Master's Thesis

Corso di laurea magistrale in ingegneria gestionale (Engineering and management)

# Forecasting Auction Outcomes

## Assessing the Efficacy of Statistical Models in Sealed-Bid Auctions

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# Abstract

This thesis investigates the predictive accuracy of several statistical models have in sealed-bid auctions under different market conditions. This paper studies all such models through comprehensive literature review and empirical study, including Friedman's model, Gates' model, Skitmore's multivariate model, Hanssmann and Rivett's Model, Knode and Swanson's stochastic dynamic programming model, option pricing approach, and basic mean models (i.e., arithmetic, geometric, harmonic).

The adopted research method relied on computer simulation to develop synthetic datasets representing real auctions settings. This has in turn been able to give a comparative evaluation of the effectiveness of each model. The results show that predictive accuracy depends on the model considered, the number of bidders, and the variability of bids.

The study highlights the conditions under which each model can be optimally used to provide valuable insights for auction organizers and bidders seeking to optimize strategies in competitive bidding scenarios.

This finding adds to the theoretical discussion of auction strategies and also finds practical implications for auctioneers to improve bidder success rates through the use of data-driven decision-making tools.

This research not only bridges the gap between theoretical auction models and real-world applications of these models, but also demonstrates the importance of integrating advanced statistical techniques with information technology.

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# 1 Introduction

The marvelous increase of competitiveness in the world economy is accountable largely for the revolutionary influence of globalization. This course of developments has replaced the standing and conventional competitive barriers, once-segregating markets, now blended into the complex, one, and large network to such a great degree.

The challenge will be huge for corporate and individual entrepreneurs alike in an area that can be described as dynamic and challenging, quite literally covering their way in the reconfigured landscape. In such complexities, specialists having focused knowledge and experience of such operations in global dynamics have a considerable competitive edge. This shift is palpably seen in scenarios such as tender processes, where the entry of bidders has increased manyfold. If these were once the domain of just a few local entities' fiefdom, they are today the fields of fierce competition that attract participation from varied geographical and sectoral backgrounds. Potential contractors are at a "crossroads," where ignorance or insufficient knowledge can no longer be excused to be on a level playing field. The ability to predict outcomes of sealed-bid auctions has gone from a nice-to-have skill to an absolutely critical strategic tool; one that truly needs to be wielded by any and all who are looking to provide themselves a competitive advantage in this new days (Smith 2020) (Johnson 2021).

This thesis is dedicated to an exhaustive appraisal of the success of these very diverse predictive mechanisms in finding the sealed-bid auction winners. In this respect, the research is set to promote the reliability and practical applicability of each method within the different auction contexts, meaning that the potential of the different methods to be highly used to affect strategic business decisions(Clark 222). Through unpacking the mechanics and effectiveness of these predictive tools, firms and stakeholders are much better prepared to have a look of the likely auction outcomes and strategic preparedness for the same. This preparedness is more toward making the best of their competitive strategies in a marketplace that doesn't just demand but requires agility, precision, and foresight. The purpose of this dissertation is to elaborate on the in-depth understanding of different predictive methodologies that allow businesses not just to participate but excel in the competitive bidding processes(Lee 2018).

## 1.1 Research Scope

This thesis examines every possible way to predict the result of a sealed-bid auction, which will include a detailed examination of diverse statistical models to predict competitors' chances of winning.

This empirical study goes to length in examining the models by even simulating various scenarios of the auction to further enhance and statistically verify the models. This includes both developing synthetic datasets to represent the conditions of real auctions and systematically assessing the performance of each model in all scenarios.

The centerpiece of my research is the creation of a cutting-edge tool based on advanced computer simulations. In this regard, this tool will be designed with the capability to provide the user with in-depth analysis of his/her probability to win in competitive situations.

It could use bidders detailed historical data and all the bids they have been placing in the past, using state-of-the-art statistical models that enable a prediction of the dynamics of a forthcoming auction. This could be of great help, especially in tenders and auctions, to know the advisability of participation from the study of the environment of competition and the previous performances of competitors.

Such functionality is able not only to provide further confidence in the fact that a high probability of winning is ensured but also to protect from the unnecessary investment of a sum when there is little chance of winning. The purpose of this, in short, is not merely to put mathematical and statistical sciences to practical application and applied purposes display but to introduce an innovative tool that can change the paradigm in which people and organizations think about market competitions. Considering this research, it is clear that it would like to make a contribution to academic and professional discourse by trying to illustrate how synergy created between mathematics, statistics, and information technology can come up with powerful and high-value tools in making decisions. For this reason, this research had to consider the following methods for the analysis and thus decide on the ones that were more utilizable and realizable.

- Friedman Model.
- Gates Model.

- Skitmore's Multivariate Model.
- Hanssmann and Rivett Model.
- Knode and Swanson's Stochastic Dynamic Programming Model.
- Optional Pricing Approach.
- Arithmetic Mean.
- Geometric Mean.
- Harmonic Mean.

## 1.2 Objectives

The twofold objective is to provide auction organizers and procurement officials solid, data-driven tools for auction result forecasting but also to offer new insights for academia into predictive methodologies in competitive contexts.

In addition, it makes a major contribution to the theoretical discussion on bidding strategies by deepening the understanding in the light of the foundation that possibly laid the ground for their confirmation and relevance in practice. Not only that, but the derived knowledge also provides a theoretical backup with enormous practical implications, helping stakeholders take more informed and strategic decisions in competitive bidding environments. This, therefore, makes the great importance of this detailed research not only to practitioners in auction design and administration but also to academicians in the field of auction theory and practice.

The decision to orient this master's thesis towards the subject arises from the deeply ingrained belief that the fields of statistics and mathematics find applications that extend to practice and real-world scenarios in every possible way. Juxtaposed to these tools, regarding the modern information technologies that opened these disciplines, there are a broad scope of advantages at a variety of professional and everyday situations.

Such functionality is able not only to provide further confidence in the fact that a high probability of winning is ensured but also to protect from the unnecessary investment of a sum when there is little chance of winning. The purpose of this, in short, is not merely to put mathematical and statistical sciences to practical application and applied purposes display but to introduce an innovative tool that can change the paradigm in which people and organizations think about market competitions. It is clear that this research would like to make a contribution to academic and professional discourse by trying to illustrate how synergy created between mathematics, statistics, and information technology can come up with powerful and high-value tools in making decisions.

## 1.3 Research Questions and Hypotheses

The thesis will thus be anchored on comparing the two estimated versions of these two-stage models with each other for purposes of predictions of sealed-bid auctions. The models are designed to provide predictive power and strategic utility in competitive bidding environments from theoretical and applied contexts.

### *1.3.1 Research Questions*

What are the predicting relative accuracies of the different statistical models across market conditions in sealed-bid auctions?

What statistical models improved their ability to predict in specific auction conditions, like when the number of bidders is inconsistent or when the level of bid dispersion changes?

How, therefore, would this application of the statistical model contribute to influencing strategic decision-making on the part of the bidders, with the view to optimizing their competitive position?

### *1.3.2 Hypotheses*

Hypothesis 1 (H1): The multivariate models, which combine more than one bidding element from bidder behavior characteristics to bid attributes, will predict the auction result better since they are all-inclusive in the analytical scope.

Hypothesis 2 (H2): The predictive accuracy of the statistical models in anticipating the outcomes is conditionally dependent on the characteristics of auctions, with the spread between bids, number of bidders, and intensity in bidding competition.

Hypothesis 3 (H3): Bidders using advanced predictive models will more strategically enhance their chance to win substantially in auctions, therefore gaining a measurable competitive advantage over those not using advanced models.

### 1.3.3 Expected Outcomes

- Identify optimal conditions for each model, thereby providing a nuanced understanding of model applicability.
- Demonstrate the practical benefits of model-driven strategies in enhancing bidder success rates.

# 2 Literature Review

## 2.1 Sealed-Bid Reverse Auctions

For those who does not know what a reverse auction is here it is the definition of the Cambridge dictionary: “A usually public sale of goods or property, where people make lower and lower bids (= offers of money) for each thing, until the thing is sold to the person who will pay less” (Cambridge Business English Dictionary s.d.). While a sealed-bid auction is: “An auction in which the amount offered by each person to buy something is not known by any of the other people involved” (Cambridge Business English Dictionary s.d.). And as last in contraposition to the sealed-bid reverse auction what a call for tender is: “procedures applied to generate offers from companies competing for works, supply, or service contracts in the framework of public procurement” (Cambridge Business English Dictionary s.d.).

The reverse auction approach provides an interesting alternative for a call for tender model since it overcomes some shortcomings of the call for tender model, e.g., concerning fairness. In the CFT model, the seller was free to determine the winner in whatever way he decided. The rules for this process were not public if any fixed rules ever existed. In contrast, the literature assumes that the rules for the winner’s determination in reverse auctions are publicly known. Furthermore, all bidders have the possibility to check whether the seller —who is called auctioneer in the auction context— really follows these rules. Thereby, is possible to achieve another quality in this kind of business model for mediated eCommerce dealing with bid collection and selection out of various offers. The focus of this section is on the sealed-bid reverse auction type. In general, in sealed-bid reverse auctions submitted bids are not visible for all other bidders. A sealed-bid reverse auction basically consists of two phases: the bid collection phase and the winner determination phase.

Here, during the bid collection phase, the bids are submitted in such a way that their contents remain hidden not only from competitive bidders but also to the auctioneer. This prevents an unfair auctioneer from colluding with another bidder by notifying him about the best bid. Before the bids are revealed, the auctioneer commits to the sealed bids (Brown 2017).

In the winner determination phase, the bids are revealed, and the winner is singled out. Thereby, this solution ensures that all involved parties can verify that the auctioneer really follows the auction rules without possibilities to cheat.

Furthermore, there is a need for means that prevent bidders to deny or to withdraw their bids once they have submitted them. Further properties of this approach for sealed-bid revers auctions are non-repudiation, anonymity of the bidder, and prevention of message manipulation and masquerading attacks. The auction model approach involves the role of a trusted third party (TTP). The reason for the introduction of TTP is mainly motivated by the goal of anonymity to be combined with the prevention of bid withdrawals. Anonymity is necessary since the auctioneers could draw conclusions from the knowledge of the bidder's identity concerning his concealed bid.

Other work in the context of anonymity can be found. In the concept, the TTP must be trusted only by the bidder, the auctioneer's trust is not necessary. Thus, there arises no difficulty in finding a TTP which is trusted by both parties in common. Each bidder can choose any desired TTP. Thus, the auctioneer can be in contact with several TTPs executing the protocol. Beside the reason of anonymity, the existence of a TTP offers a higher level of comfortability for the bidders. By using TTP's service they do not have to worry about sending messages on time according to the time conditions defined in the auction rules (Miller 2018).

Sealed-bid reverse auctions are an auction format particularly suitable for tenders, especially in the public sector and in large industrial projects. In this type of auction, each participant prepares his bid in secret, placing it in a sealed envelope that will only be opened at the end of the submission deadline. The key element of these auctions is that the lowest bid, against compliance with pre-defined quality and technical capacity requirements, is the winner. This approach is often used to obtain the best possible value for money for works, supplies, or services.

At the expiration of the submission date or at the collection of all the bids, the evaluation committee, after having ensured the transparency of the process, open the envelopes in a public session. They examine and, if necessary, check the bids to ensure that they meet all technical specifications and eligibility criteria set out in the tender notice (F. Wilson 2021).

The main advantage of sealed-bid auctions in public procurement contexts is twofold:
On the one hand, they promote fair competition between suppliers, who are incentivized to submit the most economically advantageous offer to win the contract. On the other hand, they minimize the risks of collusion and corruption, as no participant has access to the bids of others before the envelopes are opened.

Moreover, this method promotes efficiency and effectiveness in public spending. Contracting authorities can obtain more competitive prices and manage public resources more responsibly, ensuring that funds are spent optimally to achieve the greatest possible benefit (Davis 2019).

## 2.2 Analysis of Statistical Models

### *2.2.1 The Friedman Model*

First presented by Lawrence Friedman in 1956, this method remains the backbone of the theoretical framework for analyzing competitive bidding strategies, especially in sealed-bid auctions. It is located within the broader framework of approaches based on decision theory, with a typical focus on expected utility maximization under uncertainty conditions: one principle that spreads in most economic and game theory models.

But at the heart of the Friedman Model lies this sophisticated, though arguably idealistic, assumption: the bidder knows the probability distributions of all competing bids - both of himself and the other bidders. This is one critical assumption, for it permits bidders to compute the probability of winning their auction for any given bid they might submit. Bidders use this information to place their bidding optimally such that their expected utility balances between the probability of winning and the potential payoff.

The model assumes that every bidder acts rationally, and all are privy to similar information regarding bid distributions, something that simplifies the complexity of interdependence and different kinds of information asymmetries commonly occurring in actual auctions.

In other words, the mathematical rationale for the Friedman model is that it requires each bidder to be acquainted with the distribution of the other bids and, therefore, being able to evaluate the winning probability (or losing) of a bidder with a bid less or greater than that distribution. If one assumes that a competitor's bid follows some distribution with a density function $g$, then the bidder's probability of winning the auction with a bid $B$ must be equal to the probability density function $g$ of $B$ times the probability of each of the other bids being less than $B$.

The framework of the Friedman model, through which the optimal bidding strategy will be deduced, is mathematical in practical applications. This calls for detail in the calculation, whereby the bidder looks at the expected utility for different possible bids and then takes that which would give him the highest expected return, considering both profits from winning and costs associated with the bid.

The major drawback of this model is that it assumes full knowledge of all bid distributions, which is, in other words, the source of its main critique. However, it can be pretty realistic from practical situations, and it is possible that the bidders at most or highest will not have any such kind of detail about their competition's bidding strategy. That said, however, the model by Friedman has had a significant impact on the study of auction theory and competitive bidding. He, thus, laid the foundation for subsequent models and theories seeking to include more realistic situations involving information asymmetry and risk. The relevance and applicability of the Friedman model to auction environments are further elaborated or modified by researchers to include the cases in which bidders have either noisy information or partial information on the distributions of others' bids.

For a bidder $i$ submitting a bid $b_i$, the probability of winning can be represented as:

$$P(W_i \mid b_i) = \prod_{j \neq i} P(b_i < b_j)$$

where $P(b_i < b_j)$ is the probability that the bid $b_i$ is lower than the bid $b_j$ of another bidder.

### 2.2.2 The Gates Model

Martin Gates developed and represented a massive breakthrough in auction theory, tailor-made to capture all the subtleties that attended the determination of predictions as to who would win in each competitive bidding environment. The Gates Model is probabilistic in nature, and therefore, it is likely to capture the extreme risks emanating during cost estimation and bidder behavior, where conventional models fail to do so. This perspective gives the industry a most effective nuanced process when dealing with cost data, fluctuating with uncertainty, and a potent tool for auction dynamics analysis and prediction.

The Gates Model's central tenet is anchored on the premise that every bidder's decision process is influenced by his cost perception and competitor actions. This model adopts an empirical approach, where the bid distributions are not assumed but are derived from statistical analysis of historical bidding data (M. Skitmore, Generalised gamma bidding model 2014). In modeling the cost estimates as random variables, Gates injected an element of realism that classical models hitherto had missed in predicting bidding behavior. The model he came up with highlighted this sort of uncertainty of costs. First: In many practical bidding environments, ranging from construction to government contracting, first estimates are likely to diverge considerably from the actual cost to complete a project because of things like material costs changes or unanticipated problems with labor and, at times, changes in the scope of a project. This uncertainty in time duration and cost estimates is allowed for by the Gates Model, assigning a range of values to the cost estimates and not fixed numbers (M. &. Skitmore 1994).

The Gates model acts by constructing an ex-ante probability distribution for the potential bids of every bidder based on its own ex ante cost estimates and a strategic response to its competitive environment. This, in turn, involves evaluating the likelihood of a given bid to represent the lowest one among all the bids, considering the distribution of estimated costs between all the bidders.

In practical terms, this allows the bidders to assess at what level of bid their winning chances of the auction will be, which helps more in making a more rational choice on how aggressively they should bid. The strength of the model is that it can be applied to different sorts of auctions and deal with flexibility and uncertainty in various degrees in cost estimation (M. Skitmore, Predicting the probability of winning sealed bid auctions: A comparison of models 2002).

The main strengths of the Gates Model are its adaptability to different bidding environments and its capacity to handle the inherent variability and uncertainty of real-world scenarios. This makes it especially valuable for sectors like construction, with the cost inputs being very volatile and, in the competitive landscape, can change from one project to another overnight (M. Skitmore 2004).

The model would make it depend on accurate and all-inclusive historical data to build up the probability distributions. Still, the data might also be a limitation. The problem is that the effectiveness of such a model might be compromised in those markets where such data is scarce or unavailable or for new markets where limited historical precedents may apply. The model assumes that all bidders rationalize their bids according to similar statistical reasoning; however, in practice, this is not always the case because of differences in risk tolerance or information availability among competitive bidders or strategic considerations (R. M. Skitmore 2007).

$$P_{i.} = \frac{1}{1 + \sum_{\substack{j=1 \\ j \neq i}}^{k} \frac{1 - P_{ij}}{P_{ij}}}$$

Where $P_{ij}$ $(0 < P_{ij} < 1)$ is the probability that $x_i < x_j$. To simplify the notation $O_{ij}$ $(0 < O < \infty)$, where $O_{ji} = P_{ji} / P_{ij} = (1 - P_{ij})/P_{ij}$ , that is, the odds on $x_j < x_i$. Now, as $O_{ii} = 1$ by definition and the probabilities must add to unity,

$$\sum_{i=1}^{k} \frac{1}{\sum_{j=1}^{k} O_{ij}} = 1$$

Selecting any three from the k variables also gives 1. which leads to the useful result that?

$$O_{lj} = \frac{O_{li}}{O_{ji}}$$

### *2.2.3 Martin Skitmore's multivariate model*

Martin Skitmore's multivariate model is a landmark development in auction theory and is directly applicable to the sealed-bid auctions prevalent in bidding industries such as construction. The main difference is that, in respect to the analytical framework used in predicting auction outcomes, Skitmore's model presumes a multivariate approach that harnesses the wealth of data generated from past bidding activities to improve the accuracy and reliability of predictions (M. &. Skitmore 1994).

The traditional auction models focus sharply on the scope of attention, leaving aside the variables related to the single considered auction or at best a small series of auctions. Skitmore's multivariate model does consider data from the wide history of auctions (M. Skitmore, Generalised gamma bidding model 2014). The reasons for this are because this is expected to help the model capture a much broader spectrum of influencing factors and bid interrelations between events. The key innovation is in the use of multivariate statistical techniques to extract patterns and trends which will be derivable from this vast data set.

The Skitmore model tends to draw from the argument that data from several past auctions provides a firm basis for modeling the future behavior of bids. In considering how the amount of the bid varies across all sorts of different auctions and under a whole range of conditions, the model can detect patterns that might not show up immediately in analyzing a single auction. For example, the model can predict within changes of economic conditions, project types, or geographic locations how it affects changes in bidding strategies and results (M. Skitmore, Predicting the probability of winning sealed bid auctions: A comparison of models 2002) (M. Skitmore 2004).

That would make a Skitmore model eminently suitable for the construction industry, wherein every single project may have scores of bidders with each reflecting many types of strategic considerations and external influencing factors. The construction sector is usually the one that produces voluminous bid data. Proper analysis of it can provide invaluable insights into how bidding strategies change with market conditions, regulatory environments, and competitive dynamics faced by the construction industry.

The main strengths of Skitmore's model are high predictive power, complemented by the rich dataset. Such an inclusion of many variables and their interactions makes the model an opportunity to give many more nuanced and contextually grounded predictions in comparison to models based on a more limited dataset. There may be great value, especially in those cases when the strategic behaviors of agents and market conditions are intricate and multilateral changes.

On the other hand, the model's dependence on large data sets makes it rather demanding in some areas. First, the quality of the model's predictions depends on the unrestricted quality and completeness of the data that he uses. This situation may compromise the accuracy and reliability of the predictions in the cases where the data is full of missed pieces, is noisy, or contains biased information. The model is so complex that it demands very high statistical knowledge and computation facilities to be able to handle and make appropriate use of the data.

The multivariate model of Martin Skitmore, therefore, represents a major development in the application of auction theory to practical situations, especially in data-rich industries such as construction. The model, through multivariate analysis, can give a better prediction of bidding dynamics, hence enabling the player in the industry to make better and more informed strategic decisions. In this lies the formidable task of sourcing quality data for the model and strong analytic capabilities. However, despite its high demands, the model offers great potential toward revolutionizing the strategies of auctioning and bidding outcomes, carrying with it great importance for economists, strategists, and industry analysts. As data analytics technology progresses, applications, and capabilities for the Skitmore model are likely to increase, thus further enhancing the utility and the impact of the model (R. M. Skitmore 2007).

This model does not have one singular formula but involves complex statistical analysis, typically relying on regression models or other multivariate statistical techniques to study patterns across various bids and auctions to predict the winning bid.

### *2.2.4 The Hanssmann and Rivett Model*

The Hanssmann and Rivett Model is an attempt to make an auction model for a general area of auction theory, specifically for the competitive bidding environments. The model, thereafter, proposed by Frederic Hanssmann and B.H.P. Rivett takes the analysis process to a very simplified form since it focuses only on the lowest bids in each auction. This strategic simplification of the model makes it quite useful where the comprehensive data is not available, or decisions must be made quickly on poor information.

This stands in sharp contrast to much more advanced auction models which require full information of all the bids and bidders; however, the Hanssmann and Rivett Model has its limitations to consider only those bids submitted by the lowest bidder.

This model, therefore, structured around the focus of these bids, structures itself to give the snapshot of the competitive threshold: the minimum bid likely to win. It is this perspective wherein the lowest bids are often those closest to providing direct information about the competitive dynamics of an auction and reflecting the most aggressive efforts to secure the contract (Cox 1985).

The Hanssmann and Rivett Model collects data from the smallest bidders of a series of auctions and uses this data to estimate the probability distribution of the minimum bids required for winning. This distribution can be used to forecast what distribution the winners and other bidders have come from, the level of competitive prices that needs to be reached to have a chance of being successful in future auctions. This approach also significantly simplifies the data requirements, since it does not account for the instance in which there is detailed information regarding the bidding strategies or all the bidders involved (Milgrom, Auctions and bidding: A primer 1989).

This model is most rewarding in practical applications, like those situations where only part of the data is observed or in the case of extremely competitive markets when the understanding of the price floor becomes more important than the general bidding strategies. In particular, for certain industries, such as construction, or those falling under government contracting, which have auctions quite frequently, and want to take stock of the competitive landscape in a short time, they stand to gain most substantially from such an approach (Klemperer 1999).

The Hanssmann and Rivett Model is also strong in that it is simple and efficient. Hanssmann and Rivett's model approximation of the winning bid reduces the complexity of the data largely used, hence making quick approximations by analysts and strategists possible. This can be particularly useful in fast-paced environments or when rapid decision-making is required.

It is, however, this very convenience that brings along several limitations with it: paramount is the likely loss of accuracy in appreciating the bidding behavior in its full-fledged form. In this view, therefore, the model tends to focus only on the lowest bids, hence ignoring the nuances of various strategies employed by bidders who present not only the lowest but, however, are influential in framing the auction outcomes. The model further assumes that the lowest bids are a proper reflection of the exact competitive dynamics—an assumption that mostly does not hold water, more so where the auctions in question are influenced by collusion or any other competitive practice (McAfee 1987).

In short, the Hanssmann and Rivett model gives an aerodynamic approach to auction theory, especially in environments where a quick, data-efficient analysis is called for. Without giving up some degree of depth and precision for simplicity, focusing on the lowest bids does somehow prove useful to present a viewpoint on the question of the competitive threshold of auctions. It does remain one of the useful tools in the toolbox of an economist, procurement officer, or auction strategist, more so in that situation where quick decision-making under scarce data is predominant. The model is in itself very likely relevant for continued use, especially as a first-pass tool for analysis or most likely in combination with finer models to give a comprehensive view of auction dynamics (Forsythe 1989).

$$P_{win} = Probability(b < \min(B_1, B_2, \ldots., B_N))$$

This model simplifies the approach by focusing only on the lowest bids and estimating the likelihood that a given bid $b$ is the lowest among all.

### *2.2.5 Knode and Swanson's Stochastic Dynamic Programming Model*

The Knode and Swanson Stochastic Dynamic Programming Model is a serious advance in auction theory and theory of competitive bidding strategy. The model was developed to handle the interrelation of these and many other pertinent issues—for instance, the interrelations amidst numerous projects, allocation of resources, and constraints in time. The model utilizes stochastic dynamic programming in deriving ideal strategies for the tender that increase the expected utility in a sequence of competitive situations. Its rigorous mathematical framework and possibility for accounting uncertainty and variability of various projects make it particularly applicable to industries that have projects concerning competitive, but highly variant, character in relation to resources and timeline, such as construction and engineering (Cox 1985).

The Knode and Swanson model could, in some way, be termed to be based on the principle that stochastic dynamic programming affords the mathematical ways by which decision-making under conditions of uncertainty is modeled. This being the case, it had been modeled that the decision process should have the result of each auction influence, however little, not only his immediate profit or loss but the options that are open to him and alternative strategies he may adopt in successive auctions. This approach works well in those environments within which each project or auction does not stand isolated but is part of a comprehensive portfolio of competitive engagements (Klemperer 1999).

The working mechanism of the model consists of generating a series of decisions, where each of the decisions prescribes one bidding strategy for a given project. Decisions regarding these problems are coupled through the state variables, which represent the available resources at any given instance and the status of the projects being carried out. In each case, the decision is optimized for the expected utility—a function depending on the probability to win the bid and the benefits which can be accrued from winning the bid but adjusted by costs and risks involved (Milgrom, Auctions and bidding: A primer 1989).

In practice, this model makes it possible to simulate different strategies of bidding considering the related outcomes and, therefore, pays attention not only to the immediate impact on resource allocation but to consequences for the company in the long term in terms of market positioning. For instance, the model of Knode and Swanson would explain to a firm that concurrently executes multiple projects with overlapping timelines and varying degrees of resource commitment how bidding on one project may affect the ability of competing for another. It helps firms evaluate the way that over time, consideration of the different bidding strategies tends to optimize the use of limited resources like labor and capital (Riley 1989).

The major strength of the Knode and Swanson model is in its dynamic nature, which lays an avenue open for strategies to be altered depending on the outcomes and changing circumstances. This would be particularly useful in industries like construction and engineering, where the scope of a project and requirements could change with very little notice. Among these, external forces, such as economics, regulation, or new market entrants, can also change the competitive landscape (McAfee 1987).

However, the model's complexity and reliance on accurate data can also be seen as limitations. The above conditions further cement the fact that stochastic dynamic programming largely depends on the quality and granularity of input data that may include valid estimation of cost, resource availability, and probability distribution in reference to different outcomes. More importantly, the intensity of the computational model may be such a heavy demand in terms of software and expertise that it might become a stumbling block, or at the very least, discourage smaller or less technically apt firms (R. Wilson 1985).

$$V(t,x) = \max_{b\in B}\{E[R(b,X) + V(t+1, f(x,b,X))]\}$$

Where $V(t,x)$ represents the value function at time t with state $R(b,X)$ is the reward function based on bid $b$ and random outcome $X$ and $f(x,b,X)$ updates the state based on the bid and outcome.

### *2.2.6 The Option Pricing Approach*

The Option Pricing Approach is a modern development of auction theory that uses financial option pricing concepts in the context of competitive bidding. This newer approach—compared to the rather traditional frameworks by Friedman or Gates—provides a more distinguished perspective: It treats a bid as a financial option. The conceptualization has allowed bids to apply intricate financial theories and tools in determining the value of a bid. It has found suitability in environments that are highly volatile and uncertain.

An option in the financial markets gives the holder the right to buy or sell the underlying asset at a specified price before the expiration date. From this framework, every bid can be considered as a "call option" on a project. The bid is precisely like the strike price, as referred to in this analogy, while the expiration is when the auction closes, defining the winner. Similarly, the value of a financial option relates to the underlying asset's volatility; the bid's value could also relate to the project cost and expected return volatility (Hull 2016).

To evaluate bids using this approach, several steps mirroring financial option pricing techniques are involved:

- Defining Parameters:
  - Underlying Asset: This is the project being auctioned. Its 'value' could be the projected revenue or benefits derived from completing the project.

  - Strike Price: This is the amount of the bid.
  - Expiration: This is the deadline of the bid submission or auction close.
  - Volatility: This demonstrates an uncertain or risky indication of the costs and the returns involved within the project. Higher volatility suggests greater risk but also potential for reward.
- Choosing a Pricing Model:
  The most commonly used models in financial contexts are the Black-Scholes model and the binomial tree options pricing model. These require inputs such as current asset value (project value), strike price (bid amount), time until expiration (time to auction close), risk-free rate (usually some yield on a government bond), and volatility (estimated variability of project outcomes) (Hull 2016).
  Using these inputs, the Black-Scholes Model is utilized to calculate the theoretical price of an option by solving differential equations that consider the random movement of the underlying asset's price, time decay, and other factors.
- Estimating Project Volatility:
  Volatility can be estimated based on historical data of similar projects or expert forecasts. This factor has very high significance because it denotes the risk associated with the cash flow that the project is going to generate shortly.
- Risk Management:
  In this way, bidders could consider the option by employing traditional risk management techniques used in finance, such as hedging strategies or portfolio diversification principles.

The Option Pricing Approach is especially applicable to industries such as construction and oil and gas exploration, where they have been prone to cost overruns, project delays, or other risks that impact project viability. Through this approach, companies would be better placed to know the quantum of bidding given the risks and possible returns.

This also allows companies to change their bidding strategies based on the market conditions dynamically. For example, in a very volatile market, option values are high, so companies have reasons to bid higher than in stable conditions. The basic underlying concept of the option pricing approach to bid price is based on the fact that a bid can be considered as a kind of an option—a right but not an obligation to complete the project at a specific price. This analogy draws heavily on the financial derivatives theory, specifically the methods used to price options. In simple bidding terms, the "strike price" would be the bid amount when the auction ends, and the winning bidder is determined after some "expiration" (Hull 2016).

Like pricing an option on the underlying asset's volatility, a bid can be valued based on the volatility in market conditions and costs of the project. This approach is based on the time value of money, the risk-free rate, and the variability of expected returns from the project; hence, giving an all-rounded framework within which to examine the potential profitability and risks carried by the bid.

What operationally occurs in the Option Pricing Approach can be represented in steps. First, the bidder must define the parameters of the bid as an 'Option,' clearly stating what the underlying 'Asset' is (project), 'Strike Price' (bid amount), and 'Expiration date' (auction close). Finally, the bidder uses the models from financial theory to estimate "the volatility" of project returns, which could include fluctuations in the cost of materials, availability of labor, changes in regulation, or economic conditions.

Thus, it is especially applicable to industries where the projects are enormous, have very long durations, and contain high levels of uncertainty—examples of such sectors are construction, oil and gas exploration, and large-scale manufacturing. It analytically assesses the risk and potential return on a project in these sectors through the option pricing framework and, hence, allows the strategic advantage that companies are able to make better-judged decisions on when and how much to bid.

The strength of the Option Pricing Approach is the fact that it provides a very disciplined, theoretically clear framework within which to analyze the risk and return and, hence, can allow subtler, more financially appropriate strategies. It will further help the bidders make sure that they do not end up overpaying for projects where the expected return does not justify, to them, the risk involved in those projects by quantifying the risk associated with a bid and comparing it against the potential return.

But the approach has its own shortfalls related to complexity in models applied and the quality of data required. A problem may arise in using option-pricing models. Some of the required parameters—for instance, volatility and the risk-free rate—are difficult to estimate accurately, especially within industries or situations where data may be rare or exhibit high variability. Furthermore, the mathematical and financial skills required to implement such models may be a preservation of small firms or persons of less knowledge.

$$C(S,t) = S_t N(d_1) - K e^{\{-r(T-t)\}} N(d_2)$$

where:

- $d_1$ and $d_2$ are given by:
- $d_1 = \frac{\left\{log\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) * (T-t)\right\}}{\{\sigma * \sqrt{T-t}\}}$
- $d_2 = d_1 - \sigma * \sqrt{T - t}$ \]
- $C(S,t)$ is the price of the call option at time $t$
- $S_t$ is the current price of the underlying asset
- $K$ is the strike price of the option
- $T$ is the time to expiration
- $r$ is the risk-free interest rate
- $N(\cdot)$ is the cumulative distribution function of the standard normal distribution
- $\sigma$ is the volatility of the underlying asset.

### *2.2.7 Arithmetic Mean Model*

In auction cases, there is a strong justification for using arithmetic means in order to estimate the probability of winning, more so in cases where only a sample of the data is available and not the entire population. This method only considers the averages of the results from past auctions, sums the relevant data, and then divides by the number of observations.

The method has been widely spread because of its simplicity and easy interpretation of the results (Cox 1985). The averages are determined mathematically, on the assumption that each value in the data set contributes equally to the result. This is very useful, assuming the data, like past bids or their outcomes, are of equal magnitude and there is no significant skewness in the distribution of bids. The mean arithmetic could be adopted to estimate a probability of success in future auctions: having considered the historical success rate, the same could be deducted for future bids (Milgrom, Auctions and bidding: A primer 1989). Use of a sample of data for statistical calculations, therefore, means that the computed average might not tell of the central tendency well within the entire population of interest. This limitation may be applicable to auctions with time-variant bidding strategies and behaviors of the dynamics at work or that differ widely from one auction setting to another. The sample selection and the analysis of the sample need to be careful to assure that they are representative of the bidding trend (Klemperer 1999).

An adjustment might be necessary if the distribution of the past winning bids is highly skewed or contains some outliers. In such instances, it may be more appropriate to use a median or trimmed mean in order to derive a better description of central tendency that is going to offer better predictive probability to win the auction (Friedman 1956). Additional variables, including the number of auction participants, the format of the auction, and general economic conditions, would further help improve the predictive accuracy. This could, in fact, significantly affect the results of the bid, and it should be considered in combination with the arithmetic average of the previous wins to have a more accurate estimate of the probability of success (Riley 1989).

$$P_b = \frac{\sum_{\substack{i=0 \\ i \neq b}}^{N} P_{b,i}}{N}$$

Where $P_b$ is the probability of a certain bidder to be the winning one overall. $P_{b,i}$is the probability that bidder $b$ win on bidder $i$. N is the number of bidders.

### *2.2.8 Geometric Mean Model*

The use of the geometric mean in the context of auctions for the estimation of the win-probability seems to be an interesting alternative to the arithmetic mean, in all cases more particularly where the data are log-normally distributed or the percentage changes are more important than absolute differences. The geometric mean of a set of data is the product of all the values, then raised to the nth root, where n is the total number of values. This method is especially effective when dealing with data that varies on logarithmic scales, as is the case with bids in an auction (Friedman 1956).

On one hand, the geometric mean is highly influenced by all the positive values, while, on the other hand, the arithmetic mean tends to be less sensitive to outliers. This is especially desirable if the extreme bids, whether of high or low value, do not reflect general behavior among bidders. The use of the geometric mean, thus, can provide a more realistic estimate of the 'central tendency' in cases when the bids are distributed log-normally and, hence, provide a much more solid basis for predicting the probabilities of winning in future auctions (Cox 1985).

However, there comes some kind of complication, mainly with the choice for using the geometric mean. All values must be positive, since log cannot be computed in this type of averaging. This, in turn, would reduce the usability of the geometric mean in those cases where some offers had contained negative or null values. However, as is the case with an arithmetic mean, basing the estimate on a sample of data somewhat implies that the estimate may not fully represent the center tendency of the whole population of bids (Klemperer 1999). The geometric mean would be less sensitive than the arithmetic mean to such tendencies, but it would still be necessary to correct if the distribution of past winning bids was skewed or if there were important outliers.

For example, data may be log-transformed before applying the geometric mean to the contribution of extremes (Milgrom, Auction Theory 1989). Other parameters, such as the number of bidders, the type of auction, and economic circumstances, could further improve the prediction of the likelihood of winning when using the geometric mean. These factors greatly affect the bidding outcome and, therefore, the refinement of success probability estimates needs to be further considered (Riley 1989).

"Finally, the results suffer not too much from the log-normal assumption because geometric averaging is an approach giving valid and scientifically based measures to estimate the odds of winning in auction contexts, in particular, for data that are positive and log-normally distributed.". However, like all statistical methods, geometric averaging must be used in a careful and complete manner with a specification of market conditions and auction characteristics so as to be assured of accurate and reliable predictive results.

$$P_b = \sqrt[N]{\prod_{\substack{i=0 \\ i \neq b}}^{N} P_{b,i}}$$

Where $P_b$ is the probability of a certain bidder to be the winning one overall. $P_{b,i}$is the probability that bidder $b$ win on bidder $i$. N is the number of bidders.

### 2.2.9 Harmonic Mean Model

The estimation of the probability of winning through the use of the harmonic mean offers a view that is surely distinguished, especially when it wants to put side to side with data analysis characterized by great variability. The harmonic mean is actually calculated as follows: reciprocate each value within a set of data, find the average, and then find the harmonic mean by reciprocating the value back. The method works in handling datasets, whereby small values affect the mean more, for example, in bids; hence, the lowest bid counts more towards the average bid or winning (Cassady 1967).

The biggest characteristic of the harmonic mean is that it gives the least values much weight in a data set. This, to this extent, is very useful, especially during auctions, where the smallest of bids usually have higher chances of winning. The use of the harmonic mean, therefore, gives a realistic estimate of the trend of low bids and hence provides a realistic basis for the estimator to predict the probability of winning from future auctions of similar characteristics (Cox 1985).

On the other hand, some challenges were revealed when using the harmonic mean. It is only applicable to positive data, considering there is no definition for the calculation of reciprocals with negative or null values. This can only restrict its applicability to the contexts where all the bids are assuredly greater than zero. The other disadvantage is that the harmonic mean is quite sensitive to very low values, which can distort the mean significantly if not handled or filtered with care (Milgrom, Auction Theory 1989).

If the past winning bid distribution either has an overwhelming number of low values or is heavily skewed, the harmonic mean can put too much emphasis on those cases purely by its nature. Thus, the need for such corrective measures to weed out these outliers prior to the application of the harmonic mean becomes a necessity, especially in the light of the presence of bids that are unreflective of the typical behavior of the bidders.

With the inclusion of other variables, such as the total number of bids and the specific auction dynamics in general, its accuracy of measuring the estimation of the probability to win is improved. These may have far-reaching effects on the bidding results and thus should be taken into account when refining the estimates of the probability of success (Klemperer 1999).

It may, therefore, be concluded that the harmonic mean, taken into account along with other measures serving to express the mean value of bids and prices, may indeed throw some useful light on analyzing the dynamics of auctions, and more specifically, the impact of the lowest bids. It is subjected to being used like any other statistical tool: with care and complemented by robust analysis that allows an understanding of the conditions of the market and the auction.

$$P_b = \frac{N}{\sum_{\substack{i=0 \\ i \neq b}}^{N} \frac{1}{P_{b,i}}}$$

Where $P_b$ is the probability of a certain bidder to be the winning one overall. $P_{b,i}$is the probability that bidder $b$ win on bidder $i$. N is the number of bidders.

# 3 Research Methodology

This research, therefore, is out to develop a simulation of auctions to test how different approaches of auction analytics behave in an attempt to understand how the different approaches can impact the outcome of an auction under different scenarios. Some key assumptions should be introduced in order to represent the analysis manageable yet representative.

## 3.1 Key assumptions

- Number of auctions per sample:
  One of the first strong assumptions applied to the number of auctions per sample is that of 100. The decision that 100 represents 0.3% of a simulated population containing 30,000 auctions was made. This was done in an effort to achieve a balance in the case of reasonably large sample size, which would hence guarantee statistical validity but yet not be overly large to overburden this processor simulator.
- Constant participation rate:
  The other key assumption was on the participation rate of each bidder, which was assumed to be constant (equal to 1) for all bidders. This actually goes a long way in trying to simplify the auction model, for it does away with the necessity to simulate individual variations in participation rate among different bidders. This, in turn, would make it possible for us to focus more on the analysis of bidding strategies and their likely effects on auction dynamics.
- Markup range:
  With reference to markup, the analysis randomize between 80% and 100% of the auction starting price. The reasoning has been that, on average, in a descending auction, a mark-up of less than 70% of the starting value could put the bidder at an economic disadvantage and be pure loss. Therefore, has been explored an upper markup range that, in theory, would allow the bidders to keep above the breakeven point and, hence, they are willing to use a more aggressive or conservative bidding strategy based on the auction environment.

In this exhaustive study on auction dynamics, has been decided to expand and deepen the analysis by changing some key parameters directly influencing the interaction of the participants and the outcome of auctions. The aim of this stage of the study was to find out how, in general, the level of uncertainty and intensity of competition could contribute to affecting the bidding strategies and auction outcomes. In the detailed list of variables below, one has a thorough discussion that gives one a better understanding of the underlying mechanisms of auctions and also offers practical advice to participants on how to navigate such complex market scenarios. Here, there is a list of detailed variable parameters.

- Number of participants per auction:
  The quantity and type of participants in the auction is, therefore, a function of the type and size of the auction involved. In this sense, a larger number of participants implies an increased level of competitiveness in an auction, therefore, reducing the degree to which auction prices might yield profit margins while improving the allocation of resources for their fairness and efficiency.
  Simulate the auctions, where the number of participants would vary from a minimum of 2 to a maximum of 20. Such variations will allow us to take into account competitive density effects not only on bidding strategies but also on the dynamics of the final price.
- Variability of Bids:
  The variability of the bids is a good indicator of the subjective value uncertainty for the participants in the auctioned item. The larger the variability, the greater the range of rational strategies the participants are likely to be able to consider. The standard deviations of the bids were categorized into three levels.
  - Low Variability:
    The bids are presented to each other, assuming that the perception of the value of the object is similar, hence having high predictability in the outcome of the auction.

- Moderate variability:
  Bids medium difference reflecting some variety in value perception or some difference in risk strategies.
- High variability:
  In this case, very different bids are placed, which indicates high uncertainty, or, as an alternative explanation, strategic divergence of the bidders.

- Distributions of Bids:
  This distribution choice of bids matters when the implemented distribution fits very well with the reality of bidding strategies. In some sense, certain distributions could be more representative of some auction contexts: from highly competitive and predictable ones to very speculative and uncertain ones. Implementation: The bids distribution was defined in two ways:
  - Normal: Suitable for scenarios in which object value evaluations tend to be centered around a mean, with variations following a symmetrical distribution.
  - Log-normal: represents a case where there are great asymmetries in value judgment; for example, if some of the participants are willing to bid much higher than others.
- Exclusion of some methods: for sake of analysis not all the model has been used. The methods presented in the literature review Swanson's Stochastic Dynamic Programming Model and Martin Skitmore's multivariate model have not been taken into consideration due to their computation power requirements and their intrinsic difficulties.

## 3.2 Compute the probability of winning.

The bidder's probability of winning is computed by the different analytical methods. This is a very critical step in finding out the efficiency of different bidding strategies under several auction conditions.
For this, the analysis required four separate matrices that have specific functionalities during the calculation of probabilities.

### *3.2.1 Sample Markup Matrix:*

This matrix is the first originated matrix that allow us to generate all the others matrix and is the matrix that for each cell contain the percentage of the offer compared to the initial value for each bidder (column) and for each auction (row). With this matrix the Black-Scholes method is directly computed via VBA function.

### *3.2.2 Odds Matrix:*

This matrix is constructed based on past performance. In this case, the result where the auction participant won against others is recorded and relationally enumerated. For each cell of the matrix is represented as the ratio between the number of wins for the bidder in the column and each other bidder is shown in a row.

$$P = \frac{N(wins)b(column)}{N(wins)b(row)}$$

Having formed the matrix with the odds, they proceeded with the calculation of the effectiveness coefficient using the formula that the Gates method proposed. This involved dividing 1 by the sum of the values in the column for each bidder to transform the odds into the probability of winning.

### *3.2.3 Probability of Winning Matrix:*

This matrix was constructed to reflect the direct comparison of each participant's bid. The ratio between the number of times bidder 1 (column) wins on the sum of the times a bidder1 (column) and bidder 2 (row).

$$P = \frac{N(wins)b(column)}{N(wins)b(column) * N(wins)b(row)}$$

That should make it much more possible to visualize directly how one participant's bid compares with another in relative terms.

Then, to get the final winning probability for all bidders, the Friedman model the arithmetic, geometric, and harmonic average of the winning probabilities of all column elements was calculated in the matrix, respectively, leaving out those values on the diagonal line where the bidder compares itself. This allows carrying out a balanced and comparative analysis of the chance to win for each participant, considering the different dimensions of competitiveness and relative bids.

These values have been excluded, which relate to the bidders' comparison with themselves diagonally, in order that the distortions between the averages calculation of every bidder being compared to him do not occur. Then, the formulas in finding the arithmetic, geometric, and harmonic means are computed, which indicate different angles of the probability of success faced by the bidders and hence bring out different aspects of the competitiveness of the bidder.

### *3.2.4 Probability(<Min) Matrix:*

For each auction the matrix computed the probability of each bidder to win that precise auction via normal/lognormal distribution using as mean the min offer of that auction and as standard dev the std dev of the auction. Doing the mean of all the probabilities of win of each bidder for all the auction you get to the Hanssmann and Rivett coefficient.

## 3.3 Mean Absolute Error

Having calculated the winning probabilities of each bidder with a few different methodologies (Gates Model, Arithmetic Mean, Geometric Mean, Harmonic Mean, Friedman Method, Hanssmann and Rivett Method and the Option Pricing Approach), the accuracy of these estimates has been assessed based on the MAE (Mean Absolute Error) for each method and each bidder in the next section.

### *3.3.1 Calculating the absolute error (AE):*

The Absolute Error (AE) for each bidder is estimated as the mean absolute deviation between the estimated winning probability by each analytical method and the actual winning probability of that bidder. The probability of winning is actually the amount or rate of times the bidder has won regarding the sum of the total number of auctions that he has participated in. That is, the favorable cases compared to the possible cases for the entire population of auctions analyzed.
For each bidder and for each method of evaluation, the AE is calculated as follows:

$$|Method\ probability - Real\ probability|$$

### *3.3.2 Calculation of the Mean Absolute Error (MAE):*

Once the AEs for each bidder and for each method have been calculated, it now becomes necessary to aggregate them, in order to finally calculate the MAE for each method. The MAE gives a rough measure of the precision achieved by each of the estimation methods in relation to observed reality. It means it furnishes an idea of the average error committed in predicting the winning probability.

### *3.3.3 The meaning of all the absolute error for each method:*

A smaller MAE value predicts that, in general, this method of estimation will be more precise and closer to the observed reality, suggesting greater reliability of this method in terms of predicting winning probabilities. High values of MAE, on the other hand, show higher differences between the predictions and actuals, and therefore a much higher potential for improvement in the method of estimation or exploring other prediction models.

### *3.3.4 Comparative Evaluation*

After setting the initial conditions and methods of probability calculation for the win and the corresponding mistakes. The simulation phase is important in the research since it allows us to check by practice the effectiveness of the considered bidding strategies and validate the developed analytical models.

In order to run the simulation in a very optimum manner, the use of macros into the statistical software has allowed that the integrity of the results would not be compromised, we. This is critical in the automation of processes that would without a doubt be both cumbersome and intensive, especially when based on large datasets, which would require the execution of some very repetitive calculations at the same level of detail.

The use of macros reduces the time and efforts substantially spent on carrying out various complex calculations since they are automated. For example, raw data transformation, probability calculations, and errors using mathematical formulas, further processing of matrices or result tables—these are the codes set up at once and run later in an automated way over large datasets just through a simple command.

This removes any possibilities of human errors that may emanate from manual entries of data or repeating calculations.

The use of macros becomes particularly advantageous in the handling of very large volumes of data. Within this study, the use of macros is particularly useful for the organization and processing in the proper way of a huge volume of information: every single simulation performed can reach several thousand intermediate data points. This includes all filtering, sorting, and aggregation of data with special parameters in a way of generalized analysis and meaningful insight of the entire data.

Another great advantage of using macros is that they can carry out numerous simulations fast and in a methodical manner. In the current study, where each of the hundreds of simulation scenarios had to be repeated with different parameter sets, the use of macros helped us reproduce the entire process of simulation in a consistent and precise manner. Each simulation has been carried out in exactly the same way, allowing a similar result to be obtained and a strong, reliable database giving support for the final analysis. The optimized macros execute batch operations, ensuring a factor decrease in processing time. This is quite useful for project development stages where waiting times for data processing might, among other factors, be considered to put a tremendous drag on the general pace of research activity. Macros allow programming long, tedious operations to run in the dead of the night or other low-activity hours, hence helping rationalize the workflow.

So, the use of these macros in the statistical software would be a highly strategic one for the success of the research project. This would offer more efficiency, and, at the same time, it would reduce the chances of errors. It would further empower us to deal with large, complex data sets and make us capable of confidently performing huge analysis using it. All these enable us to have precise results in the simulations and analyses, and in turn ensure there is a standardization and replicable process. In the simulating phase with variable parameters of the study, 100 independent simulations has been executed for every configuration of the variable parameters. It assesses the efficiency of these processes and high-quality analytical models within varying scenarios.

Initial conditions were specifically randomized for every one of the 100 simulations run. This was in order that every run would be essentially an independent and unique test, void of possible biases that might be brought about by having fixed or predefined initial conditions. The randomization mechanism will ensure, through the composite nature of this database, that it is indeed diverse and from a wide variety of potential operational realities within the auctions.

Indeed, the robustness test of the models through these many simulations is key to making the model's prediction reliable against many, among others, the intrinsic uncertainty of the data and the market situations. This could be very important in dynamic and unpredictable environments, like auctions, where small variations in parameters would give rise to significant impacts on final outcomes.

Repetition of the simulation tests will be done, and not just robustness, but this process will also have the benefit of iteration and hence refine this models. In this process, thanks to the results of each series of simulations carried out, from the obtained ones, it will be possible to find patterns or anomalies, make parameter adjustments, and optimize techniques. This process iterates, and thus a constant rinse and repeat is given to the analysis, refining the analytical approaches to improve precision and reliability.

Each series of 100 simulation runs yields large volumes of data that are analyzed in detail and then documented. This detailed documentation helps better understand the behavior of the model under different configurations and lays a solid foundation for future research and developments. From the results, is possible to trace correlations and identify best practices for bidding strategies that lead to empirically valid recommendations for optimal bidding strategies.

Once the 100 simulations for each variable parameter configuration were completed, the next step was to calculate the Mean of the Mean Absolute Errors (MMAE) for each estimation method used. This part of the process is crucial for determining the accuracy of each method under various simulation conditions and identifying areas where the models may need further improvement.

Starting by aggregating all the MAEs resulting from each simulation for a particular configuration. Subsequently, summing all these MAE values to obtain an overall total. This total was then divided by the number of simulations, which in this case is 100, to obtain the average. This average value represents a reliable estimate of the error which is possible to expect from the estimation method when applied in conditions similar to those simulated. This process ensures the reliability of each method and refine the models to improve their accuracy and applicability in real auction settings.

The MMAEs is a special value since it gives a measure for the accuracy of the method used for the estimations. A smaller average value would thus suggest that the method would generally be more accurate; for example, it could more accurately predict likely auction results. Conversely, if this average value tends to be higher, it may signal an increasing tendency of this method to produce less accurate estimates, thus acting as one of the triggers for review and possible correction of the approach used.

This MMAEs calculation provides a benchmark for the accuracy of each method and allows a direct comparison of different estimation methods. It is possible to hence establish that the most accurate prediction techniques are in establishing particular concrete auction scenarios and provide a sound base on which recommendations for an appropriate choice of method can be given a particular need or market condition.

# 4 Results

For all the results, the evaluation methods involved capturing specific values from each data point. This systematic approach ensured that the key metrics were consistently gathered and analyzed to assess each outcome effectively.

The variables are:

- Bidders:
  - 1-20
- Variability:
  - Low
  - Medium
  - High
- Type of statistical distribution:
  - Normal
  - Lognormal
- The result data points are:
  - AVG MMS: Average Means of the Means of the Samples
  - AVG MSS: Average Means of the Standard Deviations of the Samples
  - AVG SMS: Average Standard Deviations of the Means of the Samples
  - AVG SSS: Average Standard Deviations of the Standard Deviations of the Samples
  - MMAE Gates: Mean of the Mean Absolut Errors of Gates Method
  - MMAE Arit: Mean of the Mean Absolut Errors of Arithmetic Mean Method
  - MMAE Geom: Mean of the Mean Absolut Errors of Geometric Mean Method
  - MMAE Harma: Mean of the Mean Absolut Errors of Harmonic Mean Method
  - MMAE H&R: Mean of the Mean Absolut Errors of the Hanssmann and Rivett Model
  - MMAE BS: Mean of the Mean Absolut Errors of the Option Pricing Approach

- SMAE Gates: Standard Deviation of the Mean Absolut Errors of Gates Method
- SMAE Arit: Standard Deviation of the Mean Absolut Errors of Arithmetic Mean Method
- SMAE Geom: Standard Deviation of the Mean Absolut Errors of Geometric Mean Method
- SMAE Harma: Standard Deviation of the Mean Absolut Errors of Harmonic Mean Method
- SMAE H&R: Standard Deviation of the Mean Absolut Errors of the Hanssmann and Rivett Model
- SMAE BS: Standard Deviation of the Mean Absolut Errors of the Option Pricing Approach

## 4.1 Little result changes

Before presenting all the results modification must be done. Due to what is visible in Appendix 1 all the results connected with the Friedman method has been removed, because these results have a low level of significance, and they risk making the entire research less reliable.

## 4.2 2 Bidders

Based on Appendix 2 is possible to affirm that:

- Gates Method, Arithmetic Mean, Geometric Mean and Harmonic Mean:
  The MMAE values of these four methods are equal under all conditions. It tends to indicate that its prediction accuracy slightly varies according to the tested conditions. All exhibit averages of MMAE percentages at around 3.94% to 4.16% within a normally distributed setting, indicating a fair level of accuracy despite the environment being subject to change.
- Hanssmann and Rivett Model:
  It shows the MMAE values ranging from 2.40% to 6.40%: It quite has a way of showing the distribution could be better or worse than the other under market

variability and is most pronounced under low variability in lognormal distributions (6.40%).

- Black-Scholes Model:
  It shows the highest variation in MMAE values from 2.41% to 28.27%. This model is most sensitive to changes in market conditions and works at its best under high variability scenarios in lognormal distributions (2.47%), but worst under low variability scenarios in normal distributions (28.27%).

## 4.3 3 Bidders

Based on Appendix 3 is possible to affirm that:

- Gates Method and Arithmetic Mean:
  Show wide variability of MMAE values, though the Gates method shows a bit more variability with the lognormal distribution. The arithmetic mean method looks a bit better for the lognormal conditions, particularly at medium and high variability.
- Geometric Mean and Harmonic Mean:
  Show results very close in similar performances with scenarios of slightly better accuracy in the cases of lognormal distributions. Both methods show less variation in MMAE across conditions when compared with Gates.
- Hanssmann and Rivett Model:
  It performs variably but tends to yield better accuracy under Medium and High variability in both distributions. It is much better under conditions of some, which means it is more adequate for more variable markets.
- Black-Scholes Model:
  It has the highest discrepancies in MMAE, mainly in the Normal and Lognormal cases, suggesting very high sensitivity to the underlying distribution. It performs noticeably better under Lognormal conditions.

## 4.4 4 Bidders

Based on Appendix 4 is possible to affirm that:

- Gates Method and Arithmetic Mean:
  It presents less variability in the highlight of MMAE values across conditions and is thus very consistent in accurate performance. The arithmetic method works very well in both distributions with medium and high variability.
- Geometric Mean and Harmonic Mean:
  The stated approaches yield closely similar performances, only with very slight differences in favor of the lognormal distribution, especially under medium and high variability.
- Hanssmann and Rivett Model:
  This variability in the normal distribution means that it shows a broad range of MMAE values; that is, it exhibits an excellent performance for medium and high variability. This could suggest a possible fit for variable markets.
- Black-Scholes Model:
  Experiences the largest deviations in MMAE but more significantly under the normality assumption, reflecting high sensitivity to the assumptions on the underlying distribution: it performs much better under the lognormal distribution.

## 4.5 5 Bidders

Based on Appendix 5 is possible to affirm that:

- Gates Method:
  Demonstrates low values of MMAE, suggesting good accuracy; however, the tendency is to make higher errors under high variability conditions.
- Arithmetic Mean:
  The model does well under the Lognormal distribution with medium and high variability because MMAE should be low with a good accuracy.
- Geometric Mean and Harmonic Mean:
  These methods show the same trends as Arithmetic mean with MMAE being a bit higher in the case of Normal and they present good performance.
- Hanssmann and Rivett Model:
  This can be observed in the behavior of the spread of the lowest MMAE

values: very good performance in scenarios of medium and high variability but, at the cost of increased errors, in the other scenarios.

- Black-Scholes Model:
  Exposes widely varying MMAEs, very high in the cases of the Normal distributions, but considerably more acceptable under the Lognormal conditions. This leaves a clear impression that the conclusions are strongly sensitive to the type of bid distribution.

## 4.6 6 Bidders

Based on Appendix 6 is possible to affirm that:

- Gates Method:
  It shows a low MMAE for all conditions, reflecting good predictive accuracy. The values of SMAE are also low in value, consistently reflecting reliability in the predictions.
- Arithmetic Mean:
  Appears to be the most variable of the MMAE distribution; it actually performs well under medium and high variability in the lognormal distribution. The Lognormal distribution brings out the lowest SMAE value, indicating better consistency under these conditions.
- Geometric Mean and Harmonic Mean:
  All of these methods exhibit similar trends with slightly better performance on the lognormal distribution with respect to both MMAE and SMAE, indicating good accuracy and consistency.
- Hanssmann and Rivett Model:
  Shows the lowest MMAE values under Medium and High Variability in Normal distribution, suggesting the model to perform outstandingly well under these special conditions. Very low SMAE values under Normal distribution mean high consistency.
- Black-Scholes Model:
  Displays considerable variation in MMAE; it is tending to give higher errors, especially with the Normal distribution, but the performance is hugely

improved under the Lognormal condition. The performance under the Lognormal condition is significantly better, with relatively high values of SMAE, indicating more erratic predictions compared to the other models.

## 4.7 7 Bidders

Based on Appendix 7 is possible to affirm that:

- Gates Method:
  Consistently low MMAE values across all conditions, demonstrating high accuracy with minimal variation in SMAE, indicating consistent reliability.
- Arithmetic Mean:
  Shows exceptionally lower MMAE values particularly under Medium and High variability in both distributions, but with the lowest SMAE values under Medium variability, suggesting high accuracy and reliability in more variable conditions.
- Geometric Mean and Harmonic Mean:
  Similar to the Arithmetic Mean, these methods offer good accuracy with low MMAE values under Medium and High variability. SMAE values are also low, especially under medium variability, highlighting consistent performance.
- Hanssmann and Rivett Model:
  Exhibits some of the lowest MMAE values, particularly under Medium and High variability in Normal distributions, indicating excellent performance in specific conditions. SMAE values are relatively higher, reflecting some variability in model consistency.
- Black-Scholes Model:
  Demonstrates higher MMAE values under less variable conditions but improves significantly under Medium and High variability. SMAE values, although higher than other models, indicate less consistency across different testing scenarios.

## 4.8 8 Bidders

Based on Appendix 8 is possible to affirm that:

- Gates Method:
  The MMAE values are low in the four experimental conditions, which indicates that the predictions are on target. The SMAE values are also low; hence the finding is stable and reliable.
- Arithmetic Mean:
  Excels particularly under Medium and High variability in both distributions, as can be seen with the low MMAE. This implies that the model performs very well under variable conditions. The SMAE values are also low, with the Lognormal distribution taking the lead.
- Geometric and Harmonic Means:
  Works in a very similar way to the arithmetic mean and performs well under variable conditions with very good accuracy, although the MMAE and SMAE are slightly higher than the arithmetic mean, so performance is not degraded.
- H&R Model:
  This, thus indicates high accuracy under specific conditions only with variation in model consistency being quite high. Shows among the lowest MMAE values, especially under medium and high variability in the normal distribution.
- Black-Scholes Model:
  Has the highest MMAE values, especially under the case of normal conditions, but improves under the lognormal distribution. The SMAE values for these sets are larger compared to the other models, meaning they are less consistent.

## 4.9 From 9 to 20 Bidders

All these category are put together for simplicity due to the fact that the trend is the same even if the numerical result are different.

Based on Appendix 9, Appendix 10, Appendix 11, Appendix 12, Appendix 13, Appendix 14, Appendix 15, Appendix 16, Appendix 17, Appendix 18, Appendix 19, Appendix 20 is possible to affirm that:

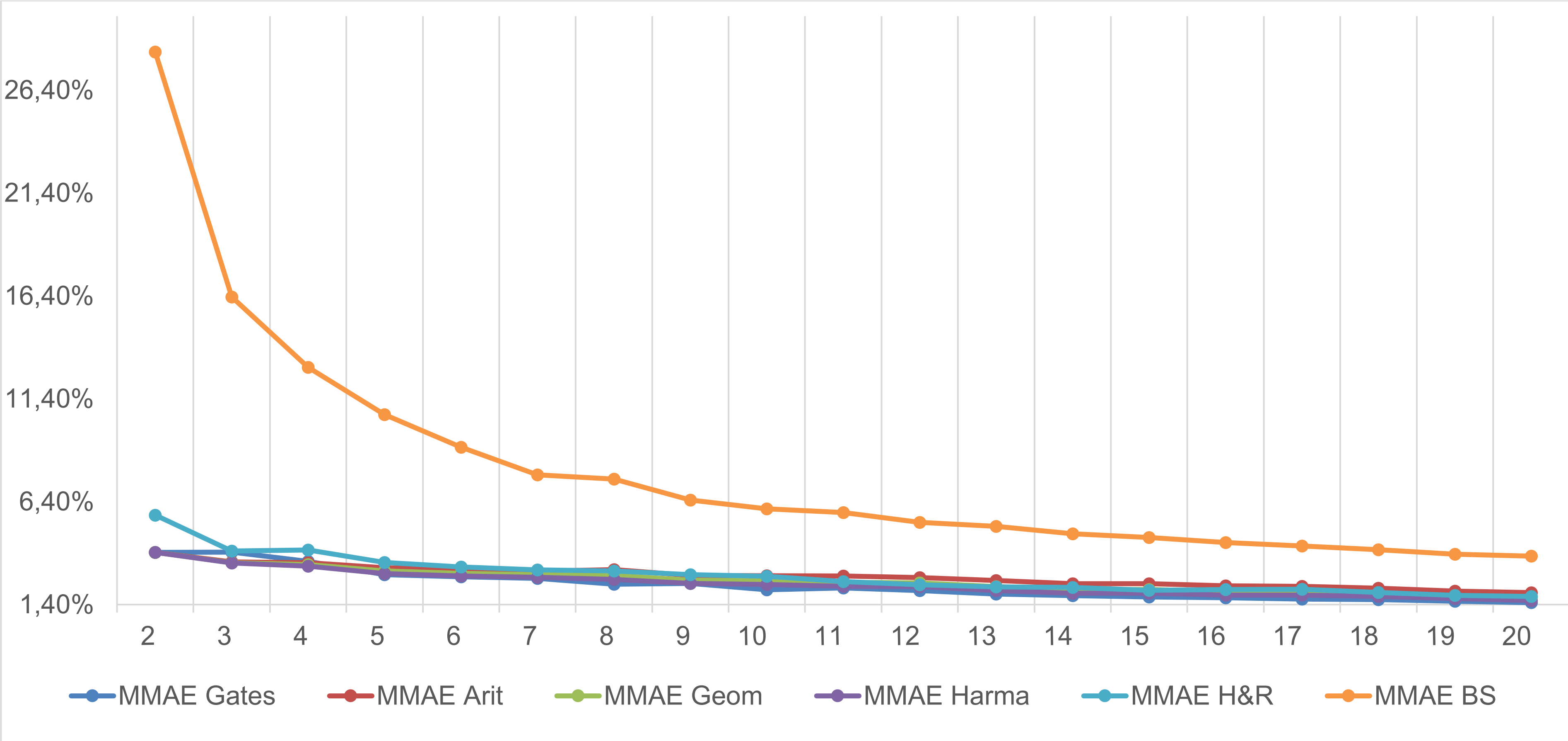
- Gates Method:
  Proves the fact of constantly low MMAE in all cases, which shows the

precision and accuracy of estimates. The SMAE is quite low as well, indicating stable and reliable performance.

- Arithmetic Mean:
  Works extremely well under Medium and High variability for both the distributions, as seen from the lowest MMAE values which implies highest accuracy. It shows very low SMAE values especially under the Medium variability. This makes it more reliable.
- Geometric Mean and Harmonic Mean:
  Similar performance to the Arithmetic Mean, with slightly higher MMAE values but still strong under Medium and High variability. Low SMAE values highlight consistent performance.
- Hanssmann and Rivett Model:
  Delivers the lowest values for MMAE under Normal distributions of variation Medium and High: for such special cases, the results are really, good. With higher SMAE, there is some variation in consistency.
- Black-Scholes Model:
  Generally provides higher MMAE values under Normal conditions but does better under Lognormal distributions. SMAE is higher than in other models, meaning less consistent in performance toward prediction.

## 4.10 Comparative evolutive analysis.

### *4.10.1 Normal & Low σ*



MMAE Gates:

It starts at the highest error rates, which reduce very fast. The Gates model has the largest reduction in error because it rapidly adapts to either new data or a changing condition with more data points. This starts off as the least accurate Gates model but makes a very great improvement later, suggesting it might be very sensitive to more data being factored in, as in the predictions really refine over time.

MMAE Arithmetic:

This is a moderate starting error, which decreases at a steady rate. The Arithmetic model shows an even, modest trend towards improvement, which would be a balanced and stable strategy for decreasing errors. It is shown not to exhibit large swings in performance, making it a solid choice for cases with steady predictive accuracy required, without large initial errors.

MMAE Geometry:

This trend is in line with the Arithmetic model but it has a gently decreasing path and an increasingly improving curve over time, albeit only slightly. A unique observation here is that the Geometric model is suggesting a performance level that is robust under varying conditions, staying near but probably giving a little more efficiency-based preference to refine its prediction relative to the Arithmetic.

MMAE Harmonic:

It starts at the same level of error rates as the Arithmetic and Geometric models and exhibits an analogous gradual decline over the same order of magnitude. It is on the level of the Arithmetic and Geometric trends, and from this perspective, it supports a well-balanced approach to accuracy. This model offers a halfway-house between the "conservatism" of the arithmetic model and the slightly more "aggressive" error-correction of the geometric model, by combining consistent performance with efficient improvement.

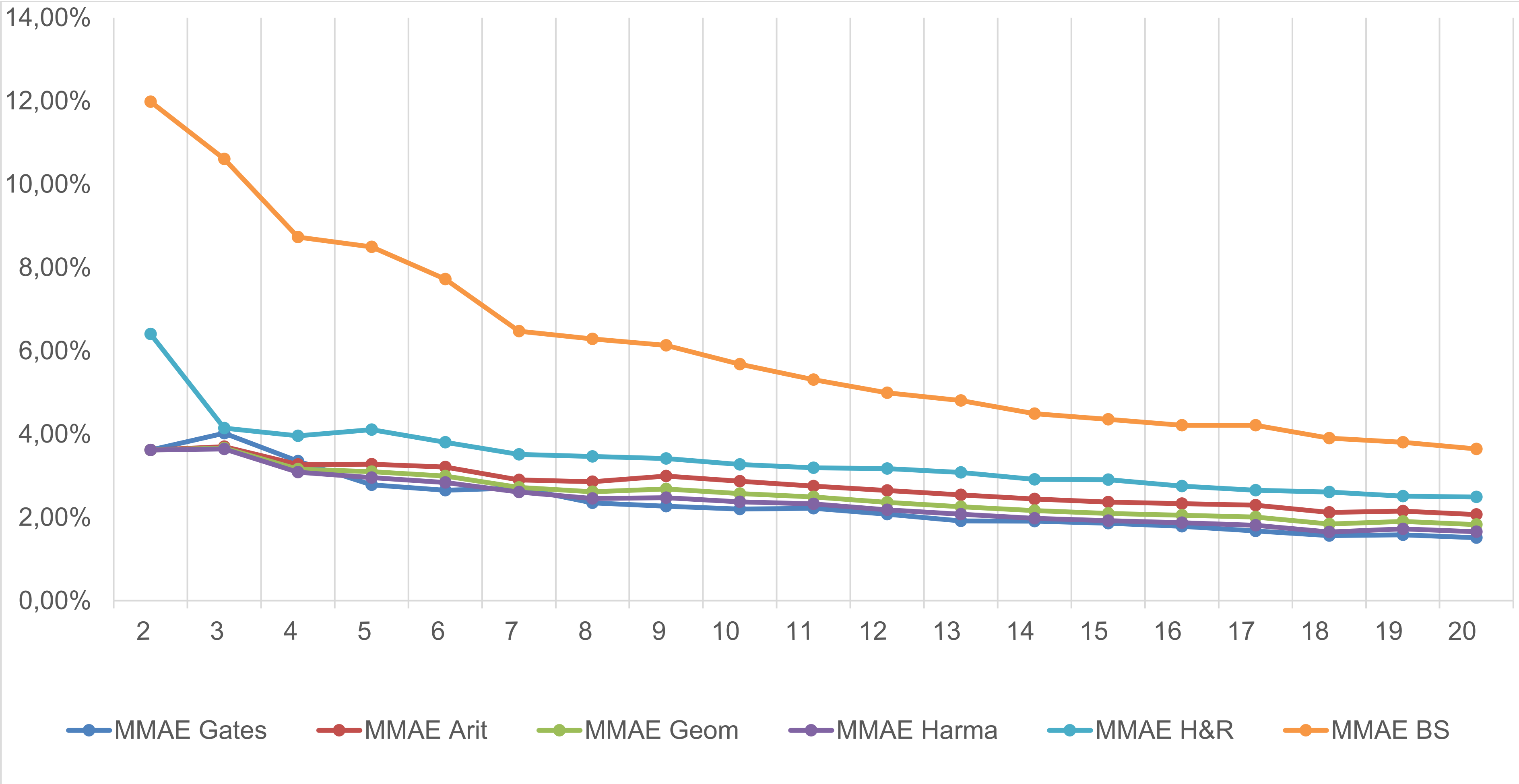
MMAE Hanssmann and Rivett:

The H&R model starts higher than most but shows a great improvement as it moves towards an approximation with the low-error models. This model can make a very big improvement from an initially larger starting error, showing that it might make use of elements that are able to compensate for little errors after an initially less accurate start, and that such a model might do well in adapting to data structures which are complex.

MMAE Black-Scholes:

This line has the greatest initial errors and drops by the highest amount, similar to the Gates model, but it does not taper down by the end as much as the others do. The trajectory of the Black-Scholes model appears to be that while it can correct itself substantially from poor initial performance, it can stabilize at a higher rate of error than the other models. This might suggest its suitability to environments that are changing rapidly, where high initial error is acceptable, but quick improvement is necessary.

### 4.10.2 Lognormal & Low σ



1 MMAE Gates:

That starts from the highest error of 26.4% when about two bidders are placed, which shows it has quite a rough start. This depicts the steepness of the fall in error rates, considering the steepness of the fall in error rates. That's where the steepness at the beginning dies away at about the 4th or 5th highest number of bidders. The rapid initial fall does seem to suggest that the complexity in the Gates model is adjusting to added data very rapidly. A reduction error rate of around 6% in the 20th bidder suggests that the complexity in the model is getting stable at this level.

2 MMAE Arithmetic:

This already peaks around 3.6%, indicating a model of relatively high accuracy at the start. The slope of the error decline remains smooth and uniform throughout the series. With that, there is always a steady decrease of the error, with the tendency to lessen the error by 2.1% constant when the 20th bidder. Thence, it points out that by doing so, the arithmetic model improves in consistency, hence offering reliability since more data is being processed.

3 MMAE Geometric:

Starting almost the same as the Arithmetic model, just off the starting mistake of the Arithmetic model, and following a track almost identical—it decreases in small and steady steps, ending ever so slightly lower than the Arithmetic one at around 1.8% by the 20th bidder. This model shows some sort of middle-ground balance between accuracy and reliability, with marginal improvements over the Arithmetic model in how it cuts down error.

4 MMAE Harmonic:

This begins just above the Arithmetic and Geometric models but below 4%, showing good early accuracy—similar to the Geometric model, which it shadows, ending at 1.7% by the 20th bidder. This places it in between the two previous, with good and consistent performances slowly evolving.

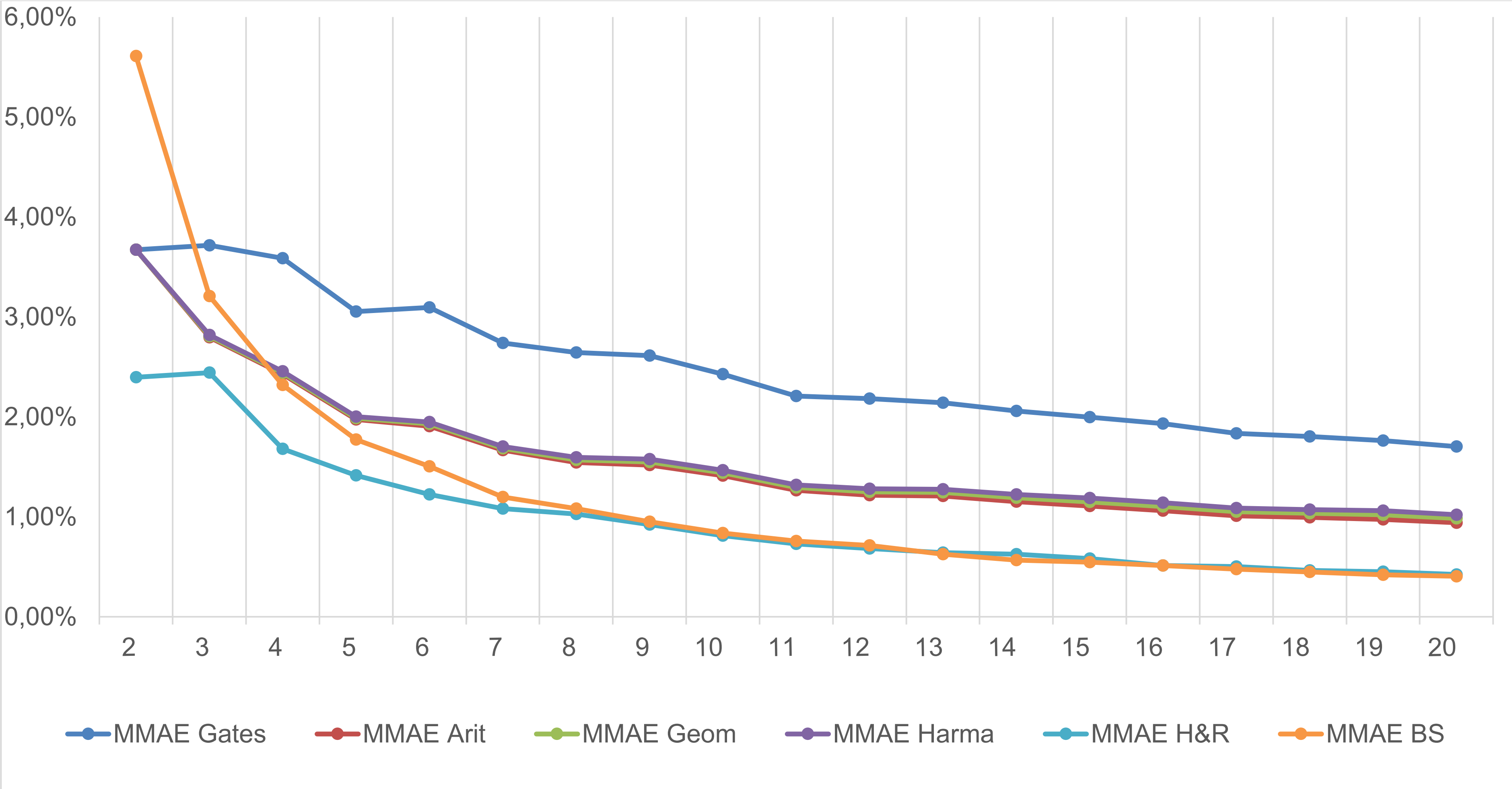
5 MMAE Hanssmann and Rivett:

Starts at about 6.4%—that means at least initial inaccuracy compared to other models except Gates. Then, dropping rapidly from 0 to 20 bidders, but tails off to level out at a fairly slow decline, reaching about 2.5% by the 20th bidder. This model shows the capability for significant improvement and stabilizes as more data is introduced.

6 MMAE Black-Scholes:

Reflects the second highest initial error, probably indicating that model fit has suffered from initial structuring or been very sensitive to the smaller numbers of data points. Like Gates' model, the fall is steep to start with but tends to level off, finishing about 3.6% by the 20th bidder; it is a pattern showing high flexibility but gets stable at a higher rate than what is observed on other models.

### *4.10.3 Normal & Medium σ*



1 MMAE Gates:

From the highest error rate at about 3.67%, it goes under 3% in the 4th bidder and then further decreases progressively from thereon. This flattens out considerably after the sharp initial drop and ends at around 1.70% by the 20th bidder. This pattern seems to suggest a very adaptive model with the initial data, quick stabilization, and keeping the reduction of error in a uniform way as more and more data points are added into the mix.

2 MMAE Arithmetic:

From a 3.67% level as the Gates model but slowing fall more slowly at first, until a 20th bidder, a 0.94%, the decline is always constant and continuous, which makes it ever so slightly less than the Gates model. This model proposes an initial accuracy that has a good chance of getting better and staying good: ideally suited to an application that may need reliability across a range of conditions.

3 MMAE Geometric:

Initiates just like the Arithmetic model and follows a closely matched pattern down. Finishing at around 0.98%, only a tad higher compared to the Arithmetic model, it shows a balanced and equable reduction of error lagging the Arithmetic model by a relatively small amount.

4 MMAE Harmonic:

It also starts around 3.67% and takes a path close to the Arithmetic and Geometric models. It exhibits a slightly steeper initial decline than the Geometric, finishing at about 0.92% by the 20th bidder. It suggests an efficient error correction, which is somewhat stronger over time than that in its closely related models.

5 MMAE Hanssmann and Rivett:

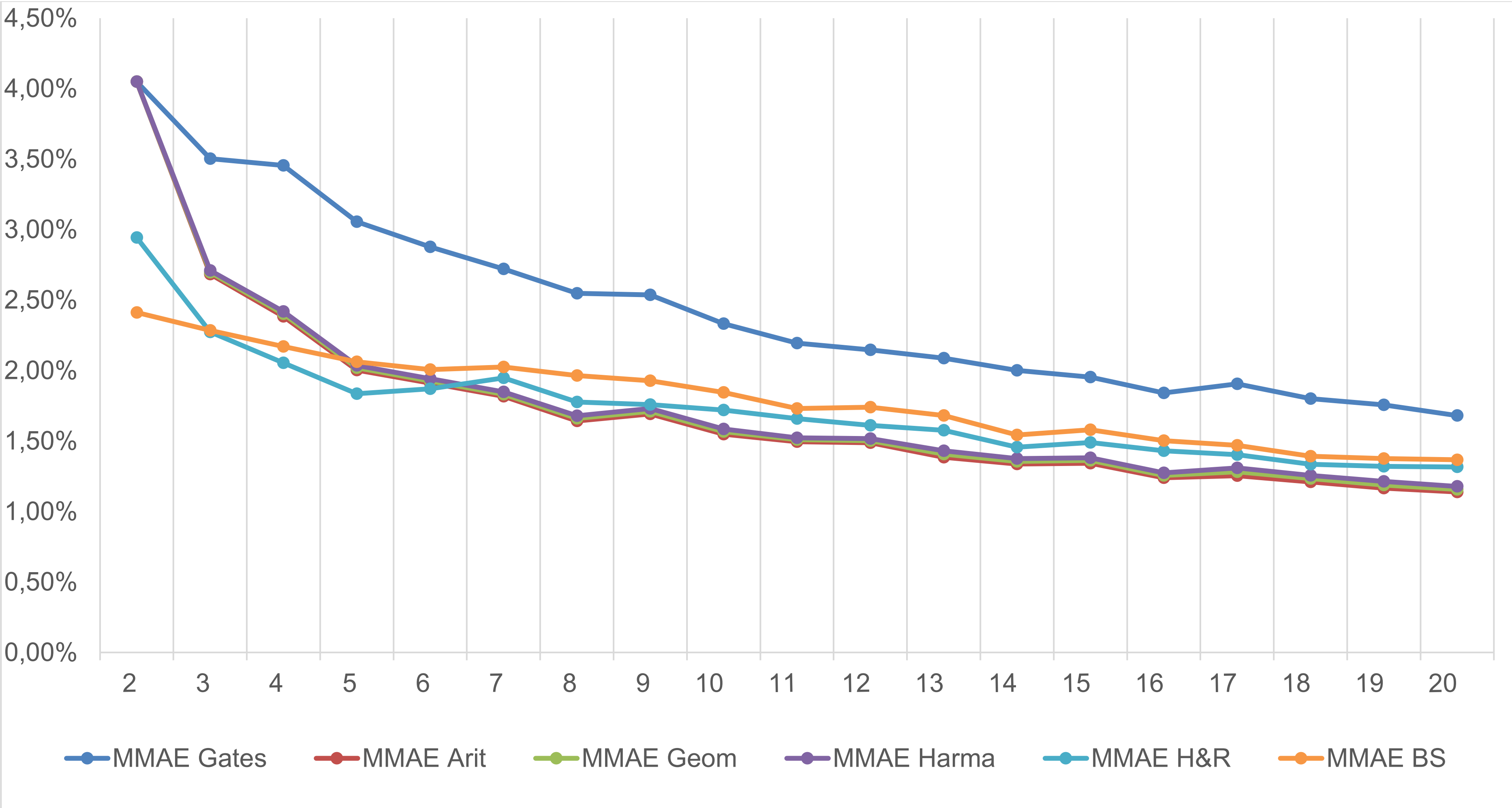
From a very low error rate of 2.40%, it clearly shows the strong initial fit of the model. It falls rapidly to stabilize under 1% by the

10th bidder and ends at 0.42% by the 20th bidder. This model seems to be showing the most rapid improvement among the models, which could emanate from the model's ability to adapt to the increase in data complexity very fast and still manage to have the lowest error rate over the other models.

6 MMAE Black-Scholes:

From the initial highest errors rate of 5.61%, which was the second highest among the initial errors after the Gates model, it drops very steeply at first and gets nearly equilibrated at a much higher level, finishing at around 1.02%. This development shows that, although the Black-Scholes model is very elastic and very quickly adapts to new data, its error level at the end of the series reached is not as low as that reached by the other models.

### *4.10.4 Lognormal & Medium σ*



1 MMAE Gates:

This model starts at the highest error rate of 4.05% and sharply decreases from the very beginning. The highest decrement flattens off slowly when compared with all other models as the auction advances towards the 20th bidder. The Gates model, therefore, would indicate a steep reduction in performance error with fast adjustment, after which stabilization in error reduction may be realized as more data became available.

2 MMAE Arithmetic:

For the Arithmetic model, it starts off at 4.05% and rapidly decreases its error rate at first but soon takes on the same shape as that seen for the Geometric and Harmonic models. Ending just below 1.5%, this is solid and steady performance, meaning that this algorithm is indicative of being able to adapt to much more data rapidly in its improvement.

3 MMAE Geometric:

The Geometric model, too, starts at around 4.05% and closely follows the trend of the Arithmetic model through all data points. Most likely, its error reduction is almost parallel to that of the Arithmetic and Harmonic models, but this does show some slight improvement over both the Harmonic and Arithmetic models, finishing just above 1%.

4 MMAE Harmonic:

Starting error at 4.05%, the Harmonic model reflects the same error reduction pathway as that of the Arithmetic and Geometric models. It gives a stable fall in the rate of errors throughout the range and ends up just higher than the Geometric model, but still below 1.5%, hence effective and balanced performance over the range of points.

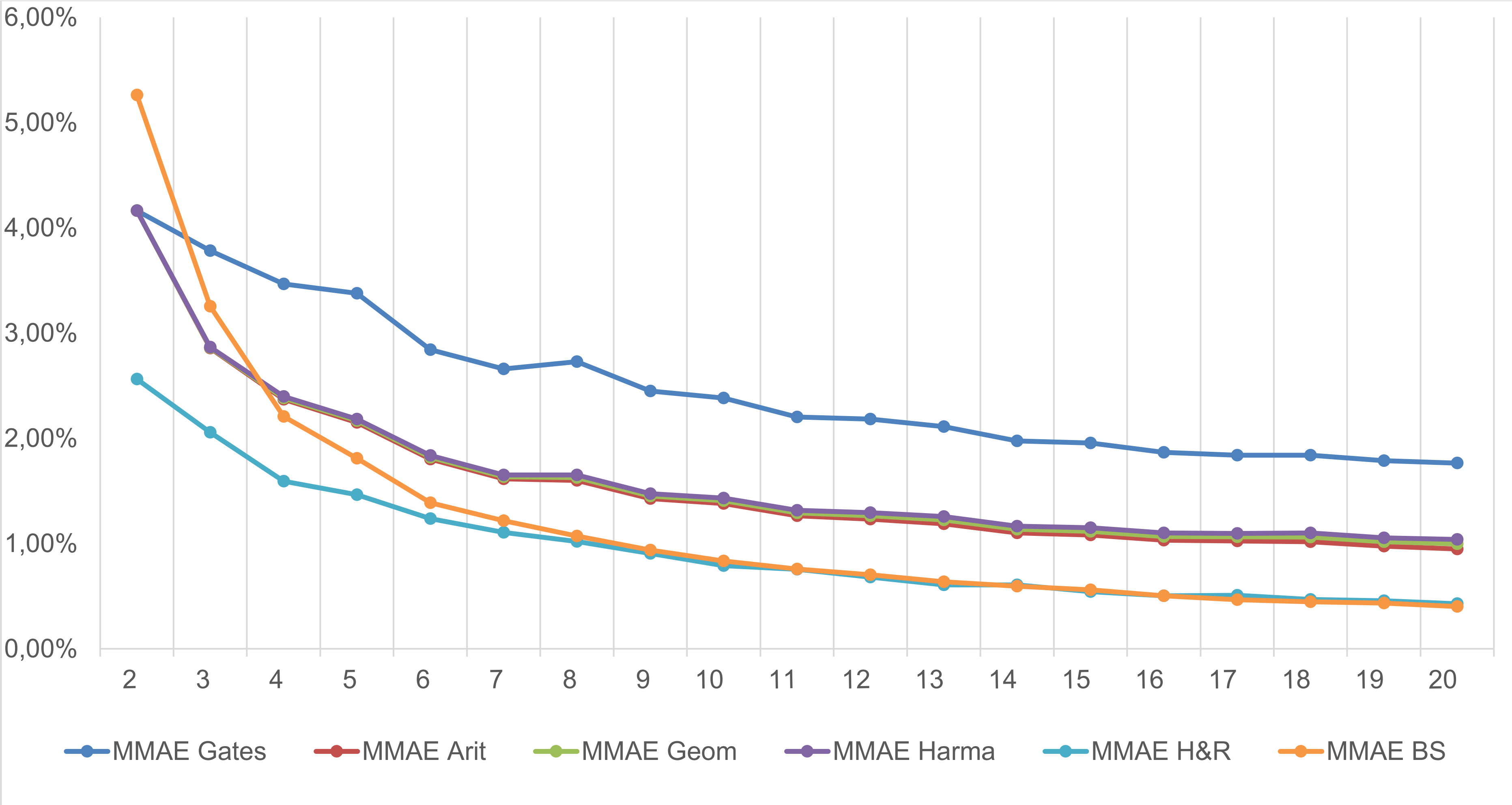
5 MMAE Hanssmann and Rivett:

Starting from 2.94%, the H&R model initially displays the lowest error rate among the first three models, but then it deviates drastically, converging with the other models around the mid-point of the series. It peaks at about 1.32%, which points out that it adopts new information relatively faster, then hits a maximum value before reaching a stable value effectively.

6 MMAE Black-Scholes:

Starting at 2.41%—just a hair under the Gates, Arithmetic, Geometric, and Harmonic models—then dropping off faster at the outset of retirement, the Black-Scholes model suggested an even quicker initial adjustment. The model will improve over time, but the rate at which it will improve will keep falling with each additional bidder until it finally converges with the H&R model around 1.18%, as shown above. Therefore, the facts display a strong competence capable of significantly reducing the errors through time.

### *4.10.5 Normal & High σ*



1 MMA Gates:

Starting from an initial error of about 4.16%, this model evidences a huge initial drop, quickly pulling down error rates as the number of bidders rises, describing a rapid adjustment to new data. The drop stabilizes post-initial drop, converzening towards a lower error rate, approximately 1.76% by the 20th bidder. This suggests high initial sensitivity to new data, quickly incorporated toward better predictive efficiency.

2 Arithmetic MMAE:

Thus, it starts at the same level as Gates but decreases more gradually without the sharp initial fall seen in the Gates model. This model always reduces the error rate, finishing slightly lower than the Gates model to 0.95%. This illustrates the improvement by the Arithmetic model, which is reliable and steady, over the increased data points towards a result in gradual and consistent error reduction.

3 MMAE Geometric:

It starts similarly to the Arithmetic model and closely follows the same path, with an incremental reduction in error of a very gradual and consistent nature to just above 1% by the 20th bidder. The performance of the Geometric model is near-equal to the Arithmetic model, showing to be a stable choice as well but a little less steep decrease than errors; therefore, this model should be chosen to reach smooth predictive accuracy improvements.

4 MMAE Harmonic:

Starts at 4.16% following the Arithmetic and Geometric models, continuing to march errors at a more consistent pace. It tends to march errors down slightly more effectively than the Geometric model through the later stages, finishing just under 1%. The Harmonic model, therefore, provides a balanced approach for error reduction, paying close alignment with other arithmetic-based models but ensuring a little better refinement at the later stages.

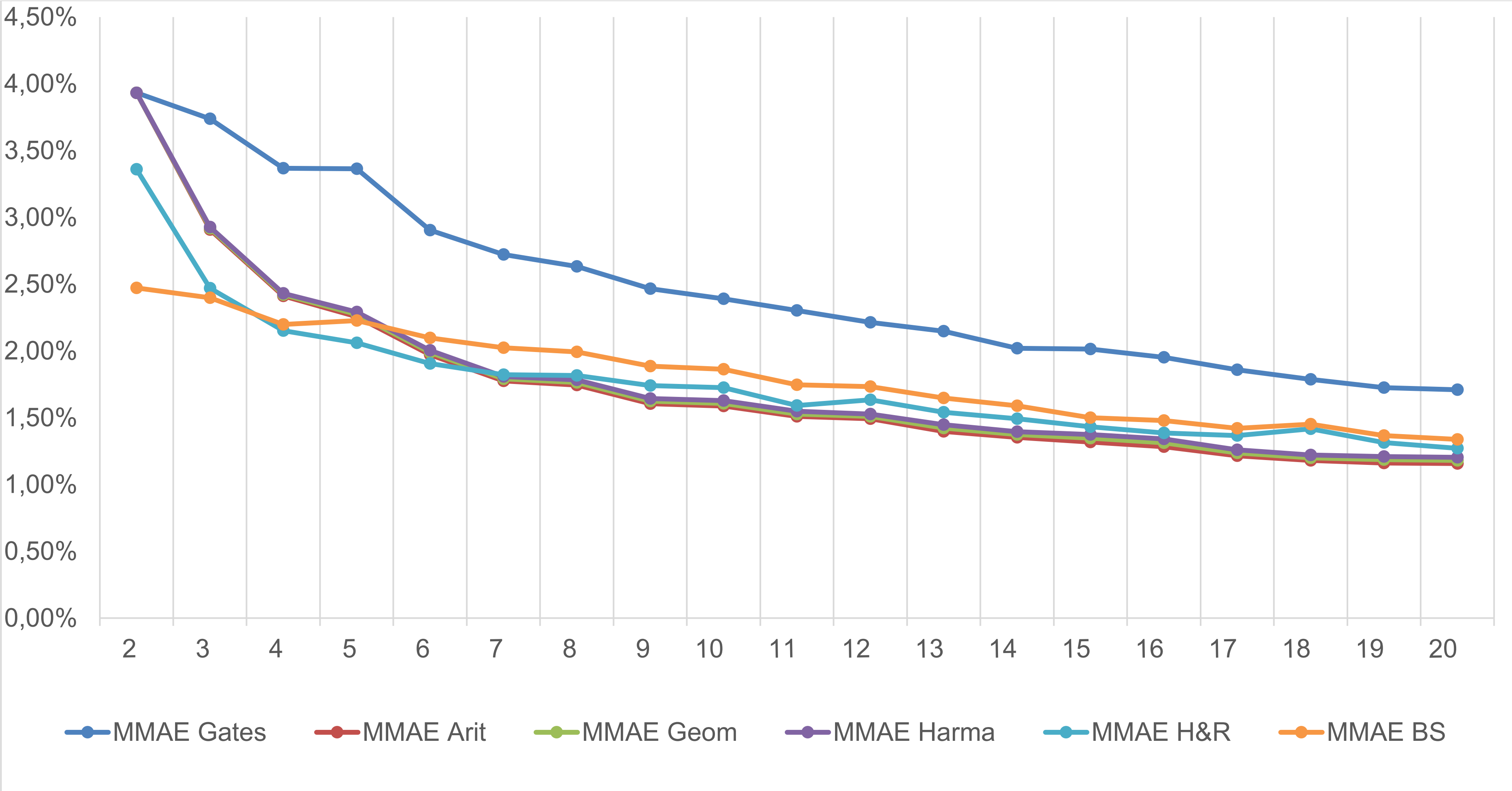
5 MMAE Hanssmann and Rivett:

Starts lower than the first three models at 2.56% and shows a relatively sharp decline initially but then flattens out as one moves through the series to end at about 0.43%. The model indicates the adaptability capability pretty well and fast in scenarios with increasing complexity but with a low error rate, therefore showing effective adaptability and precision when there are even more complex scenarios for bidding.

6 MMAE Black-Scholes:

It starts at relatively high error rates, which are the highest among other models. By the 5th bidder, it falls rapidly and eventually gets close to other models. This model shows great improvement, with an error rate ending at 0.40% by the 20th bidder. Thus, the trajectory of the Black-Scholes model is very suitable in that it does have a large capacity for the error correction process, and in a situation where initial inaccuracies should compensate for rapid improvements.

### 4.10.6 Lognormal & High σ



1 MMAE Gates:

This model entails a sharp decline of error from an initial 3.93%, rapidly reducing to under 3% by the 5th bidder, then slowly oscillating to 1.71% by the 20th bidder. The Gates model is characterized by quick early improvement; it has a slowdown toward an efficiency plateau when it reaches a lower error boundary; as such, it quickly incorporates new data to reach an efficiency plateau.

2 MMAE Arithmetic:

Starting with the Gates model, the value reads 3.93%; using the Arithmetic model to compute, it falls higher by 3.93% of the error rate in the Gates model and falls off slower, reaching at almost equal distance with the Gates model at any point. It concludes the sequence at 1.16%, showing a consistent, gradual reduction. This model provides steady error decline, fit for predictable performance applications, and has a consistent adaptation to increase data without spikes.

3 MMAE Geometric:

The Geometric model was fairly close to the Arithmetic model, starting from 3.93% and ending with 1.18%. The trajectory of the Geometric model looks a bit smoother, from point to point with less fluctuation, suggesting it may hold a much steadier decrease in error from point to point in the data, particularly suitable for continuously rising datasets.

4 MMAE Harmonic:

This also starts at 3.93% and follows very similarly to the Arithmetic and Geometric models, until finally 1.20% is reached on the 20th bidder. The reduction pattern is pretty much indistinguishable from the Geometric model, thus suggesting it also smoothest error reduction across an expanded dataset.

5 MMAE Hanssmann and Rivett:

Starting at a somewhat lower error rate of 3.36%, the H&R model evinces robust decreases, leveling off to 1.27% by the end. It showed a more aggressive decrease in the beginning than the Harmonic, Arithmetic, and Geometric ones, but it got to roughly the same final interval, which could indicate that it would be efficient for fast early adaptations followed by stable work.

6 MMAE Black-Scholes:

The Black-Scholes model starts out with the lowest initial error of 2.47% and decreases its error rapidly toward other models by the 10th bidder, ending with 1.20%. This model shows a good initial adjustment that can be quite beneficial in cases where starting accuracy is more important than the general one and then a stabilization close to the general trend of other models.

# 5 Discussion

At the end of all the result analysis is possible to conclude that there is not a unique right method for all the cases.

| Bidders | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| | Low | Medium | High | Low | Medium | High |
| 2 | Arit | H&R | H&R | Gates | BS | BS |
| 3 | Harma | H&R | H&R | Harma | H&R | BS |
| 4 | Harma | H&R | H&R | Harma | H&R | H&R |
| 5 | Gates | H&R | H&R | Gates | H&R | H&R |
| 6 | Gates | H&R | H&R | Gates | H&R | H&R |
| 7 | Gates | H&R | H&R | Harma | Arit | Arit |
| 8 | Gates | H&R | H&R | Gates | Arit | Arit |
| 9 | Harma | H&R | H&R | Gates | Arit | Arit |
| 10 | Gates | H&R | H&R | Gates | Arit | Arit |
| 11 | Gates | H&R | H&R | Gates | Arit | Arit |
| 12 | Gates | H&R | H&R | Gates | Arit | Arit |
| 13 | Gates | BS | H&R | Gates | Arit | Arit |
| 14 | Gates | BS | BS | Gates | Arit | Arit |
| 15 | Gates | BS | H&R | Gates | Arit | Arit |
| 16 | Gates | H&R | H&R | Gates | Arit | Arit |
| 17 | Gates | BS | BS | Gates | Arit | Arit |
| 18 | Gates | BS | BS | Gates | Arit | Arit |
| 19 | Gates | BS | BS | Gates | Arit | Arit |
| **20** | Gates | BS | BS | Gates | Arit | Arit |

There are some suggested methods for a certain combination of components. Watching the above table, it is observable that Gates works very well in Low condition. While for the medium and high variability condition it is suggested the Arithmetic method, The Hanssmann and Rivett model or the Black-Scholes approach.

Doing the mean of the MMAE (the MMMAE) allow us to estimate, which model works on average the best. Considering the normal distribution:

| MMMAE Gates | MMMAE Arit | MMMAE Geom | MMMAE Harm | MMMAE H&R | MMMAE BS |
|---|---|---|---|---|---|
| 2,44% | 1,98% | 1,92% | 1,89% | 1,59% | 3,49% |

The Hanssmann and Rivett model has the lower level of error overall while the Option Approach has the highest one. Considering the Lognormal distribution:

| MMMAE Gates | MMMAE Arit | MMMAE Geom | MMMAE Harm | MMMAE H&R | MMMAE BS |
|---|---|---|---|---|---|
| 2,42% | 2,08% | 2,01% | 1,98% | 2,29% | 3,23% |

The harmonic mean model has the lower level of error overall while the Option Approach has still the highest one.

Considering the merge of the two distributions:

| MMMAE Gates | MMMAE Arit | MMMAE Geom | MMMAE Harm | MMMAE H&R | MMMAE BS |
|---|---|---|---|---|---|
| 2,43% | 2,03% | 1,97% | 1,93% | 1,94% | 3,36% |

From here it is possible to say that at an average level the Harmonic mean is the method that produces less error.

As expected from the previous result the Black-Scholes approach is the one that on average produces more errors.

# 6 Conclusions

## 6.1 Impact of Results

The implications of the results of this research in sealed-bid auctions are tremendous because they largely increase the understanding of this phenomenon through advanced statistical models and simulation. In this sense, it provides more insight into which predictive models would fit better for use in auction outcome prediction under different conditions. This would be of immense help to practitioners who work in fields involving sealed-bid auctions. This will help them refine the bidding strategy and, in turn, make more practical decisions while entering real sealed-bid auctions. This may improve their winning chances. Particularly, in the light of multivariate models that provide more robust predictions across numerous diversified scenarios, it suggests that the finding in this case, that more generalized models with more independent variables and a broader set of auction characteristics, will help improve bidding accuracy.

## 6.2 Suggestions and Limitations

### *6.2.1 Proposals for Future Research:*

One of the areas that further research can be undertaken in is real-time data analytic integration into the predictive models. This is an area that can help bidders and auctioneers to alter, in real-time, their strategies in light of some live data's recent update, therefore providing a higher level of strategic depth in participation in auctions. This would have as a result the wider applicability of the results through the application of the developed models in industries that have different market conditions, so that their effectiveness and adaptability are validated.

In addition, developing new bidder behavioral models with psychological and behavioral factors embedded would help in gaining a more holistic understanding of bidder behavior, more particularly under high-stress conditions or in highly competitive markets.

### *6.2.2 Limitations of the Current Study*

It should be noted that this research has some limitations that may be addressed by future researchers. Firstly, data dependency should be considered, quality and quantity of data; if markets exist where data is either 'scanty' or not collected systematically, such data dependency can lead to a fall in the effectiveness of predictive models. Secondly, the complexity of the model should be taken into account. Some of the models demand high computational resources and highly quantitative statistical knowledge that would limit their use in small organizations or the ones not having specialized capabilities. Finally the generalizability is the last limitation. The current models have been robustly tested in a lab, but the lab can still fail to capture the full weight of messiness and unpredictability posed by real-world auctions. Future studies would be ideal to test live auction environments for the practical effect of such implementation and, if beneficial, to further refine the predictive nature of this model.

### *6.2.3 Springing Back Against Limitations*

Further work in this area could concentrate on developing some streamlined models that will need fewer resources and, thus, provide entry to small players into advanced predictive analytics. This is to consider the fact that, after the standard auction dataset framework, there would be quality improvement of the analysis and an increase in reliability for the model among several contexts.

Working together with such industry partners would also make real-world testing and model refinement based on actual auction outcomes feasible, which would both increase the practical relevance and the scientific robustness of the research. Final Thoughts This research represents a major step into the area of the use of statistical modeling to predict auction outcomes. It addresses its shortfalls and delves into the future research that has been suggested, the field maybe takes steps into further progress with respect to more sophisticated and universally applicable tools of auction prediction.

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# 8 Appendix

## 8.1 Appendix 1

| Distribu | Bidders | Variability | MMAE Gates | MMAE Arit | MMEA Geom | MMEA Harma | MMEA Fried | MMEA H&R | MMEA BS |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | L | 3,94% | 3,94% | 3,94% | 3,94% | 3,94% | 5,75% | 28,27% |
| N | 2 | M | 3,67% | 3,67% | 3,67% | 3,67% | 3,67% | 2,40% | 5,61% |
| | 2 | H | 4,16% | 4,16% | 4,16% | 4,16% | 4,16% | 2,56% | 5,26% |
| | 2 | L | 3,62% | 3,62% | 3,62% | 3,62% | 3,62% | 6,40% | 11,98% |
| LogN | 2 | M | 4,05% | 4,05% | 4,05% | 4,05% | 4,05% | 2,94% | 2,41% |
| | 2 | H | 3,93% | 3,93% | 3,93% | 3,93% | 3,93% | 3,36% | 2,47% |
| | 3 | L | 3,95% | 3,47% | 3,43% | 3,42% | 6,21% | 4,01% | 16,35% |
| N | 3 | M | 3,72% | 2,79% | 2,81% | 2,82% | 5,50% | 2,44% | 3,21% |
| | 3 | H | 3,78% | 2,86% | 2,86% | 2,87% | 5,68% | 2,06% | 3,25% |
| | 3 | L | 4,02% | 3,70% | 3,67% | 3,64% | 6,39% | 4,14% | 10,61% |
| LogN | 3 | M | 3,50% | 2,68% | 2,70% | 2,71% | 5,23% | 2,27% | 2,28% |
| | 3 | H | 3,74% | 2,91% | 2,92% | 2,93% | 5,47% | 2,47% | 2,40% |
| | 4 | L | 3,50% | 3,44% | 3,34% | 3,26% | 7,70% | 4,06% | 12,93% |
| N | 4 | M | 3,59% | 2,43% | 2,44% | 2,46% | 6,99% | 1,68% | 2,32% |
| | 4 | H | 3,47% | 2,37% | 2,38% | 2,40% | 6,70% | 1,59% | 2,21% |
| | 4 | L | 3,35% | 3,27% | 3,16% | 3,09% | 7,49% | 3,96% | 8,73% |
| LogN | 4 | M | 3,46% | 2,38% | 2,40% | 2,42% | 6,89% | 2,05% | 2,17% |
| | 4 | H | 3,37% | 2,41% | 2,42% | 2,43% | 6,52% | 2,15% | 2,20% |
| N | 5 | L | 2,86% | 3,18% | 3,03% | 2,90% | 7,92% | 3,45% | 10,63% |
| | 5 | M | 3,05% | 1,97% | 1,99% | 2,00% | 7,22% | 1,41% | 1,77% |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 5 | H | 3,38% | 2,15% | 2,17% | 2,18% | 7,82% | 1,46% | 1,81% |
| | 5 | L | 2,78% | 3,27% | 3,10% | 2,96% | 8,18% | 4,11% | 8,49% |
| LogN | 5 | M | 3,06% | 2,00% | 2,02% | 2,04% | 7,66% | 1,84% | 2,06% |
| | 5 | H | 3,36% | 2,26% | 2,27% | 2,29% | 7,84% | 2,06% | 2,23% |
| | 6 | L | 2,75% | 3,11% | 2,92% | 2,79% | 8,62% | 3,22% | 9,04% |
| N | 6 | M | 3,09% | 1,91% | 1,93% | 1,95% | 8,34% | 1,22% | 1,50% |
| | 6 | H | 2,84% | 1,80% | 1,82% | 1,84% | 7,52% | 1,24% | 1,39% |
| | 6 | L | 2,65% | 3,21% | 2,99% | 2,84% | 8,69% | 3,80% | 7,72% |
| LogN | 6 | M | 2,88% | 1,91% | 1,93% | 1,94% | 8,00% | 1,87% | 2,01% |
| | 6 | H | 2,90% | 1,97% | 1,99% | 2,01% | 8,12% | 1,91% | 2,10% |
| | 7 | L | 2,68% | 3,02% | 2,85% | 2,73% | 8,57% | 3,09% | 7,70% |
| N | 7 | M | 2,74% | 1,67% | 1,69% | 1,70% | 8,18% | 1,08% | 1,20% |
| | 7 | H | 2,66% | 1,61% | 1,63% | 1,65% | 7,99% | 1,11% | 1,22% |
| | 7 | L | 2,71% | 2,90% | 2,72% | 2,61% | 9,14% | 3,51% | 6,47% |
| LogN | 7 | M | 2,72% | 1,82% | 1,83% | 1,85% | 8,52% | 1,95% | 2,03% |
| | 7 | H | 2,72% | 1,78% | 1,79% | 1,81% | 8,72% | 1,82% | 2,02% |
| | 8 | L | 2,38% | 3,10% | 2,81% | 2,62% | 9,07% | 3,03% | 7,49% |
| N | 8 | M | 2,64% | 1,54% | 1,57% | 1,59% | 8,59% | 1,03% | 1,08% |
| | 8 | H | 2,73% | 1,60% | 1,63% | 1,65% | 8,77% | 1,02% | 1,07% |
| | 8 | L | 2,35% | 2,86% | 2,62% | 2,45% | 8,85% | 3,46% | 6,28% |
| LogN | 8 | M | 2,55% | 1,64% | 1,66% | 1,68% | 8,73% | 1,78% | 1,96% |
| | 8 | H | 2,63% | 1,74% | 1,76% | 1,78% | 8,84% | 1,82% | 1,99% |
| N | 9 | L | 2,43% | 2,81% | 2,58% | 2,42% | 9,17% | 2,85% | 6,48% |
| | 9 | M | 2,61% | 1,52% | 1,55% | 1,58% | 8,88% | 0,92% | 0,95% |

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|---|---|---|---|---|---|---|---|---|---|
| | 9 | H | 2,45% | 1,43% | 1,45% | 1,47% | 8,49% | 0,91% | 0,94% |
| | 9 | L | 2,27% | 2,99% | 2,68% | 2,47% | 9,12% | 3,41% | 6,13% |
| LogN | 9 | M | 2,54% | 1,69% | 1,71% | 1,73% | 8,62% | 1,76% | 1,93% |
| | 9 | H | 2,47% | 1,61% | 1,62% | 1,64% | 8,69% | 1,74% | 1,89% |
| | 10 | L | 2,12% | 2,81% | 2,56% | 2,37% | 8,74% | 2,78% | 6,05% |
| N | 10 | M | 2,43% | 1,41% | 1,44% | 1,47% | 8,65% | 0,81% | 0,84% |
| | 10 | H | 2,38% | 1,38% | 1,41% | 1,43% | 8,55% | 0,79% | 0,84% |
| | 10 | L | 2,20% | 2,87% | 2,57% | 2,37% | 8,78% | 3,27% | 5,68% |
| LogN | 10 | M | 2,33% | 1,55% | 1,57% | 1,59% | 8,73% | 1,72% | 1,84% |
| | 10 | H | 2,39% | 1,59% | 1,61% | 1,63% | 8,79% | 1,72% | 1,86% |
| | 11 | L | 2,20% | 2,79% | 2,48% | 2,28% | 8,71% | 2,52% | 5,87% |
| N | 11 | M | 2,21% | 1,27% | 1,29% | 1,32% | 8,39% | 0,73% | 0,76% |
| | 11 | H | 2,20% | 1,26% | 1,29% | 1,32% | 8,39% | 0,75% | 0,76% |
| | 11 | L | 2,22% | 2,75% | 2,49% | 2,32% | 8,48% | 3,19% | 5,31% |
| LogN | 11 | M | 2,19% | 1,50% | 1,51% | 1,52% | 8,65% | 1,66% | 1,73% |
| | 11 | H | 2,30% | 1,51% | 1,53% | 1,55% | 8,74% | 1,59% | 1,75% |
| | 12 | L | 2,08% | 2,72% | 2,45% | 2,27% | 8,20% | 2,36% | 5,39% |
| N | 12 | M | 2,18% | 1,22% | 1,25% | 1,28% | 8,55% | 0,68% | 0,71% |
| | 12 | H | 2,18% | 1,23% | 1,26% | 1,29% | 8,34% | 0,68% | 0,70% |
| | 12 | L | 2,08% | 2,64% | 2,36% | 2,18% | 8,17% | 3,17% | 4,99% |
| LogN | 12 | M | 2,15% | 1,49% | 1,50% | 1,52% | 8,37% | 1,61% | 1,74% |
| | 12 | H | 2,21% | 1,49% | 1,51% | 1,53% | 8,66% | 1,63% | 1,73% |
| N | 13 | L | 1,91% | 2,57% | 2,28% | 2,08% | 7,65% | 2,26% | 5,20% |
| | 13 | M | 2,14% | 1,21% | 1,24% | 1,28% | 8,23% | 0,64% | 0,63% |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **13** | **H** | 2,11% | 1,19% | 1,22% | 1,26% | 8,23% | 0,61% | 0,64% |
| | **13** | **L** | 1,91% | 2,54% | 2,25% | 2,08% | 7,66% | 3,08% | 4,81% |
| **LogN** | **13** | **M** | 2,09% | 1,38% | 1,41% | 1,43% | 8,09% | 1,58% | 1,68% |
| | **13** | **H** | 2,15% | 1,40% | 1,42% | 1,45% | 8,44% | 1,54% | 1,65% |
| | **14** | **L** | 1,83% | 2,41% | 2,13% | 1,96% | 7,14% | 2,23% | 4,84% |
| **N** | **14** | **M** | 2,06% | 1,15% | 1,19% | 1,22% | 8,20% | 0,63% | 0,57% |
| | **14** | **H** | 1,98% | 1,10% | 1,13% | 1,16% | 8,04% | 0,61% | 0,59% |
| | **14** | **L** | 1,91% | 2,44% | 2,16% | 1,97% | 7,17% | 2,91% | 4,49% |
| **LogN** | **14** | **M** | 2,00% | 1,34% | 1,36% | 1,37% | 8,00% | 1,46% | 1,54% |
| | **14** | **H** | 2,02% | 1,35% | 1,37% | 1,40% | 8,06% | 1,49% | 1,59% |
| | **15** | **L** | 1,77% | 2,41% | 2,14% | 1,95% | 6,72% | 2,09% | 4,67% |
| **N** | **15** | **M** | 2,00% | 1,11% | 1,15% | 1,19% | 8,01% | 0,58% | 0,55% |
| | **15** | **H** | 1,96% | 1,08% | 1,12% | 1,15% | 8,01% | 0,54% | 0,56% |
| | **15** | **L** | 1,86% | 2,36% | 2,09% | 1,92% | 6,65% | 2,90% | 4,35% |
| **LogN** | **15** | **M** | 1,95% | 1,34% | 1,36% | 1,38% | 7,71% | 1,49% | 1,58% |
| | **15** | **H** | 2,01% | 1,32% | 1,35% | 1,37% | 7,86% | 1,43% | 1,50% |
| | **16** | **L** | 1,74% | 2,32% | 2,04% | 1,86% | 6,38% | 2,14% | 4,42% |
| **N** | **16** | **M** | 1,93% | 1,06% | 1,10% | 1,14% | 7,76% | 0,51% | 0,51% |
| | **16** | **H** | 1,87% | 1,03% | 1,07% | 1,10% | 7,77% | 0,50% | 0,50% |
| | **16** | **L** | 1,78% | 2,33% | 2,05% | 1,87% | 6,25% | 2,75% | 4,21% |
| **LogN** | **16** | **M** | 1,84% | 1,24% | 1,26% | 1,27% | 7,48% | 1,43% | 1,50% |
| | **16** | **H** | 1,95% | 1,28% | 1,31% | 1,34% | 7,55% | 1,39% | 1,48% |
| **N** | **17** | **L** | 1,66% | 2,29% | 2,04% | 1,86% | 5,94% | 2,15% | 4,25% |
| | **17** | **M** | 1,84% | 1,01% | 1,05% | 1,09% | 7,58% | 0,50% | 0,48% |

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|---|---|---|---|---|---|---|---|---|---|
| | 17 | H | 1,84% | 1,02% | 1,06% | 1,10% | 7,43% | 0,51% | 0,47% |
| | 17 | L | 1,67% | 2,29% | 2,01% | 1,81% | 5,92% | 2,65% | 4,21% |
| LogN | 17 | M | 1,91% | 1,25% | 1,28% | 1,31% | 7,32% | 1,40% | 1,47% |
| | 17 | H | 1,86% | 1,22% | 1,24% | 1,26% | 7,46% | 1,37% | 1,42% |
| | 18 | L | 1,64% | 2,20% | 1,96% | 1,79% | 5,57% | 1,99% | 4,07% |
| N | 18 | M | 1,80% | 0,99% | 1,03% | 1,07% | 7,34% | 0,46% | 0,45% |
| | 18 | H | 1,84% | 1,02% | 1,06% | 1,10% | 7,42% | 0,47% | 0,45% |
| | 18 | L | 1,56% | 2,12% | 1,84% | 1,65% | 5,56% | 2,61% | 3,90% |
| LogN | 18 | M | 1,80% | 1,21% | 1,23% | 1,26% | 6,90% | 1,33% | 1,39% |
| | 18 | H | 1,79% | 1,18% | 1,20% | 1,22% | 7,14% | 1,42% | 1,45% |
| | 19 | L | 1,57% | 2,06% | 1,84% | 1,69% | 5,27% | 1,85% | 3,84% |
| N | 19 | M | 1,76% | 0,97% | 1,02% | 1,06% | 7,08% | 0,45% | 0,42% |
| | 19 | H | 1,79% | 0,97% | 1,02% | 1,05% | 7,27% | 0,46% | 0,43% |
| | 19 | L | 1,58% | 2,15% | 1,90% | 1,72% | 5,29% | 2,51% | 3,81% |
| LogN | 19 | M | 1,76% | 1,17% | 1,19% | 1,21% | 6,68% | 1,32% | 1,38% |
| | 19 | H | 1,73% | 1,16% | 1,19% | 1,21% | 6,51% | 1,31% | 1,37% |
| | 20 | L | 1,49% | 1,98% | 1,75% | 1,60% | 5,02% | 1,79% | 3,75% |
| N | 20 | M | 1,70% | 0,94% | 0,98% | 1,02% | 6,88% | 0,42% | 0,41% |
| | 20 | H | 1,76% | 0,95% | 1,00% | 1,04% | 6,79% | 0,43% | 0,40% |
| | 20 | L | 1,51% | 2,07% | 1,83% | 1,65% | 5,00% | 2,49% | 3,64% |
| LogN | 20 | M | 1,68% | 1,14% | 1,16% | 1,18% | 6,41% | 1,32% | 1,37% |
| | 20 | H | 1,71% | 1,16% | 1,18% | 1,20% | 6,39% | 1,27% | 1,34% |

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## 8.2 Appendix 2

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 2 | 2 | 2 | 2 | 2 | 2 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,07% | 89,82% | 89,95% | 90,57% | 89,98% | 90,16% |
| AVG MSS | 25,37% | 84,81% | 86,32% | 25,59% | 85,49% | 84,06% |
| AVG SMS | 4,59% | 4,93% | 5,23% | 4,50% | 4,90% | 4,96% |
| AVG SSS | 6,75% | 6,51% | 6,46% | 6,82% | 6,97% | 8,40% |
| | | | | | | |
| MMAE Gates | 3,94% | 3,67% | 4,16% | 3,62% | 4,05% | 3,93% |
| MMAE Arit | 3,94% | 3,67% | 4,16% | 3,62% | 4,05% | 3,93% |
| MMAE Geom | 3,94% | 3,67% | 4,16% | 3,62% | 4,05% | 3,93% |
| MMAE Harma | 3,94% | 3,67% | 4,16% | 3,62% | 4,05% | 3,93% |
| MMAE H&R | 5,75% | 2,40% | 2,56% | 6,40% | 2,94% | 3,36% |
| MMAE BS | 28,27% | 5,61% | 5,26% | 11,98% | 2,41% | 2,47% |
| | | | | | | |
| SMAE Gates | 2,81% | 2,60% | 2,86% | 2,50% | 3,30% | 2,97% |
| SMAE Arit | 2,81% | 2,60% | 2,86% | 2,50% | 3,30% | 2,97% |
| SMEA Geom | 2,81% | 2,60% | 2,86% | 2,50% | 3,30% | 2,97% |
| SMEA Harma | 2,81% | 2,60% | 2,86% | 2,50% | 3,30% | 2,97% |
| SMEA H&R | 4,96% | 2,27% | 2,25% | 6,67% | 2,80% | 3,75% |
| SMEA BS | 20,54% | 4,03% | 3,94% | 11,22% | 1,72% | 1,78% |

## 8.3 Appendix 3

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | Normal | 3 | 3 | 3 | 3 | 3 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,62% | 89,50% | 90,04% | 90,14% | 90,63% | 89,82% |
| AVG MSS | 25,77% | 85,13% | 84,67% | 24,40% | 84,89% | 84,65% |
| AVG SMS | 5,38% | 5,59% | 5,41% | 5,49% | 5,74% | 5,53% |
| AVG SSS | 8,13% | 8,18% | 8,01% | 7,60% | 7,14% | 8,02% |
| | | | | | | |
| MMAE Gates | 3,95% | 3,72% | 3,78% | 4,02% | 3,50% | 3,74% |
| MMAE Arit | 3,47% | 2,79% | 2,86% | 3,70% | 2,68% | 2,91% |
| MMAE Geom | 3,43% | 2,81% | 2,86% | 3,67% | 2,70% | 2,92% |
| MMAE Harma | 3,42% | 2,82% | 2,87% | 3,64% | 2,71% | 2,93% |
| MMAE H&R | 4,01% | 2,44% | 2,06% | 4,14% | 2,27% | 2,47% |
| MMAE BS | 16,35% | 3,21% | 3,25% | 10,61% | 2,28% | 2,40% |
| | | | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| SMAE Gates | 1,93% | 1,83% | 2,04% | 2,21% | 1,78% | 2,01% |
| SMAE Arit | 1,77% | 1,43% | 1,64% | 1,67% | 1,37% | 1,49% |
| SMEA Geom | 1,73% | 1,44% | 1,64% | 1,68% | 1,38% | 1,51% |
| SMEA Harma | 1,71% | 1,44% | 1,64% | 1,70% | 1,39% | 1,53% |
| SMEA H&R | 2,12% | 1,98% | 1,45% | 2,67% | 1,51% | 1,66% |
| SMEA BS | 8,14% | 1,74% | 1,50% | 6,00% | 1,02% | 1,23% |

## 8.4 Appendix 4

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 4 | 4 | 4 | 4 | 4 | 4 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 89,88% | 90,61% | 89,88% | 90,07% | 90,36% | 89,71% |
| AVG MSS | 24,77% | 85,64% | 85,53% | 25,24% | 85,22% | 84,01% |
| AVG SMS | 5,47% | 5,77% | 5,75% | 5,87% | 5,68% | 5,69% |
| AVG SSS | 8,86% | 8,17% | 8,30% | 8,69% | 8,33% | 8,47% |
| | | | | | | |
| MMAE Gates | 3,50% | 3,59% | 3,47% | 3,35% | 3,46% | 3,37% |
| MMAE Arit | 3,44% | 2,43% | 2,37% | 3,27% | 2,38% | 2,41% |
| MMAE Geom | 3,34% | 2,44% | 2,38% | 3,16% | 2,40% | 2,42% |
| MMAE Harma | 3,26% | 2,46% | 2,40% | 3,09% | 2,42% | 2,43% |
| MMAE H&R | 4,06% | 1,68% | 1,59% | 3,96% | 2,05% | 2,15% |
| MMAE BS | 12,93% | 2,32% | 2,21% | 8,73% | 2,17% | 2,20% |
| | | | | | | |
| SMAE Gates | 1,41% | 1,61% | 1,44% | 1,32% | 1,45% | 1,49% |
| SMAE Arit | 1,48% | 1,11% | 0,97% | 1,29% | 0,98% | 1,07% |
| SMEA Geom | 1,46% | 1,11% | 0,98% | 1,27% | 1,00% | 1,08% |
| SMEA Harma | 1,45% | 1,12% | 0,99% | 1,25% | 1,01% | 1,09% |
| SMEA H&R | 1,83% | 0,87% | 0,74% | 2,15% | 0,87% | 0,93% |
| SMEA BS | 5,50% | 0,85% | 0,86% | 3,92% | 0,80% | 0,82% |

## 8.5 Appendix 5

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 5 | 5 | 5 | 5 | 5 | 5 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,32% | 90,38% | 89,65% | 89,96% | 90,23% | 89,82% |
| AVG MSS | 25,21% | 84,61% | 84,78% | 24,59% | 85,84% | 85,75% |
| AVG SMS | 5,60% | 6,03% | 5,83% | 5,95% | 5,97% | 5,88% |
| AVG SSS | 8,54% | 8,79% | 8,73% | 8,37% | 8,26% | 8,44% |
| | | | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| MMAE Gates | 2,86% | 3,05% | 3,38% | 2,78% | 3,06% | 3,36% |
| MMAE Arit | 3,18% | 1,97% | 2,15% | 3,27% | 2,00% | 2,26% |
| MMAE Geom | 3,03% | 1,99% | 2,17% | 3,10% | 2,02% | 2,27% |
| MMAE Harma | 2,90% | 2,00% | 2,18% | 2,96% | 2,04% | 2,29% |
| MMAE H&R | 3,45% | 1,41% | 1,46% | 4,11% | 1,84% | 2,06% |
| MMAE BS | 10,63% | 1,77% | 1,81% | 8,49% | 2,06% | 2,23% |
| | | | | | | |
| SMAE Gates | 0,95% | 1,21% | 1,13% | 1,11% | 1,15% | 1,13% |
| SMAE Arit | 1,26% | 0,78% | 0,77% | 1,20% | 0,81% | 0,81% |
| SMEA Geom | 1,17% | 0,79% | 0,78% | 1,13% | 0,81% | 0,82% |
| SMEA Harma | 1,09% | 0,80% | 0,79% | 1,07% | 0,82% | 0,83% |
| SMEA H&R | 1,51% | 0,65% | 0,64% | 1,99% | 0,73% | 0,74% |
| SMEA BS | 4,11% | 0,59% | 0,57% | 2,83% | 0,68% | 0,73% |

## 8.6 Appendix 6

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 6 | 6 | 6 | 6 | 6 | 6 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,04% | 89,85% | 90,10% | 90,16% | 89,87% | 89,93% |
| AVG MSS | 25,29% | 84,78% | 85,35% | 24,71% | 85,52% | 84,62% |
| AVG SMS | 5,84% | 5,95% | 5,90% | 6,15% | 5,91% | 5,85% |
| AVG SSS | 9,08% | 8,65% | 8,51% | 8,89% | 8,75% | 8,93% |
| | | | | | | |
| MMAE Gates | 2,75% | 3,09% | 2,84% | 2,65% | 2,88% | 2,90% |
| MMAE Arit | 3,11% | 1,91% | 1,80% | 3,21% | 1,91% | 1,97% |
| MMAE Geom | 2,92% | 1,93% | 1,82% | 2,99% | 1,93% | 1,99% |
| MMAE Harma | 2,79% | 1,95% | 1,84% | 2,84% | 1,94% | 2,01% |
| MMAE H&R | 3,22% | 1,22% | 1,24% | 3,80% | 1,87% | 1,91% |
| MMAE BS | 9,04% | 1,50% | 1,39% | 7,72% | 2,01% | 2,10% |
| | | | | | | |
| SMAE Gates | 0,92% | 1,03% | 1,00% | 0,90% | 0,96% | 0,90% |
| SMAE Arit | 1,03% | 0,67% | 0,63% | 1,19% | 0,66% | 0,57% |
| SMEA Geom | 0,94% | 0,68% | 0,64% | 1,13% | 0,67% | 0,58% |
| SMEA Harma | 0,90% | 0,69% | 0,65% | 1,09% | 0,67% | 0,58% |
| SMEA H&R | 1,52% | 0,41% | 0,46% | 1,82% | 0,66% | 0,59% |
| SMEA BS | 2,53% | 0,41% | 0,39% | 2,20% | 0,61% | 0,64% |

## 8.7 Appendix 7

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|

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| Bidders | 7 | 7 | 7 | 7 | 7 | 7 |
|---|---|---|---|---|---|---|
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 89,62% | 90,02% | 90,02% | 89,72% | 89,88% | 90,17% |
| AVG MSS | 25,38% | 85,01% | 85,27% | 25,10% | 85,17% | 84,40% |
| AVG SMS | 5,81% | 5,84% | 5,91% | 6,05% | 5,79% | 6,06% |
| AVG SSS | 8,71% | 9,03% | 8,97% | 8,61% | 8,93% | 9,02% |
| | | | | | | |
| MMAE Gates | 2,68% | 2,74% | 2,66% | 2,71% | 2,72% | 2,72% |
| MMAE Arit | 3,02% | 1,67% | 1,61% | 2,90% | 1,82% | 1,78% |
| MMAE Geom | 2,85% | 1,69% | 1,63% | 2,72% | 1,83% | 1,79% |
| MMAE Harma | 2,73% | 1,70% | 1,65% | 2,61% | 1,85% | 1,81% |
| MMAE H&R | 3,09% | 1,08% | 1,11% | 3,51% | 1,95% | 1,82% |
| MMAE BS | 7,70% | 1,20% | 1,22% | 6,47% | 2,03% | 2,02% |
| | | | | | | |
| SMAE Gates | 0,82% | 0,75% | 0,84% | 0,84% | 0,90% | 0,82% |
| SMAE Arit | 0,90% | 0,48% | 0,53% | 0,91% | 0,57% | 0,53% |
| SMEA Geom | 0,88% | 0,49% | 0,55% | 0,85% | 0,58% | 0,53% |
| SMEA Harma | 0,87% | 0,49% | 0,56% | 0,79% | 0,59% | 0,54% |
| SMEA H&R | 1,31% | 0,35% | 0,37% | 1,43% | 0,58% | 0,59% |
| SMEA BS | 2,08% | 0,34% | 0,31% | 1,85% | 0,64% | 0,60% |

## 8.8 Appendix 8

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 8 | 8 | 8 | 8 | 8 | 8 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 89,98% | 89,90% | 90,21% | 90,37% | 89,96% | 89,92% |
| AVG MSS | 24,69% | 85,13% | 85,00% | 25,55% | 84,70% | 85,36% |
| AVG SMS | 6,14% | 6,09% | 5,85% | 6,13% | 5,99% | 5,77% |
| AVG SSS | 9,04% | 8,56% | 8,57% | 9,10% | 8,72% | 9,15% |
| | | | | | | |
| MMAE Gates | 2,38% | 2,64% | 2,73% | 2,35% | 2,55% | 2,63% |
| MMAE Arit | 3,10% | 1,54% | 1,60% | 2,86% | 1,64% | 1,74% |
| MMAE Geom | 2,81% | 1,57% | 1,63% | 2,62% | 1,66% | 1,76% |
| MMAE Harma | 2,62% | 1,59% | 1,65% | 2,45% | 1,68% | 1,78% |
| MMAE H&R | 3,03% | 1,03% | 1,02% | 3,46% | 1,78% | 1,82% |
| MMAE BS | 7,49% | 1,08% | 1,07% | 6,28% | 1,96% | 1,99% |
| | | | | | | |
| SMAE Gates | 0,75% | 0,77% | 0,80% | 0,71% | 0,75% | 0,76% |
| SMAE Arit | 0,90% | 0,46% | 0,45% | 0,85% | 0,47% | 0,47% |
| SMEA Geom | 0,83% | 0,48% | 0,47% | 0,77% | 0,47% | 0,48% |

| | | | | | | |
|---|---|---|---|---|---|---|
| SMEA Harma | 0,80% | 0,50% | 0,49% | 0,72% | 0,48% | 0,49% |
| SMEA H&R | 1,34% | 0,32% | 0,34% | 1,46% | 0,42% | 0,46% |
| SMEA BS | 1,84% | 0,28% | 0,30% | 1,61% | 0,45% | 0,46% |

## 8.9 Appendix 9

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 9 | 9 | 9 | 9 | 9 | 9 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,16% | 90,06% | 90,00% | 89,95% | 89,96% | 90,03% |
| AVG MSS | 24,81% | 84,85% | 84,75% | 24,79% | 84,86% | 84,98% |
| AVG SMS | 5,97% | 5,91% | 5,94% | 6,16% | 6,19% | 5,90% |
| AVG SSS | 8,92% | 8,85% | 8,88% | 9,03% | 8,90% | 8,69% |
| | | | | | | |
| MMAE Gates | 2,43% | 2,61% | 2,45% | 2,27% | 2,54% | 2,47% |
| MMAE Arit | 2,81% | 1,52% | 1,43% | 2,99% | 1,69% | 1,61% |
| MMAE Geom | 2,58% | 1,55% | 1,45% | 2,68% | 1,71% | 1,62% |
| MMAE Harma | 2,42% | 1,58% | 1,47% | 2,47% | 1,73% | 1,64% |
| MMAE H&R | 2,85% | 0,92% | 0,91% | 3,41% | 1,76% | 1,74% |
| MMAE BS | 6,48% | 0,95% | 0,94% | 6,13% | 1,93% | 1,89% |
| | | | | | | |
| SMAE Gates | 0,69% | 0,75% | 0,61% | 0,55% | 0,72% | 0,66% |
| SMAE Arit | 0,72% | 0,43% | 0,39% | 0,80% | 0,40% | 0,42% |
| SMEA Geom | 0,66% | 0,45% | 0,40% | 0,66% | 0,41% | 0,43% |
| SMEA Harma | 0,62% | 0,47% | 0,41% | 0,59% | 0,43% | 0,44% |
| SMEA H&R | 0,99% | 0,28% | 0,30% | 1,20% | 0,42% | 0,39% |
| SMEA BS | 1,49% | 0,24% | 0,23% | 1,49% | 0,45% | 0,46% |

## 8.10 Appendix 10

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 10 | 10 | 10 | 10 | 10 | 10 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,49% | 90,12% | 89,83% | 89,99% | 90,16% | 90,07% |
| AVG MSS | 24,33% | 85,08% | 85,27% | 24,81% | 85,02% | 84,65% |
| AVG SMS | 5,94% | 6,06% | 5,89% | 5,80% | 6,00% | 6,18% |
| AVG SSS | 9,03% | 8,79% | 8,81% | 9,06% | 8,88% | 8,89% |
| | | | | | | |
| MMAE Gates | 2,12% | 2,43% | 2,38% | 2,20% | 2,33% | 2,39% |
| MMAE Arit | 2,81% | 1,41% | 1,38% | 2,87% | 1,55% | 1,59% |
| MMAE Geom | 2,56% | 1,44% | 1,41% | 2,57% | 1,57% | 1,61% |

| | | | | | | |
|---|---|---|---|---|---|---|
| MMAE Harma | 2,37% | 1,47% | 1,43% | 2,37% | 1,59% | 1,63% |
| MMAE H&R | 2,78% | 0,81% | 0,79% | 3,27% | 1,72% | 1,72% |
| MMAE BS | 6,05% | 0,84% | 0,84% | 5,68% | 1,84% | 1,86% |
| | | | | | | |
| SMAE Gates | 0,55% | 0,60% | 0,55% | 0,63% | 0,64% | 0,66% |
| SMAE Arit | 0,69% | 0,33% | 0,36% | 0,68% | 0,41% | 0,39% |
| SMEA Geom | 0,62% | 0,35% | 0,37% | 0,60% | 0,42% | 0,40% |
| SMEA Harma | 0,57% | 0,36% | 0,38% | 0,57% | 0,43% | 0,41% |
| SMEA H&R | 0,94% | 0,21% | 0,25% | 1,13% | 0,43% | 0,39% |
| SMEA BS | 1,28% | 0,19% | 0,17% | 1,25% | 0,41% | 0,42% |

## 8.11 Appendix 11

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 11 | 11 | 11 | 11 | 11 | 11 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 89,95% | 90,14% | 89,90% | 90,25% | 89,78% | 90,22% |
| AVG MSS | 25,04% | 84,64% | 84,71% | 24,94% | 84,91% | 84,87% |
| AVG SMS | 6,05% | 5,93% | 5,94% | 5,99% | 6,04% | 6,00% |
| AVG SSS | 9,02% | 9,03% | 8,98% | 8,81% | 8,87% | 8,93% |
| | | | | | | |
| MMAE Gates | 2,20% | 2,21% | 2,20% | 2,22% | 2,19% | 2,30% |
| MMAE Arit | 2,79% | 1,27% | 1,26% | 2,75% | 1,50% | 1,51% |
| MMAE Geom | 2,48% | 1,29% | 1,29% | 2,49% | 1,51% | 1,53% |
| MMAE Harma | 2,28% | 1,32% | 1,32% | 2,32% | 1,52% | 1,55% |
| MMAE H&R | 2,52% | 0,73% | 0,75% | 3,19% | 1,66% | 1,59% |
| MMAE BS | 5,87% | 0,76% | 0,76% | 5,31% | 1,73% | 1,75% |
| | | | | | | |
| SMAE Gates | 0,60% | 0,54% | 0,52% | 0,51% | 0,51% | 0,50% |
| SMAE Arit | 0,66% | 0,32% | 0,29% | 0,63% | 0,33% | 0,33% |
| SMEA Geom | 0,63% | 0,33% | 0,31% | 0,58% | 0,34% | 0,34% |
| SMEA Harma | 0,61% | 0,35% | 0,32% | 0,53% | 0,35% | 0,35% |
| SMEA H&R | 0,93% | 0,19% | 0,21% | 1,08% | 0,37% | 0,31% |
| SMEA BS | 1,28% | 0,16% | 0,17% | 0,96% | 0,40% | 0,33% |

## 8.12 Appendix 12

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 12 | 12 | 12 | 12 | 12 | 12 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 89,98% | 89,87% | 90,25% | 89,97% | 89,83% | 89,90% |

| | | | | | | |
|---|---|---|---|---|---|---|
| AVG MSS | 24,82% | 85,44% | 85,09% | 24,96% | 85,05% | 85,18% |
| AVG SMS | 6,15% | 6,09% | 5,98% | 5,98% | 5,82% | 6,13% |
| AVG SSS | 8,43% | 9,09% | 8,91% | 8,81% | 9,00% | 8,85% |
| | | | | | | |
| MMAE Gates | 2,08% | 2,18% | 2,18% | 2,08% | 2,15% | 2,21% |
| MMAE Arit | 2,72% | 1,22% | 1,23% | 2,64% | 1,49% | 1,49% |
| MMAE Geom | 2,45% | 1,25% | 1,26% | 2,36% | 1,50% | 1,51% |
| MMAE Harma | 2,27% | 1,28% | 1,29% | 2,18% | 1,52% | 1,53% |
| MMAE H&R | 2,36% | 0,68% | 0,68% | 3,17% | 1,61% | 1,63% |
| MMAE BS | 5,39% | 0,71% | 0,70% | 4,99% | 1,74% | 1,73% |
| | | | | | | |
| SMAE Gates | 0,55% | 0,50% | 0,51% | 0,46% | 0,47% | 0,49% |
| SMAE Arit | 0,65% | 0,29% | 0,31% | 0,60% | 0,34% | 0,35% |
| SMEA Geom | 0,55% | 0,31% | 0,33% | 0,56% | 0,35% | 0,36% |
| SMEA Harma | 0,50% | 0,32% | 0,34% | 0,54% | 0,35% | 0,36% |
| SMEA H&R | 0,84% | 0,16% | 0,20% | 1,07% | 0,31% | 0,29% |
| SMEA BS | 1,05% | 0,15% | 0,14% | 0,97% | 0,34% | 0,30% |

## 8.13 Appendix 13

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 13 | 13 | 13 | 13 | 13 | 13 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,16% | 90,10% | 89,97% | 90,11% | 90,08% | 90,20% |
| AVG MSS | 25,03% | 84,94% | 84,80% | 24,80% | 84,67% | 85,29% |
| AVG SMS | 5,88% | 5,84% | 6,05% | 5,91% | 5,95% | 6,10% |
| AVG SSS | 8,73% | 8,92% | 8,76% | 8,75% | 8,97% | 8,84% |
| | | | | | | |
| MMAE Gates | 1,91% | 2,14% | 2,11% | 1,91% | 2,09% | 2,15% |
| MMAE Arit | 2,57% | 1,21% | 1,19% | 2,54% | 1,38% | 1,40% |
| MMAE Geom | 2,28% | 1,24% | 1,22% | 2,25% | 1,41% | 1,42% |
| MMAE Harma | 2,08% | 1,28% | 1,26% | 2,08% | 1,43% | 1,45% |
| MMAE H&R | 2,26% | 0,64% | 0,61% | 3,08% | 1,58% | 1,54% |
| MMAE BS | 5,20% | 0,63% | 0,64% | 4,81% | 1,68% | 1,65% |
| | | | | | | |
| SMAE Gates | 0,42% | 0,47% | 0,46% | 0,44% | 0,48% | 0,50% |
| SMAE Arit | 0,59% | 0,26% | 0,28% | 0,61% | 0,34% | 0,31% |
| SMEA Geom | 0,51% | 0,28% | 0,30% | 0,55% | 0,34% | 0,32% |
| SMEA Harma | 0,47% | 0,29% | 0,32% | 0,48% | 0,35% | 0,33% |
| SMEA H&R | 0,71% | 0,15% | 0,17% | 0,87% | 0,34% | 0,32% |
| SMEA BS | 0,94% | 0,14% | 0,14% | 0,93% | 0,31% | 0,32% |

## 8.14 Appendix 14

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 14 | 14 | 14 | 14 | 14 | 14 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 89,79% | 89,99% | 89,97% | 89,89% | 90,35% | 90,30% |
| AVG MSS | 25,05% | 85,29% | 85,04% | 25,28% | 85,10% | 85,18% |
| AVG SMS | 6,08% | 5,90% | 6,05% | 6,04% | 5,97% | 6,00% |
| AVG SSS | 8,82% | 8,88% | 8,89% | 8,96% | 8,70% | 8,80% |
| | | | | | | |
| MMAE Gates | 1,83% | 2,06% | 1,98% | 1,91% | 2,00% | 2,02% |
| MMAE Arit | 2,41% | 1,15% | 1,10% | 2,44% | 1,34% | 1,35% |
| MMAE Geom | 2,13% | 1,19% | 1,13% | 2,16% | 1,36% | 1,37% |
| MMAE Harma | 1,96% | 1,22% | 1,16% | 1,97% | 1,37% | 1,40% |
| MMAE H&R | 2,23% | 0,63% | 0,61% | 2,91% | 1,46% | 1,49% |
| MMAE BS | 4,84% | 0,57% | 0,59% | 4,49% | 1,54% | 1,59% |
| | | | | | | |
| SMAE Gates | 0,41% | 0,44% | 0,42% | 0,43% | 0,40% | 0,43% |
| SMAE Arit | 0,47% | 0,25% | 0,25% | 0,53% | 0,26% | 0,28% |
| SMEA Geom | 0,43% | 0,27% | 0,27% | 0,45% | 0,26% | 0,29% |
| SMEA Harma | 0,40% | 0,29% | 0,28% | 0,40% | 0,27% | 0,30% |
| SMEA H&R | 0,76% | 0,15% | 0,15% | 0,72% | 0,28% | 0,30% |
| SMEA BS | 0,85% | 0,12% | 0,12% | 0,84% | 0,30% | 0,27% |

## 8.15 Appendix 15

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 15 | 15 | 15 | 15 | 15 | 15 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,01% | 90,01% | 90,13% | 89,93% | 90,05% | 90,00% |
| AVG MSS | 25,02% | 84,93% | 84,67% | 25,21% | 84,55% | 85,32% |
| AVG SMS | 6,18% | 5,93% | 6,09% | 5,92% | 6,11% | 6,12% |
| AVG SSS | 8,75% | 8,84% | 9,00% | 8,73% | 9,04% | 8,93% |
| | | | | | | |
| MMAE Gates | 1,77% | 2,00% | 1,96% | 1,86% | 1,95% | 2,01% |
| MMAE Arit | 2,41% | 1,11% | 1,08% | 2,36% | 1,34% | 1,32% |
| MMAE Geom | 2,14% | 1,15% | 1,12% | 2,09% | 1,36% | 1,35% |
| MMAE Harma | 1,95% | 1,19% | 1,15% | 1,92% | 1,38% | 1,37% |
| MMAE H&R | 2,09% | 0,58% | 0,54% | 2,90% | 1,49% | 1,43% |
| MMAE BS | 4,67% | 0,55% | 0,56% | 4,35% | 1,58% | 1,50% |
| | | | | | | |

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| | | | | | | |
|---|---|---|---|---|---|---|
| SMAE Gates | 0,37% | 0,41% | 0,39% | 0,40% | 0,39% | 0,42% |
| SMAE Arit | 0,50% | 0,22% | 0,20% | 0,48% | 0,28% | 0,26% |
| SMEA Geom | 0,43% | 0,25% | 0,22% | 0,43% | 0,29% | 0,28% |
| SMEA Harma | 0,37% | 0,27% | 0,24% | 0,39% | 0,30% | 0,29% |
| SMEA H&R | 0,68% | 0,13% | 0,12% | 0,83% | 0,29% | 0,27% |
| SMEA BS | 0,72% | 0,10% | 0,10% | 0,67% | 0,29% | 0,26% |

## 8.16 Appendix 16

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 16 | 16 | 16 | 16 | 16 | 16 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,07% | 89,84% | 89,98% | 89,71% | 90,37% | 90,03% |
| AVG MSS | 25,12% | 85,13% | 85,05% | 25,06% | 84,78% | 85,54% |
| AVG SMS | 6,02% | 6,05% | 5,99% | 6,00% | 6,17% | 5,95% |
| AVG SSS | 8,73% | 8,78% | 8,80% | 8,95% | 8,95% | 8,79% |
| | | | | | | |
| MMAE Gates | 1,74% | 1,93% | 1,87% | 1,78% | 1,84% | 1,95% |
| MMAE Arit | 2,32% | 1,06% | 1,03% | 2,33% | 1,24% | 1,28% |
| MMAE Geom | 2,04% | 1,10% | 1,07% | 2,05% | 1,26% | 1,31% |
| MMAE Harma | 1,86% | 1,14% | 1,10% | 1,87% | 1,27% | 1,34% |
| MMAE H&R | 2,14% | 0,51% | 0,50% | 2,75% | 1,43% | 1,39% |
| MMAE BS | 4,42% | 0,51% | 0,50% | 4,21% | 1,50% | 1,48% |
| | | | | | | |
| SMAE Gates | 0,35% | 0,40% | 0,34% | 0,35% | 0,35% | 0,38% |
| SMAE Arit | 0,49% | 0,22% | 0,19% | 0,52% | 0,22% | 0,25% |
| SMEA Geom | 0,44% | 0,24% | 0,21% | 0,45% | 0,23% | 0,26% |
| SMEA Harma | 0,39% | 0,26% | 0,22% | 0,41% | 0,24% | 0,27% |
| SMEA H&R | 0,67% | 0,13% | 0,09% | 0,80% | 0,20% | 0,24% |
| SMEA BS | 0,71% | 0,09% | 0,09% | 0,67% | 0,20% | 0,24% |

## 8.17 Appendix 17

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 17 | 17 | 17 | 17 | 17 | 17 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 89,91% | 90,24% | 89,83% | 89,91% | 89,89% | 89,67% |
| AVG MSS | 25,03% | 84,74% | 84,80% | 24,59% | 84,84% | 85,38% |
| AVG SMS | 6,04% | 5,84% | 6,04% | 6,19% | 6,05% | 5,92% |
| AVG SSS | 8,94% | 8,97% | 9,11% | 8,95% | 8,91% | 8,83% |
| | | | | | | |

n

| | | | | | | |
|---|---|---|---|---|---|---|
| MMAE Gates | 1,66% | 1,84% | 1,84% | 1,67% | 1,91% | 1,86% |
| MMAE Arit | 2,29% | 1,01% | 1,02% | 2,29% | 1,25% | 1,22% |
| MMAE Geom | 2,04% | 1,05% | 1,06% | 2,01% | 1,28% | 1,24% |
| MMAE Harma | 1,86% | 1,09% | 1,10% | 1,81% | 1,31% | 1,26% |
| MMAE H&R | 2,15% | 0,50% | 0,51% | 2,65% | 1,40% | 1,37% |
| MMAE BS | 4,25% | 0,48% | 0,47% | 4,21% | 1,47% | 1,42% |
| | | | | | | |
| SMAE Gates | 0,37% | 0,33% | 0,32% | 0,33% | 0,33% | 0,31% |
| SMAE Arit | 0,47% | 0,20% | 0,18% | 0,42% | 0,25% | 0,21% |
| SMEA Geom | 0,40% | 0,23% | 0,20% | 0,34% | 0,25% | 0,22% |
| SMEA Harma | 0,36% | 0,24% | 0,22% | 0,30% | 0,26% | 0,23% |
| SMEA H&R | 0,64% | 0,11% | 0,11% | 0,61% | 0,25% | 0,21% |
| SMEA BS | 0,68% | 0,08% | 0,08% | 0,64% | 0,21% | 0,22% |

## 8.18 Appendix 18

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 18 | 18 | 18 | 18 | 18 | 18 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,32% | 90,03% | 89,86% | 90,14% | 90,07% | 89,95% |
| AVG MSS | 25,29% | 84,93% | 84,88% | 25,24% | 85,14% | 84,76% |
| AVG SMS | 6,03% | 5,92% | 6,00% | 5,86% | 5,96% | 6,06% |
| AVG SSS | 9,02% | 8,89% | 8,97% | 9,10% | 8,97% | 9,07% |
| | | | | | | |
| MMAE Gates | 1,64% | 1,80% | 1,84% | 1,56% | 1,80% | 1,79% |
| MMAE Arit | 2,20% | 0,99% | 1,02% | 2,12% | 1,21% | 1,18% |
| MMAE Geom | 1,96% | 1,03% | 1,06% | 1,84% | 1,23% | 1,20% |
| MMAE Harma | 1,79% | 1,07% | 1,10% | 1,65% | 1,26% | 1,22% |
| MMAE H&R | 1,99% | 0,46% | 0,47% | 2,61% | 1,33% | 1,42% |
| MMAE BS | 4,07% | 0,45% | 0,45% | 3,90% | 1,39% | 1,45% |
| | | | | | | |
| SMAE Gates | 0,37% | 0,28% | 0,37% | 0,31% | 0,31% | 0,29% |
| SMAE Arit | 0,38% | 0,17% | 0,20% | 0,41% | 0,21% | 0,21% |
| SMEA Geom | 0,33% | 0,19% | 0,23% | 0,35% | 0,22% | 0,21% |
| SMEA Harma | 0,30% | 0,20% | 0,25% | 0,32% | 0,23% | 0,22% |
| SMEA H&R | 0,59% | 0,09% | 0,09% | 0,58% | 0,24% | 0,23% |
| SMEA BS | 0,51% | 0,08% | 0,08% | 0,62% | 0,24% | 0,25% |

## 8.19 Appendix 19

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|

| Bidders | 19 | 19 | 19 | 19 | 19 | 19 |
|---|---|---|---|---|---|---|
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 90,08% | 89,82% | 89,91% | 90,22% | 90,22% | 89,99% |
| AVG MSS | 25,27% | 84,95% | 85,18% | 24,87% | 84,77% | 85,35% |
| AVG SMS | 5,92% | 5,99% | 6,10% | 5,94% | 5,97% | 5,98% |
| AVG SSS | 8,81% | 8,86% | 8,91% | 8,93% | 8,90% | 8,86% |
| | | | | | | |
| MMAE Gates | 1,57% | 1,76% | 1,79% | 1,58% | 1,76% | 1,73% |
| MMAE Arit | 2,06% | 0,97% | 0,97% | 2,15% | 1,17% | 1,16% |
| MMAE Geom | 1,84% | 1,02% | 1,02% | 1,90% | 1,19% | 1,19% |
| MMAE Harma | 1,69% | 1,06% | 1,05% | 1,72% | 1,21% | 1,21% |
| MMAE H&R | 1,85% | 0,45% | 0,46% | 2,51% | 1,32% | 1,31% |
| MMAE BS | 3,84% | 0,42% | 0,43% | 3,81% | 1,38% | 1,37% |
| | | | | | | |
| SMAE Gates | 0,28% | 0,33% | 0,28% | 0,30% | 0,32% | 0,34% |
| SMAE Arit | 0,40% | 0,18% | 0,16% | 0,35% | 0,20% | 0,22% |
| SMEA Geom | 0,35% | 0,20% | 0,18% | 0,33% | 0,21% | 0,23% |
| SMEA Harma | 0,31% | 0,22% | 0,19% | 0,31% | 0,22% | 0,23% |
| SMEA H&R | 0,49% | 0,08% | 0,08% | 0,58% | 0,19% | 0,19% |
| SMEA BS | 0,53% | 0,07% | 0,07% | 0,43% | 0,19% | 0,20% |

## 8.20 Appendix 20

| | Normal | | | Lognormal | | |
|---|---|---|---|---|---|---|
| Bidders | 20 | 20 | 20 | 20 | 20 | 20 |
| Variability | Low | Medium | High | Low | Medium | High |
| AVG MMS | 89,87% | 89,97% | 90,16% | 89,97% | 89,91% | 90,23% |
| AVG MSS | 25,12% | 84,97% | 84,65% | 24,97% | 84,92% | 84,86% |
| AVG SMS | 6,06% | 6,11% | 5,93% | 6,10% | 5,93% | 6,09% |
| AVG SSS | 8,94% | 8,98% | 8,83% | 8,89% | 8,99% | 8,86% |
| | | | | | | |
| MMAE Gates | 1,49% | 1,70% | 1,76% | 1,51% | 1,68% | 1,71% |
| MMAE Arit | 1,98% | 0,94% | 0,95% | 2,07% | 1,14% | 1,16% |
| MMAE Geom | 1,75% | 0,98% | 1,00% | 1,83% | 1,16% | 1,18% |
| MMAE Harma | 1,60% | 1,02% | 1,04% | 1,65% | 1,18% | 1,20% |
| MMAE H&R | 1,79% | 0,42% | 0,43% | 2,49% | 1,32% | 1,27% |
| MMAE BS | 3,75% | 0,41% | 0,40% | 3,64% | 1,37% | 1,34% |
| | | | | | | |
| SMAE Gates | 0,32% | 0,26% | 0,27% | 0,30% | 0,27% | 0,31% |
| SMAE Arit | 0,37% | 0,16% | 0,16% | 0,33% | 0,19% | 0,20% |
| SMEA Geom | 0,32% | 0,18% | 0,19% | 0,29% | 0,20% | 0,21% |

| | | | | | | |
|---|---|---|---|---|---|---|
| SMEA Harma | 0,28% | 0,19% | 0,20% | 0,28% | 0,20% | 0,21% |
| SMEA H&R | 0,57% | 0,09% | 0,09% | 0,64% | 0,17% | 0,20% |
| SMEA BS | 0,50% | 0,06% | 0,06% | 0,47% | 0,18% | 0,20% |

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