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Trajectory design for the Space Tug project: a reusable cargo spacecraft exploiting invariant manifolds.

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*A mio papà.
E alla mia mamma che è fortissima.*

Abstract

Over the past two decades, the space industry has undergone a radical transformation, shifting toward reusable concepts that have significantly reduced mission costs. Space Tugs are also part of this vision being reusable cargo spacecraft designed to provide transportation services in cislunar space. The Space Tug aims to build on the interest generated by the Artemis program and thus meet the growing demand for scalable logistics architectures to support the Lunar Gateway and future lunar surface activities. To do so, it will leverage reusability, modular mission planning, and potentially in-space refueling, reducing recurring launch demands and overall logistics costs, while meeting delivery schedules and supply requirements.

This work investigates the exploitation of gravitational dynamics within the Earth-Moon Circular Restricted Three-Body Problem (CR3BP) to improve the efficiency of the Space Tug project. The use of stable and unstable manifolds as natural transit corridors to the Lunar Gateway vicinity allows the Space Tug to minimize significantly both propellant mass and transfer time.

The proposed strategy introduces a hybrid approach that combines a coasting phase on a selected manifold with a low-thrust phase between Earth and the targeted manifold. Low-thrust propulsion is employed to transfer the spacecraft from a Supersynchronous Geostationary Transfer Orbit (sGTO) to the designed state vector required for manifold injection, as well as for targeting, rendezvous, and control maneuvers. The approach broadens the possible solutions for trajectory design compared to fully continuous low-thrust transfers and can lead to improvements while still allowing the final trajectory to be adjusted to custom requirements given by clients.

Lastly, this thesis proposes a baseline configuration for a spacecraft constellation dedicated to ensuring a sustained resupply architecture for the Lunar Gateway. A comprehensive trade-off analysis has been performed, evaluating parameters such as payload capacity, propellant consumption, and time of flight, which directly dictate delivery frequency and the required fleet size. The resulting framework offers a scalable and sustainable long-term solution to support the Lunar Gateway's operational logistics.

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Chapter 1

Introduction

1.1 Background and motivation

Our ignorance of the universe and its dynamics is one of the driving forces that compel us to study and explore it. To this end it is necessary to navigate these dynamics by understanding and leveraging them to design optimized trajectories.

The space age was initiated with the seminal launch of Sputnik, signifying a pivotal moment in history. In the space industry, technological advances and innovations have marked significant turning points. One such innovation was the development of reusable launch vehicles by private companies, such as SpaceX. Concurrently, there has been a shift in the space economy from being primarily the domain of national agencies to being dictated by the faster pace of private companies that can rely on private funding. This allows clients to set their own timelines and risk tolerance for testing. The work will concentrate on two themes: reusability and gravitational dynamics.

SpaceX was not the first to apply the concept of reusability; the Space Shuttle was already reusable, but this did not represent an economically sustainable model. However, the ability to recover and reuse the first stage of launch vehicles, combined with large-scale production, has led to a significant reduction in launch costs [1]. The cost of the Space Shuttle was \$54,500/kg, whereas the Falcon 9 can now carry a payload to Low Earth Orbit (LEO) at a cost of just \$2,720/kg. This prompts the initial question of this study: what are the advantages of making the upper stage of the launch vehicle reusable? What potential applications could be developed for such a spacecraft?

A project that addressed these questions and other interesting research points had already been developed at Politecnico di Torino under the name of Space Tug. The Space Tug is a reusable cargo spacecraft that remains in orbit and docks with a passive target (payload) that has been delivered in LEO by a launch system.

Subsequent to the completion of docking operations, the Space Tug is responsible for the transportation of the payload to its designated destination which could be a specific orbit around Earth, such as a geostationary orbit, or other desired points in space that could be feasible to reach in deeper space. In the case of the project led by Nicole Viola and et al., the architectural design of the space tug was dedicated to serving as a continuous transport system between Earth orbit and the Lunar Gateway, and thus the cislunar space.

1.2 Space Tug

The Global Exploration Roadmap[2] stated that a cislunar outpost is a necessary step on the road to deep space exploration and ultimately to Mars. Supporting such a complex orbital infrastructure highlights the distinction between crewed and uncrewed missions. Humans are transported in heavy-lift vehicles like the Space Launch System (SLS) or the Orion capsule which require high-thrust chemical propulsion to speed up the transfer of cargo and minimize radiation exposure. On the other hand, the supply of habitation modules, materials needed, logistics, and non-perishable goods is not as limited in time and allow for the use of low thrust or other trajectory design choices that would result in a time of flight on the order of months. This is where the Lunar Space Tug (LST) [3][4] becomes an important and sustainable asset. To maximize the delivered payload mass while simultaneously reducing launch costs, the LST architecture exploits an all-electric architecture, specifically utilizing a cluster of Hall Thrusters (HTs). Electric propulsion operates at a significantly higher specific impulse compared to chemical alternatives, which drastically reduces the required propellant mass.

This propulsive efficiency is fundamental to the economic viability of the reusable model and to its operations because it reduces the need for in-space refueling. Such operations would be carried out on the parking orbit, either a Low Earth Orbit (LEO), a Geostationary Transfer Orbit (GTO) or a Supersynchronous Geostationary Transfer Orbit (sGTO) [5], while the LST waits to dock with the cargo module delivered in orbit. Following the docking phase, the Tug performs the orbit-raising maneuvers towards the cislunar station executing a spiral trajectory to escape Earth's gravity and reach the Lunar Gateway. Once docked with the Lunar Gateway and delivered the payload the Space Tug starts its travel back to the parking orbit with a similar trajectory and will be ready for the next mission. The sequence of operations for the outbound leg is well summarized in this graphic used by Rimani et al in their paper[3]:

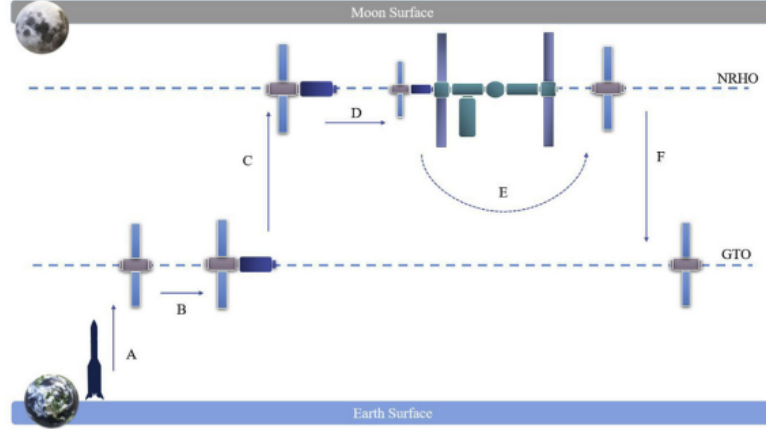


Figure 1.1: Design reference mission, for the description of each phase $A \rightarrow F$ refer to [3]

1.3 Strategic Reconfiguration and Research Rationale

In spring 2026, the Artemis program underwent a radical strategic overhaul, culminating in the official cancellation of the Lunar Gateway orbital station. The orbital infrastructure had been incorporated as a cornerstone of the program to serve as an intermediate logistics hub. It had been officially considered a key element of the project since 2019 while being conceived even earlier under the name Deep Space Gateway.

However, budgetary constraints imposed by the new administration and the necessity to expedite operational timelines led to a reorientation of the exploration strategy toward a "surface-first" approach. This transition eliminates the intermediate orbital level, thereby facilitating direct mission profiles between Earth and the lunar surface according to the primary objective of this approach which is to establish a permanent surface infrastructure.

This update to the Artemis program calls into question the central premise of this thesis, which is based on the importance of the Lunar Gateway and, consequently, the importance of a link between said infrastructure and Earth orbit. It is evident that this research project was conducted prior to the Artemis project leadership's decision. Specifically, the research was conducted in the second half of 2025, aligning with NASA's strategy at that time.

However, it can be affirmed that the utility or interest of this work is not revoked even if the Lunar Gateway is not constructed in the near future, as the trajectory designated to reach the selected orbit for the Lunar Gateway can now be regarded as a theoretical case study. The study demonstrates the feasibility of a solution

that combines the use of low-thrust maneuvers and coasting on manifolds for the transport of cargo between Earth orbit and NRHOs in cislunar space. A broad array of orbits and the corresponding manifolds that they define exist and can be used depending on their characteristics and may be of interest for a variety of lunar missions.

The present study aims to serve as a foundational starting point for other missions that prioritize a more efficient trajectory for a mission type that can afford to take up to months to reach its destination. However, it should be noted that the same strategy can yield divergent results by modifying the propulsion parameters in the low-thrust phase or by adjusting the manifold and the point at which the spacecraft enters. These changes could, for example, result in flight times shorter than those presented here as stated in the study [6]. This paper by constituted a pivotal source for the present study, which proposes a comparable notion but implements it in a distinct type of mission. This approach illustrates how a singular strategy can be utilized under varied conditions, yielding disparate outcomes based on mission parameters.

Therefore, interest in the Space Tug is not solely tied to the Lunar Gateway. In fact, this type of spacecraft is part of the evolution of the logistics market within the CLPS 2.0 (Commercial Lunar Payload Services) framework and among other private companies that are revolutionizing the aerospace sector. Furthermore, it cannot be ruled out that the administration's plans may change again in the future, reviving the idea of the Lunar Gateway or slightly modifying it into a different orbiting hub or a structural support that might eventually require logistical support.

1.4 Trajectory Design in the Cislunar Space

Traditional spacecraft trajectory design has historically relied on the patched-conic approximation and so on the two-body Keplerian mechanics. Although this approach has been more than adequate for legacy missions, it is insufficient in the low-energy cislunar regime where the gravitational perturbations of both the Earth and the Moon are comparable and must be accounted to define the motion. Cislunar trajectory design relies on the Circular Restricted Three-Body Problem (CR3BP). The CR3BP models the motion of a third body (spacecraft) of infinitesimal mass moving under the gravitational influence of two massive primaries (Earth and Moon) that revolve in circular orbits around their barycenter.

By writing the equations of motion in a rotating frame that rotates with the mean motion of the primaries (a synodic frame) the system has no explicit time dependence. In this frame, the effective potential exhibits five equilibrium points (Lagrangian or libration points) whose associated energy defines the regions where motion is possible.

The collinear libration points L_1 and L_2 act as gateways between the Earth and Lunar realms and the linearized dynamics near these points present a saddle-center-center structure. Hence, families of unstable periodic and quasi-periodic orbits exist near these points with multi-dimensional stable and unstable invariant manifolds associated with them. These manifolds represent asymptotic space tubes that can be used as natural conduits. A spacecraft placed inside or on the surface of a stable manifold tube will be guided without any energy expenditure (other than the initial insertion) to the vicinity of the associated libration point.

To improve this process, the trajectory design couples active, continuous low-thrust maneuvers, the most important of which being the slow raising of the initial orbit out of the Earth's deep gravity well, with the ballistic coasting along an invariant manifold of the CR3BP chosen on the basis of the state vector of the insertion point and the travel time along the manifold.

By employing these dynamical tubes for the multi-body transition, the active propulsive work done by the Space Tug is limited. In other words, this combination of low-thrust propulsion and manifold dynamics provides a near-optimal trajectory that reduce the propellant usage allowing to deliver a higher mass payload to cislunar space.

1.5 Thesis Objectives and mission statement

The objective of this study is to enhance the trajectory utilized in the research conducted by Jasmine Rimani et al [3]. The publication addressed the project in its preliminary stage, aiming to assess its feasibility and parameters. However, it did not undertake a comprehensive evaluation of any specific aspect of the project at a deeper design level; consequently, the trajectory employed was not optimized.

The objective of this thesis is to optimize the trajectory and assess the feasibility of employing the aforementioned invariant manifolds and to determine the potential benefits of this alternative strategy in terms of propellant mass and flight time. It is anticipated that both of these parameters will be improved.

It is important to note that the structural and systems components of the Space Tug have remained unaltered. Consequently, reference is made to the technical specifications provided in the cited paper [3], including the propulsion system. This approach eliminates the need to revise the power system's design, at least for this level of study.

A further task of this thesis will be to perform a trade-off analysis and size a fleet of space tugs based on the obtained flight times and the desired delivery frequency. The purpose of the research conducted in support of this thesis can be encapsulated in the following mission statement:

"To prove the feasibility and advantages of exploiting invariant manifolds combined with a low thrust phase to design a new trajectory for a cargo spacecraft destined to the Lunar Gateway and cislunar space. The trajectory should be optimized for the Space Tug Project and provide an improvement over the previous trajectory."

1.6 Thesis outline

The remainder of this thesis is organised as follows. Chapter 2 establishes the theoretical framework: it introduces the Circular Restricted Three-Body Problem (CR3BP), its equilibrium points and the geometry of motion, the linear dynamics in the vicinity of the collinear points, the associated families of periodic orbits and their stability through the monodromy matrix, and the invariant manifolds together with the numerical procedure adopted for their globalisation.

Chapter 3 develops the trajectory-design methodology that has been followed during the study. Starting from the low-thrust transfer strategy inherited from the MAGNETO tool, it addresses the selection of the target near-rectilinear halo orbit and of its stable manifold, the choice of the insertion point, the transposition of the injection state from the rotating to the inertial frame, the propagation of the low-thrust arc in modified equinoctial elements, and the matching of the ballistic and the propelled arcs at the patch point.

Chapter 4 presents the results and the mission analysis: the sizing of the crew provisions, the low-thrust phase results and their validation against STK, the manifold coasting output, the reconstruction of the complete outbound and inbound trajectory, the sizing of the space-tug fleet, and the comparison of the overall performance with the reference works .

Finally, Chapter 5 draws the conclusions and outlines directions for future work.

Chapter 2

Theory of the Dynamics Applied

2.1 Theoretical references

This chapter develops the theory of the three-body problem and of the dynamics that lead to the computation of the invariant manifolds. The work presented here was carried out by means of numerical codes provided by Professor S. Ross, who is also one of the advisors of this thesis; accordingly, the theoretical treatment follows the exposition given by him and his co-authors in the book *Dynamical Systems, the Three-Body Problem and Space Mission Design* [7], which constitutes one of the standard references in the field. The present chapter therefore draws extensively on that text, as it establishes the theoretical foundations needed to understand the computational tools employed in the trajectory design addressed in Chapters 3 and 4.

2.1.1 Modelling assumptions and the synodic reference frame

Consider a particle P of negligible mass, the spacecraft, m_3 , moving under the gravitational attraction of two bodies of masses m_1 and m_2 , hereafter the *primaries*, with $m_1 > m_2 \gg m_3$. In the Earth–Moon realm application like in this work, m_1 denotes the Earth and m_2 the Moon. The model relies on two idealisations:

1. *Restricted problem*: the mass of P is so small compared with m_1 and m_2 that it does not perturb their motion. The primaries therefore follow a prescribed Keplerian motion, and the problem reduces to integrating the trajectory of P in a known, time-varying gravitational field.

2. *Circular problem*: the two primaries revolve on circular orbits about their common barycenter with constant angular rate equal to their mean motion n .

Let X - Y - Z be a sidereal (inertial) frame centered at the barycenter of the primaries, with the X - Y plane being the same as their orbital plane. Because the primaries rotate uniformly, it is convenient to the work ahead to introduce a (rotating) frame x - y - z , sharing the same origin and rotating about the Z -axis with the mean motion n equal to the motion of the primaries (*synodic*), so that the primaries appear fixed in this frame. This rotating frame is composed by the x -axis directed from m_1 towards m_2 , the z -axis is parallel to Z , and the y -axis completes a right-handed triad as shown in figure 2.4. Assuming the two frames coincide at $t = 0$, the position of P in the two frames is related by

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathbf{R}(t) \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{R}(t) = \begin{pmatrix} \cos nt & -\sin nt & 0 \\ \sin nt & \cos nt & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.1)$$

Differentiating (2.1) and expressing the result in the rotating basis yields the components of the inertial velocity of P as seen in the synodic frame,

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \mathbf{R}(t) \begin{pmatrix} \dot{x} - ny \\ \dot{y} + nx \\ \dot{z} \end{pmatrix}, \quad (2.2)$$

where the terms $-ny$ and $+nx$ originate from the rotation of the frame. The decisive advantage of (2.2) is that, once the equations of motion are written in synodic coordinates, the gravitational field of the primaries becomes time-independent making the dynamical system autonomous and its theory becomes applicable.

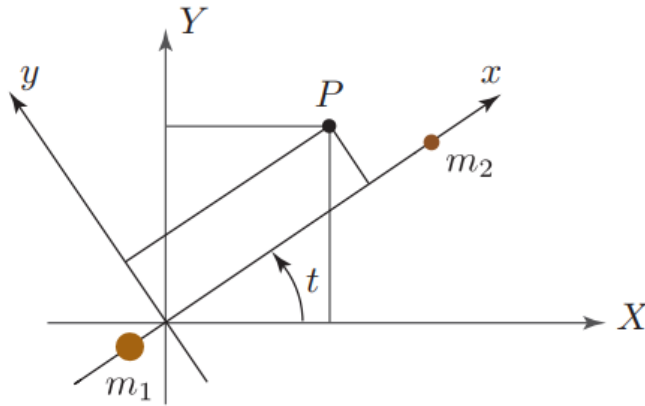


Figure 2.1: Inertial and rotating frames

2.1.2 Non-dimensionalisation and the mass parameter

It is standard to express the problem in normalized units.

- the unit of mass is the total mass of the primaries, $m_1 + m_2$;
- the unit of length is their constant separation, L ;
- the unit of time is chosen so that the primaries complete one orbital period in 2π time units.

With this choice the gravitational constant and the mean motion both reduce to unity:

$$G = 1, \quad n = 1, \quad (2.3)$$

so that the angular argument nt in (2.1) simplifies to t . Now the system is governed by a single parameter which is the *mass parameter*

$$\mu = \frac{m_2}{m_1 + m_2} \in \left(0, \frac{1}{2}\right], \quad (2.4)$$

in terms of which the normalized masses of the larger and smaller primary are respectively $1 - \mu$ and μ , respectively. This chosen scaling and frame orientation entails that the primaries are stationary in the synodic frame at the fixed positions r_1, r_2 :

$$\mathbf{r}_1 = (-\mu, 0, 0), \quad \mathbf{r}_2 = (1 - \mu, 0, 0). \quad (2.5)$$

For the Earth–Moon system considered in this work, the relevant constants are collected in Table 2.1; these will be useful in the 4th chapter when it will be necessary to link the orbit propagation studied separately in the inertial and rotating frames.

Table 2.1: Characteristic constants of the Earth–Moon CR3BP.

Quantity	Symbol	Value
Mass parameter	μ	1.215×10^{-2}
Characteristic length	L	3.850×10^5 km
Characteristic velocity	V	1.025 km/s
Characteristic time	$T/2\pi$	3.758×10^5 s

2.1.3 Equations of motion

Denoting r_1 and r_2 as the distances of P respectively from the larger and smaller primary in synodic coordinates,

$$r_1 = \sqrt{(x + \mu)^2 + y^2 + z^2}, \quad r_2 = \sqrt{(x - 1 + \mu)^2 + y^2 + z^2}. \quad (2.6)$$

making possible to define the gravitational potential energy experienced by P , in normalised units,

$$U(x, y, z) = -\frac{1 - \mu}{r_1} - \frac{\mu}{r_2} - \frac{1}{2}\mu(1 - \mu), \quad (2.7)$$

the last term being a conventional additive constant that has no effect on the equations of motion [8]. Since both distances are invariant under the rotation (2.1), the kinetic energy of P measured in the inertial frame but expressed through (2.2) in synodic coordinates results as:

$$K(x, y, \dot{x}, \dot{y}, \dot{z}) = \frac{1}{2} [(\dot{x} - y)^2 + (\dot{y} + x)^2 + \dot{z}^2]. \quad (2.8)$$

The Lagrangian of P in the rotating frame is therefore $L = K - U$, namely

$$L(x, y, z, \dot{x}, \dot{y}, \dot{z}) = \frac{1}{2} [(\dot{x} - y)^2 + (\dot{y} + x)^2 + \dot{z}^2] - U(x, y, z), \quad (2.9)$$

Applying the Euler–Lagrange equations $\frac{d}{dt}(\partial L / \partial \dot{q}_i) - \partial L / \partial q_i = 0$ to (2.9) and simplifying yields the equations of motion of the spatial CR3BP,

$$\ddot{x} - 2\dot{y} = -\bar{U}_x, \quad \ddot{y} + 2\dot{x} = -\bar{U}_y, \quad \ddot{z} = -\bar{U}_z, \quad (2.10)$$

where subscripts denote partial differentiation and

$$\bar{U}(x, y, z) = -\frac{1}{2}(x^2 + y^2) - \frac{1 - \mu}{r_1} - \frac{\mu}{r_2} - \frac{1}{2}\mu(1 - \mu) \quad (2.11)$$

is the *effective*¹ (or augmented) potential, whose planar form is recovered by setting $z = \dot{z} = 0$. An equivalent Hamiltonian description, useful in the later analysis of the motion near the libration points (Section 2.3), is obtained through the Legendre transformation. Introducing the momenta conjugate to x, y, z ,

$$p_x = \dot{x} - y, \quad p_y = \dot{y} + x, \quad p_z = \dot{z}, \quad (2.12)$$

¹An equivalent formulation, common in the mission-design literature, instead adopts a effective-potential defined with the opposite (positive) sign, $\Omega = -\bar{U}$ (also denoted U^* , see [9]), for which (2.10) reads $\ddot{x} - 2\dot{y} = \Omega_x$ and the Jacobi constant becomes $C = 2\Omega - v^2$. The two conventions differ only by sign.

the Hamiltonian of the system is

$$H(x, y, z, p_x, p_y, p_z) = \frac{1}{2} \left[(p_x + y)^2 + (p_y - x)^2 + p_z^2 \right] + U(x, y, z), \quad (2.13)$$

which generates Hamilton's equations and confirms the autonomous, six-dimensional character of the phase space.

2.1.4 The Jacobi integral

Because the Hamiltonian (2.13) does not depend explicitly on time, the energy of P is conserved along its motion. Expressed as a function of position and velocity, this energy is

$$E(x, y, z, \dot{x}, \dot{y}, \dot{z}) = \frac{1}{2} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \bar{U}(x, y, z). \quad (2.14)$$

In the celestial-mechanics literature it is customary to use the quantity $C = -2E$, known as the *Jacobi constant*²,

$$C(x, y, z, \dot{x}, \dot{y}, \dot{z}) = -2\bar{U}(x, y, z) - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2). \quad (2.15)$$

That C is indeed an integral of motion follows directly from (2.10). Differentiating the squared velocity and substituting the equations of motion,

$$\frac{d}{dt} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = 2(\dot{x}\ddot{x} + \dot{y}\ddot{y} + \dot{z}\ddot{z}) = -2(\dot{x}\bar{U}_x + \dot{y}\bar{U}_y + \dot{z}\bar{U}_z) = -2\frac{d\bar{U}}{dt}, \quad (2.16)$$

where the Coriolis terms cancel identically, so that $dC/dt = 0$.

The Jacobi constant is the only known integral of the CR3BP; the problem is non-integrable, and no further constant constrains the motion of P . For a fixed mass parameter μ , the value of C alone determines which regions of configuration space are dynamically accessible to the spacecraft, a restriction examined in detail in Section 2.2.

2.2 Equilibrium solutions and the geometry of motion

The equations of motion admit five equilibrium solutions, the libration points, in which proximity the periodic orbits and invariant manifolds exploited in this work are formed. This section locates them and, through the Jacobi constant, delimits the regions where motion is possible for the spacecraft with a fixed Jacobi constant.

²Must be noted that the Jacobi constant and the energy E have opposite signs and so as the energy of P increases the C decreases

2.2.1 The libration points

An equilibrium point of (2.10) is a point at which a particle released at rest in the rotating frame remains at rest. Setting all velocities and accelerations to zero reduces the equations of motion to $\nabla \bar{U} = 0$: the libration points are the critical points of the effective potential, whose gradient components are

$$\begin{aligned}\bar{U}_x &= -x + \frac{(1-\mu)(x+\mu)}{r_1^3} + \frac{\mu(x-1+\mu)}{r_2^3}, \\ \bar{U}_y &= y \left[-1 + \frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3} \right], \\ \bar{U}_z &= z \left[\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3} \right].\end{aligned}\tag{2.17}$$

Since the bracket in $\bar{U}_z$ is strictly positive, $\bar{U}_z = 0$ forces $z = 0$: every equilibrium lies in the orbital plane. The condition $\bar{U}_y = 0$ then splits the solutions into two families:

- for $y = 0$ the points lie on the x -axis just like the primaries. It yields the three *collinear* points: L_1 between the primaries, L_2 beyond the smaller one, and L_3 beyond the larger one. For the Earth–Moon system they are located, in nondimensional units, at $x_{L_1} \approx 0.8369$, $x_{L_2} \approx 1.1557$ and $x_{L_3} \approx -1.0051$.
- For $y \neq 0$ the bracket in $\bar{U}_y$ must vanish, which together with $\bar{U}_x = 0$ gives the closed-form solution $r_1 = r_2 = 1$; the two *equilateral* points are therefore

$$L_{4,5} = \left(\frac{1}{2} - \mu, \pm \frac{\sqrt{3}}{2}, 0 \right),\tag{2.18}$$

each forming an equilateral triangle with the primaries.

The collinear points are central to this work: L_1 and L_2 are dynamically unstable and act as the gateways governing transfer between the Earth and lunar regions.

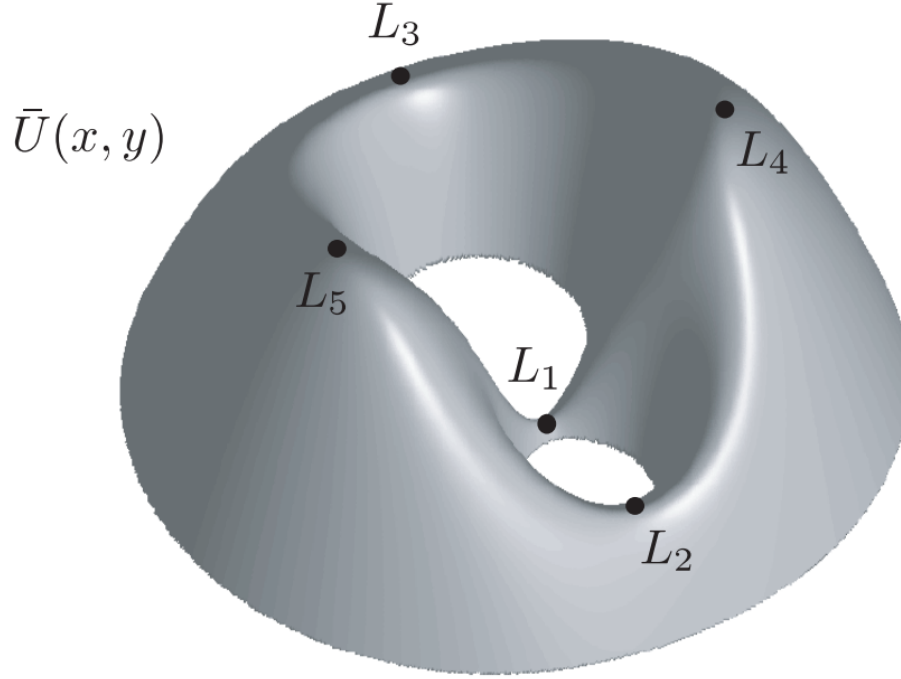


Figure 2.2: Effective potential $\bar{U}(x, y)$

2.2.2 Zero-velocity surfaces and the realms of motion

To find the regions of space where motion is possible and so $v \geq 0$ the squared velocity must be isolated in (2.15) giving $v^2 = -2\bar{U}(x, y, z) - C$. Since $v^2 \geq 0$, for a fixed value of C the spacecraft is confined to the *region of possible motion* (the Hill region)

$$\mathcal{M}(\mu, C) = \{(x, y, z) : -2\bar{U}(x, y, z) \geq C\}, \quad (2.19)$$

whose boundary, the *zero-velocity surface* $-2\bar{U} = C$, is the locus of points where the velocity is equal to zero; its planar trace is the zero-velocity curve (ZVC). The complementary set, where $-2\bar{U} < C$, is the forbidden region, which the trajectory cannot enter.

The topology of $\mathcal{M}(\mu, C)$ is controlled by the value of C relative to the Jacobi constants of the libration points, which satisfy $C_{L_1} > C_{L_2} > C_{L_3} > C_{L_4} = C_{L_5} = 3$. As C decreases from large values (i.e. as the energy increases), the accessible region grows and successive “necks” open:

- for $C > C_{L_1}$, the Earth and lunar realms are enclosed by separate ZVCs and no passage between them is possible;

- at $C = C_{L_1}$ the surface pinches at L_1 , connecting the two realms through a narrow neck;
- at $C = C_{L_2}$ a second neck opens at L_2 , connecting the lunar realm to the exterior region;
- further decrease opens the neck at L_3 and eventually removes the forbidden region entirely.

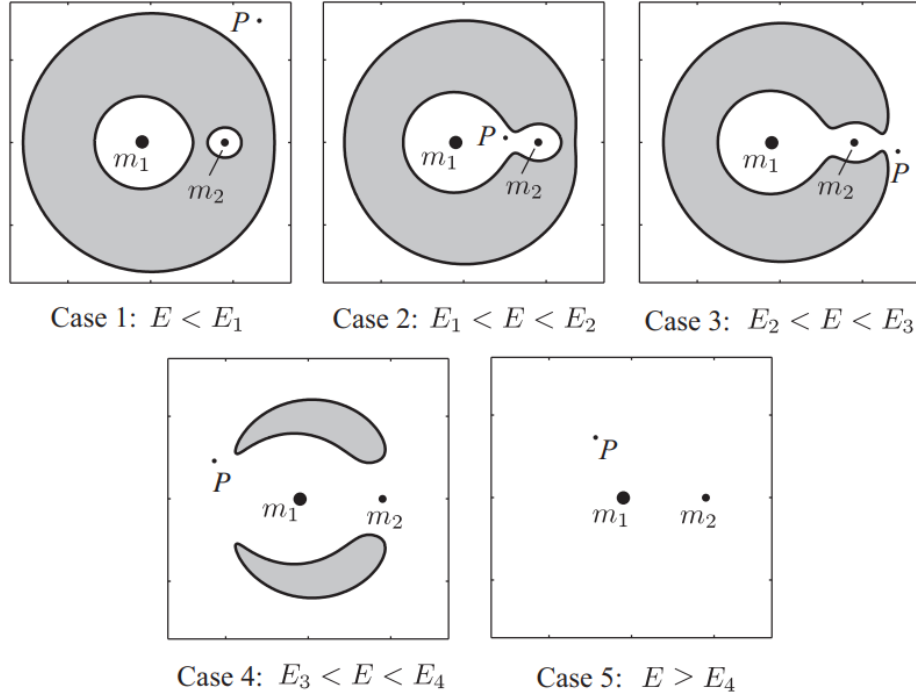


Figure 2.3: Zero Velocity Curves as shown in [7], here the parameter used is E while the cases defined above depend on C , to link these two must be used the formulation $C = -2E$

The transfer addressed in this thesis operates in the regime $C \lesssim C_{L_2}$, for which a continuous corridor through the L_1 and L_2 necks links the Earth vicinity to the lunar and Gateway region providing the natural transport channels exploited in the trajectory design that will be analyzed in the next chapter.

2.3 Linear dynamics near the collinear points

The transfer strategy relies on periodic orbits and invariant manifolds that live in the immediate vicinity of L_1 and L_2 . Their existence and their qualitative structure are already dictated by the linearisation of the equations of motion about these points, which is the subject of this section. Following the spatial treatment of [7], we work directly in the three-dimensional problem, since the orbits of interest, halo orbits, are intrinsically non-planar.

2.3.1 Linearisation about a collinear point

Since L_j ($j = 1, 2$) denote the collinear point under consideration let γ_j be its distance to the smaller primary, the unique positive root of the associated quintic equation located in Section 2.2. We translate the origin to L_j and rescale all lengths by γ_j , so that the distance between L_j and the Moon becomes unity, accordingly with the system introduced by Richardson [10]

$$\bar{x} = \frac{x - (1 - \mu) \pm \gamma_j}{\gamma_j}, \quad \bar{y} = \frac{y}{\gamma_j}, \quad \bar{z} = \frac{z}{\gamma_j}, \quad (2.20)$$

the upper sign referring to L_1 and the lower to L_2 . In these local coordinates, which we rename (x, y, z) for brevity, the gravitational potential is expanded in Legendre polynomials, and retaining the quadratic terms of the Hamiltonian (2.13) yields

$$H_2 = \frac{1}{2} (p_x^2 + p_y^2 + p_z^2) + y p_x - x p_y - c_2 x^2 + \frac{c_2}{2} (y^2 + z^2), \quad (2.21)$$

where p_x, p_y, p_z are the conjugate momenta (2.12) and c_2 is the second coefficient of the expansion. For the collinear points the coefficients are given in closed form by

$$c_n = \frac{1}{\gamma_j^3} \left[(\pm 1)^n \mu + (-1)^n \frac{(1 - \mu) \gamma_j^{n+1}}{(1 \pm \gamma_j)^{n+1}} \right], \quad (2.22)$$

again with the upper sign for L_1 and the lower for L_2 . Only c_2 enters the linear analysis; the higher-order coefficients c_n ($n \geq 3$) govern the nonlinear terms exploited for periodic orbits cited in Section 2.4. For every collinear point one has $c_2 > 1$, a property that fixes the qualitative nature of the dynamics, as shown next.

2.3.2 The saddle $\times$ center $\times$ center structure

The vertical direction in (2.21) is already decoupled from the planar ones and describes a harmonic oscillator of frequency $\sqrt{c_2}$. A linear symplectic change of coordinates brings the planar part to normal form as well, casting (2.21) into

$$H_2 = \lambda q_1 p_1 + \frac{\omega_p}{2} (q_2^2 + p_2^2) + \frac{\omega_v}{2} (q_3^2 + p_3^2), \quad (2.23)$$

in which the three constants are positive real numbers depending only on c_2 ,

$$\lambda^2 = \frac{c_2 - 2 + \sqrt{9c_2^2 - 8c_2}}{2}, \quad \omega_p^2 = \frac{2 - c_2 + \sqrt{9c_2^2 - 8c_2}}{2}, \quad \omega_v^2 = c_2. \quad (2.24)$$

The condition $c_2 > 1$ guarantees that λ , ω_p and ω_v are all real. In the normal coordinates the linearised flow separates into three uncoupled blocks,

$$\dot{q}_1 = \lambda q_1, \quad \dot{p}_1 = -\lambda p_1; \quad \dot{q}_2 = \omega_p p_2, \quad \dot{p}_2 = -\omega_p q_2; \quad \dot{q}_3 = \omega_v p_3, \quad \dot{p}_3 = -\omega_v q_3, \quad (2.25)$$

whose solutions are

$$q_1(t) = q_1^0 e^{\lambda t}, \quad p_1(t) = p_1^0 e^{-\lambda t}, \quad q_{2,3}(t) + i p_{2,3}(t) = (q_{2,3}^0 + i p_{2,3}^0) e^{-i\omega_{p,v} t}. \quad (2.26)$$

The equilibrium is thus of the type *saddle* $\times$ *center* $\times$ *center*: one hyperbolic pair $\pm\lambda$ and two elliptic pairs $\pm i\omega_p$, $\pm i\omega_v$.

2.3.3 Dynamical consequences of the saddle $\times$ center $\times$ center structure

The decomposition (2.26) contains, in linear form, every ingredient used in the remainder of this work.

The hyperbolic pair $\pm\lambda$ is the origin of the invariant manifolds. The coordinate q_1 , growing as $e^{\lambda t}$, spans the local unstable direction, while p_1 , decaying as $e^{-\lambda t}$, spans the local stable one. These two directions are the linear approximations of the stable and unstable manifolds: in Section 2.7 the manifold of a periodic orbit is generated precisely by displacing the orbit along the nonlinear continuation of such directions and integrating. Because the stable component decays forward in time, its globalisation requires backward-time propagation, the procedure adopted in this thesis. The sign of λ also makes the instability of L_1 and L_2 explicit, confirming their role as dynamical gateways.

The two elliptic pairs are the origin of the periodic orbits. The planar centre, of frequency ω_p , generates the planar Lyapunov family encircling the libration point; the vertical centre, of frequency ω_v , generates the out-of-plane oscillation. When the nonlinear terms neglected in (2.21) bring the two frequencies into resonance, the planar and vertical motions couple and bifurcate into the three-dimensional halo orbits that serve as the target orbit of the Space Tug. The frequencies in (2.24) provide the period estimates used to initialise the numerical computation of these orbits in Section 2.4.

By Moser's generalisation of the Lyapunov theorem, the qualitative picture established here for the linear system persists for the full nonlinear dynamics in a neighbourhood of L_1 and L_2 [7]. The saddle therefore yields genuine stable and

unstable manifolds, and the centres genuine periodic orbits, whose computation is the subject of the following sections.

2.4 Periodic orbits near the collinear points

The two elliptic blocks identified in the linear analysis of Section 2.3 are, by the Lyapunov centre theorem, the seeds of genuine families of periodic orbits of the full nonlinear system. This section introduces the families relevant to the mission and describes the numerical procedure used to compute them.

2.4.1 Families of periodic orbits

The planar centre, of frequency ω_p , gives rise to the *planar Lyapunov family*: a one-parameter family of planar periodic orbits encircling the libration point, whose amplitude grows as one moves away from L_j . As the in-plane amplitude increases, a critical orbit is reached at which the planar and vertical frequencies become commensurate; from this bifurcation a family of three-dimensional orbits emerges, the *halo orbits*, which combine the in-plane and out-of-plane oscillations and are symmetric with respect to the xz -plane. Each collinear point possesses a northern and a southern halo family, mirror images across the orbital plane.

Within the L_2 halo family lies the subfamily of *near-rectilinear halo orbits* (NRHOs), characterised by a close perilune passage, a large apolune excursion and a nearly stable behaviour and these are some of the motivations that led to the choice of an L_2 southern NRHO as the operational orbit of the Lunar Gateway.

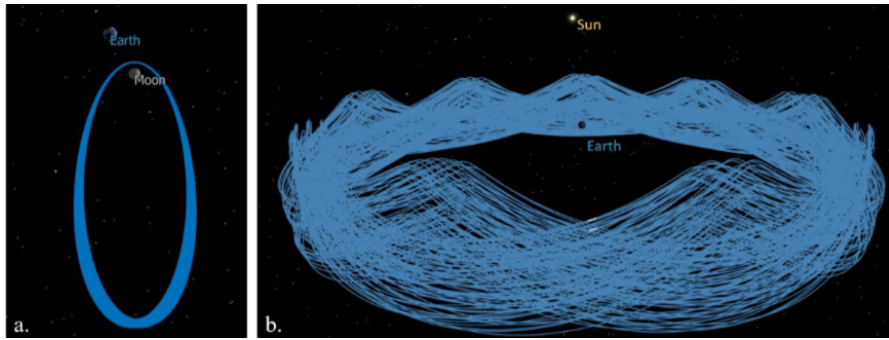


Figure 2.4: visualization of NRHO in the rotating frame (a) and the inertial frame (b). This image is taken from the work of Diane C. Davis et al.[11] by the research group of prof. Howell. These representation is taken as a reference for this study that will later show in section the NRHO computed with the data utilized for the analysis

Among the halo orbits, the near-rectilinear members are only weakly unstable and can be maintained with a very modest propellant budget, of the order of a few meters per second per year. At the same time, the large apolune excursion keeps the spacecraft out of the deep lunar gravity well and preserves a near-continuous line of sight with the Earth, which is essential for uninterrupted communications, while the close perilune passage provides convenient access to the lunar surface, and in particular to the south pole which has been selected as target site by NASA because of the presence of water ice preserved in its permanently shadowed regions[12]. A specific member of this family of orbits has been selected by imposing a 9:2 resonance with the lunar synodic period, corresponding to a period of about 6.5 days: this resonance phases the perilune passages in order to avoid prolonged eclipses by the Earth's shadow. The resulting orbit, with a perilune radius of roughly 3 250 km and an apolune radius of about 71 000 km, combines low maintenance cost, permanent Earth visibility and surface accessibility, and is for these reasons adopted as the target orbit of the transfers designed in this work.

The remainder of this chapter develops the tools required to compute such an orbit and, from it, the invariant manifolds used for the transfer.

2.4.2 The state transition matrix

The computation of periodic orbits and of their invariant manifolds rests on a single quantity: the sensitivity of a trajectory to a small change in its initial state. Consider a reference trajectory $\bar{\mathbf{X}}(t)$ and a proximal one starting from the perturbed initial state $\bar{\mathbf{X}}_0 + \delta\mathbf{X}_0$. To first order, the resulting displacement at time t is linear in the initial perturbation,

$$\delta\mathbf{X}(t) = \Phi(t, t_0) \delta\mathbf{X}_0, \quad (2.27)$$

and the matrix $\Phi(t, t_0)$ that realises this mapping is the *state transition matrix* (STM). It propagates an infinitesimal deviation from the reference trajectory forward in time and is the common tool used both the differential corrector of Section 2.4.3 and the manifold construction of Section 2.7.

Writing the equations of motion (2.10) in first-order form as $\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X})$, with $\mathbf{X} = (x, y, z, \dot{x}, \dot{y}, \dot{z})^\top$, the STM is obtained as the solution of the *variational equation*

$$\dot{\Phi}(t, t_0) = \mathbf{A}(t) \Phi(t, t_0), \quad \Phi(t_0, t_0) = \mathbf{I}_6, \quad (2.28)$$

where $\mathbf{A}(t) = \partial\mathbf{f}/\partial\mathbf{X}$ is the Jacobian of the vector field evaluated along the reference trajectory [7]. Splitting the state into its position part $\mathbf{r} = (x, y, z)$ and velocity part $\mathbf{v} = (\dot{x}, \dot{y}, \dot{z})$, and recalling that the equations of motion read $\dot{\mathbf{r}} = \mathbf{v}$ and

$\dot{\mathbf{v}} = -\nabla_{\mathbf{r}}\bar{U} + \mathbf{\Omega} \mathbf{v}$, the Jacobian acquires the block structure

$$\mathbf{A} = \begin{pmatrix} \mathbf{0}_3 & \mathbf{I}_3 \\ \mathbf{B} & \mathbf{\Omega} \end{pmatrix}, \quad \mathbf{B} = -\frac{\partial^2 \bar{U}}{\partial \mathbf{r}^2}, \quad \mathbf{\Omega} = \begin{pmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2.29)$$

whose four blocks have a direct interpretation. The upper-left block vanishes because $\dot{\mathbf{r}} = \mathbf{v}$ does not depend on position, while the upper-right identity block $\mathbf{I}_3$ expresses the purely kinematic relation that position responds to velocity. The dynamics of the CR3BP enter through the lower blocks. The block $\mathbf{B}$ is the negative Hessian of the effective potential,

$$\mathbf{B} = -\frac{\partial^2 \bar{U}}{\partial \mathbf{r}^2} = -\begin{pmatrix} \bar{U}_{xx} & \bar{U}_{xy} & \bar{U}_{xz} \\ \bar{U}_{yx} & \bar{U}_{yy} & \bar{U}_{yz} \\ \bar{U}_{zx} & \bar{U}_{zy} & \bar{U}_{zz} \end{pmatrix}, \quad (2.30)$$

and carries the gravitational and centrifugal contributions; being built from the second derivatives of $\bar{U}$, it is symmetric and is the only block that depends on the current position, which is what makes $\mathbf{A} = \mathbf{A}(t)$ time-varying along the trajectory. The block $\mathbf{\Omega}$ is constant and reproduces the terms $+2\dot{y}$ and $-2\dot{x}$ of the equations of motion, the null third row reflecting the absence of a (contribution in the out-of-plane direction).

In practice the six state equations and the thirty-six entries of (2.28) are integrated simultaneously, as a single system of forty-two ordinary differential equations, with initial conditions $\bar{\mathbf{X}}(t_0) = \bar{\mathbf{X}}_0$ and $\Phi(t_0, t_0) = \mathbf{I}_6$ [7]. Because $\mathbf{A}(t)$ depends on the reference solution, which is in general not known analytically, this integration must be carried out numerically. The resulting matrix $\Phi(t, t_0)$ is precisely the object propagated, period by period, to assess the stability of an orbit (Section 2.5) and, point by point, to grow its invariant manifolds (Section 2.7).

2.4.3 Differential correction and continuation

A halo orbit is computed by enforcing its symmetry. At a perpendicular crossing of the xz -plane the state has the form

$$\mathbf{X}_0 = (x_0, 0, z_0, 0, \dot{y}_0, 0)^\top, \quad (2.31)$$

with only the $\dot{y}$ velocity component non-zero. By symmetry, a true halo orbit returns to a second perpendicular crossing after half a period, $t = T/2$, when the trajectory must again satisfy $y = 0$ together with $\dot{x} = \dot{z} = 0$. An arbitrary guess (2.31), propagated to the next $y = 0$ crossing, will in general leave residual in-plane and out-of-plane velocities $\dot{x}, \dot{z} \neq 0$. The differential corrector removes them by adjusting the initial state along the directions provided by the STM: holding one

component fixed (for instance z_0), the residuals are driven to zero by solving the linear system relating $(\delta\dot{x}, \delta\dot{z})$ at the crossing to the admissible variations of $(x_0, \dot{y}_0)$, obtained from the relevant entries of $\Phi(T/2, 0)$ and corrected for the variation of the crossing time. The procedure is iterated until $|\dot{x}|, |\dot{z}|$ fall below a prescribed tolerance, yielding a periodic orbit and its period T . An initial guess accurate enough to start the iteration is supplied by the linear solution (2.26), or by a third-order analytical approximation [7]. A single corrected orbit is then continued into a family by taking the converged solution as the initial guess for a proximal, after a small increment of a continuation parameter (typically the out-of-plane amplitude z_0 or the Jacobi constant). Repeating this predictor–corrector step traces the entire halo family, from the small near-planar orbits up to the near-rectilinear regime containing the target NRHO.

2.5 Stability of periodic orbits: the monodromy matrix

The stability of a periodic orbit is assessed through the behaviour of the trajectories in its neighbourhood. As established in Section 2.4.2, a small perturbation $\delta\mathbf{X}_0$ of the initial state evolves, to first order, according to the state transition matrix, $\delta\mathbf{X}(t) = \Phi(t, 0) \delta\mathbf{X}_0$. For a periodic orbit Γ of period T , the natural question is whether such a perturbation grows or decays after one complete revolution; this is answered by evaluating the state transition matrix over exactly one period, which defines the *monodromy matrix*

$$\mathbf{M} \equiv \Phi(T, 0). \quad (2.32)$$

The relevance of a single period follows from Floquet theory [13]: since the Jacobian $\mathbf{A}(t)$ is itself T -periodic along Γ , one can show that $\Phi(kT, 0) = \mathbf{M}^k$, so that the displacement after k revolutions is $\delta\mathbf{X}(kT) = \mathbf{M}^k \delta\mathbf{X}_0$ [7]. The long-term growth or decay of perturbations is therefore entirely governed by the eigenvalues of $\mathbf{M}$, the *characteristic* (or Floquet) *multipliers*: a perturbation aligned with an eigenvector of $\mathbf{M}$ is reproduced after one period, scaled by the corresponding multiplier, and the orbit is unstable whenever the modulus of any eigenvalue is greater than 1. Because the CR3BP flow is Hamiltonian, $\mathbf{M}$ is symplectic, and its six eigenvalues therefore occur in reciprocal pairs $(\rho, 1/\rho)$. For the periodic orbits of interest this spectrum has a fixed structure inherited from the saddle $\times$ center $\times$ center nature of the underlying equilibrium:

- a *hyperbolic pair* of real reciprocal multipliers, $\lambda_{\max} > 1$ and $1/\lambda_{\max} < 1$, responsible for the instability of the orbit and for its stable and unstable manifolds;

- a *trivial pair* equal to unity, one multiplier being associated with the direction tangent to the orbit and the other with the conservation of the Jacobi constant;
- a *pair on the unit circle*, $e^{\pm i\theta}$, describing the bounded oscillation transverse to the orbit.

The magnitude of $\lambda_{\max}$ measures how strongly nearby trajectories depart from the orbit over one period and is commonly summarised by the stability index. Along the halo family this index decreases as the orbits become more rectilinear, and the NRHO subfamily is precisely the portion where it remains close to the marginal value, which explains the favourable station-keeping properties that motivate its operational use.

For the present work the essential output of the monodromy matrix is the hyperbolic pair: the eigenvector associated with $\lambda_{\max}$ spans the local *unstable* direction and the one associated with $1/\lambda_{\max}$ the local *stable* direction. These two eigenvectors are the starting point for the construction of the invariant manifolds.

2.6 Invariant manifolds of a periodic orbit

The *stable manifold* W^s is the set of all states that asymptotically approach the periodic orbit in forward time, while the *unstable manifold* W^u is the set of states that approach it in backward time, equivalently departing from it in forward time as defined by [14]:

$$W^s = \{\mathbf{X} : \varphi_t(\mathbf{X}) \rightarrow \Gamma \text{ as } t \rightarrow +\infty\}, \quad W^u = \{\mathbf{X} : \varphi_t(\mathbf{X}) \rightarrow \Gamma \text{ as } t \rightarrow -\infty\}, \quad (2.33)$$

where Γ denotes the periodic orbit and φ_t the flow. At each point of Γ these manifolds are tangent to the stable and unstable eigenvectors of the monodromy matrix; as the base point moves once around the orbit, each eigendirection sweeps out a two-dimensional surface with the topology of a cylinder, $S^1 \times \mathbb{R}$. The stable and unstable manifolds are thus *tubes* extending away from the orbit. These tubes are the structures that make the transfer possible. Trajectories lying on a stable tube are funnelled towards the periodic orbit at no propulsive cost beyond the initial insertion, so the tube acts as a natural conduit connecting a distant region to the vicinity of the libration-point orbit. Moreover, because each tube separates the states that transit through the neck region from those that do not, it behaves as a separatrix of the local flow. For the Space Tug, the stable manifold of the target NRHO provides exactly such a ballistic corridor towards the Gateway, and its computation is therefore the final theoretical objective of this chapter.

2.7 Numerical globalisation of the invariant manifolds

The definitions (2.33) characterise the manifolds globally but do not provide them explicitly. In practice each tube is grown numerically from its known linear approximation along the orbit, following the procedure of [7, §§4.4, 7.2].

The starting point is a reference state $\mathbf{X}_0$ on the periodic orbit and its monodromy matrix $\mathbf{M}$. Let $\mathbf{Y}^s(\mathbf{X}_0)$ and $\mathbf{Y}^u(\mathbf{X}_0)$ denote the corresponding eigenvectors, normalised to unit length; they provide the local approximation of the stable and unstable manifolds at $\mathbf{X}_0$. A first guess for a point on each manifold is obtained by displacing the reference state along these directions by a small amount ε ,

$$\mathbf{X}^s(\mathbf{X}_0) = \mathbf{X}_0 + \varepsilon \mathbf{Y}^s(\mathbf{X}_0), \quad \mathbf{X}^u(\mathbf{X}_0) = \mathbf{X}_0 + \varepsilon \mathbf{Y}^u(\mathbf{X}_0). \quad (2.34)$$

To obtain the manifold direction at any other point $\mathbf{X}(\tau)$ of the orbit, the eigenvector is transported along the trajectory by means of the state transition matrix,

$$\mathbf{Y}(\mathbf{X}(\tau)) = \Phi(\tau, 0) \mathbf{Y}(\mathbf{X}_0), \quad (2.35)$$

which propagates a perturbation aligned with the manifold from the reference point to $\mathbf{X}(\tau)$. It is necessary to renormalize the transported vector before the displacement is applied because the STM does not preserve the norm. The magnitude of ε must be small enough for the linear approximation to remain valid, yet not so small that the asymptotic nature of the manifold makes the integration time excessively long. Following the values suggested by [7], a nondimensional displacement of $\varepsilon = 10^{-6}$ is adopted here, corresponding to a position offset of about 0.385 km in the Earth–Moon system. Each seed is then propagated under the full nonlinear equations of motion to globalise the manifold: the unstable manifold is obtained by integrating *forward* in time, whereas the stable manifold, whose perturbation decays in forward time, is recovered by integrating *backward* in time. Repeating the procedure for a set of base points distributed along the orbit, and for both branches, produces the complete stable and unstable tubes. In this work both the stable and unstable manifold will be used to approach and to leave the target NRHO exploiting their natural dynamics for the outbound and inbound legs respectively.

Chapter 3

Methodology Applied

This chapter sets out the rationale and the logical thread underlying the trajectory-design choices and reasons adopted in the present work.

The starting point is the transfer employed within the MAGNETO study [3], which the present work seeks to improve .

As anticipated in the introduction of this thesis, gravitational dynamics forms what may be regarded as a natural pathway for traveling through space or what has been described in the literature as an “interplanetary superhighway” [7] along which a spacecraft can be transported at little or no propellant cost.

These preferential routes, namely the invariant manifolds introduced in Chapter 2, thus constitute the element that enables the Space Tug trajectory to save propellant and to reach the target orbit with the main engine switched off; the control thrusters must nonetheless remain active throughout the coasting phase, so as to prevent perturbations from driving the spacecraft off its nominal course. The recourse to propulsion as the source of motion cannot, however, be dispensed with: since these manifolds do not approach low Earth orbit, the spacecraft must raise its apogee and recede sufficiently from the Earth’s gravitational well to leave the two-body regime and enter the region governed by the dynamics of the three-body problem.

In order to raise the apogee of the spacecraft’s initial orbit, the strategy already adopted in the previous study [4] is retained, so that the investigation specific to this thesis may focus exclusively on the segment of the trajectory that is modified and that constitutes its innovative contribution: namely, the manner in which the invariant manifolds are exploited and combined with the low-thrust phase.

Having established the intention of employing these two distinct methodologies and of making them coexist, the first step consisted in seeking an orbit suited to the mission objective and, at the same time, yielding a manifold which, once propagated backward in time, as described in Chapter 2, exhibits the following two

features:

- it provides a trajectory that enters the region comprised between the two primaries, that is $-\mu < x < 1 - \mu$ along the x -axis of the rotating frame (with the Earth located at $x = -\mu$ and the Moon at $x = 1 - \mu$), thereby penetrating the *interior realm*, namely, in the terminology used by Ross et al.[7], the region surrounding the larger primary, which communicates with the m_2 (Moon) realm through the neck that opens around L_1 ;
- the coasting time along the manifold, from injection to arrival on the orbit, is shorter than that of the previous trajectory, thereby constituting an improvement.

The resulting trajectory envisages the spacecraft departing from a Geostationary Transfer Orbit (GTO) and performing a spiraling-up manoeuvre until the state vector required for insertion onto the selected stable manifold is attained. The trajectory subsequently proceeds along the manifold driven solely by the gravitational dynamics, until it asymptotically approaches the designated orbit. A docking phase with the Lunar Gateway then follows, whereby the Space Tug delivers its payload; upon completion of all operations, the return journey toward Earth orbit is undertaken along a trajectory symmetric to the outbound one. The return leg exploits the unstable manifold, the stable and unstable manifolds being symmetric about the x -axis [7], and is completed by a spiraling-down phase that is, however, considerably less expensive, owing to the reduced mass of the Tug after the release of the payload and the propellant expenditure of the outbound leg. The coasting arc, by contrast, is symmetric, as it is unaffected by the mass: being governed by purely ballistic dynamics, it obeys the equations of motion of the CR3BP introduced in Chapter 2 (see also [7]), in which the mass of the spacecraft does not appear so that neither the manifold nor the motion along it depends on the vehicle's mass.

Prior to undertaking the study, a literature review was carried out in order to establish whether this strategy had already been theorised and what results it had yielded. Few studies combine a hybrid trajectory of this type for the L_2 point of the Earth–Moon system; one paper in particular, however, investigates a trajectory closely resembling the one designed here. The work of Singh et al. [6] focuses precisely on an NRHO around L_2 , one of the proposed orbits being exactly the one adopted for the Lunar Gateway. Yet, whereas their study concentrates on the trajectory alone, additionally employing a higher-fidelity model, the present thesis undertakes to apply this strategy to a specific case and to investigate its logistics on the basis of the results obtained for a spacecraft with fixed data, among which an initial mass one order of magnitude higher than the 1000 kg assumed by Singh [6]. A major difference between the present thesis and the cited work therefore lies in

the fact that, here, the strategy is applied to a specific and potentially operational case, and the trajectory design is also tied to the payload budget and so the delivery frequency as well as the operations required to maintain the satellite on orbit. Further differences, together with a comparison of the results obtained, will be presented in the next chapter.

3.1 The Low-Thrust Transfer Strategy of the MAGNETO Tool

The trajectory assumed as the baseline of the present work is the fully-propelled low-thrust transfer from a Geostationary Transfer Orbit (GTO) to the target NRHO, generated by the MAGNETO tool [3] and built upon the Lunar Space Tug concept [4]. Since the vehicle is propelled by high specific-impulse Hall thrusters the manoeuvre is accomplished as a continuous orbit-raising spiral that lifts the spacecraft from the GTO up to the lunar region, with no ballistic coasting arc.

The thrust strategy adopted is a “SMART-1-like” control law, named after the homonymous ESA mission and articulated, as in [3], into three main segments: a first phase of continuous thrusting that raises the perigee up to 20 000 km, above the Van Allen belts; a second phase of intermittent thrusting that raises the apogee, performed in MAGNETO only when the manoeuvre guarantees at least 70% efficiency in raising the apogee; and a final phase of continuous thrusting that adjusts the eccentricity, the semi-major axis and the inclination so as to reach the Moon. The key to the propellant economy of this law is that most of the thrusting is concentrated in the proximity of perigee.

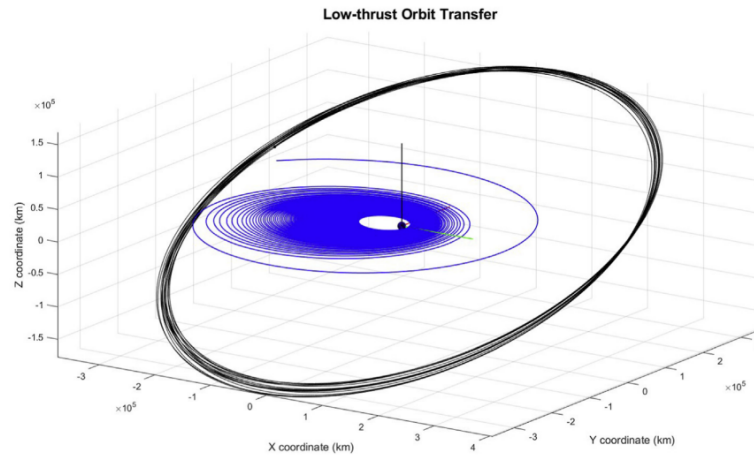


Figure 3.1: Representation of the trajectory employed in [3] and designed using a SMART-1 like control strategy

For the reference scenario, a 60-day replenishment cargo mission flown with four Hall thrusters, the optimal working point identified in [3] corresponds to a thrust of 1.05 N, a specific impulse of 2100 s and a power of 18 kW, with an initial (wet) mass of 13 645 kg. The resulting transfer requires a velocity increment of about 2.4 km/s and a transfer time of 364 days for the outbound GTO–NRHO leg. The main data characterising this baseline trajectory are summarised in the table here presented.

Parameter	Value	Unit
Initial (wet) mass	13 645	kg
Cargo mass	5400	kg
Number of Hall thrusters	4	–
Thrust	1.05	N
Specific impulse, I_{sp}	2100	s
Power	18	kW
Δv (GTO $\rightarrow$ NRHO)	2.4	km/s
Transfer time (GTO $\rightarrow$ NRHO)	364	days
Propellant mass (GTO $\rightarrow$ NRHO)	1340	kg
Propellant mass (NRHO $\rightarrow$ GTO)	800	kg

Table 3.1: Baseline GTO–NRHO trajectory of the Space Tug (MAGNETO optimal configuration [3]).

It is precisely this fully-propelled trajectory that the present work takes as its point of departure, seeking to reduce the propellant expenditure and the time of flight while affording to raise the payload mass thanks to the ballistic phase not being affected by the spacecraft mass as said.

3.2 Selection of the orbit for manifold propagation

3.2.1 Numerical Computation

Whereas Chapter 2 introduced periodic orbits from a theoretical standpoint, the target orbit is here generated at the practical level by numerically integrating the CR3BP equations of motion from a known set of initial conditions over one orbital period. The initial conditions are not constructed analytically but retrieved from the NASA/JPL Three-Body Periodic Orbits database [15], which provides, for each family of periodic orbits about the libration points of the Earth–Moon system,

the synodic state vector, the orbital period, the Jacobi constant and the stability indices. Consistently, the mass parameter adopted throughout the propagation, $\mu = 1.215058560962404 \times 10^{-2}$, is taken from the same source so as to ensure the exact reproducibility of the cataloged orbit.

In the implementation, the state vector and the period are inputs provided directly to the code by the user; the orbit is then propagated over one full period through the numerical integrator, and the monodromy matrix, together with its eigenvalues, is obtained by integrating the state-transition matrix over the period, following the procedure described in Section 2.5. This step serves both to verify that the catalogued state does reproduce a closed periodic orbit and to characterise its stability through the Floquet multipliers, thereby confirming the mildly unstable nature of the NRHO that makes its invariant manifolds exploitable.

In order to select the final orbit that will represent the arrival point for the coasting phase, it is necessary to visualise and analyse the characteristics of the manifolds that each candidate orbit provides, since the choice of the orbit is dictated by concurrent considerations: the ease of operations in the cislunar domain and the proximity to the Lunar Gateway orbit, but, at the same time, the characteristics of the manifold itself. Such trajectories must therefore be propagated first; to perform all these computations, governed by the three-body dynamics, the codes provided by the thesis supervisor, S.Ross, were employed; these belong to a library of MATLAB codes and functions developed and validated over time by him.

The invariant manifold is constructed from the same monodromy matrix employed in the stability analysis. Within the routine `orbitman`, its stable and unstable eigenvectors are extracted; a point taken along the orbit is then displaced by a small offset $\varepsilon = 10^{-6}$, in non-dimensional units, along the stable eigendirection thereby providing the seed of the manifold. This seed is globalised by numerically integrating it through the routine `int`, which applies MATLAB's `ode45` scheme to the CR3BP equations of motion `rtbp`. Since the stable manifold tends asymptotically to the periodic orbit *forward* in time, its reconstruction necessarily requires a *backward* integration: the catalogued orbit state, which physically corresponds to the arrival (terminal) condition of the ballistic coast, is therefore used as the initial condition of a backward propagation that traces the trajectory back toward the Earth region. In the implementation this is enforced by setting the integration-direction flag to `FORWARD = -1`, which reverses the sign of the vector field in `rtbp`. By iterating this procedure for N points distributed along the orbit, each generating one manifold trajectory, the set of trajectories discretises the manifold surface into its characteristic tube-like structure 3.2.

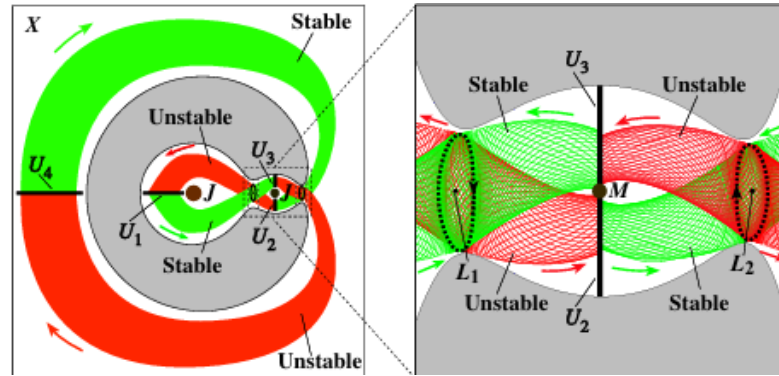


Figure 3.2: Lagrange point orbit invariant manifold tubes

3.2.2 Different families of orbits

The first choice was naturally to select an orbit coincident with the one officially adopted for the Lunar Gateway of the Artemis programme, whose data were taken from the literature [6], since this constitutes the target orbit that the Space Tug must reach in order to dock with the Lunar Gateway. For these reason from this point on the Lunar Gateway orbit will be addressed as "target orbit".

After examining the type of stable manifold generated by this orbit, by running both its positive and negative branches, and although it already yielded satisfactory results, it was decided to extend the investigation, in an attempt to identify a manifold that approached the Earth more closely or that featured a shorter coasting time.

A first idea that was explored consisted in selecting an L_1 NRHO tangent to the target orbit. Being an L_1 orbit, it is immediately apparent that it generates a far larger number of trajectories departing from the interior realm, which also reach the orbit more rapidly. It must indeed be considered that, for the NRHO of the Lunar Gateway, an L_2 orbit, a first analysis readily reveals a more substantial bundle of trajectories belonging to the exterior realm, while only a few of them develop within the interior realm. With an orbit about L_1 this proportion is reversed, the manifold trajectories developing predominantly within the interior realm.

Such an L_1 orbit therefore provides a far wider set of solutions, ranging from a manifold that approaches the Earth very closely, reaching $x = 0.213$ [non-dimensional units], with a coasting phase of about 100 days, to the opposite extreme, namely a trajectory that remains farther from the Earth, at $x = 0.743$, but requires only 26 days of travel along the manifold.

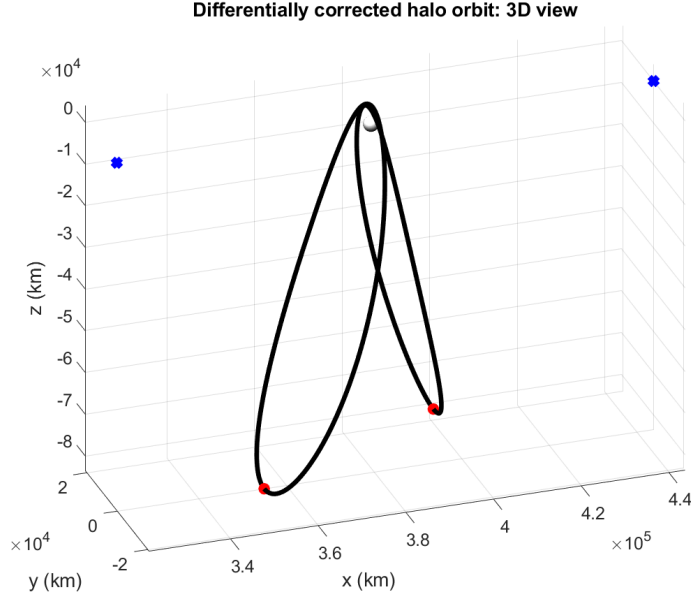


Figure 3.3: Plot representing the described orbit around L_1 (blue cross on the left) and its tangency with the target orbit around L_2 (blue cross on the right)

It would thus appear to be an excellent candidate, in view of its tangency with the target orbit. Yet, despite this spatial proximity, the transition from one orbit to the other would prove critical, since orbits about L_1 and L_2 possess opposite senses of revolution; at the point of tangency the state vector therefore exhibits opposite velocity vectors, resulting in a ΔV required for the orbit change that is excessive and unjustified. Indeed, it would not yield advantages sufficient to warrant its adoption, and it departs from one of aims of the present optimisation, namely the reduction of the propellant required for the trajectory.

Having thus discarded the idea of adopting an L_1 orbit, the possibility of selecting an L_2 orbit with a higher stability index was then assessed. Such a choice causes the trajectory to depart from (or approach) the orbit more rapidly; the outcome, however, revealed no improvement in the characteristics of the manifolds. Although some trajectories exhibited transfer times comparable to those of the target-orbit manifolds, the path they trace leads them into the exterior realm for a prolonged interval, only to subsequently cross the L_2 gateway and proceed through the interior realm toward the Earth.

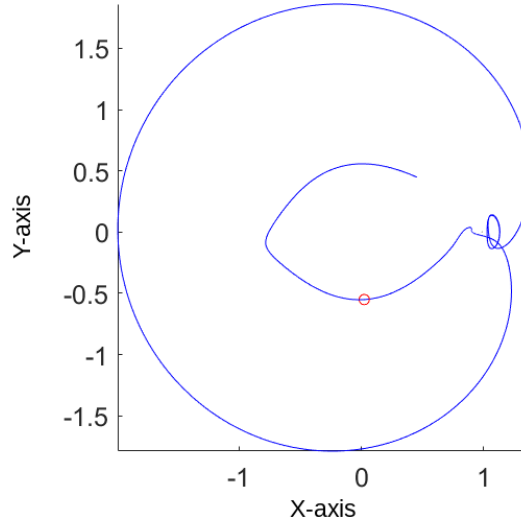


Figure 3.4: A single manifold trajectory belonging to an orbit with stability index=39 plotted in the rotating frame showing its path in the exterior and then interior realm. For a better understanding of the plot: Earth is approximately at (0,0) while Moon is at (1,0). For exact coordinates see 2.1.2 . The red circle indicates the point of closest approach to the Earth.

It was surmised that this time spent in the exterior realm would be neither efficient nor appealing in the eyes of potential customers of the Space Tug. Therefore, since here too the advantages were not such as to justify the resulting complications in the overall picture, it was decided not to investigate these orbits any further and to select the target orbit of the Lunar Gateway for the study of its manifolds.

3.2.3 Selected orbit

The analysis carried out in the previous section leads to the selection of the target orbit (which is the 9:2 synodic-resonant southern L_2 NRHO adopted for the Lunar Gateway) as the orbit whose manifold is exploited for this trajectory. Its defining parameters are collected below. The synodic state vector $\mathbf{x}_0$ and the orbital period T were retrieved from the NASA/JPL Three-Body Periodic Orbits database [15], whereas the orbit was originally identified, and its resonance and stability indices taken, from the work of Singh et al. [6]. In non-dimensional synodic coordinates,

centred at the Earth–Moon barycentre, the selected orbit is characterised by

$$\mathbf{x}_0 = \begin{bmatrix} 1.0238891897 \\ 0 \\ -1.8335389092 \times 10^{-1} \\ 0 \\ -1.0728895886 \times 10^{-1} \\ 0 \end{bmatrix}, \quad T = 1.5357215778, \quad C = 3.0446, \quad (3.1)$$

and by the two Floquet stability indices reported by Singh et al. [6],

$$\nu_1 = -1.3753, \quad \nu_2 = 0.6626. \quad (3.2)$$

Both indices lie close to unity, only ν_1 slightly exceeding it in magnitude: this confirms the *mildly unstable* character of the NRHO, which on the one hand keeps its station-keeping cost very low and, on the other, still endows the orbit with the invariant manifolds exploited throughout the remainder of this chapter.

3.3 Manifold trajectory and insertion point selection

By means of the procedure described the invariant manifolds of the selected orbit can be developed. It should be observed that a marginally different choice of the target orbit would have yielded manifolds that are different but similar for characteristics: the individual trajectories composing the tube would not be the same, and some of them might even prove slightly more performant, while remaining, on the whole, comparable to those of the adopted orbit. The manifolds would likewise be affected by a variation in the magnitude of the displacement offset ε ; this parameter, however, was deliberately kept fixed, for consistency with the reference literature [7]. For the sake of research completeness, however, an analysis of the selected orbit was also carried out for $\varepsilon = 10^{-5}$ and $\varepsilon = 10^{-7}$; as expected, this did not yield results of interest for the case studied here.

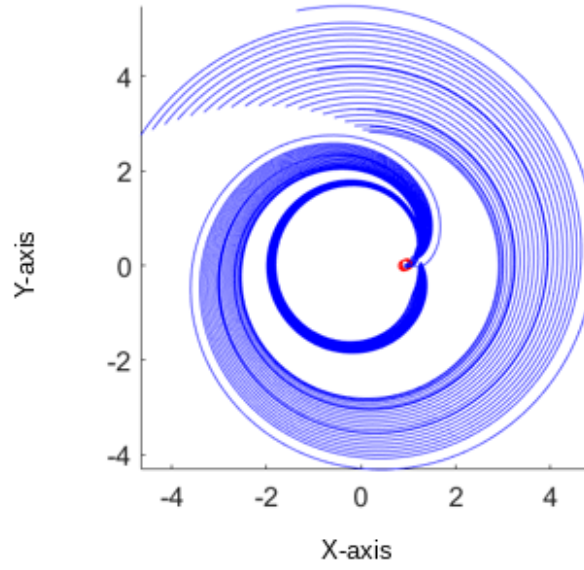


Figure 3.5: Plot of manifold generated by an offset $\epsilon = 10^{-7}$ in the rotating frame, all the trajectories develop in the exterior realm

A further methodological consideration concerns the number of base points sampled along the orbit. The analysis presented here employs 50 points; the same computation could be carried out with a larger number of points, at the cost of a considerable increase in the computation time of the code, which would render it markedly less practical. For reasons of practicality, a study of the manifolds departing from different candidate orbits was therefore first performed with 50 points and, once the target orbit had been selected, the analysis of its manifold was repeated with 1000 points for that orbit alone. It was found, however, that the best-performing and preferred trajectories did not vary appreciably, either in number or in their characteristics; the 50-point discretisation was consequently retained for all the subsequent analyses.

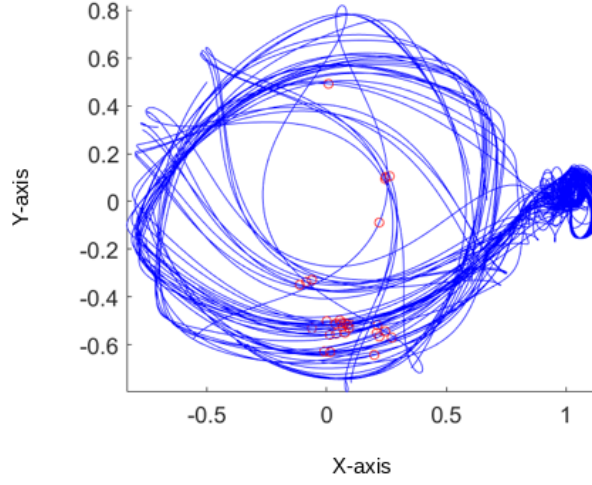


Figure 3.6: This is the result of the 1000 points discretization analysis, here are plotted only the 25 trajectories that develop in the interior realm, the red circles highlight the closest approach for each trajectory

3.3.1 Manifold trajectory

The selection of the trajectory to be followed and the insertion point, and hence of the state vector to be imposed as the arrival condition of the low-thrust phase, was carried out by means of dedicated MATLAB loops. Each discretised point was integrated backward in time, and a first screening was performed among those trajectories entering the interior realm, in particular below a threshold value of $[radius \leq 0.75 \text{ NDU}]$. It was immediately observed that only a small percentage of the trajectories belongs to the interior realm, whereas many of them, from the L_2 gateway, develop toward the exterior realm. The soundness of this analysis was validated through a comparison with the work of Singh et al. [6], whose results exhibit a very similar proportion between the trajectories and the direction they take as they depart from the Lunar Gateway orbit; the present analysis is therefore consistent with theirs as further reported in 4.6. This is best appreciated in a view given by the perspective of the x-axis, that highlights that only few trajectories develop toward the Earth.

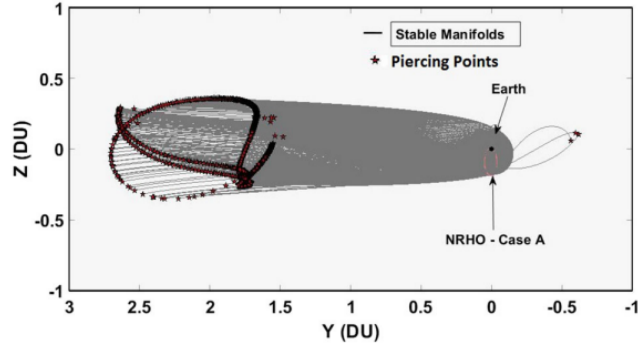


Figure 3.7: Plot shown by Singh et al. in [6] that proves the coherence of the analysis carried out with previous research

Among these few trajectories, a further selection was carried out by monitoring the distance from the Earth and, for each trajectory, selecting the point of minimum radius. From this point, the time required for the coasting phase was obtained by taking the time resulting from the backward integration up to that point and converting it from non-dimensional units to days by means of the following relation:

$$t_{\text{days}} = \frac{T_{\text{Moon}}}{2\pi} t_{\text{nd}} = \frac{27.3}{2\pi} t_{\text{nd}}, \quad (3.3)$$

Two limiting cases were then stored:

- the minimum time required for the coasting phase;
- the minimum radius, i.e. the point of closest approach to the Earth.

so that the code would immediately display the most favourable cases and allow the choice of which branch of the manifold to select. The positive branch was thus chosen, after it had been found that the negative one did not lead to equally satisfactory results. The positive branch presents only two trajectories entering below the prescribed threshold; consequently, while it had initially been intended to analyse possible trajectories with characteristics intermediate between the two limiting cases, this was no longer necessary. It should be noted that this analysis of the limiting cases was carried out for several orbits complying with the data reported in Section 3.2.3, and the orbit exhibiting the best cases, at least for the minimum-radius case, was subsequently selected, since it is recalled that one of the principal objectives is the reduction of the propellant required for the transfer, specifically thanks to the coasting phase.

Although a systematic and automated analysis of a larger number of orbits could prove useful in exploring possibly more convenient trajectories, this was not deemed necessary in the present work, as the manifolds computed for the selected orbits

did not lead to substantial differences. In any case, the purpose of the study is to demonstrate the feasibility of the project through this novel hybrid trajectory, together with its competitiveness in terms of performance; it was therefore considered sufficient to pursue the study with the data collected within the time allowed by the research.

The results obtained were thus two closely similar trajectories, of which the one featuring both the closest passage to the Earth and, at the same time, the least expensive insertion state vector was selected. The two trajectories are shown below, with the points of closest approach marked by a red circle:

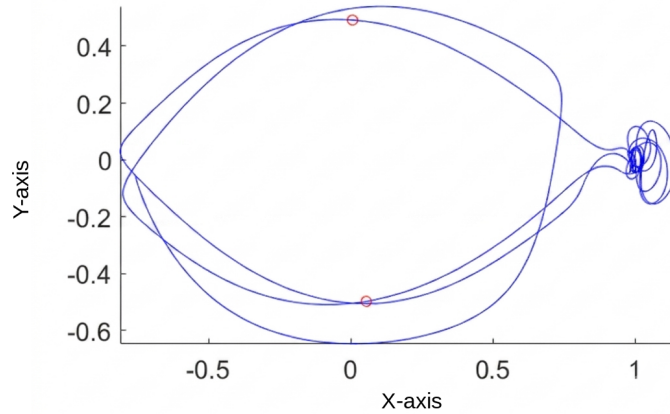


Figure 3.8: Shortlisted trajectories in the rotating frame.

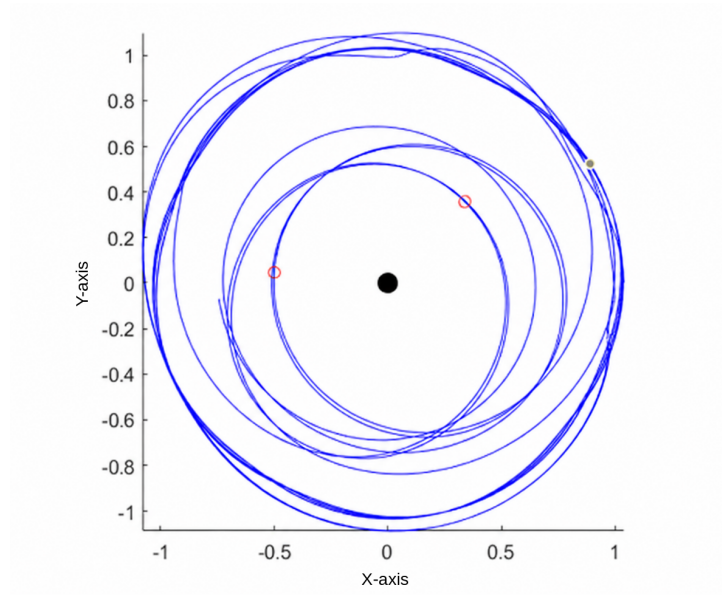


Figure 3.9: Shortlisted trajectories in the inertial frame.

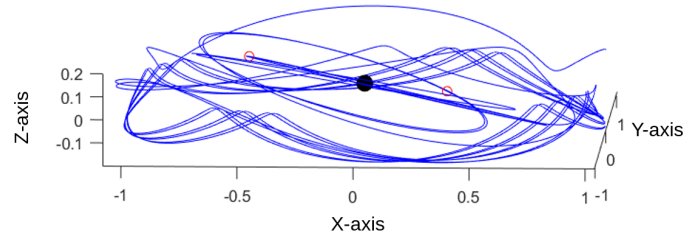


Figure 3.10: Shortlisted trajectories in the inertial frame; a different perspective that allows to appreciate the periodic motion along the z -axis.

The blue trajectory is seen to continue beyond the selected intersection point because the integration was computed over a seven-month time span, so that the evolution of the dynamics over this entire interval can be observed while the coasting phase from the insertion point forward only takes approximately 4 months.

3.3.2 Inclination

The inclination of the insertion point constitutes a further parameter taken into account in the selection of the manifold trajectory. Its relevance stems from the

fact that, being the arrival condition of the ballistic coast, it coincides with the inclination that the preceding low-thrust arc must attain (Section 3.5), since it was decided to keep the entire low-thrust phase planar, so as to reduce the propellant expenditure borne by the Space Tug and, consequently, the size of the fuel tank carried on board. The launch of the Space Tug itself, whether carrying the payload or not, lies outside the scope of the present work; what is relevant here is that, once the Tug is operational in orbit, the payload must be brought to the prescribed inclination, which is precisely that of the injection point.

The feasibility of this has been verified, and it was found that the payload mass can be delivered to the prescribed inclination and altitude by a currently operational launch vehicle. A range of masses was initially assessed, and this feasibility was subsequently confirmed once the payload mass had been fixed at 8.9 tonnes, in the light of the results that will be presented in the next chapter 4.

For the selected trajectory, the inclination of the insertion point with respect to the Earth-Centred Inertial (ECI) frame is 19° ; the transposition from the rotating frame to the inertial frame, and then to the ECI frame, will be explained in the following section.

A survey of the currently operational launch vehicles was therefore carried out. It must be recalled that the minimum inclination attainable at injection is bounded below by the latitude of the launch site, since the orbital plane must contain the launch point: a vehicle can only inject into inclinations greater than or equal to the latitude of its base. On this basis, the launchers operated by SpaceX had to be discarded, as their launch sites, Cape Canaveral and Kennedy Space Center (28.5°N), and Vandenberg (34.7°N), all lie at latitudes higher than the 19° required, so that a direct injection at the prescribed inclination is not achievable. The choice therefore fell upon the Ariane 64, operated from the Guiana Space Centre in Kourou (5.2°N): its low-latitude launch site allows direct injection at 19° , and its payload capacity is compatible with the mass to be delivered.

It should be emphasised that this consideration applies exclusively to the resupply phase, that is, to all the launches performed once the Space Tug is already operational in orbit and only the payload has to be delivered. The initial launch of the Space Tug itself lies outside of the present analysis.

3.4 Transposition of the Injection State to the Inertial Frame

The injection state selected on the manifold is expressed, at this stage, as a six-dimensional state vector in the barycentric synodic frame of the CR3BP. In order to serve as the target of the subsequent low-thrust phase, which is propagated in the Earth-Centred Inertial (ECI) frame, this state must be converted into a set of

orbital elements referred to that frame. In the code this is accomplished through a sequence of transformations, applied to the injection state in the following order.

3.4.1 From the synodic frame to the Earth-centred inertial frame

The synodic state $\mathbf{X}_{\text{syn}} = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^\top$ is referred to the larger primary, the Earth (located at $(-\mu, 0, 0)$ in the rotating frame), and translated into a non-rotating, Earth-centred frame $\mathcal{I}$ by the routine `rot2iner`. Denoting by $\theta = t + \delta$ the rotation angle, with t the non-dimensional time of the injection point and δ an offset, and recalling that the non-dimensional angular velocity of the synodic frame is unity, the transformation implemented in `rot2iner` reads

$$\mathbf{r}_{\mathcal{I}} = R_z(\theta) \begin{bmatrix} x + \mu \\ y \\ z \end{bmatrix}, \quad \mathbf{v}_{\mathcal{I}} = R_z(\theta) \begin{bmatrix} \dot{x} - y \\ \dot{y} + x + \mu \\ \dot{z} \end{bmatrix}, \quad R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (3.4)$$

The shift by μ refers the position to the Earth, while the terms $-y$ and $+x + \mu$ in the velocity account both for the rotation of the frame and for the motion of the Earth about the barycentre.

3.4.2 Redimensionalisation

The state $(\mathbf{r}_{\mathcal{I}}, \mathbf{v}_{\mathcal{I}})$ is non-dimensional. It is restored to physical units by the routine `dimensionalize` which, denoting by $a = 384\,748$ km the Earth–Moon distance and by $GM = G(m_1 + m_2)$ the total gravitational parameter of the two primaries, scales position, velocity and time as

$$\mathbf{r}_{\mathcal{I}}^{\text{dim}} = a \mathbf{r}_{\mathcal{I}}, \quad \mathbf{v}_{\mathcal{I}}^{\text{dim}} = \frac{a}{n} \mathbf{v}_{\mathcal{I}}, \quad t^{\text{dim}} = n t, \quad n = \sqrt{\frac{a^3}{GM}}. \quad (3.5)$$

The resulting position, velocity and time are expressed in metres, metres per second and seconds, respectively.

3.4.3 From the ecliptic to the equatorial ECI frame

The fundamental plane of the frame $\mathcal{I}$ is approximated with the ecliptic plane, whereas the low-thrust propagator requires the equatorial ECI frame. In the code, the two frames are related through a rotation about the vernal-equinox (x) axis by the mean obliquity of the ecliptic $\varepsilon = 23^\circ 26' 21.412''$. The equatorial-to-ecliptic

transformation is defined by the rotation matrix

$$\mathbf{eci2eclip} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon & \sin \varepsilon \\ 0 & -\sin \varepsilon & \cos \varepsilon \end{bmatrix}, \quad (3.6)$$

and the transformation from the ecliptic to the ECI frame is obtained as its transpose,

$$\mathbf{eclip2eci} = \mathbf{eci2eclip}^\top = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon & -\sin \varepsilon \\ 0 & \sin \varepsilon & \cos \varepsilon \end{bmatrix}. \quad (3.7)$$

This matrix is applied separately to the position and to the velocity vector resulting from the previous step, yielding the state expressed in the equatorial ECI frame,

$$\mathbf{r}_{\text{ECI}} = \mathbf{eclip2eci} \mathbf{r}_{\mathcal{I}}^{\text{dim}}, \quad \mathbf{v}_{\text{ECI}} = \mathbf{eclip2eci} \mathbf{v}_{\mathcal{I}}^{\text{dim}}. \quad (3.8)$$

3.4.4 Recovery of the orbital elements

From the resulting ECI Cartesian state, the classical orbital elements $(a, e, i, \Omega, \omega, \nu)$ are obtained through the matlab function `ijk2keplerian`. The elements so obtained constitute the target of the low-thrust phase: the semi-major axis a , the eccentricity e and the argument of perigee ω are retained as the terminal conditions that the low-thrust spiral must attain (Section 3.5), while the inclination i is adopted as the initial inclination of the departure orbit, in accordance with the planar low-thrust assumption introduced in Section 3.3.2.

3.5 The Low-Thrust Phase

As already discussed, the low-thrust arc of the transfer was deliberately kept adherent to the trajectory previously devised in the MAGNETO study, so as to concentrate the contribution of the present work on the second portion of the transfer, the ballistic coast along the invariant manifolds, where the proposed innovation is introduced. The low-thrust computation is therefore based on the code developed within the MAGNETO project [3], provided by the thesis supervisor, Prof. Nicole Viola.

Since the present study addresses an early design phase, aimed at assessing the feasibility of the proposed hybrid trajectory rather than at optimising its low-thrust arc, and since the bulk of the research was devoted to the invariant manifolds and to the dynamics of the three-body problem, a number of significant simplifications were introduced in the low-thrust model, as detailed in the following.

3.5.1 Dynamical model and integration

The powered arc is propagated in modified equinoctial elements (MEE). As reported in the code, this element set is useful for trajectory analysis and optimisation, is valid for circular, elliptic and hyperbolic orbits, and, unlike the classical orbital elements, exhibits no singularity for zero eccentricity and for orbital inclinations of 0° and 90° .

The state is the vector $\mathbf{y} = [p, f, g, h, k, L]^\top$ [16], whose components are related to the classical orbital elements, semi-major axis a , eccentricity e , inclination i , right ascension of the ascending node Ω , argument of perigee ω and true anomaly ν , by

- $p = a(1 - e^2)$, the semilatus rectum of the orbit;
- $f = e \cos(\omega + \Omega)$ and $g = e \sin(\omega + \Omega)$, the two components of the eccentricity vector expressed in the equinoctial frame, which together carry the eccentricity and the orientation of the line of apsides;
- $h = \tan(i/2) \cos \Omega$ and $k = \tan(i/2) \sin \Omega$, which carry the inclination and the orientation of the line of nodes;
- $L = \Omega + \omega + \nu$, the true longitude.

The evolution of $\mathbf{y}$ is governed by the equinoctial equations of motion in Gauss form, in which the perturbing acceleration enters through its radial, tangential and normal components.

The equations of motion are integrated in time by means of a Runge–Kutta–Fehlberg 7(8) scheme with adaptive step size: at each step the local truncation error is estimated against a prescribed tolerance, and the step is accordingly accepted or rejected, the step size being updated on the basis of the estimated error.

3.5.2 Perturbations

In its original form, the MAGNETO code accounts for several perturbing accelerations acting on the spacecraft, namely the Earth oblateness through the J_2 zonal harmonic, the third-body attraction of the Moon and of the Sun, the solar-radiation pressure and the atmospheric drag. Consistently with the simplified, feasibility-oriented character of the present analysis, none of these perturbations is retained: in the equations of motion they are all deactivated, so that the only acceleration acting on the spacecraft, besides the central two-body gravitational attraction, is the one produced by the thrust. Moreover, in order to account for the perturbations of the Moon and of the Sun, the implementation of an ephemeris model would have been required; this was deliberately not undertaken, although it

would also have been useful in order to determine the precise epoch of departure of the low-thrust phase and of arrival at the injection point. This option was discarded, as it was deemed to be a model that could be implemented at a later stage and whose effects would not have led to substantial variations.

3.5.3 Thrust strategy

The thrust strategy originally implemented in the MAGNETO code follows a SMART-1-like law, described in Section 3.1. In the present work this law is not adopted: the engine is instead operated with continuous thrust and is never switched off.

The thrust direction is determined by a weighted, locally-optimal steering law. For each controlled orbital element an optimal in-plane and out-of-plane orientation is defined, and the commanded thrust direction is obtained as their superposition, weighted by coefficients measuring the residual distance of each element from its target value. Each weight is defined as

$$k_\chi = \frac{\chi_f - \chi}{|\chi_f - \chi_i|}, \quad (3.9)$$

with χ_f the target value of the element, χ its current (osculating) value and χ_i its initial value, so that the weight vanishes as the element approaches its target. In the adopted configuration only the semi-major axis a and the eccentricity e are actively controlled, the eccentricity being weighted only below a prescribed semi-major-axis threshold, whereas the argument of perigee, the inclination and the right ascension of the ascending node are not, their weights are set to zero. The out-of-plane component of the thrust is also set to zero, so that the resulting steering law is purely in-plane accordingly to the strategy explained previously; the powered arc therefore drives the semi-major axis toward its target value, an operation referred to, in the code, as *orbit raising*, while adjusting the eccentricity, without performing any plane change.

The inconsistencies arising between this powered arc and the subsequent ballistic coast are addressed in the following section.

3.6 Matching of the Two Arcs at the Patch Point

In order to ensure the physical consistency of the transfer, continuity must be guaranteed between the low-thrust arc and the manifold coast at the injection point, that is, at the state vector where the former ends and the latter begins. In the present implementation this continuity is ensured by matching the orbital elements: the adopted strategy includes, at each step of the low-thrust integration cycle, a check on the current orbital elements compared to their target values.

It was not necessary, however, to perform this check on all the orbital elements. Owing to the way the mission, and, in particular, the low-thrust arc, has been set up, the first leg remains planar. Given this planarity, and therefore the absence of out-of-plane thrust (it is recalled that the perturbations which would tend to modify the orbital plane are neglected at this stage), the right ascension of the ascending node (RAAN, Ω) and the inclination (i) are not controlled: the departure orbit is indeed assumed to start from the GTO with $\Omega_i = \Omega_f$ and $i_i = i_f$, so that no control on these two elements is required. This is not an approximation but a design choice: i and Ω therefore do not need to be enforced again at the patch point, since they are preserved by construction. A further orbital element that is not considered is the true anomaly ν , since the phasing problem has been deferred to a subsequent analysis.

The elements that are checked at each iteration of the integration are therefore the semi-major axis a , the eccentricity e and the argument of perigee ω ; as soon as these fall within the prescribed tolerances in the same iteration, the integration is stopped and the code returns the time and the propellant expenditure of the powered phase. The tolerances have been kept relatively loose, consistently with the level of detail of the design phase:

- $|a - a_f| \leq 5 \text{ km}$;
- $|e - e_f| \leq 0.05$;
- $|\omega - \omega_f| \leq 0.1 \text{ rad}$.

As regards the values of the initial orbital elements, the departure orbit of the previous Space Tug study was adopted [3].

Two quantities remain unmatched. The first is the true anomaly ν : as already noted, the powered arc is designed to reach the target orbit, but not necessarily the insertion point along that orbit at the exact epoch. The second, and related, is indeed the epoch at which the two arcs connect, which has not been solved for, since no ephemeris model was employed; as discussed in Section 3.5, this is regarded as an aspect to be addressed in a future, more detailed study rather than in the present feasibility analysis. Together, the true anomaly ν and the epoch represent the remaining timing mismatch at the patch point.

Another inconsistency lies between the two dynamical models used for the transition and propagation of the low-thrust arc, i.e., two-body dynamics are used to propagate the low-thrust arc, while the coast is developed in the CR3BP. This is a simplification and a conceptual leap in the force model, rather than a physical or spatial discontinuity in the trajectory; it also stems from the conceptual manner in which the two portions of the trajectory have been developed. On the one hand, the transfer begins from the terrestrial orbit, where the Earth's gravity is dominant, and spirals up; on the other, the three-body dynamics are used to

propagate toward the Earth until the meeting point of the two phases is reached. This simplification influences the performance parameters of the low-thrust phase but not the geometric continuity of the path. It was thus deemed acceptable this early in the design.

Chapter 4

Results

4.1 Sizing of the Crew Provisions

The tools required to compute the definitive trajectory are now available; however, a decisive element of the mission design is still missing, namely the payload mass and, consequently, the mass of the spacecraft. Precisely because of this impact on the results, and owing to the thematic distance from the topics addressed in Chapters 2 and 3, it was decided to include here the following sections, which set out the rationale leading to the conceptual mission-design decision that was adopted.

The food resupply that the Lunar Gateway must necessarily receive has been selected as the primary payload. Such deliveries occur on a periodic basis and therefore align well with the concept of a fleet of space tugs devoted to repeated resupply flights. In principle, the Space Tug could also carry other types of cargo, such as scientific experiments, or the materials and equipment needed to build or expand the Lunar Gateway itself or the future lunar surface base; these too would constitute payloads well suited to the long flight times and the prolonged exposure to radiation that characterize this trajectory.

Food resupply has been chosen precisely for the aforementioned reasons of cadence and regularity of the deliveries. This does not preclude, however, that in a broader future implementation the Lunar Space Tug (LST) may be operated in parallel with other missions, such as those required to transport the crew, so that the payload may vary according to the operational needs and cargo distribution among different types of missions.

For the present purpose, this work focuses on the food resupply, which constitutes the dimensioning payload. It is therefore necessary to determine the mass of consumables needed to sustain the crew between two successive resupply flights.

Two independent questions must therefore be addressed:

- how much provisioning mass is required, as a function of the crew complement, the mission duration and the delivery cadence
- whether such provisions remain acceptable after a transfer lasting several months in the cislunar radiation environment.

The former sets a reference value for the payload; the latter establishes whether the long transfer time inherent to a low-thrust, manifold-assisted trajectory is compatible with the storability of the delivered goods.

4.1.1 Crew Complement and Mission Duration

The reference crew complement is taken from the baseline Gateway architecture. Early configurations of the station, with the Orion vehicle docked, are designed to accommodate up to four astronauts for mission durations of 30 to 90 days [17]. Unlike the International Space Station (ISS), the Gateway is not intended to be permanently crewed, but rather inhabited at least once a year, hosting crews of four for stays of up to three months at a time [18]. Consistently with the sizing tool adopted in the reference low-thrust study [3], the crew-permanence scenarios considered for the Lunar Orbital Platform–Gateway span 30, 60, 90 and 180 days.

In the present work the crew complement is fixed at its upper bound,

$$N_{\text{crew}} = 4, \quad (4.1)$$

which provides a conservative envelope for the provisioning demand. It is noted that the 180-day permanence corresponds to a prospective, extended-duration scenario, this is coherent with the goal of this work which is intended for the future operational time of the Lunar Gateway. The resupply interval is therefore retained as a free parameter in the sizing model of Section 4.1.2.

4.1.2 Consumable Demand and Provisioning-Mass Model

The per-crew consumable demand is taken from the reference sizing model [3], in which the daily requirement is assumed in accordance with ISS practice and set to

$$\dot{m} = 12.37 \text{ kg person}^{-1} \text{ day}^{-1}. \quad (4.2)$$

This aggregate figure encompasses the metabolic consumables together with the associated hygiene and packaging masses. Its order of magnitude is consistent with independently reported ISS operational data.

The provisioning mass to be delivered per resupply flight follows from a first-order material balance,

$$m_{\text{prov}} = N_{\text{crew}} \dot{m} \Delta t_{\text{resupply}}, \quad (4.3)$$

where $\Delta t_{\text{resupply}}$ is the time interval that a single delivery is required to cover, i.e. the reciprocal of the delivery frequency. Evaluating Equation (4.3) for $N_{\text{crew}} = 4$ and $\dot{m} = 12.37 \text{ kg person}^{-1} \text{ day}^{-1}$ yields the reference values reported in Table 4.1.

Table 4.1: Reference provisioning mass as a function of the resupply interval, for a four-person crew.

$\Delta t_{\text{resupply}}$ [days]	m_{prov} [kg]
30	$\approx 1.48 \times 10^3$
60	$\approx 2.97 \times 10^3$
90	$\approx 4.45 \times 10^3$
180	$\approx 8.90 \times 10^3$

The operational provisioning mass is thus not a single fixed quantity but is determined once the resupply cadence and the crew-permanence scenario are selected; the values in Table 4.1 provide the data that allows to run analysis of the time of flight and fuel consumption as a function of the payload mass and from these extract the fleet dimensioning in Section 4.5.

4.1.3 Storability of the Provisions over the Transfer

A low-thrust transfer assisted by invariant manifolds entails a transfer time of the order of several months (quantified in section 3.5). The provisions must therefore remain safe, palatable and nutritionally acceptable not only during this transit but also throughout the subsequent on-station storage until consumption, i.e. over a total interval of the order of one year. This requirement must be assessed against both the intrinsic shelf life of the space-food system and the effect of the cislunar radiation environment.

Regarding shelf life, the current space-food system already meets this requirement with margin. Foods qualified for the ISS are required to possess a shelf life of at least one year, and in practice a shelf life of two to three years is targeted, since the food may be stored for up to a year before launch [19]; thermostabilisation alone extends the shelf life to about three years, while food systems for planetary exploration are being developed to remain stable for up to five years [19]. A transfer of the order of eight months, followed by on-station storage, therefore lies well within the demonstrated envelope. This is corroborated by in-orbit data: in a long-duration experiment on the ISS, low-moisture food items maintained in orbit for 880 days showed no significant decrease in the thirty nutrients measured [20].

Regarding the radiation environment, the cislunar transfer occurs largely outside the protection of the terrestrial magnetosphere, so that the provisions are exposed to galactic cosmic rays and solar radiation at higher levels and with a different

spectrum than in low-Earth orbit, and the specific effect of this exposure on food is not yet fully characterised [21]. The available evidence nonetheless indicates that radiation is not the limiting factor over sub-year durations: in a ground simulation of the galactic-cosmic-ray environment, thermostabilised foods rich in radiation-labile nutrients were irradiated at doses of approximately 0.5 and 5 Gy and exhibited no nutritional differences with respect to the controls, either immediately or after one year of storage [21]. Conversely, the literature is not unanimous: a study of neutron secondary radiation in a deep-space-like environment reported the potential activation of meat and oxidative modification of proteins and lipids at absorbed doses between 0.01 and 4 Gy [22].

On balance, the transfer time associated with the proposed trajectory is compatible with the storability of the delivered provisions, which therefore does not constitute a feasibility constraint for the mission.

4.1.4 Mission-Design Rationale

Since the storability of the provisions does not constitute a feasibility constraint for the mission, it was decided to opt for a larger payload, and in particular for the 180-day food resupply accordingly to what will be discussed in section 4.5. This choice takes advantage of the fact that half of the trajectory exhibits no mass-dependent parameters as specified in the introduction of Chapter 3, so that an increase in the delivered mass does not affect the performance of the trajectory along the coasting arc. The mass increase must nonetheless remain compatible with the thrust available over the first, propelled arc and, in particular, with the spacecraft architecture devised in [3]; it is recalled that this architecture is retained unchanged.

A reference supply mass of approximately 8.9×10^3 kg (180 days), corresponding to the value reported in Table 4.1, was therefore adopted, while retaining the possibility of adjusting it slightly in order to better meet the delivery-frequency requirements of the fleet.

4.2 Low-Thrust Phase Results

4.2.1 Thrust Arc Validation

As stated in Section 3.5, the SMART-1-like strategy was not adopted; instead, a simplified strategy governed by the thrust weights was employed throughout the low-thrust arc. In the MAGNETO study [3] the trajectory was validated by means of GMAT, whereas in the present work the thrust phase and this slightly different strategy were validated using Ansys STK with the Astrogator propagator, in accordance with the software licences and the expertise available to the researcher.

The validation of the method was regarded as a key step to be carried out prior to the development of the trajectory, since otherwise any result obtained would have been affected by uncertainty.

As this was undertaken at an early stage of the study, when the boundary conditions of the definitive trajectory were not yet known, it was decided to validate the correctness of the method by means of a representative test mission reproducing the characteristics that were expected to occur during the low-thrust phase of the actual mission. This trajectory was initialised from a circular orbit at an altitude equal to that of a geostationary orbit, and a spiralling thrust was applied until the target values of eccentricity and semi-major axis were reached without applying any change to the orbital plane, exactly as envisaged and as effectively described in the definitive strategy of Section 3.5.

After recreating the specifics of the Space Tug on Ansys STK an equal continuous thrust of 1.025 Newton provided by 4 Hall Effect Thrusters (hence for a total thrust of 4.1 Newtons) with an $I_{sp} = 2600$ s and considering an indicative payload of 8000 kg for a total mass of 16000 kg. The trajectory has been propagated both on MATLAB and Ansys STK giving the comparable results shown in table 4.2

Table 4.2: Comparison of the low-thrust phase results between the developed code and STK/Astrogator.

Quantity	Developed methodology	STK/Astrogator	Variation [%]
Propellant mass [kg]	1228.2	1200	+2.35
Transfer time [days]	88.4	87.9	+0.57

In addition to the agreement in the final values, the two trajectories also exhibit a close visual correspondence, as shown in Figure 4.1, which provides a further qualitative confirmation of the consistency between the two methods. It is recalled that all the parameters that are not modified during the propagation of the thrust phase are kept identical for the two methods employed.

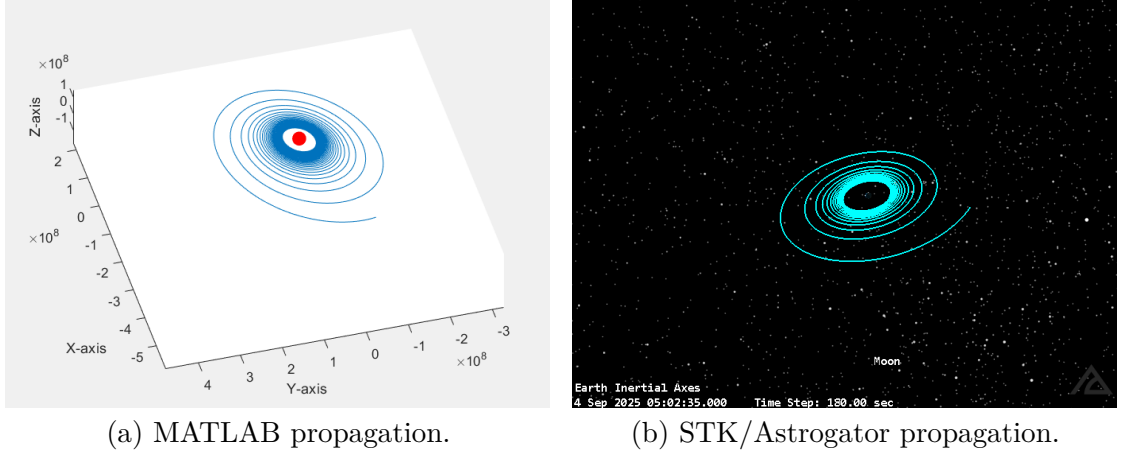


Figure 4.1: Comparison between the propagated low-thrust spiral trajectories.

4.2.2 Low-Thrust Arc

This section presents the definitive spiral of the first trajectory phase, namely the low-thrust arc. It is presented here for consistency with the sequence of events experienced by the spacecraft along the trajectory; however, it could be determined in its final form only after obtaining the target state vector to be reached, provided by the results of Section 4.3, and converting it into Keplerian elements according to the procedure described in Section 3.4. The state vector that constitutes the target of the low-thrust weighted control is therefore

$$\mathbf{x}_f = \begin{bmatrix} 0.054665 \\ -0.49754 \\ 0.10648 \\ 0.99299 \\ 0.16475 \\ 0.256 \end{bmatrix}, \quad T = -26.941, \quad (4.4)$$

where the components of $\mathbf{x}_f$ are the non-dimensional position and velocity coordinates in the rotating frame and T is the associated non-dimensional time. It should be noted that the time T carries a negative sign because the integration was performed backwards; this quantity will be employed in future studies to recover the exact instant at which the manifold must be entered by the spacecraft.

Once converted into Keplerian elements, the target orbital parameters are those reported in Table 4.3.

To reach this target point, reference was made to the possible configurations outlined in [3]; after applying such configurations to the present mission, the four-thruster, 20 kW configuration was selected, whose parameters are reported

Table 4.3: Target Keplerian elements of the low-thrust arc.

Element	Symbol	Value
Semi-major axis	a_f	2.5184×10^5 km
Eccentricity	e_f	0.2165
Inclination	i_f	19.294°
RAAN	Ω_f	41.527°
Argument of perigee	ω_f	132.07°
True anomaly	ν_f	273.56°

in Table 4.4. As regards the final selected payload mass, and consequently the required propellant mass, an iterative analysis was carried out between the fleet-sizing section and the thrust section, leading to the choice of a 180-day resupply, hence a cargo of 8900 kg.

In order to propagate the low-thrust spiral appropriately, however, the initial mass of the spacecraft must be known, which must include the dry mass, the outbound propellant mass, the inbound propellant mass and the payload. The required propellant masses were determined iteratively by means of the code and are reported in Tables 4.8 and 4.9, respectively. This leads to an initial mass of the Space Tug given by

$$m_0 = \underbrace{5900}_{\text{dry}} + \underbrace{8900}_{\text{payload}} + \underbrace{1429}_{\text{outbound prop.}} + \underbrace{446.3}_{\text{inbound prop.}} = 16675 \text{ kg}, \quad (4.5)$$

which, at this preliminary design stage, is rounded to 16700 kg.

Table 4.4: Selected propulsion configuration.

Parameter	Value
Number of thrusters	4
Power	20 kW
Specific impulse, I_{sp}	2600 s
Thrust (per thruster)	1.025 N

Taking this as the target of the thrust, which in turn determines the control weights, the spacecraft is propelled until the target values are attained, as described in Section 3.6.

The trajectory thus departs from an sGTO whose parameters were taken from the previous studies [3] and from the provided code; they are reported in Table 4.5.

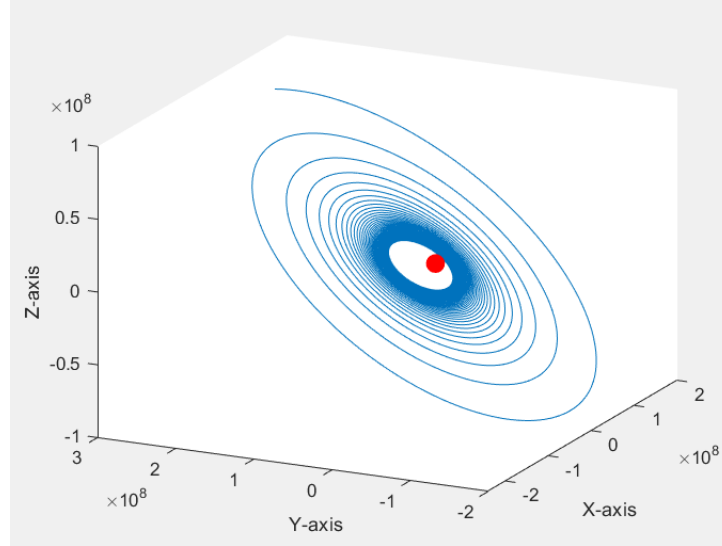
These boundary conditions, together with the weight-controlled thrust law of Equation (3.9), led to the trajectory shown in Figure 4.2.

Table 4.5: Initial sGTO orbital parameters.

Element	Symbol	Value
Semi-major axis	a_i	42165 km
Eccentricity	e_i	0.5885
Inclination	$i_i = i_f$	19.294°
RAAN	$\Omega_i = \Omega_f$	41.527°
Argument of perigee	ω_i	143.24°

Under the continuous thrust of the propelled arc, the spacecraft spirals outward, progressively raising the semi-major axis and reshaping the eccentricity until the target osculating values of a , e and ω are attained.

Throughout the arc the orbital inclination and the right ascension of the ascending node are held fixed at $i_i = i_f = 19^\circ$ and $\Omega_i = \Omega_f = 41.527^\circ$, respectively. Consistently with the planar control strategy described in Section 3.5, no out-of-plane thrust component is applied, so that the transfer develops entirely within the initial orbital plane, and neither the inclination nor the right ascension of the ascending node is a controlled or a targeted quantity. The propagation results in the spiralling trajectory depicted in figures 4.2 and 4.3

**Figure 4.2:** Planar Low-thrust arc, side view

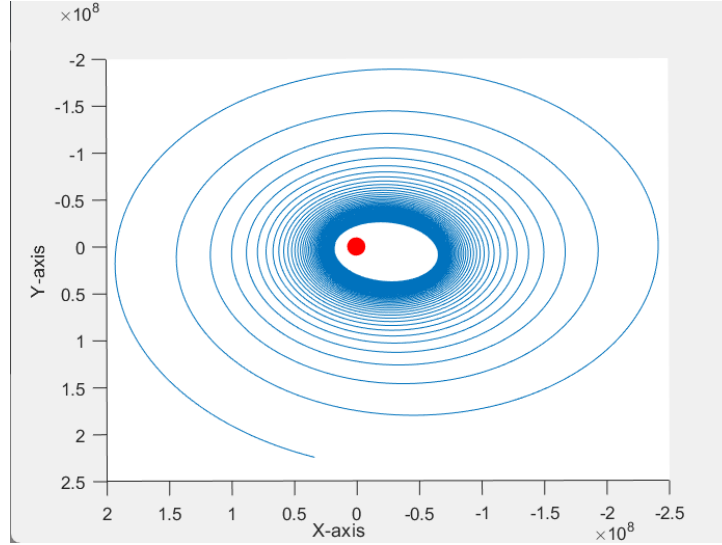


Figure 4.3: Planar Low-thrust arc, top view

The output values returned by this phase, which contribute to the construction of the complete trajectory and of the fleet in the following sections, are reported in Table 4.6.

Table 4.6: Output values of the low-thrust phase.

Quantity	Value
Time of flight	102 days
Propellant mass	1429.1 kg
Δv	2.27 km/s

It can be observed here that the Δv exhibits a value similar to the one adopted by Rimani et al. [3] for the outbound leg (while this work however is delivering an higher payload mass), and higher than the one presented in [6]. It must be borne in mind, however, that the optimisation present in [3] has been removed in favour of a continuous and simpler thrust strategy, which nonetheless leads to a considerably lower time of flight; the corresponding comparisons are presented in Section 4.6. Furthermore, the improvements have thereby been concentrated on the second part of the trajectory. This low-thrust arc is therefore still amenable to further improvement and to a reduction in propellant consumption, should a different outline of the mission and of the fleet be adopted. For the present study, this was regarded as the optimal compromise between the transfer time and the required propellant mass.

4.3 Manifold Coasting Output

As explained in Section 3.2.3, an orbit was chosen so as it provides a stable manifold trajectory approaching the Earth as closely as possible, thereby reducing the propellant consumption. Once the orbit reported in Section 3.2.3 had been selected, between the two available trajectories (Figure 3.8) the one exhibiting the minimum radius from the Earth was selected, for a comparable convenience of the orbital plane to which the state vector of the intersection point belongs.

This criterion of minimising the distance from the Earth, and hence the required propellant, was central to the choice of the coasting trajectory, but not to the low-thrust propulsion strategy; this distinction is discussed in Section 4.6.

In principle, a different insertion point along the coasting arc could also be selected; however, this would give rise to a further optimisation problem, which may be addressed in future work. For the present study, the intersection point, also referred to as the piercing point in [6], was chosen by taking the closest approach to Earth along the trajectory, propagating the trajectory backwards up to 7 months. In the following figures the piercing point is indicated by the red circle, and its state vector was presented in the previous Section 4.2.2, displayed in 4.4 together with its transposition into orbital parameters in 4.3.

Here the time reported in (4.4), expressed in non-dimensional units, is used to determine how long the spacecraft will take to reach the orbit asymptotically; here it has been converted into days according to the procedure described in equation (3.3) and reported in table 4.7, the

Table 4.7: Manifold coasting output.

Quantity	Value
Time of flight	117.1 days
Δv	0

It should be noted that the null Δv of the coasting phase does not count for any station-keeping manoeuvres to counteract the perturbations that might drive the spacecraft off its route, as these are negligible in this phase, nor any thrusting manoeuvres in the proximity of the target orbit.

The following figures present the selected trajectory belonging to the stable positive manifold, shown from several perspectives and plotted both in the rotating frame and in the inertial frame.

For all these figures, the red circle denotes the insertion point. In the rotating-frame plots, the Earth is located at $(-\mu, 0, 0)$ and the Moon at $(1 - \mu, 0, 0)$, that is, approximately at $(0, 0, 0)$ and $(1, 0, 0)$ respectively (being $\mu \approx 0.01215$ for the

Earth–Moon system), so as to provide clearer references to read the graphs. It should be recalled that, in the inertial frame, the two primaries are not fixed but revolve about the common barycenter, and therefore do not occupy constant coordinates in the corresponding plots.

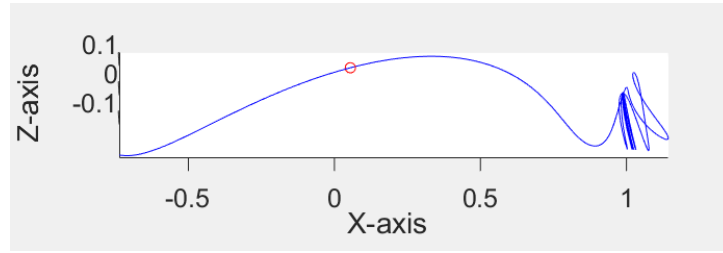


Figure 4.4: Trajectory plotted in the rotating frame

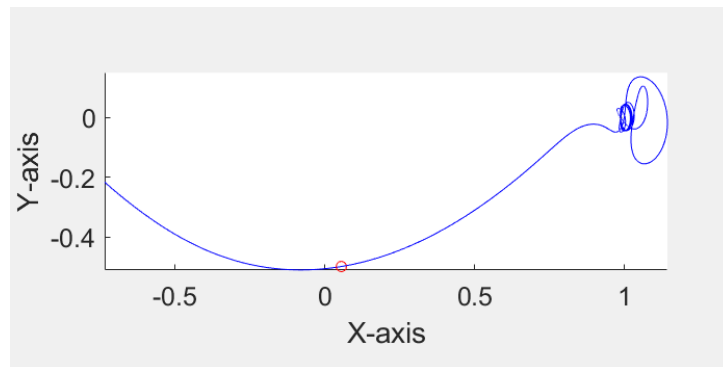


Figure 4.5: Trajectory plotted in the rotating frame, top view aligned with z -axis

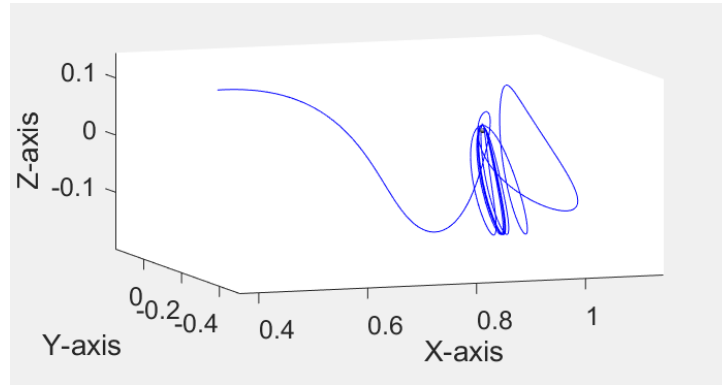


Figure 4.6: Trajectory plotted in the rotating frame, zoom on the cislunar behaviour

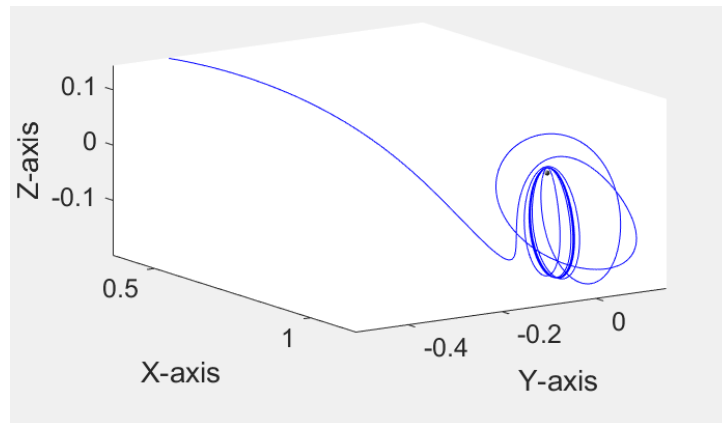


Figure 4.7: Trajectory plotted in the rotating frame, zoom in to highlight the similarity to figure 2.4(a)

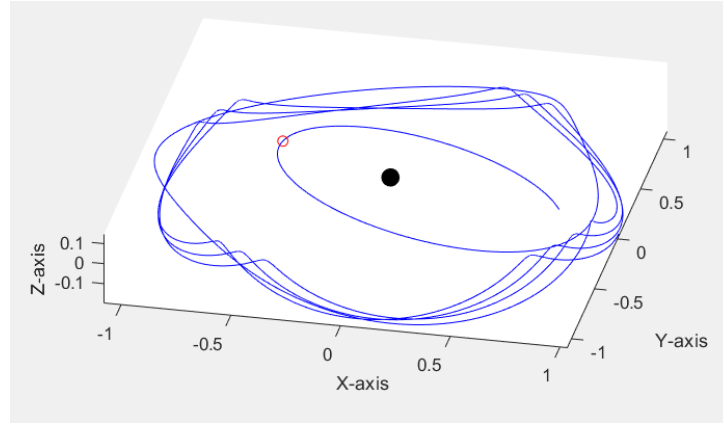


Figure 4.8: Trajectory plotted in the inertial frame

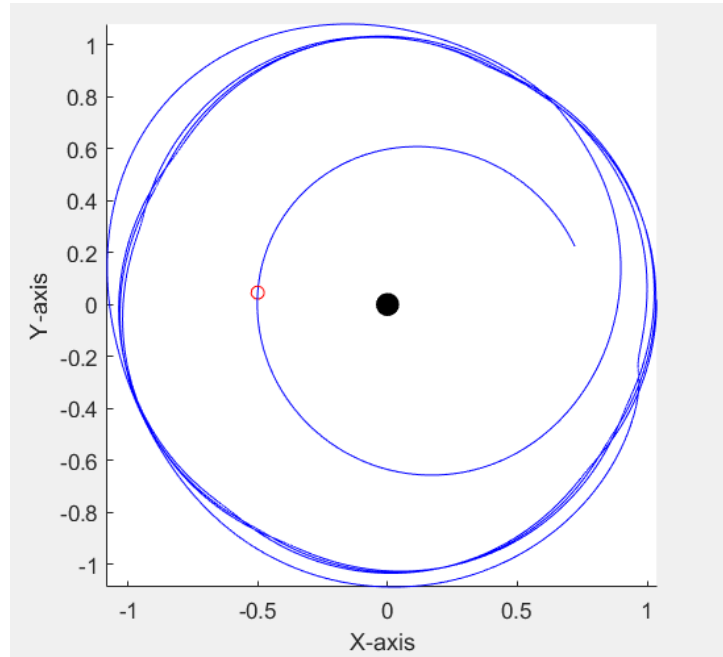


Figure 4.9: Trajectory plotted in the inertial frame, top view aligned with z -axis

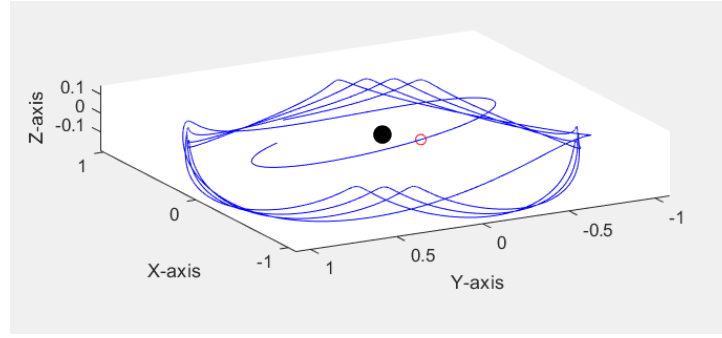


Figure 4.10: Trajectory plotted in the inertial frame, different perspective

4.4 Complete Trajectory and Return Leg

This section presents the complete trajectory, from the sGTO up to the 9:2 southern L_2 NRHO, with the intersection at the insertion point represented in the inertial frame.

Having obtained the data pertaining to the complete outbound leg, before the fleet can be organised it is required to obtain the data of the return trajectory as well. The entire problem was in fact solved iteratively, since the propellant mass of the return leg must be accounted for within the initial mass: by design, no refuelling is foreseen in cislunar space, but only in Earth orbit. The values of the return leg are therefore presented in this paragraph as well.

4.4.1 Matching at insertion point and plot in the inertial frame

To obtain the trajectory in its continuous form the intersection at the insertion point must be addressed. For such matter it was necessary to convert the state vector belonging to the rotating frame into Keplerian elements, following the method described in Section 3.4. Moreover, this meeting point is not merely physical and spatial, but also conceptual, connecting the different assumptions adopted in the analysis of the two legs, as reasoned in Section 3.6.

As demonstrated in Figures 4.11 and 4.12, which depict the two trajectories in a shared inertial reference frame, the manifold's trajectory intersects with the final orbit, wherein the space tug is situated at the culmination of the propulsion phase. It is however apparent that the intersection in the insertion point, highlighted with a red circle along the blue trajectory, does not occur at the very end of the thrust phase but along the orbit just mentioned: this is a consequence of the phasing

problem, which has not been addressed in the present study and is deferred to future work, where an ephemeris model will also be introduced as expressed in 3.6.

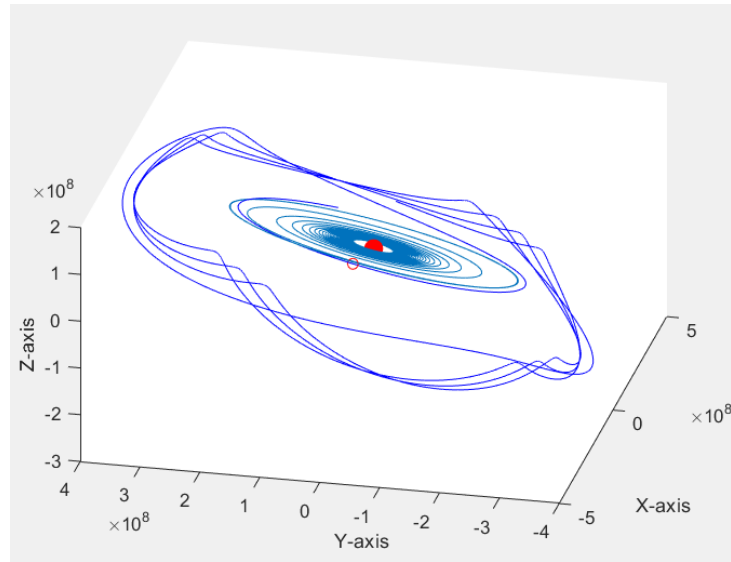


Figure 4.11: Low thrust phase and manifold coasting plotted in the same inertial frame, the red circle represents the injection point while the red sphere indicates the Earth's position

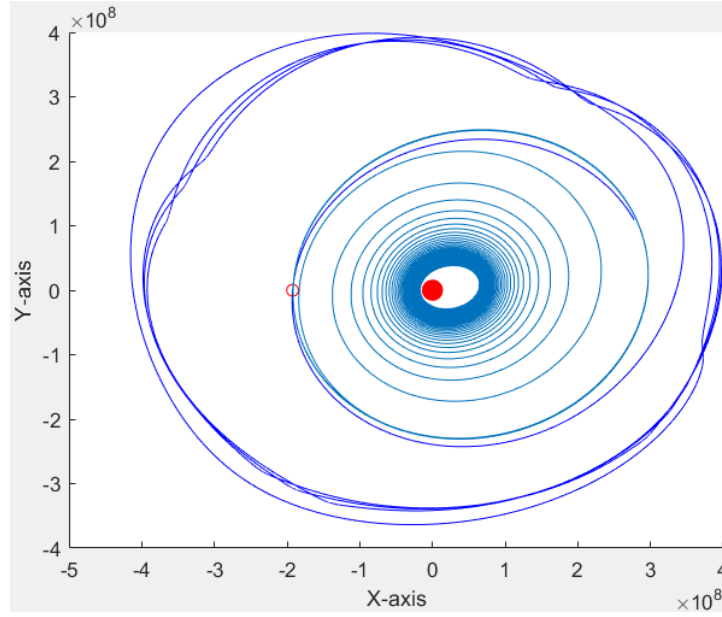


Figure 4.12: Top view of the low thrust phase and manifold coasting plotted in the same inertial frame, the red circle represents the injection point while the red sphere indicates the Earth's position

In addition to the plot here is presented a table 4.8 with the final TOF and propellant mass required by the outbound leg; these are the only parameters reported here since these have guided the decision making along the process of trajectory design,

Table 4.8: Complete outbound trajectory (sGTO $\rightarrow$ 9:2 NRHO), obtained as the sum of the low-thrust arc and the manifold coasting arc. Spacecraft initial mass = 16700 *kg* with a payload mass = 8900 *kg*

Phase	Time of flight [days]	Propellant mass [kg]
Low-thrust arc	102.3	1429.1
Manifold coasting	117.1	0
Total	219	1429.1

Note: the total time of flight is obtained as the direct sum of the two arcs, under the assumption that no loitering in the departure orbit is required. The departure epoch is scheduled so that the spacecraft reaches the manifold insertion point at the correct phase to asymptotically approach the target orbit; it is here assumed that the phasing is therefore resolved through the choice of the departure epoch rather than through an additional waiting arc.

4.4.2 Return Leg

The study of the inbound leg stems directly from the methods used to propagate the outbound leg. The manifold coasting portion follows immediately from the theory: whereas for the outbound leg a stable manifold was chosen, since the target orbit must be approached asymptotically, for the return leg it follows that the spacecraft must depart from the NRHO on which it lies and insert onto the unstable manifold.

By the well-known symmetry of the circular restricted three-body problem, under which its equations of motion are invariant with respect to the transformation $(x, y, z, \dot{x}, \dot{y}, \dot{z}, t) \rightarrow (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}, -t)$ [23], the unstable manifold of the target orbit is the mirror image of the stable one with respect to the x - z plane, combined with a reversal of the time direction. Consequently, the coasting time along the return leg is identical to that of the outbound leg (117.1 days).

The thrust phase, which for the return leg follows the manifold coasting, employs the same spiralling strategy with weighted control. In this case, however, it takes the state vector of the insertion point as the initial state, and the parking orbit designated for the refuelling and servicing operations of the Space Tug as the final state, whose orbital parameters were provided as the target for the construction of the control weights of Equation (3.9). The use of a parking orbit for the refuelling operations is consistent with the concept of operations adopted in [3], although the specific parameters of the parking orbit considered here are defined in the present work.

For the propagation of the thrust phase, the Space Tug is considered emptied of the delivered payload and of the propellant mass expended during the outbound leg. It follows that the mass is now considerably lower, consisting solely of the 5900 kg dry mass of the Space Tug and of the propellant mass required for this single leg, determined iteratively. The propagation of this spiralling-down arc yielded a required propellant mass of 446.3 kg and a duration of 33 days to reach the target parking orbit. The time of flight of the thrust phase is considerably shorter than that of the outbound leg: with the same thrusters reported in Table 4.4 but a substantially lower mass, the resulting thrust acceleration $a = T/m$ is higher, and the manoeuvre also requires a lower propulsive Δv .

Table 4.9: Complete inbound trajectory (9:2 NRHO $\rightarrow$ *sGTO* parking orbit), obtained as the sum of the manifold coasting arc and the spiraling-down thrust arc. Spacecraft wet mass = 6346 *kg*

Phase	Time of flight [days]	Propellant mass [kg]
Manifold coasting	117.1	0
Spiraling-down arc	33	446.3.
Total	150	446.3

4.5 Fleet Sizing and Optimal Resupply Interval

The importance of sizing a fleet of Space Tugs revolves around the purpose of keeping the Lunar Gateway continuously supplied. Since a single delivery covers N days of consumables, a new delivery must arrive within N days. Each tug completes a cycle of duration

$$T_{\text{cycle}} = t_{\text{out}} + t_{\text{ret}} + \tau_{\text{ops}}, \quad \tau_{\text{ops}} = 30 \text{ days}, \quad (4.6)$$

where $\tau_{\text{ops}} = 30$ days is a plausible value that has been set as a placeholder for this analysis only, and a fleet of k staggered tugs delivers once every T_{cycle}/k days. Continuity in supply therefore requires

$$k \geq \frac{T_{\text{cycle}}}{N}. \quad (4.7)$$

The return leg is identical in every case (446.3 kg, 150 days 4.9), since the tug returns empty of payload. Four resupply intervals were compared; the results are collected in Table 4.10.

Table 4.10: Fleet sizing for different resupply intervals.

Resupply	Payload [kg]	Prop./trip [kg]	T_{cycle} [days]	Fleet	Annual prop. [kg]	Prop./payload
60 days	2969	1386	365	7	8432	0.47
90 days	4453	1427	367	5	5787	0.32
135 days	6680	1572.7	378	3	4252	0.24
180 days	8906	1875.1	399	3	3802	0.21

Two trends are immediate: first, a larger payload lowers the propellant-to-payload ratio, from 0.47 at 60 days to 0.21 at 180 days of resupplies, roughly halving the propellant expended per kilogram delivered and, second, a larger

payload also requires fewer tugs, from seven down to three. Both effects stem from the fixed overhead of each round trip: the dry mass, the return propellant and, above all, the time of flight on the manifold coasting are independent of the payload, so carrying more payload per flight amortises this cost over a greater delivered mass.

A smaller fleet is preferable as it reduces the launch demand, the recurring cost and the operational complexity of the architecture. The 180-day configuration, served by three tugs, is therefore selected as optimal: it minimises simultaneously the fleet size and the propellant spent per unit payload. The 135-day case, although it also requires three tugs, is less propellant-efficient and is consequently discarded in favour of the 180-day one. It follows, therefore, that the architecture comprising three tugs and a payload sized for a 180-day resupply is the final configuration selected for the fleet studied in this thesis.

A further degree of freedom follows from the fleet geometry. Three tugs on the 180-day cycle deliver once every $T_{\text{cycle}}/k = 399/3 \approx 133$ days, that is, more frequently than the 180-day food demand strictly requires leaving about 2.3×10^3 kg of the delivered mass unused for food. Should the delivery frequency allow it, this margin could be reallocated from food to generic cargo, such as scientific experiments or hardware without enlarging either the fleet or the vehicle.

4.6 Final results and comparison with reference works

This section presents the final results of the study and compares them with the data of the previous MAGNETO trajectory and, as far as possible, with the results reported by Singh et al. [6]. The latter comparison is necessarily looser, since it refers to a different project and it relies on a higher-fidelity dynamical model; nonetheless, it constituted a pivotal reference for the present thesis. Precisely this difference between the work carried out here and that of Singh et al. represents the innovation and the identity of this research, which applies an idea already investigated by Singh et al. to an existing project with the aim of improving its trajectory.

In regard to the MAGNETO project, the objective was to enhance the trajectory in its entirety, rather than focusing on a specific parameter in isolation. Since the present study focuses on the feasibility and adaptability of this hybrid strategy to the Space Tug project, an optimum was sought for the design through trade-offs among propellant consumption, time of flight and delivered payload mass, bearing in mind that such an optimum must refer not to the single mission but to the entire architecture, as discussed in Section 4.5.

Before conducting a comparison with MAGNETO, it is imperative to acknowledge that the two systems are not equivalent in terms of propulsion system. Rather, the comparison is made at the level of mission objective, which entails the continuous resupply of the Lunar Gateway: in [3] the optimal configuration was identified as four thrusters with an I_{sp} of 2100 s, whereas the present study adopted the equally feasible architecture of four thrusters with an I_{sp} of 2600 s (Table 4.4).

This choice, together with the use of invariant manifolds, led to an improvement in the delivered payload mass (1.65 times MAGNETO's cargo) and on the transfer time (0.60 times MAGNETO's TOF). The required propellant mass, however, did not undergo a notable improvement; indeed, an increase in the propellant required for the outbound leg is observed, which is subsequently compensated by the saving on the inbound leg, when the Space Tug no longer carries the 180-day resupply payload. All the data are reported in Table 4.11.

Table 4.11: Performance comparison with previous trajectory used for MAGNETO [3].

Parameter	This work	MAGNETO
I_{sp} [s]	2600	2100
Thrust [N]	4.1	4.2
Power [kW]	20	18
Initial mass [kg]	16700	13645
Payload mass [kg]	8900	5400
<i>Outbound leg</i>		
Powered time [days]	102.3	364*
Coasting time [days]	117.1	—
Total ToF [days]	219.4	364
Propellant [kg]	1429.1	1340
Δv [km/s]	2.27	2.4
<i>Return leg</i>		
Total ToF [days]	150	—
Propellant [kg]	446.3	800
Delivered payload per kg of propellant (round trip)	4.745	2.523

* MAGNETO performs a continuous low-thrust spiral without ballistic manifold coasting; only the total transfer time is reported.

This comparison is complemented by the one with the study by Singh et al. [6], which served both as a reference and as a validation of the results obtained in the propagation of the manifolds. When propagated in the CR3BP alone, the cited paper reports a ratio between the trajectories leaving the 9:2 NRHO towards the interior realm and those directed towards the exterior realm that is similar to the

one obtained in the present study, as shown in Figure 3.7. Moreover, the trajectory selected in [6] and propagated under the CR3BP alone yields a coasting time along the manifold of **127.5 days**, which is therefore comparable with the value of **117.1 days** obtained in this work.

The study of Singh et al., however, extends the analysis further by re-propagating the manifolds in a high-fidelity model (**HFM**) that additionally accounts for the gravitational influence of the Sun and Jupiter, which considerably reduces the resulting times of flight. Accounting for these influences, together with an appropriate choice of the epoch, the manifold coasting time decreases to **57 days**; this refinement could be applied in future to the Space Tug project as well, yielding substantial benefits. In their study the thrusting time is likewise shorter, although this quantity is not directly comparable, both because of the perturbations accounted for in their model and because of the considerable difference between the two types of spacecraft.

Chapter 5

Conclusions

The work presented in this thesis demonstrates the possibility of exploiting the gravitational dynamics of the Earth–Moon system to the benefit of an existing, practical problem: the continuous resupply of the Lunar Gateway. It is shown that the invariant manifolds emanating from the near-rectilinear halo orbits of the libration points, here the L_2 point of the Earth–Moon system, constitute theoretical pathways that can be applied to a practical combination with low-thrust propulsion, and, more generally, they can become a design tool for missions conceived to take advantage of them.

In particular, the study by Singh et al. [6] had already demonstrated the feasibility of such transfers, although limited to a scientific mission. The present work broadens the scope of that idea by applying it to the Space Tug project, a cargo spacecraft remaining in orbit, considering masses one order of magnitude larger than those of the cited study, and by addressing the design of a fleet for a systematic and uninterrupted resupply.

The outcome is positive: the combination of low-thrust propulsion and manifold coasting, here treated as two separate arcs, rather than applied simultaneously to the same segment, proves both feasible and advantageous, yielding an improvement in the times of flight and in the deliverable payload. It emerged during the study that the results depend strongly on the choice of the trajectory along the manifold, and therefore on the target orbit selected, since different orbits generate different manifolds, as well as on the intersection point at which the spacecraft is inserted onto the trajectory with the appropriate state vector.

The improvements introduced are the following:

- The mass independence of the coasting phase was exploited to deliver a larger payload. With the trajectory designed in this thesis, food provisions sufficient for 180 days can be delivered to the Lunar Gateway, amounting to a total cargo of 8900 kg. Subject to the capabilities of the launchers, which must

inject the payload into a sGTO, or possibly a GTO, an even larger cargo could be carried; the present work, however, has focused on the aforementioned mass of 8900 kg. The spacecraft thereby attains an initial mass of 16700 kg and delivers to the Lunar Gateway a payload 1.65 times larger than the cargo delivered by the previous study, which did not employ invariant manifolds [3].

- By exploiting the propellant saving afforded by the manifold coasting, a continuous thrust profile could be adopted in the low-thrust spiralling phase. This carried the spacecraft from the sGTO to the insertion point in 102 days, followed by 117 days of ballistic, unpropelled flight to the target orbit. The total time of flight of the outbound journey is therefore 219 days; 145 days shorter than the previous trajectory that this work set out to improve. The return journey follows the reverse sequence of events: the spacecraft first departs from the orbit along an unstable manifold and subsequently performs a propelled spiralling-down phase towards its parking orbit. The manifold phase is symmetric to that of the outbound leg, whereas the propelled phase requires less propellant (446 kg) and is faster (33 days), owing to the reduced mass of the spacecraft, which has by then been relieved of the delivered payload.

A further result of this work is the sizing of a Space Tugs fleet based on the performance of this novel trajectory. An optimal operating point was identified with three spacecrafts, providing a delivery cadence of approximately 133 days while still carrying a payload mass corresponding to a 180-day resupply; this margin allows the food stocks to be partially replaced, where required, with scientific experiments or hardware to be delivered.

5.1 Future Work

Any study extending the present work will depend on the strategic decisions to be taken regarding the construction and commissioning of the Lunar Gateway: whereas at the time of this study the Gateway was envisaged as a cornerstone of the Artemis programme, at the time of writing it has been cancelled in favour of the more rapid development of a lunar surface base. The validity of the work carried out nonetheless remains, as it constitutes a case study of a high-payload delivery system in the cislunar environment; future studies should focus precisely on connecting the invariant manifolds to a delivery system for the lunar surface, and hence for the aforementioned base.

Furthermore, in order to complete this study within the available time and given the knowledge acquired, a number of simplifications were introduced, both in the low-thrust phase and in the treatment of the manifolds. To improve the

accuracy and efficacy of this analysis the thrust in the spiralling-up phase could be optimised with respect to specific objectives, such as propellant consumption or time of flight. The entire operations phase and its associated timescales could likewise be investigated in greater detail, and released from the assumption of a fixed inclination adopted here; a future analysis could thus quantify the impact of the orbital-plane change in the near-Earth phase. A further simplification consisted in neglecting the orbital perturbations: in order to develop a mission based on this trajectory, it is therefore imperative to analyse it within a high-fidelity model, as done by Singh et al. [6]. It is worth noting that such an analysis could yield significant benefits, possibly reducing the coasting time and thereby leading to a new optimal fleet configuration, which might operate with as few as two spacecraft.

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