Imperial College of Science, Technology and Medicine
Department of Aeronautics

Politecnico di Torino
Dipartimento di Ingegneria Meccanica e Aerospaziale

Robust and Reliability-Based Design Optimization of a Composite Floor Beam

Fabrizio Sbaraglia

Supervisors:

Professor Ferri M.H.Aliabadi
Professor Giulio Romeo

Submitted in part fulfilment of the requirements for the degree of
Master of Science in Aerospace Engineering of Politecnico
di Torino September 2018

Abstract

This thesis investigates the advantages and disadvantages of using probabilistic optimization methods in aircraft structural design.
The necessity to achieve a design insensitive to system's variations (robust) and less likely to fail (reliable) is addressed, in order to reduce costs and risk of accidents.
Mathematical formulations of Robust Design Optimization (RDO), Reliability-Based Design Optimization (RBDO) and the hybrid Robust and Reliability-Based Design Optimization (RRBDO) are presented, highlighting the differences between the concepts of robustness and reliability. In these probabilistic formulations, uncertainties due to manufacturing tolerances and material defects are considered.
Fundamentals of optimization are introduced, presenting different search methods and emphasizing the concept of multi-objective optimization. A brief review of statistics and probability basics are presented as well, in order to better understand the stochastic optimization processes.
Because of the complex nature of composite structures, the need of surrogate models to predict structural responses arose. In order to build meta-models, Design of Experiments (DOE) methods are used to determine the location of sampling points in the design space.
Monte Carlo Simulations (MCS), creating random samples, are used to propagate uncertainties from the surrogate model inputs to variations in model outputs. MCS embedded in optimization processes, are used to determine the statistical parameters of the responses and the probability of failure.
Different flowcharts for the three probabilistic methods are developed, in order to better understand, through graphical representations, the design and optimization frameworks.
To validate these frameworks and to show the different results of the various approaches, an application to a composite floor beam is considered.
Different optimization algorithms and surrogate models were compared, in order to speed up the optimization process and reduce modelling errors.
The deterministic design resulted in a not robust and not reliable design. Whereas, stochastic approaches accounting for uncertainties, resulted in enhanced robustness (Robust Design), enhanced reliability (Reliable Design) or a combination of both (Robust and Reliable Design).
Robust Design Optimization resulted globally better in terms of robustness. As for reliability based methods, either RBDO and RRBDO led to reliable designs, but the latter minimizes variability of the responses, hence resulting in a more robust design.

Acknowledgements

I would like to express profound gratitude to my supervisor Professor Ferri M.H. Aliabadi from Imperial College London, which proved to believe in me by setting goals to achieve and constantly pushing me to reach them on my own.
I would like to thank Professor Giulio Romeo from Politecnico di Torino for being helpful, promptly suggesting new material to search, giving advices and for being present also for non-academic concerns.
A sincere gratitude is towards all Imperial College London's staff, which promptly made me feel welcome and put me in an environment in which I could work at my best.
A special thank is for Postdoctoral Research Associate Hamed Farokhi, which gave me insights on most of the software used throughout the thesis. Working with him has been a pleasure either for his strong technical background and for his manners.

I would like to thank Gabriella for being the strongest and at the same time kindest, life advisor and mother, and my father, who always watches over me.
A special thank is for my strong brother Stefano and his wife Elisa, for being constantly present and supporting, and for making me uncle of a wonderful, tiny niece Gioia. I'd like to thank my sweet sister Sara which is always present and gives me unconditional love.

I'd like to express my gratitude to my awesome friends: Domenico which I admire and he never lets me down, Fabio which knows me since ages, Gianni who believes in me more than I do, Marco which is my super-chef friend, Alessio, Salvatore, my lovely cousin Alessia and the profound and sweet Cecilia.
How not to thank Giuseppe and Davide with which I experienced London, sharing thoughts and having fun in so many ways. We constantly stimulated our brains, with Domenico and Cesare as well, towards new challenges and ideas.
Finally, I'd like to thank this whole experience in London, and whoever made it possible, because opened my mind and projected me in a completely brand new, challenging future.

Dedication

To my wonderful family.

‘I cannot teach anybody anything. I can only make them think.’

Socrates

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Motivation and Objectives

As modern structures require more critical and complex designs, the need for accurate approaches to assess uncertainties in computer models, geometry, material properties, manufacturing processes, loads and operational environments has increased significantly.

A number of probabilistic analysis tools have been developed to quantify uncertainties, but the most complex systems are still designed with simplified rules and schemes, such as safety factor design.

Design optimization methods have been applied to the structural design of rockets, satellites and aircraft to the extent of, primarily, reducing the structural weight while satisfying the shrinking design requirements.

However, conventional design procedures, do not directly account for the random nature of most parameters. Thus, in order to compensate for performance variability induced by system variations, a safety factor is introduced. Unfortunately, due to lack of knowledge regarding the scatter of structural performance, especially for composite materials, safety factors specified in current design practice, may lead into a too conservative or too dangerous design [17].

The sources of uncertainty involved in a structural system life-cycle, may be classified into four stages (Fig.1.1):

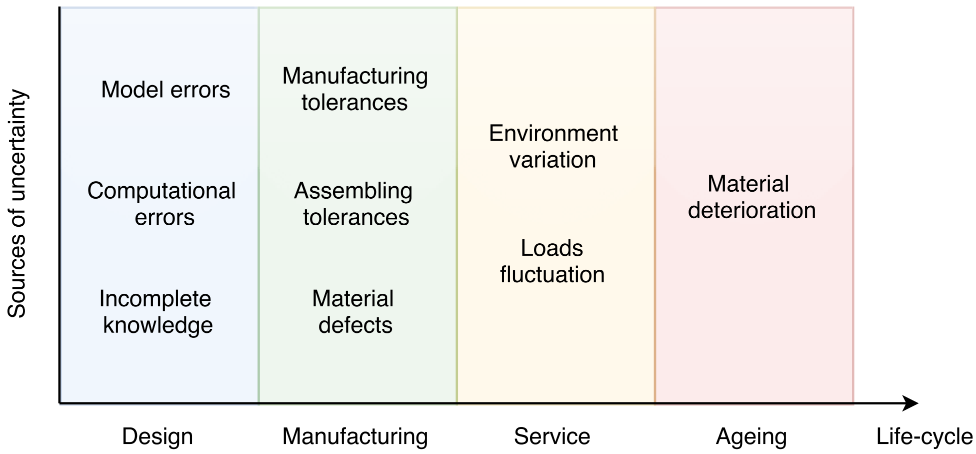


Figure 1.1: Sources of uncertainty.

- system design;
- manufacturing;
- service;
- ageing.

In the first stage, incomplete knowledge of the system and errors in the model are the main sources of uncertainty.

The manufacturing process has the potential for causing a wide range of defects in composite materials, such as fibre misalignment and porosity (presence of small voids in the matrix), leading to variations in material properties. During this stage, changes in the geometry are caused by manufacturing and assembling tolerances.

External load fluctuations and temperature variations are major sources of uncertainty in the service stage, while deterioration of material properties may become crucial to performance variability in the ageing process.

Traditional design optimization techniques tend to over-optimize, producing solution that perform well at the design point but may have poor off-design characteristics. It is important that the design ensures robustness of the structural performance, becoming less sensitive to the random variations induced in different stages of the structure's life-cycle.

In a structural design problem involving uncertainties, a structure designed using a deterministic approach may have a greater probability of failure than a structure of the same cost designed using a probabilistic approach that accounts for uncertainties. Thus becomes crucial the study of structural reliability, which is concerned with the calculation and prediction of the probability of failure at any stage during a structure's life.

The probability of the occurrence of such event is a numerical measure of the chance of its occurring. Once the probability is determined, the next goal is to choose design alternatives that improve structural reliability minimizing the probability of failure.

The main objective of this thesis is to compare two different philosophies of probabilistic optimization and design: Robust Design Optimization (RDO) and Reliability-Based Design Optimization (RBDO), developing a structured procedure to achieve an optimized design.

Particular emphasis will be placed upon the synergy between these two approaches, ultimately leading to a mixed Robust and Reliability-Based Design Optimization formulation (RRBDO).

Finally, these different optimization algorithms, combined with surrogate models, will be applied on a composite floor beam, in order to show a possible application of these design and optimization frameworks to composite components with a large number of degrees of freedom.

1.2 Literature review

Probabilistic design and optimization methods are useful techniques that improve structural performance when applied to the design of composite structures.

These procedures, as previously outlined, can be broadly classified into: Robust Design Optimization (RDO) and Reliability-Based Design Optimization (RBDO).

RDO concerns about reducing the variability of the system performance, while RBDO concentrates on finding an optimal design with low probability of failure. A comprehensive description of robust design can be found in [17], while [6] gives an application of RDO to composite panels.

To obtain a reliable and robust product, a hybrid algorithm named Robust and Reliability-Based Design Optimization (RRBDO) exploits both RDO and RBDO techniques to search for robust optima while obeying reliability-based constraints. A number of RRBDO methods have been reported for structural optimization in the past years [25] [3] [19].

Stochastic design optimization employing Monte Carlo simulation (MCS) is known as the most suitable methodology, which can directly calculate the probability of failure [27]. Lagaros et al. took into account the probabilistic constraints using MCS method combined with Latin hypercube sampling [19].

Nevertheless, these methodologies are very time-consuming and, except for simple cases, they often reach an unacceptable computational cost, due to multiple evaluations of implicit functions required to obtain the structural response. This is the one of the reasons why reliability-based design optimization is not a dominant design technique in the field of composite structures.

Surrogate methods [16] [23] [20] [12] allow the transformation of a complex implicit model into an analytic approximation that decreases in several orders of magnitude the computational cost without a significant loss of accuracy.

Metamodeling techniques have been widely used for design evaluation and optimization in many engineering applications; a comprehensive review of metamodeling applications in mechanical and aerospace systems can be found in [28].

Finally, to search for the optimal design, genetic algorithms (GA) [15] [33] are preferred, due to its extensive application in the industry, tested performance and availability in commercial software.

Chapter 2

Structural optimization

Structural optimization methods are widely used in the design of engineering structures for the purpose of improving the structural performance.

The use of structural optimization has rapidly increased during the last decades, mainly due to the developments of refined computing techniques and large-scale applications of the finite element method.

In an optimization problem, the *design variables* are the entities that define a particular design. The type of these variables (continuous, discrete, integer, mixed) is extremely important in identifying and setting up the optimal design problem.

In the search for the optimal design, the values of these entities will change over a prescribed range, bounded by lower and upper limits (*side constraints*). The set of design variables constitutes the *design vector*. The values that don't change as different designs are generated, are called *design parameters*.

The function that drives the search for the optimal design is called *objective function*. Generally, to ensure that the design will exist and interact well with its operating environment, a number of restrictions must be satisfied in a structural design problem.

These restrictions define the feasible domain in the design space and are referred to as *design constraints*.

In a design optimization problem, the objective function and the constraints are often expressed as implicit functions of the design variables and the evaluation of these functions generally involve numerical simulations such as by the finite element method.

2.1 Deterministic Optimization - DO

2.1.1 Mathematical formulation

The classical mathematical model for a deterministic optimization problem is expressed as:

$$\begin{aligned} &\mathit{find} && \mathbf{d} \\ &\mathit{minimizing} && f(\mathbf{d}) \\ &\mathit{subject\ to} && g_i(\mathbf{d}) \leq 0 \qquad (i = 1, 2, ..., k), \\ & && \mathbf{d}_L \leq \mathbf{d} \leq \mathbf{d}_U \end{aligned} \tag{2.1}$$

where $\mathbf{d}$ is the vector of design variables (*design vector*), $f(\mathbf{d})$ is the objective function, $g_i(\mathbf{d})$ is the $i - th$ inequality constraint, $\mathbf{d}_L$ and $\mathbf{d}_U$ are respectively the lower and upper bounds of the design variables (*side constraints*).

The design variables for an optimization problem involving composite structures can be: the geometrical dimensions, the number of layers, the layers orientation etc.

The objective and constraint functions can be the cost, mass, maximum displacements, maximum stresses, natural frequencies and others.

In the deterministic formulation of a structural optimization problem, the design variables and the design parameters are considered deterministic (with no scatter) and the objective function as well as the constraints are referred to their nominal values.

2.2 Robust Design Optimization - RDO

2.2.1 Introduction

As the engineering environment becomes extremely competitive, better quality products are required in industries. Unexpected deviations from the design that a designer initially intended for are caused by variations at different stages of the service life-cycle. Such scatters may not only significantly worsen the structural quality and cause deviations from the desired performance, but may also add to the structural life-cycle costs, including inspection, repair and other maintenance costs. Robust design is to prevent such phenomena, in fact it has been developed to improve product quality in industrial engineering.

To decrease the scatter of the structural performance, one possible way is to reduce or even to eliminate the scatter of the input parameters, which may either be practically impossible or add much to the total costs of the structure; another way is to find a design in which the structural performance is less sensitive to the variation of parameters without eliminating the cause of parameter variations, as in robust design.

Three stages of engineering design are identified in the literatures: conceptual design, parameter design and tolerance design. Robust design may be involved in the stages of parameter design and tolerance design.

Taguchi, who is the pioneer of robust design, said " Robustness is the state where the technology, product, or process performance is minimally sensitive to factors causing variability (either in the manufacturing or user's environment) and ageing at the lowest unit manufacturing cost ". [31]

Suh stated " Robust design is defined as the design that satisfies the functional requirements even though the design parameters and the process variables have large tolerances for ease of manufacturing and assembly. This definition of robust design states that the information content is minimized ". [30]

Box said that " Robustifying a product is the process of defining its specifications to minimize the product's sensitivity to variation ". [8]

Although different expressions are used, their meanings are similar, which is that robust design is a design insensitive to variations.

In Robust Design Optimization the concept of robustness is embedded into conventional optimization process.

2.2.2 Concept of Robustness

The concept of robustness is schematically illustrated in Fig.2.1. The horizontal axis represents the value of a structural performance (or cost) function f, which is required to be minimized. The two curves show the distributions of the occurrence frequency of the value of f corresponding to two different designs, when the system parameters are randomly perturbed from the nominal values. In the figure, μ_1 and μ_2 represents, respectively, the mean values of the performance function f for the two designs. Although the first design exhibits a smaller mean value of the cost function, the second design is desirable from the robustness point of view, since it has much less sensitivity to variations.

The principle behind the structural robust design is that, the quality of a design is justified not only by the mean value but also by the variability of the structural performance. For the optimal design of structures with stochastic parameters, one straightforward way is to define the optimality conditions of the problems on the basis of mean values of the performance function. However, the design which minimizes the expected value of the objective function may be still sensitive to the fluctuation of the variable parameters and this raises the need of robustness of the design.

Another way to understand the concept of robustness is to show the relation between the objective function and its design variable (let's suppose we have just one design variable). From Fig.2.2 can be observed that, for a prescribed variation of the input (Δx), the variation of the objective function (Δf) is larger for the design characterized by the global optimum ($\Delta f_1 > \Delta f_2$), whereas to the robust optimum corresponds a smaller variation of the objective function.

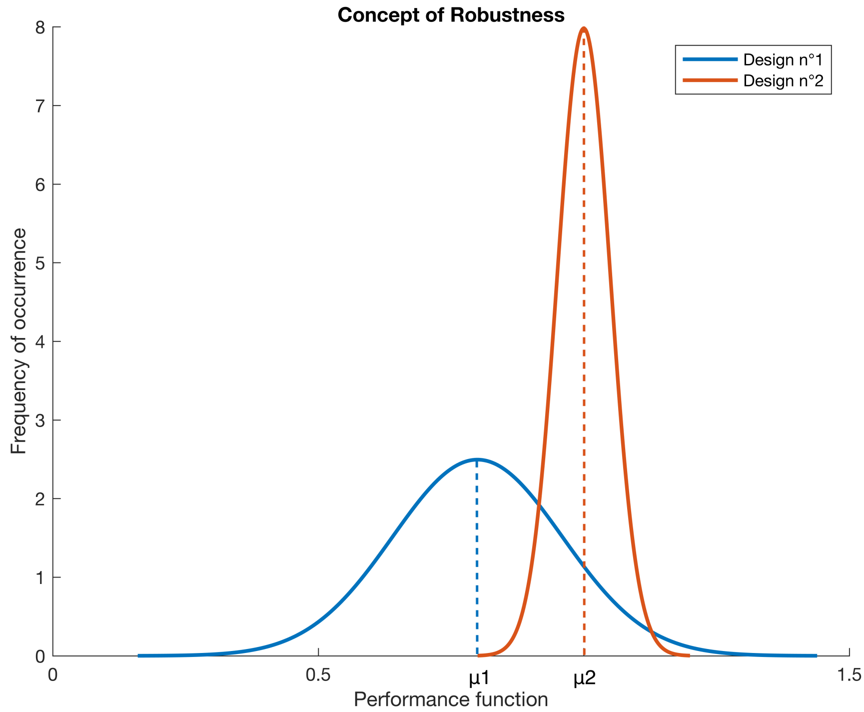


Figure 2.1: Concept of robustness.

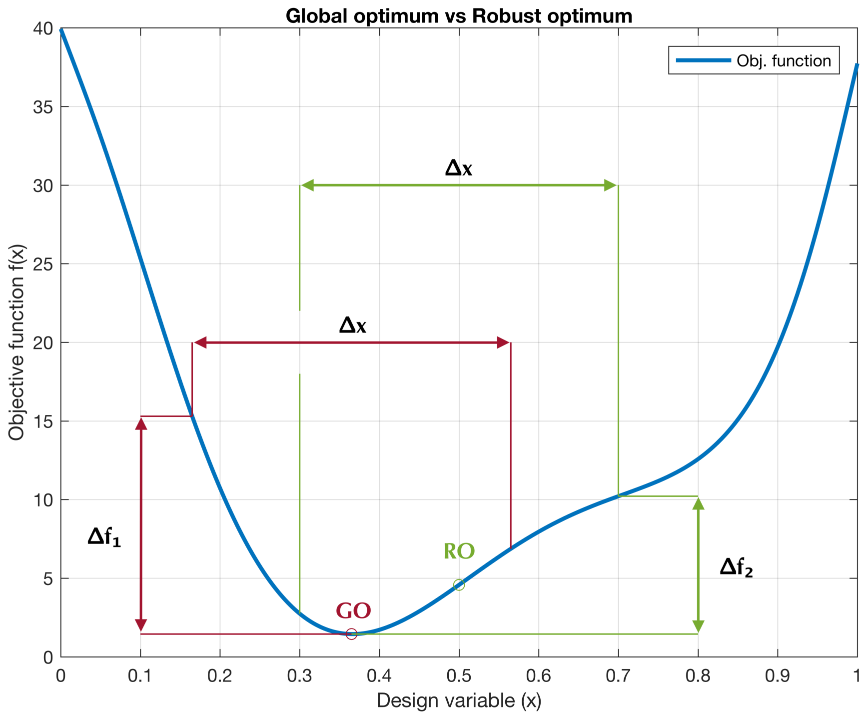


Figure 2.2: Differences between global optimum and robust optimum.

2.2.3 Mathematical formulation

The general mathematical formulation of a Robust Design Optimization problem can be stated as:

$$
\begin{aligned}
&\textit{find} && \mathbf{d} \\
&\textit{minimizing} && \{\mu\left(f(\mathbf{d})\right), \sigma\left(f(\mathbf{d})\right)\} \\
&\textit{subject to} && \mu\left(g_i(\mathbf{d})\right) + \beta_i \sigma\left(g_i(\mathbf{d})\right) \leq 0 \qquad (i = 1, 2, ..., k), \\
& && \sigma\left(h_j(\mathbf{d})\right) \leq \sigma_j^+ \qquad (j = 1, 2, ..., l), \\
& && \mathbf{d}_L \leq \mathbf{d} \leq \mathbf{d}_U
\end{aligned}
\tag{2.2}
$$

Where $\mu\left(f(\mathbf{d})\right)$ is the mean value (first statistical moment) of the objective function and $\sigma\left(f(\mathbf{d})\right)$ is the standard deviation (second statistical moment) of the objective function. Likewise $\mu\left(g_i(\mathbf{d})\right)$ and $\sigma\left(g_i(\mathbf{d})\right)$ are respectively the mean value and standard deviation of the i-th constraint function. $\sigma\left(h_j(\mathbf{d})\right)$ represents the j-th constraint on standard deviation of the response, σ_j^+ is its upper limit. The quantity $\beta_i > 0$ is a prescribed feasibility index for the i-th original constraint.

To better understand the nature of the constraints in a RDO problem a schematic representation of them is given in Fig.2.3.

In Fig.2.3, g is the structural performance function and PDF is the probability density function of g. We can observe how the first design violates the constraint $(\mu\left(g(\mathbf{d}_1)\right) + \beta\sigma\left(g(\mathbf{d}_1)\right) > 0)$, while the latter satisfies it $(\mu\left(g(\mathbf{d}_2)\right) + \beta\sigma\left(g(\mathbf{d}_2)\right) < 0)$. In this RDO formulation, the robust structural optimization problem is shown to be an optimum vector problem in which two criteria namely the statistical mean $(\mu\left(f(\mathbf{d})\right))$ and the standard deviation $(\sigma\left(f(\mathbf{d})\right))$ of the objective are to be optimized.

The feasibility index is a convenient measure of the robustness considering the design restrictions. It must not be confused with the reliability index used in RBDO employing the First Order Reliability Method (FORM) for reliability analysis, whereas in the RBDO the response

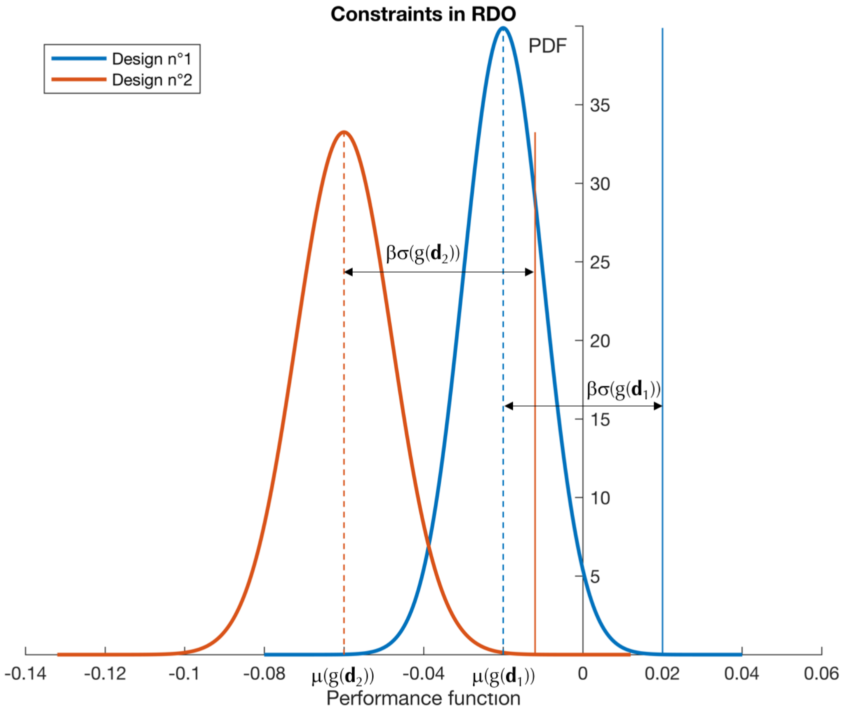


Figure 2.3: Constraints in RDO.

moments are evaluated at the most probable failure point (MPP), by which a sub-optimization loop is required for locating such a point.

2.3 Reliability-Based Design optimization - RBDO

2.3.1 Introduction

Reliability-based design optimization combines a conventional deterministic design optimization algorithm with a reliability analysis method to evaluate the design constraints that depend on the random variables of the problem.

In a RBDO problem both design variables and parameters can contain deterministic and/or random quantities.

When the occurrence of catastrophic failure of a structural component is critical in a structural system, the design optimization problem is usually defined as a problem of reliability-based design optimization.

2.3.2 Mathematical formulation

From the mathematical point of view RBDO can be defined as:

$$\begin{aligned} find \quad & \mathbf{d} \\ minimizing \quad & f(\mathbf{d}) \\ subject\ to \quad & P[g_i(\mathbf{d}) \geq 0] - P_{allow_i} \leq 0 \qquad (i = 1, 2, ..., k), \\ & \mathbf{d}_L \leq \mathbf{d} \leq \mathbf{d}_U \end{aligned} \tag{2.3}$$

where $P[g_i(\mathbf{d}) \geq 0]$ is the probability of failure (P_{f_i}), P_{allow_i} is the allowable probability of failure, which is often expressed as:

$$P_{allow_i} = \Phi(-\beta_i)$$

where Φ is the Cumulative Distribution Function of the (0,1) standardized normal distribution and β_i is called the reliability-index.

Determining the probability of failure (P_{f_i}) requires a reliability analysis, which can be exploited

either via sampling through Monte Carlo methods or through techniques such as the First Order Reliability Method (FORM) and the Second Order Reliability Method (SORM). In the First Order Reliability Method (FORM) and the Second Order Reliability Method (SORM), an additional nonlinear constrained optimization is needed to individuate the Most Probable Point of failure (MPP). In RBDO, the large number of function evaluations is computationally more expensive compared to a conventional deterministic design optimization, hence various numerical techniques have been proposed to cut down the computational cost [14].
As mentioned in [7], inadequate assumptions on the probabilistic distribution of variables and design parameters, may lead to considerable errors in the reliability analysis.

2.4 Differences between RDO and RBDO

Robust Design Optimization and Reliability-Based Design Optimization aim at incorporating random performance variations into the optimal design process, and therefore sometimes they are not clearly distinguished in the literature. The two approaches are different in some crucial aspects.
First of all, the structural robustness is assessed by the measure of the performance variability around the mean value, evaluating its standard deviation. On the other hand, reliability is concerned about the probability of failure occurrence.
RBDO is about satisfying reliability requirements under known probabilistic distributions of the input, while RDO aims at reducing the structural performances variability to unexpected variations.
In RBDO, the cost function of the problem is to be minimized under observance of probabilistic constraints. However, in RDO, the objective functions usually involve performance variations, and the design constraints can be simply defined by limits on the standard deviation of the performance function.
RDO is often achieved by reducing the performance variability (Fig.2.4), while RBDO is usually accomplished by moving the mean of the performance function (Fig.2.5).

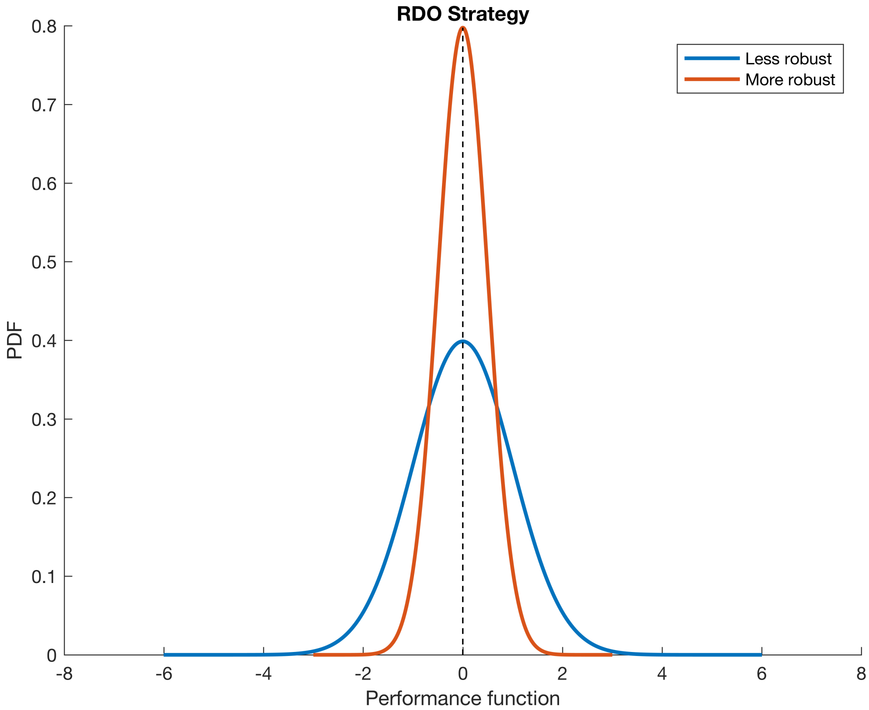


Figure 2.4: RDO strategy.

As previously stated, both methods incorporate the effect of uncertainties into the design optimization process of structural systems. The main difference between RDO and RBDO is the area of interest of the response distribution function.
RDO methods require stochastic analysis tools to approximate the influence of stochastic variations about the mean design of a system, RBDO methods generally require stochastic analysis tools that can predict the likelihood of extreme events at the tails of the response distribution (Fig. 2.6).

Additionally, RBDO requires more computational effort for its reliability analysis, while RDO usually involves only calculation of basic statistical parameters of the performance function, such as its standard deviation.
In RBDO, particular care is paid on structural safety in the case of extreme events, while in RDO more emphasis is put on the structural behaviour under everyday fluctuations of the system during the whole service life.

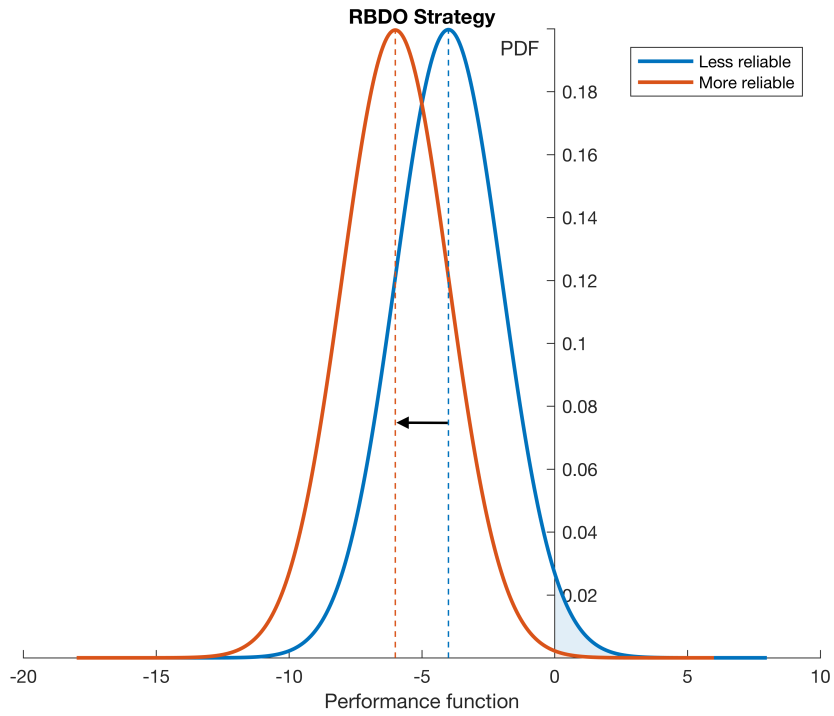


Figure 2.5: RBDO strategy.

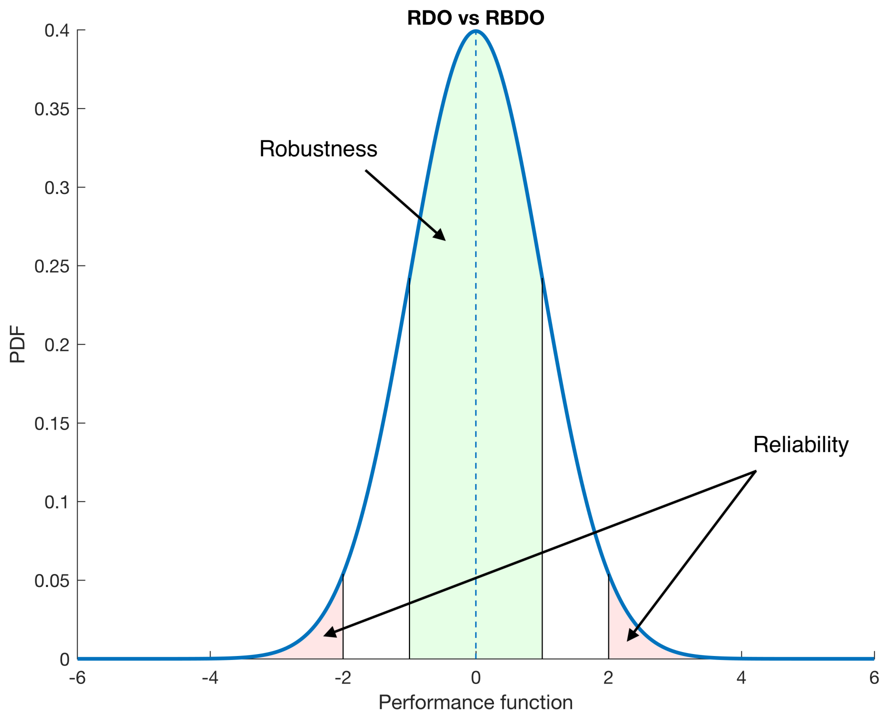


Figure 2.6: Areas of interest for RDO and RBDO.

In [35] Zang et al. presented the different domains of applicability of the two types of problems as shown in Fig.2.7.

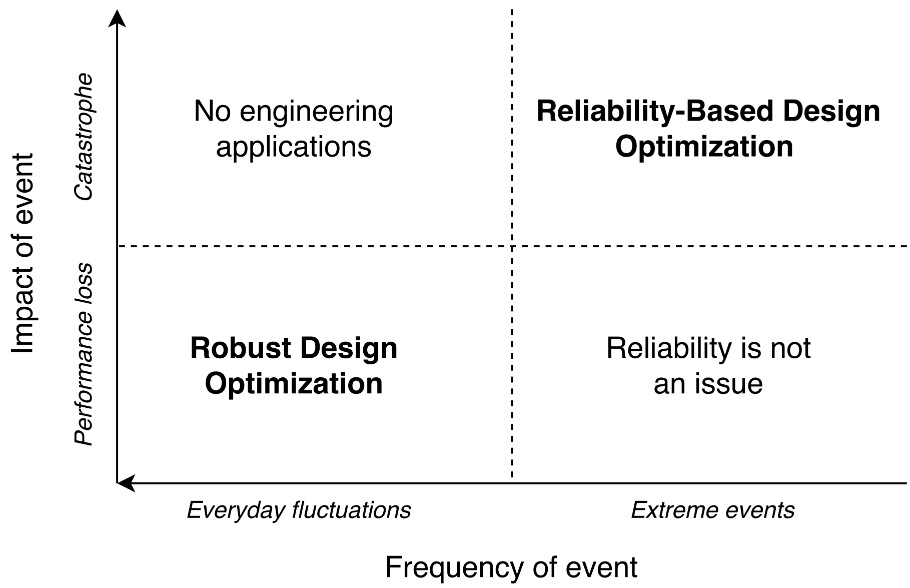


Figure 2.7: Different domains of applicability of RDO and RBDO [35].

The two major factors are the frequency of the event and the impact of the event. No system is viable if everyday fluctuations can lead to catastrophe. Instead, one would like the system to be designed such that the performance is insensitive, i.e., robust, to everyday fluctuations. On the other hand, one would like to ensure that the events that lead to catastrophe are extremely unlikely. This is the domain of reliability-based design.

In both cases, the design risk is a combination of the likelihood of an undesired event and the consequences of that event. An example of risk in the robust design optimization context is the likelihood that the aircraft design will fail to meet the aerodynamic performance targets and will consequently lose sales and perhaps even go bankrupt. An example of risk in the reliability-based design optimization context is the probability that a critical structural component will fail, which could lead to the loss of the vehicle or spacecraft, payload, and passengers, and to potential lawsuits.

2.5 Robust and Reliability-Based Design Optimization - RRBDO

2.5.1 Introduction

The need of a formulation which is both robust and reliable, rises from the high levels of reliability required from aerospace industry to guarantee safety, meanwhile designs insensitive to variations (robust) are needed to remain competitive in a market where quality standards are constantly increasing.

Hence, to obtain a reliable and robust product, a hybrid formulation of RDO and RBDO is addressed, in order to search for robust optima while obeying reliability type of constraints.

2.5.2 Mathematical model

In this mathematical model, the objective function defined for RBDO is replaced by the RDO objective functions, while fulfilling RBDO constraints.

$$
\begin{aligned}
find \quad & \mathbf{d} \\
minimizing \quad & \{\mu\left(f(\mathbf{d})\right), \sigma\left(f(\mathbf{d})\right)\} \\
subject\ to \quad & P[g_i(\mathbf{d}) \geq 0] - P_{allow_i} \leq 0 \qquad (i = 1, 2, ..., k), \\
& \mathbf{d}_L \leq \mathbf{d} \leq \mathbf{d}_U
\end{aligned}
\tag{2.4}
$$

Where $\mu\left(f(\mathbf{d})\right)$ and $\sigma\left(f(\mathbf{d})\right)$ are respectively the mean value and the standard deviation of the objective function. As seen in RBDO formulation, $P[g_i(\mathbf{d}) \geq 0]$ is the probability of failure of the i-th constraint, P_{allow_i} is its allowable probability of failure.

Chapter 3

Fundamentals of optimization

3.1 Optimization vs design

In this chapter, some optimization concepts will be introduced. Generally, the optimization procedure is integrated into the design process, so it's not always clear the difference between them [21].

Design, in general terms, can be defined as the creation of a plan and/or strategy for constructing a physical system or process.

Optimization is the process of maximizing one or more objectives without violating specified design constraints, by regulating a set of variables that influences both the objectives and the design constraints.

3.2 Optimum design problem formulation

It is generally accepted that the proper definition and formulation of a problem take roughly 50 % of the total effort needed to solve it [5]. Therefore, it is critical to follow well-defined procedures for formulating design optimization problems.

The importance of properly formulating a design optimization problem must be stressed because the optimum solution will only be as good as the formulation. Here's a five-step formulation procedure from [5]:

- Step 1 : Project/problem description;
- Step 2 : Data and information collection;
- Step 3 : Definition of design variables;
- Step 4 : Optimization criterion;
- Step 5 : Formulation of constraints.

The formulation process begins by developing a descriptive statement which describes the overall objectives of the project and the requirements that shall be met.

To develop a mathematical formulation for the problem, information on the system must be gathered. Besides, assumptions about modelling of the problem need to be made in order to formulate and solve it.

In Step 3, the number of independent design variables gives the design's degrees of freedom for the problem. Once the design variables are given numerical values we have a design of a the system.

The optimization criterion must be a scalar function whose numerical value can be obtained once a design is specified; that is, it must be a function of the design vector.

Finally, restrictions on the design must be addressed.

3.3 Optimization methods

In an optimization process, we can broadly divide the methods to search for the optimum in 3 categories:

- Gradient-based search methods;

- Direct search methods;
- Nature-inspired search methods.

3.3.1 Gradient-based search methods

These methods, as the name implies, use gradients of the problem functions to perform the search for the optimum point. Therefore, all of the problem functions are assumed to be smooth and at least twice continuously differentiable everywhere in the feasible design space.

Also, the design variables are assumed to be continuous that can have any value in their allowable ranges.

The gradient-based methods have been developed extensively since the 1950s, and many good ones are available to solve smooth nonlinear optimization problems. Since these methods use only local information (functions and their gradients at a point) in their search process, they converge only to a local minimum point for the cost function.

However, based on these methods strategies have been developed to search for global minimum points for the cost function.

3.3.2 Direct search methods

The term "direct search methods" was introduced by Hooke and Jeeves (1961) and refers to methods that do not require derivatives of the functions in their search strategy. This means that the methods can be used for problems where the derivatives are expensive to calculate or are unavailable due to lack of differentiability of the functions. However, convergence of the methods can be proved if functions are assumed to be continuous and differentiable.
The methods were developed in 1960s and 1970s. They have been employed quite regularly since then because of their simplicity and ease of use.

3.3.3 Nature-Inspired Search Methods

Nature-inspired methods also use only the function values in their search process. Problem functions need not to be differentiable or even continuous. The methods, developed since 1980s, use stochastic ideas in their search. Many methods have been developed and evaluated. It turns out that they tend to converge to a global minimum point for the cost function as opposed to a local minimum as with gradient-based methods. Another good feature of the methods is that they are more general than gradient-based methods because they can be used for problems with continuous, integer, and mixed variables. Their main drawback is that they are slower than gradient-based methods.

Some of the most common nature-inspired search methods are: Genetic Algorithms (GA), Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO).

Genetic Algorithms (GA)

Genetic algorithms loosely parallel biological evolution and are based on Darwin's theory of natural selection. The specific mechanics of the algorithm uses the language of microbiology, and its implementation mimics genetic operations.

The basic idea of the approach is to start with a set of designs, randomly generated using the allowable values for each design variable. To each design is also assigned a fitness value, usually using the objective function for unconstrained problems or a penalty function for constrained problems.

From the current set of designs, a subset is selected randomly with a bias allocated to more fit members of the set. Random processes are used to generate new designs using the selected subset of designs. The size of the design set is kept fixed. Since more fit members of the set are used to create new designs, the successive sets of designs have a higher probability of having designs with better fitness values. The process is continued until a stopping criterion is met.

To better understand how genetic algorithms work, various terms associated with the algorithm must be defined and explained:

- **Population**: The set of design points at the current iteration is called a population. It represents a group of designs as potential solution points. Np = number of designs in a population; this is also called the population size.

- **Generation**: An iteration of the genetic algorithm is called a generation. A generation has a population of size Np that is manipulated in a genetic algorithm.

- **Chromosome**: This term is used to represent a design point. Thus a chromosome represents a design of the system, whether feasible or infeasible. It contains values for all the design variables of the system.

- **Gene**: This term is used for a scalar component of the design vector; that is, it represents the value of a particular design variable.

The basic idea of a genetic algorithm is to generate a new set of designs (population) from the current set such that the average fitness of the population is improved. The process is continued until a stopping criterion is satisfied or the number of iterations exceeds a prescribed limit. Three genetic operators are used to accomplish this task:

- **Reproduction**: It's an operator where an old design is copied into the new population according to the design's fitness. There are many different strategies to implement this reproduction operator. This is also called the selection process.

- **Crossover**: Corresponds to allowing selected members of the new population to exchange characteristics of their designs among themselves. Crossover entails selection of starting and ending positions on a pair of randomly selected strings (called mating strings), and simply exchanging the string of 0s and 1s between these positions.

- **Mutation**: It's the third step that safeguards the process from a complete premature loss of valuable genetic material during reproduction and crossover. In terms of a binary string, this step corresponds to selection of a few members of the population, determining a location on the strings at random, and switching the 0 to 1 or vice versa.

The preceding three steps are repeated for successive generations of the population until no further improvement in fitness is attainable. The member in this generation with the highest level of fitness is taken as the *optimum design*.

Decisions made in most computational steps of the algorithms are based on random number generation. Therefore, executed at different times, the algorithm can lead to a different sequence of designs and a different problem solution even with the same initial conditions.
Continuity or differentiability of the problem functions is neither required nor used in calculations of the algorithms. Therefore, the algorithms are very general and can be applied to all kinds of problems.
In addition, the methods determine global optimum solutions as opposed to the local optimum ones determined by a derivative-based optimization algorithm.
These methods are relatively easy to use and program since they do not require the use of gradients of objective or constraint functions.
The main drawbacks of genetic algorithms are as follows:

1 They require a large amount of calculation for even reasonably sized problems or for problems where evaluation of functions itself requires massive calculation;

2 There is no absolute guarantee that a global solution has been obtained.

The first drawback can be overcome to some extent by the use of parallel computing. The second drawback can be overcome by executing the algorithm several times and allowing it to run longer.

3.4 Multi-Objective optimization

There are many engineering applications where the designer may want to optimize two or more objective functions simultaneously, e.g. minimizing mass while minimizing stress, minimizing the mean value and the standard deviation of mass (as seen in RDO). These are called multi-objective, multi-criteria, or vector optimization problems.

The main target in a single-objective optimization is to find a solution that minimizes the cost function. On the contrary, the process of determining a solution for a multi-objective optimization problem is slightly more complex and it is often characterized by conflicting objectives. Therefore, a multi-criteria optimization gives rise to a set of optimal solutions.

In this set, a solution is called Pareto optimal if there is no other solution that reduces at least one objective function without increasing another one.

This set of optimal solutions is known as Pareto Optimal set (Fig.3.1).

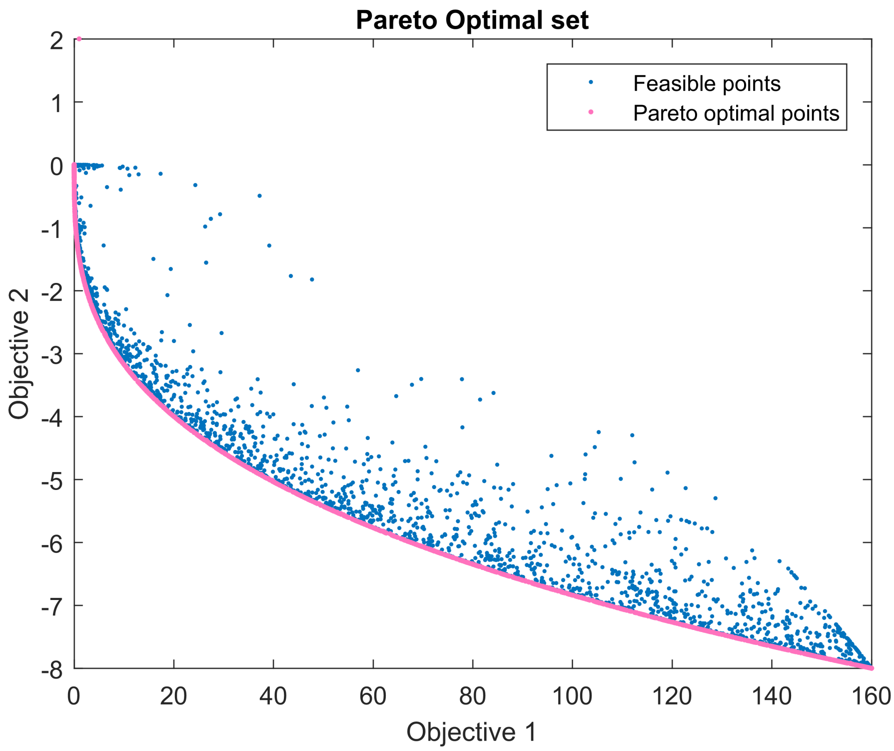


Figure 3.1: Pareto Optimal set minimizing objective 1 and 2.

One property commonly considered as necessary for any candidate solution to the multiobjective problem is that the solution is not dominated. The Pareto set consists of solutions that are not dominated by any other solution. A solution A is said to dominate B if A is better or equal to B in all attributes, and strictly better in at least one attribute [4].

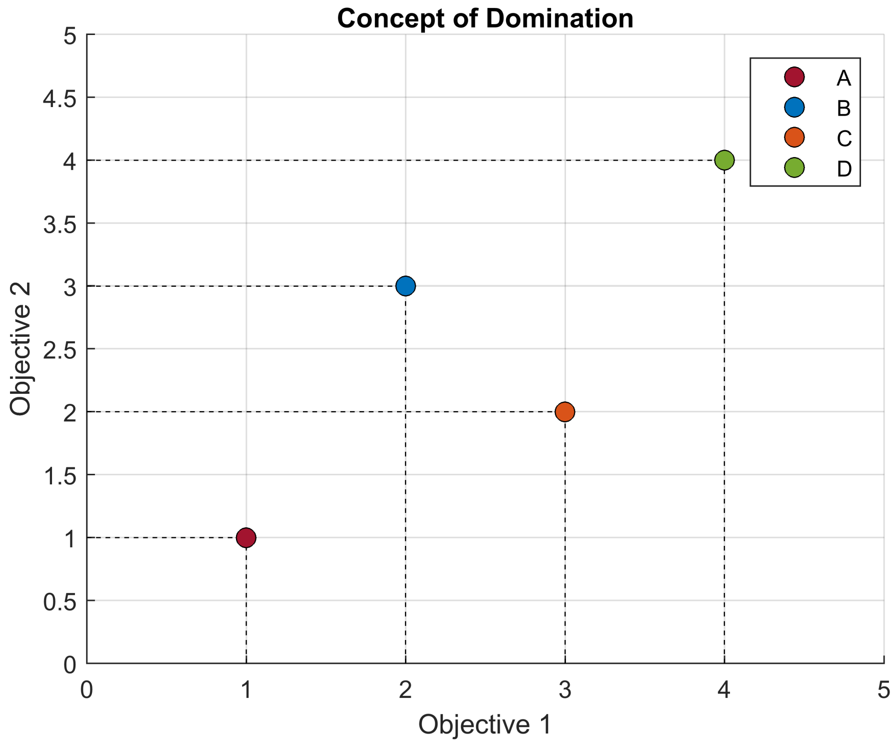


Figure 3.2: Concept of domination.

The concept of domination is illustrated in Fig. 3.2 where *Objective 1* and *Objective 1* must be minimized. Solution B dominates solution D, solution C dominates solution D, neither solution B nor C dominates each other, solution A dominates solutions B, C and D.

A predominant practice in solving multi-criteria optimization problem is by scalarization methods, in which the components of objective function vectors are combined to form a scalar objective function.

The most common scalarization method to solve multi-objective optimization problems is the *weighted sum method*:

$$S = \sum_{i=1}^{k} w_i f_i(\mathbf{d}) \tag{3.1}$$

Where S is a scalar merit function, w_i is the weighting factor for the i-th objective function $f_i(\mathbf{d})$.

As with most methods that involve objective function weights, setting one or more of the weights to 0 can result in weakly Pareto optimal points. The relative value of the weights generally reflects the relative importance of the objectives, and can be managed by the designer.

Chapter 4

Fundamentals of statistics and probability

In order to understand the probabilistic approach in the design optimization process, as well as the characterization of the design variables and parameters, some basic concepts regarding probability and statistics are compulsory.

The main difference between probability and statistics is that: *probability* deals with predicting the likelihood of future events, while *statistics* involves the analysis of the frequency of past events.

4.1 Probability density and Cumulative distribution function

A random variable X is a variable that, instead of having a single value, can assume a set of possible x values, to which is associated a given probability.

The mathematical function that describes the distribution of a random variable is called the *Probability Density Function* (PDF), $f_X(x)$ (Fig. 4.1). This function assigns a certain probability density to each value of the random variable.

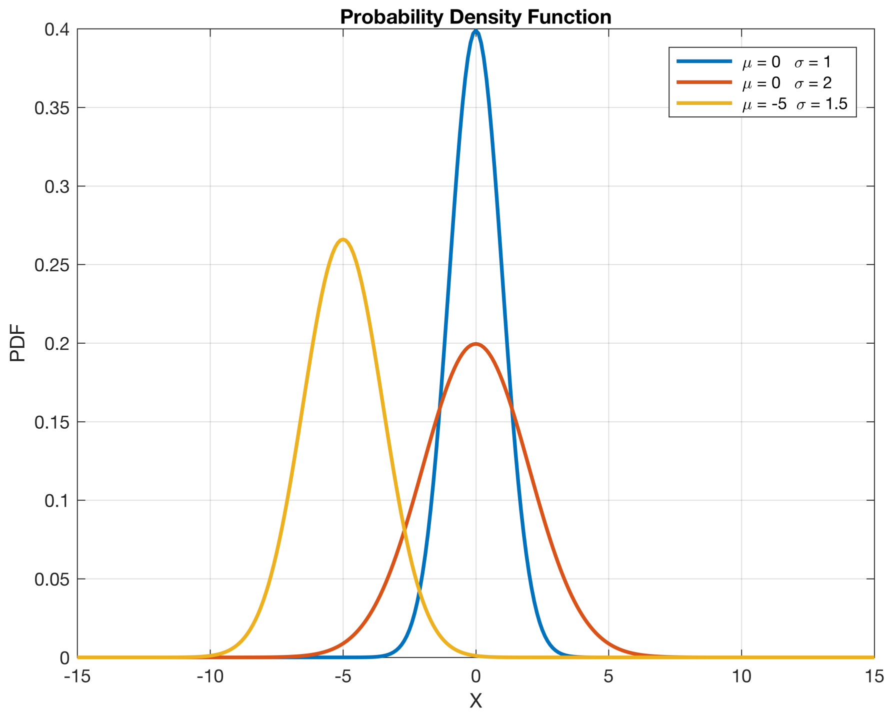


Figure 4.1: Probability density function of normal distributions.

Another way to describe the probability distribution is the *Cumulative Distribution Function* (CDF), $F_X(x)$ (Fig. 4.2). The CDF is defined for all values of a random variable X from $-\infty\ to + \infty$ and is equal to the probability that X is less than or equal to a certain value x.

For a continuous random variable, $F_X(x)$ is calculated by integrating the PDF for all values X less than or equal to x:

$$CDF = F_X(x) = \int_{-\infty}^{x} f_X(t)dt \tag{4.1}$$

Furthermore, if $F_X(x)$ is continuous, then the probability of X having a value between a and b can be calculated as:

$$F_X(b) - F_X(a) = \int_{a}^{b} f_X(x)dx \tag{4.2}$$

If the random variable X is continuous and if the first derivative of the distribution function exists, then the PDF is given by the derivative of the CDF:

$$PDF = f_X(x) = \frac{dF_X(x)}{dx} \tag{4.3}$$

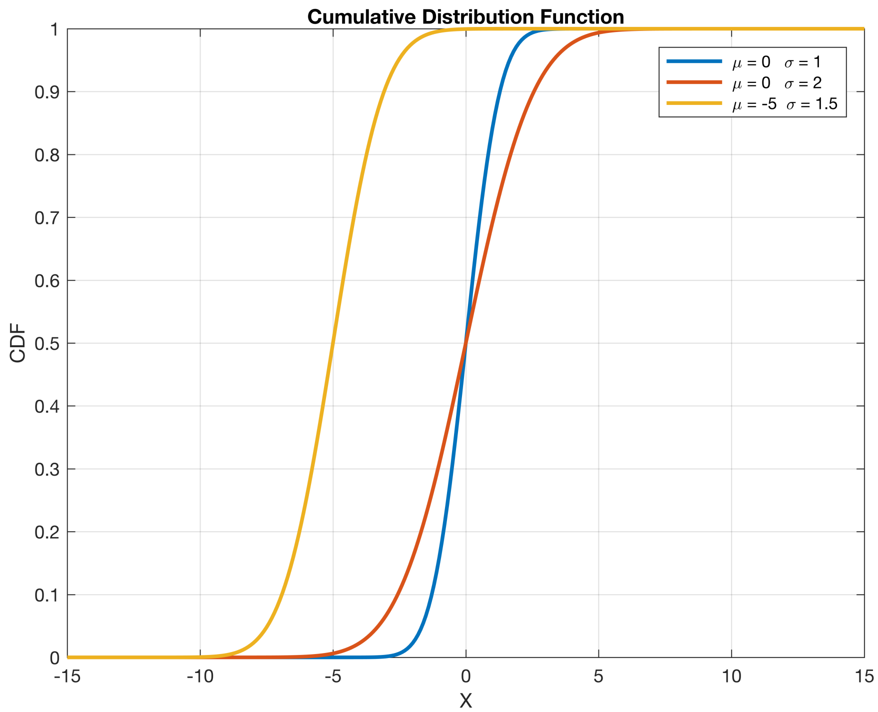


Figure 4.2: Cumulative distribution function of normal distributions.

The CDF is a non-decreasing function of x (its slope is always greater than or equal to zero), with lower and upper limits of 0 and 1 (blue curve in Fig.4.1 and 4.2).

4.2 Central measures

The population *mean* (μ), also referred as the *expected value* or *average*, is used to describe the central tendency of a random variable. This is a weighted average of all the values a random variable may take. The mean is given by:

$$\mu_X = E[X] = \int_{-\infty}^{+\infty} x f_X(x) dx \tag{4.4}$$

It's called *first moment*, since it is the first moment of area of the PDF.

Other useful central measures are the *median* and *mode* of the data: the median is the value X at which the CDF has a value of 0.5, the mode is the value of X corresponding to the peak value of the PDF (Fig.4.3).

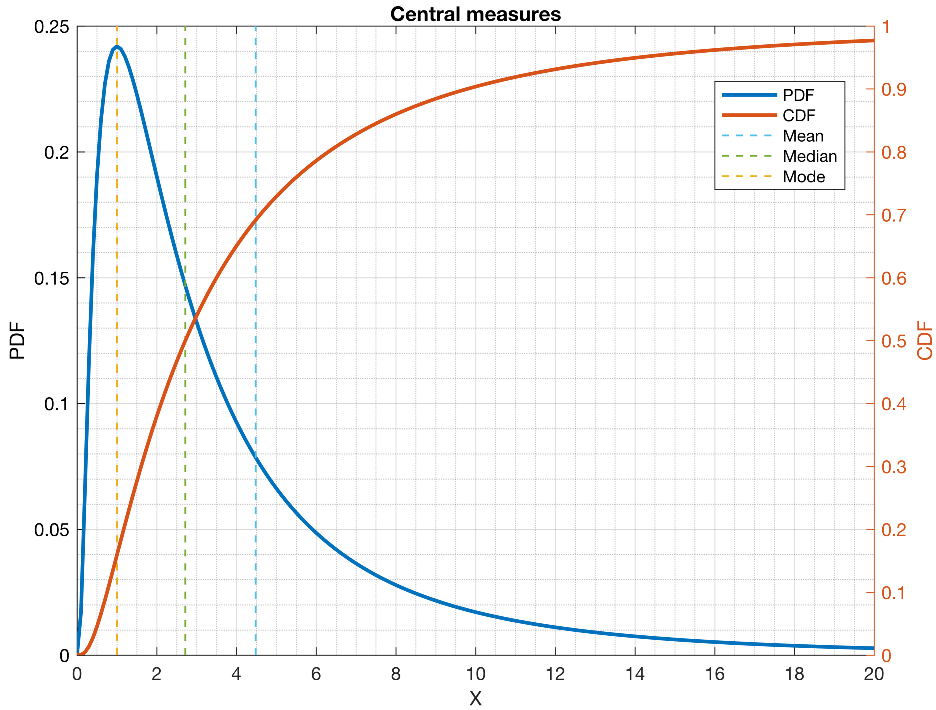


Figure 4.3: Central measures.

4.3 Dispersion measures

The *variance* $V[X]$, is the *second central moment* and is a measure of spread in the data about the mean:

$$V[X] = E[(X - \mu_X)^2] = \int_{-\infty}^{+\infty} (X - \mu_X)^2 f_X(x) dx \tag{4.5}$$

Geometrically, it represents the moment of inertia of the PDF about the mean value.

A measure of the variability if the random variable is usually given by a quantity known as the *standard deviation*:

$$\sigma_X = E[X] = \sqrt{V[X]} \tag{4.6}$$

The ratio between the standard deviation and the mean value is called the Coefficient Of Variation (COV), which indicates the relative amount of uncertainty or randomness.

$$COV = \frac{\sigma_X}{\mu_X} \tag{4.7}$$

Although the most used statistical moments are the former described, there are some other central moments worth to be mentioned.

The *skewness*, or *third central moment*, is a measure of the asymmetry of the distribution.

The *kurtosis*, or *fourth central moment*, is a measure of the steepness of the PDF.

For methods involving sampling techniques, the discrete definitions for the mean value and the variance are used:

$$\mu_X = E[X] = \frac{1}{N_s}\sum_{i=1}^{N_s} t_i \tag{4.8}$$

$$\sigma_X^2 = V[X] = E[(X-\mu_X)^2] = \frac{1}{N_s}\sum_{i=1}^{N_s}(t_i-\mu_X)^2 \tag{4.9}$$

where N_s is the number of samples.

4.4 Probability distributions

The selection ir determination of the distribution functions of random variables is known as *statistical tolerancing.*

The *central limit theorem* states that the sum of many arbitrary distribution of random variables asymptotically follows a normal distribution when the sample size becomes large. A *normal distribution*, also called *Gaussian distribution*, is often used for small coefficient of variation cases, such as Young's modulus, Poisson's ratio and other material properties [9]. The PDF of a normal distribution is given by:

$$f_X(x) = \frac{1}{\sigma_X\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu_X}{\sigma_X}\right)^2} \tag{4.10}$$

The simplest *Gaussian distribution* is called *standard normal distribution*, which is a normal distribution with 0 as mean value and 1 as standard deviation.

Other interesting distributions, from an engineering perspective, are:

- *Lognormal distribution* which is often used to describe cycles to failure, material strength;
- *Weibull distribution*, well suited for describing the weakest link phenomena, or a situation where there are competing flaws contributing to failure. It is often used to describe fracture of brittle materials and strength in composites.

Chapter 5

Design and optimization framework

5.1 Introduction

In this chapter, we'll introduce some fundamental concepts regarding mathematical modelling of complex systems and sampling techniques.
Once introduced every component of the design and optimization process, we'll outline the workflow for DO, RDO, RBDO, RRBDO.

5.2 Surrogate modelling

Many practical applications require a detailed model of the system to accurately predict the structural response to various inputs.
Optimization of such systems can be difficult, if not impossible, because evaluation of objective and constraint functions requires a large number of calculations.
Therefore, to overcome the computational difficulties, it is useful to develop simplified functions to use in the optimization process that have explicit forms in terms of the design variables.
The explicit function is a model of the model and is called a *meta-model* (or *surrogate model*).
A surrogate model can be generated by conducting experimental observations and/or numerical

simulations. Suppose that we have a mathematical model of the form:

$$f = f(\mathbf{x}) \tag{5.1}$$

where $f(\mathbf{x})$ (e.g. strength of a composite structure) does not have an explicit expression in terms of the design variables $\mathbf{x}$ (e.g. plies' orientations). The function $f(\mathbf{x})$ can be approximated by a simplified explicit function (surrogate model) using the information at some sample points $\mathbf{x}_i$. To make the meta-model, f is evaluated at k points as follows:

$$f_i = f(\mathbf{x}_i), \qquad (i = 1, .., k) \tag{5.2}$$

The meta-model is constructed using the f_i that may be obtained by experiments and/or numerical simulations (Fig.5.1).

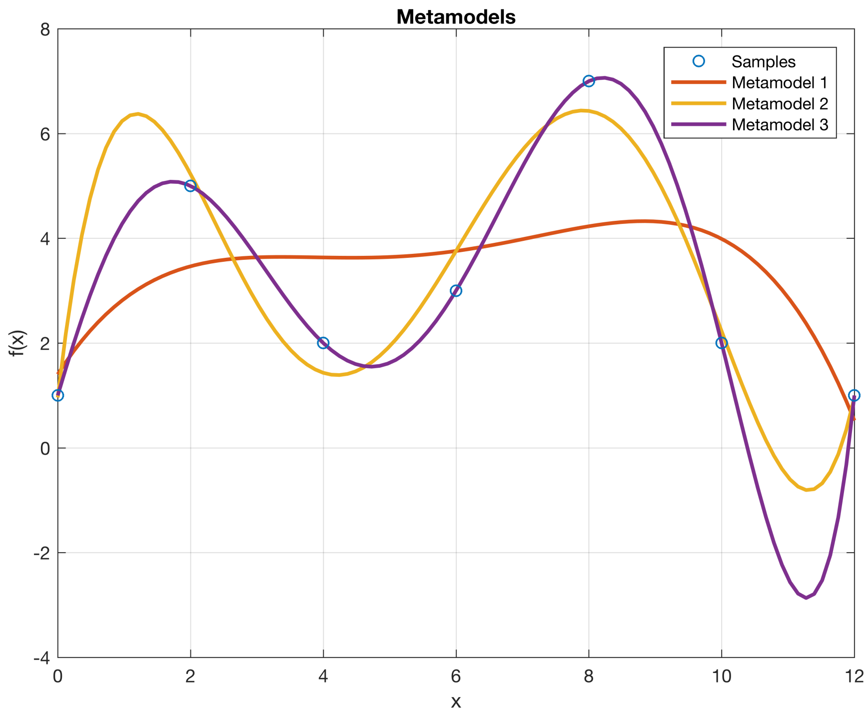


Figure 5.1: Example of metamodels.

Once a surrogate model has been developed, we can use it instead of the original model in the optimization process. Generally, the meta-model has errors due to mathematical approximations, experimental errors, or computational approximations. The error $\epsilon(\mathbf{x})$ in the meta-model is expressed as:

$$\epsilon(\mathbf{x}) = f(\mathbf{x}) - \hat{f}(\mathbf{x}) \tag{5.3}$$

where $f(\mathbf{x})$ is the original model for which we don't have an explicit expression, while $\hat{f}(\mathbf{x})$ is the constructed meta-model.

There are different global approximation techniques (meta-models) that can be created, e.g. *Response Surfaces*, *Kriging* and *Radial Basis Functions*.

However, one of the main limitations of these methods is that it is not possible to identify a priori which approximation technique is the best [16].

5.2.1 Response Surfaces

Response surfaces are normally used for model prediction in the *Response Surface Methodology* (RSM) [22].

The response surface of a model is approximated by an explicit polynomial function usually using the least squares method to minimize the error in Eq. 5.3.

The most widely used response surface approximating functions are low-order polynomials [29]. For low curvature, a first order polynomial can be used as in Eq. 5.4; for significant curvature, a second order polynomial which includes all two-factor interactions is available (Eq. 5.5):

$$\hat{f}(\mathbf{x}) = \beta_0 + \sum_{i=1}^{k} \beta_i x_i \tag{5.4}$$

$$\hat{f}(\mathbf{x}) = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k} \sum_{j=1, i<j}^{k} \beta_{ij} x_i x_j \tag{5.5}$$

As above stated, the parameters of the polynomials in Eqs. 5.4 and 5.5 are usually determined by least squares regression analysis by fitting the response surface approximations to existing data. A more complete discussion of response surfaces and least squares fitting can be found in [22].

5.2.2 Radial Basis Functions

Radial Basis Functions (RBFs) are a type of neural networks using a hidden layer of radial units. The RBF has been traditionally used in applications with large sizes of data points. In recent years, research has been conducted on creating RBF models with limited numbers of samples. An RBF model has the general form of:

$$\hat{f}(\mathbf{x}) = \sum_{i=1}^{k} \lambda_i \phi(\| \mathbf{x} - \mathbf{x}_i \|) \tag{5.6}$$

where k is the number of sampling points, $\mathbf{x}$ is a vector of design variables, $\mathbf{x}_i$ is a vector of design variables at the i-th sampling point, $(\| \mathbf{x} - \mathbf{x}_i \|)$ is the Euclidean norm, ϕ is a basis function, and λ_i is the coefficient for the i-th basis function.

The approximation function $\hat{f}(\mathbf{x})$ is actually a linear combination of some RBFs with weight coefficients. Some of the most commonly used basis functions are linear, cubic, thin-plate spline, Gaussian, multiquadric, and inverse-multiquadric (Tab. 5.1) [11].

Table 5.1: Commonly used basis functions.

Name	Symbol	Basis function
Linear	RBF-LN	$\phi(r) = r$
Cubic	RBF-CB	$\phi(r) = r^3$
Thin-plate spline	RBF-TPS	$\phi(r) = r^2 \ln(cr), 0 < c \leq 1$
Gaussian	RBF-GS	$\phi(r) = e^{-cr^2}, 0 < c \leq 1$
Multiquadric	RBF-MQ	$\phi(r) = \sqrt{r^2 + c^2}, 0 < c \leq 1$
Inverse Multiquadric	RBF-IMQ	$\phi(r) = \frac{1}{\sqrt{r^2 + c^2}}, 0 < c \leq 1$

5.2.3 Kriging

A Kriging model is a surrogate model based on a stochastic process. It was originally put forward in geostatistics from Krige, a South African mining engineer, then made its way into engineering design following the work of Sacks, et al. [26].

For a given set of sample data (input), $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_{n_s}]^T$, and the observed responses $\mathbf{y} = [y_1, y_2, ..., y_{n_s}]^T$, the expression of Kriging model that reflects the relationship between them is:

$$y(\mathbf{x}_i) = \mathbf{f}^T(\mathbf{x}_i)\beta + Z(\mathbf{x}_i), \qquad i = 1, ..., n_s \tag{5.7}$$

where $\mathbf{f}(\mathbf{x})$ is a polynomial vector of the sample $\mathbf{x}$, β is the vector of the linear regression coefficients to be estimated and $Z(\mathbf{x})$ represents errors and is assumed to be a stochastic process that follows a normal distribution with zero mean and variance σ^2 (i.e. the behaviour of $Z(\mathbf{x})$ follows a Gaussian distribution).

As seen above, Kriging models combine a global model $\mathbf{f}^T(\mathbf{x})\beta$ plus localized departures $Z(\mathbf{x})$. The $\mathbf{f}^T(\mathbf{x})\beta$ term in Eq. 5.7 is similar to a polynomial response surface, providing a "global" model of the design space, whereas $Z(\mathbf{x})$ creates "localized" deviations from the global model, so that the kriging model interpolates the sampled data points.

To estimate the stochastic process $Z(\mathbf{x})$, Kriging assumes that any two points will tend to have the same value as the distance in between approaches zero and it is the same for $Z(\mathbf{x})$ of two points. Thus, the correlation between $Z(\mathbf{x})$ of any two sample points can be expressed as a function of their spatial distance. Two of the most widely used correlation functions are the Exponential correlation function (Eq. 5.8) and the Gaussian correlation function (Eq. 5.9).

$$R(Z(\mathbf{x}_i), Z(\mathbf{x}_j)) = exp\left(-\sum_{k=1}^{m} \theta_k \mid x_i^k - x_j^k \mid\right) \tag{5.8}$$

$$R(Z(\mathbf{x}_i), Z(\mathbf{x}_j)) = exp\left(-\sum_{k=1}^{m} \theta_k \mid x_i^k - x_j^k \mid^2\right) \tag{5.9}$$

where x_i^k and x_j^k are the $k-th$ components of the two sample points $\mathbf{x}_i$ and $\mathbf{x}_j$, m denotes the number of design variables, θ_k controls the decay rate of correlation.

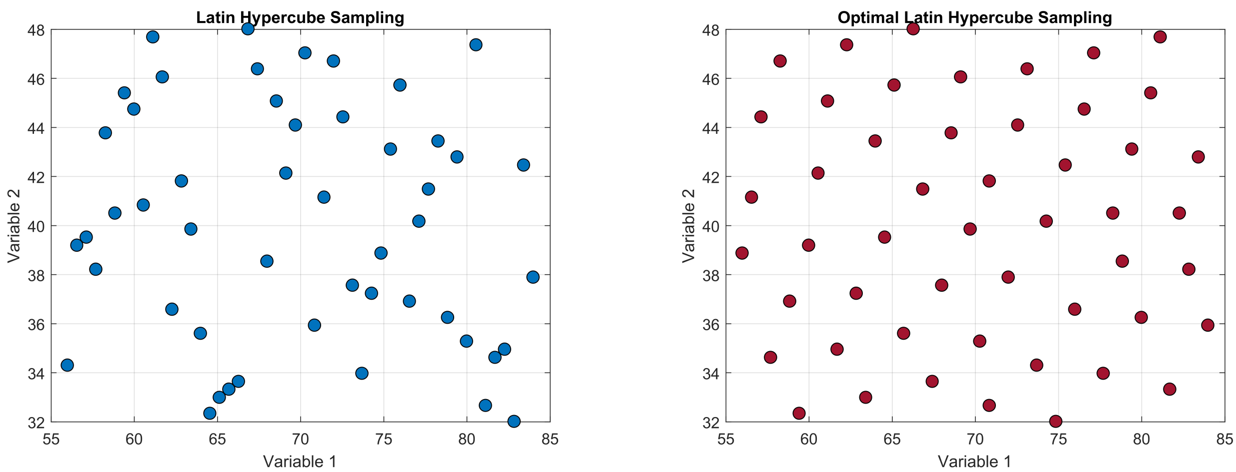
Interested readers can refer to reference [12] for a more detailed presentation of the theory, and to reference [26] for a more detailed formulation.

5.3 Design Of Experiments - DOE

To build a meta-model, Design Of Experiments methods are usually used to determine the location of sample points in the design space. DOE is a procedure with the general goal of maximizing the amount of information gained from a limited number of sample points [13].

These methods can be broadly classified into two categories: classic and modern methods. The classic DOE methods, such as full-factorial design, central composite design (CCD), Box-Behnken, were developed for laboratory experiments, with the consideration of reducing the effect of random error.

Whereas, modern methods such as Latin Hypercube Sampling (LHS), Optimal Latin Hypercube Sampling (OLHS) Orthogonal Array Design (OAD) were developed for computer based experiments.



(a) LHS (b) OLHS

Figure 5.2: Differences between LHS and OLHS.

DOE methods can manage multi-dimensional design spaces, creating a design matrix of size n_s x n_v, which explores the design space (n_s =number of samples and n_v =number of design variables).

To better understand how DOE works, in Fig. 5.2 the difference between LHS and OLHS is shown. A design of experiments created with fifty samples and two variables is presented, leading to a design matrix with dimensions 50 x 2. It can be observed that with Latin Hypercube Sampling the distances amongst design points are very different, whereas in Fig. (b) Optimal Latin Hypercube Sampling optimizes the spread of the input values (design points are separated as evenly as possible throughout the design space).

5.4 Monte Carlo Simulation

The term Monte Carlo Simulation was named after the city in Monaco (famous for its casino) where games of chance (e.g., roulette) involve repetitive events with known probabilities [3].
In general terms, the Monte Carlo method (or Monte Carlo Simulation) can be used to describe any technique that approximates solutions to quantitative problems through statistical sampling.
It belongs to the sampling category of uncertainty propagation methods. This means that, to propagate uncertainties from the (surrogate) model inputs into uncertainties in the outputs, they use random samples. Hence, it is a type of simulation that explicitly and quantitatively represents variabilities.
Monte Carlo Simulation relies on the process of explicitly representing uncertainties by specifying probability distributions of the inputs (design variables and/or design parameters). If the input describing a system are uncertain, the prediction of future performances, which obviously depend on the probabilistic input, are necessarily uncertain.
In Monte Carlo simulation, the entire system is simulated a large number (e.g. 1000) of times. Each simulation is equally likely, referred to as a realization of the system.
For each realization, all of the uncertain parameters are sampled (i.e. a single random value is selected from the specified distribution describing each parameter). The system is then simulated (given the particular set of input parameters) such that the performance of the system can be computed.
The results of the independent system realizations are assembled into probability distributions of possible outcomes, therefore the outputs are not single values, but probability distributions.
Monte Carlo analysis is suitable for cases when many stochastic variables are present, and it is a simple and straightforward method to incorporate variability in the model. The main drawback of the method is that a large number of simulations are required in order to achieve accuracy in the statistics of the response.

5.4.1 MCS in RDO

Monte Carlo methods can be embedded in the Robust Design Optimization process, where they are used to compute the mean values and the standard deviations of the response functions (5.3). First, N_s random samples are generated and stored for each design variable and/or

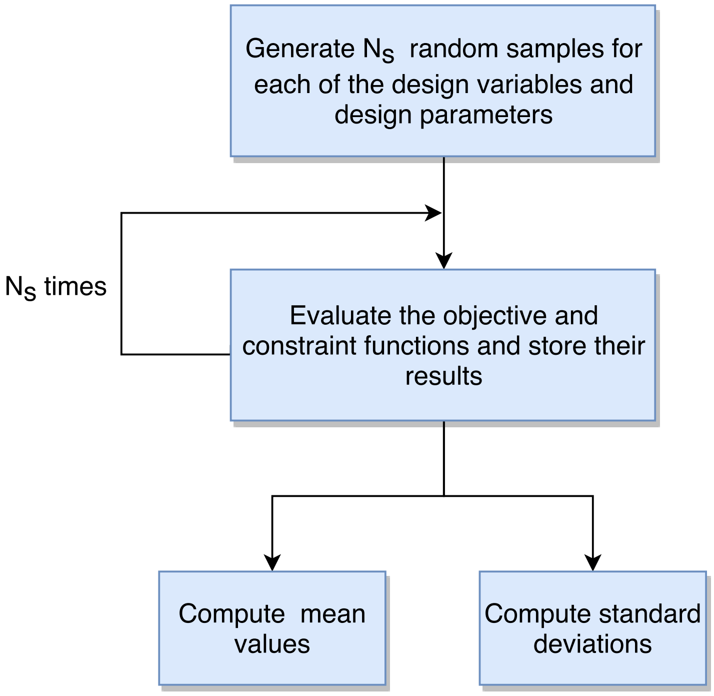


Figure 5.3: MCS in Robust Design Optimization [3].

design parameter, based on their mean and standard deviation. Then, the response functions are evaluated at each of the N_s stored points and their results are saved. Finally, the expected values and variances of the functions can be computed with Eq. 4.8 and Eq. 4.9.

The accuracy of the model depends on the number of samples (N_s) that are generated. Although it's possible to generate a large amount of samples, to get increased accuracy, the computational effort greatly rises due to an increase in function evaluations, meaning that the bigger the vector of stochastic design variables and parameters, the more inefficient is using a method such as MCS in an optimization loop.

5.4.2 MCS in RBDO

In Reliability-Based Design Optimization, Monte Carlo Simulations, instead of estimating the mean and standard deviation of the response, are used to evaluate the probability of failure, computing the probability of violating a limit state function (constraint). In 5.4, the first step

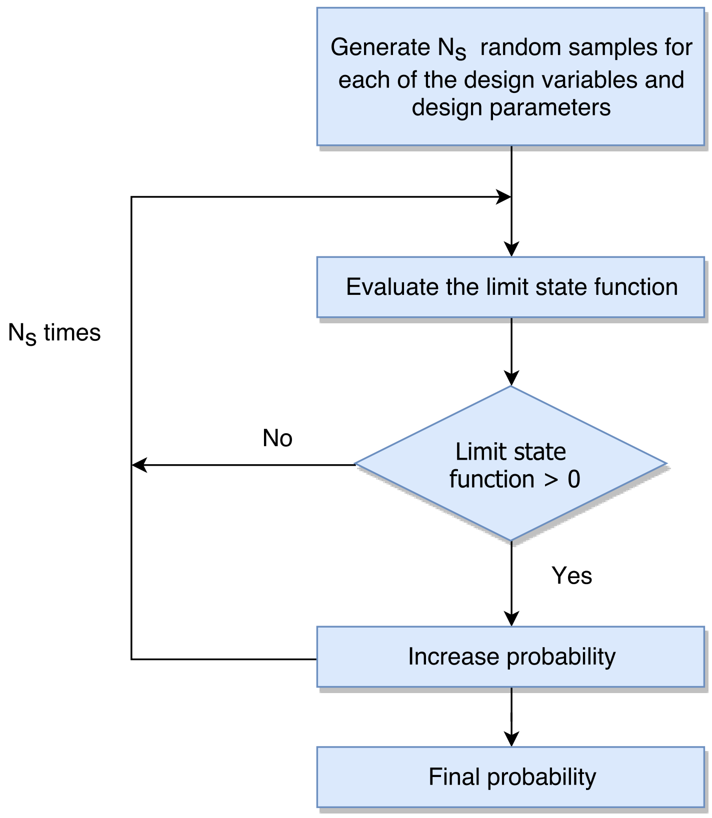


Figure 5.4: MCS in Reliability-Based Design Optimization [3].

is creating the samples of the random design variables and/or parameters, then they are substituted into the limit state function to evaluate its response.

The difference is that, instead of storing values, what is stored, is the number of times the function is greater than zero, subsequently the probability of it being higher than zero is computed according to:

$$P_f = \frac{N_{s_{failure}}}{N_s} \tag{5.10}$$

where $N_{s_{failure}}$ is the number of samples for which the limit state function was higher than zero (design points of failure) and N_s is the total number of samples.
It could be noted that, the computed probability, is referred to the function being greater than zero, but that is not the only probability that can be computed (e.g. the probability of the limit state function being lower than zero is exactly the probability of success).

5.4.3 MCS in RRBDO

In the Robust and Reliability-Based Design Optimization framework, Monte Carlo methods are used to compute both the probability of limit state violations (probability of failure) and the statistical parameters of the objective and constraint functions (5.5).

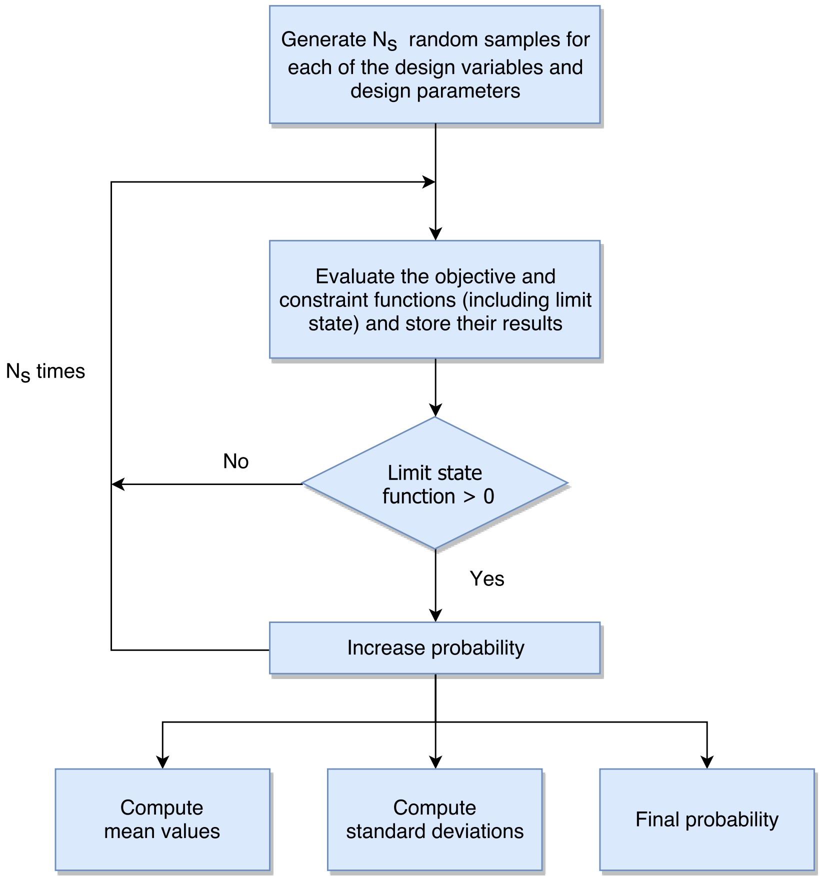


Figure 5.5: MCS in Robust and Reliability-Based Design Optimization.

5.5 Design and Optimization flowcharts

In the subsequent pages, to better understand the design and optimization processes used in this thesis, different flowcharts will be presented. It must be noted that these flowcharts are for optimization problems in which exact analytical functions don't exist, therefore creating a surrogate model is compulsory.

In these different frameworks we can observe a common section (DOE, FEM, Metamodel) and a second part (Optimization), which is slightly different for the three probabilistic approaches. The process starts with a nominal design. Once chosen the design variables, the design space is explored with a DOE, creating n_s sampling points.

Successively, the responses we are interested in, are acquired from the n_s non-linear finite element simulations.

The inputs from DOE and outputs from the FEM model, are used to create surrogate models (e.g. RSM, RBF, Kriging) for the responses. After a cross validation error analysis, the best model (with less error) is selected. If the error is not acceptable, more samples must be generated in the DOE.

In the optimization part, we set the weighting factors (according to which objective must be preferred, from high level information). The allowable probability of failure is set only for RBDO (Fig. 5.7) and RRBDO (Fig. 5.8). The RDO scheme, as we can see from Fig. 5.6, doesn't take into account the probability of failure.

Successively, it must be decided the genetic algorithm to use, then a convergence analysis (sub-loop on the deterministic problem) must be run to assess the number of population and generations to use.

Afterwords, the MCS sub-loop (different for each method) is run, leading to the mean values and standard deviations of the cost and constraint functions (RDO, RRBDO), and assessing the probability of failure (RBDO, RRBDO).

Finally, the optimization process exits with the optimal design.

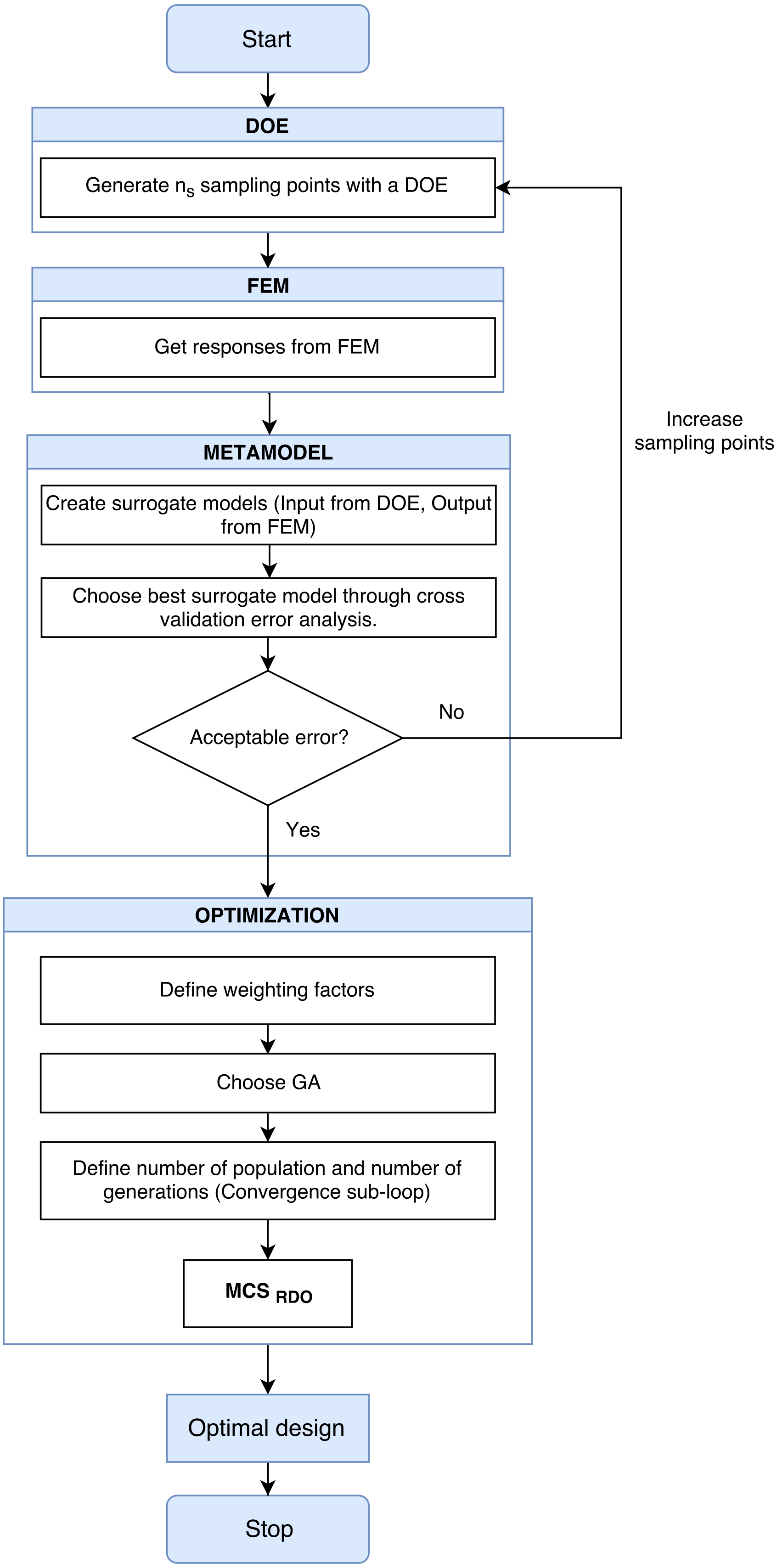


Figure 5.6: Robust Design Optimization flowchart.

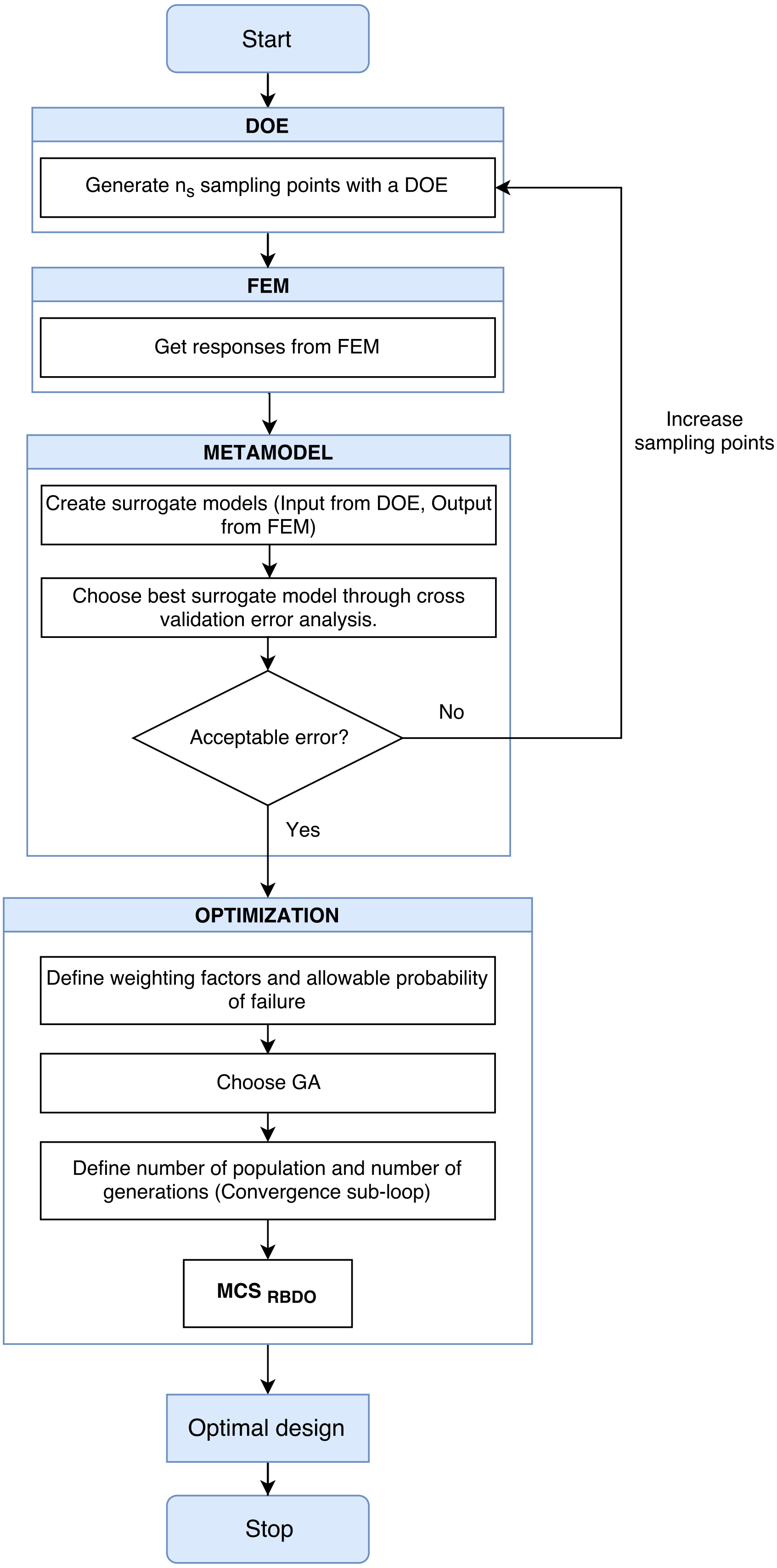


Figure 5.7: Reliability-Based Design Optimization flowchart.

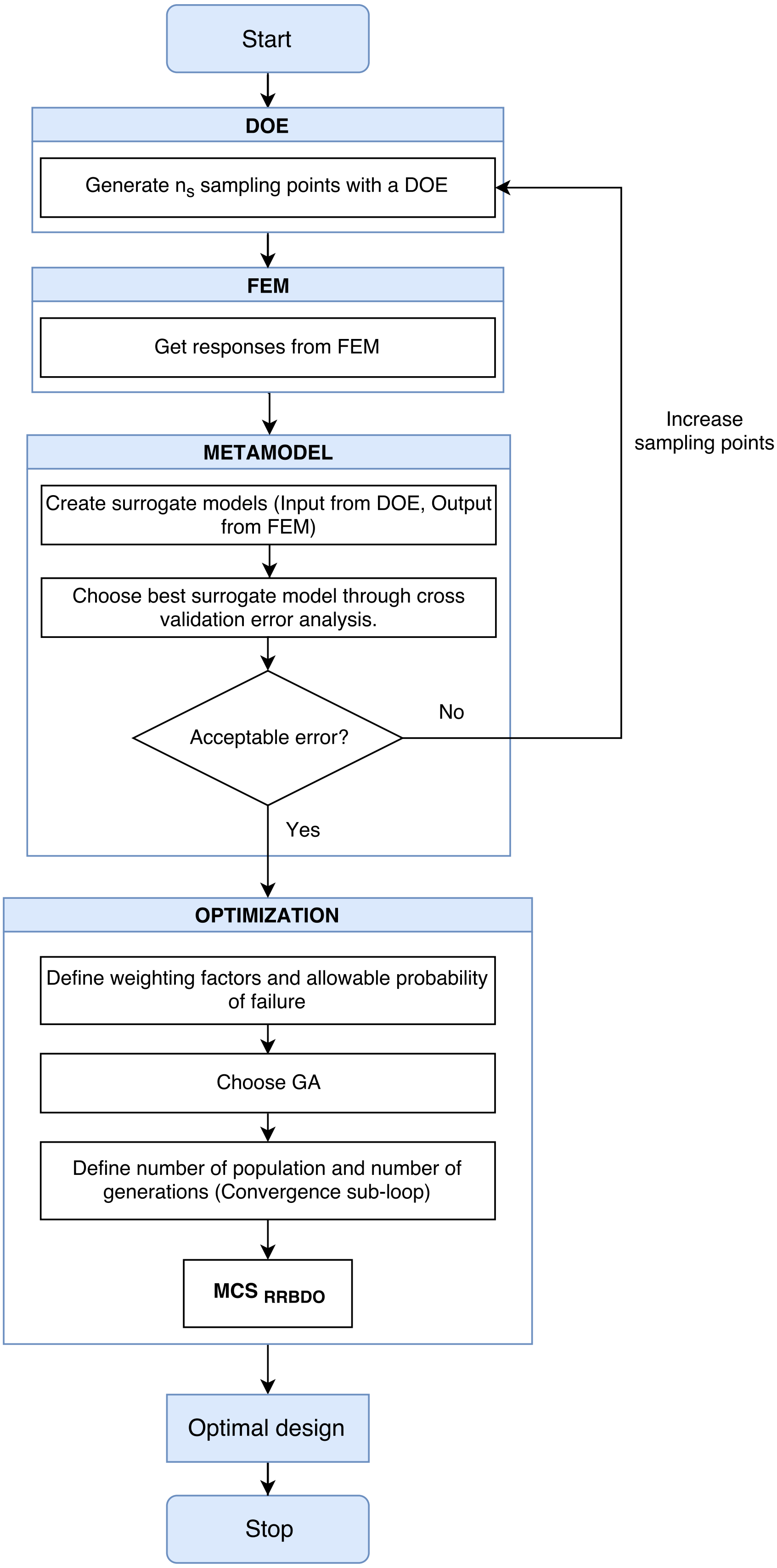


Figure 5.8: Robust and Reliability-Based Design Optimization flowchart.

Chapter 6

Composite C-Beam

6.1 Problem description

In this chapter, either to validate the optimization frameworks previously discussed and to show the differences amongst the various approaches, an application to a composite floor beam will be considered.

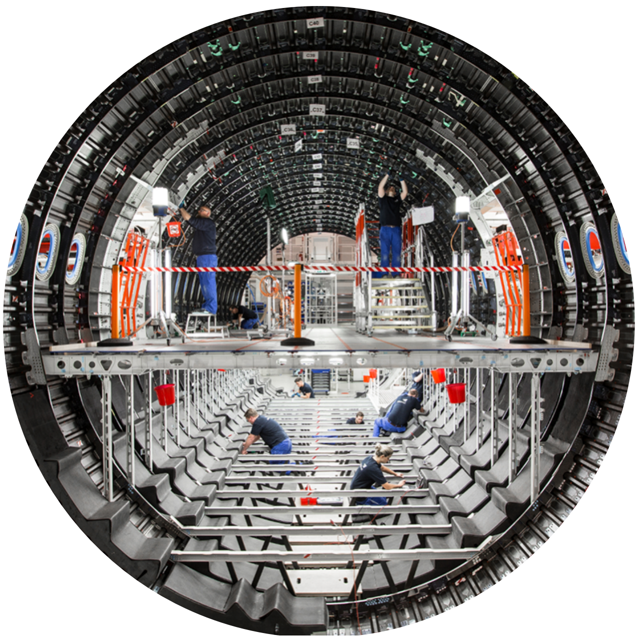


Figure 6.1: Airbus A350 floor beam structure in Aluminium-Lithium.

The analysed floor beam is a part of SHERLOC project (Structural HEalth monitoring, manufacturing and Repair technologies for Life management Of Composite fuselage).

The original structure has an overall length of 3000 mm, while the distance between struts is 2115.4 mm. Now, because of the main purpose of this thesis is to develop a framework and to show the differences between the probabilistic approaches, a composite floor beam with half of its nominal length (1057.7 mm) will be considered.

The beam is fixed at both edges (coincident with the struts), and vertically loaded by uniform increasing displacements (in correspondence of the seat rails) applied on the upper flange (Fig.6.2). The distance between struts is 1057.7mm, whereas the prescribed displacements are applied at 794.2mm and 263.5mm (seat rails' locations) from the right strut (Fig.6.4).

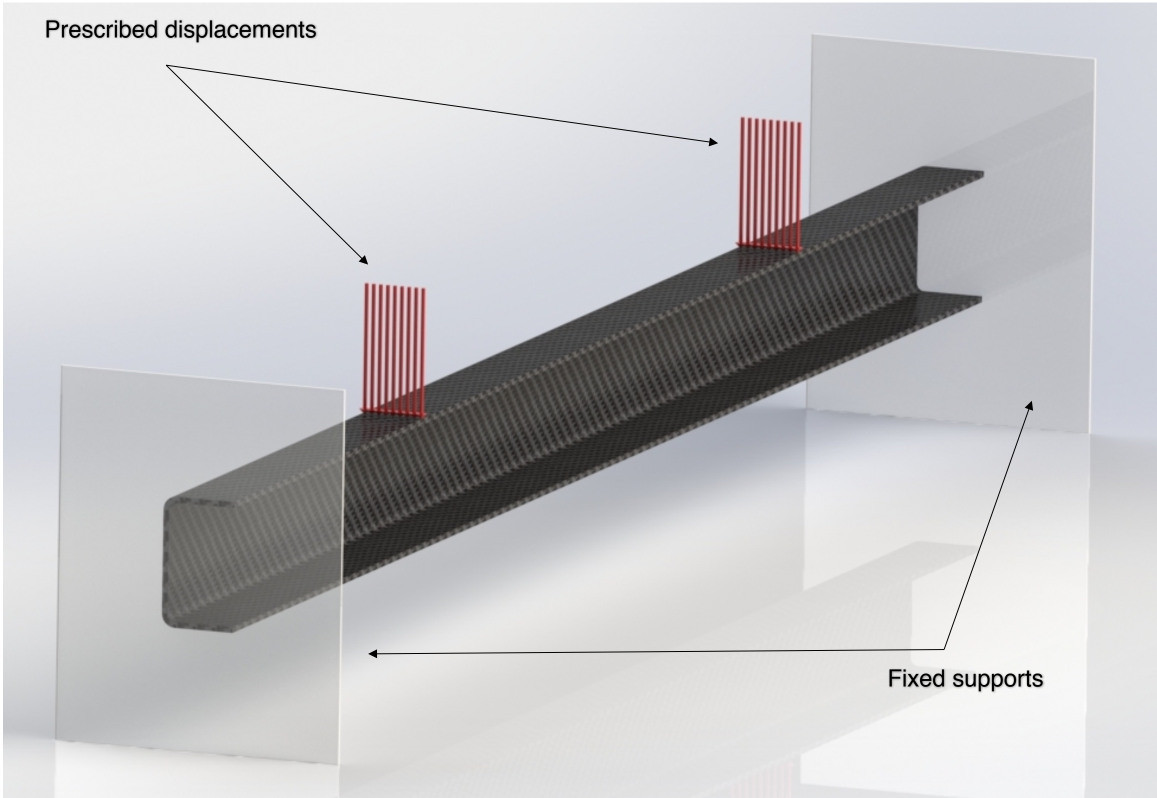


Figure 6.2: 3D structural scheme.

The realization of the FEM model and the structural analysis, will be accomplished by Simulia Abaqus™ (further explained in Subsection 6.2.3), whereas the Design Of Experiments, as well as surrogate modelling creation, Monte Carlo Simulations and the optimization via genetic algorithm, will be carried out by Simulia Isight™ and Matlab™.

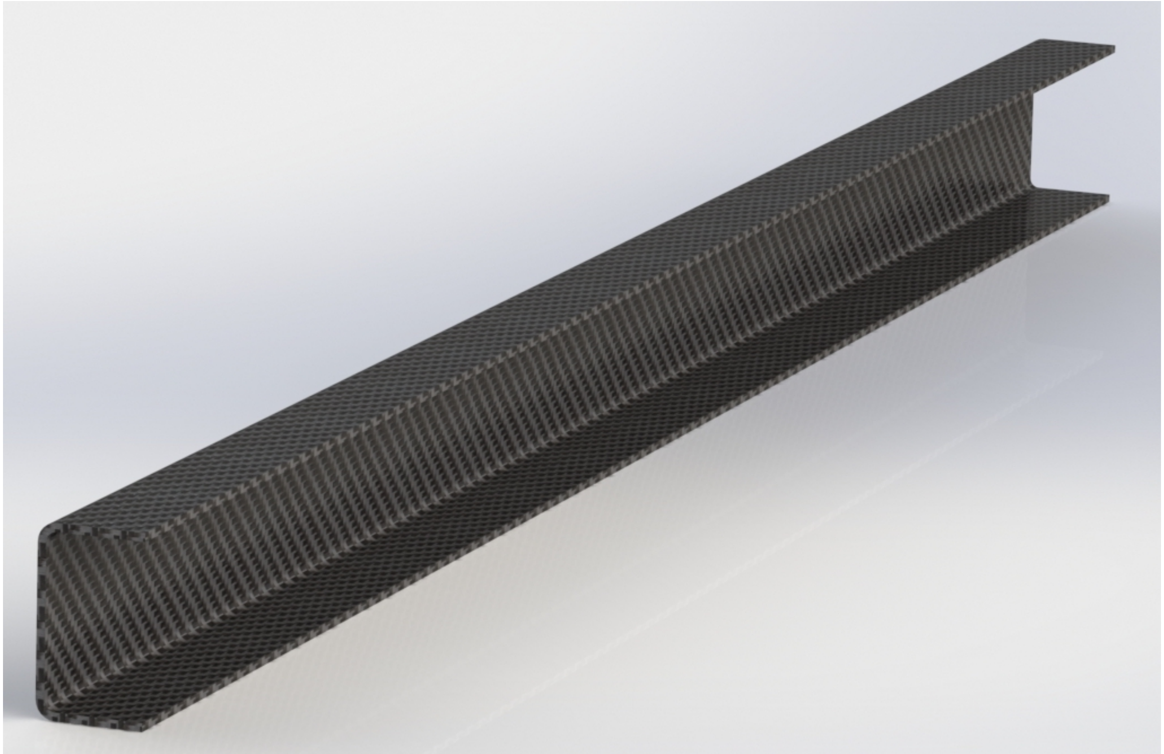


Figure 6.3: Composite floor beam.

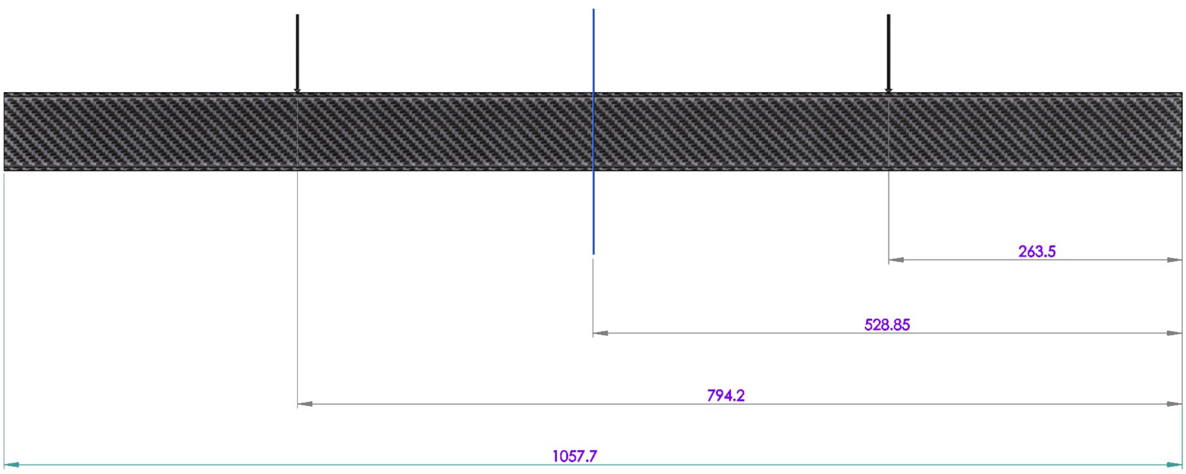


Figure 6.4: Longitudinal dimensions.

6.2 Data and information

The composite floor beam to be analysed, is a 16-ply symmetric laminate with stacking sequence $[45, 90, -45, 0, 45, 0, 0, -45]_s$ (Fig.6.7), ply thickness $t_p = 0.2mm$, hence the total thickness of the laminate is $t_l = 3.2mm$.

The fixed design parameters are the distance between struts $L = 1057.7mm$, the ply thickness, the number of plies and the layup. Whereas, the design variables of the optimization problem are the geometrical characteristics of the cross section: width, height and radius (Fig.6.5).

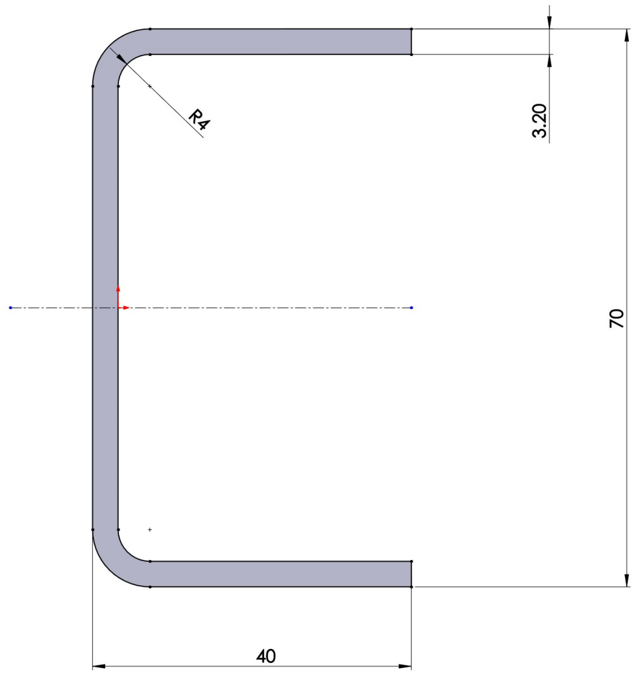


Figure 6.5: Nominal geometry of the cross-section.

The floor beam is modelled as fixed at the ends and loaded by a prescribed displacement at two loading points. As a consequence of having a "C" cross-sectional shape, the shear centre is located outside the section, so, when the beam is loaded on the flange in a vertical direction (not aligned with the shear centre), it twists as well (Fig.6.6).

This is a phenomenon that we want to avoid or, at least, to reduce at its minimum. Therefore, the goal of the optimization problem is to obtain a floor beam with minimum mass, minimum twisting angle while being able to carry as much load as possible. The latter objective (how much force the floor beam can withstand) is gathered through the value of the reaction force.

For the three probabilistic design and optimization methods (RDO, RBDO and RRBDO) and

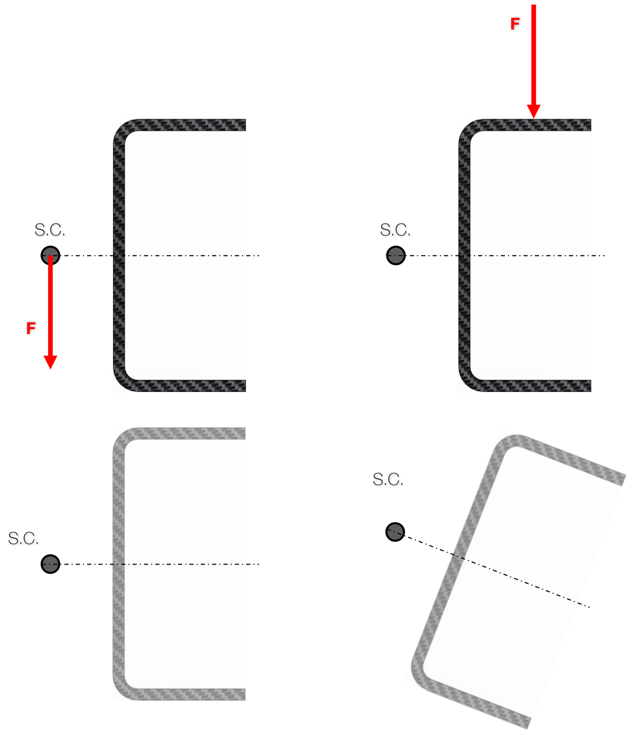


Figure 6.6: Shear centre effect on C shaped beams.

for the Deterministic Optimization, the design variables and design parameters will be the same, whereas objective functions and constraints will be different (detailed problem formulations in Subsection 6.3.1).

Table 6.1: Initial design.

Description	Value
Height $[mm]$	70
Radius $[mm]$	4
Width $[mm]$	40
Twisting angle $[°]$	27.06
Mass $[Kg]$	0.7516
Reaction Force$[KN]$	14.82

Table 6.2: Material properties.

Symbol	Value	Description
ρ $[Kg/m^3]$	1600	Density
E_{11} $[GPa]$	166	Longitudinal modulus of elasticity
$E_{22} = E_{33}$ $[GPa]$	8.1	Transversal modulus of elasticity
ν_{23}	0.45	In-plane Poisson's ratio
$\nu_{12} = \nu_{13}$	0.33	Out-of-plane Poisson's ratio
G_{23} $[GPa]$	3.1	In-plane shear modulus
G_{12}=G_{13} $[GPa]$	5.1	Out-of-plane shear modulus

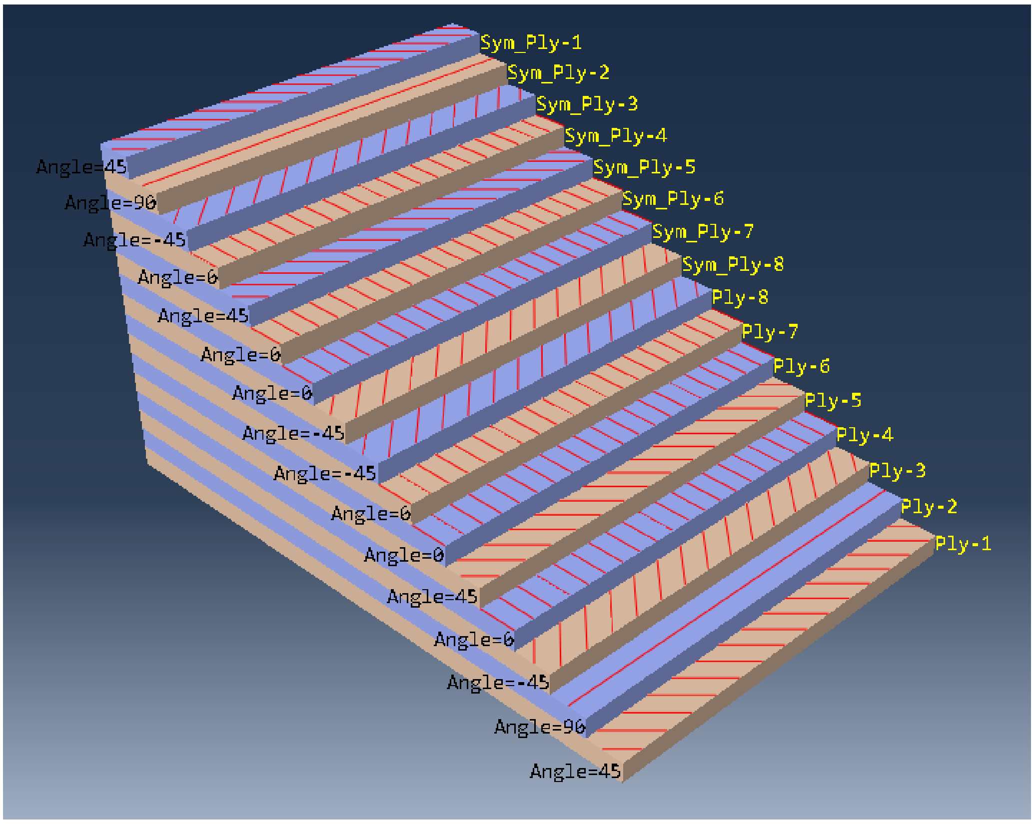


Figure 6.7: Layup.

6.2.1 Uncertainties quantification

In the previous chapters we discussed about the importance of incorporating uncertainties into the design and optimization framework. This section is about the quantification of these sources of uncertainty, explaining how they can be introduced in an optimization process.
The manufacturing tolerances are integrated in the optimization process assuming that the design variables (geometry) have a truncated normal distribution with mean, standard deviation and bounds shown in Tab.6.3. These tolerances can be included by setting the correct lower and upper bound, i.e. $\pm 0.5\%$ of the nominal dimensions of profile height and flange width (Fig.6.8 [BS EN 13706-2:2002]).

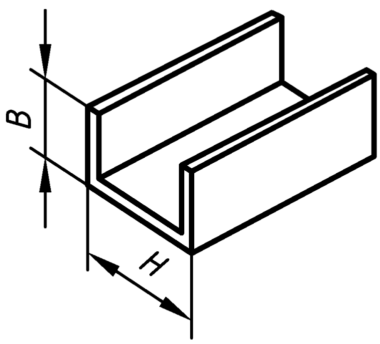
Property	Tolerance
Profile height and width of flange 	Nominal dimensions (mm) B and H: ± 0.5 % minimum ± 0.20 mm maximum ± 0.75 mm

Figure 6.8: Manufacturing tolerances.

To account for uncertainties in the material properties, they're assumed as normal distributions with a 5% of Coefficient Of Variation (ratio of the standard deviation to the mean value). The former assumption is based on the literature [34].

Table 6.3: Distribution of Design Variables.

Design Variable	Mean	Standard Deviation	Lower Bound	Upper Bound
Width $[mm]$	w	$w * 0.001$	$w - w * 0.005$	$w + w * 0.005$
Height $[mm]$	h	$h * 0.001$	$h - h * 0.005$	$h + h * 0.005$
Radius $[mm]$	r	$r * 0.01$	$r - 0.2$	$r + 0.2$

Table 6.4: Distribution of Design Parameters.

Design Parameter	Distribution	Mean	Std. Dev.
E_{11} $[GPa]$	$Normal$	166	8.3
$E_{22} = E_{33}$ $[GPa]$	$Normal$	8.1	0.405
G_{23} $[GPa]$	$Normal$	3.1	0.155
$G_{12} = G_{13}$ $[GPa]$	$Normal$	5.1	0.255

6.2.2 DOE

To create a meta-model, as examined before, a Design of Experiments must be run. In order to explore the design space, maintaining the distance between samples as evenly as possible, the Optimal Latin Hypercube Sampling technique is selected.

In Tab.6.5, are reported the minimum number of samples required for different kinds of approximation techniques. However, for fit, fidelity and exploration purposes, the recommended best practice is to double the minimum values [2].

Table 6.5: Minimum number of samples required.

RSM				RBF	Kriging
Linear	Quadratic	Cubic	Quartic		
$(n+1)$	$\left[\frac{(n+1)(n+2)}{2}\right]$	$\left[\frac{(n+1)(n+2)}{2}\right]+n$	$\left[\frac{(n+1)(n+2)}{2}\right]+2n$	$(2n+1)$	$(2n+1)$

In order to assess which meta-modelling technique is best suited to approximate our responses (comparing all of them), the RSM of fourth order (quartic) is chosen to define the number of samples required. For 7 inputs (3 design variables and 4 design parameters), the minimum sampling points required are 50. Following the best practice suggestion, this number has been doubled and a final value of 109 samples has been chosen.

6.2.3 FEM

Structural analysis

For our problem, the prediction of the collapse load is difficult because of the susceptibility of composites to the effect of through-thickness stresses. It follows that there are a number of locations in the beam and a variety of damage mechanisms which could lead to final collapse. The main damage mechanisms experienced by composites can be divided into intralaminar (fiber failure, matrix cracking or crushing and fiber-matrix shear), and interlaminar (skin-stiffener debonding) [32] [24].
For the above stated reasons, the floor beam will be analysed with non-linear explicit dynamics finite element analysis using Abaqus™ [1].

Conventional Shell vs Continuum Shell

The symmetry with respect to the middle section of the beam is exploited (applying proper constraints), hence conveniently modelling the structural part with shell elements. Abaqus™ has two different categories of shell elements: conventional shell and continuum shell.
Shell elements are used to model structures in which one dimension, the thickness, is significantly smaller than the other dimensions. Conventional shell elements use this condition to discretize a body by defining the geometry at a reference surface. In this case the thickness is defined through the section property definition. Conventional shell elements have displacement and rotational degrees of freedom.

In contrast, continuum shell elements discretize an entire three-dimensional body. The thickness is determined from the element nodal geometry. Continuum shell elements have only displacement degrees of freedom. From a modelling point of view continuum shell elements look like three-dimensional continuum solids, but their kinematic and constitutive behaviour is similar to conventional shell elements.
The continuum shell elements are general-purpose shells that allow finite membrane defor-

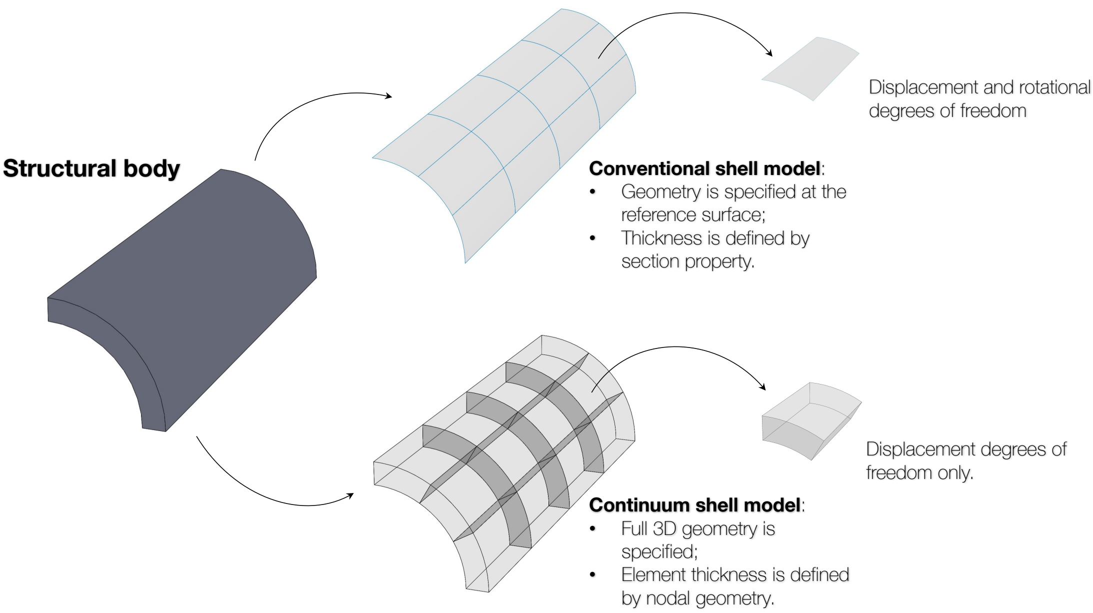


Figure 6.9: Conventional shell model vs Continuum shell model.

mation and large rotations and, thus, are suitable for non-linear geometric analysis. These elements include the effects of transverse shear deformation and thickness change. They employ first-order layer-wise composite theory, and estimate through-thickness section forces from the initial elastic moduli. Unlike conventional shells, continuum shell elements can be stacked to provide more refined through-thickness response. Stacking continuum shell elements allows for a richer transverse shear stress and force prediction.
For the above stated reasons, continuum shells are used to model our floor beam.

FEM workflow

Once set the FEM model, the non-linear structural analysis can be performed.
Because of a large number of samples (109) must be analysed, a Python™ script, parametrising all the variable inputs, is predisposed. Then, with a Matlab™ program accessing the design matrix and modifying the inputs of the Python™ script, 109 Abaqus™ input files are generated. To speed up the computing time, the simulations are run on the High Performance Computing (HPC) facilities at Imperial College London. To create the surrogate model, the information regarding the reaction force and twisting angle must be gathered from the FEM results. The

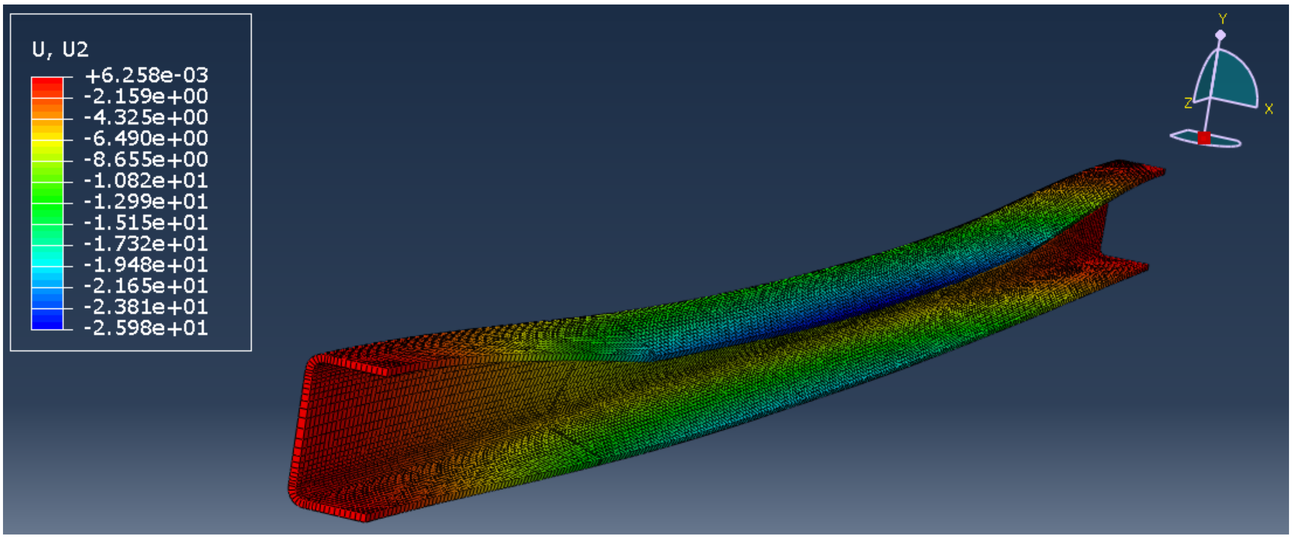


Figure 6.10: Vertical displacement of the composite floor beam.

upper flange's rotation is computed in the middle section (where is highest) evaluating the different displacements at the edges of the flange. Whereas the reaction force data is gathered in the loading point.

Here's a brief summary of the various steps to be performed:

- Step 1 : plot reaction force (RF2) and displacement (U2) in the loading point versus time, and store RF2 and time at which the structure collapses (it is when the reaction force sharply diminishes, then becomes fuzzy) (Fig.6.11);
- Step 2 : plot reaction force versus displacement, storing U2 at known RF2 (Fig.6.12);
- Step 3 : plot displacements of the border points of the middle section's upper flange (MOP: Middle Outer Point, MIP: Middle Inner Point, Fig.6.14), and storing their values at the time of collapse (previously saved). Then, with their values the rotation of the middle section can be computed (Fig.6.13).

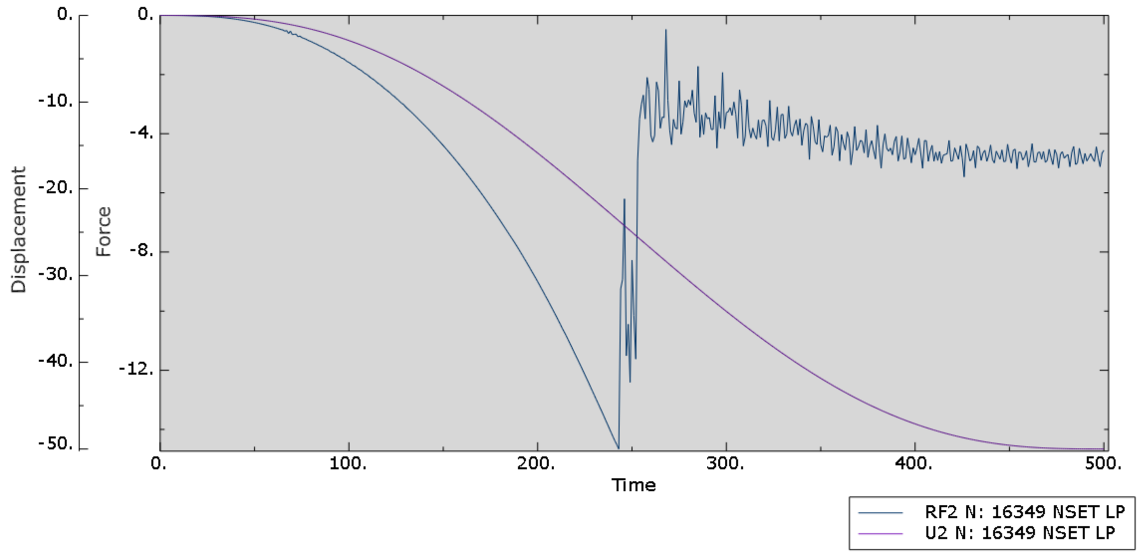


Figure 6.11: Step 1: Reaction force and displacement versus time.

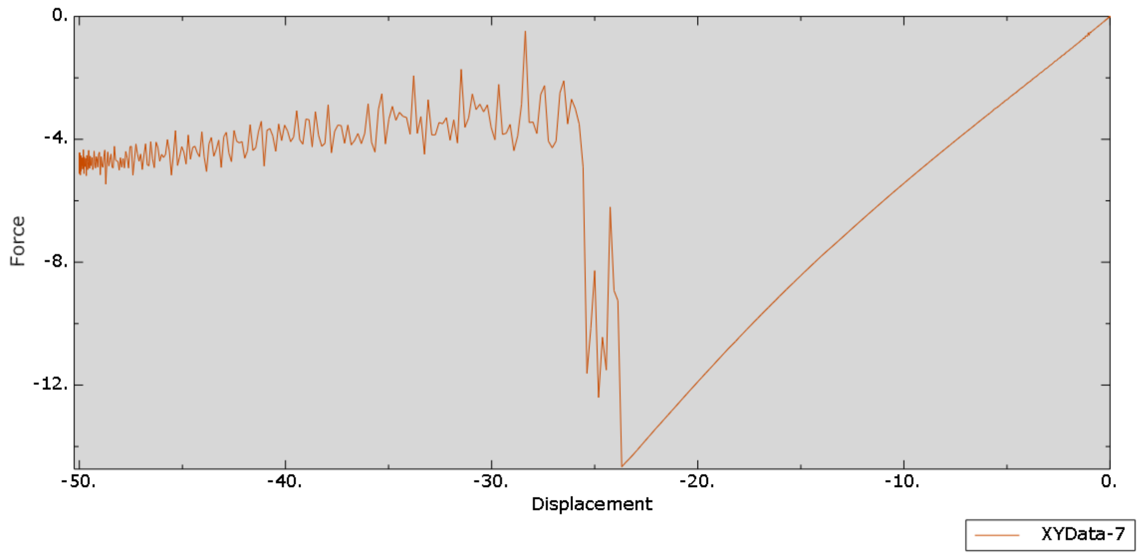


Figure 6.12: Step 2: Reaction force versus displacement.

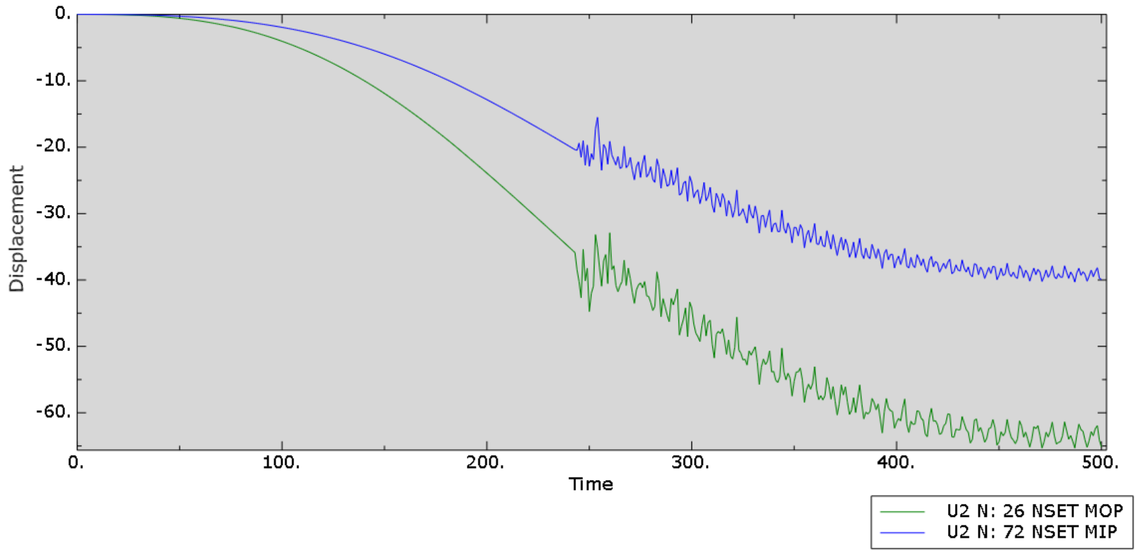


Figure 6.13: Step 3: Vertical displacement of the middle section's upper flange (border points).

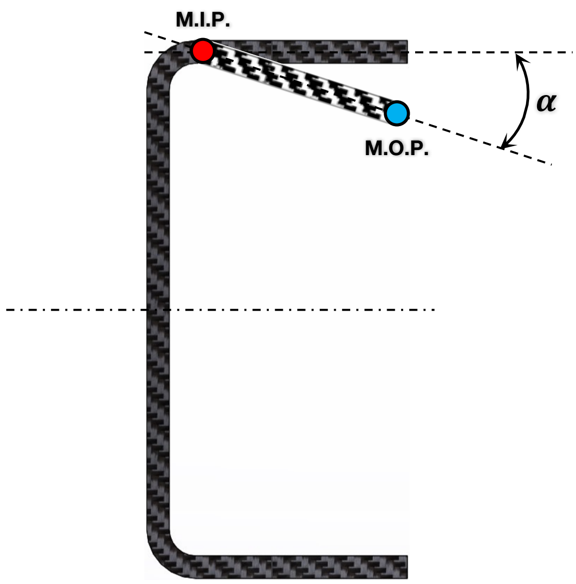


Figure 6.14: Middle section upper flange's rotation (α) computed via Middle Inner Point (M.I.P) and Middle Outer Point (M.O.P.) locations.

6.2.4 Surrogate Model

As discussed in the previous chapters, surrogate models are necessary when the problem is too complex and there are no exact analytical solutions.

For our problem, we want to create two meta-models, one relating the reaction force to the design variables and parameters, the other relating the rotation of the upper flange to the same inputs of the former surrogate model. With regard to the mass, a meta-model is not needed because the function is known.

Cross-validation error analysis

To choose which surrogate model approximates the best our sampled data, a cross-validation error analysis is run.

The compared meta-models involved in the error analysis are quartic RSM, RBF and Kriging. For cross-validation error analysis, all of the 109 data points are removed from the sampling data set, one at a time. For each of the removed points, the approximation coefficients are re-calculated, and the exact and approximate output values are compared. The removed point is then put back into the data set and the next point is removed. The points are selected randomly.

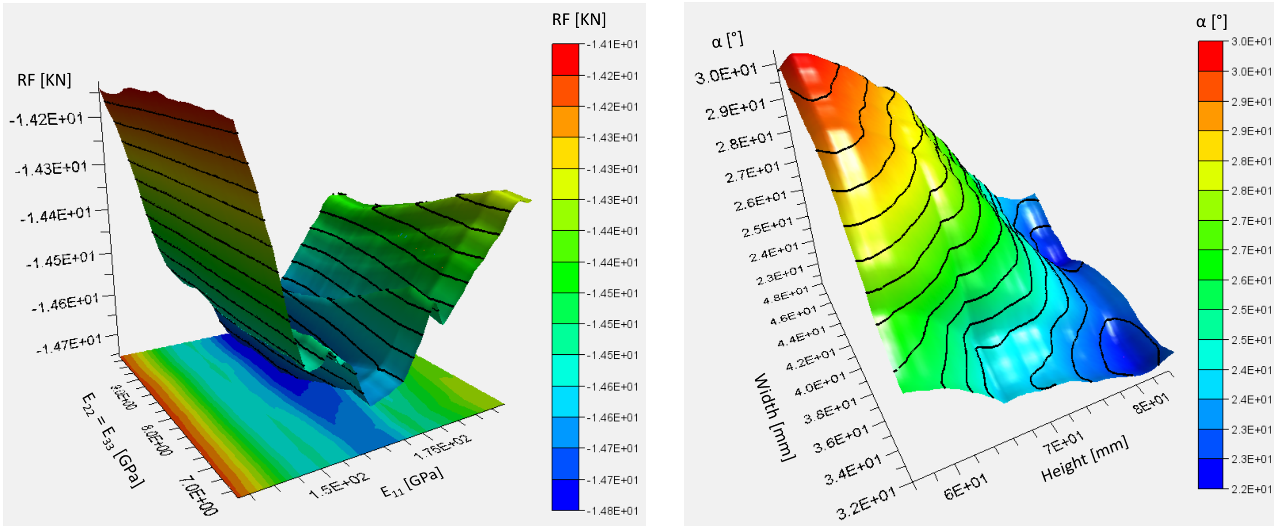
Table 6.6: Cross-validation error analysis.

	RSM	RBF	Kriging
$\epsilon(RF)$ [%]	10.42	8.169	6.517
$\epsilon(Alpha)$ [%]	5.195	4.647	4.617

In Tab.6.6, the average errors of different meta-models are presented. As can be observed, for our case, Kriging models (with exponential correlation function) have the smallest errors, either for the approximation of the reaction force and for the twisting angle.

Therefore, Kriging models (Fig.6.15) are best suited to approximate our responses.

In Fig.6.15 a, the reaction force is plotted as a function of E_{11} and $E_{22} = E_{33}$. It can be observed that, the reaction force (as can be expected) depends more on the longitudinal modulus of



(a) Kriging model for the reaction force. (b) Kriging model for the twisting angle.

Figure 6.15: Surrogate models of the responses.

elasticity rather than the transversal one.

In Fig.6.15 a, instead, can be observed the twisting angle (upper flange's rotation) as a function of height and width. Clearly, an increase in the width of the cross section corresponds to an increment of the twisting angle.

6.3 Optimization

6.3.1 Problem formulation

Deterministic Optimization - DO

In the DO problem proposed, the material properties and the design variables have their nominal values, without accounting for uncertainties.

The adopted constraints' values are the results of an on-design structural analysis ($Inputs$: $w = 40mm, h = 70mm, r = 4mm$; $Outputs : m = 0.7516Kg, RF = 14.82KN, \alpha = 27.06$ °).

It has been decided to set the values in Tab.6.7 as design constraints (for all the formulations) so that, the final design will have an overall better performance than the original floor beam, in terms of less mass, less twisting angle and higher reaction force.

Table 6.7: Deterministic Optimization problem formulation.

Description	Values
Design variables	
Width $[mm]$	$32 \leq w \leq 48$
Height $[mm]$	$56 \leq h \leq 84$
Radius $[mm]$	$3.2 \leq r \leq 8.4$
Constraints	
Mass $[Kg]$	$m \leq 0.7516$
Reaction Force $[KN]$	$RF \geq 14.82$
Twisting angle $[°]$	$\alpha \leq 27.06$
Objectives	
Description	Operation
Mass	*minimize*
Reaction Force	*maximize*
Twisting angle	*minimize*

Robust Design Optimization - RDO

Contrarily to the Deterministic Optimization previously described, this RDO problem formulation accounts for uncertainties, either for manufacturing tolerances and material properties. The statistical descriptions of design variables and parameters are not reported for clarity of exposition, because they've already been defined in Subsection 6.2.1.
In this formulation we can see that, besides the values of the objectives (actually mean values because now the objective and constraint functions are distributions), the standard deviations of the cost functions are to be minimized in order to achieve a robust design.

Table 6.8: Robust Design Optimization problem formulation.

Description	Values
Design variables	
Width $[mm]$	$32 \leq w \leq 48$
Height $[mm]$	$56 \leq h \leq 84$
Radius $[mm]$	$3.2 \leq r \leq 8.4$
Constraints	
Mass $[Kg]$	$m \leq 0.7516$
Reaction Force $[KN]$	$RF \geq 14.82$
Twisting angle $[°]$	$\alpha \leq 27.06$
Objectives	
Description	Operation
Mass	*minimize*
Reaction Force	*maximize*
Twisting angle	*minimize*
Std. Dev. Mass	*minimize*
Std. Dev. Reaction Force	*minimize*
Std. Dev. Twisting angle	*minimize*

Reliability-Based Design Optimization - RBDO

In RBDO problem formulation, which accounts for uncertainties, the original constraint related to the reaction force has been substituted with a reliability-based constraint (limit state function). In this formulation, a reliability-index $\beta = 3$ is chosen, hence the allowable probability of failure is $P_a = \phi(-\beta) = 0.135\%$. In other words, it entails (for normal distributions with only one limit) that at least the 99.865% of the population must have a reaction force higher than $RF_{lim} = 14KN$ (ultimate load). Here, conversely to RDO formulation, we don't have the standard deviations of the objective functions.

Table 6.9: Reliability-Based Design Optimization problem formulation.

Description	Values
Design variables	
Width $[mm]$	$32 \leq w \leq 48$
Height $[mm]$	$56 \leq h \leq 84$
Radius $[mm]$	$3.2 \leq r \leq 8.4$
Constraints	
Mass $[Kg]$	$m \leq 0.7516$
Twisting angle [°]	$\alpha \leq 27.06$
$P[RF < 14] - P_a \leq 0$	
Objectives	
Description	Operation
Mass	*minimize*
Reaction Force	*maximize*
Twisting angle	*minimize*

Robust and Reliability-Based Design Optimization - RRBDO

Finally, the mixed Robust and Reliability-Based Design Optimization formulation, aims at finding a robust solution (robust objectives) while satisfying reliability-based constraints. In this formulation, as well as in RBDO, a reliability-index $\beta = 3$ is adopted.

Table 6.10: Robust and Reliability-Based Design Optimization problem formulation.

Description	Values
Design variables	
Width $[mm]$	$32 \leq w \leq 48$
Height $[mm]$	$56 \leq h \leq 84$
Radius $[mm]$	$3.2 \leq r \leq 8.4$
Constraints	
Mass $[Kg]$	$m \leq 0.7516$
Twisting angle [°]	$\alpha \leq 27.06$
$P[RF < 14] - P_a \leq 0$	
Objectives	
Description	Operation
Mass	*minimize*
Reaction Force	*maximize*
Twisting angle	*minimize*
Std. Dev. Mass	*minimize*
Std. Dev. Reaction Force	*minimize*
Std. Dev. Twisting angle	*minimize*

6.3.2 Genetic Algorithm

Once set the problems' formulations, to search for the different solutions, a genetic algorithm must be chosen.

The Non-dominated Sorting Genetic Algorithm (NSGA-II) [10] is selected, because is a multi-objective technique, and deals with the high computational complexity of non-dominated sorting. Feasible solutions come first, and then infeasible solutions are sorted by increasing degree of constraint violations. Feasible solutions and every set of solutions with the same violation degree are then respectively sorted according to pareto dominance. All the solutions in a front are given the same rank value, beginning at 0 for the first front extracted, 1 for the second and so on. In this manner, solutions can be sorted according to rank.

Finally, within every group of solutions having the same rank, these results are sorted according to crowding distance. This criterion places first those solutions whose closest neighbours are farther, thus enhancing diversity.

Even though the optimization problems are all constrained optimizations, it is interesting to show how NSGA-II explores the design space and it doesn't stuck on local minima (Figs. 6.16, 6.17, 6.18, 6.19).

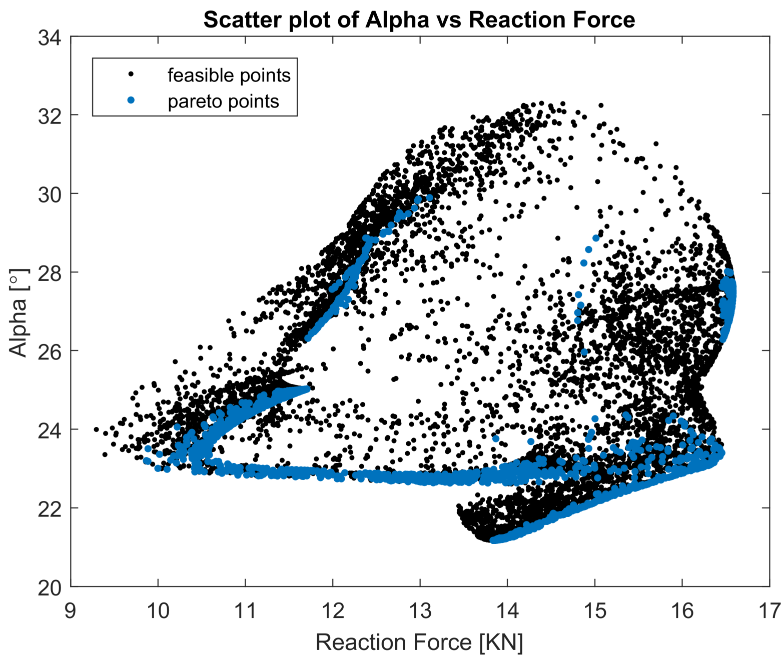


Figure 6.16: Scatter 2D plot of Twisting angle vs Reaction force, showing Pareto front.

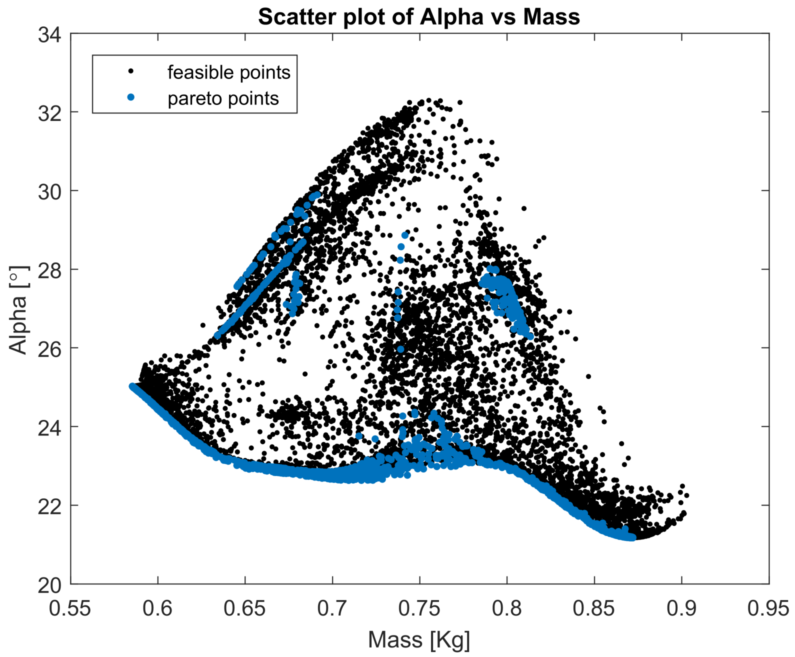


Figure 6.17: Scatter 2D plot of Twisting angle vs Mass, showing Pareto front.

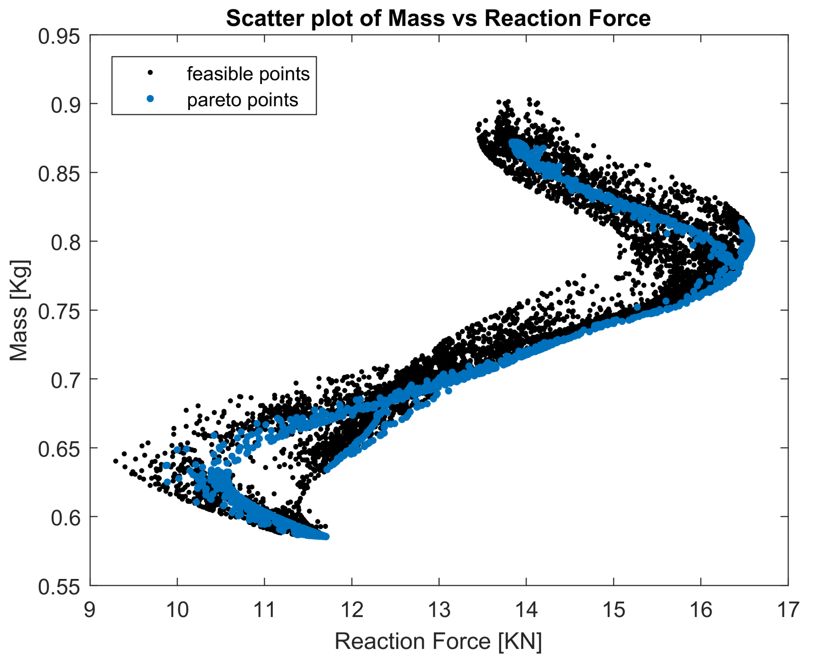


Figure 6.18: Scatter 2D plot of Mass vs Reaction Force, showing Pareto front.

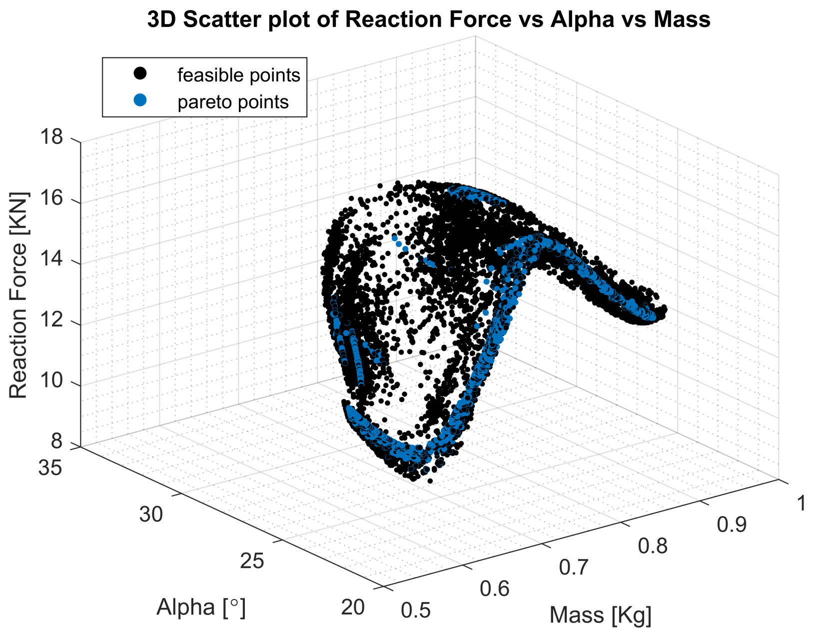


Figure 6.19: Scatter 3D plot of objectives, showing Pareto front.

Convergence analysis

Chosen the genetic algorithm, a convergence analysis is mandatory in order to get consistent results. For this purpose, the optimization is conducted on the deterministic optimization problem, which is faster to evaluate because it doesn't have the aggravating factor of considering uncertainties.

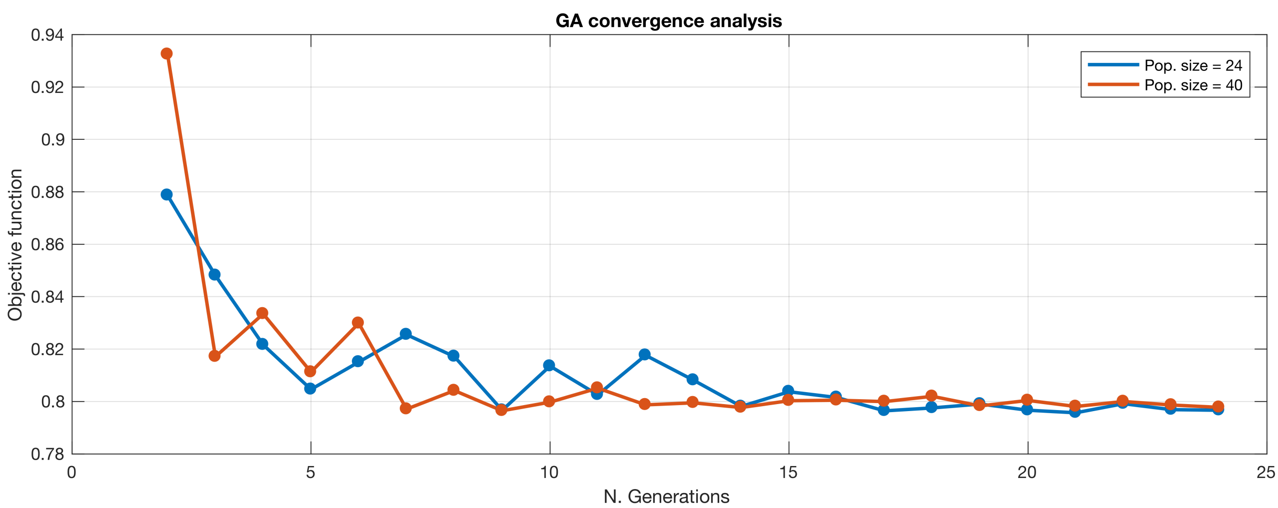


Figure 6.20: NSGA-II Convergence analysis for DO.

From Fig. 6.20 can be noticed that a genetic algorithm with higher population converges faster, but the computational time is higher as well. However, for number of generations greater than 20 both GAs are converging. In order to maintain the computing time as low as possible, a genetic algorithm with population size and number of generations equal to 24 is adopted.

6.3.3 MCS

To consider uncertainties, a certain number of realizations (N_s) of Monte Carlo Simulations must be chosen. The analysed techniques to generate these samples are (Fig.6.21):

- Simple random sampling : generates sample points by generating N_s uniformly distributed random numbers between 0 and 1 for each stochastic variable and/or parameter, obtaining corresponding values from each random variable/parameter distribution;
- Descriptive sampling : generates N_s sample points by dividing each random variable distributions into N_s interval of equal probability and randomly combining samples from these intervals for each random variable to produce design points. One point is sampled from each interval, thus ensuring a broad spread of data across the distributions of each random variable;
- Sobol sampling : is a sub-random sequence of numbers that are more uniformly distributed than both simple random and descriptive sampling.

In light of the fact above, as well as keeping a reasonable computational time, a $N_s = 1000$ realizations created with Sobol sampling technique has been chosen.

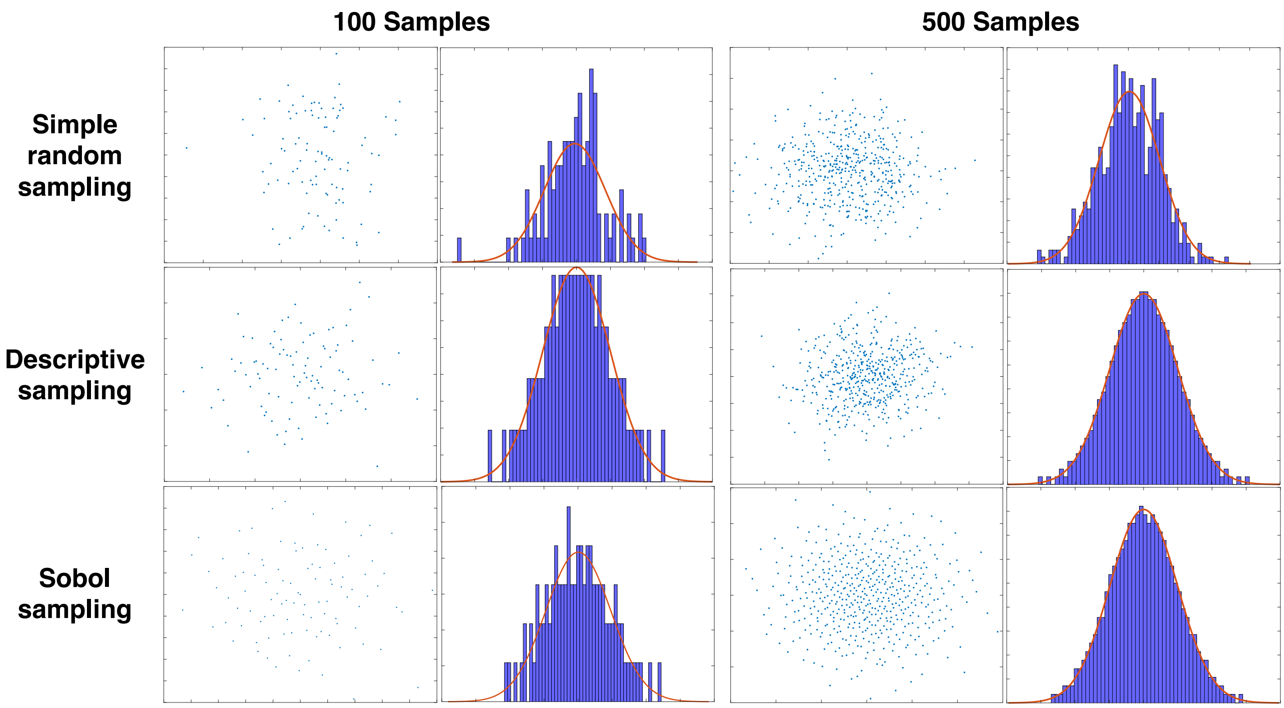


Figure 6.21: MCS sampling techniques.

6.4 Results

For the sake of clarity, the exposition of the results are structured according to each output's solutions (geometry, mass etc.), rather than different optimization approaches. In this way, it's easier to compare the various methods.

To contrast the deterministic optimization with the probabilistic ones, it has been applied (after the optimization process) variability to the deterministic design (manufacturing and material). Additionally, to gather the statistical values of the responses for the RBDO (because of their values are not stored in MCS loop), a robustness evaluation has been performed on the Reliable design.

Finally, to assess the probability of failure of the robust design (not stored during MCS loop), a reliability analysis has been performed. The hybrid robust and reliabilty-based method has all the values stored for each realization, thus no further analysis is needed.

It should be noted that, concerning the graphical representation of the results, the objectives are plotted as normal distributions with respective mean values and standard deviations.

Actually, they're not exactly normal distributions, but considering the *central limit theorem* [9], which states that the sum of many arbitrary distributed random variables asymptotically follows a normal distribution when the sample size becomes large, the assumption is reasonable. Furthermore, this is only a post-processing assumption, thus not invalidating the results.

6.4.1 Geometry

The first result we're going to discuss is the geometry of the cross section (Tab.6.11).
The genetic algorithm found that, maximizing the height of the cross section while minimizing the width of the flanges, leads to a robust design. Whereas, a reliable design is achieved minimizing the height of the cross section while maximizing the width. It's interesting to notice that the approaches that involve the minimization of the standard deviations (RDO and RRBDO) have the minimum values of fillet radius, which is the design variable with the highest coefficient of variation (Tab.6.3).
In addition, emphasizing its hybrid nature, the robust and reliability-based formulation, has values of the design variables (geometry) in between the original approaches.

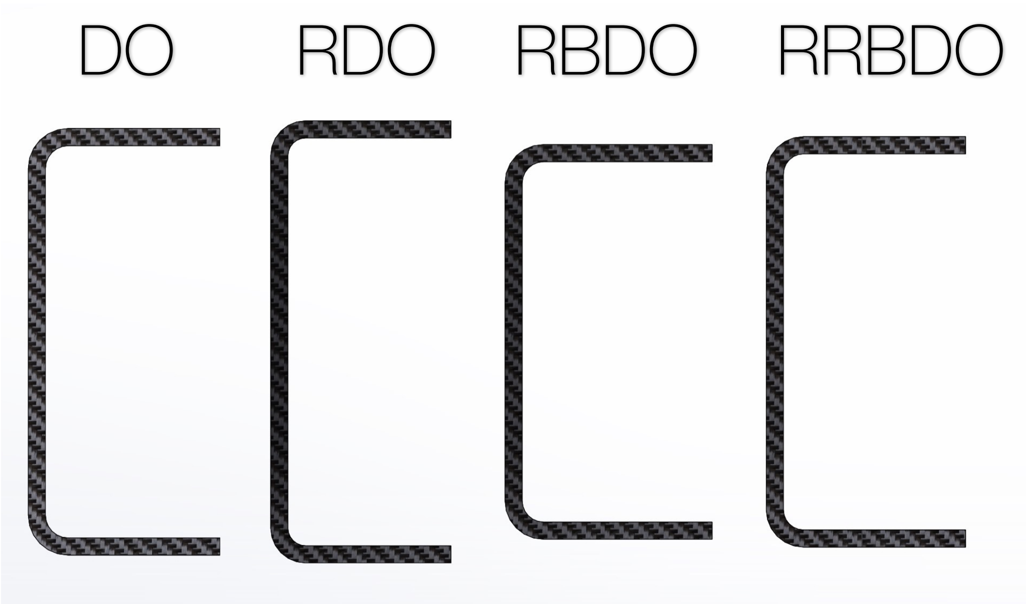


Figure 6.22: Cross-sections' geometry comparison.

Table 6.11: Geometry results.

	DO	RDO	RBDO	RRBDO
Height $[mm]$	79.01	81.88	73.10	76.02
Radius $[mm]$	4.196	3.275	3.941	3.726
Width $[mm]$	35.48	33.37	38.36	36.91

6.4.2 Mass

As for the mass, we can observe very small standard deviations, mainly because we provided the function that relates its value to only geometrical parameters, obviously. In this case, even if the uncertainties in the material properties were embedded into the optimization process, the mass response isn't affected by their variability. Here we can see that, the optimization methods for which the standard deviations were to be minimized (i.e. RDO, RRBDO) have the minimum values for those objectives. Even if it could seem counter-intuitive, those approaches have also the smallest mean values of the mass.

Table 6.12: Mass results.

	DO	RDO	RBDO	RRBDO
Mass [Kg]	0.7506	0.7475	0.7505	0.7426
σ (Mass)	6.040E-04	5.884E-04	5.992E-04	5.836E-04

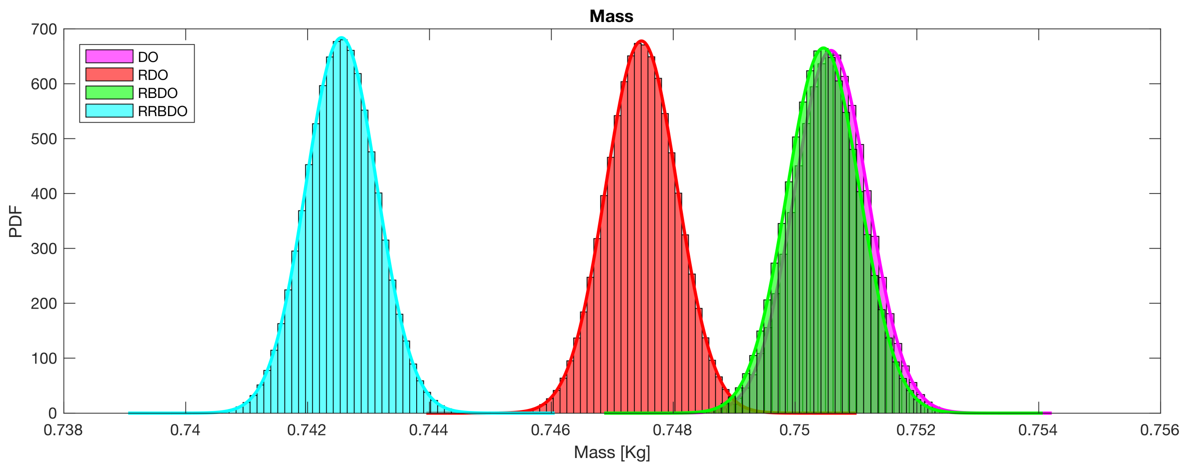


Figure 6.23: Probability distribution functions' comparison of mass results.

In Figs.6.23 and 6.24, the statistical representations of the mass for the different optimization approaches can be observed. In particular, in Fig.6.23 can be clearly noticed that all the probabilistic approaches have lower mass than the deterministic one, whereas focusing our attention on the normal curves' widths, we can observe the similar standard deviations' values. So, the design which minimizes both the mean value and the variability of mass is the robust and reliable design.

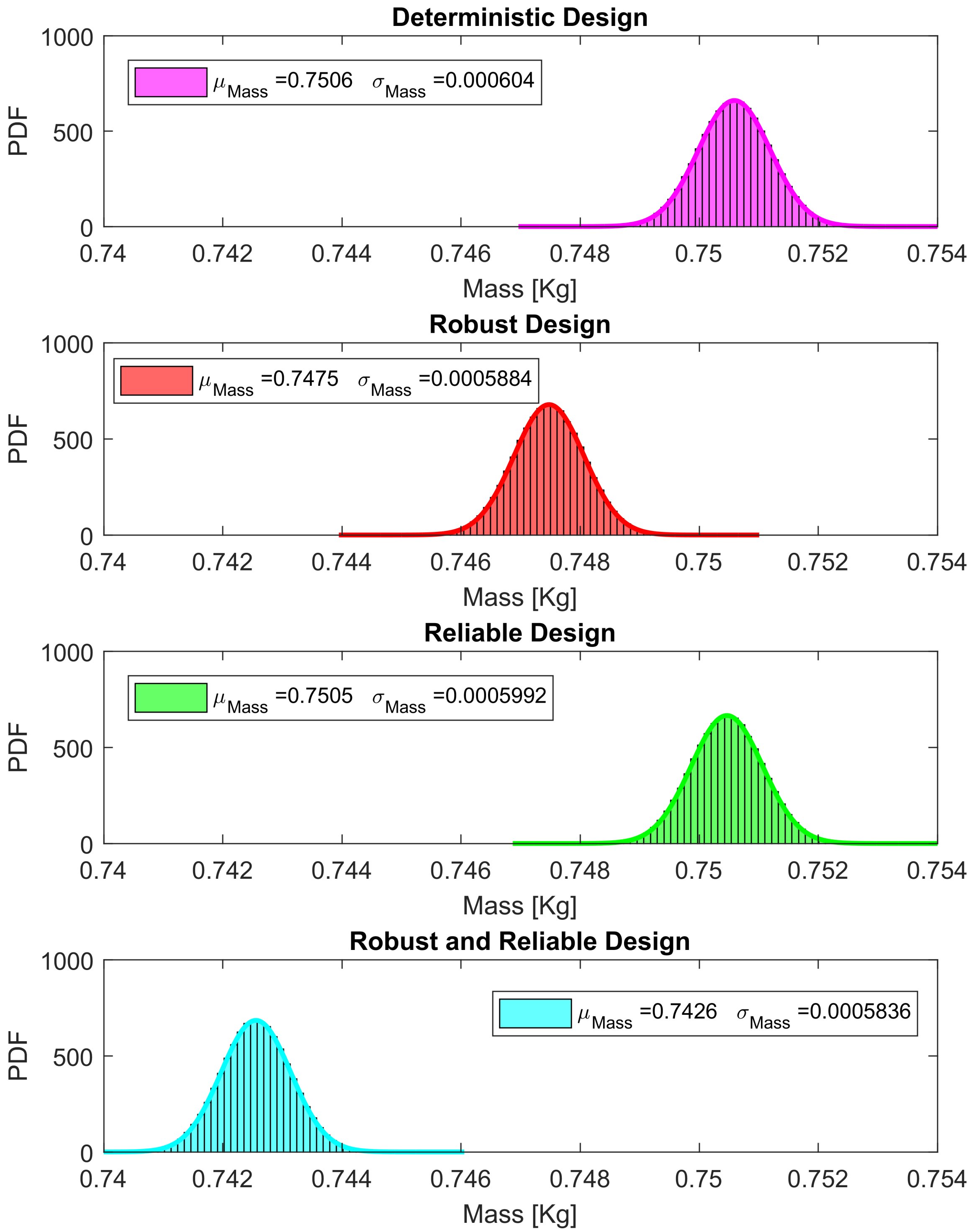


Figure 6.24: Mass results.

6.4.3 Reaction Force and Probability of failure

The results' analyses of reaction force and probability of failure are presented in conjunction, due to the relation between the failure's probability and the limit value of the reaction force (limit state constraint).

At first glance, we can see from Tab.6.13, that the RDO has a much higher value of failure's probability if compared to other methods. The rationale is that, in its formulation, it doesn't have any constraints or objectives predisposed to the minimization of failure's probability, as well as in the deterministic one. Even though the latter doesn't take into account the probability of failure, the deterministic design is more reliable than the robust.

As expected, the formulations that consider failure's probability (RBDO, RRBDO) have the smallest values of the above mentioned probability. Considering the reaction force's results, it's clear how the reliable formulation accounting for robustness (RRBDO) has the minimum standard deviation (Fig.6.25).

Table 6.13: Reaction force and probability of failure results.

	DO	RDO	RBDO	RRBDO
Reaction Force $[KN]$	15.38	14.92	15.36	14.78
σ (RF)	0.3850	0.2982	0.2130	0.1675
$P_{failure}$ [%]	0.3	8.2	0.1	0.1

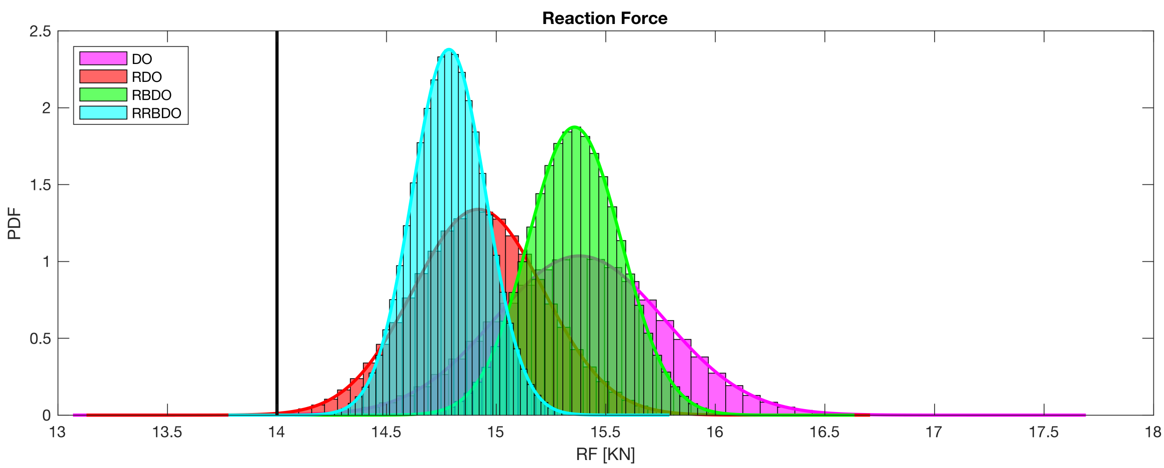


Figure 6.25: Probability distribution functions' comparison of reaction force results, showing limit state value.

In Figs.6.25 and 6.26, we can observe a concept already introduced in Chapter 2, Section 2.4, that is a more robust solution is achieved through shrinking the distribution.

Although the main aim of the robust optimization is to minimize the standard deviations of the objectives, here the robust design isn't the most robust because we have to remember that we're conducting a multi-objective optimization, that is minimizing all the standard deviations of all the objectives (not only the reaction force).

As for the design accounting for reliability (green and cyan curves), the strategy to have a more reliable design isn't just distancing the curves from the limit state value (towards right), but also diminishing the spread of the data, thus reducing the population in the tails of the distribution. In this case, the reaction force is maximum for the deterministic design, whereas the robust and reliable design (as the name suggests) has minimum variability of the reaction force and probability of failure.

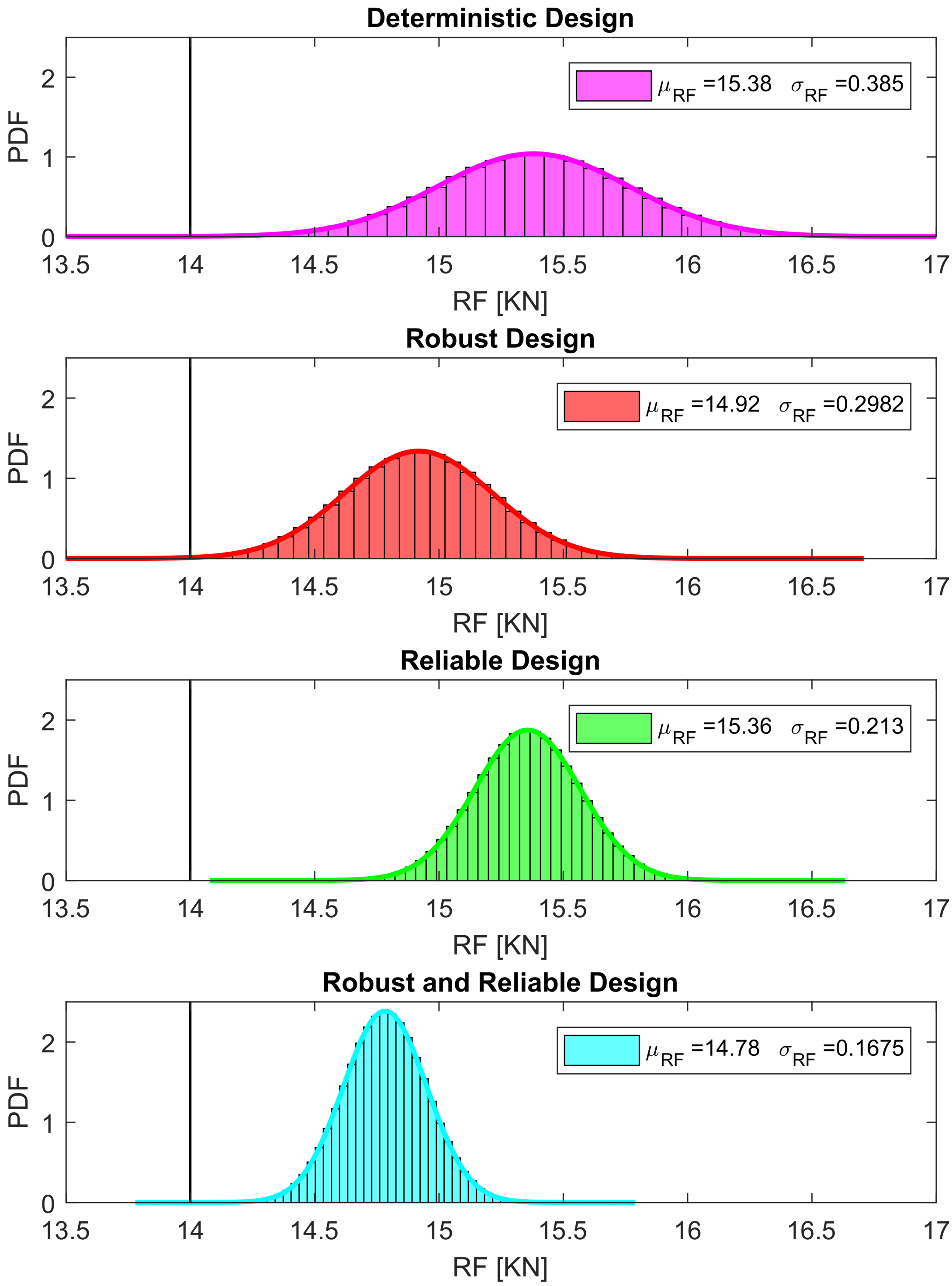


Figure 6.26: Reaction force results.

6.4.4 Twisting Angle

Concerning the rotation of the floor beam's upper flange, the design which minimizes the rotation angle is the deterministic one, while the robust design minimizes its variability. Here's clear to spot the difference between RDO (red) and RBDO (green), highlighting the fact that the latter doesn't account for minimization of objectives' variability (standard deviations) in its formulation.

From Figs.6.27 and 6.28, the red curve (robust design) evidently minimizes the variation of the rotation angle, while the cyan one (robust and reliable design) has the maximum value of twisting angle. However, it's useful to remind, that every optimized design has smaller values of twisting angle than the nominal one ($\alpha_{nominal} = 27.06°$).

Table 6.14: Twisting angle results.

	DO	RDO	RBDO	RRBDO
Twisting angle [°]	23.11	24.27	24.13	26.93
σ (Twisting angle)	0.5356	0.2456	0.9999	0.7611

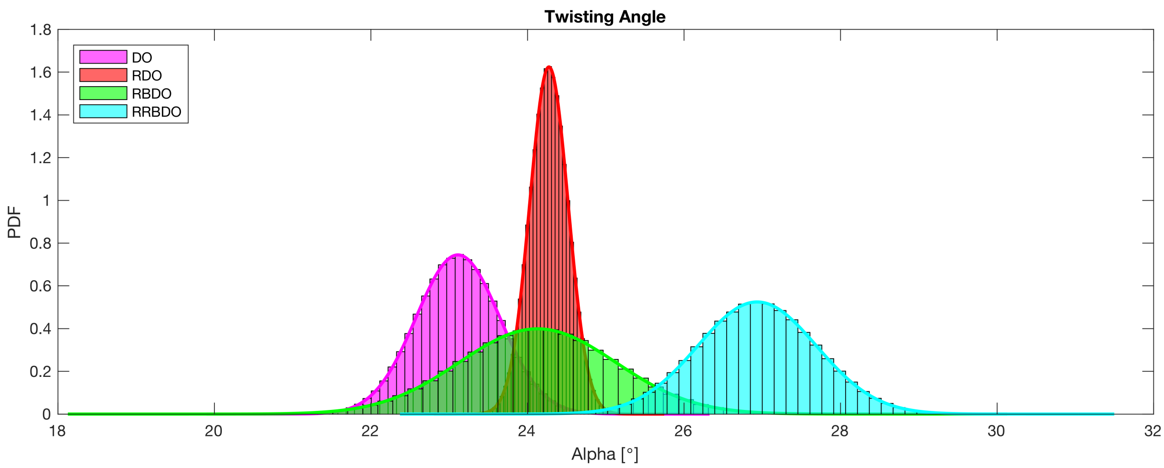


Figure 6.27: Probability distribution functions' comparison of twisting angle results.

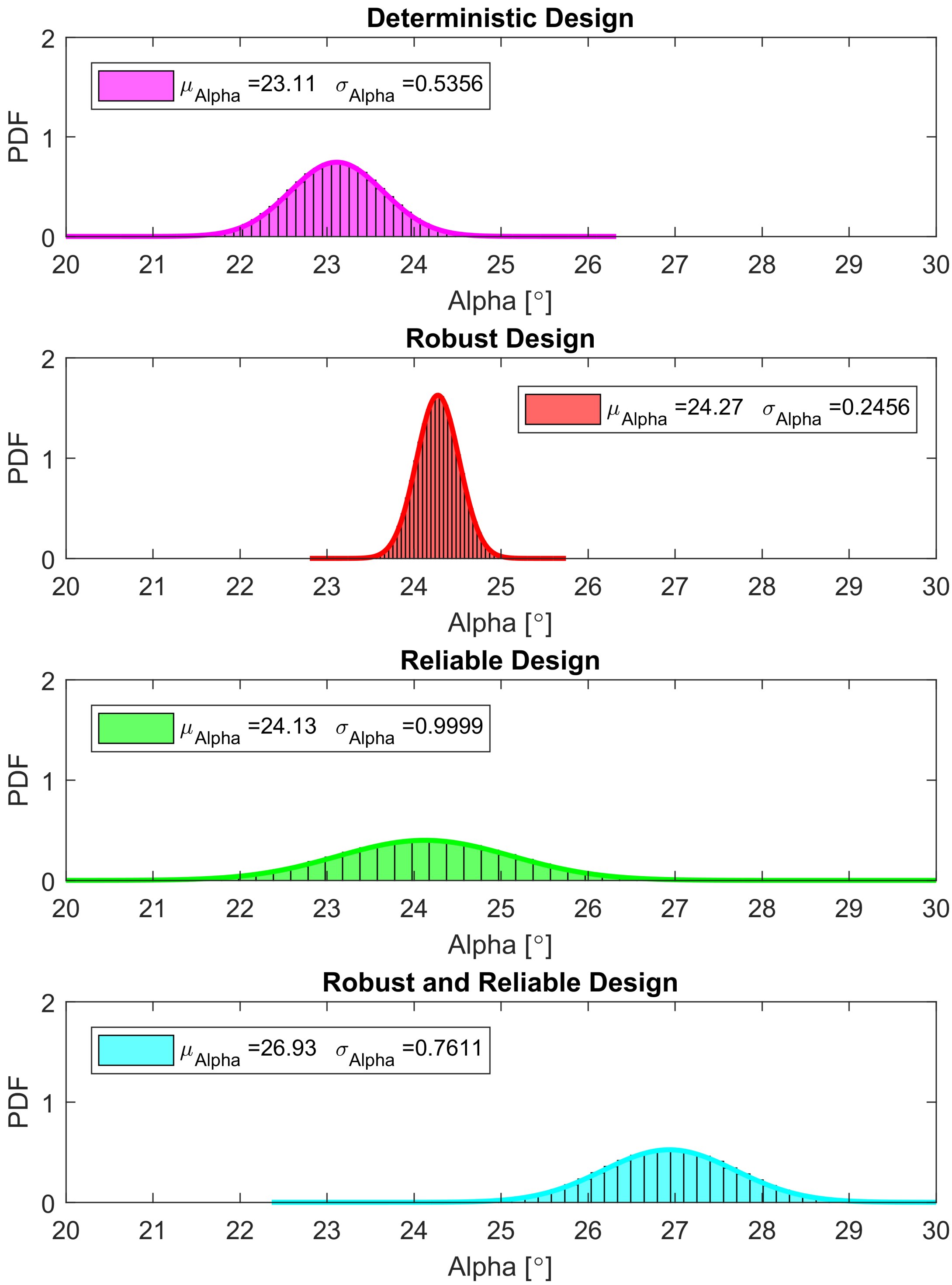


Figure 6.28: Twisting angle results.

6.4.5 Comparison

In Fig.6.29 we can observe a matrix having the various design as rows, while the different objective functions as columns.

As could be expected, the deterministic design is nor the most robust nor the most reliable, once introduced the variability in the geometry and in the material properties.

As previously mentioned regarding the reaction force (first column in Fig.6.29), the robust design has not the smallest standard deviation of the reaction force, but if we look at the other columns (mass and alpha) we can easily understand that the robust optimization found a trade-off minimizing the standard deviation of all the objectives.

Here, high level choices come into play. If, for instance, it is much more important to reduce the variability of the mass (thus geometry) due to reasons related to manufacturing costs (e.g. cut material, trim, collect and locate into mould etc. [18]), the weight factor of the mass in the multi-objective optimization function will be increased.

Finally, contrasting the reliable design (green) with the robust and reliable one (cyan), we can see that, apart from satisfying the reliable constraint, the latter has smaller overall variability.

Table 6.15: Complete results of different optimization approaches.

	DO	RDO	RBDO	RRBDO
Height [mm]	79.01	81.88	73.10	76.02
Radius [mm]	4.196	3.275	3.941	3.726
Width [mm]	35.48	33.37	38.36	36.91
Twisting angle [°]	23.11	24.27	24.13	26.93
Mass [Kg]	0.7506	0.7475	0.7505	0.7426
Reaction Force [KN]	15.38	14.92	15.36	14.78
σ (Twisting angle)	0.5356	0.2456	0.9999	0.7611
σ (Mass)	6.040E-04	5.884E-04	5.992E-04	5.836E-04
σ (RF)	0.3850	0.2982	0.2130	0.1675
$P_{failure}$ [%]	0.3	8.2	0.1	0.1

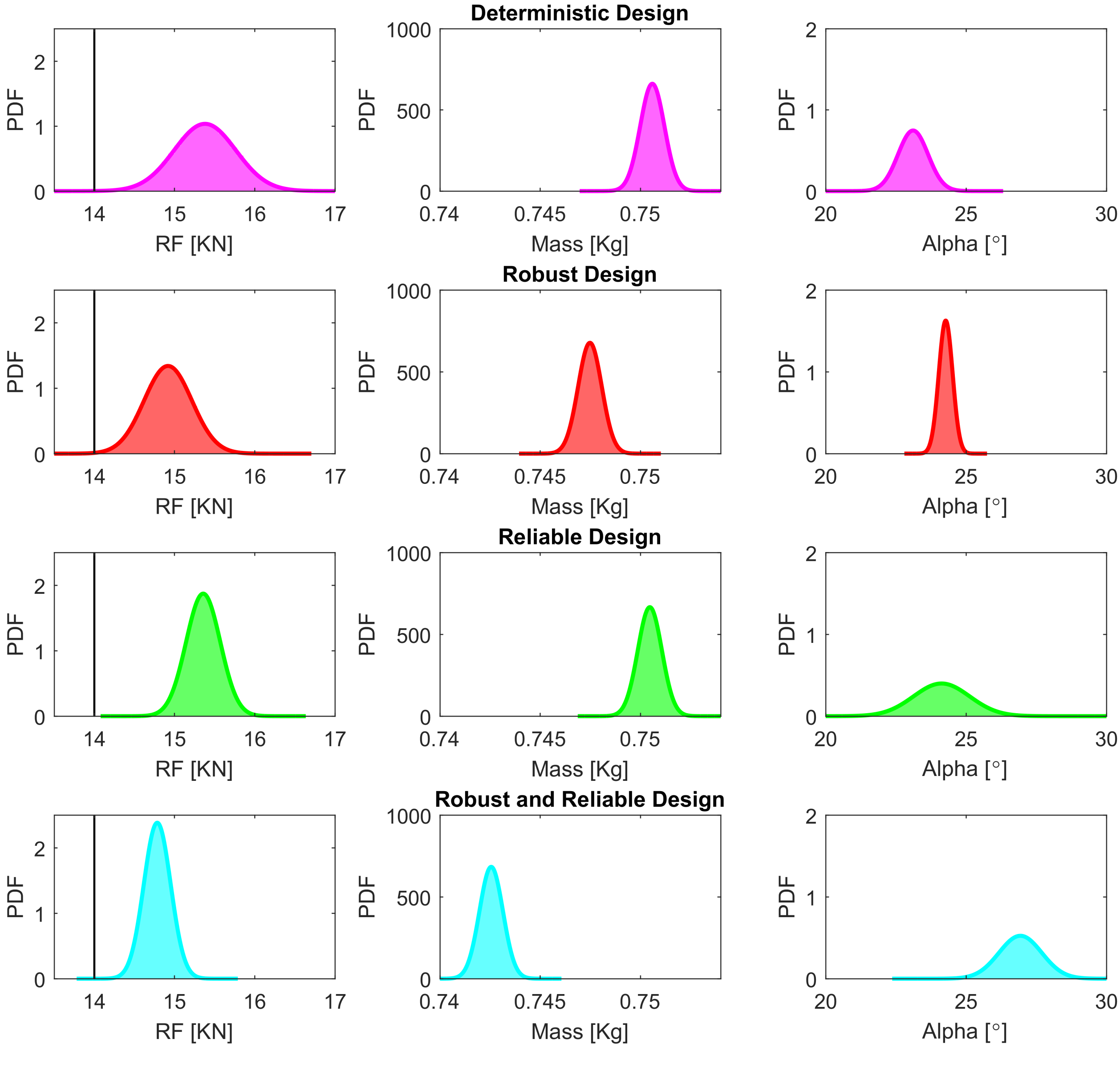


Figure 6.29: Graphical results' matrix.

Chapter 7

Conclusion

7.1 Thesis Achievements and Future Work

It this Thesis has been shown that, once introduced uncertainties due to manufacturing tolerances and material properties, the deterministic design led to a not robust and not reliable design. As extensively reminded, lack of robustness causes increasing costs throughout the structure's life-cycle, while lack of reliability may lead to incidents or even fatal accidents.
Stochastic approaches accounting for uncertainties, on the other hand, proved enhanced robustness (Robust Design), enhanced reliability (Reliable Design), or a combination of both (Robust and Reliable Design).
However, achieving a better design comes at a price, which is added complexity in process modelling, problem formulation and increased computational time. To partially overcome difficulties in process modelling, logical schemes for probabilistic optimization methods have been proposed. The need of a surrogate model to approximate the system's responses arose, and, by all of the meta-models analysed, Kriging models resulted in better approximations for either reaction force and twisting angle.
Amongst the stochastic approaches, Robust Design Optimization resulted globally better in terms of reducing performances' sensitivity to system's variations.
Both Reliability-Based Design Optimization and Robust and Reliability-Based Design Opti-

mization, satisfied the reliability constraint resulting in reliable designs. Although they're both reliable, the latter performs better in terms of robustness.

Adopting MCS to assess the probability of failure and the statistical parameters of the responses, resulted in comparable computational times for the three probabilistic methods, thus not justifying a formulation that doesn't account for either robustness and reliability.
The main disadvantage of the proposed framework is in the accuracy of the reliability assessment. Because even though MCS is easier to implement, gives an exact probability of failure and doesn't need derivatives of the objective and constraint functions, unlike different reliability methods (FORM, SORM), it requires higher number of realizations to achieve better accuracy, hence increased computational time. A comparison with different reliability analysis techniques could be interesting for future developments.

It's also interesting to observe that, in the optimization methods accounting for reliability (RBDO, RRBDO), the limit state function is related to the maximum force that the beam can withstand before collapsing (RF_{lim}), which is strongly dependent on the longitudinal modulus of elasticity (Fig.6.15). Besides, variability in the mass (thus geometry), as previously stated, is crucial for cost reduction in manufacturing processes.
For the above stated reasons, these strong correlations between responses and variable parameters could be exploited in an efficient multi-level optimization: performing a Reliability-Based Design Optimization considering only uncertainties in material properties, and a Robust Design Optimization accounting only for manufacturing tolerances.
In addition, to find the optimum design of a real composite part subjected to different types of loads, a more realistic design should include more load cases (ground loads, gust loads), hence more objective and constraint functions.
Finally, to improve the described formulations, more design variables (stacking sequence, fibres orientations) could be added to achieve better designs, at the expense of an increased design space, increased complexity and computational time.

Bibliography

[1] *Abaqus Documentation*. Dassault Systèmes, 2016.

[2] *Introduction to Isight*. Dassault Systèmes, 2016.

[3] L. Amandio, A. Marta, F. Afonso, J. Vale, and A. Suleman. Stochastic optimization in aircraft design. pages 267–272, 09 2014.

[4] J. Andersson. A survey of multiobjective optimization in engineering design johan, 2000.

[5] J. S. Arora. *Introduction to optimum design*. Academic Press, Oxford, 2011.

[6] O. Bacarreza, M. H. Aliabadi, and A. Apicella. Robust design and optimization of composite stiffened panels in post-buckling. *Structural and Multidisciplinary Optimization*, 51(2):409–422, Feb 2015.

[7] Y. Ben-Haim and I. Elishakoff. *Convex Models of Uncertainty in Applied Mechanics*. Elsevier Science Publishers, Amsterdam, 1990.

[8] G. E. P. Box and C. Fung. Is your robust design procedure robust? *Quality Engineering*, 6(3):503–514, 1993.

[9] S.-K. Choi, R. A. Canfield, and R. V. Grandhi. *Reliability-based Structural Design*. Springer, London, 2007.

[10] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A fast and elitist multiobjective genetic algorithm: Nsga-ii. *Trans. Evol. Comp*, 6(2), Apr 2002.

[11] H. Fang and M. F. Horstemeyer. Global response approximation with radial basis functions. *Engineering Optimization*, 38(4):407–424, 2006.

[12] A. I. J. Forrester, A. Sóbester, and A. J. Keane. *Engineering Design via Surrogate Modelling*. John Wiley and Sons, Ltd, 2008.

[13] A. Giunta, S. Wojtkiewicz, and M. Eldred. Overview of modern design of experiments methods for computational simulations. *AIAA*, 0649, 01 2003.

[14] R. V. Grandhi and L. P. Wang. Reliability-based structural optimization using improved two-point adaptive nonlinear approximations. *Finite Elements in Analysis and Design*, 29(1):35–48, 1998.

[15] W. Hare, J. Nutini, and S. Tesfamariam. A survey of non-gradient optimization methods in structural engineering. *Advances in Engineering Software*, 59:19 – 28, 2013.

[16] R. Jin, W. Chen, and T. Simpson. Comparative studies of metamodelling techniques under multiple modelling criteria. *Structural and Multidisciplinary Optimization*, 23(1):1–13, Dec 2001.

[17] Z. Kang. *Robust design optimization of structures under uncertainties*. PhD thesis, University of Stuttgart, 2005.

[18] C. Kassapoglou. *Design and Analysis of Composite Structures With Applications to Aerospace Structures*. John Wiley and Sons, 2013.

[19] N. Lagaros, V. Plevris, and P. Manolis. Reliability based robust design optimization of steel structures. 1:19–29, 12 2007.

[20] C. Lopez, O. Bacarreza, A. Baldomir, S. Hernandez, and M. H. Ferri Aliabadi. Reliability-based design optimization of composite stiffened panels in post-buckling regime. *Structural and Multidisciplinary Optimization*, 55(3):1121–1141, Mar 2017.

[21] A. Messac. *Optimization in Practice with MATLAB for Engineering Students and Professionals*. Cambridge University Press, Cambridge, 2015.

[22] R. H. Myers, D. C. Montgomery, and C. M. Anderson-Cook. *Response Surface Methodology: Process and Product Optimization Using Designed Experiments, fourth edition.* John Wiley and Sons, 2016.

[23] W. L. Oberkampf, S. M. DeLand, B. M. Rutherford, K. V. Diegert, and K. F. Alvin. Error and uncertainty in modeling and simulation. *Reliability Engineering and System Safety*, 75(3):333 – 357, 2002.

[24] A. C. Orifici, R. S. Thomson, R. Degenhardt, C. Bisagni, and J. Bayandor. A finite element methodology for analysing degradation and collapse in postbuckling composite aerospace structures. *Journal of Composite Materials*, 43(26):3239–3263, 2009.

[25] R. M. Paiva, C. Crawford, and A. Suleman. Robust and reliability-based design optimization framework for wing design. 52(4):711–724, 01 2014.

[26] J. Sacks, W. J. Welch, T. J. Mitchell, and H. P. Wynn. Design and analysis of computer experiments. *Statist. Sci.*, 4(4):409–423, 11 1989.

[27] G. Schueller and H. Jensen. Computational methods in optimization considering uncertainties - an overview. *Computer Methods in Applied Mechanics and Engineering*, 198(1):2 – 13, 2008.

[28] T. Simpson, J. Peplinski, P. Koch, and J. Allen. On the use of statistics in design and the implications for deterministic computer experiments. 02 1997.

[29] T. Simpson, J. Poplinski, P. N. Koch, and J. Allen. Metamodels for computer-based engineering design: Survey and recommendations. *Engineering with Computers*, 17(2):129–150, July 2001.

[30] N. P. Suh. *Axiomatic Design: Advances and Applications.* Oxford Univ. Press, New York, 2001.

[31] G. Taguchi, S. Chowdhury, and S. Taguchi. *Robust Engineering.* McGrawHill, New York, 2000.

[32] S. W. Tsai and E. M. Wu. A general theory of strength for anisotropic materials. *Journal of Composite Materials*, 5(1):58–80, 1971.

[33] M. Wahde. *Biologically inspired optimization methods: an introduction.* WIT Press, 2008.

[34] N. Yang, P. Das, J. Blake, A. Sobey, and R. Shenoi. The application of reliability methods in the design of tophat stiffened composite panels under in-plane loading. *Marine Structures*, 32:68 – 83, 2013.

[35] T. A. Zang, M. J. Hemsch, M. W. Hilburger, S. P. Kenny, J. M. Luckring, P. Maghami, S. L. Padula, and W. J. Stroud. Needs and opportunities for uncertainty- based multidisciplinary design methods for aerospace vehicles. Technical report, Langley Research Center, Hampton, Virginia, 2002.