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Experimental Identification of UAV Powertrain System

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Abstract

This thesis presents a parameter identification methodology for UAV powertrain systems consisting of a brushless DC (BLDC) motor and propeller. Accurate mathematical modeling of BLDC motor-propeller systems is essential for advanced control design, performance optimization, and fault detection in UAV applications. The identification procedure employs a least squares method to estimate the unknown parameters of the powertrain mathematical model from experimental data. The least squares approach minimizes the squared difference between measured and estimated system responses, providing optimal parameter estimates that capture the dynamic behavior of the BLDC motor-propeller combination. The identification requires experimental measurements of key system variables including rotor speed, motor current, and control inputs.

A comprehensive experimental campaign was conducted on a typical BLDC motor-propeller system used in UAV applications. The experimental data was collected under various operating conditions to ensure the identified model accurately represents the system dynamics across the entire operational envelope. The least squares algorithm was applied to determine critical parameters including motor electrical constants, mechanical inertia, and propeller aerodynamic coefficients.

The validation results demonstrate excellent agreement between the identified model and experimental data, confirming the accuracy and effectiveness of the proposed least squares identification approach for developing high-fidelity simulation models suitable for controller design and system analysis in UAV powertrain applications.

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Chapter 1

Introduction

1.1 Motivation

In recent years, Unmanned Aerial Vehicles (UAVs) have gained significant prominence in the aerospace sector, revolutionizing both civilian and military applications. The versatility and accessibility of UAV technology have enabled their deployment across different fields including aerial surveillance and reconnaissance, precision agriculture, infrastructure inspection, environmental monitoring, search and rescue operations, package delivery, cinematography, and scientific research [1]. This rapid expansion has been facilitated by advances in miniaturization, battery technology, sensor integration, and autonomous control systems, making UAVs increasingly cost-effective and operationally efficient.

UAV platforms can be broadly classified into two main categories based on their aerodynamic configuration: fixed-wing aircraft and multicopters (rotary-wing aircraft) [1]. Fixed-wing UAVs offer superior endurance and payload capacity, making them suitable for long-range missions and large-area coverage. Their propulsion systems generate forward airspeed, allowing the wings to produce lift. However, they require runways or launch systems for takeoff and landing, limiting their operational flexibility. Conversely, multicopters, including quadcopters, hexacopters, and octocopters, provide Vertical Take-Off and Landing (VTOL) capability, exceptional maneuverability, and hovering ability [1]. These characteristics make them ideal

for operations in confined spaces, urban environments, and missions requiring stationary aerial observation. Despite their advantages, multicopters face limitations in endurance and payload capacity compared to fixed-wing configurations.

Regardless of the UAV configuration, the powertrain system plays a crucial role in determining the overall performance, efficiency, and reliability of the aircraft [2]. The powertrain, typically consisting of a brushless DC (BLDC) motor, Electronic Speed Controller (ESC), and propeller, is responsible for converting electrical energy into mechanical thrust. In fixed-wing UAVs, the powertrain must provide sufficient thrust for takeoff, maintain cruise speed efficiently, and ensure stable flight throughout the mission envelope. The propeller design and motor characteristics must be optimized to achieve the desired balance between thrust generation and energy consumption, directly impacting the aircraft's endurance and range.

In multicopter systems, the importance of the powertrain is even more pronounced [1]. Each rotor-motor combination must deliver precise and rapid thrust modulation to enable attitude control, since multicopters are inherently unstable platforms that rely on continuous thrust adjustments to maintain stability. The dynamic response of the BLDC motors, particularly their ability to rapidly change speed in response to control commands, is critical for achieving stable flight and executing complex maneuvers. Furthermore, the four or more propulsion units in a multicopter must operate harmoniously, with matched performance characteristics to prevent asymmetric thrust generation that could lead to control difficulties or instability.

Accurate modeling and identification of powertrain dynamics is therefore essential for several reasons [3]. High-fidelity models enable the development and validation of advanced flight control algorithms in simulation environments before deployment on actual aircraft, reducing development time and risk.

This thesis addresses the critical need for accurate powertrain modeling by presenting a comprehensive parameter identification methodology for BLDC motor-propeller systems used in UAV applications. By employing a least squares approach with experimental data [3], this work aims to develop validated mathematical models that capture the essential dynamics of the powertrain, ultimately contributing to improved UAV design, control, and performance optimization.

1.2 BLDC Motor

In recent years, electrical machines have assumed central role within the aerospace sector [2]. Their use has become more common in both commercial transport aircraft and military applications, particularly concerning on-board systems. They play an even more pivotal role in small-scale UAVs, which have seen widespread adoption in recent years. In these small-scale applications, BLDC motors are the dominant technology, offering a wide range of advantages, including exceptional ease of regulation and control [2]. To understand why these motors are preferred in such critical applications, it is essential to analyze their internal structure and the electronic commutation principles that govern their operation.

1.2.1 Operating Principle

To fully understand the operation of the BLDC, it is helpful to first examine its predecessor, the brushed DC motor [4]. Generally, a DC motor is composed of a moving part, known as the rotor, and a stationary part, called the stator. In a brushed motor, the rotor consists of one or more coils of conductive material, while the stator is composed of permanent magnets or, alternatively, its own set of windings.

The operating principle is based on the Lorentz force, which acts upon electrons moving within a magnetic field [4]. When current flows through a coil, it generates a torque that tends to align the magnetic field of the rotor with that of the stator.

$$\vec{F}_L = i \cdot (\vec{l} \times \vec{B}) \quad (1.1)$$

In which:

- i is the current intensity [A];
- $\vec{l}$ is the length vector [m];
- $\vec{B}$ is the magnetic field vector (of the stator) [T];

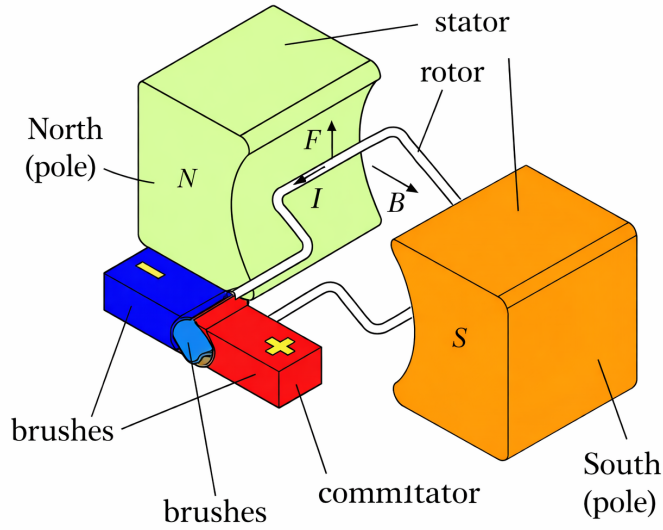


Figure 1.1: Operating principle of a DC motor [4]

As can be observed from the expression of the Lorentz force, the force is non-zero when the angle between the length vector and the magnetic field vector is different from zero. This implies that a force is generated as long as there is a misalignment between the rotor and stator magnetic fields. To ensure continuous rotation, commutation of the rotor's magnetic field is required. In brushed motors, this commutation is performed mechanically by means of the brushes [4]. Brushes represent a very simple method for commuting the rotor's magnetic field; however, they also involve significant disadvantages. The main drawbacks of mechanical commutation via brushes arise from their material composition: being made of graphite, they are subject to wear over time. Furthermore, there is a physical limit to the rotation speed, as high velocities can cause the brushes to bounce, leading to undesirable vibrations and the formation of electric arcs [2].

Brushless motors successfully resolve the typical issues associated with brushed motors [2]. BLDC motors consist of a permanent magnet rotor, while the stator is composed of conductive windings. In this type of motor, the commutation of the stator phases is performed electronically through an inverter. Typically, the stator of a BLDC motor consists of three phases which, when supplied with appropriately shifted currents, create a rotating magnetic field. The interaction between the stator's magnetic field and that of the rotor ensures that the rotor turns at a

specific angular velocity.

As previously mentioned, BLDC motors are often composed of three stator phases, which can be connected in two different configurations: Y or Δ [2]. The Y configuration is much more widely used. Crucial factors in BLDC motors are the type of stator windings and the arrangement of the rotor's permanent magnets. In fact, these elements directly impact the Back Electromotive Force (Back-EMF). Back-EMF is a voltage that opposes the supply voltage, generated across the windings [2]. This reverse voltage is caused by the change in magnetic flux through the stator coils, which is induced by the permanent magnets on the rotating rotor.

$$e(t) = K_{\Phi} \cdot \frac{d\Phi}{dt} \quad (1.2)$$

Eq.1.2 shows that the Back-EMF is proportional to the time derivative of the magnetic flux. The variation of magnetic flux in a BLDC motor depends on the winding distribution, the permanent magnet arrangement, and the angular velocity [2]. The first two factors determine the shape of the Back-EMF waveform, while the angular velocity influences its magnitude. At higher angular velocities, the magnetic flux changes more rapidly over time, resulting in a larger time derivative and, consequently, a higher induced voltage.

There are two categories of BLDC motors: those with a square flux density distribution and those with a sinusoidal distribution [2]. The former exhibits a trapezoidal Back-EMF waveform, while the latter features a sinusoidal one. In BLDC motors, understanding the Back-EMF is vital as it affects rotation smoothness and provides essential feedback for phase commutation control.

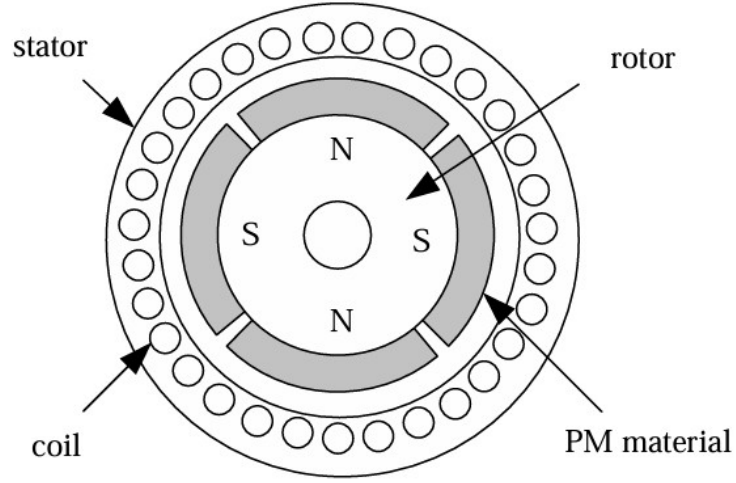


Figure 1.2: Cross section view of a BLDC motor [2]

As previously mentioned, BLDC motors lack brushes for mechanical commutation; instead, commutation must be handled electronically via an inverter [2]. In small-scale applications, the commutation of the stator phases is managed by the ESC. The ESC is powered by DC current; it uses 6 MOSFETs to commutate the stator phases, two for each phase [2]. A MOSFET (Metal-Oxide-Semiconductor Field-Effect Transistor) is a transistor that operates as an electronic switch at very high frequencies, up to 100 KHz.

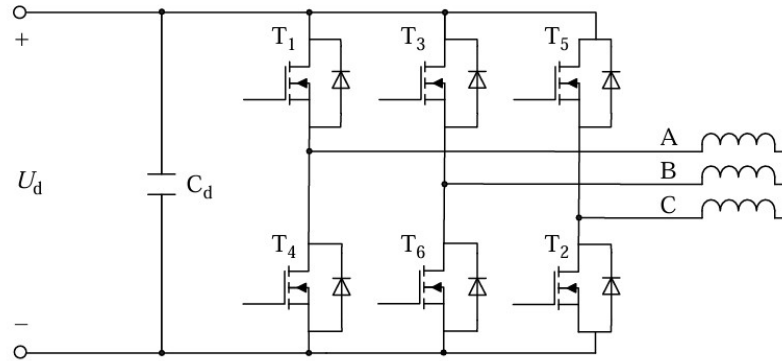


Figure 1.3: Electronic Speed Controller Scheme [2]

The integrated control unit manages these MOSFETs based on two critical

inputs [2]:

- Rotor position;
- Power required;

The power level is dictated by a PWM (Pulse Width Modulation) signal [2]. This signal consists of several pulses sent with a constant frequency and amplitude. By varying the duty cycle, it is possible to impose an average voltage across the motor terminals. This technique allows for highly efficient motor control without significant power dissipation as heat.

There are several options for measuring the rotor's position. The first is using Hall effect sensors [5]. These are active components that measure a potential difference when interacting with a magnetic field—in this case, the magnetic field of the rotor. By detecting the magnetic poles as they pass, these sensors provide the control unit with precise feedback, ensuring accurate phase commutation.

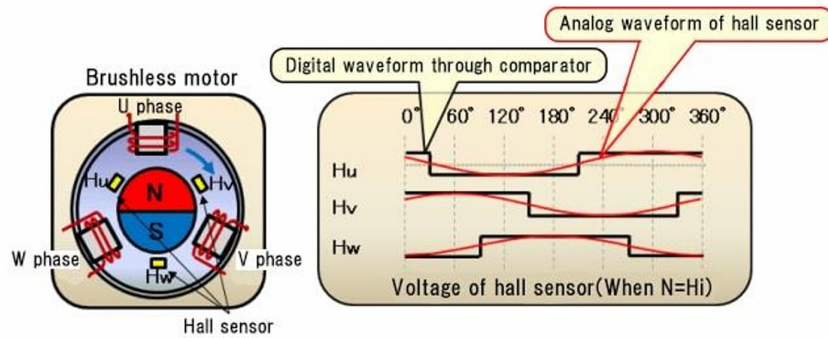


Figure 1.4: Hall sensor in BLDC motor [5]

Another method to determine the rotor position is to exploit the Back-EMF [2]. This principle is used in so-called sensorless motors, which further highlights the importance of having a known Back-EMF waveform. In drone and RC aircraft applications, the sensorless approach is preferred as it reduces overall weight, increases reliability by eliminating fragile components prone to vibration failure, and simplifies the wiring between the motor and the ESC.

1.2.2 Performance

As mentioned in the previous sections, BLDC motors are widely used in the UAV field due to their ease of control [2]. Another major advantage of this type of motor is its linear operation [4]. Indeed, it can be observed that the characteristic curve is linear. This curve can be visualized on a graph with the angular velocity (ω) on the x-axis and the mechanical torque (C) on the y-axis.

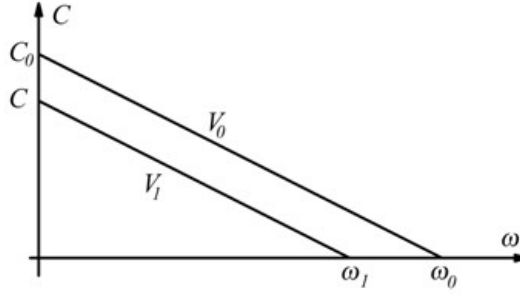


Figure 1.5: Characteristic curve of a BLDC motor [4]

As shown in Fig.1.5, the maximum torque occurs at zero speed; this is known as the stall torque. The torque decreases as the angular velocity increases due to the Back-EMF [2]. As previously mentioned, the Back-EMF increases with angular velocity because the time derivative of the magnetic flux between the stator and the rotor increases. It is also noted that the curve depends on the voltage applied to the motor terminals. In fact, as the voltage increases, the characteristic curve shifts upward. This occurs because when a higher voltage is applied, the current flowing through the stator coils becomes greater; consequently, the stator's rotating magnetic field strengthens, thereby increasing the generated torque [4]. In a practical scenario, the ESC manages this voltage increase through PWM. By increasing the duty cycle, the ESC raises the average voltage delivered to the motor terminals. This effectively shifts the entire torque-speed curve upward and to the right, allowing the motor to achieve higher angular velocities or to handle greater mechanical loads while maintaining the same battery supply. In real-world BLDC motors, the operating range is subject to specific constraints [5]. Typically, the maximum rotational speed is determined by mechanical components, such as bearings or masses subjected to centrifugal forces. Conversely, at high torque levels,

the motors can only operate for a limited time; this is necessary to prevent damage to the coil windings caused by high temperatures.

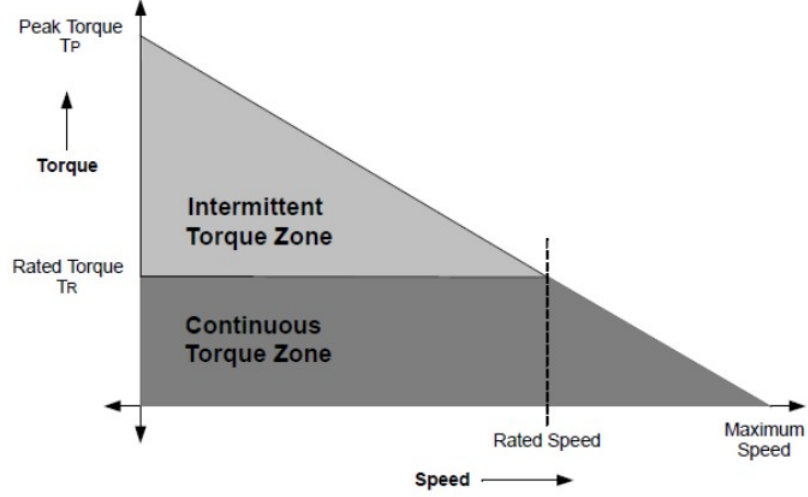


Figure 1.6: Operating area of a real BLDC motor [5]

A crucial aspect of BLDC performance is the mechanical power, defined as the product of torque and angular velocity [4].

$$P_{mech} = C \cdot \omega \quad (1.3)$$

Since torque decreases linearly as speed increases, the power output follows a parabolic curve. This means that mechanical power is zero at both stall torque and no-load speed, reaching its theoretical peak at the midpoint of the characteristic curve.

1.2.3 PID speed controller

In practical applications, the command is usually given in revolutions per minute (RPM) [2]. This means that the input must somehow be translated into a PWM duty cycle. There are two main approaches to achieve this:

- Open Loop;
- Closed Loop;

In an open loop system, the command is sent without any feedback regarding the actual speed reached by the motor [5]. In this case, the motor's operating point depends strictly on the applied load; as the load changes, the operating point moves along the characteristic curve corresponding to the average voltage applied to the motor terminals. In an open loop system, this voltage cannot change because there is no feedback on the actual speed. These systems are advantageous when the applied load is known; since they do not require feedback sensors, they prove to be more reliable in such specific cases.

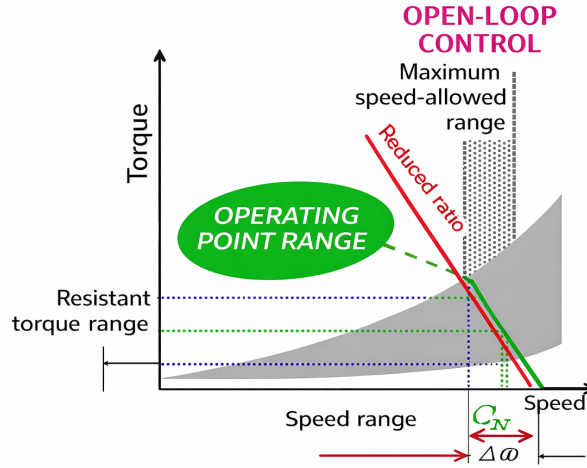


Figure 1.7: Open loop control in BLDC motor [5]

In a closed loop system, the command sent to the motor depends on the error between the target angular velocity and the actual velocity [2]. Consequently, a speed sensor is required to implement this type of control. In this way, the operating point shifts vertically, making it possible to reach the commanded speed regardless of the applied load.

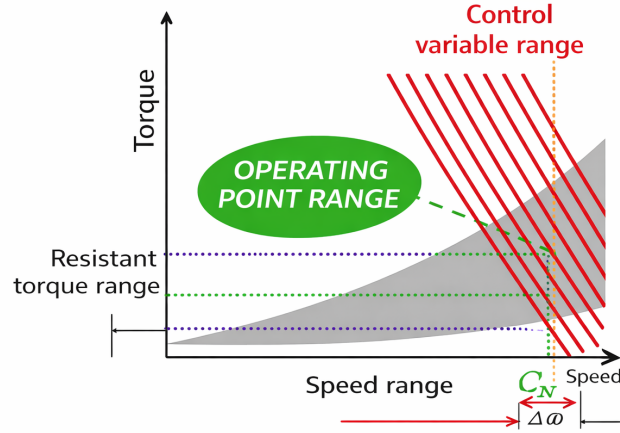


Figure 1.8: Closed loop control in BLDC motor [5]

The most widely used control system in closed loop configurations is the PID controller (Proportional-Integral-Derivative) [2]. As the name suggests, this type of controller generally consists of three contributions: one proportional to the error, one proportional to the derivative of the error, and one proportional to the integral of the error.

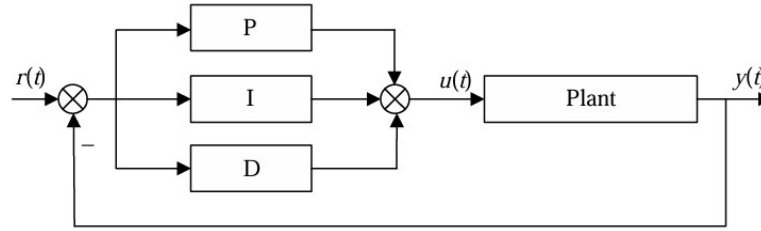


Figure 1.9: Diagram of a PID control system [2]

As shown in the fig.1.9, the command received by the plant system is the sum of these three contributions. The final output, which dictates the PWM duty cycle, is the result of combining the proportional, integral, and derivative actions to achieve

a stable and accurate motor response.

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (1.4)$$

The proportional component allows for a faster system response [2]; however, it cannot eliminate the steady-state error, as this component becomes substantially null when the error is very small. The derivative component has an anticipatory function: by deriving the error, it recognizes in advance whether the error tends to increase or decrease. The integral component allows the system to eliminate the steady-state error, as it increases even in the presence of small errors. The problem with the integral component is that it only begins to decrease once the error becomes negative. Therefore, it risks saturating the controller's output. To prevent this issue, an anti-windup filter can be used [2]. When the controller output reaches saturation, the integral contribution is halted or reset.

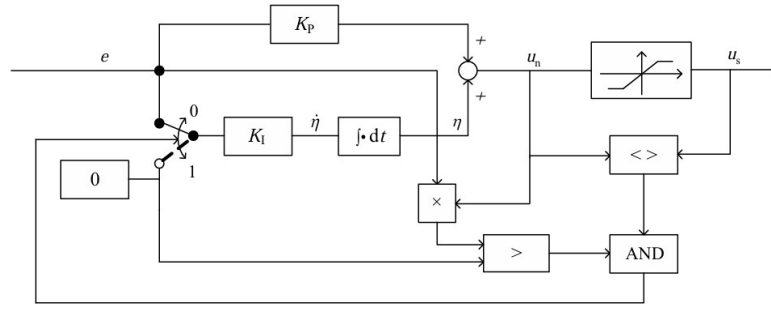


Figure 1.10: Anti wind up filter diagram [2]

1.3 Propeller aerodynamics

A propeller can be fundamentally described as a high-aspect-ratio wing, cantilevered from a central hub about which it rotates [6]. As the propeller rotates, it accelerates the air mass passing through the disk, resulting in an increase in the flow's dynamic pressure. This process gives rise to aerodynamic loads (thrust and torque) on the propeller structure.

1.3.1 Actuator disk theory

The thrust generated by the propeller can be explained through Newton's second law, which states that the resultant of the applied forces is equal to the time rate of change of momentum [6].

$$F = \frac{d(mv)}{dt} \quad (1.5)$$

In the case of a propeller, the thrust is produced by accelerating a mass of air through a small change in velocity. The theory based on this phenomenon is known as the Rankine-Froude actuator disk theory [6]. This theory treats the propeller as an actuator disk; an element composed of an infinite number of blades around the propeller hub. From a side view of the actuator disk, it is possible to consider a tubular control volume that captures all the air drawn in by the propeller and accounts for the wake contraction fig1.11.

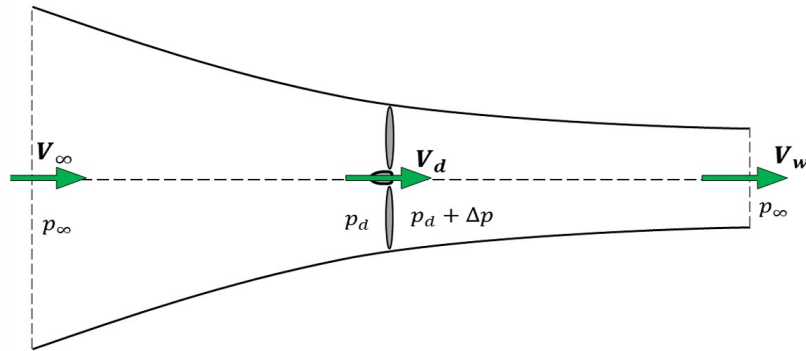


Figure 1.11: Control volume for disk actuator theory [6]

By analyzing the streamtube, we observe that air is drawn from upstream at velocity V_∞ and pressure p_∞ . As it passes through the disk, the fluid undergoes a sudden change in the total energy of the flow, through a jump in the total pressure. While the velocity increases continuously from the freestream to the far wake, the pressure experiences an instantaneous rise at the disk before gradually returning to ambient levels downstream as the flow contracts and reaches its terminal velocity [6]. Applying the Bernoulli equation from the freestream to the disk ($p_\infty + \frac{1}{2}\rho V_\infty^2 = p_1 + \frac{1}{2}\rho V_d^2$) and from the disk to the far wake ($p_2 + \frac{1}{2}\rho V_w^2 = p_\infty + \frac{1}{2}\rho V_w^2$), the pressure jump $\Delta p = p_2 - p_1$ is found to be:

$$\Delta p = \frac{1}{2}\rho(V_w^2 - V_\infty^2) \quad (1.6)$$

The thrust can be expressed both as $T = A\Delta p$ (where A is the disk area) and, from momentum theory, as $T = \rho A V_d (V_w - V_\infty)$. Equating these two expressions:

$$\frac{1}{2}\rho A (V_w - V_\infty)(V_w + V_\infty) = \rho A V_d (V_w - V_\infty) \quad (1.7)$$

Which simplifies directly to the fundamental relationship:

$$V_d = \frac{V_\infty + V_w}{2} \quad (1.8)$$

Ultimately, the thrust can be expressed as:

$$T = 2\rho A V_d (V_\infty + V_d) \quad (1.9)$$

In which:

- ρ is the air density
- A is the disk area, which could be calculated as $A = \pi r^2$
- V_d is the induced velocity caused by the propeller

1.3.2 Blade element theory (BET)

Unlike the actuator disk theory, the Blade Element Theory (BET) does not consider the propeller a single element that accelerates the flow; instead, it treats it as a collection of distinct radial elements, analyzing the aerodynamics of each one individually [6].

Specifically, BET divides the blade into numerous two-dimensional profiles (sections) [6]. This approach allows for the calculation of the infinitesimal forces and moments acting on each individual element as follows:

$$dT = dL \cos(\phi) - dD \sin(\phi) \quad (1.10)$$

$$d\tau_{aero} = r(dL \sin(\phi) + dD \cos(\phi)) \quad (1.11)$$

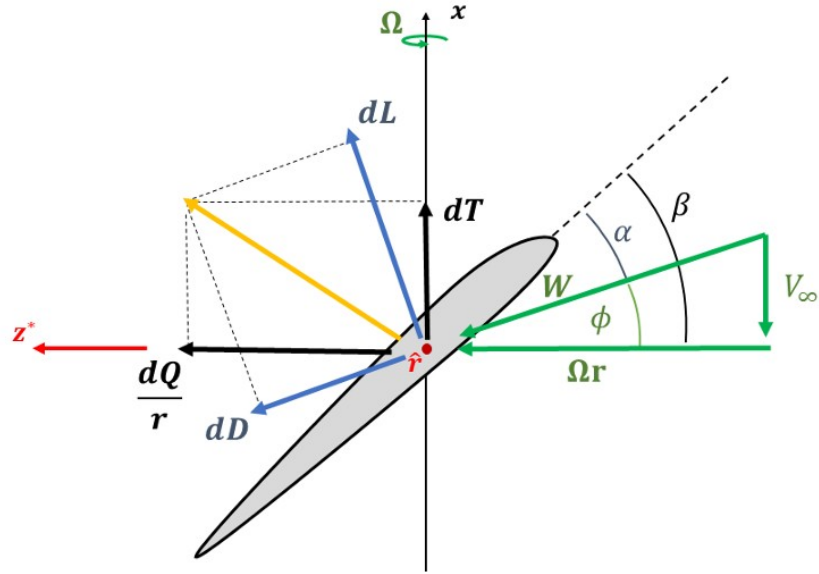


Figure 1.12: Velocity and forces decomposition for a propeller's blade element [6]

In these equations, dT and dQ represent the infinitesimal aerodynamic thrust and torque of the specific section at radius r . The terms dL and dD are the infinitesimal lift and drag forces generated by the two-dimensional airfoil section,

defined as:

$$dL = \frac{1}{2}\rho C_l c W^2 dr \quad (1.12)$$

$$dD = \frac{1}{2}\rho C_d c W^2 dr \quad (1.13)$$

Where:

- ρ is the fluid density;
- C_l and C_d are the lift and drag coefficients of the airfoil;
- c is the local chord length of the airfoil;
- W is the resultant relative velocity seen by the airfoil;
- ϕ is the inflow angle (the angle between the resultant velocity and the plane of rotation).
- β is the geometric pitch angle of the airfoil considered;
- α represents the angle of attack of the airfoil under consideration, which determines its aerodynamic performance;

The lift and drag coefficients, C_l and C_d , depend on the local angle of attack at which the airfoil operates and on the Reynolds number ($Re = \frac{cW}{\nu}$) [6]. To determine the performance of the complete propeller, these infinitesimal quantities must be integrated along the blade span from the hub to the tip and then multiplied by the number of blades (B):

$$T = B \int_0^R dT dr \quad (1.14)$$

$$\tau_{aero} = B \int_0^R d\tau_{aero} dr \quad (1.15)$$

1.3.3 Blade Element Momentum Theory (BEMT)

The Blade Element Momentum Theory (BEMT) is a hybrid aerodynamic framework that couples the Momentum Theory with the Blade Element Theory [6]. The core objective of BEMT is to determine the induced flow field at the rotor disk, which is otherwise an unknown in the standard BET.

The flow at a radial distance r is modified by the rotor's operation through the axial and tangential induction factors, a and a' [6].

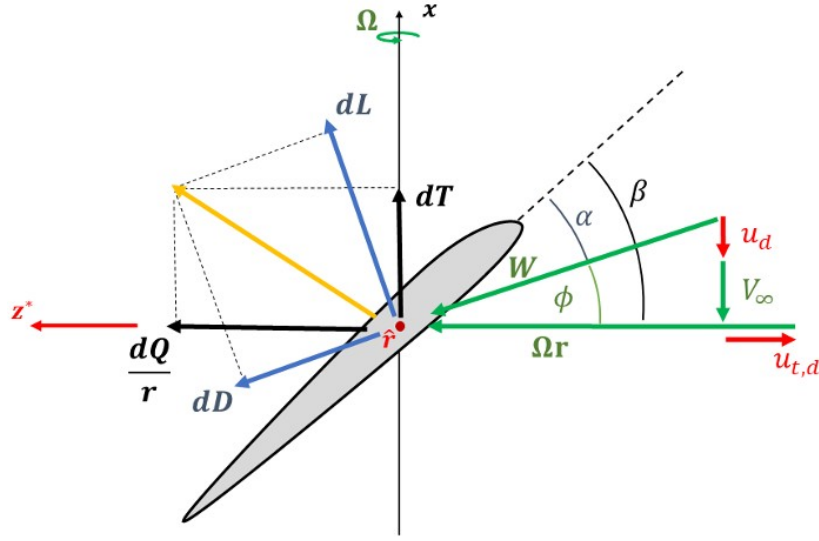


Figure 1.13: Velocity triangle and aerodynamic forces for a blade element at radius r (BEMT) [6]

These factors represent the velocity perturbations at the disk. The local velocity components and the resulting flow angle ϕ are defined as:

$$V_{axial} = V_{\infty}(1 + a), \quad V_{tangential} = \omega r(1 - a') \quad (1.16)$$

$$W = \sqrt{V_{axial}^2 + V_{tangential}^2}, \quad \phi = \arctan\left(\frac{V_{axial}}{V_{tangential}}\right) \quad (1.17)$$

BEMT operates by enforcing an equilibrium between the macroscopic momentum change of the fluid and the microscopic forces generated by the blade sections [6].

Derived from the axial and angular momentum conservation using momentum balance, the thrust (dT) and torque (dQ) are:

$$dT = 4\pi r \rho V_\infty^2 a(1+a)dr \quad (1.18)$$

$$d\tau_{aero} = 4\pi r^3 \rho V_\infty (1+a)a'\Omega dr \quad (1.19)$$

Local aerodynamic forces based on the lift (C_l) and drag (C_d) coefficients of the specific airfoil section:

$$dT = \frac{1}{2} B \rho W^2 c (C_l \cos \phi - C_d \sin \phi) dr \quad (1.20)$$

$$d\tau_{aero} = \frac{1}{2} B \rho W^2 c (C_l \sin \phi + C_d \cos \phi) r dr \quad (1.21)$$

Since C_l and C_d are non-linear functions of α , which in turn depends on a and a' , the system is solved through a fixed-point iteration for each radial station [6]. The total thrust (T) and torque (τ_{aero}) are finally obtained by integrating the converged values along the span from the hub (R_{hub}) to the tip (R_{tip}).

1.3.4 Dynamic Inflow

When a rotor generates thrust, it accelerates air downward through the rotor disk, forming a wake that extends several diameters downstream [7]. The flow field within this wake is three-dimensional, varying with streamwise distance, radial position, and blade azimuthal angle.

According to actuator disk theory (Subsection 1.3.1), the steady-state induced velocity at the rotor in hover condition is:

$$V_{iss} = \sqrt{\frac{T}{2\rho A}} \quad (1.22)$$

Where V_{iss} is the steady state induced velocity. This relation assumes the induced velocity adjusts instantaneously to thrust changes; an assumption valid only for slow, quasi-steady variations [8].

For rapid transients, however, the induced velocity exhibits a finite (non zero) response time [8, 9]. When thrust changes abruptly, the induced velocity requires time to reach its new equilibrium value. This dynamic lag stems from two fundamental

mechanisms [7, 8]:

- Apparent Mass Effect;
- Perturbation Propagation;

The slipstream contains moving fluid with inertia that resists instantaneous acceleration. The characteristic time to accelerate the wake mass scales as:

$$\tau_{wake} \propto \frac{L_{wake}}{V_i} \quad (1.23)$$

where $L_{wake} \approx 3\text{--}5D$ is the near-wake length [7].

Changes in the operating conditions (e.g., rotor speed variations) lead to disturbances in the wake that can be associated with the modification of the vorticity field [8]. These perturbations convect downstream at finite velocity $V \approx V_i + V_\infty$, hence a timescale can be defined as:

$$\tau_{prop} \approx \frac{L_{influence}}{V} \quad (1.24)$$

to traverse the aerodynamically active wake region, where $L_{influence}$ is the streamwise extent over which the wake significantly affects rotor aerodynamics.

The combined effect yields a characteristic time constant [9]:

$$\tau \approx \frac{R}{V} \quad (1.25)$$

where R is rotor radius and $V = \sqrt{V_\infty^2 + V_i^2}$ is total flow velocity through the disk.

Chapter 2

System modeling

The objective of this chapter is to describe the proposed dynamic model of a BLDC driving a fixed-pitch propeller. The model aims to capture both the electrical and mechanical dynamics of the motor, as well as the aerodynamic load from the propeller.

2.1 BLDC motor Modeling

The modeling of the BLDC motor is divided into two distinct parts. The motor circuit modeling describes how the current evolves over time, while the rotor dynamics describes how the angular velocity evolves over time.

2.1.1 Motor/ESC Circuit Modeling

The operation of the internal circuit of the BLDC motor can be expressed through a system of differential equations. A three-phase BLDC motor with Y-connected windings is considered (phases A, B and C), as illustrated in Figure 2.1.

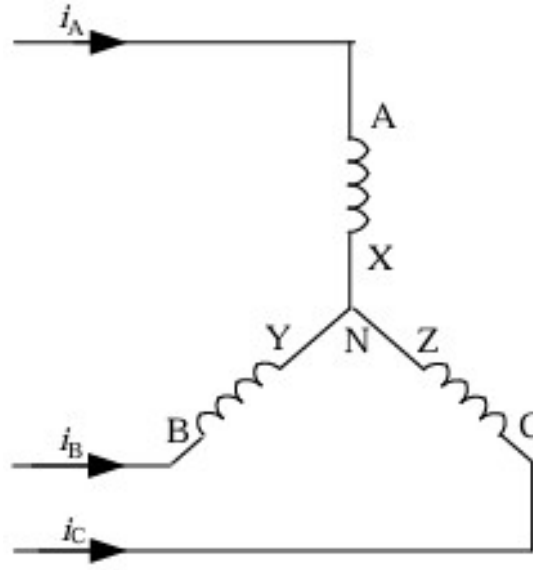


Figure 2.1: Y connection of the windings in a BLDC motor [2]

The motor model employs multiple reference frames to describe its electromagnetic behavior Figure 2.2 :

- *Stationary reference frame (abc):* The axes A, B, and C represent the fixed stator windings, spatially displaced by 120 electrical degrees from each other. These define the natural three-phase coordinate system.
- *Rotating reference frame (dq):* The direct axis (d-axis) and quadrature axis (q-axis) rotate synchronously with the rotor at angular position θ . The d-axis typically aligns with the rotor flux, while the q-axis leads by 90 electrical degrees.
- *Rotor position:* The angle θ defines the electrical position of the rotor magnetic axis (represented by axis N) relative to the phase A axis, measured in the direction of rotation.

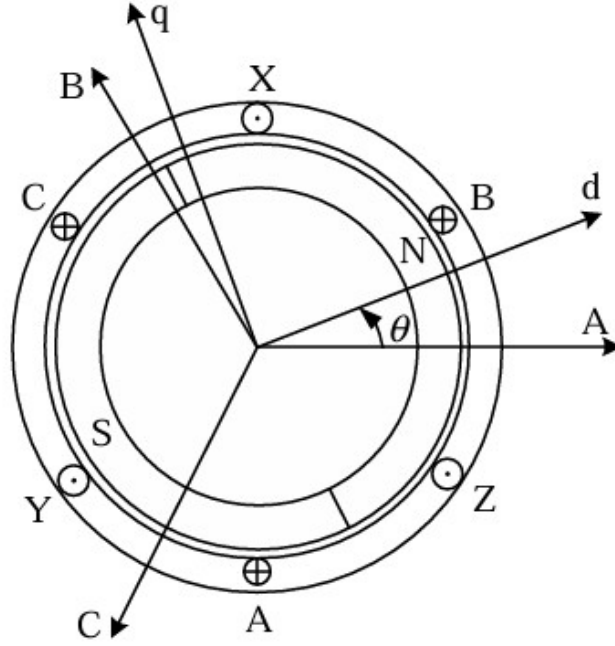


Figure 2.2: BLDC motor reference frame [2]

First of all, it is necessary to make some assumptions that allow for a less complex model while still enabling accurate predictions of the system dynamics:

1. *Magnetic linearity:* Core saturation, eddy current losses, and hysteresis losses are ignored.
2. *Ideal magnetic field:* Armature reaction is ignored, and the air-gap magnetic field distribution is assumed to be a trapezoidal wave with a flat-top width of 120 electrical degrees.
3. *Uniform conductor distribution:* The cogging effect is ignored, and the conductors are assumed to be distributed continuously and evenly on the armature surface.
4. *Ideal electronic components:* Power switches and flywheel diodes of the inverter circuit have ideal switching characteristics.

Considering a generic phase, the voltage across it can be expressed as follows:

$$u_x = R_x \cdot i_x + e_{cx} \quad (2.1)$$

where:

- u_x = phase voltage (with $x = A, B, C$)
- R_x = phase x resistance
- i_x = current in phase x
- e_{cx} = induced electromotive force (back-EMF) in phase x

In the windings, the back-electromotive force is equal to the time derivative of the magnetic flux. Taking phase A as an example, the flux can be expressed as:

$$\psi_A = L_A i_A + M_{AB} i_B + M_{AC} i_C + \psi_{pm}(\theta) \quad (2.2)$$

where:

- $L_A i_A$ = self-inductance flux (generated by its own current)
- $M_{AB} i_B$ = mutual inductance flux from phase B
- $M_{AC} i_C$ = mutual inductance flux from phase C
- $\psi_{pm}(\theta)$ = permanent magnet flux linkage (depends on rotor position θ relative to phase A axis)

The intensity of the magnetic flux ψ_{pm} depends on the magnetic field distribution of the permanent magnets in the air gap. Specifically, under the assumptions made, the radial component of the magnetic field in the air gap is trapezoidal along the stator surface. When the rotor rotates, the effective flux of the phases (e.g., phase A) will vary according to the rotor position itself with the following relationship:

$$\psi_{pm}(\alpha) = N \cdot \phi_{pm}(\alpha) \quad (2.3)$$

$$\phi_{pm}(\alpha) = \int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}+\alpha} B(\theta) \cdot S d\theta \quad (2.4)$$

where:

- α is the rotor position in θ reference frame
- $\phi_{pm}(\alpha)$ is the PM flux of phase A when the rotor position angle is α
- $B(\theta)$ PM rotor radial flux density in the air gap
- N is turns of winding
- S is product of rotor radius and effective length of conductors

By substituting into Eq.2.2 and taking its time derivative to derive the back-EMF, one obtains:

$$e_a = \frac{d\psi_a}{dt} = L_A \frac{di_A}{dt} + M_{AB} \frac{di_B}{dt} + M_{AC} \frac{di_C}{dt} + \frac{d}{dt} \int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}+\alpha} B(\theta) \cdot S d\theta \quad (2.5)$$

By evaluating the integral over the flat-top region of the trapezoidal magnetic field distribution and applying the chain rule with $\frac{d\theta}{dt} = \omega$, the back-EMF expression simplifies to:

$$e_A = k_e \cdot \omega \quad (2.6)$$

where $k_e = N \cdot S \cdot B_{avg}$ is the back-EMF constant (or voltage constant), which depends on the number of turns per phase N , the effective area S , and the average magnetic flux density B_{avg} in the air gap.

It is now possible to write a differential equation for each phase of the BLDC motor, thus obtaining a system of differential equations that describes the current behavior of the three phases A, B, and C:

$$\begin{bmatrix} u_A \\ u_B \\ u_C \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L-M \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \begin{bmatrix} e_A \\ e_B \\ e_C \end{bmatrix} \quad (2.7)$$

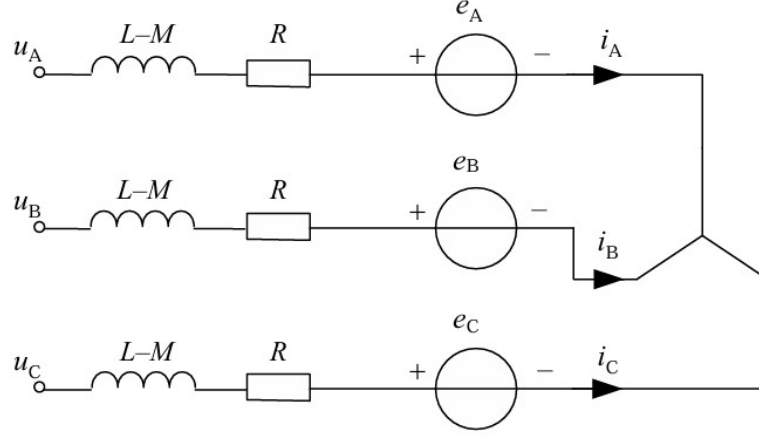


Figure 2.3: Equivalent circuit of the BLDC motor[2]

In BLDC motors with Y-connection, it is very difficult to measure the phase voltage due to the absence of a neutral point, which limits the application of the model just described. The model can also be expressed as a function of line-to-line voltages as follows:

$$\begin{bmatrix} u_{AB} \\ u_{BC} \\ u_{CA} \end{bmatrix} = \begin{bmatrix} R & -R & 0 \\ 0 & R & -R \\ -R & 0 & R \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \begin{bmatrix} L-M & M-L & 0 \\ 0 & L-M & M-L \\ M-L & 0 & L-M \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \begin{bmatrix} e_A - e_B \\ e_B - e_C \\ e_C - e_A \end{bmatrix} \quad (2.8)$$

Compared to the traditional brushed DC motor, the BLDC motor energizes its windings based on rotor position and is typically designed with three or multiple phases. However, for each energized phase winding, the principles governing back-EMF generation and electromagnetic torque production remain fundamentally the same as in conventional brushed DC motors, allowing the application of similar analytical methods. Assuming that the three-phase BLDC motor is controlled by a full-bridge driver operating in two-phase conduction mode, when phases A and B are energized simultaneously, the following relationship holds:

$$\begin{cases} i_A = -i_B = i \\ \frac{di_A}{dt} = -\frac{di_B}{dt} = \frac{di}{dt} \end{cases} \quad (2.9)$$

Consequently, the circuit equation for this conduction mode can be expressed

as:

$$u_{AB} = r_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + k_e \cdot \omega \quad (2.10)$$

Where:

- r_a — line resistance of winding, $r_a = 2R$;
- L_a — equivalent line inductance of winding, $L_a = 2(L - M)$

In two-phase conduction mode, the line voltage u_{AB} equals the PWM-modulated battery voltage: $u_{AB} = V_{bat} \cdot \delta$, where δ is the throttle command (duty cycle). The current can be considered DC because the PWM switching frequency is much higher than the motor's electrical time constant, allowing the current to be approximated by its average value during each conduction interval.

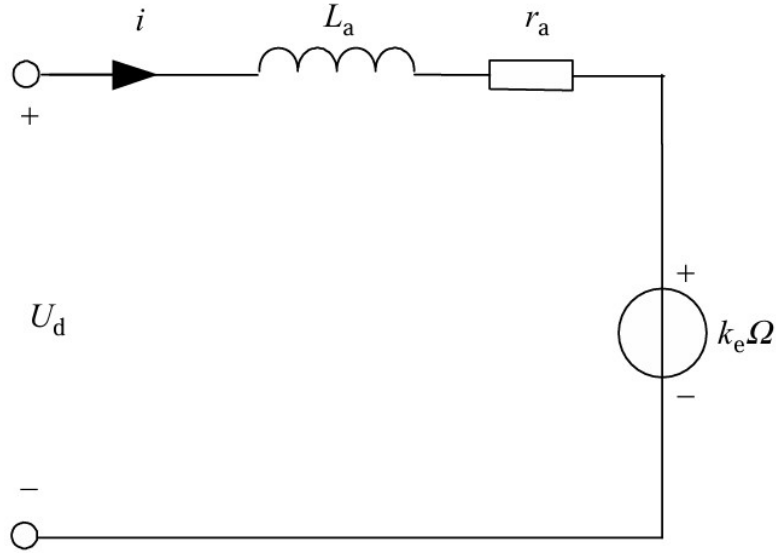


Figure 2.4: Equivalent circuit of the BLDC motor with two phase windings excited [2]

In conclusion, the differential equation describing the current evolution in the circuit can be written as:

$$L \frac{di}{dt} = \delta \cdot V_{batt} - R \cdot i - k_e \cdot \omega \quad (2.11)$$

Similar to DC motors, the electromagnetic torque analysis for a BLDC motor can be carried out using an energy-based approach. Neglecting losses in the electrical lines and electromagnetic losses, the following energy balance equation can be written:

$$P_e = e_A \cdot i_A + e_B \cdot i_B + e_C \cdot i_C = \tau_M \cdot \omega \quad (2.12)$$

In which:

- P_e is the power transferred to the motor
- τ_M is the electro-magnetic torque generated by the motor

By substituting equation for the back-EMF for each phase, one could express the electro-magnetic torque as follows:

$$\tau_M = K_T \cdot i \quad (2.13)$$

In applications where the current reaches high magnitudes, the previously assumed hypothesis of magnetic linearity is no longer valid. This phenomenon directly affects the relationship between current and electromagnetic torque. As the magnetic core approaches saturation, the magnetic flux density does not increase proportionally with the current; consequently, the torque constant K_t effectively decreases, leading to a non-linear trend.

2.1.2 Rotor and Load Dynamics

The rotational dynamics of the shaft are governed by a torque balance between the motor torque and the resistive torques from aerodynamic drag and mechanical friction. Applying Newton's second law for rotation:

$$J \frac{d\omega}{dt} = \tau_m - \tau_{\text{aero}}(\omega) - \tau_{\text{friction}}(\omega) \quad (2.14)$$

where:

- ω is the angular velocity of the shaft (rad/s),
- J is the total moment of inertia of the motor rotor and propeller assembly

($\text{kg}\cdot\text{m}^2$), representing the combined rotational inertia that resists angular acceleration,

- τ_m is the electromagnetic torque produced by the motor ($\text{N}\cdot\text{m}$), which serves as the driving torque,
- $\tau_{\text{aero}}(\omega)$ is the aerodynamic load torque generated by the propeller during operation ($\text{N}\cdot\text{m}$),
- $\tau_{\text{friction}}(\omega) = k_v\omega$ is the viscous friction torque ($\text{N}\cdot\text{m}$), representing mechanical losses in bearings and other rotating components, where k_v is the viscous friction coefficient,

The friction torque is modeled as linear viscous damping, capturing the dominant losses in motor bearings and air resistance at the motor housing, while Coulomb friction and other nonlinear effects are considered negligible at operational speeds. However, for simplicity, the friction torque τ_{friction} is neglected in the present model as it is negligible compared to the aerodynamic torque τ_{aero} at typical operating speeds, where propeller loads dominate the torque balance.

The aerodynamic torque τ_{aero} represents the load imposed by the propeller and is assumed to depend instantaneously on the current angular velocity ω alone, based on static calibration of the propeller at different steady-state speeds. This quasi-steady aerodynamic assumption implies that the torque-speed relationship established under steady-state conditions (where $d\omega/dt = 0$) remains valid during dynamic transients (where $d\omega/dt \neq 0$), and that unsteady aerodynamic effects are negligible.

This is the critical assumption we wish to analyze in a future step: that the aerodynamic torque can be computed as a function of ω alone. In reality, propeller aerodynamics exhibit unsteady behavior during rapid acceleration or deceleration, as the flow field around the blades, the induced velocity distribution, and blade loading all require finite time to adjust to changes in rotational speed. These transient effects introduce additional dependencies on the angular acceleration $d\omega/dt$ and the aerodynamic history of the flow. The validity of the quasi-steady assumption will be investigated by comparing model predictions against experimental measurements during dynamic maneuvers.

2.2 Aerodynamics model

The objective of the propeller aerodynamic model is to predict the time evolution of the aerodynamic forces generated by the propeller, namely the thrust and aerodynamic torque. The aerodynamic model employed in this study is divided into two parts: a static part and a dynamic part. The static part uses classical formulas for calculating aerodynamic forces, while the dynamic part accounts for inflow dynamics. The fundamental assumption underlying this model is that the aerodynamic forces are equal under both static and dynamic conditions when the inflow parameter is the same.

2.2.1 Static Aerodynamic Model

The static aerodynamic model provides the fundamental relationship between the propeller operating conditions and the generated aerodynamic forces. Based on the blade element momentum theory (BEMT) introduced in Section 1.3.3, the thrust and aerodynamic torque generated by the propeller can be expressed using non-dimensional coefficient formulations:

$$T(t) = \rho n^2 D^4 C_T(\lambda, \beta) \quad (2.15)$$

$$\tau_{aero}(t) = \rho n^2 D^5 C_Q(\lambda, \beta) \quad (2.16)$$

where:

- n is the angular velocity of the propeller [RPS];
- D is the propeller diameter [m];
- C_T and C_Q are the thrust and torque coefficients, respectively;
- λ is the inflow coefficient, defined as $\lambda = \frac{V_i}{\omega R}$, where V_i is the induced (inflow) velocity;

The coefficients C_T and C_Q incorporate the aerodynamic behavior of the propeller and depend on the flow conditions through the inflow coefficient λ and the geometric blade pitch angle β .

For a fixed-pitch propeller, the blade pitch angle β is constant by design and cannot be varied during operation. Consequently, the thrust and torque coefficients depend solely on the inflow coefficient λ :

$$C_T = C_T(\lambda), \quad C_Q = C_Q(\lambda) \quad (2.17)$$

According to the BEMT, as the propeller generates thrust, it induces a downward axial velocity V_i in the flow. Higher thrust production corresponds to higher induced velocities, thus larger values of λ indicate higher disk loading.

2.2.2 Dynamic Inflow Model

The model employed in this study is based on the theoretical framework developed by Peters and colleagues [8], which provides a rigorous mathematical description of unsteady rotor aerodynamics derived from potential flow theory.

The starting point for Peters' dynamic inflow theory is the representation of the propeller as an actuator disk: an infinitesimally thin, permeable disk that imparts a pressure jump to the flow passing through it. The flow is assumed to be inviscid and irrotational everywhere except at the disk itself, allowing it to be described by potential flow theory. The velocity field can be expressed as the gradient of a velocity potential Φ :

$$\mathbf{V} = \nabla \Phi \quad (2.18)$$

The velocity potential satisfies Laplace's equation in the flow domain:

$$\nabla^2 \Phi = 0 \quad (2.19)$$

The boundary condition at the actuator disk relates the pressure jump across the disk to the local blade loading. For an axisymmetric rotor in hover (no freestream velocity), the problem possesses cylindrical symmetry, and the induced velocity is purely axial. The key quantity to be determined is the axial induced velocity distribution $v(r, t)$ across the disk as a function of radial position r and time t .

In the classical steady-state momentum theory, the induced velocity is assumed uniform across the disk, and the thrust is related to the induced velocity through

momentum conservation. However, in reality, the induced velocity varies both radially and temporally. Peters' theory accounts for these variations by expanding the induced velocity in terms of spatial modes.

The induced velocity distribution across the rotor disk is expressed as a series expansion in terms of orthogonal spatial basis functions:

$$v(r, t) = \sum_{n=0}^N \lambda_n(t) \psi_n(r) \quad (2.20)$$

where $\lambda_n(t)$ are time-varying modal coefficients (inflow states), and $\psi_n(r)$ are spatial mode shapes. For an axisymmetric rotor, a common choice of basis functions is the set of orthogonal polynomials defined on the disk. Peters used a specific set of polynomial modes that satisfy orthogonality conditions weighted by the radial coordinate:

$$\int_0^R r \psi_m(r) \psi_n(r) dr = 0 \quad \text{for } m \neq n \quad (2.21)$$

The first few mode shapes are:

$$\psi_0(r) = 1 \quad (\text{uniform inflow}) \quad (2.22)$$

$$\psi_1(r) = \frac{r}{R} \quad (\text{linear variation}) \quad (2.23)$$

$$\psi_2(r) = \left(\frac{r}{R}\right)^2 - \frac{1}{2} \quad (\text{quadratic variation}) \quad (2.24)$$

The uniform mode ψ_0 represents the average induced velocity across the disk, while higher modes capture spatial variations such as radial gradients and curvature. The key insight from unsteady potential flow theory is that when the induced velocity distribution changes with time, the surrounding fluid must be accelerated, requiring additional aerodynamic work. This introduces an apparent mass effect: the rotor disk, together with the surrounding flow field, behaves as if it possesses an effective inertia that resists changes in induced velocity.

The kinetic energy associated with the induced flow can be written as:

$$KE = \frac{1}{2} \rho \int_{\text{fluid}} |\nabla \Phi|^2 dV \quad (2.25)$$

For small perturbations about a reference state, this kinetic energy can be

related to the modal coefficients through an apparent mass matrix. The temporal variation of induced velocity requires power input to accelerate the fluid, which comes from the work done by the pressure difference across the actuator disk.

Applying conservation of mass, momentum, and energy to the actuator disk with time-varying induced velocity, and projecting the resulting equations onto the spatial modes, Peters derived a system of ordinary differential equations for the modal coefficients. For the general multi-mode case, the governing equations take the form:

$$[M] \frac{d}{dt} \{\lambda\} + [L] \{\lambda\} = \{c_T\} \quad (2.26)$$

where:

- $\{\lambda\} = [\lambda_0, \lambda_1, \lambda_2, \dots]^T$ is the vector of modal inflow coefficients
- $[M]$ is the apparent mass matrix arising from fluid inertia
- $[L]$ is the aerodynamic influence coefficient matrix relating inflow to loading
- $\{c_T\}$ is the thrust loading distribution vector

The apparent mass matrix $[M]$ is derived from the kinetic energy of the induced flow and is given by:

$$M_{mn} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^R r \psi_m(r) \left[\int_{\text{fluid}} G(r, r', \theta, z) \psi_n(r') r' dr' d\theta' dz' \right] dr \quad (2.27)$$

where G is the Green's function for Laplace's equation in cylindrical coordinates. The influence coefficient matrix $[L]$ relates the induced velocity distribution to the pressure jump distribution required to sustain it. For a three-mode approximation (uniform, linear, and quadratic), Peters and He [8] derived explicit expressions for these matrices. The resulting system couples the three inflow states through their mutual aerodynamic interaction.

While the multi-mode formulation provides high fidelity representation of the spatial inflow distribution, it significantly increases model complexity. For small-scale propellers operating primarily in axial flow, and for system-level modeling

applications, the spatial variations in induced velocity are often of secondary importance compared to the temporal dynamics of the average inflow.

The uniform inflow approximation retains only the first mode ($n = 0$), assuming:

$$v(r, t) \approx \lambda_0(t) = \frac{V_i(t)}{\omega R} \quad (2.28)$$

where $V_i(t)$ is the spatially-averaged induced velocity. This reduces the system to a single scalar differential equation. Retaining only the (0,0) element of the mass and influence matrices, and normalizing, the governing equation becomes:

$$M_{00} \frac{d\lambda_0}{dt} + L_{00} \lambda_0 = C_T \quad (2.29)$$

where C_T is the total thrust coefficient. From Peters' theory, for a rotor in hover with uniform loading:

$$M_{00} = \frac{128}{75\pi} \approx 0.545, \quad L_{00} = \frac{16}{9\pi} \approx 0.566 \quad (2.30)$$

These numerical values arise from the analytical integration of the Green's function over the actuator disk domain and represent the apparent mass and aerodynamic gain for the uniform inflow mode. The apparent mass coefficient M_{00} quantifies the effective inertia of the flow field that must be accelerated when the induced velocity changes, while the influence coefficient L_{00} relates the steady-state induced velocity to the thrust loading.

Dividing equation through by L_{00} yields:

$$\frac{M_{00}}{L_{00}} \frac{d\lambda_0}{dt} + \lambda_0 = \frac{C_T}{L_{00}} \quad (2.31)$$

The ratio M_{00}/L_{00} represents a non-dimensional time scale. To obtain a dimensional time constant, this must be multiplied by a characteristic time scale of the rotor. From dimensional analysis, the natural time scale is the ratio of a characteristic length (rotor radius R) to a characteristic velocity. The appropriate velocity scale is the mean induced velocity $\bar{V}_i$. Defining the aerodynamic time constant as:

$$\tau = \frac{M_{00}}{L_{00}} \cdot \frac{R}{\bar{V}_i} \approx 0.963 \frac{R}{\bar{V}_i} \quad (2.32)$$

The term on the right-hand side, C_T/L_{00} , represents the steady-state inflow coefficient. From momentum theory applied to an actuator disk in hover, the relationship between thrust coefficient and induced velocity is:

$$C_T = 2\lambda_{ss}^2 \pi \quad (2.33)$$

where λ_{ss} is the steady-state inflow coefficient. Solving for λ_{ss} :

$$\lambda_{ss} = \sqrt{\frac{C_T}{2\pi}} \quad (2.34)$$

This represents the equilibrium induced velocity (in non-dimensional form) that would be obtained under steady-state conditions for a given thrust coefficient. Substituting this into the differential equation and rearranging:

$$\tau \frac{d\lambda_0}{dt} + \lambda_0 = \lambda_{ss} \quad (2.35)$$

which can be rewritten as:

$$\frac{d\lambda_0}{dt} = \frac{1}{\tau}(\lambda_{ss} - \lambda_0) \quad (2.36)$$

Converting from the non-dimensional inflow coefficient to the dimensional induced velocity using $\lambda_0 = V_i/(\omega R)$ and $\lambda_{ss} = V_{i,ss}/(\omega R)$:

$$\frac{d(\omega R \lambda_0)}{dt} = \frac{1}{\tau}(\omega R \lambda_{ss} - \omega R \lambda_0) \quad (2.37)$$

Assuming that the rotational speed ω varies slowly compared to the inflow dynamics (i.e., $d\omega/dt \ll \omega/\tau$), the product rule yields:

$$\omega R \frac{d\lambda_0}{dt} \approx \frac{dV_i}{dt} \quad (2.38)$$

leading to the final form of the uniform dynamic inflow equation:

$$\frac{dV_i}{dt} = \frac{1}{\tau}(V_{i,ss} - V_i(t)) \quad (2.39)$$

where:

- $V_i(t)$ is the instantaneous spatially-averaged induced velocity [m/s];

- $V_{i,ss}$ is the steady-state induced velocity corresponding to the current thrust level [m/s];
- τ is the aerodynamic time constant [s];

This first-order differential equation describes the temporal evolution of the induced velocity as an exponential approach toward the steady-state value, with the time constant τ governing the rate of response. The physical interpretation is clear: when the rotor loading changes (due to changes in rotational speed or blade pitch), the induced velocity does not adjust instantaneously but lags behind with a characteristic time scale determined by the time required for the wake to convect and establish a new equilibrium.

2.3 Model Formulation

The complete dynamic model of the powertrain is obtained by combining the electrical dynamics of the motor (Eq. 2.11), the mechanical dynamics of the shaft (Eq. 2.14), and the aerodynamic behavior of the propeller (Eqs. 2.15–2.39).

The coupling between the electrical and mechanical subsystems occurs through the electromagnetic torque τ_M and the angular velocity ω . Specifically, the motor torque $\tau_M \propto i$ depends on the current i , which evolves according to the motor circuit dynamics (Eq. 2.11). This torque drives the rotational acceleration in the mechanical equation (Eq. 2.14), where it is opposed by the aerodynamic torque τ_{aero} (Eq. 2.16) and by the viscous friction torque $\tau_{friction}(\omega)$.

For simplicity, the viscous friction torque is neglected in the final formulation, assuming that its contribution to the overall dynamics is small compared to the aerodynamic load.

The aerodynamic torque τ_{aero} depends nonlinearly on the angular velocity ω and on the induced velocity V_i , which evolves dynamically according to Eq. 2.39. The induced velocity replaces the inflow coefficient λ used in classical rotorcraft notation, with the relation $\lambda = V_i/(\omega R)$ embedded implicitly in the aerodynamic model. The steady-state value $V_{i,ss}$ required in the inflow dynamics is computed

using actuator disk theory. In this framework, the thrust is given by

$$T = 2\rho A V_{i,ss}^2,$$

and is equated to the experimental static thrust obtained from the propeller maps, which provide a relation of the form $T = f(\omega)$. Matching the two expressions yields

$$V_{i,ss} = \sqrt{\frac{f(\omega)}{2\rho A}}.$$

Thus, the aerodynamic model is fully parameterized in terms of the induced velocity V_i rather than the inflow ratio. By substituting the motor torque and aerodynamic torque expressions into the mechanical dynamics, Eqs. 2.11, 2.14, and 2.39 form a coupled nonlinear system that describes the evolution of the motor current $i(t)$, angular velocity $\omega(t)$, and induced velocity $V_i(t)$. The overall state vector can thus be defined as:

$$x = [i \quad \omega \quad V_i]^T$$

with the control input given by the PWM duty cycle δ . The main outputs of interest are the thrust force $T(t)$ (Eq. 2.15) and the aerodynamic torque $\tau_{aero}(t)$ (Eq. 2.16), which depend on the instantaneous rotational speed and the induced inflow conditions.

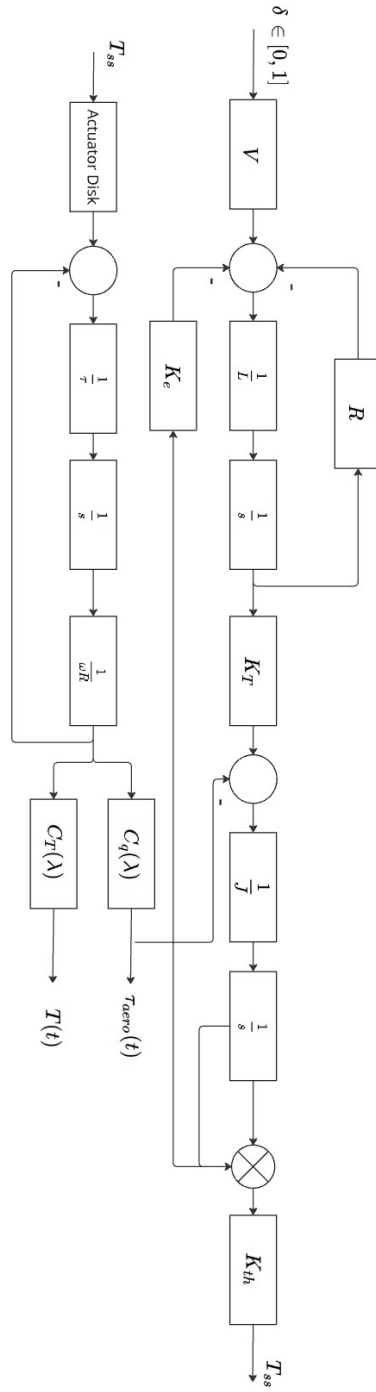


Figure 2.5: Block diagram of the model

Chapter 3

System identification

Once the dynamic model of the system under consideration has been defined in Chapter 2, it is necessary to perform a *system identification* procedure. In this context, the procedure consists of determining the parameters that constitute the dynamic model. In the present work, parameter identification is carried out based on experimental data acquired during the course of this study. Specifically, the parameters to be identified during the system identification procedure are:

- The BLDC motor/ESC internal resistance $R[\Omega]$;
- The BLDC motor/ESC internal inductance $L[H]$;
- The back EMF constant $K_e[\frac{V}{\frac{rad}{s}}]$;
- The electro-magnetic torque constant $K_T[\frac{Nm}{A}]$;
- The motor+propeller inertia $J[Kg \cdot m^2]$;
- The steady state thrust constant $K_{TH}[\frac{N}{\frac{rad^2}{s^2}}]$;
- The static maps of the non-dimensional coefficients (C_T and C_Q) as a function of λ ;

3.1 Methodology

As previously stated in the introduction of this chapter, the identification of the system parameters was conducted using a set of experimental data acquired during this study. The primary methodology employed for parameter estimation is the *Ordinary Least Squares (OLS) method*.

The *Least Squares* method is a fundamental statistical procedure used to determine the best-fit curve or line for a given set of data points by minimizing the sum of the squares of the residuals between each data point and the fitted curve.

The primary goal is to find a set of parameters θ that makes the model predictions as close as possible to the experimental observations. Consider a system that can be represented by a linear model:

$$y = \Phi\theta + \epsilon \quad (3.1)$$

Where:

- y is the vector of outputs;
- Φ (or X) is the regressor matrix;
- θ is the vector of unknown parameters to be identified;
- ϵ represents the measurement noise or modeling errors;

We define a cost function $J(\theta)$, which represents the sum of the squared residuals:

$$J(\theta) = \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \|y - \Phi\theta\|^2 \quad (3.2)$$

By squaring the residuals, we ensure that all errors are treated as positive values (preventing positive and negative errors from canceling each other out). The minimum of the cost function is found by taking the derivative of $J(\theta)$ with respect to θ and setting it to zero:

$$\frac{\partial J(\theta)}{\partial \theta} = -2\Phi^T(y - \Phi\theta) = 0 \quad (3.3)$$

Provided that the matrix $(\Phi^T \Phi)$ is non-singular (invertible), the unique solution for the estimated parameters is:

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T y$$

Therefore, parameter identification via the least squares method can be reduced to solving a linear system.

3.2 Experimental setup

The experimental activity for the parameter identification procedure was carried out across two different facilities: the Von Karman Institute for Fluid Dynamics and the laboratorio di aeronautica "M. Panetti" del Politecnico di Torino. While the core propulsion unit remains consistent between the two sites, the instrumentation and synchronization strategies were adapted to the specific requirements of each laboratory.

3.2.1 Von Karman Institute Setup

The experimental setup at VKI was specifically designed to characterize propellers manufactured via additive manufacturing techniques. The propulsion system consists of a T-Motor AT3520 KV550 brushless DC motor driven by a T-Motor 75A Electronic Speed Controller (ESC).

To measure aerodynamic performance, thrust and torque are captured using a Futek QMA150 load cell, with signals conditioned through a Futek IAA100 amplifier. The DC input current to the ESC is monitored using an ACS172 Hall-effect current sensor, maintaining consistency with the electrical sensing technology used at Politecnico di Torino.

A dedicated synchronization strategy was implemented to align rotational speed with the force and current data. An Arduino GIGA R1 WiFi is employed to interface with a SameSky AMT11 incremental encoder. The microcontroller processes the encoder pulses and converts them into an analog signal ranging from 0 to 3.3 V, which is subsequently read by an MCC 128 DAQ HAT connected to a Raspberry Pi. This configuration ensures that the RPM measurements are

perfectly synchronized with the other sampled data channels.

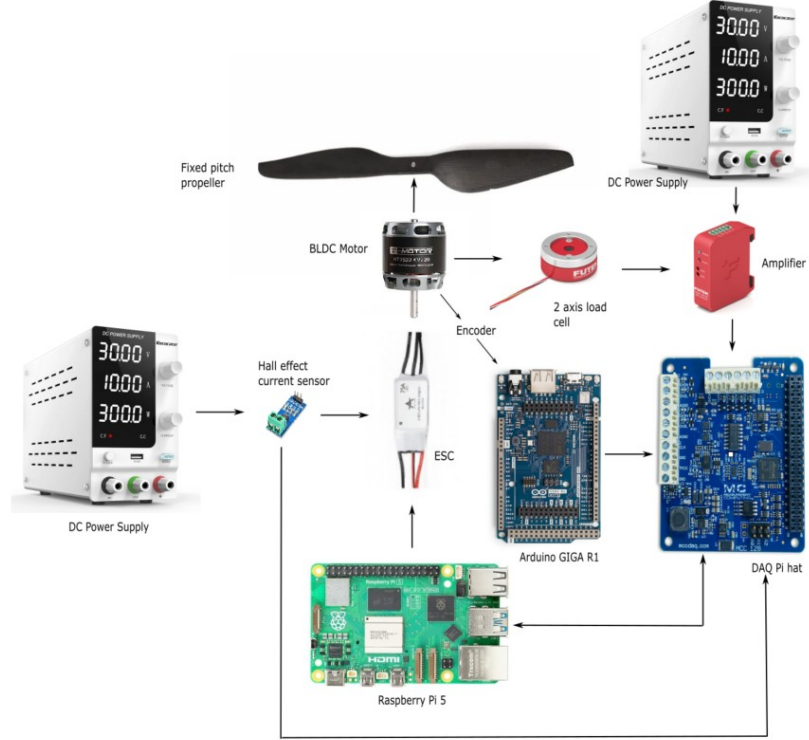


Figure 3.1: VKI Experimental setup diagram

3.2.2 Politecnico di Torino Setup

The measurement chain employed for the parameter identification procedure at Politecnico consists of a fixed-pitch propeller driven by a T-motor AT3520 KV 550 (longshaft) BLDC motor. Phase commutation for the brushless motor is managed by a T-motor AT75A 2-6S Electronic Speed Controller (ESC).

To measure the aerodynamic forces and moments generated by the propeller, a Schunk ATI-gamma six-component load cell was utilized. This sensor is characterized by a high resolution and a multi-axis measurement range. The resulting analog output signals are conditioned via a Schunk amplifier to ensure an optimal signal-to-noise ratio before acquisition.

Motor control is handled by an Arduino Uno providing a PWM signal. The micro-controller can operate the system in open-loop, using throttle percentage as input,

or in closed-loop via a PID controller to track a specific RPM setpoint. Rotational speed is monitored using a Hobbywing Hall-effect RPM sensor, powered by the Arduino's 5V pin. To measure the DC input current to the ESC, an ACS172 Hall-effect current sensor was integrated, powered at 5V by an EA-PS 2042-10 B DC power supply. The main power for the ESC is provided by a Voltcraft VSP 2410 DC power supply, configured to deliver 23V with a current limit of 20A. Data acquisition is performed using a National Instruments (NI) DAQ system comprising three modules, each capable of reading four channels. In order to capture the fast electrical transients and the high-frequency content required for accurate dynamic identification, the acquisition was performed at a sampling frequency (f_s) of $80kHz$. Finally, MATLAB and Python were utilized for data acquisition and post-processing.

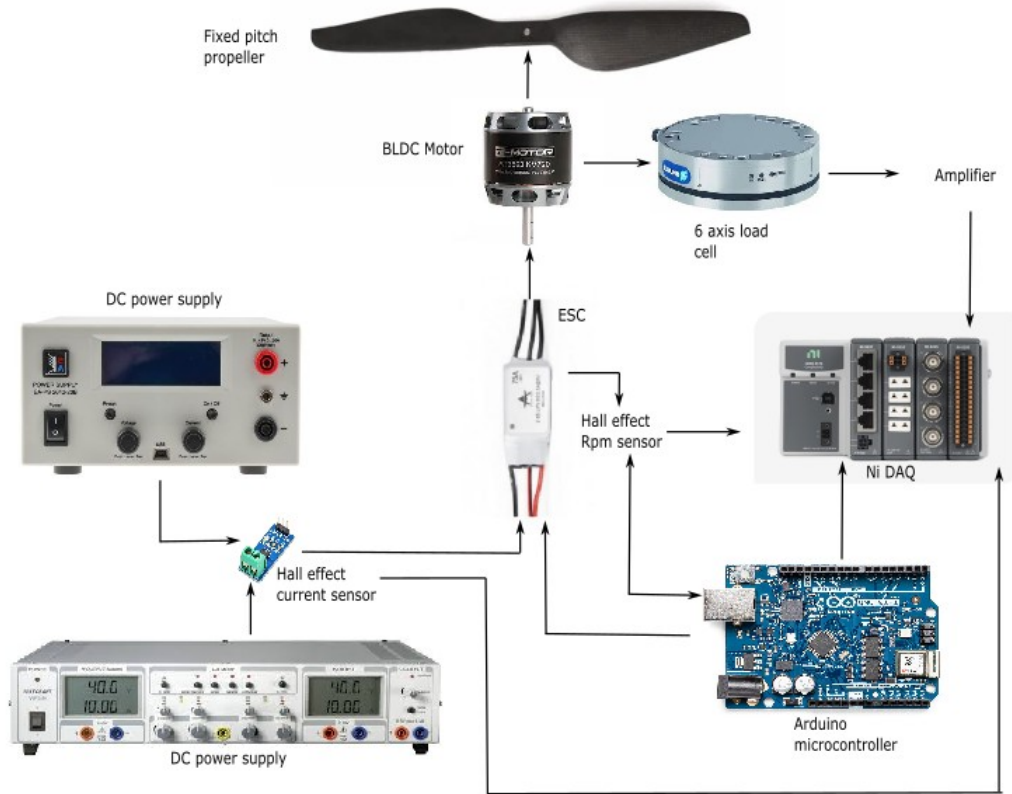


Figure 3.2: PoliTO Experimental setup diagram

3.3 Data processing

In this section, the general data analysis methodology employed in this work is presented. As detailed in Section 3.2, the experimental setup enables the acquisition of several key variables: the three force components, the three torques, the rotational velocity, the ESC input DC current, and the input PWM signal to the ESC.

The sensors integrated into the setup generate analog signals that are sampled by the DAQ system at a specific sampling frequency. Subsequently, these raw signals must be converted into International System of Units (SI).

To derive the values of the forces and moments measured by the load cell, it is necessary to convert the voltage readings into the three force and moment components. This is achieved by multiplying the voltage values obtained from the strain gauges within the load cell by the following matrix:

$$M_{calib} = \begin{bmatrix} -0.05943 & -0.01159 & 0.09636 & -7.33253 & -0.09939 & 7.17080 \\ -0.22088 & 8.36332 & -0.01148 & -4.24439 & 0.16819 & -4.13478 \\ 13.22815 & -0.15666 & 13.22535 & 0.00746 & 13.48588 & 0.00317 \\ 0.00054 & 0.09962 & -0.38091 & -0.05053 & 0.38952 & -0.04981 \\ 0.44360 & -0.00473 & -0.22071 & 0.08764 & -0.22197 & -0.08514 \\ 0.00226 & -0.23274 & 0.00441 & -0.23359 & 0.00592 & -0.22899 \end{bmatrix} \quad (3.4)$$

As observed from Eq. 3.4, the calibration matrix is non-symmetric. This is due to the fact that the strain gauge bridges within the load cell are not perfectly aligned with the corresponding force and moment components. Consequently, it is necessary to acquire all signals generated by the load cell to account for the cross-talk between the different measurement channels.

As explained in Chapter 1, the PWM command signal is a square wave with a constant frequency (50 Hz) and amplitude (5 V), where the duration of the high state (pulse width) is modulated. For the ESC used in this study, a pulse width of 1000 μ s corresponds to 0% throttle, while a pulse width of 2000 μ s corresponds to 100% throttle (represented as a value of 1). Consequently, a MATLAB routine was implemented to detect the rising and falling edges of the signal, allowing for the calculation of the pulse duration and its subsequent conversion into a throttle value.

The rotational speed is determined by processing the raw signal from the Hall-effect sensor through a dedicated MATLAB routine. The algorithm first converts the analog input into a digital signal by applying a threshold of 2.5 V, where values above this limit are normalized to a logic "high" state. To calculate the RPM, the routine identifies the rising edges of the resulting pulse train within a specific time interval. By measuring the time elapsed between consecutive pulses, the average period (T) of the signal is determined. The frequency ($f = 1/T$) is then scaled to account for the motor configuration; specifically, for the motor used in this work, the frequency is multiplied by a factor of 120/14 (where 14 represents the number of poles) to convert the pulse rate into revolutions per minute (RPM).

Finally, regarding the ESC input DC current, the calibration provided by the sensor manufacturer was utilized to convert the acquired voltage signal into Amperes. The conversion follows a specific sensitivity scale, where a voltage of 100 mV corresponds to a current of 1 A (100 mV/A). This linear relationship was implemented within the post-processing routine to derive the actual current consumption from the raw analog data.

Naturally, the acquired signals are affected by noise. To perform a sensitive procedure such as parameter identification, it is essential to utilize data that is as clean as possible. In the present work, a Butterworth low-pass filter was employed for this purpose.

The Butterworth filter is a frequency-domain-based filter characterized by a maximally flat frequency response in the passband. Its performance is defined by two primary parameters: the filter order, which determines the roll-off rate, and the cut-off frequency, which defines the boundary beyond which the signal power is attenuated. In the present work, a third-order Butterworth filter was implemented. The cut-off frequency was not chosen arbitrarily; instead, it was determined through a spectral analysis of the raw data. By analyzing the Fast Fourier Transform (FFT) of the acquired signals, it was possible to identify the frequency content of the relevant physical dynamics and distinguish it from high-frequency electrical noise and structural vibrations. This approach ensured that the cut-off frequency was set to effectively attenuate noise while preserving the integrity of the system's dynamic response.

3.4 Static identification

This test was conducted under steady-state conditions, meaning that the system reached a constant angular velocity before the data acquisition began. To ensure a stable and precise velocity throughout the duration of the test, a PID controller was employed. The static test aims to determine the torque constant (K_t) and the static maps of the thrust coefficient (C_T) and torque coefficient (C_Q) as a function of the inflow parameter (λ). Under static conditions (constant velocity), Eq. 2.14 can be rewritten as:

$$\tau_M = \tau_{aero} \quad (3.5)$$

It can be observed that the electromagnetic torque generated by the motor and the aerodynamic torque are the same. Consequently, by measuring the torque and current under static conditions, it is possible to calculate the torque constant K_t .

$$K_T = \frac{\tau_M}{i} \quad (3.6)$$

Specifically, in this work, torque and current were measured across a range of angular velocities. A linear regression was then performed on the experimental data points to determine the torque constant (K_t).

N TEST	RPM
1	1000
2	1500
3	2000
4	2500
5	3000
6	3500
7	4000
8	4500
9	5000
10	5500
11	6000

Table 3.1: Experimental parameters for closed-loop static hover tests

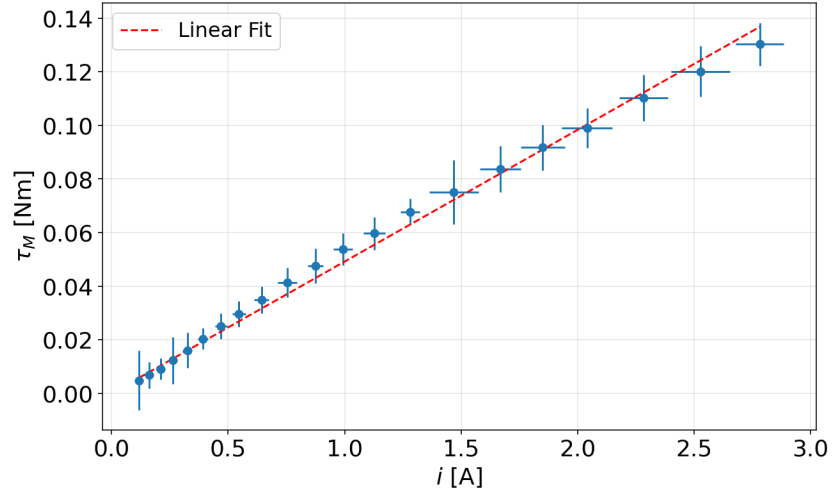


Figure 3.3: Torque as a function of the supplied current

Equation 3.6 holds under the assumption of low DC current values. At higher current levels, magnetic saturation effects occur, which tend to cause a non linearity in the motor response.

This phenomenon is characteristic of BLDC motors and is typically observable in the manufacturer’s technical specifications [10]. As illustrated in Fig. 3.4, the relationship between the electromagnetic torque and the input current exhibits a non-linear trend, primarily due to the aforementioned magnetic saturation effects.

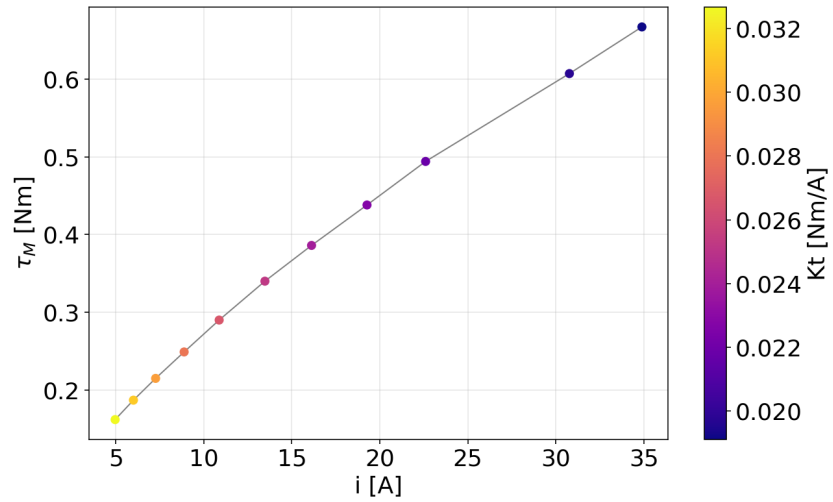


Figure 3.4: Electric motor torque data

As suggested by the data in Fig. 3.4, the trend can be better characterized by a second-order polynomial regression. The results presented in Fig. 3.5 highlight that a quadratic model significantly improves the prediction of the electromagnetic torque as a function of current, effectively capturing the slight curvature induced by magnetic saturation.

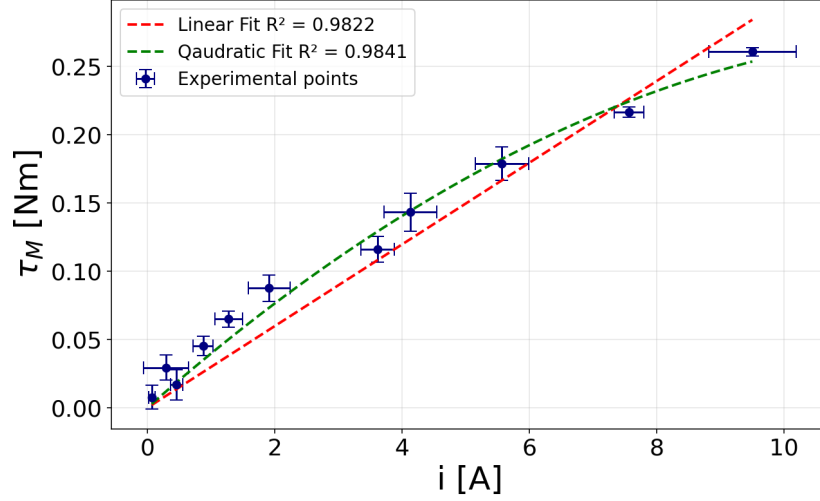


Figure 3.5: Torque as a function of the supplied current with indication of the linear (in red) and polynomial of degree 2 (in green) fits for the constant (K_T) identification

As suggested by the experimental data, the relationship between the electromagnetic torque and the current is accurately described by the following second-order polynomial regression:

$$\tau_M = 0.0412 \cdot i - 0.0015 \cdot i^2 \quad (3.7)$$

In addition to the electrical parameters, the static identification process allows for the determination of the thrust constant (K_{Th}) and the torque constant (K_{torque}). In the context of a propeller operating in static conditions both the thrust (T) and the aerodynamic torque (τ_{aero}) exhibit a quadratic dependence on the angular velocity (ω).

The relationship can be mathematically described as:

$$T = K_{Th} \cdot \omega^2 \quad (3.8)$$

$$Q = K_{torque} \cdot \omega^2 \quad (3.9)$$

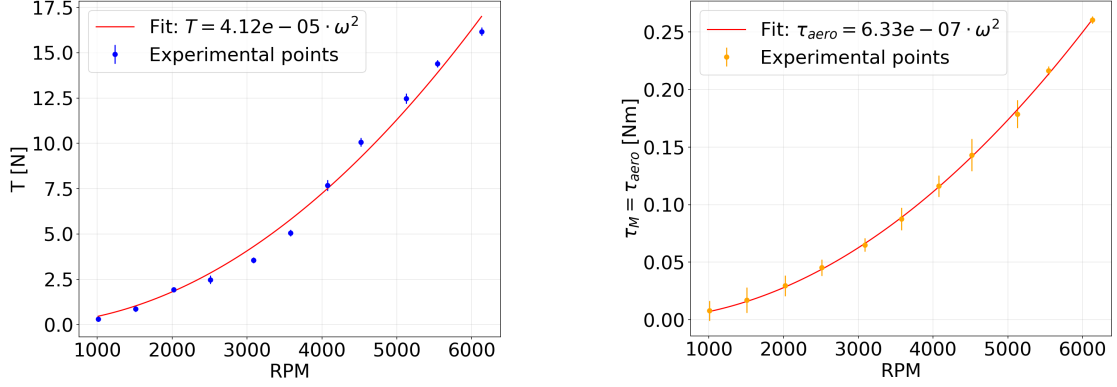


Figure 3.6: Thrust and torque characteristics under static conditions as a function of ω

Once the static thrust and torque maps have been obtained, it is possible to derive the static maps for the non-dimensional coefficients (C_T and C_Q) as a function of the inflow parameter (λ). These coefficients allow for a normalized representation of the propeller performance, making the data independent of the specific air density or rotational speed. The inflow parameter (λ) can be determined using the Actuator Disk Theory (Subsection 1.3.1). This theory relates the flow velocity through the propeller disk to the thrust produced, accounting for both the free-stream velocity and the induced velocity. In the specific case of hovering (where the free-stream velocity V_∞ is zero), the inflow parameter is determined solely by the induced velocity (v_i) required to generate thrust. According to the Actuator Disk Theory, the formula for the inflow parameter in hover is:

$$\lambda = \frac{V_i}{\omega R} = \frac{1}{\omega R} \sqrt{\frac{T}{2\rho A}} \quad (3.10)$$

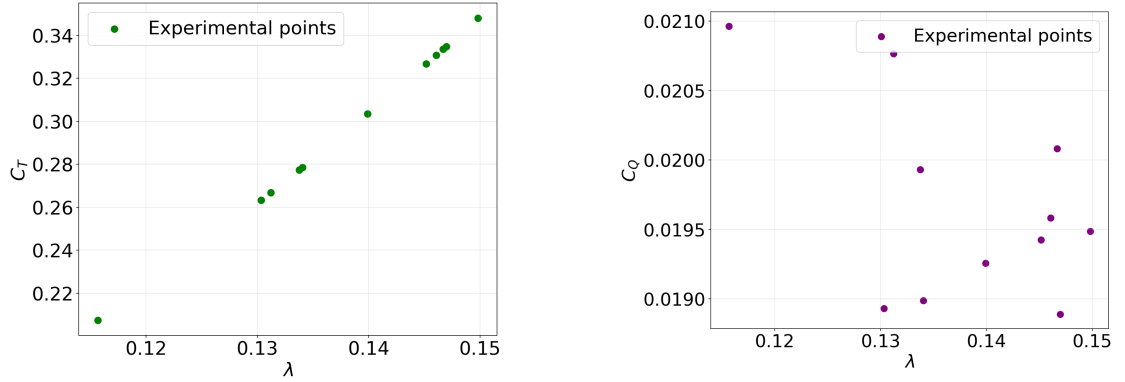


Figure 3.7: Variation of C_T and C_Q as a function of the inflow parameter λ

The data presented in Fig. 3.7 were collected under hover conditions. Since both coordinates represent non-dimensional parameters, it is theoretically expected that the data points should coincide regardless of the rotational speed. However, a discrepancy is observed; this is attributed to the structural deformation of the propeller under the influence of aerodynamic loads

3.5 Dynamic identification

The dynamic identification process allows for the characterization of the parameters governing the system's transient behavior. To determine these parameters, the Ordinary Least Squares (OLS) method was applied (Section 3.1). The experimental dataset required for the identification was collected by exciting the system with several open-loop throttle step inputs. The step input is particularly well-suited for this application, as it effectively excites both the mechanical and electrical dynamics of the system. This comprehensive excitation allows for an accurate determination of the key parameters that characterize the system's overall behavior.

N TEST	δ_{IN} [%]	δ_{FIN} [%]	t_{step} [s]
1	20	30	5
2	20	40	5
3	30	60	5
4	40	70	5
5	20	60	5
6	30	20	5
7	40	20	5
8	60	30	5
9	70	50	5
10	60	40	5
11	70	60	5

Table 3.2: Experimental parameters for open-loop dynamic identification tests (Step responses)

Regarding the electrical dynamics, the OLS method was applied to Eq. 2.11. The identification problem can be formulated in matrix form, where each row represents an individual sample. To increase the robustness of the estimation, the datasets from all experimental runs were concatenated into a single global system of equations.

$$\begin{bmatrix} \delta_1 \cdot V_{batt} \\ \delta_2 \cdot V_{batt} \\ \vdots \\ \delta_N \cdot V_{batt} \end{bmatrix} = \begin{bmatrix} \frac{di}{dt}_1 & i_1 & \omega_1 \\ \frac{di}{dt}_2 & i_2 & \omega_2 \\ \vdots & \vdots & \vdots \\ \frac{di}{dt}_N & i_N & \omega_N \end{bmatrix} \begin{bmatrix} L \\ R \\ K_e \end{bmatrix} \quad (3.11)$$

Similarly to the electrical characterization, the mechanical parameters are identified using Eq 2.14. A fundamental assumption in this phase is that the inflow dynamics is negligible. This implies that the aerodynamic flow field adjusts instantaneously to changes in rotational speed. Consequently, the aerodynamic torque can be estimated using the previously identified static relationship:

$$\tau_{aero} = K_{torque} \cdot \omega^2 \quad (3.12)$$

By substituting the aerodynamic torque expression into the motion equation, the

identification problem for N samples can be structured in matrix form:

$$\begin{bmatrix} \frac{d\omega}{dt} 1 \\ \frac{d\omega}{dt} 2 \\ \vdots \\ \frac{d\omega}{dt} N \end{bmatrix} = \begin{bmatrix} (0.0412 \cdot i_1 - 0.0015 \cdot i_1^2) & \omega_1^2 \\ (0.0412 \cdot i_2 - 0.0015 \cdot i_2^2) & \omega_2^2 \\ \vdots & \vdots \\ (0.0412 \cdot i_N - 0.0015 \cdot i_N^2) & \omega_N^2 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \quad (3.13)$$

In the matrix formulation presented, the identified parameters K_1 and K_2 are not individual physical constants but rather grouped parameters that result from dividing the entire motion equation by the total moment of inertia J .

The complete set of parameters obtained through the identification process, covering the electrical, mechanical, and aerodynamic properties of the propulsion system, is reported in Table 3.3.

Category	Parameter	Symbol	Value [Unit]
Electrical	Resistance	R	0.45 [Ω]
	Inductance	L	14 [mH]
	Back-EMF constant	K_e	0.014 [V/(rad/s)]
Mechanical	Torque constant	K_t	0.0492 [Nm/A]
	Moment of inertia	J	1.5×10^{-4} [kg·m ²]
Aerodynamic	Thrust constant	K_{Th}	4.12×10^{-4} [N/(rad/s) ²]
	Torque constant	K_{torque}	6.33×10^{-7} [Nm/(rad/s) ²]

Table 3.3: Summary of identified powertrain system parameters

Chapter 4

Model Validation

The objective of this chapter is to perform a comprehensive validation of the proposed model using experimental data. The validation process is divided into two main stages to evaluate the model's accuracy under different operating conditions. Initially, an open-loop validation is carried out by subjecting the system to a step command in throttle. This test allows for the direct evaluation of the powertrain system's intrinsic response, specifically focusing on the transition between different equilibrium points without the compensation of a controller. Subsequently, a closed-loop validation is performed. In this stage, the system is excited using various sequences of RPM step commands. This analysis is fundamental to verify the model's ability to replicate the dynamic behavior of the motor and propeller when regulated by a PID controller, particularly during transient phases and under aggressive setpoint variations.

4.1 Open Loop

To validate the accuracy of the identified parameters, a comparison between the experimental measurements and the numerical simulation results was performed under open-loop step conditions. Figures 4.1, 4.2 and 4.3 illustrate the system response to a throttle step input applied from $\delta = 0.2$ to $\delta = 0.3$ at $t = 5$ s.

The current profile shows an excellent match in both the transient peak and the steady-state values. The simulation correctly captures the high-frequency spike

occurring immediately after the step, which is primarily governed by the electrical dynamics (L and R).

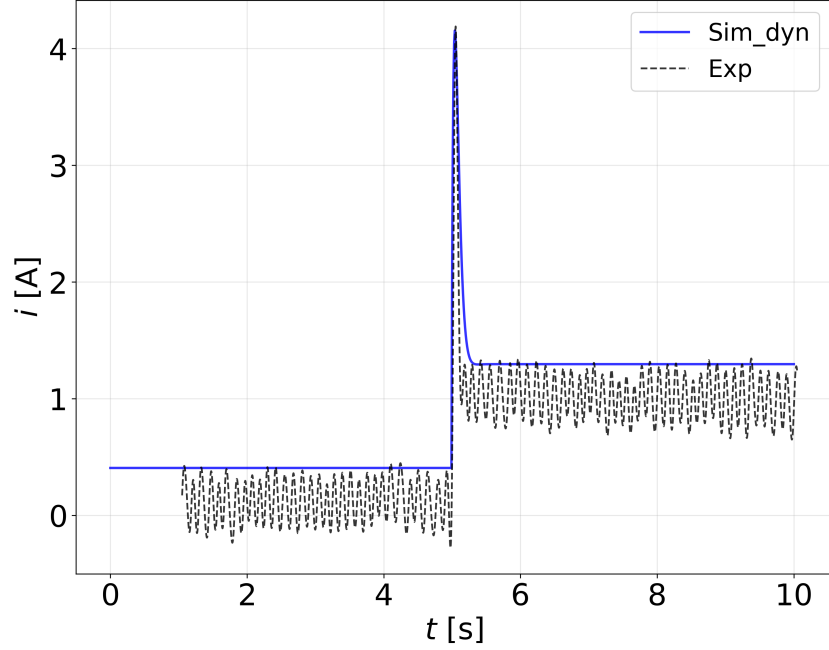


Figure 4.1: Comparison between the simulated behaviour of the current according to the proposed model (in blue) and the experimental data (in black).

The electromagnetic torque follows the expected trend, showing a sharp transient during acceleration and settling to a new steady-state value that balances the aerodynamic resistance. The slight oscillations observed in the experimental torque trace can be attributed to structural vibrations. The transition between the two equilibrium points is accurately replicated by the model. The rise time in the simulation perfectly aligns with the experimental data, confirming that the identified moment of inertia J and the assumption of negligible inflow dynamics are appropriate for this operating range.

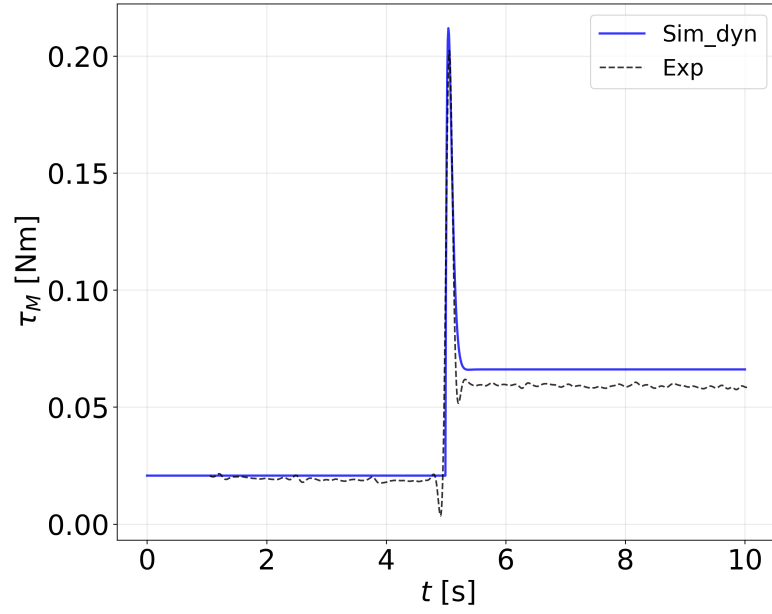


Figure 4.2: Comparison between electro-magnetic torque the simulated behaviour of the current according to the proposed model (in blue) and the experimental data (in black).

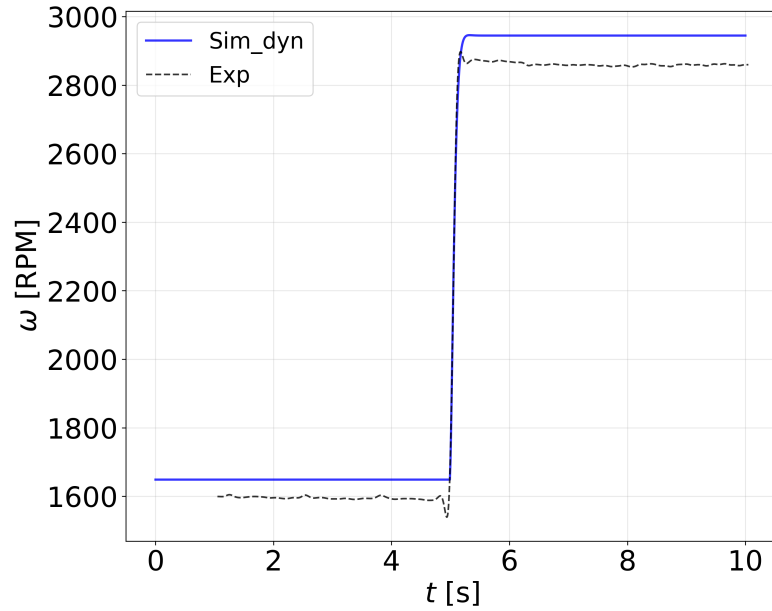


Figure 4.3: Comparison between the simulated behaviour of the angular velocity according to the proposed model (in blue) and the experimental data (in black).

4.2 Closed loop

The system can also be validated in closed-loop configuration by implementing a PID controller to regulate the motor's angular velocity. The closed-loop validation was performed by exciting the system with a sequence of consecutive step inputs. Specifically, two distinct test cases were conducted: the first utilized standard command profiles, while the second employed more aggressive setpoint variations to evaluate the controller's robustness and dynamic response.

N TEST	Time series [s]	RPM series	f_s [Hz]	t_{acq} [s]
1	0, 5, 8, 10, 12, 15	2000, 4000, 3500, 5000, 4500, 4000	80000	20
2	0, 2, 5, 7, 10, 15	4000, 2500, 3500, 2000, 3000, 5000	80000	20
3	0,1,3,4,7,8	2000,5000,6000,3000,1500,5500	80000	10

Table 4.1: Experimental matrix for closed-loop validation

Regarding the first test, simulations were conducted by modeling the electromagnetic torque both as a linear function of the current and by employing a non-linear formulation. It is observed that the linear model accurately predicts the current evolution only in the low-current regime. As the current magnitude increases, the non-linear model becomes essential to correctly capture the experimental behavior and the associated saturation effects.

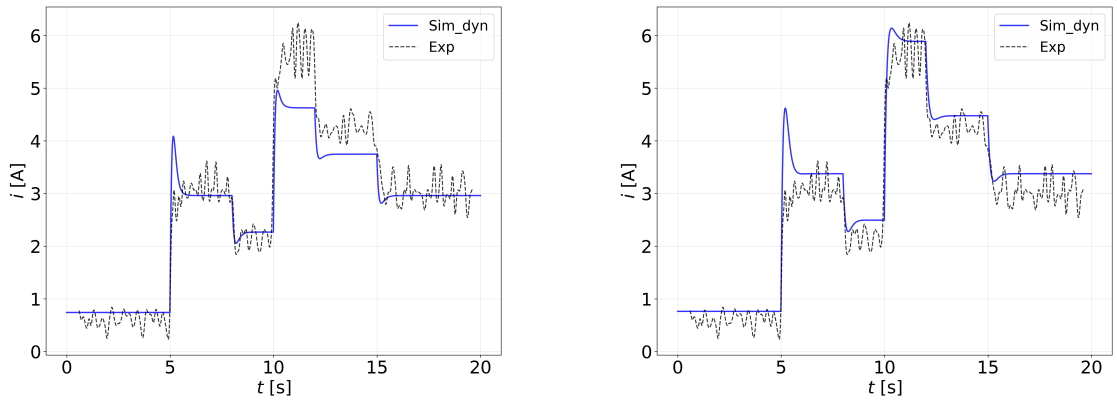


Figure 4.4: Comparison between the simulated behaviour of the current according to the proposed model (in blue) and the experimental data (in black).

By analyzing the electromagnetic torque trends, it is observed that both the linear and non-linear models accurately predict the experimental data. This agreement is a direct consequence of the closed-loop control architecture; at steady-state, the system must generate an electromagnetic torque that exactly balances the aerodynamic torque to maintain the setpoint. Consequently, the controller ensures that the required torque is produced regardless of the model type. However, a discrepancy emerges in the electrical domain: under high-load conditions, the system requires a significantly higher current than predicted by the linear model to produce the same torque, highlighting the impact of non-linearities such as magnetic saturation.

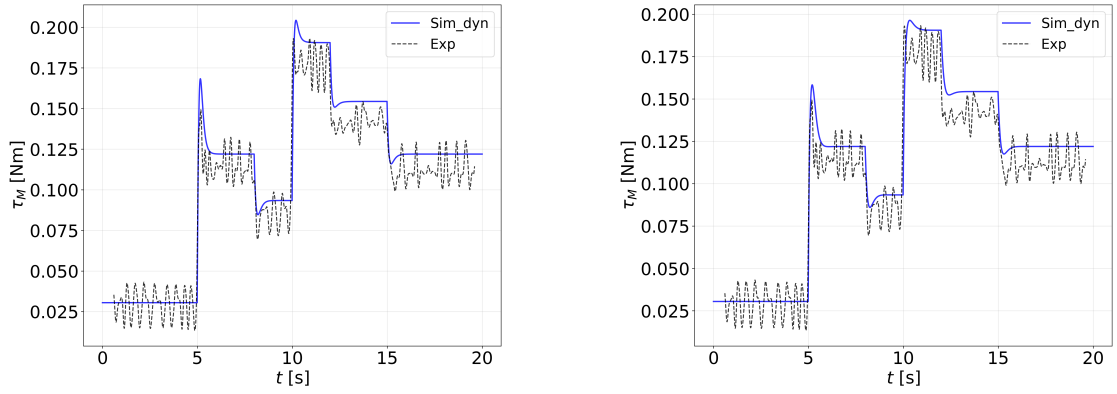


Figure 4.5: Comparison between the simulated behaviour of the electromagnetic torque according to the proposed model (in blue) and the experimental data (in black).

As illustrated in Fig. 4.6, the model accurately predicts the behavior of the angular velocity. Since the system operates in a closed-loop configuration, the steady-state values naturally coincide. However, a high degree of consistency between the experimental data and the proposed mathematical model is also observed during transient phases. This strong correlation indicates a successful identification of the parameters governing the system's dynamic response.

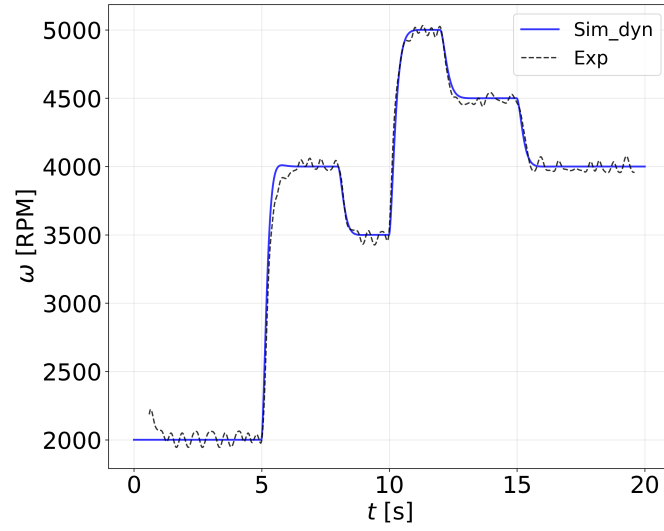


Figure 4.6: Comparison between the simulated behaviour of the angular velocity according to the proposed model (in blue) and the experimental data (in black).

Regarding the thrust generated by the propeller, simulations were conducted considering both proposed aerodynamic models: the instantaneous induced velocity model and the dynamic inflow evolution model. As shown in Figure 4.7, both models accurately predict the thrust produced by the propeller.

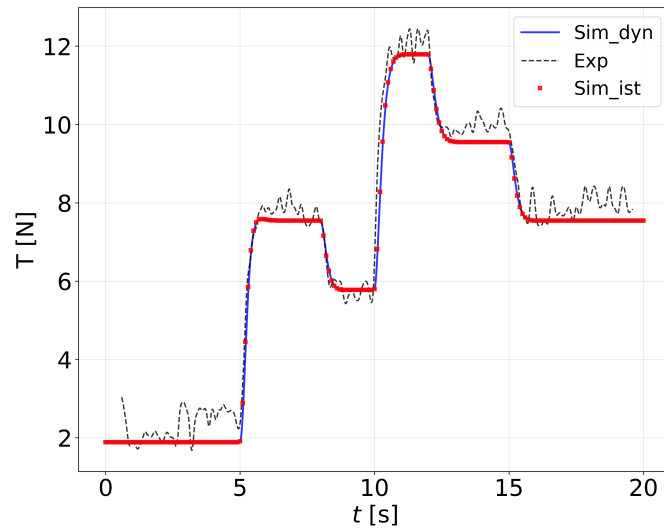


Figure 4.7: Comparison between the simulated behaviour of the thrust according to the proposed model (in blue considering dynamic inflow, in red considering instantaneous inflow) and the experimental data (in black).

As illustrated in Figure 4.7, the model effectively captures the propeller deformation effects. This is a direct consequence of the fact that the static maps (T vs. ω , τ_{aero} vs. ω , C_T vs. λ , and C_Q vs. λ) were derived experimentally. This highlights the primary advantage of employing a model based on experimental data; since these static maps are intrinsically dependent on both the geometry and the material properties of the propeller, they implicitly account for structural compliance. Indeed, a propeller with the same geometry but manufactured from a different material would exhibit a distinct deformation pattern, leading to different aerodynamic characteristics.

As demonstrated in Figure 4.8, the proposed model accurately captures the system dynamics across this second test suite. The high degree of correlation between the simulated and experimental traces—specifically in the timing of the transients and the tracking of the new equilibrium points—confirms the robustness of the identified parameters. This cross-validation proves that the model is not merely tuned to a specific dataset but provides a consistent physical representation of the propulsion system.

A similar analysis was performed for the second test case, characterized by more aggressive setpoint variations as detailed in Table 4.1. The high degree of correlation between the simulated and experimental traces—specifically in the timing of the sharp transients and the precise tracking of the new equilibrium points—confirms the robustness of the identified parameters. This cross-validation proves that the model is not merely tuned to a specific sequence but provides a consistent physical representation of the propulsion system even when subjected to higher dynamic stress.

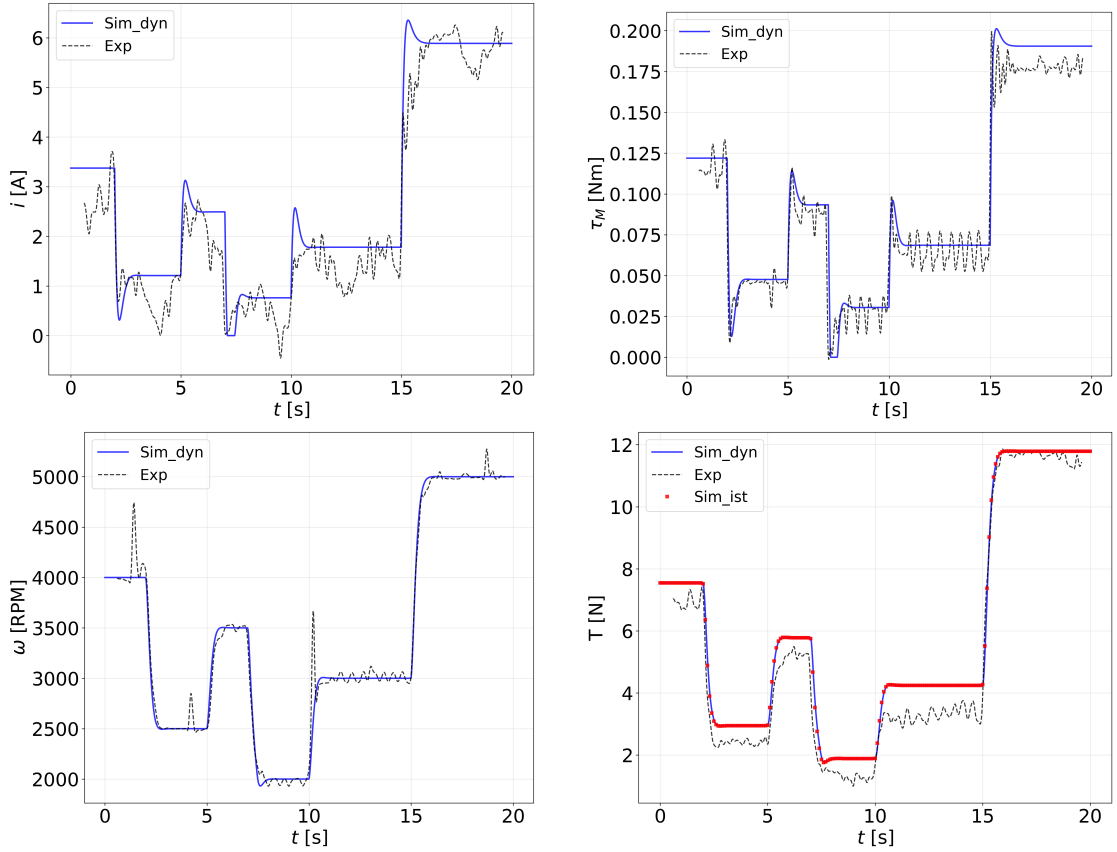


Figure 4.8: Comparison between the simulated behaviour of system according to the proposed model (in blue) and the experimental data (in black).

As illustrated in Figure 4.9, the model maintains high predictive accuracy even when subjected to the highly aggressive RPM changes of Test 3. Despite the rapid transitions between extreme setpoints (e.g., from 1500 to 5500 RPM in a single step), the simulated traces effectively track the experimental data across all physical domains. This performance confirms that the identified parameters—specifically the motor inertia and the non-linear torque-current relationship—accurately represent the system’s physics even during high-bandwidth transients where aerodynamic and electrical non-linearities are most prominent.

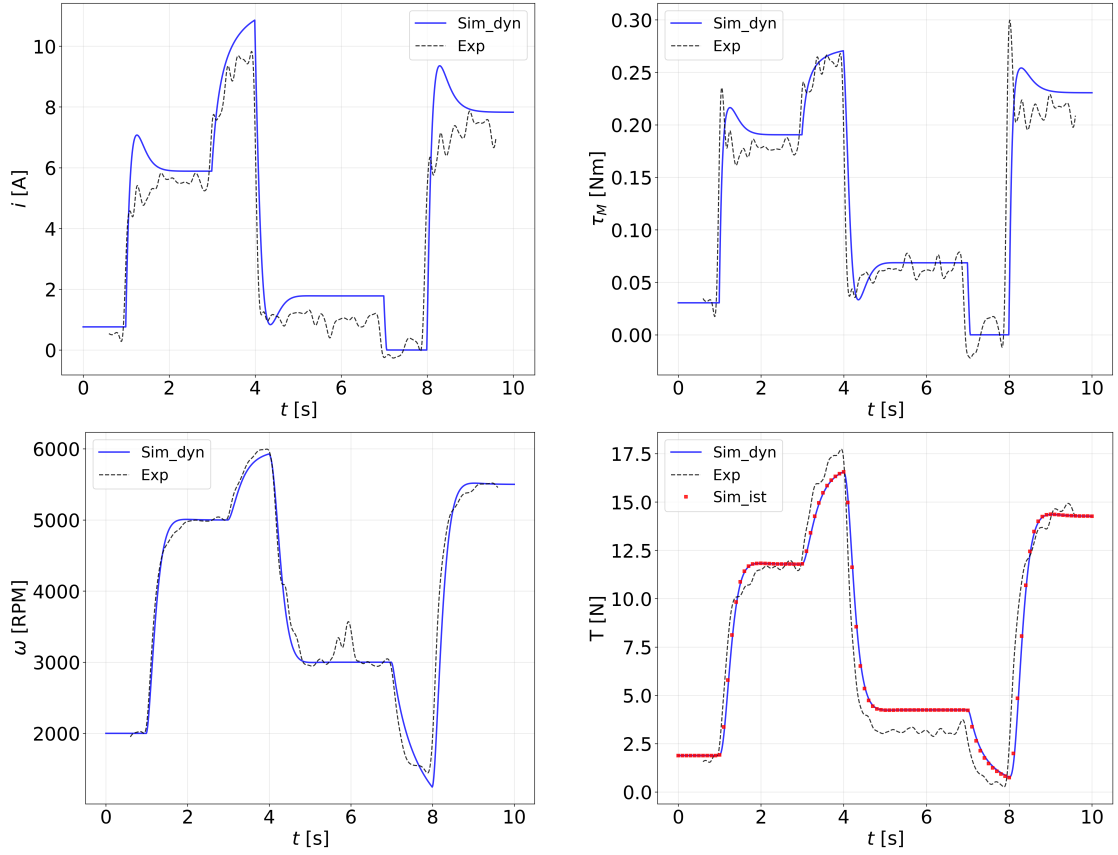


Figure 4.9: Comparison between the simulated behaviour of system according to the proposed model (in blue) and the experimental data (in black).

As observed from the three tests presented, the instantaneous aerodynamic model accurately predicts the dynamic evolution of the thrust generated by the propeller. Consequently, the time constant is not invariant during the simulation but evolves according to the operational state of the propeller. Figures 4.10, 4.11, and 4.12 illustrate the temporal evolution of the induced velocity v_i for the three conducted tests, highlighting how the wake dynamics adapt to the different RPM setpoints.

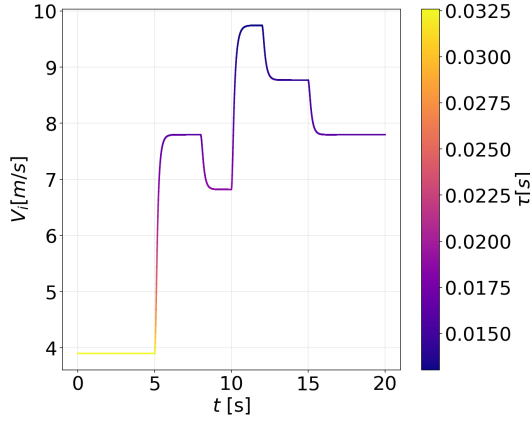


Figure 4.10: Induced velocity evolution - Test 1

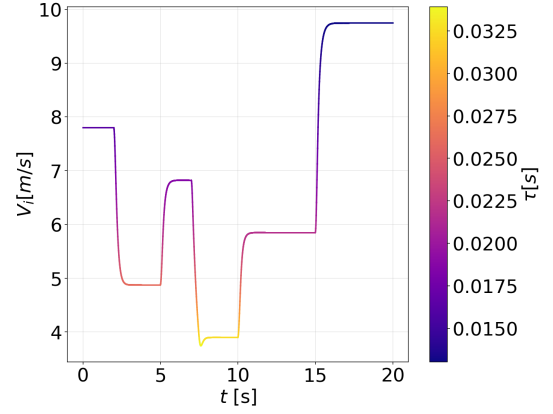


Figure 4.11: Induced velocity evolution - Test 2

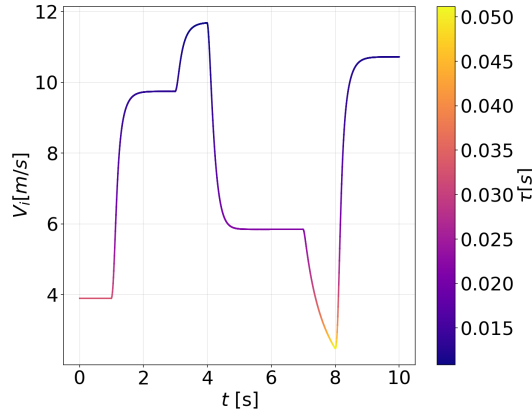


Figure 4.12: Induced velocity evolution - Test 3

Figure 4.13: Temporal evolution of the induced velocity V_i across the three validation test cases.

As detailed in Section 2.2, the steady-state induced velocity in this model is derived from the Actuator Disk Theory. According to this theory, under hover conditions, the velocity induced by the propeller depends on the disk loading (T/A). In these small-scale applications, the disk loading tends to be significant under typical UAV operating conditions, a fact primarily driven by the reduced disk area. As illustrated in Figures 4.10, 4.11, and 4.12, even in conditions where the thrust values are relatively low, the aerodynamic time constant τ remains within

the same order of magnitude as the electrical dynamics constant. This observation highlights that the bottleneck of the overall system response is represented by the mechanical dynamics of the propeller, which are governed by the moment of inertia J .

Chapter 5

Conclusions

The objective of this thesis was the development and experimental validation of a high-fidelity dynamic model for a BLDC propulsion system dedicated to small-scale UAV applications. Through a rigorous identification process and subsequent closed-loop validation, several key findings have been established.

First, the characterization of the electromagnetic torque revealed that a standard linear model is insufficient for capturing the system's behavior across its entire operating range. The introduction of a second-order polynomial regression proved essential to account for magnetic saturation effects occurring at high current magnitudes. This refinement significantly improved the model's accuracy, particularly in high-load conditions where the linear torque constant K_t would have led to a substantial underestimation of the required current.

Secondly, the aerodynamic characterization highlighted the effectiveness of using experimentally derived static maps. By incorporating $T(\omega)$ and $\tau_{aero}(\omega)$ curves obtained directly from test bench data, the model implicitly accounts for propeller deformation and aeroelastic effects that are often neglected in purely theoretical models.

Furthermore, the comparison between instantaneous and dynamic inflow models demonstrated that, for the scale of the UAV considered, the mechanical inertia J remains the primary bottleneck of the system's dynamic response. Since the aerodynamic time constant $\tau = R/V_i$ is of the same order of magnitude as the electrical dynamics, the instantaneous model provides a computationally efficient yet

highly accurate approximation for flight control purposes. Finally, the closed-loop validation across multiple test cases—ranging from standard profiles to aggressive setpoint variations—confirmed the robustness of the identified parameters. The high correlation between experimental data and numerical simulations during both transient and steady-state phases demonstrates that the proposed model is a reliable digital twin of the physical propulsion unit. The robustness of the identified parameters was confirmed through both open-loop throttle steps and aggressive closed-loop RPM sequences. The high degree of correlation between experimental traces and numerical simulations proves that the proposed model can function as a high-fidelity Digital Twin, providing a fundamental tool for Control System Design. Specifically, this mathematical framework is particularly suited for advanced strategies such as Model Predictive Control (MPC), where the ability to accurately predict future states of thrust and torque is critical for optimal trajectory tracking and constraint handling. Looking forward, several promising directions remain for extending this research beyond hover conditions. Future outlooks could involve adapting the model to account for variations in axial inflow and the complex effects of cross-flow, which are typical of forward flight and transition phases. Additionally, investigating the system’s interaction with wind gusts and atmospheric turbulence would further enhance its predictive capabilities for real-world operations. Ultimately, this work lays a solid foundation for the development of more sophisticated and resilient flight control laws, bridging the gap between theoretical modeling and experimental reality in the field of UAVs.

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