



Degree Project in Electric Power Engineering

Second cycle, 30 credits

Sensitivity Analysis of Discrete Fourier Transform-Based Phasor Estimation for Protection and Phasor Measurement Unit Applications

LORENZO RUBINETTO

Sensitivity Analysis of Discrete Fourier Transform-Based Phasor Estimation for Protection and Phasor Measurement Unit Applications

LORENZO RUBINETTO

Degree Programme in Electrical Engineering

Date: 27 mars 2026

Supervisors: Henrik Johansson, Mohsen Tajdinian, Enrico Pons

Examiner: Nathaniel Taylor

School of Electrical Engineering and Computer Science

Host company: Hitachi Energy Sweden AB

Swedish title: Känslighetsanalys av Fasvektorer Uppskattade med Diskret Fouriertransform för Tillämpning inom Skyddssystem och Fasmätningenheter

Abstract

The power system environment presents various sources of stochastic disturbances. Phasor estimation requires a chain of devices, mainly instrument transformers and phasor measurement units, whose operation is affected by such stochastic disturbances that can hinder the accuracy of the estimation itself. This thesis adopts a simulation-based sensitivity and uncertainty analysis framework to analyze how Discrete Fourier Transform-based phasor estimation during transmission system faults is impacted by current-transformer saturation, capacitive voltage transformer parameter degradation and transient behavior, and phasor measurement unit clock jitter. Furthermore, an attempt to assess which stochastic parameters are the main drivers behind these disturbances is performed, in order to lay the groundwork for future mitigation strategies. The results show that saturation in current transformers is often driven by the fault characteristics, and oversizing the current transformer can reduce the proportion of faults that cause saturation, but it only weakly improves the errors when saturation does occur. Secondly, even small parameter degradations in capacitive voltage transformer parameters can lead to relevant measurement errors, affecting specifically the magnitude of phasor estimations. However, it is incorrect to evaluate the negative effects of parameter degradation by observing variations with respect the nominal parameter values, since the performance of a capacitive voltage transformer depends on the interactions between the parameters of its capacitive divider and internal step-down voltage transformer. Lastly, the clocks normally adopted in phasor measurement units perform well with the most common synchronization mechanisms, but their performance becomes unreliable when synchronization is lost for sufficient amounts of time. Sensitivity analysis proves to be a useful instrument to assert input relevance in stochastic systems, but it is necessary to adopt second-order indices that are able to identify relevant interactions between the different parameters.

Keywords

Voltage measurement, Clocks, Current transformers, Discrete Fourier transforms, Measurement uncertainties, Phasor measurement units, Sensitivity analysis

Sammanfattning

Elkraftsystem uppvisar olika källor till stokastiska störningar som påverkar tillförlitligheten hos mätningssmetoder för fasvektorer. Denna avhandling tillämpar ett simuleringsbaserat ramverk för känslighets- och osäkerhetsanalys för att analysera hur mättnad i strömtransformatorer, parameterdegradering och transientbeteende i kapacitiva spänningstransformatorer, samt klockjitter i fasmätningssenheter påverkar prestandan hos den diskreta Fouriertransformen vid estimering av fasvektorer under fel i transmissionssystem. Vidare genomförs ett försök att identifiera vilka stokastiska parametrar som är de främsta drivkrafterna bakom dessa störningar, i syfte att lägga grunden för framtida åtgärdsstrategier. Resultaten visar att mättnad i strömtransformatorer ofta styrs av felens egenskaper, och att överdimensionering av strömtransformatorn kan minska förekomsten av mättnad, dock utan att framgångsrikt åtgärda de fel som mättnaden orsakar. För det andra kan även små parameterdegraderingar i kapacitiva spänningstransformatorer leda till betydande mätfel, särskilt genom att påverka amplituden hos de uppmätta fasvektorna. Det är dock felaktigt att bedöma de negativa effekterna av parameterdegradering genom att enbart observera variationer i förhållande till nominella parametervärden, eftersom prestandan hos en kapacitiv spänningstransformator beror på samspelet mellan parametrarna i dess kapacitiva spänningsdelare och den interna nedtransformerande spänningstransformatorn. Slutligen fungerar de klockor som normalt används i fasmätningssenheter väl tillsammans med de vanligaste synkroniseringsmekanismerna, men deras prestanda blir otillförlitlig när synkroniseringen går förlorad under särskilt lång tid. Känslighetsanalys visar sig vara ett användbart verktyg för att fastställa relevans av indata i stokastiska system, men det är nödvändigt att använda andra ordningens index som kan identifiera betydelsefulla interaktioner mellan olika parametrar.

Nyckelord

Spänningsmätning, Klockor, Strömtransformatorer, Diskret Fouriertransform, Mätosäkerhet, Fasmätningssenheter, Känslighetsanalys

Acknowledgments

I would like to thank my Hitachi Energy supervisor, Mohsen Tajdinian, and contributors, Rohit Thute and Md Tanbhir Hoq, for their help and guidance during the development of this project. Your suggestions and attitude allowed me to grow as a student and will serve as the foundation of my early professional life.

I would also like to express my gratitude to my KTH references, examiner Nathaniel Taylor and supervisor Henrik Johansson, for their availability, effort, and constructive feedback throughout the thesis period.

Lastly, I want to thank the people in my life outside of work – my girlfriend, family, friends, and university colleagues – who supported me, challenged me, and enriched my experience in the final years of my studies. Excuse me if I do not dedicate some space to each and every one of you. You know I get easily flustered.

Stockholm, March 2026

Lorenzo Rubinetto

Contents

List of Figures	xiii
List of Tables	xv
List of acronyms and abbreviations	xvii
1 Introduction	1
1.1 Background	1
1.2 Problem	3
1.3 Goals	3
1.4 Research Methodology	5
1.5 Delimitations	5
1.6 Structure of the thesis	6
2 Background	7
2.1 Phasor estimation using the Discrete Fourier Transform	7
2.1.1 Evaluation metrics for phasor estimation	9
2.2 Current transformers	10
2.2.1 Current transformer design	13
2.3 Capacitive voltage transformers	14
2.3.1 Causes of measurement errors in capacitive voltage transformers	16
2.3.1.1 Main parameters contributing to systematic errors in the case of degradation	16
2.3.1.2 Causes of error during transients	18
2.4 Clock jitter	19
2.4.1 Clock jitter in the frequency and time domains	20
2.4.2 Clock errors modeling for phasor estimation	23
2.4.2.1 Diffusion coefficients from Allan variance plots	31
2.5 Monte Carlo simulations	31
2.6 Sensitivity analysis	32

2.6.1	Classification of sensitivity analysis methods	33
2.6.2	Quality features of sensitivity analyses	34
2.6.3	Generic sensitivity analysis framework	34
2.6.4	Relevant sensitivity analysis methods	36
2.6.4.1	Borgonovo's delta	36
2.6.5	UQLab: A Framework for Uncertainty Quantification in MATLAB	39
2.7	Summary of Background concepts	40
3	Methods	41
3.1	Model and inputs definition	42
3.2	Output definition	45
3.3	Random variables assignment	45
3.4	Choice of sensitivity analysis method	50
3.5	Input sample generation and model evaluation	51
3.6	Clock errors evaluation	52
4	Results	55
4.1	Current transformers saturation analysis	55
4.1.1	Saturation presence	57
4.1.2	Maximum total vector error	61
4.1.2.1	Effects of saturation on phasor magnitude and phase estimation	63
4.1.3	Time-to-saturation	64
4.2	Capacitive voltage transformers degradation and transients analysis	66
4.2.1	Mean total vector error	66
4.2.1.1	Effects of degradation on phasor magnitude and phase estimation	70
4.2.2	Maximum total vector error	70
4.3	Clock errors analysis	72
4.3.1	Mean and maximum absolute time offsets	72
4.3.2	Mean and maximum total vector errors	74
4.3.3	Maximum frequency error	76
4.3.4	Longer holdovers	77
5	Conclusions	79
5.1	Conclusions	79
5.2	On the external validity of the results	80
5.3	Limitations	81

- 5.4 Future work 82
- References 83**
- A Taylor series approximation of the simplified total vector error expression 91**
- B Physical explanation of saturation and remanent flux and Jiles-Atherton model for ferromagnetic materials applied to current transformers 93**
 - B.1 Physical explanation of saturation and remanence in ferromagnetic materials 93
 - B.2 Jiles-Atherton model for ferromagnetic materials applied to current transformers 94
- C Ferroresonance suppression circuits 96**

List of Figures

1.1	Illustration of the device chain allowing to perform phasor estimation.	2
1.2	Conceptual framework for the application of uncertainty analysis and sensitivity analysis.	4
2.1	One-period windows utilized to compute phasor estimations at two consecutive time steps.	8
2.2	Phasor estimation of stationary signals at off-nominal frequencies.	9
2.3	Equivalent circuit of a current transformer for low-frequency alternating current.	12
2.4	Simplified equivalent circuit of a current transformer for low-frequency alternating current.	13
2.5	Equivalent circuit of a capacitive voltage transformer for low-frequency alternating current.	14
2.6	Equivalent circuit of a capacitive voltage transformer with Thévenin equivalent of the source and capacitive divider for low-frequency alternating current.	14
2.7	Basic design of an active ferroresonance suppression circuit.	16
2.8	Equivalent circuit of a capacitive voltage transformer with Thévenin equivalent of the source and capacitive divider for low-frequency alternating current with simplified secondary.	17
2.9	Digital processing of analog signals.	19
2.10	Illustration of the definition of absolute clock jitter.	21
2.11	Example of the power spectral density of clock phase fluctuations $S_{\phi}(f)$ and associated random processes.	22
2.12	Example of the Allan variance of frequency fluctuations $\sigma_y(\tau)$ and associated random processes.	23
2.13	Clock drift as a function of temperature.	24

2.14	Signal reconstruction by a phasor measurement unit.	25
2.15	Example of the effect of offset and skew corrections on a clock.	29
2.16	Sensitivity analysis framework.	37
2.17	Example of $s(X_i)$, the area corresponding to the shift between $f_Y(y)$ and $f_{Y X_i}(y)$	38
3.1	Bird's eye view on the project workflow.	42
3.2	Test system.	43
3.3	Saturation probability for each current transformer saturation voltage as a function of the number of simulations.	52
4.1	Example of the effect of saturation on current measurements and phasor estimation.	56
4.2	Histogram of the simulation time for which the total vector error for current measurements exceeds 1%.	57
4.3	Sensitivity analysis of the saturation presence.	58
4.4	Relevant probability density functions of the main variables involved in determining saturation.	59
4.5	Histogram of the maximum total vector error for current measurements in saturation cases.	61
4.6	Sensitivity analysis of the maximum total vector error for current measurements.	62
4.7	Maximum total vector error for current measurements as a function of the saturation voltage for saturation cases.	62
4.8	Histogram of the maximum magnitude error and phase error for current measurements in saturation cases.	63
4.9	Histogram of the time-to-saturation deduced from current measurements in saturation cases.	64
4.10	Sensitivity analysis of the time-to-saturation.	65
4.11	Time-to-saturation as a function of the saturation voltage.	65
4.12	Example of the effect of degradation and transients on voltage measurements and phasor estimation.	67
4.13	Histogram of the mean total vector error for voltage measurements.	68
4.14	Sensitivity analysis of the mean total vector error for voltage measurements.	68
4.15	Histogram of the values of capacitance C_2 for the simulation runs in which the mean total vector error for voltage measurements surpasses 5%.	69
4.16	Histogram of the mean phase error for voltage measurements.	70

4.17	Histogram of the maximum total vector error for voltage measurements.	71
4.18	Sensitivity analysis of the maximum total vector error for voltage measurements.	71
4.19	Histogram of the mean and maximum absolute offsets for an oven-controlled chrystal oscillator clock.	73
4.20	Sensitivity analysis of the maximum absolute offset.	74
4.21	Mean total vector error statistics for different holdovers.	75
4.22	Maximum total vector error statistics for different holdovers.	75
4.23	Maximum absolute frequency error statistics for different holdovers.	76
4.24	Mean total vector error as a function of the holdover. Long holdovers case.	77
4.25	Maximum absolute frequency error as a function of the holdover. Long holdovers case.	78
C.1	Actual design of an active ferroresonance suppression circuit.	96
C.2	Improved digital model of an active ferroresonance suppression circuit.	97
C.3	Another digital model of an active ferroresonance suppression circuit.	97

List of Tables

3.1	Test system constant parameters.	44
3.2	Current transformer constant parameters.	44
3.3	Capacitive voltage transformer constant parameters.	45
3.4	Output metrics adopted for the analysis of current transformers saturation and capacitive voltage transformers degradation and transients.	46
3.5	System random variables definition for saturation, transients, and degradation analysis.	47
3.6	Current transformer random variables definition for saturation analysis.	48
3.7	Capacitive voltage transformer random variables definition for transients and degradation analysis.	49
3.8	Diffusion coefficients for the oven-controlled crystal oscillator clock to use for the simulations.	53
3.9	Random variable definition for clock error analysis.	54
3.10	Output metrics adopted for the analysis of clock errors.	54
4.1	Probability of saturation for the different saturation voltage values.	60
4.2	Data about the linear regression model defined between the mean total vector error and mean absolute offset.	76
4.3	Data about the linear regression model defined between the mean total vector error and mean absolute offset. Long holdovers case.	77
B.1	Parameters of the PSCAD Jiles-Atherton current transformer model.	95

List of acronyms and abbreviations

AAT	All-At-a-Time
ADC	Analog-to-Digital Converter
CD	Clock Drift
CJ	Clock Jitter
CT	Current Transformer
CVT	Capacitive Voltage Transformer
DC	Direct Current
DFT	Discrete Fourier Transform
FE	Frequency Error
FSC	Ferroresonance Suppression Circuit
GNSS	Global Navigation Satellite System
HV	High-Voltage
IED	Intelligent Electronic Device
JA	Jiles-Atherton
KPI	Key Performance Indicator
LG	Line-to-Ground
LL	Line-to-Line
LLG	Double-Line-to-Ground
LLL	Three-phase
MCS	Monte Carlo Simulation
ME	Magnitude Error
MV	Medium-Voltage
OAT	One-At-a-Time
OCXO	Oven-Controlled Crystal Oscillator

PDF	Probability Density Function
PE	Phase Error
PES	Phasor Estimation
PMF	Probability Mass Function
PMU	Phasor Measurement Unit
PTP	Precision Time Protocol
RMS	Root Mean Square
ROCOF	Rate of Change of Frequency
RV	Random Variable
SA	Sensitivity Analysis
SDT	Step-down Transformer
TTS	Time-to-Saturation
TVE	Total Vector Error
UA	Uncertainty Analysis
VT	Voltage Transformer

Chapter 1

Introduction

This chapter describes the specific problem that this thesis addresses, the context of the problem, the goals of this thesis project, and outlines the structure of the thesis.

1.1 Background

Power systems are tasked with the conversion, transportation, and distribution of electrical energy from the power plants to the consumers. Their protection and control rely heavily on a chain of devices, shown in figure 1.1, that allows operators to monitor the state of the grid in real-time. Instrument transformers are the first link of the chain. They are responsible for accurately reproducing scaled-down versions of power system voltages and currents which can be easily provided to electronic devices for sampling and manipulation [1, 2]. One category of these electronic devices is **Phasor Measurement Units (PMUs)**, which estimate fundamental-frequency phasors of the input quantities in order to derive their magnitudes, phase angles, frequency, and **Rate of Change of Frequency (ROCOF)** [3, 4]. The **Discrete Fourier Transform (DFT)** is one of the numerous **Phasor Estimation (PES)** methods [5]; other methods include Kalman filter and wavelet transform-based **PESs** [6], and least square errors **PES** [7].

Practical power system environments present various sources of stochastic disturbances which can significantly affect the reliability of **PES** methods [8, 9]. Understanding how these uncertainties propagate through the **DFT**-based algorithms is vital to ensure the security, dependability, and observability of protection and **PMU** systems [9, 4]. Some sources of error affect the

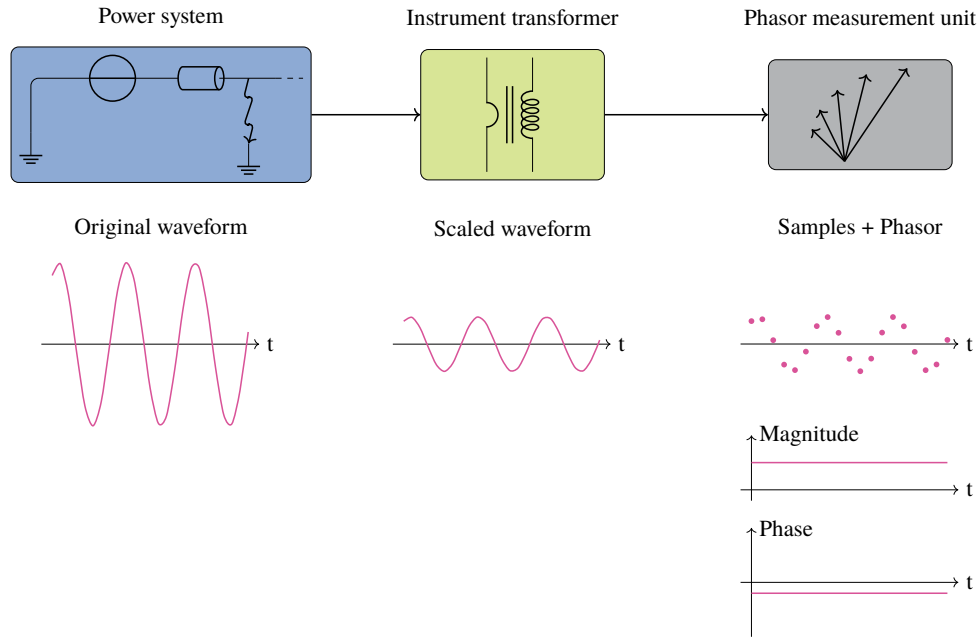


Figure 1.1: Illustration of the device chain allowing to perform **PES**. The scaled-down waveforms provided by the instrument transformers are sampled by the **PMU** and transformed into phasors.

operation of instrument transformers: for example, heavy and long-lasting fault currents can induce saturation in **Current Transformers (CTs)**, causing high relative errors in the instants following a fault [1, 10]; on the other hand, significant fault-induced transients introduce errors in voltage measurements because of the dynamic behavior of the reactive components contained in **Capacitive Voltage Transformers (CVTs)** [2, 11]. Furthermore, **PMUs** can introduce new errors during the sampling and quantization of the input signals [12, 13]. **PES** algorithms assume that **PMUs** work with a consistent sampling rate; however, this assumption is not realistic in practice, and clock errors constitute a source of error that is always present [14, 15]. All of these errors propagate through the chain and add up into the measurements available to operators and **Intelligent Electronic Devices (IEDs)**. Inaccurate phasor estimation can compromise the reliability of a protection system, both from the dependability and security standpoints, and the speed of its intervention; moreover, inaccurate phasor estimation can lead to erroneous signal reconstruction.

Given the stochastic nature of these errors, their effect cannot be fully observed and understood through a single evaluation of a deterministic model.

The randomness in system and environmental conditions can affect how the system itself reacts to a given occurrence. A **Monte Carlo Simulation (MCS)** can test the effects of different random combinations of system and environmental conditions to provide a full picture of the possible effects of an event [16, 17], such as a fault. The results can then be analyzed with stochastic analysis tools, such as **Uncertainty Analysis (UA)** and **Sensitivity Analysis (SA)**, to understand how the various sources of uncertainty propagate through the system to influence an output metric, and which sources are the most relevant in determining its uncertainty. When a measurement algorithm is of concern, the outputs to consider for the **UA** and **SA** are metrics providing information about the quality of the measurement. In the case of the **DFT** applied to **PES**, the relevant output metrics are the **Total Vector Error (TVE)**, **Magnitude Error (ME)**, **Phase Error (PE)**, and **Frequency Error (FE)** [9, 4], obtained by comparing the measurements of an ideal and non-ideal measurement chain. The combination of **UA** and **SA** can reveal which ranges of a given stochastic variable lead to substantial measurement errors. These insights directly lead to the development of adaptation or mitigation strategies and more robust **CT**, **CVT**, and clock design choices. Figure 1.2 shows the conceptual framework from a single simulation run to the **UA** and **SA** application.

1.2 Problem

Stochastic disturbances can worsen the performance of measurement algorithms. What are the main stochastic factors driving **PES** results, as well as their errors and uncertainties? How can these errors and uncertainties be minimized to improve the robustness of algorithms and hardware implementation?

1.3 Goals

The goals for the project are:

- Develop a comprehensive stochastic framework specifically tailored for **DFT**-based **PES** in power system protection and monitoring devices.
- Systematically quantify the sensitivity of key phasor parameters — including magnitude, phase angle — to a range of real-world stochastic faults. The primary sources of uncertainty to be examined are:

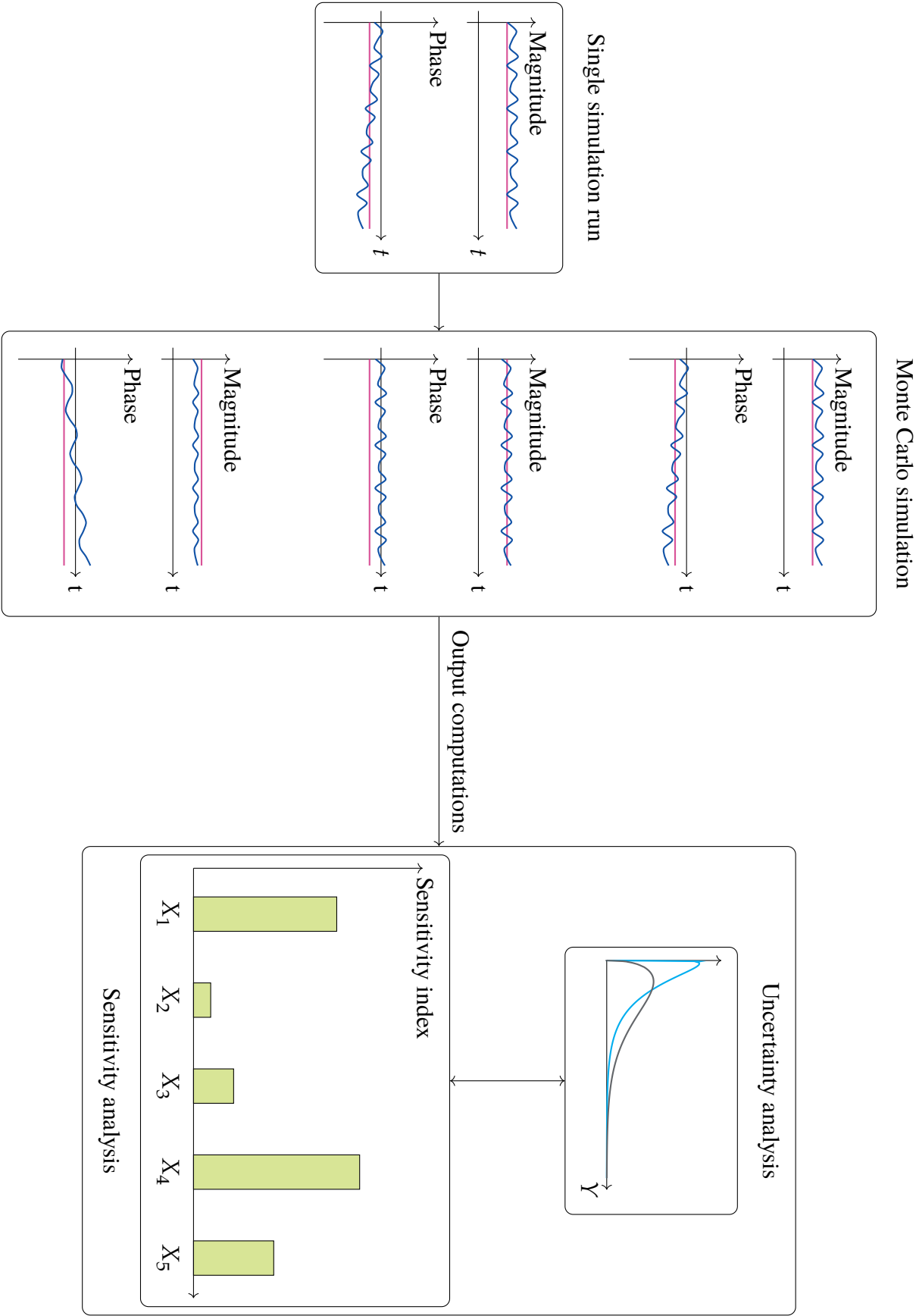


Figure 1.2: Conceptual framework for the application of UA and SA. The red and blue plots are intended to show PESs from ideal and non-ideal measurements, respectively.

- **CT Saturation**: Analyze the effects of non-linear behavior in **CTs** under high current conditions, focusing on how saturation-induced distortions propagate through **DFT** algorithms. Evaluate how **CTs** of different sizes impact the accuracy of **PESs** algorithms.
 - **CVT Transients and Parameter Degradation**: Study the impact of transient responses, resonance phenomena, and parameter variation in **CVT** on phasor accuracy and estimator stability.
 - **Sampling Clock Drift (CD) and Clock Jitter (CJ)**: Assess the consequences of timing inaccuracies – such as **CD** and random **CJ** – on the reliability of **PES**.
- Compare error sensitivity and rank drivers of uncertainty by employing simulation-based **SA**. This process will enable identification of the most influential sources of error across different scenarios.

1.4 Research Methodology

The project examines the behavior of measurement algorithms when applied to a simple test system simulated in PSCAD. Simulation is the only viable option to test a large amount of events on a system while ensuring that all possible combinations of input parameters and output cases are represented. The **MCSs** can be implemented in Python. Python interacts with PSCAD through the **PSCAD Automation Library**, which contains classes and functions to perform any PSCAD operation directly from Python scripts. The results – 4 200 simulation runs including measurements from non-ideal **CTs** and **CVTs** and their ideal counterparts – are exported from PSCAD in COMTRADE format and analyzed in MATLAB. MATLAB represents a favorable platform for this scope due to the availability of codes for **DFT** implementation and open-source frameworks for the application of **UA** and **SA**. In a later phase, the output data is manipulated to simulate the effect of clock errors on **PESs**. For **CT**, **CVT**, and clock errors analyses alike, some evaluation metrics are extracted from the comparison between the ideal and non-ideal measurements to provide suitable outputs for the **SA**.

1.5 Delimitations

This project is specifically concerned with the performance evaluation of **DFT**-based **PES** algorithms. Other possible algorithms exist and are implemented

in practice, but they are not discussed and evaluated in the thesis.

SA can be used as a design tool. However, in this project it is adopted only as an evaluation tool and applied to a pre-designed system. The simple test system is not intended to be exhaustive or representative of a real power system and all the stochastic factors that influence its behavior, but it is a simple tool to highlight the effects of some basic **Random Variables (RVs)** on the operation of **CTs** and **CVTs**. The results of the **SA** directly follow the choice of **RVs** for the stochastic inputs, and should therefore be tailored to a power system of interest.

The clock analysis adopts a model that includes master-slave synchronizations and a limited number of noise sources. Its applicability is limited by the offset correction mechanism, which is not representative of real-world behavior.

1.6 Structure of the thesis

Chapter 2 presents relevant background information about **DFT PES**, **CTs**, **CVTs**, clock modeling, **MCS**, and **SA**. Chapter 3 explains in more detail the implementation of the simulation framework and motivates the design choices. Chapter 4 presents and discusses the results for all the three simulation topics, and Chapter 5 summarizes the work and provides suggestions for future work.

Chapter 2

Background

This chapter provides basic background information about the main concepts and topics which will be subsequently employed and elaborated on during the experimental work. The level of detail is considered sufficient to justify the method choice and analysis techniques discussed in the following chapters. Section 2.1 introduces PES obtained through the DFT. Section 2.2 provides the necessary theoretical notions about CTs, and CVTs are described in section 2.3. The modeling of clock errors is introduced in section 2.4. Section 2.5 briefly introduces the basic ideas behind Monte Carlo simulations. SA, together with the framework implemented in this project for its application, is the topic of Section 2.6.

2.1 Phasor estimation using the Discrete Fourier Transform

Consider a periodic signal $x(t)$ with fundamental period $T_0 = 1/f_0$. Obtaining the phasor representation of its fundamental component, in a simple implementation, involves sampling at N equidistant time instants, separated by a sampling period $\Delta T = 1/f_s$, where f_s is the sampling frequency. The N samples constitute the *window* utilized for the PES. If $x(k\Delta T) = x[k]$ is the k -th sample, the phasor representation of the fundamental component is given by (2.1) [3]

$$\underline{X}_1 = \frac{\sqrt{2}}{N} \sum_{k=0}^{N-1} x(k\Delta T) e^{-j\frac{2\pi k}{N}} = \frac{\sqrt{2}}{N} \sum_{k=0}^{N-1} x(k\Delta T) e^{-j\omega_0 \frac{kT_0}{N}} \quad (2.1)$$

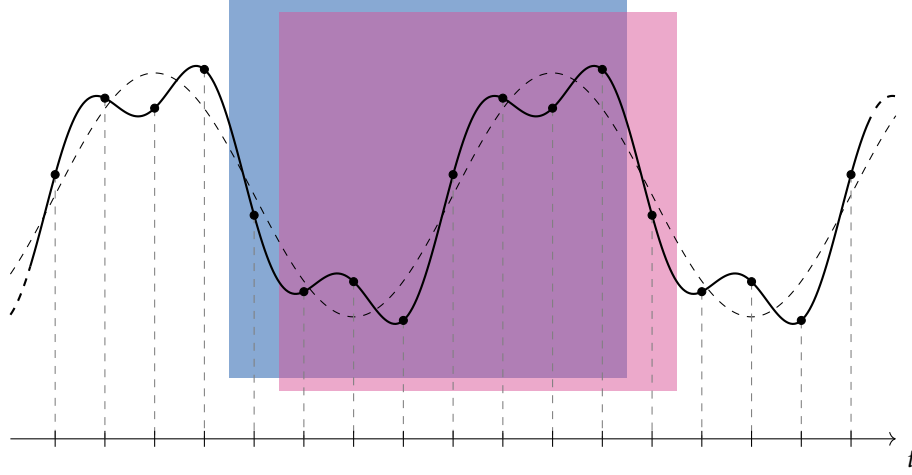


Figure 2.1: One-period windows utilized to compute **PESs** at two consecutive time steps. Example with $N = 8$. Given that two consecutive windows share $N - 1$ samples, adopting recursive updates improves the efficiency of **PES**.

where $\omega_0 = 2\pi f_0$. The magnitude of the phasor in (2.1) corresponds to the **Root Mean Square (RMS)** value of the fundamental component of $x(t)$; equivalently, (2.2) is adapted to return a phasor whose magnitude corresponds to the peak value of the fundamental component of $x(t)$:

$$\underline{X}_{l,\text{pk}} = \frac{2}{N} \sum_{k=0}^{N-1} x(k\Delta T) e^{-j\omega_0 \frac{kT_0}{N}} \quad (2.2)$$

Equation (2.2) can be implemented in practice to recursively update the estimation from the previous time step [5], using two windows that share $N - 1$ samples as shown in figure 2.1. This results in a phasor with constant magnitude and phase if the signal is stationary with nominal frequency.

The window length can be designed so that the number of samples used for a **PES** covers a multiple of the fundamental period of the signal. In particular, **DFT-based PES** can adopt half-period windows, one-period windows, or multiple-period windows. The choice of window length depends on the desired trade-off between time resolution and frequency resolution [18]: a longer window is more stable and better at identifying the frequency components of the signal and their amplitudes, but is slower in adapting to changes in the input signal. Therefore, with a constant sampling frequency, a shorter window corresponds to a less stable **PES**, although more reactive.

Otherwise, if this method is utilized to estimate phasors from signals with

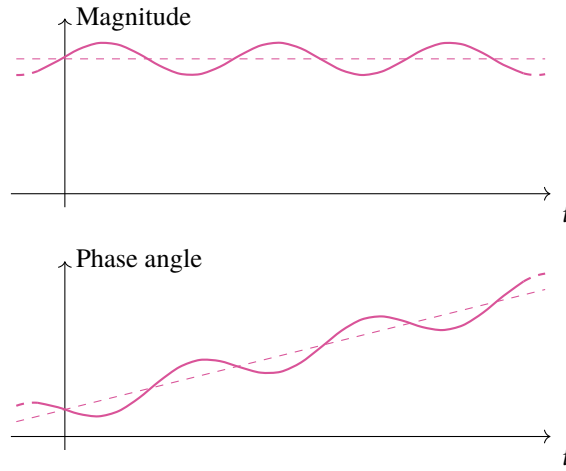


Figure 2.2: **PES** of stationary signals at off-nominal frequencies. Adapted from [8]. A gradually increasing phase angle corresponds to a signal with higher frequency than the nominal one.

off-nominal frequency, the result is a phasor with oscillating magnitude and phase [8], as shown in figure 2.2. The estimated magnitude will be slightly lower than the actual signal magnitude. The amplitude of the oscillations in figure 2.2 is exaggerated for illustrative purposes, but rarely exceeds a few percent of the actual magnitude and is basically independent of the sampling rate [8]. The same oscillations in the result happen when one tries to estimate the phasor of a nominal frequency signal with a misdesigned window, whose length is not a multiple of the signal's period. The oscillation on the magnitude and phase has a frequency corresponding to $f + f_0$, where f is the actual signal frequency and f_0 is the nominal signal frequency, and is therefore close to a second harmonic if the frequency deviation of the signal is small.

2.1.1 Evaluation metrics for phasor estimation

Given a phasor corresponding to the k -th sample of any signal, $\underline{X}[k] = X e^{j\varphi} = X_r[k] + jX_i[k]$, and its estimation $\hat{\underline{X}}[k] = \hat{X} e^{j\hat{\varphi}} = \hat{X}_r[k] + j\hat{X}_i[k]$, the **ME** and **PE** are defined as in (2.3) and (2.4) [9]:

$$\text{ME}[k] = \frac{\hat{X}[k] - X[k]}{X[k]} \quad (2.3)$$

$$\text{PE}[k] = \hat{\varphi}[k] - \varphi[k] \quad (2.4)$$

However, to have a **PE** in the domain $[-\pi, \pi]$ it can be defined as

$$\text{PE} = [(\hat{\varphi}[k] - \varphi[k] + \pi) \bmod 2\pi] - \pi \quad (2.5)$$

The **TVE** is defined to merge the **ME** and **PE** into a single scalar error that measures the complex difference between the phasor and its estimation as a fraction of the actual phasor magnitude [9]:

$$\text{TVE}[k] = \sqrt{\frac{(\hat{X}_r[k] - X_r[k])^2 + (\hat{X}_i[k] - X_i[k])^2}{X_r[k]^2 + X_i[k]^2}} \quad (2.6)$$

If the phasor and its estimation have equal magnitude, the TVE reduces to (2.7):

$$\text{TVE}[k] = \sqrt{2[1 - \cos(\hat{\varphi}[k] - \varphi[k])]} \quad (2.7)$$

Notice that the **TVE** reduces to the absolute **ME** if the phasor and its estimation are in phase. If $\hat{\varphi}[k] - \varphi[k]$ is very small, then the approximation

$$\sqrt{2[1 - \cos(\hat{\varphi}[k] - \varphi[k])]} \simeq |\hat{\varphi}[k] - \varphi[k]| \quad (2.8)$$

is valid. See appendix A for the proof.

Moving onto the definitions of frequency-related errors, the **FE** is

$$\text{FE}[k] = \hat{f}[k] - f[k] \quad (2.9)$$

where f is the real signal frequency and $\hat{f}$ is its estimation. The standard IEEE C37.118.1-2011 [4] defines the **FE** in absolute values, but [9] states that the error sign is important for reporting. In the results, the errors will be considered with or without their absolute value depending on the context.

2.2 Current transformers

CTs represent the interface between power systems and **PMUs** for current measurement. They convert nominal power system currents in the ranges of hundreds or thousands of amperes to lower currents that are manageable by measurement devices. Therefore, the performance of a measurement appliance depends on that of a **CT** [1]. The quality of a **CT** consists in its ability to correctly produce a scaled representation of the primary current i_p .

In their simplest forms, the primary and secondary sides of a **CT** correspond to two windings wrapped around a magnetic core. When the

primary side is energized, electric power is transmitted to the secondary side through electromagnetic induction: a primary electrical current generates a magnetomotive force in the magnetic core, causing a magnetic flux to flow in the core itself. The voltage at the terminals of the secondary winding is then proportional to the derivative of the magnetic flux. If a load (usually called *burden*) is connected to the secondary winding, the induced voltage causes a current to flow in the secondary winding and into the burden [10]. Ideally, if the primary current is sinusoidal, the primary and secondary currents I_p and I_s are related through the turns ratio $N = N_p/N_s$ of the CT:

$$\bar{I}_s = \frac{\bar{I}_p}{N} \quad (2.10)$$

However, magnetizing a CT requires a certain amount of current which is subtracted from the CT primary current. This current is very small during normal operation, but, when the flux density inside the magnetic core reaches a specific value, the phenomenon of saturation occurs. Saturation causes a drastic increase in the amount of magnetizing current, which can become a relevant portion of the primary current. Under these conditions, the secondary current is not tied to the primary current by a proportionality constant, and the TVE can reach values over 100% [10]. CTs are designed to operate with low flux densities during normal operating conditions; however, high fault currents cause the CT core to saturate, severely affecting its performance. Saturation can assume two different behaviors that depend on different triggering factors [10]:

- Symmetrical saturation, caused by high-magnitude symmetrical currents. It appears during the first half-cycle of the current;
- Asymmetrical saturation, caused by high transient Direct Current (DC) components resulting in an unidirectional flux build-up in the CT core. The Time-to-Saturation (TTS) depends on the fault current magnitude, fault inception angle, X/R ratio of the fault impedance, burden value, and remanent flux value. For the same symmetrical current value, saturation might be more or less heavy depending on how the DC component interacts with the remanent flux: if the DC component and remanent flux share the same sign, the amount of flux needed to reach saturation is low and so is the TTS. Vice versa, if the signs of the DC component and remanent flux are discord, the CT will saturate later and more weakly, or not saturate at all.

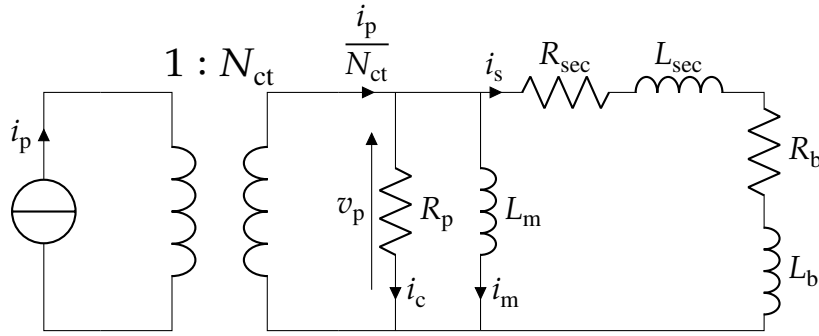


Figure 2.3: Equivalent circuit of a **CT** for low-frequency alternating current.

Consider the equivalent circuit shown in figure 2.3. The power system is represented by the primary side, while the secondary side is connected to a **PMU**. The burden represented by the **PMU** is modeled with the burden resistance R_b , which also includes the resistance of the lead wires connecting the **CT** to the **PMU**. R_{sec} and L_{sec} are the secondary resistance and inductance of the **CT**. The transversal parameters R_c and L_m model the current consumption due to iron losses and magnetization with the relative currents i_c and i_m .

The source of error is represented by the current divider on the secondary of figure 2.3. The transversal parameters are directly responsible for the difference between i_p/N and i_s , which is the output current of the **CT**; therefore, a large sum of magnetization and core loss currents (which is found during saturation) corresponds to a high measurement error*. **CTs** are designed to minimize this error during normal operating conditions. To include saturation in the circuit model, L_m is modeled as a nonlinear inductance whose value sharply drops when v_s reaches its saturation value due to a high secondary current. This causes a higher absorption of magnetization current and a higher disparity between i_p/N and i_s . It follows that a **CT** must have a high saturation voltage to be able to withstand high currents without saturating. The saturation voltage of a **CT** depends on its core material and geometric dimensions.

To continue the discussion on **CTs**, section 2.2.1 shows the approach used to design the **CTs** in this project. Appendix B discusses the physics behind the phenomena of saturation and core remanence to provide an interpretation of the **CT Jiles-Atherton (JA)** model used in the PSCAD simulations.

*The primary resistance and inductance are usually excluded from **CT** models because they are negligible and do not contribute to the measurement error.

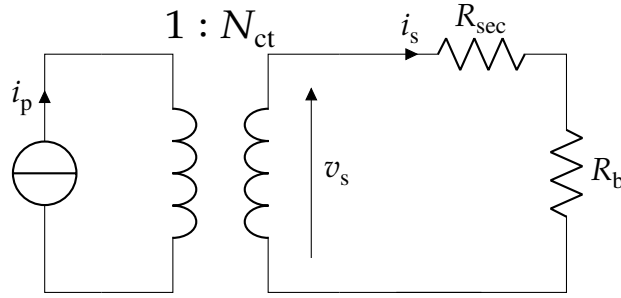


Figure 2.4: Simplified equivalent circuit of a **CT** for low-frequency alternating current.

2.2.1 Current transformer design

Since saturation negatively affects **CT** measurement accuracy, **CT** design must account for the possibility of saturation and aim to prevent its occurrence. The burden depends on the device that is connected to the secondary of the **CT**, and is therefore dependent on the application. The turns ratio N_{ct} of a **CT** depends on the burden rated current I_b and the nominal current flowing on the primary side I_{pn} :

$$N_{ct} = \frac{I_{pn}}{I_b} \quad (2.11)$$

Assuming the secondary impedance of the **CT** and the burden to be purely resistive, and neglecting the transversal parameters of the equivalent circuit as shown in figure 2.4, the secondary voltage of the **CT** is equal to

$$v_s = (R_{sec} + R_b) i_s \quad (2.12)$$

Using the turns ratio of the **CT** yields

$$v_s = (R_{sec} + R_b) \frac{i_p}{N_{ct}} \quad (2.13)$$

i_p must then be chosen as a representative fault current for the system of interest. A basic design might perhaps consider the peak three-phase fault current, that is enough to avoid the occurrence of symmetrical saturation; a more conservative approach, instead, might consider twice that amount to account for any DC component present in the fault current. An eventual oversizing depends on the time constant of the fault impedance [19], which determines the duration of the transient fault component, and on the possibility

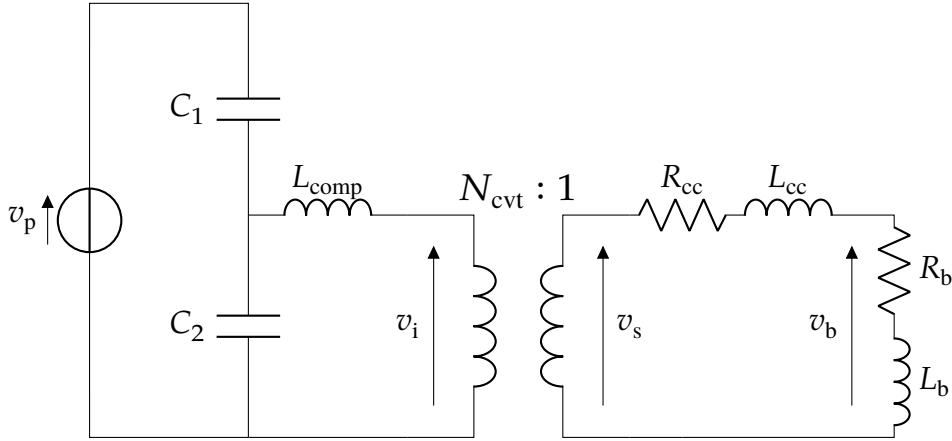


Figure 2.5: Equivalent circuit of a **CVT** for low-frequency alternating current.

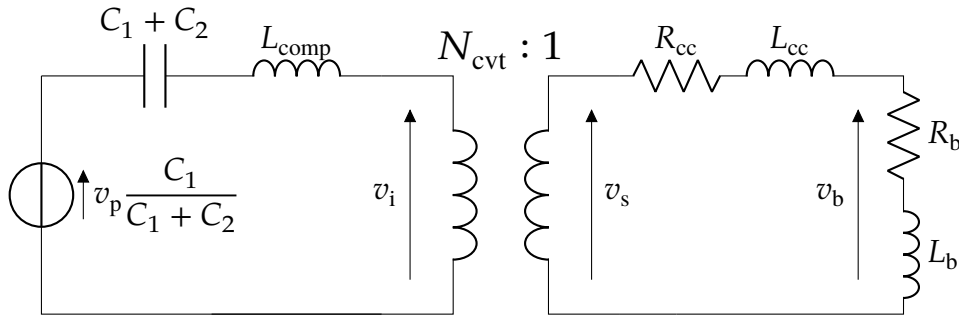


Figure 2.6: Equivalent circuit of a **CVT** with Thévenin equivalent of the source and capacitive divider for low-frequency alternating current.

that some remanent flux in the **CT** core might ease the inception of saturation even for relatively low fault currents.

2.3 Capacitive voltage transformers

Analogously to **CTs**, **CVTs** represent the interface between power systems and **PMUs** for voltage measurements. The equivalent circuit of a **CVT** is depicted in figure 2.5. This device presents two different stages of voltage conversion: the first one thanks to a capacitive divider (from v_p to v_i), and the second one through a proper **Voltage Transformer (VT)** (from v_i to v_s). To understand the relation between v_p and v_i , refer to the **CVT** equivalent circuit with the Thévenin equivalent of the source and capacitive divider for sinusoidal voltage signals, as shown in figure 2.6. Notice that if L_{comp} is designed to be resonant

with $C_1 + C_2$, then, in sinusoidal steady state,

$$\bar{V}_i = \bar{V}_p \frac{C_1}{C_1 + C_2} \quad (2.14)$$

and therefore

$$\bar{V}_s = \frac{\bar{V}_p}{N_{\text{cvt}}} \frac{C_1}{(C_1 + C_2)} \quad (2.15)$$

Notice how $\bar{V}_s$ is in phase with $\bar{V}_p$ and therefore (2.16) is also valid:

$$V_s = \frac{V_p}{N_{\text{cvt}}} \frac{C_1}{C_1 + C_2} \quad (2.16)$$

With a high burden impedance, the short-circuit parameters of the **CVT** are negligible and so is the voltage drop on them; thus, $V_b \approx V_s$. Therefore, the capacitive divider reduces the input voltage by a factor $\frac{C_1 + C_2}{C_1}$, and the **VT** by a factor N_{cvt} . The series of these two conversion stages must be designed to provide the input voltage required by the **PMU** connected to the **CVT** terminals. Notice how the burden voltage does not depend on the compensation inductance.

This design, exploiting a capacitive divider, is economically convenient for **VTs** above 138 kV [2] because it allows to use **Medium-Voltage (MV) VTs** also for **High-Voltage (HV)** systems. However, the reactive components used for the first voltage decimation are also responsible for the transient behaviors of **CVTs**, causing the output voltage not to accurately mirror the power system voltage. When the power system voltage abruptly changes, e.g. because of a fault near to the **CVT**, the energy stored in the capacitive divider and compensating inductance is reversed into the burden [11].

Another component which can harm the transient performance of a **CVT** is the **Ferroresonance Suppression Circuit (FSC)**, connected in parallel to the burden and designed to damp ferroresonance oscillations. The most basic design of an active (not purely resistive) **FSC** is shown in figure 2.7. Furthermore, the presence of such a large number of components, with their parameter uncertainties and possible deviation from the nominal values due to aging and degradation, reduces the robustness and long-term accuracy of the device.

To expand the discussion on **CVTs**, Section 2.3.1 discusses the main factors leading to measurement errors in **CVTs** in the case of parameter degradation or transient behavior. Appendix C describes other models of **FSC** and explain

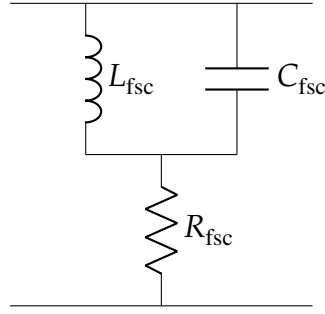


Figure 2.7: Basic design of an active **FSC**. Adapted from [20, 21]. The inductor and capacitor are sized to be resonant at the system frequency, so that the **FSC** behaves as an open circuit for the system frequency voltage components.

the reason why they were excluded from the project.

2.3.1 Causes of measurement errors in capacitive voltage transformers

Section 2.3 discusses that transients in **CVTs** and parameter degradation cause the output of the instrument transformer to not match the input voltage faithfully. Therefore, identifying the main contributors to these errors is a first step towards limiting their negative effects. Parameter degradation and transients affect the measurements in two different but complementary ways:

- aging and parameter degradation cause a systematic error that is constant during steady state;
- transients cause the output a **CVT** to diverge from its input by an amount and for a duration that depend both on power system variables and **CVT** parameters.

Sections 2.3.1.1 and 2.3.1.2 discuss the main contributors to these systematic and transients error, respectively.

2.3.1.1 Main parameters contributing to systematic errors in the case of degradation

Consider (2.16):

$$V_s = \frac{V_p}{N_{cvt}} \frac{C_1}{C_1 + C_2}$$

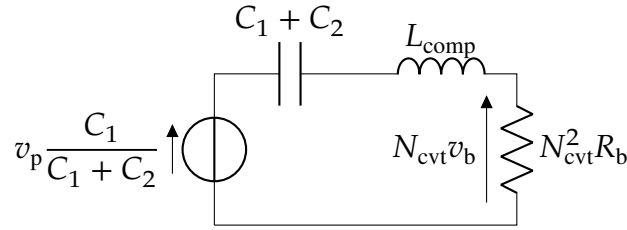


Figure 2.8: Equivalent circuit of a **CVT** with Thévenin equivalent of the source and capacitive divider for low-frequency alternating current with simplified secondary.

As stated in section 2.3, the burden voltage of a **CVT** does not depend on the short-circuit parameters of the **CVT** or the value of its compensating inductance. Qualitatively, this result can be extended to also consider the effects of aging and degradation on the **CVT** parameters. Since a well-tuned compensating inductor does not have any effect on the magnitude of the burden voltage of a **CVT**, it is reasonable to assume that a small degradation of its inductance will not be detrimental to the behavior of the device. This assumption can be confirmed by studying the equivalent circuit of figure 2.8. In sinusoidal steady-state, the burden voltage is equal to

$$\begin{aligned}
 \bar{V}_b &= \bar{V}_p \frac{C_1}{C_1 + C_2} \cdot \frac{N_{\text{cvt}}^2 R_b}{N_{\text{cvt}}^2 R_b + j\omega L_{\text{comp}} + \frac{1}{j\omega (C_1 + C_2)}} \cdot \frac{1}{N_{\text{cvt}}} \\
 &= \bar{V}_p \frac{C_1}{C_1 + C_2} \cdot \frac{jN_{\text{cvt}}^2 R_b \omega (C_1 + C_2)}{jN_{\text{cvt}}^2 R_b \omega (C_1 + C_2) - \omega^2 L_{\text{comp}} (C_1 + C_2) + 1} \cdot \frac{1}{N_{\text{cvt}}} \\
 &\simeq \bar{V}_p \frac{C_1}{C_1 + C_2} \cdot \frac{1}{N_{\text{cvt}}}
 \end{aligned} \tag{2.17}$$

where the last approximation is valid since $\omega^2 L_{\text{comp}} (C_1 + C_2) \simeq 1$ and $N_{\text{cvt}}^2 R_b \omega (C_1 + C_2) \gg 1$. Because of these factors' high difference in magnitude, it follows that a contained parameter deviation of the compensating inductance will not have any significant effect on the burden voltage. Therefore, the only parameters whose deviation will actually affect the performance of a **CVT** are the capacitive divider and the turns ratio of the internal step-down **VT**. Furthermore, the burden voltage can be expected to remain in phase with the primary voltage even if degradation is in place.

However, the deviation of one parameter from its nominal value is not

sufficient to cause measurement errors in **CVTs**. In fact, the relationship between the input and output voltages of this device depends the interactions between the two capacitances and the turns ratio of the intermediate **Step-down Transformer (SDT)** and there are scenarios in which, theoretically, it is possible that the degradation of one parameter is compensated by the degradation of the other parameters such that the overall transformation ratio remains substantially unaffected: an example is the simultaneous degradation of the divider capacitances with equal relative change of value. On the other hand, degradation that causes relative changes of capacitances that are different or even opposite, or divider degradation combined with **SDT** degradation could lead to the worst performance results.

2.3.1.2 Causes of error during transients

The following list summarizes the main factors that contribute to the magnitude and duration of the transient behavior of **CVTs** in the case of a fault [2, 11]:

1. Voltage fault inception angle: if the compensating reactor is designed to be resonant with the capacitive divider, these components together store the same amount of energy whether the primary voltage wave is at its peak or crossing one of its zeros. Therefore, the gravity of the transient depends on the discharge time constant, which is longer when the maximum energy is stored on the capacitor ($v_p = 0$ with a resistive burden).
2. Total capacitance of the capacitive divider: assuming constant burden current, the equivalent circuit of figure 2.6 shows that a higher $C_1 + C_2$ capacitance (corresponding to a lower reactance) corresponds to a lower voltage drop on the capacitors and an overall lower stored energy, resulting in transients with a lower magnitude.
3. Turns-ratio of the intermediate **SDT**: considering again a constant burden current, a higher **VT** turns ratio causes a lower current flow in the capacitive divider and compensation inductance, and an overall lower stored energy.
4. **FSC** components: the reactive components of an active **FSC** will discharge energy into the burden just like the capacitive divider components.

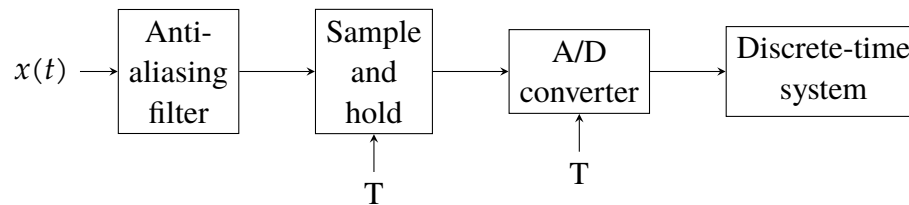


Figure 2.9: Digital processing of analog signals. Adapted from [18].

5. Burden magnitude and power factor: a higher burden causes a higher current flow and therefore an higher amount of energy to be stored in the reactive components. Furthermore, a burden which includes reactive components will also store energy and release it during the transients, altering the voltage transformation. A resistive, low burden also contributes to faster damping.
6. Composition of burden: burdens designed in parallel offer more paths for energy discharge, some of which are characterized by low resistance and therefore low damping.
7. Inherent circuit resistance: a higher series resistance would improve the overall damping of the system. However, the required values are impractical since they heavily compromise the accuracy of the device.
8. Transformer excitation current: a higher excitation current causes more energy to be stored in the reactive components on the primary side of the VT.

2.4 Clock jitter

PMUs rely on a device chain for digitalizing an analog signal. Consider the model for digital processing of analog signals in figure 2.9. The analog signal $x(t)$ is supplied to a low-pass anti-aliasing filter that limits the frequency components in the signal to one half of the sampling frequency of the **Analog-to-Digital Converter (ADC)**. Then, a sample-and-hold device, when ordered by the clock signal T , memorizes the current value of the analog signal for enough time to allow its quantization by the **ADC**. In simplified form, a sample and hold device is constituted by a capacitor, which is able to “hold” its voltage when disconnected from the rest of the circuit. The term “aperture” refers to the disconnection of this capacitor from the input signal. The capacitor should maintain a constant voltage for as long as needed for the quantization

process. Once an input sample is quantized, it can be processed by a discrete-time system, e.g., for PES inside a PMU.

The process of digitalizing analog signals injects errors in the discretized samples. In the system of figure 2.9, the overall noise is represented by the sum of quantization noise, thermal noise, and jitter [12]. Quantization noise derives from the finite precision with which an N -bit ADC can approximate its input signal: given an input range, the output of the ADC can assume only 2^N equidistant values, and the input signal is approximated to the closest of these [13, 22]. Jitter itself is the sum of Clock Jitter (CJ) and aperture jitter. Aperture jitter is due to the uncertainty about the exact instant in which a sample and hold device will activate relative to the clock signal. If the aperture does not coincide with the clock signal, the digitalized input value will differ from what was intended. The actual digitalization error depends on the errors generated by the aperture process itself and the slew rate of the input signal [23]. CJ, instead, is the deviation of a clock cycle from its nominal duration and is generally considered a random process with zero mean [14].

To continue the discussion, section 2.4.1 provides a brief mathematical foundation to understand clock errors. Subsequently, section 2.4.2 discusses the effects of clock errors on PES and proposes a model to simulate the effects of a non-ideal clock in a substation environment.

2.4.1 Clock jitter in the frequency and time domains

One of the ways to characterize CJ is as absolute jitter a , defined as

$$a_k = t_k - kT_{\text{sn}} \quad (2.18)$$

where t_k is the time at which the clock reaches a given counter value, k is the given counter value, and T_{sn} is the nominal clock period. Thus, absolute CJ represents the offset between an ideal clock and a non-ideal one. An illustration of absolute CJ is given in figure 2.10. A generic clock signal, assumed to be sinusoidal with constant amplitude, is modeled as (2.19) [15]

$$v(t) = V_0 \sin(2\pi\nu_0 t + \phi(t)) \quad (2.19)$$

The function $\phi(t)$ is referred to as *phase fluctuation*. The actual frequency of a non-ideal clock will be

$$\nu(t) = \nu_0 + \frac{1}{2\pi} \frac{d\phi(t)}{dt} \quad (2.20)$$

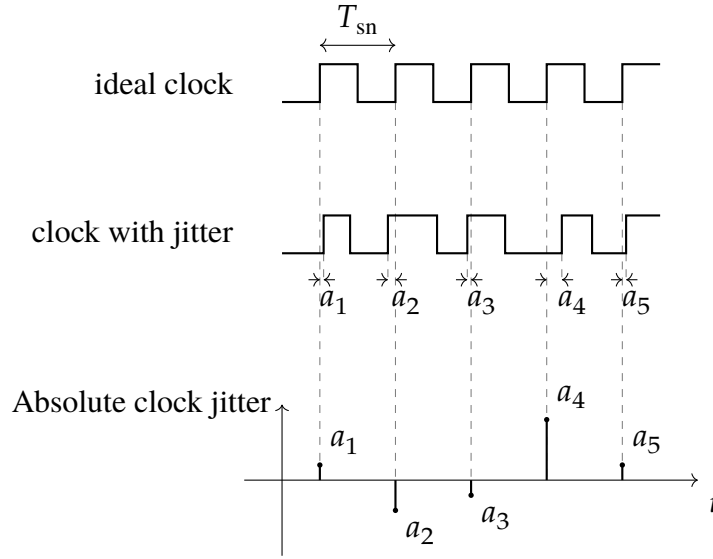


Figure 2.10: Illustration of the definition of absolute **CJ**. Adapted from [14].

Related to the phase fluctuation, the *frequency fluctuation* is defined as

$$y(t) = \frac{1}{2\pi\nu_0} \frac{d\phi(t)}{dt} \quad (2.21)$$

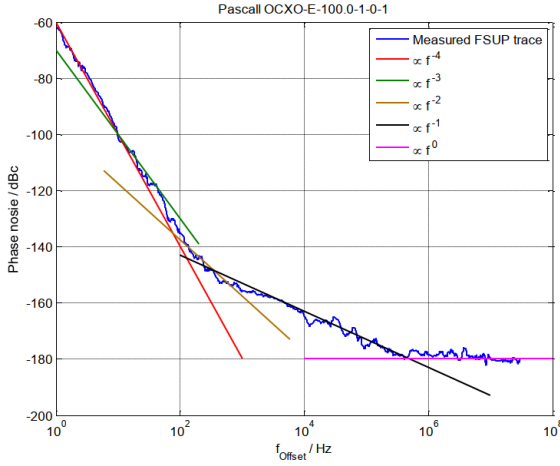
For an ideal clock, the phase and frequency fluctuations are equal to zero, meaning that the signal period is constant and related to the nominal frequency ν_0 . For non-ideal clocks, the fact that the fluctuations are not zero results in period-to-period variations that characterize **CJ**. Another useful definition is the skew,

$$\gamma(t) = \frac{\nu(t) - \nu_0}{\nu_0} \quad (2.22)$$

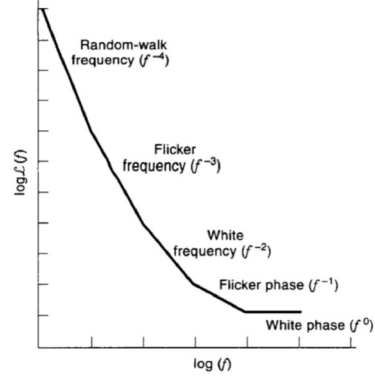
which provides a measure of the difference between the actual and nominal clock frequency caused by **CJ**.

Frequency and phase fluctuations can be analyzed in two different domains [15, 24]:

- in the frequency domain, through the spectral densities of frequency fluctuations $S_y(f)$ and phase fluctuations $S_\phi(f)$;
- in the time domain, through the two-sample variance of the frequency fluctuation $\sigma_y^2(\tau)$ over a time interval τ . This variance is normally termed the Allan variance and its corresponding standard deviation is termed the Allan standard deviation.



(a) Phase noise plot with marked up noise processes [25].



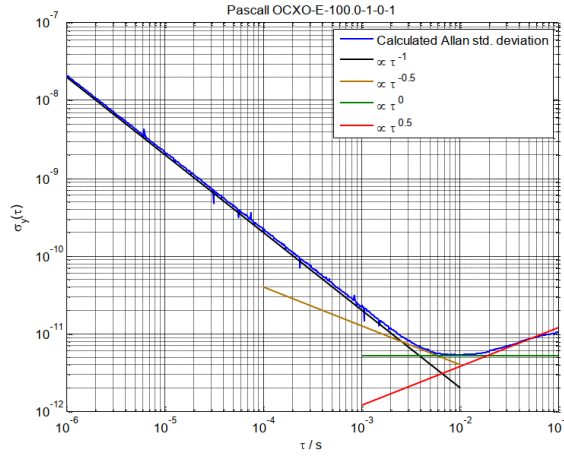
(b) Log-log plot of $\mathcal{L} = S_\phi/2$ vs. f , showing segments, reprinted from NIST Pub. 1286 [26]. Work of U.S. Govt. not subject to copyright.

Figure 2.11: Example of the power spectral density of clock phase fluctuations $S_\phi(f)$ and associated random processes.

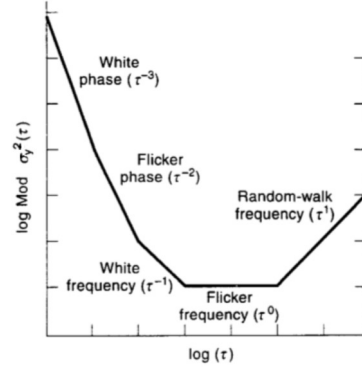
An example of the spectral density of the phase fluctuations of a commercial clock is shown in figure 2.11a. $S_\phi(f)$ is generally approximated as the sum of five different power-law random processes [27]:

$$S_\phi(f) = b_{-4} \frac{1}{f^4} + b_{-3} \frac{1}{f^3} + b_{-2} \frac{1}{f^2} + b_{-1} \frac{1}{f} + b_0 = \sum_{\beta=-4}^0 b_\beta f^\beta \quad (2.23)$$

Each one of these random processes creates noise that characterizes a specific frequency range, as shown in figure 2.11b. The phase noise of a given frequency is related to the magnitude of the corresponding frequency component from the **DFT** of the clock signal: therefore, random-walk frequency is a noise component characterized by low frequency and high-magnitude phase fluctuations; conversely, white phase noise is characterized by high frequency and low-magnitude phase fluctuations. The same five random processes can be identified by an Allan standard deviation plot, such as the one in figure 2.12a. Figure 2.12b schematically highlights the time span in which each random process is prevalent. An equivalent approximate model of the Allan variance such as (2.23) can be obtained from the analysis of stochastic differential equations. However, flicker noise cannot be modeled by a finite order state model [28], and therefore only approximate models exist that feature the remaining noise random processes, such as those contained in



(a) Allan standard deviation with marked up noise processes [25].



(b) Log-log plot of $\text{mod } \sigma_y^2(\tau)$ vs. τ showing segments, reprinted from NIST Pub. 1286 [26]. Work of U.S. Govt. not subject to copyright.

Figure 2.12: Example of the Allan variance of frequency fluctuations $\sigma_y(\tau)$ and associated random processes.

[28, 29], that derive the diffusion coefficients of Wiener processes associated with each noise component.

The spectral density of phase fluctuations and Allan variance provide complementary information. In a spectral density plot, white phase noise is identified as a random process with constant, low power, effective at the highest frequencies; in an Allan standard deviation plot, its prevalence is at low time spans, since, over high time spans, the average of white noise converges to zero. Conversely, a random walk in frequency is the manifestation of low-frequency noise that is dependent on environmental conditions, such as ambient temperature as shown in figure 2.13, and is therefore relevant over large time spans. Its effect over short time spans is negligible because its lower frequency makes it a "slow" random process compared to the others and it does not affect the frequency variance over short intervals.

2.4.2 Clock errors modeling for phasor estimation

A **PMU** relies on its internal clock to sample the signals provided by the instrument transformers and to apply timestamps to the various samples. In particular, the sampling process occurs when the clock counter reaches an integer value, usually at the end of its periods. However, non-ideal clocks operate under the effects of phase and frequency fluctuations, which

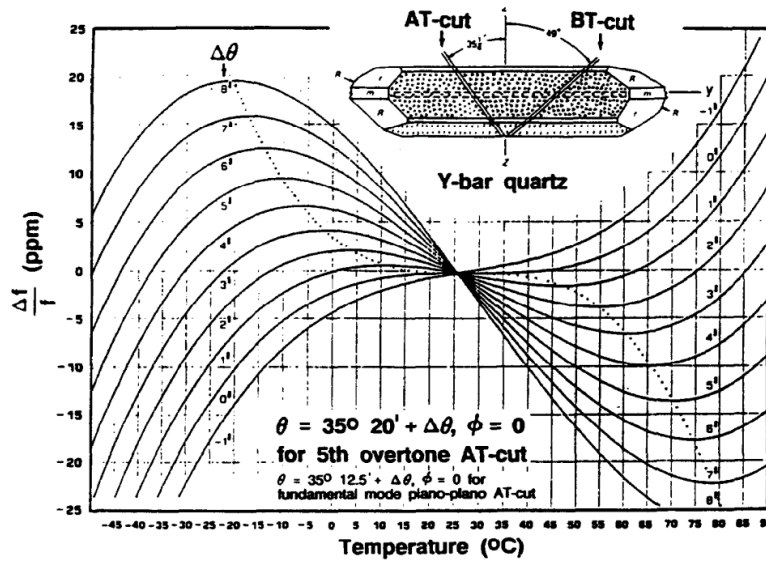


Figure 2.13: $\Delta f/f$ as a function of temperature. © IEEE. Reproduced from “Introduction to Quartz Frequency Standards”, IEEE UFFC Society, 2021 [30].

affect the actual period durations. Therefore, the sampling instants do not correspond to the nominal sampling frequency; however, **PMU** clocks are unaware of this incorrect behavior unless they communicate with a more accurate reference. With no means to estimate their own errors, **PMUs** assign timestamps as if their clocks ran perfectly. An example of how clock errors can affect timestamping operations and signal reconstruction is shown in figure 2.14, where sampling under an ideal clock is compared to sampling under a non-ideal clock. In a substation environment, **IED** clocks receive frequent synchronization messages by a master clock, which is usually connected to the **Global Navigation Satellite System (GNSS)** for accurate global synchronization. For example, when the IEC/IEEE 61850-9-3 **Precision Time Protocol (PTP)** is adopted, a slave clock will run on its own time, but with offset and skew corrections from a master clock every second [31], with the aim of maintaining the timestamp error under $1 \mu\text{s}$ [32]. To accurately model clock behavior and its effects on **PES**, it is therefore necessary to account for both the effects of **CJ**, and the corrections sent by the master clocks to the **IEDs** and **PMUs**.

An example of a clock signal is shown in (2.19). The clock counter k is

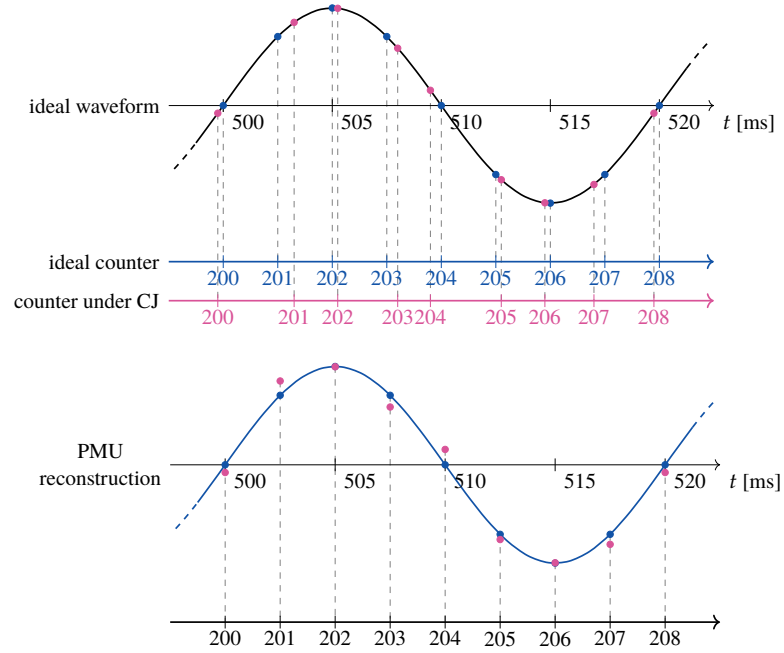


Figure 2.14: Signal reconstruction by a **PMU** under **CJ**. **PMU** with ideal clock vs. **PMU** with clock under the effect of **CJ**. Example with an ideal 50 Hz sinusoid, with a nominal sampling frequency of 8 samples per period.

incremented at the end of every period as in (2.24) [33],

$$k(t) = \left\lceil \int_0^t \nu(\tau) d\tau \right\rceil \quad (2.24)$$

and therefore the clock internal time C follows (2.25)

$$C(t) = \frac{k(t)}{\nu_0} \quad (2.25)$$

However, the period variations due to frequency and phase fluctuations cause an offset between the internal clock time and the real time:

$$\theta(t) = C(t) - t \quad (2.26)$$

Reference [34] proposes a clock and synchronization model to implement in a Kalman filter-based clock servo. However, because of the focus of synchronization mechanisms, the paper does not model clock errors other than what is necessary to implement the corrections at every synchronization. In contrast, the model developed hereafter integrates the reference model in order

to describe the evolution of the sampling instants in a **PMU** clock under the effects of clock errors and synchronization between master and slave. Since the sampling instants depend on the period durations, determining the actual sampling instants can follow from the modeling of the phase angle of a clock signal.

Consider (2.19). In this context, since the sampling instants of a clock are the results of the evolution of the whole phase angle of the oscillator output signal, the output signal itself will be defined as

$$v(t) = V_0 \sin \xi(t) \quad (2.27)$$

where $\xi(t) = 2\pi\nu_0 t + \phi(t)$. It is assumed that the clock is under the effect of **CJ** that manifests itself as white frequency noise and random-walk frequency noise. The effects of these random processes is to transform the skew and phase fluctuations into Wiener processes $\gamma_{\text{rw}}(t)$ and $\phi_{\text{rw}}(t)$, with diffusion coefficients σ_γ^2 and σ_ϕ^2 respectively. The effects of the corrections sent by the master clock at the synchronization instants are determined by $\gamma_{\text{corr}}(t)$ and $\phi_{\text{corr}}(t)$. Since $\gamma_{\text{rw}}(t)$ and $\phi_{\text{rw}}(t)$ are pure Wiener processes and do not include the effects of the corrections in any way, $\gamma_{\text{corr}}(t)$ and $\phi_{\text{corr}}(t)$ are piecewise constant over the holdover period between two synchronization messages T_{hold} and the value of the correction signals at any given instant is the sum of all the corrections applied at the previous synchronization instants. The phase angle and skew of the clock signal take the form shown in (2.28) and (2.29):

$$\xi(t) = 2\pi \int_0^t \nu(\tau) d\tau + \phi_{\text{rw}}(t) - \phi_{\text{corr}}(t) \quad (2.28)$$

$$\nu(t) = \nu_0 [1 + \gamma_{\text{rw}}(t) - \gamma_{\text{corr}}(t)] \quad (2.29)$$

Substituting (2.29) into (2.28) yields

$$\xi(t) = 2\pi\nu_0 \int_0^t [1 + \gamma_{\text{rw}}(\tau) - \gamma_{\text{corr}}(\tau)] d\tau + \phi_{\text{rw}}(t) - \phi_{\text{corr}}(t) \quad (2.30)$$

Three assumptions allow to adapt (2.30) into a discrete, recursive equation that is suitable for a simulation:

1. $\gamma_{\text{rw}}(t)$ and $\phi_{\text{rw}}(t)$ are piecewise constant for every interval $T_0 = \frac{1}{\nu_0}$: $\mathcal{H}_{T_0} \{\gamma_{\text{rw}}[k]\}(t)$, $\mathcal{H}_{T_0} \{\phi_{\text{rw}}[k]\}(t)$, where $\mathcal{H}_T \{\cdot\}(t)$ is the zero-order hold function with period T ;
2. T_{hold} is a multiple of T_0 ;

3. the corrections are sent instantaneously from the master clock and applied instantaneously by the slave clock at the end of every holdover period.

Under these assumptions, $\gamma_{\text{rw}}(t)$ and $\phi_{\text{rw}}(t)$ become random walks whose increments have variances $\sigma_\gamma^2 \cdot T_0$ and $\sigma_\phi^2 \cdot T_0$, respectively:

$$\gamma_{\text{rw}}[k+1] = \gamma_{\text{rw}}[k] + \omega_\gamma[k] \quad (2.31)$$

$$\phi_{\text{rw}}[k+1] = \phi_{\text{rw}}[k] + \omega_\phi[k] \quad (2.32)$$

$$\omega_\gamma[k] = \mathcal{N}(0, \sigma_\gamma^2 \cdot T_0) \quad (2.33)$$

$$\omega_\phi[k] = \mathcal{N}(0, \sigma_\phi^2 \cdot T_0) \quad (2.34)$$

Furthermore,

$$\begin{aligned} \int_0^t [1 + \gamma_{\text{rw}}(\tau) - \gamma_{\text{corr}}(\tau)] d\tau &= \sum_{i=0}^{k-1} \int_{iT_0}^{(i+1)T_0} [1 + \gamma_{\text{rw}}(\tau) - \gamma_{\text{corr}}(\tau)] d\tau \\ &\quad + \int_{kT_0}^t [1 + \gamma_{\text{rw}}(\tau) - \gamma_{\text{corr}}(\tau)] d\tau \\ &= \sum_{i=0}^{k-1} [1 + \gamma_{\text{rw}}[i] - \gamma_{\text{corr}}[i]] T_0 \\ &\quad + [1 + \gamma_{\text{rw}}[k] - \gamma_{\text{corr}}[k]] (t - kT_0) \end{aligned} \quad (2.35)$$

Substituting (2.35) into (2.30) and discretizing the remaining variables yields

$$\begin{aligned} \tilde{\zeta}(t) &= 2\pi \sum_{i=0}^{k-1} [1 + \gamma_{\text{rw}}[i] - \gamma_{\text{corr}}[i]] \\ &\quad + 2\pi\nu_0 [1 + \gamma_{\text{rw}}[k] - \gamma_{\text{corr}}[k]] (t - kT_0) \\ &\quad + \phi_{\text{rw}}[k] - \phi_{\text{corr}}[k] \end{aligned} \quad (2.36)$$

for every interval $t \in [kT_0, (k+1)T_0)$. Then, defining $\tilde{\zeta}[k] = \tilde{\zeta}(kT_0)$,

$$\tilde{\zeta}[k] = 2\pi \sum_{i=0}^{k-1} [1 + \gamma_{\text{rw}}[i] - \gamma_{\text{corr}}[i]] + \phi_{\text{rw}}[k] - \phi_{\text{corr}}[k] \quad (2.37)$$

Understanding that

$$\tilde{\zeta}[k+1] = 2\pi \sum_{i=0}^k [1 + \gamma_{\text{rw}}[i] - \gamma_{\text{corr}}[i]] + \phi_{\text{rw}}[k+1] - \phi_{\text{corr}}[k+1] \quad (2.38)$$

leads to the recursive equation

$$\tilde{\zeta}[k+1] = \tilde{\zeta}[k] + 2\pi [1 + \gamma_{\text{rw}}[k] - \gamma_{\text{corr}}[k]] + \omega_{\phi}[k] - \Delta\phi_{\text{corr}}[k] \quad (2.39)$$

with $\Delta\phi_{\text{corr}} = \phi_{\text{corr}}[k+1] - \phi_{\text{corr}}[k]$. Since ϕ_{corr} is piecewise constant over T_{hold} , the only k factors corresponding to $\Delta\phi_{\text{corr}}[k] \neq 0$ are those in which index $[k+1]$ corresponds to the end of a holdover period, and thus a synchronization instant. Then, extending (2.25) to also consider fractional internal clock times,

$$C[k] = \frac{\tilde{\zeta}[k]}{2\pi\nu_0} \quad (2.40)$$

and

$$\theta[k] = C[k] - kT_0 \quad (2.41)$$

The time offset between a non-ideal and an ideal **PMU** clock corresponds to a phase offset

$$\tilde{\zeta}_{\text{off}}[k] = 2\pi\nu_0\theta[k] \quad (2.42)$$

A perfect phase offset correction would therefore imply $\Delta\phi_{\text{corr}}[j] = \tilde{\zeta}_{\text{off}}[j]$, where j corresponds to the end of a holdover period. The phase offset correction $\Delta\phi_{\text{corr}}$ corresponds to a time offset correction $\Delta\theta_{\text{corr}}$:

$$\Delta\theta_{\text{corr}} = \frac{\Delta\phi_{\text{corr}}}{2\pi\nu_0} \quad (2.43)$$

The skew corrections can be inferred from the offset as well, assuming that the offset generated during an holdover can be entirely attributed to a skew variation:

$$\gamma_{\text{corr}}[j] - \gamma_{\text{corr}}[j-1] = \frac{\theta[j-1]}{T_{\text{hold}}} \quad (2.44)$$

These correction factors can be known exactly only if the master clock is aware of the state of all its slaves at all times and the communication is instantaneous. Otherwise, message latency and estimation uncertainties must be taken into account.

An example of the effect of the corrections is shown in figure 2.15. It is assumed that a non-ideal slave clock starts counting at the same instant

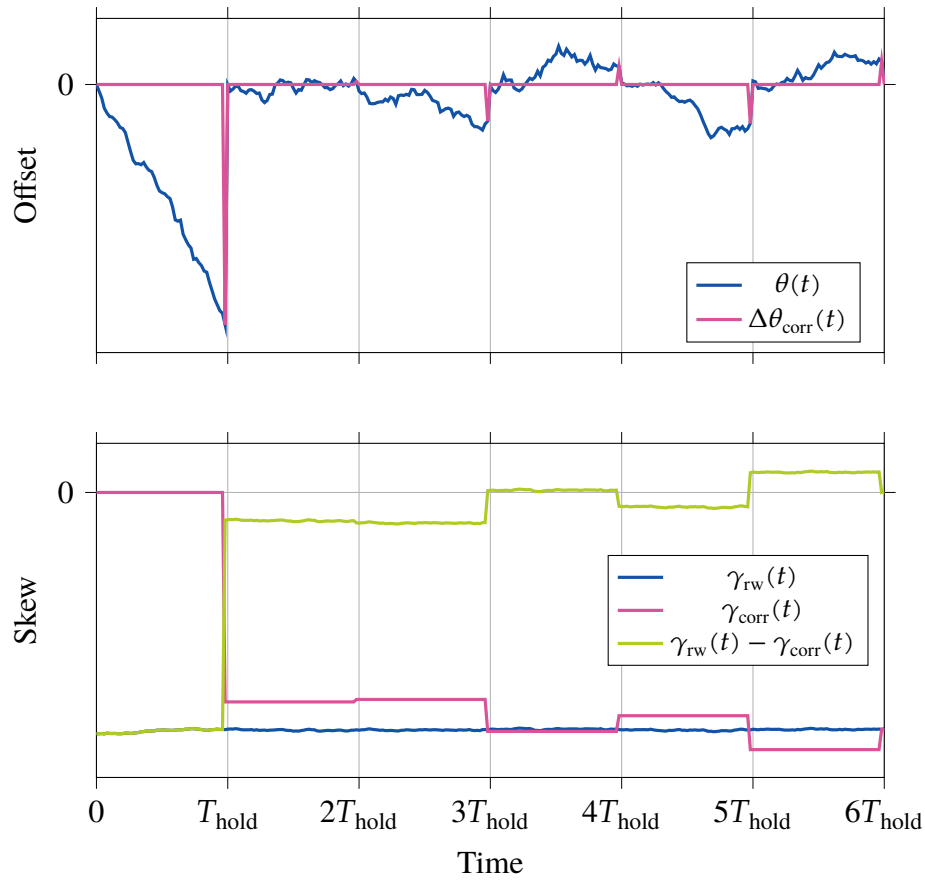


Figure 2.15: Example of the effect of offset and skew corrections on a clock.

as an ideal master clock. At the beginning, before any correction has been sent, the non-ideal clock is under the effect of a negative skew that causes the accumulation of a negative offset. At the first synchronization instant, the master clock measures the offset with the slave clock and sends a correction message to reduce the offset and compensate the skew. If the clock were left running without correction, the negative skew would continue to increase the offset indefinitely. Instead, the corrections cause the skew (green line in the bottom plot) and offset (blue line in the top plot) to always remain close to zero. A null offset means that the slave clock is running at the same rate as the master clock and the internal clock time of the slave clock is equal to the real time, leading to accurate sampling and timestamp attribution.

However, the instantaneous offset corrections cause a step in the internal clock time of the slave clock. It is necessary to differentiate between the cases in which the offset correction is positive and those in which it is negative:

- If the offset correction is positive, the internal clock time suffers a negative step. This means that the clock was counting time "in the future", and it will then come back into the past. To ensure the attribution of monotonic timestamps to the **PMU** samples, the attribution must be blocked until the internal clock time returns to its past maximum value.
- If the offset correction is negative, the internal clock time suffers a positive step, meaning that the internal clock "jumps" to a future time. If this jump causes the internal clock time to surpass a multiple of T_0 , a sample will be gathered and a timestamp will be attributed exactly at the synchronization instant. This simplification that follows from the instantaneous application of correction factors shows one limit of this model: the offset accumulation over a single holdover interval must be small enough so that the jump causes the increase of the internal clock time by one ideal period at the most. If this were not the case, there would be the risk of increasing the clock counter by multiple units because of a single correction, causing the gathering of two different samples with a single timestamp.

In reality, these limits are not encountered because the correction factors are applied gradually, and the internal clock time does not jump forward or backwards but maintains a monotonic increase with different slopes that allow the slave clock to catch up to, or wait for, the master clock.

These considerations, together with the knowledge that a **PMU** begins its sampling process at the end of a clock period, allow to define the sampling instants of a non-ideal **PMU** from the internal phase angle and internal clock time^{*}:

$$t_k = \inf \{t : \xi(t) \geq 2\pi k\} = \inf \{t : C(t) \geq kT_0\} \quad (2.45)$$

The computation of this sampling grid directly follows the computations of (2.39) and (2.40).

This model intentionally leaves frequency drift – intended as the frequency dependency on environmental variations – out of the picture because of its slow dynamics compared to the time spans of interest. Even if it were integrated into the model, its effects would be easily corrected by the skew corrections. Furthermore, this model does not apply to clocks that are dominated by flicker noise over the holdover time span, since, as

^{*}inf is the infimum, or greatest lower bound: the infimum of a subset of a set is the greatest element in the set that is less than or equal to each element of the subset [35].

stated in section 2.4.1, diffusion coefficients cannot be obtained for flicker noise that needs different modeling strategies. To complete the discussion, section 2.4.2.1 shows how the diffusion coefficients for phase and frequency fluctuations can be obtained from the Allan variance plots for different oscillators.

2.4.2.1 Diffusion coefficients from Allan variance plots

In the model of section 2.4.2, it was assumed that the phase noise was dominated by white frequency variations and random-walk frequency variations. In this case, a closed-form approximation of the Allan variance assumes the expression of

$$\sigma_y^2(\tau) = \frac{\sigma_\theta^2}{\tau} + \frac{\sigma_\gamma^2}{3}\tau \quad (2.46)$$

The method for obtaining σ_θ and σ_γ from real Allan variance plots is suggested by [28]. Referring to figure 2.12, it is evident that some noise random processes are predominant over the others for a given time span. This means that for the appropriate time spans it is possible to approximate

$$\sigma_y^2(\tau_1) \simeq \frac{\sigma_\theta^2}{\tau_1} \quad (2.47)$$

and

$$\sigma_y^2(\tau_2) \simeq \frac{\sigma_\gamma^2}{3}\tau_2 \quad (2.48)$$

However, the model is not interested in modeling a random walk in the offset through δ_θ^2 , but a random walk in the phase through δ_ϕ^2 . Since the relationship between phase variations and offset variations is known,

$$\phi = 2\pi\nu_0\theta \quad (2.49)$$

it is possible to infer

$$\sigma_\phi^2 = (2\pi\nu_0)^2\sigma_\theta^2 \quad (2.50)$$

2.5 Monte Carlo simulations

Monte Carlo methods are a class of methods that adopt random sampling to solve a generic problem [36]. The problem in question might be stochastic,

such as computing the expected value of a **RV**, or deterministic, for example, when the behavior of a complex deterministic system must be evaluated under the effect of different input parameter configurations, where each input parameter is modeled through a specific **RV**. In the second case, repeated simulation of the system behavior might be more practical than an analytic evaluation to monitor some specific output parameters.

The input parameter combinations can be sampled according to various different methods. The most straightforward one is referred to as *simple sampling*, under which the samples are collected completely at random. A **MCS** that adopts this sampling methods is referred to as *crude MCS*. More complex sampling methods exist: for example, variance reduction techniques aim at reducing the uncertainty connected to the output evaluation with respect to an analogous crude **MCS** performed with the same number of samples. Some sampling strategies, such as Latin Hypercube Sampling, ensure efficient coverage of all the input parameter ranges even with a low number of samples. Finally, Importance Sampling focuses on analyzing the specific input combinations that are the most relevant to the final result.

2.6 Sensitivity analysis

In mathematical form, a model expresses the relation between the inputs and output (or, eventually, outputs) of a system:

$$y = g(x_1, x_2, \dots, x_n) \quad (2.51)$$

where y represents the output, x_k the k -th input, and $g(\cdot)$ is a generic model equation. In this context, a model can be intended as an algorithm which aims to reproduce the behavior of a real-world system. The model definition requires a series of assumptions: its type and structure, parameter definition, boundary conditions, temporal or spatial resolution [17]. Each of these assumptions is accompanied by a degree of uncertainty. Therefore, they are considered inputs since their variations modify the behavior of the model. If the inputs' uncertainty is defined – for example, through its **Probability Density Function (PDF)** or its **Probability Mass Function (PMF)** – it is possible to let the inputs vary over their domain, assuming all their possible combinations, and observe how these variations translate to variations in the model output. This is the principle behind **UA**, which is a distinct but closely related concept to **SA**. A concise and frequently used definition of **SA** is "sensitivity analysis is the study of how the uncertainty in the model output can be apportioned to

the different sources of uncertainty in the model inputs” [16]. If **UA** handles characterization of the model output’s uncertainty, **SA** takes it one step further and determines which of the inputs are more influential towards it. In addition to helping identify input importance, the combination of **UA** and **SA** reveals which input parameter combinations lead to specific output results, which may represent optimal or suboptimal system performance. In this sense, **SA** can be a powerful tool for evaluating the need for specific mitigation strategies and improving system performance; however, in this context, its usefulness lies in the immediacy with which it helps to identify which power system variables affect **PMU** performance the most, under reasonable operating conditions.

In practice, it is not guaranteed that the input uncertainty will resemble a textbook **PDF**, or that the model is known as an explicit equation; in these cases, a **MCS** is primarily adopted for performing the model evaluation. The number of simulation runs should be sufficient to cover the whole uncertainty of the inputs with an adequate convergence level. The most common **SA** methods can be adapted and implemented to manipulate sample-based **PDFs**.

2.6.1 Classification of sensitivity analysis methods

SA methods are distinguished based on the sampling approach and the input space exploration viewpoint [17]. Sampling strategies for **SA** are divided in *One-At-a-Time (OAT)* and *All-At-a-Time (AAT)*:

- In **OAT** sampling, every model input except one is maintained constant at its baseline value. Following this approach, every variation in the model output which is encountered while varying a specific input can be attributed to that input only;
- On the other hand, in **AAT** sampling, every input parameters is varied at the same time. This approach has the advantage of evaluating the effects of the interactions between the model inputs.

Moreover, the input space exploration can be *local* or *global*:

- In local **SA**, the inputs are varied around a specific operating point of the system, without attempting to factor the whole input uncertainty;
- On the contrary, the goal of global **SA** is to consider the whole range of uncertainty in the input parameters.

Notice how the choice of the space exploration approach does not imply the choice of a specific sampling strategy. Even if the scope of local **SA** could

appear limited compared to the one of global SA, its usefulness is evident in presence of linear models (where the model behavior in the vicinity of a point is representative of the entire model) or if the modeler is interested only in the model behavior around a baseline value. Nevertheless, it is important not to generally assume that the results of a local SA could be valid even for a wider range of input parameter combinations.

2.6.2 Quality features of sensitivity analyses

Reference [37] defines the requirements that a quality SA should possess:

1. *Account for influence of scale and shape of the input parameters' uncertainty.* The PDFs of the model inputs should be reproduced as realistically as possible, in order to obtain results which are externally valid;
2. *Include multidimensional averaging,* i.e., vary the inputs according to the AAT approach to capture interactions among them;
3. *Model independence,* such that the model yields proper results regardless of the model properties, such as linearity or additivity;
4. *Interpretability,* achieved by selecting relevant input factors or grouping them together when possible.

Borgonovo [38] adds another feature to the ones discussed above, which is *moment independence*. The reference discusses how SA should account for the effects of the input uncertainty on the whole output PDF instead of referring to one of its moments, such as its variance. Variance alone is, in fact, not sufficient to fully describe variability. Furthermore, variance-based methods for SA have the disadvantage of requiring independence among input parameters for a correct evaluation of input relevance.

2.6.3 Generic sensitivity analysis framework

Reference [17] attests the necessary steps to perform a quality SA. They are subsequently explained and summarized in figure 2.16, which also emphasizes the tools used during each phase of this project.

1. *Define the purpose of the analysis.* A clear idea of the scope of the research can help the modeler discern between the relevant and irrelevant input parameters, choosing the sampling criteria, and how the input space exploration is performed.

2. *Design the model.* Also choosing the most suitable evaluation method.
3. *Define the model output.* This stage should be output-oriented, in the sense that the output should provide information that is relevant to the purpose of the analysis. Output aggregation can be considered, following the principle that a good SA should be easily interpretable (see Section 2.6.2).
4. *Identify the input parameters.* The variables adopted as input parameters for the SA may be described with continuous PDFs or discrete PMFs alike. Inputs not involved in the SA (e.g., the simulation time step) must also be fixed at their baseline values.
5. *Assign RVs to the input parameters.* This stage is crucial since the input uncertainty propagates through the simulation to define the output uncertainty. Therefore, various choices of input RVs (and their correlations) might yield substantially different results and conclusions.
6. *Choose the SA method.* Justification for the choice depends on the purpose of the analysis, characteristics of the model (e.g., linearity) and the computational burden it represents. Input constraints, such as correlation among them, is also critical.
7. *Generate the input sample.* The sampling strategy and sample size depend on the chosen SA method. The sample size also depends on the computational time of the model. There is no predetermined law that establishes the required number of samples; however, there is a trade-off between sample size and accuracy. Therefore, [17] suggests to start with a small sample size, to be enlarged in case of need.
8. *Evaluate the model.*
9. *Estimate the output uncertainty.* At this point of the analysis, it is possible to estimate a PDF of the outputs and eventually calculate some summary statistics.
10. *Extract the sensitivity measures.*
11. *Iterate SA.* The first run of the SA might have provided important information about relevant or irrelevant inputs. A subsequent analysis might start with a more accurate description of the relevant inputs, while the irrelevant ones might be discarded altogether.

12. *Make use of graphical tools to complement the analysis.* Graphical tools can be the most immediate tools to convey the results.

The steps are not as isolated as it appears from the list and figure 2.16. For example, the choice of SA method might be conditioned by the tools at hand, and the data extracted from the model might require thorough manipulation before obtaining a significant metric for the following analyses.

2.6.4 Relevant sensitivity analysis methods

SA is achievable through different methods which may or may not satisfy all the conditions described in Section 2.6.2. However, for reasons later explained in section 3.4, the Borgonovo indices were chosen as sensitivity indices to be used throughout the whole project, and their derivation is explained in the following section.

2.6.4.1 Borgonovo's delta

Borgonovo's delta is a global, AAT, and moment-independent SA method first introduced in [38]. Let

$$\underline{X} = (X_1, X_2, \dots, X_n) \in \mathbb{R}^n \quad (2.52)$$

be a set of uncertain input parameters, characterized by a PDF or a PMF. One of its realizations, $\underline{x}$, can be defined as

$$\underline{x} = (x_1, x_2, \dots, x_n) \quad (2.53)$$

The functional relationship between the output Y and the input vector $\underline{X}$ is given by g :

$$Y = g(\underline{X}) \quad g(\underline{X}) : E \subseteq \mathbb{R}^n \rightarrow \mathbb{R} \quad (2.54)$$

$F_{\underline{X}}(\underline{x})$ is the joint cumulative distribution of the various inputs X_i , and $f_{\underline{X}}(\underline{x})$ is the corresponding joint density of $\underline{X}$. $F_{\underline{X}}(\underline{x})$ and $f_{\underline{X}}(\underline{x})$ represent the modeler's state of knowledge of the inputs. Every one of them is modeled as a RV based on real-world data or assumptions. Finally, $f_Y(y)$ is the PDF of the output Y , and $f_{Y|X_i}(y)$ the conditional density of Y that is obtained when the input parameter X_i assumes a fixed value.

Now, consider the output PDF that is obtained by fixing one of the inputs at the value x_i^* . This PDF corresponds to $f_{Y|X_i}(y)$. The shift between the initial

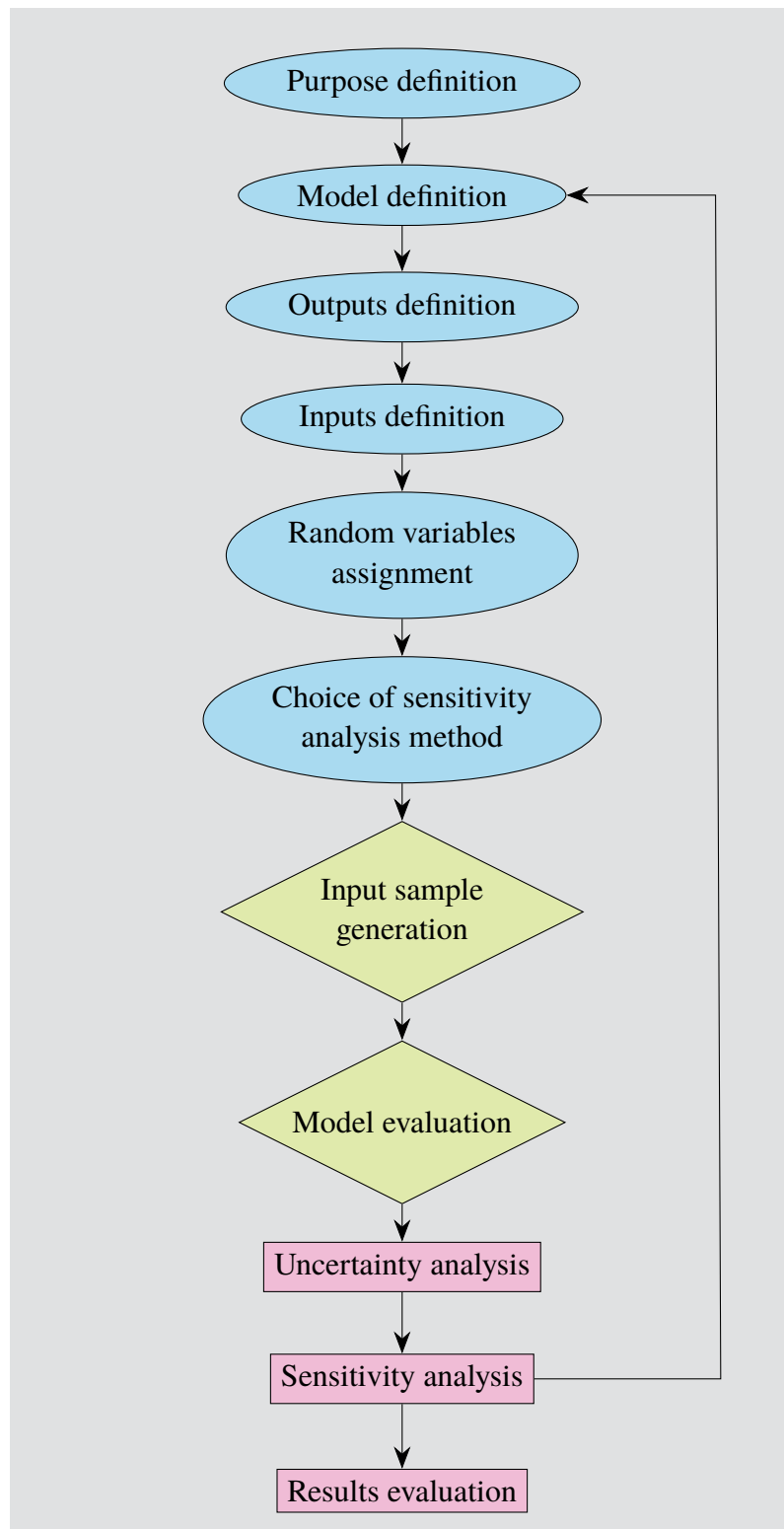


Figure 2.16: **SA** framework. The ellipses represent generic conceptual steps. The diamonds refer to operations implemented in Python/PSCAD, and the rectangles to MATLAB-based steps.

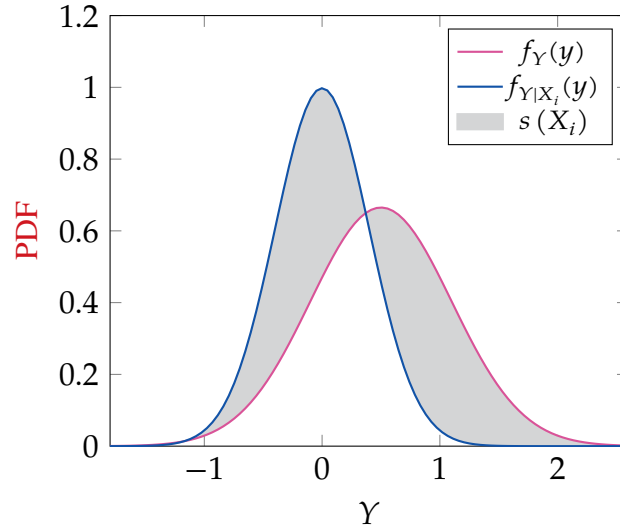


Figure 2.17: Example of $s(X_i)$, the area corresponding to the shift between $f_Y(y)$ and $f_{Y|X_i}(y)$.

PDF $f_Y(y)$ and $f_{Y|X_i}(y)$ corresponds to the area

$$s(X_i) = \int |f_Y(y) - f_{Y|X_i}(y)| dy \quad (2.55)$$

An example is shown in figure 2.17, where $f_Y(y)$ and $f_{Y|X_i}(y)$ are assumed to be Gaussian **PDFs**. The expected shift is given by

$$E_{X_i}[s(X_i)] = \int f_{X_i}(x_i) \left[\int |f_Y(y) - f_{Y|X_i}(y)| dy \right] dx_i \quad (2.56)$$

where $f_{X_i}(x_i)$ is the marginal density of input x_i . Then, the quantity δ_i , defined as

$$\delta_i = \frac{1}{2} E_{X_i}[s(X_i)] \quad (2.57)$$

is the normalized expected shift in the distribution of Y caused by X_i . This quantity can be used as a moment-independent indicator of the sensitivity of Y with respect to X_i .

The usefulness of moment-independent **SA** indexes is that, in certain cases, such as in presence of a multimodal or skewed output distribution, variance cannot describe the complete output variability [17]. Furthermore, the δ index does not require independence between the inputs for a correct evaluation of the output sensitivity.

Some relevant properties of the delta index are subsequently listed:

1. $0 \leq \delta_i \leq 1$;
2. if the output Y is independent of input X_i , then $\delta_i = 0$;
3. $\delta_{1,2,\dots,n} = 1$: the joint importance of all the input variables is equal to 1;
4. if the output Y is independent of X_j but dependent on X_i , then $\delta_{i,j} = \delta_i$;
5. $\delta_i \leq \delta_{i,j} \leq \delta_i + \delta_{j|i}$.

Refer to [38] for proofs of the aforementioned properties.

2.6.5 UQLab: A Framework for Uncertainty Quantification in MATLAB

In this project, **SA** was implemented through **UQLab** [39]. UQLab is a MATLAB-based open-source software for uncertainty quantification. As it is stated in the website, "It is made of open-source scientific modules which are smoothly connected to perform uncertainty quantification through **MCS**, **SA**, reliability analysis (computation of rare event probabilities), surrogate models (polynomial chaos expansions, Kriging, etc.), Bayesian inversion/calibration and many more."

In this project, the interest in UQLab falls onto the implemented **SA** methods:

- Sample-based methods (input/output correlation and standard regression coefficients);
- Linearization (perturbation) method;
- Screening methods (Morris elementary effects, Cotter indices);
- Moment-independent global method (Borgonovo indices);
- Sobol indices (variance-based indices);
- Generalization of Sobol indices for dependent input parameters (Kucherenko and ANCOVA indices);

The framework also allows to build models directly in MATLAB, ready to use for a **MCS** or to build metamodels through Polynomial Approximation, Polynomial Chaos Expansion, Stochastic Polynomial Chaos Expansion,

Generalized Lambda Models, Support Vector Machines, and Gaussian process modeling, by providing external input samples and the relative model evaluations. Some UQLab methods, in particular the Sobol' indices and Generalized Sobol' indices, require the construction of an internal or metamodel for the execution of the **SA**; others, such as the Borgonovo indices, can function by providing some raw external samples directly.

2.7 Summary of Background concepts

The Background chapter provides the necessary theoretical foundations to understand and interpret the results of the **SAs**. Errors introduced in instrument transformers and sampling and digitalization of analog signals affect the accuracy of **PES** results. **CT** saturation is due to high fault current magnitudes and their interactions with the signs and magnitudes of any eventual **DC** components and remanent flux densities. **CVT** errors are twofold: parameter degradation can cause systematic magnitude errors, while inaccurate transient behavior is due to the energy stored in the reactive components of the device and their time constants. Among the many errors introduced in the **ADC** of a **PMU**, the thesis focuses on clock jitter which leads to inaccurate timestamping attribution to the signal samples.

Since the aforementioned errors depend on stochastic phenomena, the chosen **SA** method is a sample-based **SA**, which adopts a **MCS** to obtain a complete statistical picture of the measurement errors and then evaluate how the output uncertainty can be apportioned to the different inputs, therefore identifying which stochastic parameters affect **PES** reliability the most.

Chapter 3

Methods

This chapter discusses and motivates the methods adopted during the experimental section of the work. A bird's eye view on the project workflow is shown in figure 3.1.

After the model definition on PSCAD, a MCS was set up in Python to generate RVs for the SA. The RVs include parameters from the test system, CTs, and CVTs, in order to observe the behavior of PES under various faults, pre-fault conditions, and instrument transformer configurations. The system parameters that were not modeled as RVs were kept constant throughout the simulation phase.

The simulation results were exported from PSCAD in COMTRADE format. The simulation time step and COMTRADE recording time step were kept fixed. The COMTRADE recording time step, in particular, was set at 100 μ s to simulate the operation of a PMU with an ideal clock running at the sampling frequency of 10 kHz, or 200 samples per cycle. Although this sampling frequency is higher than typical PMU sampling frequencies (modern devices operate with sampling frequencies of 96 or 128 samples per second [9]), a more densely sampled signal presents a lower discretization error, allowing subsequent analyzes to focus purely on the errors introduced by non-ideal instrument transformers. Then, the COMTRADE output files were read and stored in MATLAB files for PES and Key Performance Indicator (KPI) calculations. This process was sufficient to carry out the SA of different error metrics under CT saturation and CVT transients and parameter degradation: the sensitivity of the TVE between PESs from ideal and non-ideal measurements was used as the first evaluation metric in most cases, and ME and PE were examined when relevant. PES was carried out with a one-period window to balance stability and reactivity.

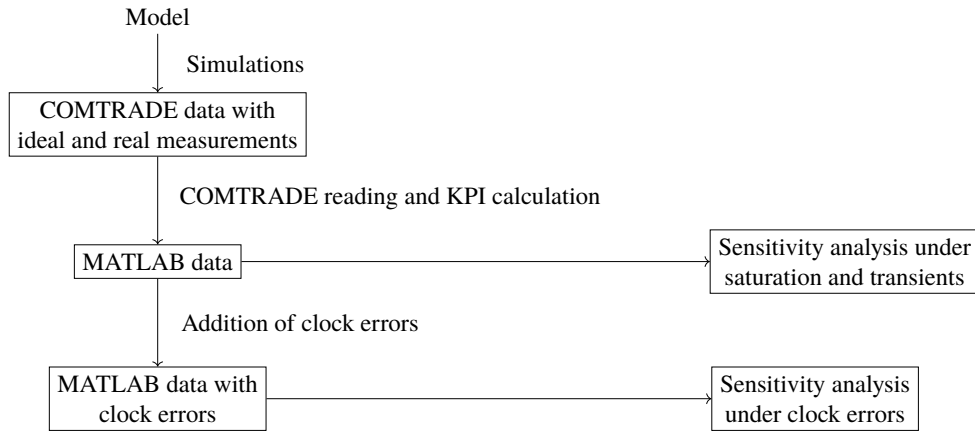


Figure 3.1: Bird's eye view on the project workflow.

In a later phase, the original MATLAB data was manipulated to simulate sampling and **PES** under the effect of a non-ideal clock with a synchronization mechanism designed according to the model of section 2.4.2. In this context, other than the **PES**-related errors, the analysis also focused on timestamp errors and **FE**. The method choice fell onto manipulating the original MATLAB data because no efficient alternative was found to inject clock errors directly into the PSCAD simulations.

For coherence with the background of section 2, the chapter is organized following the **SA** framework described in section 2.6.3 and figure 2.16. The purpose of the analysis was already defined in the introduction of chapter 1. Section 3.1 introduces the system modeled in PSCAD and provides the values of its constant parameters. Section 3.2 explains the motivations behind the particular choices of output variables, while section 3.3 focuses on the inputs of the sensitivity analysis, and motivates their choices of **RVs**. Section 3.4 explains why the Borgonovo indices are the most appropriate sensitivity indices for this context. Section 3.5 describes the simulation set up, and finally section 3.6 describes how the simulation data was manipulated for analyzing the impact of clock errors on **PES**.

3.1 Model and inputs definition

The circuitual model adopted for the simulations is reproduced in figure 3.2. It is constituted by a 230 kV transmission line supplied on both sides by non-ideal 50 Hz voltage sources, with voltages $\bar{V}_1 = V_1 \angle \varphi_1$ and $\bar{V}_2 = V_2 \angle \varphi_2$ and identical positive sequence impedances $\bar{Z}_1$ and zero sequence impedances

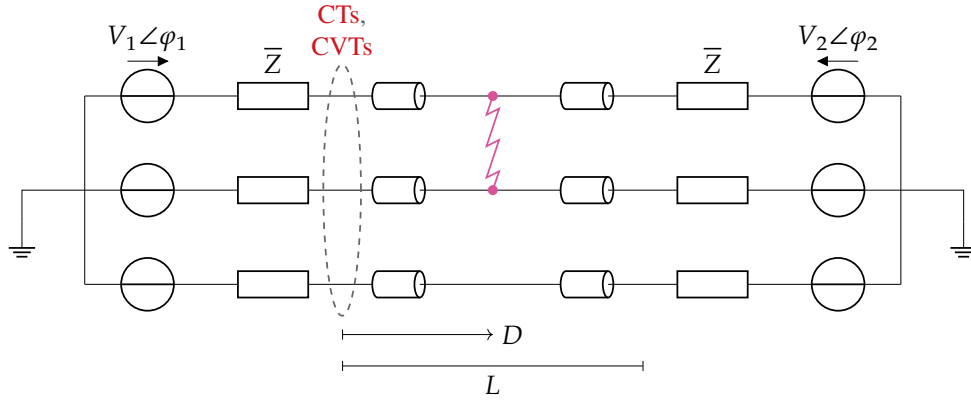


Figure 3.2: Test system.

$\bar{Z}_0$. The load angle β is the difference between the phase angles of the voltage sources, $\beta = \varphi_1 - \varphi_2$. The transmission line has parameters r_1 and x_1 and length L and is equipped with **CTs** and **CVTs** on one side to monitor current flows and voltage levels over the three phases. The simulated events are faults of different type: **Line-to-Ground (LG)**, **Line-to-Line (LL)**, **Double-Line-to-Ground (LLG)**, and **Three-phase (LLL)** with appropriate values of fault resistance R_f , at a distance D from the source $\bar{V}_1$.

The choice of parameters to be treated as **RVs** (or, conversely, to be maintained constant) reflects the common operation of a power system. Some parameters, such as the geometric and material properties of a line, remain somewhat constant, and some suitable values have been chosen; on the other hand, the electrical parameters and fault characteristics depend on the specific scenario at hand. The constant system parameters are depicted in table 3.1, while the choice of **RVs** to model the remaining parameters will be discussed in section 3.3.

The **CTs** have secondary resistances and inductances R_{sec} and L_{sec} , burden resistances R_b , and turns ratios N_{ct} , as in the equivalent circuit of figure 2.3. The core dimensions are the area A and length l . The remanent flux density in the cores is B_r and the saturation voltage is V_s . N_{ct} is designed to ensure the flow of the nominal burden current in the secondaries of the **CTs** under the maximum primary system current during normal operation. Assuming a maximum load angle $\beta = 30^\circ$ and rated system voltages of 230 kV, the maximum primary load current is close to 1 250 A, leading to a turns ratio $N_{\text{ct}} = 1\,250$ with a rated burden current of 1 A. One of the goals of the analysis was to study the behavior of the model under **CTs** with different saturation voltages and remanent flux densities; therefore, only the geometric parameters

Table 3.1: Test system constant parameters.

Category	Parameter	Value
Sequence impedances	$ \bar{Z}_1 $	5Ω
	$\angle \bar{Z}_1$	85°
	$ \bar{Z}_0 $	10Ω
	$\angle \bar{Z}_0$	80°
Line parameters	L	100 km
	r_1	$34.68 \text{ m}\Omega \text{ km}^{-1}$
	x_1	$423.37 \text{ m}\Omega \text{ km}^{-1}$

Table 3.2: CT constant parameters.

Category	Parameter	Value
Electrical parameters	R_b	$0.45 \Omega^* [40]$
	L_b	0 H
	R_{sec}	1.25Ω
	L_{sec}	2 mH
Geometric parameters	l	0.8 m
	N_{ct}	$1\,250$

that do not influence the saturation voltage and basic electrical parameters were maintained constant and are summarized in table 3.2.

The CVTs were modeled as in the equivalent circuit of figure 2.5 with the FSC of figure 2.7 in parallel to the burden. Besides the burden, assumed to be resistive with resistance R_b , and the secondary short-circuit parameters R_{cc} and L_{cc} , all other parameters were modeled as RVs to study the effects of their aging and degradation. The turns ratio N_{cvt} was designed to match the nominal system voltage with the nominal burden voltage of 110 V. The constant CVT parameters are summarized in table 3.3.

The test system was intentionally kept simple to apply the SA framework to an easily interpretable model and reduce computational burden. Furthermore, as explained in section 2.6.2, adopting a low number of random variables also eases the interpretation of the SA results by focusing strictly on the RVs used to model the measurement chain and the components directly affected by the faults. If the results were to be realistic and supported by experience or

*Including a lead wire resistance of 0.25Ω .

Table 3.3: CVT constant parameters.

Category	Parameter	Value
Electrical parameters	R_b	605 k Ω [40]
	R_{sec}	1.25 Ω
	L_{sec}	2 mH

intuition, this would serve to validate the SA framework itself.

3.2 Output definition

The inputs of the SA, as defined for this model, are the scalar values of the model parameters varied at each simulation run. The output variables are different in this regard: the measurement errors are time series over the duration of a simulation run. Therefore, to have a single scalar output for each input vector, as in the definition of (2.51), it is necessary to extract statistical measures from the output time series. The choice of which statistical measures to focus on depends on the nature of the disturbance itself. For example, saturation causes a huge temporary discrepancy between the system current and the secondary current of a CT; therefore, examining the maximum TVE, ME, and PE across the simulation runs can help to identify the cases in which saturation is the heaviest and in which way PES is impacted. Another example could be the analysis of parameter degradation in CVTs. The variation of a parameter from its nominal value leads to a stationary measurement error; therefore, the average TVE in a simulation run could be a suitable indicator of parameter degradation, while the maximum TVE might be more appropriate to evaluate transient performance resulting in instantaneous overvoltages. The PE can be analyzed to verify the analytical derivations of section 2.3.1.1. The output metrics adopted to analyze each scenario are summarized in table 3.4. The saturation presence flag was obtained as explained in section 4.1.

3.3 Random variables assignment

Since the clock errors were implemented in a later stage, the RV definition before the simulations concerned simply the variables already introduced in the previous sections. It is worth noting that this phase is crucial for a successful SA: if the RVs chosen for the simulations do not reflect the real

Table 3.4: Output metrics adopted for the analysis of **CTs** saturation and **CVTs** degradation and transients.

Scenario	Metric
CT saturation	Saturation presence flag
	Maximum TVE
	Maximum ME
	Maximum PE
CVT degradation and transients	TTS
	Mean TVE
	Max TVE
	Mean PE

world behavior of a system parameter, the output will also not be representative of the real world behavior, as discussed in section 2.6.2. The actual variables which were chosen for the simulations are introduced in table 3.5, table 3.6, and table 3.7, while the references and justifications are provided in the text.

The statistics on the frequency of each fault type come from [41]. The fault resistance is based on [42]. The article presents the use of negligible fault resistance for faults not involving the ground, while proposing a truncated Weibull for ground faults. However, the upper limit of this variable was deemed too high to be realistic, and the **PDF** was scaled to obtain a maximum resistance value of $50\ \Omega$. The fault location **RV** intends to represent the possibility that a fault may happen everywhere on the line with uniform probability; however, the extremities of the line were not included in the **RV** since PSCAD requires low simulation time steps to correctly evaluate transients on very short lines, and the adopted **RV** was deemed an appropriate compromise between speed and fidelity. The **RV** for the phase angle of voltage $\bar{V}_1$ at fault inception was chosen disregarding the fact that a fault, such as a flashover, is more likely to happen when the voltage sinusoid is closer to its peak. The load angle was kept as such to simulate realistic load conditions. Notice that the phase angle φ_2 is uniquely determined by setting the fault inception angle φ_1 and load angle β . The voltage magnitudes were modeled as a voltage control system was in place, which kept the source voltages always between $\pm 2\%$ of their nominal values of 230 kV.

The minimum and maximum **CT** saturation voltages were chosen according to the principles explained in section 2.2.1. The other intermediate voltages were adopted to better highlight the effect of different saturation

Table 3.5: System **RVs** definition for saturation, transients, and degradation analysis.

Variable	RV		Parameters
Fault type	Discrete	LG : $p = 0.7$, LL : $p = 0.15$, LLG : $p = 0.1$, LLL : $p = 0.05$	
Fault resistance R_f	Weibull + discrete	0 Ω for LL and LLL faults; Weibull with mean 12.88 Ω and mode 9.47 Ω bounded between 5 Ω and 50 Ω for LG and LLG faults	
Fault location D	Uniform		[2 km, 98 km]
Fault inception angle φ_1	Uniform		[0°, 360°)
Load angle β	Uniform		[−30°, 30°]
Voltage magnitude V_1	Uniform		[225.4 kV, 234.6 kV]
Voltage magnitude V_2	Uniform		[225.4 kV, 234.6 kV]

Table 3.6: **CT RV**_s definition for saturation analysis.

Variable	RV	Parameters
Saturation voltage V_s	Discrete	55 V, 66 V, 77 V, 88 V, 99 V, 110 V with equal probability
Remanent flux density B_r	Piecewise-uniform	$[-0.8, -0.6]$ * with $p = 0.135$, $[-0.6, -0.4]$ with $p = 0.08$, $[-0.4, -0.2]$ with $p = 0.09$, $[-0.2, 0]$ with $p = 0.195$, $[0, 0.2]$ with $p = 0.195$, $[0.2, 0.4]$ with $p = 0.09$, $[0.4, 0.6]$ with $p = 0.08$, $[0.6, 0.8]$ with $p = 0.135$

* The values are given as a proportion of the saturation flux density of the **CT**.

Table 3.7: Capacitive voltage transformer **RVs** definition for transients and degradation analysis.

Variable	RV	Parameters
Capacitance C_1	Split normal	$\mu = 0.96 \text{ nF}$, $\sigma_1 = 12 \text{ pF}$, $\sigma_2 = 24 \text{ pF}$
Capacitance C_2	Split normal	$\mu = 9 \text{ nF}$, $\sigma_1 = 0.11 \text{ nF}$, $\sigma_2 = 0.23 \text{ nF}$
Compensation inductance L_{comp}	Normal	$\mu = 1\,013 \text{ H}$, $\sigma = 12.67 \text{ H}$,
SDT turns ratio N_{cvt}	Normal	$\mu = 115.47$, $\sigma = 1.44$
FSC capacitance C_{fsc}	Normal	$\mu = 9.6 \text{ }\mu\text{F}$, $\sigma = 0.12 \text{ }\mu\text{F}$
FSC inductance L_{fsc}	Normal	$\mu = 1.055 \text{ H}$, $\sigma = 13.2 \text{ mH}$
FSC resistance R_{fsc}	Normal	$\mu = 40 \text{ }\Omega$, $\sigma = 0.5 \text{ }\Omega$

voltage values on **CT** performance. The corresponding parameters that were used in the PSCAD **JA CT** model are shown in table B.1, and were derived by assuming a saturation flux density of 1 T in all cases. Refer to section B.2 to understand the meaning behind the different parameters. Reference [43] provides the results of a statistical survey about remanence in **CTs** in 230 kV systems that can be applied directly for this work's scope.

The **CVT RVs** were defined from standard parameter values contained in industrial and academic papers. Normal distributions were deemed suitable to simulate the effects of parameter uncertainty and degradation. ABB [44] usually adopts intermediate **SDTs** with a primary voltage of $22/\sqrt{3}$ kV. With a rated burden voltage of 110 V, this voltage value corresponds to the mean of the N_{cvt} **RV** shown in table 3.7. The mean values of the C_1 and C_2 **RVs** were designed to perfectly complement the turns ratio N_{cvt} while also respecting the constraint for the total divider capacitance reported in [45]. The compensation inductance L_{comp} was chosen in order to be perfectly resonant with the capacitive divider. Lastly, the means of the **FSC** parameters were taken from [46]. The standard deviations for all parameters but the divider capacitances were chosen such that the 4σ limits corresponded to 5% of the mean values, to simulate the effects of mild degradation. For the divider capacitances, [45] reports an uncertainty of $-5\%/+10\%$ with respect to the nominal capacitance values. For this reason, these **RVs** adopt split normal distributions instead of the conventional normal symmetrical normal distribution.

All the **RVs** besides the fault type and resistance are assumed to be independent.

For the sake of interpretability, only the **RVs** relevant for a certain output metric were considered in the **SA**. For example, the **CT** saturation voltage was not taken into account during the **SA** of the **CVT**-related errors; similarly, all the **CVT** parameters were excluded from the **CT** analysis.

3.4 Choice of sensitivity analysis method

Referring to section 2.6.1, **SA** methods are classified as *local* or *global*, and **AAT** or **OAT**. However, power system operation is not organized around a vector of baseline input values, and such inputs vary together and interact with each other. Therefore, the choice of a global, **AAT SA** method is natural for this context. As listed in section 2.6.5, UQLab provides various global **AAT SA** methods that could be suitable for the case at hand. Specifically, besides screening methods, the available tool had to be chosen between

the Borgonovo indices, Sobol' indices, and generalized Sobol' indices. An attempt was made to implement the generalized Sobol' indices, which, together with the Borgonovo indices, allow dependence between the input **RVs**. However, as stated at the end of section 2.6.5, these methods require the fitting of a polynomial metamodel for their utilization in UQLab. Since these models were observed to perform poorly in the case of output **RVs** with multimodal distributions, or in the case of systems with a highly nonlinear relation between inputs and outputs, such as power system variables and **PMU** measurement errors, Borgonovo's delta became the natural choice for this project. Compared to the generalized Sobol' indices, it has the downside of not defining second order indices that highlight the effects of the interaction between **RVs**; on the other hand, being a moment-independent **SA** method, it does not limit its evaluation to the effects of a **RV** on the output variance.

3.5 Input sample generation and model evaluation

Straightforward simple sampling was preferred to other more complex sampling methods, since their implementation would have required additional time without providing significant improvements to the results of the **SA**. The number of simulation runs was fixed at 4 200, or 700 simulations for each **CT** saturation voltage. This choice was made to ensure sufficient convergence of the probability of saturation for each saturation voltage case. Figure 3.3 shows the trend of these probabilities over the simulation runs and how they tend to stabilize after 500 runs and beyond. After 700 simulations the probabilities are starting to settle around their final values. Although convergence was not yet perfect, it was considered acceptable, since most of the statistics used for the results were computed by merging the data obtained with the different saturation voltage values, using the whole entirety of the simulation runs. The consequence of using this metric as the convergence criterion is that if the saturation probabilities converge, the other metrics will converge as well, since they are computed with a higher number of samples. The final values of the probabilities, together with their confidence intervals, are shown in table 4.1.

The adopted simulation time step was 0.6 μ s. This choice depended on the **RV** assigned to the fault location as explained further in section 3.3. The simulation duration was instead fixed at 0.5 s. The fault was always initiated after the first 0.2 s of the simulation, to allow PSCAD to extinguish initial transients. Fixing the fault inception time at 0.2 s, with a 50 Hz voltage signal,

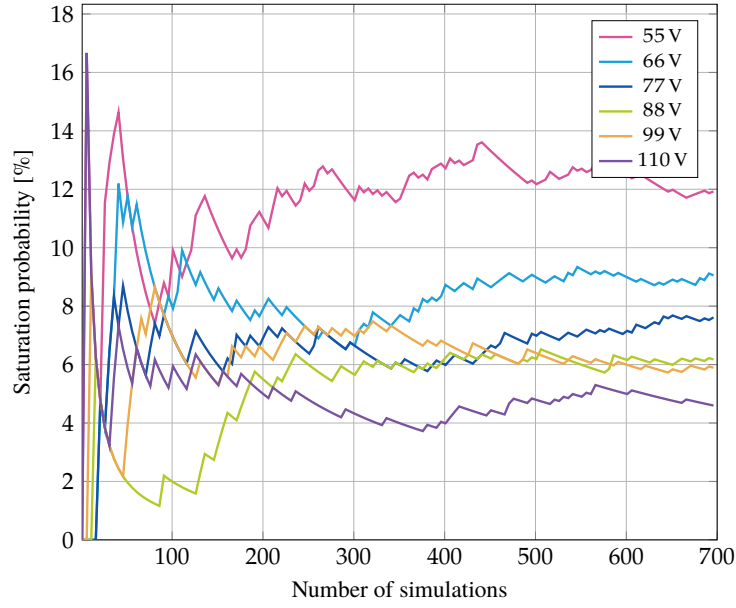


Figure 3.3: Saturation probability for each CT saturation voltage as a function of the number of simulations.

causes the initial phase angle of voltage $\bar{V}_1$ to correspond to the fault inception angle. The last 0.3 s of the simulation are enough to extinguish the fault transients, since the time constants of the source and line are approximately equal to 40 ms.

3.6 Clock errors evaluation

For this phase, the files extracted from the PSCAD simulations were downsampled to emulate the operation of an ideal clock with a sampling frequency of $\nu_0 = 1$ kHz, corresponding to 20 samples per cycle, more aligned with the usual PMU specifications [9]. The clock adopted for the analysis of the effects of CJ on PES is a Pascall Oven-Controlled Crystal Oscillator (OCXO) [47]. A reference stating the exact type of oscillators used in PMUs was not found. However, [48] describes a PMU with a Stratum 1 quality time reference, which [49] links to the usage of OCXO clocks. This particular OCXO clock was chosen for the availability of its Allan variance plot in [25], which was shown in figure 2.12a. The diffusion coefficients for this particular clock to be used for the model of section 2.4.2 were obtained with the method detailed in section 2.4.2.1 and are shown in table 3.8. The maximum reported skew for this clock is $\pm 2 \times 10^{-7}$ [47], which was used to

Table 3.8: Diffusion coefficients for the OCXO clock to use for the simulations.

τ [s]	σ_y [s]	σ_y^2 approximation	Diffusion coefficient [s ²]
10^{-6}	2×10^{-8}	$\sigma_y^2 \simeq \frac{\sigma_\phi^2}{(2\pi\nu_0)^2\tau}$	$\sigma_\phi^2 = 1.579 \times 10^{-14}$
4×10^{-2}	8×10^{-12}	$\sigma_y^2 \simeq \frac{\sigma_\gamma^2\tau}{3}$	$\sigma_\gamma^2 = 4.8 \times 10^{-21}$

set the initial conditions of the simulation.

A single simulation for the clock error analysis was performed in MATLAB according to the following structure:

1. Initially, a time grid of a specified duration corresponding to the sampling instants of a non-ideal **PMU** was determined according to (2.39) and (2.45). This duration was set to 30 minutes, chosen simply to have a wide enough window to observe the effect of repeated re-synchronizations between the master and slave clocks. The initial skew was chosen inside the range specified by the datasheet.
2. Inside the 30-minute window, an instant was randomly chosen as the starting point for the simulation, in the sense that the results of the PSCAD simulations already converted in MATLAB data were attached to these time stamps and the following ones. This simulates the fault inception after the **PMU** after has already been resynchronized numerous times.
3. The samples extracted from PSCAD were interpolated onto the sampling instants of a non-ideal **PMU** depending on the sample counter through shape-preserving piecewise cubic interpolation (pchip).
4. The values corresponding to the ideal and non-ideal time grids were compared based on counter number.

This scheme was repeated for all the PSCAD simulations, with different holdover durations from 1 s to 60 s. The random variables adopted are summarized in table 3.9. The holdover was randomized between 1 s and 60 s to understand the impact of the 1 s resynchronization interval established by **PTP** and the consequence of longer holdovers. The diffusion coefficients were not randomized since they belong to a single specific clock model. However, it is reasonable to assume that higher diffusion coefficients would lead to higher

Table 3.9: **RVs** definition for clock error analysis.

Variable	RV	Parameters
Fault initiation since PMU operation start	Uniform	[0 s, 30 min]
Initial skew	Normal	$\mu = 0, \sigma = 2 \times 10^{-7} / 4$
Holdover time	Discrete uniform	[1 s, 60 s]

Table 3.10: Output metrics adopted for the analysis of clock errors.

Scenario	Metric
Clock errors	Mean offset
	Maximum offset
	Mean TVE
	Mean absolute FE
	Maximum absolute FE

errors, since they are directly related with the variance of the skew and phase fluctuations random walks.

The outputs considered for this analysis are included in table 3.10. Both mean and maximum errors are taken into account to understand the effect of random walks in the phase and skew errors. The **FE** is also included in the study because the signal reconstructed through timing inaccuracies has a different harmonic content with respect to the original signal. The mean and maximum errors are considered in absolute form.

Chapter 4

Results

The results chapter is separated among the three realms this project focused on. In particular, 4.1 focuses on the analysis of CT errors tied to saturation, section 4.2 is concerned with the effects of degradation and transients in CVT measurements, and lastly section 4.3 looks at how clock errors can affect PMU performance.

4.1 Current transformers saturation analysis

As discussed in section 2.2, saturation is an event which affects CTs and causes relevant measurement errors. One example of the comparison between ideal and non-ideal current measurements, where the non-ideal measurements are affected by saturation, is shown in figure 4.1. In the figure, the first effects of saturation start to show around 30 ms since the fault inception and propagate until the end of the simulation. The current measurements stop showing signs of saturation at around 200 ms since the fault inception, but the PES reports errors until a later time because of the one-period window length.

As reported in section 3.2, the different metrics analyzed to study the sensitivity of CT measurements to the system conditions are going to include a flag for saturation presence, maximum TVE, ME, and PE. Saturation was detected by observing whether, during a simulation run, the TVE between the ideal and non-ideal PESs would remain higher than 1% for 15 ms or more. The use of 15 ms as a threshold is motivated by the histogram of figure 4.2, which shows the amount of time for which a TVE greater than or equal to 1% between the measurements was observed for each simulation run. The cases

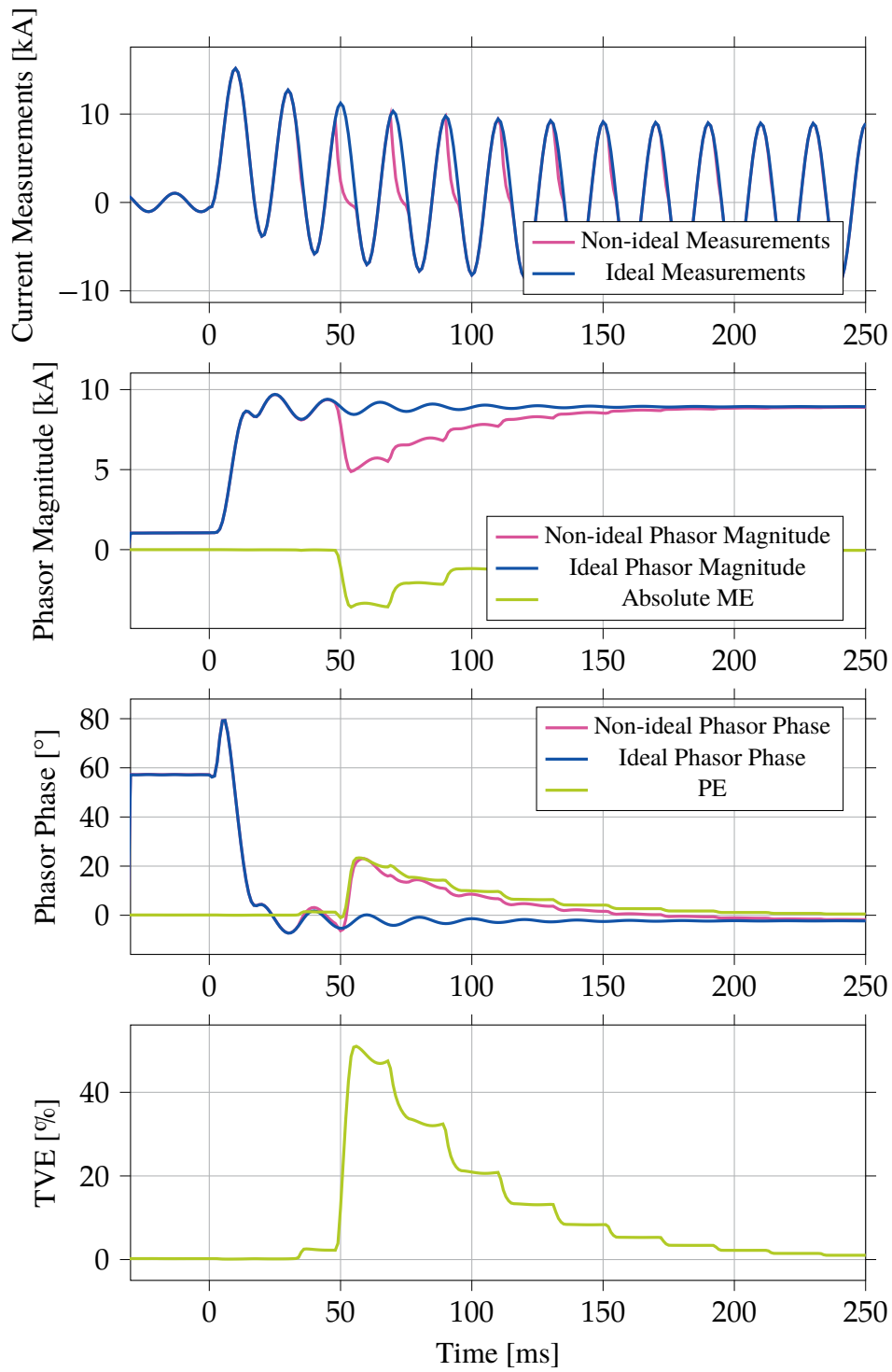


Figure 4.1: Example of the effect of saturation on current measurements and **PES**. The zeros of the horizontal axes correspond to the fault inception.

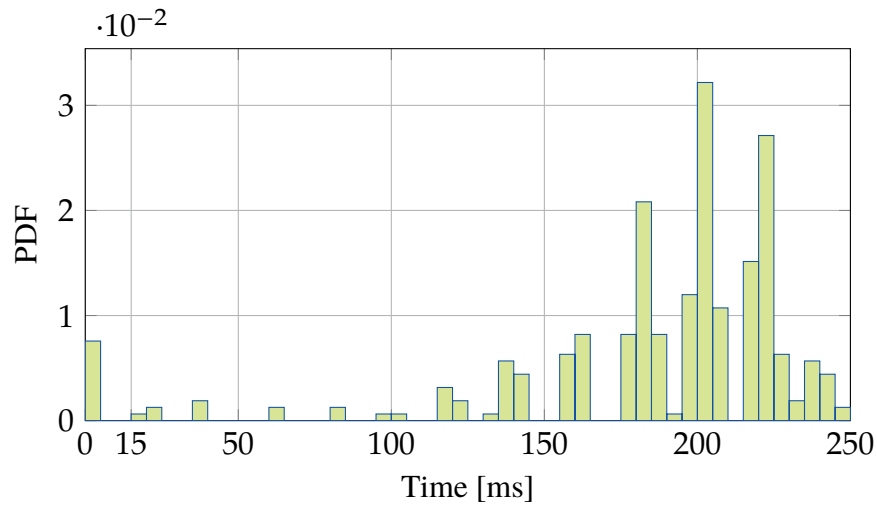


Figure 4.2: Histogram of the simulation time for which the **TVE** for current measurements exceeds 1%.

in which the **TVE** exceeded 1% for less than 15 ms showed numerical errors at the fault inception, which are not connected to saturation. In the other cases, the effects of saturation were clearly visible in the measured waveforms and relative phasors, as shown in figure 4.1.

The following sections focus more in detail on some specific output metrics that were analyzed. In particular, section 4.1.1 is concerned with the presence of saturation altogether, section 4.1.2 focuses on the maximum **TVE** and how it is distributed between **ME** and **PE**, and lastly 4.1.3 studies the distribution of **TTS** in the simulation runs.

4.1.1 Saturation presence

The method for detecting saturation explained in 4.1 revealed that saturation happened in 317 cases out of the total 4200. A **SA** was run using a flag as output, with value 1 associated to saturation cases and 0 otherwise, and the results are shown in figure 4.3. The fault type was removed from the **SA** because UQLab failed to process a categorical variable with highly different individual probabilities using its Borgonovo index algorithm. Furthermore, its close dependence with the fault resistance values allows its exclusion for the sake of interpretability. The fact that all the indices have a low absolute value means that there is no single variable that directly causes saturation by itself. However, the following variables are shown to be the most important in determining saturation:

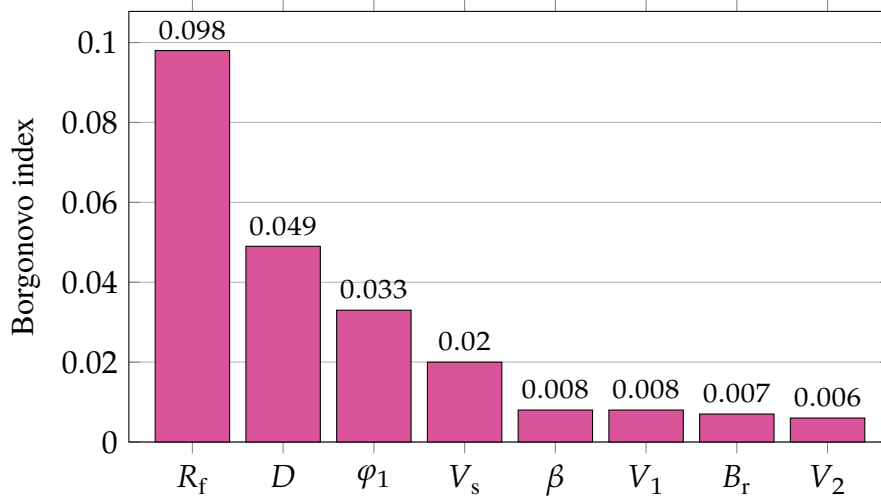


Figure 4.3: **SA** of the saturation presence.

1. Fault resistance R_f : due to its direct influence on the fault current, the fault resistance is determined to be the variable that saturation is the most sensitive to. In particular, 305 out of the 317 total cases of saturation feature a negligible fault resistance. However, the total number of cases with negligible fault resistance was 837. This symbolizes that saturation is likely to happen with negligible fault resistance, but a negligible fault resistance does not automatically result in saturation. Also, because of the dependence of the fault resistance on the fault type, this fact shows that saturation is more likely to happen for **LL** and **LLL** faults, which generate higher currents than **LG** faults.
2. Fault location D : the top histogram of figure 4.4 shows the **PDF** of the fault location **RV** in the cases where saturation appeared. This **RV** was originally sampled as uniform, but the figure shows that saturation is more likely to happen for nearby faults. This is due to the lower contribution of the line to the total fault impedance at parity of fault resistance and source impedance.
3. Fault inception angle φ_1 : since saturation is more likely to occur with negligible fault resistance, the total fault impedance in the saturation cases is mainly determined by the line and source impedances. Since these impedances are strongly reactive, the highest **DC** component in the fault current (contributing to asymmetrical saturation) is generated when the voltage angle at the fault inception is near 0° or 180° . This is

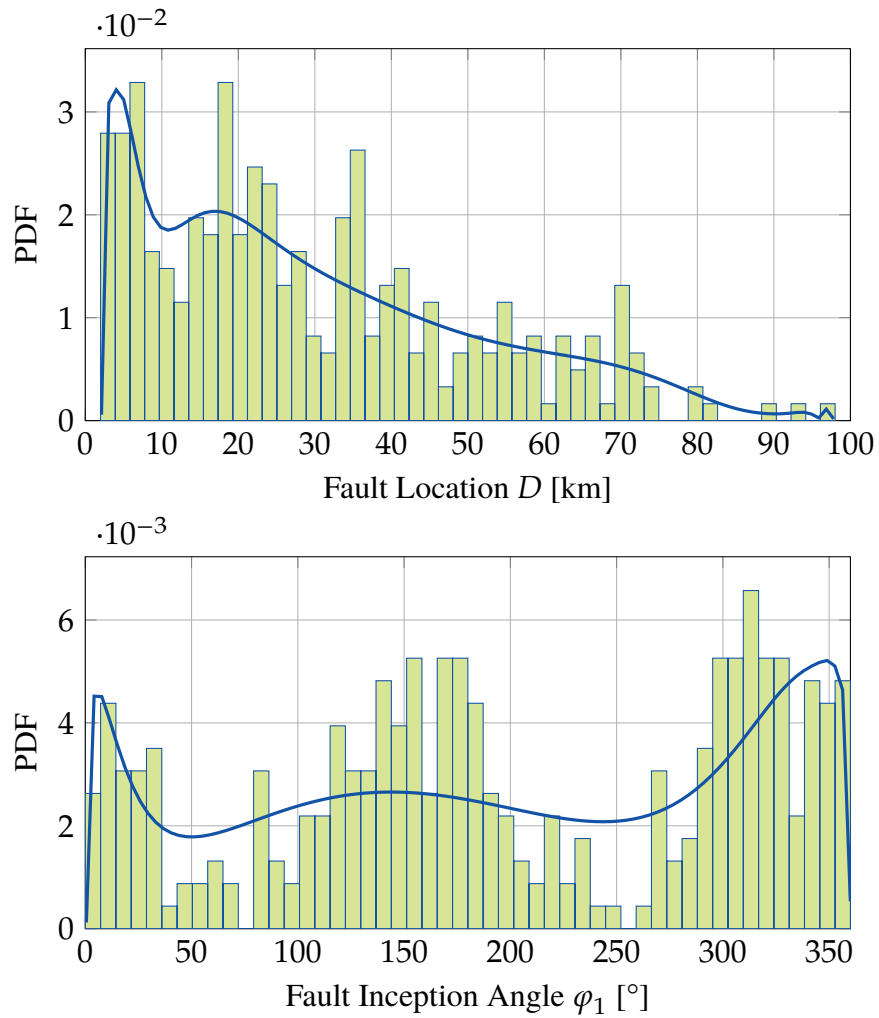


Figure 4.4: Relevant **PDFs** of the main variables involved in determining saturation.

confirmed by the bottom histogram of figure 4.4.

4. **CT** saturation voltage V_s : as expected, a higher saturation voltage lowers the probability of saturation inception. The actual statistics about probability of saturation for the different **CT** sizes are shown in table 4.1.
5. The remaining random variables show low sensitivity indices indicating negligible contribution to saturation.

This **SA** suggests that saturation is a product of the system and fault conditions, and its dependence on the saturation voltage values is secondary

Table 4.1: Probability of saturation for the different saturation voltage values.

V_s [V]	Number of saturation cases (/700)	Probability [%]	Confidence intervals [%]
55	83	11.86%	+2.63% −2.30%
66	63	9.00%	+2.37% −2.01%
77	55	7.86%	+2.25% −1.88%
88	43	6.14%	+2.04% −1.66%
99	41	5.86%	+2.01% −1.62%
110	32	4.57%	+1.82% −1.42%

compared to the fault resistances, locations, and inception angles. With the reasonable sizing philosophy applied in this project, there are still many combinations of input variables that recreate saturation conditions, which could be countered exclusively with an even more precautionary oversizing. A **MCS** such as the one applied in this project could help designers choose a suitable **CT** for a specified system according to the desired balance between performance and cost.

Some more considerations can explain the contribution of the other **RVs** to saturation:

- The load angle β determines the pre-fault load flow, which has a small contribution to the total current injected by a source in the case of a nearby fault with low fault resistance.
- The voltage magnitudes V_1 and V_2 have a direct influence on the fault current, but the low variability in their **RVs** ($\pm 2\%$ of the nominal value) causes this contribution not to be noticeable in a **SA**.
- The remanent flux density B_r represents a more singular case. Its contribution to asymmetrical saturation was highlighted in section 2.2, and the range of its **RV** spans from -80% to $+80\%$ of the saturation

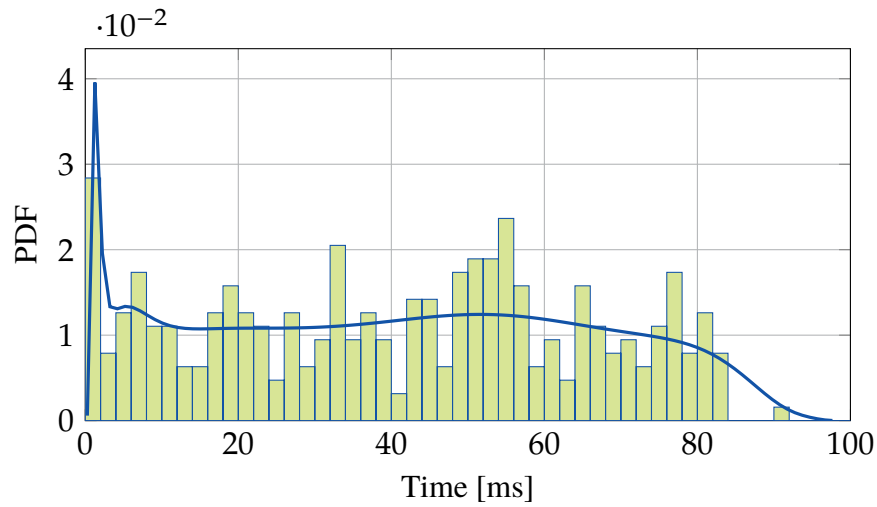


Figure 4.5: Histogram of the maximum **TVE** for current measurements in saturation cases.

flux density value. The reason for its apparent insignificance is due to its interaction with the sign of the **DC** component. A high, positive remanent flux could lead to fast and heavy saturation in case of a positive **DC** component in the fault current, or be the reason why saturation is avoided altogether in case of a negative **DC** component of the fault current. Therefore, no value of remanent flux by itself is a direct indicator of saturation. However, the importance of the interaction of the remanent flux density with the **DC** component shows the need for second-order sensitivity indices that are not covered in this project.

4.1.2 Maximum total vector error

Since saturation happens in less than 10% of the overall simulation runs, the maximum **TVE** is usually low. In particular, it is lower than 2% in 3 662 cases out of 4 200 (87.2%). The **PDF** of the maximum **TVE** in case of saturation is shown in figure 4.5. Although there is a peak in the **PDF** for low **TVE** values, saturation then leads to maximum **TVEs** that is distributed quite uniformly up to 85%.

The **SA** for the maximum **TVE** for current measurements is shown in figure 4.6. Its result is comparable to the result of the **SA** for the saturation presence since a higher maximum **TVE** happens because of saturation. Although the less significant variables are not ranked in the same order as for the saturation presence, the **SA** confirms that the most relevant variables to determine

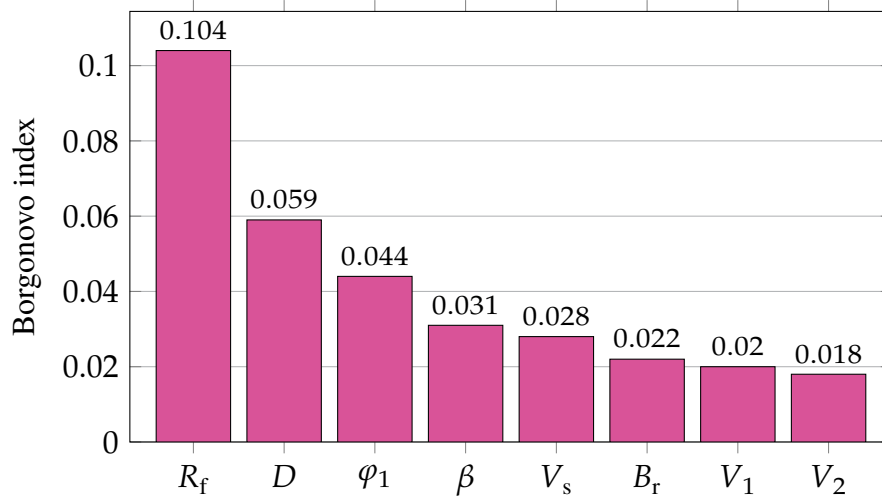


Figure 4.6: SA of the maximum TVE for current measurements.

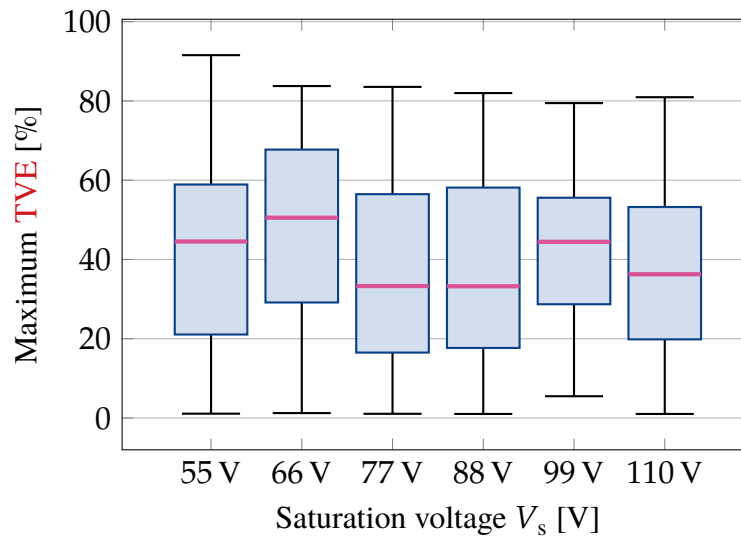


Figure 4.7: Maximum TVE for current measurements as a function of the saturation voltage for saturation cases.

saturation characteristics are still the fault resistance, fault location, and fault inception angle. The boxplot of figure 4.7 confirms the low impact that the saturation voltage holds on the maximum TVE in case of saturation, since there does not seem to be any relevant trend in the median or quartile statistics for the various saturation voltage values.

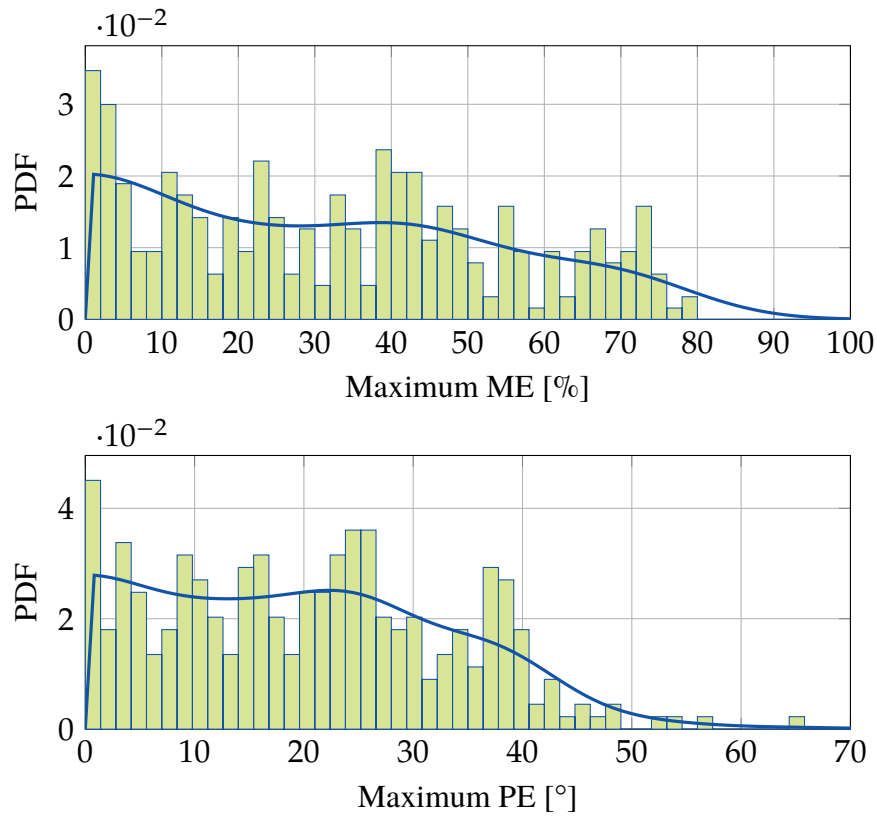


Figure 4.8: Histogram of the maximum **ME** and **PE** for current measurements in saturation cases.

4.1.2.1 Effects of saturation on phasor magnitude and phase estimation

Figure 4.1 suggests that saturation is a phenomenon which can impact negatively both the magnitude and phase of a **PES**. To study this effect in greater detail, consider the **PDFs** of the maximum **ME** and **PE** for current measurements in case of saturation shown in figure 4.8. Using (2.7), it is possible to show that a 50° **PE** between two phasor estimations of the same magnitude corresponds to a **TVE** of 84.5%. Therefore, saturation affects **ME** and **PE** similarly from a statistical point of view. Differentiating between the effects of a disturbance on magnitude and phase is important to understand which protection schemes could be affected by such disturbance.

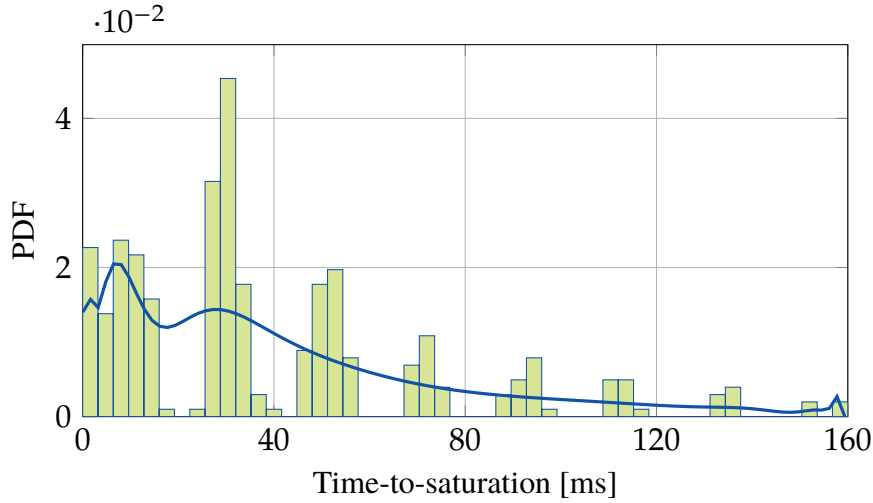


Figure 4.9: Histogram of the **TTS** deduced from current measurements in saturation cases.

4.1.3 Time-to-saturation

Analyzing the **TTS** for a **CT** can provide information about how much time a protection scheme can take to operate before the measurements are corrupted by saturation. Figure 4.9 shows the **PDF** of the **TTS** for the case at hand. It suggests that saturation can happen anywhere between the fault inception and eight fundamental frequency periods after the fault, even if around half of the cases are gathered inside the first two periods with a peak at around 30 ms. The multi-modal behavior of the **PDF** is due to the oscillations in the flux density that are produced by the symmetrical fault current component. The **SA** for the **TTS** is shown in figure 4.10. The fault resistance has been excluded from the **SA** because the **TTS** is defined only for saturation cases, and it is already known that most of the simulation runs that resulted in saturation possessed negligible fault resistance. Similarly as happened for the saturation presence and the maximum **TVE**, the fault location and fault inception angle are more important variables in driving the **TTS**. The median and quantile statistics for the **TTS** as a function of saturation voltage are shown in figure 4.11. The figure shows that the saturation voltage has no impact on the median **TTS**, but higher saturation voltages can increase the lower quartile level. The position of the lower quartiles explains the peak at 30 ms in the **PDF** of figure 4.9. Therefore, with a sufficiently high saturation voltage, the **TTS** can be expected to be lower than 30 ms for only 25% of the cases. The figure also shows how cases in which saturation inception happens later than five fundamental periods after the fault

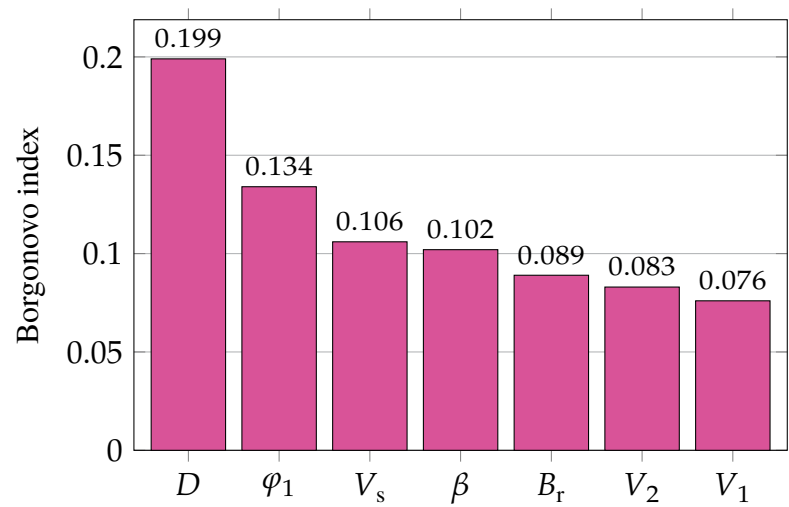


Figure 4.10: SA of the TTS.

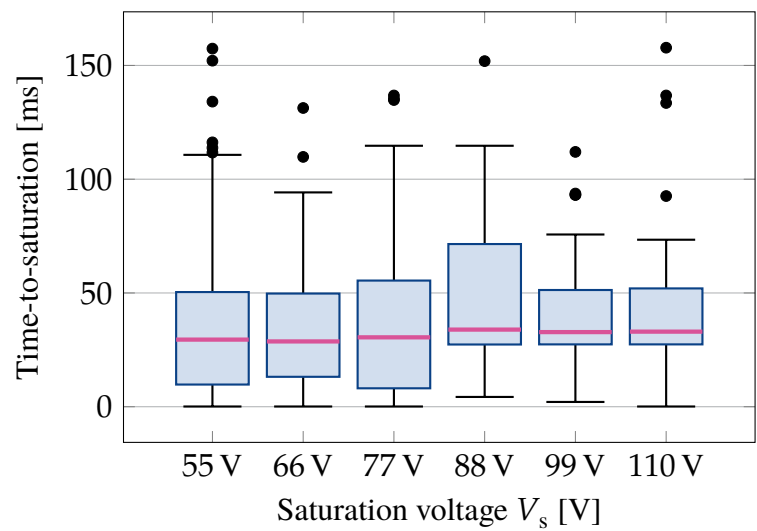


Figure 4.11: TTS as a function of the saturation voltage.

inception can be considered statistical outliers.

4.2 Capacitive voltage transformers degradation and transients analysis

Figure 4.12 shows an example of the effects of degradation and transients on **CVT** performance. What stands out is a constant, systematic magnitude error caused by the degradation which modifies the transformer ratio of the device. Furthermore, the fault transients cause an oscillation in the estimations that can temporarily worsen the performance even more. Another interesting observation is the negligible effect of degradation on the phase estimation, which confirms the assumptions obtained from the theoretical derivations of section 2.3.1.1. To isolate the effects of degradation and transients, section 4.2.1 focuses on the **SA** of mean **TVE**, and section 4.2.2 is concerned with the maximum **TVE**. Finally, in section 4.2.1.1, the effect of degradation and transients on magnitude and phase errors is discussed in greater detail.

4.2.1 Mean total vector error

The **PDF** of the mean **TVE** for voltage measurements is shown in figure 4.13. The Gaussian variations of the input **RVs** adopted a standard deviation of 1.25%, as discussed in section 3.3. However, these small variations are sufficient to obtain average **TVEs** as high as 7-8%.

The **SA** for this metric, whose indices are shown in figure 4.14, confirms the expectations of section 2.3.1.1. In fact, the capacitive divider parameters and the turns ratio of the internal **SDT** are observed to be the parameters whose degradation affects **CVT** performance the most. All other input **RVs** possess similar sensitivity indices, and therefore their contribution is not significant. The compensation inductance, whose irrelevance was established in section 2.3.1.1, scores one of the lowest sensitivity indices among all the **RVs**.

Resuming the discussion at the end of section 2.3.1.1, according to which the measurement errors of a **CVT** are due to how each parameter degrades compared to the others, this analysis would benefit from the adoption of higher-order sensitivity indices to reveal which interactions cause the strongest effects on measurement accuracy. The presented **SA** is not able to identify interactions between the input parameters at all and focuses only on the variables taken singularly. The fact that the interactions among the **RVs** are the dominant factor behind **CVT** measurement errors is confirmed by observing

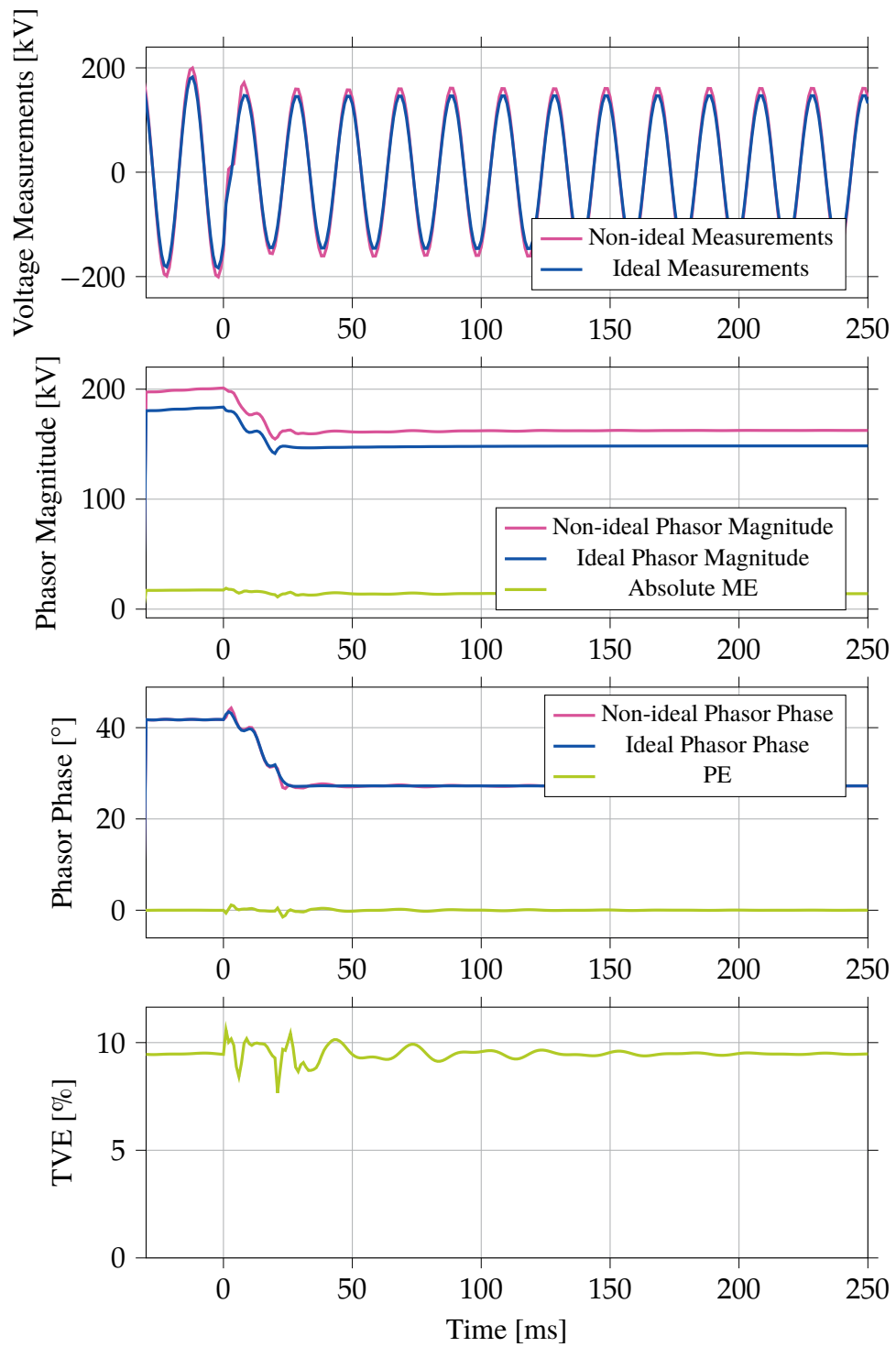


Figure 4.12: Example of the effect of degradation and transients on voltage measurements and PES. The zeros of the horizontal axes correspond to the fault inception.

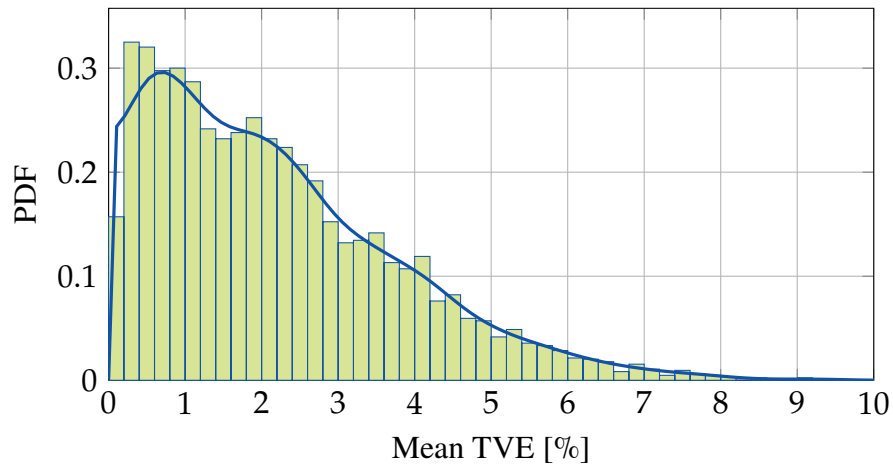


Figure 4.13: Histogram of the mean **TVE** for voltage measurements.

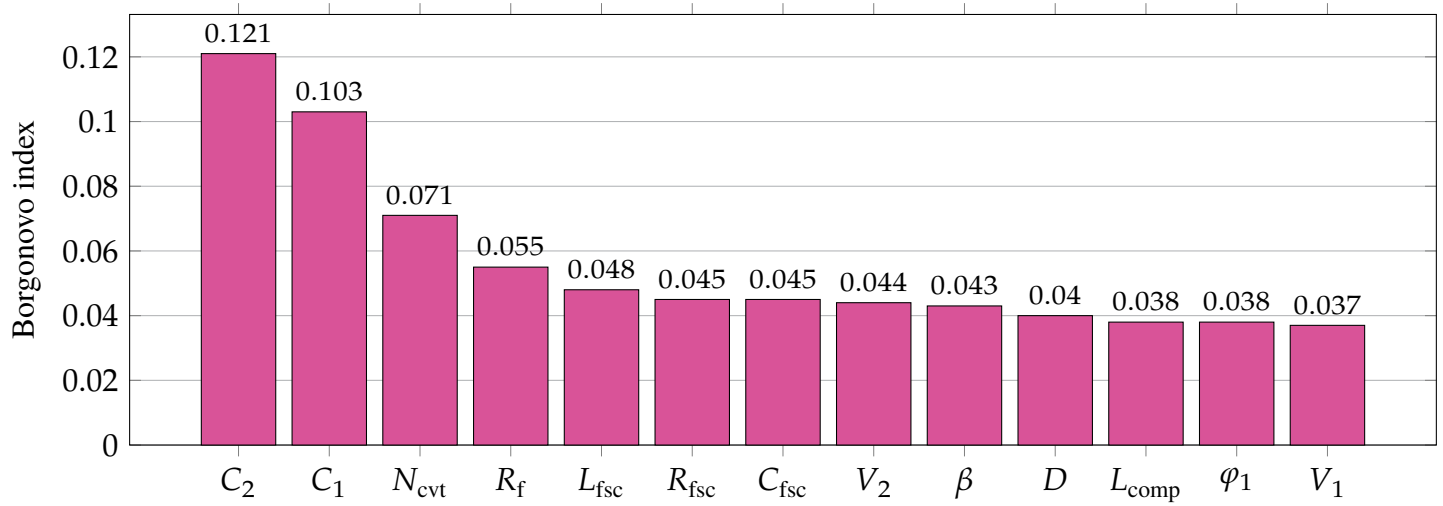


Figure 4.14: **SA** of the mean **TVE** for voltage measurements.

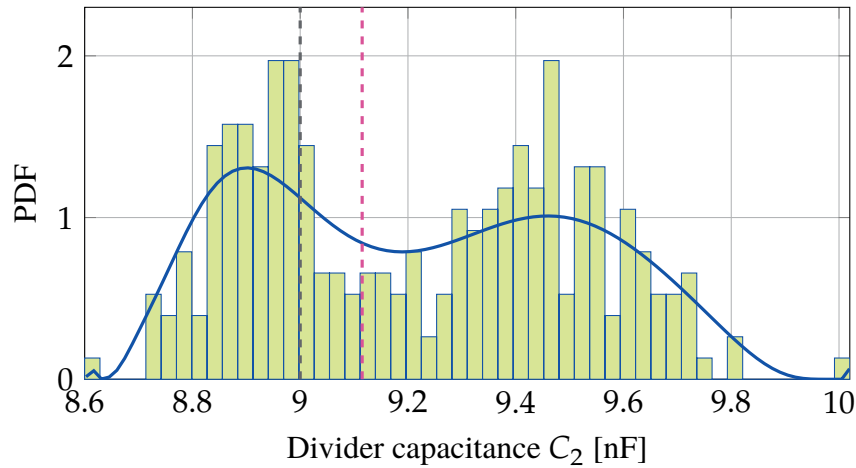


Figure 4.15: Histogram of the values of capacitance C_2 for the simulation runs in which the mean **TVE** for voltage measurements surpasses 5%. The thick dashed gray line corresponds to the nominal value of this parameter, or the mode of the original **RV**, and the red line corresponds to the median.

which parameter values were assigned to the simulations that produced the highest measurement errors. Consider the histogram shown in figure 4.15. It suggests that the highest measurement errors are not found simply when the parameter values deviate from their nominal values. If that were the case, the region associated with the highest measurement errors would be centered around the mode, corresponding to the nominal value of the parameter. The reason for this peculiarity can instead be explained by considering the properties of the involved **PDFs**. Since the standard deviations of the split-normal **RVs** assigned to the divider capacitances are higher on the right side of the **PDF**, the medians will be higher than the modes. Therefore, even if the median values correspond to a deviation from the nominal values, they represent values that are more likely to be sampled in the simulation, compared to those found in the tails. As a result, the values closest to the medians do not cause relevant errors since the deviations they represent are likely to be found in both capacitances. On the other hand, values sitting on the left side of the mode correspond to rarer occurrences that are more likely to cause significant errors even if their deviation from the nominal value is lower than for the values sitting close to the median.

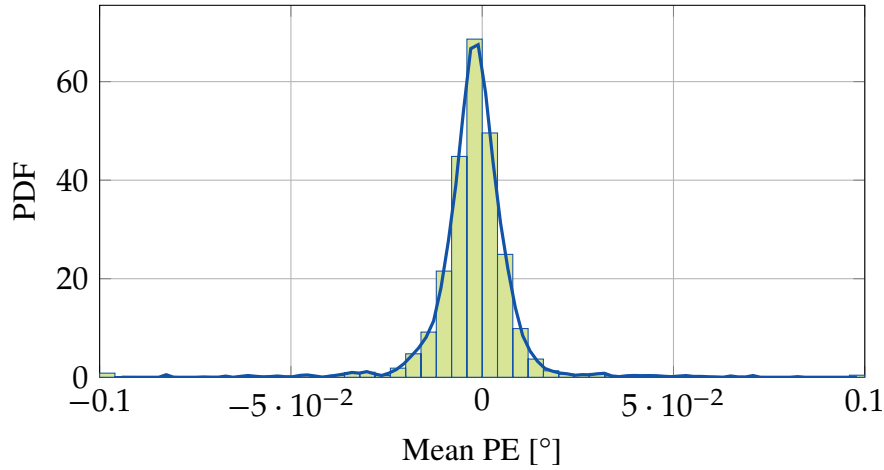


Figure 4.16: Histogram of the mean **PE** for voltage measurements.

4.2.1.1 Effects of degradation on phasor magnitude and phase estimation

As suggested by figure 4.12 and derived analytically in 2.3.1.1, **CVT** degradation has a limited effect on phase angle estimation. The statistical results of the simulations are shown in figure 4.16. In fact, the average **PEs** rarely surpass 0.1° . Therefore, the errors resulting from degradation can be purely attributed to **MEs** with good approximation.

4.2.2 Maximum total vector error

The **PDF** of the maximum **TVE** for voltage measurements is shown in figure 4.17. The maximum values in the histogram are obviously higher than those of figure 4.13, reaching values of 10-11% with some outliers at 15% and 16%. The **SA** for this metric is shown in figure 4.18. Although the divider capacitances still rank high, this analysis highlights the contribution of fault location, resistance, and inception angle in determining **CVT** performance during fault transients. The fault location has the highest impact because a fault close to the source will cause a voltage dip in correspondence of the substation. The impact of the other **RVs** is due to the factors explained in section 2.3.1.2.

However, the results of this section are heavily correlated with the results of section 4.2.1. In fact, it is easier for a **CVT** measurement to reach higher **TVE** values during a transient if the **TVE** was already high prior to the fault because of the effects of degradation. To better identify the effect of different parameter

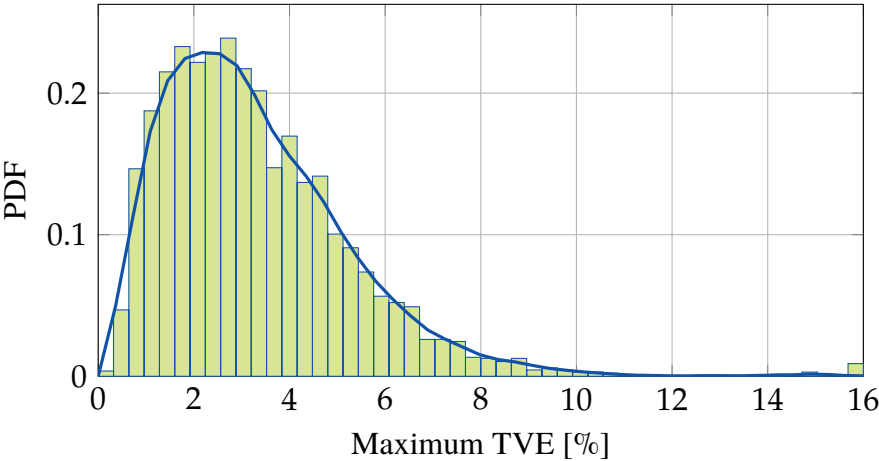


Figure 4.17: Histogram of the maximum **TVE** for voltage measurements.

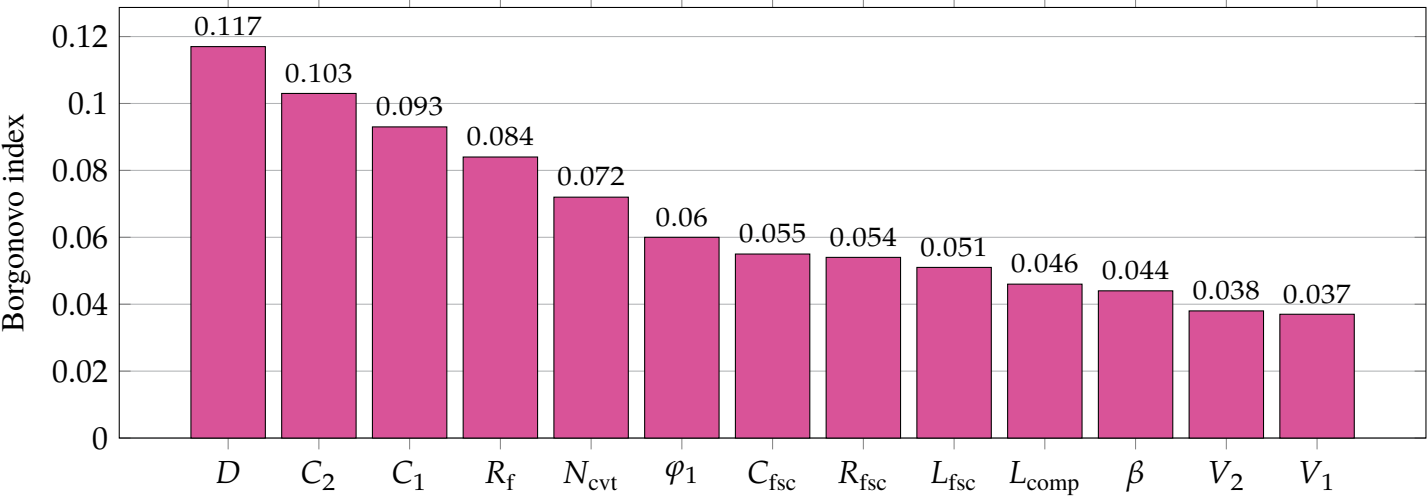


Figure 4.18: **SA** of the maximum **TVE** for voltage measurements.

values, a more suitable approach would be to compare the performance of different **CVT** designs with their nominal parameter values and see which perform the best. For example, two **CVTs** might have the same transformation ratio imposed by the capacitive divider, but, as highlighted in section 2.3.1.2, a higher total capacitance should lead to better performance. This method could effectively lead to better insights that are useful for parameter design.

4.3 Clock errors analysis

This section is concerned with the application of the clock model of section 2.4.2 to analyze the performance of clock [47] as time reference for a **PMU**, under the corrections operated based on the time offset with an ideal master clock. The errors shown in the next subsections are concerned solely with the errors introduced by timing error on signal acquisition and the other errors that affect the digital processing of an analog signal, discussed in section 2.4 are neglected.

Section 4.3.1 studies the magnitude of the offset between the internal clock time of a non-ideal clock and an ideal time reference. Section 4.3.2 focuses on **TVE** and section 4.3.3 on **FE**. Lastly, section 4.3.4 observes the behavior of the clock when not under the effects of synchronization for longer periods of time.

4.3.1 Mean and maximum absolute time offsets

The offset, defined in (2.26), measures the difference between the internal clock time of a non-ideal clock and an ideal time reference. Since the internal clock time is derived from the internal phase angle, the corrections should maintain it between close boundaries that depend on the noise characteristics of the clock and the maximum holdover. The histograms corresponding to the mean and maximum absolute offsets are shown in figure 4.19. The histogram enable two observations:

- the offsets in the simulation runs are in the order of nanoseconds. Compared to the maximum timing error limit of $1\text{ }\mu\text{s}$ imposed by [4], these errors are substantially negligible;
- the histograms for the mean and maximum absolute offsets basically coincide. This means that, over the 0.3 s period of a simulation run, the random walk noise processes in the clock model do not result in tangible variations over these time scale with the given diffusion coefficients.

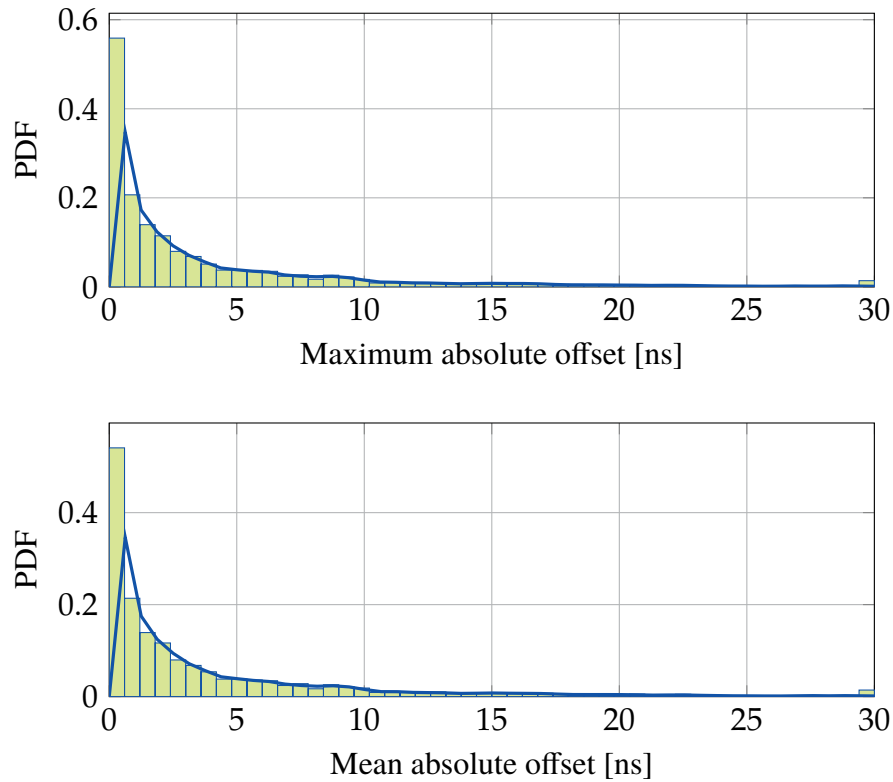


Figure 4.19: Histogram of the mean and maximum absolute offsets for an **OCXO** clock under the model of section 2.4.2 for holdovers between 1 s and 60 s.

Therefore, the adopted clock might seem too accurate for this scope. However, such a precise clock has the benefit of accurately counting even when synchronization with the master clock is lost, or the master clock stops receiving **GNSS** control.

The **SA** for the maximum absolute offset is shown in figure 4.20. The holdover is determined to be the only relevant variable in determining offsets. Assuming that the corrections are able to compensate the offset and skew well, it is reasonable that shorter holdovers, and therefore less time for the clocks to drift from their "correct" measurements, will lead to lower errors. The fact that elapsed time since the **PMU** integration is not relevant is a testament to the stability of the correction method. Lastly, the initial skew has negligible effect because it is quickly corrected by the master clock at the first correction instant. This results is observed in the **SAs** for the other considered error metrics as well; therefore, in the following sections, the **SAs** will not be shown, and all the error metrics will be shown as a function of the holdover.

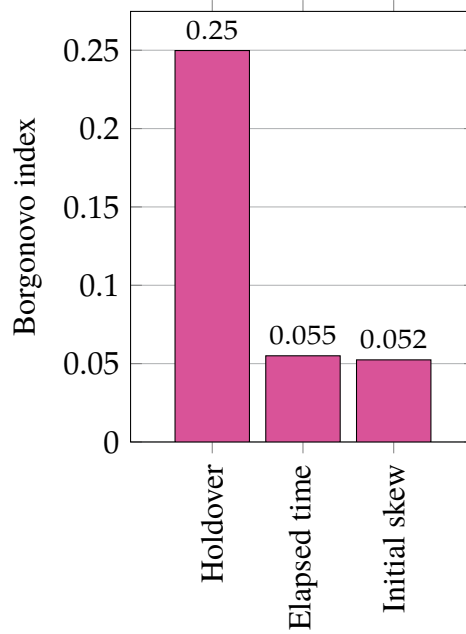


Figure 4.20: **SA** of the maximum absolute offset.

4.3.2 Mean and maximum total vector errors

Statistics for the mean and maximum **TVEs** as a function of the holdover are shown in figures 4.21 and 4.22, respectively. Since the mean absolute offset and the maximum absolute offset are statistically very similar to each other, it is reasonable to make the same observation for the **TVEs** since clock errors are the only source of error considered in this context. The errors for long holdovers are in the order of $10^{-3}\%$, and are likely translated into **PEs** since the maximum **MEs** sit at values around $5 \times 10^{-5}\%$. For the holdovers commonly found in power systems, in the order of a few seconds, the errors are absolutely negligible, especially when compared to the errors introduced by **CT** and **CVT** measurement errors.

There is an accurate linear relationship between the mean absolute **TVE** values and the mean absolute offset values,

$$\text{Mean TVE} = m \cdot (\text{mean absolute offset}) + q \quad (4.1)$$

and the parameters m , q , and the R-squared value of the linear regression are shown in table 4.2. The reason for this relationship is the fact that **PE** errors are the dominant factor in the **TVE** and that the **TVE** values are very small. Under these circumstances, approximation (2.8), in which θ is the offset, is

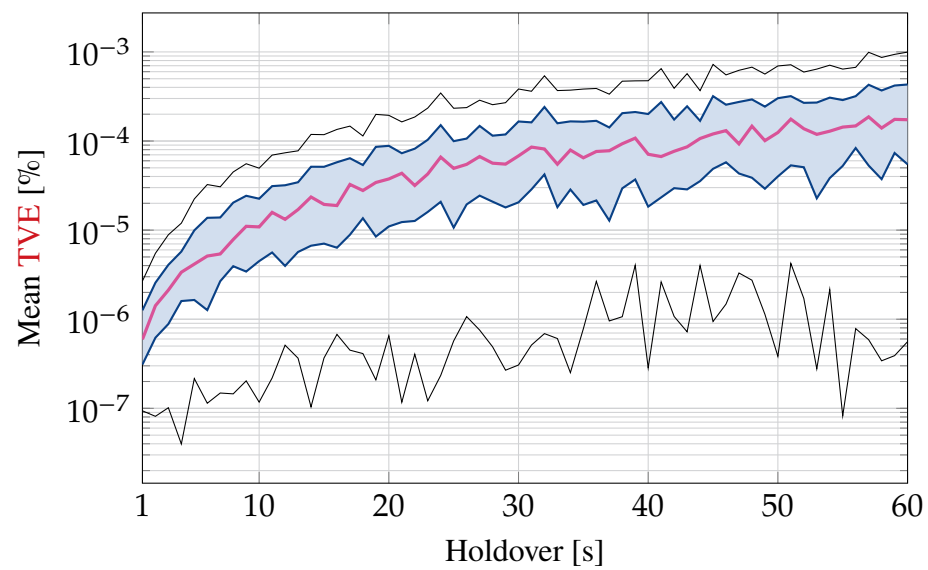


Figure 4.21: Mean **TVE** statistics for different holdovers. Median in red, quartiles in blue, whiskers in black.

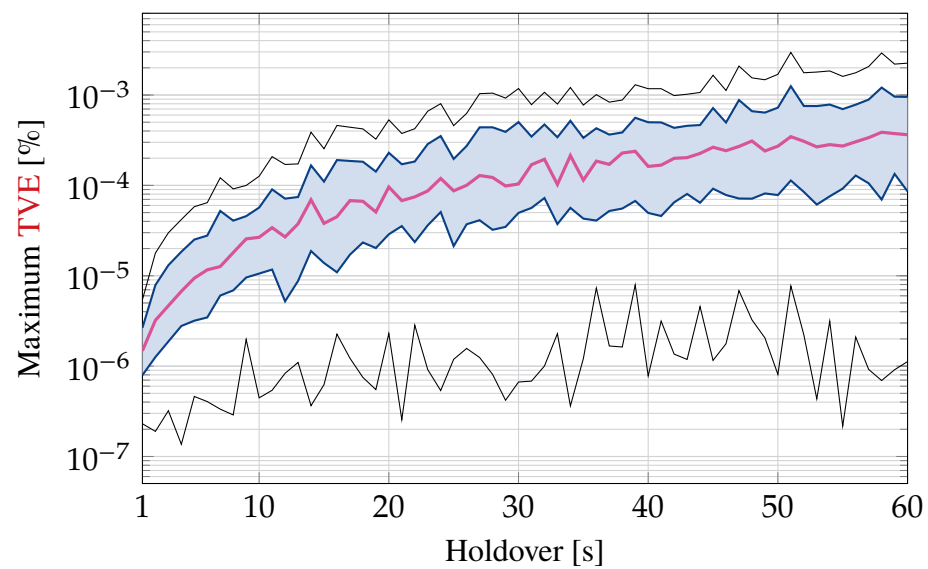


Figure 4.22: Maximum **TVE** statistics for different holdovers. Median in red, quartiles in blue, whiskers in black.

Table 4.2: Data about the linear regression model defined between the mean **TVE** and mean absolute offset.

m [s ⁻¹]	q [-]	R-squared
306.05	-1.622×10^{-9}	0.998

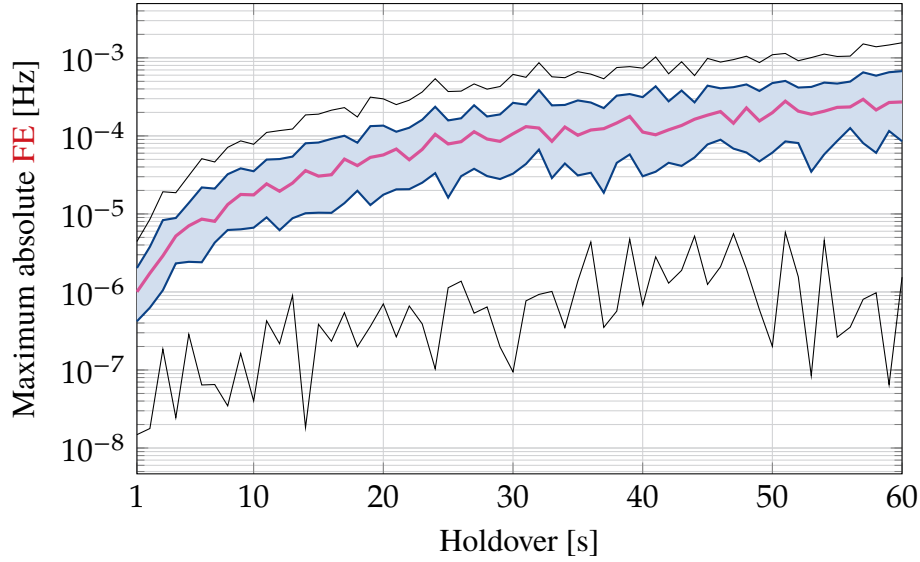


Figure 4.23: Maximum absolute **FE** statistics for different holdovers. Median in red, quartiles in blue, whiskers in black.

valid:

$$\text{TVE} \simeq |\text{PE}| = 2\pi f \theta \quad (4.2)$$

from which, with negligible q in table 4.2,

$$\frac{\text{TVE}}{\theta} = m = 2\pi f \quad (4.3)$$

which is equal to 314.16 for a 50 Hz input signal frequency, closely matching the results from table 4.2.

4.3.3 Maximum frequency error

Figure 4.23 shows the statistics for the maximum absolute **FE** as a function of the holdover. Similarly to the other results of this section, the errors are absolutely negligible for common power system holdovers, while they can reach mHz magnitudes for holdovers higher than 30 s. Once again, this statistic

Table 4.3: Data about the linear regression model defined between the mean **TVE** and mean absolute offset. Long holdovers case.

m [s^{-1}]	q [-]	R-squared
265.04	-1.0828×10^{-5}	0.949

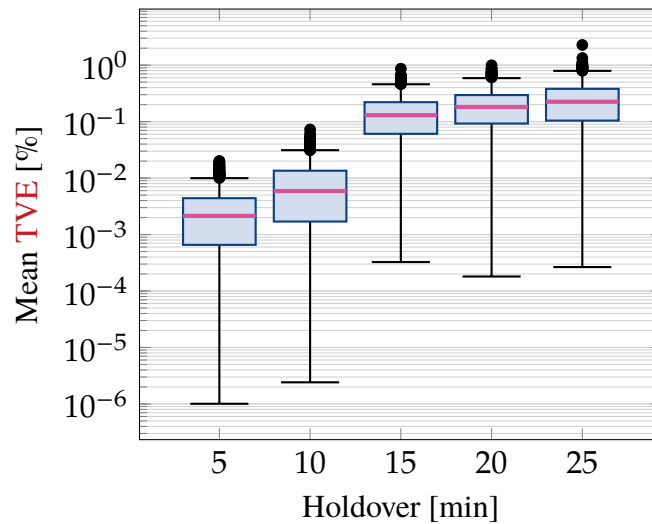


Figure 4.24: Mean **TVE** as a function of the holdover. Long holdovers case.

proves the high reliability of this clock for **PMU** applications.

4.3.4 Longer holdovers

As a way to test how the clock performs without synchronization for longer periods of time, a simulation was ran to test the behavior of the device for holdovers of 5, 10, 15, 20, and 25 minutes. As shown in table 4.3, a linear relationship between the mean absolute offset and mean **TVE** still holds, although the angular coefficient of the line is not close to the theoretical one, likely because the approximation for small **PEs** does not hold under the increased holdovers.

Figure 4.24 shows a box plot with the statistics regarding the mean **TVE** as a function of the holdover. It shows that the clock-related errors remain below 0.1% for 10 min without synchronization, but the performance starts to worsen after that, and the errors can reach 2% for holdovers of 25 min.

Figure 4.25 shows the statistics related to the maximum absolute **FE**. The frequency estimation suffers from the lack of synchronization more than the

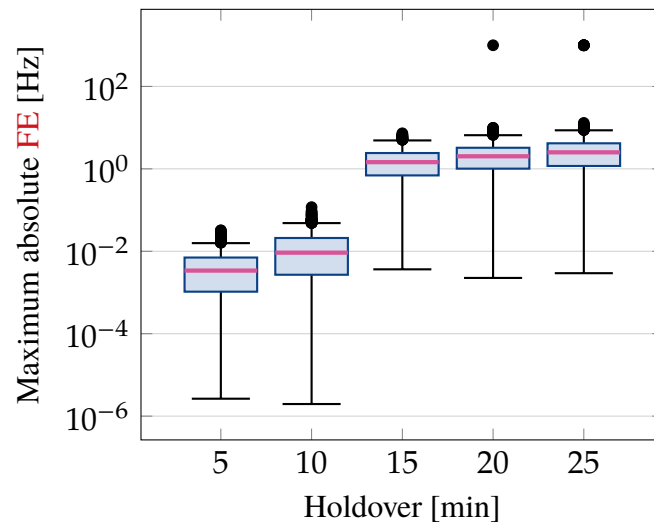


Figure 4.25: Maximum absolute FE as a function of the holdover. Long holdovers case.

PES. In fact, even holdovers as long as 5 min can lead the FE to reach tens or hundreds of mHz. For even longer holdovers, the FE becomes completely unreliable, with errors surpassing units or tens of Hz.

Chapter 5

Conclusions

This chapter provides a summary of the results and considerations about the limitations of this project and consequent future work to expand on the key findings.

5.1 Conclusions

The sensitivity of the **DFT** for **PES** to different power system disturbances was assessed by combining **SA** and **UA** applied to different metrics for measurement evaluation, such as the **TVE**, the **ME**, the **PE**, and the **FE**. The main findings for each error source, saturation, degradation and transients, and clock inaccuracies is subsequently summarized:

1. Section 4.1 showed that the presence of saturation and the maximum **TVE** reached under saturation depend mostly on system variables, such as the fault resistance, location, and inception angle, and less on **CT** saturation voltage. The consequence is that saturation cannot be avoided altogether. Unless a **CT** is hugely oversized, there will always be some combinations of fault type, location, resistance, and inception angle that result in saturation. Increasing the saturation voltage of a **CT** can reduce the probability of saturation*, but if saturation is initiated, the maximum errors do not depend on the saturation voltage. However, the **TTS** partly depends on the saturation voltage, meaning that high values of saturation

*For this reason, using a wider range of saturation voltage values for the simulations would cause the **SA** to show a higher sensitivity of the saturation presence to the saturation voltage. However, an arbitrarily wide range of values would also include some impractical values that would not be used in real power systems. This would add bias to the results, that would suggest that the saturation voltage could have a higher impact than it actually has.

voltage can reduce the probability of saturation and delay its inception in the majority of cases.

2. It has been shown that, under **CVT** parameter degradation, the **PES** errors can reach values as high as 10% because of the **CVTs** behavior under transient conditions. These errors especially affect the magnitude of the estimate and are mainly susceptible to degradation in the capacitive divider and intermediate **SDT**. However, it is not the simple variation of a parameter with respect to its nominal value that determines the error magnitude, which depends on the degradation of a component with respect to the others.
3. The clock object of this study performs well for regular power system holdovers, for which both the **PES** errors and **FEs** are limited. However, performance can suffer from lack of synchronization over a longer period of time, especially when considering frequency estimation. A **OCXO** clock might appear to over-perform under normal **PMU** operation, with offset and skew corrections every few seconds; however, an accurate clock, although expensive, is necessary to maintain acceptable performance even if **GNSS** synchronization in a substation is lost for some time.

5.2 On the external validity of the results

From the discussion of section 2.6, it should be observed that the external validity of a **SA** depends on two factors:

- whether or not the model is representative of a real power system;
- whether or not the **RVs** adopted to model the input variables are representative of the real behavior of such variables.

The input parameters can be divided into two categories: those whose behavior is common to each power system, and those characterized by other factors, such as the voltage level, ground resistance, and line ampacity. Variables belonging to the first category are the fault location and fault inception angle: as a first approximation, it can be assumed that faults might happen without preferential locations or inception angles for any power system. Variables belonging to the second category are, for example, the voltage magnitude, fault resistance, saturation voltage, and **CVT** parameters. These variables depend on the specifications of a given system. Therefore, it cannot be guaranteed

that these variables will have the same impact anywhere. For example, a transmission line that is built with precautions that minimize the probability of multiphase faults will, according to the results, encounter low saturation probability. For such a system, the contribution of the fault location to the measurement errors might be higher than that of the fault resistance. The bottom line is that the evaluation should be reproduced to determine whether or not some different conditions heavily impact the robustness of PES algorithms against line faults.

The choice of a simple test system is not an inherent limitation of the method since the test system of figure 3.2 can be representative of a real and more complex power system. In fact, if the power system under study can be simplified to produce two Thévenin equivalent circuits of the system as seen from the two ends of the line, then the test system of figure 3.2 will be suitable to study the consequences of a fault in a transmission line that is part of a more complex system*. The source impedances and nominal voltages might assume different values for the two sources depending on the system topology. The consequence of this interpretation is that the study of a complex power system might be reduced to the application of the SA framework to its individual components. This would decrease the complexity of the simulation and evaluation phases, easing the identifications of eventual weak links in a given system.

5.3 Limitations

Limitations encountered in this project concern the use of first-order sensitivity indices. In fact, it has been discussed how the CT error analysis failed to highlight the contribution of the remanent flux density to the saturation mechanism, and how the CVT error analysis could be enriched by using a tool that better captures the interactions between different RV since the effects of degradation do not depend on the parameter deviation of every single variable independently of the others.

Furthermore, the CVT analysis failed to properly identify the main parameters responsible for determining transient errors, since the results obtained in the dedicated section are biased by parameter variations due to degradation. The clock errors analysis did not exploit the full potential of a SA because of the limited number of RVs adopted in the model. Moreover, the model completely disregards the effects of drift and flicker noises, latency,

*This interpretation was suggested by the KTH supervisor, Henrik Johanssen.

and timestamp uncertainties. Drift might be neglected when evaluating the model's performance under systematic and frequent corrections, but could cause observable differences in the evaluation of clock behavior under loss of synchronization.

As pointed out in section 5.2, there are two main issues related to the external validity of the results: the fidelity with which the chosen PDFs represent the real-world behavior of the input parameters, and the effectiveness with which the model captures the dynamics of a real, more complex system.

Lastly, the results have not been validated in practice. Experimental validation would also consolidate the robustness of the SA framework adopted in the project.

5.4 Future work

Some hints for future work directly build on the limitations of this project. An immediate first step towards improving the results of this work would consist of repeating the simulations while adopting a sensitivity method that includes higher-order indices to capture interactions between all different RVs. In fact, it was discussed how the results under CT saturation, and especially CVT degradation, depend on some specific interactions between the input parameters.

Another kind of CVT analysis, where different CVT designs are compared to see which configurations perform the best under transient condition, would actually be more suitable to analyze the main drivers behind detrimental transient behavior without suffering from the bias due to the systematic errors tied to degradation. A more detailed model for clock errors analysis, including all phases of signal acquisition or considering latency and timing uncertainty in the master clock, could greatly expand the benefits of the applied methods.

Further analysis related to the external validity of the results could lead to the improvement or validation of the methods. Simulation reproduction using different RVs is suggested to evaluate the effects of the input parameters on PES in different systems. A comparative study, where the methods are applied to a complex system and to its equivalent as seen from the transmission line under study, would highlight the eventual advantages of reducing simulation complexity and lowering the number of input parameters.

Experimental validation would be hard in practice, but perhaps applicable by resorting to past fault statistics.

References

- [1] A. Y. Wu, “The Analysis of Current Transformer Transient Response and Its Effect on Current Relay Performance,” *IEEE Transactions on Industry Applications*, vol. IA-21, no. 3, pp. 793–802, 1985. doi: 10.1109/TIA.1985.349500
- [2] R. A. Hedding, “CcvT transients revisited.” [Online]. Available: <https://library.e.abb.com/public/b35cb5c34841ab38c1257943002e5dfe/CCVT%20Transients%20Revisited.pdf>
- [3] A. G. Phadke and J. S. Thorp, *Introduction*. Cham: Springer International Publishing, 2017, pp. 3–27. ISBN 978-3-319-50584-8. [Online]. Available: https://doi.org/10.1007/978-3-319-50584-8_1
- [4] “IEEE Standard for Synchrophasor Measurements for Power Systems,” *IEEE Std C37.118.1-2011 (Revision of IEEE Std C37.118-2005)*, pp. 1–61, 2011. doi: 10.1109/IEEESTD.2011.6111219
- [5] A. G. Phadke and J. S. Thorp, *Phasor Estimation of Nominal Frequency Inputs*. Cham: Springer International Publishing, 2017, pp. 29–45. ISBN 978-3-319-50584-8. [Online]. Available: https://doi.org/10.1007/978-3-319-50584-8_2
- [6] M. Kovacic, M. Jurcevic, R. Malaric, and A. Kunac, “A review of phasor estimation algorithms,” in *2020 3rd International Colloquium on Intelligent Grid Metrology (SMAGRIMET)*, 2020. doi: 10.23919/SMA-GRIMET48809.2020.9264012 pp. 109–113.
- [7] M. Tajdinian, B. Behdani, A. Goodarzi, R. Shirali, and H. R. Chamorro, “Vulnerability assessment of phasor estimation algorithms under ct saturation: A comparative analysis,” in *2022 IEEE International Conference on Environment and Electrical Engineering and 2022 IEEE Industrial and Commercial Power Systems Europe (EEEIC / I&CPS*

- Europe), 2022. doi: 10.1109/EEEIC/ICPSEurope54979.2022.9854653 pp. 1–6.
- [8] A. G. Phadke and J. S. Thorp, *Phasor Estimation at Off-Nominal Frequency Inputs*. Cham: Springer International Publishing, 2017, pp. 47–72. ISBN 978-3-319-50584-8. [Online]. Available: https://doi.org/10.1007/978-3-319-50584-8_3
- [9] —, *Phasor Measurement Units and Phasor Data Concentrators*. Cham: Springer International Publishing, 2017, pp. 83–109. ISBN 978-3-319-50584-8. [Online]. Available: https://doi.org/10.1007/978-3-319-50584-8_5
- [10] A. Hargrave, M. J. Thompson, and B. Heilman, “Beyond the knee point: A practical guide to CT saturation,” in *2018 71st Annual Conference for Protective Relay Engineers (CPRE)*, 2018. doi: 10.1109/CPRE.2018.8349779 pp. 1–23.
- [11] A. Sweetana, “Transient response characteristics of capacitive potential devices,” *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-90, no. 5, pp. 1989–2001, 1971. doi: 10.1109/TPAS.1971.292994
- [12] T. Neu, “Clock jitter analyzed in the time domain, part 1,” *Analog Applications Journal*, no. 3Q, pp. 5–9, 2010, tI Literature No. SLYT379.
- [13] W. R. Bennett, “Spectra of quantized signals,” *The Bell System Technical Journal*, vol. 27, no. 3, pp. 446–472, 1948. doi: 10.1002/j.1538-7305.1948.tb01340.x
- [14] N. Da Dalt and A. Sheikholeslami, *Understanding Jitter and Phase Noise: A Circuits and Systems Perspective*. Cambridge University Press, 2018.
- [15] D. B. Leeson, “Oscillator phase noise: A 50-year review,” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 63, no. 8, pp. 1208–1225, 2016. doi: 10.1109/TUFFC.2016.2562663
- [16] A. Saltelli, M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, and S. Tarantola, *Introduction to Sensitivity Analysis*. John Wiley & Sons, Ltd, 2007, ch. 1, pp. 1–51. ISBN 9780470725184. [Online]. Available: <https://onlinelibrary.wiley.com/doi/abs/10.1002/9780470725184.ch1>

- [17] M. Ginocchi, F. Ponci, and A. Monti, “Sensitivity Analysis and Power Systems: Can We Bridge the Gap? A Review and a Guide to Getting Started,” *Energies*, vol. 14, no. 24, 2021. doi: 10.3390/en14248274. [Online]. Available: <https://www.mdpi.com/1996-1073/14/24/8274>
- [18] A. Oppenheim and R. Schaffer, *Discrete-Time Signal Processing. 3rd Edition*. Prentice Hall, 2010.
- [19] *Measuring relays and protection equipment – Part 121: Functional requirements for distance protection*, IEC International Standard 60 255-121, 2014.
- [20] D. Tziouvaras, P. McLaren, G. Alexander, D. Dawson, J. Esztergalyos, C. Fromen, M. Glinkowski, I. Hasenwinkle, M. Kezunovic, L. Kojovic, B. Kotheimer, R. Kuffel, J. Nordstrom, and S. Zocholl, “Mathematical models for current, voltage, and coupling capacitor voltage transformers,” *IEEE Transactions on Power Delivery*, vol. 15, no. 1, pp. 62–72, 2000. doi: 10.1109/61.847230
- [21] M. Meisinger, S. Turner, A. Makki, A. Avendano, Y. Chen, R. Chowdhury, N. Fischer, R. Frye, J. Gers, L. Kojovic, Y. Liao, F. Lopez, P. McLaren, S. Meliopoulos, J. Niemira, D. Ouellette, A. Rajapakse, D. Tziouvaras, J. van de Ligt, and J. Wilson, “Mathematical models for current, voltage, and capacitively coupled voltage transformers,” IEEE Power System Relaying and Control (PSRC) Committee, Working Group I26, Tech. Rep., 2023.
- [22] W. Kester, “Taking the Mystery out of the Infamous Formula, ‘ $\text{SNR} = 6.02N + 1.76\text{dB}$ ’, and Why You Should Care,” 2009, accessed: 2026-03-24. [Online]. Available: <https://www.analog.com/media/en/training-seminars/tutorials/MT-001.pdf>
- [23] ———, “Aperture Time, Aperture Jitter, Aperture Delay Time—Removing the Confusion,” 2009, accessed: 2026-03-24. [Online]. Available: <https://www.analog.com/media/en/training-seminars/tutorials/MT-007.pdf>
- [24] J. Rutman and F. Walls, “Characterization of frequency stability in precision frequency sources,” *Proceedings of the IEEE*, vol. 79, no. 7, pp. 952–960, 1991. doi: 10.1109/5.84972

- [25] F. Ramian, *Measuring the Allan Variance: Oscillator Stability Measurement*, Rohde & Schwarz GmbH & Co. KG, München, Germany, April 2015, application Note. [Online]. Available: <https://www.rohde-schwarz.com>
- [26] F. Walls and E. Ferre-Pikal, “Frequency standards, characterization,” 1999-01-01 1999. [Online]. Available: https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id=105599
- [27] D. Howe and T. Tasset, “Clock jitter estimation based on PM noise measurements,” in *IEEE International Frequency Control Symposium and PDA Exhibition Jointly with the 17th European Frequency and Time Forum, 2003. Proceedings of the 2003*, 2003. doi: 10.1109/FREQ.2003.1275149 pp. 541–546.
- [28] C. Zucca and P. Tavella, “The clock model and its relationship with the Allan and related variances,” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 52, no. 2, pp. 289–296, 2005. doi: 10.1109/TUFFC.2005.1406554
- [29] J. Chaffee, “Relating the Allan Variance to the Diffusion Coefficients of a Linear Stochastic Differential Equation Model for Precision Oscillators,” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 34, no. 6, pp. 655–658, 1987. doi: 10.1109/TUFFC.1987.26998
- [30] J. R. Vig, “Introduction to quartz frequency standards,” 2021, educational material. [Online]. Available: <https://ieee-uffc.org/files/ieeuffcorg/2021-12/Introquartzfrequencystandards.pdf>
- [31] “Communication networks and systems for power utility automation – Part 9-3: Precision time protocol profile for power utility automation,” *IEC/IEEE 61850-9-3 Edition 1.0 2016-05*, pp. 1–18, 2016. doi: 10.1109/IEEESTD.2016.7479438
- [32] D. Ingram and B. Smellie, “Implementing PTP in Substations: Solving Electrical Substation Timing Problems,” Tekron International Ltd., White Paper, October 2014. [Online]. Available: <https://ww1.microchip.com/downloads/aemDocuments/documents/FTD/tekron/productdocuments/supportingcollateral/White-Paper-Implementing-PTP-in-Substations.pdf>

- [33] H. Kim, “Modeling and tracking time-varying clock drifts in wireless networks,” 2014. [Online]. Available: <https://api.semanticscholar.org/CorpusID:33520718>
- [34] G. Giorgi and C. Narduzzi, “Performance analysis of kalman-filter-based clock synchronization in iee 1588 networks,” *IEEE Transactions on Instrumentation and Measurement*, vol. 60, no. 8, pp. 2902–2909, 2011. doi: 10.1109/TIM.2011.2113120
- [35] “Infimum and supremum — Wikipedia, the free encyclopedia,” https://en.wikipedia.org/wiki/Infimum_and_supremum, 2026, accessed: 2026-03-24.
- [36] M. Amelin, *Monte Carlo Simulation in Engineering*. KTH Royal Institute of Technology, 2013.
- [37] A. Saltelli, “Sensitivity Analysis for Importance Assessment,” *Risk analysis : an official publication of the Society for Risk Analysis*, vol. 22, pp. 579–90, 07 2002. doi: 10.1111/0272-4332.00040
- [38] E. Borgonovo, “A new uncertainty importance measure,” *Reliability Engineering & System Safety*, vol. 92, no. 6, pp. 771–784, 2007. doi: <https://doi.org/10.1016/j.ress.2006.04.015>. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0951832006000883>
- [39] S. Marelli and B. Sudret, *UQLab: A Framework for Uncertainty Quantification in Matlab*, pp. 2554–2563. [Online]. Available: <https://ascelibrary.org/doi/abs/10.1061/9780784413609.257>
- [40] *Relion 670 Series, Phasor measurement unit RES670, Version 2.2 IEC, Technical manual*, Hitachi Energy, 2025, document ID 1MRK511408-UEN, Revision T. [Online]. Available: <https://publisher.hitachienergy.com/preview?DocumentID=1MRK511408-UEN>
- [41] R. Jegatheesan and K. Vijayakumar, *Modern power system analysis : with MATLAB applications*. Uttar Pradesh, India: Pearson India Education Services Pvt. Ltd, 2020. ISBN 93-90394-22-8
- [42] A. dos Santos and M. C. de Barros, “Stochastic modeling of power system faults,” *Electric Power Systems Research*, vol. 126, pp. 29–37, 2015. doi: <https://doi.org/10.1016/j.epsr.2015.04.015>. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S03787779615001212>

- [43] “IEEE Guide for the Application of Current Transformers Used for Protective Relaying Purposes,” *IEEE Std C37.110-2023*, pp. 1–89, 2023. doi: 10.1109/IEEESTD.2023.10132388
- [44] ABB, *Instrument Transformers – Application Guide*, 4th ed., ABB AB High Voltage Products, Ludvika, Sweden, 2015, application guide for current and voltage transformers. [Online]. Available: <https://www.abb.com>
- [45] —, *Capacitor Voltage Transformer TYD 126–550 kV: Product Leaflet*, ABB Jiangsu Jingke Instrument Transformer Co., Ltd., Suqian, Jiangsu Province, China, 2015, product leaflet, CVT oil-insulated instrument transformer. [Online]. Available: <https://www.abb.com/highvoltage>
- [46] R. L. de Andrade Reis, W. L. A. Neves, F. V. Lopes, and D. Fernandes, “Coupling capacitor voltage transformers models and impacts on electric power systems: A review,” *IEEE Transactions on Power Delivery*, vol. 34, no. 5, pp. 1874–1884, 2019. doi: 10.1109/TPWRD.2019.2908390
- [47] P. E. Limited, “OCXO SERIES – ULTRA LOW NOISE CRYSTAL OSCILLATORS,” datasheet. [Online]. Available: https://www.datasheet.live/pdfviewer?url=https%3A%2F%2Fpdf.datasheet.live%2Fdatasheets-1%2Fpascall_electronics%2FOCXO-4-0500-1-1-1.pdf
- [48] V. Communications, “<https://valiantcom.com/pmu/data-sheets/vcl-pmu-30.pdf>,” datasheet. [Online]. Available: <https://valiantcom.com/pmu/data-sheets/VCL-PMU-30.pdf>
- [49] R. Rutkowski, “NTP Stratum Levels: What You Need to Know,” 2024-07-10 2024. [Online]. Available: <https://blog.bliley.com/ntp-stratum-levels>
- [50] N. A. Spaldin, *Ferromagnetic domains*. Cambridge University Press, 2010, p. 79–95.
- [51] D. Jiles and D. Atherton, “Theory of ferromagnetic hysteresis,” *Journal of Magnetism and Magnetic Materials*, vol. 61, no. 1, pp. 48–60, 1986. doi: [https://doi.org/10.1016/0304-8853\(86\)90066-1](https://doi.org/10.1016/0304-8853(86)90066-1). [Online]. Available: <https://www.sciencedirect.com/science/article/pii/0304885386900661>

- [52] U. Annakkage, P. McLaren, E. Dirks, R. Jayasinghe, and A. Parker, "A current transformer model based on the Jiles-Atherton theory of ferromagnetic hysteresis," *IEEE Transactions on Power Delivery*, vol. 15, no. 1, pp. 57–61, 2000. doi: 10.1109/61.847229
- [53] D. Fernandes, W. Neves, and J. Vasconcelos, "Coupling capacitor voltage transformer: A model for electromagnetic transient studies," *Electric Power Systems Research*, vol. 77, no. 2, pp. 125–134, 2007. doi: <https://doi.org/10.1016/j.epsr.2006.02.007>. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0378779606000484>

Appendix A

Taylor series approximation of the simplified total vector error expression

Starting from (2.7),

$$\text{TVE}[k] = \sqrt{2[1 - \cos(\hat{\varphi}[k] - \varphi[k])]}$$

renaming $\hat{\varphi}[k] - \varphi[k]$ as $\Delta\varphi[k]$, and discarding the k notation for simplicity,

$$\text{TVE}[k] = \sqrt{2[1 - \cos \Delta\varphi]} \quad (\text{A.1})$$

it is possible to apply the Taylor series expansion to $\cos \Delta\varphi$ for small $\Delta\varphi$ values to obtain

$$\begin{aligned} \sqrt{2[1 - \cos \Delta\varphi]} &= \sqrt{2 \left[1 - \left(1 - \frac{\Delta\varphi^2}{2} + \frac{\Delta\varphi^4}{24} + O(\Delta\varphi^6) \right) \right]} \\ &= \sqrt{\Delta\varphi^2 - \frac{\Delta\varphi^4}{12} + O(\Delta\varphi^6)} \\ &= |\Delta\varphi| \sqrt{1 - \frac{\Delta\varphi^2}{12} + O(\Delta\varphi^4)} \end{aligned} \quad (\text{A.2})$$

Now defining

$$u = -\frac{\Delta\varphi^2}{12} + O(\Delta\varphi^4) \quad (\text{A.3})$$

and substituting u into (A.2) to apply the Taylor expansion of $\sqrt{1+u}$:

$$\begin{aligned}\sqrt{2[1 - \cos \Delta\varphi]} &= |\Delta\varphi| \sqrt{1+u} \\ &= |\Delta\varphi| \left(1 + \frac{u}{2} + O(u^2)\right)\end{aligned}\tag{A.4}$$

Expressing (A.4) as a function of $\Delta\varphi$ gives

$$\begin{aligned}|\Delta\varphi| \left(1 + \frac{u}{2} + O(u^2)\right) &= |\Delta\varphi| \left(1 - \frac{\Delta\varphi^2}{24} + O(\Delta\varphi^4)\right) \\ &\simeq |\Delta\varphi|\end{aligned}\tag{A.5}$$

Therefore, if the **ME** is negligible and the **PE** is very small,

$$\text{TVE}[k] \simeq |\Delta\varphi|\tag{A.6}$$

Appendix B

Physical explanation of saturation and remanent flux and Jiles-Atherton model for ferromagnetic materials applied to current transformers

This appendix serves the purpose of introducing the **JA** model for **CTs** adopted by PSCAD in section **B.2**. Before that, section **B.1** intuitively describes the physics of domain walls which the **JA** model is based on. Although not necessary for the **SA** discussion, this appendix is intended as a theoretical reference to understand the meaning behind the **CT** parameters depicted in table **B.1**.

B.1 Physical explanation of saturation and remanence in ferromagnetic materials

A ferromagnetic material is characterized by domains in which all the magnetic dipoles are aligned. In its demagnetized state, the domains possess different magnetic dipole orientations, and the total magnetization is zero; however, when an external magnetic field is applied to the material, the domains with dipoles that are parallel to the external field tend to expand, to the detriment of misaligned domains [50]. The expansion is hindered by domain wall pinning sites, which create a frictional force opposing the translation

of the domain walls. The first magnetization is reversible until one of the domain walls meets one of the pinning sites during its translation; after that, the magnetization becomes irreversible and the material cannot be demagnetized simply by removing the external magnetic field. This is the cause of the remanent flux density in the material. In general, the magnetization curve of a material depends on its crystal's structure, the characteristics of the domains, the density of the pinning sites, the energy required to detach the domain walls from the pinning sites, and the ability of the domain walls to bend when pinned. A ferromagnetic material reaches saturation once it presents a unique domain with its magnetic dipoles aligned with the external magnetic field.

B.2 Jiles-Atherton model for ferromagnetic materials applied to current transformers

The **JA** model for ferromagnetic materials defines the shape of the sigmoid hysteresis curve of a polycrystalline isotropic material based on the theory of ferromagnetic domains. The **JA** theory all the mechanisms influencing the shape of a hysteresis curve described in section **B.1** to provide a relation between the applied external magnetic field H and the material's magnetization M [51]:

$$\frac{dM}{dH} = \frac{1}{1+c} \frac{1}{\delta k / \mu_0 - \alpha (M_{an} - M)} (M_{an} - M) + \frac{c}{1+c} \frac{dM_{an}}{dH} \quad (B.1)$$

where c is a constant determined experimentally, δ is equal to 1 when $dH/dt > 0$ and -1 otherwise, k depends on the average density of pinning sites and their average energy, μ_0 is the magnetic permeability of free space, α depends on the perturbation due to interdomain coupling. Finally, M_{an} is the anhysteretic magnetization of the material, a function describing the $M(H)$ relationship in absence of pinning sites in the material.

For its usage in **CT** simulation models, (B.1) is readapted as [52]:

$$\frac{dM}{dH} = \frac{c \frac{dM_{an}}{dH_e} + \frac{M_{an} - M}{\frac{\delta k_{mod}}{\mu_0} - \frac{\alpha (M_{an} - M)}{1-c}}}{1 - \alpha c \frac{dM_{an}}{dH_e}} \quad (B.2)$$

Table B.1: Parameters of the PSCAD **JA CT** model.

V_s [V]	M_s [A m ⁻¹]	a_1	a_2	a_3	b	c	k [T]	α	β
55	1129703	4.66×10^{-8}	1.123	118.1					
66	1116183	2.6×10^{-8}	1.628	128.6					
77	1106217	3.54×10^{-8}	2.097	140.5					
88	1149081	1.76×10^{-7}	2.517	162	2	0.1	5×10^{-5}	1.3×10^{-5}	0.96
99	1137241	2.56×10^{-7}	2.979	174.8					
110	1127869	5.95×10^{-8}	3.424	188.8					

where $H_e = H + \alpha M$, and

$$k_{\text{mod}} = k \left[1 - \beta \left(\frac{M}{M_s} \right)^2 \right] \quad (\text{B.3})$$

The anhysteretic magnetization M_{an} is expressed as

$$M_{\text{an}} = M_s \frac{a_1 H_e + H_e^b}{a_3 + a_2 H_e + H_e^b} \quad (\text{B.4})$$

where M_s is the saturation magnetization of the anhysteretic M/H curve. Notice that M_{an} tends to M_s as H_e tends to infinity, with $b \geq 1$. The derivative of (B.4) with respect to H_e is

$$\frac{dM_{\text{an}}}{dH_e} = M_e \frac{a_1 a_3 + b a_3 H_e^{b-1} + (b-1)(a_2 - a_1) H_e^b}{(a_3 + a_2 H_e + H_e^b)^2} \quad (\text{B.5})$$

The parameters of (B.2), (B.4), and (B.5) are accepted by PSCAD as inputs of its **JA CT** model. These parameters can be retrieved through nonlinear curve fitting, by fixing the desired saturation voltage and flux density and core path length, as suggested in section 3.3.

Table B.1 shows the parameters of the **CTs** used in the PSCAD simulations.

Appendix C

Ferroresonance suppression circuits

FSCs are circuit components connected in parallel to the burden with the aim of adding damping to the transformers to limit overvoltages and insulation damage connected to ferroresonance. They can be either passive (resistive), or active [20]. Passive **FSCs** represent a source of loss in a **CVT**, because they absorb current even when the **CVT** is operating in steady state. To solve this issue, active **FSCs** possess a resonant LC circuit tuned to the system frequency in series with the damping resistance. The design of a **FSC** is shown in figure C.1 [20, 21]. The characteristic of this model is that its impedance is high for the system frequency, but sharply drops as soon the frequency varies slightly, allowing for effective damping of every harmonic component. The downside of using active **FSCs** is that their reactive components also discharge energy into the burden during transients, contributing to the mismatch between the primary and measured voltage.

When it comes to simulating **CVT** operation, the **FSC** of figure C.1 is

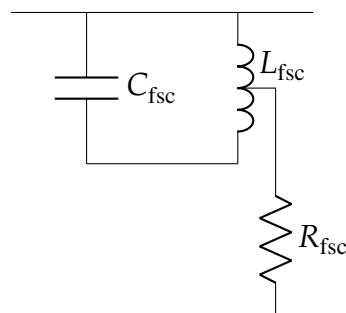


Figure C.1: Actual design of an active **FSC**. Adapted from [20, 21].

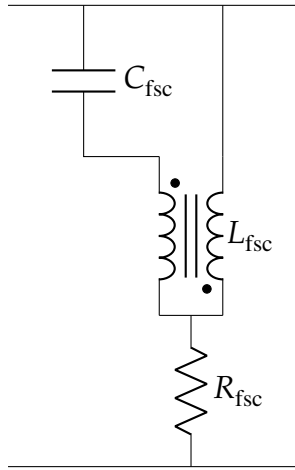


Figure C.2: Improved digital model of an active FSC. Adapted from [20, 21].

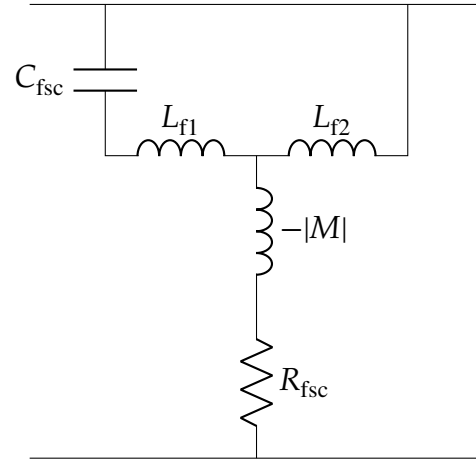


Figure C.3: Another digital model of an active FSC. Adapted from [53].

modeled as in figure 2.7 or C.2. While the basic model of figure 2.7 is simpler, its frequency response is not as representative of real FSCs as the one of figure C.2 since its impedance does not drop as sharply for frequencies close to the system frequency (see [20] and [21]). Reference [53] proposes another digital model shown in figure C.3.

However, these more accurate simulation models were excluded in favor of the simplest one because the simulations showed signs of numerical instability when inductances with negative values were introduced into the test system.

