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Zero-Chattering Sliding Mode Control for Anti-Slip and Traction Optimization of Electric High-Performance Vehicles

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Abstract

This thesis presents the design, implementation, and virtual validation of advanced wheel slip control strategies aimed at enhancing traction performance and stability in RWD electric vehicles. To overcome the high-frequency torque oscillations typical of classical non-linear methods, a chattering-free Conditional Integral Sliding Mode Control (CISMC) is developed and compared against standard PI and Traditional Sliding Mode Control (TSMC) baselines. The overall control architecture incorporates an adaptive Road Estimator Module based on the Burckhardt interpolation model, which continuously monitors vehicle kinematics and utilized friction to dynamically assign optimal reference targets. The complete control loop is integrated natively within the high-fidelity IPG Automotive CarMaker co-simulation environment via MATLAB/Simulink. The robustness and adaptive qualities of the controllers are systematically evaluated through critical testing maneuvers under non-uniform road profiles, including sudden friction transitions (μ -jumps) and asymmetrical surfaces (μ -split), with insights on sliding-mode controllers stability by Lyapunov function theory. The comparative simulation results validate the superior capability of the proposed CISMC topology, which achieves remarkable slip tracking accuracy and convergence while completely suppressing torque oscillations and chattering.

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Nomenclature

$\dot{v}_x$	Longitudinal vehicle acceleration	[m/s]
μ	Tire-road friction coefficient	[-]
μ_{max}	Peak friction coefficient	[-]
ω	Wheel rotational speed	[rad/s]
σ	Wheel slip ratio	[-]
σ_{opt}	Optimal slip ratio	[-]
φ	Boundary layer thickness.	[-]
F_x	Longitudinal tire force	[N]
F_y	Lateral force acting on wheel.	[N]
F_y	Vertical tire force	[N]
F_{aero}	Aerodynamic drag.	[N]
F_{res}	Total resistive force	[N]
F_{roll}	Rolling resistance acting on wheel.	[N]
i	Index for axle identification (R for rear, F for front)	
j	Index for wheel identification (L for left, R for right)	
M	Vehicle mass	[kg]
M_z	Self-aligning torque	[Nm]
R	Effective tire radius	[m]
v	Vehicle's speed	[m/s]

Introduction

1.1 Background and Motivation

The global automotive industry is currently undergoing a historic transition from Internal Combustion Engines (ICE) to Electric Powertrains (EP). This evolution represents a fundamental paradigm shift, particularly within the high-performance and motorsport sectors, and is not merely characterized by a change in energy source but also by a radical transformation in vehicle dynamics control capabilities. The fundamental motivation for the shift from ICE to EP in high-performance applications lies in the superior controllability of electromagnetic torque. As established by Hori et al. [1], the torque response of an electric motor is approximately two orders of magnitude faster than that of an ICE, which is constrained by intake manifold dynamics, the delays of the ignition cycle and mechanical transmission latencies, unlocking the ability to modulate torque in sub-millisecond intervals.

In a motorsport context, this increased bandwidth allows for the implementation of Optimal Slip Ratio Control. This strategy ensures that the tire-road interface remains consistently at the peak of the friction coefficient curve, maximizing longitudinal force and acceleration efficiency regardless of rapid variations in road surface conditions.

Furthermore, electrification enables a transition from centralized propulsion to distributed drive systems, such as four or two independent In-Wheel Motors (IWM). De Novellis et al. discussed how this over-actuated configuration allows for sophisticated Torque Vectoring (TV) strategies [2]: by independently modulating the torque at each wheel, the controller can generate a direct yaw moment. For a high-performance vehicle, this decouples lateral stability from longitudinal traction and, unlike traditional Electronic Stability Control (ESC) systems that rely on dissipative braking interventions, TV enhances the vehicle's ability and agility by actively assisting cornering maneuvers, thereby reducing understeer and improving transient response without sacrificing kinetic energy.

Despite these technical advantages, the implementation of high-performance traction control faces two major hurdles: the highly non-linear nature of tire dynamics and the *sensory gap* experienced by the driver.

The tire-road friction coefficient is a non-linear function of the slip ratio, and its optimal peak is a moving target influenced by tire temperature, vertical load, and road surface variability. Consequently, the controller must exhibit extreme robustness to handle these unmodeled dynamics and parameter uncertainties.

Moreover, the lack of traditional acoustic and vibrational cues in electric vehicles can impede a professional driver's ability to perceive the limit of adhesion.

Research by Corno [3] addresses this by proposing Haptic Traction Control (HTC). This system integrates the control logic with the Human-Machine Interface (HMI) by using an active accelerator pedal to provide force-feedback proportional to the wheel slip. By restoring this physical intuition, the system enables a driver-in-the-loop architecture where the human operator can modulate inputs in harmony with the automated slip regulation, ensuring safety and stability without compromising the *fun-to-drive* factor essential in motorsport.

1.2 Traction Control Challenges in Electric Vehicles

The primary objective of a Traction Control System (TCS) is to modulate the drive torque such that the friction between the tire and the road surface is maximized, preventing wheel spin during acceleration. While the hardware advantages of electric powertrains provide the necessary bandwidth for superior control, the implementation of a robust and efficient slip control strategy involves significant challenges stemming from the highly non-linear nature of tire dynamics and environmental variability.

1.2.1 The Physics of Tire-Road Interaction and Slip Control

The generation of longitudinal force between a vehicle and the ground is not a simple phenomenon of static friction, but a complex, distributed process occurring within the tire's contact patch. This interaction is the foundation of traction control, as the controller's primary task is to manage the physical states of this interface to prevent the transition from stable grip to unstable sliding.

At the heart of traction control is the concept of the wheel slip ratio (σ), which defines the relative difference between the wheel's tangential velocity and the vehicle's longitudinal velocity.

In the initial stage of torque application, the tire-road interface remains mostly in a *stick* condition, where the rubber deforms but does not slide relative to the pavement. In this linear region, the traction force increases proportionally with the slip ratio. However, as torque increases, the rear part of the contact patch begins to *slip* or *micro-slide*. This represents a transition from elastic deformation to kinetic friction.

The longitudinal force (F_x) generated at the contact patch is a non-linear function of this slip ratio; the relationship between the friction coefficient (μ) and the slip ratio is fundamentally non-linear and is often described using semi-empirical models such as the Pacejka "Magic Formula" [4]. The $\mu - \sigma$ characteristic curve typically exhibits three distinct zones:

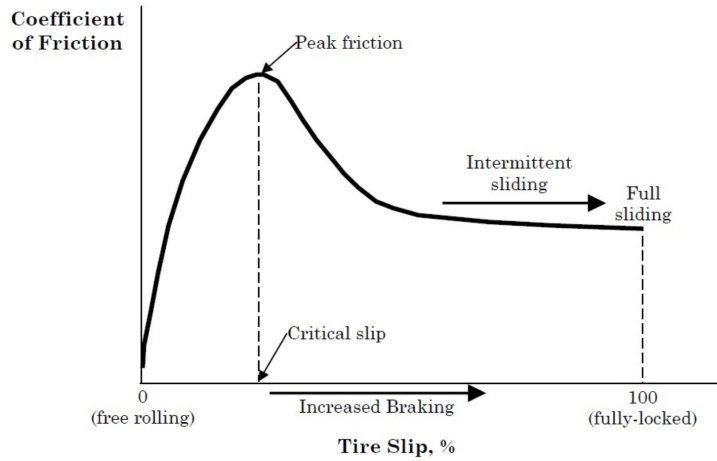


Figure 1.1: Friction coefficient as function of slip [5].

The Linear Region

At low slip values (typically $\sigma < 0.05$), the friction coefficient increases linearly. Here, the vehicle is easy to control, and the driver feels a direct correspondence between pedal input and acceleration.

The Peak Region

The curve reaches a maximum value, μ_{max} , at an *optimal slip ratio* (σ_{opt}). This point represents the physical limit of the tire's adhesion. In high-performance scenarios, maintaining the tire exactly at this peak yields the maximum possible longitudinal acceleration.

The Unstable Region

Beyond the optimal slip point, the friction coefficient begins to decrease. This is a critical instability: as the tire spins faster, it generates less force, leading

to a runaway effect where the wheel accelerates rapidly toward a *burnout* state.

Overall, there exists an optimal slip ratio where the friction coefficient reaches its maximum. Beyond this point, the tire enters the unstable region where increasing the wheel speed leads to a decrease in traction, resulting in a loss of directional stability and acceleration.

The focus of modern EV traction control, as proposed in the seminal work of Hori et al. [1], is to maintain the tire as close to this optimal point as possible. This **Optimal Slip Ratio Control** is far superior to the heuristic torque-reduction methods used in early ICE-based systems, as it directly manages the state of the tire-road interface.

A major challenge highlighted in the research by He [6] is that the optimal slip point is a moving target. The shape of the $\mu - \sigma$ curve changes drastically depending on road conditions (e.g., dry asphalt vs. wet concrete) and tire temperature. On a dry racing track, the peak might be sharp and at a relatively high slip ratio; on a wet surface, the peak is lower, flatter, and occurs at a lower slip ratio.

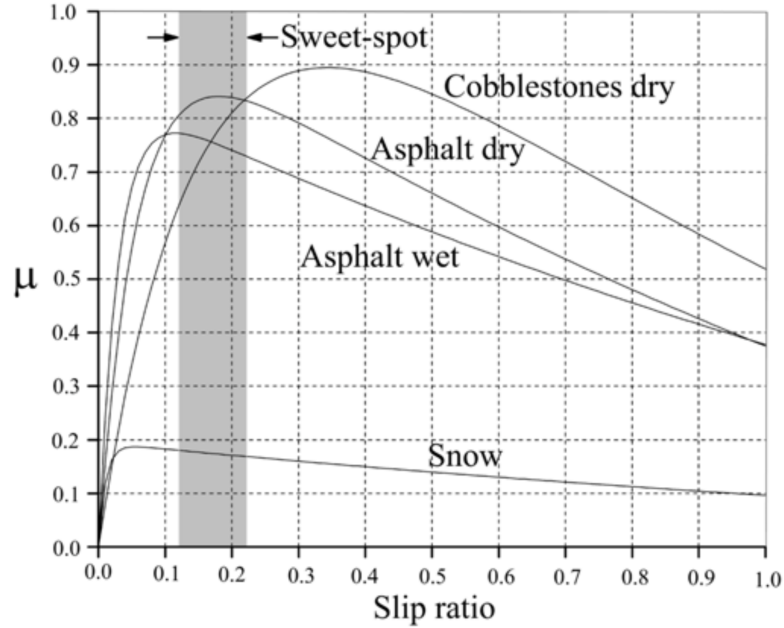


Figure 1.2: Friction coefficient as function of slip for different road surfaces [7].

Furthermore, the operational limits of a tire are fundamentally governed by the *Friction Ellipse* theory, a concept that defines the theoretical maximum resultant force that the tire-road interface can sustain. This principle points that the total

transmissible traction force is a finite resource, determined by the product of the instantaneous vertical load and the local friction coefficient. From a vectorial perspective, the magnitude of the resultant force composed of both longitudinal (F_x) and lateral (F_y) components cannot exceed this adhesion limit without inducing *macro-sliding*.

In high-performance maneuvers, this creates a critical coupling between traction and cornering stability. If a significant portion of the friction potential is consumed by longitudinal acceleration, the residual capacity for lateral force generation is proportionally diminished. Consequently, if a traction control system fails to regulate slip effectively, the saturation of the friction circle by longitudinal forces leads to a phenomenon known as *contact patch saturation*. In such a state, the tire becomes incapable of providing the necessary centripetal force for steering, resulting in a total loss of directional stability. In a motorsport context, this loss of lateral authority is particularly hazardous, as it can lead to uncontrollable understeer or oversteer during high-speed transients.

Accordingly, the strong coupling between longitudinal and lateral tire dynamics requires the traction control system to regulate slip with high precision, ensuring that the resultant tire force remains within the admissible friction-ellipse boundary. The synthesis of such a controller is further complicated by the strongly non-linear and time-varying nature of the tire-road interface. As reported by Yang et al. [4], once the slip ratio exceeds the optimal threshold ($\sigma > \sigma_{opt}$), the local system behavior transitions from a stable to an unstable regime, so that further increases in wheel speed induce a positive-feedback effect driving the tire rapidly toward full sliding.

Furthermore, as previously noted, the plant is subject to high degrees of environmental stochasticity. The friction peak is not a static parameter but a moving target that shifts according to the road surface composition, instantaneous vertical load, and the tire's thermal state [6]. Under these unsteady conditions, traditional linear control strategies, such as the Proportional-Integral-Derivative (PID) controller, often prove inadequate. Linear controllers are typically tuned for a specific operating point and lack the structural robustness required to manage the sudden gain jumps and parameter uncertainties typical of near-limit vehicle operation.

Consequently, the academic literature has explored a diverse array of advanced architectures, ranging from Model Predictive Control (MPC) to *smart paradigms* such as Fuzzy Logic (FL) and artificial Neural Networks (NN). In this context, Sliding Mode Control (SMC) emerges as a particularly compelling solution due to its inherent robustness to bounded uncertainties.

1.2.2 Slip-Based Control Strategy

The shift toward slip-based control in electric vehicles offers different advantages that are particularly critical in motorsport and high-performance environments. First and foremost, the quasi-instantaneous torque modulation capability of electromagnetic actuators allows the controller to track the optimal slip ratio even under transient conditions, such as rapid vertical load shifts during cornering. Furthermore, this strategy transforms the electric motor into a sophisticated sensor, providing a high-fidelity estimation of the tire-road interface states without the latency associated with traditional mechanical observers

Despite its theoretical superiority, slip-based control introduces several engineering trade-offs:

Robustness and Smoothness

A highly aggressive slip controller is necessary to catch sudden instabilities; however, this often leads to the phenomenon of **Chattering**, especially when using robust frameworks like Sliding Mode Control. Chattering can induce mechanical fatigue in the drivetrain and degrade the driving experience.

Estimation Lag

While the motor response is fast, the calculation of the slip ratio depends on an accurate estimation of the vehicle's longitudinal velocity (v). In motorsport, where all wheels may be near the limit of adhesion, obtaining a true ground speed reference is difficult, and errors in this estimate can lead to improper control interventions.

Sensory Gap

A highly efficient automated slip controller can effectively mask the physical limits of the car from the driver. By instantly correcting every micro-slip, the system removes the tactile cues that professional drivers use to sense the limit. This necessitates the integration of feedback mechanisms to restore the driver-in-the-loop synergy without sacrificing the safety provided by the control system.

Within this framework, the control architecture is typically structured hierarchically: a *high-level supervisor* determines the target slip ratio based on the desired vehicle dynamics and actual road conditions, while a *low-level slip regulator* modulates the motor torque to track this reference. By maintaining the tire at the peak of the $\mu - \sigma$ curve, the system effectively decouples the driver's longitudinal demand from the physical limits of the tire-road interface. This ensures that the vehicle remains in a high-performance envelope where traction and lateral stability are simultaneously optimized. Consequently, slip-based control does not replace the driver's authority but rather allows for the safe exploitation of the electric powertrain's full potential.

1.3 Project Goals and Research Questions

The preceding analysis of the tire-road interface and the advantages of electric powertrains highlights a major control challenge: maintaining the optimal slip ratio in a highly non-linear, dynamic and unsteady environment. Although Sliding Mode Control provides the robustness required for this task, its practical implementation in high-performance EVs introduces specific technical challenges that this thesis aims to address.

Problem Statement. Standard SMC is often plagued by the chattering phenomenon, where high-frequency switching of the control law excites drivetrain resonances and causes mechanical fatigue.

Furthermore, while a smoother switching function (e.g., using a saturation function) can mitigate chattering, it often introduces a steady-state error, preventing the tire from reaching the ideal peak of the friction curve. In a motorsport context, where a 1-2% deviation from the optimal slip ratio translates to lost lap time, this represents an inadmissible compromise.

Additionally, the controller must remain adaptive to varying road conditions without requiring manual re-tuning for every surface change.

Project Goals. The primary goal of this research is the design, implementation, and validation of an Adaptive Conditional Integral Sliding Mode Controller (ACISMC) tailored for a high-performance electric vehicle. The specific technical objectives are:

- **Chattering Suppression:** to implement a continuous approximation of the switching law that minimizes high-frequency torque ripples without sacrificing the response time of the electromagnetic actuators.
- **Environmental Adaptation:** to develop an adaptive algorithm capable of identifying and compensating diverse track conditions (e.g., dry vs. wet asphalt).
- **Simulation Validation:** to verify the controller's performance using a high-fidelity co-simulation environment (e.g., CarMaker and CarMaker4Simulink) under realistic racing maneuvers.
- **Comparative Performance Benchmarking:** to perform a rigorous cross-evaluation between the proposed ACISMC and baseline control strategies, such as standard PID and Traditional SMC. This objective aims to quantify the trade-offs between computational complexity and the resulting improvements in tracking accuracy and vehicle stability.

Research Questions. The investigation first examines the tracking accuracy of the slip ratio under highly dynamic conditions, evaluating how the integration of a Conditional Integral term can mathematically eliminate the residual errors inherent in standard boundary-layer Sliding Mode Control. This phase of the study aims to quantify the *performance gap* between traditional robust methods and the proposed integral-augmented architecture.

Subsequently, the analysis shifts toward the adaptive capabilities of the controller. The study investigates the algorithm’s internal mechanisms for compensating for the shifts in the tire-road friction peak, simulating transitions between heterogeneous surfaces (e.g., dry asphalt to wet concrete). The goal is to determine the controller’s ability to maintain the *optimal slip* without requiring manual calibration for environmental changes.

1.4 Thesis Structure

This dissertation is organized into five primary chapters, following a path that transitions from theoretical modeling to the synthesis and comparative validation of advanced non-linear control laws. The methodology adopted ensures that the final controller design is rooted in a deep physical understanding of the tire-road interface and is benchmarked against established industry standards.

State-of-the-Art. A systematic review of existing literature is performed to situate the research within the broader control landscape. This chapter explores diverse methodologies, ranging starting from the foundational PID regulators to more sophisticated *intelligent paradigms* such as FL and MPC. By evaluating the performance trade-offs inherent in these established strategies, the chapter identifies the specific technical gaps and limitations, particularly regarding robustness and environmental adaptability, that justify the development of advanced SMC variants as the primary focus of this dissertation.

Vehicle and Tire Modeling. This chapter establishes the analytical foundation of the research. It provides a comprehensive derivation of the longitudinal vehicle and tire dynamics, with a specific focus on the non-linearities of the tire-road interaction. By employing semi-empirical models, such as the Pacejka Magic Formula, the chapter defines the state-space representation of the plant and identifies the critical implications that these physical constraints impose on the controller’s architecture.

Baseline Controller Design. To provide a rigorous basis for comparison, this chapter details the design process of the baseline control strategies. It covers the

tuning of a standard PI and the formulation of a Traditional Sliding Mode Control.

Adaptive CISM Design. This chapter details the mathematical derivation of the Adaptive Conditional Integral Sliding Mode Controller. It explains the integration of the conditional integral term for error elimination and the development of the adaptive law used to track the variable friction peak. The stability of the proposed control law is formally analyzed using Lyapunov's direct method.

Simulation, Results, and Comparative Analysis. The final technical chapter presents the results obtained from high-fidelity co-simulations. A comparative performance analysis is conducted across the three implemented technologies, evaluating their robustness, tracking accuracy, and impact on vehicle stability under diverse track conditions.

State-of-the-Art

This chapter provides a comprehensive overview of the control strategies developed in literature for wheel slip regulation and the optimization of longitudinal vehicle dynamics. The state-of-the-art review traces the technical evolution from classical linear control, such as industry-standard PID and threshold-based logic, to more sophisticated robust non-linear strategies. Within this context, the investigation focuses on how advanced methods like Sliding Mode Control address the inherent non-linearities and parameter uncertainties of the tire-road interface that simpler linear controllers often fail to manage. Furthermore, the survey extends into modern predictive and intelligent paradigms, evaluating Model Predictive Control and Neural Network-based architectures for their capacity to handle multi-variable optimization and real-time environmental adaptation. By critically analyzing these diverse methodologies, the chapter establishes the theoretical and technical foundation necessary to justify the selection and development of the Adaptive Continuous Integral Sliding Mode Control framework presented in this work.

2.1 Control Objectives and Performance Metrics

The primary objective of a Traction Control System is to regulate the longitudinal slip ratio such that the tire-road friction capability is maintained at its maximum potential value, corresponding to μ_{max} . This task is inherently multi-objective, since it involves a delicate balance between longitudinal performance, directional stability, and drivability. From a control systems perspective, the goal is to track a time-varying *optimal slip reference* while rejecting disturbances, such as sudden changes in road surface conditions or vertical load transfers.

The definition of rigorous evaluation criteria is essential to quantify the efficacy of a traction control strategy and to provide a consistent framework for the comparative analysis presented in this chapter. Beyond the primary goal of maintaining the slip ratio at its optimal peak, several key performance indicators act as the analytical lenses through which the subsequent bibliographic review is conducted.

Specifically, the effectiveness of each control paradigm, ranging from traditional to predictive, is qualified by the following performance metrics:

Tracking Accuracy Ability of the controller to minimize the steady-state error between the actual and reference slip, particularly under non-linear working conditions.

Transient Response Quantifies controller's ability to react to abrupt variations in operating conditions (change in road friction, reference slip, etc...).

Robustness System's capability to maintain stability despite parametric uncertainties.

Chattering This metric assesses the high-frequency oscillations in the control signal, which can lead to mechanical wear and reduced passenger comfort.

2.2 Traditional Traction Control

The most straightforward approach to implementing traction control for anti-slip regulation involves the use of classical feedback loops and heuristic rules, which seek to maintain wheel slip within a stable range through relatively simple control architectures. These methods are often the first choice for industrial applications due to their low computational demand and the ease with which they can be integrated into existing vehicle hardware.

As discussed in reference [8], **PID controllers** are frequently utilized to regulate the drive torque of in-wheel electric motors by acting directly on the error between the actual and target slip. While this represents a linear and highly accessible

approach, it often comes with limitations related to non-linearities of the tire-road interface, as a single set of gains may not remain optimal across diverse road surfaces or during aggressive dynamic transitions.

The control law is typically expressed in the time domain as $T_c(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt}$, tracking the error $e = \sigma - \sigma_{ref}$.

To augment these basic structures without significantly increasing mathematical complexity, **fuzzy logic layers** are often introduced. The work in reference [9] proposes a correctional algorithm that uses FL control to handle the ambiguity of low-friction or irregular road surfaces. By translating human-like expert knowledge into linguistic rules, this method provides an intuitive way to adjust the control response to road roughness without requiring a complex analytical model of the environment.

The control action is determined by a set of *if-then rules* that map the slip error and the change in error to a gradient in correctional torque ΔT_c , used to evaluate $T_c = \int \Delta T_c dt$.

A primary technical drawback of general linear or threshold-based ASR is high slip ratio overshoot, which can reach 50% on low-adhesion roads and up to 500% during abrupt friction transitions [8]. This excessive overshoot causes the acceleration curve to jitter, negatively impacting both drivability and the speed of stabilization.

These heuristic methods do not provide a complete resolution for the sudden, transient variation of friction encountered on complex road surfaces. Further focus is required on the relationship between longitudinal force changes and the transient response of the powertrain to fully optimize recovery from slip events.

Moreover, while classical control theory can verify localized stability, these systems struggle to maintain low overshoot during the rapid non-linear transitions typical of high-performance driving.

2.3 Robust Non-Linear Control

The limitations observed in classical PID and fuzzy-based strategies highlight the need for a more rigorous mathematical framework. While heuristic and linear methods attempt to approximate the system's behavior around a nominal operating point, they often fail to account for the global dynamics of the vehicle. In fact, traction control is fundamentally a non-linear tracking problem operating in a highly uncertain environment, therefore a more robust framework is needed to provide guaranteed stability boundaries and consistent performance despite unmodeled dynamics, such as load transfers or varying powertrain response characteristics.

Within the domain of robust non-linear strategies, **Sliding Mode Control** is

distinguished by its ability to maintain invariant performance in the presence of modeling inaccuracies and external disturbances. The design objective involves two primary phases:

- **Sliding Surface Design:** a stable trajectory is defined in the state space such that when the system state is restricted to this surface, the tracking error asymptotically converges to zero.
- **Control Law Formulation:** a discontinuous control law is designed to force the system state onto the sliding surface in finite time and maintain it there.

In the reference [10], the traction control problem is addressed using a **Traditional Sliding Mode Control** framework. The primary objective is to drive the system state toward a sliding surface, typically defined as the error between the actual and reference slip ratios, $s = \sigma - \sigma_{ref}$.

The core of this strategy is the implementation of a discontinuous control law, which is realized through a switching term, often formulated as $u_{sw} = K \cdot \text{sgn}(s)$, where K is a positive gain and $\text{sgn}(s)$ is the sign function. This switching term is technically significant because it characterises the *reaching phase*, forcing the wheel dynamics to move toward the sliding surface in finite time.

However, as noted in the reference, this constant switching across the sliding surface is the source of chattering, which results in high-frequency oscillations in the motor torque command.

While highly robust against disturbances, this traditional approach can excite unmodeled dynamics in the electric drivetrain.

To address the limitations of the discontinuous law, specifically the steady-state error and the chattering, works in literature, as in reference [10] and [11], introduce an augmented version of SMC that incorporates an **integral term**. In these versions, the sliding surface $s(t)$ is redefined to include the integral of the slip error:

$$s(t) = e(t) + \lambda_i \int_0^t e(\tau) d\tau$$

Unlike the traditional law, which may allow a persistent offset if the switching gain K is too low, the integral term ensures that the tracking error asymptotically converges to zero even in the presence of constant disturbances. It provides global robustness, as the sliding surface can be designed to include the initial conditions of the system, effectively eliminating the reaching phase and its associated vulnerabilities.

Building upon the augmented integral architectures, recent research and the specific methodologies analyzed in the literature review lead to an approach specifically designed to refine the control law by smoothing the response during the most critical phases of traction regulation, named **Conditional Integral Sliding Mode Control**.

In reference [10], a solution to the chattering problem related to TSMC control is proposed, introducing a sophisticated switching logic that manages the integral term, leading to a trade-off solution between tracking precision and transient stability. Unlike a standard integral sliding surface, which is active at all times, the conditional approach operates based on the system’s proximity to the sliding surface. In this framework, the integral action is only enabled when the system state enters a predefined boundary layer surrounding the sliding surface.

By deactivating the integrator when the system is far from the sliding surface, such as during a sudden jump in friction or a large initial slip, the controller prevents the *integrator windup* typically responsible for massive overshoots observed in traditional ASR implementations.

Within the boundary layer, the integral term takes over the task of rejecting constant disturbances. This allows the discontinuous switching gain to be significantly reduced or replaced with a smoother saturation function, effectively eliminating the high-frequency vibrations in the torque command.

This evolution from the simple switching laws of control, to the integral-augmented strategies, concludes with the CISM as the most balanced solution. It provides the capability to handle the non-linearities of the tire-road interface while maintaining the control smoothness required for modern electric drivetrains.

2.4 Predictive Control

Predictive models in the automotive industry have evolved from theoretical frameworks into functional prototypes that significantly outperform traditional control systems. These operate by utilizing a mathematical model of the vehicle’s dynamics to predict future behavior and optimize control actions accordingly.

One prominent application is in traction control, where **Hybrid Model Predictive Control** models manage the nonlinear relationship of tire slip on low-friction surfaces like ice, achieving up to 20% improvement in slip regulation over standard linear methods.

In reference [12], a MPC approach specifically designed for traction control is proposed by treating the vehicle’s wheel dynamics as a hybrid system. The core of this solution lies in the recognition that tire-road interaction is highly nonlinear, making traditional linear controllers ineffective during sudden changes in surface friction. To manage this, the researchers use a **Piecewise Affine (PWA)** model, which approximates the complex, nonlinear tire-force curve by dividing it into distinct linear operating regions based on the amount of tire slip.

At each sampling interval, the MPC algorithm solves an optimization problem that minimizes a cost function, balancing the deviation from the target slip against

the effort required from the actuators, while respecting physical constraints like maximum engine torque and braking limits. Because solving these complex mathematical problems in real time is computationally expensive for standard vehicle hardware, the authors employ **Explicit MPC**. This technique moves the heavy calculations offline, partitioning the state space into various polyhedral regions where the optimal control law is pre-calculated as an affine function.

Another solution presented in [13] utilizes **Nonlinear Model Predictive Control (NMPC)** to address the highly complex and nonlinear nature of tire-road interactions. Unlike the hybrid or piecewise linear approaches, NMPC directly incorporates the nonlinear tire model into its optimization process, which allows it to more accurately represent the physical limits of the vehicle under varying conditions.

The controller accomplishes slip regulation by solving a constrained nonlinear optimization problem at every time step, minimizing the error between the actual and the reference slip while also penalizing excessive changes in motor torque, to ensure a smoother control.

A critical advantage of this NMPC approach for in-wheel motor configurations is its ability to handle the rapid torque response of electric motors. Simulation results indicated that this solution provides superior tracking performance and robustness compared to standard linear control methods, effectively preventing wheel lock-up and spinning even under extreme driving maneuvers.

2.5 Comparative Analysis

Evaluating the various control strategies for automotive traction systems reveals a clear progression from simple regulation to complex, predictive optimization. Each method offers a distinct balance of computational demand and performance accuracy.

PID controllers remain the industry standard due to their simplicity and ease of implementation. However, they struggle with the highly nonlinear and time-varying nature of tire-road friction, often leading to sub-optimal performance in extreme conditions like ice or sudden maneuvers. Linear MPC improves upon this by using a predictive model and handling system constraints, but its reliance on a single linear model makes it inaccurate when the vehicle operates outside a specific narrow range of slip .

To capture more complex dynamics, MPC with PWA models approximate nonlinearities by switching between different linear regimes. This provides a significant boost in accuracy, but also increases the mathematical complexity of the optimization problem. NMPC sits at the top of the performance hierarchy, directly

integrating nonlinear tire models and motor dynamics to provide the most precise tracking and robustness, especially for four-in-wheel motor electric vehicles.

The primary drawback of NMPC is its extreme computational cost, which often requires high-end processors or complex simplification techniques to run in real-time.

Between these extremes lie Sliding Mode Control variants. TSMC offers fast convergence and finite-time stability but can suffer from chattering. CISMIC emerges as the ideal trade-off for modern automotive applications. It combines the robustness of sliding mode control with integral action, effectively rejecting disturbances and eliminating steady-state error without the extreme computational overhead of NMPC. CISMIC provides near-optimal results by maintaining the desired slip ratio even under uncertainty, yet it remains simple enough to be implemented on standard electronic control units without the heavy optimization solvers required by predictive models. This balance of stability and manageable complexity makes CISMIC a highly practical choice for mass-market vehicle systems.

2.6 Market Analysis

For the vast majority of mass-market vehicles, the primary controller used is the PID controller. It is the industry workhorse because of its low computational weight and high reliability for standard driving conditions. In these systems, the controller monitors wheel speed sensors and, upon detecting slip, triggers either motor torque reduction or brake intervention.

In the EV segment, which now accounts for a rapidly growing portion of the mass market, the control strategy is shifting. Approximately 60% of EVs sold in 2024 utilized specialized traction modules designed for the instant torque of electric motors [14]. Because electric motors can adjust torque much faster than a mechanical engine or a hydraulic brake system, suppliers are moving toward more advanced, high-frequency control loops. While many still use sophisticated PID variants, there is a clear trend toward Predictive Traction Systems and AI-enhanced algorithms, which saw a 47% increase in adoption in 2024.

The current market trend shows a shift from *reactive systems* to *proactive architectures*, led by major OEMs and Tier-1 suppliers:

- **BYD iTAC (Intelligent Torque Adaptation Control):** BYD has redefined traction logic by moving away from traditional ABS-based wheel sensors. Their iTAC system utilizes the electric motor's internal speed sensors, which offer significantly higher resolution and faster sampling rates. Unlike conventional PID-based systems that wait for a detected slip threshold to cut power,

iTAC "anticipates" slip by monitoring motor speed fluctuations. It stabilizes the wheel in milliseconds by applying "negative torque" or millimetric torque adjustments. In real-world performance, this has demonstrated a 0.7-second improvement in 0–60 km/h acceleration on low-friction surfaces like snow compared to conventional systems.

- **Bosch Integrated Traction Control & VMM:** As a global leader, Bosch is shifting toward Vehicle Motion Management (VMM), a software suite that orchestrates brakes and motors through a centralized architecture [15]. Their logic increasingly adopts Model Predictive Control (MPC) to calculate required torque based on "road perception"—integrating cloud-based data regarding road friction and slope. This high-level integration is visible in systems like the iBooster, which coordinates slip during regenerative braking. By using predictive algorithms, Bosch avoids unwanted wheel lock-ups during energy recovery without the characteristic vibrations on the brake pedal associated with legacy ABS/TCS interventions.

The choice of controller is heavily influenced by cost and regulatory mandates such as Euro NCAP or Bharat NCAP [15]. OEMs prioritize systems that can guarantee safety across a wide range of drivers and road surfaces at the lowest possible cost per unit. This economic reality is why the high-complexity models like NMPC are currently reserved for the premium and motorsport segments, while the reliable, cost-effective PID and basic predictive models continue to rule the mass market.

Vehicle and Tires Models

The synthesis of a high-performance slip control strategy requires a rigorous physical mapping of the vehicle's motion, centered on the longitudinal degree of freedom. This section establishes the mechanical equilibrium and the kinematic relationships that govern the flow of torque from motor to tire-road interface.

To construct a high-fidelity control plant, the modeling must synthesize four deeply coupled physical domains: the rotational equilibrium of the wheel, the dynamic transfer of vertical loads, the kinematic definition of slip, and the inherently non-linear nature of tire friction. By deconstructing these interdependencies, the boundary between stable traction and uncontrolled spinning is defined, providing the controller with the analytical foundation necessary to operate precisely at available adhesion.

3.1 Vehicle Model

The dynamic model employed in this study is based on a two-track formulation and restricted to the planar vehicle motion, in order to achieve an optimal balance between physical fidelity and computational tractability for the synthesis and validation of the anti-slip control logic.

In accordance with the primary research objective, hence the regulation of longitudinal wheel slip, the dynamics associated with body roll, pitch, and vertical heave are neglected in this approximation. This modeling assumption allows for the reduction of the system's state space to 7 fundamental Degrees of Freedom (DoF):

- Longitudinal and lateral translatory motions within the $x^B y^B$ ground plane;
- Yaw motion about the vertical z^B -axis;
- Angular rotation of the four individual wheels.

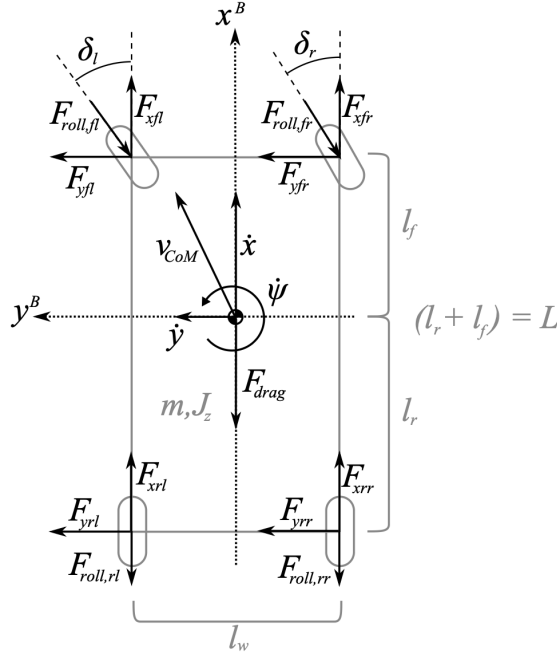


Figure 3.1: Force diagram in vehicle reference plane.

Governing equations for vehicle planar dynamics are derived from the translational force equilibrium along the longitudinal axis (x^B) combined with the rotational torque balance about the wheel axis, providing the analytical framework necessary

to evaluate the vehicle's trajectory and stability under the influence of the four individual tire-road contact forces.

Forces, torques and velocities terms in Figure 3.1 are defined below:

$F_{x_{ij}}$ Longitudinal traction force acting on ij -wheel.

F_y Lateral force acting on ij -wheel.

$F_{roll_{ij}}$ Rolling resistance acting on ij -wheel.

F_{aero} Aerodynamic drag.

3.1.1 Longitudinal Dynamics

According to Newton's Second Law, the vehicle's translation is the result of the net balance between the tractive forces generated at the four tire-road interfaces and the diverse environmental resistances opposing the motion. This global equilibrium is expressed as:

$$M\dot{v}_x = \sum_{i,j} F_{x,ij} - F_{res} \quad (3.1)$$

Where M represents the total vehicle mass and $\dot{v}_x$ is the longitudinal acceleration. While $F_{x,ij}$ is the tractive force at each driving wheel, the term F_{res} encapsulates the lumped resistance effects, primarily dominated by aerodynamic drag and rolling resistance, which, considering a flat road, can be expressed as:

$$F_{aero} = \frac{1}{2}\rho C_d A v_x^2 \quad (3.2)$$

$$F_{roll} = f_r F_z \quad (3.3)$$

F_{aero} represents the aerodynamic drag, where ρ is the environmental air density, C_d is the *aerodynamic drag coefficient*, and A is the frontal area of the vehicle. F_{roll} accounts for the rolling resistance, with f_r being the *rolling resistance coefficient*.

3.1.2 Vertical Load

A fundamental aspect of longitudinal dynamics is the redistribution of vertical loads during acceleration and braking transients. Since the tractive force that a tire can generate is strictly proportional to its vertical load $F_{z,ij}$, the controller must account for the *pitch* of the vehicle body.

Assuming a rigid chassis and neglecting aerodynamic downforce for this derivation, the vertical loads on the front and rear axles are determined by the moment

equilibrium around the contact patches:

$$\begin{cases} F_{z,F} = F_{z,F,static} - \Delta F_z \\ F_{z,R} = F_{z,R,static} + \Delta F_z \end{cases} \quad (3.4)$$

Where the generalized term for dynamic longitudinal load transfer, denoted as ΔF_z is defined as:

$$\Delta F_z = \frac{M\dot{v}_x h_g + F_{aero} h_a}{L} \quad (3.5)$$

This definition accounts for the *inertial load transfer* ($\frac{Mgb}{L}$) and the *aerodynamic pitch moment* ($\frac{F_{aero} h_a}{L}$).

The explicit definition for the vertical load is finally given by:

$$\begin{cases} F_{z,f} &= \frac{Mgb - M\dot{v}_x h_g - F_{aero} h_a}{L} \\ F_{z,r} &= \frac{Mga + M\dot{v}_x h_g + F_{aero} h_a}{L} \end{cases} \quad (3.6)$$

where a and b are the horizontal distances from the CoG to the front and rear axles, respectively, and $L = a + b$ is the wheelbase.

While the *driven wheels* contribute only to the rolling resistance and lateral guidance, the *driving* ones must manage the entirety of the driving torque output. This concentration of force on a single axle makes the system highly sensitive to longitudinal dynamic load transfer.

As the vehicle accelerates, the inertial reaction at the center of gravity induces a pitch moment that increases the vertical load on the rear axle. This phenomenon provides a physical advantage for an RWD layout: the normal force, and thus the maximum friction potential, are enhanced precisely at the wheels where the driving torque is applied.

The equations derived in this section provide the macroscopic framework of the vehicle's longitudinal motion dynamics. However, the system remains under-determined until the tractive force (F_x) is explicitly modeled. As this force is a non-linear function of the relative motion between the tire and the road, the following section is dedicated to the microscopic tire dynamics and the characterization of the friction-slip relationship.

3.2 Tyre-Road Interaction Model

This section transitions from the rigid-body dynamics of the chassis to the physics of traction force generation. By characterizing the wheel's rotational equilibrium and defining the kinematic variables that govern the tire's capacity to transmit shear forces, the non-linear plant that the traction controller must regulate is

defined. Central to this discussion is the relationship between longitudinal slip and friction coefficient, which defines the physical boundary of the vehicle's performance envelope.

3.2.1 Longitudinal Slip

While a comprehensive vehicle model requires the consideration of both longitudinal slip (σ_L) and lateral slip (σ_S) to describe combined-slip maneuvers, this thesis is specifically constrained to the longitudinal degree of freedom ($\sigma \equiv \sigma_L$), in the direction of the wheel velocity vector.

The decision to omit lateral slip dynamics from the control synthesis is motivated by the project's focus on pure longitudinal traction optimization and anti-spin regulation for Rear-Wheel Drive (RWD) architectures. By isolating the longitudinal component, the controller can more precisely manage the transmissible torque during high-acceleration transients, where the primary objective is the prevention of wheel-spin and the maintenance of the tire within its stable longitudinal friction region.

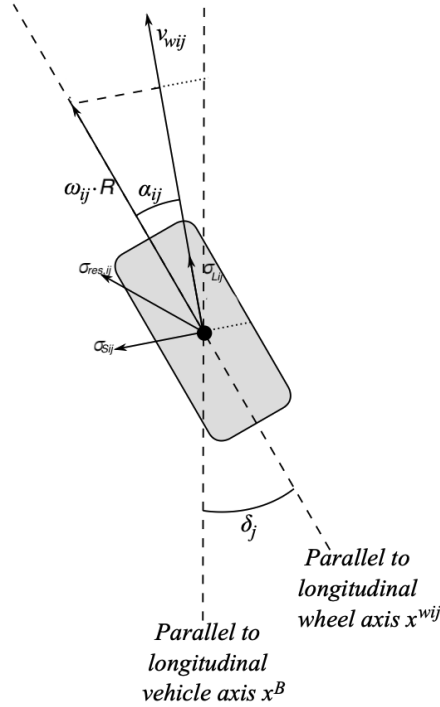


Figure 3.2: Wheel slip model.

In this context, the **longitudinal slip ratio** σ is defined as the normalized difference between the rotational velocity of the driving wheel and the average rotational velocity of the driven wheels:

$$\sigma_{Rj} = \begin{cases} \frac{\omega_{driving,Rj} - \omega_{driven,avg}}{\omega_{driving,Rj}}, & \text{traction} \\ \frac{\omega_{driving,Rj} - \omega_{driven,avg}}{\omega_{driven,avg}}, & \text{braking} \end{cases} \quad (3.7)$$

Where $\omega_{driving,Rj}$ is the rotational speed of the j -th rear wheel, and $\omega_{driven,avg}$ is the reference free rolling speed for slip ratio evaluation, retrieved as the average rotational speed of the j -th front wheels, given by $\frac{\omega_{FL} + \omega_{FR}}{2}$.

Control-oriented model definition. Slip definition is adapted to prevent mathematical instabilities that typically occur at near-zero velocities, and to account for both driving and braking regimes within a single, unified expression:

$$\sigma_{Rj} = \frac{\omega_{driving,Rj} - \omega_{driven,avg}}{\max(|\omega_{driving,Rj}|, |\omega_{driven,avg}|, \epsilon)} \quad (3.8)$$

where ϵ represents a numerical tolerance which ensures that the denominator remains non-zero during standstill conditions ($V_{vhcl}, \omega = 0$). In this context, vehicle's speed is retrieved directly from the average driven wheels rotational speed as:

$$V_{vhcl} = \omega_{driven,avg} \cdot R_F \quad (3.9)$$

3.2.2 Wheel's Dynamics

By applying the principle of angular momentum conservation to the j -th driving wheel, the governing differential equation is established as:

$$I_w \dot{\omega}_{Rj} = T_{drive,Rj} - T_{ext,Rj} - F_{x,Rj} R_R \quad (3.10)$$

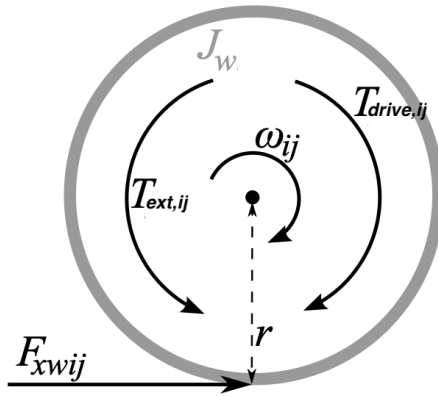


Figure 3.3: Single wheel free body diagram.

In this formulation, I_w represents the moment of inertia of the wheel assembly (including the integrated rotor in case of IWM), $T_{drive,Rj}$ is the driving torque at the wheel, provided by the electric motor, and $T_{ext,Rj}$ represents the sum of all external resistive torques, including the hydraulic braking torque $T_{b,Rj}$.

Control-oriented model definition. It is important to note that for the actual controller synthesis, the fundamental equations derived above may be further manipulated to account for specific hardware configurations and external disturbances. Furthermore, in the context of Sliding Mode Control, the resistance forces F_{res} and the non-linearities in $T_{ext,ij}$ are often lumped into a single *uncertainty term* [16]. This abstraction allows the adaptive law to compensate for exogenous factors without requiring an exact analytical measurement of every external variable.

3.2.3 Pacejka's Magic Formula and Tire Force Non-linearity

To understand the challenge of longitudinal traction control, the physical mechanism by which a pneumatic tire generates force must be examined.

Unlike the friction between two rigid bodies, tire-road friction is a result of both molecular adhesion and hysteretic deformation of the rubber. This relationship is fundamentally non-linear and is most comprehensively described by the **Pacejka Magic Formula** [17].

The dependency of the traction force generated by the wheel at a specific wheel's slip condition is described through a semi-empirical trigonometric structure:

$$F_{x,ij} = F_{z,ij} \mu_{ij} \quad (3.11)$$

$$\mu_{ij}(\sigma_{ij}) = D \sin(C \arctan(B\sigma_{ij} - E(B\sigma_{ij} - \arctan(B\sigma_{ij})))) \quad (3.12)$$

where B , C , D and E are Pacejka's coefficients, each corresponding to a specific physical attribute of the tire-road interaction:

- Stiffness Factor (B): represents the initial slope of the curve, corresponding to the elastic longitudinal stiffness of the tire carcass and tread blocks before macro-sliding begins.
- Shape Factor (C): governs the width of the peak, and determines how sharp the transition between static and dynamic friction is.
- Peak Factor (D): defines the maximum friction coefficient μ_{max} , according to the road surface condition (e.g., dry asphalt vs. icy road).
- Curvature Factor (E): accounts for the drop-off in friction after the peak, characterizing the transition into the fully developed sliding regime.

These empirical coefficients, identified for a specific tire carcass, enable the Magic Formula to reconstruct the tire's characteristic curves with high fidelity across a broad operational envelope. Although the model provides a robust approximation for most driving maneuvers, its predictive accuracy may diverge under highly non-linear or transient conditions.

These characteristic mappings analytically describe the evolution of the longitudinal force (F_x), the lateral force (F_y), and the *self-aligning torque* (M_z) as a coupled function of the state variables: vertical load (F_z), longitudinal slip (σ), sideslip angle and camber angle.

Sensitivity to Vertical Load (F_x vs. σ)

The longitudinal force is not solely a function of slip but is deeply coupled with the vertical load.

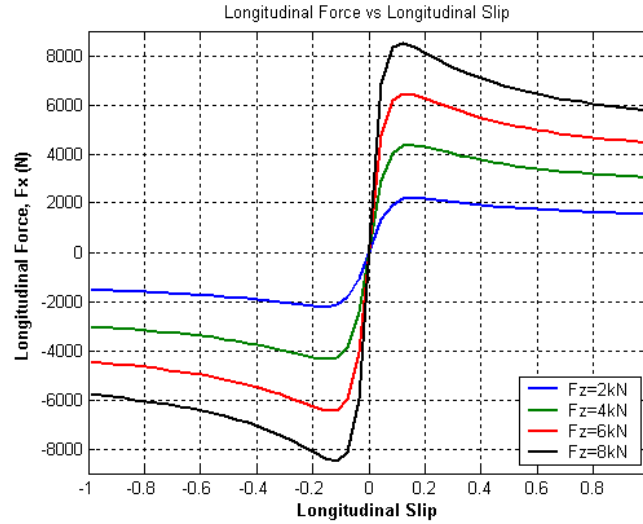


Figure 3.4: Longitudinal force and longitudinal slip mapping for various vertical loads [18].

As illustrated in the $F_x - \sigma$ map, an increase in vertical load, resulting from the longitudinal load transfer discussed in Section 3.1.1, shifts the entire curve upward. However, it is important to observe that the force does not increase linearly with F_z , but is subjected to a *load sensitivity*. This degressive behavior explains how, as the rear axle gains load during acceleration, the tire stiffness and peak force capacity change, making the plant of the traction controller time-varying and non-linear.

Control-oriented model definition. Despite the Pacejka model provides superior fidelity for offline vehicle simulations, its high degree of non-linearity and large parameter set make it computationally expensive for real-time **Optimal Slip Identification**. Specifically, it requires iterative numerical methods that are often incompatible with the fixed-step execution constraints of embedded vehicle control units. Consequently, to facilitate the design of an adaptive law capable of autonomous surface identification, the focus must shift toward a model that preserves the fundamental non-linear phenomenology of the $\mu - \sigma$ curve while offering a differentiable and analytically manageable form. For this reason, the **Burckhardt Tire Model**, which preserves the essential characteristics of the Pacejka curve with a reduced parameter set, will be utilized in Chapter 4 to identify the optimal slip and adapt the control law to varying road conditions.

3.2.4 Burckhardt Tire Model

The Burckhardt Tire Model offers a simplified alternative to the Magic Formula, replacing Pacejka's extensive parameter set with a three-coefficient exponential structure. This formulation is especially effective for adaptive control and real-time estimation, as it explicitly characterizes tire's adhesion limit through a continuous and differentiable function of the longitudinal slip [19][20]. The longitudinal friction coefficient is expressed as:

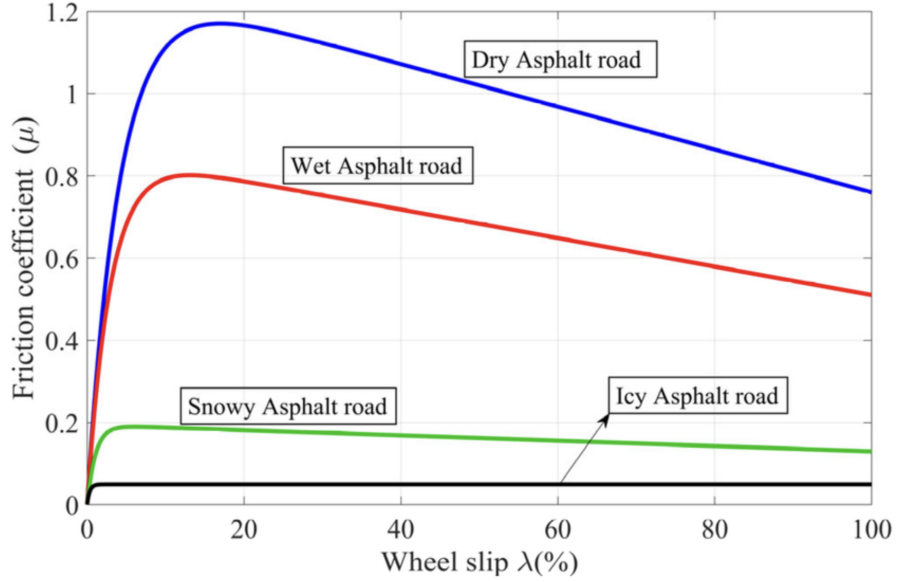
$$\mu_{ij}(\sigma_{ij}) = c_1(1 - e^{-c_2\sigma_{ij}}) - c_3\sigma_{ij} \quad (3.13)$$

Where c_1 , c_2 and c_3 are the road surface dependent coefficients defining the curve's profile. The physical significance of these parameters is defined as follows:

- c_1 : Governs the asymptotic value of the friction coefficient, effectively scaling the peak magnitude μ_{max} .
- c_2 : Dictates the exponential growth rate, representing the normalized longitudinal stiffness of the tire in the linear elastic region.
- c_3 : Introduces the slope of the sliding region, characterizing the friction drop-off beyond the saturation point.

Table 3.1: Burckhardt Tire Model Coefficients

	c_1	c_2	c_3
Asphalt, dry	1.2801	23.99	0.52
Asphalt, wet	0.857	33.822	0.347
Concrete, dry	1.1973	25.168	0.5373
Cobblestone, dry	1.3713	6.4565	0.6691
Cobblestone, wet	0.4004	33.708	0.1204
Snow	0.1946	94.129	0.0646
Ice	0.05	306.39	0


Figure 3.5: Burckhardt tire-road friction curves for different road conditions. [21].

In the initial low-slip stage, the friction coefficient increases rapidly. Physically, this represents the elastic deformation of the tire tread blocks, dominated by the term $(1 - e^{-c_2\sigma_{ij}})$.

As slip increases, the tire reaches its maximum friction potential (μ_{max}) at the optimal slip ratio (σ_{opt}). This is the mathematical ideal region where the gain from the exponential adhesion term is exactly balanced by the loss from the linear term ($c_3\sigma_{ij}$). It coincides with the condition $\partial\mu/\partial\sigma = 0$, which ensures maximum longitudinal acceleration and optimal vehicle stability.

Control-oriented model definition. The structural simplicity of the Burckhardt formulation is particularly beneficial for the synthesis of the traction control law. Friction coefficient equation is analytically differentiable, allowing for a direct derivation of the optimal slip condition. By applying the first-order necessary condition for a maximum at the friction peak ($\partial\mu/\partial\sigma = 0$), the optimal slip ratio can be retrieved in a closed-form expression.

Control Methodologies for Slip Regulation: PI and Traditional Sliding Mode Approaches

To provide a rigorous basis for comparison, this chapter details the design process of the baseline control strategies that serve as the foundation for this thesis study. The primary objective is to establish a performance benchmark by implementing two distinct control philosophies: one based on classical error-correction and the other on robust nonlinear regulation.

The first section covers the implementation of a standard PI, while the second focuses on the formulation of a TSMC, detailing the mathematical derivation of the switching law and the Lyapunov stability criteria required to ensure global convergence.

4.1 Vehicle's Architecture and Parameters

The vehicle platform under consideration employs a Rear-Wheel Drive (RWD) architecture, characterized by the integration of two independent direct-drive motors on the rear axle. Unlike traditional internal combustion powertrains, this configuration eliminates the compliance of differentials, allowing for a high-bandwidth response to torque commands.

The model under analysis involves either known or measurable quantities and unknown constant or varying parameters. The table below reports the main vehicle characteristic parameters, independent of operating condition.

Table 4.1: Main vehicle parameters.

Parameter	Symbol	Value	Unit
Vehicle Mass	m	1600	kg
Wheelbase	L	2.68	m
Distance from CG to Front Axle	l_f	1.796	m
Distance from CG to Rear Axle	l_r	1.152	m
Vehicle Front Area	A	2.156	m ²
Yaw Inertia	J_{yaw}	2700	kgm ²
Rear Wheel Radius	R_r	0.343	m
Front Wheel Radius	R_f	0.331	m
Rear i-th Wheel Moment of Inertia	J_{Ri}	1.5	kg·m ²
Front i-th Wheel Moment of Inertia	J_{Fi}	1.5	kg·m ²
Maximum Motor Torque	$T_{EM,max}$	240	Nm
Maximum Motor Power	$P_{EM,max}$	170	kW
Motor Build-up Time	τ_{EM}	$\frac{1}{2\pi \cdot 10}$	s
Gear Ratio	i_g	9	-
Efficiency	η	0.95	-
Gravity Acceleration	g	9.81	m/s ²

4.2 Problem Statement

The primary objective is to develop a robust slip control system for a RWD EV equipped with two independent motors. The controller must stabilize wheel slip at a precisely calculated optimal reference value, ensuring maximum longitudinal traction and preventing loss of control during high-torque acceleration or braking.

4.3 General Control Architecture

To ensure a rigorous and unbiased performance evaluation, the three control strategies proposed in this thesis are implemented within a unified, modular architecture composed of different *building blocks*.

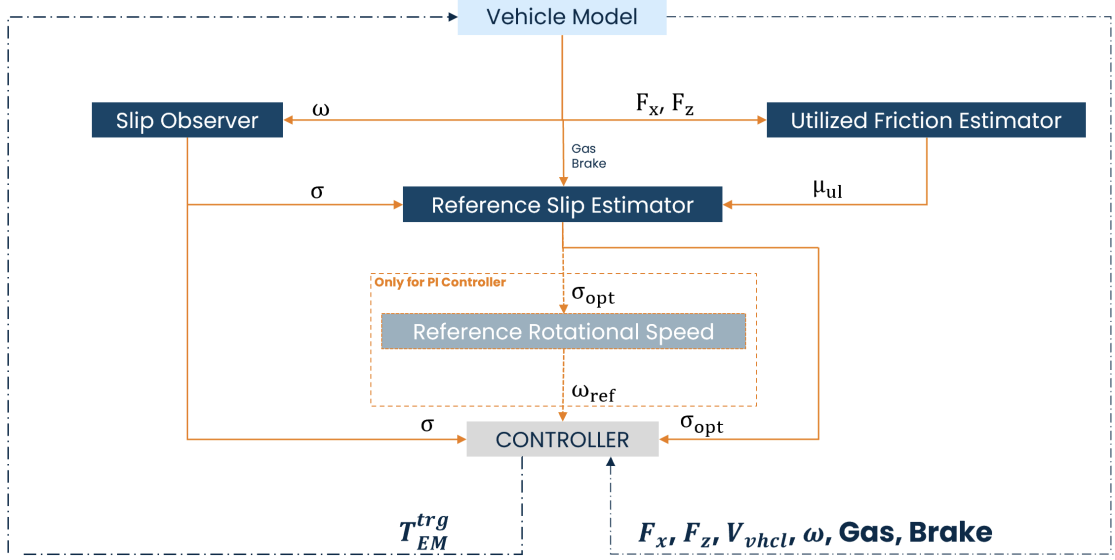


Figure 4.1: General control architecture.

Slip Observer

This block is responsible for calculating the real-time longitudinal slip (σ_{ij}) for each driving wheel by processing the front wheels rotational speed (ω_{Fj}) and the driving wheels rotational speed (ω_{Rj}). The slip is mathematically defined as in Eq.(3.8).

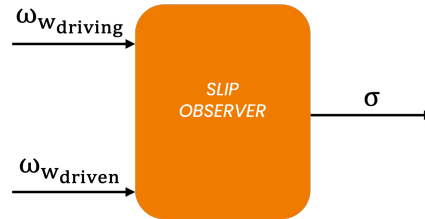


Figure 4.2: Slip observer.

Utilized Friction Estimator

It provides the controller with a direct measure of how much of the available

grip is being exploited, determining the ratio of longitudinal force to vertical load currently being exerted by the wheel.

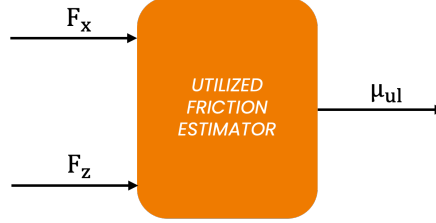


Figure 4.3: Utilized friction estimator.

Reference Slip Estimator

This serves as the target-setting module for the control system, computing the optimal slipping condition for the tire corresponding to the observed friction condition of the road.

This control loop originates with the real-time acquisition of state variables from the vehicle model, including wheels angular speed (ω), traction forces (F_x), vertical loads (F_z) and gas and brake pedal inputs (**Gas**, **Brake**). These signals are processed by the observation layer to derive the actual slip and the current friction utilization condition.

During nominal driving conditions, driver's commands are passed directly to the conventional powertrain control unit. The transition to active regulation is governed by the supervisory logic of a flag (**TCSO**N), which triggers the intervention of the controller only when a predefined threshold, determined by the divergence between actual and reference slip, is surpassed, indicating the onset of unstable wheel spin. The condition for the onset of active regulation is defined by:

$$\text{TCSO}N = (\text{abs}(\text{slip}) > 0.9 * \text{slip_target})$$

Once activated, the specific control law, whether PI, TSMC, or CISM C, assumes command of the torque distribution to mitigate the detected slip error, returning a target torque (T_{EM}^{trg}) for the Electric Motor Control Unit (MCU).

To complete the operational overview of the general control architecture, it is necessary to detail the physical interpretation of the controller's output and its interaction with the vehicle's braking system. Regardless of the core algorithmic strategy deployed, the final output of the loop is structurally defined as a net driving torque request at the wheel.

However, during active decelerating maneuvers where the system triggers traction control intervention under braking (e.g., maintaining optimal negative slip to

maximize braking force on low-adherence surfaces), a specialized torque coordination strategy is required to maintain energy efficiency. Under such conditions, if the hydraulic braking systems are actively decelerating the wheel assembly, forcing the electric motor to independently combat the primary deceleration torque would result in significant parasitic energy dissipation and unuseful electromagnetic work.

To prevent this efficiency loss, the supervisory architecture introduces a torque subtraction layer such that, when a braking command is initiated by the driver or the high-level vehicle management unit, the torque actively exerted by the hydraulic friction brakes (T_{brake}) is algebraically subtracted from the overall driving torque request computed by the controller ($T_{driving}^{req}$).

$$T_{EM}^{trg} = T_{driving}^{req} - T_{brake} \quad (4.1)$$

By implementing this equilibrium, the control loop effectively offsets the torque demands. The friction brake system handles a portion of the negative workload, allowing the electric motor to scale down its active power consumption while maintaining the precise net wheel torque necessary to track σ_{opt} . This coordination guarantees that the anti-slip control framework remains energetically optimized, eliminating antagonistic work between the hydraulic and electromagnetic actuators.

Consequently, this shared configuration allows for a rigorous comparative analysis of how each mathematical approach manages the inherent nonlinearities and control challenges, within a controlled and uniform environment.

4.3.1 Utilized Friction Estimator

The calculation is fundamentally defined by the relationship:

$$\mu_{ul} = \frac{F_x}{F_z} \quad (4.2)$$

To ensure this estimation is robust enough for real-time control, the computation algorithm implements several signal-conditioning precautions to mitigate noise and physical inconsistencies:

- The vertical force F_z is constrained to a minimum value ($F_{z,min} = 300 \text{ Nm}$) to prevent numerical instability or division by zero during dynamic weight transfers computation.
- The calculated friction candidates are clamped between predefined minimum and maximum bounds ($\mu_{max} = -\mu_{min} = 1.5$,) to keep the values within a physically plausible range.

- Confidence weights based on a threshold ratio ($k_{th} = \left(\frac{Fx}{Fz}\right)_{th} \approx 6$) are used to prioritize previous stable estimates when current data is deemed less reliable, such as during low-force scenarios.

This μ_{ul} value is a critical input for the subsequent **Reference Slip Estimator**, as it allows the system to map the current grip usage against the theoretical limits of the road surface.

4.3.2 Reference Slip Estimator

To maximize the vehicle's longitudinal performance and stability across varying track environments, a high-level **Reference Slip Estimator** block is implemented within the general supervisory architecture. This module operates as a real-time state interpreter that evaluates the current maximum adhesion capabilities of the tire and continuously computes the optimal target longitudinal slip ($\sigma_{ref} = \sigma_{opt}$).

This tracking optimization uses the analytical Burckhardt Tire Friction Model exposed in Sec. 3.2.4, which mathematically establishes the non-linear relationship between the longitudinal slip ratio and the friction coefficient ($\mu(\sigma)$) through surface-dependent parameters (c_1, c_2, c_3). To compute the optimal slip values corresponding to the seven different road conditions listed in Table 3.1, the first-order necessary condition for a maximum at the friction peak is exploited:

$$\sigma_{opt} = \arg \max_{\sigma} \mu(\sigma) \quad (4.3)$$

$$\sigma_{opt} : \frac{\partial \mu}{\partial \sigma_{opt}} = 0 \quad (4.4)$$

Performing the partial derivative with respect to slip on Eq. (3.13), it's possible to derive the formal definition of optimal slip as function of Buckhardt parameters:

$$\sigma_{opt} = \frac{1}{c_2} \ln \left(\frac{c_1 c_2}{c_3} \right) \quad (4.5)$$

From this, peak friction coefficient value can be retrieved as only function of Buckhardt parameters:

$$\mu_{max} = c_1 - \frac{c_3}{c_2} \left(1 + \ln \left(\frac{c_1 c_2}{c_3} \right) \right) \quad (4.6)$$

This allows to build a standardized dataset of peak friction coefficient and optimal slip values for the seven road conditions reported in Table 3.1:

Table 4.2: Burckhardt Tire Model Peak Friction and Optimal Slip Standardized Values

	$\sigma_{opt,k}$	$\mu_{max,k}$	k
Asphalt, dry	0.17	1.1700	1
Asphalt, wet	0.13	0.8013	2
Concrete, dry	0.16	1.0900	3
Cobblestone, dry	0.40	1.0000	4
Cobblestone, wet	0.14	0.3800	5
Snow	0.06	0.1900	6
Ice	0.04	0.0500	7

To determine which environmental condition (e.g., Dry Asphalt, Wet Asphalt, Snow, Ice, etc.) matches the physical terrain in real time, the module operates an active matching algorithm, where the instantaneous state pair (μ_{ul}, σ) forms a dynamic operating point in the friction-slip plane.

For each standard road, a *predicted friction coefficient* as function of the measured slip (σ_{Rj}):

$$\mu_{Rj}(\sigma_{Rj}) = \text{sign}(\sigma_{Rj}) \cdot c_1(1 - e^{-c_2\sigma_{Rj}}) - c_3\sigma_{Rj} \quad (4.7)$$

At each time step, the algorithm evaluates the consistency between $\mu_{ul,Rj}$ and $\mu_{Rj}(\sigma_{Rj})$ and derives a set of weight coefficients w_k that quantify the likelihood of each road model:

$$w_{k,Rj} = \frac{1}{|\mu_{Rj}(\sigma_{Rj}) - \mu_{ul,Rj}| + \varepsilon} \quad (4.8)$$

where $k = 1, 2, \dots, 7$ identifies the road condition from the ones listed in Tab. 4.2, and ε is a tolerance introduced to avoid division by zero.

Finally, the optimal slip ratio ($\sigma_{opt,Rj}$) and maximum friction coefficient ($\mu_{max,Rj}$) corresponding to the real-time operation can be obtained:

$$\mu_{max,Rj} = \frac{\sum_{k=1}^7 w_{k,Rj} \mu_{max,k}}{\sum_{k=1}^7 w_{k,Rj}} \quad (4.9)$$

$$\sigma_{opt,Rj} = \frac{\sum_{k=1}^7 w_{k,Rj} \sigma_{opt,k}}{\sum_{k=1}^7 w_{k,Rj}} \quad (4.10)$$

4.4 Design of PI Control Logic

The first baseline control strategy proposed is a Proportional-Integral (PI) controller for slip regulation. The input of the PI controller is the error between driving wheel's actual rotational speed and reference rotational speed corresponding to optimal slip condition for maximum traction. The output of the PI controller is a driving torque request at the wheel.

4.4.1 Reference Rotational Speed

The PI control block directly reacts to the error e_j between reference and actual rotational speed of the wheel:

$$e_j = \omega_{ref,Rj} - \omega_{Rj} \quad (4.11)$$

The reference rotational speed for the wheel is the one corresponding to the optimal slip ratio condition, derived from Eq.(3.7):

$$\omega_{ref,Rj} = \begin{cases} \frac{\omega_{driven,Rj}}{1-\sigma_{opt,Rj}}, & \text{traction} \\ \omega_{driven,Rj}(1 + \sigma_{opt,Rj}), & \text{braking} \end{cases} \quad (4.12)$$

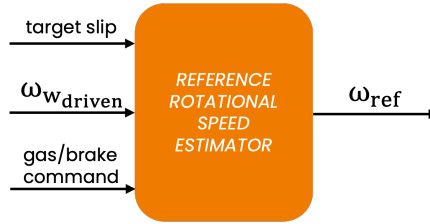


Figure 4.4: Reference rotational speed estimator.

4.4.2 Control Law

The control law for the PI control strategy is derived from the rotational equilibrium equation at the wheel of Eq. (3.10):

$$T_{drive,Rj} = I_w \dot{\omega}_{Rj} + F_{x,Rj} R_R \quad (4.13)$$

Expressing the traction force as function of vehicle speed (V):

$$F_{x,Rj} = m \dot{V} R_R \quad (4.14)$$

Differentiating according to the expression of the slip ratio as function of vehicle speed:

$$\begin{cases} \sigma_{Rj} = \frac{\omega_{Rj} R_R - V}{\omega_{Rj} R_R} \\ \dot{V} = \dot{\omega}_{Rj} R_R (1 - \sigma_{Rj}) - \omega_{Rj} R_R \dot{\sigma}_{Rj} \end{cases} \quad (4.15)$$

We can substitute Eq. (4.14) and (4.15) in Eq. (4.13) to obtain a new expression of driving torque request at the wheel:

$$T_{drive,Rj} = [I_w + m (1 - \sigma_{Rj}) R_R^2] \dot{\omega}_{Rj} - m \omega_{Rj} R_R^2 \dot{\sigma}_{Rj} \quad (4.16)$$

This definition allows for treating the driving torque request as the resultant of two main dynamics contributions:

$$T_{drive,Rj} = T_{ctrl,Rj} + \Delta T_{Rj} \quad (4.17)$$

$$\begin{cases} T_{ctrl,Rj} = [I_w + m (1 - \sigma_{Rj}) R_R^2] \dot{\omega}_{Rj} \\ \Delta T_{Rj} = -m \omega_{Rj} R_R^2 \dot{\sigma}_{Rj} \end{cases} \quad (4.18)$$

The **control torque** T_{ctrl} is the output torque of the PI, retrieved from Eq. (4.16) considering null rate for the slip ratio ($\dot{\sigma}_{Rj} = 0$). This term models the dynamics of the wheels, forcing the real-time wheel rotational speed to be the same as the reference rotational speed (ω_{ref}) corresponding to optimal slip condition.

$$\begin{cases} T_{ctrl,Rj} = k_p e_j + k_i \int e_j(t) dt \\ e_j = \omega_{ref,Rj} - \omega_{Rj} \end{cases} \quad (4.19)$$

The **torque error** ΔT between driving torque request and control torque is directly proportional to $\dot{\sigma}_{Rj}$ and is representative of the disturbances on the system and uncertainties of the parameters, related to slip dynamics.

A **compensation torque** T_{cmps} is implemented in the control law to correct the error in torque, and it's saturated considering a maximum compensation capability equal to 20% of maximum electric motor torque [8]:

$$T_{cmps,Rj} = \begin{cases} 0.2 \cdot T_{EM}^{max}, & \Delta T_{Rj} > 0.2 \cdot T_{EM}^{max} \\ \Delta T_{Rj}, & -0.2 \cdot T_{EM}^{max} < \Delta T_{Rj} < 0.2 \cdot T_{EM}^{max} \\ -0.2 \cdot T_{EM}^{max}, & \Delta T_{Rj} < -0.2 \cdot T_{EM}^{max} \end{cases} \quad (4.20)$$

Finally, the driving torque request at the wheel is obtained by sum of the two contributions:

$$T_{drive,Rj} = T_{ctrl,Rj} + T_{cmps,Rj} \quad (4.21)$$

In Fig. 4.5 a schematic representation of the control law implementation inside the control loop.

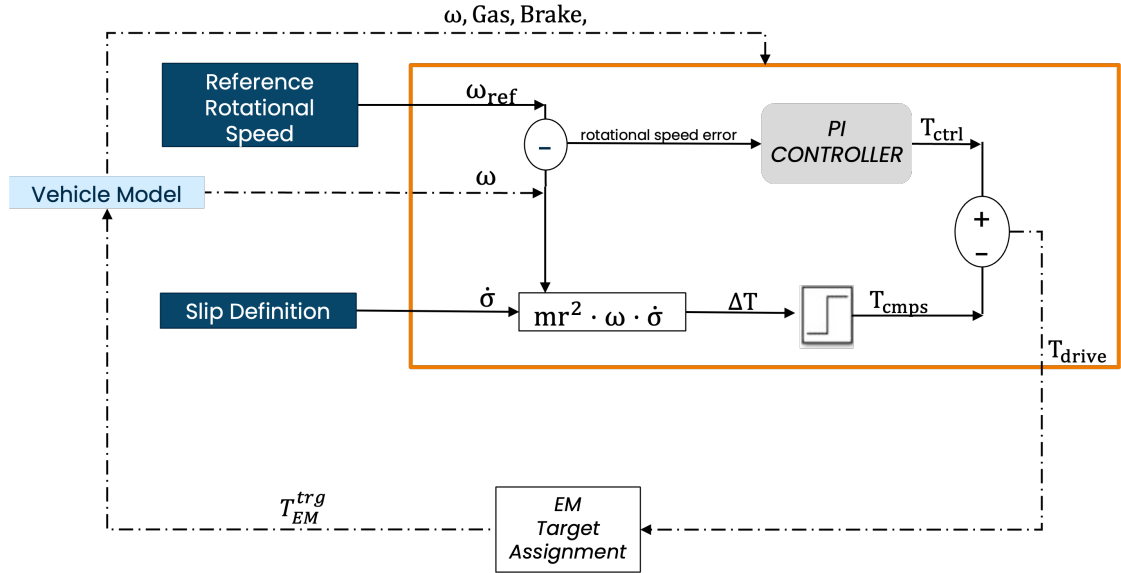


Figure 4.5: PI control scheme.

4.5 Design of Traditional Sliding Mode Control Logic

The second baseline control strategy proposed is a Traditional Sliding Mode (TSM) controller for slip regulation. TSMC works by applying a discontinuous, high-frequency switching command to force the system's state onto a predefined *sliding surface*, representing the ideal stable trajectory in the system's state-space, which forces the tracking error convergence to zero, and then maintain it there despite disturbances or model uncertainties.

4.5.1 Sliding Surface Definition

The foundational step in the synthesis of a sliding mode control strategy involves the mathematical definition of an appropriate sliding surface within the system state space. Being the primary objective of slip regulation for traction control to force the actual wheel slip to asymptotically track the optimal target reference calculated by the upper-level observation layer, in the traditional form of a SMC, the sliding surface (s) is chosen as coincident with the error in slip ratio [10]:

$$s_j \equiv e_j = \sigma_j - \sigma_{opt_j} \quad (4.22)$$

During the initial *reaching phase*, the controller applies a high-frequency switching law to force the system state trajectories toward the surface in finite time. Once the surface is reached, the system enters the *sliding phase*, where the state is strictly constrained to $s = 0$. On this ideal trajectory, the tracking error is identically zero, and the system dynamics become completely decoupled from external matched disturbances, parameter uncertainties, and the severe nonlinearities inherent to the tire-road interface.

4.5.2 Control Law

A proper control law has been defined for the TSMC, in order to guarantee controller's ability of driving the system trajectories toward the sliding surface and maintaining them within it.

As illustrated in the structural layout of the controller in Fig. 4.6, the output of the control law, which is a driving torque request at the wheel, here referenced to as the **command torque** $T_{cmd,Rj}$, is analytically decomposed into two distinct components:

$$T_{drive,Rj} = T_{cmd,Rj} \quad (4.23)$$

$$T_{cmd,Rj} = T_{eq,Rj} + T_{sw,Rj} \quad (4.24)$$

This dual formulation decouples the control action into a *continuous* model-based term and a *discontinuous* robust term, each addressing a specific phase of the sliding mode execution.

The **equivalent torque** T_{eq} is a continuous term enforcing the condition $s = 0$ in the nominal model. It is derived under the ideal nominal condition where the derivative of the sliding surface is zero ($\dot{s} = 0$). By isolating the nominal wheel dynamics, which are governed by the wheel inertia and the longitudinal tire force acting at the tire radius, the equivalent torque is formulated as:

$$T_{eq,Rj} = I_w \dot{\omega}_{Rj} + F_{x,Rj} R_R \quad (4.25)$$

This component effectively establishes a baseline torque required to balance the rotational wheel acceleration with the active longitudinal tire-road forces.

The **switching torque** T_{sw} is a robust discontinuous term designed to account for unmodeled dynamics, parametric variations, and the severe nonlinearities of the tire-road interface that deviate from the nominal model. It is mathematically defined as proportional to the sign of s and is representative of slip dynamics ($\dot{s}$):

$$T_{sw,Rj} = -K \cdot \text{sign}(s_j) \quad (4.26)$$

where K is a constant positive *switching gain*, calibrated to overcome the maximum bounded uncertainties of the system.

Because the signum function switches instantaneously between ± 1 based on the polarity of the slip tracking error, the switching law introduces a strict bang-bang behavior within the actuator command.

While this aggressive discontinuous switching mathematically guarantees deterministic convergence and absolute robustness against matched disturbances, its high-frequency chattering nature presents severe physical limitations, which will be addressed in subsequent sections.

Wheel's Dynamics

To accurately bridge the kinematic state requirements with the physical limitations of the powertrain, a dedicated wheel's angular acceleration estimator is implemented. This evaluates the required wheel angular acceleration by differentiating the slip definition of Eq. 3.7:

$$\dot{\omega}_{Rj} = \begin{cases} \frac{\dot{\omega}_{driven,j} (1-\sigma_j) + \omega_{driven,j} \dot{\sigma}_j}{(1-\sigma_j)^2}, & \text{traction} \\ \dot{\omega}_{driven,j} (1 + \sigma_j) + \omega_{driven,j} \dot{\sigma}_j, & \text{braking} \end{cases} \quad (4.27)$$

In an unconstrained state model, rapid changes in surface friction conditions or high-frequency switching from the sliding mode controller can cause this calculated

derivative to spike to physically impossible values. To prevent numerical instability and maintain absolute consistency with the vehicle's physical limits, an actuator-boundary guard is integrated into the block logic.

The maximum achievable angular acceleration is strictly bounded by the maximum electromagnetic torque available from the motor (T_{EM}^{max}) scaled through gear ratio and inertia:

$$\dot{\omega}_{Rj}^{max} = \frac{T_{EM}^{max} \cdot i_g \cdot \eta_{EM}}{I_w} \quad (4.28)$$

$$\dot{\omega}_{Rj} = \begin{cases} \dot{\omega}_{Rj}, & |\dot{\omega}_{Rj}| \leq \dot{\omega}_{Rj}^{max} \\ \dot{\omega}_{Rj}^{max} \cdot \text{sign}(\dot{\omega}_{Rj}), & |\dot{\omega}_{Rj}| > \dot{\omega}_{Rj}^{max} \end{cases} \quad (4.29)$$

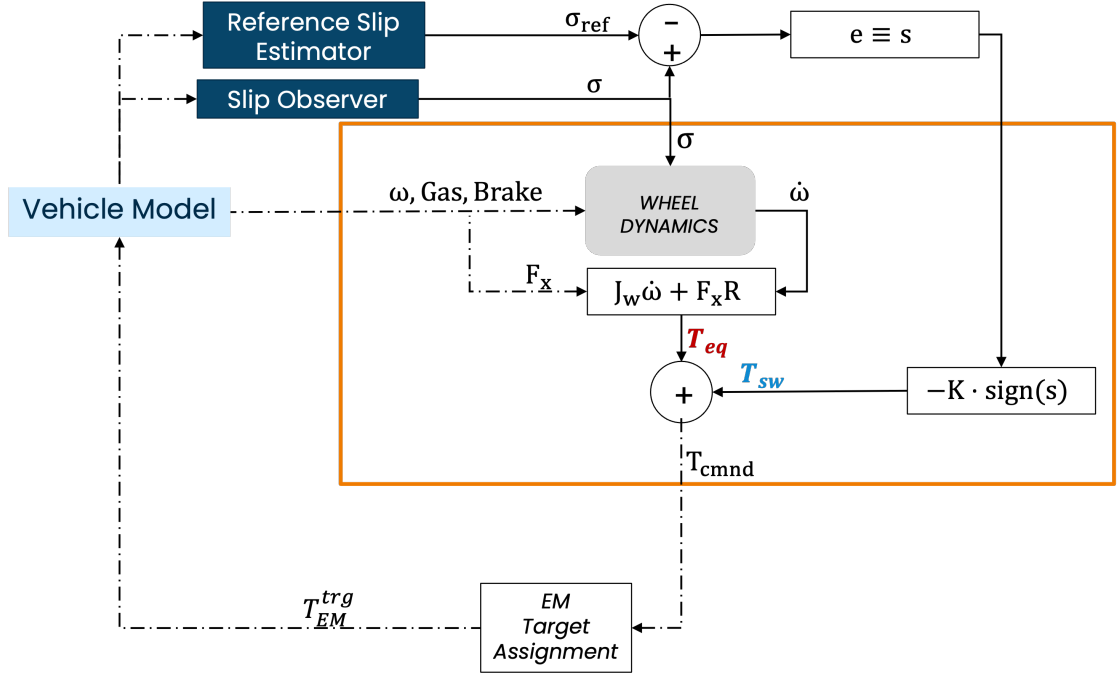


Figure 4.6: TSM control scheme.

4.5.3 Proof of Stability

To guarantee that the system state trajectory converges to the sliding surface $s = 0$ in finite time and remains strictly restricted to it, a formal stability analysis must be conducted utilizing the **Lyapunov Direct Method**.

For an individual wheel assembly, a positive-definite scalar Lyapunov candidate function is formulated [22]:

$$\begin{cases} V(s_j) = \frac{1}{2}s_j^2 > 0, & \forall s_j \neq 0 \\ V(0) = 0 \end{cases} \quad (4.30)$$

According to Lyapunov stability criteria, the system is asymptotically stable if the time derivative of the chosen candidate function is strictly negative-definite ($\dot{V}(s_j) < 0$) outside the equilibrium. This condition enforces the classic sliding reaching condition:

$$\dot{V}(s_j) = \dot{s}_j s_j < 0, \quad \forall s_j \neq 0 \quad (4.31)$$

Recalling that the traditional sliding surface is defined to coincide directly with the slip tracking error (Eq. (4.22)) its time derivative corresponds to the tracking slip error dynamics:

$$\dot{s}_j = \dot{\sigma}_j - \dot{\sigma}_{opt_j} \quad (4.32)$$

Applying nonlinear slip dynamics derived earlier with Eq. (4.26), we choose:

$$\dot{s}_j = -K \cdot \text{sign}(s_j) \quad (4.33)$$

This gives the following Lyapunov function derivative:

$$\dot{V}(s_j) = -K \cdot \text{sign}(s_j) s_j = -K \cdot |s_j| \leq 0 \quad (4.34)$$

Finally, choosing a constant positive gain ($K > 0$) for the switching term ensures asymptotic stability for the system (Eq. (4.31)).

An Advanced Approach Proposal: Conditional Integral Sliding Mode Controller

To overcome the conflicting limitations related to the baseline controllers of Chapter 4, this section proposes a Conditional Integral Sliding Mode Control (CISMC) framework. The core philosophy of the CISMC approach lies in the continuous modulation of the robust switching term based on the instantaneous state trajectory distance from the sliding surface, while simultaneously an integral control term is conditionally activated to asymptotically eliminate residual steady-state tracking errors without risking integrator wind-up during large transient maneuvers.

This multi-layered approach ensures robust slip tracking while entirely mitigating the actuator chattering typically associated with traditional sliding mode implementations.

The controller design will be carried out by initially designing a sliding surface defining the desired closed-loop behavior of the system. A stabilizing control law will then be derived assuming no parametric uncertainties.

5.1 Control Architecture

The deployment of this advanced nonlinear controller does not necessitate any modification to the external supervisory or physical plant layers. Specifically, the fundamental vehicle architecture, the geometric and mass parameters of the RWD electric platform, and the physical problem statement regarding optimal longitudinal tire-road friction utilization remain identical to those detailed for the baseline PI and TSMC implementations in the Sections 4.1, 4.2 and 4.3.

Furthermore, the general control architecture, including the feedback loops, the vehicle state observation layer, the reference slip estimation module, and the torque coordination logic for braking, is preserved in its entirety. This architectural consistency ensures a scientifically rigorous basis for comparison, isolating the evaluation of the powertrain response from external systemic variations. Consequently, the structural modification introduced in this section is confined exclusively to the internal control logic of the controller block itself, specifically reshaping how the tracking error is processed and how the motor torque command is mapped to reconcile robustness with actuator smoothness.

5.2 Sliding Surface Definition

The sliding surface is formulated, according to integral sliding mode control theory, as a weighted sum of the state error ($\sigma_j - \sigma_{opt,j}$) and the integrated state error:

$$s_j = e_j + K_1 \cdot \int_0^t e_j(\tau) d\tau - e_j(0) \quad (5.1)$$

This formulation offers two main advantages. First, it ensures that once the state trajectory is constrained to the $s_j = 0$, the tracking error is mathematically guaranteed to converge exponentially toward zero. Second, through the explicit structural inclusion of the initial condition $e_j(0)$, the controller completely eliminates the traditional reaching phase. This forces the sliding condition to be identically satisfied from the exact onset of the regulation window ($t = 0$), thereby guaranteeing absolute exponential error convergence from the very beginning of the control action, even in cases where the initial tracking error is non-zero ($e_j(0) \neq 0$) [23].

In Eq. (5.1), τ is a dummy variable used for integration, while K_1 is a constant positive gain determining the *convergence rate* [24]:

$$e_j(t) = e_j(0) \cdot e^{-K_1 t} \quad (5.2)$$

$$\Rightarrow \begin{cases} \lim_{t \rightarrow \infty} e_j(t) = 0 \\ \int_0^t e_j(\tau) d\tau = e_j(0) \cdot \frac{1}{K_1} (1 - e^{-K_1 t}) \end{cases} \quad (5.3)$$

The dynamics of the closed-loop slip control system therefore acts as a first order low pass filter with time constant equal to $1/K_1$.

5.3 Control Law

The control law considered assumes no parameter uncertainty, and it's expressed as function of two unknown functions (f and g) modeling the inverse dynamics of the plant [25]:

$$f \equiv f(\sigma_j, \omega_{driven,j}) \quad (5.4)$$

$$g \equiv g(\sigma_j, \omega_{driven,j}) \quad (5.5)$$

The dynamics of the system is described as follow:

$$\dot{\sigma}_j = f + g \cdot u_{Rj} \quad (5.6)$$

$$\dot{s}_j = \dot{e}_j + K_1 \cdot e_j = \dot{\sigma}_j - \dot{\sigma}_{opt_j} + K_1 \cdot e_j \quad (5.7)$$

This leads to the following expression of $\dot{s}$ as function of the control input u :

$$\dot{s}_j = f + g \cdot u_{Rj} + K_1 \cdot e_j - \dot{\sigma}_{opt_j} \quad (5.8)$$

Finally, the control input u :

$$u_{Rj} = -\frac{1}{g} \left(f + K_1 \cdot e_j - \dot{\sigma}_{opt_j} - \dot{s}_j \right) \quad (5.9)$$

where term $K_1 \cdot e$ enforces the condition $s = 0$ and $\dot{s}$ encompasses the discontinuous law forcing the system trajectories onto the sliding surface.

The output of the control logic, u , corresponds to a driving torque request at the wheel axle:

$$T_{drive,Rj} = u_{Rj} \quad (5.10)$$

The definition of slip dynamics ($\dot{s}$) for the CISM controller will be discussed in the next section (Section 5.3.1), where an alternative formulation of Eq. 4.33 will be proposed as a solution for the chattering problem.

5.3.1 Chattering Solution

In practical applications, because the inverse dynamics embedded within the nominal control law represents only an approximation of the true physical plant behavior, and because the vehicle is continuously subjected to exogenous disturbances and parametric uncertainties, state trajectories are constrained to the surface $s = 0$

solely by virtue of the high-gain control discontinuity. However, when dealing with a discrete-time digital processor, the ideal infinite-frequency switching can no longer be physically sustained. This limitation forces the state trajectories to oscillate aggressively across $s = 0$, giving rise to the **chattering** phenomenon. These are strongly coupled to the hardware sampling time, the magnitude of the discontinuous switching gain, and the internal unmodeled dynamics of both the vehicle system and the electromagnetic actuators.

To mitigate the resulting high control effort, the severe high-frequency oscillations in the system response and general control degradation, a virtual **boundary layer** of thickness φ is mapped symmetrically around the sliding surface, establishing a localized envelope within which the aggressive discontinuous gain is systematically smoothed. This continuous boundary layer approximation serves to smooth the switching command inside the tracking zone, thereby eliminating the high-frequency chattering. Nevertheless, this attenuation introduces an inherent trade-off, as it compromises absolute tracking precision and robustness against disturbances once the state vector resides inside the boundary layer. Consequently, the boundary layer thickness must be meticulously calibrated, selecting the minimum possible value that satisfies the physical chattering suppression requirements of the powertrain without excessively sacrificing steady-state regulation performance.

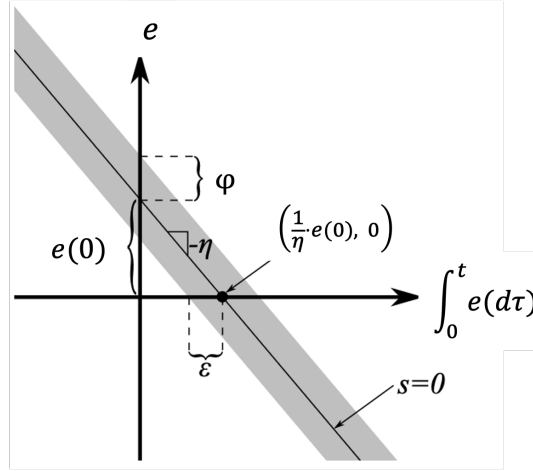


Figure 5.1: Sliding surface illustration [25].

Given fixed width φ , the discontinuous switching term is made proportional to s

employing a proper saturation function, to substitute to $sign(s)$ in Eq: (4.33):

$$\dot{s}_j = -K_2 \cdot sat\left(\frac{s_j}{\varphi}\right) \quad (5.11)$$

$$sat\left(\frac{s_j}{\varphi}\right) = \begin{cases} \frac{s_j}{\varphi}, & s_j \gg \varphi \\ sign\left(\frac{s_j}{\varphi}\right), & s_j \leq \varphi \end{cases} \quad (5.12)$$

The relationship between boundary layer thickness and tracking error is given as [25]:

$$\forall t \geq 0, \quad |e_j(t)| \leq 2 \cdot K_2 \varepsilon \quad \text{for all trajectories starting inside the boundary layer} \quad (5.13)$$

where ε is the boundary layer thickness lying on the horizontal axis illustrated in Fig. 5.1, defined as:

$$\varepsilon = \frac{\varphi}{K_2} \quad (5.14)$$

It is therefore guaranteed that:

$$\forall t \geq 0, \quad |e_j(t)| \leq 2 \cdot \varphi \quad (5.15)$$

Values of φ and K_2 are tuned in simulations reported in Chapter 5.3.3.

5.3.2 Anti Wind-up Strategy

Because the sliding surface defined in Eq. (5.1) incorporates a continuous time-integral component of the tracking error, its real-time implementation is vulnerable to **integrator wind-up**. This risk is highly pronounced in traction control loops since the actual available electromagnetic motor torque is constrained by the electric motor structural limits. When a large transient maneuver forces the tracking error to remain large over a sustained interval of time, a conventional integrator continues to accumulate, causing the virtual torque request to far exceed the physical capacity of the actuator, resulting in severe overshoots and extended settling times once the system attempts to recover from saturation.

To mitigate this operational vulnerability, a specialized conditional anti-windup framework is implemented. This scheme works by dynamically switching between distinct sliding surface definitions based on the saturation state of the actuator. The operational logic is formalized through two distinct execution windows:

$$s_j = \begin{cases} e_j + K_1 \cdot \int_{t_0}^t e_j(\tau) d\tau - e_j(t_0), & |u_{Rj}| \leq T_{drive,Rj}^{max} \\ e_j, & |u_{Rj}| > T_{drive,Rj}^{max} \end{cases} \quad (5.16)$$

As long as the calculated reference control command resides safely within the physical torque limits of the electric motor ($|u| \leq T_{drive}^{max}$), the primary integral sliding surface is employed.

The moment the torque request triggers actuator saturation ($|u| > T_{drive}^{max}$), the accumulation of the integral slip error is immediately halted.

To prevent a discontinuous step change in the sliding surface, which could riskily lead to chattering amplification, the mathematical boundaries of the integral term are dynamically shifted such that the initial slip error $e_j(0)$ is overwritten by the error at the instant of exit from saturation window ($e_j(t_0)$), and the integration lower bound (t_{in}) is re-indexed from $t_{in} = 0$ to $t_{in} = t_0$ (see Eq. (5.16)).

This ensures that the sliding surface is shifted so that the trajectories at (t_0) are located at $s = 0$ and exponential convergence is obtained immediately.

The reference physical limit for maximum driving torque is retrieved as function of electric motor limits:

$$T_{drive}^{max} = T_{EM}^{max} \cdot i_g \cdot \eta_{EM} \quad (5.17)$$

In Fig. 5.2 a schematic representation of the control law implementation inside the control loop.

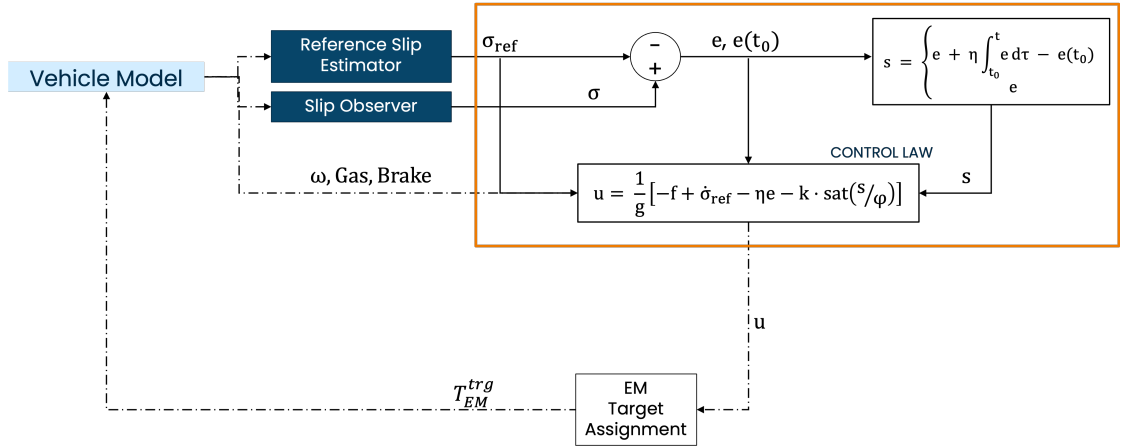


Figure 5.2: CISM control scheme.

5.3.3 Proof of Stability

Coherently with the approach adopted for the TSMC case study (Sec. 4.5.3), a formal stability analysis is conducted for the CISM controller according to Lyapunov theory.

The positive-definite scalar Lyapunov candidate function considered is:

$$\begin{cases} V(s_j) = \frac{1}{2}s_j^2 > 0, & \forall s_j \neq 0 \\ V(0) = 0 \end{cases} \quad (5.18)$$

The derivative with respect to time is:

$$\dot{V}(s_j) = \dot{s}_j s_j = \left(f + g \cdot u_{Rj} + K_1 \cdot e_j - \dot{\sigma}_{optj} \right) s_j \quad (5.19)$$

Both definitions satisfy the conditions discussed in Sec. 4.5.3, hence $V(s_j \rightarrow \infty) \rightarrow \infty$ and $\dot{V}(s_j) < 0$, given the following relationships:

$$u_{Rj} = -\frac{1}{g} \left(f + K_1 \cdot e_j - \dot{\sigma}_{optj} - \dot{s}_j \right) \quad (5.20)$$

$$\dot{s}_j = -K_2 \cdot \begin{cases} \frac{s_j}{\varphi} \\ \text{sign} \left(\frac{s_j}{\varphi} \right) \end{cases} \quad (5.21)$$

Being φ a positive constant by definition, it can be stated that:

$$\text{sign}(s_j) = \text{sign} \left(\frac{s_j}{\varphi} \right) \quad (5.22)$$

Hence to discuss about the negativity of $\dot{V}(s_j)$, the simplification $\dot{s}_j = -K_2 \cdot \text{sign}(s_j)$ and Eq. (5.20) can be substituted in Eq. (5.19):

$$\dot{V}(s_j) = -K_2 \cdot \text{sign}(s_j) \cdot s_j = -K_2 \cdot |s_j| \leq 0 \quad (5.23)$$

Finally, choosing a constant positive gain ($K_2 > 0$) for the switching term ensures $\dot{V}(s_j) < 0$, $\forall s_j \neq 0$, hence asymptotic stability for the system.

Simulations and Results

This chapter presents the virtual validation framework and the comparative performance analysis of the proposed wheel slip controllers.

To achieve seamless integration with the IPG Automotive CarMaker simulation environment, specific software adaptation blocks were developed to handle motor torque commands and state variable routing.

Subsequently, the controllers' performance is evaluated and compared through a series of critical maneuvers under varying friction conditions, including abrupt road surface transitions (μ -jumps) and asymmetrical surfaces (μ -split). Finally, the simulation results are presented and analyzed with a specific focus on slip tracking accuracy, motor torque smoothness, and the suppression of powertrain torsional oscillations, highlighting the distinct performance advantages offered by the CISM strategy.

6.1 Simulation Environment

To evaluate the proposed control algorithms under realistic conditions, a high-fidelity simulation environment is established by interfacing MATLAB/Simulink with IPG Automotive CarMaker. Within this framework, CarMaker handles the complex, non-linear physical plant dynamics, including the multi-body chassis, a detailed powertrain configuration, aerodynamic forces, and dynamic tire models, while the complete control logic runs entirely within Simulink.

6.1.1 CarMaker for Simulink

The validation environment is built as a co-simulation architecture using CarMaker for Simulink (CM4SL).

CM4SL establishes a real-time co-simulation loop where the high-fidelity multi-body chassis and tire dynamics run natively within CarMaker, while the customized control algorithms execute entirely inside MATLAB/Simulink. Communication is synchronized at each simulation time step using dedicated S-Functions, allowing the control loop to read real-time kinematic states directly from the CarMaker Data Dictionary (via **Read CM Dict** blocks). Concurrently, the calculated controller torque commands are directly injected back into the virtual powertrain assembly using a Model Plug-in layer (via **Write CM Dict** blocks), overriding the nominal vehicle actuators to close the loop under realistic unmodeled physical constraints.

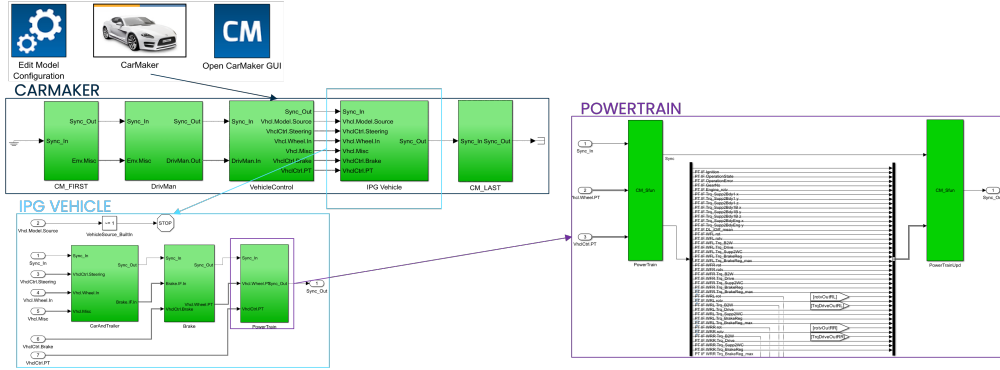


Figure 6.1: CM4SL environment.

Within this co-simulation architecture, the developed slip control logic is embedded directly inside the **IPGVehicle > Powertrain** subsystem. Rather than delegating torque generation to CarMaker's native internal powertrain or driver models, the customized Simulink plug-in bypasses the default actuator commands, overwriting target torque values for the electric motor, whenever the controller state is active.

Simulations within CarMaker4Simulink environment have been carried out with a timestep of 0.001 s.

6.1.2 Actuator Target Assignment and Modified Powertrain Subsystem

Once the core tracking loop within the Traction Control System computes the raw corrective action, the output is routed to the EM Target Assignment layer ($T_{EM}^{req} = \frac{T_{drive}^{req}}{\eta_{EM} \cdot i_g}$). The theoretical controller output is scaled and saturated according to the physical operational boundaries of the electric motors. The validated electromagnetic request is then explicitly assigned to the dictionary variable `PT.Control.Motor.Trq_trg`.

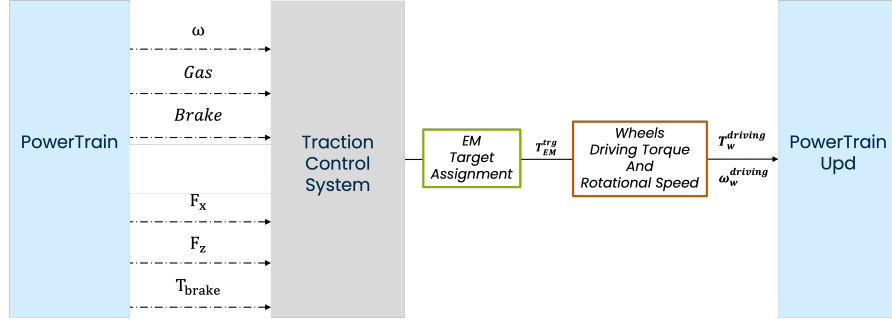


Figure 6.2: Actuator target assignment and Modified Powertrain subsystem

However, to ensure that the overall vehicle plant model seamlessly executes the controller's request, the default torque transmission paths inside CarMaker must be bypassed. This is achieved by routing the torque target through a customized **Modified Powertrain** block configuration located between the native **PowerTrain** and **PowerTrain Upd** interfaces.

the distance between the actual wheels' slipping state and the optimal slip value: control-loop is enabled whenever $\sigma_j > \sigma_{th,j} = 0.9 \cdot \sigma_{opt,j}$, in order to initiate the control action just before entering an unstable condition.

A dedicated decision block continuously reads the functional state of the controller (ON or OFF):

- Controller OFF: The decision block passes the nominal, unmodified variables directly from the default CarMaker Powertrain Interface.
- Controller ON: The loop instantly switches to inject the modified, controller-derived kinetic and kinematic quantities directly into the PowertrainUpd Interface.

By establishing this overriding sequence, the simulation ensures that the high-fidelity chassis model updates its longitudinal tire forces based strictly on the optimized slip control requests, preventing any internal algebraic loops or parameter conflicts from disrupting the co-simulation layer.

6.2 Selected Test Maneuvers and Simulation Scenarios

The scenarios tested to evaluate and compare the performance of the proposed controllers have been selected to replicate operational conditions that typically challenge vehicle stability and traction control systems.

Three distinct test benchmarks have been designed:

Low Friction

Full gas pedal maneuver starting from approximately 10 km/h, followed by a braking maneuver. Road friction coefficient is $\mu_{road} = 0.3$.

Mu-jump

Constant gas pedal ($\approx 70\%$), starting from approximately 10 km/h. Road friction coefficient varies from $\mu_{road,high} = 1$ to $\mu_{road,low} = 0.3$. Speed of the vehicle is not over 20 Km/h when the change in road friction is encountered.

Mu-split

Constant gas pedal ($\approx 70\%$), starting from approximately 10 km/h. Road friction coefficient varies from $\mu_{road,high} = 1$ to $\mu_{R_{road},low} = 0.3$ for the right wheels and $\mu_{L_{road},low} = 0.1$ for the left wheels. Speed of the vehicle is not over 20 Km/h when the change in road friction is encountered.

6.3 Low Friction Maneuver

The first test profile begins with an initial constant-speed cruising phase at $V_{vhcl} \approx 10$ km/h (≈ 2.8 m/s), followed by a 100% full gas pedal step input initiated at approximately 4.5 seconds after simulation starts, demanding maximum available acceleration torque. Finally at $t \approx 14.5$ s a braking demand is commanded.

The road is characterized by uniform low friction coefficient $\mu_{road} = 0.3$, representative of packed snow or extremely wet cobblestones, and the Reference Slip Estimator identifies a constant optimal longitudinal slip peak of $\sigma_{opt} = 0.1038$ for both rear wheels, during the traction phase, switching to a negative sign target $\sigma_{opt} = -0.0976$ during the braking phase.

Results shown in this section refer to the simulation being stopped as soon as $V_{vhcl} = 0.8$ m/s after the braking phase, to avoid loss of information given by poor mathematical definitions fidelity at such low speed.

6.3.1 Traction Control System OFF

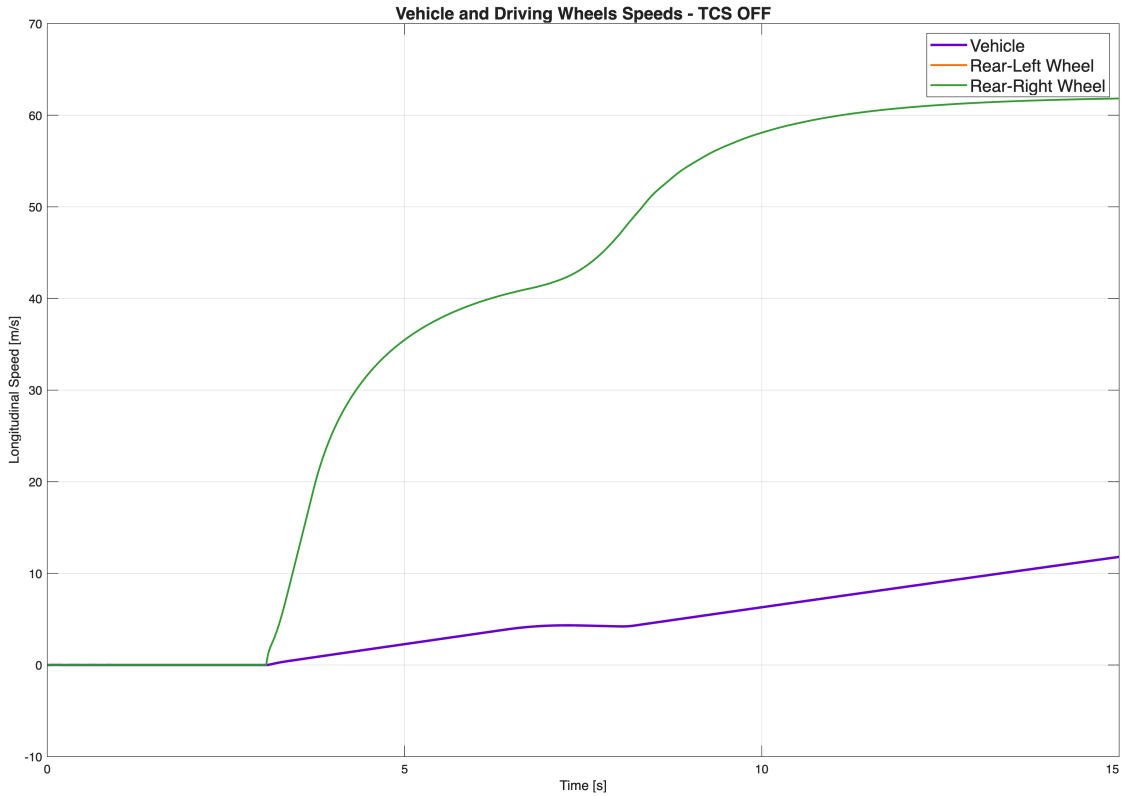


Figure 6.5: Vehicle and driving wheels' speed with traction control OFF.

Without an active traction control system, the full-throttle tip-in causes immediate, severe spin on both wheels.

The wheels accelerate exponentially, peaking at a peripheral speed of over 62 m/s, while the actual vehicle velocity increases at a slower rate due to the loss of effective force transmission.

Braking phase is not initiated due to complete loss of adherence at the tire-ground contact.

Comparing actual longitudinal slip against reference optimal value for both wheels (Fig. 6.6), slip increase is visible as soon as pedal input is applied ($t \approx 4.5$ s): it bypasses the stable linear region and jumps straight to an initial spike reaching full spin ($\sigma = 1$) and maintaining a value above 0.8 afterwards.

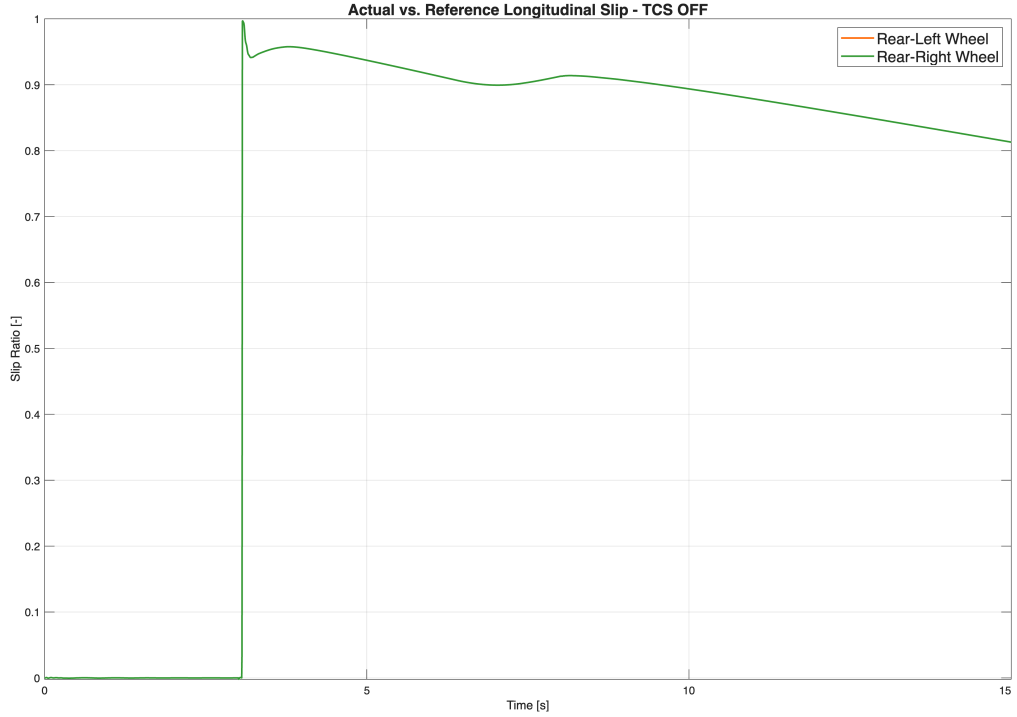


Figure 6.6: Actual vs. reference longitudinal slip with traction control OFF.

The consequences of uncontrolled slip are confirmed by the utilized friction coefficient plot in Figure 6.7, showing how the adherence capabilities of the tire-road interface are never efficiently exploited ($|\mu_{ul}| < |\mu_{road}|$) as the tire is operating deep within the unstable sliding region.

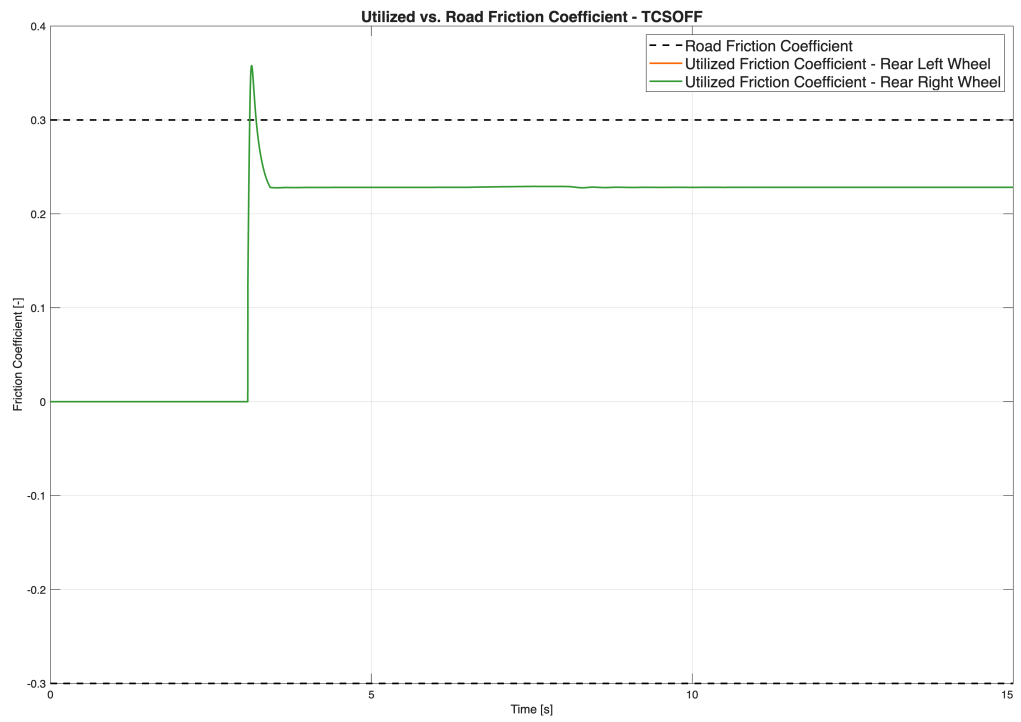


Figure 6.7: Utilized vs. road friction coefficient with traction control OFF.

6.3.2 PI Controller

Figure 6.8 illustrates the binary operational state of the PI activation block. As expected, the controller switches from an idle state (0) to an active state (1) immediately at $t \approx 4.5$ s as the wheel slip crosses the activation threshold ($\sigma_j > \sigma_{th,j}$). The PI controller remains fully engaged throughout both the traction and braking phases.

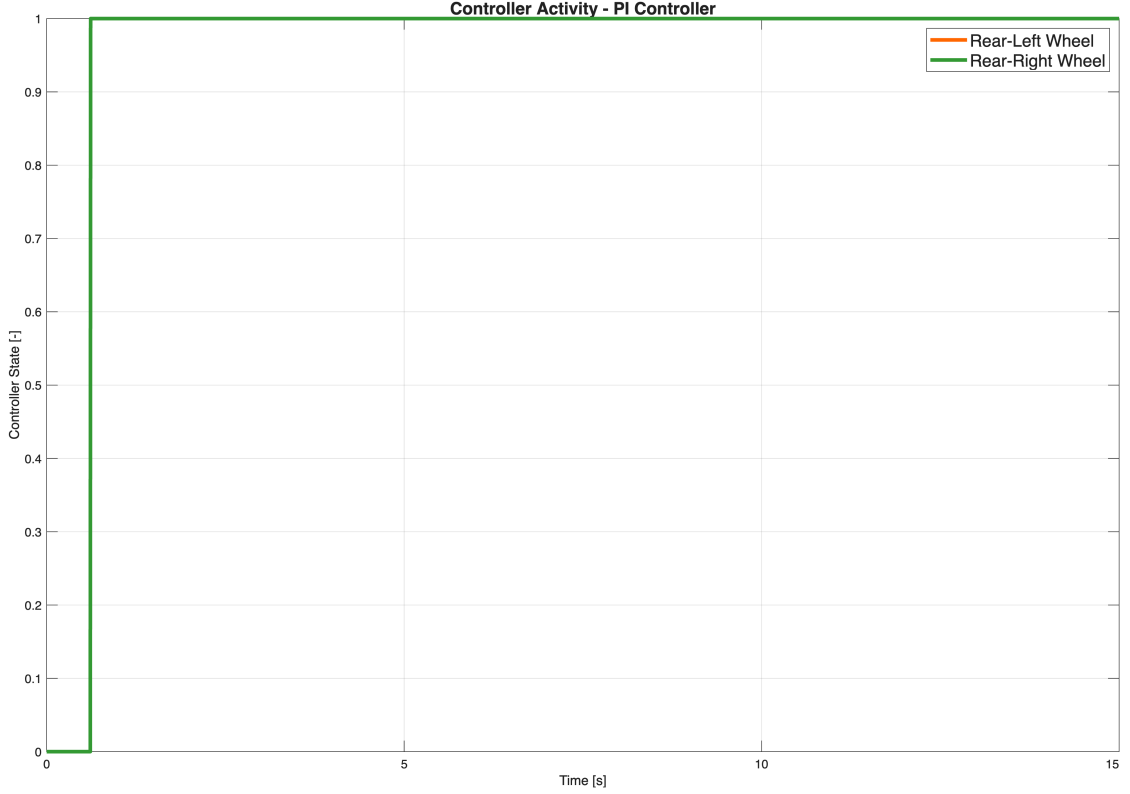


Figure 6.8: PI controller activity.

Under active PI control, the vehicle's longitudinal behavior is significantly improved. As plotted in Figure 6.9, the driving wheel speeds no longer diverge into uncontrolled spin; instead, they are held at a regulated distance above the vehicle speed curve in the V-t plane. This tight velocity coupling matches the target optimal slip (σ_{opt}) velocity profile.

The system remains stable during steady-state acceleration, and the control loop successfully prevents wheel lock-up during the sudden traction-to-braking transition at $t \approx 14.5$ s.

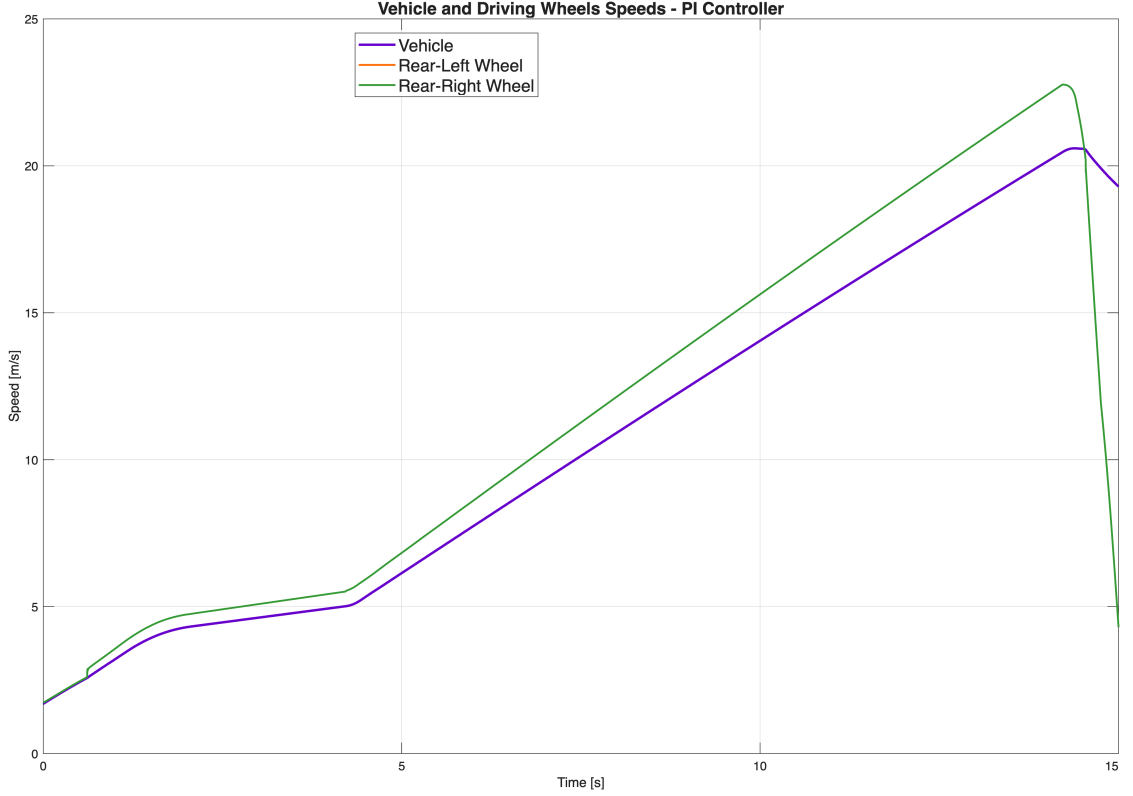


Figure 6.9: Vehicle and driving wheels' speed with PI.

The slip tracking performance is verified in Figure 6.10. The PI controller successfully contains the initial slip spike, bringing the longitudinal slip magnitude to match the optimal value of 0.1038 during the traction phase, and cleanly transitions to -0.0976 during the deceleration phase.

Looking closely at the controller's tracking performance (Fig 6.11), longitudinal slip tracking error is kept to zero for the whole simulation, despite a negligible overshoot to $e_{max} < 0.02$ in correspondence of the transition phase ($t \approx 14.5$). Wheel's rotational speed tracking error ($\omega_j - \omega_{ref,j}$) follows the same trend, with magnitude of overshoot ranging from 0.3 to -0.9 rad/s approximately. This behavior is caused by the physical wind-up of the integral channel before the controller can adapt to the rapid change in torque direction.

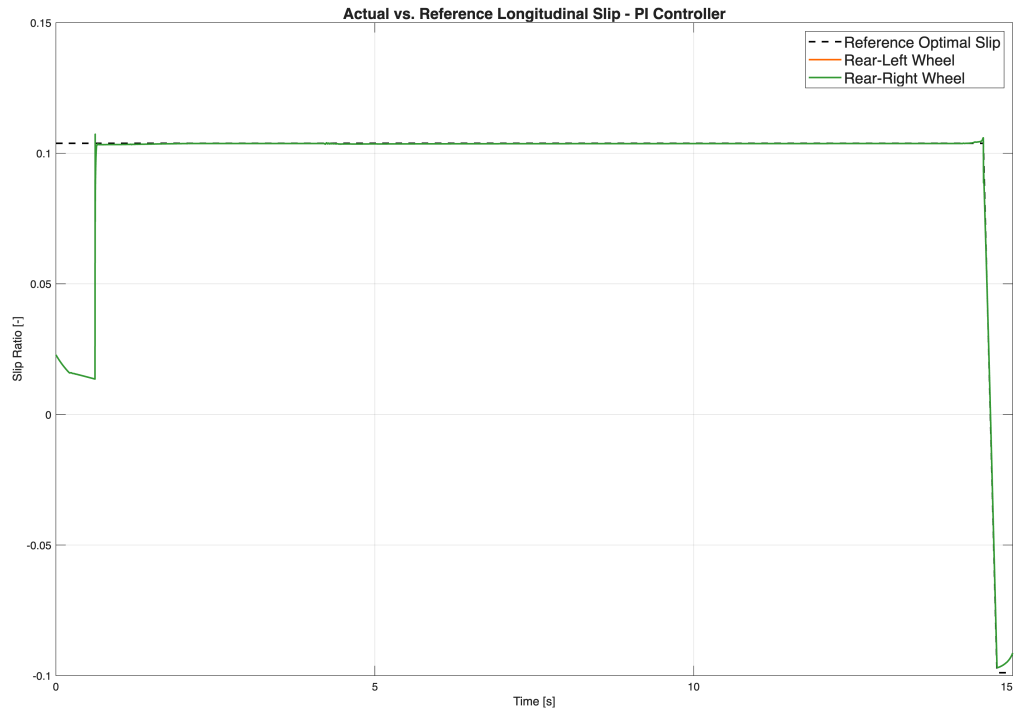


Figure 6.10: Actual vs. reference longitudinal slip with PI.

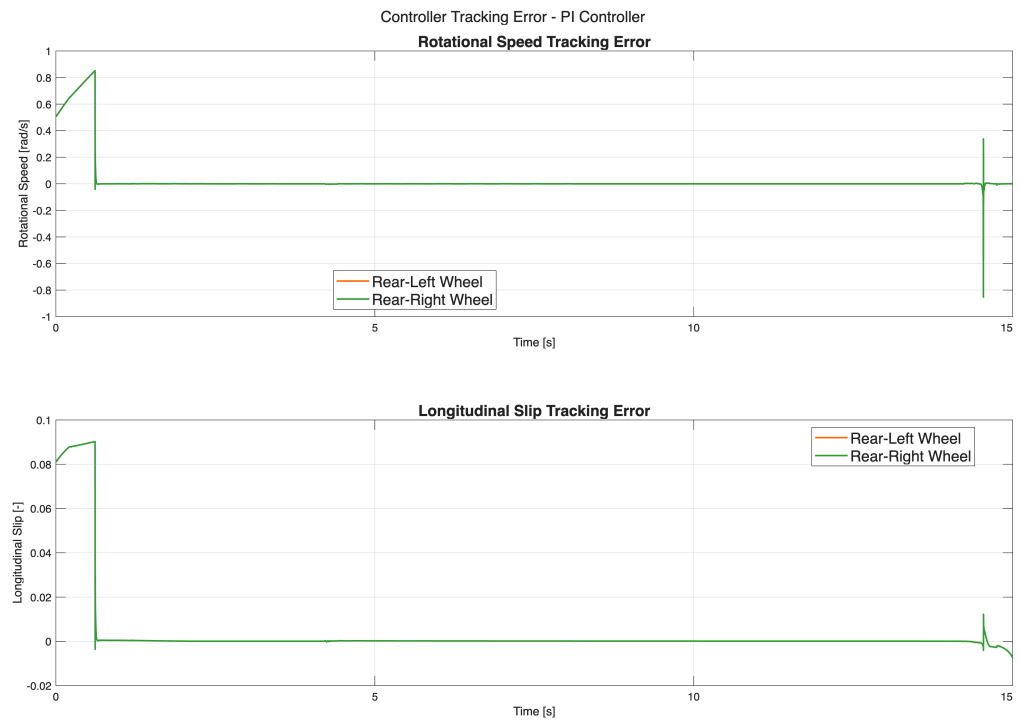


Figure 6.11: PI controller tracking error.

Controller's capability to enhance friction utilization at the tire-ground contact is reported in the Figure 6.12, where $|\mu_{ul}| \geq |\mu_{road}|$ for both wheels throughout the whole controller's intervention. Unlike the TCS OFF configuration, the system fully exploits the available adherence of the tire-road interface during both acceleration and deceleration, drastically improving overall safety and acceleration metrics.

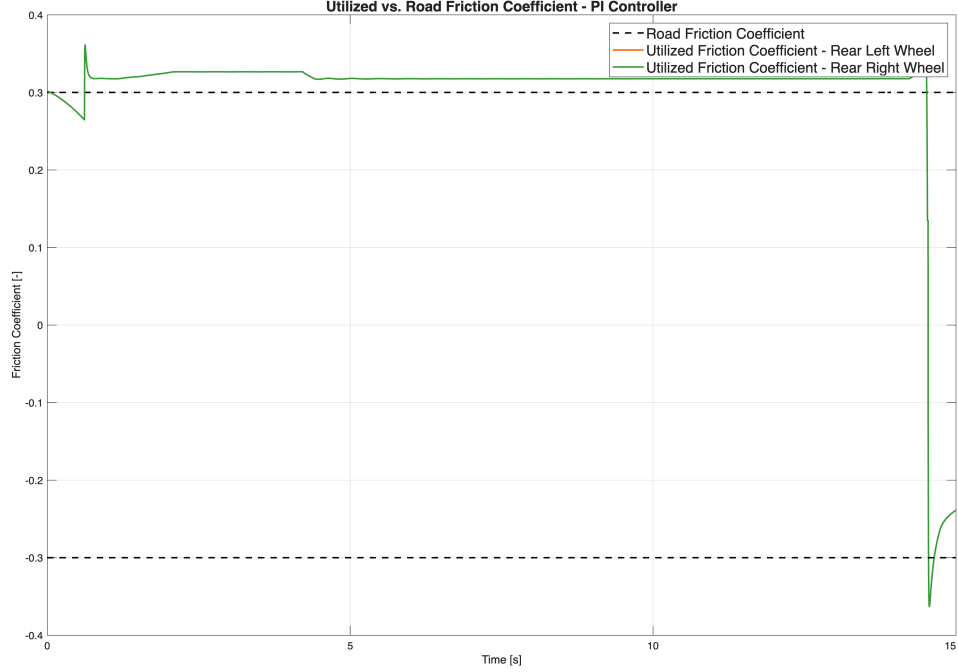


Figure 6.12: Utilized vs. road friction coefficient with PI.

Controller's Output

As previously discussed in Sec 4.4, the control logic outputs a driving torque request at the wheel, which is further manipulated for electric motors target setting.

The calculated torque request generated by the PI control logic is reported in Figure 6.13 and Figure 6.14. The total command torque is mathematically decomposed into the primary Control Torque and an auxiliary Compensation Torque serving as an anti-windup layer.

As highlighted in the zoomed-in version of controller's output plot in Figure 6.14, the compensation torque remains strictly zero during regular operations. Even, during the extreme reference sign inversion at $t \approx 14.5$ s, the control torque never hits its saturation bounds, hence the anti-windup compensation loop doesn't activate.

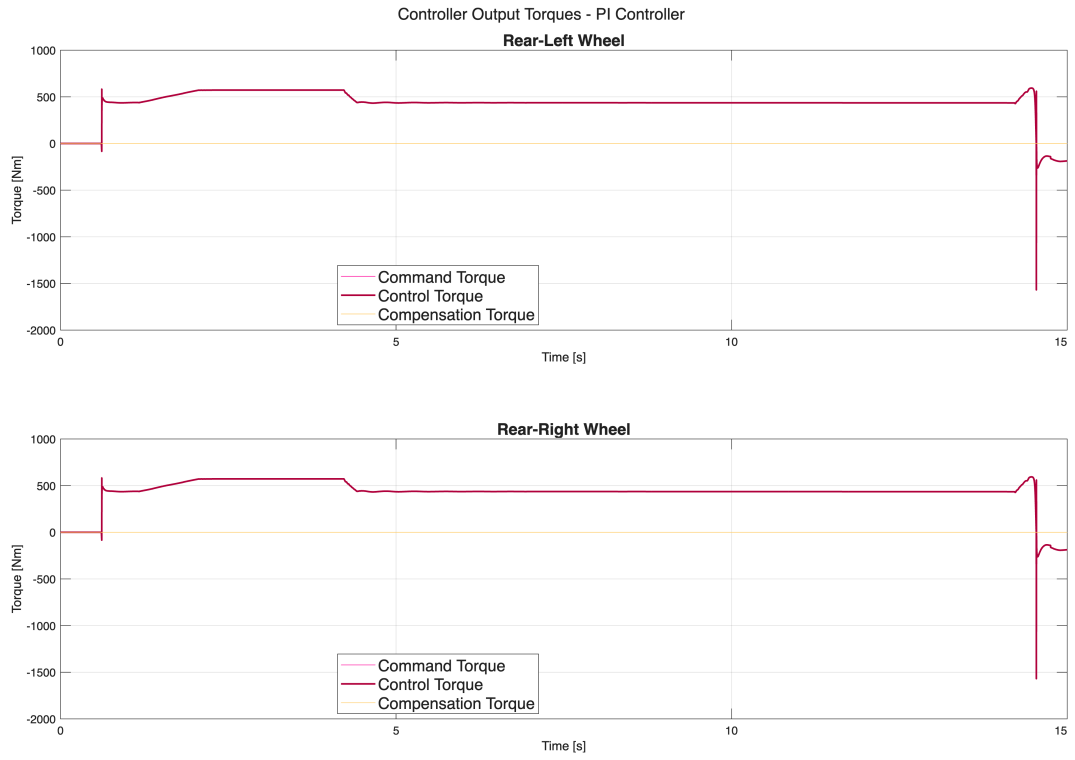


Figure 6.13: PI controller output torques.

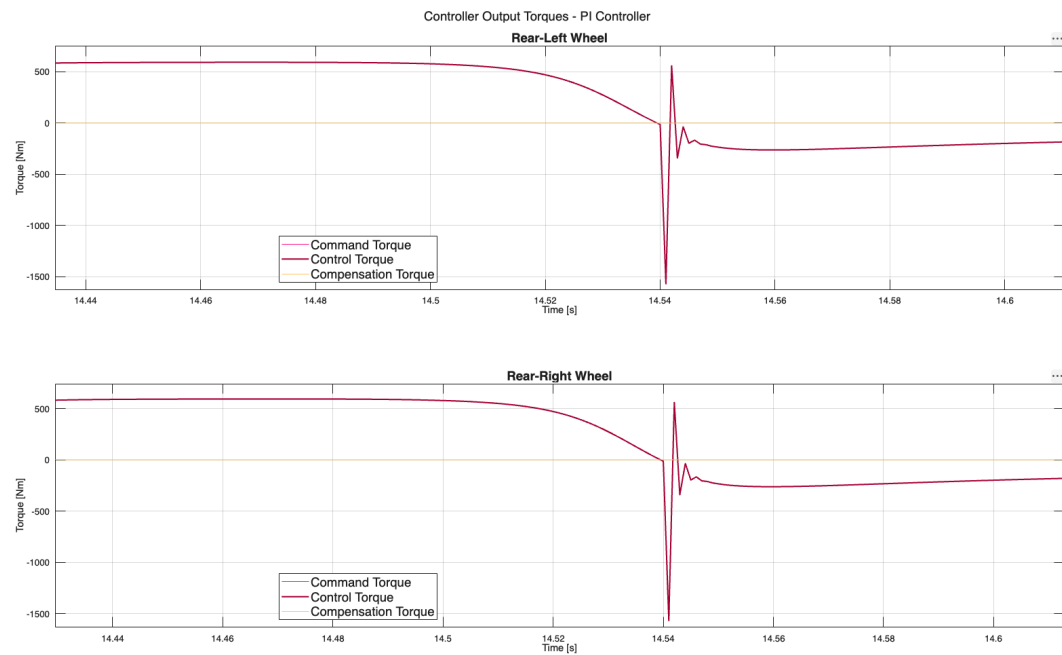


Figure 6.14: PI controller output torques, detail.

Finally, target electric motor torque commands and actual physical motor torque outputs are detailed in Figure 6.15.

The resulting target torque for the electric motor is a smooth steady profile, and is easily tracked by the EM torque.

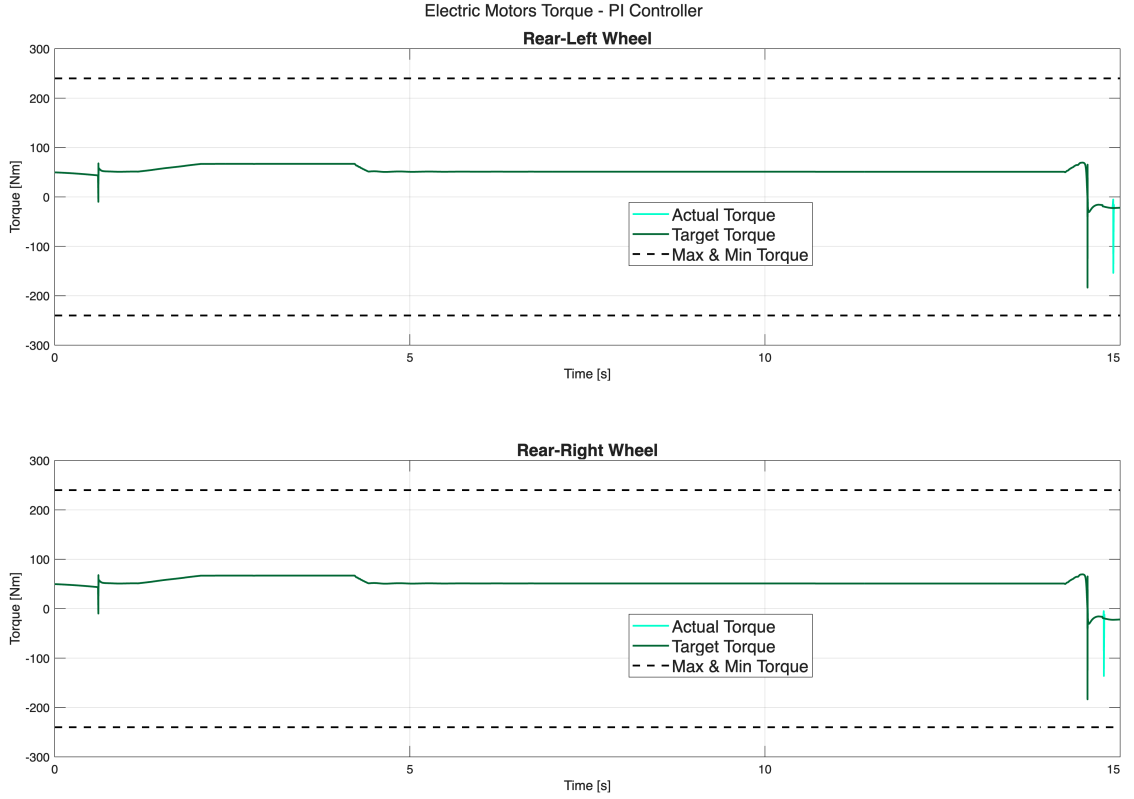


Figure 6.15: Electric motors torque with PI controller.

However, during the torque reversal phase at $t \approx 14.5$ s, the reference signal demands an instantaneous change in sign: this step change forces the target torque to change at a frequency out of the maximum physical motor envelope, hence the actual motor torque tries to track this extreme demand with an instantaneous step response, according the hardware capability limits, inevitably resulting in a short-time oscillation, before stabilizing into a steady braking profile.

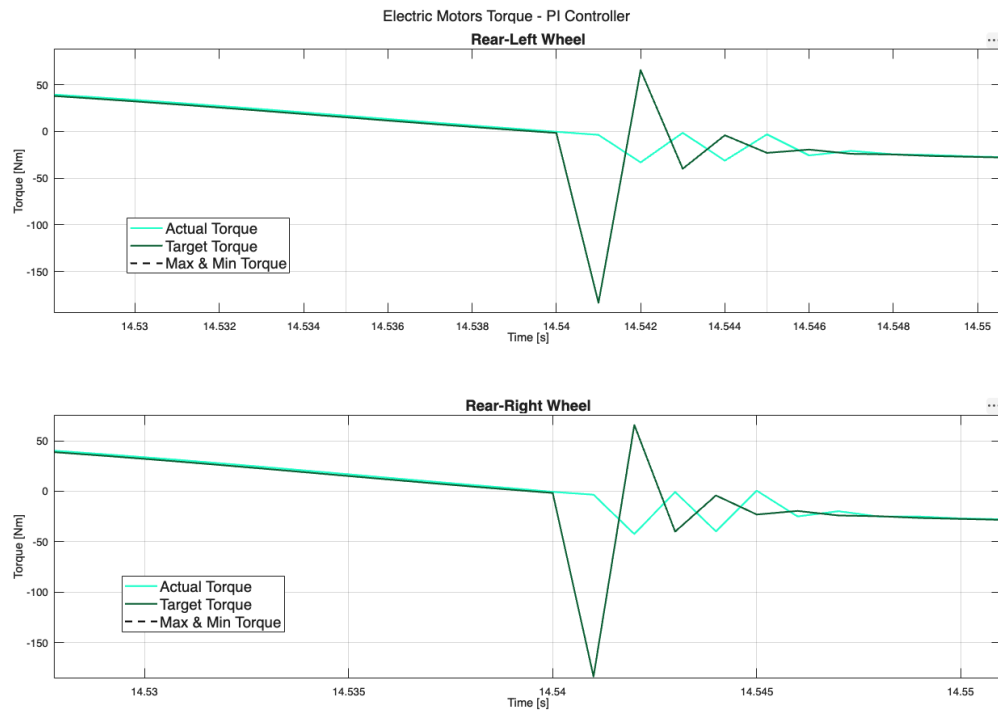


Figure 6.16: Electric motors torque with PI controller, detail.

6.3.3 TSM Controller

The primary objective of this architecture is to exploit the robust, non-linear discontinuous reaching laws of sliding mode theory to ensure faster transient response and absolute invariance to model uncertainties compared to the baseline PI controller.

The controller utilizes an identical switching threshold mechanism to the PI layout, mapping the idle state to 0 and full engagement to 1. The loop triggers instantaneously at $t \approx 1$ s, remaining active throughout the entire high-slip regime.

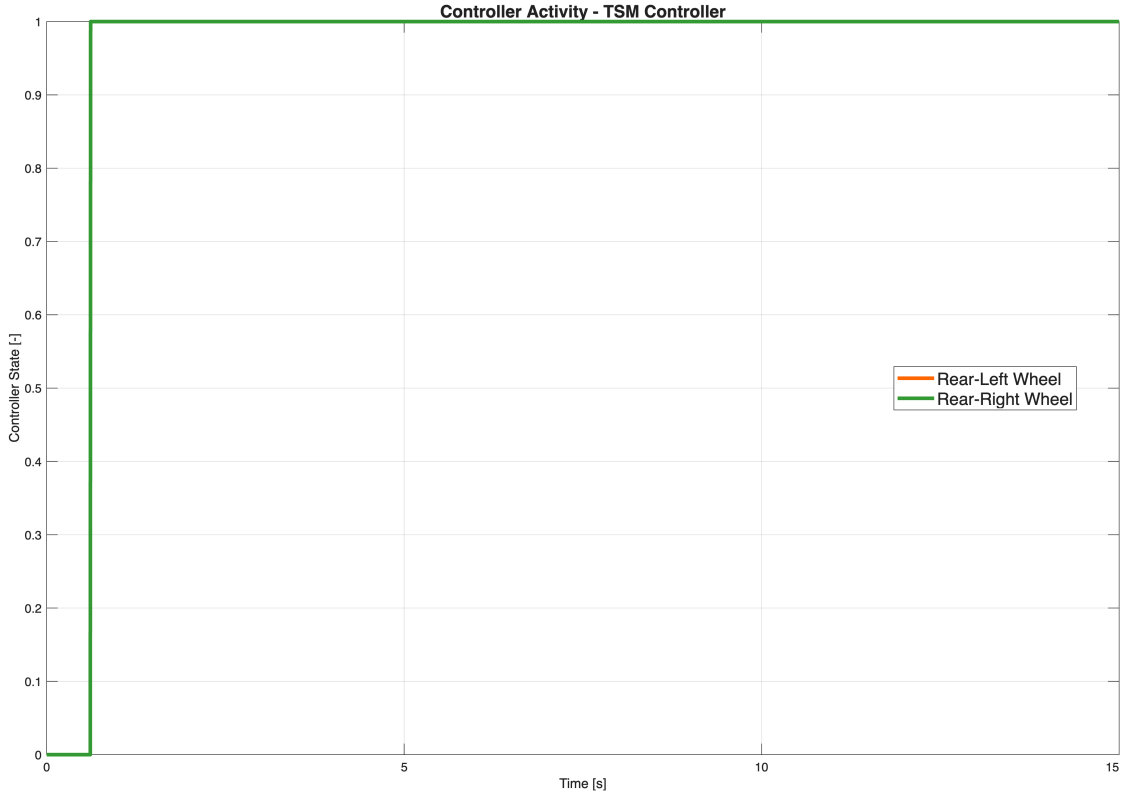


Figure 6.17: TSM controller activity.

Figure 6.18 illustrates the vehicle longitudinal speed and the tangential wheel velocities.

The application of the non-linear switching manifold completely alters the transient response compared to the PI baseline.

The TSMC limits the initial spin surge instantaneously, locking the wheel velocity tightly onto the tracking line prescribed by the optimal reference slip.

However, a close inspection of the wheel speed curves during steady-state traction reveals high-frequency, low-amplitude oscillations directly superimposed onto the velocity trajectories.

This kinematic ripple is the physical manifestation of high-frequency actuator switching.

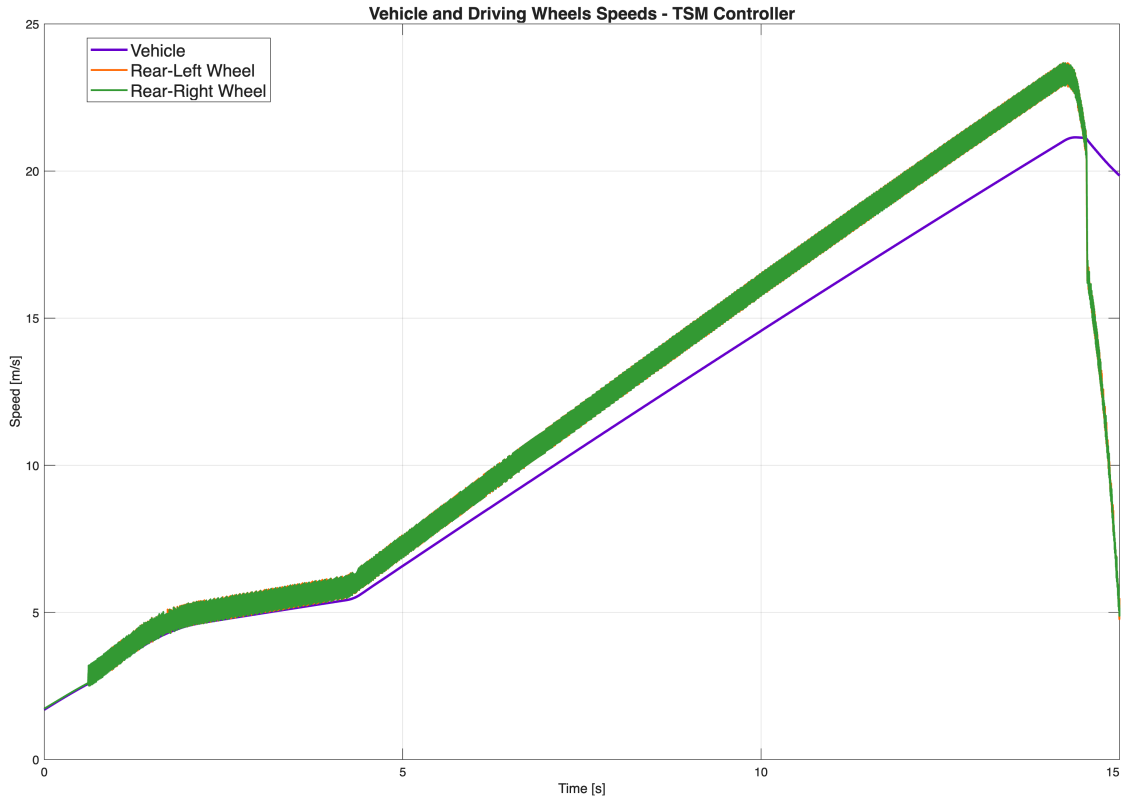


Figure 6.18: Vehicle and driving wheels' speed with TSMC.

This tracking behavior is further confirmed by the longitudinal slip development plotted in Figure 6.19.

TSMC demonstrates a remarkably steep reaching phase, forcing the longitudinal slip to hit the optimal reference boundary ($\sigma_{opt} = 0.1038$) within milliseconds, even during the aggressive torque reversal at $t \approx 14.5$ s, where the slip profile switches instantly to match the negative reference.

Unlike the PI controller, which suffered from integral wind-up lag, the TSMC crosses the zero-slip line with practically zero transient delay. Despite this exceptional speed, the slip never fully stabilizes at a smooth constant value; instead, it exhibits continuous high-frequency chatter directly around the reference optimal slip curve.

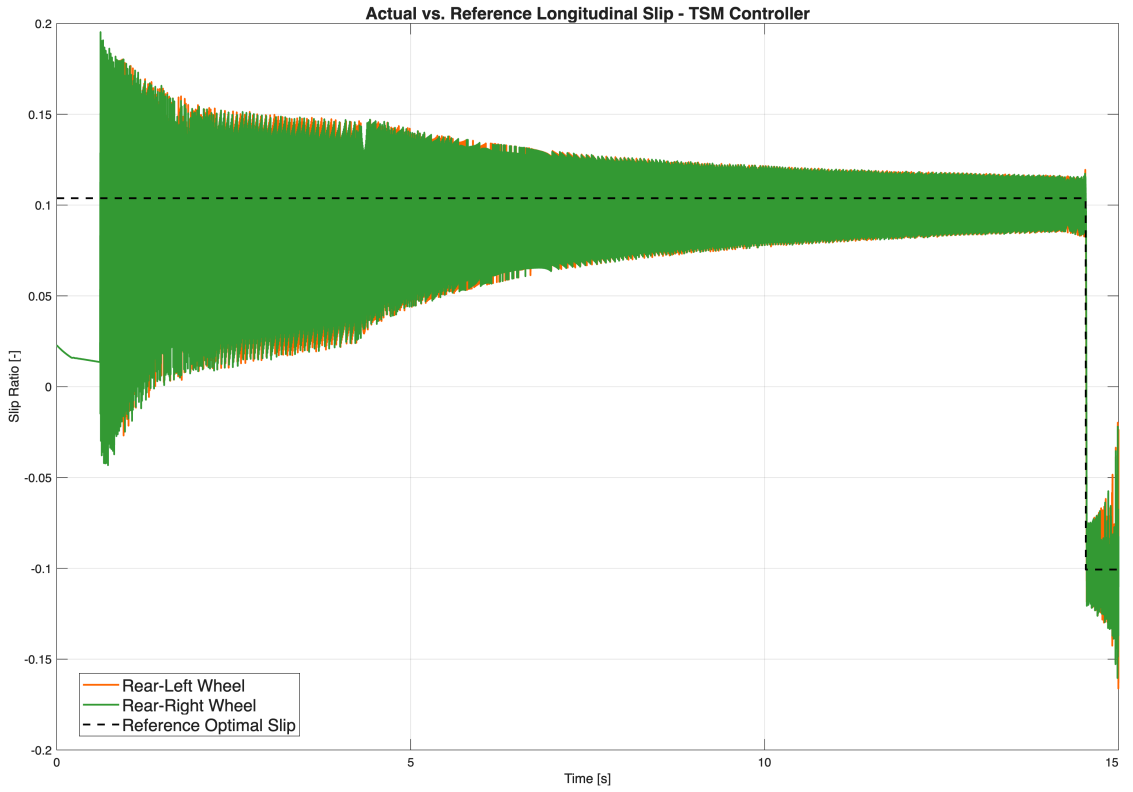


Figure 6.19: Actual vs. reference longitudinal slip with TSMC.

To quantify this tracking capability, Figure 6.20 isolates the raw longitudinal slip tracking error.

The tracking error metrics highlight the trade-offs of the traditional sliding mode approach.

During the severe traction-to-braking step transition at $t \approx 14.5$ s, the peak slip tracking error magnitude is again approximately 0.2.

Because the traditional sliding control law employs a discontinuous sign function ($sign(s)$), the tracking error does not asymptote to a clean zero. Instead, it enters a continuous, high-frequency chattering around the zero-error axis, with a persistent average residual oscillation amplitude of approximately ± 0.05 .

Moreover, it's visible how the intensity of chattering is inversely proportional to vehicle speed. While at low speeds ($V_{vhcl} < 5$ m/s) the maximum error amplitude is in the order of 0.14, at high speeds ($V_{vhcl} > 12$ m/s) the system is more stable, guaranteeing a lower amplitude of oscillations ($e_{max} \approx 0.02$).

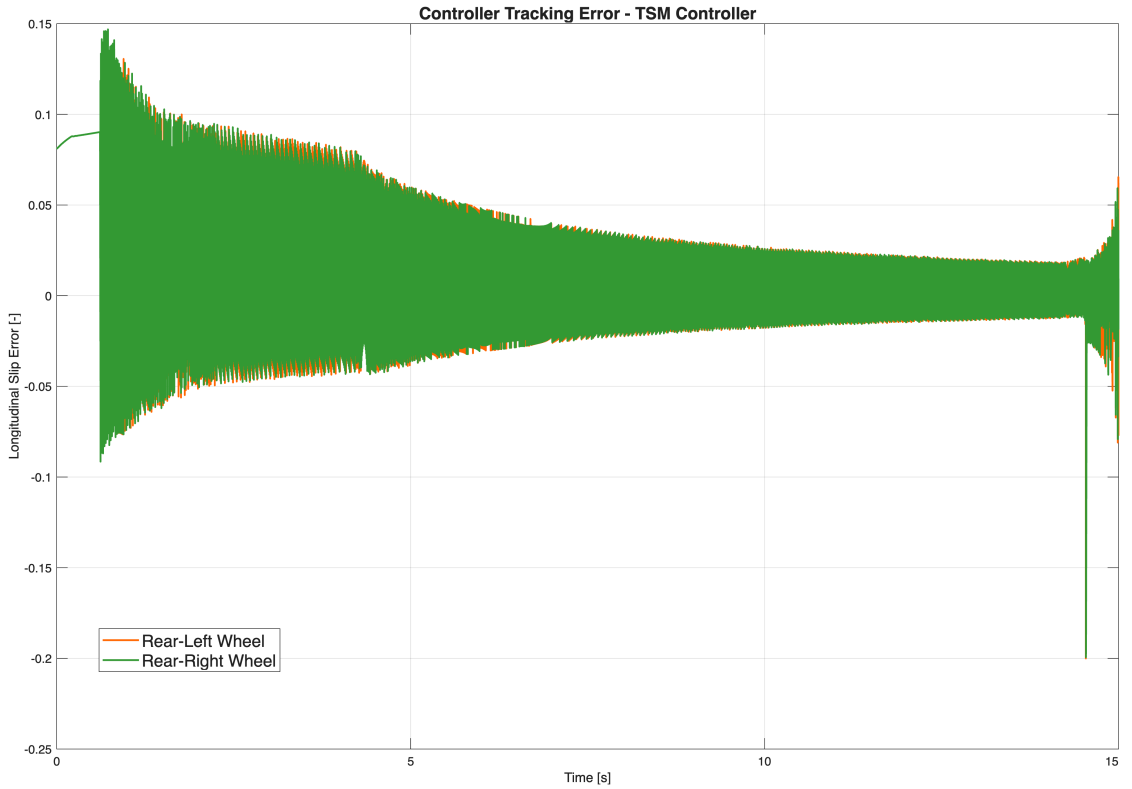


Figure 6.20: TSM controller tracking error.

As reported in Figure 6.21, ensuring that the longitudinal slip is restricted to the peak zone ($\sigma = 0.1038$), the TSMC maximizes force transmission, maintaining the utilized friction coefficient close or higher than the road nominal one.

Utilized friction coefficient magnitude is always higher than road theoretical friction's one, and also higher than in the PI case, confirming a higher capability of the TSM controller of maximizing adherence at tire-ground contact.

The rapid tracking response during the torque reversal guarantees that the tire contact patch switches from maximum acceleration force to maximum braking force almost instantaneously. However, matching the slip fluctuations, the calculated friction forces exhibit high-frequency noise, which can introduce mechanical vibrations.

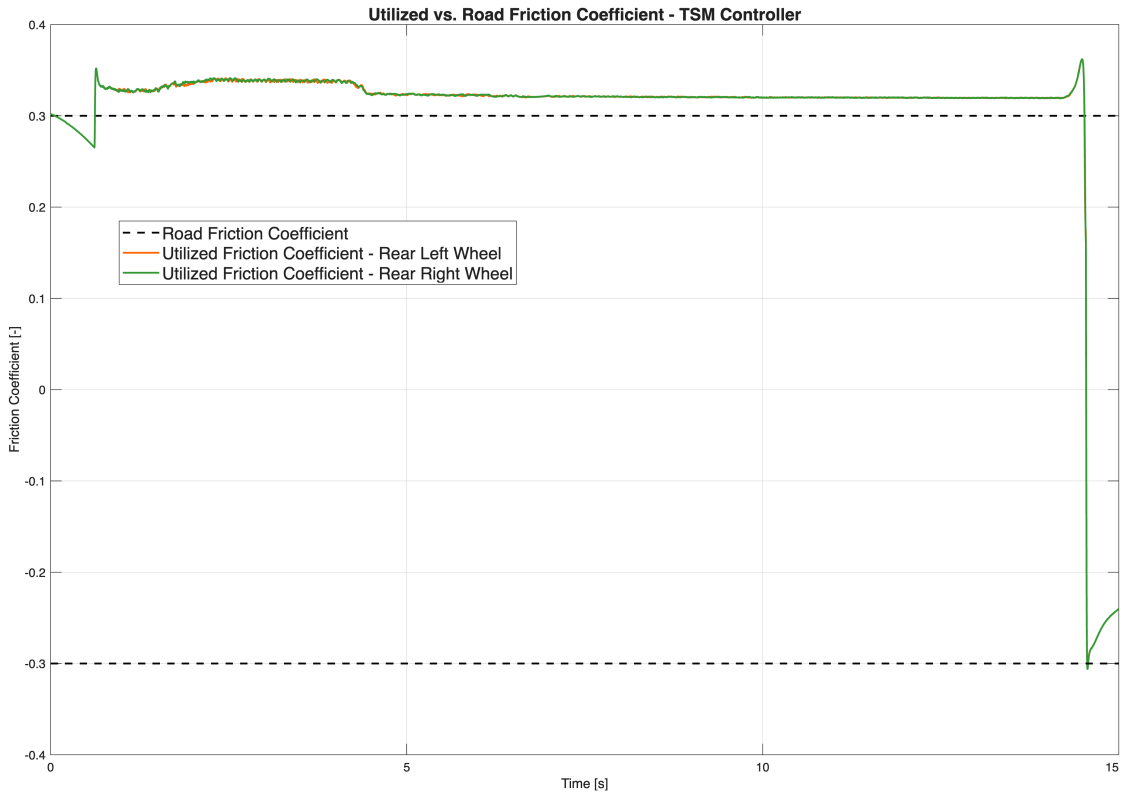


Figure 6.21: Utilized vs. road friction coefficient with TSMC.

Controller's Output

The underlying cause of the high-frequency kinematic fluctuations is identified by examining the controller output variables. Figure 6.22 illustrates the computed command torque request.

To counteract unmodeled drivetrain dynamics and external disturbances, the switching gain must be set to a high value. Consequently, the controller command signal switches back and forth between its extreme operational limits.

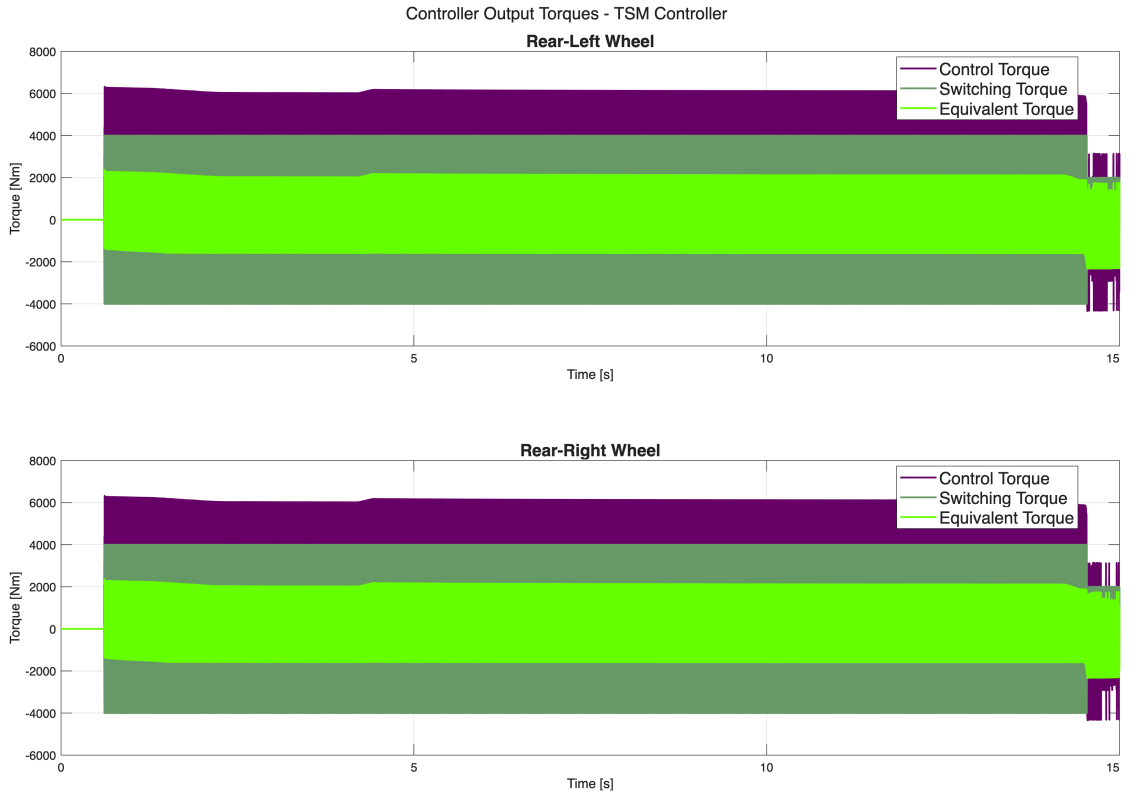


Figure 6.22: TSM controller output torques.

This severe chattering behavior is continuously sustained during both the steady-state traction phase ($t = 5 \text{ s} \div 14 \text{ s}$) and the braking phase ($t = 14 \text{ s} \div 15 \text{ s}$).

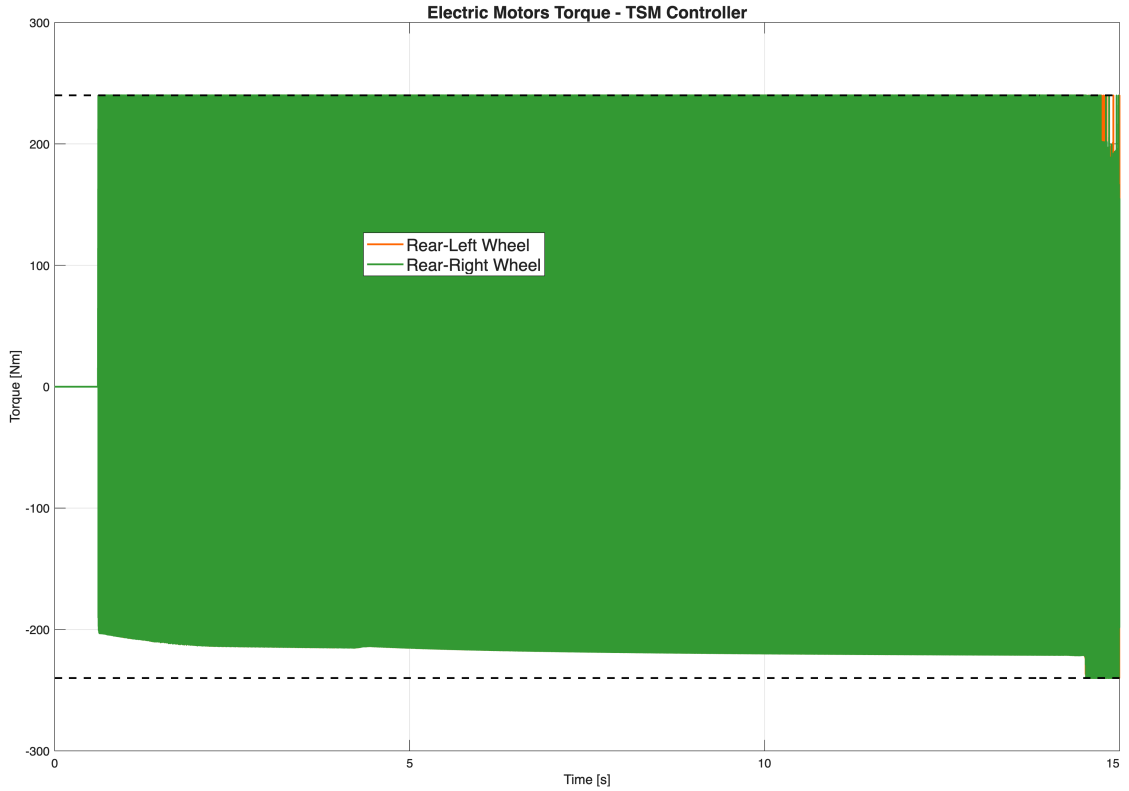


Figure 6.23: Electric motors torque with TSM controller.

Figure 6.23 displays the target electromagnetic motor torque command set for the electric motors.

Because modern permanent magnet synchronous motors possess high-bandwidth power electronics, the physical actuator actively attempts to execute this high-frequency chattering profile.

As a result, the actual motor torque output continuously oscillates over a massive envelope. While this switching enforces absolute mathematical robustness on the slip tracking layer, it is physically unacceptable in a production vehicle.

In a real powertrain layout, high-frequency torque chattering of this magnitude would excite the torsional resonant modes of the drive shafts, generate severe acoustic noise, accelerate mechanical fatigue of the reduction gears, and cause high thermal losses in the power inverter stages.

Controller's Stability

To confirm the theoretical stability proofs discussed in Section 4.5.3 with direct numerical simulation, Figure 6.24 illustrates the real-time tracking of the sliding

manifold surface s alongside its corresponding computed time derivative $\dot{V}(s) = s \dot{s}$ for both rear wheels.

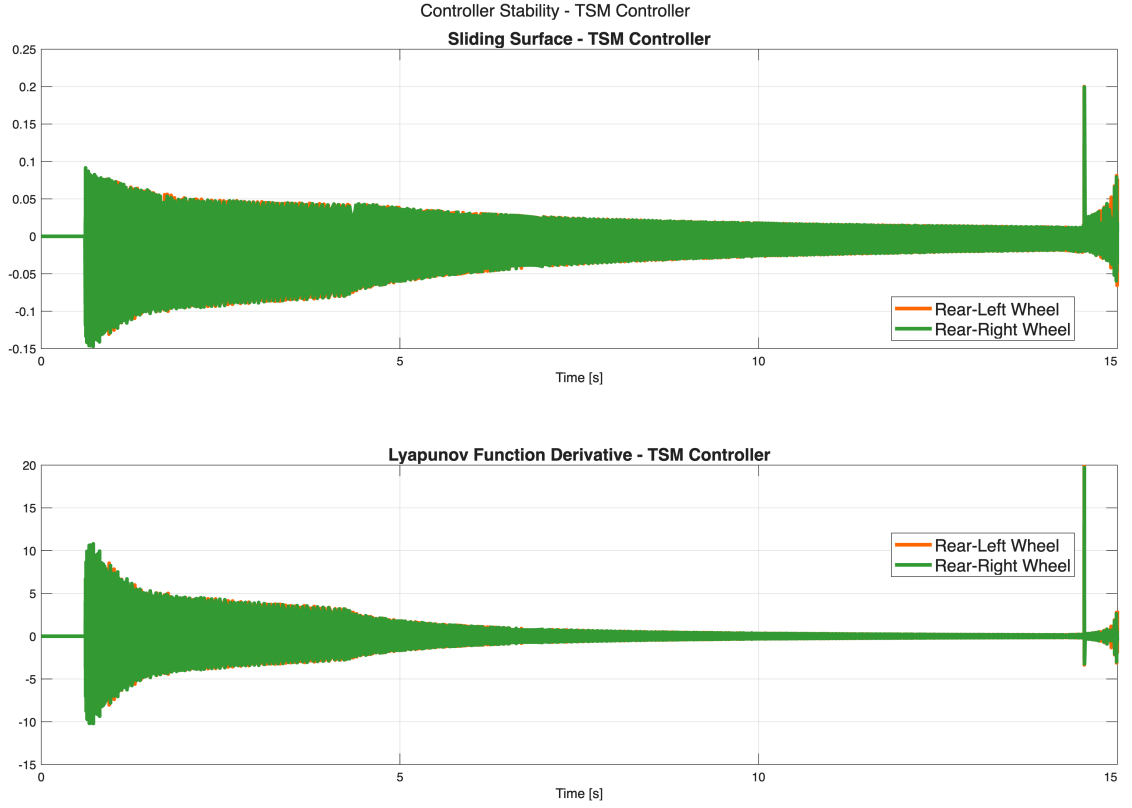


Figure 6.24: TSM controller's stability analysis.

Before controller's intervention, both variables obviously rest at zero. The severe torque demand caused by gas pedal input induces an instantaneous perturbation in wheel kinematics, forcing the sliding surface variable to spike away from the ideal manifold ($s = 0$). Concurrently, the Lyapunov derivative exhibits an immediate, sharp negative impulse, plunging deeply into the negative-definite region.

This behavior strictly validates the reaching condition: it proves that the non-linear control law reacts to the tracking error by generating an immediate corrective action that forces the system trajectory back toward the sliding surface.

After this reaching phase, rather than settling smoothly at absolute zero, both s and $\dot{V}$ exhibit continuous, high-frequency chattering around the zero axis. Because the controller relies on a discontinuous $sign(s)$ term, it overcorrects at every sampling interval. Consequently, the sliding surface oscillates rapidly across zero with a bounded, residual amplitude.

Matching this behavior, the Lyapunov derivative oscillates rapidly in phase with

the switching control law. Because $\dot{s}$ switches signs every time s crosses zero, their product chatters symmetrically around the zero line.

While the system remains macroscopically stable and bounded within this tight limit cycle, this continuous chattering represents high-frequency energy that directly causes the torque oscillations seen in the actuator outputs.

6.3.4 CISM Controller

The core objective of this advanced architecture is to overcome the physical limitations of the previous two setups, aiming to match the instantaneous, robust tracking response of the TSMC while completely eliminating the high-frequency actuator chattering that compromises powertrain integrity.

The operational profile of the CISM loop is shown in Figure 6.25

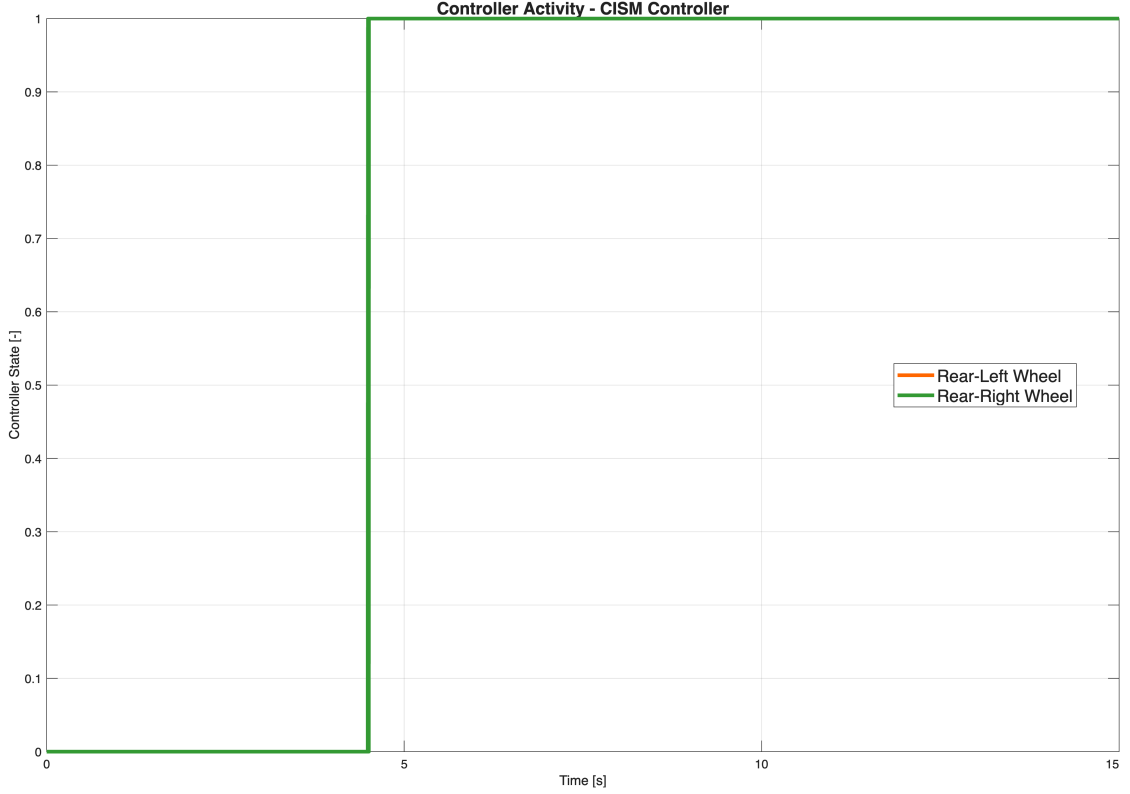


Figure 6.25: CISM controller activity.

Mirroring the previous control configurations, the threshold switch registers an immediate transition from an idle state (0) to full enforcement (1) at $t \approx 4.5$ s when the driver's full-throttle step input occurs.

The controller remains continuously active, handling the localized slip dynamics through both traction and braking regimes.

Figure 6.26 shows vehicle longitudinal speed alongside individual rear wheels tangential velocities.

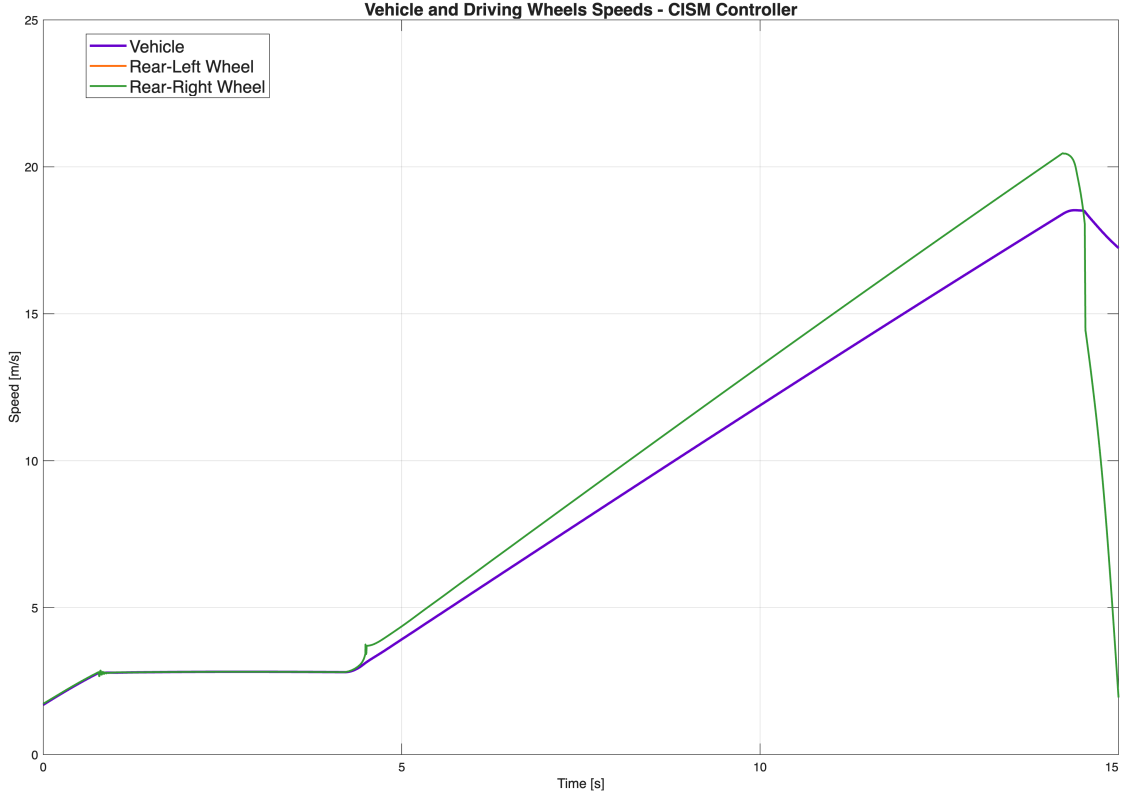


Figure 6.26: Vehicle and driving wheels' speed with CISM.

Speed profiles are altered by the introduction of the conditional integral sliding surface.

At $t \approx 4.5$ s, the rear wheels velocities rise smoothly above the vehicle velocity baseline with almost zero overshoot or transient spin spikes.

Crucially, unlike the TSMC speed profiles, the velocity lines under CISM are perfectly smooth and continuous during steady-state acceleration ($t = 5$ s $\div$ 14 s). The high-frequency ripple is entirely suppressed, demonstrating that the boundary layer smoothing successfully translates to clean kinematic wheel behavior.

This tracking capability is confirmed by the longitudinal slip development reported in Figure 6.27.

The CISM C achieves excellent tracking performance. When the full throttle step is applied, the actual slip converges exponentially onto the optimal target ($\sigma_{opt} = 0.1038$), despite an instantaneous overshoot to 0.17. Once the sliding manifold is reached, the slip remains tightly locked on the target reference without any of the high-frequency oscillatory chatter seen in the TSMC.

During the sharp torque inversion at $t \approx 14.5$ s, the tracking remains stable. Thanks to the conditional integration rule, the integral term is frozen the moment the actuator saturates during the rapid torque reversal, preventing integration wind-up. When the system exits saturation to track the braking target ($\sigma_{opt} = -0.0976$), the slip converges cleanly to the new value with negligible lag and zero undershoot.

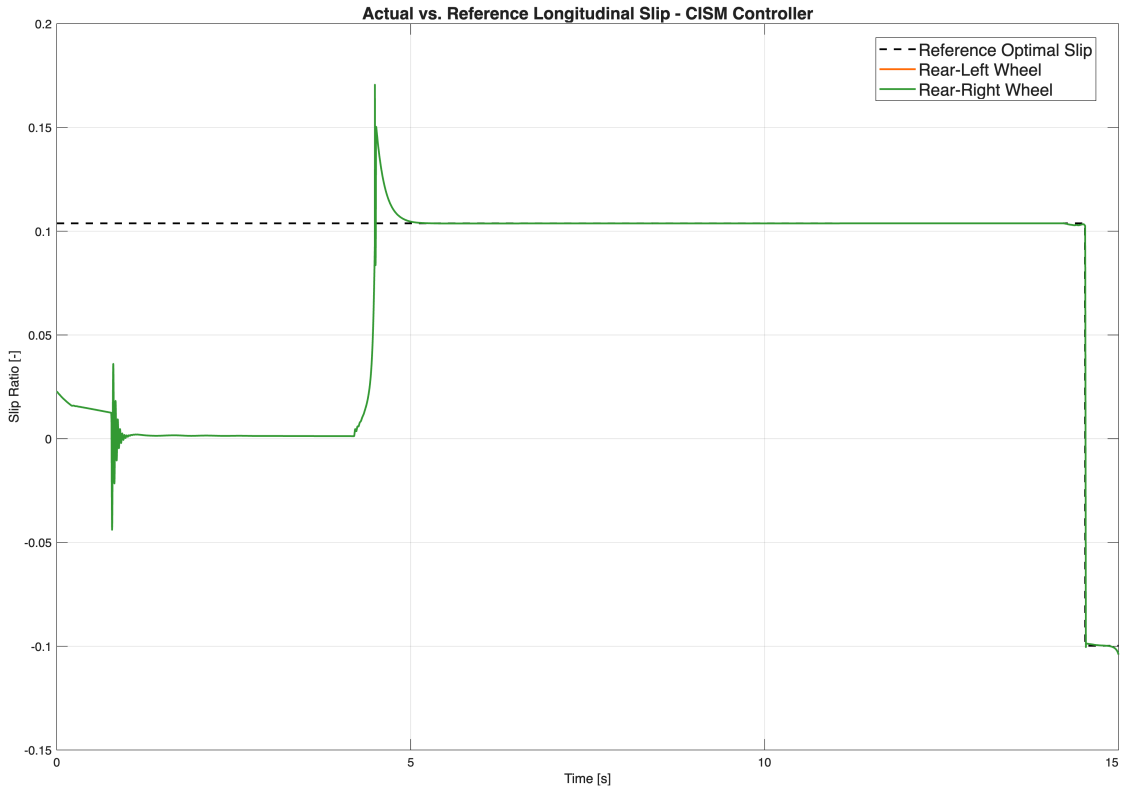


Figure 6.27: Actual vs. reference longitudinal slip with CISM C.

Because the controller stabilizes the tire slip ratio precisely at the absolute peak of the road profile ($\sigma = 0.1038$), as shown in Figure 6.28, utilized friction coefficient is held continuously close to and above its theoretical physical maximum.

By avoiding the tracking fluctuations of the PI and the high-frequency chattering forces of the TSMC, the CISM C maintains a steady force application at the contact patch, ensuring predictable, maximized acceleration performance.

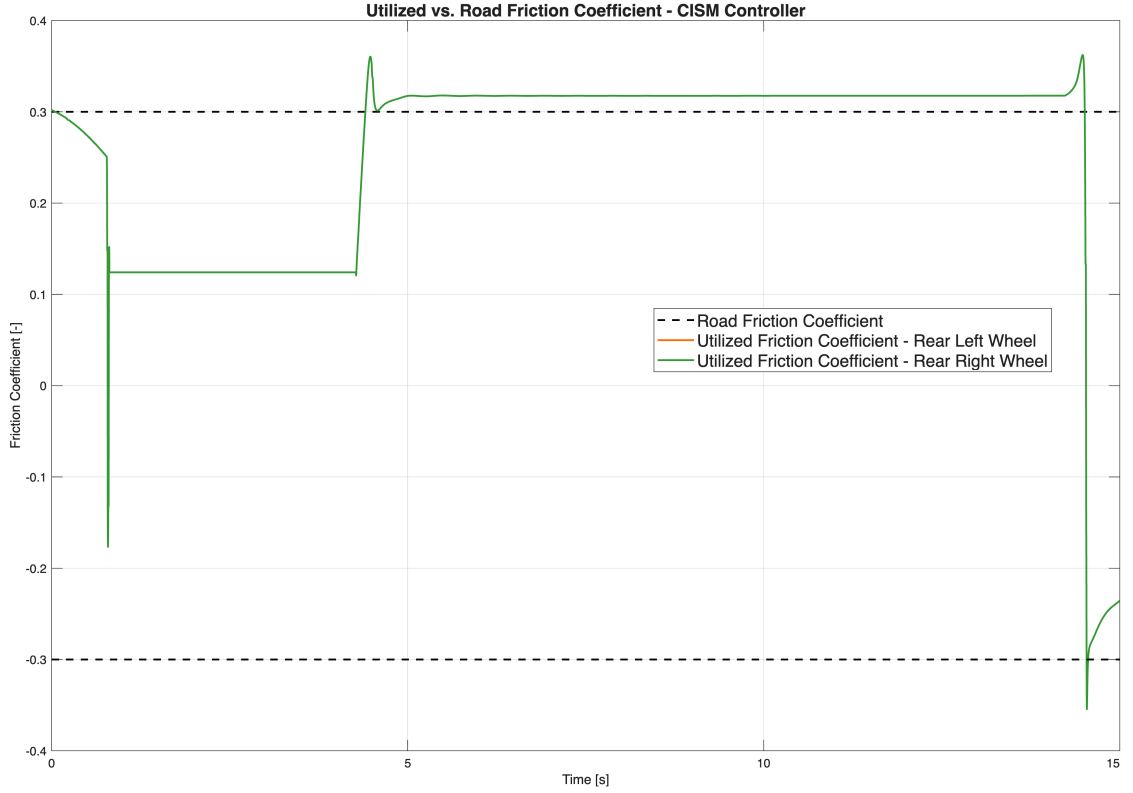


Figure 6.28: Utilized vs. road friction coefficient with CISM C.

Controller's Output

Chattering elimination can be visualized by analyzing the controller's internal outputs of Figure 6.29.

As discussed in Section 5.3, the control input, u_{ref} , corresponds to a driving torque request at the wheel axle. This signal is saturated accordingly to the physical limit for maximum driving torque (see Eq. (5.17)) and translated into a target torque command for the electric motor, as seen in detail in Section 6.1.2.

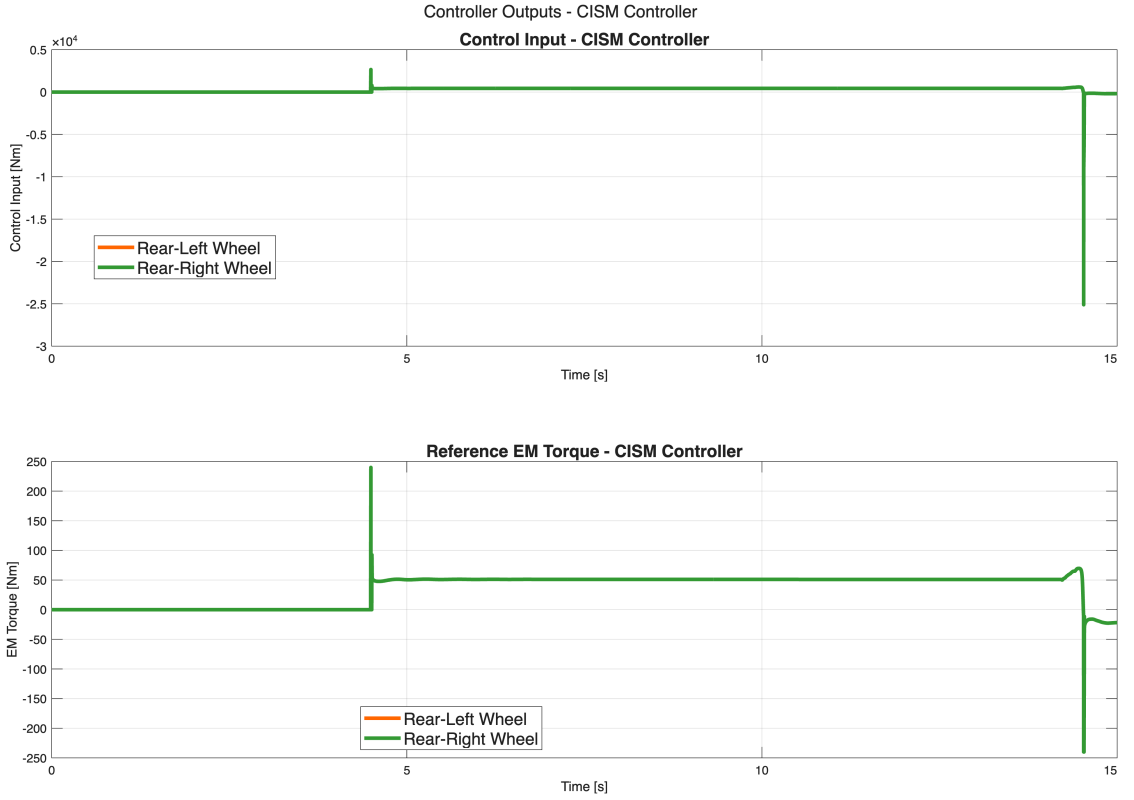


Figure 6.29: CISM controller's output torques.

As explained in Section 5, the CISM logic avoids chattering by replacing the discontinuous sign function with a continuous saturation function inside a predefined boundary layer (φ). The output torque profile displayed in detail in Figure 6.30 reflects this design, providing clean, smooth changes in torque demand during both the steady acceleration and steady braking regimes.

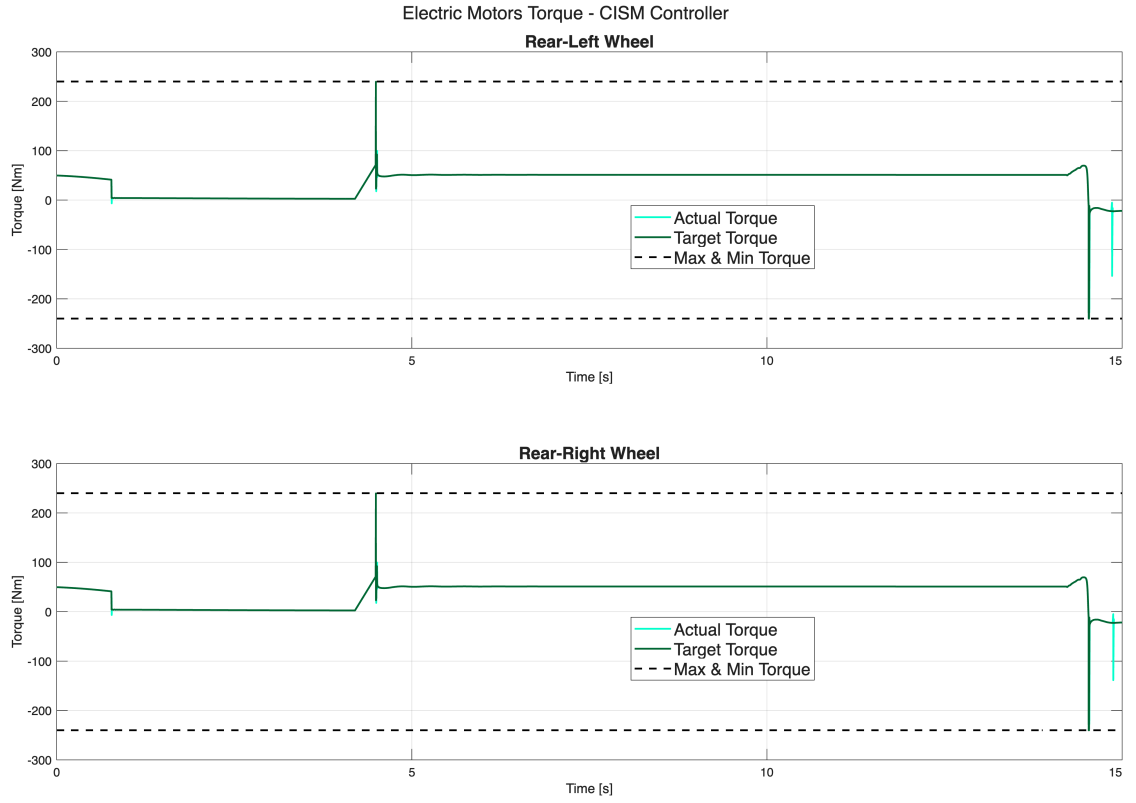


Figure 6.30: Electric motors torque with CISM controller.

The target torque command sent via the `PT.Control.Motor.Trq_trg` is entirely free of high-frequency switching noise.

During traction ($t = 5 \text{ s} \div 14 \text{ s}$), the torque command remains a continuous signal below 100 Nm per wheel.

During the traction-to-braking transition at $t \approx 14.5 \text{ s}$, the controller demands a smooth, rapid drop to full negative braking saturation before settling into a stable deceleration torque profile. Since the command signal is smooth, the actual physical torque delivered by the in-wheel motors matches the target exactly.

Controller's Stability

To evaluate how the CISMC resolves the physical limitations of the traditional formulation, Figure 6.31 and 6.32 isolates the internal components of the system inverse dynamics in the control law (functions f and g), and the time-domain evolution of the sliding surface s .

Considering the operational time window within which the controller's state is ON ($t \gtrapprox 4.5$), despite the initial sudden torque surge perturbing the wheel plant kinematics, causing a brief, localized deviation, the sliding surface is held at zero. Because the CISMC replaces the discontinuous sign function with a continuous saturation interpolation function inside a predefined boundary layer, the control law smoothly handles the perturbation, forcing the tracking error to converge exponentially back toward the ideal trajectory ($s = 0$) within milliseconds of the initial tip-in.

By allowing the control law to interpolate continuously within the boundary layer, the high-frequency switching phenomenon is completely eliminated. Within this layer, the system behaves similarly to a high-gain linear controller where the conditional integral term continuously estimates and compensates for the unmodeled dynamics and external disturbances. This structural change ensures that the sliding surface tracking remains smooth and precise, preventing the high-frequency ripple from reaching the wheel speed trajectories.

During the rapid transition from full acceleration to an aggressive braking demand at $t \approx 4.5$ s, the reference slip target shifts instantly from a positive to a negative value. This step change creates a large tracking error, causing s to experience a transient spike.

The sliding variable briefly exits the boundary layer ($|s| > \varphi$), which automatically switches the controller back to its robust sliding mode reaching law ($\text{sign}(\frac{s}{\epsilon})$) to drive the error down rapidly.

Simultaneously, the conditional logic detects that the motor actuators have hit their physical saturation limits and temporarily freezes the integral state. This prevents integration wind-up and ensures that as soon as s is pulled back inside the boundary layer, the tracking variable hooks cleanly back onto the zero axis without any undershoot or secondary oscillations.

While function f represents the autonomous, unforced dynamics of the wheel slip state, which scales based on the current vehicle velocity and estimated wheel deceleration trends, function g maps how effectively the electromagnetic motor torque translates into slip acceleration. Both functions match the macroscopic physical phases of the vehicle maneuver. They change smoothly during steady acceleration and steady braking, remaining entirely free of high-frequency noise.

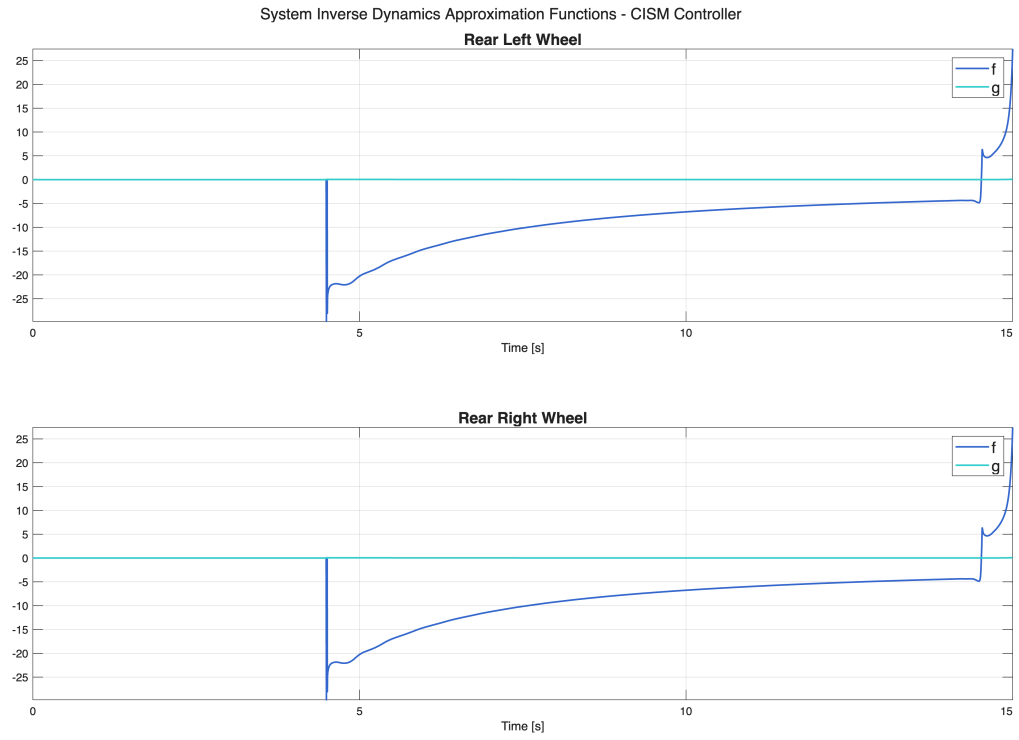


Figure 6.31: CISM system inverse dynamics functions.

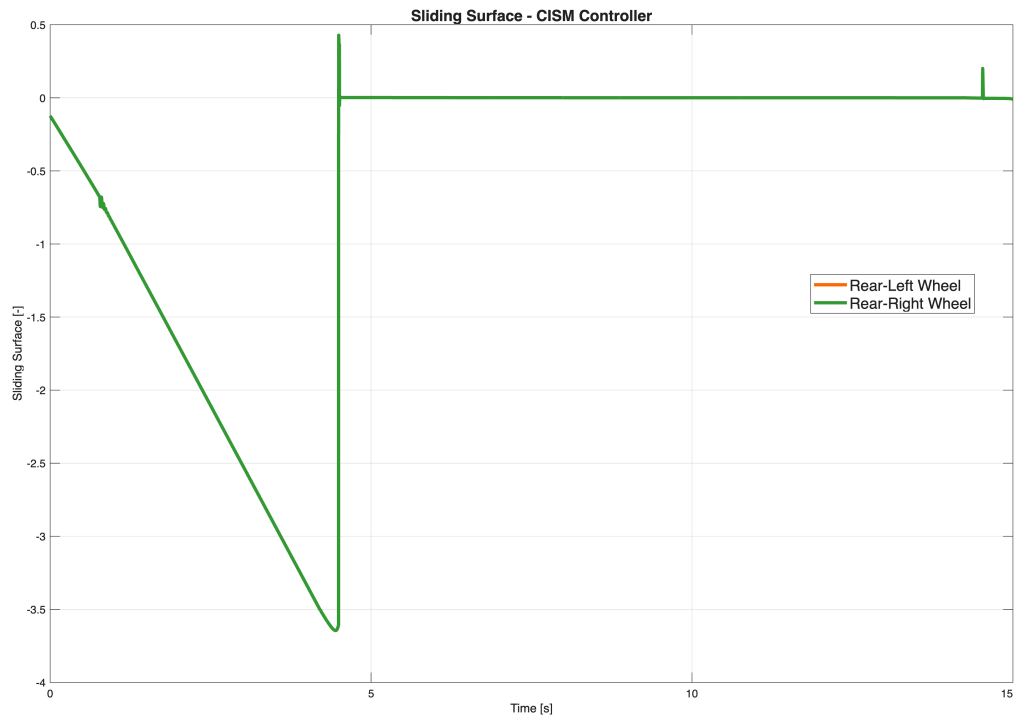


Figure 6.32: CISM controller sliding surface.

6.4 Mu-jump Maneuver

The second testing scenario evaluates the transient response, adaptability, and stability of the vehicle when encountering a sudden decrease in road surface adherence. The maneuver is simulated under a constant 70% gas pedal input, initiated from an initial cruising velocity of $V_{vhcl} \approx 10 \text{ km/h}$.

To evaluate the worst-case transient response, the surface step transition occurs precisely while the vehicle velocity is low ($V_{vhcl} \approx 20 \text{ km/h}$), maximizing the potential for severe wheel spin if the control loop fails to instantly re-stabilize the plant.

The road is characterized by an initial dry asphalt section followed by snowy/wet cobbleston, and again wet asphalt. Inherently, friction coefficient varies in the range $\mu_{road} \in [1, 0.3, 0.5]$, and the Reference Slip Estimator identifies optimal longitudinal slip peaks in the range $\sigma_{opt} \approx 0.10 \div 0.14$

6.4.1 Traction Control System OFF

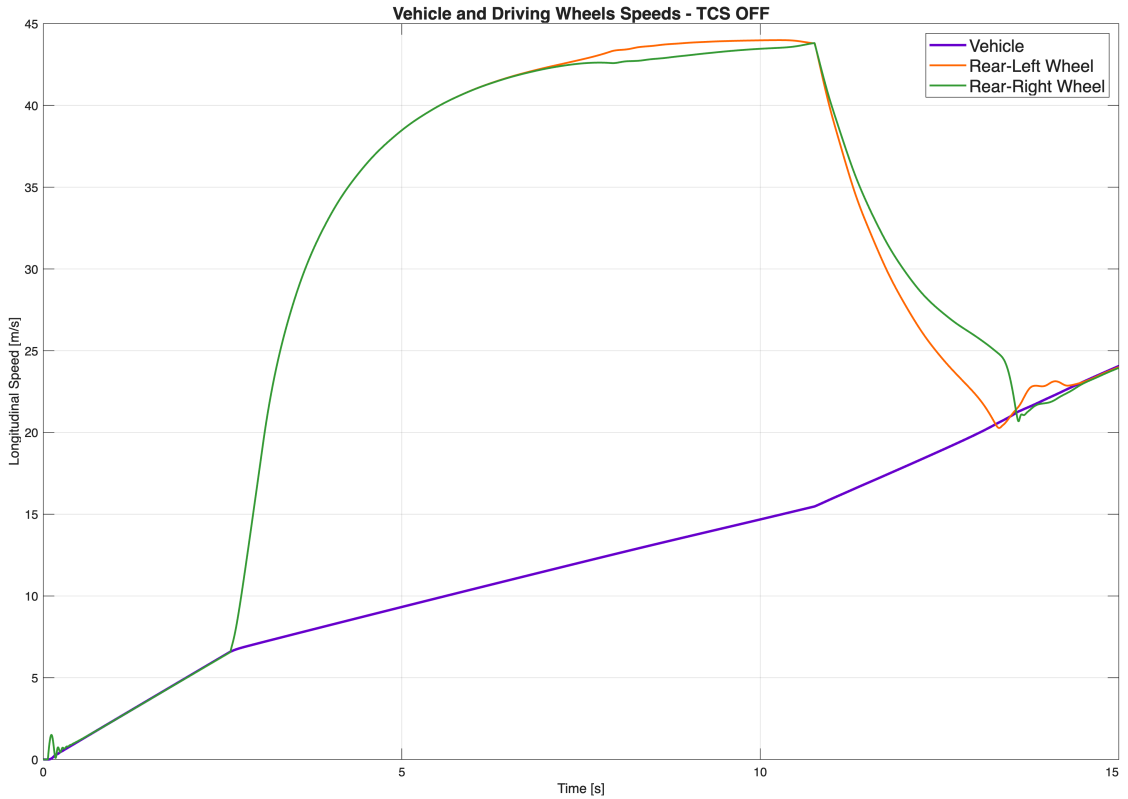


Figure 6.33: Vehicle and driving wheels' speed with traction control OFF.

When the maneuver is tested with the Traction Control System completely deactivated, on the initial high-adherence surface, the constant 70% throttle command produces stable vehicle acceleration, with the wheel speeds tracking the vehicle velocity curve.

However, the instant the rear wheels cross the friction bump at $t \approx 1.5$ s, the abrupt reduction in available tire-road friction lowers the admissible longitudinal force, causing the rear tires to exceed the adhesion limit.

Since the electric motor continues to output an unconstrained, high driving torque, the excess energy goes entirely into accelerating the wheel, hence the tangential wheels' speeds spike exponentially, shooting from under 24 km/h (6.6 m/s) to a severe spinning state over 100 km/h (28 m/s) within a few seconds, up to almost 160 km/h (44 m/s).

This divergence can be visualized analysing longitudinal slip behavior in Figure 6.34.

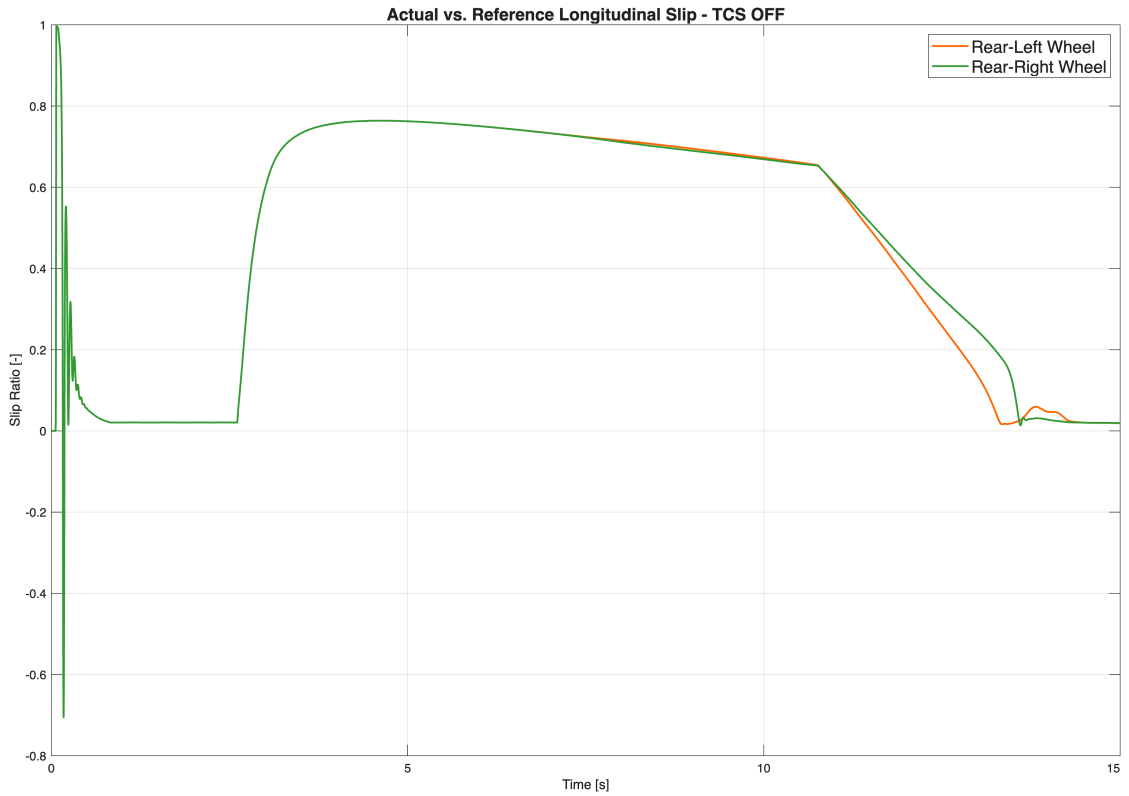


Figure 6.34: Actual vs. reference longitudinal slip with traction control OFF.

Ignoring initial spikes in slip given by maneuver start, prior to the transition, the longitudinal slip is bounded within the stable region ($\sigma < 0.01$). At the friction jump ($t \approx 2.6$ s), the slip breaks through the stable limit and rises monotonically, saturating near a full spinning state of $\sigma \approx 0.76$. At this operating point, the tire is completely decoupled from efficient force generation.

The severe consequences of this transition on force transmission is highlighted in Figure 6.35, which tracks the utilized friction coefficient.

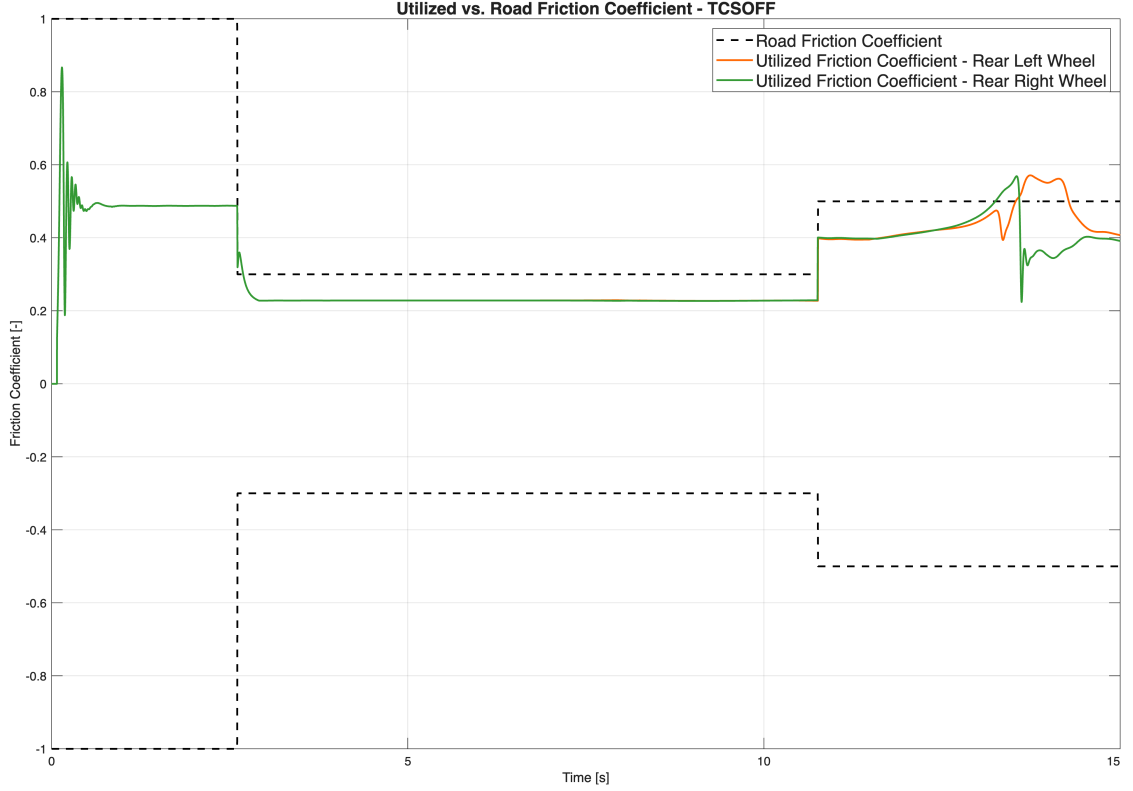


Figure 6.35: Utilized vs. road friction coefficient with traction control OFF.

Upon crossing the μ -jump, the utilized friction drops abruptly: since the slip simultaneously shoots up to the unstable region, the tire operates on the downward tail of the low-friction curve, utilizing only a fraction ($\mu_{ul} \approx 0.23$) of the road physical limit.

6.4.2 PI Controller

PI activation history is illustrated in Figure 6.37.

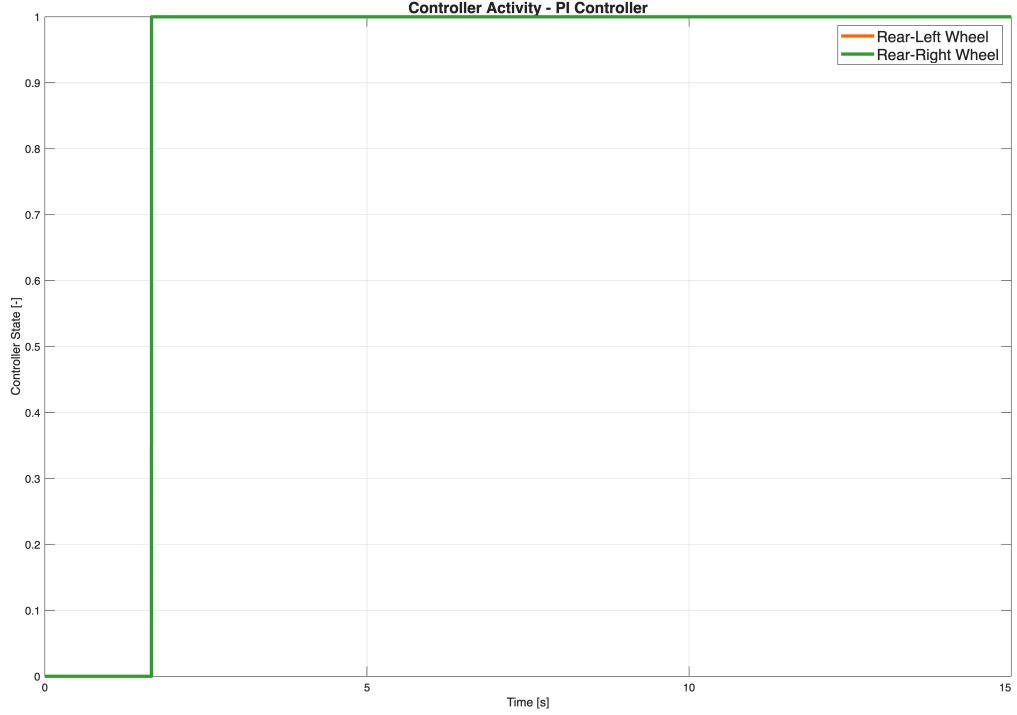


Figure 6.36: PI controller activity.

On the initial dry asphalt section ($\mu = 1.0$), the high adherence allows the tire contact patches to transfer the demanded powertrain torque within the stable linear elastic micro-slip zone. Consequently, the longitudinal slip remains below the activation threshold, and the PI status is OFF.

The sudden reduction in road adherence causes an immediate slip departure, consequently the control loop registers this threshold breach instantly, switching to full engagement (1).

Figures 6.37 and 6.38 display the resulting wheel peripheral speeds, vehicle longitudinal velocity, and longitudinal slip comparison to optimal target.

Upon entering the low-friction zone, the Reference Slip Estimator updates the optimal slip reference from $\sigma_{opt}^{\mu=1} = 0.1410$ to $\sigma_{opt}^{\mu=0.3} = 0.1038$.

Because the linear PI lacks of predictive action, it relies entirely on the feedback error accumulation. As a result, the driving wheel speeds experience a transient surge: the longitudinal slip exhibits a visible overshoot, peaking at $\sigma = 0.12$.

The controller takes approximately 26 ms to stabilize the slip onto the target.

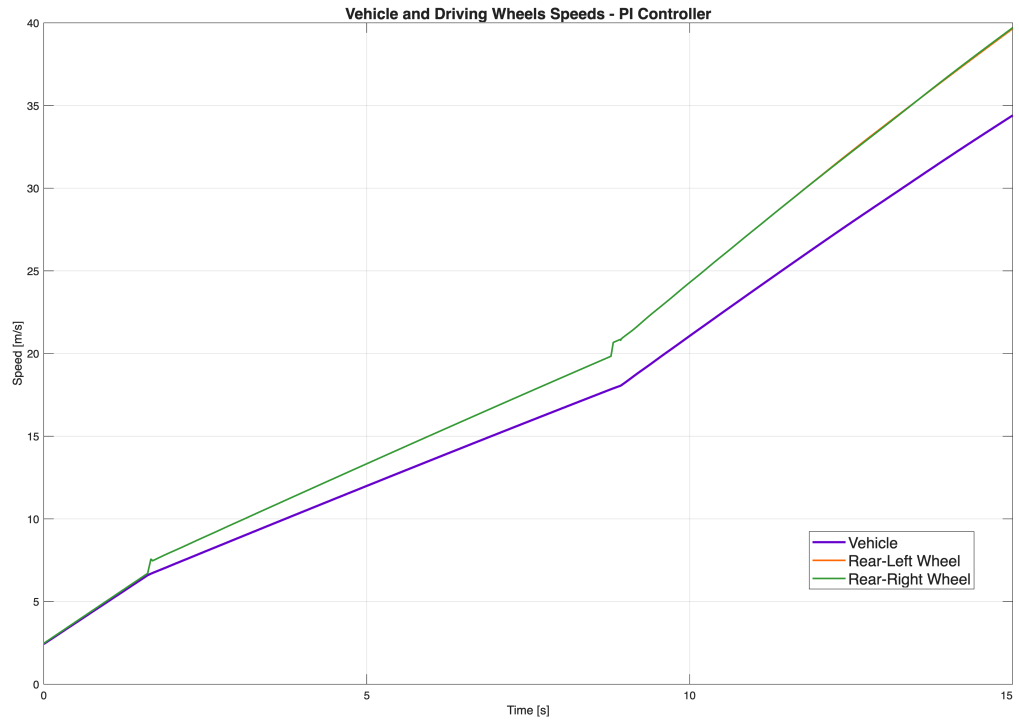


Figure 6.37: Vehicle and driving wheels' speed with PI.

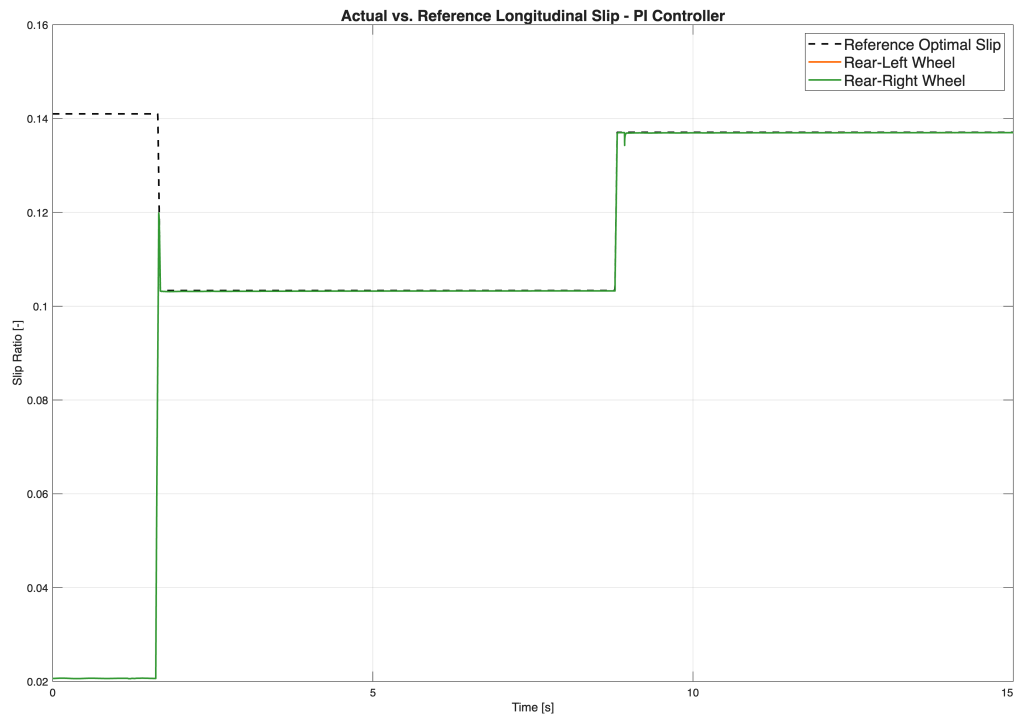


Figure 6.38: Actual vs. reference longitudinal slip with PI.

When entering the intermediate road friction segment ($\mu = 0.5$), the supervisor updates the target slip to $\sigma_{opt}^{\mu=0.5} = 0.1370$. Due to the sudden increase in road capacity, an instantaneous drop in wheel speed and a corresponding undershoot in longitudinal slip down to $\sigma = 0.1342$ is visible. This happens because PI control loop briefly ramps up the torque to exploit the higher friction limit, achieving then steady-state convergence onto the new reference line.

Figure 6.39 details the isolated wheel speed error.

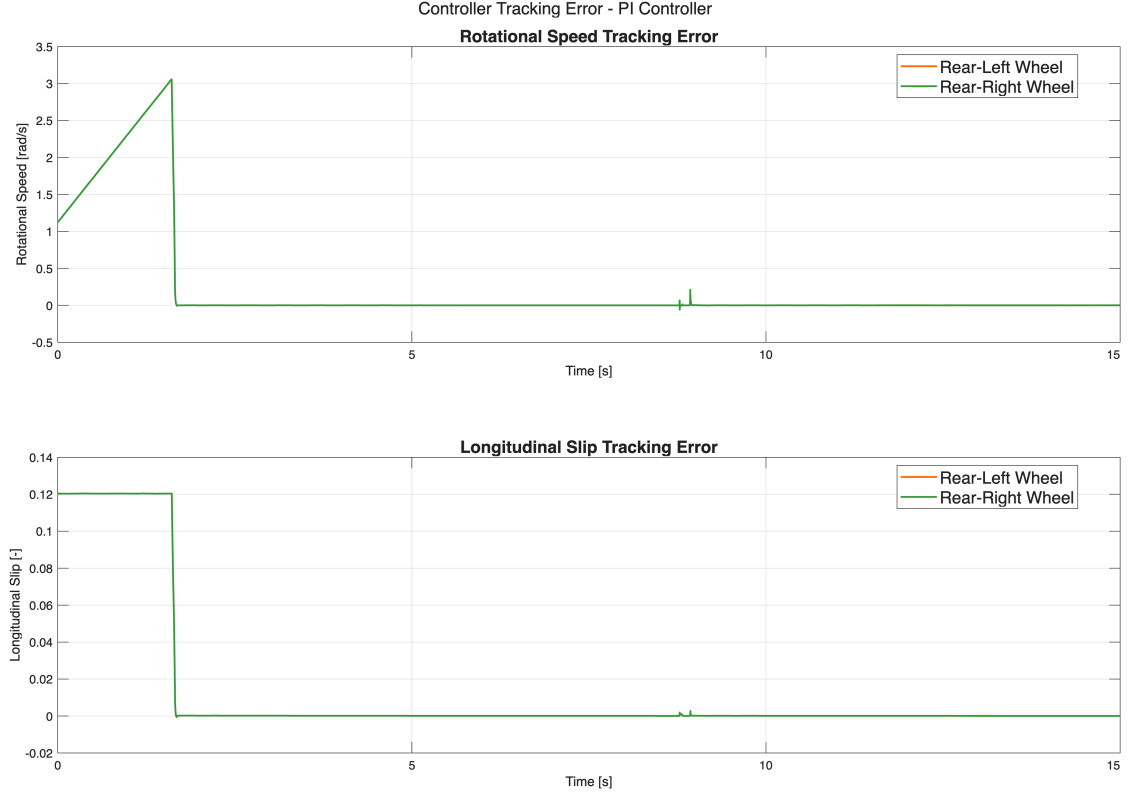


Figure 6.39: PI controller tracking error

Considering the operating window of the controller ($t \approx 1.5 \div 15$ s), since the first severe μ -jump, the longitudinal slip tracking error is held constant at zero for the whole maneuver.

The only exception is made for a slight rise in longitudinal slip error of $e_\sigma \approx 0.002$, corresponding to an overshoot in rotational wheel speed error of $e_\omega \approx 0.2$ m/s, in coincidence of the rise in road friction coefficient from $\mu = 0.3$ to $\mu = 0.5$.

Figure 6.40 displays the utilized friction coefficient generated at the driving wheels.

The PI controller provides a massive improvement compared to the unstable, open-loop configuration.

During the steady-state windows of both the low-friction zone ($t = 1.5 \div 8.5$ s) and intermediate-friction zone ($t = 8.5 \div 15$ s), the utilized friction coefficient overcomes the road nominal friction coefficient, guaranteeing a maximization of tire traction capability.

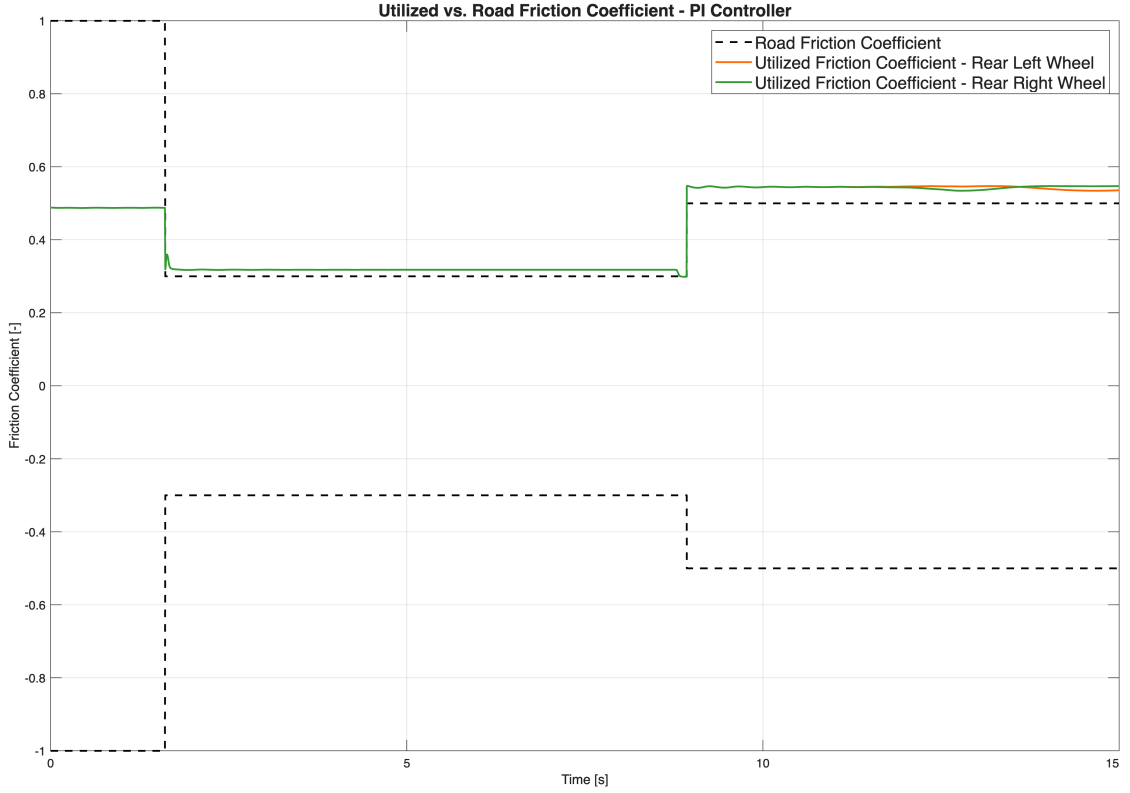


Figure 6.40: Utilized vs. road friction coefficient with PI.

Controller's Output

Figure 6.41 reports the internal parts of the calculated torque command for both rear wheels.

While accelerating on dry asphalt ($\mu = 1$), for the majority of the time the wheel slip stays below the activation threshold, hence the control loop is inactive, so the command torque is zero.

Once a threshold breach is detected, while approaching the end of the dry asphalt segment, the controller activates, injecting a torque command (≈ 500 Nm) to bring the wheel to the desired slipping condition ($\sigma_{opt} = 0.1410$). After few milliseconds there is the jump in surface friction down to $\mu = 0.3$, which lowers the target slip down to $\sigma_{opt} = 0.1038$. This makes the torque command decrease to finally stabilize to $T_{cmd} \approx 450$ Nm for the whole low-friction segment.

When the driving wheels cross the boundary from the low-friction surface to the higher adherence intermediate section ($\mu = 0.5$) at $t = 8.5$ s, the road can instantly support a higher longitudinal tractive force. The sudden increase in road grip acts as an external mechanical load, quickly pulling the wheel speeds down. This causes the actual longitudinal slip to instantly drop below the reference target (see Fig. 6.38).

This kinematic drop creates a spike in tracking error (see Fig. 6.39). Because a standard PI controller is purely reactive, it responds directly to this localized error signal, hence as a result, the target torque jumps from its previous low-friction holding value of approximately 400 Nm and climbs toward a new steady-state value of roughly 750 Nm. Once the wheel speeds accelerate enough to match the new optimal slip target ($\sigma_{opt} = 0.1370$), the tracking error returns to zero, and the torque command stabilizes cleanly.

As the T_{ctrl} coming from the PI never overcomes saturation limits, $T_{cmps} = 0$ for the whole simulation.

The behavior described is obviously matched by the EM torque signals, as displayed in Figure 6.42.

As the torque profile requested by the controller is smooth and within actuator's feasibility limits, the electric motors easily manage to match the target imposed.

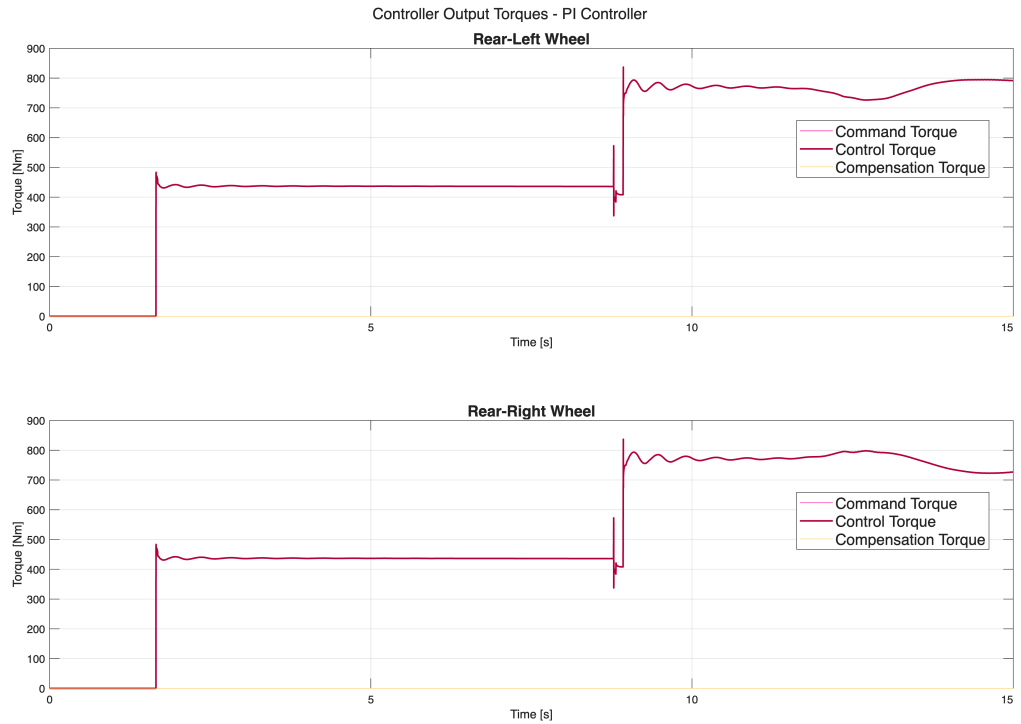


Figure 6.41: PI controller's output torques.

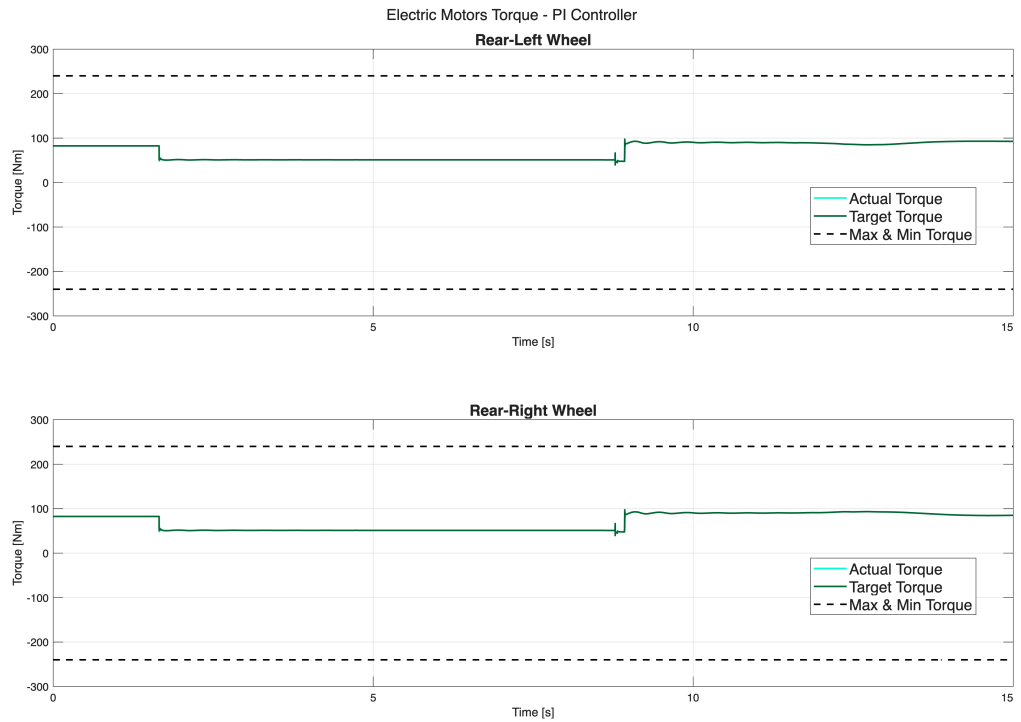


Figure 6.42: Electric motors torque with PI controller.

6.4.3 TSM Controller

The operational status of TSM control loop is tracked in Figure 6.43.

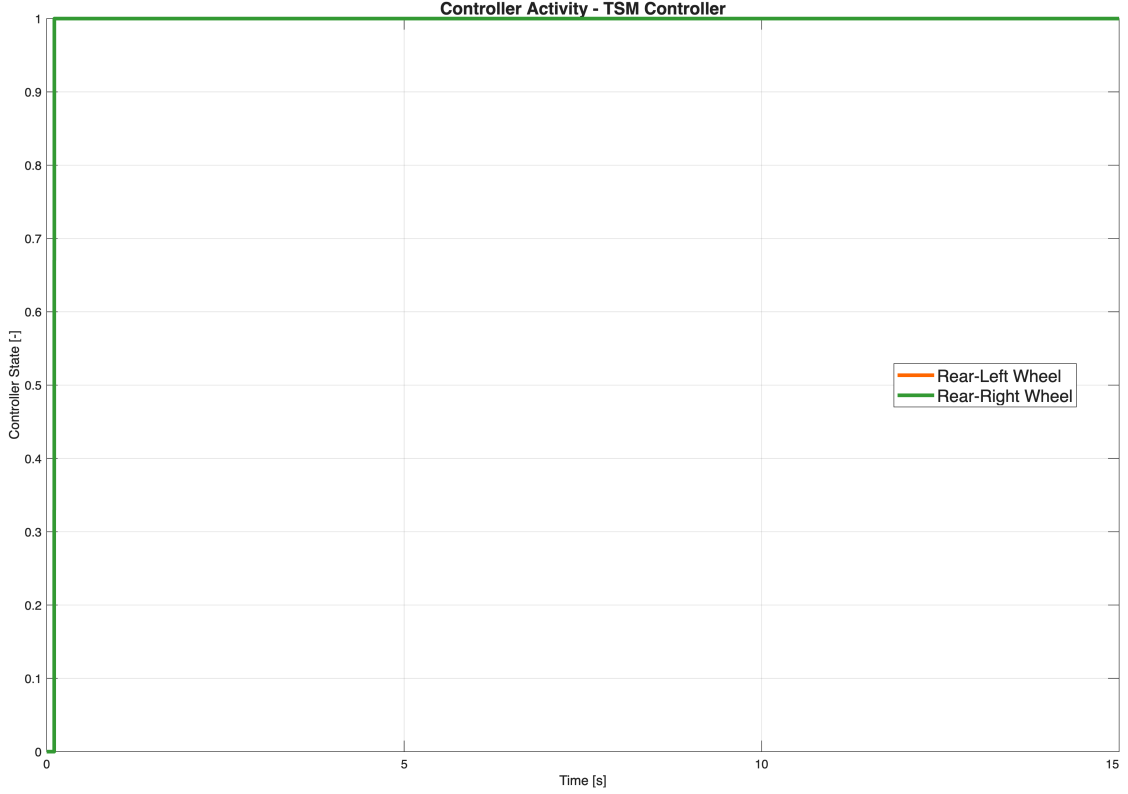


Figure 6.43: TSM controller activity.

Again, for the very first portion of the dry asphalt section, the controller’s status sits flat at 0. As soon as the slip ratio crosses the engagement threshold, the status flag steps cleanly to 1, and remains locked at full engagement for the remainder of the simulation window.

Figure 6.44 and 6.45 illustrate the vehicle and wheel velocities alongside the longitudinal slip tracking trajectories.

TSM controller reacts more aggressively than the PI.

The wheel velocities exhibit negligible macroscopic overshoot, completely suppressing the severe wheel spinout seen in the open-loop configuration, and matching the vehicle speed profile with a fixed distance between curves, corresponding to the target slip.

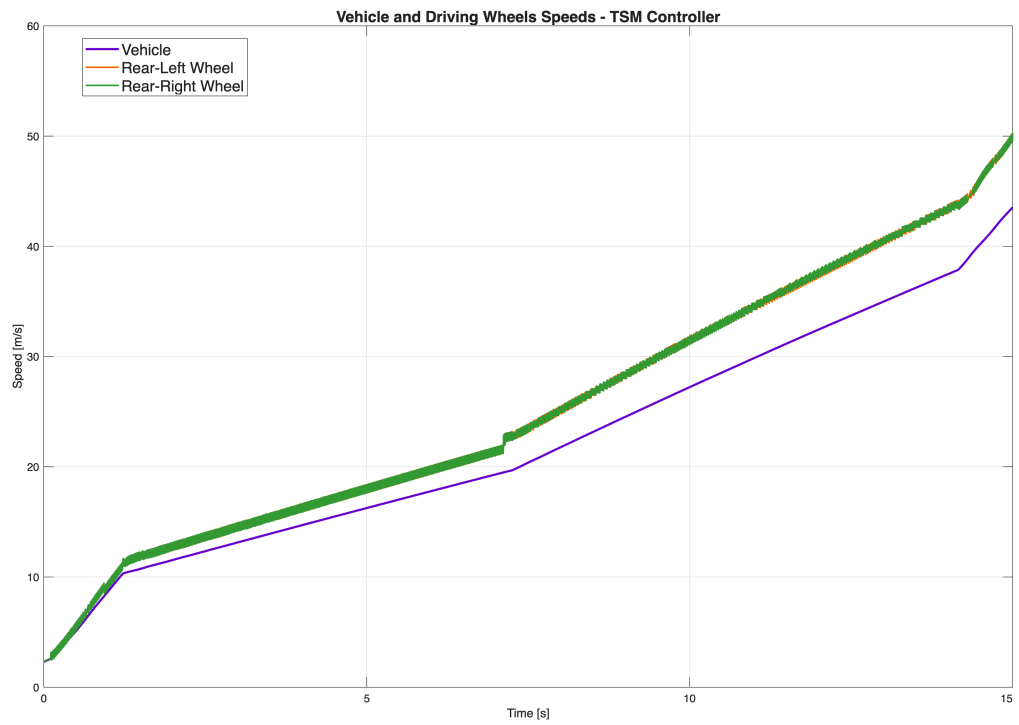


Figure 6.44: Vehicle and driving wheels' speed with TSMC.

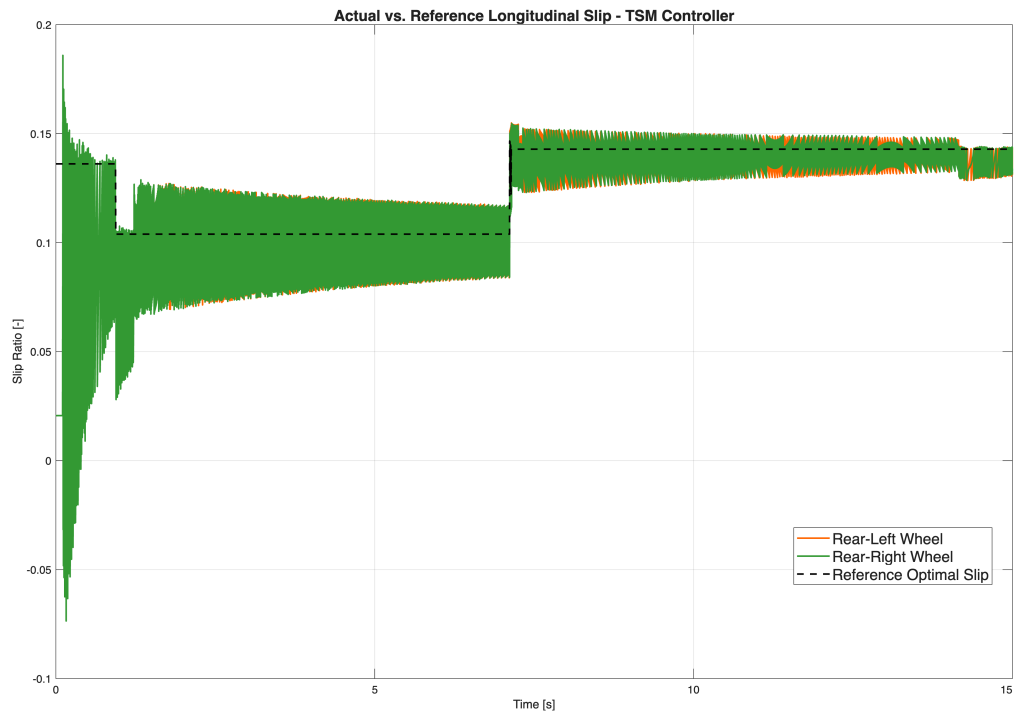


Figure 6.45: Actual vs. reference longitudinal slip with TSMC.

During controller's intervention window, the actual slip curves never form a perfectly flat line. Instead, they exhibit a continuous, dense, high-frequency oscillation around the reference targets. This is a direct visual indicator of sliding mode chattering, confirming that the state trajectories are continuously crossing back and forth over the sliding surface as the control law switches between its upper and lower bounds.

The actual longitudinal slip curves for the rear wheels converge onto the updated target within milliseconds, completely suppressing the severe wheel spinout seen in the open-loop configuration.

Since the controller is activated fews milliseconds after the start of the simulation, for $t < 1$ s the longitudinal slip profile displays severe chattering with high amplitude of oscillations. This is mainly caused by the fact that the vehicle is travelling at low speed, influencing stability of mathematical definitions in the model, and that the controller is trying to bring the wheels at a condition of higher slip to match the reference $\sigma_{opt} = 0.1410$.

As the road surface switches from $\mu = 1$ to $\mu = 0.3$, the reference slip target steps down to 0.1038, and the actual longitudinal slip curves for both the rear-left and rear-right wheels track this change rapidly, with only a small tracking undershoot down to approximately 0.05.

When transitioning to the intermediate-friction section ($\mu = 0.5$), the reference slip target steps up to 0.1370. Unlike a linear controller which would suffer from tracking delays, the non-linear switching law forces the actual slip to jump upward instantly.

The tracking error plot directly reflects the underlying longitudinal slip dynamics, revealing a reduced tracking capability at lower vehicle velocities. Within this initial low-speed regime, the system exhibits a maximum tracking overshoot of 0.20 and undershoot of -0.05 . As the vehicle velocity increases, the amplitude of these slip oscillations progressively decays, indicating enhanced tracking stability.

By ensuring instant convergence to the optimal slip zones, the TSMC maximizes longitudinal force transmission. The utilized friction coefficient drops directly from its asphalt value to stabilize on a flat plateau above 0.3 on the low-grip surface. The moment the vehicle crosses the boundary at $t = 8.5$ s, steps upward instantly to hit a flat plateau above 0.5. Because the slip tracking is tight, the friction allocation is remarkably improved, though it is superimposed with a low-amplitude, high-frequency noise ripple matching the slip chattering.

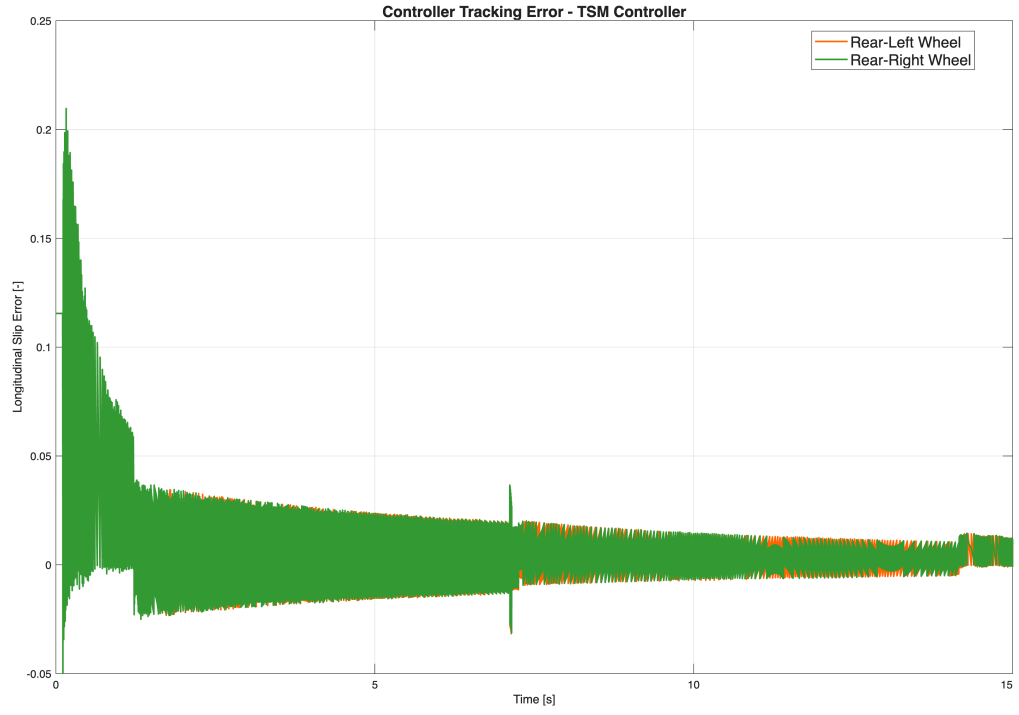


Figure 6.46: TSM controller's tracking error.

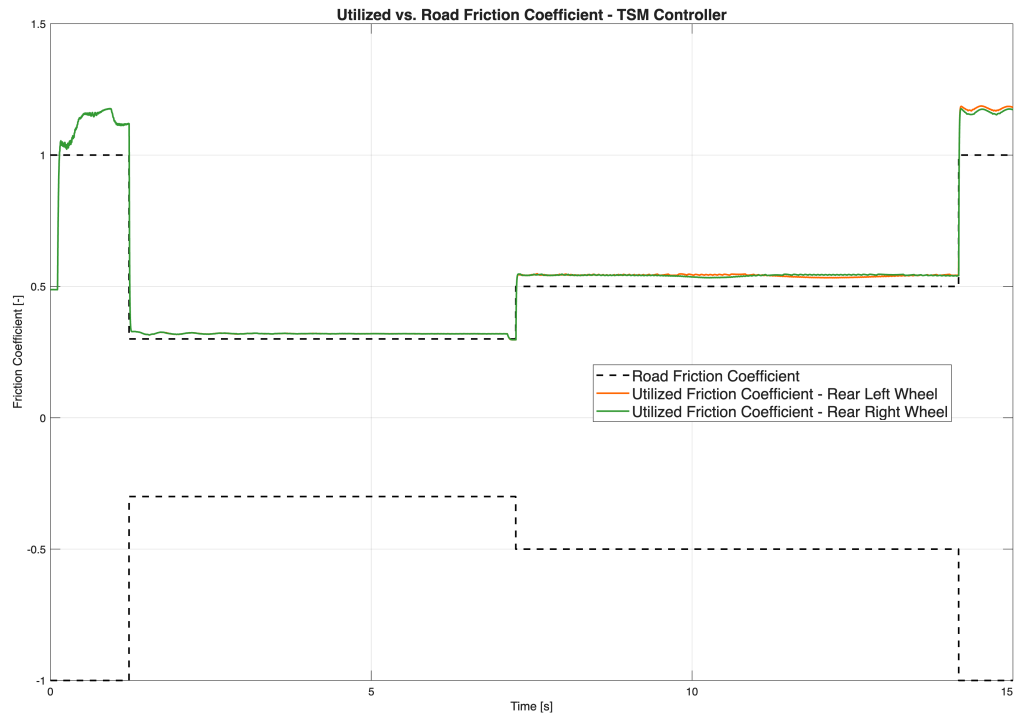


Figure 6.47: Utilized vs. road friction coefficient with TSMC.

Controller's Output

The internal torque calculation plot in Figure 6.48 that the primary Control Torque command switches violently between its upper and lower bounds to maintain the sliding state.

When the vehicle hits the intermediate-friction surface ($\mu = 0.5$), the controller shifts its macroscopic operating point to match the higher tractive capability. However, the high-frequency switching remains active.

As illustrated in Figure 6.49, the Target Torque command and the Actual Torque output of the electric motors oscillate rapidly across a huge peak-to-peak switching band between tires and motors feasibility limits.

While this aggressive switching behavior enforces mathematical tracking invariance against road variations, it represents an unacceptable physical penalty in a real vehicle application, directly justifying the need for a smoothed, chattering-free sliding mode variant.

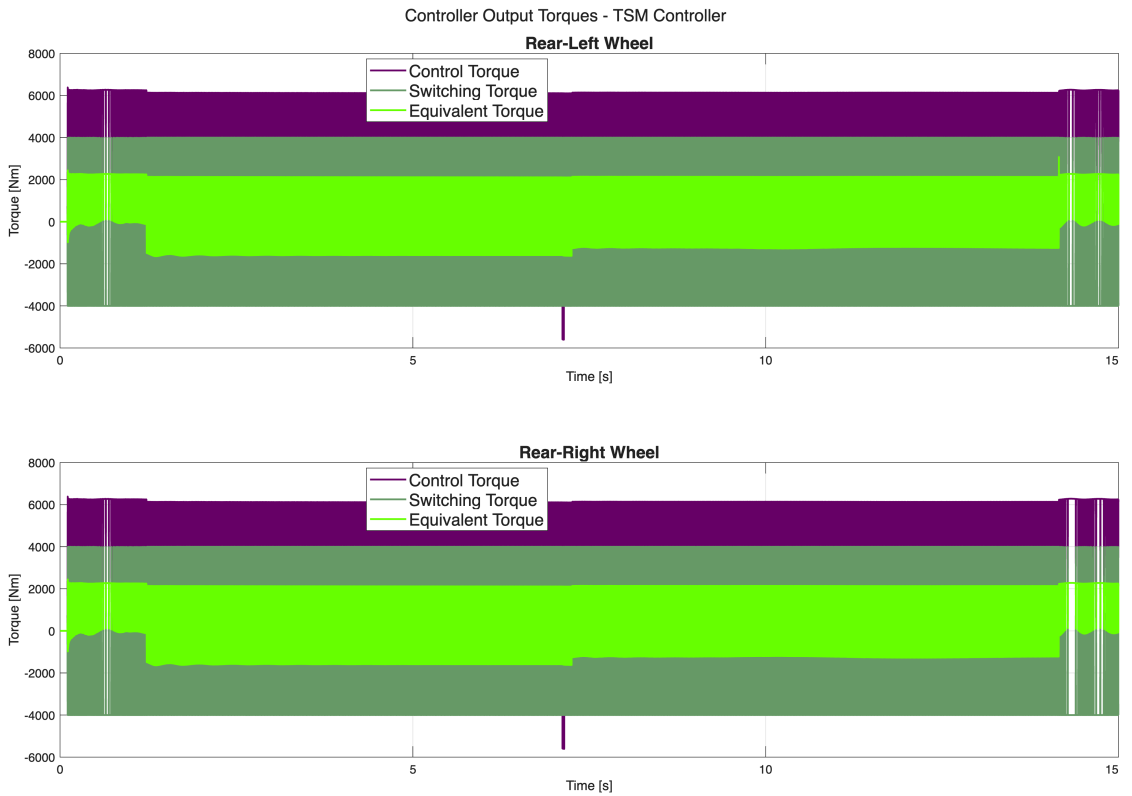


Figure 6.48: TSM controller output torques.

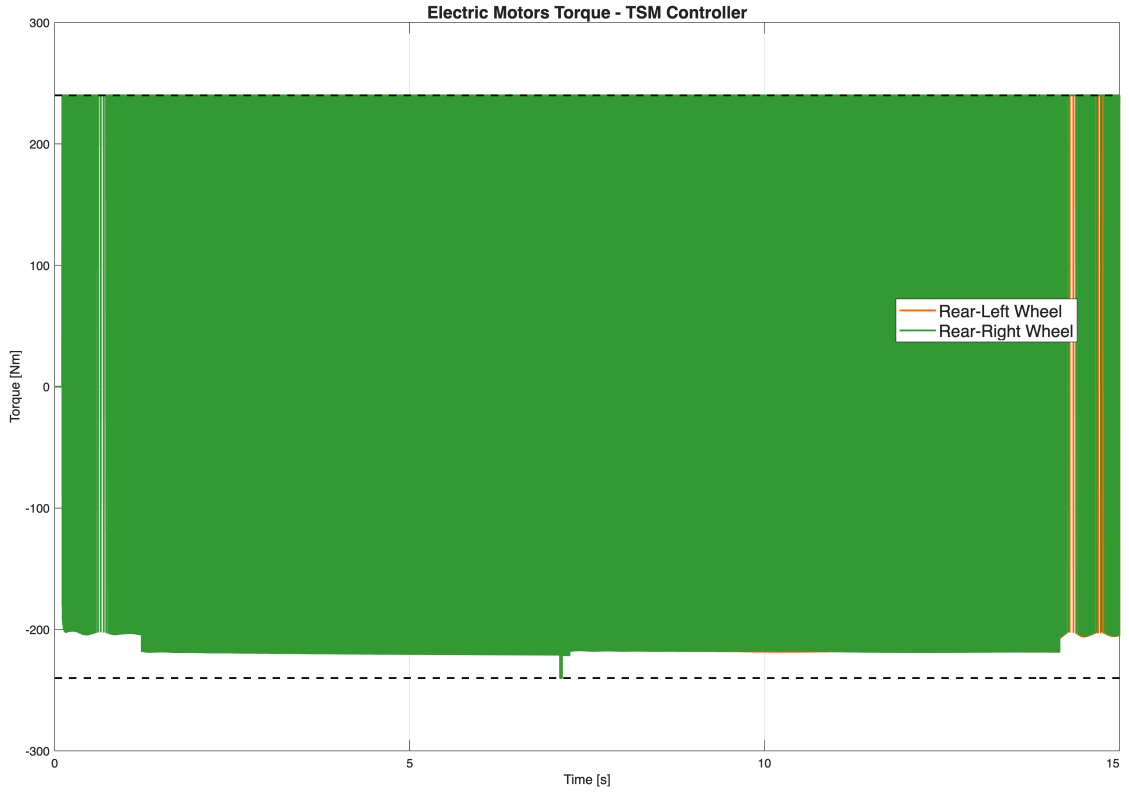


Figure 6.49: Electric motors torque with TSM controller.

Controller's Stability

To evaluate the stability and switching behavior of the TSMC, Figure 6.50 tracks the sliding surface s and the calculated Lyapunov function derivative ($\dot{V}(s) = \dot{s}s$) for both rear wheels.

For the portion of controller's activity along the high-friction surface, sliding surface reaches its maximum overshoot (0.05) and undershoot (-0.2) values, and chatters around an equilibrium value of approximately 0.03, an offset from the ideal condition $s = 0$ caused by system instability at low speeds.

The instant the vehicle hits the low-friction boundary, the driving wheels accelerate rapidly, forcing the sliding surface to spike violently up to a positive peak of approximately 0.02, from this point on chattering around the equilibrium $s = 0$. As travelling speed increases, the amplitude of chattering oscillations decreases, as the system and control loop stabilize.

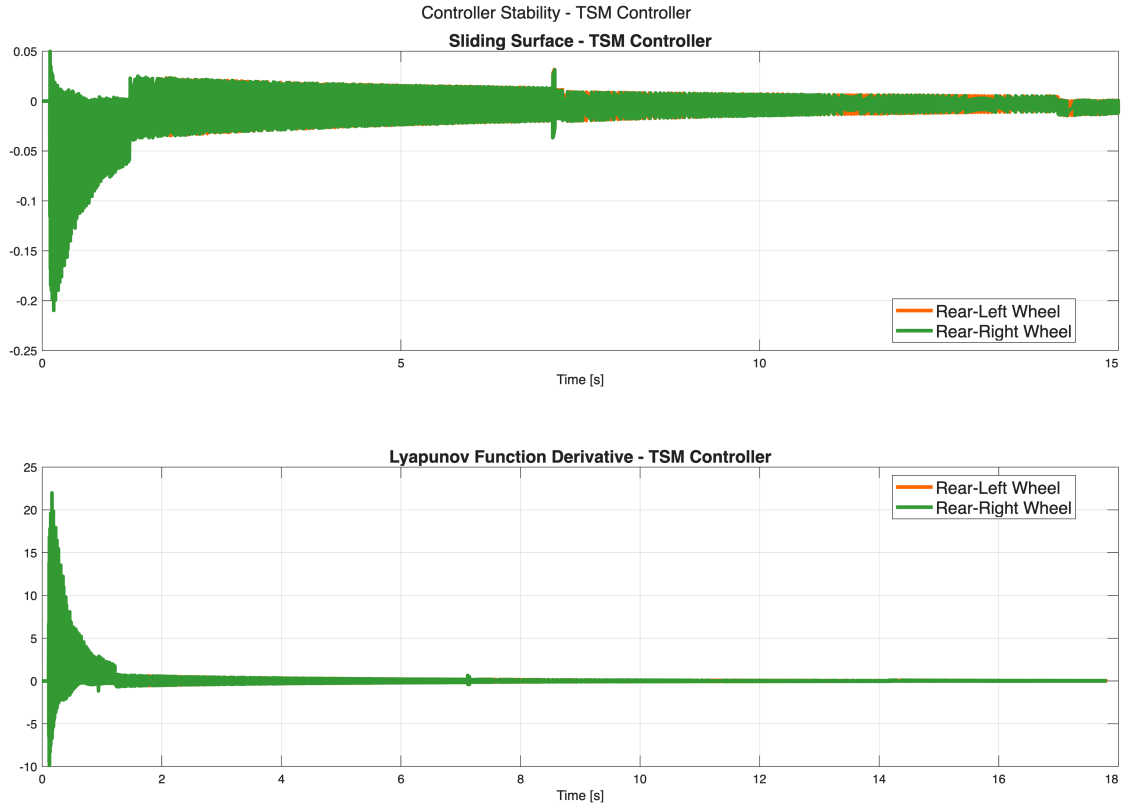


Figure 6.50: TSM controller's stability analysis.

Lyapunov function derivative plot confirms that the non-linear control law generates an aggressive corrective energy response that forces the perturbed state trajectories back toward the sliding surface. Because the discontinuous control law ($\text{sign}(s)$) switches signs at every solver step, the derivative $\dot{s}$ alters its direction continuously. As a result, $\dot{V}$ exhibits rapid, symmetric high-frequency chattering directly across the zero trajectory, between 0.15 and -0.15 .

6.4.4 CISM Controller

The dynamic activation state history of the CISM loop is recorded in Figure 6.51. Identical to the baseline PI configuration, At $t \approx 1.5$ s the sudden loss of road friction forces the slip ratio to cross the activation threshold, shifting the controller's state instantly to 1 (active). The CISM remains fully engaged for the rest of the simulation window to manage the changing road friction.

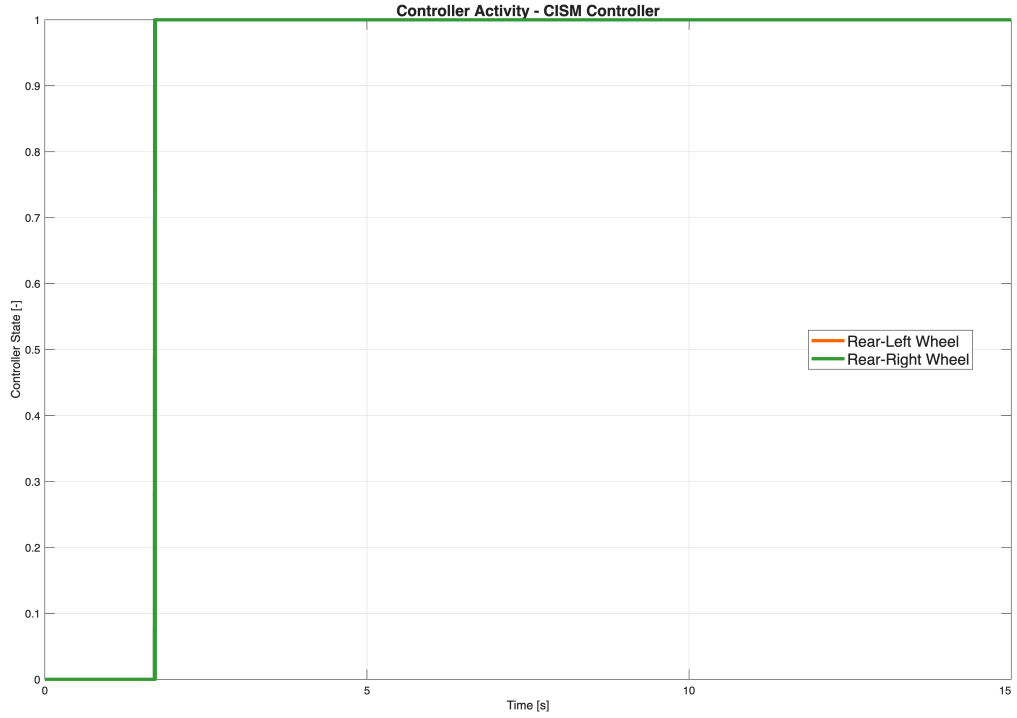


Figure 6.51: CISM controller activity.

Figure 6.52 and 6.53 illustrate the vehicle and wheel velocities alongside the resulting longitudinal slip tracking trajectories.

When entering the low-friction zone ($\mu = 0.3$), the reference slip target steps down to 0.1038. The CISM layout handles this severe transition with good damping, despite an instantaneous overshoot up to 0.18. The driven wheel velocities experience negligible macroscopic overshoot or uncontrolled spinout. The actual longitudinal slip curves for both rear wheels converge exponentially onto the reference line within approximately 0.5 seconds.

When transitioning to the intermediate-friction surface, the controller manages to track the higher slip reference (0.1370) with zero overshoot and null delay, locking cleanly onto the new target condition.

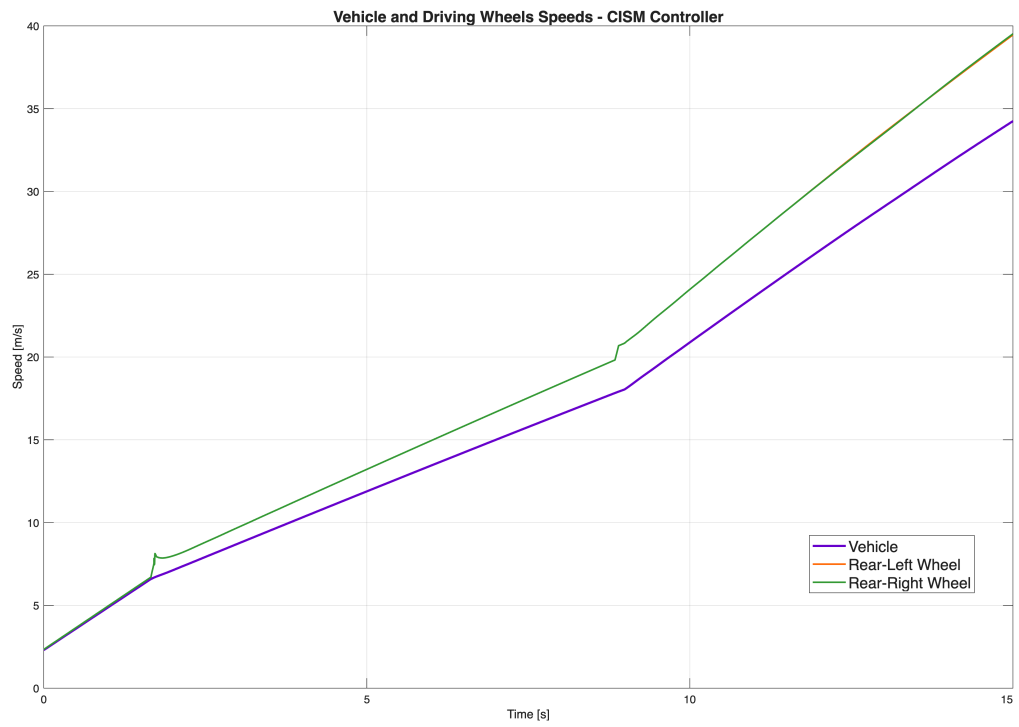


Figure 6.52: Vehicle and driving wheels' speed with CISM.

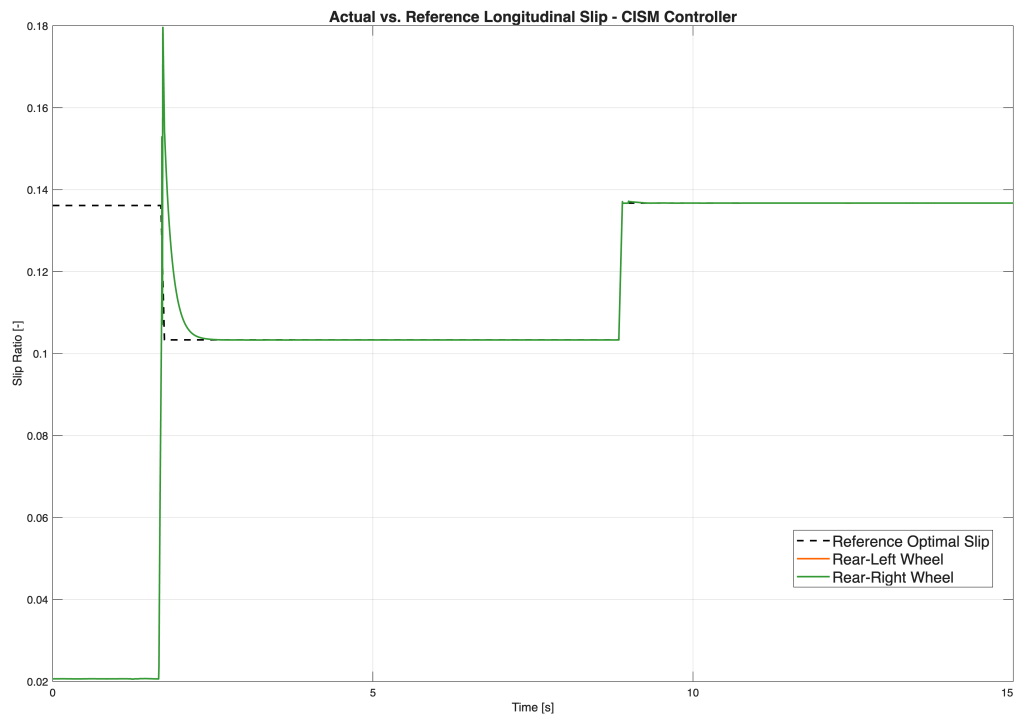


Figure 6.53: Actual vs. reference longitudinal slip with CISM.

Crucially, unlike the fuzzy and wide oscillation bands seen under traditional SMC, both the wheel velocities and longitudinal slip lines remain perfectly thin, smooth, and ripple-free during controller's active phase ($t = 1.5 \div 15$). This confirms the successful chattering suppression performance of the CISMCM.

The tracking performance of the controller is plotted in Figure 6.54.

At the initial friction drop ($t \approx 1.5$ s), the longitudinal slip tracking error experiences an isolated, brief positive peak limited to a maximum of 0.06, matching the robust transient performance of the TSMCM but without its steady-state noise.

Once the transient perturbations are rejected, the slip tracking error converges directly to a flat, noise-free absolute zero line. By allowing the integral term to operate within the boundary layer, the controller completely eliminates the persistent chattering error observed in the traditional SMC layout.

Figure 6.55 illustrates the evolution of the utilized friction coefficient at the rear-left and rear-right wheels.

Within controller's intervention time window ($t = 1.5 \div 15$ s) the utilized friction coefficient is always higher than road nominal one.

The moment the vehicle crosses onto the $\mu = 0.3$ section, the available road friction drops instantly. Under CISMCM control, the utilized friction coefficient drops directly to a transient low of 0.29 for a fraction of a second before recovering sharply, stabilizing onto a perfectly flat, noise-free line at 0.32.

When transitioning to the section at $\mu = 0.5$, the road capacity steps upward and μ_{ul} displays a fast and sharp transient response, shooting up to 0.54, where it stabilizes.

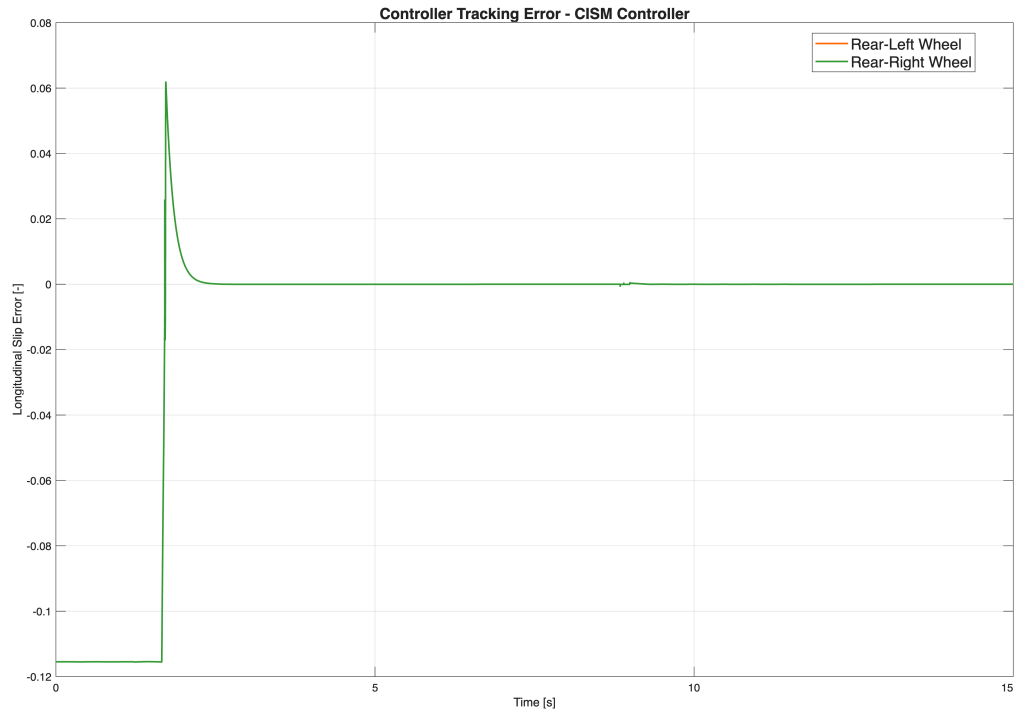


Figure 6.54: CISM controller's tracking error.

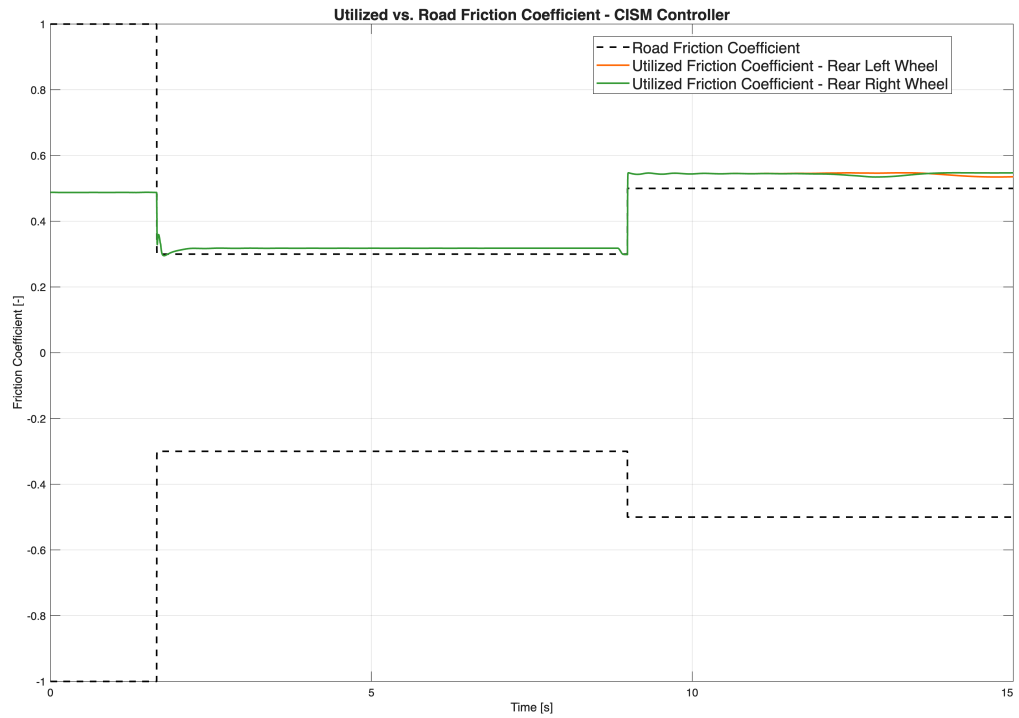


Figure 6.55: Utilized vs. road friction coefficient with CISM.

Controller's Output

The underlying control inputs driving this performance are reported in Figure 6.56.

The control input (u_{ref}) behaves as a smooth, continuous signal, as the conditional integration block manages the actuator bounds safely.

At $t \approx 8.5$ s, the controller increases its torque command to exploit the intermediate grip surface, stepping up smoothly from a low-friction holding value of approximately 50 Nm to a new steady-state plateau of roughly 100 Nm per wheel.

As illustrated in Figure 6.57, the Target Torque command line and the Actual Torque output line overlap perfectly, tracking the demands with no visible lag. Most importantly, the huge high-frequency switching seen under traditional SMC are completely eliminated. The actual motor torque line remains perfectly smooth and quiet through both the $\mu = 0.3$ and $\mu = 0.5$ surfaces.

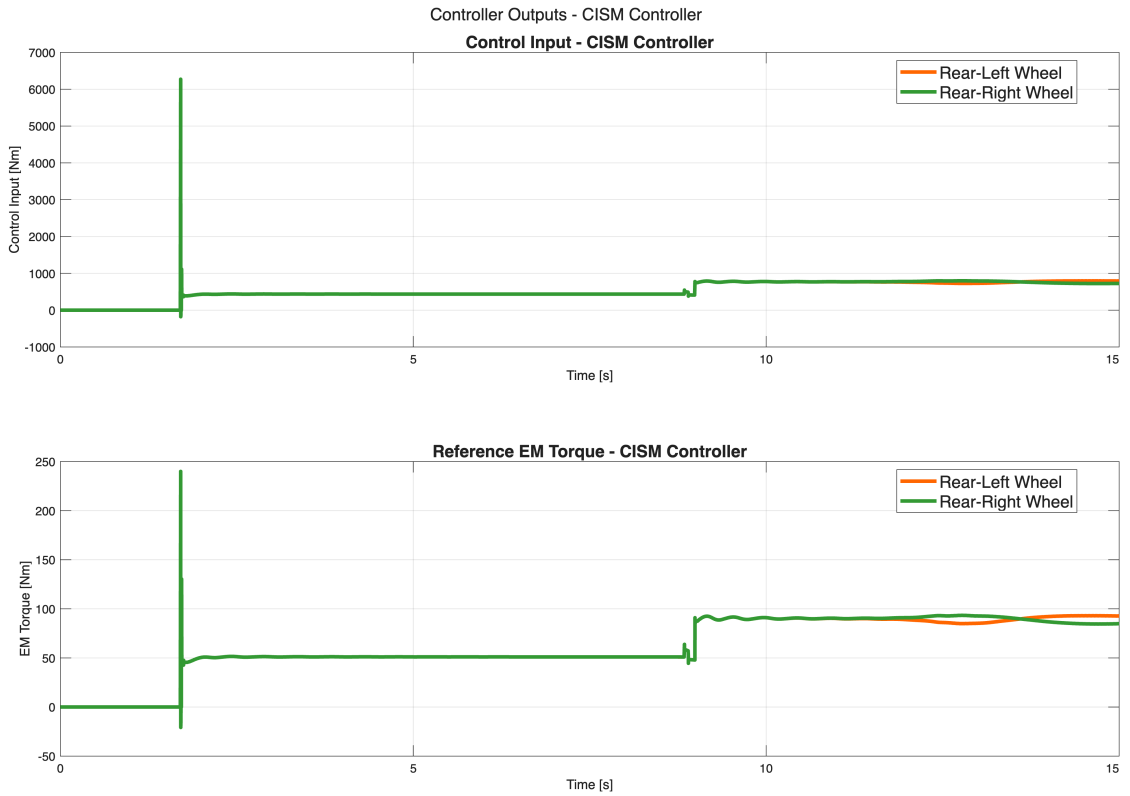


Figure 6.56: CISM controller's output torques.

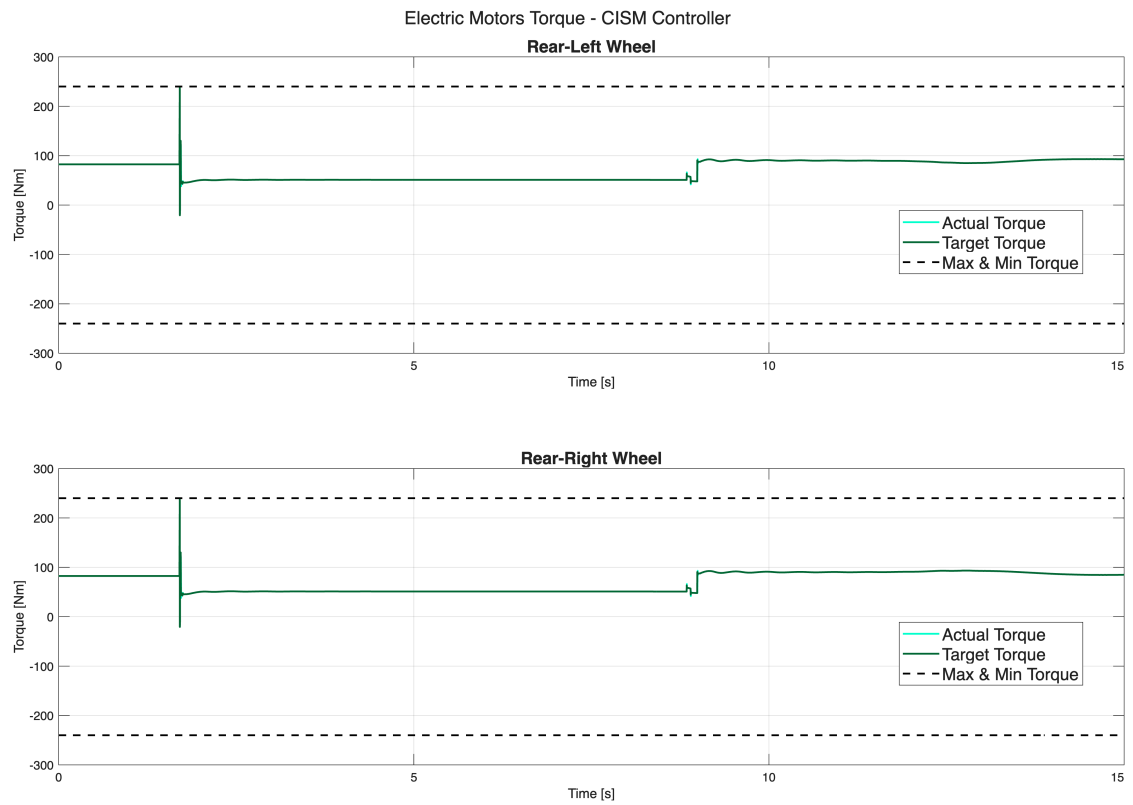


Figure 6.57: Electric motors torque with CISM controller.

Controller's Stability

As stated previously, the smooth control output achieved by the CISMCM relies on the real-time estimation and cancellation of the vehicle's non-linear dynamics through functions f and g .

Function f and g evolves smoothly without high-frequency oscillations, showing step-like transitions exactly synchronized with the surface boundaries.

While these model-based terms absorb the deterministic non-linear variations of the plant, the auxiliary switching block is relieved of high-gain duties, ensuring a highly stable control command.

In Figure 6.59 tracking metrics for the sliding surface are reported.

At the initial high-to-low μ -jump ($t \approx 1.5$ s), the sudden acceleration of the driving wheels forces the sliding surface into an instantaneous positive spike peaking at approximately 0.1.

The most significant improvement over the traditional sliding mode architecture is seen in the fact that, once the transient spike is suppressed, s returns to and stays flat at exactly zero for the whole simulation.

The controller completely eliminates the high-frequency chattering seen in traditional SMC, proving that the conditional integral term successfully handles the remaining unmodeled errors asymptotically, guaranteeing a noise-free, highly accurate slip regulation.

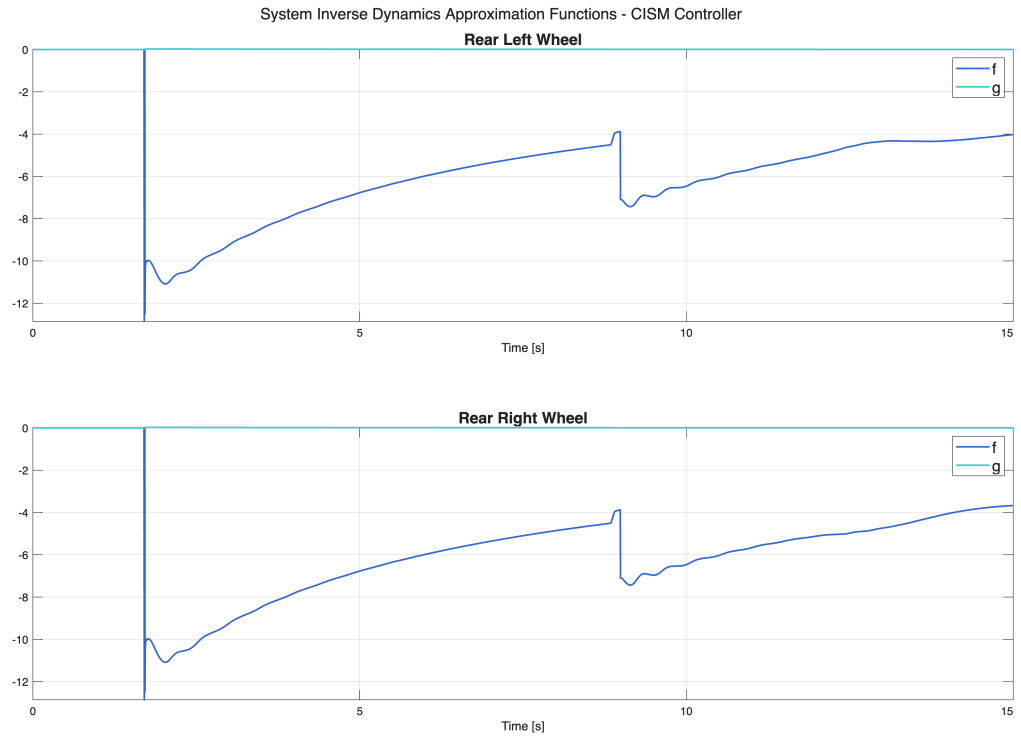


Figure 6.58: CISM system inverse dynamics functions.

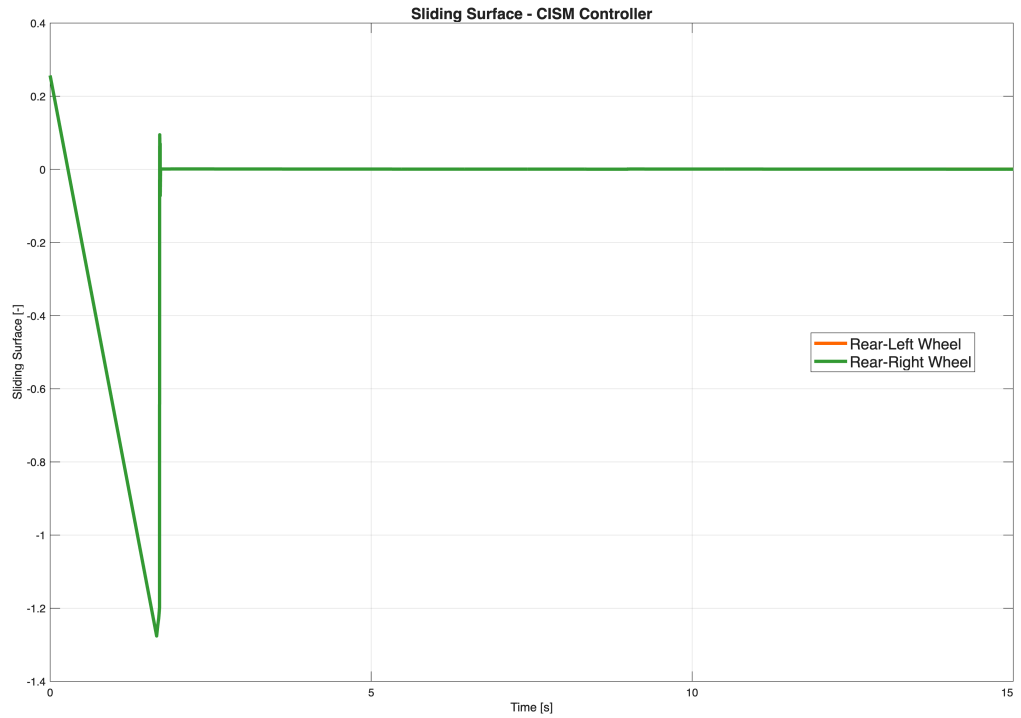


Figure 6.59: CISM controller stability.

6.5 Mu-split Maneuver

To establish a clear benchmark for evaluating the active control strategies, this section analyzes the vehicle's open-loop response during an asymmetric transition to a μ -split surface.

Under a constant 70% gas pedal demand, the vehicle crosses a road boundary where the left wheels encounter a low-grip profile ($\mu_L = 0.3$) and the right wheels drop onto an ultra-low adherence profile ($\mu_R = 0.1$).

6.5.1 Traction Control System OFF

The vehicle and wheels velocity profiles under open-loop operation are illustrated in Figure 6.60.

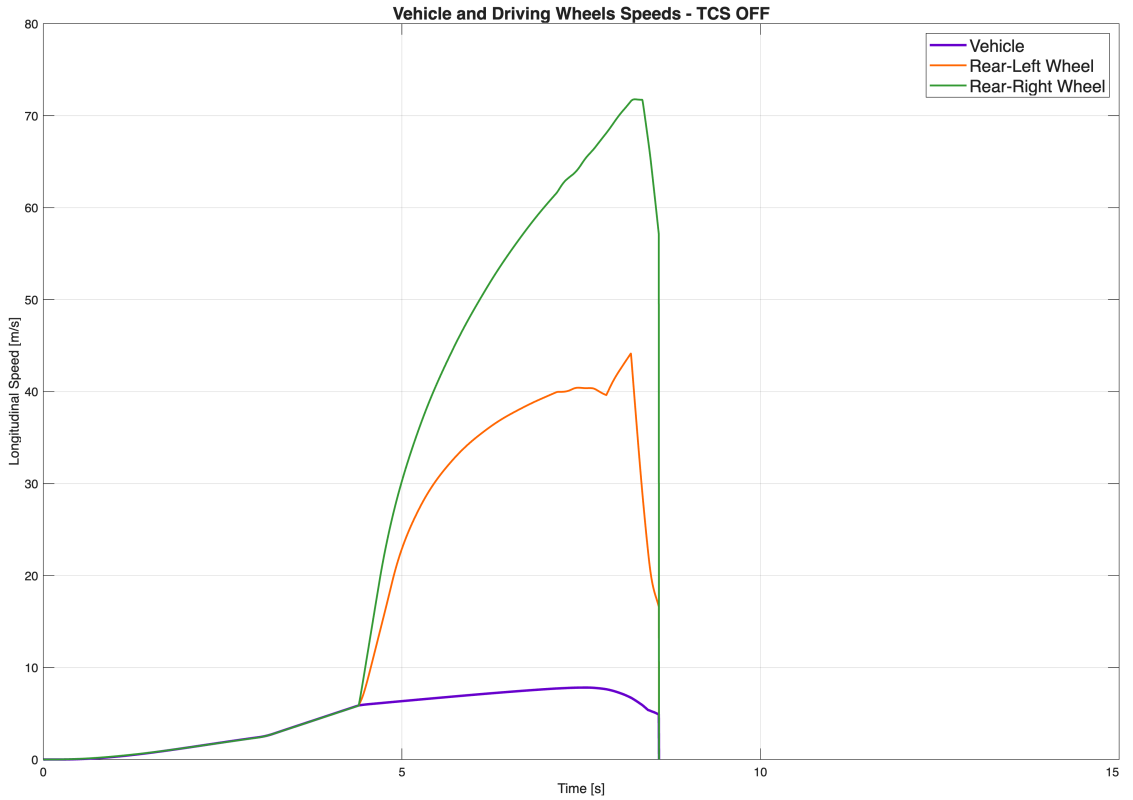


Figure 6.60: Vehicle and driving wheels' speed with traction control OFF.

Prior to the transition, the vehicle accelerates uniformly on dry asphalt. Both the rear-left and rear-right wheel tangential speeds perfectly match the longitudinal velocity of the vehicle body.

The instant the vehicle crosses the friction boundary at low speed ($V_{veh} \approx 20$ km/h), the absence of a traction control system causes an immediate, loss of kinematic stability. Because a constant 70% throttle command continues to inject power into the driving axles, the wheels immediately accelerate beyond the physical limits of the road.

Due to the severe asymmetry of the road, the two driven wheels diverge rapidly. The right wheel, sitting on the most critical $\mu = 0.1$ surface, spins out more violently, peaking at approximately 72 m/s by $t \approx 8.5$ s. The left wheel, experiencing a slightly higher grip level ($\mu = 0.3$), also breaks away completely but climbs at a lower rate, reaching a peak near 44 m/s.

Because the wheel speeds diverge so massively from the actual vehicle velocity, the vehicle completely loses stability and simulation is ended prematurely by the software, not managing to complete the maneuver.

Figure 6.61 confirms this unstable behavior by mapping the longitudinal slip ratios of both wheels.

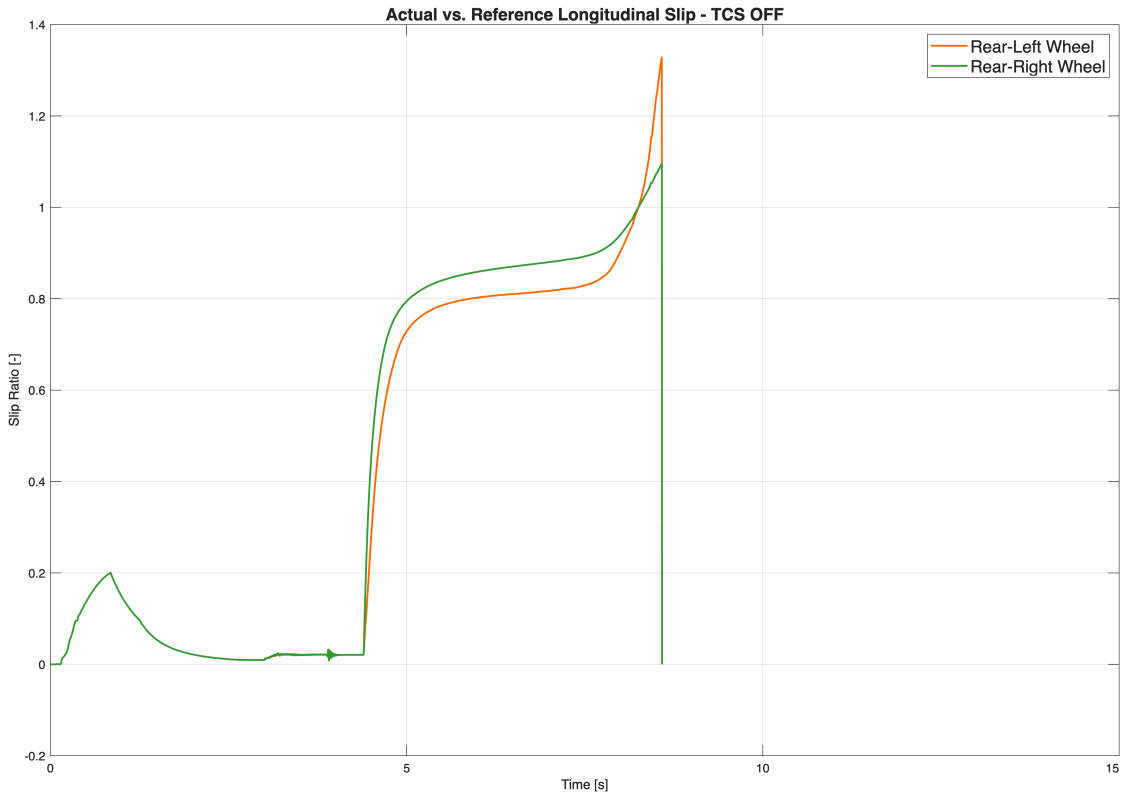


Figure 6.61: Actual vs. reference longitudinal slip with traction control OFF.

Both slip curves experience an immediate exponential growth at $t = 4.5$ s. Left wheel slip shoots straight to the maximum saturation boundary of 1.0 within less than 4 s from the transition point. Right wheel follows a slightly delayed but equally unstable trajectory, reaching full spinning condition at the same time.

The physical consequences of this uncontrolled spinout on force generation are recorded in Figure 6.62, which tracks the utilized road friction coefficient.

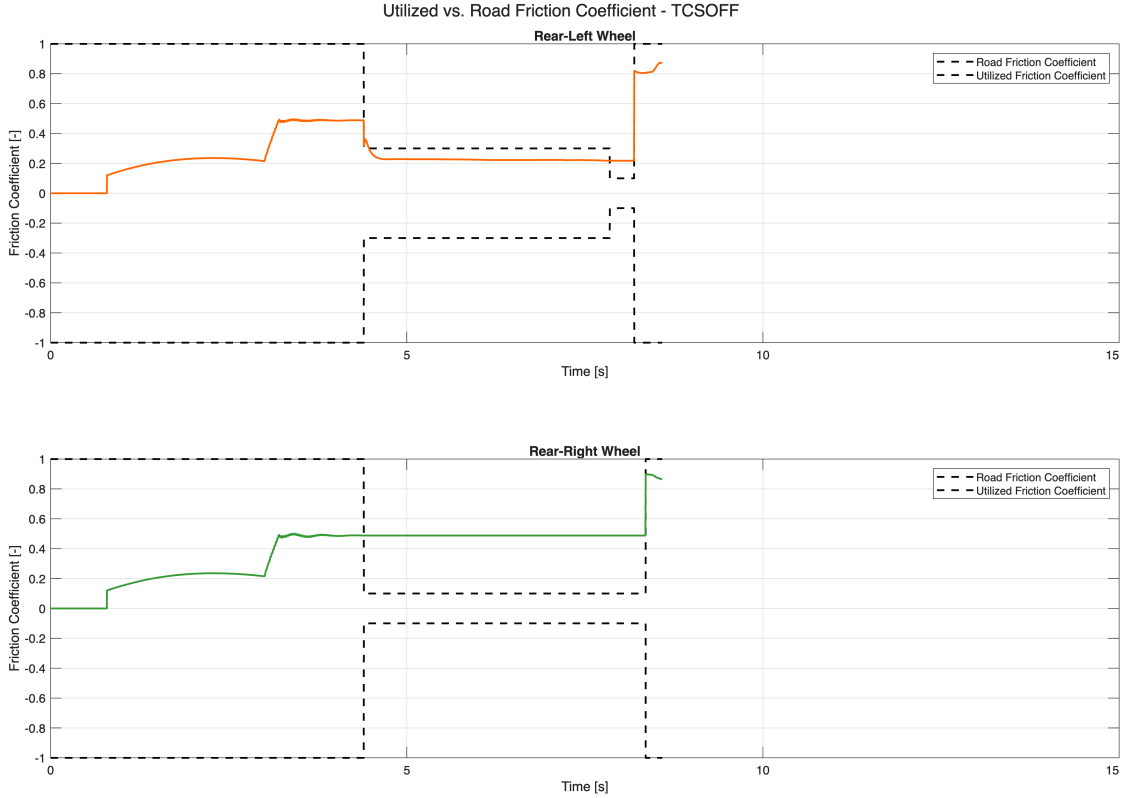


Figure 6.62: Utilized vs. road friction coefficient with traction control OFF.

Because the tire operates within the unstable region of the $\mu - \sigma$ curve, the available tractive force continuously degrades. Furthermore, the massive difference in force production between the right and the left track introduces a severe, uncontrolled yaw moment across the chassis. In a physical vehicle, this highly asymmetric force distribution would instantly spin the car out of its lane.

6.5.2 PI Controller

This section evaluates the tracking capabilities, transient errors, and structural limitations of the PI architecture during the highly asymmetric μ -split event.

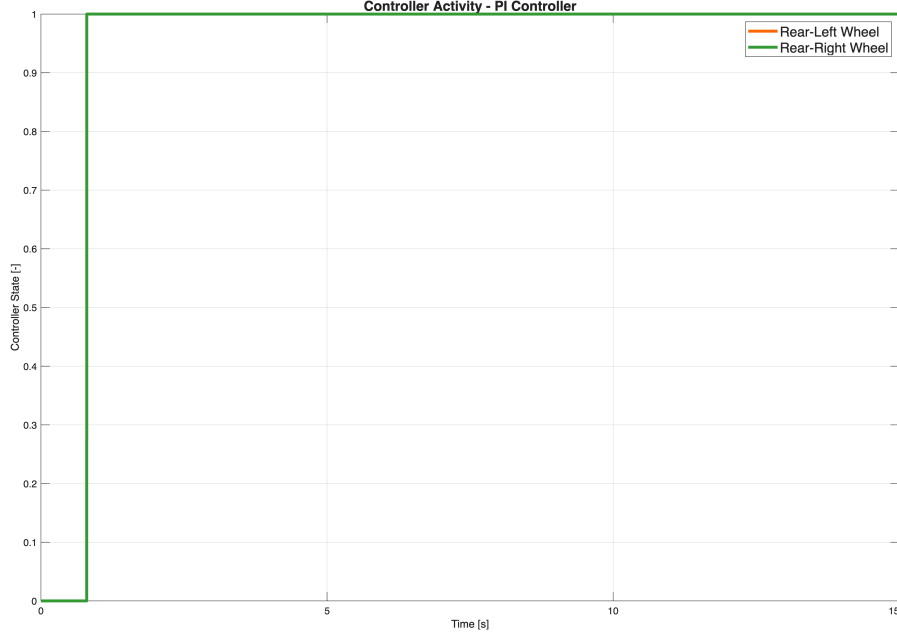


Figure 6.63: PI controller activity.

On dry asphalt ($\mu=1.0$), both wheels operate below the activation slip threshold, keeping controller's status at 0. The moment the vehicle crosses the friction boundary, the sudden slip departure causes controller's activation at $t \approx 1$.

Figure 6.64 and 6.65 capture the resulting vehicle and wheels tangential velocities and longitudinal slip profiles.

For the rear-left wheel ($\mu = 0.3$), at the friction bump the reference slip target steps down to 0.0935. The actual slip shoots upward dramatically, peaking at an undershoot of 0.07 and overshoot of 0.1290 at $t \approx 1$ s before the integral gain accumulates enough error to decelerate the wheel.

For the rear-right wheel the reference slip target steps down to 0.0568. It undergoes a smaller, yet distinct overshoot up to 0.09.

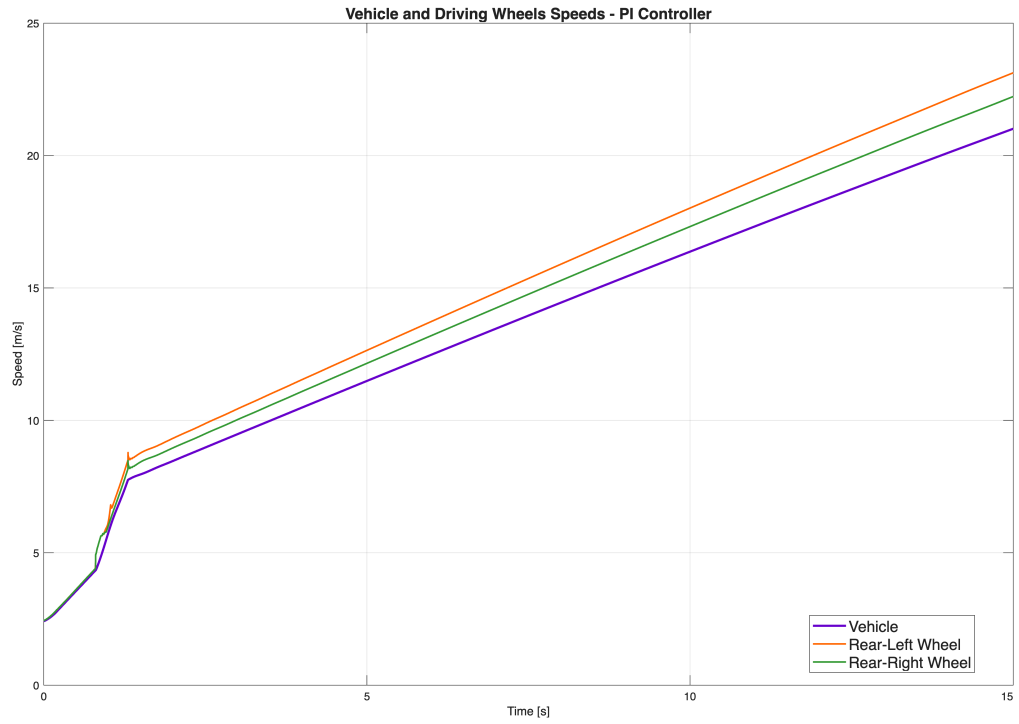


Figure 6.64: Vehicle and driving wheels' speed with PI.

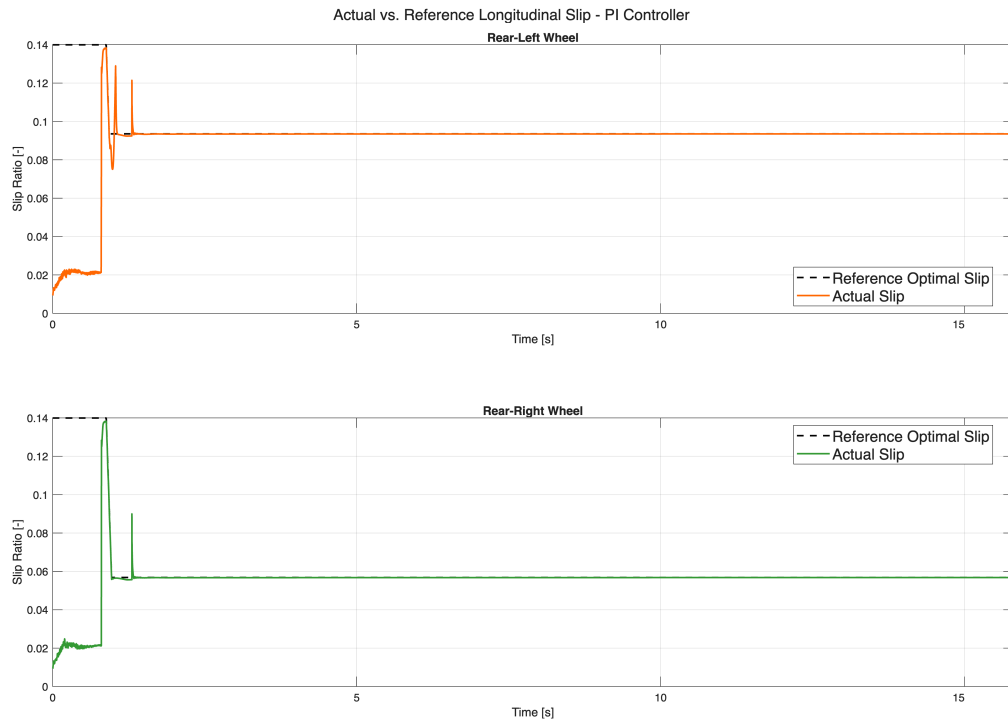


Figure 6.65: Actual vs. reference longitudinal slip with PI.

Both slip metrics eventually converge to their reference target by $t \approx 1.4$ s. However, because the slip targets for the surfaces are unequal, a continuous wheel speed difference develops. By $t = 6$ s, the rear-right wheel travels at roughly 13.7 m/s while the rear-left wheel at 13.2 m/s to maintain that optimal slip ratio on the slicker road patch.

Controller's tracking deviations are reported in Figure 6.66.

Mirroring the slip plots, the longitudinal slip tracking error spikes to +0.017 and -0.035 on the left wheel and -0.033 on the right wheel. The corresponding physical wheel speed error peaks at 0.375 and -0.9 rad/s for rear-left and -1 rad/s for rear-right.

Tracking error stabilizes after 0.8 seconds, lying flat for the rest of the simulation.

Figure 6.67 illustrates the resulting utilized friction coefficient generated at each tire contact patch.

Once the PI loop stabilizes the slip onto the optimal target ($t > 1.5$ s), the utilized friction curves settle onto two distinct, flat plateaus: rear-left wheel stabilizes perfectly at $\mu = 0.30$, fully exploiting the maximum tractive capacity of the snow track, while for the rear-right wheel the controller manages to maximize friction utilization up to $\mu \approx 1$.

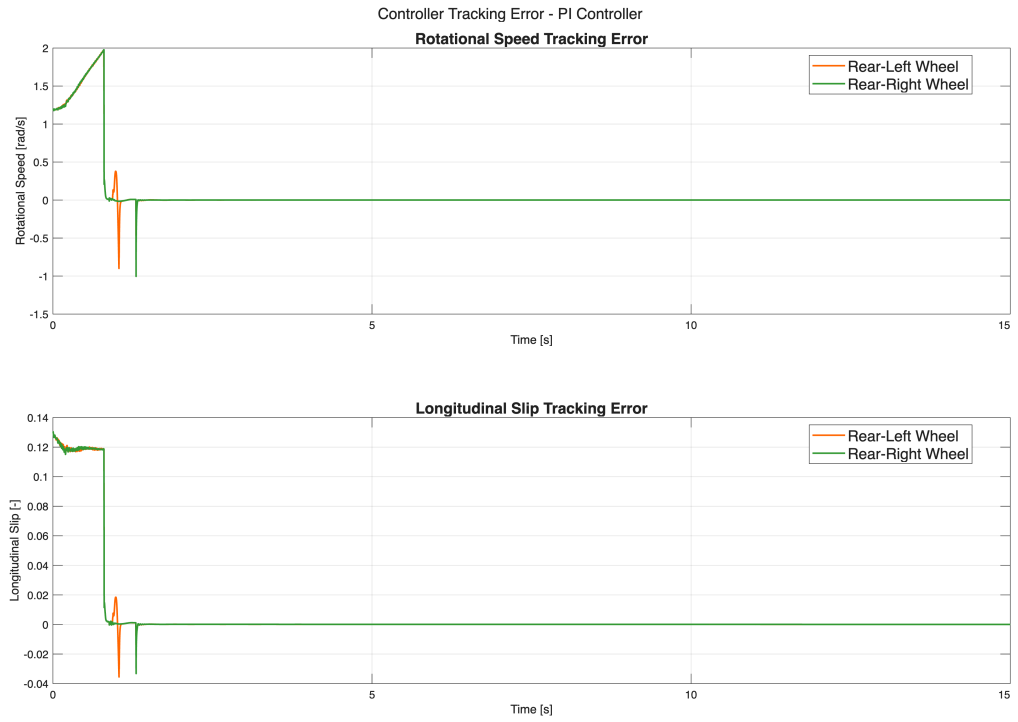


Figure 6.66: PI controller tracking error.

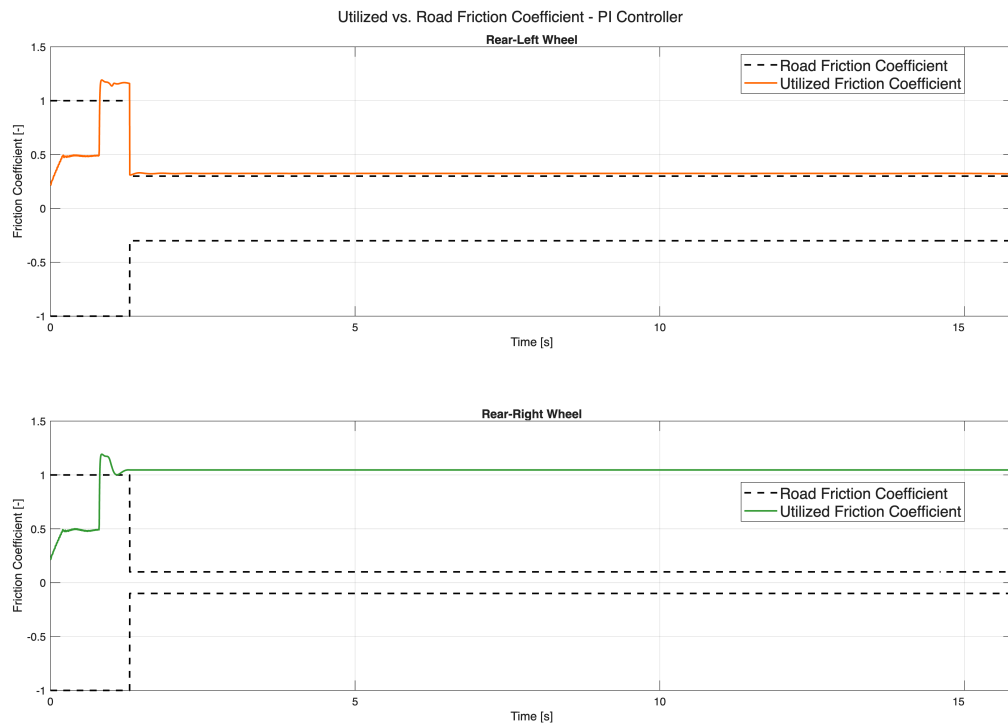


Figure 6.67: Utilized vs. road friction coefficient with PI.

Controller's Output

The internal torque composition plot in Figure 6.68 clearly shows the decoupled operation of the anti-windup loops.

At the instant of activation, the primary Control Torque for both wheels spikes instantly proportionally to the slip error. T_{cmd} saturates, reaching feasibility limits for both wheels.

Limit overcoming is more aggressive in the case of the left wheel, for which Compensation Torque (T_{cmps}) doesn't manage to bring the final Control Torque (T_{ctrl}) within the predefined boundaries.

The rear-right channel experiences a less severe saturation window, demanding a smaller contribution from its matching anti-windup compensation path.

After this brief transition phase, output torque stabilizes to 435 Nm for the left and 143 Nm for the right wheels.

Figure 6.69 tracks the resulting target and actual electromagnetic torques. Both electric motors track their respective controller commands with negligible lag.

On the ultra-low-grip right track ($\mu = 0.1$), the motor stabilizes to provide a holding torque of approximately only 20 Nm.

On the low-grip left track ($\mu = 0.3$), the torque must be limited less severely, holding a steady-state value of 50 Nm.

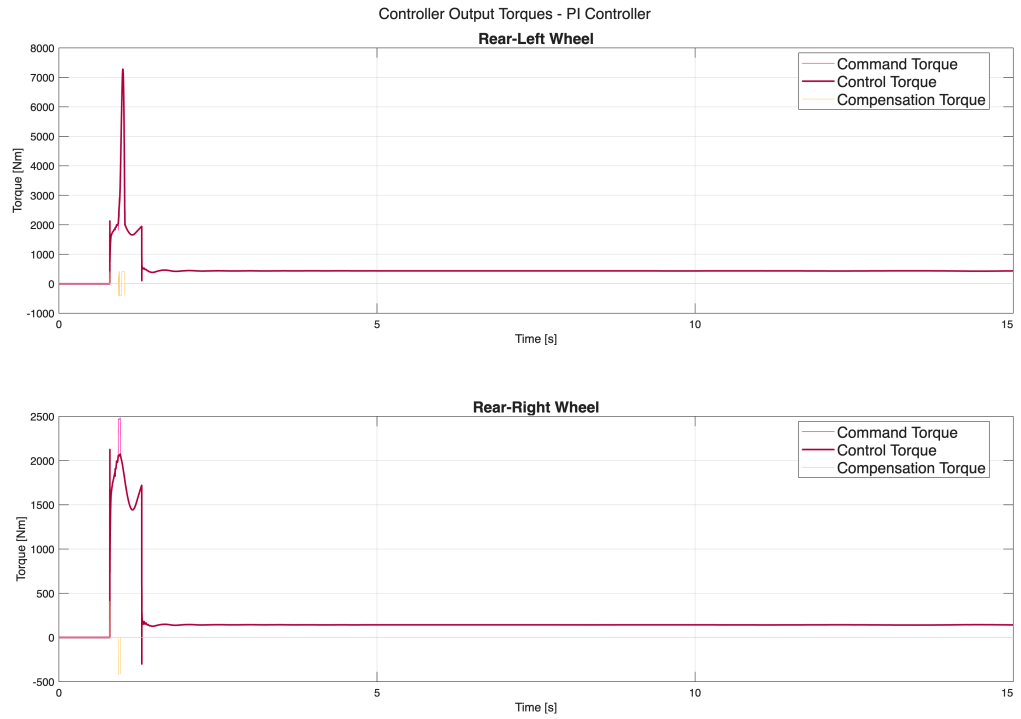


Figure 6.68: PI controller's output torques.

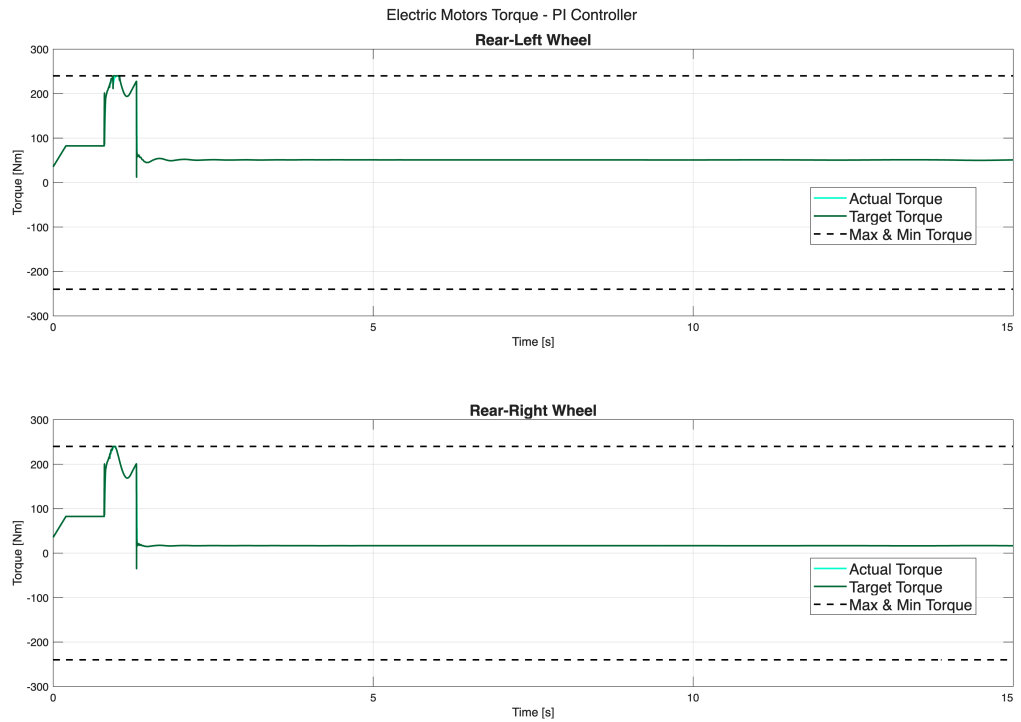


Figure 6.69: Electric motors torques with PI controller.

6.5.3 TSM Controller

This section investigates the closed-loop performance of the TSMC architecture under the asymmetric μ -split condition.

The activation history of the control loop is shown in Figure 6.70.

On the majority of the dry asphalt section ($t < 1$ s), both wheels operate within the stable linear elastic micro-slip regime, holding the activation flags flat at 0 (inactive). At $t \approx 1$ controller switches to full engagement (1) simultaneously on both wheels.

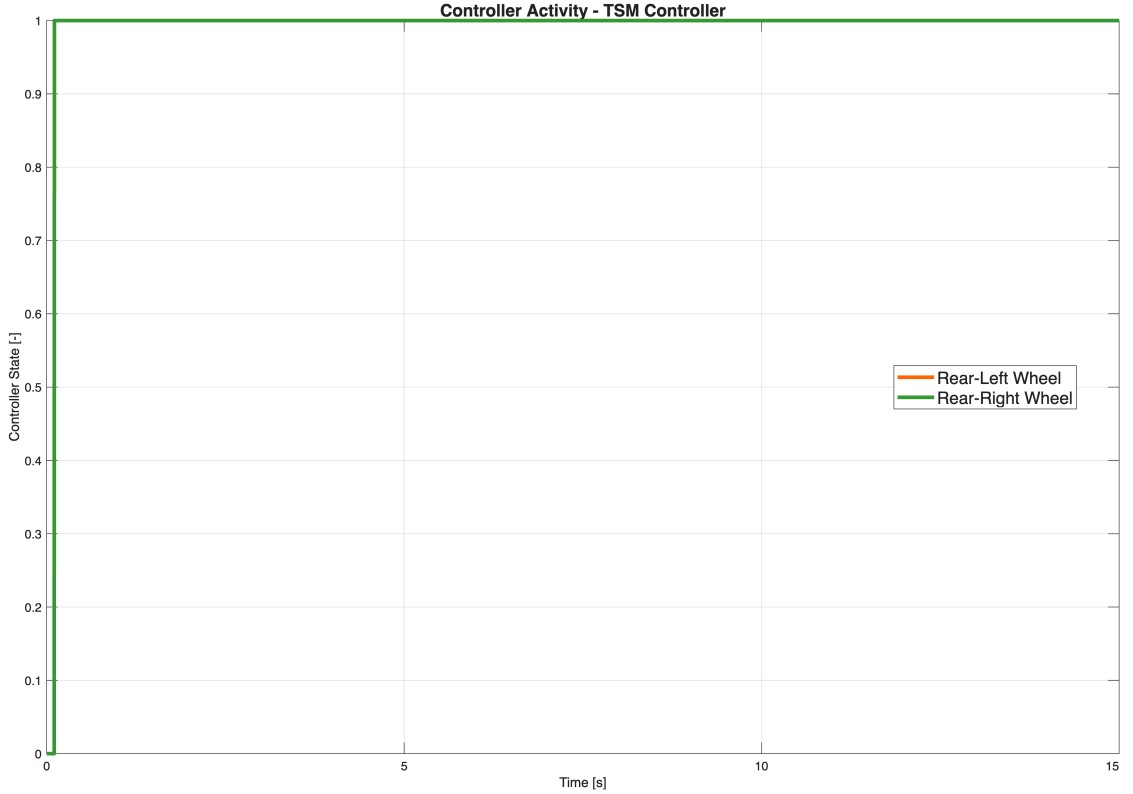


Figure 6.70: TSM controller activity.

Figure 6.71 and 6.72 detail the vehicle speed, wheels tangential speeds and longitudinal slip behaviors.

The moment the vehicle encounters the friction bump ($t \approx 1$ s), the TSMC demonstrates its robust sliding invariance. Unlike the PI controller, there is no large macroscopic wheel speed runaway or delayed recovery, and both driving wheels are aggressively constrained by the switching logic.

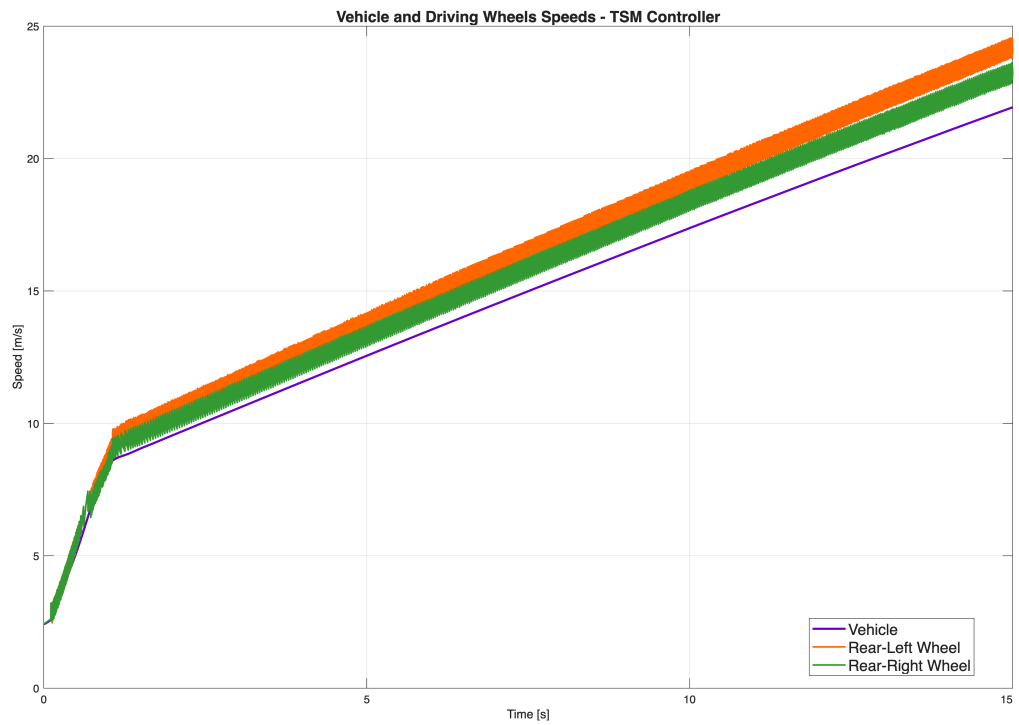


Figure 6.71: Vehicle and driving wheels' speed with TSMC.

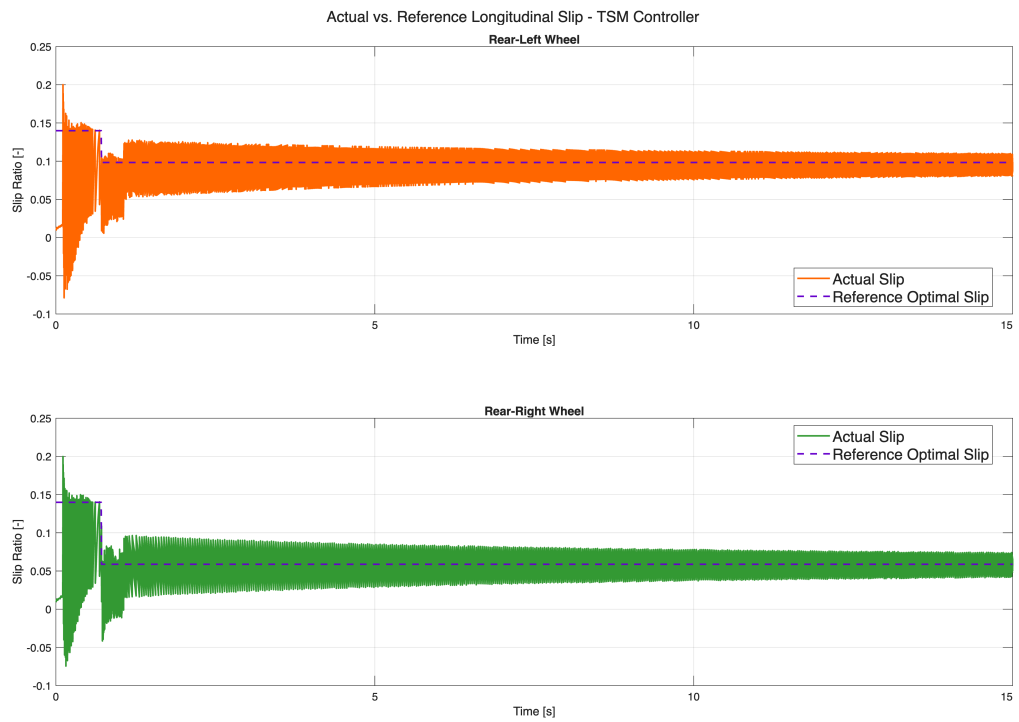


Figure 6.72: Actual vs. reference longitudinal slip with TSMC.

The actual longitudinal slip trajectories for the rear-left and rear-right wheels converge onto their target line within milliseconds, experiencing a very minor, transient overshoot that is quickly contained (≈ 0.2).

While macroscopic tracking is tight, both wheel speed lines and slip lines exhibit a distinct high-frequency chattering across the sliding surface $s = 0$.

Controller's tracking performance is reported in Figure 6.73.

At the transition point ($t \approx 1$ s), the slip tracking error begins its reaching phase, exhibiting an isolated positive impulse bounded to a maximum of $+0.07$ on both tracks, initiating the sliding mode phase after only 0.8 seconds.

Outside of this initial transition peak, the errors do not settle to a smooth zero line. Instead, the slip tracking error chatters continuously with a persistent average amplitude of roughly ± 0.02 .

Figure 6.74 reports the resulting utilized friction coefficient on the two asymmetric tracks.

TSMC manages to achieve a maximized road capacity utilization.

The rear-left tire drops onto a perfectly flat plateau slightly above 0.30, fully exploiting the maximum tractive capability of the track.

The rear-right tire stabilizes onto a higher plateau at approximately 0.8, maximizing adherence in the lowest-friction segment.

Both plateaus are superimposed with a low-amplitude, high-frequency noise ripple matching the underlying slip chattering frequency.

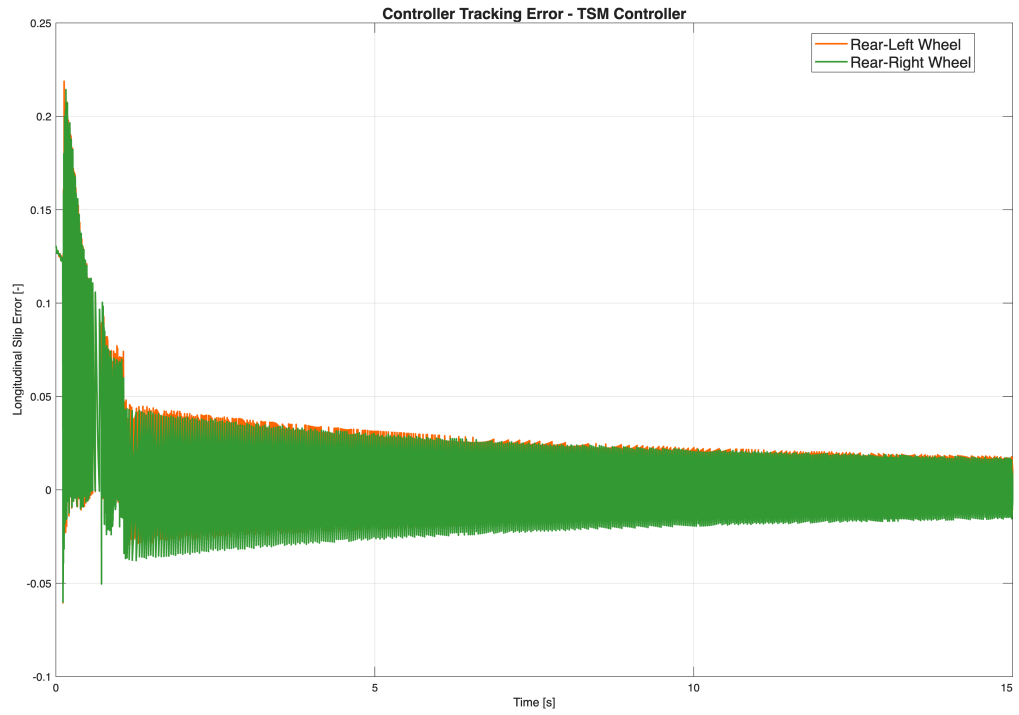


Figure 6.73: TSM controller tracking error.

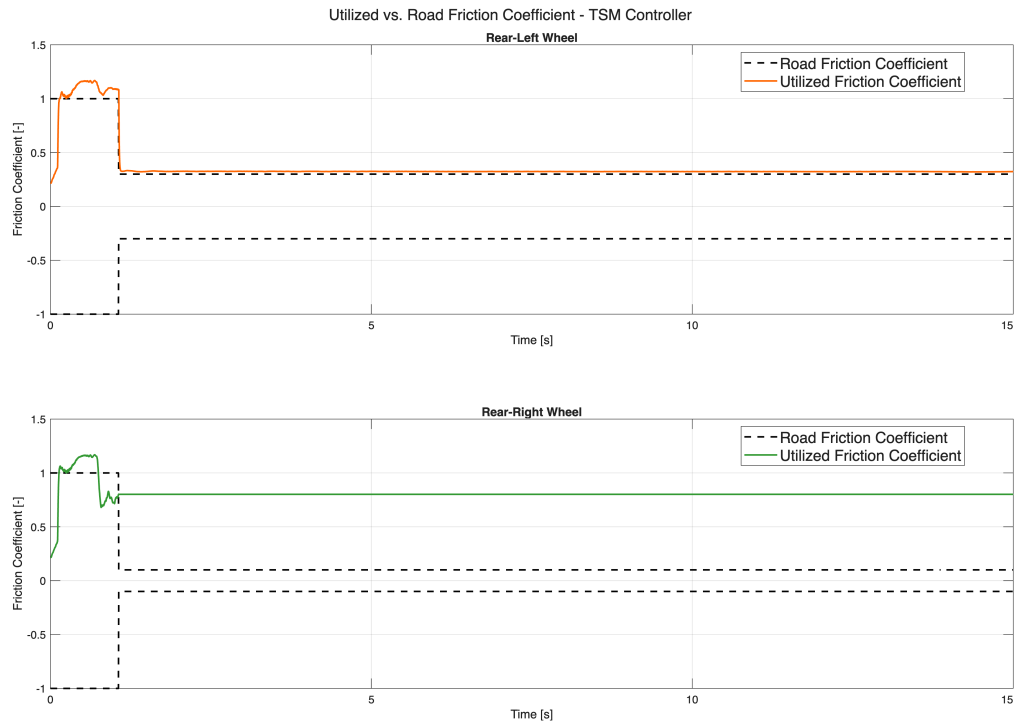


Figure 6.74: Utilized vs. road friction coefficient with TSMC.

Controller's Output

As shown in the torque decomposition plot in Figure 6.75, the primary Control Torque commands switch violently between their upper and lower limits.

When hitting the friction bump, the controller successfully decouples the macroscopic torque operating points to match the unequal road limits.

The average torque request is forced into a wide chattering band that oscillates heavily between 6100 and -1900 Nm, with higher frequency of oscillations for the left wheel.

As testified by Figure 6.76, electric motors are forced to execute these massive, high-frequency oscillations.

While the TSMC effectively manages the asymmetric traction drop and eliminates the severe transient overshoots, the resulting actuator chattering represents an unacceptable physical penalty.

High-frequency torque switching of this magnitude across both axles would generate heavy drivetrain vibrations and severe mechanical stress on the half-shafts and reduction gears, directly highlighting the need for a boundary-layer smoothing technique.

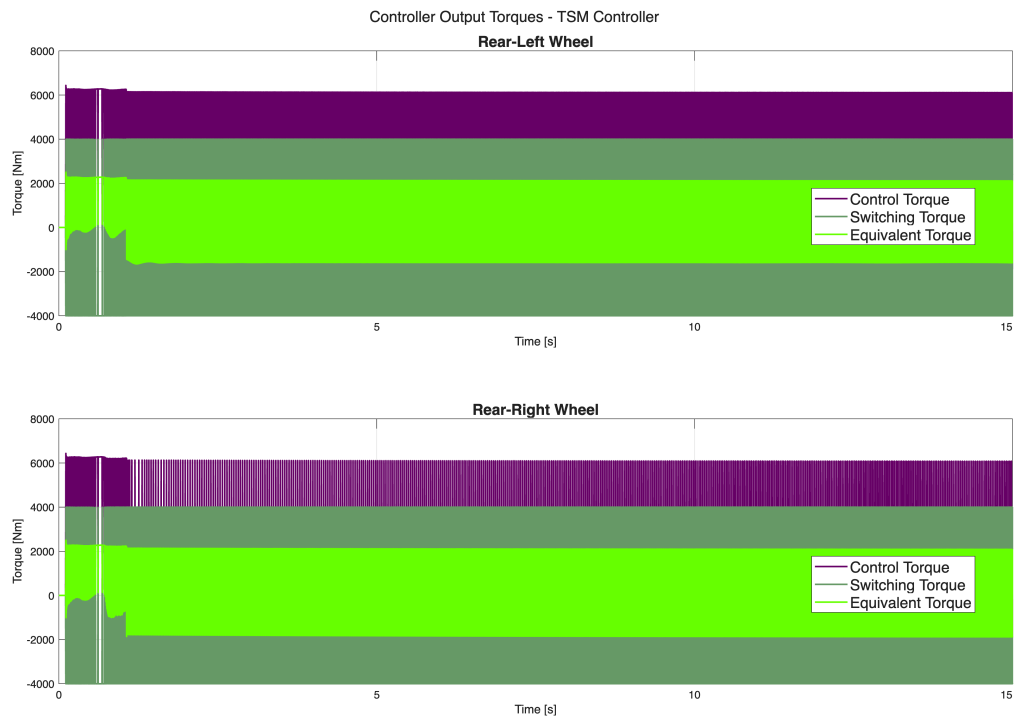


Figure 6.75: TSM controller output torques.

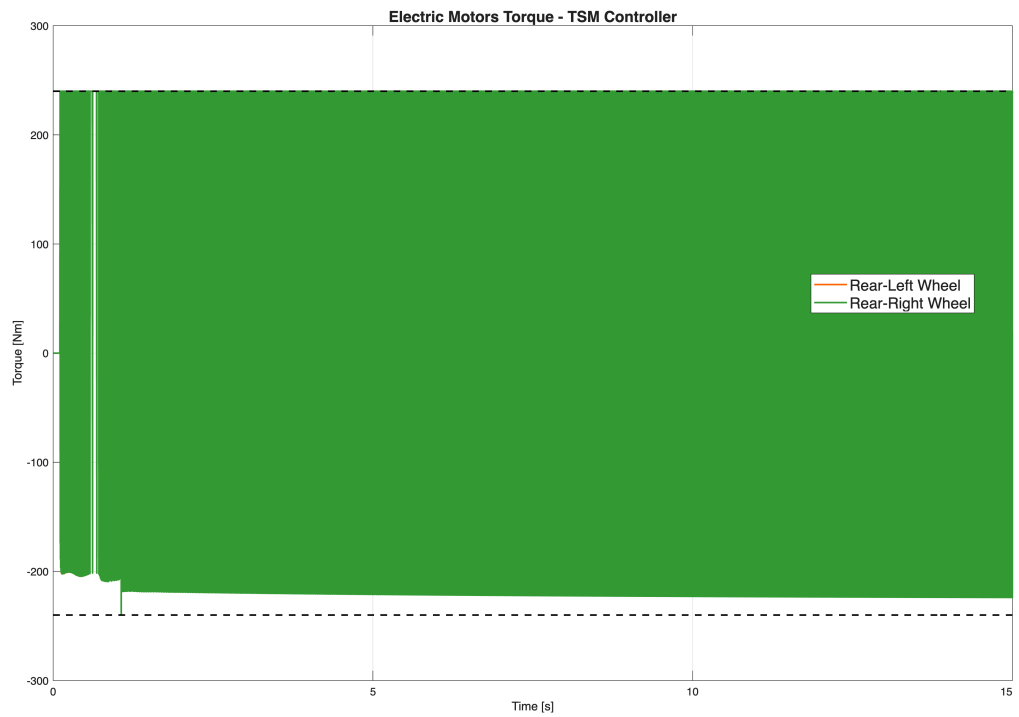


Figure 6.76: Electric motors torque with TSM controller.

Controller's Stability

Sliding surface trajectories are collected in Figure 6.77.

The sliding variable s oscillates symmetrically directly around the $s = 0$ axis for the entire duration of the active phase, with amplitude ± 0.022 .

Concurrently, the Lyapunov derivative chatters rapidly across the zero line, alternating between ± 0.4 . This confirms that while the system remains globally stable and bounded, the discontinuous $\text{sign}(s)$ term prevents asymptotic convergence.

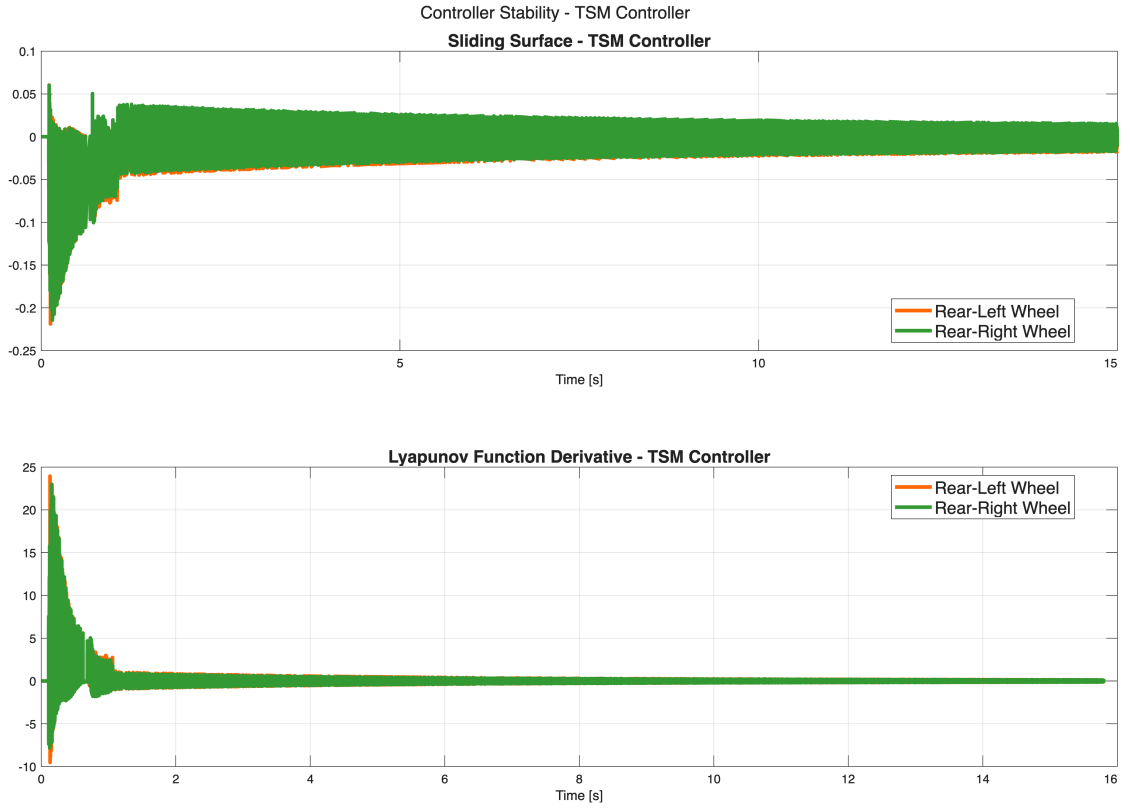


Figure 6.77: TSM controller stability.

6.5.4 CISM Controller

This section analyzes the performance of the proposed CISM strategy under the asymmetric μ -split maneuver.

The activation timing of each wheel controller is tracked in Figure 6.78.

On the dry asphalt section, both wheels operate within the stable linear regime, keeping the activation flags flat at 0. At $t \approx 1.5$ s, the sudden loss of road friction forces the slip ratio to cross the activation threshold, shifting the flags instantly to 1. Both control channels remain fully engaged for the remainder of the simulation window.

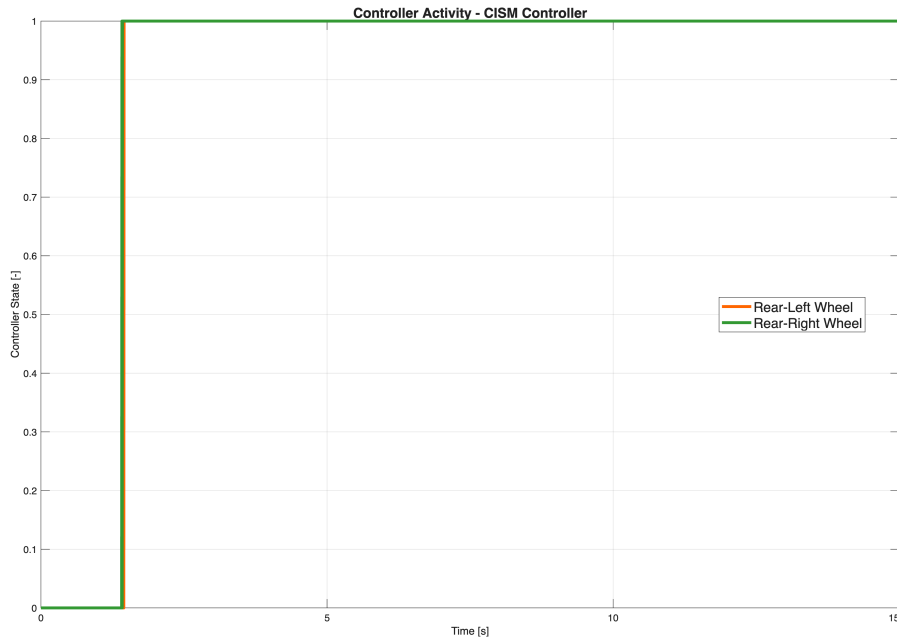


Figure 6.78: CISM controller activity.

Figure 6.78 and 6.79 illustrate the vehicle and wheels tangential velocities and longitudinal slip tracking trajectories.

The CISM layout handles the split friction with remarkable damping capability. The driven wheel velocities experience almost zero macroscopic overshoot or uncontrolled spinout, safely matching the vehicle velocity trajectory.

As shown in Figure 6.80, the reference slip targets for both wheels step down to 0.1038 (left) and 0.0604 (right).

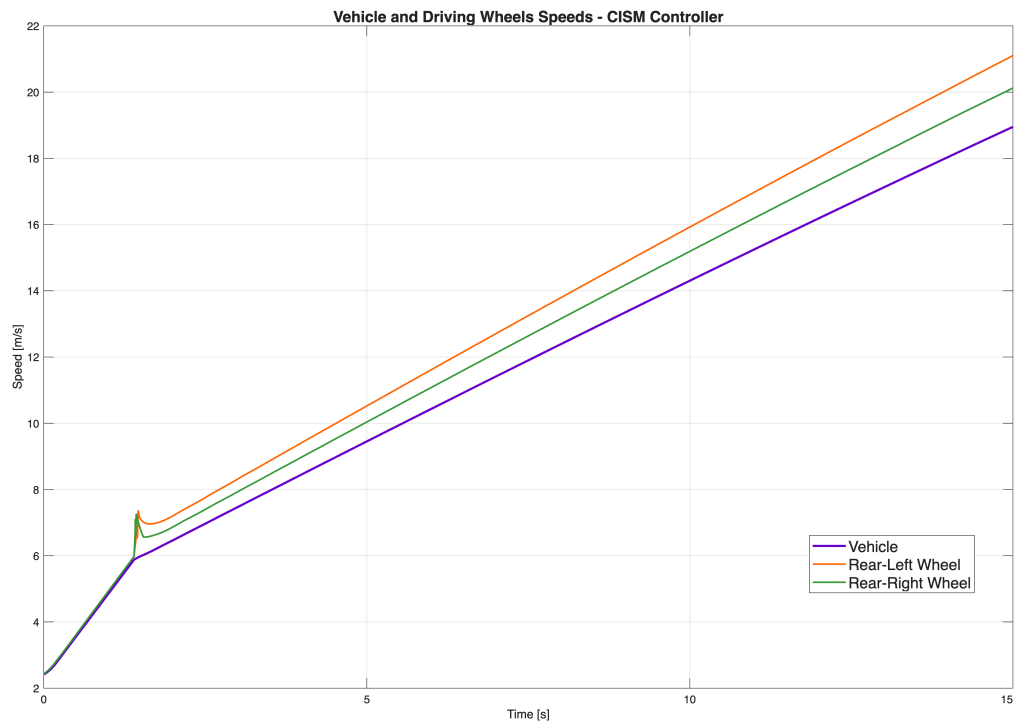


Figure 6.79: Vehicle and driving wheels' speed with CISM.

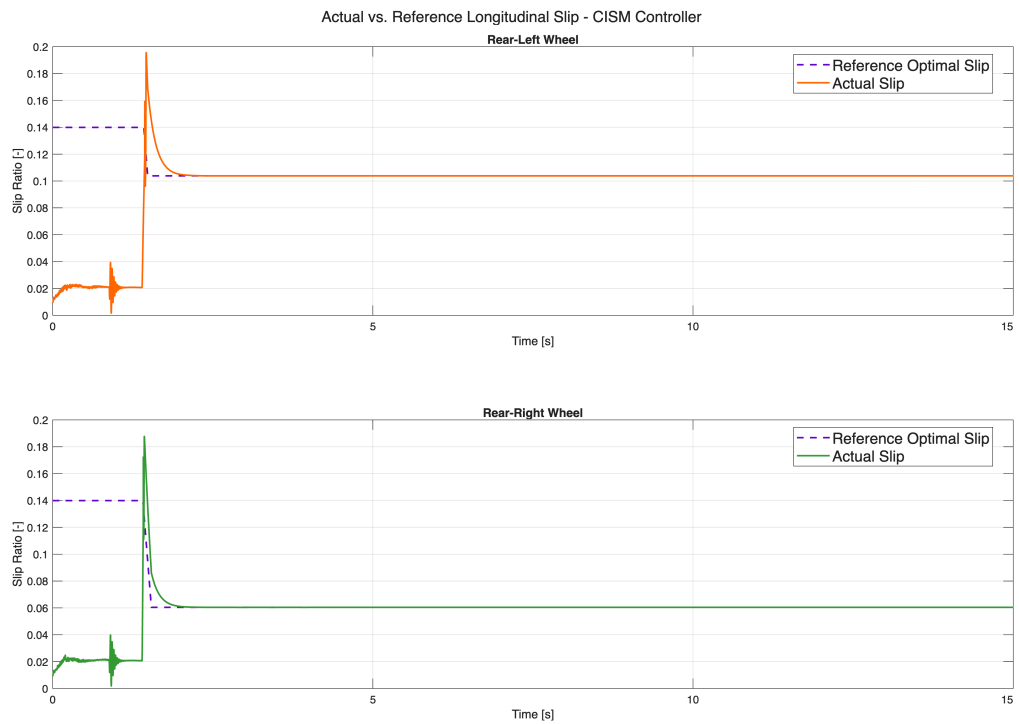


Figure 6.80: Actual vs. reference longitudinal slip with CISM.

Driven by the non-linear sliding law, the actual longitudinal slip trajectories, after initial overshoots up to 0.1958 (for the rear-left) and 0.1878 (for the rear-right), converge exponentially onto the reference lines within a 0.6 seconds.

Crucially, unlike the fuzzy and wide oscillation bands seen under traditional SMC, both the wheel velocities and longitudinal slip lines remain perfectly thin, smooth, and ripple-free during all active phases.

Controller tracking performance is reported in Figure 6.81.

At the initial friction drop ($t \approx 1.5$ s), the longitudinal slip tracking error experiences an isolated, brief overshoot to 0.0781 on both wheels, matching the robust transient performance of the TSMC but without its steady-state chattering and noise.

The key benefit of the conditional integration block is seen after the initial overshoot. Once the transient perturbations are rejected, the slip tracking error converges directly to a flat, noise-free absolute zero line.

By allowing the integral term to operate within the boundary layer, the controller completely eliminates the persistent chattering error observed in the traditional SMC layout.

By keeping the slip tracking tightly locked onto the optimal reference, the CISMIC ensures a perfectly smooth friction profile, perfectly matching the road friction coefficients.

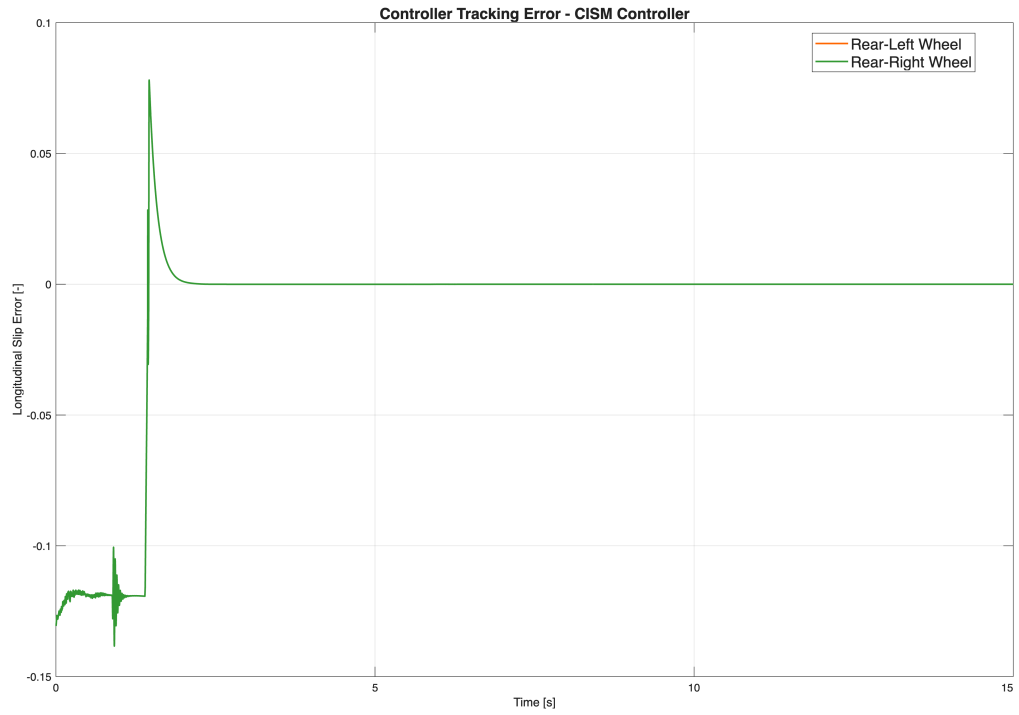


Figure 6.81: CISM controller tracking error.

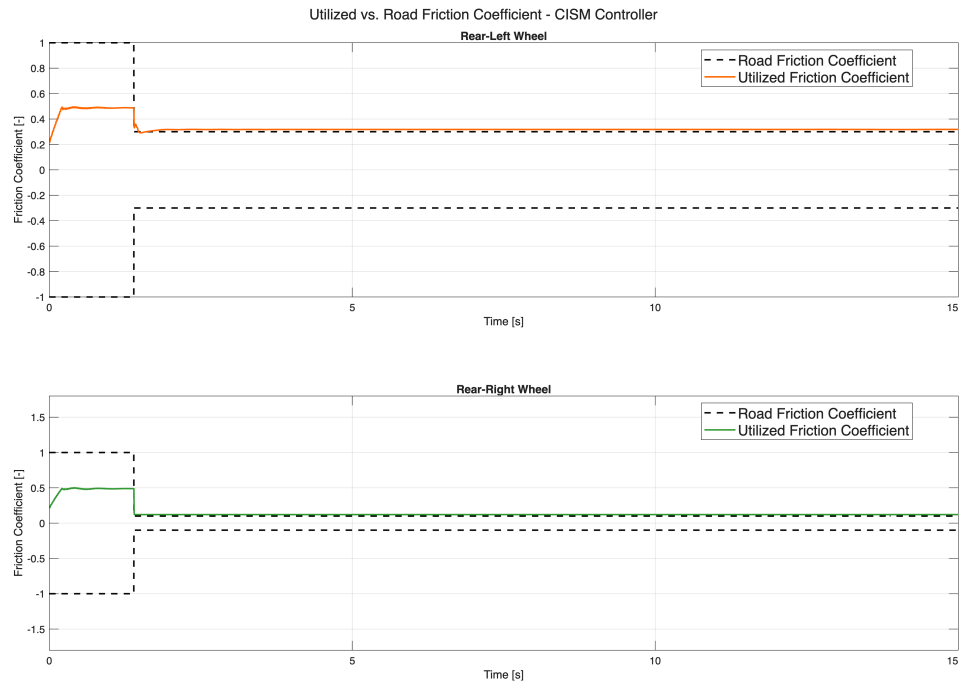


Figure 6.82: Utilized vs. road friction coefficient with CISM.

Controller's Output

The underlying control inputs driving this controller's performance are reported in Figure 6.83 and 6.84.

The primary Control Torque commands behave as smooth, continuous signals. The conditional integration block manages the actuator bounds safely; during the initial transition window, the torque request spikes to its upper limit, where the anti-windup logic freezes the integrator to prevent numerical saturation growth. Afterwards, torque request stabilizes at 140 Nm for right wheel and 426 Nm for left wheel, coherently to the higher traction capability at $\mu = 0.3$.

The resulting actual electromagnetic torques track the respective controller's target with negligible lag.

On the low-grip left track, electric motor torque stabilizes at 50 Nm, while on the ultra-low-grip right track the torque is limited severely, holding a perfectly smooth steady-state value of only 16.5 Nm.

Most importantly, the huge high-frequency switching bands seen under traditional SMC are completely eliminated, as the actual motor torque lines remain perfectly smooth and quiet through the entire maneuver.

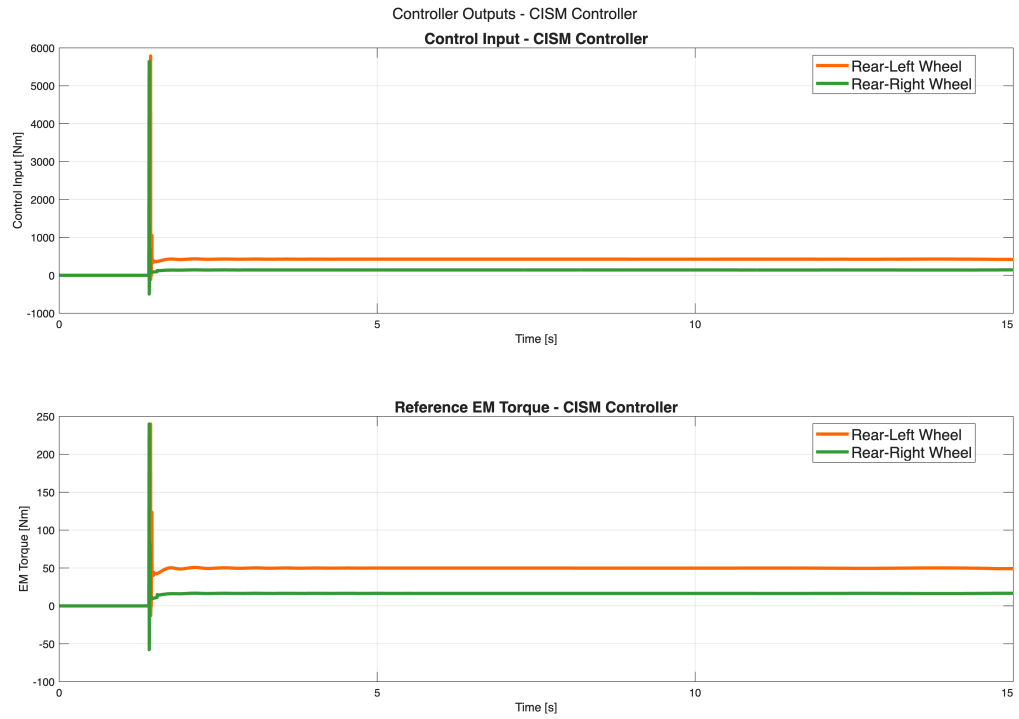


Figure 6.83: CISM controller output torques.

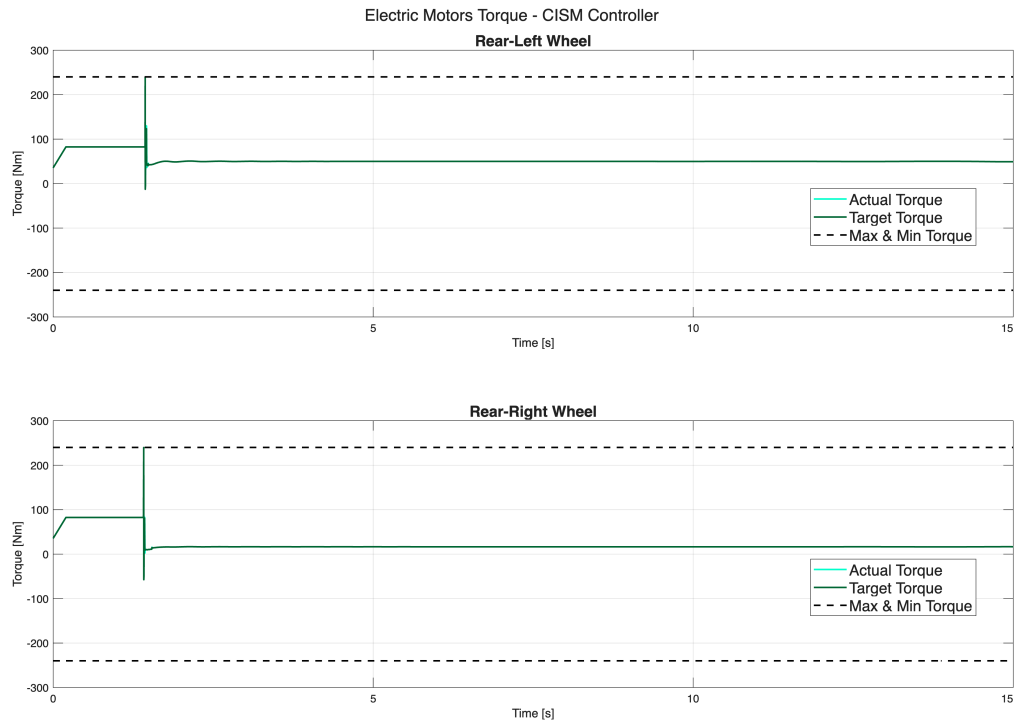


Figure 6.84: Electric motors torque with CISM controller.

Controller's Stability

Figure 6.85 tracks the behavior of the internal system dynamics approximation functions f and g .

Both functions mimics the dynamics of the vehicle's maneuver. In correspondence of the friction bump, they adapt smoothly to capture the changing wheel slip acceleration trends on the snow and ice sections.

Because these model-based feedforward terms handle the non-linear plant dynamics, the auxiliary sliding law only needs to reject minor residual errors, keeping the total actuator demand stable.

The numerical analysis of controller's stability is visualized in Figure 6.86, tracking the sliding surface s .

Considering the time window during which the controller intervenes, the sudden slip perturbation in correspondence of the friction bump causes an overshoot up to 0.17. This spike is suppressed instantaneously, allowing the tracking states to enter the continuous boundary layer ($|s| \ll \varphi$) so that the discontinuous switching law is smoothly interpolated inside this layer, allowing the controller to completely eliminate the high frequency chattering seen in traditional SMC.

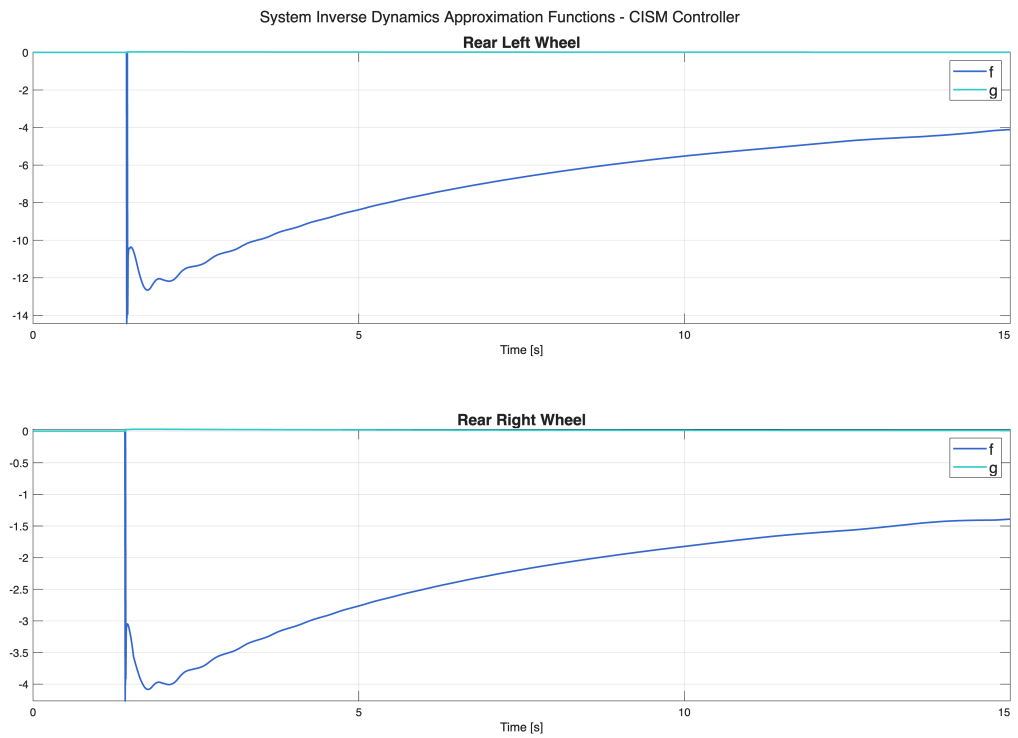


Figure 6.85: CISM system inverse dynamics functions.

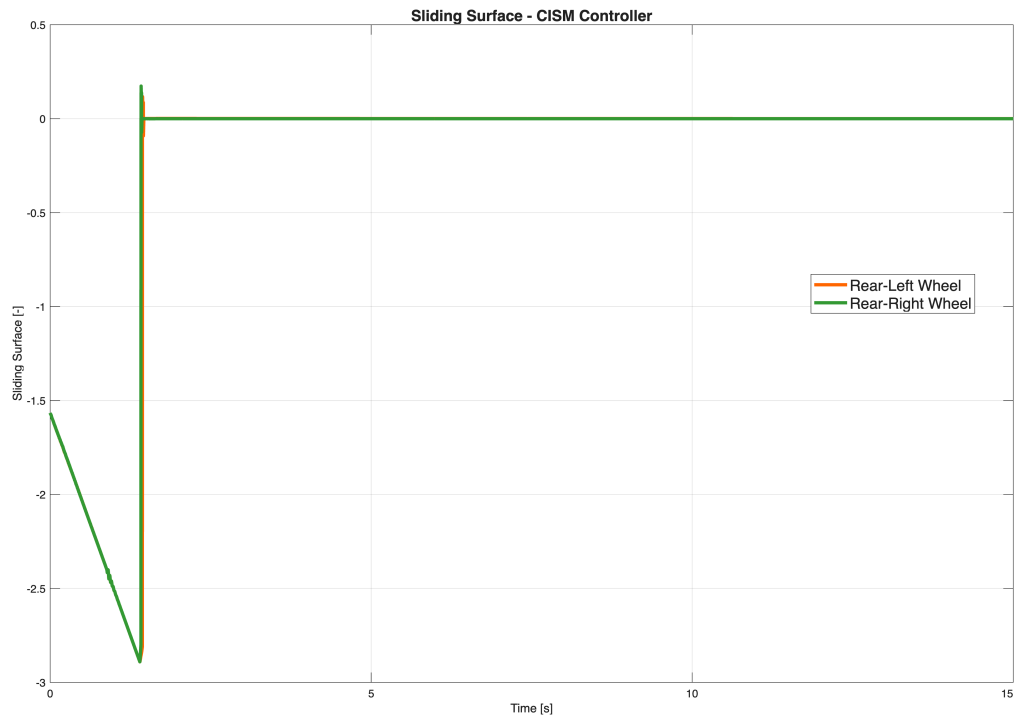


Figure 6.86: CISM controller stability.

6.6 Performance Comparison

This section provides a comprehensive comparative analysis of the three control architectures evaluated throughout this thesis.

While the previous section examined each controller independently under specific transient conditions, this comparative discussion evaluates their global performance across the three selected maneuvers.

The objective is to evaluate the fundamental trade-offs inherent to each control methodology. Specifically, the analysis focuses on the balance between transient robustness under non-linear system variations, steady-state regulation accuracy, and the physical feasibility of the control inputs, together with the overall vehicle's performance improvement. By assessing these characteristics simultaneously, this section demonstrates how the architectural enhancements introduced in the CISMCMC successfully reconcile the limitations of both classical linear control and conventional non-linear sliding mode techniques.

Table 6.1 resumes and collects the parameters selected during controller's design, for each architecture implemented.

Table 6.1: Parameters and gains of the implemented controllers (PI, TSMC, and CISMCMC).

	Symbol	Value
PI Controller		
Proportional Gain	k_p	2000
Integral Gain	k_i	300
TSMC Controller		
Sliding Gain	K	4000
Braking Sliding Gain	K_{brake}	2000
CISMCMC Controller		
Sliding Gain traction	K_1	8
Sliding Gain braking	$K_{1\text{brake}}$	350
Reaching Gain traction	K_2	15
Reaching Gain braking	$K_{2\text{brake}}$	50
Boundary Layer traction	φ	0.02
Boundary Layer braking	φ_{brake}	0.06

6.6.1 Low Friction Maneuver

When the full acceleration step is commanded, the open-loop system (TCS OFF) encounters an immediate, catastrophic wheel spinout, with the longitudinal slip rapidly saturating at $\sigma = 1$. This response confirms that the available engine torque vastly exceeds the physical tractive capacity of the road.

When active control is engaged, all three architectures (PI, TSMC, and CISMIC) demonstrate high effectiveness in suppressing the slipping runaway state. As evidenced by the longitudinal slip tracking trajectories, each controller successfully intervenes at the exact onset of the acceleration step without any visible operational delay. Both the linear PI loop and the non-linear sliding mode variants force the longitudinal slip curves to intercept and lock onto the optimal target slip ($\sigma_{opt} = 0.1038$).

The traditional sliding mode controller (Fig. 6.19) fails to achieve a smooth, asymptotic convergence. Due to the discontinuous switching nature of the sign function, the actual longitudinal slip profile is trapped in a high-frequency chattering cycle.

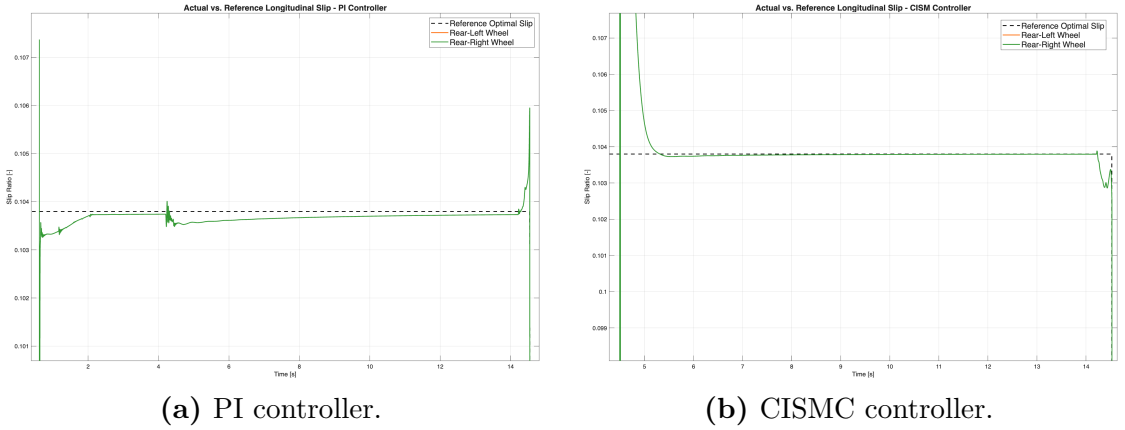


Figure 6.87: Zoomed-in longitudinal slip comparison, traction.

The PI controller (Fig. 6.87a) provides a cleaner steady-state longitudinal slip signal than the TSMC. Because it is a linear loop relying on continuous error accumulation, it does not induce high-frequency chattering. The slip profile appears as a thin line; however, it exhibits minor low-frequency waves and tracking deviations due to its pure reliance on reactive gains without non-linear plant compensation.

The CISMIC (Fig. 6.87b) demonstrates its superiority by combining the advantages of both layouts. The actual longitudinal slip trajectory tracks the reference target as an exceptionally thin, smooth, and ripple-free line. By implementing a boundary layer dual logic, the high-frequency chattering seen in TSMC is completely

suppressed. Simultaneously, the conditional integration block actively suppresses residual unmodeled plant mismatches, forcing the tracking error to a noise-free absolute zero line that outperforms the PI's low-frequency tracking variance.

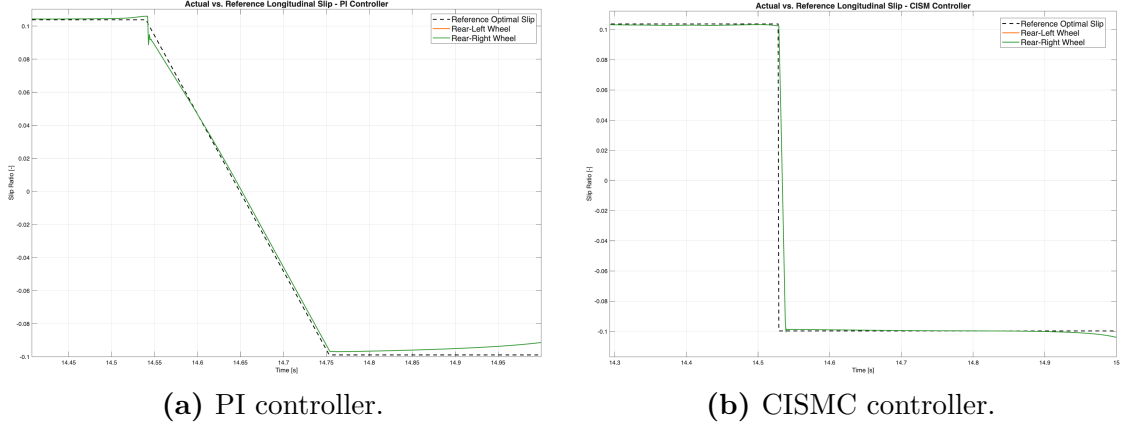


Figure 6.88: Zoomed-in longitudinal slip comparison, braking.

In the PI layout (Fig. 6.88a), the transition down to the braking reference line introduces a visible settling delay. The linear controller requires a finite time to process the sudden error sign inversion and discharge its integrator history, resulting in a slightly wider transient trajectory before locking back onto the negative reference.

In the CISM layout (Fig. 6.88b), the step-down is executed with extreme precision. The moment the target drops, the sliding surface variable s exits the boundary layer, immediately freezing the integrator and switching the control law back to its high-gain sliding mode reaching state. This non-linear drive pulls the slip down exponentially. Once inside the negative boundary layer, the conditional integrator re-engages smoothly, ensuring that the actual slip locks onto the braking target with zero undershoot or secondary settling waves, maintaining optimal deceleration tracking until the simulation is ended.

The parameters, summarized in Table 6.2, isolate the specific operational trade-offs across the simulation.

The macroscopic kinematic metrics, specifically **Top Speed** (V_{vhcl}^{max}) and distance required to bring the vehicle speed to 0.8 m/s normalized by the top speed (in the table referenced to as **Normalized Braking Distance**) highlight how traction management directly translates into vehicle performance.

The impact of integration management is explicitly evident during the deceleration phase initiated at $t \approx 14.5$ s. The CISM minimizes the normalized braking

distance yielding a direct safety improvement over the classical PI architecture, with a reduction of 0.26%.

The quality of the regulation loop is measured via the **Root Mean Square Error** (RMSE) of the longitudinal slip and the **Settling Time** evaluated under a strict $\pm 1\%$ tolerance band.

Even though the CISMIC achieves steady-state convergence slower than PI, the linear PI framework requires almost 5s to fully decay its initial setting oscillations and clamp perfectly onto the reference target.

TSMC settling time is not specified, as the oscillations never fully lay within the $\pm 1\%$ tolerance band.

The CISMIC minimizes the tracking error, yielding a low RMS value(0.0054) normalized with respect to σ_{peak} comparable to the PI (0.0011), and extremely lower than the TSMC (0.0294).

Moreover, **Torque Standard Deviation** is extremely high for the TSMC, which is a behavior representative of the chattering phenomenon characterizing this control law, while it's kept the lowest for the CISMIC.

To qualitatively evaluate how friction generation is managed at the tire-road interface, it's possible to analyze the maximum and average friction deviation indexes (**Max $\Delta\mu$** and **Average $\Delta\mu$**).

While CISMIC manages to achieve results comparable to the PI strategy, the average is strongly influenced by the initial overshoot at the regulation start, which decreases the overall percentage value, bringing the wheel to a higher slip condition, hence losing adherence at the tire-road interface.

Low- μ Maneuver Comparative Results

Performance Indicator	PI	TSMC	CISMC
Normalized Braking Distance [m]	0.7814	0.7838	0.7794
Top Speed [m/s]	20.5964	21.1458	18.5244
Avg. Longitudinal Acceleration [m/s ²]	1.5584	1.5720	1.5720
Settling Time [s]	0.015	-	0.2940
<i>Wheel Slip & Friction</i>			
Slip RMS [-]	0.0011	0.0294	0.0054
σ_{peak} [-]	0.1074	0.1953	0.1707
Max $\Delta\mu$ - Rear Left [%]	20.4199	17.4330	17.5712
Avg $\Delta\mu$ - Rear Left [%]	6.7141	8.8810	5.7574
Max $\Delta\mu$ - Rear Right [%]	20.4199	17.4330	17.5712
Avg $\Delta\mu$ - Rear Right [%]	6.7141	8.9103	5.7574
<i>Torque & Actuation Metrics</i>			
Torque Std. Deviation - Rear Left [Nm]	4.8792	224.2332	2.6084
Torque Std. Deviation - Rear Right [Nm]	4.8793	224.2257	2.6084
Chattering Frequency [Hz]	-	409.8814	-
Control Effort [(Nm) ²]	3.06×10^9	1.54×10^{11}	2.98×10^9

Table 6.2: Performance comparison between PI, TSMC, and CISMC controllers during Low- μ simulation.

Finally, with comparable or better results, CISMC guarantees a reduction in **Control Effort** (energy of the signal computed as $\|u\|_{L_2}^2$) of -98% with respect to TSMC and -2.6% with respect to PI.

6.6.2 Mu-jump Maneuver

When the sequential multi-step friction transitions are encountered under a continuous acceleration demand, the open-loop system immediately suffers from severe instability at the first friction dropout boundary. The sudden mismatch between the requested drivetrain torque and the collapsed tractive capability causes the longitudinal slip to uncontrollably diverge to $\sigma = 0.8$.

When active control is engaged, all three architectures (PI, TSMC, and CISMIC) demonstrate high adaptability in rejecting these abrupt surface disturbances.

Both the linear PI loop and the CISMIC force the longitudinal slip curves to intercept and track the step-changing optimal targets corresponding to each road segment, while the TSMC fails to achieve a smooth convergence.

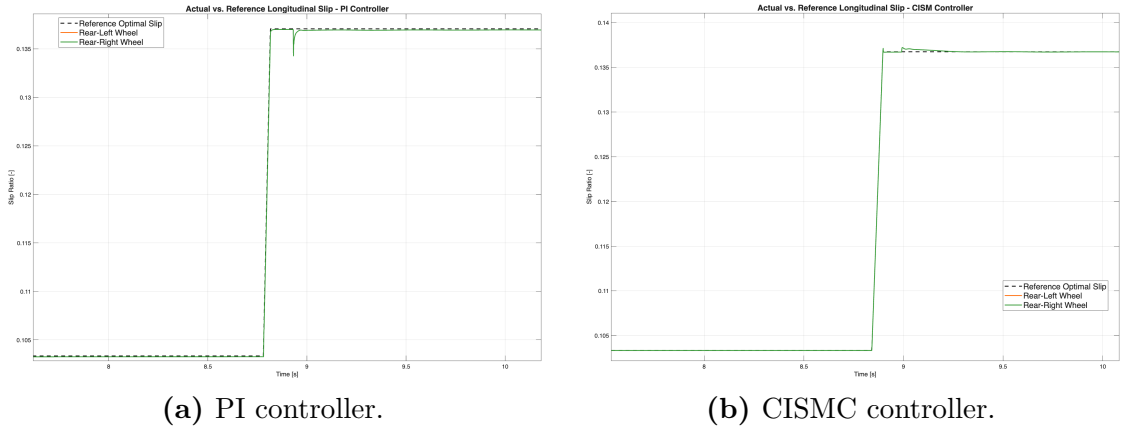


Figure 6.89: Zoomed-in longitudinal slip comparison.

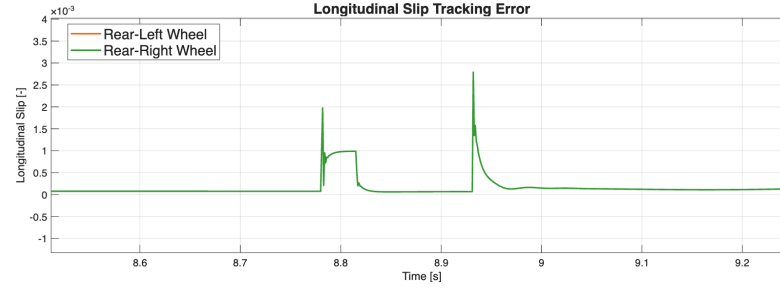
The performance indexes, systematically organized in Table 6.3, break down the specific control capabilities during non-uniform road travel.

While TSMC strategy allows to reach highest **Top Speed** and **Average Acceleration**, PI and CISMIC is comparable, with lowest **Settling Time** for the PI case - difference probably given by the fact that the CISMIC is disadvantaged by the presence of a higher overshoot (σ_{peak}) at the beginning of the regulation.

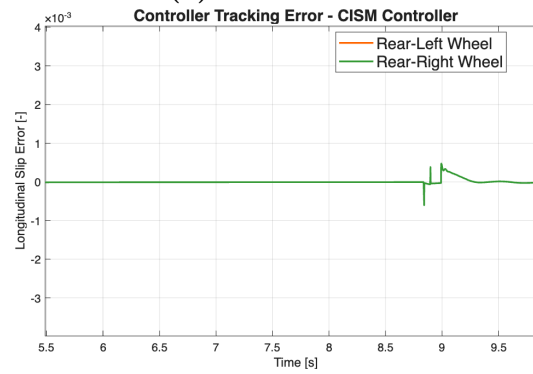
However, the linear PI regulation relies entirely on continuous error accumulation without non-linear plant compensation, which makes it reactive and prone to prolonged low-frequency setting waves during sharp friction step variations.

Figure 6.90 highlights the tracking error curve in correspondence of the friction jump, for the PI and CISMIC cases. Here it's visible how CISMIC manages to keep

the tracking error lower under surface adherence variation, manifesting a higher adaptability to system's variability.



(a) PI controller.



(b) CISM controller.

Figure 6.90: Zoomed-in tracking error comparison.

Since the primary physical limitation of sliding mode strategies is their compatibility with real electromagnetic actuators, this engineering trade-off can be quantified by the **Torque Standard Deviation**.

As predictable, TSMC displays the highest value among the three controllers, while CISM manages to keep it the lowest, providing a solution to the structural limitations of the TSMC, and a performance improvement with respect to PI.

To mathematically evaluate how effectively each control loop exploits the available adherence limit across the transient friction steps, the maximum and average friction deviation indexes (**Max** $\Delta\mu$ and **Average** $\Delta\mu$) are analyzed, certifying how closely the actual tire operational point matches the peak of the local friction curve.

CISM manages to achieve higher tire-road adherence and friction utilization with respect to the linear PI controller, manifesting more efficient traction exploitation at the road.

μ -Jump Maneuver Comparative Results

Performance Indicator	PI	TSMC	CISMC
Top Speed [m/s]	48.4583	60.9005	48.0719
Avg. Longitudinal Acceleration [m/s ²]	2.3289	2.9046	2.3064
Settling Time [s]	0.011	-	0.319
<i>Wheel Slip & Friction Metrics</i>			
Slip RMS [-]	1.9161×10^{-4}	0.0192	0.0040
σ_{peak} [-]	0.1370	0.1860	0.1796
Max $\Delta\mu$ - Rear Left [%]	13.5266	273.5032	14.4233
Avg $\Delta\mu$ - Rear Left [%]	7.1305	7.8987	6.9777
Max $\Delta\mu$ - Rear Right [%]	13.5266	273.5033	14.4233
Avg $\Delta\mu$ - Rear Right [%]	6.9104	7.9701	6.7602
<i>Torque & Actuation Metrics</i>			
Torque Std. Deviation - Rear Left [Nm]	24.7233	207.0892	24.0148
Torque Std. Deviation - Rear Right [Nm]	25.8619	208.3752	25.0774
Chattering Frequency [Hz]	-	409.9878	-
Control Effort [(Nm) ²]	1.14×10^{10}	3.49×10^{11}	1.12×10^{10}

Table 6.3: Performance comparison between PI, TSMC, and CISMC controllers during μ -Jump simulation.

Finally, with comparable or better results, CISMC guarantees a reduction in **Control Effort** (energy of the signal computed as $\|u\|_{L_2}^2$) of -97% with respect to TSMC and -1.75% with respect to PI.

These collected quantitative indicators mathematically validate the architectural superiority of the CISMC, which breaks the structural trade-off between non-linear robustness and actuator longevity under highly transient friction conditions.

6.6.3 Mu-split Maneuver

In this scenario, the left and right wheels encounter drastically different friction conditions at the road interface, generating a massive destabilizing yaw moment that tests the differential tracking performance of the powertrain.

When the jumps in friction are encountered by the wheels, the open-loop system immediately suffers from severe instability, uncontrollably reaching full spin condition in few seconds.

As evidenced by the longitudinal slip tracking trajectories (Fig. 6.65, 6.72, 6.80), each controller successfully handles the sudden grip variations, preventing runaway wheel spin during drops in friction and avoiding prolonged slip undershoots when the road surface grip partially recovers.

The performance metrics which allow to provide a comparative analysis of the three controllers tested are reported in Table 6.4.

As for the simulations discussed previously, the TSMC allows to reach highest top speed, while CISMV vehicle performance is comparable to PI one, also in terms of average acceleration.

μ -Split Maneuver Comparative Results

Performance Indicator	PI	TSMC	CISMV
Top Speed [m/s]	21.7326	22.6205	19.6712
Avg. Longitudinal Acceleration [m/s ²]	0.9670	0.9561	0.9612
Settling Time [s]	0.51	-	0.29
<i>Wheel Slip & Friction Metrics</i>			
Slip RMS [-]	0.0018	0.0221	0.0040
σ_{peak} [-]	0.1383	0.2004	0.1958
Max $\Delta\mu$ - Rear Left [%]	19.1808	260.1806	15.2440
Avg $\Delta\mu$ - Rear Left [%]	8.5150	8.3081	5.6937
Max $\Delta\mu$ - Rear Right [%]	946.3363	701.5474	20.7246
Avg $\Delta\mu$ - Rear Right [%]	902.2839	642.4977	20.7246
<i>Torque & Actuation Metrics</i>			
Torque Std. Deviation - Rear Left [Nm]	29.1013	223.5607	2.8698
Torque Std. Deviation - Rear Right [Nm]	32.4636	229.4097	0.5686
Chattering Frequency [Hz]	-	411.6896	-
Control Effort [(Nm) ²]	6.51×10^9	1.86×10^{11}	2.63×10^9

Table 6.4: Performance comparison between PI, TSMC, and CISMV controllers during μ -Split simulation.

During the μ -Split maneuver the CISM C succeeds in obtaining the lowest settling time and **Slip RMS** among the three control strategies.

Best results in terms of friction utilization are achieved by the PI and TSMC, but the CISM C stays comparable, guaranteeing anyway a substantial improvement for traction at the tire-road interface.

Major strength points of the CISM C strategy are reported for what concerns the **Torque and Activation Metrics**.

CISM C allows for the lowest **Torque Standard Deviation**, which is one order of magnitude lower than the PI and two orders of magnitude lower of the TSMC.

This architectural superiority is confirmed also by the **Control Effort** metrics. CISM C is realized with the lowest control effort between all three control strategies implemented, with a reduction of -99% with respect to TSMC and -60% with respect to PI.

6.7 Robustness to Parameters Variability

Since in physical automotive applications control robustness against plant parameter variations is a critical factor for vehicle safety and stability, and real-time parameters such as the effective wheel radius, longitudinal tire forces and vertical loads fluctuate continuously due to centrifugal tire growth, tread wear, and load transfer during acceleration or braking, to test the structural resilience and superiority of the proposed CISM control loops, a sensitivity analysis was conducted.

For the three maneuvers exposed in the previous sections, all three controllers have been analyzed under the bounded variability of the following quantities reported in Table 6.5.

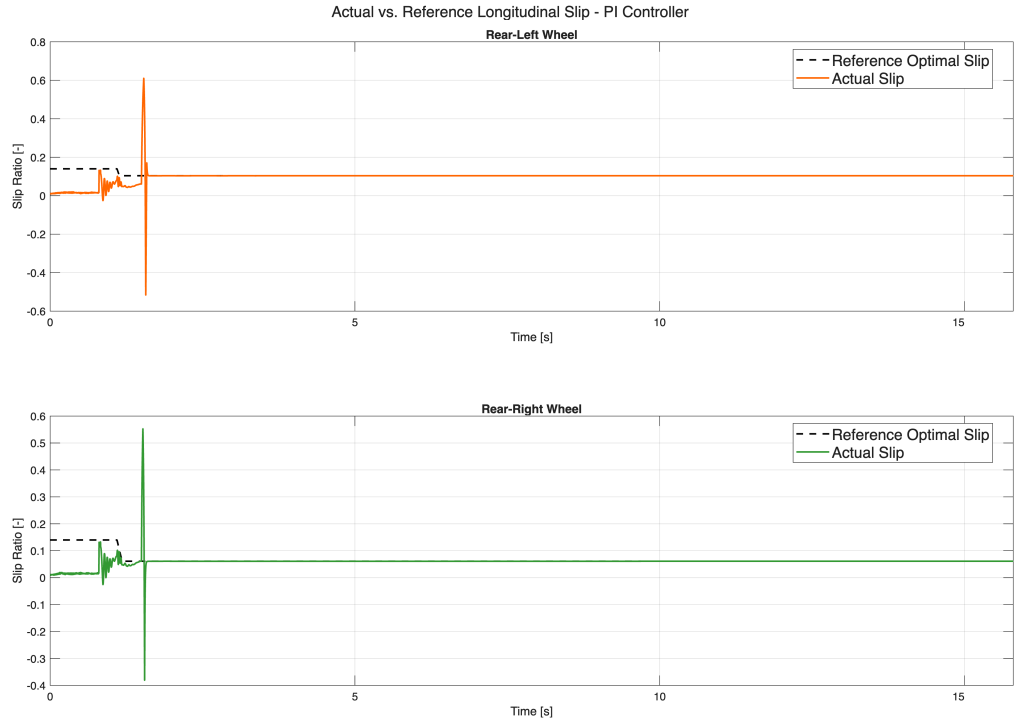
Parametric Variations			
Physical Parameter	Nominal	Minimum	Maximum
Effective Wheel Radius [m]	0.343	0.2944	0.3046
Longitudinal Tire Force [N]	Dynamic	−1374	6593
Vertical Load [N]	Dynamic	3252	4487

Table 6.5: Range of parametric perturbations applied during simulations.

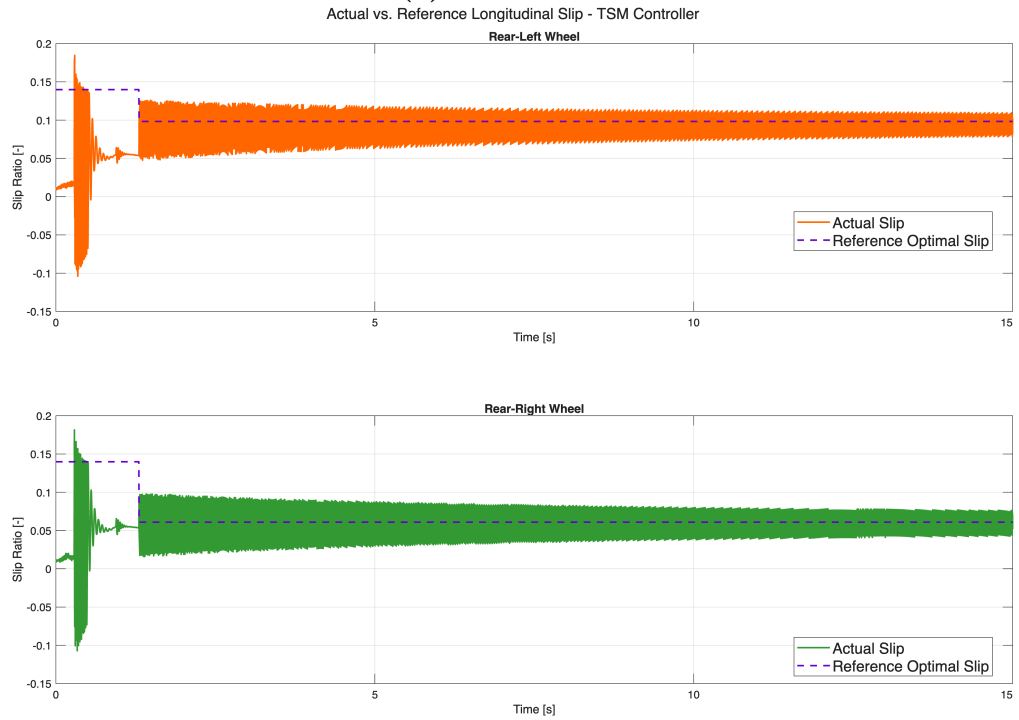
As demonstrated by the evaluation conducted across Sections 6.3, 6.4 and 6.5, all controllers demonstrated a satisfactory capability to adapt to the abovementioned parameters variability, without any performance degradation. Hence, to further investigate controllers' robustness, the asymmetric μ -split simulation campaign, recognized as the most dynamically critical operational profile due to its severe cross-track disturbance, was replicated for all three controllers, considering an increase of the total vehicle mass from its nominal value of 1600 kg up to an overloaded condition of 2100 kg (a +31.25% mass increase). This severe unmodeled plant perturbation directly shifts the vehicle's longitudinal inertia, and consequently the system's dynamics and response.

The resulting longitudinal slip profiles are illustrated in Figure 6.91a and 6.91b for the PI and TSMC, and in Figure 6.92 for the proposed CISM solution.

As evidenced by the longitudinal slip curves in the PI simulation plot, the linear architecture undergoes a severe performance degradation under non-nominal mass conditions. Because the PI loop relies solely on fixed feedback gains without inertia compensation, the altered vehicle momentum completely saturates the error-integration capacity.



(a) PI controller.



(b) TSM controller.

Figure 6.91: Longitudinal slip profile comparison with $m = 2100$ kg.

The actual slip on the low-adherence patch exhibits high-amplitude, low-frequency tracking waves, drifting aggressively away from the optimal reference line and failing to prevent localized wheel spinning. This unstable behavior demonstrates that the linear framework cannot guarantee vehicle drivability when the plant parameters stray significantly from the nominal calibration point.

Conversely, the traditional TSMC proves its inherent non-linear robustness by forcing the longitudinal slip to remain bounded around the optimal target despite the 500 kg mass mismatch.

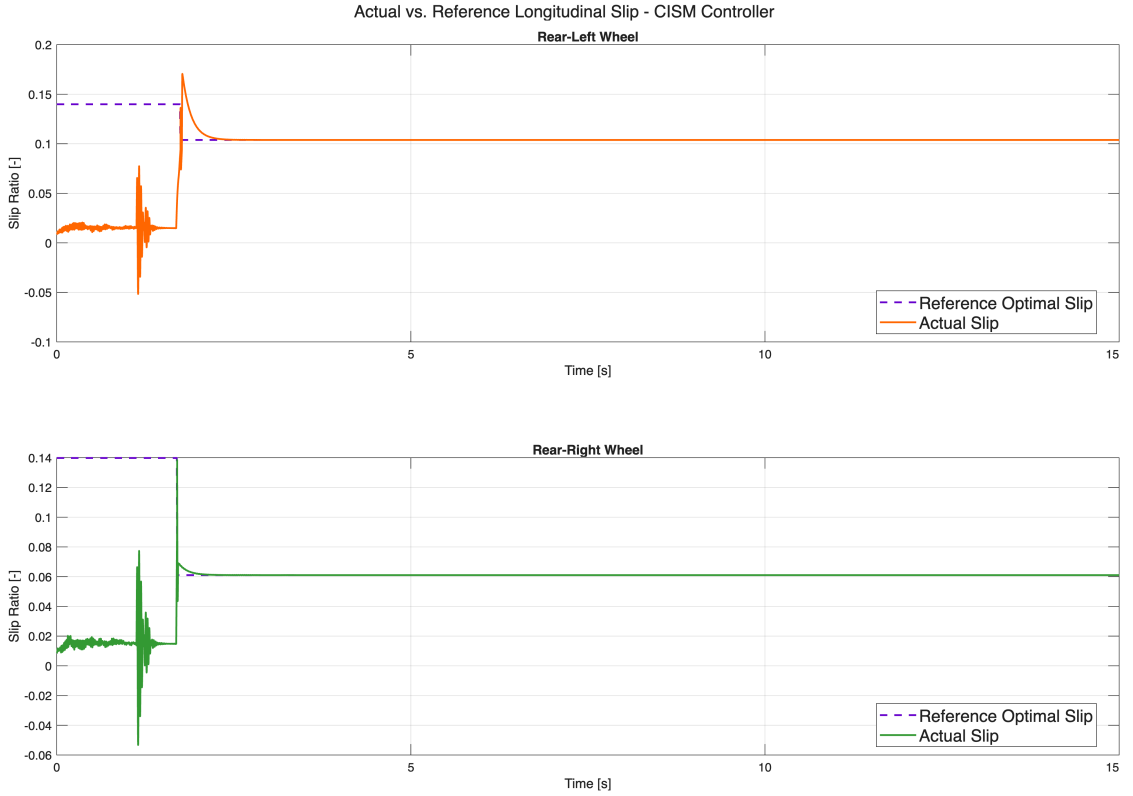


Figure 6.92: Longitudinal slip profile with CISM controller, with $m = 2100$ kg.

The proposed CISM achieves the superior control compromise under severe uncertainty.

As shown in the CISM longitudinal slip plot, despite the system operating with an unmodeled mass of 2100 kg, the actual longitudinal slip curves on both the rear-left and rear-right wheels converge seamlessly and rapidly onto the optimal slip targets. The control logic safely absorbs the parameter mismatch without tracking lag or signal distortion. Most importantly, the tracking line remains perfectly clean and noise-free, completely preserving the zero-chattering behavior of the torque output.

6.8 Robustness to Compliant Driveshafts and Time Delay Constraints

To thoroughly investigate the industrial feasibility of the proposed CISMV topology, the co-simulation framework was upgraded to include severe hardware-representative non-idealities.

In real-world automotive traction control, assuming an infinitely rigid connection between the electric motor and the wheel hub introduces a significant modeling gap. To prove that the CISMV is suitable for a vehicle-in-the-loop implementation, the control loop was stress-tested by incorporating the following non-idealities:

- **Torsional Drivetrain Compliance:** the rear half-shafts were modeled as compliant spring-damper systems with physical torsional stiffness (k_{DS}) and torsional damping (c_{DS}).
- **Actuation and Communication Latency:** a communication delay of 5 ms was introduced, by means of a low-pass filter, to replicate CAN bus propagation bottlenecks and inverter power electronics settling times.

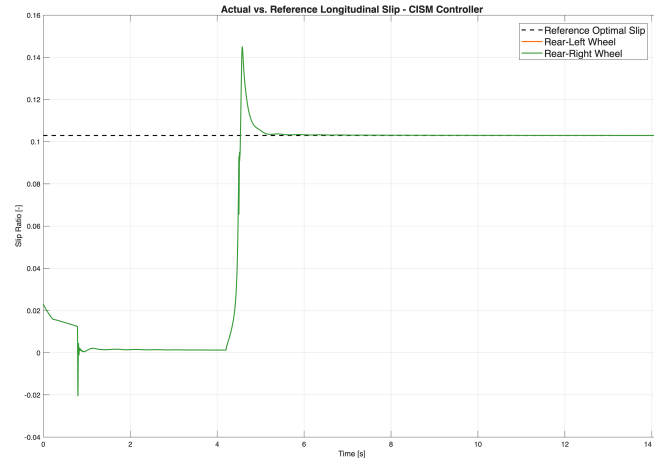
The CISMV was subjected to these combined constraints across all three operational scenarios to evaluate its tracking performance and chattering-rejection capabilities.

Real Powertrain Constraints

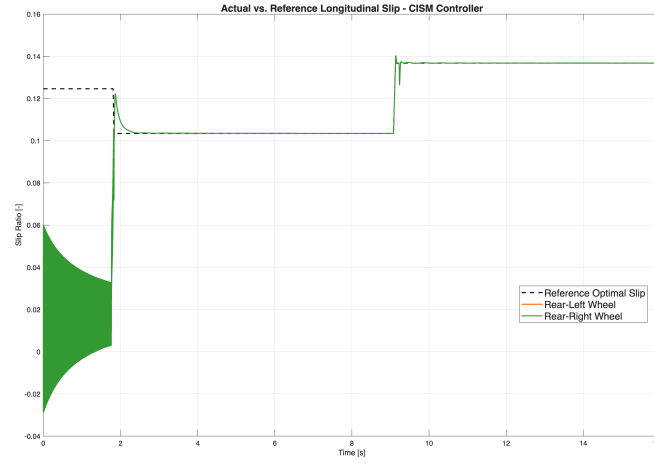
Parameter		Value	Unit
Drive Shaft Torsional Stiffness	k_{DS}	804583.66	Nm/rad
Drive Shaft Torsional Damping	c_{DS}	286.48	Nms/rad
Equivalent Filter Time Constant	τ_{lpf}	0.005	s
Filter Cut-off Frequency	$f_{cut-off_{lpf}}$	200	Hz

Table 6.6: Physical constraints and drivetrain non-idealities implemented for the real-world validation framework.

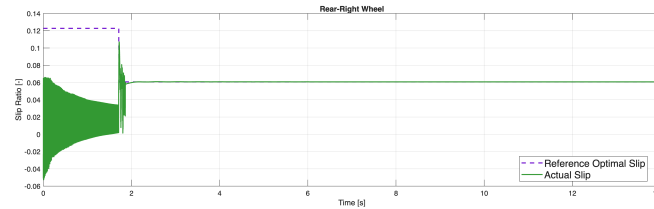
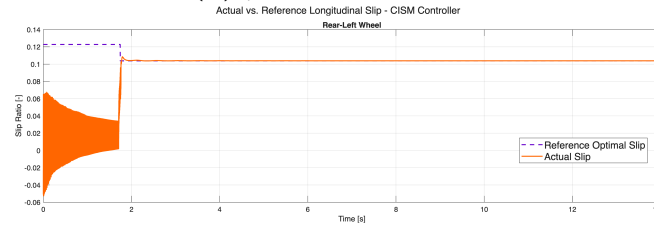
The proposed CISMV demonstrates an exceptional capacity for tracking persistence, establishing a highly robust operational boundary that remains uncompromised across the distinct road-adherence profiles.



(a) Low- μ maneuver.

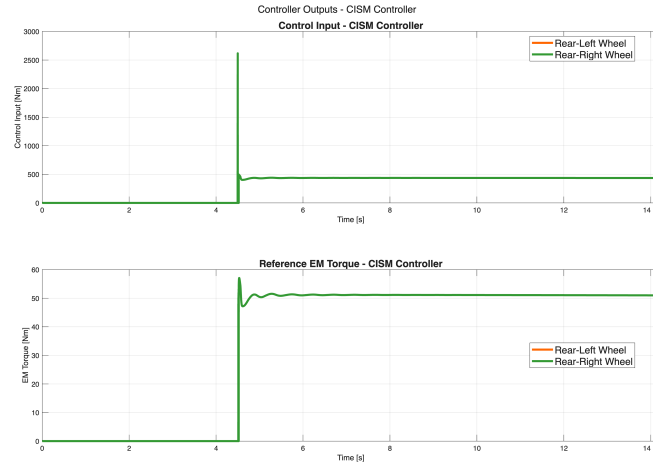


(b) μ -jump maneuver.

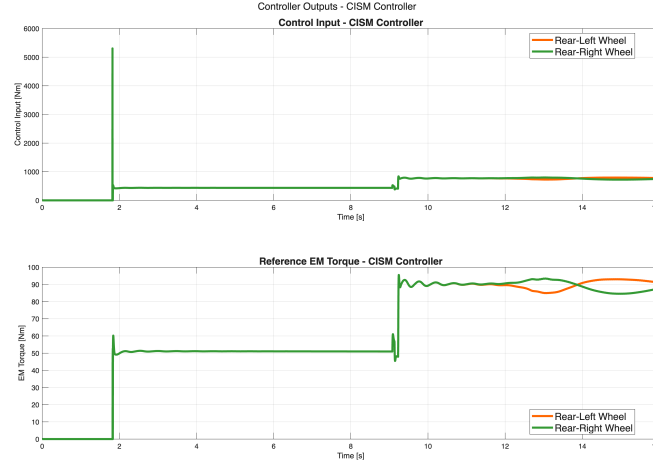


(c) μ -split maneuver.

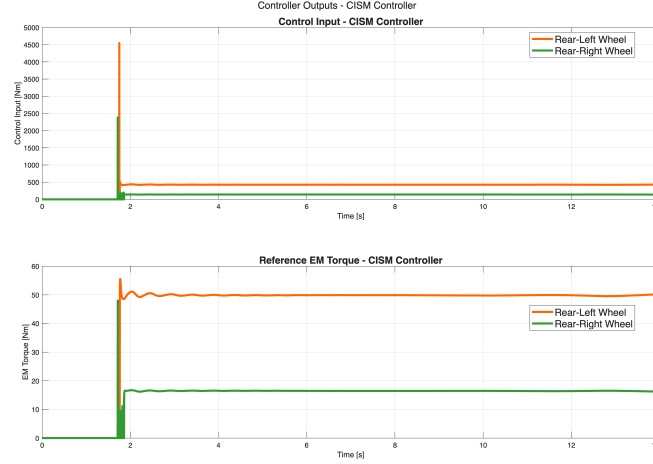
Figure 6.93: CISM longitundinal slip profile comparison with real-world constraints.



(a) Low- μ maneuver.



(b) μ -jump maneuver.



(c) μ -split maneuver.

Figure 6.94: CISM controller outputs comparison with real-world constraints.

The controller successfully decouples the wheels' slipping dynamics from the high-frequency parasitic dynamics injected by transmission compliance and actuation lag.

The conditional integration mechanism dynamically manages sudden transient errors without entering anti-windup saturation or generating overshoot cascades.

The actuator control effort and corresponding electric motor responses illustrated in Figure 6.94 confirm the chattering-free nature of the CISM C under realistic hardware latencies, with complete absence of high-frequency switching or destructive torque reversals across all configurations, and settling into a smooth, chatter-free steady-state command.

By maintaining highly precise, chatter-free slip regulation across all transient and steady-state events, the CISM C proves that robust non-linear tracking can coexist with actuator longevity.

Conclusions

7.1 Research Synthesis and Key Contributions

This thesis has successfully addressed the foundational engineering trade-off inherent to high-performance EV traction control: the reconciliation of non-linear tracking robustness with powertrain actuator longevity. While classical linear control architectures may fail to handle severe tire-road friction dropouts without excessive slip overshoot, Traditional Sliding Mode Control introduces high-frequency torque ripples (*chattering*) that can induce incompatibilities with actuators' physical limits and trigger drivetrain resonances.

The primary contribution of this work is the development and virtual validation of a Conditional Integral Sliding Mode Control topology integrated with an adaptive Burckhardt-based Road Estimator Module. By introducing the implementation of a conditional integration block managed by a continuous saturation interpolation within a properly designed boundary layer, high-frequency chattering was entirely extinguished, while completely preventing anti-windup saturation lag during aggressive torque reversals.

7.2 Quantitative Performance Verification

The proposed control architecture was systematically benchmarked against a standard industrial PI controller and Traditional Sliding Mode Control layout across three critical operating conditions within a high-fidelity IPG CarMaker co-simulation environment:

- **Uniform Low- μ Maneuver:** the CISMCMC guaranteed exponential convergence onto the optimal slip target during full-acceleration and braking demands, yielding smooth, noise-free steady-state tracking lines.
- **Transient μ -Jump Maneuver:** during sudden changes in road friction, the CISMCMC successfully prevented the critical wheel spinouts seen in the open-loop

configuration. Crucially, the CISMIC minimized the total Control Effort to 1.12×10^{10} , representing a massive 97% **reduction in actuator energy stress** compared to the unmodulated TSMC and a 1.75% improvement over the linear framework.

- **Asymmetric μ -Split Maneuver:** Under cross-track friction imbalances ($\mu_L = 0.1, \mu_R = 0.3$), the CISMIC successfully decoupled the independent rear axle dynamics, allowing the high-grip tire to safely maximize its tractive throughput while suppressing localized wheel spinning on the ice-like patch.

These results mathematically confirm that the proposed CISMIC topology breaks the structural compromise between non-linear stability boundaries and actuator physical preservation.

7.3 Future Outlook and Research Directions

While the virtual validation of the proposed control loop demonstrates high efficacy, several compelling pathways remain open for future academic and industrial extension:

1. **Integration with Full-Vehicle Lateral Dynamics:** The current slip control configuration operates primarily on an independent wheel-by-wheel longitudinal layer. Future work could focus on scaling this topology to cooperate with an upper-level Torque Vectoring and Electronic Stability Control supervisor, actively modulating independent yaw moments to decouple lateral cornering stability from longitudinal traction constraints.
2. **Driver Intent Integration in Reference Slip Estimation:** The baseline reference slip architecture focuses primarily on maximizing the instantaneous road friction coefficient. To enhance subjective drivability, future updates could incorporate a refined estimation block that blends environmental adherence limits with the driver's torque request from the accelerator and brake pedals. By dynamically mapping the driver's intent into the reference generation profile, the controller can allow controlled, transient slip variations during aggressive maneuvers to improve responsiveness, while maintaining absolute safety boundaries under steady-state tracking.
3. **Regenerative Braking Optimization and Energy Harvesting:** Since the concept vehicle utilizes independent motors, the ABS control law can be expanded into a multi-objective optimization framework during deceleration. Future research could explore a dynamic actuator blending algorithm designed to maximize energy recovery into the high-voltage battery pack. The sliding manifold will act as the core supervisor, distributing the high-frequency slip

control effort primarily to the electric motors for regenerative harvesting, and smoothly blending in the hydraulic friction brakes only when satisfying extreme braking torque demands or handling low-speed actuator saturation limits.

4. **Hardware-in-the-Loop (HIL) Validation:** To bridge the gap between software-in-the-loop (SIL) co-simulations and real-world implementation, the control code will be deployed on an automotive micro-controller (e.g., dSPACE) interfaced with a physical electric motor testbench, exposing the sliding manifolds to physical signal noise, measurement latencies, and actuator constraints.

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List of Acronyms

ACISMC Adaptive Conditional Integral Sliding Mode Controller

CISMC Conditional Integral Sliding Mode Control

EP Electric Powertrain

ESC Electronic Stability Control

FL Fuzzy Logic

HMI Human-Machine Interface

HTC Haptic Traction Control

ICE Internal Combustion Engine

IWM In-Wheel Motor

MPC Model Predictive Control

NMPC Non-linear Model Predictive Control

PID Proportional-Integral-Derivative

PWA Piecewise Affine

RWD Rear-Wheel Drive

SMC Sliding Mode Control

TCS Traction Control System

TSMC Traditional Sliding Mode Control

TV Torque Vectoring

Definitions

Chattering

A phenomenon characteristic of Variable Structure Systems (VSS) and Sliding Mode Control (SMC) consisting of high-frequency, finite-amplitude oscillations of the system state trajectory around the sliding manifold. This behavior is primarily caused by the discontinuous nature of the switching control law in the presence of non-ideal factors, such as finite sampling frequencies, actuator delays, or unmodeled high-frequency dynamics.

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