Effect of the Orientation of Rock discontinuities on Wellbore Stability

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REFERENCE
NOMENCLATURE

c'_w = Cohesion of weakness planes
P_f = Pore pressure
P_{slip}^w = (Jaeger criterion) Mud pressure needed to prevent slip

\beta_w = Angle between line indicating the distance (radial) & plane of weakness

\phi_w = Friction angle (weakness plane)

\sigma_1 = Maximum principal stress
\sigma_2 = Minimum principal stress
\sigma_\theta = Tangential stress
\sigma_r = Radial stress

\sigma_{max} = Maximum principal far-field stress
\sigma_{min} = Minimum principal far-field stress

\beta_{w\text{crit}} = Critical inclination

\theta_{dip} = Dip angle
\theta_{str} = Dip direction
\theta_{IA} = Wellbore azimuth

i = angle of inclination

\gamma = Angle between the principal direction of the stress and the direction of the bigger value of one of \sigma_z, \sigma_\theta.

\sigma_{m,2} = mean stress
CHAPTER ONE

INTRODUCTION

1.0. STATEMENT OF PROBLEM

Over the years, the increase in drilling cost of operation has been a serious challenge for industry experts in the oil and gas industry. The presence of discontinuities in rocks which can range from bedding plane to faults has led to rocks portraying an anisotropic strength. Rocks exhibiting strength anisotropy cause serious stability problems during the process of drilling especially as it relates to the phenomenon of sliding along a weakness plane.

Several studies have been done to estimate the minimum mud pressure to prevent sliding along these weakness planes. The general consensus is that there will be an improvement of the stability when the wellbore is drilled at a normal or near normal to the bedding planes according to Wilson et al (1999). However, in some cases, drilling has to be carried out along directions that are not favourable to the stability.

There is a great need to investigate the simultaneous effect of the dip angle and dip direction angles of the weakness planes on the stability of wellbores and how it affects the minimum mud pressure needed to prevent slip failure.

1.1. OBJECTIVES OF THE STUDY

To investigate the effect of dip angle and dip direction on stability in weakness planes
To investigate the minimum mud pressure needed to avoid slip failure in a weakness plane
To investigate the critical condition for failure in Artificial rocks

1.2. METHODOLOGY

The idea of this thesis is to investigate wellbore stability as it relates to the inclination of the weakness planes for a wellbore that is drilled along a principal direction with an anisotropic far-field stress considering the variation of pressure with dip angle and the dip direction.. The basic fundamental equation is the Jaeger (1960) weakness plane model characterized by two main parameters which are the friction angle and the cohesion. it is the most widely used model for investigating anisotropic rock strength.
The idea of this thesis is to investigate wellbore stability as it relates to the inclination of the weakness planes for a wellbore that is drilled along a principal direction with an anisotropic far-field state of stress considering the variation of mud pressure with dip and the dip direction angles. The basic fundamental approach to analyse this issue is the Jaeger (1960) weakness plane model which is characterized by two main strength parameters: the friction angle and the cohesion of the weakness planes. This model is the most widely used in the oil and gas industry because of its very simple. This thesis seeks to analyse the stability of a wellbore with the Jaeger model a by analysing the following aspects:

1) Parametric analysis of the effect of the variation of the dip angle with a constant dip direction and obtaining a value of the mud pressure needed to prevent slip
2) Parametric analysis of the effect of the variation of the dip directions with a constant dip angle and obtaining a value of the mud pressure needed to prevent slip failure
3) Analysis on the inclinations of the discontinuities that can lead to a critical condition as it relates to the friction angles of the planes.
4) The thesis also highlights a new solution for the angle between the normal direction of a weakness plane and that of the direction of the maximum principal stress ($\beta_w$)
5) Data used for analysis is from the wellbores drilled in the Perdenales Field in Venezuela with graphs indicating the wellbore stability as a function of different wellbore azimuth
CHAPTER TWO
WELLBORE INSTABILITY

The failure of a wellbore is majorly due to a collapse of the wall of the borehole as a result of the changes in the formation and also the stress redistribution in the rock around the wellbore. The challenges relating to wellbore instability accumulate over a period of time with indicators such as borehole wall breakages as early symptoms. Other indicators include transport of damaged pieces into the annulus and the ultimate effect is that we experience challenges such as wash-out, stuck pipe and also tight hole.

2.2. Causes of Wellbore Instability

The causes of wellbore instability can be divided into two major classes which are man-made or natural causes as indicated by the figure below according to Lawrence et al. 2014

Factors that lead to wellbore instability. (Lawrence et al. 2014)

**Natural Factors**
- High Pore Pressures
- Weak rocks
- Bedding Planes
- Fractured Zones

**Man-Made factors**
- Drillstring Vibration
- Temperature
- Well Inclination
- Wash out

2.2.1. Natural Factors

The occurrence of bedding planes usually results in the failure of a wellbore due to tensile or shear failure of the weakness planes (Tan et al. 1999). According to Wu et al (2010) they posited that the strength of the bedding planes is stronger for the intact material than that of the strength along the bedding planes.
2.2.2. Man-Made Factors

Thermal gradient occurs down the borehole and this leads to a difference in temperature between the drilling fluid and the formation. This usually leads to water undergoing thermal expansion which is usually really larger than that of the rock. (A magnitude if 5-10 times higher). Choi et al (1998) volume expansion of the fluid in the pore occurs due to the heating of the formation which ultimately leads to an increase in pore pressure. The pore pressure increase combining with the thermal stress ultimately causes the borehole to ne unstable.

Tensile or shear failure results due to the mud pressure not being high enough to act as a support for the wellbore. It must be noted also that an excess of mud pressure can also cause hydraulic fracture.

The activity of drilling leads to a change in the concentration of stress in the vicinity of the wellbore. This extent of the concentration of stress is dependent of the orientation of the wellbore and that of the in-situ stress and also the magnitude (Bradley 1979). It is also mentioned that the determining factor for the necessary mud weight is the wellbore stability analysis with the underlying assumption of the rock being linear elastic, homogenous and also isotropic.

In order to investigate the stable nature of a wellbore that is inclined, Aadnoy (1988) posited a solution in order to model isotropic materials. He posited that ignoring the effect of anisotropy does affect and leads to major errors during the stability of wellbore analysis.

A 3-D criterion (Anisotropic) combined with a 3-D stress model was posited by (Ong and Roegiers (1993). They proved that two major influencers of wellbore stability are the rock strength anisotropy and also the in-situ stress differential.

Aoki et al (1993) and Zou et al (1996) used numerical methods in studying the concept of wellbore stability in anisotropic rocks. It was discovered that anisotropy plays a major role in determining the proper mud weights.

Skelton et al 1995 posited from their observations that wellbore stability is improved by drilling normal to the bedding plane compared to drilling close to parallel which leads to serious problems as it relates to stability.
Okland and Cook (1998) highlighted the importance of an ‘attack angle’ when dealing with weak planes in analysing issues dealing with wellbore stability.

Wilson et al (1999) investigated the Perdenales field in Venezuela and posited from their observations that there was a significant increase in wellbore stability when the wellbore was normal to the bedding planes compared to other orientations which cause major challenges and problems of instability.

(Chen et al., 2003) highlighted the significant increase in drilling costs is a major reason why wellbore stability is of prime concern for the oil industry at large.

Brehm et al. (2006) investigated stability of wellbore of a rock with weak bed in Shenzi field (Gulf of Mexico). They posited that when low angles of attack are considered, there was an increase in the instability but that it was less significant when drilling occurred normal to the bedding plane.

Aadnoy et al, (2009) posited that two determining factors affect rock failure along a bedding plane. They are the normal stresses and also the angle between the bedding plane and the wellbore.

Wu and Tan (2010) analysed failure in weak bedding planes using shale as material. They posited that in shale, serious wellbore stability problems and also stuck pipe problem may arise due to the effect of weak planes.

Younessi and Rasouli, (2010) posited that during the drilling procedure, it must be noted that the target reservoir extends through different rocks ranging from large faults to weak bedding planes.

Santarelli et al., 1992; Zhang et al., 2006; Chen et al. 2003; Faulkner et al., 2006, Fontana et al., 2007; Younessi and Rasouli, 2010) proved that overbalance drilling causes a serious challenge to wellbore stability due to the possible reactivation of the fractures e.g. joints. This strength reduction can cause sliding along the fractures. It is observed that drilling fluids enters into the fractures which thereby leads to a reduction in the rock shear strength. This entrance of the drilling fluid is caused by the situation of the mud pressure being higher than that of the formation.
Asadi et al. (2010) investigated fluid injection at high pressures. They posited that surface geometry and pressure plays a key role in the size of the zone that is damaged when considering a fault that has been reactivated.

Sagy et al., 2007; Asadi et al., 2010; Rasouli and Hosseinian, 2011) posited and highlighted the effect of morphology on the rock hydro-mechanical response.

Lu et al (2013) posited that wellbore stability can be significantly affected by a porous flow thereby making reducing wellbore stability.

Fekete et al (2014) explained that the appropriate trajectory for a drilled well is best known by determining the attack angle in order to prevent slip and shear failure.
2.3. ROCK FAILURE IN RELATION TO WELLBORES

During the drilling process there occurs an imbalance in the rock strength and also the stress which leads to instability caused by the failure (tensile or compressive) of the wall borehole. The in-situ stress controls the stability of the borehole. A typical borehole can experience tensile failure due to the pressure caused by the, mud induced stress at the wall of the borehole which is typically greater than the strength of the rock.

Wellbore pressures as it relates to rock stresses help to describe the instability of a wellbore. The following are the main components:

- Along the radius of the wellbore we have the stress component (radial) ($\sigma_r$).
- Round the wellbore circumference we have the hoop stress ($\sigma_\theta$) (tangential).
- There is also a shear stress component and an axial stress which acts parallel to the path of the well. ($\sigma_z$). (Lawrence 2012)

ROCK MATRIX (ISOTROPIC): SHEAR FAILURE AND HYDRAULIC FRACTURE

Fig 2: a) Insitu & b) Borehole stresses        Lawrence et al (2014)
FIG 3: Shear tension for a vertical borehole Lawrence et al (2014)

If a body that is rigid is acted upon by a normal stress as shown the figure 2a above the outcome is that we have a generation of a shear and a normal stress within the body considered.

If we consider the figure 2b above we have a plane that is imaginary and also at angle $\theta$ to the stress $\sigma_1$ which would have a normal stress $\sigma$ and a shear stress $\tau$. The shear stress acts by a sliding effect of the surfaces of the imaginary plane relative to one another while the normal stresses act by drawing the surface of the plane together. A critical point to note is that in a situation where the shear strength (induced) is more than the shear strength of the rock, the resulting occurrence is a shear failure in the rock,

To avoid this the failure in shear, the shear-stress state that is gotten as a result of the gap between the stress components should not be higher than the shear strength failure envelope.

Failure modes of a rock considering different angle of orientation and a variation of confining pressures is an important consideration in the development of a failure criterion. It is important to note under triaxial compression, the mode of failure of a typical anisotropic rock is affected by the stress orientation while for an isotropic rock it is way more complicated.

Tensile Failure leads to fracturing and in other to avoid that the hoop stress should not be lower to an extent that is leads to be tensile and is exceedingly above the tensile strength of the rock. Radial stresses are known to increase with the wellbore pressure due to mud weight and also hoop stress reduces with mud weight which leads to serious stability problems.

Drilling activities in a particular formation leads to a change in the state of stress and this leads to the redistribution of the stress around the wellbore. This state of the stress that is redistributed can be more than the strength of the rock and this may lead to failure.
Below are two types of failure in rocks:

1) Tensile Failure:
   This type of failure can be divided into two types as it relates to the principal stresses. When the mud pressure is higher than normal, Hydraulic Fracturing occurs. Exfoliation takes place the pore pressures is higher than that of the mud pressure due to the deformation of the matrix in an undrained condition. For failure to not occur the mud pressure must exist between in a safe window of the mud pressures.

2) Shear Failure:
   Shear Failure will occur when there is a mud pressure that is not sufficient to act as a support for the borehole while helical or elongated shear failure occurs when the mud pressure is too high.

Hydraulic fracturing is used in deep wells for the determination of the minimum in-situ principal stress. For a case of vertical hydraulic fracture, it is induced within a borehole which is perpendicular to that of the minimum horizontal stress.

Fig 5: Maximum and Minimum Principal stresses (Naturalfractures.com)
ROCK STRENGTH ANISOTROPY: TRANVERSELY WEAKNESS PLANES

As posited by several studies, a great number of rocks have an anisotropic behaviour as they are affected by the strength anisotropy. Rock anisotropy is as a result of two factors

1) The orientation of the microstructure
2) The presence of a weakness plane.

Rock anisotropy can be divided into intrinsic and structural anisotropy. Intrinsic anisotropy refers to the fact that the material (homogenous) exhibits different mechanical properties in the different directions. Structural anisotropy has to deal with the localized discontinuities in the weakness planes. Triaxial tests indicate that the rock strength will change in relation to the orientation of the loading while the maximum strength occurs in a situation where the axial loading is almost normal or parallel to the plane of weakness.

The plane of weakness model posited by Jaeger (1960). The failure along the intact rock material and also failure along the discontinuity. The model is primarily based on the coulomb criterion.

Fig 6: Plane of weakness model

The plane of weakness model posited by Jaeger (1960) has two main parameters which are the cohesion and friction angle parameters. It Is worthy to note that this criterion Is the most used in the industry in the prediction of rock strength anisotropy. Tien and Kuo (2001) posited a new criterion by adopting the Hoek and Brown criterion but their criterion is close to that of the extended criterion of the model posited by Jaeger (1960).
2.4.1. CONDITIONS FOR WEAK BEDDING PLANE FAILURE

Wellbore failure is caused by the following factors

- The orientation of the wellbore and the orientation of the Insitu stress
- The in-situ stress magnitude
- The orientation of the bedding plane and the position of failure on the wall of the borehole.

The stress conditions that cause failure are expressed below

![Diagram showing conditions for wellbore failure](image_url)

Fig 7: Plug bedding plane and the wellbore position (Aadnoy_)2009

The two major conditions as indicated in the figure above

- $\sigma_x < \sigma_y$ the borehole fails at Case A
- $\sigma_y < \sigma_x$ the borehole fails at Case B

It must be noted that this holds true if there is an occurrence of a big contrast in stress between $\sigma_x$ and $\sigma_y$. In cases where we have a little contrast in stress we might consider other failures that will occur which also depends on the plane of weakness.

Particles arrangement is basically as a result of the load applied orientation as it relates to the bedding plane. The anisotropy of rock is due to combination or the result of this process and it is essentially how the strength of a rock is affected by the weak bedding planes.
Aadnoy, Chenevert et al (1987) posited that anisotropy is based on the weakness of the plane and also the plane orientation as it relates to the force that is applied.

**FIG 8:** Angle between Normal to bedding plane and maximum principal stress  Jaeger and cook (1979)

The figure above indicates a triaxial test with a bedding plane at an angle $\beta$ to the maximum stress that is applied.

**Fig 9a:** Transversely isotropic specimen with bedding planes in triaxial Fig 9b: Rock peak strength variation with angle $\beta$ in the triaxial test at constant confining stress by test  Zhang (2013)
2.5. BEDDING PLANE

The discussion on the stability analysis of a wellbore and its role in field development is based on two major facts which are simply economic analysis and consideration and also the development of horizontal wells. The effect of the instability of a wellbore ranges from lost circulation to closure of the hole characterized by tensile failure and compressive failure respectively.

The instability of the wellbore leads to a huge increase in the drilling costs. There are two operational factors that are considered in the prevention of wellbore instability and they are the weight of the drilling mud and the composition of the mud. It has been observed that drilling in greatly deviated and horizontal wells are subjected to problems related to instability.

The following parameters are considered when analysing the collapsing and fracturing of the borehole: Insitu stress in terms overburden stress, horizontal stress (maximum), horizontal stress (minimum); bedding and weak-plane directions. Taking note of the Insitu stress which are influenced by the rock weight and also the lateral restraints. This is the mainly used rock failure criteria in wellbore stability analysis.

Aadnoy (1988) did an analysis on the impact of the strength of the rock, the elastic properties of the rock etc during the modelling of boreholes that are typically inclined. It was observed that during certain conditions, rock materials usually fail along a weakness plane. It was also observed that the strength of the rock is high when a vector force is at an angle that high in relation to the bedding. Therefore, for angles that are low e.g. 15°, the compressive strength is reduced therefore ultimately there is failure along the bedding planes. The instability of the wellbore is basically a compressive failure which is as a result of the enlargement of the wellbore which ultimately leads to different problems such as collapse of the hole and major hydraulic issues.
The Figure 11 indicates that the relationship between the stress direction (horizontal) and that of the direction of the maximum slip failure is not parallel. There is an angle ($\psi$) to that of the maximum stress directions (horizontal) and that of the minimum stress direction.

The failure caused by the slip failure as it relates to the weakness planes is shown in the diagram and indicated by the Red part while the blue part indicates the failure area as a result of both the failure due to shear and also the slip failure in the plane of weakness.

The Bedding plane is defined as the surface that separates a layer from another layer or a bed from another bed in a typically stratified rock.
A bed by definition is the sediment layer that is separate and different from another Layer. Their size can range from 1cm-3m in terms of thickness. These beds are also different from one another in terms of their texture and also weathering resistance. Collapse Failure is posited by (Aadnoy 1988) has having a weakness plane in the range of $100 < \gamma < 400$

2.5.1. ATTACK ANGLE, OPTIMUM WELL PATH AND DIFFERENT PARAMETERS RELATED TO BEDDING PLANE

The basic rock types are igneous rock, metamorphic rocks and also sedimentary rocks. There are three major characteristics in term of physical structure and they are the

1) Strength
2) Anisotropy and
3) Durability

Anisotropic rocks fail by three major techniques

1) An occurrence of shear failure or shear faulting across a plane of anisotropic
2) A slip or a plastic flow along a plane of anisotropy
3) Kinking

The overall nature of the failure is dependent on the orientation of the sample and also the confining pressure
Attack Angles

Islam (2010) explains using the diagram above that the instability of the wellbore can be investigated using varied attack angles between the weak bedding planes and the loading. The Fig. 12a illustrates a vertical wellbore drilled at an angle 45deg to the weak bedding plane. Drilling a well in such a setting is considered to be the highest risk of mechanical borehole stability.

The Fig 12b illustrates a deviated wellbore which is parallel to the weak bedding plane. Mechanical borehole stability is indeed a serious adverse result of a well being drilled along a bedding plane.

The remaining models shown in Fig. 12c deviated well at an angle ≥ 70Deg to the bedding plane and Fig. 12d horizontal well are relatively less challenging with respect to material failure.
2.5.2. Relation between borehole direction and borehole failure

By definition, a strike of a bed is a line that indicates the intersection of the bed with a particular horizontal plane.

The Dip is simply the angle measured between the horizontal and a planar feature. It measured as a perpendicular to the strike direction. It can also be defined as the inclination angle measured as a right angle to the strike. In a 3-D space the Dip and Strike play a major role in analysing the orientation of a plane. The angle of dip is measured in degrees.

Fig 13: Diagram showing dip, strike and attack angle (Islam et al 2010)

Attack angle is the angle between the wellbore and the bedding plane, it’s normally taken as acute angle. Attack angle 90deg means wellbore is perpendicular to bedding plane. 0° means the Wellbore is parallel to the bedding.

Attack angle is extremely important because, if favourable conditions exist (100 < Attack < 300) plane of weakness may occur at tremendous low load condition.
Failure plane means in what plane the wellbore/specimen will fail. One can analyse failure plane by Mohr-coulomb and tri-axial test (under different load condition) and can be determined angle of fracture ($\alpha$) from a specimen.
2.6. ROCK FAILURE CRITERIA

In rock failure there are different criteria to be considered. The Mohr-Coulomb Criterion and it is most applied criterion in the geomechanics industry. Based on the maximum stress criterion (normal), when the principal normal stress (maximum) reaches the uniaxial compressional strength or uniaxial tensile strength failure occurs. (Amamoo2012).

The borehole stresses play a major role in determining the stress-strain reaction used in modelling a formation under a loaded condition. Lawrence et al (2014). A common assumption is the formations is isotropic, linear elastic and also homogenous. Generally, in analysis of wellbore stability we assume a linear elastic model.

Stability simply means formation stability when a load or stress is applied. There are three main principal stresses
1) Vertical stress
2) Minimum horizontal stress
3) Maximum horizontal stress

Fig 15; Three Main Principal Stresses
As a result of movements of tectonic plates within the earth crust, stresses are induced in reservoir rocks and a state of equilibrium is attained with time. Drilling activities are the primary cause for an alteration in this equilibrium.

Over the years, many criteria have been posited for failure in rocks. These criteria of failure help to understand the in-depth knowledge of the strength of the rock and also the wellbore
stress limits. The knowledge of these factors is vital in the prediction and estimation of the instability of wellbores with the Mohr-Coulomb criterion being the most popular.

2.6.1. MOHR-COULOMB FAILURE CRITERION

This criterion takes its basis from the Mohr’s circle and also the maximum normal stress criterion (Coulomb). The criterion posits that the moment the Mohr’s circle at a point in a particular body is not enveloped by the Mohr’s circle for both the uniaxial compressional strength and also the uniaxial tensile strength, failure is likely to occur. (Amamoo 2012).

This criterion posits that rock strength is not affected by the intermediate principal stress. Jaeger (1960) explains through his analysis bedding failure due to the different loading situations. Shear failure is modelled using the Jaeger’s criterion by using the Mohr Coulomb failure model with a variation of the cohesive strength and also the internal friction angle while considering the inclination of the bedding plane relative to the loading.

The criterion is based on the effective normal stress and also the shear stress indicated by the equation below

\[ \tau = c + \sigma \tan \phi \]  

(6)

Where

- \( c \) = Cohesion
- \( \phi' \) = internal friction angles
We have
\[ \tau = \frac{1}{2} (\sigma_1 - \sigma_3) \sin 2\beta \] while
\[ \sigma = \frac{1}{2} (\sigma_1 + \sigma_3) + \frac{1}{2} (\sigma_1 - \sigma_3) \cos 2\beta \]

The Mohr Coulomb can be demonstrated in terms of the Principal stresses \( \sigma_1 \) and \( \sigma_3 \) which are the maximum and minimum principal stresses.

\[ \sigma_1 = \sigma_c + q\sigma_3 \quad \text{----------------------------- (7)} \]

Zhang et al (2010) posits that the expression of the parameters \( q \) and \( \sigma_c \) is given by the equation below

\[ q = \tan^2 \left( 45 + \frac{\phi}{2} \right) = \frac{1 + \sin \theta}{1 - \sin \theta} \quad \text{----------------------------- (8)} \]

\[ \sigma_c = 2c \tan \left( 45 + \frac{\phi}{2} \right) = \frac{2c \cos \theta}{1 - \sin \theta} \quad \text{----------------------------- (9)} \]

According to Gholam et al (2014), the required mud weight to prevent failure can be obtained.
2.7. WELLBORE PRESSURE CALCULATED WITH CRITERION
JAEGER CRITERION

The theory of weakness plane posited by Jaeger (1960) explains that a body fails in a shear form. The Jaeger theory serves a ground and foundational theory which takes into the consideration that the weakness place is characterised by a shear strength that is limited and also is noted by the Coulomb criterion.

\[ \tau_w = C_w' + \sigma_3 \tan \phi_w' \]

\[ c_w' = \text{Cohesion} \]

\[ \phi_w' = \text{Friction Angle} \]

A bedding PLANE CD plane indicated in the figure below shows an important angle called \( \beta \) where this defines the angle between the normal direction of CD and also the maximum principal stress \( \sigma_1 \).

we get the Failure condition for a weakness plane CD as indicated in below.

\[
(\sigma_1 - \sigma_3) \text{slip} = \frac{2(C_w' + \sigma_3 \tan \phi_w')}{(1 - \tan \beta_w \tan 2\beta_w) \sin 2\beta_w} \\
...............(24)
\]

\[ \sigma_1 = \text{Maximum Principal stress} \quad \sigma_3 = \text{Minimum Principal Stress} \]
As stated above, the Jaeger (1960) criterion posits that the material experiences shear failure. The theory is indeed a generalization of Mohr-Coulomb failure theory. It identifies a body that is isotropic has a set of parallel weakness planes or possesses a single plane.

**Expression for Matrix failure**

\[ \tau = \tau_o + \sigma \tan \phi \]  \hspace{1cm} (1)

\( \tau_o \) = cohesive strength

\( \tan \phi \) = coefficient of friction.

**Expression for Failure along the plane of weakness**

\[ \tau = \tau_w + \sigma \tan \phi_w \]  \hspace{1cm} (2)

Jaeger (1960) posited two major failure criteria with respect to anisotropic rocks and this was based on the Mohr-Coulomb theory as it relates to isotropic rocks.

Jaeger’s single plane of weakness takes into account the plane of weakness as it relates to an isotropic body. He also posited the ‘’ Variable shear strength’’ theory which posits that cohesive strength of the rock \( \tau_o \) is also a function of the anisotropic stress orientation.

Compression failure is as a result of shear failure cause slip (inter-granular). Shear strength is a result of the relation between the rock grains friction and also the cohesion.

Another criterion was posited by Walsh (1964) which identifies that there is an occurrence of the non-random Griffith cracks(Oriented) in materials that close under a loading force.

Hoek (1964) is posited a failure criterion, which is essentially a slight modification of that of the Griffith theory.

**Expression for the Shear Failure Gradient in the weakness planes**

In order to prevent shear failure or collapse at the wellbore in the bedding planes, the mud weight (minimum) is required while the mud weight (maximum) needed to prevent the failure due to tension or hydraulic fracturing is required.

According to Peng et al (2007) the required mud pressure needed to prevent wellbore stability can be adopted from the Kirsch’s solution which is express by

\[ P_W = \frac{\sigma_H + \sigma_h - 2(\sigma_H - \sigma_h) \cos 2\beta - UCS \beta + (q - 1)P_P}{q + 1} \]
Where \( q = \frac{1 + \sin \theta}{1 - \sin \theta} \)  

**Expression for Initiation fracture pressure and angle in a transversely isotropic rock**

In order to determine the initiation fracture pressure in the vicinity of the wellbore, it is assumed that the initiation of the fracture starts when the tangential stress (effective) magnitude is equal to that of the tensile strength of the rock.

\[ \sigma_\theta - P_p = -\sigma_t \]

A good estimation of the wellbore pressure that meets the strength criterion (tensile) is indicative of the location and position of the initiation of the fracture in the vicinity of the wellbore. \( P_{wminimum} \) is therefore the fracturing pressure at the point that is the weakest in the vicinity of the well which is the fracture initiation point. Serajian et al (2011)

**Expression for \( P_{wslip} \) using the general expression of \( \sigma_\theta \) at the wall of the borehole**

Here we combine the Jaeger criterion with the Kirsch solution for the general expression of \( \sigma_\theta \) at the wall of the borehole. This helps to give us the Mud pressure minimum to prevent the occurrence of slip failure.

\( \sigma_\theta \) is given by the expression below

\[ \sigma_\theta = \sigma_{Max} + \sigma_{Min} - 2(\sigma_{Max} - \sigma_{Min}) \cos 2\theta - P_w \quad \text{............... (25)} \]

Expressing equation above in terms of effective stress

\[ \sigma'_\theta = \sigma_{Max} + \sigma_{Min} - 2(\sigma_{Max} - \sigma_{Min}) \cos 2\theta - P_w - P_f \quad \text{............... (26)} \]

For easy derivation, we assume \( S = \sigma_{Max} + \sigma_{Min} - 2(\sigma_{Max} - \sigma_{Min}) \cos 2\theta \)

Hence \( \sigma'_\theta = S - P_w - P_f \quad \text{............... (27)} \)

We also consider \( \sigma_\theta = \sigma_1 \)

**From equation 24**

\[ \sigma'_1 = \frac{2(\sigma'_w + \sigma'_t \tan \theta'_w)}{(1 - \frac{\tan \theta'_w}{\tan \theta'_p}) \sin 2\theta_w} + \sigma'_3 \]
\[
\sigma'_1 = \frac{2(C'_w + \sigma'_3 \tan \theta'_w)}{1 - \tan \theta'_w \tan \beta'w} \sin 2\beta_w + P_w - P_f \quad \text{............... (28)}
\]

Equating

\[
S - P_w - P_f = 2 \left(\frac{C'_w + \sigma'_3 \tan \theta'_w}{1 - \tan \theta'_w \tan \beta'w} \sin 2\beta_w\right) + P_w - P_f
\]

We have

\[
S = \frac{2(C'_w + \sigma'_3 \tan \theta'_w)}{1 - \tan \theta'_w \tan \beta'w} \sin 2\beta_w + 2P_w \quad \text{............... (29)}
\]

Assuming \((1 - \frac{\tan \theta'_w}{\tan \beta'w}) \sin 2\beta_w = D\) and multiplying all parts of the equation by \(D\) we have,

\[
DS = 2(C'_w + \sigma'_3 \tan \theta'_w) + 2P_wD \quad \text{............... (30)}
\]

Making \(P_{w}^{\text{slip}}\) the subject of the formula we have the expression for the minimum mu
dpressure to prevent slip which is

\[
P_{w}^{\text{slip}} = \frac{S(1 - \frac{\tan \theta'_w}{\tan \beta'w}) \sin 2\beta_w - 2C'_w + 2P_f \tan \theta'_w}{2(\frac{\tan \theta'_w}{\tan \beta'w}) \sin 2\beta_w + 1} \quad \text{............... (31)}
\]
2.8. Wellbore stability considering new solution for $\beta_w$ where dip direction and dip angle affects its value.

If we consider the normal to a plane in a coordinate system, we consider the direction vector that is normal direction of weak bedding plane.

$$\mathbf{n} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$

The plane is characterized by the Dip Direction $\Theta_{str}$ and the Dip angle $\Theta_{dip}$ therefore

$$a_1 = \sin \Theta_{dip} \cos \Theta_{str}$$
$$a_2 = \sin \Theta_{dip} \sin \Theta_{str}$$
$$a_3 = \cos \Theta_{dip}$$

Direction of the vector of maximum principal stress $\sigma_{\theta}$ in a coordinate system with respect to the geographic north is given as

$$\mathbf{M} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k} \quad \text{where } \sigma_z < \sigma_{\theta}$$

$$b_1 = -\cos I_A \cos \Theta \cos \gamma - \sin I_A \cos \Theta \cos \gamma + \cos I_A \sin \Theta \sin \gamma$$
$$b_2 = -\sin I_A \cos \Theta \cos \gamma + \cos I_A \cos \Theta \cos \gamma + \sin I_A \sin \Theta \sin \gamma$$
$$b_3 = +\sin I \sin \Theta \cos \gamma + \cos I \sin \gamma$$

$\gamma$ = Angle between the principal direction of the stress and the direction of the bigger value of one of $\sigma_z, \sigma_{\theta}$.

Along a principal direction we know that $\gamma = 0$

$I = Inclination angle$

Combining the rock failure criteria and the well bore stability considering the direction vector we derive the formula for the new value of $\beta$ as indicated below.

$$\cos \beta = \frac{\mathbf{n} \cdot \mathbf{m}}{||\mathbf{n}|| \cdot ||\mathbf{m}||} = \frac{|a_1b_1 + a_2b_2 + a_3b_3|}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

.......................... (33)
CHAPTER THREE

EFFECT OF VARIATION OF ORIENTATION OF WEAKNESS PLANE IN WELLBORE STABILITY

3.0. Investigation of mud Pressure minimum to avoid tensile failure considering the effect of dip direction and dip angle by Parametric analysis

1) We investigate by making a parametric analysis of the variation of $P_{w_{\text{slip}}}$ with Dip direction $\theta_{\text{str}}$ while keeping the Dip angle constant and the variation of $P_{w_{\text{slip}}}$ with Dip angle $\theta_{\text{dip}}$ while keeping the Dip direction constant $\theta_{\text{str}}$, using data from Perdenales field Venezuela.

2) We also investigate the critical conditions using Artificial rock date from Tien et al(2006) by keeping $C_w$ and varying the friction angle by increasing the value from the initial.

3.1. Solving Procedures for wellbore stability analysis considering dip direction and Dip angle

Fig 19: Solving Procedure for wellbore stability analysis
DATA ANALYSIS

Data used for analysis of wellbore stability considering both dip direction and Dip angle

TABLE 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Horizontal stress ($\sigma_H$)</td>
<td>45.4MPa</td>
</tr>
<tr>
<td>Minimum horizontal stress ($\sigma_h$)</td>
<td>34.8MPa</td>
</tr>
<tr>
<td>Cohesion strength of weak bedding planes ($c_w$)</td>
<td>2.07MPa</td>
</tr>
<tr>
<td>Internal Friction Angle of weak bedding planes ($\phi_w$)</td>
<td>26.6 MPa</td>
</tr>
<tr>
<td>Dip Direction range</td>
<td>0-360 Degree</td>
</tr>
<tr>
<td>Dip Angle</td>
<td>0-90 Degree</td>
</tr>
<tr>
<td>Wellbore Azimuth ($\Theta_{IA}$)</td>
<td>315 Degree</td>
</tr>
</tbody>
</table>

Analysing the influencing parameters on wellbore stability

- Dip Angle
- Dip Direction are compared in this case
Effects of changing dip angle (θdip) with constant Dip direction (θstr) on Pwslip

In analysing the effect of a changing dip angle with constant dip direction, Table 1 data is used together with the following

I (angle of inclination) = 45 Degree

Dip angle (θdip) = (0-90°)

Dip Direction (θstr) = (0-360°)

Wellbore Azimuth (ΩIA) = 315°

The graph of mud pressure PwSLIP against Dip angle (θdip) with each dip directions (θstr) shows different irregular points but equal values at (θdip) = 0 and (θdip) = 90.

We can observe that the maximum pressures are spread from 0<θdip<90 and 0<θstr<360 as there is a change in Φw with the different change in dip angle at constant direction.

We can also observe that the highest mud pressure is at (θdip) = 40 and (θstr) = 330 while we can conclude that the lowest mud pressure peak is at (θdip) = 90
It is also observed for dip directions higher than 60°, higher pressures were observed for the downdip wells.

The critical tensile condition is also observed at $\beta_{\text{crit}} = 58.3$.

Maximum mud pressures is also observed at $\theta = 0$ and $\theta = 90$.

Considering the fact that a ratio of the maximum horizontal stress to minimum horizontal stress is quite low = 1.303, the mud pressure peaks are in a very short range close to each other.

**Change in Configuration ($I = 45^\circ, \theta_{IA} = 90^\circ$)**
Effects of changing dip Direction (θstr) with constant Dip angle(0dip) on Pwslip (I = 45° θIA = 315°)

Fig 21: Minimum mud pressures to prevent slip failure VS Dip direction (0str) with different Dip angles(0dip)

In analysing the effect of a changing dip direction with constant dip angle, Table 1 data is used together with the following

I (angle of inclination) = 45 Degree

Dip angle (0dip) = (0-90°)

Dip Direction (0str) = (0-360°)

Wellbore Azimuth (ΩIA) = 315°

The graph of mud pressure Pwslip against dip directions (0str) with each Dip angle (0dip) shows different irregular points but equal values at (0str) = 0 and (0str) = 360.

We can observe that the maximum pressures are spread from 0<0dip<90 and 0<0str<360 as there is a change in Bw with the different change in dip direction at constant dip angle.
We can also observe that the highest mud pressure is at ($\theta_{str}$) = 0 and ($\theta_{str}$) = 360 while we can conclude that the lowest mud pressure peak is at ($\theta_{str}$) = 240 and ($\theta_{dip}$) = 50.

Considering the fact that a ratio of the maximum horizontal stress to minimum horizontal stress is quite low = 1.303, the mud pressure peaks are in a very short range close to each other.

**Change in configuration with ($I=90^\circ \ \Theta_{ia}=315^\circ$)**
3.2. ARTIFICIAL ROCK

CASE 1

In analysing the effect of a changing dip direction with constant dip angle, Table 1 data is used together with the following:

I (angle of inclination) = 45 Degree

Dip angle \((\theta_{\text{dip}}) = (0-90^\circ)\)

Dip Direction \((\theta_{\text{str}}) = (0-360^\circ)\)

Wellbore Azimuth \((\Theta_{IA}) = 315^\circ\)

The cohesion strength \((c'_w)\) is set at 4MPa and Internal friction angle \((\Theta'_{w})= 29\) and it is observed that the minimum pressure occurs at \((\theta_{\text{dip}}) =0\) or \(90\) while the maximum Pwslip is slightly reduced compared to the results obtained with a lower cohesion strength \((c'_w)\) and friction angle.
ARTIFICIAL ROCK

CASE 2 (Increased Friction angle = 31 Deg, Constant cohesive strength= 4Mpa)

In analysing the effect of a changing dip angle with constant dip direction, Table 1 data is used together with the following. The cohesion strength ($c'_w$) is set at 4MPa and Internal friction angle ($\Theta'_w$) = 31 and it is observed that the minimum pressure occurs at $\theta_{dir}=240$ and $\theta_{dip}=50$ while the maximum Pwslip is slightly reduced compared to the results obtained with a lower cohesion strength ($c'_w$) and friction angle in case 1.
CHAPTER FOUR
RESULT & DISCUSSION

The results of the analysis are the following:

**Effects of changing dip angle (θdip) with constant Dip direction (θstr) on Pwslip** with I (degree of inclination) = 45 Degree & Wellbore Azimuth (ΘIA) = 315°. We can observe that the maximum pressures are spread from 0<θdip<90 and 0<θstr<360 as there is a change in $\beta_w$ with the different change in dip angle at constant direction. Also, the critical tensile condition is also observed at $\beta_{w_{crit}} = 58.3$

**Effects of changing dip Direction (θstr) with constant Dip angle (θdip) on Pwslip** (I = 45°, $\Theta_{IA}=315^\circ$) with I (degree of inclination) = 45 & Degree Wellbore Azimuth (ΘIA) = 315°. We can observe that the maximum pressures are spread from 0<θdip<90 and 0<θstr<360 as there is a change in $\beta_w$ with the different change in dip direction at constant dip angle. We can also observe that the highest mud pressure is at (θstr) = 0 and (θstr) = 360 while we can conclude that the lowest mud pressure peak is at (θstr) = 240 and (θdip) = 50

In analysing the effect of a changing dip direction with constant dip angle for **Artificial Rock**, Table 1 data is used together with the following : I (degree of inclination) = 45 Degree, Dip angle (θdip) = (0-90°), Dip Direction (θstr) = (0-360°) & Wellbore Azimuth (ΘIA) = 315°. The cohesion strength ($c'_w$) is set at 4MPa and Internal friction angle ($\Theta'_w$) = 29 and it is observed that the minimum pressure occurs at (θdir) = 0 or 90 while the maximum Pwslip is slightly reduced compared to the results obtained with a lower cohesion strength ($c'_w$) and friction angle.

For **Artificial Rock** with the cohesion strength ($c'_w$) is set at 4MPa and Internal friction angle ($\Theta'_w$) = 31 and it is observed that the minimum pressure occurs at (θdir) = 240 and (θdip) = 50 while the maximum Pwslip is slightly reduced compared to the results obtained with a lower cohesion strength ($c'_w$) and friction angle in case 1.
CHAPTER FIVE
CONCLUSION

The idea behind this work is to develop a parametric analysis around the concept of the simultaneous effect of the dip angle and dip direction on the stability of a weakness as regards slip failure. The work involves a development of the minimum mud pressure needed to avoid slip failure along a weakness plane. It involves graphical comparison of the relationship between the Mud pressure minimum and dip angle with varying dip directions from 0° - 360° and also the comparison of the relationship between mum pressure minimum to avoid slip and the dip directions while varying the dip angle. An investigation was also done for the critical condition for failure in case of an Artificial rock.

The weakness plane model (Jaeger, 1960) was investigated in terms of the critical condition for the highest minimum mud pressure to prevent slip along the weakness planes. This critical condition identifies, in a given case, a critical inclination of the weakness planes as a function of the friction angle of the planes.

The results obtained from the studies shows the importance of the dip direction and dip angle in the parametric analysis of a wellbore affected by a weakness plane in terms of Effects of changing dip angle (θdip) with constant Dip direction(θstr) on Pwslip with I (angle of inclination) = 45 Degree & Wellbore Azimuth (ΘIA) = 315. Effects of changing dip Direction (θstr) with constant Dip angle(θdip) on Pwslip (I =45° ΘIA=315°) with I (angle of inclination) = 45 & Degree Wellbore Azimuth (ΘIA) = 315°. Also in analysing the effect of a changing dip direction with constant dip angle for Artificial Rock with a changing friction angle. This thesis is a Parametric analysis of the effect of the variation of the dip angle with a constant dip direction and obtaining a value of the mud pressure needed to prevent slip. Parametric analysis of the effect of the variation of the dip directions with a constant dip angle and obtaining a value of the mud pressure needed to prevent slip failure.

Analysis on the inclinations of the discontinuities that can lead to a critical condition as it relates to the friction angles of the planes and the thesis also highlights a new solution for the angle between the normal direction of a weakness plane and that of the direction of the maximum principal stress (\( \beta_w \)). Data used for analysis is from the wellbores drilled in the Perdenales Field in Venezuela with graphs indicating the wellbore stability as a function of different wellbore azimuth.
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