RACING MOTORCYCLE DYNAMIC ANALYSIS MODELS

Pre and post processing software

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u	Forward velocity
ϕ	Roll angle
ϕ'	Roll rate
ϕ''	Roll acceleration
ψ	Yaw angle
ψ'	Yaw rate
ψ''	Yaw acceleration
τ_{gear}	Engaged gear transmission ratio
τ_s	Crown pinion ratio
τ_p	Primary shaft ratio
cx	Aerodynamic resistance coefficient
P	Air density
s	Frontal area
a	Longitudinal distance between the front wheel axis and the COG
b	Longitudinal distance between the rear wheel axis and the COG
hg	Height of the center of mass
wb	Wheelbase
m	Mass of the motorcycle
λ	Sideslip angle
I_x	Roll moment of interita relative to the COG
I_y	Pitch moment of interita relative to the COG
I_z	Yaw moment of interita relative to the COG
I_{xz}	Roll-yaw product of interita relative to the COG
rt	Average toroidal radius
N_f	Normal front force
N_r	Normal rear force
F_f	Lateral front force
F_r	Lateral rear force
S	Thrust (global)
I_e	Parameters for wheels gyroscopic effect

1 Abstract

This document will propose different models to analyze racing motorcycle behavior in both pre and post processing cases. The first part is about motorbike's dynamic models for lap time simulation and setup variation analysis. The second one that is about data analysis; proposing a fast method to understand how the rider drives along the circuit and how much confidence he has with the motorbike

Proposed dynamic models do not have the aim of defining optimal trajectory of a motorbike along the track or the ideal lines over the circuit. The goal is to be able to predict and evaluate how an upgrade to the motorbike or a setup tuning can influence the lap time or the equilibrium of the motorbike during a lap. It permits to evaluate both engine and chassis modifications giving information about forces, grip condition and the lap time value. There will be no controller to simulate the rider behavior or any sort of optimization of that since this is not the aim of the model.

Second part is about data analysis. The goal of the proposed software is to compare different riders' driving style and performances without taking the whole log for every one and manually compare their speed and suspension data. Furthermore, that software can make the work of the data analysist easier and smarter, trying to auto highlight strong and weak points of the rider.

2 Introduction

Software that will follow on have been created with the goal of optimize bike and riders performance during a race weekend. Both software share a common approach that refers to an ideal rider behavior. Imagine perfectly knowing how the whole system should works, engine, suspensions, chassis, gearbox and tires. Real riders never replicate this ideal condition but their data are always analyzed taking in consideration what would have been the ideal behavior. Obviously, ideal riders should be on the edge of the mechanical and physical limits so that every acceleration or deceleration is limited from the actual grip or other limits as wheelie or capsizing. Real riders can use a few things to control the bike; these are throttle, brakes and the steer while the roll motion that is a child of the rider position over the bike controls the equilibrium of the system. While for bike controls an ideal profile of usage can be defined it is not the same for the roll motion. This strictly depends on the riding style of the rider and it is very tricky to define if one is better than another since it deeply depends on the rider feeling.

Defining an optimal strategy for controlling the roll motion is a hard and complex thing. The usual approach is to balance the bike and put it in the stable zone at the beginning, then, once it is stable, control the body movement to exploit the best grip and so the speed. Many problems figure out on that strategy, for example the body cannot be considered as a point mass but should at least be something similar to a solid with homogeneous distributed weight. Since this control part requires a lot of time to be tuned and to be simulated, the approach used here starts from real roll data.

Therefore, starting from an ideal behavior on throttle and brake that is shared between all riders and all motorcycles the software are created. Many choices made during the creation depends on how ideal riders should drive. This type of approach, made in pre and post processing analysis, takes to coherent data outputs. It means that predictive analysis output will indicates where the limit condition is if the bike is driven in a certain way, while, post processing analysis will give an idea of how far from the ideal and from the actual best the rider is.

3 Phenomena description

Both parts regard the analysis of a motorcycle, here, different mathematical simulation models are proposed. Their goal is to have a prediction of what will happen in the reality while, the rider confidence software performs a post process analysis to what happened in the real track. Since they are focused on the same phenomena, this chapter offers a description of them in order to clarify everything as much as possible and to better understand some choices that have been made during the software development. Understanding what is happening on the motorcycle at a low hardware level have been a key point to correctly design the software and make their output coherent having always in mind the goal of best performance.

3.1 System definition

Motorcycle is an unstable system that is kept in the stable zone by its rider that acts as a feedback controller acting on throttle, brake, steer, roll and by moving himself on the seat influencing the center of gravity position so the forces and the torques acting. Reproducing the driver behavior with a math model has always been very tricky since there are many variables that must be taken in consideration. Furthermore, there is no optimal strategy to adopt due to the fact that it depends on parameters related to the riding style of a specific riders and it must be noted that it changes from one to another rider depending on how he feels the bike, his sensation about the weight, the height and the grip perceived.

For these reasons, usually, simplified bike models do not take in consideration the rider behaviour, imposing a fixed centre of gravity, supposed to be on the sprung masses.

One way to perform the analysis is to divide the riding in different phases, for example

- Uniform rectilinear motion
- Uniform accelerated motion

Inside these two main groups other subdivisions can be done by separating braking and acceleration phases from the turning ones.

Actually, in the real world these phases happen together in a mixed way so that the result is that the bike turns while it's braking or accelerating.

Neutral phase also occurs; the bike is going on a coast down condition, no brake and no throttle situation. This phase should be avoided since the rider is not controlling the bike and it reacts to every road input in a uncontrolled way. For these reasons, usually neutral phase is not developed in mathematical models, considering also that ideal rider should have zero neutral time to optimize lap timing.

Starting from these basic consideration, analyzing a flying lap on a track all the described phases can be easily recognized. That means this is a possible way to model the bike status during the lap. Now every condition will be examined in a deeper way.

3.1.1 Kinematic of a motorcycle

First of all, a kinematic focus is necessary, then an explanation of dynamical phenomena is given. (Cossalter, 2006) Motorcycles are made of a great variety of mechanical parts, including some complex ones but from a strictly kinematic point of view, a motorcycle can be defined as simply a spatial mechanism composed of five rigid bodies:

- The chassis assembly (chassis, tank, engine, gearbox)
- The front assembly (the fork, the steering head and the handlebars),
- The rear assembly(rear wishbone, rear shock)

- The front wheel,
- The rear wheel.

Note that if rigid suspension are used, rear frame and chassis is a unique rigid body. These rigid bodies are connected by revolute joints (two wheel axis, steering axis, rear wishbone revolute joint). Each revolute joint inhibits five degrees of freedom in the spatial mechanism, while each wheel-ground contact point leaves three degrees of freedom free. Considering the hypothesis of the pure rolling of tires on the road to be valid every wheel can only rotate in three directions: the contact point (1DOF) the intersection axis of the motorcycle and road plane (2DOF) and around the axis passing through the contact point and the center of the wheel.

The control of these degrees of freedom is up to the rider and, depending on his riding style he will do a combination of moves resulting in his interpretation of the turn.

These considerations have been made assuming that the tires move without slippage. However, in reality, the tire movement is not just a rolling process.

The generation of longitudinal forces (driving and braking forces) and lateral forces requires some degree of slippage in both directions, longitudinally and laterally, depending on the road conditions. The number of degrees of freedom is therefore seven:

- Forward motion
- Rolling motion
- Handlebar rotation
- Longitudinal front slip
- Longitudinal rear slip
- Lateral front slip
- Lateral rear slip

Rigid model proposed in the following will actually have locked handlebar so the effective DOF are six. But when linearizing sideslip angles and ignoring small displacements of the wheels' contact points (with respect to the curvature radius) the global effect of sideslip is taken in consideration and it is the yaw acceleration and its integrated value so that in the end for a rigid model it is considered to have 3 degrees of freedom.

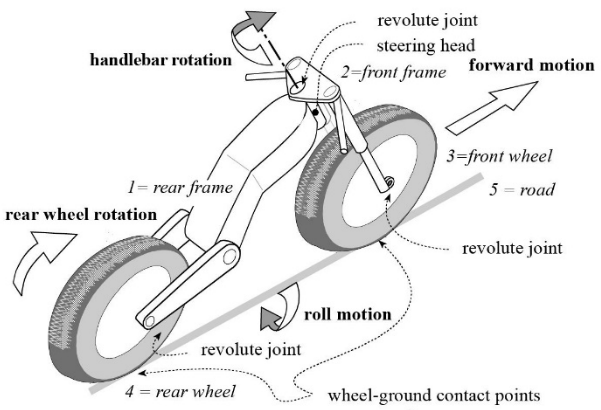

Figure 1, bike multibody

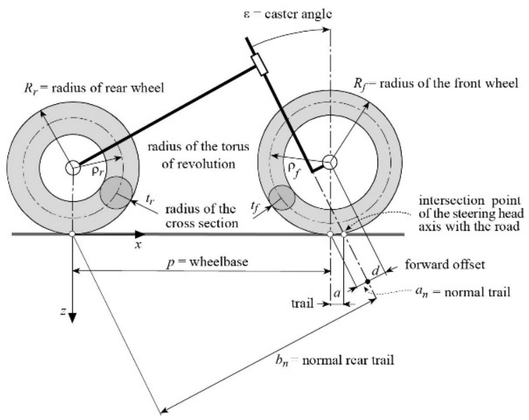

Figure 2, motorcycle base quotes

3.1.2 Acceleration

This phase starts on the exit of a corner with the bike that usually is still at a high lean angle, the rider tries to accelerate as fast as possible rapidly increasing the throttle creating a little bit of spinning on the rear tyre that is strictly connected to the oversteering behaviour that helps the bike close the corner. On the contrary too much spinning means losing time and destroying the tyre carcass and so the performance degrade increase.

Acceleration phase continue also on the straight and actually ends only when the rider stars braking. During that phase the bike is like "sitting" on its rear, trying to wheelie with the rider acting to keep the front wheel down in contact with the ground. That means acceleration generate a compression on the rear suspension, magnitude and velocity of that movement depends on geometry spring and damper settings. Observing this phenomena at very little time steps, it can be considered a uniformly accelerated motion and so all the deriving equations can be used.

Generally the acceleration is quite low during the initial spinning phase and rapidly increase while the bike gains grip, obviously the lower the gear, the higher is the acceleration.

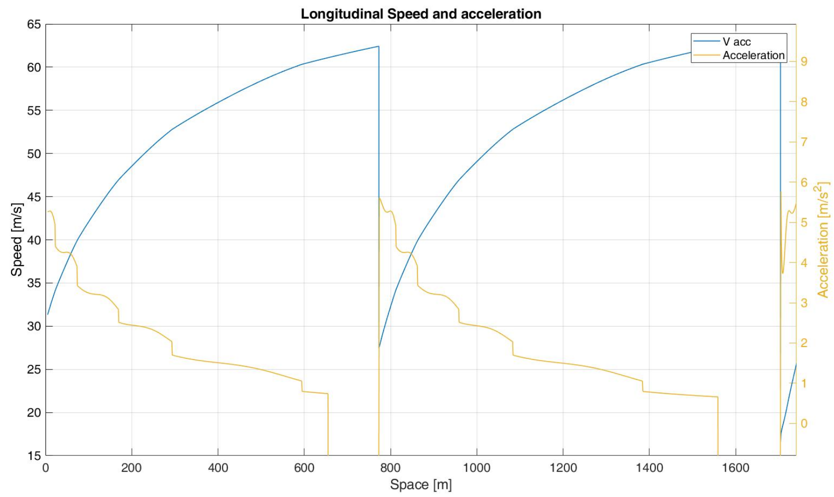

Figure 3, Example of speed profile and acceleration

3.1.3 Braking

When the rider approach a corner, he usually have to slow down in order to be able to turn the bike correctly. As soon as he starts braking, he also moves up and rear on seat to increase the aerodynamic resistance by increasing the frontal area and avoiding the capsizing moving backwards the whole COG position. Weight transfer plays a key role in this phase, the higher is the vertical load on the braking tire the higher is the maximum deceleration achievable. Usually riders, especially on small bikes, use only the front brake since as said before, due to weight transfer, the front wheel is much more loaded than the rear one and so it is able to generate a higher longitudinal force. This is also the reason behind the bigger disk and caliper on the front than on the rear.

Ideal braking consist in reaching as soon as possible the peak pressure on the brake in order to better exploit all the available weight transfer and then slowly decrease the pressure in a linear way avoiding brutal change of vehicle balance that means losing time since the rider is not braking at the maximum of the bike possibilities.

When braking the front fork compresses with a certain velocity depending on the shock absorber settings. The amount of compression is related to the springs mounted and to the preload settings adopted. Some riders prefer to have a fast fork in compression to take advantage of the load transfer while others prefer a slow fork in compression to have a more predictable behavior of the motorbike.

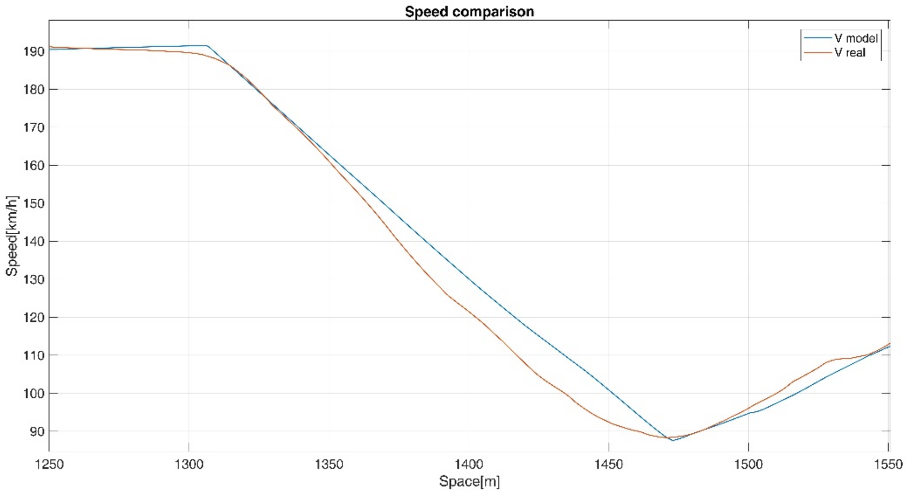

Figure 4, Example of a brake phase. (San Donato turn, Mugello circuit)

This phase is often critical since in the final part the vertical load on the front wheel is reducing and the bike is already at high lean angle. If the rider insist in asking for longitudinal and turning forces together, the result is that the front wheel will “close” towards the center of the corner and the bike will crash since the rider cannot control it anymore, unless you are Marc Marquez. As for the acceleration phase, if the analysis is performed with small time steps (hundredth of a second) the motion can be considered as uniformly accelerated motion.

Figure 5, Marc Marquez save. 64° of lean angle at 133 km/h. No grip at all on the front wheel

3.1.4 Neutral phase

When the rider is not accelerating or braking the rider is going on neutral phase and generating the so-called neutral time. Usually, this phase occurs between the braking and the accelerating ones. It should be as smaller as possible since, as said before, it is very risky and time losing.

Riding a motorbike means find the correct compromise between all these phases so that the rider is able to go fast without taking many risks on every lap. Pay attention to the fact that every rider drives in a different way and for example one that is able to bring a lot of speed at the corner center will have more neutral phase than the one who brakes more before entering the corner. This is related to their feeling and to the fact that beginning the corner at a lower speed usually means the rider feels more confident when he opens the throttle so less time will pass between braking and accelerating. On the contrary, a rider who slows down less will enter the corner at a higher speed but he will not be able to open the throttle as soon as the other one. Since exists a maximum speed at which the corner can be done, that strictly depends on the grip condition and so on the maximum lateral force developed on the contact points. Actually, if he is going at a speed that is very near to the maximum one he will not be able to generate neither any longitudinal force nor acceleration or brake ones since he is at the limit of the tire and so he is forced to do a little bit of neutral time. Note that probably, the time spent to cover the corner will be the same or very similar but very different phenomena are happening on the bike and the one doing neutral phase will always be closer to the limit than the other one.

These basic considerations on what is happening on the bike while it is going over the track will be very useful to better understand some choices made during the software developments. Mathematical predictive models as well as the post analysis have the final goal of helping riders to bring out the best performance and to help technicians to find out problems on the bike and provide some possible solutions or improvements to be tested. These software tries to optimize the bike behavior on the track, firstly in a predictive way that give as output an ideal lap of that bike on a specific track with all the related variables about acceleration, speed, suspension behavior and so on. Secondly, with a post analysis that puts in evidence strong and weak points of the rider highlighting how and where he can gain time.

3.1.5 Suspension motion

(Cossalter, 2006) Explain that motorcycle without suspension moving over uneven ground presents difficulties in steering because of the loss of wheel grip on the road, and because of rider discomfort. The tires easily absorb small bumps on the ground, but for adequate absorption of larger bumps, the motorcycle needs appropriate suspension. A motorcycle with suspension, from a dynamics point of view, can be considered as a rigid body connected to the wheels with elastic systems (front and rear suspension). The rigid body constitutes the sprung mass (chassis, engine, steering head, rider), while the masses attached to the wheels are called unsprung masses. Suspension has to satisfy the following three purposes:

- Allow the wheels to follow the profile of the road without transmitting excessive vibration to the rider. This purpose concerns rider comfort that is the isolation of the sprung mass from the vibration generated by the interaction of the wheels with road irregularities.
- Ensure wheel grip on the road plane in order to transmit the required driving, braking and lateral forces.
- Ensure the desired trim of the vehicle under various operating conditions (acceleration, braking, entering and exiting turns).

The degree of required comfort or performance varies according to the use of the vehicle. Obviously, with racing vehicles, comfort is less important than the motorcycle’s capacity to keep the wheels in contact with the ground and to assume the desired trim. However, in other vehicles the suspension is expected to serve other purposes. For example, in off road vehicles the suspension serves to isolate the sprung mass from continuous impact generated by vehicle jumps. For this reason, suspension in off-road vehicles has greater wheel travel than in touring vehicles, and more so than in racing vehicles. As for the trim, it should be highlighted that it depends on the stiffness of the suspension and on the loads. The load can be quite variable in motorcycles (one or two passengers, possibly with baggage); and furthermore, load transfer between the front and rear wheel occurs in both acceleration and braking.

4 Lap time simulation

Over the years many different approaches to this theme have been developed. Simulation studies determine whether the motorcycle can perform maneuvering tasks such as path following with a specified velocity profile. Although open-loop simulation can be used to analyze automobile handling, a virtual motorcycle rider is needed to stabilize the roll mode. Without feedback to provide appropriate steering and throttle/brake inputs, the simulated motorcycle usually falls quickly. Actually, it has developed a model providing a virtual rider based on a simplified motorcycle model is applied to a commercial multibody code motorcycle model. The control strategy is based on a receding horizon scheme that uses preview information on the path to be followed. The development and use of a virtual rider is described in (V.Cossalter, 1999). The multibody approach provides very precise data and requires a lot of time in both creation and execution. Commercial and private software provides code for that goal.

The proposed method here is focused on high performance manoeuver used in racing competition. Riders, over their years of racing, develop the ability to get the optimal trajectory to improve their speed during cornering. The line depends on their riding style, the bike setup, the grip condition and the ground conditions. For example, unless the simulation use a 3D laser scanned track as input it is impossible to replicate the waves and the little holes of the track. Ideal rider can provide an ideal trajectory based on the speed optimization but it is not so obvious that real riders, for the reasons said before, will be able to replicate these lines. This type of analysis is not focused on get out the ideal best of that bike; it provides a reference for comparison with reality, given some fixed points as input so that riders and technicians can use it for comparison.

Due to its stable nature, models based on racing cars have been developed before than the motorcycle ones; using very different approach, from a quasi-static definition to a nDOF model producing racing line and the speed profile. Due mainly to instability issues, the exploration of maximum performance for motorcycle models is more recent. In (V.Cossalter, 1999) optimal control techniques are used to define and assess a notion of motorcycle maneuverability. The cost function uses penalty functions to address constraints such as the width of the road. Using a combination of penalties, (V.Cossalter, 1999) produces plausible approximate race lines. A more direct attack on the maximum performance and minimum lap time problem for motorcycles is presented in (E. Bertolazzi, 2005), which also uses penalty functions to handle inequality constraints. The optimal solution is found by solving a discretized two-point boundary value problem expressing the first-order optimality conditions.

Here, due to our goals, a simpler approach has been used. At the beginning, the bike is a point mass vehicle and is following a fixed path that is directly derived from the raw data collected when the bike was running on track. Therefore, the model tries to replicate real speed profile. Then, by increasing number and complexity of constrains, instead of fixing the trajectory the roll and the speed at the center of the curve will be fixed and used as inputs. In the next pages three types of models are shown, they are based on the same working principle but with increasing complexity. They are

- Point mass vehicle, 1 DOF
- Rigid body vehicle, 3 DOF
- Hybrid model, sprung unsprung masses, 7 DOF

4.1 Preliminary calculation

Before starting raw data must be processed to obtain a usable dataset. The database consist in a set of raw data of some laps on every circuit of the world championship. First of all the length of the track must be determined so that the useless parts of the data can be taken away

The method used to correctly determine track length consist in comparing curvature peaks and the very first approximation of the speed profile, if two peaks remains in the threshold used for the definition so the distance between these two peaks is the track length. Since the raw data consist in a little more than one lap this comparison usually finds more than one useful result. In that case, an average value is taken as the track length. In addition to that, some other security check are performed in order to avoid that a circuit with similar curves can be considered in the wrong way.

Defining the track length from a curvature peak means that also the starting and the finish line will be at the center of the corner. Note that all the proposed models are made like that and so every figure below showing a profile will not start from the real finish line but from the last corner or the second last.

The approach used to determine the speed profile used for determining the length consists in extracting from the curvature a very first speed, simply taking in consideration the centrifugal force and the aerodynamic one.

$$V = \sqrt[4]{\left(\left(\frac{Fy_0}{m}\right) * R\right)^2 + \left(\frac{1}{2}\rho S C_x\right)^2} \tag{1}$$

In this specific case, Fy_0 is considered constant but this is not true, in the following pages deeper explanation on that variable will be given.

4.1.1 Magic Tire formula

As stated (Saccon, 2006), to better describe the longitudinal and lateral tire force coupling, we briefly describe how tire forces are modelled using Pacejka's magic formula [3]. The magic formula is a set of equations relating load, slip ratio, slip angle, and camber angle, denoted by *Fz*, κ, α, and φ , respectively, to the longitudinal force, side force, and aligning moment. These equations use a clever composition of trigonometric functions to provide a family of parameterized functions for fitting empirical tire data. The original formula developed for car tires has become standard in that context. The extension to motorcycle tires necessitates substantial changes to accommodate the different roles of sideslip and camber forces in the two cases (R.S. Sharp, 2004), (Lot, 2004) Tire forces and moments are produced through a combination of geometry and slip. The camber angle is the angle between the wheel plane and the line perpendicular to the road surface. Due to the shape of a motorcycle tire, nonzero camber results in a lateral force called camber thrust. Additional tire forces and moments are produced by slip between the tire and road surface. Roughly speaking, slip occurs when the velocity vector of the contact point between tire and road is different from the velocity vector determined by the speed and heading of the wheel. The slip angle α is the angle between the wheel's actual direction of travel and the direction toward which it is pointing, while the slip ratio κ provides a no dimensional description of the relative motion between the tire and the road surface (Pacejka, 2002). The slip ratio is nonzero when the tire's rotational speed is greater or less than the free-rolling speed. Magic formula parameter values for a given tire are used to calculate the steady-state force and moment system for realistic operating conditions. Additional features for modelling dynamic effects are developed (Pacejka, 2002) but are not discussed here. In the magic formula scheme, the cases of pure longitudinal slip and pure lateral slip are treated separately and then combined using loss functions that characterize the reduction of forces in combined slip. In pure longitudinal slip, the slip angle α and camber angle φ are set to zero so that the tire does not generate any lateral force. The longitudinal force *Fx*0 in pure slip is then a function of the slip ratio κ and the normal load *Fz*. This function is given by

$$Fx_0(\kappa, Fz) = D_x \sin[C_x \arctan(B_x\kappa - E_x * (B_x\kappa - \arctan(B_x\kappa))) \tag{2}$$

Even if magic formulas are well known from long time ago (the first Pacjeka model was made in the late 80's), it is very difficult to derive the coefficients correctly since they are empirical and require a lot of time and tuning. Dataset about the tire used in this model contains all the coefficients regarding lateral dynamics so, in this presentation the magic formula will be used only for that one. Longitudinal forces will be considered at constant slip with a fixed coefficient that will roughly represent the grip condition of the track.

From the measurements made in laboratory

$$F_{x_0} = 1.3 * Fz \tag{3}$$

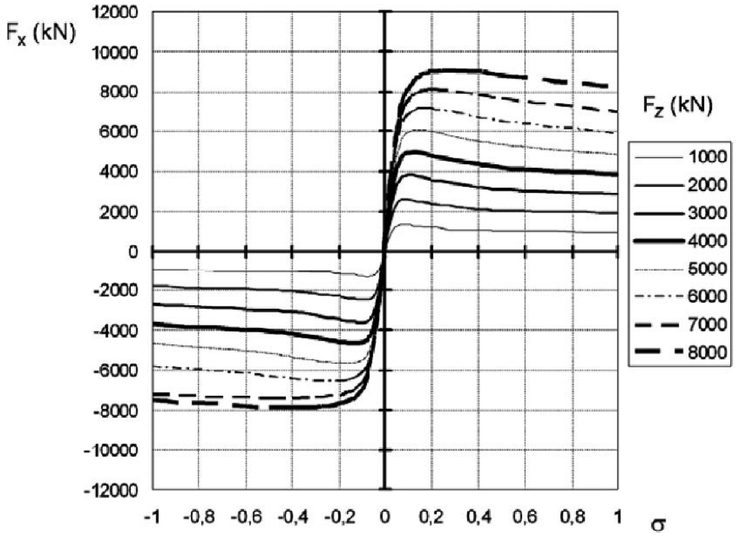

Figure 6, longitudinal force, varying slip

A typical plot under constant load is depicted in figure above . Note that the longitudinal force linearly increases with increasing slip ratio κ up to a maximum longitudinal force followed by a significant drop. When the tire is forced to work beyond the peak, the rider experiences a sudden loss of grip as the slip dynamics transition from a stable region with positive slope to an unstable region. Physically, the tire spins up rapidly under power as the shear force decreases under increasing slip ratio while the engine torque remains nearly constant. In pure lateral slip, the slip ratio κ is set to zero so that the tire does not generate any longitudinal force. The lateral force in pure lateral slip, which is a function of the slip angle α, the camber angle φ, and the normal load *Fz*, has the form

$$\begin{aligned} Fy_0(\alpha, \phi, Fz) = D_y \sin[&C_y \arctan(B_y\alpha - E_y * (B_y\alpha - \arctan(B_y\alpha))) \\ &+ C_\phi \arctan\,(B_\phi\phi - E_\phi * B_\phi\phi - \arctan(B_\phi\phi))) \end{aligned} \tag{4}$$

In the real world, the tire works quite always in mixed condition with longitudinal and lateral slip angles. To simulate this condition the tire ellipse will be used so that the forces that can be generated are directly connected and limited each other. Pay attention that every Fz defines an ellipse, the one plotted here is the maximum one that contains all the others.

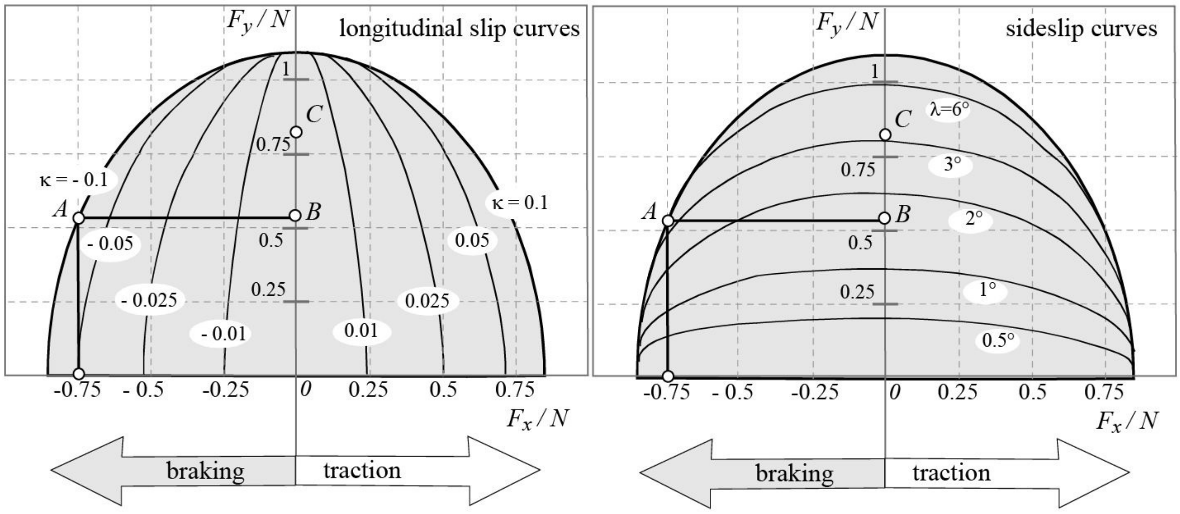

Figure 7, traction ellipse

$$\left(\frac{Fy}{Fy_0}\right)^2 + \left(\frac{Fx}{Fx_0}\right)^2 = 1 \tag{5}$$

In order to make easier the use of these magic formulas, two 3D lookup table have been created, one for the front and one for the rear tire. So knowing the vertical force acting on the tire, the roll angle and the sideslip one the maximum lateral force can be obtained.

Linearizing the motorcycle behavior while cornering and making some assumption, an estimation of the sideslip angle can be done.

- Corner radius >> bike wheelbase
- Steering angle < ± 3°
- Roll Angle < 45°

Starting from the lateral force equation, sideslip angles can be written as

$$\lambda = \frac{1}{k_\lambda} * \left(\frac{F_y}{F_z} - k_\phi * \phi\right) \tag{6}$$

Which is approximatively

$$\lambda \simeq \frac{1 - k_\phi}{k_\lambda}\phi \tag{7}$$

Since developed bike models are not taking in consideration the steering angle, the sideslip have been considered the same for the front and the rear tires. Note that racing bikes can often reach roll angle higher than 45 degrees and so the definition of sideslip does not follow the linearization proposed. For that reason, after a fine tuning of the model, the maximum lateral forces of the tire have been increased from 0 to 10% more with a parabolic behavior from 45 to 60 degrees of roll. So the tire will follow always the pacjeka model but for roll angle higher than 45 degrees, the maximum lateral force will be multiplied by a factor, that goes from 1 to 1.1, following g a parabola curve depending on the roll angle.

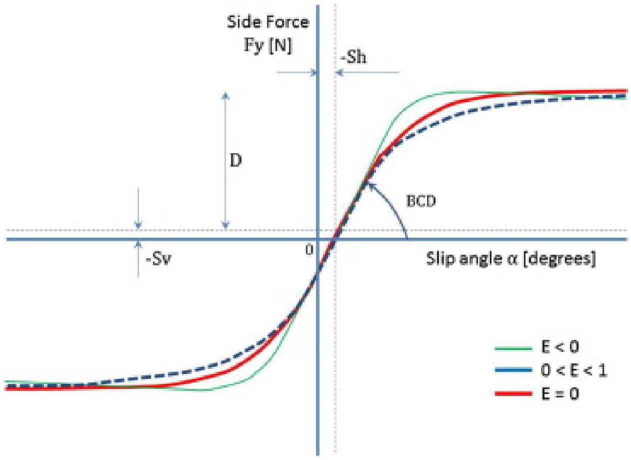

Figure 8, Typical Pacjeka force behavior with respect to sideslip variation

Below are reported the two ellipse of the rigid model, one per each tire. Note that the ellipse plotted is the one relative to maximum normal force developed during the simulation and so to its relative longitudinal and lateral forces.

As you can see in the figures below, there are some points outside the ellipse. That means the bike is actually in an adherence zone the ellipse cannot model, where a nonlinear model should be used (like Pacjeka). Every point on the maps has coordinate (Fx, Fy) given by the equations output. Points outside the ellipse represent an instant where no traction can be exerted since the bike is on the limit, in the meantime, it slows down, the roll angle or the normal load changes and the working points come back into the ellipse. Note that grip limits are calculated only for one tire per time, the rear one in acceleration and the front one in braking.

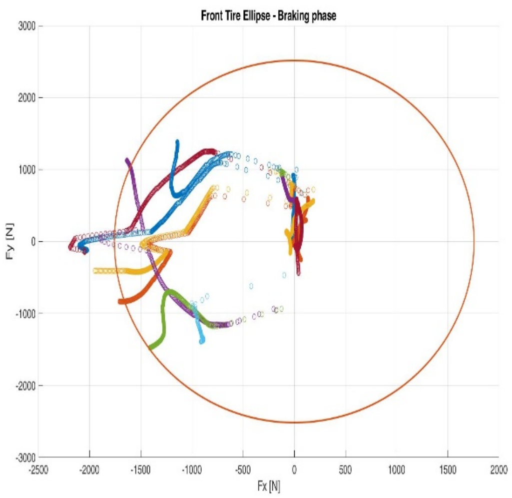

Figure 9, Front Tire Ellipse

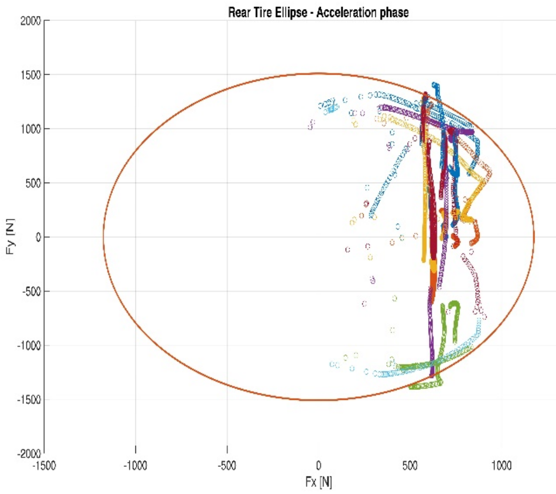

Figure 10, Rear Tire Ellipse

4.1.2 Minimum speed points

Once the track length is determined, the next point is to define the speed at the center of the corner since it will be the start of the acceleration phase and the end of the braking one. The method used is similar to the one above.

Suppose that the front and the rear tires are always at the static load condition so:

$$Fz_f = mg * \frac{b}{wb} \tag{8}$$

$$Fz_r = mg * \frac{a}{wb} \tag{9}$$

The system is mainly subjected to two forces, a longitudinal one that depends on the aero drag and on the engine power and a lateral one that is strictly connected to the centripetal force. In that specific case the only power requested to the engine is the one to maintain constant speed so, there is no acceleration.

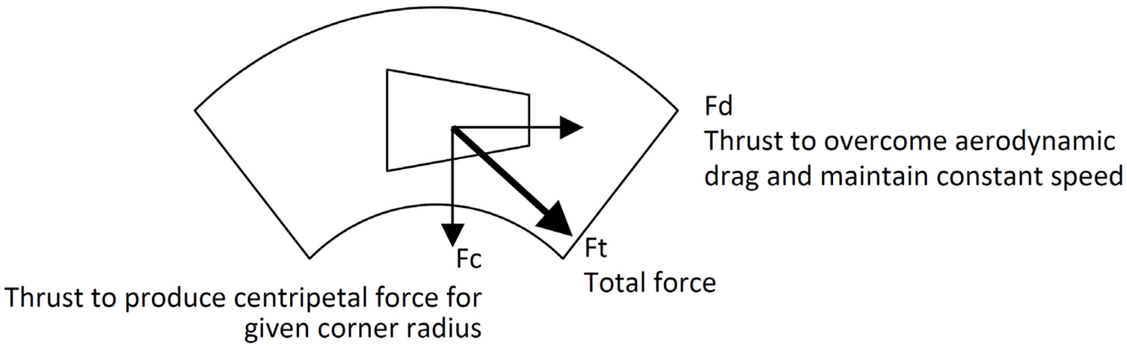

Figure 11, point mass forces

Tires must transmit at least these forces in order to keep the bike in the stable zone.

$$F_t = \sqrt{F_c^2 + F_d^2} = \mu * Fz_{tot} \tag{10}$$

Since there is no acceleration, F_d *is equal to the aerodynamic drag.*

$$F_d = \frac{1}{2}\rho S C_x V^2 \tag{11}$$

Centripetal force is defined as

$$F_c = \frac{mV^2}{R} \tag{12}$$

Where m is the mass of the vehicle, V its speed and R the corner radius. Since the trajectory is an input, R is known. We are interested at the speed at the center of the corner and that indicates a precise moment when the rider should switch from braking to acceleration phase. Suppose that exists an infinite small instant of neutral phase that corresponds to the center of the corner. That means in this moment longitudinal force is approximatively equal to zero. So, F_c is equal to the Pacjeka one, at that roll angle and static vertical load. V is so obtainable by

$$V_{nc} = \sqrt[4]{\frac{Fy_0^2}{(m\sigma)^2 + \left(\frac{1}{2}\rho S C_x\right)^2}} \tag{13}$$

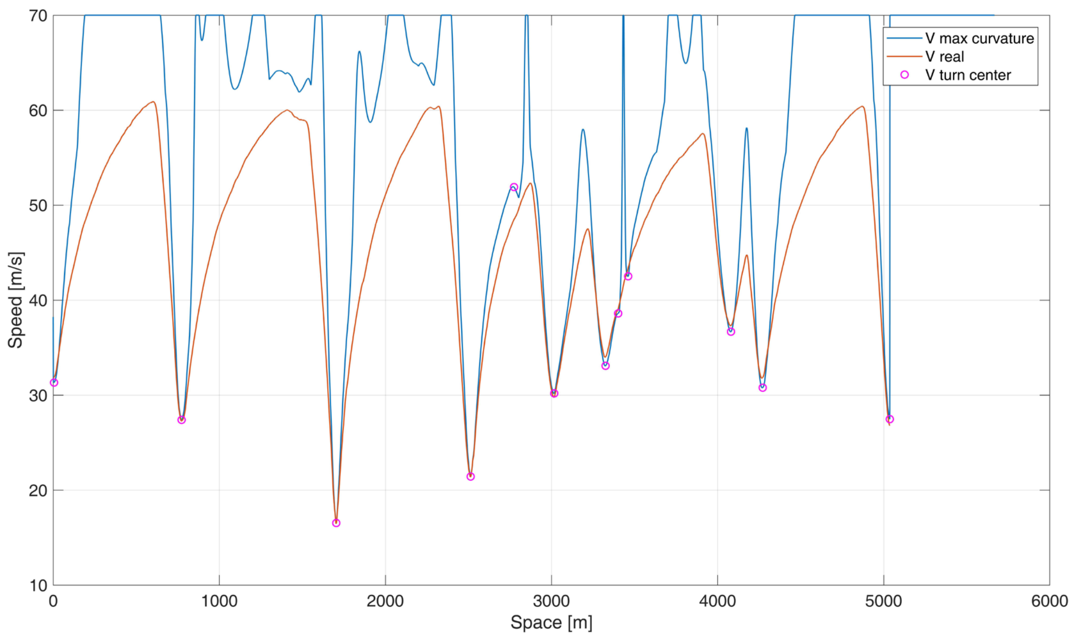

Figure 12, curvature dependent speed

Minimum points of that speed profile are the ones of interest. A specific function to find minimums has been created. It checks that local minimums have corresponding curvature above a certain threshold; finally yet importantly, it checks corresponding speed of that minimum is below a defined one. User can easily set speed and curvature threshold before launching the simulation. The function has two output, one is the speed of the minimum, and the other is the distance at which it has been found, relative to the first corner center. At this point, the result is a table containing the speed and the distances of corner's centers. Now, every part of the track between two minimums is going to be analyzed.

4.1.3 Engine model

In order to simulate the engine output here it is used a lookup table that, given an rpm as input return the maximum available torque as output.

To improve time simulation also a polynomial curve has been calculated, so that the program do not interpolate the lookup table at every step but just use the polynomial form to have a correct estimation. In this case an 8-th grade polynomial has been used and so the available torque will be

$$T = c_{f9} + c_{f8}\omega_{eng} + c_{f7}\omega_{eng}^2 + c_{f6}\omega_{eng}^3 + c_{f5}\omega_{eng}^4 + c_{f4}\omega_{eng}^5 + c_{f3}\omega_{eng}^6 + c_{f2}\omega_{eng}^7 + c_{f1}\omega_{eng}^8 \tag{14}$$

$$acc = \frac{1}{m_{eq}}\tau_{gear}\tau_p\tau_s * \frac{T}{R_{rw}} - \frac{1}{2}\rho C_x S v^2 \tag{15}$$

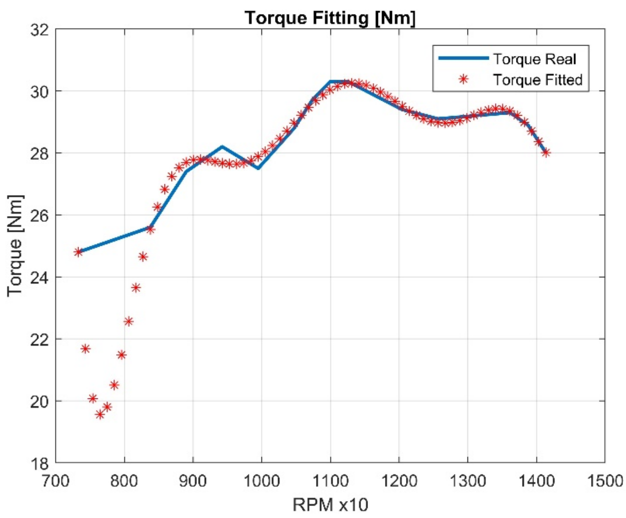

Figure 13, engine torque curve

Engine torque is then transmitted to the ground via the primary shaft, gearbox and the crown-pinion ratios. On every step, included the first one, engaged gear is calculated from the max speed achievable for any gear. Every gear defines a specific speed range where it can be used, limited by the engine rev limiter, which for the engine used in this case is set to 13500 RPM. Once the gear is determined a security check is performed in order to be sure that the engine rev speed is lower than the limit, if the revving speed is higher than the limit it means that the gear engaged must be the higher one.

$$\omega_{eng} = u * \frac{\tau_p \tau_{cp} \tau_{gear}}{R_{wr}} \tag{16}$$

Figure below shows the maximum speed achievable for each gear at every engine speed

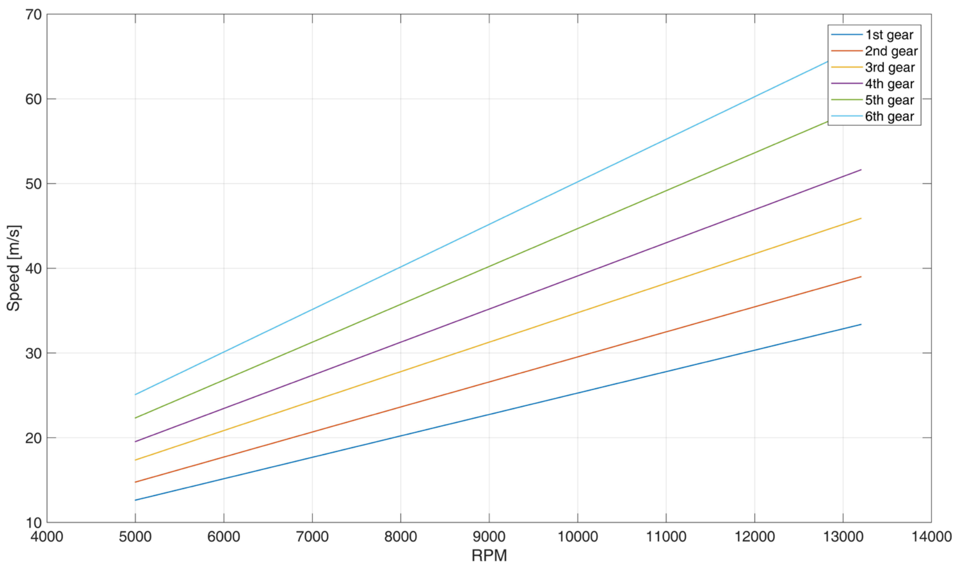

Figure 14, max speed per each gear

Gearbox is actuated every time the speed overcomes the maximum achievable for that gear so that engine speed is always lower than the rev limiter. The model can also be useful to evaluate different rev point to change gear in order to optimize lap time and engine performance.

4.1.4 Equivalent mass

When analyzing vehicle dynamics, especially during acceleration phase, inertia of rotating parts can not be ignored. They add an inertia component to the whole system that is usually calculated as an increment of mass. Taking in consideration rotating parts as wheels, engine crankshaft, gearbox shafts and the engaged gear. The term m_{eq} is the equivalent or apparent mass of the vehicle, i.e., the mass of an object that, when moving at the same speed as the vehicle, has the same total kinetic energy.

$$m_{eq}(\tau_{gear}) = m + \left(\frac{I_{wr}}{R_{wr}}\right)^2 + cI\left(\frac{\tau_s\tau_p\tau_{gear}}{R_{rw}}\right)^2 + ccI\left(\frac{\tau_{gear}\tau_s}{R_{rw}}\right)^2 + csI\left(\frac{\tau_s}{R_{rw}}\right)^2 \qquad (17)$$

Since it depends on the engaged gear, the results will be six equivalent masses, as you can see from the formula the higher the τ of the higher is the mass. Higher τ corresponds to lower gear and to higher revving speed where even small inertias play an important role

gear	1st	2nd	3rd	4th	5th
Equivalent mass	171.32 Kg	167.57 Kg	165.22 Kg	163.70 Kg	162.86 Kg

Table 1, equivalent masses

4.1.5 Models working principles

As said before, this model is not going to give the optimal trajectory on the track but, once it is has been fixed is going to optimize the possible speed.

Models provide different solution to calculate two speed profiles, one for acceleration and one for deceleration. For every part between two speed minimum found in (13) a distance has been defined. Therefore, the acceleration and braking phases are performed until the distance traveled is equal to the distance between two minimums.

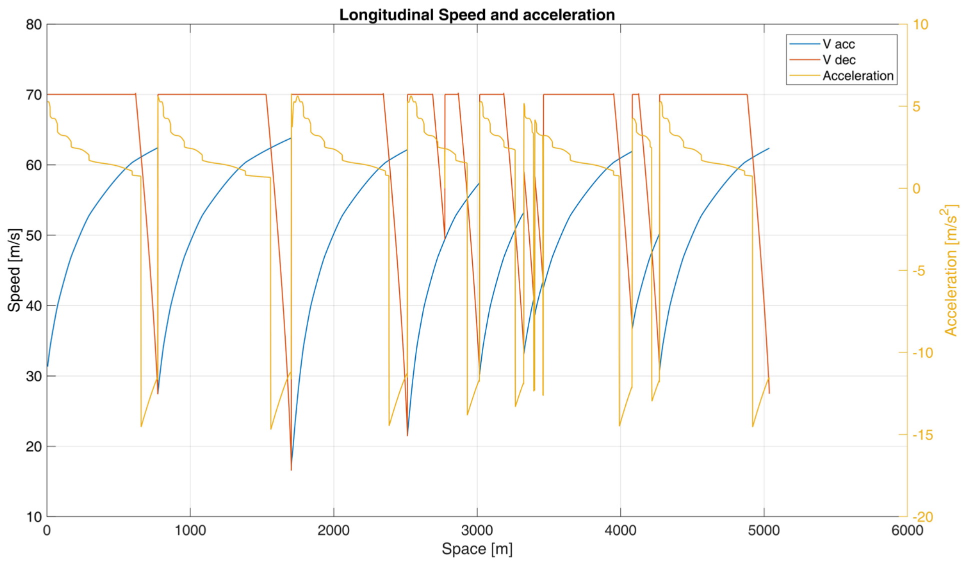

Figure 15, example of speed profile intersection, together with their relative acceleration

Sometimes can happen that one minimum actually is above the maximum real velocity the bike can reach in that point. For example, the bike is accelerating at the maximum of its possibility, limited only by the engine, and the speed it is carrying is lower than the maximum allowed by the trajectory it means that for the bike taken in consideration the minimum is not a real limit. Probably it could became a limit for a more powerful bike. It is a situation that clearly happens in every circuit where different class lap during the same weekend. It is very easy to find a turn that small classes make in full throttle while superbikes or motogp bikes must reduce throttle or even brake.

Once speed profiles are obtained, the minimum between the two is performed and the intersection of the two lines will be the braking point. The key point of that analysis is to compare the two profiles in space base in order to be sure the comparison is made on the same part of the track.

This is the way speed profile is calculated, various models proposed just add limitations and constraints to the acceleration and deceleration phases but the way the lap time is calculated remains the same.

Once determined the maximum acceleration possible given all the constraints, speed is calculated using uniform accelerated motion formulas

$$v = v_i + a(t - t_i) \tag{18}$$

To make easier the comparison in space base the simulation has been divided in step of one meter and then by inverting (18) the dt of the step is calculated, note that the delta time obtained is then used in the next step to determine the speed achievable.

Physical phenomena happening during braking has been described in (4.1.2), models proposed are not analyzing what's going on in braking pads, caliper, oil pressure and on the disk. Here it is just proposed a way to calculate the maximum deceleration possible with increasing grade of complexity. This model make as an assumption a simplification: riders brake only with the front brake so the grip condition in braking is analyzed only for the front wheel, supposing the rear one is just rolling on the ground. Actually, this description of the phenomena is very similar to what really happens in the reality since the rear brake is used only for stabilizing the bike in the beginning of the braking phase or during the turn exit to better close the trajectory.

Suppose to analyze the braking phase of the first turn, the model starts from the speed at the center of the corner and simulate an acceleration with magnitude equal to the optimal one that can be limited by grip and capsizing. To better understand how the model works, just imagine the bike going backwards from the center of the turn one to the center of the last turn. It is accelerating at its maximum but instead of being limited by the engine power it is constrained by capsizing and grip. Once the space traveled is equal to the distance between the first and the last corner all the data obtained are flipped and now are ready to be compared with the acceleration ones.

It must be noted that this process is very tricky and on point mass model it's very fast and easy to run but on three dof rigid model and eight dof model, yaw and roll speed and acceleration must be corrected in the right way or data will be useless and more importantly, unreal.

4.1.6 Ground slope

Data recorded from the GPS provide, together with latitude and longitude provides the altitude value. Knowing the space between one point and another, the slope of that part can be calculated. In this way, we got the track slope for every meter and it will be used to calculate its influence on the longitudinal and vertical equilibrium.

$$slope = \text{atan}\left(\frac{\Delta z}{\Delta x}\right) \tag{19}$$

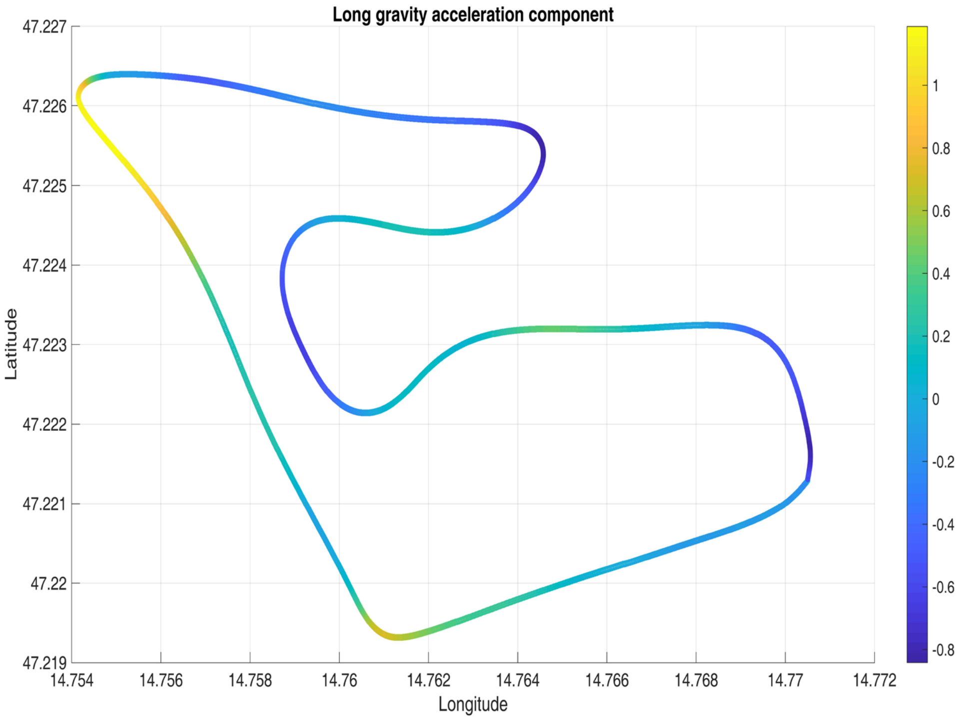

Figure 16, gravity acceleration's longitudinal component map

4.2 Point mass model

Starting from the first corner center the script has auto determined, the model firstly calculate all the acceleration phases speed. Start from the minimum at the center of the turn and traveling the exact space to the next turn. Once the space traveled is equal to the distance between two corners, it switch to the next track zone. The result is the one shown below

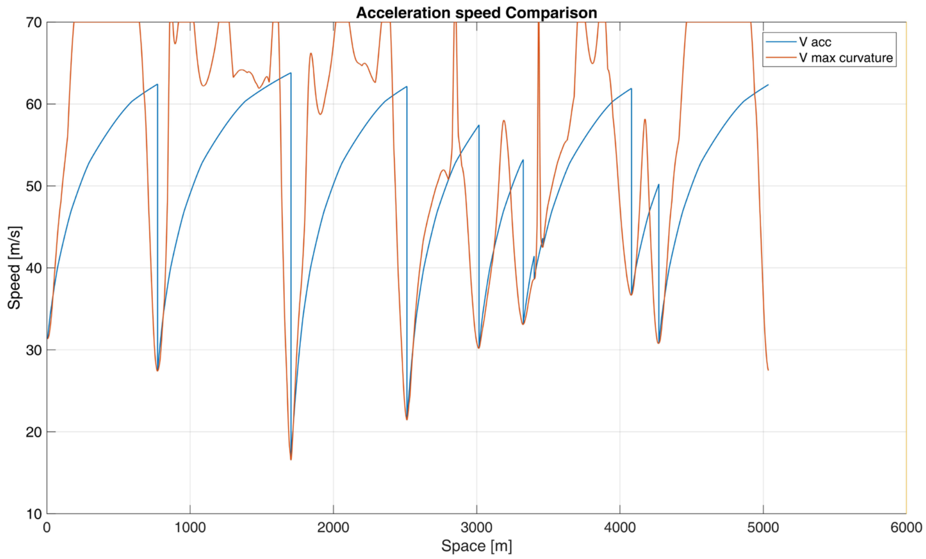

Figure 17, comparison between maximum speeds calculated

Figure above shows the comparison between the maximum speed achievable trough engine acceleration at WOT (Wide Open Throttle) and the maximum speed allowed by the trajectory if the load on the wheels is the static one. The blue line is relative to engine acceleration, as you can see at this phase of the calculations, there is no braking phase modelled.

In this case the model accelerate, limited only by the engine available torque. There is no constraint about grip or wheelie, the engine is giving its maximum torque so it is supposed to be used always at full throttle. Once it reaches RPM limit it changes gear and switch to the next one. This type of model, even if it's very simple permits to evaluate the benefit of different gear ratios or analyse the gear changing point.

The same process is made for the braking phase so the script starts from the every corner centre speed and go backwards, accelerating at the maximum acceleration permitted by the static capsizing shown below (20). Once the space is travelled is equal to the distance between the starting corner centre and the one before the model stops and switch to next part of the track. During the braking phase the front area of the bike is considered to be 30% higher since the rider stand up to increase the aero resistance and move the COG position backwards to balance the weight transfer.

$$acc = \frac{1}{m_{eq}}\left(\tau_{gear}\tau_p\tau_s * \frac{T}{R_{rw}} - \frac{1}{2}\rho C_x S v^2\right) - g * \sin(slope) \tag{20}$$

$$dec = g * \frac{wb - b}{h} + \frac{1.3}{m}\frac{1}{2}\rho S C_x V^2 + g * \sin(slope) \tag{21}$$

Also in this phase the motion is considered to be in uniform acceleration in every step so equation (18). In both acceleration and deceleration phases, as shown in (20) (21) the influence of the ground slope is considered

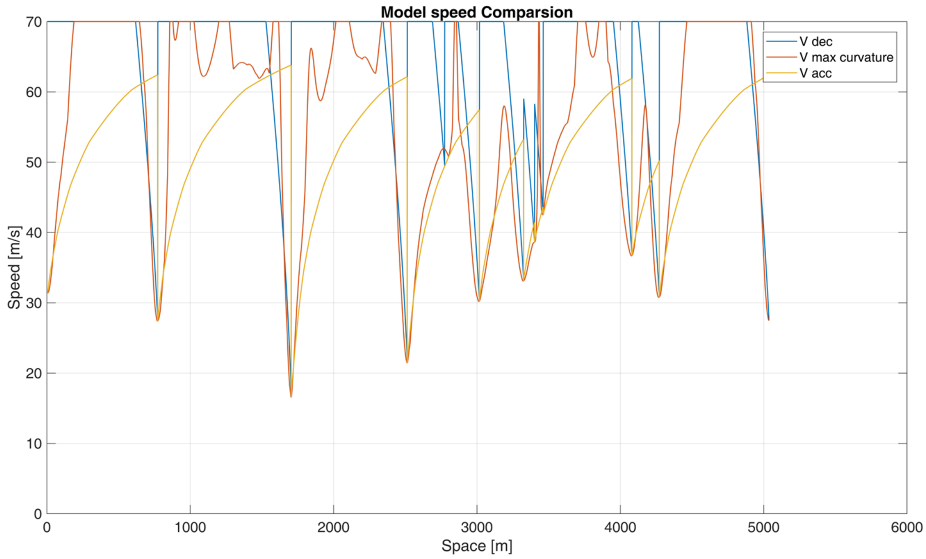

Figure 18, acceleration, deceleration speed profiles and max trajectory speed

Once the speed profile has been determined, the lap time calculation is performed. Knowing the track length that is calculated in (5.1) all the data are then truncated at the point where the distance travelled is higher or equal to the track length. Taking the last value of the truncated time vector the lap time is obtained.

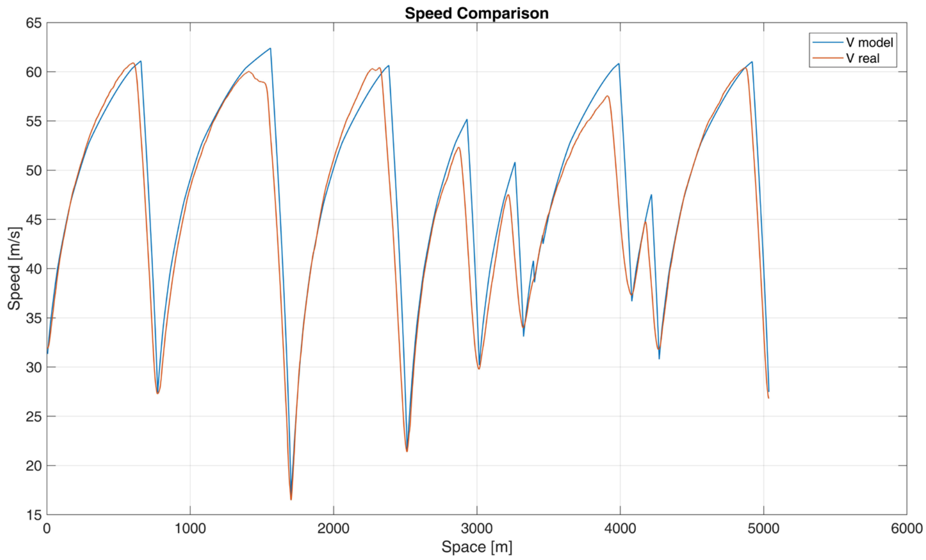

Figure 19, Speed comparison between reality and point mass model

4.3 3 DOF Rigid model

The dynamic model proposed directly derives from the one proposed by Cossalter in (V.Cossalter, 1999), it focuses on the gross motion only and do not include steering angle or steering torque but these simplification are not deeply affecting the simulation and allows to solve the boundary value problem keeping it to a reasonable number. A quasi-steady state approach is used on the YZ plane since we are not going to consider a lateral acceleration and not even a lateral velocity in the vehicle body reference system but considered sideslip angles are not zero. They are determined as shown before in (5.1.1) to calculate the lateral grip limit but they are supposed to be zero at the contact point so that forces are always parallel to X and Y axes. From an inertial reference system, that is the one used to analyse the trajectory the model is following the system will have a velocity along x and y direction directly depending on the yaw angle.

$$\dot{x} = v_x \cos\psi \tag{22}$$

$$\dot{y} = v_x \sin\psi \tag{23}$$

(Saccon, 2006) propose a quasi-steady state approach that is imposing roll acceleration equal to zero so that roll rate instantly goes to a fixed value for every step analysed and a simplified version of the model fixing also the yaw acceleration to zero as the roll one. Therefore, the model becomes a four DOF where the only unknowns are longitudinal and lateral acceleration together yaw rate and roll rate.

The model is composed of six equations:

- Three force equilibrium equations;
- Three moment equilibrium equations: equilibrium around the *X* -axis (roll), the *Y* -axis (pitch) and the *Z*-axis(yaw).
- Cossalter model is based on 8 state variable that here have been reduced to 6, this is a comparison table

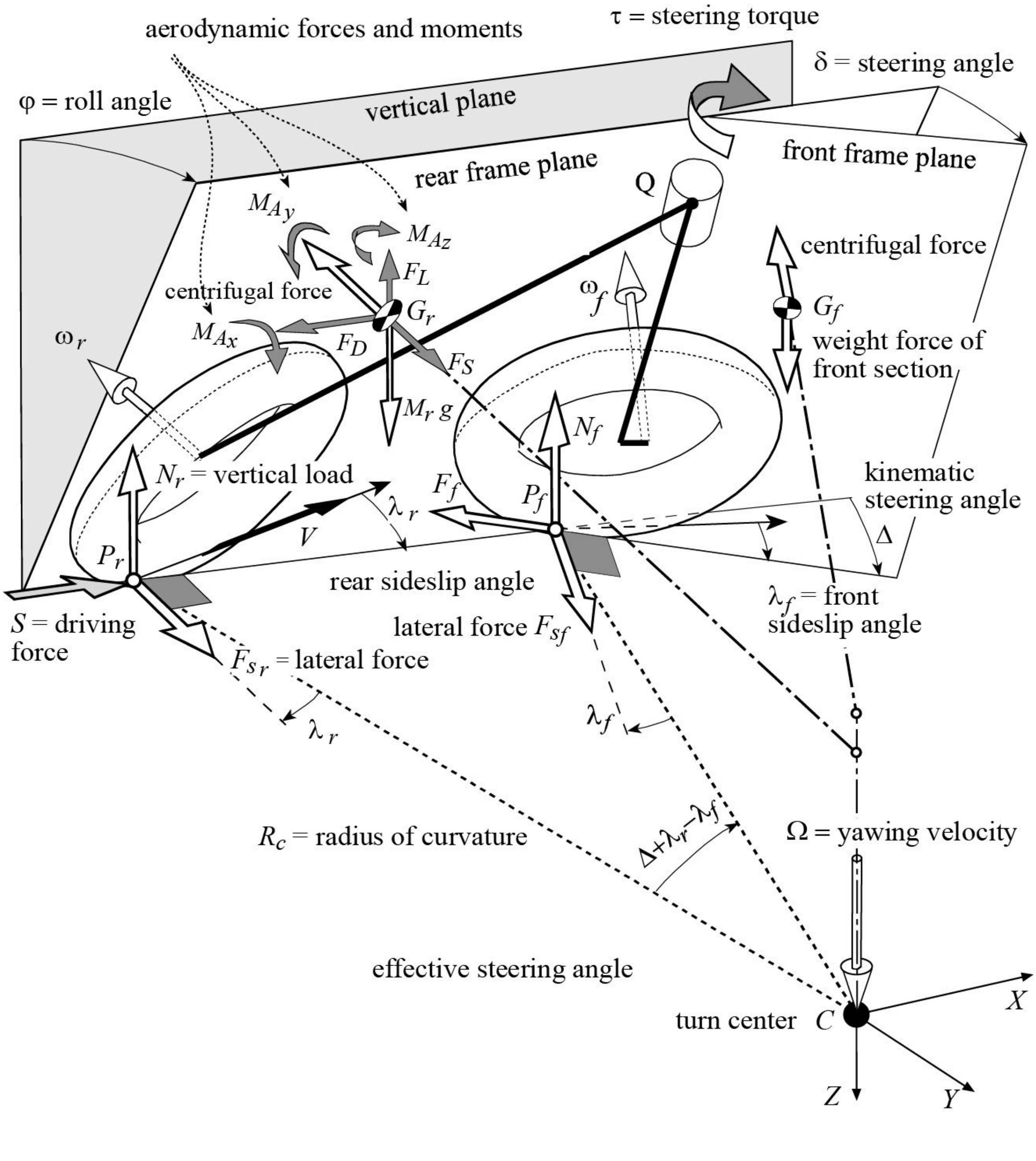

Figure 20, Steady turning bike forces

Variable	Cossalter model	Proposed model
Longitudinal velocity	Unknown	Child of maximum acceleration
Lateral velocity	Unknown	Always zero
Roll motion	Unknown	Input variable
Yaw motion	Unknown	Unknown
Pitch motion	Neglected	Neglected
Front normal load	Unknown	Unknown
Rear normal load	Unknown	Unknown
Front longitudinal force	Child of brake strategy	Unknown
Front lateral force	Child of steer torque	Unknown
Rear longitudinal force	Child of the acceleration strategy	Unknown
Rear lateral force	Unknown	Unknown

Table 2, equations variables

As shown in the table there are seven unknown in the proposed model but, actually, longitudinal force is calculated only on the tire that we are interested in, depending on the phase we are analysing. Therefore, during acceleration it will be the rear longitudinal force while during brake it will be the front one. Since steering actions are not taking in consideration, front and rear frame of the bike always remain on the same plane and so there is no need of creation of two separate reference system as (Cossalter, 2006). But as proposed in (Cossalter, 2006), the system is supposed to be symmetric with respect to the X-Z plane hence its inertial characteristics are represented by the following four terms:

- I_{xg} mass center moment of inertia about X axis (roll moment of inertia);
- I_{yg} mass center moment of inertia about Y axis (pitch moment of inertia);
- I_{zg} mass center moment of inertia about Z axis (yaw moment of inertia);
- I_{xzg} mass center inertia product about X - Z -axes.

Small displacement of the wheels' contact points are neglected. The gyroscopic effect of the wheels and flywheel is taken in consideration, except for the contribution due to the steering rate, which is definitely small compared to the other angular rates. It is considered that the lateral and vertical displacements of driver torso do not occur, like as the pilot is thought of as rigidly connected to the motorcycle. The road is considered even but with longitudinal slope, as in the pure longitudinal model since we are essentially committed to the evaluation of the ability of the vehicle in exploit the best grip condition and transform it to acceleration. For the same reason, the suspensions are not modelled, which again means that the effect of pitch and heave due to suspension stroke is not evaluated for now. The whole system is subject to the forces depicted in Figure 16, i.e., gravity, aerodynamic drag, reactions normal to the road plane, lateral forces on tyres, and thrust. The tyre forces are applied to the point where the wheels meet the road plane, taking into account the radii of the tyre variation depending on the cross section and so on the roll, except for the fact that pitch and front frame rotation are neglected.

$$m\{\dot{u} - h\sin(\phi)\ddot{\psi} - b\dot{\psi}^2 - 2h\dot{\psi}\phi\cos(\phi)\} = -F_{ax} + S - mg\sin(slope) \tag{24}$$

$$m\{\dot{\psi}u + h\cos(\phi)\ddot{\phi} + b\ddot{\psi} - h\dot{\phi}^2\sin(\phi) - h\dot{\psi}^2\sin(\phi)\} = F_f + F_r \tag{25}$$

$$m\{(h-rt)\dot{\phi}^2\cos(\phi) + (h-rt)\sin(\phi)\ddot{\phi}\} = -N_f - N_r + mg\cos(slope) \tag{26}$$

$$\begin{aligned} I_x\ddot{\phi} - I_{xz}\cos(\phi)\ddot{\psi} + (I_z - I_y)\dot{\psi}^2\cos(\phi)\sin(\phi) + I_e\dot{\psi}u\cos(\phi) \\ = (rt_f - h)F_f\cos(\phi) + (rt_r - h)F_r\cos(\phi) - (rt_f - h)N_f\sin(\phi) \\ - (rt_r - h)N_r\sin(\phi) - rt_fF_f - rt_rF_r \end{aligned} \tag{27}$$

$$\begin{aligned} I_y\sin(\phi)\ddot{\psi} - I_e\dot{u} + I_{xz}\dot{\psi}^2 - I_{xz}\psi^2\cos(\phi) + (I_x + I_y + I_z)\dot{\psi}\dot{\phi}\cos(\phi) \\ = aF_f\sin(\phi) - bF_r\sin(\phi) + aN_f\cos(\phi) - bN_r\cos(\phi) \\ + (rt\cos(\phi) - rt + h)S \end{aligned} \tag{28}$$

$$\begin{aligned} I_z\cos(phi)\ddot{\psi} - I_{xz}\ddot{\phi} - I_e\dot{\phi}u + (-I_x + I_y - I_z)\dot{\psi}\dot{\phi}\sin(\phi) + I_{xz}\dot{\psi}^2\cos(\phi)\sin(\phi) \\ = aF_f\cos(\phi) - bF_r\cos(\phi) - aN_f\sin(\phi) + bN_r\sin(\phi) \\ - rtS\sin(\phi) \end{aligned} \tag{29}$$

4.3.1 Algorithm procedure

As for the point mass model the track is divided in different part and trims depend on the corner center. The first step is considered to be in steady state and no acceleration is performed, therefore the static equilibrium is found. And so the six unknowns (two normal loads, two lateral forces, one longitudinal and the yaw acceleration)

Starting from the vertical loads, magic formulas and the grip ellipse a grip-limited acceleration is found so from (5)

$$Fx_r = Fx_{0r}\sqrt{1 - \left(\frac{Fy_r}{Fy_{0r}}\right)^2} \tag{30}$$

Therefore, the grip coefficient is determined and so the maximum torque permitted

$$\mu = \frac{\sqrt{Fx_r^2 + Fy_r^2}}{Fz_r} \tag{31}$$

$$I_{eng} = \dot{\omega}_{eng} * (cI + \frac{ccI}{\tau_p^2} + \frac{csI}{\tau_p^2\tau_{gear}^2}) \tag{32}$$

$$Tlim_{grip} = \frac{\mu Fz_r * R_{wr} + acc_{wr}I_{wr}}{\tau_{gear}\tau_p\tau_s} + I_{eng} \tag{33}$$

Then the minimum between (33) and (14) is performed and this will be the torque limit in every condition of the acceleration phase. By the way, minimum acceleration depends on other parameters like wheelie condition and the fact that all the equilibrium condition must be satisfied.

$$a_{lim1} = \frac{S}{m_{eq}} - g * \sin(slope) \tag{34}$$

$$a_{\underset{wheelie}{\lim}} = g\cos(\phi) * \frac{b}{hg} - \frac{1}{2\,m_{eq}}\rho C_x SV^2 - g * \sin(slope) \tag{35}$$

$$acc_{lim3} = \frac{1}{m_{eq}}\tau_{gear}\tau_p\tau_s * \frac{T}{R_{rw}} - \frac{1}{2}\rho C_x Sv^2 - g * \sin(slope) \tag{36}$$

Obviously, the torque value appearing in (36) must be the result of the minimum between (33) and (14). The output of that calculation will be the maximum acceleration possible and so the maximum speed achievable trough (18). Acceleration and speed value obtained will be used as input in the next step of the simulation and so on.

The same approach is used in the braking phase, so Pacjeka and the grip ellipse are calculated for the front tire and kinematic limit is now related to capsizing (21) instead of the wheelie. Since the model is now depending on six equations they must be adapted to the fact that roll and yaw are now related to a bike that is going backwards, therefore these are the useful equations

$$m\{\dot{u} + h\sin(\phi)\,\ddot{\psi} + b\dot{\psi}^2 + 2h\dot{\psi}\phi\cos(\phi)\} = -F_{ax} + S - mg\sin(slope) \tag{37}$$

$$m\{-\dot{\psi}u + h\cos(\phi)\,\ddot{\phi} - b\ddot{\psi} + h\dot{\phi}^2\sin(\phi) + h\dot{\psi}^2\sin(\phi)\} = F_f + F_r \tag{38}$$

$$m\{(h - rt)\dot{\phi}^2\cos(\phi) + (h - rt)\sin(\phi)\,\ddot{\phi}\} = -N_f - N_r + mg\cos(slope) \tag{39}$$

$$\begin{aligned} I_x\ddot{\phi} + I_{xz}\cos(\phi)\,\ddot{\psi} - (I_z - I_y)\dot{\psi}^2\cos(\phi)\sin(\phi) - I_e\dot{\psi}u\cos(\phi) \\ = (rt_f - h)F_f\cos(\phi) + (rt_r - h)F_r\cos(\phi) - (rt_f - h)N_f\sin(\phi) \\ - (rt_r - h)N_r\sin(\phi) - rt_fF_f - rt_rF_r \end{aligned} \tag{40}$$

$$\begin{aligned} -I_y\sin(\phi)\,\ddot{\psi} - I_e\dot{u} - I_{xz}\dot{\psi}^2 + I_{xz}\psi^2\cos(\phi) - (I_x + I_y + I_z)\dot{\psi}\dot{\phi}\cos(\phi) \\ = aF_f\sin(\phi) - bF_r\sin(\phi) + aN_f\cos(\phi) - bN_r\cos(\phi) \\ + (rt\cos(\phi) - rt + h)S \end{aligned} \tag{41}$$

$$\begin{aligned} -I_z\cos(phi)\,\ddot{\psi} - I_{xz}\ddot{\phi} - I_e\dot{\phi}u - (-I_x + I_y - I_z)\dot{\psi}\dot{\phi}\sin(\phi) - I_{xz}\dot{\psi}^2\cos(\phi)\sin(\phi) \\ = aF_f\cos(\phi) - bF_r\cos(\phi) - aN_f\sin(\phi) + bN_r\sin(\phi) \\ - rtS\sin(\phi) \end{aligned} \tag{42}$$

Note that the only input of that model is the roll motion, not the trajectory. Actually, the curvature is now a child of the six equations and the limit velocity calculated in (13) is now a part of the matrix composed by the equations. It is like having a controller that acts on the roll and controls it perfectly replicating reality.

Gyroscopic torque

Motorcycle dynamics incorporate a variety of gyroscopic effects, which may be broken down according to the second axis of rotation b − b:

- **Yaw gyroscopic effects**: where axis b − b passes through the turn center of the path and is perpendicular to the roadway;
- **Roll gyroscopic effects**: where axis b − b is the straight line lying in the plane of the roadway which passes through the tire contact points; Neglected for the sake of simplicity
- **Steering gyroscopic effects:** where axis b − b is the steering head axis. Neglected since the bike model has no steer.

The gyroscopic torque generated by the rotation of the wheels has the effect of straightening the wheel, if yaw rate is small with respect to the speed of rotation of the wheels:

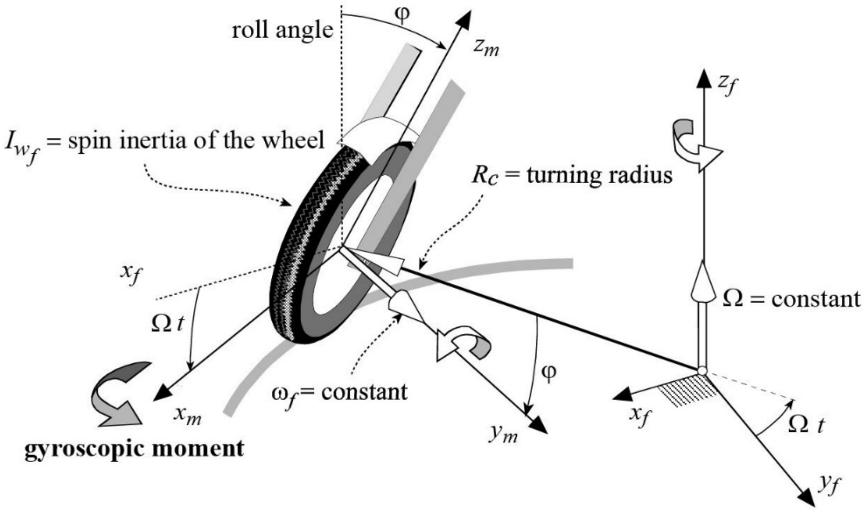

Figure 21, wheels gyroscopic moment

$$M_g \cong I_{w_f} \omega_f \Omega \cos(\phi) \tag{43}$$

This torque must be accounted when considering the equilibrium with respect to X axis.

4.3.2 Gearbox optimization

Knowing the track characteristic it is possible to perform a gearbox optimization. One of the motorcycle taken in consideration during analysis is regularly racing in the Italian speed championship and, due to technical regulamentations, it is not allowed to change individual gear ratio but only the crown pinion ratio.

In addition to that, also the gear change point is obviously free to be chosen. Note that if the track would have been completely flat and the bike has no roll, optimal gear change point can be easily defined since they are calculated once all the gear ratio and the power curve is given. Actually, reality is much different since the bike rolls and so the effective rolling radius that means that different bike speed can correspond to the optimal gear changing point. Furthermore, also circuit slope must be taken in consideration since there can be point where could be better to anticipate or delay the shifting.

The model is able to manage all this information and can give a suggestion about correct gear ratio and about correct shifting. The best thing to do is to start and use the optimal gear change point and analyze the output, comparing also with old data about speed and relative rpm. Actually, there is no optimizer of the lap time in the proposed model acting on available crown pinion ratio and on engine rpm. By the way, is possible to try different crown pinion combination and gear shifting points, observing how the lap time is varying. Usually we are talking about differences in the order of half a second between the best and the worst case.

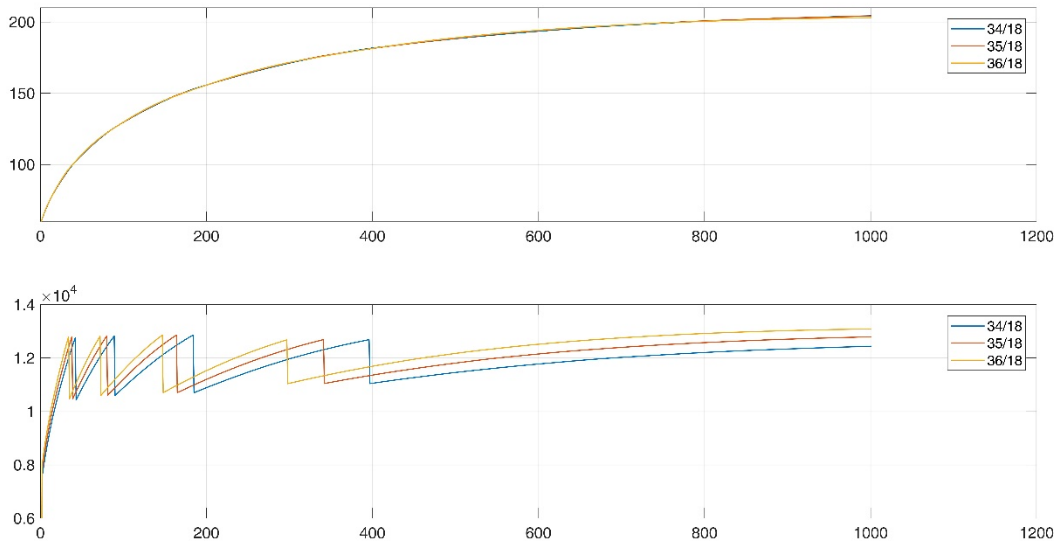

Figure 22, crown pinion ratio comparison

4.3.3 Model output

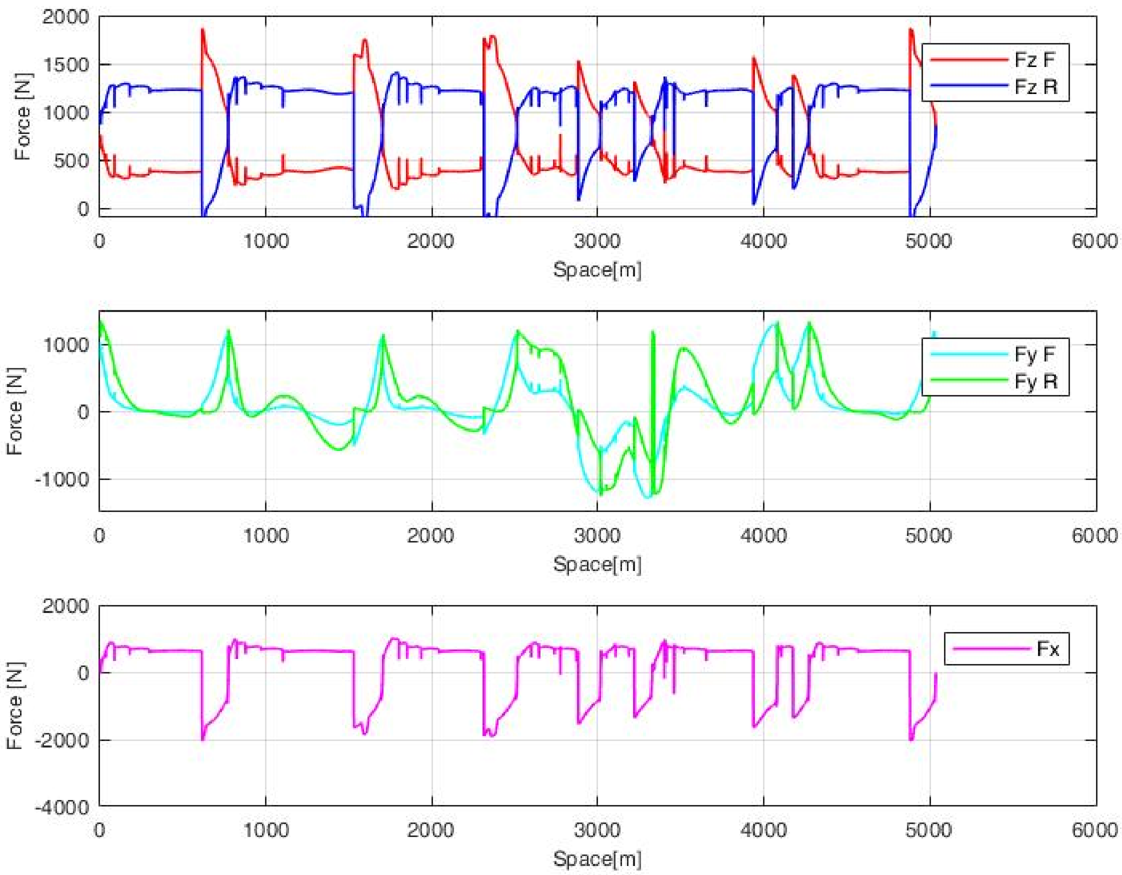

Figure 23, forces applied on the ground

Figure above reports all the forces the bike exchanges with the ground during a lap simulation. As you can see the graph is characterized by abrupt changes in longitudinal loads and that reflects, with an almost zero delay in to the vertical and lateral forces. This behavior is strictly related to the fact that switch between acceleration and braking phases happens instantaneously. It is like having a rider that is able to start braking at the maximum possible force in the same moment he is closing the throttle. Since it is a rigid model, the load transfer happens instantaneously and that is the reason why vertical loads have something like discontinuity on the braking points. The same happens during accelerations but since the difference between the two acceleration is lower in magnitude, the discontinuity is smaller. Note also that since the braking is modelled to be on the limit of capsizing there is a certain phase for every braking one that has rear vertical load equal or similar to zero; this is not a problem since during that phase only the front wheel is exerting longitudinal force. Due to the fact that vertical load is almost zero, rear wheel cannot exert any lateral force during braking phase but as soon as it gains grip you can see that part of the lateral force also goes on the real wheel. Analyzing the shape of Fx and Fz forces it is interesting to highlight the little load transfer that happens in every gear change, as you can see every time the bike is accelerating and changes gear the rear loose a part of the vertical load and it is transferred to the front. The problem of the discontinuity, finds a partial solution in the next proposed model where, after the calculation of braking and acceleration phases, a global lap is performed giving smoothed acceleration results as input, the following chapter will explain it in details.

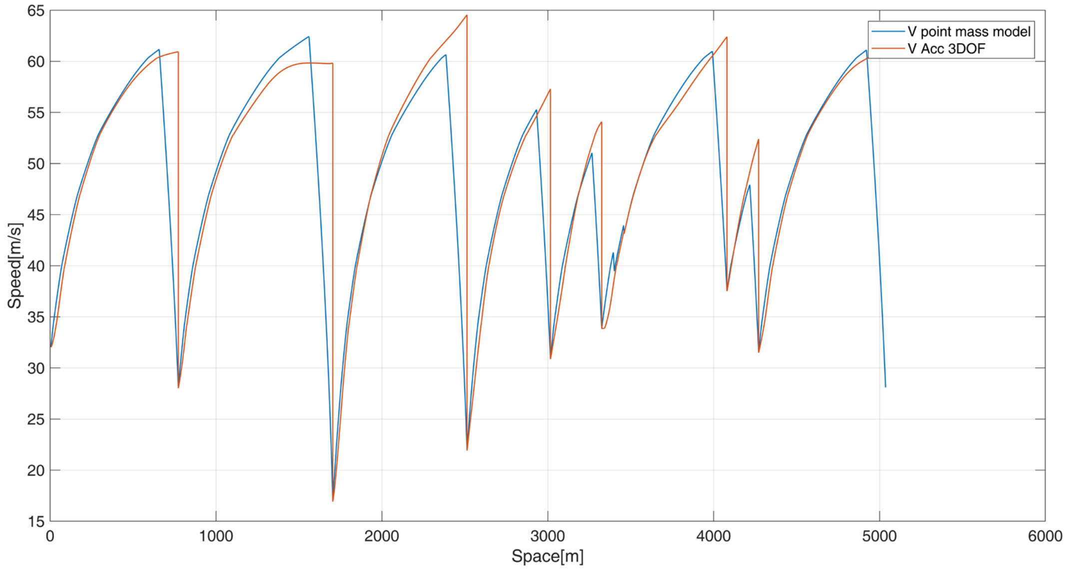

Figure 24, comparison between speed of 3dof model and point mass one

Figure above shows a comparison between the speed profile of the point mass model and the profile calculated by the 3 dof model. As you can see the model containing lateral dynamics is always slower than the point one. This result derives from the fact that there is a grip limit slowing down acceleration and deceleration phases. Figure 17 is a zoom on the first part of the track, first braking zone approaching the 90 degrees right turn, shown in figure 18

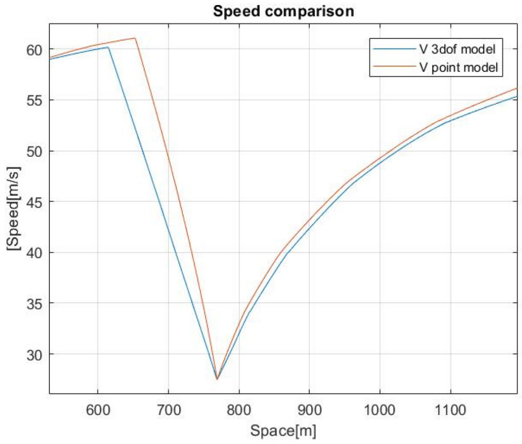

Figure 25, first corner model differences

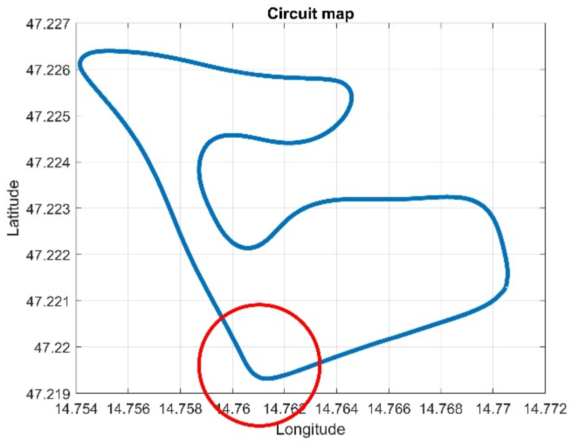

Figure 26, track map

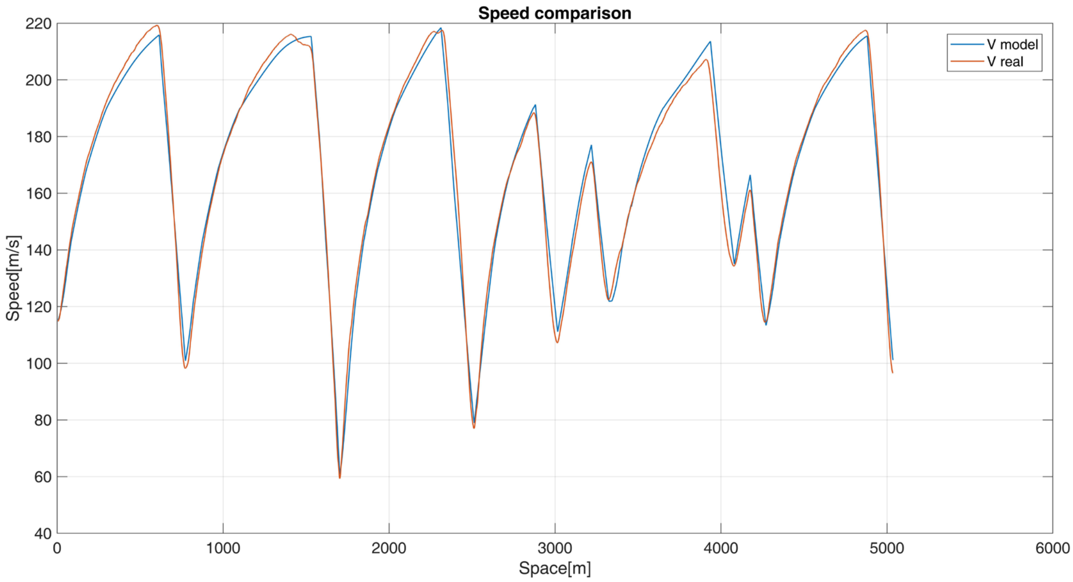

Figure 27, speed comparison 3DOF vs real profile

Figure above shows a comparison between the real speed profile and the 3DOF one, it is important to note that the model is able to correctly replicate reality.

Figure below shows the yaw acceleration in blue and the yaw rate in yellow. These parameters are of key importance since they actually represent the trajectory the bike is following

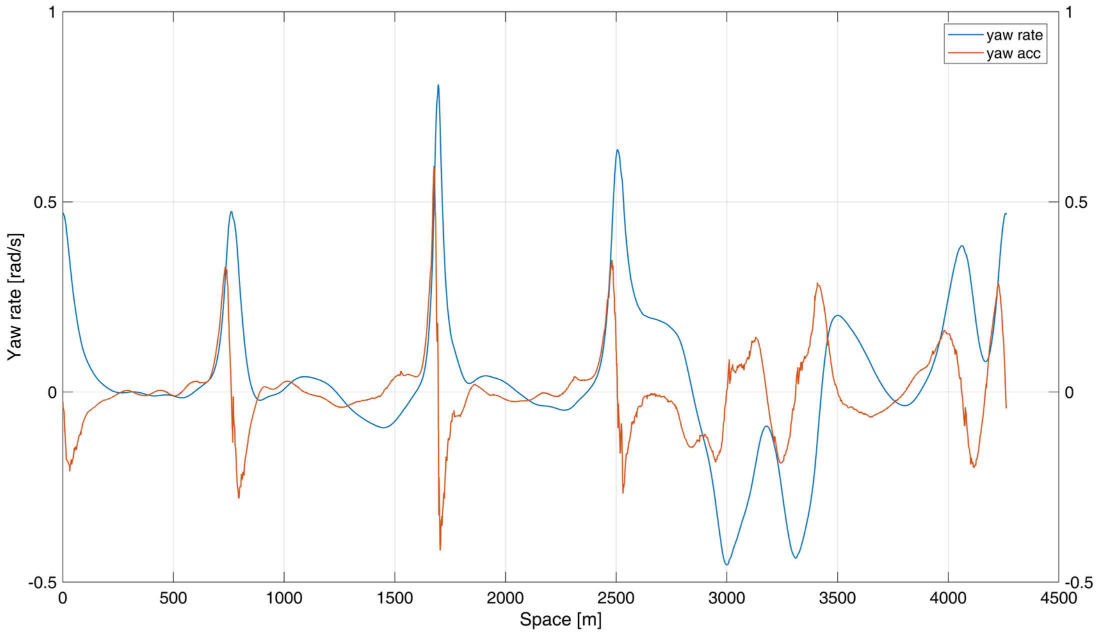

Figure 28, yaw acceleration and yaw rate

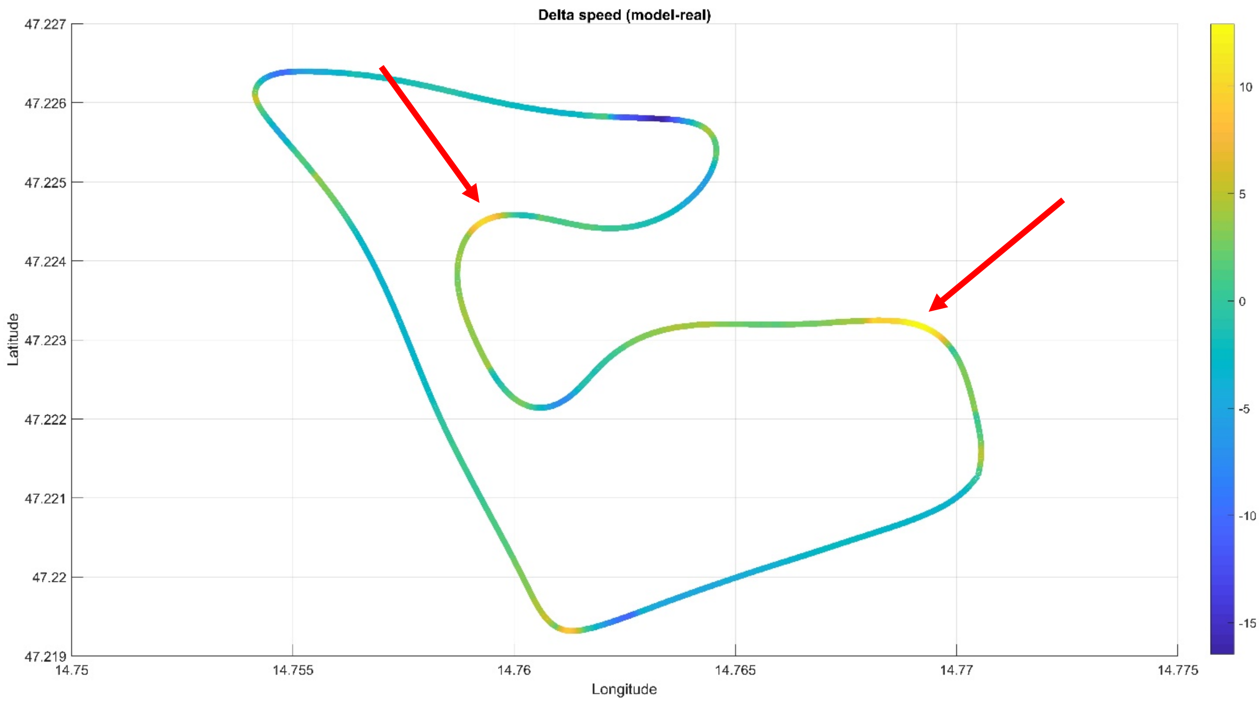

Figure 29, delta speed map

Figure above reports the delta speed in (km/h). It is interesting to highlight the final part of the track, colored in yellow. That means the model is accelerating more than the real rider is. In This part of the track, due to its shape real rider usually use less braking force while the model keeps accelerating at the grip limit.

It is important to say that this part of the circuit has a transversal grade that is not modelled in the proposed case but is a limit for real riders that are leaning to the center of the corner to close the turn but the ground is pushing them to the outside as shown from the picture below.

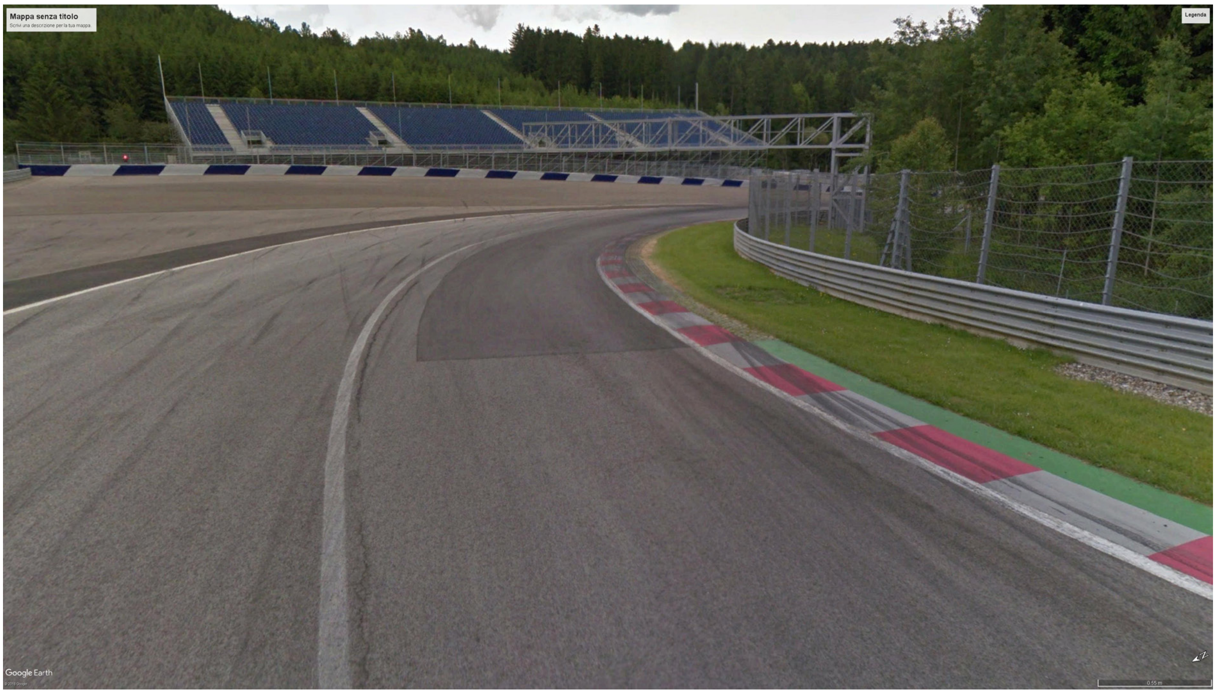

Figure 30, turn 7 Red Bull Ring

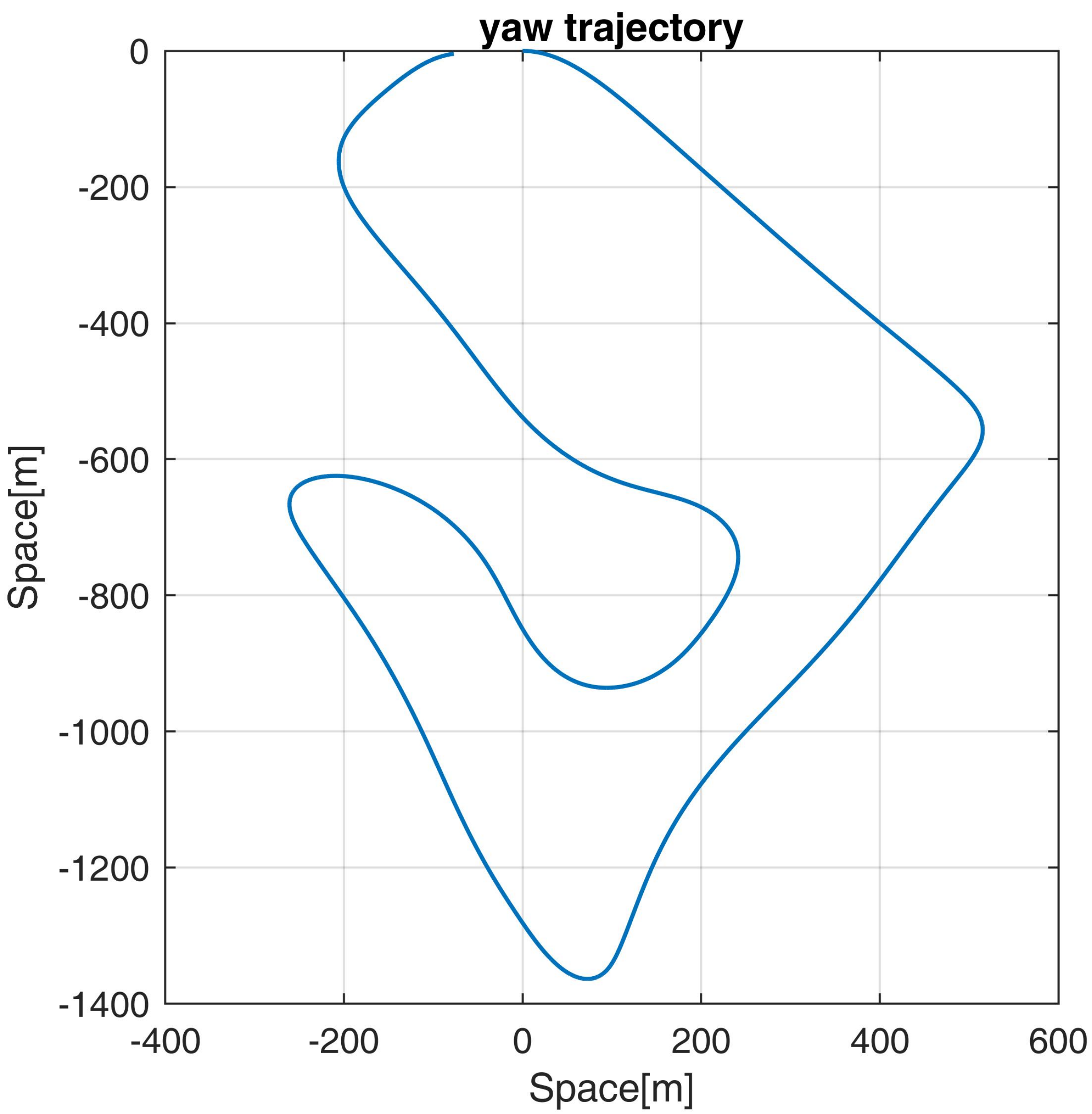

Figure 31, yaw trajectory of 3DOF model

Rider	Lap Time
Real rider	1’37.871
Point mass model	1’31.210
3 DOF rigid model	1’37.153

Table 3, models lap time comparison

Apart from the speed consideration, 3DOF model also carries information about lateral dynamics. As said before the only input of the model is the roll angle while the yaw derives from the calculations. Below you can see the yaw acceleration, yaw rate and the yaw angle along the lap simulation and the trajectory followed by the model.

4.3.4 Results

Below are shown some results of the models on different world championship tracks, mixed type containing a huge variety of corners and straight, hills and dip. For the sake of presentation only the trajectory and the speed profile will be reported

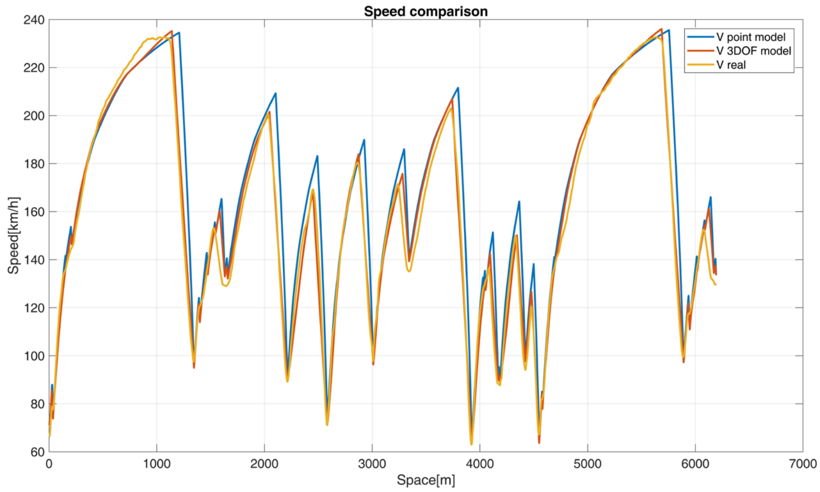

Figure 32, Barcelona circuit real and model speed comparison

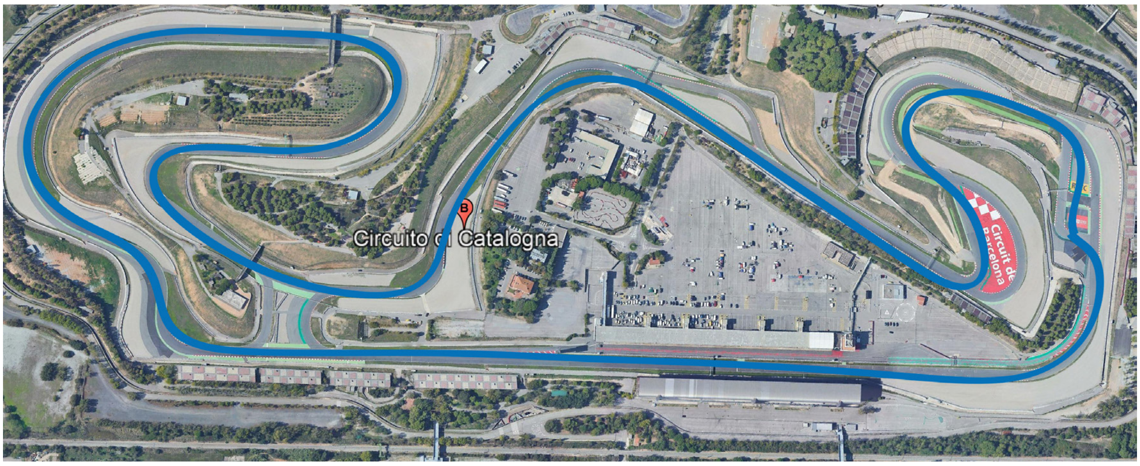

Figure 33, Barcelona trajectory of the model

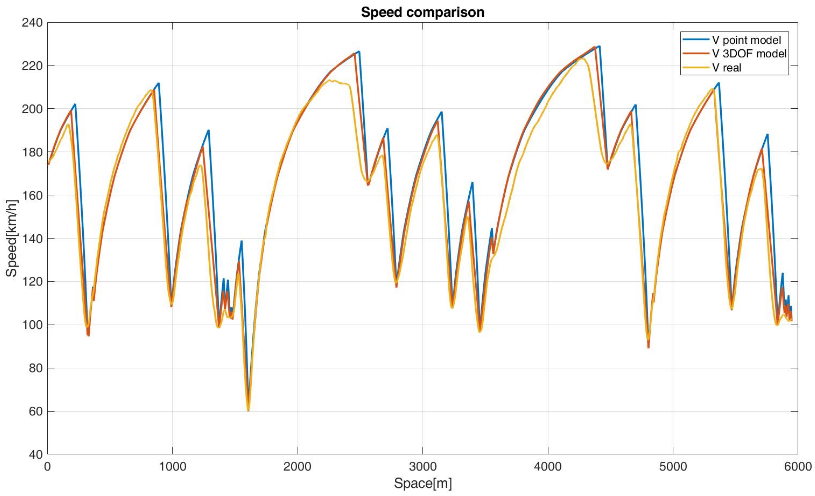

Figure 34, Assen circuit real and model speed comparison

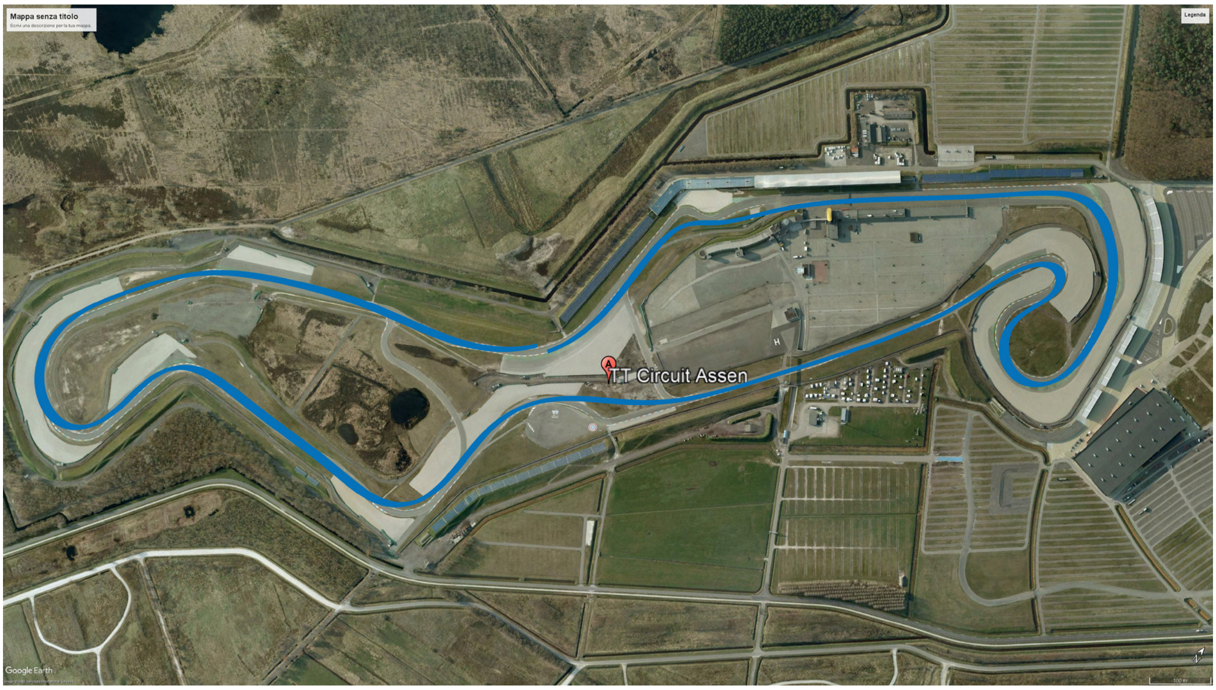

Figure 35, , Assen trajectory of the model

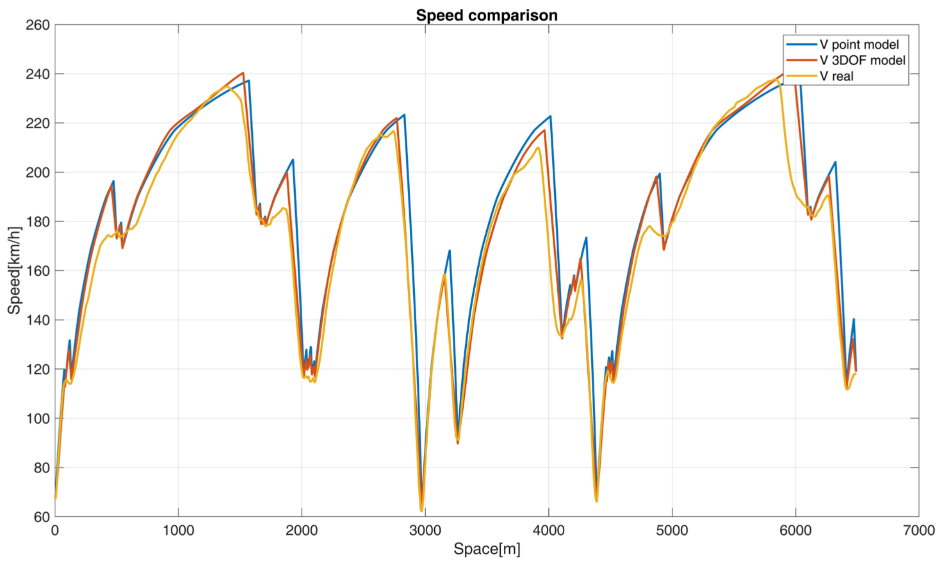

Figure 36, Phillip Island circuit real and model speed comparison

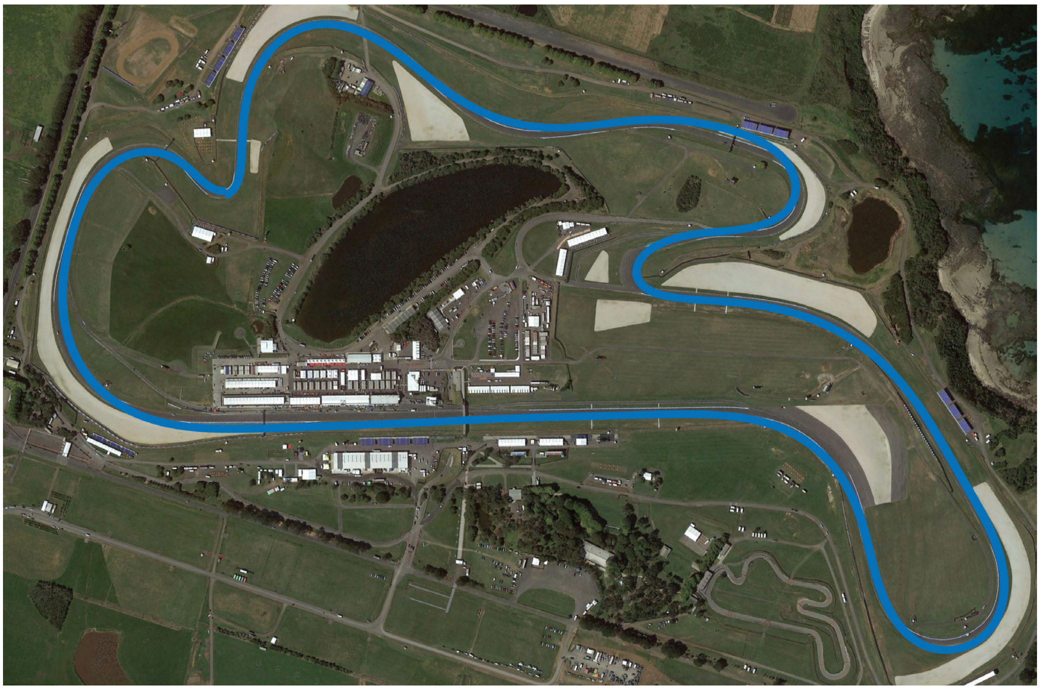

Figure 37,Phillip Island trajectory of the model

Results below refers to another machine and as you can see they are feasible with reality

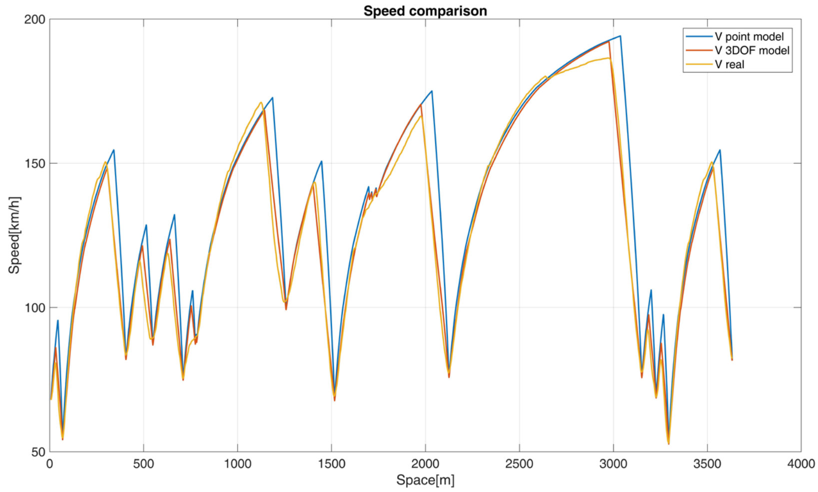

Figure 38, Cremona circuit real and model speed comparison

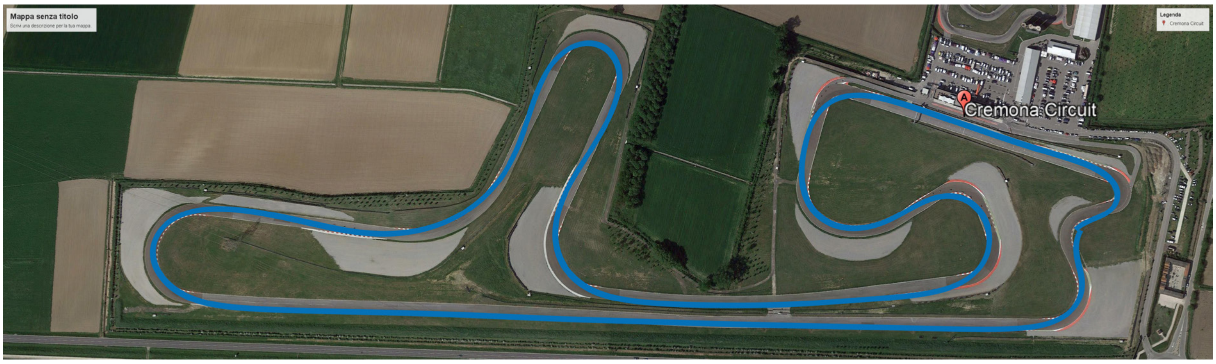

Figure 39,Cremona circuit trajectory of the model

4.4 5DOF model

The most complex model here proposed introduce suspension motion and so heave and pitch motion are now allowed. Summary of the possible motion

- Longitudinal acceleration
- Vertical acceleration (Heave)
- Rotation about X axis (Roll)
- Rotation about Y axis (Pitch)
- Rotation about Z axis(Yaw)

Note that lateral acceleration is still not allowed in this model since we consider always a pure longitudinal acceleration

4.4.1 Preliminary consideration

Introducing suspension motion means the six equations pack used before is not more valid and must be adapted. The bike in analysis is equipped at the front with an upside down telescopic fork. This type of fork is characterized by low inertia around the steering axis. Friction forces represents its greatest disadvantage when forces are applied orthogonal to the axis along which the sliders run - for example, in braking and on curves. In braking, because of the load transfer, the telescopic fork compresses, as the rear suspension is unloaded and extending until its rest position; thus, the vehicle pitches forward. Racing bikes are often equipped with fork that permits variable caster angles. A smaller angle of inclination of the fork causes a reduction of the value of the trail and so a bike a little less stable on the straight and in high speed corners but much more easy to turn in chicanes and on tight corners or direction changes. On the other hand, a higher inclination angle means an increase of the trail that makes the bike more stable in high-speed turns and on the straight while, it remains harder to turn in tight corners (Cossalter, 2006). On the rear, the bike is equipped with link-rocker system that permits to obtain the desired stiffness curve; this type of design derives from the four bar linkage but permits an easier construction.

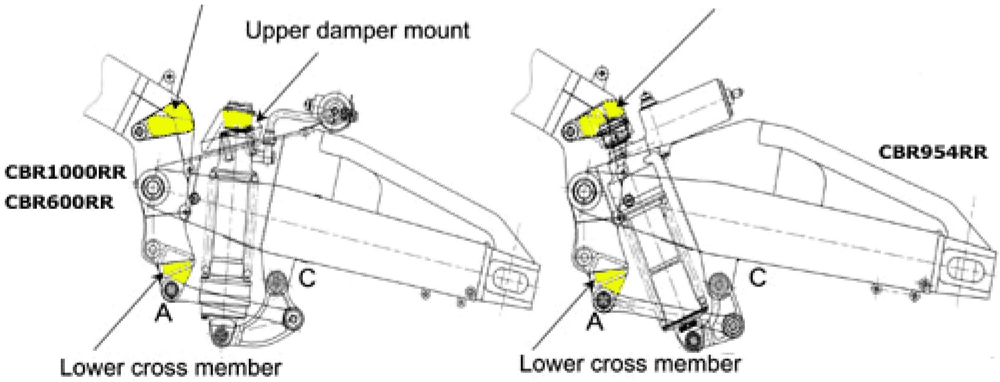

Figure 40, example of rear suspension kinematics

4.4.2 Reduced suspension stiffness

Choosing front and rear suspension characteristics (stiffness, damping, preload) is a complex process that must take in consideration many parameters. For example: the weight of the rider and the motorcycle, the COG position or the distribution of the loads on the wheels, the characteristics of stiffness and vertical damping of the tires, the geometry of the motorcycle, the conditions of use, the road surface, the braking performance, the motor power, the driving technique, etc.

For the study of in-plane dynamics, it is appropriate to reduce the real suspension to equivalent suspension, represented by two vertical spring-damper units that connect the unsprung masses to the sprung mass. The parameters defining equivalent suspension are:

- reduced stiffness
- reduced damping
- dependence of the reduced stiffness on the vertical displacement (progressive /degressive suspension)
- Maximum travel and preload

4.4.2.1 Front suspension

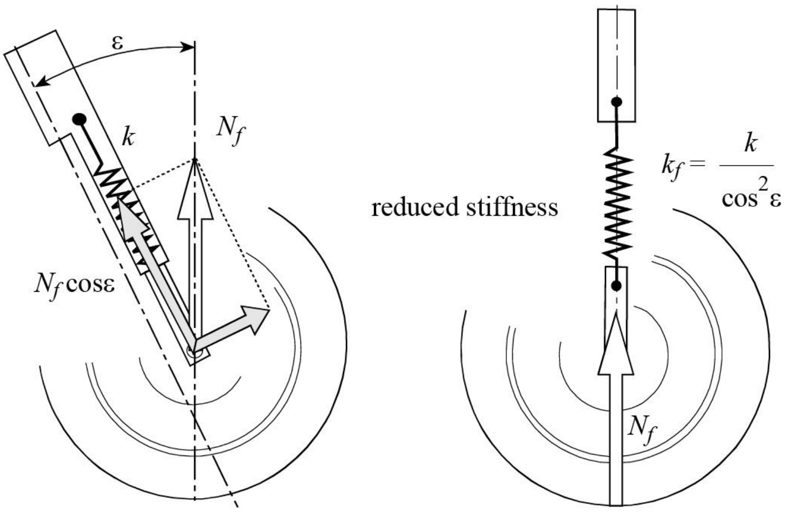

Figure 41, reduced front stiffness

Reduced stiffness and damping are quite easy to define for the front fork, the equivalent spring has a stiffness that permits to exert the same force as the real one but with a different displacement that is the vertical one in both cases

$$K_f = \frac{k}{\cos^2(\epsilon)} \tag{44}$$

$$C_f = \frac{c}{\cos^2(\epsilon)} \tag{45}$$

(44) Refers to the equivalent damping, same vertical velocity, different forces exerted by the real and the equivalent dampers.

4.4.2.2 Rear Suspension

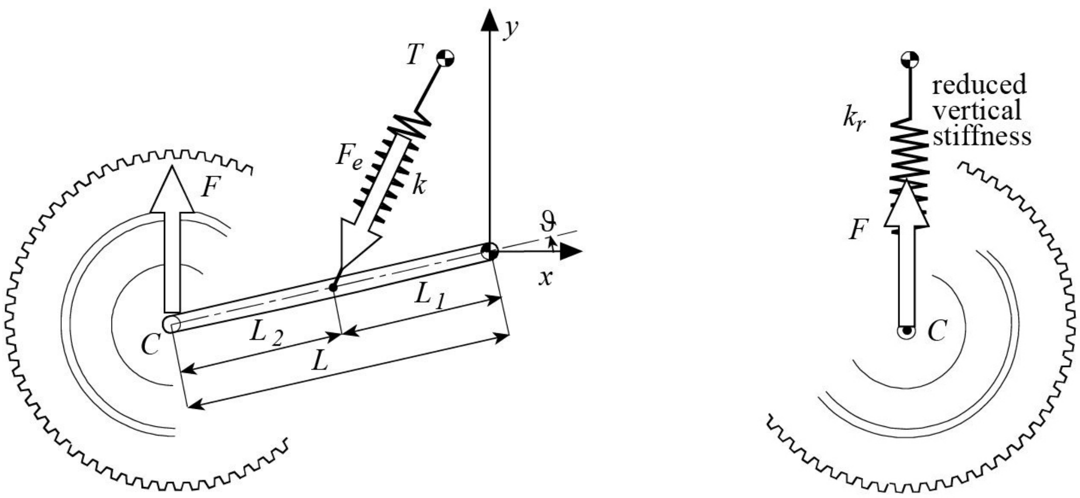

Figure 42, reduced rear stiffness

Reduced rear suspension forces depends on a velocity ratio instead of geometric one. The ratio between wheel stroke speed and the shock stroke speed.

$$\tau_m, y_c = \dot{L}_m / \dot{y}_c \tag{46}$$

$$k_r = \frac{\delta}{\delta y_C} F \cong k\, \tau_m^2, y_C \tag{47}$$

$$c_r = \frac{F}{\dot{y}_C} = c\, \tau_m^2, y_C \tag{48}$$

This ratio is not constant and depends on the suspension stroke. Since its variations are minimal it will be considered constant, regardless of the stroke. It is in the range between 0.75 and 0.65 so 0.7 will be used.

4.4.3 Damper table

In order to easily and correctly model the damper behaviour different lookup table have been created. As shown in the table below, Damper data coming from OEM is a table where, knowing the velocity at which the damper is going a force response can be calculated. The table below has two speed column, the positive one is related to the compression phase while the negative one is for the extension phase.

Vel compression	C1_6	C1_7	Vel rebound	R6_6	R6_7
0	0	0	0	0	0
0.001	24	24	0.001	25	25
0.005	63	60	0.005	97	87
0.01	80	78	0.01	161	144
0.02	99	95	0.02	269	244
0.03	109	107	0.03	350	323
0.05	125	123	0.05	458	432
0.07	136	134	0.07	544	517
0.1	153	151	0.1	652	624
0.12	163	161	0.12	724	694
0.15	181	179	0.15	822	793
0.2	211	209	0.2	982	950
0.25	239	237	0.25	1124	1094
0.3	266	264	0.3	1262	1231
0.4	319	317	0.4	1512	1479
0.5	370	368	0.5	1741	1707
0.6	425	422	0.6	1986	1942
0.7	478	475	0.7	2208	2167
0.8	529	526	0.8	2425	2384
1	635	633	1	2835	2803

Table 4, Example of damper look up table

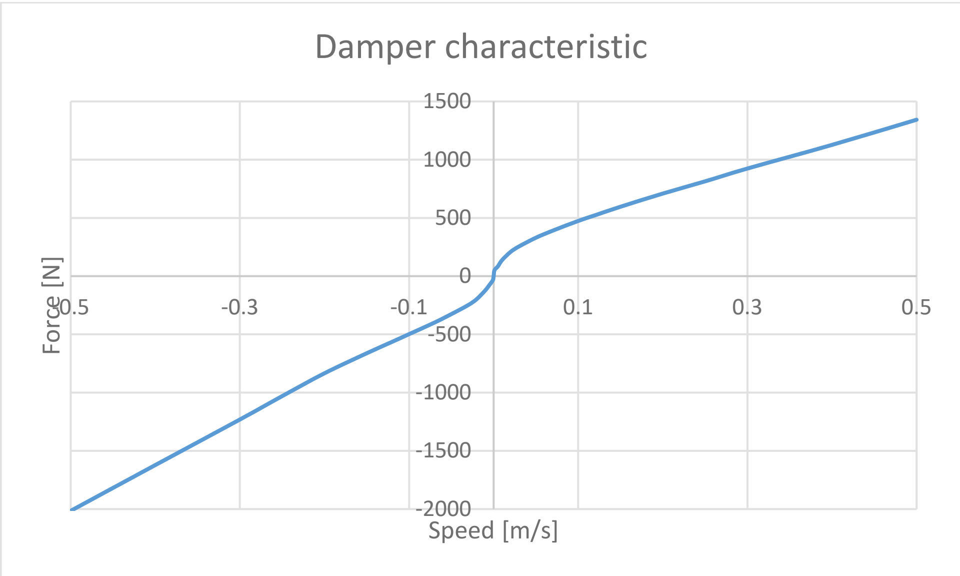

Figure 43, damper characteristics

Actually, instead of this proper table, the model use the damper table deriving from it

$$\beta = \frac{Force}{vel} \tag{49}$$

In this way, a new lookup table (Speed, Damping. 1-D Table) is created and it is used in this way.

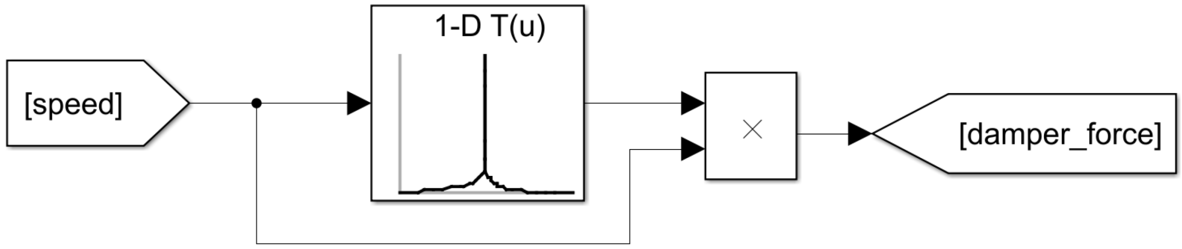

Figure 44, model damping force calculation

4.4.4 The model

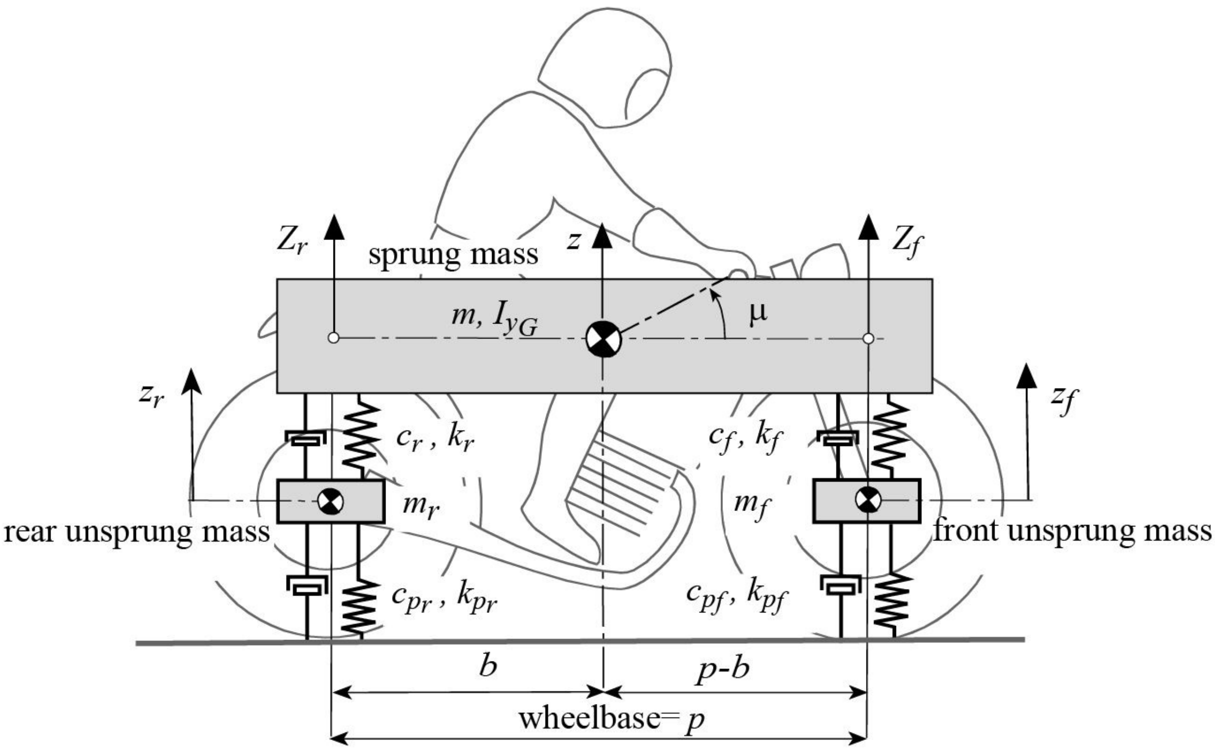

Figure 45, 4 DOF model scheme

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_{yg} & 0 & 0 \\ 0 & 0 & m_f & 0 \\ 0 & 0 & 0 & m_r \end{bmatrix} \begin{Bmatrix} \ddot{z} \\ \ddot{\mu} \\ \ddot{z}_f \\ \ddot{z}_r \end{Bmatrix} + \cos(\phi) \begin{bmatrix} k_f + k_r & ak_f - bk_r & -k_f & -k_r \\ ak_f - bk_r & a^2 k_f + b^2 k_r & -ak_f & bk_r \\ -k_f & -ak_f & k_f + k_{tf} & 0 \\ -k_r & bk_r & 0 & k_r + k_{tr} \end{bmatrix} \begin{Bmatrix} z \\ \mu \\ z_f \\ z_r \end{Bmatrix} = 0 \tag{50}$$

This model puts together a plane-based dynamics to analyse vertical behaviour of the bike and the lateral dynamics seen in the previous chapter. The result is an eight-equation model composed by the four in (48) plus (24, 25, 28, 29). Basically pitch and vertical equilibrium equations are now child of the XZ plane dynamics while others equations still consider the bike as a rigid system which, especially along the Y direction is a valid approximation. Note that (48) is not containing the dampers equations. Actually, (48) is used only to calculate the initial condition of the model every time it is run while during normal calculations also the damper contribution is taken in consideration. Differently from the two rigid model shown before, this one is based on several differential equation containing variable damping coefficients. In order to correctly represent the model all the equations have been transferred to Simulink instead of pure matlab code.

The script works in a similar way as before, every part of the track is travelled in acceleration and deceleration and minimum velocity is taken. Knowing roll angle in every point of the track the initial conditions are calculated and then Simulink model is run, giving only the roll angle as input.

Note that Simulink outputs are time based data and in order to compare the same part of the track they must be converted to space base one. For simplicity, the conversion has been made on one-meter step interpolation. When the global speed profile is calculated also all the others parameters associated are taken from the correct simulation for example if a step, has the speed profile coming from the acceleration phase that is the minimum one, also the acceleration, yaw, forces and so on will be the ones coming from that step of the simulation.

Once the speed profile has been determined a final run is performed, this time the focus is on the yaw calculation, the resulting acceleration from the previous launch is now given as an input and the model run in one all the track length since it has longitudinal acceleration and roll as inputs. This last run is very useful to analyse the suspension behaviour, their stroke and speed from the histograms. Obviously since there is no ground modelled, suspension velocity depends only on the longitudinal and pitch accelerations and so on the sprung mass motions. Despite it is little affecting the lap time (tenth of a second), it is very important to analyse the output histogram and check differences between one damper configuration and another. See figure 48.

4.4.5 Suspension analysis

Suspension histogram reports the speed range where the suspension is working. The aim is to let the suspension work at low speed In order to let the rider be able to ride comfortably the bike. Actually, if a suspension is working at high speed it means that abrupt trim changes are happening and that is always an unwanted situation. Furthermore, when suspensions work at high speed it means that they are not being slowed down by the hydraulics and consequently they will easily reach their limit compression that is often a critical zone due to the high frictions acting between various parts. Shape of the graph as shown in figure below. Damper setting, together with the spring one are key element during the race weekend since they are main elements to connect the rider to the ground so first of all they have to filter ground asperities, dip and hollows.

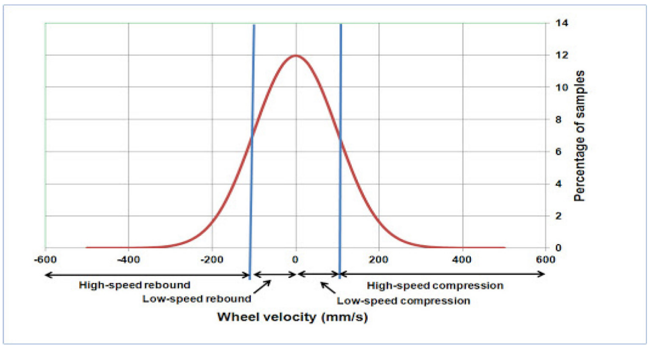

Figure 46, ideal histogram shape

Racing dampers allows different regulations, usually they have three or four way to be set up.

- Compression at slow speed
- Compression at high speed
- Rebound at low speed
- Rebound at high speed

These four ranges can be regulated by screw or bolt. Usually when they switch from one configuration to the other one is quite easy to hear a sound similar to a click. Generally, default configuration is with the valves completely closed so that increasing the number of “click” the damper will be softer. To sum up: high number of click correspond to a softer setting while a lower one correspond to a stiffer one, this rule refers generally to all regulation. Here we do not have any information about clicks but several damper configuration where the first number of the spec refers to the slow speed so it transpose all the stiffness curve up or down depending on the number while the second number refers to hypothetical click so the higher it is the softer the damper is.

	Front Set		Rear Set		Lap time
	Compression	Rebound	Compression	Rebound	
STIFFER	C7_9	R6_6	C23_9	R3F_21	1’36.18
	C7_9	R6_6	C23_9	R3F_9	1’35.93
	C7_9	R6_6	C23_9	R24F_6	1’36.00
	C8_6	R6_6	C23_9	R3F_9	1’36.07
	C8_6	R7_6	C23_9	R3F_9	1’36.88

Table 5, damper variation and relative lap times

Having done some test about dampers settings it’s interesting to note that even if modifications have been made I order to optimize the histogram shape, lap time didn’t suffers too much of that changes. Apart from the last one that significantly increased (0.8 seconds). Yellow bars corresponds to the stiffer configuration and watching the histogram it would be obvious to say that the yellow one should guarantee lower lap time but this is not true. This issue is related to vertical load. Last setup (front rebound R7_6) is too much stiff, what happens is that exiting from corners the front comes up so fast that the bike is approaching the wheelie limit and so the rider reduce the acceleration and that leads to a lap time increase.

Together with damper setting, also springs can be changed. Racing motorcycle are equipped with two spring in the front fork that works in parallel and one spring on the rear. Usually for practical reasons, especially on small bikes on the front the springs are asymmetrical in order to get every stiffness step between one set of springs and another. For example, suppose to have one set of 8kN/m and one of 7kN/m by adopting one spring per set the resulting one will be 7.5kN/m. Stiffer springs means smaller suspension travel, that obviously influences bike trim and so the behaviour of the load transfer and the effective grip.

It is important to note that despite the changes made, lap time did not have big oscillations and that is important since it permits to evaluate little modifications to the bike setup. The fact that lap time always remains in an acceptable range around the real one means the model is able to correctly replicate reality and let evaluate bike setup before trying in the reality.

All these tests and simulations on suspensions make sense due to the fact that at the beginning the model has been validated from the lap time before and form the real suspension data then.

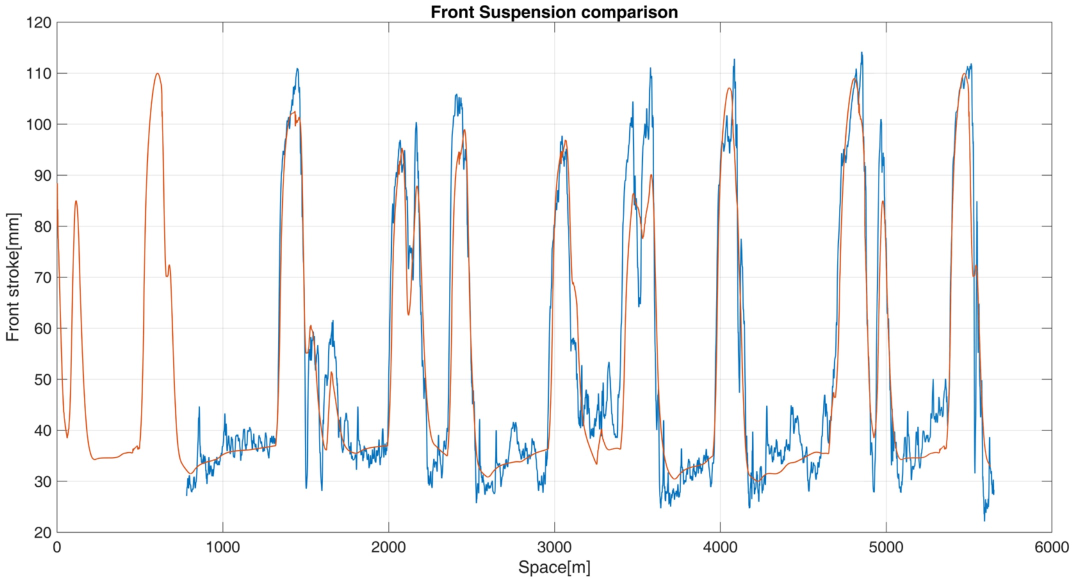

Figure 47, suspension stroke comparison, blue line is the real data, red one the model output

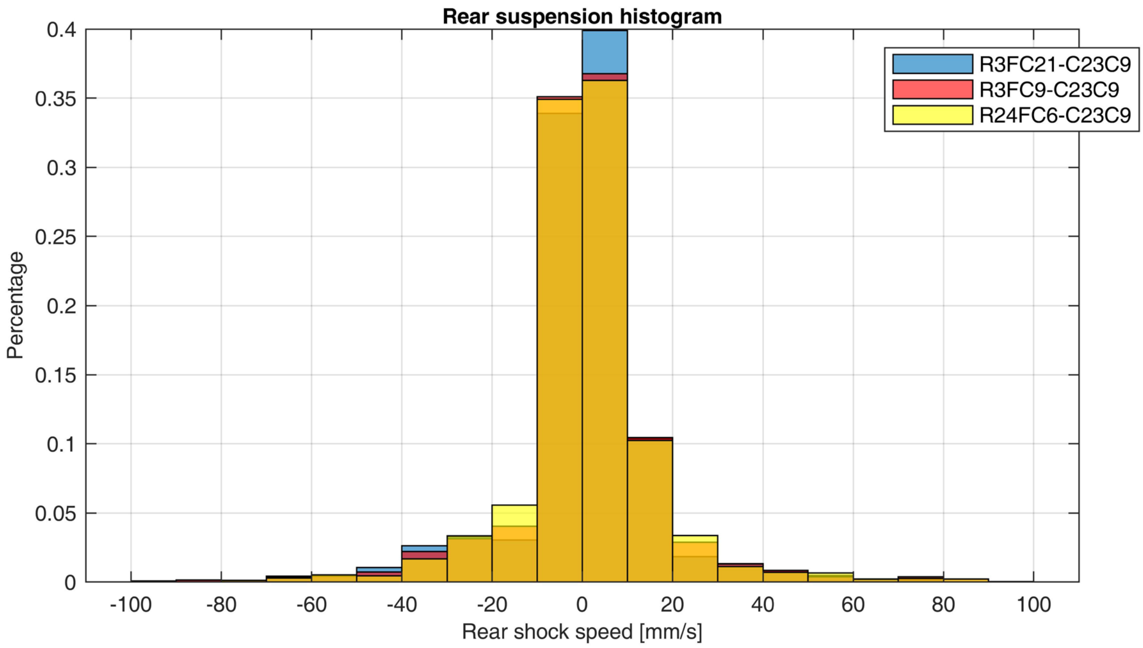

Figure 48, rear suspension histogram variation

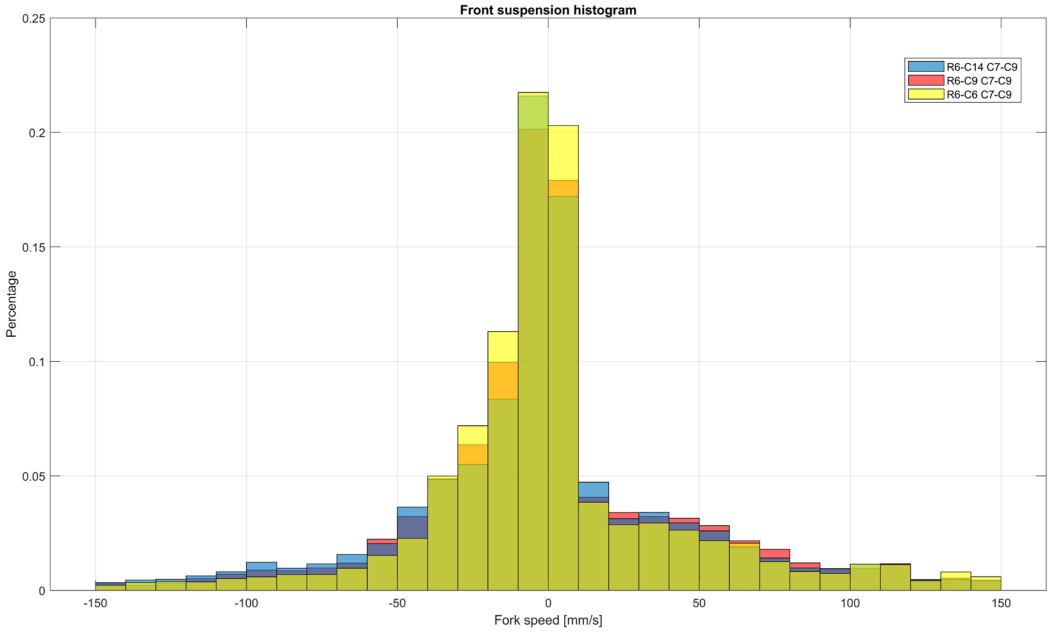

Figure 49, front suspension histogram variation

4.5 Conclusion

This software permits to have a prediction of what will be the behaviour of the bike along a given circuit. It has been developed for all types of racing bikes but, during the work, only data from Moto3 and premoto3 bikes were used. The software has been created with the aim of helping the bike setup during the weekend and to evaluate possible upgrades to the motorbike during the developing phase. The interesting thing is that every modification is not evaluated in a static way based on structural analysis but in dynamics and can be tried over real circuit, starting from real data inputs. It helps to have an idea of what is the gain an upgrade can bring. To do so, it is important that the models output are quite close to real one since that is a sort of validation of the model and let the prediction be quite reliable.

At the beginning of the work, it was made the choice of preferring a quick model to a very precise one. Actually, the rigid model scripts are running in less than twenty second each. Targeting at the beginning a fast running speed has the consequence of limiting many other factor (line optimization and rider control mainly) but permits to try a lot of different setup since they run in a few seconds

Furthermore, suppose having a rider that prefers the bike set in a certain way that can be with stiffer front or rear springs, or with the front that is higher than the nominal condition. Knowing that, the software permits to evaluate possible modification to this condition, changes that will improve the bike behaviour without removing the original trim wanted by the rider. Eight degree of freedom model is more focused on how the rider wants the bike to react and to let him ride at its best while rigid model is more focused on the lap time optimization, showing where riders can gain something.

Supposing to have data from different riders, the model can run on both their lines and tells what the best line between them is and how to exploit the bike performance at its best.

As last consideration, it is interesting to note that all three models report results that are feasible with the reality. This last model have been also a kind of new approach in the motorbike modelling consisting in decouple the vertical motion of the sprung mass from the lateral behaviour. Like as the chassis remain infinite rigid along the Y direction and is flexible along Z. It is, at least notable, that despite this simplification, the output of the suspension model is quite near to the real one. That underline the fact that real component has almost no frictions deriving from the lateral forces acting on the fork or the rear shock.

5 Rider Confidence

5.1 Introduction

Data analysis has always been a key element of competition, during a race weekend it is very helpful for understanding how and what modifications are required to correctly tune the bike setup. Data analyzer software basically provides raw data coming from all the sensors installed on the bike, more complex and expensive software provides also the opportunity to create math channels and evaluate variables that cannot be physically measured, for example by integrating or deriving the sensor output. Above these basic considerations, commonly used software allow to compare different laps from the same or different rider by superimposing the signal coming from the same sensor as shown in the figure below. This kind of approach is useful to understand meter by meter and with a very high level of detail what is happening on the bike and permit, to compare riders' data meter by meter along the track. On the other hand, this much-focused approach requires time to be done and a very skilled person to correctly understand the signals.

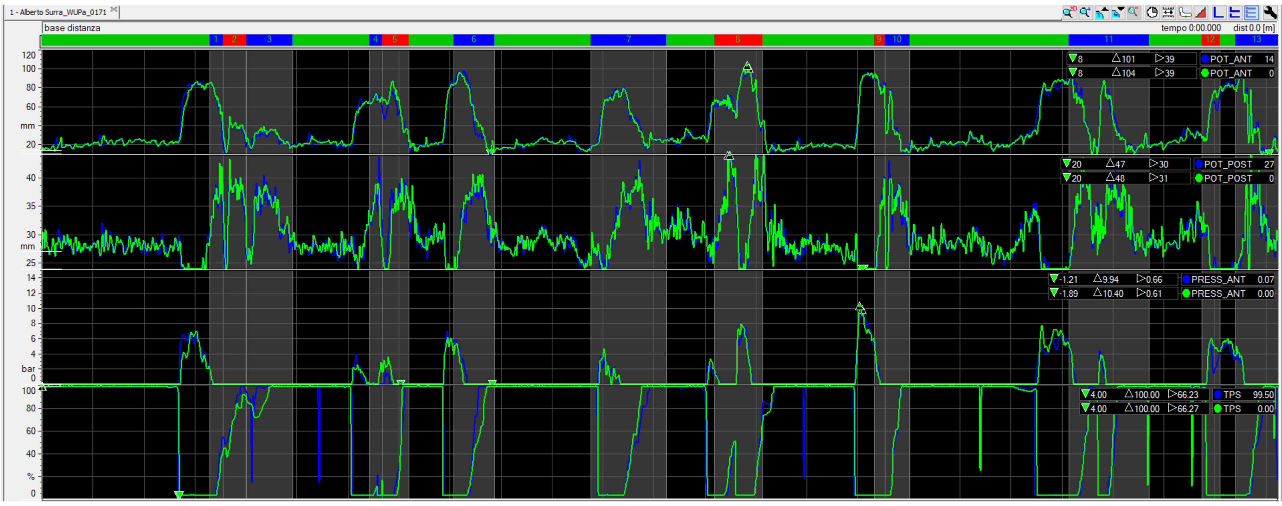

Figure 50, classical data analysis interface

The aim of rider confidence is to compare major track parts, for example turns, and associate to that some parameters showing how the rider is driving and how much confident he is feeling in every part of the turn: before(braking), during(turning), exit(accelerating). All values are then referred to the time spent on that part of the track that obviously is parameter that we want to minimize. In the end, the final output of the program will be an excel file where we can compare all riders on all turns of all laps of selected sessions. Basically, the software provides a simple and intuitive feature extraction that tries to give an idea of how the rider is driving and what are strong and weak points along the track.

5.2 Channel definition

In order to evaluate confidence some new math channels must be created. This is the first key point of the analysis and what the software does is combining some sensor signals to create a value that indicates how the rider is able to do both things at the same time. It's important to say that this value often has no physical meaning. For example braking and rolling, accelerating and rolling or how much faster is reaching the correct brake pressure or how much fast he is opening the throttle. Generally, the aim is to compare the riders with respect to an ideal behaviour. Ideal rider should do few moves, very short and straight movement. This is the reason why some parameters are analysed in function of their speed instead of their pure magnitude

This step must be done taking also in consideration what are the conditions that maximize bike efficiency (ie. some bikes need a lot of weight transfer and braking hard while others prefer less braking and more speed carrying along the turn). For that reason the software, provides and easy way to add, modify and remove channels.

First of all it is very important to say that every channel has always zero value apart from the zone where we are interested in, mainly these are transient one so that it is easier to compare the transition phases between one lap and another and between riders. Finally, these are the channels used:

- Time spent: report time spent to travel that specific part of the track. The lower the better.
- Average speed: report average speed along that specific part of the track. The higher the better.
- Neutral distance: report the distance travelled with no gas and no brake. The lower the better.
- Braking distance: report the distance travelled with brake pressure higher than zero. The lower the better.
- Partial throttle distance: report the distance travelled with Throttle Position Sensor (TPS) value is in the range of 4.5% (the one used for idle revving) and 95%.The lower the better but its interpretation is strictly related to the average speed
- Roll-brake confidence: this is a math channel created by multiplying brake pressure by the roll angle. The aim is to evaluate how much the rider is able to brake with a certain angle of roll. The higher it is the late the rider can brake since he is "bringing" the brake phase as closer as possible to the apex. The higher the better.
- Velocity in braking and rolling: this is a math channel created by multiplying the derivative of the brake pressure by the derivative of the roll. The goal is to evaluate how much time the rider spend to reach the peak pressure and lean the bike to the desired angle. Taking these two parameters as standalone parameters, we are always looking for the maximum value of that. The higher the better but deeply depends on riding style
- Throttle confidence: this is a math channel created by reporting the TPS position when it is in the range of 4.5% and 95%. This channel will be very important in the evaluation phase since the speed at which rider is able to reach the WOT condition is very important. The higher the better.
- Roll-Throttle confidence: this is a math channel created by multiplying the throttle confidence channel by the roll angle. A high value of that parameter means that the rider has high confidence in opening the throttle even if it is at a high roll angle. The higher the better.
- Speed-Throttle confidence: this is a math channel created by multiplying the throttle confidence channel by the bike speed. A high value of that parameter means that the rider has high confidence in opening the throttle even if it is going at high speed, the goal is to catch the fact that rider is able to accelerate during high speed corner. The higher the better.
- Pitch confidence: this is a math channel created by multiplying the brake pressure by the fork speed. The combination of these two value gives an idea of how much the rider feels confident in braking when the front fork is going in high compression and he has to travel many meters in with the bike that is pitching through the front. Note that instead of the pure fork position we are interested in the speed of the fork since is important for the rider to be able to do the right brake pressure regardless of what the fork is doing. As said in (5.4.5) when the fork is working well the majority of its working range is in the slow speed zone therefore, if the fork is correctly set up a high value of the pitch confidence parameter means the rider is able to do high brake pressure in the transient when the fork is compressing. Both slow speed and slow pressure will result in low values so the higher the parameter is the better the rider is feeling.
- Pitch-Roll confidence: this is a math channel created by multiplying the fork speed by the derivative of the roll. This is strictly related to the pitch confidence one; actually this channel has value different from zero only where also the pitch confidence has non-zero value. The aim of that channel is to analyse the precise moment where the rider is loading the front fork and it is going down while he is also leaning the bike. As said before the factors appearing in channel equation are all speeds and that means this channel is mainly focused on transient moment. A high value means the rider is able to fast load the fork and at the same time, he is rolling the bike, which means he is combining well two

actions that are usually made in rapid succession. Therefore, the higher the value the better it is but deeply depends on riding style.

5.3 Track division

The next step is to select the parts in which we are splitting the track. In order to help the user the software provides a map of the track for every lap done and it is divided in parts where the rider is at full gas (straight) and parts where he is braking or in a situation of partial throttle. Then manually selecting the splitting points a reference coordinate vector will be created and this will be used for all the analysis on that track. The key point of that phase is to select zones of the track where it is supposed that riders can lose or gain time. This process is done only one time per track in order to compare the same zones over the sessions and over the years.

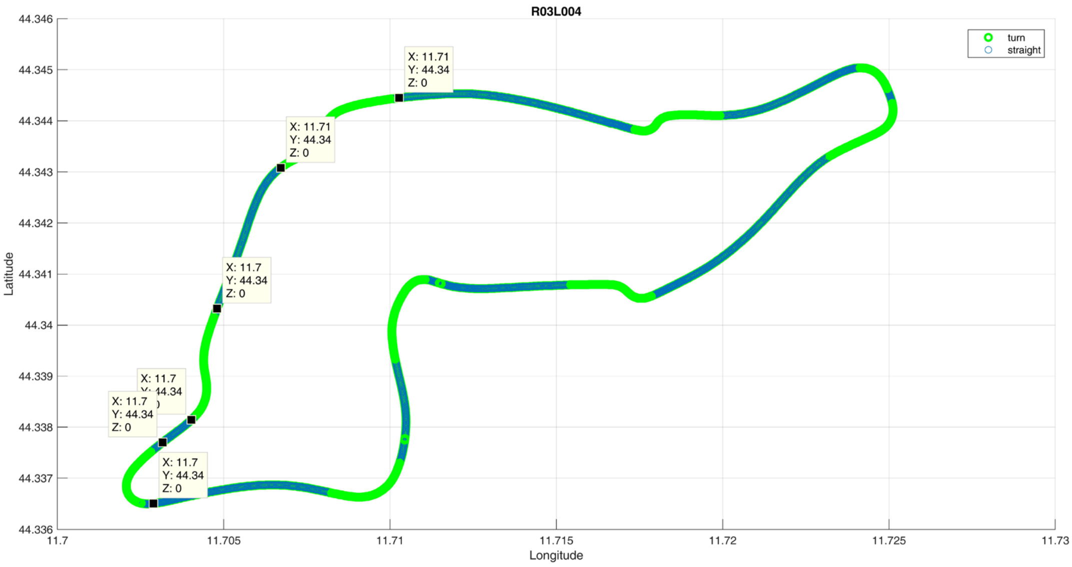

Figure 51, track section creation

5.4 Feature extraction

Since the final goal is to simplify the analysis and compare more laps together, on every section created before, a feature extraction is now performed. For example, we can have a sum, a max, a min or an average of a particular channel in a specific section. That means we are no more evaluating the bike and rider behaviour meter by meter but what is happening in the whole section. It is very important to have a very user friendly software in that part since track by track or rider by rider the key point to analyse can be different. In order to read and understand the analysis in an easier way, every value is then normalized with respect to the max of that feature in that section. Therefore, if a rider reaches a peak value of a confidence parameter during a session (ie. during a FP) that will be a "1" and the reference for that feature for all riders until someone will reach a higher value of that feature. Consequently, all other values of the column will be between "0" and "1". Again, it must be clear that some of these features have no physical meaning since they are the result of mathematical operations on channels that have mixed physical units. The point is to summarize in a few numbers how the rider is driving along the track and if he is improving session by session. It permits to evaluate the progression of a rider along the sessions since normalization can be done also for one rider only comparing every turn of every session.

Here it is the list of extracted features:

- Time: reports the last value of the time vector that starts at the first meter of the analysed section
- Neutral phase: reports the last value of the cumulative space, in meters, made with no gas and no brake in that section
- Brake distance: reports the last value of the cumulative space, in meters, made with brake pressure higher than zero in that section
- Partial throttle distance: reports the last value of the cumulative space, in meters, made with throttle higher than the minimum and lower than the maximum in that section
- Average speed is the mean speed of the section
- Brake and roll confidence: there are reported both the max value and the cumulative one. The maximum indicates how far from the apex the rider is doing the peak brake pressure. Usually the peak pressure is made at the beginning of the braking phase, when the bike is still straight. A high max value means that the rider is retarding the braking phase trough the corner centre and so it is able combine the peak brake pressure with a certain roll. On the other hand, the cumulative one, indicates how much a rider is braking while leaning. It is an instant parameter to understand how much he is mixing the two actions.
- Velocity in braking and rolling: only the cumulative value of that channel is reported since we are interested in the speed of reaching the peak brake pressure and the speed of gaining high roll angle over all the section. The cumulative take also in consideration if the rider slowly reach the goal roll angle since it will result in small value (sum of very small delta) while a fast lean or a high delta brake pressure results in a higher cumulative. This parameter must be read in strictly correlation with the brake and roll one since it indicates how the rider reach the magnitude values that are considered in the brake and roll channel.
- Pitch confidence: from that channel both maximum and cumulative sum are extracted. A high maximum value means both delta brake pressure and fork speed have high values and that is possible only if the rider is rapidly reaching the peak pressure and he has confidence in loading the front fork with no hesitation. The cumulative one indicates how long is the phase where the fork is compressing and the brake pressure increasing, it means the rider has no fear in continuously asking for braking force while approaching the corner. Both these two parameters are strictly related to the construction of modern racing bikes; actually, to best exploit the longitudinal force the tire can provide it needs to be loaded a lot. Therefore, the rider must use all the load transfer to brake in the best way.
- Pitch-roll confidence: of that channel it is interesting to analyse the cumulative sum since it is representative of how the rider is able to perform meters while varying the brake pressure or loading the front fork and at the same time increase the roll. Note that since it is a cumulate value, if the rider has a hesitation and he is like playing with brake or with roll, this will result in a low value of that feature. Suppose to brake and approach a corner at a certain moment and then you realize that you are anticipating too much the corner so you reduce roll angle and then you go through the corner. The roll derivative signal will be higher and lower than zero over the section and the cumulative value will obviously depend on that. This means the rider was not so confident in the line he chose and so in the moves he makes.
- Throttle confidence: for that channel is interesting to analyse both maximum value and the cumulative one, the max value indicates if the rider has a certain speed when opening the throttle. It is a main parameter of how the rider feels the bike when accelerating, obviously an ON-OFF behaviour is ideal, it would have infinite derivative in that case. The cumulative, again will report high values for riders that do not have hesitation in accelerating and, more important, is not having reduction on throttle position. A rider that slightly increase throttle will have a low value on the max

parameter but a high one on the cumulative one. The best thing is very likely a situation that stays in the middle; anyway, the time spent to travel the section will judge which the best approach is.

- Speed-throttle confidence: here only the maximum value is extracted since this channel is thought to analyse a precise situation that is high speed cornering. Obviously, in that type of corner the rider must stay for long time with throttle open but depending on their feelings they can go at wide open throttle or not. This is the key point of the analysis, understand how many meters he is doing at partial throttle and catch how much faster he is able to reach the WOT condition even if he is already going at high speed. Therefore, both a rider that brings a lot of speed at the corner apex and the one that enters slowly but opens the gas faster will have a high value of this parameter.
- Roll-Throttle: maximum and cumulative values are here extracted, highlighting if the rider feels confident in opening the gas at high roll angle and how much strong the confidence is over the corner. A high peak value means the rider has no fear in rapidly opening the throttle, even if he is at high lean angle. The sum, on the other hand, reports if a rider is accelerating with no hesitation while he is with the bike rolled.

5.5 Final output

The figure below shows the final output of the software where we got a table with coloured cells that allow us to quickly understand the data. There are columns where the lower the value is the better it is (ie. Time, neutral phase, partial throttle distance, brake distance) while there are others where a higher value is preferred (ie. Throttle, braking and pitch confidence). Note that there are columns that have not been normalized since they actually have a physical meaning for examples the distance or time ones. Every column correspond to a feature extracted from the channels described before and every row corresponds to a lap. Every section defined in 6.3 has a table like the one below in a dedicated sheet, a last sheet is then used to plot the circuit map to make the analysis easier.

To better understand tables will follow in the document some abbreviation explanation:

- Dist. = distance;
- Brk = brake;
- Avg = average;
- Thrt = throttle;

Rider 1	Time	nuetral_phase	dist_brk	dist_partial	avg_speed	brk_roll_max	brk_roll_sum	dbrk_droll_sum
	6.90	2.85	142.46	80.51	137.84	0.89	0.61	0.62
	6.95	5.37	141.60	77.24	138.21	0.73	0.54	0.95
	6.90	2.86	135.92	78.13	139.47	0.70	0.63	0.55
	6.90	2.83	140.47	81.76	138.77	0.62	0.58	0.58
	6.90	0.00	151.54	78.71	138.10	0.73	0.69	0.48
	7.30	0.00	166.47	82.06	132.43	1.00	1.00	0.83
Rider 2								
	7.25	22.29	164.49	72.81	132.17	0.40	0.36	1.00
	7.15	17.30	144.12	71.63	133.70	0.28	0.29	0.51
	6.95	22.98	129.75	66.23	137.28	0.37	0.36	0.37
	6.85	11.96	126.54	77.41	138.82	0.39	0.40	0.63
	6.85	20.53	130.55	66.28	139.67	0.35	0.37	0.39
	6.85	11.98	134.67	75.76	138.19	0.33	0.40	0.48

pitch_max	pitch_sum	pitch_roll_sum	thrt_max	thrt_sum	thrt_speed_max	thrt_roll_max	thrt_roll_sum
1.00	0.80	0.32	0.88	1.00	0.22	0.94	1.00
0.90	0.72	0.37	0.81	0.98	0.40	0.87	0.94
0.95	0.78	0.21	0.84	0.91	0.55	0.89	0.95
0.84	0.80	0.10	0.95	0.99	0.73	0.93	0.98
0.85	0.57	0.18	0.97	0.93	0.85	0.98	0.98
0.66	0.83	0.39	1.00	0.74	0.94	0.88	0.71
0.42	0.87	1.00	0.96	0.89	0.19	0.79	0.81
0.71	0.75	0.27	0.90	0.93	0.35	0.93	0.90
0.64	0.95	0.33	0.81	0.76	0.51	1.00	0.82
0.75	1.00	0.19	0.88	0.93	0.68	0.78	0.90
0.70	0.95	0.55	0.76	0.69	0.83	0.98	0.76
0.85	1.00	0.24	0.75	0.82	1.00	0.91	0.86

Table 6, example of rider confidence output

This figure reports a typical output of the software, two parts of the table are actually merged horizontally, here they were divided into two parts in order to make it printable in a user-friendly size.

At the beginning, values that should be analysed are the time spent per section and the brake distances. Comparing these two values is quite easy and immediate to understand if a rider is taking the wrong line or he is misreading the turn. The table below shows an example of a that situation

Rider 1	Time	nuetral_phase	dist_brk
	10.55	6.42	144.86
	10.40	0.00	141.62
	10.40	0.00	153.07
	10.45	0.00	187.31
	10.55	7.36	181.52
	10.25	8.08	163.54
Rider 2			
	10.25	31.90	126.46
	10.10	55.34	90.61
	10.20	36.44	101.87
	10.15	29.33	115.59
	10.05	39.86	107.58
	10.10	49.19	110.77

Table 7, comparison on Acque minerali turn

Note that, apart from the time value, the big difference is in the brake distance, as if rider one is making almost the double of the space with brake pushed. It is important to highlight that this behaviour is constant over the laps, that means there is a different approach of the turn by the two riders. Since from the few data we are quite sure of that difference, let us check GPS recorded map and the result reported in the figure below confirm the situation. There is a huge difference between their trajectories as shown below.

Rider one is taking the blue line and prefer to do less meters and seems to anticipate too much the turn and not being able to accelerate correctly since he takes the wrong line closing too much the trajectory

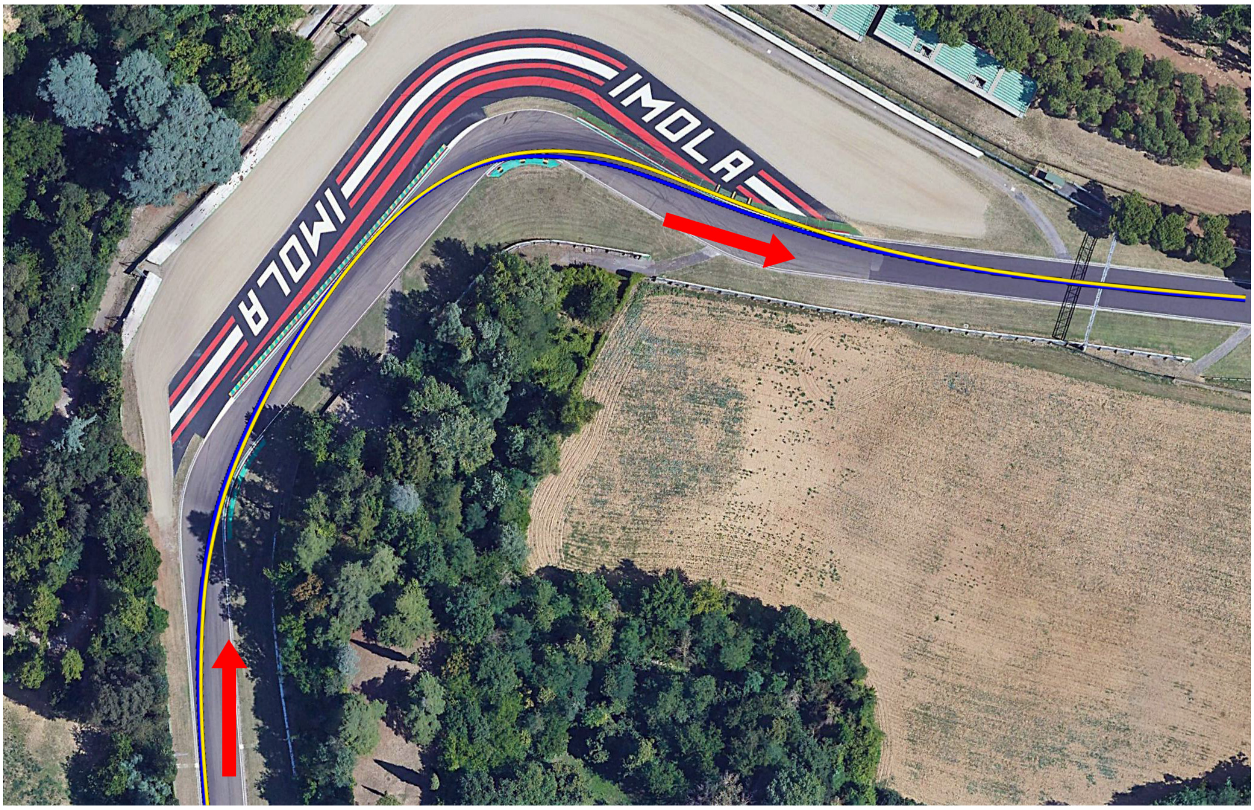

Figure 52, line comparison. Red arrow show the travel direction. Blue - rider 1, yellow rider 2.

Rider 1	Time	nuetral_phase	dist_brk	dist_partial	avg_speed	brk_roll_max	brk_roll_sum
	8.45	1.99	164.25	47.54	159.07	0.67	0.58
	8.45	0.64	150.54	10.55	159.03	0.73	0.61
	8.50	0.00	158.85	20.78	158.83	0.71	0.58
	8.45	0.00	182.31	31.96	160.56	0.96	1.00
	8.30	0.00	165.28	10.84	162.56	1.00	0.79
	8.25	0.00	165.25	26.94	162.55	0.75	0.72
Rider 2							
	8.50	6.37	176.44	67.91	157.63	0.56	0.41
	8.45	14.70	163.41	72.65	158.96	0.40	0.30
	8.50	18.13	145.37	0.00	158.34	0.43	0.32
	8.40	14.58	141.18	92.02	159.31	0.38	0.32
	8.40	10.98	147.32	5.27	160.18	0.74	0.42
	8.30	11.19	160.31	0.00	161.96	0.53	0.48

Table 8, Confidence parameters, braking Rivazza turn

The table above puts in evidence an interesting thing, as you can see rider 1 brakes for more meters and has higher braking parameters; that means he is also reaching higher brake pressure than rider 2. Despite braking harder and longer, rider one has also higher average speed. This is possible only if rider one starts braking at a higher speed. Note also that rider 2 travels many meters in neutral phase, it means he is using a lot of engine brake and that explain the lower average speed. Actually, rider two made his best time when he have been braking for more meters. The conclusion is that on that turn is better to brake a little more at the corner entry in order to be able to accelerate before instead of bringing a lot of speed at the corner apex but not being able to accelerate correctly. Throttle confidence parameters shown below confirm that assumption since rider one has higher value for every parameter.

Rider 1	thrt_max	thrt_sum	thrt_roll_max	thrt_roll_sum
	0.96	0.36	0.88	0.48
	0.98	0.13	0.63	0.12
	1.00	0.13	0.85	0.16
	0.99	0.34	1.00	0.48
	0.93	0.13	0.90	0.17
	0.92	0.22	0.96	0.30
Rider 2				
	0.88	0.07	0.77	0.09
	0.77	0.06	0.78	0.09
	0.00	0.00	0.00	0.00
	0.92	1.00	0.79	1.00
	0.79	0.07	0.52	0.06
	0.00	0.00	0.00	0.00

Table 9, Confidence parameters, exiting Rivazza turn

The table below refers to the first turn of Mugello circuit, note that despite having higher braking confidence parameters, rider one needs more time to travel the turn and this is due to the fact that is going slower (about 5 km/h). This case puts in evidence that rider one, probably has more confidence in braking but he also has a little more fear in bringing speed to the corner center, that means the higher value are related to the fact that they last more time in that section for rider one than for rider two. From these fast analysis a lot of information have been rapidly caught and so transferred to the riders

Rider 1	Time	nuetral_phase	dist_brk	dist_partial	avg_speed	brk_roll_max	brk_roll_sum	dbrk_droll_sum
	7.45	5.19	139.88	86.72	123.05	0.49	0.70	0.31
	7.45	0.00	177.57	63.65	126.36	0.95	1.00	0.83
	7.25	10.50	132.75	71.09	127.09	0.85	0.82	0.51
	7.30	20.69	127.58	58.18	126.05	1.00	0.83	1.00
	7.20	21.37	132.91	73.82	127.92	0.76	0.74	0.39
	7.35	2.43	143.89	81.10	125.94	0.94	1.00	0.99
Rider 2								
	7.30	10.14	132.85	50.24	130.30	0.64	0.78	0.47
	7.30	10.49	120.06	72.68	128.98	0.57	0.87	0.42
	7.50	7.37	123.87	75.60	124.87	0.58	0.76	0.37
	7.10	10.75	120.18	64.87	131.91	0.54	0.76	0.73
	7.05	19.23	119.01	61.92	131.63	0.51	0.65	0.37
	7.10	10.97	126.37	121.69	130.77	0.79	0.73	0.56

Table 10, Confidence parameters, San Donato turn

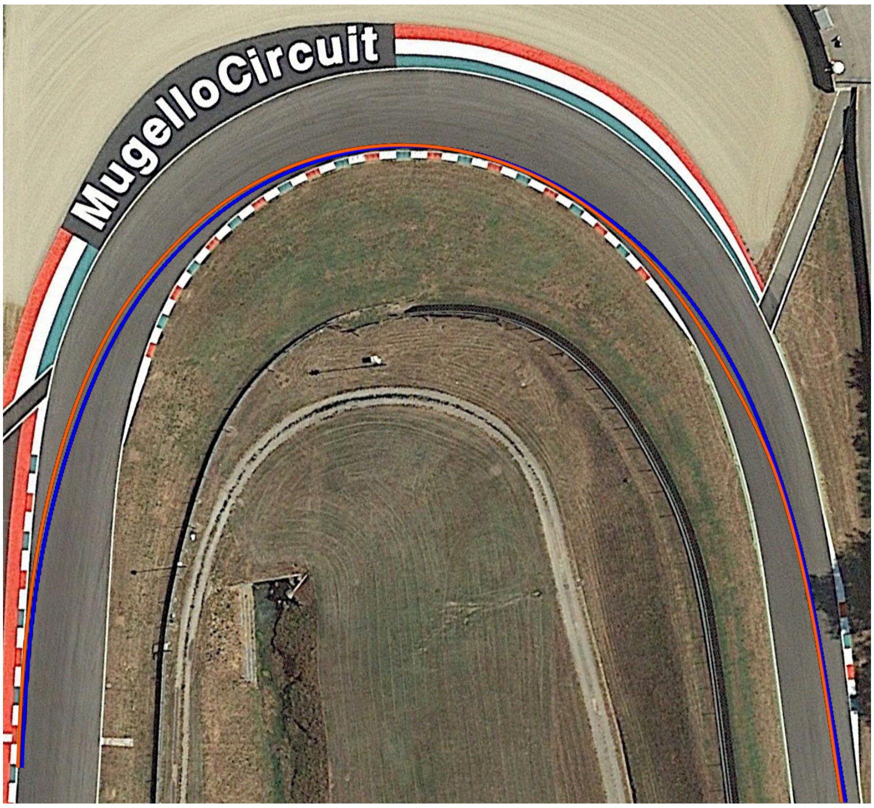

Figure 53, first corner of Mugello circuit

Table below refers to the Correntaio turn in Mugello circuit. It is a 180° turn and the ground is quite downhill. Corner exit speed is much more important than the entry one since there is a considerable full throttle part after that turn.

Rider 1	Time	nuetral_phase	dist_partial	thrt_max	thrt_sum	thrt_speed_max	thrt_roll_max	thrt_roll_sum
	8.10	33.65	83.25	0.94	0.78	0.24	0.64	0.97
	8.05	7.98	119.66	0.99	0.96	0.36	0.58	0.73
	8.00	5.62	143.78	0.97	1.00	0.51	0.66	0.91
	8.10	0.00	110.43	0.99	0.82	0.63	0.51	0.85
	8.00	11.12	88.98	0.98	0.64	0.77	0.68	0.78
	8.10	7.35	116.36	0.98	0.95	0.91	0.49	0.87
Rider 2								
	8.00	21.19	105.87	0.92	0.72	0.26	0.54	0.89
	8.25	15.64	103.07	0.99	0.71	0.40	0.47	0.73
	8.00	16.71	115.70	0.87	0.80	0.55	0.86	1.00
	7.90	9.10	110.82	0.90	0.77	0.70	1.00	0.97
	8.00	14.66	99.05	1.00	0.86	0.86	0.79	0.99
	7.95	20.48	108.11	0.97	0.86	1.00	0.69	0.92

Table 11, Confidence parameters, Correntaio turn

Therefore, analyzing corner exit parameters that are the throttle confidence ones, interesting things come out. First of all both riders travel similar distances with partial throttle and both have equal parameters on the throttle max and sum value but there are big differences in mixed parameters, especially on the throttle and roll ones. That means rider two is able to open the throttle with the bike still at a high lean angle, he has a lot of confidence in opening the throttle at the corner center. He has no fear in rapidly open the throttle (high max value) and then is able to constantly increase the throttle position utill the wide open throttle condition (high cumulative value). Last but not least, note that throttle speed parameter is usually higher for rider two, since they have equal pure throttle confidence it means that rider two is going at higher speed than rider one.

Below it is proposed the analysis of the last chicane of imola circuit (Variante bassa). They are quite slow and tight corners in rapid succession.

Rider 1	Time	nuetral_phase	dist_brk	dist_partial	avg_speed	pitch_max	pitch_sum	pitch_roll_sum
	7.60	0.00	147.65	54.91	102.02	0.86	0.67	0.77
	7.50	0.00	140.91	59.97	103.59	0.77	0.84	0.86
	7.55	0.00	140.97	47.55	103.69	0.96	0.66	0.78
	7.50	0.00	172.35	67.29	104.20	1.00	0.45	0.79
	7.50	0.00	156.96	60.62	104.64	0.79	0.01	0.77
	7.55	0.00	158.21	69.09	103.91	0.48	0.24	0.73
Rider 2								
	7.70	21.01	157.65	39.63	101.67	0.65	0.47	0.64
	7.40	23.71	152.93	40.15	104.92	0.60	0.75	0.74
	7.40	23.97	140.15	42.44	105.60	0.57	0.74	0.68
	7.35	21.25	146.90	38.55	106.31	0.58	1.00	0.94
	7.50	10.32	149.73	57.51	104.72	0.73	0.78	1.00
	7.30	27.22	149.42	37.72	107.13	0.63	0.82	0.91

Table 12, confidence parameters, entering Variante bassa turn

A few consideration must be done analyzing these data. First of all both rider travel similar distances in braking phase and that lead to the next brake parameter that are the pitch ones. Look that rider two has higher cumulative parameters in pure pitch and mixed roll pitch values but lower maximum value. That means, rider one is able to load the front fork rapidly and at the beginning of the braking phase while rider two is loading it slowly but he is carrying this load trough the apex of the first turn. In the direction change then, he exploit the fork dynamics to let the bike front lift a little, recover part of the compression and let the bike turn easily. On the other hand rider one approaches the turn in a different way, he prefers to keep the bike loaded a lot and turn using the throttle, as you can see he covers almost the double of the space than rider one in partial throttle. That indicates that he is accelerating in the middle of the chicane to let the bike turn easily.

Apart from the time spent, that says that rider two is faster, is interesting to note that the correct behavior is the one used by rider one. Actually, at the end of the session, rider two reported tired muscles and less force in the arms after only few laps, while rider one was not affected by this issue. The software highlighted the cause of that problem that is the fact that rider two is not using throttle to turn the bike. As you can see below, throttle parameters reward rider one since as said before all the variables analyzed are though taking in consideration how the ideal behavior should be. By the way, despite having higher throttle parameters, rider one results to be slower in that part of the track but, it is very important to say that in qualifying and most important during the race the correct approach in riding the bike lead him to the top position.

Marfurt	thrt_max	thrt_sum	thrt_speed_max	thrt_roll_max	thrt_roll_sum
	0.83	0.52	0.27	0.74	0.58
	0.77	0.54	0.41	0.74	0.57
	1.00	0.51	0.51	0.63	0.48
	0.90	1.00	0.78	0.97	1.00
	0.89	0.62	0.80	0.91	0.60
	0.91	0.73	0.96	1.00	0.83
Surra					
	0.76	0.78	0.28	0.61	0.77
	0.84	0.79	0.42	0.59	0.80
	0.85	0.83	0.57	0.66	0.81
	0.85	0.84	0.67	0.66	0.80
	0.92	0.82	0.87	0.72	0.94
	0.63	0.65	1.00	0.53	0.75

Table 13, confidence parameters, exiting Variante bassa turn

5.6 Conclusion

Rider confidence software wants to give data analyst another instrument to understand what the rider is doing on the bike and let it be easy to understand. So, the output has been thought with that layout, key parameters before, corner entry, apex point and corner exit. Colors and time reference rapidly help the user in getting on what is going on during the lap. Furthermore, the opportunity to see all the laps of the selected sessions together permits to have a rough evaluation of where the rider is improving or where he is very constant, him strong and weak points.

One of the focus during the software development has been the creation of a code that is easy to modify. This is of key importance since parameters' definition can change rapidly from one track to another or over the years depending on the analyst experience. The installation of new sensors on the bike can also lead to definition of new interesting channels. The high flexibility of the software permits to adapt it also for car racing; obviously, the channel definition would change but the working scheme would remain the same. It can be adapted to control engine parameters, pressure, temperature and working points. The one proposed here is just one of the possible configuration and analysis type the program can perform.

As seen during this presentation, that software cannot be considered as a complete replacement of the classical data analysis but the merge of the two output easily leads to find out problems on riders' ride. It is an attempt to transform in numbers part of the rider feelings and understand where it feels strong and where not.

Suppose to do a procedure like the one about the analysis on the acque minerali turn. Without this software the analysis would have started from a random lap of one rider, then the other and eventually the comparison between them. At this point, the difference in speed between them would have been noticed and so a similar analysis performed taking the data from the throttle position and from brake pressure. Then, the difference in braking points should be noted and only at this point it would be clear that they are braking very differently. This method is surely correct but requires time, especially if you want to perform it lap by lap.

Here, a faster approach is proposed, not analyzing the lap meter by meter but using section defined before. Summarizing hundreds of meters in a number obviously makes faster the analysis but also less precise. That kind of approach is also useful since the parameters we are analyzing are almost about rider feelings not only related to bike's sensors. Using it during a proper race weekend has been very helpful in better understanding riders' feeling, giving sometimes an advice to bring out correctly their sensation and sometimes in predicting what their report would have been.

6 Conclusions

The development of this thesis has been for me a completely new adventure. Coming from automotive studies, at the beginning, my knowledge about motorcycle dynamics was almost zero; create something that could simulate it was a big motivation for me. The fact that in the end I have been able to use these software for something useful during a race weekend is the biggest satisfaction I could ask when I started.

Nowadays, simulation and fast data analysis is becoming more and more important to save money and time during the development of a product and even more in racing world to anticipate what will be the correct choice. Think for example that during a Formula one race there are many simulations going in parallel to predict what could be the best strategy to adopt. Despite being of less importance, simulation in motorcycle racing cannot be overlooked; rider still plays the major role in riding a motorcycle, especially when they are young and unexperienced as the one analysed in this document.

Thinking about future work on these programs, I think it would be very useful to create something that automatically check the rider confidence parameters highlighting issues, where data are discordant or there is much difference between riders or also between laps. About the simulation model, the first thing I would like to improve is the braking phase, putting on the equation of the force exerted given a brake pressure so that putting an ideal or real profile, effective braking behaviour can be simulated. It would be also useful to develop a predictive model that catch the optimal braking point instead of determine it by the velocity profile intersection. Eventually, even a sort of rider controller to follow a given roll profile can be implemented.

In a race weekend, time is always not enough between a session and another and find a way to speed up things is always useful. In some way, even if I am new to this world I made this work with the precise intention of make it ready and usable even when there is no time.

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Appendix A

Here below is reported the point mass model script and its output

Point mass model

```
tic
set(groot,'defaultfigureposition',[100 100 500 400])
set(groot,'defaultLineLineWidth',1.5)
set(groot,'defaultAxesFontSize',14)
clear all
close all
clc
```

Model setup

```
Pac_min=200;          %Avoid pacjeka formula failure at zero slip
coeff_mu=1.13;        %correction coefficient to Fy0
coeff_nc=1.30;        %correction coefficient to Vnc
limit_centroc=50;     %speed threshold [m/s] to select turn center speed
limit_curv=0.004;     %curvature threshold to select corner center
setfreb='R6_14';      %front rebound setting
setfcomp='C7_14';     %front compression setting
setrreb='R24F_10';    %rear rebound setting
setrcomp='C23_9';     %rear compression setting
track='cremona_alby';
```

Motorcycle data

```
bikedata_TWP
Fz1s=m*g*bg/wb;
Fz2s=m*g*ag/wb;
Fy0=2200;
```

Import the data

```
[~, ~, raw] = xlsread(strcat('C:\Users\carlo\Desktop\',track),strcat(track),'A3:K9500');
raw(cellfun(@(x) ~isempty(x) && isnumeric(x) && isnan(x),raw)) = {''};
```

Replace non-numeric cells with NaN

```
R = cellfun(@(x) ~isnumeric(x) && ~islogical(x),raw); % Find non-numeric cells
raw(R) = {NaN}; % Replace non-numeric cells
```

Create output variable

```
data = reshape([raw{:}],size(raw));
```

Create table

```
Circui = table;
```

Allocate imported array to column variable names

Circui.Time = data(:,1);
Circui.Dist = data(:,2);
Circui.Curvature = smooth(data(:,3),70);
Circui.V_GPS = data(:,4);
Circui.Latitude = data(:,5);
Circui.Longitude = data(:,6);
Circui.Alt = data(:,7);
Circui.Lunghezza = data(:,8);
Circui.delta_spazio = data(:,9);
Circui.spazio_cumulato = data(:,10);
Circui.rollio = atan(((Circui.V_GPS/3.6).^2.*Circui.Curvature)/g);

Clear temporary variables

```
clearvars data raw R;
Circuit=rmmissing(Circui);
```

Preliminary calculations

```
[PicchiM,PosizioniM]=findpeaks(Circuit.Curvature);
[PicchiL,PosizioniL]=findpeaks(-Circuit.Curvature);
[tabella]=[[PosizioniM; PosizioniL] [PicchiM; PicchiL]];
tab_ord=sortrows(tabella);
Posizione=tab_ord(:,1);
Picchi=tab_ord(:,2);
indices=find(abs(Picchi)<0.005); %value to detect turns
Picchi(indices)=[];
Posizione(indices)=[];
load pac_r.mat
load pac_f.mat
```

Torque definition

```
rpmm=xlsread('C:\Users\carlo\Desktop\CIV\prove_motore_modello\2WP_2018.1.xlsx','A:A')*(2*pi/60);
eff=0.85;%rpm
torquee=eff*xlsread('C:\Users\carlo\Desktop\CIV\prove_motore_modello\2WP_2018.1.xlsx','C:C');
%torque
A=[rpmm torquee];
[B,ida]=unique(A(:,1));
rpm=A(ida,1);
torque=A(ida,2);
```

Equivalent masses definition

gear ratio

```
ts=35/18;            %crown/pinion
tp=73/24;            %primary ratio
t1=32/13;            %gear ratio I
t2=32/16;            %gear ratio II
t3=28/17;            %gear ratio III
t4=26/19;            %gear ratio IV
t5=25/21;            %gear ratio V
tau=[t1 t2 t3 t4 t5];
rpm_max=[13000 13000 13000 13000 13200]*2*pi/60;
v_max=zeros(1,length(tau));
me=zeros(1,length(tau));
for xj=1:length(tau)
    v_max(xj)=1.87*rpm_max(xj)*60/(2*pi)*60/1000*1/(ts*tp*tau(xj))*1/3.6;
```

```
me(xj)=m+Iwr*(1/rw)^2+cI*((ts*tp*tau(xj))*1/rw)^2+ccI*((tau(xj)*ts)*1/rw)^2+csI*((ts)^2*1/rw)^2;
end
```

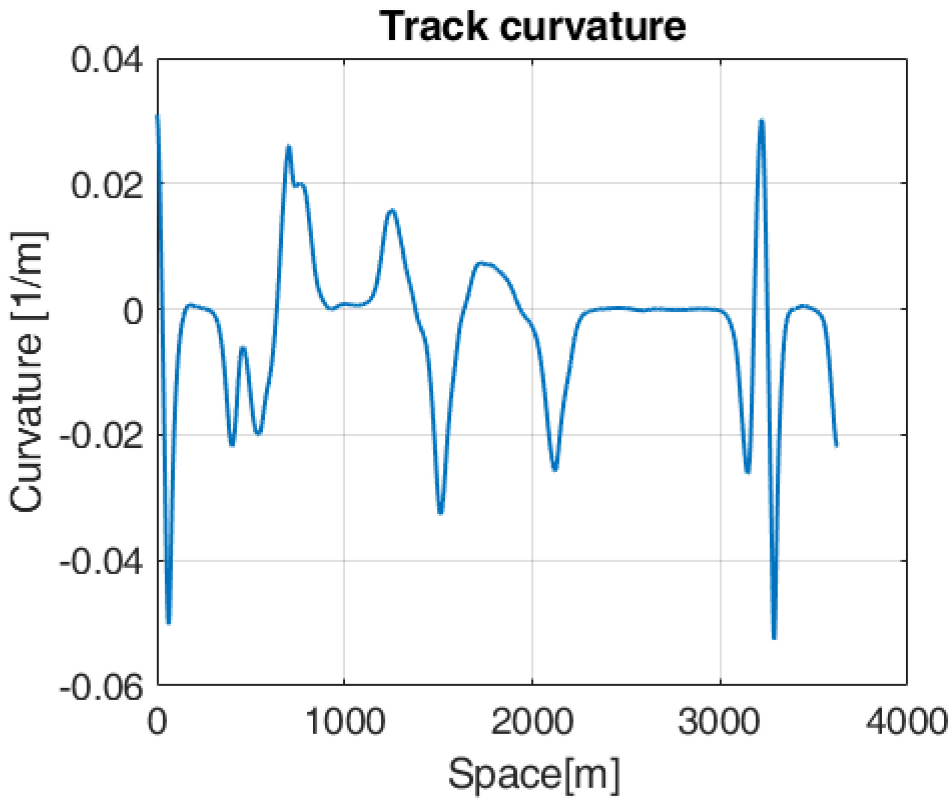
Track lenght determination, first speed profile

```
v_lim=70;
v_nc=nthroot((Fy0^2)./(m^2*Circuit.Curvature.^2+b^2),4);
vel1=min(v_lim,v_nc);
dt=vel1.^-1;
Dis=cumsum(vel1.*dt);
[v_centrocurva,step_centrocurva_raw]=findpeakssV2(vel1,Circuit.Curvature,limit_centroc,limit_curv);
n=1;
dummy=0;
len=zeros(1,length(Picchi));
for i=1:length(Picchi)
    flag=0;
    for j=7:length(Picchi) % index starts from 7 since every track has at least seven corners
        if Picchi(j)>Picchi(i)*0.92 && Picchi(j)<Picchi(i)*1.08
            len(i)=Posizione(j)-Posizione(i);
            flag=1;
        end
    end
    if i==1 && flag~=0
        dummy(n)=len(i);
    end
    if i>1 && flag~=0 && dummy(n)~=0
        if len(i)~=0 && len(i)>0.99*dummy(n) && len(i)<1.03*dummy(n)
            n=n+1;
            dummy(n)=len(i);
        end
    elseif i>1 && flag~=0 && dummy(n)==0
        dummy(n)=len(i);
    end
end
if dummy(n)~=0
step_circuito=round(mean(dummy));
else
    step_circuito=len(1);
end
```

Real variables and input preparation

```
TT=(0:0.02:300);
v_real=Circuit.V_GPS(step_centrocurva_raw(1):step_centrocurva_raw(end))/3.6;
acc_real=movmean(diff(v_real)/0.02,10);
acc_real(length(acc_real)+1)=acc_real(end);
spazio_cumulato_real=Circuit.spazio_cumulato(step_centrocurva_raw(1):step_centrocurva_raw(end))-
Circuit.spazio_cumulato(step_centrocurva_raw(1));
curvatura_tracciato=Circuit.Curvature(step_centrocurva_raw(1):step_centrocurva_raw(end));
deriv_curvatura=movmean(diff(curvatura_tracciato)/0.02,10);
deriv_curvatura(length(deriv_curvatura)+1)=deriv_curvatura(end);
delta_z=diff(movmean(Circuit.Alt(step_centrocurva_raw(1):step_centrocurva_raw(end)),10));
delta_z(length(delta_z)+1)=delta_z(length(delta_z));
slope=smooth(atan(delta_z./Circuit.delta_spazio(step_centrocurva_raw(1):step_centrocurva_raw(end)))
,50);
comp_acc_grav=g.*sin(slope);
rollio_real=smooth(Circuit.rollio(step_centrocurva_raw(1):step_centrocurva_raw(end)),40);
rollio_real=Circuit.rollio(step_centrocurva_raw(1):step_centrocurva_raw(end));
roll_rate_real=movmean(diff(rollio_real)/0.02,10);
```

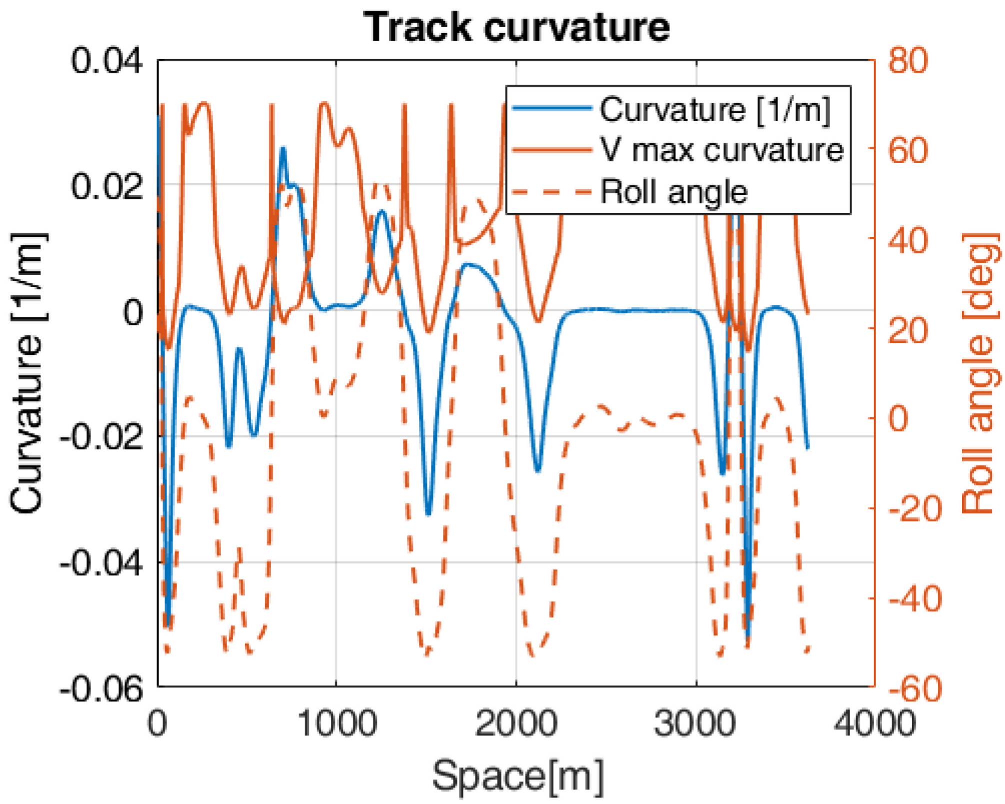
```
roll_rate_real(length(roll_rate_real)+1)=roll_rate_real(end);
roll_acc_real=movmean(diff(roll_rate_real)/0.02,10);
roll_acc_real(length(roll_acc_real)+1)=roll_acc_real(end);
Time=TT(1:length(curvatura_tracciato));
yaw_acc_real=acc_real.*curvatura_tracciato+v_real.*deriv_curvatura;
yaw_primo_real=v_real.*curvatura_tracciato;
yaw_real=cumtrapz(Time,yaw_primo_real);
[tab_cur]=[spazio_cumulato_real curvatura_tracciato Time'];
figure
plot(spazio_cumulato_real,curvatura_tracciato);
hold on
grid on
xlabel('Space[m]')
ylabel('Curvature [1/m]')
title('Track curvature')
```

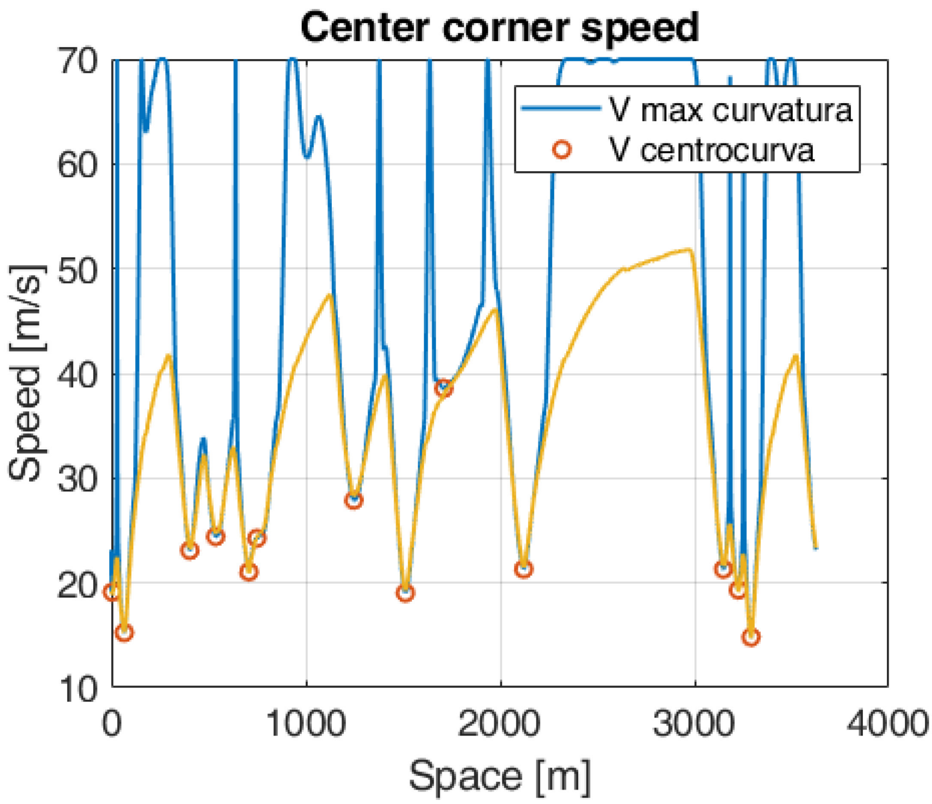


Curvature dependent speed profile

```
spazio_cumulato_nc(1)=0;
swap=vel1(1);
clear v_nc vel1 phi
vel1(1)=swap;
n=1;
%using real space lenght to be sure to travel at least the required
%length(V_nc>>Vreal)
for i=2:length(spazio_cumulato_real)
    phi=interp1(tab_cur(:,1),rollio_real,spazio_cumulato_nc(i-1));
    lambda_r(i)=lininterp2((-1.13:0.01:1.13),linspace(1,2300,100),alpha_r,phi,Fz2s);
    lambda_f(i)=lininterp2((-1.13:0.01:1.13),linspace(1,2300,100),alpha_f,phi,Fz1s);
    Fy0_r(i)=max(Pac_min,abs(coeff_mu*lininterp3(linspace(-2,2,100),(-
1.13:0.01:1.13),linspace(1,2300,100),Fpac_r,lambda_r(i),phi,Fz2s)));
    Fy0_f(i)=max(Pac_min,abs(coeff_mu*lininterp3(linspace(-2,2,100),(-
1.13:0.01:1.13),linspace(1,2300,100),Fpac_f,lambda_f(i),phi,Fz1s)));
    cur(i)=interp1(tab_cur(:,1),tab_cur(:,2),spazio_cumulato_nc(i-1));
```

```
    v_nc(i)=coeff_nc*nthroot((Fy0_f(i)+Fy0_r(i))^2/((m*cur(i))^2+b^2),4);
    vel1(i)=min(v_lim,v_nc(i));
    dt=vel1(i)^-1;
    acc_nc(i)=(vel1(i)-vel1(i-1))/dt;
    spazio_cumulato_nc(i)=spazio_cumulato_nc(i-1)+vel1(i)*dt;
    if spazio_cumulato_nc(i)>spazio_cumulato_real(end)
        break
    end
end
yyaxis right
plot(spazio_cumulato_nc,vel1)
ylabel('Roll angle [deg]')
hold on
plot(spazio_cumulato_real,rollio_real*180/pi)
legend('Curvature [1/m]','V max curvature','Roll angle')
[v_centrocurva,step_centrocurva_nc]=findpeakssV2(vel1,cur,limit_centroc,limit_curv);
clear dummy
dummy=0;
for i=2:length(v_centrocurva)
    if step_centrocurva_nc(i)<step_centrocurva_nc(i-1)+5
        dummy=dummy+1;
        indice(dummy)=i-1;
    end
end
if dummy~=0
    v_centrocurva(indice)=[];
    step_centrocurva_nc(indice)=[];
end
v_centrocurva_c=v_centrocurva;
pos_centrocurva=spazio_cumulato_nc(step_centrocurva_nc);
figure
plot(spazio_cumulato_nc,vel1,spazio_cumulato_nc(step_centrocurva_nc),v_centrocurva_c,'o')
grid on
hold on
plot(spazio_cumulato_real,v_real);
legend('V max curvatura','V centrocurva');
title('Center corner speed')
xlabel('Space [m]')
ylabel('Speed [m/s]')
for i=2:length(step_centrocurva_nc)
    step_raw(i-1)=spazio_cumulato_nc(step_centrocurva_nc(i))-
spazio_cumulato_nc(step_centrocurva_nc(i-1));
end
```



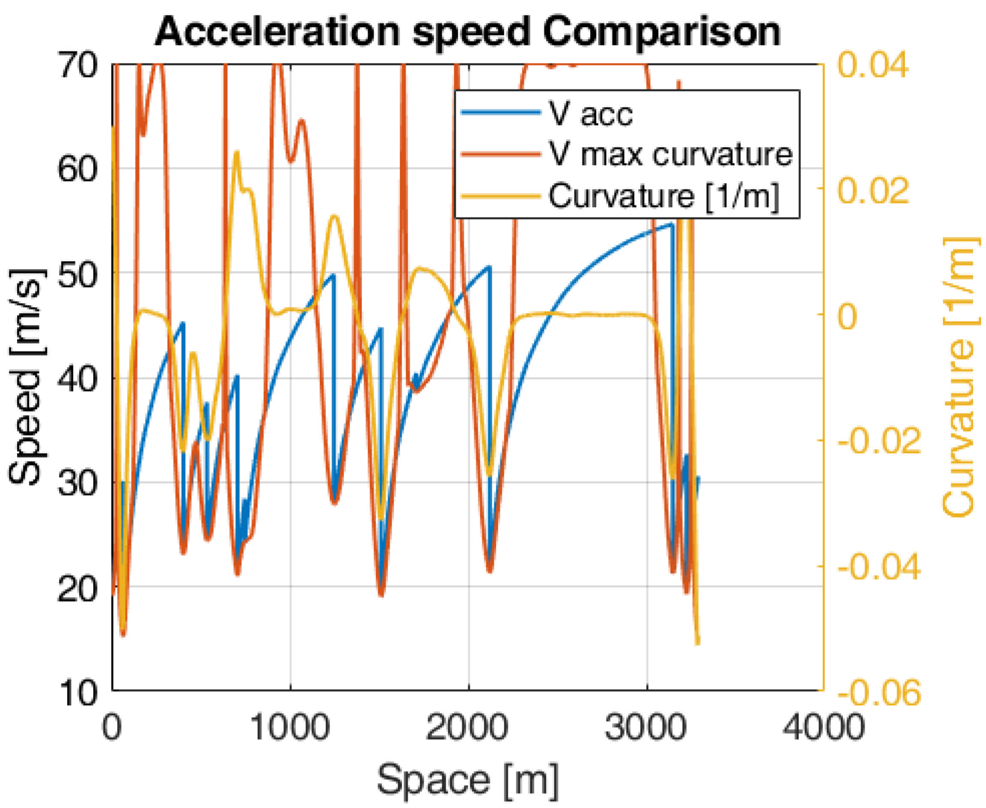


Acceleration phase

```
for j=1:length(step_raw)
    clear vz ap
    step=round(step_raw(j));
    r1=linspace(12,22,10);
    r2=linspace(33,24,10);
    mx=(max(r2)-min(r2))/(min(r1)-max(r1));
```

```
    const=(max(r2)-min(r2))/(min(r1)-max(r1))*min(r1)-max(r2);
    r22in=mx*min(r1)-const;
    ttauin=r22in/min(r1);
    we=(v_centrocurva_c(j)*(ttauin*tp*ts))/rw;
    pri=min(r1);
     while we > 1382 && pri<24
      pri=pri+1;
      r22in=mx*min(r1)-const;
      ttauin=r22in/pri;
      we=(v_centrocurva_c(j)*(ttauin*tp*ts))/rw;
     end
    Tz(1)=interp1(rpm,torque,we);
    ap(1)=1/(me(1)*kr)*(ttauin*tp*ts*Tz(1)/(rw)-b*v_centrocurva_c(j)^2);
    vz(1)=v_centrocurva_c(j);
    vz2(1)=v_centrocurva_c(j);
    if vz(1)<v_max(1)
        z=1;
    elseif vz(1)>v_max(1) && vz(1)<v_max(2)
        z=2;
    elseif vz(1)>v_max(2) && vz(1)<v_max(3)
        z=3;
    elseif vz(1)>v_max(3) && vz(1)<v_max(4)
        z=4;
    elseif vz(1)>v_max(4) && vz(1)<v_max(5)
        z=5;
    end
    gear(1)=z;
    clear limite spazio_singolo_step dt gear
    limite(1)=0;
    dt(1)=vz(1)^-1;
    spazio_singolo_step(1)=0;
    for n=2:1500
        if limite(end)<step_raw(j)
            comp=interp1(tab_cur(:,1),comp_acc_grav,pos_centrocurva(j)+limite(end));
            we1(n)=(vz(n-1)*tau(z)*ts*tp)/rw;
            Tz(n)=interp1(rpm,torque,we1(n));
            ap(n)=1/(me(z)*kr)*(tau(z)*tp*ts*Tz(n)/(rw)-b*vz(n-1)^2-comp);
            vz(n)=vz(n-1)+ap(n)*dt(n-1);
            vz2(n)=sqrt(vz2(n-1)^2+2*ap(n));
            if vz(n)<v_max(1)
                z=1;
            elseif vz(n)>v_max(1) && vz(n)<v_max(2)
                z=2;
            elseif vz(n)>v_max(2) && vz(n)<v_max(3)
                z=3;
            elseif vz(n)>v_max(3) && vz(n)<v_max(4)
                z=4;
            elseif vz(n)>v_max(4) && vz(n)<v_max(5)
                z=5;
            end
            gear(n)=z;
            dt(n)=vz(n)^-1;
            limite(n)=limite(n-1)+1;
            spazio_singolo_step(n)=1;
        else
            if j~=length(step_raw)
                spazio_singolo_step(n-1)=pos_centrocurva(j+1)-
(pos_centrocurva(j)+limite(length(limite)-1));
            end
```

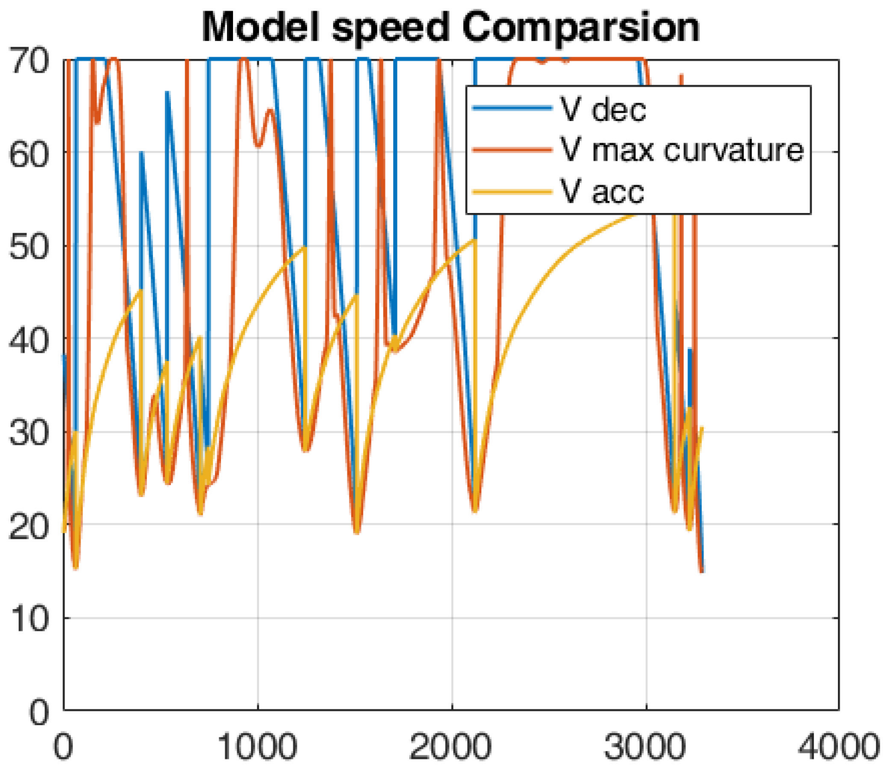
```
            if v_centrocurva_c(j+1)>=vz(n-1)
                v_centrocurva_c(j+1)=vz(n-1);
            end
            break
        end
    end
    Vo{j} = vz;
    SP{j} = spazio_singolo_step;
    Torq{j} = Tz;
    vet_acc{j}=ap;
    tempo{j}=dt;
    gears{j}=gear;
end
vopt_a=cell2mat(Vo);
metri_acc=cell2mat(SP);
acc_raw=cell2mat(vet_acc);
tempo_acc=cell2mat(tempo);
spazio_percorso_acc=cumsum(metri_acc)+pos_centrocurva(1);
vel1_acc=interp1(spazio_cumulato_nc,vel1,spazio_percorso_acc);
curve=interp1(spazio_cumulato_real,curvatura_tracciato,spazio_percorso_acc);
figure
plot(spazio_percorso_acc,vopt_a,spazio_percorso_acc,vel1_acc)
xlabel('Space [m]')
ylabel('Speed [m/s]')
grid on
hold on
yyaxis right
plot(spazio_percorso_acc,curve)
ylabel('Curvature [1/m]')
legend('V acc','V max curvature','Curvature [1/m]')
title('Acceleration speed Comparison')
```



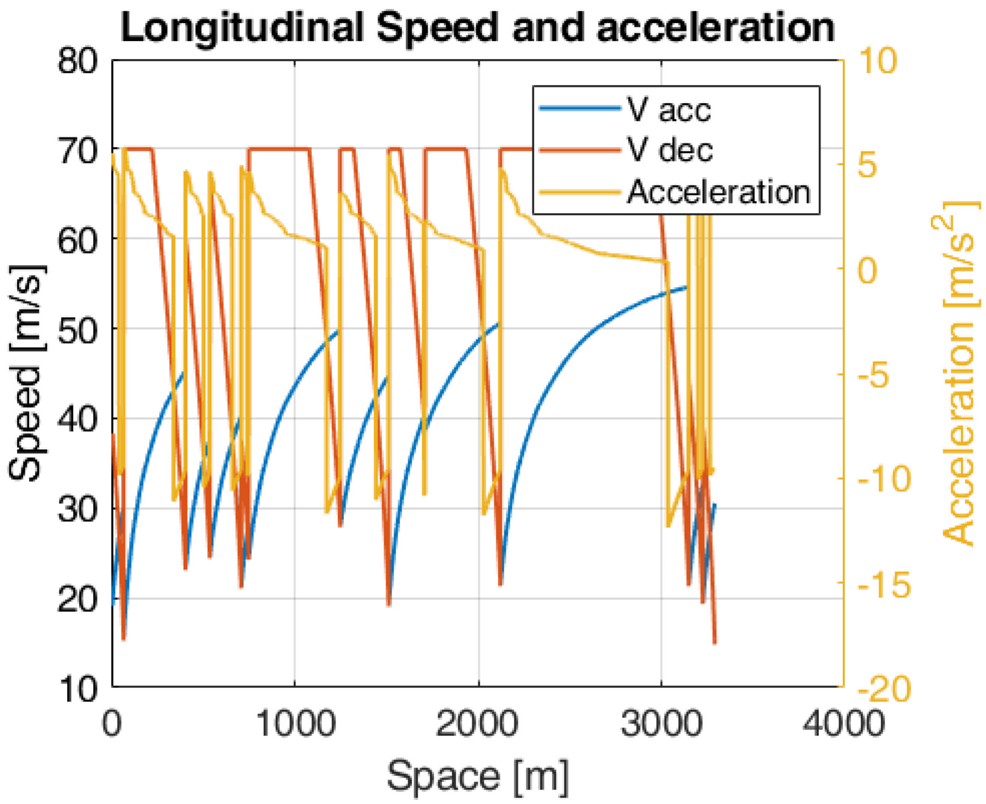
Braking phase

```
clear SP x index spazio_percorso_raw dt tempo
for jj=1:length(step_raw)
    clear limite vz spazio_singolo_step a2 a4 spazio_percorso vopt dt
    spazio_singolo_step(1)=0;
    vz(1)=v_centrocurva_c(jj+1);
    limite(1)=0;
    dt(1)=vz(1)^-1;
    for i=2:1500
        if limite(end)<step_raw(jj)
            acc_ribaltamento(i)=(g*(wb-bg)/hg+0.5*ro/m*S*1.3*Cx*vz(i-1)^2);
            a2(i)=acc_ribaltamento(i);
            if vz(i-1)<70
                vz(i)=vz(i-1)+a2(i)*dt(i-1);
                dt(i)=vz(i)^-1;
            elseif vz(i-1)>=70
                vz(i)=70;
                a2(i)=0;
                dt(i)=1/70;
            end
            limite(i)=limite(i-1)+1;
            a4(i)=a2(i)/vz(i);
            spazio_singolo_step(i)=1;
        else
            spazio_singolo_step(i-1)=pos_centrocurva(jj+1)-
(pos_centrocurva(jj)+limite(length(limite)-1));
            break
        end
    end
    vd{jj}=flip(vz); %backward integration
    SP{jj} = flip(spazio_singolo_step);
    dec_vet{jj}=flip(a2);
    tempo{jj}=flip(dt);
end
vopt_d=cell2mat(vd);
metri_dec=cell2mat(SP);
spazio_percorso_dec=cumsum(metri_dec)+pos_centrocurva(1);
tempo_dec=cell2mat(tempo);
vel1_dec=interp1(spazio_cumulato_nc,vel1,spazio_percorso_dec);
figure
plot(spazio_percorso_dec,vopt_d,spazio_percorso_dec,vel1_dec)
grid on
hold on
plot(spazio_percorso_acc,vopt_a)
ylim([0 70])
legend('V dec','V max curvature','V acc')
title('Model speed Comparsion')
clear vopt acc x index gears
for i=1:length(vopt_d)
    if vopt_d(i)<=vopt_a(i)
        vopt_raw(i)=vopt_d(i);
        vet_tempo_raw(i)=tempo_dec(i);
    else
        vopt_raw(i)=vopt_a(i);
        vet_tempo_raw(i)=tempo_acc(i);
    end
    if vopt_raw(i)<v_max(1)
       z=1;
       elseif vopt_raw(i)>v_max(1) && vopt_raw(i)<v_max(2)
       z=2;
```

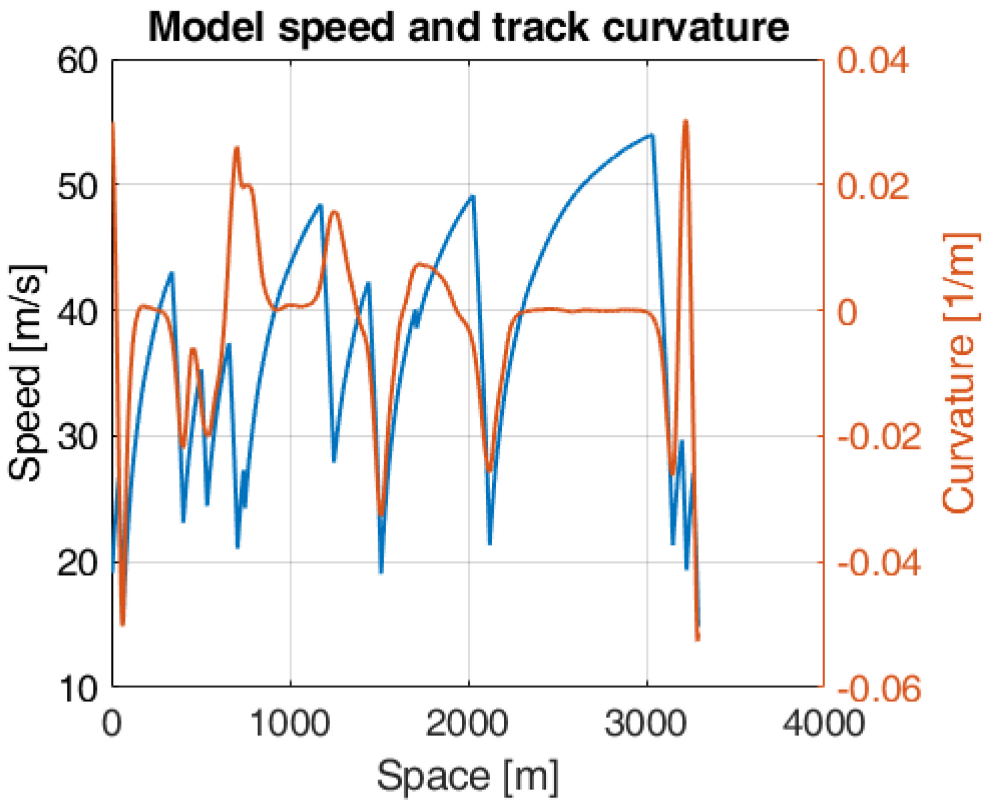
```
        elseif vopt_raw(i)>v_max(2) && vopt_raw(i)<v_max(3)
        z=3;
        elseif vopt_raw(i)>v_max(3) && vopt_raw(i)<v_max(4)
        z=4;
        elseif vopt_raw(i)>v_max(4)
        z=5;
    end
    gears_raw(i)=z;
end
acc_raw=diff(vopt_raw)./vet_tempo_raw(1:length(vet_tempo_raw)-1);
acc_raw(length(acc_raw)+1)=acc_raw(length(acc_raw));
[spazio_all,index]=unique(spazio_percorso_acc);
acc=acc_raw(index);
vopt=vopt_raw(index);
gears=gears_raw(index);
vet_tempo=vet_tempo_raw(index);
clear curve
curve=interp1(spazio_cumulato_real,curvatura_tracciato,spazio_all);
figure
plot(spazio_percorso_acc,vopt_a,spazio_percorso_dec,vopt_d)
ylabel 'Speed [m/s]'
xlabel 'Space [m]'
hold on
yyaxis right
plot(spazio_all,acc)
ylim([-20 10])
ylabel 'Acceleration [m/s^2]'
grid on
legend('V acc','V dec','Acceleration')
title('Longitudinal Speed and acceleration')
figure
plot(spazio_all,vopt)
grid on
xlabel('Space [m]')
ylabel('Speed [m/s]')
yyaxis right
plot(spazio_all,curve)
ylabel('Curvature [1/m]')
title('Model speed and track curvature')
```



Model speed Comparsion
V dec
V max curvature
V acc
70
60
50
40
30
20
10
0
0
1000
2000
3000
4000



Longitudinal Speed and acceleration
V acc
V dec
Acceleration
Speed [m/s]
Acceleration [m/s^2]
Space [m]
80
70
60
50
40
30
20
10
10
5
0
-5
-10
-15
-20
0
1000
2000
3000
4000

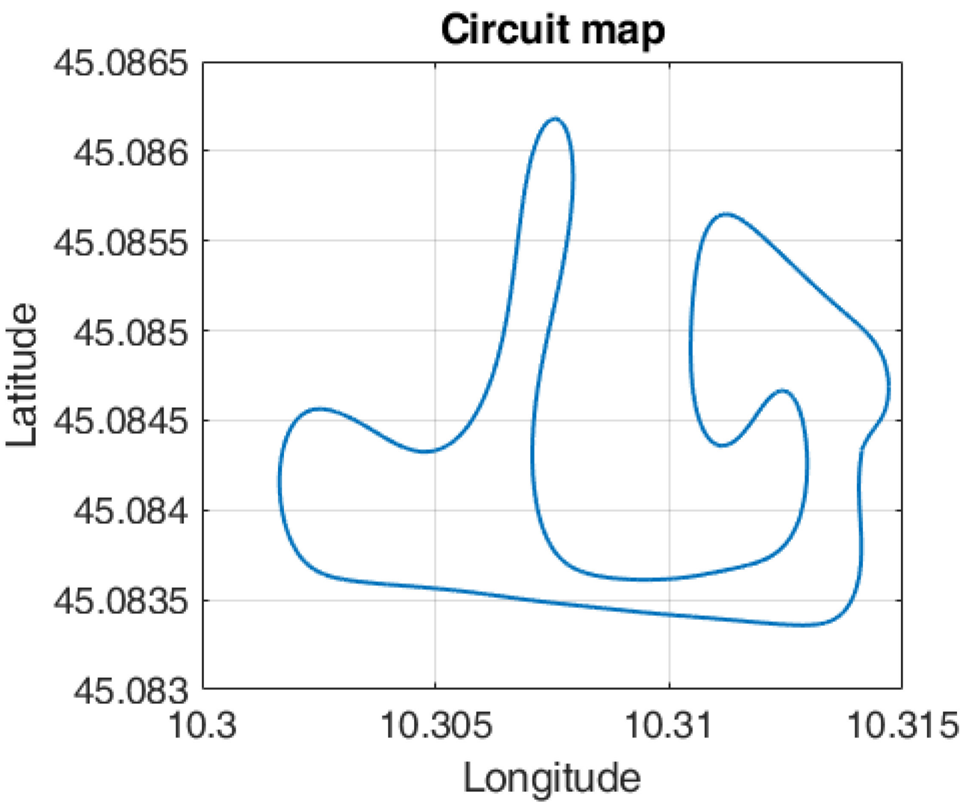


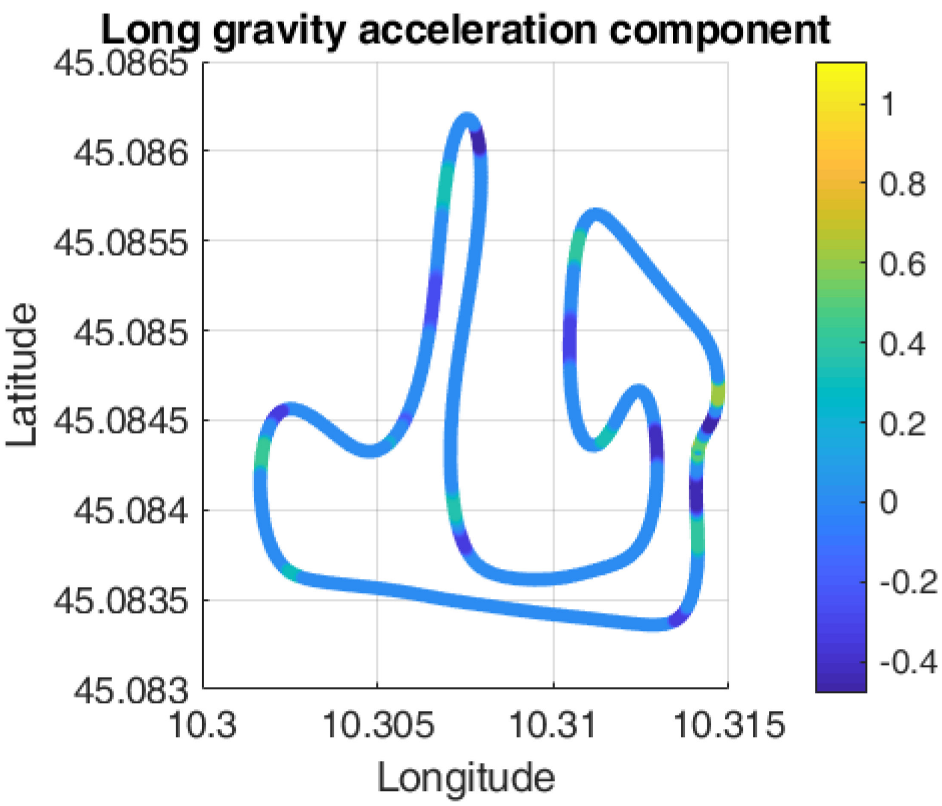
Laptime calculation

```
Lunghezza_real=spazio_cumulato_real(step_circuito);
tlap_real=Time(step_circuito);
zzz=step_circuito;
windowsize=70;
b=(1/windowsize)*ones(1,windowsize);
a=1;
Lat_f=filtfilt(b,a,Circuit.Latitude(Posizione(1):Posizione(1)+step_circuito-1));
Long_f=filtfilt(b,a,Circuit.Longitude(Posizione(1):Posizione(1)+step_circuito-1));
[tab_coordinate]=[spazio_cumulato_real(1:step_circuito) Lat_f Long_f];
figure
plot(Long_f,Lat_f)
grid on
title('Circuit map')
xlabel 'Longitude'
ylabel 'Latitude'
for i=1:length(vopt)
    if spazio_all(i)<=Lunghezza_real
       v_media=mean(vopt(1:i));
    elseif spazio_all(i)>Lunghezza_real
        break
    end
end
spazio_cumulato_tracciato=spazio_all(1:i);
step_circuito=i;
tlap=sum(vet_tempo(1:step_circuito));
disp(strcat('Point mass model lap time_', num2str(tlap), ' sec'));
disp(strcat('Real lap time_', num2str(tlap_real), ' sec'));
figure
scatter(Long_f,Lat_f,20,comp_acc_grav(1:zzz));
grid on
title('Long gravity acceleration component')
```

```
xlabel 'Longitude'
ylabel 'Latitude'
colorbar
```

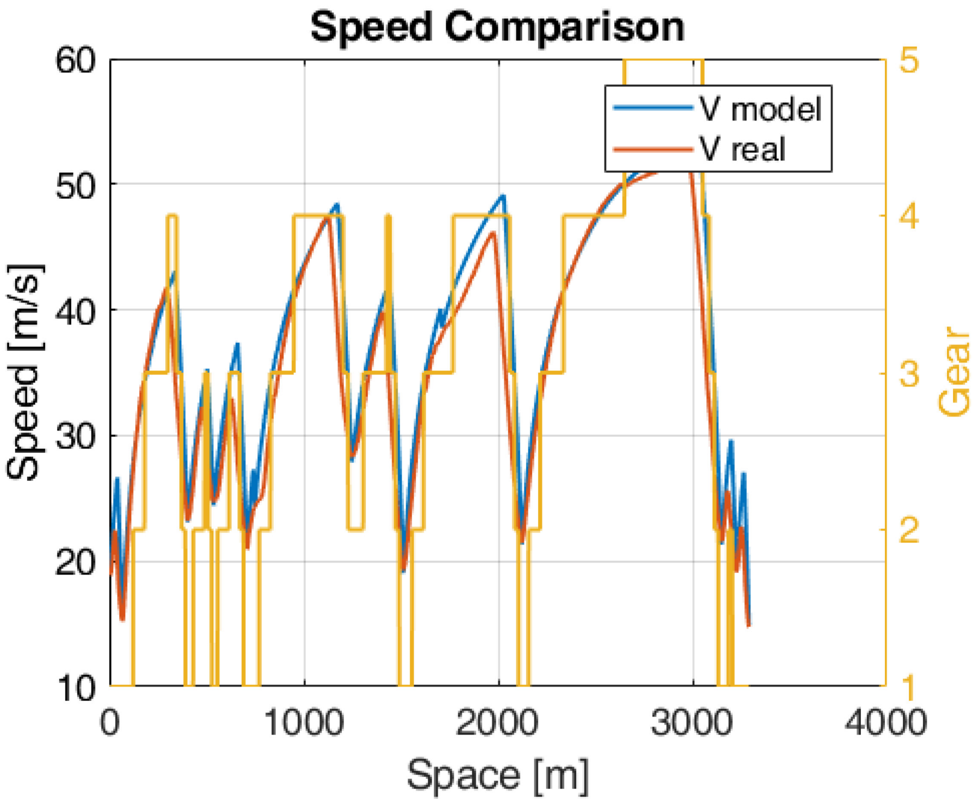
```
Point mass model lap time_88.0926 sec
Real lap time_95.58 sec
```





Real speed comparison

```
clear v_real
v_real=interp1(spazio_cumulato_real,Circuit.V_GPS(step_centrocurva_raw(1):step_centrocurva_raw(leng
th(step_centrocurva_raw))),spazio_all);
figure
plot(spazio_all,vopt,spazio_all,v_real/3.6)
ylabel('Speed [m/s]')
yyaxis right
plot(spazio_all,gears);
title('Speed Comparison')
grid on
legend('V model','V real')
xlabel('Space [m]')
ylabel('Gear')
toc
```




```
Elapsed time is 21.342898 seconds.
```

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