



**Politecnico
di Torino**

Department of Energy "Galileo Ferraris"

Degree in Electric Engineering

Master Degree Thesis

**Design and Multiphysics
Analysis of a
Multi-Three-Phase Motor
with C-Core Stator**

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Abstract

The work focuses on the design and Multiphysics Analysis of a C-core Multi-Three-Phase surface permanent magnet motor. The Multi-phase structure represents a good alternative to meet the requirements of reduced dimensions, high efficiency, low vibrations and noise.

The aim of this thesis is to investigate the electromagnetic performance of the C-core SPM using Finite Element Analysis (FEA).

Simulations are performed in COMSOL Multiphysics under both no-load and on-load conditions.

In the no-load study, zero current is imposed in the stator windings; the induced voltage waveform and its harmonic content is obtained.

For the on-load study, stator currents are imposed with phase shifts determined by the winding geometry. Key outputs include current and voltage waveforms, torque, torque ripple and air-gap radial magnetic flux density.

For both operating conditions, the spatial distribution of the magnetic flux density (B) is mapped throughout the motor.

A comparative validation is performed using magnetostatic simulations in FEMM, where a one third of the motor is simulated.

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Introduction

The global electric motor industry is experiencing exponential growth, driven by the pursuit of higher power density and reliability and reduced local emissions. In recent years, permanent magnet motors have fulfilled these purposes; they can be classified as Surface Permanent Magnet (SPM) and Interior Permanent Magnet (IPM) Motors.

In particular, the SPM motor has been widely applied due to its simple rotor structure and manufacturing convenience. Some improvement needs to be achieved with electromagnetic and thermal design: the choice of the materials composing permanent magnets and stator and rotor core, as well as windings, is fundamental in order to reduce losses caused by hysteresis and eddy current.

The aim of this work is to design two different configurations of the stator of a SPM multi-three-phase motor and analyze which one offers the best performance

The work thesis is structured as:

- **Chapter 1**, introduces the materials composing permanent magnets and Soft Magnetic Composites, and consequentially core power losses, how they are defined and calculated.
- **Chapter 2**, introduces permanent magnet machines, focusing on SPM and multi-three-phase motor.
- **Chapter 3**, illustrates the choices and steps behind the design of the C-shaped core SPM.
- **Chapter 4**, focuses on the simulation in Comsol Multiphysics and in FEMM of the motor under different conditions.
- **Chapter 5**, reports the data and results of the simulations.
- **Chapter 6**, conclusions.

Chapter 1

Magnetic Materials and Power Losses

1.1 Permanent Magnets: materials' properties

The materials composing permanent magnets are hard ferrimagnetic and ferromagnetic materials. These materials differ from other magnetic materials due to their relative permeability μ_r much greater than 1 and M magnetic dipole moment per unit volume of material higher than 0.

The magnetization M is defined as:

$$M = (B/\mu_0) - H \quad (1.1)$$

In ferromagnetic materials, the magnetic field is strengthened within the material, where permanent dipoles are rigidly aligned parallel to the external field (situation that persists below the Curie temperature T_c). These dipoles are grouped into Weiss domains, whose magnetic moments are discordant for null or very low magnetic flux density B values.[22]

As the induction increases, the dimensions of domains aligned with the field increases (the atomic moments are forced to align with the magnetic field), leading to a rigid rotation of other domains.

Materials exhibiting ferromagnetic behavior are: iron, nickel, cobalt, and some of their alloys.

Ferrimagnetic materials consist of ions with different magnetic moments and exhibit intermediate effects between ferromagnetic and antiferromagnetic materials, which dipole coupling is anti-parallel.

The behavior of a magnetic material is defined by the magnetization curve, B as a function of the magnetic field H . By gradually increasing the magnetic field on a demagnetized sample of ferromagnetic material, a characteristic curve is obtained as shown in the figure; this curve is known as the initial magnetization characteristic.

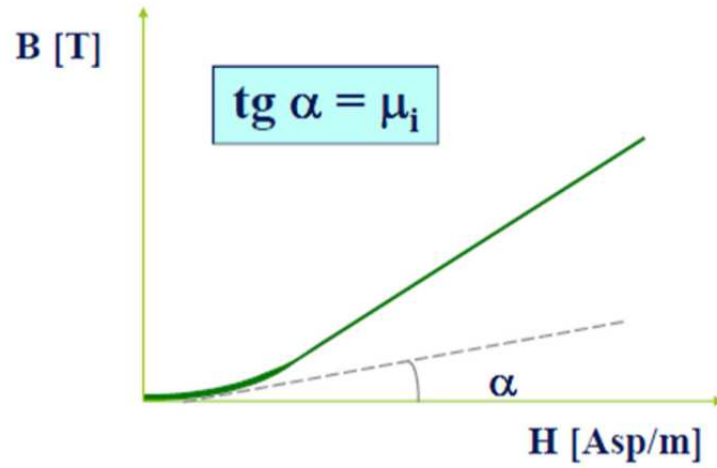


Figure 1.1: Initial magnetization characteristic [22].

The slope of the characteristic is very large in the central section and then decreases. In particular, beyond a saturation magnetic flux density value B_s , ranging between 1.4 T and 2.2 T, the slope becomes constant, showing a linear relationship between B and H , corresponding to the vacuum permeability μ_0 . [22]

Another characteristic of ferromagnetic materials defines the irreversible nonlinear response of B to an imposed magnetic field H , obtained by magnetizing and then demagnetizing the material. It presents different domain structures, starting from the unmagnetized state at the origin, the magnetic field is varied from a positive to a negative limit value, and then returns to the starting value. What is obtained is a closed curve called a hysteresis loop; this phenomenon can be explained by the fact that for a zero induction value, not all domains return to their initial orientation, resulting in losses proportional to the area of the loop.

The important properties to define are: B_r , remanent flux density corresponding to a zero magnetic field; H_c , coercive field corresponding to a zero induction value.[7]

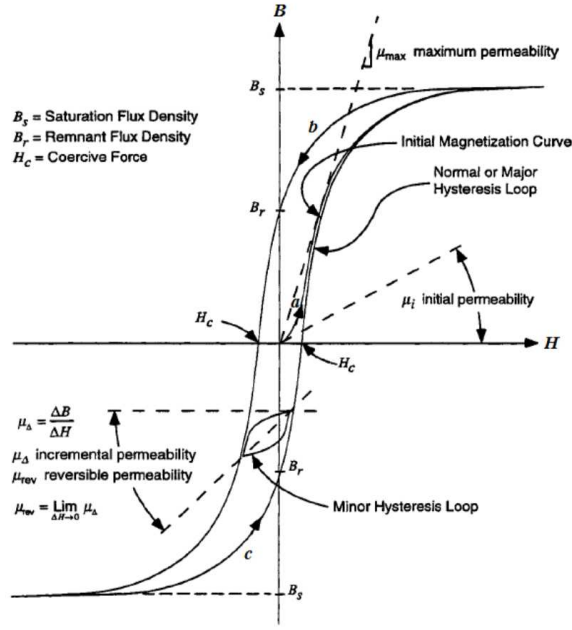


Figure 1.2: Hysteresis Loop [27].

The loop combines information on intrinsic magnetic property, the spontaneous magnetization M_s which is the maximum value of the field dependent magnetization achieved when a large value of external magnetic field is applied, and two extrinsic properties B_r and H_c , which depend on bulk shape, surface roughness, microscopic structure and thermal behavior. [10]

Based on the hysteresis loop, it is possible to define soft and hard ferromagnetic materials. Hard magnetic materials have broad, square loops with high H_c and B_r , and they are suitable for permanent magnets since, once magnetized by applying a field sufficient to saturate the magnetization, they remain in a magnetized state when the field is removed or when in presence of a reverse field.

The materials used include Nickel, not in its natural state but processed and as a constituent of alloys (AlNi); Cobalt, which exhibits high magnetic hysteresis losses, also as a constituent of alloys (AlNiCo, Samarium-Cobalt); Ferrites, non-metallic ferrimagnetic materials consisting of iron oxides and a bivalent metal, similar to ceramic materials in terms of hardness and fragility, and also have very low conductivity and therefore reduced losses. Ferrites are used in high frequency applications, from a few kilohertz to well over 80 MHz.

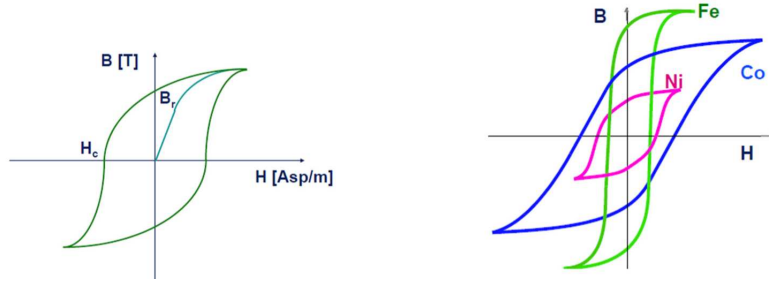


Figure 1.3: Hysteresis loop: different hard ferromagnetic materials [22].

The magnetization of AlNiCo derives from the FeCo constituent of the alloy, which has a composition corresponding to the highest known B_s and a high T_c . On the other hand, its weakness is the anisotropy, not sufficient to induce a robust coercitivity.

Ferrites exhibits high coercitivity but low magnetization and are made with inexpensive elements, easy to process. As oxides, they have excellent corrosion stability; however, their low magnetization requires large volumes of material to generate enough flux.

Another material that compose permanent magnets are rare earth materials, such as Neodymium and Samarium, whose availability and cost is not uniform. For example, the higher-temperature performance requirement leads the Nd-based permanent magnet alloy composition to improve its high-temperature resistance to demagnetization, quantified by the permanent loss of its magnetic flux density B from an opposing magnetic field H . In general, RE-based magnets are prone to rapid decomposition in wet, warm or corrosive environments if not adequately encased in a protective coating.[23]

For PM applications, the local magnetization direction needs to be stabilized in order to provide a uniaxial magnetic state and it is achieved by developing magnetic anisotropy: a single preferred direction of the magnetization with respect with the physical body of the magnet.

The most important source of anisotropy is magnetocrystalline anisotropy, which dictates how the magnetic energy depends on the magnetization directions relative to the crystal axes and inhibits the collapse or switching of M in a reverse H .

In a sufficient large reverse field, the magnetization will start to reverse; the field correspondent to a $B(H)=0$ defines the coercitivity. [23]

Hard materials undergo intense and unidirectional magnetization, allowing them to lock magnetic domains in position even after the exposure to strong external magnetic fields. The ability of magnets to sustain a magnetic field is called stability. Once the initial magnetizing field is removed, the domains remain in their position until external forces act upon them, exceeding those holding them in place. The energy required to alter the field produced by the magnets varies greatly for each type of material.

Factors influencing stability include time, temperature, reluctance variations, demagnetizing fields, radiation, stress, and vibrations.

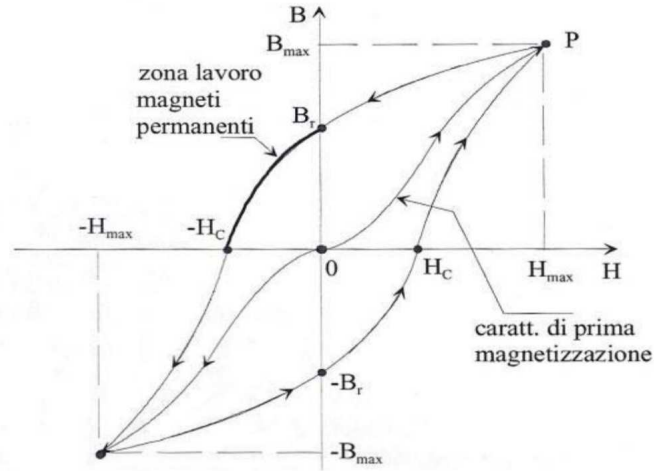


Figure 1.4: Working region of PM [3].

As shown in the figure, permanent magnets operate in the second quadrant, within a working region between B_r and H_c , with a considerable margin with respect to the non-linear characteristic.

Starting from the saturation, the field is reduced creating regions of uniform reverse magnetic polarization, magnetic domains, which highly depend on the microstructure of the magnet. At the coercive field, a number of those reverse domains, separated by the domain walls, are formed so that $\mathbf{B}$ average to zero.

If the grains forming the domains present highly irregular boundaries, lower energy sites for domain reversal are provided, reducing H_c . [23]

The work than can be performed by PM is quantified by the maximum energy product $(BH)_{max}$, determined by the hysteresis loop.

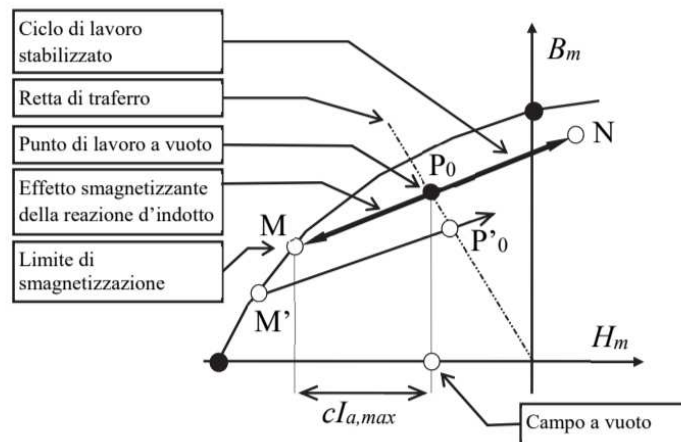


Figure 1.5: Stabilization loop [9].

Once the excitation structure is mounted, the permanent magnet undergoes stabilization cycles consisting of magnetizations and demagnetizations around the operating point.

The portion of the magnet that tends to demagnetize, operates within the stabilization loop (recoil line) up to a limit value of flux density. This value should never drop below a minimum value, otherwise, a more or less extensive portion of the magnet starts working on the main hysteresis loop (normal line). When the stress stops, the operating points return to the load line following a lower cycle: the magnets undergo an irreversible demagnetization process, with total or partial loss of their characteristics.

The adoption of Samarium-Cobalt magnets limits the risk of demagnetization, as the stabilization loop coincides with the main hysteresis loop. [9]

The working point is determined by the intersection between the recoil line and the load line of the magnet. It is identified by the coordinates (B_d, H_d) as in the graph.

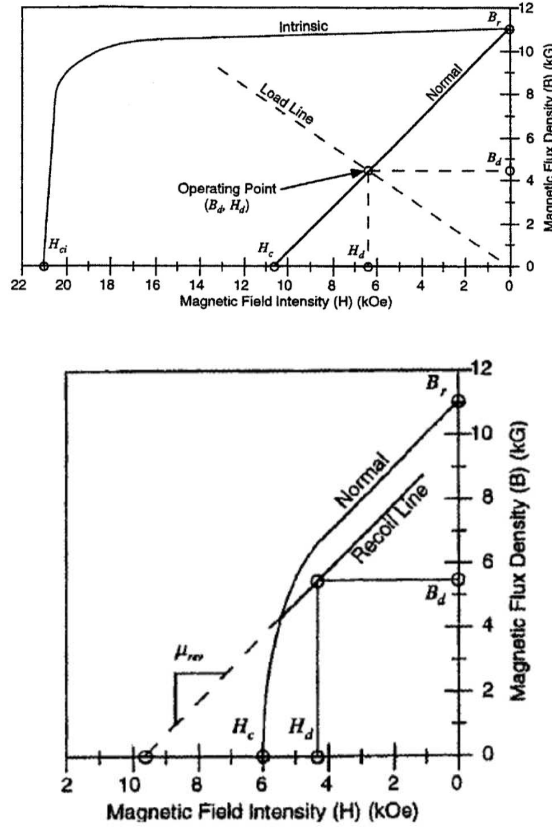


Figure 1.6: Examples of stabilization loop, with working point, recoil line [27].

1.2 SMC Material

Since the magnetic core of electrical machines is source of iron losses, there is a constant request to improve the efficiency which leads to the adoption of more

performing materials. [28]

Soft magnetic composites (SMCs) can be described as high purity ferromagnetic powder particles individually coated with a electrically insulating film, that can be organic (*resin*) or inorganic (*metallic oxide* or *ceramic insulation*) typology. [11]

This forms a magnetically active material that is at the same time electrically resistive to prevent Joule losses. [30]

Soft magnetic parts produced by powder metallurgy processes can be broadly divided into:

- *Sintered magnetic cores*, manufactured by the conventional PM process. The flux density of iron-based materials is strongly proportional to the purity and density of the sintered part.

- *Powder compressed magnetic cores*, manufactured without sintering. Powder cores are made from magnetic powder particles approximately $100\ \mu\text{m}$ in size, which are insulated individually.

The mixture is then compacted and heat treated. In this case, the heat treatment must be performed at a temperature which will not destroy the inorganic insulating layer or the organic resin binder.

The main difference between SMCs and Powder Metallurgy component production is sintering, which is avoided by SMCs through the heat treatment.

SMC components are normally manufactured by conventional powder compaction techniques combined with new ones, such as two step compaction, warm compaction, multi-step and magnetic annealing. *Warm compaction* and *two step compaction* methods increase magnetic induction, magnetic permeability and density and decrease core losses.

These techniques are followed by a heat treatment at low temperature, to avoid the destruction of the insulating layer between the iron particles, to relieve the stress, reduce hysteresis losses and improve permeability.

SMCs are magnetically and thermally isotropic due to their powdered nature [1], creating key benefits.

Compared to traditional laminated steel cores, these composite materials present several advantages, such as the three dimensional field path design of magnetic circuits; low contribution of eddy current loss due to the individually insulated particles; excellent cost efficiency due to the compact design requiring less material and simple net shaping production process with minimum waste; improved thermal characteristics; flexible machine design and assembly. [1]

They also present improved high frequency performance, which lead to relatively low total core loss at medium and high frequencies.

At low frequencies the two materials are comparable but at high frequencies, in order for laminations to compete with SMCs, the losses need to be reduced by making them thinner and introducing additives. This however would increase the cost of manufacturing process and hence would not be convenient.

SMCs materials offer high remanent magnetization, high resistivity, low coercivity and high Curie temperature. [30]

High flux density and high saturation magnetization are useful in applications where high torque is required; low iron losses, high strength and high thermal conductivity are for applications where high efficiency and high speed are required. [1] SMC has also certain limitations: the mechanical performances, in terms of strength are weak [28]; higher hysteresis loss resulting from the domain structure and from residual stress if not properly relaxed by heat-treatment of the pressed component.

The properties and performance of the SMC material are directly proportional to the mix and the component density that is related to the actual pressure of compaction: high pressure means small gaps between the particles leading to higher permeability and saturation flux density.

The performance at a given frequency is determined by the iron grain size: the smaller the better performance at high frequencies.

The magnetic, electrical and mechanical characteristics depend on the preparation and processing of the components: the purity, shape and size of particles.

1.3 Power Losses

When a variable field is present, energy must be provided to allow continuous changes in domain orientation. Describing a hysteresis loop, this energy is not fully returned but instead dissipates, equal to the area enclosed by the cycle itself. Therefore, in a ferromagnetic material subjected to variable flux, it is important to consider the phenomena of hysteresis losses and eddy current losses, which are proportional to the frequency.

In order to determine the power losses due to hysteresis p_i it is used the Steinmetz equation:

$$p_i = k_i \cdot f \cdot B_m^\eta \quad (1.2)$$

in which k_i is the hysteresis loss coefficient; B_M is the maximum flux density value; η can assume values in a range between 1.6 and 2.2, increasing with B .

Eddy currents are induced by a moving conductor in a constant magnetic field or in a stationary conducting material subjected to a time-dependent magnetic field. [25] They can be also introduced during the magnetization process by the motion of magnetic domain walls.

Losses due to eddy currents are created by the thermal heat they generate and by the currents flowing on planes orthogonal to the direction of the flux density vector, variable over time:

$$p_c = k_c \cdot f^2 \cdot B_M^2 \quad (1.3)$$

in which f is the frequency; k_c eddy current loss coefficient, it depends on the resistivity of the material and on the cross-section area perpendicular to the direction of B .

Total losses, core losses, corresponds to the sum of hysteresis losses and eddy currents and, with good approximation, they can be defined as:

$$p_f = k_f \cdot B_M^2 \quad (1.4)$$

According to the core loss performances of iron material, the coefficients can be determined with the curve fitting method.[34]

At low frequencies the hysteresis loss is the main core loss part and can be reduced by large particle size, high purity of the iron and stress relieving heat treatment.

Eddy current loss can be minimized in different ways:

- increasing the resistivity through the addition of Si to the iron powder;
- using thinner laminates;
- using insulated iron powder, which gives a relatively high resistivity. The small non-magnetic distances between every particle act as air gaps and decrease the permeability of the bulk material.

The value of losses in a ferromagnetic material is provided through the loss cipher p_m , which represents the specific loss occurring when a material is subjected to a sinusoidal alternating magnetic field at a frequency of 50 Hz with fixed maximum flux density. Defining G the weight of the material under a flux density of 1 T, the losses are:

$$P_m = k \cdot p_m \cdot G[W] \quad (1.5)$$

in which $k > 1$, the empirical coefficient, and considers how the magnetic core is mounted. For induction values different from 1 T, as first approximation:

$$P_m = k \cdot p_m \cdot G \cdot B^2[W] \quad (1.6)$$

Losses in magnetic performance from environmental degradation, especially for RE-based PM, must be controlled.

1.3.1 Eddy current losses

PM losses have been investigated in several studies, as results hysteresis losses in PM materials have no significant influence because they operate in the second quadrant of the $B(H)$ curve without crossing J-axis. In particular, rare earth PMs exhibit less losses than ferrite ones. [25] The eddy current loss in magnets, compared to other losses, may be small but can have significant effect on the permanent magnet overheat which deteriorate their performance and cause irreversible magnet demagnetization. [13]

In the ideal case, where the air-gap field contains only synchronized space harmonics, those losses can be neglected. But in real cases, the magnetic flux density in the air-gap has spatio-temporal variation, hence unsynchronized space harmonics are generated. [25]

Exists different sources of PM losses:

- *Slotting effect*

- *Winding distribution*

Space harmonics of MMF induce losses in the PM due to flux variation. Losses are almost double in concentric alternate teeth winding (single-layer winding) compared to concentric all teeth winding (two-layer winding).

Eddy currents are related to the asynchronous components of the MMF spatial harmonics, and the low order of unsynchronized ones induces a large amount of losses.

Combinations with a large number of poles and a small number of slots are characterized by large rotor losses.

Windings with different turns per coil side, in fractional-slot PMSMs, lead to reducing and/or canceling some space harmonics, resulting in lower rotor losses by the armature reaction field.

[25]

Eddy current losses can be classified into two groups: resistance-limited for low frequency and relatively large skin depth

$$\delta = \sqrt{\frac{2\rho}{\omega \cdot \mu}}; \quad (1.7)$$

inductance-limited or skin-limited for high frequency and relatively small skin depth.

[13]

Generally, these losses results from slotting effect, winding magnetomotive force (MMF) space harmonics, time harmonics in current.

Time harmonics in the non-sinusoidal current can be the cause of eddy current losses due to their magnetic fields asynchronous with the rotor; while MMF space harmonics produce asynchronous components of armature magnetic field due to the winding distributions and the air gap permeability harmonics caused by the stator slotting, inducing losses in the magnets. This eddy current loss is described as the resistances in the equivalent circuit of the machine.[13]

The sum of losses in magnets is not equal to the total magnet losses when the current time harmonics and winding MMF space harmonics have the same orders; the interaction between harmonics of different orders has no influence on the total magnet loss, but makes the losses in different pieces of magnets different.

Studies found a simplified method for calculating magnet eddy current loss due to the slot-opening, in which the flux density B is calculated by using the relative air-gap permeance. This method consists in employing a 2-D magnetic field and the Carter coefficient, which accounts the influence of slotting on the average flux density.[13]

The slotting effect on the magnet loss due to armature reaction is commonly neglected, but The procedure of the original relative permeance method consists, firstly, in obtaining the flux density in magnets B_m by using a 1-D slotless model:

$$B_m = \frac{h_{pm}}{h_{pm} + \mu_r g} B_r \quad (1.8)$$

in which g represents the physical air-gap distance while g' the effective air-gap length; h_{pm} magnet thickness and B_r the magnet remanence.

Secondly, the conformal map is applied to derive the relative air-gap permeance. The magnetic field accounting for slotting effect is obtained by the modulation of the slotless one with relative permeance. Hence, the average eddy current loss per magnet due to slotting can be expressed as:

$$P_{pm} = \frac{lh_{pm}B_m^2v_r^2b_{pm}}{2\sigma_{pm}} \sum_{n=1,2,3,\dots} K_n^2 \left[1 - \left(\frac{\tau_t}{\pi nb_{pm}} \sin \frac{\pi nb_{pm}}{\tau_t} \right)^2 \right] \quad (1.9)$$

in which p is the pole number; τ_t is the slot-pitch; n is the order of Fourier series of modulation functions; b_{pm} magnet width; l is the axial length of magnet; Ω is the rotor angular speed; $v_r = \Omega R_s$ is the rotor surface linear speed; σ_{pm} is the magnet resistivity; R_s is the stator bore radius.

K_n is the Fourier series coefficient of the modulation functions and is expressed as:

$$K_n = 0.5 \left[1 - \frac{1}{\sqrt{1 + (b_0/2/g')^2}} \right] \frac{4}{\pi n} \sin(1.6\pi b_0/\tau_t) \left[0.5 + \frac{(nb_0/\tau_t)^2}{0.7815 - 2(nb_0/\tau_t)^2} \right] \quad (1.10)$$

in which b_0 is the slot opening width.

No segmentation is used and all poles are electrically separated; the total magnet loss due to slotting is:

$$P_{pmt} = 2pP_{pm} \quad (1.11)$$

The original method's accuracy deteriorates with the skin depth reduction, in particular considering the calculation of the flux density, hence a 2-D analytical method can be applied for analysis of magnet loss.

The magnetic field harmonics:

$$B_n = K_B(n)f_{Br}(n) - \mu_0 M_n \frac{np}{(np)^2 - 1} \left(\frac{r}{R_r} \right)^{np-1} + \mu_0 M_n \frac{npA_{3n}}{(np)^2 - 1} \quad (1.12)$$

in which K_B is; f_{Br} is; M_n is and A_{3n} is.

The effective air-gap length is modified by using the Carter coefficient $K_c = \frac{\tau_t}{\tau_t - \gamma \cdot g'}$, not considered in the original model:

$$g_e = g + (K_c - 1)g' \quad (1.13)$$

and the modified stator bore radius:

$$R_{se} = R_s - (K_c - 1)g' \quad (1.14)$$

in which $\gamma = \frac{4}{\pi} \left[\frac{b_0}{2g'} \arctan \left(\frac{b_0}{2g'} \right) - \ln \sqrt{1 + \left(\frac{b_0}{2g'} \right)^2} \right]$ The 2-D method uses $B_m = \sqrt{B_1^2 + B_3^2 + B_5^2 + \dots B_n^2}$, hence the total magnet loss is:

$$P_{pmt} = \frac{V_{pm}\Omega^2 R_s^2 K_{Bm} K_{sp}}{2\sigma_{pm}} \quad (1.15)$$

which considers $K_{Bm} = \sum_{n=1,3,5,\dots} B_n^2$, the permanent field; V_{pm} , the magnet volume; $K_{sp} = \sum_{n=1,2,3,\dots} K_n^2 \left[1 - \left(\frac{\tau_t}{\pi n b_{pm}} \sin \frac{\pi n b_{pm}}{\tau_t} \right)^2 \right]$, how the slotting effect is represented.

Since there is little difference in magnet loss between different pole numbers, if the slot number is fixed, the open-circuit magnet loss is mostly determined by the relative air-gap permeance.

The magnet eddy current loss greatly increases when the slot-opening increases; the variation trend is mostly determined by the modulation effect of slotting K_{sp} . [13]

Magnet loss due to armature reaction MMF Harmonics

There is an analytical model able to calculate the magnet loss due to armature reaction:

$$P_{en} = P_{cn} + P_{an} \quad (1.16)$$

The parameters involved in these equations are α , the pole arc; q , the phase number; J_n , the amplitude of eddy current; p_s , the fundamental number of pole pairs of stator winding; N_s , number of series turns per phase.

$$P_{cn} = \frac{q^2 \mu_0^2 \alpha J_n^2}{8 \sigma_m n^2 p_s^2} (np_s \mp p_r)^2 \Omega^2 \left[(R_s/R_m)^{2np_s} R_s^2 R_m^2 F_n + (R_s/R_r)^{2np_s} R_s^2 (R_r^2 - R_m^2) + (R_s/R_r)^{2np_s} \frac{R_s^2 R_r^2}{2np_s + 2} (1 - (R_m/R_r)^{2np_s+2}) \right] / \left(1 - \left(\frac{R_r}{R-s} \right)^{2np_s} \right)^2 \quad (1.17)$$

$$P_{an} = -\frac{q^2 \mu_0^2 \alpha J_n^2}{8 \sigma_m n^4 p_s^4} (np_s \mp p_r)^2 \Omega^2 \left[(R_s/R_r)^{np_s} \frac{R_s^2 R_r^2}{np_s + 2} (1 - (R_m/R_r)^{np_s+2} + (R_s/R_m)^{np_s} R_s R_m^2 G_n \right]^2 \frac{\sin^2(np_s \alpha / 2)}{(R_r^2 - R_m^2)[1 - (R_s/R_r)^{2np_s}]^2} \quad (1.18)$$

$$F_n = \begin{cases} \frac{(R_r/R_m)^{-2np_s+2}}{-2np_s+2}, & np_s \neq 1 \\ \ln \left(\frac{R_r}{R_m} \right), & np_s = 1 \end{cases} \quad (1.19)$$

$$G_n = \begin{cases} \frac{(R_r/R_m)^{-np_s+2}}{-np_s+2}, & np_s \neq 2 \\ \ln \left(\frac{R_r}{R_m} \right), & np_s = 2 \end{cases} \quad (1.20)$$

$$J_n = (2N_s I_m / \pi R_s K_{wn}) K_{sov} \quad (1.21)$$

In which I_m , peak value of phase current; K_{wn} , winding factor; K_{dn} , winding distribution factor; K_{pn} , winding pitch factor.

$$K_{wn} = K_{dn}K_{pn} \quad (1.22)$$

It is possible to define the slot opening factor as:

$$K_{sov} = \sin(nb_0/2R_s)/(nb_0/2R_s) \quad (1.23)$$

The values of this factor depend on the slot opening due to slot-pitch ratio, stator bore radius and MMF harmonic order. [13]

As, said, the slot opening variation has significant influence on K_{sov} , especially the one of the 7th harmonic, which produces the majority of losses, and the 5th harmonic, which produces the torque.

Slotting reduces the magnet loss due to armature reaction, but has almost no influence on the fundamental MMF harmonics.

Eddy Current Losses Reduction: Segmentation

Several techniques are used to reduce eddy current losses in magnets, such as adding a copper shielding layer, slot or PM skewing and the total or partial magnet segmentation.

The segmentation consists in dividing each rotor pole into two or more elementary magnet blocks with the same magnetization direction. The blocks are inserted beside each other with certain angular space, β . [4]

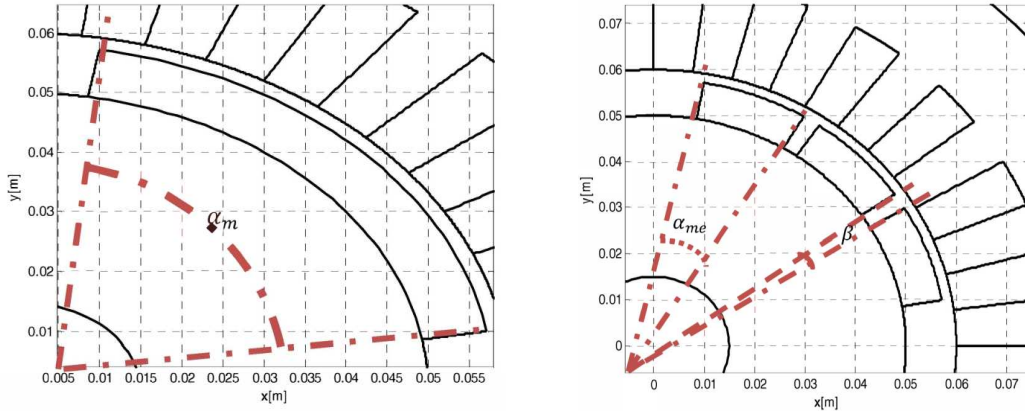


Figure 1.7: The magnet has a span α_m , which is split into N_s blocks, obtaining a span of each elementary blocks of α_{me} [4].

Segmentation can be applied either radially or axially, or together, depending on the application and losses. [12]

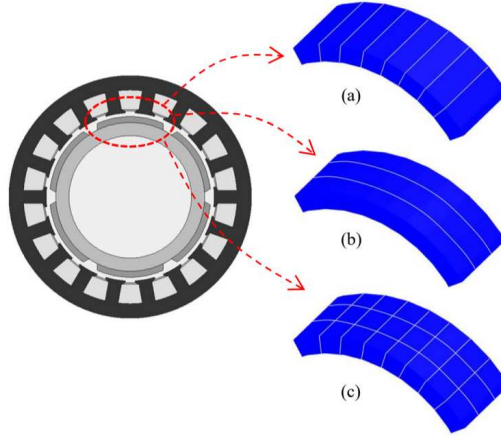


Figure 1.8: (a) radial segmentation, (b) axial segmentation, (c) both [12].

As demonstrated in previous studies the segmentation is only valid with the satisfaction of some parameters:

- Neglected skin effect: due to the high frequency harmonics involved in eddy currents induction, the losses are induced in a narrow area at the edge of the magnets, the skin depth. If considerably smaller than the magnet dimension losses may increase by segmentation. Hence, it is necessary to operate considering only resistance-limited eddy currents losses, characterized by low frequency components and leading to uniformly distributed losses across the magnet [20];
- Homogeneous magnetic flux density over the PM width;
- Neglected end effect.

Out of these conditions this method may increase rather than reduce the losses. [4]

Considering circumferential segmentation, studies as shown that higher the number of segments is, lower are the eddy current losses.

1.3.2 Eddy Current Losses Reduction: Copper shielding layer

Another effective method to reduce the eddy current losses is improving the rotor structure by adding a copper shielding layer. Although it may generate additional losses, the shielding effect on the harmonic content of the magnetic field is reported. [33]

Papers state that the copper shield reduces the losses at high speeds, while the segmentation is preferred for low speed.

This method can produce different results, depending on different parameters:

- the placing of the copper shield and its thickness. Having the copper shield outside the rotor (*External*) can suppress the harmonics in the whole rotor and

magnet but the losses may increase since the shield is located near the airgap. A solution is to increase the thickness of the shield, also in order to control the possible mechanical failure.

Having the copper shield between the sleeve and the magnets (*Internal*) can still suppress the harmonics, especially low order high amplitude time-harmonics, and therefore reduce the overall rotor loss. At lowest speed, where the space-harmonics have large skin depth, losses are not reduced.

The thickness of the copper shield is selected based on the skin depth of the current time harmonics, assuming that the space one has little effect, or based on the space-harmonics assuming that the current is purely sinusoidal.

The thickness is subtracted from the sleeve thickness, causing small reduction in the mechanical retaining capability of the sleeve. [2]

- airgap thickness;
- stator space-harmonics (slotting effect);
- frequencies value.

Chapter 2

Permanent Magnet Machines

In recent years, permanent magnet machines have been widely adopted in various electric drive applications because of their higher power density and improved reliability.

They are classified as Surface Permanent Magnet (SPM) and Interior Permanent Magnet (IPM) Machines, according to the PM installation site. [17]

The SPM machine has been widely applied in the traditional industry, owing to its simple rotor structure and manufacturing convenience.

The efficiency of the motor and the torque density is the key to optimization; the improvement needs to be completed by electromagnetic and thermal design.

To further improve the continuous torque capability, cooling structure is improved by increasing the dissipation path of winding.

It exists a relationship between the pole number and tooth height on the electromagnetic characteristics of external rotor PM motor. [19]

The IPM machine exhibits many different behaviors compared to the SPM machine, which is mainly caused by the rotor saliency and the better flux weakening ability owing to the improved d-axis inductance. The IPM machine shows the highest efficiency at the rated speed because of the utilization of the reluctance torque in addition to the PM torque.

Regarding the SPM machines with RE and ferrite PMs, the replacement of the RE reduces the torque, but it is compensated by the reluctance one and the increase of the ferrite PM volume. The gap of the torque ability between the RE and ferrite PM machines can be filled by a slight increase of the axial length. [17]

The results of studies show that the IPM machine has a better overload capability over the entire speed range and the SPM machine is affected by extra-joule loss for suppressing the PM flux at high speed. Furthermore, the PM volume of the IPM machine for the EV application is comprehensively studied to enlarge the constant power range. [17].

2.0.1 Multi-phase machines

Multi-phase machines are the subject of considerable interest in different fields of application. By dividing the required power between multiple phases, more than the conventional three, higher power levels can be obtained and power electronic converters with limited power range can be used to drive the multi-phase machine. Traditionally used to reduce the stator phase current, without increasing the voltage per phase, in large power applications, they are now been used also in the field of traction applications for their reliability, fault tolerant capability, weight and volume reduction, together with high efficiency.

Another advantage in the realization of multi-n-phases machine is the chance to have different voltage levels on each n-phase system. This solution is surely interesting and it has been widely considered if proper windings and phase shift between the various n-phase system is chosen.

Multi-phase machines with sinusoidal winding distribution present an airgap field distribution with lower space-harmonic content. This compensation of the harmonic components on the Magneto Motive Force MMF has a direct impact on the torque performances. In fact, by increasing the number of phases over three, it is possible to increase the torque density and to reduce the torque ripple amplitude. Furthermore, the noise emanated from a machine are reduced, so as the stator copper losses.

The additional degrees of freedom in multi-phase system enables to inject harmonic currents or supply multi-motors from a single inverter. The converter design in a multi-phase solution can results in an easier integration of power converters and motors active parts from the cooling point of view.

[26]

There are induction and synchronous multi-phase machines; a synchronous machine may be with permanent magnet excitation, with field winding, or of reluctance type.

The difference between three-phase machines and multi-phase machines can be shown through the design of the stator windings.

Three phase machines are normally designed with a distributed stator winding that gives near-sinusoidal MMF distribution, and is supplied with sinusoidal currents. Nevertheless, spatial MMF distribution is never perfectly sinusoidal and some spatial harmonics are inevitably present.

Multi-phase machines show more versatility: the stator winding can be designed to yield either near-sinusoidal or quasi-rectangular MMF distribution, by using distributed or concentrated windings, for all ac machine types. Near-sinusoidal MMF distribution requires use of more than one slot per pole per phase; as the number of phases increases it becomes progressively difficult to realize it. [18]

Multi-phase machines are divided into symmetrical or asymmetrical machines, depending on the electrical displacement between two adjacent phases. A symmetrical machine has a spatial displacement between any two consecutive stator phases equals $\alpha=2\pi/n$. This will always be the case if the number of phases is an odd prime number.

When the number of phases is an even number or an odd number that is not a prime

number, the machine can be seen as k sets, each having a subphases (where $n=a \cdot k$). Typically, $a=3$ (three-phase sets), $k=2,3,4,5,\dots$. In such a case, the spatial displacement between the first phases of the two consecutive a subphase windings is $\alpha=\pi/n$, leading to an asymmetrical distribution of magnetic winding axes in the cross section of the machine. [5]

In a concentrated winding machine, a possibility of enhancing the torque production by stator current harmonic injection exists. Given the odd phase number n , all odd harmonics in between one and n can be used to couple with the corresponding spatial MMF harmonics to yield additional average torque components. Torque enhancement by stator current harmonic injection is one possible use of the additional degrees of freedom, offered by the fact that only two currents are required for flux and torque control due to the fundamental stator current component. [18]

In summary, in these machines, the phases are grouped in separated stars of n -phases.

The possible single star fault is supported with a reduced performance of the drive and, typically, the phase per unit current ripple is reduced. If the n -phase stars are supplied separately, a fault tolerant drive can be easily achieved because each star can work separately, considering a fault both in the converter or in the stator.

The multi- n -phase machine as a combination of 3-phase stars, may be more attractive in an industrial environment due to the wide availability of 3-phase inverter solutions both in the power and in the control area. In fact for multi-phase structures, it is possible to realize converters as a combination different three phase converters with an equal subdivision of the currents. [14]

Chapter 3

Design of the motor

3.1 Geometric sizing and selection of materials

3.1.1 C-shaped core stator

A magnetic core is a magnetic material with high permeability. The fundamental purpose of any magnetic core is to provide an easy path for flux in order to facilitate flux linkage between two or more magnetic devices (magnets, transformers and electrical machines), and consequentially increase the magnetic flux density. [16]

A C-shaped core is a stationary component in an electric motor or generator that is obtained by combining multiple, independent, continuous, C-shaped segments of laminated steel, and stacking them together.

The defining feature of a C-shaped core stator is its modularity, allowing to reduce iron mass and overall weight.

The design is used to improve manufacturing, thermal and magnetic performance, in comparison with a single continuous ring.

C-shaped core stator offers several benefits, primarily due to its segmented design:

- It provides a *larger slot area* for the stator winding.
It allows pre-wound coils to be easily placed around the pole pieces during assembly, which simplifies the manufacturing process, leading to an increase of the value of the copper fill factor, compared to traditional designs where winding occurs after the core is assembled.
- *Material Efficiency*: stamping individual C-shaped segments from steel sheets minimizes material waste compared to obtaining a complete annular shape from a wider sheet.
- *Improved Performances*: since the design helps optimize the use of material and of the winding region, it offers higher power density, reduced core losses (hysteresis losses and eddy current losses), effective flux focusing (which improves magnetic utilization) and better thermal management.

On the other hand, the c-core design presents also some disadvantages:

- *Higher manufacturing costs*, compared to standard shapes.
C-cores can simplify winding but the fabrication with specific materials and assembling components to minimize gaps is challenging.

- *Increased cogging torque*: C-core designs can exhibit larger cogging torque compared to other topologies.

Cogging torque is an unwanted pulsating torque produced in a permanent magnet machine in order to break the magnetic attraction between the rotor mounted permanent magnets and the stator teeth.

Those teeth can produce a reluctance torque variation as the rotor rotates and changes position.

Cogging torque is an undesired effect that lower torque's quality and that contributes to the machine output ripple, produces vibration and mechanical noise.

This irregular torque is especially evident in applications with low inertia and speed.

Cogging torque can be minimized through mechanical means by optimizing the number of magnetic poles and teeth, or by skewing the lamination stack, or by skewing and shaping the permanent magnets to make their transition between stator teeth more gradual. [31]

- *Performance constraints*: the open structure of C-cores can cause higher leakage flux compared to closed cores, and material stresses during production can increase core losses.

The performance can be limited also by magnetic saturation issues.

C-shaped stator core: Geometric Sizing

The motor referred to in this thesis is a multiphase surface permanent magnet motor, which stator is composed of 27 c-shaped cores. The standard geometry of the single c-shaped core has been obtained by measuring 27 cores from a lot with a calipers and then calculating, using Excel, the mean value of each dimension.

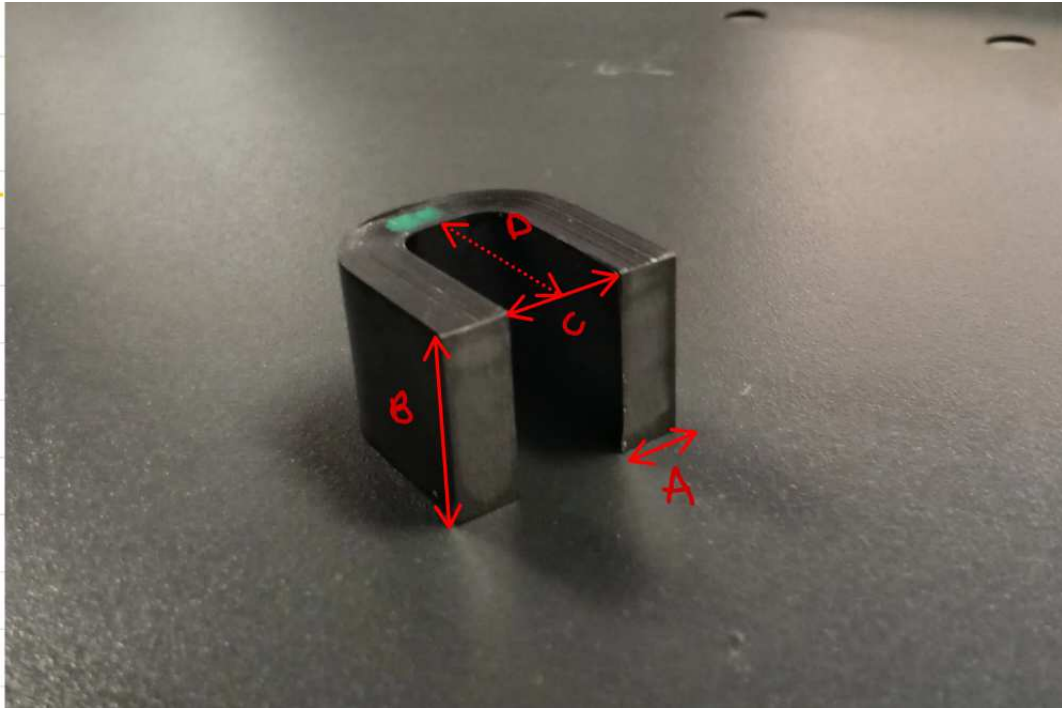


Figure 3.1: C-shaped core and the measures needed for geometry.

As shown in the picture 3.1, the useful dimensions are A,B,C,D, and in the following tables all the measures are listed.

Table 3.1: 27 cores' measures.

Number of cores	A [mm]	B [mm]	C [mm]	D [mm]
1	5	14.98	9.37	15.87
2	4.89	14.93	9.09	15.40
3	4.95	15.03	9.47	15.80
4	4.90	14.97	9.47	15.86
5	4.96	14.94	9.24	15.75
6	4.86	14.95	9.37	15.32
7	4.87	14.93	9.38	15.46
8	4.98	14.98	9.27	15.61
9	4.98	14.98	9.42	15.58
10	4.80	14.93	9.61	15.82
11	4.89	14.98	9.28	15.46
12	5.03	14.98	9.30	15.49
13	5.06	14.93	9.35	15.42
14	4.83	15.37	9.51	15.95
15	4.95	14.94	9.23	15.93
16	4.94	14.96	9.42	15.77
17	4.91	14.93	9.54	15.89
18	5.02	14.97	4.96	15.68
19	4.86	14.95	9.62	15.87
20	5	14.97	9.11	15.52
21	4.93	14.93	9.41	15.48
22	4.83	14.97	9.34	15.91
23	4.80	14.98	9.62	15.73
24	4.87	14.93	9.35	15.64
25	4.90	14.95	9.34	15.37
26	4.90	14.95	9.39	15.84
27	4.98	14.95	9.34	15.44

Table 3.2: Medium values.

A_{med}	B_{med}	C_{med}	D_{med}
4.92	15.17	9.38	15.67

The dimension A corresponds to the thickness of the C-shaped stator core; the dimension B corresponds to the depth of the stator core; the dimension C corresponds to the length of the slot window (circumferential direction) and the dimension D corresponds to the height of the slot window (radial direction).

Other stator core's parameters worth mentioning for this study are the following:

- length stator core

$$length_n = C_{med} + A_{med} \cdot 2 \quad (3.1)$$

- height stator core

$$height_n = D_{med} + A_{med} \quad (3.2)$$

- Stator inner radius Rs_{int}
- Stator outer radius Rs_{out}
- Number of slots N_{slots}
- In rotating field electrical machines, the windings are typically distributed within slots facing the air gap. The slots are regularly shifted by the same angle α , known as the slot pitch. The slot pitch is defined as a mechanical angle and is expressed in degrees by the following equation:

$$\alpha_n = \frac{360^\circ}{N_{slots}} \quad (3.3)$$

Table 3.3: Stator Core Parameters.

length_n	height_n	Rs_{int}	Rs_{out}	N_{slots}	α_n
19.22 mm	20.59 mm	83 mm	123.59 mm	27	13.33 °

C-shaped stator core: Selection of Material

The material chosen for the C-shaped stator cores is a grain oriented electrical steel(GO). GO are the key materials for high-efficiency transformers and rotating electrical machines. They have different magnetic properties depending on the direction of the magnetic flux, and therefore it is effective in improving the performance. The main difference between GO and non-oriented electrical steel (NO), which has been widely used for the materials of the stator and rotor core of the motors, is that NO has uniform magnetic properties regardless of the direction of the flux flowing through the core. [32]

GO materials are usually used in power distribution networks as transformer cores. The main magnetic properties required are high permeability along the rolling direction, which enable compact designs, and low core losses. Eddy currents are reduced by stacking strips of thin, electrically isolated material, while hysteresis losses arises from the formation of domains and grains not oriented in the rolling direction; hence the grains must be oriented in the direction. [29]

The properties of the GO material chosen for this project thesis are listed in the following tables:

- non linear BH curve values;
- relative magnetic permeability μ_r , from the curves;

- electrical conductivity;
- laminated in plane.

Table 3.4: Stator Core Material Properties.

Parameter	Value	Unit of Measurement
σ	2	MS/m
Lam thickness	0.1	mm
Lam fill factor	0.96	

Table 3.5: Stator Core Non Linear BH Curve Data.

B [T]	H [A/m]
0.000000	0.000000
0.018000	1.000000
0.022000	1.200000
0.028000	1.500000
0.035000	1.800000
0.045000	2.200000
0.060000	2.700000
0.080000	3.300000
0.110000	4.000000
0.160000	5.000000
0.220000	6.000000
0.310000	7.500000
0.420000	9.000000
0.580000	11.000000
0.760000	13.500000
0.980000	16.500000
1.200000	20.000000
1.420000	25.000000

B [T]	H [A/m]
1.580000	30.000000
1.720000	38.000000
1.820000	48.000000
1.900000	60.000000
1.960000	75.000000
2.010000	95.000000
2.050000	120.000000
2.080000	150.000000
2.110000	190.000000
2.130000	240.000000
2.150000	300.000000
2.170000	380.000000
2.190000	480.000000
2.210000	600.000000
2.230000	800.000000
2.250000	1000.000000
2.260000	1500.000000
2.270000	2000.000000
2.285000	3000.000000
2.300000	5000.000000
2.310000	8000.000000
2.320000	10000.000000

3.1.2 Stator Windings

Stator windings consist of insulated conductive wire coils placed within the slots of the stator core. Their fundamental aim is to generate the rotating magnetic field that interacts with the rotor to produce electromagnetic torque.

The windings can be manufactured using:

- *Copper* is the predominant conductor material for motor windings in most applications, due to its excellent electrical conductivity, reduced losses and improved efficiency. Copper's ductility allows for tight coil winding without fracturing, while maintaining structural integrity under electromagnetic forces;

its high thermal conductivity facilitates heat dissipation from the winding to cooling systems. Its heaviness and reduced efficiency at high frequencies, however, can limit its use in lightweight or high-speed systems.

- *Aluminum* conductors represent approximately 10% of motor winding applications and it is used primarily in cost-sensitive or weight-critical applications. While aluminum offers lower electrical conductivity, compared to copper, it provides different advantages:
 - Weight Reduction: crucial for aerospace, automotive, and portable equipment.
 - Material Cost: aluminum typically costs less than copper per unit volume, though more material is required for equivalent conductivity.
 - Corrosion Resistance: aluminum's natural oxide layer provides inherent corrosion protection in certain environments.

Nevertheless, aluminum's lower conductivity necessitates larger conductor cross-sections for equivalent performance, potentially increasing weight and cost advantages. Aluminum also hardens more easily than copper, complicating the winding process.

- *Silver* is a very high conductive material, used in ultra-high-performance or cryogenic motors where maximum conductivity justifies the extreme cost.
- *High frequency application materials*: Carbon nanotubes (CNTs), Copper-CNT Hybrids, Copper-Clad Aluminum (CCA).
- *High power density application materials*: Iron-Cobalt Alloys and superconductors.

[21]

There are two winding methods for the interior permanent-magnet synchronous motor: *concentrated winding* and *distributed winding*. Both have merits and demerits, and both influences the motor's performance.

In concentrated type of winding all the winding turns are wound together in series to form one multi-turn coil, wound around a single tooth and so all the turns have the same magnetic axis.

The concentrated windings configuration of PMSM is currently becoming more popular than a distributed windings configuration due to its good and cheap manufacturability, higher power density and lower losses.

Concentrated windings have low manufacturing cost and short end-coil dimensions. This makes the layout much more compact, with fewer turns and shorter copper paths.

Due to the minor use of wire, copper losses are lower, which can increase efficiency.[15]

In distributed type of winding, all the winding turns are arranged in several full-pitch or fractional pitch coils. These coils are then inserted into slots spread along

the air gap periphery, spread across multiple slots per pole per phase.

The coils are inserted into dedicated slots machined along the inner circumference of the laminated stator core. Each slot may contain either one coil side (single-layer winding) or two coil sides (double-layer winding) placed radially.

Coils are insulated, fundamental is phase-to-phase insulation, which consist in additional insulating sheets placed between coils of different phases sharing the same slot (required especially in double-layer windings).

Windings can be divided into two main classes:

- *Single-layer windings*, which have very limited use, as they cause issues with coil overhang (end windings) and often require the use of different coils to facilitate manufacturing. Furthermore, they do not allow for short-pitching, which is used to reduce airgap flux distortion and back EMF harmonics. Single-layer windings are characterized by the fact that each slot contains only one active side. Active sides spaced one pole pitch τ_p apart are assigned opposite conventional current directions, since the current distribution along the stator must tend toward a sinusoidal shape with a period of $2\tau_p$. The active sides are connected to each other in the end windings, allowing current to flow through the phases and generate the magnetic field.
- *Double-layer windings*, the most widely used type, are characterized by the fact that two active sides are placed in the same slot. The end winding construction is simple, and it is possible to use identical coils and to implement short-pitching (chording). The winding can be realized with multi-turn coils. The pitch can be:
 - Full-pitch, with multi-turn coils: each coil comprises two active sides spaced one pole pitch apart, one in the upper layer and one in the lower layer (radial division). The coils of each phase are arranged without overlapping. The connections between coils can be either series connections, within the same pole-phase group, or parallel connections, provided they have the same induced voltage across their terminals.
 - Short-pitched winding, with $\mathbf{n}$ number slots, $\mathbf{p}$ pole pairs, and $\mathbf{q}$ slots per pole per phase. It is obtained from the full-pitch winding by shifting the bottom layer (radial division) by n_r slots while leaving the end connections between active sides unchanged. q can be an integer or a fraction. Fractional- q windings can be:
 - * Distributed type ($q > 1$), which exhibit a high content of spatial harmonics and are used only when an integer- q winding cannot be realized;
 - * Concentrated type ($q < 1$), either single-layer (alternating wound and unwound teeth) or double-layer (all teeth wound).

Short-pitched windings with fractional $q \ll 1$ also exhibit a high content of spatial harmonics and are used for motors with a high number of poles and limited diameters.[8]

Stator Windings: Layout and Phase Distribution

In surface permanent magnet (SPM) machines, the stator windings consist of insulated copper coils placed within the stator slots. Their primary function is to carry alternating currents that interact with the magnetic field produced by the permanent magnets mounted on the rotor surface. This interaction generates electromagnetic torque according to the Lorentz force principle.

Unlike in induction machines, the rotor magnetic field is provided by the magnets themselves; therefore, the stator windings do not need to induce rotor currents. Instead, they create a rotating magnetomotive force (MMF) that spatially aligns with or leads the rotor magnetic field to produce torque. [9]

In this thesis project, the number of slots is $N_{slots}=27$, the number of poles $N_{poles}=30$, the total phases are $m=9$ and the synchronous rotational speed $n=1000$ rpm and so the number of slots per pole per phase is

$$q = \frac{n}{N_{poles} \cdot m} = 0.1 \quad (3.4)$$

. Taking into account what was explained in the previous definition of winding configuration, the motor under analysis features double-layer, concentrated, short-pitched stator windings with tangential partitioning. Concentrated windings, because the coils are not distributed over multiple slots, but instead embrace individual stator teeth blocks.

It is possible to define the following parameters, in mechanical degrees:

$$\alpha_n = \frac{360^\circ}{N_{slots}} = 13.33^\circ \quad (3.5)$$

$$\tau_p = \frac{360^\circ}{N_{poles}} = 12^\circ \quad (3.6)$$

Slot pitch angle is defined as the angular peripheral distance between the two sides of a coil, measured in number of slots.

Pole pitch angle refers to the angular distance between the centers of two adjacent poles. A pole pitch angle is always 180° electrical regardless of the number of poles on the machine.

Double-layer, because each slot houses inside two active sides of two different phases, geometrically arranged side-by-side in the tangential direction. Short-pitched, because the geometric slot pitch angle α_n (200° electrical) does not coincide with the ideal pole pitch angle (180° electrical).

Since $q \ll 1$, the winding is fractional. This configuration with a very low q is typical of spm with a very high pole count, designed to deliver high torque at low speeds and to achieve an extremely reduced torque ripple.

In the study of the operation of common three-phase rotating AC machines, reference is usually made to the axes of the stator and rotor windings for the description of conductor distributions, magnetic coupling coefficients, etc. The treatment of the real machine can be quite complicated and 'uneconomical', also considering the understanding of the fundamental operating characteristics.

In many cases, it can be helpful to study a machine with a simpler winding structure that is electromechanically equivalent to the real one and, hopefully, also energetically.

Under certain simplifying assumptions, the operations of transforming a machine into another one that is electromagnetically equivalent can be carried out through the use of different types of linear operations on the electromagnetic quantities used for studying the machine.

Clarke Transformation

Clarke transformation is a three-phase/bi-phase transformation.

Given a generic vector X on a plane characterized by three unit vectors n_1, n_2, n_3 ; an arbitrary triplet of values x_1, x_2, x_3 represented in matrix form by the algebraic vector X_{123} (an ordered n-tuple of numbers) can be completely represented by the geometric vector X (and therefore spatial) in the plane n_1, n_2, n_3 and by the value x_0 (indicates the presence of a zero-sequence (homopolar) component) according to the following scheme:

$$X_{123} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leftrightarrow X; x_1 + x_2 + x_3 = x_0$$

This set of three phases values can be decomposable in:

$$\begin{cases} x_1 = x_1^* + \frac{x_0}{3} \\ x_2 = x_2^* + \frac{x_0}{3} \\ x_3 = x_3^* + \frac{x_0}{3} \end{cases}$$

where

$$\sum_{k=1}^3 x_k^* = 0 \quad (3.7)$$

The starred values constitute the balanced component of the three-phase system. The construction of the vector X leads to the following expression:

$$X = \frac{2}{3}(x_1 n_1 + x_2 n_2 + x_3 n_3) \quad (3.8)$$

Only the balanced component of the original three-phase set of values x_1, x_2, x_3 is capable of defining the vector X .

Vice versa, the vector X is capable of representing only the balanced components of any three-phase set of values.

In the case of generic unbalanced three-phase systems, the correspondence between a vector and a generic three-phase set of values cannot be a bijective correspondence.

For the description of the geometric vector X , the basis consisting of the three unit vectors n_1, n_2, n_3 is redundant. In fact, to identify the vector X , a basis consisting of two orthogonal unit vectors $\mathbf{a}, \mathbf{b}$ is sufficient.

For convenience, the pair of new unit vectors $\mathbf{a}, \mathbf{b}$ are chosen so that the unit vector $\mathbf{a}$ coincides with unit vector n_1 and unit vector $\mathbf{b}$ is rotated positively by 90° compared to unit vector $\mathbf{a}$. The following vector relations between the two vector bases are obtained below.

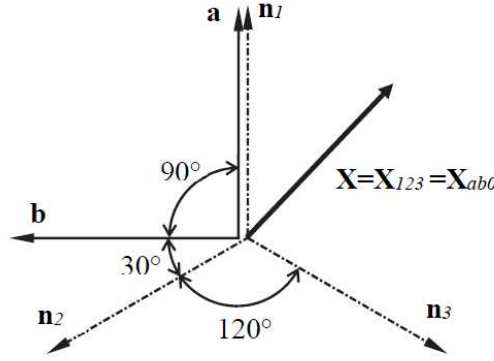


Figure 3.2: Vector base n_1, n_2, n_3 and vector base $\mathbf{a}, \mathbf{b}$.

Looking at the picture, using trigonometry it is possible to obtain these equations:

$$\begin{cases} n_1 = a \\ n_2 = -\frac{1}{2}a + \frac{\sqrt{3}}{2}b \\ n_3 = -\frac{1}{2}a - \frac{\sqrt{3}}{2}b \end{cases}$$

Recalling equation (3.8), it yields to:

$$\begin{cases} x_a = \frac{2}{3}(x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3) \\ x_b = \frac{2}{3}(\frac{\sqrt{3}}{2}x_2 - \frac{\sqrt{3}}{2}x_3) \\ x_0 = x_1 + x_2 + x_3 \end{cases}$$

These equations represent the core of the *three-phase/bi-phase transformation*. By adopting the **Amplitude Invariance Convention** (meaning that the amplitudes of the dq quantities correspond directly to the amplitudes of the phase quantities), the transformation can be summarized in matrix representation through the

matrix $\mathbf{T}$ and its reverse $\mathbf{T}^{-1}$ (bi-phase/three-phase transformation).

$$T = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \quad T^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -1/2 & \sqrt{3}/2 & 1 \\ -1/2 & -\sqrt{3}/2 & 1 \end{bmatrix}$$

This transformation is essential for simplifying both analysis and current control.

Park Transformation

In some cases, it may be useful to represent the electrical and magnetic quantities of an electrical machine by referring their representative vectors to a system of axes that is to a system of axes that is movable with respect to the fixed axes of the system.

Given a generic vector $\mathbf{X}$, defined through its components on an orthogonal system of axes 'ab', in order to find its components on a pair of axes ' α, β ' rotated by an angle θ defined as positive in the counterclockwise direction with respect to the reference frame 'ab'.

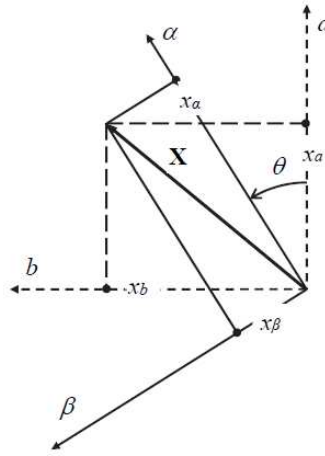


Figure 3.3: Vector base $\alpha\beta$ and vector base ab .

From this figure, it is evident that:

$$\begin{cases} x_{\alpha} = x_a \cos\theta + x_b \sin\theta \\ x_{\beta} = -x_a \sin\theta + x_b \cos\theta \end{cases}$$

The coordinate transformation is commonly referred to as a **rotation transformation**. The matrix that defines this transformation and its reverse are:

$$R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad R_\theta^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

SPM useful equations

The electrical and magnetic equations relevant to the study of the machine's behavior are those related to the **a** and **b** axes. The homopolar axis is generally uninfluential and independent, and for this reason it will be neglected in the remainder of the study.

The transformation concerns only the stator a and b axes.

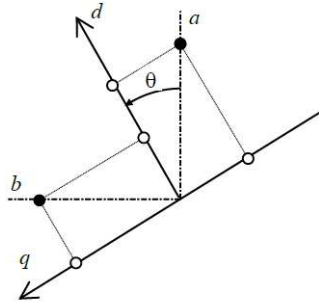


Figure 3.4: Stator a and b axes rotation.

Combining the Clarke and Park transformations to define the currents, yields the direct $(abc) \rightarrow (dq)$ transformation:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$

The introduction of the Clarke and Park transformation and d,q axes is fundamental to explain how the phases of the spm motor under study are disposed and why.

The transformation is defined with a positive sign convention for the q-axis component, consistent with the clockwise positive direction of rotation adopted in this work.

In terms of calculation, it is appropriate to start from a three-phase case.

The currents of a three-phase set are defined

$$\begin{cases} i_A = \sin(\omega t); \\ i_B = \sin(\omega t - \frac{2\pi}{3}); \\ i_C = \sin(\omega t + \frac{2\pi}{3}). \end{cases}$$

where I is the peak current amplitude and ω is the electrical angular frequency. Substituting the sinusoidal phase currents into the Park transformation gives:

$$i_d = \frac{2}{3}I \left[\sin\theta\cos\theta + \sin\left(\theta - \frac{2\pi}{3}\right)\cos\left(\theta - \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{2\pi}{3}\right) \right] \quad (3.9)$$

Using the trigonometric identity

$$\sin(x)\cos(x) = \frac{1}{2}\sin(2x) \quad (3.10)$$

:

$$i_d = \frac{1}{3}I \left[\sin(2\theta) + \sin\left(2\theta - \frac{4\pi}{3}\right) + \sin\left(2\theta + \frac{4\pi}{3}\right) \right] = 0 \quad (3.11)$$

, because the sum of three sinusoidal terms shifted by $2\pi/3$ is zero.

For the i_q component:

$$i_q = \frac{2}{3}I \left[\sin\theta\sin\theta + \sin\left(\theta - \frac{2\pi}{3}\right)\sin\left(\theta - \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{2\pi}{3}\right)\sin\left(\theta + \frac{2\pi}{3}\right) \right] \quad (3.12)$$

Using the trigonometric identity

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \quad (3.13)$$

:

$$i_q = \frac{1}{3}I \left[3 + (\cos(2\theta) + \cos(2\theta - \frac{4\pi}{3}) + \cos(2\theta + \frac{4\pi}{3})) \right] = I \quad (3.14)$$

Thus:

$$i_d = 0; |i_q| = I \quad (3.15)$$

However, this result depends on a specific assumption: the phase A current is expressed as a sinusoidal function of ωt (i.e., with zero phase offset), which implicitly assumes that the magnetic axis is aligned with the d-axis of the synchronous reference frame.

With this alignment, the sinusoidal definition ensures that the entire current amplitude is projected onto the q-axis.

This is exactly the condition required for **maximum torque production** in a SPM motor, where the torque is proportional only to i_q .

The demonstration is below.

The equations of the SPM in the d,q axes, in explicit form, using p to denote the derivative of the flux linkage and pp to denote $\frac{N_{poles}}{2}$, are:

$$\begin{cases} v_d = R_s i_d + p\lambda_d - pp\omega\lambda_q \\ v_q = R_s i_q + p\lambda_q + pp\omega_r\lambda_d \end{cases}$$

$$\begin{cases} \lambda_d = L_d i_d + \lambda_{mp} \\ \lambda_q = L_q i_q \end{cases}$$

And the electromagnetic Torque is given by:

$$T = \frac{3}{2} \frac{N_{poles}}{2} (\lambda_d i_q - \lambda_q i_d) \quad (3.16)$$

By replacing the fluxes definition, it is obtained:

$$T = \frac{3}{2} \frac{N_{poles}}{2} [i_d i_q (L_d - L_q) + \lambda_m i_q] \quad (3.17)$$

The first part of the formula depends on the magnetic isotropy of the motor ($L_d = L_q$), while the second depends on permanent magnets.

Consequently, the torque is:

$$T = \frac{3}{2} \frac{N_{poles}}{2} \lambda_m i_q, \quad (3.18)$$

confirming that only the current on q-axis create Torque.

If the phase currents were instead defined with a cosine function while keeping the same alignment, the current would be entirely on the d-axis, producing zero torque. Therefore, the choice of the sine function is not arbitrary but a deliberate design choice to achieve the desired torque production.

The SPM considered in this work, as explained in the Multi-phase machines, is a multi-n-phase SPM motor, a combination of 3-phase stars, specifically a triple three-phase solution.

The same reasoning made with the 3-phase case can be generalized to the triple-three-phase system. The explicit derivation of the 9-phase Park matrix is not reported here for the sake of brevity, as the analytical development is more complex than the 3-phase case. However, the transformation is implemented internally by COMSOL Multiphysics, as the underlying principle is identical.

In order to configure the disposition of the phases in relation to the concentrated winding it is used the convention: *clockwise is positive* and *counterclockwise is negative*. The phase A1 is taking as the reference at 0° electrical and the winding sequence is: A1 – B3 – A2 – C1 – A3 – C2 – B1 – C3 – B2 (repeated three times over the 27 slots).

It is recalled that the correlation between the mechanical angle and the electrical angle is:

$$\theta_{el} = \theta_m \cdot N_{poles}/2 \quad (3.19)$$

Thereby, to clarify the relationship between the physical slot positions and the electrical spatial phase shifts the following table is provided:

Table 3.6: Phases Disposition.

Phase	α_m	α_{el}	α_{el} Reduced
A1	0°	0°	
B3	13.33°	200°	-200°
A2	26.66°	400°	-40°
C1	40°	600°	120°
A3	53.33°	800°	-80°
C2	66.66°	1000°	80°
B1	80°	1200°	-120°
C3	93.33°	1400°	40°
B2	106.66°	1600°	-160°

The phases belonging to the same three-phase set (A1, B1, C1), (A2, B2, C2), and (A3, B3, C3) are each separated by 120° electrical. The homologous phases (A1, A2, A3), (B1, B2, B3), and (C1, C2, C3) exhibit successive shifts of 40°.

The phasor diagram of the motor is shown in the figure below.

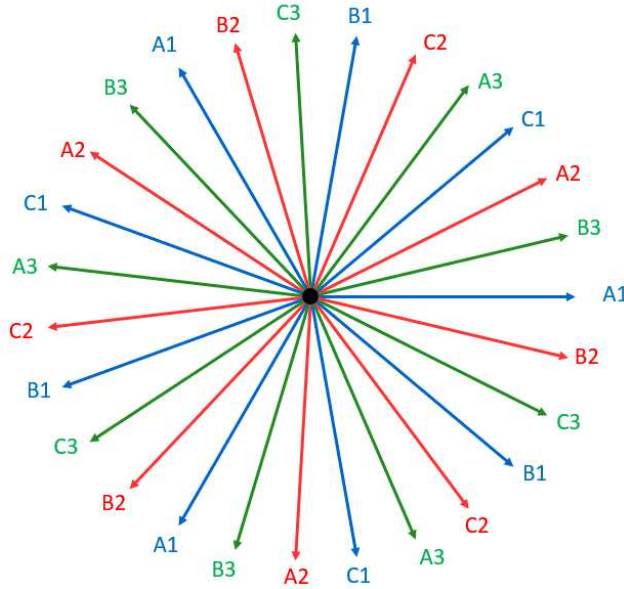


Figure 3.5: Phasor Diagram Configuration.

From the diagram, it can be easily observed that starting phase A1 (the reference phase) of the SPM motor is aligned with the d-axis, as explained before, and the other phasors are arranged counterclockwise with the correct phase shifts.

Each phase occupies 6 half-slots, corresponding to 3 full coil, so that each of the 27

slots of the motor are occupied.

Prior to delineating the phase currents, several parameters such as electrical speed expressed in rad/s , electrical frequency f_{el} , and peak current I_{pk} must be defined.

$$f_{el} = \frac{n \cdot N_{poles}/2}{60} \quad (3.20)$$

$$\omega_{el} = \frac{n \cdot 2\pi \cdot N_{poles}/2}{60} \quad (3.21)$$

Table 3.7: Motor's Parameters.

Parameter	Value	Unit of Measurement
n	1000	rpm
f_{el}	250	Hz
ω_{el}	104.72	rad/s
I_{pk}	25	A

In the following, all electrical phase shifts, previously given in degrees, are converted into radians for the mathematical formulation. The conversion is carried out according to $\theta[rad] = \theta[^\circ] \cdot \pi/180$. The phase currents are, therefore, expressed as sinusoidal functions of time with their respective phase shifts given in radians and considering a default offset.

$$\left\{ \begin{array}{l} I_{A1} = I_{pk} \cdot \sin(\omega_{el} \cdot t + \alpha_{offset}) \\ I_{B3} = I_{pk} \cdot \sin(\omega_{el} \cdot t + \alpha_{offset} - \frac{2\pi}{3} - \frac{80\pi}{180}) \\ I_{A2} = I_{pk} \cdot \sin(\omega_{el} \cdot t + \alpha_{offset} - \frac{40\pi}{180}) \\ I_{C1} = I_{pk} \cdot \sin(\omega_{el} \cdot t + \alpha_{offset} + \frac{2\pi}{3}) \\ I_{A3} = I_{pk} \cdot \sin(\omega_{el} \cdot t + \alpha_{offset} - \frac{80\pi}{180}) \\ I_{C2} = I_{pk} \cdot \sin(\omega_{el} \cdot t + \alpha_{offset} + \frac{2\pi}{3} - \frac{40\pi}{180}) \\ I_{B1} = I_{pk} \cdot \sin(\omega_{el} \cdot t + \alpha_{offset} - \frac{2\pi}{3}) \\ I_{C3} = I_{pk} \cdot \sin(\omega_{el} \cdot t + \alpha_{offset} + \frac{2\pi}{3} - \frac{80\pi}{180}) \\ I_{B2} = I_{pk} \cdot \sin(\omega_{el} \cdot t + \alpha_{offset} - \frac{2\pi}{3} - \frac{40\pi}{180}) \end{array} \right.$$

Stator Windings: Geometric Sizing

The slot window dimensions, as reported in the table of the magnetic core section, are **length_window** = 9.38 mm and **height_window** = 15.67 mm. On the basis of these values, the geometric dimensions of the coils are defined. The SPM motor adopts double-layer windings, with two active sides arranged side by side, in the tangential direction, in each slot. Each coil has 44 turns. Accordingly, the table below lists the dimensions of each half-coil within the stator slots.

Table 3.8: Stator Windings Dimension: Half Coil.

Parameter	Value	Unit of Measurement
w_coil	4.36974	mm
h_coil	13.6325	mm
w_coil clearance	0.25	mm
h_coil clearance	0.6	mm

The two clearance values represent the minimum distance in air between two conductive parts, measured along the shortest path, one for the tangential dimension and one for the radial dimension.

Stator Windings: Selection of Material

The material chosen for the concentrated windings is standard copper, characterized by different parameters:

- a linear BH curve;
- an electrical conductivity σ ;
- by a relative magnetic permeability slightly lower than unity, both in the x-direction e in the y-direction, since it is a diamagnetic material;
- consequently to the previous point, copper has a zero coercitive field, as it exhibits no magnetic hysteresis.

Table 3.9: Stator Windings Material Properties.

Parameter	Value	Unit of Measurement
σ	45.4618531287401	MS/m
μ_{rx}	1	
μ_{ry}	1	
H_c	0	A/m

3.1.3 Permanent Magnets

As said in the chapter , permanent magnets are hard ferrimagnetic or ferromagnetic materials, characterized by high magnetocrystalline anisotropy. These properties allow the material to retain residual magnetic flux B_r in the absence of an externally applied field, resisting demagnetization. Depending on the composition and manufacturing process, it is possible to differentiate classes of materials, such as neodymium-iron-boron (NdFeB), Samarium-Cobalt (SmCo), ferrites, and AlNiCo magnets, each with specific characteristics regarding thermal stability, corrosion resistance, and critical demagnetizing field. The property of a permanent magnet, besides the residual remanent flux B_r , is the coercive field H_c .

Permanent Magnets: Geometric Sizing

In the section concerning stator windings, two fundamental parameters for the analysis of permanent magnet machines — namely, the pole pitch angle τ_p and the number of poles N_{poles} — have already been defined.

In the SPM motor under investigation, the magnets are arranged in a circular pattern, forming an annular ring whose inner boundary is defined by the rotor outer radius and whose outer boundary is defined by the stator inner radius minus the airgap thickness (i.e., the outer radius of the magnet volume).

Due to the presence of non-magnetized sectors separating adjacent north and south poles, the magnet does not extend over the entire pole pitch, but only over a fraction thereof.

Indeed, the effective angular width of the magnet is denoted as α_{pm} , while the angular portion corresponding to the airgap between adjacent poles is denoted as α_a . The pole pitch angle is therefore divided into two contributions:

$$\tau_p = \alpha_{pm} + \alpha_a \quad (3.22)$$

In the following table are listed the values of this two parameters, together with the geometric dimensions and the inner and outer boundaries.

Table 3.10: Permanent Magnets Parameters.

Parameter	Value	Unit of Measurement
m_{length}	10.67	mm
$m_{thickness}$	4.50	mm
Rm_{int}	78	mm
Rm_{out}	82.50	mm
N_{poles}	30	
α_{pm}	10.80	°
α_a	1.2	°

Since the stator inner radius is 83 mm, while the outer radius of the magnet ring is 82.5 mm, it follows that the airgap is 0.5 mm, a value resulting from a specific design choice.

For the geometric description of the individual magnet, the parameter m_{length} is the circumferential length of the magnet (i.e., the angular arc length occupied by the magnet along the tangential direction); $m_{thickness}$ is the radial thickness of the magnet. These quantities are representative of the amount of active magnetic material and directly affect the airgap flux and, consequently, the electromagnetic performance of the motor.

Permanent Magnet: Selection of Material

The material chosen for the permanent magnets is MPN40SH, a Neodymium Iron Boron (Nd-Fe-B) pm type. Among the permanent magnets currently available on the market, Nd-Fe-B magnets feature the highest $(BH)_{max}$. They combine excellent magnetic performance with relatively low production costs and abundant availability of raw materials. The stoichiometric composition of Nd₂Fe₁₄B provides a favorable balance between magnetic induction and magnet size. In recent years, Nd-Fe-B alloys have received significant research interest, as they have made it possible to produce permanent magnets with properties that rival or surpass those of SmCo (Samarium–Cobalt) magnets.[6]

The permanent magnet is characterized by a linear BH curve, the other magnetic properties are listed in the table below:

Table 3.11: Permanent Magnets Material Properties.

Parameter	Value	Unit of Measurement
σ	0.6944444444444444	MS/m
H_c	936499.51614133	A/m
μ_{rx}	1	
μ_{ry}	1	

3.1.4 Rotor

Rotor: Geometric Sizing

The rotor is configured as an annular ring placed within the magnet ring. Hence, the rotor outer radius corresponds to the inner radius of the magnets. Although not separated by air gaps like the magnets, the rotor exhibits a radial tooth pattern with angular extensions of 10.8° and 1.2° , which are repeated periodically over the entire circumferential length. The rotor geometric parameters are summarized in the table below.

Table 3.12: Rotor Parameters.

Rr_{int}	Rr_{out}	y_{mr}
73.50 mm	78 mm	14 mm

The circumferential length of the rotor yoke, y_{mr} , corresponds to the arc subtended by the angular width of 10.8° , calculated at the mean radius of the rotor, which is 75.75 mm. This yields a value of approximately 14 mm.

Rotor: Selection of Material

The material chosen to compose the rotor is M980-50DallNew_TH. Its properties include:

- non linear BH curve;
- electrical conductivity;
- relative magnetic permeability μ_r , obtained from the curves.

Table 3.13: Rotor Non Linear BH Curve Data.

B [T]	H [A/m]
0.000000	0.000000
0.005200	7.450000
0.014200	14.890000
0.049000	29.790000
0.119900	44.680000
0.269100	59.570000
0.466200	74.470000
0.831600	111.700000
1.030500	148.940000
1.343500	297.870000
1.403000	372.340000
1.480200	558.510000
1.518700	744.680000
1.587100	1489.360000
1.625600	2234.040000
1.653200	2978.720000
1.700600	4468.080000
1.741400	5957.440000
1.776600	7446.800000
1.851600	11170.200000
1.927500	15875.700000
2.121900	34457.000000
2.217400	68914.000000
2.270100	103530.000000
2.321500	139499.000000
2.368100	175468.000000
2.400000	200535.000000
2.500000	279715.000000

B [T]	H [A/m]
2.600000	359054.000000
2.700000	438551.000000
2.800000	518049.000000
2.900000	597587.000000
3.000000	677141.000000

Table 3.14: Rotor Material Properties.

Parameter	Value	Unit of Measurement
σ	2	MS/m

3.2 Modelling on Comsol Multiphysics

3.2.1 C-shaped core stator and Windings

Geometry and Layout Phase Windings

The C-core stator geometry was modeled within COMSOL Multiphysics according to the dimensional data provided in the C-shaped stator core: Geometric Sizing 3.1.1 section.

The C-core stator was modelled with its lower-left corner (A) fixed at the global origin (0,0).

To accurately represent the rounded curvature of the C-shaped core, the built-in Fillet operation was employed on both the internal and external corner vertices. This operation replaces sharp vertices with smooth, rounded corners, characterized by a constant radius. A fillet radius of $r = 0.8 \text{ mm}$ was uniformly applied to all selected vertices.

The resulting geometry, after the fillet operation has been applied, is shown in the figure. The black points underline the original baseline geometry.

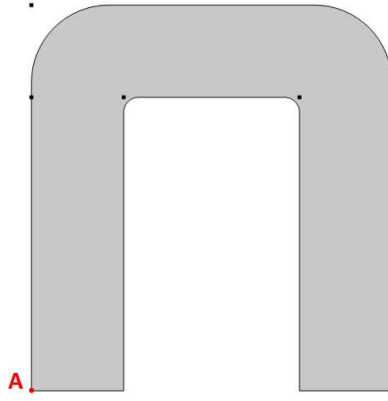


Figure 3.6: C-core on COMSOL.

Following the core construction, the windings were placed inside the core. Owing to their double-layer configuration and tangential side-by-side arrangement, they were modelled as two identical rectangular regions (3.1.2), separated by an air clearance.

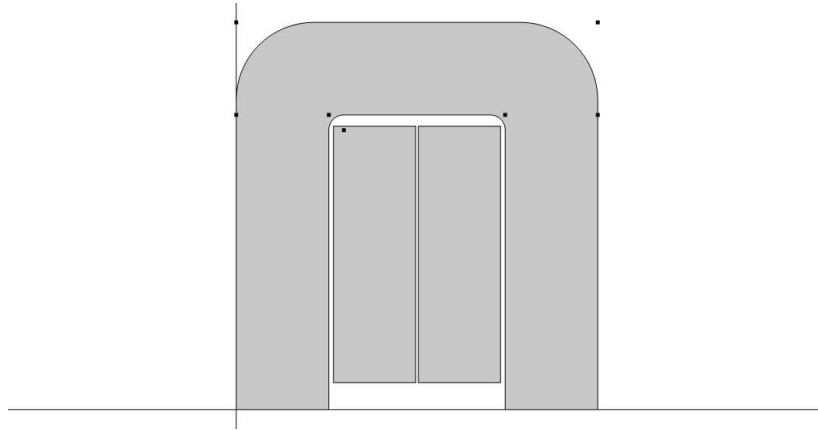


Figure 3.7: C-core and windings on COMSOL.

The final SPM motor stator was then generated by applying a series of geometric transformations in the sequential order reported below:

- Move: a translation was applied using a displacement vector defined as $d = (-Rs_{int} - \text{length_n}, \text{nthickness} + \text{length_window}/2)$;
- Rotation 1: a 90° rotation was executed about the $(-Rs_{int}, \text{nthickness} + \text{length_window}/2)$. The purpose of this operation is to align the core such that its horizontal diameter, lying along the global x-axis, exactly splits the core into two symmetrical halves;
- Rotation 2: a final rotation was performed over the angular interval range $(\alpha_n, \alpha_n, 360 - \alpha_n)$, with the centre of rotation set at the origin. This last step yields the complete stator geometry of the SPM machine.

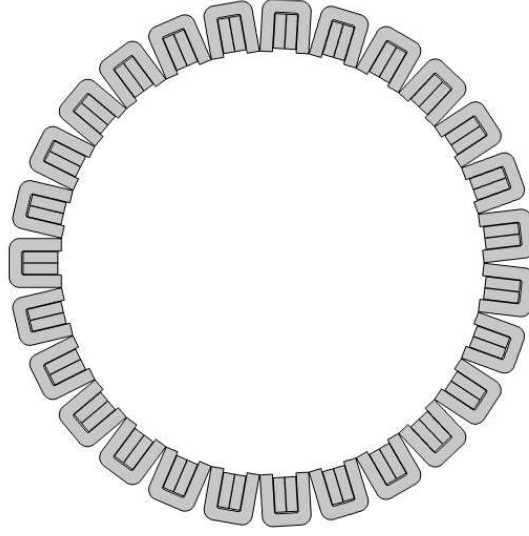


Figure 3.8: Complete stator geometry.

It is worth noting that α_n denotes the slot pitch, namely the angular offset between consecutive slots. Therefore, the rotational transformation is employed to distribute the 27 slots along the circumference defined by the stator inner radius Rs_{int} .

As a result of the final rotation step, the core is positioned so that, in the positive xy-quadrant, the x-axis symmetrically bisects the stator geometry, i.e., it divides the stator cross-section into two mirror-image halves.

This symmetric arrangement is not incidental, but a prerequisite for aligning the **phase A1** with the **d-axis** of the ***dq reference frame***. Such alignment is crucial for consistently defining the phase current and for correctly setting up the subsequent electromagnetic field simulations.

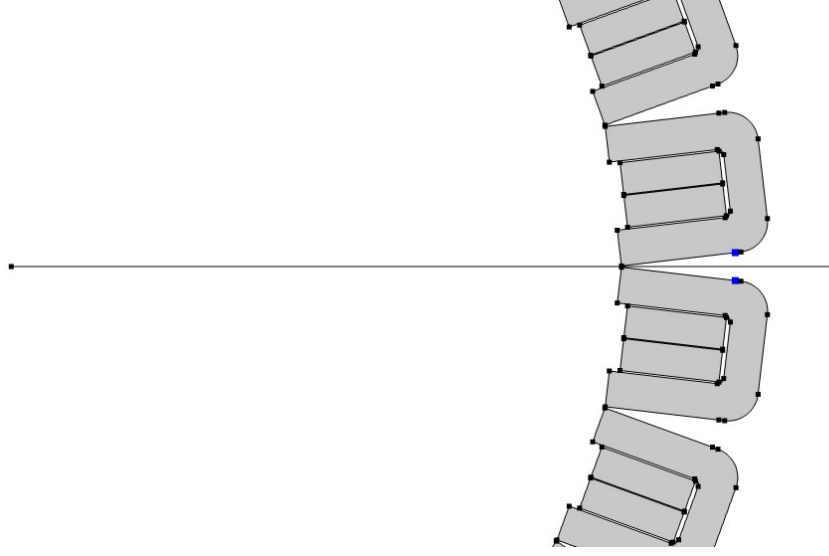


Figure 3.9: Highlight x-axis, which symmetrically splits the gap between two adjacent cores. As evidence, the coordinates relative to the origin of the point above the x-axis are (99.11848166, 1.909867219) [mm] and of the point below the x-axis are (99.11848166, -1.909867219) [mm].

In order to define the current excitations and assign them correctly to the corresponding phase regions, nine *Coil* nodes were created in the COMSOL model, one for each phase. Each Coil node was applied to the respective winding domain. For each coil, the 'Coil Group' option was selected, and the number of turns was set to $N_{turns}=44$.

The conductor model was set to *Homogenized Multiturn*, which is the appropriate choice in COMSOL for this type of winding representation, as it allows the definition of series-connected coil domains and the specification of the number of turns.

3.2.2 Permanent Magnets and Rotor

The permanent magnets and the rotor, as previously illustrated, are two concentric annular regions defined by their respective inner and outer radii. These regions were subdivided using the Partition operation in COMSOL. The partition was performed in two steps: first, a set of lines was created with an angular spacing defined by the range (12, 12,360-12) [°], i.e. 30 lines spaced by 12 °.

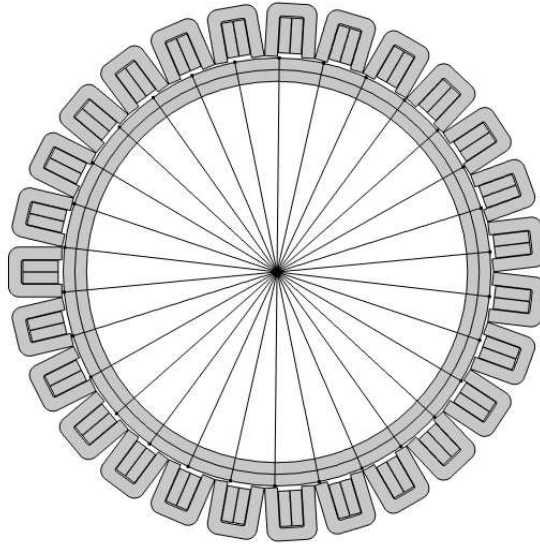


Figure 3.10: First step of partition.

Subsequently, a second set of lines was introduced with a finer angular resolution, defined by range (1.2, 12, 360) [°]. This operation resulted in a subdivision of the pole pitch angle into alternating sectors of 10.8 ° (occupied by the magnet) and 1.2 ° (occupied by air).

The rotor region was also partitioned following the same angular pattern, although it is not physically separated by air gaps; this was done to maintain a consistent subdivision between the rotor and the magnet domains.

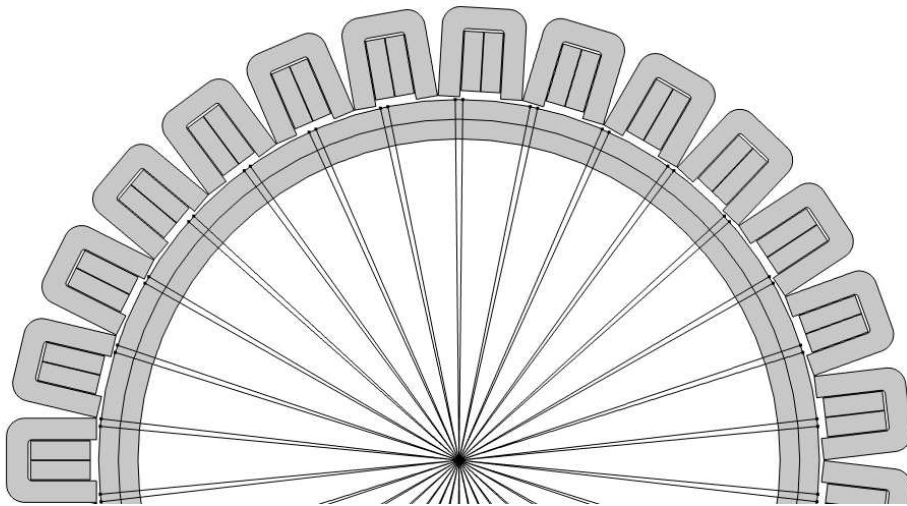


Figure 3.11: Second step of partition.

Finally, the Partition operation was used to further split the magnet sectors into

North and South polarisation regions, according to a certain magnetisation pattern.

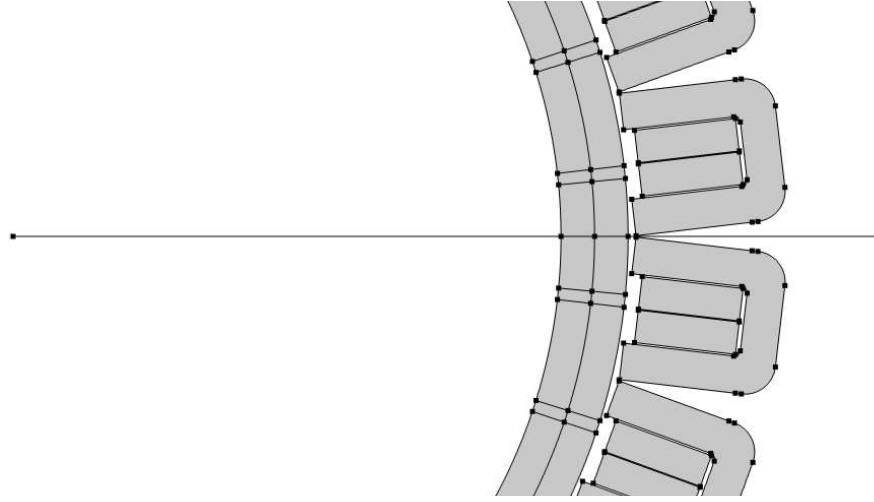


Figure 3.12: Highlight x-axis (aligned with d-axis, which symmetrically splits the N-magnet.

As in the picture, the d-axis, in addition to being the symmetry axis bisecting the airgap between the two adjacent stator cores, also passes through the centre of the magnet located at that angular position. This magnet was assigned a North polarisation. The remaining North and South polarisation pattern for the other magnets was then defined accordingly, following the alternating sequence determined by the angular periodicity of the machine.

The final steps in the construction of the SPM motor geometry consisted of adding two supplementary annular domains. The first is a support structure surrounding the stator, with an outer radius $R_{s_{out}}$. The second fills the central void defined by the rotor inner radius. Both regions are treated as air in the model, since they are not part of the active magnetic circuit but are necessary to close the geometry and to properly define the boundaries for the magnetic field solution.

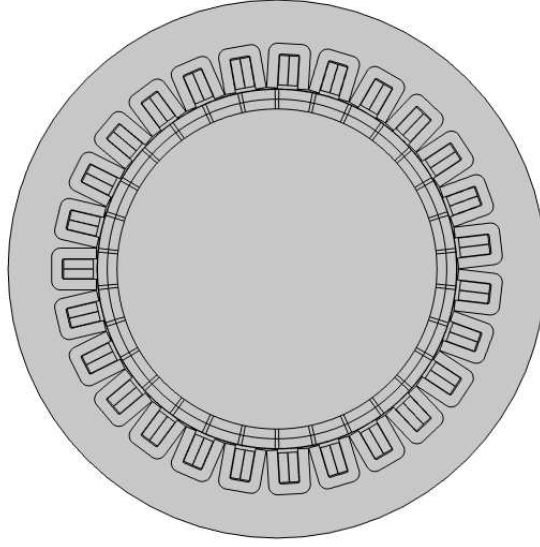


Figure 3.13: SPM motor final geometry.

The individual geometric domains forming rotor and stator were combined using the *Form Union* operation, which merges all objects into a single geometry while preserving internal boundaries between different domains. This approach ensures a conforming mesh across interior boundaries and maintains the continuity of the magnetic field by default, as required for magnetic field simulations.

When the *Form Union* operation is applied, the circular boundary that defines the airgap is assigned to the rotor domain. This ensures that the sliding interface between the rotor and the stator is properly positioned at the airgap boundary.

Finally, the entire geometry was finalized using the *Form Assembly* operation. This step is necessary when using the Rotating Machinery, Magnetic interface, as it treats the geometry as a collection of separate objects and automatically generates identity pairs at the touching boundaries, allowing relative motion between the rotor and stator domains.

3.2.3 Physics Setup: Rotating Machinery, Magnetic Interface and Mesh Generation

The electromagnetic field distribution is computed using the Rotating Machinery, Magnetic (rmm) interface. This physics interface is specifically designed for rotating electrical machines and provides the necessary framework for modelling the relative motion between rotor and stator, as well as the magnetic field equations. This subsection describes the key subnodes and settings used in the present model.

An **Identity Boundary Pair** is implemented to couple the airgap boundaries belonging to the rotor (source) and the stator (destination).

This procedure establishes a connection between the two separate parts of the assembly, enforcing the continuity of the magnetic vector potential across the airgap and allowing the solver to treat the airgap as a coupling interface without explicitly

modelling the air layer as a separate domain.

The source selection was assigned to the outer boundaries of the rotor, while the destination selection was assigned to the inner boundaries of the stator.

Magnetic Insulation and **Continuity** conditions are applied to this pair under the Rotating Machinery, Magnetic interface.

The Magnetic Insulation condition ($\mathbf{n} \times \mathbf{A} = 0$) was also applied to the outer boundary of the stator support. Physically, this imposes a zero tangential component of the magnetic vector potential, which is equivalent to a perfectly magnetic wall. This assumption considers the leakage flux outside the magnetic core negligible.

The **Ampère's Law** node is applied to all domains of the model. For the air regions, a linear constitutive relation with $\mu_r = 1$, isotropic is used.

For the rotor and stator cores, the BH constitutive relation is used as magnetization model. The magnetic behaviour is defined by the magnetic field norm and the magnetic coenergy density too. The electrical conductivity is specified in the conduction model. All material properties are taken directly from the material definition.

For the permanent magnets, the **Conductive Magnet** subnode is used. Regarding the BH constitutive relation, the magnetization model is defined by the remanent flux density and the recoil permeability. For the J_c - E constitutive relation, the electrical conductivity is specified. All material properties are taken directly from the material definition.

The electromagnetic torque is computed using the **Force Calculation** node, which integrates the Maxwell stress tensor over the selected domains. The torque axis is set as the z-axis, with the rotation point at the origin (0,0).

The finite element mesh discretizes the computational domain into smaller elements, directly affecting the accuracy and convergence of the numerical solution. A finer mesh is required in regions where high field gradients are expected, such as the airgap, while coarser elements can be used in less critical areas. The mesh was generated using the following settings.

Global mesh settings:

- Mesh size calibration: General Physics;
- Predefined size: Normal.

Free Triangular node:

- Smoothing across removed control entities: Enabled;
- Number of iterations: 4;
- Maximum element depth to process: 4;
- Tessellation method: Automatic.

Boundary Layers node (applied to the airgap and stator support domains):

- Smooth transition to interior mesh: Enabled;

- Number of iterations: 4;
- Maximum element depth to process: 6;
- Handling of sharp corners: Splitting;
- Split for angles greater than: 240° ;
- Maximum angle per split: 100° ;
- Maximum layer decrement: 2;
- Trim for angle less than: 50° .

Boundary Layer Properties subnode (applied to the identity airgap boundaries):

- Number of layers: 5;
- Stretching factor: 1.2;
- Thickness specification: Automatic;
- Thickness adjustment factor: 1.

3.2.4 Materials Selection

The material properties used in the COMSOL model are those defined in section 3.1. In the COMSOL environment, these materials were assigned to the corresponding geometric domains using the '*Materials*' node under the Component tree.

For each material, the relevant physical properties were specified according to the material type. Specifically, for the linear materials (copper windings and permanent magnets), the electrical conductivity, the relative permeability, and, in the case of the magnets and coercivity were defined. For the non-linear stator and rotor cores, the magnetic behaviour was described by means of the B-H curve and electrical conductivity.

The C-shaped stator cores are physically laminated with a non-standard pattern: the laminations are arranged as concentric C-shaped layers in the xy-plane, rather than stacked along the axial direction. Since COMSOL only supports standard planar laminations (stacked along the out-of-plane axis), this specific geometry could not be modelled with the built-in lamination feature. Therefore, the cores were treated as solid (massive) bodies in the simulation. This simplification was deemed acceptable for the purpose of this study, although it may lead to an overestimation of eddy current losses.

For the windings, the standard 'Copper' material from the COMSOL material library was selected, using the built-in material properties as provided by the software.

For the stator and rotor cores, which exhibit non-linear magnetic behaviour, a custom non-linear material was defined. The B-H curve data were imported from external ".txt" files into the respective 'BH Curve' sub-nodes of the material definitions.

The imported data were first validated using the 'Curve Checker' feature to ensure monotonicity and correct slope behaviour. Additionally, the effective B-H curves were computed using the built-in 'Effective Nonlinear Magnetic Curves Calculator' app available in the AC/DC Module.

For the permanent magnets, a linear B-H characteristic was assumed. The material was defined as a custom variant of 'NdFeB', with properties (including remanent flux density, coercivity and relative permeability) adapted to the specific grade used in this study.

All material assignments were performed at the domain level, and the material properties were linked to the respective physics interfaces (Magnetic Fields and Ampere's Law) through the predefined COMSOL multiphysics couplings.

Chapter 4

Finite Element Analysis (FEA) Simulation Setup: FEMM and COMSOL

4.1 COMSOL Simulations

Before introducing the studies, the rotational motion is defined using the Rotating Domain sub-node, which is available under the Moving Mesh node.

The rotor domains (i.e. the permanent magnets, the rotor core, airgap and the inner air region) were selected as the rotating domains. The rotation type was set to *Specified Rotational Velocity*, with the rotation velocity defined as *Constant Angular Velocity*. The **angular velocity** was specified as: $n \cdot 2\pi/60$ [rad/s]. The **initial angle** was set to 0 rad and the rotation axis base point was set at the origin ($x=0$, $y=0$) [m].

A multi-step simulation procedure was adopted for methodological reasons. In the first stage, a Stationary study was performed without activating the Loss Calculation node. This provided the initial magnetic field distribution, the torque-vs-angle characteristic, and the spatial maps of flux density B and current density J over the entire geometry, serving as a consistent initial condition for the subsequent time-dependent simulations.

Then, the first Time Dependent study was run without losses. This was done to compute the load current and voltage waveforms, the torque ripple and the air-gap flux density under nominal operating conditions, without the additional complexity introduced by the loss models. As in the stationary study, both B and J surface maps are obtained.

A second Time Dependent study was run under no-load conditions (zero current impressed to the windings) to obtain the sinusoidal back-EMF and its harmonic content.

These loss-free simulations were performed first to establish a baseline reference for the machine performance and to validate the model behaviour before introducing the loss computation. They also provide a consistent set of results against which

the loss-inclusive simulations can be compared.

Finally, a Time Dependent study was run with the Loss Calculation node active, including a Time to Frequency Losses study step. This allowed the computation of resistive losses (Joule losses) in all conductive domains from the time-domain solution. In this final study, currents, voltages, torque ripple, airgap flux density (both norm and radial components), as well as the surface plots of magnetic flux density B and current density J , were also re-computed, now including the effect of losses. This enabled a direct comparison with the loss-free cases, highlighting the impact of eddy currents on the motor performance.

The loss-free studies were therefore essential not only for obtaining the baseline performance but also for providing a consistent initial condition and a reference solution against which the loss-inclusive results could be evaluated.

The initial values of the variables solved for the time dependent studies were taken from the Stationary study solution, which provides a consistent and converged initial condition for the transient simulation.

4.1.1 Stationary Study

The Stationary study was set up to compute the magnetic field distribution and the torque as a function of the rotor position. The study solves for the dependent variables defined in the Rotating Machinery, Magnetic physics interface.

The main solver settings and the parametric sweep configuration adopted in the present study are described below.

Values of dependent variables:

- Initial values of variables solved for: Physics controlled;
- Values of variables not solved for: Physics controlled.

Adaptation and error estimates: None.

The sweep is defined over the parameter α_{offset} (which is defined in the model parameters as 0 rad), using the range: range(0, 5, 360)[deg]. This parametric sweep allows the evaluation of the torque as a function of the angular position.

Parametric Sweep (subnode):

- Use parametric solver: Automatic;
- Default solver sequence generation: Using global parameters;
- Scaling method: Automatic;
- Residual scaling method: Automatic.

Stationary Solver (subnode):

- Nonlinear method: Automatic (Newton);

- Linear Solver: Direct;
- Relative Tolerance: 1e-6;
- Initial damping factor: 1;
- Minimum damping factor: 1e-4;
- Restriction for step-size update: 10;
- Restriction for step-size increase: 1;
- Use recovery damping factor: Automatic;
- Recovery damping factor: 0.75;
- Update automatic scale factors in weights: On;
- Termination technique: Tolerance;
- Maximum number of iterations: 100;
- Termination criterion: Solution or residual;
- Residual factor: 1000.

4.1.2 No Load Time Dependent Study - No Losses

The no load analysis has the purpose of characterising the induced phase voltages and evaluating the harmonic content of the back-electromotive force (back-EMF).

The study is divided into two-step simulation procedure.

In the **first step**, a stationary parametric study is performed with the rotor at standstill and zero current imposed in all windings. The rotor mechanical angle is swept over one pole pitch in order to compute the flux linkage for each phase as a function of the rotor position. From the resulting flux-angle curves, the flux-null angles are identified for each phase, corresponding to the rotor positions where the flux derivative with respect to the angle is maximum and therefore the induced voltage reaches its maximum.

In the **second step**, a time-dependent study is set up with the rotor rotating at synchronous speed, again with zero current in all windings and with a fixed initial rotor position. The induced voltage for each phase is then extracted, providing the time-domain waveforms of the no-load phase voltages, essential for determining the amplitude, phase shift and harmonic distortion of the induced voltages.

As a qualitative validation of the field solution, the spatial distribution of the magnetic flux density is examined through a surface plot over the entire motor domain, which allows the identification of saturation regions, flux paths and the effectiveness of the magnetic insulation boundary condition.

First Step Solver Settings Values of dependent variables, both under physics control.

Adaptation and error estimates: None.

The sweep is defined over the parameter α_{offset} (which is defined in the model parameters as 0 rad), using the range: range(0, 5, 360)[deg]. This parametric sweep allows the evaluation of the torque as a function of the angular position.

Parametric Sweep (subnode):

- Use parametric solver: Automatic;
- Default solver sequence generation: Using global parameters;
- Scaling method: Automatic;
- Residual scaling method: Automatic.

The sweep is defined using the range: range(0, 0.1,12) [deg].

Stationary Solver (subnode):

- Nonlinear method: Automatic (Newton);
- Linear Solver: Direct;
- Relative Tolerance: 0.001;
- Initial damping factor: 1;
- Minimum damping factor: 1e-4;
- Restriction for step-size update: 10;
- Restriction for step-size increase: 1;
- Use recovery damping factor: Automatic;
- Recovery damping factor: 0.75;
- Update automatic scale factors in weights: On;
- Termination technique: Tolerance;
- Maximum number of iterations: 100;
- Termination criterion: Solution or residual;
- Residual factor: 1000.

Second Step Solver Configuration

Initial rotor position (Rotation angle): - 6 [deg].

Solver settings (subnode):

- Time range: $\text{range}(0, 1/f_{el}/100, 1/f_{el})$ (1 period at 0.004 s, with a fixed time step);
- Time to store: steps taken by solver closest to output times;
- Nonlinear method: Automatic (Newton);
- Jacobian Update: on every iteration;
- Time stepping: manual;
- Solver Type: Implicit;
- Linear Solver: Direct;
- Method: BDF, maximum order 2 and minimum order 1;
- Initial time step: 1e-7 s;
- Event Tolerance: 0.01;
- Consistent Initialization: Backward Euler with the default values;
- Error Estimation: exclude algebraic;
- Relative tolerance: 1e-6;
- Scaling method: Automatic;
- Residual scaling method: Manual, 1;
- Solver: MUMPS;
- Damping factor: 1;
- Termination technique: Tolerance;
- Maximum number of iterations: 50;
- Tolerance Factor: 0.2;
- Termination criterion: Solution.

To extract the harmonic content of the phase voltages, a Discrete Fourier Transform was applied to the time-dependent solution using the FFT feature available in the Results node. The frequency spectrum was selected, and the amplitude scale was set to Divided by number of frequencies.

4.1.3 On Load Time Dependent Study - No Losses

The Time-Dependent study under on-load conditions was performed to evaluate the spm motor behaviour when nominal currents are impressed in the windings. This simulation provides the transient electromagnetic response of the motor.

The study solves the time-dependent magnetic field equations using the same physics interface defined in the stationary study. The currents are imposed as sinusoidal functions with the appropriate phase shifts, as defined in the Coil nodes. The rotor speed is kept constant at the rated value, specified in the Rotating Domain node.

Solver settings (subnode):

- Time range: range(0.0001, $1/f_{el}/100$, $1/f_{el}$) (1 period at 0.004 s, with a fixed time step);
- Values of variable not solved: from Stationary Study, *Parameter Value*: $\alpha_{offset}=0$ [deg];
- Time to store: steps taken by solver closest to output times;
- Nonlinear method: Constant (Newton);
- Jacobian Update: on every iteration;
- Time stepping: manual;
- Solver Type: Implicit;
- Linear Solver: Direct;
- Method: BDF, maximum order 2 and minimum order 1;
- Steps taken by solver: strict;
- Initial time step: 1e-8 s;
- Event Tolerance: 0.01;
- Consistent Initialization: Backward Euler with the default values;
- Error Estimation: exclude algebraic;
- Relative tolerance: 1e-7;
- Scaling method: Automatic;
- Residual scaling method: Manual, 1;
- Solver: MUMPS;
- Damping factor: 1;
- Termination technique: Tolerance;
- Maximum number of iterations: 50;

- Tolerance Factor: 0.1;
- Termination criterion: Solution.

4.1.4 Time Dependent Study - Time To Frequency Losses

The final Time-Dependent study is performed with the Loss Calculation subnode active and it is run over two electrical periods to ensure steady-state conditions. This study computes the electromagnetic losses, specifically the resistive contributions in the stator cores, rotor, permanent magnets, and windings, all due to induced eddy currents in these conductive domains.

The Loss Calculation node is configured for resistive heating and applied to the ferromagnetic and magnet regions, while the winding losses are computed directly from the current distribution defined by the Coil nodes.

The study itself includes an additional Time to Frequency Losses study step, which is added after computing the Time Dependent study. The Time to Frequency Losses step performs a forward Fast Fourier Transform (FFT) from the time domain to the frequency domain. It takes the input data from the last steady-state period of the time-dependent solution and computes the cycle-averaged loss density. The loss is then obtained by summing the contributions from the different harmonic frequencies.

Time Dependent Study Solver settings (subnode):

- Time range: $\text{range}(0, 1/f_{el}/1000, 2/f_{el})$ (1 period at 0.004 s, with a fixed time step);
- Values of variable not solved: from Stationary Study;
- Time to store: steps taken by solver closest to output times;
- Nonlinear method: Automatic (Newton);
- Jacobian Update: on every iteration;
- Time stepping: manual;
- Solver Type: Implicit;
- Linear Solver: Direct;
- Method: BDF, maximum order 2 and minimum order 1;
- Steps taken by solver: strict;
- Event Tolerance: 0.01;
- Consistent Initialization: Backward Euler with the default values;
- Error Estimation: exclude algebraic;
- Relative tolerance: 0.01;

- Scaling method: Automatic;
- Residual scaling method: Manual, 1;
- Solver: MUMPS;
- Damping factor: 1;
- Termination technique: Tolerance;
- Maximum number of iterations: 100;
- Tolerance Factor: 0.2;
- Termination criterion: Solution.

Time to Frequency Loss

- Input Study: Time Dependent Losses;
- Start Time: 0 [s];
- End Time: $2/f_e l$;
- Number of Harmonics: 6;

4.1.5 Post Processing

The following post-processing operations are performed to extract the relevant quantities from the simulation results.

To extract the **harmonic content** of the phase voltages, a Discrete Fourier Transform is applied using the FFT feature available in the Results node. First, the coil voltage is evaluated using a Global Evaluation from the time-dependent solution, which returns a table of discrete values at each time step. These tabulated data, which represent the back-EMF waveform over time, are then used directly as input to the FFT function to perform the harmonic analysis. In this node, the frequency spectrum is selected, and the amplitude scale is set to: Divided by number of frequencies.

The **air-gap flux density** is extracted using two different approaches. The first approach computes the normal component of the flux density directly on the airgap domain, using the expression $n_x \cdot B_x + n_y \cdot B_y$, where n_x and n_y are the normal vectors of the selected surface. The results are then plotted against the arc length along the selected boundaries. The second approach uses a 2D parametrized curve representing the centre of the airgap. The curve is defined as follows:

- Name: s;

- Minimum: 0;
- Maximum: 2π ;
- Expression for x: $(Rs_{int} - airgap/2) \cdot \cos(s)$;
- Expression for y: $(Rs_{int} - airgap/2) \cdot \sin(s)$;
- Snapping: none.

Along this curve, the radial component is computed as: $B - r = B_x \cdot \cos(s) + B_y \cdot \sin(s)$, and plotted against the mechanical angle $s \cdot (180 \cdot \pi)$ [rad]. Both methods yield equivalent results for an ideal SPM machine.

Following the loss calculation, the **cycle-averaged loss density Q** (Q/m^3) is computed using the data from the second electrical period of the Time-Dependent solution with the Loss Calculation node active, ensuring that the machine was operating in steady-state conditions.

To obtain the total losses in Watts for each motor component, a Surface Integration node is used under Derived Values, selecting separately the stator core, rotor core, permanent magnets and windings. Since the model is 2D planar, the integration expression is defined as:

$$rmm.Qrh \cdot depth_n \cdot 1e - 3$$

where $depth_n$ is the motor stack length expressed in millimeters.

The factor 1e-3 converts the depth to meters, ensuring the correct units for the volume integration.

The Surface Integration operates on a surface (m^2). Multiplying the loss density Q (W/m^3) by the depth (m) converts it into losses per unit surface (W/m^2). The integration over the surface then yields the total losses in Watts (W) at each time instant of the solution.

Additionally, the surface area of each component is computed using the same integration approach (using the expression 1), providing the corresponding reference areas for the loss density plots.

The **average torque** is computed using a Global Evaluation node. The torque variable $rmm.Tax_0$ is evaluated over the second electrical period of the time-dependent solution with the Loss Calculation node active, still ensuring that the machine operates in steady-state conditions.

The Data Series Operation is set as follows: Normalization: None, Transformation: Average, Method: Auto.

The mechanical **power output** is calculated from the average torque and the rotational speed using the following equation:

$$P_{out} = T_{avg} \cdot \omega$$

where T_{avg} is the average torque in $N \cdot m$ and ω is the angular velocity in rad/s. The angular velocity is obtained from the rotational speed n (in rpm) as expressed in the chapters before.

The total losses P_{loss} are obtained by summing the individual contributions from the stator core, rotor core, permanent magnets and windings. The input electric power is then calculated as:

$$P_{in} = P_{out} + P_{loss}$$

Finally, the motor efficiency is determined as:

$$\eta = (P_{out}/P_{in}) \cdot 100$$

4.2 FEMM Simulations - Magnetostatic

In addition to the COMSOL model, a second finite element model was developed using FEMM (Finite Element Method Magnetics). This model was built by another researcher who exploited the geometric periodicity of the machine to model only one third of the motor, taking advantage of the symmetry of the geometry.

The geometrical dimensions of the FEMM model differ slightly from those of the COMSOL model. In particular, the stator cores are smaller, occupying less space, and both the inner and outer stator radii are reduced. The airgap is at a radius of 80.1567 mm.

Consequently, the radii of the permanent magnets and the rotor are also smaller. Additionally, the rotor has a larger radial thickness: 4.5 mm in the COMSOL model compared to 5.49 mm in the FEMM model.

A magnetostatic analysis ($f = 0$) was performed in FEMM, both under no-load and under loaded conditions.

It is noted that, unlike the COMSOL model, where the current excitations are set directly in the physics interfaces, in the FEMM model the phase currents are not embedded in the .fem file. They are instead injected through Lua scripts, which enable parametric control of the current amplitudes and phase shifts, facilitating the analysis of different operating points without the need to manually modify the geometry file.

4.2.1 No Load Study

In the no-load simulation, a Lua script was used to extract the air-gap flux density and to compute the torque, which was found to be approximately zero, as expected for a no-load condition.

```

open("Cshape_vuoto.fem")
mi_saveas("temp_prova_vuoto.fem")

-- 1. AZZERAMENTO AUTOMATICO DI TUTTE LE CORRENTI A VUOTO (0 AMPERE)
mi_modifycircprop("fase1", 1, 0)   mi_modifycircprop("fase1n", 1, 0)
mi_modifycircprop("fase2", 1, 0)   mi_modifycircprop("fase2n", 1, 0)
mi_modifycircprop("fase3", 1, 0)   mi_modifycircprop("fase3n", 1, 0)
mi_modifycircprop("fase4", 1, 0)   mi_modifycircprop("fase4n", 1, 0)
mi_modifycircprop("fase5", 1, 0)   mi_modifycircprop("fase5n", 1, 0)
mi_modifycircprop("fase6", 1, 0)   mi_modifycircprop("fase6n", 1, 0)
mi_modifycircprop("fase7", 1, 0)   mi_modifycircprop("fase7n", 1, 0)
mi_modifycircprop("fase8", 1, 0)   mi_modifycircprop("fase8n", 1, 0)
mi_modifycircprop("fase9", 1, 0)   mi_modifycircprop("fase9n", 1, 0)

mi_analyze()
mi_loadsolution()

-- 2. CALCOLO DELLA COPPIA DI COGGING (Corona circolare Gruppo 2 + Gruppo 3)
mo_seteditmode("area")
mo_clearblock()
mo_groupselectblock(2) -- Ferro rotore + aria interna
mo_groupselectblock(3) -- Magneti permanenti

local coppia_vuoto_terzo = mo_blockintegral(22)
local coppia_vuoto_totale = coppia_vuoto_terzo * 3

local file_v = openfile("coppia_vuoto_cogging.txt", "w")
write(file_v, "Coppia di cogging a vuoto in questo istante: " .. coppia_vuoto_totale .. " Nm\n")
closefile(file_v)

-- 3. TRACCIAMENTO DELL'ARCO AL TRAFERRO PER IL PROFILO DI B A VUOTO (Raggio 80.1567 mm)
mo_seteditmode("contour")
mo_clearcontour()

local raggio_vero = 80.1567
local ang_max = 120 * pi / 180
local passi_arco = 40

for p = 0, passi_arco do
    local x = raggio_vero * cos(p * (ang_max / passi_arco))
    local y = raggio_vero * sin(p * (ang_max / passi_arco))
    mo_addcontour(x, y)
end

-- Salva i 100 punti di B a vuoto (puro magnete) per il confronto con COMSOL
mo_makeplot(1, 100, "B_traferro_vuoto.txt", 1)

```

Figure 4.1: Script Lua No-Load Simulation.

4.2.2 On Load Study

In the loaded simulation, a Lua script was again used to extract the air-gap flux density and to compute the torque, which was obtained via an angular sweep.

```

open("Cshape.fem")
mi_saveas("temp_sweep_360.fem")

local I_max = 25
local gruppo_rotore = 3
local sfas_120 = 2 * pi / 3
local sfas_40 = 2 * pi / 9

local file_sweep = openfile("tabella_sweep_coppia.txt", "w")
write(file_sweep, "Angolo_wt_Gradi\tCoppia_Totale_Nm\n")

print("Risoluzione sweep a 360 gradi in corso...")

-- Proviamo 12 angoli diversi (passi di 30 gradi) per mappare tutto il periodo elettrico
for passo = 0, 12 do
    local wt = passo * (pi / 6)
    local wt_deg = passo * 30

    -- Formule correnti stabili con mi_modifycircprop
    mi_modifycircprop("fase1", 1, I_max * sin(wt))
    mi_modifycircprop("fase1n", 1, -I_max * sin(wt))
    mi_modifycircprop("fase3", 1, -I_max * sin(wt - sfas_40))
    mi_modifycircprop("fase3n", 1, I_max * sin(wt - sfas_40))
    mi_modifycircprop("fase5", 1, -I_max * sin(wt - 2 * sfas_40))
    mi_modifycircprop("fase5n", 1, I_max * sin(wt - 2 * sfas_40))

    mi_modifycircprop("fase2", 1, I_max * sin(wt - 2 * sfas_40 - sfas_120))
    mi_modifycircprop("fase2n", 1, -I_max * sin(wt - 2 * sfas_40 - sfas_120))
    mi_modifycircprop("fase7", 1, -I_max * sin(wt - sfas_120))
    mi_modifycircprop("fase7n", 1, I_max * sin(wt - sfas_120))
    mi_modifycircprop("fase9", 1, -I_max * sin(wt - sfas_40 - sfas_120))
    mi_modifycircprop("fase9n", 1, I_max * sin(wt - sfas_40 - sfas_120))

    mi_modifycircprop("fase4", 1, I_max * sin(wt + sfas_120))
    mi_modifycircprop("fase4n", 1, -I_max * sin(wt + sfas_120))
    mi_modifycircprop("fase6", 1, -I_max * sin(wt + sfas_120 - sfas_40))
    mi_modifycircprop("fase6n", 1, I_max * sin(wt + sfas_120 - sfas_40))
    mi_modifycircprop("fase8", 1, -I_max * sin(wt + sfas_120 - 2 * sfas_40))
    mi_modifycircprop("fase8n", 1, I_max * sin(wt + sfas_120 - 2 * sfas_40))

```

Figure 4.2: Script Lua On-Load Simulation - Sweep Part 1.

```

mi_analyze()
mi_loadsolution()

-- Selezione corona circolare
mo_seteditmode("area")
mo_clearblock()
mo_groupselectblock(2)
mo_groupselectblock(3)

local coppia_terzo = mo_blockintegral(22)
local coppia_totale = coppia_terzo * 3

write(file_sweep, wt_deg .. "\t" .. coppia_totale .. "\n")
mo_close()
end

closefile(file_sweep)

-- Lasciamo l'ultimo calcolo sull'arco reale del traferro isolato a 80.1567 mm
mi_analyze()
mi_loadsolution()
mo_seteditmode("contour")
mo_clearcontour()
local raggio_vero = 80.1567
local ang_max = 120 * pi / 180
for p = 0, 40 do
    local x = raggio_vero * cos(p * (ang_max / 40))
    local y = raggio_vero * sin(p * (ang_max / 40))
    mo_addcontour(x, y)
end
mo_makeplot(1, 100, "B_traferro_carico_corretto.txt", 1)
mo_close()

```

Figure 4.3: Script Lua On-Load Simulation - Sweep Part 2.

To obtain the terminal voltage, the flux linkage value was first extracted from the 'Properties' section of the results, and the voltage was then calculated analytically from the flux linkage derivative.

```

open("Cshape.fem")
mi_saveas("temp_carico_30deg.fem")

local I_max = 25
local wt = 30 * pi / 180 -- Configurato a 30 gradi in radianti!

local sfas_120 = 2 * pi / 3
local sfas_40 = 2 * pi / 9

-- 1. CALCOLO DELLE CORRENTI ALL'ANGOLO DI PICCO DI 30 GRADI
local I1 = -I_max * sin(wt)
local I3 = -(-I_max * sin(wt - sfas_40))
local I5 = -(-I_max * sin(wt - 2 * sfas_40))

local I2 = -I_max * sin(wt - 2 * sfas_40 - sfas_120)
local I7 = -(-I_max * sin(wt - sfas_120))
local I9 = -(-I_max * sin(wt - sfas_40 - sfas_120))

local I4 = -I_max * sin(wt + sfas_120)
local I6 = -(-I_max * sin(wt + sfas_120 - sfas_40))
local I8 = -(-I_max * sin(wt + sfas_120 - 2 * sfas_40))

-- 2. APPLICAZIONE AUTOMATICA SUI CIRCUITI SENZA ERRORI DI LINEA
mi_modifycircprop("fase1", 1, I1)    mi_modifycircprop("fase1n", 1, -I1)
mi_modifycircprop("fase2", 1, I2)    mi_modifycircprop("fase2n", 1, -I2)
mi_modifycircprop("fase3", 1, I3)    mi_modifycircprop("fase3n", 1, -I3)
mi_modifycircprop("fase4", 1, I4)    mi_modifycircprop("fase4n", 1, -I4)
mi_modifycircprop("fase5", 1, I5)    mi_modifycircprop("fase5n", 1, -I5)
mi_modifycircprop("fase6", 1, I6)    mi_modifycircprop("fase6n", 1, -I6)
mi_modifycircprop("fase7", 1, I7)    mi_modifycircprop("fase7n", 1, -I7)
mi_modifycircprop("fase8", 1, I8)    mi_modifycircprop("fase8n", 1, -I8)
mi_modifycircprop("fase9", 1, I9)    mi_modifycircprop("fase9n", 1, -I9)

-- 3. GENERAZIONE MESH E AVVIO SOLUTORE
mi_analyze()
mi_loadsolution()

print("\n--- ANALISI A 30 GRADI COMPLETATA! ---")
print("La finestra dei risultati neri è aperta.")
print("Clicca sull'icona della Tabella dei Circuiti per leggere i Flux Linkage!")

```

Figure 4.4: Script Lua On-Load Simulation.

4.2.3 Post Processing

For both no-load and on-load conditions, the direct results from FEMM allow the visualisation of the flux density map (B) and the current density map (J) over the entire 2D motor geometry.

Chapter 5

Comparative Analysis of FEA Results

5.1 Comsol Results

5.1.1 No Load Study - No Losses

The no load magnetic flux density distribution is evaluated at the end of the electrical period ($t = 0.004$ s), as shown in Figure 5.1.

At this instant, the rotor has returned to its initial angular position, with the d-axis (which bisects the north magnet) aligned with the magnetic axis of phase A1. Since the study is performed under no load conditions, the magnetic field is generated exclusively by the permanent magnets on the rotor. The flux lines leave the north magnet, cross the air gap, enter the stator teeth, travel through the yoke, and return to the south magnet. The resulting flux density reaches a maximum value of approximately 2.19 T, and is concentrated in the stator teeth corresponding to the phase A1 winding.

This concentration is the spatial signature of the maximum flux linkage between the rotor magnets and the stator winding at this specific rotor position.

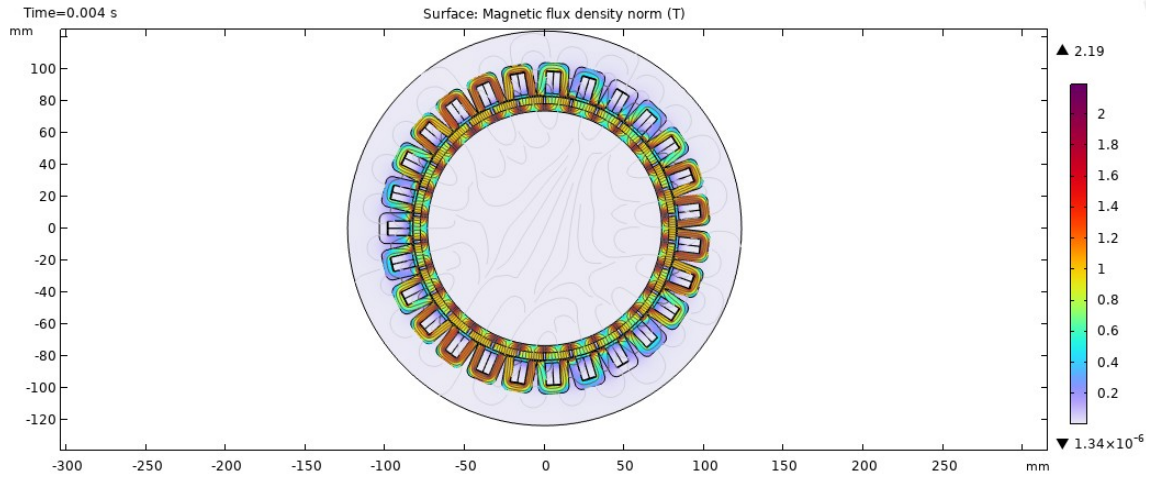


Figure 5.1: Magnetic Flux Density Distribution - No Load.

Figure 5.2 presents the no load phase voltages waveforms. A transient is observed at the beginning of the simulation, after which the coil voltages settle into a steady-state sinusoidal pattern.

The peak value of the phase voltages is 34.134 V.

This steady-state sinusoidal waveforms is the expected result for an SPM motor under no-load conditions, as the back-EMF generated by the permanent magnets is inherently sinusoidal.

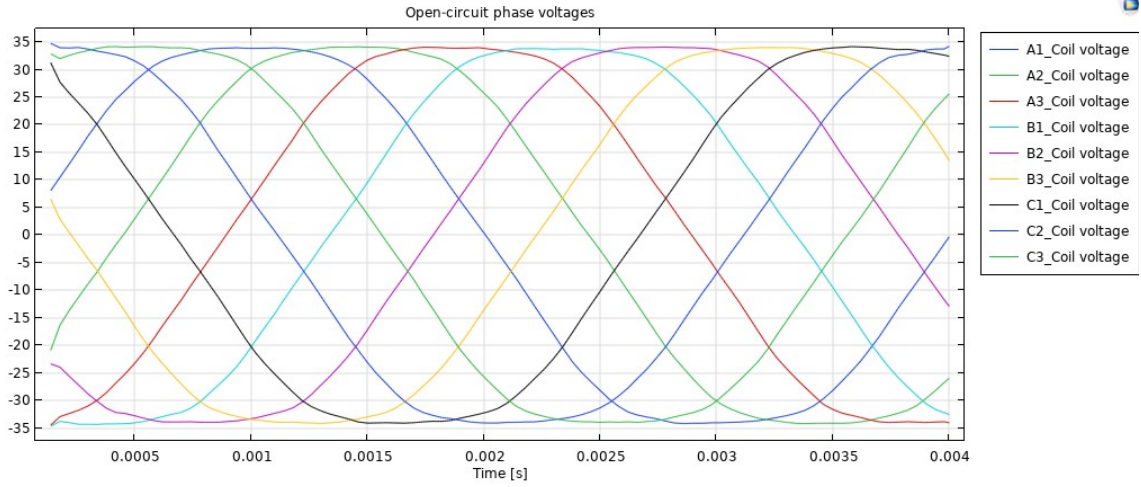


Figure 5.2: Open Circuit Phase Voltages.

The harmonic content of the phase A1 voltage is analysed using a Fast Fourier Transform (FFT). The FFT spectrum, shown in Figure 5.3, reveals a dominant peak at approximately 259 Hz, with an amplitude of 26.384 V. This corresponds to the fundamental electrical frequency (expected value: 250 Hz), with a slight deviation due to the finite resolution of the FFT analysis.

A second peak is observed at approximately 777 Hz, corresponding to the third harmonic (expected value: 750 Hz). Beyond the third harmonic, no significant higher-order components are present. This confirms that the multi-three-phase SPM motor configuration effectively reduces the harmonic content of the back-EMF, resulting in a near-sinusoidal voltage waveform.

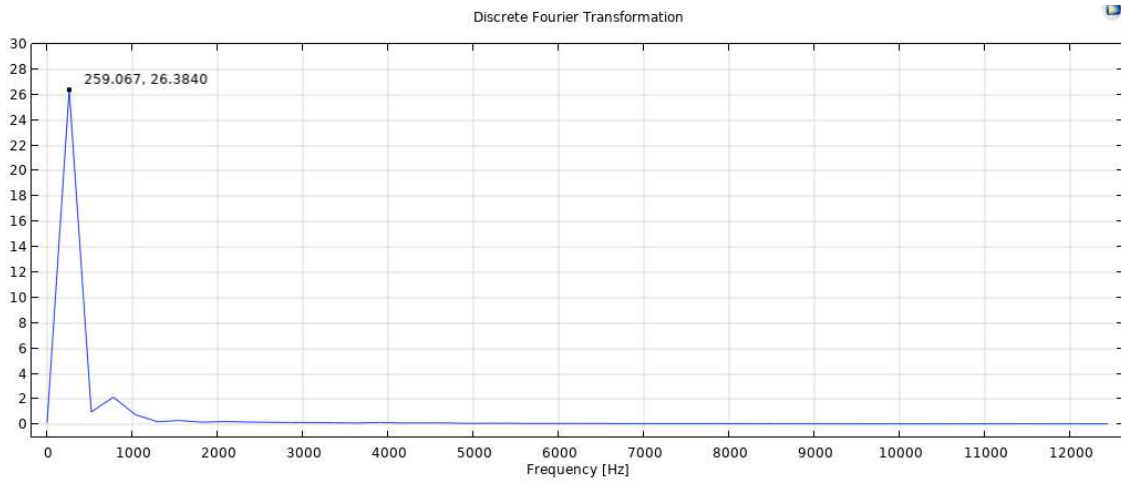


Figure 5.3: FFT with Discrete Fourier Transformation tool – Phase A1.

5.1.2 Load Study - No Losses

Under load, current flows through the stator windings, and the magnetic field distribution is no longer determined purely by the permanent magnets, but by the interaction between the permanent magnet field and the armature reaction field generated by the stator currents.

This results in a shift of the flux density peak relative to the no load case.

As shown in Figure 5.4, the maximum concentration of B is translated by two slot pitch angles in the counter-clockwise direction with respect to the no load distribution.

This translation is evaluated at the same instant as the no load case, i.e., at the end of an electrical period ($t = 0.004$ s), ensuring that the rotor is in the same angular position in both simulations.

This shift is the physical manifestation of the armature reaction: the stator current, which is oriented along the q-axis (in quadrature with the d-axis of the magnets), produces a magnetic field that interacts with the magnet field, effectively translating the resulting flux pattern.

The interaction between these two fields is the fundamental mechanism responsible for the production of electromagnetic torque.

The maximum flux density in this case is approximately 2.81 T.

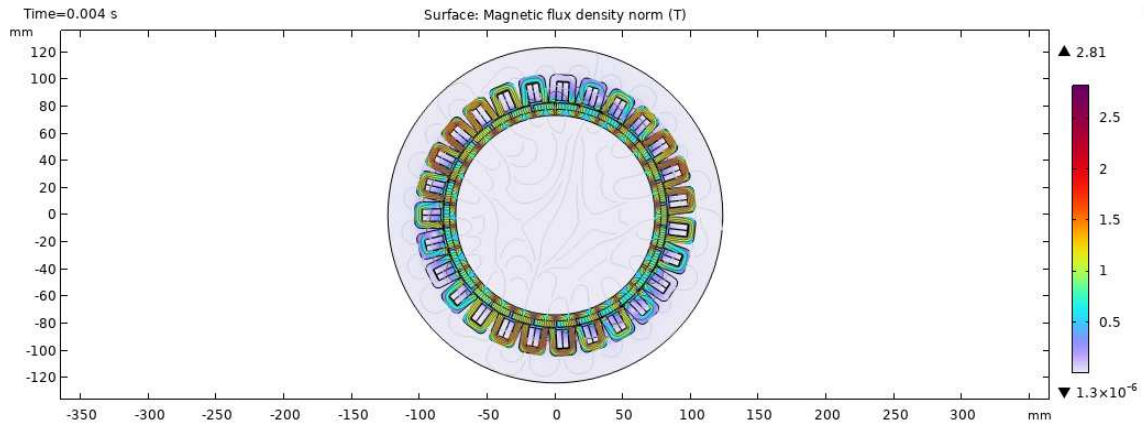


Figure 5.4: Magnetic Flux Density Distribution-Load.

The voltage waveform under load deviates from the ideal no load sinusoidal shape due to the armature reaction and the presence of current harmonics.

The peak value of the phase voltage under load is approximately 46.5 V. This deviation is a direct consequence of the interaction between the stator currents and the magnetic field, which modifies the back-EMF profile.

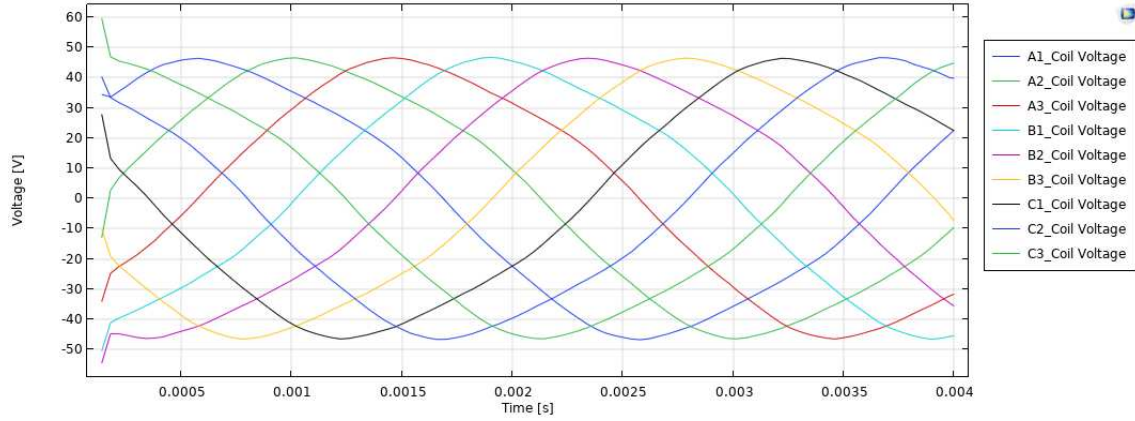


Figure 5.5: Sinusoidal Voltage Phases Waveforms - Load Time Dependent Study.

The phase current waveforms are presented in Figure 5.6. Since the windings are current-controlled, the currents are imposed directly by COMSOL Multiphysics and follow ideal sinusoidal profiles. The peak value is approximately 24.996 A. The currents are defined as functions of time, with the appropriate electrical phase shifts.

The plot is in the time domain, and the values assumed by each phase over time are consistent with the imposed current definitions, confirming the correct phase sequence and timing of the multi-phase system.

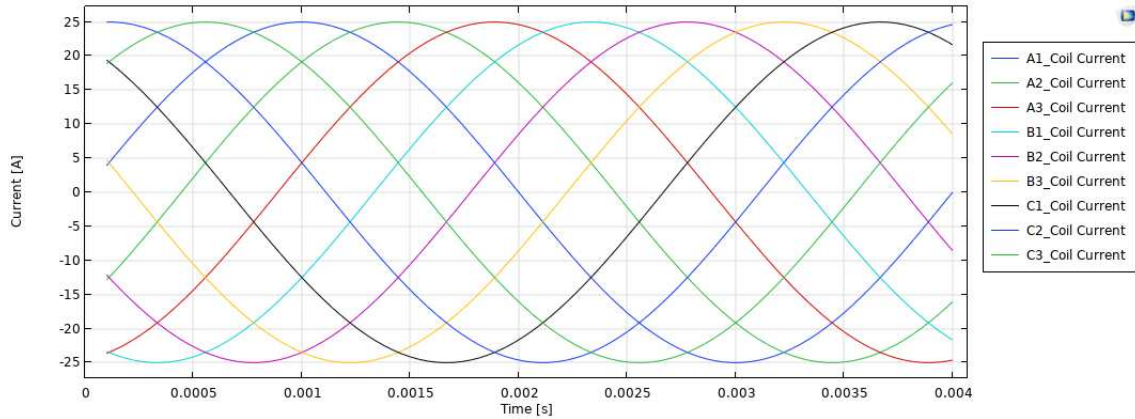


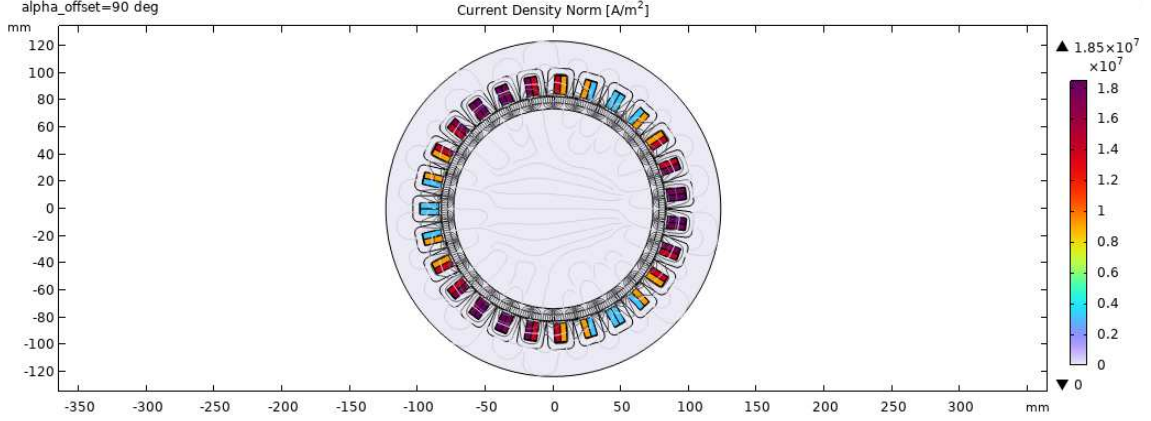
Figure 5.6: Sinusoidal Current Phases Waveforms - Load Time Dependent Study.

The surface current density plot provides a direct visualization of the stator current distribution in the copper conductors.

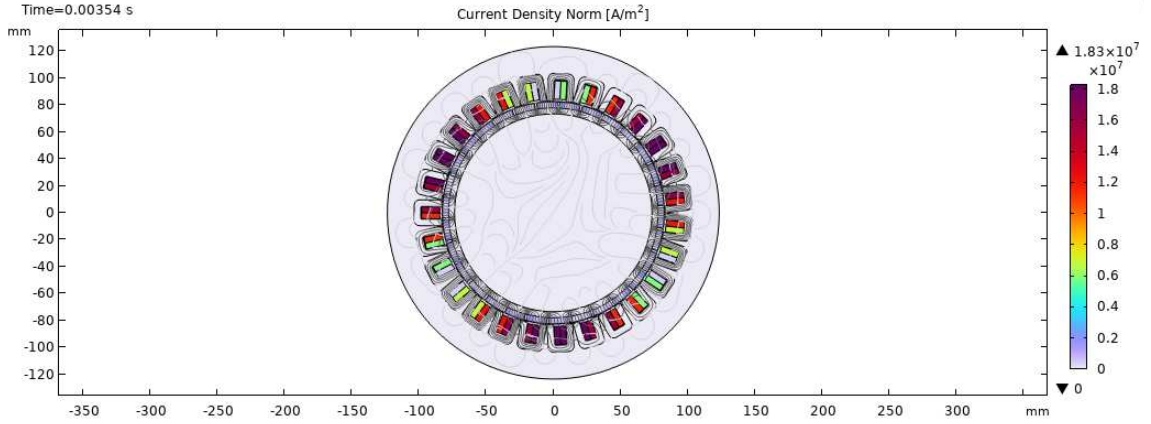
In the stationary study, at $\alpha_{offset} = 90^\circ$ mechanical, the highest current density regions are concentrated around the windings of phase A1. This is simply a consequence of the imposed current definitions: at $t = 0$ s, the phase A1 current is defined as $I_{A1} = I_{pk} \cdot \sin(90^\circ) = I_{pk}$, while the other phases carry lower currents due to their phase shifts.

In the Time-Dependent study, which is initialized from the Stationary solution with $\alpha_{offset} = 0^\circ$, the rotor rotates and the phase currents evolve over time according

to their sinusoidal profiles. At the snapshot $t = 0.00354$ s, the peak of the current density distribution is observed to shift in the counter-clockwise direction. This shift is exactly as expected from the phase current waveforms in the figure 5.6: at this instant, the current distribution is such that the phase carrying the maximum instantaneous current is no longer A1, but A2 and C1. The region of maximum current density therefore travels along the stator periphery as the rotor rotates, following the evolution of the phase currents.



(a) Current Density Distribution- Stationary Study.



(b) Current Density Distribution - Time Dependent Study.

The airgap flux density distribution is evaluated both in the stationary and in the time-dependent studies. Figure 5.8 shows the normal flux density along the arc length in the stationary case, while Figure 5.9 presents the radial flux density as a function of the mechanical angle in the time-dependent solution.

In both cases, the flux density waveform exhibits a periodic pattern. This periodicity is a direct consequence of the spatial distribution of the permanent magnets on the rotor surface, which alternate north and south poles along the circumference, and of the stator teeth, which introduce local variations in the magnetic reluctance. The peak values, approximately ranging between 1.2 T and 1.4 T, are consistent with the flux density levels observed in the magnetic flux density map of the SPM

motor.

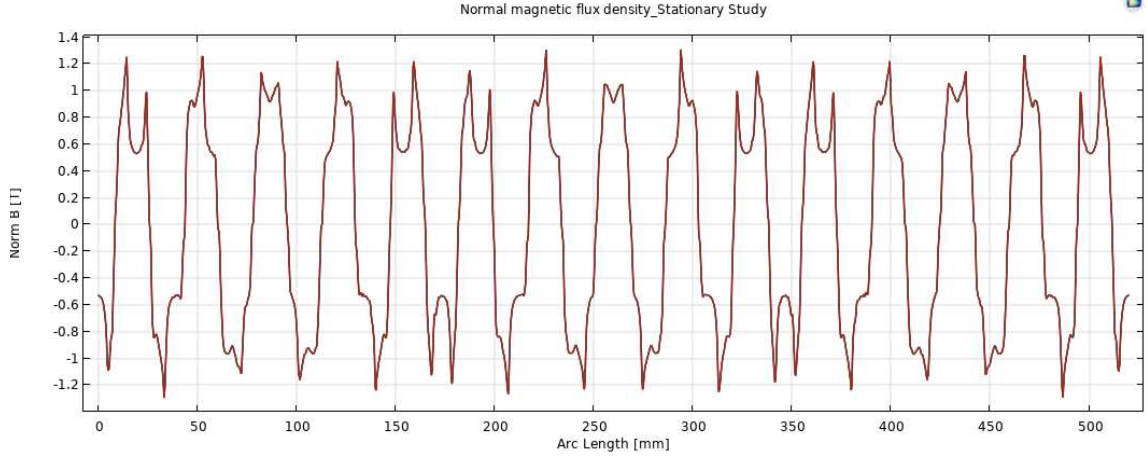


Figure 5.8: Airgap Normal Magnetic Flux Density - Stationary Study.

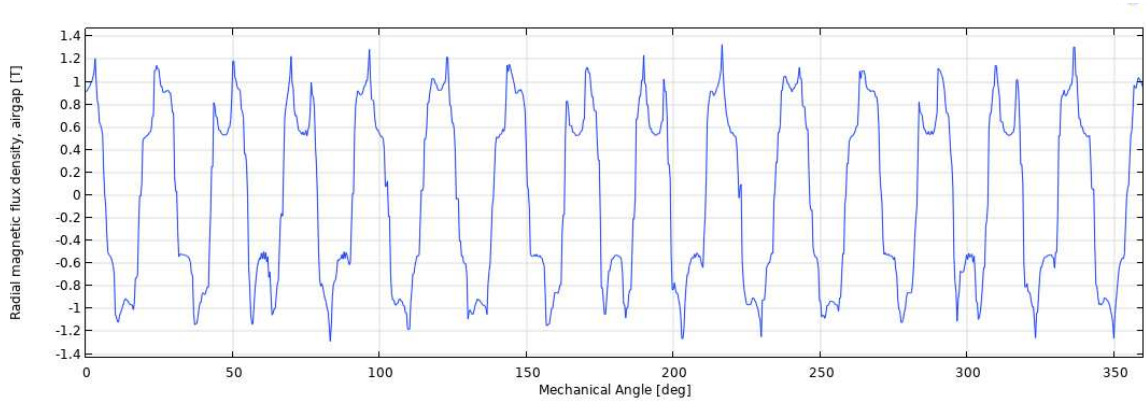


Figure 5.9: Airgap Radial Magnetic Flux Density vs. Mechanical Angle [deg].

The torque characteristic as a function of the current angle α_{offset} is presented in Figure 5.10. The torque is computed over a full mechanical rotation, from 0° to 360° .

As described in the subsection 3.1.2, the phase A1 has been aligned with the d-axis of the rotor. With this design choice, at $\alpha_{offset} = 0^\circ$, the stator current is injected in quadrature with the rotor field, i.e., along the q-axis. Since the electromagnetic torque in an SPM motor is proportional only to i_q , this condition yields the maximum torque. The graph confirms this behavior: the peak torque occurs at $\alpha_{offset} = 0^\circ$, validating the correct implementation of the winding layout and the current angle definition. The peak values is 37.6188 Nm.

Following this reasoning, at $\alpha_{offset} = 90^\circ$, the current is injected along the d-axis. In an SPM motor, the d-axis current does not contribute to torque production. Therefore, this condition results in zero torque.

The sinusoidal variation of the torque with α_{offset} is fully consistent with the theoretical expectations.

It should be noted that this load study is performed without losses, so the torque values represent the ideal loss-free operating condition.

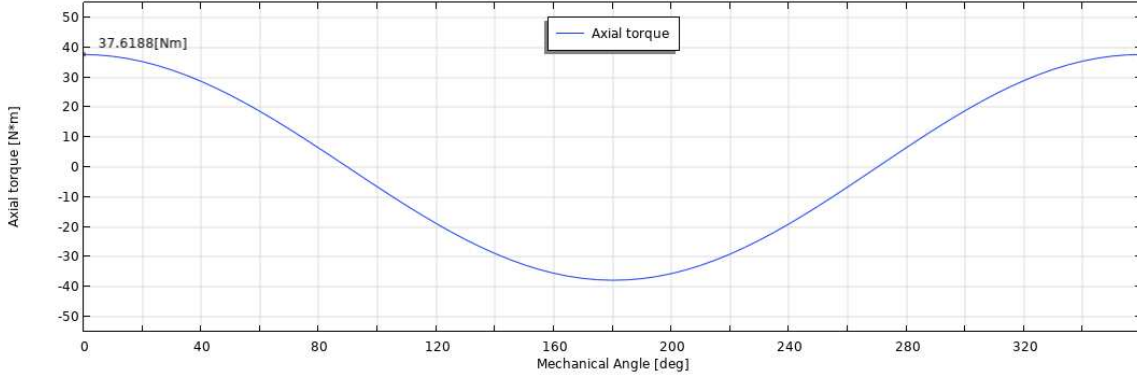


Figure 5.10: Force Calculation: Axial Torque - Stationary Study

The torque ripple is evaluated over one electrical period, with the angular position defined as:

$$\theta = n \cdot \frac{2\pi}{60} \cdot t \cdot \frac{180}{\pi}$$

where n is the rotational speed in rpm and t is the time.

This angle represents the instantaneous mechanical position of the rotor.

The resulting torque waveform is presented in Figure 5.11.

A transient is observed at the beginning of the simulation, from 0° to approximately 0.84°, during which the torque rises from zero to its steady-state value. This initial transient is due to the starting conditions of the time-dependent simulation, which is initialized from the stationary solution at $\alpha_{offset} = 0^\circ$. This condition corresponds to the maximum torque operating point, as confirmed by the stationary torque characteristic.

After this brief transient, the torque reaches its steady-state value and oscillates around the mean value of approximately 37.6188 Nm, which corresponds to the maximum torque obtained in the stationary study.

The torque ripple, defined as the difference between the maximum and minimum torque values in steady-state conditions, is: $T_{ripple} = T_{max} - T_{min} = 38.6252 - 36.6131 = 2.0121$ Nm.

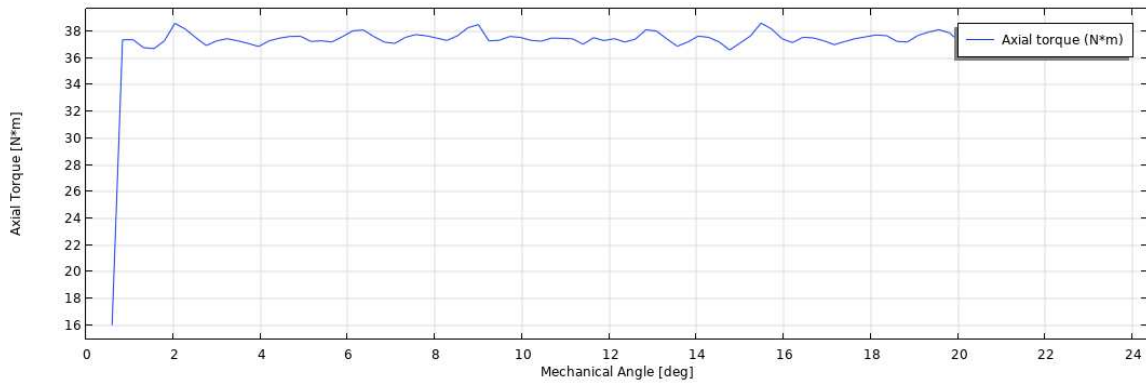


Figure 5.11: Axial Torque: Ripple - Time Dependent Study.

5.1.3 Load Study - Losses

The stator and rotor cores of the SPM motor under investigation are not laminated. Consequently, the conventional iron loss models on COMSOL (such as Steinmetz or Bertotti), which are specifically formulated for laminated cores, are not applicable.

Instead, the losses in the ferromagnetic cores are computed directly using the Resistive heating model. This choice is physically consistent with the solid nature of the components and provides a direct calculation of the eddy current losses, without relying on empirical coefficients that would be inappropriate for a solid core.

The magnetic flux density distribution under load with losses is presented below. The maximum value of B is approximately 2.54 T, which is lower than the corresponding loss-free Time-Dependent case (2.81 T).

In addition, the flux density appears more evenly distributed across the stator teeth and yoke compared to the loss-free case.

This behavior is attributed to the eddy currents induced in the conductive stator core and magnet domains, which generate opposing magnetic fields that dampen local saturation peaks and promote a more uniform flux distribution.

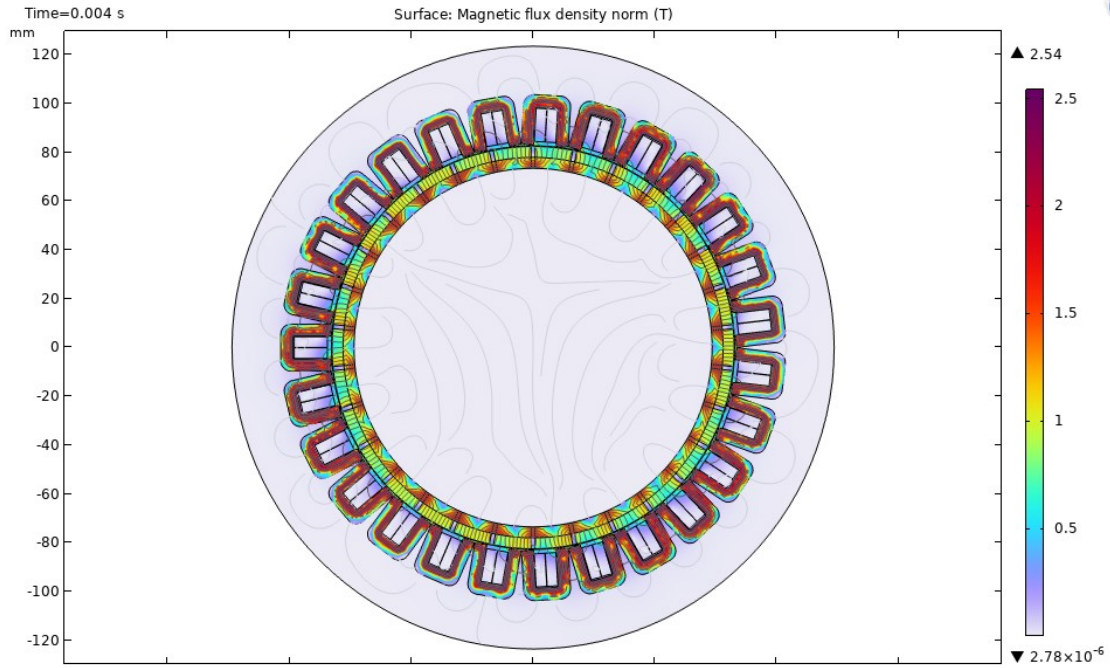


Figure 5.12: Magnetic Flux Distribution - Losses.

The current density distribution for the time loss-inclusive case is shown in Figure 5.13, with the snapshot taken at the same time instant as the loss-free study. Compared to the loss-free case, the peak of the current density shifts in the counter-clockwise direction, aligning with the windings of phase C2. This shift does not coincide with the instantaneous phase currents imposed by the source waveforms, as the presence of eddy currents, induced by the time-varying magnetic field in the conductive stator core and magnets, alters the local current distribution. As a result, the peak of J is displaced from its expected location.

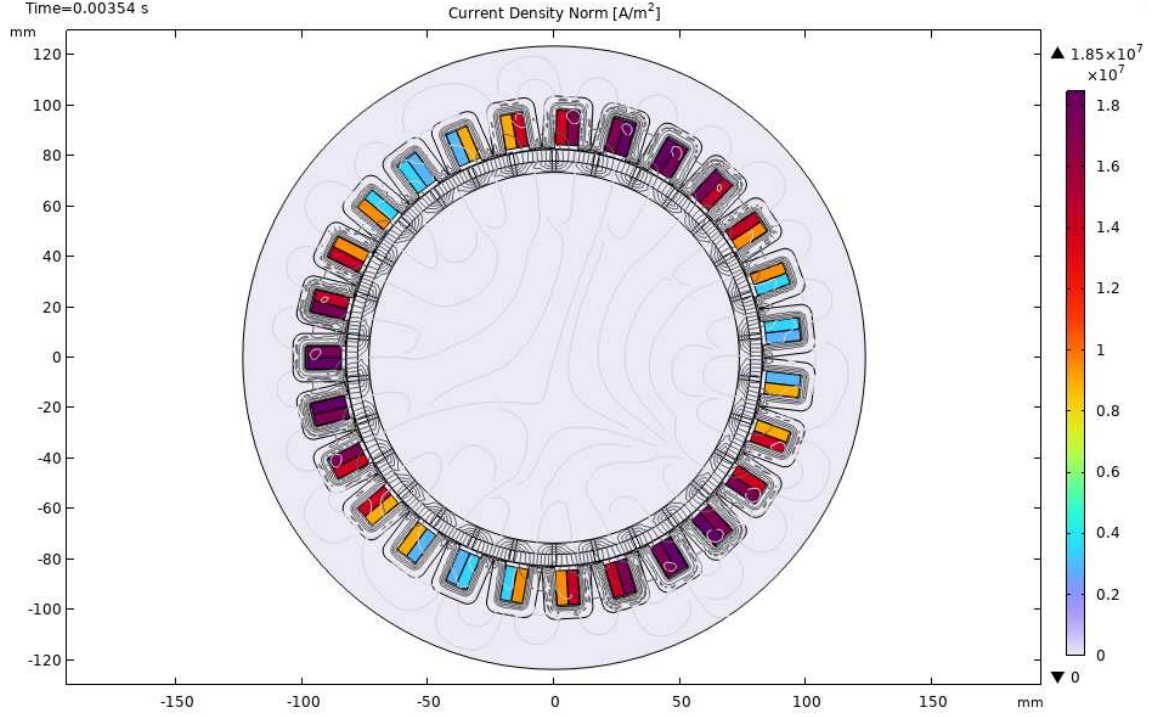


Figure 5.13: Current Density Distribution - Losses.

The normal and radial airgap flux density distributions are presented in Figures 5.14 and 5.15, respectively, with both snapshots taken at $t = 0.004$ s, corresponding to one full electrical period.

The normal flux density waveform exhibits a periodic pattern which is compatible with the loss-free case, confirming that the fundamental flux distribution is preserved. In contrast, the radial component shows a more densely peaked waveform. This behavior is attributed to the presence of eddy currents which generate opposing magnetic fields that primarily affect the radial component of the flux density, since this is the component that crosses the air gap and is subject to the most rapid spatial variations.

The normal component is less influenced by these local perturbations. Consequently, the radial flux density exhibits a more distorted and peaky waveform, whereas the normal component remains largely unaffected.

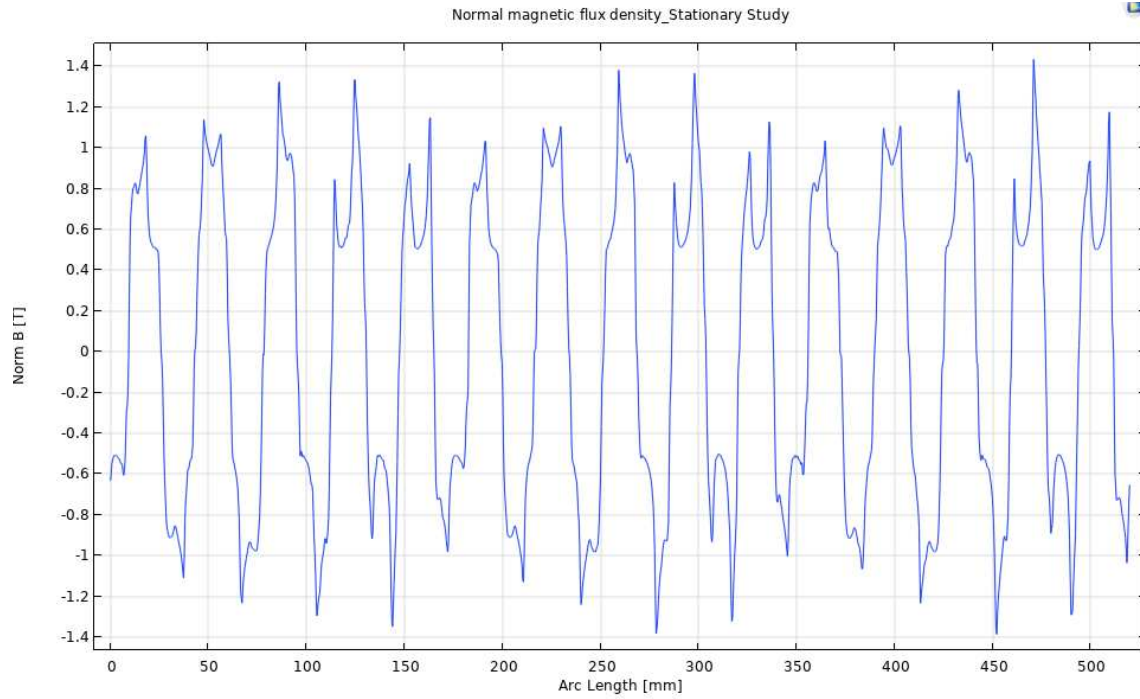


Figure 5.14: Airgap Normal Magnetic Flux Density - Losses.

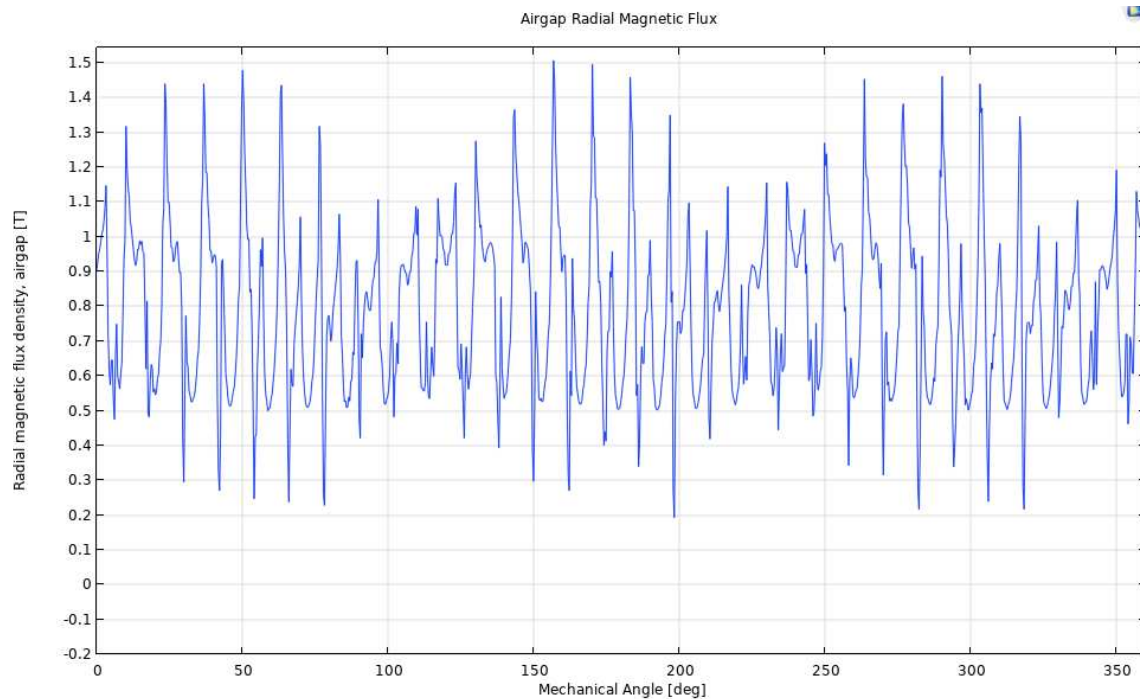


Figure 5.15: Airgap Radial Magnetic Flux Distribution - Losses.

The phase voltage waveforms are plotted over the two electrical periods of the time-dependent simulation.

Compared to the loss-free case, the waveforms show significantly higher distortion and a reduced peak value of 37 V circa (from 46.5 V in the loss-free case).

This is a direct consequence of the induced currents in the conductive domains, which smooth the voltage amplitude while introducing spectral content that distorts the waveform.

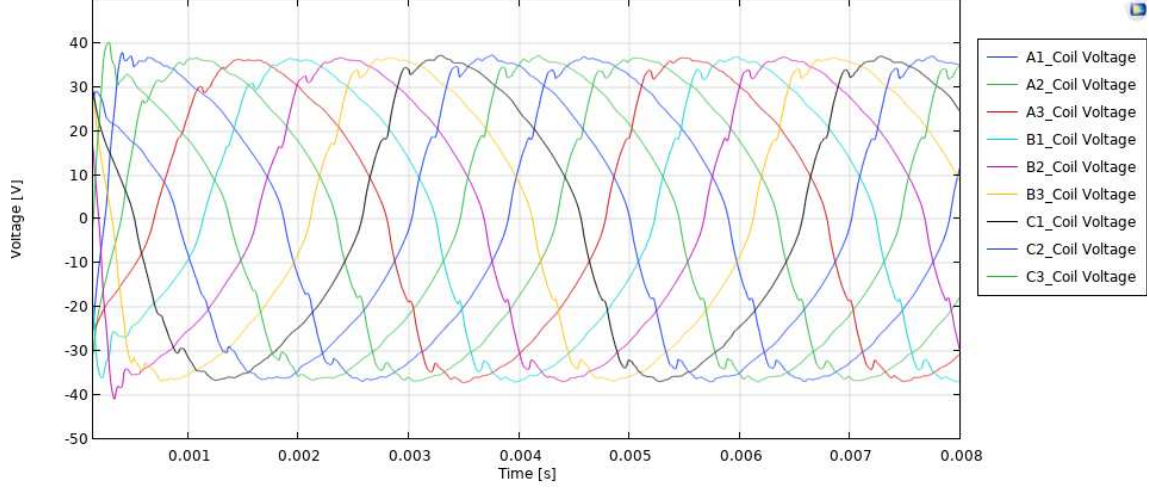


Figure 5.16: Sinusoidal Voltage Phases Waveforms - Losses.

The torque ripple for the loss-inclusive case is plotted over two electrical periods, with the angular position defined as in the loss-free Time-Dependent study. The x-axis spans twice the range consistently with the extended simulation time.

A pronounced initial transient is observed: following the start-up peak, the torque does not immediately reach its steady-state value, but gradually decreases until it settles around 10° mechanical. This extended transient behavior alter the dynamic response of the machine, delaying the establishment of steady-state operation.

Once the steady-state is reached, the torque oscillates around its mean value of 16.33 Nm with a peak-to-peak ripple of approximately: $T_{\text{ripple}} = T_{\text{max}} - T_{\text{min}} = 17.5301 - 14.3247 = 3.2054$ Nm.

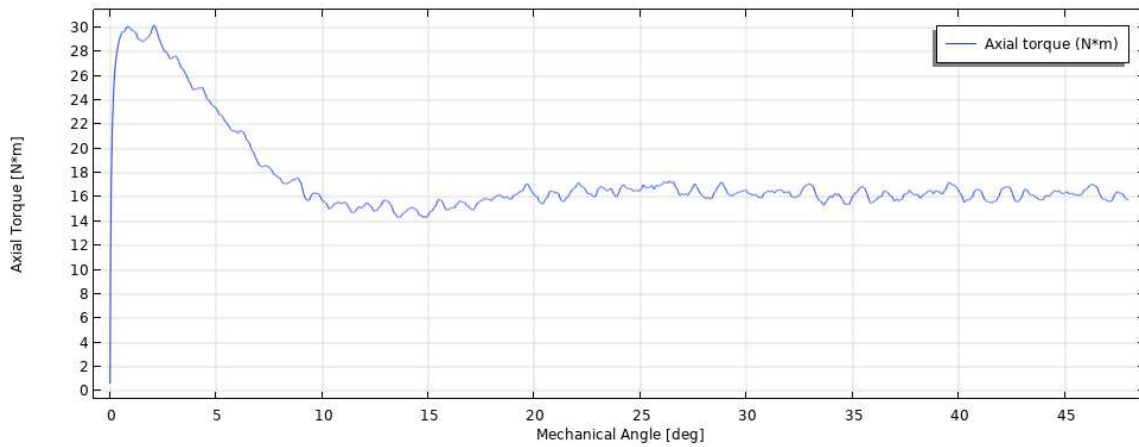


Figure 5.17: Axial Torque Ripple - Losses.

Figure 5.18 shows the distribution of the cycle-averaged loss density Q (W/m^3) at the end of the two electrical periods simulation, corresponding to $t = 0.008$ s. The highest loss concentrations are observed in the windings and in the permanent magnets. In the windings, the loss density distribution closely follows the pattern of the current density J , as expected from the resistive nature of these losses. However, the loss density plot alone is not directly indicative of the total losses per component, since Q is a volumetric quantity (W/m^3) and does not account for the actual volume of each domain.

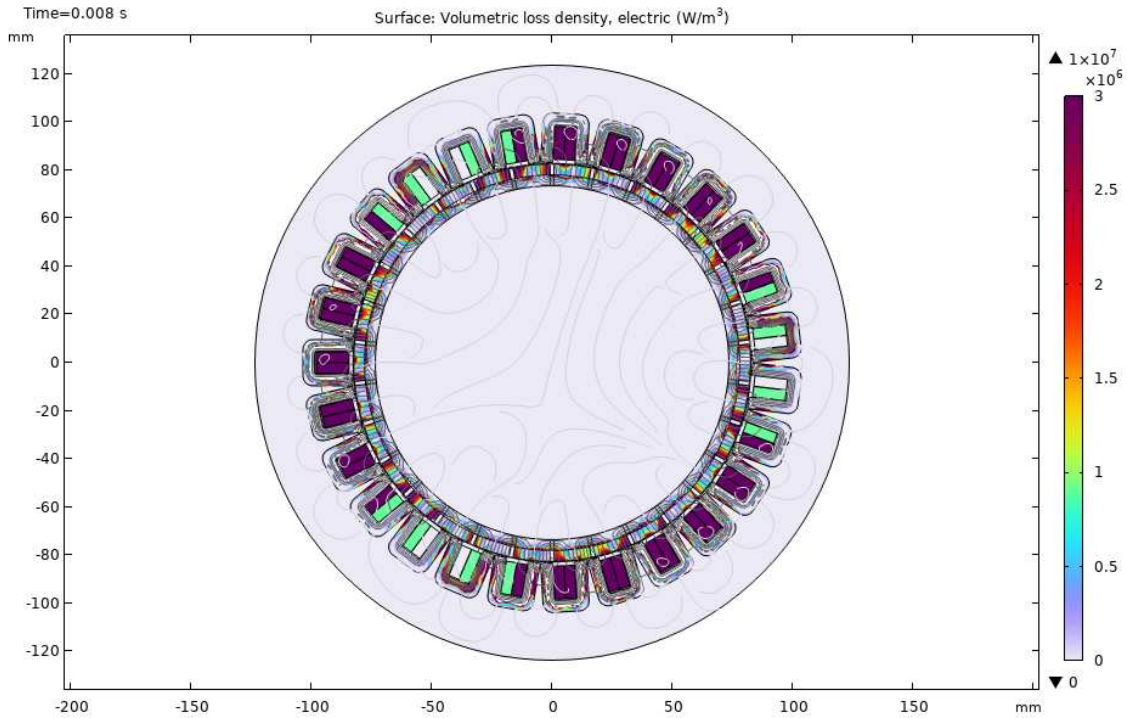


Figure 5.18: Losses Distribution.

The total losses for each component is obtained by integrating the loss den-

sity over the surface of each domain and multiplying by the motor stack depth $depth_n = 15.17mm$, as explained in the post processing of Chapter 4. Since the same depth is applied to all components, the surface area provides a direct reference for the loss distribution.

Table 5.1: Component losses and corresponding surface areas.

Component	Losses [W]	Surface area [m ²]
Stator C-cores	2052.14	0.006519019
Windings (copper)	187.73	0.003216806
Permanent magnets	36.59	0.002043084
Rotor	25.58	0.002142768

The loss distribution across the motor components is consistent with the expected behavior for an SPM motor, with the exception of the stator core losses, which are significantly higher than typical values.

This is due to the fact that the C-shaped stator cores are modeled as non-laminated solid iron. In a real laminated SPM motor, the eddy current losses would be substantially reduced due to the insulation layer between each lamination.

The present approach, however, allows the study of the worst-case condition of a solid core.

The rotor core exhibits much lower losses, approximately 25 W compared to 2052 W in the stator. This difference is explained by the different nature of the magnetic flux variation in the two components.

In the stator, the flux is strongly alternating, being continuously crossed by the rotating field of the permanent magnets, which induces substantial eddy currents.

In contrast, the rotor consists of a solid iron ring with surface-mounted permanent magnets; since the magnets rotate with the rotor, the magnetic field is nearly stationary in the rotor reference frame.

Usually flux variations arise mainly from spatial harmonics due to winding distribution and stator slotting, as well as from temporal harmonics if the phase currents are non-sinusoidal [35].

In the motor under study, the currents are sinusoidal, so temporal harmonics are absent, and the rotor losses are primarily due to spatial harmonics. These harmonics are considerably smaller than the fundamental alternating field in the stator, and consequently the resulting losses are only a small fraction of the stator ones. This is consistent with the literature, which states that rotor eddy current losses in SPM machines are often neglected in first approximation, since the rotor rotates in synchronism with the fundamental field [35].

In order to evaluate the efficiency of the SPM motor under study, it is fundamental to first calculate the power contributions.

The mechanical output power is calculated as:

$$P_{\text{out}} = T \cdot \omega = T \cdot \frac{2\pi n}{60}$$

Recalling that $T = 16.33 \text{ Nm}$, $n = 1000 \text{ rpm}$ and

$$\omega = \frac{2\pi \cdot 1000}{60} = 104.72 \text{ rad/s}$$

The total losses are the sum of the component losses:

$$P_{\text{loss}} = P_{\text{stator}} + P_{\text{windings}} + P_{\text{magnets}} + P_{\text{rotor}}$$

The input power and efficiency are then:

$$P_{\text{in}} = P_{\text{out}} + P_{\text{loss}}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

Table 5.2: Losses and Efficiency.

Parameter	Value
Mechanical power output P_{out}	1710 W
Total losses P_{loss}	2302.04 W
Input power P_{in}	4012.04 W
Efficiency η in%	42.6%

The resulting efficiency of approximately 42.6% is lower than typical values for SPM motors, which usually exceed 90%. This low efficiency is a direct consequence of the solid stator cores, which leads to very high eddy current losses. Nevertheless, this analysis provides a quantitative assessment of the performance penalty associated with a non-laminated core design.

5.2 FEMM Results

The results presented in this section are obtained using the open-source finite element software FEMM (Finite Element Method Magnetics). For this study, the

simulation is set up in magnetostatic mode ($f = 0$ Hz), as the study at non-zero frequency was not feasible.

This difficulty arises from the combined effect of the dense mesh and the presence of highly non-linear ferromagnetic materials. In such conditions, when current is applied, the magnetic material is driven into strong saturation. The Newton-Raphson solver, which attempts to correct the field solution at each iteration, then begins to oscillate between two states, failing to converge towards a stable solution.

The FEMM studies are also performed with zero electrical conductivity assigned to all conductive domains.

This choice does not affect the magnetic field or flux linkage calculation, since in magnetostatics eddy currents are not induced in the absence of time-varying fields. This setup allows a direct comparison with the loss-free studies carried out in COMSOL, where eddy current effects are neglected.

The non-linear B-H curves of the rotor and C-shaped stator core materials are however retained in the FEMM model, ensuring a consistent representation of the magnetic saturation behaviour.

Furthermore, these simulations are obtained from the geometry designed by another student, who modeled only one third of the motor, exploiting the periodicity of the motor.

5.2.1 No Load Study

The no-load flux density distribution obtained from FEMM is shown in Figure 5.19.

At the initial rotor position, the d-axis, which bisects the north magnet, is aligned with the magnetic axis of phase A1. Since the study is performed in magnetostatic mode under no load conditions, the magnetic field is generated exclusively by the permanent magnets on the rotor.

The resulting flux density reaches a maximum value of approximately 1.5 T, and is concentrated in the stator teeth corresponding to the phase A1 windings.

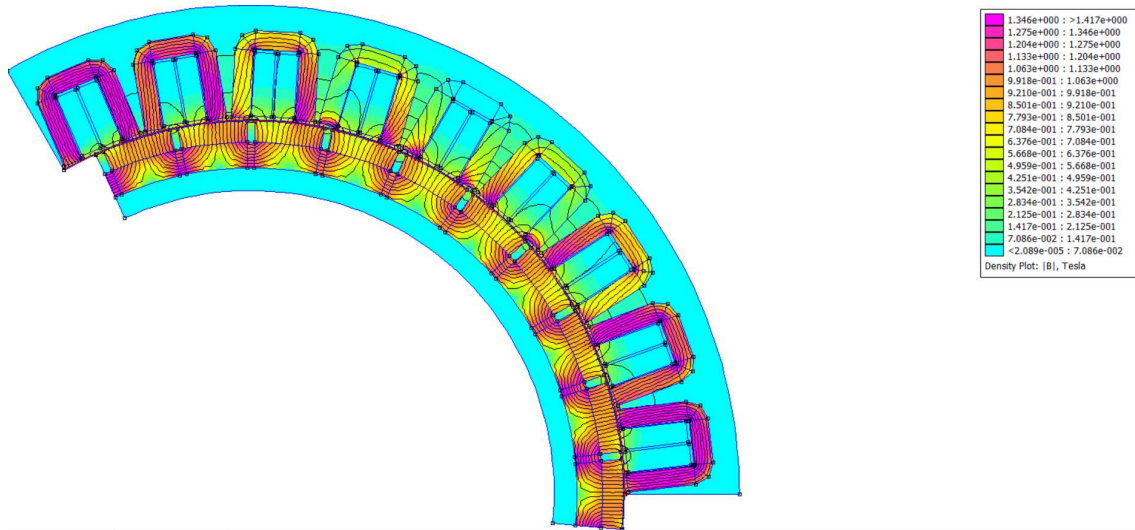


Figure 5.19: Magnetic Flux Density Distribution - No Load.

The no load airgap flux density distribution obtained from FEMM is presented in Figure 5.20, plotted along the arc length of the periodic sector. The waveform exhibits a periodic pattern, with a mirrored sequence in the first and second halves.

It starts from zero and returns to zero at the end of the arc.

The peak values are consistent with the airgap flux density levels visible in the B-field plot.

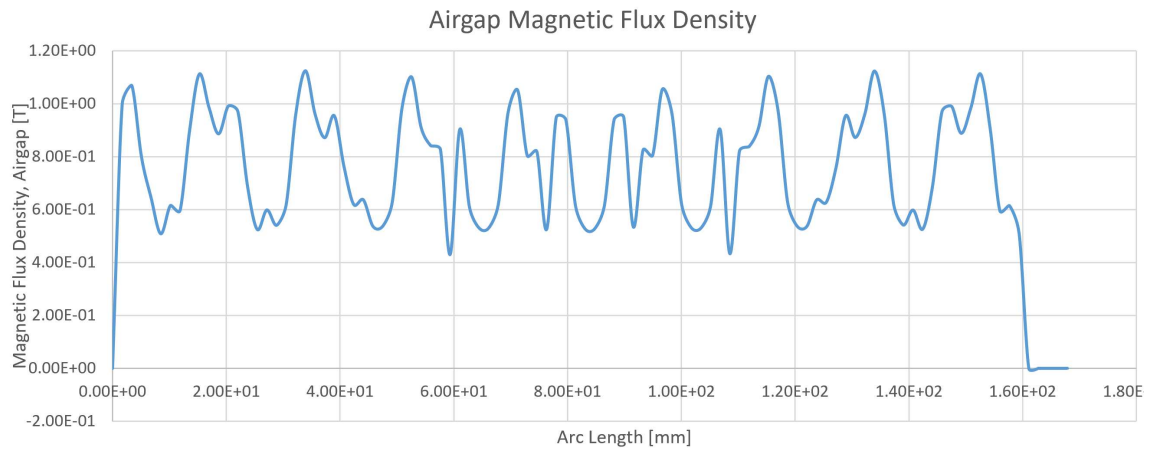


Figure 5.20: Airgap Normal Magnetic Flux Density - No Load.

The cogging torque obtained from the FEMM no load simulation is approximately 0.019 Nm. This value is effectively negligible compared to the rated torque of the motor, confirming the good quality of the design in terms of cogging torque reduction.

5.2.2 Load Study

The flux density distribution under load is shown in Figure 5.21. The peak values reach approximately 2.27 T, higher than in the no load case. In contrast to the no load condition, the distribution appears less ordered and does not concentrate on a specific group of windings and stator teeth. The flux density is spread across a wider region, without a clearly defined localized concentration on a particular set of windings.

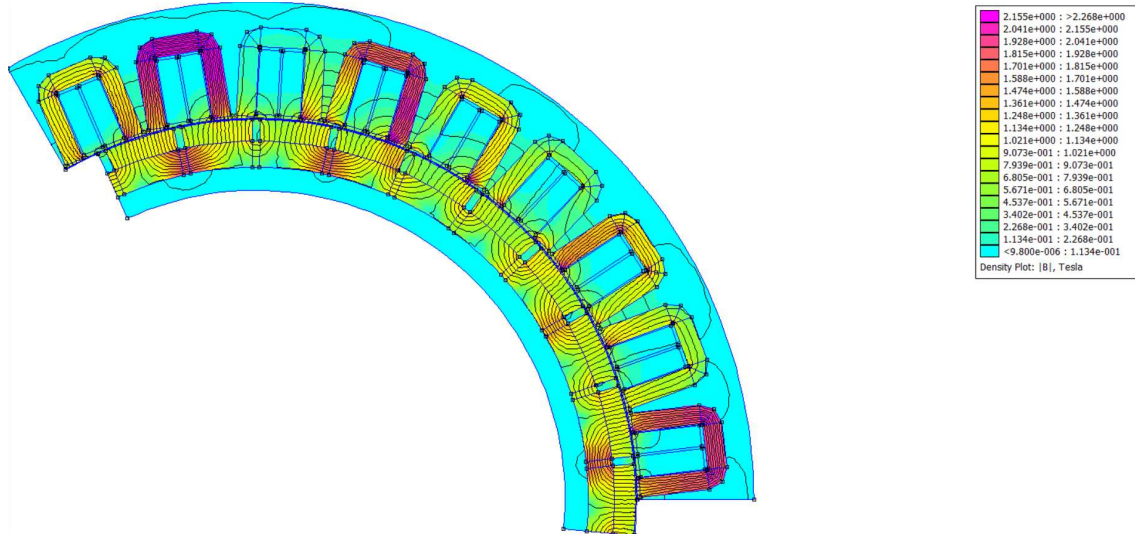


Figure 5.21: Magnetic Flux Density Distribution - Load.

The airgap flux density distribution under load obtained is shown in Figure 5.22, plotted along the arc length of the periodic sector.

Unlike the no load case, the waveform does not exhibit a clear periodic pattern. Nevertheless, it starts from zero, reaches positive peaks, and returns to zero at the end of the arc.

The peak values are higher than in the no load case, consistent with the increased flux density levels expected under load conditions.

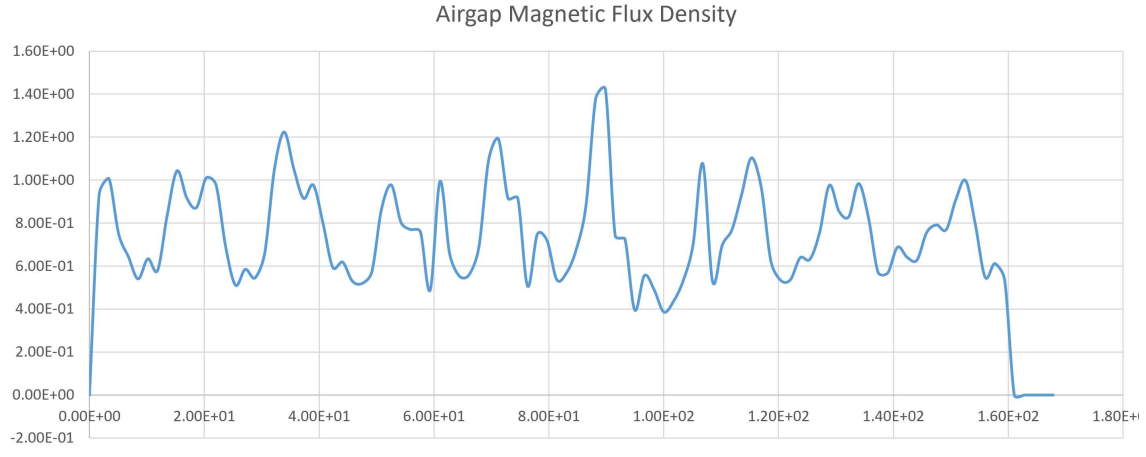


Figure 5.22: Airgap Normal Magnetic Flux Density - Load.

A parametric sweep over the full 360° mechanical rotation is performed in FEMM to evaluate the torque characteristic of the motor. The rotor is rotated in steps of 30° mechanical.

The resulting torque values are summarized in the following table. The maximum torque of approximately 19.91 Nm is obtained at a rotor angle of 30° mechanical.

Table 5.3: Torque values from FEMM sweep over 360° mechanical.

Angle [$^\circ$]	Torque [Nm]
0	17.8665
30	19.9135
60	16.5503
90	8.5718
120	-1.6090
150	-11.2194
180	-17.4787
210	-18.5614
240	-15.0535
270	-7.8699
300	1.4349
330	11.0880
360	17.8665

To evaluate the induced voltage, an additional study is performed with the rotor

fixed at 30° mechanical, corresponding to the maximum torque position. The phase with the highest flux linkage is found to be phase 8, with a value of 0.00648857 Wb. The corresponding RMS voltage per phase is calculated as:

$$V = \omega \cdot \lambda \cdot N_{\text{ph}}$$

With $\omega = 104.72 \text{ rad/s}$, $\lambda = 0.00648857 \text{ Wb}$, and $N_{\text{turns}} = 44$ turns:

$$V = 104.72 \times 0.00648857 \times 44 \approx 29.90 \text{ V}$$

5.3 Comparative Analysis of Stationary results; Comsol Multiphysics vs FEMM

The results obtained with FEMM are compared with the corresponding COMSOL No Losses solutions.

Since the FEMM study is magnetostatic, the comparison is performed with the COMSOL stationary study for the flux density distribution, the airgap flux density, and the maximum torque value (obtained via a sweep over the rotor position).

For the on load voltage, the comparison is performed at a single operating point. The rotor is fixed at the position corresponding to the maximum torque. The flux linkage is then extracted from the FEMM solution for each phase, and the phase with the highest flux linkage is selected for the voltage calculation.

The corresponding COMSOL voltage is extracted from the transient waveform by converting the angular position into time using the rotational speed.

It should be noted, however, that both the magnetostatic analysis and the time dependent referring COMSOL studies, do not include dynamic effects such as eddy currents. In particular, FEMM does not include either the armature reaction under time-varying conditions.

5.3.1 No Load Studies Comparison

The FEMM no load flux density distribution confirms the same concentration pattern observed in the COMSOL no load solution: the flux is concentrated in the stator cores corresponding to the phase A1 windings, which is the spatial signature of the maximum flux linkage between the rotor magnets and the stator winding at the initial rotor position.

However, the peak values obtained from FEMM are lower than those from COMSOL. This discrepancy is attributed to the different mesh densities and the reduced geometry modeled in FEMM (one third of the machine with periodic boundary conditions), which may affect the local saturation levels.

5.3.2 On Load Studies Comparison

The differences observed between the FEMM and COMSOL flux density distributions under load can be also attributed to the different modeling approaches: whereas, FEMM operates at 0 frequency and models only one third of the motor. This simplification can smooth the flux distribution compared to the full geometry. In contrast, COMSOL solves the time-dependent problem, including the rotation of the field and its temporal variation. Furthermore, the non-linear B-H curves of the ferromagnetic materials, combined with the applied stator currents, may lead to saturation in a non-uniform manner, which is handled differently by the two solvers. Consequently, the FEMM distribution appears more distributed and less concentrated on a specific set of windings than the COMSOL solution.

A comparison between the FEMM and COMSOL normal airgap flux density distributions, plotted as functions of the respective arc length, reveals a difference in sign. In COMSOL, the normal component exhibits both positive and negative values, as the flux lines cross the air gap in opposite directions depending on the magnet polarity. In FEMM, the flux density along the arc is consistently positive, due to the different conventions used for the normal component and the periodic boundary conditions.

This discrepancy is not indicative of an error, but rather reflects the different definitions and modeling approaches adopted in the two software environments. Both representations are physically consistent and correctly describe the magnetic field distribution in the SPM motor.

For the comparison of the on load voltage, the FEMM solution is evaluated at the rotor position corresponding to maximum torque (30° mechanical). The phase with the highest flux linkage, identified as phase 8, is selected for the voltage calculation. The corresponding phase voltage is then computed from the flux linkage using the rotational speed.

On the COMSOL side, the voltage value at the same rotor position is extracted from the time-domain phase voltage waveform. The angular position is converted into time using the relation $t = \theta/\omega$, with $\omega = 104.72$ rad/s, which yields $t = 0.005$ s.

The time-dependent loss-free COMSOL simulation, however, covers one electrical period (0.004 s). Although the voltage waveform is not perfectly sinusoidal, it is periodic, therefore, the voltage value at $t = 0.001$ s is equivalent to that at $t = 0.005$ s, corresponding to the same rotor position.

The difference between the FEMM and COMSOL values (29.90 V vs 18 V) is significant. This discrepancy can be attributed to several factors, most of them already described: the magnetostatic nature of FEMM, which solves the field at a single rotor position with idealized conditions, while COMSOL includes the full transient behavior; the different geometry and mesh resolutions, leading to different flux linkage and voltage predictions.

Despite this difference, FEMM results remain a useful validation of the COMSOL model, as they provide a consistent magnetostatic reference for the same operating point.

Regarding the torque, the FEMM sweep gives a maximum value of approximately

19.91 Nm at 30° mechanical, while the COMSOL stationary study gives approximately 33 Nm at the same rotor position. This discrepancy is consistent with the voltage difference.

In addition to the factors already discussed, the numerical approach used in FEMM to handle rotor motion contributes to the reduction of the peak flux linkage. Specifically, FEMM employs a "sliding band" method, which allows the rotor to move without remeshing, thus avoiding numerical noise associated with mesh regeneration. However, this approach relies on interpolation across a fixed band of elements in the airgap, which can introduce a slight smoothing effect on the field solution. [24]

This smoothing attenuates the peak flux linkage, affecting both the induced voltage and the torque.

Chapter 6

Conclusions

In this thesis, a C-core multi-three-phase Surface Permanent Magnet (SPM) motor was developed and modeled using COMSOL Multiphysics.

Subsequently, a second Finite Element simulation environment, FEMM, was introduced, in which only one third of the motor had already been modeled by exploiting geometric periodicity.

The main objective of the work was to compare the simulation results, as well as the usability and physical consistency of the two software tools in a real-world design context.

The comparison was carried out under both no load and load conditions, considering flux density distribution, airgap flux density, torque characteristic, and induced voltage.

The results show that both tools are capable of reproducing the machine behavior. However, significant differences emerge, attributable to non-identical geometries, different boundary conditions applied in the two software (in particular with respect to the air gap), and different mesh discretizations.

These discrepancies are particularly evident in the load flux density distribution, where FEMM shows a more spread pattern compared to the more concentrated distribution obtained in COMSOL. The airgap flux density comparison also reveals a difference in sign conventions, which reflects the different definitions adopted by the two solvers rather than a physical inconsistency.

The torque and voltage values also differ at the maximum torque position.

This is attributed to the combined effect of the simplified one-third geometry in FEMM, the different mesh resolutions, and the numerical smoothing introduced by the sliding band interpolation, which reduces the effective flux linkage. Despite these differences, both models provide physically consistent descriptions and remain valid for comparative purposes.

FEMM proved to be a valid solver for rapid iterations and preliminary parametric studies, such as evaluating the mapping of magnetic flux density B under both no-load and load conditions, air-gap magnetic flux density values, and torque and voltage calculation.

This was possible thanks to its essential interface and solution speed. However, when

simulating a complex motor with materials characterized by a strongly non-linear B-H curve, the analysis in the non-magnetostatic regime led to convergence issues. Moreover, the FEMM model, being limited to a one-third sector with periodic boundary conditions, provides a simplified representation of the machine, which may affect the local saturation levels and the voltage prediction.

COMSOL, on the other hand, supported the computational complexity by providing a more detailed analysis, allowing both stationary and time-dependent studies to be solved. This was achieved at the expense of higher setup complexity and significantly longer computation times.

The stationary study confirmed the design choice of aligning the phase A1 axis with the d-axis of the rotor, resulting in a torque characteristic consistent with the expected behavior.

Furthermore, the data obtained from the COMSOL loss free simulations confirmed that the multi-three-phase SPM motor under study exhibits low harmonic content of the back EMF under no-load conditions and so the voltage waveforms are nearly sinusoidal.

Under load, the voltage waveforms deviate from the ideal sinusoidal shape due to the interaction between the current harmonics and the armature reaction, while the torque ripple remains low relative to the average torque, confirming the good quality of the design. The imposed phase currents are perfectly sinusoidal, as they are directly imposed by the current-controlled model. The flux density and current density distributions obtained from the various studies consistently confirm the expected spatial behavior of the machine, validating the geometric assumptions and the winding configuration.

Finally, the activation of the Loss Calculation node allowed the evaluation of the eddy current losses in all the domains. The loss-inclusive study, in fact, was performed using the resistive heating model, which does not account for hysteresis losses.

Compared to the no losses case, both the flux density and current density distributions are modified by the induced eddy currents. The qualitative pattern of the normal airgap flux density remains similar, while the radial component exhibits a more densely peaked waveform, confirming that eddy currents primarily affect the radial flux component.

The voltage waveforms show higher distortion and a reduced peak value, as a result of the induced currents in the conductive domains.

The torque response exhibits a more pronounced initial transient, delaying the establishment of steady-state operation. Once reached, the torque ripple remains low. Since the C-shaped stator core is modeled as non-laminated solid iron, the stator losses dominate the loss distribution. This is consistent with the physics of eddy currents induced in a solid conductive material under alternating magnetic flux. The rotor losses remain significantly lower, as the magnetic field is nearly stationary in the rotor reference frame. The losses in the permanent magnets and windings are also low. The resulting efficiency is consistent with the worst-case scenario of a non-laminated design.

For future developments, it would be interesting to extend the analysis to a 3D configuration in COMSOL to capture axial effects and end-winding contributions. Additionally, a suitable procedure to describe the non-standard lamination of the C-core stator could be implemented, enabling a more realistic assessment of the iron losses and improving the accuracy of the efficiency prediction.

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