue should be specified which is prediction horizon; it is the main parameter in mpc controller. The prediction horizon influences the final performance of the controller and the computation cost. For autonomous driving car it is quiet important to fulfill the path tracking in real time, otherwise the vehicle is easily to meet the dangerous situation due to the delay of the time.

Chapter 2 Vehicle and tire model

Autonomous driving car need to know the car model such as kinematic or dynamic model to fulfill the function of path planning and path tracking. From the first chapter we have insight that the better known of vehicle dynamic model especially when design the path planning system putting dynamic model in it, the better the path tracking results will present. It means that the reliable vehicle model is not only precondition of designing of mpc controller, but also basement of path tracking.

When design mpc controller it is important to consider driving situation of autonomous driving car. In order to have proper autonomous driving motion control we must choose proper variable to have precise prediction of vehicle motion changing.

Motion of vehicle on the road is very complicated, in order to have right descriptions of car motion; one of the useful approaches is complex system of differential equations, choosing different variable representing motion of car.

Adding vehicle kinematic or dynamic model inside mpc controller, we can confirm aim of control for the controller. Especially in motion planning, by the help of linearization and simplification of vehicle model, requirement of real time performance can be met.

The linearization and simplification are important approach to solve the car model such as tire elliptical model and point mass model of car. In this chapter we present some models of the car, their assumptions and constraints.

2.1 kinematic model

Kinematic is kind of theory represents the motion of points and group of objects, neglecting the force that have effects on the motion.

Kinematic is always in direction of geometry of motion to make the research, considering as a branch of mathematics. Kinematics problem begins by describing the geometry of the system and declaring the initial conditions of any known values of position, velocity and/or acceleration of points within the system.[1]

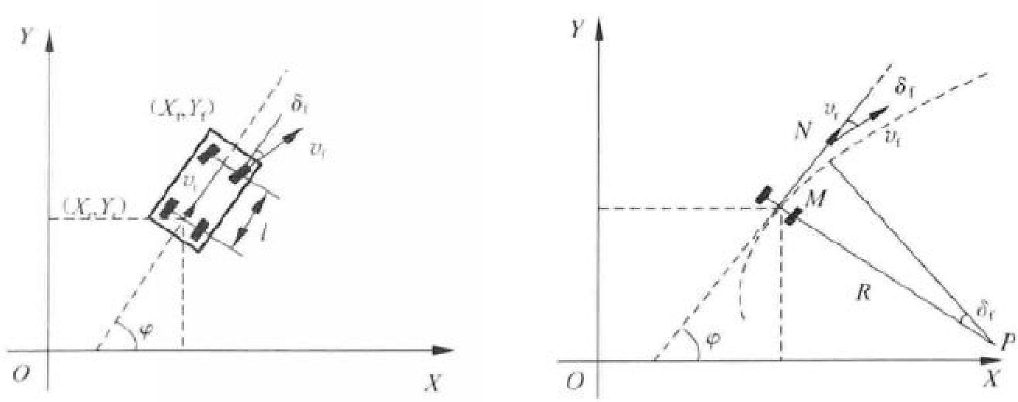


figure 2 kinematic model

As figure 2. showed,$(X_f Y_f)$, $(X_r Y_r)$ represent the coordinate of center point of front axles and rear axles, φ represent yaw angle of vehicle,δ_f is steering angle of front axle, $V_r\ is\ speed\ of$ rear axle, V_f is speed of front axle, l is wheel base of vehicle.

R is radius of trajectory of the center of mass of rear wheel, P is Instantaneous center of rotation, M is center point of rear axle, N is center point of rear axle.

At the center point of rear axle M, the speed is:

$$V_r = \dot{X}_r \cos\varphi + \dot{Y}_r \sin\varphi$$

The kinematic constrains of front and rear axles are:

$$\dot{X}_f \sin(\varphi + \delta_\mathrm{f}) - \dot{Y}_f \cos(\varphi + \delta_\mathrm{f})$$

$$\dot{X}_r \sin\varphi - \dot{Y}_r \cos\varphi = 0$$

Combined with the upper 2 parts of formula we can get:

$$\begin{cases} \dot{X}_r = V_r \cos\varphi \\ \dot{Y}_r = V_r \sin\varphi \end{cases}$$

Considering the geometry relation of front and rear axles:

$$\begin{cases} X_f = X_r + l\cos\varphi \\ Y_f = Y_r + l\sin\varphi \end{cases}$$

the yaw rate is :

$$\omega = \frac{V_r}{\mathrm{l}} \tan\delta_\mathrm{f}$$

ω is yaw rate of the car, at the same time from yaw rate the radius of curvature R and steering angle of front axle can get:

$$\begin{cases} \mathrm{R} = \dfrac{V_r}{\omega} \\ \delta_f = \tan^{-1}(l/R) \end{cases}$$

Finally the kinematic model is :

$$\begin{bmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ \tan\delta_f/l \end{bmatrix} V_r$$

The model can be described in a more uniform way which is state space matrix. Here state variable is $[X_r\ Y_r\ \varphi]$ and the control variable is $[V_r\ \omega]$. The new model is :

$$\begin{bmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{bmatrix} V_r + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega$$

2.2 dynamic model

Vehicle dynamic mainly used to analysis the vehicle suspension effect including quarter car model and the drivability of car. In drivability analysis focused on the longitudinal and lateral dynamic issues.[15]

In this thesis we want use mpc controller finishes the path tracking function, as well as to simplify vehicle dynamic model to reduce computation cost, in order to confirm real time.[16]

Using kinematic model is not reliable especially when the speeds are increased and path curvatures are changing. But in such condition, dynamic model has good performance in path tracking performance.

The dynamic model of the car is very complicated and has more degree of freedom to handle. At the same time the system is nonlinear discontinuous.

Before build vehicle dynamic model we need purpose some ideal assumption:

1. The road is considered as flat surface, the motion is planar.
2. Neglect the effect of motion of suspension
3. Consider the linear tire model neglecting decoupled effect in 2 directions.

4. The load transfer is not into consideration
5. Neglect aerodynamic force.

2.2.1 bicycle model

Based on above 5 ideal assumptions, there are 3 direction of motion for planar vehicle which are longitudinal, lateral and yaw motion of the car.

As illustrated in figure 3, coordinate system oxyz is fixed in the vehicle; xoz is midplane of left and right. The original point is in center of gravity of the car. Coordinate system OXY fixed in the road.

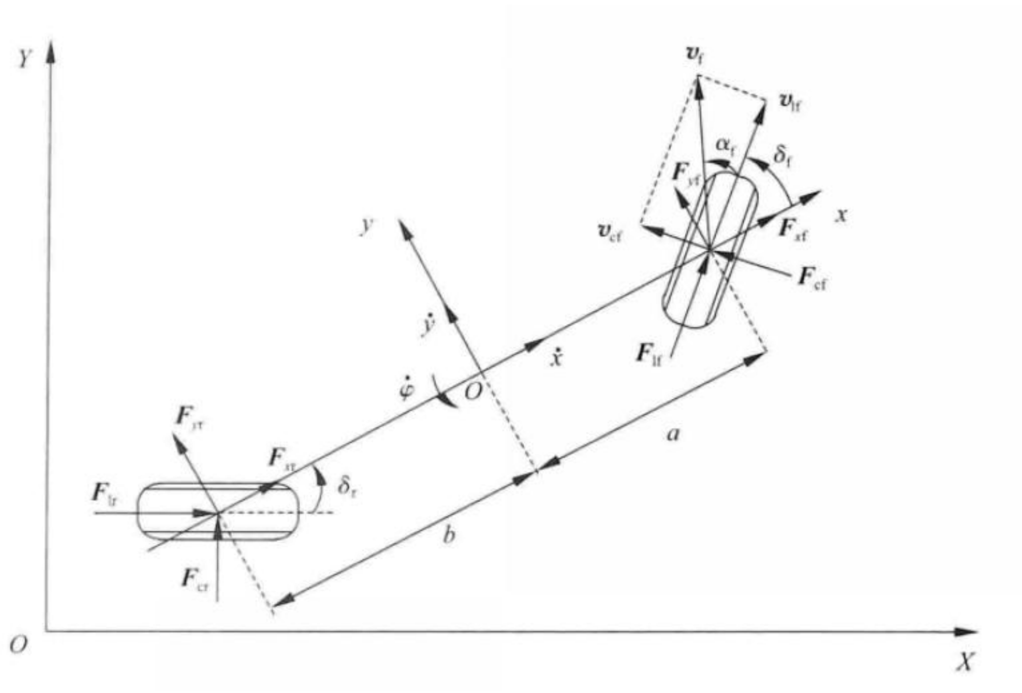


figure 3 bicycle model

About the force in figure:

F_{lf} F_{lr} are the longitudinal force of front and rear tire

F_{cf} F_{cr} are the lateral force of front and rear tire

F_{xf} F_{xr} are the tire force in x direction

F_{yf} F_{yr} are the tire force in y direction

Applying Newton's second law of motion along the X-axis ,Y-axis and Z-axis:

$$m\ddot{x} = m\dot{y}\dot{\varphi} + 2F_{xf}+2F_{xr}$$

$$m\ddot{y} = m\dot{x}\dot{\varphi} + 2F_{yf}+2F_{yr}$$

$$I_z\ddot{\varphi} = 2a\,F_{yf} - 2b\,F_{yr}$$

Where m and I_z denote the vehicle mass and yaw inertia, respectively.$\dot{x}$and $\dot{y}$ denote vehicle longitudinal and lateral velocities, respectively, and$\dot{\varphi}$ is the turning rate around a vertical axis at the vehicle's centre of gravity.

The longitudinal and lateral tire force components in the vehicle body frame are modeled as follows:

$$F_{xf} = F_{lf}\cos\delta_f - F_{cf}\sin\delta_f$$

$$F_{xr} = F_{lr}\cos\delta_r - F_{cr}\sin\delta_r$$

$$F_{yf} = F_{lf}\sin\delta_f - F_{cf}\cos\delta_f$$

$$F_{yr} = F_{lr}\sin\delta_r - F_{cr}\cos\delta_r$$

The longitudinal and lateral tire forces are given by Pacejka's model.

They are nonlinear functions of the tire slip angles α,slip ratios σ, normal forces Fz and friction coefficient between the tire and road μ:

Fl = fl(α, σ, Fz, μ),

$$Fc = fc(\alpha, \sigma, Fz, \mu).$$

The slip angles are defined as follows:

$$\alpha = \tan^{-1}\frac{\nu_c}{\nu l}$$

where the tire lateral and longitudinal velocity components are computed from:

$$\upsilon_l = \upsilon_y \sin\delta + \upsilon_x \cos\delta$$

$$\upsilon_c = \upsilon_y \cos\delta - \upsilon_x \sin\delta$$

The velocity component can be calculated from:

$$\upsilon_{yf} = \dot{y} + a\dot{\varphi} \qquad \upsilon_{yr} = \dot{y} - b\dot{\varphi}$$

$$\upsilon_{xf} = \dot{x} \qquad \upsilon_{xr} = \dot{x}$$

The slip ratios s is approximated as follows:

$$s = \begin{cases} \dfrac{r\upsilon\omega_t - 1}{\upsilon} (\upsilon > r\omega_t, \nu \neq 0) \\ 1 - \dfrac{\upsilon}{r\omega_t} (\upsilon < r\omega_t, \omega_t \neq 0) \end{cases}$$

The friction coefficient μ is assumed to be a known constant and is the same at all four wheels. We use the static weight distribution to estimate the normal force on the wheels. They are approximated as:

$$F_{zf} = \frac{bmg}{2(a+b)} \qquad F_{zr} = \frac{amg}{2(a+b)}$$

Finally consider the vehicle fixed coordinate system and inertial frame.

$$\dot{Y} = \dot{x}\sin\varphi + \dot{y}\cos\varphi$$

$$\dot{X} = \dot{x}\cos\varphi + \dot{y}\sin\varphi$$

2.2.2 Tire model

Making research on vehicle dynamic, the longitudinal and lateral force with aligning torque has effect on drivability and handling stability. Due to the reason of complex behavior of tire, the dynamic model is non linear. So the important issue to generate dynamic model is choosing reliable and useful tire model.

Pacejka purposes the magic formula which uses trigonometric function to represent longitudinal and lateral force [22 23]. Finding relationship between side slip angel and force in different direction, the characteristics of tire can be presented.

The usual form of magic formula is:

$$Y(X) = D\sin(C\arctan(B\Theta(X))) + S_v,$$

Where Y is output of function which can be longitudinal or lateral tire force. B,C,D,Sv are the coefficients correlate to the experiment data. In the following part we show some typical example of tire plots using pacejka formula.

Figure 4 shows the lateral tire force as function of side slip angle at different value of friction coefficient.

Figure 5 shows the longitudinal tire force as function of slip ratio at different value of friction coefficient.

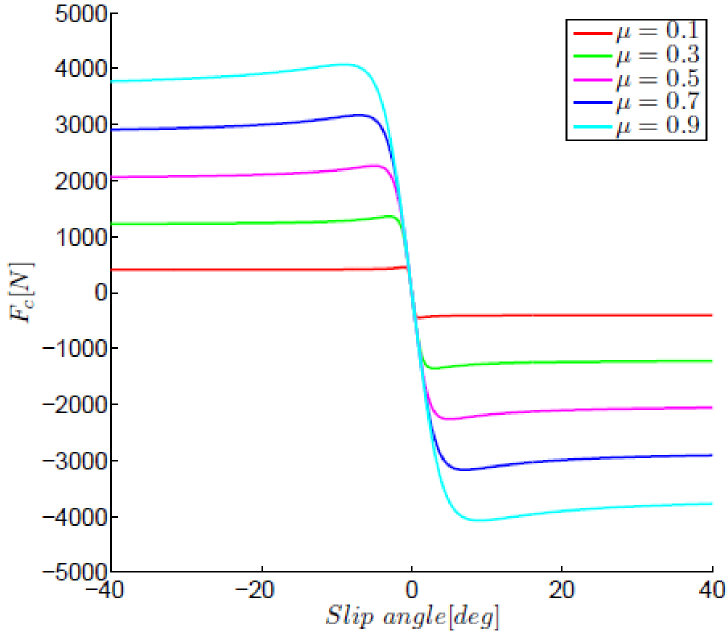


figure 4 lateral force

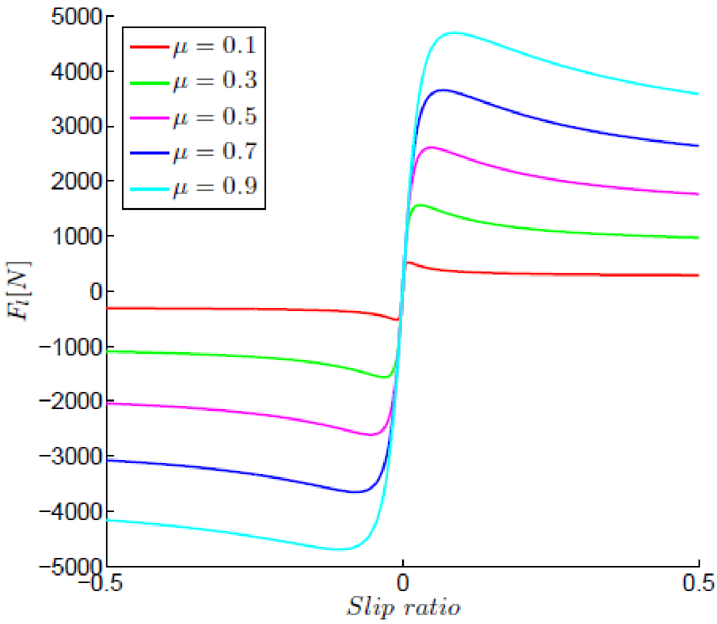


figure 5 longitudinal force

Using pacejka formula for controller designed is also too much complicated, which means the more simplified model needs to be presented. From the figure we can easily find that based on the assumption of side slip angle or slip ratio is very small, the tire force is in the linear region. The formula can be changed to:

$$F_C = C_\alpha \alpha$$

Where C_α is cornering stiffness coefficient of tire, it is related to frication coefficient and normal forceF_z.

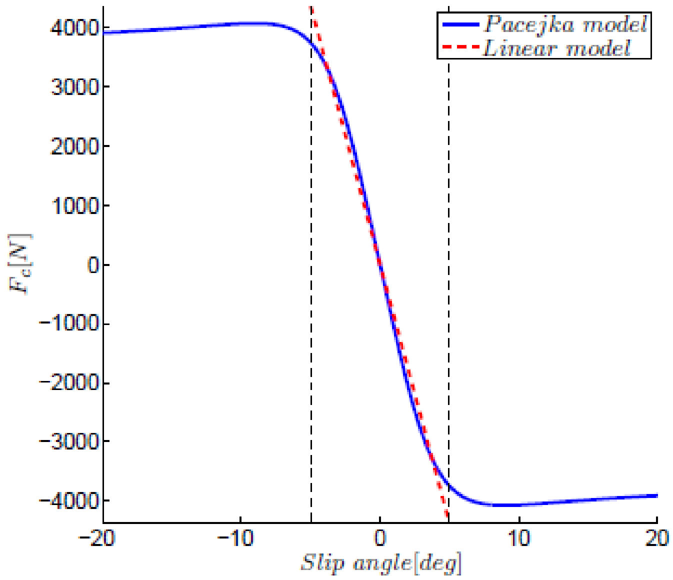


figure 6 linear tire force

Figure6 shows difference between pacejka model and linear model. The linear tire model is the simplest one which can be used in dynamic model to design mpc controller without damage to the performance of path tracking.

2.2.3 linear bicycle model.

To linear dynamic model, based on the small angle assumption, approximate all trigonometric function using first order Taylor expansion:

$$\cos\theta = 1, \sin\theta = \theta, \tan\theta = \theta$$

After that the side slip angle are:

$$\alpha_f = \frac{\dot{y}+a\dot{\varphi}}{\dot{x}} - \delta_f, \qquad \alpha_r = \frac{\dot{y}-b\dot{\varphi}}{\dot{x}}$$

Combined equations ,the lateral tire force are:

$$F_{cf} = C_f(\frac{\dot{y}+a\dot{\varphi}}{\dot{x}} - \delta_f) \qquad F_{cr} = C_r(\frac{\dot{y}-b\dot{\varphi}}{\dot{x}})$$

Substitute the equation we get the linear dynamic equation.

$$m\ddot{y} = -m\dot{x}\dot{\varphi} + 2[C_f\left(\frac{\nu_y + a\dot{\varphi}}{\nu_x} - \delta_f\right) + C_r\frac{\nu_y - b\dot{\varphi}}{\nu_x}]$$

$$I\ddot{\varphi} = 2aC_f\left(\frac{\nu_y + a\dot{\varphi}}{\nu_x} - \delta_f\right) + 2bC_r\frac{\nu_y - b\dot{\varphi}}{\nu_x}$$

Chapter 3 MPC controller for lateral control

This chapter the general knowledge of mpc is presented to show the reason why that the controller is using mpc theory. Following section is to show an example of simple controller tracking a desired path. Inside controller the vehicle model is kinematic model because the reduction of complexity at first.

The rest of this chapter is to illustrate the controller that used commonly to fulfill path tracking function based on dynamic vehicle model.[17 18 19 20 21]

3.1 overview of model predictive control

MPC is a method to control process with a variety of constrains. Chemical industry firstly introduce to factory since 1980s. Nowadays the use of this control theory has expanded to power electronics and autonomous driving car, because of the advantage of current timeslot optimization while keeping future time slots in account.

Inside mpc control process there are 3 critical steps: prediction model, rolling optimization and feedback correction.

Model Predictive Control optimizes the output of a plant over a finite horizon in an iterative manner (Refer to Figure 7). At time step k, the initial value of current plant state is known, the control input is calculated for finite time steps in future k = t + 0T; t + 1T; :::; t + pT ,where p is previewed prediction horizon steps. During calculation the problem has transferred to an open loop, constrained, finite time one. In practical situations, only the first value of control sequence could be the input to the system. Because of the model simplification and added

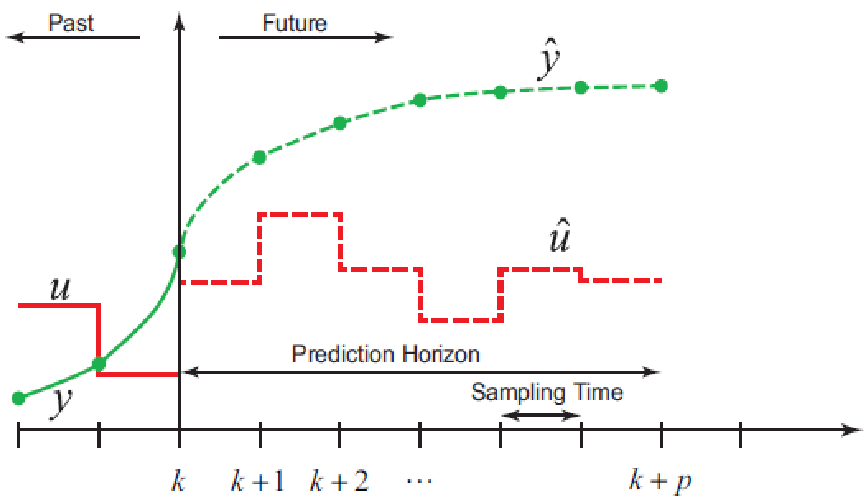


figure 7 mpc algorithm

disturbances or other kind of noise which can cause error between the predicted output and the actual process output.

Thus only the first step of the control strategy is applied to the plant and the plant state is measured again to be used as the initial state for the next time step. This feedback of the measurement information to the optimizer adds robustness to the control. The plant state is sampled again and the whole process is repeated again with the newly acquired states. The prediction time window [t + 0T; t + 1T; :::; t + pT] shifts forward at every time step (reason why MPC is also known as Receding Horizon Control.

Figure8 shows that general mpc used for autonomous driving car. The 3 main components are dynamic optimizer, the vehicle model, and the cost function and constraints.

The output from the mpc controller will be the input to the vehicle; here mpc controller is for lateral control which means the output is steering angle. Usually to control a vehicle we need also 2 more parameter the throttle and the brake which will be discussed in next

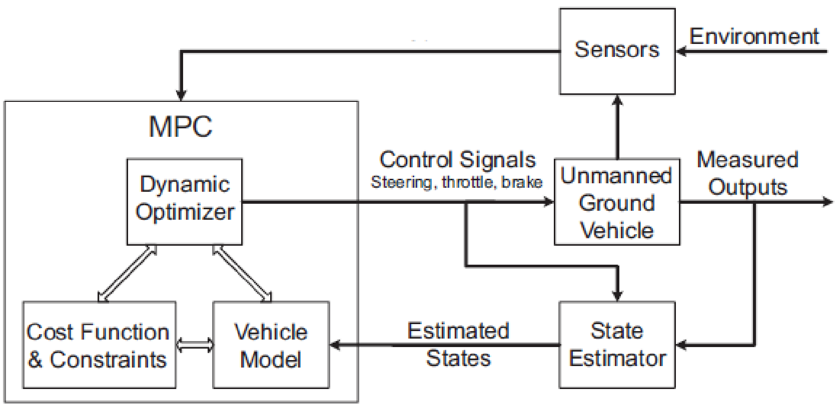


figure 8 MPC controller

chapter. The ground vehicle can be simulated in simulink by a block. The state estimator gives all the indication of the vehicle in which represents all state of vehicle. The output of this block will be the input to mpc as the new initial condition of next time step calculation.

The task of sensor is to give information of environment which may be the boundaries of 2 roads, the position of obstacle, and the presence of other cars.

3.2 LTI model predictive control algorithm

LTI model prediction control algorithm is based on LTI model as prediction model which is also common used model in model predictive control. Compared with nonlinear model predictive control the advantage is that computation is simple and better real time performance. For autonomous driving car the real time performance of algorithm is important issue to consider. Due to that reason here the introduction of LTI model predictive control algorithm is presented.

As said before in following parts we divide 3 topics to discuss: prediction function, optimization solution and feedback.

(1)prediction function:

Consider following discrete linear model:

$$x(k+1) = A_{k,t}x(k) + B_{k,t}u(k)$$

We can set as :

$$\xi(k|t)=\begin{bmatrix} x(k|t) \\ u(k-1|t) \end{bmatrix}$$

A new state space equation:

$$\xi(k+1|t)=\widetilde{A}_{k,t}\xi(k|t)+\widetilde{B}_{k,t}\Delta u(k|t)$$

$$\eta(k|t)=\tilde{C}_{k,t}\xi(k|t)$$

All the matrix is defined as:

$$\widetilde{A}_{k,t}=\begin{bmatrix} A_{k,t} & B_{k,t} \\ 0 & I_m \end{bmatrix}, \widetilde{B}_{k,t}=\begin{bmatrix} B_{k,t} \\ I_m \end{bmatrix}$$

$$\tilde{C}_{k,t}=\begin{bmatrix} C_{k,t} & 0 \end{bmatrix}$$

In order to simplify calculation, purpose assumption:

$$\widetilde{A}_{k,t}=\widetilde{A}_t, k=1,\cdots,t+N-1$$

$$\widetilde{B}_{k,t}=\widetilde{B}_t, k=1,\cdots,t+N-1$$

Considering prediction horizon is Np, control horizon is Nc. The output of control sequence and the output of system in prediction horizon is calculated by:

$$\xi\left(t+N_p\middle|t\right)=\widetilde{A}_t^{N_p}\xi(t|t)+\widetilde{A}_t^{N_p}\widetilde{B}_t\Delta u(t|t)+\cdots+\widetilde{A}_t^{N_p-N_c-1}\widetilde{B}_t\Delta u(t+N_c|t)$$

$$\eta\left(t+N_p\middle|t\right)=\tilde{C}_t\ \widetilde{A}_t^{N_p}\xi(t|t)+\tilde{C}_t\ \widetilde{A}_t^{N_p-1}\widetilde{B}_t\Delta u(t|t)+\cdots$$
$$+\tilde{C}_t\ \widetilde{A}_t^{N_p-N_c-1}\widetilde{B}_t\Delta u(t+N_c|t)$$

To make the relation more clearly, set system output in future time as matrix:

$$\mathrm{Y(t)} = \Psi\xi(\mathrm{t}|\mathrm{t}) + \Theta\Delta\mathrm{U(t)}$$

$$\boldsymbol{Y}(t) = \begin{bmatrix} \boldsymbol{\eta}(t+1 \mid t) \\ \boldsymbol{\eta}(t+2 \mid t) \\ \cdots \\ \boldsymbol{\eta}(t+N_c \mid t) \\ \cdots \\ \boldsymbol{\eta}(t+N_p \mid t) \end{bmatrix} \qquad \boldsymbol{\Psi}_t = \begin{bmatrix} \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t \\ \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t^2 \\ \cdots \\ \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t^{N_c} \\ \cdots \\ \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t^{N_p} \end{bmatrix} \qquad \Delta \boldsymbol{U}(t) = \begin{bmatrix} \Delta \boldsymbol{u}(t \mid t) \\ \Delta \boldsymbol{u}(t+1 \mid t) \\ \cdots \\ \Delta \boldsymbol{u}(t+N_c \mid t) \end{bmatrix}$$

$$\boldsymbol{\Theta}_t = \begin{bmatrix} \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{B}}_t & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t \widetilde{\boldsymbol{B}}_t & \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{B}}_t & \boldsymbol{0} & \boldsymbol{0} \\ \cdots & \cdots & \ddots & \cdots \\ \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t^{N_c-1} \widetilde{\boldsymbol{B}}_t & \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t^{N_c-2} \widetilde{\boldsymbol{B}}_t & \cdots & \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{B}}_t \\ \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t^{N_c} \widetilde{\boldsymbol{B}}_t & \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t^{N_c-1} \widetilde{\boldsymbol{B}}_t & \cdots & \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t \widetilde{\boldsymbol{B}}_t \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t^{N_p-1} \widetilde{\boldsymbol{B}}_t & \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t^{N_p-2} \widetilde{\boldsymbol{B}}_t & \cdots & \widetilde{\boldsymbol{C}}_t \widetilde{\boldsymbol{A}}_t^{N_p-N_c-1} \widetilde{\boldsymbol{B}}_t \end{bmatrix}$$

From equation we know state variable and output in prediction horizon is calculated by the current state $\xi(t|t)$ and control increment $\triangle U$, which is the function of prediction.

(2) optimization solution

In fact control increment is unknown for us only after setting target requirement and solves it. Finally we can get the control sequence in prediction horizon.

$$J(k) = \sum_{j=1}^{N} \chi^{\Gamma}(k+j|k)Q\chi^{\Gamma}(k+j|k) + u^{\Gamma}(k+j-1|k)Ru \quad (k+j-1|k)$$

Setting target function like upper one, by the help of some method we can transfer it to quadratic programming problem. Quadratic programming (QP) is the process of solving a special type of mathematical optimization problem—specifically, a (linearly constrained) quadratic optimization problem, that is, the problem of optimizing (minimizing or maximizing) a quadratic function of several variables subject to linear constraints on these variables. Quadratic programming is a particular type of nonlinear programming.

Equation set control quantity as state quantity in target function but with some disadvantage such as no possibility to set constraint to control increment. So if we change the control increment as state quantity. Optimized target function:

$$J(\xi(t)), u(t-1), \Delta U(t) = \sum \left\| \eta(t+i|t) - \eta_{ref}(t+i|t) \right\| 2_Q + \sum \| \Delta u(t+i|t) \| 2_R$$

Q and R is weight matrix ,the whole equation is used for tracking the desired path. At the same time it must based on some constraints such as :

Control quantity constraint:

$$u_{min}(t+k) \le u(t+k) \le u_{max}(t+k), k = 0,1,\cdots,N_c - 1$$

Control increment constraint:

$$\Delta u_{min}(t+k) \le \Delta u(t+k) \le \Delta u_{max}(t+k), k = 0,1,\cdots,N_c - 1$$

Output constraint:

$$y_{min}(t+k) \le y(t+k) \le y_{max}(t+k), k = 0,1,\cdots,N_c - 1$$

From here the whole optimization solution is finished, solving all these equation with different constraints, the control sequence in future time can be calculated.

(3) Feedback

After solving all the equation, the control increment sequence in future time :

$$\Delta U_t^* = \left[\Delta u_t^*, \Delta u_{t+1}^*, \cdots, \Delta u_{t+N_c-1}^*\right]$$

Based on model predictive control theory, setting the first one in the sequence as the input control to the system.

$$u(t) = u(t-1) + \Delta u_t^*$$

The system execute control input till next time step. In next time step the system based on current information predict future output.

3.3Nonlinear model predictive control

In order to use linear model predictive control, the first requirement is to have linearized vehicle dynamic model. Apart from the nonlinear model predictive, linear model predictive control is a second choice. As the research is going on, nonlinear model predictive control is used in more common area.

For a nonlinear system, consider the model like that:

$$\xi(t+1) = f(\xi(t), u(t))$$

$$\xi(t) \in \chi, u(t) \in \Gamma$$

Where f is transfer function of system,ξ is n dimensional state variable is m dimensional control variable,χ is state vatiable constraint,Γ is control variable constraint.

Set f(0,0)is a stable point of system, and also the control target of system. For any time horizon N, consider following target function J_N:

$$J_N(\xi(t), U(t)) = \sum_{k=t}^{t+N-1} l(\xi(k), u(k) + P(\xi(t+N))$$

Where U(t) is control sequence in horizon N, $\xi(t)$ is all the state variable sequence under control input sequence. Inside the target function J_N the first item represents tracking ability, the second item presents constraints.

Combined with model and target function, nonlinear model predictive control is to solve the problem under all kind of constraints in every time step. For example:

$$
\begin{aligned}
\min \quad & J_N(\xi_t, U_t) \\
\text{s.t} \quad & \xi_{k+1,t} = f(\xi_{k,t}, u_{k,t}), \quad k = t,\cdots,N-1 \\
& \xi_{k,t} \in \chi \qquad k = t,\cdots,N-1 \\
& u_{k,t} \in \Gamma \qquad k = t,\cdots,N-1 \\
& \xi_{k,t} = \xi(t) \\
& \xi_{N,t} \in \chi_{fin}
\end{aligned}
$$

If we get the results from equation satisfy all requirement and constrains, the optimal control sequence U(t) will meet. Based on the model predictive control theory, setting the first item, which is inside the control sequence, as input to the controlled system.

In next time step, system gets new current state and solves the equation; continue setting first item as input to system. For any nonlinear system during calculation there are N(n+m) variable, nN nonlinear constraints, also including control increment constraints and output constraints.

So the more dof of dynamic system the more calculation power needed. In our case vehicle dynamic model is 3 dof model, the advantage of nonlinear model predictive control is still there. Otherwise for high dof vehicle model it needs some simplification, to obey the requirement in real time.

3.4 numerical method for nonlinear system

In nonlinear model predictive control, by the nonlinear model, current state and control sequence, predict the future state of system.

It is an iteration process, but with unknown control sequence. Due to that reason it needs an iteration equation to have an approximate solution of differential equation.

Usually there are 2 methods, one is Euler method, and the other is Runge-Kutta method.

The Euler method (also called forward Euler method) is a first-order numerical procedure for solving ordinary differential equations (ODEs) with a given initial value. It is the most basic explicit method for numerical integration of ordinary differential equations.

The Euler method is a first-order method, which means that the local error (error per step) is proportional to the square of the step size, and the global error (error at a given time) is proportional to the step size. The Euler method often serves as the basis to construct more complex methods.

For a differential equation:

$$y'(t) = f(t, y(t))$$

Using Euler method, the result in time n+1 can be calculated from time n. the iteration equation is

$$y_{n+1} = y_n + hf(t_n, y_n)$$

But usually this method has a very low level of precision. So in some condition we try to use another method.

the Runge–Kutta methods are a family of implicit and explicit iterative methods, which include the well-known routine called the Euler Method, used in temporal discretization for the approximate solutions of ordinary differential equations.

Usually the method is :

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$t_{n+1} = t_n + h$$

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf(t_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$

$$k_3 = hf(t_n + \frac{h}{2}, y_n + \frac{k_2}{2})$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

Here one important issue needs to be specify, the more precision the results we require, the lower the computation speed.

3.5 dynamic model in path coordinate

In order to control autonomous driving cars it is important to represent dynamic model in to path coordinate. Based on constant longitudinal speed assumption, the relationship between path r(s) and yaw rate can be found:

$$r(s) = \kappa(s) v_x.$$

After that the lateral acceleration derived as:

$$\dot{v}_y(s) = \kappa(s) v_x^2.$$

$$\begin{aligned} \ddot{e}_{cg} &= (\dot{v}_y + v_x r) - \dot{v}_y(s) \\ &= \dot{v}_y + v_x(r - r(s)) \\ &= \dot{v}_y + v_x \dot{\theta}_e. \end{aligned}$$

$$\dot{e}_{cg} = v_y + v_x \sin(\theta_e)$$

Where e_{cg} is orthogonal distance of center of gravity to the desired path. θ_e is defined as $\theta - \theta_p$

$$\begin{aligned} \ddot{e}_{cg} - v_x \dot{\theta}_e &= \frac{-(c_f + c_r)}{m v_x}(\dot{e}_{cg} - v_x \theta_e) \\ &\quad + \left[\frac{\ell_r c_r - \ell_f c_f}{m v_x} - v_x\right](\dot{\theta}_e + r(s)) + \frac{c_f}{m}\delta \\ \ddot{e}_{cg} &= \frac{-(c_f + c_r)}{m v_x}\dot{e}_{cg} + \frac{c_f + c_r}{m}\theta_e \\ &\quad + \frac{\ell_r c_r - \ell_f c_f}{m v_x}\dot{\theta}_e + \left[\frac{\ell_r c_r - \ell_f c_f}{m v_x} - v_x\right] r(s) + \frac{c_f}{m}\delta \end{aligned}$$

$$
\begin{aligned}
\ddot{\theta}_e + \dot{r}(s) &= \frac{\ell_r c_r - \ell_f c_f}{I_z v_x}(\dot{e}_{cg} - v_x \theta_e) \\
&\quad + \frac{-\left(\ell_f^2 c_f + \ell_r^2 c_r\right)}{I_z v_x}(\dot{\theta}_e + r(s)) + \frac{\ell_f c_f}{m}\delta \\
\ddot{\theta}_e &= \frac{\ell_r c_r - \ell_f c_f}{I_z v_x}\dot{e}_{cg} + \frac{\ell_f c_f - \ell_r c_r}{I_z}\theta_e \\
&\quad + \frac{-\left(\ell_f^2 c_f + \ell_r^2 c_r\right)}{I_z v_x}(\dot{\theta}_e + r(s)) + \frac{\ell_f c_f}{m}\delta - \dot{r}(s).
\end{aligned}
$$

The state space model in tracking error variables is therefore given by:

$$
\begin{bmatrix} \dot{e}_{cg} \\ \ddot{e}_{cg} \\ \dot{\theta}_e \\ \ddot{\theta}_e \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(c_f+c_r)}{m v_x} & \frac{c_f + c_r}{m} & \frac{\ell_f c_f - \ell_f c_f}{m v_x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{\ell_r c_r - \ell_f c_f}{I_z v_x} & \frac{\ell_f c_f - \ell_r c_r}{I_z} & \frac{-(\ell_f^2 c_f + \ell_r^2 c_r)}{I_z v_x} \end{bmatrix} \begin{bmatrix} e_{cg} \\ \dot{e}_{cg} \\ \theta_e \\ \dot{\theta}_e \end{bmatrix}
+ \begin{bmatrix} 0 \\ \frac{c_f}{m} \\ 0 \\ \frac{\ell_f c_f}{I_z} \end{bmatrix} \delta + \begin{bmatrix} 0 \\ \frac{\ell_r c_r - \ell_f c_f}{m v_x} - v_x \\ 0 \\ \frac{-(\ell_f^2 c_f + \ell_r^2 c_r)}{I_z v_x} \end{bmatrix} r(s)
$$

Chapter4 Longitudinal control

After talking about longitudinal control, it is mandatory to analysis the longitudinal control, this chapter is focus on how to use PID control theory to design our longitudinal controller.

First the general introduction of PID is presented, after that the desired velocity profile is calculated as the reference for controller [12 15]. Since the longitudinal and lateral dynamic of the vehicle is coupled, the simplest part, which is elliptical tire model, is derived to have general understanding on it.

4.1 Overview of PID theory

PID controller is composed of 3 parts: proportional part, integral part and derivative part. It mainly used for linear time invariant system.

PID controller is the most common used controller in industry control area. The general theory is about control loop feedback technology. It takes the state value from system and compared it with a reference one.

Based on this error applies a correction take in to account proportional, integral, and derivative terms (denoted P, I, and D respectively), hence the name.

The main target of controller is to let the output of system as same as the desired value. PID controller adjusts input according to the history data and error to make system more stable.

In some application there is no mandatory to use all 3 parts of controller. Usually sometimes only PI or PD or only P parts are presented in controller.

Figure 9 shows the basic principles of theory and how the 3 parts are applied to the system. According to the error value which is the difference between desired setpoint and process output, controller generate control input under the algorithm of pid and this input try to minimize the error value

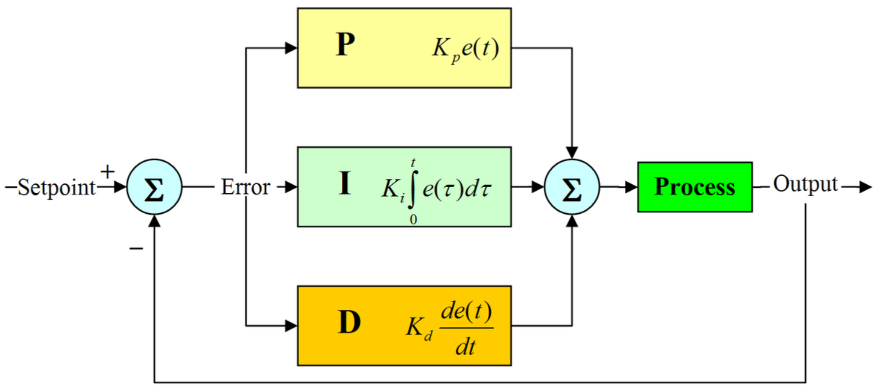


figure 9 PID control

P means proportional to the current value of error. By the help of gain factor"k" directly multiplied the error value, using proportional response to compensate system.

I is represents past value of error and integrate it them over time. Due to the condition that after proportional control there is a still the error between current state and desired one which is called residual error. The I parts try to eliminate residual error. When the residual error turn to be 0, the integral will not continue increase.

D is a part taking in to account the future trend of error. By the help of derivation of error value, the mission of this part is to reduce change rate of error value to make it more stable.

The total function can be expressed as :

$$u(t) = K_{\mathrm{p}} e(t) + K_{\mathrm{i}} \int_0^t e(t')\,dt' + K_{\mathrm{d}} \frac{de(t)}{dt},$$

Where $\mathrm{K_p}$ denote as proportional terms, $\mathrm{K_i}$ denote as integral terms and $\mathrm{K_d}$ denote as derivative part.

4.2 introduction of longitudinal control

After talking about the lateral control of the autonomous driving cars, it is also important to spend time in longitudinal control, since that the vehicle can’t drive with constant speed.

These days a lot of work and research has developed to solve challenge in security problem hence that even in today there are also millions of people are injured because of car accident. It is the reason that every car maker is trying to use ADAS to reduce this big number. In other words the ADAS is also the low level of autonomous driving cars.

One of the common used method is that multi point preview model[24], this model divide the desired path ahead of vehicle to some points based on the deviation between the point and current lane generate signal of steering angle and throttle/ brake.

An active cruise control (ACC) is also used by a majority of vehicles. The main target is to keep vehicle speed and safety distance between cars. An mpc based control algorithm is generated [25] with the advantage of minimal fuel consumption.

Since the vehicle is a system work that every parts have deeply relationship with each other.

Here we just specify some important parts such as:

The longitudinal and lateral motions are kinematic and dynamic coupling, because of the yaw motion.

When the vehicle is accelerating or cornering, there are longitudinal and lateral accelerations. Due to that the load transfer is happening and the vehicle is no more in static state.

The constraint of tire force which called the elliptical model of the tire is coupled in longitudinal and lateral direction.

It is complex to consider all the effects simultaneously. It is necessary to have a nonlinear dynamic model to capture all the effects. Since in the lateral controller which is developed base on mpc theory spends a lot of time to do calculation. If in longitudinal controller also the nonlinear model is used which will be a big challenge for the data processor inside the car. While the real time performance is the first priority of total system.

Due to the reason of simplification, here the longitudinal controller is developed by PID theory, using the previewed curvature of path in front of vehicle, getting data from camera, calculating the desired velocity while cornering. Here the lateral and longitudinal coupled effects have effects on the maximal friction force of tire. Based on the desired velocity and current one, The output of longitudinal controller is signal of throttle or brake pedal to simulate the driver and it is convenient to directly send this signal to ECU inside autonomous driving cars.

4.3 desired velocity generation

When the vehicle is cornering, it is important to clearly know the relationship between the steering angel and centrifugal force. If the tire cannot generate enough force to balance the centrifugal and aerodynamic force, the vehicle will out of the control which we called under steering/over steering.

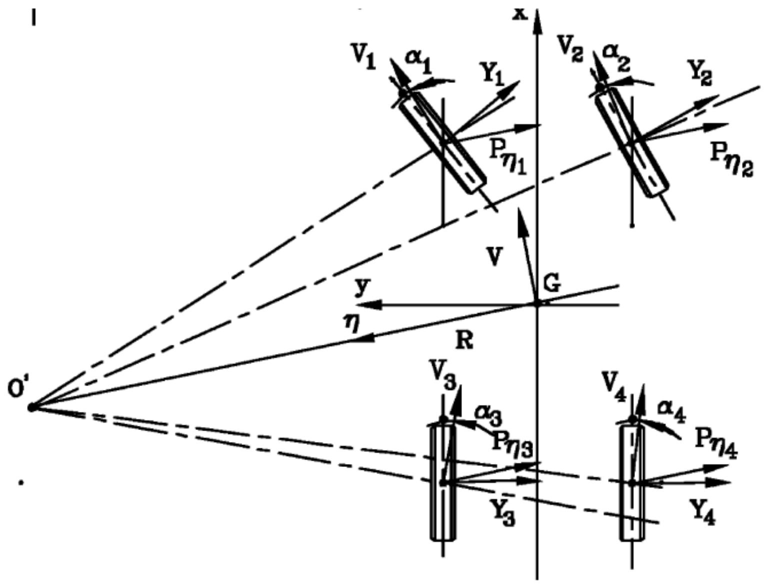


figure 10 force distribution in 4 wheels

From figure 10 shows：we assume that the vehicle is driving in flat road and neglect aerodynamic force because that the simple model we want to have.

Taking into account the centrifugal force and tire frication force in η direction to finish the equilibrium equation.

$$\frac{mV^{\wedge}2}{R} = \sum P_{\eta}$$

Here we assume P_{η} are conflated with the tire lateral cornering forceF_y. The vertical force on each tire are assumed to be equal which means all the tire have same frication coefficient μ_y. Based on all these assumption, the approach is referred to ideal steering.

$$F_Z = \sum F_Z$$

$$F_Z = mg$$

Combined with all the upper equation

$$\frac{mV^2}{R} = \mu_y mg$$

$$\frac{1}{R} = K$$

Where k denotes the curvature of road. Thus finally we can get the desired velocity:

$$V_{x-max} = \sqrt{\frac{g\mu}{|k|}}$$

To make conclusion of this section, from curvature of the road the maximal velocity is calculated. It is one of the constraints of longitudinal controller otherwise the vehicle will go out of the lanes.

The maximal velocity is reference speed of PID controller, time by time the controlled calculated the error between current speed and maximal one. Based on the error controller generate signal of throttle/brake pedal to control the speed of the car.

4.4 Coupled effect between longitudinal and lateral direction.

After getting the maximal velocity, it is mandatory to make research on the relationship between coupled effect of longitudinal and lateral direction. In the past the force is generated separately, only the lateral force to control of the steer of the vehicle and investigate the performance of autonomous driving cars.

But now due to the presence of longitudinal controller, the tire forces are generated in both directions at the same time. The condition itself is changed and the constraints and relationship need to investigate to make sure the good results.

From the research when increase the longitudinal force of the tire that

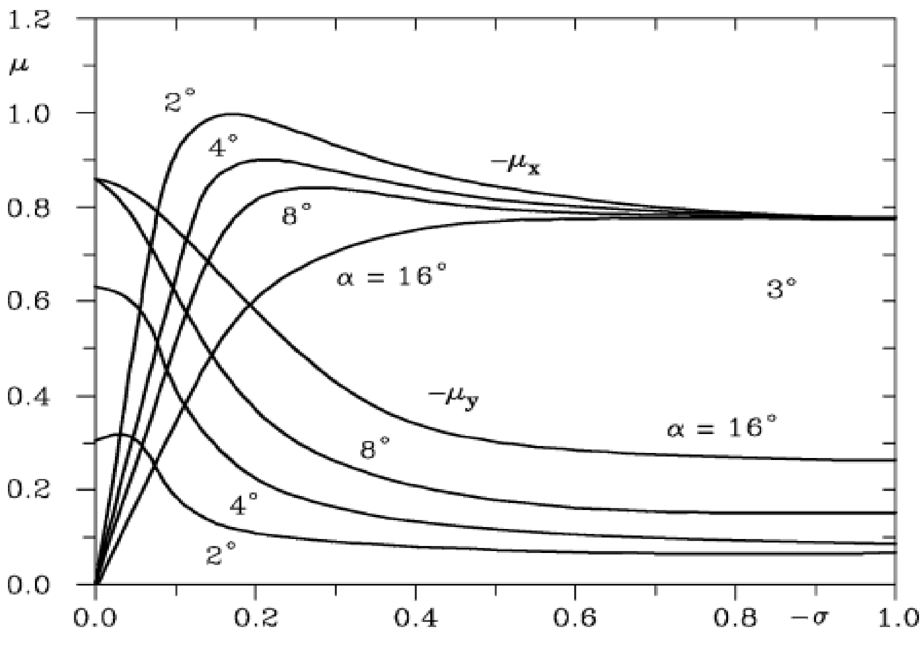


figure 11 frication coefficient as function of slip ratio has certain side slip angle, the force in lateral direction is decreases. The result is same as showed in figure11

By applying the driving or braking torque to the tire with a certain side slip angle, the shape of frication coefficient is changed along different side slip angle.

A set of experiment set longitudinal force is in x-axis and lateral force is in y-axis with constant side slip angle. The results is showed in figure 12

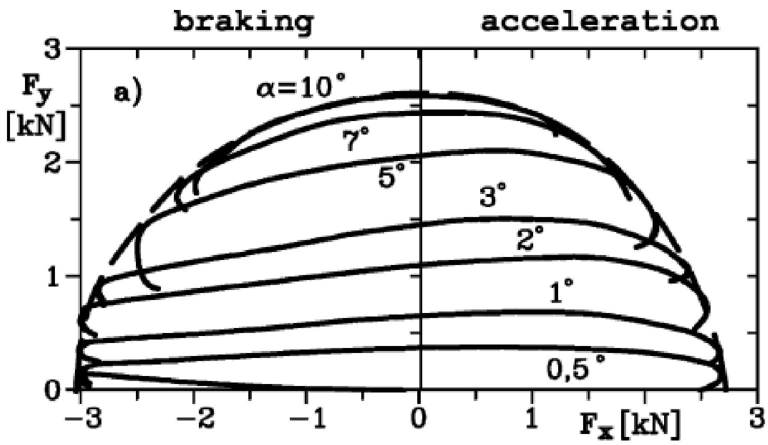


figure 12 longitudinal and lateral force based on experiment data

Apart from the sideslip angle there are some other parameters influencing the curvy of force such as the speed of the car, the tire and road condition. The different material and distribution of different layers have effects also.

Due to the reason of simplification, a model is purposed to approximate the curve. Using elliptical approximation the tire behavior is represented by the equation:

$$\left(\frac{F_y}{F_{yo}}\right)^2 + \left(\frac{F_x}{F_{xo}}\right)^2 = 1$$

Where F_{yo} are the lateral force at the given side slip angle with no longitudinal force applied. The F_{xo} is maximal longitudinal force with no lateral force which means no side slip angle

Using cornering stiffnessC_0 substitute lateral force:

$$\left(\frac{C\alpha}{C_0\alpha}\right)^2 + \left(\frac{F_x}{F_{xo}}\right)^2 = 1$$

Finally the elliptical model of tire is generated to show the couple effect of lateral and longitudinal direction in figure 13

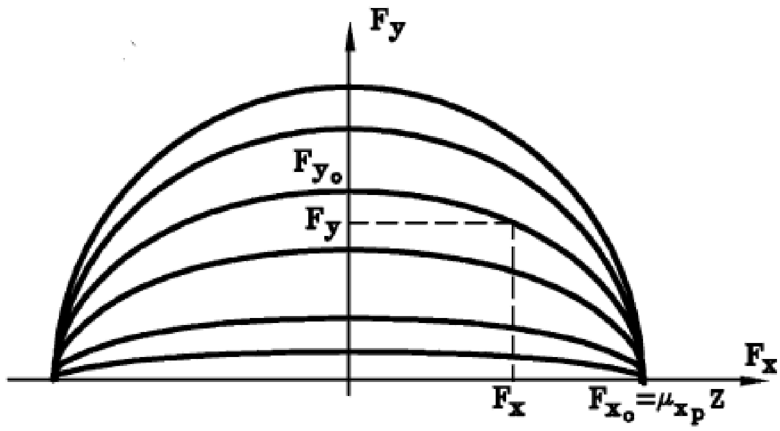


figure 13 elliptical model

Based on the output of mpc controller, the steering angle and side slip angle of tire, the lateral force need to apply to the tire is calculated by:

Slip angles:

$$\alpha_f = tan^{-1}(\frac{v_y + l_r r}{v_x}) - \delta$$

$$\alpha_r = tan^{-1}(\frac{v_y - l_r r}{v_x})$$

Lateral forces:

$$F_{yf} = C_f \alpha_f$$

$$F_{yr} = C_r \alpha_r$$

From lateral force substitute it in to elliptical model; the available longitudinal force is generated. The available longitudinal force divide the mass of the vehicle is the acceleration limit of the vehicle. The limit is another constraint of PID controller.

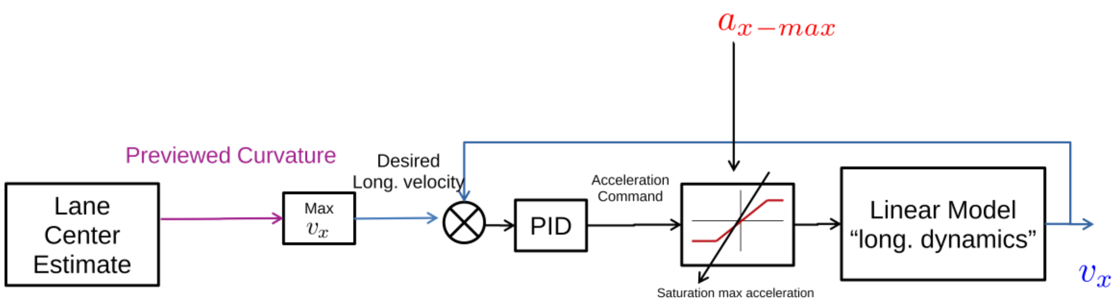


figure 14 sequence of longitudinal control

In figure14 it presents the sequence of the data. Starting from the lane center estimate block, which will be discussed detailed in next chapter, the curvature of the path is calculated. After due to the prediction characteristic of mpc controller, the previewed curvature is current curvature plus the value that curvature derivative multiplies the prediction horizon. Based on the maximal previewed curvature the maximal reference velocity is generated, sending it to the PID controller, according to the difference between the desired velocity and current velocity, the output is acceleration signal. The saturation rate limit considers maximal longitudinal force available applied to the tire. Taking in to account the couple effect in 2 directions, the system put first priority into lateral control otherwise the vehicle will go out of the lanes. Inside linear model longitudinal dynamic block, by the help of simple transfer function, the input signal which is acceleration transfers to output signal which is speed. Using this speed re-send it to mpc controller updates all the sate inside model. The total architecture of longitudinal and lateral controller is showed in following figure.

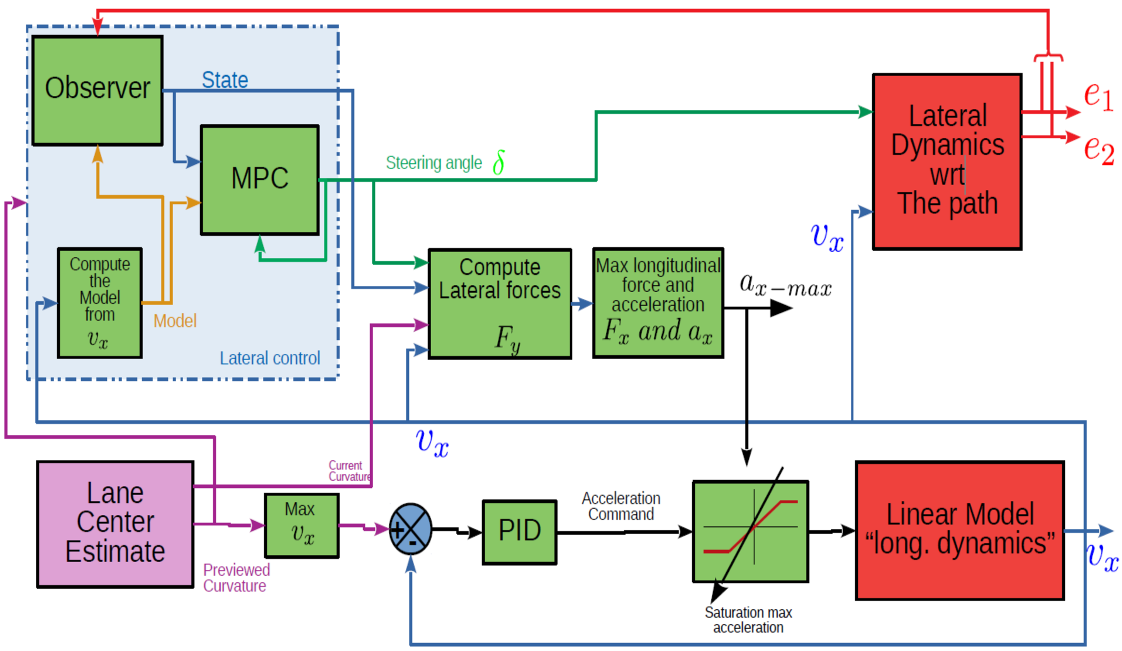


figure 15 total architecture of control algorithm

Chapter 5 Results

This chapter introduces the simulation environment based on matlab and simulink to show the general function of blocks. The test bench developed in simulink by the help of some example provided by Matlab Company. It is a basement for our process giving the opportunity to test the control algorithm performance.

Carsim model is also as s-function introduced to our system because that it is more comprehensive model taking into account the aerodynamic force and some dynamic effects of the vehicle. Let the performance more reliable.

Here first we show the results only with mpc controller to see the performance about the autonomous driving car in constant speed. Second Carsim math model representing the whole vehicle system is introduced into simulink, to see the effects of vehicle dynamic such as load transfer. After that the results is related to condition including both longitudinal and lateral controller.

Inside Carsim it provides a procedure of lane keeping function which is same as our project. It provides an opportunity to have benchmarking with Carsim. The same scenario is generated both in Carsim and simulink. The results from 2 methods are compared.

5.1 simulink environment

Using matlab to simulate the autonomous driving cars is because that the powerful computation model and a variety of toolbox that can be directly used in our project, such as model predictive control toolbox and automated driving system toolbox.

The automated driving system toolbox provides algorithms and tools to design and simulate autonomous driving systems. The system give possibility to generate different scenario including all traffic information, simulate the entire sensor that embedded in the car and deal with data.

figure 16 presents the total environment of our simulink model which

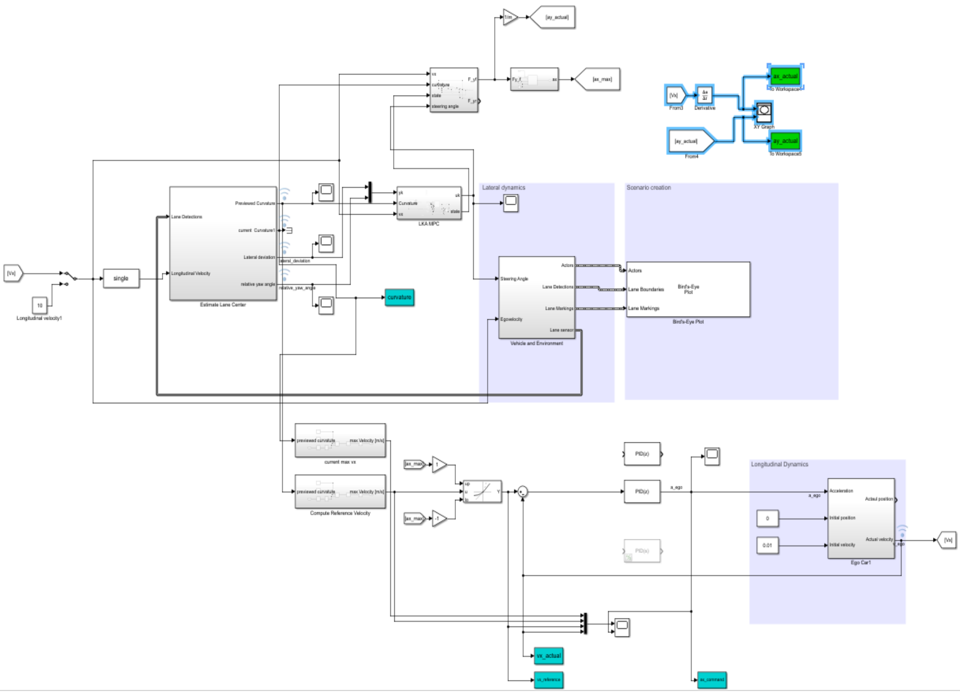


figure 16 total simulink environment

can be divided into some parts. As discussed before there are longitudinal and lateral control parts, the vehicle and environment parts, estimate lane center part to simulate the sensor getting the data of vehicle path.

The general ideal of our system is to keep the vehicle in its lane and follow the curved road by controlling the front steering angle. This goal is achieved by driving the lateral deviation e1 and the relative yaw angle e2 to be small. As figure 17 shows:

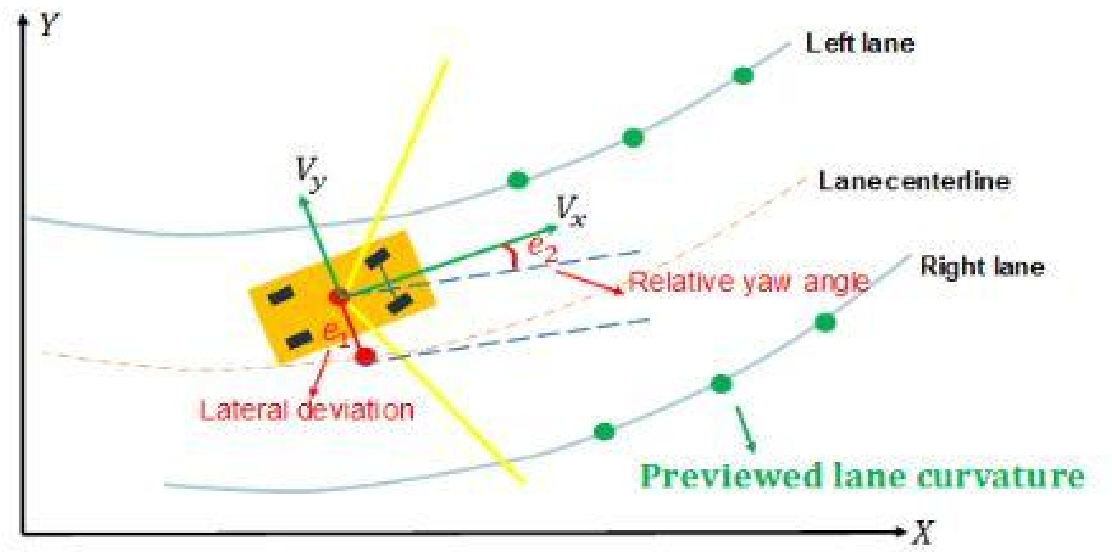


figure 17 lateral deviation and relative yaw angle

Controller calculates a steering angle for the ego car based on the following inputs:

1. Previewed curvature (derived from Lane Detections)
2. Ego longitudinal velocity
3. Lateral deviation (derived from Lane Detections)
4. Relative yaw angle (derived from Lane Detections)

Using the function representing the vehicle dynamic model, giving the steering angle and the longitudinal velocity, we can have the vehicle state during driving. The output can be:

1. Longitudinal position of the car(X coordinate)
2. Lateral position(Y coordinate)
3. Longitudinal velocity
4. Lateral velocity
5. Yaw angle
6. Yaw rate

All these data are used to estimate current state of the autonomous driving car and re-send it to our controller to correct the control variable in next step time that we have discussed more detailed in chapter 2.

About the vehicle and environment parts

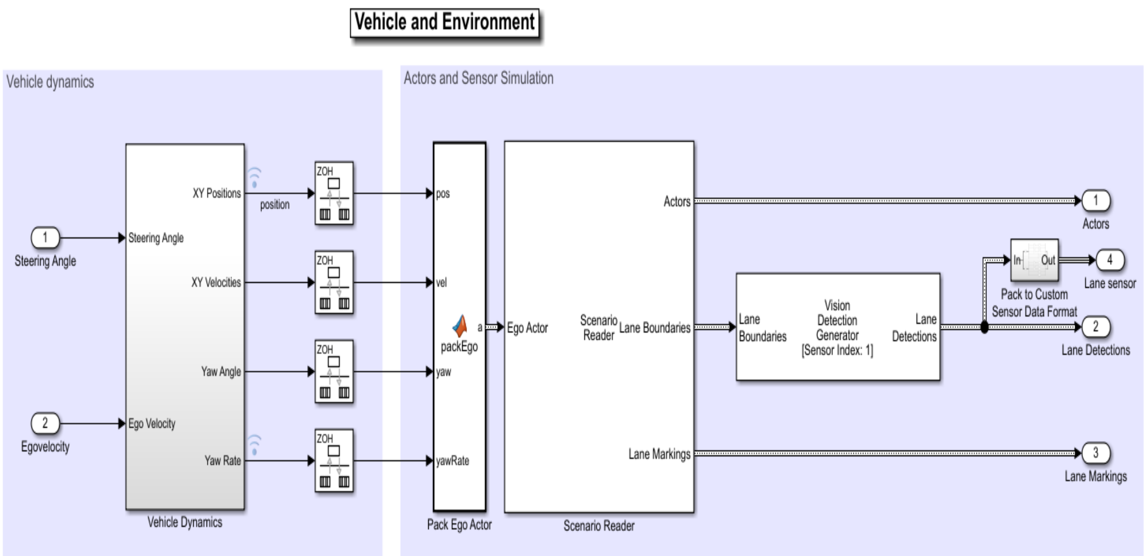
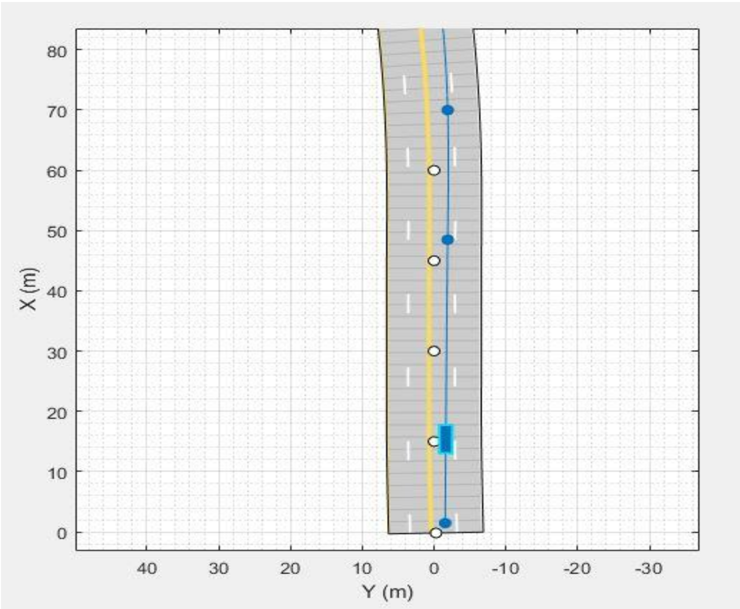
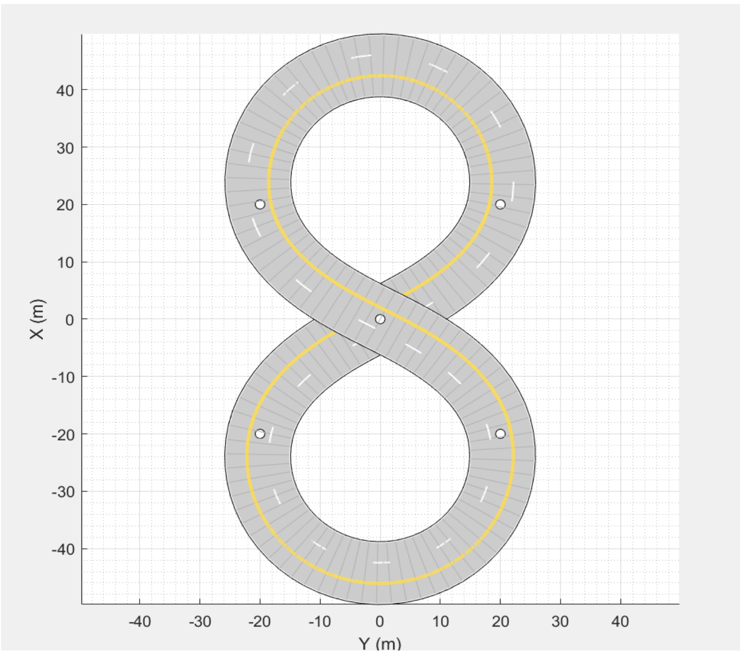


figure 18 vehicle and environment part

Giving the scenario reader to test different scenario generates the ideal left and right lane boundaries based on the position of the vehicle. From lane detection, the previewed curvature, the lane markings, the relative yaw angle and the lateral deviation can be followed to get.

The scenario, which including the information of road , the path lines, the ego car which is controlled by controller, and the other car as obstacle with its drive path, use to test the performance of control algorithms. Following figure presents different scenario.





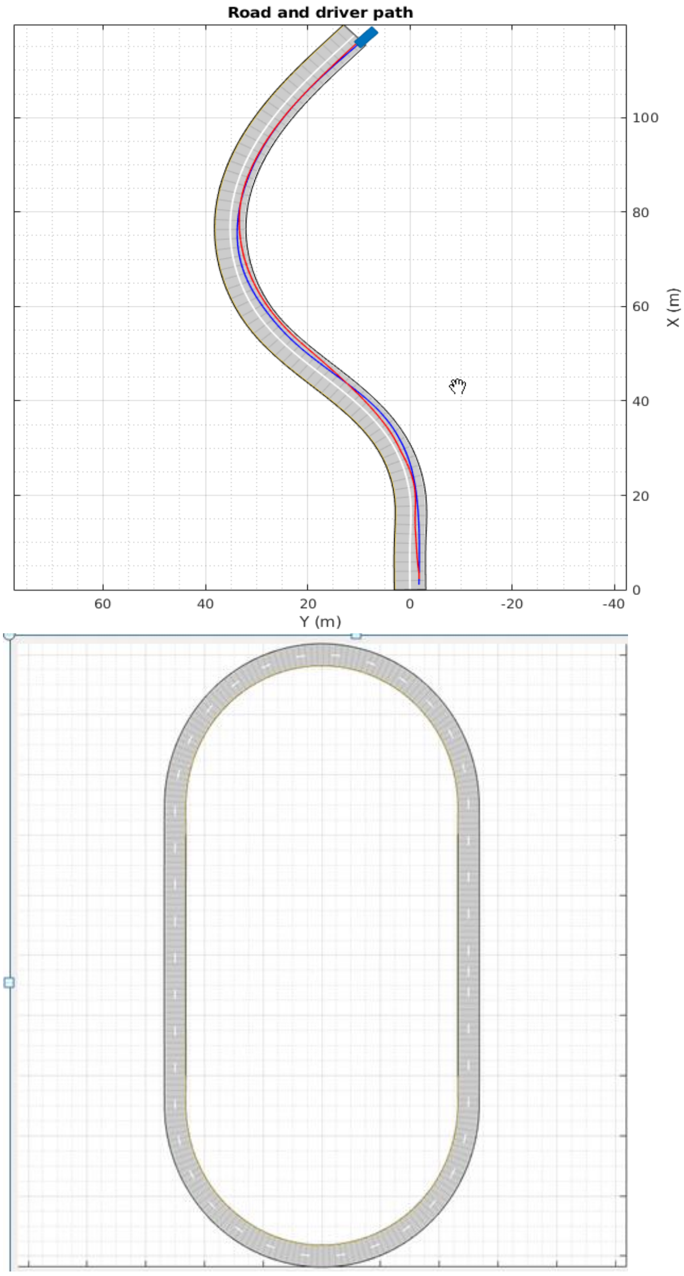


figure 19 different scenario

5.2 cooperation simulation with Carsim

Carsim is the software developed by Mechanical Simulation Corporation; it mainly used to simulate the dynamic behavior of vehicle. The software simulates the response results with respects to the inputs such as driver command, the road condition and the aerodynamic force.

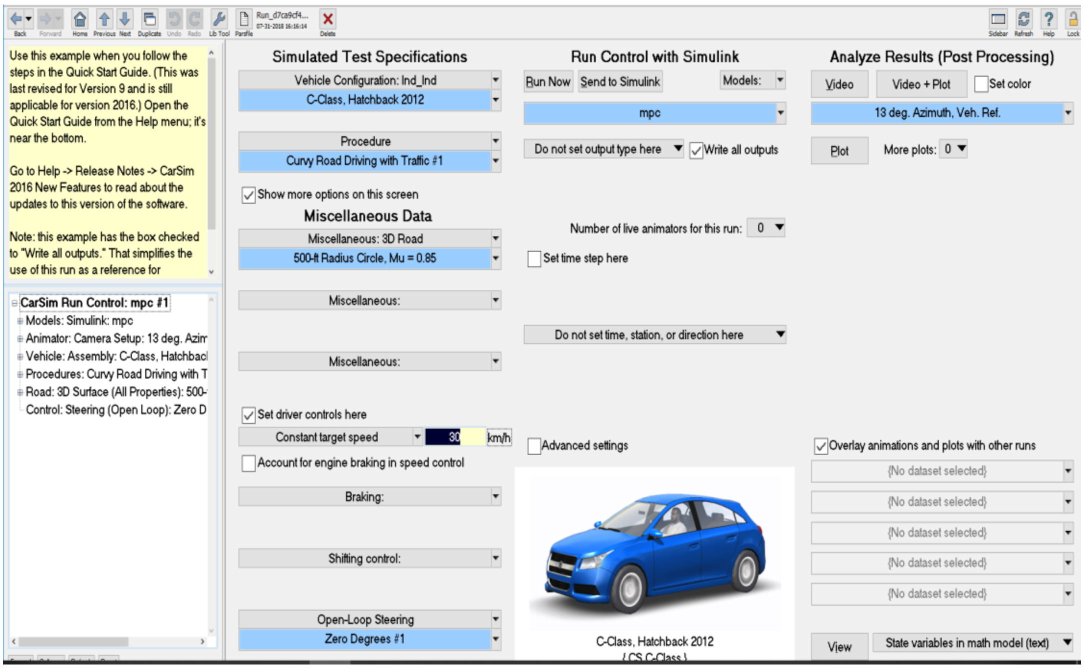


figure 20 carsim configuration

The vehicle parameter data getting from Carsim is showed in table 5.1. Inside Carsim vehicle model it defines vehicle dimension parameters, vehicle mass and moment of inertia in different direction. In mpc controller the vehicle bicycle model is 3 dof.

Table 5.1

Parameter	value
M:total mass	1412kg
lf :distance form GC to front axle	1.015m
lr:distance from GC to rear axle	1.89m
J:vehicle yaw moment of inertia	1536.7kgm^2

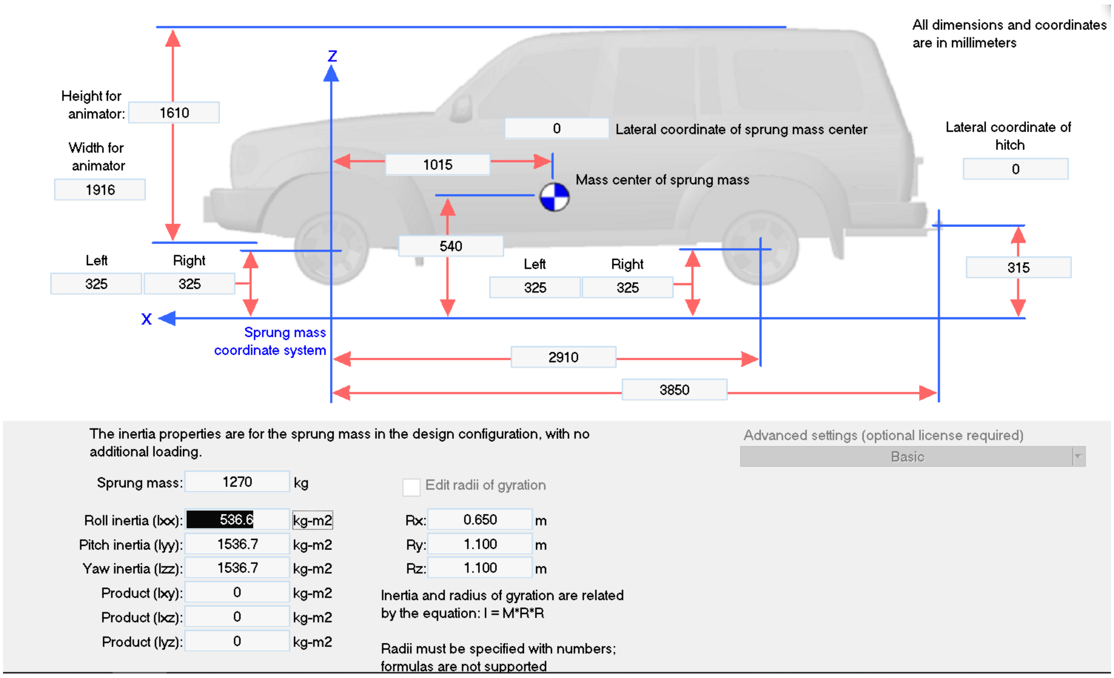


figure 21 vehicle dimension

After that it is mandatory to set sending vehicle model as s-function to simulink. Specify the input and output of s-function. The figure 22 23 presents the general configuration.

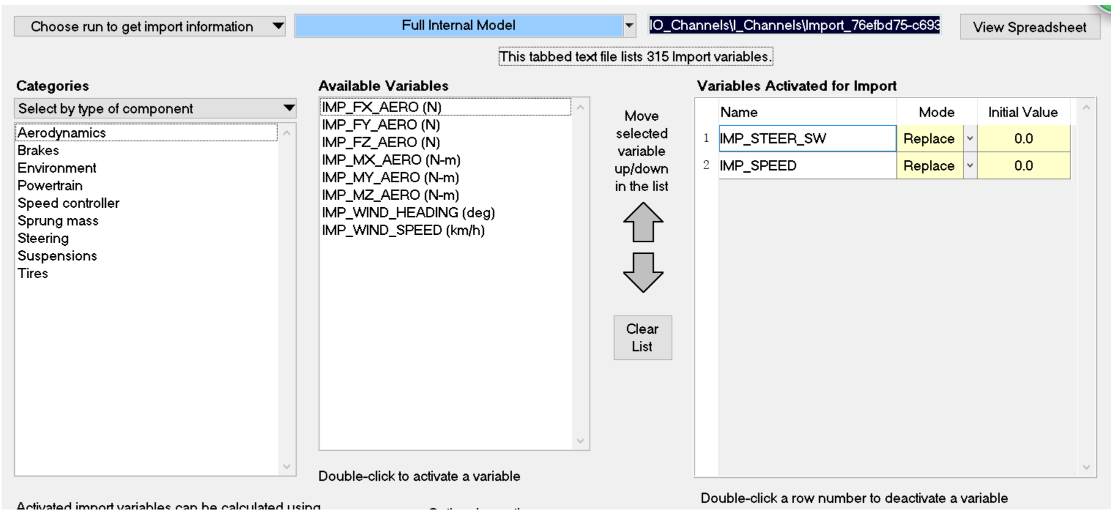


figure 22 input set

Input signal set:

1: steering angle 2: Vehicle speed

One thing must be mentioned here before the input signal of steering angle is representing the steering angle of tire. But in Carsim it is defined by the angle of steering wheel. So the transmission ration is introduced.

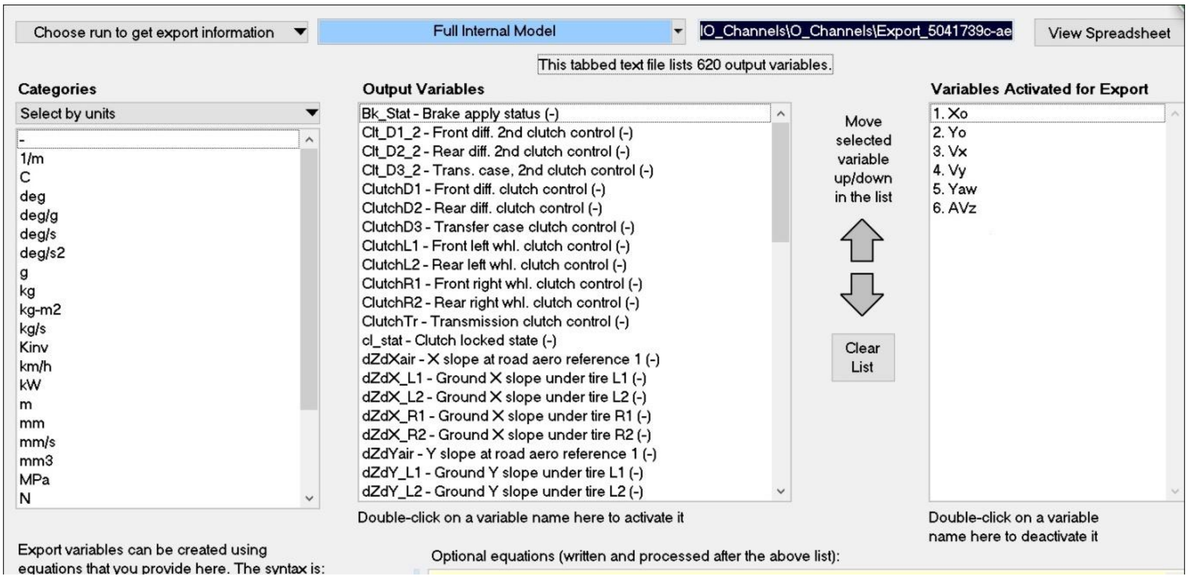


figure 23 output set

Output signal set：

1. Longitudinal position 2.Lateral position

3. Longitudinal velocity 4.Lateral velocity

5. Relative yaw angle 6.Yaw rate

Finally we change the vehicle state function to Carsim s-function, to have more comprehensive and reliable data. As figure 24:

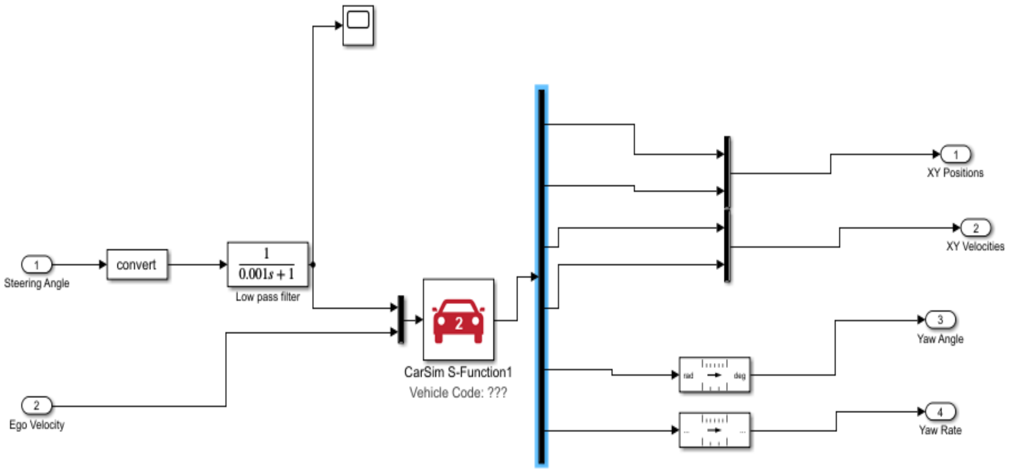


figure 24 carsim s-function

5.3 Benchmarking with Carsim

Here the procedure we choose lane keeping normal driving, to compare the results

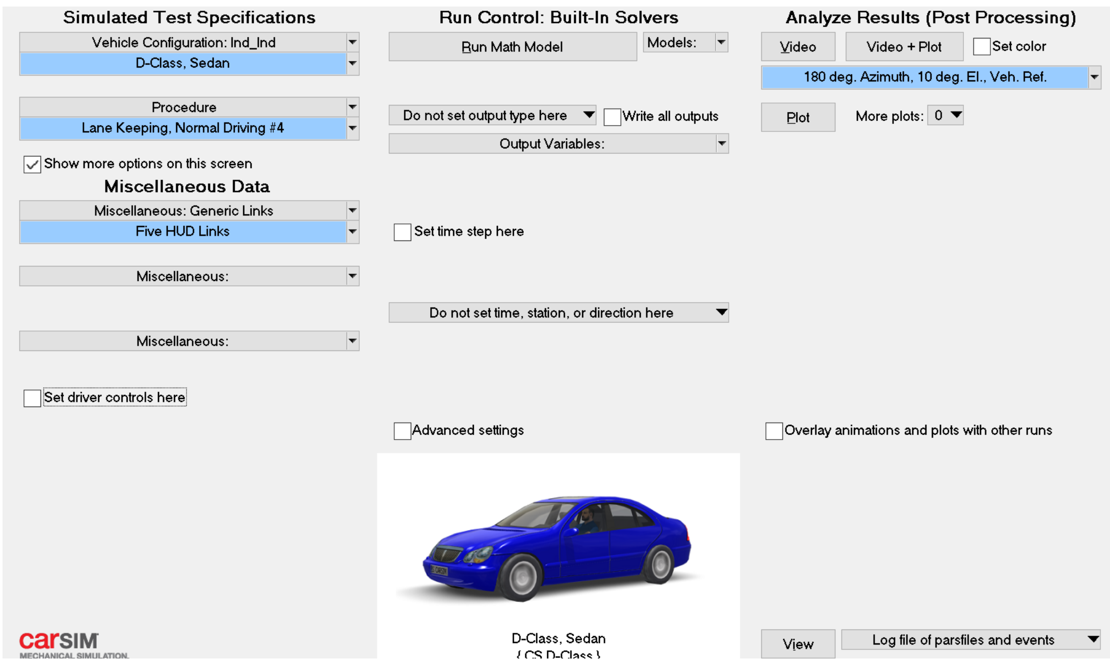


figure 25 carsim lane keeping function

The speed controller can calculate target speed as a function of curvature in the reference target path, combined with driver aggressiveness

The acceleration limits used to determine target speed are based on a skill level and aggressiveness limits

Skill 0 : Ax and Ay are not combined. The target speed is adjusted to allow acceleration either longitudinally or laterally, but never both at once.

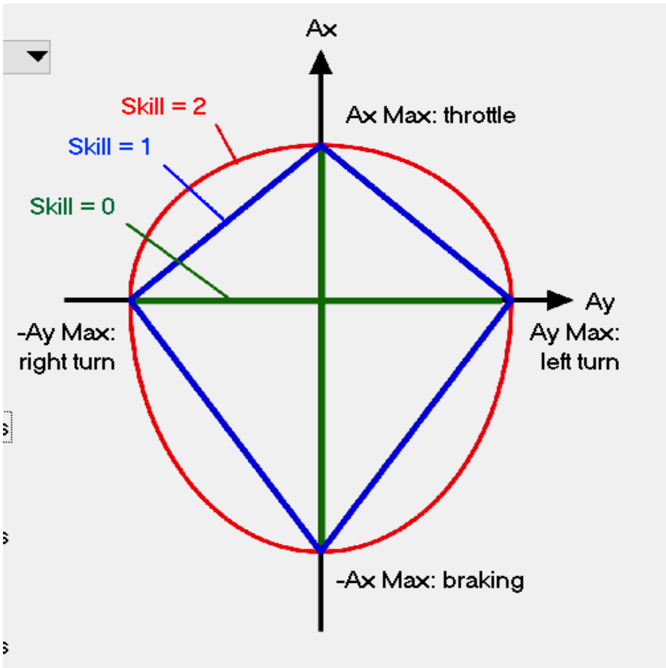


figure 26 acceleration constraints

Skill 1: Ax and Ay are combined using straight lines, allowing some combination of lateral and longitudinal acceleration. However, the combined acceleration does not make use of as much available friction as is used in pure longitudinal or pure lateral acceleration.

Skill 2: Ax and Ay are combined using a friction ellipse, providing a consistent use of available friction regardless of the direction of the total acceleration vector.

After that talking about the path preview length set the data as same as camera configuration and characteristic of lane detection function.

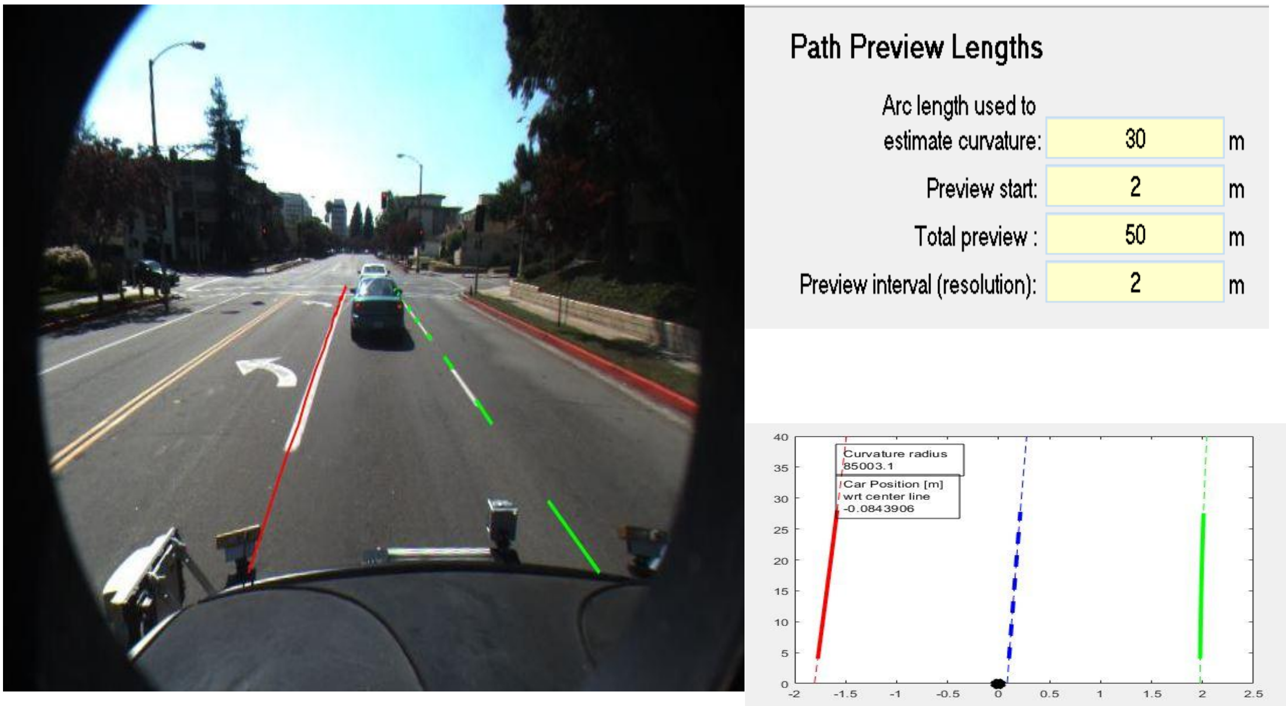


figure 27 preview length

Previewing of the target path is configured with four length parameters

1.Arc length used to estimate curvature: Length of path segment used to calculate curvature at the mid-point of the segment

2. Preview start :The portion of the reference path that is previewed

starts this distance in front of the origin of the vehicle sprung mass coordinate system (typically the origin is at the center of the front axle)

3. Total preview: This defines the portion of the reference path that is previewed. Longer distances sometimes give better results for complicated paths combined with aggressive acceleration settings.

4. Preview interval: Interval for calculating path curvature and target speed over the preview path

Finally generate the same scenario both in matlab and Carsim to compare the results

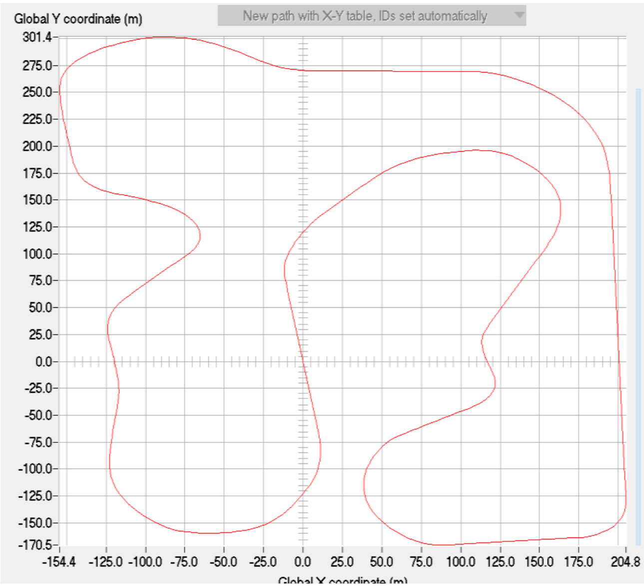


figure 28 scenario in carsim

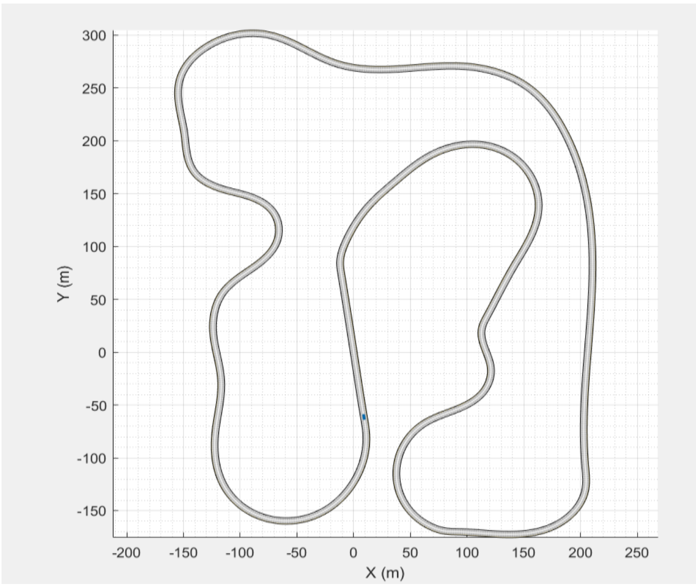
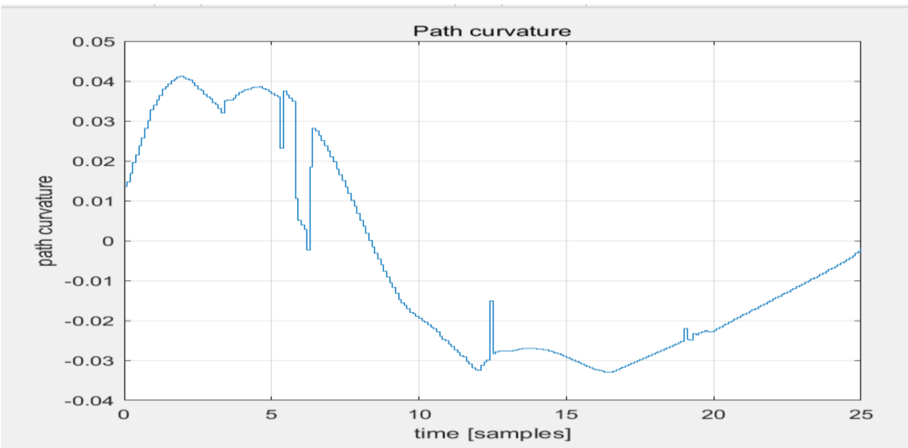
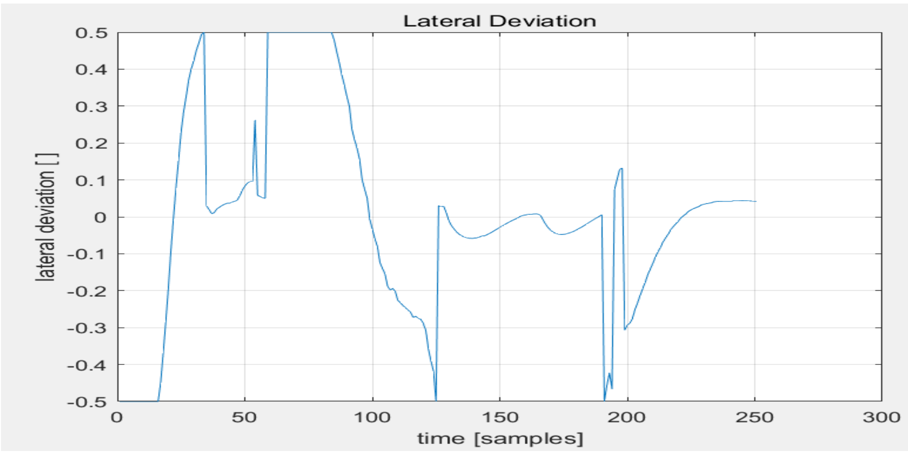


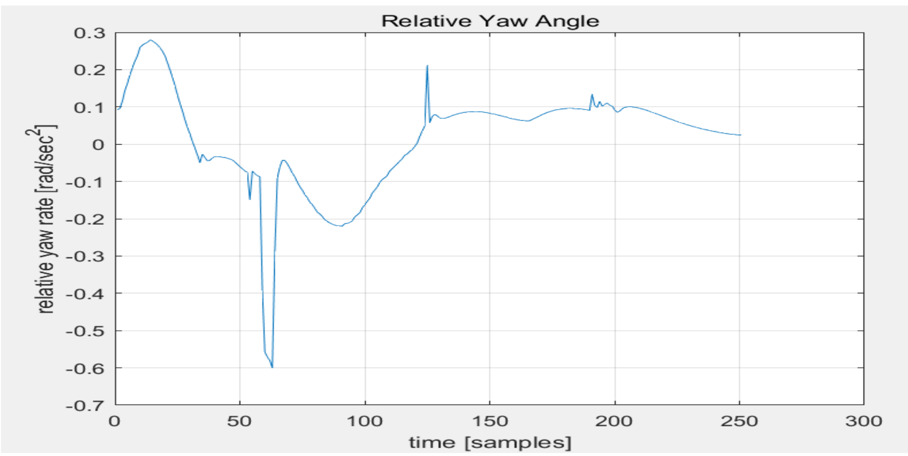
figure 29 scenario in matlab

5.4performance and final result plots

5.4.1 lane keeping assistant system only with lateral control







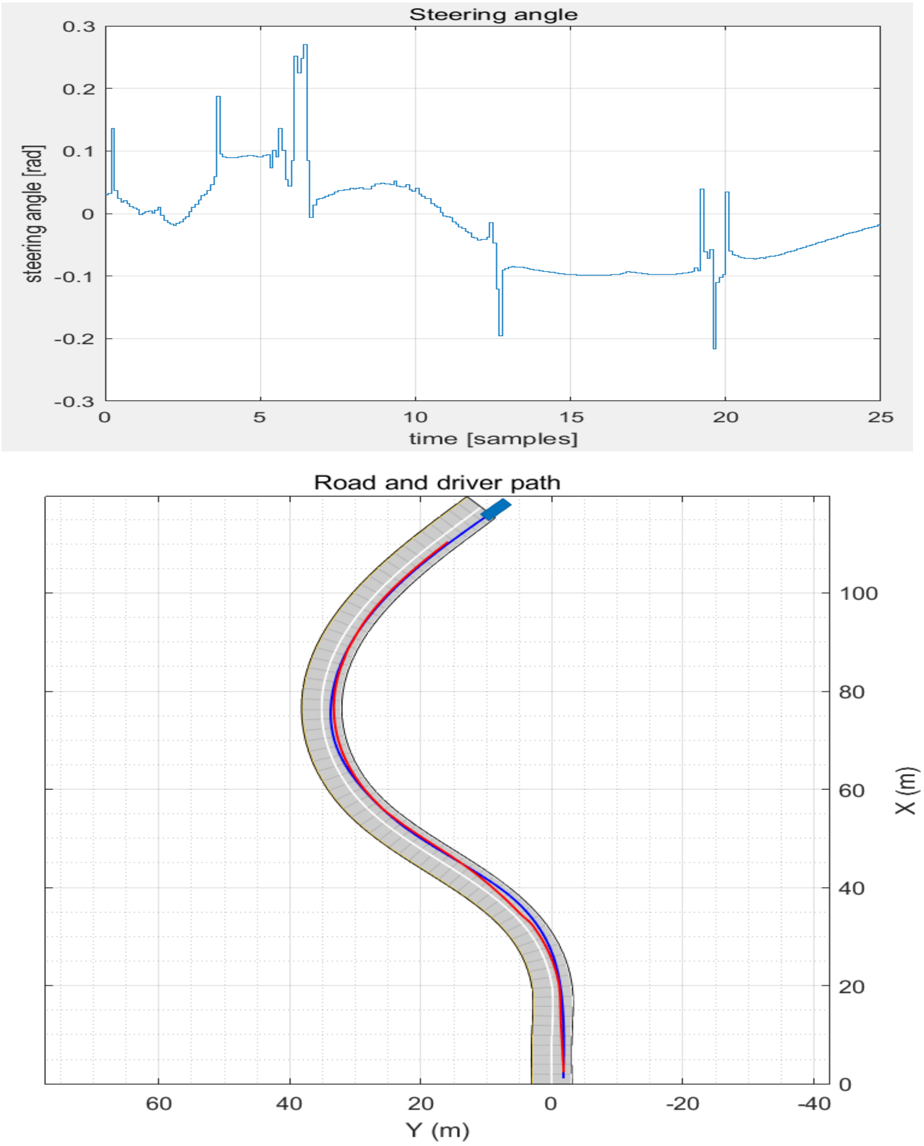
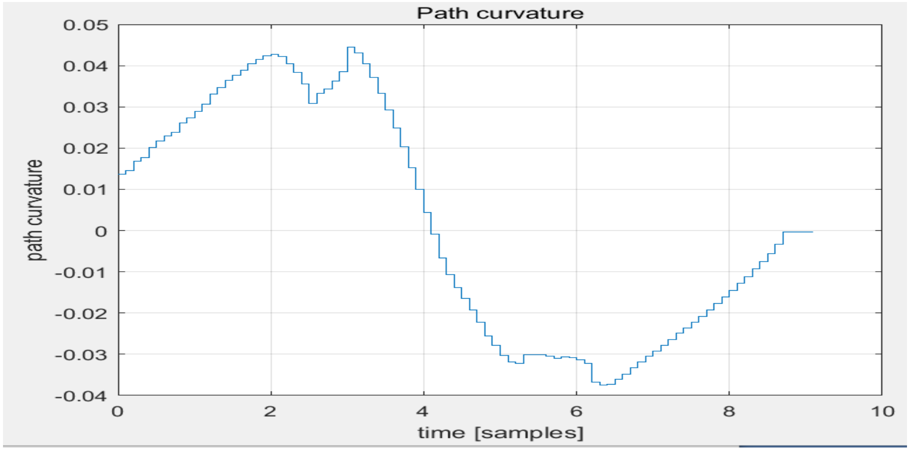


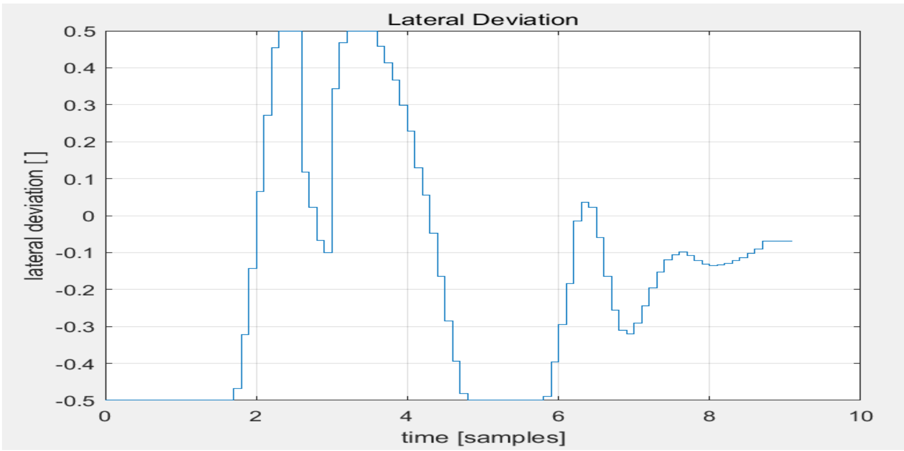
figure 30 performance plot A

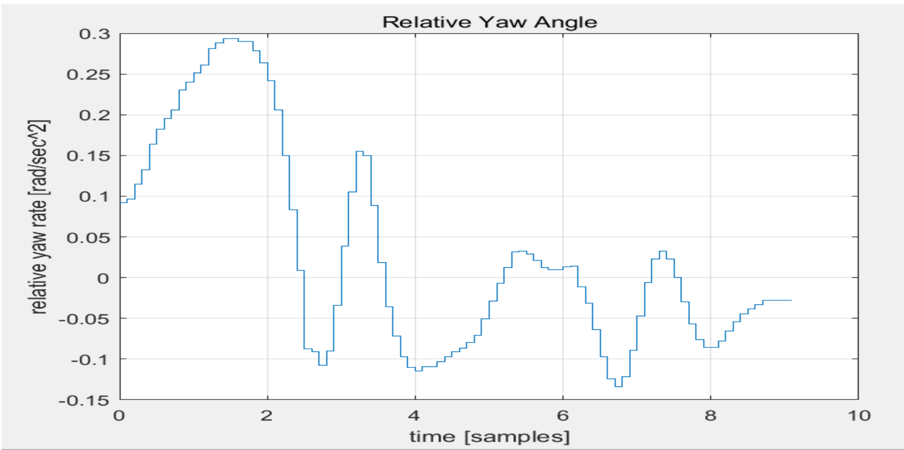
From figure 30, the results of controller are presented such as curvature, lateral deviation, relative yaw angle, steering angle and driver path of autonomous car. In the driver path plot the blue line represents center line of two boundaries the red line is path of vehicle.

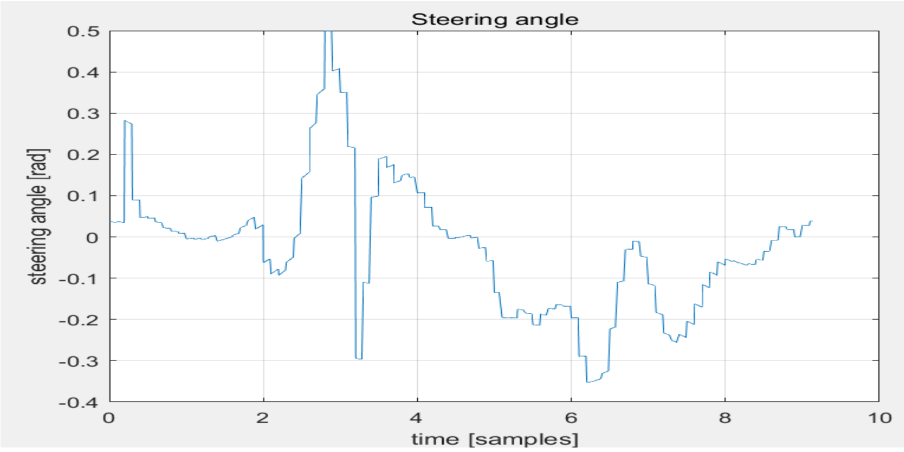
The controller performance is good enough to achieve lane keeping requirement. The steering angle is in the range of [-0.2 +0.3] rad.

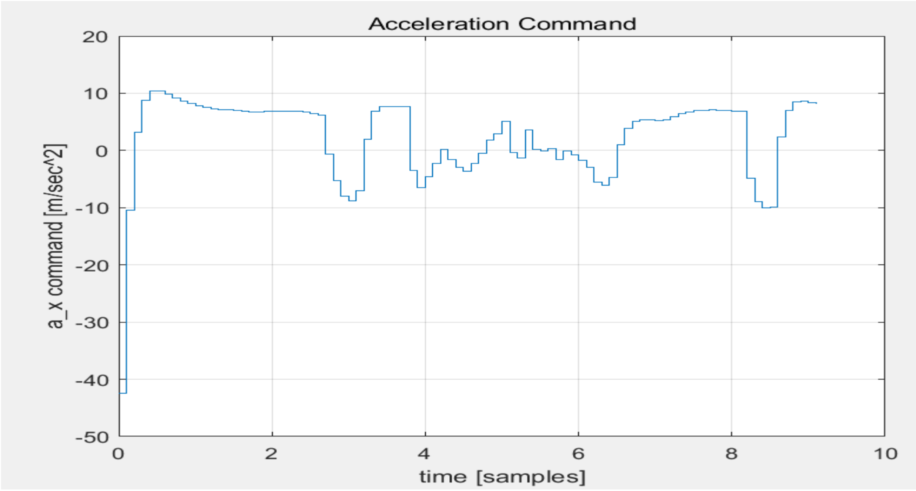
5.4.2 lane keeping assistant system with both longitudinal and lateral control.

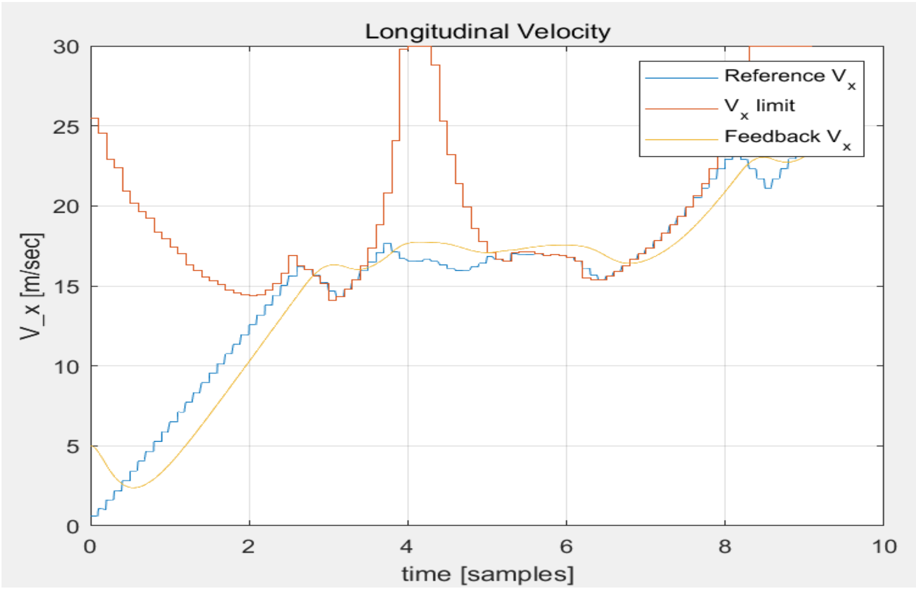







Steering angle
steering angle [rad]
time [samples]


Acceleration Command
a_x command [m/sec^2]
time [samples]


Longitudinal Velocity
V_x [m/sec]
time [samples]
Reference V_x
V_x limit
Feedback V_x

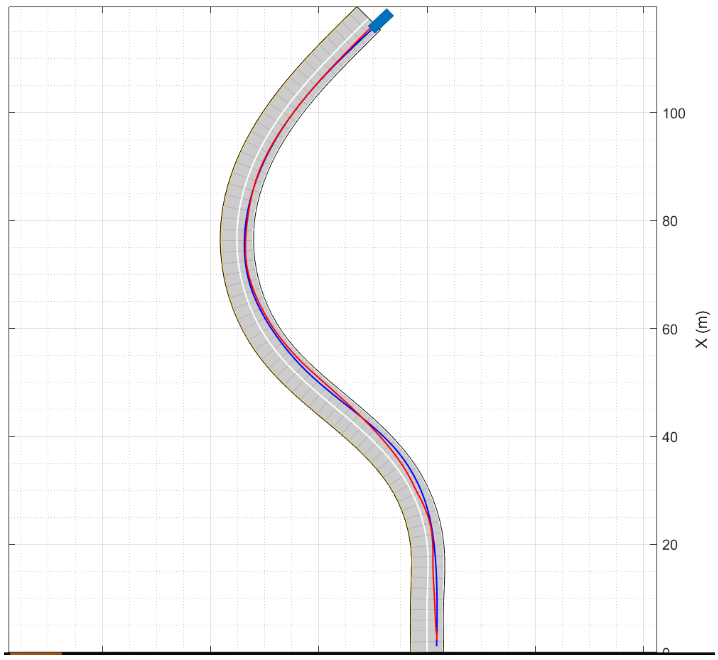
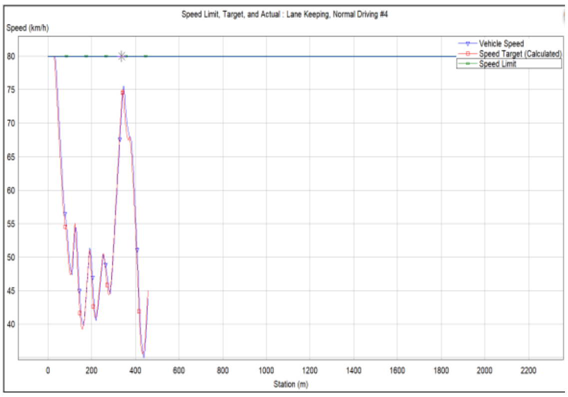
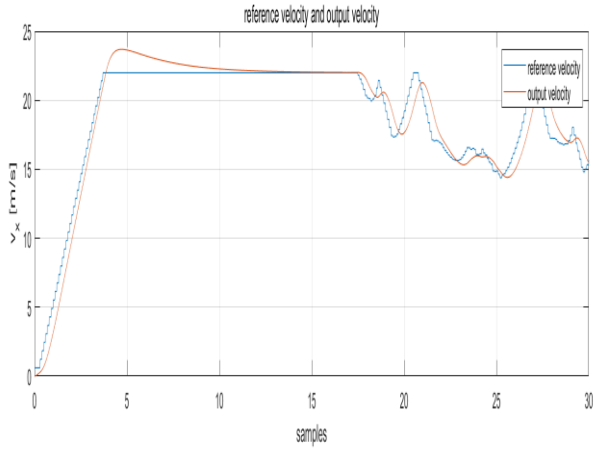


figure 31 performance plot B

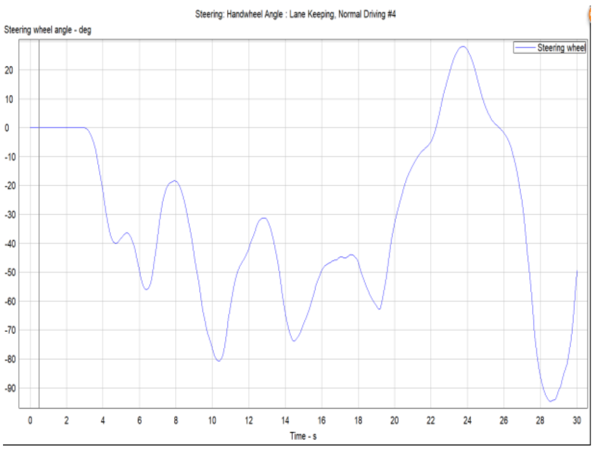
Apart from items introduced before here the acceleration command and longitudinal velocity are included. Based on the curvature the reference velocity is calculated and autonomous driving car follows it. Since the velocity is not constant as before when cross curvature the speed is higher than before. As the result of it the steering angle is in the range of [-0.4 +0.5]

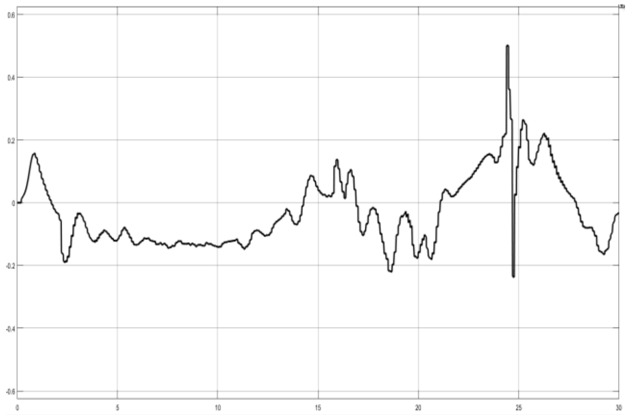
5.4.3 benchmarking with Carsim



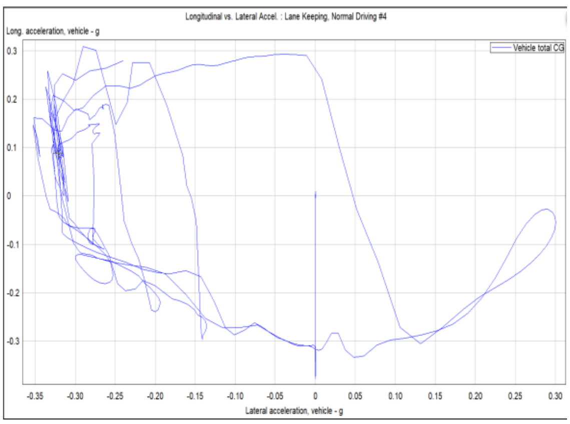


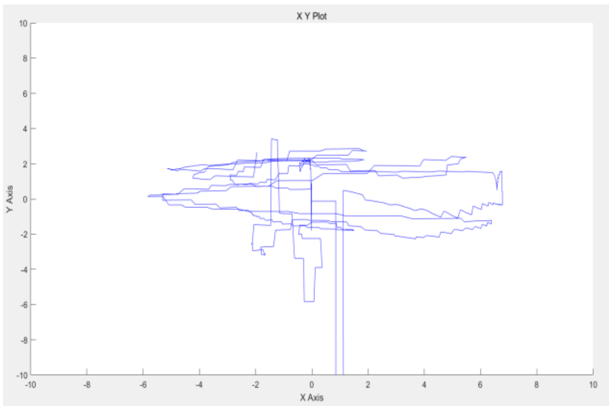
Speed comparison



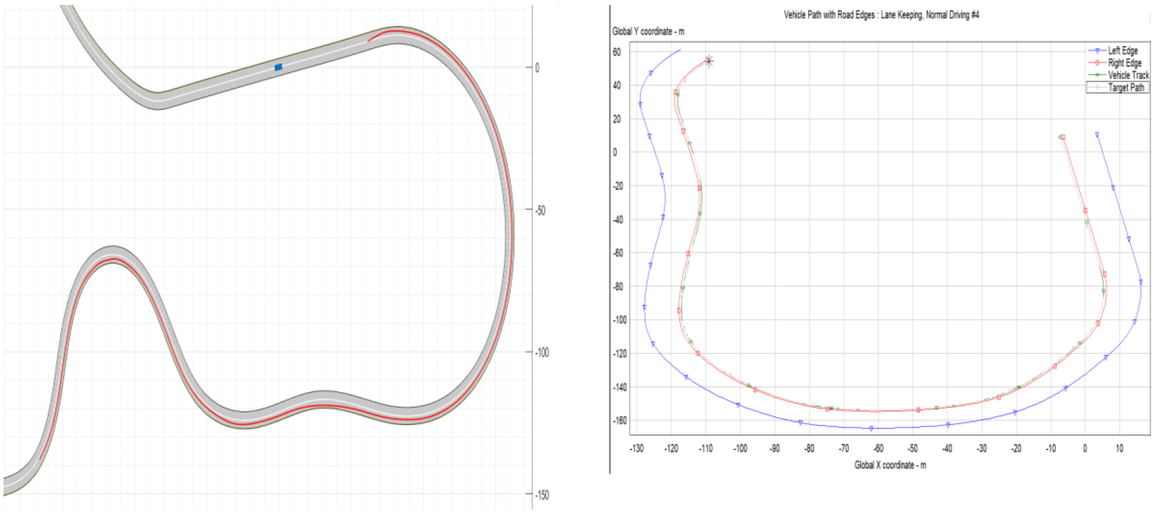


Steering angle comparison





G-G plot comparison



Vehicle path comparison

figure 32 benchmarking reslts comparision

G-G plot, which represents the time history of longitudinal and lateral acceleration, is a main indicator of formula racing car. Here the results of 2 software is quiet same.

Due to the reason of software default set in Carsim the speed profile is starting from speed limit which is 80 km/h since in matlab it starts from 0km/h. For the steering angle matlab has smoother maneuver than Carsim.

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