POLITECNICO DI TORINO

MASTER's Degree in Mechatronic Engineering



Set-Membership Data-Driven Control of a Steer-by-Wire System

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October 2025

Abstract

Advancements in automotive technology are directed towards by-wire systems, that are all those functionalities where mechanical components can be replaced by electronic actuators. Some of the many advantages of this approach are the absence of mechanical limitations, reduced weight and improved safety. The absence of steering shaft and mechanical linkages allows more design flexibility in the whole vehicle architecture; in particular, more freedom in the design of advanced control strategies, such as lane keeping, autonomous parking and active steering.

In this thesis, I had the possibility to design and validate a controller for a Steerby-Wire system intended for an autonomous vehicle for Bylogix S.r.l, a company located in Grugliasco (TO).

The objective was to develop a direct data-driven controller using a Set-Membership Identification approach which leverage convex relaxation techniques for polynomial optimization to design the controller. This method avoids the need for explicit system modelling by relying on input-output data and prior assumptions on the bound of the measurement noise. The controller parameters were computed as the Chebyshev center of the feasible controller parameter set, which is the optimal point-wise estimate for the considered problem. This kind of approach gave the opportunity to focus on the actual control problem rather than the identification of the plant, that is often a very time-consuming and expensive process, that can itself lead to secondary issues in the control design phase. However, a first stage of study of the system was necessary to make the correct a-priori assumptions needed for the formulation of the problem. Then, the design problem is formulated and solved using the experimental data acquired on the vehicle under study. The obtained controller is then tested on a vehicle simulator implemented in Matlab-Simulink environment.

Future research could explore data-driven design of alternative and more advanced controller structures, such as MPCs or nonlinear neural network-based controllers.

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Acronyms

ADAS Advanced Driver Assistance Systems.

DDDC Direct Data-Driven Control.

DLC Double Lane Change.

DOF degree of freedom.

ECU electronic control unit.

EE equation-error.

EFCPS Extended Feasible Controller Parameter Set.

EIV errors-in-variables.

FCPS Feasible Controller Parameter Set.

FPS Feasible Parameter Set.

IMC Internal Model Controller.

IMU Inertial Measurement Unit.

LMI Linear Matrix Inequality.

LS Least Squares.

 \mathbf{LP} Linear Programming.

MVOBB minimum volume outer-bounding box.

POP polynomial optimization problem.

SbW Steer-by-Wire.

 ${\bf SDP}$ semidefinite programming.

 ${\bf SI}$ System Identification.

 ${\bf SISO}$ single-input single-output.

SMC Slide Model Controller.

 \mathbf{SMI} Set-Membership Identification.

Chapter 1

Introduction

1.1 Context and Motivation

The origins of modern control theory are typically placed in the 1960's with the introduction of state-space model representation by Kalman. In modern control theory, there is a wide variety of control techniques that have one thing in common. First, a model of the plant is needed and, based on this model, the chosen control technique can be applied. However, it is not always possible to possess a good model of the plant under consideration, and so an approximated one has to be used instead under the assumption that it is a good approximation of the real plant. The other situation is when the plant model obtained is a high-order function representation, so a high-order controller has to be employed to apply a control action. However, in practice, high-order controllers are greatly restricted by physical realization and added production cost.

According to [1] a full definition of what data-driven controllers are:

"Data-driven control includes all control theories and methods in which the controller is designed by directly using on-line or off-line I/O data of the controlled system or knowledge from the data processing but not any explicit information from mathematical model of the controlled process, and whose stability, convergence, and robustness can be guaranteed by rigorous mathematical analysis under certain reasonable assumptions."

This design process meets the drive-by-wire technology that is widely used in the automotive sector today. Mechanical components are being replaced by mechatronics systems, where hardware and software are integrated. First striking examples of this evolution are the debut of digitally controlled fuel injection combustion engines in 1979 and digitally controlled antilock brake systems (ABS) in 1978. [2] Markets today are requiring more sophisticated functionalities and vehicle performances. Thus, by an accurate design of the controller under target, it

is possible to achieve those results, both in Advanced Driver Assistance Systems (ADAS) and autonomous driving scenarios.

1.2 Problem Definition

The specific control problem addressed in this thesis is the yaw dynamics of a vehicle that is equipped with an Steer-by-Wire (SbW) system. The steering wheel angle is the input of interest, and the main controlled output is the vehicle yaw rate. It is not straightforward to design a good controller for this kind of system. Model-based classical methods rely on the accurate identification of the plant dynamics. However, these models can be both expensive and time-consuming to achieve by means of lengthy experimental tests, and can cause model-plant mismatch issues when the vehicle is operated in a different scenario.

Within this viewpoint, direct data-based control techniques are an attractive alternative because they make direct utilization of experimental input-output data for designing the controller without passing through explicit plant identification. This can potentially reduce development time as well as the danger of modeling errors, if robustness against bounded uncertainties is explicitly addressed.

1.3 Objectives of the Thesis

The main objective of this work is the design, implementation, and validation of a Steer-by-Wire system controller following a Direct Data-Driven Control approach in a Set-Membership Identification framework.

The following are specific objectives of the thesis.

- Formulate the yaw rate control problem for a Steer-by-Wire system in a Set-Membership framework.
- Estimate bounded uncertainties from experimental data gathered on the vehicle of interest.
- Synthesize a Direct Data-Driven Controller from experimental collected data.
- Validate the controller performance through realistic simulation of normal driving manoeuvrers and emergency manoeuvrers in MATLAB environment.

Thesis outline

The remainder of the work is organized as follows. Chapter 2 gives the background on Steer-by-Wire systems, their merits, limitations, and control requirements. Chapter 3 provides the theoretical background of Set-Membership Identification

and Direct Data-Driven Control. Chapter 4 provides a step-by-step derivation of the controller form, explains the reasoning behind the design choices, and how the parameters were computed. As well as a brief discussion about signal noise and filtering strategies adopted in this work. Chapter 5 documents the system modeling of the system under study, needed to perform the validation phase. Chapter 6 documents the simulation environment, scenario-based testing, and validation of the proposed controller. Lastly, Chapter 7 concludes the thesis contributions and gives the potential directions for future research.

Chapter 2

Background on Steer-by-Wire Systems

2.1 Overview of Automotive Steering Systems

Conventional automotive steering systems rely on mechanical linkages to transmit the driver's input to the wheels. Typically, a steering wheel is connected to a steering column, which in turn is mechanically coupled to a rack-and-pinion assembly. This mechanism converts the rotational motion of the steering wheel into a lateral displacement, thereby steering the front wheels. Such systems are inherently mechanical, with components including rods, pivots, joints, and gearboxes.[3]

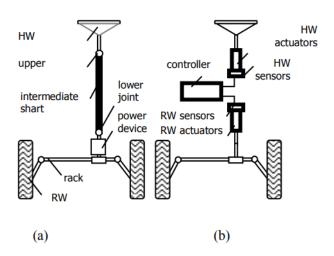


Figure 2.1: Comparison between a conventional steering system (a) and a SbW system (b) [4]

The modern market in the automotive field demands improved safety, reduced weight, and improved flexibility in vehicle design, leading to the development of Steer-by-Wire (SbW) systems. These kinds of system aim to eliminate the mechanical connection between the steering wheel and the wheels, replacing it with electronic sensors, actuators, and controllers. In a SbW system commonly a steering angle sensor and a torque sensor are located on the steering wheel or pinion. These two sensors collect data and transmit them to an ECU responsible for computing the torque profile required for the mission. Generally, it is also required to generate a reaction force on the steering wheel to give realistic feedback to the driver, allowing a more comfortable driving experience.[8] This paradigm shift offers new opportunities in vehicle dynamics optimization, driving autonomy, and human-machine interaction, although it introduces new challenges in terms of system safety and reliability that must be investigated.

2.2 Advantages and Challenges of Steer-by-Wire

The transition to SbW technology introduces numerous advantages. Chief among these is the reduction in mechanical complexity, which leads to significant weight savings. The absence of a steering shaft and mechanical linkages enables greater flexibility in vehicle architecture and interior design, particularly for electric and autonomous vehicles. Moreover, SbW systems facilitate the integration of advanced control strategies for features such as lane keeping, parking assistance, and active steering [5].

From a safety perspective, SbW systems enhance both passive and active safety. For instance, the ability to decouple the driver's input from the wheels allows for more intelligent control interventions during emergency maneuvers. Furthermore, they enable fault-tolerant architectures where redundant actuators and sensors can be incorporated for improved reliability.

Nevertheless, several technical challenges must be addressed. The design of robust and reliable control algorithms is crucial, given the absence of a mechanical fallback. Additionally, replicating the natural tactile feedback traditionally provided by mechanical connection between steering wheel and vehicle tires, often referred to as "road feel", requires sophisticated haptic feedback systems. These must be carefully tuned to ensure a realistic and intuitive driving experience. Finally, the whole safety, stability and reliability of SbW systems must be rigorously validated to meet regulatory standards, particularly in safety-critical applications like autonomous driving. [6] [7]

2.3 Control Requirements and Constraints

In SbW systems, control plays a central role in both functionality and safety. The electronic control unit (ECU) must interpret the driver's steering intention from sensor measurements and generate actuator commands that steer the wheels accordingly. Even in an autonomous vehicle, in an emergency situation, the driver should be able to take complete control of the guide immediately. This process must account for actuator dynamics, sensor noise, road disturbances, and hardware limitations.

Key control requirements include tracking accuracy, responsiveness, and stability under varying driving conditions. Constraints are imposed by physical limits of actuators (e.g., maximum torque, response rate), safety-related redundancies, and the need for fail-operational capabilities in case of component failure. In addition, the system must integrate seamlessly with other vehicle subsystems, such as braking and traction control, particularly in modern drive-by-wire architectures.

A robust and reliable control framework must also ensure compliance with automotive safety standards such as ISO 26262, which require formal validation, redundancy, and fault-detection mechanisms.

In [8] OH et al. developed a control algorithm for a SbW to improve driver's steering feel by generating reactive torque on the steering wheel and a front wheel motor control to improve vehicle's manuverability and stability. The system was modeled with bond graph theory. In [9] the same issue was studied by Fahami et al., but the authors proposed a LQR controller to generate the force feedback torque and the variable steering ratio to vary the feedback torque in different driving situations. The effectiveness of this method was experimentally validated in an HIL environment. In [10] the authors Zhu et al. estimated the parameters for a SbW system by using an ARX algorithm and then designed an Internal Model Controller (IMC) and a Slide Model Controller (SMC) for active steering. They proved to improve vehicle stability and anti-interference capability under different road conditions. In [11] a robust SbW control strategy is investigated to improve vehicle yaw stability under asymmetric disturbances, like μ -split braking, side wind forces or unilateral loss of tire pressure. A model regulator steering control architecture is used and the problem is formulated as mixed sensitivity design. Linear and non-linear single track vehicle simulations demonstrate effective yaw disturbance rejection and stable performances.

Chapter 3

Fundamentals on Set-Membership Data-Driven Control

3.1 Introduction to Set-Membership Methods

In many real-world control applications, including automotive systems, precise models of system dynamics are either unavailable or too complex to derive. Traditional system identification approaches rely on stochastic assumptions about the nature of disturbances and measurement noise. In contrast, Set-Membership methods offer an alternative by considering uncertainty in a deterministic and bounded framework.

The core idea of Set-Membership Identification (SMI) is that, if the disturbance and measurement noise are known to lie within certain a priori bounds, then the set of all models that are consistent with the observed data can be determined. This set is referred to as the Feasible-Parameter Set (FPS).

Formally, let us consider a linear regression model of the form:

$$y(k) = \varphi^{\top}(k)\theta + d(k) \tag{3.1}$$

where $y(k) \in \mathbb{R}$ is the output at time k, $\varphi(k) \in \mathbb{R}^n$ is the regressor vector constructed from past inputs and outputs, $\theta \in \mathbb{R}^n$ is the parameter vector to be estimated, and d(k) is a disturbance term. The disturbance d(k) is assumed to be unknown but bounded, i.e.,

$$|d(k)| \le \delta, \quad \forall k \tag{3.2}$$

Given a collection of measured data points $\{y(k), \varphi(k)\}_{k=1}^N$, the FPS is defined as:

$$\Theta_N = \left\{ \theta \in \mathbb{R}^n : |y(k) - \varphi^\top(k)\theta| \le \delta, \quad \forall k = 1, \dots, N \right\}$$
 (3.3)

This set represents all possible parameter vectors θ that are consistent with the measured data and the assumed disturbance bounds. As more data are collected, the FPS typically shrinks, yielding tighter descriptions of the system's dynamics.

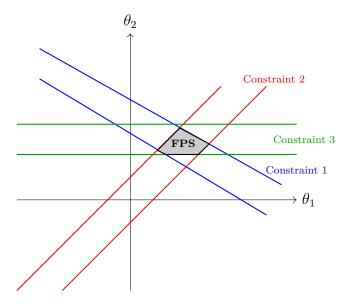


Figure 3.1: Feasible Parameter Set (FPS) in a two variable space as the bounded intersection of linear constraints. Each constraint is represented by two parallel lines enclosing the FPS.

In [12] Milanese and Vicino reviewed the main results of Set-Membership approach applied to different practical problems. The traditional approach in most estimation problems is to make assumptions on the noise affecting the available data, described statistically by a probability density function. However, this could not always be the case and by making this kind of assumptions there could be a loss of relevant information for the estimation process. Instead, in Set-Membership approach the only assumptions made about the noise are two; the noise enters the problem in an additive way respect to the available data and the noise is known to be bounded. This kind of description in many practical cases is more realistic and less demanding.

Set-Membership methods are particularly well-suited for applications where robustness is a critical requirement. Since the identified model lies within a known set of admissible models, it is possible to design controllers that explicitly account for uncertainty and guarantee robust performance across all models in the set. Compared to classical least-squares identification, which provides point estimates, Set-Membership approaches provide guaranteed bounds on the model parameters. This makes them inherently more reliable in safety-critical systems such as Steerby-Wire (SbW), where unexpected behaviour due to unmodelled dynamics can be unacceptable.

In the following sections, we will explore how these principles are extended and utilized in both identification and control, particularly in the presence of disturbances, modelling uncertainty, and real-time data availability.

3.2 Robust Identification in a Set-Membership Framework

In the field of system identification and control, robustness refers to the ability of algorithms to maintain performance and guarantee stability in the presence of uncertainty. Uncertainty may originate from measurement noise, external disturbances, unmodelled dynamics, or incomplete system knowledge. A robust identification strategy explicitly accounts for such uncertainty during model estimation, thereby producing models that are reliable under real-world operating conditions.

In the context of SMI, robustness is achieved by assuming that all uncertainties including noise and disturbances are bounded, without requiring a stochastic description. Rather than estimating a single parameter vector, the objective is to compute the FPS, which contains all models consistent with the available data and the assumed bounds on uncertainty. [13]

As new input-output data are collected, each data point imposes a strip constraint in the parameter space, corresponding to the set of all parameter vectors that could have generated that output within the assumed uncertainty bounds. The FPS is formed by the intersection of all such constraints and, in the linear case, takes the shape of a convex polytope. This approach ensures that the true parameter vector, under correct assumptions, remains within the FPS at all times. Additionally, the FPS shrinks with increasing data, thereby refining the model's precision while preserving robustness guarantees.

Modelling Approaches in Robust Identification

Robust identification within the Set-Membership framework can be applied across a range of model structures, typically categorized as follows:

• White-box models: These models are derived directly from first-principles physics, and all model parameters are assumed to be known exactly. In robust

identification, white-box models are often used as validation references rather than identification targets.

- Gray-box models: In this approach, the model structure is based on physical insight, but some parameters are unknown and must be estimated from data. The Set-Membership framework is particularly well-suited here, as it enables parameter estimation while explicitly respecting bounded uncertainty and structural knowledge.
- Black-box models: These models rely purely on Data-Driven techniques, with no explicit connection between model parameters and physical interpretation. SMI provides a safe and interpretable approach for estimating such models by ensuring all feasible models lie within known uncertainty bounds, even in the absence of physical insight.

Comparison with Probabilistic Methods

Set-Membership identification offers a fundamentally different philosophy from stochastic estimation techniques such as least squares or Kalman filtering. Instead of assuming noise distributions (e.g., Gaussian), it relies solely on deterministic bounds. This distinction leads to several practical advantages:

- No distributional assumptions: Performance and guarantees hold regardless of the shape of the noise or disturbance distributions, provided the bounds are valid.
- Worst-case guarantees: The resulting FPS contains all models consistent with the data, offering strong robustness in safety-critical systems.
- Natural incorporation of prior knowledge: Physical bounds, saturation effects, and known parameter limits can be directly embedded in the identification process.

Overall, robust identification within a Set-Membership framework provides a rigorous and conservative foundation for modelling dynamic systems under uncertainty. This approach is especially effective in applications such mechatronic systems, where physical bounds are known, safety is critical, and data may be limited or noisy.

3.3 Data-Driven Control Principles

Classical control design methodologies typically rely on the availability of a precise mathematical model of the plant, derived either from first-principle equations or through parametric identification techniques. However, in many practical systems, such as automotive SbW actuators, accurately capturing system dynamics via physical modelling can be time-consuming and prone to significant structural uncertainty. In these contexts, Data-Driven Control has emerged as a compelling alternative paradigm. Instead of relying on an explicit model, Data-Driven Control leverages measured data directly to design the control law, enabling more flexible and adaptive control synthesis in uncertain and evolving environments.

The core principle of Data-Driven Control is to use input-output data collected from the real system to either infer a control-oriented representation or to directly compute the control action that ensures desired closed-loop behavior. This can be achieved through various methodologies, including but not limited to virtual reference feedback tuning (VRFT), direct feedback control, subspace methods, or predictive control frameworks such as DeePC (Data-enabled Predictive Control) [14]. In model-implicit approaches, data is used to reconstruct internal system dynamics or behavioral descriptions without estimating traditional parametric models. This is particularly beneficial when the structure of the system is partially unknown or difficult to capture accurately.

When combined with SMI, Data-Driven Control techniques gain a critical robustness dimension. The FPS, computed recursively from data under bounded uncertainty assumptions, serves as the basis for control synthesis. Rather than optimizing performance for a single nominal model, the controller is designed to guarantee stability and performance for all models consistent with the available data, i.e., for all parameters $\theta \in \Theta_k$ [15]. This ensures that the resulting closed-loop system is robust to model uncertainties and maintains safe operation under bounded disturbances.

Despite its advantages, Data-Driven Control also presents several challenges. Guaranteeing robust stability in the presence of incomplete or noisy data requires conservative design assumptions, potentially limiting performance. Moreover, online implementation of such controllers demands efficient computational strategies, especially when the parameter space or input-output data dimension is large.

Nevertheless, the ability to synthesize robust controllers directly from operational data, while accounting for uncertainties in a non-probabilistic framework, makes Data-Driven Control a powerful and increasingly adopted tool in modern control engineering. In safety-critical applications like SbW systems, where reliability and adaptability are paramount, this paradigm provides a practical and theoretical grounded foundation for control design.

Chapter 4

Control Design for Steer-by-Wire

4.1 Requirements for Steering Control

The design of the controller was carried out within a model-matching framework, in which the desired performance specifications are first expressed in terms of a reference model $M(q^{-1})$. The objective of the identification and synthesis process is to obtain a closed-loop system whose behaviour closely matches that of the reference dynamics. This approach provides a systematic way of translating abstract control requirements into concrete and measurable performance indices.

For automotive steering systems, particularly those based on Steer-by-Wire technology, the definition of performance requirements is strongly motivated by both safety considerations and driver comfort. Unlike conventional mechanical steering systems, where the dynamics are inherently constrained by the physical linkages, electronic steering systems allow the control designer to directly shape the transient and steady-state response. This flexibility, however, also requires careful specification of the desired behaviour to ensure that safety-critical manoeuvrers can be performed reliably.

In the present work, the following key requirements were imposed on the reference model:

- Rise time. The system is required to achieve the commanded steering angle within a very short time window. A fast rise time ensures that the vehicle can respond promptly to steering commands, which is essential for maintaining stability in sudden manoeuvrers such as obstacle avoidance or lane-change operations.
- Minimum overshoot. Overshoot in the steering response must be avoided

as much as possible. Excessive oscillations or transient deviations from the target trajectory can result in abrupt or unintended lateral vehicle motions. In safety-critical scenarios, such as high-speed driving or emergency manoeuvrers, these oscillations could amplify driver inputs and lead to loss of control.

- Settling time. The transient phase of the steering response should decay rapidly so that the system converges to the desired trajectory without delay. A short settling time is not only beneficial for trajectory tracking but is also crucial in high-speed or emergency scenarios where prolonged transients may compromise safety and reduce the overall stability margin of the vehicle.
- Zero steady-state error. The closed-loop system must be able to track constant steering inputs with zero steady-state error. This requirement guarantees that the vehicle's actual yaw rate or steering angle converges exactly to the desired reference in steady operating conditions, thereby eliminating long-term deviations. In practical terms, zero steady-state error ensures accurate trajectory tracking, lane-keeping, and driver confidence in the steering response.

From a theoretical point of view, this property can be enforced by including an integrator in the controller transfer function. In fact, according to the final value theorem, for a step reference input r(k), the steady-state error is given by

$$|e_{\infty}| = \lim_{k \to \infty} |r(k) - y(k)| = \lim_{z \to 1} (z - 1)|e(z)|$$

Thus, in order to achieve $e_{\infty} = 0$, the closed-loop transfer function must contain at least one pole at z = 1, which corresponds to integral action. In the context of steering control, this ensures that constant steering commands are tracked without bias, preventing cumulative errors that could otherwise result in significant lateral displacement during long-duration manoeuvrers.

These requirements reflect the expectation that an autonomous steering system should provide a response that is at least comparable to, and ideally faster and more precise than, that of a skilled human driver. Whereas human drivers typically exhibit reaction and actuation delays on the order of several tenths of a second, electronic steering controllers can achieve significantly faster responses. This capability, however, must be balanced with the requirement of smooth and well-damped transients, since overly aggressive dynamics may compromise ride comfort or even destabilize the vehicle.

4.2 Direct Data-Driven Control Design using Set-Membership Techniques

The basis of this work was based on [12] [13] [24] [23]. Here, a general formulation of the problem is presented and then, later in the discussion, the specific details of the case under study are explained in depth.

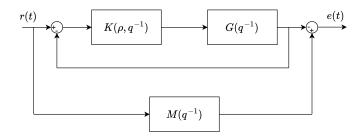


Figure 4.1: Block scheme of feedback control system compared to reference model

The problem is formulated as a DDDC design of a single-input single-output (SISO) system in presence of errors-in-variables (EIV) noise. Given the feedback control scheme in figure 4.1, where $G(q^{-1})$ is the unknown plant, $K(\rho, q^{-1})$ is the controller to be identified and $M(q^{-1})$ is the reference model of the system, we state the following assumptions¹:

1. The controller is a discrete time linear time-invariant system of order n. The order n is fixed and the controller has the following structure

$$K(\rho, q^{-1}) = \frac{\sum_{j=n+1}^{2n+1} \rho_j q^{-j-n-1}}{1 + \sum_{j=1}^{n} \rho_j q^{-j}}$$

2. The noise enters the problem with an EIV structure, meaning that the both the input and the output signals are affected by noise. As discussed at length in Chapter 3, it is reasonable to assume that the noise in unknown but bounded.

¹Note that q^{-1} is the backward-shift operator.

So that

$$\tilde{u}(t) = u(t) + \epsilon(t)$$

$$\text{with } |\epsilon(t)| \le \Delta_{\epsilon} \ \forall t$$

$$\tilde{y}(t) = y(t) + \eta(t)$$

$$\text{with } |\eta(t)| \le \Delta_{\eta} \ \forall t$$

 \tilde{u} and \tilde{y} are respectively the measured input signal and the measured output signal, ϵ and η are the noises affecting the signals and u and y are the "true" signals unaffected by noise.

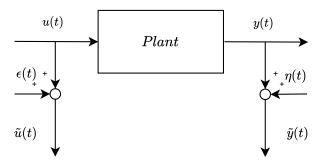


Figure 4.2: EIV model structure.

3. A set of N samples of the input-output sequence was collected to perform the parameter estimation.

By looking at the block diagram in figure 4.1, it is trivial to derive that the condition to minimize the error e(t) between the actual feedback system and the reference model $M(q^{-1})$ is the following

$$\frac{M(q^{-1})}{1 - M(q^{-1})}u(t) = K(\rho, q^{-1})y(t)$$
(4.1)

Now, putting all these informations together the Feasible Controller Parameter Set (FCPS) can be formalized.

$$\mathcal{D}^{K} = \{ K(\rho, q^{-1}) : \frac{M}{1 - M} u(t) = K(\rho, q^{-1}) y(t), \forall t = 1, ..., N,$$
$$||\tilde{u} - u||_{q} \leq \Delta_{\epsilon}, \ \forall t = 1, ..., N,$$
$$||\tilde{y} - y||_{q} \leq \Delta_{\eta}, \ \forall t = 1, ..., N \}$$

From now on in the discussion the notation indicating that the condition should hold for all the sequence samples t=1,...,N is implied. Since some variables involved in the equation cannot be simplified they must be included into the FCPS and, so, the Extended Feasible Controller Parameter Set (EFCPS) is defined.

$$\mathcal{D}_{\rho,u,y}^{K} = \{ \rho \in \mathbb{R}^{2n+1}, u \in \mathbb{R}^{N}, y \in \mathbb{R}^{N} : \frac{M}{1-M} u(t) = K(\rho, q^{-1}) y(t), \\ ||\tilde{u} - u||_{q} \leq \Delta_{\epsilon}, \\ ||\tilde{y} - y||_{q} \leq \Delta_{\eta} \}$$

By choosing the minimum volume outer-bounding box (MVOBB) we can define the intervals were the identified parameters should lie, as:

$$CPUI = [\underline{\rho}_i, \overline{\rho}_j], \ j = 1, ..., 2n + 1$$

where

$$\underline{\rho}_j \in \min \rho_j$$

s.t. $(\rho, u, y) \in \mathcal{D}_{\rho, u, y}^K$

and

$$\overline{\rho}_j \in \max \rho_j$$

s.t. $(\rho, u, y) \in \mathcal{D}_{\rho, u, y}^K$

So, the problem can be rewritten in standard form. For the minimum, we have

$$\underline{\rho}_i \in \min \rho_j$$

s.t.
$$(\rho, u, y) \in \mathcal{D}^K = \{K(\rho, q^{-1}) : \frac{M(q^{-1})}{1 - M(q^{-1})} u(t) = K(\rho, q^{-1}) y(t),$$

$$||\tilde{u} - u||_q \le \Delta_{\epsilon}, ||\tilde{y} - y||_q \le \Delta_{\eta} \}$$

And, analogously for the maximum

$$\overline{\rho}_{j} \in \max \rho_{j}$$
s.t. $(\rho, u, y) \in \mathcal{D}^{K} = \{K(\rho, q^{-1}) : \frac{M(q^{-1})}{1 - M(q^{-1})} u(t) = K(\rho, q^{-1}) y(t),$

$$||\tilde{u} - u||_{q} \leq \Delta_{\epsilon}, ||\tilde{y} - y||_{q} \leq \Delta_{\eta} \}$$

The optimization problem involves bilinear constraints, which make the formulation inherently non-linear and therefore non-convex. This non-convexity represents a significant challenge: while convex problems can be efficiently solved to global optimality using well-established algorithms, non-convex problems generally admit only local minima. A local solution, however, may be far from the true global optimum, thereby undermining the performance and robustness of the resulting controller design.

To address this limitation, a convex relaxation strategy is adopted. The central idea of convex relaxation is to approximate the original non-convex feasible set with a convex outer approximation, in particular a minimum volume outer-bounding box (MVOBB). By doing so, the relaxed problem becomes convex and can be solved efficiently with global guarantees. Although the solution of the relaxed problem does not necessarily coincide with the exact solution of the original non-convex formulation, it is guaranteed to lie within the original feasible set. Moreover, the relaxed problem provides a tractable surrogate that preserves feasibility while ensuring a bound on the optimal cost. This property is especially appealing in the context of data-driven control, where computational tractability and robustness are of fundamental importance.

One strategy is to carry out the relaxation within the framework of polynomial optimization. Specifically, bilinear terms can be reformulated as polynomial expressions and then relaxed through semidefinite programming (SDP) techniques. This approach make it possible to convert the original non-convex formulation into a convex problem solvable by existing SDP solvers.

This implementation can be performed using the SparsePOP package, a numerical tool specifically developed for large-scale sparse polynomial optimization. SparsePOP exploits the sparsity structure of the problem to significantly reduce computational complexity, thus enabling the solution of problems that would otherwise be intractable. It applies Lasserre's hierarchy of semidefinite relaxations to approximate the solution but does so by taking advantage of sparsity patterns to maintain scalability. The solver can be run within a MATLAB environment, which provided the framework for both problem formulation and analysis of the results.

4.3 Filter Design for Yaw Rate signal

Due to the experimental nature of this work, particular challenges emerged during the data acquisition phase. Especially, the yaw rate signal provided by the Inertial Measurement Unit (IMU) was found to be significantly affected by noise. This issue has been widely reported in the literature, as inertial sensors are inherently sensitive to mechanical vibrations and electrical interference. As a result, appropriate filtering strategies are often required to obtain reliable measurements. Some of the most

studied filtering strategies are Kalman filters, complementary filters and adaptive filters. However, also new more advanced approaches are being studied, for instance in [26] a direct approach with Set-Membership methods is proposed for the optimal design of filters for linear parameter-varying systems directly from experimental data. The method has been applied to yaw rate with satisfactory results. In [27] the problem of yaw rate characteristic changes at different vehicle velocity is directly addressed. Together with an intelligent PI controller is introduced the use of a Kalman filter to address this specific issue.

In this thesis, a different filtering strategy is proposed instead. The main objective was to design a filter that can attenuate high-frequency noise without inducing delay and at the same time preserving meaningful information, a key aspect of the estimation process. Therefore, a Forward-Backward filter was chosen. The filter performs zero-phase digital filtering by processing the data in both the forward and reverse directions. After filtering the data in the forward direction, the function matches initial conditions to minimize startup and ending transients, reverses the filtered sequence, and runs the reversed sequence back through the filter. The formulation and effectiveness of this method is discussed in [28]. First, an exact method is presented as well as an approximated one for faster computation on large scale problems. Then, a series of noise sequences are examinated and multiple test were conducted to compare results. The filter algorithm is also implemented on the Signal Processing Toolbox in MATLAB with the function filtfilt, making it ready-to-use and easy to integrate with the already existing code for pre-processing of experimentally collected data for identification.

4.4 Direct Data-Driven Control formulation for a Steer-by-Wire system

Given the general formulation of DDDC, the specific problem considered in this thesis concerns the control of a SbW system. Several critical aspects must be addressed to ensure a realistic and reliable problem formulation. First, the dynamics of a steering system are inherently non-linear. Although linearity assumptions can sometimes be made for low-speed manoeuvres or small steering angles, such simplifications are not valid in this context, since the objective is to design a controller suitable for autonomous driving applications, where the system must operate reliably across a wide range of driving scenarios and dynamic conditions, including emergency manoeuvrers.

Another fundamental consideration concerns the quality of the measured output signal. As discussed in Section 4.3, the yaw rate, which constitutes the main output variable of the system, is highly sensitive to measurement noise due to the limitations of the IMU employed. Accurate noise handling and filtering are

therefore essential to guarantee meaningful identification and consistent controller performance.

A further crucial requirement of the DDDC approach is the persistence of excitation of the input signal. This condition ensures that the experimental data contains sufficient information to accurately estimate the controller parameters. However, achieving persistently exciting inputs in real-vehicle experiments is non-trivial, as it may involve safety issues and practical limitations on the manoeuvres that can be executed. Moreover, the amount of collected data directly affects the numerical complexity of the optimization problem; increasing the dataset size can improve estimation accuracy but also significantly raise the computational complexity. Consequently, a trade-off must be found between the informativeness of the input signal and the feasibility of the data-driven optimization.

Considering all these aspects, an equation-error (EE) noise structure was adopted in this work. The chosen structure provides a robust framework for formulating the DDDC problem in a way that allow the designer to account for the main sources of uncertainty and nonlinearity inherent to SbWapplications.

More precisely, given a reference model $M(q^{-1})$ and an unknown plant the $G(q^{-1})$, the data-driven controller is designed according to the model matching problem presented in Figure 4.1 but with a slightly different formulation described here below.

4.4.1 Equation error formulation of the DDDC problem

Given the collected input and output samples, \tilde{u} and \tilde{y} respectively, let us compute the signal \tilde{s} as follows:

$$\tilde{s}(t) = L_r(q^{-1})\tilde{u}(t)$$

where

$$L_r(q^{-1}) = \frac{M(q^{-1})}{1 - M(q^{-1})}$$

By exploiting condition (4.1) the last equation can be rewritten as

$$\tilde{s}(t) = K(\rho, q^{-1})\tilde{y}(t) + e(t), \quad |e(t)| \le \Delta e$$

Where the equation error e(t) is added in order to account for the cumulative effects of (i) measurement noise affecting the collected data \tilde{u}, \tilde{y} , and of (ii) the nonlinear vehicle dynamics implicitly neglected in the linear control setting considered in this work. The equation error e is here assumed, according to the

set-membership theory, to be unknown but bounded by a real constant Δe . By some trivial algebraic passages, the last two conditions can be rewritten as

$$|\tilde{s}(t) + \rho_1 \tilde{s}(t-1) + \dots + \rho_n \tilde{s}(t-n) - \rho_{n+1} \tilde{y}(t) - \dots - \rho_{2n+1} \tilde{y}(t-n)| \le \Delta e(t)$$

and, consequently, we can define the FCPS as:

$$\mathcal{D}_{\rho}^{K} = \{ \rho \in \mathbb{R}^{2n+1} : |\tilde{s}(t) + \rho_{1}\tilde{s}(t-1) + \dots + \rho_{n}\tilde{s}(t-n) - \rho_{n+1}\tilde{y}(t) - \dots - \rho_{2n+1}\tilde{y}(t-n) | \leq \Delta e(t), \forall t = 1, \dots, N \}$$

Since, as discussed, the equation errors account cumulatively for different sources of uncertainty, it is quite difficult to derive the value of the bound Δe from the available *a-priori* information on the system and the measurement noise. Therefore, here we look for the controller parameter vector ρ which minimizes the bound on the actual equation error. The optimization problem providing such a controller is obtained by introducing a slack variable γ as follows:

$$\rho^* = \arg\min_{\rho,\gamma} \gamma$$
s.t.
$$\{\tilde{s}(t) + \rho_1 \tilde{s}(t-1) + \dots + \rho_n \tilde{s}(t-n) - \rho_{n+1} \tilde{y}(t) - \dots - \rho_{2n+1} \tilde{y}(t-n) \leq \gamma, \\ \forall t = 1, \dots, N$$

$$\tilde{s}(t) + \rho_1 \tilde{s}(t-1) + \dots + \rho_n \tilde{s}(t-n) - \rho_{n+1} \tilde{y}(t) - \dots - \rho_{2n+1} \tilde{y}(t-n) \geq -\gamma, \\ \forall t = 1, \dots, N\}$$

This optimization problem is a Linear Programming (LP) problem. This kind of problems are convex, because the controller parameters to be estimated enters the model equation linearly. Consequently, the problem can be solved through a linear regression algorithm, making it more manageable in a computational point of view.

Chapter 5

System Modelling and Measurements Setup

5.1 Mathematical Model of a Steer-by-Wire System

In traditional control design, the first step for the design of any controller is the definition of the plant under study and its operating boundaries. As outlined in the previous chapters, a SbW system differs from conventional steering systems by the fact that mechanical linkages between steering wheel and front wheels are eliminated and replaced by an ECU and electrical actuators. The function of this ECU is to control the motor responsible for the steering action based on sensor feedback. In the vehicle under study, a steering angle sensor and a yaw rate sensor are installed. These two sensors allow the implementation of closed-loop control strategies.

As discussed in the previous chapter, one of the main reasons that leads to the choice of designing a controller through SMI techniques is to avoid dealing with exact plant identification. However, a reference plant representation was required in order to test and validate the controller performances. For this purpose a simple model, widely used and studied in the literature for similar systems, was selected. The specific parameters in the model were not directly measured but instead estimated from the available experimental data. The simulated outputs obtained from this estimated model were then compared against the actual measured responses. Once a satisfactory level of agreement between the two was observed, the model was considered sufficiently accurate to be used as a reference for validation. It is crucial to underline once again that the estimated plant was not directly involved in the design of the controller but solely used as a reference for the simulations conducted

in the validation phase.

In this work, following the description in article [16], the standard bicycle model was used to model the dynamics of the vehicle described by the transfer function $G_s(s)$, taking as input variable the steering angle δ and as output the yaw rate $\dot{\psi}$ of the vehicle. The transfer function $G_s(s)$ written in terms of the vehicle physical parameters is given by:

$$G_s(s) = \frac{b_0 + b_1 s}{a_0 + a_1 s + a_2 s^2} \tag{5.1}$$

where

$$b_0 = \frac{c_f c_r (l_f + l_r) v}{r_\delta}$$

$$b_1 = \frac{c_f l_f m v^2}{r_\delta}$$

$$a_0 = c_f c_r (l_f + l_r)^2 + (c_r l_r - c_f l_f) m v^2$$

$$a_1 = \left(c_f (J_z + l_f^2 m) + c_r (J_z + l_r^2 m) \right) v$$

$$a_2 = J_z m v^2$$

The symbol m denotes the vehicle mass; J_z is the moment of inertia around the vertical axis; l_f and l_r are the distances between the vehicle center of gravity and the front and rear axles, respectively; v is the vehicle speed; and r_δ is the ratio between the steering-wheel angle and the front-wheel angle. The parameters c_f and c_r are the so-called front and rear cornering stiffness coefficients, respectively, which are used to describe the tire behaviour.

A linear model of the tire has been assumed, where the lateral tire force is described as the product of the cornering stiffness and the wheel sideslip angle.

5.2 Uncertainties and Disturbances in the System

In order to perform the measurements of the system variables, two sensors are installed in the vehicle. In particular, for the electronic steering control device a steering wheel angle sensor is needed. Furthermore, for the control algorithm a Inertial Measurement Unit (IMU) is needed in order to track the yaw rate, which is the output variable under study. Each sensor introduces a disturbance in the system due to noise, quantization effects, and possible sensor bias or drift, that needs to be taken in account to perform a correct analysis.



Figure 5.1: Accelerator Sensor Bosch MM7.10¹

These disturbances are modelled as uncertainties on the acquired signals. So, the signals are affected by a noise that, however, is bounded and measurable. When datasheet specifications were not available or insufficient, these bounds were empirically estimated from stationary or repeated measurements. This bounded-error model forms the foundation of the Set-Membership approach used in the subsequent identification phase.

5.3 Experimental Setup for System Identification

The datasets used for system identification were collected during a series of controlled driving experiments on a test track. The experiments included standard manoeuvres such as a double lane change, as described in ISO 3888-1 [22] and sinusoidal profiles at different frequencies. These scenarios were selected to excite the system dynamics sufficiently, ensuring persistent excitation of the signals under study.

Measurements were acquired using an onboard data acquisition system via CAN connected to the steering angle sensor and yaw rate sensor from the IMU. Data were sampled at 100 Hz to capture relevant dynamics without aliasing. All signals were time-synchronized and preprocessed to remove offsets, outliers, and static noise.

The input of the system is defined as the steering angle signal, while the output

¹https://www.bosch-motorsport.com/content/downloads/Raceparts/en-GB/ 245667595336397707.html

²https://dcemotorsport.com/#how-it-works

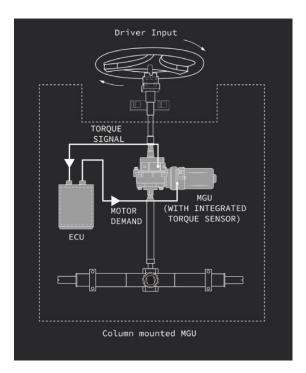


Figure 5.2: Functional scheme of the steering motor with integrated torque sensor and ECU²

is the measured yaw rate. These signals were normalized and segmented into batches for offline identification. Known signal bounds were also extracted during preprocessing to define the admissible uncertainty sets required by the identification algorithm.

³https://dcemotorsport.com/ultra-mgu-with-autonomous-control/



Figure 5.3: EPAS01 motor system for both manual and autonomous steering operations 3

Chapter 6

Simulation and Experimental Validation

6.1 Simulation Framework and Implementation

In order to test and validate the proposed control strategy, a series of experiments were carried out in a MATLAB/Simulink environment.

The Steer-by-Wire (SbW) system plant was modelled by the transfer function $G_s(s)$, as described in Chapter 5, and then discretized using a zero-order hold method, since the actuator is controlled by a digital system. The controller was designed with the Set-membership Direct Data-Driven approach described in the thesis and using data experimentally collected on the real vehicle. Such a controller was implemented in discrete-time and evaluated in closed-loop using the single-track linearized model G_s to model the plant.

In simulation, different reference profiles of yaw rate were used, such as a steep steer, a sinusoidal steer profile, and a standard Double Lane Change, as described in ISO 3888-1 [22]. Those profiles were chosen to evaluate robustness and performance under different possible driving conditions.

All simulations were carried out using a sample time of $T_s = 0.1s$, compatible with the data acquisition system on the vehicle. The model was also discretized with a sampling period equal to the one used in the experimental data acquisition system, in order to guarantee consistency between simulated and measured responses.

To proper design the yaw-rate reference signal ψ_r to be tracked by the control system, the same approach as in [16] was applied. The idea is to compute the reference yaw rate as

$$\dot{\psi}_r(t) = g(t) = h^{-1}(\delta(t), v_x(t))/v_x(t)$$

where the function $h(a_y, v_x(t))$ is the steering diagram of the uncontrolled vehicle

which, for any fixed velocity $v_x(t)$, relates the lateral acceleration $a_y(t)$ and the steering angle $\delta(t)$ at steady state. The steering diagram $\delta(t) = h(a_y(t), v_x(t))$ of the vehicle considered in this paper is shown in Figure 6.1 for the case of $v_x = 100$ km/h. Since it is easy to show that, at constant longitudinal velocity v_x , the lateral acceleration $a_y(t)$ and the yaw rate $\dot{\psi}(t)$ at steady state satisfy the equation

$$a_y(t) = v_x \dot{\psi}(t)$$

it turns out that the function g(t) is, for any approximately constant velocity v_x , a good approximation of the static mapping which relates the steering angle $\delta(t)$ and the yaw rate $\dot{\psi}(t)$ of the passive vehicle at steady state. Thus, the proposed reference yaw rate, which, at steady state, equals g(t), preserves the steady-state behaviour of the vehicle.

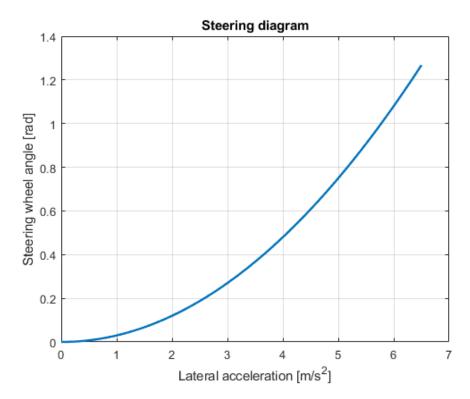


Figure 6.1: Vehicle steering diagram considered corresponding to $v_x = 100 \text{ km/h}$

6.2 Performance Evaluation in Different Scenarios

The performance of the designed controller was evaluated under various reference tracking scenarios to assess its robustness and accuracy. Each test focused on tracking performance, control smoothness, and noise sensitivity.

• Steep steer reversal test. The system was subjected to a sudden 90° steering wheel command, which highlights the transient response, overshoot, and settling time of the controlled system. As emerged in the simulation, the proposed controlled ensures quick response and zero steady-state error.

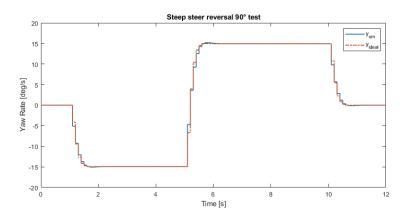


Figure 6.2: Yaw Rate response to a steer reversal test of 90°: (blue) SM controller, (red) (yellow) reference.

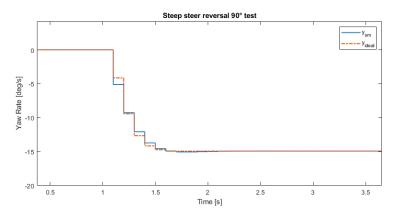


Figure 6.3: Detail of previous figure

• Double Lane Change (ISO 3888-1). A smooth, double transition in

steering was simulated to emulate a standardized emergency manoeuvrer. This scenario validated the controller's ability to maintain stability and responsiveness in emergency conditions. This manoeuvre is of particular interest because it propose a realistic scenario in which an obstacle should be avoided at high speed. Also, in this scenario the proposed controller performed accordingly to the requested goals.

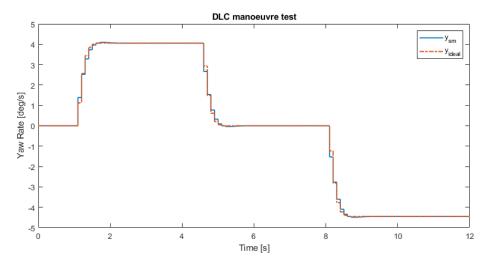


Figure 6.4: Yaw Rate response to standard DLC manoeuvrer: (blue) SM controller, (red) reference.

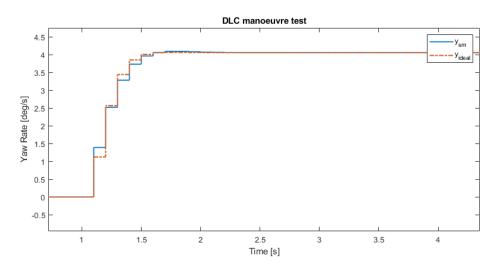


Figure 6.5: Detail of previous figure

• Sinusoidal steer. A continuous sine wave input tested the frequency-domain

performance of the controller. This scenario is the most realistic in a normal use of a vehicle. The test is needed to ensure a smooth and realistic feel during the driving. Also, in this case the controller tracks successfully the reference signal.

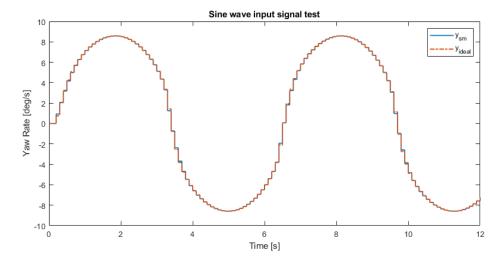


Figure 6.6: Yaw Rate response to a sinusoidal input: (blue) SM controller, (red) reference.

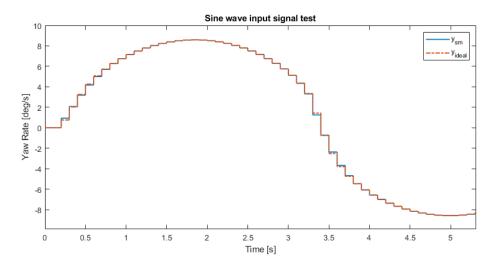


Figure 6.7: Detail of previous figure

In all scenarios, the control signal remained bounded and smooth, indicating that the controller is both accurate and implementable in real-time applications.

The impact of measurement noise was limited, thanks to the use of bounded-error modelling and noise-aware identification.

Chapter 7

Conclusions and Future Work

7.1 Limitations and Open Challenges

Despite, the main objectives of this thesis have been achieved, several limitations that deserve discussion were encountered.

A first critical issue concerns the interpretation of the yaw rate signal, which in this study was selected as the system output. Commercially available IMU sensors are subject to significant measurement noise, which complicates the use of yaw rate as a reliable feedback variable. As discussed in the previous chapters, a filter was implemented to preserve the meaningful components of the signal while attenuating noise. Nevertheless, the problem is not trivial, and alternative approaches could be explored. For instance, one possibility would be to reformulate the control problem in a way that does not rely directly on yaw rate measurements. A second limitation relates to the data-driven nature of the proposed methodology. Since the controller synthesis relies heavily on the available experimental data, its performance is dependent by the quality, variety, and coverage of the dataset. In particular, the data considered in this work were collected under limited operating conditions. Extending the acquisition campaign to include safety-critical manoeuvres, higher vehicle speeds, and more diverse road conditions would allow a more comprehensive assessment of robustness and generalizability.

7.2 Future Research Directions

An important aspect for future work concerns the validation of the proposed controller. While this thesis relied on simulation and experimental data analysis,

a natural extension would be the implementation of Hardware-in-the-Loop (HIL) experiments, which allow testing the controller in real time under realistic conditions without the risks associated with full vehicle deployment. Ultimately, on-road testing will be necessary to confirm the controller performance and robustness in real driving scenarios.

Another key point is that the work was made under the assumption that the steering system operates in isolation. In this work, the focus was deliberately restricted to the yaw rate control problem in order to assess the feasibility of the data-driven approach. However, in an autonomous driving context the steering subsystem interacts with other safety-critical subsystems, such as braking and traction control. Investigating the coupling between steering and braking, particularly during high-speed or emergency manoeuvrers, would provide a more comprehensive understanding of vehicle dynamics and controller integration. On the other hand, other alternative and more advanced controller structures could be investigated, such as MPCs or non-linear neural network-based controllers.

In summary, future research should extend the validation framework through HIL and on-road experiments, explore multi-subsystem interactions, and address robustness under more diverse and safety-relevant scenarios, as well as different control strategies can be adopted. These directions are essential to bridge the gap between proof-of-concept demonstrations and the deployment of data-driven controllers in safety-critical automotive applications.

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