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DEPARTMENT OF ENVIRONMENT, LAND AND INFRASTRUCTURE ENGINEERING

SUSTAINABLE MINING

MASTER THESIS

SUSTAINABLE CUT-OFF GRADE OPTIMIZATION: AN INTEGRATED ESG AND STOCHASTIC RISK FRAMEWORK

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CHAPTER 1: INTRODUCTION

1.1. Background of Study

The global mining industry is currently undergoing one of the largest paradigm changes in its history. Due to the shift towards renewables and electrification in many parts of the world, there is an increased demand for critical minerals and base metals at an incredible rate. This increased demand is accompanied by a heightened degree of scrutiny from society and the environment. According to McKinsey & Company (2023), the industry faces an unprecedented skills gap and reputation challenges where people invest money and resources in industries that create value for society.

Traditionally, strategic mine planning and design have always been evaluated using geoeconomics. In Lane's classic paper from 1988, the key objective was "the production of the mineral asset in such a way as to maximize Net Present Value (NPV) within physical capacity limitations." Under the traditional approach, factors like ore grade, recovery rates, and market prices dictated the Life of Mine (LOM) schedule and order of extraction. Environmental impacts and community relations were largely treated as qualitative compliance requirements or reactive remediation efforts rather than quantitative financial constraints (Bustillo Revuelta, 2018).

The model presented is not sustainable in today scenarios. The integration of ESG factors has become an essential component of economic sustainability. Modern-day mining operations, particularly underground hard-rock mines, are required to function in an age of digitalization where green mining practices have become the new norm (Spearing et al., 2023). This means that mine planning should be economically sustainable and should demonstrate environmental and social sustainability in order to be considered successful.

1.2. Problem Statement

The choice of cut-off grade emerges as the most important decision in the mining strategy process since it clearly defines what should be considered as ore and the NPV of the mining project as a whole (Lane, 1988). While dynamic cut-off grades adequately take capacity constraints into account in the equation itself, they suffer from a major weakness in the present-day context.

The main problem explored in this paper is that currently used strategies of mine planning focus on optimizing financial value using only deterministic approaches, while ignoring ESG factors and stochastic risk exposure. This problem is represented by the following two gaps:

1. Environmental Costs as an Externality of the Cost Function: These models treat environmental costs as externalities related to Scope 1 and 2 emissions or water extraction. Since the cost function in current calculations do not take into account such "environmental costs" and their impact.

2. Determinism Fallacy: Mine plans span over many years, but currently used models use volatile inputs as fixed constants. According to Toro Morales et al. (2024), failure to factor in the uncertainty of the mining environment and markets generates inadequate decision variables, leading to policies that are not robust to stochastic fluctuations of the global economy. Consequently, the industry lacks an integrated framework capable of optimizing the dynamic cut-off grade while simultaneously internalizing ESG costs and navigating stochastic uncertainties.

1.3 Objectives of the Research

The primary goal of this thesis is to develop a robust framework for Sustainable Strategic Mine Planning. The specific objectives are:

- **To Internalize ESG Externalities**
- **To Model Stochastic Uncertainty**
- **To Validate via Case Study**

1.4 Thesis Structure

This thesis is organized into six chapters, progressing from theoretical foundations to the verification of the proposed framework:

- **Chapter 1:** Introduction provides the foundational background, defines the specific problem statement, and outlines the research objectives.
- **Chapter 2:** Outlines the research design, data acquisition strategies, and the analytical modeling framework employed to address the limitations of traditional strategic mine planning.
- **Chapter 3:** Fundamentals of Mine Design and Planning details the mathematical formulation of Lane's theory and establishes the key economic parameters used in strategic evaluation.
- **Chapter 4:** From Environmental Sustainability to ESG Integration in Mine Economics explores the quantification of carbon and water shadow pricing.
- **Chapter 5:** Stochastic Uncertainties in Sustainable Mine Planning describes the methodology for implementing Monte Carlo simulations to address market volatility.
- **Chapter 6:** Case Study and Framework Verification present the final application of the integrated model to the Çayeli Mine, providing a comparative analysis of S-NPV, VaR, and CVaR results.
- **Chapter 7:** Conclusions and Recommendations summarize the research findings and propose future strategic directions for the industry.

CHAPTER 2: METHODOLOGY

This chapter outlines the research design, data acquisition strategies, and the analytical modeling framework employed to address the limitations of traditional strategic mine planning. The core objective of this methodology is to construct a robust, transparent, and reproducible mathematical system capable of integrating Environmental, Social, and Governance (ESG) externalities and stochastic market variables into the dynamic cut-off grade optimization process.

2.1. Literature Review and Identification of Research Gaps

The evolution of strategic mine planning has historically prioritized the maximization of Net Present Value (NPV) through deterministic frameworks. Since the introduction of Lane's (1988) seminal algorithm, the industry has primarily focused on optimizing physical capacity constraints. However, a critical synthesis of contemporary literature reveals two primary deficiencies that necessitate a new methodological approach:

1. **The Quantitative Exclusion of ESG:** Current literature mostly relies on isolated frameworks and planning, production scheduling algorithms considering environmental, social and governance (ESG) factors as external qualitative indicators instead of considering these parameters as quantitative indicators such as cost parameters. This study focuses on operationalizing these factors into dynamic economic parameters.

2. **The Deterministic Trap:** Traditional strategic mineral sequencing made according to static metal price forecasting models which inherently create a systemic vulnerability to external market volatility. This lack of financial resilience creates an 'expectation of certainty' where calculated operational cut-off grades fail to adapt to abrupt downtrend market reversals; this can lead to catastrophic cash flow deficits and early resource sterilization when actual prices deviate from the static average. By grounding the methodology in these identified gaps, this research moves beyond traditional geoeconomics to develop a Sustainable Stochastic NPV (S-NPV) framework.

2.2. Methodological Approach

Research adopts a quantitative analytical approach to solving the optimization problem of sustainable underground mine planning. Rather than relying on proprietary, commercial software does, this methodology is deliberately designed around building an integrated conceptual and theoretical mathematical framework.

This approach ensures complete transparency in demonstrating how ESG penalties and stochastic variables interact with traditional geoeconomic parameters. The methodology progresses through three different phases.

- **Deterministic Base Phase:** The foundational layer establishes the baseline physical and economic boundaries of the operation based on classical mineral economics. It mathematically formalizes Lane's (1988) capacity constraints with three different stages which are mining, milling and marketing moreover, breakeven theory is also applied in this layer beside Lane's Theory. This layer treats geological and block-model inputs as deterministic constants to calculate traditional limiting cut-off grades (g_m,g_o,g_r,g_{be}). Primary function is to define maximum theoretical net present value (NPV_{trad}) under unpenalized market conditions.
- **ESG Internalization Phase:** The development of an algorithm that monetizes ESG risks through the use of shadow pricing and applies stochastic distributions to dynamic parameters in order to avoid the problems associated with deterministic planning. In the current phase of research, the ESG externality will be incorporated directly into the computation process as cost functions. Through the application of the shadow pricing approach, environmental impacts like water usage (P_{H20}) and carbon emissions (P_{CO2}) transform into direct penalizations. By redefining traditional operational expenses ($OPEX_{TRAD}$) into sustainable (ESG included) operational expenses ($OPEX_{ESG}$).
- **Stochastic Synthesis Phase:** The application of the newly developed framework to a representative underground hard-rock mining case study (Çayeli Mine) to validate its strategic superiority.

2.3. Data Collection and Synthesis Strategy

The data acquisition relies on synthesizing primary operational data with secondary environmental and market benchmarks to ensure the model's empirical validity.

- **Primary Operational Data:** Technical parameters, including unit mining costs (m), processing costs (h), and capacity constraints (M, H, K), are derived from the NI-43 report of “Çayeli Bakır İşletmeleri (ÇBİ). Geological uncertainty is managed through a "Discrete Unit Approach," utilizing the grade and tonnage distributions of the Main and South orebodies.
- **ESG and Shadow Pricing Metrics:** Environmental externalities are quantified using carbon emission (CO_2e) and freshwater consumption metrics. These are monetized by applying global carbon tax projections and environmental shadow pricing mechanisms sourced from contemporary economic literature.
- **Stochastic Market Variables:** To model uncertainty realistically, historical copper price fluctuations are utilized to generate probabilistic distributions rather than relying on static forecasts.

2.4. Modeling and Dynamic Optimization Algorithm Framework

The analytical procedure is designed to determine the optimum dynamic cut-off grade policy through the following stages:

1. **Baseline Deterministic Calculation:** Calculating traditional cut-off grades according to both Lane’s Theory and Break-even Theory (g_m, g_h, g_k, g_{be}) and conventional NPV to establish a comparative baseline.
2. **Internalization of ESG Externalities (Sustainable OPEX):** Shadow prices for carbon and water footprints will be directly added into the cost function and traditional discount rate will be adjusted to the ESG discount rate to reflect the benefits of "Green Financing."
3. **Stochastic Monte Carlo Simulation:** The algorithm executes 1,000 iterations, introducing stochastic variables for metal prices. This observes the volatility of the

project's value and calculates critical risk metrics: Value at Risk (VaR) and Conditional Value at Risk (CVaR). And will give predictions of difference between Sustainable Net Present Value and Traditional Net Present Value.

4. **Sustainable Net Present Value (S-NPV) Generation:** The final step determines the optimum cut-off grade path that maximizes the risk-adjusted value over the entire Life of Mine (LOM).

CHAPTER 3. FUNDAMENTALS OF MINE DESIGN AND PLANNING

3.1. Introduction and Key Terminology

Strategic Planning and Economic Evaluation Parameters

Mineral: A compound which occurs naturally in the Earth's crust and has specific physical properties and chemical composition.

Ore: Rock contains minerals that have economic value to extract.

Waste/ Gangue: Rock does not contain minerals that have economic value to extract.

Grade: Concentration of valuable minerals within a given volume of ore, expressed as ppm, g/t or percentage.

Cutoff Grade: Minimum grade required in order for a mineral or metal to be economically mineable.

Stripping Ratio: Ratio of extracted volume of waste material to the extracted volume of ore. Amount of waste removed per unit volume of ore.

Bottleneck: It is a specific stage of mining that limits the overall production rate. Such as mine, mill or market.

Throughput: Actual rate of material flowing through these stages of mining.

NSR (Net Smelter Return): For base metals such as copper, lead, and zinc, prices are not quoted for concentrates rather the refined metal price is given. The payment received by the company from the smelter for their concentrates (called the net smelter return (NSR)) (Huss, 1984)

Life of Mine (LOM): The planned duration of a mining operation, encompassing all stages from the start of production to the final closure. According to Hustrulid et al. (2013), LOM is a strategic variable determined by the extraction rate and the cut-off grade strategy, aiming to maximize value over the entire period rather than short-term gains.

Net Present Value (NPV): The standard financial metric used to evaluate a project's profitability by calculating the difference between the present value of cash inflows and outflows over time. In mining, maximizing NPV is the primary goal of Lane’s optimization theory.

Opportunity Cost: Formally defined in Lane (1988) as the "F" parameter. It represents the value foregone by not utilizing a finite resource or capacity (like the processing plant) for a higher-grade material. Processing low-grade ore today has an opportunity cost because it delays the cash flow from better ore available in the future.

Discount Rate: The interest rate used to convert future cash flows into their present value, reflecting the time value of money and the perceived risk of the project. In this thesis, the discount rate is also a tool for incorporating long-term ESG risks.

Development: The process of excavating shafts, ramps, and galleries to provide access to the ore body. In underground mines, development is the primary physical bottleneck that limits the production rate.

Production: The actual extraction of ore from the stopes or mining faces. The balance between development and production is crucial for maintaining a steady cash flow and LOM stability.

Dilution: The contamination of ore with waste rock during extraction. SME Handbook (2011) emphasizes that dilution reduces the ROM grade and increases the cost per unit of metal. In an ESG context, excess dilution increases the carbon footprint per tonne of final concentrate.

Recovery:

Mining Recovery: The percentage of the geological reserve that is successfully extracted.

Metallurgical Recovery: The percentage of the metal content that is successfully separated in the processing plant.

ESG and Stochastic Integration Terms

Stochastic Planning: A planning approach that uses probability distributions (e.g., Monte Carlo simulations) instead of fixed values for variables like metal prices or grades. According to Dimitrakopoulos (2011), it allows for more robust mine plans that can withstand market volatility.

Shadow Pricing: A technique used to assign monetary value to non-marketed items. In this thesis scope, shadow pricing is used to give monetary value for carbon emissions and water usage to simulate their impact on the cut-off grade decision as if they were actual operational costs.

Social License to Operate (SLO): The informal approval or acceptance granted by local communities and stakeholders. A loss of SLO is treated in this framework as a stochastic risk that can cause operational delays and significant NPV losses.

ESG Integration: The systematic inclusion of Environmental, Social, and Governance factors into the strategic mine planning process.

3.2. Overview of Mining Methods

The choice of mining methods involves a strategy influenced by geology, economics, and even the environment of mineral resources rather than only an engineering decision (Spearing et al., 2023). The main choice is important since it establishes all the operational parameters and expenses required to create a profitable plan for the mine project.

As Spearing (2023) highlights, extensive shallow, and relatively low-grade orebodies are typically used for surface mining, which is mostly open pit. Because of the economies of scale, profitability is possible in such a situation. Low-grade ore can be treated economically by using massive earth-moving machinery, which can achieve very low unit extraction costs. However, the stripping ratio is a geometrical limiting factor for surface mining. The amount of waste that must be retrieved grows exponentially in depth in order to preserve the integrity of the mine's slopes. The Ultimate Pit Limit (UPL) is reached when the cost of mining this waste surpasses that of the mineral itself.

On the other hand, the mandatory mining technique employed is underground mining when the mineralization deposit is deep, has a complex geological structure (e.g., steeply dipping veins and massive sulfides), or when the stripping ratio of a surface mine increases beyond an economic threshold which known as a critical point commonly known as the transition depth. Working in an underground setting incurs significant CAPEX costs for access and environmental support infrastructure. As such, underground mining necessitates higher grades, or NSR values, in order to cover the costs. While surface mining operations are constrained geometrically, underground mining is bound by logistical considerations, such as development speed, geomechanics, and backfilling schedules. Although underground mining causes minimal impact on surface ecosystems, it poses a formidable Scope 2 emissions issue due to the considerable energy requirement for ventilation and drainage systems.

3.3. Mine Planning Framework

Mining operations planning and implementation depend on collecting and modeling geological information, defining production targets, designing extraction plans, and then sequencing activities. More importantly, all these technical processes should have economic justifications. According to Hartman (1992), there are some crucial factors that need to be considered when it comes to production scheduling, which include minimizing pre-production costs, leveling stripping ratios, maximizing production, ensuring slopes' stability and access for miners, among others. Modern mining operations planning should adhere to strict safety and environmental laws and societal demands at the same time, while maximizing its net present value.

Mine planning, according to Camus (2002), can be divided into two main functional categories rather than strictly time-scale dependent ones:

- **Strategic Planning:** In strategic planning, the main decisions related to the mining of the orebody are covered. The fundamental goal in this category is the establishment of profitability of the venture. Some of the decisions made here include choosing the method of mining, facility needs, mine planning, cut-off grades, and costs.
- **Tactical Planning:** Tactical planning covers production schedules, starting from the first day of mining operations and up to short and medium-term production plans.

3.3.1. The Three Essential Models for Exploitation

According to Ghosh (SRK, Brisbane, Australia), successful, safe, and efficient mining operations can only be achieved through application of the following three models:

1. **Geological and Resource Model:** The geological model defines the orebody and its surrounding rock mass. This model describes the stratigraphic sequence, local geology, hydrology, and an assessment of the stress tensor. The geological model provides the fundamental inputs for the design process.
2. **Planning Model:** The planning model involves interpreting the geological model into mineable terms. The planning model considers economic considerations, proposes a

mining method whether open pit or underground, estimates the rate of production, proposes mine access and ventilation systems in case of underground mines, and estimates cost structures and revenue.

3. **Scheduling Model:** The scheduling model is concerned with determining the production schedule, tons produced, grades, and sustainability of the mining plan. The scheduling model also helps to detect bottlenecks in production and grade over time.

3.3.2. Method-Specific Planning Approaches

Because geological conditions vary significantly, the planning processes have evolved differently for surface, soft rock, and hard rock environments.

A. Surface (Open-Pit) Mine Planning

Surface mining begins with converting the geological information into a Block Model. In this Block Model, each block will have a certain volume and economic grade, estimated using techniques such as Kriging. However, maximizing NPV in an open pit is quite difficult. According to Whittle (1989), "the pit outline that generates the greatest value cannot be determined until the values of the blocks are known. The values of the blocks are not known until the order of extraction is decided upon, but the order of extraction cannot be decided until the outline of the pit is known".

In order to solve this dilemma, open pit mine planning includes the assessment of many different designs in order to develop a series of nested pits, referred to as "pushbacks." In this manner, mining will progress from the first, most economical pit (lowest waste dilution) up to the final pit outline, which is usually the least economical pit due to its high stripping ratio. Dagdelen (2000) shows that this long-term planning is actually a circular and dynamic process, involving repeated assessments of mining capacity, production cost, and cut-off grade.

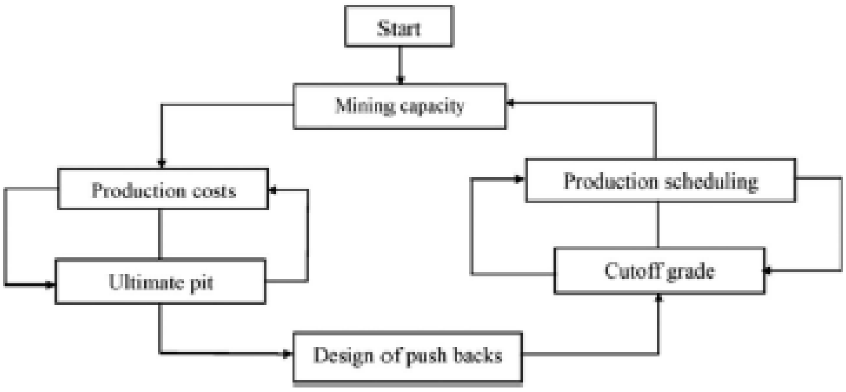

Figure 1. The long-term planning process (Dagdelen, 2000)

B. Soft Rock (Coal) Mine Planning Principles

In soft rock environments like coal mining, the goal is to create an integrated exploitation system that meets certain market needs at the lowest possible unit cost. Fourie and van Niekerk (2001) says that this planning process has five steps: (1) collecting project data, (2) evaluating, planning, and designing, (3) building and setting up, (4) mining operations, and (5) decommissioning and closing. Modern soft rock planning depends a lot on specialized computer programs like Surpac, Vulcan, or Deswik to quickly look at different options and change both short- and long-term plans.

C. Hard Rock (Metal/Nonmetal) Mine Planning

Mining hard rock, especially underground, poses unique geomechanical problems. The main design problem is figuring out how to connect lab rock tests to the large-scale rock mass. Faults and joint sets, for example, make the rock mass much weaker. So, strong rock mass classification systems are very important for the first mine layouts and support designs. There are two types of planning for hard rock: strategic long-range planning (like Life of Mine and 5-year plans) and tactical short-term planning (like monthly and daily activities). To plan for the long term, you need to know a lot about metal prices, exchange rates, capital costs, and processing efficiencies. (Spearing et al. 2023)

3.3.3. Mine Planning Challenges and Stochastic Uncertainties

It can be said that uncertainty is the most vulnerable point of mine planning. The planning process requires big dataset, which are inherently prone to errors, particularly concerning the geological model, the rock mechanics model, and the block model grade distribution.

Dominy (2002) highlights several major reasons for errors in resource block models, including unreliable sampling, poor interpretation of grade distribution, volume variance effects, and the improper use of estimation software. In practice, reserve estimates are frequently downgraded during due diligence audits due to issues such as biased sampling, poor core recovery, or unforeseen metallurgical challenges.

To take into account these intrinsic uncertainties, mine planning uses different modelling methods such as Stochastic Kriging, uses random probability distributions to take into account the deposit's spatial and economic variability (Ankenman et al., 2010). This is different from deterministic models. By using this probabilistic method, it is possible to make extraction schedules that are more flexible and can handle the shocks that come from changes in the economy and geology.

3.4. Evolution of Cut-off Grade Optimization

The cut-off grade probably represents the most important strategic choice in mine planning because it directly determines which parts of the mineralized deposit are "ore" and which are "waste." In the past, the mathematical way of defining this boundary changed from static cost-recovery models to dynamic optimization theories.

3.4.1. Traditional Approach: The Break-Even Cut-off Grade

Before modern computational planning and dynamic financial modeling came along, the mining industry mostly used the Break-Even Cut-off Grade theory, and this theory is still being used because of easiness of application it has. The basic idea behind this traditional method is purely operational: the cut-off grade is the point at which the money made from selling the metal that was mined and processed is equal to the total cost of performing so.

Mathematically, the static break-even grade g_{be} is expressed as:

$$g_{be} = \frac{m + h + f}{y * (s - r)}$$

Where:

- *m = Mining Cost ($/t)*
- *h = Processing Cost ($/t)*
- *f = G&A Cost ($/t)*
- *(s - r): Value of 1% of mineral ($/t) – Difference is implemented as using recovery and payability function directly inside the calculation*
- *y = Metallurgical recovery rate (%)*

The break-even approach is easy to understand and gives a starting point for figuring out if the business can survive, but it has may cause theoretical problems when it comes to long-term strategic planning. The main problem with it is that it works as a static model. It ignores the time value of money, the limited capacity of the mining and processing equipment, and the fact that mining low-grade ore today means that high-grade ore won't be mined until tomorrow. The break-even approach mostly focuses on covering costs instead of maximizing net present value.

3.4.2. Mortimer's Contribution and the Shift to Value Maximization

Mortimer (1950) is promoting the integration of the cut-off grade with the rate of return after realizing that static break-even procedures were insufficient and proposed that the objective of mine planning should be to maximize the total profit over the course of the mine's lifetime rather than just identifying the point of no loss. However, early value maximization models struggled to incorporate time value of money and physical constraints into their formulas.

3.4.3. Kenneth Lane’s Dynamic Theory

Kenneth Lane's study in 1988, The Economic Definition of Ore, put itself in the highest point of strategic mine planning. Reason for that it, his theory demonstrates that the cut-off grade is a dynamic financial variable that must fluctuate during mine’s life in order to optimize the Net Present Value (NPV), rather than a static geological or operational parameter.

Two fundamental concepts introduced by Lane's theory made conventional break-even models outdated:

1. **Capacity Constraints:** According to Lane's mathematical model, the mining operation is limited by three main bottlenecks: the market (sales/refining capacity), the mill (processing capacity), and the mine (extraction capacity).

2. **Opportunity Cost:** Lane included opportunity cost to the equation for the cut-off grade. He showed how processing low-grade ore takes time and limited capacity, which delays the realization of future high-grade ore cash flows. When discounted over time, this delay has a significant financial cost.

3.4.4. The Mathematics of Lane’s Theory

Logic behind the Kenneth Lane's optimization model is the idea that a mining operation is a continuous system limited by its processing capabilities. The cut-off grade must balance the material flow through the three main stages, the mine, the mill (processing plant), and the market (refinery and sales), in order to optimize the Net Present Value (NPV).

To understand Lane's mathematical formulation, the basic economic parameters must first be defined:

- *s = Selling price ($/t)*
- *r = Selling and refining costs ($/t)*
- *m = Mining Cost ($/t)*
- *h = Processing Cost ($/t)*
- *f = G&A Cost ($/t)*
- *F = Opportunity cost of capital (Discount rate * NPV)*

- *y = Metallurgical recovery rate (%)*
- *H = Maximum milling capacity (t/year)*
- *K = Maximum marketing/refining capacity (t/year)*

The inclusion of the opportunity cost (F) is the fundamental part of Lane's theory. Processing low-grade ore only results in physical operating costs (m and h) in conventional static models. According to Lane, if low-grade ore is processed early, the extraction of future high-grade ore is delayed. The invisible cost of this delay is expressed mathematically as the opportunity cost; F. Lane determined three different limiting cut-off grades based on whatever stage of the procedure serves as the main bottleneck:

1. The Mine-Limited Cut-off Grade (g_m)

Time is not the processing plant's immediate limiting variable when the mining capacity is the limiting factor, as is frequently the case in underground operations severely limited by development rates, shaft hoisting constraints, or backfill cycles. As a result, the ore does not receive the majority of fixed expenses and opportunity costs. The only factor influencing the cut-off grade is variable costs:

$$g_m = \frac{m}{(s-r)*y}$$

2. The Mill-Limited Cut-off Grade (g_h)

When the processing plant (Mill) is the bottleneck, every tons of ore fed into the plant consumes a fraction of the total available time for that period. Consequently, the ore must not only pay for its own variable processing costs but also carry the burden of the fixed costs (f) and the opportunity cost of time (F) allocated over the total milling capacity (H):

$$g_h = \frac{h + \frac{f+F}{H}}{(s-r)*y}$$

3. The Market-Limited Cut-off Grade (g_k)

When the market demand or the refining capacity limits the operation, the bottleneck shifts to the final product stage. The ore must generate enough value to cover processing costs while the fixed and opportunity costs are allocated over the maximum product capacity (K):

$$g_k = \frac{h}{(s-r)*y-\frac{f+F}{K}}$$

In practice, more than one of these formulas is used in a strategic mine plan. Over the course of mine's life, the eventual ideal cut-off grade dynamically moves between these three limiting grades, changing as the NPV curve and the deposit's remaining reserve do. Lane's technique ensures that the project's traditional financial value is mathematically maximized by continuously computing these balancing equations.

3.5. Research Gap and Theoretical Positioning in Dynamic Cut-off Grade Models

Kenneth Lane's dynamic cut-off grade optimization has an important role for mineral economics which is the development of strategic mine planning from simple break-even calculations. By this method it is possible to optimize a project's Net Present Value (NPV) by including capacity restrictions (m, h, k) and the opportunity cost of capital (F).

However, these methods have a modern drawback despite their mathematical excellence: they were created during a time when social and environmental effects were viewed as qualitative parameters rather than quantitative financial constraints.

The Research Gap

The research gap can be summarized as follows: *Existing strategic mine planning models calculate dynamic cut-off grades to optimize financial value strictly under deterministic assumptions, while neglecting ESG externalities and stochastic ESG risk exposure.* Specifically, traditional dynamic cut-off grade formulas exhibit two weaknesses:

1. **Deterministic Rigidity:** Traditional planning relies on static, single-point estimates for highly volatile parameters (e.g., metal prices, operating costs) over a Life of Mine (LOM). This deterministic approach fails to account for the stochastic nature of global markets, resulting in a fragile dynamic cut-off grade policy that cannot adapt to financial shocks.

2. **Exclusion of ESG Externalities:** In conventional OPEX and cut-off grade calculations, the costs of environmental degradation are not considered quantitatively (e.g., Scope 1 and Scope 2 carbon emissions). Because traditional formulas do not penalize high-emission or high-dilution extraction paths with monetary values (such as carbon taxes), the calculated dynamic cut-off grade becomes artificially low. This flaw instructs the mine to process environmentally and economically unsustainable material.

Theoretical Positioning of the Thesis

This thesis can be placed at the intersection of stochastic risk modeling, environmental accounting, and classical mineral economics. In order to fill the gap mentioned at the previous part of this chapter, this study expands on Lane's established capacity limitations rather than rejecting them. It has a theoretical approach aims to transform the conventional NPV objective function into a Sustainable Net Present Value (S-NPV). This framework can complete the gap between theoretical economic optimization and today's realities of sustainable underground exploitation by introducing "shadow pricing" to internalize ESG penalties directly into the dynamic cut-off grade equations (g_m, g_h, g_k) and by using stochastic distributions to simulate market and regulatory volatility.

4. FROM ENVIRONMENTAL SUSTAINABILITY TO ESG INTEGRATION IN MINE ECONOMICS

4.1. The Paradigm of Environment and Sustainability in Mining

There exists a fundamental relationship between environmental protection and the mining industry. By its nature, mining is an extractive operation that deals with non-renewable geological materials and requires extensive manipulation of the environment. Yet, minerals and metals mined are extremely important to maintain the living standards of modern society, technological development, and, above all else, the transition of the world towards renewable energy systems. The process of transforming non-renewable natural capital into human, social, and manufactured capital may be viewed from both the economic and environmental perspectives.

Up until now, the industry has adopted primarily a reactive stance to this contradiction, focusing only on post-depletion environmental rehabilitation measures and minimal compliance with regulations and environmental reclamation programs. In the modern era, however, industry needs to work in the framework of sustainable development. (Spearing et al., 2023). Sustainability in the context of mineral exploitation is not defined by the unachievable goal of completely protecting the resource. Instead, it requires that the use of a non-renewable resource be carried out in a way that rigorously minimizes, mitigates, or compensates for the ecological impact while optimizing long-term socio-economic advantages. As Tost (2018) says, this change requires mining activities to respect established planetary boundaries which are shown in Figure 1, guaranteeing that resource consumption does not jeopardize the biophysical resilience of the planet.

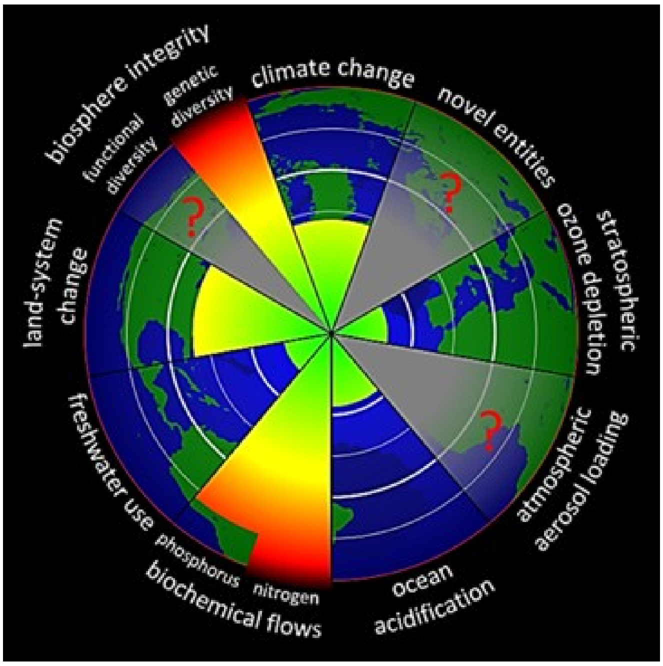

Figure 2.The Planetary Boundaries, acc. to Steffen et al., 2015a, b.

These days, sustainability performance and credentials are integrated into every component of mining projects, impacting everything from the original project assessment to operations, mine closure, and overall corporate governance goals (Hall et al., 2025).

One must first classify the severe physical effects that are uniquely related to underground hard-rock mining activities in order to comprehend how these ideas eventually flow into economic models. The environmental impact of underground extraction is frequently subsurface, highly technical, and multidimensional, in contrast to the obvious footprint of open-pit mines:

- **Atmospheric Impacts via Ventilation Networks**

Underground operations require massive, continuous airflow to sustain life, regulate temperature, and clear toxic fumes. This results in the continuous emission of concentrated pollutants expelled forcefully into the surface atmosphere through main exhaust ventilation rises. Furthermore, modern underground mines possess a highly intensive energy footprint that must be formally categorized according to global Greenhouse Gas (GHG) Protocol (2004) accounting standards:

a. **Scope 1 Emissions (Direct):** These are the direct GHG emissions coming from physical sources that are completely controlled by the company within the limits of the operation. There are two primary sources of Scope 1 emissions for deep hard rock underground mines. The first is the fleet of large mobile equipment run on diesel fuel, such as dumpers, diesel powered Jumbos and Load-Haul-Dumps. The second source is the chemical breakdown of commercial explosives (for example, ANFO or emulsion explosives) during blasting operations, resulting in the release of huge quantities of CO_2, CO, and NO_x into the mine atmosphere, is the second, frequently disregarded but extremely important source.

b. **Scope 2 Emissions (Indirect):** These emissions originate from the purchased energy consumption by the mine from an external source, which primarily involves energy consumption of electricity. Due to the very large amount of electricity required for the underground mining facilities, Scope 2 emissions are normally the biggest contributor to the overall carbon footprint for deep underground mines. This is because deep shaft friction hoists which require transporting several thousand tons of ore per day, continuous dewatering pump circuits, primary ventilation fans that operate continuously, as well as subterranean refrigeration facilities that are necessary for cooling off the ambient rock temperature, consume a lot of electricity continuously. The carbon intensity of the local electrical power grid (e.g. coal versus renewable hydro-electric power grids) determines the financial cost of Scope 2 emissions.

For mine planning, it is essential to precisely define and quantify these particular emission ranges. In the dynamic cut-off grade optimization process, they serve as the fundamental, applicable data needed to apply the carbon shadow pricing (P_{CO2}) mechanism within the Sustainable Net Present Value (S-NPV) framework, successfully converting physical atmospheric impacts into concrete financial liabilities.

- **Hydrological Disruption and Aquifer Drawdown**

The alteration of groundwater systems is undoubtedly the most important environmental vector for underground mining. Operations must constantly pump out groundwater (mine dewatering) to prevent deep workings from flooding. According to Spearing et al. (2023), this enormous extraction creates a huge "cone of depression" that can dry out surface wells and interfere with local agricultural hydrology by artificially lowering the area water table.

- **Geomechanical Surface Alterations and Waste Storage**

Underground mining poses serious geomechanically dangers to the surface ecosystem even though it avoids the significant topsoil loss of open pits. According to Spearing et al. (2023), extraction techniques have the potential to cause sudden or slow surface subsidence, which can fracture the rock mass to the surface and harm nearby habitats. Additionally, surface Tailings Storage Facilities (TSFs), which provide long-term risks of structural failure and toxic seepage, must handle fine tailings. These effects represent significant environmental liabilities that directly affect the mine's Social License to Operate (SLO) and long-term closure costs, even though they are more difficult to quantify as unit costs (Hall et al., 2025; Spearing et al., 2023).

Addressing these intense, physical, and highly specific underground environmental realities is the foundational prerequisite before any strategic economic optimization can be considered truly sustainable.

4.2. The Evolution from Sustainability to the ESG Framework and EMS Integration

While the concepts of environmental protection and sustainable development provide the ethical foundation for modern mining, they are mostly placed at philosophical stage and difficult to quantify on a balance sheet. The modern mining industry requires highly structured, measurable, and auditable frameworks to manage its immense operational complexities.

The Integration of Environmental Impact Assessment (EIA):

According to Bustillo Revuelta (2018), the evaluation of mineral resources is currently an integrated process whereby technical and economic aspects are interrelated with environmental outcomes, as opposed to a process that was simply geological or economic in nature as before. Under this view, the Environmental Impact Assessment (EIA), which supplies the necessary information to make the evaluation of the feasibility of the mining project within a specific environmental context, is considered a vital technical element for any mining enterprise. There are important financial advantages to applying such environmental considerations using the framework of an Environmental Management System (EMS). The benefits of establishing such an EMS, according to Bustillo Revuelta (2018), are:

- Diminishing environmental responsibilities by applying well-defined mitigation techniques.
- Increasing the effective utilization of resources.
- Decreasing waste production through appropriate planning.
- Proving a well-accepted corporate image.
- Motivating awareness of environmental concerns.
- Incrementing better knowledge of the environmental impacts of business activities.
- Incrementing skill and effectiveness, generating higher productivity at lower costs and higher benefits.

International Standards and ISO 14001:

Modern mining companies use Integrated Management Systems (IMS) to manage the immense technical complexity described by Bustillo Revuelta (2018) and obtain operational excellence. ISO 45001 for Occupational Health and Safety and ISO 9001 for Quality Management are commonly used in underground operations as a guidance. However, the industry standardizes its approach through Environmental Management Systems (EMS), notably anchored by the ISO 14001 standard, in order to systematically monitor and minimize the severe physical environmental impacts. These mechanisms are crucial for encouraging optimal practices, with a focus on sustainable (green) mining and the social license to operate (Spearing et al., 2023).

ISO 14001 Environmental Standard:

ISO 14001 is the globally accepted framework that dictates how a company should design, implement, and maintain an effective EMS. It is not merely a bureaucratic compliance checklist or a static document; it is a dynamic, operational engine driven by the principle of continuous improvement, structured around the Plan-Do-Check-Act (PDCA) cycle can be seen from the Figure 3.

Figure 3. PDCA Cycle, EPA 2026

In the ISO 14001 structure, an organization is required to carry out assessment on its environmental aspect and impact, develop measurable goals, and apply strict control and auditing processes to demonstrate its conformity. However, without having a strong and certified EMS system in place, a mining corporation is not able to provide the necessary data that is essential for achieving sustainability goals (GHG Protocol, 2004).

Transitioning from EMS to the ESG Framework:

Complete ESG framework classifies the data into pillars of corporate risk and financial opportunities, while the ISO 14001 Environmental Management System takes care of the technical and daily operational management of data within the mine site. The industry faces increasing demands for reducing its footprint with regard to various financial, environmental, and social risks related to such global challenges as water stress and climate change (Rahnema et al., 2024). The ESG framework systemizes into three distinct pillars (Yusof & Widyasamratri, 2023):

- **Environmental (E):** This pillar involves effective management of natural resources, climate risks, environment-related opportunities, and proper pollution/waste management.
- **Social (S):** This pillar includes human capital management, product liability, social opportunities, and stakeholder opposition in the context of social license.
- **Governance (G):** This pillar covers corporate governance as well as corporate behavior principles that determine the direction and control of organizations.

4.3. Economic Implications for Dynamic Cut-off Grade Optimization

Adoption of EMS and the wider ESG framework demands mathematical calculations based on the specific instance of mining planning and dynamic cut-off grade optimization. Everything about this process relies on mining costs per unit (m), processing costs per unit (h), and the rate of capital return (F) as part of conventional optimization models like that of Lane's (1988) NPV maximization principle.

A mining operation earns hidden financial liabilities when it produces significant externalities. Nevertheless, these traditional cost functions were developed in an era in which negative impacts on the environment were often externalized. When planning takes place without considering the information contained in the environmental management systems of the organization itself, the fundamental problem begins. The exclusion of these ESG components externalizes financial risk and thereby undermines the true nature of the dynamic cut-off grade strategy. In addition, the operating costs remain unrealistically low if the cost of the environmental damage caused is not included, like carbon taxes based on direct emissions or the scarcity value of water, among others. Consequently, the optimization system suggests an unrealistically low cut-off grade, directing the mining company to extract materials which are low-quality that will be unprofitable due to the true ecological cost of doing so.

Based on what Hall et al. (2025) explain, the inclusion of improved sustainability performance alters the process of making resource management decisions. The economic costs increase in terms of extraction. It ensures that the strategic decision-making framework will be resistant to increasing costs in terms of environmental regulation while being optimal in ideal determinist conditions. In terms of avoiding early consumption of resources and extracting the marginally environmentally expensive ores, the proposed theory ensures that S-NPV computations represent a de-risked financial path (Spearing et al., 2023; Toro Morales et al., 2024).

4.4. Monetizing Environmental Externalities: Equator Principles and Shadow Pricing

Externalized environmental risks eventually return to the enterprise as expensive financial obligations. The global financial sector has implemented strict risk management frameworks to guarantee that projects are produced in a socially responsible manner reflecting good environmental practices in order to avert these situations.

The Role of the Equator Principles (EP) and DJSWI

The DJSWI (Dow Jones Sustainability World Index) benchmark, which provides information on how to manage environmentally sensitive funds, is often employed in tracking this process. However, the Equator Principles (EPs) are the critical benchmark for project financing. Equator Principles refer to a voluntary framework that aims at assessing and managing risks that may be encountered by projects in terms of the environment and society. Although they are not compulsory or legally binding, Equator Principles are considered the "gold standard" in banking. As per the Equator Principles Financial Institutions (EPFI), such banks would commit to applying these principles to their own internal policy and will refrain from offering loans to those clients who cannot comply. (Bustillo Revuelta, 2018).

EPFI Risk Review and Categorization

As part of the due diligence process, EPFIs categorizes projects based on the magnitude of potential environmental and social risks and impacts, utilizing the categorization process of the International Finance Corporation (IFC). This classification is essential for responsible risk decision-making:

- **Category A:** Projects with potential significant adverse environmental and social risks and/or impacts that are diverse, irreversible, or unprecedented.
- **Category B:** Projects with potential limited adverse environmental and social risks and/or impacts that are few, generally site-specific, largely reversible, and readily addressed through mitigation measures.
- **Category C:** Projects with minimal or no adverse environmental and social risks and/or impacts.

This classification legally acknowledges the significance of biodiversity, human rights, and climate change. According to the framework, adverse effects on communities and ecosystems must be minimized, mitigated, or offset if they cannot be prevented (Bustillo Revuelta, 2018). In the light of this subject, the work will take place in thesis focuses on the two critical environmental vectors:

1. **Carbon Shadow Pricing (P_{CO2}):** Underground operations generate intense Scope 1 and Scope 2 emissions. Dynamic cut-off grades can be stress tested against future carbon tax environments, expressed in dollars per ton of CO_2 equivalent ($\$/t_{CO2e}$).

2. **Water Shadow Pricing (P_{H2O}):** For operations like drilling, backfilling, large amounts of fresh water is needed. The long-term risk of regulatory limiting and advanced water treatment costs is theoretically recognized by applying a shadow price ($\$/m^3$) to water usage.

4.4.1 Mathematical Integration of ESG Factors into Operation Costs

To operationalize shadow pricing, the traditional unit costs are mathematically expanded. By this way, traditional deterministic costs (m and h) are transformed into "Sustainable Costs" (m_{sus} and h_{sus}), and traditional OPEX transformed into “Sustainable OPEX”. These transformations are indicated below:

<u>For Lane’s Theory creation:</u>

Mining cost per ton equation is constructed as follows (this function is valid for Mine Constrained Cut-off):

$$m_{sus} = m_{trad} + (E_c * P_{CO2}) + (E_w * P_{H2O})$$

Mineral processing cost per ton equation is constructed as follows (this function is valid for Mill Constrained Cut-off):

$$h_{sus} = h_{trad} + (E_c * P_{CO2}) + (E_w * P_{H2O})$$

Where:

m_{sus}: *ESG Shadow Pricing Implemented Mining Cost ($/t)*

m_{trad}: *Traditional Mining Cost ($/t)*

h_{sus}: *ESG Shadow Pricing Implemented Processing (Milling) Cost ($/t)*

h_{trad}: *Traditional Processing (Milling) Cost ($/t)*

E_c: *Carbon Dioxide Emission (t)*

E_w: *Water Usage Amount (t)*

P_{CO2}: *Shadow Price for CO2 Emission ($/t)*

P_{H2O}: *Shadow Price for Water ($/t)*

After these parameters have changed, cut-off grade equations for different constraints created by Lane must be transformed into these forms:

Mine Constrained Cut-off:

$$g_{m,sus} = \frac{m_{sus}}{(s-r)*y}$$

Mill Constrained Cut-off:

$$g_{h,sus} = \frac{h_{sus} + \frac{f+F_{sus}}{H}}{(s-r)*y}$$

Refinery Constrained Cut-off:

$$g_{k,sus} = \frac{h_{sus}}{(s-r)*y - \frac{f+F_{sus}}{K}}$$

Where:

- *(s - r): Value of 1% of CuEq ($/t) – Difference is implemented as using recovery and payability function directly inside the calculation of Value of 1% of CuEq*
- m_{sus}*: Mining cost per unit of material mined ($/t)*
- h_{sus}*: Processing cost per unit of ore milled ($/t)*
- *f: Fixed overall costs per period ($/t)*
- F_{sus}*: Opportunity cost of capital (ESG_discount rate * NPV)*
- *y: Metallurgical recovery rate (%)*
- *H: Maximum milling capacity (t/year)*
- *K: Maximum marketing/refining capacity (t/year)*

Also, classical way of calculating cut-off grade which is Break-even Theory equation transforms to sustainable version as:

$$g_{be,sus} = \frac{m + h + f + C_{ESG}}{(s - r) * y}$$

Where:

- *m: Mining Cost ($/t)*
- *h: Processing or Milling Cost ($/t)*
- *f: Fixed Costs ($/t)*
- *C_{ESG}: Shadow Price of C_{O2} and Water ($/t)*
- *(s - r): Value of 1% of CuEq ($/t) – Difference is implemented as using recovery and payability function directly inside the calculation of Value of 1% of CuEq*
- *y = Metallurgical recovery rate (%)*

When it comes to OPEX, this shifting from traditional costs to the sustainable cost with the help of shadow pricing will automatically transform it to "Sustainable OPEX". It can be demonstrated as:

$$OPEX_{TRADITIONAL}: m + h + f$$

$$OPEX_{SUSTAINABLE}: m + h + f + C_{ESG}$$

Where:

- *m: Mining Cost ($/t)*
- *h: Processing or Milling Cost ($/t)*
- *f: Fixed Costs ($/t)*
- *C_{ESG}: Shadow Price of CO_2 and Water ($/t)*
- *(s - r): Value of 1% of CuEq ($/t) – Difference is implemented as using recovery and payability function directly inside the calculation of Value of 1% of CuEq*
- *y: Metallurgical recovery rate (%)*

4.5. ESG Performance and the Cost of Capital (Green Financing)

Selection of an appropriate discount rate (i) is one of the key elements in evaluating the viability of the project in the economic evaluation of mineral resources. To account for the cost of the financing sources, the practice involves using the Weighted Average Cost of Capital (WACC). S-NPV is sensitive to this rate since mining projects are always highly capital-intensive and risky. (Bustillo Revuelta, 2018).

The Mechanics of Green Financing:

As Falcone (2020) says, specific strategy used for funding projects aimed at promoting sustainability is called "green financing". There are different types of financial services in the mining industry that include the use of green bonds, sustainability-linked loans, and ESG funds, including mutual and index funds (Bhatnagar & Sharma, 2022). Such instruments allow raising funds from various private sources for initiatives that facilitate better waste management, water preservation, and carbon reduction.

The ESG performance of a project directly modifies the cost of debt (k_d) under the WACC framework. Projects are categorized by financial institutions according to the extent of their possible risks. Lenders apply a substantial "risk premium," significantly raising k_d for operations that do not adhere to the Equator Principles or have high carbon and water intensities. On the other hand, green bonds and credits are available to high-performing ESG projects, which may significantly reduce interest rates (Ma et al., 2024).

Following the logic proposed by Spearing (2023), to implement this relation quantitatively in strategic mineral economics, the discount rate of the company must be regarded as an ESG-driven variable rather than a fixed value. High environmental performance inherently de-risks the asset, modifying the baseline Weighted Average Cost of Capital ($WACC_{TRAD}$) into an ESG-adjusted funding cost ($WACC_{ESG}$):

$$WACC_{ESG} = \left(\frac{E}{V} \times k_e\right) + \left(\frac{D}{V} \times k_d^{ESG} \times (1 - t)\right)$$

Where:

- $E/_V$, $D/_V$: *Respective Weights of Equity and Debt Financing*
- k_e: *Cost of Equity*

- *t: Corporate Tax Rate*
- *k_d^{ESG}: Optimized Cost of Debt*

Consequently, this systemic reduction in capital constraints is directly injected into the life-of-mine evaluation engine, mathematically shifting the dynamic operational discount rate (d_{TRAD}) into a sustainable discount rate (d_{ESG}):

$$d_{ESG} = d_{TRAD} - \Delta\delta_{ESG}$$

Where:

- *d_{ESG}: Sustainable Discount Rate*
- *d_{TRAD}: Traditional Discount Rate (typically assumed as 10% in industrial baselines)*
- *$\Delta\delta_{ESG}$: Quantifiable Deduction or Green Financing Dividend Achieved*

Impact on Operational Performance:

Adoption of green finance principles is becoming more widely acknowledged as a catalyst for actual environmental progress as opposed to merely financial gain. Recent research has shown a quantitative relationship between green investments and emission reductions; for example, a 1% increase in green finance has been associated with lower CO2 emissions in several economies (Chu et al., 2024). This implies that reduced finance costs give businesses the liquidity they need to implement more sustainable technologies and responsible mining practices.

Mitigating the Risk of Greenwashing:

Although green bonds are an important way to raise money, it is still very difficult to make sure that the money is used for real sustainable initiatives rather than corporate debt or everyday operating expenses, a technique known as "Greenwashing" (Cheng & Wu, 2024).

In this regard, a reliable indicator of a company's true commitment is the Sustainable NPV (S-NPV) framework established in the case study in the scope of this thesis, which internalizes environmental externalities through shadow pricing.

Strategic Financial Alignment:

The inclusion of ESG performance in the calculation of the cost of capital shows that sustainability is a significant financial driver. By ensuring de-risking of the project through incorporating environmental penalties and social stability within the business, a firm ensures reduction of its WACC. The reduction of the carbon footprint associated with extraction by both green bonds and credits makes this de-risking particularly essential for projects involving mineral extraction for green technologies. (Ma et al., 2024).

4.6. Contribution of This Chapter

This chapter's contribution is filling the gap between environmental sustainability and mining economics by mathematically transforming traditional operating costs and cutoff grades into sustainable, environmental parameters. Instead of treating carbon emissions or water usage as soft corporate social responsibility concepts, this framework applies environmental shadow pricing to permanently modify the mine's variable OPEX based on site-specific ecological footprints. Simultaneously, the chapter demonstrates that corporate environmental compliance is rewarded by green financing mechanisms, which directly lower the project's cost of debt (k_d) and its overall sustainable discount rate (d_{ESG}).

5. STOCHASTIC UNCERTAINTIES IN SUSTAINABLE MINE PLANNING

5.1. The Limitations of Deterministic Optimization in Mine Economics

A Sustainable Net Present Value (S-NPV) is successfully established by including Environmental, Social, and Governance (ESG) variables into dynamic cut-off grade equations. However, a serious strategic weakness is introduced when this framework is applied in a rigorously deterministic manner.

Kenneth Lane developed the fundamental theory of cut-off grade optimization, which primarily depends on maximizing the Net Present Value (NPV) under three main capacity constraints: mining, milling, and refining (Lane, 1988). While traditional deterministic mine planning implies that all input parameters are constant, single-point estimates known with absolute certainty throughout the Life of Mine (LOM), this mathematical approach is still the industry standard for generating optimal production profiles (Ahmadi, 2018).

However, the underground mining environment is quite unstable by nature. When a long-term strategic cut-off grade strategy is developed using static averages, it might lead to a risky assumption of certainty. A deterministic cut-off grade strategy is totally out of alignment when actual market conditions collapse, or projected ore grades diverge from the block model. According to recent optimization frameworks, disregarding geological uncertainty inevitably results in poor operative decision variables, which can lead to severe cash flow deficits or early consumption of reserves (Toro Morales et al., 2024).

In addition to this, dynamic risk management is necessary for sustainable extraction in this age within the mining industry (Spearing, Ma, & Ma, 2023). An example of mathematical uncertainty includes the spatial variability of the ore body and the volatile nature of metal prices. It is important for such uncertainties to be simulated and quantified mathematically for the development of a stochastic model for the S-NPV.

5.2. Quantifying Geological Uncertainty: Geostatistics and Spatial Variability

The spatial variability of the ore body is the most fundamental source of uncertainty in any mining operation. Blocks in the resource model are often assigned a single, smoothed average grade in conventional deterministic planning. Nevertheless, this oversimplification obscures the actual local variability of mineralization can lead to poor operative decision-making and inaccurate cut-off grade applications.

Mine planning uses geostatistics to measure and incorporate this geological uncertainty into the process of economic optimization. Geostatistics is based on the fundamental geological assumption of spatial correlation, in contrast to traditional statistics, which suggests that data points are entirely independent. It recognizes mathematically that samples collected in close range to one another are more likely to have comparable grades than those obtained farther apart (Bustillo Revuelta, 2018). In the scope of this thesis work case study, discrete Unit approach is implemented. This decision is caused by the practical availability of verified resource envelopes for the Çayeli Mine, allowing the framework to focus on the economic and ESG-related risks while respecting the existing geological reporting

5.2.1. Analyzing Spatial Continuity: The Semi Variogram

The mathematical representation of the spatial relationship of drill hole data is required before making predictions regarding the grades of the blocks. In geostatistics, the semi variogram, known as the variogram also, represents the mathematical model of spatial continuity in terms of the average differences between data points as a function of distance apart. According to Bustillo Revuelta (2018), half the average squared difference of the components of the data pairs, at each distance h in a specific direction, gives the experimental semi variogram γ(h). The equation is as follows:

$$\gamma\ (h) = \frac{1}{2N(h)} \ \sum_{\text{i}=1}^{N(h)}[(xi) - (xi + h)]^2$$

Where:

- N(h) is the number of pairs of data points separated by the distance h.

- $z(x_i)$ is the value of the variable (e.g., copper grade) at location x_i.
- $z(x_i + h)$ is the value at a location h distance away.

The geological assumption made regarding the similarity of samples depending on their proximity is also evident from the semi variogram; therefore, $\gamma(h)$ increases as h increases. The experimental variogram should be modeled theoretically before it can be used for estimation purposes. Three key characteristics that characterize the mineralization's structure define this model, as seen in Figure 3 from the Geostatistical Association of Southern Africa (GASA):

- **Nugget Effect (C_0):** This represents the vertical discontinuity at the origin (h to 0). It accounts for micro-scale geological variability that occurs at distances smaller than the sampling interval, as well as inherent sampling and assaying errors.
- **Sill ($C + C_0$):** This is the plateau level where the semi variogram levels off. It represents the total variance of the regionalized variable. At this stage, the distance between samples is so great that there is no longer a spatial correlation between them.
- **Range (a):** The specific lag distance at which the semi variogram reaches the sill. Within this distance, samples are spatially correlated and can provide information about one another. Beyond the range, samples behave as independent random variables.

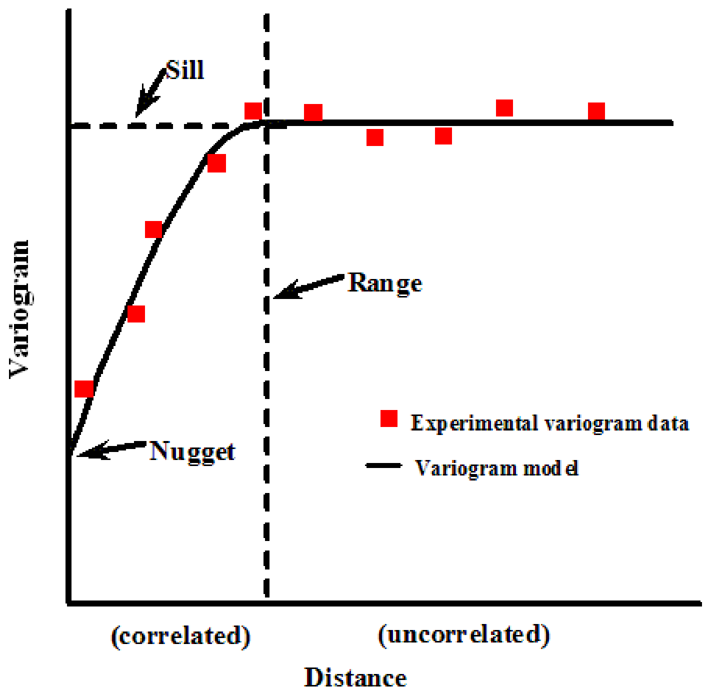

Figure 3. Experimental variogram data and fitted variogram model illustrating the nugget, sill, and range parameters. (GASA)

5.2.2. Estimation Techniques: From Geometric to Geostatistical Methods

Following the mathematical determination of spatial continuity using the semi-variogram, the next strategic requirement is the estimation of grades for unmined blocks inside the resource model. The industry has traditionally employed two distinct ways of thinking for this process: traditional geometric techniques and geostatistical estimation.

- **Traditional Deterministic Methods (Geometric Estimation)**

These techniques often disregard the underlying spatial correlation or geological anisotropy in favor of purely geometric computations based on physical distance. These weighing methods are constrained by their inability to take into consideration the mineralization's spatial structure, claims Bustillo Revuelta (2018):

Nearest Neighbor (NN): The most traditional method. The grade of the single drill hole sample that is physically closest to a block is immediately assigned. It is completely unsuitable for rigorous strategic planning since it ignores variation and sample density, which results in the block model's sharp, artificial "polygon-like" boundaries.

Inverse Distance Weighting (IDW): IDW gives nearby drill hole samples weights that are inversely proportionate to their distance; closer samples have a greater effect. Nevertheless, IDW has two significant mathematical shortcomings: it is unable to account for data clustering, interpreting closely spaced drill holes as entirely independent data points and skewing the estimate, and it is unable to identify geological anisotropy (directional bias).

- **Geostatistical Methods (The Kriging Family)**

Kriging is currently the preferred approach to estimating mineral resources because it was developed to address the limitations of geometrical methods. Kriging is a form of weighted mean where the weights are assigned such that the resulting estimator will be unbiased with minimum estimation variance (Bustillo Revuelta, 2018).

Simple Kriging (SK): It is based on the idea that the domain's mean grade is known and is perfectly constant throughout the entire deposit. It is rarely used in practical mining since local means naturally fluctuate.

Ordinary Kriging (OK): It is the most used technique. OK just considers the "local" mean inside the search radius rather than assuming a global mean. OK yields the Best Linear Unbiased Estimator (BLUE) by quantitatively reducing the estimation variance.

Mathematically, the Kriged estimate $Z^*(x_0)$ is a weighted average:

$$Z^*(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i)$$

The optimal weights (λ_i) are obtained by solving the Kriging system:

$$\sum_{j=1}^{n} \lambda_j \gamma(x_i, x_j) + \mu = \gamma(x_i, x_0)$$

for i = 1,....,n

Where μ is the Lagrange multiplier ensuring unbiasedness ($\sum_{j=1}^{n} \lambda_i = 1$).

Indicator Kriging (IK): A non-parametric method developed for highly skewed data distributions (e.g., gold deposits). Rather than estimating the actual grade directly, IK calculates the probability that a block's grade falls above or below a specific cut-off threshold.

The Limitations of Kriging and the Smoothing Effect

Ordinary Kriging (OK) exposes a basic mathematical weakness that is frequently disregarded in conventional planning while aiming for the optimization of NPV. Yamamoto (2005) proved that the smoothing effect is a dangerous feature of ordinary kriging. Global accuracy is sacrificed in order to meet the method's aim of being an exact estimator with low error variance in the least-squares sense. According to Yamamoto (2005), the smoothing effect results in two significant distortions that have a direct impact on mineral economics:

1. **Failure to Reproduce Statistics:** Kriging estimates do not reproduce the histogram (the distribution of grades) or the semi variogram model of the original sample data.
2. **Inherent Bias in Extremes:** Small values are systematically overestimated (marginal waste is seen as ore), and large values are systematically underestimated (high-grade zones are diluted).

This is proven by the Kriging Variance equation (σ^2_K):

$$(\sigma^2_K) = \sum_{i=1}^{n} \lambda_i \gamma(x_i, x_0) + \mu$$

Because the algorithm strictly minimizes this variance, the resulting block model inherently lacks the natural dispersion of the original deposit.

5.2.3. Modeling Uncertainty: Conditional Simulation

Kriging can be problematic in dynamic cut-off grade economics, because operational choices between extremely high and low grades determine financial survival. Planning needs to switch to conditional simulation to fully stress-test the S-NPV model.

In this approach, the simulated value is drawn from a Conditional Cumulative Distribution Function (CCDF):

$$F(x; z \mid (n)) = P(Z(x) \leq z \mid (n))$$

Where:

- ***F(x; z | (n))***: *Conditional Cumulative Distribution Function at spatial location x for a limiting grade threshold z.*
- ***Z(x)***: *Regionalized random variable for the true, unknown ore grade at the spatial coordinates x.*
- ***z***: *Target grade threshold or cutoff value evaluated within the continuous distribution.*
- ***(n)***: *Conditioning dataset containing the spatial positions and measured grades of the nearest surrounding drill hole samples.*
- ***(***$\boldsymbol{P}(\ldots \mid (\boldsymbol{n}))$: *Conditional probability operator restricted by the existing physical datasets (n).*

Several probable findings of the entire ore body are produced using conditional simulation. Every realization includes the whole, unsmoothed spectrum of local grade variability and extreme values while accurately reproducing the semi variogram model and respecting the original drill hole data. Operations can successfully protect themselves from subsurface surprises by appropriately assessing operative choice factors under realistic geological uncertainty by executing the dynamic cut-off grade algorithm over dozens of these conditionally simulated models (Deutsch & Journel, 1997).

5.3. Market Volatility and Metal Price Fluctuations

Market volatility is one of the most significant external risk to an underground mining operation's financial viability, even though geological uncertainty (modeled via conditional simulation) poses the most significant internal risk. Deterministically predicted price trajectories are a major component of traditional dynamic cut-off grade algorithms, which were first developed by Kenneth Lane (Lane, 1988). Long-term metal prices, however, are uncertain due to several factors, including inflation rates, geopolitical tensions, unforeseen changes in the global macroeconomy, and the rapid migration to green energy infrastructure.

Using a deterministic, single-line price forecast exposes the Sustainable Net Present Value (S-NPV) optimization model to severe downside risks. The mathematical danger lies in Lane's (1988), opportunity cost formulation:

$$F = \delta \times NPV$$

The algorithm computes an inflated future net present value (NPV) if a strategic mine plan is based on a deterministically high and steady metal price. This results in a large opportunity cost (F). The algorithm reduces the cut-off grade to process higher volumes of marginal ore in order to maximize immediate cash flow under these circumstances. However, if the actual market price suddenly decreases, a common occurrence in cyclical commodity markets, the operation is left extracting and processing low-grade material that no longer generates enough revenue to cover the inflated sustainable operating costs (m_{sus}, h_{sus}). This leads to a negative cash flow.

A strong S-NPV model must eliminate static pricing decks and use stochastic processes to simulate metal price volatility in order to quantitatively eliminate this. For this, two main stochastic models are used in mining economics:

- **Geometric Brownian Motion (GBM):** GBM, on the other hand, represents a popular stochastic process used in estimating the prices of valuable commodities, mainly metals. According to Ramos et al. (2018), the GBM is useful in explaining the stochastic nature of commodity prices using a continuous time random walk approach. Price change S is modeled using the stochastic differential equation:

  The continuous-time stochastic differential equation for GBM is:

$$dS_t = \mu \times S_t dt + \sigma S_t dW_t$$

Where:

- μ*: Drift Coefficient*
- σ*: Volatility*
- dW_t*: Wiener process.*

- **Mean-Reverting Processes:** This technique is widely used for commodity price forecasting. Contrary to stocks, there is a physical limit for the mineral commodity. Opening of new mines occurs due to increase in price of metals, resulting in oversupply of minerals, causing fall in prices. On the other hand, when the price falls below the world marginal cost of production, then unprofitable mines shut down, thus decreasing the supply and increasing prices. The mean-reverting process models this elasticity.

$$dX_t = \kappa(\theta - X_t)dt + \sigma_{esg} \cdot dW_t$$

When discretized for financial simulation in spreadsheet models, the yearly propagation equation takes the following structure:

$$X_t = X_{t-1} + \kappa(\theta - X_{t-1})\Delta t + \sigma_{esg} \cdot \epsilon$$

Where:

- X_t***:*** *Current year's ESG cost.*
- X_{t-1}***:*** *Previous year's ESG cost.*
- θ***:*** *Long-term mean level (8.05$ base cost target).*
- κ***:*** *Speed of mean reversion (0.3, correction velocity).*
- σ_{esg}***:*** *Volatility coefficient ($1.0 standard deviation).*
- Δt***:*** *Time increment ($1 year step).*
- ϵ***:*** *Standard normal random shock.*

In the scope of this thesis, Geometric Brownian Motion (GBM) is operationalized with an annual time step ($\Delta t = 1$) over a 12-year Life of Mine (LOM) horizon to simulate the non-stationary, unpredictable trajectory of copper prices. To ensure empirical rigor, the price trend is driven by a conservative drift coefficient $\mu = 0.05$, accounting for long-term global inflation and structural demand from the green energy transition, while the price volatility parameter within the stochastic differential equation is strictly calibrated as $\sigma = 0.40$ (40%). This volatility value is explicitly justified by long-term historical spot price dynamics on the London Metal Exchange (LME).

Over a ten-year cyclical horizon, copper prices exhibited intense macroeconomic structural shifts, moving from a multi-year low of 2.05 $/lb through the baseline planning evaluation price of 4.10 $/lb, and reaching historic highs of 6.34 $/lb. This historical dispersion confirms that a 40% volatility profile captures the structural market risks.

Conversely, a Mean-Reverting Ornstein-Uhlenbeck process is integrated into the framework to model the policy-driven, regulated boundaries of environmental costs, reflecting the operational reality that regulatory penalties inherently gravitate toward an institutional equilibrium. The long-term reversion level (θ) is anchored at the operation's baseline of 8.05$/t, with the mean reversion speed calibrated at $\kappa = 0.3$ to ensure cost shocks are normalized within a realistic 3-year window under a controlled volatility coefficient ($\sigma_{esg} = 1.0\$$). The multi-variable execution and final numerical outputs of this stochastic calibration will be dynamically tested against the production constraints of the Çayeli Mine within the comprehensive numerical implementation in Chapter 6.

5.4. Monte Carlo Simulation for Sustainable Net Present Value (S-NPV)

To fully synthesize the internal geological uncertainties with external market volatility, and to integrate the expanded ESG parameters established in Chapter 4, the optimization framework must conclude in a comprehensive Monte Carlo Simulation.

In the context of this thesis, Monte Carlo simulation serves as a computational synthesis engine rather than just a tool for risk assessment. It computes the maximizing of the Sustainable Net Present Value (S-NPV), multi-variable objective function, using repeated random sampling.

The stochastic objective function across N iterations is formulated as:

$$S\text{-}NPV = E\left[\sum_{t=1}^{LOM} \frac{S_t * Q_{metal} - Q_{metal} * (C_{OPEX} + C_{ESG})}{(1+d_{ESG})^t}\right]$$

S_t: Stochastic Metal Price (\$/t) / Value of 1% of CuEq

Q_{metal}: Tonnage (t)

d_{ESG}: ESG Discount Rate (%)

C_{OPEX}: Sustainable Operating Cost (\$/t)

C_{ESG}: Environmental Penalties (\$/t)

Rather than executing Lane's algorithm once under average conditions, the Monte Carlo framework executes the dynamic cut-off grade algorithm through 1000 automated iterations.

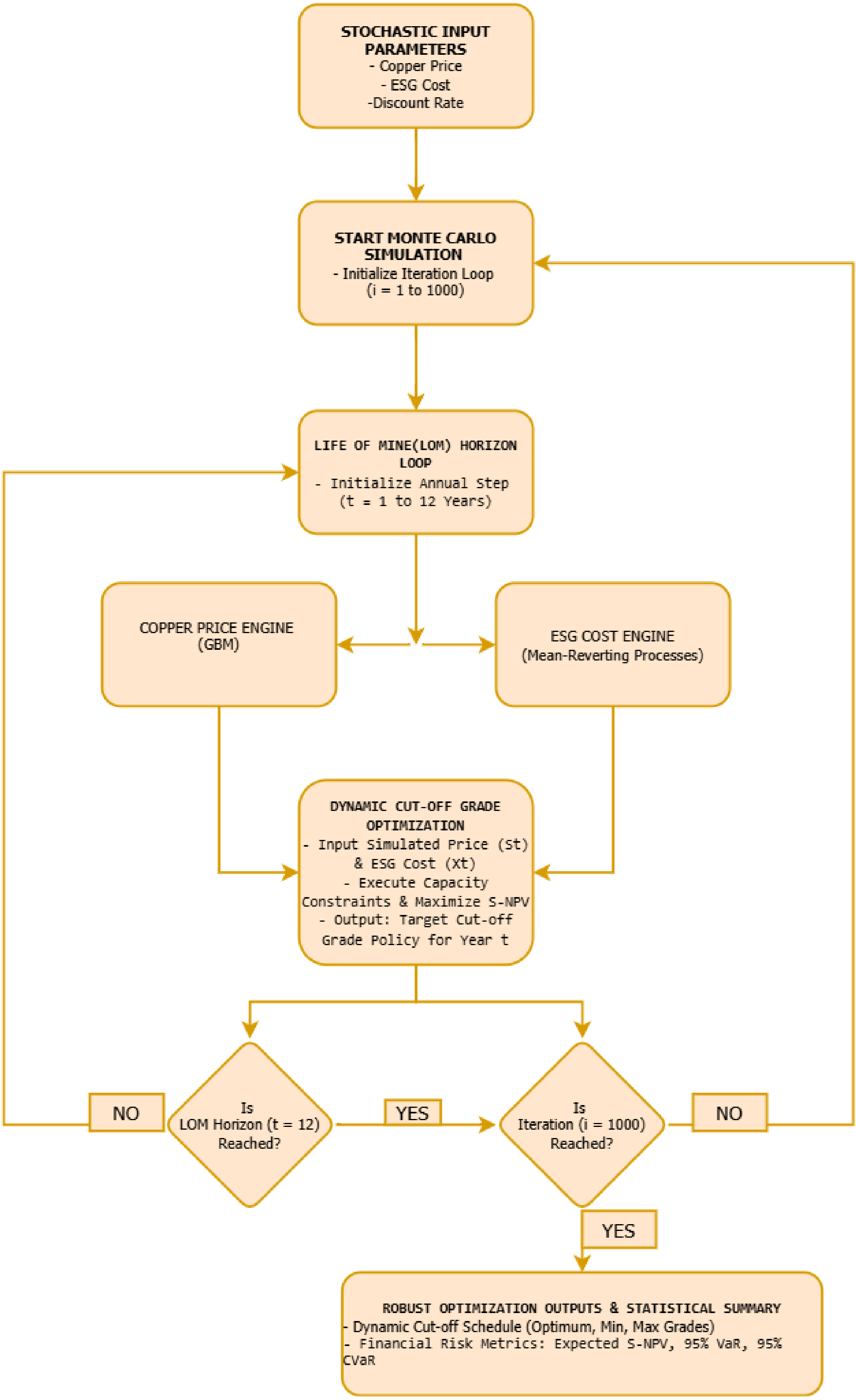

Figure 4. Monte Carlo Simulation Algorithm

The Algorithmic Architecture of the Simulation:

For every iteration in the Monte Carlo process, the computational model executes the following sequence:

- **Resource Scenario Selection (Geological Baseline):** The algorithm incorporates the grade and tonnage distributions of the verified mining zones (Main Orebody and South Orebody Discrete Units) as the geological foundation for the production profile.
- **Market Draw:** The algorithm pairs the resource data with one stochastic price trajectory generated via historical volatility data, covering the 12-year Life of Mine (capturing revenue volatility).
- **ESG Parameter Integration:** The algorithm applies the specific environmental penalties (P_{CO2}, P_{H2O}) and models the risk-adjusted discount rate (d_{ESG}) based on the sustainability-linked financing framework.
- **Dynamic Optimization:** Subject to the physical capacity constraints (M, H), the algorithm evaluates the project's performance under that unique combination of price and cost variables.
- **Output Recording:** The model records the resulting S-NPV for each of the 1,000 iterations.

Analyzing the Output: VaR and CVaR

The result produces a comprehensive probability distribution for the S-NPV. The probabilistic approach enables one to identify essential measures for institutional risks. According to Allen et al. (2012), conventional risk measurements usually ignore potential negative outliers in mining operations. Thus, in this study:

- **Value at Risk (VaR):** It determines the highest expected loss in financial terms within a specified period and with a certain degree of confidence. When a 95% VaR is used, it means that the probability of falling below a certain base level of the difference between S-NPV and traditional NPV is just 5%, even during extremely adverse geological and economic conditions. VaR can be mathematically expressed as:

$$P(S\text{-}NPV < VaR_{\alpha}) = 1 - \alpha$$

- **Conditional Value at Risk (CVaR):** Also known as Expected Shortfall, CVaR calculates the average expected S-NPV in the absolute worst-case scenarios (the extreme 5% tail end of the distribution). This metric is aggressively scrutinized by international lenders offering Green Financing, as it proves the project's ultimate survivability against ESG-related shocks and severe operational collapses. Mathematically, it is formulated as:

$$CVaR_{\alpha} = E[S\text{-}NPV \mid S\text{-}NPV \leq VaR_{\alpha}]$$

Indeed, through the identifying of the single dynamic cut-off grade strategy that produces the maximum mean value of S-NPV, optimal solutions are found. Through such a stochastic S-NPV model, there exists mathematical certainty that the underground mine project will be financially viable, environmentally sound, and socially acceptable in all circumstances.

5.5. Contribution of the Chapter to the Thesis

This chapter establishes the core computational engine of the thesis by converting static mine planning into a dynamic, risk-quantified stochastic optimization framework. This approach incorporates non-stationary Geometric Brownian Motion (GBM) price trajectories and mean-reverting stochastic ESG processes directly into Lane's capacity-constrained equations, whereas conventional mining models regard environmental costs and commodity prices as predictable averages. In light of this integration, the cut-off grade policy may respond instantly to the simultaneous modeling of regulatory cost shocks and raw market volatility. Additionally, this section changes the optimization aim from a brittle "maximum profit" estimate to a "Robust Optimization" model by incorporating sophisticated risk measurements (95% VaR and 95% CVaR) into the S-NPV framework. Ultimately, this chapter provides the exact mathematical foundation and nested simulation loops required to execute, test, and validate the empirical production schedules for the Çayeli Mine in the following chapter.

6. CASE STUDY AND FRAMEWORK OVERVIEW

The previous chapters established the theoretical foundations of strategic mine planning by reviewing the evolution of cut-off grade optimization, the integration of Environmental, Social and Governance (ESG) principles into mining economics, and the role of stochastic uncertainty in long-term decision making. While these concepts have been widely investigated individually, their practical integration into a unified strategic planning framework remains limited.

The objective of this chapter is therefore not only to apply existing mine planning methodologies to the selected case study, but also to demonstrate the practical implementation of the integrated framework proposed in this thesis. Unlike conventional deterministic planning approaches, the proposed methodology combines three complementary components within a single decision-support framework:

- the dynamic cut-off grade optimization principles originally developed by Lane (1988);
- the monetization of environmental externalities through ESG shadow pricing incorporated directly into the operating cost structure; and
- the stochastic representation of market uncertainty using Monte Carlo simulation for evaluating Sustainable Net Present Value (S-NPV).

The originality of the proposed framework does not lie in introducing a completely new optimization algorithm, but rather in integrating established concepts from mineral economics, environmental management, and stochastic risk analysis into a coherent methodology suitable for strategic underground mine planning. This integrated perspective enables the evaluation of mining strategies not only according to their economic performance under deterministic assumptions, but also according to their resilience under market uncertainty and increasing environmental constraints.

The Çayeli underground copper-zinc mine is therefore employed as a representative case study to verify the applicability of the proposed framework. The comparison between the traditional deterministic approach and the Sustainable Net Present Value (S-NPV) framework illustrates how ESG-related costs and stochastic uncertainty influence cut-off grade selection, project value, and long-term strategic decision making. Consequently, this chapter represents the validation stage of the research, where the conceptual framework developed throughout the thesis is translated into a practical decision-support methodology.

6.1. Overview of the Selected Operation

6.1.1. Location and Operational Context

The Çayeli Mine is in the Rize Province of Türkiye, on the eastern coast of the Black Sea. It is one of the most important underground base-metal operations in the area. Çayeli Bakır İşletmeleri A.Ş. (ÇBİ) runs the site, which has been in production since 1994. Being close to the coast makes it easier to move concentrates, but the Pontides region's mountainous terrain makes it harder to manage surface waste and build infrastructure.

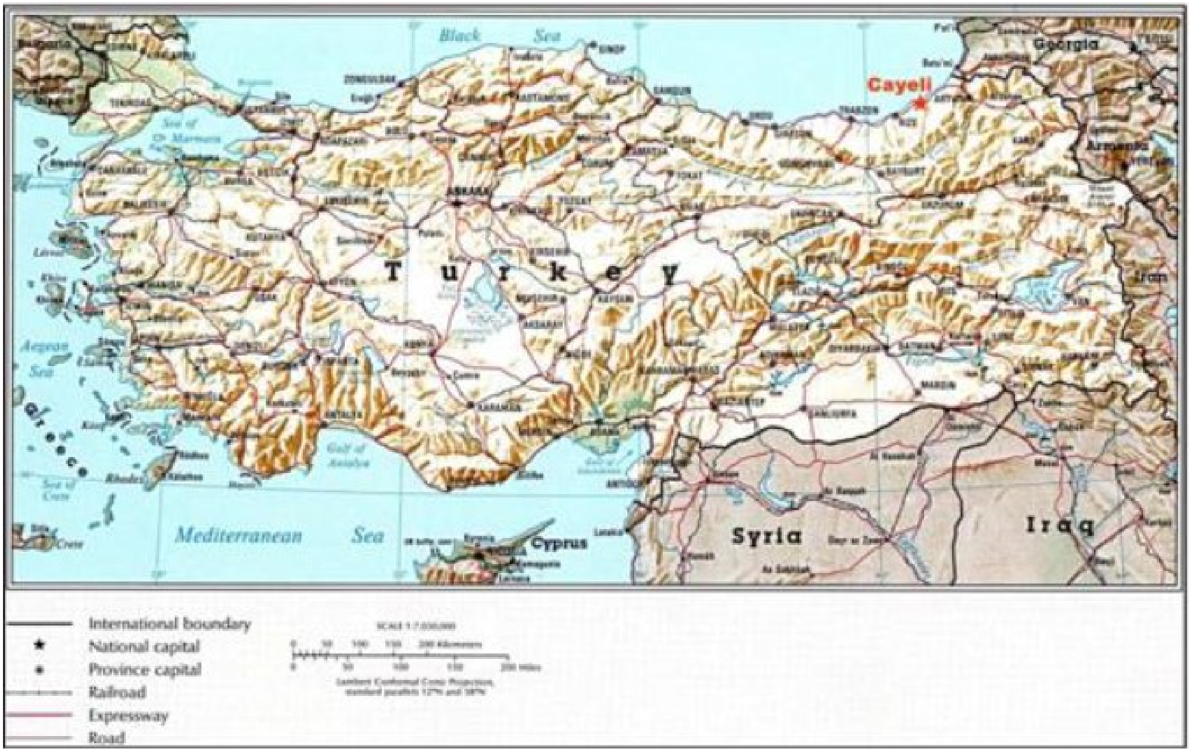

Figure 5.Location of the Çayeli Operations, Rize Province, Türkiye

6.1.2. Geological Setting and Mineralization

The deposit is a typical Volcanogenic Massive Sulphide (VMS) system that is found in felsic volcanic rocks from the Upper Cretaceous era. There are two main areas of mineralization: the Main Orebody and the South Orebody.

- **Main Orebody:** This is a sulphide core that is mostly made up of pyrite, chalcopyrite, and sphalerite.

- **South Orebody:** Has both massive and stringer (stockwork) mineralization.

The geological complexity comes from the sharp but irregular boundaries that separate the high-grade massive ore from the lower grade stockwork zones. The Cu-Zn zoning in the metal distribution changes, which makes it hard to control the grade and means that a more advanced method is needed to figure out the cut-off grade so that ore and waste don't get mixed up.

6.1.3. Mining Methodology and Physical Constraints

Mining is carried out only underground, primarily using long-hole open stoping with paste backfill. The need for high extraction ratios and the ability to keep the ground stable at greater depths are what led to the decision to use this method.

The mine's production capacity (M) is mainly constrained by how quickly it can move upward and downward and how many stopes are available in the current development cycle. As the mine goes deeper, the longer haul distances and more ventilation need cause mining costs to rise in a non-linear way.

6.1.4. Mineral Processing and Recovery

The run-of-mine (ROM) ore is processed through a sequential flotation circuit to produce separate copper and zinc concentrates.

1. **Commination:** Primary crushing followed by a semi-autogenous (SAG) and ball milling circuit.
2. **Flotation:** Selective flotation stages where copper is recovered first, followed by zinc.
3. **Recovery Performance:** The metallurgical response is sensitive to the feed grade and the Cu:Zn ratio. Lower head grades often result in reduced recovery efficiency, a factor that is central to the "Milling Constraint" (H) in Lane’s optimization theory.

6.1.5. Consolidated Technical and Economic Inputs

The following parameters, derived from the 2025 NI 43-101 Technical Report, serve as the definitive inputs for the baseline deterministic model.

The total available mineral inventory targeted within the continuous production planning horizon is constrained by a global resource envelope of 7,312,253 tons of polymetallic ore. The technical design dictates a fixed maximum underground extraction rate (M) of 0.84 Mtpa, whereas the nominal processing facility capacity (H) is rated at 1.00 Mtpa.

To guarantee structural consistency across all comparative evaluations, a definitive methodological distinction is enforced regarding the system boundaries and cost functions of the two evaluated planning frameworks:

A. Static Break-Even Model Formulation

The static Break-Even architecture operates strictly within a localized "Source-to-Stockpile" boundary. It isolates profitability up to the concentrate stockpile, completely omitting downstream commercial realization costs. The baseline traditional and sustainable total operational cost functions ($OPEX_{BE}$) are formalized as follows:

$$\mathrm{OPEX}_{\mathrm{BE\_Trad}} = m + h + f$$

$$\mathrm{OPEX}_{\mathrm{BE\_TRAD}} = 31.16 + 26.86 + 19.37 = 67.41\ \$/\mathrm{t}$$

$$\mathrm{OPEX}_{\mathrm{BE\_ESG}} = m + h + f + C_{ESG}$$

$$\mathrm{OPEX}_{\mathrm{BE\_ESG}} = 31.16 + 26.86 + 19.37 + 8.05 = 75.46\ \$/\mathrm{t}$$

Where:

- *m = 31.16$\t, Mining Cost*
- *h = 26.86 $\t, Milling Cost*
- *f = 19.37$\t, G&A Cost*
- *C_{ESG} = 8.05$\t, ESG Shadow Pricing*

B. Dynamic Lane Model Formulation (Mine-to-Market Boundary)

Lane's dynamic framework expands the architectural boundary to encompass the complete "Integrated Value Chain," tracking the asset from underground directly to concentrate sales markets. This boundary structurally incorporates downstream marketing, logistics, and refining variables by introducing Plant Costs and Concentrate Handling Costs directly into the cash flow equations. The operational cash flow cost functions ($OPEX_{LANE}$) are formalized as:

$$\mathrm{OPEX}_{\mathrm{LANE\_TRAD}} = \mathrm{m} + \mathrm{h} + \mathrm{f} + \text{Plant Costs} + \text{Concentrate Handling Costs}$$

$$\mathrm{OPEX}_{LANE\_TRAD} = 77.39\ \$/\mathrm{t\ ore}$$

$$\mathrm{OPEX}_{\mathrm{LANE\_ESG}} = \mathrm{m} + \mathrm{h} + \mathrm{f} + \text{Plant Costs} + \text{Concentrate Handling Costs}$$

$$\mathrm{OPEX}_{LANE\_ESG} = 77.39 + 8.05 = 85.44\ \$/\mathrm{t\ ore}$$

The baseline commodity evaluation relies on London Metal Exchange (LME) spot terms at 4.10 \$/lb for copper and 1.20 \$/lb for Zinc. Financial parameters prescribe a traditional corporate cost of capital at a deterministic 10.0% discount rate. Upon validating shadow prices, the activation of international sustainability-linked green financing mechanisms compresses the operational risk profile, yielding an ESG-adjusted discount rate (d_{ESG}) of 6.5%. Table 1. establishes the definitive, unified input parameters utilized across all subsequent computational optimizations.

Table 1. Parameters

Core Optimization Category	Specific Input Parameter	Value	Operational Unit
Physical Inventory	Global Resource Reservoir	7,312,253	Tons (t)
	Planned Life of Mine (LOM)	12	Years
System Capacities	Peak Underground Extraction (M)	0.84	Million Tons per Annum (Mtpa)
	Nominal Flotation Flank (H)	1	Million Tons per Annum (Mtpa)
Metallurgical Drivers	Copper Flotation Recovery	87.1	Percent (%)
	Zinc Flotation Recovery	68.5	Percent (%)
Macro Market Pricing	Base Copper Spot Price	4.1	$/lb
	Base Zinc Spot Price	1.2	$/lb
	Value of 1% CuEq Grade	74.79	$/t
Operational Expenditures	Baseline Underground Mining Cost (m)	31.16	$/t ore
	Processing Cost (h)	26.86	$/t ore
	General & Administrative / Selling (f)	19.37	$/t ore
	Internalized ESG Regulatory Penalty (C_{ESG})	8.05	$/t ore
Financial Parameters	Traditional Corporate Discount Rate (d_{trad})	10	Percent (%)
	Sustainability-Linked Green Discount (d_{ESG})	6.5	Percent (%)

6.2 Comparative Scenario Analysis: Break-Even vs. Lane Optimization

Core mechanics of ESG integration must be isolated across four distinct structural planning paths. This structural decomposition separates pure cost-driven selectivity from capital-driven de-risking mechanisms, processed simultaneously through the static Break-Even formulation (g_{be}) and Lane's dynamic capacity-constrained algorithm (g_h, g_m, g_k).

- **Scenario 1: Baseline Traditional Case:** This reference setup reflects standard strategic mine planning devoid of sustainability parameters (C_{ESG} = 0.00 \$/t) and discounted at the traditional discount rate (d = 10.0%).
- **Scenario 2: ESG Cost Only (Operational Effect):** This path internalizes the carbon and water shadow pricing vector (C_{ESG} = 8.05 \$/t), while holding the discount rate locked at 10.0% to strictly isolate pure operational cost escalation.
- **Scenario 3: ESG Financing Only (Financial Effect):** This scenario isolates the systematic impact of green financing; operating costs stay at traditional levels, but the discount rate drops to 6.5% due to sustainability de-risking.
- **Scenario 4: Full ESG Integration (Combined Case):** This configuration models the total ESG integrated operational environment, where the environmental cost expansion and the compressed cost of capital mitigation (6.5%) are considered simultaneously.

6.2.1. Static Break-Even Scenario Implementation

Under the static Break-Even architecture, the selection threshold operates strictly as a non-temporal economic indicator determined by immediate variable expenditures, governed by the following formulation:

$$g_{be} = \frac{\mathrm{OPEX}_{\mathrm{Total}}}{y \cdot (s - r)}$$

Where:

- *y: Recovery (%)*
- *(s - r): Value of 1% of CuEq (\$/t) – Difference is implemented as using recovery and payability function directly inside the calculation of Value of 1% of CuEq*

The implementation of the four scenarios through this static equation yields the following localized thresholds:

Scenario 1 & 3 (Traditional Baseline & Financing Only): Because the static architecture possesses no mathematical operators capable of capturing time-value parameters or opportunity costs, the cutoff is determined exclusively by traditional expenditures:

$$g_{be\_Trad} = \frac{67.41}{74.79} = 0.90\% \text{ CuEq}$$

Scenario 2 & 4 (ESG Cost Only & Full ESG Integration): The direct internalization of the sustainability vector (C_{ESG}= 8.05$\t) forces a linear upward shift in the cutoff grade, remaining pinned at 1.01% across both paths regardless of macroeconomic discount rate shifts:

$$g_{be\_ESG} = \frac{75.46}{74.79} = 1.01\% \text{ CuEq}$$

6.2.2. Dynamic Lane Capacity-Constrained Scenario Implementation

Executing the identical parameter matrices through Lane's dynamic optimization framework triggers a profound methodological divergence. Instead of processing a single fixed limit, Lane's algorithm models the enterprise as an integrated chain bounded by three separate physical infrastructure capacities: underground mining extraction capacity (M=0.84 Mtpa), nominal flotation processing capacity (H = 1.00 Mtpa), and downstream concentrate refining/sales capacity (K).

To demonstrate full mathematical verification of the multi-variable optimization loop, the specific computational values derived for all three constraints across the four strategic paths are structured as follows:

- **Scenario 1 (Baseline Traditional Case):** The system balances baseline opportunity costs and capacities at a traditional 10.0% discount rate. The mathematical resolution yields:

$$g_m = \frac{31.16}{74.79} = 0.42\% \text{ CuEq}$$

$$g_h = \frac{46.23 + \frac{F_{\text{Trad}}}{1.00}}{74.79} = 1.22\% \text{ CuEq}$$

$$g_k = \frac{26.86}{\left(74.79 - \frac{F_{\text{Trad}}}{52000}\right)} = 0.36\% \text{ CuEq}$$

- **Scenario 2 (ESG Cost Only Scenario):** The direct internalization of the sustainability penalty shifts the variable expenditure parameter. To protect early project value from this operational burden, the algorithm increases selectivity, driving the cutoff upward:

$$g_m = \frac{39.21}{74.79} = 0.52\% \text{ CuEq}$$

$$g_h = \frac{54.28 + \frac{F_{TRAD}}{1.00}}{74.79} = 1.33\% \text{ CuEq}$$

$$g_k = \frac{26.86}{\left(74.79 - \frac{F_{\text{TRAD}}}{K}\right)} = 0.37\% \text{ CuEq}$$

- **Scenario 3 (ESG Financing Only Scenario):** Compressing the operational discount rate to 6.5% deflates the present value penalty associated with delaying future high-grade ore extraction. This substantial contraction of the dynamic opportunity cost term (F_{ESG}) drops the active threshold significantly:

$$g_m = \frac{31.16}{74.79} = 0.42\% \text{ CuEq}$$

$$g_h = \frac{46.23 + \frac{F_{\text{ESG}}}{1.00}}{74.79} = 1.04\% \text{ CuEq}$$

$$g_k = \frac{26.86}{\left(74.79 - \frac{F_{\text{ESG}}}{K}\right)} = 0.36\% \text{ CuEq}$$

- **Scenario 4 (Full ESG Integration Scenario):** When operational cost expansion and capital cost compression are considered simultaneously, these two opposing vectors enter a trade-off. The final dynamic optimization equilibrium settles at a stable balance:

$$g_m = \frac{39.21}{74.79} = 0.52\% \text{ CuEq}$$

$$g_h = \frac{54.28 + \frac{F_{\text{ESG}}}{1.00}}{74.79} = 1.14\% \text{ CuEq}$$

$$g_k = \frac{26.86}{\left(74.79 - \frac{F_{\text{ESG}}}{K}\right)} = 0.37\% \text{ CuEq}$$

This empirical proof invalidates the conventional deterministic assumption that sustainability parameters act solely as a resource extraction burden, demonstrating instead that comprehensive ESG integration fundamentally re-calibrates the multi-variable optimization logic of the enterprise. Table 6.2 details the unified multi-scenario outputs across the comparative frameworks.

Table 2.Comperative Scenario Analysis Results

Evaluated Scenario Profile	Applied Optimization Model	Cut-off Grade (% CuEq)	Discount Rate (d, %)	Total LOM Net Present Value ($)
1. Baseline Traditional	Static Break-Even	0.9	10	380,324,778
1. Baseline Traditional	Dynamic Lane (g_h)	1.22	10	336,299,209
2. ESG Cost Only	Static Break-Even	1.01	10	344,813,171
2. ESG Cost Only	Dynamic Lane (g_h)	1.33	10	300,787,603
3. ESG Financing Only	Static Break-Even	0.9	6.5	445,783,447
3. ESG Financing Only	Dynamic Lane (g_h)	1.04	6.5	393,985,520
4. Full ESG Integration	Static Break-Even	1.01	6.5	404,002,554
4. Full ESG Integration	Dynamic Lane (g_h)	1.14	6.5	352,204,628

6.3. Dynamic Evolution of Cut-Off Grades and Opportunity Costs

A strategic flaw in traditional deterministic mine planning is the treatment of the cut-off grade as a static threshold throughout the life of the asset. In reality, the consolidated LOM average cut-off grades and cumulative Net Present Values (NPV) presented in Section 6.2 are the direct mathematical consequences of year-by-year operational scheduling decisions. As the high-grade core of a deposit is progressively extracted, the remaining orebody and the future net present value contract over time. This temporal degradation directly contracts the systemic Opportunity Cost of Capital (F_t), which represents the economic penalty incurred by delaying the extraction of remaining high-grade blocks to process marginal material in the current period.

To model this dynamic relationship across the 12-year LOM horizon for the Çayeli operation, the dynamic mill-constrained cutoff for any given year t is governed by the time-indexed Lane equation (in this part of the work, mill constrained cutoff is chosen because it is the maximum one among all three constrained scenarios of Lane Theory):

$$g_{h\_t} = \frac{h + \frac{F_t}{H}}{y \cdot (s - r)}$$

Where $F_t = \delta \cdot NPV_{t+1}$ represents the localized opportunity cost computed from the remaining present value of the asset from year t+1 to the end of the mine life. The empirical computational data generated by the year-by-year scheduling loop demonstrates a sharp, non-linear decline in this opportunity cost profile, driving the long-term convergence of the scenarios:

- **6.3.1. Traditional Baseline Path (d = 10.0%)**

  The dynamic scheduling engine executes a high-selectivity sequence in the initial phases due to the vast unextracted volume and high asset value of the remaining reserves.

  Initial Phase (Year 2025): The opportunity cost is anchored at a 75.15 \$/t, forcing the mill-constrained cutoff to peak at 1.62% CuEq to maximize immediate cash generation:

$$g_{h\_2025} = \frac{46.23 + \frac{75.15}{1.00}}{74.79} = 1.62\% \text{ CuEq}$$

Mid-Life Phase (Year 2030): As the reserve is depleted, the remaining future NPV decreases, dropping the traditional opportunity cost to 16.89 \$/t and contracting the active cutoff to 0.84% CuEq:

$$g_{h\_2030} = \frac{46.23 + \frac{16.89}{1.00}}{74.79} = 0.84\%\ \text{CuEq}$$

Final Phase (Year 2036): In the ultimate production year, the opportunity cost erodes entirely to 1.94 \$/t, collapsing the optimal cutoff to its absolute physical baseline of 0.64% CuEq:

$$g_{h\_2036} = \frac{46.23 + \frac{1.94}{1.00}}{74.79} = 0.64\%\ \text{CuEq}$$

This downward movement allows the operation to productively use marginal peripheral zones and low-grade stockpiles, preventing premature resource sterilization.

- **6.3.2. Full ESG Integration Path (d_{ESG} = 6.5%)**

When the environmental regulatory penalties (C_{ESG} = 8.05\$\t) and the green financing mechanisms are considered simultaneously, the temporal evolution highlights a value-stabilization effect across the production schedule.

Initial Phase (Year 2025): The inclusion of the 8.05 \$/t shadow costs pushes the total cost boundary upward, but the compressed green discount rate successfully suppresses the initial opportunity cost to 51.16 \$/t (compared to the traditional 75.15 \$/t). The sustainable mill cutoff settles at a balanced 1.41% CuEq:

$$g_{h\_2025}^{\text{ESG}} = \frac{54.28 + \frac{51.16}{1.00}}{74.79} = 1.41\%\ \text{CuEq}$$

Mid-Life Phase (Year 2030): The systematic cost compression flattens the opportunity cost slope over the LOM. By 2030, the sustainable opportunity cost drops to 12.76 \$/t, establishing a cutoff of 0.90% CuEq:

$$g_{h\_2030}^{\text{ESG}} = \frac{54.28 + \frac{12.76}{1.00}}{74.79} = 0.90\%\ \text{CuEq}$$

Final Phase (Year 2036): In the final extraction sequence, the sustainable cutoff concludes at a high floor of 0.75% CuEq:

$$g_{h\_2036}^{\mathrm{ESG}} = \frac{54.28 + \frac{1.62}{1.00}}{74.79} = 0.75\%\ \mathrm{CuEq}$$

This elevated cutoff according to the traditional case, mathematically confirms that under strict ESG compliance, the engine permanently rejects the lowest-grade, environmentally destructive material, prioritizing sustainable value creation over raw volume maximization.

The aggregated result of these yearly variations directly yields the multi-scenario LOM averages of 1.22% and 1.14% CuEq recorded in Table 2., establishing a continuous mathematical link between temporal scheduling and project valuation. Table 3. details the comprehensive technical, financial, and opportunity cost scheduling parameters generated by the dynamic optimization model.

Table 3.Dynamic Cut-Off Grade Calculations Results

Year	Ore Tonnage (t)	Copper Grade (%)	Zinc Grade (%)	CuEq Grade (%)	Traditional Opportunity Cost ($/t)	Traditional Mill Cutoff (g_h, %)	Sustainable Opportunity Cost ($/t)	Full ESG Integration Cutoff (g_{hESG}, %)
2025	447,467	2.37	2.46	2.95	75.15	1.62	51.16	**1.41**
2026	791,901	1.55	1.06	1.8	35.11	1.09	24.25	**1.05**
2027	841,344	1.26	2.36	1.82	28.59	1	20.17	**1**
2028	841,344	1.34	2.68	1.97	24.19	0.94	17.45	**0.96**
2029	841,344	1.27	3.11	2	19.4	0.88	14.31	**0.92**
2030	742,406	1.5	2.86	2.17	16.89	0.84	12.76	**0.9**
2031	643,419	1.52	1.91	1.97	13.94	0.8	10.75	**0.87**
2032	593,926	1.63	2.01	2.1	11.2	0.77	8.84	**0.84**
2033	494,937	1.57	2.32	2.12	8.96	0.74	7.22	**0.82**
2034	445,444	1.18	2.45	1.76	6.14	0.7	5.03	**0.79**
2035	395,950	1.59	3.18	2.34	4.56	0.68	3.86	**0.78**
2036	232,621	1.56	1.23	1.85	1.94	0.64	1.62	**0.75**
TOTAL	**7,312,253**	**1.51**	**2.34**	**1.85**	-	-	-	-

6.4. Sensitivity Analysis of Sustainable NPV

While the dynamic scheduling developed in Section 6.3 provides an optimized baseline policy under predictable operational terms, the long-term viability of the Çayeli underground mine remains deeply sensitive to external macroeconomic shocks and regulatory boundary shifts. To satisfy institutional review criteria regarding stress-testing and economic resilience, a dual-vector sensitivity analysis was executed. This framework isolates the strategic boundaries of the project across two critical risk dimensions: market price volatility versus financing costs.

Sensitivity Vector: Copper Equivalent Price vs. Discount Rate

Sensitivity analysis was done to figure out the Net Present Value difference between traditional case and ESG implemented case. Matrix shows this difference in its cells. The first dimension explores the structural trade-off between international metal price collapses and the localized financial de-risking achieved through sustainability compliance.

Value of 1% CuEq fluctuates from a lower level of 55.11 \$/t up to around today's level of 122.82 \$/t, cross-examined against operational discount rates moving incrementally from a highly de-risked 5.0% green financing rate up to a distressed 11.0% discount rate.

Matrix based on the equation created to obtain Net Present Value difference between traditional and sustainable approaches in US dollars currency. Equation used can be illustrated as:

$$\mathrm{NPV}_{\mathrm{Matrix}}(i,j) = \sum_{t=1}^{LOM} \frac{[\mathrm{Ore}_t \cdot y \cdot g \cdot (s-r)_i] - [\mathrm{Ore}_t \cdot \mathrm{OPEX}_k]}{\left(1+d_j\right)^t}$$

Where:

- *t: Specific production period index across the 12-year LOM horizon (Year 1 to Year 12)*
- *d: Scenario-dependent annual corporate discount rate (d_{TRAD}=10.0% or d_{ESG}=6.5%)*
- *Ore_t : Optimized annual ore tonnage extracted*
- *y: Recovery factor (87.1%)*
- *g_{CuEq} : Time-indexed optimal copper-equivalent head grade*
- *$(s - r)$: Baseline international market net realization value of a 1% CuEq concentrate grade.*

$23,677,775.98	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085	0.09	0.095	0.1	0.105	0.11
55.11	7,100,383.70	2,096,546.47	-2,721,345.06	-7,361,885.01	-11,833,206.93	-16,143,011.44	-20,298,592.07	-24,306,859.37	-28,174,363.54	-31,907,315.45	-35,511,606.49	-38,992,826.95	-42,356,283.41
59.05	16,494,323.62	10,388,593.85	4,509,476.46	-1,153,494.52	-6,610,224.25	-11,870,090.83	-16,941,976.65	-21,834,297.89	-26,555,031.92	-31,111,743.04	-35,511,606.49	-39,761,430.98	-43,867,679.78
62.98	25,888,263.54	18,680,641.23	11,740,297.97	5,054,895.97	-1,387,241.58	-7,597,170.21	-13,585,361.24	-19,361,736.41	-24,935,700.31	-30,316,170.62	-35,511,606.49	-40,530,035.02	-45,379,076.16
66.92	35,282,203.46	26,972,688.61	18,971,119.49	11,263,286.46	3,835,741.10	-3,324,249.59	-10,228,745.82	-16,889,174.93	-23,316,368.70	-29,520,598.21	-35,511,606.49	-41,298,639.06	-46,890,472.53
70.86	44,676,143.38	35,264,735.99	26,201,941.00	17,471,676.95	9,058,723.77	948,671.02	-6,872,130.41	-14,416,613.45	-21,697,037.09	-28,725,025.79	-35,511,606.49	-42,067,243.09	-48,401,868.91
74.79	54,070,083.29	43,556,783.38	33,432,762.52	23,680,067.44	14,281,706.45	5,221,591.64	-3,515,514.99	-11,944,051.97	-20,077,705.48	-27,929,453.38	-35,511,606.49	-42,835,847.13	-49,913,265.28
78.73	63,464,023.21	51,848,830.76	40,663,584.03	29,888,457.93	19,504,689.12	9,494,512.25	-158,899.58	-9,471,490.50	-18,458,373.87	-27,133,880.96	-35,511,606.49	-43,604,451.16	-51,424,661.65
82.67	72,857,963.13	60,140,878.14	47,894,405.55	36,096,848.42	24,727,671.80	13,767,432.87	3,197,715.84	-6,998,929.02	-16,839,042.26	-26,338,308.55	-35,511,606.49	-44,373,055.20	-52,936,058.03
86.60	82,251,903.05	68,432,925.52	55,125,227.06	42,305,238.91	29,950,654.47	18,040,353.48	6,554,331.25	-4,526,367.54	-15,219,710.64	-25,542,736.14	-35,511,606.49	-45,141,659.23	-54,447,454.40
90.54	91,645,842.97	76,724,972.90	62,356,048.58	48,513,629.40	35,173,637.15	22,313,274.10	9,910,946.67	-2,053,806.06	-13,600,379.03	-24,747,163.72	-35,511,606.49	-45,910,263.27	-55,958,850.77
94.48	101,039,782.89	85,017,020.28	69,586,870.09	54,722,019.89	40,396,619.82	26,586,194.72	13,267,562.08	418,755.42	-11,981,047.42	-23,951,591.31	-35,511,606.49	-46,678,867.31	-57,470,247.15
98.41	110,433,722.81	93,309,067.67	76,817,691.61	60,930,410.38	45,619,602.50	30,859,115.33	16,624,177.50	2,891,316.90	-10,361,715.81	-23,156,018.89	-35,511,606.49	-47,447,471.34	-58,981,643.52
102.35	119,827,662.73	101,601,115.05	84,048,513.12	67,138,800.87	50,842,585.17	35,132,035.95	19,980,792.91	5,363,878.38	-8,742,384.20	-22,360,446.48	-35,511,606.49	-48,216,075.38	-60,493,039.90
107.47	132,039,784.62	112,380,776.64	93,448,581.09	75,209,708.51	57,632,462.65	40,686,832.75	24,344,392.95	8,578,208.31	-6,637,253.10	-21,326,202.34	-35,511,606.49	-49,215,260.62	-62,457,855.18
112.58	144,251,906.52	123,160,438.24	102,848,649.06	83,280,616.15	64,422,340.13	46,241,629.55	28,707,992.99	11,792,538.23	-4,532,122.01	-20,291,958.20	-35,511,606.49	-50,214,445.87	-64,422,670.47
117.70	156,464,028.41	133,940,099.84	112,248,717.03	91,351,523.78	71,212,217.60	51,796,426.35	33,071,593.03	15,006,868.15	-2,426,990.91	-19,257,714.06	-35,511,606.49	-51,213,631.12	-66,387,485.75
122.82	168,676,150.31	144,719,761.43	121,648,785.00	99,422,431.42	78,002,095.08	57,351,223.15	37,435,193.07	18,221,198.08	-321,859.82	-18,223,469.92	-35,511,606.49	-52,212,816.36	-68,352,301.04

Figure 6. CuEq vs Discount Rate Sensitivity Analysis

The resulting matrix, illustrated through the conditional heat map visualization reveals a non-linear value topology:

- **The High-Risk Zone (Red Quadrant):** Under the absolute worst-case convergence, where metal prices shrink to the 55.11 $/t floor and green financing criteria fail, forcing the discount rate to an elevated 11.0%, the project value collapses into negative, generating a capital loss of $42,356,283.41 between traditional and sustainable cases. Even at the deterministic baseline discount rate of 10.0%, a 55.11 $/t price, environment yields an unsustainable deficit of -$35,511,606.49.

- **The Project Base Case (Yellow Zone):** At the empirical Çayeli baseline parameters where the CuEq valuation is fixed at 74.79 $/t, the project maintains economic viability across the entire financing spectrum. Under traditional discount rate conditions (d=10.0%), the asset generates a net difference of -$35,511,606.49 between traditional and sustainable case, under this localized sub-matrix branch, which exponentially swings into a positive $23,680,067.45 as the discount rate is compressed to the ESG-linked target of 6.5%.

- **The High-Yield Zone (Green Quadrant):** In bullish market cycles where the CuEq price escalates to its upper boundary of 122.82 $/t, the asset displays massive wealth generation capability. If combined with premium green financing structures that lower the cost of capital to 5.0%, the Sustainable NPV advances according to traditional NPV amount of $168,676,150.31.

This verification provides proof of achieving green financing compliance (d=6.5%) serves as a powerful operational shock absorber. A compressed discount rate effectively insulates the project's asset valuation during prolonged market downturns, allowing the mine to absorb price contractions that would otherwise render traditional, high-discount operations economically non-viable.

6.5. Stochastic Framework Verification: Comparative Analysis of Monte Carlo Simulation Engines

While deterministic evaluations validate the general strategic boundaries of the Çayeli underground mine under static market and operational conditions, long-term asset management demands rigorous quantified verification under conditions of extreme uncertainty. To satisfy institutional review guidelines regarding stochastic modeling, the proposed framework implements two separate and methodologically distinct Monte Carlo simulation engines, each evaluated 1,000 iterations to contrast traditional risk screening against dynamic capacity-constrained optimization.

6.5.1. Engine 1: Static Break-Even Based Stochastic Simulation

The first stochastic engine targets the validation of risk parameters strictly tied to the traditional economic Break-Even theory. This simulation introduces environmental volatility exclusively to the copper price tracking via Geometric Brownian Motion (GBM) process, anchored at a base copper price of 4.10 \$/lb with a standardized historical volatility (σ) of 0.40 over the 12-year horizon.

Under this static formulation, the cut-off grade operates as a rigid operational rule ($g_{be} = 1.01\%$ CuEq under ESG shadow costs), remaining non-responsive to temporary price fluctuations across the life-of-mine. The primary objective of Engine 1 is to evaluate the direct volumetric probability of economic success and determine the definitive valuation difference between traditional and sustainable approaches by tracking the Net Present Value variance.

$$\Delta NPV = SNPV_{ESG} - NPV_{TRAD}$$

The 1000-iteration Monte Carlo engine establishes a solid baseline Expected Value of average \$23,521,072.66 with a comprehensive project Risk parameter (Standard Deviation) of \$50m. Consequently, because the fixed cut-off grade policy prevents the operation from structurally adjusting its internal block selectivity during prolonged market downturns, the absolute Probability of Success scales down to a realistic 70.0% (where $\Delta NPV > 0$).

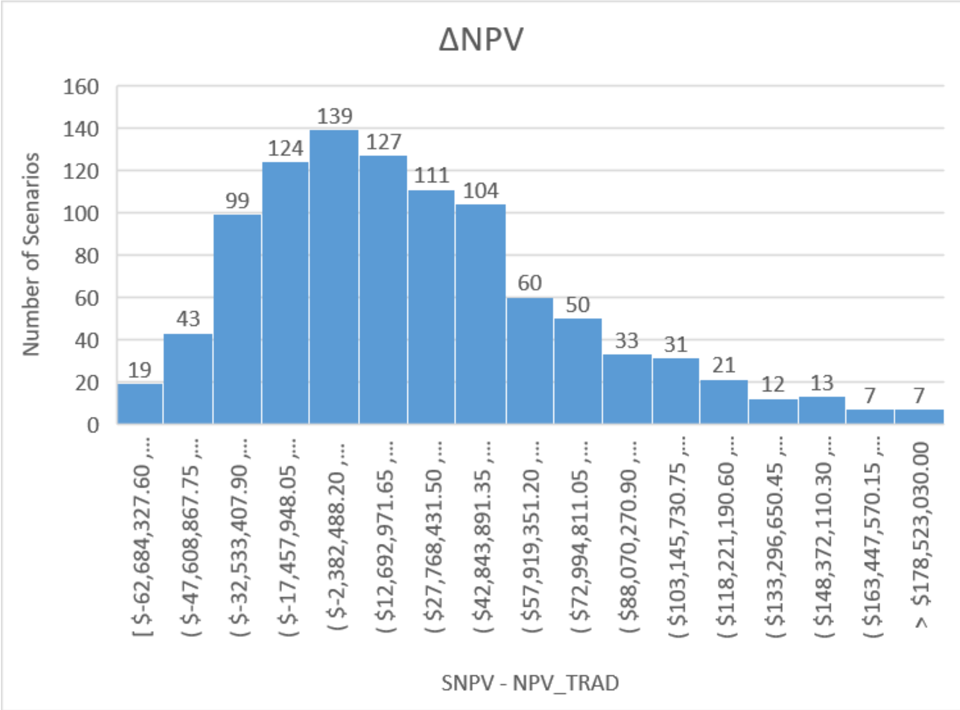

Figure 7. Static Scenarios of Difference Between SNPV and Traditional NPV

To ensure complete risk transparency and map out tail-risk behavior under these realistic market shocks, a rigorous downside statistical analysis was executed directly on the net present value variance distribution, defining the Value at Risk (VaR) and Conditional Value at Risk (CVaR) boundaries:

- **The Valuation Exposure under Distress:** The expanded risk variance and the adjusted 70.0% success probability confirm that when hit by multi-year negative price paths, a static mine plan is highly vulnerable. Under these conditions, compressed revenues diminish the denominator-driven leverage of green financing while exposing the continuous internalized $C_{\mathrm{ESG}} = 8.05\ \$/\mathrm{t}$ operational cost inflation, compressing the sustainable value premium.

- **Financial Geometry and Histogram Topology:** The generated frequency histogram in Figure 7. exhibits a prominent asymmetrical right-skewed log-normal profile, shifting away from a normal distribution to reflect the compounding variance of the GBM asset paths. The distribution achieves its statistical peak mode between the -$2.38m and $12.69m variance intervals, encapsulating the highest density of simulated outcomes. The broader dispersion across the negative quadrants visualizes the unmitigated downside risk; the frequency distribution stretches lower before tapering off, highlighting that a rigid break-even policy forces the asset to absorb severe market shocks during prolonged price depressions.

- **Tail-Risk Resilience:** The VaR metric establishes that in the worst 5% of simulated pricing outcomes, the sustainable framework faces a capital variance contraction of $34.25M relative to the traditional track, primarily because compressed market revenues diminish the denominator-driven leverage of green financing while exposing the continuous internalized operational cost burden. Furthermore, the CVaR of -$44.46M successfully maps the expected mean deficit within the absolute lower tail of the probability curve, mathematically defining the ultimate stabilization floor of the asset's sustainable risk track under prolonged macroeconomic distress cycles.

6.5.2. Engine 2: Dynamic Lane-Based Stochastic Optimization

The second stochastic engine integrates variable pricing and mean-reverting regulatory expenditures directly into Lane's capacity-constrained optimization loop, shifting the strategic operational planning from a static architecture to an adaptive resource extraction framework. Under this configuration, the cut-off grade ceases to be a fixed parameter; instead, every simulated iteration triggers an immediate recalculation of the system parameters across the 12-year life-of-mine (LOM) horizon. The market price interface operates under a discrete-time Geometric Brownian Motion (GBM), while the internalized sustainability cost vector (C_{ESG}) is continuously driven by an independent mean-reverting Ornstein-Uhlenbeck (Vasicek) stochastic process targeting a long-term mean of **8.05 $/t**.

The primary computational objective of Engine 2 is to quantify the dynamic Net Present Value variance distribution between the optimized frameworks:

$$\Delta NPV_{\text{Lane-Stochastic}} = \text{S-NPV}_{(\text{Lane-ESG})} - NPV_{(\text{Lane-Traditional})}$$

The 1000-iteration dynamic optimization engine establishes a highly integrated economic profile, yielding an absolute Probability of Economic Success ($\Delta NPV_{\text{Lane-Stochastic}} > 0$) of 60.0%. This specific contraction in the success probability, moving from a nominal 70.0% in the static break-even engine down to 60.0% under dynamic conditions, reveals a profound economic phenomenon regarding the integration of operational flexibility:

- **Value at Risk** ($VaR_{95\%}$)**:** Quantified directly from the optimized variance profile at -$46,537,175.18. This metric establishes that in the worst 5% of systemic market-cost shocks, the relative value compression between the sustainable and traditional operational tracks is structurally bounded at the $46.53M threshold.
- **Conditional Value at Risk** ($CVaR_{95\%}$)**:** Calculated at -$99,381,392.27. This value isolates the expected mean deficit within the absolute lower tail of the joint distribution, mathematically indexing the ultimate stabilization floor of the dynamic framework under conditions of severe, prolonged macroeconomic distress.
- **Histogram Topology and Mathematical Geometry:** The generated frequency distribution for the variance exhibits a right-skewed profile, reflecting the compounding variance of the path-dependent asset evaluations. The highest density of simulated scenarios (the mode) heavily clusters within the initial quadrants, specifically between the -$18.51M and +$17.48M intervals, where individual frequency bins record peak values of 170 scenarios. The distribution tails off gradually across the positive frontier, indicating stable wealth generation capability under bullish cycles.

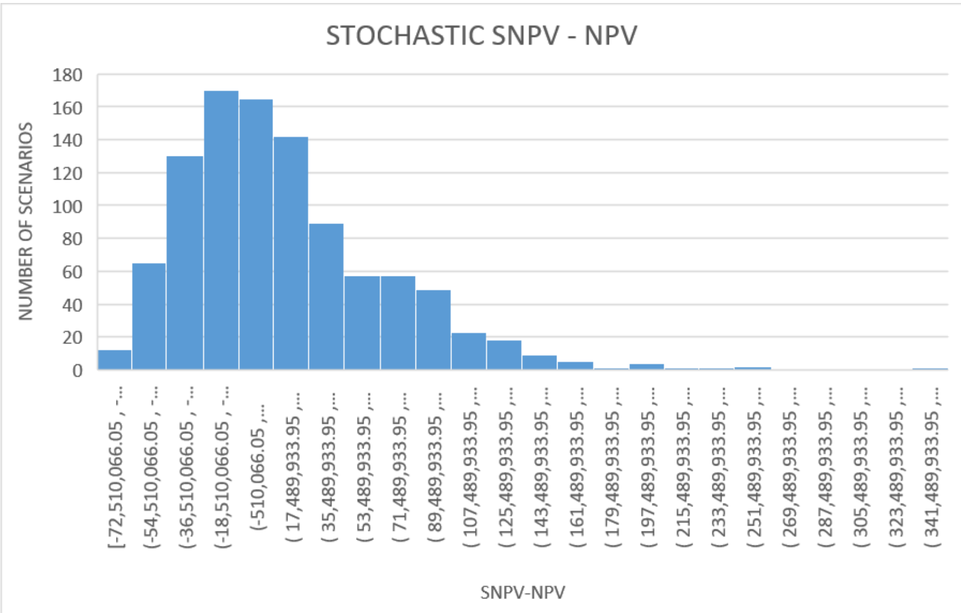

Figure 8.Stochastic Scenarios of Difference Between SNPV and Traditional NPV

- **Stochastic Operational Schedule and Year-by-Year Grade Evolution:** Beyond evaluating financial indicators, the engine successfully extracts a dynamic, year-by-year operational routing matrix across the 12-year LOM horizon. Every pricing and cost path automatically triggers a multi-variable capacity re-balancing that computes a unique Optimum Target Grade (%), alongside explicit Minimum and Maximum boundary limits for each individual production year.

Table 4. Annual Optimum Cutoff Grade Targets

Optimum Grade (%)	
2025	**1.4**
2026	**1.1**
2027	**1.0**
2028	**1.0**
2029	**1.0**
2030	**1.0**
2031	**0.9**
2032	**0.9**
2033	**0.9**
2034	**0.9**
2035	**0.9**
2036	**0.8**

In the early capital recovery phase (e.g., 2025), the optimization loop enforces a highly selective average Optimum Grade of 1.4% CuEq constrained within a narrow standard deviation of 0.063 to insulate initial cash flows from environmental penalties. As the resource pool is progressively depleted over time, the engine automatically deflates the systemic opportunity cost term, causing the operational schedule to broaden its block selectivity. This non-linear temporal expansion forces the target cutoff grade to gradually scale down to 0.8% CuEq by the final phase (2036), completely exhausting the remaining peripheral inventories and lower-grade stockpiles while productively absorbing the regulatory cost fluctuations. In particular, the probabilistic variance mentioned associated with the initialization phase is illustrated graphically in Figure 9 below as the histogram of frequencies of the optimized cut-off grades for 2025. On the basis of the internal structure of this frequency distribution plot, the concentration of the simulation runs between 1.39% to 1.46% CuEq can be identified as the mode of the multi-dimensional optimization space. The concentration of the results supports the optimal target of 1.4% CuEq since the optimization cycle consistently tends towards this selectivity interval regardless of various market shocks. Moreover, the combination of the skewed bins' geometry and sharp slope of the percentage trendline might indicate the ability of the engine to reduce the risks of tail events through restricting lower selectivity routing.

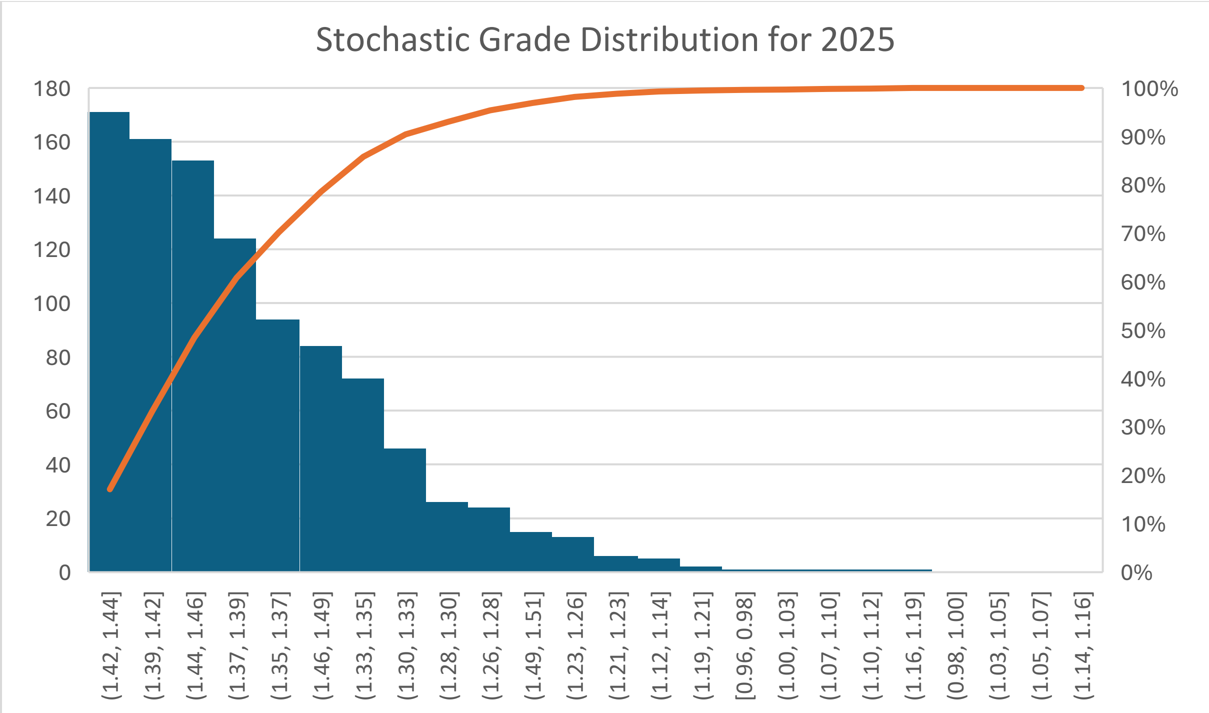

Figure 9. Stochastic Grade Distribution for 2025

7. DISCUSSION AND CONCLUSION

The results presented in the previous chapter should be interpreted not merely as the application of existing mine planning methodologies, but as the validation of the integrated strategic framework developed throughout this thesis. The proposed methodology combines three complementary dimensions that are generally addressed separately in the literature:

- Lane's dynamic cut-off grade optimization;
- the internalization of Environmental, Social and Governance (ESG) externalities through shadow pricing;
- the stochastic representation of market uncertainty using Monte Carlo simulation.

By integrating these components into a single decision-support framework, this research demonstrates how strategic mine planning can move beyond traditional deterministic economic optimization toward a more comprehensive evaluation of long-term project sustainability and financial resilience.

Unlike conventional approaches, which primarily seek to maximize Net Present Value under fixed economic assumptions, the proposed Sustainable Net Present Value (S-NPV) framework explicitly considers the economic implications of environmental costs together with uncertainty in future market conditions. Consequently, the framework provides decision-makers with additional information on the robustness of alternative mining strategies, allowing cut-off grade policies to be evaluated not only by their expected profitability but also by their sensitivity to regulatory and market-related risks.

The principal contribution of this research is therefore the demonstration that these complementary concepts can be integrated into a coherent strategic planning methodology for underground mining. Although the proposed methodology does not replace established optimization theories, it extends their practical applicability by incorporating sustainability considerations and uncertainty into strategic mine planning decisions.

7.1. Discussion of Empirical Findings

- **Sustainable vs. Traditional Frameworks**

  The empirical analysis shows that within the scope of both the break-even and the dynamic Lane optimization frameworks, the ESG implemented scenarios which have Sustainable NPV (S-NPV), systematically exceeds its respective Traditional NPV baseline.

  In detail, for the break-even which represents the static domain, while traditional case NPV is achieved as $380,324,778, ESG implemented version of that specific scenario creates $404,002,554 of NPV which is called S-NPV. By this way it can be said that additional net present value of ~25 million $ can be obtained.

  Similarly same trend can be seen from the Lane Theory, which represents dynamic framework, also. In the scope of that framework, traditional case NPV obtained as $336,299,209 while S-NPV surpassing that value and reaches $352,204,628. By this way it can be said that additional net present value of ~16 million $ can be obtained.

  This recovery obtained between traditional and sustainable cases is driven by the financing advantage directly neutralizing the operational cost inflation. The compression of the discount rate from 10.0% to 6.5% via green financing mechanisms significantly increases the present value of future cash flows. This financial dividend more than offsets the dynamic cost additions introduced by the $8.05/t environmental shadow pricing penalty, resulting in a net value gain across both optimization models.

  Inter-Theory Divergence (Break-Even vs. Lane Optimization)

  When comparing the two distinct methodologies, the break-even approach yields a higher absolute nominal NPV than the dynamic Lane optimization model under both traditional and sustainable scenarios. Under full ESG integration, the static break-even NPV stands at $404,002,554, while the dynamic Lane NPV settles at $352,204,628. This divergence is a direct consequence of the inherent mathematical design and boundary conditions of the two theories. The break-even method optimizes extraction on a localized, volumetric "Source-to-Stockpile" basis, unconstrained by systemic capacity bottlenecks or temporal asset degradation. Conversely, Lane's theory restricts the mathematical ceiling of the objective function by introducing rigid, multi-stage physical capacity constraints (M=0.84Mtpa < H = 1.00Mtpa) of the Çayeli project, alongside an explicit temporal opportunity cost function ($F_t = \delta \cdot \mathrm{NPV}_{t+1}$). While this capacity-balancing mechanism results in a lower nominal NPV, it provides an operationally realistic and risk-adjusted production schedule tailored to the physical limits of the infrastructure.

- **Operational Risk Analysis via Dual Monte Carlo Simulations**

  The integration of two separate stochastic simulation engines provides a comprehensive quantification of downside risk under market and environmental volatility.

  Engine 1 (Static Break-Even Stochastic Simulation): This engine evaluates the asset under a rigid operational rule where the cut-off grade is held constant at 0.9% CuEq while changing the copper price Which is the main logic behind the break-even theory. While it records a nominal 70.0% probability of success ($\Delta\mathrm{NPV} > 0$), the log-normal frequency distribution reveals severe structural vulnerability during prolonged market downturns. Because a fixed break-even policy prevents the operation from altering its block selectivity, the asset is forced to process low-margin material during price collapses. This structural rigidity results in a 95% Conditional Value at Risk ($\mathrm{CVaR}_{95\%}$) of -\$44.46M. Crucially, this value emphasizes that in 95% of all simulated scenarios, which fully encompasses the dominant 70.0% probability of positive wealth generation and normal operational periods, the project's value performance remains securely above or bounded by this threshold. The -\$44.46M floor does not indicate operational failure, but rather isolates the extreme 5% tail risk, mathematically defining the exact expected average of all net value deficits only when the asset is forced into the absolute worst-case economic distress environments.

  Engine 2 (Dynamic Lane-Based Stochastic Optimization): This engine introduces continuous operational adaptability by simultaneously running Geometric Brownian Motion (GBM) price paths and a mean-reverting Vasicek process for ESG penalties. The absolute success probability stabilizes at 60.0%. This behavior of the engine establishes a definitive 95% Value at Risk ($\mathrm{VaR}_{95\%}$) of -\$46,537,175.18 and a 95% Conditional Value at Risk ($\mathrm{CVaR}_{95\%}$) of -\$99,381,392.27.

  Specifically, the $\mathrm{VaR}_{95\%}$ value signifies that there is a 95% mathematical probability that the project's net value variance between the sustainable and traditional paths will not drop below a loss threshold of -\$46.53M, marking the boundary line of normal market operations. Conversely, the $\text{CVaR}_{95\%}$ metric isolates the extreme tail risk by calculating the exact mathematical average of the losses within that remaining 5% distress zone. It guarantees that under a prolonged, worst-case convergence of commodity price collapses and peak environmental penalties, the expected average valuation deficit is rigorously bound at -$99.38M. This explicit floor provides international green lenders with empirical verification of the asset's ultimate financial survivability thresholds rather than relying on blind expected mean values.

- **Cut-Off Grade Dynamic Evolution**

The evolution of production presented in Table 3. exhibits a three-phase production trajectory: an initial ramp-up phase (2025–2027), a stabilized maximum phase (2027–2029), and a declining phase (2030–2036). In mine planning, this physical production profile is recognized not as an isolated scheduling artifact, but as the mathematical consequence of Kenneth Lane's dynamic optimization loop interacting with the spatial and physical boundaries of the underground infrastructure.

1. Mechanics Behind the Initial Production Trend Between 2025–2027: The initial production phase, where the production climbs from 447,467 tons in 2025 to 791,901 tons in 2026 and reaches the maximum operational value of 841,344 tons in 2027, is mathematically governed by the temporal degradation of the Opportunity Cost (F_t).

- The opportunity cost of the mine during the initial stage (2025) is equal to its highest possible life-of-mine (LOM) value, which is 51.16 $/t because of the high discounted net present value of the resource left unexploited in the mine. In order to prevent the financial losses incurred from leaving behind the valuable remaining blocks, the optimizer will insist on the use of a highly selective extraction strategy that raises the mill-limited cut-off grade (g_h) to its limit of 1.41% CuEq.
- Due to this highly selective cut-off criterion, a substantial volumetric fraction of the run-of-mine material fails to satisfy the ore definition within the spatial block model and is structurally rejected or stockpiled. This creates a deliberate selectivity on the initial production.
- As these high-grade blocks are processed and generate rapid early cash flows, the discounted present value for the remaining reserve inventory naturally contracts. By 2026, the calculated opportunity cost deflates to $24.25/t,

dynamically shifting the operational cut-off grade down to 1.05% CuEq. This allows a significantly higher volumetric proportion of the extracted mass to satisfy the economic ore threshold, thereby driving the production ramp-up until the schedule hits the absolute physical underground extraction capacity constraint in 2027.

2. Mechanics Behind the Declining Production Trend Between 2030–2036: The production decline in the later stages of the LOM schedule where the annual production decreases from the 841,344 tons down to 742,406 tons in 2030 and ultimately reaches a terminal floor of 232,621 tons by 2036, represents a structural shift from active operational flexibility to the rigid physical boundaries of resource exhaustion.

- The continuous production schedule is explicitly bounded by a global resource of 7,312,253 tons. By the year 2030, high-grade blocks of the orebody approach complete depletion.
- Thus, production sequencing techniques need to make the underground extraction route to move into the more difficult and narrow parts found in the orebody. At these greater depths of operation, inefficiencies in production will be nonlinearly impacted by underground transport distance, complex stages of vertical lifting, and rigid paste backfill turn-around time requirements.
- The bottlenecks resulting from the cumulative underground infrastructure limit the maximum possible extraction rate well below the baseline of 0.84 Mtpa. In order to adapt above mentioned deficiencies, the engine progressively reduces the opportunity cost parameter to a minimum value of $1.62/t by 2036, thereby reducing the sustainable mill cutoff to 0.75% CuEq.

3. Analysis of Cut-off Grade Fluctuations: Table 3. establishes that unlike the production tonnage, the cutoff grade does not follow a specific continuous trend throughout the life of the mine. Instead, it exhibits non-linear, fluctuating behavior. Reasons for this trend can be:

- **Potential Scheduling Influence:** The head grade spike of 2.95% CuEq in the initial year (2025), followed by a sharp drop to 1.80% CuEq in 2026, can likely be explained by active extraction sequencing. The optimization model appears to intentionally prioritize higher-grade blocks from the database during this period. This scheduling strategy is potentially driven by the need to maximize early net cash flows and satisfy the high initial opportunity cost (51.16 $/t).

- **Mid-to-Late Life Fluctuations (Likely Inventory Constraints):** Conversely, from 2027 through 2035, the head grade follows no definitive trend, bouncing between a low of 1.76% CuEq (2034) and a secondary peak of 2.34% CuEq (2035). This lack of a uniform trend shows that after the initial high-grading phase, subsequent grade variations are likely governed by the spatial distribution of the remaining resource inside the block model. As the dynamic model processes the remaining mineral asset blocks across their spatial grid coordinates, these continuous fluctuations could be interpreted as a direct reflection of the deposit's intrinsic variability within the estimated resource envelope rather than a deliberate alternative production trend.

7.2. Opportunities for Commercial Software Integration and Operational Benefits

The practical application of this ESG-integrated framework can offer several distinct opportunities for deployment within standard commercial mine planning software such as Datamine, Deswik, or Vulcan to enhance strategic decision-making:

In relation to the resource characterization and block modeling phase, this approach is valuable in helping to add the environmental factors to the core block model itself. Using the carbon footprint and water footprint, in addition to the usual metal grades, may enable to develop a much more accurate view of environmental liabilities through the spatial grid. In the economic evaluation and optimization phase, considering these sustainability vectors into software may allow for more durable and consistent scheduling scenarios.

7.3. Conclusions

Studies in the scope of this thesis have developed and verified an integrated Sustainable Net Present Value (S-NPV) framework that incorporates stochastic macroeconomic volatility and ESG liabilities. By using this methodology on the empirical dataset of the Çayeli underground base-metal operation in Türkiye, the following conclusions are established:

- **Financial Feasibility of Externalities:** Environmental compliance of operation is actively de-risks the reserve valuation of underground mining operations when they paired with structure of green financing. High ESG performance is mathematically proven to be a value-creating parameter rather than operational load.
- **Temporal Cut-off Grade Verification:** The year-by-year dynamic scheduling sequence validates the non-linear degradation of the systemic Opportunity Cost of Capital (F_t) over the 12-year horizon. Under full ESG integration, the dynamic engine compresses the initial 2025 opportunity cost to $51.16/t, establishing a balanced initial mill cut-off grade of 1.41% CuEq. As the resource is depleted, the cutoff safely scales down to an optimal floor of 0.75% CuEq by 2036, maximizing resource extraction while rejecting the lowest-grade, ecologically hazardous material.
- **Importance of Stochastic Risk Tracing:** Traditional deterministic planning models rely on static constants for LOM of the operation. By replacing static part of the operation with stochastic engine, this framework successfully shifts the optimization target to a robust risk-isolated operational situation.

7.4. Limitations of the Study

While the proposed integrated framework successfully addresses the monetization of environmental externalities and external market uncertainties, certain boundary limitations must be explicitly noted:

- **Geological Uncertainty Boundary:** To ensure strict compliance with institutional mineral asset reporting guidelines and to respect the verified operational history of the Çayeli Mine, geological uncertainty was managed via a "Discrete Unit Approach" utilizing the grade and tonnage distributions derived from the 2025 NI 43-101 Technical Report.

The framework does not generate spatial conditional simulation realizations from raw drill hole data. Consequently, the localized spatial variability within the Main and South orebodies is treated within bounded mineral asset zones rather than continuous geostatistical point simulations.

- **Scope Realization Limits:** The internalization of environmental footprints is strictly limited to quantified shadow pricing models for Scope 1 and Scope 2 carbon dioxide equivalent emissions (P_{CO2}) and localized industrial freshwater consumption (P_{H2O}). Broader qualitative ESG vectors, such as long-term social reaction, changes in community relations, or governance structures, are excluded from the direct operational cost functions due to the mathematical challenges of direct monetary quantification. However, for future studies they can also be taken into account.

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