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# **Heat Transfer in Air Through Additively Manufactured Channels**

**Experimental Evaluation of Nusselt Number Via a Coupled**

**Thermal-Electric STAR-CCM+ Model**

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# Abstract

This thesis focuses on heat transfer in additively manufactured cooling channels. Additive manufacturing (AM) allows for the design of complex cooling schemes for aerospace and turbomachinery applications, improving efficiency and reducing carbon emissions. The characteristic surface roughness of AM components also increases heat transfer at the cost of increased pressure losses. Therefore, the aim of this work is to develop an experimental approach for evaluating heat transfer enhancement with air as cooling medium, utilising an existing test rig which generates heat into an AM test coupon by joule heating and an inverse coupled thermal-electric STAR-CCM+ numerical model. Test coupons were fabricated via laser powder bed fusion (LPBF) and exhibited different inlet geometries as well as different hydraulic diameter and number of sub-channels. The heat source term in the heat transfer equation was computed by evaluating the current into the test object either via outlet air temperature measurement or direct electrical power measurement by considering two equivalent energy balance equations. Experimental results for single-channel test objects showed different values of Nusselt number for the two methods, while testing on a six-channel stainless steel test coupon has revealed a higher degree of overlap upon comparison with results obtained for single-channel test object of same hydraulic diameter and material. Further testing on multi-channeled test objects (or highly conductive materials) is thus needed to better understand this phenomenon and allow for future thermal performance investigation of joule-heated AM components.



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# List of Symbols

| Symbol          | Description                                           | SI Unit                          |
|-----------------|-------------------------------------------------------|----------------------------------|
| $A$             | Cross-sectional area                                  | $\text{m}^2$                     |
| $c_p$           | Specific heat at constant pressure                    | $\text{J kg}^{-1} \text{K}^{-1}$ |
| $C$             | Coefficient in entry length correlation (Bhatti-Shah) | —                                |
| $C_f$           | Fanning friction factor                               | —                                |
| $D_{cf}$        | Constricted flow diameter                             | $\text{m}$                       |
| $D_h$           | Hydraulic diameter                                    | $\text{m}$                       |
| $e_i$           | Specific internal energy                              | $\text{J kg}^{-1}$               |
| $e$             | Absolute surface roughness                            | $\text{m}$                       |
| $\dot{E}_{gen}$ | Rate of energy generation within control volume       | $\text{W}$                       |
| $\dot{E}_{in}$  | Rate of energy entering control volume                | $\text{W}$                       |
| $\dot{E}_{out}$ | Rate of energy leaving control volume                 | $\text{W}$                       |
| $\dot{E}_{st}$  | Rate of change of stored energy in control volume     | $\text{W}$                       |
| $\underline{E}$ | Electric field vector                                 | $\text{V m}^{-1}$                |
| $f$             | Darcy friction factor                                 | —                                |
| $\underline{g}$ | Gravitational acceleration vector                     | $\text{m s}^{-2}$                |
| $\bar{h}$       | Convective heat transfer coefficient                  | $\text{W m}^{-2} \text{K}^{-1}$  |
| $h_{cu}$        | Heat transfer coeff. at copper-test object interface  | $\text{W m}^{-2} \text{K}^{-1}$  |
| $h_p$           | Heat transfer coeff. at parker-test object interface  | $\text{W m}^{-2} \text{K}^{-1}$  |
| $I$             | Electric current                                      | $\text{A}$                       |
| $\underline{J}$ | Electric current density vector                       | $\text{A m}^{-2}$                |
| $k$             | Thermal conductivity (general)                        | $\text{W m}^{-1} \text{K}^{-1}$  |
| $k_f$           | Thermal conductivity of the fluid                     | $\text{W m}^{-1} \text{K}^{-1}$  |
| $k_s$           | Equivalent sand-grain roughness                       | $\text{m}$                       |



| Symbol                | Description                                        | SI Unit            |
|-----------------------|----------------------------------------------------|--------------------|
| $k_s^+$               | Roughness Reynolds number                          | —                  |
| $k_{s,smooth}^+$      | Critical roughness Reynolds number (smooth regime) | —                  |
| $k_{s,rough}^+$       | Critical roughness Reynolds number (rough regime)  | —                  |
| $k_{in}, k_{out}$     | Inlet/outlet pressure loss coefficients            | —                  |
| $L$                   | Length of test object or tube                      | m                  |
| $L_i$                 | Mechanical work per unit mass                      | J kg <sup>-1</sup> |
| $\dot{m}$             | Mass flow rate                                     | kg s <sup>-1</sup> |
| $n$                   | Exponent in velocity profile law (Prandtl)         | —                  |
| $\phi$                | Exponent in entry length correlation (Bhatti-Shah) | —                  |
| $Nu$                  | Nusselt number                                     | —                  |
| $\overline{Nu}$       | Average Nusselt number                             | —                  |
| $Nu_x$                | Local Nusselt number                               | —                  |
| $Nu_{Dfd}$            | Nusselt number for fully developed conditions      | —                  |
| $Nu_{enh}$            | Nusselt number enhancement ( $Nu/Nu_0$ )           | —                  |
| $Nu_0$                | Smooth Nusselt number (Gnielinski correlation)     | —                  |
| $\underline{n}$       | Normal vector to a boundary                        | —                  |
| $p$                   | Pressure                                           | Pa                 |
| $P_{atm}$             | Atmospheric pressure                               | Pa                 |
| $P_{diff} / \Delta P$ | Differential/Total pressure drop                   | Pa                 |
| $\mathcal{P}$         | Power (mechanical)                                 | W                  |
| $\Pi$                 | Power (electrical)                                 | W                  |
| $P_w$                 | Wetted perimeter                                   | m                  |
| $Pr$                  | Prandtl number                                     | —                  |
| $Q$                   | Thermal energy per unit mass                       | J kg <sup>-1</sup> |
| $Q_{fluid}$           | Heat transferred to the fluid                      | W                  |
| $Q_{losses}$          | Heat losses (environment, fittings, etc.)          | W                  |
| $\dot{q}$             | Volumetric heat generation rate                    | W m <sup>-3</sup>  |
| $q''$                 | Heat flux per unit area                            | W m <sup>-2</sup>  |
| $q_s''$               | Surface heat flux                                  | W m <sup>-2</sup>  |
| $r$                   | Radial coordinate                                  | m                  |
| $r_0$                 | Radius of circular pipe/channel                    | m                  |
| $R$                   | Electrical resistance                              | $\Omega$           |
| $R_{in}, R_{out}$     | Inlet/outlet parker resistance                     | $\Omega$           |
| $R_{tot}$             | Total electrical resistance                        | $\Omega$           |

| Symbol            | Description                                    | SI Unit                    |
|-------------------|------------------------------------------------|----------------------------|
| $R_{TO}$          | Electrical resistance of test object           | $\Omega$                   |
| $Re / Re_D$       | Reynolds number (based on hydraulic diameter)  | —                          |
| $Re_x$            | Local Reynolds number (based on $x$ )          | —                          |
| $t$               | Time                                           | s                          |
| $T$               | Temperature                                    | K or $^{\circ}\text{C}$    |
| $T_{cu,in/out}$   | Copper bar temperature (inlet/outlet)          | K or $^{\circ}\text{C}$    |
| $T_{f,i}$         | Local mean fluid temperature in channel $i$    | K or $^{\circ}\text{C}$    |
| $T_{in}, T_{out}$ | Inlet/outlet bulk fluid temperature            | K or $^{\circ}\text{C}$    |
| $T_i$             | Generic surface/wall temperature               | K or $^{\circ}\text{C}$    |
| $T_{i,exp}$       | Experimental surface temperature at sensor $i$ | K or $^{\circ}\text{C}$    |
| $T_{i,model}$     | Model surface temperature at sensor $i$        | K or $^{\circ}\text{C}$    |
| $T_m$             | Mean fluid temperature                         | K or $^{\circ}\text{C}$    |
| $T_p$             | Parker surface temperature                     | K or $^{\circ}\text{C}$    |
| $T_s$             | Surface temperature                            | K or $^{\circ}\text{C}$    |
| $T_{\infty}$      | Free stream fluid temperature                  | K or $^{\circ}\text{C}$    |
| $u, v$            | Velocity components ( $x, y$ direction)        | $\text{m s}^{-1}$          |
| $u_m$             | Mean fluid velocity                            | $\text{m s}^{-1}$          |
| $u_{max}$         | Maximum fluid velocity                         | $\text{m s}^{-1}$          |
| $u_{\tau}$        | Friction velocity                              | $\text{m s}^{-1}$          |
| $V$               | Voltage                                        | V                          |
| $V_{in/out}$      | Inlet/outlet parker voltage                    | V                          |
| $V_{meas}$        | Measured voltage across test section           | V                          |
| $x$               | Axial coordinate / Distance from entrance      | m                          |
| $x_{fdh,l}$       | Hydrodynamic entry length (laminar)            | m                          |
| $x_{fdh,t}$       | Hydrodynamic entry length (turbulent)          | m                          |
| $x_{fdt,l}$       | Thermal entry length (laminar)                 | m                          |
| $x_{fdt,t}$       | Thermal entry length (turbulent)               | m                          |
| $y$               | Coordinate normal to the wall                  | m                          |
| $\alpha$          | Thermal diffusivity                            | $\text{m}^2 \text{s}^{-1}$ |
| $\delta$          | Boundary layer thickness (velocity)            | m                          |
| $\delta_t$        | Boundary layer thickness (thermal)             | m                          |
| $\Delta T$        | Temperature difference ( $T_{out} - T_{in}$ )  | K                          |
| $\epsilon$        | Absolute roughness height                      | m                          |
| $\epsilon_{inst}$ | Instrumental error                             | (as per measurement)       |
| $\epsilon_{rand}$ | Random error                                   | (as per measurement)       |

| Symbol             | Description                     | SI Unit                           |
|--------------------|---------------------------------|-----------------------------------|
| $\eta$             | Thermodynamic cycle efficiency  | —                                 |
| $\mu$              | Dynamic viscosity               | Pa s                              |
| $\nu$              | Kinematic viscosity             | m <sup>2</sup> s <sup>-1</sup>    |
| $\rho$             | Density                         | kg m <sup>-3</sup>                |
| $\sigma_b$         | Stefan-Boltzmann constant       | W m <sup>-2</sup> K <sup>-4</sup> |
| $\sigma$           | Standard deviation (statistics) | as per<br>measurement             |
| $\sigma_e$         | Electrical conductivity         | S m <sup>-1</sup>                 |
| $\underline{\tau}$ | Viscous stress tensor           | Pa                                |
| $\tau_s$           | Wall shear stress               | Pa                                |

# Chapter 1

## Introduction

This chapter provides the scientific motivation behind the following work, alongside a presentation of its industrial application and the limitations to be considered.

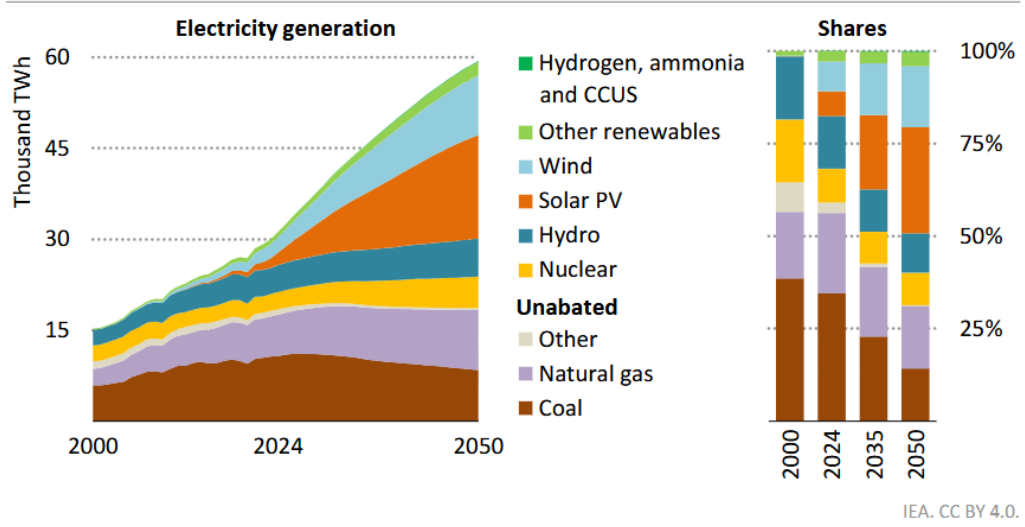
### 1.1 Motivation

The 2025 World Energy Outlook [1] published by the International Energy Agency (IEA) highlights how from 2015 to 2024 global electricity demand rose by 2.9 % every year, with the trend continuing until 2035.

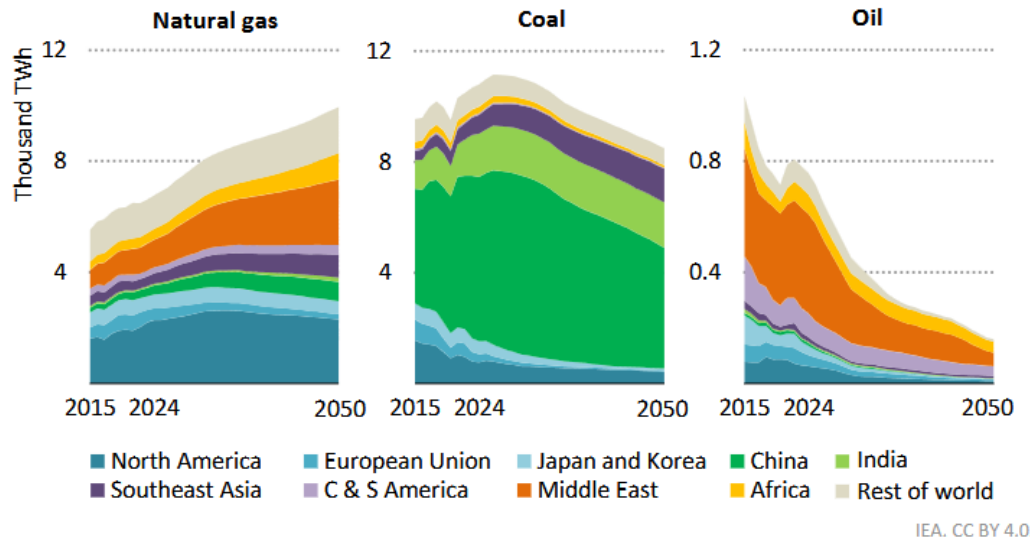
Amongst other sources as seen in Figures 1.1 and 1.2, natural gas remains the second largest source of electricity generation worldwide through 2035, projected to increase by more than 45 % from 2024 to 2035.

Concurrently, data from IEA 2025 Global Energy Review [2] denotes how natural gas emissions of  $CO_2$  increased by 2.5 % in 2024; the same data shows natural gas is the main contributor of emissions growth worldwide.

In order to meet the 1.5 °C global warming goal set by the Paris Agreement [3], it is then instrumental to improve the efficiency of gas turbines in converting natural gas into the required electrical power with fewer  $CO_2$  emissions.

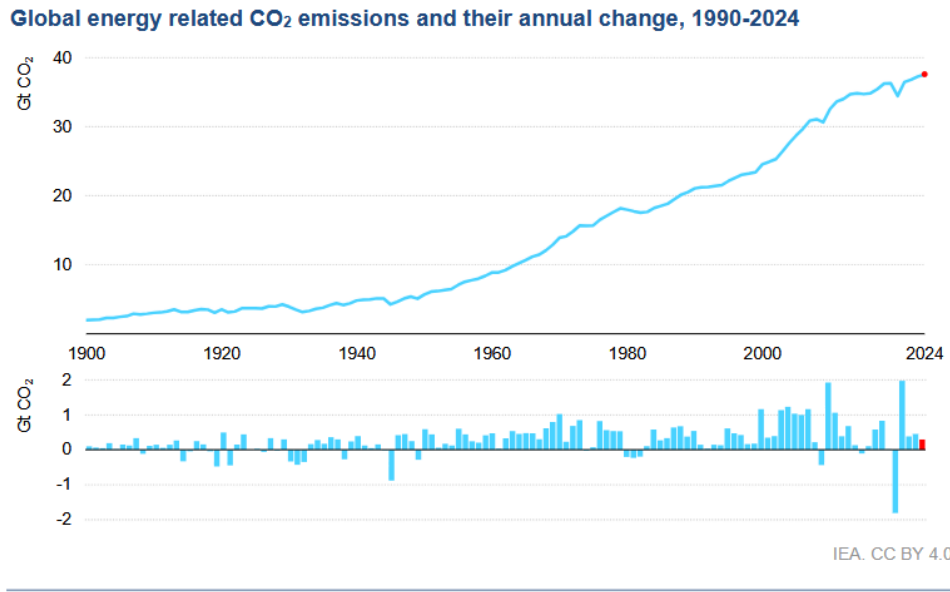


**Figure 1.1:** Electricity generation worldwide divided by source according to IEA Current Policies Scenario [1].



**Figure 1.2:** Electricity generation worldwide divided by source and region according to IEA Current Policies Scenario [1].

The work presented in this thesis is thus motivated by increasing the efficiency of modern gas generator systems, focusing on Additive Manufactured turbine cooling channels.



**Figure 1.3:** Global energy-related  $CO_2$  emissions and their annual change [2].

## 1.2 Industrial background

Gas turbines are thermal machines able to produce mechanical power by expanding hot combustion gases in a turbine. The physical principle employed in the conversion of internal energy (or enthalpy) into power is the First Principle of Thermodynamics [4]:

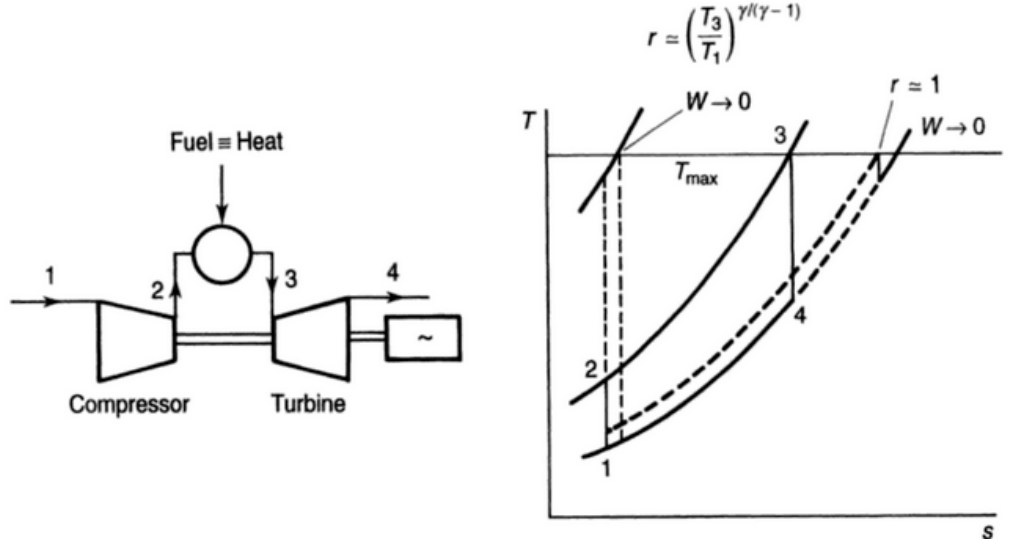
$$L_i + Q = \Delta h \quad (1.1)$$

Where  $h = c_p T$ , under the assumption of a perfect gas, represents the enthalpy contained in a gas per unit mass and  $Q$  and  $L_i$  are respectively the thermal energy per unit mass and mechanical work per unit mass exchanged in the transformation. Considering that a gas turbine as in Figure 1.4, much alike a Turboshift aircraft engine, is comprised of a compressor, combustion chamber, turbine and one or several shafts, it is possible to represent the pertaining thermodynamic cycle as a Joule Brayton cycle [4].

The thermodynamic cycle efficiency  $\eta$ , under the assumption of an ideal Joule-Brayton cycle, can be written as:

$$\eta = \frac{L_{tot}}{Q} = 1 - \frac{T_4}{T_3} \quad (1.2)$$

Where  $L_{tot}$  is the net work obtained by the machine. Thus, for a fixed turbine outlet temperature  $T_4$  the efficiency of the cycle increases as the Turbine Inlet



**Figure 1.4:** Schematization of a simple gas turbine and corresponding ideal Joule Brayton cycle [4].

Temperature  $T_3$  increases; consequently, modern gas turbines operate with hot-gas temperatures that may exceed the allowable metal temperature of turbine blades [5].

To comply with this requirement inner blade cooling schemes have been developed over the years [6], and recent advancements in additive manufacturing techniques [7] have allowed for the implementation of more complex geometries with reduced costs compared to traditional tool-based procedures.

In addition, a common feature of AM materials is the increased surface roughness, increasing heat transfer at the cost of increased pressure losses.

As analytical and numerical models constitute lower fidelity approaches compared to an experimental program, and often require increased computational power and complexity for rough surfaces applications [8], experimental programs are needed to guide future turbine blade designs correlating heat transfer enhancement and pressure drop increase to AM material roughness.

### 1.3 Objectives and Limitations

The primary objective of this work is to assess an experimental inverse methodology for estimating local and average heat-transfer coefficients and Darcy friction factors in selected smooth and additively manufactured test coupons, within the transitional and turbulent Reynolds-number ranges accessible to the rig.

A new methodology will be presented and assessed using smooth reference channels

and comparisons with established internal-flow correlations before being applied to AM samples.

In conducting the work presented in this thesis, the following limitations need to be considered:

- The duration of the project was 7 months, including 1 month of overlap with the previous student operating the test rig, limiting the amount of new test coupons that could be tested after manufacturing.
- Improvements over preexisting methodology were considered, implemented and validated, further limiting the number of test coupons presented in this work.
- With the nature of this study being experimental, CFD based flow simulations won't be included in this thesis. Instead, a solid thermal-electrical inverse model will be considered.
- Due to limitations in methodology which will be expanded in subsequent chapters, results for the Nusselt number and Darcy friction factor have been obtained only for the transitional and turbulent flow regimes.
- No direct roughness estimation was conducted on available test objects during the project's period and preexisting values were not available for all test objects, thus interpretations in terms of relative roughness should be treated as qualitative and not as quantitative evidence.



## Chapter 2

# Literature Review

In this chapter a brief review of existing literature in the fields of momentum and heat transfer over rough walls will be provided.

### 2.1 Momentum transfer

The earliest studies focusing on the influence of surface roughness on the pressure drop in pipes date back to over a century ago. Darcy [9] conducted experiments on pipe flow with differing roughness, deducing higher pressure drop with increased roughness. Later Fanning [10] suggested a correlation for pressure drop in water flow including the effects of surface roughness.

Both defined a nondimensional number for evaluating pressure losses in fluid flow: the Darcy friction factor  $f$  and the Fanning friction factor  $C_f$ .

The former provides a nondimensional form for the pressure drop across a pipe:

$$f = \frac{2\Delta p D_h}{L \rho u_m^2} \quad (2.1)$$

Where  $\Delta p$  is the total pressure drop in the tube of length  $L$  and hydraulic diameter  $D_h$  and  $\rho$  and  $u_m$  are respectively the fluid's density and mean velocity. The latter provides a nondimensional form for the wall shear stress:

$$C_f = \frac{\tau_s}{\frac{1}{2} \rho u_m^2} \quad (2.2)$$

Where  $\tau_s$  is the shear stress at the wall and  $\rho$  and  $u_m$  are the fluid's density and mean velocity.

Nikuradse [11] defined a model for surface roughness with uniformly sized sand grains and researched roughness effects on pressure drop for various Reynolds numbers and diameters. The main findings were that surface roughness has

reduced effect in laminar flow and a considerable one in turbulent flow, and that a higher roughness yielded increased friction and pressure drops.

Moreover, he identified three flow regimes based on the roughness Reynolds number, defined as

$$k_s^+ = \frac{u_\tau k_s}{\nu}, \quad (2.3)$$

where  $k_s$  is the equivalent sand-grain roughness,  $u_\tau$  is the friction velocity, and  $\nu$  is the kinematic viscosity of the fluid. These comprise the hydraulically smooth regime, in which the roughness elements do not appreciably affect momentum transfer; the transitionally rough regime, in which both the Reynolds number and the relative roughness influence the flow; and the fully rough regime, in which momentum transfer is governed primarily by surface roughness.

Later, Colebrook [12] conducted several experiments on roughness and flow friction factor in pipes of various diameters, linings and roughness types. This work resulted in the definition of the well-known Colebrook-White nonlinear equation, which Moody [13] would employ to plot the Darcy friction factor as a function of Reynolds and relative roughness, defined as the ratio of equivalent sand grain roughness  $k_s$  on tube diameter.

In following years, extensive research has been produced building upon Moody's work, mainly in defining the limitations of the Colebrook correlation. For instance, Allen et al. [14] found the Moody diagram might significantly overestimate pressure losses in the transitionally rough region and LaViolette [15] proposed an alternative to the Moody diagram which generated a solution without iterative procedures for any kind of pipe flow.

Taylor et al. [16] reported the limitations of the Moody diagram for relative roughness greater than 5 %, proposing an alternative for relative roughness up to 14 % and suggesting classical heat and momentum transfer theories might need to be revised when researching channels with diameters smaller than 1 mm.

On this subject Kandlikar [17] suggested a correction for the microchannel diameter based on roughness height  $k$  deemed the constricted flow diameter  $D_{cf}$  and reformulated the Darcy friction factor definition for laminar flow and Colebrook-White's equation employing it.

Lastly, substantial research has been conducted on providing alternatives to Colebrook-White's equation that do not necessitate an iterative procedure. For instance, Flack and Schultz [18] proposed a new correlation for the friction drag coefficient  $C_f$  in rough pipes, including various roughness ranges and types.

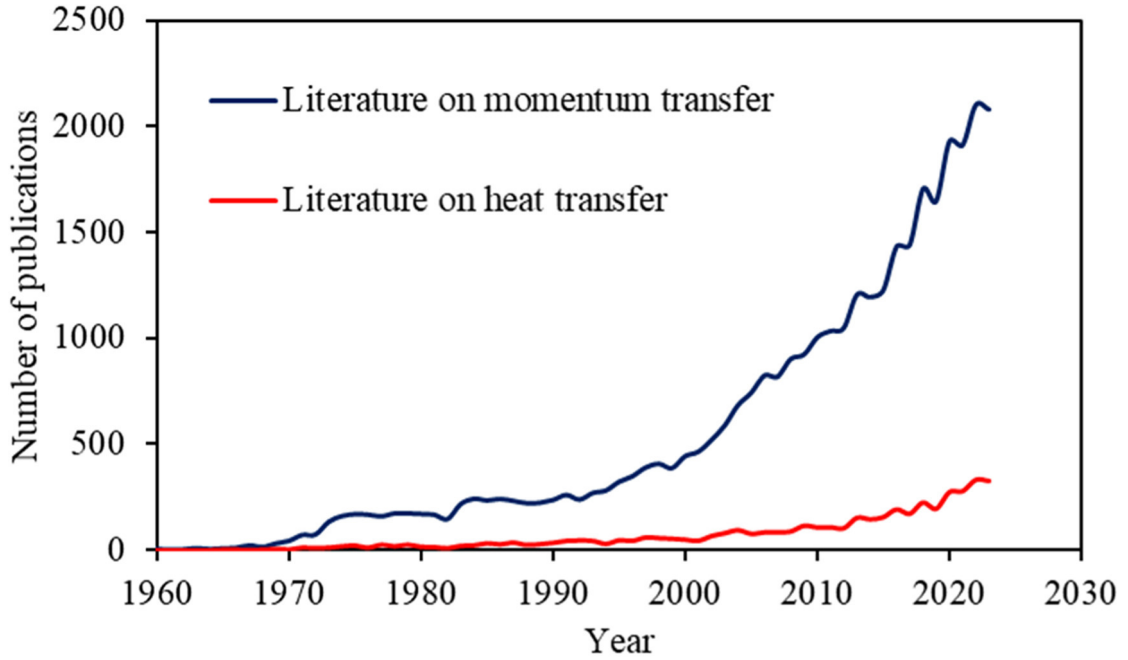
## 2.2 Heat transfer

The first documented work on heat transfer with rough walls was conducted by Cope [19] focusing on 2D roughness elements and using air. Then Dipprey and Sabersky

[20] investigated heat transfer and friction factor increase in rough geometries using water as a cooling medium and joule heating as heat generation.

Their findings evidenced increases in heat transfer coefficient as high as 270 percent, accompanied by larger friction factor increases.

However, as illustrated in Figure 2.1, in the following years research on rough wall heat transfer has substantially lagged behind that pertaining momentum transfer, only to resurface in more recent times. This surge in interest is likely



**Figure 2.1:** Publications over the years on heat and momentum transfer over rough walls [8].

due to advancements in high fidelity numerical models and computational power, increasing progress in additive manufacturing processes and increasing concern over innovative solutions in energy efficiency and reduced carbon footprint [8].

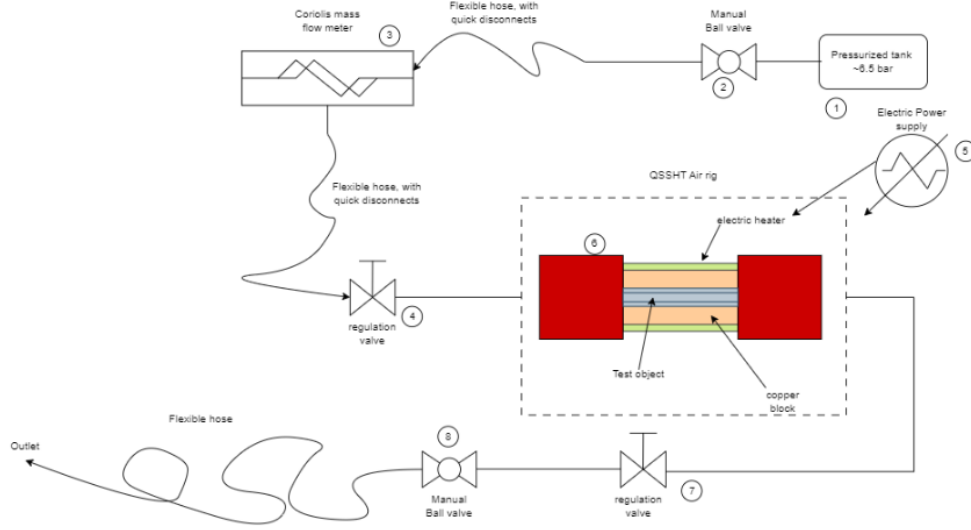
Penn state has recently conducted studies on heat transfer and pressure losses in additive manufactured test coupons [21]. The test objects presented wavy structures and were manufactured employing direct metal laser sintering (DMLS).

The research concluded that test objects exhibiting short wavelength channels yielded high pressure losses without increases in heat transfer, while long wavelength ones offered good thermal performance with less of a penalty in pressure drop.

Additional work on heat transfer in additively manufactured channels was conducted by Siemens Energy AB via the Fluid Dynamics laboratory in its R&D department. In particular, Pagani [22] and Madonia [23] obtained Nusselt number and friction

factor results for selective laser melting manufactured test objects, using air as cooling medium and heat conduction from a copper block encasing the test coupon as heat source.

The experimental setup they used, the quasi-steady state heat transfer rig (qssht), is presented in Figure 2.2. 6.5 bar pressurised air from a reservoir enters the test



**Figure 2.2:** Flow chart schematics of the qssht test rig [24].

object as it's heated via conduction by a copper block, also heated via electrical current.

A manual ball valve would allow for air to flow into the system while the regulating valves would allow to modify inlet air pressure into the system.

A Coriolis mass flow meter would record the mass flow  $\dot{m}$ , PT100 temperature sensors would record both the inlet and outlet air temperatures and thermocouples would measure the copper's core temperature.

This setup allowed both for a simplified, live acquisition of the average Nusselt number  $\bar{Nu}$  along the test object utilising in series radial thermal resistances and a more accurate post-processing procedure via FEM, utilising the software C3D.

Key limitations of the rig were the difficulty in evaluating the heat transfer coefficient between test object and copper block, which then defines heat transfer between test object and cooling fluid; the inaccessibility of the test object's surface for local measurements; the limited number of test points that can be evaluated during a test due to long transient state periods.

As such, further research was conducted on obtaining Nusselt number results employing internal energy generation in the test section.

On this subject, Vieira [24] set up an experimental rig which allowed for the joule heating of the test objects via a transformer and a pair of copper plates as electricity conduits, and validated it with smooth test objects on the basis of Gnielinski's correlation [25].

The results obtained were significantly less dependent on heat transfer coefficient specification on the thermal contact, on the basis of the much reduced contact surface. However, further testing was required to refine the methodology, quantify its repeatability, and obtain reliable rough-wall heat-transfer data. The present work addresses this need by further developing the Joule-heating approach and applying it to selected additively manufactured test coupons, using a coupled thermo-electrical STAR-CCM+ model to estimate local and average Nusselt numbers together with Darcy friction-factor measurements.

## Chapter 3

# Theoretical Background

This chapter provides the theoretical background necessary to understand the work presented in this thesis. It covers the fundamental concepts of heat transfer and internal flows and will provide an overview of the nondimensional parameters that will be focused on in the following chapters.

### 3.1 Fundamentals of Heat Transfer

Heat transfer is the process of energy transfer from one body to another or within the same body in virtue of a temperature difference [26]. Based on the mechanism and medium through which such energy is transported, heat transfer can happen through three different modes: conduction, convection and radiation.

- **Conduction** is the transfer of thermal energy by diffusion through a solid medium or a stationary fluid, the heat rate per unit area  $q''$  in a 3D Cartesian reference frame can be expressed by Fourier's law [26]:  $q'' = -k\nabla T$ .

Where  $k$  is the medium thermal conductivity and  $\nabla T$  is the 3D temperature gradient.

- **Convection** is the transfer of thermal energy through a moving fluid, the heat rate per unit area  $q''$  can be expressed by Newton's law of cooling [26]:  $q'' = h(T_s - T_\infty)$ .

Where  $h$  is the medium convective heat transfer coefficient and, under the assumption of fluid flow over a flat plate,  $T_s$  is the surface temperature and  $T_\infty$  the free stream temperature.

- **Radiation** is the transfer of thermal energy through electromagnetic waves, the heat rate per unit area of an ideal black body can be expressed by Stefan-Boltzmann law[26]:  $q'' = \sigma_b T_s^4$ .

Where  $\sigma_b$  is Stefan-Boltzmann's constant and  $T_s$  the black body's external surface temperature.

For the purposes of this thesis, we shall only focus on heat conduction and *forced convection*. That is, convection in forced fluid flow, such as in a pipe or channel.

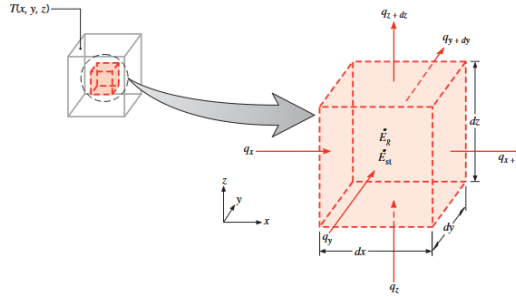
### 3.1.1 Conduction

Conduction is the transport of energy in a medium due to a temperature gradient, and the physical mechanism is one of random atomic or molecular activity[27].

One of the main purposes of heat conduction is to determine the temperature distribution in a medium, the knowledge of which allows then to determine other important quantities such as the heat fluxes via Fourier's law.

To accomplish this task, it is first necessary to derive an appropriate partial differential equation that describes such temperature distribution under the given boundary conditions. Considering the following assumptions:

- The medium is homogeneous and isotropic, with no bulk motion.
- The medium is incompressible.
- Uniform specific heat  $c_p$  and thermal conductivity  $k$ .



**Figure 3.1:** Differential control volume for heat conduction analysis[27].

The temperature distribution in cartesian coordinates and at a given time  $T(x, y, z, t)$  can be determined by applying the principle of conservation of energy to a differential control volume  $dV = dx * dy * dz$  such as in Figure 3.1:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \quad (3.1)$$

where  $\dot{E}_{in}$  and  $\dot{E}_{out}$  are the net energy rates entering and leaving the control volume,  $\dot{E}_{gen} = \dot{q}dV$  is the energy generated within the control volume and  $\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dV$  is the rate of change of thermal energy stored in the control volume.

Applying Fourier's law to express the incoming and outgoing heat rates in the 3D Cartesian reference frame, the following differential equation can be derived:

$$\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (3.2)$$

where  $\alpha = \frac{k}{\rho c_p}$  is the thermal diffusivity of the medium.

This equation is known as the Heat Transfer Equation [27]. For steady state conditions, the rate of change of thermal energy stored in the volume is zero and the equation reduces to:

$$\nabla^2 T + \frac{\dot{q}}{k} = 0 \quad (3.3)$$

The boundary conditions that typically accompany the Heat Transfer equation are:

- **Dirichlet boundary condition**, where the temperature is specified at the boundary:  $T = T_s$ . Where  $T_s$  is the specified temperature at the boundary.
- **Neumann boundary condition**, where the heat flux is specified at the boundary:  $q_n'' = -k \nabla T \cdot \underline{n} = q_s''$ . Where  $q_s''$  is the specified heat flux and  $\underline{n}$  is the normal vector to the boundary.
- **Robin (or Convection) boundary condition**, where a convective heat transfer coefficient  $h$  and ambient temperature  $T_\infty$  are specified at the boundary:  $q_n'' = -k \nabla T \cdot \underline{n} = h(T_s - T_\infty)$ .

Of particular interest for this thesis is the case of a solid medium with internal heat generation given by an electrical current:

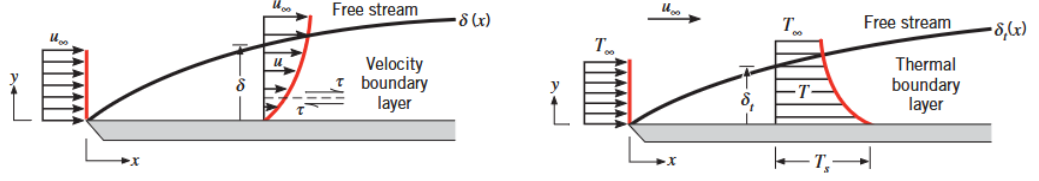
$$\dot{q}V = RI^2 \quad (3.4)$$

where  $V$  is the total volume of the medium,  $R$  is its electrical resistance and  $I$  is the current flowing through it.

### 3.1.2 Convection

Convection includes energy transfer by both the bulk fluid motion (advection) and the random motion of fluid molecules (conduction or diffusion) [26]. These two mechanisms interact in the boundary layer.





**Figure 3.2:** Example of the kinematic and thermal laminar boundary layers developing on a flat plate[27].

When a moving fluid of velocity  $u_\infty$  and temperature  $T_\infty$  comes into contact with a flat, smooth surface at temperature  $T_s$ , a thin layer of fluid called boundary layer forms near the surface, where the velocity and temperature gradients develop. For the case of a flat plate, outside of the boundary layer, the fluid can be considered inviscid and the temperature field can be considered uniform, with velocity and temperature equal to the free stream values  $u_\infty$  and  $T_\infty$  respectively [26]. The thickness of the boundary layer  $\delta$  or  $\delta_t$  is defined as the distance from the surface to the point where the velocity or temperature reaches 99% of its free stream value.

Inside of the boundary layer, both momentum and energy are transferred in virtue of a gradient of velocity and temperature being present. And it is thus possible to define both *shear stresses*  $\tau$  and heat fluxes  $q''$  between parallel layers of fluid. In particular, at the plate wall we can define:

$$\tau_s = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} \quad (3.5)$$

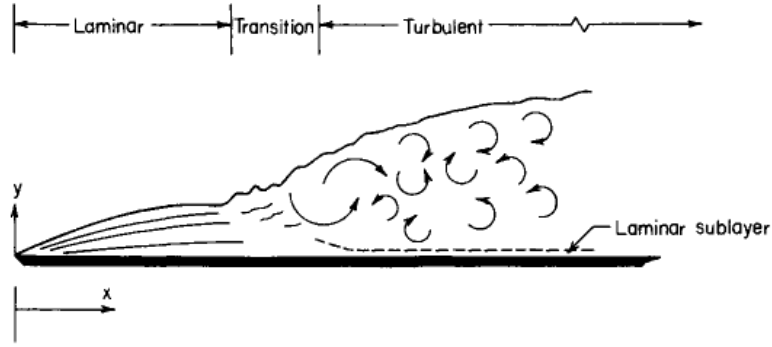
$$q_s'' = -k_f \left. \frac{\partial T}{\partial y} \right|_{y=0} \quad (3.6)$$

Equation (3.6) was obtained considering that heat transfer at the wall happens by diffusion as the velocity at the wall is zero[26]. If we combine equation (3.6) and Newton's law of cooling, we can relate the convective heat transfer coefficient  $h$  to the thermal conductivity of the fluid  $k_f$  and the temperature gradient at the wall:

$$h = -\frac{k_f}{T_s - T_\infty} \left. \frac{\partial T}{\partial y} \right|_{y=0} \quad (3.7)$$

Considering flow over a flat plate such as in figure 3.3, three flow regimes can be identified: laminar, transitional and turbulent.

- **Laminar flow:** characterized by highly ordered flow, where fluid particles follow streamlines. Diffusion is the dominant mechanism of momentum and energy transfer.
- **Transitional flow:** a regime where the flow transitions from laminar to turbulent, characterized by the presence of vortices and instabilities.
- **Turbulent flow:** characterized by chaotic and irregular motion, where fluid particles move in a random manner. Now turbulent mixing is the dominant mechanism of momentum and energy transfer.



**Figure 3.3:** Example of the three flow regimes developing on a flat plate[26].

Transition to turbulence is due to disturbances in the flow that grow in magnitude downstream, until inertial forces overcome the viscous damping forces and the flow structures break down. External disturbances, such as surface roughness, can trigger this transition prematurely [28].

The nondimensional parameter used to weigh the relative importance of inertial and viscous forces and to predict transition from laminar to turbulent flow is the Reynolds Number. For flow over a flat plate, the local Reynolds number is defined as:

$$Re_x = \frac{\rho u_\infty x}{\mu} \quad (3.8)$$

Where  $x$  is the distance from the leading edge of the plate.

Turbulent flow enhances momentum and energy transfer, this is due to the mixing effect provided by the turbulent eddies and results in higher wall shear stresses  $\tau_s$  and heat transfer coefficients  $h$ .

Therefore, in terms of velocity and temperature gradients:

$$\begin{aligned} \left| \frac{\partial u}{\partial y} \right|_{y=0, \text{turbulent}} &> \left| \frac{\partial u}{\partial y} \right|_{y=0, \text{laminar}} \\ \left| \frac{\partial T}{\partial y} \right|_{y=0, \text{turbulent}} &> \left| \frac{\partial T}{\partial y} \right|_{y=0, \text{laminar}} \end{aligned} \quad (3.9)$$

## 3.2 Nondimensional Parameters

To relate experimental data obtained for a model system to a real system, it is often convenient to describe the physical phenomena in terms of nondimensional parameters.

For heat transfer, such parameters are obtained by expressing the governing differential equations in nondimensional form or by considering the Buckingham  $\pi$  theorem.

For convective heat transfer, the governing differential equations are the Boundary Layer Equations, which are derived from the conservation equations of mass, momentum and energy.

Considering the differential control volume shown in Figure 3.4, the conservation laws of mass, momentum and energy written in differential form are:

$$\text{Continuity equation : } \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0 \quad (3.10)$$

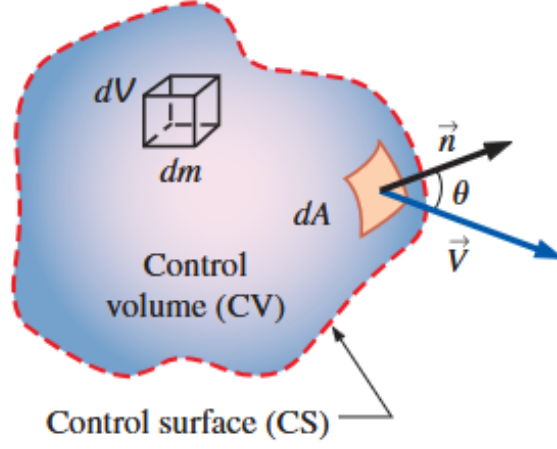
$$\text{Conservation of momentum : } \frac{\partial(\rho \underline{u})}{\partial t} + \nabla \cdot (\rho \underline{u} \otimes \underline{u}) = -\nabla p + \nabla \cdot \underline{\tau} + \rho \underline{g} \quad (3.11)$$

$$\text{Conservation of energy : } \frac{\partial(\rho e_i)}{\partial t} + \nabla \cdot (\rho e_i \underline{u}) = -\nabla \cdot \underline{q} + \underline{\tau} \cdot \nabla \underline{u} - \nabla p \cdot \underline{u} \quad (3.12)$$

where  $\rho$  is the density,  $\underline{u}$  is the velocity vector,  $p$  is the pressure,  $\underline{\tau}$  is the viscous stress tensor,  $\underline{g}$  is the gravitational acceleration,  $e_i$  is the specific internal energy and  $\underline{q}$  is the heat flux vector.

To obtain the boundary layer equations, the following assumptions will be made to simplify the conservation equations:

- The fluid is incompressible.
- The flow is steady, meaning that the time derivatives are zero.
- Body forces are negligible.



**Figure 3.4:** Example of a differential control volume[29].

- The fluid is Newtonian, meaning that the viscous stress tensor is proportional to the rate of strain.
- Diffusion in the x-direction is negligible compared to diffusion in the y-direction.

Then for laminar flow the boundary layer equations can be written as:

$$\text{Continuity equation : } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.13)$$

$$\text{Momentum equation : } u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad (3.14)$$

$$\text{Energy equation : } u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3.15)$$

If we then express the boundary layer equations in nondimensional form, using [26]:

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{u_\infty}, \quad v^* = \frac{v}{u_\infty}, \quad T^* = \frac{T - T_\infty}{T_s - T_\infty}, \quad p^* = \frac{p}{\rho u_\infty^2} \quad (3.16)$$

We obtain:

$$\text{Continuity equation : } \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \quad (3.17)$$

$$\text{Momentum equation : } u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (3.18)$$

$$\text{Energy equation : } u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{RePr} \frac{\partial^2 T^*}{\partial y^{*2}} \quad (3.19)$$

In addition to the Reynolds number, we obtain another nondimensional parameter being the *Prandtl number*, which is defined as:

$$Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k_f} \quad (3.20)$$

The Prandtl number is the ratio of momentum diffusivity to thermal diffusivity, and it indicates the relative thickness of the velocity and thermal boundary layers[26]. For  $Pr < 1$ , the thermal boundary layer is thicker than the velocity boundary layer, while for  $Pr > 1$ , the opposite is true.

Recalling again the hypothesis of a smooth, flat plate if we insert the nondimensional form of the velocity gradient at the wall into Equation (3.5), we can relate the Reynolds number  $Re$  to the skin friction coefficient  $C_f$ :

$$C_f = \frac{\tau_s}{\frac{1}{2}\rho u_\infty^2} = \frac{2}{Re} \left. \frac{\partial u^*}{\partial y^*} \right|_{y^*=0} \quad (3.21)$$

Since for a prescribed geometry the gradient of the nondimensional velocity at the wall is only a function of the Reynolds number and the nondimensional axial coordinate  $x^*$ , the skin friction coefficient can be expressed as a function of these two parameters:

$$C_f = f(Re, x^*) \quad (3.22)$$

If we insert the nondimensional form of the temperature gradient at the wall into Equation (3.7), we can relate the convective heat transfer coefficient  $h$  to the Nusselt number  $Nu$ :

$$Nu = \frac{hL}{k_f} = - \left. \frac{\partial T^*}{\partial y^*} \right|_{y^*=0} \quad (3.23)$$

Since for a prescribed geometry the gradient of the nondimensional temperature at the wall is only a function of the Reynolds number, the Prandtl number and the nondimensional axial coordinate  $x^*$ , the Nusselt number can be expressed as a function of these three parameters:

$$Nu = f(Re, Pr, x^*) \quad (3.24)$$

Thus, as long as the the physical system shares the same geometry and the same similarity parameters  $Re$  and  $Pr$ , the Nusselt number will be comparable and the convective heat transfer coefficient can be readily determined.

Although the aforementioned dimensionless parameters were derived for the case of a flat plate, they can be applied to other geometries as well, provided a suitable characteristic length  $L$  is chosen. As a matter of fact, in this work we will be focusing on the case of internal flows, where the characteristic length is typically chosen as the hydraulic diameter  $D_h$  of the channel.

It must be reminded however that for rough surfaces and internal flows surface roughness and entry effects also play a part in the definition of the relevant nondimensional parameters.

### 3.3 Internal flows

Internal flows are confined within a solid boundary such as in a channel, and as a result not only is it necessary to consider whether the flow is laminar or turbulent, but also the flow regime in terms of whether the flow is fully developed or developing.

As such, we can identify two distinct regions in an internal flow: the entry region and the fully developed region.

For the purposes of this thesis, we will be focusing on the case of a circular pipe, with hydraulic diameter  $D_h$  as the characteristic length:

$$D_h = \frac{4A}{P_w} \quad (3.25)$$

With  $A$  being the cross-sectional area and  $P_w$  its wetted perimeter.

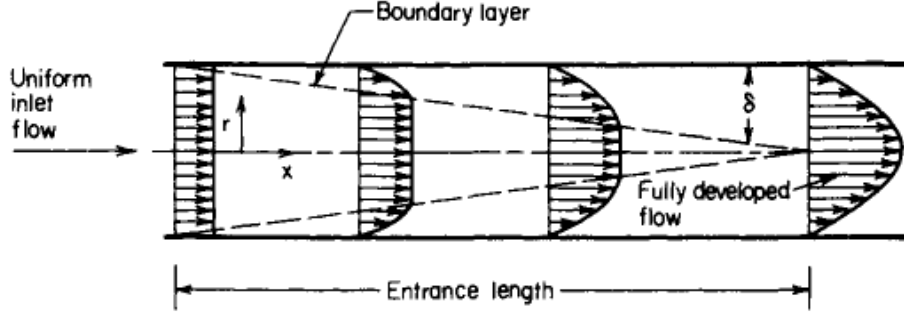
We will consider both the fluid mechanics and the heat transfer aspects of the flow.

#### 3.3.1 Fluid Mechanics

Considering a steady state and incompressible flow of uniform velocity  $u(0, r)$  entering a circular, smooth pipe of diameter  $D_h$  such as in figure 3.5 , a kinematic boundary layer will begin to develop in the radial direction starting from the pipe wall (where velocity is zero). As the flow moves downstream, the boundary layer grows at the expense of the inviscid core region until a certain distance from the entrance  $x_{fdh}$  is reached. At this point, the boundary layer thickness  $\delta$  is equal to the radius of the pipe and the flow is said to be fully developed.

When the flow is fully developed, viscous effects extend over the entire cross section, the transverse velocity component is zero the axial velocity profile is uniform in the channel axis direction:

$$\frac{\partial u}{\partial x} = 0, \quad v = 0 \quad (3.26)$$



**Figure 3.5:** Example of the hydrodynamic entry and fully developed regions in an internal laminar flow[26].

For *laminar flow* the entry region length  $x_{fdh,l}$  can be computed from knowledge of the Reynolds Number [27]:

$$Re_D = \frac{\rho u_m D_h}{\mu} \quad (3.27)$$

It is important to note that in the absence of a free stream velocity  $u_\infty$  for internal flows a mean velocity  $u_m$  is ind referred to:

$$u_m = \frac{\dot{m}}{\rho A} \quad (3.28)$$

Where  $\dot{m} = \int_A \rho u(r) dA$  is the mass flow across the channel cross section. Thus we obtain  $x_{fdh}$  as:

$$\frac{x_{fdh,l}}{D_h} \approx 0.05 Re_D \quad (3.29)$$

By applying the momentum conservation equation to an annular differential element of surface  $dA = 2\pi r dr$  it is possible to compute the velocity profile for fully developed laminar flow:

$$u(r) = -\frac{1}{4\mu} \left( \frac{\partial p}{\partial x} \right) r_0^2 \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right] \quad (3.30)$$

Hence the velocity profile is parabolic.

Knowledge of the mean velocity (for example via mass flow measurement) also allows to compute the pressure gradient for fully developed laminar flow by applying Equation 3.26:

$$u_m = -\frac{r_0^2}{8\mu} \frac{\partial p}{\partial x} \quad (3.31)$$

It is then possible to compute the axial velocity at any radial position from the centerline.

Instead of the skin friction coefficient  $C_f$ , for internal flows it is more common to refer to the Darcy friction factor  $f$ :

$$f = \frac{-\left(\frac{\partial p}{\partial x}\right) D_h}{\rho u_m^2/2} \quad (3.32)$$

The Darcy friction factor thus represents a nondimensional form of the pressure drop necessary to sustain motion across the channel, and it is an important experimental output for the purposes of this work.

For laminar flow, it can be computed analytically by substituting Equation (3.27) and (3.31) into (3.32):

$$f = \frac{64}{Re_D} = f(Re_D) \quad (3.33)$$

$f$  is thus a constant in the fully developed region. Although this result was obtained for a smooth pipe or channel, from Equation (3.32) we can infer that the friction factor is not a function of surface roughness for laminar flows.

From Equation 3.31, computing the shear stress at the wall  $\tau_s$  and substituting into the definition of skin friction coefficient  $C_f$  yields:

$$f = 4C_f \quad (3.34)$$

It has been found [30] that for internal flows in smooth channels transitional behaviour begins from  $Re_D = 2300$  and fully turbulent behaviour takes place from  $Re_D = 10000$ .

It must be noted that again, external factors like sudden turns and surface roughness can anticipate transitional behaviour [28].

Once turbulence sets in, analytical approaches are significantly more complex and often experimental correlations are preferred.

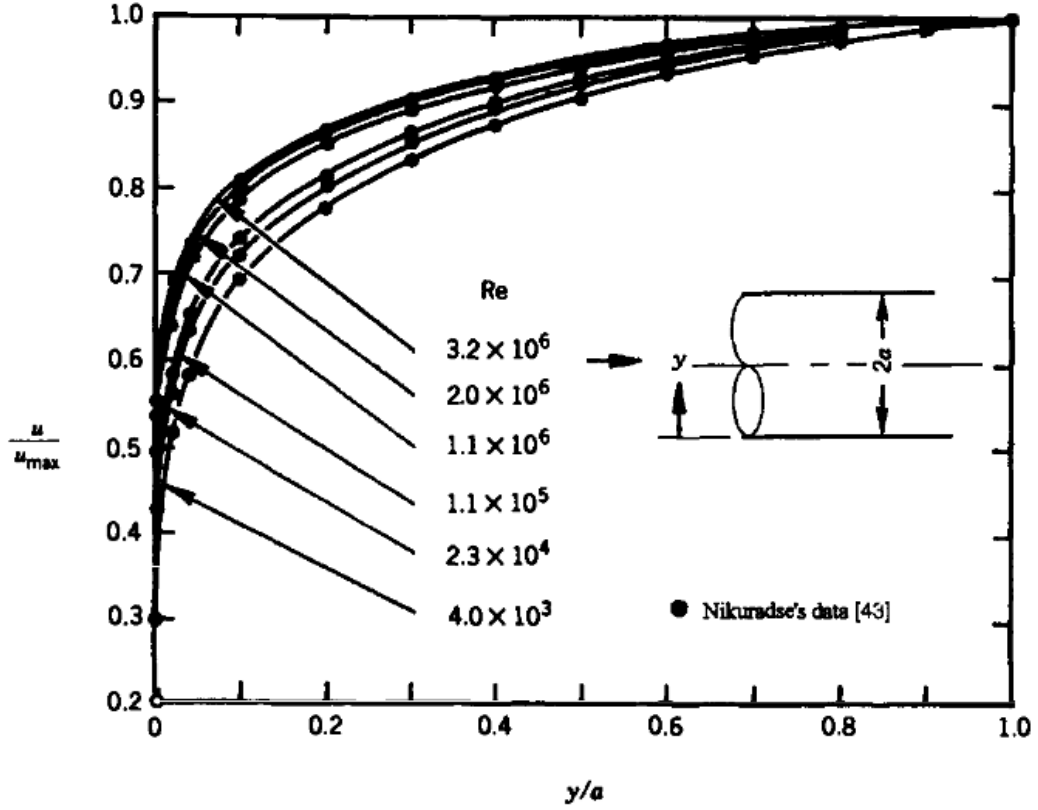
For turbulent flow the entry region length  $x_{fdh,t}$  is much smaller than its laminar counterpart  $x_{fdh,l}$  and as an approximation the following range is taken into account:

$$10 < \frac{x_{fdh,t}}{D_h} < 60 \quad (3.35)$$

A solution was proposed by Prandtl [31] for the velocity profile in the fully developed region:

$$\frac{u}{u_{max}} = \left(\frac{r_0 - r}{r_0}\right)^{\frac{1}{n}} \quad (3.36)$$





**Figure 3.6:** Velocity profile for fully developed turbulent flow [26] .

Where the coefficient  $n$  depends on the Reynolds number  $Re_D$ . Nikuradse [32] obtained several values of  $n(Re)$  experimentally and the resulting turbulent velocity profiles can be observed in Figure 3.6.

For turbulent flow, the friction factor  $f$  is a function of both the  $Re_D$  and the channels relative roughness  $\frac{e}{D_h}$ , where  $e$  is a parameter indicating absolute surface roughness:

$$f = f(Re_D, e/D_h) \quad (3.37)$$

Experimental correlations have thus been obtained correlating the friction factor  $f$  to both  $Re_D$  and relative roughness, one of which by Colebrook-White [12]:

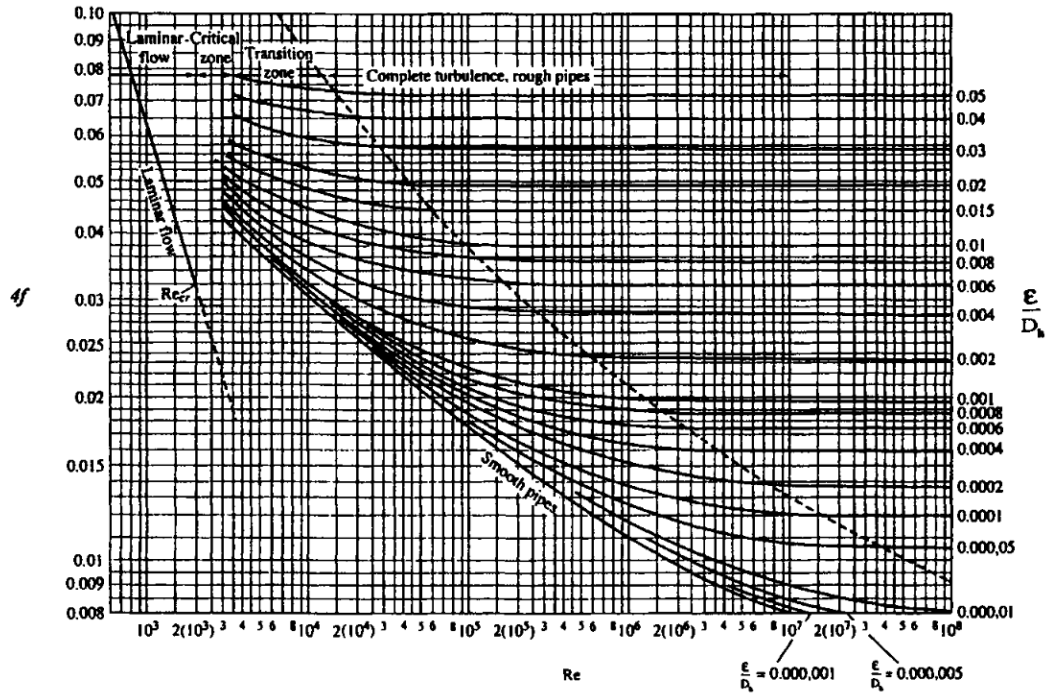
$$\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left[ \frac{e/D_h}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right] \quad (3.38)$$

Another one by Petukhov [33] correlates the friction factor to the Reynolds Number for smooth circular pipes:

$$f = [0.790 \ln(Re_D) - 1.64]^{-2} \quad (3.39)$$

It must be noted that Equation (3.39) is valid for  $3000 < Re_D < 5 * 10^6$ .

Analytical and experimental solutions for  $f(Re_D, e/D_h)$  for both laminar, transitional and turbulent flow are summarized by Moody [13] in the diagram of Figure 3.7.



**Figure 3.7:** Darcy Friction factor for fully developed flow in a circular tube [26] .

Once  $f$  is then obtained either from the Moody Diagram [13] or Equations (3.33), (3.38) and (3.39), from knowledge of the mass flow  $\dot{m}$  it is possible to express the pressure drop  $\Delta p$  associated with the channel length  $L$ .

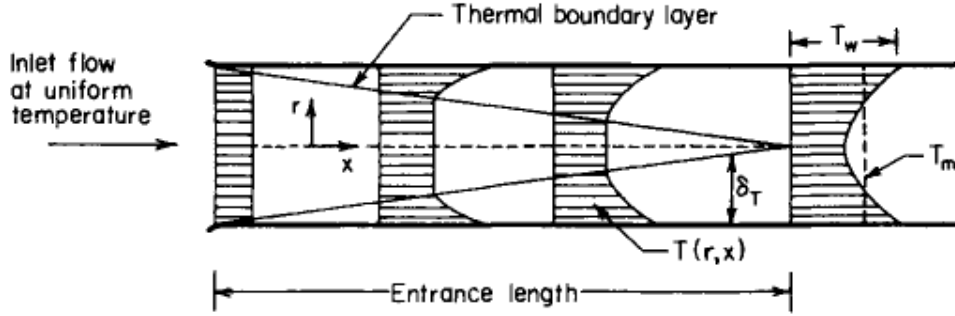
Then the pump/fan power needed to sustain the flow assuming the ideal case can be expressed as:

$$\mathcal{P} = \Delta p * \frac{\dot{m}}{\rho} \quad (3.40)$$

### 3.3.2 Heat Transfer

If we consider a steady state and incompressible flow of uniform temperature  $T(0, r)$  entering a circular, smooth pipe of diameter  $D_h$  the inner surface of which is at a temperature  $T_s$  different than  $T$  then a thermal boundary layer will begin to develop in the radial direction such as in Figure 3.8 and convection heat transfer will occur.

If the wall temperature  $T_s$  is kept constant in the streamwise direction, or if a constant wall heat flux  $q_s''$  flows into or out of the fluid, then it will also be possible to distinguish a thermal entry region and a thermally fully developed region.



**Figure 3.8:** Entry and fully developed region for internal flow in a channel or pipe[26].

For laminar flow, the thermal entry length  $x_{fd,t}$  was found [27] to be dependent on both  $Re_D$  and  $Pr$ :

$$\frac{x_{fd,t}}{D_h} \approx 0.05 Re_D Pr \quad (3.41)$$

For air it is reasonable to assume a constant value of the Prandtl number:

$$Pr \approx 0.71 \quad (3.42)$$

Thus  $x_{fd,t} < x_{fd,h}$  and thermal development is faster than hydrodynamic development.

It is important to keep in mind that thermally fully developed conditions don't imply a constant temperature profile  $T(r)$ . As a matter of fact, if there is heat transfer then the mean temperature  $T_m$  of the fluid will either increase or decrease downstream.

We define the mean temperature so that:

$$\dot{m} c_p T_m = \int_A \rho u c_p T dA \quad (3.43)$$

And for the case of constant heat flux  $q_s''$ , of particular importance for this work as it is representative of cooling of joule-heated channels, the mean temperature is

linear.

In fact, if we consider the energy balance of an annular differential element  $dS = \pi D_h dx$  with negligible conduction in the axis direction:

$$dq = \dot{m}c_p((T_m + dT_m) - T_m) = \dot{m}c_p dT_m = q''\pi D_h dx \quad (3.44)$$

Which yields:

$$\frac{\partial T_m}{\partial x} = \frac{q''\pi D_h}{\dot{m}c_p} \quad (3.45)$$

Then the function  $T_m(x)$  is obtained by integrating along the channel length:

$$T_m(x) = \frac{q''\pi D_h}{\dot{m}c_p}x + T_0 \quad (3.46)$$

Where  $T_0$  is the inlet temperature of the undisturbed flow. This result is valid for entry length and fully developed region alike.

The mean temperature can then be used to apply Newton's Law of Cooling to express the uniform heat rate:

$$q_s'' = h(T_s - T_m) \quad (3.47)$$

Thus we define the condition for thermally fully developed flow to be one of constant nondimensional temperature difference:

$$\frac{\partial}{\partial x} \left( \frac{T_s - T}{T_s - T_m} \right) = 0 \quad (3.48)$$

Which means the relative shape of the temperature profile  $T(r)$  no longer changes along the axial direction.

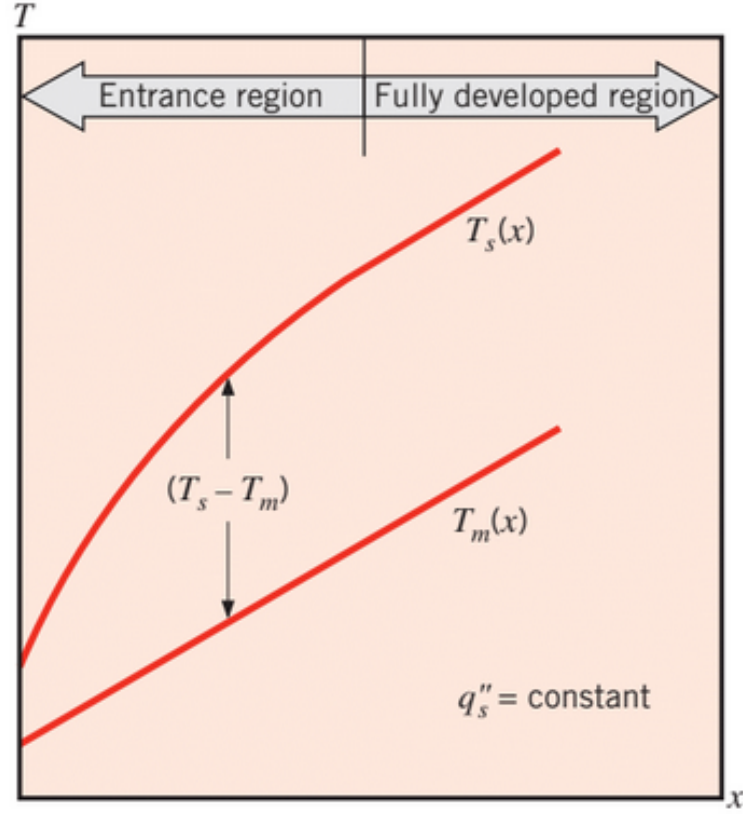
Fully developed flow implies several important properties. For example derivating the nondimensional temperature difference with respects to the radial coordinate, assuming fully developed conditions (that is, Equation (3.48) is satisfied) yields:

$$\frac{\partial}{\partial r} \left( \frac{T_s - T}{T_s - T_m} \right) = const \Rightarrow -\frac{\frac{\partial T}{\partial r}}{T_s - T_m} = const \quad (3.49)$$

Evaluating the radial derivative at the wall by expressing Equation (3.6) and substituing Newton's law of cooling (Equation (3.46)) yields:

$$\frac{h}{k_f} = const \Rightarrow Nu_x = const \quad (3.50)$$

Demonstrating the local Nusselt number  $Nu_x$ , for both laminar and turbulent flow, is a constant in the fully developed region for a fluid with constant thermal



**Figure 3.9:** Mean fluid temperature  $T_m$  and surface temperature  $T_s$  along the channel length for uniform heat rate  $q''_s$  [27].

properties.

A constant  $Nu_x$  in turn implies, with constant  $q''_s$ :

$$T_s - T_m = \text{const} \Rightarrow \frac{\partial T_s}{\partial x} = \frac{\partial T_m}{\partial x} \quad (3.51)$$

So the temperature gradient in the axis direction is the same for both mean and surface temperature.

Substituting Equation (3.51) into (3.48) also implies:

$$\frac{\partial T}{\partial x} = \frac{\partial T_m}{\partial x} \quad (3.52)$$

The axis temperature gradient is thus the same for the radial temperature distribution  $T(r)$  as well. Figure 3.9 displays some of the aforementioned properties.

As can be observed,  $T_s$  will increase in the entry region as the developing boundary

layer implies decreasing local heat transfer coefficient  $h_x$ .

Assuming fully developed, laminar flow and applying the energy balance formula [27]  $q = \dot{m}c_p(T_{out} - T_{in})$  and Equation (3.30) to an annular differential element of radius  $r$  and assuming negligible axial diffusion, it is possible to obtain the temperature profile  $T(r)$  analytically:

$$T(r) = T_s(x) - \frac{2u_m r_0^2}{\alpha} \frac{\partial T}{\partial x} \left[ \frac{3}{16} + \frac{1}{16} \frac{r^4}{r_0^4} - \frac{1}{4} \frac{r^2}{r_0^2} \right], \quad q_s'' = \text{const} \quad (3.53)$$

This result allows to obtain the mean temperature via Equation (3.43), and from (3.45) we obtain:

$$T_m(x) - T_s(x) = -\frac{11}{48} \frac{q_s'' D_h}{k_f} \quad (3.54)$$

Combining with Equation (3.47) yields:

$$h = \frac{48}{11} \frac{k_f}{D_h} \Rightarrow Nu_D = 4.36 \quad (3.55)$$

Thus the Nusselt number is a constant in the laminar, fully developed region, independent of  $Re_D$  and  $Pr$ . Such a result is obtained also considering uniform surface temperature  $T_s$ , for which condition  $Nu_D = 3.66$  is obtained [26]. As was seen previously for the fluid mechanics of turbulent flow, turbulent heat transfer for internal flows is substantially more difficult to describe analytically and experimental correlations are often employed.

As of entry region length, it is assumed [27]:

$$\frac{x_{fdt,t}}{D_h} > 10 \quad (3.56)$$

Assuming fully developed turbulent flow, there is very little difference in obtained Nusselt number for either constant temperature or constant heat rate boundary condition [26] and Dittus and Boelter [34] provided a simple empirical law for smooth channels:

$$Nu_D = 0.023 Re_D^{0.8} Pr^{0.4} \quad (3.57)$$

The latter correlation being valid for  $Re_D > 10000$ . A more accurate experimental correlation was obtained by Gnielinski [25]:

$$Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{0.5}(Pr^{\frac{2}{3}} - 1)} \quad (3.58)$$

Where  $f$  is the Darcy friction factor and can be obtained by the Moody Diagram or by Equation (3.39).

Equation (3.58) is valid for  $Re_D > 3000$  and  $0.6 < Pr < 10^5$  and it must also be

noted that it is verified for smooth channels and assumes fully developed conditions. Since Equation (3.58) assumes fully developed conditions, considering flow in a smooth pipe described by certain  $f$ ,  $Re_D$  and  $Pr$ :

$$\overline{Nu_D} > Nu_D \quad (3.59)$$

Where  $Nu_D$  is the value for the Nusselt number computed via Equation (3.58) and  $\overline{Nu_D}$  is the average Nusselt number of the pipe including entry effects.

For the purposes of this thesis, it is important to consider the effects of simultaneous thermal and hydrodynamic development, as the fluid of interest is air ( $Pr \approx 0.71$ ) and the test objects on which experiments will be carried out are short ( $L/D_h \approx 50 - 70$ ).

A correlation was suggested by Bhatti and Shah [35] for the estimation of the mean Nusselt number:

$$\frac{\overline{Nu_D}}{Nu_{Dfd}} = 1 + \frac{C}{(x/D_h)^\phi} \quad (3.60)$$

Where  $Nu_{Dfd}$  is the Nusselt number for fully developed conditions and  $C$  and  $\phi$  are coefficients dependent on inlet configuration. Such coefficients were obtained for air by Mills [36] and for a squared entrance and constant heat rate:

$$\begin{aligned} C &= 2.4254 \\ \phi &= 0.676 \end{aligned} \quad (3.61)$$

The results of Equation (3.60) are presented in Figure 3.10. While in Figure 3.11 it is possible to observe turbulent entry lengths for various conditions as formulated by Notter and Sleicher [37]. In conclusion it must be noted that the theoretical framework laid in this chapter is needed for understanding the physics of the problem and will be taken into account as a frame of reference for the following experimental methodology and results; however, the correlations exposed in this section are valid for smooth geometries, and additively manufactured geometries often present considerable surface roughness.

As a consequence, experimental values of  $Nu_D$  and  $f$  will be compared, respectively, to Gnielinski and Moody's results, but the mode with which those coefficients will be obtained is largely independent of previous correlations.



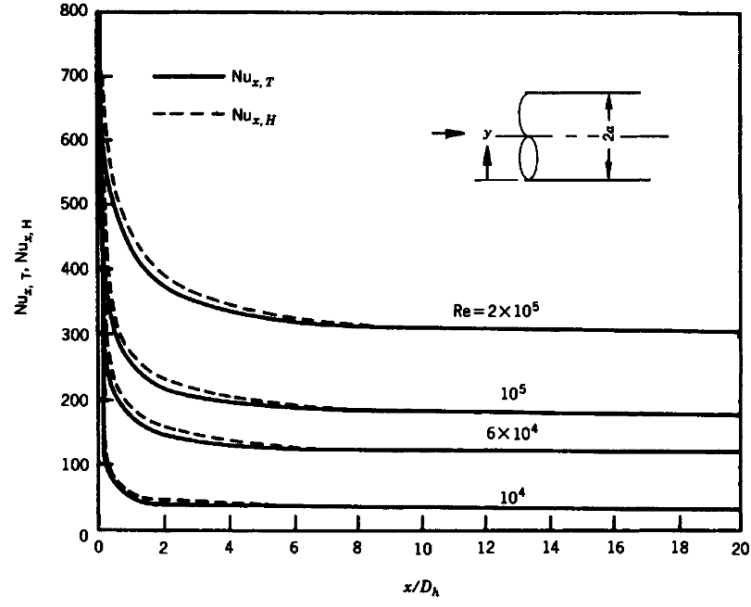


Figure 3.10: Local over mean nusselt number in the entry region[26].

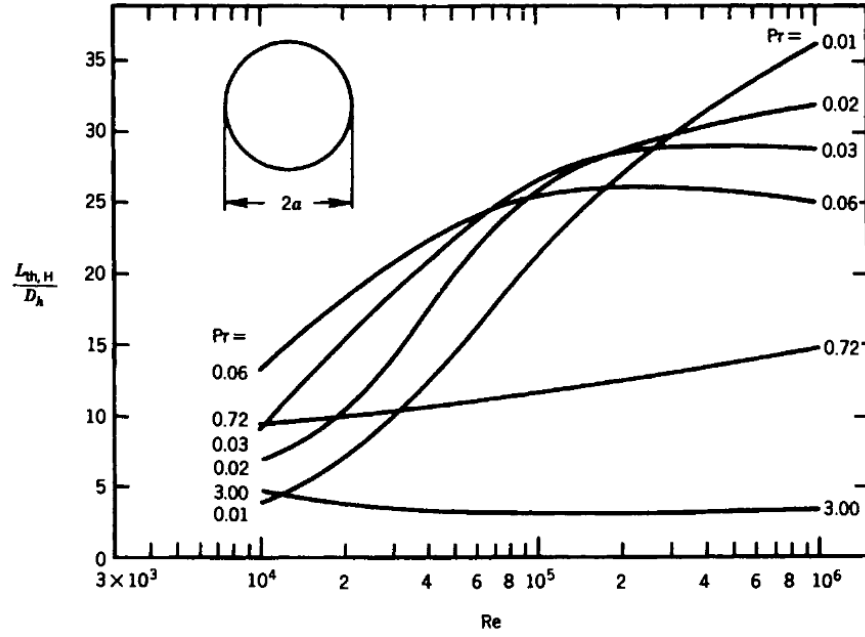


Figure 3.11: Turbulent thermal entry region length over  $Re_D$  and  $Pr$ [26].

## 3.4 Additive Manufacturing

Additive Manufacturing is the process of manufacturing a part directly from CAD models by selectively depositing, sintering, or combining material in one layer over the next [38].

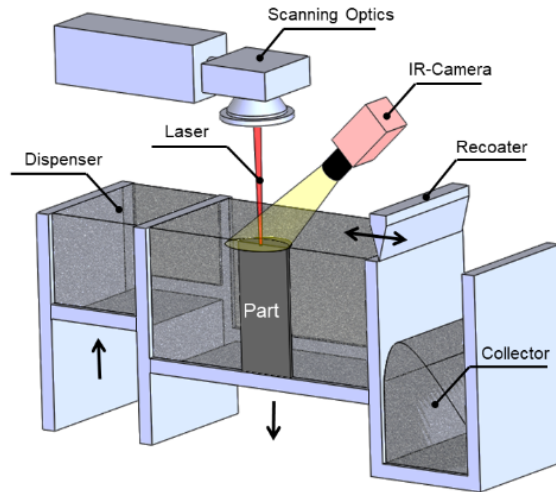
The several sub-processes of AM differ from each other on the basis of the required material and the finished product's mechanical and thermal properties; however, a similarity is found in the fact that AM removes the need of a tool or mold by allowing the building of a near net shape component layer by layer.

### 3.4.1 Powder Bed Fusion Methods

Of the several sub-processes of AM, powder bed fusion (PBF) processes are the most relevant in the context of this thesis, as they are commonly used in metal component production [39].

PBF fabricates 3D objects by melting powders of powdered feedstock using a laser (Selective Laser Melting SLM), electron beam (Electron Beam Melting EBM) or intense infrared lamp.

In both SLM and EBM, a focused energy source scans the cross section of the component and locally melts the powders as well as partially remelts the previous layer to allow for the binding of contiguous layers. As the cross section is scanned, the build platform is lowered and additional powder is deposited from a dispenser (see Figure 3.12 ) to increment the part vertically.



**Figure 3.12:** Selective laser melting on a Powder Bed Fusion architecture for additive manufacturing [40].

SLM processes produce components with higher surface finish and lower porosity compared to previous, sintering based, processes [38]. However, the internal stresses left in the metal due to the thermal gradients introduced require post build heat treatment [38]. Moreover, energy losses due to powder reflectivity need to be considered.

EBM processes are more energy efficient compared to SLM as the metal powder is melted by the friction from the electrons striking the layer's surface; moreover, the high energy electron beam maintains a high temperature in the build chamber, reducing thermal gradients and internal stresses in the part [38].

EBM needs a vacuum environment to prevent scattering from collisions with gas particles and oxidation of the hot metal pool.

Materials fabricated with EBM can also be fully dense and have similar mechanical properties as the equivalent cast materials [38].

Key limitations however include high expense, relatively small build volumes and the necessity of a pathway to ground for the electrons, limiting EBM processes to the manufacturing of metallic part [38].

### **3.4.2 AM process parameters**

The goal of additive manufacturing is producing a defect-free part. For PBF methods like EBM and SLS those can be grouped in two categories [41]:

- Pores and voids
- Cracks and delaminations

The onset of such defects can not only render the part unusable, but also disrupt the manufacturing process itself. As such, the appropriate combination of process parameters needs to be chosen.

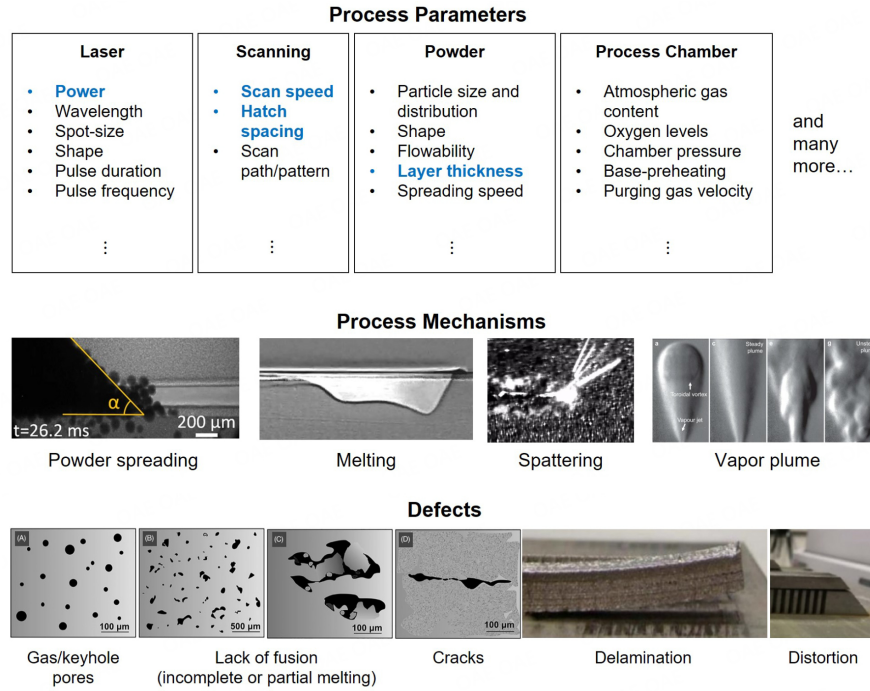
Regarding porosity, the key aspects to consider are the pore's morphology and formation mechanism. Irregularities in the pores (such as sharp edges) act as stress concentrators and porosity due to localised lack of fusion is detrimental to the material's mechanical properties [41].

It also needs to be noted that a complete lack of porosity is impractical, as the formation of pores and voids might be linked to factor such as power fluctuation, particle packing and spatter, all of them largely intractable [42].

Cracks and delaminations are linked to the process' thermal cycling [43]: as the energy source heats up the powder surface, a heated zone forms undergoing thermal expansion and a compressive stress region is formed at the contact between the heated zone and the surrounding material at a lower temperature; when the energy source is removed, the previously heated zone undergoes shrinkage and said

shrinkage is restrained by previous plastic strain, leading to tensile residual stress. Macroscale residual stresses are most responsible for surface integrity of AM parts [43] and can cause delaminations, cracks and distortions. In order to mitigate the effects of residual stresses, using the lowest effective energy source and preheating of the powder bed's base plate have been suggested [41].

Process parameters not only influence the frequency and severity of defects, but can also be manipulated to influence material properties [39]. Figure 3.13 presents a selection of process parameters, process mechanisms and material defects resulting from PBF processes.



**Figure 3.13:** A selection of process parameters for SLM [41].

### 3.4.3 AM Induced Surface Roughness

Surface roughness is the most important feature of AM materials for the purpose of the presented work. One of the major factors which affect surface roughness is the inherent "layer by later" approach of AM processes, and in terms of process parameters, contour parameters (such as laser/electron beam power and scanning speed) have been found to be the most impactful [44].

Table 3.1 summarizes how several other process parameters influence surface roughness:

| Parameter            | Surface Roughness                                |
|----------------------|--------------------------------------------------|
| Power                | Increases if power increases                     |
| Scanning speed       | Either increases or decreases if speed increases |
| Energy density       | Decreases if energy density increases            |
| Powder particle size | Either increases or decreases if size decreases  |
| Number of remelts    | Decreases if number of remelts increases         |

**Table 3.1:** A selection of AM Process parameters and their effect on Surface Roughness [45].

The main aspect of AM surface roughness to be considered in this work is its interaction with turbulent fluid flow.

Considering the roughness Reynolds number  $k_s^+$ :

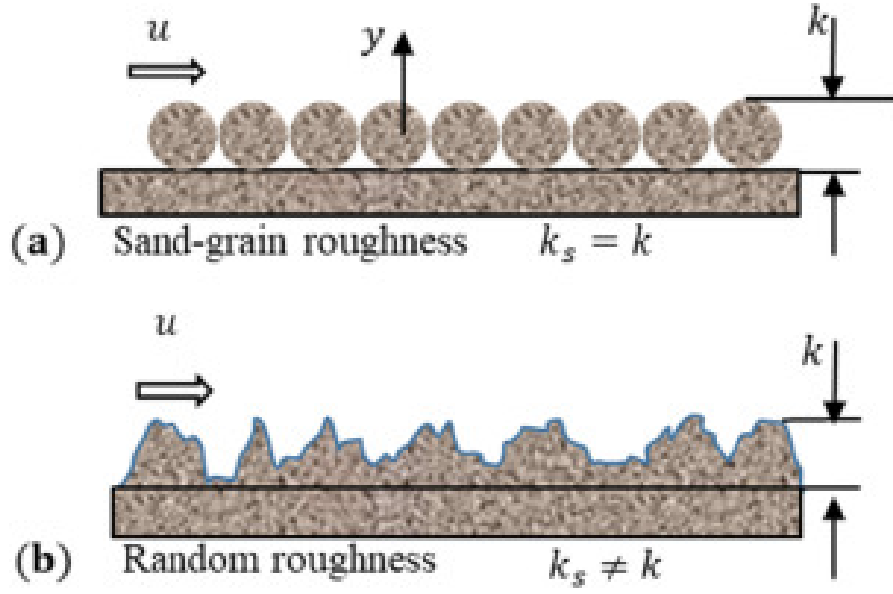
$$k_s^+ = \frac{k_s u_\tau}{\nu} \quad (3.62)$$

Where  $k_s$  is the equivalent sand-grain roughness, for the evaluation of which several methods and correlations are available in literature [46],  $\nu$  is the kinematic viscosity and  $u_\tau = \sqrt{\frac{\tau_s}{\rho}}$  the friction velocity, and two critical values  $k_{s,smooth}^+$  and  $k_{s,rough}^+$ , three different roughness regimes are defined [46]:

- **Hydraulically smooth regime:**  $k_s^+ \leq k_{s,smooth}^+$  and the roughness elements are embedded in the viscous sublayer and skin friction and drag coefficients are not modified.
- **Transitionally rough regime:**  $k_{s,smooth}^+ \leq k_s^+ \leq k_{s,rough}^+$  and both viscous and pressure forces influence skin friction and drag coefficients.
- **Fully rough regime:**  $k_s^+ \geq k_{s,rough}^+$  and the rough elements protrude in the fully turbulent flow region, the viscous sublayer is destroyed by the turbulent mixing at the wall and the pressure loss becomes independent of fluid viscosity, as a result quantities like the Darcy friction factor for bounded flows become functions of solely relative roughness.

$k_{s,smooth}^+$  and  $k_{s,rough}^+$  depend on roughness type and several values are available in the literature [46].

Figure 3.14 shows an example of ideal sand-grain roughness and random roughness. In the former case, the sand-grain height  $k$  may define the sand-grain roughness  $k_s$ , however this is not the case for the latter situation, where one parameter cannot describe it anymore.



**Figure 3.14:** Roughness characterisation for ideal sand-grain roughness and random roughness [46].

It has been found [8] that both the density and topology of roughness elements influence turbulent heat transfer over rough walls. In particular, roughness density affects the impingement of flow over roughness elements, enhancing heat transfer in the fully rough regime and until sheltering effects prevail [8]. Because direct roughness measurements are not available for all coupons, interpretations in terms of relative roughness should be treated as qualitative and not as quantitative evidence.

# Chapter 4

## Methodology

This chapter will showcase the methodology adopted to obtain experimental results in terms of both Nusselt number and Darcy friction factor: it will provide an overview of the instrumentation employed, testing procedures and post processing methods.

### 4.1 Experimental setup

The laboratory setup employed to acquire the desired data can be subdivided into three components:

- **Joule heating rig:** comprised of an air loop and an electrical loop, it provides airflow from a reservoir and heat generation via a step-down transformer.
- **Measurement instrumentation:** the instruments employed to acquire live data.
- **Test Objects:** the additively manufactured test coupons for which Nusselt Number and Friction Factor data will be acquired.

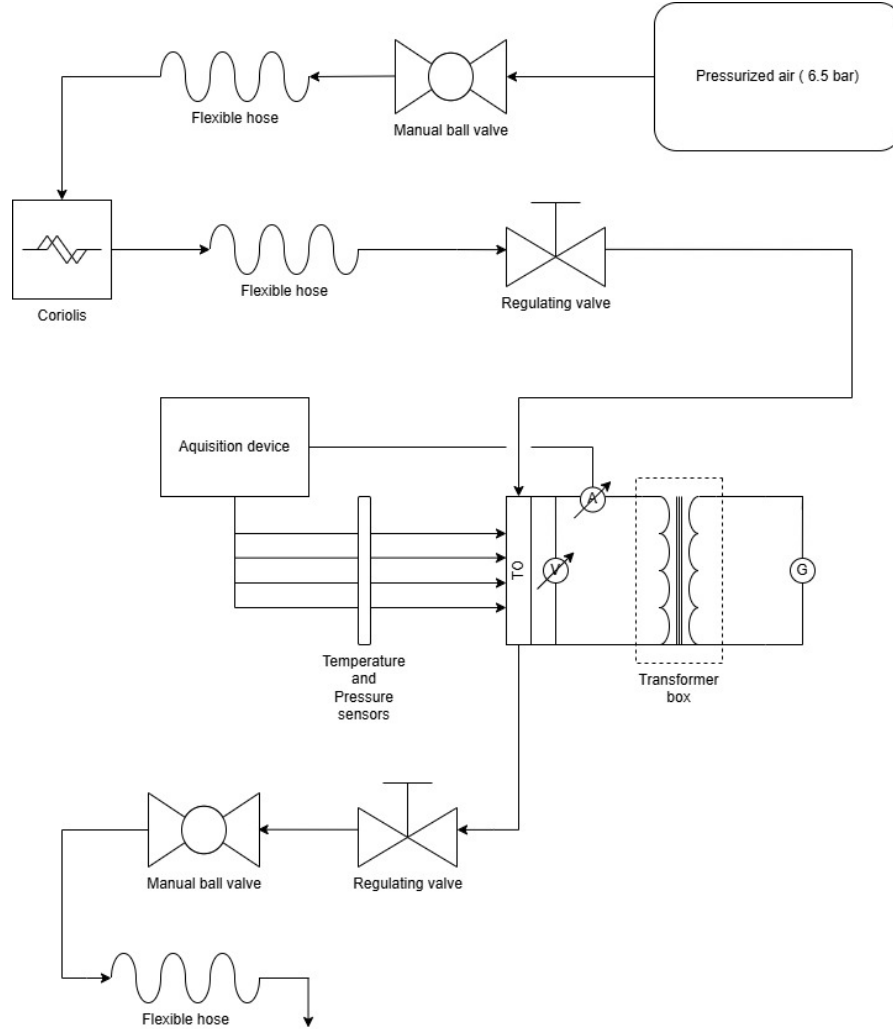
This section will thus provide a detailed description of each component, and how these interact during laboratory experiments.

#### 4.1.1 Joule Heating air rig

The Joule Heating test rig is the component of the setup meant to generate the desired physical conditions for either Nusselt number or friction factor evaluation. When conducting the former, it provides both internal heat generation in the test coupon via joule heating and pressurized air for forced convection heat transfer. It will be seen that the experimental model will employ a constant wall heat flux

boundary condition in the center of the test coupon [26].

For friction factor results, only pressurized air will be provided instead, and a different hydraulic piping configuration will be adopted so as to not disturb air flow for more accurate pressure measurement.



**Figure 4.1:** Flow chart schematics of the joule heating test rig air loop.

In Figure 4.1 a flow chart of the Joule Heating test rig is presented and an overview of its main elements can be as follows:

- **Manual ball valves:** adjust air flow through the system, the outlet one is kept fully open and the inlet one is only opened when air flow has to be provided. It is then kept fully open to provide air at its highest pressure (6.5 bar).

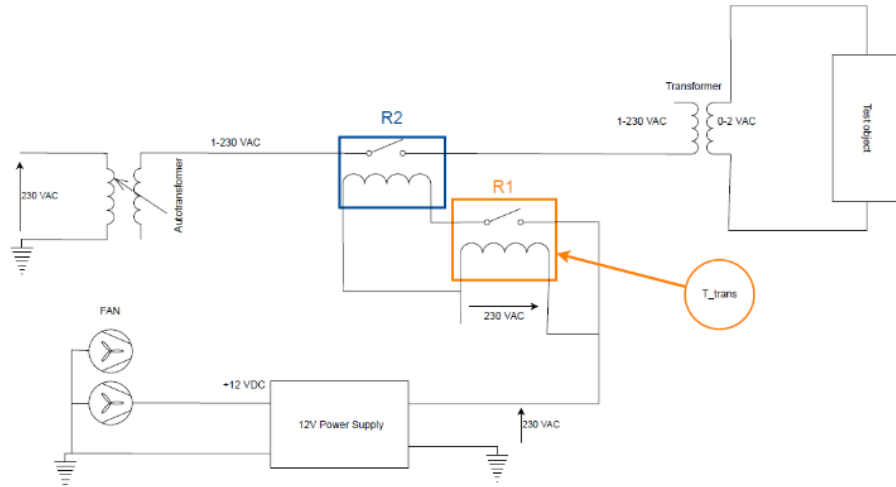


- **Flexible hoses:** they connect the ball valves to either the mass flow meter (inlet) or the remainder of the closed air loop of the laboratory.
- **Regulating valves:** positioned at the inlet and at the outlet of the test coupon, they can be adjusted to regulate mass flow and inlet air pressure. During testing, the inlet regulating valve is kept fully open, while the outlet one is adjusted to regulate mass flow at the highest inlet pressure.
- **Coriolis:** it provides the measurement of the mass flow rate into the system.
- **Acquisition devices:** they collect data from temperature, current, voltage or pressure sensors and relay it to a computer program for live data acquisition.
- **Transformer box:** provides voltage to the test object.

It should also be noted that apart from the flexible hoses, the remainder of the hydraulic system is made up of metal connectors, to allow for the safe passage of the pressurized air.

The electrical circuit employs 50 Hz AC current relayed by a fixed ratio step-down transformer, in a range of 0-400 A for a voltage range of 0-2 V.

More detailed schematics for the transformer box are presented in Figure 4.2.

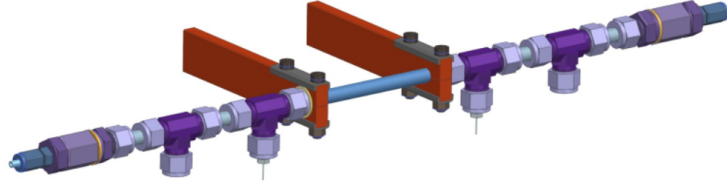


**Figure 4.2:** Circuit schematics of the transformer box.

The test object itself is then connected to the hydraulic piping via Parker connections: they consist in a steel nut that can be screwed to the test section prior application of a steel ring and cone; when screwing the nut in via a wrench, by plastic deformation of the said cone, the Parker nut will be fixed in place and will

be able to connect to the remainder of the hydraulic system.

The test object's connection to the transformer box is accomplished via electrical cable: one or two turns are made around the transformer core and then said cable is fixed with bolt and nut to a copper bar, which is in turn clamped to the test object via bolt and nut.



**Figure 4.3:** CAD drawing of Nusselt number Joule heating rig [47].



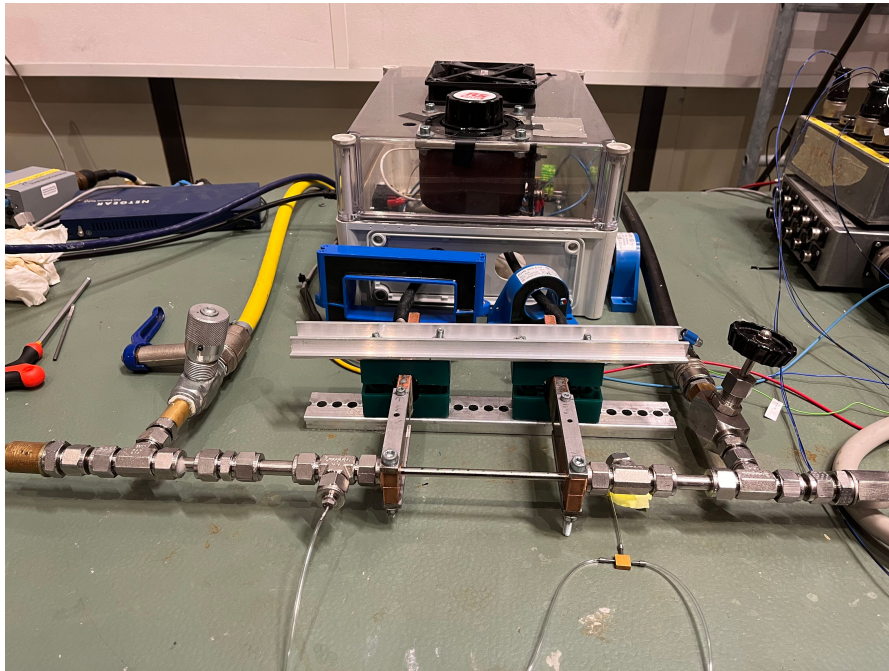
**Figure 4.4:** CAD drawing of Darcy friction factor rig [47].

Figure 4.3 and 4.4 showcase CAD drawings of the testing rig setup for Nusselt number and friction factor respectively.

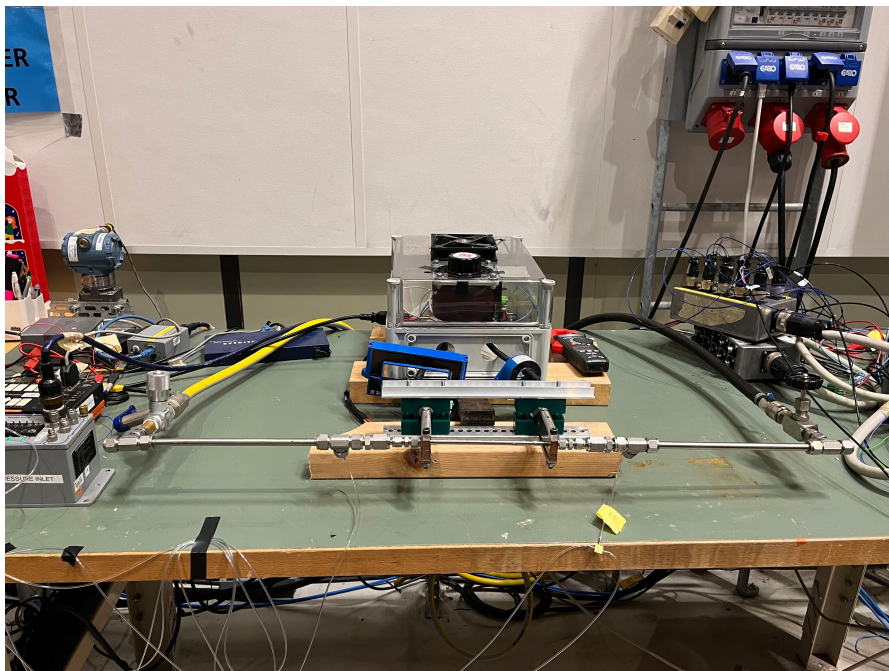
It is important to note that the difference in hydraulic connectors at the inlet and outlet of the test object between the two setups is due to the need to include mean temperature sensors for a Nusselt test, and to provide undisturbed flow pressure measurements for a Darcy test. Figure 4.5 displays a typical setup for a Nusselt number test, before the addition of temperature sensors and insulating material. Both the inlet and outlet regulating valves can be seen as well as the transformer box and the copper bars connecting the test object to the power supply system. A support structure comprised of two beams and two pairs of plastic clamps keeps the test object in place and prevents excessive deviation of the air flow.

Figure 4.6 instead displays a typical setup for a friction factor test. This setup is simpler than its heat transfer counterpart, as it will be demonstrated only mass flow and pressure measurements will be needed for post processing in this case. The test object is again fixed to the transformer box for added stability.

As will be shown in a following subsection, the test objects have a machined and an unmachined part, the latter displaying an external diameter of either 10 mm or



**Figure 4.5:** Experimental Setup for Nusselt number test.



**Figure 4.6:** Experimental Setup for Darcy friction factor test

12 mm. Thus, inlet and outlet piping for both Nusselt and Darcy tests have to be chosen in accordance with the test section itself.

#### 4.1.2 Measurement Instrumentation

The data that needs to be acquired during testing for either Nusselt or Darcy tests include:

- **Static Pressures** :  $P_{in}, P_{out}, P_{atm}, P_{diff}$
- **Mass flow rate**:  $\dot{m}$
- **Bulk temperatures**:  $T_{in}, T_{out}, T_{cu,in}, T_{cu,out}$
- **Surface temperatures**:  $T_i, T_{p,in}, T_{p,out}$ ,
- **AC Voltage and current**:  $V, I$

Where  $T_i, T_p, T_{cu}$  respectively refer to a generic test object external surface temperature, a Parker surface temperature (either inlet or outlet) and a copper bar temperature (either inlet or outlet).

$T_{in}, T_{out}$  refer to inlet and outlet mean air temperature.

The AC voltage and current  $V, I$  instead refer to the voltage across the test object and the electrical current flowing through it. Table 4.1 summarises the devices employed to gather such data:

| Variable             | Device            | Accuracy            | Unit |
|----------------------|-------------------|---------------------|------|
| Mass flow rate       | Coriolis (10-21)  | $\pm 0.56\%$        | g/s  |
| Pressure             | PSI 9116          | $\pm 1\%$ (100 PSI) | kPa  |
| Atmospheric pressure | Rosemount 3051CA1 | $\pm 1\%$ (100 PSI) | kPa  |
| Surface Temperature  | PT100             | $\pm 0.15$ °C       | °C   |
| Bulk Temperature     | PT100             | $\pm 0.05$ °C       | °C   |
| AC Current           | Biltema 15-338    | $\pm 2\% + 0.08$    | A    |
| AC Voltage           | Fluke 179         | $\pm 1\% + 0.3$     | mV   |

**Table 4.1:** Measurement devices and relative uncertainty.

The mass flow rate measurement is taken via a coriolis device before the inlet to the test section. Two different devices have been employed in the gathering of data: coriolis 10 for experimental points between 0 and 0.9 g/s and coriolis 21 for experimental points over 0.9 g/s. Coriolis 10 has better precision for lower mass flow rates compared to coriolis 21.

Both instruments are connected via USB port to a computer for live monitoring of

data.

Static pressure measurements are taken at the inlet and outlet of the test object, via the PSI 9116. The latter converts the deformation of an internal diaphragm into a pressure reading: thus, in addition to  $P_{in}$  and  $P_{out}$ , it also relays the  $P_{diff}$  between two given points by converting the net diaphragm deformation into a net pressure differential. Each channel in the PSI is tied to a certain end of scale, these being 2.5 PSI, 15 PSI and 100 PSI.

The PSI is then connected to a computer for live monitoring via ethernet connection.

Temperature measurements are taken via PT100 sensors: these come in either a probe type, or a wire soldered to a flat rectangular surface. The former are employed to take mean fluid temperatures  $T_{in}$  and  $T_{out}$  and copper bar temperatures  $T_{cu,in}$  and  $T_{cu,out}$ , while the latter take surface temperature measurements only:  $T_i, T_{p,in}, T_{p,out}$ .

The device employed to relay such temperature measurements to the live data monitor program is the Keysight Agilent 34972A.

The electrical measurements are taken via handheld devices: the multimeter is applied across the copper bars to measure  $V$  and the ammeter directly to either cable of the transformer.

The atmospheric pressure is taken by the Rosemount 3051CA1 transducer, and in turn this value is connected to a computer for live monitoring via the Datascan 7220 device.

### 4.1.3 Test Objects

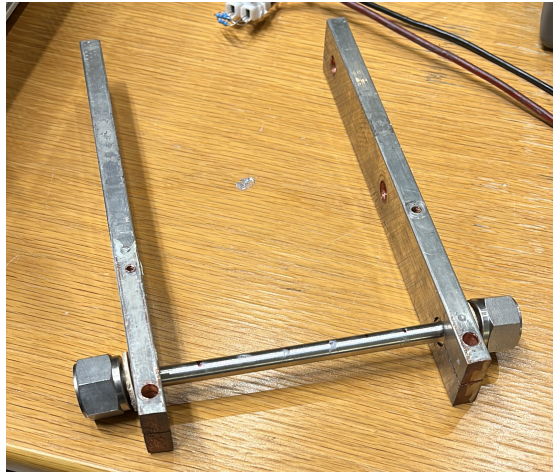
The test coupons considered in this work consist in cylindrical bodies with internal micro-channels of varying geometry and number. The materials vary from superalloys to stainless steel and all are made by PBF additive manufacturing, with the exception of those used for rig validation, which exhibit smooth internal geometries. The lengths vary between 150 mm and 200 mm and can be consulted in Table 4.2. The external diameter of AM coupons is not uniform, as those are machined in the middle to increase electrical resistance and heat generation.

Thus, the diameter of the unmachined portion of each test object ranges from 10 mm (for single-channel test objects) to 12 mm (for multichannel test objects). Its length is usually 25 mm for each side, to accommodate both the Parker connection (15 mm of length) and the copper bar (10 mm of length).

Figure 4.8 displays a test object alongside its connectors to both the air and the electrical loop of the test rig. Cork insulator rings are placed between Parker and copper busbar to prevent the electrical current from flowing towards the exterior. It is important to note that during testing the voltage measurement is taken across the copper bars to provide for an easier setup.



**Figure 4.7:** Example of a machined test object. In this case (Inconel 939), the machined portion is longer, as it also includes the length of the copper bar.



**Figure 4.8:** Assembly of test object (Haynes 282), Parker connectors and copper bars.

In turn, this calls for reducing the electrical resistance of the Parker section: a digital torque wrench set at 20 Nm is thus employed to fix the copper bars to the test object via dedicated clamps. As a result, the residual resistance is in the order of  $10^{-5} \Omega$ .

The reduction in residual resistance also allows for better accuracy of the numerical



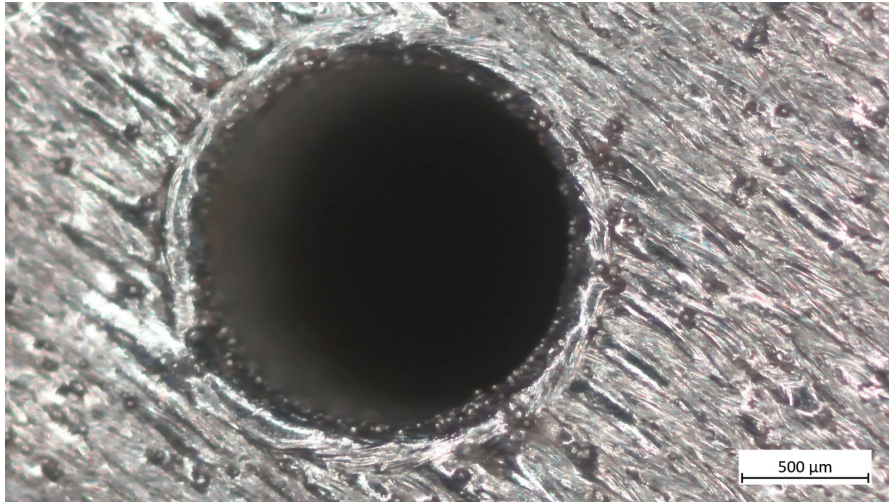
model, as will be shown in a later section.

Finally, each test object is defined by its hydraulic diameter. As both the Nusselt number and the Darcy friction factor are sensitive to it, rather than relying on its nominal value a more accurate estimation is accomplished with the use of a microscope image.

Usually at least two microscope images will be taken, with different zooms, of both the inlet and outlet cross-section. Then with the aid of a MATLAB script a spline will be traced on the outline of the internal circumference on the image and values for perimeter, area and diameter of such spline interpolation will be computed.

An average of the values obtained on the different images will be considered for the final hydraulic diameter chosen for each test object.

When dealing with multichannel test objects, an average over each sub-channel will also be considered. Figure 4.9 displays an example of a microscope image for geometry evaluation.



**Figure 4.9:** Microscope image of a test object's cross-section for geometry evaluation.

The following Table 4.2 then summarises the test objects for which experimental results will be presented in this work.

The test objects labeled "smooth" have only been tested for rig validation purposes and were not produced via AM, as a result they have little surface roughness and can be modeled as smooth channels. Furthermore, they have not been machined. The section geometry is circular for each test object but Haynes 282, which displays a square 2x2 mm inner channel.

| Test Object | Hydraulic diameter [mm] | Length [mm] | n° of channels |
|-------------|-------------------------|-------------|----------------|
| Haynes 282  | 2.096                   | 150         | 1              |
| Inconel 939 | 2.131                   | 150         | 1              |
| SS 6x1.25   | 1.304                   | 170         | 6              |
| SS 1x1.25   | 1.270                   | 170         | 1              |
| SS (smooth) | 4.035                   | 200         | 1              |
| Al (smooth) | 3.280                   | 150         | 1              |

**Table 4.2:** List of test objects for which experimental data has been produced.

## 4.2 Testing Procedures

The following section will present an overview of the testing procedures followed for obtaining data for both Nusselt number and Darcy friction factor evaluation.

### 4.2.1 Nusselt Number Test procedure

After setting up the test object on the rig like in Figure 4.5 and ensuring the copper bar clamps have been tightened via digital wrench, the following actions are carried out before proceeding with the testing:

1. Ensure the copper clamps have been tightened with the desired torque.
2. Perform a leakage test: this consists in letting air flow in the system and checking for air leaks, eventually spraying soap on critical connections and searching for bubbles.
3. Place the surface PT100 sensors on the test object.
4. Place the probe PT100 in the copper plates.
5. Place the insulating material on the rig and test object.

The fluid temperature PT100 are usually already screwed in place in the hydraulic circuit while the surface PT100 are placed in the following fashion:

- 8 or 9 "channel" sensors on the test object's outer surface.
- 1 sensor each on the inlet and outlet Parker surface.

The PT100 on the test object are placed with constant spacing considering the length of the machined section, with each sensor placed parallel to the axis of the coupon.



The number of sensors depends on test object length as it has to both provide sufficient resolution for the following post-processing procedure and ease of installation. For all Nusselt number results presented in this work and with the sole exception of Inconel 939, testing has been conducted employing 9 sensors.

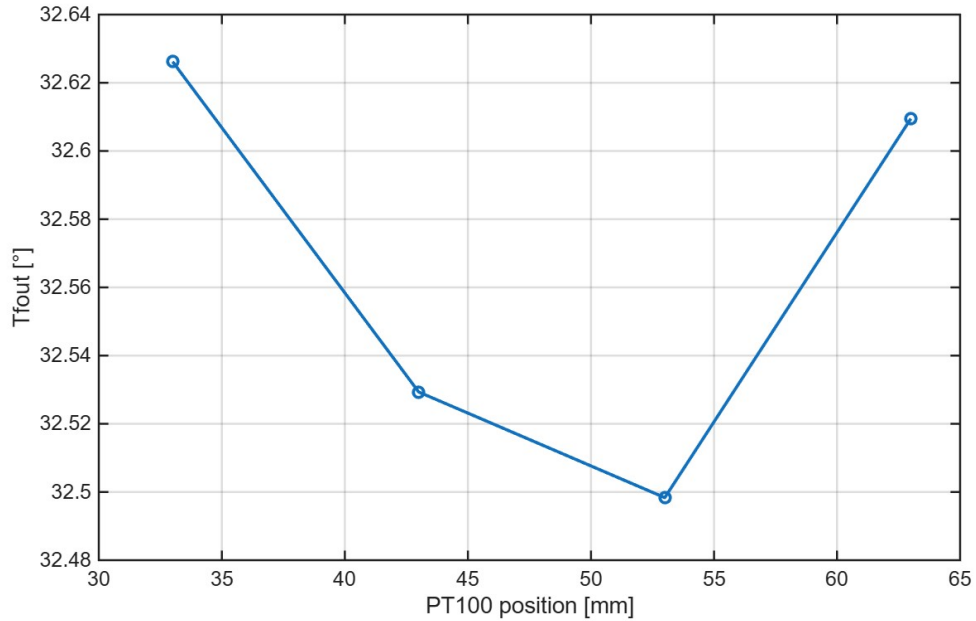
Each sensor is first dipped in Arctic mx4 thermal paste and then kept in place with a first layer of vinyl commercial tape, another layer of tape ensures the PT100 surface adheres to the test object as closely as possible.

The number of channel sensors is usually chosen to ensure good resolution for the experimental axial temperature distribution, and can vary depending on the available sensors and length of the test object.

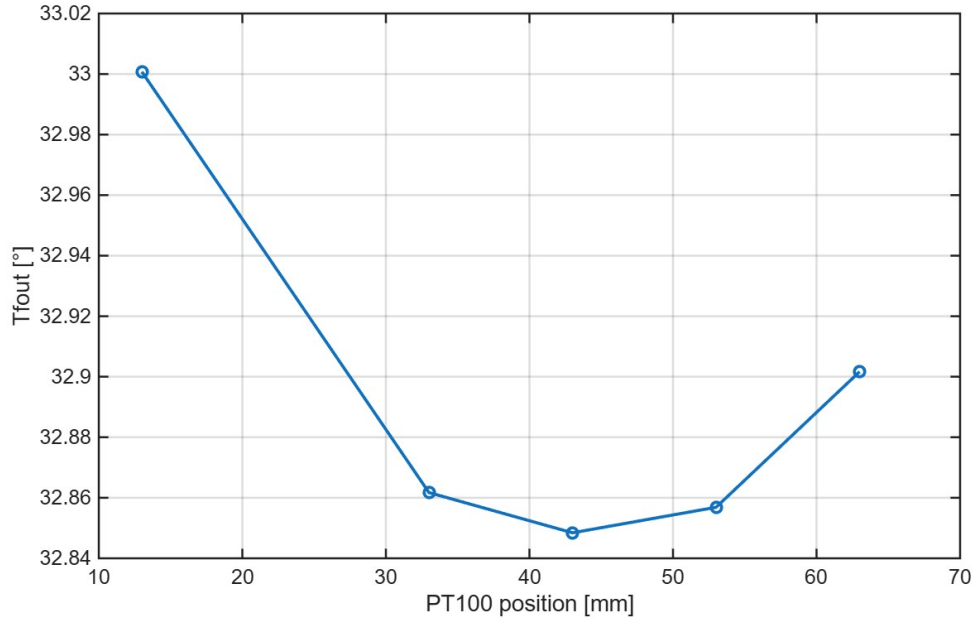
The same procedure is taken with regards to the Parker sensors.

The copper probe PT100 are inserted in the copper bars into pre-drilled holes. In virtue of the material's high thermal conductivity, the temperature distribution in the copper bar is fairly uniform, reducing the temperature reading sensitivity to the holes' placement.

Regarding the mean fluid temperature PT100, in order to minimize the  $T_{out}$  reading sensitivity to the "depth" of the outlet PT100, a flow mixer is added either upstream or downstream of the  $P_{out}$  pressure tap. Figures 4.10 and 4.11 display  $T_{out}$  values



**Figure 4.10:** Outlet fluid Temperature reading related to sensor depth (10 mm rig).



**Figure 4.11:** Outlet fluid Temperature reading related to sensor depth (12 mm rig).

obtained for both the 10 mm and 12 mm test rigs. In each test Reynolds number and electrical power have been kept fixed, the only variable being the length of the exposed PT100 probe.

As can be seen, the different  $T_{out}$  values obtained in both cases fall under a threshold of  $0.2\text{ }^{\circ}\text{C}$ .

It must be noted however that for the 12 mm rig, a flow mixer with a single row of fins has been employed, while the 10 mm rig was equipped with a double finned one, reducing the maximum depth achievable by the probe. Figure 4.12 shows an example of a double finned flow mixer.

Figure 4.13 shows the Joule heating rig previous application of all temperature sensors.

The neoprene insulating material is placed on either ends of the rig hydraulic piping and on the test object itself. To prevent measurement errors from channel PT100, a second layer of insulator is also placed around the test object. This has the added benefit of reducing thermal losses at the external surface of the test object, thereby making it reasonable to approximate this surface as adiabatic in the numerical model and operate the experimental model at a constant wall heat flux boundary condition.

To keep the insulation system in place, plastic zip ties are wrapped around each



**Figure 4.12:** Double finned outlet flow mixer.

layer and closed shut.

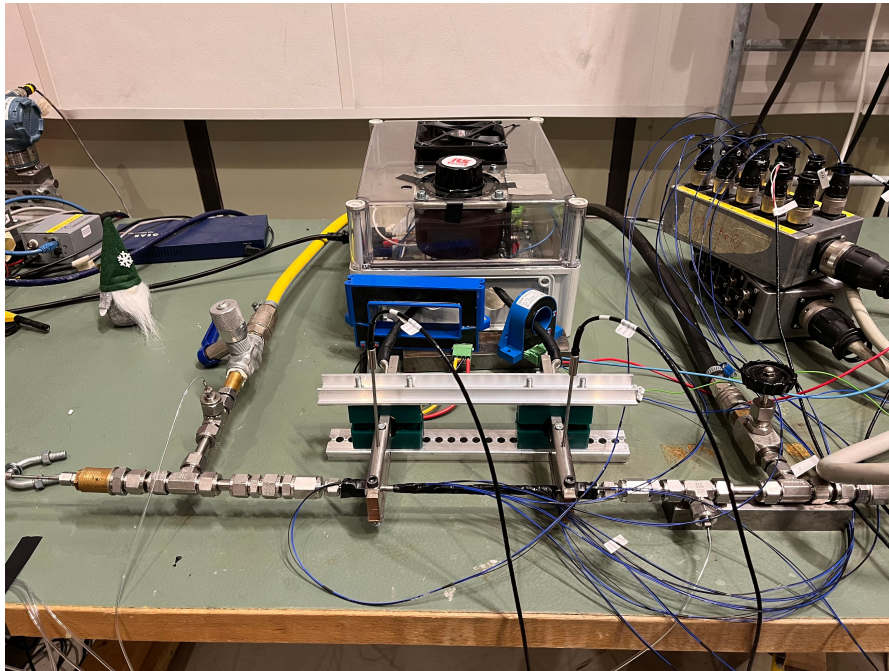
Figure 4.14 shows an example of a completed Nusselt number test setup. Once the setup is completed, it is possible to begin data acquisition via Rigview.

Rigview allows to manage and plot live data coming from the instrumentation thanks to the aforementioned ethernet and usb connections.

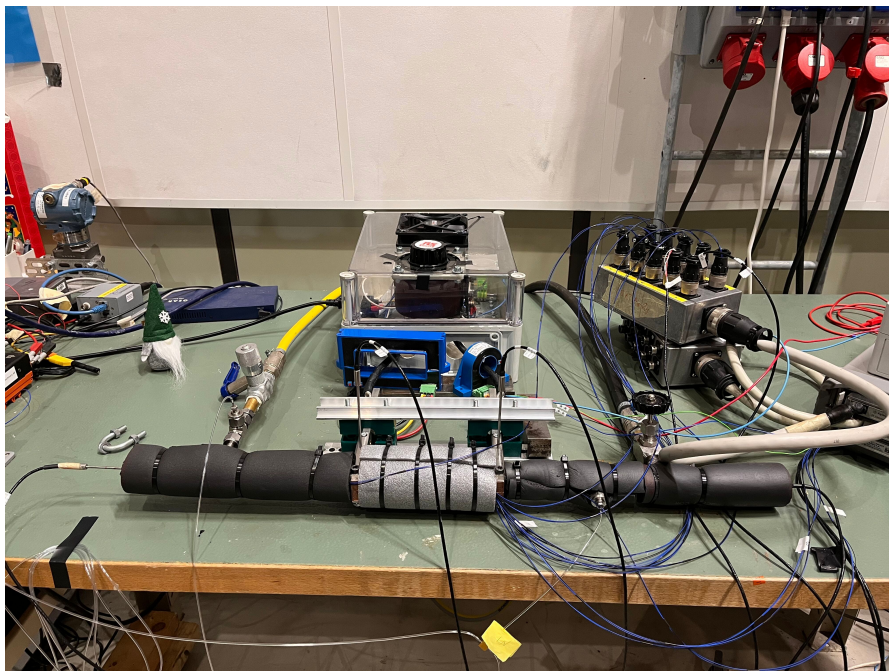
At the same time, it has been found practical to monitor quantities such as the Reynolds number and the inlet/outlet temperature difference via an excel sheet, which elaborates Rigview live data via the Scada/Temas live software.

The objective of a Nusselt test is to obtain temperature, mass flow and current/-voltage measurements for a certain Reynolds number measured considering inlet air conditions. The typical routine for each test point is summarised as follows:

1. Adjust mass flow so as to reach the desired Reynolds number.
2. Adjust the electrical power into the test section via the transformer valve, so as to maintain  $\Delta T = T_{out} - T_{in} \approx 10 - 20$  °C.
3. Monitor each temperature via Rigview plot, and wait until semi-steady state conditions are achieved.



**Figure 4.13:** Joule heating air rig after temperature sensor placement



**Figure 4.14:** Complete Nusselt number test setup

4. Acquire the data for the selected Reynolds via Rigview.
5. Measure via handheld devices voltage and current through the test section.
6. Acquire inlet and outlet Parker resistances, typically in the  $10^{-5} \Omega$  range if the copper bars have been fixed to the test object according to the procedure showcased in section 4.1.3.

Regarding point 3, a minimum threshold for the length of the transient state between two consecutive Reynolds points has been defined as the criterion for achieving semi steady state.

It has been defined as the time necessary for the test object's surface temperatures  $T_i$  to reach a rate of change of less than  $0.1^\circ\text{C}$  per minute.

However it must be noted that without a method for monitoring the first derivative of each measurement, this definition remains mostly based on previous experiences. Regarding point 6, the inlet and outlet Parker resistances  $R_{in}$  and  $R_{out}$  are measured first by acquiring the inlet/outlet Parker voltages  $V_{in}$  and  $V_{out}$  and dividing by the measured current  $I$ :

$$R_{in/out} = \frac{V_{in/out}}{I} \quad (4.1)$$

Thus for each Reynolds point Rigview will output a line on a .DIF file containing the measurements in table 4.3. It must be noted that pressure measurements won't be included in the post-processing and are only taken for safety purposes, and ensuring the flow is incompressible.

|                                        |                  |
|----------------------------------------|------------------|
| Mass flow rate                         | g/s              |
| Inlet air temperature $T_{in}$         | $^\circ\text{C}$ |
| Outlet air temperature $T_{out}$       | $^\circ\text{C}$ |
| Test object wall temperature(s) $T_i$  | $^\circ\text{C}$ |
| Inlet Parker temperature $T_{p,in}$    | $^\circ\text{C}$ |
| Outlet Parker temperature $T_{p,out}$  | $^\circ\text{C}$ |
| Inlet copper temperature $T_{cu,in}$   | $^\circ\text{C}$ |
| Outlet copper temperature $T_{cu,out}$ | $^\circ\text{C}$ |
| Inlet air pressure $P_{in}$            | kPa              |
| Outlet air pressure $P_{out}$          | kPa              |
| Differential pressure $P_{diff}$       | kPa              |
| Atmospheric pressure $P_{atm}$         | kPa              |

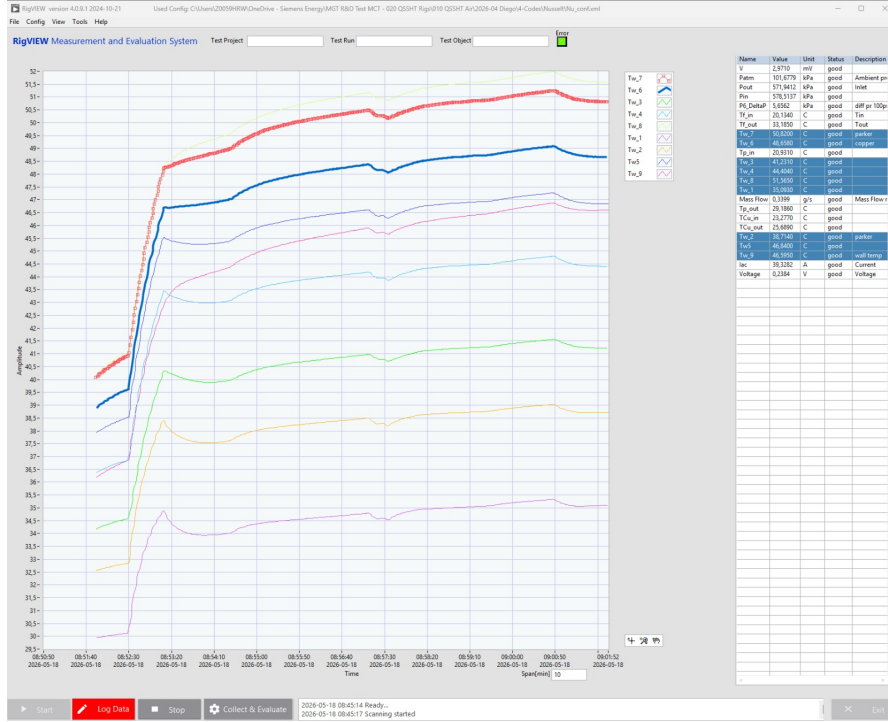
**Table 4.3:** Rigview outputs for every test point of the Nusselt number test procedure.

At the same time on a separate excel sheet the quantities reported on table 4.4



|                                    |          |
|------------------------------------|----------|
| AC Voltage                         | mV       |
| AC Current                         | A        |
| Inlet Parker resistance $R_{in}$   | $\Omega$ |
| Outlet Parker resistance $R_{out}$ | $\Omega$ |

**Table 4.4:** Electrical quantities taken for each test point.



**Figure 4.15:** Rigview interface for a Nusselt number test.

are recorded. Figure 4.15 displays the live data to be monitored during a Nusselt number test.

By plotting the desired temperatures together, it is possible to follow their evolution over time and determine that steady state conditions are satisfied once their rate of change over time has sufficiently decreased.

Despite the picture only showing the test object's outer wall temperatures, it is important to monitor several different temperature measurements, as transient periods differ.

Nevertheless, considering joule heating results in a constant wall heat flux boundary condition, an often effective first check on the quality of the experimental setup is accomplished by checking that the wall temperatures increase in a linear fashion in the flow direction for a certain Reynolds.

Obviously, as thermal contacts are present at the interface with Parkers and copper bars, a monotone linear temperature distribution cannot be accomplished.

As mentioned in subsection 4.1.2, two different coriolis are used to measure mass flow rate, thus the Reynolds pertaining to  $\dot{m} = 0.9$  g/s marks the coriolis switch, and a test point at that same Reynolds with either instruments will be taken.

A Nusselt test is usually concluded once either of these conditions are met:

- The maximum mass flow rate that can flow through the rig has been reached (around 3.8 g/s).
- The outlet Mach number has exceeded the value of 0.25

Although no definite bounds are placed on the Reynolds number spread for each test, and a Nusselt test can be carried out either decreasing the mass flow from a starting high Reynolds, or increasing it from a starting low Reynolds.

However attention must be paid in regards to wall temperatures: an excessively low Reynolds compared to the  $\Delta T$  chosen might cause the adhesive tape to melt and/or catch fire or damage the PT100.

As a result Nusselt tests do not include results in the laminar region, for in that case the limited heat transfer will lead to excessively high temperatures if coupled with  $\Delta T \approx 10$  °C.

### 4.2.2 Friction Factor Test procedure

Considering the setup in Figure 4.6 and compared to a Nusselt test, the setup for a Friction factor test is simpler as it only requires checking the system for leakages, that in turn are now more impactful on the final result.

No more sensors need to be placed on the rig/test object and it is immediately possible to begin taking testdata via Rigview, which now should display and load into a .DIF file the quantities from Table 4.5.

It must be noted that although no direct air temperature measurement is taken, every Darcy test still includes an ambient air reading for air density evaluation, typically from the Coriolis device.

|                                  |     |
|----------------------------------|-----|
| Mass flow rate                   | g/s |
| Inlet air pressure $P_{in}$      | kPa |
| Outlet air pressure $P_{out}$    | kPa |
| Differential pressure $P_{diff}$ | kPa |
| Atmospheric pressure $P_{atm}$   | kPa |

**Table 4.5:** Rigview outputs for every test point of the friction factor test procedure.

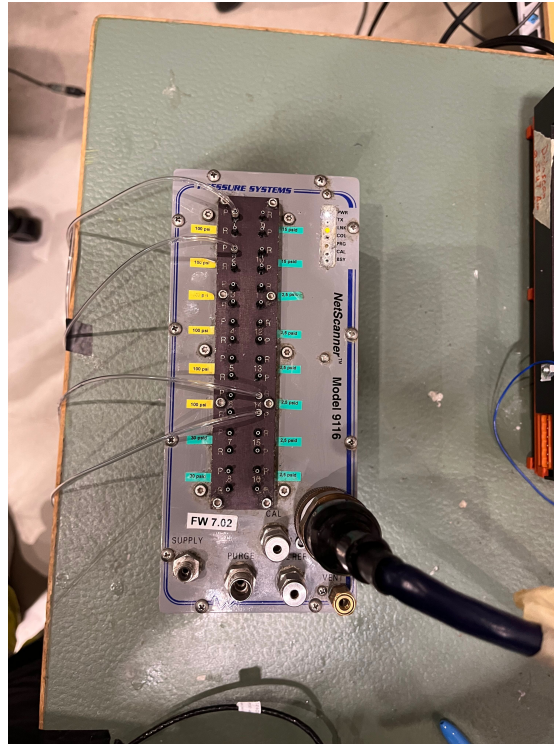
The procedure for the acquisition of a test point is again similar to a Nusselt test:

1. Adjust mass flow so as to reach the desired Reynolds number.
2. Monitor  $P_{diff}$  and mass flow rate, wait until semi-steady state conditions are achieved.
3. Acquire the data for the selected Reynolds via Rigview.

For a Darcy test however better care must be taken in ensuring the flow is not compressible (outlet Mach  $< 0.24$ ), and updating the PSI channel reading the differential pressure  $P_{diff}$ .

For example, when increasing Reynolds,  $P_{diff}$  also increases and its value can approach the maximum value that can be read by the PSI while that measurement resides in a certain channel.

Considering that the PSI channels are capped at either 2.5 PSI ( $\approx 17.5$  kPa), 15 PSI ( $\approx 100$  kPa) or 100 PSI ( $\approx 690$  kPa), when starting at a low Reynolds it might be useful to begin with  $P_{diff}$  at 2.5 PSI, and then change it to 15 PSI.



**Figure 4.16:** Pressure channels in a PSI. In the figure  $P_{out}$  and  $P_{in}$  are respectively placed in the first and second channels (both capped at 100 PSI) while  $P_{diff}$  is placed in channel 14 (capped at 2.5 PSI).



### 4.3 Nusselt Number Evaluation

This section provides a detailed description of the procedures employed to obtain the desired outputs from the "raw" test data collected following the indications of the previous section.

The average Nusselt number as a function of Reynolds for the test object is obtained via a STAR-CCM+ model of the test coupon, parameterized by a Java solver. The peculiarity of this method is the limited amount of flow assumptions: only the solid metal is modeled and the Nusselt number is obtained by convergence to the experimental model by updating the test object's channel boundary conditions.

#### 4.3.1 Star-CCM+ model

The partial differential equation that is to be solved is Equation 3.3 (the Heat transfer equation) coupled with the generalised Ohm's Law [48]:

$$\underline{J} = \sigma_e * \underline{E} \quad (4.2)$$

Where  $\underline{J}$  indicates the electrical current density distribution,  $\sigma_e$  is the material's electrical conductivity and  $\underline{E}$  is the electrical field driving the current through the test object.

Solving this differential equation for a certain voltage applied to the test object gives knowledge of the current density, which in turn yields the volumetric heat source term  $\dot{q}$ :

$$\dot{q} = \underline{J} \cdot \underline{E} \quad (4.3)$$

The model is parameterised in order to reach a convergence criterion after a number of simulations where Equations 3.3 and 4.1 are solved via 3D finite volumes. This is repeated for every test point and Reynolds number.

The necessary steps to build a numerical model of a test object are summarised as follows:

1. **Geometry definition:** A CAD of the test section is drawn via Siemens NX and imported into STAR-CCM as .igs or .prt file. The surface subdivision necessary to define boundary conditions is performed directly on the CAD before importing on STAR-CCM.
2. **Boundary definition:** Once the geometry has been defined in STAR-CCM, each surface is assigned a boundary condition.
3. **Material definition:** Density, electrical and thermal conductivity of the test object material are assigned to the the model. STAR-CCM allows for the addition of temperature-dependent properties by adding tabled data.

4. **Mesh definition:** A finite volume mesh is then generated on the test object geometry. The mesh is generated via the STAR-CCM+ automated volume mesher [48], aiming for a total cell count approaching  $\approx 10^5$ , as the test objects considered in this work are of similar dimensions.

Figure 4.17 presents the geometry of a test object as seen in STAR-CCM. The PT100 contact surfaces are separated from the rest of the external machined surface in order to evaluate an average model temperature over such areas.



**Figure 4.17:** Test object geometry for STAR-CCM+ evaluation.

The boundary conditions of the model are assigned differently based on if the boundary is external or internal:

- **External boundary:** Divided into contact surfaces with PT100 sensors, copper bars and Parkers; external machined surface and inlet and outlet cross sections.
- **Internal boundary:** Divided into "channels" centered around each PT100 location. The length of such channels is equal except for the first and last ones.

As the PDE to be solved is coupled, each boundary will be assigned a thermal and an electrical boundary condition. Table 4.6 summarises the boundary conditions applied to the model.

The heat transfer coefficients used to model the thermal contact at the interface between test object and copper busbar ( $h_{cu}$ ) or Parker ( $h_p$ ) are defined as constants:

- $h_{cu} = 2587 \frac{W}{m^2 K}$
- $h_p = 400 \frac{W}{m^2 K}$

| Boundary                        | Thermal B.C                              | Electrical B.C            |
|---------------------------------|------------------------------------------|---------------------------|
| Inlet Busbar surface            | Thermal contact ( $h_{cu}, T_{cu,in}$ )  | Current ( $I, R_{in}$ )   |
| Outlet Busbar surface           | Thermal contact ( $h_{cu}, T_{cu,out}$ ) | Current ( $-I, R_{out}$ ) |
| Inlet Parker (fitting) surface  | Thermal contact ( $h_p, T_{p,in}$ )      | Insulator                 |
| Outlet Parker (fitting) surface | Thermal contact ( $h_p, T_{p,out}$ )     | Insulator                 |
| PT100 surface                   | Adiabatic                                | Insulator                 |
| External machined surface       | Adiabatic                                | Insulator                 |
| Inlet cross section             | Adiabatic                                | Insulator                 |
| Outlet cross section            | Adiabatic                                | Insulator                 |
| Interior channel                | Convection ( $h_i, T_{f,i}$ )            | Insulator                 |

**Table 4.6:** Boundary conditions assigned to the model

While the surface temperatures at the interface are the recorded  $T_{cu,in/out}$  and  $T_{p,in/out}$  for the relevant test point.

The value for  $h_{cu}$  has been obtained under laboratory testing conditions by Santos [49], using water as cooling medium, for the stainless steel 6 channel test object.

The value for  $h_p$  has been chosen considering the thermal resistance of the plastic Parker cone and ring.

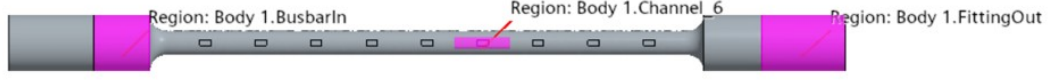
The inner channel heat transfer coefficients  $h_i$  and fluid temperatures  $T_{f,i}$  are initialised and then iterated to satisfy the convergence criterion by the Java script. Similarly the current  $I$  is initialised and then iterated for each test point, while the resistances  $R_{in}$  and  $R_{out}$  are defined as the averages of the recorded values during testing and kept constant. The STAR-CCM resistance specification implies the resistance is uniformly distributed across the surface, thus high values of  $R_{in}$  and  $R_{out}$ , indicating uneven contact between busbar and test object, might lead to modeling errors as in that case localised heat generation would occur.

The variables presented in this section, alongside others pertaining to fluid properties (air thermal conductivity, Reynolds) and channel geometry (Area, hydraulic diameter) are defined as parameters by the user in STAR-CCM+ and set and updated by the Java script at each iteration until convergence is met.

### 4.3.2 Java solver

The model is parameterized via a Java solver, which will find the desired unknowns (heat transfer coefficients and air bulk temperatures for each channel) by matching the model temperatures, obtained as weighted averages of the PT100 surface temperatures, to the recorded experimental ones.

The solver reads the test data from Table 4.3 and 4.4 alongside the values for  $h_{cu}$  and  $h_p$ , then enters a for cycle (with number of iterations equal to the number of



**Figure 4.18:** Example of external and internal boundary regions in a STAR-CCM+ model (Haynes 282).

experimental test points) where the following actions are taken:

1. Set Parameters in the model ( $Re$ ,  $D_h$ ,  $h_i$ ,  $T_{f,i}$ ,  $I$ ,  $R_{in}$ ,  $R_{out}$ , etc.).
2. Solve for the current density and temperature distribution
3. Enter an inner while cycle
4. After the while cycle convergence criterion is met, perform 10 more simulations
5. Compute the local Nusselt numbers  $Nu_i = \frac{h_i D_h}{k_{f,i}}$
6. Print the output files

The inner while cycle is meant to update  $I$ ,  $h_i$  and  $T_{f,i}$  in order to meet a convergence criterion, where the error is defined as:

$$error = \sum_i^{n_{channels}} |\Delta T_i| \quad (4.4)$$

Where

$$\Delta T_i = T_{i,exp} - T_{i,model} \quad (4.5)$$

Where  $T_{i,exp}$  are the experimental surface temperature measurements on the test object and  $T_{i,model}$  are the model surface averages over the PT100 surfaces defined in the previous subsection.

Thus, as long as  $error > 0.1$ :

1. Update the current to satisfy the test object's energy balance
2. Evaluate the errors between experimental and modeled surface temperatures
3. Update  $h_i$  based on the local relative errors
4. Update  $T_{f,i}$  based on the logarithmic mean method

5. Update the error
6. Perform 1 simulation

It should be noted that the Java script takes into account temperature-dependant fluid properties ( $c_p, k_f, \mu$ ) by polynomial interpolation with the following coefficients:

- $c_p = a_0 + a_1 T + a_2 T^2 \quad \frac{J}{KgK} \Rightarrow a_0 = 1085.80; a_1 = -0.4445; a_2 = 0.0007;$
- $k_f = k_0 + k_1 T \quad \frac{Wm}{K} \Rightarrow k_0 = 0.0048; k_1 = 7.1e - 05$
- $\mu = m_0 + m_1 T \quad Pa * s \Rightarrow m_0 = 4.65e - 06; m_1 = 4.66e - 08$

The reference temperature for  $\mu$  is the experimental inlet air temperature  $T_{in}$  for each test point, while for  $c_p$  the average between  $T_{in}$  and  $T_{out}$  is considered and for  $k_{f,i}$  the local mean temperature  $T_{f,i}$  from point 4 of the while cycle procedure. It must be noted that the aforementioned coefficients were obtained by interpolating each property value over the following range of temperatures:

$$T = [273, 373] \quad K \quad (4.6)$$

The electrical current  $I$  is first initialised in the outer loop based on the recorded experimental value, but in the inner while cycle it is updated to satisfy the energy balance, which can be defined in two ways:

- **Air temperature method:**  $R_{tot} * I^2 = Q_{fluid} + Q_{losses}$
- **Electrical power method:**  $R_{tot} * I^2 = P_{electric} = V_{meas} * I_{meas}$

Where:

- $Q_{fluid} = \dot{m} c_p (T_{out} - T_{in})$  where  $T_{out}$  and  $T_{in}$  (and  $\dot{m}$ ) are measured values.
- $Q_{losses}$  is the total heat transferred to copper busbars and Parkers in the model.
- $R_{tot} = R_{in} + R_{out} + R_{TO}$  with  $R_{TO} = \frac{V}{I}$

Note that with the air temperature method,  $Q_{fluid}$  is constant for each while iteration and with the electrical power method only  $R_{TO}$  varies in the while cycle as both  $V$  and  $I$  change with each iteration.

$V$  is used both in  $R_{TO}$  evaluation as the voltage drop between copper bars and in  $P_{electric}$  as the total voltage drop across the test object: if  $R_{in}$  and  $R_{out}$  are sufficiently small as discussed in the previous sections this simplification is acceptable.

Obviously,  $V \neq V_{meas}$  as  $V$  indicates the modeled value and  $V_{meas}$  is the measured

value and is kept constant. Same applies to  $I$  and  $I_{meas}$ .

Since two methods are available to define the modeled current, the heat source term in the heat transfer equation will vary between the methods. As a result, the average Nusselt number obtained will also vary.

The channel heat transfer coefficients are first initialised in the outer for cycle according to Dittus-Boelter's equation (Equation 3.57):

$$h_i = 0.023 Re^{0.8} * Pr^{0.4} * \frac{k_f}{D_h} \quad (4.7)$$

Where  $D_h$  is the hydraulic diameter of the test object and  $k_f$  is the fluid thermal conductivity as a function of the average temperature between the experimental  $T_{in}$  and  $T_{out}$ . As the equation suggests, the initial guess keeps the heat transfer coefficient uniform across all channels.

In the while cycle,  $h_i$  is updated based on a local relative error, as such it will either increase or decrease after each iteration based on the difference between the numerical model and the experimental setup:

$$h_i = h_i * (1 - u * \frac{\Delta T_i}{T_{i,exp}}) \quad (4.8)$$

Thus, if the model surface temperature  $T_{i,model}$  is lower than the recorded value, the inner channel htc in the model will decrease. Viceversa, if the model is found to be "hotter" than the experimental setup, the htc in the model increases and along with it local heat transfer.

$u$  is the under relaxation factor and increases the oscillation amplitude of  $h_i$  for each iteration.

The local mean fluid temperature  $T_{f,i}$  is first initialised in the outer for loop as:

$$T_{f,i} = T_{in} + \frac{i}{n_{channels}} * (T_{out} - T_{in}) \quad (4.9)$$

And in the while cycle it is updated considering the logarithmic mean temperature method:

$$T_{f,i} = \frac{T_{out,i} - T_{in,i}}{\ln \frac{T_{metal} - T_{out,i}}{T_{metal} - T_{in,i}}} + T_{metal} \quad (4.10)$$

Where:

- $T_{in,i}$  is the channel inlet mean temperature
- $T_{metal}$  is the average solid temperature between the internal and external radius
- $T_{out,i} = (T_{in,i} - T_{metal}) * e^{\frac{-h_i * A_i}{\dot{m} * c_p}} + T_{metal}$  is the channel outlet mean temperature

With  $A_i$  being the wetted surface of each internal channel. Once the convergence criterion is met, the numerical model's PT100 surfaces temperatures will match the experimental temperatures taken during the test and the following output files will be produced:

1. **Results:** a .csv file containing the local nusselt numbers  $Nu_i$  for each  $Re$
2. **Log:** a .csv file containing the full heat transfer report from STAR-CCM in terms of model surface heat fluxes and model current and voltage.
3. **Temperatures:** a .csv file containing the modeled average temperatures over the test object surfaces and the model mean temperatures



**Figure 4.19:** Temperature distribution on the stainless steel multi-channelled test object for a certain Reynolds number, as reconstructed by STAR-CCM.

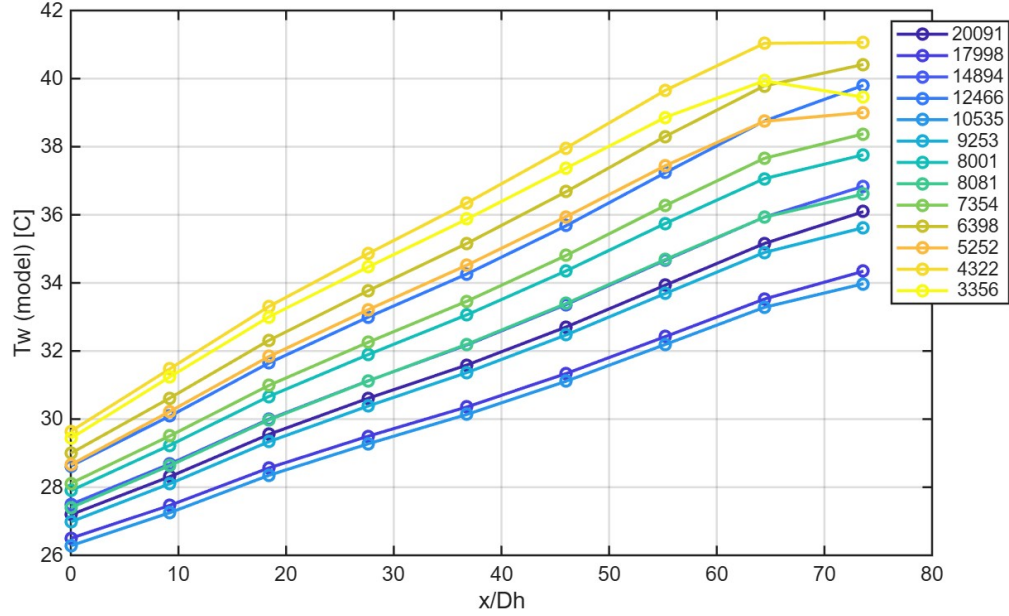


**Figure 4.20:** STAR-CCM monitor plot for the model and experimental temperatures over solver iterations.

Figure 4.20 shows how once the convergence criterion is met, values for local Nusselt number are obtained such that the model matches the experimental setup in terms

of sensor surface temperatures.

Figure 4.21 shows the modeled axial surface temperature distribution for the multichannel stainless steel test object.



**Figure 4.21:** Model axial temperature distribution for the multichannel stainless steel test object over different Reynolds.

Sensitivity to misalignment between experimental and model sensor placement needs to be kept in mind, especially for sensors placed close to the outlet section of the test object, which Figure 4.19 shows is often subjected to elevated temperature gradients.

Thus, the use of 2x1 mm dedicated surfaces in the model in place of points immersed in the volume mesh is explained.

## 4.4 Friction factor evaluation

The Darcy friction factor is computed by directly processing test data from a .DIF file as in Table 4.5.

A python script reads geometrical specifications from a .JSON file in terms of total length  $L$ , hydraulic diameter  $D_h$ , surface area  $A$ , outer unmachined diameter  $D_{cu}$ , number of channels  $N$  and rig inner diameter  $D_{rig}$ .

Further, the .DIF file is read and the air densities at the inlet and outlet are



computed via the ideal gas law:

$$\rho_{in/out} = \frac{P_{in/out}}{RT} \quad (4.11)$$

Where  $R$  is the ideal gas constant divided by molar mass,  $T$  is an appropriate ambient air temperature and  $P_{in/out}$  are the absolute inlet/outlet air pressures.

Further, the dynamic viscosity  $\mu$  is computed via Sutherland's law:

$$\mu = \frac{T_0 + S}{T + S} * \left(\frac{T}{T_0}\right)^{1.5} * \mu_0 \quad (4.12)$$

Where  $T_0 = 293.15$  K,  $S = 110.56$  K and  $\mu_0 = 1.856e - 5$  Pa\*s. At this point it is possible to evaluate the Reynolds number:

$$Re = \dot{m} * \frac{D_h}{\mu A} \quad (4.13)$$

And the inlet and outlet air velocities:

$$u_{in/out} = \frac{\dot{m}}{\rho_{in/out} A} \quad (4.14)$$

Since the inlet and outlet air pressures are taken respectively 10mm upstream and downstream of a sudden contraction and a sudden expansion, appropriate coefficients  $k_{in}$  and  $k_{out}$  derived by the procedures found in Idelchik [50] are employed to obtain the true pressure differential across the test objects inlet and outlet:

$$\Delta P_{loss} = 0.5 * k_{in} * \rho_{in} * u_{in}^2 + 0.5 * k_{out} * \rho_{out} * u_{out}^2 \quad (4.15)$$

Then it is possible to compute the pressure differential simply as:

$$\Delta P = P_{diff} - \Delta P_{loss} \quad (4.16)$$

Where  $P_{diff}$  is the differential pressure recorded by the PSI.

Then the Darcy friction factor is computed as:

$$f = \frac{2 * D_h * \Delta P}{L * (\rho_{in} u_{in}^2 * 0.5 + \rho_{out} * u_{out}^2 * 0.5)} \quad (4.17)$$

Noting that the term  $\rho u_m^2$  has been averaged over the inlet and outlet values.

## 4.5 Error Analysis

In this section a method for evaluating the uncertainty associated to the estimation of both the Darcy friction factor and the Nusselt number is presented, alongside the results of such procedures.

### 4.5.1 Darcy friction factor uncertainty

The total uncertainty associated to a value for  $f$  is computed via the standard error propagation formula, considering that the friction factor is computed directly from Equation 3.32 and the following direct measurements:

$$f = \frac{2D_h\Delta P}{L\rho u^2} = f(\dot{m}, D_h, L, \Delta P) \quad (4.18)$$

As such, the total uncertainty may be evaluated as [51]:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial \dot{m}}\delta\dot{m}\right)^2 + \left(\frac{\partial f}{\partial D_h}\delta D_h\right)^2 + \left(\frac{\partial f}{\partial L}\delta L\right)^2 + \left(\frac{\partial f}{\partial \Delta P}\delta\Delta P\right)^2} \quad (4.19)$$

Where  $\delta\dot{m}$ ;  $\delta D_h$ ;  $\delta L$ ;  $\delta\Delta P$  are the absolute errors associated to each direct measurement (mass flow, hydraulic diameter, test object length and pressure differential) and are computed considering both instrumental and random errors for each measurement for a particular test, according to the Root Sum Square method [51]:

$$\delta R = \sqrt{(\epsilon_{inst})^2 + (\epsilon_{rand})^2} \quad (4.20)$$

Where  $\delta R$  indicates the generic absolute error.

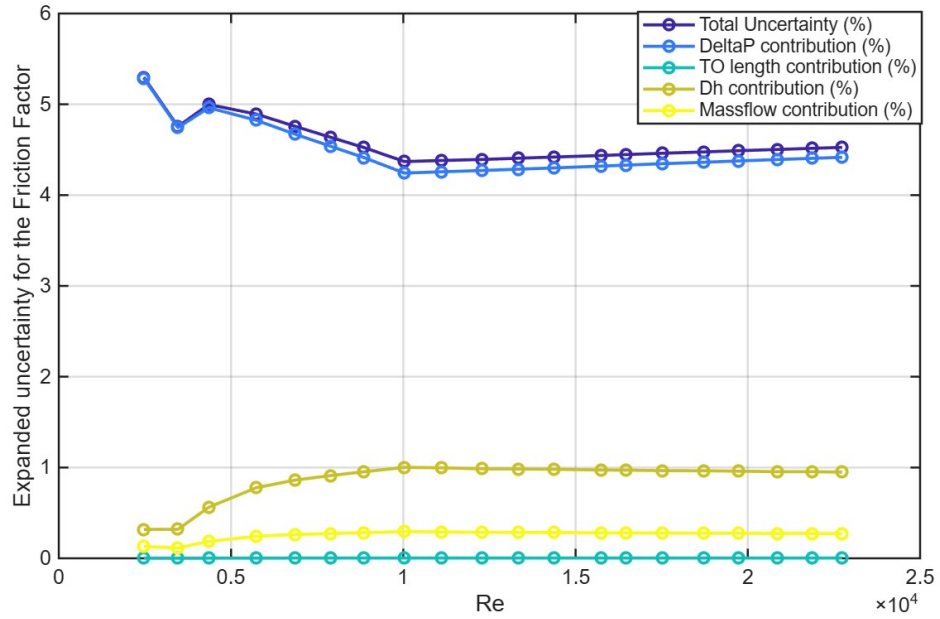
The systematic errors associated to instrument accuracy  $\epsilon_{inst}$  are taken from Table 4.1 while the random errors  $\epsilon_{rand}$  are computed from repeated measurements taken during test conditions according to:

$$\epsilon_{rand} = \frac{\sigma}{\sqrt{N}} \quad (4.21)$$

Where  $\sigma$  represents the standard deviation of the sample and  $N$  the sample size. Before proceeding with the results showcase the following remarks are to be kept in mind:

1. The sample size  $N$  for the random error estimation of every measurement (such as mass flow, diameter, etc.) is limited to  $N = 10 - 15$ . Due to time constraints encountered during laboratory work, more accurate (stochastic) procedures have not been carried out and only one reference Reynolds number was considered.
2. The density  $\rho$  has been assumed constant and equal to the inlet value  $\rho_{in}$ . Such simplification is acceptable as long as compressibility effects are negligible and during testing compressible flow is seldom approached considering the  $M_{out} < 0.24$  requirement.

3. Since the inlet and outlet loss coefficients were obtained from handbook correlations for idealized geometries, an additional uncertainty contribution was assigned to the pressure-loss correction. In the absence of a dedicated calibration for the actual fittings and pressure-tap arrangement, this contribution was conservatively estimated as 15% of the correction term.
4. Due to the limited number of repeated measurements and to the fact that the random contribution was evaluated at a single reference Reynolds number, the resulting uncertainty should not be interpreted as a complete statistical characterization of the entire experimental range. Rather, the values reported in the following should be regarded as representative estimates of the expected order of magnitude of the uncertainty, useful to identify the dominant error sources while acknowledging that local variations with mass flow rate, pressure differential and test-object geometry may still be present.

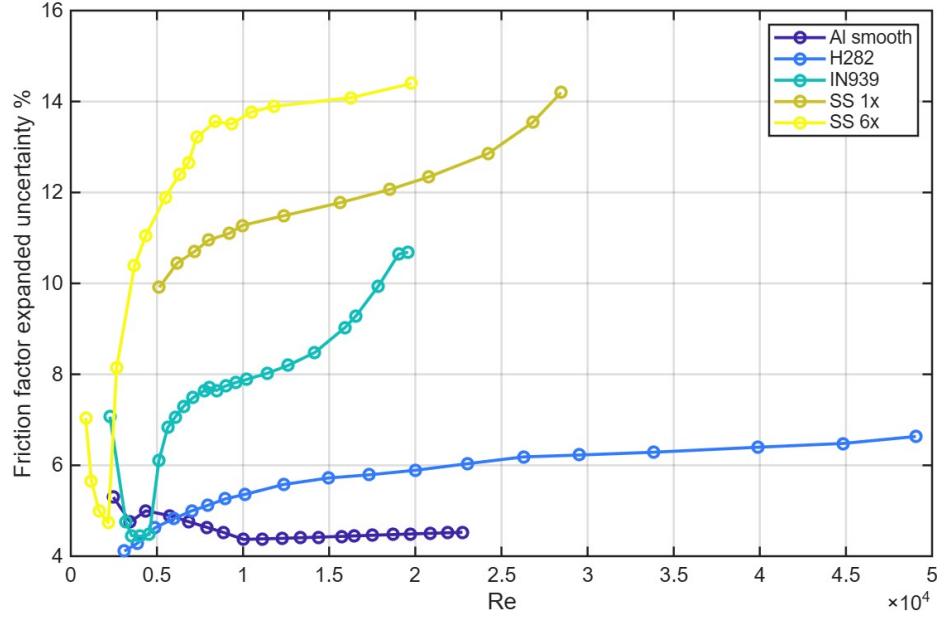


**Figure 4.22:** Total friction factor uncertainty contributions for the Al (smooth) test object.

Figure 4.22 showcases the uncertainty analysis conducted on the Al 3mm smooth test object employed for the friction factor validation of the rig.

The average total uncertainty on  $f$  for the turbulent region is approximately 4.58 % and the main contribution is given by the uncertainty on the  $\Delta P$  measurement. Such Measurement becomes less accurate as the transition region is approached,

thus the total uncertainty on the friction factor increases accordingly.



**Figure 4.23:** Total friction factor uncertainty for all test objects.

Figure 4.23 showcases the total friction factor uncertainty across all test objects considered in this work. As can be observed, for test objects tested close to the transition region, a local increase in the uncertainty is often encountered, while the total error tends to plateau in the fully rough region, with the outlier being IN939.

#### 4.5.2 Nusselt number uncertainty

As the Nusselt number is obtained from post-processing of the test data instead of direct computation of a formula (in opposition to the friction factor), the total error is estimated as a root sum square of the percentage error from the variables it depends on according to:

$$Nu = \frac{hD_h}{k_f} = f(h, D_h, k_f) \quad (4.22)$$

Thus the total percentage error on the Nusselt number can be computed as:

$$\delta Nu(\%) = \sqrt{(\delta h(\%))^2 + (\delta D_h(\%))^2 + (\delta k_f(\%))^2} \quad (4.23)$$

The percentage error on air thermal conductivity was assumed 1 %.

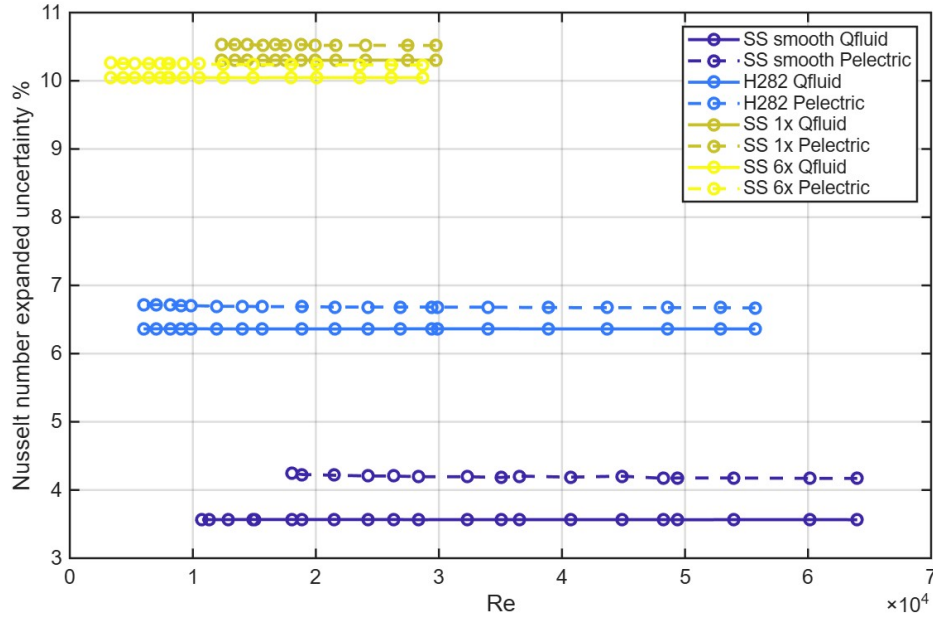
The percentage error on the heat transfer coefficient can be expressed in two

separate ways based on the chosen post-processing method as described in section 4.3.2:

- **Air Temperature:**  $\delta h(\%) = \sqrt{(\delta T_w(\%))^2 + (\delta \dot{m}(\%))^2 + (\delta \Delta T(\%))^2}$
- **Electrical power:**  $\delta h(\%) = \sqrt{(\delta V(\%))^2 + (\delta I(\%))^2}$

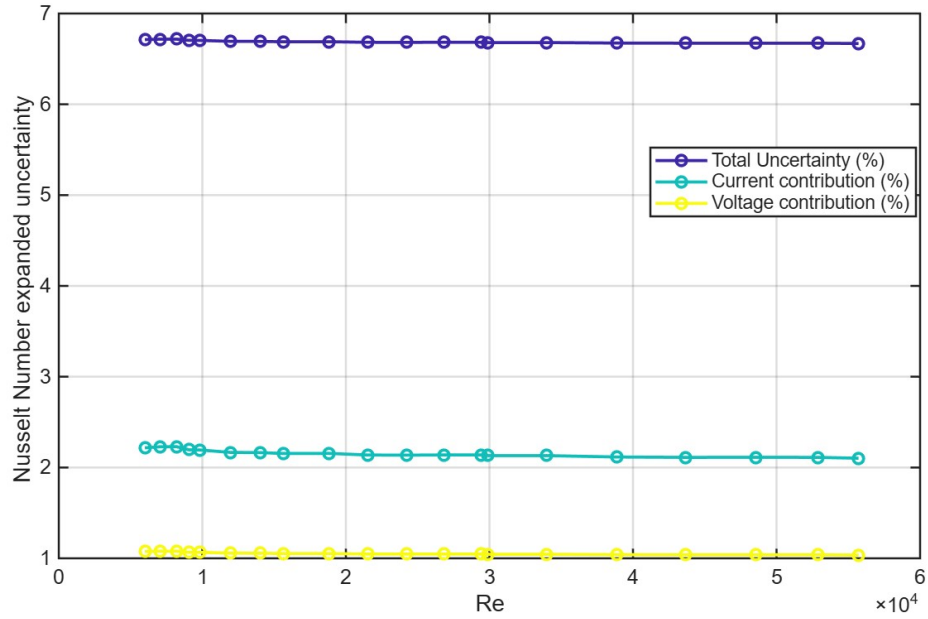
The total percentage error on each measurement was obtained by evaluating the total error considering Equation 4.18 and Table 4.1, then it follows, considering a certain measurement  $R_i$ :

$$\delta R(\%) = \frac{\delta R}{R_i} * 100 \quad (4.24)$$



**Figure 4.24:** Nusselt number uncertainty for all test objects presented in this work tested with 9 external surface temperature sensors. Both methods of heat transfer coefficient evaluation are considered.

Figure 4.24 presents the results of the error analysis conducted on the test objects for which Nusselt number results are presented in this work. Inconel 939 has been excluded from the comparison as the experiment was conducted employing 8 external temperature sensors instead of 9: as such, since the definition of the total temperature error in Equation (4.4) is not normalised on the basis of number of sensors, the resulting Nusselt number values would be affected by fewer errors



**Figure 4.25:** Nusselt number percentage error contributions from current and voltage measurements for Haynes 282.

compared to the other test object, which were all tested for 9 sensors. Figure 4.25 displays the contributions of only current and voltage to the definition of the total percentage error  $\delta Nu(\%)$ . As the instrument uncertainty for both measurements is defined as a percentage of the measured value, the instrument error contribution increases as that value decreases.

Thus, to limit such inaccuracies it is advisable to not exceed a minimum  $\Delta T$  of 10 degrees when recording a test point.

# Chapter 5

## Results

This chapter showcases the experimental results obtained for the test objects presented in Table 4.2. Each section provides data in terms of both Nusselt number and Darcy friction factor.

As air properties are largely constant with temperature [26], it is implied that all Nusselt number results showcased in this chapter are obtained at a constant Prandtl:  $Pr = 0.71$ .

### 5.1 Rig Validation

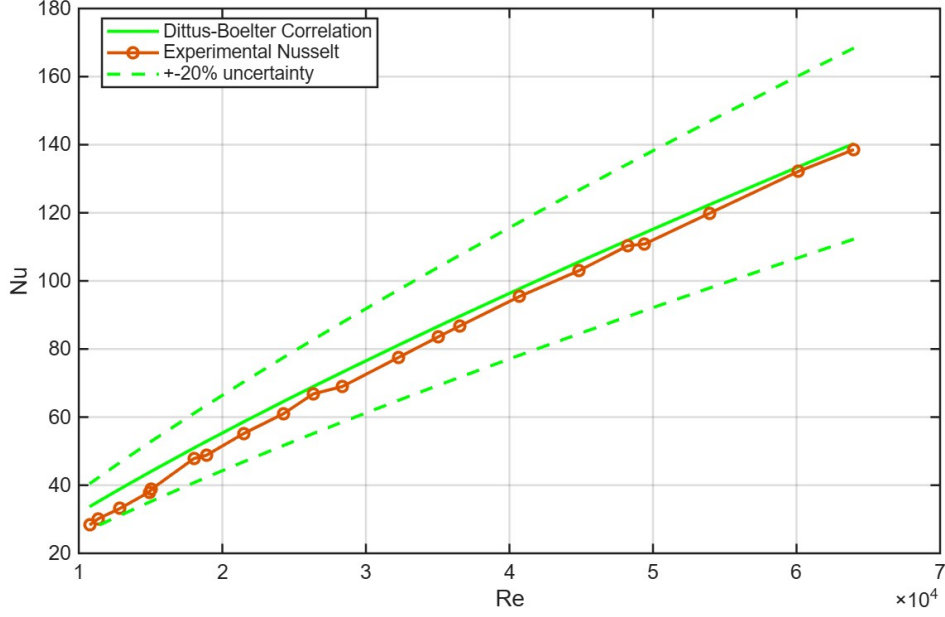
This section presents results obtained for smooth test objects. Both a hydraulic and a thermal validation will be presented. Experimental results will be compared with pre-existing correlations and the validity of subsequent results obtained for rough geometry will then be assessed.

#### 5.1.1 SS 4mm x 200mm

The following Figures 5.1, 5.2 and 5.3 showcase the results obtained for the thermal validation of the rig, for a test performed on a Stainless steel smooth pipe of dimensions available from Table 4.2.

The experimental results have been obtained considering the outlet air temperature method for updating the electrical current and have been compared to values obtained via Equations (3.57) and (3.58). Regarding Equation 3.58, the Gnielinski correlation, the friction factor was computed via Equation (3.39).

Considering Figure 5.2, the experimental average Nusselt number  $\overline{Nu}$  is within a 10 % uncertainty threshold with reference to the fully developed Nusselt number computed via Gnielinski's correlation. For Reynolds above  $Re \approx 20000$  the experimental curve is above the theoretical line, while the values obtained for  $Re \approx 15000$



**Figure 5.1:** Average Nusselt number over Reynolds obtained for a stainless steel smooth pipe, compared to the average Nusselt computed from Dittus-Boelter’s correlation.

and under approach the 10 % error line below Gnielinski.

In particular it will be seen also with rough geometries that with the methodology at hand, as the Reynold number approaches values below  $Re = 10000$ , the experimental Nusselt number computed via the outlet air temperature method falls below even the respective Gnielinski line computed for the same Reynolds interval and a smooth friction factor.

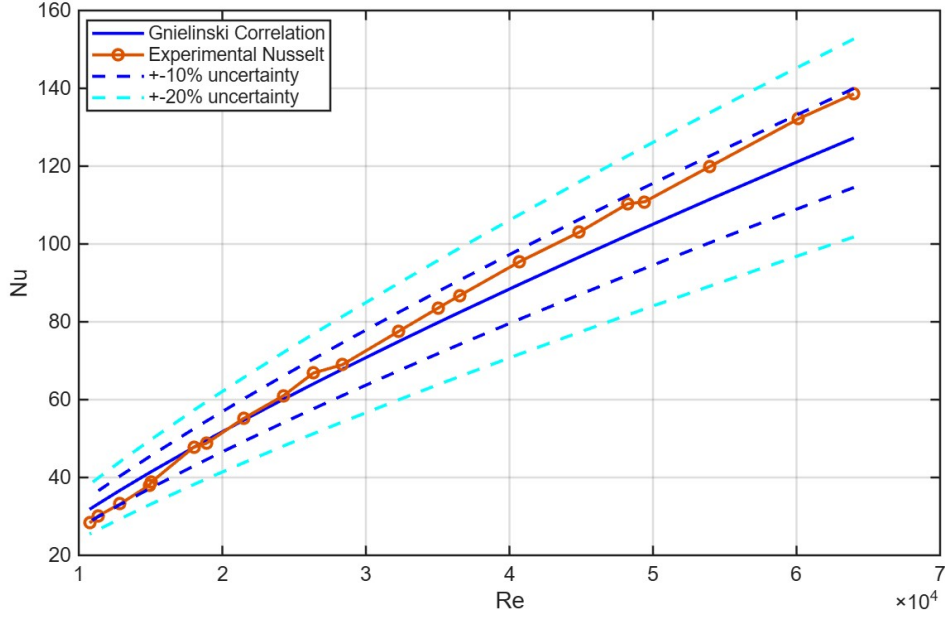
Such behaviour was not observed when evaluating the Nusselt number via the electrical power method, as will be shown for the rough test objects.

Lastly, Figure 5.3 showcases the local Nusselt number obtained for each channel subdivision of the test object. From the second to the second-last channel the local Nusselt number  $Nu_x$  for all Reynolds considered approaches a constant value, while at the extremities the value decreases.

Although the value for  $Nu_x$  obtained with the method presented is sensitive with respect to surface temperature sensor placement as seen in Equation (4.8), values obtained by Vieira [24] for the same experimental setup and methodology presented in this work showcase a similar trend for a smooth stainless steel test object.

It must be noted that the x-axis in the figure represents the distance between the first and last PT100 placement point, as such  $x/D_h = 0$  does not coincide with the





**Figure 5.2:** Average Nusselt number over Reynolds obtained for a stainless steel smooth pipe, compared to the average Nusselt computed from Gnielinski's correlation.

inlet section, rather with the first channel location along the axis of the test object.

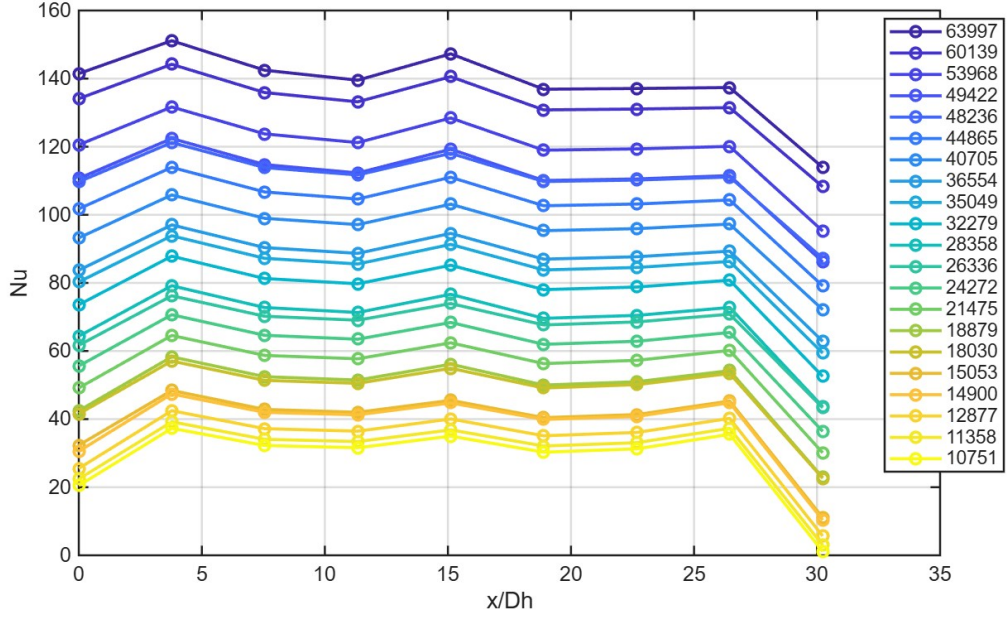
### 5.1.2 Al 3mm x 150mm

The following section showcases the results obtained for the hydraulic validation of the rig, for a test performed on an unmachined smooth aluminum channel of dimensions available from Table 4.2.

The experimental friction factor has been compared to results obtained from laminar theory and Equation (3.39).

As obtaining values in the laminar region has been found to be difficult with the current method, due to the differential pressure reading  $P_{diff}$  approaching even negative values as  $Re \approx 2300$ , only values pertaining the transition and turbulent regions have been showcased, as per the limitations introduced in Chapter 1.

As can be seen from Figure 5.4, the friction factor in the transitional region increases as turbulent structures become more prevalent and at  $Re \approx 10000$  and higher it starts to match the Petukhov line from Equation (3.39).



**Figure 5.3:** Local Nusselt number as a function of Reynolds obtained for a stainless steel smooth pipe.

## 5.2 AM test objects

This section showcases the experimental results in terms of Nusselt number and Darcy friction factor obtained for the AM test coupons of dimensions available from Table 4.2.

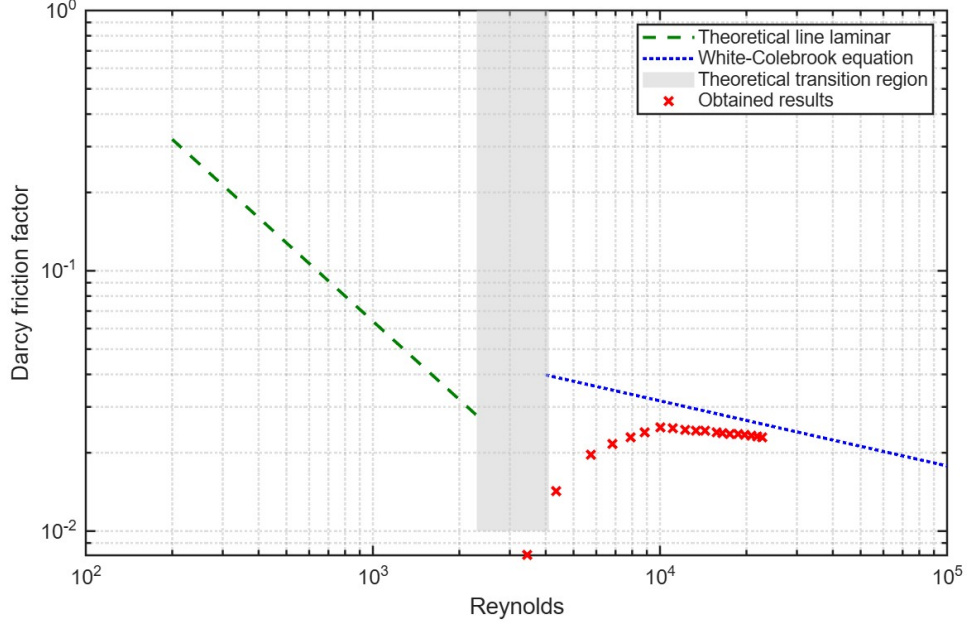
### 5.2.1 Nusselt Number results

Figures 5.5, 5.8, 5.14 and 5.11 display the local Nusselt number obtained for each Reynolds number considered during laboratory testing.

Compared to the validation results, Figure 5.3, the values for  $Nu_x$  show a higher tendency to stabilize around a constant value, with the notable exception of the mono-channel stainless steel test object, whose  $Nu_x$  values are displayed in Figure 5.14.

Again, sensor placement during the particular test might influence the shape of  $Nu_x$  correlation obtained for the same test object. And it must be reminded that in each figure the x-axis is defined as the distance between the first and last sensor placement point.

Figures 5.6, 5.9, 5.12 and 5.15 display the average Nusselt number enhancement



**Figure 5.4:** Darcy friction factor for a smooth test object

over Gnielinski against Reynolds.

The Nusselt number enhancement  $Nu_{enh}$  was defined such that:

$$Nu_{enh} = \frac{\overline{Nu}}{Nu_0} \quad (5.1)$$

Where  $\overline{Nu} = \frac{\sum Nu_x}{N}$  is the average experimental Nusselt number and  $Nu_0$  is the smooth Nusselt number as per Gnielinski's correlation, using the same Reynolds obtained for the respective experimental point and Pethukov's friction factor.

As anticipated in Chapter 4, the two methods of model electrical current evaluation result in different results in terms of Nusselt number: the Outlet air temperature method seems to produce  $Nu(Re)$  that rapidly decay in value as Reynolds approaches the transition region and plateau in the high turbulent region; in contrast, the electrical current method produces "flatter" correlations with reduced oscillations.

Moreover, for every Reynolds:

$$Nu_{el} > Nu_{T_{out}} \quad (5.2)$$

Where  $Nu_{el}$  and  $Nu_{T_{out}}$  respectively refer to the experimental Nusselt obtained for either the electrical power or outlet temperature methods.

Figures 5.7, 5.10, 5.16 display the offset between the model electrical power  $\Pi_{model}$  for the  $T_{out}$  method and the measured electrical power  $\Pi_{measured}$  during testing conditions for the mono-channel test objects.

For each Reynolds spread, it stands:

$$\Pi_{measured} > \Pi_{model} \quad (5.3)$$

A possible explanation for this offset is that the heat losses represented in the numerical model do not include all the thermal paths present in the experimental setup. In this case, the power balance imposed by the model can be written as

$$\Pi_{model} = Q_{fluid} + Q_{losses} < \Pi_{measured}, \quad (5.4)$$

where  $Q_{losses}$  denotes the losses accounted for through the prescribed contact boundary conditions. Any additional losses through the fittings, insulation or external surfaces would therefore make the heat source term used in the model lower than the electrical power supplied during the test.

According to the solver logic described in Chapter 4, a lower heat source term leads to lower estimated local heat-transfer coefficients, and therefore to lower values of  $Nu_x$ , while still allowing the model to match the measured surface temperatures  $T_i$ . Since no independent heat-loss calibration was performed, this interpretation should be considered as a plausible explanation of the observed offset rather than a direct quantification of the missing losses

The offset from Equation (5.4) also approaches the orders of magnitude of the total heat transfer into air as Reynolds decreases. For example, for the Stainless steel mono-channel AM test object:

| $Re$  | $Q_{fluid}$ [W] | $\Pi_{measured} - \Pi_{model}$ [W] |
|-------|-----------------|------------------------------------|
| 12345 | 3.1688          | 1.5082                             |
| 13402 | 3.4888          | 1.7203                             |
| 14422 | 3.7483          | 1.4624                             |
| 21640 | 5.6708          | 0.8756                             |
| 24033 | 5.8964          | 0.6279                             |
| 27465 | 6.1493          | 0.8479                             |

**Table 5.1:** Offset between measured and model power for a single-channeled stainless steel test object

As such, for lower Reynolds such unaccounted losses become as relevant as the total measured heat transfer in the medium.

It is then interesting to note that for the stainless steel multi-channeled test object such offset is much reduced, as shown in Figure 5.13. In this case, the measured

$Q_{fluid}$  is higher as heat exchange happens through six micro channels instead of one, and the sum  $Q_{fluid} + Q_{losses}$  makes it so that:

$$\Pi_{model} \approx \Pi_{measured} \quad (5.5)$$

With modeling errors in  $Q_{fluid}$  remaining but with diminished weight compared to a single-channeled test object.

As a consequence of Equation (5.5), the Nusselt number values obtained via both methods are closer, as seen in Figure 5.12, for  $Re > 10000$ . For the low Reynolds region however, the weight of the offset might still cause differences between the two methods.

It must be reminded however, that modeling errors are not the only ones affecting the term  $Q_{losses}$ , as evaluating an experimental point in unsteady conditions, when the copper and Parker surface temperatures are still in transient state, might also affect its value. However, post-processing unsteady state data might affect the measured  $Q_{fluid}$  more, as a greater weight is put in such term in evaluating the current via the outlet temperature method:

$$I = \sqrt{\frac{Q_{fluid} + Q_{losses}}{R_{tot}}} \quad (5.6)$$

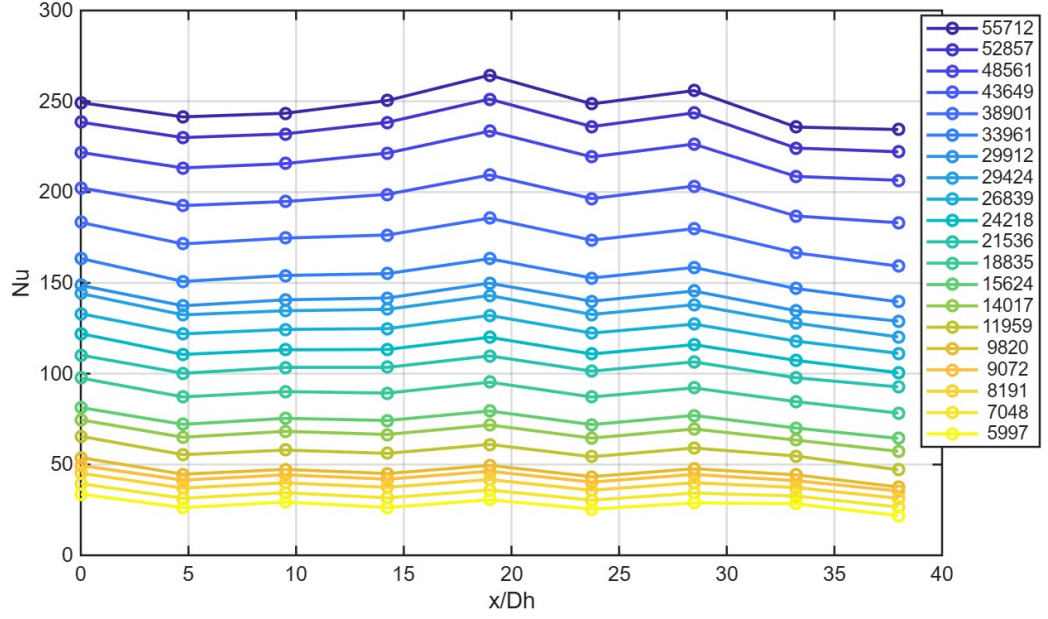
As  $Q_{fluid} > Q_{losses}$  especially at high Reynolds.

Moreover, a comment must be made regarding the measurement uncertainties in Figure 5.12. The model errors alone would result in Equation (5.2) being verified for every Reynolds also for this multi-channeled case, however for high Reynolds the offset between model and measured electrical is such that Equation (5.2) is inverted.

Such behaviour might be explained by measurement uncertainties, which for the Nusselt number evaluation are reported in Figure 4.24 of Chapter 4.

| $Re$  | $Q_{fluid}$ [W] | $ \Pi_{measured} - \Pi_{model} $ [W] |
|-------|-----------------|--------------------------------------|
| 3356  | 4.8801          | 0.6869                               |
| 4322  | 7.1044          | 0.4448                               |
| 5252  | 7.7466          | 0.5709                               |
| 14894 | 22.1092         | 0.5564                               |
| 17998 | 23.0560         | 0.6510                               |
| 20091 | 28.0433         | 0.1976                               |

**Table 5.2:** Offset between measured and model power for a multi-channeled stainless steel test object



**Figure 5.5:** Local Nusselt number obtained for Haynes 282 as a function of Reynolds, electrical power method.

### 5.2.2 Friction factor results

Figures 5.17, 5.18 and 5.19 display the experimental results obtained in terms of Darcy friction factor for each Reynolds considered during laboratory testing.

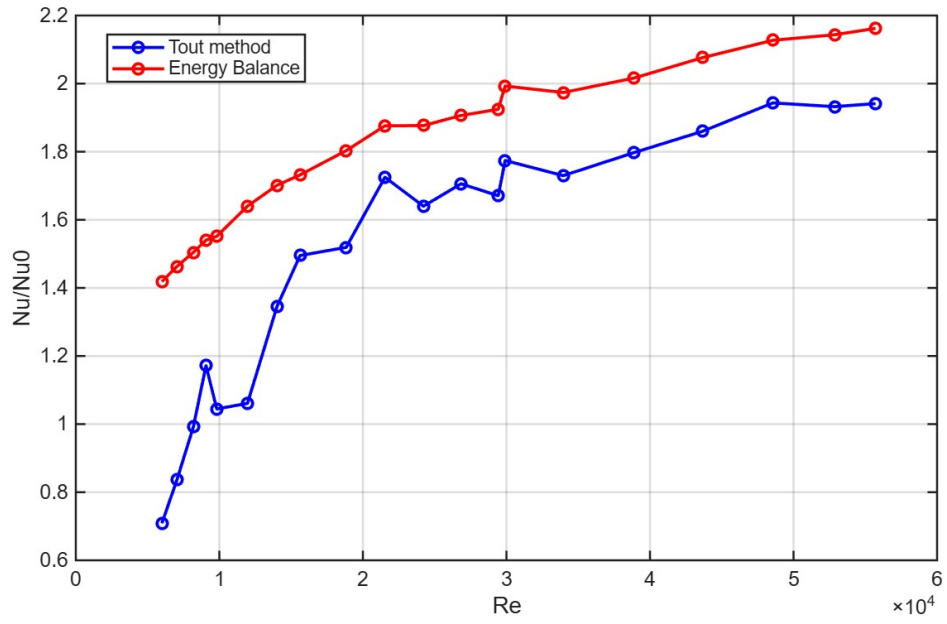
In the case of Haynes 282 and IN939, the friction factor increases with Reynolds across the transition region and stabilises to a constant value in the fully turbulent region.

Inconel 939 displays a higher friction factor compared to Haynes 282 in the turbulent region, a result supported by the former showing increased relative roughness compared to the latter [52].

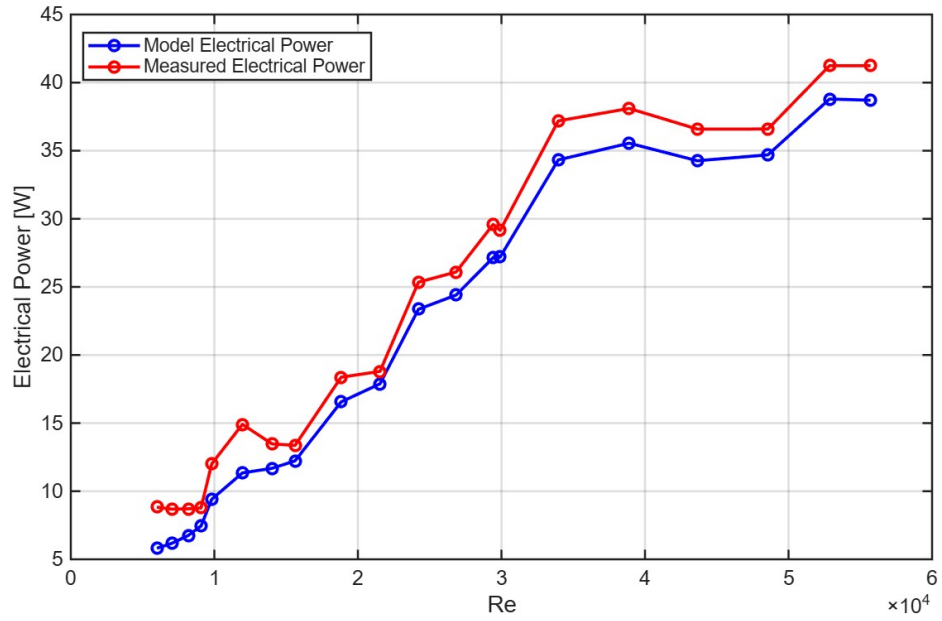
Data for the single-channeled stainless steel test object was obtained only in the turbulent region, while the multi-channeled stainless steel was also tested in the transitional and laminar regions.

Interestingly, from Figure 5.19 it appears the Darcy friction factor obtained for the multi-channeled test object is higher than its single-channeled counterpart for all the Reynolds considered in the turbulent region: as the hydraulic diameter for both coupons is the same, with relative roughness and Reynolds being similar one would expect similar values of  $f$ , thus it is likely that the current procedure may not account for additional pressure losses caused by flow partitioning.

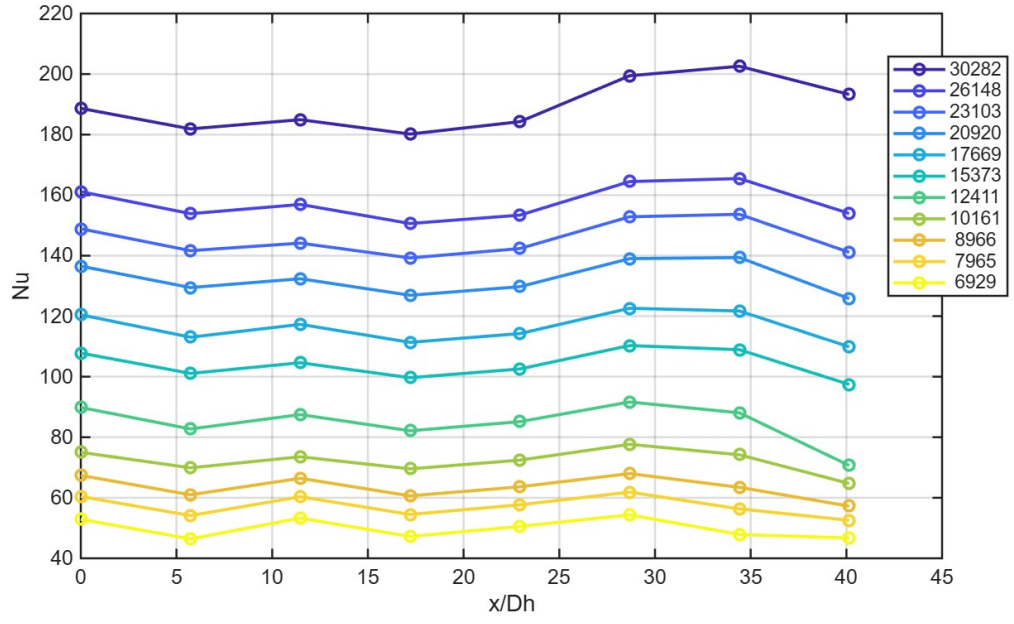
The same friction factor behaviour between single and multichannel test objects



**Figure 5.6:** Average Nusselt number obtained for Haynes 282.



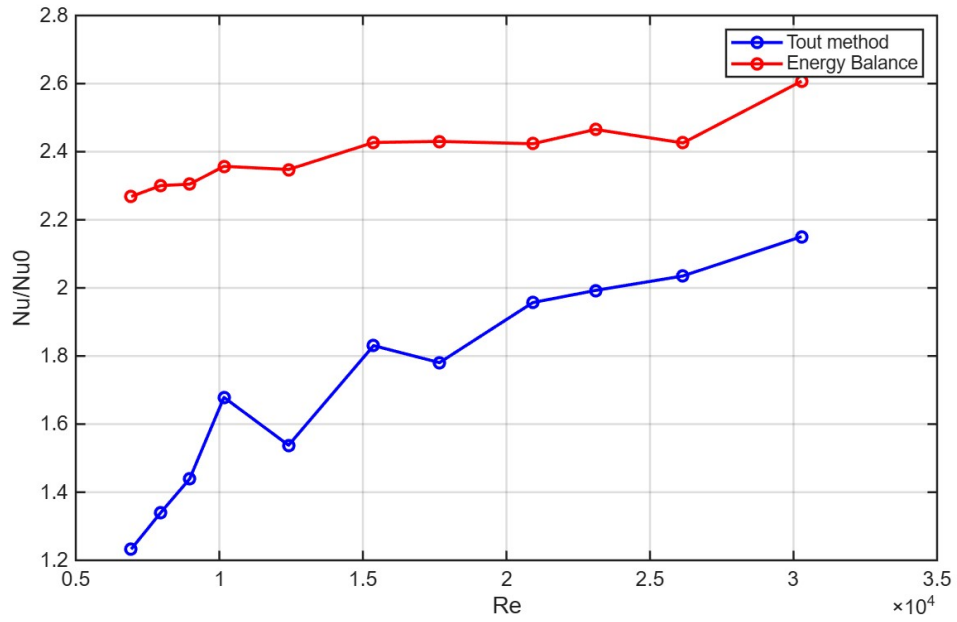
**Figure 5.7:** Comparison of modeled and recorded electrical power for Haynes 282.



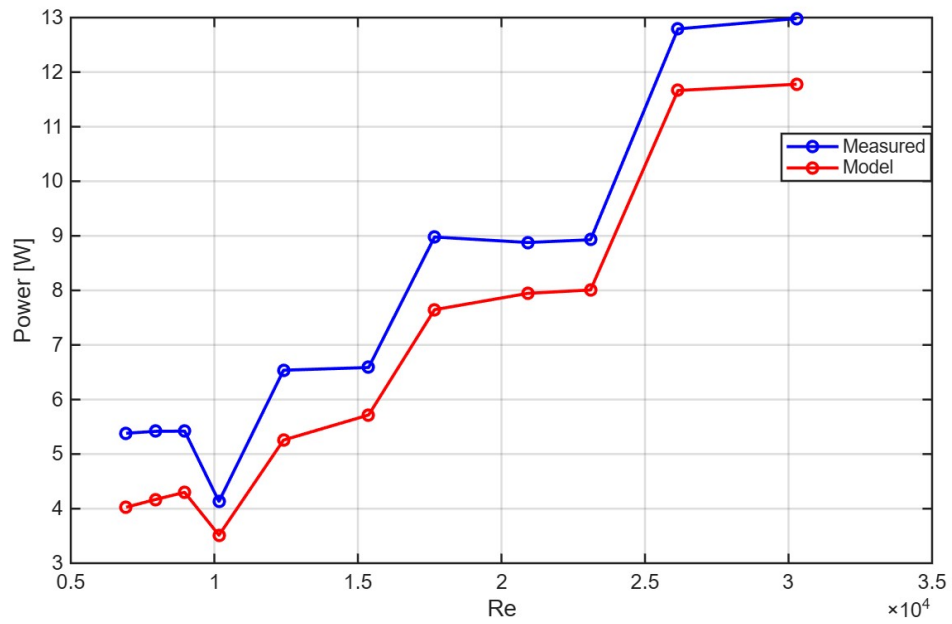
**Figure 5.8:** Local Nusselt number obtained for Inconel 939 as a function of Reynolds, electrical power method.

was observed by Santos [49], employing water as medium.

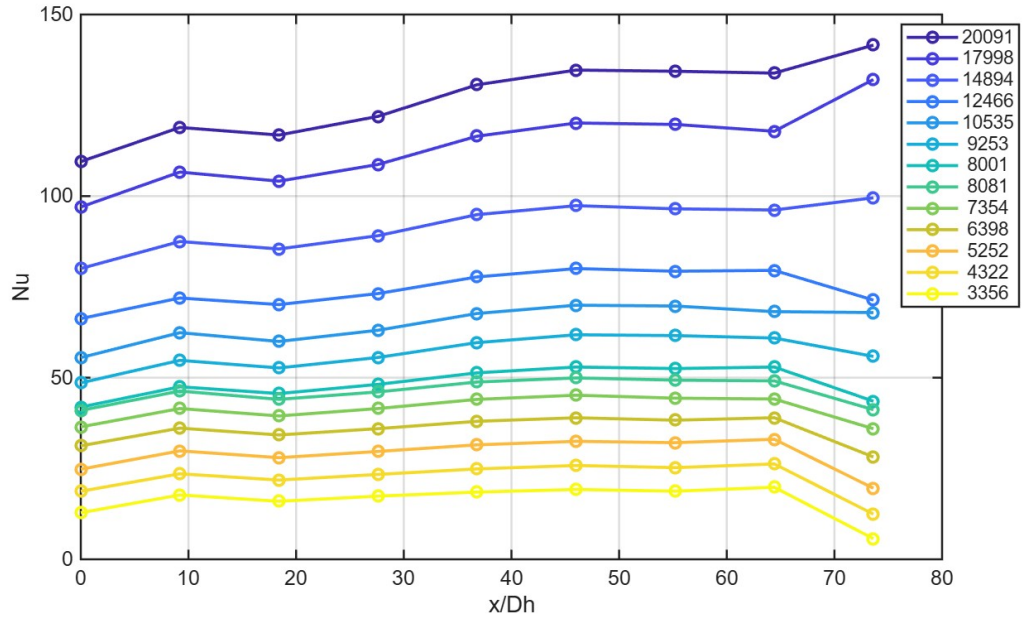




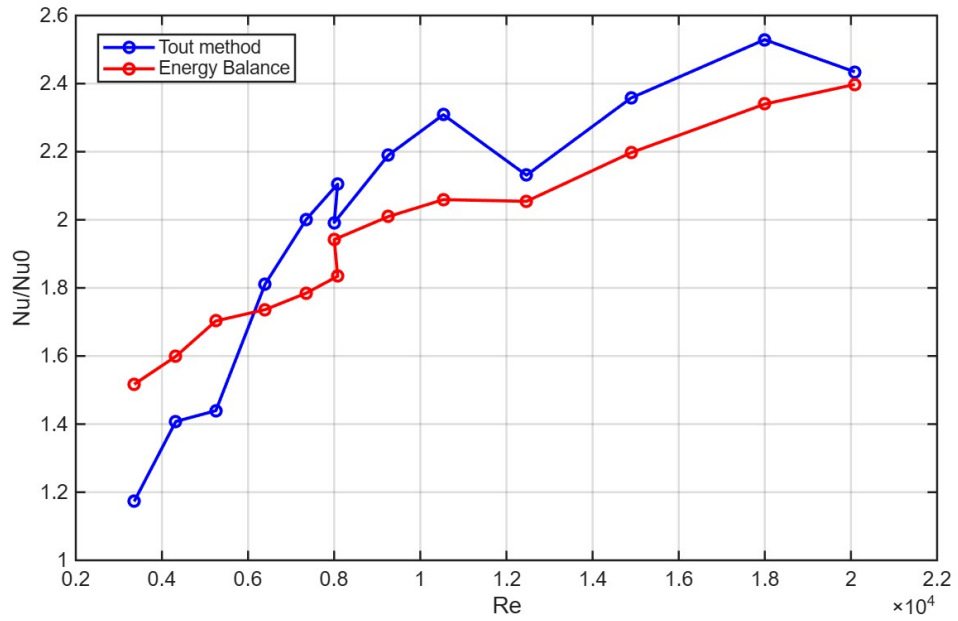
**Figure 5.9:** Average Nusselt number obtained for Inconel 939.



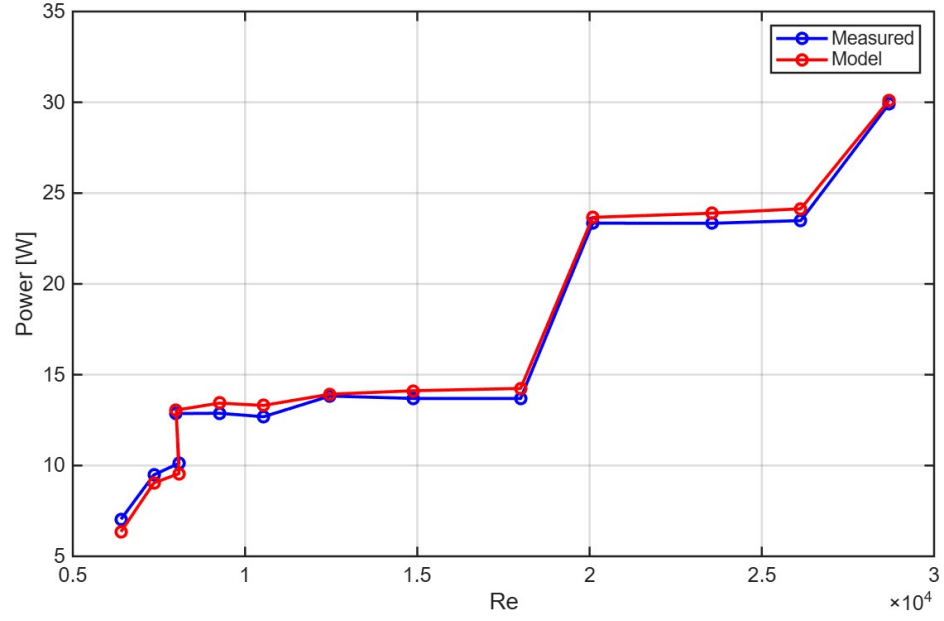
**Figure 5.10:** Comparison of modeled and recorded electrical power for Inconel 939.



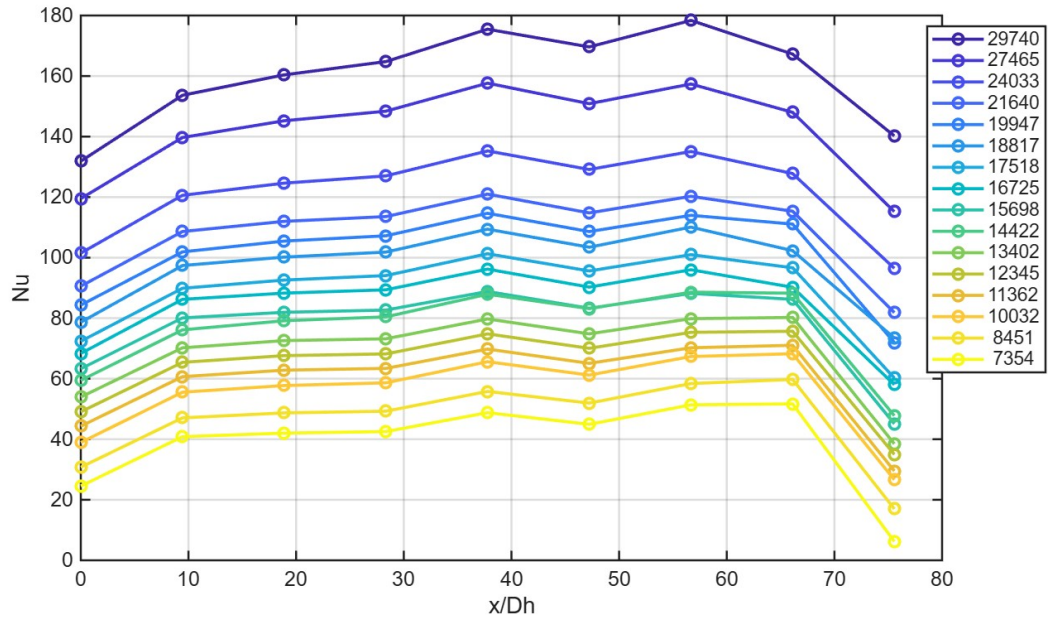
**Figure 5.11:** Local Nusselt number obtained for a six channel stainless steel test object as a function of Reynolds, electrical power method.



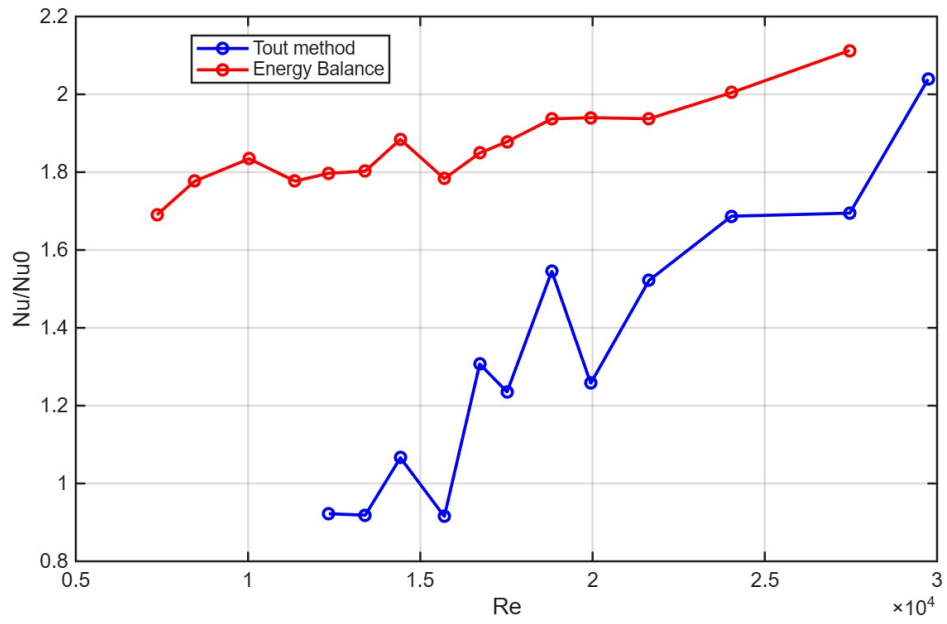
**Figure 5.12:** Average Nusselt number obtained for a six channel stainless steel test object.



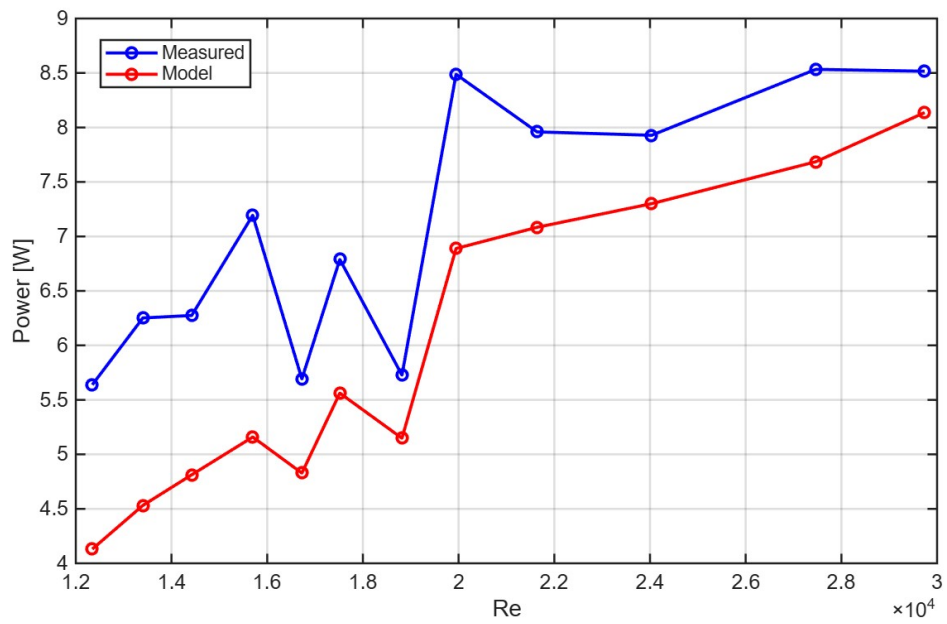
**Figure 5.13:** Offset between modeled and recorded electrical power for a six channel stainless steel test object.



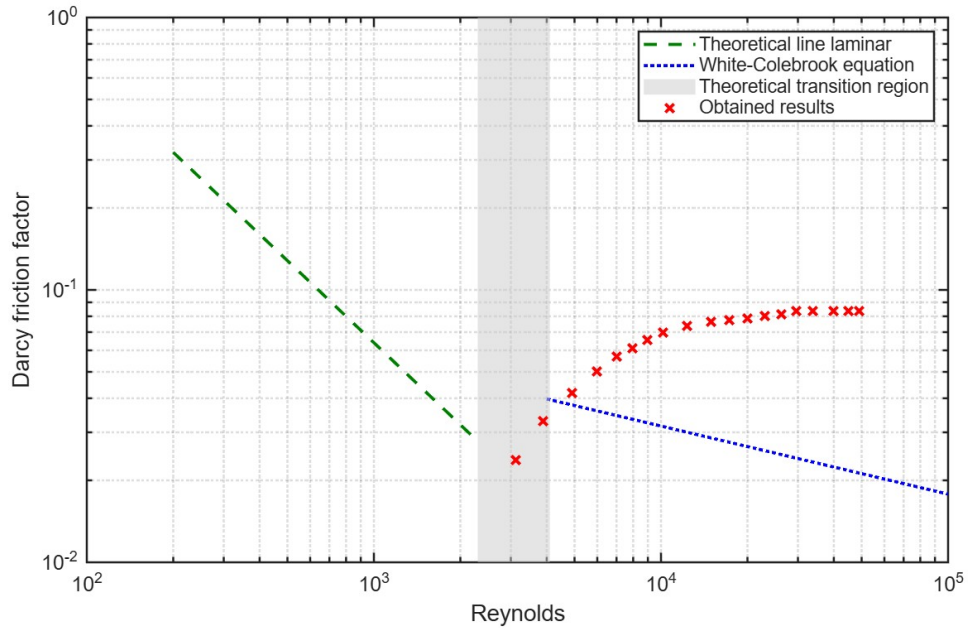
**Figure 5.14:** Local Nusselt number obtained for a mono-channel stainless steel test object as a function of Reynolds, electrical power method.



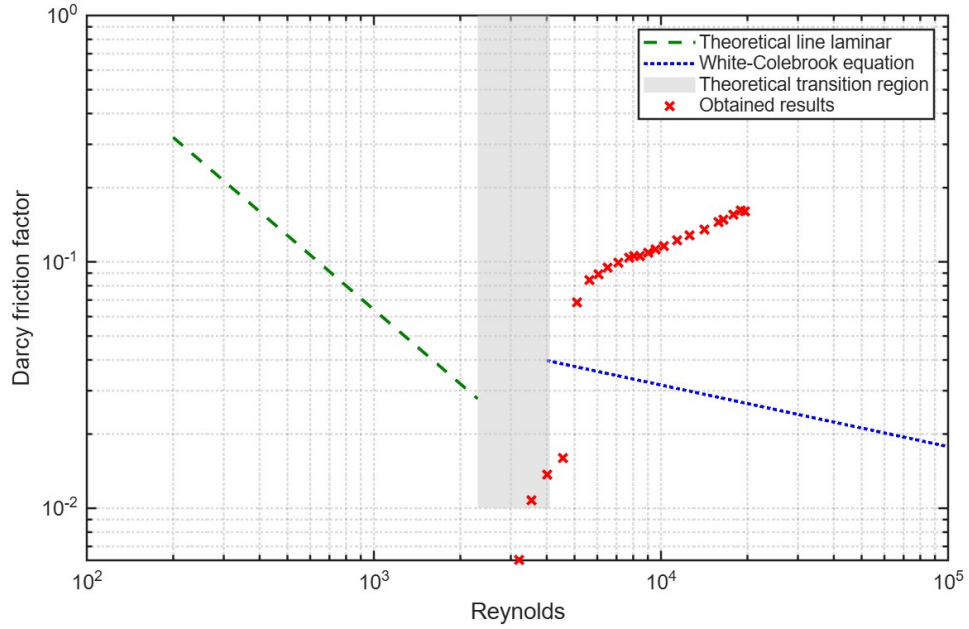
**Figure 5.15:** Average Nusselt number obtained for a mono-channel stainless steel test object.



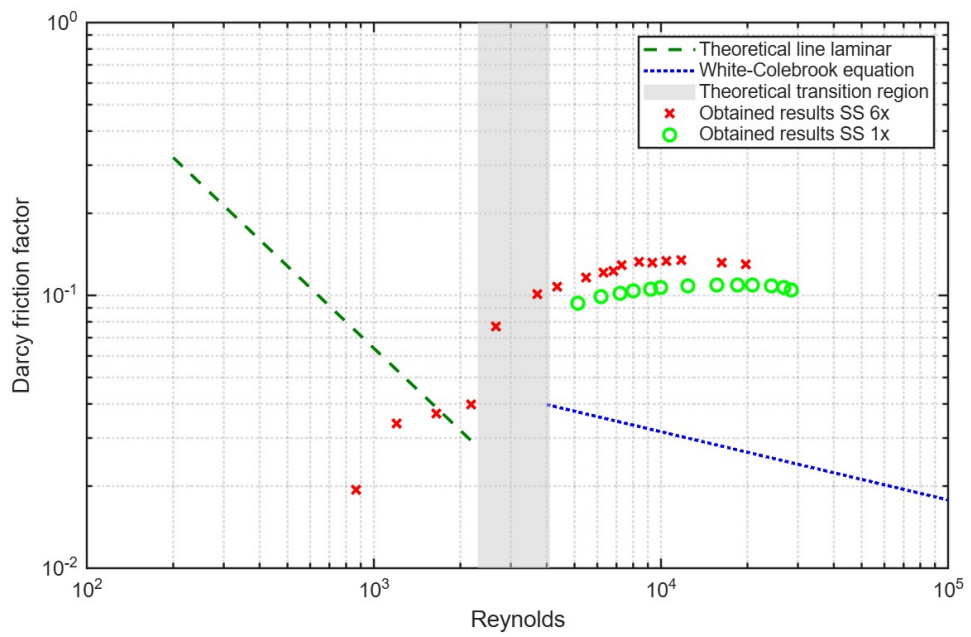
**Figure 5.16:** Offset between modeled and recorded electrical power for a mono-channel stainless steel test object.



**Figure 5.17:** Darcy friction factor obtained for Haynes 282.



**Figure 5.18:** Darcy friction factor obtained for the Inconel 939 test object.



**Figure 5.19:** Darcy friction factor comparison between a six channel and a mono-channel stainless steel test object.

## Chapter 6

# Conclusions and Future Work

In this chapter, a brief review of the findings of this thesis will be presented, alongside suggestions and considerations for future research resulting from the experience developed during laboratory work.

### 6.1 Conclusions

The proposed inverse methodology provides preliminary estimates of local and average Nusselt numbers, but the results remain sensitive to the heat-source closure, contact-boundary assumptions and unmodelled heat losses.

For both methods, the average Nusselt number increases with Reynolds and a comparison between Haynes 282 and Inconel 939 also allows for considerations on the heat transfer effects of relative roughness, as Nusselt results for the latter are higher compared to the former.

Further, a comparison for Nusselt results obtained for the stainless steel test objects have shown the current methodology for Nusselt evaluation is highly dependent on the heat source term: as heat transfer with the cooling medium increases (via increased thermal conductivity or a higher number of sub-channels), the heat source terms from the two methods converge, and so does the Nusselt number enhancement.

Further testing on multi-channeled test objects might thus provide valuable data to obtain more uniform Nusselt number results for this methodology.

Results obtained in the turbulent region confirm the dependence of  $f$  on relative roughness, while friction factor evaluation for laminar flow has proven to be difficult with the current method.

A comparison of the two stainless steel test object results also raises interest in

evaluating possible additional pressure losses due to flow subdivision.

The current pressure-measurement configuration is not sufficiently accurate for laminar validation; therefore, hydraulic validation is restricted to transitional/turbulent conditions.

## **6.2 Future work**

On the basis of the laboratory experience developed during the research period and the results presented in this work, the following suggestions on conducting future research on this matter and with the presented methodology can be considered:

- Adopting automatic current and voltage meters, as manual measurements might exhibit additional data dispersion based on individual technique.
- Considering conducting a Montecarlo (or similar) based error analysis to better evaluate experimental data, especially to better compare the Nusselt number results for the different evaluation methods.
- Testing additional multi-channeled test objects, to substantiate the findings of this work and move towards a uniform Nusselt number result.
- Considering implementing a more robust criterion for assessing steady state conditions during testing, possibly based on live monitoring of first derivatives of the quantities of interest.
- As the temperature sensor placement on the surface of the test object proves critical in the successful conduction of a Nusselt number test, considering implementing guides for uniformed sensor positioning.
- Improving the current rig to allow for laminar regime testing for Darcy friction factor evaluation.



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