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**Offshore vertical wind extrapolation: literature review
and unified assessment of most-based profile
formulations**

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Dictionary of Symbols and Variables

This dictionary summarizes the main symbols and variables used throughout the thesis. The same symbol may appear in a local equation with a specific contextual meaning; in those cases, the local definition given near the equation remains the reference definition.

Symbol	Name or meaning
A	Rotor-swept area.
$A_{s,f}(z)$	Profile-shape function used for stability scheme s and profile formulation f .
$A_{\text{GRYN}}(z)$	Gryning-type profile-shape function.
C_d	Drag coefficient.
C_p	Power coefficient.
$e_m(t, z)$	Error of model combination m at timestamp t and height z .
f	Coriolis parameter, computed as $f = 2\Omega \sin \varphi$.
g	Gravitational acceleration.
l	Local or effective mixing length.
L	Monin–Obukhov length.
L_{Ri}	Monin–Obukhov length inferred from a Richardson-based stability parameter.
L_{SL}	Surface-layer length scale in the Gryning-type formulation.
L_{MBL}	Middle-boundary-layer or mixed-layer length scale in the Gryning-type formulation.
L_{UBL}	Upper-boundary-layer length scale in the Gryning-type formulation.
$L_{\text{SL},\text{N}}$	Neutral surface-layer length scale.
$L_{\text{MBL},\text{N}}$	Neutral middle-boundary-layer length scale.
$L_{\text{UBL},\text{N}}$	Neutral upper-boundary-layer length scale.
m	Model combination, defined by one stability scheme and one profile formulation.
N	Number of valid samples used in an error metric or selection bin.
$N_{m,z}$	Number of valid samples for model combination m at height z .
N_{used}	Number of samples used after filtering or model selection.
$N_{\text{tot}}, N_{\text{total}}$	Total number of available samples in the corresponding group.

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Symbol	Name or meaning
$N_{>40}$	Number of samples above the 40 m s ⁻¹ filtering threshold.
n	Iteration index in the numerical closure.
p	Atmospheric pressure.
p_0	Reference pressure used in the potential-temperature definition.
P	Extracted wind-turbine power or atmospheric pressure, depending on the local context.
q	Specific humidity used in thermodynamic equations.
Q_H	Sensible heat flux.
R	Gas constant for dry air.
SST	Sea-surface temperature.
Ri	Richardson number used in empirical stability relations.
Ri_g	Gradient Richardson number.
Ri_B	Bulk Richardson number.
s	Empirical stability-correction scheme.
stab(t)	Atmospheric stability class assigned to timestamp t .
t	Timestamp.
T	Absolute temperature.
T_{2m}	Air temperature at 2 m.
T_{proxy}	Temperature proxy computed from 2 m air temperature and sea-surface temperature.
T_v	Virtual temperature.
U	Mean wind speed.
$U(z)$	Mean wind speed at height z .
U_{hub}	Hub-height wind speed.
U_{ref}	Reference wind speed measured at the near-surface reference height.
U_{model}	Wind speed predicted by a model formulation.
U_{LiDAR}	Wind speed measured by the LiDAR system.
$U_{s,f}(z, t)$	Wind speed predicted by stability scheme s and formulation f at height z and timestamp t .
u'	Turbulent fluctuation of the streamwise wind component.
w'	Turbulent fluctuation of the vertical wind component.
$\overline{u'w'}$	Reynolds momentum flux.
$\overline{w'\theta'}$	Turbulent potential-temperature flux.
$\overline{w'\theta'_v}$	Turbulent virtual-potential-temperature flux or buoyancy-related heat flux.
u_*	Friction velocity.

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Symbol	Name or meaning
$u_{*,0}$	Near-surface friction velocity in the Gryning-type formulation.
$u_*(z)$	Height-dependent friction-velocity scale in the Gryning-type formulation.
V	Cross-wind horizontal velocity component used in Richardson-number definitions.
x	Auxiliary variable used in the Businger–Dyer unstable function.
$\mathbf{x}(t)$	Input vector at timestamp t .
y	Auxiliary variable used in the Jimenez unstable convective function.
z	Height above the surface.
z_0	Aerodynamic roughness length.
$z_{0,\min}$	Minimum roughness length allowed in the numerical closure.
z_i	Boundary-layer height or diagnostic boundary-layer scale.
$z_{i,\min}$	Minimum boundary-layer-height scale used to avoid degenerate numerical cases.
z_r	Reference height used in Richardson-based estimation of L .
z_{ref}	Reference height used to anchor the wind-profile calculation.
α	Empirical constant; used as the Charnock constant in roughness equations and as a decay exponent in the Gryning-type formulation when locally defined.
α_{Ch}	Charnock constant used in the numerical roughness update.
β_m	Empirical momentum constant in stable Businger–Dyer-type correction.
γ_m	Empirical momentum constant in unstable Businger–Dyer-type correction.
Δ	Finite difference or change in a variable.
Bias	Mean signed model error.
RMSE	Root mean square error.
MAE	Mean absolute error.
ε_L	Convergence tolerance for the Monin–Obukhov length.
ζ	Monin–Obukhov stability parameter, defined as z/L .
ζ_{ref}	Stability parameter evaluated at the reference height.
ζ_{Ri}	Richardson-derived stability parameter.
θ	Potential temperature.
θ_v	Virtual potential temperature.
$\overline{\theta}_v$	Mean virtual potential temperature.
κ	von Kármán constant.
ρ	Air density.

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Symbol	Name or meaning
σ	Standard deviation.
τ_0	Surface shear stress.
$\phi_m(\zeta)$	Dimensionless momentum-gradient function.
$\psi_m(\zeta)$	Integrated momentum stability correction.
$\psi_{m,s}$	Integrated momentum stability correction associated with scheme s .
ξ	Dummy integration variable used in the definition of ψ_m .
Ψ_m	Alternative notation sometimes used for the integrated momentum correction.
Ω	Angular rotation rate of the Earth.
$\Omega_{m,z}$	Set of valid timestamps used for model combination m at height z .
φ	Latitude.
c	Empirical coefficient in the Lange boundary-layer-height correction.
c_p	Specific heat of air at constant pressure.
$c_m(\zeta)$	Integrated momentum correction notation used in the Jimenez formulation.
$c_h(\zeta)$	Integrated heat correction notation used in the Jimenez formulation.
$c_{h,m}^K$	Kansas-type component in the Jimenez formulation.
$c_{h,m}^C$	Convective component in the Jimenez formulation.
a, b, d	Empirical coefficients used in stability-correction functions.
a_m, a_h	Momentum and heat empirical constants in the Jimenez formulation.
$\mathcal{Z}$	Set of evaluated heights.
$\mathcal{S}$	Set of empirical stability-correction schemes.
$\mathcal{F}$	Set of profile formulations: LOG, Lange, and GRYN.

Abstract

Methods to improve vertical extrapolation of offshore wind speeds are examined for scenarios where only near-surface measurements are available and hub-height data are limited. The analysis focuses on heights relevant to modern wind turbines, ranging from 30 to 200 m, and typical offshore environments. The study highlights the substantial influence of air stability, sea-surface roughness, and boundary-layer structure on the mean wind profile.

A unified computational pipeline is developed to estimate vertical wind profiles by applying Monin–Obukhov Similarity Theory (MOST) alongside several stability-dependent formulations. The methodology integrates a classical logarithmic profile with stability corrections, a reformulation incorporating a boundary-layer height term, and a Gryning-type correction based on an empirical mixed-layer length. Four widely used stability functions, namely Businger–Dyer, Jiménez, Holtslag–de Bruin, and Brutsaert, are implemented within their respective validity ranges. Surface roughness is addressed using offshore-specific parameterisations, including a Charnock-based formulation related to friction velocity.

The models are evaluated using offshore LiDAR buoy measurements collected over a multi-year period from 2022 to 2024, with consistent quality control and error statistics based primarily on root mean square error (RMSE) and Bias, stratified by height, time of day, and stability regime. Results indicate that no single formulation performs optimally across all regimes. Instead, model effectiveness systematically varies with stability and height, necessitating an operational selection strategy. Conditional performance tables inform a rule-based adaptive scheme that selects the model–correction combination with the lowest RMSE in each context, while Bias is retained to diagnose systematic overestimation or underestimation.

In summary, this thesis presents a reproducible methodology for constructing offshore wind profiles and a practical decision framework that integrates Monin–Obukhov Similarity Theory, reanalysis data, and LiDAR observations for resource assessment and operational applications.

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Keywords: offshore wind; vertical wind profile; Monin–Obukhov similarity theory; atmospheric stability; roughness length; LiDAR validation; model selection; adaptive strategy.

1 Introduction

Offshore wind energy is an important component of the transition toward low-emission electricity generation. Compared with onshore sites, offshore areas often provide broader available space, lower terrain-induced roughness, and persistent wind conditions over large marine regions. This makes offshore wind especially relevant in regions with high maritime resources, such as the North Sea. However, the value of an offshore site depends not only on the wind available near the sea surface, but also on the accuracy with which this wind can be estimated at turbine-relevant heights.

A central challenge in offshore wind resource assessment is the limited availability of direct wind measurements at hub height and across the rotor-swept layer. Tall offshore masts are expensive, and long-term measurements at multiple heights are still less common than near-surface observations. For this reason, vertical extrapolation remains necessary when wind information is obtained from low-level measurements, buoys, or reanalysis products. [17, pp. 219–236], [15, pp. 587–617], [18, pp. 935–948]

This extrapolation is not straightforward because the offshore wind profile is controlled by several atmospheric and surface processes. Wind speed varies with height due to momentum exchange between the atmosphere and the sea surface. This exchange is affected by aerodynamic roughness, wave state, thermal stratification, air–sea temperature differences, humidity effects, and the depth of the atmospheric boundary layer. [2, pp. 99–105], [15, pp. 587–617], [1, pp. 1043–1052] Atmospheric stability is particularly important, since unstable conditions tend to enhance turbulent mixing, while stable conditions tend to suppress it and increase vertical shear. [21, pp. 347–399], [22, pp. 83–88] Therefore, the same near-surface wind speed may lead to different hub-height estimates depending on stability, height, and model formulation.

The practical problem addressed in this thesis is whether offshore vertical wind-speed profiles can be reconstructed from limited input information while remaining physically traceable, numerically reproducible, and consistent with independent LiDAR observations. The work focuses on formulations based on Monin–Obukhov Similarity Theory (MOST), because MOST links wind shear, surface exchange, roughness, and atmospheric stability within a physically based framework. At the same time, the thesis recognizes that MOST-based formulations may not perform equally well at all heights or under all stability regimes, especially when the evaluated levels approach the upper part of the turbine rotor layer.

The main objective of this thesis is to evaluate reduced-input offshore wind-speed extrapo-

lation methods based on MOST and related profile reformulations. More specifically, the work constructs vertical wind-speed profiles from near-surface wind and auxiliary meteorological inputs, compares different MOST-based formulations combined with empirical stability corrections, evaluates the results against LiDAR observations using RMSE, Bias, and MAE, and defines rule-based selection criteria according to height, stability class, and hour of day.

The analysis uses offshore LiDAR buoy measurements from the IJmuiden Ver measurement campaign in the North Sea. [19] The evaluated profile formulations include the logarithmic MOST profile, the Lange-type formulation with a boundary-layer height contribution, and the Gryning-type extension based on an empirical mixed-layer length scale. These formulations are combined with the Businger–Dyer, Jiménez, Holtslag–de Bruin, and Brutsaert stability corrections. The purpose is not to identify a universal formulation, but to determine which model combination performs better under specific offshore conditions.

The thesis is organized to move from theoretical formulation to computational implementation and model validation. The literature review presents the physical and empirical basis of offshore wind-speed extrapolation, including the logarithmic profile, offshore roughness, atmospheric stability, MOST corrections, and profile extensions beyond the surface layer. The methodology then describes the computational pipeline used to process the input data, estimate the required physical parameters, construct the modelled wind-speed profiles, and match the results with LiDAR observations.

The results are analysed through both numerical metrics and graphical diagnostics. Model performance is evaluated using RMSE, Bias, and MAE, first in a general comparison and then under conditional groupings based on height, stability regime, and hour of day. In addition, the vertical wind-speed profiles are plotted to compare the mean modelled profiles with the corresponding LiDAR observations, while the error distributions are used to identify systematic overestimation, underestimation, and dispersion at different heights. This combination of metrics and visual analysis makes it possible to assess not only which model performs best overall, but also how its performance changes across the vertical profile and under different atmospheric conditions. Finally, the conclusion summarizes the main findings, discusses the limitations of the reduced-input approach, and suggests possible future improvements.

2 Literature review

The central physical problem of this thesis is the vertical variability of wind speed in the marine atmospheric boundary layer. Offshore wind-energy applications require information at hub height and across the rotor-swept layer, but measurements are often available only close to the sea surface. This creates a practical gap between the height where the wind is measured and the height where the turbine extracts energy. The gap matters because wind speed controls resource characterization, energy-yield estimates, load assessment, and the preliminary design of offshore wind projects. [17, pp. 219–236], [15, pp. 587–617]

The first practical objective of vertical extrapolation is therefore to provide an estimate of the wind speed at hub height, here denoted as U_{hub} . For preliminary resource assessment, U_{hub} can be used as the representative inflow velocity at the centre of the rotor. This gives a first approximation of the wind resource and allows simplified power or power-curve based calculations when only limited vertical information is available. This dependence becomes explicit in energy-yield estimation. In simplified form, the power extracted from the flow is commonly related to wind speed through the turbine power relation

$$P = \frac{1}{2} \rho A C_p U_{\text{hub}}^3, \quad (2.1)$$

where P is the extracted power, ρ is the air density, A is the rotor-swept area, C_p is the power coefficient, and U_{hub} is the hub-height wind speed used in this simplified approximation. The cubic dependence on U_{hub} means that a small bias in extrapolated wind speed can produce a much larger bias and, consequently, an incorrect estimation of expected energy.

This hub-height approximation is useful, but it is not equivalent to describing the whole rotor-swept flow. Modern offshore rotors span a broad vertical range, so the lower and upper parts of the rotor may experience different wind speeds, shear, veer, and turbulence conditions. For this reason, the full vertical profile $U(z)$ remains relevant beyond the first approximation based on U_{hub} . It supports the interpretation of wind shear across the rotor disk, which is relevant for loads, fatigue, and control. [10, Art. 012052] For this reason, the problem is not only mathematical. It is directly connected to the engineering reliability of offshore resource assessment.

The challenge is that the atmosphere is not a static or uniform medium. Instead, vertical shear changes with turbulent momentum exchange, thermal stratification, surface roughness,

wave conditions, and the depth of the boundary layer. On land, terrain roughness, obstacles, and topography often dominate the lower boundary. Offshore, the surface is smoother, but it is mobile and thermally coupled to the atmosphere through the sea surface. This difference makes offshore wind profiles physically distinct from onshore profiles and motivates the use of stability-aware formulations. Thus, the same wind speed measured near the sea surface can correspond to different wind speeds aloft, depending on the turbulent and thermodynamic state of the lower atmosphere.

This chapter reviews the physical basis used in this thesis to extrapolate offshore wind speed from a low reference height to turbine-relevant heights. The review is organized from the simplest formulation to the more complete ones used later in the methodology. First, the neutral logarithmic profile is introduced as the baseline case. Then, the two surface parameters that control this profile, the friction velocity u_* and the aerodynamic roughness length z_0 , are discussed. After that, atmospheric stability is introduced through the Monin–Obukhov length L and through the stability correction functions. Finally, the chapter discusses the empirical formulations for the integrated correction ψ_m , the practical limits of MOST, and the reformulations that introduce additional boundary-layer scales.

In this way, the chapter provides the theoretical background for the comparison performed in this thesis. The central issue is not only how to compute a wind profile, but also how to observe, validate, and register the results of implementing different formulations under the same offshore conditions. This motivates the unified comparison between the classical logarithmic correction, the formulation with a boundary-layer height term, and the Gyrning-type extension applied later in the work.

2.1 Neutral logarithmic wind profile

The neutral logarithmic profile is the simplest starting point for the vertical extrapolation problem. It represents the mean wind profile in the atmospheric surface layer when thermal stratification has a negligible effect on turbulent mixing. This does not mean that thermal processes are already represented by the neutral profile. It means the opposite: buoyancy effects are neglected as a first approximation, so the vertical shear is mainly controlled by mechanical turbulence generated by the interaction between the flow and the surface. [16, pp. 163–187], [21, pp. 347–399]

The formulation assumes that the flow is horizontally homogeneous and approximately stationary. It also assumes that the turbulent momentum flux varies little with height inside the surface layer. Under these hypotheses, the mean wind speed at height z can be written as

$$U(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right). \quad (2.2)$$

Here, u_* is the friction velocity, κ is the von Kármán constant, and z_0 is the aerodynamic roughness length. The height z_0 acts as the aerodynamic boundary condition of the profile. Equation (2.2) therefore states that the neutral wind profile depends on one velocity scale and one roughness scale. [16, pp. 163–187], [21, pp. 347–399]

This formulation has a clear physical interpretation. The friction velocity u_* represents the momentum exchange between the atmosphere and the surface, while the aerodynamic roughness length z_0 represents the aerodynamic effect of the lower boundary. For this reason, the neutral logarithmic profile is useful as a baseline formulation. It provides a simple and physically transparent reference from which more complete formulations can be introduced.

However, the neutral logarithmic profile is also limited. It does not represent the effect of heating or cooling at the surface, and it does not include additional boundary-layer scales that may become relevant above the surface layer. For this reason, the neutral profile is used here as the reference formulation from which the stability-corrected and extended profiles are introduced in the following sections. [22, pp. 83–88], [8, pp. 251–268]

2.2 Velocity scale and shear stress: u_*

Before introducing empirical stability corrections, it is useful to clarify the meaning of the friction velocity. The notation u_* can be misleading because it has units of velocity, but it is not a wind speed measured at a specific height. It is a velocity scale constructed from the shear stress exerted by the turbulent flow on the lower boundary. In this sense, u_* summarizes how strongly momentum is being transferred between the atmosphere and the surface.

Surface shear stress is the tangential stress associated with this momentum exchange. It should not be interpreted only as molecular friction at the interface. In the atmospheric boundary layer, the dominant contribution comes from turbulent eddies that move air parcels upward and downward. When faster air from aloft is transported downward and slower air near the surface is transported upward, vertical fluctuations carry horizontal momentum. This turbulent transport produces a Reynolds stress, which appears in the mean flow as a shear stress. Over the sea, this stress is transmitted to a moving surface, so it is also linked to waves and sea-state-dependent roughness. [21, pp. 151–186], [6, pp. 2311–2328]

In the surface layer, turbulence therefore dominates the vertical transport of momentum. This transport appears as a turbulent flux associated with velocity fluctuations, and the reference quantity is the surface shear stress, denoted by τ_0 . As a first approximation, this flux varies little with height within the surface layer. [21, pp. 151–186], [6, pp. 2311–2328], [22, pp. 83–88] The friction velocity u_* is defined by

$$\tau_0 = \rho u_*^2, \quad (2.3)$$

where ρ is the air density. Equivalently, $u_* = \sqrt{\tau_0/\rho}$. This definition explains why the variable

is expressed in meters per second: it is the square root of a stress per unit density. It establishes a velocity scale linked to the momentum flux and provides the basis of similarity relations in the surface layer. [21, pp. 347–399]

In practice, τ_0 and u_* can be obtained through three common approaches.

(i) From measured turbulent flux (eddy covariance). When turbulent measurements are available, the Reynolds flux is estimated directly. In the surface layer, the approximation

$$\overline{u'w'} \approx -u_*^2, \quad (2.4)$$

holds, which allows u_* to be estimated as $u_* \approx \sqrt{-\overline{u'w'}}$ when $\overline{u'w'} < 0$. [21, pp. 151–186], [3, Art. 1197]

(ii) From a bulk formula (drag law). In surface parameterizations, shear stress is related to the mean wind by

$$\tau_0 = \rho C_d U^2, \quad (2.5)$$

where C_d is the drag coefficient and U is the wind speed at a reference height. Thus, u_* is obtained as $u_* = \sqrt{C_d} U$. [6, pp. 2311–2328], [17, pp. 219–236], [22, pp. 83–88]

(iii) From the logarithmic profile. Another alternative uses the mean profile in the surface layer and isolates u_* as

$$u_* = \frac{\kappa U(z)}{\ln(z/z_0)}. \quad (2.6)$$

This expression corresponds to the neutral case presented in Eq. (2.2). When stability is included later, the denominator is modified by the integrated correction function $\psi_m(z/L)$. In this project, this profile-based approach is the basis used to estimate u_* and extrapolate wind speed to other heights. Here, only the neutral framework is presented. The stability correction and the full calculation procedure are detailed in the following sections and in the methodology. [16, pp. 163–187], [21, pp. 347–399], [17, pp. 219–236]

2.3 Aerodynamic roughness of the sea

In offshore applications, the aerodynamic roughness length is usually much smaller than over land. However, it is not necessarily constant. The sea surface changes with wind speed, wave development, fetch, and air–sea interaction. For this reason, offshore roughness is often represented through specific parameterizations rather than by a fixed land-type roughness value. Among these parameterizations, the Charnock relation is widely used because it links z_0 directly

to the friction velocity and therefore to the surface momentum exchange. [6, pp. 2311–2328], [2, pp. 99–105]

Main methods for z_0 and practical implications

Roughness models mainly differ in the *set of inputs* they require and in how well they can represent variations in sea state and coastal influence. In broad terms, the main implications and challenges are [13]:

- **Constant coefficient (fixed roughness).** This is the simplest and most robust choice when little additional information is available. It ignores the dependence of z_0 on wind and waves, and therefore does not represent rapid variations in sea state. Still, for some resource-assessment applications, this approximation may be sufficient and tends to behave relatively stably under non-stationary conditions. In offshore studies, a commonly adopted fixed value is $z_0 = 0.2 \text{ mm}$, or $2 \times 10^{-4} \text{ m}$, as in the WAsP model. In addition, sensitivity analyses often test ranges of z_0 spanning orders of magnitude, for example from 2×10^{-5} to $2 \times 10^{-3} \text{ m}$, to quantify the impact of this assumption on the extrapolated profile. [13], [2, pp. 99–105]
- **Charnock model, dependent on u_* .** This model is a classical parameterization for the sea surface, linking the aerodynamic roughness length z_0 directly to the shear stress at the air–sea interface. The central idea is to use u_* , defined from the surface stress as in Eq. (2.3). The Charnock relationship expresses roughness as

$$z_0 = \alpha \frac{\tau_0}{\rho g}. \quad (2.7)$$

Substituting τ_0 yields

$$z_0 = \alpha \frac{\rho u_*^2}{\rho g}.$$

After canceling ρ , the expression simplifies to

$$z_0 = \alpha \frac{u_*^2}{g}, \quad (2.8)$$

where g is gravity and α is the Charnock constant. [6, pp. 2311–2328] In offshore studies, a typical value is $\alpha = 0.0144$. [17, pp. 219–236]

- **Wave age model, dependent on sea state.** This class seeks to include explicit information about the wave field, typically through a wave-age measure associated with the spectral peak. The practical implication is that the model can represent roughness variations linked to wave development, but it requires wave variables, such as peak phase speed or

peak frequency, that are not always available in wind studies. In addition, the literature discusses that spurious correlations can arise when common variables enter both sides of the fitted relationships. [13]

- **Fetch-dependent model.** This approach replaces the need to directly measure wave variables with a geometrically available proxy, the *fetch*. The challenge is that the effective *fetch* depends on wind direction and coastal geometry, and calibration relationships typically assume ideal conditions for wind propagation relative to the coastline. Thus, applicability may be limited in scenarios with complex coastlines or rapid changes in wind direction. [13]

In this thesis, the Charnock relationship, Eq. (2.8), is adopted as the main model for z_0 because it offers a balance between physical basis and input availability, requiring only u_* and physical constants. The other formulations are described here to contextualize alternatives and to show that, in offshore studies, different z_0 choices can be tested depending on data availability. [13]

Finally, it is worth noting that the estimation of u_* and, therefore, of z_0 through the Charnock model depends on the surface-layer parameterization and on atmospheric stability. Under stable conditions, parameterization choices and limitations of flux–profile relationships can influence model performance and parameter consistency. [22, pp. 83–88], [3, Art. 1197], [12, pp. 898–918] These aspects are discussed in more detail in the sections dedicated to MOST limitations under atmospheric stability.

2.4 Thermodynamic basis of atmospheric stability

The introduction presented atmospheric stability in intuitive terms, as the tendency of the atmosphere to resist, favour, or remain approximately indifferent to vertical air motions. For the purposes of wind-profile modelling, this idea must now be expressed more formally. The standard framework is based on the behaviour of an air parcel that is displaced vertically in a stratified atmosphere. The question is whether that parcel tends to return toward its initial level, continue moving away from it, or remain in approximate equilibrium after the displacement. [21, pp. 169–172]

A useful first approximation is to treat the parcel displacement as an adiabatic process. This means that the parcel exchanges little or no heat with the surrounding air during the motion, so its temperature changes mainly because of expansion or compression associated with pressure changes. If a parcel rises, the ambient pressure decreases, the parcel expands, and its temperature decreases. If it sinks, the ambient pressure increases, the parcel is compressed, and its temperature increases. This adiabatic framework is fundamental because it allows the parcel temperature to be compared with the environmental temperature at the new height. [21,

pp. 7–8]

The atmosphere itself is characterized by an environmental temperature gradient, or environmental lapse rate, which describes how temperature varies with height. For an unsaturated parcel undergoing adiabatic motion, the temperature changes approximately according to the dry adiabatic lapse rate,

$$\Gamma_d = -\frac{dT}{dz} \approx 9.8 \text{ K km}^{-1}. \quad (2.9)$$

If moisture condensation occurs, latent heat is released and the parcel cools more slowly, so the relevant lapse rate becomes the moist adiabatic lapse rate, which is smaller than the dry value and varies with thermodynamic conditions. The comparison between the environmental lapse rate and the rate at which a displaced parcel cools or warms is the basis of formal atmospheric-stability analysis. [21, pp. 7–8, 169–172]

In formal terms, the atmosphere is statically stable if a vertically displaced parcel becomes denser than its surroundings and tends to return toward its original position. It is statically unstable if the displaced parcel becomes less dense than the surrounding air and therefore continues to move away from its initial level. A neutral state corresponds to the limit in which the displaced parcel has nearly the same density as the environment and experiences little or no restoring buoyancy force. Thus, atmospheric stability is fundamentally a buoyancy problem: it depends on whether vertical displacement creates a restoring force, an amplifying force, or almost no net force at all. [21, pp. 169–172]

Figure 1 illustrates this parcel-based interpretation in a simplified way. In the figure, the cloud-shaped symbols represent idealized air parcels or air masses, not real clouds. If the displaced air mass is colder than the surrounding air, it is denser and tends to sink back toward its original level. This corresponds to a stable configuration. If the displaced air mass is warmer than the surrounding air, it is less dense and tends to rise further, which represents an unstable configuration. When both temperatures are approximately equal, buoyancy produces little net acceleration and the state is represented as neutral. This illustration should be read as a conceptual aid. The formal stability diagnosis used later in this chapter is based on potential temperature, virtual potential temperature, and the Monin–Obukhov stability parameter.

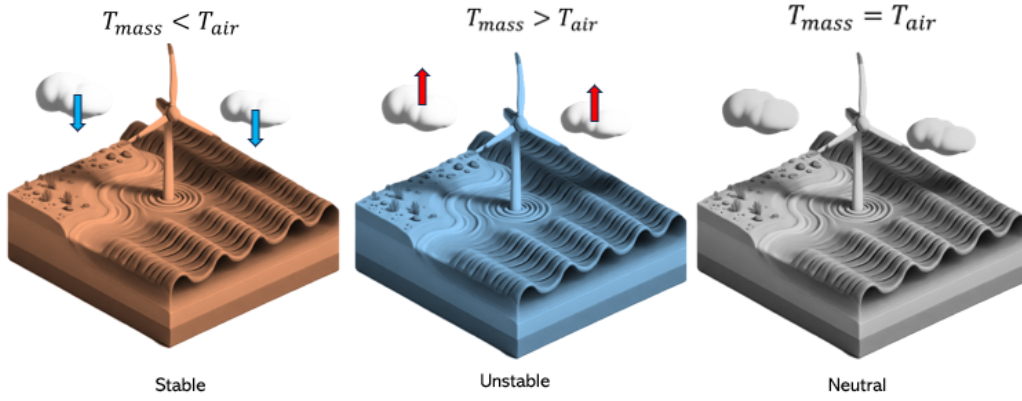


Fig. 1: Schematic representation of atmospheric stability regimes and their thermodynamic interpretation.

For a compressible atmosphere, a more convenient variable than temperature itself is the potential temperature. The potential temperature θ is defined as the temperature that an air parcel would have if brought adiabatically to a fixed reference pressure p_0 , usually taken as 1000 hPa:

$$\theta = T \left(\frac{p_0}{p} \right)^{R/c_p}, \quad (2.10)$$

where T is the absolute temperature, p is the parcel pressure, R is the gas constant for dry air, and c_p is the specific heat at constant pressure. Because θ is conserved under dry adiabatic motion, it provides a direct way to diagnose static stability in a vertically varying pressure field. [21, pp. 7–8, 169–172]

2.4.1 Virtual temperature and virtual potential temperature

The previous definition uses dry-air thermodynamics. In marine environments, however, moisture can be important because humid air is less dense than dry air at the same temperature and pressure. For this reason, buoyancy calculations often use the virtual temperature T_v . Virtual temperature is the temperature that dry air would need to have in order to present the same density as moist air at the same pressure. For unsaturated air, a common approximation is

$$T_v \simeq T (1 + 0.61q), \quad (2.11)$$

where q is the specific humidity. The corresponding virtual potential temperature is written as

$$\theta_v \simeq \theta (1 + 0.61q). \quad (2.12)$$

This variable is especially relevant for Monin–Obukhov theory because the buoyancy flux depends on density effects, not only on sensible-temperature fluctuations. Thus, the formal definition of the Monin–Obukhov length uses $\overline{w'\theta'_v}$ rather than only $\overline{w'\theta'}$. When humidity information is unavailable, using θ instead of θ_v is an approximation. This approximation can be acceptable as a practical closure, but it should be stated clearly in offshore applications, where air–sea humidity exchange may modify the effective buoyancy flux. [21, pp. 7–8, 347–399], [6, pp. 2311–2328]

Using potential temperature, the formal criterion becomes especially clear. As a first local criterion, an increase of potential temperature with height indicates static stability, whereas a decrease indicates static instability. An approximately constant potential-temperature profile is often associated with neutral or well-mixed conditions. However, this criterion must be interpreted with care in the boundary layer, because the local lapse rate alone is not always sufficient to determine the stability state. The turbulent buoyancy flux and the structure of the whole layer may also be required. [21, pp. 169–172]

This formulation is particularly useful in boundary-layer meteorology because it expresses stability in a way that separates genuine thermal stratification from temperature changes produced only by compression and expansion. [21, pp. 7–8, 169–172]

For offshore wind applications, these concepts matter because thermal stratification directly affects turbulence intensity, vertical momentum exchange, and wind shear. A weakly mixed stable layer can sustain strong vertical gradients in wind speed, whereas unstable stratification tends to redistribute momentum more efficiently and reduce shear. The thermodynamic description given above therefore provides the physical basis for the similarity framework used later in this chapter. Monin–Obukhov Similarity Theory translates these buoyancy effects into practical profile corrections through the Monin–Obukhov length and the associated stability functions. [21, pp. 347–399]

In offshore wind-speed extrapolation, this connection is especially important because the logarithmic profile is most appropriate under neutral conditions, while departures from neutrality require stability corrections. Previous offshore studies show that atmospheric stratification strongly affects wind-speed profiles and that applying stability corrections can improve the prediction of vertical wind profiles relative to a purely neutral logarithmic formulation. [17, pp. 219–236], [10, Art. 012052]

2.5 Atmospheric stability, Monin–Obukhov parameter, and MOST correction of the wind profile

The previous section established the physical meaning of atmospheric stability in terms of buoyancy, potential temperature, virtual potential temperature, and vertical mixing. For wind-profile modelling, this physical concept must be translated into a parameter that describes

how thermal stratification modifies turbulent exchange in the surface layer. Monin–Obukhov Similarity Theory provides this link by comparing the effects of mechanical shear production and buoyancy on turbulence. This comparison leads to the Monin–Obukhov length L , which acts as the characteristic stability scale of the surface layer. [16, pp. 163–187], [21, pp. 347–399]

The neutral logarithmic profile is only a baseline approximation. The offshore atmosphere is not always neutral, and the vertical exchange of momentum depends strongly on atmospheric stability. Under unstable conditions, buoyancy enhances turbulent mixing and the vertical wind shear tends to be weaker. Under stable conditions, buoyancy suppresses turbulent mixing and the wind profile can become steeper. In MOST, these effects are represented through the Monin–Obukhov length L and through the integrated momentum correction function ψ_m . The role of ψ_m is to modify the logarithmic profile according to the thermal stratification of the surface layer. [16, pp. 163–187], [21, pp. 347–399]

The Monin–Obukhov length can be written as

$$L = -\frac{u_*^3 \bar{\theta}_v}{\kappa g \overline{w' \theta'_v}}, \quad (2.13)$$

where u_* is the friction velocity, κ is the von Kármán constant, g is gravity, $\bar{\theta}_v$ is the mean virtual potential temperature, and $\overline{w' \theta'_v}$ is the turbulent flux of virtual potential temperature. This definition shows that L depends on both mechanical turbulence, represented by u_* , and thermal forcing, represented by the buoyancy flux.

The sign of L follows from the sign of the turbulent buoyancy flux. When the heat flux is upward, buoyancy contributes to turbulence generation and L is negative. This indicates an unstable tendency. When the surface layer is stably stratified, buoyancy suppresses vertical motion and L is positive. This indicates a stable tendency. When $|L|$ becomes very large, buoyancy effects are weak relative to mechanical mixing, and the flow approaches the neutral limit of surface-layer similarity. However, the strength of the stability effect in the wind-profile formulation is not determined by the sign of L alone. It depends on the ratio between the height of interest and the Monin–Obukhov length. [16, pp. 163–187], [21, pp. 347–399]

For this reason, MOST introduces the dimensionless stability parameter

$$\zeta = \frac{z}{L}, \quad (2.14)$$

where z is the height above the surface. This parameter compares the height of interest with the stability length scale. In practical terms, ζ organizes the effect of thermal stratification into a single dimensionless variable. Values of ζ close to zero indicate near-neutral behaviour. Negative values indicate an unstable tendency, where buoyancy tends to enhance turbulent mixing. Positive values indicate a stable tendency, where buoyancy suppresses vertical exchange

and generally increases vertical wind shear. Thus, ζ , rather than L alone, is the practical variable used in MOST stability functions. [21, pp. 347–399], [10, Art. 012052]

Several classification schemes can be built from ζ . A common three-class convention in wind-energy and surface-layer applications groups the flow as

- **Unstable:** $\zeta < -0.01$,
- **Neutral:** $-0.01 \leq \zeta \leq 0.01$,
- **Stable:** $\zeta > 0.01$.

This convention should not be interpreted as a universal physical definition of stability. It is an operational classification that defines a narrow near-neutral interval around $\zeta = 0$. The physical basis is that the sign of L indicates a stable or unstable tendency, while the magnitude of z/L indicates whether the buoyancy effect is strong enough to modify the surface-layer profile appreciably. [4]

Under this convention, very large positive or negative values of L are treated as near-neutral because $\zeta = z/L$ becomes close to zero. For example, at a conventional reference height of $z = 10$ m, the interval $|\zeta| \leq 0.01$ corresponds approximately to $|L| \geq 1000$ m. Thus, a very large negative value of L does not imply a strongly unstable profile, and a very large positive value of L does not imply a strongly stable profile. Both cases approach the neutral limit because the buoyancy contribution is weak compared with mechanical mixing.

After defining ζ , MOST describes how stability modifies the mean wind profile. The first effect appears in the local vertical wind shear. This modification is expressed through the dimensionless momentum gradient function $\phi_m(\zeta)$, defined as

$$\frac{\kappa z}{u_*} \frac{\partial U}{\partial z} = \phi_m(\zeta). \quad (2.15)$$

This equation can be interpreted as a normalized form of the wind shear. When $\phi_m = 1$, the neutral logarithmic shear is recovered. When $\phi_m > 1$, the shear is larger than in neutral conditions. When $\phi_m < 1$, the shear is smaller than in neutral conditions. Thus, ϕ_m is the empirical gradient function that represents how stability changes the local wind shear. [21, pp. 347–399], [22, pp. 83–88]

The wind speed $U(z)$, however, is not obtained from the local gradient alone. It is obtained by integrating the shear from the roughness length to the height of interest. For this reason, the gradient correction ϕ_m leads to an integrated stability correction, denoted here by ψ_m . With the sign convention adopted here, the relation between the two functions is

$$\psi_m(\zeta) = \int_0^\zeta \frac{1 - \phi_m(\xi)}{\xi} d\xi, \quad \psi_m(0) = 0. \quad (2.16)$$

This convention implies that ψ_m is usually negative under stable conditions and positive under unstable conditions. As a result, the term $-\psi_m$ increases the wind speed predicted above the reference level in stable conditions and reduces it in unstable conditions, relative to the neutral logarithmic profile.

The corresponding MOST-corrected logarithmic wind profile is

$$U(z) = \frac{u_*}{\kappa} \left[\ln\left(\frac{z}{z_0}\right) - \psi_m\left(\frac{z}{L}\right) + \psi_m\left(\frac{z_0}{L}\right) \right]. \quad (2.17)$$

Since z_0 is usually much smaller than the measurement heights considered in wind-energy applications, the term $\psi_m(z_0/L)$ is often neglected. This gives the simplified form

$$U(z) = \frac{u_*}{\kappa} \left[\ln\left(\frac{z}{z_0}\right) - \psi_m\left(\frac{z}{L}\right) \right]. \quad (2.18)$$

This expression is the stability-corrected extension of the neutral logarithmic profile. The neutral profile is recovered when $\zeta \rightarrow 0$, because $\psi_m(0) = 0$. [16, pp. 163–187], [21, pp. 347–399], [10, Art. 012052]

In words, L defines the stability scale of the surface layer. The parameter $\zeta = z/L$ expresses how relevant that stability is at height z . The function $\phi_m(\zeta)$ corrects the local vertical wind shear. The function $\psi_m(\zeta)$ is the integrated correction used in the wind profile. The result is the stability-corrected wind speed $U(z)$.

MOST defines this structure, but it does not prescribe a unique empirical form for ϕ_m or ψ_m . These functions must be specified from observations, theoretical constraints, or surface-layer parameterizations. For this reason, different formulations appear in the literature. Businger–Dyer, Jiménez et al., Holtslag–de Bruin, and Brutsaert-type schemes can be understood as different ways of translating the same stability parameter $\zeta = z/L$ into a momentum correction for the wind profile. This distinction is central for the comparison performed in this thesis, because two formulations may use the same reference wind speed, the same roughness length, and the same Monin–Obukhov length, but still predict different wind speeds aloft. [16, pp. 163–187], [21, pp. 347–399], [10, Art. 012052]

2.6 Empirical formulations for the integrated momentum correction

The previous section defined the role of ψ_m in the MOST-corrected wind profile. The use of ψ_m introduces a methodological difficulty. The same measured reference wind speed can lead to different extrapolated velocities depending on how stability is estimated and how the correction function is parameterized. Several empirical formulations are available in the literature, including Businger–Dyer, Jiménez et al., Holtslag–de Bruin, and Brutsaert-type corrections. These formulations should not be interpreted only as different mathematical

expressions. They also reflect different assumptions about turbulent exchange, stability range, and the behaviour of the surface layer. This distinction is important because stable regimes are especially difficult to model, as turbulence can become weak, intermittent, and affected by processes not fully represented by classical MOST. [22, pp. 83–88], [3, Art. 1197]

This section gives the empirical formulations used in the comparison. The notation is kept consistent throughout the thesis: the integrated momentum correction is written as ψ_m . Some references use alternative symbols, such as Ψ_m or c_m . When those quantities represent the same integrated correction to momentum, they are treated here as equivalent notation. This avoids introducing different symbols for the same physical role.

The variables are obtained in the same way for all schemes. The height z is the level at which the wind profile is evaluated, and the Monin–Obukhov length L is estimated from the available observations or parameterizations. Then $\zeta = z/L$ is computed and inserted into the selected expression for ψ_m . The quantities u_* and z_0 do not define the empirical correction directly, but they enter the wind profile through Eq. (2.18). For this reason, the following subsections only specify the empirical mapping from ζ to ψ_m .

2.6.1 Businger–Dyer formulation

The Businger–Dyer formulation uses different integrated functions under unstable and stable stratification. For unstable conditions, $\zeta < 0$, the integrated momentum correction is commonly written as

$$\psi_m(\zeta) = 2 \ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - 2 \arctan(x) + \frac{\pi}{2}, \quad x = (1 - \gamma_m \zeta)^{1/4}. \quad (2.19)$$

For stable conditions, $\zeta > 0$, a linear form is adopted:

$$\psi_m(\zeta) = -\beta_m \zeta. \quad (2.20)$$

The parameters β_m and γ_m control the strength of the correction and depend on the selected empirical constants. In the computational implementation of this thesis, the Businger–Dyer branch uses the constants reported in the code appendix. The important point for the literature review is that the scheme produces a positive correction under instability and a negative correction under stability, consistently with the sign convention of Eq. (2.18). [21, pp. 347–399], [10, Art. 012052]

The neutral case is not introduced as an additional empirical formulation. It is recovered as the limit $\zeta \rightarrow 0$. In this limit, the integrated correction vanishes, so that $\psi_m(0) = 0$, and the wind profile reduces to the neutral logarithmic formulation.

2.6.2 Jimenez et al. formulation

Jiménez et al. present the surface-layer correction through integrated functions usually denoted by $c_m(\zeta)$ and $c_h(\zeta)$. In the notation of this thesis, the momentum function is used as $\psi_m(\zeta) \equiv c_m(\zeta)$, with the same physical role as the integrated correction in Eq. (2.18). [12, pp. 898–918]

For unstable conditions, the formulation combines a Kansas-type contribution with a convective contribution. In compact form, this can be written as

$$c_{h,m}(\zeta) = \frac{c_{h,m}^K(\zeta)}{1 + \zeta^2} + \frac{\zeta^2 c_{h,m}^C(\zeta)}{1 + \zeta^2}, \quad (2.21)$$

where $c_{h,m}^K$ is the Kansas-type component and $c_{h,m}^C$ is the convective component. For the convective contribution, the integrated form can be written as

$$c_{h,m}^C(\zeta) = \frac{3}{2} \ln\left(\frac{y^2 + y + 1}{3}\right) - \sqrt{3} \arctan\left(\frac{2y + 1}{\sqrt{3}}\right) + \frac{\pi}{\sqrt{3}}, \quad y = (1 - a_{h,m}\zeta)^{1/3}. \quad (2.22)$$

The constants a_m and a_h differ for momentum and heat and are defined by the authors. For stable conditions, Jiménez et al. adopt integrated functions that tend to limit excessive growth under strong stability:

$$c_m(\zeta) = -a \ln[\zeta + (1 + \zeta^b)^{1/b}], \quad (2.23)$$

with an analogous expression for $c_h(\zeta)$ using its own empirical constants. This saturation-like behaviour is important because stable regimes often produce larger spread among surface-layer functions than unstable regimes. [12, pp. 898–918]

2.6.3 Holtslag–de Bruin formulation

The Holtslag–de Bruin formulation is mainly used for neutral and stable stratification. It should not be treated as a general unstable correction in this thesis. For stable conditions, the integrated momentum correction includes an exponential term and can be written as

$$\psi_m(\zeta) = -a\zeta - b\left(\zeta - \frac{c}{d}\right)\exp(-d\zeta) - \frac{bc}{d}, \quad \zeta > 0. \quad (2.24)$$

A commonly used set of coefficients is $a = 1$, $b = 2/3$, $c = 5$, and $d = 0.35$, which gives

$$\psi_m(\zeta) = -\zeta - \frac{2}{3}\left(\zeta - \frac{5}{0.35}\right)\exp(-0.35\zeta) - \frac{10}{1.05}. \quad (2.25)$$

This form is constructed so that the correction approaches zero near the neutral limit and becomes negative under stable conditions. Its practical role is to avoid applying an unstable correction to a scheme whose main use is associated with neutral and stable surface layers. [9],

[10, Art. 012052]

2.6.4 Brutsaert-type formulation

The Brutsaert-type correction used here modifies the stable branch through a piecewise function of ζ . For $\zeta > 0$, the integrated momentum correction is written as

$$\psi_m(\zeta) = \begin{cases} -\beta_m \zeta, & 0 < \zeta \leq 1, \\ -\beta_m \ln(\zeta) - \beta_m, & \zeta > 1. \end{cases} \quad (2.26)$$

The first branch is linear for weak and moderate stability, while the second branch reduces the growth of the correction under stronger stability. In the implementation of this thesis, the unstable Brutsaert branch is not introduced as a separate new unstable formula. Instead, the unstable side follows the same general unstable correction used for the Businger-type branch, while the Brutsaert distinction is applied to the stable regime. This choice keeps the comparison focused on how different stable corrections affect the extrapolated profile. [10, Art. 012052]

The four empirical schemes therefore differ in the way they transform ζ into ψ_m . This is why they can lead to different profiles even when u_* , z_0 , and L are held fixed. In the methodology, these formulations are combined with the LOG, Lange, and GRYN profile families and then evaluated against LiDAR observations under the same filtering and validation procedure.

2.7 Indirect estimation of the Monin–Obukhov length using Ri_g and Ri_B

The previous section showed that MOST-based wind-profile corrections require the stability parameter $\zeta = z/L$. When turbulent fluxes are available, L can be obtained directly from its definition. However, direct turbulent-flux measurements are often unavailable in offshore wind applications. For this reason, indirect methods are frequently used to infer an equivalent stability scale from mean wind and temperature observations. [10, 15]

A key point is that Richardson numbers are not definitions of the Monin–Obukhov length. They are stability diagnostics based on the competition between thermal stratification and vertical wind shear. They can indicate whether buoyancy tends to suppress or enhance vertical mixing, but they do not provide L directly. An estimate of L can only be obtained if an additional empirical or theoretical relationship is assumed between a Richardson number and the Monin–Obukhov stability parameter $\zeta = z/L$.

The gradient Richardson number is a local stability diagnostic. It compares the local vertical

gradient of virtual potential temperature with the local vertical shear of the horizontal wind:

$$Ri_g = \frac{\frac{g}{\theta_v} \frac{\partial \overline{\theta_v}}{\partial z}}{\left(\frac{\partial \overline{U}}{\partial z}\right)^2 + \left(\frac{\partial \overline{V}}{\partial z}\right)^2}. \quad (2.27)$$

Here, Ri_g measures the local balance between thermal stratification and wind shear. Positive values generally indicate stable stratification, while negative values indicate unstable stratification. However, Ri_g remains a Richardson-number diagnostic. It becomes an indirect estimate of L only after it is related to $\zeta = z/L$. [21, pp. 175–179]

In practice, local gradients are rarely available. A common alternative is to approximate them with finite differences between two measurement levels. This gives the bulk Richardson number:

$$Ri_B = \frac{g}{\theta_v} \frac{\Delta \theta_v \Delta z}{(\Delta U)^2 + (\Delta V)^2}, \quad (2.28)$$

where $\Delta \theta_v$ is the virtual-potential-temperature difference between two heights, Δz is their vertical separation, and ΔU and ΔV are the differences in the two horizontal wind components. If only wind-speed magnitude is available, the denominator is sometimes approximated by $(\Delta |\mathbf{U}|)^2$, although this neglects directional shear. Unlike Ri_g , Ri_B describes the mean stability of a finite layer, so its value depends on the selected height interval. [21, pp. 175–179]

Several wind-energy studies use empirical relations to convert Richardson-based quantities into $\zeta = z/L$. For example, Holtslag et al. [10] describe Richardson-based methods in which the Richardson number is first mapped to z/L . For the gradient-Richardson method, the relation is written as

$$\left(\frac{z}{L}\right)_{\text{RI-Grad}} = \begin{cases} \frac{Ri}{1 - 5Ri}, & Ri \geq 0, \\ Ri, & Ri < 0. \end{cases} \quad (2.29)$$

For the bulk-Richardson method, the corresponding relation is

$$\left(\frac{z}{L}\right)_{\text{RI-Bulk}} = \begin{cases} \frac{10Ri}{1 - 5Ri}, & Ri \geq 0, \\ 10Ri, & Ri < 0. \end{cases} \quad (2.30)$$

After $\zeta = z/L$ has been estimated, the associated Monin–Obukhov length can be inferred as

$$L_{\text{Ri}} = \frac{z_r}{\zeta_{\text{Ri}}}, \quad (2.31)$$

where z_r is the reference height associated with the empirical relationship and ζ_{Ri} is the

Richardson-derived stability parameter.

2.8 Practical limits of MOST and alternative reformulations

A second limitation concerns height. MOST was developed for the atmospheric surface layer, where turbulent fluxes of momentum and heat can be treated as approximately constant with height. Modern offshore wind turbines operate across a vertical range that can extend beyond this strict surface-layer domain. As a result, a formulation that performs well near the surface may become less reliable at higher levels. This motivates alternative or extended formulations that introduce boundary-layer information into the wind profile. Examples include corrections based on the boundary-layer height and the profile extension proposed by Gryning et al. These approaches aim to preserve the physical basis of MOST while accounting for the fact that the rotor-swept layer is not always contained within the lowest part of the atmospheric boundary layer. [8, pp. 251–268]

Despite being widely used, the classical MOST formulation expressed in Eq. (2.18) was conceived for the atmospheric surface layer. In this domain, turbulent fluxes of momentum and heat are assumed to be approximately constant with height, which supports the direct link between turbulent scales and the mean wind gradient. Observational evidence and applied wind-energy studies indicate, however, that the strict validity of this framework tends to weaken when the extrapolation reaches heights far above the lower surface layer. This limitation is especially relevant for modern offshore turbines, because the rotor-swept layer can extend into heights where additional boundary-layer scales influence the wind profile. [16, pp. 163–187], [8, pp. 251–268]

Offshore, the same limitation is combined with marine-specific processes. The vertical profile can reflect air-mass changes, coastal transition, wave state, and air–sea thermal contrast. These processes can modify stability and vertical structure along the fetch, which reinforces the need to treat L and ψ_m with caution in real applications. Even when MOST remains an operational reference, a single scale z/L may not be sufficient to describe the profile shape above the surface layer. [15, pp. 587–617], [6, pp. 2311–2328]

The main difficulty appears under stable conditions. Stable stratification suppresses vertical motion and can make turbulence weak, intermittent, and sensitive to non-stationarity. In this regime, different flux–profile relationships and different stability functions often diverge more strongly than under unstable conditions. The same body of evidence also reports the relevance of low-level jets, gravity waves, and shallow turbulent layers, all of which can violate the assumptions behind a simple surface-layer profile. [22, pp. 83–88], [3, Art. 1197]

These limitations do not invalidate MOST. They define its domain of interpretation. The classical stability-corrected logarithmic profile remains physically transparent because it shows where the main uncertainties enter the problem: u_* , z_0 , L , and the selected ψ_m . At the

same time, the vertical range of offshore turbines motivates reformulations that introduce additional boundary-layer information. Two lines are considered in this thesis. The first is the Lange correction, which preserves the MOST logarithmic structure but adds a term based on the boundary-layer height z_i . The second is the Gryning-type extension, which introduces a composite boundary-layer mixing length and separates surface-layer, middle-boundary-layer, and upper-boundary-layer scales. The order adopted below follows this logic: the simpler Lange correction is introduced first, and the more complete Gryning extension is discussed afterward. [14, pp. 1–15], [8, pp. 251–268]

2.8.1 Lange et al.: boundary-layer height correction

The previous section explained why the classical MOST profile may become insufficient above the surface layer. This limitation is especially important offshore, where hub heights can extend well beyond the range where constant-flux assumptions are most reliable. Lange et al. proposed a practical correction to account for this limitation in offshore wind-speed extrapolation. The correction preserves the MOST logarithmic structure, but adds an additional shear contribution related to the boundary-layer height. In this thesis, this formulation is referred to as the Lange correction. [14, pp. 1–15], [15, pp. 587–617]

The physical motivation is direct. MOST represents stability effects through the parameter z/L . This parameter describes the balance between mechanical shear and buoyancy near the surface. However, it does not explicitly describe the finite vertical extent of turbulent mixing. When the target height becomes a significant fraction of the atmospheric boundary layer, the distance to the upper part of the mixed region can influence the wind profile. For this reason, an additional vertical scale is introduced through the boundary-layer height z_i . This scale represents the approximate depth of the layer where the flow remains coupled to the surface by turbulent transport. In the numerical framework of this thesis, z_i is not measured directly. It is introduced as a diagnostic boundary-layer scale, estimated from the friction velocity and the Coriolis parameter as

$$z_i = \max\left(0.1 \frac{u_*}{|f|}, z_{i,\min}\right), \quad (2.32)$$

where $f = 2\Omega \sin \varphi$ is the Coriolis parameter and $z_{i,\min}$ is a small lower bound used to avoid degenerate numerical cases. This expression should not be read as a universal definition of boundary-layer height. It is an operational estimate used to introduce the missing vertical scale required by the Lange formulation. The implementation details are presented later in Sec. 3.7.2.

The practical form used in offshore applications can be written as

$$U(z) = \frac{u_*}{\kappa} \left[\ln\left(\frac{z}{z_0}\right) - \psi_m\left(\frac{z}{L}\right) + c \frac{z}{z_i} \right], \quad (2.33)$$

where u_* is the friction velocity, κ is the von Kármán constant, z_0 is the roughness length, L is the Monin–Obukhov length, and ψ_m is the integrated stability correction. The term cz/z_i is the additional boundary-layer contribution. The parameter c is empirical. It controls how strongly the finite depth of the boundary layer modifies the shear. In the implementation of this thesis, this coefficient is taken as $c = 4.0$, following the operational formulation adopted in the profile comparison.

Equation (2.33) should be interpreted carefully. The first two terms are the standard MOST profile. They describe the logarithmic shear and the stability correction. The last term is not another empirical stability function. It is an additional correction that introduces information about the finite depth of mixing. Thus, the Lange formulation does not replace ψ_m . It adds a boundary-layer contribution on top of the MOST-corrected logarithmic profile.

When the wind speed is known at a reference height z_{ref} , the expression is more consistently written in anchored form:

$$U(z) = U(z_{\text{ref}}) + \frac{u_*}{\kappa} \left[\ln\left(\frac{z}{z_{\text{ref}}}\right) - \psi_m\left(\frac{z}{L}\right) + \psi_m\left(\frac{z_{\text{ref}}}{L}\right) + c \frac{z - z_{\text{ref}}}{z_i} \right]. \quad (2.34)$$

This form is useful for applied extrapolation. It guarantees that the profile passes through the measured wind speed at z_{ref} . It also avoids interpreting the near-surface reference value as a model prediction. In this thesis, this point is important because the profile is anchored at the low measurement height, while the validation is performed aloft.

The role of z_i is central in this formulation. If z_i is large, the ratio z/z_i is small and the additional correction remains weak. In this case, the profile approaches the classical MOST-corrected logarithmic form. If z_i is small, the same target height becomes closer to the top of the turbulent layer. The correction then becomes stronger and the predicted vertical shear can increase. This behaviour is physically reasonable, because a shallower boundary layer limits vertical momentum exchange. However, it also makes the method sensitive to the estimate of z_i .

This sensitivity is one of the main limitations of the Lange correction. The boundary-layer height is rarely measured directly in offshore resource studies. It is often estimated from parameterizations, reanalysis products, or simplified scaling relations. Errors in z_i can therefore propagate directly into the extrapolated wind speed. The problem becomes more important under stable conditions, when the boundary layer may become shallow and turbulence can be weak or intermittent. Under low wind speeds, small values of u_* can also produce small estimates of z_i in diagnostic formulations. In that case, the correction term can become large and may overcorrect the profile. [14, pp. 1–15]

The Lange correction is therefore best understood as a practical extension of MOST. It is useful because it introduces a missing physical scale. It remains simple enough for offshore engineering applications. At the same time, it depends strongly on how z_i is estimated. For

this reason, it should not be treated as universally superior to the classical MOST profile. Its value must be tested against observations, especially at heights between 100 and 200 m.

This interpretation defines its role in the present thesis. The Lange formulation is not used as a replacement for the LOG formulation. It is compared against LOG and against the Gryning-type extension under the same input data, stability classification, and validation metrics. This comparison allows the results to show when the additional boundary-layer height term improves the extrapolation, and when it introduces excessive shear.

2.8.2 Gryning et al. (2007): introducing boundary-layer scales into the wind profile

The extension proposed by Gryning et al. [8] starts from a simple idea. The wind profile above the surface layer becomes dependent on additional scales. The reason is physical and also mathematical. In the surface layer, MOST describes the mean gradient based on surface scales. This framework becomes less informative when the height of interest grows. In particular, the turbulence that transports momentum has a finite depth. This depth begins to influence the shear when z is not small. [8, pp. 254–256]

The new term that anchors this change is z_i . Mathematically, z_i enters as a maximum vertical scale of the problem. Physically, z_i represents a characteristic height of the atmospheric boundary layer. It delimits the region where the flow remains strongly coupled to the surface. This coupling occurs via turbulent transport of momentum and heat. When stratification is more stable, vertical mixing tends to weaken. In that case, the effective turbulent layer can become shallower. When stratification is less stable, mixing can occupy a deeper layer. Thus, z_i can vary in time and with the thermal regime. This variation changes which heights are close to the relative top of mixing. As a consequence, the mean wind gradient can change with stability and height. [21, pp. 347–399], [22, pp. 83–88]

The formulation of Gryning et al. incorporates z_i without abandoning an analytical profile. The starting point is a first-order closure for the mean gradient. The gradient is written as a function of a local mixing length l . [8, p. 253]

$$\frac{du}{dz} = \frac{u_*}{\kappa l}. \quad (2.35)$$

In this expression, u_* is the friction velocity, and κ is the von Kármán constant. The term l acts as the scale that controls the local efficiency of turbulent mixing. The next step is to recognize that momentum transport does not need to be constant with height. By introducing a characteristic top z_i , the authors allow u_* to decrease with z . [8, p. 253]

$$u_*(z) = u_{*,0} \left(1 - \frac{z}{z_i} \right)^\alpha. \quad (2.36)$$

Here, $u_{*,0}$ represents the friction velocity near the surface. The exponent α controls the shape of the vertical decay. At this step, z_i appears as a geometric condition of turbulent flow. It transforms the formulation from a purely local law into a law with boundary-layer memory.

The second change is the definition of l as a composite effective scale. Instead of assuming a single length proportional to z throughout the column, the model separates regions. The surface layer has its own scale, the middle part has another, and the vicinity of the top has another. The combination is done by inverse summation, so that the smallest scale dominates locally. This avoids piecewise transitions and produces a smooth variation with height. [8, p. 254]

$$\frac{1}{l} = \frac{1}{L_{SL}} + \frac{1}{L_{MBL}} + \frac{1}{L_{UBL}}. \quad (2.37)$$

Before including stability, the authors define the scales under neutral conditions. In the neutral surface layer, the scale grows linearly with height.

$$L_{SL,N} = z. \quad (2.38)$$

In the middle part of the boundary layer, the scale stops being proportional to z and tends toward an almost constant value. This parameter is denoted $L_{MBL,N}$ and must be provided by parametrization or by observational analysis. [8, p. 254] In the upper region, the scale is linked to the distance to the top and decreases as z approaches z_i . [8, p. 254]

$$L_{UBL,N} = (z_i - z). \quad (2.39)$$

With these definitions, the inverse sum makes $L_{SL,N}$ dominate near the surface. It also makes the term associated with $L_{UBL,N}$ grow in importance as z approaches z_i . This is the mathematical mechanism that translates the effect of finite depth into the gradient. [8, pp. 254–255]

Stability enters as a correction applied to the scales, not as a change in the profile structure. The idea is to multiply each neutral scale by an appropriate stability factor. For applied wind-profile extrapolation, this thesis uses the resulting middle-boundary-layer length scale, written as

$$L_{MBL} = \frac{u_* / |f|}{\left[-2 \ln \left(\frac{u_*}{|f| z_0} \right) + 55 \right] \exp \left[-\frac{1}{400} \left(\frac{u_*}{|f| L} \right)^2 \right]}. \quad (2.40)$$

This expression links the mixed-layer scale to friction velocity, the Coriolis parameter, roughness length, and atmospheric stability. It is not an independent measurement of boundary-layer depth. It is an applied parameterization used to introduce the missing scale between the surface layer and the upper boundary layer. [8, pp. 254–260]

Using this effective scale, the practical Gryning-type shape adopted in the profile comparison

can be written as

$$A_{\text{GRYN}}(z) = \ln\left(\frac{z}{z_0}\right) + 5\frac{z}{L_{\text{MBL}}}. \quad (2.41)$$

This expression keeps the logarithmic dependence on roughness, but replaces the direct empirical stability correction by a boundary-layer length-scale contribution. For this reason, the Gryning-type formulation is treated in this thesis as a reformulation of the profile shape, not simply as another empirical function ψ_m . [8, pp. 251–268]

3 Methodology

3.1 Methodological problem and general strategy

In offshore resource assessment, wind speed is often available near the sea surface, while the required information is located within the rotor-swept layer. This limitation is more critical for modern turbines, whose relevant heights can extend from approximately 30 to 200 m. The first operational target of the methodology is therefore to estimate U_{hub} , which can be used as a first approximation for hub-height resource and power calculations. At the same time, the algorithm stores the complete set of extrapolated velocities at all evaluated heights, because U_{hub} alone does not describe the full shear structure across the rotor-swept layer. [10, Art. 012052]

The literature review established the physical basis of this problem. It was discussed the neutral logarithmic profile, the role of u_* and z_0 , the Monin–Obukhov stability parameter, and the main limitations of MOST above the surface layer. For this reason, the present chapter does not repeat the theoretical derivations. It explains how those formulations are used, combined, filtered, and validated in this thesis.

The methodology follows a controlled comparison strategy. The same input data, filtering rules, convergence criteria, and validation metrics are applied to all model combinations. This allows the results to isolate the effect of the selected stability function and profile family. The objective is not to prove that one formulation is universally best. Instead, the objective is to identify which formulation performs better under specific heights, stability regimes, and hours of the day.

The workflow is organized in five steps. First, near-surface wind measurements and open meteorological data are aligned in time. Second, the missing surface-layer parameters are estimated through an iterative physical closure. Third, the converged parameters are used to generate vertical profiles with three profile families. Fourth, the extrapolated speeds are compared with LiDAR observations at several heights. Fifth, the error statistics are transformed into conditional performance tables and rule-based selection methods. In addition to this main workflow, a height-matched CERRA2 benchmark is constructed to compare the LiDAR-anchored MOST-based estimates with an open reanalysis-based wind-speed product. This benchmark is not part of the MOST physical closure; it is used only as an external reference for the final model-performance comparison.

3.2 Data sources, input variables, and availability criterion

The methodology intentionally represents a reduced-input scenario. This choice reflects many preliminary offshore studies, where complete turbulent-flux measurements are not available. For this reason, the workflow combines one low-level offshore wind observation with open meteorological variables obtained from reanalysis data.

The main observational reference used in this thesis is the IJmuiden Ver metocean campaign, whose public data portal is available through the RVO offshore wind database.¹ The campaign included two floating LiDAR buoy sites, IJV A1 and IJV B2. However, only the measurements from IJV B2 are used as the observational reference in this thesis. This choice was made to maintain a single, internally consistent validation dataset and to avoid combining two nearby but distinct buoy records in the model-performance evaluation. In addition, IJV B presented slightly higher data availability over the 24-month campaign, both in terms of system availability and post-QC data availability. Therefore, IJV B2 was selected as the primary reference time series, while IJV A1 is mentioned only to describe the original measurement campaign and is not included in the model-performance calculations. [19]

The main-anchor coordinates of the IJV B2 buoy used in this thesis are reported in Table 1.

Table 1: Coordinates of the IJV B2 LiDAR buoy

Site	Latitude	Longitude	Nominal water depth
IJV B2	52°53'07'' N	3°41'01'' E	26.0 m LAT

The IJV B2 buoy provides the near-surface wind speed used as the lower boundary condition of the extrapolated profile. In this thesis, the wind speed measured at 3.7 m above mean sea level is denoted as U_{ref} . The higher LiDAR wind-speed measurements are not used as model inputs, but only as independent validation levels.

The evaluated heights are

$$\mathcal{Z} = \{3.7, 30, 41, 70, 100, 120, 140, 160, 180, 200\} \text{ m.} \quad (3.1)$$

The 3.7 m level is therefore an input level, not an independent validation level. The remaining heights are used to evaluate the vertical extrapolation.

The open meteorological variables were obtained from the ERA5 post-processed daily statistics on single levels from 1940 to present, made available through the Copernicus Climate Change Service Climate Data Store. [5] ERA5 products are commonly used to represent coupled air-sea variables such as sea-surface temperature and surface heat fluxes, including in recent studies focused on mesoscale sea-surface temperature and latent heat-flux coupling. [11] In

¹<https://offshorewind.rvo.nl/page/view/5c06ac88-c12f-4903-89f3-27d66937b7e9/general-information-ijmuiden-ver->

this thesis, the ERA5 data were extracted for a spatial domain covering the IJV B2 measurement area and later reduced to the buoy point of interest. The detailed temporal harmonization, spatial extraction, and filtering steps are described in Section 3.4.

The 2 m air temperature and sea-surface temperature are used to define an operational temperature proxy,

$$T_{\text{proxy}} = \frac{T_{2m} + SST}{2}. \quad (3.2)$$

This proxy is used in the thermodynamic preprocessing step to compute the potential temperature, following the definition introduced in the literature review. The purpose of using T_{proxy} is to represent an intermediate temperature scale between the near-surface atmosphere and the sea surface, without introducing an additional measured temperature level.

The main input variables and their sources are summarized in Table 2.

Table 2: Main input variables used in the extrapolation workflow and their data sources.

Variable	Source	Role in the methodology
U_{ref} at 3.7 m	IJV B2 buoy	Lower boundary condition for the extrapolated wind-speed profile.
LiDAR wind speeds at 30–200 m	IJV B2 buoy	Validation of the extrapolated wind speeds at turbine-relevant heights.
Atmospheric pressure	IJV B2 buoy meteorological sensor	Thermodynamic input used in the physical closure.
2 m air temperature T_{2m}	ERA5 / CDS	Input for the temperature proxy and potential-temperature calculation.
Sea-surface temperature SST	ERA5 / CDS	Input for the temperature proxy and representation of the air–sea thermal contrast.
Instantaneous surface sensible heat flux Q_H	ERA5 / CDS	ERA5-derived heat-flux input used for the estimation of the Monin–Obukhov length L .

CERRA2 wind-speed data are not used as input to the MOST-based extrapolation workflow. They are introduced only as an external reanalysis benchmark for comparison with the LiDAR observations and with the MOST-based estimates. The construction of this height-matched CERRA2 benchmark is described separately in Section 3.9.

This data structure affects the interpretation of the results. The friction velocity u_* , the aerodynamic roughness length z_0 , and the Monin–Obukhov length L are not directly measured. Instead, they are estimated through the iterative physical closure described in Section 3.5. Thus, the thesis evaluates an accessible, reproducible extrapolation workflow rather than the maximum predictive performance achievable with complete offshore flux, wave, and turbulence measurements. The ERA5-derived information used in this thesis contains modified Copernicus Climate Change Service information. Neither the European Commission nor ECMWF is responsible for any use that may be made of the Copernicus information or data it contains.

3.3 Model combinations evaluated

The comparison combines two modelling choices. The first choice is the empirical formulation used for the integrated stability correction ψ_m . The second choice is the vertical profile formulation used after the physical closure has converged.

Four stability schemes are evaluated:

$$\mathcal{S} = \{\text{Businger}, \text{Jimenez}, \text{Holtslag}, \text{Brutsaert}\}. \quad (3.3)$$

Three profile formulations are evaluated:

$$\mathcal{F} = \{\text{LOG}, \text{Lange}, \text{GRYN}\}. \quad (3.4)$$

The symbols in $\mathcal{F}$ are reported labels used in the thesis tables and discussion. LOG denotes the MOST profile with integrated stability correction. Lange denotes the boundary-layer-height correction proposed by Lange et al., where a boundary-layer-height contribution is added to the MOST-corrected profile. GRYN denotes the Gryning-type extension based on an effective mixed-layer length scale [8].

Each evaluated model is defined as

$$m = (s, f), \quad s \in \mathcal{S}, \quad f \in \mathcal{F}. \quad (3.5)$$

The complete set contains up to twelve combinations. The Holtslag formulation is not treated as a general unstable correction, because its main use is associated with neutral and stable conditions [9, 10]. When a combination produces a non-physical or non-finite result, that sample is stored as missing.

3.4 Pre-processing and temporal alignment

The LiDAR observations and meteorological inputs are first temporarily aligned. The LiDAR wind measurements are provided at 10-minute resolution. They are aggregated to hourly means to match the temporal resolution of the meteorological forcing. After aggregation, all variables are aligned by timestamp.

Only the temporal intersection between the datasets is used. No artificial gap filling is applied during model evaluation. For each timestamp t , the input vector is

$$\mathbf{x}(t) = [U_{\text{ref}}(t), \theta(t), Q_H(t), P(t), T_{2m}(t), \text{SST}(t)]. \quad (3.6)$$

Before the iterative closure, each sample is screened through basic physical filters. The

reference wind speed must be finite and greater than 0.1 m s^{-1} . The potential temperature, pressure, and sensible heat flux must also be finite. Cases with $Q_H = 0$ are excluded because they create a non-convergent update for L . These filters remove undefined states before the profile calculation.

3.5 Iterative closure of u_* , z_0 , and L

The core of the methodology is the estimation of u_* , z_0 , and L . These variables are not independent. The wind-profile equation requires all three quantities. The roughness length depends on u_* through the Charnock relation. The Monin–Obukhov length depends on u_*^3 and on the sensible heat flux.

For each timestamp and stability scheme, the loop starts from prescribed initial values. The initial roughness is

$$z_0^{(0)} = 2 \times 10^{-4} \text{ m}. \quad (3.7)$$

This value is used only as the first guess. The first value of L is assigned with the sign implied by Q_H . This helps the first evaluation of ψ_m occur on the appropriate stable or unstable branch.

At iteration n , the reference stability parameter is

$$\zeta_{\text{ref}}^{(n)} = \frac{z_{\text{ref}}}{L^{(n)}}. \quad (3.8)$$

The friction velocity is updated by isolating the stability-corrected profile at the reference height:

$$u_*^{(n+1)} = \frac{\kappa U_{\text{ref}}}{\ln(z_{\text{ref}}/z_0^{(n)}) - \psi_{m,s}(\zeta_{\text{ref}}^{(n)})}. \quad (3.9)$$

This step forces the profile closure to remain tied to the measured near-surface wind speed.

The roughness length is updated with the Charnock relation already introduced in the literature review:

$$z_0^{(n+1)} = \max\left(\alpha_{\text{Ch}} \frac{(u_*^{(n+1)})^2}{g}, z_{0,\text{min}}\right), \quad (3.10)$$

where $\alpha_{\text{Ch}} = 0.0144$ and $z_{0,\text{min}} = 10^{-6} \text{ m}$. The lower bound avoids zero roughness and undefined logarithms.

The Monin–Obukhov length is then updated with the heat-flux approximation used in this thesis:

$$L^{(n+1)} = -\frac{\rho c_p \theta |u_*^{(n+1)}|^3}{\kappa g Q_H}. \quad (3.11)$$

This expression is the operational form used in the computation. The theoretical definition of L was already presented in Section 2.5.

Convergence is defined as

$$|L^{(n+1)} - L^{(n)}| < \varepsilon_L, \quad \varepsilon_L = 0.01 \text{ m}. \quad (3.12)$$

The maximum number of iterations is 1000. If the loop does not converge, the sample is rejected for that model combination. The same rejection is applied when u_* , z_0 , L , or the denominator of Eq. (3.9) becomes non-finite or non-physical.

A good initial estimate of L is important because the correction function depends on z/L . If the first guess has the wrong sign, the algorithm may evaluate the wrong stability branch. If the first guess is too close to zero, the correction may become unrealistically strong. For this reason, the initial value is chosen with a moderate magnitude and with the sign implied by the sensible heat flux.

3.6 Atmospheric stability classification

Atmospheric stability is classified only after the iterative loop has converged. This order avoids using a prescribed class to force the solution. The final stability parameter is

$$\zeta_{\text{ref}} = \frac{z_{\text{ref}}}{L}. \quad (3.13)$$

The operational classes used in the calculations are

$$\text{stab}(t) = \begin{cases} \text{unstable}, & \zeta_{\text{ref}} < -0.1, \\ \text{neutral}, & -0.1 \leq \zeta_{\text{ref}} \leq 0.1, \\ \text{stable}, & \zeta_{\text{ref}} > 0.1, \\ \text{undefined}, & L \text{ not finite.} \end{cases} \quad (3.14)$$

This threshold is an operational choice of the numerical analysis. It should not be read as a universal theoretical boundary. The general meaning of ζ and the narrower near-neutral convention are discussed in Section 2.5.

The class is diagnosed separately for each empirical scheme. Therefore, two schemes can classify the same timestamp differently. This is acceptable within the present framework, because each scheme produces its own converged u_* , z_0 , and L .

3.7 Generation of vertical wind profiles

After convergence, the final values of u_* , z_0 , and L are used to compute the vertical wind-speed profiles $U(z)$. The theoretical expressions for the three profile formulations are given in the

literature review. Therefore, this section only explains how they are used in the algorithm.

For each empirical stability correction in $\mathcal{S}$, the algorithm first estimates a consistent set of u_* , z_0 , and L . These values are then used to generate the three profile formulations defined in $\mathcal{F}$: LOG, Lange, and GRYN. The wind-speed profile is computed at all heights in $\mathcal{Z}$, with the reference level treated as a boundary condition:

$$U(z_{\text{ref}}, t) = U_{\text{ref}}(t). \quad (3.15)$$

This means that the model output at $z_{\text{ref}} = 3.7$ m is not independently predicted. Instead, it is fixed equal to the measured reference wind speed. The vertical extrapolation is therefore evaluated only at the remaining LiDAR heights, where the profile formulations actually differ.

3.7.1 LOG formulation

The LOG formulation follows the stability-corrected logarithmic MOST profile introduced in Eq. (2.18). In the algorithm, this formulation is evaluated for each empirical stability correction using its corresponding integrated function ψ_m . It represents the classical MOST-based profile and does not include any explicit boundary-layer-height contribution. For this reason, the LOG formulation is used as the reference MOST formulation against which the Lange and Gryning reformulations are compared.

3.7.2 Lange correction

The Lange correction follows the profile introduced in Eq. (2.33) and uses the diagnostic boundary-layer scale defined in Eq. (2.32). In the output files, this formulation is stored with the short label `zi`. This label is used only for traceability in the computed columns and does not indicate a separate empirical stability correction. It identifies the Lange reformulation, in which the MOST-corrected logarithmic profile receives an additional boundary-layer-height contribution through z_i . In the methodology, this correction is applied after the convergence of u_* , z_0 , and L .

3.7.3 Gryning formulation

The Gryning-type formulation follows the mixed-layer scale and profile shape introduced in Eq. (2.40) and Eq. (2.41). In the algorithm, the effective mixed-layer length scale L_{MBL} is evaluated only when $u_* > 0$, $z_0 > 0$, the Coriolis parameter is non-zero, and L is finite. If these conditions are not satisfied, the corresponding Gryning output is stored as missing. In this formulation, stability affects the final wind-speed profile through L_{MBL} . For this reason, the Gryning formulation is treated as a profile reformulation rather than only another empirical ψ_m correction.

3.8 Output structure and framework for result analysis

After the vertical wind profiles are generated, the methodology moves from profile construction to result evaluation. At this stage, two requirements must be satisfied. First, the calculated profiles must remain consistent with the reference wind speed used as input. Second, the outputs must be organized in a way that allows each model combination to be validated, compared, and later used in a rule-based selection strategy.

The reference-height anchoring defined in Eq. (3.15) is applied to all profile families. This step ensures that every calculated profile starts from the same measured wind speed at z_{ref} . As a result, the comparison does not include artificial differences at the input height. The validation therefore focuses only on the vertical extrapolation above the measured level, where the model formulations actually differ.

For each model combination, the output file stores both the converged physical variables and the corresponding wind-speed estimates. The stored diagnostics include u_* , z_0 , L , z_i , L_{MBL} , the stability class, and $U(z)$ at each evaluated height. The column names identify the stability scheme, the profile family, and the height. This structure keeps the workflow traceable from the input data to the final validation metrics.

The validation then compares each estimated wind speed with the LiDAR wind speed at the same timestamp and height. Only timestamps available in both datasets are used. This avoids temporal interpolation and ensures that each comparison refers to the same atmospheric condition. The error is computed as the difference between the modelled wind speed and the LiDAR observation:

$$e_m(t, z) = U_m(t, z) - U_{\text{LiDAR}}(t, z). \quad (3.16)$$

With this convention, positive errors indicate overestimation, while negative errors indicate underestimation. This sign convention is kept fixed throughout the analysis, so that bias and error distributions can be interpreted consistently.

Before computing the metrics, the same filtering procedure is applied to all model combinations. Only finite paired samples are retained. In addition, all modelled or observed wind-speed values above 40 m s^{-1} are discarded. These values are treated as extreme or physically unreliable for the present validation, since they can be associated with measurement artefacts, numerical instabilities, or non-representative events for the comparison framework. A quantile-based filter is then applied to reduce the influence of very long distribution tails. This prevents isolated numerical artefacts from dominating the comparison, while preserving the central behaviour of each model.

After this filtering step, the number of valid samples is denoted by N . Thus, N represents the number of usable model–LiDAR pairs available for a given model combination, height, stability regime, or hour of day. If N differs between heights, empirical correction functions, or

MOST reformulations, this difference is a direct consequence of the filtering procedure and of the availability of valid paired data in each subset. It does not indicate that a different statistical method was applied to each model. For this reason, N is always reported together with the error metrics, so that the robustness of each comparison can be interpreted correctly.

The validation uses RMSE and Bias as the main metrics, with MAE retained as an auxiliary measure of typical absolute error. Bias is used to identify systematic deviation. It is defined as the mean signed error:

$$\text{Bias}_{m,z} = \frac{1}{N_{m,z}} \sum_{t \in \Omega_{m,z}} e_m(t, z), \quad (3.17)$$

where $\Omega_{m,z}$ is the filtered set of valid samples for model combination m and height z . A positive Bias means that the model tends to overestimate the LiDAR wind speed. A negative Bias means that the model tends to underestimate it.

RMSE is used to measure the overall magnitude of the error:

$$\text{RMSE}_{m,z} = \sqrt{\frac{1}{N_{m,z}} \sum_{t \in \Omega_{m,z}} e_m(t, z)^2}. \quad (3.18)$$

This metric gives stronger weight to large deviations and remains expressed in m s^{-1} . Therefore, it can be interpreted directly as a wind-speed error.

MAE is also computed as

$$\text{MAE}_{m,z} = \frac{1}{N_{m,z}} \sum_{t \in \Omega_{m,z}} |e_m(t, z)|. \quad (3.19)$$

Thus, Bias describes the direction of the systematic error, RMSE highlights the magnitude of the error with stronger sensitivity to large deviations, and MAE represents a more direct estimate of the typical absolute error.

For model selection, the main metric is RMSE. The selected model is the one that produces the lowest RMSE within the analysed context. Bias is not used as the primary ranking variable, because positive and negative errors can compensate in the mean. However, it is essential for interpretation, because two combinations with similar RMSE may have different systematic tendencies. For this reason, the results always report Bias together with RMSE whenever a best combination is selected. The number of valid samples N is also retained to avoid giving the same confidence to well-sampled and poorly sampled bins.

The same metrics are computed for all model combinations and all evaluated heights. This keeps the comparison symmetric. When a model does not produce a valid result for a given timestamp, that timestamp is excluded from the corresponding metric. For this reason, sparse bins are interpreted with caution, especially when the selection is conditioned simultaneously

by height, stability, and hour of day.

After the error metrics are computed, the results are organized at different aggregation levels. The purpose is to determine whether model performance is constant or context-dependent. This step is necessary because a model that performs well on average may not be the best option at every height, stability regime, or hour of the day.

The first aggregation level evaluates the global performance of each combination. This produces a general ranking across the complete dataset. The second level separates the results by height, showing how extrapolation error evolves as the target level moves farther from the reference measurement. The third level separates the results by atmospheric stability, allowing stable, neutral, and unstable regimes to be analysed independently. The fourth level separates the results by hour of day, which captures possible diurnal patterns in model performance.

The analysis also considers combined contexts. For example, performance can be evaluated by height and stability together, or by height, stability, and hour simultaneously. This structure is useful because the best model may depend on more than one factor. A formulation may work well at low heights under neutral conditions, but not at high heights under stable conditions. The conditional organization therefore transforms the validation from a single global ranking into a context-dependent assessment.

The stability class used in each grouping is the class diagnosed by the same empirical scheme used in that model combination. This avoids mixing stability labels produced from different estimates of L . It also keeps the comparison internally consistent, since each formulation carries its own stability diagnosis and profile correction.

A direct comparison by stability regime can be misleading if all regimes are treated as equally frequent. In reality, stable, neutral, and unstable conditions do not occur with the same frequency. A model may perform very well in a rare regime, but this does not necessarily imply strong performance over the full time series. For this reason, the thesis also evaluates conditional performance together with the observed frequency of stability conditions.

This frequency information gives context to the RMSE and Bias values. A regime that appears frequently affects the practical relevance of a model more strongly than a rare regime. A rare regime, on the other hand, can still be important if it produces large errors or strong systematic Bias. For this reason, the interpretation combines three pieces of information: RMSE, Bias, and sample support. This produces a more realistic assessment than an isolated regime ranking, because it connects model error with how often the corresponding atmospheric condition occurs.

The final step converts the conditional performance tables into rule-based model selection policies. The purpose of these policies is not to create a new wind-profile model. Their purpose is to choose, for each situation, the model combination that performed best during validation. Each policy is defined by the variables used to describe the decision context. A simple policy may select one fixed model for the entire dataset. A more detailed policy may choose a

model depending on height. Other policies may also include stability regime, hour of day, or combinations of these variables. For each context, the selected model is the one with the lowest RMSE, while Bias and N are retained to assess systematic deviation and robustness.

This approach converts the validation results into an operational lookup table. For example, if the target height, hour, and stability class are known, the policy identifies which empirical stability function and profile family should be used. This is useful because the results show that no single formulation is expected to dominate under all conditions. However, more detailed policies also require more caution. As the number of decision groups increases, the number of samples in each group can decrease. This may produce unstable choices in rare contexts. For this reason, the policy analysis must consider RMSE, Bias, and coverage together. A highly detailed policy may reduce RMSE in the validation dataset, but it may also depend on sparse bins. A simpler policy may produce slightly larger errors, but may be more robust and easier to apply.

In this thesis, the rule-based policies are interpreted as decision-support tools. They summarize the conditional behaviour observed in the IJmuiden Ver dataset. They do not prove that the same lookup table is universally valid for every offshore site. Instead, they show how model selection can become more adaptive when sufficient validation data are available.

The same validation logic is also used to evaluate the external CERRA2 benchmark described in Section 3.9. In that case, the benchmark is not used for rule-based model selection, but only as an independent reanalysis-based comparison against the LiDAR observations and the MOST-based estimates.

3.9 Construction of the CERRA2 height-matched benchmark

An additional benchmark was constructed using CERRA2 wind-speed data. The CERRA2 data were obtained from the Copernicus Climate Change Service Climate Data Store [20]. The purpose of this benchmark is to compare the LiDAR-anchored MOST-based extrapolation workflow with an open reanalysis product commonly used when local measurements are limited. CERRA2 is not treated as an observational reference. Instead, it is used as an independent gridded reanalysis dataset to be evaluated against the LiDAR measurements.

The use of a reanalysis benchmark follows the practical motivation discussed by Gandoin and Garza [7]. Their work highlights that open atmospheric datasets are valuable in offshore applications because they provide spatially complete and accessible wind fields. However, such products should still be validated against high-quality offshore measurements before being interpreted as representative of local wind conditions.

In this thesis, the available CERRA2 wind speeds at 10 and 100 m were used to construct wind-speed estimates at the LiDAR validation heights. For each timestamp, a power-law

exponent was computed from the CERRA2 wind speeds at 10 and 100 m:

$$\alpha_{\text{CERRA2}} = \frac{\ln(U_{100}/U_{10})}{\ln(100/10)}. \quad (3.20)$$

The CERRA2 wind speed at a target height z was then obtained from

$$U_{\text{CERRA2}}(z) = U_{10} \left(\frac{z}{10} \right)^{\alpha_{\text{CERRA2}}}. \quad (3.21)$$

For target heights between 10 and 100 m, this procedure represents an interpolation. For target heights above 100 m, the same fitted power law is extended as an extrapolation. The resulting CERRA2-derived wind speeds were then matched with the LiDAR observations using the same timestamp and height. This produced a height-matched CERRA2 benchmark that could be directly compared with both the LiDAR observations and the MOST-based model estimates.

The error definition used for CERRA2 follows the same sign convention adopted for the MOST-based models in Eq. (3.16):

$$e_{\text{CERRA2}}(t, z) = U_{\text{CERRA2}}(t, z) - U_{\text{LiDAR}}(t, z). \quad (3.22)$$

Therefore, negative Bias indicates that CERRA2 underestimates the LiDAR wind speed, while positive Bias indicates overestimation.

4 Results

4.1 Overview of the result analysis

This chapter presents the validation of the vertical wind-speed profiles computed with the empirical stability corrections and MOST-based formulations described in the previous chapters. The objective is not only to identify which formulation produces the lowest error, but also to understand how each stability correction modifies the vertical structure of the wind profile. For this reason, the first part of the results is organized by empirical stability correction.

The analysis begins with the Businger–Dyer, Brutsaert, Holtslag, and Jimenez corrections. Each correction is first evaluated in an aggregated form, grouping the logarithmic formulation, the Lange correction, and the Gryning reformulation. This provides a general view of the behaviour of each stability correction before separating the effect of each profile formulation. The comparison is made against the LiDAR observations at the same height and timestamp.

The error is defined as the modelled wind speed minus the LiDAR wind speed. Positive errors therefore indicate overestimation, while negative errors indicate underestimation. This convention is used throughout the chapter to interpret the vertical profiles and the error distributions. The analysis focuses on where each correction tends to overestimate or underestimate the observed wind speed, how this tendency changes with height, and whether the error distribution remains centred around zero.

RMSE is used as the main metric because it quantifies the average magnitude of the error in physical units. Bias is used as a complementary metric to identify systematic deviations. This distinction is important because a small Bias does not necessarily indicate a more accurate model. Positive and negative errors can compensate each other, while the RMSE and the spread of the distribution remain large.

The reference level at 3.7 m is not treated as an independent validation height. It is used as the lower boundary condition for the vertical extrapolation. Therefore, the discussion focuses on the extrapolated levels from 30 to 200 m. At each height, the results are interpreted in terms of mean profile behaviour, error dispersion, RMSE, and Bias.

This structure allows the analysis to move gradually from a general interpretation of each empirical correction to a more specific comparison between the logarithmic, Lange, and Gryning formulations. In this way, the results show whether the behaviour observed for each correction is intrinsic to the stability function itself or mainly caused by the MOST-based formulation used

to construct the vertical profile.

For clarity, the chapter is divided into two main parts. The first part keeps the empirical stability correction fixed and analyses Businger–Dyer, Brutsaert, Holtslag, and Jimenez separately. Within each correction, the grouped behaviour is discussed first, and the internal response of LOG, Lange, and Gryning is then separated. This first block therefore explains how each stability correction behaves before any cross-correction selection is introduced.

The second part changes the interpretation of the problem. Instead of analysing one correction at a time, all empirical corrections and MOST-based formulations are considered together as possible candidates. This allows the results to be interpreted as model-selection policies, where the best combination may depend on height, stability regime, hour of day, or their combined effect.

4.2 Part I: Fixed empirical stability-correction analysis

This first part evaluates the empirical stability corrections one at a time. In each case, the stability correction is kept fixed, so the discussion focuses on the behaviour introduced by Businger–Dyer, Brutsaert, Holtslag, or Jimenez. The grouped profile is analysed first because it provides a compact view of the correction as a whole. After that, the same correction is separated into its LOG, Lange, and Gryning profile formulations to identify whether the observed behaviour is caused by the stability function itself or by the vertical reformulation used with it.

The end of this first part marks the transition from interpretation to selection. Until that point, the results are read as fixed-correction diagnostics. The following part then treats all model–formulation combinations together and evaluates how they can be mixed in rule-based strategies.

4.2.1 Businger–Dyer results

The first empirical correction analysed is the Businger–Dyer formulation. In this analysis, the logarithmic formulation, the Lange correction, and the Gryning reformulation are not separated. Instead, all profiles computed with the Businger–Dyer stability function are grouped together. This choice provides a general assessment of the behaviour of the empirical correction itself, before isolating the effect of each MOST reformulation.

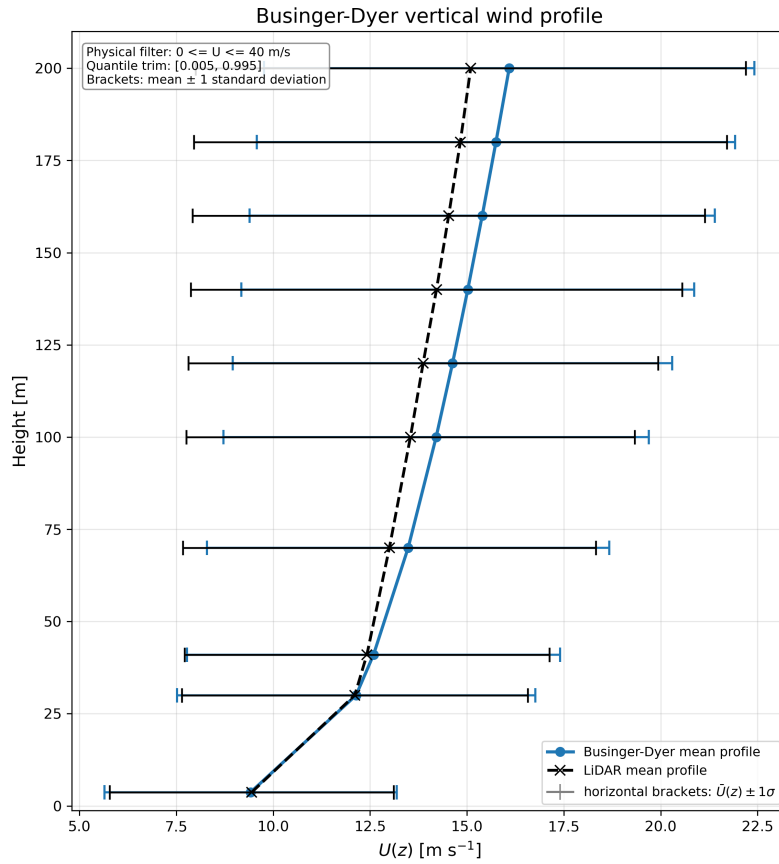


Fig. 2: Mean wind-speed profile for the Businger–Dyer correction.

Figure 2 shows that the Businger–Dyer profile follows the observed LiDAR profile with reasonable consistency in the lower part of the domain. As the height increases, the modelled profile gradually moves to the right of the observed profile. This indicates a mild tendency to overestimate wind speed at the upper levels. The tendency is not abrupt, but it becomes more visible as the extrapolation distance from the reference height increases.

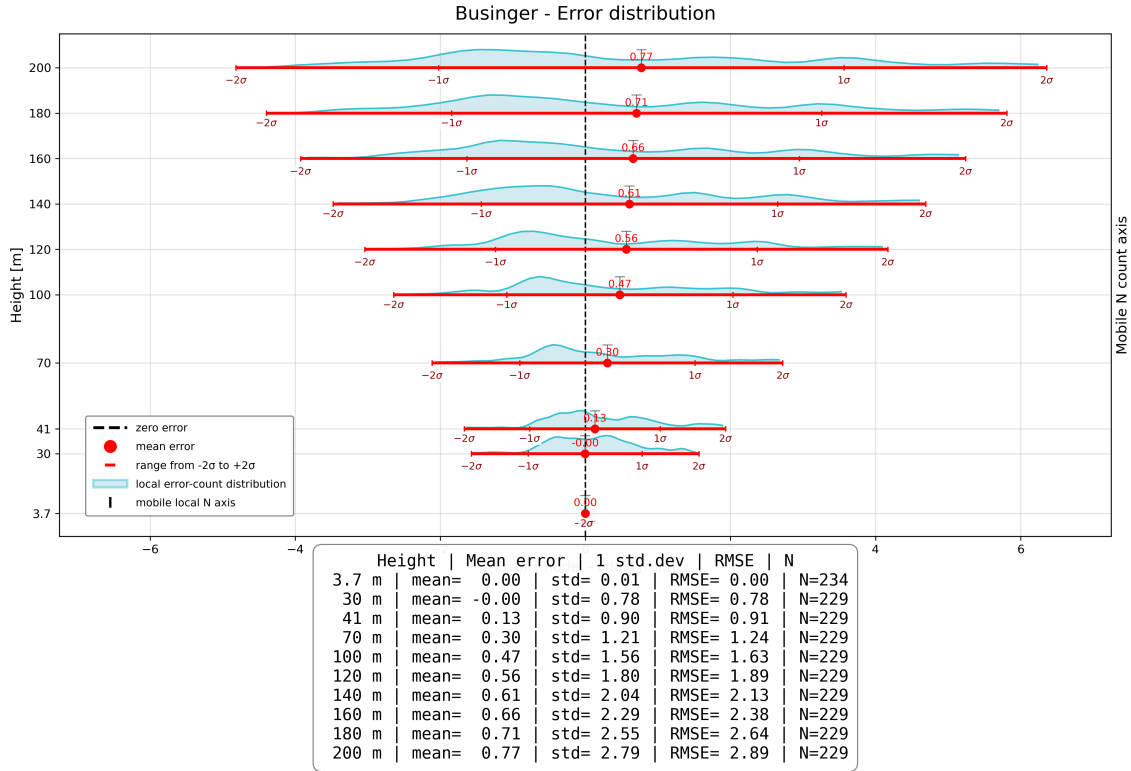


Fig. 3: Filtered error distribution for the Businger–Dyer correction.

Excluding the reference height at 3.7 m, the Businger–Dyer correction gives an overall weighted RMSE of 1.96 ms^{-1} and an overall weighted Bias of 0.47 ms^{-1} .

The same behaviour appears in the error distributions shown in Figure 3. The mean error remains relatively close to the zero-error line, which suggests that the Businger–Dyer correction does not introduce a strong systematic bias. However, the displacement of the mean toward positive values at higher levels indicates that the overestimation is persistent. Thus, the formulation remains balanced in its central behaviour, but it becomes slightly too energetic in the upper part of the profile. The distribution shape also provides useful information. At the lowest extrapolated level, the error distribution suggests a possible bimodal structure. This may occur because the grouped Businger–Dyer subset still contains profiles produced by different vertical reformulations, as well as different atmospheric regimes. At higher levels, the distributions are more clearly dominated by a single central mode, although some asymmetry and secondary shoulders remain present. For this reason, the model behaviour should not be interpreted only from the mean error.

The uncertainty of the prediction increases with height. This is visible both in the widening of the horizontal brackets in Figure 2 and in the broader error distributions in Figure 3. This

behaviour is expected in vertical extrapolation, since the target levels become progressively farther from the reference measurement. Overall, the Businger–Dyer correction behaves as a stable baseline formulation. Its main limitation is not a strong abrupt bias, but a gradual increase in positive Bias and error dispersion with height.

Comparison between Businger–Dyer MOST, Lange, and Gryning formulations

The grouped analysis of the Businger–Dyer correction can be further clarified by separating the three MOST-based formulations. Figure 4 shows the same empirical stability correction applied through the classical logarithmic formulation, the Lange boundary-layer-height correction, and the Gryning reformulation. This separation is important because the apparent mean behaviour of Businger–Dyer results from three distinct vertical responses.

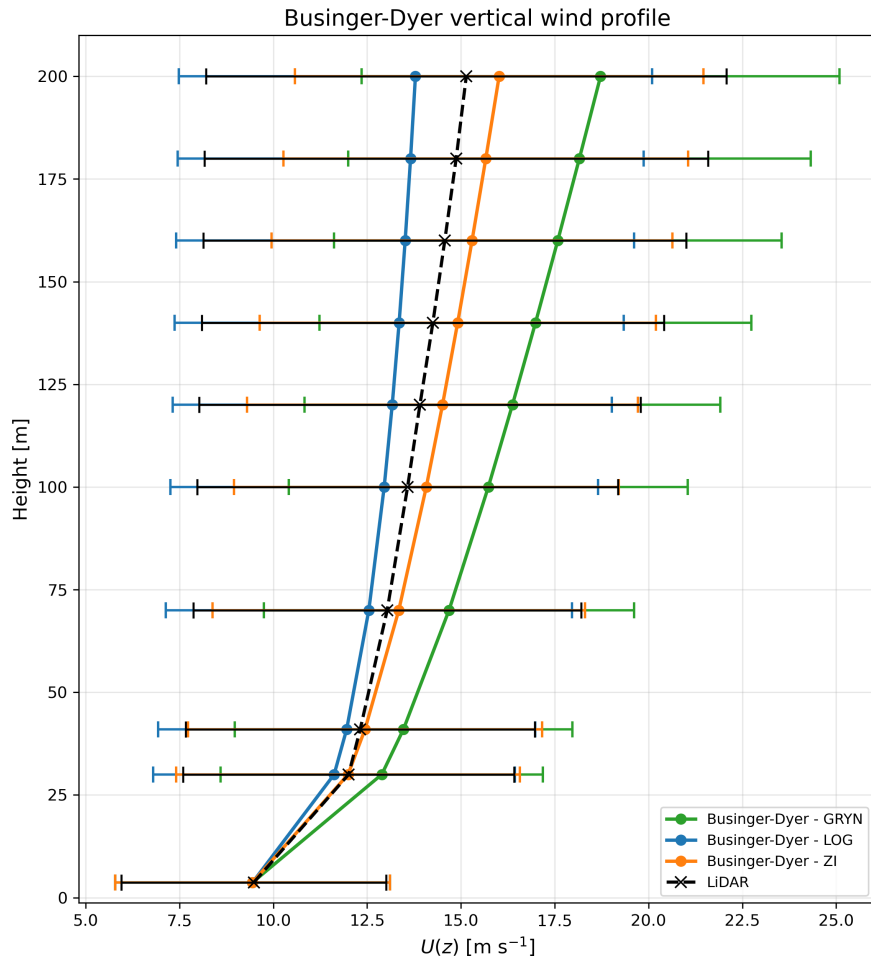


Fig. 4: Businger–Dyer profiles by MOST formulation.

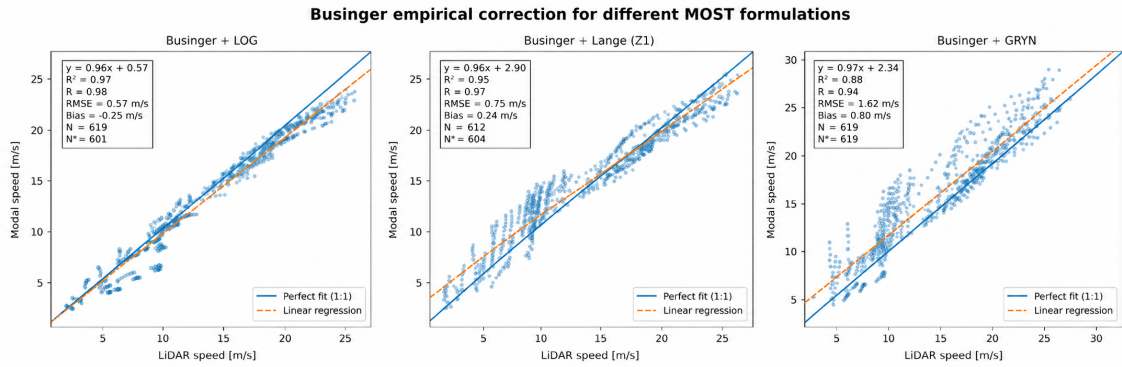


Fig. 5: Scatter-plot comparison for the Businger–Dyer empirical correction applied with different MOST formulations.

Figure 5 provides a complementary point-wise view of the Businger–Dyer behaviour by comparing the modelled wind speed against the LiDAR observations for the three MOST-based formulations. The LOG and Lange formulations show the most compact distributions around the perfect-fit line, confirming that most matched height–timestamp pairs are reproduced with limited scatter. However, the Lange formulation presents a stronger upward displacement of the regression line at lower and intermediate speeds, while the Gryning formulation shows the largest dispersion and the clearest tendency to overestimate. This supports the interpretation obtained from the vertical profiles: the grouped Businger–Dyer behaviour is not produced by a uniform response of all formulations, but by a relatively balanced LOG response combined with a more energetic behaviour from Lange and, especially, Gryning.

When the formulations are analysed separately, the LOG formulation is the most consistent option over most of the profile. It does not reproduce the slight overestimation seen in the grouped profile. Instead, it progressively underestimates the observed LiDAR wind speed as height increases. Even with this negative Bias, LOG gives the lowest RMSE from the intermediate levels upward. The Lange formulation behaves differently. It gives the lowest RMSE at the two lowest extrapolated levels, where its Bias remains small and close to zero. However, above the lower part of the profile, Lange becomes progressively more energetic than the LiDAR profile. The Gryning formulation shows the strongest deviation, since it systematically overestimates the upper part of the profile. This behaviour is also visible in Figure 5, where the Gryning panel presents a wider scatter cloud and a regression line displaced above the perfect-fit reference.

Table 3: Height-dependent Businger–Dyer formulation selected by RMSE after filtering.

Height [m]	Best formulation	N_{used}	N_{tot}	$N_{>40}$	RMSE	Bias
30	LOG	6072	6079	7	1.188	-0.621
41	LOG	6072	6079	7	1.328	-0.690
70	LOG	6072	6079	7	1.575	-0.847
100	LOG	6072	6079	7	1.729	-0.942
120	LOG	6072	6079	7	1.808	-0.997
140	LOG	6073	6079	6	1.872	-1.041
160	LOG	6073	6079	6	1.923	-1.073
180	LOG	6073	6079	6	1.967	-1.099
200	LOG	6073	6079	6	2.000	-1.113

Excluding the reference height at 3.7 m, the corrected Businger–Dyer best-formulation strategy gives an overall weighted RMSE of 1.73 m s^{-1} and an overall weighted Bias of -0.94 m s^{-1} . The total sample fraction is $N_{\text{used}}/N_{\text{tot}} = 54652/54711$.

The height-dependent Businger–Dyer strategy improves the grouped result by reducing the overall weighted RMSE from 1.96 m s^{-1} to 1.73 m s^{-1} . However, the selected formulation is LOG at all evaluated heights, meaning that this strategy is not a truly mixed formulation in this case. The improvement in RMSE is accompanied by a persistent negative Bias, indicating systematic underestimation of the LiDAR wind speed.

4.2.2 Brutsaert results

The second empirical correction analysed is the Brutsaert formulation. As in the previous subsection, the logarithmic formulation, the Lange correction, and the Gryning reformulation are not separated at this stage. Instead, all profiles computed with the Brutsaert stability function are grouped together. This choice provides a general assessment of the behaviour of the empirical correction itself, before isolating the effect of each MOST reformulation.

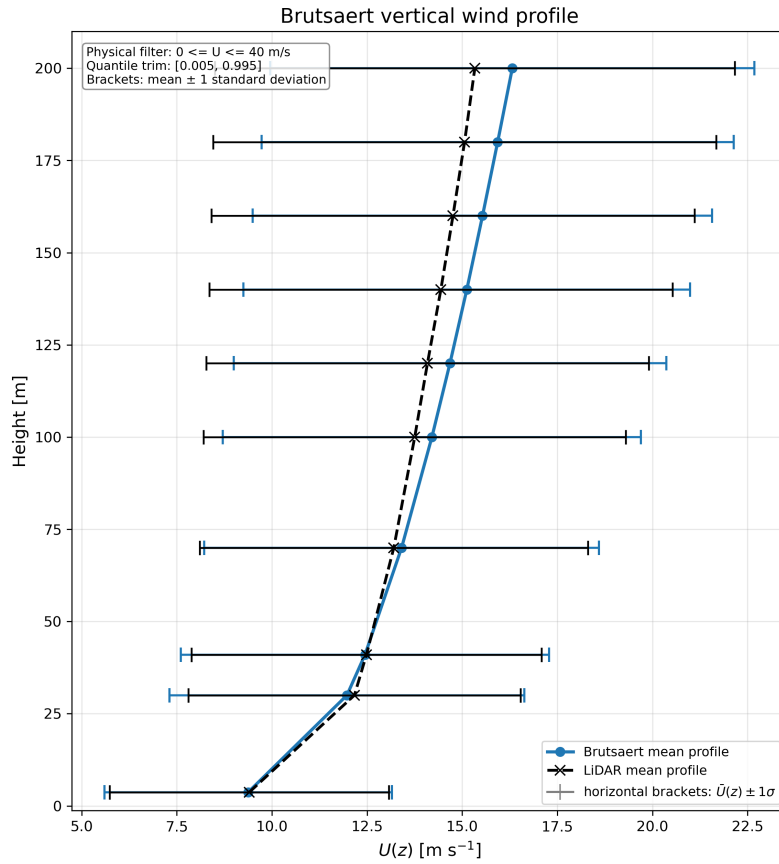


Fig. 6: Mean wind-speed profile for the Brutsaert correction.

Figure 6 shows that the Brutsaert profile follows the observed LiDAR profile closely near the reference level and in the lowest extrapolated levels. At 30 and 41 m, the modelled profile remains very close to the LiDAR profile, with only small deviations. However, above approximately 70 m, the modelled curve gradually moves to the right of the observed profile. This indicates a progressive tendency to overestimate wind speed in the upper part of the profile. The overestimation is not abrupt. It develops gradually as the extrapolation height increases.

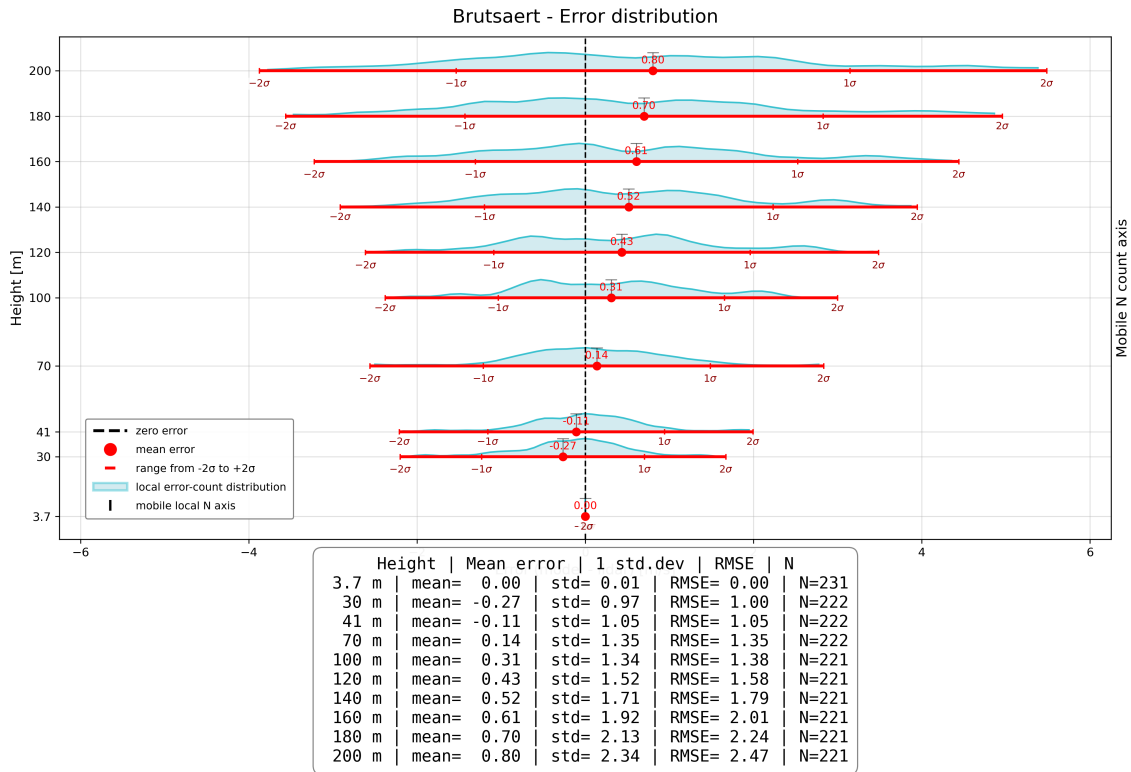


Fig. 7: Filtered error distribution for the Brutsaert correction.

Excluding the reference height at 3.7 m, this correction gives an overall weighted RMSE of 1.72 ms^{-1} and an overall weighted Bias of 0.35 ms^{-1} .

The same behaviour appears in the error distributions shown in Figure 7. The mean error is slightly negative at the lowest extrapolated levels and becomes positive from the middle of the profile upward. This transition from weak underestimation to systematic overestimation is an important feature of the Brutsaert correction. It indicates that the central error is not uniform along the profile. The model is well balanced near the lower levels, but it becomes increasingly too energetic as height increases. The uncertainty of the prediction also increases clearly with height, as shown by the widening brackets and the broader error distributions.

Overall, the Brutsaert correction behaves as a reasonable formulation in the lower part of the profile, but it shows a clear tendency toward positive Bias at higher levels. Its main limitation is the progressive increase in both mean error and error dispersion with height. For this reason, the Brutsaert correction should be interpreted with caution when applied to turbine-relevant heights, especially above 100 m.

Comparison between Brutsaert MOST, Lange, and Gryning formulations

The Brutsaert correction presents a more balanced comparison between LOG and Lange than Businger–Dyer. Figure 8 shows that the LOG and Lange profiles remain relatively close to the LiDAR curve over most heights. The Gryning profile, however, separates progressively to the right and becomes the most energetic formulation. This indicates that the grouped Brutsaert behaviour is strongly affected by the upper-level response of the Gryning formulation.

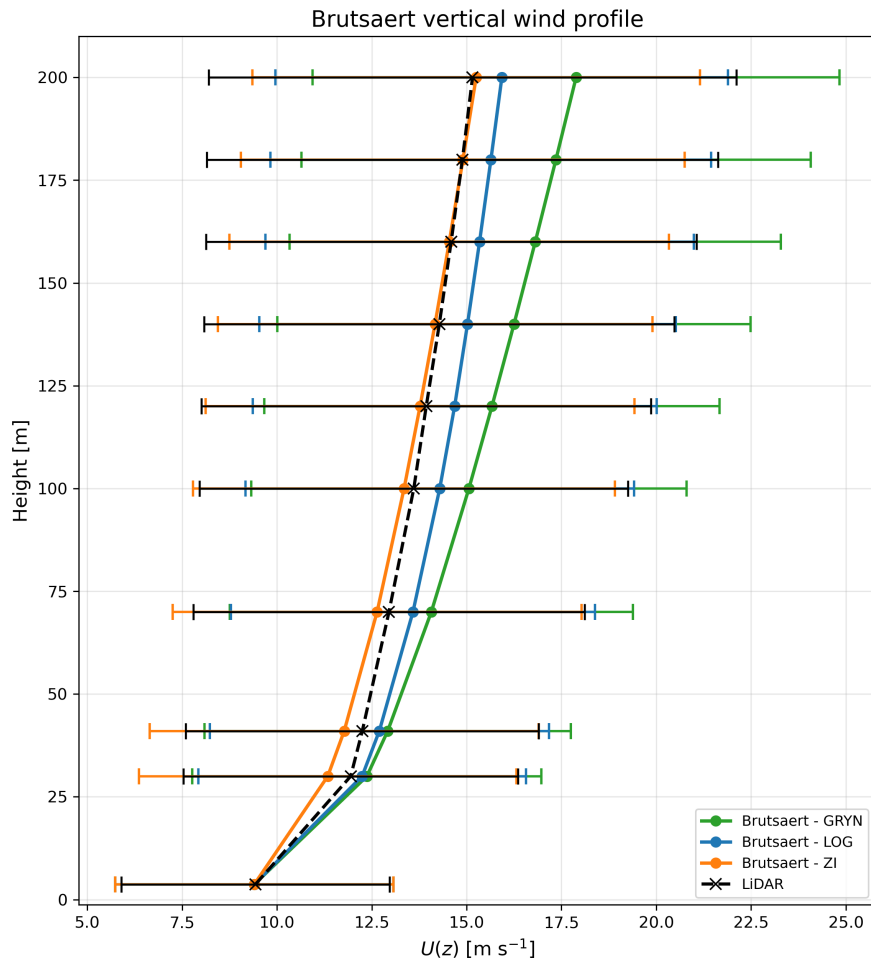


Fig. 8: Brutsaert profiles by MOST formulation.

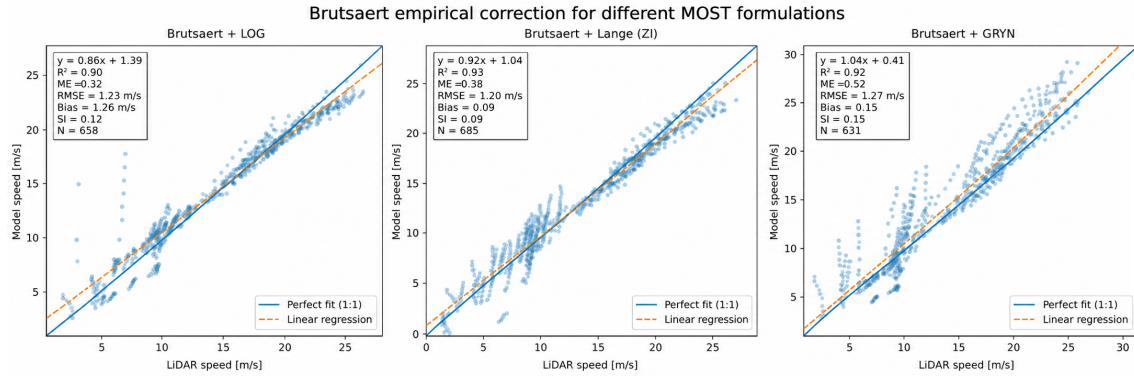


Fig. 9: Scatter-plot comparison for the Brutsaert empirical correction applied with different MOST formulations.

Figure 9 shows that the Brutsaert correction produces a more balanced behaviour between the LOG and Lange formulations. Both panels remain close to the perfect-fit line, with a relatively compact scatter distribution and no strong systematic displacement of the regression line. This indicates that, for Brutsaert, the main difference between LOG and Lange is not a complete change in the model response, but a moderate adjustment in the distribution of errors. The Gryning formulation, in contrast, presents a broader scatter cloud and a clearer tendency toward positive deviations, especially at higher wind speeds. Therefore, the grid confirms that Brutsaert is most reliable when combined with LOG or Lange, while the Gryning reformulation is the main source of upper-level overestimation in the grouped behaviour.

The LOG formulation is selected at most heights. It gives the lowest RMSE at the lowest level and through most of the upper profile. The Lange formulation remains competitive and is selected at specific heights, especially where it reduces the error relative to LOG. The Gryning formulation is selected only locally and becomes increasingly too energetic at higher levels. Thus, the Brutsaert correction is most reliable when used with LOG or Lange, while the Gryning reformulation requires caution in the upper part of the profile. This interpretation is supported by Figure 9, where LOG and Lange show a more compact agreement with the LiDAR observations, while Gryning introduces a larger spread around the regression line.

Table 4: Height-dependent Brutsaert formulation selected by RMSE after filtering.

Height [m]	Best formulation	N_{used}	N_{tot}	$N_{>40}$	RMSE	Bias
30	LOG	5714	6078	364	0.850	-0.048
41	LOG	5714	6078	364	0.961	0.065
70	LOG	5714	6078	364	1.448	0.334
100	Lange	6072	6078	6	2.039	-0.374
120	Lange	6072	6078	6	2.102	-0.182
140	Lange	6072	6078	6	2.170	0.019
160	Lange	6072	6078	6	2.249	0.234
180	Lange	6072	6078	6	2.339	0.454
200	Lange	6072	6078	6	2.443	0.687

Excluding the reference height at 3.7 m, the corrected Brutsaert best-formulation strategy gives an overall weighted RMSE of 1.94 m s^{-1} and an overall weighted Bias of 0.13 m s^{-1} . The total sample fraction is $N_{\text{used}}/N_{\text{tot}} = 53574/54702$.

In the corrected Brutsaert table, the height-dependent strategy does not reduce the overall RMSE relative to the grouped result. Instead, it gives an overall weighted RMSE of 1.94 m s^{-1} , compared with 1.72 m s^{-1} for the grouped Brutsaert result. Its main advantage is the smaller overall Bias, which remains close to zero. Therefore, this strategy should be interpreted as a Bias-balancing choice rather than as an RMSE improvement.

4.2.3 Holtslag results

The Holtslag formulation presents a different behaviour from the previous empirical corrections. Figure 10 shows that the modelled profile remains very close to the LiDAR profile over most of the vertical range. The largest visible differences occur in the lower extrapolated levels, especially between 30 and 70 m. In this part of the profile, the Holtslag curve is slightly displaced to the left of the LiDAR curve. This indicates that the model underestimates the observed wind speed near the lower levels.

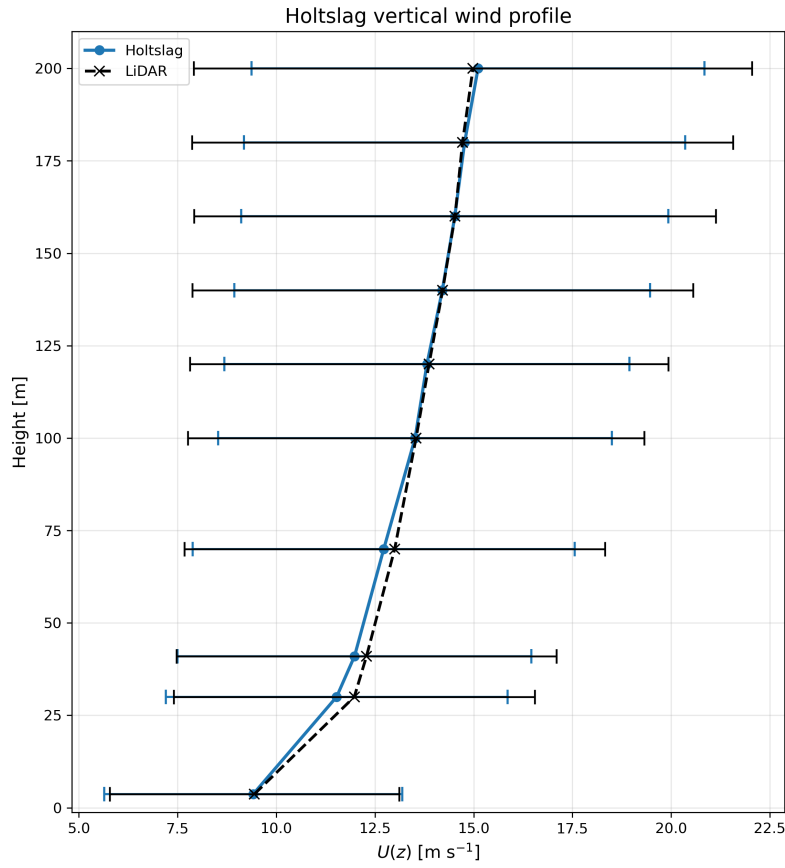


Fig. 10: Mean wind-speed profile for the Holtslag correction.

From 100 m upward, the agreement between the Holtslag profile and the LiDAR profile improves in terms of mean wind speed. The two curves become progressively closer, and at the highest levels they are nearly superposed. This behaviour suggests that the Holtslag correction does not produce a strong mean overestimation in the upper part of the profile. Instead, the mean modelled profile remains slightly below or very close to the observed LiDAR profile.

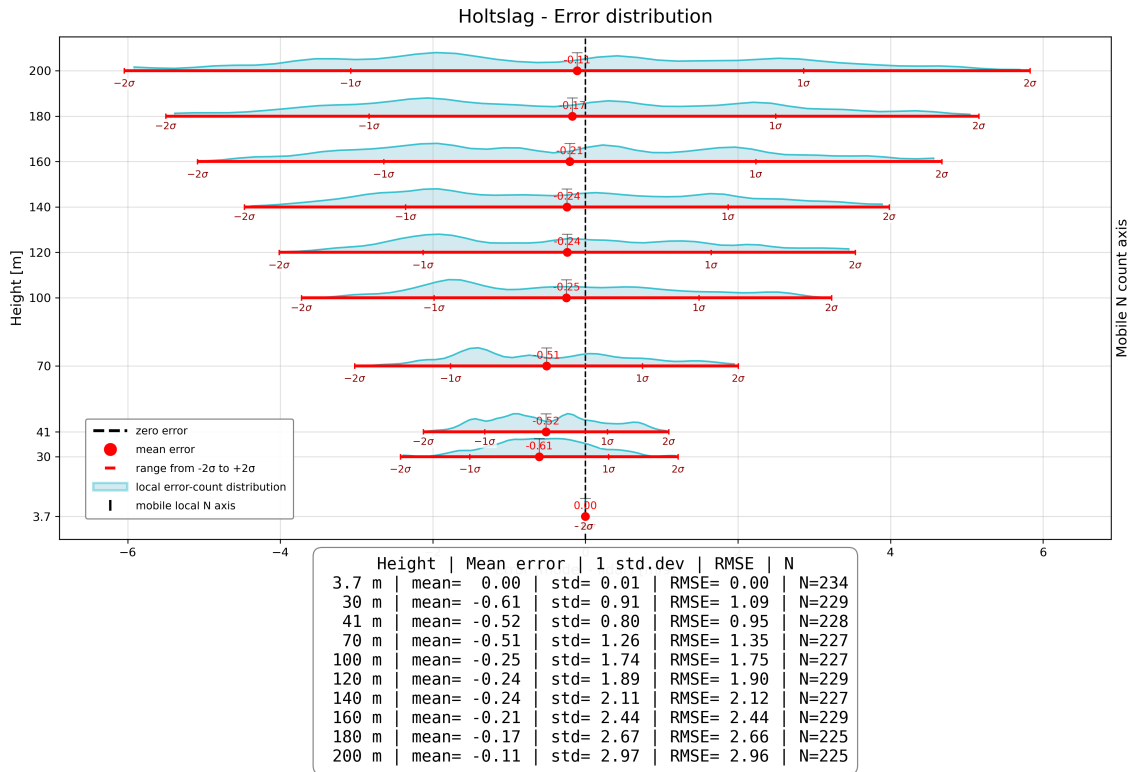


Fig. 11: Filtered error distribution for the Holtslag correction.

Excluding the reference height at 3.7 m, this correction gives an overall weighted RMSE of 2.02 m s^{-1} and an overall weighted Bias of -0.32 m s^{-1} .

The error distributions confirm that the mean error is negative at all evaluated heights. The magnitude of this negative Bias decreases gradually above 100 m. Therefore, the Holtslag correction does not show a continuous increase in overestimation with height. The opposite trend is observed, since the mean underestimation becomes weaker as height increases. However, this improvement in Bias does not mean that the model becomes more accurate at higher levels. The RMSE and the standard deviation increase with height, showing that the main limitation of Holtslag is the progressive widening of the error distribution.

Overall, the Holtslag correction provides a mean vertical profile that is very close to the LiDAR profile, especially above 100 m. Its systematic error is negative over the full height range, but the magnitude of this underestimation decreases with height. The main weakness is the strong growth of dispersion and RMSE toward the upper levels.

Comparison between Holtslag MOST, Lange, and Gryning formulations

The Holtslag correction shows the clearest separation between the three MOST-based formulations. Figure 12 shows that LOG, Lange, and Gryning produce different vertical responses. This explains why the grouped Holtslag profile can appear close to the LiDAR mean while still presenting a large RMSE at upper levels.

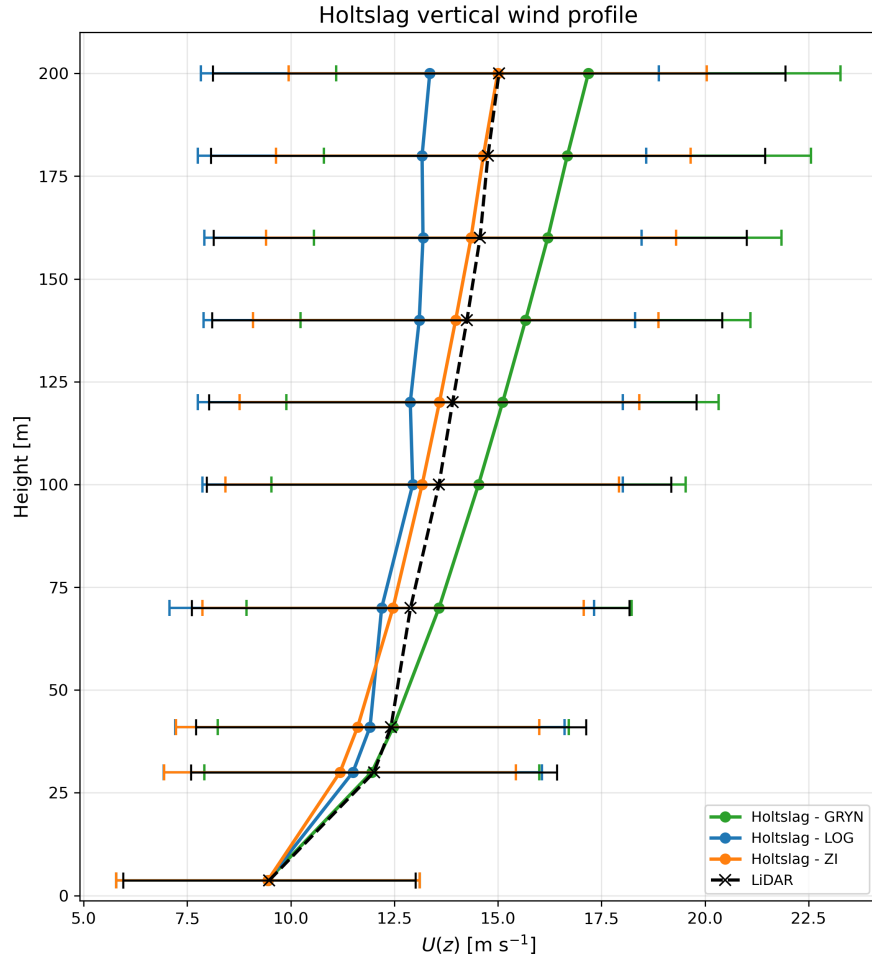


Fig. 12: Holtslag profiles by MOST formulation.

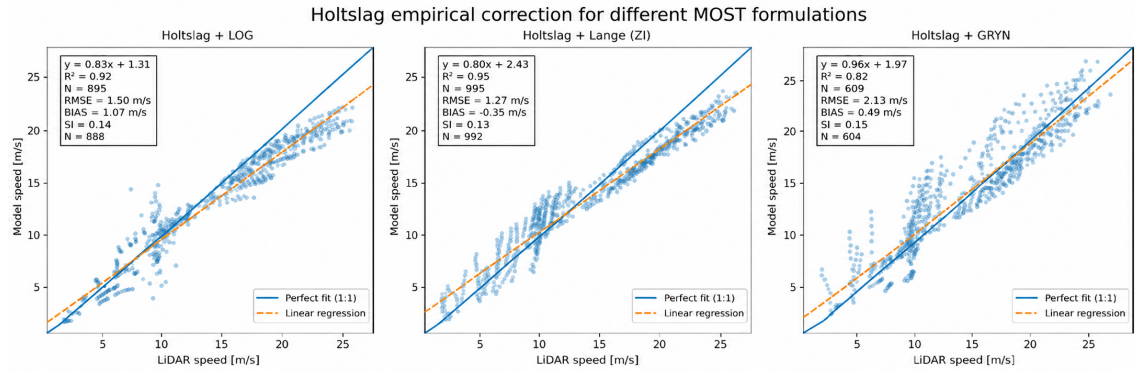


Fig. 13: Scatter-plot comparison for the Holtslag empirical correction applied with different MOST formulations.

Figure 13 reinforces that the Holtslag correction is strongly dependent on the selected MOST formulation. The LOG and Lange panels both preserve a clear linear relation with the LiDAR observations, but the regression lines indicate a tendency to remain below the perfect-fit line at higher wind speeds, which is consistent with the negative Bias discussed previously. The Gryning formulation behaves differently: it reduces part of this underestimation in some ranges, but it also increases the dispersion of the points and becomes less stable at higher speeds. This explains why the grouped Holtslag profile can appear close to the LiDAR mean while still presenting relevant RMSE values. The grid therefore supports the interpretation that the grouped response results partly from compensation between formulations rather than from a uniformly accurate behaviour.

The LOG formulation systematically underestimates the LiDAR profile above the lowest levels. The Lange formulation becomes the best option for most of the vertical domain and is selected from 70 m upward. Its Bias remains negative, but its magnitude decreases with height. The Gryning formulation behaves in the opposite direction. It is useful only in the lowest part of the profile and becomes too energetic aloft. Therefore, the grouped Holtslag profile is partly the result of compensation between underestimation from LOG and overestimation from Gryning. The scatter-grid comparison in Figure 13 gives additional support to this interpretation, since the three panels do not show a uniform displacement relative to the perfect-fit line.

Table 5: Height-dependent Holtslag formulation selected by RMSE after filtering.

Height [m]	Best formulation	N_{used}	N_{tot}	$N_{>40}$	RMSE	Bias
30	GRYN	5177	6078	901	1.135	-0.416
41	GRYN	5177	6078	901	1.302	-0.273
70	Lange	6069	6078	9	1.707	-0.417
100	Lange	6069	6078	9	1.909	-0.140
120	Lange	6069	6078	9	2.037	0.054
140	Lange	6069	6078	9	2.169	0.257
160	Lange	6069	6078	9	2.300	0.473
180	Lange	6069	6078	9	2.436	0.695
200	Lange	6069	6078	9	2.579	0.929

Excluding the reference height at 3.7 m, the corrected Holtslag best-formulation strategy gives an overall weighted RMSE of 2.03 m s^{-1} and an overall weighted Bias of 0.15 m s^{-1} . The total sample fraction is $N_{\text{used}}/N_{\text{tot}} = 52837/54702$.

The height-dependent Holtslag strategy gives an overall weighted RMSE of 2.03 m s^{-1} , which is very close to the grouped Holtslag result. Therefore, the mixed selection does not substantially improve the RMSE in this case. Its main effect is to reduce the magnitude of the overall Bias, which becomes close to zero.

4.2.4 Jimenez results

The Jimenez formulation presents a clear tendency toward positive error with height. Figure 14 shows that the modelled profile remains close to the LiDAR profile near the reference level and at the lowest extrapolated heights. At 30 m, the two profiles are almost superposed. At 41 m, the model is already slightly displaced to the right of the LiDAR curve. This indicates a weak overestimation in the lower part of the profile.

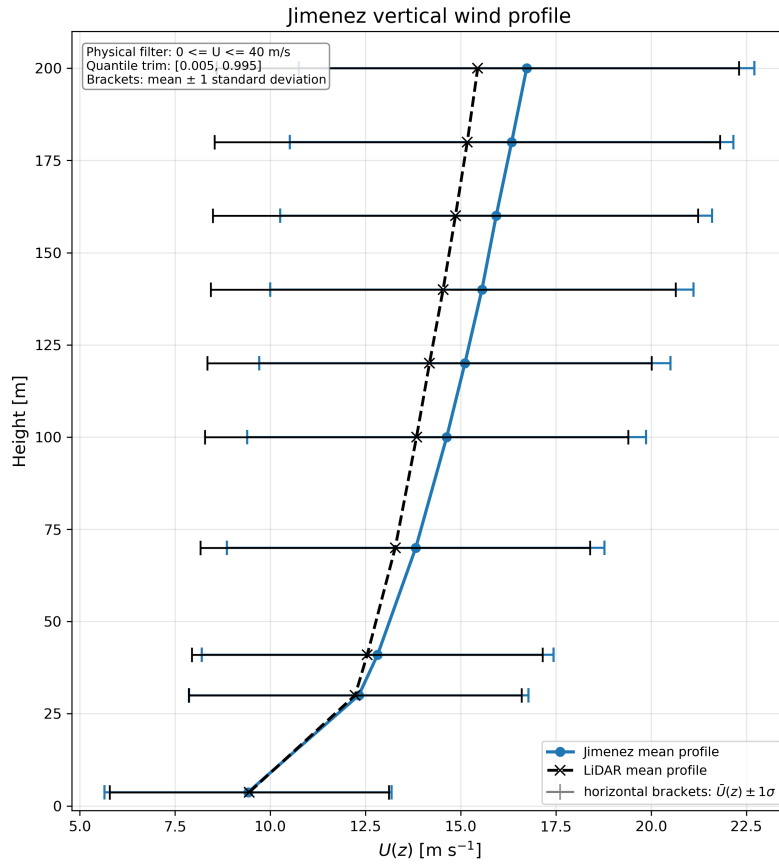


Fig. 14: Mean wind-speed profile for the Jimenez correction.

From 70 m upward, the separation between the Jimenez profile and the LiDAR profile becomes progressively clearer. The modelled curve remains systematically to the right of the observed curve. This means that the Jimenez correction increasingly overestimates the LiDAR wind speed as height increases. The difference is moderate around 70–100 m, but it becomes more visible above 120 m. At 200 m, the modelled mean profile is clearly higher than the observed LiDAR mean.

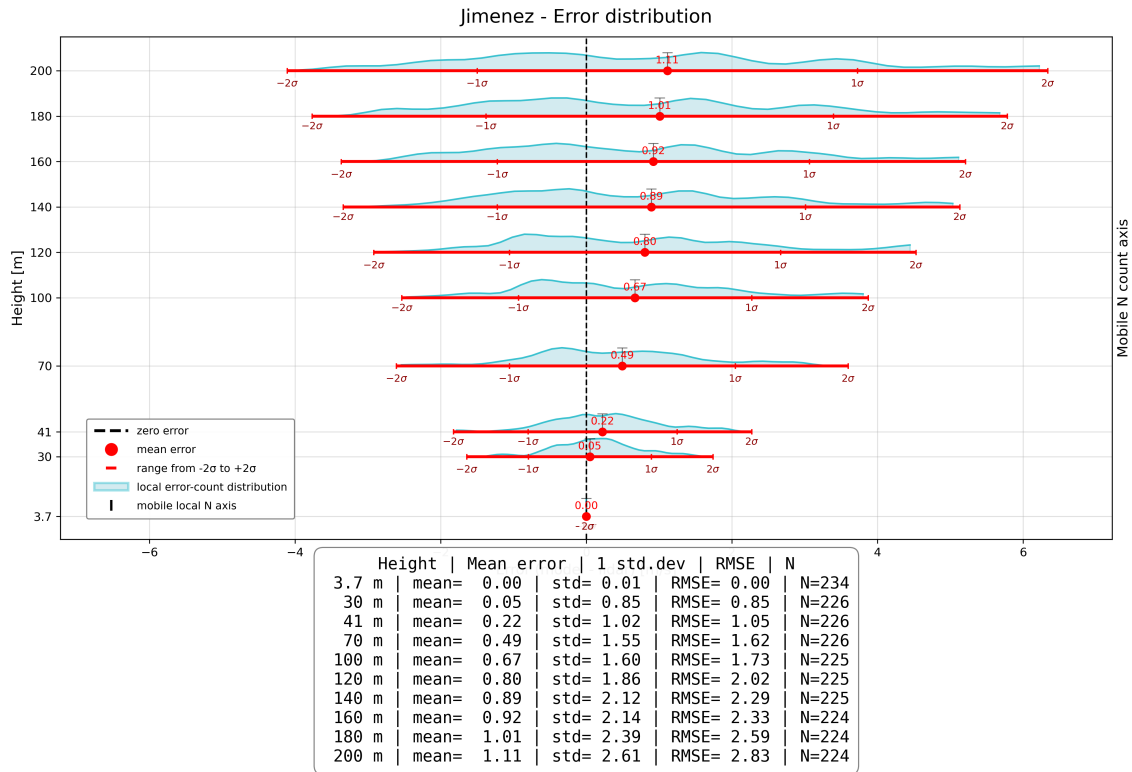


Fig. 15: Filtered error distribution for the Jimenez correction.

Excluding the reference height at 3.7 m, this correction gives an overall weighted RMSE of 1.92 ms^{-1} and an overall weighted Bias of 0.64 ms^{-1} .

The error distributions confirm that the mean error is positive at all evaluated heights. This behaviour indicates that the mean vertical shear produced by the grouped Jimenez profiles is stronger than the shear observed by the LiDAR. The result is a profile that becomes increasingly too energetic at turbine-relevant heights. The RMSE also increases with height, which shows that the Jimenez correction does not only overestimate the mean profile at higher levels. It also becomes more uncertain as the extrapolation distance increases.

Overall, the Jimenez correction reproduces the lower part of the profile well, especially at 30 and 41 m. However, it develops a systematic positive Bias above 70 m. The overestimation grows almost continuously with height, and the dispersion also increases toward the upper levels. For this reason, Jimenez behaves as an energetic correction in the grouped analysis and should be interpreted with caution above 100 m.

Comparison between Jimenez MOST, Lange, and Gryning formulations

The Jimenez correction changes substantially depending on the MOST-based formulation. Figure 16 shows that the Gryning profile is systematically the most energetic option. The LOG and Lange profiles remain closer to the LiDAR profile, but their relative performance changes with height. This makes the Jimenez case more sensitive to the choice of vertical formulation than the previous corrections.

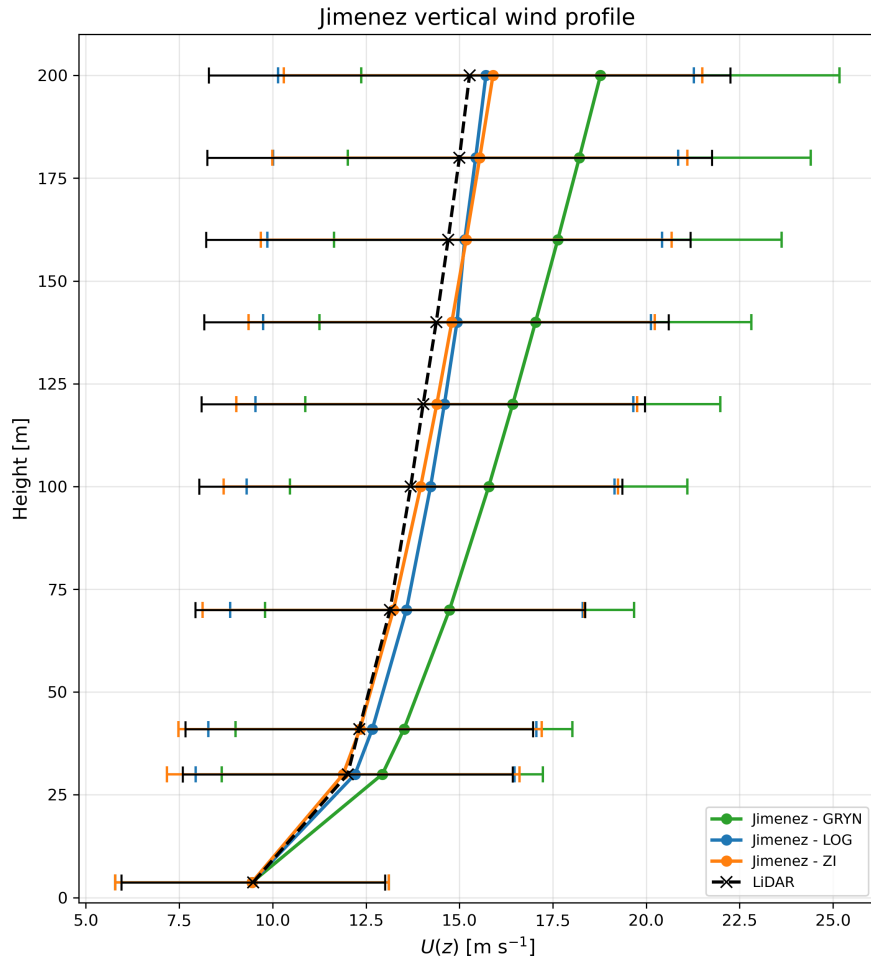


Fig. 16: Jimenez profiles by MOST formulation.

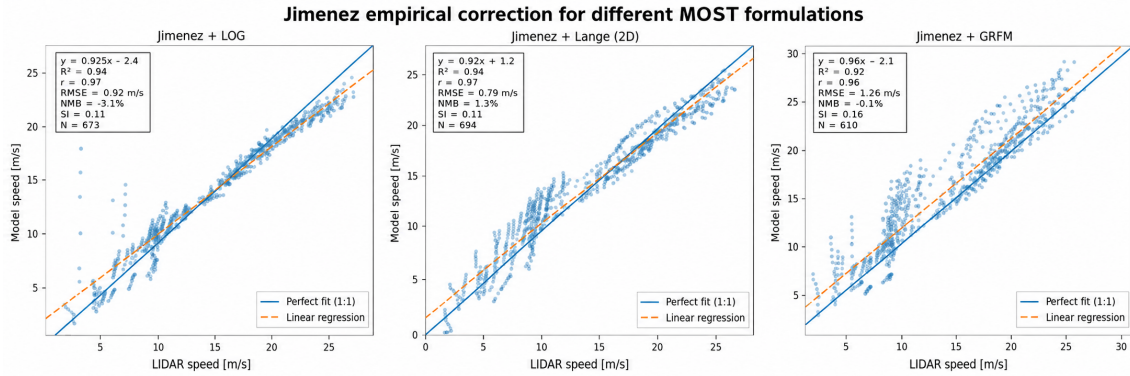


Fig. 17: Scatter-plot comparison for the Jimenez empirical correction applied with different MOST formulations.

Figure 17 shows that the Jimenez correction remains highly correlated with the LiDAR observations for all three MOST-based formulations, but the position of the regression lines and the dispersion of the points change significantly between them. The LOG and Lange formulations present the most controlled behaviour, with scatter clouds that remain relatively close to the perfect-fit line. Nevertheless, the Lange formulation introduces a more energetic response in part of the distribution, which is consistent with the positive Bias observed in the vertical-profile analysis. The Gryning formulation shows the clearest deviation, with a broader scatter distribution and a stronger tendency to overestimate the LiDAR wind speed. Thus, the grid confirms that Jimenez is one of the empirical corrections most sensitive to the choice of vertical formulation, and that its upper-level overestimation is mainly amplified when combined with Gryning.

The Lange formulation is the best option from the lowest extrapolated levels up to the middle part of the profile. It gives a good lower- and mid-level response, although the Bias becomes progressively positive with height. At the highest levels, LOG becomes preferable. This indicates that the Lange correction improves the lower and intermediate profile, while the classical logarithmic formulation gives a better centred response aloft. The Gryning formulation is the weakest option for Jimenez, since it systematically overestimates the LiDAR profile and becomes increasingly too steep with height. The same tendency is visible in Figure 17, where the Gryning panel shows the broadest spread and the strongest displacement from the perfect-fit line.

Table 6: Height-dependent Jimenez formulation selected by RMSE after filtering.

Height [m]	Best formulation	N_{used}	N_{tot}	$N_{>40}$	RMSE	Bias
30	LOG	5783	6079	296	0.838	-0.111
41	LOG	5783	6079	296	0.926	-0.013
70	LOG	5783	6079	296	1.288	0.218
100	LOG	5783	6079	296	1.842	0.491
120	LOG	5783	6079	296	2.269	0.674
140	Lange	6070	6079	9	2.483	0.652
160	Lange	6070	6079	9	2.621	0.874
180	Lange	6070	6079	9	2.763	1.102
200	Lange	6070	6079	9	2.914	1.341

Excluding the reference height at 3.7 m, the corrected Jimenez best-formulation strategy gives an overall weighted RMSE of 2.15 m s^{-1} and an overall weighted Bias of 0.59 m s^{-1} . The total sample fraction is $N_{\text{used}}/N_{\text{tot}} = 53195/54711$.

For Jimenez, the height-dependent strategy gives an overall weighted RMSE of 2.15 m s^{-1} and an overall weighted Bias of 0.59 m s^{-1} . Therefore, in the corrected table, the height-dependent selection should not be interpreted as an improvement relative to the grouped result or to the best single formulation. Instead, it shows that changing the MOST formulation only as a function of height is not sufficient to improve the Jimenez correction in this dataset.

4.3 Part II: Mixed formulation and adaptive selection

The second part no longer treats the empirical stability corrections as isolated groups. Instead, the candidate set is expanded to include all model–formulation combinations. This means that Businger–Dyer, Brutsaert, Holtslag, and Jimenez can be compared directly, together with the LOG, Lange, and Gryning formulations. The purpose is to test whether a single fixed option is sufficient, or whether a mixed policy can reduce RMSE by selecting different combinations under different conditions.

The term mixed strategy is used here in an operational sense. It does not define a new physical profile model. It defines a rule-based selection procedure that chooses among the existing formulations according to the validation context, such as height, stability class, hour of day, or combinations of these variables.

4.3.1 Global results by height

This section identifies the best-performing model combination at each height using RMSE as the primary selection metric. RMSE is adopted because it provides a direct measure of the average

error magnitude in physical units. This is suitable for interpreting wind-speed extrapolation, since the error is expressed in m s^{-1} and can be compared directly across heights. Bias is used as a complementary diagnostic to indicate whether each selected combination tends to overestimate or underestimate the LiDAR observations. When two combinations present similar RMSE values, the absolute value of Bias is useful to interpret which option has the smaller systematic deviation.

The results reported in this section were computed after applying the 99% filtered error distribution. This filtering reduces the influence of extreme numerical artefacts while preserving the central behaviour of each model. The reference height of 3.7 m is not included in the validation because it is used as the boundary condition of the profiles. Therefore, the analysis focuses only on the extrapolated levels from 30 to 200 m.

Table 7: Best overall model combination by height.

Height [m]	Model–MOST formulation	N	RMSE	Bias	Bias	MAE	Mean U_{model}	Mean U_{LiDAR}
30	Holtslag–GRYN	6456	1.407	-0.302	0.302	1.048	9.591	9.893
41	Businger–LOG	7646	1.538	-0.567	0.567	1.133	8.750	9.316
70	Businger–LOG	7619	1.678	-0.677	0.677	1.237	9.084	9.761
100	Businger–LOG	7589	1.742	-0.733	0.733	1.292	9.307	10.040
120	Businger–LOG	7580	1.785	-0.775	0.775	1.329	9.413	10.189
140	Businger–LOG	7566	1.811	-0.809	0.809	1.355	9.505	10.314
160	Businger–LOG	7559	1.839	-0.838	0.838	1.382	9.581	10.419
180	Businger–LOG	7559	1.871	-0.869	0.869	1.409	9.645	10.514
200	Businger–LOG	7561	1.901	-0.893	0.893	1.435	9.701	10.595

Excluding the reference height at 3.7 m, the height-dependent best-formulation strategy gives an overall weighted RMSE of 1.74 m s^{-1} and an overall weighted Bias of -0.72 m s^{-1} .

Table 7 shows that the best formulation depends only weakly on height after the lowest validation level. At 30 m, the lowest RMSE is obtained by Holtslag–GRYN, with an RMSE of 1.407 m s^{-1} and a Bias of -0.302 m s^{-1} . This indicates a small systematic underestimation at this height. From 41 m upward, however, Businger–LOG becomes the best combination for every evaluated level.

The dominance of Businger–LOG above 41 m suggests that the classical MOST-corrected logarithmic formulation provides the most stable height-dependent performance in this dataset. Its RMSE increases gradually with height, from 1.538 m s^{-1} at 41 m to 1.901 m s^{-1} at 200 m. This trend is expected because extrapolation uncertainty grows as the target height moves farther from the reference measurement. The increase is progressive rather than abrupt, which indicates that the formulation remains relatively consistent across the full validation range.

The Bias values are negative at all heights. This means that even the best RMSE-based

combinations tend to underestimate the LiDAR wind speed on average. For Businger–LOG, the magnitude of the negative Bias increases with height, from -0.567 m s^{-1} at 41 m to -0.893 m s^{-1} at 200 m. This systematic underestimation is also visible when comparing the mean model speed with the mean LiDAR speed. For example, at 200 m, the mean model speed is 9.701 m s^{-1} , while the mean LiDAR speed is 10.595 m s^{-1} .

These results lead to two main conclusions. First, if a height-dependent policy is allowed, Holtslag–GRYN is the preferred option only at 30 m. Second, for the rest of the profile, Businger–LOG is the most reliable RMSE-based choice. However, the persistent negative Bias indicates that the selected formulations still underestimate the observed wind speed, especially at the highest levels. Therefore, the best model by RMSE should not be interpreted as unbiased. It should be interpreted as the formulation that minimizes the overall error magnitude while still showing a systematic tendency toward underprediction.

4.3.2 Policy by stability and height

While the previous subsection focused on a single global ranking that aggregates all heights and stability regimes, practical applications often require a more detailed view. In particular, the performance of each empirical model and MOST reformulation can change substantially with both height and stability. This section therefore introduces a policy by stability and height, analysing all 12 combinations across the unstable, neutral, and stable regimes for the full extrapolation range (30–200 m). By doing so, it reveals how the aggregate ranking decomposes into regime-dependent patterns and identifies which combinations are most effective under specific combinations of height and stratification.

Policy by stability and height: neutral regime

For the stability–height policy, the best model is selected independently for each atmospheric stability regime and each height. In this subsection, only the neutral regime is considered. The selection criterion is the lowest RMSE. Bias is then reported as a diagnostic of systematic error. Negative Bias indicates that the selected model underestimates the LiDAR wind speed, while positive Bias indicates overestimation.

Table 8: Best model combination by height under neutral conditions.

Height [m]	Model–MOST formulation	N	RMSE	Bias	Bias	MAE	Mean U_{model}	Mean U_{LiDAR}
30	Jimenez–LOG	234	1.115	0.076	0.076	0.802	10.385	10.309
41	Brutsaert–LOG	1254	1.162	0.248	0.248	0.864	11.541	11.293
70	Brutsaert–LOG	1254	1.288	0.387	0.387	0.980	12.270	11.883
100	Brutsaert–LOG	1254	1.419	0.539	0.539	1.098	12.831	12.292
120	Businger–LOG	4166	1.495	-0.403	0.403	1.128	11.533	11.936
140	Businger–LOG	4859	1.555	-0.479	0.479	1.171	11.650	12.129
160	Businger–LOG	5541	1.615	-0.549	0.549	1.218	11.717	12.267
180	Businger–LOG	5539	1.662	-0.590	0.590	1.252	11.802	12.392
200	Businger–LOG	5538	1.704	-0.622	0.622	1.284	11.878	12.500

Excluding the reference height at 3.7 m, the neutral-regime height-dependent strategy gives an overall weighted RMSE of 1.57 m s^{-1} and an overall weighted Bias of -0.41 m s^{-1} .

Table 8 shows that the neutral-regime policy is not controlled by a single formulation across all heights. At 30 m, the best RMSE is obtained by Jimenez–LOG. Between 41 and 100 m, Brutsaert–LOG gives the lowest RMSE. From 120 m upward, Businger–LOG becomes the best-performing option.

The Bias also changes sign with height. At 30 m, Jimenez–LOG presents a small positive Bias of 0.076 m s^{-1} , indicating a weak overestimation. Between 41 and 100 m, Brutsaert–LOG also overestimates the LiDAR wind speed, with Bias increasing from 0.248 to 0.539 m s^{-1} . Above 120 m, the selected Businger–LOG formulation systematically underestimates the LiDAR observations. The magnitude of this underestimation increases with height, from -0.403 m s^{-1} at 120 m to -0.622 m s^{-1} at 200 m.

These results indicate that the best neutral-condition policy changes with height. The lower part of the profile favours Jimenez–LOG and Brutsaert–LOG, while the upper part favours Businger–LOG. This confirms that the best model under neutral conditions cannot be selected only from the global ranking. It must also consider the target height.

Policy by stability and height: stable regime

The same RMSE-based selection is then applied to the stable regime. In this case, the interpretation requires additional care because the number of valid samples is smaller at several heights. This is expected because stable conditions are less frequent in the dataset and because the iterative closure is more sensitive when turbulence is weak. Therefore, the selected model is still the one with the lowest RMSE, but the corresponding value of N must be considered when interpreting the robustness of each choice.

Table 9: Best model combination by height under stable conditions.

Height [m]	Model–MOST formulation	N	RMSE	Bias	Bias	MAE	Mean U_{model}	Mean U_{LiDAR}
30	Jimenez–GRYN	21	1.900	-1.635	1.635	1.640	3.507	5.142
41	Holtslag–GRYN	25	2.179	-1.840	1.840	1.883	3.491	5.330
70	Holtslag–GRYN	6	1.962	-1.757	1.757	1.757	3.122	4.879
100	Holtslag–GRYN	6	2.158	-1.956	1.956	1.956	3.198	5.154
120	Holtslag–GRYN	6	2.216	-2.031	2.031	2.031	3.238	5.269
140	Holtslag–Lange	586	2.733	-1.731	1.731	2.221	5.166	6.897
160	Businger–Lange	237	2.580	-1.507	1.507	2.135	5.299	6.806
180	Businger–Lange	273	2.535	-1.242	1.242	2.072	5.733	6.975
200	Holtslag–Lange	588	2.442	-1.031	1.031	1.957	5.970	7.001

Excluding the reference height at 3.7 m, the stable-regime height-dependent strategy gives an overall weighted RMSE of 2.56 m s^{-1} and an overall weighted Bias of -1.39 m s^{-1} .

Table 9 shows that stable conditions produce a different selection pattern from the neutral regime. At 30 m, the lowest RMSE is obtained by Jimenez–GRYN. Between 41 and 120 m, Holtslag–GRYN gives the lowest RMSE. At higher levels, the selected formulation alternates between Holtslag–Lange and Businger–Lange.

The Bias is negative at all heights. This means that, under stable conditions, even the best RMSE-based combinations systematically underestimate the LiDAR wind speed. The magnitude of this underestimation is also larger than in the neutral regime. For example, at 120 m, Holtslag–GRYN has a Bias of -2.031 m s^{-1} , while at 200 m Holtslag–Lange has a Bias of -1.031 m s^{-1} . This confirms that stable stratification is the most difficult regime for the profile formulations tested in this thesis.

The sample size must also be considered. Some of the best RMSE values between 70 and 120 m are based on only six valid samples. These selections should therefore be interpreted as local best values within the available filtered dataset, not as robust universal recommendations. At 140–200 m, the selected models have larger sample support, which makes the interpretation more reliable. Overall, the stable-regime policy suggests that additional boundary-layer corrections can become useful, but the persistent negative Bias shows that the stable profiles remain systematically underestimated.

Policy by stability and height: unstable regime

The unstable regime is evaluated using the same RMSE-based policy. However, one methodological restriction is applied. Holtslag is not selected under unstable conditions because this formulation is not used as a valid unstable-regime correction in this thesis. Therefore, the unstable policy is selected only among the physically admissible combinations.

Table 10: Best model combination by height under unstable conditions.

Height [m]	Model–MOST formulation	N	RMSE	Bias	Bias	MAE	Mean U_{model}	Mean U_{LiDAR}
30	Businger–GRYN	110	1.094	-0.191	0.231	0.834	4.687	4.877
41	Businger–GRYN	169	1.140	-0.168	0.220	0.897	4.824	4.992
70	Brutsaert–GRYN	56	1.112	-0.457	0.457	0.869	5.266	5.723
100	Businger–GRYN	227	1.220	0.050	0.159	0.978	5.106	5.056
120	Brutsaert–GRYN	121	1.240	-0.342	0.342	0.941	5.401	5.743
140	Brutsaert–GRYN	171	1.273	-0.377	0.377	0.972	5.358	5.734
160	Brutsaert–GRYN	204	1.290	-0.342	0.342	1.002	5.497	5.839
180	Brutsaert–GRYN	204	1.317	-0.300	0.300	1.028	5.562	5.862
200	Businger–GRYN	175	1.371	0.256	0.296	1.125	5.505	5.249

Excluding the reference height at 3.7 m, the unstable-regime height-dependent strategy gives an overall weighted RMSE of 1.25 m s^{-1} and an overall weighted Bias of -0.18 m s^{-1} .

Table 10 shows that the unstable-regime policy is dominated by the Gryning-type formulation. At 30 and 41 m, Businger–GRYN gives the lowest RMSE. At 70 m, the selected model changes to Brutsaert–GRYN. At 100 m, Businger–GRYN becomes the best option again. Between 120 and 180 m, Brutsaert–GRYN gives the lowest RMSE, while at 200 m the best admissible combination is Businger–GRYN.

Compared with stable conditions, the unstable regime presents lower RMSE values. The selected RMSE values remain between 1.094 and 1.371 m s^{-1} across the full height range. This indicates that the tested formulations reproduce unstable profiles more consistently than stable profiles. The Bias is also smaller in magnitude than in the stable regime. Most selected combinations slightly underestimate the LiDAR wind speed, although Businger–GRYN shows weak overestimation at 100 and 200 m.

These results suggest that, under unstable conditions, the additional vertical structure introduced by the GRYN family improves the RMSE-based selection. This behaviour is different from the neutral regime, where LOG formulations dominate most of the height range. It is also different from the stable regime, where the best models still present large negative Bias. Therefore, the stability–height policy confirms that the best formulation depends strongly on the atmospheric regime. A single global model is not sufficient to describe the best RMSE-based choice across neutral, stable, and unstable conditions.

4.3.3 Policy per stability condition, height, and hour

The most detailed policy combines three sources of conditional variability: stability regime, target height, and hour of day. This policy is more specific than the previous tables because it allows the selected combination to change not only with height or stability separately, but with

their simultaneous occurrence during each hourly window. For consistency with the rest of this thesis, the selection criterion is the lowest RMSE within each valid bin. Bias is then retained to diagnose whether the selected combination tends to overestimate or underestimate the LiDAR observations. The number of valid samples is also reported whenever this policy is visualized, because the joint conditioning can create sparse bins.

The policy is based on the physical magnitude of the error, represented by RMSE, and on the sign of the systematic deviation, represented by Bias. This makes the interpretation direct, because both quantities are expressed in ms^{-1} . The selected model in each bin is therefore the one that minimizes the time-resolved wind-speed error, while the associated Bias indicates whether that selection remains systematically high or low.

The profile grids shown below are used as qualitative support for the RMSE–Bias interpretation. They help visualize whether each formulation reproduces the observed vertical structure of the LiDAR data. However, the figures do not define the ranking by themselves. A model can appear visually close to the mean profile and still perform poorly in RMSE if its time-resolved errors are large. Similarly, a model can have a competitive RMSE while retaining a non-negligible Bias. For this reason, visual profile agreement is interpreted together with the selection maps reported for each stability regime.

Neutral conditions. The traditional LOG formulation remains the most consistent family. The neutral profile grid shows that the classical MOST shape follows the LiDAR mean profile more closely than the Lange and GRYN reformulations over most of the vertical range. This behaviour is expected, because neutral stratification reduces the importance of additional stability and boundary-layer corrections. In this regime, the reformulated profiles tend to become more linear with height and can produce stronger departures from the LiDAR profile, especially at upper levels.

The height–hour selection map confirms this visual behaviour. Most selected cells belong to LOG-based combinations, although the empirical stability correction changes with height and hour. Businger–LOG and Brutsaert–LOG appear repeatedly, while other combinations occur only in more localized parts of the map. The Bias pattern also shows a gradual transition with height. At lower levels, the selected combinations remain close to the LiDAR observations, while at higher levels the selected models tend to become slightly low on average. Therefore, the neutral policy is controlled mainly by the robustness of the LOG profile, with secondary adjustments associated with the empirical correction and the hour of day.



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Stability condition: Neutral										
Hour [UTC]	30	41	70	100	120	140	160	180	200	Total
00	0.760 0.112 226/226	0.843 0.202 226/226	1.018 0.317 226/226	1.180 0.461 226/226	1.286 0.556 226/226	1.407 0.647 226/226	1.549 0.764 226/226	1.629 -0.889 251/251	1.670 -0.916 251/251	1.309 0.114 2084/2084
01	0.629 -0.227 234/234	0.703 -0.246 234/234	0.907 -0.399 234/234	1.110 -0.523 234/234	1.231 -0.605 234/234	1.328 -0.672 234/234	1.399 -0.720 234/234	1.463 -0.764 234/234	1.502 -0.775 234/234	1.182 -0.548 2106/2106
02	0.721 0.133 217/217	0.801 -0.282 240/240	0.992 0.373 217/217	1.164 0.517 217/217	1.265 0.617 217/217	1.367 -0.638 240/240	1.430 -0.674 240/240	1.473 -0.703 240/240	1.531 -0.729 240/240	1.233 -0.179 2068/2068
03	0.659 -0.211 230/230	0.714 -0.237 230/230	0.917 -0.356 230/230	1.062 -0.427 230/230	1.144 -0.481 230/230	1.214 -0.522 230/230	1.276 -0.562 230/230	1.325 -0.593 230/230	1.367 -0.606 230/230	1.103 -0.444 2070/2070
04	0.815 -0.294 246/246	0.870 -0.305 246/246	0.996 -0.405 246/246	1.119 -0.461 246/246	1.221 -0.514 246/246	1.292 -0.555 246/246	1.359 -0.592 246/246	1.420 -0.629 246/246	1.460 -0.656 246/246	1.193 -0.490 2214/2214
05	0.743 -0.217 244/244	0.792 -0.232 244/244	0.957 -0.344 244/244	1.118 -0.433 244/244	1.206 -0.500 244/244	1.273 -0.550 244/244	1.343 -0.601 244/244	1.397 -0.638 244/244	1.442 -0.663 244/244	1.167 -0.464 2196/2196
06	0.742 0.102 237/237	0.787 0.205 237/237	0.934 -0.317 237/237	1.065 0.405 237/237	1.155 0.551 237/237	1.227 -0.582 237/237	1.305 -0.593 237/237	1.343 -0.629 237/237	1.419 -0.721 237/237	1.132 -0.425 2133/2133
07	0.797 -0.222 243/243	0.835 -0.228 243/243	0.936 -0.304 243/243	1.042 -0.369 243/243	1.108 -0.415 243/243	1.158 -0.452 243/243	1.219 -0.495 243/243	1.294 -0.528 243/243	1.365 -0.565 243/243	1.100 -0.398 2187/2187
08	0.748 -0.188 243/243	0.784 -0.191 243/243	0.883 -0.257 243/243	0.988 -0.315 243/243	1.045 -0.348 243/243	1.096 -0.394 243/243	1.154 -0.432 243/243	1.217 -0.477 243/243	1.269 -0.506 243/243	1.035 -0.345 2187/2187
09	0.743 0.186 216/216	0.833 0.297 216/216	0.962 -0.312 243/243	1.049 -0.379 243/243	1.108 -0.409 243/243	1.172 -0.454 243/243	1.254 -0.491 243/243	1.343 -0.541 243/243	1.411 -0.569 243/243	1.124 -0.310 2133/2133
10	0.681 -0.217 243/243	0.723 -0.224 243/243	0.807 -0.294 243/243	0.879 -0.354 243/243	0.961 -0.386 243/243	1.044 -0.448 243/243	1.133 -0.498 243/243	1.218 -0.550 243/243	1.311 -0.590 243/243	0.995 -0.394 2187/2187
11	0.746 0.132 220/220	0.823 -0.279 251/251	0.948 -0.367 251/251	1.050 -0.436 251/251	1.130 -0.495 251/251	1.203 -0.551 251/251	1.282 -0.603 251/251	1.361 -0.657 251/251	1.431 -0.699 251/251	1.135 -0.447 2228/2228
12	0.772 0.035 221/221	0.856 0.122 221/221	1.013 0.258 221/221	1.162 0.409 221/221	1.248 -0.580 250/250	1.325 -0.644 250/250	1.400 -0.689 250/250	1.469 -0.734 250/250	1.536 -0.770 250/250	1.237 -0.315 2134/2134
13	0.677 0.172 212/212	0.754 -0.187 212/212	0.931 0.356 212/212	1.092 0.505 212/212	1.219 0.693 245/245	1.283 0.792 245/245	1.347 0.833 245/245	1.416 0.879 245/245	1.468 0.921 245/245	1.180 -0.266 2073/2073
14	0.695 0.172 213/213	0.751 -0.223 232/232	0.914 -0.328 233/233	1.051 -0.402 233/233	1.148 -0.471 233/233	1.224 -0.519 233/233	1.316 -0.579 233/233	1.387 -0.614 233/233	1.447 -0.644 233/233	1.136 -0.406 2076/2076
15	0.779 -0.156 237/237	0.825 -0.160 237/237	0.954 -0.267 237/237	1.100 -0.358 237/237	1.184 -0.412 237/237	1.276 -0.475 237/237	1.352 -0.524 237/237	1.415 -0.570 237/237	1.484 -0.618 237/237	1.177 -0.393 2133/2133
16	0.687 -0.144 250/250	0.741 -0.164 250/250	0.904 -0.266 250/250	1.074 -0.360 250/250	1.166 -0.421 250/250	1.257 -0.476 250/250	1.320 -0.523 250/250	1.385 -0.562 250/250	1.434 -0.597 250/250	1.138 -0.390 2250/2250
17	0.691 -0.154 259/259	0.747 -0.176 259/259	0.930 -0.289 259/259	1.096 -0.384 259/259	1.208 -0.447 259/259	1.303 -0.500 259/259	1.368 -0.540 259/259	1.442 -0.583 259/259	1.500 -0.618 259/259	1.177 -0.410 2331/2331
18	0.663 -0.141 249/249	0.720 -0.154 249/249	0.905 -0.268 249/249	1.085 -0.364 249/249	1.188 -0.435 249/249	1.272 -0.484 249/249	1.365 -0.551 249/249	1.422 -0.584 249/249	1.479 -0.617 249/249	1.157 -0.400 2241/2241
19	0.735 -0.180 240/240	0.784 -0.201 240/240	0.972 -0.323 240/240	1.131 -0.429 240/240	1.234 -0.505 240/240	1.325 -0.574 240/240	1.408 -0.629 240/240	1.476 -0.674 240/240	1.506 -0.695 240/240	1.206 -0.468 2160/2160
20	0.818 0.172 219/219	0.890 0.274 232/232	1.085 0.412 232/232	1.278 0.596 219/219	1.395 0.693 219/219	1.502 0.792 232/232	1.570 0.753 232/232	1.634 0.792 232/232	1.691 0.826 232/232	1.356 -0.269 2049/2049
21	0.722 0.211 214/214	0.827 0.297 214/214	1.022 0.433 214/214	1.198 0.573 214/214	1.315 0.656 214/214	1.435 0.740 214/214	1.562 0.833 214/214	1.650 0.787 237/237	1.717 -0.833 237/237	1.326 0.211 1972/1972
22	0.790 0.101 223/223	0.861 -0.313 240/240	1.053 0.304 223/223	1.190 0.441 223/223	1.293 0.518 223/223	1.399 0.596 223/223	1.521 0.696 223/223	1.673 0.802 223/223	1.740 -0.929 240/240	1.320 0.232 2041/2041
23	0.715 0.136 216/216	0.820 0.220 216/216	1.010 0.347 216/216	1.203 0.504 216/216	1.324 0.597 216/216	1.459 -0.744 236/236	1.531 -0.809 236/236	1.601 -0.855 236/236	1.665 -0.901 236/236	1.312 -0.193 2024/2024
Total	0.732 -0.053 5552/5552	0.796 -0.124 5655/5655	0.956 -0.143 5643/5643	1.104 -0.119 5630/5630	1.200 -0.220 5692/5692	1.288 -0.406 5748/5748	1.367 -0.436 5748/5748	1.440 -0.596 5796/5796	1.498 -0.693 5813/5813	1.186 -0.313 51277/51277

Legend (selected stability-correction + MOST formulation):

■ Businger-LOG ■ Businger-Lange ■ Businger-GRYN
■ Brutsaert-LOG ■ Brutsaert-Lange ■ Brutsaert-GRYN
■ Holtslag-LOG ■ Holtslag-Lange ■ Holtslag-GRYN
■ Jimenez-LOG ■ Jimenez-Lange ■ Jimenez-GRYN

Cell content: RMSE (top), Bias (middle), and N_{used}/N_{total} (bottom). Last row and last column show totals.

Fig. 19: Neutral-regime model selection by height and hour.

Stable conditions. The interpretation is more delicate. The profile grid shows that several LOG and Lange formulations can produce excessive vertical shear, especially above the lower part of the profile. This behaviour is visible as a strong increase of the modelled wind speed with height. In contrast, the GRYN reformulation often produces a more curved profile and avoids the strongest high-level acceleration. However, this does not mean that GRYN fully

solves the stable-regime problem. In many cases, it reduces the excessive shear but shifts the model towards systematic underestimation.

The selection map reflects this compromise. GRYN and Lange formulations appear frequently, but the selected combinations still retain a predominantly negative Bias. This means that even the best RMSE-based choices tend to remain below the LiDAR observations in stable conditions. The stable regime also has the weakest sample support among the three stability classes. A non-negligible fraction of model outputs exceeds the 40 m s^{-1} screening threshold and is therefore counted through the $N_{\text{used}}/N_{\text{total}}$ ratio. For this reason, stable-regime selections should not be interpreted only from the lowest RMSE. They must also be read together with Bias and sample support.

Physically, this result is consistent with the known difficulty of representing stable offshore profiles. Stable stratification can produce strong wind shear, intermittent turbulence, and partial decoupling between near-surface and upper-level winds. These processes are difficult to reproduce with simplified MOST-based formulations. Thus, the stable policy identifies the best available combination in each bin, but it also shows the limits of the tested formulations under stable conditions.

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Mean Profiles U(z) | Stability = Stable

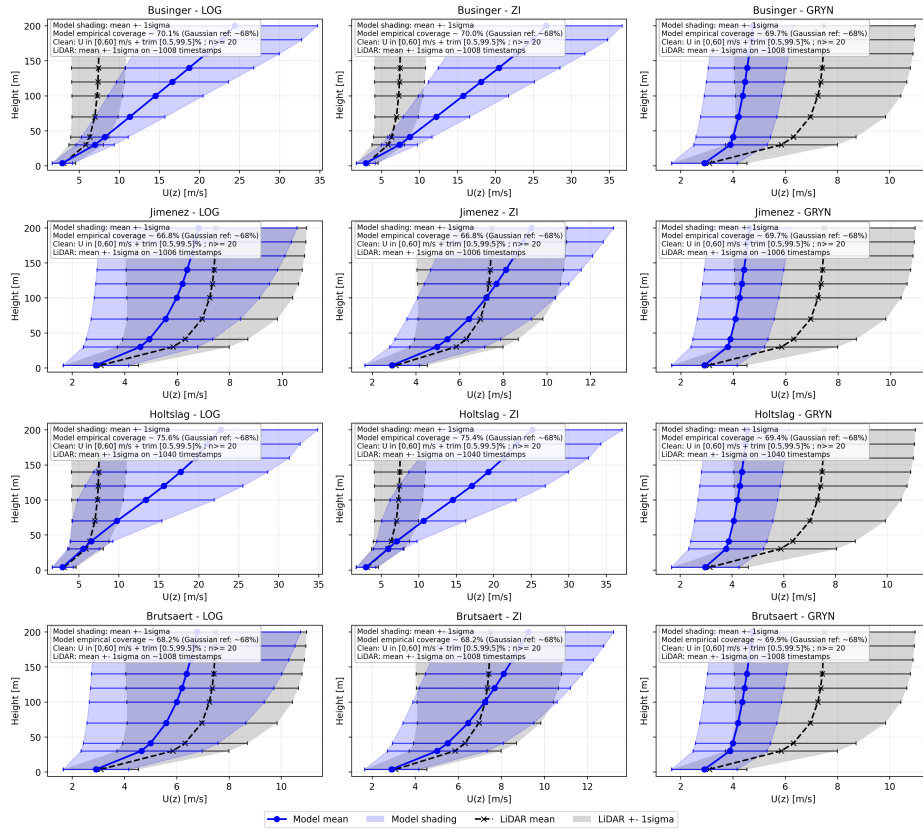


Fig. 20: Mean vertical profiles under stable conditions.

Stability condition: Stable										
Hour [UTC]	30	41	70	100	120	140	160	180	200	Total
00	0.973 -0.156 24/32	1.411 0.250 24/32	3.301 1.715 24/32	3.828 -2.954 40/40	3.795 -2.779 40/40	3.694 -2.526 40/40	3.585 -2.255 40/40	3.478 -1.977 40/40	3.346 -1.683 40/40	3.272 -1.515 312/336
01	1.379 -0.338 22/37	1.560 -0.465 25/37	2.922 0.893 25/37	3.709 -2.909 37/37	3.693 -2.765 37/37	3.626 -2.575 37/37	3.543 -2.319 37/37	3.393 -2.026 37/37	3.241 -1.725 37/37	3.127 -1.581 294/333
02	1.161 0.023 26/35	1.155 -0.324 29/35	2.278 1.132 29/35	3.827 -3.071 36/36	3.792 -2.880 36/36	3.686 -2.632 36/36	3.519 -2.334 36/36	3.382 -2.062 36/36	3.209 -1.757 36/36	3.076 -1.562 300/321
03	1.608 -0.524 27/32	1.761 -0.169 27/32	3.002 1.281 27/32	3.807 -2.999 34/34	3.776 -2.771 34/34	3.683 -2.471 34/34	3.563 -2.197 34/34	3.381 -1.900 34/34	3.226 -1.596 34/34	3.206 -1.516 285/300
04	1.033 0.091 23/30	1.376 0.519 23/30	2.823 1.674 27/29	3.690 -2.814 31/31	3.665 -2.622 31/31	3.546 -2.378 31/31	3.373 -2.020 30/30	3.170 -1.683 30/30	2.953 -1.399 31/31	3.003 -1.208 257/273
05	0.947 -0.567 26/30	0.991 -0.366 26/30	1.800 1.038 26/30	3.340 -2.470 33/33	3.358 -2.252 33/33	3.331 -2.011 33/33	3.232 -1.731 33/33	3.100 -1.390 33/33	2.979 -1.037 33/33	2.773 -1.237 276/288
06	1.267 -0.013 27/34	1.304 -0.289 27/34	2.502 1.201 27/34	3.025 -2.104 34/34	3.003 -1.896 34/34	2.963 -1.677 34/34	2.881 -1.435 35/35	2.822 -1.127 35/35	2.762 -0.802 35/35	2.593 -0.973 288/309
07	1.976 -0.599 16/27	2.257 -0.213 16/27	3.152 -2.507 29/29	3.144 -2.270 29/29	3.025 -2.064 29/29	2.874 -1.799 29/29	2.730 -1.509 29/29	2.638 -1.176 29/29	2.536 -0.838 29/29	2.738 -1.458 235/257
08	1.545 -0.740 13/24	1.794 -0.580 13/24	2.903 0.598 13/24	3.041 -2.495 25/25	2.922 -2.210 25/25	2.770 -1.899 24/24	2.583 -1.546 24/24	2.404 -1.202 24/24	2.246 -0.849 24/24	2.521 -1.224 185/218
09	1.355 -0.420 16/25	1.495 -0.042 16/25	3.007 -2.394 27/27	2.805 -1.872 27/27	2.613 -1.521 27/27	2.421 -1.182 27/27	2.296 -0.847 27/27	2.225 -0.548 27/27	2.163 -0.263 27/27	2.335 -1.023 221/239
10	2.014 0.018 11/22	3.039 -2.532 26/26	3.238 -2.430 26/26	2.946 -1.960 26/26	2.702 -1.623 26/26	2.459 -1.286 26/26	2.253 -0.960 26/26	2.120 -0.673 26/26	2.055 -0.429 26/26	2.582 -1.333 219/230
11	2.003 0.233 7/26	2.355 0.172 8/26	3.202 -2.525 26/26	3.015 -2.158 26/26	2.837 -1.867 26/26	2.693 -1.627 26/26	2.568 -1.392 26/26	2.437 -1.159 26/26	2.332 -0.953 26/26	2.628 -1.253 197/234
12	2.098 0.681 7/22	3.042 -2.543 23/23	3.328 -2.663 23/23	3.278 -2.421 23/23	3.138 -2.197 23/23	2.976 -1.979 23/23	2.808 -1.752 23/23	2.669 -1.539 23/23	2.491 -1.275 23/23	2.898 -1.755 191/206
13	1.140 0.013 12/25	1.528 0.413 12/25	2.756 -2.138 27/27	2.401 -1.786 27/27	2.183 -1.547 27/27	2.040 -1.319 27/27	1.871 -1.032 27/27	1.741 -0.827 27/27	1.610 -0.549 27/27	1.982 -1.333 213/239
14	1.107 0.046 16/30	1.980 0.650 15/31	2.460 -1.807 33/33	2.299 -1.636 30/30	2.132 -1.426 30/30	1.963 -1.185 30/30	1.804 -0.908 30/30	1.691 -0.651 30/30	1.604 -0.373 30/30	1.938 -0.816 244/274
15	1.250 -0.387 23/36	1.467 0.015 23/36	2.480 -1.875 36/36	2.635 -1.777 36/36	2.610 -1.638 36/36	2.507 -1.429 36/36	2.385 -1.197 36/36	2.267 -0.944 36/36	2.178 -0.716 36/36	2.248 -1.105 298/324
16	1.152 -0.049 18/36	1.809 0.022 21/35	2.741 -2.190 37/37	2.868 -2.082 37/37	2.908 -1.912 37/37	2.896 -1.688 37/37	2.901 -1.499 37/37	2.810 -1.278 37/37	2.745 -1.059 37/37	2.611 -1.316 298/330
17	1.942 0.018 21/33	2.657 0.667 21/33	2.886 -2.348 36/36	2.857 -1.136 36/36	2.828 -1.972 36/36	2.776 -1.760 36/36	2.681 -1.505 36/36	2.639 -1.272 36/36	2.576 -1.020 36/36	2.670 -1.289 294/318
18	1.683 -0.151 24/43	2.431 0.324 24/43	3.095 -2.464 44/44	3.111 -2.286 44/44	3.035 -2.115 44/44	2.921 -1.932 44/44	2.795 -1.708 44/44	2.691 -1.501 44/44	2.599 -1.295 44/44	2.743 -1.467 356/394
19	2.274 0.026 20/41	2.806 -2.282 42/42	3.129 -2.398 43/43	3.005 -2.203 43/43	2.889 -2.049 43/43	2.774 -1.902 43/43	2.710 -1.750 43/43	2.675 -1.573 43/43	2.643 -1.391 43/43	2.779 -1.732 363/384
20	1.546 -0.097 21/37	1.991 0.287 21/37	3.112 -2.426 39/39	3.089 -2.294 39/39	3.046 -2.172 39/39	2.987 -2.046 39/39	2.927 -1.889 39/39	2.852 -1.708 39/39	2.795 -1.512 39/39	2.764 -1.558 315/347
21	1.714 -0.321 28/42	2.126 0.127 28/42	3.254 -2.523 42/42	3.280 -2.409 42/42	3.247 -2.284 42/42	3.179 -2.100 42/42	3.083 -1.893 42/42	2.979 -1.674 42/42	2.852 -1.448 42/42	2.905 -1.614 350/378
22	1.435 -0.696 28/43	1.509 -0.509 28/43	2.412 0.607 28/43	3.407 -2.657 43/43	3.404 -2.515 43/43	3.375 -2.338 43/43	3.314 -2.134 43/43	3.247 -1.945 43/43	3.156 -1.749 43/43	2.910 -1.549 342/387
23	1.218 -0.081 25/41	1.468 0.317 25/41	3.167 1.747 25/41	3.693 -2.862 43/43	3.752 -2.718 40/40	3.744 -2.563 40/40	3.675 -2.357 40/40	3.544 -2.112 40/40	3.409 -1.838 40/40	3.213 -1.381 318/366
Total	1.531 -0.203 481/783	1.958 -0.244 543/789	2.892 -0.956 719/805	3.226 -2.386 821/821	3.168 -2.192 818/818	3.078 -1.970 817/817	2.971 -1.721 817/817	2.860 -1.465 817/817	2.751 -1.203 818/818	2.777 -1.371 6651/7285

Legend (selected stability-correction + MOST formulation):

Businger-LOG Businger-Lange Businger-GRYN
 Brutsaert-LOG Brutsaert-Lange Brutsaert-GRYN
 Holtslag-LOG Holtslag-Lange Holtslag-GRYN
 Jimenez-LOG Jimenez-Lange Jimenez-GRYN

Cell content: RMSE (top), Bias (middle), and N_{used}/N_{total} (bottom). Last row and last column show totals.

Fig. 21: Stable-regime model selection by height and hour.

Unstable conditions. The behaviour is different from the stable case. The profile grid shows smaller separation between modelled and observed mean profiles. The selected combinations also present lower RMSE values and Bias values with smaller magnitude. This indicates that the tested formulations reproduce unstable profiles more consistently than stable profiles in the present dataset.

Chapter 4. Results

The selection map is more heterogeneous than in neutral conditions. At lower levels, Lange formulations appear frequently. At intermediate heights, Brutsaert–Lange becomes relevant in several hourly windows. At higher levels, LOG and GRYN combinations also appear depending on the hour. This pattern suggests that, under unstable stratification, no single formulation controls the whole height–hour domain. Instead, the best response depends more strongly on the interaction between height and time of day.

Holtslag is not selected in the unstable map because it is not treated as a valid unstable-regime correction in this thesis. This choice avoids assigning physical meaning to a correction outside its intended stability range. Overall, the unstable regime gives the most favourable conditional performance among the three stability classes. Its Bias is slightly positive, while the neutral and stable regimes tend to be negative on average. This means that unstable selections are closer to being centred around the LiDAR observations, although some height–hour variability remains.

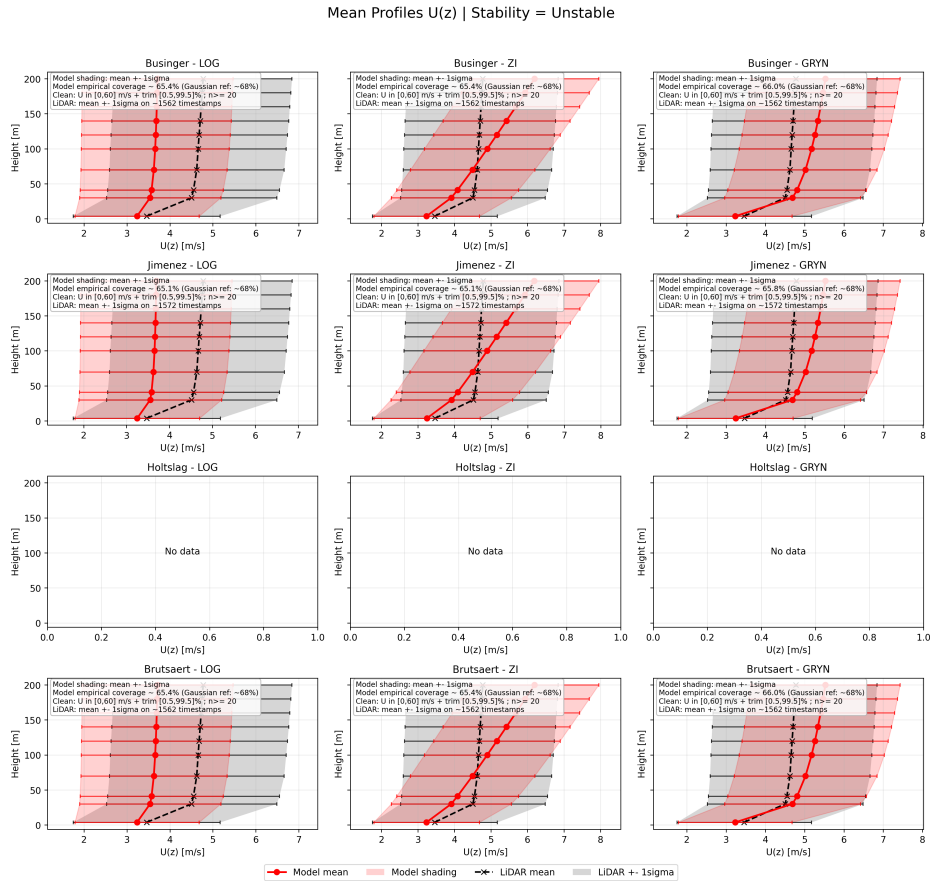


Fig. 22: Mean vertical profiles under unstable conditions.

Chapter 4. Results

Stability condition: Unstable										
Hour [UTC]	30	41	70	100	120	140	160	180	200	Total
00	1.016 -0.187 60/60	1.031 -0.039 60/60	1.046 -0.136 79/79	1.054 -0.031 79/79	1.053 -0.010 79/79	1.085 0.016 79/79	1.108 0.013 79/79	1.129 0.017 79/79	1.167 0.032 79/79	1.081 -0.032 673/673
01	0.826 -0.164 67/67	0.823 -0.001 67/67	0.772 -0.100 71/71	0.835 0.315 71/71	0.970 0.048 71/71	1.000 0.077 71/71	1.039 0.117 71/71	1.084 0.155 71/71	1.126 0.178 71/71	0.953 0.074 673/673
02	0.818 0.153 54/54	0.888 0.350 54/54	0.960 0.115 62/62	1.029 0.224 76/76	1.123 0.342 62/62	1.187 0.407 62/62	1.253 0.038 76/76	1.292 0.074 76/76	1.338 0.104 76/76	1.130 0.191 598/598
03	1.027 0.125 63/63	1.042 -0.191 65/65	1.066 -0.235 71/71	1.073 0.179 71/71	1.166 0.447 71/71	1.229 0.180 65/65	1.262 -0.083 71/71	1.345 -0.071 71/71	1.425 -0.080 71/71	1.192 0.030 619/619
04	1.032 0.047 55/55	1.079 -0.314 60/60	1.081 -0.134 60/60	1.015 -0.370 28/28	0.991 -0.312 28/28	0.954 -0.257 28/28	0.965 -0.220 28/28	0.991 -0.182 28/28	1.082 -0.148 28/28	1.034 -0.192 343/343
05	0.967 0.124 58/58	1.017 -0.256 60/60	0.997 -0.166 79/79	1.064 0.262 79/79	1.196 0.526 79/79	1.270 0.019 79/79	1.328 0.042 79/79	1.396 0.076 79/79	1.438 0.089 79/79	1.209 0.088 671/671
06	0.981 -0.184 87/87	1.024 -0.117 87/87	0.979 -0.020 87/87	1.091 0.436 87/87	1.180 0.194 87/87	1.233 0.241 87/87	1.273 0.308 87/87	1.318 0.323 87/87	1.357 0.345 87/87	1.168 0.163 783/783
07	0.890 -0.122 64/64	0.948 -0.357 86/86	0.952 0.060 86/86	1.104 0.501 86/86	1.225 0.346 64/64	1.288 0.295 86/86	1.339 0.339 86/86	1.405 0.350 86/86	1.435 0.372 86/86	1.199 0.203 730/730
08	0.894 -0.116 73/73	0.909 0.053 73/73	0.892 0.013 87/87	1.030 0.468 87/87	1.115 0.210 87/87	1.166 0.252 87/87	1.222 0.280 87/87	1.295 0.296 87/87	1.344 0.321 87/87	1.116 0.206 755/755
09	0.986 0.224 61/61	1.039 -0.065 69/69	1.048 -0.069 88/88	1.153 0.405 88/88	1.277 0.195 87/87	1.336 0.235 87/87	1.391 0.274 87/87	1.445 0.291 87/87	1.505 0.326 87/87	1.268 0.207 741/741
10	1.168 0.105 59/59	1.188 -0.158 65/65	1.117 -0.139 84/84	1.182 0.317 84/84	1.303 0.603 84/84	1.357 0.168 84/84	1.389 0.219 84/84	1.429 0.231 84/84	1.470 0.280 84/84	1.202 0.192 712/712
11	1.142 0.166 57/57	1.160 -0.518 88/88	1.087 -0.090 88/88	1.178 0.347 88/88	1.304 0.621 88/88	1.403 0.151 87/87	1.437 0.194 87/87	1.478 0.228 87/87	1.517 0.245 87/87	1.315 0.148 757/757
12	1.004 -0.153 66/66	1.019 0.044 66/66	0.984 -0.049 87/87	1.089 0.385 87/87	1.232 0.676 87/87	1.284 0.257 86/86	1.316 0.295 86/86	1.351 0.337 86/86	1.376 0.373 86/86	1.202 0.257 737/737
13	0.915 0.364 59/59	1.037 -0.539 59/59	0.998 -0.105 92/92	1.096 0.334 92/92	1.214 0.613 92/92	1.237 0.895 59/59	1.270 0.923 59/59	1.273 -0.930 59/59	1.299 0.945 59/59	1.149 0.138 630/630
14	1.057 0.215 58/58	1.067 -0.052 65/65	1.025 -0.057 76/76	1.109 0.365 76/76	1.219 0.643 76/76	1.267 0.144 75/75	1.285 0.166 75/75	1.332 0.174 75/75	1.360 0.180 75/75	1.202 0.201 651/651
15	1.056 0.149 55/55	1.040 -0.089 60/60	1.051 -0.152 75/75	1.094 0.272 75/75	1.214 0.535 75/75	1.320 0.262 60/60	1.364 0.084 75/75	1.383 0.094 75/75	1.393 0.140 75/75	1.228 0.146 625/625
16	1.119 -0.057 49/49	1.155 0.128 49/49	1.179 -0.251 68/68	1.172 0.190 68/68	1.245 0.456 68/68	1.301 0.123 58/58	1.336 0.185 58/58	1.341 0.231 58/58	1.388 0.061 68/68	1.255 0.121 544/544
17	0.945 0.191 45/45	0.987 -0.144 49/49	0.965 0.010 63/63	1.057 0.231 63/63	1.168 0.067 63/63	1.187 0.101 63/63	1.212 0.147 63/63	1.226 0.210 63/63	1.239 0.210 63/63	1.124 0.122 535/535
18	0.924 -0.216 49/49	0.934 -0.103 49/49	0.935 0.010 61/61	1.031 0.231 49/49	1.090 0.293 49/49	1.102 0.345 49/49	1.160 0.408 49/49	1.185 0.443 49/49	1.212 0.476 49/49	1.066 0.204 453/453
19	0.912 0.253 47/47	0.989 -0.045 50/50	0.977 -0.011 67/67	1.080 0.435 67/67	1.141 0.272 49/49	1.154 0.322 49/49	1.210 0.384 49/49	1.246 0.427 49/49	1.272 0.434 49/49	1.110 0.269 476/476
20	1.044 -0.192 61/61	1.059 -0.003 61/61	1.045 -0.106 69/69	1.093 0.072 69/69	1.110 -0.024 69/69	1.158 0.002 69/69	1.201 0.011 69/69	1.259 0.032 69/69	1.296 0.032 69/69	1.146 -0.034 605/605
21	0.995 -0.211 60/60	0.991 -0.038 60/60	1.081 0.042 60/60	1.147 0.181 60/60	1.204 0.241 60/60	1.260 0.304 60/60	1.296 0.331 60/60	1.361 0.367 60/60	1.382 0.389 60/60	1.199 0.178 540/540
22	0.893 -0.115 63/63	0.907 -0.156 75/75	0.888 -0.013 76/76	1.024 0.394 76/76	1.120 0.059 75/75	1.180 0.077 75/75	1.239 0.108 75/75	1.283 0.128 75/75	1.329 0.144 75/75	1.111 0.073 665/665
23	0.780 -0.028 64/64	0.821 0.151 64/64	0.946 0.329 64/64	1.058 0.155 76/76	1.095 0.207 76/76	1.129 0.232 76/76	1.157 0.255 76/76	1.210 0.266 76/76	1.240 0.246 76/76	1.070 0.204 648/648
Total	0.977 0.004 1434/1434	1.011 -0.074 1541/1541	1.006 -0.058 1806/1806	1.083 0.287 1788/1788	1.176 0.328 1732/1732	1.227 0.141 1687/1687	1.269 0.138 1722/1722	1.312 0.161 1722/1722	1.351 0.173 1732/1732	1.168 0.127 15164/15164

Legend (selected stability-correction + MOST formulation):

■ Businger--LOG ■ Businger--Lange ■ Businger--GRYN
■ Brutsaert--LOG ■ Brutsaert--Lange ■ Brutsaert--GRYN
■ Holtslag--LOG ■ Holtslag--Lange ■ Holtslag--GRYN
■ Jimenez--LOG ■ Jimenez--Lange ■ Jimenez--GRYN

Cell content: RMSE (top), Bias (middle), and N_{used}/N_{total} (bottom). Last row and last column show totals.

Fig. 23: Unstable-regime model selection by height and hour.

Considering the three stability-conditioned strategies together, and weighting each regime by its total sample support, the final overall RMSE is 1.42 m s^{-1} and the final overall Bias is -0.33 m s^{-1} . The total sample fraction is $N_{used}/N_{total} = 73092/73726$.

Overall, the combined policy by stability, height, and hour should be read as an adaptive selection tool based on RMSE, Bias, and sample support. The best model in a given bin is the one with the lowest RMSE. Bias then indicates whether that selected model is systematically high or low relative to the LiDAR observations. Finally, $N_{\text{used}}/N_{\text{total}}$ indicates how much confidence should be placed in the selected rule. This structure keeps the validation in physical wind-speed units and avoids reducing model performance to a dimensionless hit-rate indicator.

The main conclusion from this policy is that the best formulation depends strongly on the stability regime. Neutral conditions favour the classical LOG family. Stable conditions remain the most problematic and require cautious interpretation because of larger Bias and reduced sample support. Unstable conditions show the most favourable overall behaviour, but with a more heterogeneous selection across height and hour. Thus, the adaptive policy is useful because it exposes where each formulation works, but it also makes clear that stable offshore profiles remain the main limitation of the tested MOST-based approach.

4.4 Comparison between policies and choice of the operational metric

Table 11 summarizes the main levels of model selection used in this chapter. The first block compares the empirical stability corrections when the MOST-based formulations are grouped. The second block keeps each empirical correction fixed, but allows the MOST formulation to change with height. The final block allows both the empirical correction and the MOST formulation to vary according to height, hour, stability, or their combined effect. This structure shows the transition from fixed formulations to adaptive selection policies.

Table 11: Summary of the evaluated model-selection strategies.

Model or selection rule	RMSE	Bias	Highlights
Filtered results comparing only the empirical atmospheric-stability corrections, with LOG, Lange, and GRYN grouped together			
Businger–Dyer	1.96	0.47	Moderate positive Bias. The correction remains relatively balanced near the central profile, but the error dispersion increases with height.
Brutsaert	1.72	0.35	Best grouped empirical correction in terms of RMSE. It behaves well at lower levels, but progressively overestimates aloft.
Holtslag	2.02	-0.32	The mean profile can appear close to LiDAR, but the RMSE remains high due to large error dispersion.
Jimenez	1.92	0.64	Energetic grouped behaviour. The positive Bias increases with height, indicating systematic upper-level overestimation.
Filtered results selecting the best MOST formulation by height for each fixed empirical stability correction			
Businger–Dyer	1.73	-0.94	LOG is selected at all evaluated heights in the corrected table. The RMSE improves relative to the grouped case, but the strategy introduces persistent underestimation.
Brutsaert	1.94	0.13	LOG is preferred at lower heights, while Lange becomes selected above 100 m. The overall Bias remains close to zero, but RMSE increases relative to the grouped result.
Holtslag	2.03	0.15	GRYN is selected at the lowest levels and Lange dominates above 70 m. The overall Bias is small, but the RMSE remains high.
Jimenez	2.15	0.59	LOG is selected up to 120 m and Lange above 140 m. This is the weakest corrected height-dependent strategy among the four empirical corrections.
Filtered adaptive policies allowing both the empirical stability correction and the MOST formulation to change			
By height	1.74	-0.72	The best selection is mostly controlled by Businger–LOG above the lowest validation level, with a persistent negative Bias.
By hour	1.32	-0.06	This strategy gives the lowest RMSE among the intermediate policies and strongly reduces the mean Bias, but some hourly bins have limited sample support.
By stability and height	1.63	-0.45	The selection improves the physical conditioning of the model choice. Neutral and unstable regimes perform better, while stable conditions remain limiting.
By stability, height, and hour	1.42	-0.33	This is the most detailed adaptive strategy. It balances stability, height, and temporal variability, while keeping the final result weighted by sample support.

The first result is that the grouped empirical-correction analysis remains competitive. When LOG, Lange, and GRYN are grouped within each stability correction, Brutsaert gives the lowest RMSE among the fixed empirical corrections, followed by Jimenez and Businger-Dyer. This does not mean that grouping the MOST formulations is physically preferable. Rather, it indicates that the main source of error is not explained only by changing the MOST formulation at different heights. The empirical correction itself and the atmospheric state still play an important role in the final error.

This point becomes clearer in the second block of Table 11. When each empirical correction is kept fixed and the best MOST formulation is selected by height, the improvement is not systematic. For Businger–Dyer, the height-dependent selection reduces the RMSE relative to the grouped result. However, the same behaviour is not observed for Brutsaert, Holtslag, and Jimenez. For these corrections, selecting LOG, Lange, or GRYN only as a function of height does not improve the overall result. In some cases, the RMSE becomes higher than in the grouped empirical-correction analysis.

This means that the mixed MOST formulation should not be interpreted as a universal replacement for the individual profile formulations. It is an optimization procedure that can be useful in specific cases, but height alone is not sufficient to determine the best vertical formulation. The vertical level explains part of the error growth, because extrapolation uncertainty increases with distance from the reference height. However, the results show that this is not the only controlling factor. The performance also depends on the stability regime and on time-dependent boundary-layer conditions.

The adaptive policies in the third block confirm this interpretation. The policy based only on height gives an overall RMSE of 1.74 m s^{-1} and a Bias of -0.72 m s^{-1} . This is not better than the best grouped empirical correction and remains affected by systematic underestimation. Therefore, selecting among all 12 combinations only by height does not fully solve the model-selection problem. The result suggests that the best formulation cannot be determined from the target height alone.

The hourly policy gives a stronger improvement, with an overall RMSE of 1.32 m s^{-1} and a Bias close to zero. This indicates that the hour of day captures part of the variability that is not represented by height alone. Even in offshore conditions, where the diurnal cycle is weaker than over land, the hour can still act as a proxy for changes in air–sea temperature contrast, turbulence intensity, and boundary-layer evolution. For this reason, the hourly conditioning improves the selection more effectively than a purely height-dependent strategy.

The policies that include atmospheric stability also support this conclusion. When the selection depends on both stability and height, the RMSE decreases relative to the height-only policy. When stability, height, and hour are combined, the final policy gives an overall RMSE of 1.42 m s^{-1} and a Bias of -0.33 m s^{-1} . This is not the lowest RMSE among all adaptive policies, but it is the most physically complete rule because it combines vertical position, atmospheric

regime, and temporal variability. It therefore provides a more interpretable compromise between numerical performance and physical consistency.

Overall, the results show that the main limitation is not simply the choice between LOG, Lange, and GRYN at a given height. The performance depends on how each formulation interacts with atmospheric stability and time-dependent offshore boundary-layer conditions. A height-dependent MOST selection can improve some cases, but it does not represent the full variability of the wind profile. The most useful strategies are those that include information about the atmospheric state, especially stability and hour of day. For this reason, the final interpretation should not be based on a single universal formulation. A fixed formulation can be used as a practical baseline, but adaptive selection becomes more appropriate when height, stability, and temporal information are available.

4.5 External benchmark against CERRA2

The construction of the height-matched CERRA2 benchmark was described in Section 3.9. This section compares CERRA2-derived wind speeds with LiDAR observations and MOST-based formulations. The objective is to determine whether the proposed LiDAR-anchored extrapolation framework provides a closer representation of the observed offshore wind profile than the open reanalysis-based benchmark.

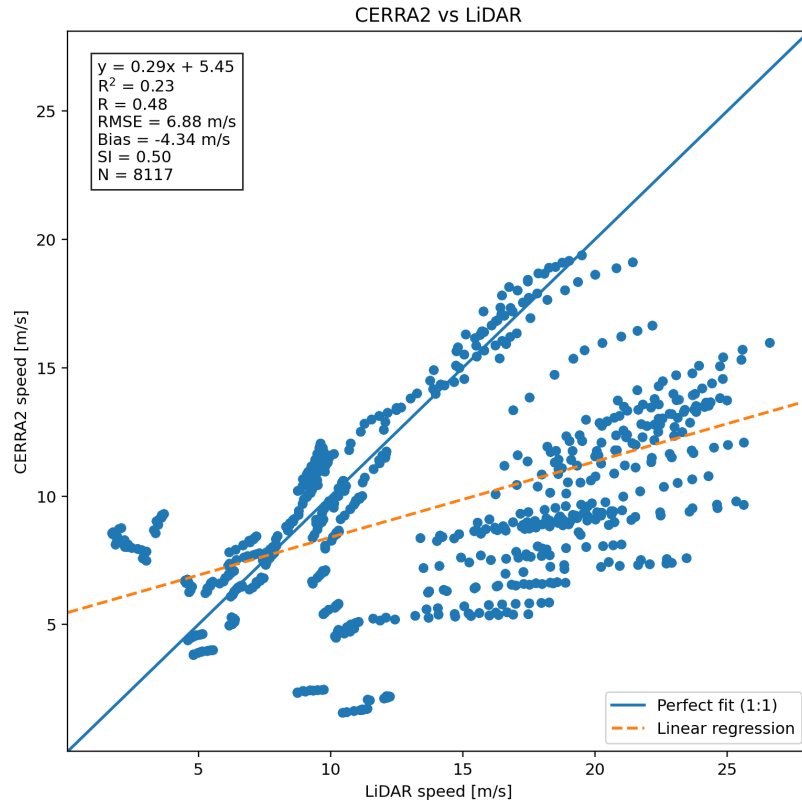


Fig. 24: Comparison between CERRA2-derived wind speed and LiDAR wind speed for the matched validation dataset.

Figure 24 shows that the CERRA2-derived wind speeds present a broad scatter relative to the LiDAR observations and a clear tendency toward underestimation. The fitted regression line remains far from the 1:1 reference line, indicating that CERRA2 does not reproduce the recorded variability as closely as the LiDAR-anchored MOST formulations. For the global matched dataset, the comparison resulted in an RMSE of 6.88 m s^{-1} , a Bias of -4.34 m s^{-1} , and an R^2 of 0.23, confirming the poor agreement between CERRA2 and the LiDAR observations. These values indicate that CERRA2 systematically underestimates the LiDAR wind speed and exhibits substantially larger dispersion, reinforcing the conclusion that it does not match the observed profile well.

This behaviour is consistent with the caution raised by Gandoin and Garza [7], which is relevant here because the present comparison likewise uses a field derived from reanalysis against LiDAR observations. Although their work focused on open reanalysis products in a wider offshore context, its main methodological message remains relevant here: reanalysis datasets are useful because they provide spatially complete and accessible wind fields, but they should not be interpreted as direct observations lacking local validation. In the present

comparison, this limitation is evident in the systematic negative Bias of CERRA2 relative to the LiDAR measurements, which supports the conclusion that reanalysis alone is insufficient here.

5 Conclusion

5.1 General outcome and methodological contribution

This thesis addressed a practical problem in offshore wind-resource assessment: the need to estimate wind speed at turbine-relevant heights when direct measurements are only available near the sea surface. This limitation is especially relevant for modern offshore turbines, whose rotor-swept area can range from approximately 30 to 200 m. In this context, the main question was whether a reduced-input methodology could reproduce the observed vertical wind profile with sufficient reliability for preliminary offshore applications.

The results show that reduced-input vertical extrapolation is feasible, but only when the modelling approach is interpreted as conditional rather than universal. The proposed methodology generated vertical wind-speed profiles that reproduced the main structure of the LiDAR observations over the evaluated height range. The agreement was not perfect at every timestamp, and the errors increased with height and atmospheric complexity. However, the mean profiles, RMSE values, and Bias diagnostics indicate that MOST-based formulations can describe the dominant offshore wind-profile behaviour when the missing surface-layer quantities are estimated through a consistent physical closure.

A first contribution of this work was the development of a complete computational framework for this reduced-input problem. The algorithm starts from the measured near-surface wind speed and combines it with open meteorological variables required for stability and thermodynamic closure. The iterative loop estimates the friction velocity u_* , aerodynamic roughness length z_0 , and Monin–Obukhov length L in a coupled way. This coupling is necessary because these quantities are not independent: the wind-profile equation depends on u_* and z_0 , the roughness length is updated through the Charnock relation, and L depends strongly on u_* and on the sensible heat flux. By repeating this procedure until convergence, the methodology provides a physically traceable set of variables for each timestamp.

A second contribution was the unified evaluation of different empirical stability corrections and MOST-based profile reformulations under the same dataset, numerical workflow, and validation criteria. After convergence, the algorithm computed three profile families: the classical logarithmic MOST formulation, the Lange-type boundary-layer-height correction, and the Gryning-type reformulation. These profile families were combined with the Businger–Dyer, Brutsaert, Holtslag, and Jimenez empirical stability corrections. This allowed the thesis to

compare formulations that are often discussed separately in the literature, but not always tested under the same operational framework.

A third contribution was the construction of a validation structure that separates the input boundary condition from the extrapolated part of the profile. The wind speed measured at 3.7 m was used as the lower boundary condition. Thus, the model output at this height was fixed to the measured value U_{ref} , while the validation focused on the higher LiDAR levels. This distinction is important because the objective was not to reproduce the input height, but to evaluate whether the models could represent the observed increase in wind speed with height. The comparison therefore focused on the heights at which the profile formulations differ.

Finally, an external benchmark based on CERRA2 was included. The height-matched CERRA2 dataset was constructed from the available reanalysis wind speeds at 10 and 100 m and compared against the same LiDAR observations used to validate the MOST-based formulations. This benchmark was not treated as a reference for observation. Instead, it was used as an independent open-data product against which the proposed LiDAR-anchored framework could be evaluated.

Overall, the methodology provides a unified and reproducible framework for testing MOST-based offshore wind-profile formulations under limited-data conditions. Its contribution is not only the computation of vertical profiles, but also the organisation of the modelling process into a transparent sequence of physical closure, profile generation, validation, conditional model comparison, and external benchmarking.

5.2 Main findings and model-selection strategy

The results showed that RMSE alone is not sufficient to interpret model performance. Bias was essential for identifying systematic overestimation and underestimation. Some model combinations produced acceptable error magnitudes while still showing persistent underestimation at upper levels. Other combinations behaved more energetically and tended to overestimate the LiDAR wind speed. These tendencies are relevant for offshore wind-resource assessment because systematic errors in hub-height wind speed can directly affect energy-yield estimates and uncertainty evaluation. For this reason, the results were interpreted using both RMSE and Bias.

A central conclusion is that there is no single universally best model. The performance depends on the target height, atmospheric stability, hour of the day, and selected profile formulation. This confirms that model choice should not be treated as a fixed decision valid for all offshore conditions. A formulation that performs well in one regime can become less reliable in another. Neutral conditions were generally more compatible with classical logarithmic formulations, while unstable conditions often benefited from the Gryning-type structure. Stable conditions remained the most difficult regime, even after conditional model selection.

The comparison between selection strategies reinforces this interpretation. Height-only selection was useful because it accounted for the vertical dependence of the error, but it did not fully capture the variability in model performance. The hourly strategy reduced the overall error and produced a Bias closer to zero, indicating that time of day can act as a useful proxy for changes in the offshore boundary layer. The most complete strategy was the adaptive selection based on stability condition, height, and hour together. This strategy does not select a single model for the full dataset. Instead, it selects the empirical stability correction and the profile formulation according to the atmospheric and vertical context. This is one of the main conclusions of the thesis: offshore wind extrapolation is better addressed through conditional model selection than through a single fixed correction.

This conclusion is especially relevant when atmospheric stability is known or can be estimated with reasonable confidence. In that case, the recommended approach is to segment the model choice by stability regime, target height, and hour of day. If stability information is unavailable or considered too uncertain, a less complete but still useful alternative is to organise the model choice by height and time window. The results show that hourly grouping can capture some of the variability not captured by height alone. Therefore, even when stability classification is uncertain, the model-selection process should not necessarily return to a single universal formulation.

The stable regime deserves particular attention. The tested formulations performed worse under stable conditions than under neutral or unstable conditions. Even the best RMSE-based selections often retained a negative Bias, meaning that the LiDAR wind speed was systematically underestimated. This suggests that the tested MOST-based models do not fully reproduce the vertical structure of stable offshore profiles. In stable cases, the observed wind profile can present strong low-level acceleration followed by a smoother logarithmic-like behaviour aloft. Several formulations failed to capture this transition with sufficient accuracy. This result indicates that stable offshore stratification remains a major limitation of simplified MOST-based extrapolation and should be prioritised in future modelling work.

The CERRA2 benchmark reinforced the value of the LiDAR-anchored approach. For the matched validation dataset, CERRA2 presented an RMSE of 6.88 m s^{-1} , a Bias of -4.34 m s^{-1} , and an R^2 of 0.23. This indicates broad scatter relative to the LiDAR observations and a systematic underestimation of the observed wind speed. In contrast, the MOST-based formulations produced substantially lower RMSE values, especially for the best-performing combinations. Therefore, for this offshore validation case, the reduced-input MOST framework reproduced the observed wind profile more closely than the open reanalysis-based benchmark.

5.3 Methodological limitations and interpretation of results

The results must be interpreted within the methodological scope of the thesis. The framework was intentionally designed for a reduced-input scenario. Consequently, several key quantities were estimated rather than directly measured. The friction velocity u_* , aerodynamic roughness length z_0 , Monin–Obukhov length L , diagnostic boundary-layer height z_i , and Gryning mixed-layer scale L_{MBL} all depend on assumptions and parameterizations. Therefore, the results should be interpreted as the performance of an accessible, physically motivated extrapolation framework, not as the maximum accuracy achievable with complete offshore instrumentation.

The thermodynamic closure also involved simplifications. The potential temperature was computed from an operational temperature proxy based on T_{2m} and SST , rather than from a direct in situ temperature profile at the wind-reference height. This approximation was necessary within the reduced-input framework, but it introduces uncertainty in the stability closure. In addition, humidity effects were not included in the main stability closure. This can be relevant in marine environments, where humidity fluxes may modify the effective buoyancy flux and, consequently, the estimated stability state.

The validity of MOST also becomes less certain as the extrapolation extends beyond the surface layer. This limitation is one of the reasons why the classical logarithmic formulation was compared with the Lange and Gryning reformulations. The comparison does not assume that MOST is universally valid over the full 30–200 m range. Instead, it tests how different MOST-based extensions behave when applied to turbine-relevant offshore heights. This is particularly important under very stable conditions, where intermittent turbulence, shallow turbulent layers, gravity waves, and low-level jets may occur. These processes are not explicitly resolved by the present methodology and help explain why stable offshore profiles remain difficult to reproduce with simplified profile models.

Other limitations are related to the validation design and data structure. The validation was performed at discrete LiDAR heights. Therefore, the thesis evaluated error at specific levels rather than by integrating the full vertical profile. This was appropriate for the available dataset and for the comparison performed here, but it does not fully represent the continuous shape of the wind profile across the rotor-swept layer. A future improvement would be to compute an integrated profile error by interpolating between the evaluated heights. This would allow the comparison to account for the complete vertical structure of $U(z)$, rather than only the measured levels.

The iterative closure was tested at one offshore site. The IJmuiden Ver dataset provides a valuable and realistic offshore validation case, but it cannot prove that the same selection rules are universal. The adaptive strategies derived in this thesis should therefore be interpreted as site-specific evidence and as a methodological demonstration. Future studies should repeat the same workflow at other offshore locations, with different wind climates, wave conditions,

thermal regimes, and boundary-layer structures. This would show whether the same conditional patterns remain valid or whether the selection rules need to be recalibrated for each site.

The methodology also used fixed numerical thresholds and prescribed parameter choices. This includes convergence criteria, filtering limits, roughness assumptions, stability-class boundaries, and the simplified division between unstable, neutral, and stable regimes. These choices were necessary to build a reproducible workflow, but they are not unique to it. Future work should test the sensitivity of the results to these thresholds. The stability classification could also be refined by distinguishing weakly unstable, strongly unstable, near-neutral, weakly stable, and very stable conditions. This would be especially useful for stable regimes, where turbulence can become intermittent and where classical MOST assumptions are more fragile.

The comparison at the timestamp level remains limited by the nature of the available data. The LiDAR observations and the open meteorological inputs do not necessarily represent the same temporal and spatial scales. Even after temporal harmonisation, part of the atmospheric variability can be smoothed, shifted, or lost. For this reason, the most robust interpretation of the results comes from aggregated metrics, mean profiles, RMSE, and Bias distributions, rather than from individual timestamp predictions. Future applications should use datasets with identical temporal resolution and more complete collocated measurements whenever possible.

The comparison with CERRA2 must also be interpreted carefully. It is not a fully symmetric comparison between two equivalent modelling approaches. The MOST-based profiles are anchored to the local LiDAR wind speed measured at 3.7 m, while CERRA2 is an independent gridded reanalysis product whose wind speeds were interpolated or extrapolated to the validation heights using a fitted power law. Part of the CERRA2 error may therefore be related to spatial representativeness, temporal smoothing, the original reanalysis wind field, and the additional height-matching step. For this reason, the result should not be interpreted as a general rejection of CERRA2. Rather, it shows that, for this specific offshore validation case, reanalysis data alone did not reproduce the local LiDAR wind profile with the same fidelity as the LiDAR-anchored MOST-based extrapolation framework.

These limitations do not invalidate the results. They define the domain in which the results should be interpreted. The thesis evaluates physically motivated formulations under accessible input conditions and is therefore most appropriate for preliminary offshore wind-resource assessment and for identifying when simple profile models become insufficient.

5.4 Final remarks and future perspectives

Despite the limitations discussed above, the thesis demonstrates that limited-data offshore wind extrapolation can be made more reliable when the modelling process is transparent, physically structured, and conditionally evaluated. The work shows that the problem should

not be reduced to choosing the best empirical stability correction in isolation. The correction function, the profile formulation, the target height, the stability regime, and the time of day interact with each other. A model can only be judged properly when these interactions are considered.

The final conclusion is threefold. First, the proposed workflow extrapolated offshore wind speed from a near-surface reference level to turbine-relevant heights with strong agreement with LiDAR observations. This supports the feasibility of reduced-input vertical extrapolation for a preliminary offshore wind resource assessment. Second, the best modelling strategy is adaptive. No single empirical correction or MOST reformulation should be assumed to be universally optimal. The most defensible approach is to select the formulation based on the atmospheric regime, target height, and temporal context, while always interpreting the results alongside RMSE, Bias, and sample support. Third, the comparison with CERRA2 shows that open reanalysis products are valuable external benchmarks, but they should not be interpreted as direct substitutes for local offshore measurements. In this thesis, CERRA2-derived wind speeds systematically underestimated the LiDAR observations, whereas LiDAR-anchored MOST-based formulations reproduced the observed profile more accurately.

In this sense, the main contribution of the thesis is not only the identification of individual best-performing combinations. Its main contribution is the development of a unified, reproducible framework for testing, comparing, and selecting MOST-based offshore wind-profile formulations under limited-data conditions. The methodology helps quantify when simple profile models are sufficient, when they become biased, and when more conditional selection is needed. This provides a practical basis for future offshore wind studies that must estimate hub-height or rotor-layer wind speed without complete vertical measurements.

Future work should extend the framework in four directions. First, the validation should be repeated at additional offshore sites to test whether the conditional patterns found for IJmuiden Ver remain valid under other metocean and boundary-layer conditions. Second, the error assessment should include an integrated profile metric, so that the complete vertical structure of $U(z)$ is evaluated rather than only discrete heights. Third, the stability classification should be refined, especially in stable conditions, where the current three-class division may mask important differences between weakly and very stable regimes. Fourth, future applications should include additional offshore processes when data are available, such as humidity effects, direct turbulent-flux measurements, measured boundary-layer height, and wave-state information.

These developments would not replace the reduced-input framework proposed in this thesis. Rather, they would allow its assumptions to be tested more deeply and its applicability to be expanded. The present work therefore provides a methodological basis for future offshore wind extrapolation studies that need to balance physical realism, data availability, and practical applicability.

References

- [1] R. J. Barthelmie, A. M. Sempreviva, and S. C. Pryor. The influence of humidity fluxes on offshore wind speed profiles. *Annales Geophysicae*, 28:1043–1052, 2010. doi: 10.5194/angeo-28-1043-2010.
- [2] Rebecca J. Barthelmie. Evaluating the impact of wind induced roughness change and tidal range on extrapolation of offshore vertical wind speed profiles. *Wind Energy*, 4: 99–105, 2001. doi: 10.1002/we.45.
- [3] Giacomo Casasanta, Roberto Sozzi, Igor Petenko, and Stefano Argentini. Flux–profile relationships in the stable boundary layer—a critical discussion. *Atmosphere*, 12(9):1197, 2021. doi: 10.3390/atmos12091197.
- [4] Andrew Clifton, Julie K. Lundquist, Neil Kelley, George Scott, David Jager, and Scott Schreck. Characterizing inflow conditions across the rotor disk of a utility-scale wind turbine. Technical Report NREL/PO-5000-53816, National Renewable Energy Laboratory, 2012.
- [5] Copernicus Climate Change Service, Climate Data Store. ERA5 post-processed daily statistics on single levels from 1940 to present, 2024. URL <https://cds.climate.copernicus.eu/datasets/derived-era5-single-levels-daily-statistics>. Accessed on 05-Jul-2026.
- [6] James B. Edson and Christopher W. Fairall. Similarity relationships in the marine atmospheric surface layer for terms in the TKE and scalar variance budgets. *Journal of the Atmospheric Sciences*, 55(11):2311–2328, 1998. doi: 10.1175/1520-0469(1998)055<2311:SRITMA>2.0.CO;2.
- [7] Rémi Gandoin and Jorge Garza. Underestimation of strong wind speeds offshore in ERA5: evidence, discussion and correction. *Wind Energy Science*, 9:1727–1745, 2024. doi: 10.5194/wes-9-1727-2024.
- [8] Sven-Erik Gryning, Ekaterina Batchvarova, Burghard Brümmer, Hans Jørgensen, and Søren Larsen. On the extension of the wind profile over homogeneous terrain beyond

References

- the surface boundary layer. *Boundary-Layer Meteorology*, 124:251–268, 2007. doi: 10.1007/s10546-007-9166-9.
- [9] A. A. M. Holtslag and H. A. R. De Bruin. Applied modeling of the nighttime surface energy balance over land. *Journal of Applied Meteorology and Climatology*, 27(6):689–704, 1988. doi: 10.1175/1520-0450(1988)027<0689:AMOTNS>2.0.CO;2.
- [10] Maarten C. Holtslag, Wim A. A. M. Bierbooms, and Gerard J. W. van Bussel. Estimating atmospheric stability from observations and correcting wind shear models accordingly. *Journal of Physics: Conference Series*, 555(1):012052, 2014. doi: 10.1088/1742-6596/555/1/012052.
- [11] Xuwei Hu, Yuan Feng, Jiahao Liu, Yuanxiang Xu, and Shengyu Song. Long-term prediction of mesoscale sea surface temperature and latent heat flux coupling using the itransformer model. *Sensors*, 25(3):985, 2025. doi: 10.3390/s25030985.
- [12] Pedro A. Jiménez, Jimy Dudhia, J. Fidel González-Rouco, Jorge Navarro, Juan P. Montávez, and Elena García-Bustamante. A revised scheme for the WRF surface layer formulation. *Monthly Weather Review*, 140(3):898–918, 2012. doi: 10.1175/MWR-D-11-00056.1.
- [13] Bernhard Lange, Jørgen Højstrup, Søren Ejling Larsen, and Rebecca J. Barthelmie. Comparison of sea surface roughness models for offshore wind power utilisation. In *Offshore Wind Energy (CD-ROM)*, Brussels, 2002. European Wind Energy Association (EWEA). EWEA Offshore Wind Energy Special Topic Conference, Brussels, Belgium, 10–12 Dec 2001.
- [14] Bernhard Lange, Søren E. Larsen, Jørgen Højstrup, and Rebecca J. Barthelmie. The wind speed profile at offshore wind farm sites. In *Proceedings of the OWEMES 2003 Conference*, pages 1–15, 2003.
- [15] Bernhard Lange, Søren Larsen, Jørgen Højstrup, and Rebecca J. Barthelmie. The influence of thermal effects on the wind speed profile of the coastal marine boundary layer. *Boundary-Layer Meteorology*, 112(3):587–617, 2004. doi: 10.1023/B:BOUN.0000030652.20894.83.
- [16] A. S. Monin and A. M. Obukhov. Basic laws of turbulent mixing in the atmospheric surface layer. *Trudy Geofizicheskogo Instituta Akademii Nauk SSSR*, 24:163–187, 1954. In Russian.
- [17] Manuel Motta, Rebecca J. Barthelmie, and Preben Vølund. The influence of non-logarithmic wind speed profiles on potential power output at danish offshore sites. *Wind Energy*, 8(2):219–236, 2005. doi: 10.1002/we.146.

References

- [18] Mike Optis, Nicola Bodini, Mithu Debnath, and Paula Doubrawa. New methods to improve the vertical extrapolation of near-surface offshore wind speeds. *Wind Energy Science*, 6: 935–948, 2021. doi: 10.5194/wes-6-935-2021.
- [19] RPS Australia West Pty Ltd. Supply of metocean data for ijmuiden ver investigation area: 24-month measurement and validation report. Technical Report 100-CN-REP-2163, Rev. 1 (MMF03707.000 | TN328326), RPS, 2024. Report pages cited in-text (e.g., p. 3, p. 6, p. 7, p. 9, p. 15, p. 16, p. 21, p. 27, p. 29) refer to the pagination printed in the document footer/header.
- [20] S. Schimanke, M. Ridal, P. Le Moigne, L. Berggren, P. Undén, R. Randriamampianina, U. Andrea, E. Bazile, A. Bertelsen, P. Brousseau, P. Dahlgren, L. Edvinsson, A. El Said, M. Ginton, S. Hopsch, L. Isaksson, R. Mladek, E. Olsson, A. Verrelle, and Z. Q. Wang. CERRA sub-daily regional reanalysis data for Europe on pressure levels from 1984 to present, 2021. Accessed on 07-Jul-2026.
- [21] Roland B. Stull. *An Introduction to Boundary Layer Meteorology*. Springer, Dordrecht, 1988.
- [22] Esa-Matti Tastula, Boris Galperin, Semion Sukoriansky, Ashok Luhar, and Philip Anderson. The importance of surface layer parameterization in modeling of stable atmospheric boundary layers. *Atmospheric Science Letters*, 16(1):83–88, 2015. doi: 10.1002/asl2.525.