POLITECNICO DI TORINO

Master's Degree in Mechatronic Engineering

Master's Degree Thesis

Advanced algorithms for the simulation of Li-ion batteries

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July 2024

Abstract

Reducing fossil fuel consumption requires efforts on the improvement of energy storage technologies. The Li-ion cell is the only technology that allows to do that in the most efficient ways. Bat-man is a novel technology that provides very accurate diagnosis of lead acid batteries. The purpose of the thesis is to understand and predict the behaviour of the Li-ion batteries. To do this, in absence of a real battery to charge and discharge, a Simulink model is built for the simulation. This model is a current controlled voltage generator and has high accuracy but also high complexity because of the presence of 3 RC groups. To reduce the complexity of this, a new model with 1 RC group for the identification is built using the data collected from the previous one. Then the parameters obtained from the identificated model have been approximated with functions. Finally there is the validation step, whose purpose is to test the two models. The two approaches of validation are open-loop and closed-loop. The closed-loop utilizes the kalman filter in order to directly control the states and to correct the errors.

Acknowledgements

“Today is difficult, tomorrow is much more difficult, but the day after tomorrow is beautiful”
Jack MA

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Acronyms

SOC

state of charge

SOH

state of health

OL

open loop

CL

closed loop

EKF

extended kalman filter

Chapter 1

Introduction

Only about half the incoming solar energy is absorbed by the Earth's surface. The rest is scattered back and to some extent absorbed by the atmosphere (clouds included), or reflected by the ground. The Earth itself radiates at wavelengths much longer than those of solar radiation. This Earth radiation, unlike the solar radiation, is strongly absorbed in the atmosphere. The absorption is mainly caused by water vapor and clouds, but also by some trace gases. Only a very small part of the radiation emitted by the ground escapes directly to space. In this way the atmosphere is heated, and returns energy to the Earth's surface, where it is again absorbed and re-radiated. Thus a remarkable exchange of thermal energy takes place between the ground and the lower atmosphere. These processes, somewhat misleadingly called the "green house" effect, are responsible for the relatively high mean surface temperature on the Earth, of about 14°C. If the Earth had no atmosphere the corresponding temperature would be about -18°C. The term "greenhouse effect" is now often used to name a predicted increase in the temperature of the lower atmosphere as a consequence of man's release of CO_2 and other trace gases to the atmosphere. This predicted additional effect will in the following be referred to as an increasing "green house effect" or "green house" warming. In the Earth's atmosphere dry air consists of nearly 78% (by volume) nitrogen (N_2), about 21 % oxygen (02) and about 1% argon (Ar). In humid air the water vapor content varies from about 3% in the tropics to a small fraction of this quantity in the polar regions. Carbon dioxide ($C0_2$) is just a trace component, with a concentration of about 0.035% (= 350 ppm), but it plays an important part in plant and animal life processes. The concentration of CO_2 varies with time and place. It has been found, for example, that the concentration may double during one single day over a wheat field (Fergusson, 1985). During the 1970s and the first half of the 1980s several climatic model computations predicted that for a hypothetical doubling of the average atmospheric CO_2 concentration during the next 60 years, the average global temperature will increase by 1 to 5°C (see e.g. review by Braathen et al., 1989), that the polar regions will warm more than the lower latitudes, up to 8 to 10°C (Schneider, 1975; Manabe and Wetherald, 1980), and that the seasonal variations will be greatest in the north polar regions (Ramanathan et al., 1979). These models also predicted considerable changes in the geographical distribution of precipitation. At the end of the 1980s more sophisticated models revised the earlier predictions substantially, decreasing the net impact on the climate and changing its geographical distribution. A recent study estimated a 1.2°C increase in the surface and tropospheric temperature due to doubling the atmospheric CO_2 (Lorius et al., 1990), assuming no feedback processes. To the present natural global atmospheric flow of CO_2, man's buming of fossil carbon may add somewhere between 0.1 and 3.6%, according to different estimates. CO_2 is one of about 40 trace "greenhouse gases" present in the atmosphere (Ramanathan et al., 1985). Water vapor contribute the most to the total "greenhouse effect" of the atmosphere of about ISO W/m2 (Raval and Ramanathan, 1989). According to Kondratyev (1988) H_2O contributes about 62%, CO_2 21.7%, O_3 7.2%, N_2O 4.2%, CH_4 2.4%, and other gases 2.4% to the mean "greenhouse effect" of the atmosphere. A doubling of CO_2 would increase its "greenhouse" contribution to about -4 W/m^2 (Raval and Ramanathan, 1989 a). Landsberg (1974) estimated that only 3% decrease in atmospheric water vapor, and a 1% increase in cloudiness can compensate the warming from an antlcipated CO_2 doubling (other conditions held constant). As a whole, the influence of clouds on atmospheric temperature is still an unsolved problem (e.g. Schlesinger and Mitchell, 1987). The predictions of CO_2 doubling are based on an assumption that all past human activities have contributed about 21 % of the current atmospheric CO_2, the level of which is supposed to be 25% higher than in the pre-industrial period (IPCC, 1990). This assumption is based on glacier studies. As will be seen later on these studies do not provide a reliable basis for such an estimate. The level of atmospheric CO_2 depends on constantly changing thermodynamic equilibria between its sources and sinks. Oceanic flows of this gas in and out of the global atmosphere are important for the CO_2 budget. Even very small natural fluctuations of these oceanic flows can mask the man-made CO_2 inputs into the global atmosphere. Several studies have suggested that radiative heating of 4 W/m^2 caused by the doubling of atmospheric CO2 would lead to a global warming of 3.5 to 5°C (Hansen et aL, 1984; Wilson and Mltchell, 1987; Washington and Meehl, 1984; Wetherald and Manabe, 1988). The total present global mean warming due to all trace "green house" gases added by man of about 2 W/m^2, is below the estimated natural variation of about ± 5 to 10 W/m^2 in the global net radiation (Raval and Ramanathan, 1989). The positive (warming) forcing by clouds is about 30 W/m^2 (Raval and Ramanathan, 1989). This "greenhouse effect" of clouds is approximately fifteen times larger than that resulting from a hypothetical doubling of CO_2 (increase from -2 to 4 W/m^2). A new estimate of Ramanathan et al. (1989 b) suggests that "the CO_2 concentratlon In the atmosphere has to be increased more than two orders of magnitude to produce a "greenhouse effect" comparable to that of clouds". The negative cloud forcing (due to a high albedo at the upper cloud surface) is about -50 W/m^2. From such figures one gets a net cloud forcing of about -20 W/m^2 (30 minus 50), i.e. much higher than the claimed man-made positive forcing of CO_2 doubling. Ramanathan et al. (1989 b) demonstrated that the clouds have a large net cooling effect on the Earth, which will offset the possible increasing "greenhouse effect" warming. This is because an increase in the global temperature will increase the amount of clouds in the troposphere, introducing a strong negative radiative feedback. It is claimed that the total past anthropogenic "green house" forcing (due to CO_2 and other trace gases) between 1850 and 1985 should cause a global surface warming of 0.8 to 2.4°C (Ramanathan et aL, 1989 bl. However, no such warming has been observed, which may indicate that the estimate of the "green house gases" increase is incorrect or that the negative cloud forcing of about -20 W/m^2 (or some other negative forcings) is sufficiently large to stabilize the increasing "green house effect" warming. This latter supposition was confirmed by Slingo (1989) who found that the radiative forcing by doubled CO_2 concentrations can be balanced by modest increases in the amount of low clouds. Wigley et al. (1989) revived the idea posed by Mitchell (1975) that SO2 derived cooling may offset considerably the "greenhouse" warming. SO_2 originates from dimethylsulfide from the oceans (Charlson et al., 1987), volcanic emissions, and from man-made sources. The cooling effect is partly due to the absorption of incoming solar radiation by sulfuric acid in the stratosphere and partly due to an increase of cloud condensation nuclei in the atmosphere. The latter effect is due to H_2SO_4 and sea-salt aerosols (Latham and Smith, 1990) and serves to "brighten" clouds (increase their albedo), thereby reflecting part of the incoming solar radiation back into space. Satellite data now confirm that the stratocumulus clouds (one of the most common cloud types on Earth, and the variety most likely to be affected by an increasing number of condensation nuclei) are indeed considerably brighter in the lee of regions of major anthropogenerated SO_2 emissions (Cess, 1989). Wigley (1989) argued that man-made SO_2 is sufficiently large to offset significantly the global warming that might result from the "greenhouse effect", and to cool the Northern Hemisphere relative to the Southern Hemisphere, because most of the man-made SO_2 emissions occur in the Northern Hemisphere. Wigley (1989) supposed that the man-made SO_2–derived (negative) forcing might explain the inconsistency between General Circulation Model (GCM) predictions of current warming and observations. To substantiate this Wigley cited two sulfate records from ice collected in southern Greenland showing up to threefold increase during the twentieth century. The temperature in this region is high enough to allow summer melting, which may lead to changes in chemical composition of snow and ice (Jaworowski et al., 1992). However, seven other studies in the Arctic and five studies in Antarctica demonstrated no increase of sulfate or acidity in snow and ice during the past century. These studies indicate that there were covariations of the sulfate content in precipitation from the Southern and Northern Hemisphere in relation to major volcanic events, and that during the last decades the concentration of sulfate in precipitation in the Arctic was similar to that in Antarctica (Jaworowski, 1989). Thus Wigley's hypothesis Is not substantiated. A main feature of the predictions of almost all climate models is a relatlvely large warming at high latitudes. Therefore polar regions may be assumed to be the most promising ones for detection of any current increasing "greenhouse effect" warming. Temperature and to some degree glacier records can be used to check these model predictions. A discussion of computer modelling with the help of GCM is, however, beyond the scope of this report. Cess et al. (1989) compared 14 different models of this kind and showed that from the same input data the models produced results which varied greatly, i.e., both cooling and warming of the climate. Also in Cess et al. (1991) the net effect of snow feedback produced by 17 models differed markedly, ranging from cooling to warming. In this latter paper it was demonstrated that the conventional explanation that a warmer Earth will have less snow cover, resulting in a darker planet, absorbing more solar radiation, is overly simplistic. As will be seen from the discussion below, the hypothesis of an imminent climatic change is based on data subject to serious uncertainties and inconsistencies. These uncertainties should be factored into policy decisions in view of the staggering costs of implementation of "anti-greenhouse" decisions on a global scale. In the United States alone these costs may reach 3.6 trillion US dollars (Passel, 1989). Implementation of the CO_2 tax of 500 US dollars per metric ton of carbon would increase the price of crude oil about 3.7 times (to more than 60 dollars per barrel) and of utility coal about 8.3 times (to more than 276 dollars per short ton) (Anonymous, 1992). This might have serious negative social consequences both for developed and third world countries reaching beyond the 21st century. Such consequences should be weighed against the very uncertain predictions of environmental effects of an increase in atmospheric CO_2. In most cases scientists are aware of the weak points of their basic assumptions and simplifications needed to interpret the results of measurements or to create models. However, these uncertainties are mostly ignored or banished to a subordinate clause when the results are presented by politicians or mass media. In the process of forming environmental policy the preliminary hypothesis are transformed into "reliable facts" when presented to the public. The magnitude of "normal" natural reservoirs, fluxes, and variations are not presented and not compared to claimed "abnormal" anthropogenic contributions. A more balanced view is certainly needed. The most important basis of the hypothesis of man-made climatic warming due to buming of fossil carbon fuels are the measurements of CO_2 in air and in glacier ice, hydrogen and oxygen isotopes in glacier ice, carbon isotopes in tree rings, and 100-150 years long atmospheric temperature records. In this paper we critically review these measurements and their interpretations, in order to test the nowadays widely accepted postulate that "the change in atmospherie CO_2 is not just a fluctuation of nature, but is predominantly the consequence of the activities of mankind - chiefly the burning of fossil fuels such as coal, gas and oil" (Rowland and Isaksen, 1988). We also compare the quantities of anthropogenic contributions with known fluxes of natural reservoirs, and discuss air temperature and glacier balanee records, which should reveal signals of an increasing "greenhouse effect". The present energy economy based on fossil fuels is at a serious risk due to a series of factors, including the continuous increase in the demand for oil, the depletion of non-renewable resources and the dependency on politically unstable oil producing countries. Another worrying aspect of the present fossil fuel energy economy is associated with CO_2 emissions, which have increased at a constant rate, with a dramatic jump in the last 30 years, the CO_2 level has almost doubled passing from 1970 to 2005, this resulting in a rise in global temperature with associated series of dramatic climate changes. The urgency for energy renewal requires the use of clean energy sources at a much higher level than that presently in force. The CO_2 issue, and the consequent air pollution in large urban areas, may be only solved by replacing internal combustion engine (ICE) cars with ideally, zero emission vehicles, i.e. electric vehicles (EVs) or, at least, by controlled emission vehicles, i.e. full hybrid electric vehicles (HEVs) and/or plug-in electric vehicles (PHEVs). This replacement can be done with the use of $li-ion$ batteries. Lithium batteries are characterized by high specific energy, high efficiency and long life. These unique properties have made lithium batteries the power sources of choice for the consumer electronics market with a production of the order of billions of units per year. These batteries are also expected to find a prominent role as ideal electrochemical storage systems in renewable energy plants, as well as power systems for sustainable vehicles, such as hybrid and electric vehicles. However, scaling up the lithium battery technology for these applications is still problematic since issues such as safety, costs, wide operational temperature and materials availability, are still to be resolved. Nowadays, the most important problem is that the cycle life of a package of batteries when it leaves the factory is 20% less than expected. In the work of this thesis, the use of advanced algorithms is done to analize carefully the behaviour of the $li-ion$ batteries in order to guarantee the maximum duration of their cycle life and this is done by constantly monitoring the SoH.

Chapter 2

Lithium ion battery

2.1 Introduction

Lithium ion batteries are made up of four main components: the anode or negative electrode usually made of graphite, the cathode or positive electrode usually made of cobalt oxide and lithium, the separator or layer of generally plastic material which prevents the electrodes are touching each other and the electrolyte. The electrolyte is usually a solution of lithium perchlorate in ethylene carbonate and is the body within which the lithium ions move. When the battery is charged, the lithium ions move from the cathode to the anode, which transfers the electrons through the cathode closing the circuit, in fact this process causes an oxidation reaction in the anode and a reduction reaction in the cathode. The reverse cycle happens when we discharge the battery, with the lithium ions returning from the anode to the cathode, thus generating electricity. Among the advantageous characteristics of this type of battery we find: high density of available energy, large quantities of current for high-power applications high power), low self-discharge capacity (no programmed cycle to maintain battery life), no memory effect. The battery is the set of multiple cells (or elements) with metal collectors and sheets of polymeric material, connected to each other in series or parallel.

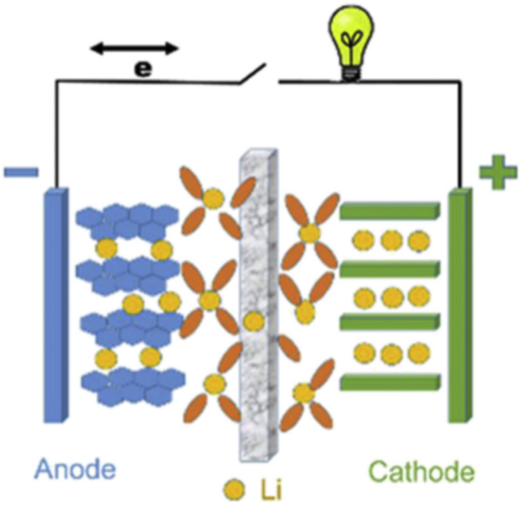

Figure 2.1: Schematic of Li-ion battery (LIB)

2.2 Advantages

2.2.1 High energy density

Thanks to the higher energy density compared to traditional batteries, lithium ion batteries have the ability to store greater quantities of energy in limited volumes and weights, making the devices that use them lighter and more compact

2.2.2 Long duration

Lithium batteries have a very long life compared to other technologies, with a high number of charge/discharge cycles before losing performance. This makes them suitable for a wide spectrum of applications, from mobile telephony to energy storage for renewable energy

2.2.3 Low memory effect

Unlike other battery technologies, lithium batteries are not subject to the memory effect, i.e. the gradual loss of maximum charge capacity following partial cycles. This makes them more reliable and long-lived over time.

2.3 Safety

Despite the numerous advantages presented, lithium ion batteries have a series of problems related to safety aspects. When their use is abused, for example internal short circuit induced by an accident or strong mechanical impact, external short circuit following a fault, overloading of the battery beyond the maximum voltage specified in the technical data sheet, excessive currents during the charging phases and of discharge, the cells in lithium ion batteries can undergo a process called thermal runaway in which there is a rather sudden increase in the temperature of the single cell and generation of flammable gases capable of setting them on fire.

2.3.1 Electrical abuse

Electrical abuse can result from overloading a cell to a voltage that is too high (generally > 4.2 V) or if a higher current is passed. In this case there is a loss of battery capacity, or the formation of dendritic structures which can lead to short circuits and the production of heat due to the Joule effect, with local or generalized overheating of the battery or of the single cell and therefore the possibility of thermal runway. . The over-discharge effect, however, occurs when the voltage drops below a given minimum value (generally <2 V), following which the dissolution of the current collector and during charging can occur, of micro-short circuits.

2.3.2 Mechanical abuse

Mechanical abuse is related to external phenomena due to the manipulation of batteries such as accidental punctures, falls, crushing, etc.

The consequences can be immediate, but also delayed, depending on the extent of the mechanical abuse and its impact on the individual battery cells. Mechanical fatigue, like aging, depends on the internal changes that the interface undergoes during the charging and discharging phase of the cell, which can lead to cell breakage with consequent thermal release.

2.3.3 Thermal abuse

Thermal abuse depends on the insufficient dispersion of heat that is created internally to the individual cells or due to a high external heat flow such as exposure to direct sunlight or exposure to open flames.

General or localized overheating of the cell or battery can thus lead to thermal runaway resulting in a fire. Both mechanical and electrical abuse are therefore expressed in thermal effects which can range from swelling of the cell, to loss of sealing with leakage of chemical substances making up the cell, to the emission or expulsion of solvents or other products with or without fire, venting, until the cell breaks with relative explosion and fire.

Batteries can however be equipped with protection devices both at cell and battery level. The first level of protection is however inherent in the design choices aimed at guaranteeing the best heat exchange conditions and maintaining the cells at a constant temperature, inside the working window, while the second level of protection is given by devices external to the cell , but present in the battery which vary depending on the various types of construction.

2.4 Conclusions

Lithium-ion batteries represent a key technology in energy storage solutions, thanks to their high energy density, lack of memory effect and low self-discharge rate. However, it is essential to understand the specific characteristics of these batteries to optimize their use and extend their useful life. Their versatility makes them suitable for a wide range of applications, from microelectronics to automotive, playing a crucial role in contemporary technological evolution.

Chapter 3

Simulation

3.1 Simulink model

The simulink model is built according to several rules to perform the simulation of the discharge of the Li-ion battery for all the input currents from 0.5C to 2C. In Figure 3.1 we can see that first there is a Discharge unit composed by a constant pulse current, which is a simple constat value, and a Variable pulse current composed by a state flow chart that simulates the variable square wave in amplitude and period. Then these two inputs are sent to a switch block controlled by a constant to decide which of these two will be sent in input to the cell model. The input is still filtered by another switch block in which is decided that if the State of Charge of the battery is positive then the input is free to go in the cell model 3RC, otherwise the input will be the constat value put in the simulink model. Then there is the Cell model 3 RC: in this block there is the MATLAB function that describes the differential equations that rules the battery with all the errors parameters. Finally the output of the Cell model 3RC block,which is the terminal voltage of the battery, is monitored by a scope. The simulation of the discharge of the batteries for all the currents cases are performed thanks to a MATLAB paramgen file, in which all the parameters of the simulink model are assigned to a specific value in this script and then the file generated by this script is sent in input to the simulink program to perform the desired simulation.

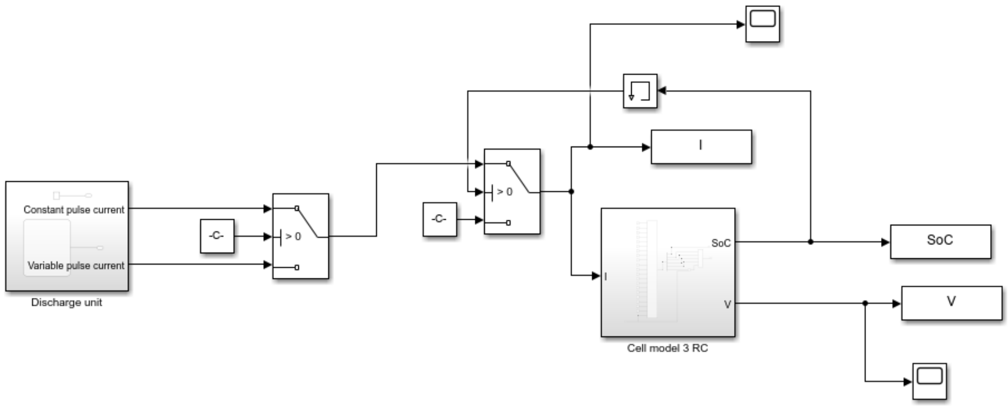

Figure 3.1: Simulink model of the battery

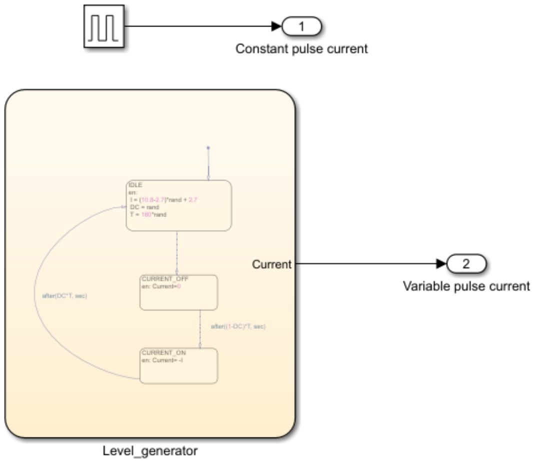

Figure 3.2: Discharge unit

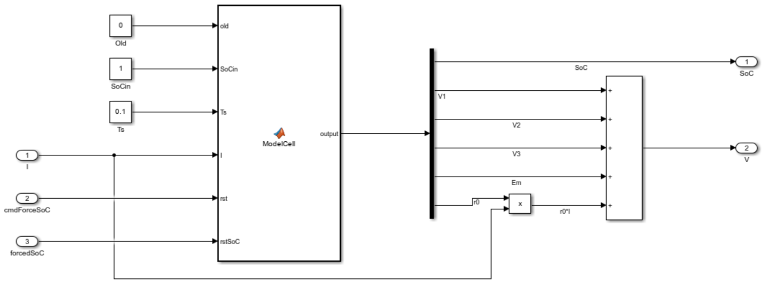

Figure 3.3: ModelCell 3RC

3.2 Mathematical model

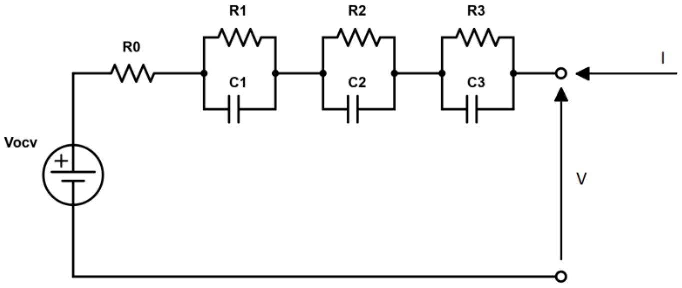

Figure 3.4: Circuital scheme of the 3RC group battery

The circuital scheme in Figure 3.4 can be mathematical written as:

$$V_T = V_{OCV} + R_0 I + V_1 + V_2 + V_3 \tag{3.1}$$

Using Laplace we resolve the three dynamical voltages:

$$V(s) = \frac{R}{sCR+1} I(s) \tag{3.2}$$

$$V(s) \cdot sCR + V(s) = RI(s) \tag{3.3}$$

We define the time constant τ:

$$\tau = \frac{1}{RC} \tag{3.4}$$

So we can write the equation as:

$$V(s) \cdot \frac{s}{\tau} + V(s) = RI(s) \tag{3.5}$$

Then trasforming in time domain:

$$V(t) = -\frac{1}{\tau} \cdot \frac{dV(t)}{dt} + Ri(t) \tag{3.6}$$

To resolve that equation we integrate:

$$V(t) = \int_{t_0}^{t_1} (-\tau V(t) + \tau Ri(t)) dt + V(t_0) \tag{3.7}$$

In real world time is discretized so we consider a sample time T_s:

$$V(k+1) = V(k) + [-\tau V(k) + \tau Ri(k)] T_s \tag{3.8}$$

Now it's possible to write the 3 state space equation associated to the circuit:

$$\begin{cases} V_T(k) = V_{OCV} + R_0 I(k) + V_1(k) + V_2(k) + V_3(k) \\ V_1(k+1) = V_1(k) + [-\tau_1 V_1(k) + \tau_1 R_1 i(k)]T_s \\ V_2(k+1) = V_2(k) + [-\tau_2 V_2(k) + \tau_2 R_2 i(k)]T_s \\ V_3(k+1) = V_3(k) + [-\tau_3 V_3(k) + \tau_3 R_3 i(k)]T_s \end{cases} \tag{3.9}$$

3.3 Discharging simulation

From the simulations we can see that the difference between old and new battery is the transient: in the old one the transient requires a little more time to reach the standard value and this can be seen by the curve of the graphs that is much more rounded than the new one. The other difference is that in the old one the downward spike is much more accentuated than in the new one.

- New battery

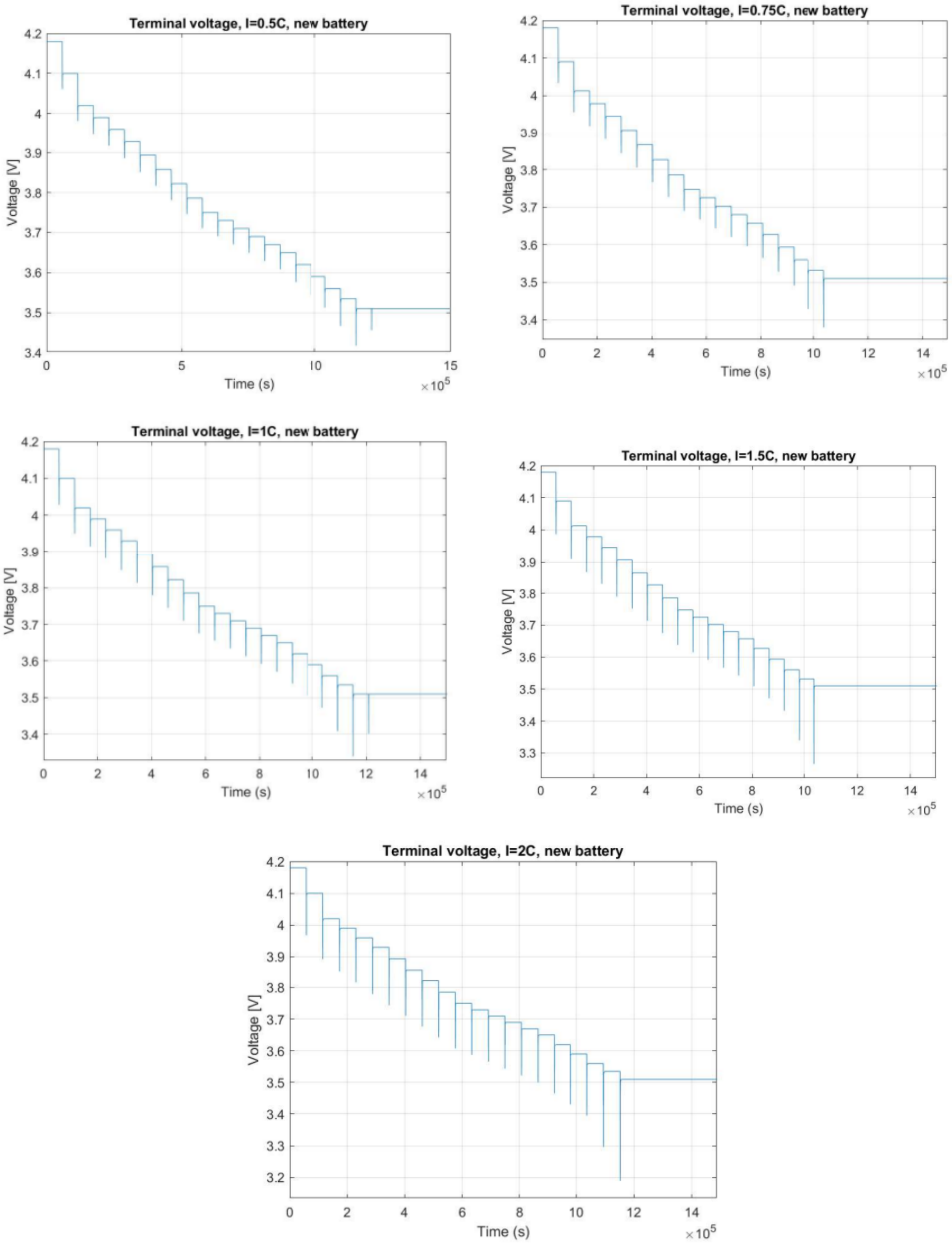

Figure 3.5: New battery, discharge simulation

- Old battery

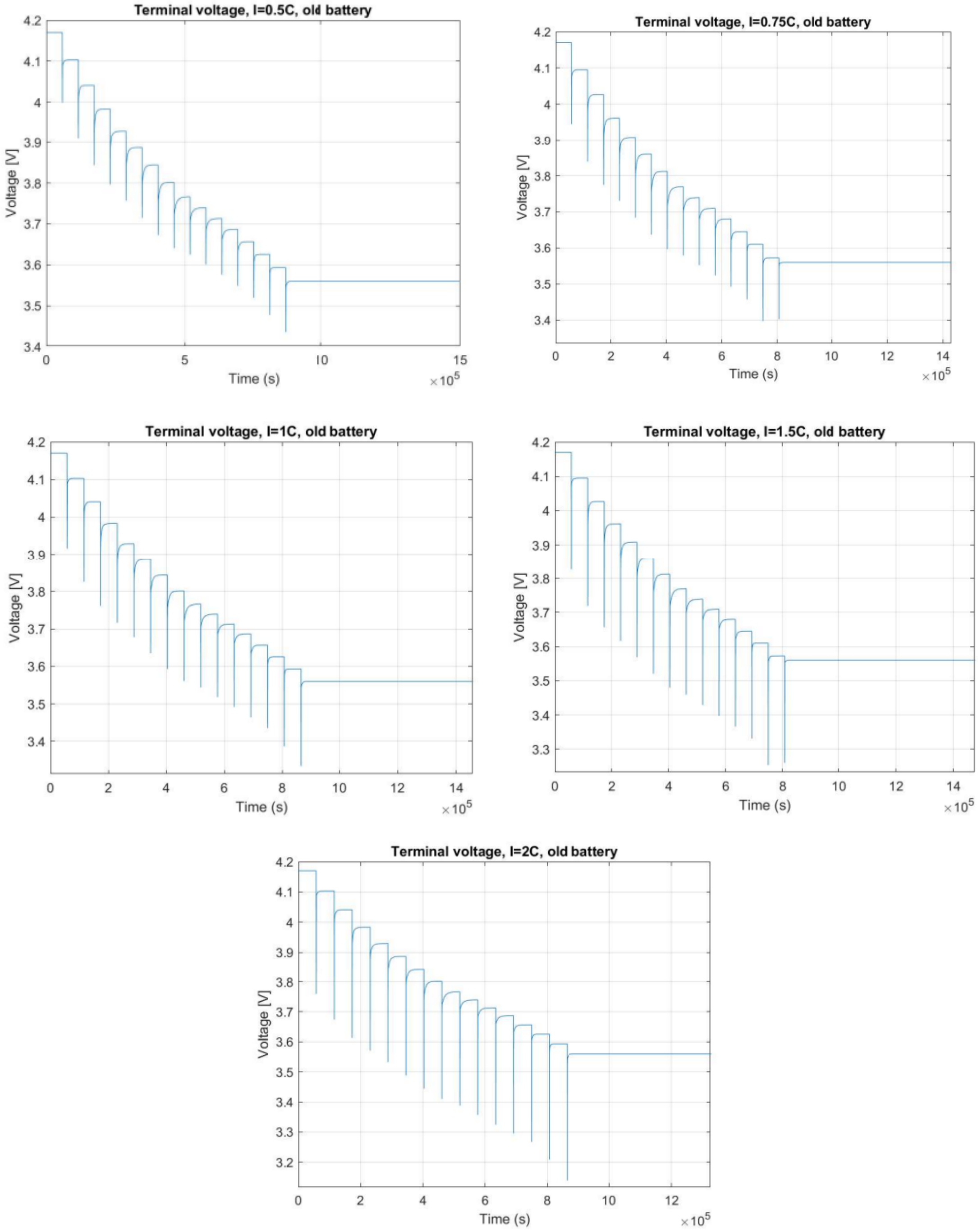

Figure 3.6: Old battery, discharge simulation

Chapter 4

Identification

In the Identification model we pass from 3RC group to 1RC group because of the complexity of the 3RC, in fact a human mind cannot analyze the cross dependency of 8 parameters that depend on other 3: we have Vocv(no-load voltage),R0(series resistance),and parameters of RC groups (R1,R2,R3,C1,C2,C3) that depend on I(discharging current), SoC (state of charge) and SoH (state of health). So we take only 1 RC group and the problem now is trying to match the new model with the old one and this can be done using the Least-Square method so we take input data outgoing from the 3RC(V,I,SoC,SoH) model and we identify the new model with 1RC.

4.1 Mathematical model

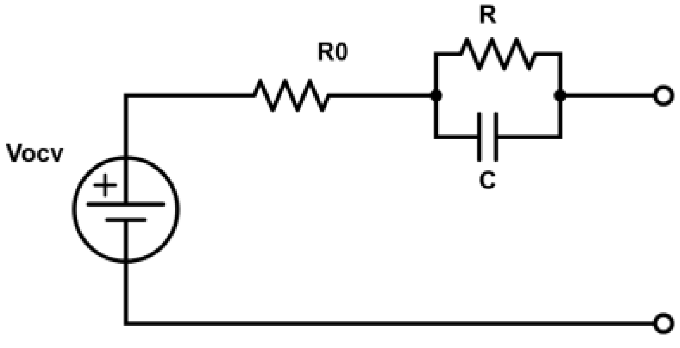

Figure 4.1: Circuital scheme of the 1RC group battery

The state space representation of the system is:

$$\begin{cases} V_T(k) = V_{OCV} + R_0 i(k) + V(k) \\ V(k+1) = V(k) + [-\tau V(k) + \tau R i(k)]T_s \end{cases} \tag{4.1}$$

The problem must be written according to the LS theory: V(k) and V(k-1) must be expressed as functions of known data:

$$V(k) = V_T(k) - V_{OCV}(k) - R_0 I(k) \tag{4.2}$$

$$V(k-1) = V_T(k-1) - V_{OCV}(k-1) - R_0 I(k-1) \tag{4.3}$$

Substituting in the second equation:

$$\begin{aligned} V_T(K) - V_{OCV} - V_T(k-1) + V_{OCV}(k-1) \\ = (R_0 + R\tau T_s)I(k) - R_0 I(k-1) - \tau T_s[V_T(k-1) - V_{OCV}(k-1) - R_0 I(k-1)] \end{aligned} \tag{4.4}$$

Direct dynamical model identification:

$$\begin{aligned} V_T(k) - V_{OCV}(k) - V_T(k-1) + V_{OCV}(k-1) \\ = (R_0 + R\tau T_s)I(k) + (R_0\tau T_s - R_0)I(k-1) - (V_{OCV}(k-1) - V_T(k-1))\tau T_s \end{aligned} \tag{4.5}$$

Defining the three slack variables:

$$\begin{cases} \alpha = R_0 + R\tau T_s \\ \beta = R_0\tau T_s - R_0 \\ \gamma = \tau T_s \end{cases} \tag{4.6}$$

4.2 Least square

The vector of known values:

$$Y = \begin{bmatrix} V_T(2) - V_{OCV}(2) - V_T(1) + V_{OCV}(1) \\ ... \\ V_T(k) - V_{OCV}(k) - V_T(k-1) + V_{OCV}(k-1) \end{bmatrix} \tag{4.7}$$

And the vector:

$$X = \begin{bmatrix} I(2) & I(1) & V_{OCV}(1) - V_T(1) \\ ... & ... & ... \\ I(k) & I(k-1) & V_{OCV}(k-1) - V_T(k-1) \end{bmatrix} \tag{4.8}$$

The overall identification problem can be written as:

$$Y = X \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \tag{4.9}$$

And the original parameters are expressed as function of the three slack variables:

$$R0 = \frac{\beta}{\tau T_s - 1} \tag{4.10}$$

$$R = \frac{\alpha - R_0}{\tau T_s} \tag{4.11}$$

$$\tau = \frac{\gamma}{T_s} \tag{4.12}$$

4.2.1 Parameters estimated LS

- New battery

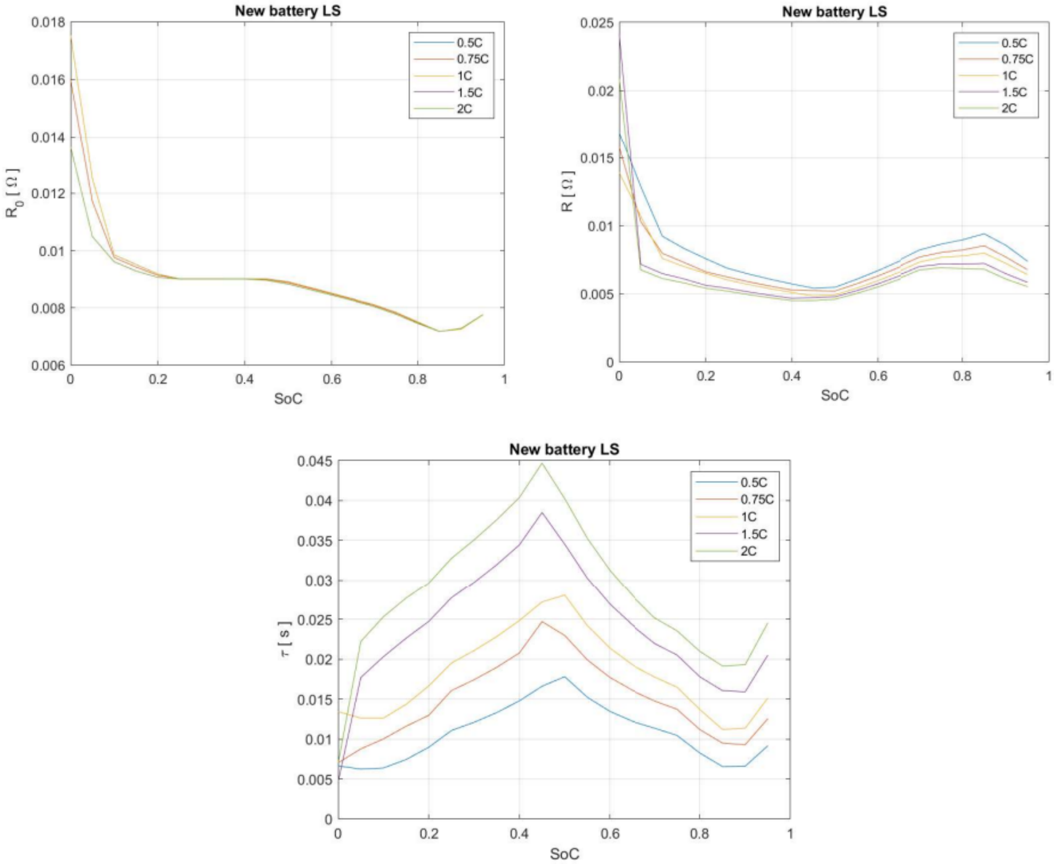

Figure 4.2: Dynamical parameters of the new battery, LS

- Old battery

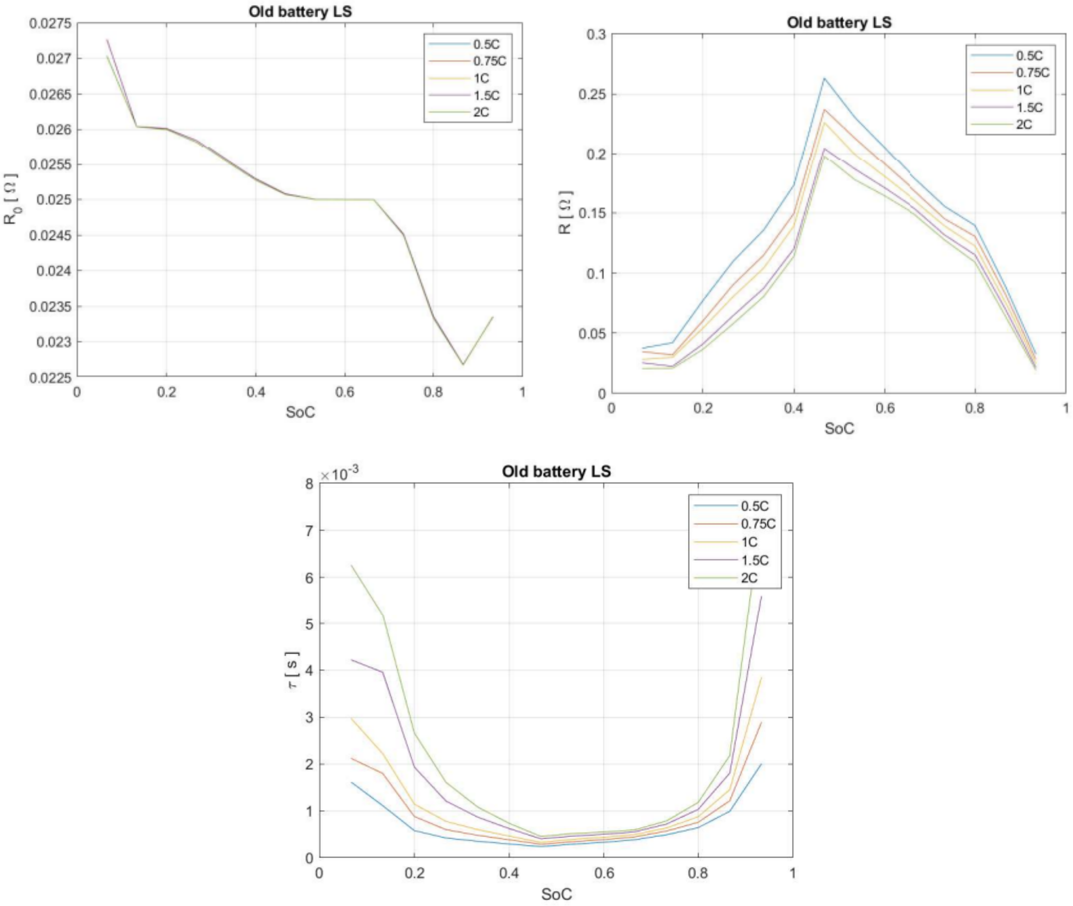

Figure 4.3: Dynamical parameters of the old battery, LS

4.3 Recursive least square

The same procedure can be done by a recursive least square algorithm, in particular in this case we use the RLS-2:

- Time update:

$$R(t) = (1 - \frac{1}{t})R(t-1) + \frac{1}{t}\phi(t)\phi(t)^T \tag{4.13}$$

- Algorithm gain:

$$K(t) = \frac{1}{t}R(t)^{-1}\phi(t) \tag{4.14}$$

- Prediction error:

$$\epsilon(t) = y(t) - \phi(t)^T\hat{\theta}_{t-1} \tag{4.15}$$

- Estimate update:

$$\hat{\theta}_t = \hat{\theta}_{t-1} + K(t)\epsilon(t) \tag{4.16}$$

4.3.1 Parameters estimated RLS-2

- New battery

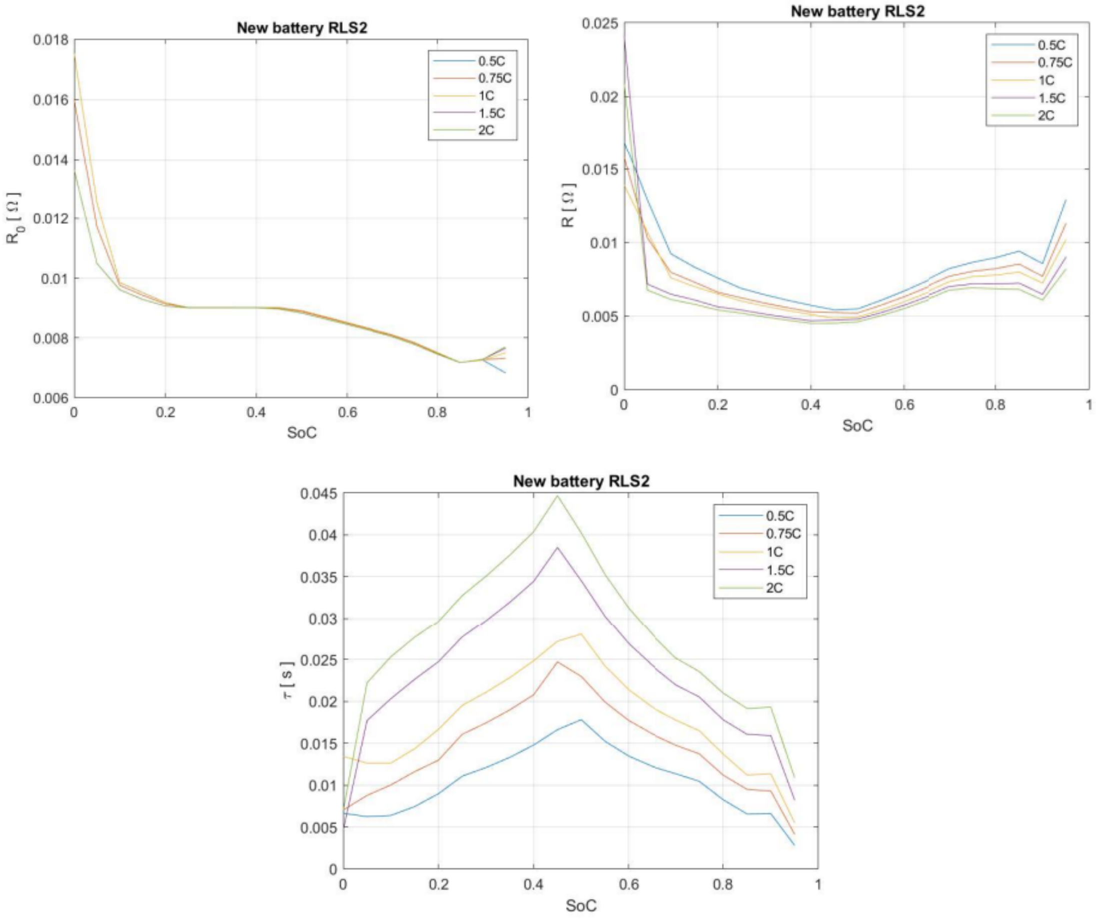

Figure 4.4: Dynamical parameters of the new battery, RLS-2

- Old battery

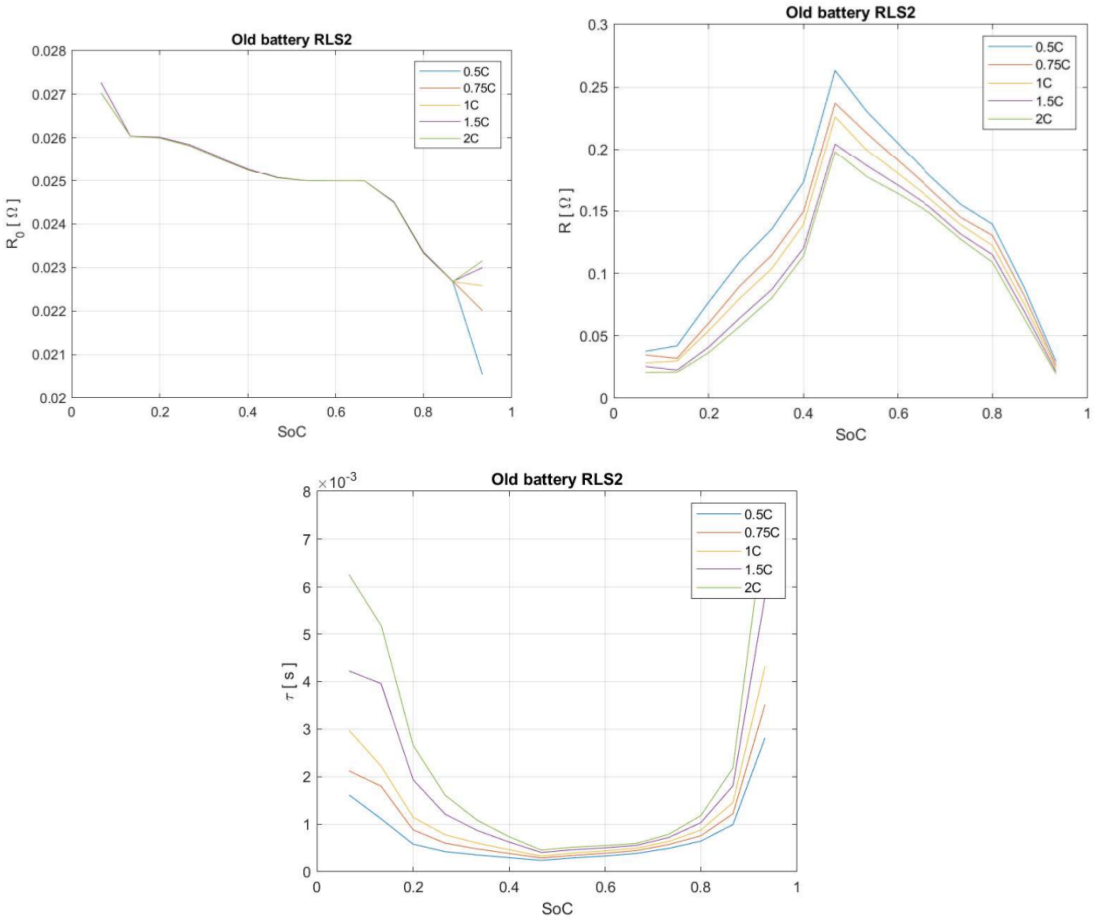

Figure 4.5: Dynamical parameters of the old battery, RLS-2

The convergence between parameters estimated with LS and parameters estimated with RLS-2 is quite good.

Chapter 5

Approximation

In this chapter we try to approximate the parameters obtained from the LS one with appropriate functions:

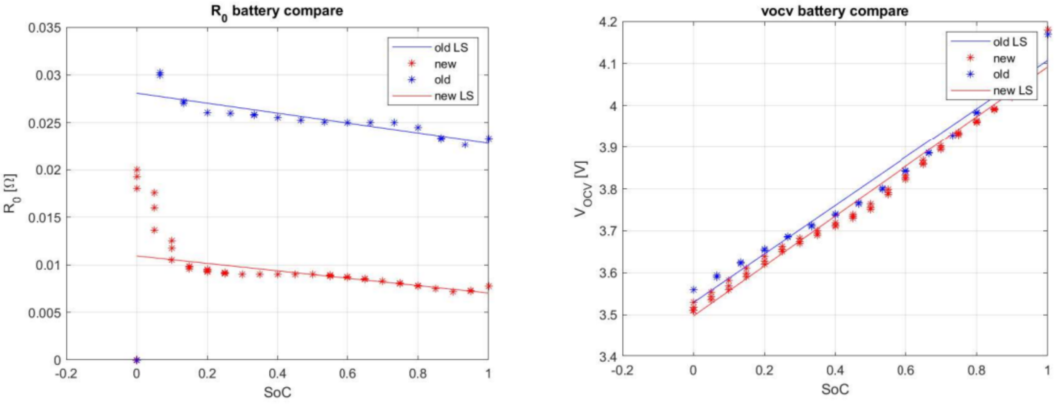

Figure 5.1: R_0 and V_{OCV} approximations

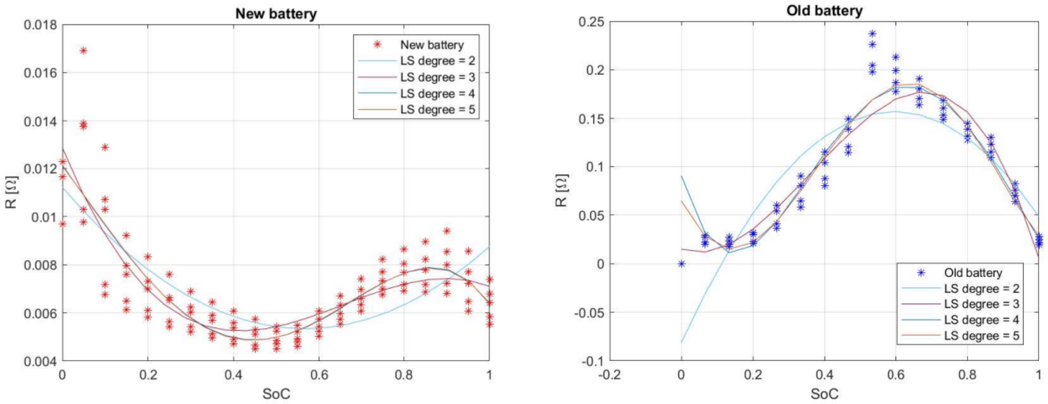

Figure 5.2: R new and old battery

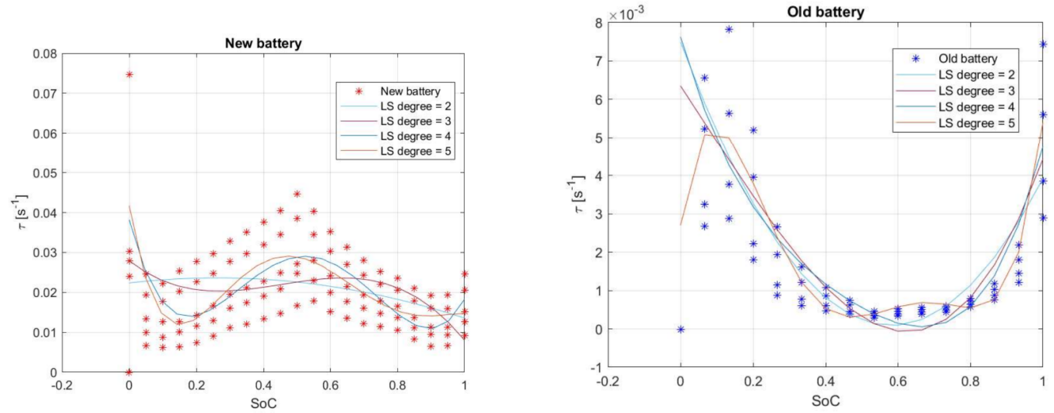

Figure 5.3: τ new and old battery

Best approximations:

- R_0 and V_{OCV}: linear
- R new and old battery: cubic
- τ new and old battery: quadratic

Chapter 6

Validation

6.1 Open loop validation

The aim of the OL approach is to understand if I did a good job with the identification so it is useful in this sense, obviously this can be demonstrated by the error that is the difference between the output of the two models, the original one and the identificated one: if the error is low, the system is quite similar to the identificated one, otherwise we'll need another approach, the CL one that we'll see in the next chapter. Obviously the error will be very big because of the approximation done to build the identificated system.

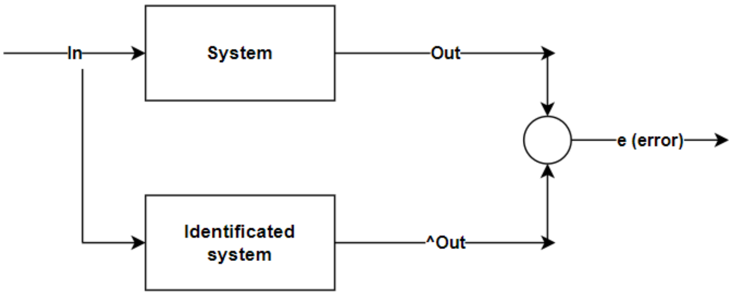

Figure 6.1: Block scheme of the open loop control

6.1.1 Mathematical model

The state space model used in this approach is:

$$\begin{cases} V(k+1) = V(k) + [-\tau V(k) + \tau R i(k)] T_s \\ SoC(k+1) = SoC(k) + \frac{T_s}{3600 \cdot Capacity} i(k) \\ V_T(k) = V(k) + V_{OCV} + R_0 i(k) \end{cases} \tag{6.1}$$

6.1.2 Discharge simulation

Constant pulse current

- New battery

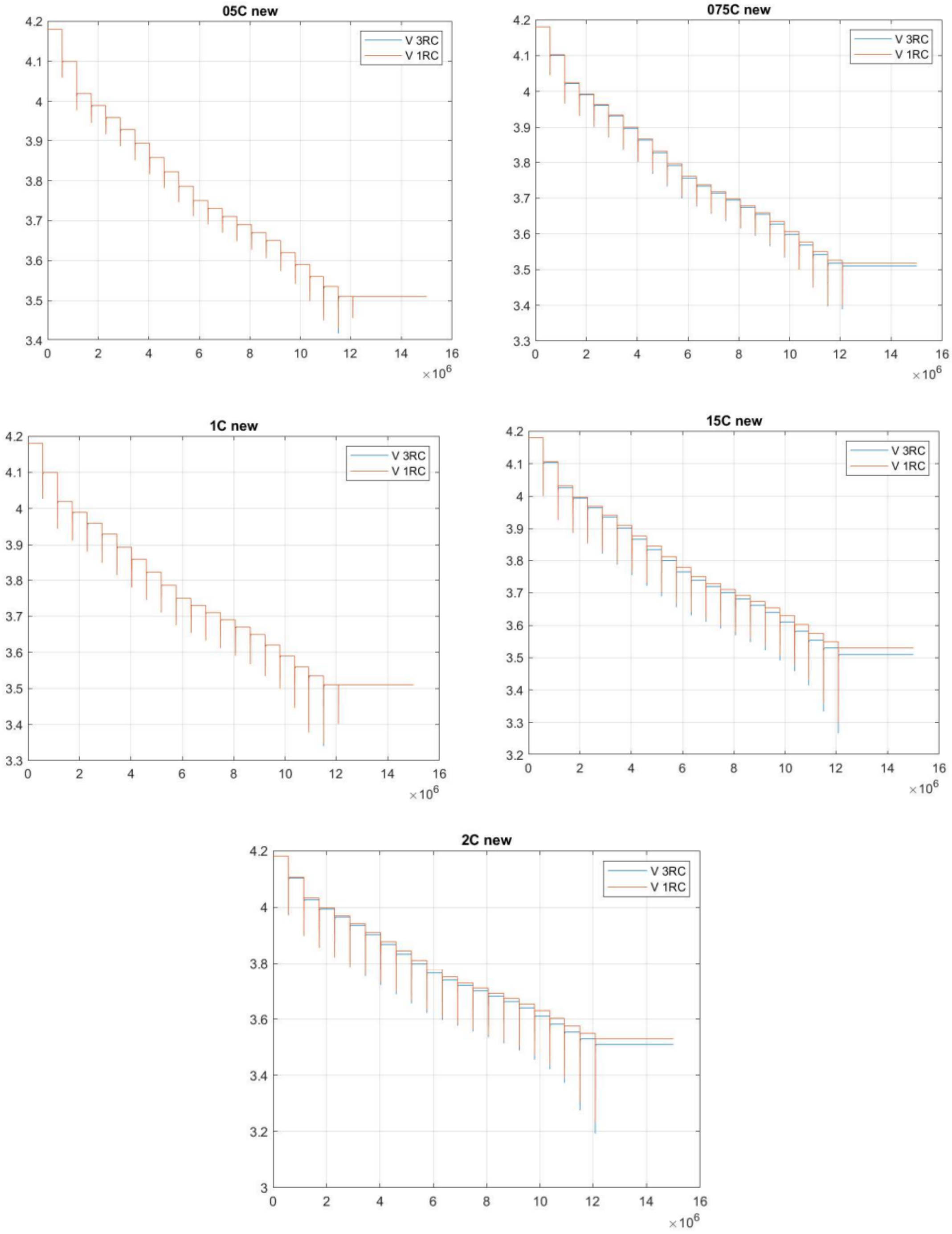

Figure 6.2: New battery, constant discharge simulation 3RC-1RC OL

- Old battery

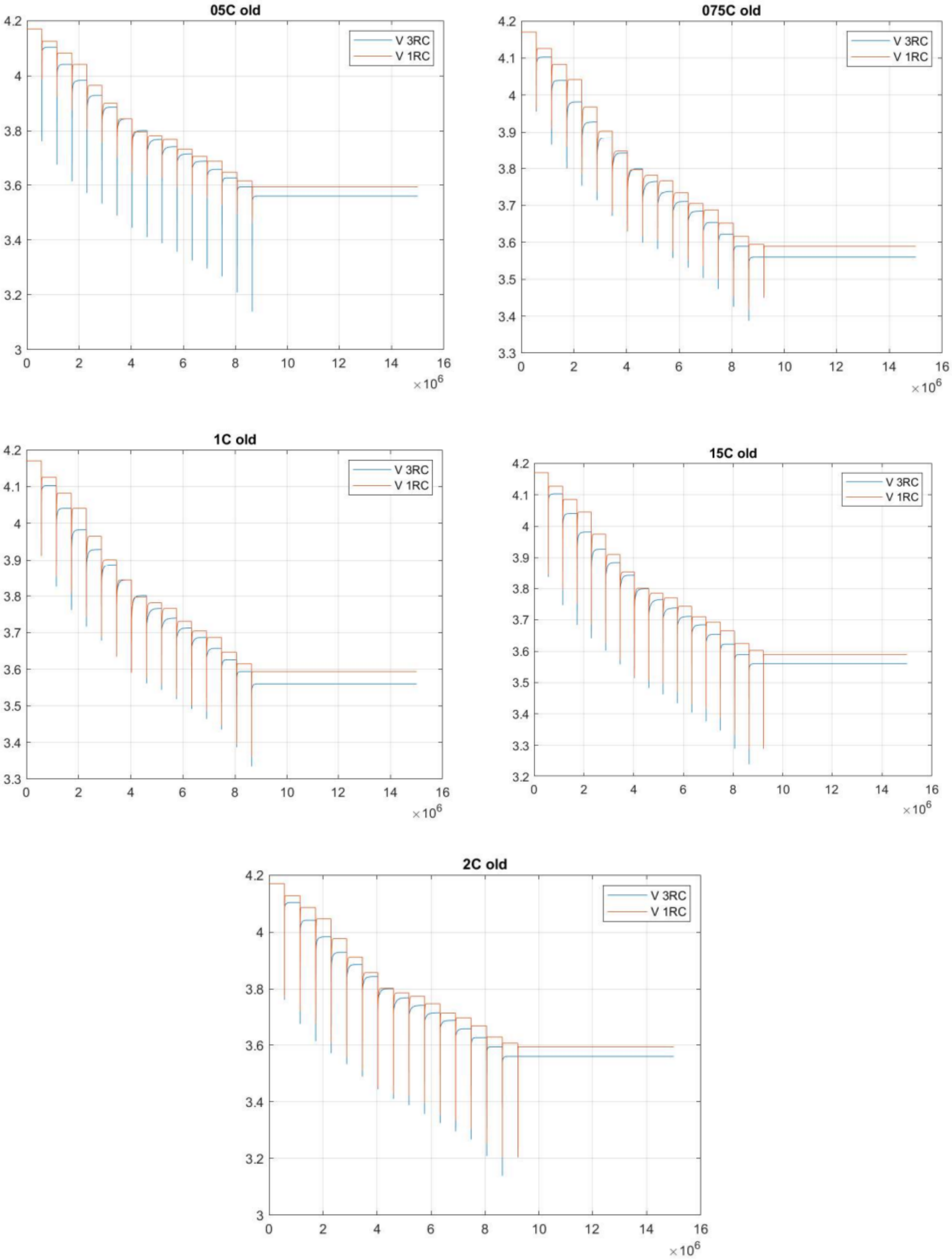

Figure 6.3: Old battery, constant discharge simulation 3RC-1RC OL

As we can see from the graphs, the error between the 3RC and 1RC discharge simulation is very big, especially for the old battery

RMSE

Current	New	Old
0.5C	$5.7844e^{-04}$	0.0319
0.75C	0.0057	0.0305
1C	$6.8816e^{-04}$	0.0304
1.5C	0.0142	0.0341
2C	0.0142	0.0356

Table 6.1: RMSE of constant pulse current discharge simulation OL

Variable pulse current

- New battery

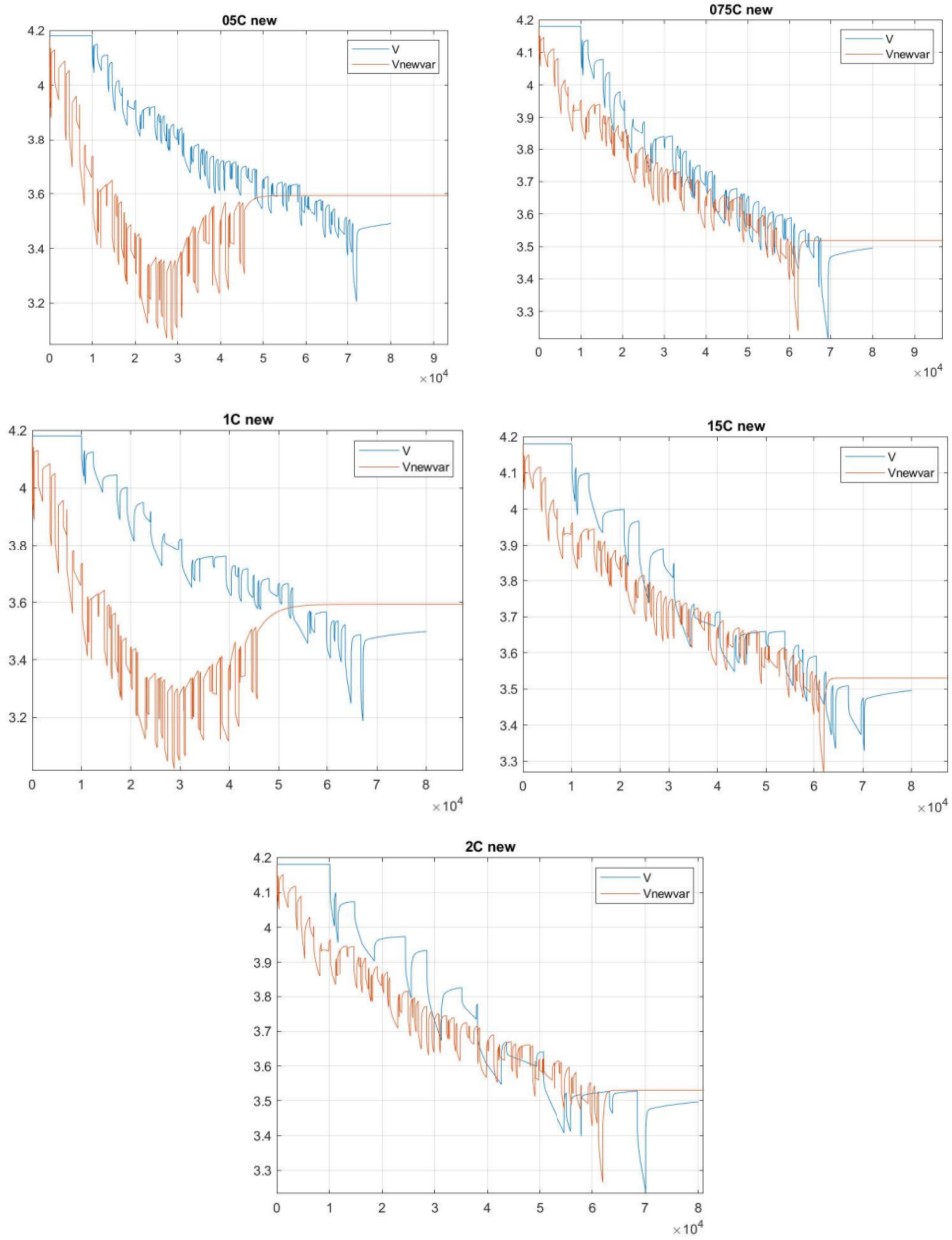

Figure 6.4: New battery, variable discharge simulation 3RC-1RC OL

- Old battery

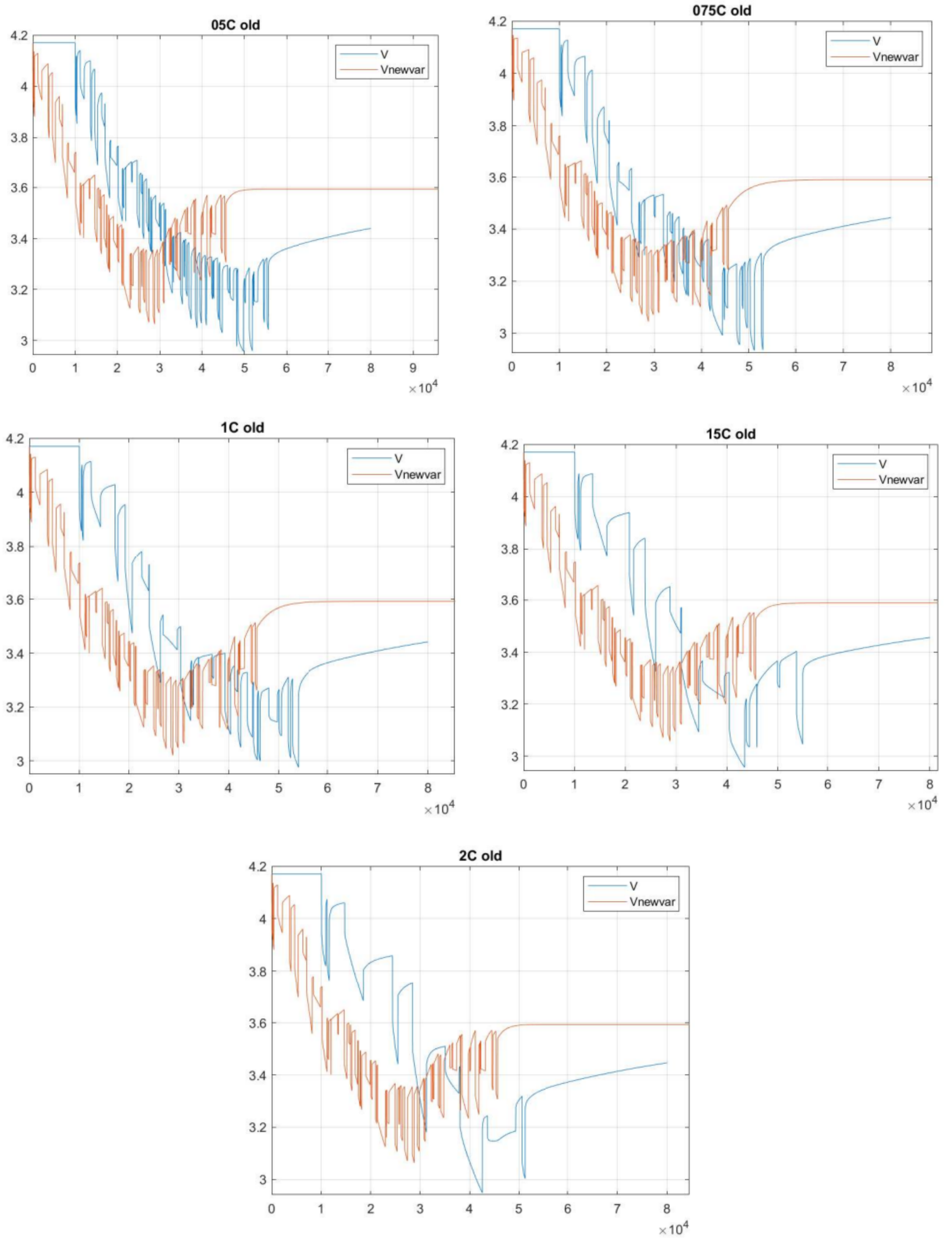

Figure 6.5: Old battery, variable discharge simulation 3RC-1RC OL

6.2 Closed loop validation

The advantage of the CL approach is that the connection from OUT to Kalman Filter allows me to control directly the states and to compensate the errors done during the identification. The Kalman Filter compensate the noise but in our case we consider noise the errors on the parameters and then we do a tuning on the input states. With the Kalman Filter put in the CL approach the outputs obtained have a very small margin of error.

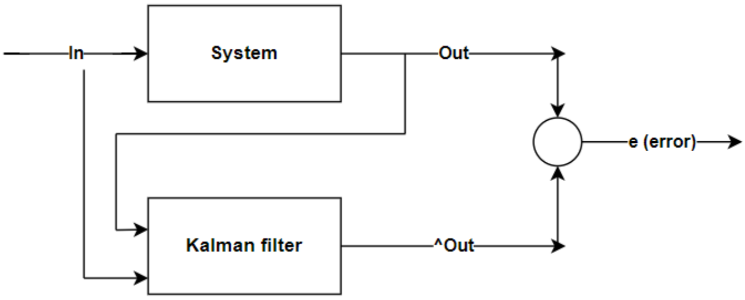

Figure 6.6: Block scheme of the closed loop control

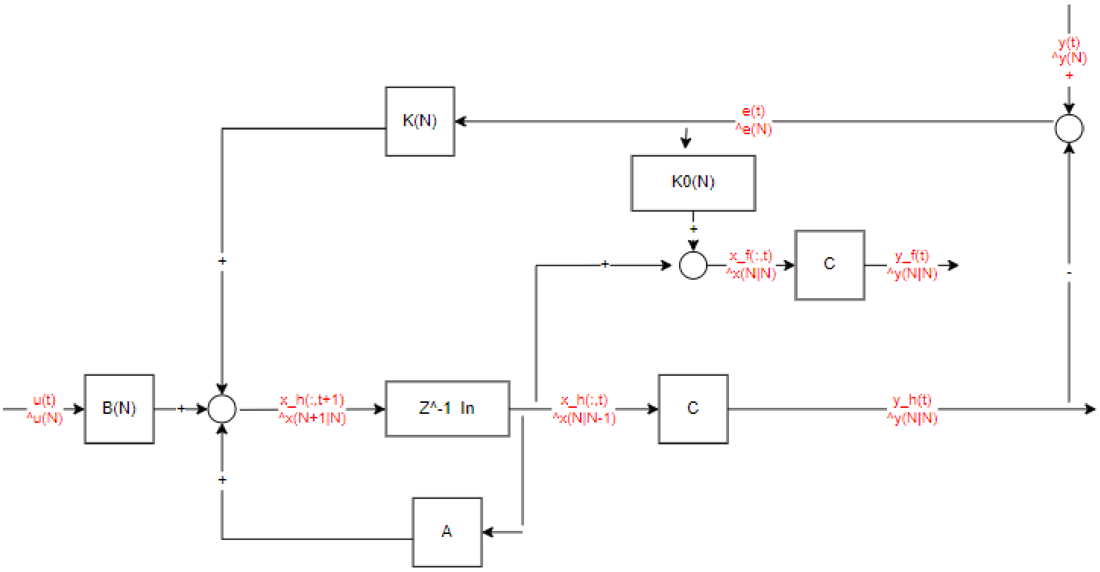

Figure 6.7: Kalman filter in standard form

6.2.1 Mathematical model

The state space model is:

$$\begin{cases} x_1(k+1) = x_1(k) + [-\tau x_1(k) + \tau R u(k)]T_s \\ x_2(k+1) = x_2(k) + \frac{T_s}{3600 \cdot Capacity} u(k) \\ x_3(k+1) = x_1(k+1) + V_{OCV} + R_0 u(k) \\ y(k) = x_3(k) \end{cases} \tag{6.2}$$

Substituting $x_1(k+1)$ with the first relation in the model, the third equation becomes:

$$\begin{aligned} x_3(k+1) &= x_1(k) + [-\tau x_1(k) + \tau R u(k)]T_s + V_{OCV} + R_0 u(k) \\ &= [1 - \tau T_s]x_1(k) + [\tau R T_s + R0]u(k) + V_{OCV} \end{aligned} \tag{6.3}$$

Extended kalman filter (EKF)

The transition matrices are defined as:

$$\bar{A}(t) = \frac{\partial f(\cdot)}{\partial x} \tag{6.4}$$

$$\bar{B}(t) = \frac{\partial f(\cdot)}{\partial u} \tag{6.5}$$

$$\bar{C}(t) = \frac{\partial h(\cdot)}{\partial x} \tag{6.6}$$

$$\bar{D}(t) = \frac{\partial h(\cdot)}{\partial u} \tag{6.7}$$

The transfer functions are determined as:

$$f(x, u) = \begin{bmatrix} x_1[1 - \tau T_s] + \tau R T_s u \\ x_2 + \frac{T_s}{3600 \cdot Capacity} u \\ [1 - \tau T_s]x_1 + [\tau R T_s + R_0]u + V_{OCV} \end{bmatrix} \tag{6.8}$$

$$h(x) = x_3 \tag{6.9}$$

The partial derivatives are calculated:

$$\frac{\partial f_1}{\partial x_1} = 1 - \tau T_s, \frac{\partial f_1}{\partial x_2} = 0, \frac{\partial f_1}{\partial x_3} = 0 \tag{6.10}$$

$$\frac{\partial f_2}{\partial x_1} = 0, \frac{\partial f_2}{\partial x_2} = 1, \frac{\partial f_2}{\partial x_3} = 0 \tag{6.11}$$

$$\frac{\partial f_3}{\partial x_1} = 1 - \tau T_s, \frac{\partial f_3}{\partial x_2} = 0, \frac{\partial f_3}{\partial x_3} = 0 \tag{6.12}$$

$$\frac{\partial f}{\partial u_1} = \tau T_s R \tag{6.13}$$

$$\frac{\partial f}{\partial u_2} = \frac{T_s}{3600 \cdot Capacity} \tag{6.14}$$

$$\frac{\partial f}{\partial u_3} = \tau T_s R + R_0 \tag{6.15}$$

$$\frac{\partial h}{\partial x_1} = 1, \frac{\partial h}{\partial x_2} = 0, \frac{\partial h}{\partial x_3} = 1 \tag{6.16}$$

$$\frac{\partial h}{\partial u} = 0 \tag{6.17}$$

and the Jacobians:

$$A = \begin{bmatrix} 1 - \tau T_s & 0 & 0 \\ 0 & 1 & 0 \\ 1 - \tau T_s & 0 & 0 \end{bmatrix} \tag{6.18}$$

$$B = \begin{bmatrix} \tau T_s R \\ \frac{T_s}{3600 \cdot Capacity} \\ \tau T_s R + R_0 \end{bmatrix} \tag{6.19}$$

$$C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \tag{6.20}$$

$$D = \begin{bmatrix} 0 \end{bmatrix} \tag{6.21}$$

Riccati equations:

$$K_0 = PC'(CPC' + V2)^{-1} \tag{6.22}$$

$$K = (APC' + V12)(CPC' + V2)^{-1} \tag{6.23}$$

$$P = APA' + V1 - K(CPC' + V2)K' \tag{6.24}$$

Kalman filter in standard form:

$$y_h = Cx_h \tag{6.25}$$

$$e_k = y - y_h \tag{6.26}$$

$$x_h = Ke_k + Bx + Ax_h \tag{6.27}$$

$$x_f = K_0 e_k + x_h \tag{6.28}$$

$$y_f = Cx_f \tag{6.29}$$

6.2.2 Discharge simulation

Constant pulse current

- New battery

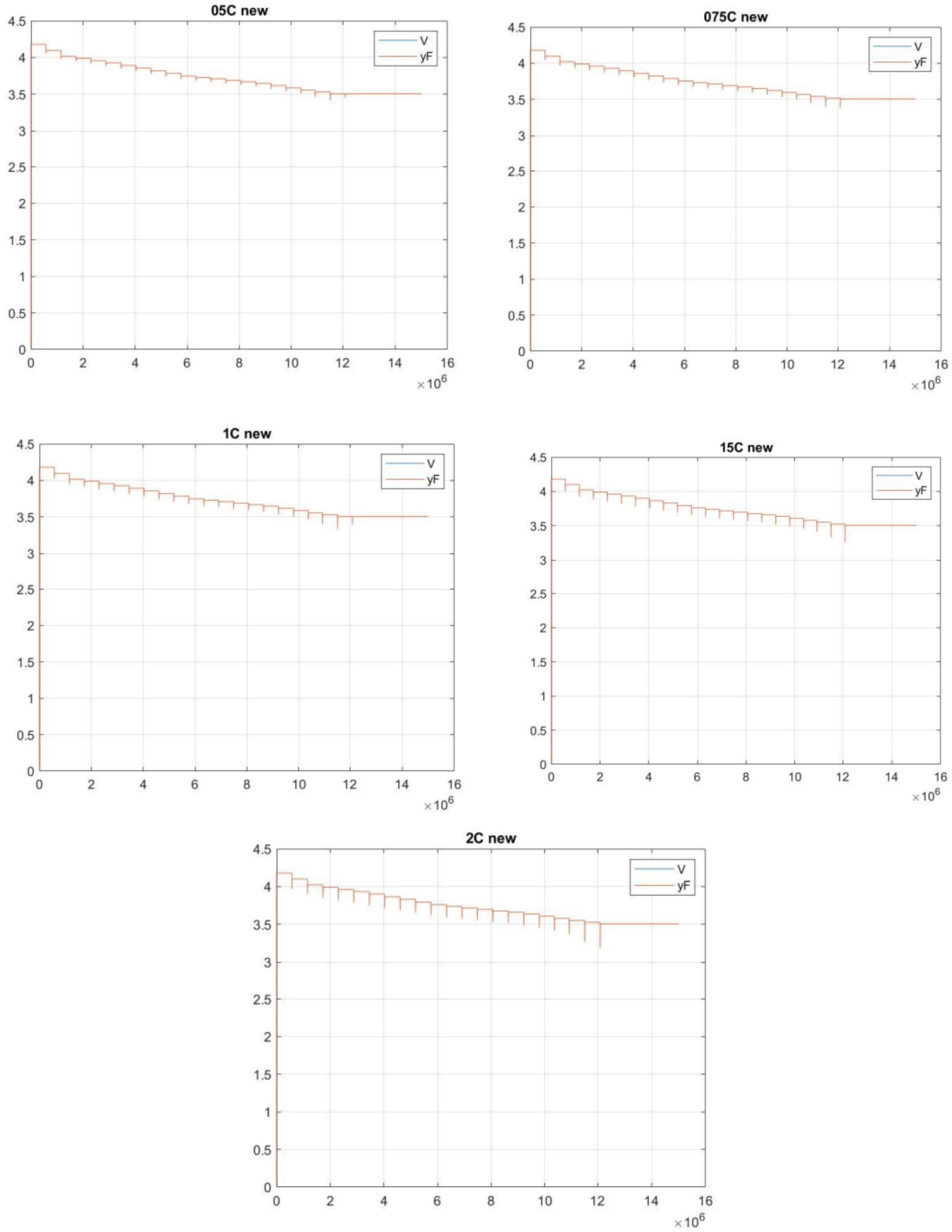

Figure 6.8: New battery, constant discharge simulation 3RC-1RC CL

- Old battery

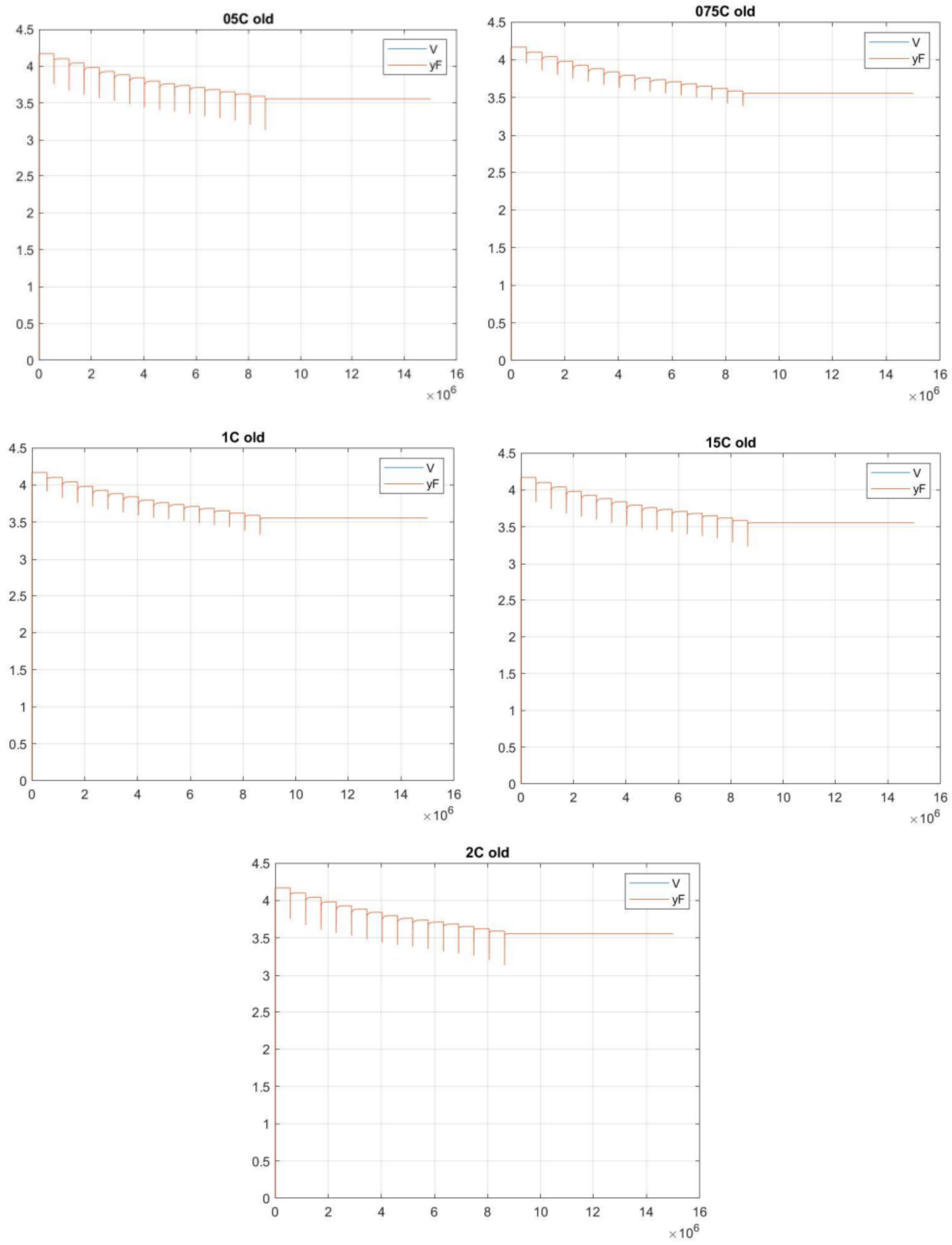

Figure 6.9: Old battery, constant discharge simulation 3RC-1RC CL

As we can see from the graphs, the error between the 3RC and 1RC discharge simulation is very low.

RMSE

Current	New	Old
0.5C	0.0045	0.0045
0.75C	0.0015	0.0027
1C	0.0042	0.0031
1.5C	0.0000	0.0039
2C	0.0013	0.0045

Table 6.2: RMSE of constant pulse current discharge simulation CL

Variable pulse current

- New battery

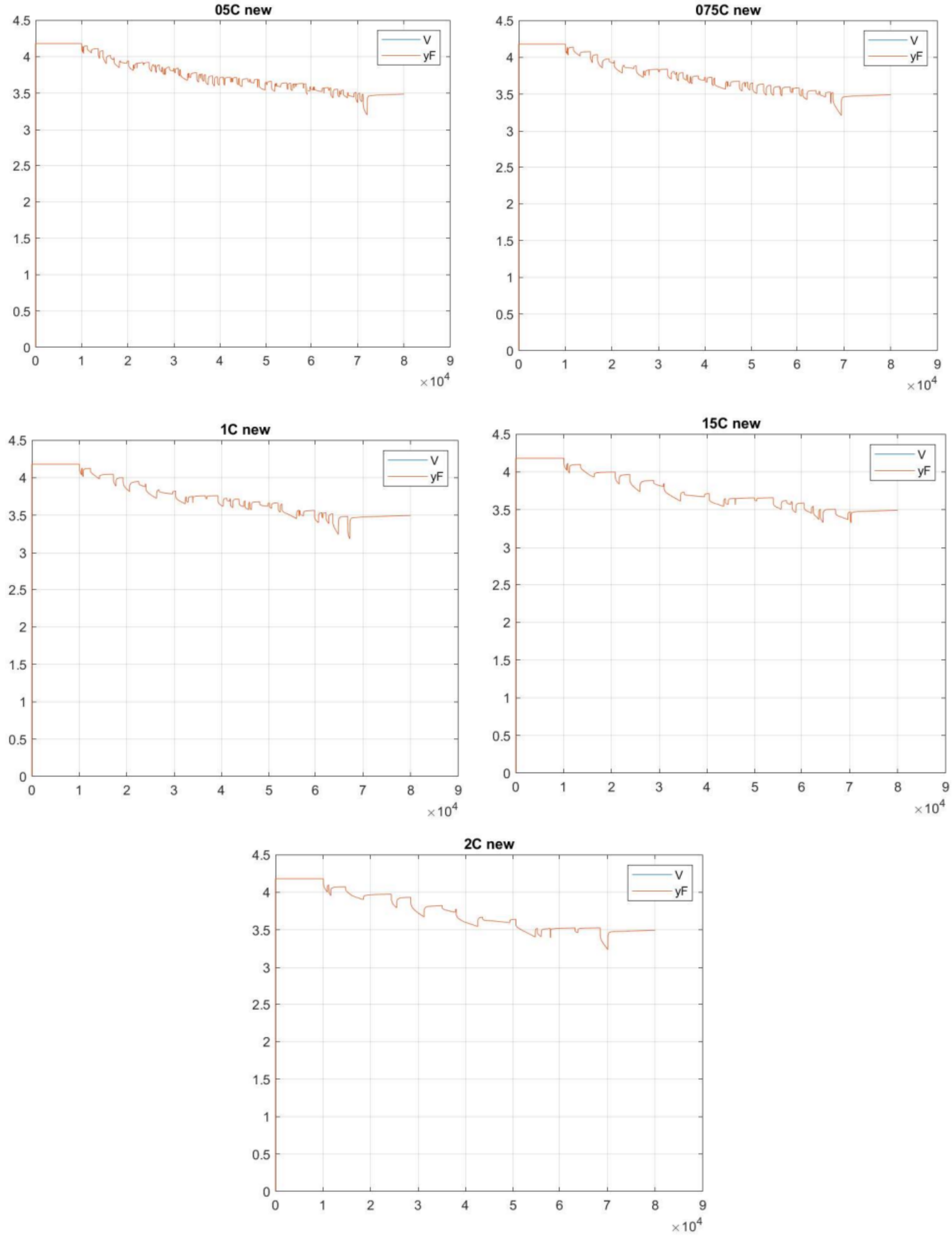

Figure 6.10: New battery, variable discharge simulation 3RC-1RC CL

- Old battery

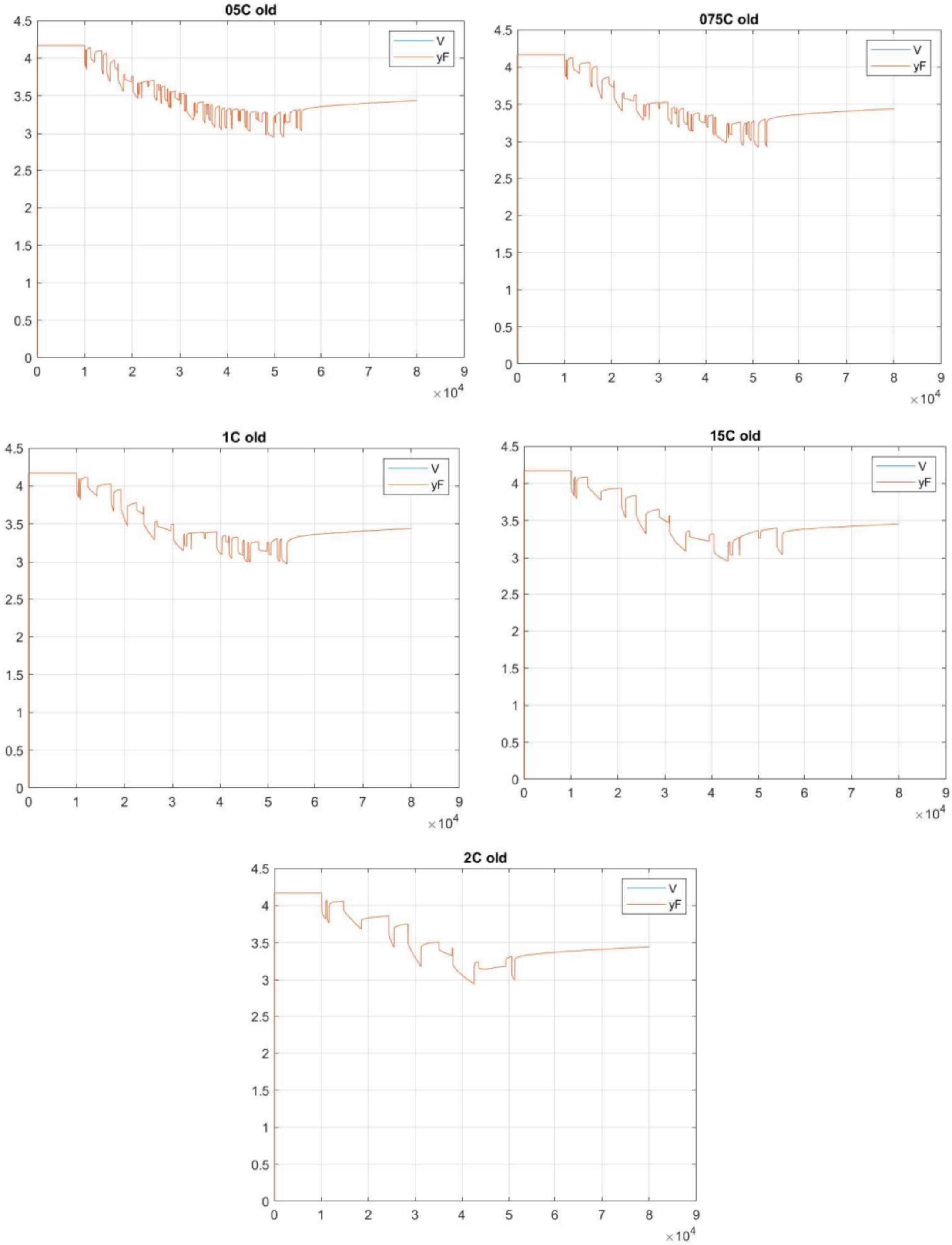

Figure 6.11: Old battery, variable discharge simulation 3RC-1RC CL

Appendix A

Data generation

A.1 Battery model

```
function output  = ModelCell(e_Em, e_R0, e_R1, e_R2, e_R3, e_C1, e_C2, e_C3, Capacity, Em_LUT, R0_LUT, R1_LUT, R2_LUT, R3_LUT, C1_LUT, C2_LUT, C3_LUT, SOC_LUT, SoCin, Ts, I)

persistent SoC;
if isempty(SoC)
    SoC = SoCin;
end

persistent V1;
if isempty(V1)
      V1 = 0;
end

persistent V2;
if isempty(V2)
      V2 = 0;
end

persistent V3;
if isempty(V3)
      V3 = 0;
end

      Em_LUT_1 = (1+e_Em).*Em_LUT;
      R0_LUT_1 = (1+e_R0).*R0_LUT;
      R1_LUT_1 = (1+e_R1).*R1_LUT;
      R2_LUT_1 = (1+e_R2).*R2_LUT;
      R3_LUT_1 = (1+e_R3).*R3_LUT;
```

```
    C1_LUT_1 = (1+e_C1).*C1_LUT;
    C2_LUT_1 = (1+e_C2).*C2_LUT;
    C3_LUT_1 = (1+e_C3).*C3_LUT;


r0 = interp1(SOC_LUT, R0_LUT_1, SoC, 'linear', 'extrap');
r1 = interp1(SOC_LUT, R1_LUT_1, SoC, 'linear', 'extrap');
r2 = interp1(SOC_LUT, R2_LUT_1, SoC, 'linear', 'extrap');
r3 = interp1(SOC_LUT, R3_LUT_1, SoC, 'linear', 'extrap');
c1 = interp1(SOC_LUT, C1_LUT_1, SoC, 'linear', 'extrap');
c2 = interp1(SOC_LUT, C2_LUT_1, SoC, 'linear', 'extrap');
c3 = interp1(SOC_LUT, C3_LUT_1, SoC, 'linear', 'extrap');
Em = interp1(SOC_LUT, Em_LUT_1, SoC, 'linear', 'extrap');

V1 = V1 + (-(1/(r1*c1))*V1 + (1/c1)*I)*Ts;
V2 = V2 + (-(1/(r2*c2))*V2 + (1/c2)*I)*Ts;
V3 = V3 + (-(1/(r3*c3))*V3 + (1/c3)*I)*Ts;

SoC = SoC + I*Ts/(Capacity*3600);

output = [SoC, V1, V2, V3, Em, r0];

end
```

A.2 Parameters file

```
clear all
close all
clc

Amplitude_Pulse_Gen = -10.8;
Period_Pulse_Gen = 57600;
Pulse_Width_Pulse_Gen = 0.15625;
Phase_delay_Pulse_Gen = 57600;

Amp_I_level_gen = 10.8 - 2.7;
Off_I_level_gen = 2.7;
Amp_T_level_gen = 180;
Current_off_level_gen = 0;

Current_mode = 1;
Current_zero = 0;
Old_mode = 1;
SoCin_mode = 1;
Ts_func_mode = 0.1;
```

```
Ts_mode = 0.1;

e_Em = 0.*randn(1,7);
e_R0 = 0.*randn(1,7);
e_R1 = 0.*randn(1,7);
e_R2 = 0.*randn(1,7);
e_R3 = 0.*randn(1,7);
e_C1 = 0.*randn(1,7);
e_C2 = 0.*randn(1,7);
e_C3 = 0.*randn(1,7);

if Old_mode == 0

    Capacity = 5.4;
    Em_LUT = [3.51 3.56 3.65 3.75 3.93 4.02 4.18];
    R0_LUT = [0.02 0.01 0.009 0.009 0.008 0.007 0.008];
    R1_LUT = [0.006 0.003 0.0035 0.0032 0.004 0.0027 0.0029];
    t1 = [10 12 15 12 20 15 12];
    R2_LUT = [0.0025 0.0017 0.0013 0.0012 0.0021 0.0025 0.0026];
    t2 = [25 40 75 125 80 100 110];
    R3_LUT = [0.025 0.013 0.007 0.003 0.007 0.012 0.005];
    t3 = [1000 1250 1100 850 1000 1400 1100];

else

    Capacity = 4.05;
    Em_LUT = [3.56 3.61 3.68 3.78 3.94 4.07 4.17];
    R0_LUT = [0.032 0.026 0.026 0.025 0.025 0.022 0.024];
    R1_LUT = [0.003 0.005 0.0052 0.0045 0.0075 0.004 0.007];
    t1 = [15 10 12 60 150 20 18];
    R2_LUT = [0.005 0.01 0.02 0.14 0.065 0.01 0.003];
    t2 = [150 400 10000 6500 2500 150 120];
    R3_LUT = [0.075 0.035 0.16 0.2 0.175 0.13 0.025];
    t3 = [4000 2100 6500 6400 8000 3500 2000];

end

C1_LUT = t1./R1_LUT;
C2_LUT = t2./R2_LUT;
C3_LUT = t3./R3_LUT;

SOC_LUT = [0 0.1 0.25 0.5 0.75 0.9 1];

save('param.mat')
```

Appendix B

Identification

B.1 Static parameters

```
close all
clear
clc

load V.mat
load SoC.mat
load I.mat

Ts = 0.1;
j = 1;
cnt = 0;

for i = 2:length(I)
    if I(i)-I(i-1) < 0
        cnt = cnt+1;
        index(j) = i-1;
        j = j+1;
    end
end

index(j) = index(j-1) + index(1);
index(j+1) = index(j) + index(1);

j = 1;

for i = 1:length(index)-1
    current(i,:) = (I(index(j):index(j+1)-1))';
    voltage(i,:) = (V(index(j):index(j+1)-1))';
    vocv_05C(i) = V(index(j));
```

```

    indx_vect(i,:) = index(j):index(j+1)-1;

    state_of_charge(i,:) = (SoC(index(j):index(j+1)-1))';
    state_of_charge_vocv_05C(i) = SoC(index(j));

    j = j+1;
end

[y,yy] = size(state_of_charge);

for i = 1:y
    vocv_new(i,:) = interp1(state_of_charge_vocv_05C, vocv_05C, state_of_charge(i,:), 'linear', 'extrap');
end

for i = 1:y
  for j = 2:yy

      Y(j-1,1) = (voltage(i,j)-vocv_new(i,j)-voltage(i,j-1)+vocv_new(i,j-1));
      X1(j-1,1) = current(i,j);
      X2(j-1,1) = current(i,j-1);
      X3(j-1,1) = vocv_new(i,j-1)-voltage(i,j-1);

  end

    X = [X1 X2 X3];

    vec = X\Y;

    tau_05C(i) = vec(3)/Ts;
    R0_05C(i) = vec(2)/(tau_05C(i)*Ts - 1);
    R_05C(i) = (vec(1)-R0_05C(i))/(tau_05C(i)*Ts);

end

media_charges_05C = state_of_charge_vocv_05C(1:length(tau_05C));
```

B.2 RLS2 parameters

```
load V.mat
load SoC.mat
load I.mat

```

```

Ts = 0.1;
j = 1;
cnt = 0;

for i = 2:length(I)
    if  I(i)-I(i-1) < 0
        cnt = cnt+1;
        index(j) = i-1;
        j = j+1;
    end
end

k = length(index)-1;
j = 1;

for i = 1:k

           current(i,:) = (I(index(j):index(j+1)))';
           voltage(i,:) = (V(index(j):index(j+1)))';
           vocv(i) = V(index(j));

           state_of_charge(i,:) = (SoC(index(j):index(j+1)))';
           state_of_charge_vocv(i) = SoC(index(j));

           j = j+1;
end

for i = 1:k
     vocv_new(i,:) = interp1(state_of_charge_vocv, vocv, state_of_charge(i,:), 'linear', 'extrap');
end

[y,yy] = size(state_of_charge);
sum_charges = zeros(k,1);
media_charges_05C_new = zeros(k,1);

for i = 1:k
    for l = 1:yy
    sum_charges(i) = sum_charges(i) + state_of_charge(i,l);
    end
    media_charges_05C_new(i) = sum_charges(i)/yy;
end

Ru = eye(3);
RLS2 = zeros(3,yy);

          for i = 1:k
              for j = 2:yy
```

```

                        Y(j-1,i) = (voltage(i,j)-vocv_new(i,j)-voltage(i,j-1)+vocv_new(i,j-1));
                        X1(j-1,i) = current(i,j);
                        X2(j-1,i) = current(i,j-1);
                        X3(j-1,i) = vocv_new(i,j-1)-voltage(i,j-1);
                        X_tot = [X1(j-1,i);X2(j-1,i);X3(j-1,i)];

                        Ru = (1- 1/j)*Ru + (1/j)*(X_tot*X_tot');
                        K = (1/j)*(Ru\X_tot);
                        e = Y(j-1,i)-X_tot'*RLS2(:,j-1);
                        RLS2(:,j) = RLS2(:,j-1)+K*e;
                end

                  W{i} = RLS2(:,j);
                  tau_05C_new(i) = W{i}(3)/Ts;
                  R0_05C_new(i) = W{i}(2)/(tau_05C_new(i)*Ts - 1);
                  R_05C_new(i) = (W{i}(1)-R0_05C_new(i))/(tau_05C_new(i)*Ts);


           end
```

Appendix C

Approximation

C.1 R0 and Vocv

```
clear all
close all
clc

load("R0_05C.mat")
R0_LUT(1,:) = R0_05C;
load("R0_075C.mat")
R0_LUT(2,:) = R0_075C;
load("R0_1C.mat")
R0_LUT(3,:) = R0_1C;
load("R0_15C.mat")
R0_LUT(4,:) = R0_15C;
load("R0_2C.mat")
R0_LUT(5,:) = R0_2C;
load("media_charges_05C.mat")
SOC_LUT = media_charges_05C;

load("R0_05C_old.mat")
R0_LUT_old(1,:) = R0_05C_old;
load("R0_075C_old.mat")
R0_LUT_old(2,:) = R0_075C_old;
load("R0_1C_old.mat")
R0_LUT_old(3,:) = R0_1C_old;
load("R0_15C_old.mat")
R0_LUT_old(4,:) = R0_15C_old;
load("R0_2C_old.mat")
R0_LUT_old(5,:) = R0_2C_old;
load("media_charges_05C_old.mat")
SOC_LUT_old = media_charges_05C_old;
```

```

Y = [];
A = [];

Y_old = [];
A_old = [];

for i = 1:5
    Y = [Y;R0_LUT(i,:)'];
    A = [A;media_charges_05C'];

    Y_old = [Y_old;R0_LUT_old(i,1:end-1)'];
    A_old = [A_old;media_charges_05C_old(1:end-1)'];


end

A = [A, ones(22*5,1)];
b = A\Y;

A_old = [A_old, ones(15*5,1)];
b_old = A_old\Y_old;

x = [0:1];
y = b(1)*x + b(2);

x_old = [0:1];
y_old = b_old(1)*x_old + b_old(2);

m_coeff = [b_old(1);b(1)];
q_coeff = [b_old(2);b(2)];

SoH = [0.75;1];
SoH = [SoH, ones(2,1)];

b_coeff_m = SoH\m_coeff;
b_coeff_q = SoH\q_coeff;

y_coeff_m = b_coeff_m(1)*x + b_coeff_m(2);
y_coeff_q = b_coeff_q(1)*x + b_coeff_q(2);

figure
for i = 1:5
    plot(SOC_LUT,R0_LUT(i,:),'r*')
    plot(SOC_LUT_old,R0_LUT_old(i,:),'b*')
    plot(x,y,'r')
    plot(x_old,y_old,'b')
    legend('old LS','new ','old','new LS')
    hold on
```

```
end

 xlabel("SoC")
 ylabel("R_0 [\Omega]")
 title("R_0 battery compare")
grid on

figure
    plot(x,y_coeff_m)
    hold on
    plot(x,y_coeff_q)
    legend('m','q')

 xlabel("SoH")
 ylabel("Coeff")
 title("R_0 battery compare")
grid on



load("R_05C.mat")
R_LUT(1,:) = R_05C;
load("R_075C.mat")
R_LUT(2,:) = R_075C;
load("R_1C.mat")
R_LUT(3,:) = R_1C;
load("R_15C.mat")
R_LUT(4,:) = R_15C;
load("R_2C.mat")
R_LUT(5,:) = R_2C;
load("media_charges_05C.mat")
SOC_LUT = media_charges_05C;

load("R_05C_old.mat")
R_LUT_old(1,:) = R_05C_old;
load("R_075C_old.mat")
R_LUT_old(2,:) = R_075C_old;
load("R_1C_old.mat")
R_LUT_old(3,:) = R_1C_old;
load("R_15C_old.mat")
R_LUT_old(4,:) = R_15C_old;
load("R_2C_old.mat")
R_LUT_old(5,:) = R_2C_old;
load("media_charges_05C_old.mat")
SOC_LUT_old = media_charges_05C_old;

Y = [];
A = [];

```

```

Y_old = [];
A_old = [];

for i = 1:5
    Y = [Y;R_LUT(i,:)'];
    A = [A;media_charges_05C.^2'];


    Y_old = [Y_old;R_LUT_old(i,:)'];
    A_old = [A_old;media_charges_05C_old.^2'];


end

A = [A, ones(22*5,1)];
b = A\Y;

A_old = [A_old, ones(16*5,1)];
b_old = A_old\Y_old;

x = linspace(0,1,100);
y = b(1)*x.^2 + b(2);

x_old = linspace(0,1,100);
y_old = b_old(1)*x_old.^2 + b_old(2);

m_coeff = [b_old(1);b(1)];
q_coeff = [b_old(2);b(2)];

SoH = [0.75;1];
SoH = [SoH, ones(2,1)];

b_coeff_m = SoH\m_coeff;
b_coeff_q = SoH\q_coeff;

y_coeff_m = b_coeff_m(1)*x + b_coeff_m(2);
y_coeff_q = b_coeff_q(1)*x + b_coeff_q(2);




figure
for i = 1:5
    plot(SOC_LUT,R_LUT(i,:),'r*')
    plot(SOC_LUT_old,R_LUT_old(i,:),'b*')
    plot(x,y,'r')
    plot(x_old,y_old,'b')
    legend('old LS','new ','old','new LS')
```

```
    hold on
end

 xlabel("SoC")
 ylabel("R [\Omega]")
 title("R battery compare")
grid on


figure
    plot(x,y_coeff_m)
    hold on
    plot(x,y_coeff_q)
    legend('m','q')

 xlabel("SoH")
 ylabel("Coeff")
 title("R battery compare")
grid on


load("tau_05C.mat")
tau_LUT(1,:) = tau_05C;
load("tau_075C.mat")
tau_LUT(2,:) = tau_075C;
load("tau_1C.mat")
tau_LUT(3,:) = tau_1C;
load("tau_15C.mat")
tau_LUT(4,:) = tau_15C;
load("tau_2C.mat")
tau_LUT(5,:) = tau_2C;
load("media_charges_05C.mat")
SOC_LUT = media_charges_05C;

load("tau_05C_old.mat")
tau_LUT_old(1,:) = tau_05C_old;
load("tau_075C_old.mat")
tau_LUT_old(2,:) = tau_075C_old;
load("tau_1C_old.mat")
tau_LUT_old(3,:) = tau_1C_old;
load("tau_15C_old.mat")
tau_LUT_old(4,:) = tau_15C_old;
load("tau_2C_old.mat")
tau_LUT_old(5,:) = tau_2C_old;
load("media_charges_05C_old.mat")
SOC_LUT_old = media_charges_05C_old;

Y = [];
A = [];
```

```

Y_old = [];
A_old = [];

for i = 1:5
    Y = [Y;tau_LUT(i,:)'];
    A = [A;media_charges_05C.^2'];

    Y_old = [Y_old;tau_LUT_old(i,:)'];
    A_old = [A_old;media_charges_05C_old.^2'];


end

A = [A, ones(22*5,1)];
b = A\Y;

A_old = [A_old, ones(16*5,1)];
b_old = A_old\Y_old;

x = [0:1];
y = b(1)*x.^2 + b(2);

x_old = [0:1];
y_old = b_old(1)*x_old.^2 + b_old(2);

m_coeff = [b_old(1);b(1)];
q_coeff = [b_old(2);b(2)];

SoH = [0.75;1];
SoH = [SoH, ones(2,1)];

b_coeff_m = SoH\m_coeff;
b_coeff_q = SoH\q_coeff;

y_coeff_m = b_coeff_m(1)*x + b_coeff_m(2);
y_coeff_q = b_coeff_q(1)*x + b_coeff_q(2);





figure
for i = 1:5
    plot(SOC_LUT,tau_LUT(i,:),'r*')
    plot(SOC_LUT_old,tau_LUT_old(i,:),'b*')
    plot(x,y,'r')
    plot(x_old,y_old,'b')
    legend('old LS','new ','old','new LS')
```

```
    hold on
end

xlabel("SoC")
ylabel("\tau [s^{-1}]")
title("tau battery compare")
grid on

figure
    plot(x,y_coeff_m)
    hold on
    plot(x,y_coeff_q)
    legend('m','q')

xlabel("SoH")
ylabel("Coeff")
title("tau battery compare")
grid on




load("vocv_05C.mat")
vocv_LUT(1,:) = vocv_05C;
load("vocv_075C.mat")
vocv_LUT(2,:) = vocv_075C;
load("vocv_1C.mat")
vocv_LUT(3,:) = vocv_1C;
load("vocv_15C.mat")
vocv_LUT(4,:) = vocv_15C;
load("vocv_2C.mat")
vocv_LUT(5,:) = vocv_2C;
load("state_of_charge_vocv_05C.mat")
SOC_LUT = state_of_charge_vocv_05C;

load("vocv_05C_old.mat")
vocv_LUT_old(1,:) = vocv_05C_old;
load("vocv_075C_old.mat")
vocv_LUT_old(2,:) = vocv_075C_old;
load("vocv_1C_old.mat")
vocv_LUT_old(3,:) = vocv_1C_old;
load("vocv_15C_old.mat")
vocv_LUT_old(4,:) = vocv_15C_old;
load("vocv_2C_old.mat")
vocv_LUT_old(5,:) = vocv_2C_old;
load("state_of_charge_vocv_05C_old.mat")
SOC_LUT_old = state_of_charge_vocv_05C_old;

Y = [];
```

```
A = [];

Y_old = [];
A_old = [];

for i = 1:5
    Y = [Y;vocv_LUT(i,:)'];
    A = [A;state_of_charge_vocv_05C'];

    Y_old = [Y_old;vocv_LUT_old(i,:)'];
    A_old = [A_old;state_of_charge_vocv_05C_old'];


end
A = [A, ones(22*5,1)];
b = A\Y;

A_old = [A_old, ones(16*5,1)];
b_old = A_old\Y_old;

x = [0:1];
y = b(1)*x + b(2);

x_old = [0:1];
y_old = b_old(1)*x_old + b_old(2);

m_coeff = [b_old(1);b(1)];
q_coeff = [b_old(2);b(2)];

SoH = [0.75;1];
SoH = [SoH, ones(2,1)];

b_coeff_m = SoH\m_coeff;
b_coeff_q = SoH\q_coeff;

y_coeff_m = b_coeff_m(1)*x + b_coeff_m(2);
y_coeff_q = b_coeff_q(1)*x + b_coeff_q(2);

figure
for i = 1:5

    plot(SOC_LUT,vocv_LUT(i,:),'r*')
    plot(SOC_LUT_old,vocv_LUT_old(i,:),'b*')
    plot(x,y,'r')
    plot(x_old,y_old,'b')
    legend('old LS','new ','old','new LS')
    hold on
end

```

```
 xlabel("SoC")
 ylabel("V_{OCV} [V]")
 title("vocv battery compare")
grid on

figure
    plot(x,y_coeff_m)
    hold on
    plot(x,y_coeff_q)
    legend('m','q')

 xlabel("SoH")
 ylabel("Coeff")
 title("vocv battery compare")
grid on
```

C.2 Rnew

```
clear all
close all
clc

load("R_05C.mat")
R_LUT(1,:) = R_05C;
load("R_075C.mat")
R_LUT(2,:) = R_075C;
load("R_1C.mat")
R_LUT(3,:) = R_1C;
load("R_15C.mat")
R_LUT(4,:) = R_15C;
load("R_2C.mat")
R_LUT(5,:) = R_2C;
load("media_charges_05C.mat")
SOC_LUT = media_charges_05C;


MSE = zeros(1,4);
x = linspace(0,1,22);

cnt = 1;

figure


for i = 1:5
```

```


        plot(SOC_LUT,R_LUT(i,:),'r*')

  legendinfo{cnt} = '';
  cnt = cnt + 1;

        hold on
 end
legendinfo{1} = 'New battery';


 for degree = 2:5

y = 0;

Y = [];

b = 0;

y_2 = zeros(5,22);

A = cell(5,degree);
Aones = cell(5,1);

ind = 1;

  for i = 1:5

        Y = [Y;R_LUT(i,1:end-1)'];


     for j = 1:degree

        A{i,j} = SOC_LUT(1:end-1).^(degree+1-j)';

        Aones{i} = ones(length(R_LUT)-1,1);


     end
 end

  Aarr = cell2mat(A);
  Aarrones = cell2mat(Aones);

  Aarr = [Aarr, Aarrones];

  b = Aarr\Y;

```

```
  for j = 1:degree+1

      y = y + b(j).*x.^(degree+1-j);
  end


plot(x,y)

legendinfo{cnt} = ['LS degree = ' num2str(degree)];
cnt = cnt + 1;

hold on
grid on

 for j = 22:-1:1

        y_2(:,ind) = y(j);

        ind = ind + 1;

  end

grid on

for i = 1:5
    for j = 1:21
        MSE(degree-1) = MSE(degree-1) + (y_2(i,j)-R_LUT(i,j))^2;
    end
end

 MSE_med(degree-1) = MSE(degree-1)/(i*j);
 RMSE(degree-1) = sqrt(MSE_med(degree-1));

 end

 xlabel("SoC")
 ylabel("R [\Omega]")
 title("New battery")


 legend(legendinfo)
```

C.3 Rold

```
clear all
```

```
close all
clc

load("R_05C_old.mat")
R_LUT_old(1,:) = R_05C_old;
load("R_075C_old.mat")
R_LUT_old(2,:) = R_075C_old;
load("R_1C_old.mat")
R_LUT_old(3,:) = R_1C_old;
load("R_15C_old.mat")
R_LUT_old(4,:) = R_15C_old;
load("R_2C_old.mat")
R_LUT_old(5,:) = R_2C_old;
load("media_charges_05C_old.mat")
SOC_LUT_old = media_charges_05C_old;


MSE = zeros(1,4);
x = linspace(0,1,16);

cnt = 1;

figure

for i = 1:5


          plot(SOC_LUT_old,R_LUT_old(i,:),'b*')

    legendinfo{cnt} = '';
    cnt = cnt + 1;

          hold on
 end
legendinfo{1} = 'Old battery';

 for degree = 2:5

y = 0;

Y = [];

b = 0;

y_2 = zeros(5,16);

A = cell(5,degree);
Aones = cell(5,1);

```

```
ind = 1;

  for i = 1:5

         Y = [Y;R_LUT_old(i,1:end-1)'];


     for j = 1:degree

        A{i,j} = SOC_LUT_old(1:end-1).^(degree+1-j)';

        Aones{i} = ones(length(R_LUT_old)-1,1);


     end
 end

   Aarr = cell2mat(A);
   Aarrones = cell2mat(Aones);

   Aarr = [Aarr, Aarrones];

   b = Aarr\Y;

  for j = 1:degree+1

     y = y + b(j).*x.^(degree+1-j);
  end


plot(x,y)

legendinfo{cnt} = ['LS degree = ' num2str(degree)];
cnt = cnt + 1;

hold on
grid on

 for j = 16:-1:1

       y_2(:,ind) = y(j);

       ind = ind + 1;

  end

grid on

for i = 1:5
```

```
    for j = 1:16
        MSE(degree-1) = MSE(degree-1) + (y_2(i,j)-R_LUT_old(i,j))^2;
    end
end

 MSE_med(degree-1) = MSE(degree-1)/(i*j);
 RMSE(degree-1) = sqrt(MSE_med(degree-1));

 end

 xlabel("SoC")
 ylabel("R [\Omega]")
 title("Old battery")

 legend(legendinfo)
```

C.4 τ new

```
clear all
close all
clc

load("tau_05C.mat")
tau_LUT(1,:) = tau_05C;
load("tau_075C.mat")
tau_LUT(2,:) = tau_075C;
load("tau_1C.mat")
tau_LUT(3,:) = tau_1C;
load("tau_15C.mat")
tau_LUT(4,:) = tau_15C;
load("tau_2C.mat")
tau_LUT(5,:) = tau_2C;
load("media_charges_05C.mat")
SOC_LUT = media_charges_05C;


MSE = zeros(1,4);
x = linspace(0,1,22);

cnt = 1;

figure

for i = 1:5

```

```

         plot(SOC_LUT,tau_LUT(i,:),'r*')

   legendinfo{cnt} = '';
   cnt = cnt + 1;

         hold on
 end
legendinfo{1} = 'New battery';

 for degree = 2:5

y = 0;

Y = [];

b = 0;

y_2 = zeros(5,22);

A = cell(5,degree);
Aones = cell(5,1);

ind = 1;

  for i = 1:5

         Y = [Y;tau_LUT(i,1:end-1)'];


     for j = 1:degree

        A{i,j} = SOC_LUT(1:end-1).^(degree+1-j)';

        Aones{i} = ones(length(tau_LUT)-1,1);


     end
 end

   Aarr = cell2mat(A);
   Aarrones = cell2mat(Aones);

   Aarr = [Aarr, Aarrones];

   b = Aarr\Y;

  for j = 1:degree+1

```

```
    y = y + b(j).*x.^(degree+1-j);
  end


plot(x,y)

legendinfo{cnt} = ['LS degree = ' num2str(degree)];
cnt = cnt + 1;

hold on
grid on

 for j = 22:-1:1

        y_2(:,ind) = y(j);

        ind = ind + 1;

  end

grid on

for i = 1:5
    for j = 1:21
        MSE(degree-1) = MSE(degree-1) + (y_2(i,j)-tau_LUT(i,j))^2;
    end
end

 MSE_med(degree-1) = MSE(degree-1)/(i*j);
 RMSE(degree-1) = sqrt(MSE_med(degree-1));

 end

 xlabel("SoC")
 ylabel("\tau [s^{-1}]")
 title("New battery")

 legend(legendinfo)
```

C.5 τ old

```
clear all
close all
clc

```

```
load("tau_05C_old.mat")
tau_LUT_old(1,:) = tau_05C_old;
load("tau_075C_old.mat")
tau_LUT_old(2,:) = tau_075C_old;
load("tau_1C_old.mat")
tau_LUT_old(3,:) = tau_1C_old;
load("tau_15C_old.mat")
tau_LUT_old(4,:) = tau_15C_old;
load("tau_2C_old.mat")
tau_LUT_old(5,:) = tau_2C_old;
load("media_charges_05C_old.mat")
SOC_LUT_old = media_charges_05C_old;


MSE = zeros(1,4);
x = linspace(0,1,16);

cnt = 1;

figure

for i = 1:5


         plot(SOC_LUT_old,tau_LUT_old(i,:),'b*')

   legendinfo{cnt} = '';
   cnt = cnt + 1;

         hold on
 end
legendinfo{1} = 'Old battery';

 for degree = 2:5

y = 0;

Y = [];

b = 0;

y_2 = zeros(5,16);

A = cell(5,degree);
Aones = cell(5,1);

ind = 1;

  for i = 1:5
```

```

          Y = [Y;tau_LUT_old(i,1:end-1)'];


      for j = 1:degree

         A{i,j} = SOC_LUT_old(1:end-1).^(degree+1-j)';

         Aones{i} = ones(length(tau_LUT_old)-1,1);


      end
 end

   Aarr = cell2mat(A);
   Aarrones = cell2mat(Aones);

   Aarr = [Aarr, Aarrones];

   b = Aarr\Y;

  for j = 1:degree+1

     y = y + b(j).*x.^(degree+1-j);
  end


plot(x,y)

legendinfo{cnt} = ['LS degree = ' num2str(degree)];
cnt = cnt + 1;

hold on
grid on

 for j = 16:-1:1

       y_2(:,ind) = y(j);

       ind = ind + 1;

  end

grid on

for i = 1:5
    for j = 1:16
        MSE(degree-1) = MSE(degree-1) + (y_2(i,j)-tau_LUT_old(i,j))^2;
```

```
        end
end

 MSE_med(degree-1) = MSE(degree-1)/(i*j);
 RMSE(degree-1) = sqrt(MSE_med(degree-1));

 end

 xlabel("SoC")
 ylabel("\tau [s^{-1}]")
 title("Old battery")

 legend(legendinfo)
```

Appendix D

Open-loop validation function

```
function output = ModelCell(old, SoCin, Ts, I)

persistent e_Em;
if isempty(e_Em)
    e_Em = 0;
end

persistent e_R0;
if isempty(e_R0)
    e_R0 = 0;
end

persistent e_R;
if isempty(e_R)
    e_R = 0;
end



persistent e_C;
if isempty(e_C)
    e_C = 0;
end


persistent First_time;
if isempty(First_time)
    First_time = true;
```

```
    if First_time == true

        if old == 0

         e_Em = 0.*randn(1,21);
         e_R0 = 0.*randn(1,21);
         e_R = 0.*randn(1,21);
         e_C = 0.*randn(1,21);

        else

         e_Em = 0.*randn(1,15);
         e_R0 = 0.*randn(1,15);
         e_R = 0.*randn(1,15);
         e_C = 0.*randn(1,15);

        end

    end
    First_time = false;
end

persistent SoC;
if isempty(SoC)
    SoC = SoCin;
end

persistent V;
if isempty(V)
    V = 0;
end

persistent Capacity;
if isempty(Capacity)
    if old == 0
         Capacity = 5.4;
    else
         Capacity = 4.05;
    end
end

persistent Em_LUT;
if isempty(Em_LUT)
    if old == 0
         Em_LUT = [3.51 3.5350 3.56 3.59 3.62 3.65 3.67 3.69 3.71 3.73
    3.75 3.7860 3.8220 3.8580 3.894 3.93 3.96 3.99 4.02 4.1 4.18];
    else
          Em_LUT = [3.5933 3.6256 3.6567 3.6866 3.7131 3.7396 3.7664
   3.8013 3.8440 3.8867 3.9293 3.9833 4.0411 4.1033 4.1700];
```

```
    end
end

persistent R0_LUT;
if isempty(R0_LUT)
    if old == 0
        R0_LUT = [0.02 0.0175 0.0125 0.0099 0.0095 0.0092 0.0090 0.0090 0.0090 0.0090 0.0090 0.0089 0.0087 0.0085 0.0083 0.0081 0.0078 0.0075 0.0072 0.0073 0.0078];
    else
        R0_LUT = [0.0300 0.0270 0.0260 0.0260 0.0258 0.0255 0.0253 0.0251 0.0250 0.0250 0.0250 0.0245 0.0233 0.0227 0.0234];
    end
end


persistent R_LUT;
if isempty(R_LUT)
    if old == 0
        R_LUT = [0.0097 0.0169 0.0129 0.0092 0.0083 0.0076 0.0069 0.0065 0.0061 0.0057 0.0054 0.0055 0.0061 0.0067 0.0074 0.0082 0.0086 0.0090 0.0094 0.0086 0.0074];
    else
        R_LUT = [0.0203 0.0174 0.0207 0.0365 0.0579 0.0804 0.1140 0.1980 0.1777 0.1640 0.1488 0.1275 0.1090 0.0640 0.0193];
    end
end


persistent tau_LUT;
if isempty(tau_LUT)
    if old == 0
        tau_LUT = [0.0747 0.0066 0.0062 0.0064 0.0074 0.0090 0.0111 0.0121 0.0133 0.0148 0.0166 0.0178 0.0152 0.0135 0.0122 0.0114 0.0105 0.0083 0.0066 0.0066 0.0092];
    else
        tau_LUT = [0.0066 0.0078 0.0052 0.0027 0.0016 0.0011 0.0007 0.0005 0.0005 0.0005 0.0006 0.0008 0.0012 0.0022 0.0074];
    end
end



persistent C_LUT;
if isempty(C_LUT)

    C_LUT = 1./(tau_LUT.*R_LUT);
end

```

```


persistent SOC_LUT;
if isempty(SOC_LUT)
    if old == 0
        SOC_LUT = [0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95 1];
    else
         SOC_LUT = [0 0.1 0.15 0.2 0.3 0.35 0.4 0.55 0.6 0.65 0.7 0.75 0.8 0.9 1];
    end
end



    Em_LUT_1 = (1+e_Em).*Em_LUT;
    R0_LUT_1 = (1+e_R0).*R0_LUT;
    R_LUT_1 = (1+e_R).*R_LUT;
    C_LUT_1 = (1+e_C).*C_LUT;


r0 = interp1(SOC_LUT, R0_LUT_1, SoC, 'linear', 'extrap');
r = interp1(SOC_LUT, R_LUT_1, SoC, 'linear', 'extrap');

c = interp1(SOC_LUT, C_LUT_1, SoC, 'linear', 'extrap');

Em = interp1(SOC_LUT, Em_LUT_1, SoC, 'linear', 'extrap');

V = V + (-(1/(r*c))*V + (1/c)*I)*Ts;

SoC = SoC + I*Ts/(Capacity*3600);

output = [SoC, V, Em, r0];

end
```

Appendix E

Closed-loop validation

E.1 Kalman new

```
close all
clear all
clc

load I
load V
load R
load R0
load TAU
load VOCV
load SOC

I_LUT(1,:) = I_05C_new;
I_LUT(2,:) = I_075C_new;
I_LUT(3,:) = I_1C_new;
I_LUT(4,:) = I_15C_new;
I_LUT(5,:) = I_2C_new;

V_LUT(1,:) = V_05C_new;
V_LUT(2,:) = V_075C_new;
V_LUT(3,:) = V_1C_new;
V_LUT(4,:) = V_15C_new;
V_LUT(5,:) = V_2C_new;

R0_LUT(1,:) = R0_05C;
R0_LUT(2,:) = R0_075C;
R0_LUT(3,:) = R0_1C;
R0_LUT(4,:) = R0_15C;
R0_LUT(5,:) = R0_2C;
```

```

R_LUT(1,:) = R_05C;
R_LUT(2,:) = R_075C;
R_LUT(3,:) = R_1C;
R_LUT(4,:) = R_15C;
R_LUT(5,:) = R_2C;

TAU_LUT(1,:) = tau_05C;
TAU_LUT(2,:) = tau_075C;
TAU_LUT(3,:) = tau_1C;
TAU_LUT(4,:) = tau_15C;
TAU_LUT(5,:) = tau_2C;

VOCV_LUT(1,:) = vocv_05C;
VOCV_LUT(2,:) = vocv_075C;
VOCV_LUT(3,:) = vocv_1C;
VOCV_LUT(4,:) = vocv_15C;
VOCV_LUT(5,:) = vocv_2C;

SOC_LUT(1,:) = media_charges_05C;
SOC_LUT(2,:) = media_charges_075C;
SOC_LUT(3,:) = media_charges_1C;
SOC_LUT(4,:) = media_charges_15C;
SOC_LUT(5,:) = media_charges_2C;

 Capacity = 5.4;
 Ts = 0.1;


     Bv1 = 0.001*[10 1 1]';
     V1 = Bv1*Bv1';
     V2 = 0.00001;
     V12 = 0;

     n = 3;




 for i = 1:5

     xf{i} = zeros(3,length(I_LUT(i,:)));
     xh{i} = zeros(3,length(I_LUT(i,:)));
     yf{i} = zeros(length(I_LUT(i,:)),1);


     P{i} = eye(n);

     yR = 0;
```

```
    yR0 = 0;
    yTAU = 0;
    yVOCV = 0;

    YR = [];
    YR0 = [];
    YTAU = [];
    YVOCV = [];


    A1 = [];
    A2 = [];
    A3 = [];


    bR = 0;
    bR0 = 0;
    bTAU = 0;
    bVOCV = 0;


    YR =  [YR;R_LUT(i,1:end-1)'];
    YR0 = [YR0;R0_LUT(i,1:end-1)'];
    YTAU = [YTAU;TAU_LUT(i,1:end-1)'];
    YVOCV = [YVOCV;VOCV_LUT(i,1:end-1)'];


    A1 = [A1;SOC_LUT(i,1:end-1).^3'];
    A2 = [A2;SOC_LUT(i,1:end-1).^2'];
    A3 = [A3;SOC_LUT(i,1:end-1)'];


    AR = [A1, A2, A3, ones(21,1)];
    AR0 = [A3, ones(21,1)];
    ATAU = [A2, A3, ones(21,1)];
    AVOCV = [A3, ones(21,1)];


    bR = AR\YR;
    bR0 = AR0\YR0;
    bTAU = ATAU\YTAU;
    bVOCV = AVOCV\YVOCV;


    VOCV{i}(1) = 4.18;
    R0{i}(1) = 0.0078;
    R{i}(1) = 0.0074;
    TAU{i}(1) = 0.0092;
    SoC{i}(1) = 1;
```

```


for  t = 2:length(I_LUT(i,:))


    VOCV{i}(t) = bVOCV(1)*SoC{i}(t-1) + bVOCV(2);

    R0{i}(t) = bR0(1)*SoC{i}(t-1) + bR0(2);

    R{i}(t) = bR(1)*SoC{i}(t-1).^3 + bR(2)*SoC{i}(t-1).^2 + bR(3)*SoC{i}(t-1) + bR(4);

    TAU{i}(t) = bTAU(1)*SoC{i}(t-1).^2 + bTAU(2)*SoC{i}(t-1) + bTAU(3);

    SoC{i}(t) = SoC{i}(t-1) + I_LUT(i,t)*Ts/(Capacity*3600);


    A{i} = [1-TAU{i}(t)*Ts 0 0; 0 1 0; 1-TAU{i}(t)*Ts 0 0];
    B{i} = [TAU{i}(t)*Ts*R{i}(t) Ts/(3600*Capacity) TAU{i}(t)*R{i}(t)*Ts + R0{i}(t)]';
    C{i} = [0 0 1];
    D{i} = 0;


    K0{i} = P{i}*C{i}'*inv(C{i}*P{i}*C{i}'+V2);
    K{i} = (A{i}*P{i}*C{i}'+V12)*inv(C{i}*P{i}*C{i}'+V2);
    P{i} = A{i}*P{i}*A{i}' + V1 - K{i}*(C{i}*P{i}*C{i}'+V2)*K{i}';

    xh{i}(1,t) = xf{i}(1,t-1) + (-TAU{i}(t)*xf{i}(1,t-1) +    TAU{i}(t)*R{i}(t)*I_LUT(i,t))*Ts;
    xh{i}(2,t) = xf{i}(2,t-1) + Ts/(3600*Capacity)*I_LUT(i,t);
    xh{i}(3,t) = xf{i}(1,t-1) + VOCV{i}(t) + R0{i}(t)*I_LUT(i,t);

    yh{i}(t,1) = C{i}*xh{i}(:,t);
    e_k{i}(t) = V_LUT(i,t) - yh{i}(t,1);
    xh{i}(:,t+1) = K{i}*e_k{i}(t) + B{i}*I_LUT(i,t) + A{i}*xh{i}(:,t);
    xf{i}(:,t+1) = K0{i}*e_k{i}(t) + xh{i}(:,t);
    yf{i}(t,1) = C{i}*xf{i}(:,t);


end
RMSE(i) = sqrt(mean(e_k{i}));
end

x_f = cell2mat(xf);
x_h = cell2mat(xh);
y_f = cell2mat(yf);
```

```
y_h = cell2mat(yh);
```

E.2 Kalman old

```
close all
clear all
clc

load I
load V
load R
load R0
load TAU
load VOCV
load SOC

I_LUT(1,:) = I_05C_old;
I_LUT(2,:) = I_075C_old;
I_LUT(3,:) = I_1C_old;
I_LUT(4,:) = I_15C_old;
I_LUT(5,:) = I_2C_old;

V_LUT(1,:) = V_05C_old;
V_LUT(2,:) = V_075C_old;
V_LUT(3,:) = V_1C_old;
V_LUT(4,:) = V_15C_old;
V_LUT(5,:) = V_2C_old;

R0_LUT(1,:) = R0_05C_old;
R0_LUT(2,:) = R0_075C_old;
R0_LUT(3,:) = R0_1C_old;
R0_LUT(4,:) = R0_15C_old;
R0_LUT(5,:) = R0_2C_old;

R_LUT(1,:) = R_05C_old;
R_LUT(2,:) = R_075C_old;
R_LUT(3,:) = R_1C_old;
R_LUT(4,:) = R_15C_old;
R_LUT(5,:) = R_2C_old;

TAU_LUT(1,:) = tau_05C_old;
TAU_LUT(2,:) = tau_075C_old;
TAU_LUT(3,:) = tau_1C_old;
TAU_LUT(4,:) = tau_15C_old;
TAU_LUT(5,:) = tau_2C_old;
```

```

VOCV_LUT(1,:) = vocv_05C_old;
VOCV_LUT(2,:) = vocv_075C_old;
VOCV_LUT(3,:) = vocv_1C_old;
VOCV_LUT(4,:) = vocv_15C_old;
VOCV_LUT(5,:) = vocv_2C_old;

SOC_LUT(1,:) = media_charges_05C_old;
SOC_LUT(2,:) = media_charges_075C_old;
SOC_LUT(3,:) = media_charges_1C_old;
SOC_LUT(4,:) = media_charges_15C_old;
SOC_LUT(5,:) = media_charges_2C_old;

 Capacity = 4.05;
 Ts = 0.1;


     Bv1 = 0.001*[10 1 1]';
     V1 = Bv1*Bv1';
     V2 = 0.00001;
     V12 = 0;

     n = 3;


 for i = 1:5

     xf{i} = zeros(3,length(I_LUT(i,:)));
     xh{i} = zeros(3,length(I_LUT(i,:)));
     yf{i} = zeros(length(I_LUT(i,:)),1);

     P{i} = eye(n);


     yR = 0;
     yR0 = 0;
     yTAU = 0;
     yVOCV = 0;

     YR = [];
     YR0 = [];
     YTAU = [];
     YVOCV = [];


     A1 = [];
     A2 = [];
     A3 = [];

```

```

    bR = 0;
    bR0 = 0;
    bTAU = 0;
    bVOCV = 0;


    YR =  [YR;R_LUT(i,1:end-1)'];
    YR0 = [YR0;R0_LUT(i,1:end-1)'];
    YTAU = [YTAU;TAU_LUT(i,1:end-1)'];
    YVOCV = [YVOCV;VOCV_LUT(i,1:end-1)'];



    A1 = [A1;SOC_LUT(i,1:end-1).^3'];
    A2 = [A2;SOC_LUT(i,1:end-1).^2'];
    A3 = [A3;SOC_LUT(i,1:end-1)'];


    AR = [A1, A2, A3, ones(15,1)];
    AR0 = [A3, ones(15,1)];
    ATAU = [A2, A3, ones(15,1)];
    AVOCV = [A3, ones(15,1)];

    bR = AR\YR;
    bR0 = AR0\YR0;
    bTAU = ATAU\YTAU;
    bVOCV = AVOCV\YVOCV;

    VOCV{i}(1) = 4.18;
    R0{i}(1) = 0.0078;
    R{i}(1) = 0.0074;
    TAU{i}(1) = 0.0092;
    SoC{i}(1) = 1;


for t = 2:length(I_LUT(i,:))

    VOCV{i}(t) = bVOCV(1)*SoC{i}(t-1) + bVOCV(2);

    R0{i}(t) = bR0(1)*SoC{i}(t-1) + bR0(2);

    R{i}(t) = bR(1)*SoC{i}(t-1).^3 + bR(2)*SoC{i}(t-1).^2 + bR(3)*SoC{i}(t-1) + bR(4);

    TAU{i}(t) = bTAU(1)*SoC{i}(t-1).^2 + bTAU(2)*SoC{i}(t-1) + bTAU(3);

    SoC{i}(t) = SoC{i}(t-1) + I_LUT(i,t)*Ts/(Capacity*3600);
```

```

        A{i} = [1-TAU{i}(t)*Ts 0 0; 0 1 0; 1-TAU{i}(t)*Ts 0 0];
        B{i} = [TAU{i}(t)*Ts*R{i}(t) Ts/(3600*Capacity) TAU{i}(t)*R{i}(t)*Ts + R0{i}(t)]';
        C{i} = [0 0 1];
        D{i} = 0;

        K0{i} = P{i}*C{i}'*inv(C{i}*P{i}*C{i}'+V2);
        K{i} = (A{i}*P{i}*C{i}'+V12)*inv(C{i}*P{i}*C{i}'+V2);
        P{i} = A{i}*P{i}*A{i}' + V1 - K{i}*(C{i}*P{i}*C{i}'+V2)*K{i}';

        xh{i}(1,t) = xf{i}(1,t-1) + (-TAU{i}(t)*xf{i}(1,t-1) + TAU{i}(t)*R{i}(t)*I_LUT(i,t))*Ts;
        xh{i}(2,t) = xf{i}(2,t-1) + Ts/(3600*Capacity)*I_LUT(i,t);
        xh{i}(3,t) = xf{i}(1,t-1) + VOCV{i}(t) + R0{i}(t)*I_LUT(i,t);

        yh{i}(t,1) = C{i}*xh{i}(:,t);
        e_k{i}(t) = V_LUT(i,t) - yh{i}(t,1);
        xh{i}(:,t+1) = K{i}*e_k{i}(t) + B{i}*I_LUT(i,t) + A{i}*xh{i}(:,t);
        xf{i}(:,t+1) = K0{i}*e_k{i}(t) + xh{i}(:,t);
        yf{i}(t,1) = C{i}*xf{i}(:,t);


end
RMSE(i) = sqrt(mean(e_k{i}));
end

x_f = cell2mat(xf);
x_h = cell2mat(xh);
y_f = cell2mat(yf);
y_h = cell2mat(yh);
```