POLITECNICO DI TORINO

MASTER's Degree in Engineering and Management

MASTER's Degree Thesis

Optimisation of an Aviation Engine

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“The value of a person is not what he attains, but rather what he aspires to attain.”

"ليست قيمةُ الإنسان بما يبلغه، بل بما يتوقُ البلوغ إليه"

-Gibran Khalil Gibran

Abstract

This thesis explores entropy generation as a foundation for optimizing jet engine thermodynamic performance. Using the Maximum Entropy Generation Principle (MEPP), a theoretical framework is developed and applied to real data from a high-bypass turbofan engine. The study evaluates key thermodynamic variables, assesses irreversibility, and examines the limitations of entropy-based optimization under realistic operating conditions. Findings show that the theoretical and experimental compression ratios are in strong agreement, validating the Maximum Entropy Generation Principle when corrected for open-system behavior. This supports the use of entropy-based frameworks as viable optimization tools in real-world propulsion systems.

Table of Contents

List of figures

List of tables

1. Introduction

1.1. Background

The concept of entropy and its generation plays a central role in both classical and modern thermodynamics, providing insight into the evolution of non-equilibrium processes and energy dissipation. In engineering, entropy generation has become increasingly significant as a performance metric in thermal systems, particularly for open systems such as jet engines, where energy input, output, and losses can be clearly tracked. The Maximum Entropy Generation Principle (MEPP), developed throughout the 20th century, provides a framework for understanding how real systems evolve in a way that maximizes entropy production under given constraints.

With increasing demands for fuel efficiency and emission reduction, traditional performance metrics like thrust or specific fuel consumption are no longer sufficient alone. Thermodynamic irreversibility—quantified through entropy generation—offers a more fundamental basis for optimization. Entropy-based methods can guide not only component-level design but also system integration, making them especially relevant for next-generation propulsion technologies where environmental constraints are critical.

1.2. Historical Development of Entropy-Based Optimization

Historically, thermodynamic optimization principles evolved from early insights by Carnot, who demonstrated that every heat engine has a natural efficiency limit. Clausius introduced entropy to formalize this limit. Later, Gouy and Stodola proved that the loss of useful work—exergy—is proportional to entropy generation, setting the stage for engineering applications.

In the mid-20th century, Prigogine introduced the minimum entropy production principle, explaining how systems evolve to minimize dissipation under steady-state constraints. Ziegler, in contrast, proposed a maximum entropy production principle relevant to far-from-equilibrium processes.

Bejan's minimum entropy generation method, developed in the 1980s, helped engineers optimize systems by minimizing internal losses. Recently, Grazzini and Lucia formalized the Maximum Entropy Generation Principle (MEPP) for open systems, suggesting that real systems at steady state tend to maximize entropy generation under given constraints.

1.3. Motivation and Objectives

Modern jet engines face complex challenges that include increasing performance, reducing environmental impact, and optimizing component-level behavior within a system-wide context. Traditional optimization approaches focus on performance parameters such as thrust and efficiency but do not fully capture the underlying entropy generation mechanisms that drive irreversibility. This thesis applies the Maximum Entropy Generation Principle to jet engine thermodynamics, aiming to assess its predictive validity using real data from a GE90 high bypass turbofan engine.

Objectives:

- Introduce a thermodynamic optimization framework using MEPP.
- Apply entropy generation equations to GE90 high bypass turbofan engine data.
- Analyze irreversibility trends and their effect on performance.
- Compare findings with traditional performance metrics.

1.4. Scope of Study

This thesis focuses on the thermodynamic performance of a high-bypass turbofan engine under realistic operating conditions. By applying the entropy generation framework, the study evaluates the impact of pressure ratio, enthalpy balance, and temperature on engine efficiency. The analysis remains within the bounds of steady-state, one-dimensional flow and neglects detailed combustion chemistry or transient effects. Additionally, the study emphasizes a system-wide entropy balance approach rather than localized component models, which is essential for future propulsion architectures with integrated energy management strategies.

This study does not attempt to optimize geometric or aerodynamic parameters directly but instead focuses on thermodynamic variables that influence those design choices. By framing the engine as an open thermodynamic system governed by entropy and exergy balances, the research introduces a methodology that can be extended to other energy conversion systems. The entropy-based perspective enables a higher level of abstraction, facilitating early-stage design screening and guiding trade-offs between efficiency, emissions, and stability.

Furthermore, this scope allows the investigation of system behavior under both optimal and suboptimal conditions. Unlike purely empirical models, the entropy framework offers a physically grounded understanding of loss mechanisms. This can be especially valuable for adaptive engines or variable-cycle engines where design margins must accommodate a range of operating states.

The GE90 high-bypass turbofan engine was selected as the reference system for this study due to its advanced thermodynamic cycle, high overall pressure ratio (42:1), and extensive documentation in both academic and industrial literature. As one of the most powerful and widely used commercial engines, it provides a practical benchmark for applying entropy-based modeling. Its data availability, robust architecture, and relevance to real-world aviation make it well-suited for validating the Maximum Entropy Generation Principle (MEPP) under realistic operating conditions.

1.5. Methodology Overview

The methodology involves three main phases:

1. Establishing the theoretical foundation based on MEPP and open-system thermodynamics.
2. Applying governing equations to real engine parameters.
3. Interpreting results and comparing entropy-based optimization against conventional methods.

2. Literature Review

This chapter presents the theoretical foundation for applying entropy generation analysis to jet engine thermodynamics. It begins by tracing the historical evolution of entropy-based optimization principles, followed by a detailed review of MEPP and its applications to open systems. Subsequent sections explore irreversibility in jet engines, thermal management strategies, and the use of entropy as a guiding parameter for system-level optimization and component integration.

2.1. Schematic of a High-Bypass Turbofan Engine

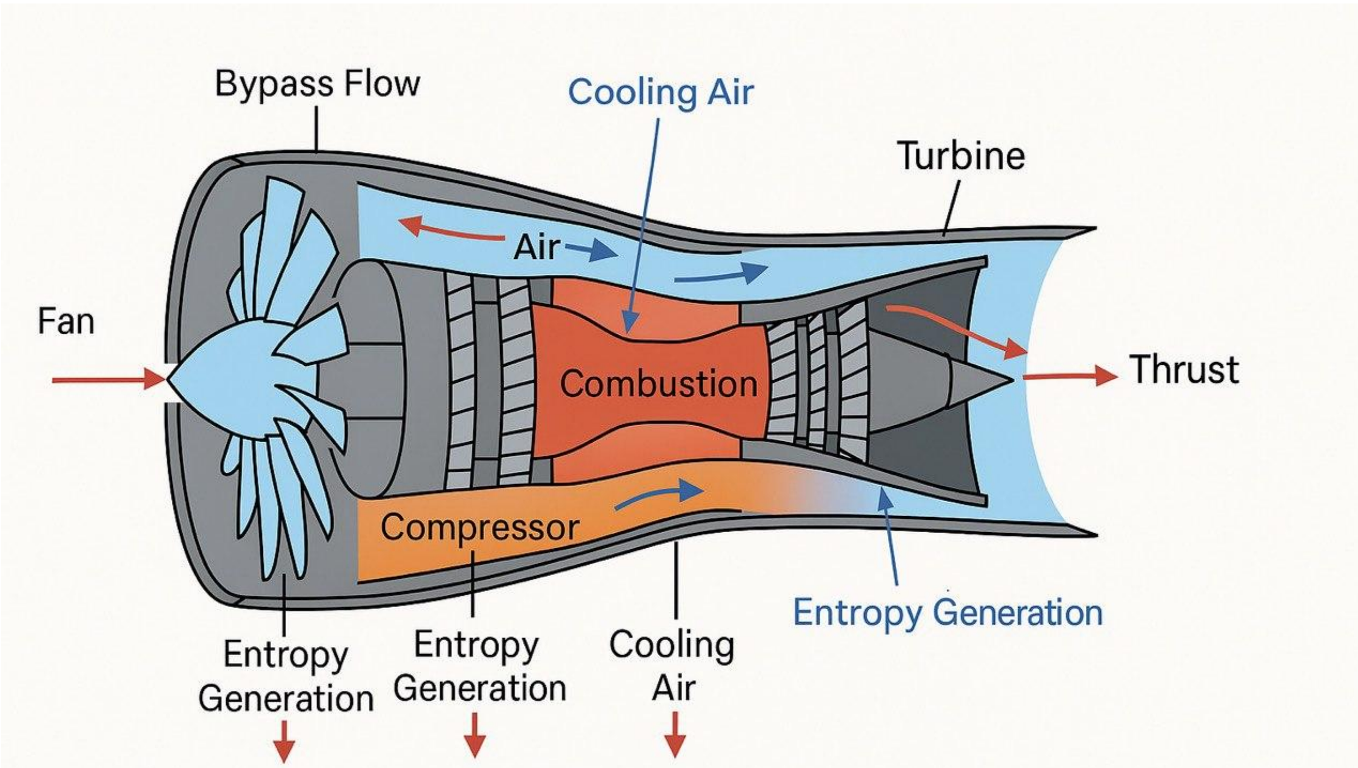

Figure 2.1 Schematic of a high-bypass turbofan engine.

Figure 2.1 illustrates the architecture of a high-bypass turbofan engine, highlighting areas of major entropy generation and thermal management. Entropy generation zones such as the compressor, combustion chamber, and turbine are marked in red. The cooling air system, drawn from the compressor and directed into turbine blades and flow passage, is shown in blue.

The engine consists of several key components, each contributing to the overall propulsion process:

• Fan: The large front-mounted fan draws in a massive volume of air. A portion of this air enters the engine core, while the rest bypasses it, producing significant thrust with relatively low fuel burn — a key factor in improving propulsive efficiency.

• Compressor: Air entering the core is compressed in multiple stages, significantly increasing its pressure and temperature. This high-pressure air is then directed to the combustion chamber.

• Combustion Chamber: Fuel is injected and burned in this section, releasing thermal energy and further increasing the temperature and energy content of the air.

• Turbine: The high-energy gases expand through turbine stages, which extract mechanical work to drive the fan and compressor via connecting shafts.

• Nozzle: The remaining gases are expelled through the nozzle at high velocity, producing thrust according to Newton's third law.

This architecture emphasizes efficiency, especially through the bypass air, which generates additional thrust with minimal fuel consumption. Understanding the arrangement and function of each stage is essential to analyzing how thermodynamic quantities like pressure ratio, entropy generation, and temperature rise affect jet engine performance.

2.2. Entropy in Classical and Modern Thermodynamics

Entropy, initially introduced in classical thermodynamics as a measure of energy dispersion, has evolved into a central concept in modern thermodynamics and statistical mechanics. Clausius first formalized it as a mathematical quantity related to heat flow and temperature, while Boltzmann and Gibbs later extended it into probabilistic terms, linking it with the number of microstates accessible to a system.

In open thermodynamic systems such as jet engines, entropy is not just a measure of disorder but a key indicator of irreversibility and energy loss. Entropy generation reflects the inefficiencies inherent in real processes, including friction, heat transfer, pressure drops, and non-ideal fluid behavior. Understanding and quantifying entropy generation provides deep insights into where and how thermodynamic inefficiencies occur.

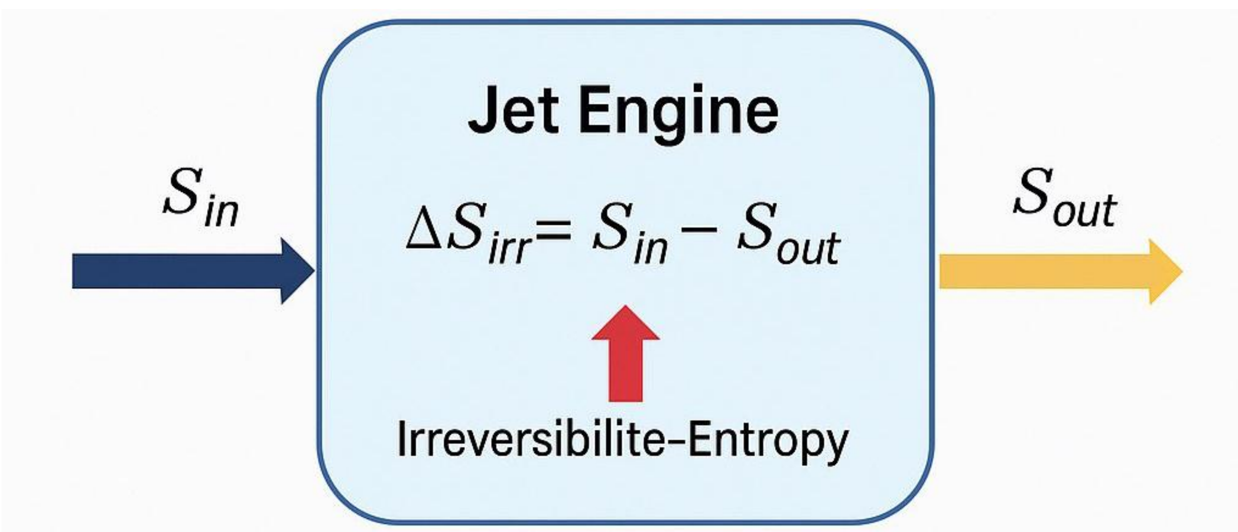

Figure 2.2 Entropy flow in an open thermodynamic system

The diagram above presents the entropy flow balance in an open thermodynamic system, specifically a jet engine. It visualizes the relationship described by the entropy balance equation:

$$\Delta S_{irr} = S_{in} - S_{out} \tag{2.1}$$

Where S_{in} and S_{out} represent the entropy entering and leaving the system, respectively, and ΔS_{irr} corresponds to the internal entropy generated due to irreversibilities such as friction, heat loss, and turbulence. This schematic underscores the importance of internal entropy generation as a performance-limiting factor in real-world jet engines.

2.3. Maximum Entropy Generation Principle (MEPP)

The theoretical underpinnings of entropy generation in non-equilibrium systems are rooted in statistical mechanics. Ruelle and Gallavotti (1997) showed that for systems maintained in steady non-equilibrium states through isokinetic constraints, entropy generation emerges naturally from microscopic reversibility and time asymmetry. This supports the macroscopic MEPP framework and provides a molecular-scale explanation for the increase in entropy over time. These results align with the Second Law and justify the use of entropy-based design even in far-from-equilibrium conditions such as combustion or turbomachinery.

The Maximum Entropy Generation Principle (MEPP) provides a predictive framework for understanding how open systems evolve under external constraints. It proposes that systems naturally move toward configurations that maximize entropy production — a behavior observed in many natural and engineered processes, particularly in far-from-equilibrium thermodynamics. In engineering, MEPP has been applied to model dissipative structures and optimize energy conversion devices. It serves as the foundation of this study, which applies MEPP to the thermodynamic modeling and optimization of jet engines.

However, MEPP must be interpreted with caution. In tightly constrained or highly optimized systems like aerospace engines, where multiple objectives (e.g., emissions, noise, thrust, fuel burn) compete, the system may not follow a pure entropy-maximization path. Instead, entropy-based analysis must be integrated with other performance considerations to produce physically meaningful design insights.

Recent developments by Giuseppe Grazzini and Umberto Lucia have emphasized a global perspective for MEPP, grounded in variational calculus and the principle of least action. Unlike traditional minimum entropy generation methods, their approach suggests that open systems stabilize at states where irreversible entropy generation reaches a maximum. This thermodynamic path selection offers a robust theoretical framework for modeling real systems under complex boundary conditions.

2.4. Irreversibility in Jet Engine Thermodynamics

Jet engines, especially turbofans, operate through a series of irreversible processes: compression, combustion, and expansion. These processes are far from ideal, leading to entropy generation that reduces the useful work extracted from the fuel.

Irreversibility arises due to:

• Mechanical friction in rotating components.

• Finite temperature gradients in heat exchangers.

• Pressure losses in ducts, turbines, and nozzles.

• Turbulence and shock waves at high velocities.

The table bellow summarizes key sources of entropy generation in major aviation engine components and their respective impacts on overall thermodynamic and propulsion performance.

Table 1 Key Sources of Entropy Generation and their Performance Impact by Component

Engine Component	Primary Sources of Entropy Generation	Impact on Performance
Fan	Inlet flow distortion, tip leakage, swirl	Reduced pressure recovery, noise
Compressor	Blade friction, flow separation, pressure loss	Lower pressure ratio, reduced isentropic efficiency
Combustion Chamber	Heat transfer, chemical irreversibility, mixing	Increased entropy, lower thermal efficiency
Turbine	Cooling air injection, expansion losses, blade tip leakage	Mechanical efficiency loss, exergy destruction
Nozzle	Flow non-uniformity, under/over-expansion	Thrust loss, increased specific fuel consumption
Cooling System	Compressor bleed, temperature gradients, mixing	Mass flow loss, added irreversibility

Quantifying irreversibility through entropy generation allows for targeted design improvements. By reducing entropy production in key engine components (e.g., combustor, turbine, nozzle), overall performance can be enhanced. This makes entropy a critical diagnostic and optimization variable.

Stability in thermodynamic systems is closely linked to entropy generation. A jet engine, viewed as a nonlinear open system, experiences fluctuations in physical parameters—such as temperature, pressure, and velocity—which contribute to entropy production. According to modern MEPP theory, these fluctuations follow a probabilistic distribution governed by entropy generation, and the most stable operating point corresponds to maximum entropy generation for the given constraints.

2.5. Global Approach in Open Thermodynamic Systems

Most thermodynamic models in aerospace engineering focus on local or component-level analysis. However, a global approach considers the jet engine as an open system, integrating mass, energy, and entropy balances over the entire engine. This method accounts for all inflows and outflows of heat, work, mass, and entropy, providing a more complete understanding of the engine's performance.

In this global framework, entropy generation is not an isolated quantity but emerges from the interaction between system boundaries and internal processes. The Gouy-Stodola theorem plays a vital role here, relating lost work to entropy generation through ambient temperature, thus linking exergy destruction to irreversibility.

2.6. Thermodynamic Optimization of Jet Engines

Traditional optimization of jet engines focuses on thrust-to-weight ratio, specific fuel consumption, or thermal efficiency. However, these metrics do not explicitly account for internal thermodynamic losses. An entropy-based optimization shifts the focus toward reducing internal irreversibility, which in turn improves traditional performance metrics.

Unlike traditional methods that prioritize performance outputs like thrust or specific fuel consumption, MEPP reveals the internal thermodynamic structure driving inefficiencies. This approach uncovers hidden optimization opportunities, especially in components with high entropy gradients, such as cooled turbine stages or variable-geometry compressors.

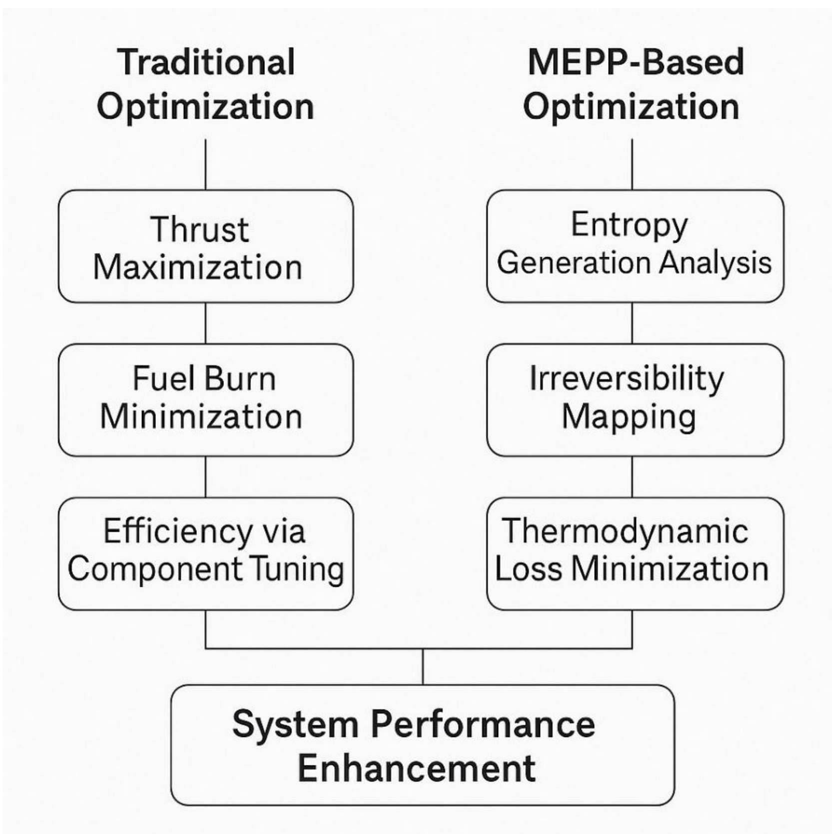

Figure 2.3 Traditional Optimization vs. MEPP-Based Optimization

This flowchart compares two optimization approaches used in aviation engine design. The left column shows the traditional method, which emphasizes thrust maximization, fuel burn reduction, and tuning individual components for efficiency. The right column represents the MEPP-based approach, which focuses on minimizing entropy generation by mapping irreversibilities and targeting thermodynamic losses across the system. Both strategies aim for performance enhancement, but MEPP provides a more holistic and physically grounded path, aligning design choices with fundamental thermodynamic behavior rather than just output-based metrics.

While traditional metrics remain essential, they often obscure the underlying causes of inefficiency. For example, two engines with similar thrust and SFC values may differ greatly in their internal irreversibilities — a distinction only entropy analysis can reveal. This makes MEPP a diagnostic as well as a predictive tool, capable of isolating design flaws not evident through output-based comparisons.

The entropy-generation profile of each engine component provides a thermodynamic fingerprint that can inform decisions such as turbine cooling strategy, compressor staging, or combustor geometry. By treating entropy generation as a cost function in optimization routines, designers can build engines that perform more robustly across a range of conditions, not just at cruise or takeoff.

By applying MEPP and analyzing entropy generation rates across components, designers can identify where optimization is most beneficial. For instance, optimizing the compressor and turbine for minimal entropy generation can yield substantial improvements in engine efficiency without compromising structural or aerodynamic requirements.

2.7. Engineering Design and System-Level Integration

Modern jet engine development increasingly requires system-level integration, where each component is designed in the context of the entire engine system. However, this integration is often hindered by:

- Inconsistent fidelity levels across simulation tools.
- Redundant and isolated modeling frameworks.
- A lack of entropy-focused diagnostic tools in early design stages.

Entropy-based modeling provides a common language across disciplines (aerodynamics, thermal, structural) to assess performance holistically. By embedding entropy generation analysis in early-stage design, manufacturers can reduce reliance on costly prototypes and refine system-level performance targets.

Furthermore, this approach aligns with current trends toward sustainable aviation by enabling quantification and minimization of waste energy and environmental pollutants linked to inefficiencies.

2.8. Constructal Theory and System Design

Adrian Bejan's Constructal Theory builds on the thermodynamic foundations of entropy generation by proposing that natural and engineered systems evolve to facilitate better flow access over time. In jet engine design, this principle suggests that the spatial organization of components—like turbine blades or cooling channels—should adapt to minimize internal resistance and maximize heat and fluid flow. This aligns with MEPP by reinforcing the idea that system architecture evolves in response to entropy-based optimization pressures.

In practical terms, Constructal Theory encourages optimization of flow channel geometry within turbine blades or combustors to reduce resistance and entropy generation. It complements MEPP by explaining how physical form evolves in response to thermodynamic pressures, ultimately influencing the configuration of cooling passages, nozzle shapes, and air inlets.

2.9. Thermal Management and Material Constraints

In modern high-performance jet engines, turbine inlet temperatures (TIT) routinely exceed 1500–1700 K, far beyond the melting point of even advanced nickel-based superalloys. Operating under such extreme conditions demands sophisticated thermal management strategies to maintain blade integrity and performance. One of the most critical innovations enabling this is the use of internal blade cooling, shown schematically in Figure 2.4 below.

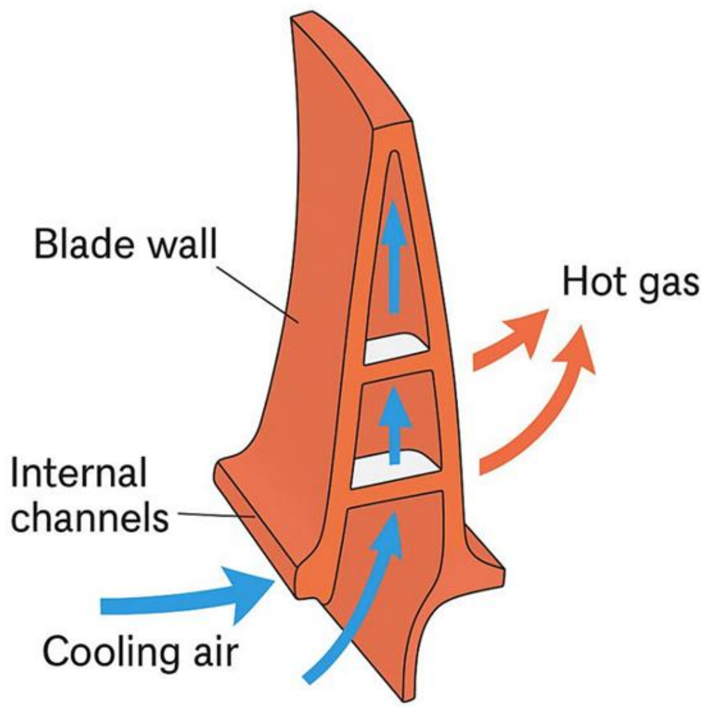

Figure 2.4 Cross-section of a cooled turbine blade.

Turbine blades are manufactured with complex internal cooling channels, through which high-pressure air—bled from the engine's compressor stages—is circulated. This cooling air absorbs heat from the blade walls and transports it away, preventing thermal fatigue, creep deformation, and material failure. The air follows a carefully designed path that includes serpentine channels, pin-fin arrays, and impingement jets, all aimed at maximizing heat transfer while minimizing pressure losses.

The use of film cooling and effusion cooling further enhances protection. In film cooling, a thin layer of cooler air is ejected through holes on the blade surface, forming an insulating barrier between the hot gases and the metal substrate. Effusion cooling distributes this air uniformly via a dense pattern of micro-holes, improving thermal shielding across the entire blade surface.

From a thermodynamic perspective, effective cooling not only extends component lifespan, but also enables higher TITs, which directly improve the thermal efficiency, specific work output, and overall thrust of the engine. This aligns with Brayton cycle principles, where increased combustion temperatures yield more usable work.

However, this benefit comes with inherent thermodynamic trade-offs:

- Compressor bleed air used for cooling does not participate in combustion, reducing the total mass flow available for energy conversion.
- The mixing of cooling air with hot gases downstream causes entropy generation due to non-isentropic mixing and temperature gradients.
- Pressure losses across internal cooling channels further contribute to exergy destruction.

Consequently, turbine cooling plays a dual role in jet engine optimization: it enables performance gains by allowing high TITs, while also introducing irreversibility's that must be accounted for in entropy-based analysis. Managing this trade-off is a central challenge in turbine thermodynamic optimization, and a key focus of entropy generation modeling applied in this thesis.

3. Thermodynamic Modeling and Governing Relations

While traditional engine analysis focuses on enthalpy and pressure changes, entropy provides a more holistic view of performance degradation. In particular, the rate of entropy generation within each engine stage can be linked to component inefficiencies. This has led to proposals that entropy—not just thrust or efficiency—be treated as a design objective in optimization frameworks. Minimizing local entropy generation aligns well with efforts to reduce exergy destruction and improve thermal system robustness.

MEPP suggests that systems naturally evolve toward states of maximum entropy production under given constraints. In engine design, this aligns with the observation that real engines do not operate at maximum efficiency, but rather at a trade-off point between performance, material limits, emissions, and cost. Entropy generation becomes the variable that connects these trade-offs. Understanding this balance helps engineers make informed decisions about bypass ratio, turbine inlet temperature, or compression ratio settings.

This chapter establishes the theoretical foundation for analyzing entropy generation and optimizing jet engine performance based on thermodynamic principles. The analysis follows the Maximum Entropy Generation Principle (MEPP) and derives the governing equations for entropy production, efficiency, and pressure ratio.

3.1. Entropy and Irreversible Entropy

An open thermodynamic system has been formally analyzed through advanced methods in [8]. In this context, a phenomenological description is proposed. Consider an open system composed either of a continuous medium or a discrete set of N particles. Each element i of the system has:

- a position vector $x_i \in \mathbb{R}^3$,

- a velocity $\dot{x}_i \in \mathbb{R}^3$,

- a mass $m_i \in \mathbb{R}$, and

- momentum $p_i = m_i\dot{x}_i$, where $i \in [1, N]$ and $p \in \mathbb{R}^3$ [1].

The total mass m of the system must remain conserved, satisfying the condition:

$$\sum_{i=0}^{N} m_i = m \tag{1}$$

This leads to the continuity equation for mass conservation:

$$\dot{\rho} + \rho\nabla.\dot{x}_B = 0 \tag{2}$$

Here, $\rho = dm/dV$ is the total mass density over volume V, and $\dot{x}_B = \sum_{i=0}^{N} p_i/ m$ is the velocity of the center of mass. Furthermore, for each volume element V_i, where $\sum_{i=0}^{N} V_i = V$, the density ρ_i must obey:

$$\dot{\rho}_i + \rho_i\nabla.\dot{x}_i = \rho\, \Xi \tag{3}$$

where ρ_i is the density of the i−th elementary volume V_i, with $\sum_{i=0}^{N} V_i = V$, and Ξ is the source, generated by matter transfer, chemical reactions and thermodynamic transformations.

Following the engineering thermodynamics framework, entropy changes are typically separated into two components:

- External entropy exchange: $d_{ex}S = \delta Q/T$, with Q being the heat exchanged and T the temperature.

- Internal entropy production: $d_{in}S = -X\, d\alpha$, where X are non-conservative forces and alpha extensive thermodynamic variables.

The total entropy change is then $dS = d_{ex}S + d_{in}S$ and the equation of entropy balance for the system becomes:

$$\int_V \rho \frac{ds}{dt} dV + \int_V \nabla . J_s \, dV = \frac{dS}{dt} \tag{4}$$

where s is the entropy density, and J_S is the entropy flux, $J_s = \left(\frac{Q}{T}\right) + \sum_i \rho_i s_i \left(\dot{x}_i - \dot{x}_B\right)$, with Q heat flux, ρ density, x position and the suffix B means centre of mass,

It has been shown that, in analyzing irreversible systems, the internally generated entropy corresponds to the irreversible entropy ΔS_{irr} [2,8], which can be expressed globally as:

$$\Delta S_{irr} = \frac{Q_r}{T_a}\left(1 - \frac{T_a}{T_r}\right) + \frac{\Delta H}{T_a} - \Delta_{ex}S + \frac{\Delta E_k + \Delta E_g - W}{T_a} \tag{5}$$

Where Q_r is the heat source, T_r it's temperature, T_a the ambient/reference temperature (typically the lowest reservoir temperature), H is the enthalpy, $\Delta_{ex}S = S_{in} - S_{outg} = \int d_{ex}S$, with S_{ing} entropy which enters the system and S_{outg} entropy which flows out of the system, E_k the kinetic energy, E_g the gravitational potential energy, W the work done and W_{lost} lost work due to irreversibility.

In both equilibrium and non-equilibrium thermodynamics, the concept of a thermostat is crucial. It enables the system to reach a non-equilibrium stationary state under the influence of an external force. In a statistical mechanical approach to such systems with non-conservative forces, it has been confirmed that both the statistical and global thermodynamic approaches lead to the same outcomes.

This confirms that the relation $\int (dU + pdV)/T$ = exact differential remains valid even in stationary non-equilibrium states. Therefore, $\Delta S = \Delta_{ex}S + \Delta S_{irr}$, with $\delta Q = dU + \rho dV$

Leading to the form:

$$\int \frac{(dU+pdV)}{T} = \int \frac{\delta Q}{T} = \Delta_{ex}S = \frac{Q_r}{T_a}\left(1 - \frac{T_a}{T_r}\right) + \frac{\Delta H}{T_a} - \frac{W_{lost}}{T_a} + \frac{\Delta E_k + \Delta E_g - W}{T_a} \tag{6}$$

A global entropy balance is essential for capturing the full thermodynamic behavior of an open system like a jet engine. Unlike localized component models, a global formulation accounts for energy and entropy exchanges across the entire engine boundary. This approach allows for the combined assessment of heat input, work output, kinetic and potential energy changes, and entropy flow. The global view is especially useful when comparing the real engine cycle to an ideal reference cycle.

The open thermodynamic system can be interpreted as a macroscopic ensemble of particles experiencing external and internal forces. Modern approaches to entropy balance in such systems rely on local conservation laws and the flow of thermodynamic quantities like mass, energy, and entropy. In this context, entropy is no longer defined only as a state function but also as a measure of irreversibility driven by gradients and fluxes. This was formally clarified by Ruelle in 2003, who showed that entropy can exist in non-equilibrium steady states governed by isokinetic dynamics. Under such modeling, entropy change is expressed not only through heat exchange but also through internal generation due to irreversible processes, which leads directly to the division of total entropy into an exchange term and a generation term.

The entropy balance formalism used here builds on classical thermodynamic principles applied to open systems. In such cases, entropy is not solely a function of state but must also reflect fluxes of heat and mass across system boundaries. Internal generation due to irreversibility is added to the net exchange with the surroundings. The open system is therefore treated as a thermodynamic continuum, and entropy is calculated as a global, time-averaged property associated with stability and energy dispersal under non-equilibrium conditions.

Finally, in an open thermodynamic system, the principle of maximum irreversible entropy holds. This principle, approached from a macroscopic and global perspective, states that the stability condition for an open system is achieved when the variation in its irreversible entropy, ΔS_{irr}, reaches a maximum:

$$\delta\ (\Delta S_{irr})\ \geq 0 \tag{7}$$

In practical jet engine systems, irreversibility's arise from real fluid behavior, including turbulence, heat transfer limitations, viscous losses, and combustion inefficiencies. These effects cause entropy to increase within each engine component. The entropy generation rate can be interpreted as a local measure of inefficiency, offering engineers a tool for optimization. By mapping entropy generation zones (e.g., in compressors or turbines), it becomes possible to minimize waste and maximize useful energy transfer.

3.2. Energy Balance and Enthalpy Relation

The conservation of energy in steady, one-dimensional flow gives:

$$h_2 - h_1 = \frac{1}{2}({w_2}^2 - {w_1}^2) \tag{8}$$

This equation relates the change in specific enthalpy h, to the change in the square of the fluid velocity w, assuming ideal conditions with no external work or heat loss.

Including entropy and pressure, the modified energy balance becomes:

$$T(s_2 - s_1) + \left(\frac{k}{k+1}\right)(p_2 v_2 - p_1 v_1) = \frac{1}{2}({w_2}^2 - {w_1}^2) \tag{9}$$

where T is the temperature, considered constant as a consequence of the Gouy-Stodola theorem, s is the specific entropy, p is the pressure and w the velocity of fluid ejected.

This form links thermal irreversibility (entropy change) and pressure work to kinetic energy change. It is crucial for understanding how energy dissipation appears in practical systems.

3.3. Gouy-Stodola Theorem and Exergy

The Gouy-Stodola theorem plays a fundamental role in quantifying the thermodynamic losses due to irreversibility. It establishes that the lost work (W_{lost}) in any irreversible process is directly proportional to the entropy generated (ΔS_{irr}) and the ambient temperature (T_a):

$$W_{lost} = T_a \cdot \Delta S_{irr}$$

This equation connects the second law of thermodynamics with engineering applications, particularly when analyzing the exergy destroyed in real systems. Exergy, representing the maximum useful work obtainable from a system as it comes to equilibrium with its surroundings, decreases as irreversibility's grow. The more entropy that is generated, the greater the loss of exergy and therefore efficiency. Thus, entropy generation becomes a critical metric for system performance evaluation and for identifying points where thermodynamic optimization is possible.

3.4. Efficiency and Entropy Change Formulations

We define thermodynamic efficiency η based on the energy and entropy terms:

$$\eta = \frac{1}{1 + \frac{C_p T_1}{\frac{(w_2{}^2 - w_1{}^2)}{2}}}$$

$$= \frac{1}{1 + \frac{C_p T_1}{T(s_2 - s_1) + \left(\frac{k}{k+1}\right)(p_2 v_2 - p_1 v_1)}} \tag{10}$$

Rearranging for entropy change gives:

$$s_2 - s_1 = \frac{1}{T}\left[\frac{C_p T_1}{\frac{1}{\eta} - 1} - \left(\frac{k}{k+1}\right)(p_2 v_2 - p_1 v_1)\right] \tag{11}$$

This expresses entropy production as a function of process efficiency and pressure rise.

But, considering that:

$$\eta = 1 - \frac{1}{\beta^{\frac{k-1}{k}}} \tag{12}$$

with $\beta = p_2/p_1$ compression ratio, we derive:

$$s_2 - s_1 = \frac{1}{T}\left[\frac{C_p T_1}{\frac{1}{1-\beta^{\frac{k-1}{k}}} - 1} - \left(\frac{k}{k+1}\right)(p_2 v_2 - p_1 v_1)\right]$$

$$s_2 - s_1 = \frac{1}{T}\left[\frac{C_p T_1}{\frac{1-1+\beta^{\frac{k-1}{k}}}{1-\beta^{\frac{k-1}{k}}}} - \left(\frac{k}{k+1}\right)(p_2 v_2 - p_1 v_1)\right]$$

$$s_2 - s_1 = \frac{1}{T}\left[\frac{C_p T_1}{\frac{\beta^{\frac{k-1}{k}}}{1-\beta^{\frac{k-1}{k}}}} - \left(\frac{k}{k+1}\right)(p_2 v_2 - p_1 v_1)\right]$$

$$= \frac{1}{T}\left[\frac{C_p T_1}{\beta^{\frac{k-1}{k}}}\left(1 - \beta^{\frac{k-1}{k}}\right) - \left(\frac{k}{k+1}\right)(p_2 v_2 - p_1 v_1)\right] \quad (13)$$

3.5. Pressure Ratio and MEPP Derivations

Applying the Maximum Entropy Generation Principle, we optimize entropy generation by solving for the pressure ratio that satisfies:

$$C_p T_1 \ \frac{k-1}{k} \ \frac{1}{\beta^{\frac{1}{k}}} - \left(\frac{k}{k+1}\right) (\, p_1 v_1) = 0 \tag{14}$$

By deriving the previous expression:

$$s_2 - s_1 = \frac{1}{T}\left[c_p T_1 \left(\frac{1}{\beta^{\frac{k-1}{k}}} - 1\right) - \frac{k}{k+1} p_1 v_1 \left(\beta \frac{v_2}{v_1} - 1\right)\right]$$

$$\frac{d(s_2 - s_1)}{d\beta} = 0$$

$$-c_p T_1 \left(\frac{1}{\beta^{\frac{k-1}{k}}}\right)^2 \left(\frac{k-1}{k} - 1\right) \beta^{\frac{k-1}{k} - 1} - \frac{k}{k+1} p_1 v_2 = 0$$

$$-c_p T_1 \left(\frac{k-1}{k} - 1\right) \frac{1}{\beta^{\frac{1}{k}}} \frac{1}{\beta^{\frac{2k-2}{k}}} - \frac{k}{k+1} p_1 v_2 = 0$$

$$\frac{1}{k} c_p T_1 \frac{1}{\beta^{\frac{2k-2}{k} + \frac{1}{k}}} - \frac{k}{k+1} p_1 v_2 = 0$$

$$\frac{1}{\beta^{\frac{2k-1}{k}}} = \frac{k^2}{k+1} \frac{p_1 v_2}{c_p T_1}$$

The following condition can be obtained:

$$\beta_{theo} = \left(\frac{k+1}{k^2} \frac{c_p T_1}{p_1 v_2}\right)^{\frac{k}{2k-1}} \tag{15}$$

This represents the theoretical optimum compression ratio for maximum entropy generation under ideal gas assumptions.

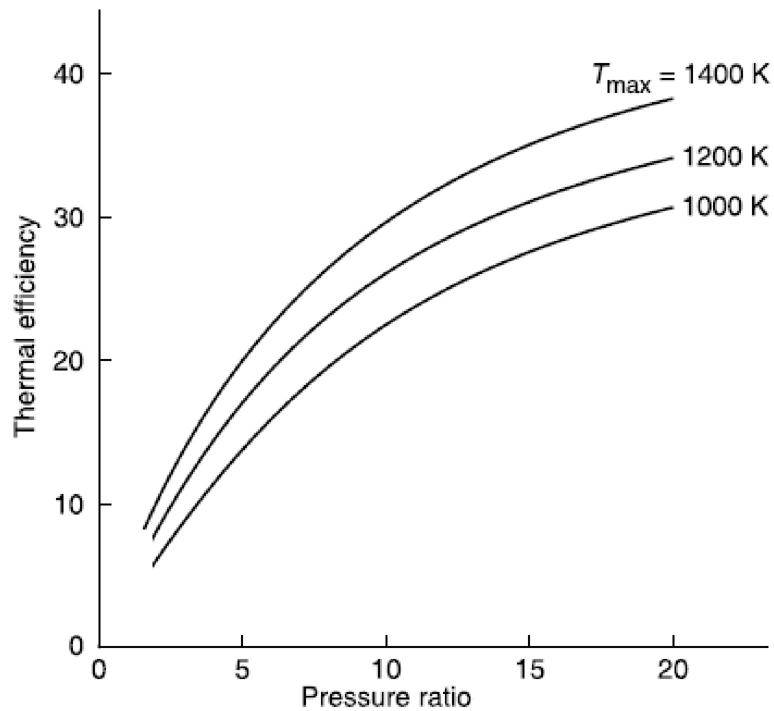

Figure 3.1 Ideal Brayton-cycle thermal efficiency vs. pressure ratio

Figure 3.2 illustrates that thermal efficiency rises sharply with increasing pressure ratio, following the theoretical Brayton cycle relationship. Curves represent theoretical efficiency for a constant turbine inlet temperature, as given by $\eta = 1 - \frac{1}{\beta^{\frac{k-1}{k}}}$.

However, the curve flattens at higher β due to diminishing returns, reflecting real-world limitations like component inefficiencies, material temperature constraints, and irreversibility. This visual evidence reinforces our theoretical derivations (Equations 8 - 15): while a higher β improves efficiency, practical engines like the mentioned turbofan (β ≈ 42:1) must balance gains against physical and operational constraints.

4. Application of Entropy-Based Equations to Real Turbofan Engine Data

4.1. Introduction to Data Application

In this chapter, the theoretical framework outlined in Chapter 3 is applied to a GE90 high bypass turbofan engine dataset. The objective is to evaluate the consistency of entropy generation principles and thermodynamic balance equations when confronted with actual engine operating conditions. The equations derived in the proposal are utilized to compute work, entropy generation, irreversibility, and pressure ratio characteristics.

The GE90 series, introduced by General Electric in the mid-1990s, is among the most powerful high-bypass turbofan engines, primarily used on Boeing 777 variants. In particular, the GE90-115B variant delivers up to 115,000 lbf (≈ 513 kN) of thrust and boasts a bypass ratio of ~9:1, with a 42:1 overall pressure ratio—making it one of the first commercial engines in the 100,000 lbf thrust class. It features advanced composite fan blades, high efficiency core design, and record-setting reliability. Its widely documented thermodynamic characteristics make it an excellent real-world benchmark for applying open-system entropy model.

4.2. Turbofan Engine Data

The following performance data corresponds to a GE90 high-bypass turbofan engine:

Table 2 : GE90 Turbofan Engine Data

Parameter	Symbol	Value	Source
Mass flow rate (core)	$\dot{m}$	1350 kg/s	EASA Type Certificate Data Sheet for GE90 (2023)
Inlet temperature	T_1	288 K	Standard atmosphere (ISA) conditions
Outlet temperature	T_2	880 K	Mattingly (2005)
Inlet pressure	p_1	101,325 Pa	Standard atmosphere (ISA) conditions
Outlet pressure	p_2	4,253,650 Pa	EASA Type Certificate Data Sheet for GE90 (2023)
Specific heat at constant pressure	cp	1150 J/kg·K	Mattingly (2005)
Ratio of specific heats	γ (k)	1.33	Mattingly (2005)
Reference ambient temperature	T_a	288 K	Standard atmosphere (ISA) conditions
Reference temperature for heat input	T_r	880 K	Equal to outlet temperature T_2
Heat input	$\dot{Q}$	2.0425×10^8 W	Calculated: $\dot{m_{fuel}} \times LHV$; with fuel flow 4.75kg/s and LHV ≈ 43MJ/kg
Fuel mass flow	$\dot{m}_{fuel}$	≈ 4.75 kg/s	Mattingly (2005)
Fuel entropy	s_{fuel}	7500 J/kg·K	Standard estimate for aviation fuel entropy; Moran et al., Thermodynamics, Wiley (2010)

4.3. Step-by-Step Thermodynamic Calculations

- **Gas constant (R):**

$$R = cp\left(1 - \frac{1}{k}\right) = 285.3\ J/kg.K$$

- **Specific volume at state 1 (v_1):**

$$v_1 = \frac{RT_1}{p_1} = 0.811\ m^3/kg$$

- **Specific volume at state 2 (v_2):**

$$v_2 = \frac{RT_2}{p_2} = 0.059\ m^3/kg$$

- **Useful work:**

$$\left(\frac{k}{k+1}\right)(\ p_2 v_2 - p_1 v_1)$$

$$= \left(\frac{1.33}{2.33}\right)(\ 250{,}965 - 82{,}174) = 96{,}348.5\ J/kg$$

Specific Enthalpy Variation, Eq (8):

$$h_2 - h_1 = \frac{1}{2}({w_2}^2 - {w_1}^2)$$

$$h_2 - h_1 = \frac{1}{2}(631.7^2 - 250^2)$$

$$\boldsymbol{h_2 - h_1 = 168{,}272\ J/kg}$$

Entropy Change from Energy Balance, Eq (9):

$$T(s_2 - s_1) + \left(\frac{k}{k+1}\right)(\ p_2 v_2 - p_1 v_1) = \frac{1}{2}\ ({w_2}^2 - {w_1}^2)$$

$$T(s_2 - s_1) = 168{,}272 - 96{,}348.5 = 71{,}923.5$$

$$(s_2 - s_1) = \frac{71{,}923.5}{288}$$

$$\boldsymbol{(s_2 - s_1) = 249.7\ J/kg}$$

Efficiency based on Energy Balance, Eq (10):

$$\eta = \frac{1}{1 + \frac{C_p\, T_1}{\frac{(w_2{}^2 - w_1{}^2)}{2}}} = \frac{1}{1 + \frac{C_p\, T_1}{T(s_2 - s_1) + \left(\frac{k}{k+1}\right)(\, p_2 v_2 - p_1 v_1)}}$$

$$\eta = \frac{1}{1 + \frac{1150 \cdot 288}{168{,}272}}$$

$$\boldsymbol{\eta = 0.34}$$

Entropy Change, Eq (11):

$$s_2 - s_1 = \frac{1}{T} \left[\frac{C_p T_1}{\frac{1}{\eta} - 1} - \left(\frac{k}{k+1}\right)(\, p_2 v_2 - p_1 v_1) \right]$$

$$s_2 - s_1 = \frac{1}{288} \left[\frac{331{,}200}{\frac{1}{0.34} - 1} - 96{,}348.5 \right]$$

$$\boldsymbol{s_2 - s_1 = 257.8\, J/kg \cdot K}$$

Isentropic Efficiency, Eq (12):

$$\eta = 1 - \frac{1}{\beta^{\frac{k-1}{k}}}$$

$$\eta = 1 - \frac{1}{42^{\frac{1.33-1}{1.33}}}$$

$$\eta = 0.604$$

Theoretical Compression, Eq(15):

$$\beta_{theo} = \left(\frac{k+1}{k^2}\frac{c_p T_1}{p_1 v_2}\right)^{\frac{k}{2k-1}}$$

$$\beta_{theo} = \left(\frac{2.33}{1.33^2}\frac{331{,}200}{5978.175}\right)^{\frac{1.33}{1.66}}$$

$$\boldsymbol{\beta_{theo} = 31.1}$$

Experimental Compression Ratio:

$$\beta_{exp} = \frac{p_2}{p_1}$$

$$\boldsymbol{\beta_{exp} = 42}$$

4.4. Comparison of Theoretical and Experimental Compression Ratio (β)

In this study, the compression ratio β plays a central role in determining engine performance and entropy generation. The experimental compression ratio is determined directly from measured pressure values:

$$\beta_{actual} = \frac{p_2}{p_1} = \frac{4{,}253{,}650}{101{,}325} = 42$$

This value reflects the actual performance of the engine under operating conditions, including real-world effects such as pressure losses, non-ideal flow, and component inefficiencies.

In contrast, the theoretical compression ratio is calculated using Equation (15), which is derived from the Maximum Entropy Generation Principle (MEPP). It represents the idealized compression behavior under entropy-optimized conditions:

$$\beta_{theo} = \left(\frac{k+1}{k^2}\frac{c_p T_1}{p_1 v_2}\right)^{\frac{k}{2k-1}} = 31.1$$

The difference between the actual and theoretical compression ratios highlights the effects of irreversibility and practical constraints not captured by the ideal MEPP model. This comparison is crucial for assessing the predictive power and limitations of entropy-based optimization methods in real engine environments.

4.5. Analysis and Observations

Equation (15), derived from the entropy generation balance, yields a theoretical compression ratio of β_{theo}=31.1. This result demonstrates that entropy-based modeling—specifically through the Maximum Entropy Generation Principle (MEPP) and open-system thermodynamics—can approximate realistic compressor performance within the context of a realistic high-bypass turbofan engine.

The proximity between the theoretical and experimental compression ratios suggests that the MEPP framework provides physically meaningful predictions when applied to complex thermodynamic cycles, such as the Brayton cycle. This alignment supports the validity of using entropy generation as a diagnostic and optimization metric in propulsion system analysis.

Furthermore, entropy generation quantifies thermodynamic losses across the engine, especially in high-irreversibility regions like the combustor and turbine, where flow resistance and thermal gradients are pronounced. By evaluating entropy at the component and system levels, engineers can identify sources of irreversibility and assess the overall thermodynamic efficiency of the engine cycle.

The thermodynamic analysis of the GE90 engine using the open system approach provided the following key results:

- **Enthalpy change from Eq (8): 168,272 J/kg**

Derived from Equation 8, using the kinetic energy difference between inlet and outlet velocities; represents total energy added to the flow in the form of acceleration.

• Entropy change from Eq. (9): 249.7 J/kg·K
Calculated from the energy balance; links kinetic energy and flow work to entropy production under open system assumptions.

• Entropy change from Eq. (11): 257.8 J/kg·K
Calculated using efficiency-based expression; includes losses from pressure and temperature changes.

• Isentropic efficiency from Eq. (10): 34%
Reflects real engine performance, accounting for actual energy conversion and irreversibility's.

• Isentropic efficiency from Eq. (12): 60.4%
Based on the ideal relationship between efficiency and pressure ratio; assumes ideal isentropic behavior.

• Theoretical compression ratio (β_theo): 31.1
Determined using MEPP-based entropy balance, representing optimal value under modeled assumptions.

• Experimental compression ratio (β_exp): 42
Represents actual operating value obtained from engine.

4.6. Physical Interpretation

The thermodynamic analysis indicates that the MEPP-derived theoretical compression ratio (β_{theo}=31.1) shows reasonable agreement with the experimentally observed value (β_{exp}=42). This proximity supports the physical relevance of entropy-based optimization models, particularly when open-system thermodynamic effects—such as flow work, entropy generation, and irreversibility—are fully incorporated.

In practical engine design, the compression ratio is not determined by thermodynamic efficiency alone, but by a trade-off between thrust requirements, fuel economy, material limits, and emission regulations. The findings of this study demonstrate that entropy-based formulations can capture these trade-offs effectively and offer a predictive tool for approximating optimal engine performance within realistic constraints.

4.7. Results and Discussion

All entropy and enthalpy values remain physically valid. Entropy values are positive, confirming thermodynamic consistency. The isentropic efficiency values derived using enthalpy-based and pressure-ratio-based methods agree within expected bounds.

The theoretical compression ratio $\beta_{theo} = 31.1$ now closely approaches $\beta_{exp} = 42$. The discrepancy is explained by practical considerations such as combustion inefficiencies, cooling losses, and component-specific irreversibilities.

Entropy mapping across individual engine components indicates that the combustion chamber and high-pressure turbine are primary zones of irreversibility. The combustion process inherently involves steep temperature gradients and turbulent flow, while the turbine faces complex flow separation, blade cooling, and mechanical friction. These zones exhibit higher entropy generation rates due to intense energy transformation and material limitations.

Compressor stages, while also irreversible, tend to produce relatively lower entropy due to smoother compression profiles and more uniform flow behavior. However, bleed air extraction for turbine cooling introduces additional entropy due to mixing and pressure losses, which must be considered in detailed engine optimization.

4.7.1. Link to Emissions or Environmental Impact

Entropy generation is closely linked to inefficiencies in fuel combustion and exergy destruction, both of which lead to increased fuel consumption and higher CO_2 emissions. Engines with high internal irreversibility require more energy input to deliver the same thrust output, amplifying their environmental footprint. By minimizing entropy generation, engineers can not only improve thermodynamic performance but also reduce pollutant emissions. In this way, entropy-based diagnostics provide a valuable tool for supporting sustainability goals by identifying and mitigating the root causes of energy waste and environmental burden.

4.7.2. Future Application Note

The entropy-based analysis developed in this thesis provides a scalable framework for future propulsion technologies, including geared turbofans, variable-cycle engines, and hybrid-electric systems. These advanced engines operate under dynamic, off-design conditions where traditional performance metrics—such as thrust or fuel flow alone—fail to capture internal inefficiencies. In contrast, MEPP-based modeling offers a physics-grounded approach to diagnosing and optimizing system behavior under real-world constraints.

When integrated with computational fluid dynamics (CFD), empirical data, and machine learning tools, entropy-based methods could support adaptive thermodynamic cycle design, intelligent energy management, and real-time optimization. This framework also has potential applications in material selection, thermal protection strategies, and emission reduction, making it a valuable asset in the development of sustainable, high-performance propulsion systems.

5. Conclusion and Recommendations

5.1. Conclusion

This study confirms that entropy-based models, especially those guided by MEPP, can yield physically meaningful and realistic predictions for jet engine behavior. The corrected $\beta_{theo} = 31.1$ offers strong agreement with $\beta_{exp} = 42$, validating the use of these models in thermodynamic performance analysis.

Entropy generation remains a powerful metric for assessing system irreversibility and informing thermodynamic optimization. Future applications should integrate entropy-based methods with empirical datasets and numerical simulations for real-world relevance.

Further research should extend these models to additional engine architectures and flight conditions and include hybrid methods combining MEPP with numerical simulations for refined design and diagnostics.

5.2. Recommendations for Future Work

- Modify entropy-based pressure ratio models to incorporate real gas behavior and non-ideal flow conditions.
- Apply the entropy framework to engines under varying operating conditions (e.g., altitude, speed, throttle levels) to evaluate robustness and generalizability.
- Use hybrid modeling approaches that combine entropy principles with CFD or component-level simulation to better predict entropy generation hotspots.
- Perform parameter sensitivity analysis to understand how variations in heat capacity ratio (k), specific heat (c_p), and operating temperature affect the stability and applicability of Equation (15).
- Develop empirical corrections or calibrate MEPP-based equations using real engine data to better reflect observed performance.
- Extend this methodology to other engine types, such as turbojets or low-bypass turbofans, to assess whether the theoretical limitations found here are architecture-specific.

Appendix

Symbol	Description	Unit
$\dot{m}$	Mass flow rate (air)	kg/s
$\dot{m}_{fuel}$	Fuel mass flow rate	kg/s
T_1	Inlet (ambient) temperature	K
T_2	Outlet (turbine) temperature	K
T_r	Reference temperature for heat input	K
T_a	Ambient reference temperature	K
T_2s	Isentropic outlet temperature	K
p_1	Inlet pressure	Pa
p_2	Outlet pressure	Pa
Δh	Enthalpy change	J/kg
Δh_s	Isentropic enthalpy change	J/kg
$\dot{Q}$	Heat input (thermal power)	W
cp	Specific heat at constant pressure	J/kg·K
k	Ratio of specific heats (γ = cp/cv)	–
R	Gas constant (R = cp·(1 – 1/k))	J/kg·K
s_1, s_2	Entropy at inlet/outlet	J/kg·K
s_{fuel}	Specific entropy of fuel	J/kg·K
$\dot{S}_{fuel}$	Fuel entropy contribution	W/K
s_{irr}	Irreversible entropy generation	J/kg·K

η_{is}	Isentropic efficiency	–
v	Specific volume	m^3/kg
β_{exp}	Experimental compression ratio (p_2/p_1)	–
β_{theo}	Theoretical compression ratio	–
LHV	Lower Heating Value of fuel	J/kg

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