POLITECNICO DI TORINO

Corso di Laurea Magistrale
in Automotive engineering

Master degree Thesis

Influence of tyre pressure on vehicle dynamics

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Chapter 1

Introduction

1.1 Target of the thesis

The work aim is to characterize the performance of a vehicle when different inflation pressure are applied to the tires.
The tools used to accomplish this task are two simulation software: CDtire and CarMaker. The first one is used to characterize the behaviour of the tire stand alone and the second is used for the simulations of the whole vehicle.
The work is divided in two different steps: first we characterize the tire though various simulations done in CDtire and we store the results in look-up tables based on the simulation variables; second we made a simulink model implemented in CarMaker that uses the results stored in the simulations done before. In this way we have obtained a look-up table based model which has the characteristics of the tire simulated in CDtire.
In CarMaker there is also the pacejka 6.1 model that allow a comparison with the results obtained with our model (in fact pacejka 6.1 is the new model that consider as a variable of the inflation pressure).

1.2 State of the art

The literature about the effect of the inflation pressure on the tire performance it is not very wide. The main results that researches highlights are the effect of the pressure on the vertical stiffness, longitudinal and lateral stiffness and the rolling resistance.

1.2.1 Optimum slip ratio

The tire inflation pressure is a crucial parameter in tire performance behavior and numerous studies conducted on this field. For what regards the optimum slip, a research done by Taiyuan University of Technology and University of Huddersfield[1] proposed an improved optimum slip prediction that takes into account the tire pressure. Starting from the Liu[2] and Bian[3] formulation

$$\lambda = \frac{u - \omega R_e}{u} \tag{1.1}$$

$$\lambda_{op,nom} = \lambda_0 + 0.165 \lg\left(\frac{64}{u}\right) + 0.01\delta^{1.5} \tag{1.2}$$

Where $\lambda_{op,nom}$ is the optimal slip ratio, λ_0 is the road surface friction coefficient, u is wheel velocity, and δ is the tire slip angle.

The improved one takes into account the inflation pressure with a second order equation. In fact fitting the results obtained from the magic formula, we obtain a correlation index of 0.982.

$$\lambda_{op} = \lambda_{op,nom} + p_{pi1} dp_i + p_{pi2} dp_i{}^2 \tag{1.3}$$

The result in the following figure are of a 205/60-R15 tire under a vertical load of 5000N.

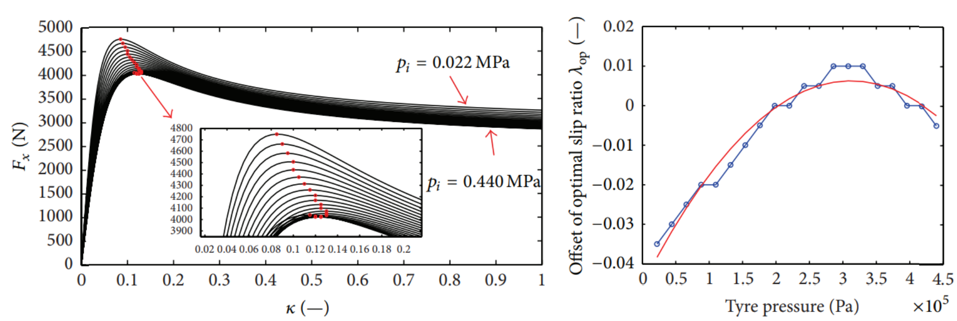


Figure 1.1: Left: longitudinal force vs slip at different inflation pressure; Right: offset from the optimal slip ratio vs inflation pressure, continuos line is the theoretical

1.2.2 Numerical simulations using STI model

A research made at *Clemson University* studied the effect of the pressure on the vehicle handling characteristics. The variation of longitudinal, lateral and self-aligning torque are numerically computed using the STI model. Finally using CarSim software the results are integrated and a quadruple lane change test is made for different inflation pressure and load conditions.

The STI model is a model based on the tire/road interface contact patch that affects the longitudinal and lateral stiffness of the tire. The model considers the tire inflation pressure effect on the contact patch lenght that influence the longitudinal and lateral stiffness. The length of the contact patch is

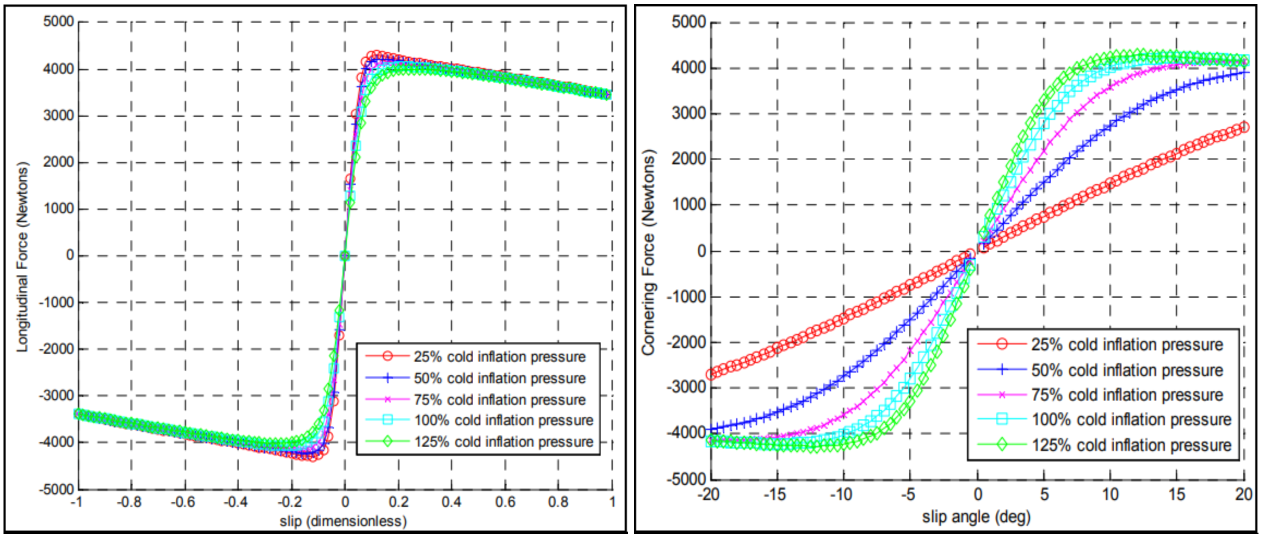
$$a_p = a_{p0}\left(1 - K_a \frac{F_x}{F_z}\right) \tag{1.4}$$

where a_{p0} is the length at static conditions that contains the pressure effect, F_x and F_z are respectively the longitudinal and vertical force acting on the tire and K_a is a coefficient. The longitudinal and lateral stiffness coefficient are then calculated as

$$K_{lon} = \frac{1}{{a_{p0}}^2} F_z \left(\frac{CS}{FZ}\right) \tag{1.5}$$

$$K_{lat} = \frac{1}{{a_{p0}}^2} \left(A_0 + A_1 F_z - \frac{A_1}{A_2} {F_z}^2\right) \tag{1.6}$$

where K_{lon} and K_{lat} are the longitudinal and lateral stiffness, and the other are parameters pressure dependent.
The equation for the computation of the forces and moments are much more complicated and take into account the large values of slip and sideslip angles, but we are not going to present them since it is out of the scope of the work. Now the look-up tables are built offline for the various operating scenario and stored.
In 1.2b and 1.3 we can see some results for a load of 5231 N at different inflation pressure: we can see that the pick longitudinal force and longitudinal stiffness are not affected so much while the lateral stiffness and the pick position (for the self-aligning moment) change significantly. The tire parameters of the STI model used are for a P185/70 R13 tire.



(a) Longitudinal force vs slip ratio

(b) Lateral force vs slip angle

Figure 1.2: Forces exchanged with different pressures

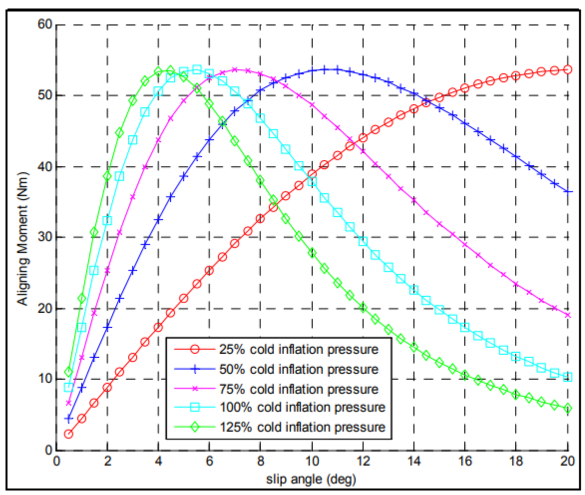


Figure 1.3: Self-aligning moment versus slip angle with different pressures

The maneuvers simulated in CarSim is a quadruple lane change with a sedan (150kW engine, automatic transmission, front wheel drive, rack/pinion power steering) at 90 kph target speed.
The tests are done at different pressure conditions with three different load case: standard (config A) with m=1,370kg, CG at (1.11m longitudinal from front axle, 0.21m above axle), passenger (config B) with m=1,624.5kg, CG at (1.18m, 0.24m), and rear with trunk load (config C) with m=1,643kg, CG at (1.49m, 0.24m). In the table below we can see the results about the maximum values obtained in the simulation.

In figure 1.5 we can see the maximum values of steering angle δ, sideslip angle of the vehicle(β), sideslip angle of the front left tire α_{LF} and sideslip angle of the rear left tireα_{LR}

Case	Tire Loc	Tire %T_{dp}	Max(δ)	Max(β)	Max(α_{LF})	Max(α_{LR})
1A	All	100	68.2°	3.4°	3.3°	4.0°
1B			65.5°	3.8°	3.7°	4.4°
1C			64.1°	5.4°	4.1°	6.1°
2A	All	70	104.2°	6.1°	4.5°	6.7°
2B			104.0°	6.9°	5.2°	7.4°
2C			146.4°	12.8°	5.5°	13.6°
3A	LF, RF	70	94.5°	3.7°	4.6°	4.3°
3B			92.3°	4.2°	4.9°	4.7°
3C			86.8°	5.6°	4.6°	6.0°
4A	LR, RR	70	77.7°	6.4°	4.5°	7.2°
4B			83.1°	7.6°	5.0°	8.5°
4C			sat (*)	56.0°	52.5°	57.3°
5A	RF, RR	70	85.1°	4.5°	3.9°	5.1°
5B			84.3°	5.0°	4.2°	5.6°
5C			82.0°	7.9°	4.6°	7.4°
6A	LF, RR	70	81.8°	4.4°	3.9°	5.0°
6B			81.1°	5.0°	4.4°	5.6°
6C			81.2°	7.9°	4.9°	8.7°
7A	All	40	230.3°	15.8°	8.3°	16.7°
7B			276.5° (*)	19.4°	7.7°	20.3°
7C			sat (*)	160.3° (*)	178.7° (*)	161.1° (*)
8A	LF, RF	40	185.1°	4.6°	9.4°	5.2°
8B			186.0°	5.0°	9.6°	5.6°
8C			143.4°	6.3°	7.7°	6.8°
9A	LR, RR	40	sat (*)	170.4° (*)	179.9° (*)	179.3° (*)
9B			sat (*)	179.1° (*)	178.4° (*)	179.2° (*)
9C			sat (*)	178.5° (*)	179.4° (*)	179.6° (*)
10A	RF, RR	40	114.9°	6.2°	4.6°	6.9°
10B			115.2°	7.0°	5.2°	7.6°
10C			sat (*)	76.3° (*)	98.2° (*)	80.6° (*)
11A	LF, RR	40	107.3°	5.8°	5.1°	6.5°
11B			108.0°	6.7°	5.8°	7.2°
11C			193.9°	15.5°	7.3°	16.4°

Figure 1.4: Peak values of steering wheel angle(δ), sideslip angle of the vehicle(β), sideslip angle of the front left tire α_{LF}, sideslip angle of the rear left tireα_{LR}. The parameter $\%T_{dp}$ indicates the percentage of inflation pressure and the values signed with * means that in that conditions the vehicle lose control

during a quadruple lane change. The load case are examined at different pressure conditions and we can see that the best condition is at the nominal inflation pressure, in which we have less steering effort and lower sideslip angles.

1.2.3 Simulations using pacejka 6.1 model

We have seen some simulation results starting from the STI model that characterizes the tire behaviour. An article in *Journal of Dynamic Systems, Measurement, and Control* show the results obtained in a reaserch with different inflation pressure, using the pacejka 6.1 model to characterize the tire.
The different inflation pressures tested are determined based on the best value of comfort, ride, handling and fuel consumption; for each case in fact they will have a better value of pressure that is different for each one: we will see that the best pressure to optimize fuel consumption is far from the best value for comfort, ride or handling.

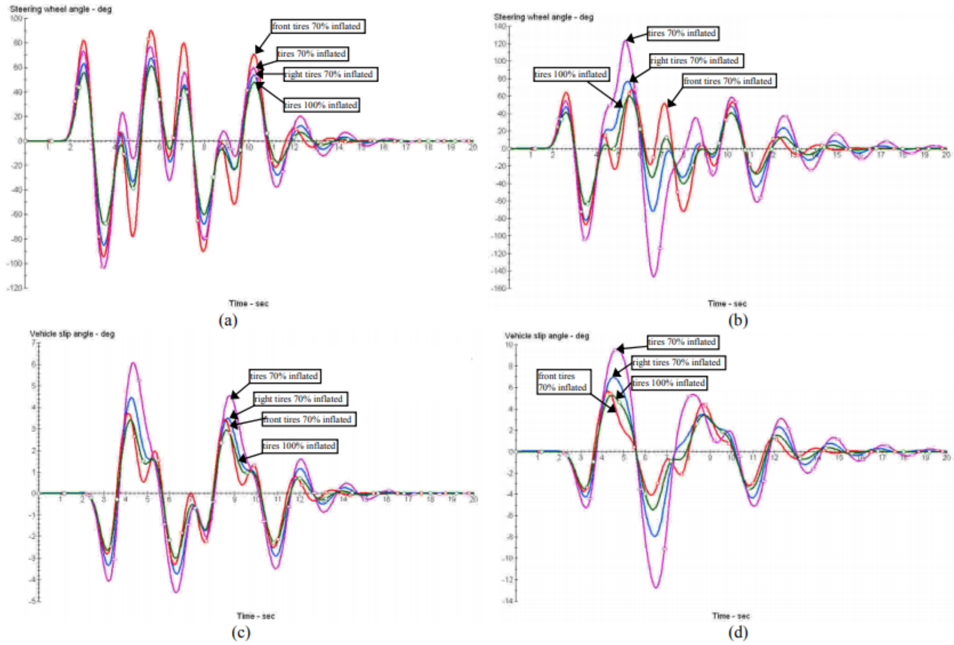


Figure 1.5: Steering wheel and vehicle slip angles for a quadruple lane change. Plot (a) and (c) are for A configuration while (b) and (d) are for C configuration.

First of all we will talk about handling. The model used to study the problem is the quarter car model (1.6) and we have to find the best value of k_u that is directly related to the inflation pressure.

The analytical relation between vertical stiffness and pressure (from pacejka formulation) is the following

$$k_t = (1 + q_{Fz3}dp_i)(q_{Fz1} + 2q_{Fz2}\rho) \tag{1.7}$$

where q_{Fz} are fitting parameters that strictly depends on the tire, ρ and dp_i are

$$dp_i = \frac{p}{p_0} - 1 \tag{1.8}$$

$$\rho = R_\Omega - R_l \tag{1.9}$$

The parameters of the quarter-car model, as seen in Fig. 1, are sprung mass m_s, suspension stiffness k_s, suspension damping coefficient (c_s), tire/wheel mass (i.e., unsprung mass) (m_u), and tire vertical stiffness (k_t). Considering a sinusoidal excitation

$$y = Y\sin(\omega t) \tag{1.10}$$

we can define a trasmissibility function

$$A_z = \frac{a_{zs}}{Y} \tag{1.11}$$

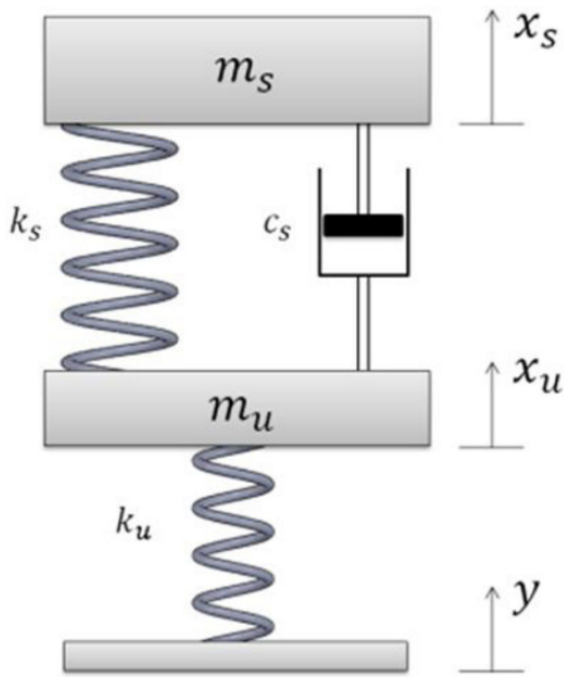


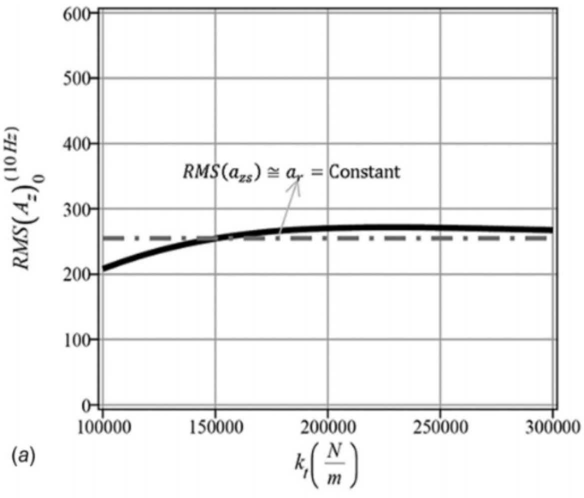
Figure 1.6: Quarter car model of the vehicle

and its power spectral density

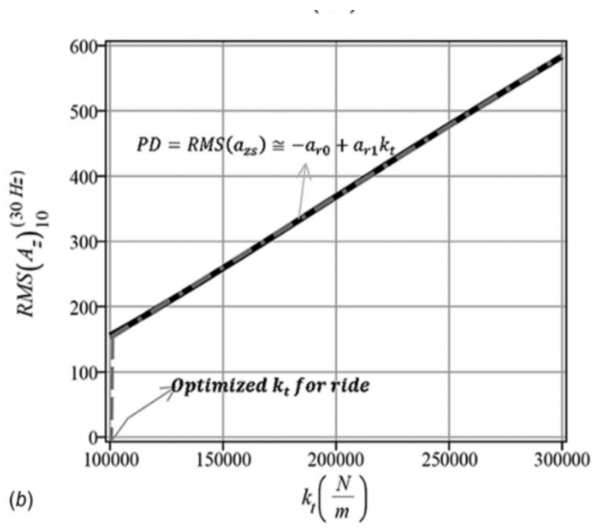
$$RMS(A_z) = \sqrt{\frac{1}{\omega_f - \omega_i}\int_{\omega_i}^{\omega_f} {A_z}^2 d\omega} \tag{1.12}$$

that would be a measure of confort. The lower the value of $RMS(A_z)$ means the lower the value of A_z that means the lowest vertical excitation of sprung mass a_{zs}.

For what concern transmissibility, in figure 1.7 we can see that we can notice some differences only at very high frequencies. While if we look at the RMS (figure 1.7a) we can see that in the low frequency range (0-10 hertz) it is still not pressure dependent, while at higher frequency range (10-30 hertz) it is proportional with it (figure 1.7b).



(a) Transmissibility from 0 Hz to 10 Hz



(b) Transmissibility from 10 Hz to 30 Hz

Figure 1.7: Forces exchanged with different pressures

The handling behaviour is mainly characterized by the lateral stiffness of the tire since it is responsible of the lateral force building; it is clear to understand that we are going to find the value of inflation pressure that maximize the lateral stiffnessc.
Remembering that the lateral stiffness is defined as

$$C_{F\alpha} = \left(\frac{\delta F_y}{\delta \alpha}\right)_{\alpha=0} \tag{1.13}$$

From the magic formula 6.1 we would have that is formulated as

$$C_{F\alpha} = P_{KY1}\Big(1 + P_{KY1} dp_i\Big) F_{z0} \sin\Big\{2 \arctan\Big[\frac{F_z}{P_{KY2}(1 + P_{KY2} dp_i) F_{z0}}\Big]\Big\} \tag{1.14}$$

where P's terms are parameters of the specific tire, F_{z0} is the nominal load of that tire and F_z is the actual load. Solving equation (4) for dp_i we have

$$dp_i \cong \frac{1}{q_{Fz3}}\left(\frac{k_t}{q_{Fz1}} - 1\right) \tag{1.15}$$

in this way we can have a direct relation between lateral stiffness and vertical.
In figure **??** we can see the relation between the two. The phisical meaning of the plot could be explained thinking about the contact patch of the tire: it is maximum for a certain value of pressure and tend to decrease for outer value; since the lateral stiffness is proportional to the contact patch the plot has the same shape.

Similarly to the handling, for accelerating/braking analysis we are going to investigate the value of the longitudinal stiffnes of the tire and the optimum tire inflation pressure would we that one that maximizes this value.

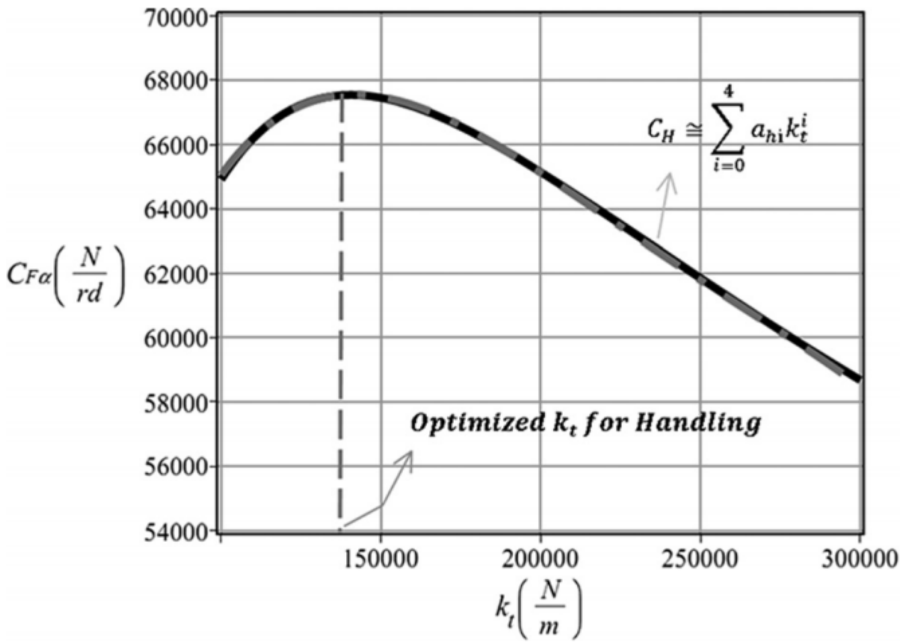


Figure 1.8: Lateral stiffness vs vertical stiffness of the tire

We recall the definition of longitudinal stiffness

$$C_{F\lambda} = \left(\frac{\delta F_x}{\delta \lambda}\right)_{\lambda=0} \tag{1.16}$$

where λ is the slip and F_x the longitudinal force. Based on the pacejka 6.1 we have

$$C_{F\lambda} = F_z(P_{KX1} + P_{KX2}df_z) \exp P_{KX2}df_z(1 + P_{PX3}dp_i + P_{PX4}{dp_i}^2) \tag{1.17}$$

where P's terms are parameters of the specific tire and df_z is the normalized change of the load, defined as

$$df_z = \frac{F_z - F_{z0}}{F_{z0}} \tag{1.18}$$

As before we can substitute dp_i with equation 1.15 to obtain a direct relation with the vertical stiffness.
In figure 1.9 we can see the longitudinal vs vertical stiffnes graph. At the end we have the fuel consumption. The fuel consumption is affected by the tire rolling resistance, which is the resistive force to be overcome to mantain a tire speed.
We do not need to know exaclty the relation between fuel consumption and rolling resistance force, we just take in mind that the lower the rolling resistance the lower the consumption. So we are searching the value of pressure that minimizes it.
As before in formula we can define the rolling resistance coefficient as

$$f_{rr} = f_{rr0}(dp_i + 1)^{-\alpha}(df_z + 1)^{\beta} \tag{1.19}$$

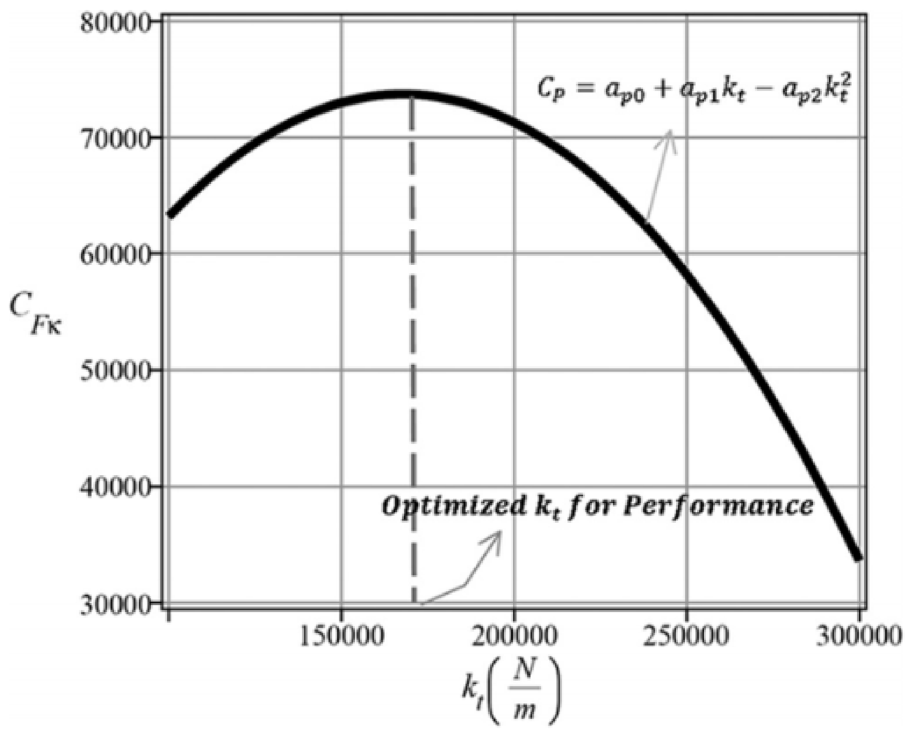


Figure 1.9: Lateral stiffness vs vertical stiffness of the tire

and substituting dp_i with equation 1.15 we find the relation with vertical stiffness k_t.
The figure shows the rolling resistance coefficient versus vertical stiffness: the optimum value corresponds to the maximum value of pressure. Finally with CarSim software they made a double lane change simulation at $33\,m/s$ longitudinal speed. The simulation is made for the same tire (195/65R15) at the four different optimized pressure seen before plus an optimized value that corresponds to a trade off among them. The results shows a consistent change between the various tested vertical stiffness of the tire.

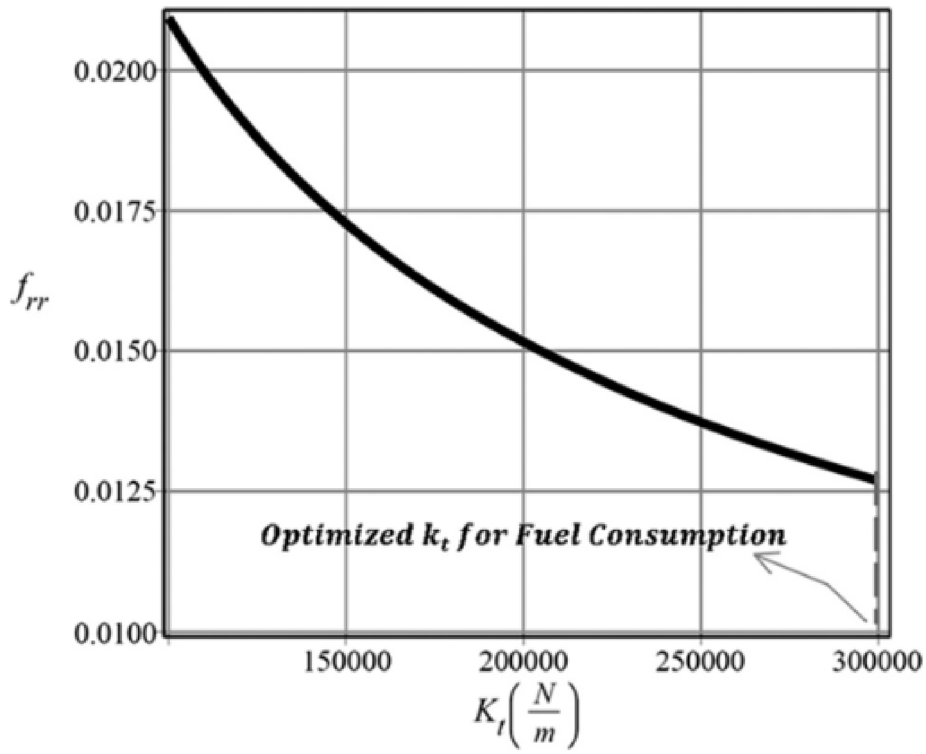


Figure 1.10: Rolling resistance coefficient vs vertical stiffness of the tire

1.3 Pacejka 6.1

In CarMaker there are several tire models available and one of them is Pacejka 6.1. It is the latest version of pacejka's models and the peculiar characterics is that it takes into account the inflation pressure of the tire.
This model will be a term of comparison of results that we will find in the CDtire based model in order to see the differences between the two.

1.3.1 Reference axis

Forces and moments are referred in the tire road contact point, considering a tire as an infinetily thin disk in the middle plane of simmetry of it (1.12).

The general formula of pacejka is

$$Y(X) = D\sin[C\arctan\{B(X+S_H) - E(\,B(X+S_H) - \arctan B(X+S_H)\,)\,\}\,] - S_V \quad (1.20)$$

where:

- $Y(X)$ represents F_x, F_y or M_z
- D is the peak value

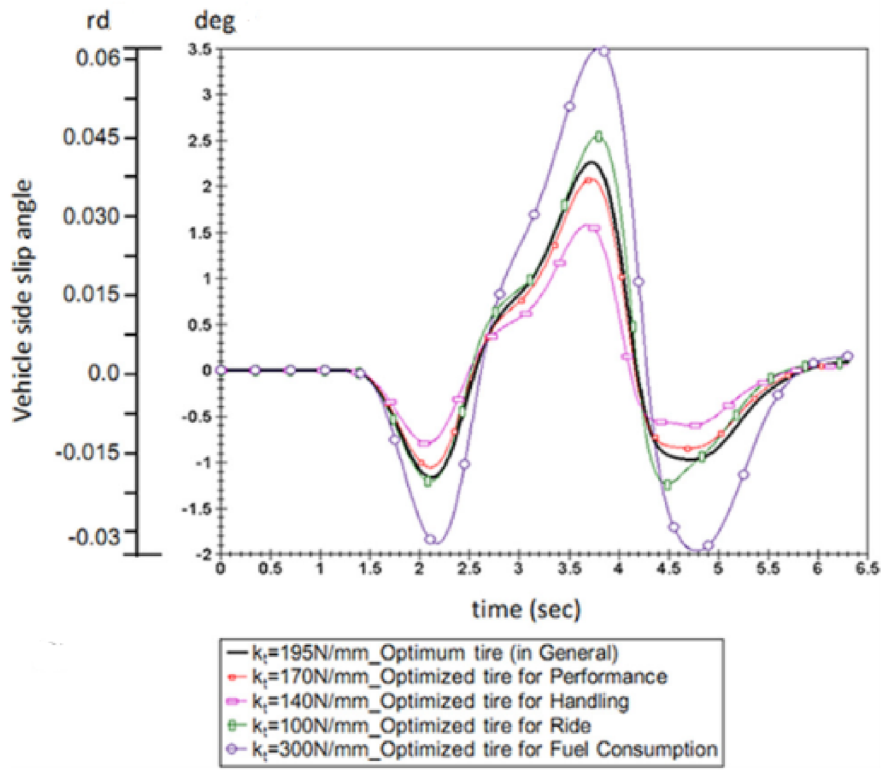


Figure 1.11: Sideslip angle of the vehicle vs time

- C is the shape factor
- B is the stiffness factor
- E is the curvature factor
- S_V and S_H are respectively the vertical and horizontal shift

Clearly the formula above is only a general formula. When we need to characterize a special tire the magic formula equation contains p, q, r, s and λ parameters.
The tire different nominal load is accounted with a scaling factor and is introduced the normalized change in vertical load

$$F'_{z0} = \lambda_{F_{z0}} F_{z0} \tag{1.21}$$

$$df_z = \frac{F_z - F'_{z0}}{F'_{z0}} \tag{1.22}$$

Moreover it is introduced the normalized change in inflation pressure

$$dp_i = \frac{p - p_0}{p_0} \tag{1.23}$$

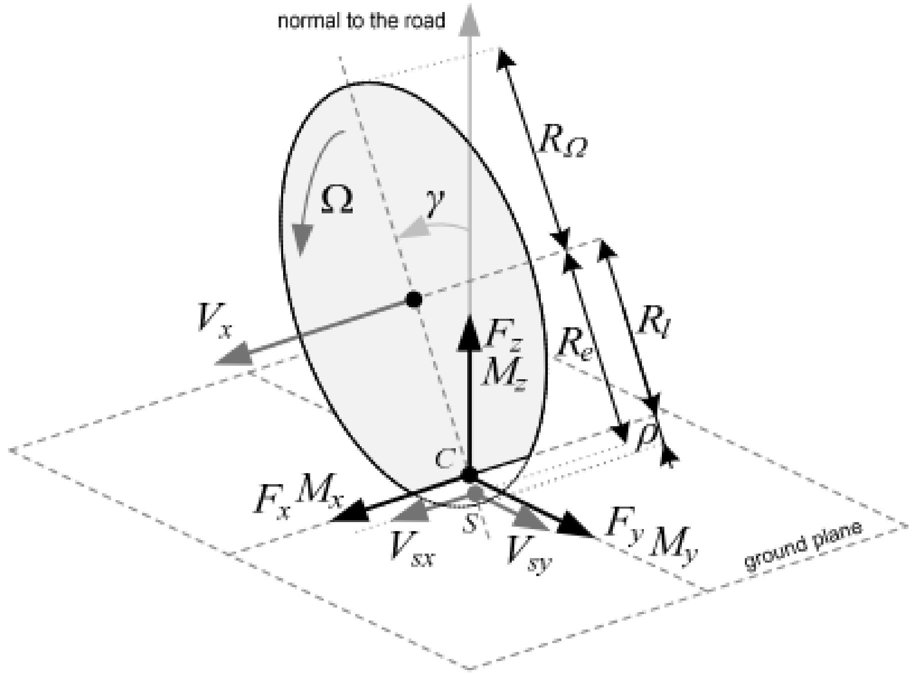


Figure 1.12: Reference tire parameters

1.3.2 Dependance on inflation pressure

The main characteric of version 6.1 is the possibility of taking into account the inflation pressure variations. The effective rolling radius, defined as the theoretical radius in free rolling conditions, is one of the characteristics affected by the inflation pressure.

$$R_e = \frac{V_x}{\omega} \tag{1.24}$$

The effective rolling radius is estimated by:

$$R_e = R_\omega - \rho_{F_{z0}}(F_{reff}\,\rho_d + D_{reff}\arctan(B_{reff}\,\rho_d)) \tag{1.25}$$

where F_{reff}, D_{reff} and B_{reff} are adimensional coefficient typical of the tire, while $\rho_{F_{z0}}$ and ρ_d are defined as

$$\rho_{F_{z0}} = \frac{F_{z0}}{C_z} \tag{1.26}$$

$$\rho_d = \frac{\rho}{\rho_{F_{z0}}} \tag{1.27}$$

The term F_{z0} refers to the nomimal vertical load of the tire; ρ is the tire vertical deflection that is calculated considering a constant vertical stiffness C_z

$$\rho = \frac{\rho}{\rho_{F_{z0}}} \tag{1.28}$$

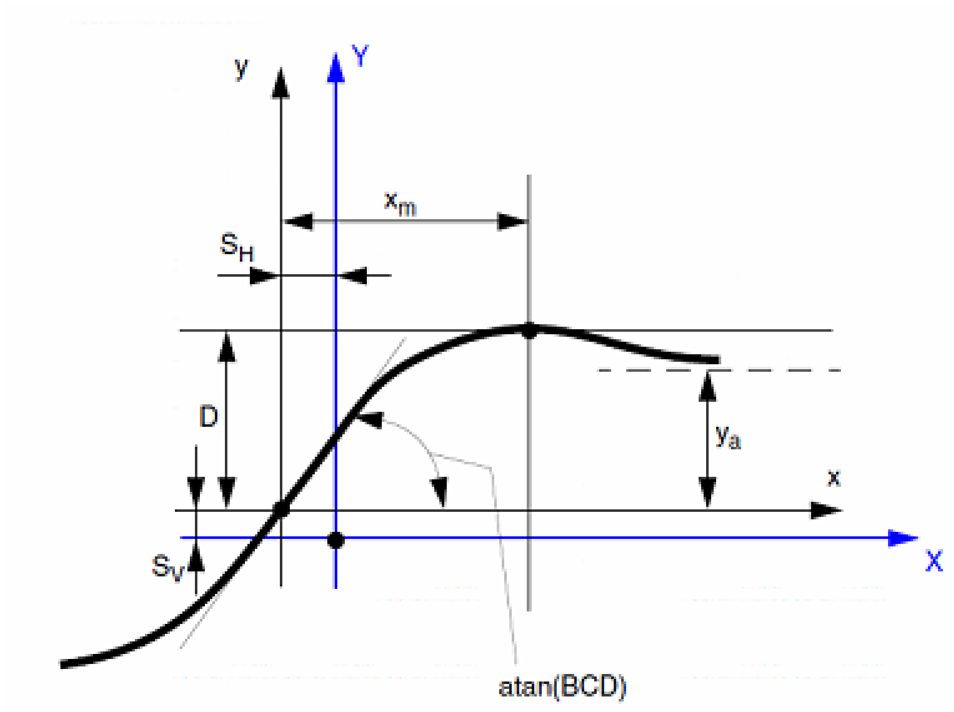


Figure 1.13: Magic formula plot

that is obviousy related with the inflation pressure whith the following formula

$$C_z = C_{z0}(1 + p_{Fz1} dp_i) \tag{1.29}$$

where C_{z0} is the vertical stiffness at the nominal pressure and p_{Fz1} an adimensiol factor to take into account the pressure effect on vertical stiffnes.
Furthermore we can calculate the free rolling radius R_ω, based on the free non rolling radius R_0, the reference speed V_0 and the rotational speed ω

$$R_\Omega = R_0\Big(q_{re0} + q_{v1}\Big(\frac{\omega R_0}{V_0}\Big)^2\Big) \tag{1.30}$$

The vertical force will be calculated then

$$F_z = \Big(1 + qv2\frac{R_0}{V_0}|\Omega| - \Big(\frac{q_{Fcx}F_x}{F_{z0}}\Big)^2 - \Big(\frac{q_{Fcy}F_y}{F_{z0}}\Big)^2\Big)\Big(qFz1\frac{\rho}{R_0} + q_{Fz2}\Big(\frac{\rho}{R_0}\Big)^2\Big)\Big(1 + p_{Fz1}dp_i\Big)F_{zo} \tag{1.31}$$

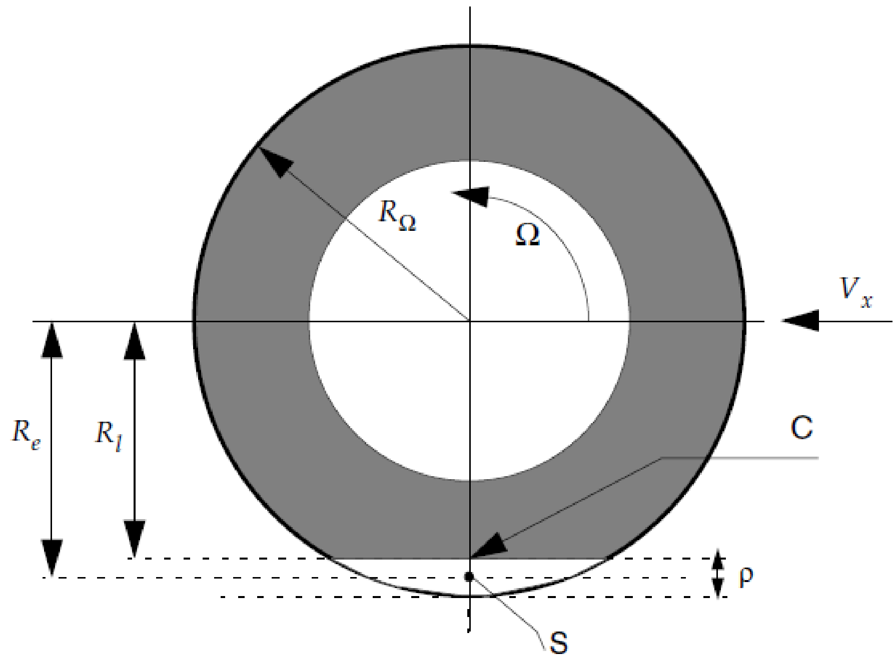


Figure 1.14: Effective rolling radius

1.3.3 Longitudinal and lateral forces

The pacejka 6.1 model define copletely the tire behaviour for what concern forces, moments, effective rolling radius, loaded radius, rolling resistance moments ecc. We do not report all this stuff, but we only report the equation concerning the longitudinal and lateral forces, the interaction between them and the transient behaviour, since they the most interesting for the vehicle dynamics.
The calculation of the longitudinal and lateral forces needs the definition of two more parameters: longitudinal and lateral slip, respectively λ and α.
In the equation that we are going to present, we only define those parameters that include a pressure effect in order to have a comparison with the result that we will find.
As we say before, the pure longitudinal and lateral force (without interaction between them) are defined based on the longitudinal and lateral slip, as reference to figure 1.12 we define them as

$$\lambda = -\frac{V_x - \omega R_e}{V_x} \tag{1.32}$$

$$\alpha = \arctan \frac{V_{sy}}{|V_x|} \tag{1.33}$$

Consequently it is defined the longitunal force with the equation 1.32, in which Y is now F_x and X is λ

$$F_{x0} = D_x \sin[C_x \arctan\{B_x\lambda - E_x(\, B_x\lambda - \arctan B_x(\lambda)\,)\,\}\,] - S_{Vx} \tag{1.34}$$

in which the pressure effect is accounted in B_x and D_x term

$$B_x \propto \frac{1 + p_{px1}\, dp_i + p_{px2}\, {dp_i}^2}{C_x D_x + \epsilon_x} \tag{1.35}$$

$$D_x \propto 1 + p_{px3}\, dp_i + p_{px4}\, {dp_i}^2 \tag{1.36}$$

Similarly we define the pure lateral force

$$F_{y0} = D_y \sin[C_y \arctan\{B_y\alpha - E_y(\, B_y\alpha - \arctan B_y(\alpha)\,)\,\}\,] - S_{Vy} \tag{1.37}$$

and the terms containing pressure dependance

$$B_y \propto \arctan\left(\frac{F_z}{(p_{Ky2} + p_{Ky5}{\gamma_y}^2)(1 + p_{py2}dp_i){F_{zo}}'}\right)(1 + p_{py1}dp_i) \tag{1.38}$$

$$D_y \propto 1 + p_{py3}\, dp_i + p_{py4}\, {dp_i}^2 \tag{1.39}$$

If we consider a cobined longitudinal and lateral force, it is clear that the forces developed by the tire would not be the same: in fact generally the forces developed decreases when we have a combined effect.
For what concerns the combined effect of the longitudinal and lateral forces, we have the following equations

$$F_x = G_{x\alpha} F_{x0} \tag{1.40}$$

$$F_y = G_{y\lambda} F_{y0} + S_{Vy\lambda} \tag{1.41}$$

in which F_{x0}, F_{y0} are respectively the longitudinal and lateral forces calculated without interaction; $G_{x\alpha}$, $G_{y\lambda}$ and $S_{Vy\lambda}$ are coefficient dependent on slip and lateral slip. However in pacejka 6.1 these three coefficient do not depend on inflation pressure.

1.3.4 Transient behaviour

It is well known that the forces developed in the tire/ground interface have a transient effect. The transient effect is visible like a lag in the development of the forces, moments. This lag is modeled with a first order differential equation for the slip and sideslip angle of the tire as follow

$$L_x \dot{u} + |V_x| u = -L_x (V_x - \omega R_e) \tag{1.42}$$

$$L_y \dot{v} + |V_x| v = L_y V_{sy} \tag{1.43}$$

in which L_x and L_y represents the longitudinal and lateral relaxation lengths, u and v are introduced as longitudinal and lateral deformation of the tire.
Instead of λ and α the equation written before will then be calculated with the following parameters

$$\lambda' = \frac{u}{L_x} sgn(V_x) \tag{1.44}$$

$$\alpha' = \arctan \frac{v}{L_y} \tag{1.45}$$

The pressure effect is taken into account in the longitudinal and lateral stiffness that are inversely proportional to the relaxation lenghts

$$C_x \propto 1 + p_{cfx3} dp_i \tag{1.46}$$

$$C_y \propto 1 + p_{cfy3} dp_i \tag{1.47}$$

and then the relaxation lengths would be

$$L_x \propto \frac{1}{C_x} \tag{1.48}$$

$$L_y \propto \frac{1}{C_y} \tag{1.49}$$

Chapter 2

CDtire simulations

In this chapter we will see how we build the simulations and gather the results from them.
The approach used to characterize the tire is divided in three steps:

- calculate the steady state pure longitudinal and lateral forces and moments
- calculate two coefficients that take into account the interaction between them
- characterize the transient behaviour with the relaxation length first order differential equation model, calculating the best fitting values that better aproximize the transient behaviour

The CDtire software is modeled as a simulink black box with intputs and outputs. The inputs are the spindle orientation, the rotation and translation position and speed of it, and the inflation pressure. The outputs are the forces and moments occurring at wheel and road. Since we see that the longitudinal and lateral slip are not an input, we need to espress the related quantities with them to obtain a direct relation, but it is not an issue; the issue is to have the vertical load as an input, since in CDtire is an output: the solution is to have a controller that follow the vertical load and it will have as input variable the distance of the spindle from ground.

After the simulations, the results are processed (we will see) and then collected in 3D look-up tables in order to be used afterward in CarMaker for simulink.

2.1 CDtire overview

CDtire is a tire model that compute, as we have already said, forces and moments acting on spindle and tire/road contact interface in driving conditions.
The road surface model is a 3D model in which are defined the mesh points and the friction coefficient; the interpolation between the mesh points is linear. The road surface model that we use is a flat road with constant friction coefficient. For what concern the tire, is modeled with 3D shells that have different characteristics based on the component of the tire (e.g. the belt and sidewalls are with different shell parameters).
The model properties are stored in a parameter file and can be suited for the tire we want

to simulate. The parameter file that we use is for a tire with dimensions 205/55R16.
The model is then implemented in a simulink block, that is the actual tool that we will use. In fig 2.2 we can see the Simulink CDtire block. The inputs/outputs of the blocks are

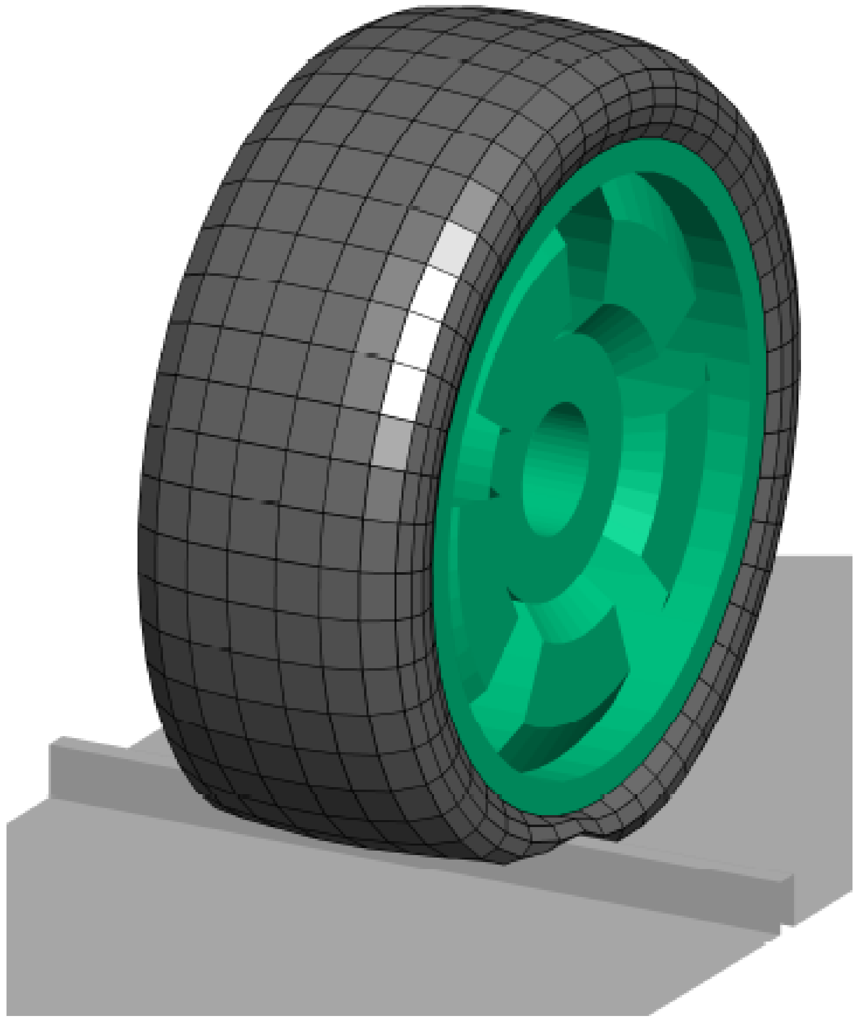


Figure 2.1: CDtire mesh

referred to two reference frame(figure 3.3), indicated as

- $< W >$ that is the fixed world reference frame
- $< S >$ that is the reference frame fixed at the spindle

The inputs:

- `sp.angl[alpha; beta; gamma],rad` : trasformation angles from reference frame $< S >$ to $< W >$
- `sp.pos<W>[x; y; z],m` : position of the spindle center

- sp.rvel<W>[x; y; z],rad/s : rotational speed of the spindle
- sp.vel<W>[x; y; z],m/s : speed of the spindle center
- rim_rel.angl,rad : rotational position of the rim
- rim_rel.rvel,rad/s : rotational speed of the rim
- tire.pres,kPa : inflation pressure

The outputs:

- wc.frc<W>[x; y; z], N : forces exchanged at the spindle
- wc.trq<W>[x; y; z], Nm : torques exchanged at the spindle
- rd.frc<W>[x; y; z], N : forces exchanged between tire and road

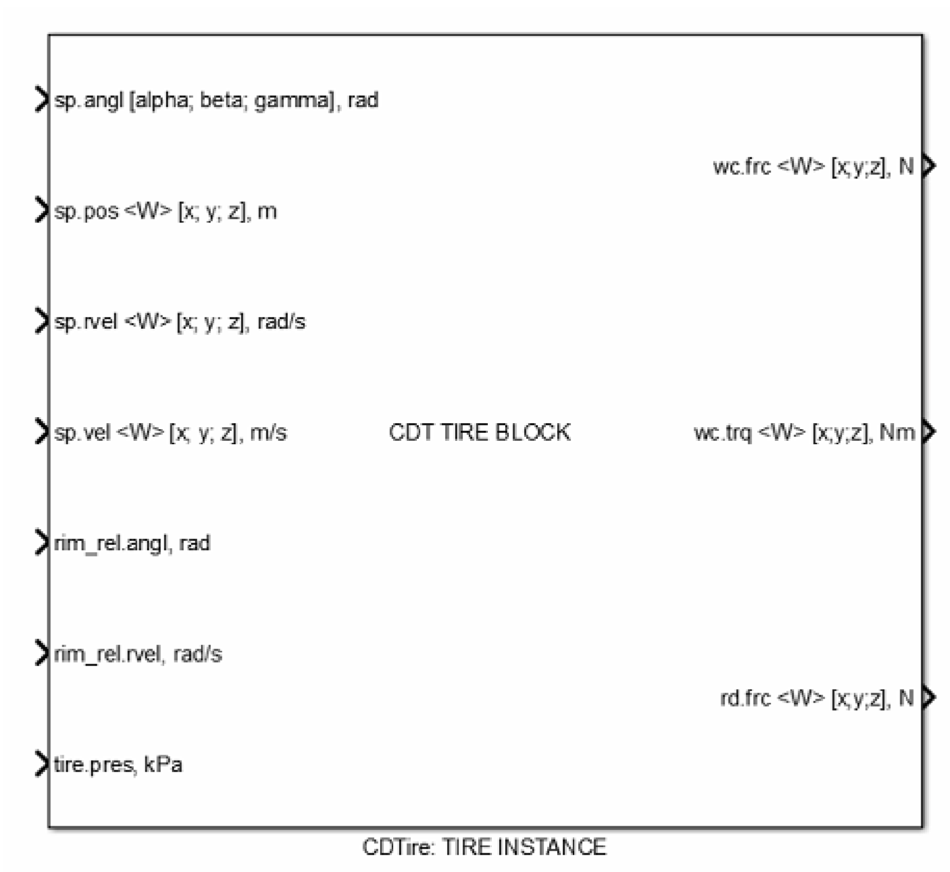


Figure 2.2: CDtire Simulink Block

Just to give an idea, it must be mentioned that the simulation time of the block is very long, in fact a second of simulation in the CDtire environment corresponds to two minutes and a half in the reality more or less.

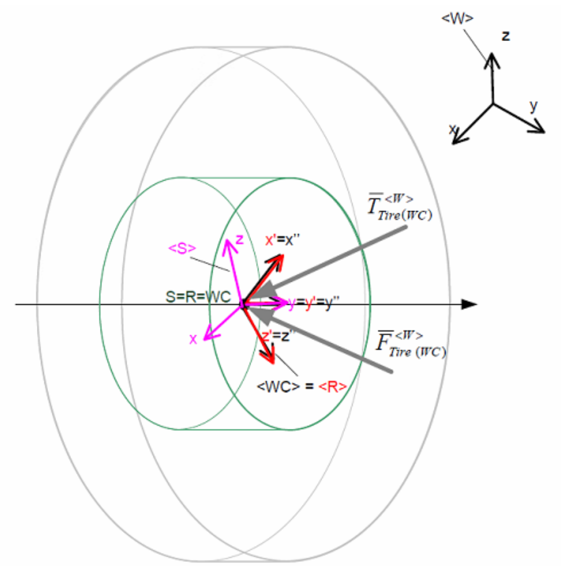


Figure 2.3: Reference frames of CDtire

2.1.1 Simulation parameters

We remind that that the simulation quantity that we need to store in the look-up tables are: longitudinal force F_x, lateral force F_y, rolling resistance force F_r, self aligning moment M_z, effective radius R_{eff} and loaded radius R_L.
The parameters by which we calculate the quantity above are the kinetic characteristic of the tire such as slip, slip angle, load, longitudinal speed and obviously the inflation pressure. In formula:

$$F_x = f(\lambda, \alpha, F_z, V_x, p) \tag{2.1}$$

$$F_y = f(\lambda, \alpha, F_z, V_x, p) \tag{2.2}$$

$$F_y = f(\lambda, \alpha, F_z, V_x, p) \tag{2.3}$$

$$F_r = f(\lambda, \alpha, F_z, V_x, p) \tag{2.4}$$

$$M_z = f(\lambda, \alpha, F_z, V_x, p) \tag{2.5}$$

$$R_{eff} = f(\lambda, \alpha, F_z, V_x, p) \tag{2.6}$$

$$R_L = f(\lambda, \alpha, F_z, V_x, p) \tag{2.7}$$

As we said not all the parameter match the inputs of the block, but they could be derived from them. In fact the parameters λ and α are defined as follows

$$\lambda = \frac{\omega R_{eff} - V_x}{V_x} \tag{2.8}$$

$$\alpha = \arctan \frac{V_y}{V_x} \tag{2.9}$$

in which giving as inputs λ, α and V_x we can derive the rotational speed ω and the lateral speed V_y, that are the two block inputs that we need for the CDtire block. In formula

$$\omega = \frac{V_x \lambda + V_x}{R_{eff}} \tag{2.10}$$

$$V_y = V_x \tan \alpha \tag{2.11}$$

The effective rolling radius is defined, as we have already seen, as the ratio between the longitudinal speed and the angular speed of the rim in free rolling conditions

$$R_{eff} = \frac{V_x}{\omega} \tag{2.12}$$

F_z is the vertical load applied to the rim. The parameter p is obviously the inflation pressure.

In the simulations we adopted a finite number of values of the parameters and not a continuos variyng of them, this for two reasons: because it would not be possible with more than one parameter, then we risk in a continuos variyng parameters to have dynamic effect that for the moment we do not want to account for.
The operative ranges of the parameters that we choose are

$$\begin{aligned}
\lambda &= -100\% \div +100\% && (2.13)\\
\alpha &= 0^\circ \div 35^\circ && (2.14)\\
F_z &= 2kN \div 6kN && (2.15)\\
V_x &= 10\frac{km}{h} \div 250\frac{km}{h} && (2.16)\\
p &= 1bar \div 3bar && (2.17)\\
& && (2.18)
\end{aligned}$$

The reasons for this values are quite easy to be explained:

- for λ -100% is the wheel totally locked and more than $+100\%$ would be not meaningful because we have already passed a very unstable region of the tire;
- α is assumed with simmetric behaiour with negative value, that is true until the thread of the tire is simmetric;
- F_z are choosen taking into account the maximum theoretical lateral acceleration, given by a friction coefficient equal to one; in formula:

  $$F_{yMax} = mg\mu_y \tag{2.19}$$

  knowing the maximum lateral force we can calculate the maximum load variation ΔF_z that take on the tires of the vehicles

  $$|\Delta F_z| = F_{yMax}\frac{cog}{t} \tag{2.20}$$

where cog and t are respectively the heigth of the center of gravity and the track of the vehicle. This force is then the sum of the front plus the rear tires that we assume for sake of simplicity equal. In this way on a single tire

$$|\Delta F_{z_{tire}}| = \frac{|\Delta F_z|}{2} \tag{2.21}$$

Assuming the following likely values

$$\mu_y \approx 1 \tag{2.22}$$
$$t \approx 1.6\, m \tag{2.23}$$
$$cog \approx 0.4\, m \tag{2.24}$$
$$m \approx 1600\, kg \tag{2.25}$$

we would have

$$|\Delta F_{z_{tire}}| \approx 2000\, N \tag{2.26}$$

- V_x
- p interval is choosen as that are the minimum and maximum value that can be calculated with CDtire

The values are evenly spaced, except for what concern λ and α that are spaced in a logaritmic way cause we want to have more accuracy with low values of them, since high values are conditions of tire instability.

2.2 F_x-F_y-M_z simulations

The simulations of the forces and moments are made initially without taking into account the combined effect of lateral and longitudinal slip: we made the simulations varying only the longitudinal or lateral slip.
The longitudinal speed V_x is maintained constant since we can suppose that it does not affect the results (we choose arbitrarly the value of $50\,km/h$). Moreover since we want to calculate pure longitudinal and lateral forces and moments, we have null sideslip angle for longintudinal force and null slip for lateral force and self-aligning moment. In formula

$$F_x = f(\lambda, \alpha = 0, F_z, V_x = 50\,km/h, p) \tag{2.27}$$

$$F_y = f(\lambda = 0, \alpha, F_z, V_x = 50\,km/h, p) \tag{2.28}$$

$$M_z = f(\lambda = 0, \alpha, F_z, V_x = 50\,km/h, p) \tag{2.29}$$

The simulation is made giving a step function to the parameters; the time of each step is set such that is sufficient to stabilize the output and so give the steady state value.

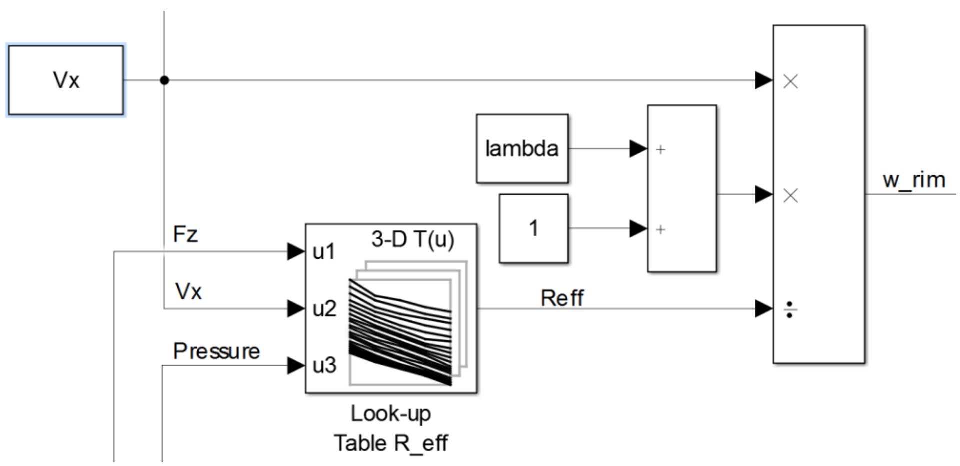


Figure 2.4: The model shows how to set the proper rotational speed of the hub given the longitudinal speed and the tire slip. The look-up table of the effective radius is calulated in previous simulations.

Except from the pressure (p) the other quantities are derived values. Once we know the parameters and the value that must have, we have to relate them to the inputs of our simulation.
As we have already seen in equation 2.10 and 2.11 we can easily relate the longitudinal and lateral slip with the rotational and lateral speed of the tire.

We will see afterward how to compute the value of R_e.
Now there is an issue with the vertical force (F_z) since it is not an input, but an output of our simulink block. So, how can we impose it? Considering that the vertical force depends on the loaded radius, the solution we adopted is to use a PI controller on the force that has as input the loaded radius (figure 2.5).

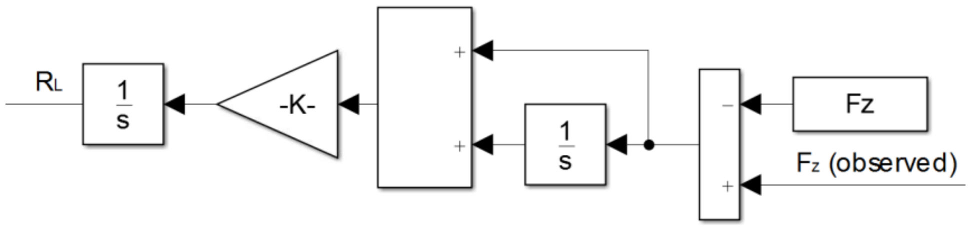


Figure 2.5: Controller used to follow a given F_z. The exit is the loaded radius since it is the variable to be controlled

2.3 R_e-F_L-F_r simulations

As we say before we need to calculate the effective radius (R_e) in order to define the rotational speed. The loaded radius (R_L) and the rolling resistance (F_r) are not necessary in the previous simulation, while are necessary in CarMaker for simulink.
The parameters used to characterize theese value are now different; in particular we can understand very clearly that now the factor that has a negligible effect are the longitudinal and lateral slip.
The parameters that has the most influence are the vertical load (F_z), longitudinal speed (V_x) and clearly the pressure (p). In formula

$$R_{eff} = f(\lambda = 0, \alpha = 0, V_x, F_z, p) \tag{2.30}$$

$$R_L = f(\lambda = 0, \alpha = 0, V_x, F_z, p) \tag{2.31}$$

$$F_r = f(\lambda = 0, \alpha = 0, V_x, F_z, p) \tag{2.32}$$

We need now to find out the method to calculate theese quantities.
For what concern the effective rolling radius (R_e), recalling the definition, we know that represents the hypothetical radius that will have a rigid disk having the same longitudinal free rolling speed of the tire.
The free rolling speed of the tire corresponds to the condition in which we have zero longitudinal force (F_x) and so we must set the simulation with another PI controller that acts on the rotational speed and follow the zero longitudinal force.
For the loaded radius R_L it is much more simple, since we impose the vertical load and we measure the vertical distance from the road of the hub which corresponds exactly to the loaded radius.

For what concern the rolling resistance (F_r) we need to use another CDtire simulink block in wich now we have as input the forces and moments and as output the rotation and translation speeds. Now, knowing that we have a driving wheel, we follow a certain value of rotational speed of the rim ω, which control the driving torque. The equilibrium condition represents

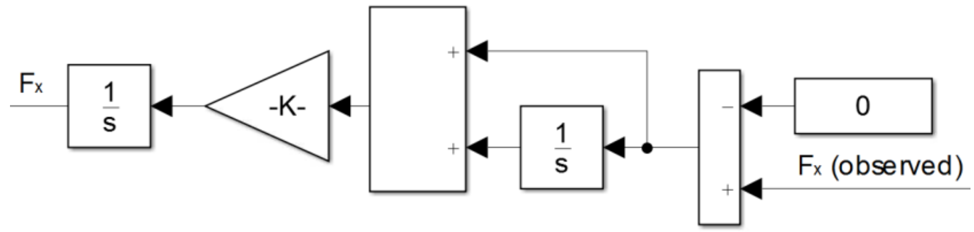


Figure 2.6: The model shows how to set the proper rotational speed of the hub given the longitudinal speed and the tire slip. The look-up table of the effective radius is calulated in previous simulations.

the torque needed to overcome the friction. We can easily obtain the rolling resistance force as

$$F_r = \frac{T_y}{R_L} \tag{2.33}$$

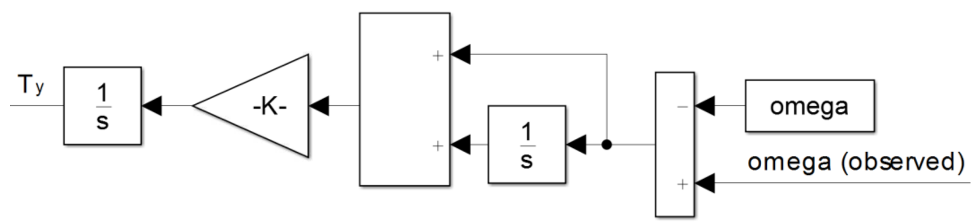


Figure 2.7: The model shows how to set the proper rotational speed of the hub given the longitudinal speed and the tire slip. The look-up table of the effective radius is calulated in previous simulations.

2.4 Look-up tables

Once we have the results of the six simulations we can build the look-up table.
Firts of all we have to collect in a 3D matrix each result, in which the values are ordered based on the parameters in row, column and depth positions. Making a resum, the six look-up tables are

- F_x longitudinal force (in x coordidate in reference frame FrW)
- F_y lateral force (in y coordidate in reference frame FrW)
- M_z self-aligning torque (in z coordidate in reference frame FrW)
- R_e effective radius as defined before

F_x					
	2 kN	3 kN	4 kN	5 kN	6 kN
0	...	...	...	...	...
0.001	...	...	...	...	...
$\vdots$	...	...	...	...	...
1	...	...	...	...	...

R_{eff}					
	2 kN	3 kN	4 kN	5 kN	6 kN
10 $\frac{km}{h}$	...	...	...	...	...
35 $\frac{km}{h}$	...	...	...	...	...
$\vdots$	...	...	...	...	...
250 $\frac{km}{h}$	...	...	...	...	...

F_y					
	2 kN	3 kN	4 kN	5 kN	6 kN
0°	...	...	...	...	...
0.01°	...	...	...	...	...
$\vdots$	...	...	...	...	...
35°	...	...	...	...	...

R_L					
	2 kN	3 kN	4 kN	5 kN	6 kN
10 $\frac{km}{h}$	...	...	...	...	...
35 $\frac{km}{h}$	...	...	...	...	...
$\vdots$	...	...	...	...	...
250 $\frac{km}{h}$	...	...	...	...	...

M_z					
	2 kN	3 kN	4 kN	5 kN	6 kN
0°	...	...	...	...	...
0.01°	...	...	...	...	...
$\vdots$	...	...	...	...	...
35°	...	...	...	...	...

F_r					
	2 kN	3 kN	4 kN	5 kN	6 kN
10 $\frac{km}{h}$	...	...	...	...	...
35 $\frac{km}{h}$	...	...	...	...	...
$\vdots$	...	...	...	...	...
250 $\frac{km}{h}$	...	...	...	...	...

Table 2.1: Example of F_x look-up table

- R_l loaded radius: defined as the vertical distance between the wheel hub center and the road (in z coordidate in reference frame FrW)
- F_r rolling resistance (in x coordidate in reference frame FrW)

The third dimension is represented by the inflation pressure

In the following we will see the plot of the look-up tables. Naturally since we have 3D look-up table we must fix a dimension, so we choose to show the plot at three different pressure at a fixed load of 4000 N.
In figure 2.8 we can see the longitudinal force: the maximum value is not affected by the

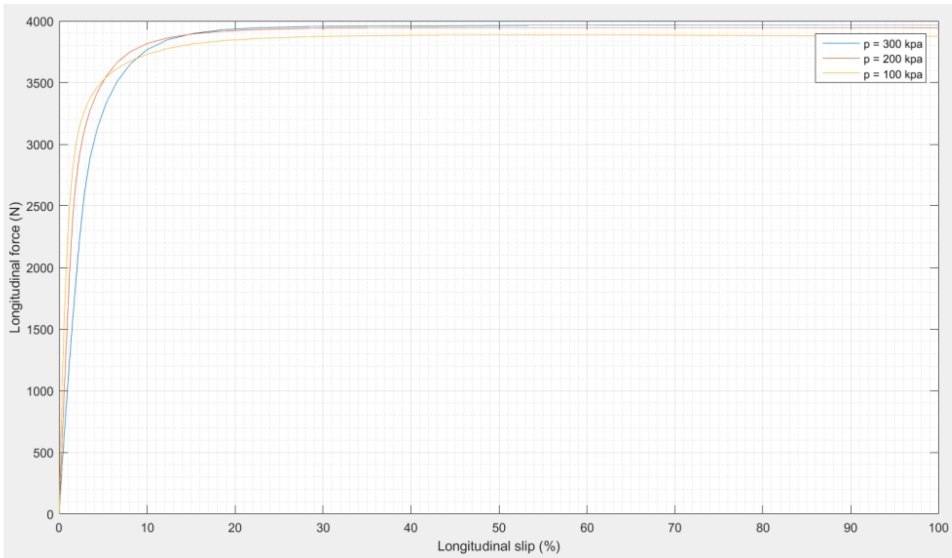


Figure 2.8: Longitudinal force vs slip at a load of 4000 N

inflation pressure. The same is for the lateral force (figure 2.9) while for the self-aligning moment we notice an important effect of the inflation pressure

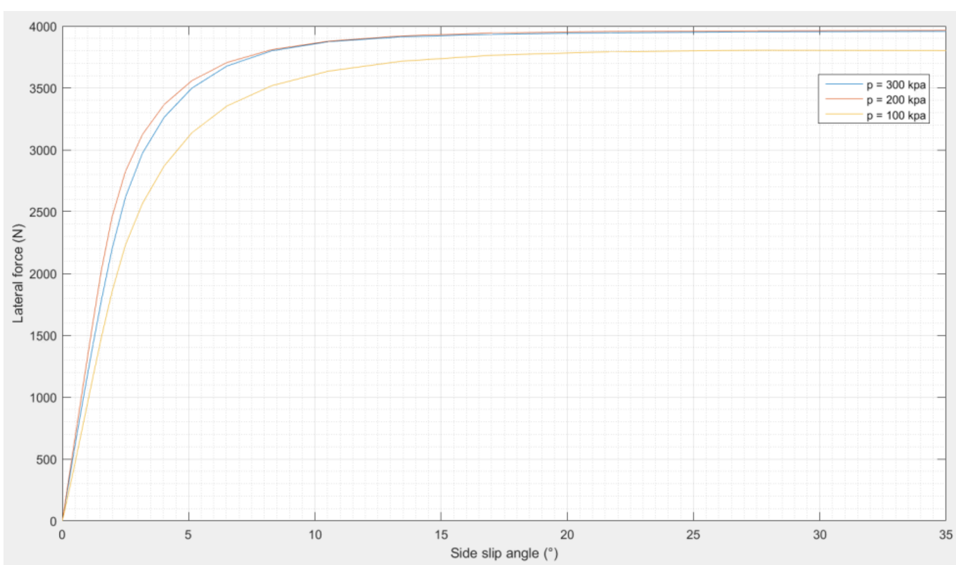


Figure 2.9: Lateral force vs side slip angle at a load of 4000 N

The effective rolling radius and the loaded radius present the expected behaviour: an increase with the longitudinal speed due to the centrifugal effect and an increase with pressure.
The rolling resistance force show an expected increase with the longitudinal speed and a

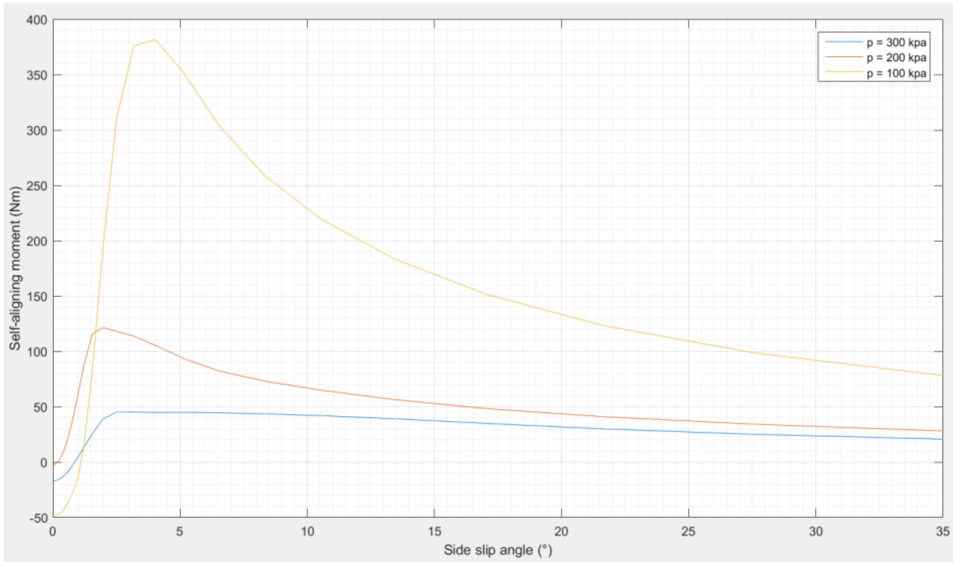


Figure 2.10: Self-alignig moment vs side slip angle at a load of 4000 N

decrease with pressure, probably due to the decrease of tire/ground contact patch.

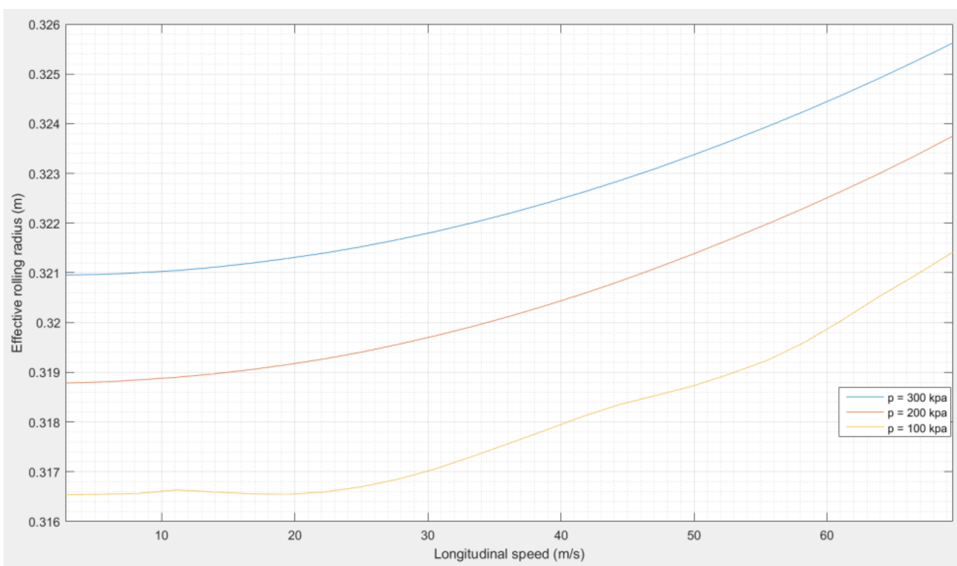


Figure 2.11: Effective rolling radius vs longitudinal speed at a load of 4000 N

In figure 2.14 and 2.15 we can see the longitudinal and lateral sfiffnesses vs the inflation pressure at different load: the longitudal one is always maximum at minimum pressure for all the loads, while the lateral change the maximum value position with respect to load: at minimum load we have the maximum value for the minimum pressure and increasing the load increases the pressure of the maximum.

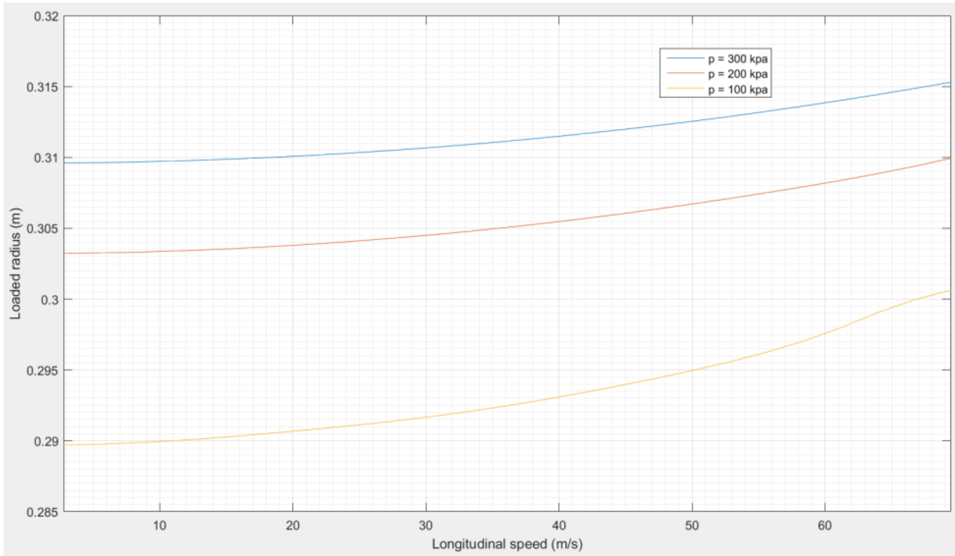


Figure 2.12: Loaded radius vs longitudinal speed at a load of 4000 N

2.5 Transient behaviour

As we said we take into account the transient behaviour.
In fact the values evaluated in the look-up tables are in steady state condition and we need to take into account the transient behaviour. The model used is the same of the pacejka model: the ralaxation length concept. Recalling equations 1.42 and 1.43 we define the two time constants as

$$\tau_x = \frac{L_x}{V_x} \tag{2.34}$$

$$\tau_y = \frac{L_y}{V_x} \tag{2.35}$$

In our system we decide to apply this time lag τ directly on the force and so the equation would be

$$\frac{dF_{trans}}{dt} = \frac{F_{steady} - F_{trans}}{\tau} \tag{2.36}$$

where L_{rel} is a general relaxation lenght.

Obviously, the the relaxation length cannot be directly measured in CDtire. In order to determine it we made a simulation in which we impose a sine sweep first on λ (for the determination of the longitudinal relaxation length) and later on α (for the determination of the lateral relaxation length), we store the results, and we find out the best fitting parameters τ that aproximize best the results obtained in the simulation. In order to find the best fitting value of τ we look at the value that makes the lowest root mean square of the difference between the simulated forces of CDtire and the forces modeled.
The steady state forces are given by the look-up tables, and since we are in a linear range

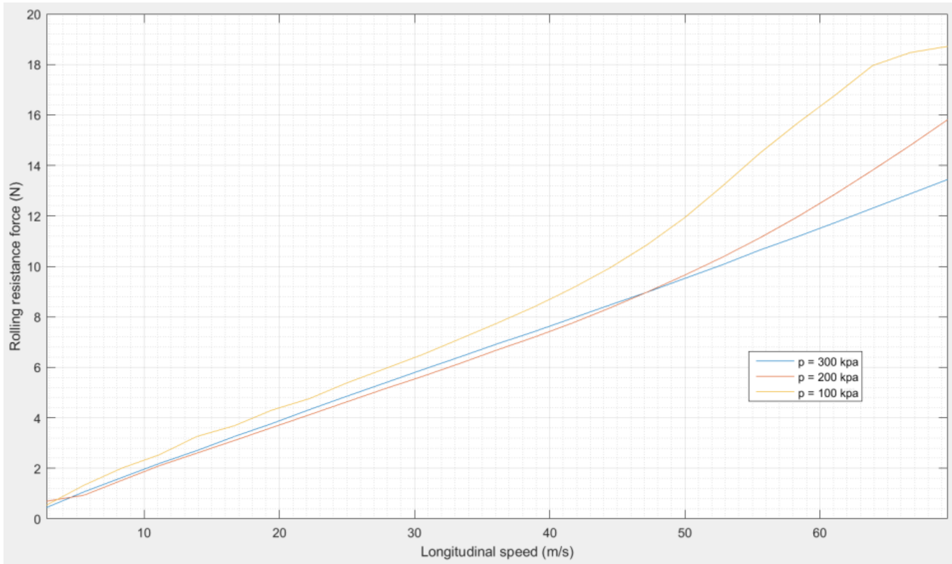


Figure 2.13: Rolling resistance force vs longitudinal speed at a load of 4000 N

we can directly apply the sine sweep on the forces. In formula

$$F_{x-steady} = C_\lambda \lambda \sin(\omega t^2) \tag{2.37}$$

$$F_{y-steady} = C_\alpha \alpha \sin(\omega t^2) \tag{2.38}$$

Implementing a system as in figure 2.16 we can evaluate the best value of tau, that corresponds to the value of minimum RMS^2.
Afterward we can easily calculate the relaxation lengths since

$$\tau_x = \frac{L_x}{V_x} \tag{2.39}$$

$$\tau_y = \frac{L_y}{V_x} \tag{2.40}$$

2.6 Interaction between longitudinal and lateral forces

It is well known that when we don't have a pure slip or sideslip angle, the tire behaviour changes. In formula:

$$F_x(\lambda, \alpha = 0) \neq F_x(\lambda, \alpha) \tag{2.41}$$
$$F_y(\lambda = 0, \alpha) \neq F_y(\lambda = 0, \alpha) \tag{2.42}$$

The approach we have used in order to take into account this effect is to use two different coefficient

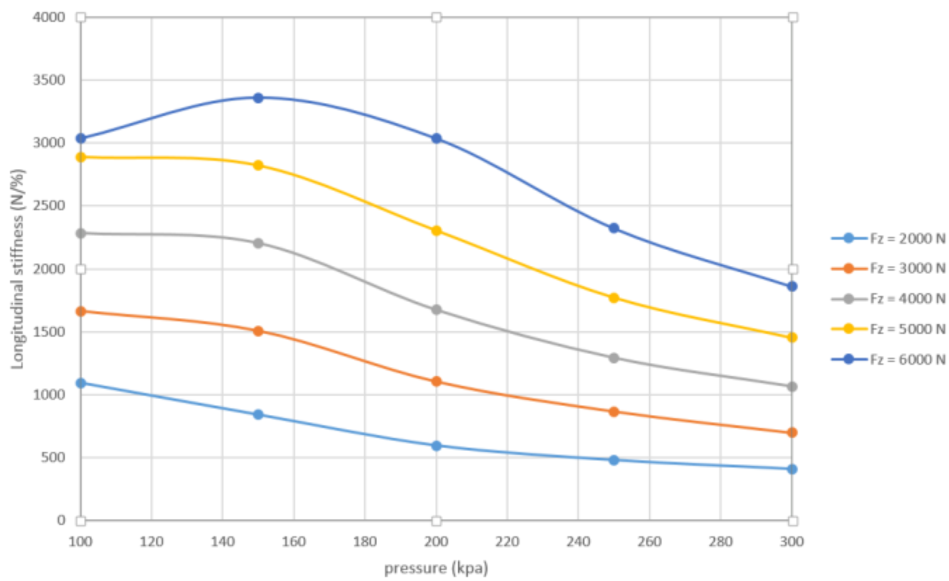


Figure 2.14: Longitudinal stiffness vs pressure

$$k_x(\lambda, \alpha = 0) = \frac{F_x(\lambda, \alpha)}{F_x(\lambda, \alpha = 0)} \tag{2.43}$$

$$k_y(\lambda = 0, \alpha) = \frac{F_y(\lambda, \alpha)}{F_y(\lambda = 0, \alpha)} \tag{2.44}$$

As before the coefficient is calculated for the five pressure defined before at a load of 4000 N, assuming the value is not dependent on load.

As we can see in figure 2.19 and 2.20 we obtain an inversion of the tendency of k_x at $\lambda \approx 1,87\,\%$. This value of longitudinal slip is more or less in the correspondence of the end of the linear behaviour of the tire.
In figure 2.21 we can see F_x vs slip for 0 sideslip angle.

As we can see in 2.23 and 3.14 the beahaviour of k_y is equal for α from 0 to 1.96°. Also this sideslip angle value corresponds to the end of the linear characteristics of the tire.
For higher alpha values there is an increase tendency as we can see in figure 3.14.

It is worth to note that we have obtained the various coefficients for the five different values of pressure, but as in the pacejka model there is not dependance on the pressure values.

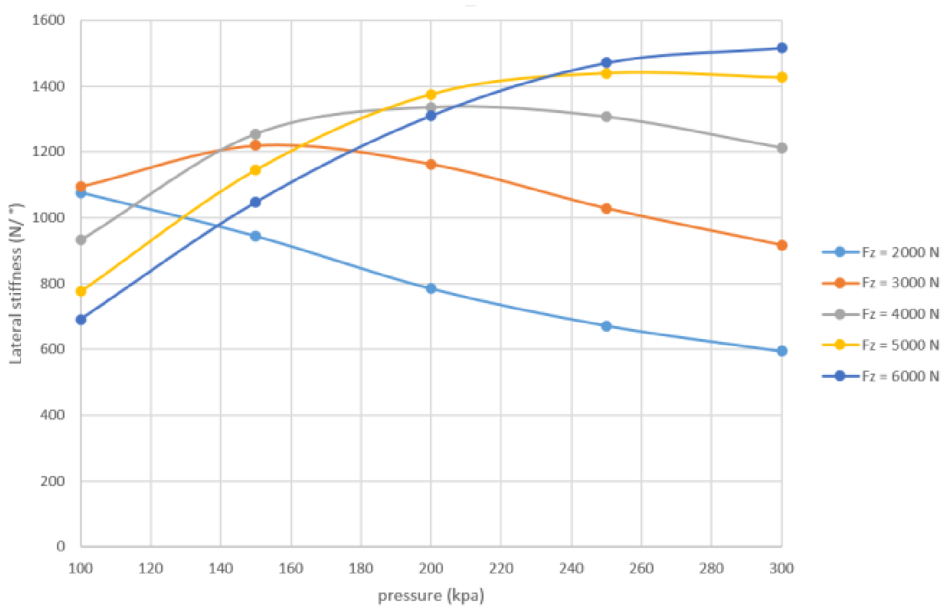


Figure 2.15: Lateral stiffness vs pressure

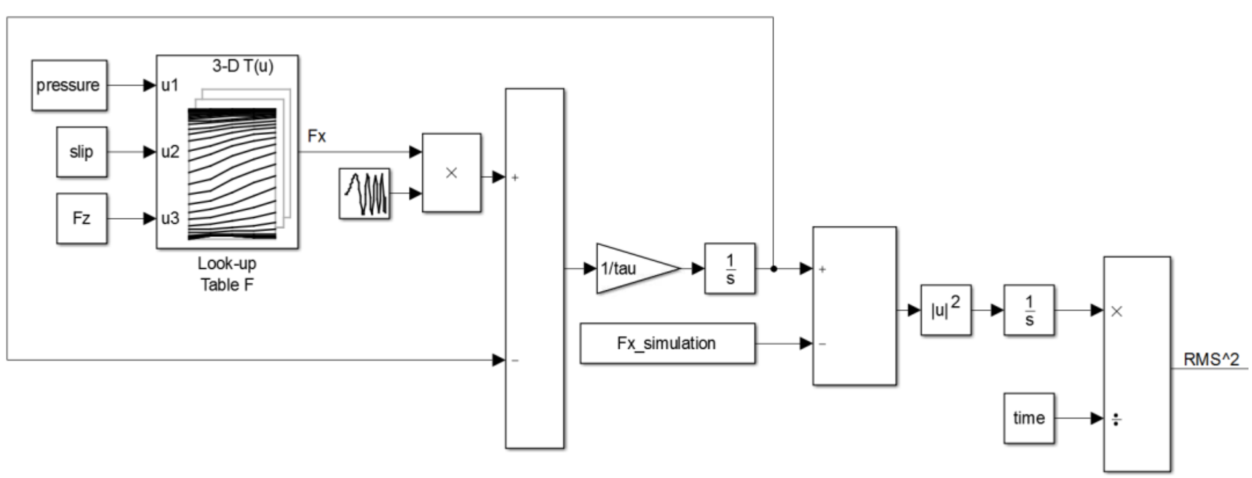


Figure 2.16: Simulink model for the calculation of the squared RMS of the difference between analytic and simulation data.

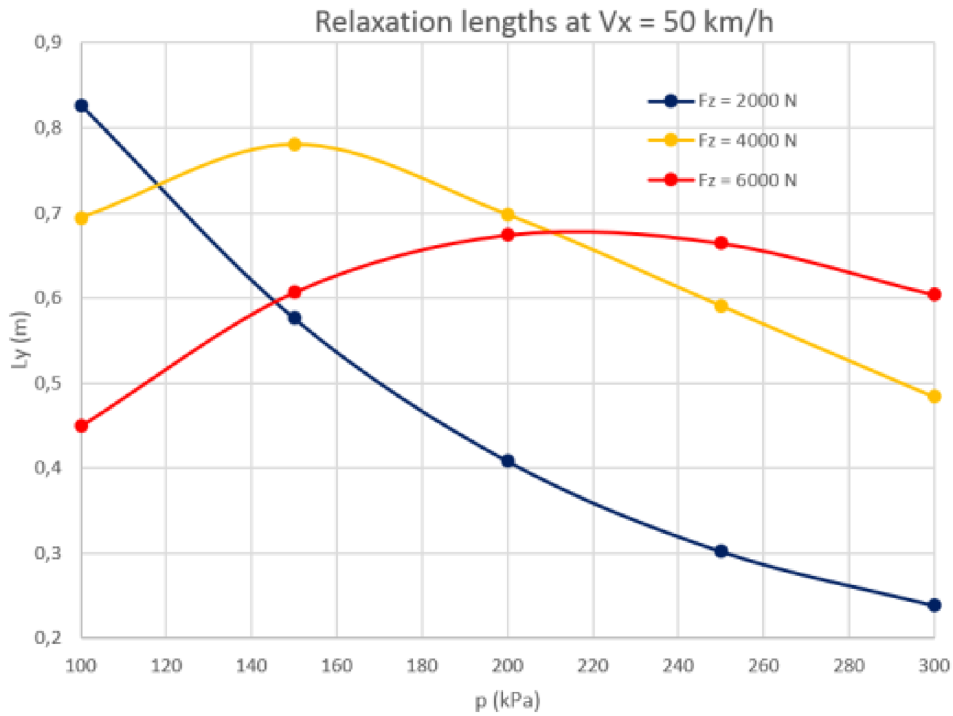


Figure 2.17: Lateral relaxation lengths at 50 kmph.

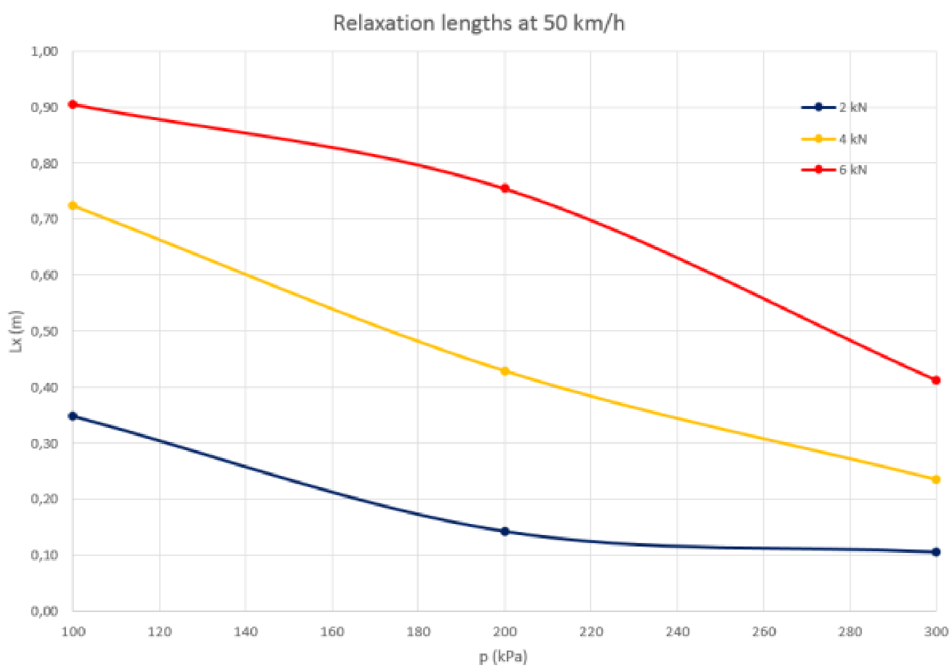


Figure 2.18: Longitudinal relaxation length vs inflation pressure for three loads

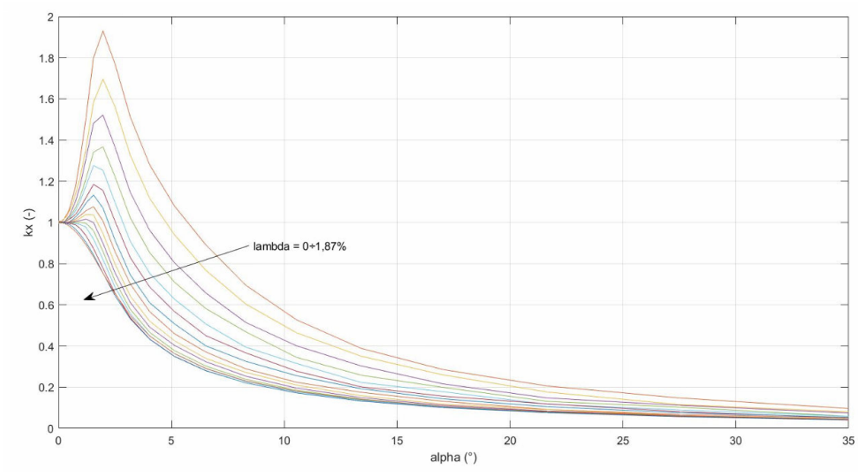


Figure 2.19: k_x values vs. sideslip angles at $p = 200\,kpa$

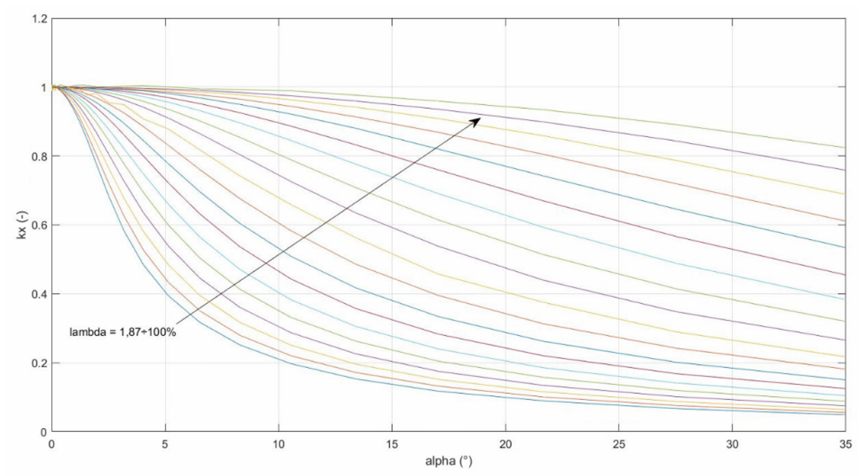


Figure 2.20: k_x values vs. sideslip angles at $p = 200\,kpa$

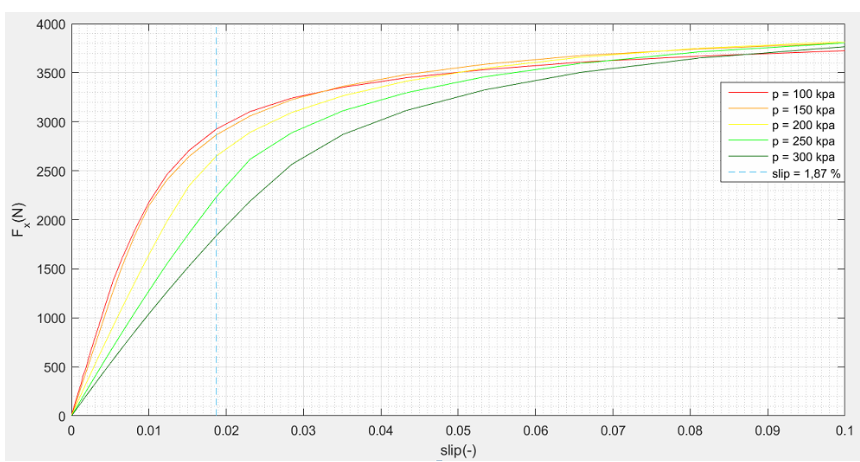


Figure 2.21: F_x vs. slip $\alpha = 0$. The dashed vertical line represents the slip point at which we have the inversion (1.87%)

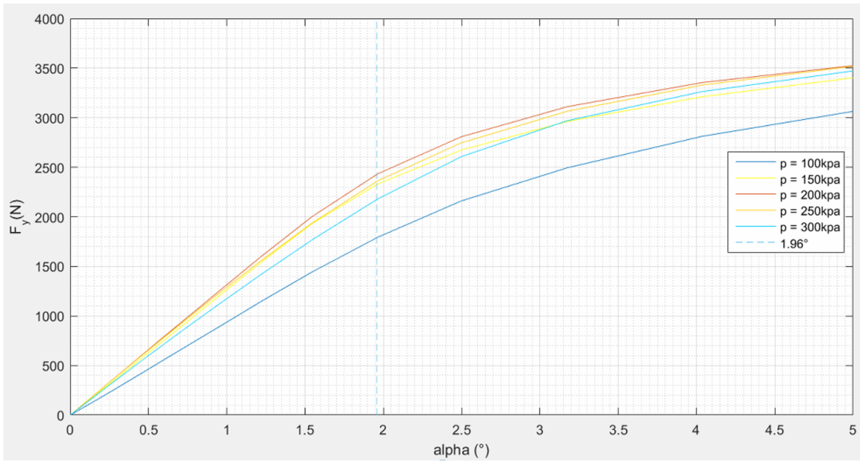


Figure 2.22: F_y vs. alpha with $\lambda = 0$. The dashed vertical line represents the alpha point of 1.96°

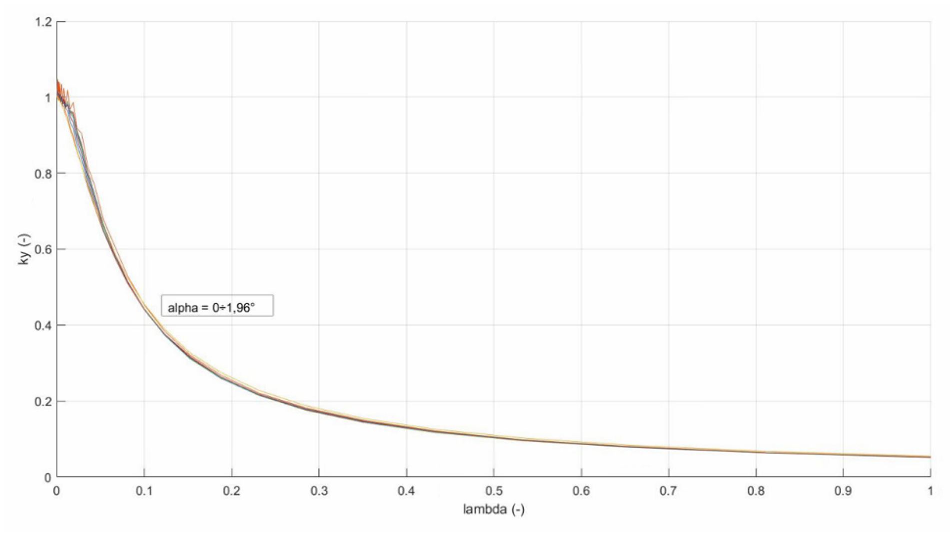


Figure 2.23: k_y vs slip at $p = 200\,kpa$ for different alpha values

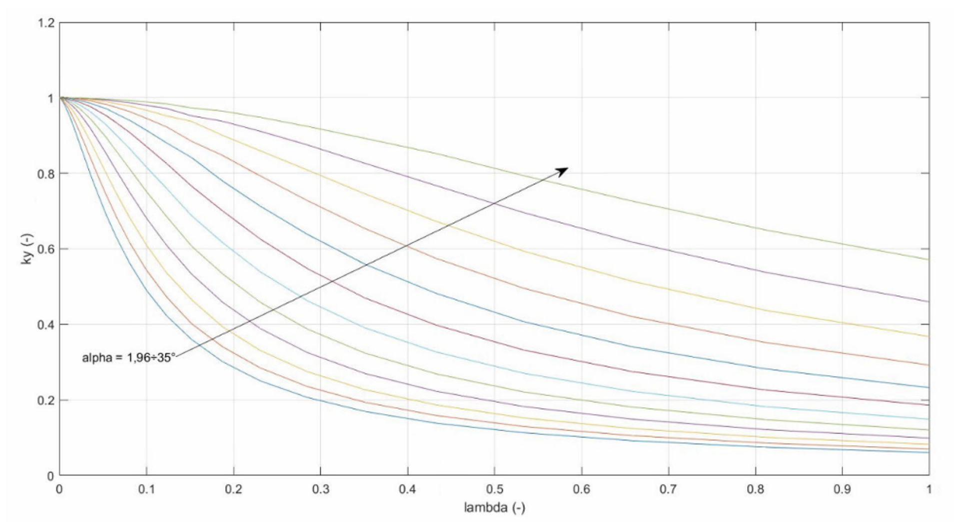


Figure 2.24: k_y vs slip at $p = 200\,kpa$

Chapter 3

CarMaker simulations

Once we have all the look-up tables, we have defined the interaction between longitudinal and lateral forces and we have described the transient behavaiour, we can implement the results in the simulink model and make some simulation. In the following we describe the tire model made in CarMaker.

3.1 CarMaker for simulink overview

CarMaker for simulink is an extension of CarMaker that enables us to modify the model used in the simulations. In other words CarMaker is a complete simulator of the vehicle, with complete subsystems (engine system, braking system...) that enables us to make our own simulink subsystem.
For example supposing that we want (it is our own case) to have a particular tire, with certain characterics that we cannot find in tires model available in CarMaker: we can create our own tire model in CarMaker for simulink and make simulations with that particular tire model created.
Worth to note that we don't have to create a complete vehicle model with all the subsystems included, because the software is able to simulate in a mixed way: we can so modify how many system we want and run a simulation that will work using the created subsystems and the one already present in basic subsystem.

In figure 3.1 we can see how it is presented the CarMaker for simuling overall model: going deeper in the block "*IPG Vehicle*" we will find the tires subsystems (one for each wheel).
Exploring for example the one of the front left wheel we will see how it is made the model: we have a set of inputs and outputs, as for CDtire. The inputs are calculated and given by the previous blocks and are used by our model to calculate the outputs.

Before giving an explanation of the inputs and outputs we need to define the CarMaker reference frame that we can see in fig; the x axis is longitudinal to the vehicle whith positive direction forward the vehicle; the z axis is perpendicular to the road positive towards the sky and y is defined consequently from the right hand rule. The position of the reference frame is in centre of the contact patch of the wheel, but it would not be relevant for our

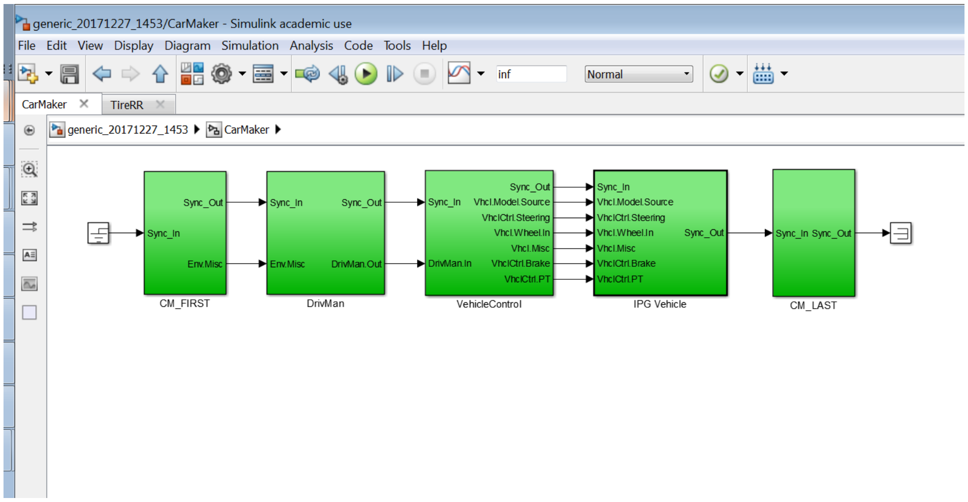


Figure 3.1: CarMakerSimulinkOverview

purposes.
In figure 3.2 we can see the inputs and outputs of the CarMaker tire subsystem.

Inputs:

- `Sync_in` : *The Sync_In port is an important concept for CarMaker for Simulink, in particular:*
  - *It guarantee the proper order of execution for the CarMaker blocks.*
  - *It let the user choose exactly when a CarMaker dictionary variable is accessed with a Read CM Dict or Write CM Dict block. For e.g., this way it is possible to read the most up to date value of a variable (and not the value calculated in the previous cycle) or override the value of a variable only after it has been calculated internally by CarMaker*[1]
- `Load` : vertical load $F_z\,[N]$ along z (reference frame FrW) in the tire/road contact point
- `vxtrans` : longitudinal speed $v_x\,[\frac{m}{s}]$ along x (reference frame FrW) of the hub center
- `vytrans` : lateral speed $v_y\,[\frac{m}{s}]$ along y (reference frame FrW) of the hub center
- `Rim_rotv` : rotational speed $\omega\,[\frac{rad}{s}]$ along y (reference frame FrC) of the wheel hub
- `Rim_turnv` : rotational speed $\dot{\psi}\,[\frac{rad}{s^2}]$ along z (reference frame FrW) of the rim around vertical axis z
- `InclinAngle` : angle $\gamma\,[rad]$ along x (reference frame FrW) of perpendicular of the road and median plan of the wheel

[1] text from `QuickStartGuide.pdf` (*CarMaker for Simulink 6.0*)

- `muRoad` : friction coefficient [-] between road and tire

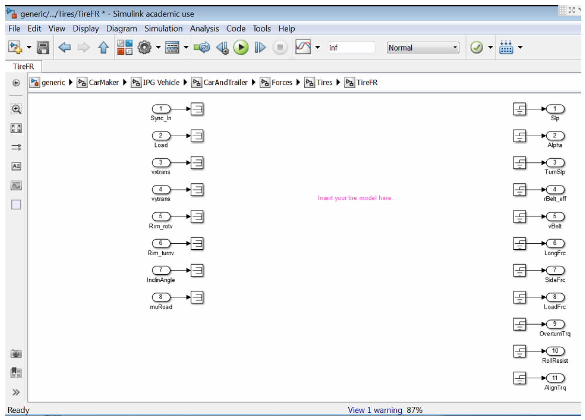


Figure 3.2: Simulink subsystem of tire in CarMaker

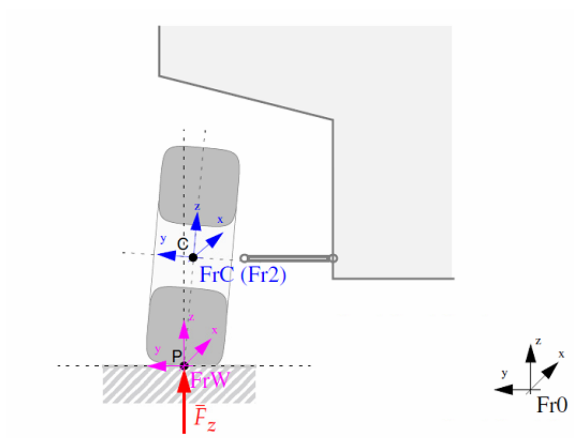


Figure 3.3: Reference frames in simulink

Outputs:

- `Slp` : longitudinal slip $\lambda\,[\,rad\,]$ of tire
- `Alpha` : sideslip angle $\alpha\,[\,rad\,]$
- `TurnSlp` : rotational speed of the tire around the vertical axis divided the longitudinal speed
- `rBelt_eff` : distance in $[\,m\,]$ between contact point and hub center
- `vBelt` : speed in $[\,fracms\,]$
- `LongFrc` : longitudinal force along x in $[\,N\,]$

- `SideFrc` : lateral force along y in $[N]$
- `LoadFrc` : vertical force along z in $[N]$
- `OverturnTrq` : torque along x in $[Nm]$
- `RollResist` : rolling resistance
- `AlignTrq` : sef-aligning torque along z in $[Nm]$

3.2 CarMaker for simulink tire model

The first thing to do is to calculate the inputs of the look-up tables that we remember are:

- λ: longitudinal slip
- α: slip angle
- F_z: vertical load
- V_x: longitudinal speed of the wheel
- p: inflation pressure

The only inputs that must be truly calculated are λ and α, since the other are inputs of the tire subsystem. From equations 1.32 and 1.33 we calculate the the two slip as we see in figure 3.4 and 3.5.

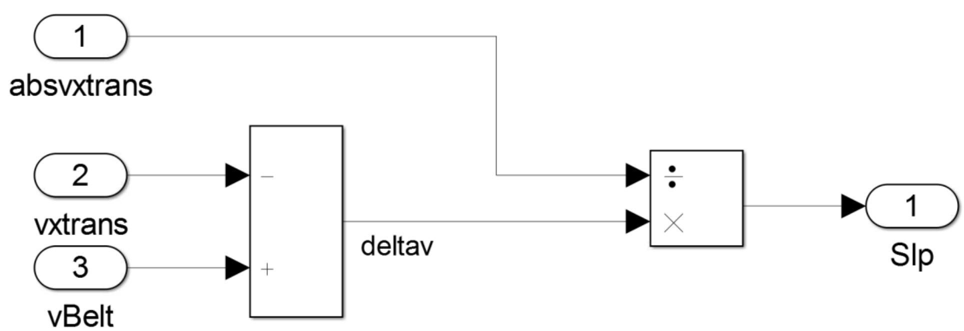


Figure 3.4: Slip calculation

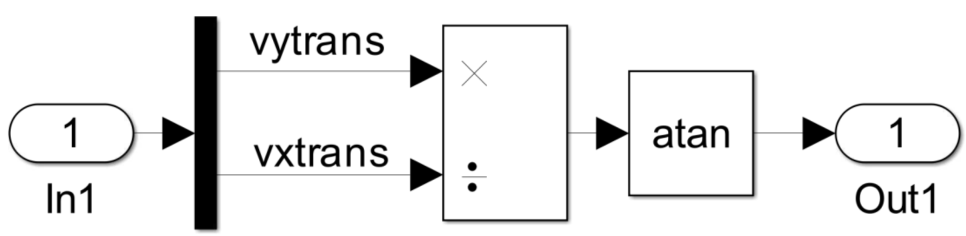


Figure 3.5: Alpha calculation

Now we have all the necessary inputs of the look-up tables and we can build the model of the longitudinal and lateral forces.
The models take into account the interaction between forces and the transient of the the tire as we have already seen. In figure 3.7 and 3.6 we can see how the models look like.

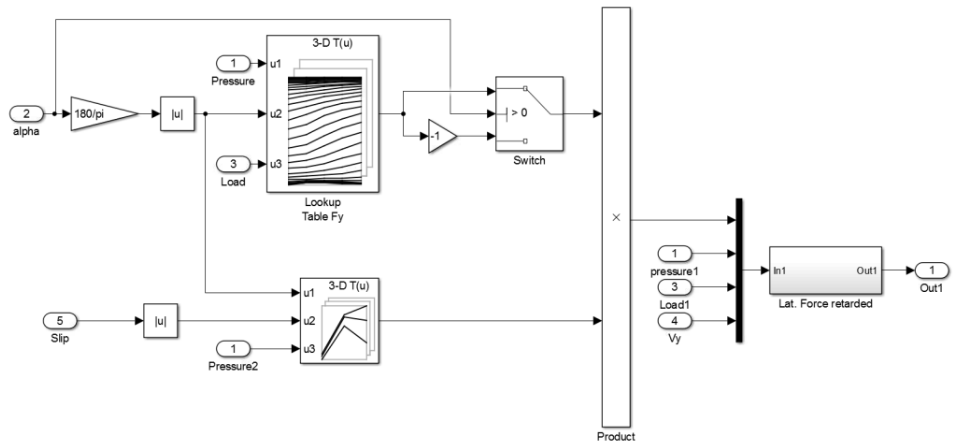


Figure 3.6: Lateral force model

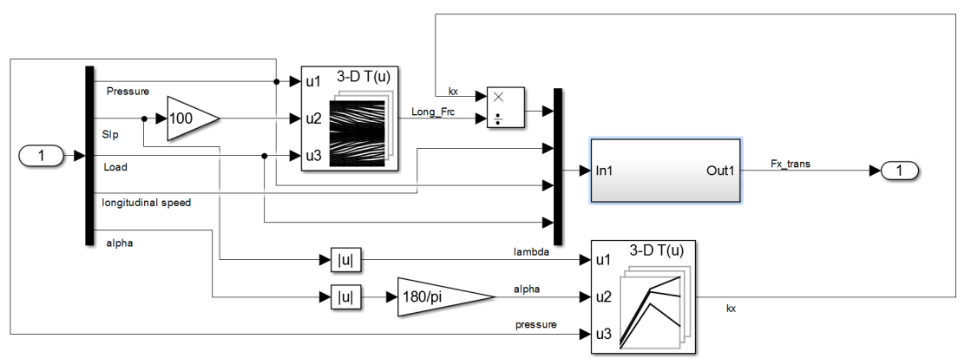


Figure 3.7: Longitudinal force model

In figure 3.9 and 3.8 we can see look-up tables of effective rolling radius, rolling resistance and self-aligning torque. The self-aligning torque is also affected by a dynamic effect such as the longitudinal and lateral force, but the effect on the overal vehicle dynamic can be negligible.

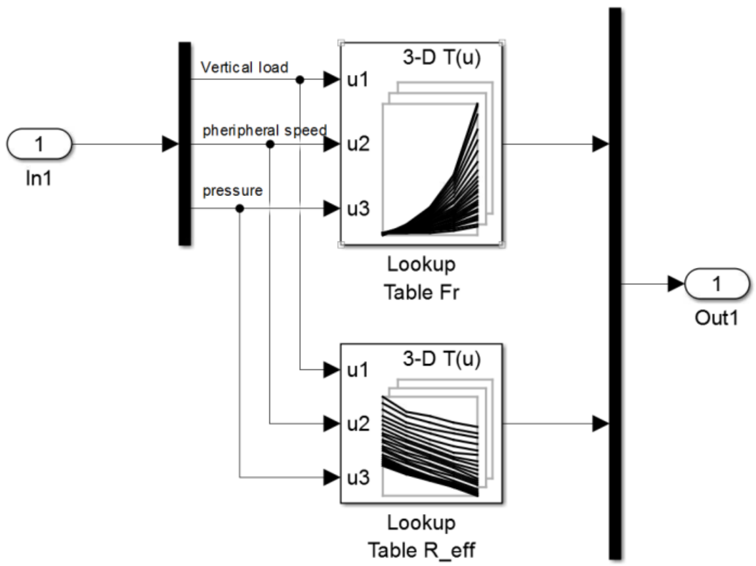


Figure 3.8: Effective rolling radius and rolling resistance force

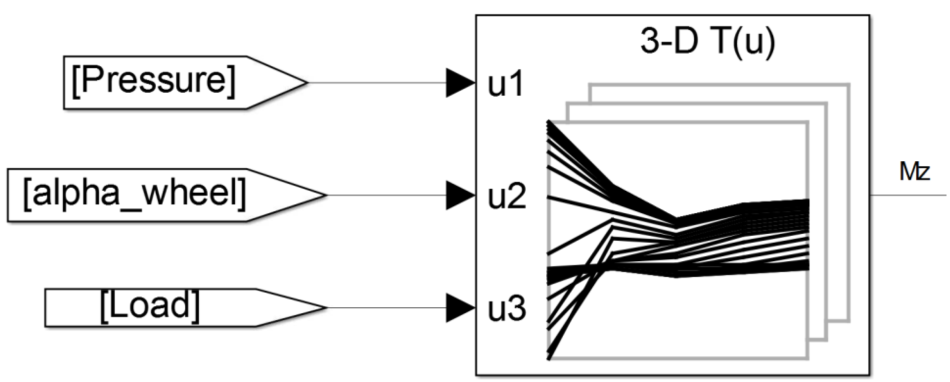


Figure 3.9: Self aligning torque

3.3 Step Steer maneuver

The first simulation we made is a step steer maneuver; the maneuver is done on a flat road with 0 slope.
The condition of the test are the following:

- Test duration 12 s
- Step steer maneuver start 5 s
- Longitudinal speed constant 50 $\frac{km}{h}$
- Steering wheel angle final value 100°
- Tire inflation pressure 200 kpa

The simulation is carried out with both our tire model and with the Pacejka 6.1 model of CarMaker.
In the following figures we can see the comparinon between the results. In figure 3.10 and

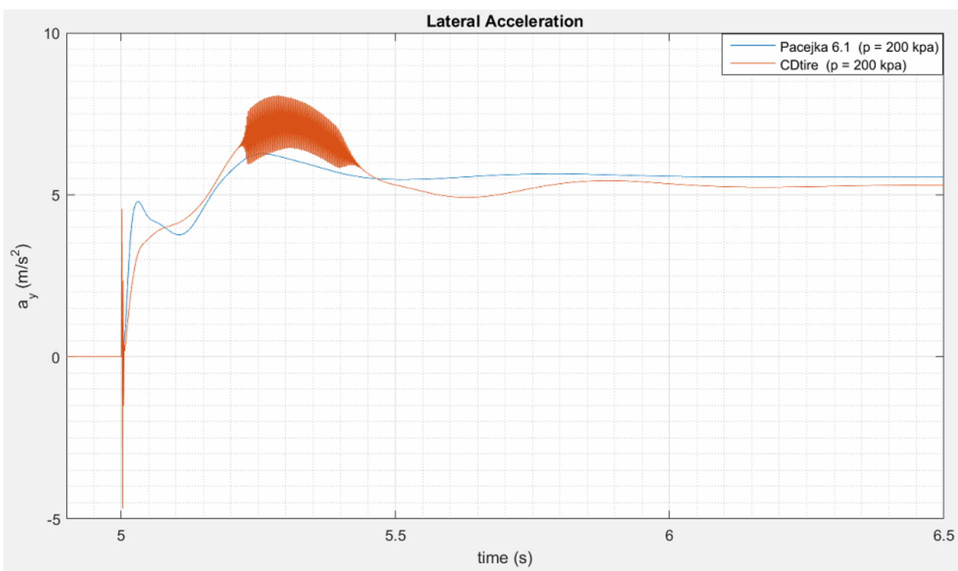


Figure 3.10: Step Steer Lateral acceleration

3.15 we see an unexpected vibration resonance.

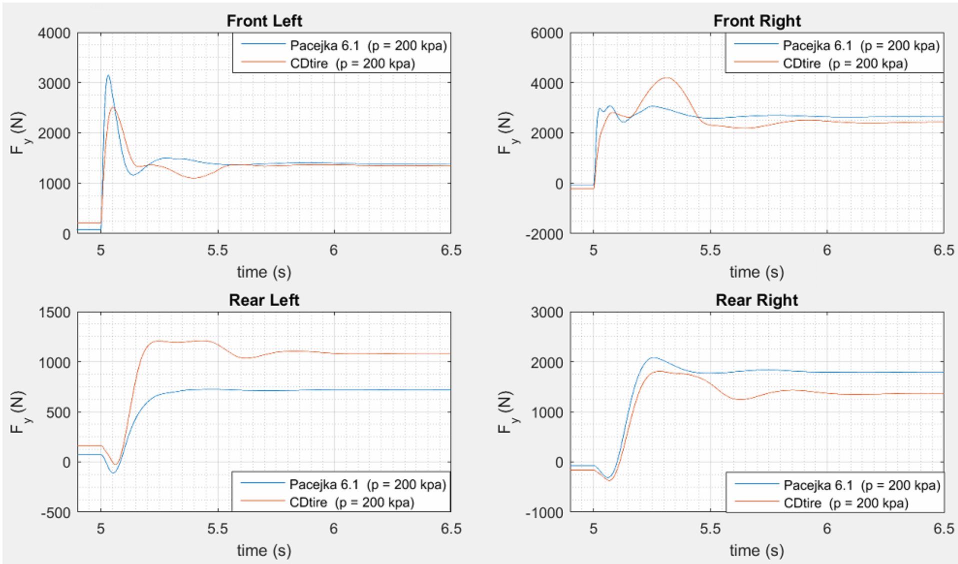


Figure 3.11: Step Steer Side Forces

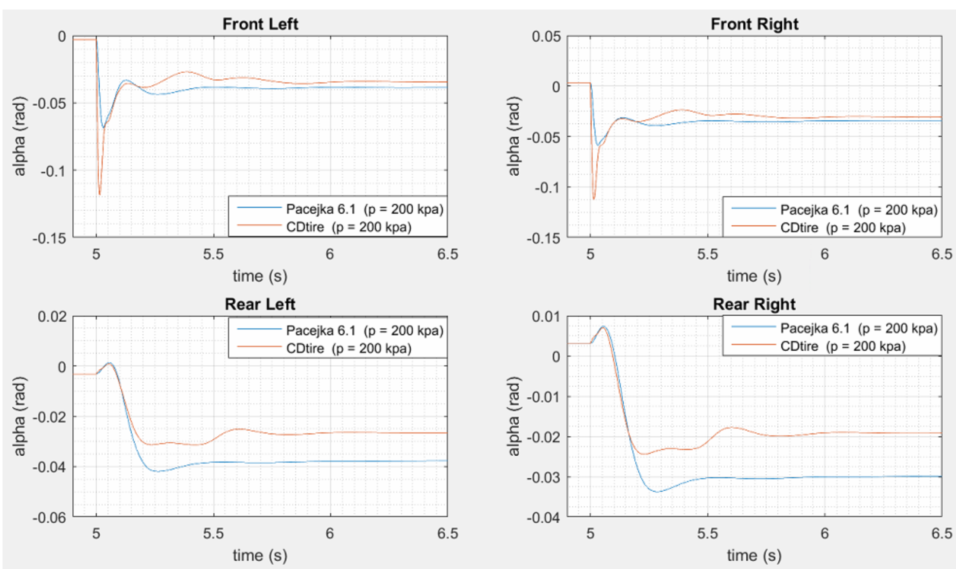


Figure 3.12: Step Steer Side Slip Angles (wheels)

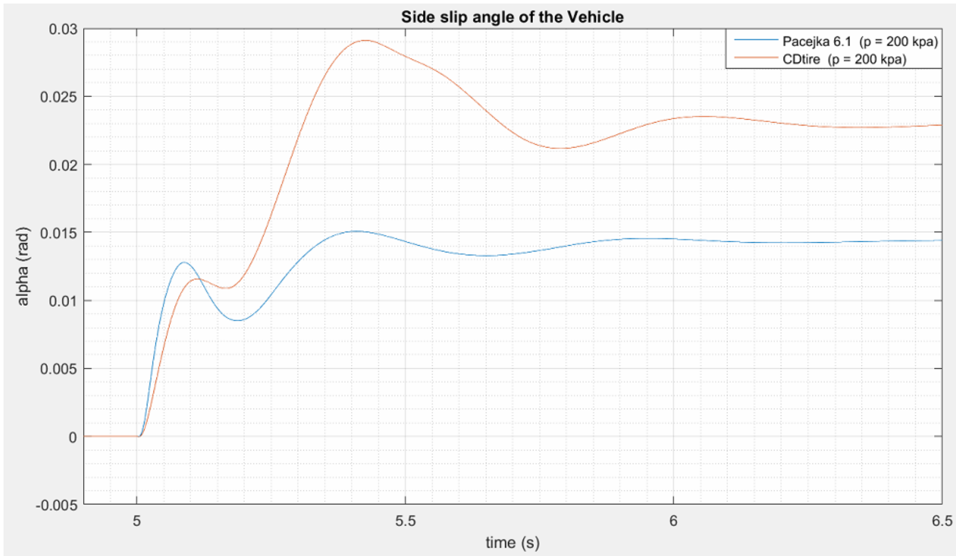


Figure 3.13: Step Steer Side Slip Angles (vehicle)

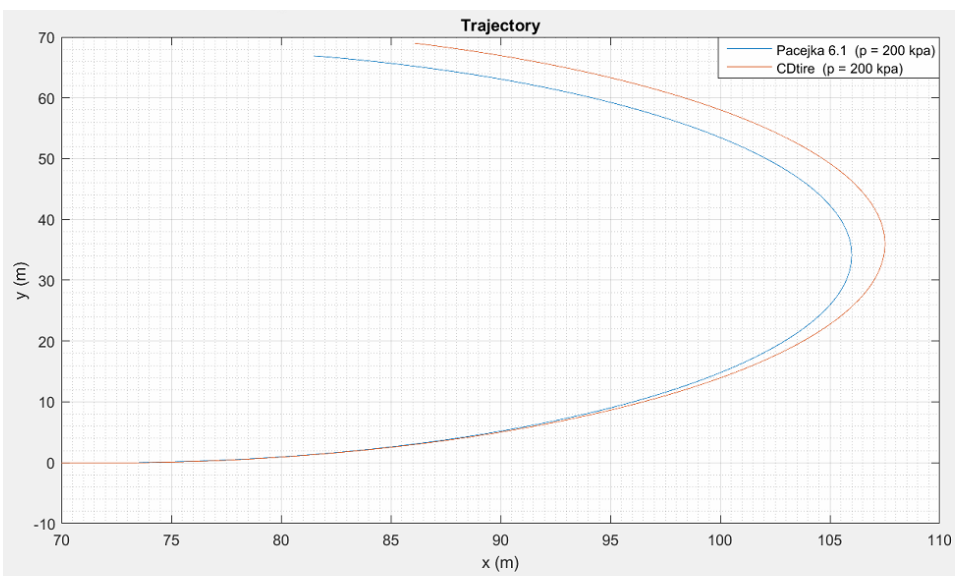


Figure 3.14: Step Steer Trajectory

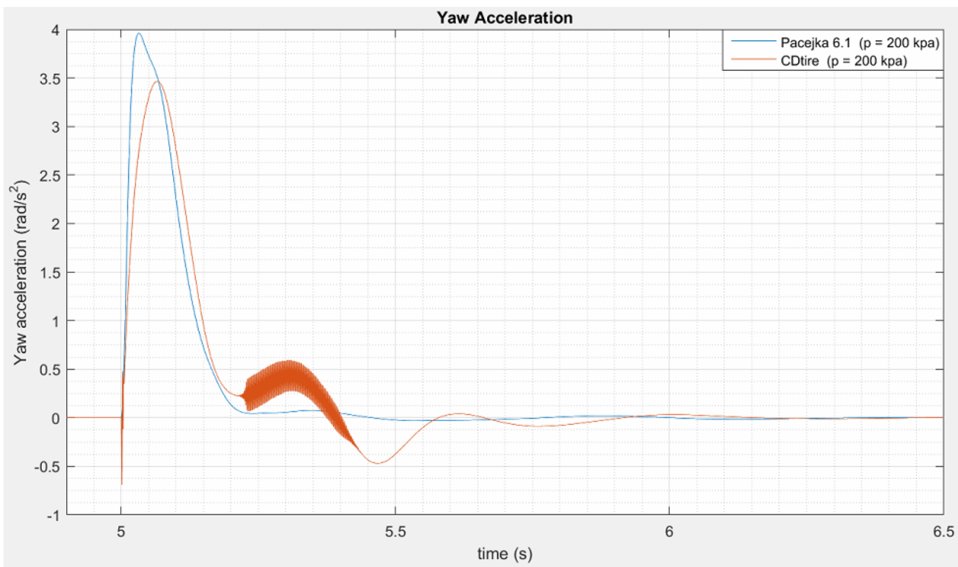


Figure 3.15: Step Steer Yaw Acceleration

Chapter 4

Conclusion

The final CarMaker simulations are a lane change and a slamom done at different inflating pressure, to evaluate the dynamic different behaviour of the vehicle.
The first is done at 65 kph with the gas pedal released during the manevuer; the second is done at 50 kph. The car is a BMW 5 series 140 kW of maximum power. The inflation pressure tested are 100, 200 and 300 kpa.
The lateral acceleration plot (both in figure 4.2 and 4.6) show clearly lag effect for what

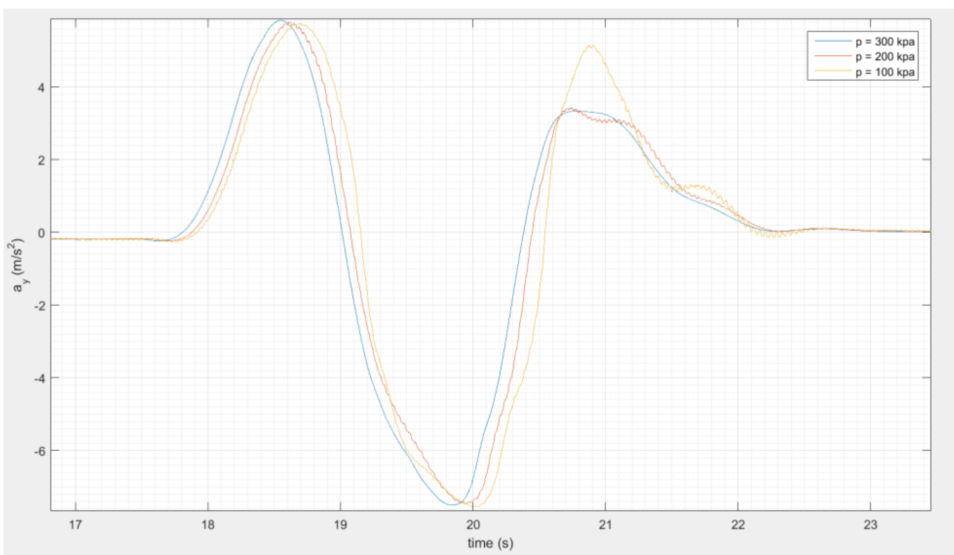


Figure 4.1: Lateral acceleration

concern the minimum inflation pressure. This could be due to the larger value of relaxation lengths. In both 4.2 and 4.7 we see the different side slip angles of the vehicle. The first thing we notice it is the much larger value for the minimum inflating pressure; in figure we notice also a different sign for the minimum pressure.
For what concern the steering effort it is clear from both the plot (figures 4.3 and 4.8) that it is higher for the lower pressure. The yaw acceleration (figure 4.4) presents some vibrations unexpected, probably due to discontinuity of the look-up tables.
The trajectory does not present relevant differences (figure 4.5).

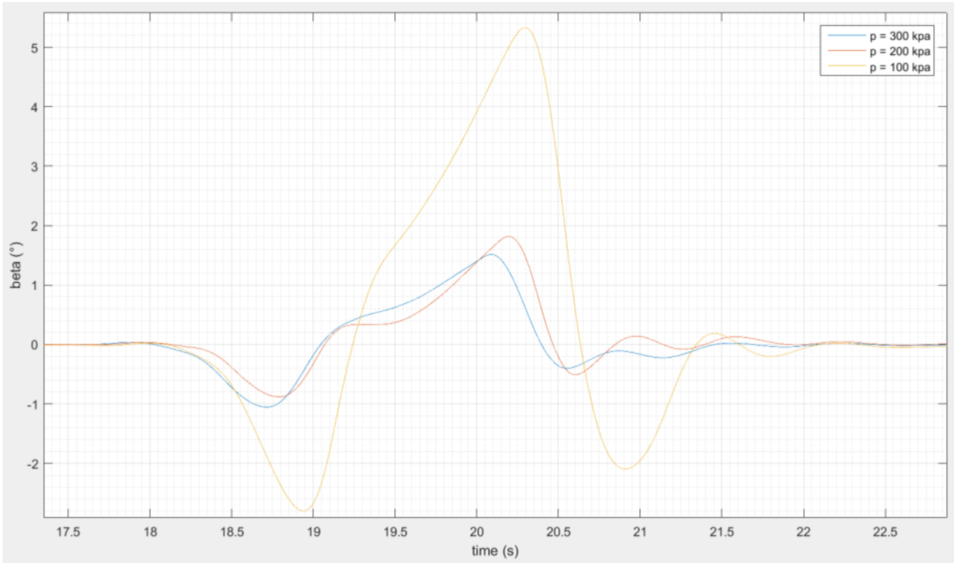


Figure 4.2: Sideslip angle of the vehicle

The main relevant results of the work are certainly the change in the lateral stiffness coefficient and the lateral relaxation length: we see that the pressure effect is not negligible in the calculation of this quantities. From this final simulations the conclusions are: a lower stability of the vehicle due to higher sideslip angles and a larger steering effort, both resulting from the lower value of pressure tested.

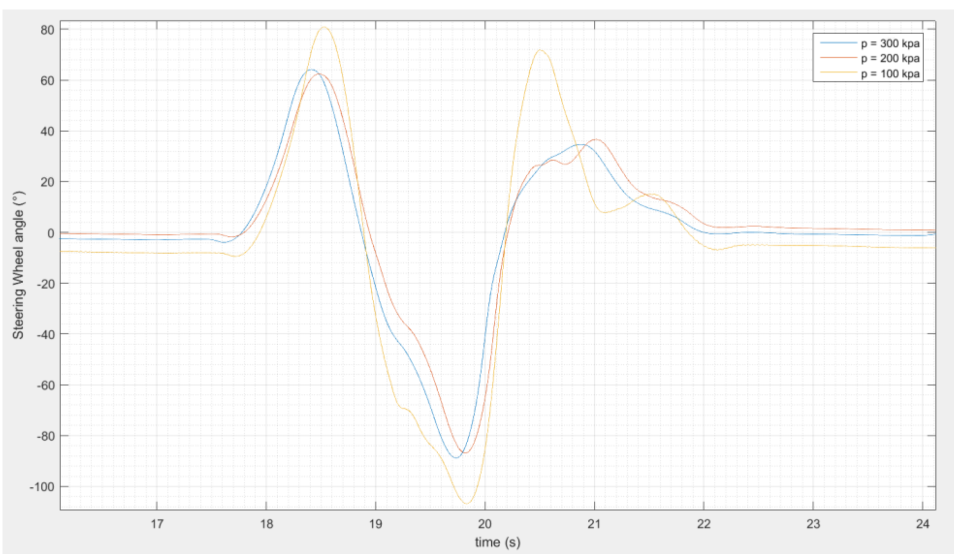


Figure 4.3: Steering wheel angle

For what concern the different trajectory (figure 4.5) of the vehicle there are no significative changes.

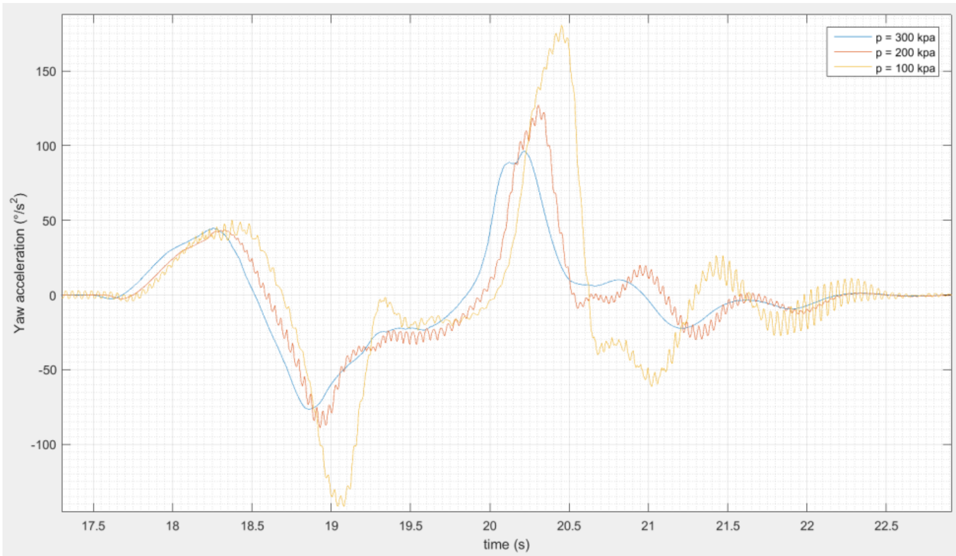


Figure 4.4: Yaw acceleration

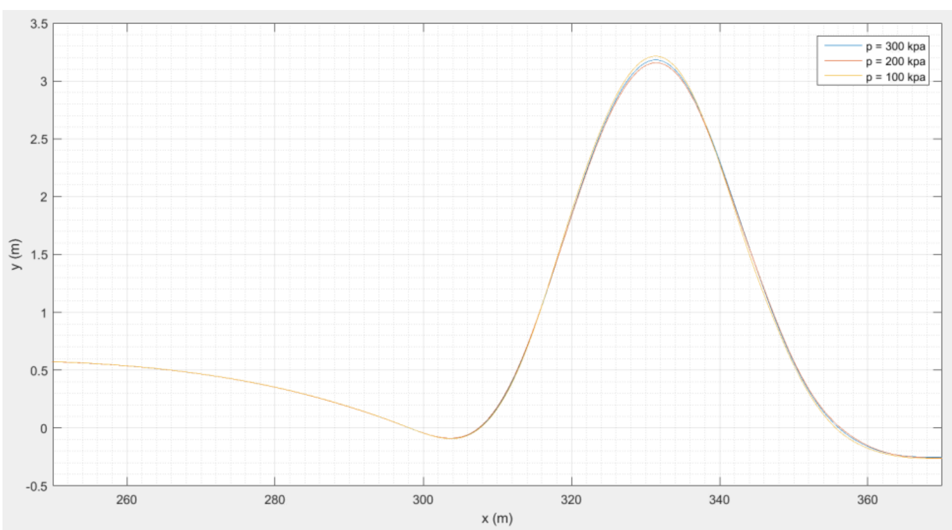


Figure 4.5: Trajectory of the vehicle

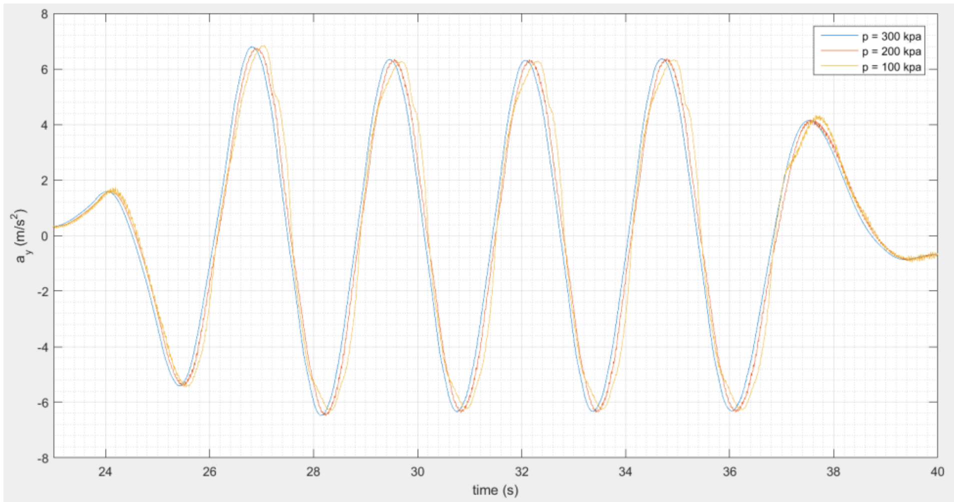


Figure 4.6: Lateral acceleration

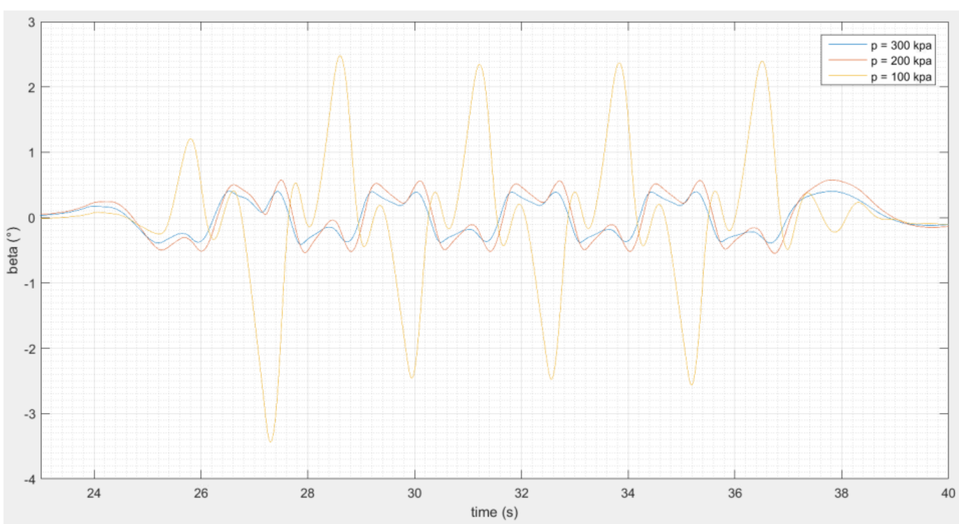


Figure 4.7: Sideslip angle of the vehicle

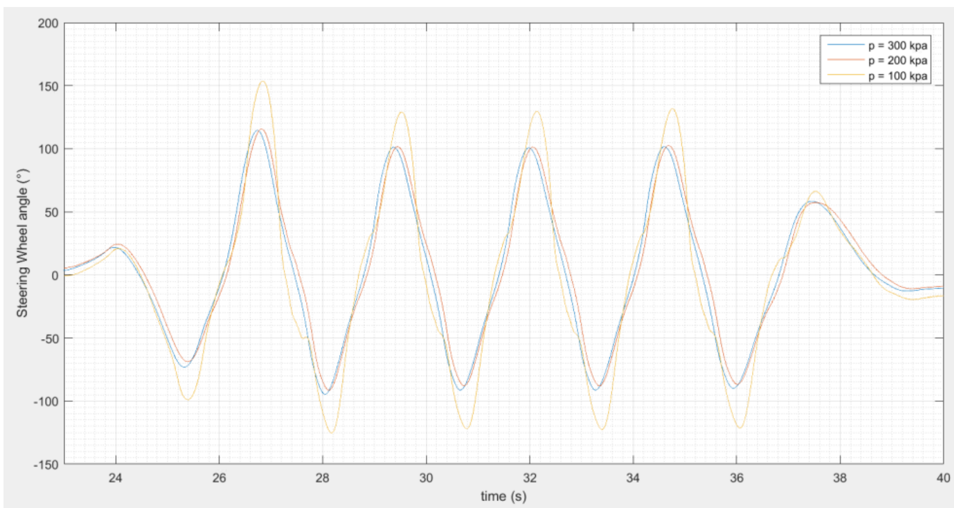


Figure 4.8: Steering wheel angle

Bibliography

[1] Guoxing Li,1,2 TieWang,1 Ruiliang Zhang,1 Fengshou Gu,1,2 and Jinxian Shen1. *An Improved Optimal Slip Ratio Prediction considering Tyre Inflation Pressure Changes.* 1 Department of Vehicle Engineering, Taiyuan University of Technology, Taiyuan 030024, China.
2 Centre for Efficiency and Performance Engineering, University of Huddersfield, Huddersfield HD1 3DH, UK.

[2] Z. Liu. *Mathematical models of tire-longitudinal road adhesion and their use in the study of road vehicle dynamics.* Journal of Beijing Institute of Technology, no. 2, pp. 193204, 1996.

[3] M. Bian, *Simplified tire model for longitudinal road friction estimation.* Journal of Chongqing University of Technology (Natural Science), vol. 26, no. 1, pp. 15, 2012.

[4] Schmeitz, A. J. C., Besselink, I. J. M., De Hoogh, J., and Nijmeijer, H., 2005, *Extending the Magic Formula and SWIFT Tyre Models for Inflation Pressure Changes.* 10th International VDI Congress, Hannover, Germany, pp. 201225.

[5] Amir Soltani (Islamic Azad University), Avesta Goodarzi (University of Waterloo), Mohamad Hasan Shojaeefard(Iran University of Science and Technology), Khodabakhsh Saeedi(University of Waterloo), *Optimizing Tire Vertical Stiffness Based on Ride, Handling, Performance, and Fuel Compsumption Criteria.* Journal of Chongqing University of Technology (Natural Science), vol. 26, no. 1, pp. 15, 2012.

[6] Besselink, I. J. M., Schmeitz, A. J. C., and Pacejka, H. B., 2010, *An Improved Magic Formula/Swift Tyre Model That can Handle Inflation Pressure Changes,* Veh. Syst. Dyn.,48(S1), pp. 337352.

[7] E. Kuiper and J. J. M. Van Oosten, *The PAC2002 advanced handling tire model.*

[8] Mark Reiter, John Wagner, *Automated Automotive Tire Inflation System Effect of Tire Pressure on vehicle handling* Mechanical Engineering Department, Clemson University, Clemson, SC 29634 USA

[9] F. Grappe, R. Candau, B. Barbier, M. D. Hoffman, A. Belli and J.-D. Rouillon (1999), *Influence of tyre pressure and vertical load on coefficient of rolling resistance and simulated cycling performance* Ergonomics, 42:10, 1361-1371, 10 nov 2010.