POLITECNICO DI TORINO

Dipartimento di Ingegneria Meccanica e Aerospaziale

The Master of Science in Mechanical Engineering

Master Thesis

"Optimization of a Mathematical Model for the Stiffness Calculation of a Cradle for Front Suspension Axle"

Superadvisor
Prof. Alessandro Fasana

Candidate
Ozgun Tugce Ibis

Internship Tutor,
FCA, R&D Chassis & Vehicle Dynamics
Eng. Andrea Taurino

Academic Year 2017/2018

ABSTRACT

The aim of this study is to identify the main parameters that should be considered for stiffness analysis of a cradle (mechanical cross member) for the front suspension axle and optimization of a mathematical model for the stiffness calculation of this specific suspension part of a car. So that this numerical formulation can be used to create a tool which can be used for verification and manufacturing of a mechanical component without affecting its reliability.

The study consist of two stages;

In the first part, the main features of the suspension of a car, with particular reference to the front suspension, is represented and the geometry of the cradle model is described. The cradle design of ***Jeep Renegade*** is used as a reference model. Based on this reference model, three simplified cradle models, created with different geometries, are introduced. Subsequently fundamentals of Finite Element Analysis method is explained and the discretization errors and convergence methods are mentioned briefly.

In the second part, the analytical results obtained by FEM analysis compared with the experimental results obtained by non-FEM analysis and the mathematical model, created as an alternative calculation method, is presented. At this point, for CAD modelling and simulations, NX software program was used.

Finally the possible future developments are discussed. This study is an initial attempt to introduce a nonconventional future tool which may enable to make mathematical computations with higher accuracy and less error percentage in a very short time, compared to FEM analysis and can be used for verification and manufacturing of a mechanical component in a more efficient way with improving the parts reliability.

This study was conducted in ***FCA Group - R & D Chassis & Vehicle Dynamics Department.*** Data used for this study were obtained from the current cradle models situated in FCA database.

All the experimental processes and the statistical approaches carried out for the implementation of the mathematical model of stiffness analysis are presented in this study. All the numerical formulations were developed and optimized based on DFFS (Design for Six Sigma) methodology.

Table of Contents

List of Figures

List of Tables

Chapter 1

Vehicle Suspension System

1.1 Introduction

The chassis of a car is composed of *the structural frame*, *the suspension system*, *the steering system*, and their connection between *the tires & wheels* basically. The suspension system is one of these major systems in a vehicle which can be seen in **Figure 1**.

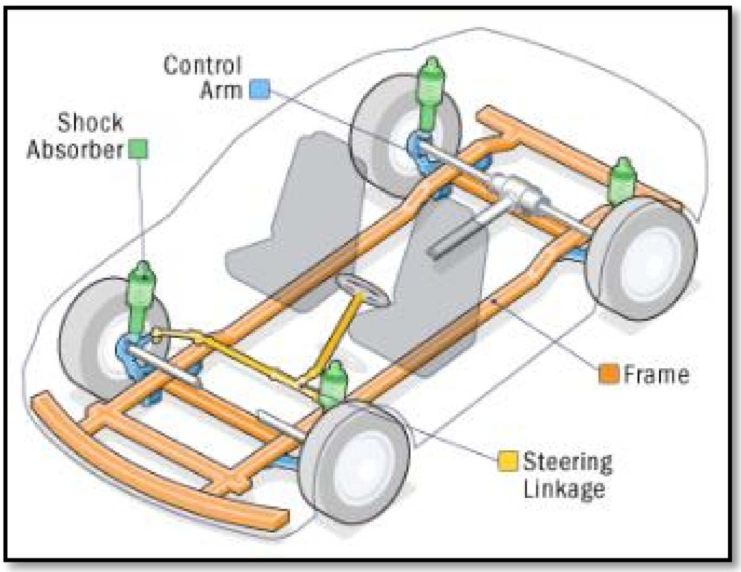


Figure 1: Chassis of a Car Body [8]

The suspension system is a combination of elastic and damping elements which connects the tire to the main frame (chassis) of the vehicle. Its main duty is to ensure the contact between the wheel and the road and minimize the vertical forces which may be forwarded to the passenger compartment.

Ensuring the wheel-road surface contact and tire load fluctuation is significant in order to provide a reliable ride and handling performance. Moreover the wheels should be maintained in the proper position in order to ensure the steering control during maneuver.

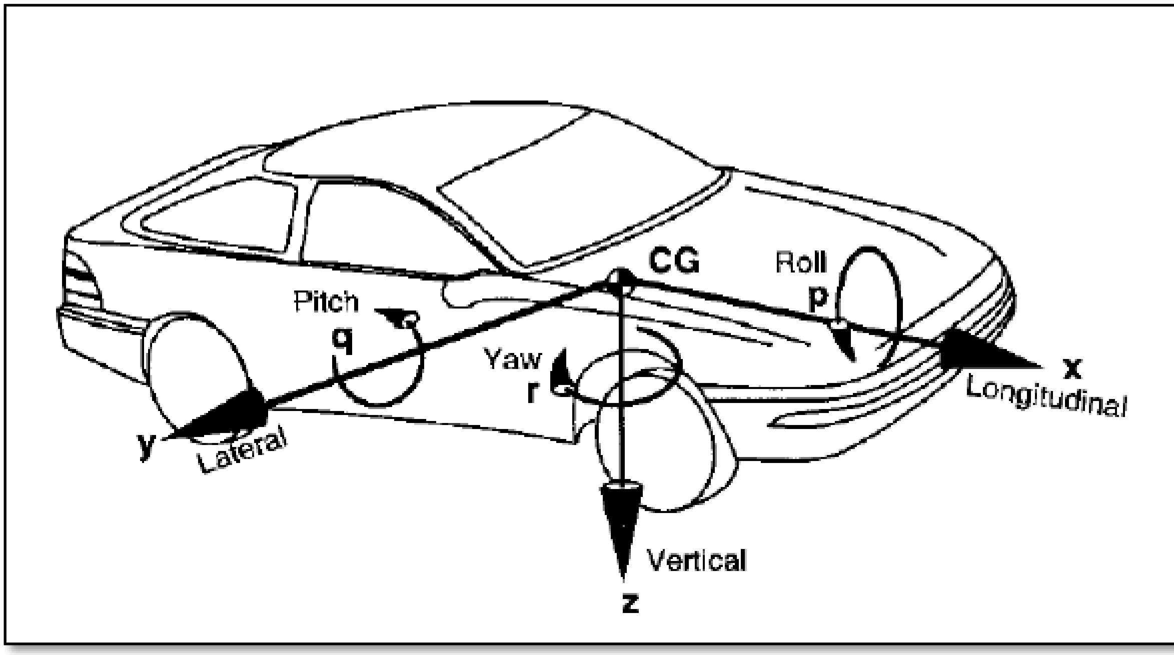


Figure 2: Vehicle Axis System [9]

In **Figure 2**, the orthogonal axes system fixed to a vehicle is illustrated. It can be seen that the horizontal x-axis points forward and it is in the longitudinal plane of symmetry while the y-axis points the driver's RHS and z- axis points downward. The rotations are stated as "*rolling*" about x-axis, "*pitching*" about y axis and "*yawing*" about z-axis. [9]

The basic components of a suspension system are shown in **Figure 3** as follows:

- Control Arm (a movable lever that fastens the steering knuckle to the vehicle frame or body)
- Control Arm Bushing (a sleeve that allows the control arm to move up and down on the frame)
- Strut Rod (prevents the control arm from swinging to the front or rear of the vehicle)
- Ball Joint (a swivel joint that allows the control arm and steering knuckle to move up and down, as well as side to side)
- Shock Absorber or Strut (keeps the suspension from continuing to bounce after spring compression and extension)
- Stabilizer Bar (limits body roll of the vehicle during cornering)

- Spring (supports the weight of the vehicle, permits the control arm and wheel to move up and down)

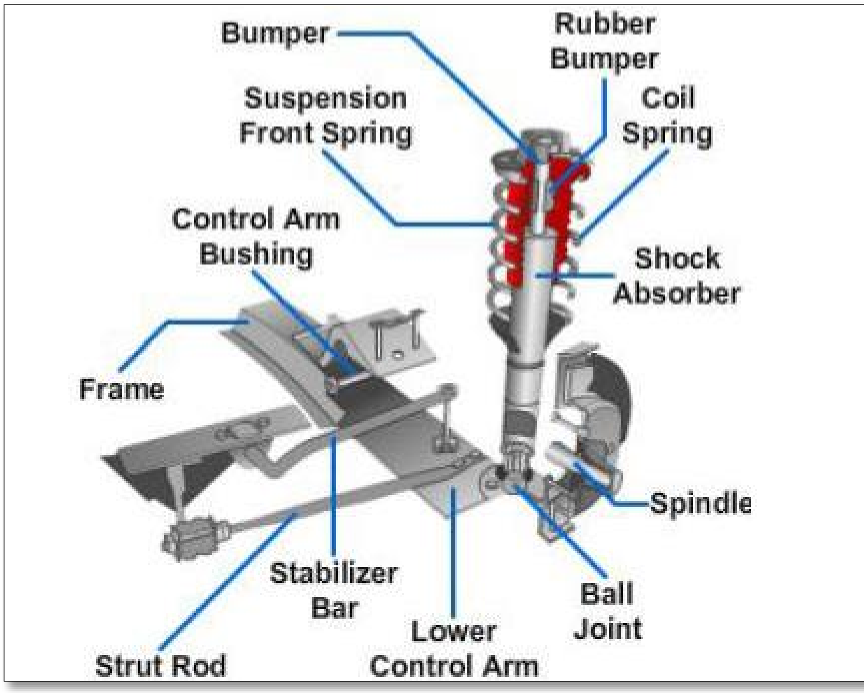


Figure 3: The basic components of a suspension system [11]

Springs

Springs are used to absorb wheel vibration and decrease the overall oscillations on the vehicle.

The springs are located in the car between the wheels and the frame. Based on their location, the total mass can be defined as *sprung mass* and *unsprung mass*.

Sprung mass is the mass which is supported by the springs (suspended mass) while the unsprung mass is the mass between the road and the suspension springs (unsuspended mass).

Springs are great for energy absorption but yet not enough for a whole suspension system as the energy needs to be dissipated. For this reason, dampers are needed as well.

Dampers (Shock Absorbers)

Unwanted spring motion can be dissipated by using dampers in the suspension system. Shock absorbers decrease the magnitude of the vibration by converting the kinetical energy into heat energy which can be dissipated through hydraulic fluid.

Basically the shock absorbers is located between the wheel and the car. It works in two cycles; compression and extension. As the piston moves downward during compression, it compresses the hydraulic fluid in the chamber below the piston. As the piston moves upward the top of the pressure tube during extension movement, the fluid in the chamber moves above the piston (**Figure 4**).

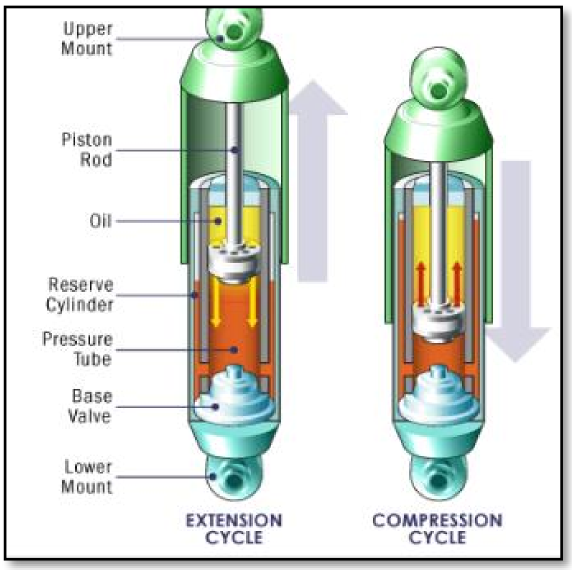


Figure 4: Twin-Tube Shock Absorber [8]

Strut is another damping structure. It is basically a shock absorber which is placed inside a coil spring. It provides structural support for the vehicle suspension and supports the weight and dampening action just as the shock absorber (**Figure 5**).

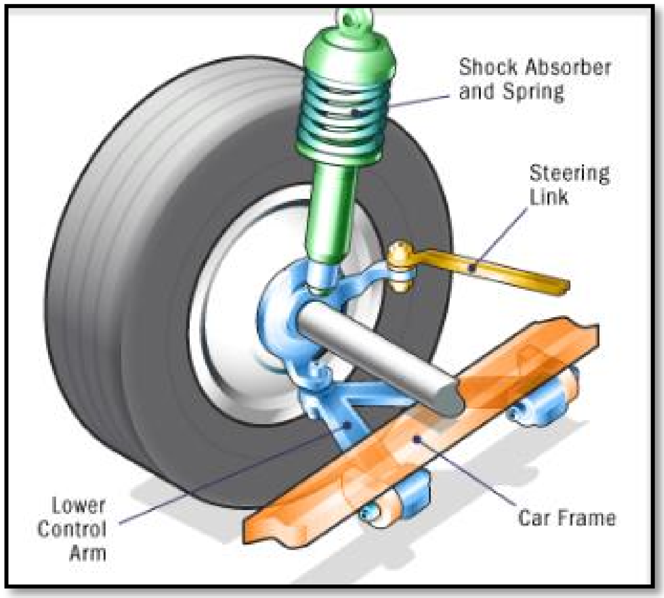


Figure 5: Basic Strut Design for a Suspension System [8]

Shocks and struts are important components in the suspension system as they improve the tire-road surface contact and enhance handling/braking performance by controlling the vehicle-weight transfer from side to side and front to back.

Anti-sway Bars

Anti-sway (anti-roll) bar is a metal rod that is used with shock absorbers (struts) and spans the entire axle to increase the stability of the vehicle by providing additional support. It is used to connect each side of the suspension together.

When there is a vertical (up and down) movement on the wheel, anti-sway bar reduces the risk of concussion by transferring the movement to the other wheel. Moreover anti sway bar prevents the rollover during cornering and creates a smoother ride.

1.2 Front Suspension System

The four wheels of a car work together in two independent systems; the two wheels connected by the front axle and the two wheels connected by the rear axle. Which means that a car can have different types of suspension on the *front* and *back*. As having a front wheel drive (FWD) layout is quite common, usually the front axle is towing and it is in charge of controlling the steering of the vehicle. On the other hand, rear suspension is relatively smaller in order to leave more space for other components located on the rear axle, i.e. fuel tank, spare wheel, exhaust pipe, exhaust silencer. For this particular study, it is enough to understand the principal behavior of the front suspension system.

For the front suspension system, another classification can be made by further questioning whether the rigid axle is linked to the front wheels or not. In case of having the front wheels permitted to move independently from the front axle, the arrangement is called *independent front suspension system*. However when the suspension system have a connection between the rigid front axle and the front wheels, the arrangement is called *dependent front suspension system*. The McPherson strut, developed by Earle S. MacPherson of General Motors in 1947, is the most widely used front suspension system, especially in cars of European origin.

1.3 McPherson Suspension Model

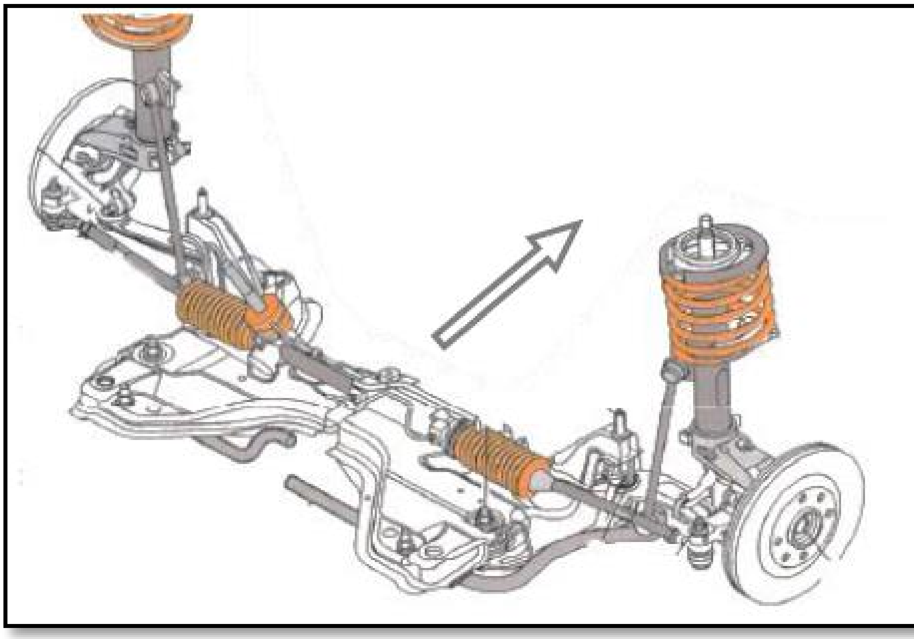


Figure 6 : McPherson Front Suspension System

In this independent front suspension setup, the front wheels are allowed to move independently.
The McPherson strut combines a shock absorber and a coil spring into a single unit and provides a more compact and lighter suspension system for front wheel drive (FWD) vehicles.

McPherson Front Suspension System (**Figure 6**) is the most widely used suspension layout for the front axle. It is possible to observe this kind of a layout on A and B segment cars.

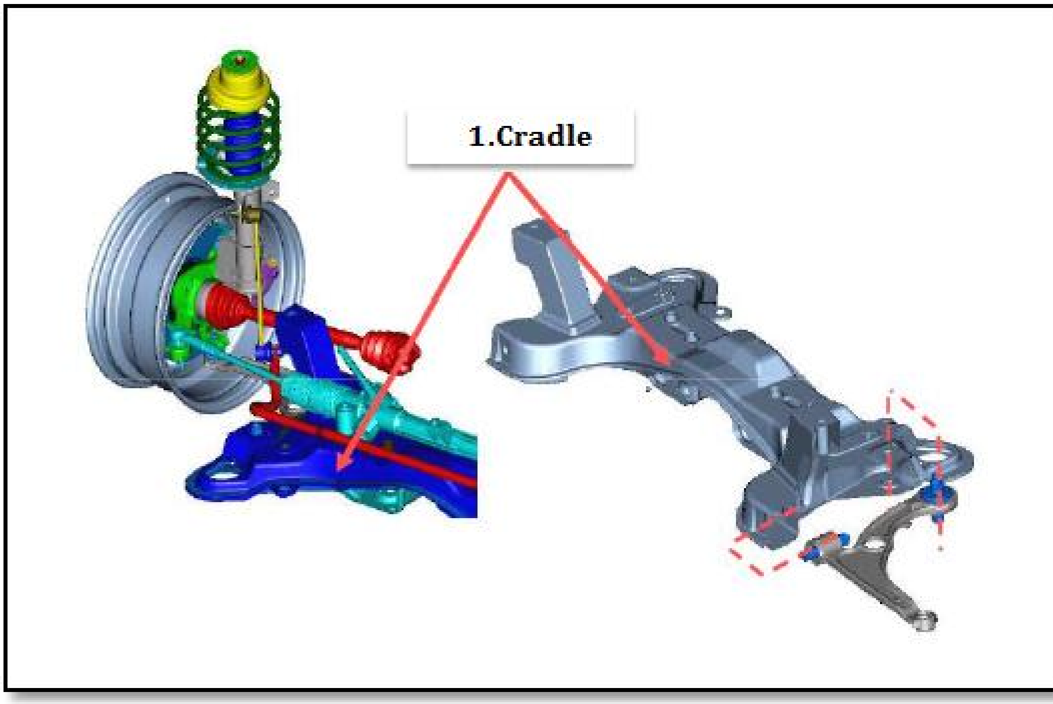


Figure 7 : Cradle (*Cross-member*) in the McPherson Suspension Model-I

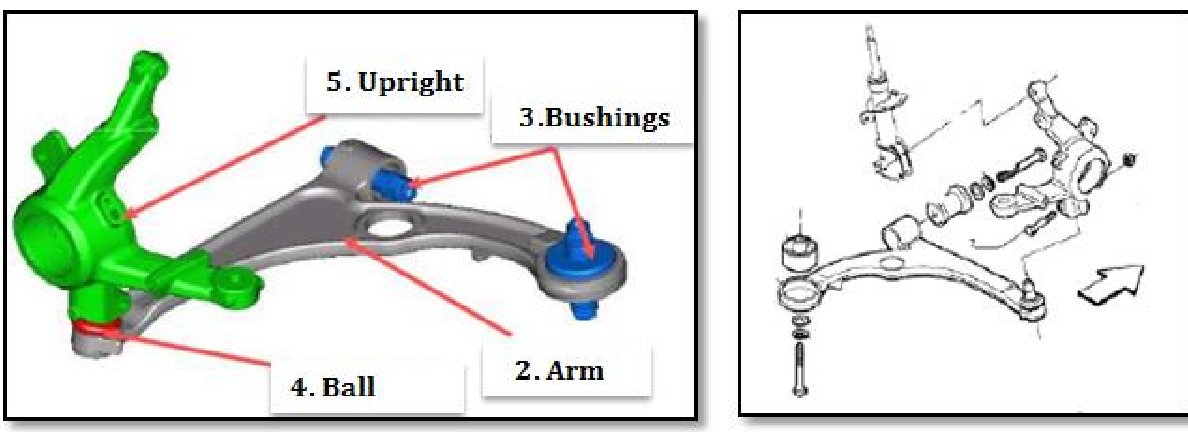


Figure 8 : Description of McPherson Suspension Model-II

Referring to **Figure 7** & **Figure 8** it can be seen that, ***the cradle*** *(cross-member)* (1) is connected to the ***lower arm*** (2) in two points by the ***elastic bushings*** (3) which is connected to the ***upright*** (5) by a ***ball joint*** (4).

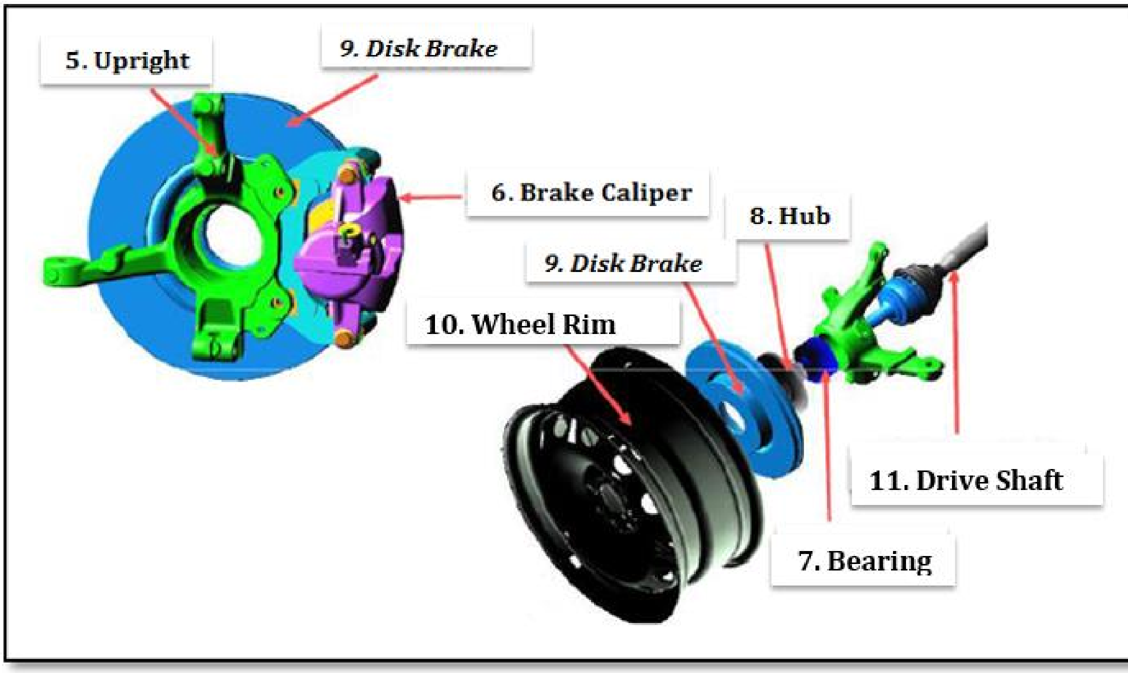


Figure 9 : Description of McPherson Suspension Model-III

In **Figure 9**, it is illustrated that the ***upright*** (5) is the component which is not only necessary for the fixing of the ***brake caliper*** (6) but also important for the housing of the outer ring of the ***wheel bearing*** (7).

The inner ring of the ***wheel bearing*** (7) is attached to the ***hub*** (8) while the hub is coupled with the ***disk brake*** (9) and the ***wheel rim*** (10). In order to allow the transmission motion, the connection between the ***hub*** (8) and the ***drive shaft*** (11) is made by splined coupling.

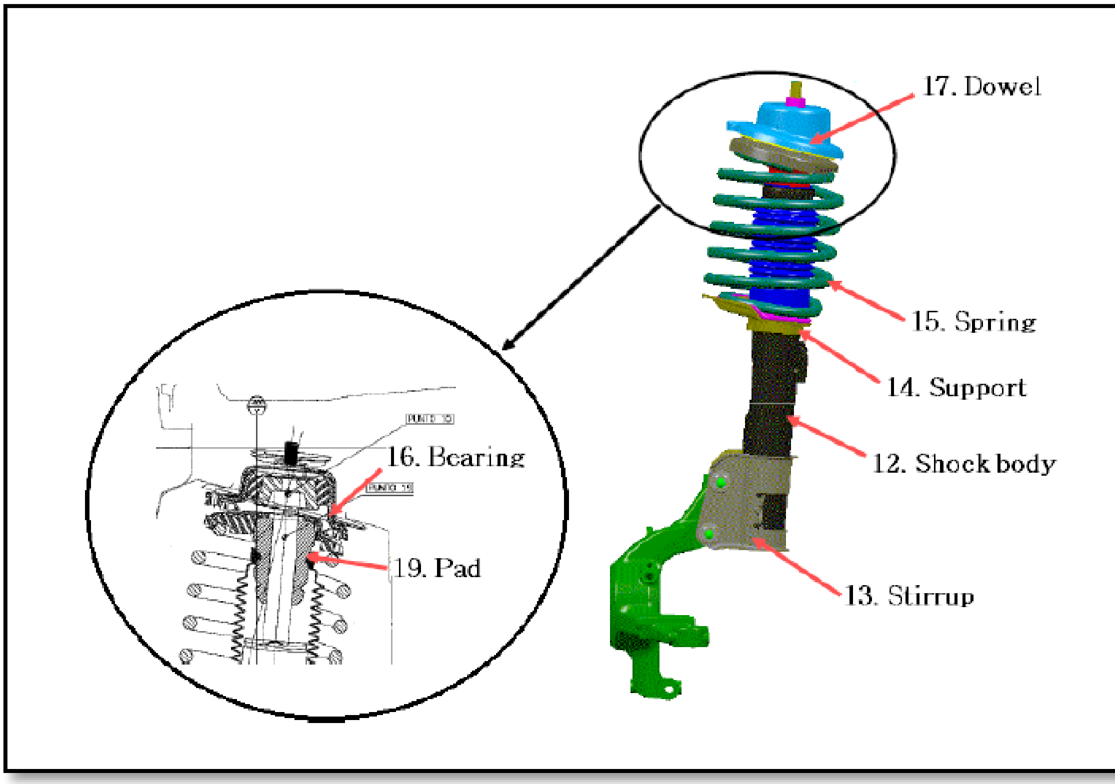


Figure 10 : Shock Absorber in the McPherson Suspension Model-IV

Figure 10 indicates specifically the structural part which generates the damping action for the vibration arisen by the vertical movement of the wheel created due to the unevenness on the road surface.

If **Figure 10** and **Figure 9** is considered together, ***the damper*** shock body (12), on its upper part, connected to the ***lower spring support*** (14) and on its lower part, it is attached rigidly to the ***upright*** (5, on Figure 9) through ***the stirrup bracket*** (13).

Again in **Figure 10**, ***the dowel*** (17) is the interference component between the absorbing part and the car body and its main function is minimizing the oscillations. ***The pad*** (19) is used to avoid metal to metal direct contact.

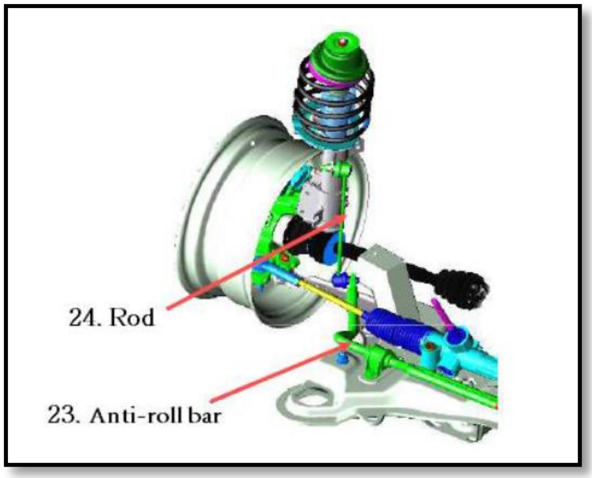


Figure 11 : Front Wheel in the McPherson Suspension Model-V

In **Figure 11**, it can be seen that the anti-roll bar is hinged to the cradle in the front suspension, in order to allow the rotational motion around its axis, and connected to the body of the shock absorber through the rod.

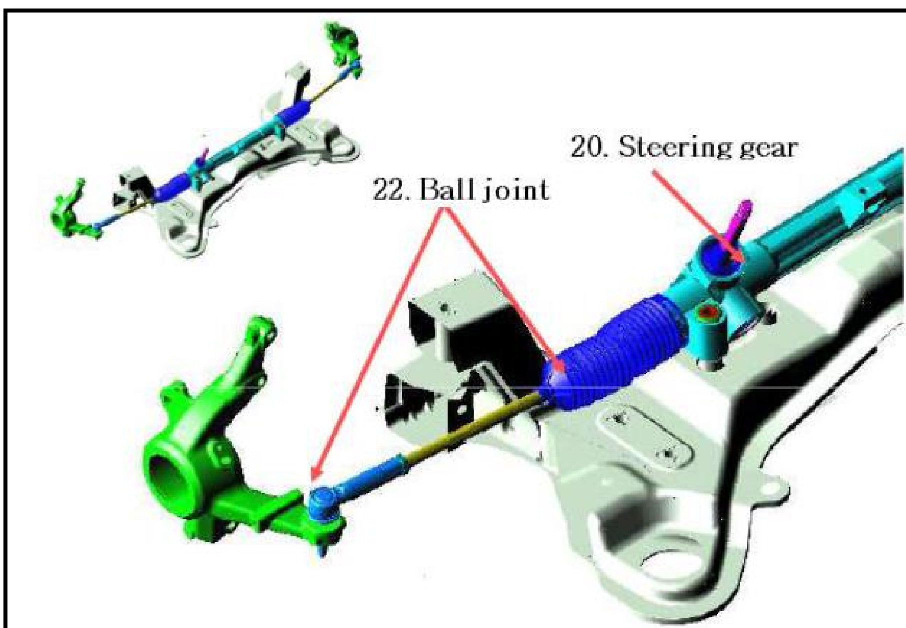


Figure 12 : Steering Gear in the McPherson Suspension Model-VI

In **Figure 12**, it is shown that the steering gear is also bolted on the cradle.

1.4 Cradle (Cross Member Part)

The Cradle (Cross Member) performs 3 main functions in the front suspension assembly:

• ***Creating Support:*** Cradle is a structural component of the vehicle, usually made of Al or steel (depends on the vehicle type), and bolted across the underside of a monocoque / unibody motor vehicle, to support the underside of the car, the engine and transmission.

• ***Providing Stiffness***: The stiffness of the cradle can increase the whole rigidity of the car body. The cross member frame has to be strong enough to cope with the loads applied on it to be able to provide a convenient suspension, ensure a proper handling and keep the body panels in alignment. The cradle part must not deflect, and have a torsional strength high enough to resist twisting.

• ***Absorbing the Vibration***: Cradle part minimizes the oscillation on the car body with the shock absorbers. It is also effective to improve the comfort in the passenger compartment.

Besides these main functions, other objectives can be; to provide a better elimination of vibrations resulting from the road and, to transfer and absorb the loads resulting from a car crash.

The cradle is basically made of either two different sheet metals, stamped and welded together, or two sheet metals made from the same material. Usually these two symmetric sheet metals are the same, and made from aluminum. It can be manufactured by using different technologies such as shell casting, low pressure casting, die casting method.

The cradle supports and transfers the whole body loads transmitted through the attached members. The connection between the attachments and the cradle can be a rigid connection, an elastic connection or a mixed type connection.

It allows mounting the suspension on another separate line other than the main production line.

The cradle, being a component much smaller than the whole body, allows good control of the dimensional tolerances for the positioning of the anchor points.

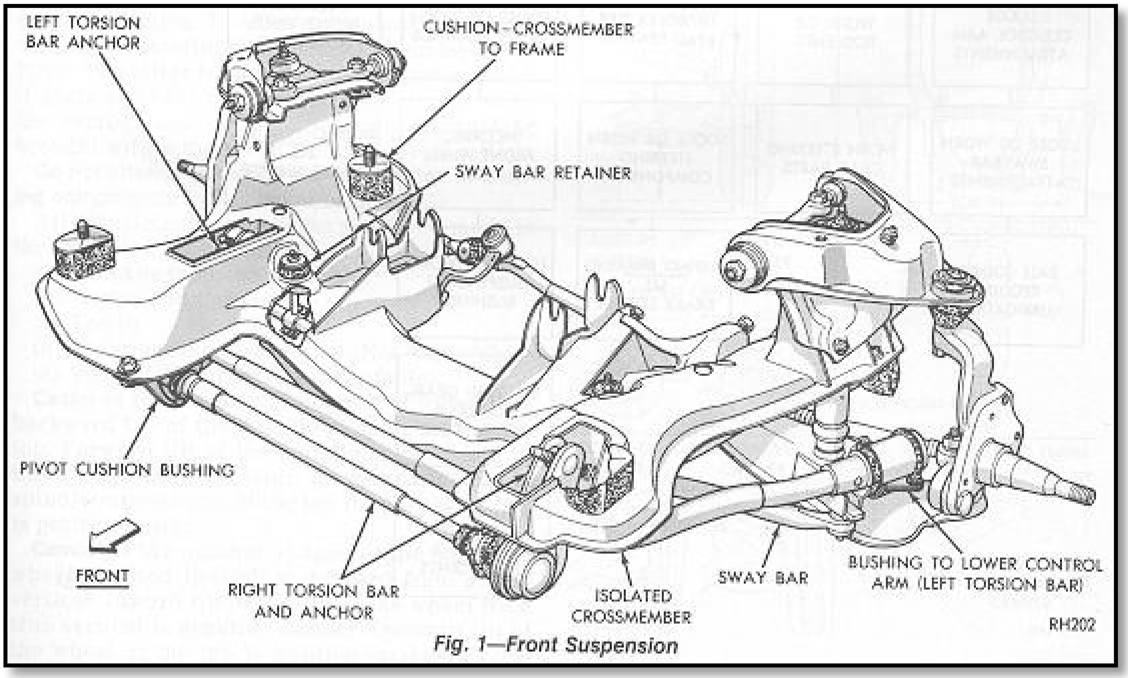


Figure 13 : Cradle (Cross Member) Model of a Car

Chapter 2

Jeep Renegade – History and the Development

2.1 Introduction of Jeep Renegade

Jeep Renegade is a B segment car, developed in 2014 by the FCA Group. In the development process, the structural composition of the Jeep Renegade chassis was inspired by previous models. For instance the cradle model (whole cross-member group) for the front suspension system is generated by following the design guidelines indicated for the previous Grande Punto and Fiat 500 models and similar structural solutions were adopted to this new model during the chassis design. On the other hand Fiat Giulietta -third model of the same group- differs from those two models in terms of structural design based on having lower mass for constitutional parts.

The weight of the vehicle is a significant factor for the design stage, which influences the dimensions and the power requirements of the various structural components. Moreover for a SUV design, several performance requirements in terms of drivability and comfort should be considered in order to respond the customer needs.

Differences can be underlined more clearly by making a comparison between the structural elements of the three models mentioned above, in order to understand how the choices of cradle model and various components were made and what the functionality of each structural element is.

Figure 14 : Jeep Renegade 4x4 2.0 Multi-jet II from Geneva Auto Fair 2015

2.2 Cradle Design

In the design phase of the cradle model, the input data and the layouts of the assembly models for various structural components are already fixed such as;

- Swing arm fixing
- Guide box fastening
- Stabilizer bar fixing
- Motor drive position
- Exhaust pipe passage

Having all those components with specified mechanical properties (i.e. stiffness, corrosion resistance, thermal resistance, torsional strength, and impact energy) enables revisions and let the designers to make required alterations in a certain frame to optimize the overall design and improving quality in case of performance and vehicle comfort.

If different Fiat models are compared, Fiat Panda has one of the first cradle structure models which had been developed and then used as a design guideline for the following chassis design models. Later, for the Fiat 500 model, the current design used for Fiat Panda model was enriched with the addition of two attachment points at the front part and C brackets on the left and right for the third load line on the opposite side.

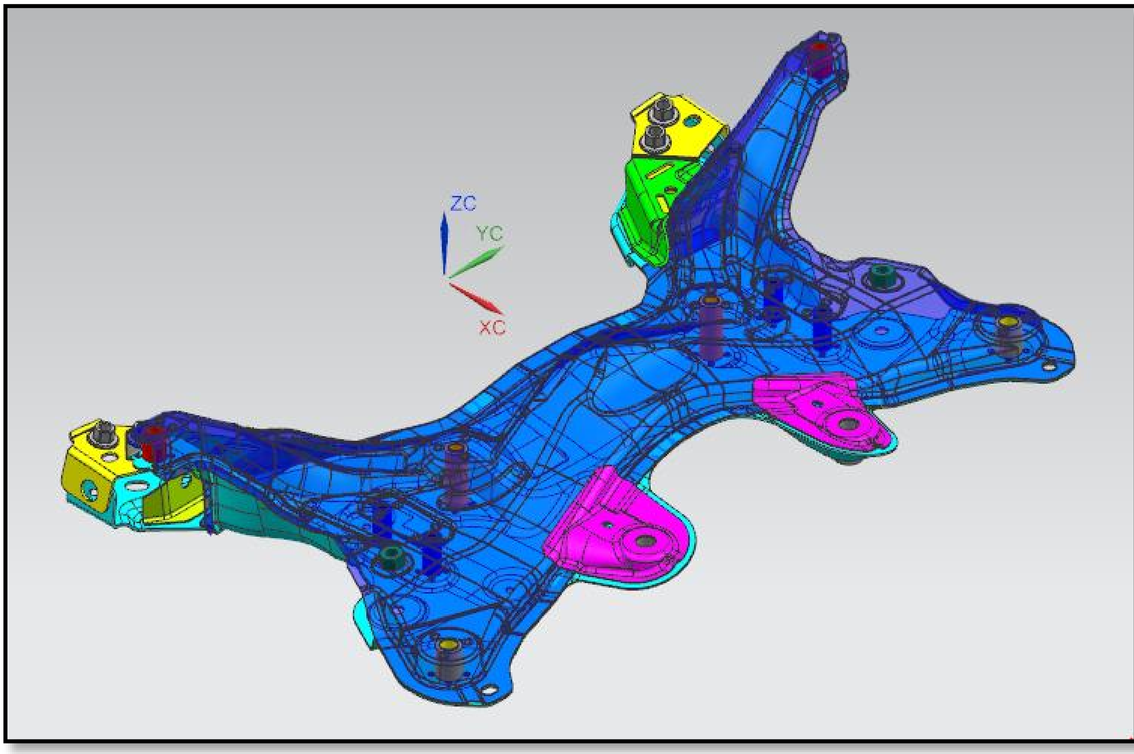


Figure 15 : CAD of Cradle Model for Fiat 500 (2005)

For the cradle, the dynamic interactions between different parts is defined through its structural components such as; the top shell on the cradle (mechanical cross member), the bottom shell (flat bar) and the right and left attachments in order to make upper and lower shell assembled (with necessary reinforcements in between). Side settings and the C brackets on the left and right are designed for absorbing the energy in case of front impact loading.

Figure 16 (a): Cradle Model of Fiat 500-Lower Shell

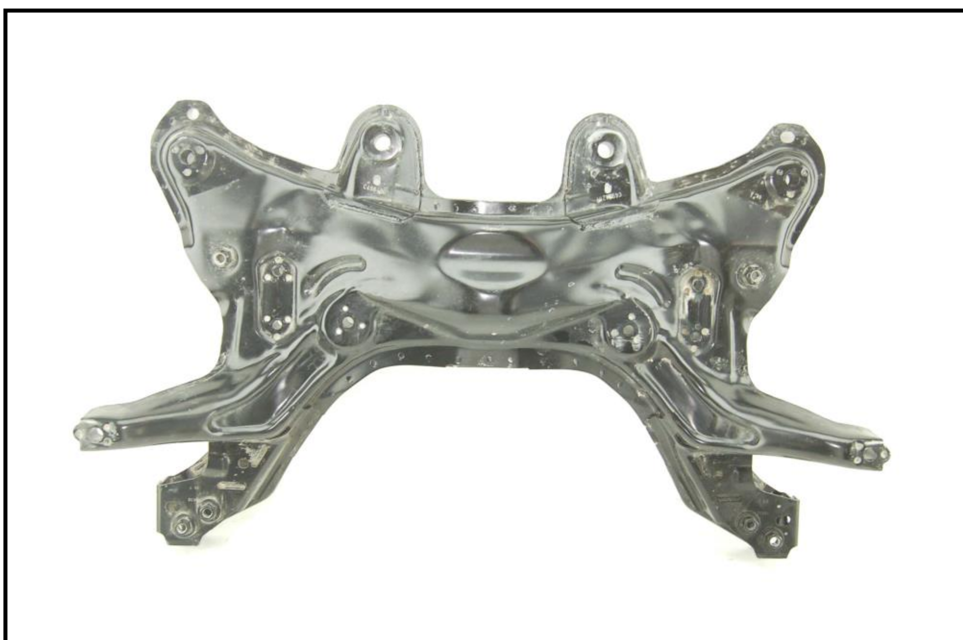


Figure 17 (b): Cradle Model of Fiat 500-Upper Shell

A modern development of cradle can be seen on Fiat Grande Punto model;

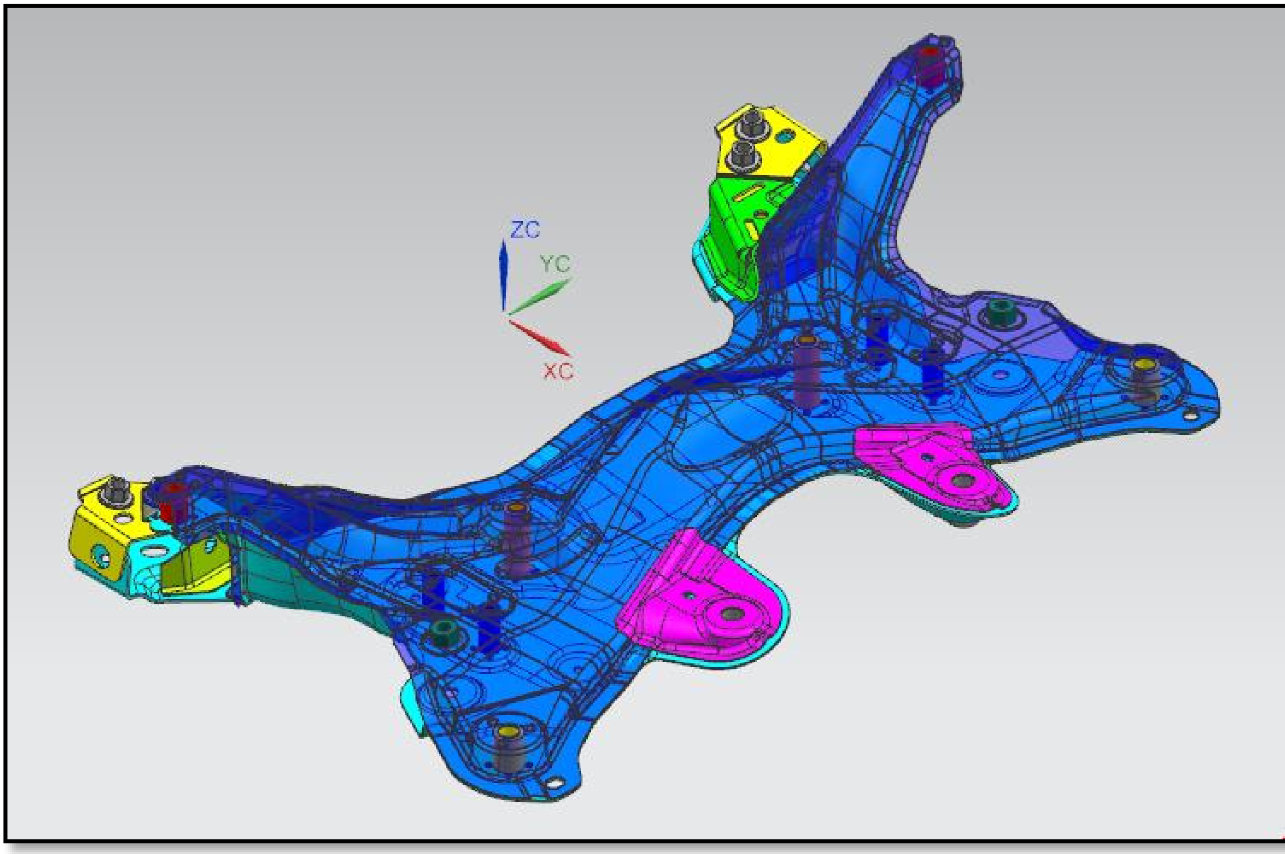


Figure 18 : CAD of Cradle Model for Fiat Grande Punto (2007)

Figure 19 : Cradle Model of Fiat Grande Punto-Lower Shell

Giulietta was designed in 2010 with an improved structural design. As it is a sport car, the design concept was different. In this case, the cradle has a non-conventional, contemporary design compared to first two models.

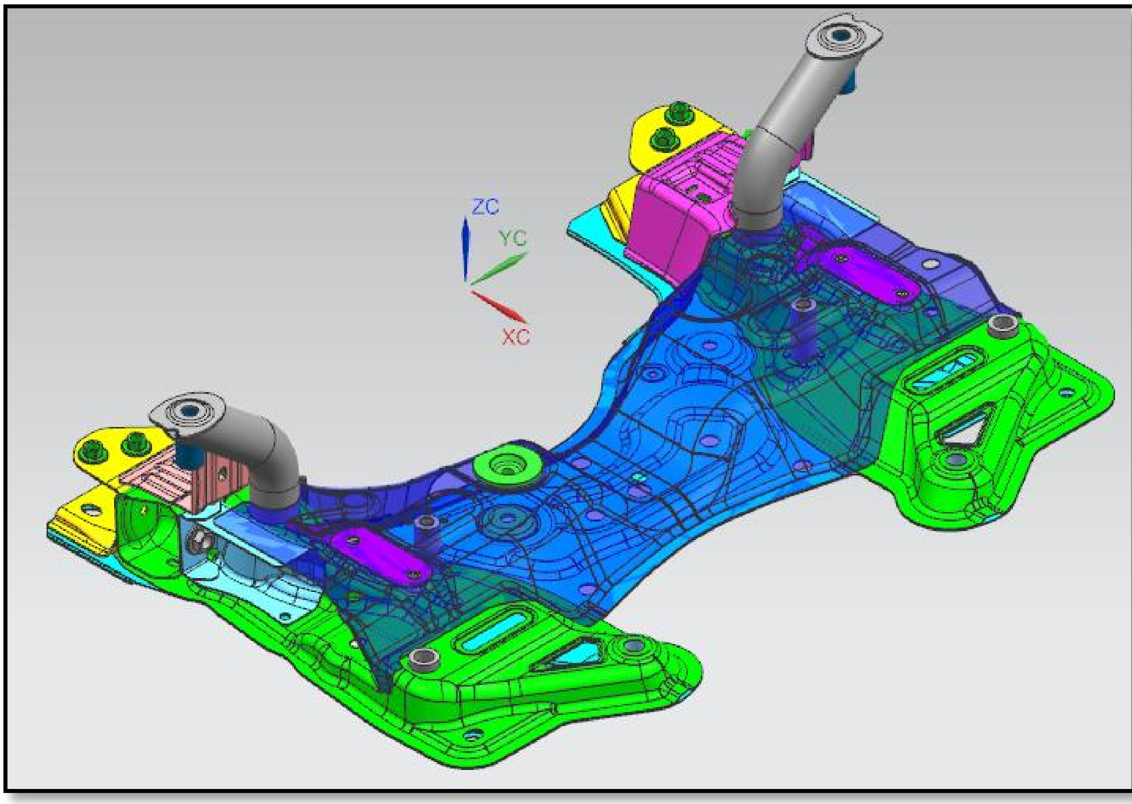


Figure 20: CAD of Cradle Model for Fiat Giulietta (2010)

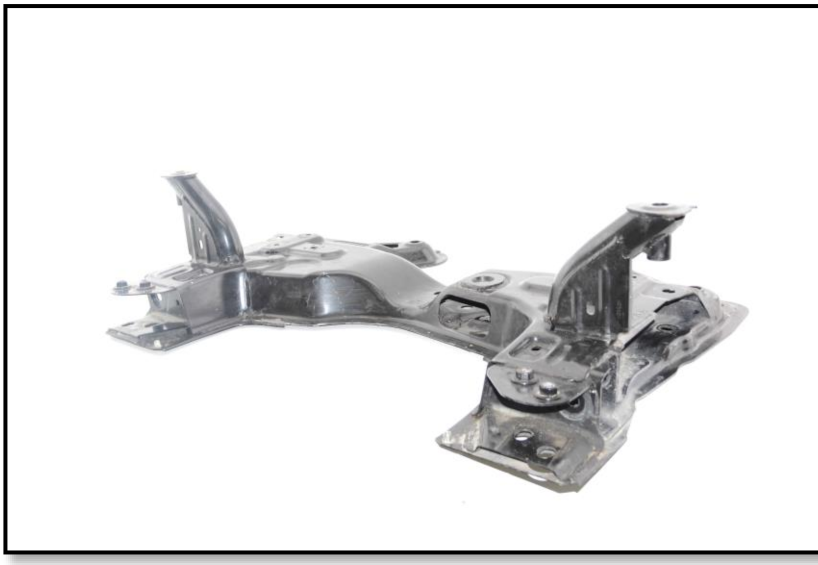


Figure 21 : Cradle Model of Fiat Giulietta-Lower Shell

Cradle design for the Jeep Renegade is the most recent one, produced in 2016. It aims to eliminate the compact and continuous shapes on the cradle surface and introduces a more linearized shape. This innovative design provides possible reduction of body attachment points on the cradle thanks to its linear design and ensures the ability to reduce high-speed shocks.

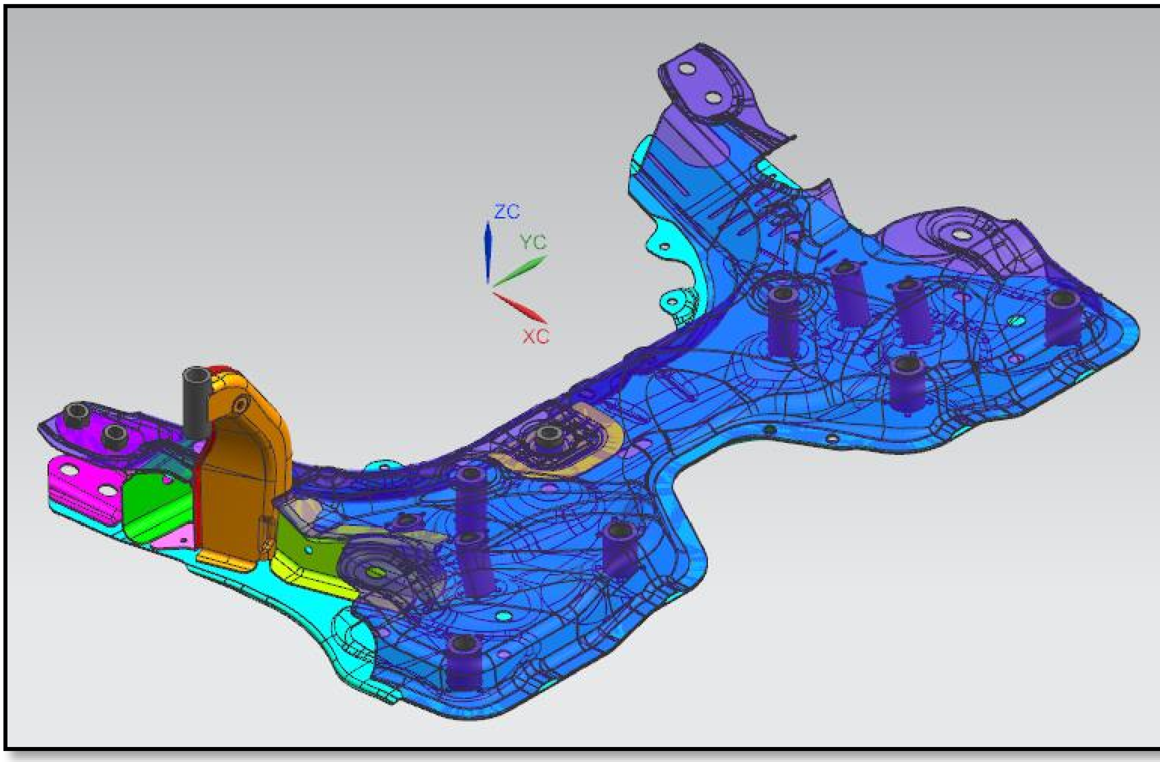


Figure 22 : CAD of Cradle Model for Jeep Renegade (2016)

Figure 23 : Cradle Model of Jeep Renegade-Lower Shell

When a cradle design is made there are 4 main considerations;

- Design of the Upper Shell of the Cross Beam and Its Positioning (in order to increase the stiffness of the cross member part)
- Body Attachment Points
- Addition of Longitudinal and Third Load Lines for Impact Resistance (in order to increase the total crashworthiness and increase the resistance of the cross member for impact loading in case of a crash)
- Design of the Control Arm

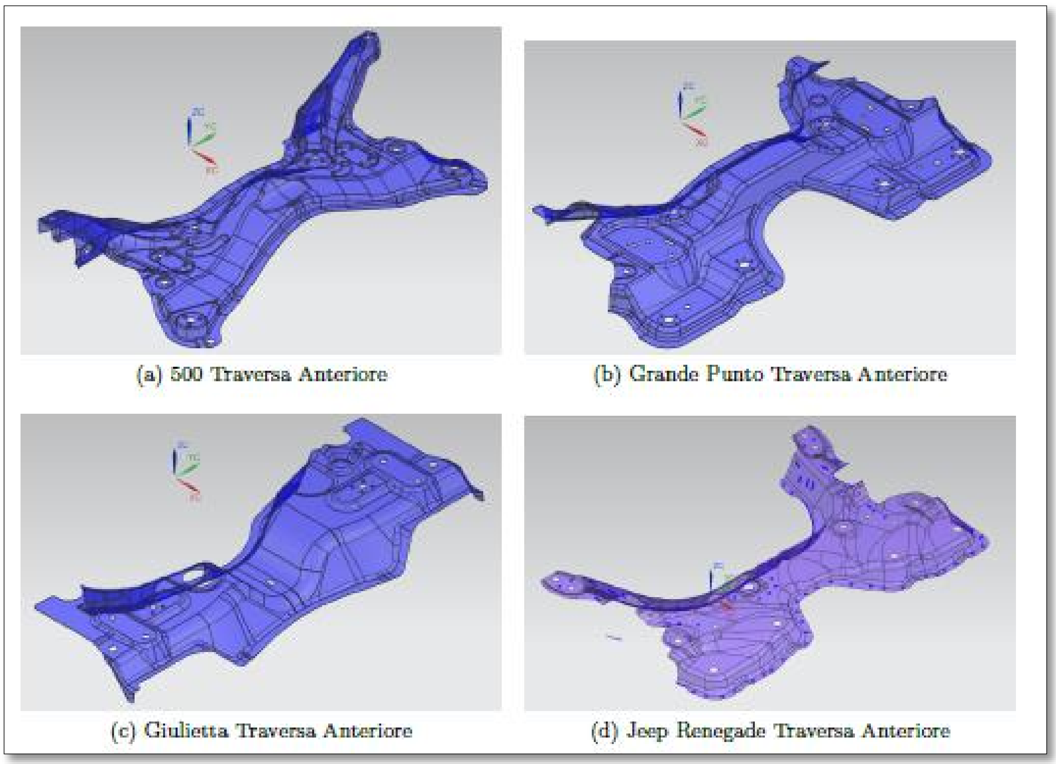


Figure 24: Front Cradle – Upper Shell Design

As it can be seen in **Figure 24**, the design layout evolved with time. The surface area is very large for Fiat 500 and Grande Punto models, and the concave profiles on positive z axis was preferred for these two designs, while the cradle design in Giuletta and Jeep Renegade models are more modern with a flat surface. The Giuletta and Jeep Renegade cradle designs are the more rigid ones since the structure is more compact. However the final design choice is directly related with the market interest.

Another important consideration while designing the cradle profile is the positioning of the exhaust pipes coming out of the engine.

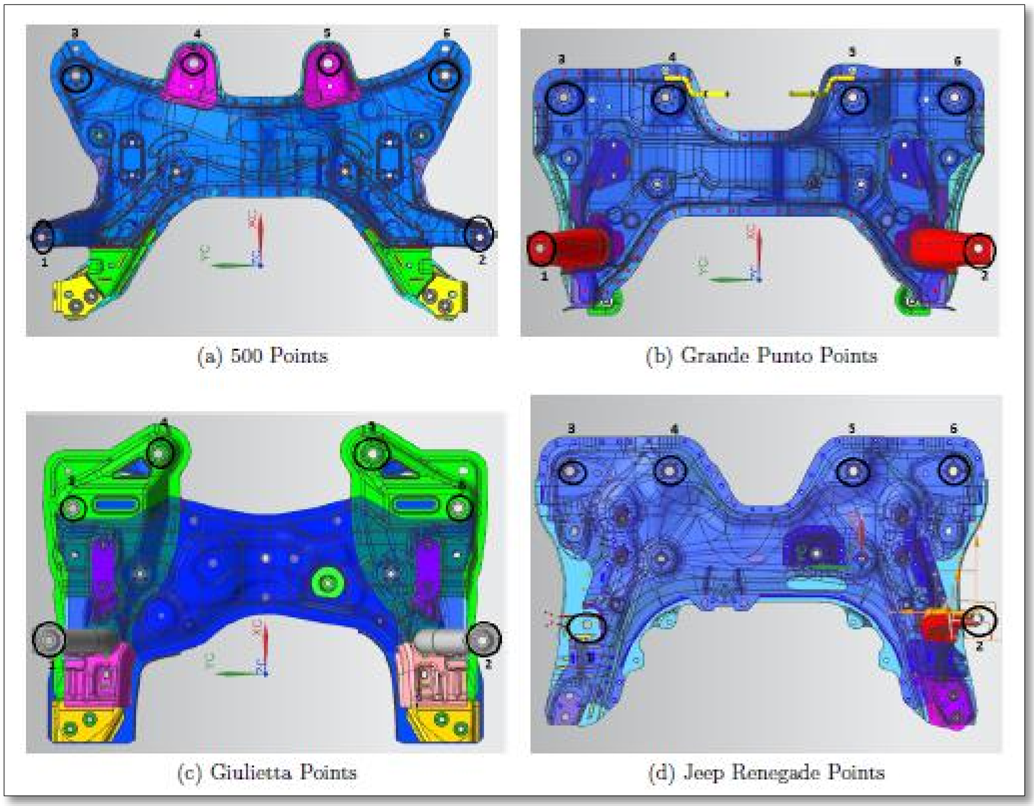


Figure 25: Six Attachment Points on the Cradle for Different Models

6 attachment points (2 lower attachment points and 4 upper attachment points), which restricts the cradle motion are shown in **Figure 25**.

In the following chapters, these 6 attachment points will be represented again, as constraint points, on the simplified cradle models created for the numerical stiffness calculations.

			Grande Punto	*500*	*Giulietta*	*Jeep Renegade*
	Peso vettura [kg]		470,55	412,46	538,4	571,02
	Peso traversa [kg]		14,47	13,75	19,04	18,05
	Ingombro [mm]	x	500	560	656	610
		y	940	935	982	935
		z	220	200	256	255
	Interasse candelieri [mm]					888
	N° Punti attacco a scocca		6	6	6	6
	Materiali [GPa]		E206	E206	E206	E206
	Misuse?		no	si	si	si
	PT1 [mm]	x				-21,6
		y				-367,7
		z				-117,1
RIGIDEZZE	PT2 [mm]	x				240,6
		y				-384,9
		z				-105,4
	PT3 [mm]	x				19,84
		y				-113,5
		z				-51,48

Table 1: Properties of the Cradle (Mechanical Cross-Member) Designs for Fiat 500, Grande Punto, Giulietta and Jeep Renegade Models

Chapter 3

Finite Element Analysis (FEA) Method

3.1 Introduction

Finite Element Method (FEM) is a numerical method for solving engineering and mathematical physics problems. Typical FEM application areas include mechanical-structural analysis, heat transfer, fluid flow, mass transport, and electromagnetic potential.

In more detail, the law of physics for space-time dependent problems are usually expressed in terms of partial differential equations (PDEs). The analytical solution of these problems generally requires the solution to boundary value problems for partial differential equations. However for most of the geometries and mathematical problems, these PDEs cannot be solved with analytical methods. At this point, an approximation can be written for these equations, based on different types of *discretization*. These discretization methods approximate the PDEs with numerical model equations and enable them to be solved by using numerical methods. By this numerical method, the solutions are an approximation of the real solutions of the PDEs. Thus the finite element method (FEM) is used to compute such approximations. [10]

The finite element method formulation of the problem results in a system of algebraic equations. The method yields approximate values of unknowns at discrete number of points over the domain. To solve the problem, FEM subdivides a large problem into smaller, simpler parts that are called *finite elements*. The simple equations that model these finite elements are then assembled into a larger system of equations that models the entire problem. This provides considerable simplifications for the analysis. However this implies a certain degree of *approximation*. FEM then uses variational methods to approximate a solution by minimizing an associated error function. [2]

The subdivision of a whole domain into simpler parts has several advantages, such as:

- Accurate representation of complex geometry

- Inclusion of dissimilar material properties
- Easy representation of the total solution
- Capture of local effects

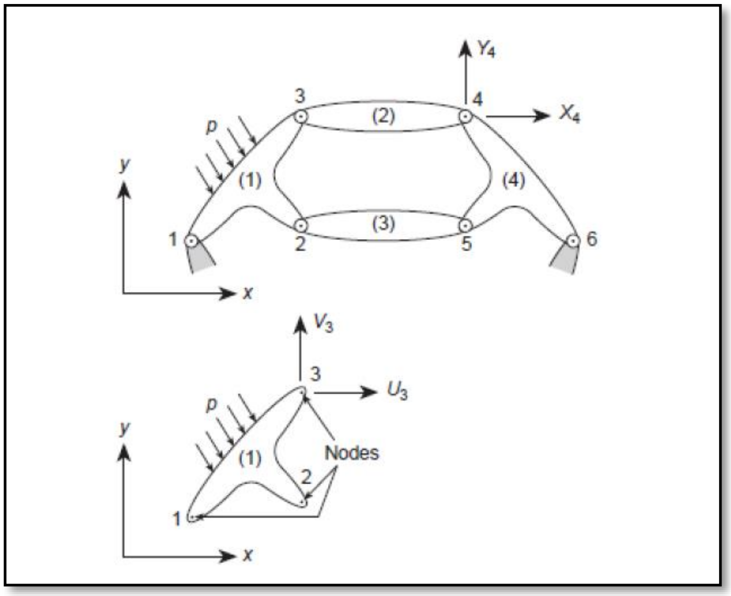


Figure 26 : Generic Discretization into Elements of a Structural Component

For the mathematical calculations, the modelling in FEA is done by taking the whole body as continuous and homogenous. The properties are considered the same for each point of the model. Later, *the body is divided into a series of elements* that have finite volume and simple geometry such as triangle, square or cube. Although, these elements are not intermeshing between them and considered as they are linked to each other through finite number of points and the displacements.

Defining the displacement of these series of elements is easy. The displacement of their internal points depends on the displacements of the respective nodes. These displacement functions of respective nodes are called *shape functions.*

A certain number of degrees of freedom (DOF) characterizes each node. Each degree of freedom has a physical meaning depending on either the type of the body movement (i.e. translation, rotation) or the type of the physical phenomenon that is being evaluated (i.e.

temperature, pressure, and displacement). The number of DOF for each node characterizes the behavior of the single element [1].

3.1.1 Structure of NX Software

NX software is an advanced high-end CAD/CAM/CAE. It is one of the most used design programs thanks to its versatility and computational power. Among other tasks, it is mainly used for *design* (parametric and direct solid/surface modelling), *engineering analysis* (static, dynamic, electro-magnetic and thermal analysis by using the finite element method; and fluid analysis by using the finite volume method), *and manufacturing – after design operations* and *machining modules*.

Its utilization for this study is divided into three main areas; Computer Aided Design (CAD modelling) of parts, Finite Element Analysis (FEA) and estimation of deformations and tensions of constructed models (FEM Simulation).

3.2 Setting of FEM Models & Stiffness Validation

For the analysis of the mechanical cross member (cradle) through Finite Elements Method, a virtual model for the component should be introduced. For this reason while the CAD modelling of the real part was constructed, a simplified cradle model was created. By simplification of this complex geometry both the FEM analysis and building the mathematical stiffness formulation related to this part, became much easier. As the aim of this study is constructing a mathematical expression for the stiffness analysis which is close enough to the stiffness value calculated conventionally by FEM, virtualizing the exact geometries has not been considered as crucial at this stage.

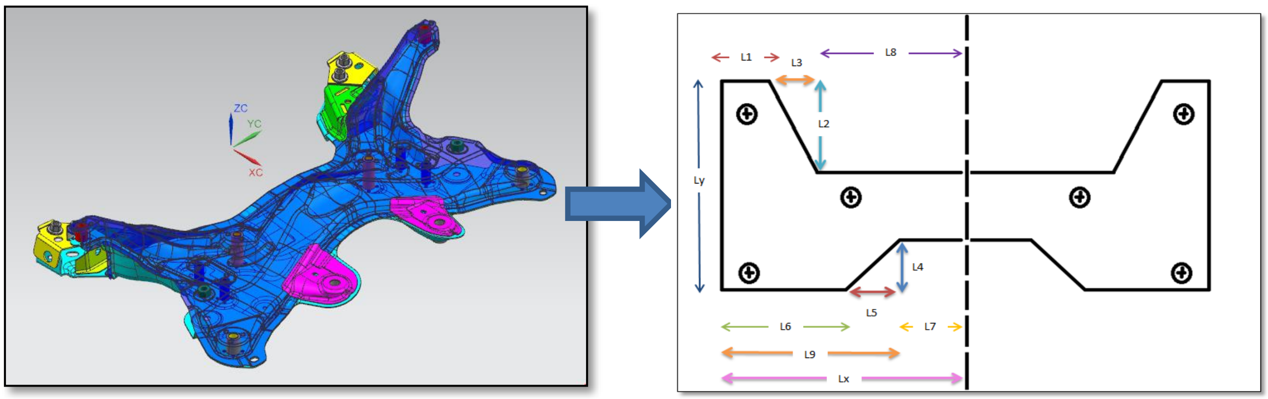
Meshing and Assumptions in the Modelling

Meshing of a component is used for the division of the part into finite elements. Mesh /Grid generation approximates a geometric domain that needs to be discretized. The elements chosen for dividing the cradle are 2D quadrilateral meshes in different dimensions depending on the thickness of various parts of the whole geometry.

The displacement undergone by each element is a function of displacement in the individual nodes. The number of nodes on each side of the element determines the interpolating function of displacement. Two nodes per side give a linear displacement. A linear field may not be sufficient for discretization of a complex geometry but for sheet metals (as the simplified cradle models have) , using four node plane elements gives accurate results in the analysis as the thickness variation is not in a significant range.

The linearity of the displacement field implies a uniform deformation for each element, based on uniform stress-strain distribution. In addition to this, to be able to have a more precise result from the analysis, based on this uniform distribution, the division should be split into small parts for each element, especially for the areas that have a large variation of stress distribution and a wide range of deformation change (due to the existing notches and discontinuities on the model). However NX software design is capable of automatic recognizing of critical geometric zones in the model and separating the elements. Hence the software can automatically increase the number of nodes on the critical areas for a better adaption.

3.2.1 Simplified Cradle Models



In order to make the calculations and the formation of the stiffness formulation easier, the actual cradle geometry is simplified as it is illustrated above. The model is composed of two symmetrical sheet metal geometries with a certain sheet metal thickness. Then the part will be completed with the addition of filler material on the inner side (which is neglected for the stiffness calculation of the cradle).

After simplifying the actual geometry, the dimensions were also arranged and three different models were constructed as CAD models for FEM analysis.

The study starts with the FEM analysis of these three simplified models in order to use the results as experimental values. Since a new stiffness formulation is being created for this specific cradle part as an alternative method, some values are needed to verify the result of this new mathematical formulation. Also in order to check the accuracy of this new mathematical formulation, for the simulations three different thickness values are considered as control factors. Because it is known that when the thickness increases, the stiffness should also increase.

In other words, the first model has the dimensions of the actual cradle geometry, while the second model has nearly half of the actual dimensions and the third model has nearly twice of the actual dimensions. Also the overall geometry is slightly altered for different models.

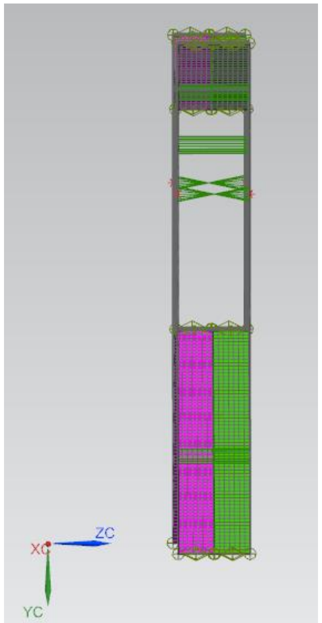
Each of these three simplified geometries also have three different values for sheet metal thickness. Consequently there are 9 FEM simulations for each cradle models (Three simulations for Model I for three different sheet metal thickness values, three simulations for Model II for three different thickness values and three simulations for Model III for three different thickness values, nine simulations in total).

3 different sheet metal thicknesses for 3 different cradle models for stiffness calculations FEM on X / Y / Z DIRECTIONS ;										
	MODEL I			MODEL II			MODEL III			
	Thickness 1	Thickness 2	Thickness 3	Thickness 1	Thickness 2	Thickness 3	Thickness 1	Thickness 2	Thickness 3	mm
Kx Value with FEM analysis	#	#	#	#	#	#	#	#	#	N/mm
Ky Value with FEM analysis	#	#	#	#	#	#	#	#	#	N/mm
Kz Value with FEM analysis	#	#	#	#	#	#	#	#	#	N/mm

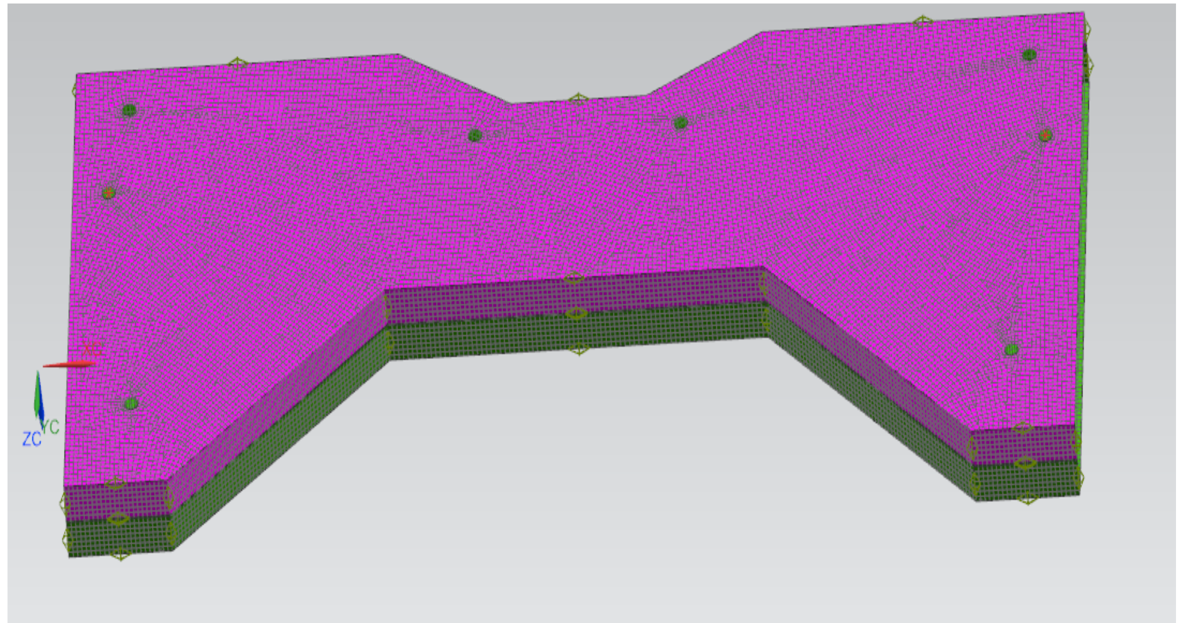
Table 2: Finite Element Analysis Table

For each simulation, after creating 3D modelling by specifying the bushing points and the application point for the applied loads the following steps were completed in order to start finite element analysis for computing the stiffness value;

- Assembly of two symmetric sheet metal parts
- Creation of the elements (2D mesh generation)
- Assignment of the properties (material selection, default thickness specification)
- Connecting the elements (1D connection from node to node, feature edge nodes)
- Indication of the constrains (fixed transition constrain assignment for 6 bushing points)
- Specification for applied forces and boundary conditions

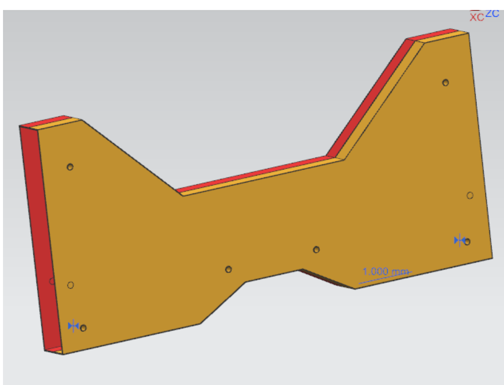


(1)

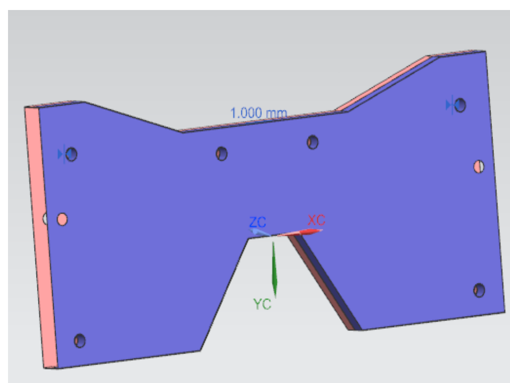


(2)

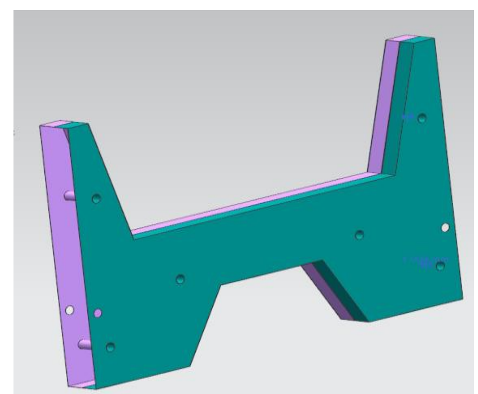
Figure 27: The meshing for the 3D model and fixing the constraints on NX Software (1) and (2)



Model I



Model II



Model III

Figure 28: Three simplified cradle models (Model I & Model II & Model III)

3.2.2 Numerical Calculation of Stiffness for Simplified Cradle Models

After meshing the whole geometry it is necessary to add and connect nodes separately, especially the ones which generates constrain forces. In the cradle there are six constrains points. These six points binds the whole cradle to these six bushings, and then all six degrees of freedom in space become locked. After specifying the constrain points, when the FEM simulation begins, program extracts the corresponding values of deformation in the constrain points, in all three directions (fixed constrain transmission is used for the simulations).

The applied load is given as an input to the program. The rigidity estimation is done during the simulation with respect to Hooke's Law;

$$k_i = \frac{F_i}{\delta_i} \quad with \quad i = \{x, y, z\}$$

Where *F* is the applied force specified as an input, δ is the displacement obtained with FEA as an output (since the material selection and mechanical properties specified in advance) and finally *k* is the estimated stiffness value by using the Hooke's Law while the subscript *i* identifies the directions for; the applied force, displacement and estimated stiffness.

Based on the industrial confidentiality reasons, the stiffness of the component will not be disclosed. To assert the accuracy of the study, the obtained results with the mathematical stiffness formulation will be represented as a percentage by taking FEA results as reference.

The FEA results of the simplified cradle models are used for verification of the formulation results, since the stiffness values of the exact model cannot be shared by FCA.

Chapter 4

Alternative Calculation Method

4.1 Design for Six Sigma (DFSS) Methodology

4.1.1 Introduction

Design for Six Sigma is an industry systematic methodology which is used during design and redesign of products and services according to the demands and expectations of the customer. The methodology is used thanks to the data offered by ASI Consulting Group, LLC [4] in FCA group. Sigma (σ) is a statistical metric that corresponds to a certain amount of dpm, defects per million (Table…).

2σ	308537	dpm
3σ	66807	dpm
4σ	6210	dpm
5σ	233	dpm
6σ	3.4	dpm

Table 3: σ unit

DFSS methodology is used for;

- Optimization of the design process in order to obtain 6 σ performance
- A more systematic approach to procure the final result, the resources or the trade offs.
- Possibility to provide better products and services which customers wants and desire to pay for it
- Eliminating the gap between product characteristics provided by an ongoing process and customer requirements
- Eliminating the waste of time that is spent on estimation about the shortcomings of existing systems before starting to search for a new method of redesign

In order to obtain the best results, it is decided to use the DFSS technique to build the mathematical stiffness formulation by identifying the control factors associated with the multiplication and exponential coefficient of the formulation. The formulation model will give different results for different approximation of the stiffness values.

DFSS is a methodology which is used as a strategy of optimization for production and design in various industries. Many corporations have used DFSS for a long period of time. Clearly, this approach is not owned by FCA Group and therefore it is protected by copyright.

D*esign* ***f****or* ***S****ix* ***S****igma* ***(DFSS)*** is an improvement system used to develop new processes or products at Six Sigma quality levels.

The purpose of using this methodology in this work is to maximize the ***Signal-to-Noise (S/N) ratio*** and get the ***mean (β) value*** closed to nominal value as much as possible when the mathematic formulation model is designed.

4.1.2 IDDOV

DFSS methodology is divided into five stages; Identify, Define, Develop, Optimize and Verify. By following these steps, it is easier to reach the production of a product/system in the shortest possible time and with the minimum number of risks associated with a poorly functioning or its deviation with respect to the request of the customer.

3.1.2.1 Identify

This is the phase where the customer/system requirements is identified and customer needs are prioritized, and those needs are translated into design requirements. It is the most important phase of DFSS since all of the future activities of the projects depends on this phase. Once the requirements are identified, the complete project plan can be made. Ideally, the project plan consists of the scope of the project, objectives, project milestones and the budget.

In our case, the project plan was identified in this step.

3.1.2.2 Define

This is the phase where the product requirements are clearly defined. In this phase, the customer/system needs and wants are translated into verifiable requirements. The primary tool used for this purpose is Quality Function Deployment (QFD). QFD is a method by which the customer needs or wants are converted into specific corporate goals so that product designers are aware of what exactly they should do.

In this study, only the specification and requirements of the design was specified in this step.

3.1.2.3 Develop

This is the phase where a feasible concept is developed to meet the customer/system requirements. If the concept developed is found to be unreasonable, then other alternatives are assessed. In this phase, any potential product failure is identified and thus, eliminated. The usual tools used during this phase are TRIZ (the theory for inventive problem solving), Pugh (a technique for evaluating and developing concepts), and FMEA (Failure Mode and Effective Analysis). However these tools are mostly used for production and manufacturing systems.

In this study, only the mathematical formulation is developed and the calculation errors are decreased after analyzing the results. The formulation is redesigned when it is needed for further improvements.

3.1.2.4 Optimize

This is the phase the design is optimized in order to achieve the maximum output obtained from the developed concept. In this phase, the critical design variables and functional parameters are determined to ensure utmost customer/system satisfaction.

In our case, the results were optimized and the best result were obtained for the project.

3.1.2.5 Verify

Once optimization of the design is done, it is validated against established process controls and a complete cost-benefit analysis is done. In this phase, testing is done to verify that the product/design meets all the predefined conditions. Also, all the unexpected side effects are eliminated. The three step process to ensure that the product is tested is to:

i) Verify the capability of the manufacturing process by verifying the capability of the manufacturing personnel, training processes, manufacturing processes, equipment, measurement systems, etc.

ii) Conduct a prototype testing where the product are released to customers and the quality of the product is thus determined.

iii) Conduct a pilot production run to verify the capability of manufacturing processes to deliver quality products at low costs.

The role of IDDOV in many companies mainly; GE, Motorola, and Hyundai is notable. The IDDOV framework has helped all these companies to remodel their processes and to rebuild new products on the basis of DFSS.

4.2 Testing Method: Orthogonal Array

Orthogonal array testing is a black box testing technique that is a systematic, statistical way of software testing [5]. It is used when the number of inputs to the system is relatively small, but too large to allow for exhaustive testing of every possible input to the systems. It is particularly effective in finding errors associated with faulty logic within computer software systems. Orthogonal arrays can be applied in user interface testing, system testing, regression testing, configuration testing and performance testing. The permutations of factor levels comprising a single treatment are so chosen that their responses are uncorrelated and therefore each treatment gives a unique piece of information. The net effects of organizing the experiment in such treatments is that the same piece of information is gathered in the minimum number of experiments.

Orthogonal Vectors

Orthogonal vectors exhibit the following properties:

- Each of the vectors conveys information different from that of any other vector in the sequence, i.e., each vector conveys unique information therefore avoiding redundancy.
- On a linear addition, the signals may be separated easily.
- Each of the vectors is statistically independent of the others, i.e., the correlation between them is nil.
- When linearly added, the resultant is the arithmetic sum of the individual components.

Consider a system which has 3 parameters and each of them has 3 values. To test all the possible combinations of these parameters (i.e. exhaustive testing) we will need a set of $3^3 = 27$ test cases. But instead of testing the system for each combination of parameters, we can use an orthogonal array to select only a subset of these combinations. Using orthogonal array testing, we can maximize the test coverage while minimizing the number of test cases to consider.[4]

Orthogonal Array

Test case ↓	Parameter 1	Parameter 2	Parameter 3
1	1	1	3
2	1	2	2
3	1	3	1
4	2	1	2
5	2	2	1
6	2	3	3
7	3	1	1
8	3	2	3
9	3	3	2

Table 4: L9 Orthogonal Array Structure

Given that assumption, the table shows the set of nine combination of parameters which are sufficient to catch the fault, considering the interaction of the input parameters, which is very effective and economical. The array is orthogonal, because all possible pair-wise combinations between parameters occurs only once.

The given L9 Orthogonal Array assess result of test cases as follows [12]:

Single Mode Faults - Single mode faults occur only due to one parameter. For example, in above orthogonal array if test cases 7, 8 and 9 show error, we can expect that value 3 of parameter 1 is causing the error. Likewise we can detect as well as isolate the error.

Double Mode Fault - Double mode fault is caused by the two specific parameters values interacting together. Such an interaction is a harmful interaction between interacting parameters.

Multimode Faults - If more than two interacting components produce the consistent erroneous output, then it is a multimode fault. Orthogonal array detects the multimode faults.

Benefits

- ✓ Testing cycle time is reduced and analysis is simpler.
- ✓ Test cases are balanced, so it's straightforward to isolate defects and assess performance. This provides a significant cost savings over pair-wise testing.

In this study, as there were always 8 control factors, for all three formulations the same L18 orthogonal array structure was used for iterations.

Chapter 5

Construction of Mathematical Model

5.1 Starting Idea

The aim of this study is to develop an alternative calculation method to evaluate the stiffness of the mechanical cross member in the front suspension system, without using FEA method in order to decrease the processing time on FEM simulations.

When it is needed to make an evaluation for the mechanical properties on pre-design stage, in order to select the best design between several options, using FEM analysis for each design options can be a quite waste of time. At this point, an alternative mathematical calculation method can be used which is able to give a faster computational result than a complete FEM simulation.

Thus, this alternative calculation method can be used to make the first elimination between the proposed designs. After separating the reasonable designs and eliminating the unsuitable ones among a whole range of proposed designs, the mechanical properties for the most convenient designs can be evaluated one more time with FEM simulations, in order to obtain a final result with a higher accuracy. In this way the simulation time will be decreased.

On the front suspension, the stiffness value of the cradle part is very significant. This mechanical property of the cradle design must be evaluated primarily. With this purpose, in this study it is decided to generate a mathematical formulation *which can be used only for this specific cradle part*.

The starting point is defining the stiffness parameters. After making a little literature review (That was only done to have a basic idea about stiffness concept, literature review is not stated as a part of this study as the purpose was creating a new unique formula which cannot be found in the literature.) and brainstorming, it is decided to take *Elastic Modulus*, *Moment of Inertia*, *Thickness of the Sheet Metal*, *Cross Sectional Area* and *Length* of the cradle as main parameters for the stiffness formulation.

5.2 Mathematical Calculation of Stiffness for Simplified Cradle Models

First of all, three simplified cradle models with different dimensions were created on NX software. Then, in order to see the effect of thickness value of the cradle model on resultant stiffness clearly and make a proper comparison between the FEA results and Non-FEA results, for the same cradle model the simulations were done with three different sheet metal thicknesses values (Th1 / Th2 / Th3).

Thus;

- For one direction only, the total number of simulations for stiffness values of one cradle model (with three different thickness values; Th1 / Th2 / Th3) is ***3***.

- For all three directions (X / Y / Z) the total number of simulations for stiffness values of one cradle model is ***9*** (3x3).

- For all three directions (X / Y / Z) the total number of simulations for stiffness values of all three cradle models (Model I / Model II / Model III) is ***27*** (3x3x3).

Then an initial load applied on these 3D models on X, Y, Z directions and FEA simulation was run to determine the displacements on X, Y, Z directions. Through this simulation results, the stiffness on X, Y, Z directions can be calculated by using ***Hooke's law***;

$$F = k\underline{X}$$

In which; ***F*** is the applied force, ***k*** is the constant factor for stiffness and $\underline{X}$ is the displacement. As the displacement can be detected through simulations with the applied forces on three different directions the same relation can be expressed as;

$$\underline{X} = \frac{F}{k}$$

After defining stiffness values on all three directions (X/Y/ Z) for three different simplified cradle models (Model I / Model II / Model III), the table for Stiffness with FEM Analysis Method was created as;

3 different sheet metal thicknesses for 3 different cradle models for stiffness calculations FEM on X / Y / Z DIRECTIONS ;										
	MODEL I			MODEL II			MODEL III			
	Thickness 1	Thickness 2	Thickness 3	Thickness 1	Thickness 2	Thickness 3	Thickness 1	Thickness 2	Thickness 3	mm
Kx Value with FEM analysis	#	#	#	#	#	#	#	#	#	N/mm
Ky Value with FEM analysis	#	#	#	#	#	#	#	#	#	N/mm
Kz Value with FEM analysis	#	#	#	#	#	#	#	#	#	N/mm

Table 5 : Finite Element Analysis Table

Then, by using the main formulation parameters for the stiffness - which are *Elastic Modulus*, *Moment of Inertia*, *Thickness of the Sheet Metal*, *Cross Sectional Area* and *Length* of the cradle- the mathematical formulation model was developed and iterations were made to designate the multiplication and exponential constants. In order to verify and optimize the mathematical model, the following ratio was used;

$$R = \frac{\text{Non} - \text{FEA Result}}{\text{FEA Result}}$$

Mathematic formulation constants were validated in order to get this ratio R ≈ 1.

In this study, when the mathematical formulation was constructed by using DFSS methodology, ***Nominal is the Best*** approach was considered, the mean value of R was expected as 1 and when the mathematical formulation was evaluated, the target was increasing the Signal to Noise ratio as much as possible by decreasing the standard variation, and having the mean value close to 1 as much as possible.

5.2.1 **First Formulation (Cross Sectional Area Calculation on the Cradle Models)**

In the first formulation cross sectional areas of the cradle model was considered as a stiffness parameter. As the stiffness must increase when the cross sectional area increases, it is considered as directly proportional to stiffness.

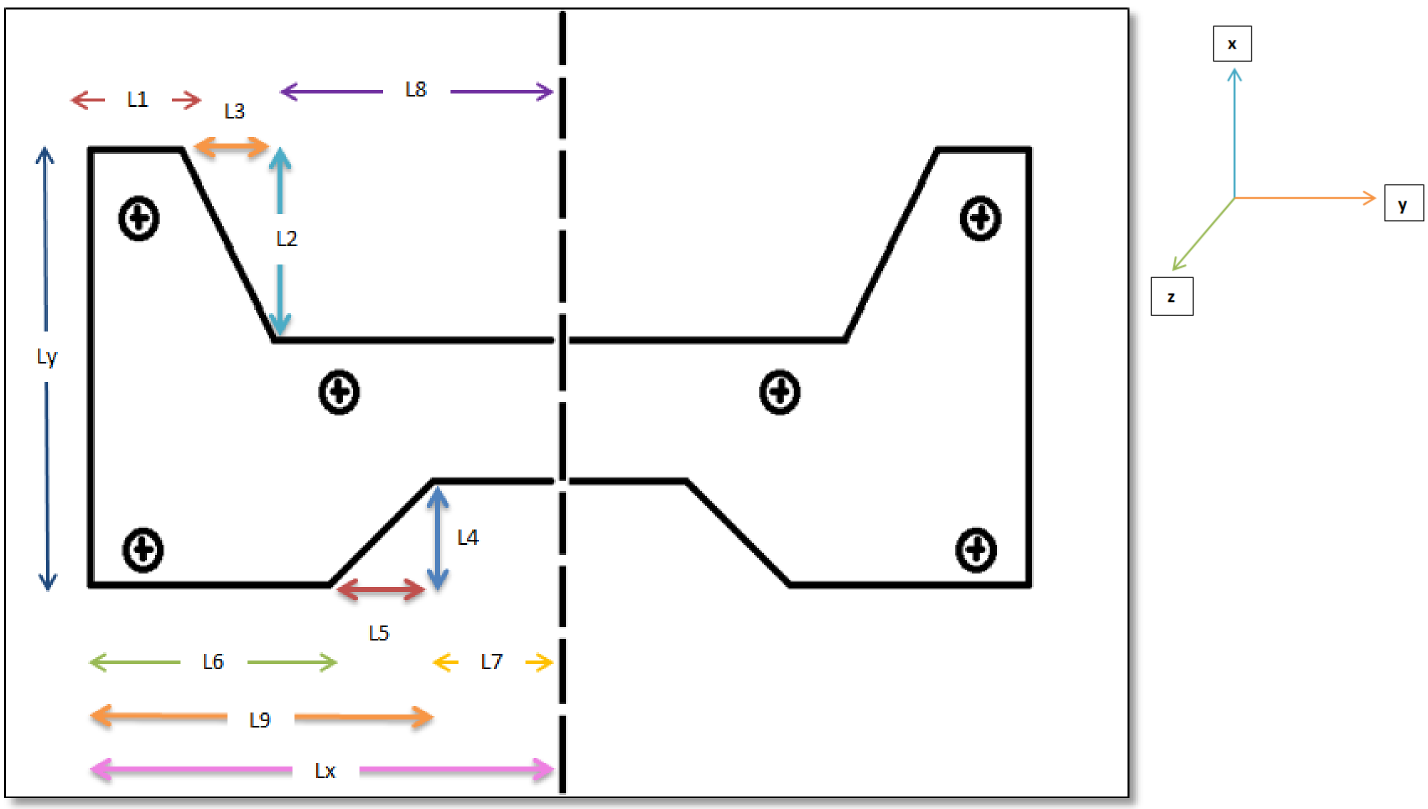


Figure 29 : Simple Geometry of Cradle Models

The cross sectional areas for three different simplified cradle models were calculated on excel with ds=5mm with the following formulations;

Cross Sectional Area on X Direction;

$$(L_1+\sum_{n=1}^{N}[L1+(\frac{L3}{N}.n)]+[(L_5+L_6+L_7).(L_y-\frac{(L2+L4)}{ds})]+\sum_{k=1}^{K}[L9+(\frac{L5}{K}.k)]).\mathbf{th}.2$$

Where;

$N = (\frac{L2}{ds} - 1)$ $K = (\frac{L4}{ds} - 1)$ and $L_9 = (L_5 + L_6)$

Cross Sectional Area on Y Direction;

$$((Ly. L_1) + \sum_{i=1}^{P} [Ly - (\frac{L2}{M} . i)] + [\sum_{i=(P+1)}^{M} [Ly - (\frac{L2}{M} . i)] - \sum_{r=1}^{R} [(\frac{L4}{R} . r)] + [(L_y + L_2 + L_4.) . L_7]) . \mathbf{th} . 2$$

Where;

$M = (\frac{L3}{ds} - 1)$ $R = (\frac{L5}{ds} - 1)$ and $p = (\frac{L6-L1}{ds})$

For this formulation, moment of inertia was not calculated. Only *Elastic Modulus*, *Thickness of the Sheet Metal*, *Cross Sectional Area* and the *Length* of the cradle were considered as main parameters for the formulation;

$$K = \frac{E . s^3 . A_{cr}}{L_x . 10^4}$$

$$K = \frac{x_1 . E^{Y1} . x_2 . (s^3)^{Y2} . x_3 . (A_{cr})^{Y3}}{x_1 . (L_x)^{Y4} . 10^4}$$

Initial Formulation

X_1, X_2, X_3, X_4 and Y_1, Y_2, Y_3, Y_4 coefficients are defined respectively Multiplication Coefficients and Exponential Coefficients of these four parameters (*Elastic Modulus*, *Thickness of the Sheet Metal*, *Cross Sectional Area* and the *Length*).

The control factors are the coefficients that can be modified in order to optimize the formulation design. Since there are 8 control factors in this formulation (4 multiplication and 4 exponential coefficients that we can switch), "***L18" orthogonal arrays structure*** is used for validation and optimization of the mathematical formula.

As there were always 8 control factors, for all three formulations the same L18 orthogonal array structure (**Table 6**) was used for iterations.

However, the formulation constructed above failed and the required stiffness values couldn't be reached by this approach. At this point, in order to improve the first formulation, another approach was developed.

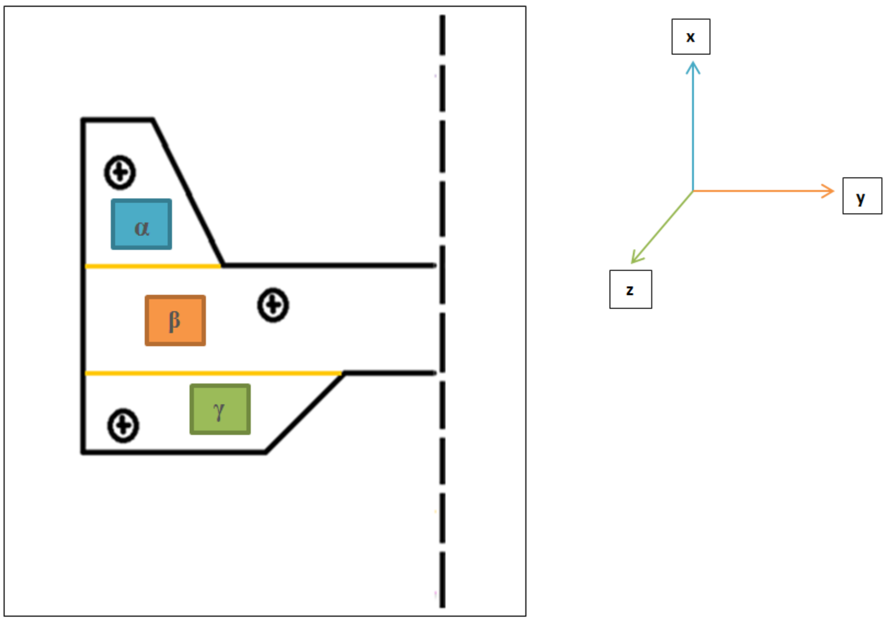


Figure 30: Half of the Symmetrical Simplified Cradle Model Divided into Three Areas for First Formulation

As it can be noticed in the cross-sectional area formulations above, in order to make the calculations easier, the half of the cradle model was considered to calculate the cross sectional areas at first, then the calculation was simply multiplied by two. When the half cradle was considered, it was also realized that there are three main geometries in the half cradle (two quadrilaterals and one rectangle in the middle).

The change in the cross sectional areas was also different in these three different geometries. Consequently, it is decided to calculate the cross sectional area changes for three different geometries on the half cradle and construct the formulation by having control factors for each ***cross sectional area*** values on three different geometries denoted by; **α, β** and **γ.** These three different geometries will be explained more in detail on the following formulations.

$$K = \frac{E \,.\, x_1 . L_1^{Y1} . (x_2 . \alpha^{Y2} x_3 \,.\, \beta^{Y3} \,.\, x_4 . \gamma^{Y4}) . 2 . S^3}{2 \,.\, L_x \,.\, 10^4}$$

First Formulation

	x1	x2	x3	x4	y1	y2	y3	y4
1	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*
2	*1*	*1*	*2*	*2*	*2*	*2*	*2*	*2*
3	*1*	*1*	*3*	*3*	*3*	*3*	*3*	*3*
4	*1*	*2*	*1*	*1*	*2*	*2*	*3*	*3*
5	*1*	*2*	*2*	*2*	*3*	*3*	*1*	*1*
6	*1*	*2*	*3*	*3*	*1*	*1*	*2*	*2*
7	*1*	*3*	*1*	*2*	*1*	*3*	*2*	*3*
8	*1*	*3*	*2*	*3*	*2*	*1*	*3*	*1*
9	*1*	*3*	*3*	*1*	*3*	*2*	*1*	*2*
10	*2*	*1*	*1*	*3*	*3*	*2*	*2*	*1*
11	*2*	*1*	*2*	*1*	*1*	*3*	*3*	*2*
12	*2*	*1*	*3*	*2*	*2*	*1*	*1*	*3*
13	*2*	*2*	*1*	*2*	*3*	*1*	*3*	*2*
14	*2*	*2*	*2*	*3*	*1*	*2*	*1*	*3*
15	*2*	*2*	*3*	*1*	*2*	*3*	*2*	*1*
16	*2*	*3*	*1*	*3*	*2*	*3*	*1*	*2*
17	*2*	*3*	*2*	*1*	*3*	*1*	*2*	*3*
18	*2*	*3*	*3*	*2*	*1*	*2*	*3*	*1*

Table 6 : 3^n Orthogonal Arrays – L18 (2^1 x 3^7) Structure for Robust Optimization

ITERATION #2	Control Factor Level 1	Control Factor Level 2	Control Factor Level 3	
x1	1,75	2		Multiplication Coefficients
x2	7	7,25	7,5	
x3	9	9,25	9,75	
x4	6,25	6,75	7	
y1	3	3,15	3,25	Exponentional Coefficients
y2	1,66	1,75	1,8	
y3	1,8	1,9	2	
y4	1,75	1,85	1,9	

Table 7 : Concerted Control Factors for 1st Formulation

The control factors of this model converges as it can be seen on the table above. However the signal to noise ratio increased to 2.75 dB from 2.65 dB.

									Mean	STD	S/N Ratio	
19	1,75	7,5	9	7	3	1,66	1,8	1,9	0,8791926	0,640556	2,750534	Optimal Design
20	3	6	9	7	3	1,8	1	1,75	0,1427983	0,105167	2,656895	Initial Design
									Gain =	0,0936387	dB	:)

Figure 31: The improvement on S/N ratio for the First Mathematical Model

The improvement on S/N ratio was not good enough and in the further iterations S/N ratio started to decrease. For this reason, the model was set aside. As the model could not displayed a successful signal to noise ratio after the first two directions, formulation for Z direction was not calculated. The study continued by generating another formulation.

The factorial effects calculations on the table below stand for the calculations only on X direction. As the first formulation was not sufficiently good, the calculation for Y and Z directions were not necessary at this point.

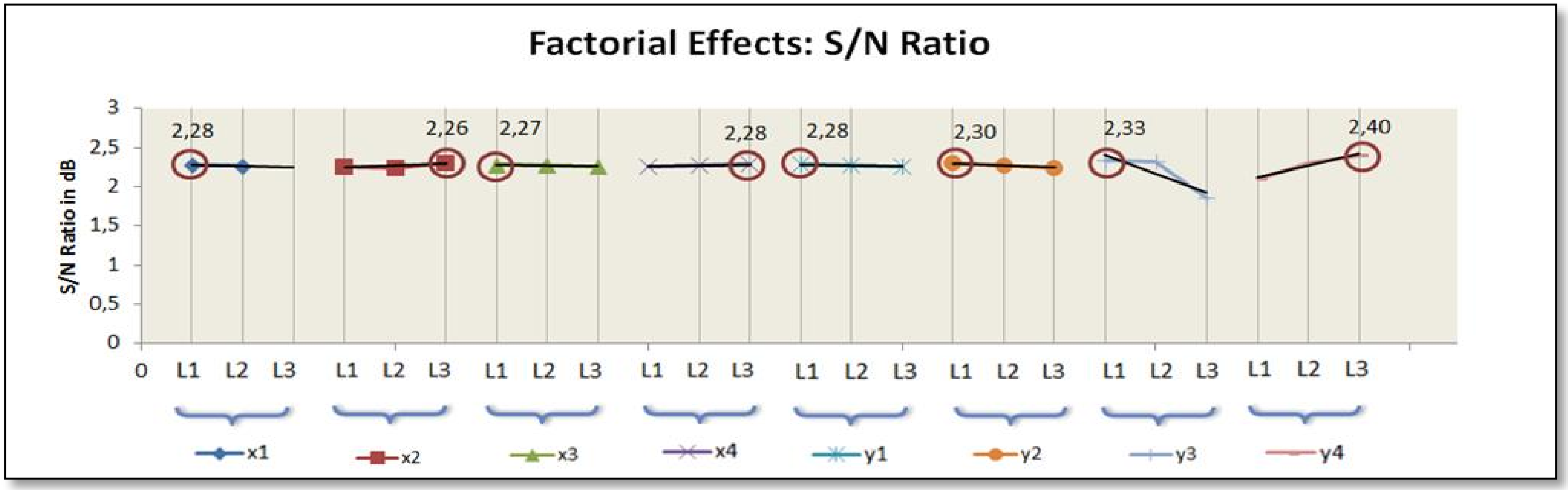


Table 8: Factorial Effect for S/N Ratio in the First Formulation

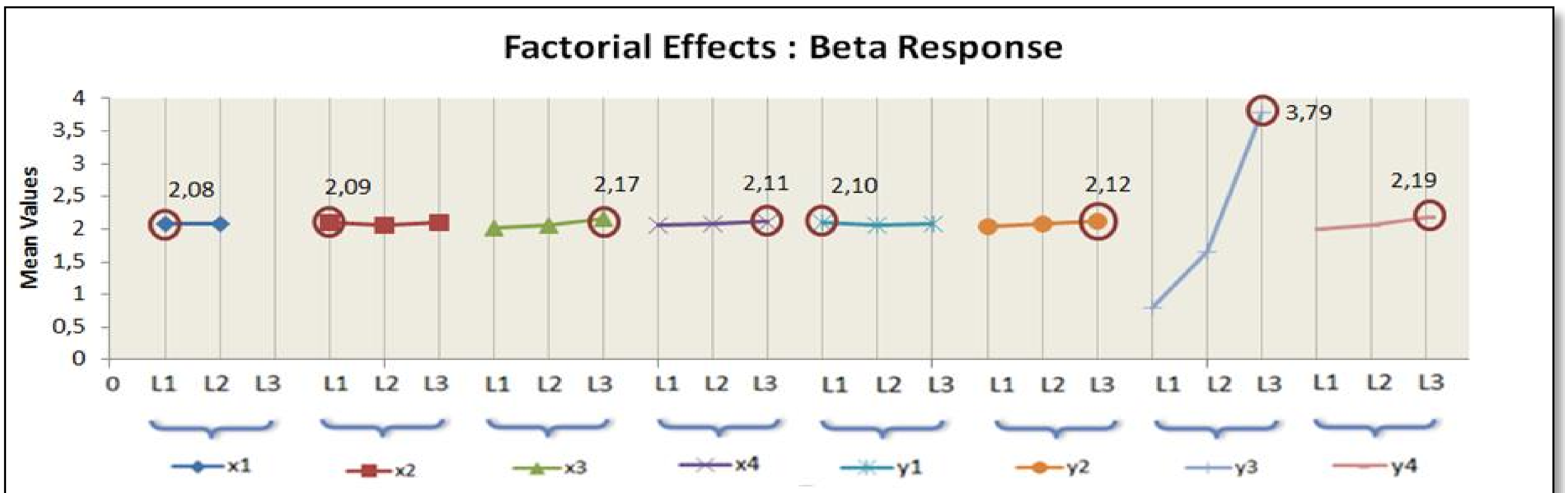


Table 9: Factorial Effect for Mean Value in the First Formulation

5.2.2 **Second Formulation**

In this step it was desired to calculate and focus on *moment of inertia* of the cradle as well. The cross sectional area calculation is replaced with *cross sectional moment of inertia calculation*. As it can be seen on **Figure 32**, half of the simple cradle model geometry was divided into three on X direction. Then moment of inertia was calculated for every cross section.

Based upon having the cradle geometry symmetric on y-axis, the cross sectional moment of inertia calculations were done only for half of the cradle and then simply multiplied by two.

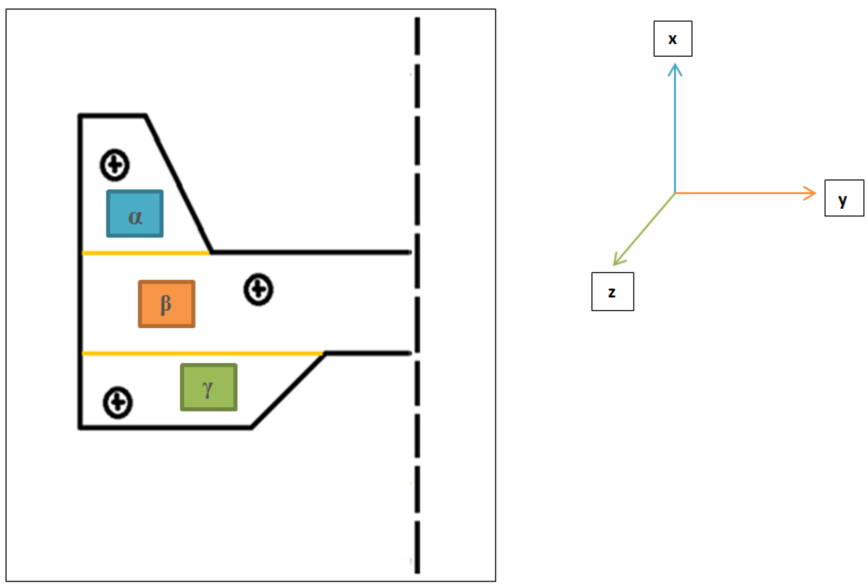


Figure 32: Half of the Symmetrical Simplified Cradle Model Divided into Three Areas for Second Formulation

As different geometries in the cradle model can affect the stiffness value differently, the cradle considered as summation of those three different parts. ***Cross sectional moment of inertia*** on X direction calculated for these three areas are named as **α, β** and **γ**. And in the stiffness calculation these three parts, of half cradle model contains different multiplication and exponential coefficients as control factors as follows;

$$K = \frac{E.\,2.\,(x_1.\,\alpha^{Y1}.\,x_2.\,\beta^{Y2}\,x_3.\,\gamma^{Y3}).\,x_4.\,\text{"CC"}^{Y4}\;.\,S^3}{2.\,L_x\,.10^5}$$

Second Formulation

Again there are 8 control factors on the formulation and L18 orthogonal array structure was used for iterations.

In this second formulation with elastic modulus **E**, cross sectional moment of inertias **α, β and γ**, thickness of the sheet metal cubed S^3 and length of the cradle L_x, we have another new term "**CC**" that stands for "*Constrain Constant*". As there are six constrain points in the whole cradle, somehow the effect of these constrain forces should also be considered for the mathematical stiffness formulation.

For constrain constant some proportional values were considered in order to make a simple approach. If only the half of the symmetric cradle geometry is considered, it is known that there are three constrain points (as shown on **Figure 33**). It can be seen that when these three constrain points are connected, that will create a triangular area (as illustrated on **Figure 34**). Starting from this, a proportional expression can be derived by considering the constrain loads.

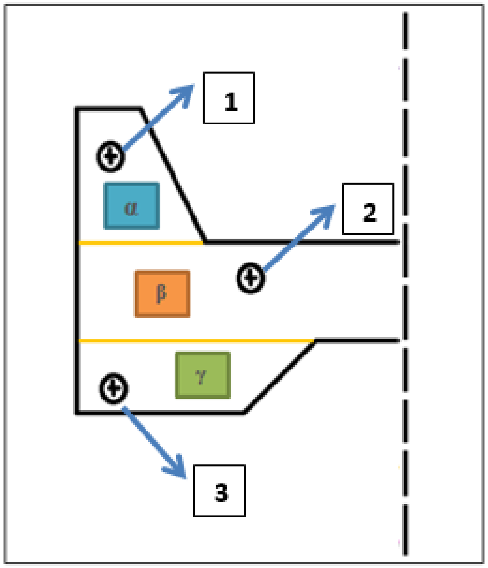


Figure 33

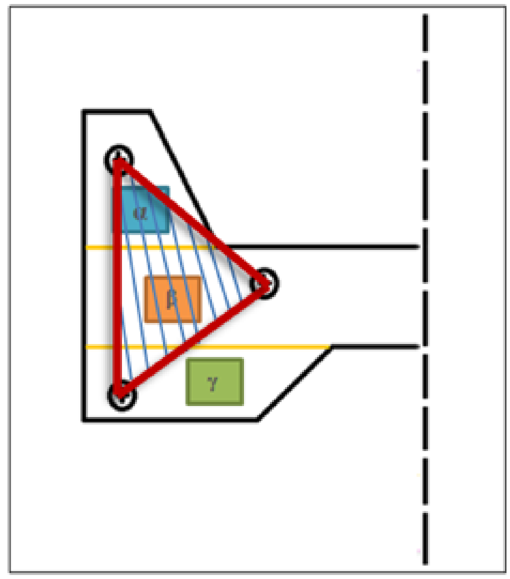


Figure 34

If we specify *the force application point* with "⊗ ", *the center of gravity of the constrain triangle* with "★" and *the center of gravity of the whole cradle* with "★" , **d1** is the distance between the force application point and the center of gravity of the constrain triangle, **d2** is the distance between the center of gravity of the constrain triangle and the center of gravity of the whole cradle and AITI is the area of the constrain triangle.

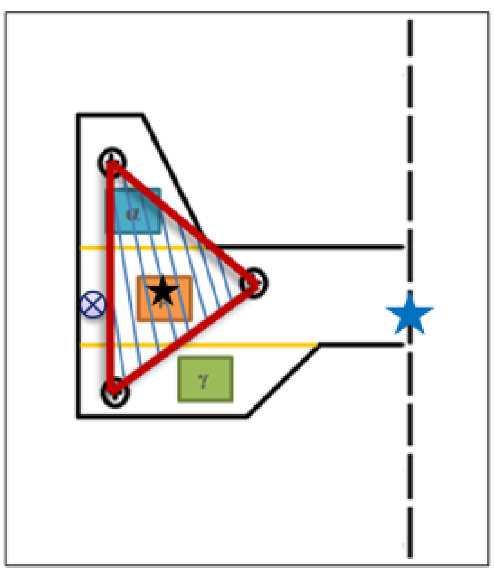


Figure 35

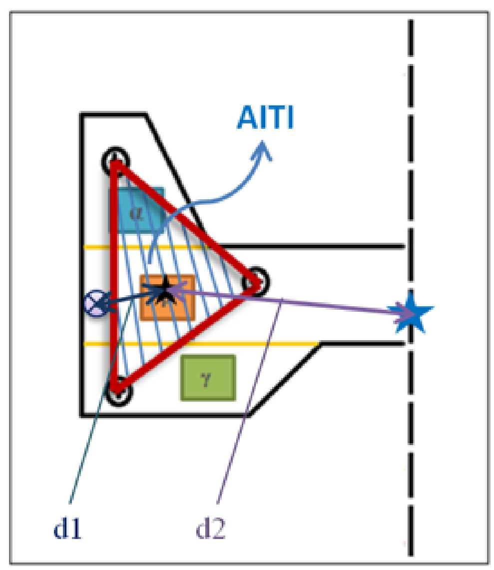


Figure 36

By considering these quantities a correlation can be built as follows;

1. It can be stated that the more the area of the triangle **AITI** increases, the more the cradle will be constraint, hence the design will be more stiff.

2. Similarly the shorter the distance d_2, the stiffer the design will be.

3. And the longer the distance d_1, the stiffer the design will be.

On the grounds of this correlation, constrain constant **"CC"** is written as;

$$\text{"CC"} = \frac{R \cdot d_2}{\text{AITI}} \quad \text{where} \quad R = \frac{d_1}{(d_1 + d_2)}$$

ITERATION #4	Control Factor Level 1	Control Factor Level 2	Control Factor Level 3	
x1	40	50		Multiplication Coefficients
x2	300	400	500	
x3	15	17	19	
x4	0,47	0,53	0,6	
y1	0,42	0,44	0,5	Exponentional Coefficients
y2	0,049	0,05	0,056	
y3	0,29	0,35	0,44	
y4	0,0043	0,0045	0,0052	

Table 10 : Iteration Chart for Second Formulation

The control factors of this model converge as it can be seen on the table above. However the signal to noise ratio increased to 1.65 dB from 1.56 dB.

									Mean	S/N Ratio	Beta Response	
19	50	500	19	0,53	0,42	0,05	0,44	0,0043	0,91307	0,754685	1,654748854	Optimal Design
20	6	100	25	2	0,12	0,05	0,2	0,02	4,27847	3,571457	1,56886517	Initial Design
										Gain :	0,085883683	dB

Figure 37: The improvement on S/N ratio for the Second Mathematical Model

In this second formulation, the improvement on S/N ratio was not good enough neither, and similarly in the further iterations S/N ratio started to decrease. As a result, the model was set aside. The study continued by constructing the next third formulation.

The factorial effect calculations below stand for only the calculation on direction X again. As the formulation was not sufficiently good, the calculations for Y and Z directions were not necessary at this point.

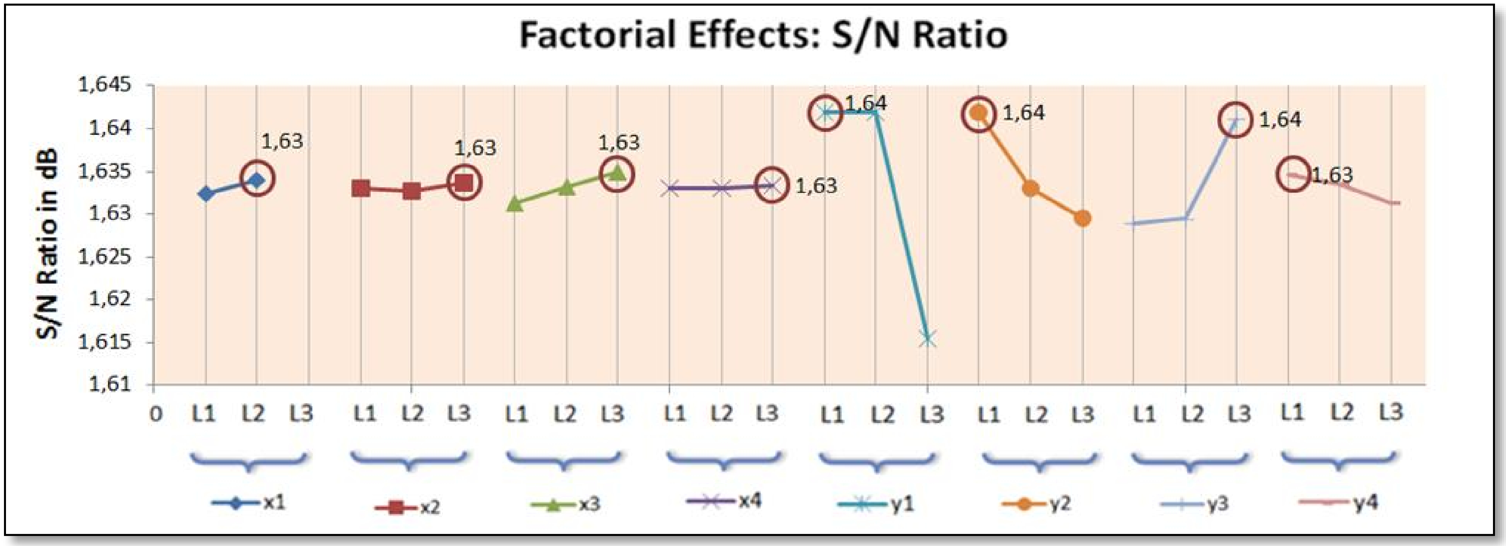


Table 11: Factorial Effect for S/N Ratio in the Second Formulation

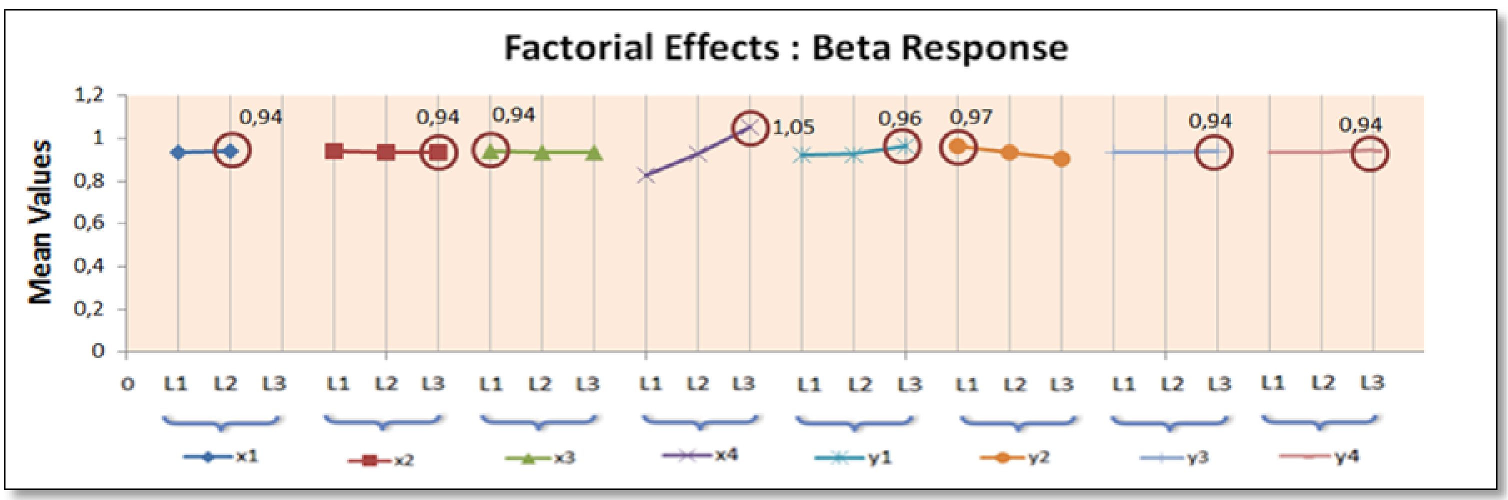


Table 12: Factorial Effect for Mean Value in the Second Formulation

5.2.3 **Third Formulation**

In this last formulation, instead of considering the effect of moment of inertia change in three different geometries, the total value of the moment of inertia was considered as one single parameter.

In the previous formulations, the idea was observing the change in one specific property (first *cross sectional area* change / then *moment of inertia* change) on certain parts of design (**α, β and γ**). In other words, instead of focusing on only the change in one certain parameter depending on the different cross sectional geometries on the design, this time the focus is on the total change in the parameters itself . Thus there are no **α, β and** γ expressions anymore. $\mathbf{I_y}$ stands for the ***total cross sectional moment of inertia*** value for the whole cradle model which is defined as a sum.

$$K = \frac{E.\ x_1.I_y^{\ Y1}.\ x_2.(\text{“CC”})^{\ Y2}\ x_3.\ S^{\ Y3}}{x_4.\ (\text{Width})^{Y4}.\ 10^N}$$

Third Formulation

ITERATION #5	Control Factor Level 1	Control Factor Level 2	Control Factor Level 3		
x1	150	190		Multiplication Coefficients for:	Moment of Inertia
x2	3,5	3,69	3,75		Dimensionless Quantity
x3	0,98	1,15	1,21		Sheet Metal Thickness
x4	126	128	129		Length of the Model
y1	0,0051	0,0052	0,0053	Exponentional Coefficients for:	Moment of Inertia
y2	2,27	2,28	2,3		Dimensionless Quantity
y3	1,8	1,9	2		Sheet Metal Thickness
y4	8,1	8,2	8,3		Length of the Model

Table 13: Iteration Chart for Third Formulation

These calculation was made for Y direction. The control factors of this model converges as it can be seen on the table above. Eventually *the signal to noise ratio* reached **10.24 dB** in the fifth iteration with a *mean value* equal to **1.00**.

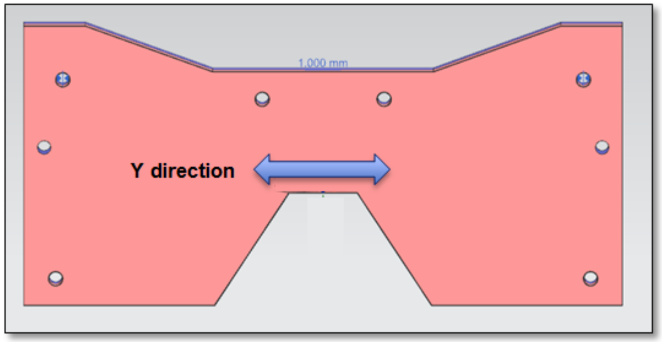
									Mean	STD	S/N Ratio	
19	*190*	*3,75*	*1,15*	*129*	*0,0051*	*2,28*	*1,8*	*8,2*	**0,9253111**	**0,2844193**	**10,24657**	Prediction
19	***206***	*3,75*	*1,15*	*129*	*0,0051*	*2,28*	*1,8*	*8,2*	**1,003232**	**0,3083704**	**10,24657**	Confirmation

Figure 38: The improvement on S/N ratio for the Third Mathematical Model

Since the signal to noise ratio obtained with this third formulation is adequately high, the calculations were made in all three directions (Y, X and Z) with this final formulation.

These numbers above stands for only calculations on Y direction. Then after, direction X and Z are also calculated as presented in the following part.

Third Formulation – Calculations on Y Direction



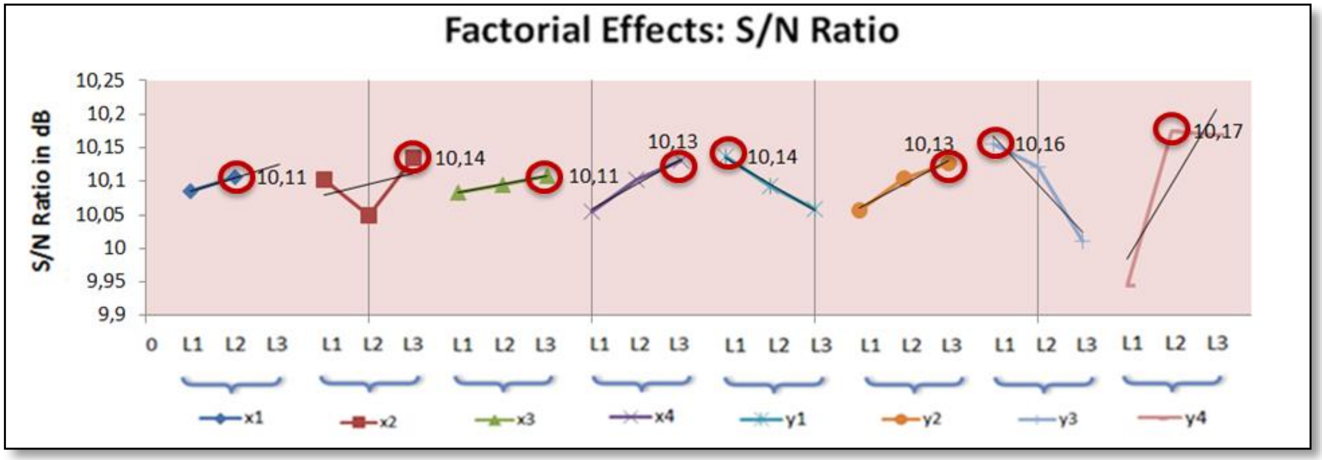


Table 14: Factorial Effect for S/N Ratio in the Third Formulation Direction Y

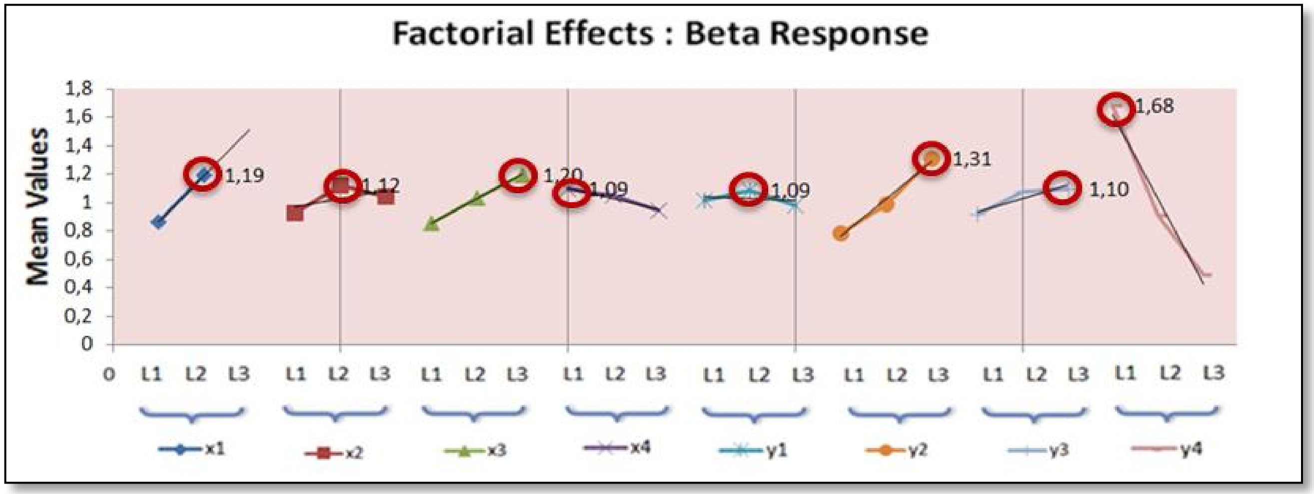
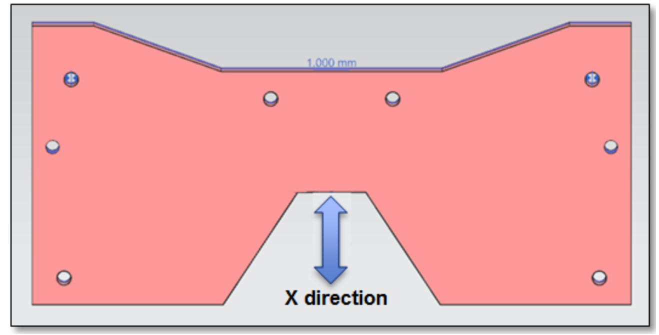


Table 15: Factorial Effect for Mean Value in the Third Formulation Direction Y

Third Formulation – Calculations on X Direction



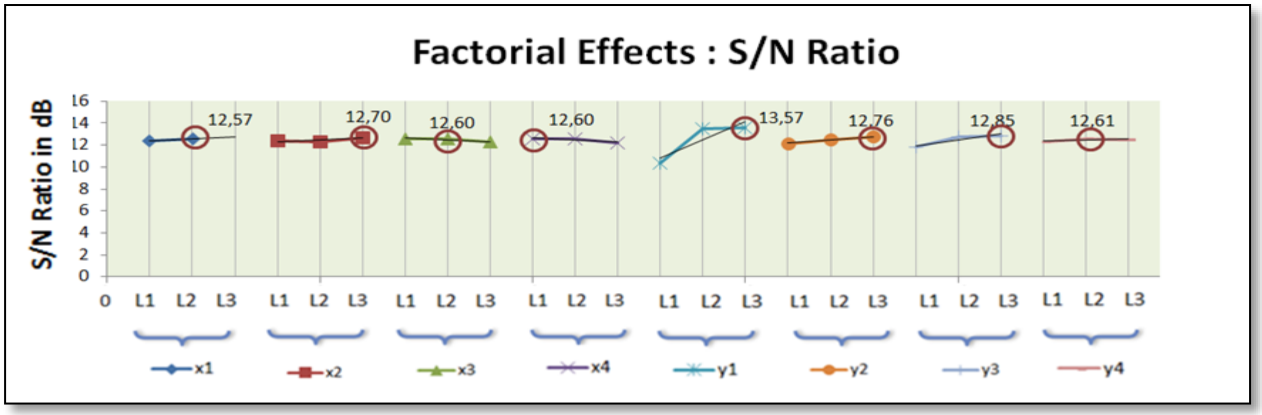


Table 16: Factorial Effect for S/N Ratio in the Third Formulation Direction X

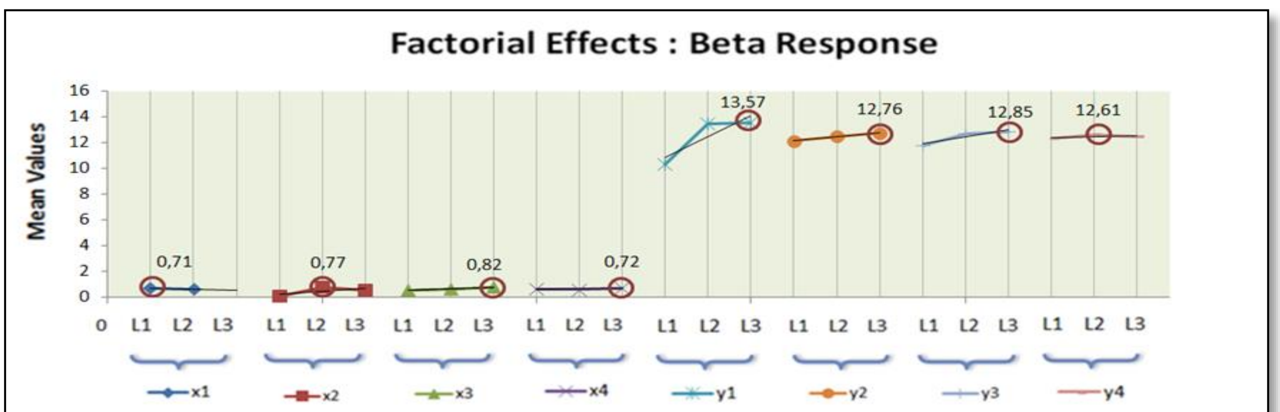
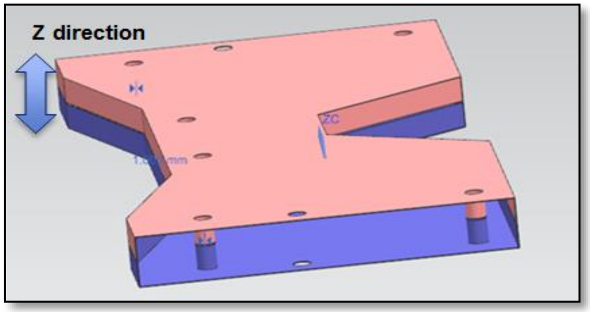


Table 17: Factorial Effect for Mean Value in the Third Formulation Direction X



Third Formulation – Calculations on Z Direction

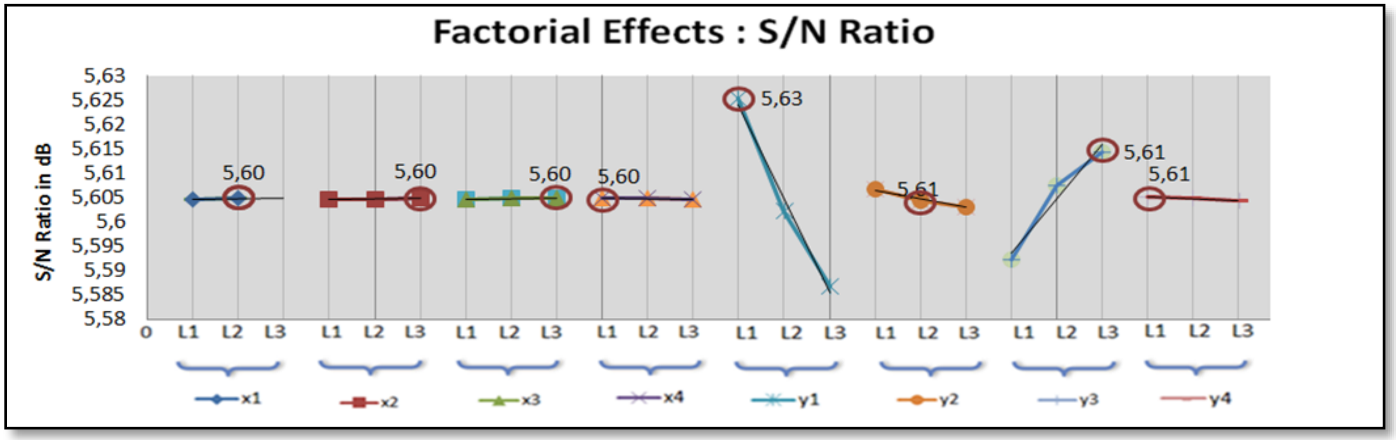


Table 18: Factorial Effect for S/N Ratio in the Third Formulation Direction Z

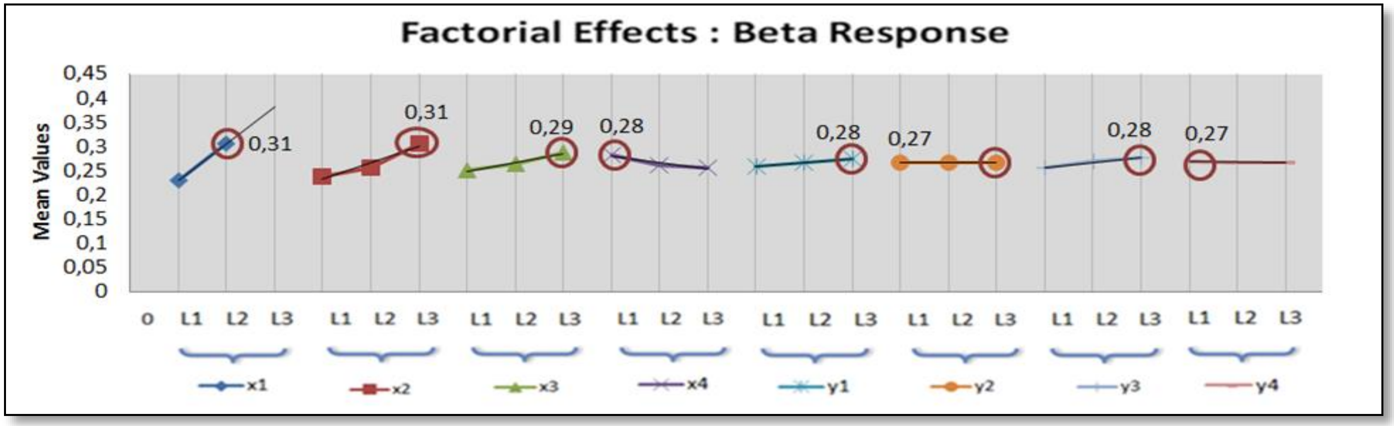


Table 19: Factorial Effect for Mean Value in the Third Formulation Direction Z

Final Formulations After Tuning – Y Direction

	x1	x2	x3	x4	y1	y2	y3	y4	Mean	STD	S/N Ratio	
19	190	3,75	1,15	129	0,0051	2,28	1,8	8,2	0,9253111	0,2844193	10,246573	Prediction
20	206	3,75	1,15	129	0,0051	2,28	1,8	8,2	1,003232	0,3083704	10,246573	Confirmation
FINAL	206	3,75	1,15	129	0,0051	2,28	1,8	8,2	0,971685	0,1272758	17,655591	After Tuning

Figure 39 : The improvement on S/N ratio for the Third Mathematical Model Y Direction

Tuning operation was done with the pre-defined functions indicated as follows;

	MODEL I	MODEL II	MODEL III
Thickness I (1.5 mm)	(1.3030) . (Result of Th 1)	(1.3030) . (Result of Th 1)	(1.3030) . (Result of Th 1)
Thickness II (2 mm)	(1.0336) . (Result of Th 2)	(1.0336) . (Result of Th 2)	(1.0336) . (Result of Th 2)
Thickness III (3mm)	(0,74575) . (Result of Th 3)	(0,74575) . (Result of Th 3)	(0,74575) . (Result of Th 3)

Table 20: Final Formulation after Tuning Operation Direction Y

Final Formulations After Tuning – X Direction

	x1	x2	x3	x4	y1	y2	y3	y4	Mean	STD	S/N Ratio	
19	1	5,3	98	23	0,52	0,82	0,89	0,98	1,97220278	0,4198658	13,436821	Prediction
20	1	5	120	25	0,49	0,7	0,99	0,98	0,69990307	0,1372543	14,150237	Confirmation
FINAL	1	5	120	25	0,49	0,7	0,99	0,98	0,78000973	0,143912	14,680058	After Tuning

Figure 40: The improvement on S/N ratio for the Third Mathematical Model X Direction

Tuning operation was done with the pre-defined functions indicated as follows;

X direction	MODEL I	MODEL II	MODEL III
Thickness I (1.5 mm)	(1,5503) . (Result of Th 1)	(1,5503) . (Result of Th 1)	(1,5503) . (Result of Th 1)
Thickness II (2 mm)	(1.4171) . (Result of Th 2)	(1.4171) . (Result of Th 2)	(1.4171) . (Result of Th 2)
Thickness III (3mm)	(1.2403) . (Result of Th 3)	(1.2403) . (Result of Th 3)	(1.2403) . (Result of Th 3)

Table 21: Final Formulation after Tuning Operation Direction X

Final Formulations After Tuning – Z Direction

	x1	x2	x3	x4	y1	y2	y3	y4	Mean	STD	S/N Ratio	
19	0,2	0,0036	2,4	0,228	0,079	0,0048	2,9	0,0007	0,371896	0,1943827	5,635270677	Prediction
20	0,44	0,0018	2,4	0,228	0,079	1,229	2,9	0,0007	4,395309	0,4520828	19,75542821	Confirmation
Final	0,44	0,0018	2,4	0,228	0,079	1,229	2,9	0,0007	1,001209	0,1029801	19,75542821	After Tuning

Figure 41: The improvement on S/N ratio for the Third Mathematical Model Z Direction

Tuning operation was done with the pre-defined functions indicated as follows;

Z direction	MODEL I	MODEL II	MODEL III
Thickness I (1.5 mm)	(4,39) . (Result of Th 1)	(4,39) . (Result of Th 1)	(4,39) . (Result of Th 1)
Thickness II (2 mm)	(4,39) . (Result of Th 2)	(4,39) . (Result of Th 2)	(4,39) . (Result of Th 2)
Thickness III (3mm)	(4,39) . (Result of Th 3)	(4,39) . (Result of Th 3)	(4,39) . (Result of Th 3)

Table 22: Final Formulation after Tuning Operation Direction Z

5.3 Verification

As a consequence, the third formulation is the most robust mathematical model which satisfies Design For Six Sigma conditions in the best way with a high signal to noise ratio and a mean value around the target value, 1.

After making the tuning operation mentioned before, the formulation can be adapted for any cradle model. Final formulation after the tuning operation reaches to a signal to noise ratio of 17.65 dB for calculations on Y direction while it reaches to 14.68 dB for calculations on X direction and 19.75 dB for calculations on Z direction.

However, even with the final formulation the signal to noise ratio is only between 14-19 dB for calculations stiffness values in three directions. Which means the accuracy of the formulation can be improved with the future work.

Chapter 6

Evaluation of the Final Results

As a result of this research, the third formulation is the one which gives the most satisfying signal to noise ratio and mean value around 1.00. Although the finite element values and created mathematical formulation give quite close results, still the accuracy of the mathematical formulation is not as high as finite element method, which was expected.

In the beginning it was assumed that the stiffness value should be linearly increasing with the increasing sheet metal thickness. However on Y direction mathematical formulation on the third model didn't follow this proportional increase. This happened because of the non-constant ratio between the FEA results.

For FEA results, as the ratio between the stiffness results for different thickness values are not constant, it is quite hard to reach a mean value around 1 with a low sigma by a mathematical formulation.

Since in the mathematical formulation thickness increases with "$x_1.(s^3)^{y3}$", where $\mathbf{x_1}$ and $\mathbf{y_3}$ are constants when S increases as 1.5mm-2mm-3mm the stiffness ratio between different thicknesses in one model will be constant.

On the other hand, it can be seen that highest signal to noise ratio is achieved on direction Z. The reason for that is the constant ratio between stiffness values for different thicknesses also in the FEA results. As finite element method also have a constant increasing ratio and the stiffness increases proportionally with the increasing thickness, it is easier to lower the standard variation for direction Z.

Chapter 7

Conclusion and Future Developments

The purpose of this study is to obtain a ***K value*** for the stiffness analysis of a *cradle* -can be also named as *mechanical cross member*- for the front suspension system of a car -*Fiat Jeep Renegade* model specifically- by using a mathematical formulation.

As it is mentioned before, the motive to lead us to search for an alternative method instead of FEA method is to decrease the processing time in the finite element simulations during the pre-design stage and provide an alternative quicker evaluation system that enables a faster selection between a large variety of design options. Then, in the following stage, the selected top designs among a large variety of pre-designed models, can be processed to evaluate the final mechanical properties and design conditions by using conventional FEA method.

This study is the first step of this alternative methodology and it will be the basis for the following research work. With this study, only the mathematical stiffness calculation is constructed. However the signal to noise ratio still can be improved to increase the accuracy of the current formulation and it can be adapted to other types of cradle models (by developing more efficient pre-defined functions).

Moreover, after the proper mathematical formulation is created, a user interface software supposed to be designed which is capable of evaluating different cradle models with this mathematical model much quicker than the finite element simulations. It will be significant to develop an optimizer system capable of choosing the best design options among the pre-design ones. At this point a project team can be created which includes mechanical engineers, designers and computer engineers. This team can work with IT management and Calculus department. With this team-working, it is possible to create a user interface with an encrypted mathematical stiffness model which may also be capable of calculating stiffness value of other mechanical members in the suspension system of a car.

References

[1] William F., Milliken Douglas, L. Milliken *Race Car Vehicle Dynamics*, Society of Automotive Engineers, 1995.

[2] Daryl L. Logan, *A first course in the finite element method*, Cengage Learning. ISBN 978-0495668251, 2011.

[3] O.C. Zienkiewicz, CBE, FRS, FREng, R.L. Taylor *The Finite Element Method*, International Centre for Numerical Methods in Engineering, 2000.

[4] 7. ASI Consulting Group, LLC*,* *Design for Six Sigma*, FCA Group

[5] Pressman, Roger S, *Software Engineering: A Practitioner's Approach* (6th ed.). McGraw-Hill. ISBN 0-07-285318-2, 2005.

[6] Phadke, Madhav S. *Planning Efficient Software Tests*. Phadke Associates, Inc. Numerous articles on utilizing Orthogonal Arrays for Software and System Testing.

[7] Dustin, Elfriede. "*Orthogonally Speaking*" (PDF).

Web edition:

[8] *How Car Suspensions Work* by William Harris, 2015 from

http://auto.howstuffworks.com/car-suspension.htm

[9] *Suspension Systems and Component*s Smith, 2002 from

http://web.iitd.ac.in/~achawla/public_html/736/15-Suspension_systems_and_components_v2.pdf

[10] *An Introduction to the Finite Element Method*, 2014 from

https://www.comsol.com/multiphysics/finite-element-method

[11] David L. Heiserman, Editor Copyright © Sweet Haven Publishing Services from http://www.free-ed.net/sweethaven/MechTech/Automotive01/AutomotiveSystems.asp?iNum=122

[12] *Orthogonal Array Testing Strateg*y (OATS) Technique. Jeremy M. Harrell Quality Assurance Manager Seilevel, Inc. , 2001 from

http://www.51testing.com/ddimg/uploadsoft/20090113/OATSEN.pdf

ACKNOWLEDGMENT

I would first like to express my gratitude to my thesis advisor Professor Alessandro Fasana from the Department of Mechanical and Aerospace Engineering in Politecnico di Torino. I want to thank him for guiding me, giving me the opportunity to work with him and trusting me with my work.

I would also like to thank Andrea Taurino, my thesis guide. He followed me during the whole process. I would also like to thank the experts in FCA who were involved in the validation of this research project. Without their passionate participation and input, the evaluation could not have been successfully conducted.

I want to express my gratitude to my best friends for their accompany throughout these years. Thanks to my classmates Andrea Zucchi, Juan Pablo Keymer Hevia, Tintu Mathew, Sanjar Okilbekov, Nikhil Rajan and Bakhtiyor Ismailov for showing me their own culture and offering me their friendship. In particular I would like to thank Craig Mclennaghan who always supported me and encourage me during this process. I must express my very profound appreciation to my family, for providing me with unfailing support and continuous encouragement throughout my years of study, and through the process of researching and writing this thesis. This accomplishment would not have been possible without them. I would like to thank my sister Özge Ilgın Ibis for all the good thoughts and positive energy she is always sending to me. Finally, I would like to express my gratitude to my parents Naile Ibis and Nejmi Ibis, they have always believed in me and have supported my dessitions. Thank you for your inconditional love.

Ozgun Tugce Ibis