POLITECNICO DI TORINO
TECHNISCHE UNIVERSITÄT WIEN

Master's Degree in Environmental and Land Engineering

Track: Climate Change

Master's Degree Thesis

The Evolution of the Water Quality Dynamics in Europe through Socio-Hydrological Modelling: Model Development

Supervisor

Prof. Alberto VIGLIONE

Co-Supervisor

Prof. Günter BLÖSCHL

Candidate

Matilde ROSSI

DATA LAUREA

October, 2024

Acknowledgments

I would like to thank, in first place, my supervisor Professor Alberto Viglione for his great help and constant attention and enthusiasm with which he guided me through this master thesis. A further thank is due to Professor Günter Blöschl from TU Wien where I carried out my thesis and where he supported me with interest and passion.

I want also to thank all the extraordinary colleagues from TU Wien and their warm hospitality with which I was received during my three months of visiting. In particular, I'm so grateful to Dušan, Riccardo and Luigi because they have been not only colleagues, but true friends with which this experience has been unforgettable.

A special thanks to my dear friends Coralie, Alessia and Serena who, during these two intense years of study, have backed me up and always sustained me. Sharing this moment with you simply adds that little extra to this special day.

Finally, I have to thank my parents who, not only allowed me to study at Politecnico of Turin, but especially because they continuously support all my choices and my student career, conveying me trustfulness and passion.

Abstract

Nowadays, the water quality management has become a crucial concern all around the world and particularly in Europe. It is negligible remarking the importance of water in natural and human environment, but its quality and so its evolution through time could be fundamental aspects to understand its dynamics. The evolution of water quality in Europe from 1900 to today has been characterized by significant changes influenced by various factors such as industrialization, urbanization, agricultural practices, technological advancements, and environmental regulations. Tracking water quality dynamics in Europe from 1900 to the present involves understanding various historical, environmental, and socio-economic factors that have influenced water quality over time as internal and external forcings.

For this reason, the aim of this thesis is that one to develop a qualitative socio-hydrological model able to reproduce the water quality evolution in Europe over the 20th and the 21st centuries and, moreover, to foresee its changes over the imminent future. It is important to highlight the background of this analysis: we look at a socio-hydrological model, an unconventional hydrological study that accounts for the water-human interactions and relationships.

Consequently, in the following chapters, the socio-hydrological model is worked out following a step-by-step procedure: 1) The main variable, the water quality, is analysed considering its evolution over the past years in Europe. A general statement of the phenomenon is given, the interactions between internal and external variables, the domain and the scale of the phenomenon are defined; 2) The perceptual model is built up by developing the causal loop diagram which defines the feedbacks between each variables: water quality, environmental awareness, industrial polluting, economic health, demographic change and climate change; 3) The internal state variables are chosen and studied deeply; 4) Causal factors must be assessed and the basic relationships between each variable are work out; 5) The functional relationships are built using intuition, data analysis, related studies and consensus principles.

In conclusion, the main goal of this thesis is to model the water quality evolution over time and so, be able to understand the strongest and weakest interactions between each variable in order to predict the possible future challenges for its management and also for the human health. Moreover, the model reveals that the most

effective strategy to improve the water quality is not merely enacting green laws, setting limits, or investing in sustainable policies, as these actions primarily impact the economy rather than reducing pollution. If the State aims to swiftly address emerging environmental issues, it must focus on enhancing eco-friendliness by prioritizing green and sustainable technologies and production methods.

Moreover, the socio-hydrological model describes the impact of demographic and climate change. In the future, demographic change may either delay or accelerate already known outcomes and has a more pronounced effect on industrial pollution production than on water quality. On the other hand, climate change is a forward-looking variable; its impacts are not apparent in the past or present. Therefore, this qualitative model underscores the importance of studying climate change and implementing mitigation and adaptation measures. Although its impacts may not seem alarming now, the potential future consequences could be much more severe.

TABLE OF CONTENTS

1. Introduction

1.1. The importance of Socio-hydrology

“Socio-hydrology: the new science of people and water that is aimed at understanding the dynamics and co-evolution of coupled human-water systems” (Sivapalan et al., 2012). This is the basic definition of the socio-hydrology: Socio-hydrology is a field that integrates humans and their activities as essential components of water cycle dynamics, with the primary aim of predicting the interactions between them. This emerging discipline shares significant similarities with the already established field of eco-hydrology, which focuses on the co-evolution of vegetation and water availability. While eco-hydrology examines how vegetation and water availability evolve together, socio-hydrology explores how people organize themselves within their environment, particularly concerning water availability. In traditional hydrology, human activities related to water resource management are typically viewed as external factors influencing the water cycle. Today, understanding the co-evolution of interconnected human-water systems and improving predictive capabilities is crucial for ensuring the sustainable management of water, as outlined in Target 6 of the United Nations 2030 Sustainable Development Goals. The development process of socio-hydrology can be summarized in the following figure:

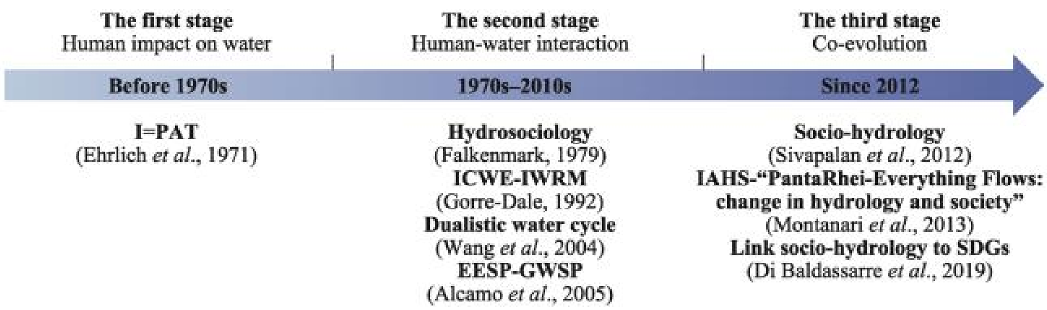


Figure 1.1: The development process of socio-hydrology (Xia et al, 2022)

For most of human prehistory, societies adjusted to the ever-changing hydrological cycle with minimal impact on it. However, following the Agricultural and Industrial Revolutions, human activities began to significantly affect hydrology on both local and global scales. These impacts include water consumption for agriculture, river engineering, changes in land use, pollutant discharge, and anthropogenic climate change, among others. These human-induced hydrological changes triggered changes in human behaviour related to water use, management, and governance, as deeply discuss in *"Coevolution and Prediction of Coupled Human-Water Systems"* by Tian et al. in the *Panta Rhei Book* (2023). Human-induced climate change, population growth, rapidly evolving lifestyles, and the increasing globalization of economic production have accelerated the pace of change. These factors are creating significant challenges in managing water resources at local, regional, and global levels. Effectively understanding and addressing these challenges requires considering the interconnected nature of human-water systems, considering the co-evolution of humans and water (Sivapalan et al., 2012).

The unintended consequences of water management activities typically become apparent only over the long term. However, advancements in technology have significantly shortened this response time. Human use of water resources and alterations to landscapes are increasingly accelerating the dynamics of the water cycle, impacting it on both local and global scales, as well as across timeframes ranging from decades to centuries (Vörösmarty et al., 2000).

Figure 1.2 presents three crucial aspects for understanding and predicting the coevolution of coupled human-water systems: (a) multiscale coupled human-water system structures and their co-evolutionary dynamics, (b) water-related human well-being outcomes that emerge across physical scales and governance levels, and (c) the normative values and goals of individuals and whole societies with respect to water use, conservation, and sustainability (Sivapalan et al., 2014).

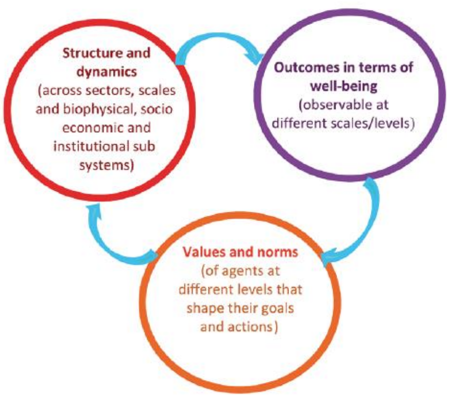


Figure 1.2: Key elements of the water management analysis framework under the new paradigm of coupled human-water systems (Sivapalan et al., 2014).

Several frameworks are used to study the interplay between water and society. They represent a wide variety of approaches to study coupled human-water systems and they can be summarized in eight frameworks (see Figure 1.3): Socio-hydrology (SH), Hydroeconomic Modelling (HM), Large Scale Hydrology (LS), Economics (EC), Physical Geography and Spatial Science (PG), Ecological Economics (EE), Institutionalism (IN) and Critical Geography (CG).

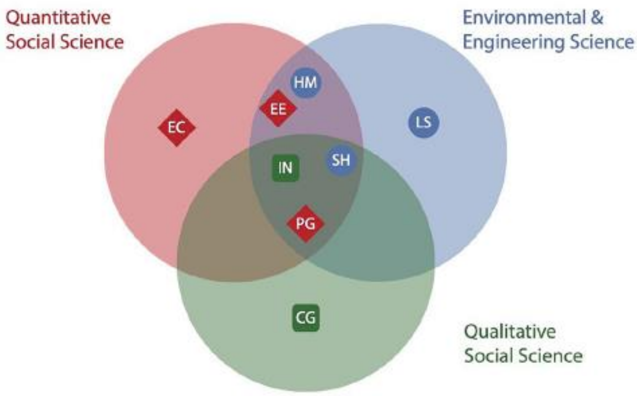


Figure 1.3: Considered frameworks by disciplinary category (Tian et al., 2023)

Socio-hydrology, as a science centred on the interaction between human and water systems, strives to give equal attention to both subsystems by focusing on their two-way feedback loops. Human systems are often modelled through three distinct yet interconnected social variables, which are believed to coevolve with hydrological systems: infrastructure systems (technology), socioeconomic systems (economy), and institutional/regulatory systems (norms, culture, and values). Since socio-hydrology originally developed from efforts to understand interactions across different time scales, the study of temporal dynamics plays a significant role in its research, as it's possible to see in the following Figure 1.4.

In terms of its paradigm and perspective, socio-hydrology is a positive science that aims to develop a generalized understanding of the dynamics within coupled human-water systems (Troy et al., 2015). In natural sciences, understanding is typically achieved through hypothesis-driven research, facilitated by experiments with controlled boundary conditions. However, in the study of coupled human-water systems, defining these boundary conditions can sometimes be challenging. As a result, socio-hydrologic research may often lean towards being exploratory rather than strictly hypothesis-driven (Blöschl, 2017).

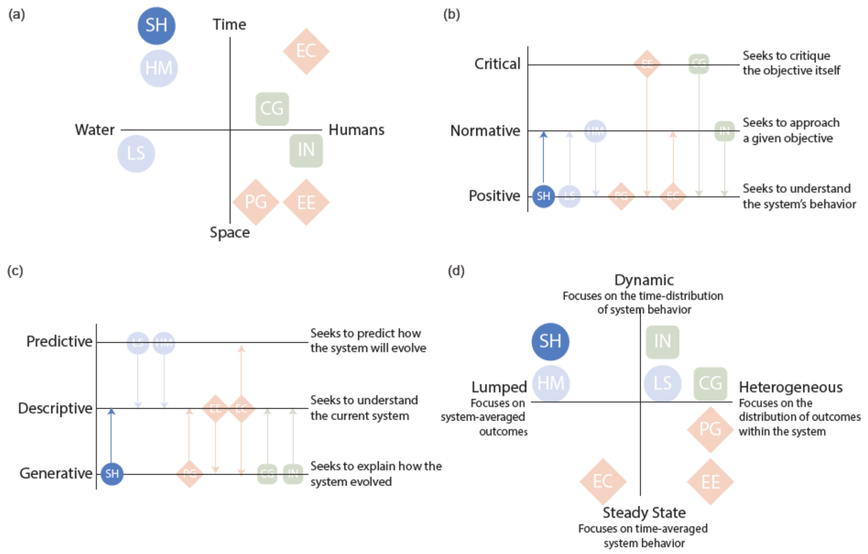


Figure 1.4: Typology placement for Socio-hydrology: (a)Components; (b)Paradigm; (c)Perspective; (d)Aggregation (Tian et al., 2023)

It is also often inductive, rather than deductive, in that it develops models to generate (rather than test) hypotheses by interpreting persistently observed phenomena. In terms of aggregation, as it's possible to understand by the previous figure, socio-hydrology focuses on dynamic phenomena that are based on the time-distribution of system behaviour. However, with increasing attention to heterogeneity in space, socio-hydrology studies have also expanded beyond aggregate (lumped) performance, focusing on system-averaged outcomes.

Xia Jun et al. in their work "Developing Socio-Hydrology: Research Progress, Opportunities, and Challenges" (2022) introduce the question of the opportunities and challenges for the development of socio-hydrology. One potential opportunity lies in the interdisciplinary nature of socio-hydrology, which intersects with various other disciplines and theories, thereby strengthening interdisciplinary theoretical systems. For instance, complex system science is frequently applied in socio-hydrological modelling, while hydro-economics addresses water resource allocation, a critical factor in the interactions between social and natural systems. Another opportunity is the role of socio-hydrology in supporting Integrated Water Resources Management (IWRM) in alignment with Sustainable Development Goals (SDGs). There is an urgent need for effective and scientifically-based water resource management plans, as enhancing the sustainable management of catchment water resources is crucial for addressing the water crisis. However, the discipline of socio-hydrology faces several significant challenges. One major issue is how to overcome difficulties in socio-hydrological modelling and enhance the scientific rigor and accuracy of these models. These challenges include: Human variables, which include social, economic, and political systems, are highly complex and uncertain, complicating the modelling process; Given the long time frames and large spatial scales involved, it is challenging to gather sufficient data for model operation, calibration, and validation; There is often a delay between social impacts and their effects on natural systems, making it difficult to capture and predict interactions accurately; Human societies change rapidly, while changes in the natural environment occur more slowly. This disparity makes it challenging to synchronize and integrate the two systems in models. To address these challenges, improving the capability of models to utilize multisource data effectively is essential (Xia Jun et al., 2022).

1.2. An Overall Vision of the Phenomenon

A socio-hydrological model is a very powerful tool for the analysis of water-human systems and for forecasting their future dynamics. For these reasons, several examples of socio-hydrological studies can be found in literature and their topics vary widely. The 80.81% of the scientific studies are articles (Herrera-Franco et al., 2021) and their development shows an exponential growth from 2012. The contribution by different countries facilitates the understanding of the relationship between knowledge and its institutions.

Socio-hydrology is such widely spread all around the world and is becoming so important for researchers that it can be linked with several topics. The most popular fields of studies are flood risk management, droughts, groundwater and, more in general, water supply and water resources. For example, T.H.M van Emmerik et al. (2014) build a socio-hydrologic model to understand the competition of water between agriculture development and environmental health in the Murrumbidgee River Basin (Australia); or D. Liu et al. (2015) published an article regarding a conceptual socio-hydrological model of interactions between social, economic and ecological systems in the Trim River Basin (China); moreover, Xi Chen et al. (2016) developed a socio-hydrological model for the Kissimmee River Basin (Florida) to simulate the interactions between community interests and hydrology; finally, an urban socio-hydrological model was built by Bin Li et al. (2019) for exploration of Beijing's water sustainability challenges, showing a transformation from productive to domestic water use. However, these are only few of the articles about this new science and citing all of the past studies will be a very time-expensive work. Anyway, the related literature is quite lacking of models about the water quality dynamics and its interactions with other socio-economic variables. One of these models is here analysed to understand better feedbacks and relationships present. This article is “Exploring water quality management with a socio-hydrological model: a case study from Burkina Faso” by G. Carr et al. (2022), where a socio-hydrological model has been created to capture the interplay between various factors, including local organizations' awareness of water quality issues, land use in riparian zones, agricultural practices, and suspended sediment concentration, which serves as an indicator of water quality. The model generates scenarios for both the current state and potential future pathways, exploring how different riparian land management strategies could lead to improved water quality.

In a broader sense, socio-hydrology could be increasingly utilized to study the evolution of water quality and its interactions with various factors. Protecting water quality has become a crucial issue in modern times, leading to the development of new technologies for water purification, wastewater treatment, and other synergistic technologies. Despite these advancements, new water quality problems have emerged since the mid-20th century, including eutrophication, inadequate water treatment facilities, and pollution from chemical and microbiological substances. Historically, legislation often focused on preserving water quality to enhance supply. Today, water quality remains a significant challenge for both scientists and governments. This challenge is driven by factors such as population growth, insufficient resources and infrastructure, the use of unconventional water sources, the proliferation of emerging pollutants like antibiotics and microplastics, and the impacts of climate change and variability. In response to these persistent and new challenges, human experience has led to the development of strategies and measures aimed at protecting water quality and ensuring the sustainability of water supplies for the future (N. Angelakis et al., 2022). Such actions are: revisit and recognize the chronological evolution of water quality technologies as an essential guarantee against water quality preservation through the ages; revise water resource management programs within an integrated and holistic approach; provides links between climate change and water quality in order to understand how this changing climate parameters influence the spreading of pollutants to water bodies and even to humans.

Water quality dynamics are inherently complex, making it essential to construct a socio-hydrological model to gain a deeper understanding of these dynamics. While the construction of such a model might seem straightforward, it requires careful analysis and adherence to a precise methodology. The process begins by examining the broad field of water quality and identifying all potential influencing variables. Each of these variables is assessed to understand their effects and interactions. The model incorporates not only environmental factors but also political, economic, and social variables to provide a comprehensive view. Ultimately, a dynamic and holistic approach enables the model to evaluate both the evolution of water quality over time and space, as well as the strength and impacts of various influencing factors. By analysing past practices and solutions, we can gain insights that inform current and future strategies for managing water quality (N. Angelakis et al., 2022).

1.3. Scope and Operational Framework

The main goal of this study is to develop a qualitatively socio-hydrological model able to shape the water quality evolution from 1900 until the closest future and to explain the operational steps to obtain it. In fact, most of the socio-hydrological studies in literature presents the final model and the related results, without showing the steps before them. For this reason, this thesis would be like a help manual which drives the reader in a step-by-step building process of the model and let to understand the complexity and research behind it. In order to do so, the article by M. Sivapalan and G. Blöschl (2015) is taken as a guide to develop this model: "Time Scale Interactions and the Coevolution of Humans and Water" provides the steps necessary to be followed and the directions for the model development.

First of all, the phenomenon has to be described and analysed, also doing some literature research and the domain and the scale must be clarified. A brief history of the problems has to be written in order to understand all of the possible interaction between the variables. Secondly, the perceptual model is built for the description of the system and, so, the causal loop diagram for the representation of each feedback. The causal loop diagram is very useful in order to understand links, chains and possible connections between variables and it's dynamic, so it can be modified whenever required. Then, the internal state variables are chosen and analysed in time and space and, consequently, causal factors affecting them are considered. Finally, the functional relationships between each variable are computed and the final model developed. For them, ordinary differential equations are formulated and, using the “R” software, computed. This software is very powerful because it can easily resolve mathematical systems of equations and show results in real-time. Once having found the most suitable ones, “R” plots the results and computes also the strength of each relationships. The previously indicated article considers two more steps, the parameter estimation and the model validation and uncertainty. These ones aren’t developed in this thesis due to the qualitatively nature of the model and for the fact that they will be quite time consuming for a three months’ work. At the end, some conclusions and considerations are shown and an interesting question is made: how much impactful are the demographic change and the climate change on the water quality over time? The socio-hydrological model will be able to answer to this question and show interesting results that could be very useful both in the present and in the future.

2. The Socio-hydrological Model Structure

This entire work is based on the paper “Time Scale Interactions and the Coevolution of Humans and Water” by M. Sivapalan and G. Blöschl (2015), where the authors present a coevolutionary view of hydrologic systems, spinning around feedbacks between environmental and social processes working across different time scales. The term "coevolution" refers to the simultaneous adaptation of closely interacting animal or plant populations, where each population exerts a significant selective influence on the other (Ehrlich and Raven, 1964). This concept is often represented mathematically using dynamical systems. In this framework, the change in a system's state over time is determined by its current state, with future states evolving in a predictable manner based on the present state. A coevolutionary dynamical system could be simply written as:

$$Fast\colon\ \varepsilon\frac{dx}{dt} = f(x, X\colon \theta)$$

$$Slow\colon\ \frac{dX}{dt} = g(x, X, Y\colon \phi)$$

$$Coupled\ Slow\colon\ \frac{dY}{dt} = h(X, Y\colon \psi)$$

Equation 2.1: Fast, Slow and Coupled slow equations of a coevolutionary dynamical system

where x is the variable that varies on a fast time scale, X is the same variable but it varies on a slow time scale and Y is a different variable coupled with X and also operates in a slow time scale. The interaction between the first and second equations in a model represents the relationships between different time scales, while the coupling between the second and third equations captures the coevolutionary process. The parameters θ, ϕ, and ψ are vectors representing different system parameters, and ε denotes the ratio of small- to large-time scales. This parameter ε is introduced to align the functions f and g to the same order of magnitude. Slow-fast systems illustrate feedback mechanisms across varying time scales and between different variables. Positive feedback occurs when an increase in one variable, say x, leads to an increase in another variable, Y, and vice versa, thereby amplifying any fluctuations. Conversely, negative feedback occurs when an increase in x leads to a decrease in Y, stabilizing the system by damping fluctuations. These feedback mechanisms are captured in the functions f, g, and h.

An essential feature of dynamical systems is the presence of equilibrium points, where the state variables remain constant over time. These equilibrium points can be categorized as stable, unstable, or saddle points, based on the eigenvalues of the linearized equations near these points. In coevolutionary hydrological systems involving humans, the overall structure is typically divided into four subsystems: Natural Resources System includes rivers, lakes, and aquifers. It generally experiences gradual degradation if overused; Infrastructure System, comprising canals, reservoirs, and wells, evolves slowly over time; Socioeconomic System encompasses water-related human activities. It is initially based on the exploitation of natural resources, leading to short-term dynamics. Over the long term, however, the accumulation of wealth can drive technological advancements and population growth through natural increase and migration; Institutional System includes administration, legislation, and regulation. It is responsible for making decisions that balance short-term human water use with long-term environmental needs.

Given the complexities of feedback loops, non-linearities, thresholds, and heterogeneities, various models have been developed to represent the interactions between human and natural systems. These models include: Stylized Models (Anderies, 2000) which are simplified representations that capture essential features of the system; Agent-Based Models (Evans and Kelley, 2004; An, 2012; Noel, 2015) that simulate the actions and interactions of individual agents to explore complex dynamics. Comprehensive System-of-Systems Models (Yaeger et al., 2014) that are detailed models that integrate multiple subsystems to represent the entire coupled human-nature system. One of the models used to better understand the dynamics and the complex behaviour of a hypothetical coupled human-nature system is the simple, stylized model known as Wonderland (Sanderson, 1994; Milik et al.,1996). In this model, humans utilize natural capital (resources) to generate economic output, which in turn leads to pollution. This pollution can deplete natural capital and potentially constrain long-term economic growth. The model is formulated as a system of differential equations, incorporating various parameters as scaling factors. By computing different scenarios, the model helps visualize the potential future responses of the entire system.

Dynamic models of human-water interactions, such as Wonderland, share many similarities with models of natural systems. However, they incorporate additional elements related to technology, socio-economics, and institutions. The inclusion of human factors introduces variability in values and preferences regarding water management over time and space, a topic thoroughly examined by Elshafei et al. (2015). The evolution of societal preferences and values is a crucial aspect of modern water resource management. To illustrate the diverse range of emergent phenomena resulting from human-water feedbacks, Figure 2.1 is provided, highlighting the involved time scales (Sivapalan and Blöschl, 2015).

Change of Values or Preferences in Time	Difference of Values or Preferences in Space: No Difference of Values or Preferences in Space	Difference of Values or Preferences in Space: Difference of Values or Preferences in Space/Place
No change in values or preferences in time	Collapse of system (e.g., dry out of Aral Sea due to short-term irrigation and long-term water balance and economy) [*Cai et al.*, 2002] Resource capture by the elite (e.g., building of Narmada Dam, India, and loss of livelihood of locals due to short-term business interests and long-term resource depletion) Lock-in of groundwater depletion (e.g., groundwater overexploitation in India due to subsidized energy, interaction of short-term pumping at no cost, and long-term water balance) Increase in vulnerability due to overreliance on technology (e.g., collapse of the Mayas due to drought and dependence on reservoirs [*Lucero*, 2002])	Upstream-downstream conflicts (e.g., Mekong hydropower development affecting livelihood of downstream rural/fishery communities due to short-term flow disruption and long-term environmental degradation and economic development) Large-scale water transfers (e.g., South-North project, China, associated with short-term flow disruption and long-term food security) Operation of hydropower reservoir for flood mitigation (e.g., Orlik reservoir upstream of Prague, short-term loss of electric power, and long-term flood risk reduction)
Change in values or preferences in time	River training and restoration (e.g., Sacramento river management due to short-term flooding and long-term change in environment) Levee effect (e.g., settlement pattern in the Netherlands due to short-term flood protection and long-term flood plain encroachment) Efficiency paradox (e.g., increase in water consumption in the Tarim basin, China, in spite of increased water use efficiency as a result of increase of agricultural land, short-term irrigation, and economic gains and long-term policy changes)	Adaptation: peak water paradox (e.g., increasing upstream water extraction and downstream degradation in the Murrumbidgee basin, Australia, short-term economic decisions, and long-term environmental damage) Virtual water trade (e.g., food trade from South America to Asia, short-term economics, and long-term food security) Coastal hypoxia (e.g., hypoxic zone in Gulf of Mexico resulting from agriculture in Mississippi basin, short-term profits, and long-term receiving water quality degradation)

Figure 2.1: Emergent phenomena arising from human-water feedbacks (Sivapalan and Blöschl, 2015)

At this point of the analysis, in order to generate and test hypotheses about mechanisms causing these phenomena, it is useful to frame and model them in a quantitative or qualitative way. The framing and modelling of socio-hydrological systems can vary in detail based on their intended purpose. Models may range from highly detailed, comprehensive representations of specific locations to more stylized versions that abstract the system with less detail but aim to capture the holistic aspects of the entire system in a general manner. The development of socio-hydrological models follows steps akin to those used in coupled dynamic environmental models, with additional considerations specific to socio-hydrology. Similarly, the process for developing coupled dynamic environmental models is similar to that for typical hydrological models, but with additional factors to account for. The previously indicated steps are 7 and are described in the following list (Sivapalan and Blöschl, 2015):

-Step 1 (Phenomenon, domain, scale): This step involves formulating a broad description of the phenomenon and defining the boundaries of the problem, both spatially and in terms of the governing variables. The phenomenon is typically derived from narratives provided by stakeholders and experts from various disciplines. A crucial aspect of socio-hydrological models is determining which processes to include internally and which to treat as external forces, prescribed as boundary conditions. Ideally, external forces should remain unaffected by the system's behaviour and should be well-defined and known.

-Step 2 (The perceptual model): This step involves creating a perceptual model to describe the system, which includes formulating working hypotheses about the underlying causes of the phenomenon. Given the importance of feedbacks between processes and scales, it is often helpful to begin the perceptual model by drawing causal loop diagrams that illustrate these feedbacks. These diagrams can be based on problem narratives or preliminary data analyses. For example, Figure 2.2 shows a causal loop diagram illustrating the levee effect (Di Baldassarre et al., 2015), which depicts the feedback loops between people and floods in an urban setting, including the impact of levee construction. At this stage, the goal is to conceptually identify the key components of the system and their interactions. The specific state variables will be defined in a later step (Step 3).

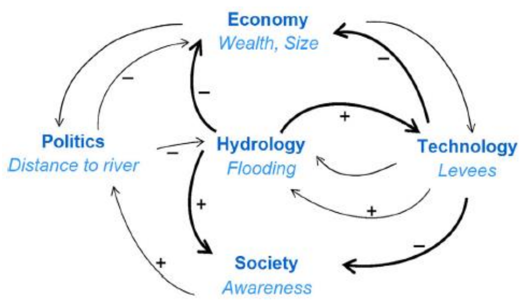


Figure 2.2: Causal loop diagram showing feedbacks between different variables (DI Baldassarre et al., 2013)

-Step 3 (Choice of state variables): State variables are fundamental to the model, making their selection a crucial step in the framing process. Whether a variable is treated as variable or fixed depends on the model's nature and the phenomenon being studied. A common strategy is to begin with a simple model and introduce additional variables only as needed to accurately capture the phenomenon of interest. Classifying variables as "fast" or "slow" can assist in this process, helping to organize and manage the complexity of the model.

-Step 4 (Causal factors that affect state variables): Causal factors influencing each state variable can include external factors (such as precipitation, a country's GDP, or legal conditions), other state variables, or even the state variable itself. The causal loop diagrams developed in Step 2 help guide the selection of these causal factors. These causal factors, also known as external variables, are incorporated into the model as external inputs or forces that affect specific variables within the system without being influenced by the system's behaviour.

-Step 5 (Functional relationships by which causal factors affect state variables): The next step involves specifying the functional relationships that describe feedbacks between state variables and the effects of external forcings. These relationships can be conceptualized through intuition, data analysis (when appropriate data is available), literature on related studies, or consensus principles (such as logistic growth) (Elshafei et al., 2014). Functional relationships between values and human behavioural responses can be determined through choice experiments, where experts or stakeholders provide insights via questionnaires. These functional relationships may operate across different time scales, corresponding to the time scales of the state variables they influence. The equations must be dimensionally consistent, and dimensional analysis can help keep these relationships compact. Non-dimensionalizing the equations may also reduce the number of parameters (Viglione et al., 2014). This step is the most complex and time-consuming, as it forms the foundation of the entire model and dictates how all results are derived.

The above steps are those ones developed in this master thesis but the socio-hydrological modelling frame include also two more steps. As said before, Step 6 and Step 7 aren't implemented in this work because of the lack of time and data availability. However, their explanations aren't negligible and it's thus showed in the following section.

-Step 6 (Parameter estimation): The complexity of coupled processes in socio-hydrological models presents significant challenges for parameter estimation, much more so than in traditional hydrological models (Brun et al., 2001). One approach to managing these difficulties is to treat slow processes as constant when estimating parameters for fast processes, and vice versa, assuming that fast processes average out when estimating parameters for slow processes. For a hydrological model, parameters are typically estimated in the usual manner, treating other components as external factors. Similarly, economic parameters would be estimated from economic data while assuming hydrological conditions are fixed. Parameters related to the dynamics of human values could be estimated using data from stated preference methods or surveys of media sources (Elshafei et al., 2015). After estimating the parameters for the individual components, the entire model is reassembled. This process is then repeated with a focus on the feedback interactions between the model components.

-Step 7 (Model validation and uncertainty): The approach of disassembling and then reassembling the model can also serve as a validation strategy. Individual components of the model can be validated using the standard split-sample method, where one part of the dataset is used for parameter estimation and the remaining part is used for validation. For validating the entire reassembled model, there are two main scenarios: Repeatable Phenomena means that if the socio-hydrological phenomenon of interest occurs repeatedly in space or time, it can be validated using data from different periods or locations. For example, interactions between drought and settlement might be observed in various periods at the same location, or shifts in environmental values might be seen at different locations in the same period. In such cases, data from these different instances can be used to validate the model; Non-Repeatable Phenomena means that if the phenomenon is not repeatable, it will be challenging to validate the model in the traditional sense. In this situation, the model may have limited predictive power beyond the specific case study it was developed for, as the phenomenon cannot be observed again under the same conditions. In summary, validation strategies vary depending on whether the socio-hydrological phenomenon is repeatable or unique, impacting the model's generalizability and predictive capabilities.

After this brief explanation of what is socio-hydrology, its importance and use and which are the main steps to build a socio-hydrological model, it’s time to implement them and work out the system that will be able to replicate the water quality evolution. It’s important to highlight again that this model will be a qualitative one: no real data are implemented, just articles and papers, and the final result must be considered such a good example from which real data can be used and quantitatively discussed. A driven step-by-step procedure is then developed, the final results are computed and thus conclusions are discussed in the last chapter.

3. Preliminary Steps for the Model Formulation

3.1. Phenomenon, Domain and Scale

This chapter refers to the *"Step 1"* described in the paper "Time Scale Interactions and the Coevolution of Humans and Water" by M. Sivapalan and G. Blöschl (2015). Developing a general statement of the phenomenon and setting boundaries require a deep research and study of the main object. Each variable has to be analysed by narratives, papers and articles in order to understand their nature and relationships between each other. So, the main problem under study is how the water quality evolves over time, from the 20th to the 21st century in Europe and try to forecast its features in the future. The internal state variables considered for this study are: the water quality, the industrial polluting activity, the economic health and the environmental awareness. In addition, two external forcing interact with the system, such as the demographic change and the climate change. The first step requires to describe all the variables in general (supporting with papers and reads) and then display their evolution in this time span. So, let's start the analysis from the main variable, the water quality:

-Water Quality: The quality of any body of surface water is a function of either or both natural influences and human activities. Without human influences, water quality would be determined by weathering of bedrock minerals, by atmospheric processes, by hydrological factors and biological processes etc. As a result, water in the natural environment contains many dissolved substances and non-dissolved particulate matter (*"Water Quality for Ecosystem and Human Health" by UNEP)*. Water can also contain substances that are harmful to life, such as mercury, lead, cadmium, pesticides, organic toxins and radioactive contaminants. So, the availability of water and its physical, chemical and biological composition affect not only the ecosystem services, but also the human activities. In fact, water can be used for human consumption and public water supply, in agriculture and aquaculture, in industry, for recreation and for electrical power generation.

Water quality is not a fixed state and cannot be determined by a single measurement. Instead, it fluctuates over time and across different locations, requiring regular monitoring (A. Tzanakakis et al, 2020). The typical chemical, physical and biological parameters measured are: temperature (affecting the speed of chemical reactions and the solubility); dissolved oxygen (influencing inorganic chemical reactions and requiring for aerobic organism metabolism); pH and alkalinity (linked with biological productivity); turbidity and suspended solids (affecting the clarity); salinity and specific conductance; nutrients (nitrogen, phosphorous, silica); metals (mercury); organic matter.

The evolution of water quality over time is also influenced by advancements in treatment technologies. Water supply and treatment methods have been developed and refined alongside urbanization. Additionally, the positive impact of water quality and sanitation on human health, particularly in terms of life expectancy, is well recognized. In many European countries, the significant increase in life expectancy, especially after World War II, is likely attributed to improvements in drinking water quality and hygiene practices (N. Angelakis et al., 2022). However, since the mid-20th century, new challenges in water quality have arisen, including eutrophication, advancements in water treatment technologies, and issues related to chemical and microbiological pollution. In numerous European rivers, phosphorus concentrations have decreased significantly between the periods 1987-1991 and 1992-1996 (Figure 3.1). The decreases were generally attributed to improvements in wastewater treatment and the reduction of phosphorus in detergents (S.C. Nixon et al, 2000).

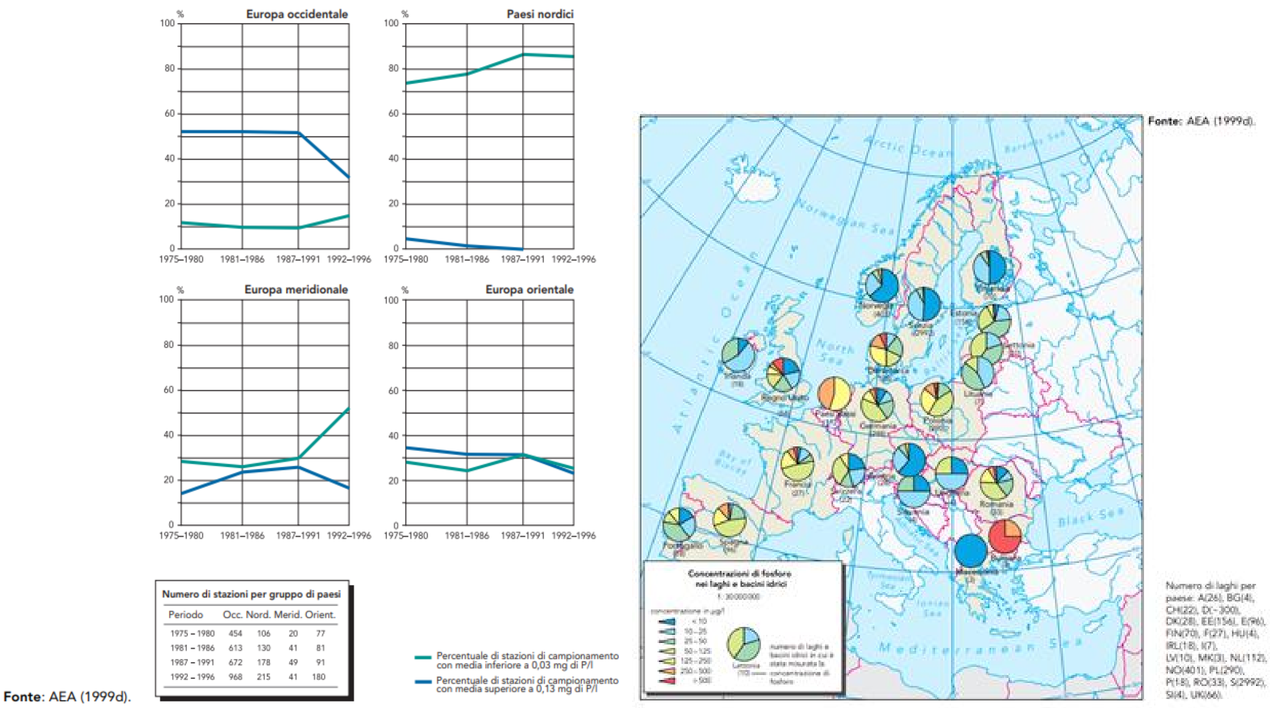


Figure 3.1: Evolution and distribution of the average phosphorous concentration in Europe (S.C. Nixon et al, 2000)

The natural level of nitrates in groundwater is generally less than 10 mg NO3 /L. High levels are due exclusively to human activities, particularly to the use of nitrogen fertilizers and manure, although also local pollution from urban or industrial sources can be significant. Major problems due to presence of pesticides in water underground have been reported by Austria, Cyprus, Denmark, France, Hungary, Republic of Moldova, Norway, Romania and the Republic Slovakian (Figure 3.2).

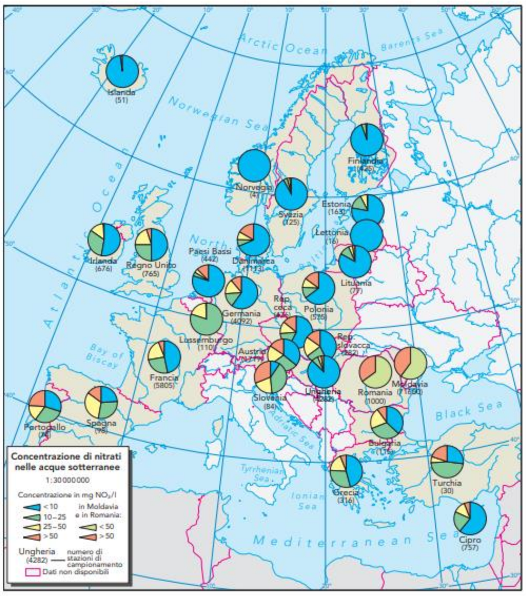


Figure 3.2: Nitrates concentration in the groundwater (Source: AEA 1998)

An attempt to improve the water quality was introduced by the European Union (EU), in 2000, with the Water Framework Directive (WFD) with the title "Directive 2000/60/EC of the European Parliament and of the Council establishing a framework for the Community action in the field of water policy" was adopted. The WFD required to assess water management status and the quality of water resources by using a plethora of water quality-related parameters (e.g., specific nutrients and organic compounds). Overall, the evolution of water quality in Europe from 1900 to today reflects a transition from widespread pollution and degradation to greater awareness, regulation, and investment in sustainable water management practices. The current issues and challenges with regard to the management and protection of water quality include global changes in population and urbanization, lack of infrastructure, use of nonconventional water resources, spreading of emerging pollutants and contaminants (e.g., antibiotics and microplastics), and climatic variability impacts. In fact, increased drought events due to changes in climate characteristics may further exacerbate water scarcity and water supply. An opposite effect of climate change includes intensive precipitations causing extreme hydrological events and the spreading of pollutants. Due to climate change, wet areas become wetter and dry areas become drier but, most importantly, rainfall is very intense. Most existing treatment systems were not designed to operate under very intense rainfall events. Anyway, the influences of climate change will be discussed later in the external variables section. For all these reasons, the water quality variable will be affected by climate change and industrial polluting production. In order to better understand the condition of the water quality in Europe, the WISE Water Framework Directive Quality Elements map containing information from the 2nd River Basin Management Plans (RBMPs) is reported in Figure 3.3. The map shows the ecological status or potential of surface water bodies based on their quality elements status value.

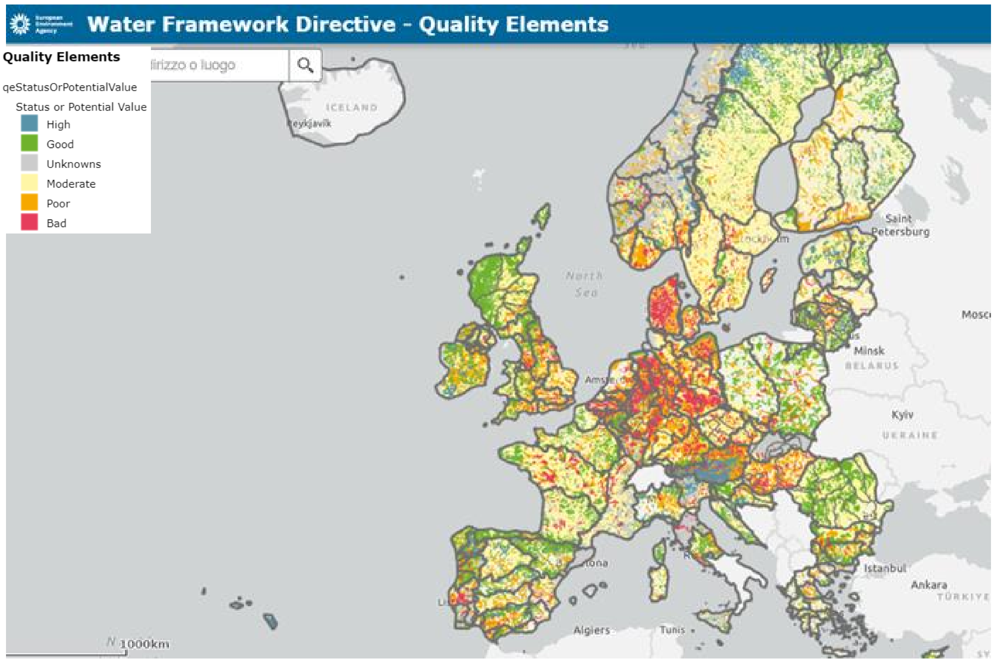


Figure 3.3: Ecological status of surface water bodies in Europe (Source: European Environmental Agency)

-Environmental Awareness: The very term ‘environmentally conscious’ refers to being aware of the environment and the threats to the biosphere. Being environmentally conscious comes hand in hand with taking responsibility for solving environmental issues. However, environmentally conscious thinking is not enough: practical action is also needed. Global warming and the danger of a potential climate catastrophe are widely known. Still, the collective action against industry practices, destructive customer behaviours, and other driving factors of global warming are not adequately addressed. Hence, the goal is not only to educate the public but also to encourage others to act. This is where environmental awareness comes into the picture. According to the European Environment Agency, environmental awareness is defined as follows: *"The growth* *and development of awareness, understanding, and consciousness toward the biophysical environment and its problems, including human interactions and effects. Thinking ecologically or in terms of ecological consciousness."* So, as possible to understand, the environmental awareness isn't a physical variable, but more a conceptual one. Its quantification will be possible only considering the number of policies, laws, polluting limits or the investments in more green production technologies and innovation.

The environmental awareness evolution can be summarised in five main steps (S. Juuti and S. Katko, 2005):

1- Early 20th Century when industrialization and urbanization led to increasing pollution of water bodies. Basic sanitation measures were spurred due to concerns about waterborne diseases and early water quality laws were often limited in scope and primarily focused on preventing obvious sources of contamination;

2- Post-World War II Era when rapid industrial expansion and population growth exacerbated water pollution problems, leading to visible degradation of water bodies. In response, governments began enacting more comprehensive water quality laws and regulations, such as the U.S. Federal Water Pollution Control Act (1948), which was later expanded and strengthened as the Clean Water Act (1972). Advances in analytical techniques allowed for better monitoring of water quality parameters;

3- The Environmental Decade in 1970s saw a surge in environmental awareness and legislative action, such as The Clean Water Act (CWA) of 1972 in the United States established the basic structure for regulating pollutant discharges into U.S. waters;

4- Globalization of Environmental Issues in 1980-1990 when international cooperation on water quality issues increased with the adoption of agreements like the Helsinki Convention on the Protection and Use of Transboundary Watercourses and International Lakes (1992). Efforts to address specific water quality challenges, such as eutrophication in lakes and rivers, gained traction, leading to the development of targeted regulations and management strategies;

5- 21st Century in which water quality management has become increasingly integrated with broader environmental and sustainability goals. Laws and regulations continue to evolve to address emerging water quality challenges, such as emerging contaminants (e.g., pharmaceuticals, microplastics) and the impacts of climate change on water availability and quality.

In the following Figure 3.4, the evolution of the European monitoring network is showed to better understand the increasing of the environmental awareness over time (L. Beck et al., 2010):

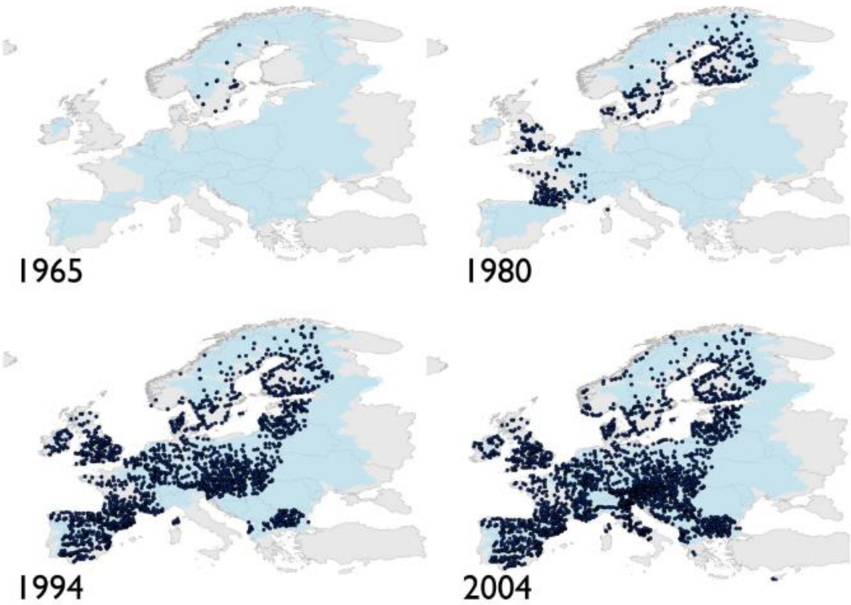


Figure 3.4: Evolution of the European water quality monitoring network from 1965 to 2004 (L. Beck et al., 2010)

Moreover, a timeframe of the main laws and policies regarding the water quality and showing the growth of the environmental awareness is reported, by S. Kirschke et al., 2020.

-Clean Water Act (CWA) (United States): The primary federal law in the United States governing water pollution. It regulates the discharge of pollutants into the nation's surface waters and sets water quality standards for surface waters.

-European Water Framework Directive (EWFD): A European Union directive that establishes a framework for the protection and management of water resources across Europe. It sets environmental objectives for surface waters and groundwater and requires member states to develop river basin management plans.

-Water Framework Directive (WFD) (United Kingdom): Legislation that implements the European Water Framework Directive in the United Kingdom, aiming to improve and protect water quality in rivers, lakes, estuaries, and coastal waters.

-Safe Drinking Water Act (SDWA) (United States): Legislation that regulates the quality of drinking water in the United States, including standards for contaminants and requirements for water treatment and distribution systems.

For all these reasons, the environmental awareness will be influenced by the water quality and by the economic health variables.

-Industrial Polluting Production: This variable is very impactful on the water quality evolution. In fact, although harmful substances have both natural and human-produced sources, the last ones far overshadow natural sources. This variable reflects all the industrial activities that, with them production spread contaminants polluting water bodies. As we can see from the figure below, the industrial sector is that one with the largest number of consequences on the environment (by UNEP):

Consequence:	Sedimentation	Eutrophication	Thermal pollution	Dissolved oxygen	Acidification	Microbial contamination	Salinization	Trace metals contamination	Mercury	Non-metallic toxins	Pesticides	Hydrocarbons	Micronutrient depletion
Sector:													
Agriculture	✓	✓	✓			✓	✓	✓			✓		
Urban use	✓	✓	✓	✓		✓	✓	✓			✓	✓	
Forestry	✓	✓	✓								✓		
Hydroelectric power generation and water storage	✓	✓	✓	✓					✓				✓
Mining	✓	✓	✓	✓	✓			✓		✓			
Industries	✓	✓	✓	✓	✓		✓	✓	✓	✓		✓	

Figure 3.5: Relationship between human activity by economic sector and consequences of these activities to aquatic ecosystem (by UNEP)

Since 1900, the evolution of industrial pollution has been marked by notable shifts in industrial practices, technological progress, regulatory measures, and changing societal attitudes toward environmental protection. Below is an overview of how industrial pollution production has developed over the past century (S. Juuti and S. Katko, 2005):

1- Early 20th Century (1900-1940s) witnessed rapid industrialization in many parts of the world, industrial processes during this period often lacked environmental controls, resulting in significant pollution of air, water, and soil. Common pollutants included smoke and particulate matter from factories, untreated wastewater from industrial processes, and toxic chemicals released into the environment.

2- Post-World War II Era (1940s-1960s) saw further industrial expansion and technological development, leading to increased production and pollution. Advances in manufacturing processes and the widespread use of synthetic chemicals contributed to new forms of pollution, including air emissions of sulphur dioxide and nitrogen oxides, as well as toxic industrial waste. Growing concerns about the health and environmental impacts of pollution, along with high-profile environmental disasters such as the Minamata mercury poisoning in Japan (1956-1960s), spurred calls for stronger regulation.

3- The Environmental Decade (1970s) marked a turning point in the regulation of industrial pollution, with the emergence of the modern environmental movement and the enactment of landmark legislation in many countries. The Clean Air Act (1970) and the Clean Water Act (1972) established comprehensive regulatory frameworks for controlling industrial emissions and wastewater discharges.

4- Technological Advances and Regulatory Expansion (1980s-1990s) saw significant technological advancements in pollution control technologies and industrial processes. Regulations governing industrial pollution became more stringent, with a focus on pollution prevention, waste minimization, and the adoption of cleaner production methods. International cooperation on environmental issues increased during this period, leading to the negotiation of agreements such as the Montreal Protocol (1987) to address ozone depletion and the Basel Convention (1989) on the control of transboundary movements of hazardous wastes.

5- 21st Century when industrial pollution continues to be a significant environmental concern, particularly in emerging economies experiencing rapid industrialization. Efforts to address industrial pollution have increasingly focused on promoting sustainable production practices, resource efficiency, and the adoption of clean technologies.

Overall, the evolution of industrial pollution production since 1900 reflects a complex interplay of technological, economic, regulatory, and social factors. While significant progress has been made in reducing industrial pollution in many parts of the world, challenges remain in achieving sustainable industrial development and protecting human health and the environment from the impacts of industrial activities. For all the above reasons, this variable will be affected by the demographic change and economic health variables. In the Figure 3.6, the trend of pollutants released into water and the gross value added (GVA) from industry sector in the EU-27 from 2010 to 2021 is showed:

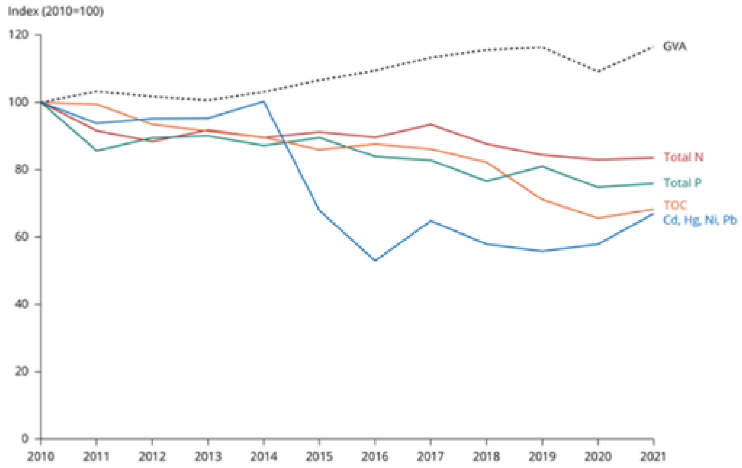


Figure 3.6: Trend of pollutant releases into water in the EU-27 from 2010 to 2021 by using 2010 releases values as reference. In addition, gross value added (GVA) from the industry sector is added (Source: EEA EU)

-Economic Health: This concept was initially challenging to define and was often identified as the capital of a country. In this context, "capital" is more commonly associated with funds used for productive or investment purposes. Companies use capital to finance the ongoing production of goods and services, with the aim of generating profit. They invest their capital in various assets and activities to create value (*"Capital: Definition, How It's Used, Structure and Types in Business" by Marshall Hargrave, 2024*). Capital investment enables research and development, which is the initial step in bringing new products and services to market. By acquiring additional or improved capital goods, companies can enhance labour productivity, becoming more efficient in their operations. Upgrading to newer equipment or facilities allows for faster production of more goods, thereby boosting efficiency. This increase in efficiency contributes to economic growth and results in a higher gross domestic product (GDP) for the country. GDP represents the total monetary or market value of all finished goods and services produced within a country during a specific period (*"How Capital Investment Influences Economic Growth" by Ross, 2024*). However, quantifying the capital invested in industrial pollution production and establishing a clear relationship between these two variables is challenging. As a result, this variable is often referred to as "economic health." Why? Because countries globally assess their prosperity, and one of the most common, though imperfect, measures of a country's prosperity is gross domestic product (GDP). While GDP and its fluctuations are widely used as indicators of a nation's overall economic output, economists have long acknowledged that GDP is an incomplete measure of economic health. For instance, GDP does not account for pollution production, nor does it consider factors like population health, education, and other aspects tied to overall well-being. Because of this, various governments and organizations have started following alternative measurements such as the Human Development Index, Better Life Index and Genuine Progress Indicator (*"Beyond GDP: Three Other Ways to Measure Economic Health" by Praew Grittayaphong, 2023)*. For example, the HDI emphasizes people and their capabilities, instead of economic growth alone, for assessing the development of a country. This index consists of three categories: health, education and standard of living. HDI takes a more comprehensive approach than GDP growth with a view of the economy that includes human development. However, the indicator is not perfect because it does not reflect other important human development factors such as inequalities, empowerment or poverty, as the program website points out.

The Better Life Index (BLI) identifies as essential to health relate to material living conditions (housing, income, jobs) and quality of life (community, education, environment, governance, health, life satisfaction, safety and work-life balance). It allows to personalized rankings but not to make comparisons over time.

The Genuine Progress Indicator (GPI) is designed to measure the health of a country by considering economic, environmental and social factors. The economic aspect may include variables such as personal expenditures and income inequality. For the environmental aspect, the index mostly includes factors such as ozone depletion and climate change. Crime, family breakdown, and more are included in the social aspect. The benefits include costs like pollution, values like volunteering. Some limitations come up for the subjective of some variables. This variable will be affected by the industrial polluting production (positively) and by the environmental awareness variables (negatively).

Figure 3.7: Three economic health measures beyond GDP (Source: Federal Reserve Bank of St. Louis)

Then, after the explanations of all the internal variables of the system, it's time to describe the two external variables, or external forcing, that will be added at the model only in the final step. As mentioned before, the external forcing are variables which influence only few of the internal variables and their evolution doesn't affect the whole system. These two external variables are the demographic change and the climate change.

-Demographic Change: This variable refers to the change in the population density and so to the population growth over time. In comparison with the other variables, this is the easiest to be defined and even to be quantified and it will be affected only the industrial polluting production. The world population will approach 10 billion in the next 30 years, and a significant proportion of this growth will take place in developing countries. Today, more than half of the world population is living in urban areas, particularly in highly dense cities; by 2050, more than two-thirds of the population will live in urban areas (A. Tzanakakis et al., 2020). On the basis of the facts that the available fresh water supplies on earth will remain the same, being unevenly distributed, these urban areas will become water-stressed, enhancing the conflicts among users, particularly among the urban, agricultural, and industrial sectors.

At the same time, significant impacts are expected on availability and quality of water resources, quality of soil resources, potential water demand increases, flood intensity and frequency, and ecosystem functioning and derived services impacts that are tightly inter-connected and influenced by background climate and topography characteristics of the areas as well as by human activities. The demographic change refers to the shifts and trends in the population characteristics of a specific region, country, or the world as a whole. These changes can occur due to various factors such as: population growth or decline (death rates and birth rates); age structure (children density, working-age adults, elderly individuals); migration patterns; fertility rates; mortality rates; urbanization; population composition.

In the following Figure 3.8, the total population and the annual population change in EU-27 from 1960 to 2070 is showed:

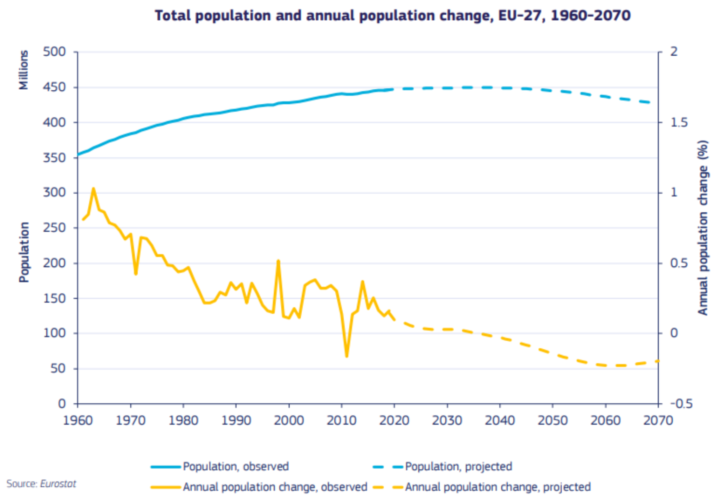


Figure 3.8: Total population and annual population change in EU-27 (Source: Eurostat)

-Climate Change: This is the last variable analysed and very complex one to be deeply defined and quantified. For this model, the term climate change refers to the entire phenomenon: climate change refers to significant and lasting changes in Earth's climate patterns over an extended period. These changes are largely driven by natural processes and, more recently, by human activities such as the burning of fossil fuels, deforestation, and industrial activities that release greenhouse gases into the atmosphere. "Climate change" refers to a range of phenomena, including global warming, shifts in precipitation patterns, changes in the frequency and intensity of extreme weather events (like hurricanes, droughts, and heatwaves), melting polar ice caps and glaciers, rising sea levels, and alterations in ecosystems and biodiversity distribution. The impacts of climate change are extensive, affecting both natural and human systems, including agriculture, water resources, public health, infrastructure, economies, and geopolitical stability. Increased droughts due to changing climate conditions may further intensify water scarcity and challenges in water supply, posing a significant threat to climate-vulnerable regions now and in the future (N. Angelakis et al., 2021). An opposite effect of climate change includes intensive precipitations causing extreme hydrological events and the spreading of pollutants. Due to climate change, wet areas become wetter and dry areas become drier but, most importantly, rainfall is very intense. Most existing treatment systems were not designed to operate under very intense rainfall events. Precipitation is expected to increase at higher lat. An increase in the ambient temperature and changes in the frequency and duration of precipitation is expected to change the intensity, frequency and duration of flood and drought events. Inland waters will be influenced by these changes, altering the water temperature, flow regimes and water levels. Warmer water will decrease the saturation of dissolved oxygen, increasing the likelihood of anoxia. Warmer temperature will increase the decomposition rate of organic material, increasing the BOD. These changes could accelerate the biological productivity as well as increase plant biomass (algae bloom). At the same time, increased floods could damage treatment plants increasing the risk of contamination. On the other side, in areas undergoing droughts concentration of pollutants will increase and also the salinity, so the dilution effects will be reduced.

In the following Figure 3.9, a scheme of the possible climate effects is reported:

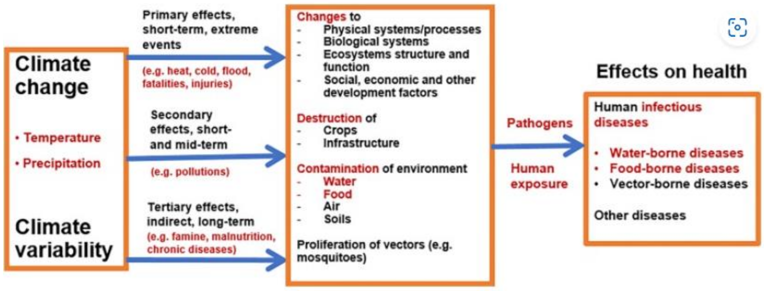


Figure 3.9: Possible future effects of climate change (Source: EEA EU)

In the end, as a conclusion of this section, it’s extremely important to recap how all the previous described variables have been changing from 1900. So, the following time-line is built:

- 1900: industrialization and urbanization across Europe leads to widespread pollution of water bodies from industrial effluents and domestic sewage. At the beginning of the 20th century, water quality in Europe was generally poor, particularly in industrialized regions where rapid urbanization and industrial growth led to extensive pollution of water bodies. Waterborne diseases spread out such as cholera and typhoid, resulting in high mortality rates and public health crises. Limited regulations were minimal, and there was limited awareness of environmental issues.

-Early to Mid-20th Century: the two World Wars had significant impacts on water quality due to industrial production, destruction of infrastructure, and contamination from military activities. After World War II, there was a focus on rebuilding infrastructure, including water treatment plants and sewage systems, which improved water quality in some areas. Environmental movements began to emerge, advocating for cleaner water and stricter pollution controls.

- Late 20th Century: Governments began implementing environmental regulations to control pollution discharges into water bodies. The European Union (EU) played a significant role in setting standards and directives for water quality management. During the late 20th century, there was a growing emphasis on environmental protection and sustainable development in Europe. requirements regarding the agricultural irrigation and aquifer recharge as well as monitoring actions. Advancements in water treatment technologies, such as chlorination, filtration, and sewage treatment, improved the quality of drinking water and wastewater discharge. Advents in water treatment technology, such as the adoption of biological treatment processes and improved filtration methods, helped to enhance the quality of treated wastewater. The “Nitrate Directive” was developed and acts under the WFD. It concerns the protection of waters against pollution caused by nitrates from agricultural sources. Many polluted rivers and lakes underwent clean-up efforts, aided by government initiatives and public awareness campaigns. However, rapid industrialization and population growth continued to place pressure on water resources, resulting in ongoing pollution and degradation of water quality. Stricter regulations were enacted to address water pollution, including the implementation of the European Union's Urban Wastewater Treatment Directive in 1991, which aimed to improve the collection and treatment of urban wastewater.

-21st Century: the WFD, adopted in 2000, established a framework for the protection and management of water resources across Europe, setting objectives for achieving good ecological status in water bodies. Countries invested in upgrading water treatment infrastructure to meet WFD standards and ensure safe drinking water for their populations. Concerns grew over emerging contaminants such as pharmaceuticals, microplastics, and chemical residues from agriculture, prompting research and monitoring efforts to address these issues. Climate change brought about changes in precipitation patterns, temperature, and hydrological cycles, affecting water availability and quality in various regions of Europe.

- Present: efforts to monitor and manage water quality continue, with a focus on achieving the objectives set by the WFD and addressing emerging challenges such as climate change and emerging contaminants. Public awareness of water quality issues remains high, and there is increasing public participation in water management initiatives and environmental conservation efforts.

3.2. Perceptual Model

In this chapter, which represents the *"Step 2"* of the paper by Sivapalan and Blöschl (2015), a perceptual model is computed: it describes the system and the working hypotheses about the causes of the phenomenon. A causal loop diagram (CLD), then, is built as it is a graphical representation of the causal relationships among variables in a system. All the feedbacks are worked out by the previous careful analysis and also the following figures are used as guidelines for the causal loop diagram:

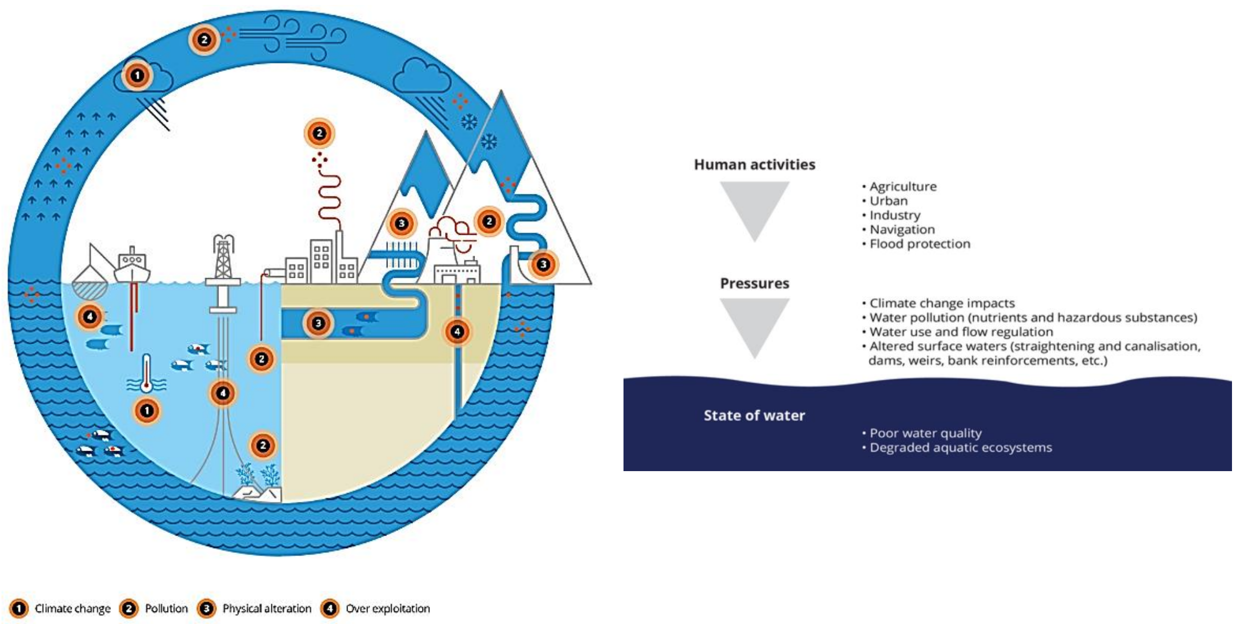


Figure 3.10: Main issues affecting the water quality (left) and relationship between driving force, pressures and state of the art (right) by EEA EU

Then, for all the variables, the main feedbacks are worked out and described in the following lists. The red arrows represent negative feedbacks, instead the green arrows stand for positive effects.

Water Quality (**W**):

- ↑ Industrial polluting production → ↑ Contaminant concentration → ↓ Water quality

- ↑ Climate change → ↑ Precipitation, Drought, Flood, Temperature, Sea Level → ↑ Contaminant concentration → ↓ Water quality

Industrial Polluting Production (P):

- ↑ Economic health → ↑ Capital investment → ↑ Industrial activity → ↑ Industrial pollution

- ↑ Demographic change→ ↑ Goods requirement → ↑ Industrial activity → ↑ Industrial Pollution

Economic Health (C):

- ↑ Industrial pollution→ ↑ Industrial activity → ↑ Production market → ↑ Capital → ↑ Economic health

- ↑ Environmental awareness → ↑ Limits and laws in the polluting investments → ↓ Economic health

Environmental Awareness (E):

- ↑ Water quality → ↑ Human health → ↓ Environmental problems → ↓ Environmental Awareness

- ↑ Economic health → ↑ Human health and well-being → ↑ Concerns and investments for the nature → ↑ Environmental awareness

The feedbacks concerning the two external variables are described in a more detailed way. Remember that they don't affect the loop but influence only the water quality and the industrial polluting production.

Demographic Change (D) can have significant impacts on industrial activities in various ways:

1) Labour force availability and composition: aging populations and declining birth rates can lead to a shrinking labour force, which may pose challenges for industries reliant on skilled workers; Changes in the demographic composition, such as shifts in age groups or gender ratios, can affect the skills, preferences, and availability of labour, influencing workforce recruitment, training, and management practices in industries;

2) Consumer demand and preferences: demographic shifts, such as population growth, urbanization, and changes in income levels, can influence consumer demand for goods and services, driving shifts in market demand and product preferences; Industries may need to adapt their production processes, product offerings, marketing strategies, and distribution channels to meet the changing needs and preferences of different demographic groups.

3) Regional development and urbanization: demographic changes, including urbanization and rural depopulation, can impact the spatial distribution of economic activities and industrial clusters; Urbanization

may lead to the concentration of industries in urban areas, driven by factors such as access to markets, infrastructure, skilled labour, and agglomeration economies.

4) Technological innovation and automation: demographic shifts, such as aging populations and labour shortages, can incentivize industries to invest in technological innovation and automation to increase productivity, reduce labour costs, and address workforce challenges; Industries may adopt advanced manufacturing technologies, robotics, artificial intelligence, and other automation solutions to streamline production processes and enhance competitiveness in the face of demographic changes.

5) Regulatory and policy implications: demographic trends, such as population aging and changing social needs, can influence government policies and regulations affecting industrial activities; Policy responses may include incentives for industries to invest in sectors related to aging populations (e.g., healthcare, eldercare services) or measures to address environmental and social concerns associated with industrial development.

6) Supply chain dynamics: demographic changes can affect supply chain dynamics, including sourcing, logistics, and distribution networks, as industries adapt to changes in consumer demand, labour availability, and market conditions; Industries may reassess supply chain strategies, diversify sourcing options, and optimize distribution channels to respond to demographic shifts and emerging market opportunities.

Climate Change (**C_c**) can have significant effects on water quality through various pathways. Here are some ways in which climate change affects water quality:

1) Changes in precipitation patterns: climate change can alter precipitation patterns, leading to more frequent and intense rainfall events in some regions and prolonged droughts in others. Heavy rainfall can increase surface runoff, erosion, and sedimentation, resulting in higher concentrations of sediment and suspended solids in rivers and lakes, which can degrade water quality and aquatic habitats.

2) Temperature increases: rising temperatures can affect water temperature, which influences the growth rates of aquatic organisms, metabolic rates, and the solubility of gases and pollutants in water. Higher water temperatures can promote the growth of harmful algal blooms, which produce toxins that can contaminate water supplies and harm aquatic ecosystems.

3) Changes in streamflow and hydrology: changes in precipitation patterns and temperature can alter hydrological regimes, including streamflow, river discharge, and groundwater recharge rates. Reduced streamflow during droughts can lead to higher concentrations of pollutants in water bodies due to decreased dilution, while increased streamflow during heavy rainfall events can flush pollutants into waterways, affecting water quality.

4) Sea level rise and salinity intrusion: sea level rise, driven by climate change, can lead to saltwater intrusion into coastal aquifers and estuaries, resulting in increased salinity levels in surface water and groundwater. Higher salinity can degrade water quality, affect freshwater ecosystems, and impact drinking water supplies, agriculture, and industrial processes dependent on freshwater resources.

5) Changes in water chemistry: climate change can alter the chemistry of water bodies, including pH levels, nutrient concentrations (e.g., nitrogen, phosphorus), and dissolved oxygen levels. Increased temperatures and nutrient runoff can exacerbate eutrophication, leading to algal blooms, oxygen depletion, and habitat degradation in affected water bodies.

6) Extreme weather events: climate change is associated with an increase in the frequency and intensity of extreme weather events, such as hurricanes, floods, and storms. These events can cause physical damage to infrastructure, release contaminants from industrial sites, wastewater treatment plants, and agricultural fields, and lead to waterborne diseases and contamination of drinking water sources.

Then, the causal loop diagram can be built. It is a powerful tool for understanding the relationships between each variable and how they interact with each other. For these reasons, it is a dynamic system that can be modified along the process if more possible feedbacks are discovered. In fact, the first causal loop diagram didn’t consider any feedbacks between the economic health and the environmental awareness. But, during the working process, interesting links came up between these two variables and so the final causal loop diagram was defined.

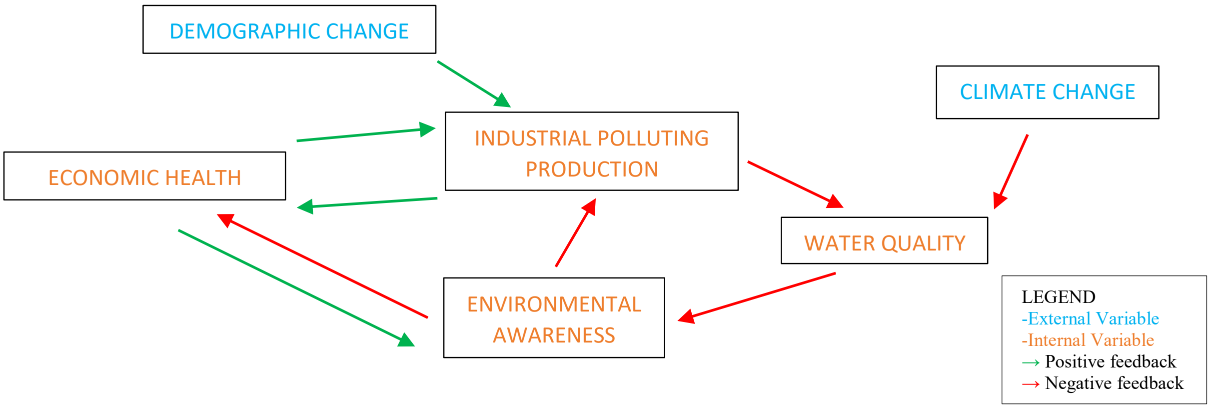


Figure 3.11: Final causal loop diagram of the model

3.3. The Choice of the State Variables and the Causal factors affecting them

In this chapter, *"Step 3"* and *"Step 4"*, related to the paper of Sivapalan and Blöschl (2015), are described in details and Chapter 4 is completely reserved to them because their definition is quite short after having built the causal loop diagram. As indicated in Chapter 2, in Step 3 the state variables are finally chosen. Their domains, initial conditions and boundary conditions must be established in order to set the limits for the following functional relationships. Here, they are described in mathematical terms, due to the fact that their historical evolution is widely analysed in the previous chapter. So, the internal state variables, as introduced in the causal loop diagram, are summarised in the next Table 3.1:

Variable	**Symbol**	**Domain**	**Initial Conditions**	**Boundary Conditions**
Water Quality	W [-]	$0 \leq W \leq 1$	W(t=1900) ≈1	$\lim_{t\to\infty} \frac{dW}{dt} = 1$
Environmental Awareness	E [-]	0≤E≤1	E(t=1900) ≈0	$\lim_{t\to\infty} \frac{dE}{dt} = 1$
Industrial Polluting Production	P [-]	$0 \leq P \leq 1$	P(t=1900) ≈0	$\lim_{t\to\infty} \frac{dP}{dt} = 0$
Economic Health	C [-]	$0 \leq C \leq 1$	C(t=1900) ≈0	$\lim_{t\to\infty} \frac{dC}{dt} = 1$

Table 3.1: Internal state variables properties

As showed in the previous table, all the internal variables are bounded between 0 and 1 and are dimensionless. This choice is quite clever and lets to build a qualitative model that could be widely developed with real data afterwards. Moreover, the values covered by each quantity haven't physical meanings, but represent only the quality of the variable over time, like an index: 0 means the worst condition and 1 represents the best state for the variable. This assumption allows a simplifier and clearer computation of the variables' evolution.

In the next Step 4, the causal factors affecting the state variables are then outlined. In particular, this section refers to the setting of relationships between each quantity and info are provided by the causal loop diagram. Right now, the external forcings are introduced and linked with the others. Notably, the demographic change is called *D* and the climate change is called C_c The internal variables will be frequently written as temporal derivative in order to describe their evolution over time. Therefore, considering Figure 3.11, a similar system of Equation 2.1 is computed:

$$
\begin{cases}
\frac{dW}{dt} = f(W, P, C_c) & \text{(a)} \\
\frac{dE}{dt} = h(E, W, C) & \text{(b)} \\
\frac{dP}{dt} = g(P, E, C, D) & \text{(c)} \\
\frac{dC}{dt} = q(C, P, E) & \text{(d)}
\end{cases}
$$

Equations 3.1: System of ordinary differential equations of the internal variables

Each variable is computed as a function of itself and other quantities and this function is called *f, h, g, q*. Before introducing the Step 5, furthermore it's useful to create a time-series in which, for all the internal variables, the evolution is shaped over time. For the sake of clarity, the period under study is divided into three slots and the results are reported in the following Table 3.2:

Period	1900-1950	1950-2000	2000-present
Variable Evolution	$\frac{dW}{dt}$<0	$\frac{dW}{dt}$=0 (min) and $\frac{dW}{dt}$>0	$\frac{dW}{dt}$>0
	$\frac{dE}{dt}$=0 (cost=0)	$\frac{dE}{dt}$>0	$\frac{dE}{dt}$=0 (cost=1)
	$\frac{dP}{dt}$>0	$\frac{dP}{dt}$=0 (max) and $\frac{dP}{dt}$<0	$\frac{dP}{dt}$=0 (cost=0)
	$\frac{dC}{dt}$>0	$\frac{dC}{dt}$>0	$\frac{dC}{dt}$=0 (cost=1)

Table 3.2: Evolution of the internal variables in the three indicated periods

As it's possible to notice, in the Table 3.2 the external variables don't show up: their influences in the model will be computed by the non-stationary analysis of two parameters (speed and equilibrium parameters, see Chapter 4 for more details).

Finally, the initial hypotheses regarding each variable are explained and reported in the following list:

-Water Quality *W*: if *W*=0, it means that the quality is the worst one due to the highest concentration of pollutants. The effects on the water bodies are very harmful and natural life in danger. So, industrial polluting production *P*≈1; if *W*=1, it means that the quality is the best one and no contaminants are present in the water bodies. The ecosystem service and life are in safe and no threats are acting. So, industrial polluting production *P*≈0.

For these reasons, the hypotheses are: $\frac{dW}{dt} > 0\ if\ P\ very\ low$ & $\frac{dW}{dt} < 0\ if\ P\ very\ high$

-Environmental Awareness *E*: if *E*=0, it means that the interest of population/government/politics in the environment doesn't exist due to several causes: or because the water quality is too good, so there is no reason for worrying about the nature, or because the economic health of the country is equally very bad and the state can't spend time and economic resources on environmental issues. So, *C*≈0 and *W*≈1; if *E*=1, it means that people/statesman are very concerned about the environmental health and they are spending efforts for its protection. At the same time, the economic health of the state is good enough to be able to invest also on the natural mitigation and adaptation. So, *C*≈1 and *W*≈0.

For these reasons, the hypotheses are: $\frac{dE}{dt} > 0 \; if \; W \; very \; low, C \; very \; high$ & $\frac{dE}{dt} < 0 \; if \; W \; very \; high, C \; very \; low$

-Industrial Polluting Production *P*: if *P*=0, it means that the industrial production has no impacts in terms of pollutants. The contaminants released during the industrial activity is equal to zero. Moreover, it could mean that also the economic health of the country is very low and so the state can't invest in more industrial production for its citizens. This minimum value is due to the effective improvements of environmental awareness, which is even stronger. So, *C*≈0 and E≈1; if *P*=1, it means that the industrial production is very intense but in a non-sustainable way. It pollutes at the maximum rates, so the environmental laws/limits/efforts are absent or even inefficient. At the same time, this high polluting production is funded by the flourishing economic health. So, *C*≈1 and *E*≈0.

For these reasons, the hypotheses are: $\frac{dP}{dt} > 0 \; if \; E \; very \; low, C \; very \; high$ & $\frac{dP}{dt} < 0 \; if \; E \; very \; high, C \; very \; low$

-Economic Health C: if *C*=0, it means that the economic health of the country is the minimum one and no money/financing/funding are available. For this reason, also the industrial polluting production is very low, because no investments are possible. On the other hand, the environmental awareness of the population is quite high and also it contributes on settings laws/limits to reduce the capital invested for the polluting production. So, *E*≈1 and *P*≈0; if *C*=1, it means that the state is very prosperous and the well-being very high. The industrial productive rates are quite fast and so the pollutants are spread into the water bodies. On the other hand, the environmental awareness strength is very weak to reduce the investments on the polluting production. So, *E*≈0 and *P*≈1.

For these reasons, the hypotheses are: $\frac{dC}{dt} > 0 \; if \; E \; very \; low, P \; very \; high$ & $\frac{dC}{dt} < 0 \; if \; E \; very \; high, P \; very \; low$

These first hypotheses will be validated in the next chapters through the model development. They are now formulated because are the starting point of the entire study and put the basis for the equations' formulation.

4. Functional Relationships for the Model Formulation

This chapter is one of the most important of the entire model framing because it explains the mathematical steps for the final system computation and it refers to the *"Step5"* of the previously indicated paper. The so called "functional relationships" refer to a system of ordinary differential equations that describes the dynamical evolution over time of the state variables and lets to analyse the interactions between them. But before computing the final model, several preliminary studies must be assessed, such as phase planes computation, equilibrium analysis, parameters estimation, stationary and no-stationary cases. For this purpose, the Chapter 4 is divided into 5 more sub-chapters to go deeper in each passage.

4.1. The Development of the First Set of Equations

As suggested by Sivapalan and Blöschl (2015), it's advisable starting the equation building with few variables and then adding the remaining ones. Then, the water quality W, the environmental awareness E and the industrial polluting production P are chosen and relationships between them are built. Such quantities are considered also in Milik et al. (1996) analysis and suggestions on how construct the system are taken from here. Thereby, the causal loop diagram (Figure 3.11) is considered to establish these relationships and design the suitable set of equations. Reminding Table 3.1, the following characteristics are reported:

$$0 \leq W, E, P \leq 1$$

$$[W, E, P] = [-]$$

Initial conditions: W(t=1900) =1; E(t=1900)=0.2; P(t=1900)=0.01

The ordinary differential equations describing water quality and environmental awareness evolution are the following ones:

$$\begin{cases} \dfrac{dE}{dt} = \alpha\,(1 - E - W)\,E \\[2ex] \frac{dW}{dt} = \beta\,(1 - W - P) \end{cases}$$

Equation 4.1: ODE describing the relationship and evolution of environmental awareness and water quality variables

Equation 4.1 is then used for understanding the behaviour of the environmental awareness in relation with the water quality. The evolution of people's awareness regarding the environment depends on the environmental awareness itself and on the water quality. More the population is aware of environmental health, smaller its change over time. In the same way, higher is the water quality, better is the water bodies health, so people are less worried about environmental issues. This explanation is supported by the analytical solving reported in the followed system:

$$\begin{cases} \frac{dE}{dt} > 0 \rightarrow E > 0 \; or \; E < 1 - W \\ \frac{dE}{dt} < 0 \rightarrow E < 0 \; or \; E > 1 - W \\ \frac{dE}{dt} = 0 \rightarrow E = 0 \; or \; E = 1 - W \end{cases}$$

Equation 4.2: Sign analysis of the environmental awareness's time derivative

The environmental awareness's ODE reported in Equation 4.1 is described by a logistic growth: the factor "*E*" sets the minimum value at 0, instead the "*1-E*" factor fixes the maximum value at 1. Moreover, thanks to the document *"An Introduction to Dynamical Systems and Chaos"* by M. Spiegelman (2013), the Equations 4.1 can be analysed using the phase plane: it is a graph where the axes represent two variables of which the relationship has to be assessed. Inside the graph, arrows are plotted and their direction and length stand for to which value and how much fast a variable change over time. Inspection of the phase plane gives an immediate feeling for how this problem will evolve. The field of arrows can be seen as a flow field like currents in a river. If the Equation 4.1 is solved at any initial water quality and environmental awareness, it would also track out a trajectory in the graph, such that at any point, the phase plane arrows would be tangent to the trajectory. Solving by hands the previous system of ODE can be very difficult, for this reason the software "*R*" is implemented and the following phase plane between *W* and *E* is plotted in Figure 4.1. As it's possible to notice, in the phase plane a red dot appears: this point represents the stable equilibrium point, which is at 0.5 [-]. It means that, at the equilibrium, if *P*=0.5 [-], also the other two variables reach this value. All the arrows point the red dot and their length refers to the strength on reaching it. The phase plane is the graphical representation of the ODE's resolution. In addition, two parameters show up in the system: α [yr^{-1}] and β [yr^{-1}] are speed parameters and change the speed with which the environmental awareness and the water quality reach the equilibrium point, respectively. At the beginning, they are set as: α=0.05 yr^{-1} and β=0.04 yr^{-1} and then they will be studied in the next section.

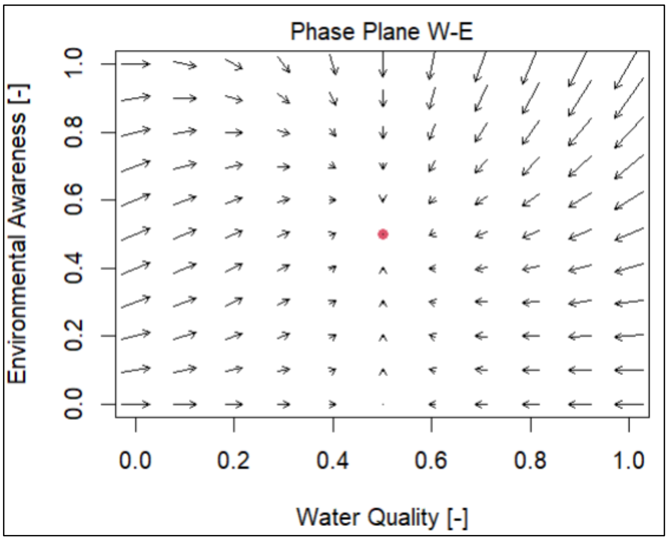


Figure 4.1: Phase plane between environmental awareness and water quality with α=0.05 yr^{-1}, β=0.04 yr^{-1}, P=0.5

Proceeding with the analysis, the ordinary differential equations describing water quality and industrial polluting production evolution are the following ones:

$$\begin{cases} \frac{dW}{dt} = \beta\,(1 - W - P) \\ \frac{dP}{dt} = \gamma\,(1 - P - E) \end{cases}$$

Equation 4.3: ODE describing the relationship and evolution of water quality and industrial polluting production variables

Equation 4.3 is then used for understanding the behaviour of the industrial polluting production in relation with the water quality. The evolution of the health of the water bodies depends on the water quality itself and on the pollutants released by the industrial production. More the industrial activities are flourishing, higher the concentration of pollutants in the water bodies, lower is the water quality. This explanation is supported by the analytical solving reported in the system of Equation 4.4. The ODE of water quality reported in Equation 4.3 is described as a logistic growth, with the maximum value equal to 1. Also, in this case, the phase plane is plotted: the value at the equilibrium of *E* is set equal to 0.5 [-] and the speed parameters are γ=0.1 yr^{-1} and β=0.04 yr^{-1}. As it's possible to appreciate from Figure 4.2, higher values of pollution push water quality down (arrows go down) and lower levels of pollution rise the water quality (arrows go up).

$$\begin{cases} \frac{dW}{dt} > 0 \rightarrow W < 1 - P \\ \frac{dW}{dt} < 0 \rightarrow W > 1 - P \\ \frac{dW}{dt} = 0 \rightarrow W = 1 - P \end{cases}$$

Equation 4.4: Sign analysis of the water quality's time derivative

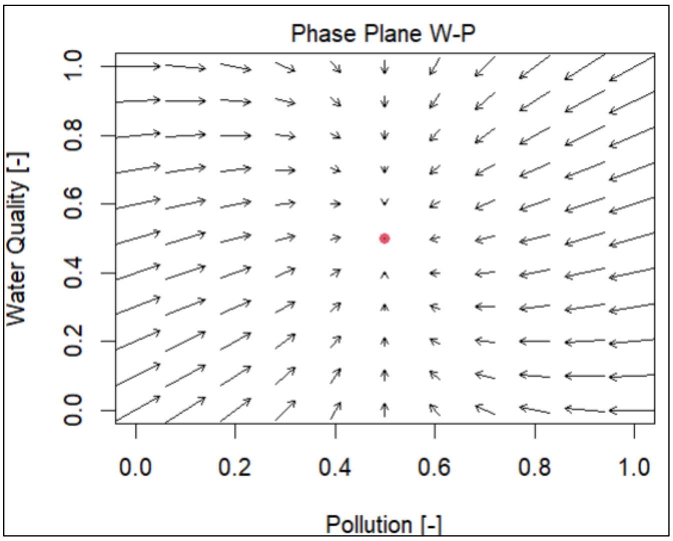


Figure 4.2: Phase plane between water quality and industrial polluting production with γ=0.1 yr^{-1}, β=0.04 yr^{-1}, E=0.5

Finally, the relationship between the environmental awareness and the industrial polluting production is analysed. The ordinary differential equations describing environmental awareness and industrial polluting production evolution are the following ones:

$$\begin{cases} \frac{dP}{dt} = \gamma\,(1 - P - E) \\ \frac{dE}{dt} = \alpha\,(1 - E - W)\,E \end{cases}$$

Equation 4.5: ODE describing the relationship and evolution of industrial polluting production and environmental awareness variables

Equation 4.5 is then used for understanding the behaviour of the industrial polluting production in relation with the environmental awareness: the change of the pollution depends on the pollution itself and on the population's awareness on the natural environment. If the industry produces a lot, the pollutants released increase, and then the derivative doesn't change too much over time. The same process works for the environmental awareness: the more the population is aware of the levels of pollution, the strongest the efforts to reduce the pollutants flowing into rivers. Also, in this case, the ODE for pollution is defined as a logistic growth and the maximum value is set equal to 1. The following system describes how the derivative's sign changes over time:

$$\begin{cases} \frac{dP}{dt} > 0 \rightarrow P < 1 - E \\ \frac{dP}{dt} < 0 \rightarrow P > 1 - E \\ \frac{dP}{dt} = 0 \rightarrow P = 1 - E \end{cases}$$

Equation 4.6: Sign analysis of the industrial polluting production's time derivative

Also, in this case, the phase plane is plotted: the value at the equilibrium of W is set equal to 0.5 [-] and the speed parameters are γ=0.1 yr^{-1} and α=0.05 yr^{-1}. As it's possible to appreciate from Figure 4.3, higher values of awareness push pollution down (arrows go down) and lower levels of awareness rise the pollutants' spreading (arrows go up).

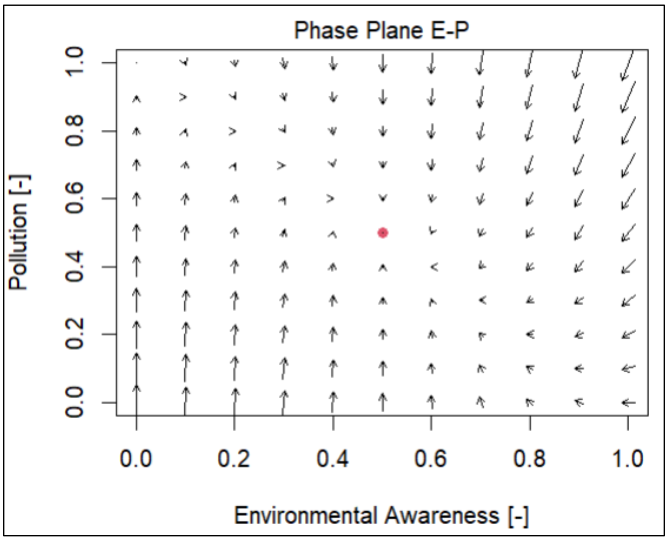


Figure 4.3: Phase plane between industrial polluting production and environmental awareness with γ=0.1 yr^{-1}, α=0.05 yr^{-1}, W=0.5

The last step is to resolve the whole system of the ordinary differential equations and plot their evolution in the time period 1900-2050. The values of the speed parameters aren't influence right now and they will be set in the last chapter. So, the following system of Equation 4.7 is computed inside "*R*" with the function

"`> library(deSolve)`":

$$
\begin{cases}
\dfrac{dE}{dt} = \alpha\,(1 - E - W)\,E \\[2ex]
\dfrac{dW}{dt} = \beta\,(1 - W - P) \\[2ex]
\dfrac{dP}{dt} = \gamma\,(1 - P - E)
\end{cases}
$$

Equation 4.7: Ordinary differential equation for the first three variables of the model: environmental awareness, water quality and industrial polluting production

The results are plotted in the following graph, also produced in "*R*":

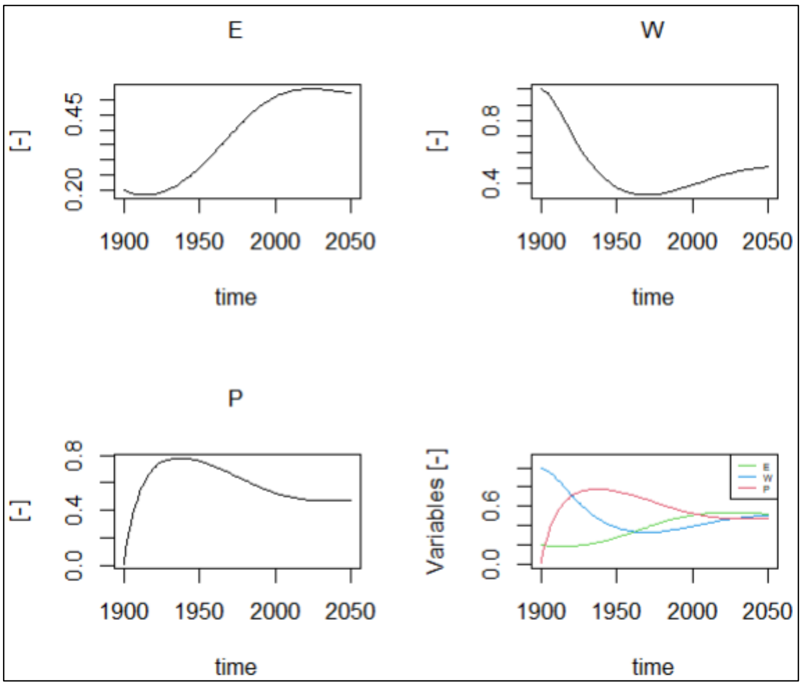


Figure 4.4: Evolution of the state variables over time

All the variables reach an equilibrium value of 0.5 [-] in 2050, yet. This value isn't the desirable one and so it will be fixed by adding the equilibrium's parameters, as discussed in the following section. Moreover, the shape of each curve is quite similar to the hypothesized one: the water quality has a minimum in 1960, while the industrial polluting production reaches a maximum in 1950. The environmental awareness increases slowly with a logistic growth, until stabilizing at 0.5 [-]. It improves the slope in the same year when the water quality reaches its minimum, as expected. Furthermore, the environmental health approaches the minimum value after the pollution has grown until its maximum. This delay over time is due the different value of the speed parameters: $\gamma > \beta$, so the water quality will reach its minimum after the industrial polluting production has reached its maximum.

In conclusion, the system reported in Equation 4.7 is made by three different ordinary differential equations which are quite similar: they are described with a logistic growth with a maximum equal to 1 and 0 as the minimum value for the environmental awareness. This construction let the variables to be bounded inside the limits fixed at the beginning of this chapter. Moreover, another important suggestion given by Sivapalan and Blöschl (2015) is to make the system as linear as possible, so all the derivatives are built in a very similar way.

4.2. Equilibrium Analysis

Before going ahead with the addition of the economic health variable inside the model, it's firstly important to understand how the equilibrium point changes and which are the parameters influencing it. Figures 4.1, 4.2, 4.3 show that the equilibrium point at an infinite time is, for all the variables, at 0.5 [-]. This means that, whatever happens before, the final value for each quantity in the future will be 0.5 [-], as reported in Figure 4.4. This result isn't suitable nor desirable for this model, so the equilibrium analysis is necessary to calibrate the final value.

There are different fixed points that can occur inside a system (M. Spiegelman, 2013): stable fixed points are attractors in that any trajectory that starts near them will eventually end up at the steady state fixed point after some time; unstable fixed points are repellers and everything flies away from them; the saddle node is essentially unstable but has a stable manifold, that is parts of phase space are attracted to the fixed point until they get close enough and then fly away along the unstable manifold. In this case, the point at the equilibrium is a stable fixed point because it's reached at the steady state condition.

Therefore, the equilibrium point is computed by setting the derivative equal to 0, so by imposing that no temporal variation occurs anymore. Some more knowledges about how solve a system for the equilibrium analysis are taken from the document "*Differential Equations, Chapter 4*" by Bristol University (2013).

Taking the system in Equation 4.7 and equalizing the equations to 0, the stable fixed point is worked out: the final value expected is 0.5 [-]:

$$\begin{cases} \frac{dE}{dt} = \alpha\,(1 - E - W)\,E = 0 \\ \frac{dW}{dt} = \beta\,(1 - W - P) = 0 \\ \frac{dP}{dt} = \gamma\,(1 - P - E) = 0 \end{cases}$$

Equation 4.8: Equilibrium analysis of the original system

The previous system has the following solutions:

Unstable point B: *B (E=0; W=0; P=1)*

Stable point A: *A (E=0.5; W=0.5; P=0.5)*

In order to change this final value of 0.5 [-], we need just to put at least one parameter inside each derivative equation. A first attempt is multiplying *W*, *P* and *E* variables in $\frac{dE}{dt}, \frac{dW}{dt}, \frac{dP}{dt}$, respectively, for a dimensionless coefficient, like in the next Equation 4.9:

$$\begin{cases} \frac{dE}{dt} = \alpha\,(1 - E - \varepsilon\,W)\,E \\ \frac{dW}{dt} = \beta\,(1 - W - \lambda\,P) \\ \frac{dP}{dt} = \gamma\,(1 - P - \chi\,E) \end{cases}$$

Equation 4.9: ODEs' system with equilibrium parameters ε, λ and χ

Just proceeding as before, the new equilibrium point will be:

$$\begin{cases} \alpha\,(1 - E - \varepsilon\,W)\,E = 0 \quad \rightarrow \quad E = \frac{1 + \lambda\varepsilon - \varepsilon}{1 + \lambda\chi\varepsilon} \\ \beta\,(1 - W - \lambda\,P) = 0 \quad \rightarrow \quad W = \frac{1+\lambda\chi-\lambda}{1+\lambda\chi\varepsilon} \\ \gamma\,(1 - P - \chi\,E) = 0 \quad \rightarrow \quad P = \frac{1 + \chi\varepsilon - \chi}{1 + \lambda\chi\varepsilon} \end{cases}$$

Equation 4.10: New equilibrium point for the state variables

Then, the new stable fixed-point *A* is: $A\left(E = \frac{1+\lambda\varepsilon-\varepsilon}{1+\lambda\chi\varepsilon}, W = \frac{1+\lambda\chi-\lambda}{1+\lambda\chi\varepsilon}, P = \frac{1+\chi\varepsilon-\chi}{1+\lambda\chi\varepsilon}\right)$

These three parameters ε, χ and λ are called "*equilibrium parameters*" and are defined as following:

$$0 \le \varepsilon, \chi, \lambda \le 1$$

$$[\varepsilon, \chi, \lambda] = [-]$$

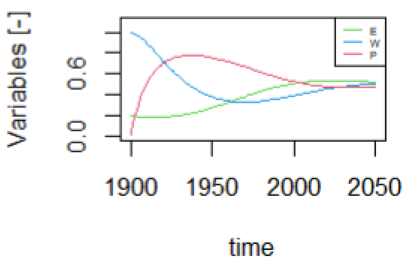
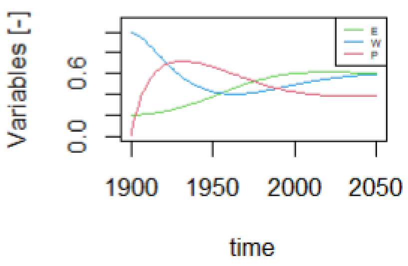
If these parameters are, at the same time, equal to 1, then the same equilibrium point as in Equation 4.8 is obtained. However, just changing theirs values the desired equilibrium point can be assessed. Before finding this final value, each parameter has to be described and defined correctly. Usually, when a new parameter is introduced, it could be useful to give it a name to better understand its function inside the system.

-Disregard ε [-]: this parameter multiplies the water quality variable inside the environmental awareness derivative. It’s called disregard because it represents the level of negligence of population in front of water quality’s issues. In fact, if this value approaches 1, the strength of the water quality in changing the environmental awareness value over time has the same weight of the environmental awareness itself. On the other hand, if the disregard of people for natural diseases is very low ($\varepsilon \approx 0$), then the influence of the water quality on the environmental awareness is absent. In the Equation 4.11, the effects of the disregard on the environmental awareness growth is described:

$$\begin{cases} \frac{dE}{dt} > 0 \rightarrow growing \rightarrow E > 0 \; or \; E < 1 - \varepsilon W \\ \frac{dE}{dt} < 0 \rightarrow decreasing \rightarrow E < 0 \; or \; E > 1 - \varepsilon W \\ \frac{dE}{dt} = 0 \rightarrow E = 0 \; or \; E = 1 - \varepsilon W \end{cases}$$

Equation 4.11: Influences of disregard on the time evolution of the environmental awareness variable

To better appreciate its power, the next Table 4.1 is computed with the evolution over time of variables:

ε=1 [-]; λ=1 [-]; χ=1 [-]	**ε=0.7 [-]; λ=1 [-]; χ=1 [-]**
	

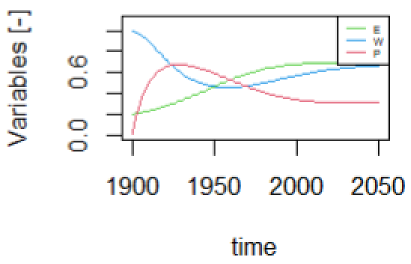
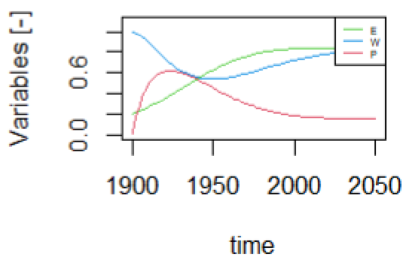
ε=0.5 [-]; λ=1 [-]; χ=1 [-]	ε=0.2 [-]; λ=1 [-]; χ=1 [-]
	

Table 4.1: Model evolution with changing in disregard ε values

As it's described in Equation 4.10, the variable mostly affected by this parameter at the equilibrium is the environmental awareness. In fact, the Table 4.1 supported this hypothesis: the higher the disregard is, the lower the environmental awareness at an infinite time. Moreover, this parameter affects also the industrial polluting production, whose equilibrium point value decreases as the disregard goes down, too. All the variables have a difference of 0.333 [-] in their final values at the equilibrium. The next Table 4.2 reports the final values at the equilibrium for the variables, affected by the ε changes:

Parameters' Values	**State Variables at the Equilibrium [-]**
ε=1 [-]; λ=1 [-]; χ=1 [-]	E=0.5 W=0.5 P=0.5
ε=0.7 [-]; λ=1 [-]; χ=1 [-]	E=0.588 W=0.588 P=0.412
ε=0.5 [-]; λ=1 [-]; χ=1 [-]	E=0.667 W=0.667 P=0.333

ε=0.2 [-]; λ=1 [-]; χ=1 [-]	E=0.833 W=0.833 P=0.167

Table 4.2: Variation of the equilibrium values with respect the disregard changes

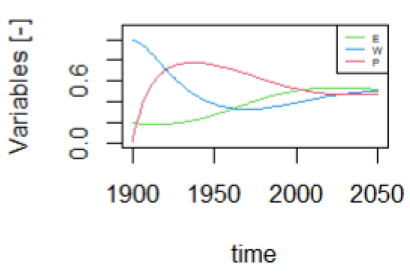
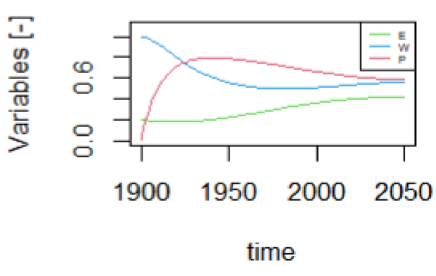
-Harmfulness λ [-]: this parameter is called harmfulness because, as reported in Equation 4.10, it multiplies the industrial polluting production inside the water quality derivative. It means that the higher is the threat of the pollutants spread by the industrial activity, the lower will be the water quality enhancement over time. If the harmfulness approaches 0, the health of the water bodies will be the maximum one, even if the industrial polluting production is at its highest value. This fact explains the meaning of λ: it quantifies the hazard of the pollutants spread by the industries and not the amount of them.

The following system helps to better visualize the influence of the harmfulness on the water quality evolution over time:

$$\begin{cases} \frac{dW}{dt} > 0 \rightarrow growing \rightarrow W < 1 - \lambda P \\ \frac{dW}{dt} < 0 \rightarrow decreasing \rightarrow W > 1 - \lambda P \\ \frac{dW}{dt} = 0 \rightarrow W = 1 - \lambda P \end{cases}$$

Equation 4.12: Influences of harmfulness on the time evolution of the water quality variable

As did before, the evolution of the model and a table with the final values of the variables at the equilibrium are reported next:

ε=1 [-]; λ=1 [-]; χ=1 [-]	**ε=1 [-]; λ=0.7 [-]; χ=1 [-]**
	

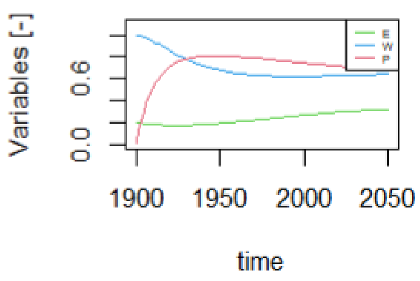
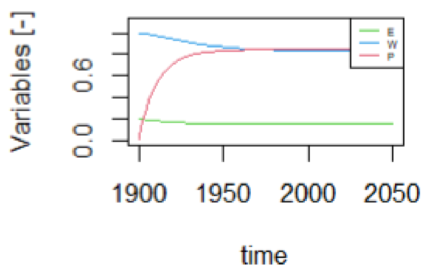
ε=1 [-]; λ=0.5 [-]; χ=1 [-]	ε=1 [-]; λ=0.2 [-]; χ=1 [-]
	

Table 4.3: Model evolution with changing in harmfulness λ values

Parameters' Values	State Variables at the Equilibrium [-]
ε=1 [-]; λ=1 [-]; χ=1 [-]	E=0.5 W=0.5 P=0.5
ε=1 [-]; λ=0.7 [-]; χ=1 [-]	E=0.412 W=0.588 P=0.588
ε=1 [-]; λ=0.5 [-]; χ=1 [-]	E=0.333 W=0.667 P=0.667
ε=1 [-]; λ=0.2 [-]; χ=1 [-]	E=0.167 W=0.833 P=0.833

Table 4.4: Variation of the equilibrium values with respect the harmfulness changes

If the harmfulness of the pollutants is very low, the population isn't worried about the water bodies health and, in fact, the water quality is very good even if the amount of pollutants released by industries is high.

-Eco-friendliness χ [-]: this parameter appears into the pollution derivative and multiplies the environmental awareness variable. It's called eco-friendliness because represents the efforts of politics in improving sustainable technologies for the industrial production. It regards the investments spent for green technologies in the industrial sector. So, it's expected that when these investments are large, the pollutants are few in water bodies and the environmental health is very good. From the next Table 4.5, it's possible to understand that if the eco-friendliness approaches 0, the pollutants reach their maximum value and the water quality drops down until 0. This last feature lets the environmental awareness to grow despite the green investments are absent. As before, the sign of the pollution derivative is studied with the following system:

$$\begin{cases} \dfrac{dP}{dt} > 0 \rightarrow growing \rightarrow P < 1 - \chi E \\ \frac{dP}{dt} < 0 \rightarrow decreasing \rightarrow P > 1 - \chi E \\ \dfrac{dP}{dt} = 0 \rightarrow \ P = 1 - \chi E \end{cases}$$

Equation 4.13: Influences of eco-friendliness on the time evolution of the industrial polluting production variable

In the following Table 4.5 and Table 4.6 all the previous indicated characteristics are shown:

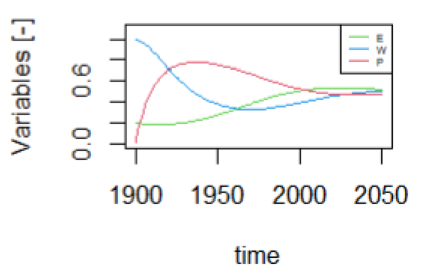
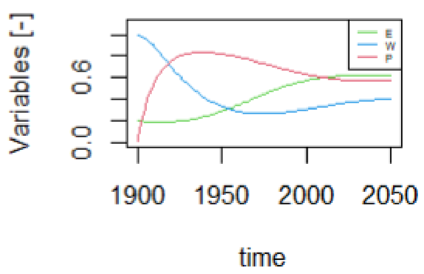
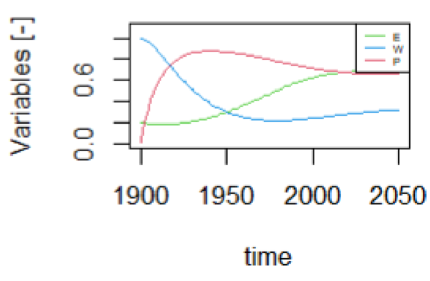
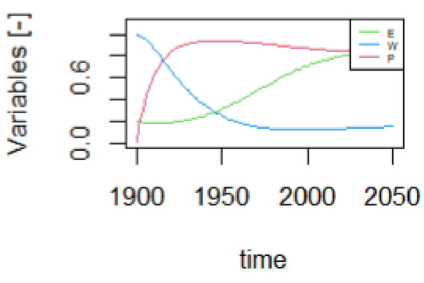
ε=1 [-]; λ=1 [-]; χ=1 [-]	**ε=1 [-]; λ=1 [-]; χ=0.7 [-]**
	
ε=1 [-]; λ=1 [-]; χ=0.5 [-]	**ε=1 [-]; λ=1 [-]; χ=0.2 [-]**
	

Table 4.5: Model evolution with changing in eco-friendliness χ values

Parameters' Values	State Variables at the Equilibrium [-]
ε=1 [-]; λ=1 [-]; χ=1 [-]	E=0.5 W=0.5 P=0.5
ε=1 [-]; λ=1 [-]; χ=0.7 [-]	E=0.588 W=0.412 P=0.588
ε=1 [-]; λ=1 [-]; χ=0.5 [-]	E=0.667 W=0.333 P=0.667
ε=1 [-]; λ=1 [-]; χ=0.2 [-]	E=0.833 W=0.167 P=0.833

Table 4.6: Variation of the equilibrium values with respect the eco-friendliness changes

In conclusion from the analysis above, it's interesting to notice that each equilibrium parameter, even though having different meanings, has the same effects on the equilibrium point value respectively of the variable of interest. It means that if ε is 0.7 [-], its effect on the environmental awareness is the same as that one on the water quality, if λ is 0.7 [-] too. Another important observation is that these parameters are the only ones able to change the equilibrium, so their choices are fundamental for the final result of the model. The last observation is that the holistic nature of a socio-hydrological model can be experienced also in the formulation of the equilibrium point: each of the equilibrium parameters influences all the state variables, although with different weight. This involves to multiple changes at the same time for the unlike variables. After understanding the meaning and the power of these equilibrium parameters, it's time now to add the last state variable of the system: the economic health.

4.3. The Inclusion of the Economic Health Variable into the Model

In order to complete the model with all its components, the last internal variable must be considered. It is the economic health *C.* The definition of this variable was quite complex due to its political, social and economic nature. At the beginning, it was defined as *"capital"*, then as *"economic wellness"*, but none of these names could deeply described the real meaning of this variable. It's more appropriate call its economic health. In fact, if we think about the capital we refer to the money and the richness of a country. For this reason, it can't be bounded between 0 and 1 because the wealth of a nation could grow unlimitedly. In order to link the capital with the industrial polluting production and the environmental awareness, it's important to find a more appropriate name to this economic variable. Generally, the capital refers to the GDP of a nation, but it isn't enough to describe the relationships with *E* and *P*. Although GDP and its changes are the most popular indicators of a nation's overall economic output, economists have long recognized that it is an imperfect measure of overall economic health. For example, GDP does not account for the production of pollution or consider the health and education of the population and other factors tied to health. Because of this, various governments and organizations have started following alternative measurements such as the Human Development Index, Better Life Index and Genuine Progress Indicator, as described in Chapter 3.

All these aspects lead to change the causal loop diagram in the final one (see Figure 3.11) and build two more feedbacks between the economic health and the environmental awareness. The economic health influences positively the industrial polluting production because the higher is the economic richness, the more the funding are available to increase the production. Vice versa, the larger the industrial production, the more items are sold and so more money enriches the nation and so the economy. These are the two positive feedbacks between *C* and *P*. But, at the same time, if the ecosystem services are at risk, the environmental awareness is quite high and so the government is more aware about the bad water quality and also about all the possible causes for the diseases. For this fact, the environmental awareness has negative feedback both for the industrial polluting production and for the economic health. In particular, that part of the *E* affecting the amount of released pollutants refers to all the sustainable technological innovations, instead that part, linked with the economic health, concerns the policies and laws gained to limit the money spent for the industrial polluting production. These are the other two negative feedbacks between *E* and *P* and between *E* and *C*. The last feedback added to the new causal loop diagram is negative and relates *C* and *E*. If the economic health is high, the nation has enough funding also to improve the researches and efforts on improving the natural health and so the environmental awareness increases too. On the other hand, if the state is quite poor, it hasn't enough money to take care also of the environment and, for this reason, the environmental awareness decreases because it's not funded by the state. A poor governance prefers taking care of its population instead on promoting or spending money on environmental improvements. Before adding the economic health to the system, the domain and dimension must be defined. They are the same as the other variables:

$$0 \leq C \leq 1$$

$$[C] = [-]$$

Initial conditions: C(t=1900) =0.1

The economic health derivative is built as a logistic growth, so the entire system of ODEs is built in a similar and simple way. The following system (Equation 4.14) shows how the economic health derivative is defined and how the other two variables, *P* and *E*, are affected by this addition:

$$\begin{cases} \frac{dC}{dt} = \theta\, C\, (1 - C - \eta E)\, P & (a) \\ \frac{dE}{dt} = \alpha\, (1 - E - \varepsilon\, W)\, E\, (C - \psi) & (b) \\ \frac{dP}{dt} = \gamma\, (1 - P - \chi\, E)\, C & (c) \end{cases}$$

Equation 4.14: ODEs after the addition of the economic health variable

The Equation 4.14 (a) describes how the economic health derivative evolves over time depending positively by the industrial polluting production and negatively by the environmental awareness. A new parameter η is considered to balance the power of the environmental awareness inside the economic health evolution. The higher this parameter is, the slower the economic health will grow over time.

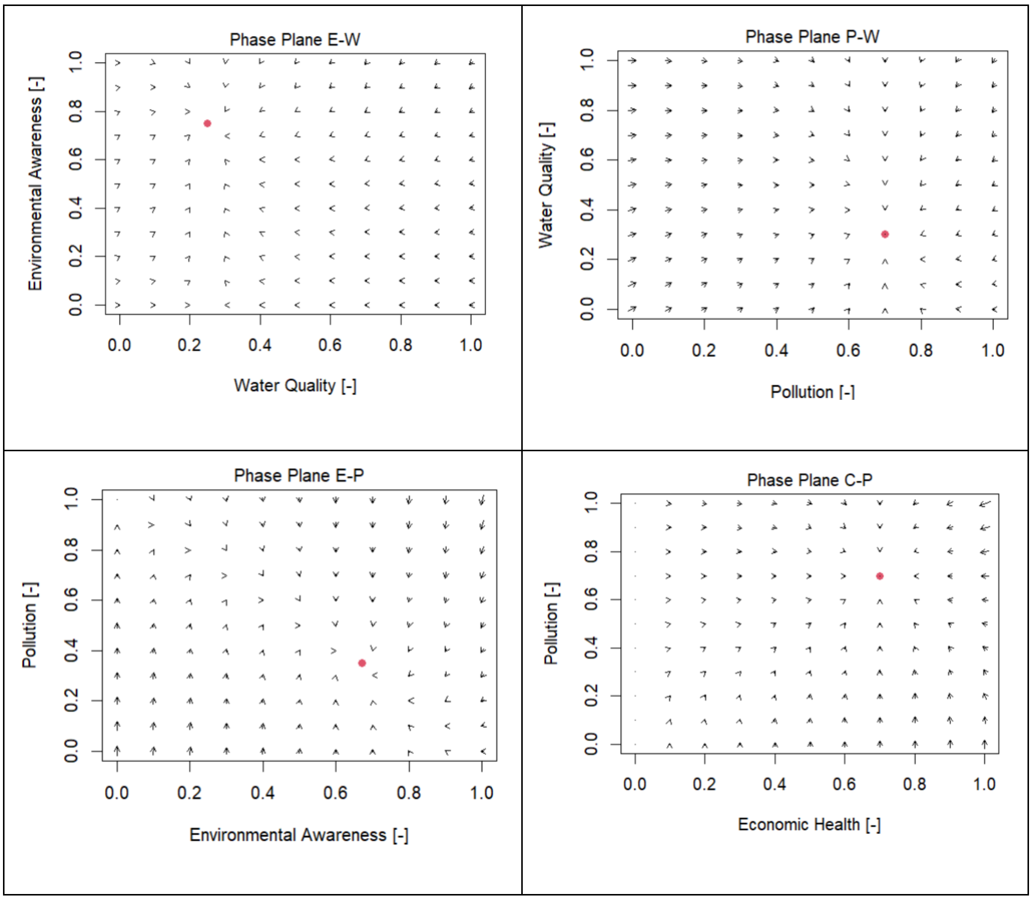
The Equation 4.14 (b) shows how the environmental awareness derivative is affected by the economic health and, in particular, a factor $(C-\psi)$ multiplies the whole equation. This term represents the positive feedback between the economic health and the environmental awareness. In fact, if C increases, also the derivative of E will grow over time. The new parameter ψ pushes down this factor and it will be described in the next section.

Finally, the Equation 4.14 (c) is the same as the original ODE for the industrial polluting production, but the economic health is considered inside as simply a multiplicative factor. In this way, when the economic health is positive, also the industrial polluting production grows positively over time.

As did in the previous section, in order to better appreciate the relationships between these three variables, the phase planes are computed using "R" and reported in the following Table 4.7. The parameters are set randomly and they don't refer to the final ones. They will be analysed in the next chapter:

Speed Parameters $[yr^{-1}]$*:* $\alpha=0.05;\ \beta=0.04;\ \gamma=0.1;\ \theta=0.2$

Equilibrium Parameters [-]: $\varepsilon=1;\ \lambda=1;\ \chi=1;\ \eta=1;\ \psi=0$



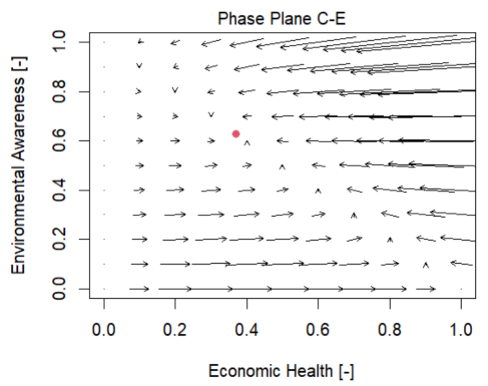


Table 4.7: Phase planes showing the feedbacks between each variable

The first three phase planes show the same relationships reported in Figure 4.1, Figure 4.2 and Figure 4.3. So, when the water quality is very good, the environmental awareness aims to low values; if the industrial polluting production is quite elevated, the water health degrades; when the environmental awareness is very strong, the pollution is decreased a lot. On the other hand, the last two phase planes describes the new feedback introduced by the presence of the economic health variable. A high economic health leads to an increasing of the industrial production, and so of the pollutants. If C=0, the pollution derivative doesn't change over time, so there is an unstable equilibrium point. At the same time, if the health of the nation is good, this status lets also the environmental awareness to grow. This last phase plane is that one more complex because it describes two opposite feedback that happens at the same time.

Combining together all the above equations, the system of ordinary differential equations now includes all the state variables and finally the model is complete. The system of ODEs and the evolution over time of the four variables are reported in Equation 4.15 and Figure 4.5, respectively. The speed parameters and the equilibrium parameters are the same used for the phase planes and they will be assessed later.

$$\begin{cases} \frac{dE}{dt} = \alpha\,(1 - E - \varepsilon\,W)\,E\,(C - \psi) \\ \frac{dW}{dt} = \beta\,(1 - W - \lambda\,P) \\ \frac{dP}{dt} = \gamma\,(1 - P - \chi\,E)\,C \\ \frac{dC}{dt} = \theta\,C\,(1 - C - \eta E)\,P \end{cases}$$

Equation 4.15: ODEs system of the final model

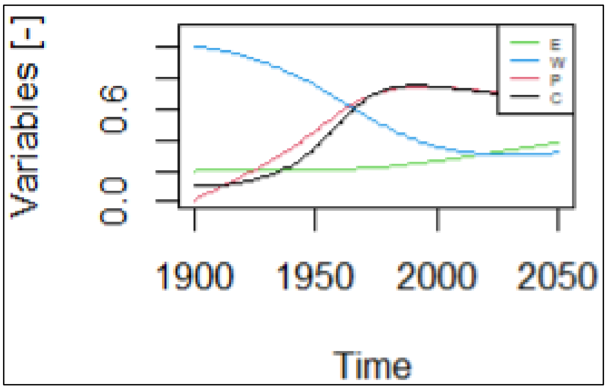


Figure 4.5: Evolution over time of the variables inside the final model

Before finding the suitable values of the coefficients, the last step of this Chapter 4.3 is to analyse the equilibrium again and then determine the nature of the two new parameters η and ψ. The equilibrium analysis is carried out as before, so equalling to zero the system of Equation 4.15. For a sake of simplicity, only the fixed stable equilibrium point is computed and it's called A:

$$A\left(E=\frac{1+\lambda\varepsilon-\varepsilon}{1+\lambda\chi\varepsilon}, W=\frac{1+\lambda\chi-\lambda}{1+\lambda\chi\varepsilon}, P=\frac{1+\chi\varepsilon-\chi}{1+\lambda\chi\varepsilon}, C=1-\eta\frac{1+\lambda\varepsilon-\varepsilon}{1+\lambda\chi\varepsilon}\right)$$

The equilibrium point is the same as before adding the economic health variable into the system, but now it includes also the fixed stable point for the economic quantity. In particular, this point drifts apart 1 by a factor proportional to the value of E at the equilibrium. The higher the equilibrium point of the environmental awareness, the lower the final value of the economic health, by a factor η. The stable fixed point of C would be 1 (as the logistic growth will lead to), if the environmental awareness contribution didn't exist. In order to understand the importance of η and ψ, let's analyse the sign of the time derivatives of C and E:

$$\begin{cases} \frac{dC}{dt}>0 \rightarrow growing \rightarrow C<1-\eta E \text{ or } C>0 \text{ or } P>0 \\ \frac{dC}{dt}<0 \rightarrow decreasing \rightarrow C>1-\eta E \text{ or } C<0 \text{ or } P<0 \\ \frac{dC}{dt}=0 \rightarrow maximum \rightarrow C=1-\eta E \text{ or } C=0 \text{ or } P=0 \end{cases}$$

Equation 4.16: Sign's analysis of the economic health variable C

$$\begin{cases} \frac{dE}{dt} > 0 \rightarrow growing \rightarrow C > \psi \text{ or } E>0 \text{ or } E<1-\varepsilon W \\ \frac{dE}{dt} < 0 \rightarrow decreasing \rightarrow C < \psi \text{ or } E<0 \text{ or } E>1-\varepsilon W \\ \frac{dE}{dt} = 0 \rightarrow maximum \rightarrow C = \psi \text{ or } E=0 \text{ or } E=1-\varepsilon W \end{cases}$$

Equation 4.17: Sign's analysis of the environmental awareness variable E

Then, how can be defined the new parameters η and ψ? Both these coefficients are dimensionless and are bounded between 0 and 1 in order to be consistent with the others and to make sure that the variables stay inside their limits:

$$0 \leq \eta, \psi \leq 1$$

$$[\eta, \psi] = [-]$$

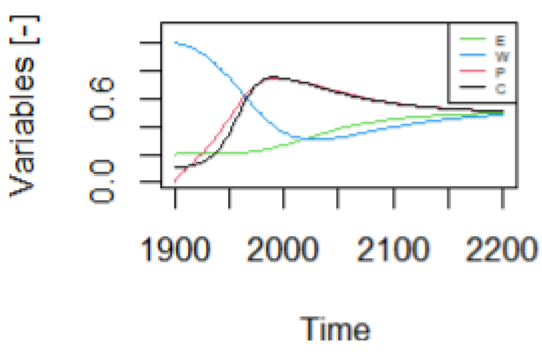
For the definition of the equilibrium parameter η, let's look at Equation 4.15 and Equation 4.16: this coefficient directly affects only the economic health variable and determines the strength of the environmental awareness on decreasing the value of the economic health over time. Moreover, it shows up in the equilibrium point formulation of *C* by diverting it from 1. So, this parameter concerns that part of the environmental awareness able to reduce or limit the influence of the economic health on the industrial polluting production and it represents the percentage of the environmental awareness focusing into policies and taxes able to reduce the economic health invested on the industrial polluting activities. For all these reasons, this parameter η is called "*financial sustainability"*. It's very important to highlight the similarity with the *"eco-friendliness χ"*: both these coefficients represent the strength of the environmental awareness on reducing the pollution in the water bodies, but the State could decide to invest directly on sustainable green technologies for the industrial production (here the *eco-friendliness* χ) or it could just improve its knowledges for promoting green laws, policies and guidelines directly affecting the economic health (here the *financial sustainability* η). Anyway, in the next Chapter 4.4, an accurate analysis of their meanings will be worked out.

At this point, let's use the software "*R"* to understand the power of the *financial sustainability* on the whole model. The evolution curves are plotted for different values of η. As before, the other speed and equilibrium parameters are set randomly and only in the next section they will be defined properly:

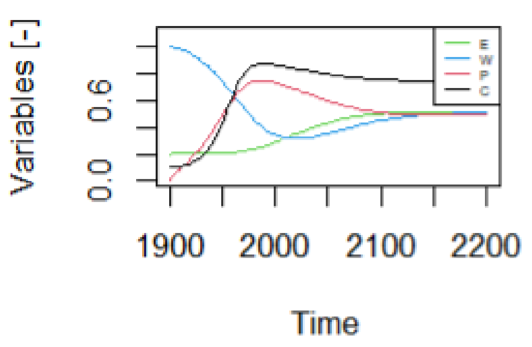
Speed Parameters [yr^{-1}]: α=0.05; β=0.04; γ=0.1; θ=0.2

Equilibrium Parameters [-]: ε=1; λ=1; χ=1; ψ=0.3

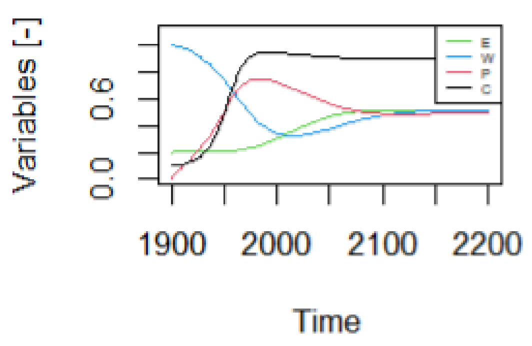
η=1 [-]



η=0.5 [-]



η=0.2 [-]



η=0.05 [-]

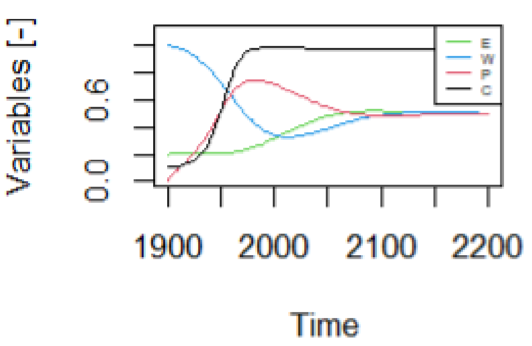


Table 4.8: Evolution of the variables through time for different values of the financial sustainability η

In this case, the analysis is let running until 2200 in order to see better the final values at the equilibrium. From Table 4.8, the above hypotheses are validated: the value of η affects only the equilibrium of the economic health. If its value is the highest one (η=1), the final values reached by all the variables is 0.5 [-], as expected. On the other hand, if the financial sustainability decreases, it means that the State doesn't make any efforts in laws or green policies to limit the economic health used to finance the industrial production. So, the economic health is free to grow and stimulate the polluting production. On the other hand, when the State is concerned about the environment but put its efforts only in the technological improvements (on χ, the *eco-friendliness*), the economic health reaches a final value higher than before. An interesting conclusion that comes up from the previous analysis is the footprint of the environmental awareness on the industrial polluting production. What is important to understand is that spending money on laws and green policies isn't enough to reduce the pollution and have significant effects on the water quality. In fact, when the environmental awareness is basically based on the financial sustainability, it means that $\eta>\chi$, so the financial sustainability should play a bigger role in the reduction of the pollution through the control of the economic health, but it doesn't happen. The eco-friendliness of the environmental awareness, so the efforts of the governance spent on the technological improvements rather than on laws or policies, is the key to appreciate in the future a decisive reduction of pollution in the surface bodies. In order to validate this strong statement, let's have a look to the previous graphs. When the economic health increases we expect to see a similar increasing also for the industrial polluting production, but it doesn't occur. The pollution keeps its original value due to the power of the eco-friendliness and so of the efforts of the environmental awareness on technologies. At the same time, when the economic health is heavily reduced by the financial sustainability (so the state spends lots of money for laws and economic policies to improve the natural capital), the system shows only the reduction of the economic heath, but the pollution doesn't change.

Going on with the analysis, for the definition of the parameter ψ, let's look at Equation 4.15 and Equation 4.17: this coefficient doesn't play any roles in the equilibrium, but it influences the time necessary to reach the equilibrium itself. In fact, this coefficient appears inside the factor $(C-\psi)$ which multiplies the other terms inside the environmental awareness derivative. Then, ψ is the value of the economic health (the lowest limit) above which efforts for the environmental improvements are practicable and so the awareness of the State about the environment is stimulated to grow. At the same time, this coefficient is also the limit below which the knowledges of people about natural condition decreases because the State isn't as flourished as before and the actions necessary to improve the natural capital are too costly to be afforded. So, ψ is called "*environmental threshold budget*".

Like for the other parameters, the software *"R"* is useful to study the dynamics of each variable over time with different values of ψ. The following speed and equilibrium parameters quantities are implemented:

Speed Parameters $[yr^{-1}]$: $\alpha=0.05; \beta=0.04; \gamma=0.1; \theta=0.2$

Equilibrium Parameters $[-]$: $\varepsilon=1; \lambda=1; \chi=1; \eta=1$

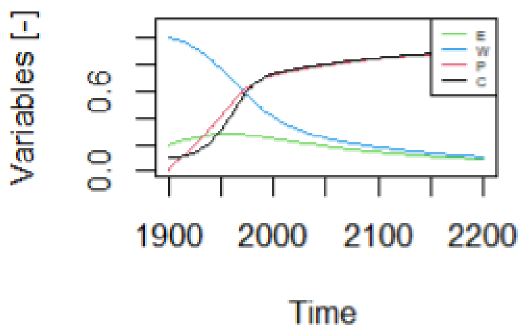
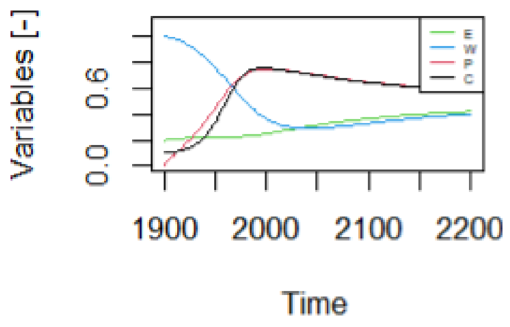
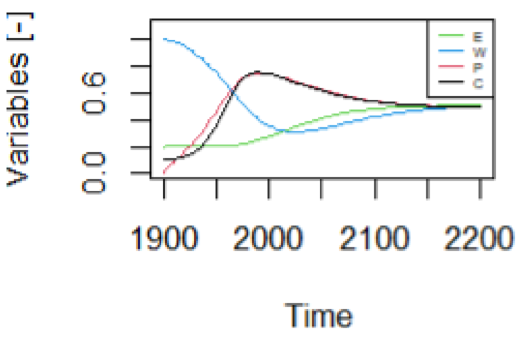
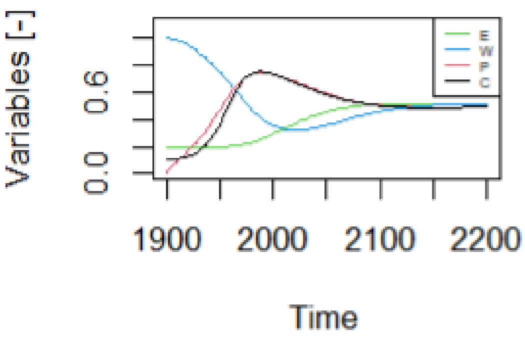
<table>
<tr><th>ψ=1 [-]</th><th>ψ=0.5 [-]</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><th>ψ=0.2 [-]</th><th>ψ=0.05 [-]</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Table 4.9: Evolution of the variables through time for different values of the environmental threshold budget ψ

As Table 4.9 shows, if the environmental threshold budget is the maximum one (=1), the environmental awareness only decreases over time and finds its equilibrium at 0. This case is quite unusual and it's the worst case, but it's important to understand the meaning of this parameter ψ: the amount of investments required to solve the natural problems are very large and higher than the economic health, so the State prefers taking care of the population rather than trying improving the environmental quality. For this reason, the industrial polluting production evolution is always positive and the water quality only decreases over time. The opposite case, the most optimistic one, is when the environmental threshold budget is very low (=0.05) and so the economic health is more easily higher than ψ. The governance is flourished and can take care about the environmental quality, also because the cost for it is affordable. Being the state's wealth quite good, the environmental awareness increases because there are the possibilities on investing on policies or technologies.

However, only in these two extreme scenarios, the equilibrium changes and the variables reach different values from the original ones. What is important to understand is that the higher is the environmental threshold budget, the lower the State is willing to take care also of the nature because the economic health is very low. On the other hand, the larger is the wealth of the people, the higher the considerations for the environment, whatever the *ψ* value. Regarding the equilibrium, it is reached quicker or slower depending on *ψ*. The higher its value is, the more slowly the system reaches the equilibrium and vice versa. Meaningfully, if the efforts required to improve the environment are huge, the State needs more time to aim the desirable results.

At this point, the last left step is the analysis of the speed parameters and the choice of reasonable values for them. Then, the model will be complete and significant results will be discussed.

4.4. Definition of Equilibrium and Speed Parameters' Values

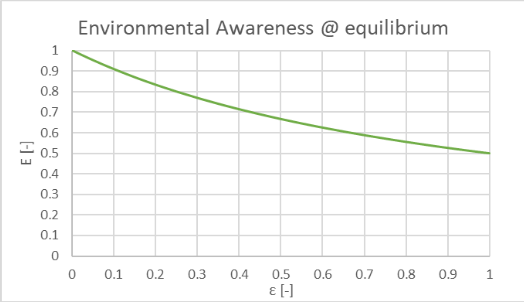
This chapter regards the estimation of the values of the equilibrium parameters and the study of the roles of speed coefficients inside the model. In order to find the most suitable equilibrium point for all the variables, it would be useful having a wide prospective of how the equilibrium parameters *ε, λ, χ* and *η* really affect each quantity of the system. Remembering the coordinates of the equilibrium point *A*:

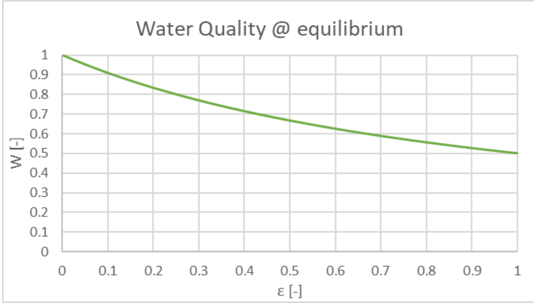
$$A\left(E = \frac{1+\lambda\varepsilon-\varepsilon}{1+\lambda\chi\varepsilon}, W = \frac{1+\lambda\chi-\lambda}{1+\lambda\chi\varepsilon}, P = \frac{1+\chi\varepsilon-\chi}{1+\lambda\chi\varepsilon}, C = 1-\eta\frac{1+\lambda\varepsilon-\varepsilon}{1+\lambda\chi\varepsilon}\right)$$

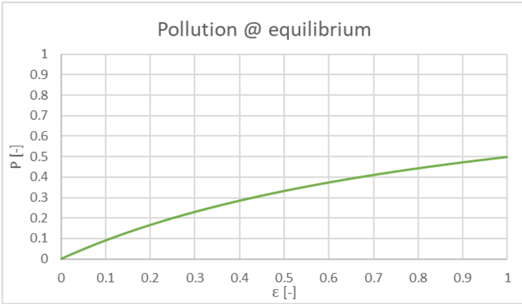
Using "*Excel",* the following graphs are computed: they show how the value at the equilibrium for each variable changes due to different values of the equilibrium parameters. When a coefficient varies, the others are fixed and equal to 1. So, for example, the value at the equilibrium of the environmental awareness is plotted for different values of *ε,* but the other coefficients are set equal to 1. This procedure is repeated for all the variables and all the equilibrium coefficients.

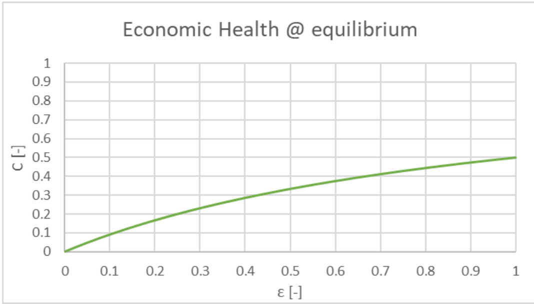
ε [-]

Disregard variation



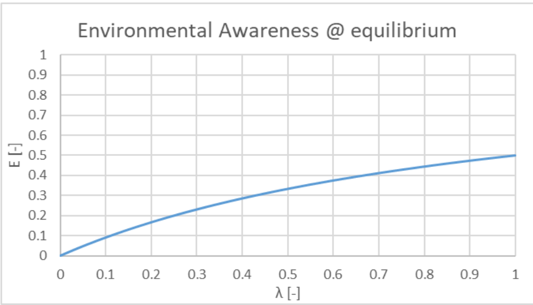


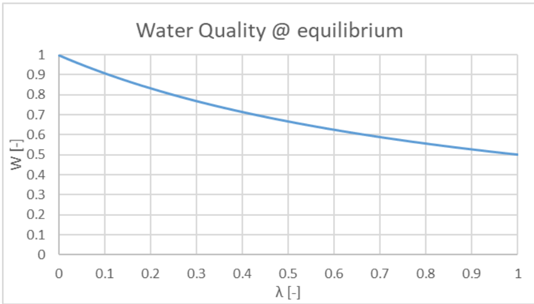


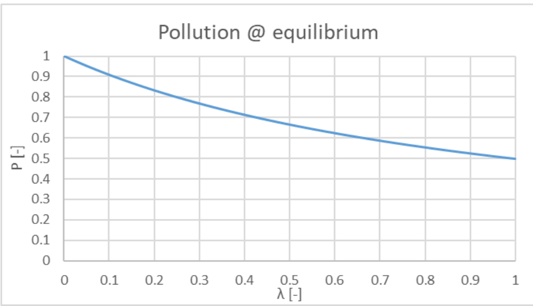


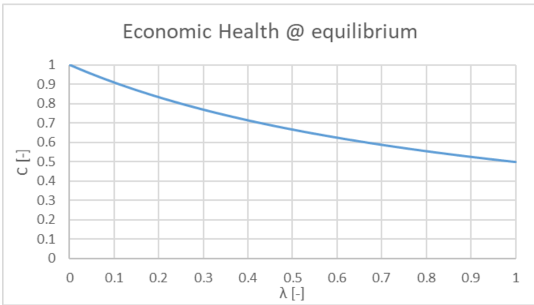
λ [-]

Harmfulness variation









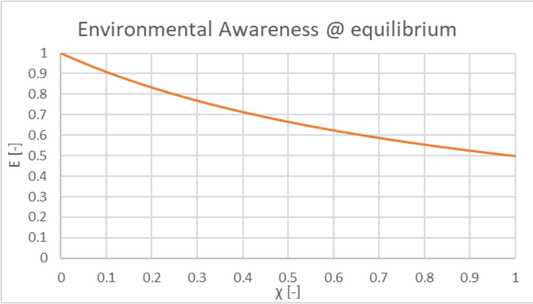
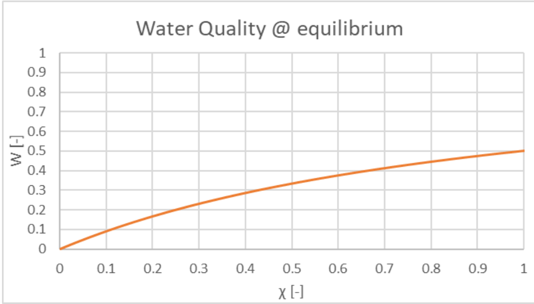
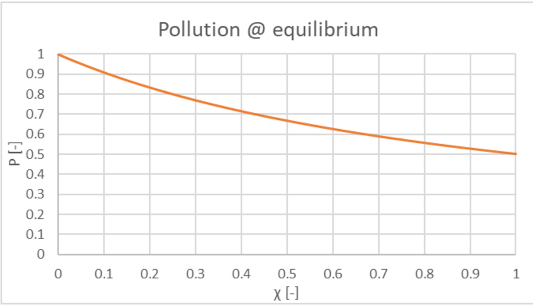
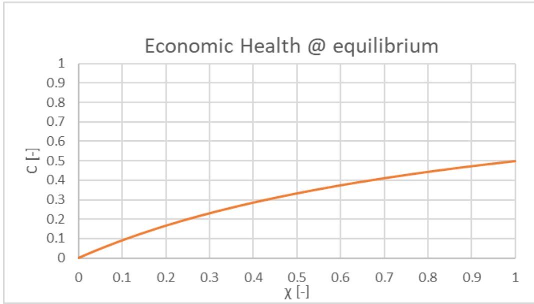
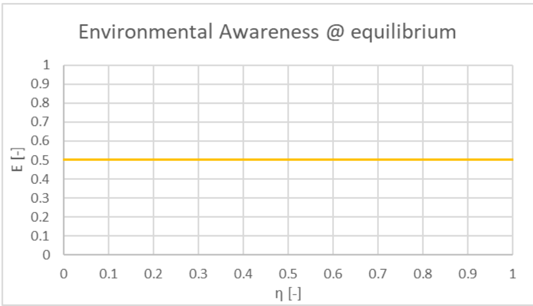
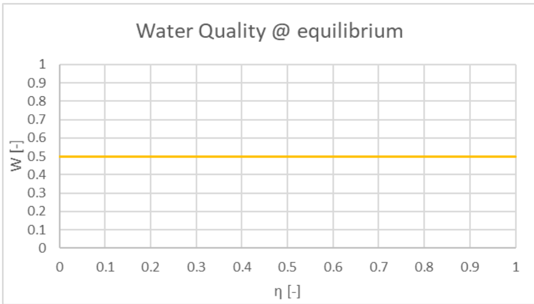
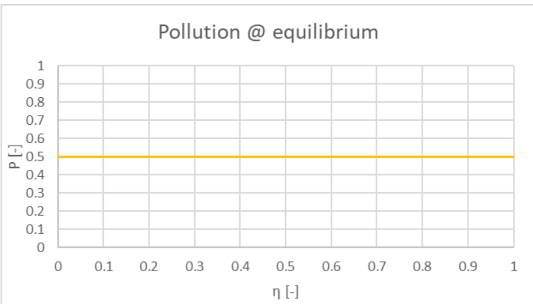
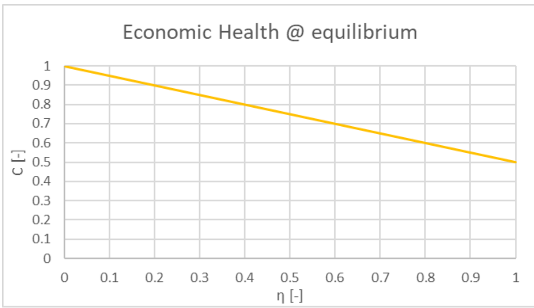
χ [-]	Eco-Friendliness variation		
			
η[-]	Financial Sustainability		
			

Table 4.10: Equilibrium analysis for all the state variables and all the equilibrium parameters

From the previous Table 4.10, the real power of the equilibrium parameters can be appreciated. In fact, this analysis is very useful in order to predict immediately the equilibrium values of the state variables, depending on the chosen condition. For example: the green curves represent which is the final value for each internal quantity just picking a random value of the disregard *ε*, knowing that the other parameters are set equal to 1.

So, if ε=0.1 [-] is taken, the equilibrium value of each variable will be: E=0.9 [-], W=0.9 [-], P=0.1 [-] and C=0.1 [-]. The same discussion counts for the left coefficients.

In particular, a positive growth of the disregard ε (when $\chi=\lambda=\eta=1$) leads to negative effects for the whole system in the future. Actually, the awareness of the Nation about the environment decreases and also the health of the ecosystem services gets worst. At the same time, the concentrations of contaminants in the water bodies and the economic health increase. This means that an uncontrolled growing indifference about the environment will carry to serious natural problems.

Moreover, an increasing of the harmfulness λ (when $\chi=\varepsilon=\eta=1$) leads to a deterioration of the water quality, even if the industrial polluting production decreases. So, this parameter has the strongest effects on the water quality, rather than on pollution: it regards the threats of contaminants and even if their concentration is low due to the environmental awareness's actions, pollutants are so impactful that not even laws or technologies can save the water quality.

Then, a positive growth of the eco-friendliness χ (when $\varepsilon=\lambda=\eta=1$) leads to the best results among all the possible scenarios. Actually, the environmental awareness's value at the equilibrium decreases because a water quality's improvement occurs. This is due to the implementation of green technologies which powerfully act on the pollutants. At the same time, the nation is flourish and the economic health's final value increases with positive change of the eco-friendliness.

The last parameter analysed is the financial sustainability η. When $\varepsilon=\lambda=\chi=1$ and η increases, the only variable affected at the equilibrium is the economic health. Specifically, as the financial sustainability value changes to higher level, the environmental awareness, water quality and industrial polluting production keep their equilibrium values at 0.5[-]. So, the more are the efforts of the environmental awareness on laws and green policies, the larger is the reduction of the economic health of the State, but not producing any effects on the pollution. Then, this is the main different between the eco-friendliness χ and the financial sustainability η: if the aim is achieving a better water quality in the future, the most effective solution is investing on green technologies for the industrial activities rather than improving laws or sustainable policies. This last solution leads only to a decreasing of the economic health and a constant maintenance of pollutants and water quality.

Get to this point of the study, the only terms not analysed yet are the speed parameters. These are α, β, γ and θ and, inside this model, their characteristics are the following:

$$(\alpha, \beta, \gamma, \theta) \geq 0$$

$$[\alpha, \beta, \gamma, \theta] = [yr^{-1}]$$

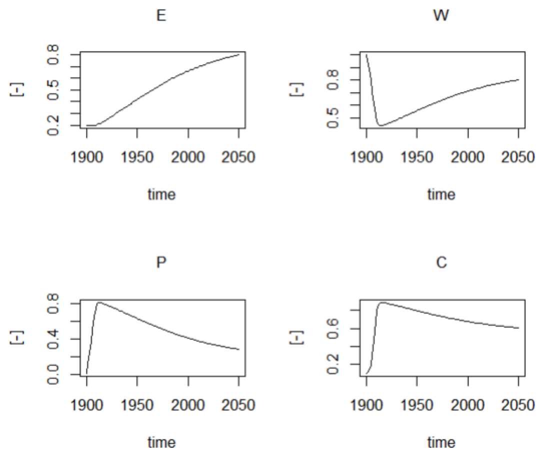
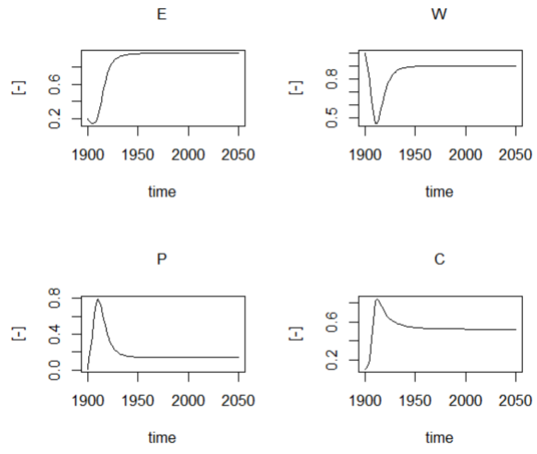
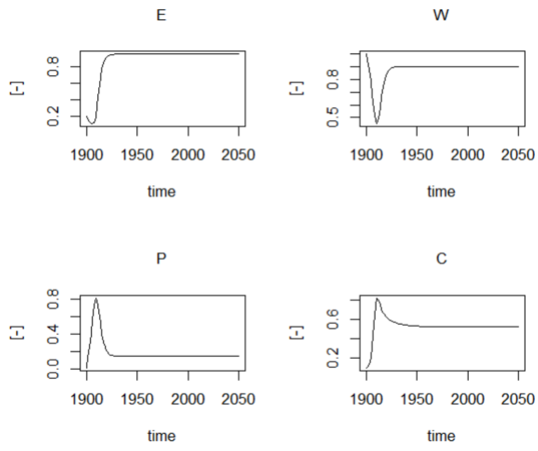
Their dimension is the inverse of time, so they control the speed with which the variables evolve and reach the equilibrium. In order to start the analysis, just recall the ordinary differential equations' system:

$$\begin{cases} \frac{dE}{dt} = \alpha\,(1 - E - \varepsilon\, W)\, E\,(C - \psi) \\ \frac{dW}{dt} = \beta\,(1 - W - \lambda\, P) \\ \frac{dP}{dt} = \gamma\,(1 - P - \chi\, E)\, C \\ \frac{dC}{dt} = \theta\, C\,(1 - C - \eta E)\, P \end{cases}$$

Equation 4.18: ODEs' system of the final model

They aren't affecting the equilibrium point value, but only the time through which the quantities claim this final value. So, the hypothesis is that if one of these parameters changes its value, the relative variable will delay or accelerate its evolution. In the next tables, the model evolution is computed for different values of these coefficients and the time necessary to reach the minimum, maximum and equilibrium values is computed in a time period from 1900 to 2200. When a speed parameter is analysed, the other three are equal to 1 yr^{-1}.

-α, Environmental Awareness's Speed: as shown in Equation 4.18, it changes the speed with which the environmental awareness reaches the equilibrium.

Speed Parameter [yr^{-1}]	Variables [-]	Equil. [-] Year	Min [-] Year	Max [-] Year	
α=0.05	E W P C	after 2200	0.193-1906 0.436-1915 0.01-1900 0.1-1900	0.929-2200 1-1900 0.809-1912 0.887-1915	
α=0.5	E W P C	0.955-2060 0.901-1981 0.140-1997 0.522-2030	0.143-1906 0.457-1912 0.01-1900 0.1-1900	0.955-2060 1-1900 0.784-1909 0.843-1912	
α=1	E W P C	0.955-1948 0.901-1945 0.140-1948 0.522-2018	0.102-1906 0.475-1909 0.01-1900 0.1-1900	0.955-1948 1-1900 0.809-1909 0.802-1912	

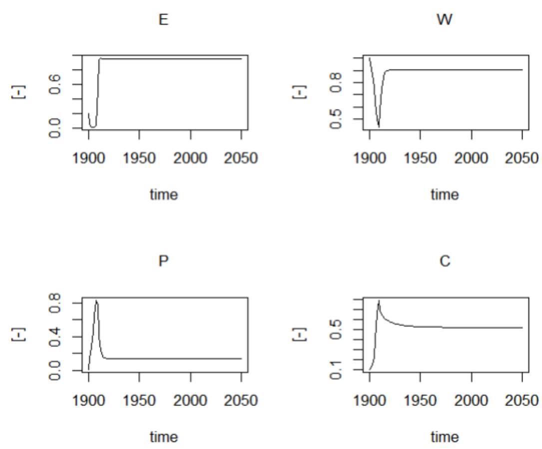
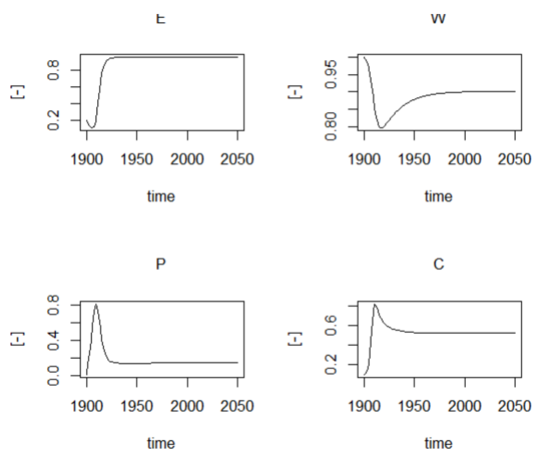
α=5	E W P C	0.955-1921 0.901-1921 0.140-1924 0.522-2015	0.007-1906 0.429-1909 0.01-1900 0.1-1900	0.963-1912 1-1900 0.783-1909 0.796-1909	

Table 4.11: Speed parameter α analysis

Regarding the time required to reach the equilibrium point, the largest difference is recorded for the environmental awareness variable with a delta of 82 years. The maximum and minimum values aren't too much anticipated or delayed by α increasing, but the biggest delta occurs for the environmental awareness variable with -0.041 [-]. So, the quantity much more affected by this parameter's dynamics is the awareness of people about the natural capital, as expected.

-β, Water Quality Speed: as shown in Equation 4.18, it changes the speed with which the water quality reaches the equilibrium.

Speed Parameter [yr^{-1}]	**Variables [-]**	**Equil. [-] Year**	**Min [-] Year**	**Max [-] Year**	
β=0.05	E W P C	0.955-2009 0.901-2057 0.140-2009 0.522-2003	0.102-1906 0.795-1918 0.01-1900 0.1-1900	0.957-1936 1-1900 0.809-1909 0.805-1912	

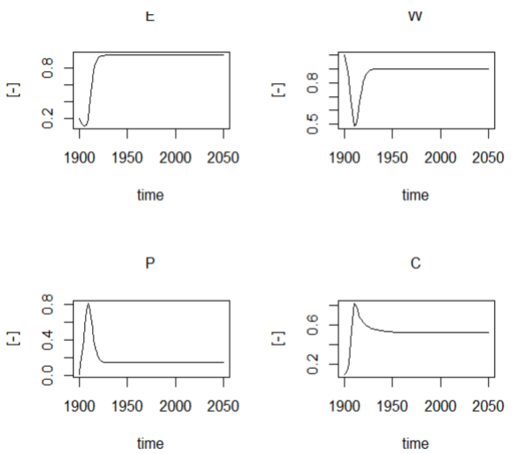
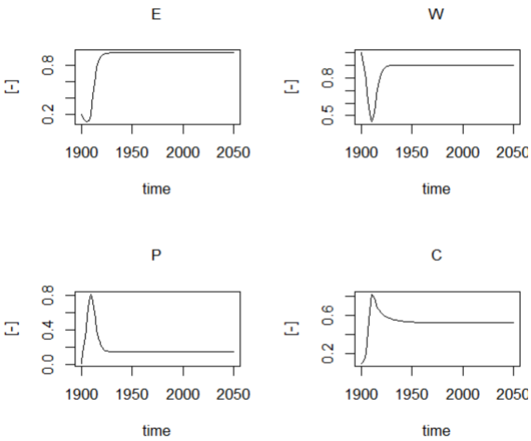
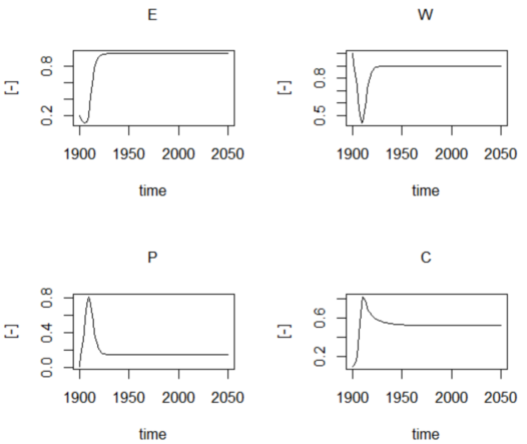
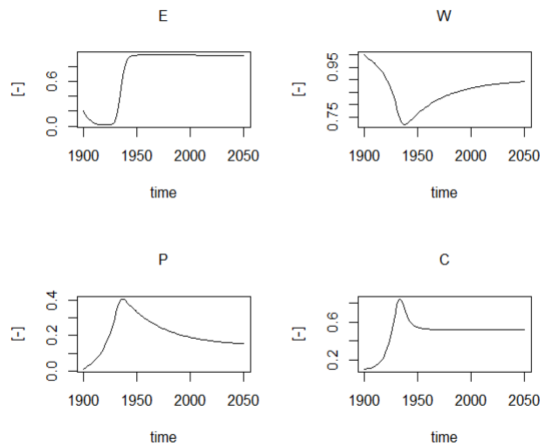
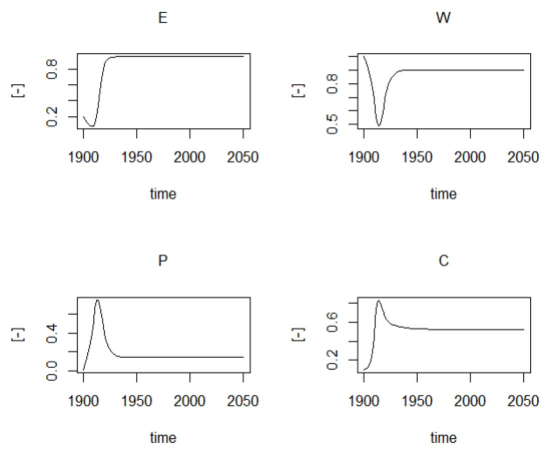
β=0.5	E W P C	0.955-1948 0.901-1945 0.140-1936 0.522-2018	0.102-1906 0.493-1912 0.01-1900 0.1-1900	0.955-1948 1-1900 0.809-1909 0.802-1912	
β=1	E W P C	0.955-1948 0.901-1945 0.140-1948 0.522-2018	0.102-1906 0.475-1909 0.01-1900 0.1-1900	0.955-1948 1-1900 0.809-1909 0.802-1912	
β=5	E W P C	0.955-1948 0.901-1945 0.140-1951 0.522-2018	0.102-1906 0.437-1909 0.01-1900 0.1-1900	0.955-1948 1-1900 0.809-1909 0.802-1912	

Table 4.12: Speed parameter β analysis

Regarding the time required to reach the equilibrium point, the largest difference is recorded for the water quality variable with a delta of 112 years. The maximum and minimum values aren't too much anticipated or delayed by β increasing, but the biggest delta occurs for the water quality variable with -0.3 [-]. So, the quantity much more affected by this parameter's dynamics is the natural capital health, as expected.

-γ, Industrial Polluting Production Speed: as shown in Equation 4.18, it changes the speed with which the industrial polluting production reaches the equilibrium.

Speed Parameter [yr^{-1}]	**Variables [-]**	**Equil. [-] Year**	**Min [-] Year**	**Max [-] Year**	
γ=0.05	E W P C	0.955-2121 after 2200 after 2200 0.522-2136	0.01-1921 0.718-1936 0.01-1900 0.1-1900	0.960-1960 1-1900 0.405-1936 0.844-1933	
γ=0.5	E W P C	0.955-1945 0.901-1951 0.140-1957 0.522-2018	0.08-1909 0.499-1915 0.01-1900 0.1-1900	0.955-1945 1-1900 0.740-1912 0.799-1915	

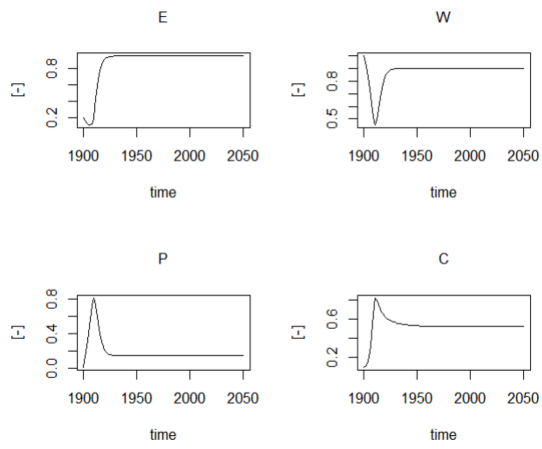
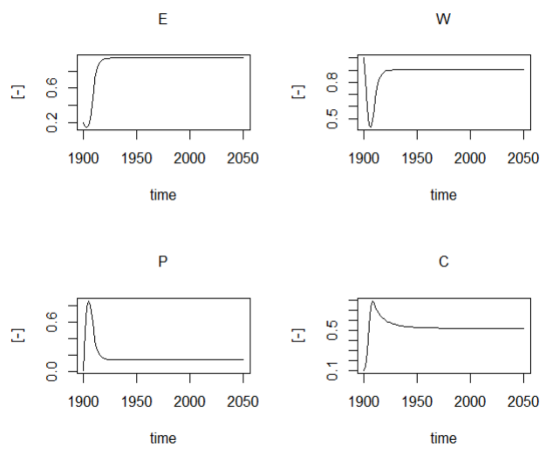
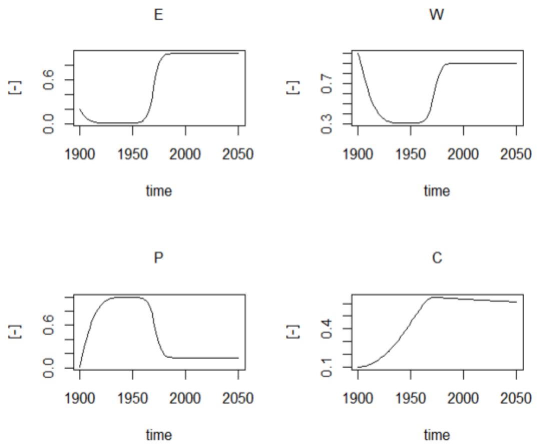
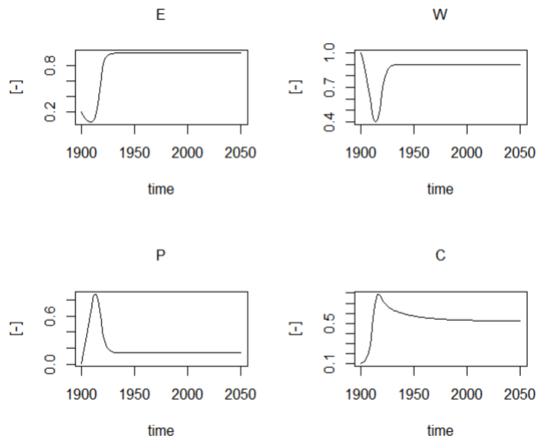
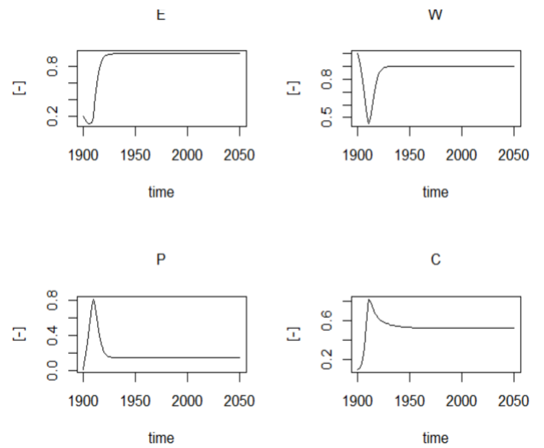
γ=1	E	0.955-1948	0.102-1906	0.955-1948	
	W	0.901-1945	0.475-1909	1-1900	
	P	0.140-1948	0.01-1900	0.809-1909	
	C	0.522-2018	0.1-1900	0.802-1912	
γ=5	E	0.955-1945	0.141-1903	0.955-1945	
	W	0.901-1939	0.427-1906	1-1900	
	P	0.140-1945	0.01-1900	0.796-1906	
	C	0.522-2018	0.1-1900	0.775-1909	

Table 4.13: Speed parameter γ analysis

Regarding the time required to reach the equilibrium point, the largest difference is recorded for the industrial polluting production variable with a delta of 249 years. The maximum and minimum values are quite anticipated or delayed by γ increasing: the maximum of *P* is anticipated of 24 years and increased of 0.335 [-]. So, the quantity much more affected by this parameter's dynamics is concentration of contaminants due to the industrial production, as expected.

-θ, Economic Health Speed: as shown in Equation 4.18, it changes the speed with which the economic health reaches the equilibrium.

Speed Parameter [yr^{-1}]	Variables [-]	Equil.[-] Year	Min [-] Year	Max [-] Year	
θ=0.05	E W P C	0.955-2000 0.901-2000 0.140-2003 after 2200	0.002-1936 0.302-1945 0.01-1900 0.1-1900	0.955-2000 1-1900 0.997-1942 0.650-1972	
θ=0.5	E W P C	0.955-1948 0.901-1945 0.140-1948 0.522-2030	0.07-1909 0.416-1915 0.01-1900 0.1-1900	0.955-1948 1-1900 0.868-1912 0.783-1915	
θ=1	E W P C	0.955-1948 0.901-1945 0.140-1948 0.522-2018	0.102-1906 0.475-1909 0.01-1900 0.1-1900	0.955-1948 1-1900 0.809-1909 0.802-1912	

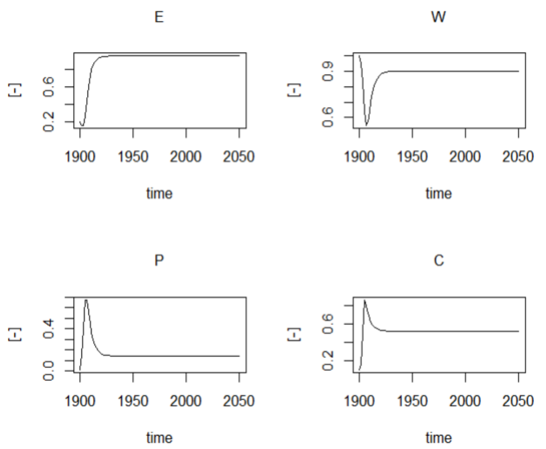
θ=5	E	0.955-1951	0.159-1903	0.955-1951	
	W	0.901-1945	0.546-1906	1-1900	
	P	0.140-1951	0.01-1900	0.669-1906	
	C	0.522-1945	0.1-1900	0.799-1906	

Table 4.14: Speed parameter θ analysis

Regarding the time required to reach the equilibrium point, the largest difference is recorded for the economic health variable with a delta of 170 years. The maximum and minimum values are quite anticipated or delayed by θ increasing: the maximum of *C* is anticipated of 57 years and increased of 0.133 [-]. So, the quantity much more affected by this parameter's dynamics is the economic health of the nation, as expected.

In conclusion of this analysis, the increasing of the speed parameters affects almost the time required to reach the equilibrium point of the relative variable. In particular, the largest anticipation is recorded for the pollution variable and it's of 249 years. In general, the minimum and maximum values of each variable aren't so shifted in time, but their values could increase or decrease. In particular, the biggest variation in the maximum value occurs for *P* and it's of 0.335 [-] and the largest anticipation is of 57 years for the maximum of the economic health. So, as assumed at the beginning, the speed parameters affect only the time necessary to aim the equilibrium point: they will be adjusted in the next chapter to find the most suitable speed to shape the model evolution.

4.5. The Stationary and No-Stationary Case: External Variables Introduction

All the internal variables and the equilibrium and speed parameters have been defined and discussed in the previous chapters. For these reasons, the system of the ordinary differential equations can be considered complete and the socio-hydrological model deeply defined. However, in order to give a realistic example on how build such a model, the stationary case has to be computed.

-Stationary Case Model: Stationary case of a system means that the model works in steady state conditions, so the parameters have constant values in time. The next graphs (Figure 4.6 and Figure 4.7) show the evolution over time of the internal variables and the dynamics of the whole model: each parameter is chosen based on the most reliable and suitable results and on the information got by papers and literature research. In the Table 4.15 these values are reported and they are calculated over a time period from 1900 to 2400 to appreciate when the equilibrium is reached. Moreover, to better visualise the final result, the ordinary differential equations are reported again in the next system:

$$
\begin{cases}
\frac{dE}{dt} = \alpha\,(1 - E - \varepsilon\,W)\,E\,(C - \psi) \\
\frac{dW}{dt} = \beta\,(1 - W - \lambda\,P) \\
\frac{dP}{dt} = \gamma\,(1 - P - \chi\,E)\,C \\
\frac{dC}{dt} = \theta\,C\,(1 - C - \eta E)\,P
\end{cases}
$$

Equation 4.19: ODEs' system of the final model

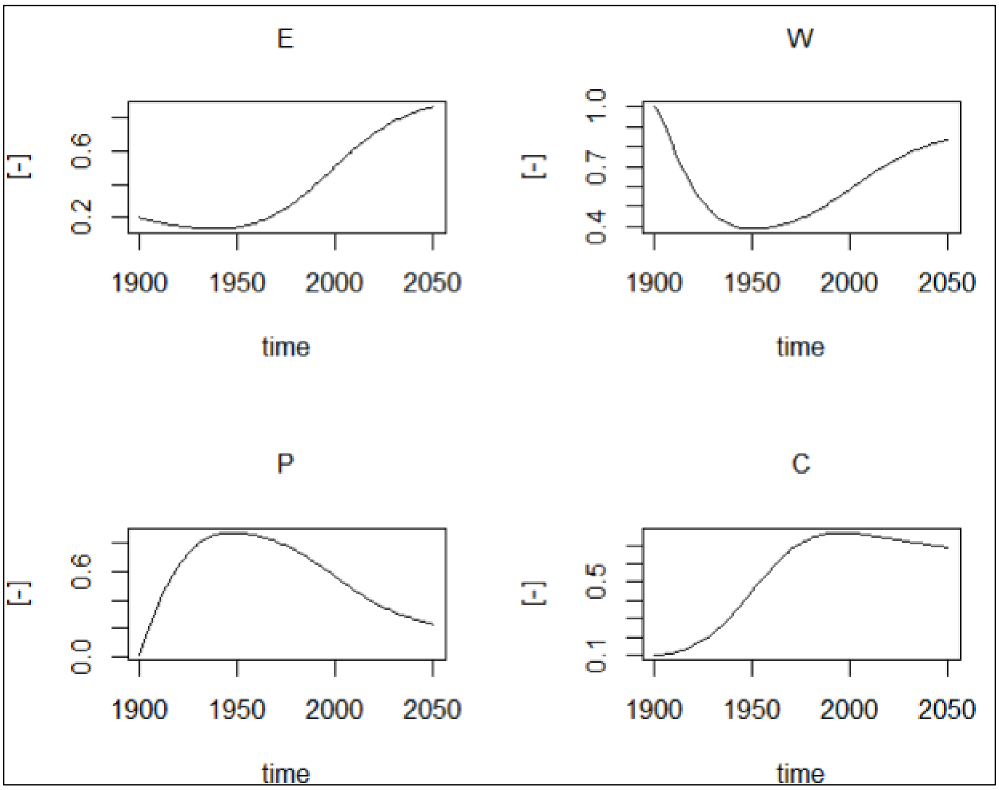


Figure 4.6: Evolution of the internal variables in the final socio-hydrological model

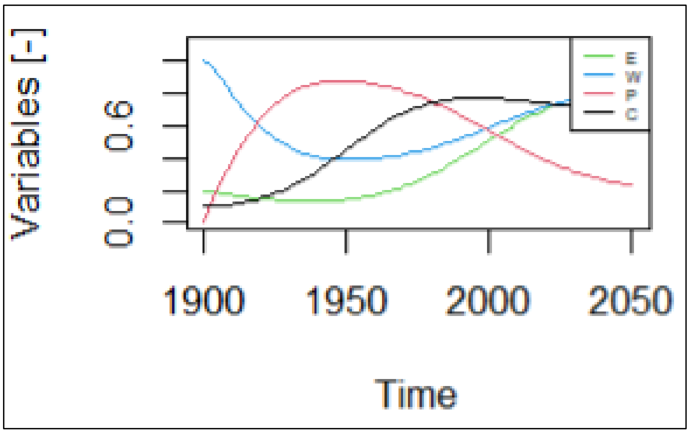


Figure 4.7: Dynamics of the whole model in which all the state variables are plotted together

Speed Parameters [yr^{-1}]	Equilibrium Parameters [-]	State Variable	Initial Value [-]	Max [-] Year	Min [-] Year	Equilibrium Value [-] Year
α=0.1	ε=0.05	E	0.2	0.955 2339	0.133 1935	0.955 2339
β=0.4	λ=0.7	W	1	1 1900	0.388 1950	0.901 2263
γ=0.6	χ=0.9	P	0.01	0.874 1945	0.01 1900	0.140 2223
θ=0.07	η=0.5; ψ=0.3	C	0.1	0.770 1995	0.1 1900	after 2400

Table 4.15: Speed and Equilibrium Parameters analysis inside the socio-hydrological model

In addition, the equilibrium point A is computed, resolving the system reported in Equation 6.18, and its coordinates are the following ones:

$$A\left(E = \frac{1+\lambda\varepsilon-\varepsilon}{1+\lambda\chi\varepsilon}, W = \frac{1+\lambda\chi-\lambda}{1+\lambda\chi\varepsilon}, P = \frac{1+\chi\varepsilon-\chi}{1+\lambda\chi\varepsilon}, C = 1-\eta\frac{1+\lambda\varepsilon-\varepsilon}{1+\lambda\chi\varepsilon}\right)$$

$$A\,(E = 0.955\,[-], W = 0.901[-], P = 0.140[-], C = 0.522[-])$$

An accurate discussion of all the model will be given in the Chapter 5, but some considerations can be foreseen: the disregard ε is very low, in comparison with the harmfulness λ and the eco-friendliness χ, which is the highest parameter. The financial sustainability η is also quite high and the environmental threshold budget is affordable. The fastest speed parameter is the industrial polluting production and the slowest one is the economic health. It makes sense because the spread of contaminants by industrial activities is a pretty quick phenomenon and, at the same time, the dynamics of the economic health of a Nation requires a lot of time. For these reasons, the variable that reaches first the equilibrium point is the industrial polluting production and the last one aims the stable fixed point is the economic health. Moreover, the parameters values are set in such a way to obtain logical results, considering the literatures and previous studies. The hopeful equilibrium point for the environmental awareness is 0.955 [-] and that one for the water quality is 0.901 [-], which are both encouraging results. The lowest value in a sustainable future is for the polluting, with 0.140 [-], instead the economic health sees a satisfactory equilibrium value for both a green and economical prosperous future. Anyway, a deeper analysis of the socio-hydrological model dynamics is shown in the next Chapter 5.

-No-Stationary Case Model: the no-stationary case is developed to study how the system evolves when one or more parameters change over time. In this case, the speed or the equilibrium coefficients aren't constant values, but they are defined with a specific mathematical function. Due to the no-steady conditions, the changing speed parameters and the equilibrium ones can be linked with the external variables' presence. So, the two external variables are added to the model as functions reproducing the variations over time of the speed and equilibrium parameters. In particular, as shown in Figure 3.11, the causal loop diagram defined two external forcings: the demographic change and the climate change. The demographic change has a positive feedback on the industrial polluting production, instead the climate change has a negative feedback on the water quality. To the first external variable is linked the variation of the industrial polluting production speed parameter γ and to the second external forcing is linked the variation of the harmfulness λ. Before showing the results, let's introduce each of these variables and understand their behaviours.

The *DEMOGRAPHIC CHANGE (D)* refers to change in the population composition and density. In fact, the larger is the population size inside a Nation, the higher is the goods demand and so the industrial production. Then, the pollution could increase due to these reasons and the water quality may deteriorates. On the other hand, if the demographic change isn't so relevant, there are less people requiring for more goods and so also the industrial polluting production is negatively affected. In order to build the mathematical function shaping the demographic change dynamics, some scenarios are taken from literature and previous studies. As it's shown in Figure 4.8, the demographic change has been analysed by "*Eurostat"* in a time period from 1990 to 2060 and different scenarios are plotted. The coloured curves refer to more optimistic or pessimistic forecasting of the population growth: this graph is used in order to understand which kind of function can better reproduce these paths and a logistic growth is chosen as the best option.

The system of ODEs in Equation 4.19 is rewritten by considering the no-stationary case and changing γ in γ_1, the industrial polluting production's speed parameter in the no-steady conditions. The new system is reported in the next Equation 4.20:

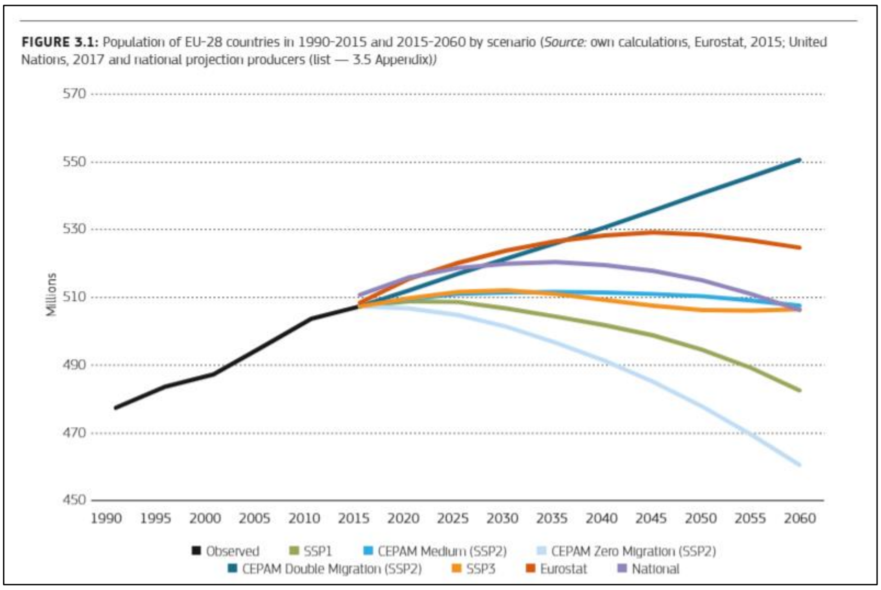


Figure 4.8: Demographic change evolution and scenarios in Europe from 1990 to 2060, by Eurostat

$$
\begin{cases}
\frac{dE}{dt} = \alpha\,(1 - E - \varepsilon\, W)\, E\, (C - \psi) \\
\frac{dW}{dt} = \beta\,(1 - W - \lambda\, P) \\
\frac{dP}{dt} = \gamma_1\,(1 - P - \chi\, E)\, C \\
\frac{dC}{dt} = \theta\, C\,(1 - C - \eta E)\, P
\end{cases}
$$

$$where:\ \gamma_1(t) = \gamma + \frac{\kappa - \gamma}{1+e^{-\mu(t-t_0)}}\ [yr^{-1}]$$

Equation 4.20: ODEs' system with the no-stationary γ_1 speed parameter

The system is the same as before but a no-stationary parameter is considered in the industrial polluting production derivative: γ_1 depends on time t and its evolution is described by a logistic function. Several factors appear on its equation: t_0 [yr] is the time at which the inflection point occurs; μ [yr^{-1}] is the rate of growth; κ [-] is the carrying capacity; γ [yr^{-1}] is the initial value of this parameter in 1900 and is the same of the stationary case (0.6 [yr^{-1}]). Then, four main scenarios are built, changing the values of the factors in the γ_1 expression, and then the four different pathways try to mimic the influence of first external forcing: the demographic change *D*. What is expected from this analysis is that the no-stationarity of the speed parameter γ_1 changes the speed with which the pollution variable reaches the equilibrium. The four different scenarios are called with arbitrary names in order to differentiate them and remember their characteristics.

1.1) The Bio-Bomb: in this first scenario, the world's population continues growing steadily, fuelled by high birth rates, improvements in medical care, and increased longevity. This could lead to challenges related to natural resource management, pressure on health services, and accelerated urbanization and, for these reasons, this scenario is called "*Bio-Bomb*". The expression of the speed parameter γ_1 is the next one:

$$\gamma_1(t) = \gamma + \frac{\kappa - \gamma}{1 + e^{-\mu(t-t_0)}} = 0.6 + \frac{9.6 - 0.6}{1 + e^{-0.02(t-2055)}}$$

Equation 4.21: No-stationary speed parameter γ_1 for the Bio-Bomb scenario

Using *"R"*, the model is computed with these values and the following figures are produced:

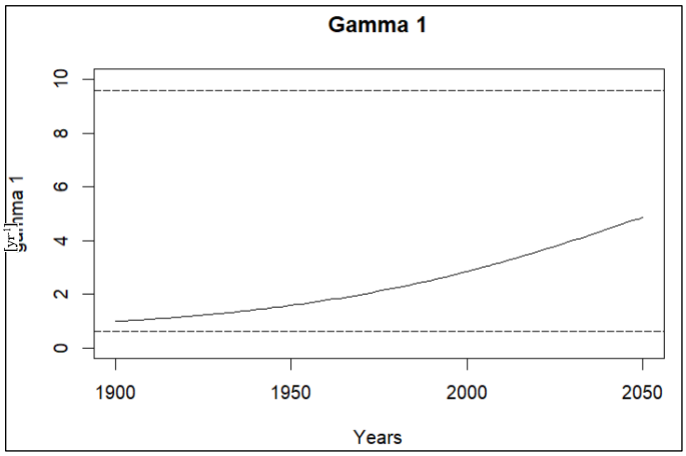


Figure 4.9: Evolution over time of the no-stationary speed parameter γ_1

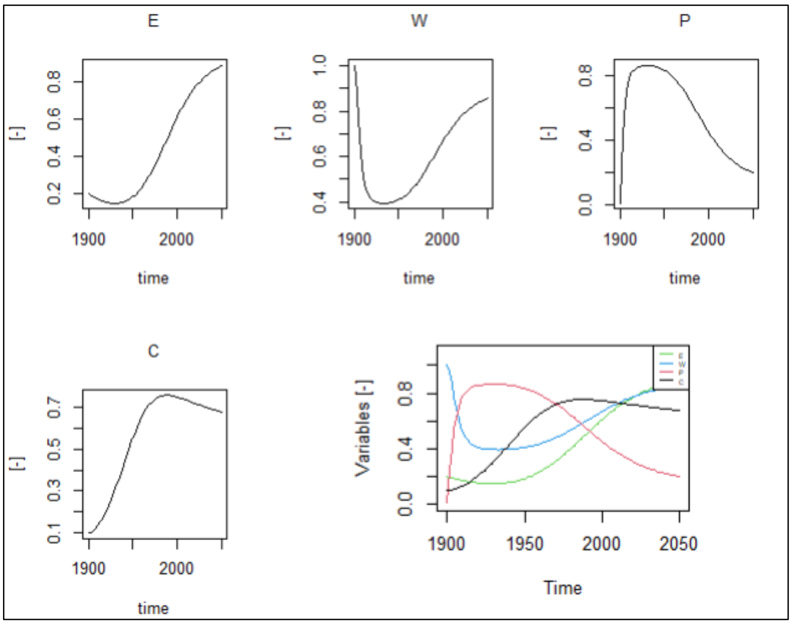


Figure 4.10: Evolution of the model after the no-stationary speed parameter γ_1 addition

The next Table 4.16 shows the parameters and the values of the minimum, maximum and equilibrium point for each variable. Moreover, the differences are computed compared to the original system.

Speed Parameters [yr^{-1}]	**Equilibrium Parameters [-]**	**State Variable**	**Max [-] Year**	**Δ**	**Min [-] Year**	**Δ**	**Equilibrium Value [-] Year**	**Δ**
α=0.1	ε=0.05	E	0.955 2329	-10yr	0.149 1930	+0.02 -7yr	0.955 2329	-10yr
β=0.4	λ=0.7	W	1 1900	/	0.395 1935	+0.007 -15yr	0.901 2247	-16yr
γ=0.6	χ=0.9	P	0.865 1930	-0.01 -17yr	0.001 1900	/	0.140 2304	-20yr
θ=0.07	η=0.5, ψ=0.3	C	0.755 1986	-0.01 -9yr	0.1 1900	/	0.522 after 2400	same

Table 4.16: Results of the Bio-Bomb scenario

Due to the unbounded population growth, the industrial polluting production reaches the equilibrium point faster, with an anticipation of about 20 years. Regarding the largest difference in values, it's for the min/max of the environmental awareness that both increase of 0.02. Instead, the highest anticipation concerns the minimum and the maximum values of water quality and pollution, respectively of 15 years and 17 years.

1.2) Very Slow Population Stabilization: this second scenario implies a slowdown in world population growth, with a stabilization of birth rates and a decrease in mortality. Birth control policies, improved education and economic opportunities for women could be implemented. This could lead to greater environmental sustainability and less pressure on public services, but it could also lead to challenges related to an aging population and a shrinking workforce. These changes happen quite slowly and, for these reasons, this scenario is called "*Very Slow Population Stabilization*". The expression of the speed parameter γ_1 is the next one:

$$\gamma_1(t) = \gamma + \frac{\kappa - \gamma}{1 + e^{-\mu(t-t_0)}} = 0.6 + \frac{4.6 - 0.6}{1 + e^{-0.04(t-2010)}}$$

Equation 4.22: No-stationary speed parameter γ_1 for the Very Slow Population Stabilization scenario

Using *"R"*, the model is computed with these values and the following figures are produced:

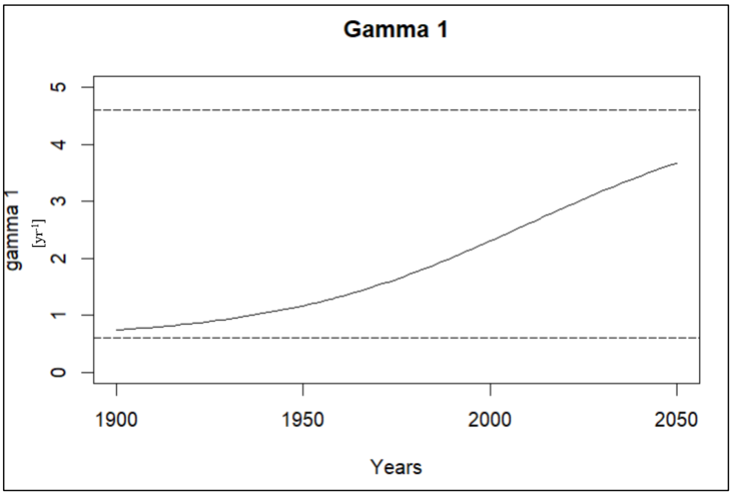


Figure 4.11: Evolution over time of the no-stationary speed parameter γ_1

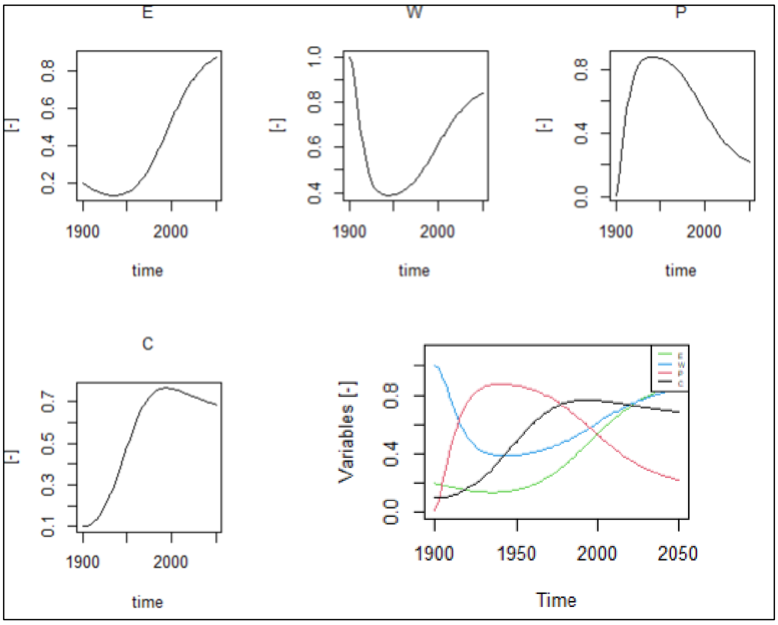


Figure 4.12: Evolution of the model after the no-stationary speed parameter γ_1 addition

The next Table 4.17 shows the parameters and the values of the minimum, maximum and equilibrium point for each variable. Moreover, the differences are computed compared to the original system.

Speed Parameters [yr^{-1}]	**Equilibrium Parameters [-]**	**State Variable**	**Max [-] Year**	**Δ**	**Min [-] Year**	**Δ**	**Equilibrium Value [-] Year**	**Δ**
α=0.1	ε=0.05	E	0.955 2335	-4yr	0.136 1935	+0.003 -2yr	0.955 2335	-4yr
β=0.4	λ=0.7	W	1 1900	/	0.387 1945	-0.001 -5yr	0.901 2257	-6yr
γ=0.6	χ=0.9	P	0.876 1940	+0.002 -7yr	0.01 1900	/	0.140 2316	-8yr
θ=0.07	η=0.5, ψ=0.3	C	0.766 1995	-0.004	0.1 1900	/	0.522 after 2400	same

Table 4.17: Results of the Very Slow Population Growth scenario

If the population grows very slowly, the model shows that the variables require less time to reach the equilibrium than the previous case. The biggest equilibrium advance regards the industrial polluting production, with an anticipation of 8 years. Moreover, the minimum/maximum values are brought forward too and the largest time-shift concerns the pollution with 7 years. The peak of industrial production increases of about 0.002 [-] and the water quality deteriorates of 0.001 [-].

1.3) Very Fast Population Stabilization: this third scenario implies that the population grows quite quickly because the births are very abundant or the deaths are less common. In this scenario, births are less controlled and the State looks after more to old people. This scenario could be similar to the unbounded population growth (the first scenario), but some differences are expected.

$$\gamma_1(t) = \gamma + \frac{\kappa - \gamma}{1 + e^{-\mu(t-t_0)}} = 0.6 + \frac{4.6 - 0.6}{1 + e^{-0.07(t-2000)}}$$

Equation 4.23: No-stationary speed parameter γ_1 for the Very Fast Population Stabilization scenario

Thnks to *"R"*, the model is computed with the previous equation of γ_1 and the following figures are produced:

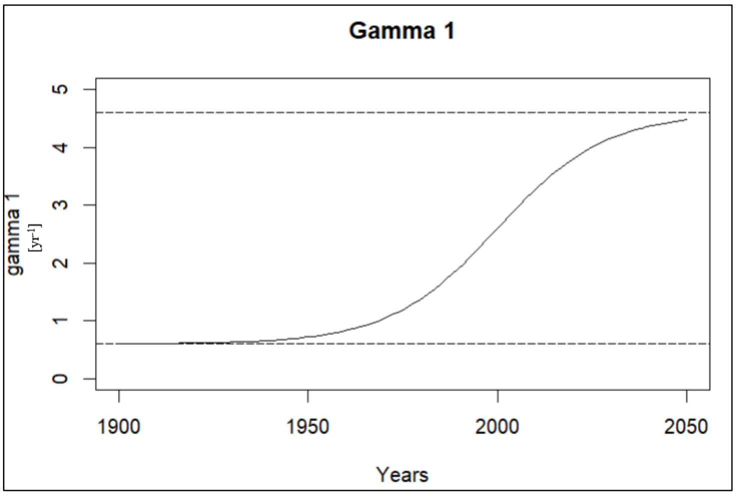


Figure 4.13: Evolution over time of the no-stationary speed parameter γ_1

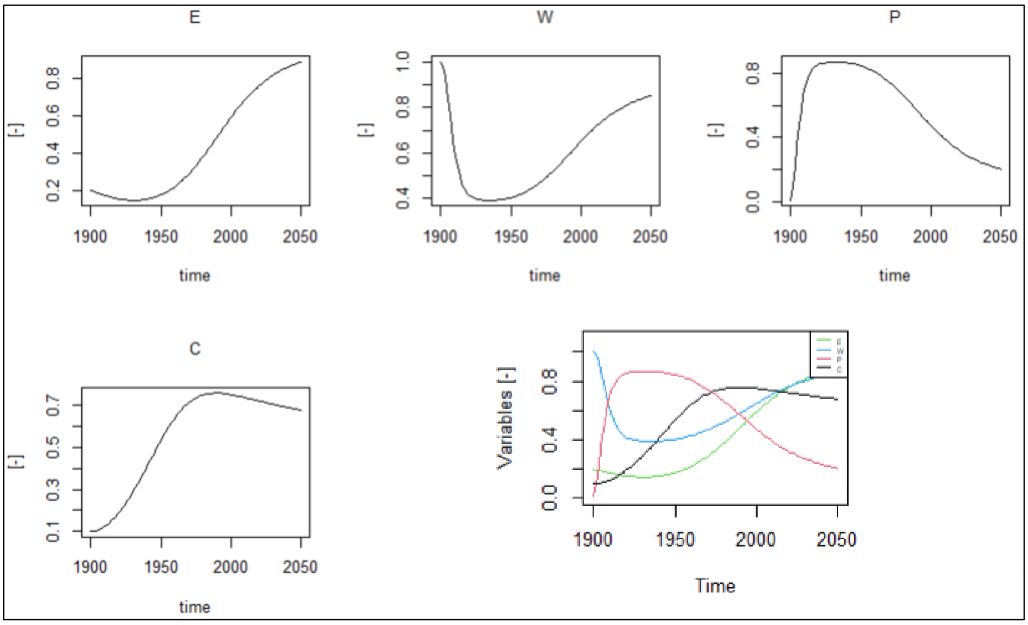


Figure 4.14: Evolution of the model after the no-stationary speed parameter γ₁ addition

The next Table 4.18 shows the parameters and the values of the minimum, maximum and equilibrium point for each variable. Moreover, the differences are computed compared to the original system.

Speed Parameters [yr^{-1}]	Equilibrium Parameters [-]	State Variable	Max [-] Year	Δ	Min [-] Year	Δ	Equilibrium Value [-] Year	Δ
α=0.1	ε=0.05	E	0.955 2334	-5yr	0.145 1930	+0.012 -7yr	0.955 2334	-5yr
β=0.4	λ=0.7	W	1 1900	/	0.391 1935	+0.003 -15yr	0.901 2255	-8yr
γ=0.6	χ=0.9	P	0.869 1930	-0.005 -17yr	0.01 1900	/	0.140 2314	-10yr
θ=0.07	η=0.5, ψ=0.3	C	0.758 1991	-0.012 -4yr	0.1 1900	/	0.522 after 2400	same

Table 4.18: Results of the Very Fast Population Growth scenario

This scenario could seem very similar to the "*Bio-Bomb*" scenario but in this case the time required to reach the equilibrium is shorter than before. Here, the variables reach the equilibrium with an anticipation of maximum 10 years, instead in the *"Bio-Bomb"* one the maximum was 20 years. Moreover, the maximum and minimum values of industrial polluting production and water quality are advanced of 17 years and 15 years, respectively. The difference in the maximum and minimum values aren't so relevant.

*1.4) Catastrophic Event: i*n this last scenario, events such as full-scale wars, global pandemics, natural catastrophes, or environmental disasters could drastically affect the world's population. These events could cause sudden decreases in population or radical changes in geographical distribution. Such a scenario could lead to deep economic, social and political turmoil.

$$\gamma_1(t) = \gamma + \frac{\kappa - \gamma}{1 + e^{-\mu(t-t_0)}} = 0.6 + \frac{0.3 - 0.6}{1 + e^{-0.06(t-1920)}}$$

Equation 4.24: No-stationary speed parameter γ_1 for the Catastrophic Event scenario

Implementing on "*R*" the previous equation of γ_1, the next figures are computed and the Table 4.19 is filled with the final results.

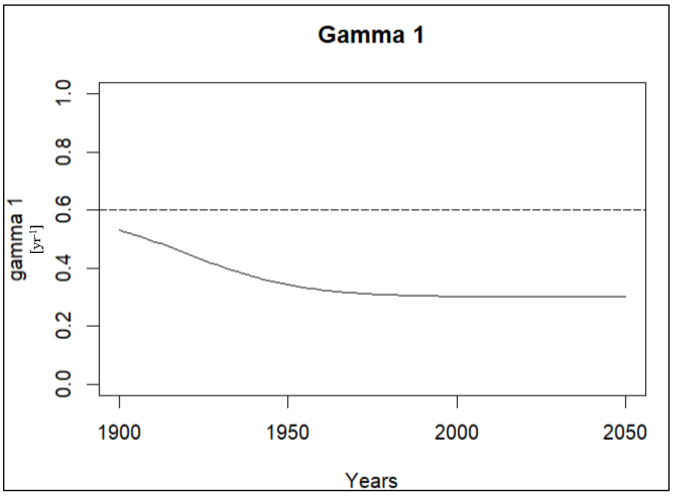


Figure 4.15: Evolution over time of the no-stationary speed parameter γ_1

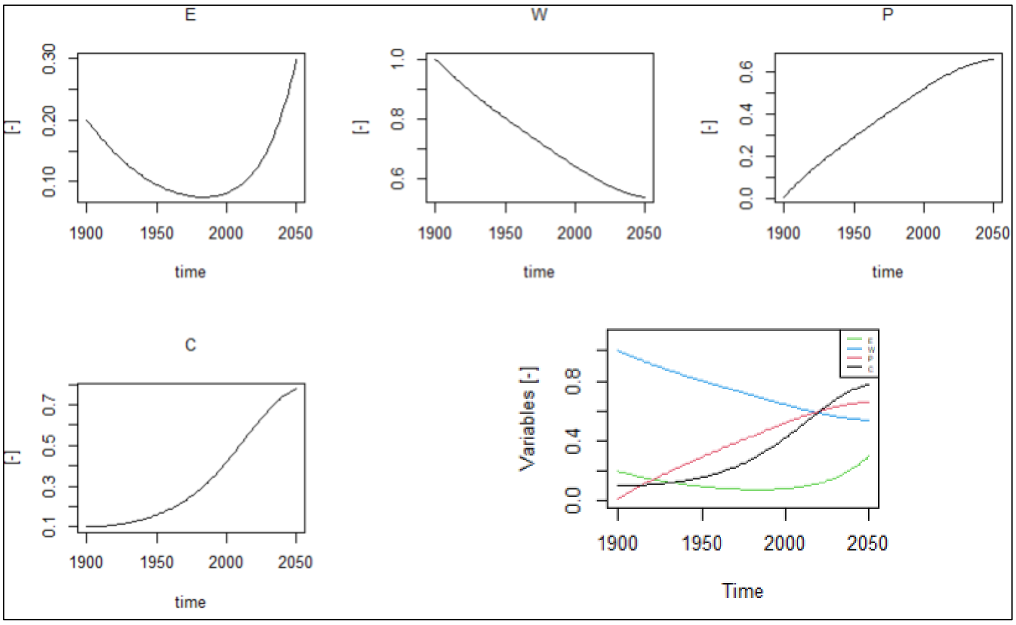


Figure 4.16: Evolution of the model after the no-stationary speed parameter $\gamma_1$addition

Speed Parameters [yr^{-1}]	**Equilibrium Parameters [-]**	**State Variable**	**Max [-] Year**	**Δ**	**Min [-] Year**	**Δ**	**Equilibrium Value [-] Year**	**Δ**
α=0.1	ε=0.05	E	0.955 >2400	+100yr	0.075 1986	-0.06 +49yr	0.955 >2400	+100yr
β=0.4	λ=0.7	W	1 1900	/	0.536 2062	+0.15 +112yr	0.901 >2400	+200yr
γ=0.6	χ=0.9	P	0.662 2056	-0.21 +109yr	0.01 1900	/	0.140 >2400	+100yr
θ=0.07	η=0.5, ψ=0.3	C	0.791 2062	+0.02 +66	0.1 1900	/	0.522 after 2400	same

Table 4.19: Results of the Catastrophic Event scenario

This scenario is the most optimistic because the values are heavily delayed of 100 yr. In particular, if the population decreases over time, the water quality increases and its minimum value (+0.21 [-]) is delayed of 112 years. At the same time, the industrial polluting production decreases its maximum value (-0.21 [-]) and postpones it by 109 years. Regarding the equilibrium, all the variables reach theirs after 200 years. This scenario is the only one which shows positive effects for all the variables, in particular a reduction of the polluting production and an enhancement of the water quality. However, the final equilibrium values are the same for all the scenarios and, in this case, they are only delayed over time, but not improved.

In conclusion, the variability of the speed parameter γ_1 over time produces several effects on the model: the biggest impacts are related to the time required to reach the equilibrium and the minimum and maximum values of the pollution and the water quality. It makes sense because the variability of γ reflects the impact of the demographic change in the model and in particular in the industrial polluting production. For this reason, the larger and faster is the demographic change, the bigger and faster will be the differences in the values of the variables. Finally, the main difference between an unbounded population growth and a very fast growth rate is that in the first case the anticipations of equilibrium are larger than in the second case. Moreover, a very fast population growth rate is able to brought forward the maximum and minimum values of the variables as much as the unbounded scenario. This means that the "Bio-Bomb" scenario produces more effects on the future, rather than in the current present.

As anticipated before, the least and last external variable that must be considered inside the model is the climate change. In the next analysis, this external forcing will be studied as a no-stationary case of the harmfulness parameter λ.

The *CLIMATE CHANGE (C_c)* refers to a long-term alteration of temperature and typical weather patterns in a specific location or across the entire planet. It encompasses various observed effects related to shifts in Earth's local, regional, and global climates. These changes are driven by both natural processes and human activities and they could be droughts, floods, intense precipitation, warmer temperature, etc. Anyway, whatever change is chosen, its feedback on the water quality is always the same: a negative effect on it. The droughts accumulate the contaminants ad so increases the concentration of pollutants; the floods help contaminants to spread around and pollute wider areas; intense precipitation also lead to a flowing of pollutants; higher temperature changes the chemistry composition of the surface water and exacerbates the algae bloom.

Like for the demographic change, also the effects of this new external forcing are modelled considering the no-stationarity of a parameter: the harmfulness λ is chosen to implement this analysis. This coefficient affects the equilibrium point and in particular the final value of the water quality variable. In fact, the climate change variable influences negatively the water quality and so increases the harmfulness in the natural capital. This external variable can't be linked with the disregard ε coefficient or with the eco-friendliness χ coefficient because the first one refers to population awareness about the environment and the second one to the industrial polluting production.

In order to build the mathematical function shaping the climate change dynamics, some scenarios are taken from literature and previous studies. As it's shown in Figure 4.17, some of the climate change effects have been deeply analysed by *IPCC* Scenarios. Instead, Figure 6.18 shows the *RCPs*, the *"Representative Concentration Pathways":* they are climate change scenarios used to project future greenhouse gas concentrations. These pathways describe anticipated greenhouse gas concentrations (not emissions) and have been formally adopted by the Intergovernmental Panel on Climate Change (IPCC).

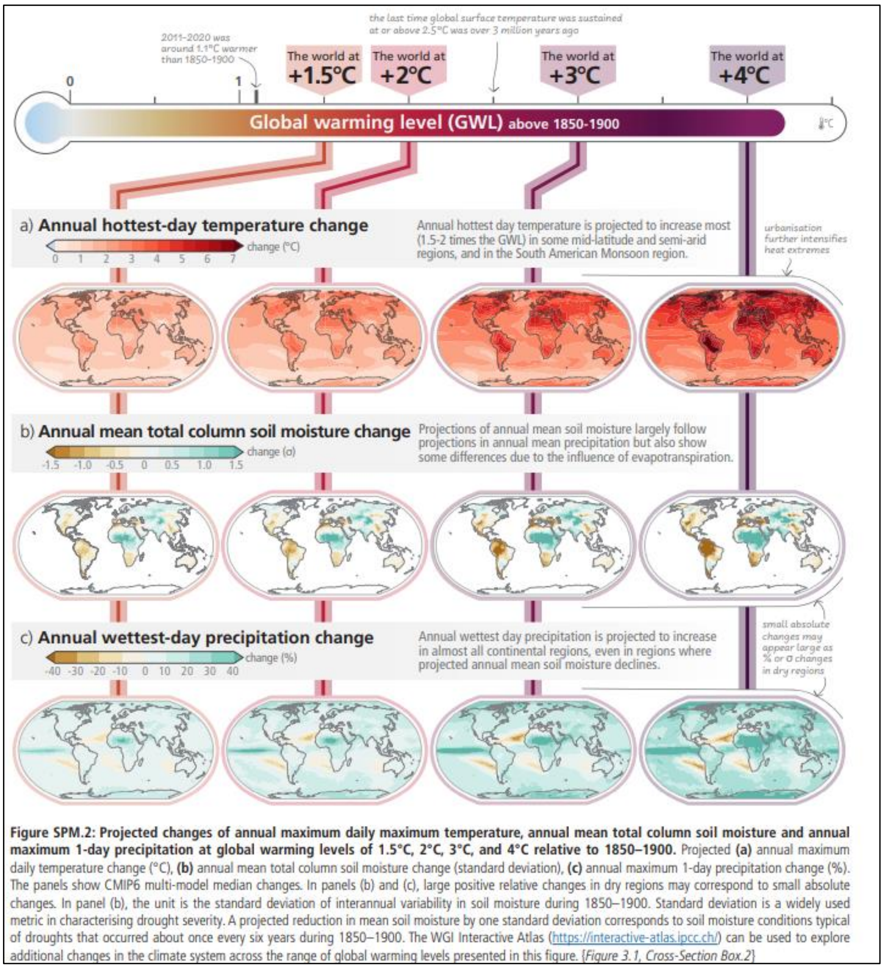


Figure 4.17: Projected changes of different climate change effects (by Interactive Atlas IPCC)

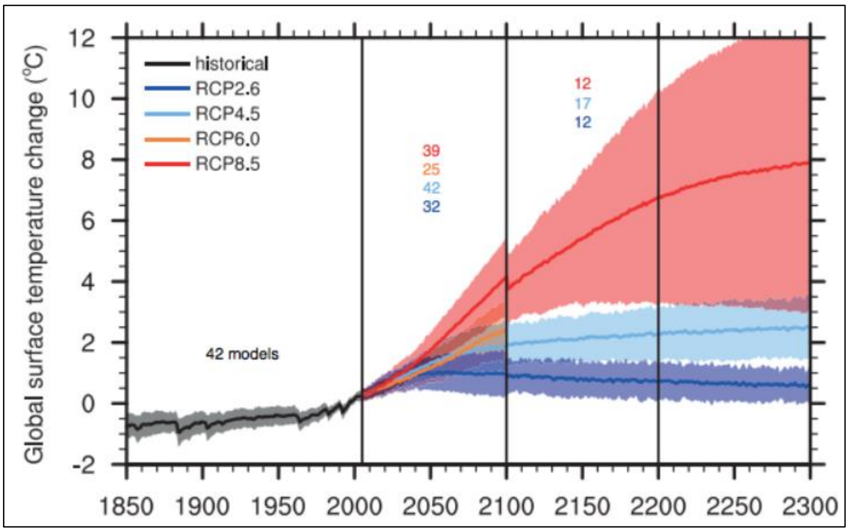


Figure 4.18: RCPs for the global surface temperature change (by IPCC Report)

Figure 4.18 is used in order to understand which kind of function can better reproduce these paths and a logistic growth is chosen as the best option. So, the system of ODEs in Equation 4.19 is rewritten by considering the no-stationary case and changing λ in λ_1, the harmfulness parameter in the no-steady conditions. The new system is reported in the next Equation 4.25:

$$\begin{cases} \frac{dE}{dt} = \alpha\,(1 - E - \varepsilon\, W)\, E\,(C - \psi) \\ \frac{dW}{dt} = \beta\,(1 - W - \lambda_1\, P) \\ \frac{dP}{dt} = \gamma\,(1 - P - \chi\, E)\, C \\ \frac{dC}{dt} = \theta\, C\,(1 - C - \eta E)\, P \end{cases}$$

$$where:\ \lambda_1(t) = \lambda + \frac{\phi - \lambda}{1 + e^{-\rho(t - t_0)}}\ [-]$$

Equation 4.25: ODEs' system with the no-stationary λ_1 speed parameter

The system is the same as the original one but a no-stationary parameter is considered in the water quality derivative: λ_1 depends on time *t* and its evolution is described by a logistic function. Several factors appear on its equation: t_0 [yr] is the time at which the inflection point occurs; ρ [yr^{-1}] is the rate of growth; ϕ [-] is the carrying capacity; λ [-] is the initial value of this parameter in 1900 and is the same of the stationary case (0.7 [-]). As pointed out before, λ_1 changes over time and so the equilibrium points do the same. The equilibrium point A must be upgraded with the new definition of λ_1:

$$A\left(E = \frac{1 + \lambda_1 \varepsilon - \varepsilon}{1 + \lambda_1 \chi \varepsilon}, W = \frac{1 + \chi \lambda_1 - \lambda_1}{1 + \lambda_1 \chi \varepsilon}, P = \frac{1 + \chi \varepsilon - \chi}{1 + \lambda_1 \chi \varepsilon}, C = 1 - \eta \frac{1 + \varepsilon \lambda_1 - \varepsilon}{1 + \lambda_1 \chi \varepsilon}\right)$$

Then, four main scenarios are built, changing the values of the factors in the λ_1 expression, and then the four different pathways try to mimic the influence of first external forcing: the demographic change C_c. What is expected from this analysis is that the no-stationarity of the harmfulness λ_1 changes the equilibrium values of all the variables, but with the biggest effects for the water quality's one. The four different scenarios are called with the same names of the *RCPs* in order to differentiate them and remember their characteristics. All the analyses are carried out in a time period from 1900 to 2400 to better appreciate the hypothesized differences in the equilibrium point.

2.1) RCP 2.6-Rapid Climate Action Scenario: in this first scenario, unprecedented and immediate actions are taken to reduce greenhouse gas emissions and limit global warming to well below 2 °C, in line with the targets of the Paris Agreement. Governments, businesses, and individuals implement ambitious measures to decarbonize the economy, such as phasing out fossil fuels, implementing carbon pricing mechanisms, investing in renewable energy infrastructure, and promoting sustainable consumption and production patterns. This scenario results in mitigating the most severe impacts of climate change, preserving ecosystems, protecting biodiversity, and ensuring a more sustainable future for generations to come. The expression of harmfulness parameter λ_1 is the next one:

$$\lambda_1(t) = \lambda + \frac{\phi - \lambda}{1 + e^{-\rho(t-t_0)}} = 0.7 + \frac{1.7 - 0.7}{1 + e^{-0.05(t-2030)}}$$

Equation 4.26: No-stationary harmfulness parameter λ_1 for the RCP 2.6

Using *"R"*, the model is computed with these values and the following figures are produced:

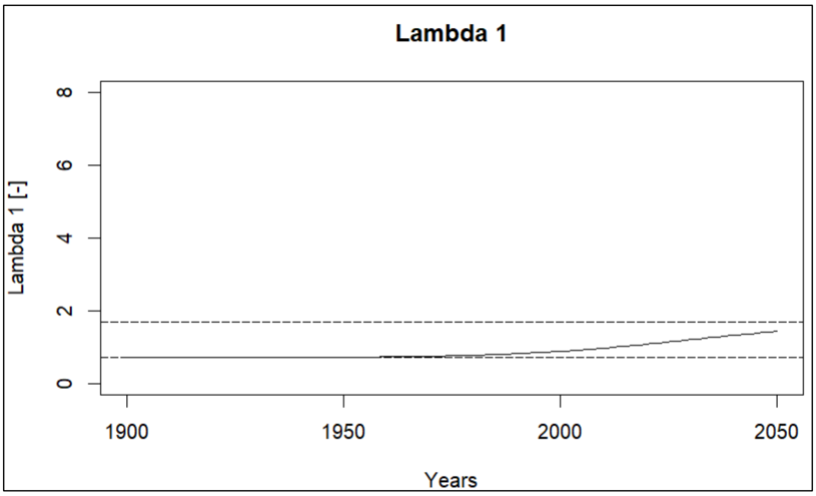


Figure 4.19: Evolution over time of the no-stationary harmfulness parameter λ_1

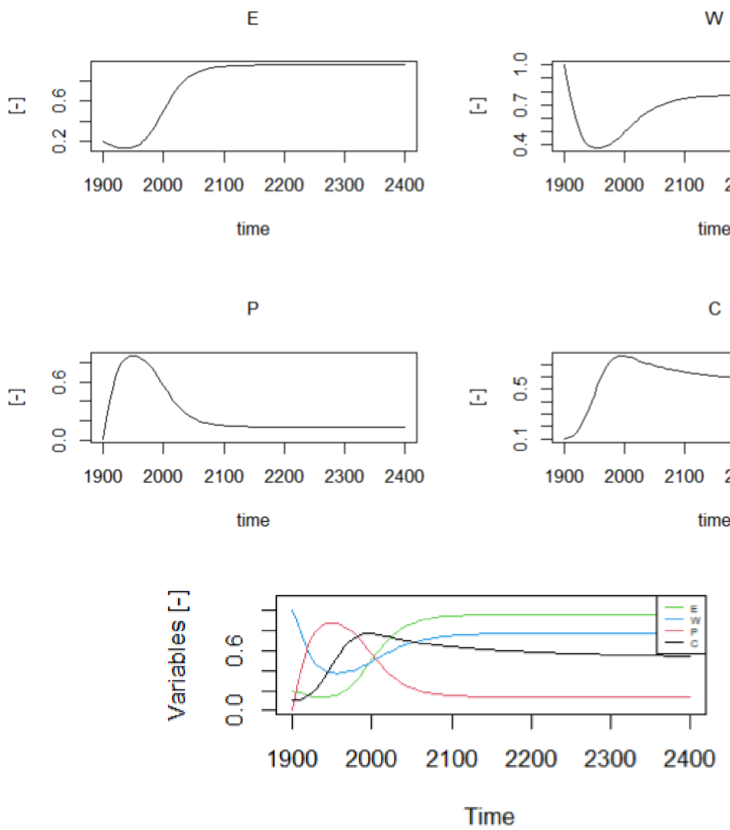


Figure 4.20: Evolution of the model after the no-stationary harmfulness parameter λ_1 addition

The next Table 4.21 shows the parameters and the values of the minimum, maximum and equilibrium point for each variable. Moreover, the differences are computed compared to the original system.

Speed Parameters [yr^{-1}]	**Equilibrium Parameters [-]**	**State Variable**	**Max [-] Year**	**Δ**	**Min [-] Year**	**Δ**	**Equilibrium Value [-] Year**	**Δ**
α=0.1	ε=0.05	E	0.961 2289	+0.007 -50yr	0.132 1935	-2yr	0.961 2289	+0.007 -50yr
β=0.4	λ=0.7	W	1 1900	/	0.373 1955	-0.015 +5yr	0.771 2350	-0.13 +87yr

γ=0.6	χ=0.9	P	0.874 1945	-0.001 -2yr	0.01 1900	/	0.135 2263	-0.006 -61yr
θ=0.07	η=0.5, ψ=0.3	C	0.791 2062	+0.02 +66	0.1 1900	/	0.519 after 2400	-0.002 same

Table 4.20: Results of the RCP 2.6 scenario

This scenario should be the most optimistic one because it represents a highly ambitious pathway requiring immediate and substantial action to mitigate climate change effectively. It involves significant transformation in energy systems, land use, and economic policies to achieve its goals. As shown in the previous Table 4.20, the biggest differences from the original model is the equilibrium point value: even if this scenario is the most positive one, the final point value of the water quality decreases of 0.13 [-]. Regarding the minimum or maximum values and the time shifting, they aren't so affected by this external forcing and the largest difference is a decreasing of 0.015 [-] for the minimum value of the water quality. Moreover, it seems that also the time for reaching the equilibrium point is influenced by the no-stationarity of the harmfulness λ_1: the water quality gains it final value with a delay of 87 years. Instead, the environmental awareness and the industrial polluting production reach their equilibrium more quickly than before, of about 50/60 years.

2.2) RCP 4.5-Sustainable Development Scenario: in this second scenario, significant efforts are made to reduce greenhouse gas emissions through international cooperation, policy interventions, technological advancements, and changes in behaviour. The global community transitions to renewable energy sources, adopts sustainable land use practices, promotes energy efficiency, and invests in carbon capture and storage technologies. As a result, the rate of temperature increase is limited to below 2 °C, although some impacts of climate change are still felt. The expression of harmfulness parameter λ_1 is the next one:

$$\lambda_1(t) = \lambda + \frac{\phi - \lambda}{1 + e^{-\rho(t-t_0)}} = 0.7 + \frac{2.7 - 0.7}{1 + e^{-0.05(t-2050)}}$$

Equation 4.27: No-stationary harmfulness parameter λ_1 for the RCP 4.5

Using *"R"*, the model is computed with these values and the following figures are produced:

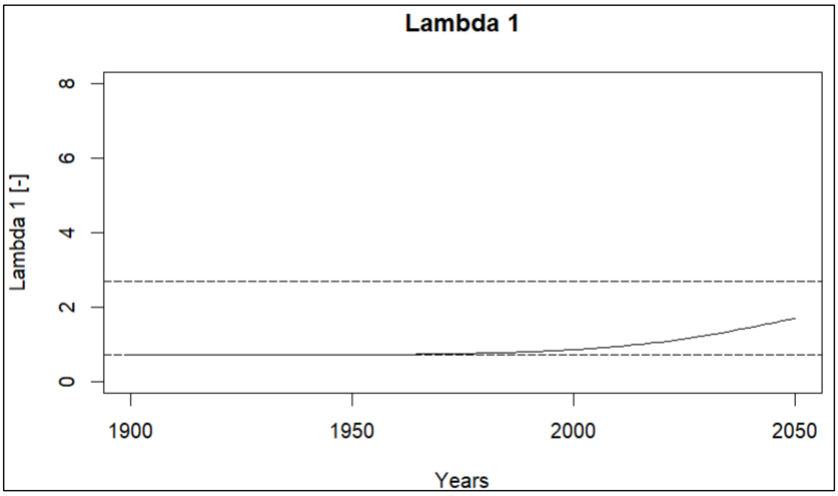


Figure 4.21: Evolution over time of the no-stationary harmfulness parameter λ_1

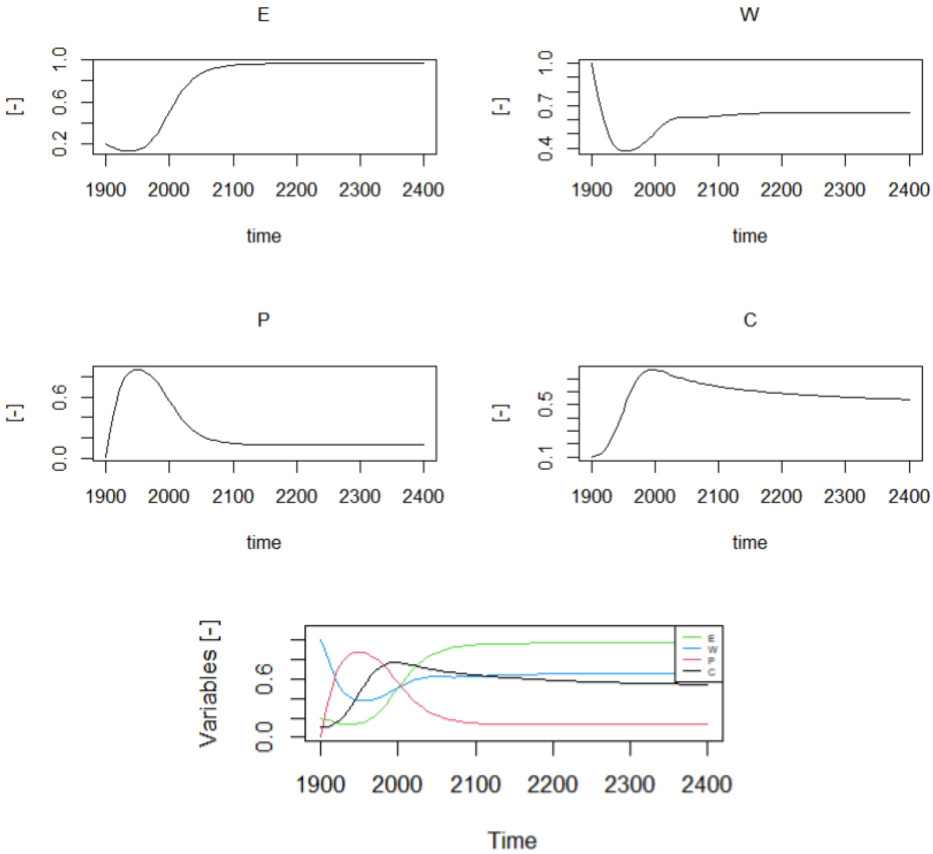


Figure 4.22: Evolution of the model after the no-stationary harmfulness parameter $\lambda_1$addition

The next Table 4.22 shows the parameters and the values of the minimum, maximum and equilibrium point for each variable. Moreover, the differences are computed compared to the original system.

Speed Parameters [yr^{-1}]	Equilibrium Parameters [-]	State Variable	Max [-] Year	Δ	Min [-] Year	Δ	Equilibrium Value [-] Year	Δ
α=0.1	ε=0.05	E	0.967 2284	+0.013 -55yr	0.133 1935	+0.001 -2yr	0.967 2284	+0.013 -55yr
β=0.4	λ=0.7	W	1 1900	/	0.377 1955	-0.01 +5yr	0.651 2370	-0.251 +107y
γ=0.6	χ=0.9	P	0.874 1945	-0.001 -2yr	0.01 1900	/	0.129 2344	-0.01 +20yr
θ=0.07	η=0.5, ψ=0.3	C	0.770 1996	+1yr	0.1 1900	/	0.516 after 2400	-0.005 same

Table 4.21: Results of the RCP 4.5 scenario

This second scenario represents a pathway that balances mitigation efforts with economic and technological feasibility, aiming for stabilization of emissions and radiative forcing to moderate levels. It necessitates significant but manageable changes in policies, technologies, and behaviours to achieve a sustainable future. Table 4.21 shows the results of its implementation: the biggest effects concern the water quality whose equilibrium value goes even down of 0.251 [-]. Moreover, it is delayed of 107 years, instead the environmental awareness and pollution experience an increment and decrement of their equilibrium values of about 0.01, respectively. The minimum and maximum values aren't affected by this scenario, both in terms of time-shifting and differences in the quantities.

2.3) RCP 6-Buisness as Usual Scenario: in this third scenario, greenhouse gas emissions continue to rise at current or similar rates due to limited action to mitigate climate change. The global average temperature increases well beyond 2 °C compared to pre-industrial levels by the end of the century, leading to severe and widespread impacts. Extreme weather events become more frequent and intense, including heatwaves, storms, floods, and droughts. Sea levels rise, leading to coastal flooding and erosion, displacing millions of people. Ecosystems face significant disruption, with loss of biodiversity and habitat degradation. Agriculture and food security are threatened, leading to economic instability and social unrest. The expression of harmfulness parameter λ_1 is the next one:

$$\lambda_1(t) = \lambda + \frac{\phi - \lambda}{1 + e^{-\rho(t-t_0)}} = 0.7 + \frac{4.7 - 0.7}{1 + e^{-0.04(t-2080)}}$$

Equation 4.28: No-stationary harmfulness parameter λ_1 for the RCP 6

Using *"R"*, the model is computed with these values and the following figures are produced:

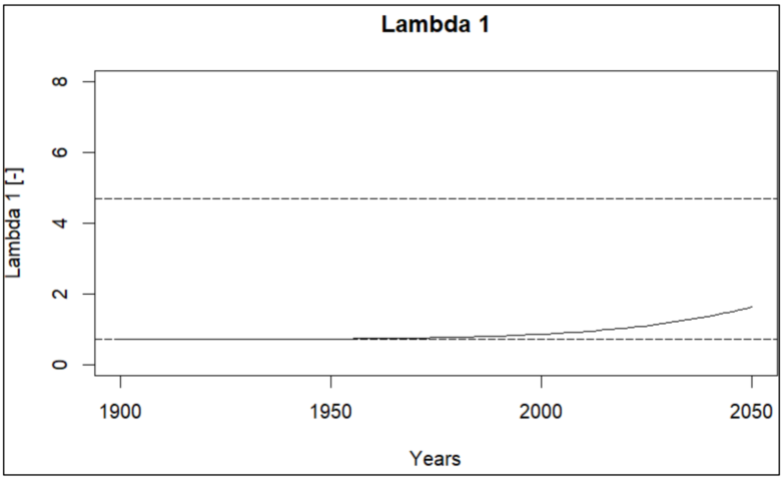


Figure 4.23: Evolution over time of the no-stationary harmfulness parameter λ_1

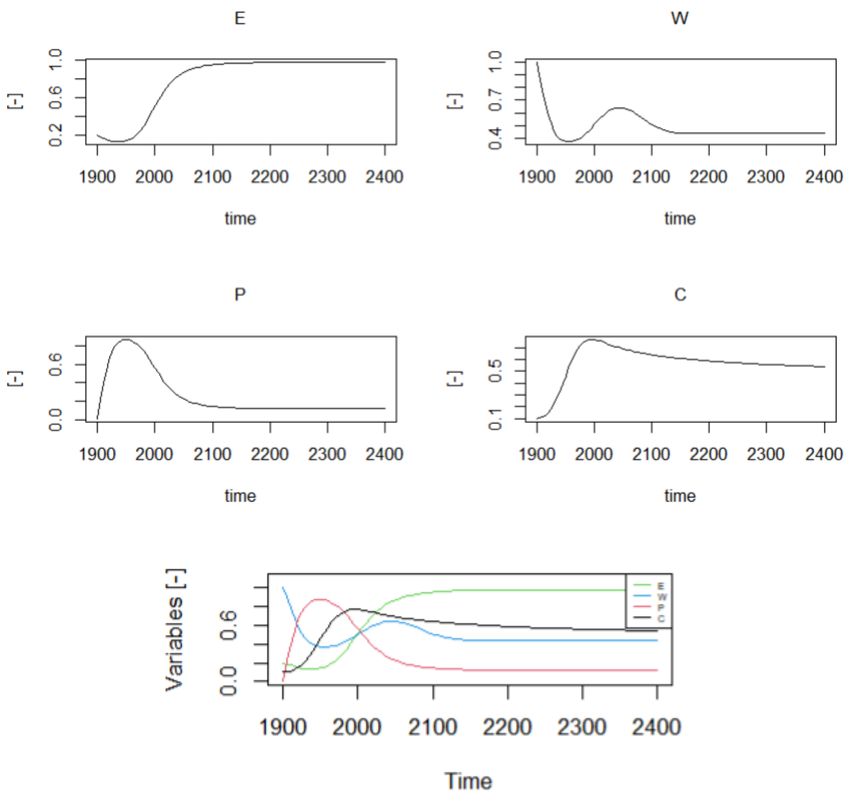


Figure 4.24: Evolution of the model after the no-stationary harmfulness parameter $\lambda_1$addition

The next Table 4.23 shows the parameters and the values of the minimum, maximum and equilibrium point for each variable. Moreover, the differences are computed compared to the original system.

Speed Parameters [yr^{-1}]	Equilibrium Parameters [-]	State Variable	Max [-] Year	Δ	Min [-] Year	Δ	Equilibrium Value [-] Year	Δ
α=0.1	ε=0.05	E	0.978 2334	+0.02 -5yr	0.132 1935	-2yr	0.978 2334	+0.02 -5yr
β=0.4	λ=0.7	W	1 1900	/	0.369 1955	-0.02 +5yr	0.437 2344	-0.46 +81yr
γ=0.6	χ=0.9	P	0.874 1945	-0.001 -2yr	0.01 1900	/	0.119 2354	-0.02 +30yr
θ=0.07	η=0.5, ψ=0.3	C	0.769 1996	-0.001 +1yr	0.1 1900	/	-0.01 after 2400	same

Table 4.22: Results of the RCP 6 scenario

This third scenario represents a pathway with moderate-high emissions and a stabilization approach that allows for continued but more controlled use of fossil fuels, aiming for stabilization of radiative forcing at higher levels. It involves moderate changes in policies, technologies, and behaviours to achieve its goals, balancing between mitigation efforts and economic feasibility. Even then, the worst conditions refer to the water quality: its equilibrium value decreases a lot, of 0.46 [-] and it's delayed of 81 years. The environmental awareness and pollution's equilibrium point don't change and aren't too much time-shifted. Regarding the maximum and minimum values, they aren't affected by this scenario too much and their values are quite the same. Finally, the water quality curve has a special shape: a minimum point occurs in 1950, as for the other scenarios, but then after the maximum in 2050, the values go down again and stabilize after 2100. So, the recovery in 2000 is an unstable point.

2.4) RCP 8.5-Climate Catastrophe Scenario: in this last scenario, global efforts to address climate change are insufficient, and tipping points are crossed, triggering irreversible and catastrophic impacts. This could include the melting of polar ice caps, the release of methane from thawing permafrost, and the collapse of major ecosystems such as coral reefs and rainforests. Extreme weather events become so frequent and severe that they overwhelm society's ability to adapt, leading to widespread disruption, food and water shortages, mass migrations, and geopolitical conflicts. The global economy faces collapse, and efforts to mitigate the impacts of climate change become increasingly futile. This is the most pessimistic scenario. The expression of harmfulness parameter λ_1 is the next one:

$$\lambda_1(t) = \lambda + \frac{\phi - \lambda}{1 + e^{-\rho(t-t_0)}} = 0.7 + \frac{7.7 - 0.7}{1 + e^{-0.04(t-2110)}}$$

Equation 4.29: No-stationary harmfulness parameter λ_1 for the RCP 8.5

Using *"R"*, the model is computed with these values and the following figures are produced:

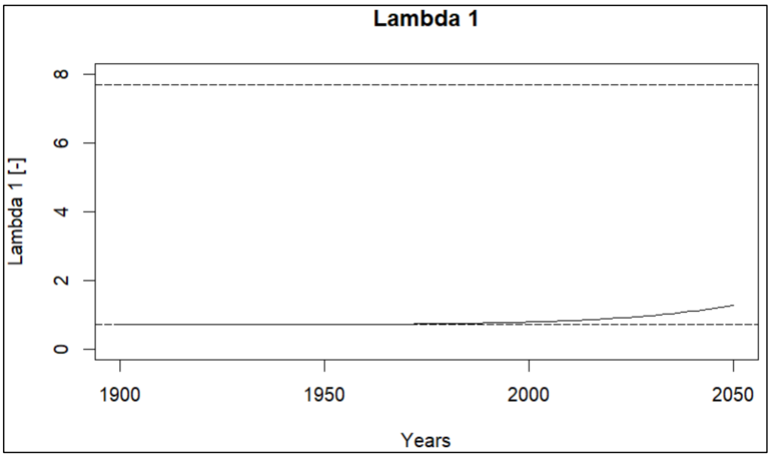
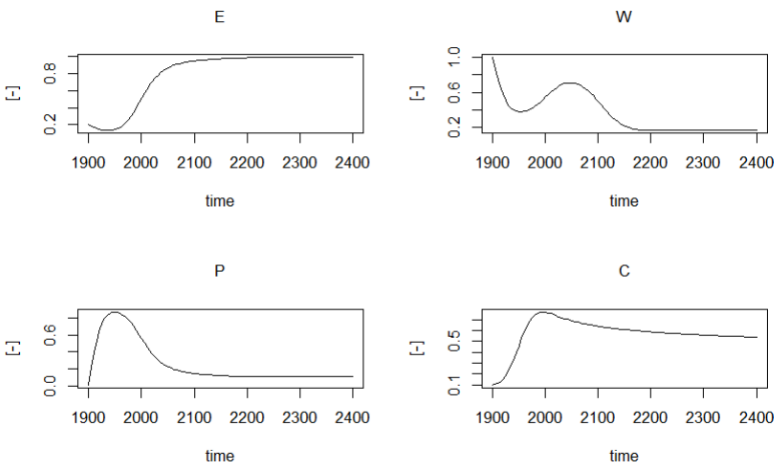


Figure 4.25: Evolution over time of the no-stationary harmfulness parameter λ_1



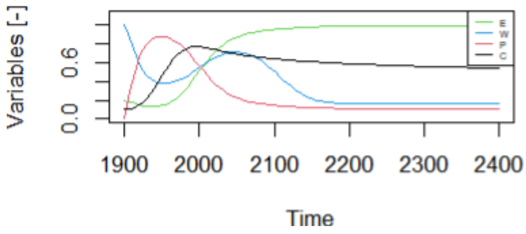


Figure 4.26: Evolution of the model after the no-stationary harmfulness parameter $\lambda_1$addition

The next Table 4.24 shows the parameters and the values of the minimum, maximum and equilibrium point for each variable. Moreover, the differences are computed compared to the original system.

Speed Parameters [yr^{-1}]	Equilibrium Parameters [-]	State Variable	Max [-] Year	Δ	Min [-] Year	Δ	Equilibrium Value [-] Year	Δ
α=0.1	ε=0.05	E	0.991 2329	+0.04 -10yr	0.132 1935	-2yr	0.991 2329	+0.04 -10yr
β=0.4	λ=0.7	W	1 1900	/	0.165 2208	-0.22 +258yr	0.171 2364	-0.73 +101yr
γ=0.6	χ=0.9	P	0.874 1945	-0.001 -2yr	0.01 1900	/	0.107 2374	-0.03 +50yr
θ=0.07	η=0.5, ψ=0.3	C	0.770 1996	+1yr	0.1 1900	/	0.504 after 2400	-0.02 same

Table 4.23: Results of the RCP 8.5 scenario

This last scenario represents a "business-as-usual" scenario where no significant efforts are made to mitigate climate change. It highlights the potential consequences of continued high emissions, emphasizing the need for urgent and substantial action to avoid such severe outcomes. Again, the water quality's equilibrium point is the worst affected and, in this scenario, experiments the largest decreasing: -0.73 [-] with a delay of 101 years. The other equilibrium points are quite the same as the original model and also the minimum and maximum values keep the same amount. The biggest difference is in the minimum value of the water quality, which decreases of 0.2 [-] and it's delayed of 258 years. Moreover, as it's possible to appreciate from the last graph in Figure 4.23, the water quality variable undergoes two unstable points: the first point (a minimum in 1950) is followed by a recovery in 2000, then the maximum in 2050 is followed by a decreasing in 2100.

In general, what is important to underline is the fact that the climate change variable has more effetcs on the future, rather than on the present. Actually, the min or the max values of the water quality and the other variables aren't so affected. On the other hand, the final value reached by the water quality at the equilibrium is deeply influenced: the higher the impacts and delayed over time, the lower the equilibrium value gained by the water quality. Another intersting fact is that the other state variables are less affetced by the climate change: it acts directly on the water quality and all the impacts on it have indirectly consequences on the environmental awareness and the industrial polluting production. Due to the fact that the water quality, in these scenarios, is going to decrease, the environmental awareness is always anticipated and its equilibrium value increases a little bit. Regarding the pollution, it isn't so influenced by the climate change because all the impacts are firstly mitigated by the environmental awareness. For this reason, its equilibrium values sligthly decreases nor delayed. For these scenarios, the maximum decreasing of the water quality in the future is forecasted as -0.73, so quite high and harmfull.

5. Discussions and Conclusions

5.1. The Evolution of the Water Quality

This chapter is entirely devoted to the comprehensive description of the socio-hydrological model. The system of equations needs to be deeply understood and the model evolution analysed, relating the results with the main events reported in the previously viewed literatures. In the previous Chapter 4 all the steps necessary to build the model were explained, but the actual meanings of the equations must be assessed. First of all, the system of ordinary differential equations is reported, again, in the next Equation 5.1:

$$\begin{cases} \frac{dE}{dt} = \alpha\,(1 - E - \varepsilon\,W)\,E\,(C - \psi) \\ \frac{dW}{dt} = \beta\,(1 - W - \lambda\,P) \\ \frac{dP}{dt} = \gamma\,(1 - P - \chi\,E)\,C \\ \frac{dC}{dt} = \theta\,C\,(1 - C - \eta E)\,P \end{cases}$$

Equation 5.1: The socio-hydrological model as system of ordinary differential equations

As discussed above, all the internal variables' derivatives are described with a speed parameter and an equilibrium parameter. Moreover, their evolution is defined by a logistic growth and all of them are bounded between 0 [-] and 1 [-] and dimensionless. The coefficients are chosen based on the shapes expected by literatures and the conclusive results are shown in the next Table 5.1:

Speed Parameters [yr^{-1}]	Equilibrium Parameters [-]	State Variable	Initial Value [-]	Max [-] Year	Min [-] Year	Equilibrium Value [-] Year
α=0.1	ε=0.05	E→quite slow	0.2	0.955 2339	0.133 1935	0.955 2339
β=0.4	λ=0.7	W→quite fast	1	1 1900	0.388 1950	0.901 2263
γ=0.6	χ=0.9	P→fast	0.01	0.874 1945	0.01 1900	0.140 2223
θ=0.07	η=0.5; ψ=0.3	C→slow	0.1	0.770 1995	0.1 1900	0.522 after 2400

Table 5.1: Main parameters and values of the internal state variables for the final socio-hydrological system

Starting the analysis from the first column, the speed parameters refer to the velocity by which the evolution of the variable over time is governed. The higher is the value of the coefficient, the faster the selected variable will reach the equilibrium. As reported in the table above, industrial polluting production has the quickest dynamic over time because its speed parameter γ has the highest value (0.6 yr^{-1}) and, in fact, it reaches before all the others its final value. This particular evolution suits the industrial polluting production because the spreading of contaminants in the environment is characterised by fast processes.

The second fastest variable is the water quality: in fact, its speed parameter β is equal to 0.4 yr^{-1} and it reaches the equilibrium point in 2263 (40 years after the industrial polluting production). The health of the environmental capital heavily depends on the density of the contaminants in the water bodies, so, its evolution should be slower than its cause.

Then, the environmental awareness changes over time with a velocity lower than the water quality because its dynamics is related to this variable. So, the environmental awareness has a slow response and changes need more time compared to the spread of pollutants into the surface bodies. Actually, as it's possible to see in the real world, when a natural disease happens or an environmental problem pops up, actions for limiting the consequences require a long time to be effective.

Finally, the economic health has a very slow response and it needs after 2400 to reach the equilibrium point. This behaviour depends to the fact that all the economic or political mechanisms are quite complex and their effects are valuable in the long term. All these values are selected based on literature facts: the industrial polluting production should be the first stabilized variable and with the lowest value because it is pushed down by several factors (laws, policies, green technologies) that heavily change the concentration of contaminants. As a consequence, the water quality is able to gain its equilibrium only after the pollution variable has stabilized and this final value is very high. So, according to the model, the forecast of the water quality is pretty good: it will reach 0.901 [-].

Moving to the equilibrium parameters' analysis, they are added inside the ordinary differential equations' system and they are useful in order to give weight to each variable affecting the time derivative. As it's possible to appreciate from Equation 5.1, four equilibrium coefficients are selected as multiplicative factor of the internal variables and their main function is not only to quantify the power of each relationship between the quantities, but also to determine the final equilibrium value. Solving the system in Equation 5.1, the equilibrium point A results as followed:

$$A\left(E=\frac{1+\lambda\varepsilon-\varepsilon}{1+\lambda\chi\varepsilon}, W=\frac{1+\lambda\chi-\lambda}{1+\lambda\chi\varepsilon}, P=\frac{1+\chi\varepsilon-\chi}{1+\lambda\chi\varepsilon}, C=1-\eta\frac{1+\lambda\varepsilon-\varepsilon}{1+\lambda\chi\varepsilon}\right)$$

Each variable's value is computed in the same way: in the numerator a specific coefficient prevails and the denominator is the same for all the values. Starting the analysis from the environmental awareness' derivative, its evolution over time is influenced by the coefficients ε, the *disregard*. It is set equal to 0.05 [-] and it multiplies the water quality variable. It's called disregard because it represents the negligence towards the water quality of the social awareness. The highest the carelessness of people towards the environmental health, the largest the value of this coefficient. So, in this model the disregard of people is considered sufficiently low and, thanks to this fact, the equilibrium value of the environmental awareness quantity is very high: 0.955 [-].

The next equilibrium parameter to be studied is λ, the harmfulness. This coefficient multiplies the industrial polluting production variable inside the water quality's time derivative. So, its effect is to give the correct weight to P in changing the evolution of the water quality over time. It's set equal to 0.7 [-], so the model is pretty rough about the harmfulness of the contaminants produced by the industrial production. This coefficient indeed represents the percentage of dangerousness of the released pollutants in the environment. The more considerable their threat is, the more the health of the water bodies is degraded. The fact that the harmfulness λ is pretty high brings to an enhancement of the awareness of society with respect to the environment, so it allows the water quality's equilibrium value to remain high and the pollution's one is kept low. It's significant to notice that λ refers to the likelihood of the pollutants in deteriorating the water bodies and not to their quantity: so, even if their concentration is negligible, the harmfulness could be very high. This particular value of 0.7 [-] lets the water quality variable to reach its equilibrium point at 0.901 [-].

The third parameter to be interpreted is χ, the eco-friendliness. It multiplies the environmental awareness variable inside the industrial polluting production's time derivative. So, this coefficient quantifies how much society is concerned about the industrial polluting production and how many efforts it implements to reduce it. These efforts are directly related with the technological innovations improved to make the industrial production more sustainable and efficient. The eco-friendliness χ is set equal to 0.9 [-] and it means that the percentage of the environmental awareness on the industrial polluting production's reduction through green technologies is very relevant. The more the improvements on sustainable technologies are applied to the industrial production, the lower the pollutants will be and so the better the water quality will become. This value of the eco-friendliness coefficient leads the industrial polluting production to stay constant at 0.140 [-] at the equilibrium point.

Another equilibrium parameter which is applied directly to the environmental awareness variable is the financial sustainability η. It is set equal to 0.5 [-] and, as it's possible to appreciate from the equilibrium point *A,* it affects the final value of the economic health variable. In fact, the final point of *C* has a quite similar formulation of the complementary of the environmental awareness's equilibrium point: the financial sustainability is the percentage of the of the final value of *E* subtracted to 1. In other words: the financial sustainability represents the amount of the knowledges of the society spent in laws, policies or new sustainable economic guidelines, so the percentage of the environmental awareness concerns about that part of the economic health that supplies the industrial polluting production. The value of η equal to 0.5 [-] brings the equilibrium point of the economic health at 0.522 [-], which is pretty well-balanced. When the society puts all its efforts on promulgating green laws, setting limits or emitting policies, the only effective consequence obtained is the reduction of the economic health and, as a secondary effect, the depletion of the industrial polluting production.

One of the most significant conclusions that comes up is the footprint of the environmental awareness on the pollution. What is important to understand is that spending efforts on laws, policies, limits implementation isn't enough to reduce the pollution and to have effective results on the water quality. In fact, when the environmental awareness is basically based on the financial sustainability, it means that $\eta > \chi$, so the financial sustainability should play a bigger role in the reduction of the pollution through the control of the economic health. But it isn't like this. The eco-friendliness of the environmental awareness, so the efforts of the state spent on the technological improvements rather than on laws or policies, is the key to see in the future a practical reduction of pollution. In order to validate this strong statement, let's have a look to the previous graphs in Figure 4.8. When the economic health increases (so η is low), a similar increasing also for the industrial polluting production is expected to happen, but it doesn't. The pollution stays to its original value due to the power of the eco-friendliness and so of the efforts of the environmental awareness on green technologies. At the same time, when the economic health is heavily reduced by the financial sustainability (so the state spends lots of efforts on laws and economic policies to improve the natural capital, η is high), the model shows only the reduction of the economic heath variable, but the industrial polluting production doesn't change.

The last parameter to be analysed is the environmental budget ψ. This coefficient is dimensionless as the equilibrium parameters, but it doesn't play any roles on the final values of the variables. So, it can be described as a speed coefficient. The environmental budget is set equal to 0.3 [-] and it appears on the environmental awareness's time derivative. Its meaning is the following one: if the amount of money necessary to solve the natural problems is considerable (ψ is high) and larger than the economic health ($\psi > C$), the State prefers taking care of the population rather than the environmental quality because the required efforts for the natural capital are to expensive to be held. In this case, the industrial polluting production increases and the water quality degrades over time. The opposite case is when the environmental budget is quite low and so the economic health has more chance to be higher (ψ<C). The State is flourished and so it is able to think about the environmental quality, also because the cost for it isn't so expensive. Regarding the equilibrium, it is reached quicker or slower depending on the ψ. Higher its value is, slowly the system reaches the equilibrium and vice versa. Meaningfully, if the efforts required to improve the environment are consistent, the State needs more time to reach the desirable results.

In order to remember the links between the variables and the coefficients, the causal loop diagram is shown and the parameters are added. Inside each variables' box, the speed parameters are shown and the equilibrium coefficients are reported next to the correspondent arrow representing the relationship.

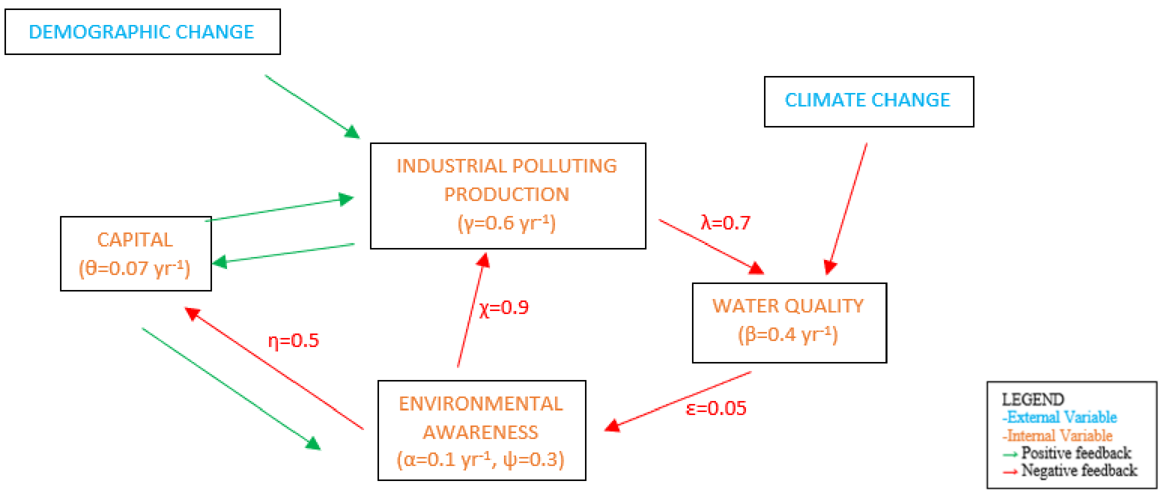


Figure 5.1: Causal loop diagram with all the equilibrium and speed parameters

With these steady values of the speed and equilibrium coefficients, the socio-hydrological model evolves over time in the so called "stationary way". Then, computing the system in Equation 5.1 produces the following model's evolution in a period from 1900 to 2050:

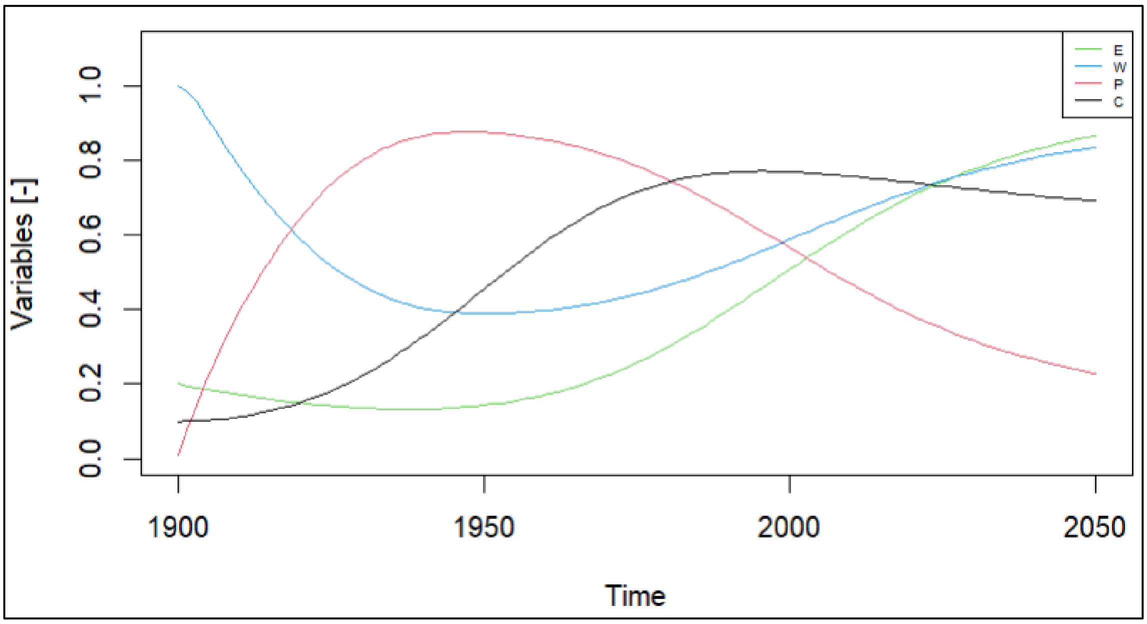


Figure 5.2: Evolution of the socio-hydrological model from 1900 to 2050

In order to better understand its evolution, the whole model's evolution is split into four time intervals. These ranges are selected based on the information taken from literatures and for each of them the state variables are analysed in the stationary condition. Imaging to subdivide the Figure 5.2 into four time ranges, the next analysis is carried on:

- From 1900 to 1950: the industrialization and urbanization across Europe leads to widespread pollution of water bodies from industrial effluents and domestic sewage. Waterborne diseases spread out resulting in high mortality rates and public health crises. Limited regulations were minimal, and there was limited awareness of environmental issues. The, the two World Wars had significant impacts on water quality due to industrial production, destruction of infrastructure, and contamination from military activities.

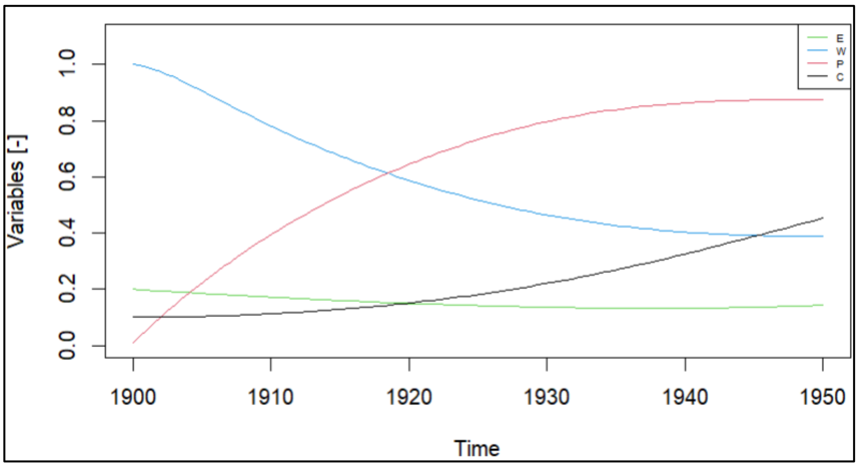


Figure 5.3: Model's evolution from 1900 to 1950

Specifically, the single variable is studied in this time interval:

- Water quality: the quality of the water bodies at the beginning of the 1900 is quite high because of the poor industrial and urban development. In fact, the initial value of the water quality variable in 1900 is set equal to 1 [-]. Then, there was a deterioration of the natural capital due to widespread pollution into water bodies and in 1950 the water quality variable reaches the lowest value of 0.388 [-].

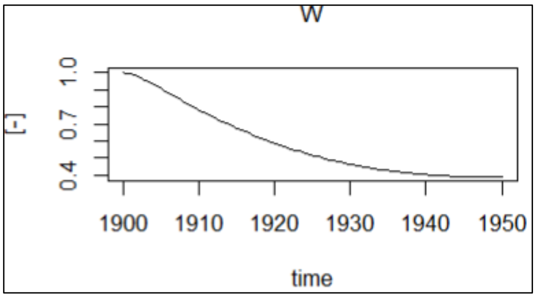


Figure 5.4: Water quality variable's evolution in the first time interval from 1900 to 1950

- Environmental Awareness: industrialization and urbanization led to increasing pollution of water bodies. Basic sanitation measures were spurred due to concerns about waterborne diseases and early water quality laws were often limited in scope and primarily focused on preventing obvious sources of contamination. For these reasons, the environmental awareness variable starts from 0.2 [-] in 1900, reaches its minimum value of 0.133 [-] in 1935 and then begins to increase after that time. Originally the aware of people was limited and slightly focus on the environment. Actually, this variable decreases from 1900 until 1935 because the environmental budget ψ is higher than the economic health, so the State is more worried about the imminent war or other political problems rather than worrying about the decreasing water quality.

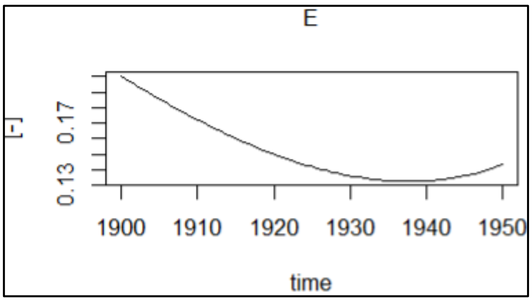


Figure 5.5: Environmental awareness variable's evolution in the first time interval from 1900 to 1950

- Industrial Polluting Production: witnessed rapid industrialization in many parts of the world, industrial processes during this period often lacked environmental controls, resulting in significant pollution of air, water, and soil. In 1900 the industrial polluting production recorded the lowest value of 0.01 [-] and then, after the fast industrialization and the two World Wars, this variable reached its highest value of 0.874 [-] in 1945.

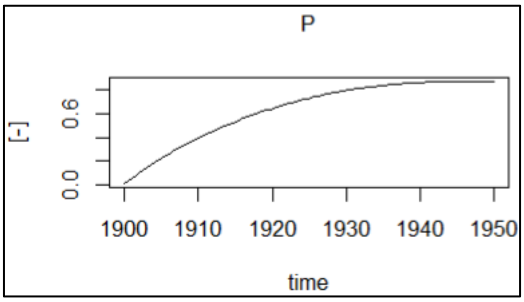


Figure 5.6: Industrial polluting production variable's evolution in the first time interval from 1900 to 1950

- Economic Health: in this time interval, the economic health variable starts from 0.1 [-] as minimum value and then increases. This is due to the fact that the industrialization is flourishing and it has positive effect on the economic health of the state. Even if two world wars occur, this variable still increases because the environmental awareness's power is too low.

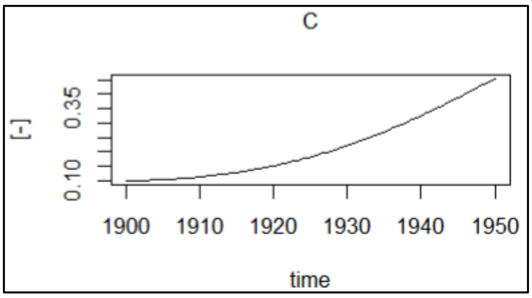


Figure 5.7: Economic health variable's evolution in the first time interval from 1900 to 1950

- From 1950 to 1980: after World War II, there was a focus on rebuilding infrastructure, including water treatment plants and sewage systems, which improved water quality in some areas. Environmental movements began to emerge, advocating for cleaner water and stricter pollution controls.

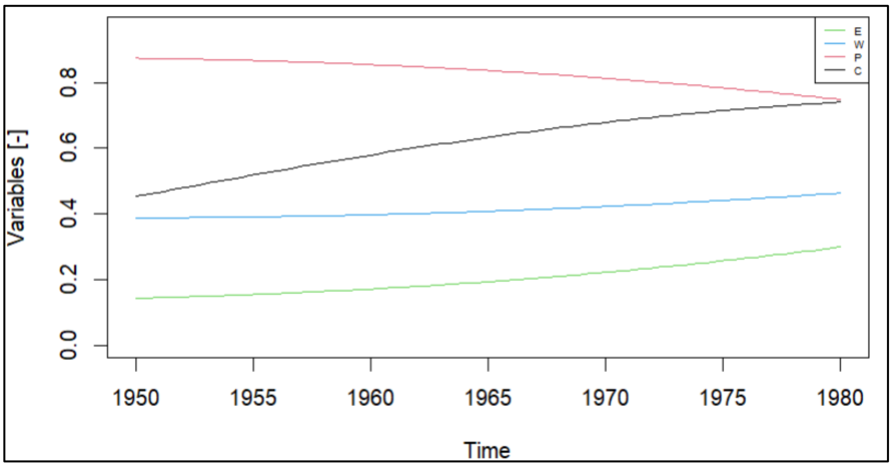


Figure 5.8: Model's evolution from 1950 to 1980

  - Water Quality: in this time period, water quality grows from 0.388 [-] to 0.464 [-] and this enhancement is due to the development of water treatment plants and sewage systems.

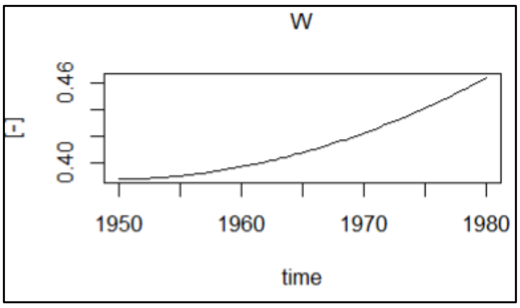


Figure 5.9: Water quality variable's evolution in the second time interval from 1950 to 1980

  - Environmental Awareness: Post-World War II Era when rapid industrial expansion and population growth exacerbated water pollution problems leads to visible degradation of water bodies. In response, governments began enacting more comprehensive water quality laws and regulations, such as the U.S. Federal Water Pollution Control Act (1948), which was later expanded and strengthened as the Clean Water Act (1972). Advances in analytical techniques allowed for better monitoring of water quality parameters. The Environmental Decade in 1970s saw a surge in environmental awareness and legislative action, such as The Clean Water Act (CWA) of 1972 in the United States established the basic structure for regulating pollutant discharges into U.S. waters. For these facts, the environmental awareness passes from 0.143 [-] to 0.298 [-] in just 30 years.

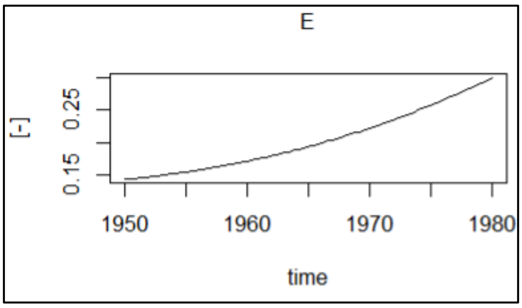


Figure 5.10: Environmental awareness variable's evolution in the second time interval from 1950 to 1980

- Industrial Polluting Production: growing concerns about the health and environmental impacts of pollution, along with high-profile environmental disasters such as the Minamata mercury poisoning in Japan (1956-1960s), spurred calls for stronger regulation. In the 1970s the Environmental Decade marked a turning point in the regulation of industrial pollution, with the emergence of the modern environmental movement and the enactment of landmark legislation in many countries. The Clean Air Act (1970) and the Clean Water Act (1972) established comprehensive regulatory frameworks for controlling industrial emissions and wastewater discharges. This variable decreases from 0.874 [-] to 0.748 [-].

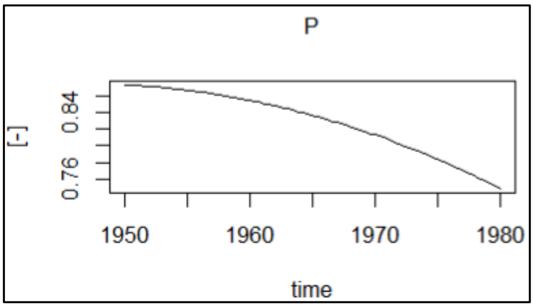


Figure 5.11: Industrial polluting production variable's evolution in the second time interval from 1950 to 1980

- Economic Health: in this time range, the economic health variable grows from 0.453 [-] to 0.740 [-]. This increasing is due to the re-building phase after the two world wars. It's interesting to notice that in this time interval P and C have different behaviour: the industrial polluting production decreases and the economic health still increases. The reason is that industries are even producing but the amount of pollutants released are heavily reduced due to the incoming environmental laws. So, the economic health grows too.

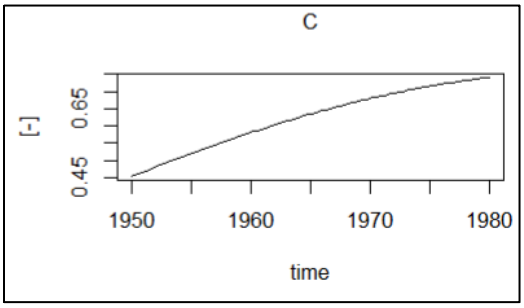


Figure 5.12: Economic health variable's evolution in the second time interval from 1950 to 1980

- From 1980 to 2000: governments began implementing environmental regulations to control pollution discharges into water bodies. The European Union (EU) played a significant role in setting standards and directives for water quality management. During the late 20th century, there was a growing emphasis on environmental protection and sustainable development in Europe. Advents in water treatment technology, such as the adoption of biological treatment processes and improved filtration methods, helped to enhance the quality of treated wastewater. The "Nitrate Directive" was developed and acts under the WFD. Stricter regulations were enacted to address water pollution, including the implementation of the European Union's Urban Wastewater Treatment Directive in 1991, which aimed to improve the collection and treatment of urban wastewater.

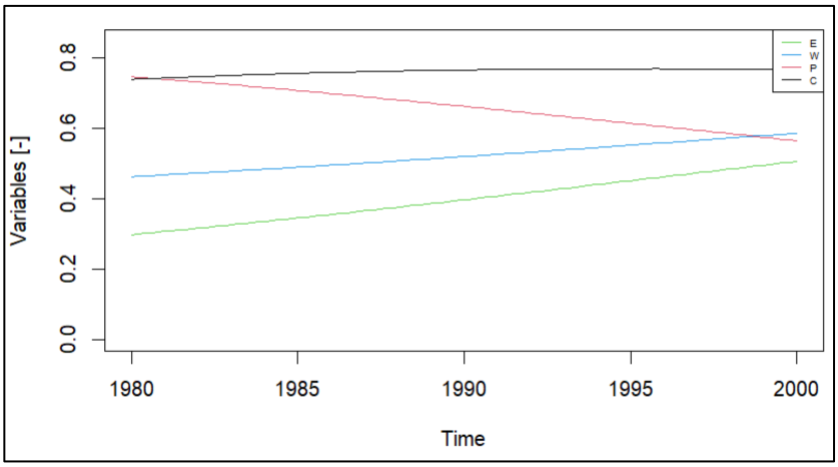


Figure 5.13: Model's evolution from 1980 to 2000

- Water Quality: in this period, the water quality enhances its growth: it passes from 0.464 [-] to 0.587 [-] in only 20 years, so the health of the surface bodies is improving quickly. This is because the early born environmental policies are right now having effect on the water.

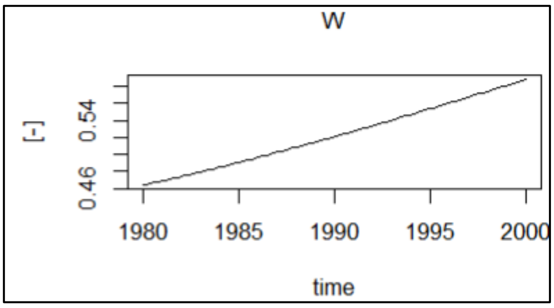


Figure 5.14: Water quality variable's evolution in the third time interval from 1980 to 2000

- Environmental Awareness: globalization of environmental issues in 1980-1990 when international cooperation on water quality issues increased with the adoption of agreements like the Helsinki Convention on the Protection and Use of Transboundary Watercourses and International Lakes (1992). Efforts to address specific water quality challenges, such as eutrophication in lakes and rivers, gained traction, leading to the development of targeted regulations and management strategies. For these reasons, this variable grows from 0.298 [-] to 0.508 [-].

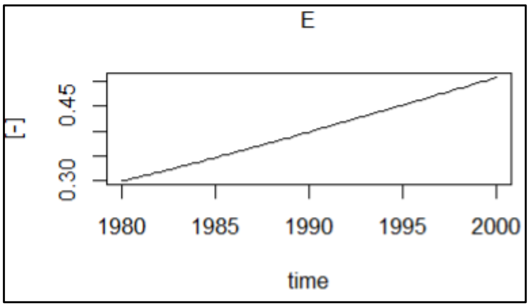


Figure 5.15: Environmental awareness variable's evolution in the third time interval from 1980 to 2000

- Industrial Polluting Production: from 1980 to 1990 technological advances and regulatory expansion saw significant technological advancements in pollution control technologies and industrial processes. Regulations governing industrial pollution became more stringent, with a focus on pollution prevention, waste minimization, and the adoption of cleaner production methods. International cooperation on environmental issues increased during this period, leading to the negotiation of agreements such as the Montreal Protocol (1987) to address ozone depletion and the Basel Convention (1989) on the control of transboundary movements of hazardous wastes. This variable decreases from 0.748 [-] to 0.564 [-].

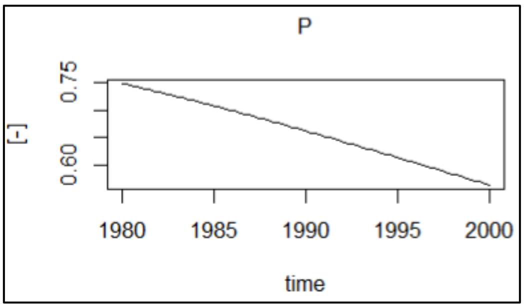


Figure 5.16: Industrial polluting variable's production evolution in the third time interval from 1980 to 2000

- Economic Health: During this this time range, the economic health variable grows less slowly than before: in 20 years goes from 0.740 [-] to 0.768 [-] and it touches its maximum value of 0.770 [-] in 1995. What happens is that during this period the environmental awareness is becoming stronger and stronger and so its power is acting not only on the pollution, but also in the economic health.

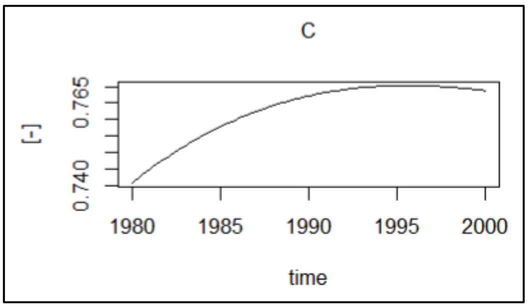


Figure 5.17: Economic health variable's evolution in the third time interval from 1980 to 2000

From 2000 to 2100: this last time interval shows how the state variables evolve from the present ages until the near future, giving also a possible forecasting of the water quality evolution. The WFD, adopted in 2000, established a framework for the protection and management of water resources across Europe, setting objectives for achieving good ecological status in water bodies. Countries invested in upgrading water treatment infrastructure to meet WFD standards and ensure safe drinking water for their populations. Concerns grew over emerging contaminants prompts research and monitoring efforts to address these issues. Climate change brought about changes in precipitation patterns, temperature, and hydrological cycles, affecting water availability and quality in various regions of Europe. In the present ages, efforts to monitor and manage water quality continue, with a focus on achieving the objectives set by the WFD and addressing emerging challenges such as climate change and emerging contaminants. Public awareness of water quality issues remains high, and there is increasing public participation in water management initiatives and environmental conservation efforts.

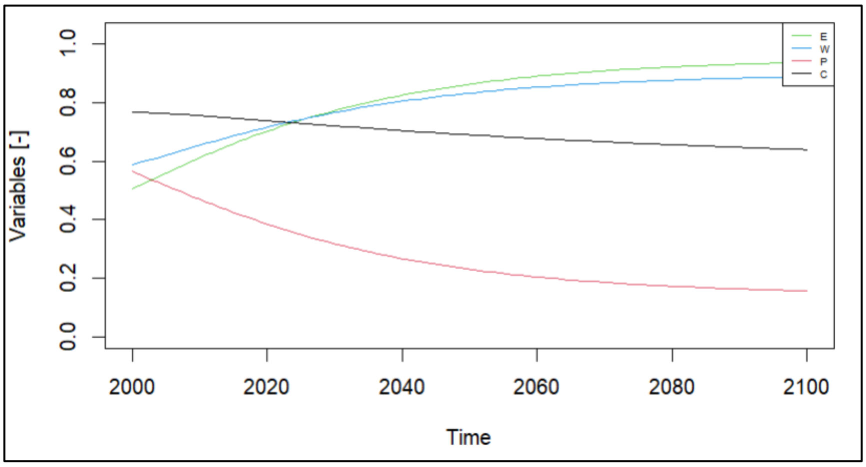


Figure 5.18: Model's evolution from 2000 to 2100

- Water Quality: based on the model results, the water quality increases from 0.587 [-] to 0.889 [-], with a delta of 0.302 [-], and it's projected to grow again until stabilizing at higher value. So, the health of the water bodies seems to be safe, but, despite all, the population never has to lower the guard because the water quality dynamic has a very complicate equilibrium and a little change could lead to the most dangerous effect.

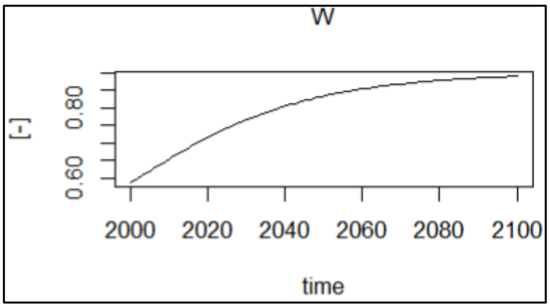


Figure 5.19: Water quality variable's evolution in the forth time interval from 2000 to 2100

- Environmental Awareness: water quality management has become increasingly integrated with broader environmental and sustainability goals. Laws and regulations continue to evolve to address emerging water quality challenges, such as emerging contaminants (e.g., pharmaceuticals, microplastics) and the impacts of climate change on water availability and quality. The environmental awareness variable still grows from 0.506 [-] to 0.938 [-] , with a delta of 0.432 [-], and keeps increasing until 2100.

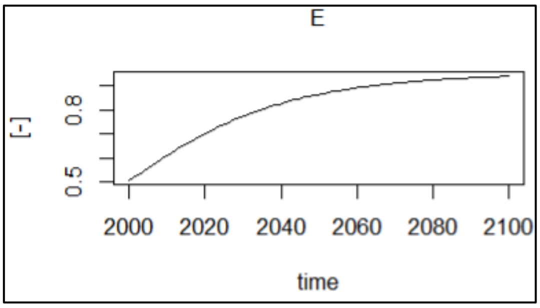


Figure 5.20: Environmental awareness variable's evolution in the forth time interval from 2000 to 2100

- Industrial Polluting Production: industrial pollution continues to be a significant environmental concern, particularly in emerging economies experiencing rapid industrialization. Efforts to address industrial pollution have increasingly focused on promoting sustainable production practices, resource efficiency, and the adoption of clean technologies. For these reasons, the industrial polluting practices are pushed down and also in the near future, they will be kept at low values. This variable decreases from 0.565 [-] to 0.156 [-] with a negative delta of 0.409.

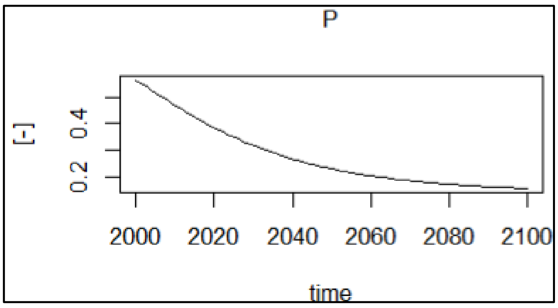


Figure 5.21: Industrial polluting variable's production in the forth time interval from 2000 to 2100

- Economic Health: the economic health keeps decreasing from 0.768 [-] to 0.639 [-] with a negative delta of 0.129 [-]. It means that the economic health is pushed down not only by the reduction of the industrial polluting production but also by the more powerful sustainable policies.

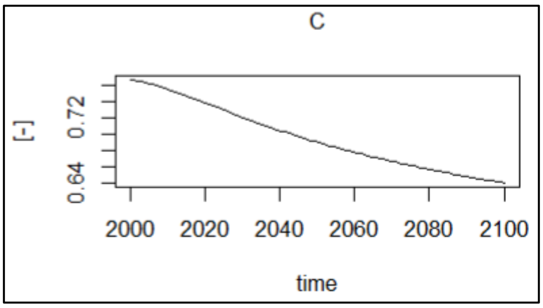
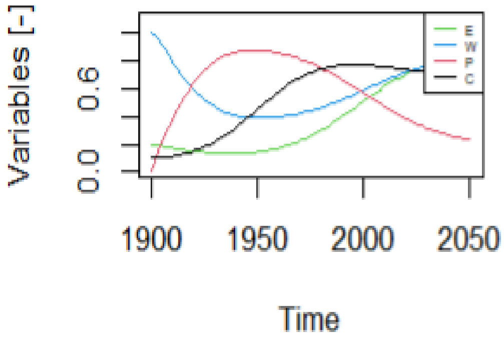
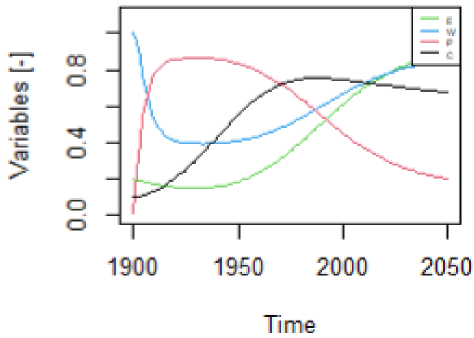
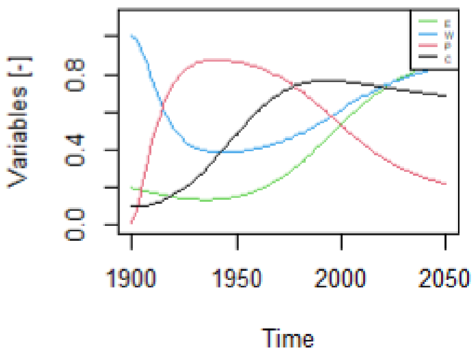
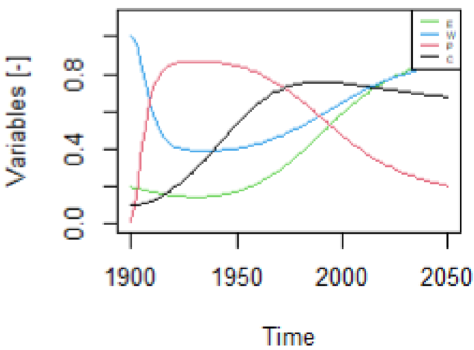


Figure 5.22: Economic health variable's evolution in the forth time interval from 2000 to 2100

The previous analysis concerns the stationary case of the socio-hydrological model. As discussed in Chapter 4.5, the evolution of the water quality is studied also considering the variability of some coefficients which in the stationary case were constant. In particular, the steady pollution's speed parameter γ is changed into γ_1 which changes over time as a logistic growth (see Equation 4.20). The evolution over time of this coefficient shapes the presence of one of the external variables: the demographic change. Building four possible scenarios for the parameter γ_1 ("The Bio-Bomb", "Slow Population Stabilization", "Fast Population Stabilization", "Catastrophic Event"), the next Table 5.2 is created in order to summarize the results.

MODELS	RESULTS	COMMENTS
Original model		
1-The Bio-Bomb		κ=9.6 [-] μ=0.02 [yr^{-1}] t_0=2055 [yr]
2-Slow Population Stabilization		κ=4.6 [-] μ=0.03 [yr^{-1}] t_0=2010 [yr]
3-Fast Population Stabilization		κ=4.6 [-] μ=0.07 [yr^{-1}] t_0=2000 [yr]

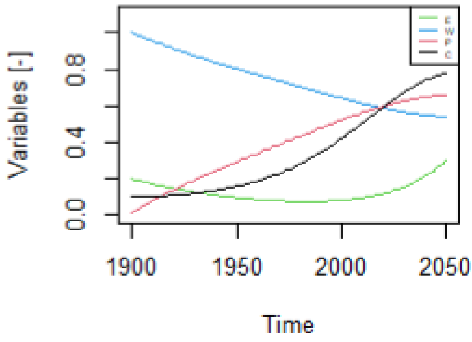
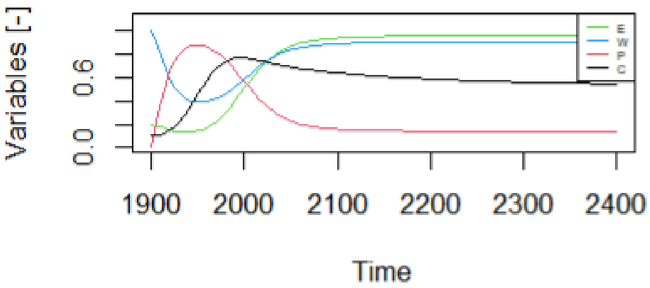
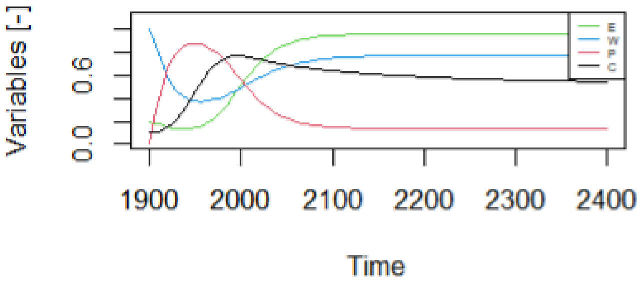
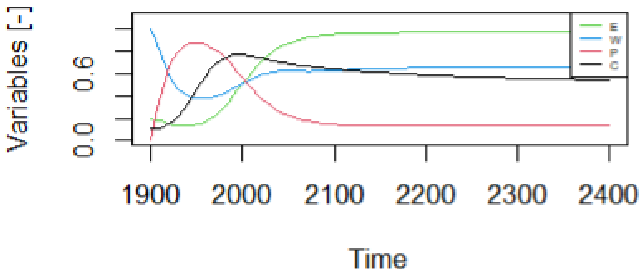
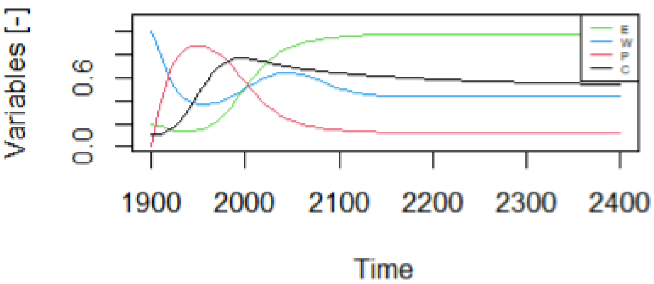
<table>
<tr>
<td>4-Catastrophic Event</td>
<td>
</td>
<td>κ=0.3 [-]
μ=0.06 [yr^{-1}]
t_0=1920 [yr]</td>
</tr>
</table>

Table 5.2: Evolution of the socio-hydrological model for different formulation of the no-stationary pollution speed's function γ_1

The model reveals the following things: the demographic change just shifts the curves over time and it means that it affects the time required to reach the equilibrium. It doesn't influence the final value of the equilibrium, but just the time. Moreover, the shifting affects also the maximum and minimum values of each variables, both in terms of time and in terms of quantities. The biggest differences shown in the previous Table 5.2 are for the pollution and then for the water quality. So, firstly a possible demographic change is able to curb or speed up the evolution of water quality and pollution. Afterwards, it could also influence the minimum and the maximum values of the state variables, especially for W and P. In conclusion, if a big change in population size occurs, its consequences are experienced simultaneously and their effects are carried on also in the future. Moreover, the impacts of a possible demographic change can be predicted also for the future because the final results of each variables are the same as the original model but shifted over time.

Finally, another no-stationary case is built: the harmfulness λ is no more constant and it evolves over time as a logistic growth, becoming λ_1. So, the dangerousness of the released pollutants changes and its effects regard the equilibrium values for each variable. In fact, as explained in Figure 5.2, λ represents the weight of the industrial polluting production on the water quality evolution over time and it influences the final value of the variables. As did before, the variability of λ_1 can shape another external variable, the climate change. As before, four scenarios are built for different shapes of λ_1 functions and the results are reported in the next Table 5.3. They are made considering the RCPs formulated by the IPCC:

<table>
<tr><th>MODELS</th><th>RESULTS</th><th>COMMENTS</th></tr>
<tr><td>Original model</td><td>
</td><td></td></tr>
<tr><td>1-RCP 2.6</td><td>
</td><td>ϕ=1.7 [-]
ρ=0.05 [yr^{-1}]
t_0=2030 [yr]
λ_1 stable in 2050
ΔW_{eq}=-0.13</td></tr>
<tr><td>2-RCP 4.5</td><td>
</td><td>ϕ=2.7 [-]
ρ=0.05 [yr^{-1}]
t_0=2050 [yr]
λ_1 stable in 2100
ΔW_{eq}=-0.25</td></tr>
<tr><td>3-RCP 6</td><td>
</td><td>ϕ=4.7 [-]
ρ=0.04 [yr^{-1}]
t_0=2080 [yr]
λ_1 stable in 2150
ΔW_{eq}=-0.46</td></tr>
</table>

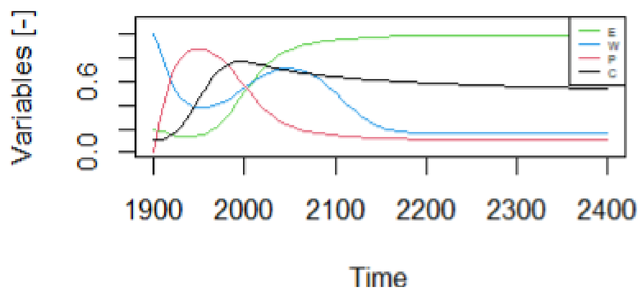
4-RCP 8.5		ϕ=7.7 [-] ρ=0.04 [yr^{-1}] t_0=2110 [yr] λ_1 stable in 2200 ΔW_{eq}=-0.73

Table 5.3: Evolution of the socio-hydrological model for different formulation of the no-stationary harmfulness λ_1

As it's possible to see from the results shown in Table 5.3, the climate change variable has different effects on the model. This external forcing influences mostly the water quality rather than pollution, and these impacts regard the final equilibrium values. The time isn't influenced so much as for the demographic change, apart from the water quality that changes not only its final value but also its time required for the equilibrium. For these reasons, the climate change variable is a future oriented forcing and it doesn't shift the curves, it changes the equilibrium values. With the most pessimistic scenario, the water quality at the equilibrium can be reduced of 0.73 points and it could have dangerous impacts on the environment. The main problem of climate change is that its effects become significant only for future generations and not in the present days. This external variable has the biggest effects on the water quality rather than on the industrial polluting production variable because it's very difficult to be predicted for the future due to the fact that in the present few clues are acting, so the consequences will be greater than for the demographic change. So, the model unveils that the worst effects on the health of the water bodies come from the actual climate change phenomenon, instead a possible demographic change will influence more the spreading of the pollutants.

5.2. Conclusions

The major aim of this master thesis is to provide guidelines about how build a socio-hydrological model for the evolution of the water quality in Europe from 1900 until present ages. As widely described in the Chapter 1.1, the socio-hydrology is a relatively recent science but its spreading is very fast due to its strong power. Actually, in traditional hydrology, human activities are typically described as boundary conditions, or external forcings, to the water systems. Following the increased hydrological challenges due to human-induced changes, the limitation of traditional hydrology started to be overcome by accounting for the mutual interactions between water and society and by advocating for greater connection between social science and hydrology. For these reasons, a new science was born in 2012 thanks to Murugesu Sivapalan: socio-hydrology is an interdisciplinary field studying the dynamic interactions and feedbacks between water and people and it includes the historical study of the interplay between hydrological and social processes, comparative analysis of the co-evolution and self-organization of human and water systems in different cultures, and process-based modelling of coupled human-water systems.

As it's possible to understand, developing a socio-hydrological model requires a large amount of data from different fields (hydrology, society, technology, politics and economy) and it could be quite time consuming to develop. In socio-hydrological modelling, the holistic understanding the complete system is the main objective. This approach makes the model's development quite complicated: for all the variables inside the system the relationships must be fully described by differential equations. In this way, the data have to be accurate and it needs a careful investigation. Sometimes could happen that data from economy or politics are pretty hard to find and the model may be not so reliable.

The socio-hydrological model of this thesis work describes the evolution of the water quality considering feedbacks from industrial polluting production, environmental awareness, economic health, demographic change and, finally, climate change. Gathering all these quantitative data from several different fields isn't so simple, especially for a three months work. Consequently, the socio-hydrological model developed must be consider as a qualitative one: in order to compute it, papers, articles, books and literature were reviewed and, based on this information, some possible relationships and feedbacks between the state and external variables were built. So, the final goal is the formulation of a qualitative socio-hydrological model ready to be implemented also with real numerical data, when they are available.

Notwithstanding that the model gives qualitative results (all the variables are bounded between 0 and 1 and dimensionless), the results are very significant: the water quality variable is such sensitive that just changing the values of the coefficients produces big consequences in the outputs. For example, from the model emerges the significant differences between the environmental awareness-pollution relationship and environmental awareness-economic health feedback. The establishment of two coefficients helps to understand these links: the model explains that the eco-friendliness (represents by χ) is the tool through which the environmental awareness tries to reduce the polluting production by improving more sustainable technologies. On the other hand, another tool for the State in controlling the contaminants is acting directly on the economic health and it's represented by the financial sustainability η. However, the model shows that the most effective strategy is not publishing green laws, setting limits or investing in sustainable policies because it has consequences mostly on the economy rather than on the pollution. If the State wants to act quickly on the incoming environmental diseases, it must improve the eco-friendliness (represents by χ) and so putting its efforts on green and sustainable technologies and production.

The last significant conclusion born by the socio-hydrological model is the impact of the demographic change and the climate change. The demographic change, represented by the no stationarity of the pollution's speed parameter γ_I, has evident effects in the maximum and minimum values of the state variables over time, so they can be appreciated in the past and present ages. In the future the demographic change just delays or anticipates the already known consequences. Moreover, the demographic change affects more the industrial polluting production rather than the water quality. However, the climate change is a forward-looking variable: its effects aren't touchable in the past nor present ages, the minimum and maximum values of the variables are the same, but the worst consequences happen in the future with the most pessimistic scenarios for the water quality variable. Anyway, this qualitative model explains that the climate change phenomenon must be studied and present mitigation and adaptation improvements must be taken because, even if its impacts are not so worrying now, the future consequences could be the dangerous ones.

In the end, socio-hydrological models could be used to anticipate what trajectories might occur in the coming decades, depending on the present condition of a human-water system. Models can, later on, be used in policy formation and decision making, whereas it could be really useful.

6. Bibliography

- An, L. (2012), *Modelling human decisions in coupled human and natural systems: Review of agent-based models. Ecological modelling*, 229, 25-36, https://doi.org/10.1016/j.ecolmodel.2011.07.010
- Anderies, J. M. (2000), *On modelling human behaviour and institutions in simple ecological economic systems*, Ecol. Econ., 35(3), 393–412, https://doi.org/10.1016/S0921-8009(00)00221-4
- Angelakis A. N. et al. (2022), *Water Quality Focusing on the Hellenic World: From Ancient to Modern Times and the Future*, https://doi.org/10.3390/w14121887
- Beck, L. (2010), *Environmental, political, and economic determinants of water quality monitoring in Europe,* https://doi.org/10.1029/2009WR009065
- Bin, L. (2019), *An Urban Sociohydrologic Model for Exploration of Beijing's Water Sustainability Challenges and Solution Spaces*, https://doi.org/10.1029/2018WR023816
- Blöschl, G. (2017), *Debates—Hypothesis testing in hydrology: Introduction. Water Resources Research*, 53(3), 1767-1769, https://doi.org/10.1002/2017WR020584
- Bristol University (2013), *Lecture motes for Applied Partial Differential Equations 2 (MATH20402), Lecturer: Dr Valeriy Slastikov,* https://people.maths.bris.ac.uk/~maajh/ODEs/chap4.pdf
- Brun, R.et al. (2001), *Practical identifiability analysis of large environmental simulation models*, Water Resour. Res., 37(4), 1015–1030, https://doi.org/10.1029/2000WR900350
- Carr, G. et al (2022), *Exploring water quality management with a sociohydrlogical model: a case study from Burkina Faso,* https://doi.org/10.1080/02626667.2021.2020276
- Chen, X., D. Wang, F. Tian, and M. Sivapalan (2016), *From channelization to restoration:*
- Di Baldassarre, G. (2015), *Debates—Perspectives on socio-hydrology: Capturing feedbacks between physical and social processes, https://doi.org/10.1002/2014WR016416*
- Ehrlich, P. R., and P. H. Raven (1964), *Butterflies and plants: A study in coevolution, Evolution, 18, 586–608,* https://doi.org/10.2307/2406212
- Elshafei, Y. (2014), *A prototype framework for models of socio-hydrology: identification of key feedback loops and parameterisation approach,* Hydrol. Earth Syst. Sci., 18, 2141–2166, https://doi.org/10.5194/hess-18-2141-2014, 2014

- Elshafei, Y. (2015), *A model of the socio-hydrologic dynamics in a semiarid catchment: Isolating feedbacks in the coupled human-hydrology system,* https://doi.org/10.1002/2015WR017048
- European Environmental Agency, https://www.eea.europa.eu/en/analysis
- Eurostat Database, https://ec.europa.eu/eurostat/web/main/data/database
- Evans, T. P., and H. Kelley (2004), Multi-scale analysis of a household level agent-based model of landcover change, J. Environ. Manage.,72(1), 57–72, https://doi.org/10.1016/j.jenvman.2004.02.008
- Grittayaphong, P. (2023), *Beyond GDP: Three Other Ways to Measure Economic Health", https://www.stlouisfed.org/open-vault/2023*
- Hargrave, M. (2024*), Capital: Definition, How it's Used, Structure, and Types in Business*
- Herrera-Franco, G. et al. (2021), *Approach on water-energy-food (WEF) nexus and climate change: A tool in decision-making processes,* https://doi.org/10.1016/j.envdev.2023.100858
- IPCC, *Climate Change 2022 Impacts, Adaptation and Vulnerability*, https://www.ipcc.ch/report/sixth-assessment-report-working-group-ii/
- Kirschke, S. et al. (2020), *Capacity challenges in water quality monitoring: understanding the role of human development,* https://doi.org/10.1007/s10661-020-8224-3
- Liu, D., F. Tian, M. Lin, and M. Sivapalan (2015), *A conceptual socio-hydrological model of the co-evolution of humans and water: Case study of the Tarim River basin, western China*, Hydrol. Earth Syst. Sci., 19, 1035–1054, https://doi.org/10.5194/hess-19-1035-2015, 2015.
- Milik, A., A. Prskawetz, G. Feichtinger, and W. C. Sanderson (1996), Slow-fast dynamics in Wonderland, Environ. Model. Assess., 1(1-2), 3–17, https://doi.org/10.1007/BF01874842
- Nixon, S. C. et al (2000), *Uso sostenibile dell'Acqua in Europa? Stato, prospettive e problemi*, https://www.eea.europa.eu/it/publications/water_assmnt07
- Noel, P. H. C. (2015), *Studying Coupled Human and Natural Systems From a Decentralized Perspective: The Case of Agent-Based and Decentralized Modelling*, MS thesis, 62 pp., Dep. of Civ. Environ. Eng., Univ. of Ill., Urbana.
- Petri S. Juuti and Tapio S. Katko (2005), *Water, Time and European Cities- History Matters for the Futures*

- Ross, S. (2024), *How Capital Investment Influences Economic Growth",* https://www.investopedia.com/ask/answers/032615
- Sanderson, W. C. (1994), *Simulation models of demographic, economic, and environmental interactions, in Population—Development— Environment*, pp. 33–71, Springer, N. Y, https://pure.iiasa.ac.at/3932
- Sivapalan, M., and G. Blöschl (2015), *Time scale interactions and the coevolution of humans and water,* Water Resour. Res., 51, 6988–7022, https://doi.org/10.1002/2015WR017896
- Sivapalan, M., H. H. G. Savenije, and G. Blöschl (2012), *Socio-hydrology: A new science of people and water*, Hydrol. Processes, 26,1270–1276, https://doi.org/10.1002/hyp.8426
- Sivapalan, M., M. Konar, V. Srinivasan, A. Chhatre, A. Wutich, C. A. Scott, J. L. Wescoat, and I. Rodriguez-Iturbe (2014), Socio-hydrology: Use-inspired water sustainability science for the Anthropocene, Earth's Future, 2, 225–230, https://doi.org/10.1002/2013EF000164
- *Sociohydrologic modelling with changing community preferences in the Kissimmee River Basin, Florida*, Water Resour. Res., 52, 1227–1244, https://doi.org/10.1002/2015WR018194
- Spiegelman, M. (2013), *An Introduction to Dynamical Systems and Chaos, pdf*
- Troy, T. J., M. Pavao-Zuckerman, and T. P. Evans (2015b), *Debates—Perspectives on socio-hydrology: Socio-hydrologic modelling: Trade-offs, hypothesis testing, and validation*, Water Resour. Res., 51, 4806–4814, https://doi.org/10.1002/2015WR017046
- Tzanakakis, V. A. et al (2020), *Water Supply and Water Scarcity*, https://doi.org/10.3390/w12092347
- UNEP Global Environmental Monitoring System Water Programme (2008), *Water Quality for Ecosystem and Human Health",* http://www.gemswater.org/
- Van Emmerik, T. H. M. et al (2014), *Socio-hydrologic modelling to understand and mediate the competition for water between agriculture development and environmental health: Murrumbidgee River basin, Australia*, Hydrol. Earth Syst. Sci., 18, 4239–4259, https://doi.org/10.5194/hess-18-4239-2014

- Viglione, A. et al. (2014), *Insights from socio-hydrology modelling on dealing with flood risk–roles of collective memory, risk-taking attitude and trust*, J. Hydrol., 518, 71–82, https://doi.org/10.1016/j.jhydrol.2014.01.018
- Vörösmarty, C. J., & Sahagian, D. (2000), *Anthropogenic disturbance of the terrestrial water cycle,* Bioscience, 50(9), 753-765, https://doi.org/10.1641/0006-3568(2000)050 [0753: ADOTTW]2.0.CO;2
- Xia, J., Dong, Y. & Zou, L. (2022), *Developing socio-hydrology: Research progress, opportunities and challenges*. J. Geogr. Sci. 32, 2131–2146, https://doi.org/10.1007/s11442-022-2040-3
- Yaeger, M. A. et al. (2014), *An integrated modelling framework for exploring flow regime and water quality changes with increasing biofuel crop production in the US Corn Belt*, Water Resour. Res., 50, 9385–9404, https://doi.org/10.1002/2014WR015700