

# Politecnico di Torino

Taguchi Method: Design of Experiments



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# Introduction

This thesis is aimed at this method: can we use the non-significant variables (setting to proper values), in order to minimize VARIATION of the output (Y) when the significant variables variate a little around their set values?

## 1. Taguchi methods

### A) Taguchi's Robust Design Method

Taguchi methods are statistical methods, also can be called robust design methods, developed by Genichi Taguchi to improve the quality of manufactured goods, and recently widely used in many periods. Technical statisticians are looking forward to the goals and improvements that brought by Taguchi methods, particularly by Taguchi's development of designs for studying the variation, but have criticized the inefficiency of some of Taguchi's proposals.

From 1960 on, Taguchi methods have been used for improving the quality of Japanese products which leads a colossal improvement. Within the 1980's, lots of companies in North American finally realized that the American methods for ensuring quality were not better than the Japanese methods. The American methods for quality assurance relied greatly depended on examining products when they rolled off from the conveyers of the production line and weeding out those products which did not fall within a certain acceptable tolerance range. However, Taguchi quickly pointed out that inspection didn't make a difference in improving a product quality which must be considered under design phase for a product. Companies in the North American and Europe only recently began adopting Taguchi's robust design approaches to improve product quality and design robustness.

The main purpose of Robust Design is to determine the best combination

of design parameters so that the system response has minimal dispersion around the required value for any combination of disturbance factor values.

In industrial practice, it is normally sought to limit the effect of such disturbing variables with the use of statistical techniques and error detection directly in production, which is the procedure indicated by Taguchi, or during operation / the use of a product. Designing a product that will then be hardly kept under statistical control during the implementation phase, means investing in poor quality, precluding the possibility of improving the product, increasing the costs for control.

A correct method to control (in the sense of guiding) the quality of the product before it enters production, is to distinguish the right combination of design parameters, often numerous, which allows to make the product and the production process relatively insensitive to variations and influences of disturbing factors. The technique that most promises success, to obtain the desired results, in the sense now said, is robust design.

It is worth pointing out that the Robust Design only tends to dispel the undesired effects instead of removing the causes, not orienting to cross a theoretical model that describes the phenomenon under examination, but only to dig out the optimal levels of the factors studied. This approach starts with the identification of the control factors that have more influence on the variance and after those that influence the average response (fig.1).

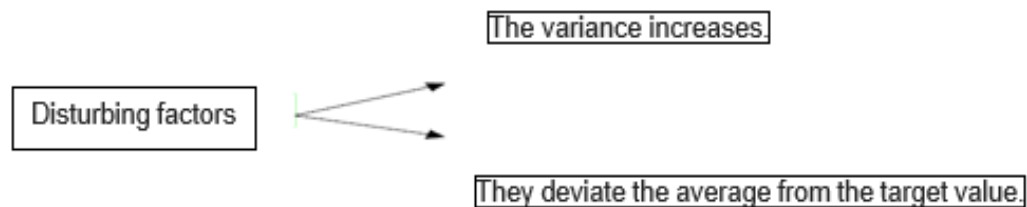


Fig.1

Within the design phase, with "optimization of tolerances", reducing the variability on the product and process quantities that have greater influence on the product's qualitative characteristics and leaving more freedom to those sizes that, instead, have little weight on the desired output, thus reducing costs where this phase is related to industrialization, for which we are going to study the factors whose levels have different costs, starting from higher cost levels (fig.2).

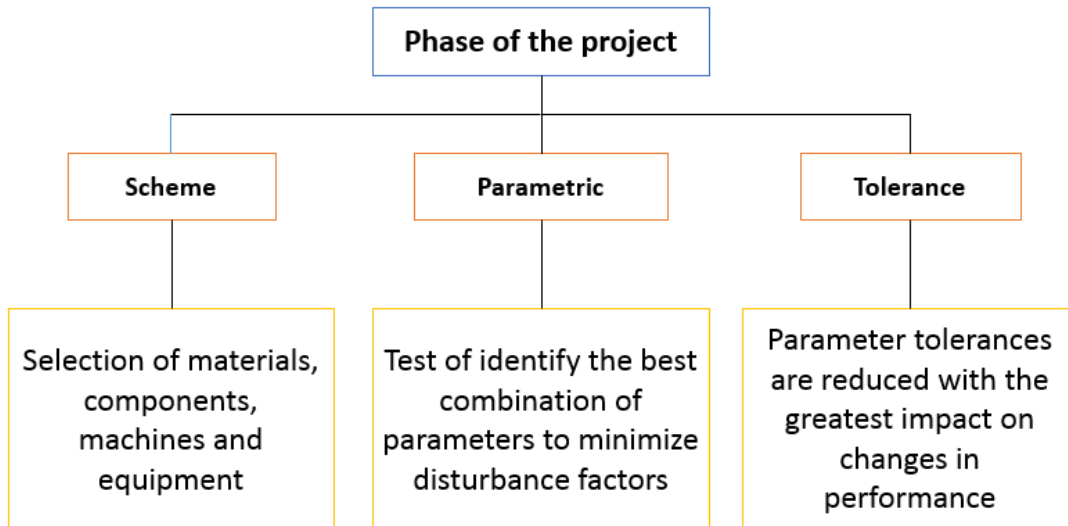


Fig.2

The variation of the output, a physical quantity or any measurable parameter, of a process - it has been said - is conditioned by many factors, whose influence is difficult to evaluate with the sole analysis of experimental data. It may be appropriate then to manipulate the values assigned to the disturbing factors to be able to determine the influence on the output. In this way, the shunt induced artificially by the sources of variation is studied, rather than the one induced by the case which is the one induced by the random error.

In the choice of parameters, it is very important to identify and divide the control factors from those of disturbance or uncontrollable, because they must be treated differently, even if this division is often subjective and linked to the knowledge of the phenomenon.

The optimization of the parameters is aimed at finding the optimal values of the controllable factors, so as to have maximum performance at the lowest cost, regardless of the values of the levels of the disturbance factors.

In addition to the aspect of improvement of the performance characteristics, it is also important that the reliability and safety of the system being designed is important. The search for possible failures in the preliminary phase to the realization of the prototype and the pre-series is of great economic impact for the manufacturer, allowing to save on the continuous changes to the project resulting from the return of data from the field.

The Robust Design can be summarized in the following points:

- Define the problem and what is to be achieved;

- Identify the system and the objective response to be optimized;
- Identify the variables that affect the response (quality characteristics) and the causes of undesired variability, effects and failure mode;
- Identify control factors and levels;
- Identify disturbance factors and levels;
- Define the conditions, the test criteria and the measurement methods for the experiments;
- Define the experiment plan;
- Perform the experiments and collect the data;
- Analyze the data;
- Identify VCF (Variability Control Factors) and TCF (Target Control Factors);
- Determine the combination of optimal levels of the various factors;
- Forecasting results with estimation techniques;
- Check the forecast with a verification experiment;
- Possible reiterate the steps related to the identification of factors and levels and perform new experiments.

## **B) Taguchi's Quality Loss Function**

Quality control in industrial production processes has largely benefited from statistical methods for assessing the effects of different sources of variability.

From the use of the classical methods of descriptive statistics, based largely on the use of graphs, to that of statistical inference, we moved on to the planning and analysis of experiments when the objectives of the quality system have expanded to include the entire production cycle.

The basic tools for the statistical planning of experiments, performed according to the methods foreseen for the analysis of the results and of the aims pursued, were essentially developed by the Anglo-Saxon school in the first third of the century.

In a few decades, the analysis of experimental data in the presence of uncertainties and the structures of the main families of experimental plans were established on a solid mathematical basis.

To this imposing body of knowledge on the statistical methodologies applied to experimentation, in the '40s it began to rely also on industry to solve problems related to quality.

In fact, due to the needs related to industrial applications, where it is often

necessary to quickly analyze the effects of numerous factors, fractional plans of different types have been developed and tabulated extensively, together with the related data analysis methodologies.

But, while the wave of economic prosperity after World War II, the quality issue was not receiving much attention in the US, Japan, struggling to rebuild, the exposed American techniques and first popularized by Deming, Juran and from then others, are studied and assimilated and the imperative need to significantly improve the quality of the product is recognized at management level.

The commitment of people like Ohno, Shingo and Ishikawa lead to establish on solid ground the Japanese movement of quality to which it gives an innovative contribution of undeniable Taguchi validity that had been exposed in its initial work environment, to the most interesting technical and methodological developments US.

Taguchi is credited with having laid the methodological basis of "robust design", highlighting in a very innovative way the costs of non-quality, albeit within the tolerance ranges, and having strongly stimulated the systematic use of the planned experimentation in the industrial field.

To put into practice the philosophy of "robust design", he proposed simplified application procedures, in order to favor a wider diffusion in the industrial field, even in environments where electronic calculators were not yet available (in the 70s).

From the beginning of the 80s onwards, the "design of experiment" and "robust design" methodologies are the subject of growing interest from the industrial world where their applications provide results of increasing importance, with the growth of dissemination and application strategies to be refined.

Its greatest contribution was to link engineering activities with statistical methods, in order to achieve rapid improvements in costs, quality and on-time deliveries by optimizing the design of products and production processes.

The Taguchi methods are based on the reduction of sensitivity to changes in external factors and internal parameters in order to optimize the costs of quality, performance and tolerances making them applicable in all industrial sectors.

According to Taguchi: "quality is the loss that a product causes to the

company after it has been sent to the customer, in addition to losses caused by its intrinsic function".

When a quality characteristic deviates from the target value, it causes a loss; in other words it is the antithesis of quality. Quality simply means no variability or very small variations from the objective value.

In order to measure the quality, Taguchi delimited the mass loss function. The quality loss function is a continuous function, which is defined as the deviation between the design parameters and the ideal value or the target value, as shown in Fig 3b. Taguchi's view on the nature of the quality loss function represents the basic paradigm shift of the manufacturer's good way of considering the product (especially in the United States). The traditional methods adopted by American manufacturers, such as SONY American certification, have been using "step function" to ensure that their performance falls within the upper and lower limit specifications shown in Fig 3a.

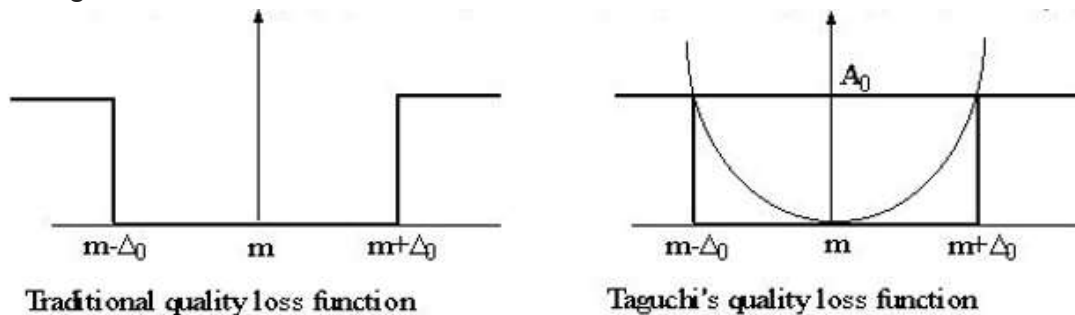


Fig. 3

In order to minimize the loss, the traditional method is to monitor process variables in production process, and adjust process to reduce manufacturing defects, so that the response parameters fall within the specified range. This method increases the cost of the manufacturing process and does not improve the quality of the product. Using the Taguchi method, the average response must be adjusted, and must reduce the variance to minimize losses. By using the off-line quality control technology products and process engineers can reduce the use of online change; quality control technology in the production process, technology and production engineers can be reached on average response adjustment. In Taguchi's concept, quality improvement methods are considered; however, it is the ultimate goal to integrate the product quality into the product at the design stage (i.e. off-line).

In order to reach the ideal product quality through design, Taguchi proposed a three stage process: system design, parameter design and tolerance design. The system design is the concept and synthesis of the products or processes which to be used. In the system design stage, the



design team uses the new ideas, concepts and knowledge in the field of science and technology to determine the correct combination of materials, parts and process and design factors to meet the functional and economic specifications. In order to improve the quality at this level, it is necessary to innovate, so it is not usually improved. In parameter design, we analyze the variables of the system by experiments to estimate how products or processes respond to the "controllable noise" in the system, and parameter design is the gist of Taguchi method. The parameter design is related to the search for the appropriate design factor level to make the system less sensitive to the change of the uncontrollable noise factor, even if the system is robust. In this way, the performance of the product is better and the loss of the customer is reduced.

The last step of Taguchi's robust design method is tolerance design. When product or process tolerances are established to minimize the sum of manufacturing and life cost of products or processes, tolerance design will occur. In the stage of tolerance design, only after the design stage of the parameter can the tolerance to the most influential factors be adjusted, and the quality target value has not been reached. Most engineers tend to combine quality with better tolerances, but restricting tolerance will increase the cost of products or processes, because it requires better materials, components or machinery to achieve more stringent tolerances. Taguchi's parametric design method can improve the product quality instead of demanding better materials or parts, and can improve quality and reduce cost (or at least keep the same). The next section will discuss the design of the parameters in detail.

The guiding concept is to economically achieve quality, low variability in the durability of functional performance. In particular, Taguchi focuses on reducing variability generated by non-controllable factors that call noise factors. The noise, or disturbance, can be external or internal. The external sources of disturbance (outer noise) are the deviations of the environmental conditions, the internal ones (inner noise) are the deviations of the characteristics of their nominal values due to processing imperfections or their deterioration.

The first parameters that must be considered in the parameter design phase are those that are easy to modify and at low cost.

If the Quality Improvement approach is maintained, then after the identification of the causes of variability we try to control the sources of variability, we will obtain a product that will work very well in a controlled environment (such as of a laboratory), but which will be subject to failures and malfunctions in a real environment. Hence the effort to try to reduce the

effects of sources of variability rather than controlling the sources themselves.

The main purpose in parameter design is to reduce the variability of the response and, as I said, this can be done by trying to reproduce in the experimental environment the variability of the process or of the operating environment.

The layout of the experimental plan for Taguchi consists of two matrices, the first one, CFA (Control Factor Array), related to the control factors (inner array) and the second NFA (Noise Factor Array), and related to the noise factors (outer array). The columns of the CFA and the NFA represent the noises and the lines a specific combination of the factor levels. The combination of the two matrices constitutes the experimental plan. The characteristic of the method is in the selection of noise factor levels instead of experimentation, leaving the choice of these levels to chance. When the distribution of noise factors is known, Taguchi also indicates the levels to be used. When it is impossible to select an appropriate level of a noise factor it is convenient to repeat the test several times.

The combination of the CFA and the NFA is accomplished by executing, in random order, each row of the CFA combined with each row of the NFA. The results of the tests, that is the observations on the system's response, will be used to elaborate statistical measures (Performance Measure), whose analysis will estimate the averages and minimize the effects of noise factors on the response.

A measure of variability (Noise Performance Measure) will serve to identify those control factors that act on variability. An analysis of the means (Target Performance Measure) will be used to identify and set those factors that act on the average.

For the measurement of noise many indices have been proposed, among which Taguchi prefers those called "Signal to noise ratios" or signal / noise ratios.

The designer can thus only intervene on some control factors called "project parameters" which are defined in the terminology of Taguchi:

- noise control factors, which are used to control the reject of the response;
- signal factors, which are used to check the average response value;
- neutral factors, which do not influence either the average or the waste, so that they can be kept at the levels for which the cost is minimal.

## 2. Procedure

When a quality characteristic deviates from the target value, it causes a loss; in other words it is the antithesis of quality. Quality simply means no variability or very small variations from the target value.

The approach of Taguchi, on the other hand, is based on the definition of robust design: "factors that can't be controlled in the use of a product are greater than those that can be controlled".

A product is all the more robust the more it is able to maintain the nominal performance as a result of uncontrollable factors.

In order to better focus this difference between the two approaches, a passage taken from: "Taguchi, Clausing: Robust Quality; HBR, jan-feb 1990".

The loss can be represented in terms of a mathematical relationship through the use of Taylor series development with starting point  $x_0$ :

$$L(x) = L(x_0) + L'(x_0)(x - x_0) + \frac{L''(x_0)}{2}(x - x_0)^2 + \frac{L'''(x_0)}{3!}(x - x_0)^3 + \dots$$

Where:

$x$  is the actual performance of the product;

$x_0$  is the nominal performance of the product;

$L(x)$  is the loss function that depends on the deviation  $|x - x_0|$ .

With the hypothesis of null loss in nominal conditions ( $L(x_0) = 0$ ), request of a minimum in the loss function ( $L'(x_0) = 0$ ) and neglecting the terms superior to the second order (when the difference between  $x$  and  $x_0$  is very small) you will get a quadratic form of the loss function:

$$L(x) \approx \frac{L''(x_0)}{2}(x - x_0)^2 = K(x - x_0)^2$$

$$K = \frac{C}{\Delta^2}$$

Where:

$\Delta$  is the maximum deviation tolerated for  $x$  with respect to  $x_0$ ;

$C$  is the cost supported when this value is exceeded.

The equation thus obtained is the equation of a parabola.

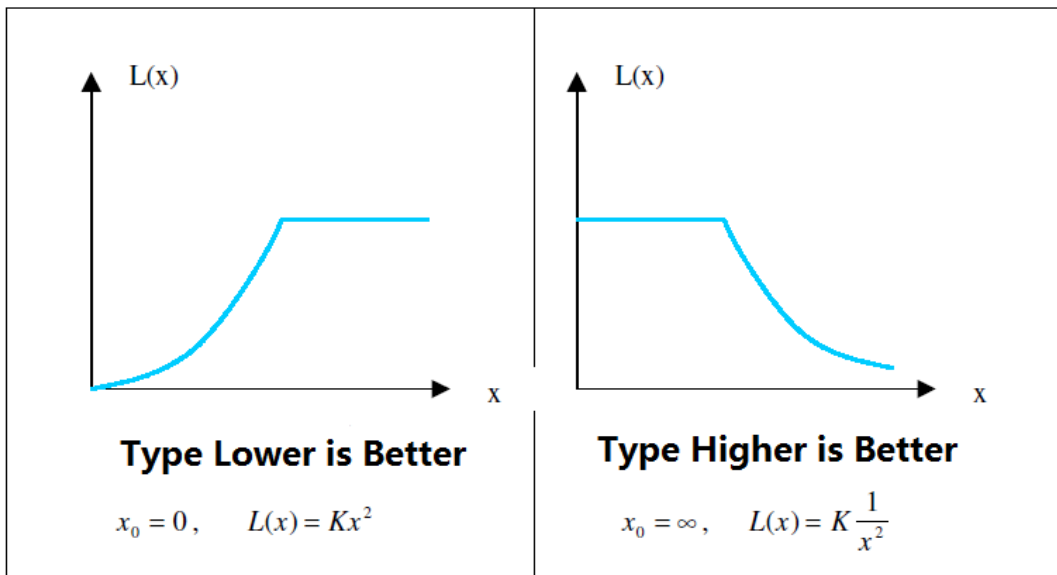
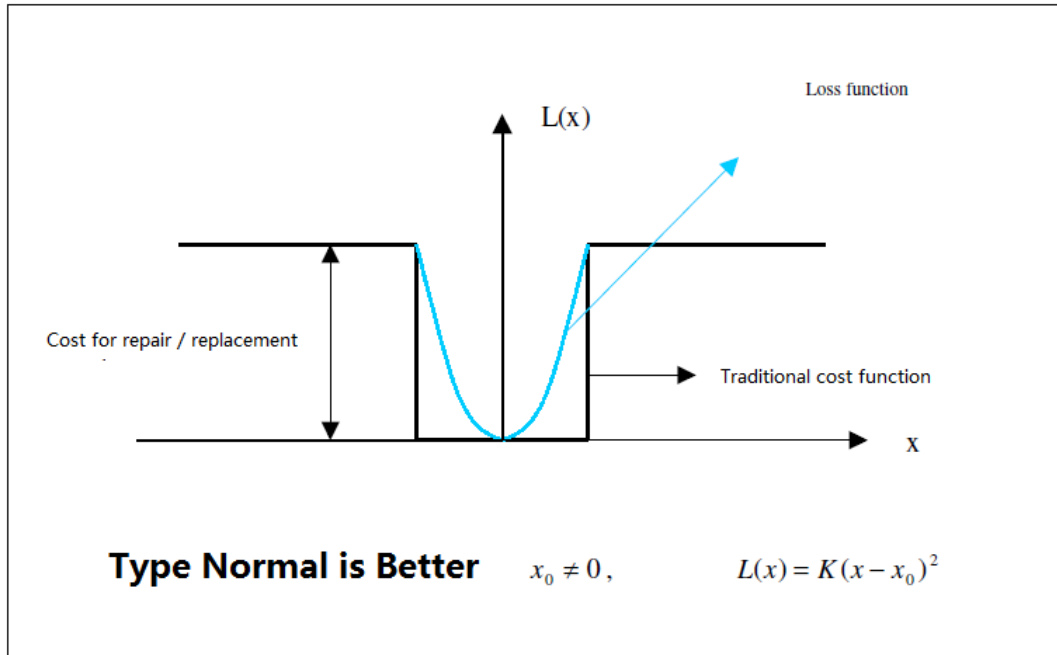


Fig.4

If we consider  $x$  a random variable, the loss function becomes a random function:

$$x \longrightarrow X = \hat{\mu} + Z, \quad Z \approx N(0, \sigma^2)$$

Where:

$\hat{\mu}$ , media controlled by statistical quality control;

$Z$ , robust design.

Three cases will be considered below:  $x_0 \neq 0$ ,  $x_0 = 0$ ,  $x_0 = \infty$ , for each of them

the average value will be calculated.

1. Case  $x_0 \neq 0$ : the loss function will be  $L(x) = K(x - x_0)^2$

$$\begin{aligned}
 E[L(x)] &= K \cdot E[(x - x_0)^2] \\
 &= K \cdot E[x^2 + x_0^2 - 2xx_0] \\
 &= K \cdot E[x^2 + \mu^2 - \mu^2 + x_0^2 - 2xx_0] \\
 &= K \cdot \{E[x^2] + \mu^2 - \mu^2 + x_0^2 - 2x_0\mu\} \\
 &= K \cdot \{[E[x^2] - \mu^2] + (\mu - x_0)^2\} \\
 &= K \cdot \{(\mu - x_0)^2 + Var(x)\}
 \end{aligned}$$

2. Case  $x_0 = 0$ : the loss function will be  $L(x) = Kx^2$ .

$$\begin{aligned}
 E[L(x)] &= K \cdot E[(x - x_0)^2] \\
 &= K \cdot E[x^2 + x_0^2 - 2xx_0] \\
 &= K \cdot E[x^2 + \mu^2 - \mu^2 + x_0^2 - 2xx_0] \\
 &= K \cdot \{E[x^2] + \mu^2 - \mu^2 + x_0^2 - 2x_0\mu\} \\
 &= K \cdot \{[E[x^2] - \mu^2] + (\mu - x_0)^2\} \\
 x_0 &= 0 \\
 &= K \cdot \{\mu^2 + Var(x)\}.
 \end{aligned}$$

3. Case  $x_0 = \infty$ : the loss function will be obtained by changing the variable.

$$y^2 = \frac{1}{x^2}, \quad L(x) = K \cdot \frac{1}{x^2}$$

With the help of the Taylor series, with initial point is, brought up to second order we will get the average of the loss function:

$$\frac{1}{x^2} = \left(\frac{1}{x^2}\right)_{x=\mu} + D\left(\frac{1}{x^2}\right)_{x=\mu} (x-\mu) + D^2\left(\frac{1}{x^2}\right)_{x=\mu} (x-\mu)^2$$

$$E[L(x)] = K \cdot \left[ \frac{1}{\mu^2} + 3 \frac{Var(x)}{\mu^4} \right]$$

Summary table:

$x_0 \geq 0$	$L(x)$	$E[L(x)]$
$x_0 \neq 0$	$K(x-x_0)^2$	$K[(\mu-x_0)^2 + Var(x)]$
$x_0 = 0$	$Kx^2$	$K[\mu^2 + Var(x)]$
$x_0 = \infty$	$K \frac{1}{x^2}$	$K \left[ \frac{1}{\mu^2} + 3 \frac{Var(x)}{\mu^4} \right]$

Table1

To identify the most robust combination of the control factor levels, the monotonic signal-to-noise ratio function is analyzed, which highlights the effects of the factors on the variability of the x performance. The signal-to-noise ratio is reciprocally proportional to the cost evaluated with the loss function. The greater the relationship, the more robust its performance.

It is therefore possible to state that the signal-to-noise ratio gives a sense of how close the design is to the optimal performance of a product or process.

<b>case</b>	<b>Max of SNR</b>
$x_0 \neq 0$ (Fig.5 a)	$10 \text{Log} \frac{\bar{x}}{S}$
$x_0 = 0$ (Fig.5 b)	$-10 \text{Log} \frac{\sum_i x_i^2}{n}$
$x_0 = \infty$ (Fig.5 c)	$-10 \text{Log} \frac{\sum \frac{1}{x_i^2}}{n}$

Table 2

The case  $x_0=0$  is used every time when you want to check the losses due to the impurities in order to reduce the effect; instead,  $x_0=\infty$  will be preferred when you are interested in power.

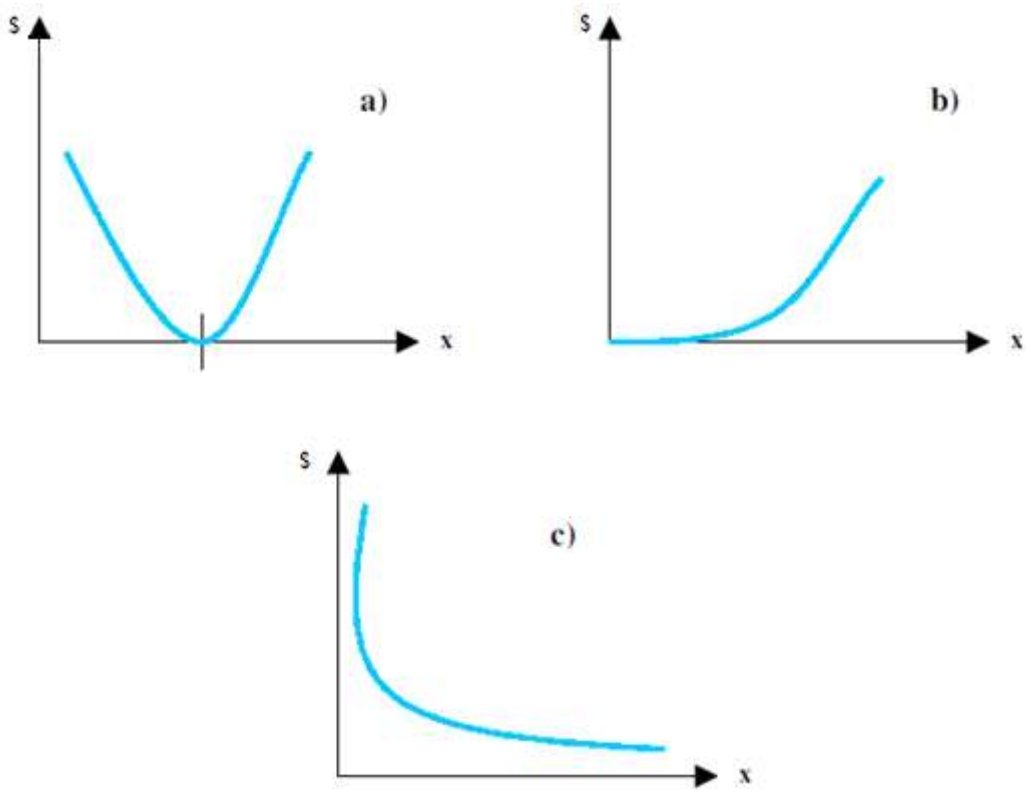


Fig.5

### **3. Design of Experiments**

Design of experiments (DOE) is a mature technique which is used to experimentally characterize and improve the performance of a physical system and which enables designers to determine simultaneously the individual and interactive effects of many factors that could affect the output results in any design. DOE also provides an adequate insight of interaction among every design elements; therefore, it gives assistance to turn any reference design into a robust one. Generally, DOE helps to point out the sensitive parts and sensitive areas in designs which will cause problems in Yield. Then designers are able to solve these problems and produce robust and higher yield designs prior going into production.

The fundamental importance for the success of an experiment is the programming of the same not only from the point of view of the magnitudes involved and the levels of the factors, but for all the activities and tools necessary for the survey and analysis of the data. The effect of the results of an experiment is clearly conditioned by the timing of obtaining the results, which must be available in time for any operational decision to modify or innovate a process. The complexity of programming must conform to the level of complexity of the system under study and the type of economic constraints to which it is subject.

The "design of experiments" techniques allow sampling the parameter space in a minimum and optimal number of points in order to derive the elements useful for the construction of mathematical models, which are therefore empirical and non-phenomenological models.

In the past few decades, it has also been used for product design, which evolves into various approaches such as robust design. More recently, DOE has become the mainstay of the "Improve" phase of the Define-Measure-Analyze-Improve-Control cycle in Six Sigma, as it dedicates the most efficient and effective ways to reveal possible cause-and-effect relationships in the subject of study.

The basic problem of "design of experiments" is to decide which set of measurement points is most suitable in the particular situation of interest.

In some applications, data on the response are in a discrete form either because assessment of performance is routinely reported in non-numeric or because of the fact that there is little necessity or practical meaning in having a response accurately measured. Some examples are the use of letters or indices to describe the extent of Wi-Fi coverage, strength of signals, degree of



air pollution, amount of error of hit of projectiles, or level of performance of prototypes. In fact, in the service industry, it is common to have customer feedback on some variant of choosing on a range of 'Strongly Agree, Agree, Neutral, Disagree, Strongly Disagree', sometimes simply represented as 'on a scale of 1 to 5, and so on. In many oral, written, or performance examinations, letter grades of A, B, C, and so forth are commonplace.

With the more extensive acceptance and popularity of the statistical approach and some labeled tools such as those of Six Sigma or projects related to Data Science, reports of demonstration or actual studies on systems which do not furnish themselves to responses that evaluated on a continuous scale which have appeared with increasing frequency. Even without the Six Sigma label, studies of non-physical subjects such as service systems and related surveys are now ubiquitous, using DOE as the fundamental of analytical for measures related to improvement of service quality. This paper aims at bringing to the attention of well-intentioned users of DOE a consequential issue that is seldom addressed in for example on-the-job training in industry.

The basic problem of "design of experiments" is to decide which set of measurement points is most suitable in the particular situation of interest.

The question of the location of the experimental points for the estimation of an empirical model is apparently a vicious circle, because if the behavior of the function was known beforehand, it would be possible to make a good choice of the measuring points.

But the determination of the function is accurately the object of the investigation.

Fortunately, this circularity is not paralyzing, especially when the experiments can be conducted sequentially, so that the information obtained in a group of experiments influences the choice of experiments of the next group.

The simplest and most frequently used scheme for each considered value in the past in the industrial field consists in carrying out one or more tests, or level, of the parameter assumed as independent, maintaining constant every other condition.

Then in order to evaluate the effects of the other parameters, the procedure is repeated for each considered parameter.

The excessive simplicity for this kind of procedure to proceed with uni-variate is accompanied by two essential disadvantages, which make its use seldom justifiable.

The first disadvantage is that the use of experimental results is poor, particularly when it is necessary to perform replications to measure the variance of the quality characteristic; the second disadvantage is that indications are obtained about the effect of each parameter valid only for one and only one particular combination of levels of all the others.

In other words, lacking any estimate about the extent of the interactions within the various parameters, the effect of changing two or more parameters at the same time can't be predicted.

The "design of experiments" is not only on account of a multivariate approach, where more than one parameter is changed simultaneously from one experiment to another. Once the system parameter levels have been set, the maximum information can be obtained with a complete factorial plan, which provides for the testing of all the possible combinations of the levels of the various parameters and which allows the estimation of models containing all the terms of interaction between the parameters involved.

The complete factorial plans are the easiest form among the multivariate experimental plans contemplated by the "design of experiments" techniques.

On the other hand it is not necessary in these cases to resort to models including all the terms of interaction between each parameter, since generally those higher order interactions are listed on decreasing entity.

We then resort to "fractional factorial" plans that exploiting a priori knowledge or "working hypothesis" related to the experimental environment where we operate, allow us to estimate the "non-negligible" parameters of the model instead of utilizing the costly complete plans.

The construction of a fractional factorial plan is carried out respecting another important property: the orthogonality.

This allows to estimate each term of the model in a statistically uncorrelated or "independent" way from the others.

There is then a whole series of experimental plans, not necessarily orthogonal, (centered compound planes, star planes, optimal planes, etc.) which present peculiarities sometimes important for the efficient estimation of non-linear models.

The "design of experiments" techniques consent to achieve exceptional efficiencies in the derivation of data given by the experimentation, tending the

experiential modeling of systems with extreme complexity both for the number of parameters involved and for non-linearity behavior.

The basic requirements for the DOE are as follows:

- Experiments must have clearly defined objectives;
- The effects of the factors must not be hidden by other variables: The use of appropriate models and schemes helps to divide the effects of interest from those due to uncontrolled variables and simplifies the analysis of the results;
- The experiment must be performed by randomizing the sequence of tests;
- The experiment must provide an estimate of the variance of the experimental error by replicating some tests.

Randomization of the tests (Randomization)

Performing tests of the experiment in random order gives greater confidence that the experimental disturbances and errors are distributed in a normal way between the data and allows the use of tests of significance of the factors according to appropriate confidence intervals.

Repetition of the tests (Replication)

The repetition of the tests is useful for measuring the accuracy and repeatability of the observations with the aim of balancing the effect of unknown variables or disturbing factors that can act differently from one replica to another.

Subdivision of the experiment (Blocking)

A homogeneous portion of the experimental material or environment is defined as "block". For example, elements taken from a given batch will typically be more uniform than those taken from different lots.

Even the experimental observations collected during a limited time will have less dispersion than those collected over several days. If a blockage is made on the day, it means that the variability due to the test day is of little importance, or that one does not want to study the effect due to its variability.

The need to subdivide the experiment may derive from the batch supply of the experimental material, or from the duration of the tests or from the difficulty of setting a factor level.

The presence of disturbance factors therefore urges the choice of the experimental plan so that:

1. The possible influence due to the disturbance factors does not influence the factors of primary interest;
2. Some information on the magnitude of the effects of the disturbance variables can be obtained.

### A) Parametric plans:

The simplest experimental plans are the parametric plans (also called "one factor at a time"), in which the effect of each factor is assessed on the responses at different levels, while maintaining the other fixed factors.

To conduct a parametric plan you can proceed in one of the following ways:

1. An experiment can be conducted in which, at each execution, the level of one factor is varied at a time, keeping the others constant as shown in Table 3 (for two-level factors). The limit of this method is clear: if there are interactions between one factor and another, these can't be evaluated; moreover, the high number of combinations ( $2^k$  for  $k$  factors at two levels) makes the experiment extremely long and expensive.

Execution	Factor	Level	Measurement
1	A	-	xx
2	A	+	xx
3	B	-	xx
4	B	+	xx

Table 3

2. You can evaluate variations of several factors but one at a time; with 4 factors the five combinations shown in table 4 are obtained. In reality, even if this reduces the number of combinations from  $(2^k)$  to  $(k + 1)$ , the interactions between the  $K$  factors can't yet be evaluated.

Combination	A	B	C	D	Measurement
1					xx
2					xx
3					xx
4					xx
5					xx

Table 4

It is noted that in the four cases in which the factor A is at the "-" level, the factor is 3 times "-" and only 1 time at the "+" level, whereas when A "+" B is only at the "

For balancing, on the other hand, at the "-" level of factor A, B would be many times at the "+" level as often as in the "-" level. When A is "+", B should again take the two levels "+" and "-" in equal numbers.

3. A third approach to the problem is to change all the factors at the same time, as indicated in the table in the case of 4 factors.

Combination	A	B	C	D	Measurement
1	-	-	-	-	xx
2	+	+	+	+	xx

Table 5

In this case there are only two combinations (regardless of how many factors are), but the possibility of evaluating the effect of the individual factors is lost: for example, if two factors act in opposite fashion, the fact that the result on the answer is nothing could suggest that they do not characterize the experiment.

### B) Complete factorial plan

The limits of the three previous methods are overcome by a type of DoE already established and widespread: the complete factorial experiment (Full Factorial Experiment).

With this method, regardless of the number of factors, it is possible to have a balanced plane in which each factor can be estimated independently from any other and it is also possible to evaluate all the interactions of any order.

This plan is used to study two or more factors at various levels when the interaction between factors can be significant. The number of tests is  $y^k$ . For example, if you have two factors with three levels each, you will need to conduct  $3^2$  tests to get the full factorial plan.

In general, if for the  $i$ -th factor we have a number of levels  $n(i)$  then the number of trials will be the product of all  $n(i)$ .

In the analysis of data we speak of main effects and interaction.

If the factor has more than two levels, it is called main components: linear, quadratic, cubic, etc.

The fact is intuitive thinking that between two points (two levels) passes a straight line from which it is possible to understand the linear effect, between three points passes a second degree curve or the quadratic effect is estimated.

In the phase of choosing the number of levels and their spacing, we try to obtain as much information as possible on the effect of the factor in the study.

The problem presented by the complete factorial experiment is that of being very "cumbersome" even if extremely precise and detailed; already with a 4-factor experiment and ten replicates, 160 answers are obtained which, without adequate computer support, make the statistical analysis heavy.

If then the number of repetitions or factors increases further the described method can be practically inapplicable for the time and the cost involved.

When the factorial has a high number of tests to be conducted in non-homogeneous conditions, the so-called block factorial plan is used.

The complete set of combinations of factors and levels is subdivided into subsets (blocks) sacrificing the estimation of higher order interactions (which usually have less effect).

The partition of the set of the factorial tests the blocks is made using suitable subdivisions so as to safeguard the estimation of the main factors and interactions and the levels corresponding to the interaction not to be estimated among the factors in combination with each level of the factors to be estimated.

When the experimental material or environment can be conveniently divided into homogeneous blocks, randomized block planes are used in which the tests are conducted in random order within each block.

### C) Analysis of multi non-significant factors

Source: Said Jahanmir NIST Ceramics Division

Description: Measurements from high performance ceramics experiment with the goal of characterizing the effect of grinding parameters sintered reaction-bonded silicon nitrated.

Data
X1: Table Speed
X2: Feed Rate
X3: Wheel Grit
X4: Direction
X5: Batch
Y: Strength
ORDER: Run Order

Table 6

This example is used to demonstrate a full factorial experiment.

X1	X2	X3	X4	X5	Y
-1	-1	-1	-1	-1	680.45
1	-1	-1	-1	-1	722.48
-1	1	-1	-1	-1	702.14
1	1	-1	-1	-1	666.93
-1	-1	1	-1	-1	703.67
1	-1	1	-1	-1	642.14
-1	1	1	-1	-1	692.98
1	1	1	-1	-1	669.26
-1	-1	-1	1	-1	491.58
1	-1	-1	1	-1	475.52
-1	1	-1	1	-1	478.76
1	1	-1	1	-1	568.23
-1	-1	1	1	-1	444.72
1	-1	1	1	-1	410.37
-1	1	1	1	-1	428.51
1	1	1	1	-1	491.47
-1	-1	-1	-1	1	607.34
1	-1	-1	-1	1	620.8
-1	1	-1	-1	1	610.55
1	1	-1	-1	1	628.04
-1	-1	1	-1	1	585.19
1	-1	1	-1	1	586.17
-1	1	1	-1	1	601.67
1	1	1	-1	1	608.31
-1	-1	-1	1	1	442.9
1	-1	-1	1	1	434.41
-1	1	-1	1	1	417.66
1	1	-1	1	1	510.84
-1	-1	1	1	1	392.11
1	-1	1	1	1	343.22
-1	1	1	1	1	385.52
1	1	1	1	1	446.73

Table 7

First of all, we need to be done is to distinguish the significance of each factor related to the output.

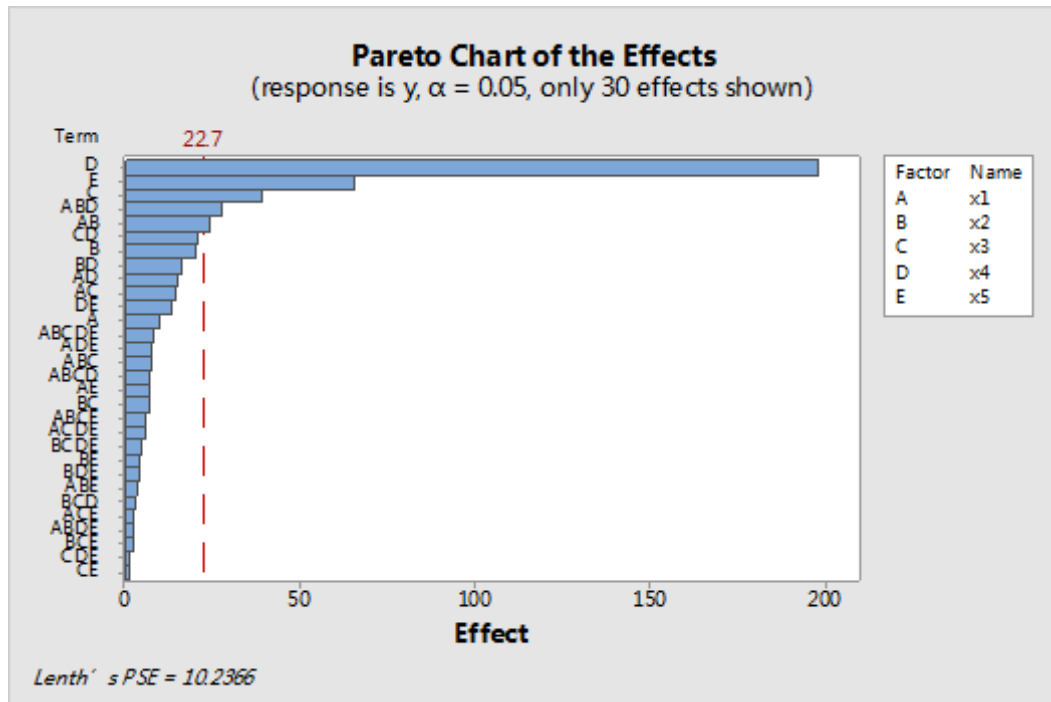


Fig.6

It's obviously that C, D, E, AB and ABD are significant. To have more clearly understanding of each factor's significance, focus only on the significant factors and re-plot the Pareto Chart of the Effects.

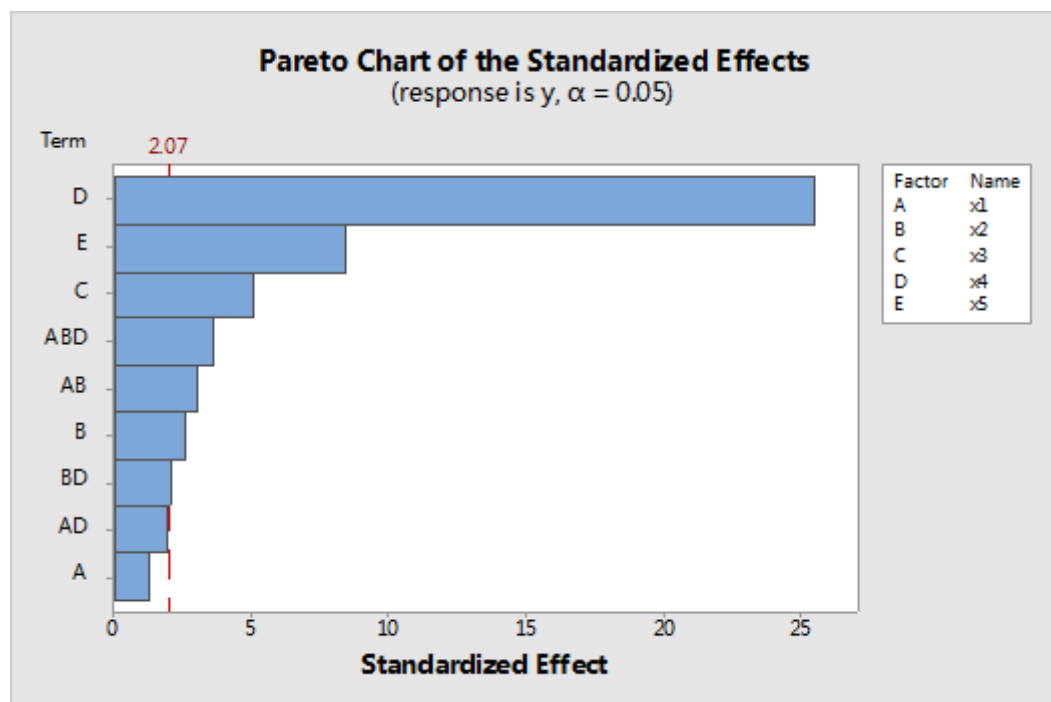


Fig.7



Now we clearly distinguish that ABD is significant only depends on the D. Then we try to optimize the results by changing the non-significant factors which is exactly what we purpose to discuss. Hypothesis: x2 equals to 1.

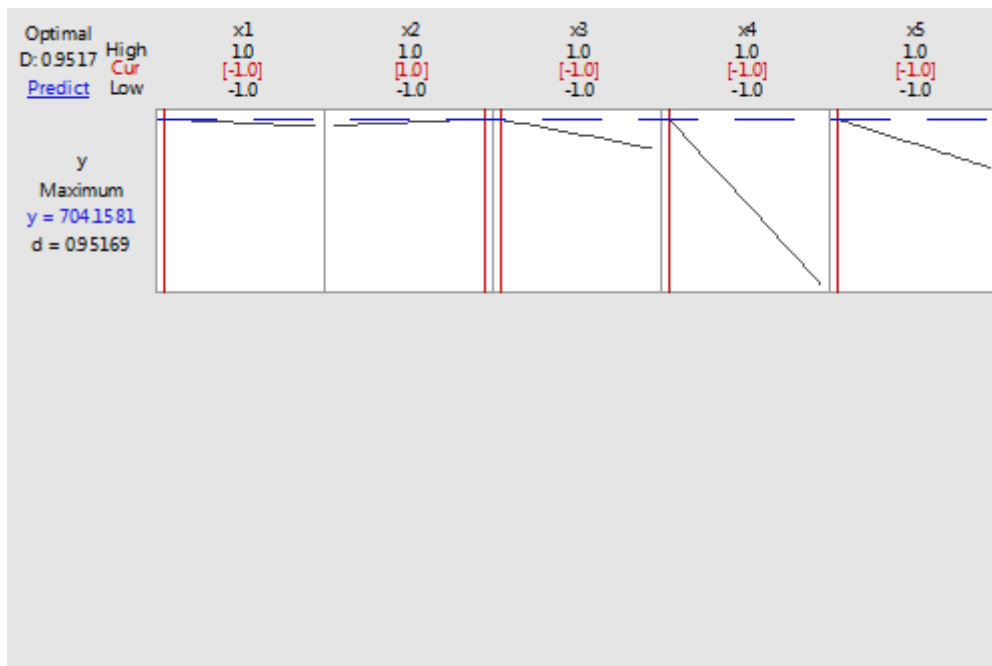


Fig.8

What was shown in fig.8 is the maximize result of output Y for each factor locates in a specific value. Shown in the figure, the maximum value of Y is 704.1581 as x4 change from -1 to 1 the Y reduce to 479.9356 shown in fig.9. Difference is 224.225.

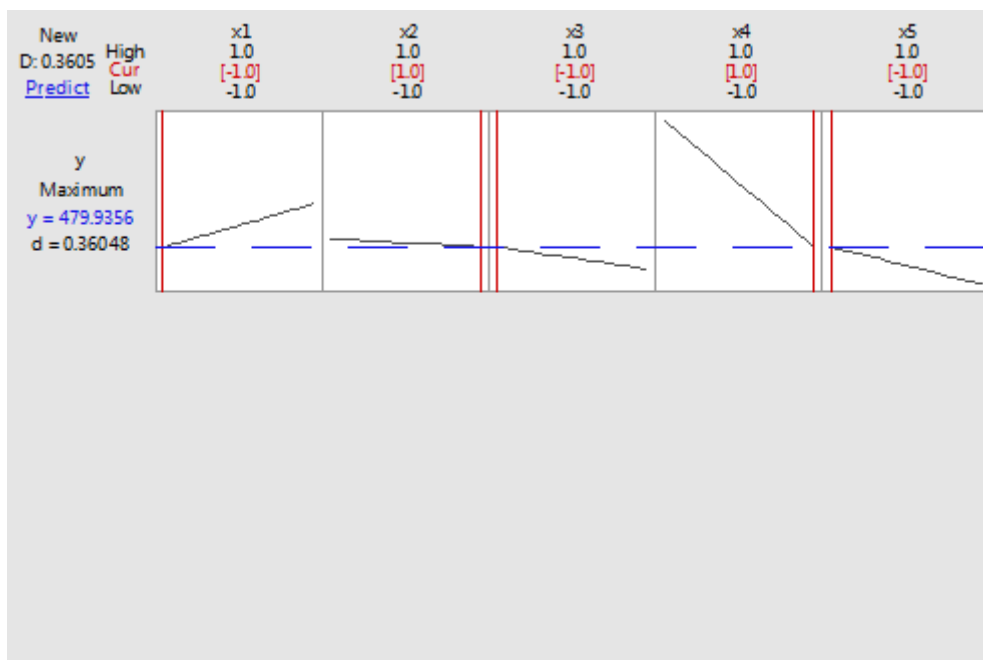


Fig.9

As we change the value of  $x_1$  from -1 to 1. The maximum value of  $Y$  is now changing to 695.4581.

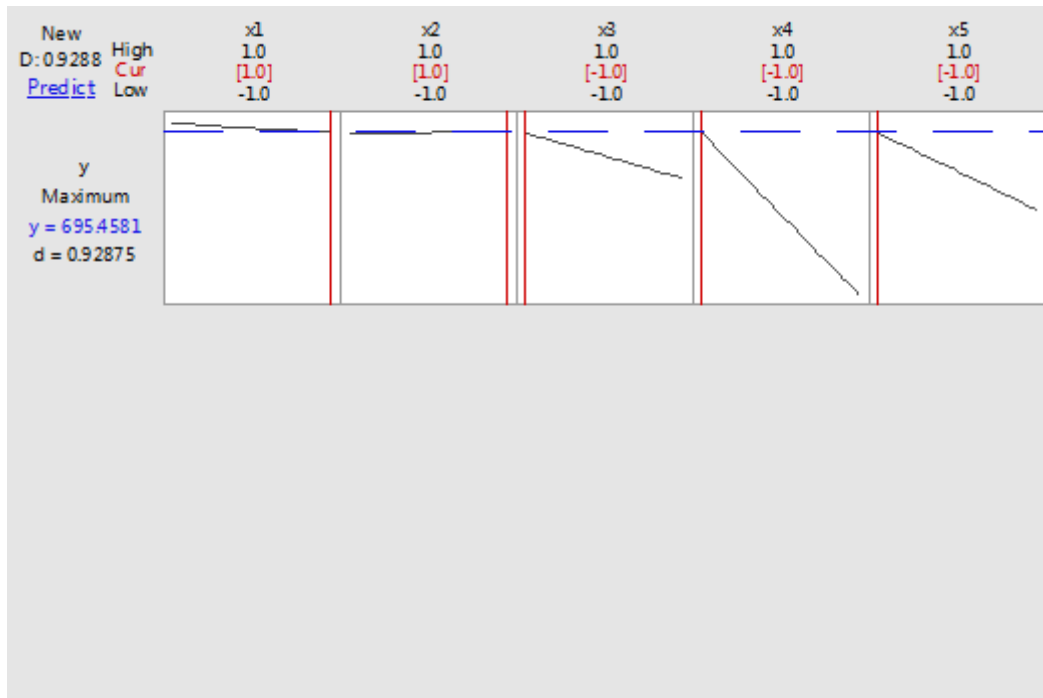


Fig.10

Under this condition, as we change  $x_4$  from -1 to 1. Maximum of  $Y$  will reduce to 556.6406 shown in fig.11. With difference equals to 138.8175 which is lower than the difference of  $x_1$  equals to -1.

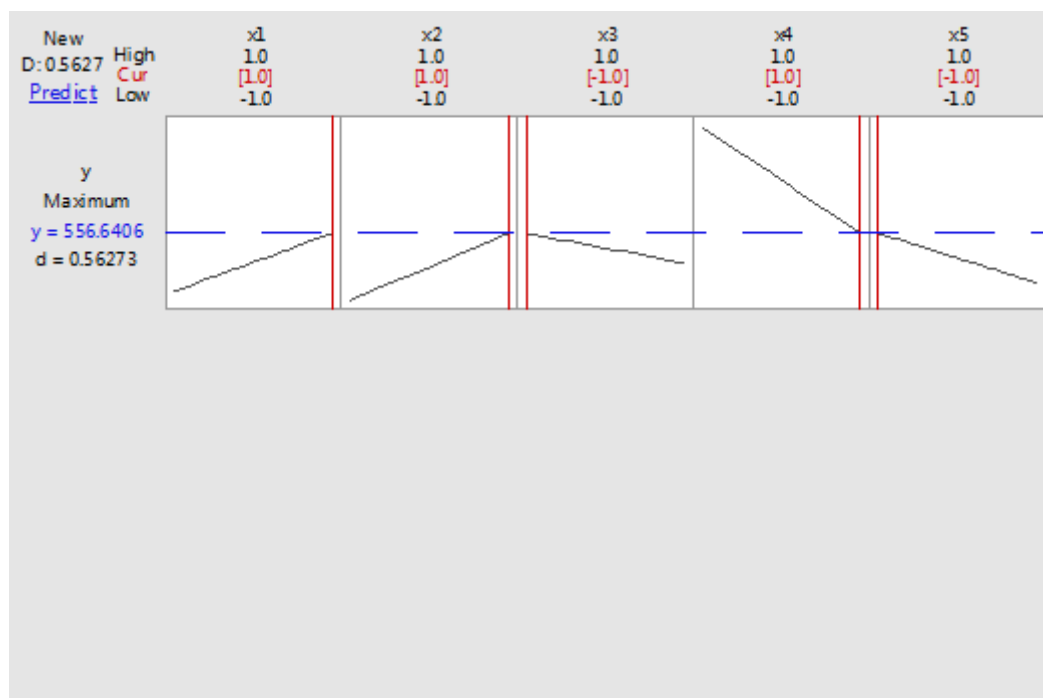


Fig.11

Now reverse the hypothesis, the x2 is now changed to -1.

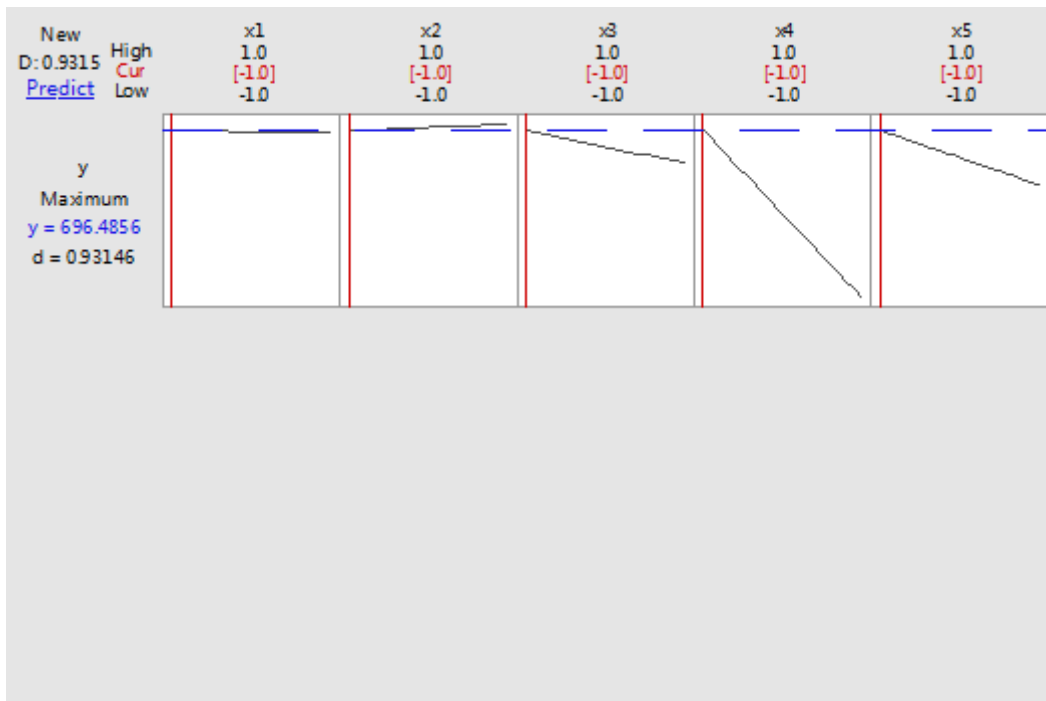


Fig.12

What was shown in fig.12 is the maximize result of output Y for each factor locates in a specific value. Shown in the figure, the maximum value of Y is 696.4856 as x4 change from -1 to 1 the Y reduce to 495.1506 shown in fig.13. Difference is 201.335.

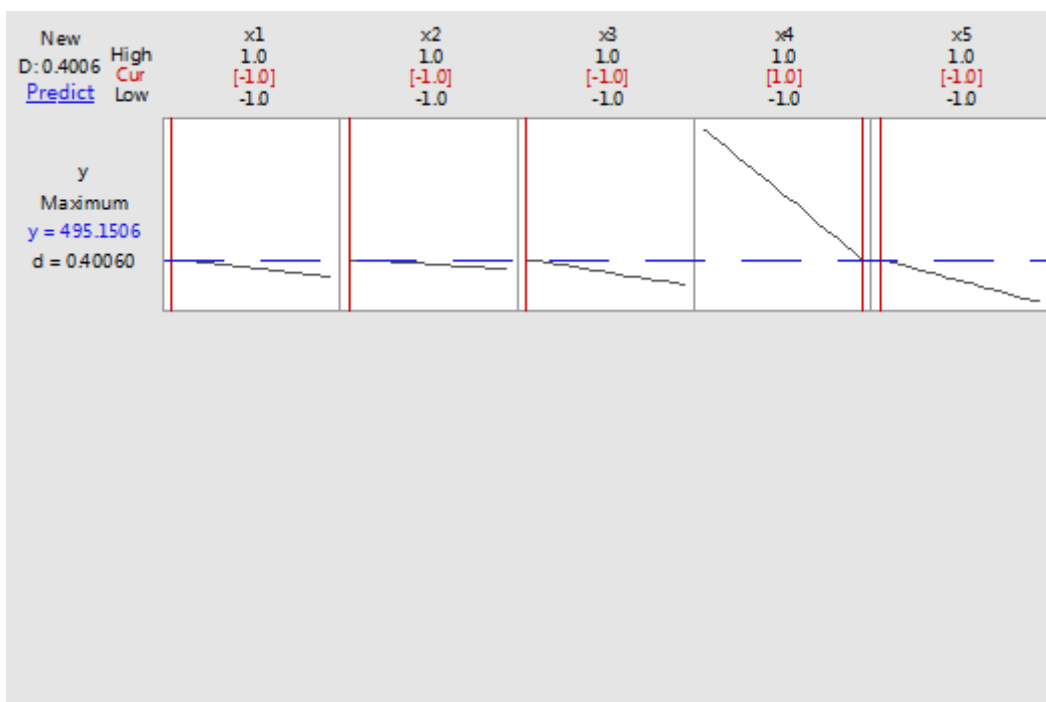


Fig.13

As we change the value of  $x_1$  from -1 to 1. The maximum value of  $Y$  is now changing to 695.2206 shown in fig.14.

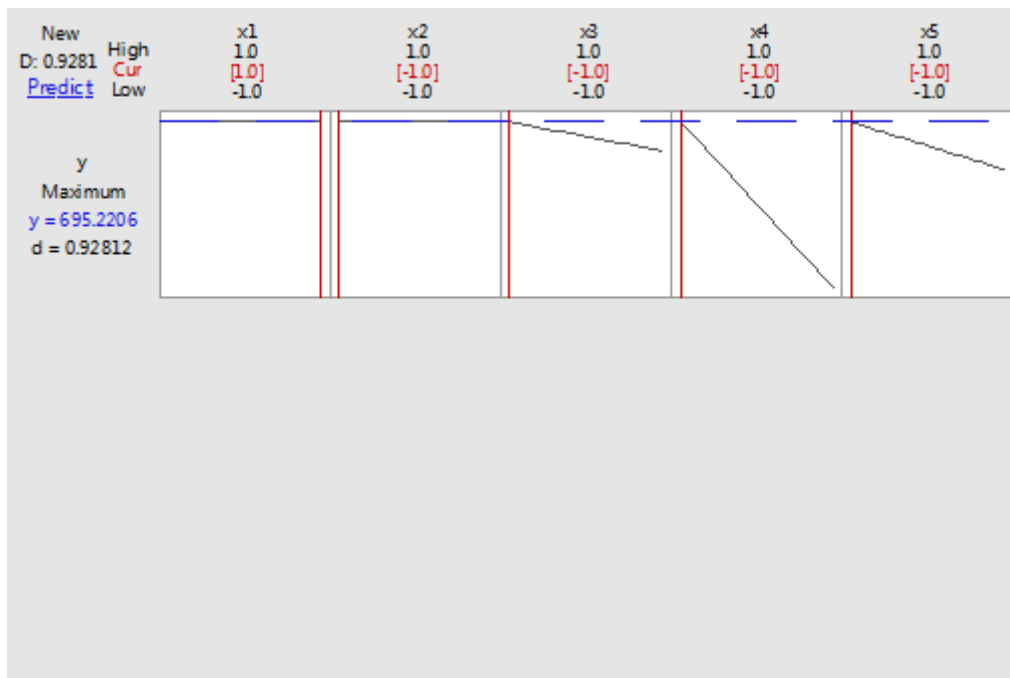


Fig.14

Under this condition, as we change  $x_4$  from -1 to 1. Maximum of  $Y$  will reduce to 468.2031 shown in fig.15. With difference equals to 227.0175 which is higher than the difference of  $x_1$  equals to -1. In this case shows that  $x_2$  equals to 1 is optimal.

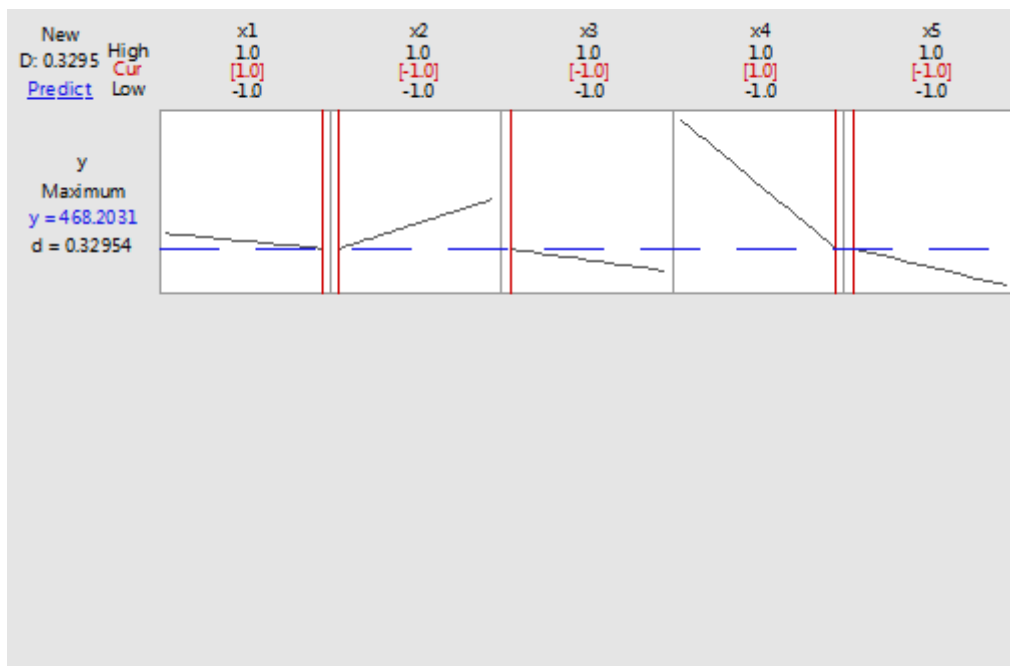


Fig.15

#### D) Analysis of single non-significant factor

The experiment in this case is the analysis of silver by flame AAS to set up the experimental design the following steps need to be carried out:

A	Flame Height above Base (mm)
B	Flame Stoichiometry
C	Acetic Acid (%)
D	Lamp Current (mA)
E	Wavelength (nm)
F	Slit Width (nm)

Table 8

A	B	C	D	E	F	Signal
-1	-1	-1	-1	-1	-1	95
1	-1	-1	-1	1	-1	41
-1	1	-1	-1	1	1	63
1	1	-1	-1	-1	1	113
-1	-1	1	-1	1	1	59
1	-1	1	-1	-1	1	114
-1	1	1	-1	-1	-1	121
1	1	1	-1	1	-1	88
-1	-1	-1	1	-1	1	107
1	-1	-1	1	1	1	53
-1	1	-1	1	1	-1	75
1	1	-1	1	-1	-1	73
-1	-1	1	1	1	-1	60
1	-1	1	1	-1	-1	97
-1	1	1	1	-1	1	105
1	1	1	1	1	1	53
0	0	0	0	0	0	79
0	0	0	0	0	0	74
0	0	0	0	0	0	77

Table 9

Same as before, first of all, we need to be done is to distinguish the significance of each factor related to the output.

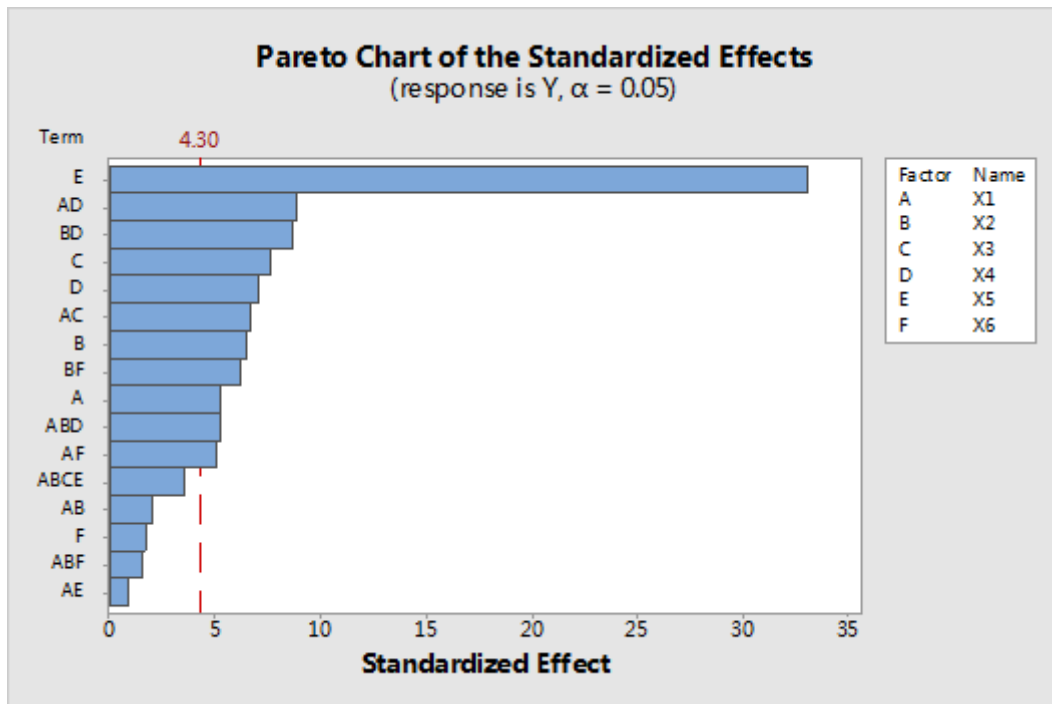


Fig.16

Clearly distinguish from the graph of Pareto Chart of the Standardized Effects that factor F is non-significant, which is different from the previous.

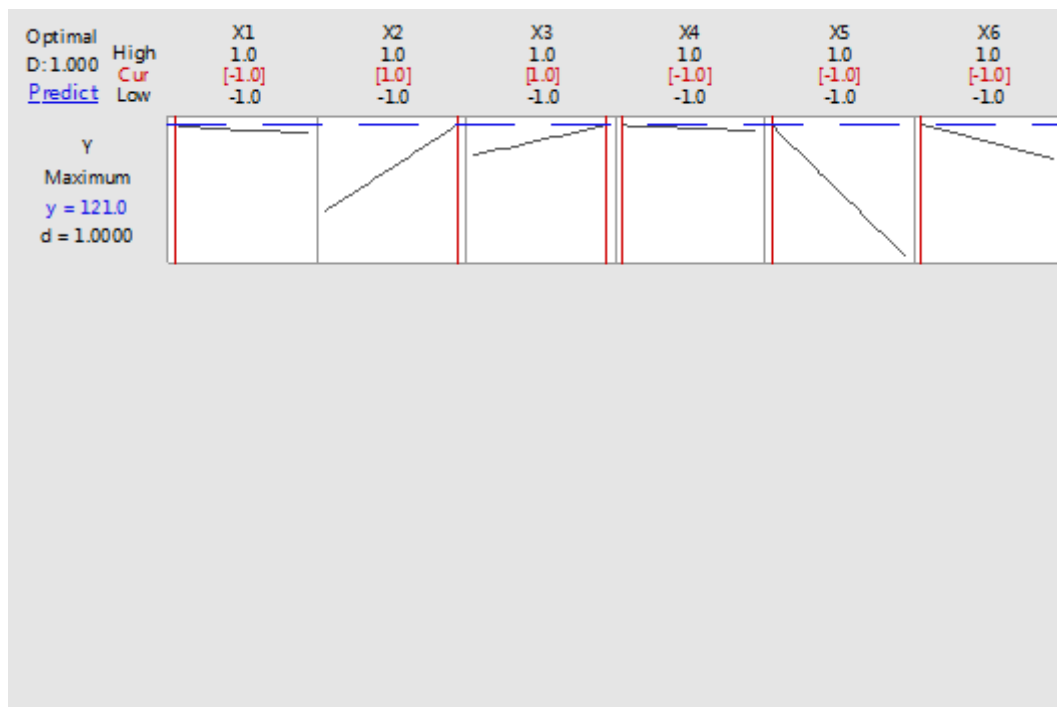


Fig.17

As we can see from fig.17 the maximum value of Y is 121. Then, we change x5 from -1 to 1. The value of maximum of Y will be changed to 66.9583 shown in fig.18 with difference equals to 54.0417.

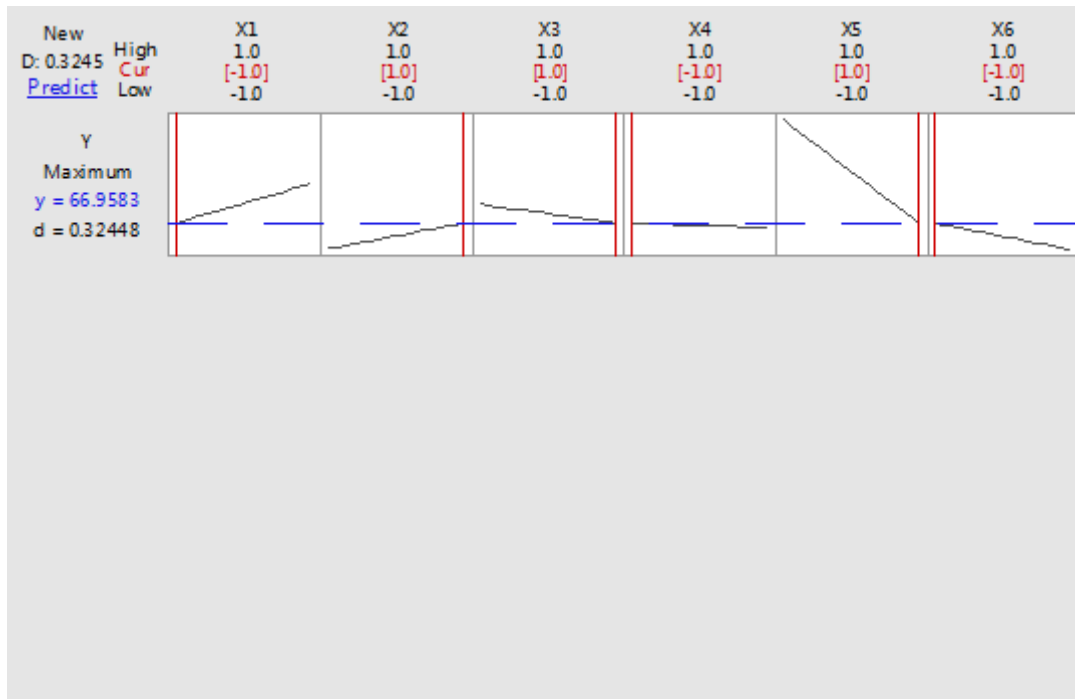


Fig.18

Then we change x1 from -1 to 1. The maximum value of Y is 117.2083 with x5 returning to -1.

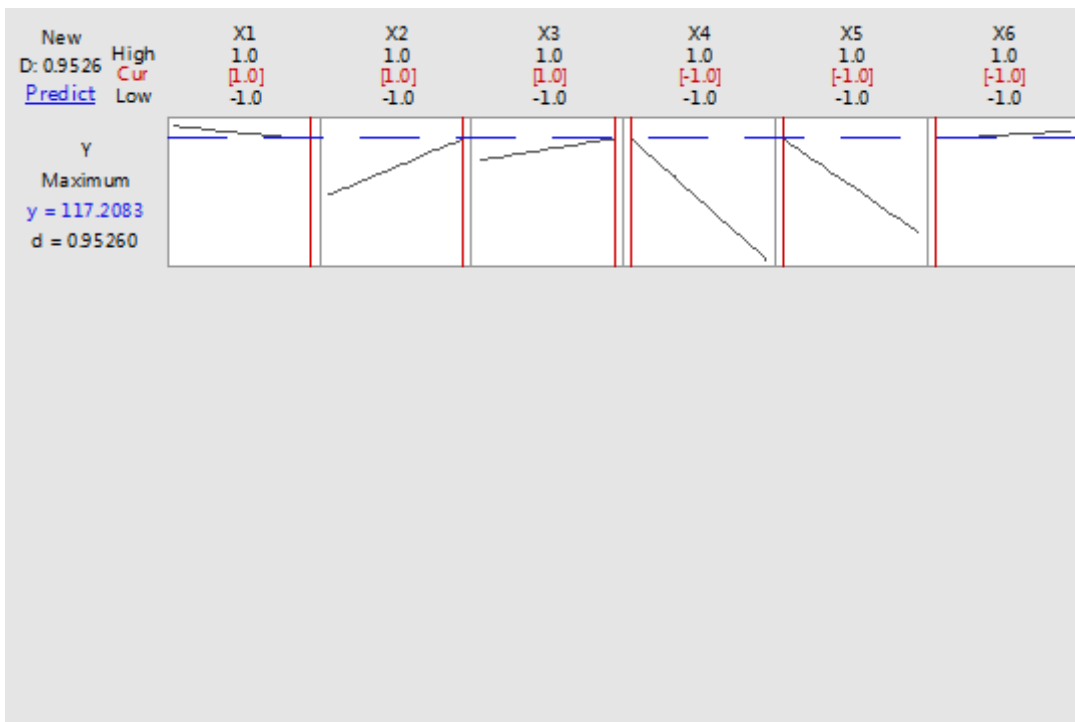


Fig.19

Then change the value of  $x_5$  to 1 the maximum value of  $Y$  is now changed to 88 with difference equals to 29.2083. Which is quite smaller than before which leading more stable output.



## **4. Conclusion.**

Designed of experiments are an advanced and powerful analysis tool which is used in projects. A professional experimenter can filter out non-significant process factors and find out significant factors. These factors can be used to control response properties in a process done by the engineer and to the exact specification of the needed product and service.

A well-designed experiment is able to economize project time and moreover solve critical problems which are still invisible in the whole processes. Teams will learn what factors have great effects and what factors don't have according to observation and evaluation of the interactions of factors.

Instead of neglecting the non-significant factors as usual, but now we focus on them. According to the statistical analyses shown before, it's obviously that indeed the non-significant factors can improve the stability of the output by reducing the variation of the output. Meanwhile it doesn't require to redesign the whole operation process which will cost a lot, but just to minor adjust the environments which relate to the considered non-significant factors to achieve the aim of reducing the variation of the outputs.

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