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Altman Z-Score Indicators

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Introduction

In an era marked by economic volatility and uncertainty, the ability to identify and mitigate financial crises has become imperative for businesses and policymakers alike. One prominent tool in this domain is the Altman Z-Score model, developed by Professor Edward I. Altman in the late 1960s. This model has since become a cornerstone in assessing a company's financial health and forecasting potential distress. Although considered dated, the Altman Z-score remains the benchmark against which most of the bankruptcy or default prediction models are evaluated. It continues to be widely utilized by both financial market practitioners and academic scholars for various purposes. This paper aims to examine the evolution of the Altman family of bankruptcy prediction models, including their extensions and diverse applications in financial markets and managerial decision-making.

The Altman Z-Score model synthesizes multiple financial ratios into a single score, providing a quantitative measure of a company's bankruptcy risk. By analyzing key financial indicators such as liquidity, profitability, leverage, solvency, and activity, the model assigns weights to each variable and aggregates them to produce a composite score. This score serves as a proxy for the likelihood of financial distress within a specified timeframe, typically one to two years.

The significance of the Altman Z-Score model extends beyond its predictive capabilities. Its simplicity, adaptability, and robustness have made it a staple in financial analysis across industries and geographic regions. Moreover, its empirical validation and widespread adoption in academia and industry underscore its utility and reliability as a tool for assessing corporate financial health.

While the original Z-Score model was developed over five decades ago, its relevance persists in today's dynamic and evolving business landscape. However, it is imperative to acknowledge the multifaceted nature of financial distress prediction. The dynamic interplay of market dynamics, industry trends, regulatory changes, and macroeconomic factors can exert significant influence on a company's financial health, rendering traditional models susceptible to inherent limitations.

Therefore, this thesis aims to be a literature review of the Z-Score Altman model in contemporary financial analysis, looking at its applicability nowadays from different perspectives. By examining its strengths, weaknesses, and applicability in different contexts, this study seeks to contribute to the ongoing discourse on financial distress prediction. Additionally, it will explore potential enhancements or alternative approaches to augment the model's predictive power and adaptability in today's complex business environment.

To give some context, in the recent past history there have been 6 main global crisis that affected all the world, and not a single company or sector: the Great Depression (1929 – 1939), the World War II (1939 – 1945), the Oil Crisis (1973), the Dot-Com Bubble Burst (2000-2001), the Global Financial Crisis (2007-2008) and the COVID-19 Pandemic (2020), which will be analyzed deeper in the last chapter. Each of them has been triggered by different factors and dynamics, initiating bankruptcy proceedings against companies above the historical average. Looking at granular cases instead, the literature defines three main types of crise: the latent, the manifest and the acute crise, each of them with different severity grades. A company crise is usually caused by financial mismanagement, change in market dynamics, operational issues, and strategic missteps. Some of these causes are due to poor management, others to external factors.

In case of insolvency for a company, different scenarios may occur, depending on various characteristic. For the purpose of this thesis, that will analyze the American Z score model behavior in Italy, it is important to underline that in various jurisdictions, different bankruptcy methods are employed to manage the affairs of financially troubled businesses. These methods include Chapter 7 and Chapter 11 bankruptcy in the United States, as well as Concordato Preventivo, Fallimento, and Amministrazione Straordinaria in Italy. Each method serves distinct purposes, ranging from liquidation to reorganization, and offers avenues for debtors and creditors to resolve financial challenges and achieve equitable outcomes.

In the United States, in case of a Chapter 7 bankruptcy, a trustee is appointed to oversee the sale of the company's assets, which are then distributed to creditors according to a priority ranking. Chapter 11 bankruptcy instead allows a company to restructure its debts and operations while remaining in business.

In Italy, bankruptcy proceedings are governed by the Italian Bankruptcy Law (Legge Fallimentare) and are primarily regulated by the Italian Civil Code and the Italian Bankruptcy Law. Concordato Preventivo is a pre-bankruptcy agreement between a debtor and its creditors aimed at restructuring the debtor's debts and avoiding bankruptcy; Fallimento instead is the formal declaration of bankruptcy by a court, initiated when a debtor is unable to pay its debts; Amministrazione Straordinaria is a special insolvency procedure applicable to large companies of strategic importance to the national economy, which allows for the temporary suspension of the company's obligations and the appointment of a special commissioner to manage the company's affairs and develop a restructuring plan.

These definitions are fundamental to properly understand the Chapter 2 of this paper, where for a sample of Italian manufacturing companies a Z score will be analyzed in 2019 and 2021. For both the years, the population will be divided in three subgroups: the first percentile of companies with lowest Z' score, the last percentile of companies with highest Z' score, and the population in the remaining 98 percentiles. The main purpose is to understand the different behavior among the population, especially on the tails of the distribution, which are fundamental to keep into consideration in an Italian context. Results will show that an overall decrease in Z' score happened in that time frame, with a major impact for companies which were already declared as insolvent by the model in 2019. The Chapter will also give a framework of the manufacturing Italian industry, as well as government extraordinary interventions during Covid-19, which will allow the reader to better interpret the results.

Chapter 1 focuses instead on the Altman Z Score Model through a detailed exploration of its origins, methodology, and empirical applications.

By traversing these chapters, the reader will gain a holistic understanding of financial crises, from theoretical frameworks to practical applications. Moreover, this thesis aims to spark further inquiry and debate in the field, paving the way for continued advancements in crisis management and risk mitigation strategies.

Chapter 1

Altman Z Score Model

1.1 Introduction to the Credit Score System

Credit scoring systems aimed at assessing a firm's repayment likelihood can trace their origins back to the medieval era, particularly during the Crusades when travelers required "loans" for their expeditions. These systems gained prominence later in the United States, notably during the nation's westward expansion, as companies and entrepreneurs played pivotal roles in economic growth. In the 1800s, lending institutions typically evaluated rudimentary financial information, primarily subjective or qualitative in nature, focusing on ownership, management variables, and collateral. It wasn't until the early 1900s that rating agencies and financially oriented corporate entities introduced univariate accounting measures and industry peer group comparisons with rating designations.

These innovative techniques allowed analysts to compare individual corporate entities' financial performance metrics with reference databases of time series (same entity) and cross-section (industry) data, emphasizing the critical role of data and databases in meaningful diagnostics. In the realm of credit scoring, data reigns supreme, and the success of models in capturing the probability of default hinges on their applicability to databases of varying sizes and relevance.

The original Altman Z-score model (Altman 1968) was developed based on a sample of sixty-six manufacturing companies, categorized into bankrupt and nonbankrupt firms, with an additional holdout sample of fifty companies. In the absence of electronic databases, researchers and analysts had to construct their own databases from primary (annual report) or secondary (Moody's and S&P industrial manuals and reports) sources. While researchers today often have access to databases containing thousands, even millions, of firms, the significance of databases remains paramount. Notably, Moody's and S&P made significant investments in acquiring extensive databases, underlining their importance in credit assessment.

1.2 Z Score Origins

Altman incorporated Linear Discriminant Analysis (LDA) into his research, a method originally proposed by Ronald Fisher in 1936 for classifying objects into predefined populations. Although not as widely recognized as linear regression, this technique, as elucidated by Altman, has found applications across diverse disciplines since its introduction. Initially employed in the realms of biology and behavioral sciences, its adoption in the financial sector came later.

Altman recognized the potential of LDA and leveraged it to formulate his own model, which sought a balance between precision and simplicity. This model aimed to ascertain whether a company falls within one of two predefined groups: the first comprising healthy companies and the second comprising those that have faced failure.

The fundamental principle of this statistical method is as follows: establish a criterion for categorizing companies into either the financially stable or financially distressed group, with the goal of minimizing estimation errors. Instead of relying on a single financial statement indicator to determine the parameter that distinguishes between these groups, a set of accounting ratios is employed. These ratios are appropriately weighted and condensed into a statistical index known as the "score."

To elaborate, the score's value is derived through the formulation of a function, referred to as the discriminant function. This function incorporates independent variables comprising various balance sheet indicators, each assigned specific weighting coefficients. Importantly, the discriminant analysis ensures the objective attribution of these coefficients.

The resulting discriminant function is configured as follows:

$$S_j = v_1X_{1j} + v_2X_{2j} + \ldots + v_nX_{nj}$$

Where:

S_j = score of the j-th company

v_i = coefficient of the variable X_i

X_{ij} = descriptive variable of the i-th characteristic for the j-th company, each of the measured parameters must be considered several times over a period of time.

It is important to consider that each of the objects has its own peculiarities, which vary over time. The parameters considered cannot be values considered only once or analyzed only once, abstract, but rather must be somehow contextualized with respect to the others and considered in their "changing" value with respect to the

others.

Let's examine two pre-defined samples, denoted as A and B, each with a known size. The size of the first sample is N_A, and the second sample has a size of N_B. Now, by introducing the variables X_A e X_B, representing the matrices of observations on the variables, with dimensions $N_A \times n$ for the first sample and $N_B \times n$ for the second sample; $\overline{X_A}$ and $\overline{X_B}$ the vectors containing the means of the variables for each sample; $\bar{x} = \frac{N_A}{N} \cdot \bar{x}_A + \frac{N_B}{N} \cdot \bar{x}_B$ the column vector representing the combined observations, where it's evident that the sum of N_A and N_B equals N, the variable x serves as our reference point for analyzing the i-th variable, which will undergo examination for its variance and covariance; and W the $n \times n$ variance and covariance matrix.

We then pinpoint our coefficient denoted by ai in the ultimate formula, and it will be determined as follows:

$$a_i = \pi \cdot r^2 \cdot i(\overline{x_A} - \overline{x_B})' V^{-1}$$

The final value of the score will then be:

$$S_j = (\overline{x_A} - \overline{x_B})' V^{-1}$$

while the average score of population A, denoted as the score of A or S_A, is expressed as:

$S_A = (\overline{x_A} - \overline{x_B})' V^{-1} \overline{x_j}$ as well as S_B.

The rule of linear classification can thus be articulated in terms of distances among scores: the j-th enterprise is assigned to population A if:

$$|S_j - S_A| < |S_j - S_B|$$

otherwise, it is assigned to B population.

In geometric terms, the Linear Discriminant Analysis is represented in *Figure 1*, in 2 variables, 2 populations case.

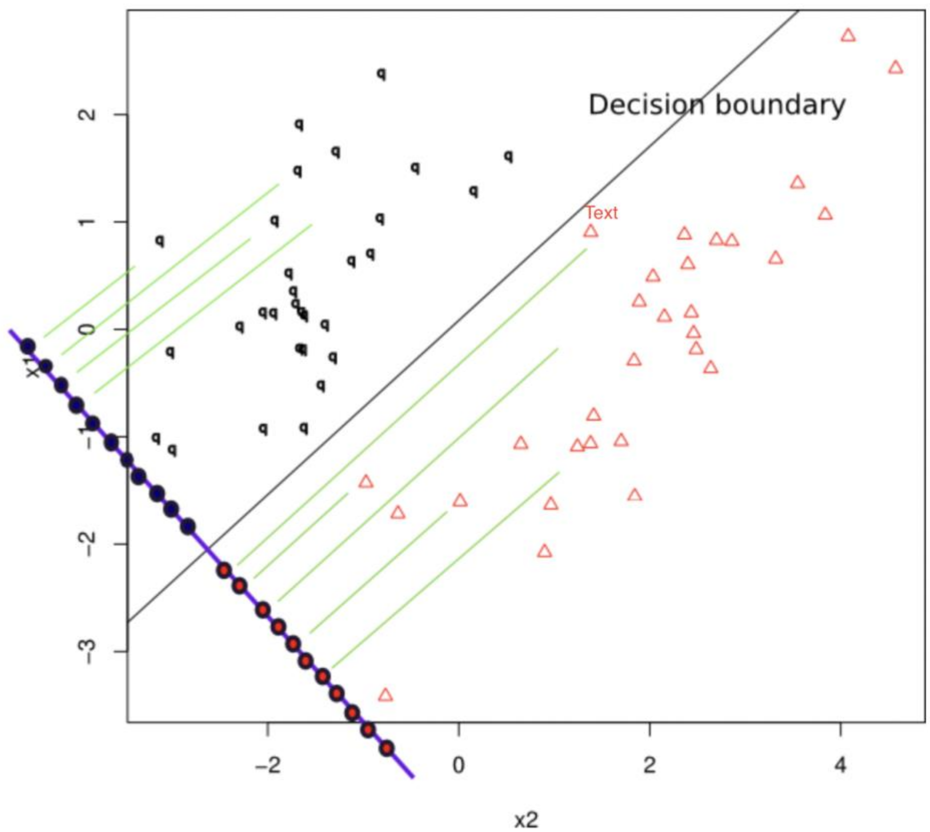


Figure 1 - Linear Discriminant Analysis – Source: R.A. Fisher, The use of multiple measurements in taxonomic problems, 1936.

In the above figure, we observe two populations, A and B, plotted on the X_1 and X_2 axes. Notably, there's a central line, referred to as Decision Boundary. For now, our focus will be on this line, as it divides the space into two portions, facilitating a classification of points in proximity with minimal attribution errors (which we will touch upon later). This implies that this line possesses the property of offering a clearer designation for points close to it between the two sets.

At the base of the axis, there's another line perpendicular to the first one. This represents the optimal discriminant function, considering the characteristics X1 and X2 of the two groups. The businesses to be classified are represented by the points on the analyzed line, making their classification more straightforward compared to considering their 2 characteristics separately.

In this analysis, the only subjective aspect lies in the choice of variables X to observe in the businesses for classification, while the weights are determined by considering the characteristics of the two populations.

1.3 Classification Error

Being the Linear Discriminant Analysis a model, as we pointed out in the previous chapter, it will always be affected by errors. The concept and purpose of models is to minimize them, but they will always occur at some point. Taking as example the Figure 1, a classification error occurs if a point classified as q is in the reality a *triangle* point.

To summarize, when applying the linear discriminant model to predict business crises, two potential errors arise: incorrectly classifying a healthy company as unhealthy (False Positive) or incorrectly classifying an unhealthy company as healthy (False Negative).

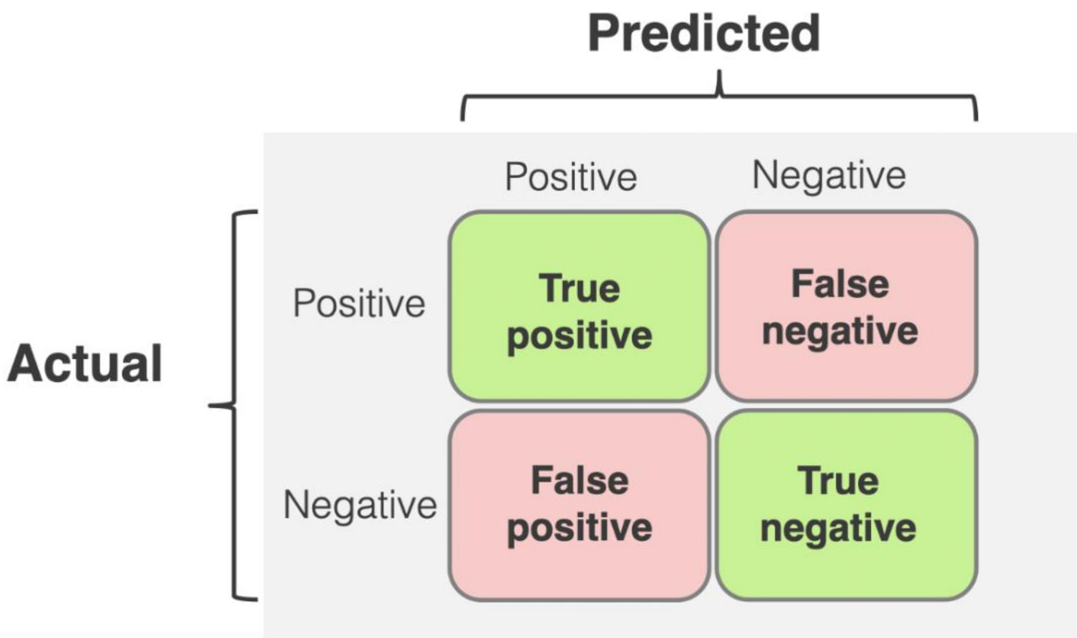


Figure 2 - Confusion Matrix. Source: EvidentlyAI

Considering the primary objective of the model, the error of the second type is decidedly more severe. In such a case, a bank making an evaluation mistake risks granting funding to a potentially insolvent business. Similarly, an entrepreneur aiming to assess their production process might erroneously perceive it as sound when urgent corrections are necessary.

Conversely, the error of incorrectly classifying a healthy company as abnormal would be less costly. This rationale leans towards preferring a more "cautious" model, one that more readily classifies a healthy company as unhealthy, rather than a model that, in moments of uncertainty, tends to categorize the subject among the healthy businesses.

Every classification model (e.g. a model where the aim is to classify a dataset among different groups) has to deal with classification errors, and they are also used to compute the efficiency of the model (e.g. the area under the ROC curve for the standard logistic regression).

1.4 The study

In his pursuit of creating a comprehensive model to determine a company's potential for failure, Altman aims to encapsulate all necessary information in a single value. The American economist's objective is clear – to develop a model that should be rapid and simple, catering to individuals with limited mathematical and statistical knowledge. This ingenious approach harmonized the demands of academics seeking precision and practitioners desiring an easily manageable tool.

Altman emphasized that his model is not probabilistic but descriptive comparative. Its purpose is to identify trends in financial indicators in the years preceding insolvency for both healthy and troubled companies.

During Altman's time, one widely-used method for analyzing a company's financial health was the ratio analysis. Despite its popularity among analysts, Altman notes that ratio analysis, focusing on individual balance sheet indices, did not receive favorable acknowledgment from academics. While it demonstrated effectiveness, particularly in stability analysis over a discrete time frame preceding financial distress, its limitations were evident: the methodology's univariate nature and emphasis on individual signals of impending problems make it "susceptible to faulty interpretation and potentially confusing outcomes" (E. I. Altman, 1970).

Consider an enterprise with low profitability and/or solvency; it might be perceived as financially vulnerable. However, if it exhibits a high level of liquidity, the situation may not be as dire. Altman emphasizes the necessity of a model that can provide a holistic view of a company's situation, going beyond the limitations of ratio analysis.

As Altman first wrote in 1968, and as appears increasingly evident even in the late 1990s, scholars seem to be leaning towards phasing out ratio analysis as a method for evaluating business performance. Theorists are critical of arbitrary rules of thumb, such as company ratio comparisons, which have been widely embraced by practitioners. With esteemed members of the academic community questioning the relevance of ratio analysis, one might wonder if it's relegated solely to the realm of practicality. The aim of Altman was to reconcile traditional ratio analysis with the more rigorous statistical techniques favored by academics in recent years, rather than completely abandoning it. Alongside our primary focus on corporate bankruptcy, Altman also aims to assess the efficacy of ratio analysis as an analytical tool.

It's worth noting that much of the foundational research for this paper was conducted in 1967, and subsequent studies have discussed the Z-Score model and its effectiveness, including adaptations made in 1995 for credit analysis of emerging market corporations.

Altman's goal is to evolve existing models by combining different measures into a unique and meaningful model. This task, however, presents challenges, especially in selecting the indices that will be part of the model and the method used to integrate these singular and separated values into a unified whole.

Altman's initial studies were published in 1968 in "The Journal of Finance" under the title "Financial Ratios, Discriminant Analysis, and the Prediction of Corporate Bankruptcy" and will be crucial for the purpose of this paper.

To better understand the reasons behind Altman's aim, it is worth to compare the Traditional Ratio Analysis with the Multivariate Discriminant Analysis, then chosen by Altman for his model, as well described in the "PREDICTING FINANCIAL DISTRESS OF COMPANIES: REVISITING THE Z-SCORE AND ZETA® MODELS" paper, written by Edward I. Altman and published by the New York University in July 2000.

1.5 Traditional Ratio Analysis

The examination of operational and financial challenges within companies has been a topic well-suited for analysis using financial ratios. Before the emergence of quantitative performance measures, organizations were established to provide qualitative assessments of the creditworthiness of specific merchants.

One of the seminal works in the realm of ratio analysis and bankruptcy classification was conducted by Beaver (1967). His analysis of various bankruptcy predictors through univariate methods paved the way for subsequent multivariate approaches by him and others. Beaver identified several indicators capable of distinguishing between matched samples of failed and non-failed firms up to five years before failure. Despite initial skepticism towards multivariate analysis, subsequent studies, including the Z-Score model, embraced this approach. Deakin (1972) further explored Beaver's variables using multivariate discriminant models.

These studies suggest a clear potential for ratios to predict bankruptcy. Generally, ratios measuring profitability, liquidity, and solvency emerged as the most significant indicators. However, the exact order of their importance remains unclear, as different studies often highlight different ratios as the most effective predictors.

While these studies offer valuable insights into the performance and trends of specific metrics, their practical application for assessing bankruptcy potential is questionable. In most cases, the methodology relied on univariate analysis, emphasizing individual signals of impending issues. Such an approach to ratio analysis is susceptible to misinterpretation and potential confusion. For example, a firm with a poor profitability or solvency record may be deemed a potential bankruptcy risk, but its above-average liquidity may mitigate concerns. This potential ambiguity in comparing the relative performance of different firms underscores the limitations of univariate analysis.

An appropriate extension of these studies would be to build upon their findings and integrate multiple measures into a comprehensive predictive model. In doing so, the strengths of ratio analysis as an analytical technique would be emphasized rather than diminished. Key questions in this endeavor include determining the most important ratios for detecting bankruptcy potential, assigning appropriate weights to selected ratios, and objectively establishing these weights.

1.6 Multivariate Discriminant Analysis

Altman's exploration into predicting corporate bankruptcy involved the consideration of four potential tools (E. I. Altman and A. Saunders, 1997): the linear probability, the *logit*, the *probit*, and the discriminant analysis models.

After careful consideration of the nature of the problem and the purpose of this analysis, Altman chose multiple discriminant analysis (MDA) as the appropriate statistical technique. Although not as popular as regression analysis, MDA has been utilized in a variety of disciplines since its first application in the 1930s. During those earlier years, MDA was used mainly in the biological and behavioral sciences. In recent years, this technique has become increasingly popular in the practical business world as well as in academia.

MDA is a statistical technique used to classify an observation into one of several a priori groupings dependent upon the observation's individual characteristics. It is used primarily to classify and/or make predictions in problems where the dependent variable appears in qualitative form, for example, male or female, bankrupt or nonbankrupt. Therefore, the first step is to establish explicit group classifications. The number of original groups can be two or more. After the groups are established, data are collected for the objects in the groups; MDA in its most simple form attempts to derive a linear combination of these characteristics which "best" discriminates between the groups. If a particular object, for instance, a corporation, has financial ratios which can be quantified for all of the companies in the analysis, the MDA determines a set of discriminant coefficients. When these coefficients are applied to the actual ratios, a basis for classification into one of the mutually exclusive groupings exists. The MDA technique has the advantage of considering an entire profile of characteristics common to the relevant firms, as well as the interaction of these properties. A univariate study, on the other hand, can only consider the measurements used for group assignments one at a time. Another advantage of MDA is the reduction of the analyst's space dimensionally, that is, from the number of different independent variables to G-1 dimension(s), where G equals the number of original *a priori* groups. This analysis is concerned with two groups, consisting of bankrupt and nonbankrupt firms. Therefore, the analysis is transformed into its simplest form: one dimension.

Altman's discriminant function, as introduced with Fisher's technique, aimed at combining individual financial indices into a unified value Z in the form of:

$$Z = a_1x_1 + a_2x_2 + \dots\dots + a_nx_n$$

where independent variables x_i assume the values of various financial indicators, assessing a company's potential for failure, and a_i the weights assigned to them by the model.

The economist's approach involved a thorough investigation to identify suitable predictive variables, ensuring the discrimination's feasibility. Altman grappled with the question of whether there were significant differences between healthy and distressed companies that would facilitate accurate discrimination and the construction of a precise yet user-friendly model (E. I. Altman, 1968).

When utilizing a comprehensive list of financial ratios in assessing a firm's bankruptcy potential, there is reason to believe that some of the measurements will have a high degree of correlation or collinearity with each other. While this aspect is not serious in discriminant analysis, it usually motivates careful selection of the predictive variables. It also has the advantage of potentially yielding a model with a relatively small number of selected measurements which convey a great deal of information. This information might very well indicate differences among groups, but whether or not these differences are significant and meaningful is a more important aspect of the analysis. The primary advantage of MDA in dealing with classification problems is the potential of analyzing the entire variable profile of the object simultaneously rather than sequentially examining its individual characteristics (E.I. Altman, 2000).

Just as linear and integer programming have improved upon traditional techniques in capital budgeting, the MDA approach to traditional ratio analysis has the potential to reformulate the problem correctly. Specifically, combinations of ratios can be analyzed together to remove possible ambiguities and misclassifications observed in earlier traditional ratio studies. Altman's unique approach involved customizing Fisher's technique through a meticulous analysis of a sample of companies. This approach tested the direct and immediate descriptive potential of various indices and their combinations, evaluating their ability to discern the lived health state of a company. The details of this analysis reveal Altman's commitment to creating a model that is not only precise but also accessible to a broad audience operating in the economic domain.

1.7 The Z Score Model

Altman embarked on developing a specialized application of Fisher's model, commencing with the creation of a company sample for testing and constructing his unique model. In this process, Altman stressed the importance of forming a sample with entities sharing similar characteristics, a crucial factor in identifying the descriptive power of discriminating variables. As previously mentioned, many of the indices considered in ratio analysis were interrelated, influencing each other. It is precisely this aspect that allowed Altman to choose a relatively low number of indices, ensuring simplicity of use and interpretation for his method and avoiding the ambiguities that plagued older tools.

The initial set comprises 66 corporations, evenly divided with 33 entities falling into each of the two categories. The first group, labeled as Group 1, consists of manufacturers that have filed for bankruptcy under Chapter X of the National Bankruptcy Act between 1946 and 1965. Although a 20-year timeframe isn't ideal due to shifts in average ratios over time, it was the only available option due to data constraints. Acknowledging the lack of complete homogeneity within this group, mainly due to variations in industry and size, a meticulous selection of non-bankrupt firms was made for Group 2.

Group 2 consists of a paired sample of manufacturing firms chosen through a stratified random method, where firms were stratified based on industry and size, with assets ranging between \$1 and \$25 million. While the mean asset size of Group 2 firms (\$9.6 million) slightly exceeded that of Group 1, an exact match in asset size between the two groups was deemed unnecessary. Additionally, Group 2 firms were still operational at the time of analysis, and the data collected aligned with those compiled for bankrupt firms, extracted from financial statements dated one year prior to bankruptcy.

An essential consideration was determining the asset-size range for sampling. The decision to exclude both small firms (under $1 million in total assets) and very large companies from the initial sample was primarily influenced by the asset range of firms in Group 1. Furthermore, bankruptcy occurrences in large asset-size firms were rare before 1966, but increased notably after 1970, with several major bankruptcies like Penn Central Railroad emerging. Industrial bankruptcies of substantial magnitude also surged after 1978, with over 100 Chapter 11 bankruptcies exceeding $1 billion recorded since the enactment of the existing Bankruptcy Code in 1978.

It's often argued that financial ratios inherently adjust statistics by size, effectively mitigating much of the size effect. The Z-Score model, discussed later, appears sufficiently robust to accommodate large firms.

Moving forward, Altman carefully considered which indices could best facilitate a clear separation of the two groups. These variables initially numbered 22 and were classified into five macro-categories: Liquidity, Profitability, Financial Leverage, Solvency, and Assets.

While the Beaver study (1967) identified the cash flow to debt ratio as the most effective single ratio predictor, it was omitted from the 1968 study due to inconsistencies in depreciation and cash flow data. Nonetheless, the results obtained were still superior to those achieved by Beaver with his single best ratio. Cash flow metrics were integrated into the ZETA model tests, as discussed later.

Altman's criteria for selecting ratios were based on their popularity in pre-study literature and their potential relevance. Additionally, Altman introduced new indices specifically developed for this study.

Each of the 22 indices underwent evaluation for its individual contribution and predictive ability within the model. Furthermore, each index was incorporated into a function to assess its contribution within a more complex context, considering its behavior in relation to other indices and the correlations between them.

Contrary to expectations, the study revealed that within this multivariate function, the most significant ratios were not the same as those accorded greater importance in univariate analysis. Altman attached crucial importance to the interaction these ratios would have with each other when selecting the optimal function through an iterative study. In particular, to arrive at a final set of variables, the following procedures are employed: assessment of the statistical significance of alternative functions, including determination of the relative contributions of each independent variable; examination of intercorrelations among relevant variables; evaluation of predictive accuracy of different profiles; and finally, judgment of the analyst.

The final function took the following form:

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 0.999X_5$$

Before delving into an analysis of individual variables, it is essential to draw the reader's attention to the values assumed by the different weights. These weights already provide insights into which variables will have a greater impact on determining the final Z score, highlighting the variables with higher discriminatory power.

1.8 The five Independent Variables

$$\boldsymbol{X_1 = \frac{Working\ Capital}{Total\ Assets}}$$

where: *Working Capital = Current Assets - Current Liabilities*

With *Current Assets* is meant all the resources on a company's balance sheet that are expected to be converted into cash or used up within one year, such as Cash and Cash Equivalents, Accounts Receivable, Inventory, Prepaid Expenses, Short-Term Investments, Notes Receivable (Short-Term). On the contrary, *Current Liabilities* are all the obligations that a company is expected to settle within one year or its normal operating cycle, whichever is longer, such as Accounts Payable, Short-Term Debt, Accrued Liabilities, Income Taxes Payable, Dividends Payable, Unearned Revenue, Notes Payable (Short-Term), Current Portion of Long-Term Debt.

The ratio of working capital to total assets, commonly examined in studies addressing corporate issues, serves as an indicator of the firm's net liquid assets in relation to its overall capitalization. Working capital offers insight into the firm's liquidity and size attributes. Typically, a firm encountering sustained operating losses will witness a reduction in current assets relative to total assets. Among the three liquidity ratios assessed, this particular ratio emerged as the most insightful. The other two liquidity ratios examined, namely the current ratio and the quick ratio, were found to be less beneficial and susceptible to erratic patterns in certain failing firms.

A higher X_1 indicates a larger proportion of assets financed by working capital, which is generally considered favorable in terms of short-term financial strength.

To give some context, the table below summarize X_1 value in different industries:

Industry	**X1 value range**	**Comment**
Retail	0.2 – 0.5	Lower X1 ratios due to the nature of their operations, where inventory turnover is crucial.
Technology	0.5 – 0.8	Relatively higher X1 ratios as they often have lower inventory levels and faster cash conversion cycles.
Manufacturing	0.3 – 0.6	Moderate X1 ratios, as they typically require a balance between working capital and production needs.
Service	0.6 - 1	Higher X1 ratios due to lower inventory requirements and faster cash turnover.
Healthcare	0.4 – 0.7	Moderate X1 ratios, reflecting the need for a balance between liquidity and operational demands.

Table 1 - X_1 average value across industries. Source: SUBS

$$\boldsymbol{X_2 = \frac{Retained\ Earnings}{Total\ Assets}}$$

This ratio assesses the proportion of a company's total assets that is financed by its retained earnings, reflecting the contribution of internally generated profits to its asset base.

Retained earnings represents the cumulative amount of reinvested earnings and/or losses of a company throughout its entire lifespan. This account is also known as earned surplus. It's important to acknowledge that the retained earnings account can be influenced by corporate quasi-reorganizations and stock dividend declarations. Although these factors aren't evident in this study, significant reorganizations or stock dividends could introduce bias, necessitating appropriate adjustments to the accounts.

This metric, reflecting cumulative profitability over time, can be considered a "new" ratio, as mentioned earlier. The firm's age is implicitly taken into account in this ratio. For instance, a relatively young company might exhibit a lower RE/TA ratio due to insufficient time to accumulate profits. Consequently, it could be argued that young firms might be somewhat disadvantaged in this analysis, with a relatively higher likelihood of being classified as bankrupt compared to older firms, all other things being equal. However, this mirrors the real-world scenario where failure rates are notably higher in a company's early years. In 1993, around 50% of all failing firms did so within the first five years of operation (Dun & Bradstreet, 1994).

Moreover, the RE/TA ratio also serves as an indicator of a company's leverage. Firms with a high RE relative to TA have financed their assets through retained profits rather than relying heavily on debt.

When analyzing X_2, it's important to consider the historical context. Examining trends in retained earnings relative to total assets over time provides insights into the company's financial trajectory. Consistent growth in retained earnings may signal financial strength and prudent financial management.

On the flip side, a declining X_2 ratio over time may raise concerns. It could indicate challenges in the company's ability to generate and retain earnings, potentially impacting its overall financial stability.

Industry	**X_2 value range**	**Comment**
Retail	0.1 – 0.3	Lower X_2 ratios as they typically operate with thinner profit margins and may rely more on external financing.
Technology	0.2 – 0.4	Moderate X_2 ratios, reflecting a mix of internal and external financing.
Manufacturing	0.3 – 0.5	Moderate to higher X_2 ratios, as they often need to reinvest profits for capital expenditures.
Service	0.4 – 0.6	Higher X_2 ratios, indicating a reliance on retained earnings for operational needs.
Healthcare	0.2– 0.4	Moderate X_2 ratios, reflecting a balance between internal and external financing.

Table 2 - X_2 average value across industries. Source: SUBS

$$\boldsymbol{X_3 = \frac{EBIT}{Total\ Assets}}$$

Where *EBIT* stands for *Earnings Before Interest and Taxes*, which is a measure of a company's operating profit, representing its profitability from core business operations before accounting for interest expenses and income taxes.

This ratio represents the genuine productivity of the company's assets, unaffected by tax implications or leverage considerations. Given that a company's viability hinges on the earning capacity of its assets, this ratio seems especially relevant for investigations into corporate collapse. Moreover, insolvency in a bankruptcy context arises when the total liabilities surpass a fair assessment of the company's assets, with valuation based on asset earning potential. A higher value indicates that the company is more efficient in generating profits relative to its total asset base. This efficiency is a positive signal, suggesting that the company is effectively managing its resources to produce earnings.

On the other hand, a lower may imply lower profitability in relation to the total assets employed. This could be due to various factors such as operational inefficiencies, increased operating expenses, or lower-than-expected revenues.

Industry	**X_3 value range**	**Comment**
Retail	0.05 – 0.15	Lower X3 ratios due to the nature of the industry, where profit margins tend to be thinner.
Technology	0.10 – 0.25	Moderate X3 ratios, reflecting their potential for higher-profit margins.
Manufacturing	0.08 – 0.20	Moderate to higher X3 ratios, indicating their ability to generate earnings from their asset base.
Service	0.15 – 0.30	Higher X3 ratios, as their assets are often more human-capital-intensive.
Healthcare	0.10 – 0.20	Moderate X3 ratios, reflecting the balance between profitability and asset utilization.

Table 3 - X_3 average value across industries. Source: SUBS

$$\boldsymbol{X_4 = \frac{Market\ Value\ of\ Equity}{Total\ Liabilities}}$$

X_4 plays a significant role by examining the relationship between the market value of equity and the book value of total liabilities. This ratio, calculated as the market value of equity divided by the book value of total liabilities, offers insights into the market's perception of a company's financial position. Equity comprises the collective market worth of all classes of stock, encompassing both preferred and common shares, while liabilities encompass both current and long-term obligations. This metric indicates the extent to which the company's assets can depreciate in value (assessed by the combined market value of equity and debt) before liabilities surpass assets, leading to insolvency.

A higher X_4 is generally interpreted as a positive signal. It signifies that the market values the company's equity more favorably than its accounting liabilities. This optimism suggests that investors have confidence in the company's ability to generate future cash flows and view its financial position more optimistically.

In summary, X_4 adds a market-based dimension to the Altman Z-Score, capturing the market's perception of a company's financial strength in relation to its liabilities, which was overlooked in many previous failure studies. For example, Altman explains that a company with a market value of equity worth $1,000 and a debt of $500 could withstand a two-thirds decline in equity value before becoming insolvent. Similarly, the same company but with an equity value of $250 would become insolvent with a one-third decline in equity value (E. I. Altman, 1968).

The reciprocal of X4 is a slightly adapted version of a variable effectively employed by Fisher (1959) in studying corporate bond yield-spread differences. It also proves to be a more reliable predictor of bankruptcy compared to a similar but more commonly used ratio: net worth divided by total debt (based on book values). Later, Altman will substitute the book value of net worth with market value to formulate a discriminant function for privately held firms (Z') and for non-manufacturers (Z").

Recent models, such as the KMV approach, primarily rely on the market value of equity and its volatility. The market value of equity acts as a proxy for the company's asset values.

Industry	**X4 value range**	**Comment**
Retail	0.6 – 2.2	Lower X4 ratios reflecting potential volatility and lower market valuations compared to other sectors.
Technology	2.5 – 2.5	Higher X4 ratios, due to the market's favorable perception of their innovative potential and growth prospects.
Manufacturing	0.8 – 2.5	Moderate X4 ratios, reflecting a balance between market confidence and industry stability.
Service	1 – 2.8	Moderate X4 ratios, influenced by factors like intellectual property and brand value.
Healthcare	2.2 – 2	Higher X4 ratios, given the market's confidence in the sector's stability and long-term demand.

Table 4 - X_4 average value across industries. Source: SUBS

$$\boldsymbol{X_5 = \frac{Total\ Sales}{Total\ Assets}}$$

In the Altman Z-Score framework, X_5 flows as a measure of operational efficiency, encapsulating how well a company transforms its total assets into revenue. Emphasizing the crucial link between operations and financial performance, X_5 integrates seamlessly into a comprehensive evaluation of a company's health.

The capital turnover ratio is a commonly used financial metric that demonstrates the firm's ability to generate sales from its assets. It serves as an indicator of management's effectiveness in navigating competitive environments. Despite being the least impactful ratio when considered individually, this final ratio holds significant importance due to its distinctive correlation with other variables in the model. In fact, it wouldn't have been included based solely on its individual statistical significance. However, its relationship with other variables contributes substantially to the overall discriminatory power of the model. Nevertheless, there exists considerable variability among industries in terms of asset turnover, prompting the specification of an alternative model (Z") that excludes X5 at a later stage.

Industry	**X5 value range**	**Comment**
Retail	0.2 – 0.6	Lower X5 ratios due to potentially lower asset turnover in a sector with significant inventory and physical assets.
Technology	0.5 – 1	Higher X5 ratios, indicating efficient utilization of assets to generate revenue, given the innovative and high-growth nature of the sector.
Manufacturing	0.3 – 0.7	Moderate X5 ratios, due to the nature of the business where high assets are needed to produce.
Service	0.6 – 2.2	Higher X5 ratios, particularly for those relying on intellectual capital and service-driven revenue streams.
Healthcare	0.4 – 0.8	Moderate X5 ratios, considering the capital-intensive nature of the sector and the importance of efficient asset utilization.

Table 5 - X_5 average value across industries. Source: SUBS

Identifying the five indicators as independent variables in the function, Altman established the five weighting coefficients. These coefficients represent a weighted value assigned to each of the five variables, amplifying the distinctions between companies under normal balanced conditions and those in a distressed situation, as already highlighted by the financial indicators included in the model.

1.9 The Results

The average value assumed by the individual variables, and their respective F ratio, for groups G1 and G2 is as follows:

Variable	Average Value Healthy Companies	Average Value Failed Companies	F ratio
X1	− 6,1%	41,4%	32,50
X2	−62,6%	35,5%	58,86
X3	−31,8%	15,3%	26,56
X4	40,1%	247,7%	33,26
X5	150,0%	190,0%	2,84

Table 6 - Healthy vs Failed companies variables values. Source: NYU

In the table above, we can discern the individual contributions of various selected variables to the model. It's apparent that the ratio values for financially stable companies consistently remain positive and notably higher than those associated with distressed companies. Consequently, a company's potential for financial distress inversely correlates with its discriminatory score, indicating a lower score for higher risk.

Regarding a specific variable, its values are quite similar between the two groups (150.0% for one and 190.0% for the other). When conducting a sequential, univariate analysis, the contribution of this variable might go unnoticed.

This brings us to a segment of Altman's methodology in determining the array of variables applied in the final discriminant function. He initially assessed their significance individually before delving into a collective analysis. Notably, the index with comparatively weaker performance in univariate analysis is strategically assigned one of the higher weights in the ultimate formula.

A method to assess the overall effectiveness of the model is through the F-value, which represents the ratio of the sums-of-squares between-groups to the within-groups sums-of-squares. Maximizing this ratio serves to increase the separation between the means of the groups while simultaneously reducing the dispersion of individual points (firm Z-values) around their respective group means. This test, commonly known as the F-test, is logically suitable because the goal of the MDA is to identify and utilize variables that effectively distinguish between groups and exhibit consistency within groups.

In the original two-group sample, the group means are as follows:

Group 1 = -0.29, F = 20.7

Group 2 = +5.02, F_{4n} (0.01) = 3.84

The significance test therefore rejects the null hypothesis that the observations originate from the same population. The means of variables measured one financial statement prior to bankruptcy and the resulting F-statistics were presented in Table 6. Variables X_1 through X_4 are all statistically significant at the 0.001 level, indicating substantial differences among groups in these variables. Variable X_5, however, does not exhibit a significant difference among groups, and its inclusion in the variable profile remains unclear. Unilaterally, all ratios suggest higher values for the nonbankrupt firms, and all discriminant coefficients demonstrate positive signs, as expected. Hence, a firm's distress potential is inversely proportional to its discriminant score.

Considering the following table:

Variable	Scale Vector	Importance Order
X1	3.29	5
X2	6.04	4
X3	9.89	1
X4	7.42	3
X5	8.41	2

Table 7 - Variables importance order. Source: NYU

As previously highlighted, the true significance of a variable is not observable when considered in isolation. Altman, therefore, examines the coefficients for their corrected values based on the relative contribution of each variable. Upon observing the table, it becomes evident that, in terms of importance for discrimination, variables X_3, X_5, and finally X_4 are most useful.

The profitability index is undoubtedly the key contributor to distinguishing healthy companies from those on the verge of failure. This is not surprising, as the profit of a company on the brink of failure is likely to be negligible or even absent.

Let's shift our attention to variable X_5. Its importance in a multivariate context is emphasized, making it the second most significant in the studied model. The reason for this result is likely the strongly negative correlation observed between X_3 and X_5 in the group of failed companies (equal to -0.78). This negative correlation is confirmed in studies conducted on groups after those used to develop the model.

This result underscored the greater significance of a negative correlation, rather than a positive one, as it carries additional information beyond a correlation greater than zero.

This negative correlation can be attributed to the fact that failed companies tend to be characterized by assets whose value tends to deteriorate due to cost-cutting strategies, leading to a reluctance to renew and replace old company assets. Additionally, their financial size tends to shrink over time through cumulative losses, rendering any sales momentum ineffective.

Once the values of various ratios are calculated, it becomes possible to determine the Z score for each company in the two groups or any company under analysis. After establishing the average Z score for the two considered groups, it is suggested to identify a range of Z scores. Using this range, the health status of a given company can be classified by calculating its score, facilitating a determination of its overall health.

1.10 Altman's Analysis

To analyze the results, check the model and improve it, Altman applied a process constituted by 6 steps: control of outcomes on the initial sample, by looking at the confusion matrix, giving a 95% precision to the model; predictive ability two years before failure, by looking at the confusion matrix on a 2-year period (83% precision); potential errors or deviations and validation techniques, by executing a t-test on a subsample for 5 times, which confirmed the effectiveness of the model; examination of a secondary sample of failed companies, by testing the model on a sample of failed companies different from the first one (96% precision); examination of a secondary sample of healthy companies, same as previous point but tested on healthy companies (79% precision), this step will give to Altman the idea on how to define a gray area, that will be treated later; long-term accuracy, which shows a reliability of the model up to two years before bankruptcy.

As highlighted in point 5, once it was confirmed that his model could provide satisfactory answers, Altman contemplated the most appropriate way to develop the model to make it usable for anyone wanting to assess a company's health, whether it be banks, analysts, or others. It was necessary to identify a discriminant value that would serve as a dividing line between failed and healthy companies, known as the cutoff point or Z intermediate. Companies with a Z score below a certain threshold would be classified as insolvent, while those with a higher Z score would be considered healthy.

Analyzing the data from his samples, Altman observed that companies with a Z score above 2.99 could unquestionably be classified as healthy, while those with a Z score below 2.81 were certainly in crisis. Inevitably, there were intermediate results that couldn't unequivocally indicate a company's imminent failure or health. It became necessary to establish guidelines for companies classified in the gray zone. To do this, the situation of companies falling into the "overlap" area had to be reexamined. Within the range of uncertainty, the goal was to identify the values leading to the fewest misclassifications. The gray area, encompassing Z scores between 2.81 and 2.99, presented a zone of uncertainty where errors of Type 1 or 2 could occur.

Within this gray area, Altman identified a critical value for discrimination. After conducting additional tests on different samples, he found that the most critical value fell between 2.67 and 2.68. The selected critical value was 2.675, indicating that companies with a Z score below this value could be considered potentially insolvent, while those with a Z score above it belonged to the group of healthy companies.

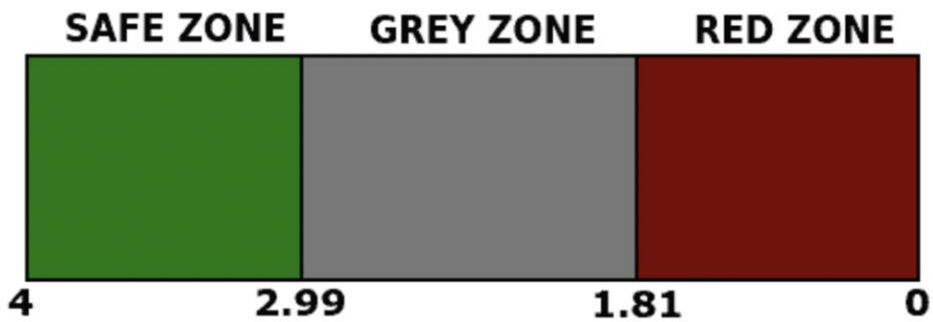


Table 8 - Z score areas. Source: Z Table

In three consecutive assessments, Altman examined 86 distressed companies spanning from 1969 to 1975, followed by 110 bankruptcies from 1976 to 1995, and subsequently 120 from 1997 to 1999. Altman discovered that employing the Z-Score model with a cutoff score of 2.675 yielded accuracy rates ranging between 82% and 94%. Subsequent tests conducted up to 1999 consistently demonstrated that the Z-Score model's accuracy in predicting distressed firms hovered around 80-90%, based on data extracted from one financial reporting period before bankruptcy.

However, the incidence of Type II error, which entails misclassifying firms as distressed when they do not go bankrupt, escalated significantly. As much as 15-20% of all firms, including 10% of the largest ones, exhibited Z-Scores below 1.81. Recent examinations indicate a noticeable rise in the average Z-Score, climbing from the 4-5 level during 1970-1995 to nearly 10 by 1999, primarily propelled by a substantial surge in stock prices impacting X_4. Altman recommends adopting the lower boundary of the "zone of ignorance" (1.81) as a more realistic cutoff Z-Score than 2.675, despite the latter yielding the lowest overall error in the original assessments.

By 1999, over 20% of U.S. industrial firms in the Compustat data tapes had Z-Scores below 1.81. Up to this juncture, sample companies were selected either based on their bankruptcy status (Group I) or their resemblance to Group I in all aspects except economic well-being. Instead misclassifying an healthy firm as bankrupt exemplifies a Type II error. An exceedingly rigorous evaluation of the discriminant model's efficacy would involve identifying a large sample of firms encountering earnings difficulties and observing the Z-Score's classification outcomes.

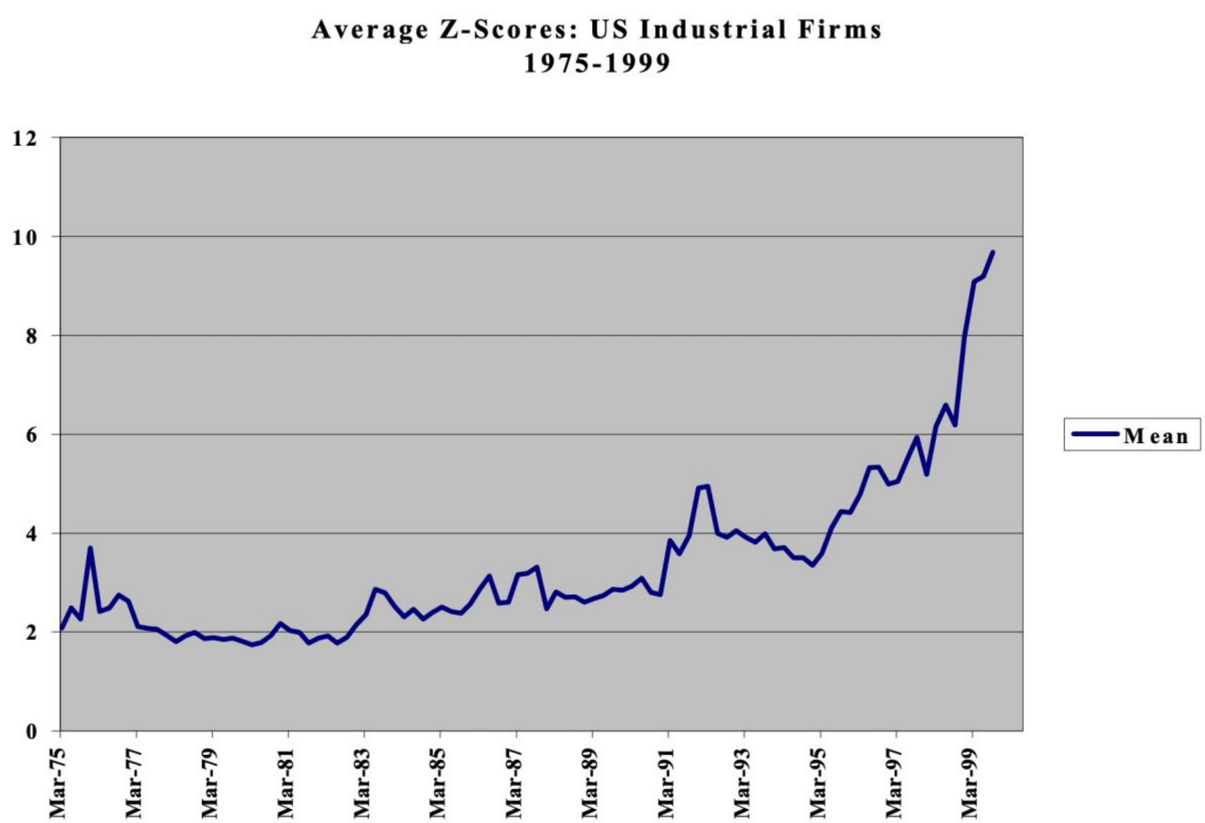


Table 9 - US Industrial Firms Average Z Scores 1975-1999. Source: Osler and Hong, 2000

To conduct such a test, a sample of 66 firms is chosen based on net income (deficit) reports from the years 1958 and 1961, with 33 from each year. Over 65% of these firms experienced two or three years of negative profits in the previous three years. Selected regardless of their asset size, the only criteria for inclusion were that they were manufacturing firms suffering losses in 1958 or 1961. These companies are then assessed by the discriminant model to ascertain their bankruptcy potential.

Results reveal that out of the 66 firms, 14 are classified as bankrupt, with the remaining 52 accurately classified. Thus, the discriminant model correctly identifies 79% of the sample firms. This percentage is particularly noteworthy considering these firms constitute a secondary sample of below-average performance. The t-test for the significance of the result is 5=4.8, significant at the 0.001 level. Of the 14 misclassified firms in this secondary sample, 10 have Z-Scores falling between 1.81 and 2.67, indicating less definitive bankruptcy predictions compared to the vast majority in the initial sample of bankrupt firms. Roughly one-third of the 66 firms in this last sample have Z-Scores within the entire overlap area, underscoring the efficacy of the selection process in identifying firms exhibiting signs of profitability deterioration.

Although these tests are based on data from over 40 years ago, they underscore the robustness of the model, still relevant in the year 2000. The preceding outcomes offer significant evidence of the reliability of conclusions drawn from both the initial and holdout samples of firms. A pertinent extension would involve evaluating the overall efficacy of the discriminant model over a longer period preceding bankruptcy.

To address this, data are collected for the 33 original firms from the third, fourth, and fifth years before bankruptcy. It is anticipated, a priori, that as the lead time increases, the relative predictive capability of any model would diminish. This was corroborated by earlier univariate studies and holds true for the multiple discriminant model. However, as will soon be evident, the more recent model exhibits higher accuracy over a longer timeframe.

Based on the results, it is posited that the Z-Score model effectively forecasts failure up to two years before distress, with accuracy substantially decreasing as the lead time extends. A trend analysis of the individual ratios in the model reveals two crucial conclusions: firstly, all observed ratios exhibit a deteriorating trend as bankruptcy approaches, and secondly, the most significant changes in the majority of these ratios occur between the third and second years before bankruptcy. This underscores the importance of integrating information from individual ratio measurement trends with the analytical findings of discriminant analysis.

Year Prior To Failure	Original Sample (33)	Holdout Sample (25)	1969-1975 Predictive Sample (86)	1976-1995 Predictive Sample (110)	1997-1999 Predictive Sample (120)
1	94% (88%)	96% (92%)	82% (75%)	85% (78%)	94% (84%)
2	72%	80%	68%	75%	74%
3	48%	-	-	-	-
4	29%	-	-	-	-
5	36%	-	-	-	-

Classification & Prediction Accuracy
Z-Score (1968) Failure Model*

*** Using 2.67 as cutoff score (1.81 cutoff accuracy in parenthesis)**

Table 10 - Z Score Failure Model performance 1968. Source: NYU

Table 10 above presents the results of testing the Z-Score model across various sample periods over the past 30 years. In each test, the Type I accuracy using a cutoff score of 2.67 ranged from 82-94%, based on data from one financial statement prior to bankruptcy or default on outstanding bonds. Notably, in the most recent test involving 120 firms defaulting on their publicly held debt during 1997-1999, the accuracy rate was an impressive 94%.

1.11 Z' and Z'' scores

Altman, following criticism from the academic community regarding the lack of sophistication in financial ratios as a method to assess a company's health, combined various indices with linear discriminant analysis. The goal was to test the predictive capability of this technique in anticipating corporate failures. Altman aimed to determine if integrating these indices into a multivariate context would yield greater statistical relevance compared to their more commonly sequential use.

Results from this test were encouraging. The employed model demonstrated exceptional precision in predicting failures, accurately identifying critical conditions for 94% of the first group of companies and assigning the correct group membership for 95% of all companies. The model also performed satisfactorily for subsequent groups, which did not contribute to the model's creation, making their classification more challenging.

Notably, the model could identify companies destined for failure as early as two years preceding the event. However, beyond the two-year mark, the model's reliability diminished, becoming less credible. Altman's test, building on prior studies by other authors, revealed a tendency to reverse classifications beyond the third year.

A significant limitation of the model is its applicability to specific types of companies - those in the manufacturing sector, publicly traded, with easily accessible financial information. Altman acknowledged this limitation and, in the concluding comments of his initial study, expressed the intent to develop a more versatile Z score applicable to a broader range of companies, particularly smaller ones not listed on the stock exchange and more prone to financial distress.

The subsequent models, Z' score and Z'' score, appear to have originated in Altman's mind at the conclusion of his first article, where he explicitly acknowledged the limitations of the initial model with the intention to address them. Although the process to develop these models paralleled that of the Z score, the results were more versatile in terms of applicability. This acknowledgment marked the study's conclusion with an awareness of the need to refine the model while recognizing the excellent starting point for subsequent model development.

The first pain point he addressed was about companies being publicly traded. In order to change the model, the variable $X_4 = \frac{Market\ Value\ of\ Equity}{Total\ Liabilities}$ needed to be changed. This model is applicable to private manufacturer companies.

The new variable in the model is the following:

$$X_4^* = \frac{Book\ Value\ of\ Equity}{Total\ Liabilities}$$

And the coefficients for the variables in this model is slightly different than the Z-Score model (Altman, 1983):

$$Z' = 0.717X_1 + 0.847X_2 + 3.107X_3 + 0.420X_4^* + 0.998X_5$$

Where:

- X_1 = Working Capital/Total Assets
- X_2 = Retained Earnings/Total Assets
- X_3 = Earnings Before Interest and Taxes/Total Assets

- X_4^* = Book Value of Equity/Total Liabilities

- X_5 = Sales/Total Assets

- Z' = Overall Index

Looking at the weights, all of them have been decreased, so at first glance it seems more difficult for a company to be in the green zone using this index. However, also the range has been changed accordingly: firms with a Z' value exceeding 2.90 are categorized as non-bankrupt, while those with index values ranging from 2.23 to 2.90 fall into the gray area. If the index value is less than 2.23, it indicates that companies are facing a challenging situation and are classified as being at high risk of bankruptcy. As can be seen, the gray area is now broader than before.

The model's accuracy in classifying bankrupt firms was 90.9%, and for non-bankrupt firms, it was 97.0% (E.I. Altman, 1983).

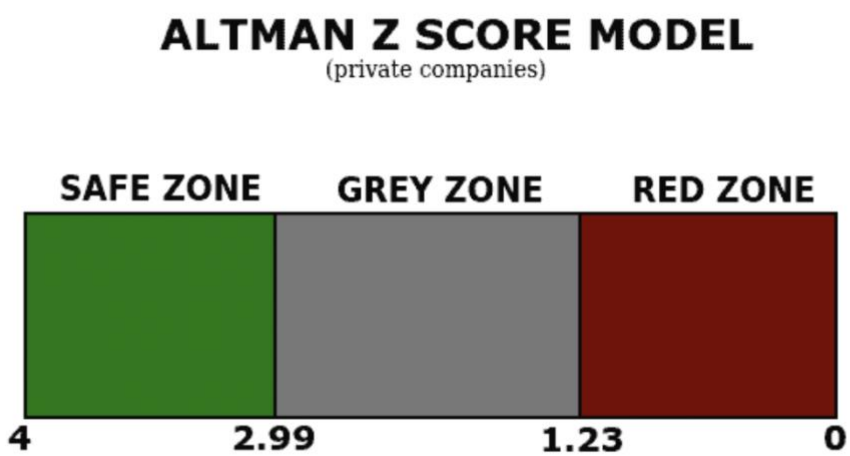


Figure 3 - Z' score areas. Source: Z Table

The last Altman's model, the so-called Z'' score, wants to have a more general application, for both private manufacturing and non-manufacturing companies. To do so, Altman removed the variable $X_5 = \frac{Total\ Sales}{Total\ Assets}$ to minimize the potential industry impact. The new model has the following form (E.I. Altman, J. Hartzell and M. Peck, 1995):

$$Z'' = 6.56X_1 + 3.26X_2 + 6.72X_3 + 2.05X_4^* + 0.998X_5$$

Where:

- X_1 = Working Capital/Total Assets
- X_2 = Retained Earnings/Total Assets
- X_3 = Earnings Before Interest and Taxes/Total Assets
- X_4^* = Book Value of Equity/Total Liabilities
- Z'' = Overall Index

When the Z'' value surpasses 2.60, companies are categorized as non-bankrupt. If their index value falls within the range of 2.10 to 2.60, they fall into the gray area classification. When index values dip below 2.10, companies face challenging circumstances and are labeled at a high risk of bankruptcy. The model exhibited a 97% accuracy in classifying non-bankrupt firms and a 90.9% accuracy for bankrupt firms.

A constant of 3.25 has been added to this last version for emerging market companies.

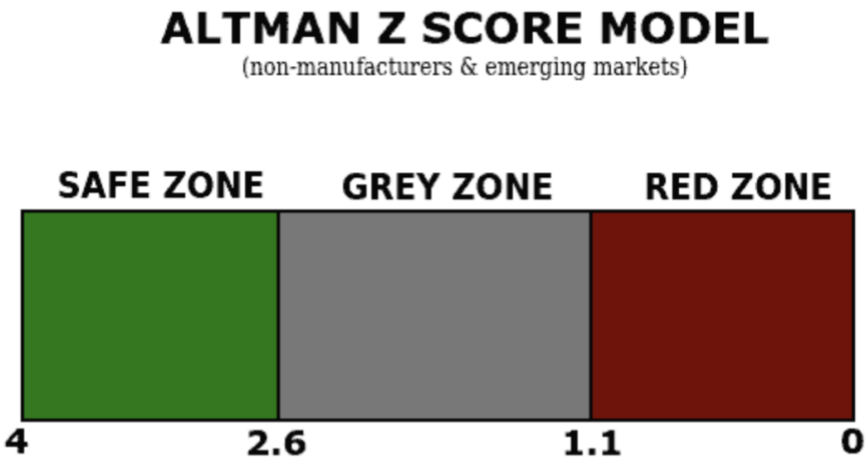


Figure 4 – Z'' score areas. Source: Z Table

By applying the Z" score, Altman and Hotchkiss in 2006 outlined a correspondence between the results obtained through the model and the scores assigned by the U.S. rating agency Standard & Poor's. This procedure involved calculating the average Z" score for the population of companies in each Standard & Poor's rating class (taking into consideration the 3.25 constant in the model).

	Healthy Companies									
Rating	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-
Z" Score	>8.15	8.15	7.60	7.30	7.00	6.85	6.65	6.40	6.25	5.83
Rating	BB+	BB	BB-	B+	B	B-	CCC+	CCC	CCC-	D
Z" Score	5.65	5.25	4.95	4.75	4.50	4.15	3.75	3.20	2.5	<2.75

Grey Area	Insolvent Companies

Table 11 - Credit S&P Rating and Z'' Score – Source Altman E.I. Hartzell J. Peck M.

1.12 Other Bankruptcy Models

1.12.1 S Score

In 1978, Springate introduced a bankruptcy prediction model known as the S-Score, which utilizes the Multiple Discriminant Analysis (MDA) technique. By incorporating various financial ratios, Springate's model boasts an impressive 92.5% accuracy rate in foreseeing financial distress.

The S-Score is calculated using the following formula:

$$S\ Score = 2.03X_1 + 3.07X_2 + 0.66X_3 + 0.4X_4$$

Here's a breakdown of the components:

- X_1: Represents the ratio of working capital to total assets.
- X_2: Denotes the ratio of profit before interest and taxes to total assets.
- X_3: Reflects the ratio of profit before tax to current debt.
- X_4: Indicates the ratio of sales to total assets.

To interpret the S-Score effectively, a cutoff value of 0.862 is applied (as indicated by Rahayu, 2017). This cutoff delineates three distinct zones: the "Distress" Zone (S < 0.862), the "Grey" Zone (0.862 < S < 2.062) and the "Safe" Zone (S > 2.062). Companies falling within the first zone are in a precarious financial state, teetering on the brink of insolvency: the likelihood of bankruptcy looms large, necessitating urgent attention and strategic interventions. Entities within the second range are grappling with financial instability that demands proactive management. While not yet in imminent danger of bankruptcy, swift and effective measures are required to navigate through the uncertainty. Failure to address these challenges promptly may exacerbate the situation, potentially leading to insolvency.

Companies positioned in the safe zone instead enjoy robust financial health, with minimal risk of bankruptcy. Their sound financial standing signifies effective management practices and prudent decision-making, mitigating the threat of insolvency.

The S-Score, with its nuanced categorization and high predictive accuracy, serves as a valuable tool for stakeholders in assessing the financial viability and risk exposure of companies.

By comparing the variables in the two models it is clear that in both the S-Score and Altman Z-score models, the ratio of working capital to total assets serves as a measure of liquidity and short-term financial health. A higher ratio indicates a stronger ability to cover short-term obligations with current assets. Therefore, a higher value contributes positively to the overall score in both models. Regarding the EBIT to Total Assets ratio, in the S-Score model reflects profitability relative to total assets, capturing the efficiency of utilizing assets to generate profits before accounting for interest and taxes. Similarly, Altman's Z-score includes a profitability component, albeit in a slightly different form. Altman's model uses earnings before interest and taxes (EBIT) relative to total assets to assess operational profitability. Both ratios aim to evaluate the company's ability to generate profits from its asset base, with higher values indicating better financial performance. In the S-Score model, the Profit Before Tax to Current Debt ratio assesses the company's profitability in relation to its current debt obligations, providing insights into its ability to service debt with pre-tax earnings. Altman's Z-score does not directly include a ratio that specifically measures the relationship between profitability and current debt. However, it incorporates various ratios related to profitability and debt to assess overall financial health. While the specific formulation differs, both models seek to evaluate the company's ability to manage its debt obligations in light of its profitability.

Finally, Sales to Total Assets ratio in the S-Score model represents the efficiency of asset utilization in generating sales revenue. Altman's Z-score does not include a direct measure of sales to total assets. Instead, it focuses on profitability, liquidity, and solvency ratios to assess financial health. However, the efficiency of asset utilization indirectly affects profitability, which is a key component of the Altman Z-score.

To conclude, comparing the two models, we find that while they share some common themes, such as assessing liquidity, profitability, and solvency, they differ in the specific variables used and their formulations. The Altman Z-score may offer a broader perspective by including additional ratios like market value of equity to total liabilities, which captures market sentiment, while the S-Score provides a focused analysis on profitability, liquidity, and debt management.

1.12.2 Model Zmijewski (X-Score)

Zmijewski employs ratio analysis to assess a company's performance, leverage, and liquidity. Central to this model is the profound consideration of debt levels as the foremost determinant of bankruptcy risk (Rudianto, 2013).

Zmijewski's approach offers formulas tailored for various business types, exemplified by the X-Score formula:

$$X\ Score = -4.3 - 4.5X_1 + 5.7X_2 - 0.004X_3$$

Here's a breakdown of the variables:

- X_1: Represents the ratio of earnings after taxes to total assets.
- X_2: Reflects the ratio of total debt to total assets.
- X_3: Denotes the ratio of current assets to current liabilities.

According to the evaluation standards cited by Fanny and Saputra (2000) in Peter & Yoseph (2011), a higher value correlates with an augmented likelihood of bankruptcy. Consequently, if the calculation yields a positive value using this model, it signals an elevated risk of bankruptcy for the company. Conversely, a negative value suggests a lower probability of bankruptcy (Rudianto, 2013).

Diving in deep in the variables and comparing them with Altman's ones: Earnings After Taxes to Total Assets assesses profitability relative to the total asset base, indicating how effectively the company generates earnings from its assets after accounting for taxes. Similarly, the Altman Z-score model includes a profitability component, where earnings before interest and taxes (EBIT) to total assets is used to gauge operational profitability. Both ratios aim to evaluate the company's ability to generate profits from its asset base, with higher values indicating better financial performance.

Total Debt to Total Assets is a measure of the proportion of total debt relative to the total asset base, providing insights into the company's leverage level. In contrast, the Altman Z-score model also incorporates leverage metrics, such as the ratio of total liabilities to total assets, to assess the company's solvency and risk of bankruptcy. Both models consider the relationship between debt and assets as a critical factor in evaluating financial health, with higher leverage ratios indicating higher financial risk.

About the Current Assets to Current Liabilities, while the Altman Z-score model does not include a direct measure of current assets to current liabilities, it assesses liquidity through ratios such as working capital to total assets and current assets to total liabilities. Both models aim to assess the company's ability to meet its short-term financial obligations, with higher ratios indicating better liquidity and lower risk of financial distress.

Comparing the two models, we find that while they share similar themes in assessing profitability, leverage, and liquidity, they differ in the specific variables used and their formulations. The Altman Z-score model offers a broader perspective by including additional ratios such as market value of equity to total liabilities, while Zmijewski's X-Score model provides a focused analysis on profitability, leverage, and liquidity.

1.12.3 Model Grover (G-Score)

The Grover model emerged from a comprehensive overhaul and reevaluation of the Altman model. In 1968, Jeffrey S. Grover embarked on this endeavor by augmenting the Altman model with thirteen additional financial ratios. His study spanned from 1982 to 1996, involving a sample of 70 companies, evenly split between those that declared bankruptcy (35) and those that did not (35). The outcome, as documented by Jeffrey S. Grover in 2001, is encapsulated in the following equation (J.S. Grover, 2001):

$$G\ Score = 0.057 + 2.650X_1 + 3.404X_2 - 0.016X_3$$

Where:

- X_1: Represents the ratio of working capital to total assets.
- X_2: Denotes the ratio of earnings before interest and taxes to total assets.
- X_3: Reflects the ratio of net income to total assets.

In Grover's model, companies deemed bankrupt are identified by a score equal to or lower than -0.02, while companies classified as non-bankrupt possess a score equal to or higher than 0.02.

By comparing the variables with the Z score model, it is enhanced that for the Working Capital to Total Assets ratio, it assesses the efficiency of a company's working capital utilization relative to its total assets. Similarly, the Altman Z-score model incorporates a measure of liquidity, which evaluates the adequacy of a company's working capital to meet its short-term obligations.

The Earnings Before Interest and Taxes to Total Assets ratio in the G-Score model reflects the profitability of a company in relation to its total asset base, measuring its ability to generate earnings before interest and taxes. Similarly, the Altman Z-score model includes a profitability component, which assesses operational profitability through the ratio of earnings before interest and taxes to total assets.

The Net Income to Total Assets variable in Grover's G-Score model evaluates the profitability of a company by comparing its net income to its total assets. The Altman Z-score model does not include a direct measure of net income to total assets, but it assesses profitability through various ratios such as earnings before interest and taxes to total assets.

Comparing the two models, we find that they share similar themes in assessing liquidity, profitability, and financial health. However, they differ in the specific variables used and their formulations. The Altman Z-score model incorporates additional ratios such as market value of equity to total liabilities, which capture market sentiment, while Grover's G-Score model provides a focused analysis on liquidity and profitability.

1.12.4 Taffler Model

In 1983, Taffler introduced the Taffler model, aiming to forecast the likelihood of manufacturing companies encountering bankruptcy within the London Stock Exchange during the period from 1969 to 1976, as referenced by Widiasmara and Rahayu in (2019). The Taffler model incorporates four key financial ratio elements: pre-tax earnings in relation to current obligations, the ratio of current assets to total liabilities, the proportion of total assets represented by current liabilities, and post-tax net income as a fraction of total assets. This model demonstrates an impressive accuracy rate of 95.7% in predicting companies prone to bankruptcy, achieving a flawless 100% accuracy rate for those deemed unlikely to face bankruptcy. Comparatively, the Taffler model surpasses other predictive models such as Altman, Springate, and Grover, exhibiting a 96% accuracy rate and a mere 4% error rate (Prakoso, 2022).

The formula for the Taffler model is outlined as follows:

$$Z_{Taffler} = 3.20 + 12.18X_1 + 2.50X_2 - 10.68X_3 + 0.0289X_4$$

Within the Taffler framework, when the T value falls below 0.2, the company is categorized into the distress zone, indicating a susceptibility to bankruptcy risk. Conversely, when the T value exceeds 0.2, the company is identified as financially stable and is considered not at risk of bankruptcy.

While both Altman and Taffler models evaluate liquidity, profitability, and financial stability, they differ in the specific variables used. The Taffler model emphasizes earnings and liquidity ratios, while the Altman Z-score incorporates a broader set of metrics covering profitability, liquidity, solvency, and market valuation.

1.12.5 Models Performance Comparison

In evaluating the four bankruptcy prediction models of the previous chapters, and comparing them with the Z Score, it is evident that each model has its own merits and reported levels of accuracy. Among these models, the Z score developed by Altman, stands out as a widely recognized and extensively studied tool for bankruptcy prediction. Its historical performance has consistently demonstrated high accuracy levels, typically ranging from 82% to 94% in various empirical studies.

Comparatively, while the other models may also yield competitive accuracy rates, they typically demonstrate accuracy rates ranging from 70% to 85%.

The decision to utilize the Z score over other models in the next chapter analysis is also rooted in its established reputation for accuracy and its extensive validation in academic literature. With its demonstrated reliability across different datasets and time periods, the Z score emerges as a preferred tool for assessing the likelihood of corporate distress and bankruptcy.

1.13 Conclusion

Altman's initial model can be viewed as a preliminary attempt to test a novel idea and assess its validity. While the first model is reliable and accurate, Altman recognizes its limitations, particularly in excluding a significant number of companies from investigation. The first Z-score, as previously mentioned, is tailored exclusively for publicly traded companies assumed to be larger and specific to manufacturing industries. This automatically leaves out a substantial portion of businesses in the market, especially the smaller enterprises that are more susceptible to financial distress.

The subsequent models, namely the Z' score or the Z'' score, seem to have originated from Altman's desire to address the limitations explicitly mentioned in his initial article. The approach to developing these models mirrors that of the original Z-score but yields more broadly applicable results.

All the three models resulted in great results in terms of precision and reliability, with a 82-94% range accuracy rate.

Comparing Altman's model with the S score, Zmijewski, Grover and Taffler models, it results to be more accurate, as well as more famous in the industry.

Chapter 2

Z' score application on Italian manufacturing companies

2.1 Introduction

The purpose of the following section is to analyze the Altman model behavior within the Italian manufacturing industry, and how it and its variables have been affected by Covid 19 pandemic.

The section will include a chapter to understand why Altman tailored its model to the manufacturing industry, then the main difficulties and possible threats to apply the model to Italian companies, instead of US ones will be described, followed by a chapter will introduce the Italian manufacturing industry, and its contribution in terms of value added to the economy of the country.

Then, before delving into the study, the dataset of companies will be defined, first from an operational data gathering point of view, then looking at the features of it. Finally, results will be interpreted and analyzed.

The study will take data from manufacturing Italian companies in 2019 and 2021, for each year the firms will be divided in three groups: the first percentile of firms with the lowest Z' score, the last percentile of firms with the highest Z' score, and the rest of the population. These three groups will be analyzed, enhancing the main differences among them between 2019 and 2021.

Since most of the manufacturing Italian companies are not public, the model used for this analysis will be the Z' score, described in the section 2.9 of this paper.

2.2 Altman Choice

Before diving in deep on the empiric test, it is important to understand why Altman chose to tailor the Z score to the manufacturing sector in the late 20th century. Altman's choice was not arbitrary but rather based on several factors inherent to this sector, as capital intensity, because manufacturing companies often require significant investments in fixed assets, such as machinery, equipment, and infrastructure. These capital-intensive nature exposes them to higher financial risk, particularly if they are unable to generate sufficient returns to cover their fixed costs. The manufacturing sector is also highly cyclical, meaning its performance is closely tied to economic cycles. During economic downturns, demand for manufactured goods typically decreases, leading to revenue declines and potential financial distress for companies operating in this sector.

Operating Leverage: Manufacturing companies often exhibit high operating leverage, where a large portion of their costs is fixed. This means that small changes in revenue can lead to disproportionate changes in profitability, amplifying the impact of economic downturns or adverse market conditions. Effective management of working capital is another crucial factor for manufacturing companies due to the need to finance inventory, receivables, and payables. Inefficient working capital management can strain liquidity and solvency, increasing the likelihood of financial distress. Manufacturing companies are subject to a high competitive environment with thin profit margins, especially in commoditized markets. Companies must continuously innovate, optimize operational efficiencies, and manage costs to remain competitive, failing which they may face financial difficulties. The Supply Chain plays a fundamental role, since manufacturing companies are often part of complex supply chains, relying on suppliers for raw materials, components, and logistics services. Disruptions in the supply chain, whether due to natural disasters, geopolitical tensions, or unexpected events, can adversely impact manufacturing operations and financial stability.

Given these inherent characteristics of the manufacturing sector, Altman recognized the need for a robust financial tool to assess the creditworthiness and bankruptcy risk of companies operating in this industry.

2.3 Altman Z-Score Model in an Italian context

While the Altman Z-Score model has proven effective in assessing the financial health of US companies, its application to Italian manufacturing companies introduces several challenges and considerations. The decision to apply the model to Italian manufacturing companies necessitates a careful evaluation of the differences in financial reporting standards, business practices, and market dynamics between the two countries. The first factor to take into consideration relates to the Accounting Standards and Practices: US Generally Accepted Accounting Principles (GAAP) and International Financial Reporting

Standards (IFRS) adopted in Italy. Variations in accounting standards, terminology, and treatment of financial items can affect the calculation and interpretation of the financial ratios used in the Z-Score model. Furthermore, Italy's business culture and institutional framework differ from those of the United States, influencing financial reporting practices, corporate governance norms, and investor behavior. Factors such as the prevalence of family-owned businesses, the role of government intervention, and the importance of relationships in business dealings may impact the relevance and reliability of financial data used in the Z-Score calculation. Regarding market structure and dynamics, the Italian manufacturing sector exhibits unique characteristics in terms of market structure, competitive landscape, and industry dynamics compared to its US counterpart. Variations in sectoral composition, market concentration, supply chain relationships, and regulatory environments can affect the financial performance and risk profiles of Italian manufacturing companies, potentially influencing the predictive accuracy of the Z-Score model. From a data perspective point of view, accessing to comprehensive and reliable financial data for Italian manufacturing companies may pose challenges due to differences in disclosure requirements, data availability, and transparency levels compared to US firms. Limited access to historical financial information, inconsistent reporting practices, and data gaps could impact the robustness and effectiveness of the Z-Score model in predicting bankruptcy risk for Italian companies. Finally, a key difference is played by economic environment, since macroeconomic conditions, including factors such as inflation rates, interest rates, exchange rates, and fiscal policies, may differ significantly from those of the United States. Variations in economic cycles, industry-specific trends, and external shocks could influence the financial stability and performance of Italian manufacturing firms, requiring adjustments or modifications to the Z-Score model to account for these contextual factors.

2.4 The Italian Manufacturing Industry

The Italian manufacturing sector holds a significant position in both the national economy and the global market. Renowned for its tradition of craftsmanship, innovation, and specialization in high-quality products, Italy's manufacturing industry plays a pivotal role in the country's economic landscape.

Italy has a rich industrial history that dates to the late 19th century when the country underwent rapid industrialization, particularly in the northern regions. The emergence of sectors such as textiles, automotive, machinery, fashion, and design propelled Italy into becoming one of the world's leading manufacturing hubs. Over the years, the sector has evolved, adapting to technological advancements, globalization, and changing market demands.

As of 2024 the Italian manufacturing sector continues to be a vital component of the national economy. It encompasses a diverse range of industries, each contributing uniquely to Italy's industrial prowess. Notable sectors include automotive, where Italy is home to renowned automotive brands such as Fiat Chrysler Automobiles (FCA), Ferrari, Lamborghini, and Maserati. The automotive industry contributes significantly to both manufacturing output and exports. Fashion and Textiles, where Italy is globally recognized for its luxury fashion brands, including Gucci, Prada, Armani, and Versace. The textile industry, particularly in regions like Lombardy and Tuscany, is renowned for its craftsmanship and high-quality products. In the Machinery and Equipment industry, Italy is esteemed for its innovation and specialization in machinery and equipment for various industries, including agriculture, construction, and manufacturing. Italy is also synonymous with design excellence, with companies like Alessi, Kartell, and Poltrona Frau leading the way in furniture and interior design. The food and beverage industry is another cornerstone of Italian manufacturing, encompassing renowned brands in wine, pasta, olive oil, and cheese.

According to the World Bank, the Italian manufacturing sector accounted for 14.92% of the country's GDP in 2022.

Italy's manufacturing output totaled 169.3 billion USD in 2022, reflecting its substantial contribution to the national economy.

Despite the big importance of the services industry, the Italian manufacturing sector remains a cornerstone of the country's economy, embodying a tradition of innovation, quality, and specialization.

Italy: TOP Industries

2,640,039 registered companies

	Industry	Number of Companies	Market Share
•	Services	661,510	25.1%
•	Retail Trade	517,819	19.6%
•	Finance, Insurance, and Real Estate	351,753	13.3%
•	Construction	339,366	12.9%
•	Manufacturing	275,643	10.4%
•	Wholesale Trade	211,075	8.0%
•	Transportation, Communications, Electric, Gas, and Sanitary...	122,343	4.6%
•	Agriculture, Forestry, and Fishing	121,340	4.6%
•	Unknown industry	22,398	0.8%
•	Public Sector	13,971	0.5%
•	Mining	2,821	0.1%

Source: HitHorizons.com, data accessed in December 2023

Figure 4 - Italian Industries Market Share Percentage

As enhanced by figure 4 above, as of December 2023, the manufacturing industry occupies the 5th position in terms of market share, with over 275,000 existing companies out of 2,640,039 across all the sectors.

2.5 The sample

The databases used for both 2019 and 2021 have been downloaded from Orbis, filtering for companies which are based in Italy, which have a balance sheet available in 2019/2021 (depending on the year of selection needed), which have NAICS 2017 codes equal to 31, 32 or 33. This last filter allowed to select companies from the Manufacturing industry, from:

- 31 - Manufacturing
    - 311 - Food Manufacturing
    - 312 - Beverage and Tobacco Product Manufacturing
    - 313 - Textile Mills
    - 314 - Textile Product Mills
    - 315 - Apparel Manufacturing
    - 316 - Leather and Allied Product Manufacturing
- 32 - Manufacturing
    - 321 - Wood Product Manufacturing
    - 322 - Paper Manufacturing
    - 323 - Printing and Related Support Activities
    - 324 - Petroleum and Coal Products Manufacturing
    - 325 - Chemical Manufacturing
    - 326 - Plastics and Rubber Products Manufacturing
    - 327 - Nonmetallic Mineral Product Manufacturing
- 33 - Manufacturing
    - 331 - Primary Metal Manufacturing
    - 332 - Fabricated Metal Product Manufacturing
    - 333 - Machinery Manufacturing
    - 334 - Computer and Electronic Product Manufacturing

- 335 - Electrical Equipment, Appliance, and Component Manufacturing
- 336 - Transportation Equipment Manufacturing
- 337 - Furniture and Related Product Manufacturing
- 339 - Miscellaneous Manufacturing

These filters gave a result of 248,282 companies in 2019, and 279,409 in 2021.

After that, both the databases have been cleaned due to null values, dropping the number of companies whose data are available, and so Z' score can be computed, to 101,700 and 107,480 respectively in 2019 and 2021.

```
import pandas as pd
df = pd.read_excel('2019-f.xlsx', sheet_name='Risultati')
df = df[~df.apply(lambda row: (row == 'n.d.').any(), axis=1)]
df = df[~df.apply(lambda row: (row == 'n.s.').any(), axis=1)]
df.to_excel('2019-cleaned.xlsx', index=False)
```

Figure 5 - Data Cleaning using Python.

2.6 Z' Score in 2019

The initial 2019 sample was composed by 248,282 companies. An initial data cleaning has been necessary for "data garbage", where the fields assumed null values, dropping the total number of companies to 101,700.

Then, public companies have been deleted from the sample, reaching the final number of 101,525 firms.

Altman Z’ Score has been computed for each company, defined as:

$$Z' = 0.717X_1 + 0.847X_2 + 3.107X_3 + 0.420X_4^* + 0.998X_5$$

Where:

- X_1 = Working Capital/Total Assets
- X_2 = Retained Earnings/Total Assets
- X_3 = Earnings Before Interest and Taxes/Total Assets
- X_4^* = Book Value of Equity/Total Liabilities
- X_5 = Sales/Total Assets
- Z' = Overall Index

To remind it, Z’ Score has been used because most of the Italian companies in the Manufacturing industry are not public.

The overall picture in 2019 is enhanced in the following table, where over 60% of the Italian companies operating in the manufacturing industry are marked as insolvent in the following two years, while only 20% of them can be considered as capable to repay their debts.

Zone	Number of Companies	Percentage of Total
Red	21,037	20.72%
Grey	61,724	60.80%
Green	18,764	18.48%

Table 12 - Z' Score zone distribution 2019.

As enhanced by the box plot below, the dataset is very dispersive, and an important duty is to understand if the outliers are driven by incorrect data, and in this case can be removed, or are real values, which have to be taken into consideration. In Italy, the Gross Domestic Product (GDP) faces challenges due to the presence of tail companies, which exert downward pressure on the overall economic output of the country. These tail companies, often small and less productive entities, can significantly impact Italy's GDP growth and economic performance (Bank of Italy, 2020). Additionally, tail companies usually operate in sectors characterized by low productivity or stagnant growth, such as traditional crafts, small-scale manufacturing, which are relevant to our analysis.

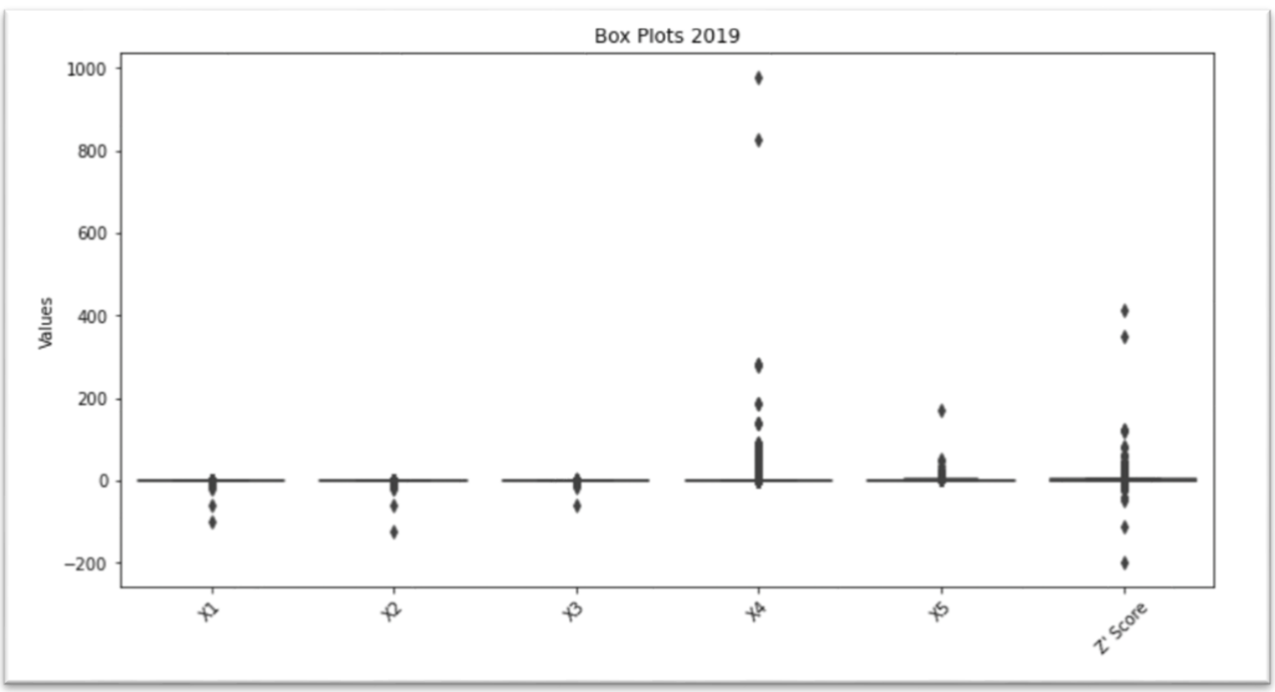


Figure 6 – Variables Box Plots 2019

It is fundamental to remind here the importance of the weights in the variables. For example, at equal value between X_3 and X_4, the former will drive the value of the score up (or down) more than 7 times compared to the latter, due to the difference weights: 3.107 and 0.420 respectively.

To address this characteristic, it is important to analyze the box plots among the absolute value of each variable multiplied by its weight, since this is what really determines the final value in the Z' score.

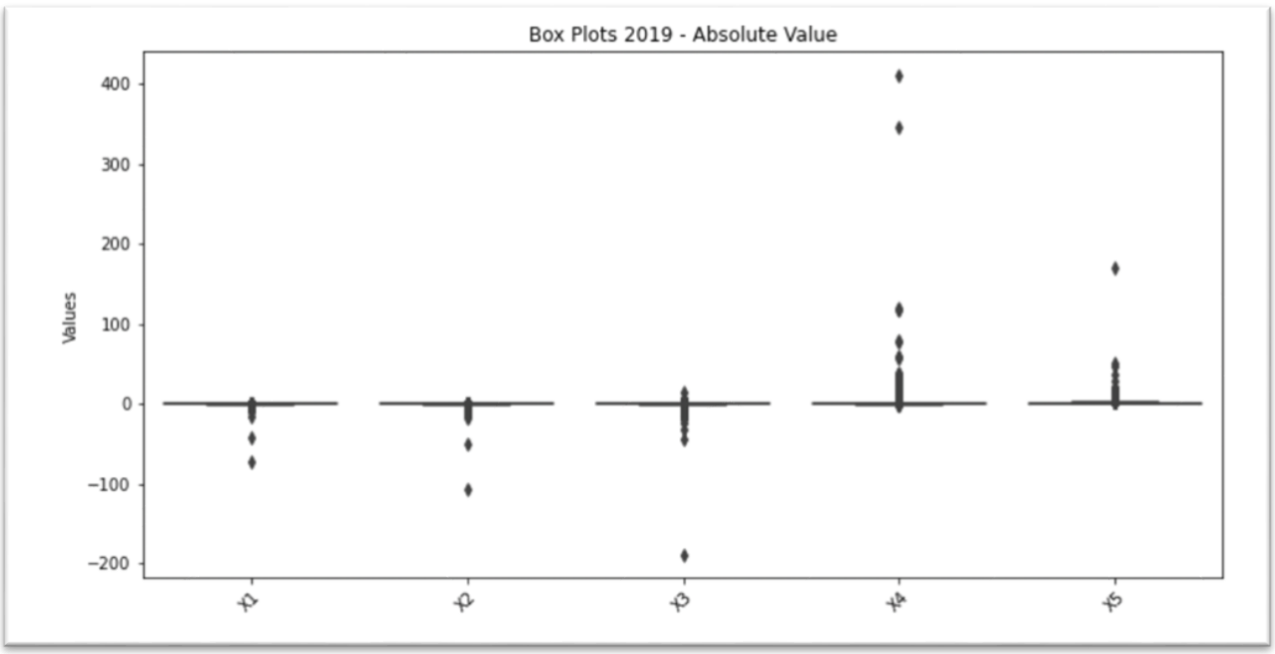


Figure 7 - Box Plots 2019 Absolute weight values.

As remarked by the box plot above and comparing it with Figure 6, it seems that X_3, X_4, and X_5 play a fundamental role in the tails of the Z' score distribution.

Since the dataset is large, it would be almost impossible to look at cases one by one to determine if those are real outliers or data garbage. A this point of the analysis, two approaches are available that leads to different paths: the first one is to make assumptions on the outliers (e.g. they are all data garbage), choose a cut off percentile for both the upper and lower tail, and exclude them from the analysis; the second approach is to divide the dataset in three parts, the two tails and the middle part, analyze each independently, and compare them then with the 2021 results. For the purpose of this analysis, remarking the importance of the tails in an Italian context, this latter approach will be chosen. However, we will take out evident outliers, that would affect the tail analysis as well.

Since the purpose is to look at the extreme tails, the dataset will be divided in these three groups: first 1% Z’ score quantile, last 1% Z’ score quantile, and rest of the population.

The dataset representing the first quantile contained initially 1016 companies. After looking at companies whose Z’ Score is lower than -100, as enhanced in Figure 6, two rows were found, and taken out from the dataset as evident data garbage (total liabilities major than 30 times total assets).

	mean	**std**	**min**	**median**	**max**
Number of Employees	17	86	1	4	2,389
Total Revenues	2,025,645	9,002,411	541	176,536	130,957,125
Total Assets	2,944,294	11,123,652	1,899	274,694	136,786,696
Current Assets	1,500,760	5,618,318	343	155,082	66,088,503
Current Liabilities	3,132,988	13,452,894	0	333,750	237,299,748
EBIT	(859,373)	4,014,919	(71,716,050)	(60,796)	2,135,790
Retained Earnings	(1,512,861)	8,952,713	(208,247,295)	(89,490)	3,308,717
Equity Book Value	(1,082,036)	7,231,969	(175,247,295)	(64,849)	6,162,497
Total Liabilities	4,026,330	16,401,390	1,662	411,363	288,934,293

Table 13 - Main Statistics for first percentile balance sheet data in 2019. All values are expressed in EUR except for the "Number of Employees" variable.

	mean	std	min	median	max
X1	-0.75	1.25	-20.28	-0.53	0.98
X2	-0.85	1.55	-20.43	-0.44	0.38
X3	-0.46	0.74	-13.91	-0.31	0.46
X4	-0.25	0.3	-0.95	-0.26	2.45
X5	1.09	1.34	0	0.75	18.93
Z' Score	-1.7	2.9	-47.29	-0.86	-0.32

Table 14 - Main Statistics for first percentile Z score ratios in 2019.

As enhanced by Table 12, the dataset is extremely dispersive, showing a standard deviation over mean ratio major or equal than four for each variable. As expected, given its big weight in the Z' Score formula, the EBIT negative value is one of the keys driven of a low (even negative in this case) score. A -859,373 € mean value means a high risk of insolvency, dragging down the respective Z' score values. For what concerns the Equity Book Value variable, being that negative can be explained by a total value of debt greater than the total value of assets, revealing a distressful situation from a solvency point of view. It is interesting to denote that these companies seem to be small-medium enterprises, with no more than a couple of dozens number of employees on average, and 50% median of them with only 4 employees.

Table 13 shows instead the key characteristics of the Z' score variables, enhancing some possible outliers in the dataset, as a Z' Score equal to -47.29, or a X1 value equal to -20.28, as well as a X5 ratio assuming a 0 value, revealing no sales in a full year. However, as enhanced before, these cases are possible in an Italian context, where tail companies exist.

It is important to denote that all the Z' scores in the first percentile are negative, with a maximum value of -0.32.

Applying the same process for the upper tail, the initial dataset was composed by 1016 companies, dropping to 1011 after taking out evident outliers. The main statistics are the following:

	mean	std	min	median	max
Number of Employees	24	221	1	4	6,710
Total Revenues	25,627,100	280,990,177	10,921	667,061	6,179,996,000
Total Assets	15,230,579	126,484,260	1,327	588,261	3,127,228,000
Current Assets	9,212,369	98,904,433	0	358,671	2,938,635,000
Current Liabilities	2,809,127	35,438,126	0	67,996	959,300,000
EBIT	4,478,257	102,483,829	(23,088,698)	60,170	3,244,003,500
Retained Earnings	11,423,046	96,652,723	(334,182)	291,601	2,107,912,000
Equity Book Value	12,021,916	98,693,887	(102,037)	351,603	2,157,912,000
Total Liabilities	3,208,663	37,363,941	49	95,710	969,316,000

Table 15 - Main Statistics for last percentile balance sheet data in 2019. All values are expressed in EUR except for the "Number of Employees" variable.

	mean	std	min	median	max
X1	0.51	0.36	-3.5	0.56	1
X2	0.61	0.4	-3.87	0.75	0.99
X3	0.26	0.36	-3.88	0.17	5.14
X4	10.27	16.49	-0.76	6.32	188.22
X5	3.06	4	0	1.74	51.65
Z' Score	9.05	6.57	5.94	7.14	83.76

Table 16 - Main Statistics for last percentile Z score ratios in 2019.

Surprisingly from Table 14, the median number of employees in the last percentile is equal to the one in the first quantile. EBITs in the last percentile are positive in mean, contributing to a greater Z' score. A key aspect to denote, is a low median in total liabilities equal to less than 100,000 €.

Looking at Table 15, the main drawer of credit solvency is represented by the variable X_4 with a median value of 6.32, enhancing big equity book value over total liabilities values for companies with high Z' scores, as reflected by an average low total liabilities value in Table 14. This companies also have a higher sales turnover rate, with sales almost tripling on average their total assets in one year, enhancing an agile organization within these companies. Z' scores assume high values, with a 7.14 as median.

The middle population (whose Z' score is comprised between the 2^{nd} percentile and the 98^{th} ones), is composed by 100,509 companies, and its main statistics are as follow:

	mean	std	min	median	max
Number of Employees	30	208	1	9	31,984
Total Revenues	9,117,141	112,657,012	160	1,316,762	25,153,398,000
Total Assets	9,264,273	99,885,024	2,248	1,249,171	19,378,194,000
Current Assets	5,691,968	49,125,005	164	857,790	8,088,167,000
Current Liabilities	3,866,582	41,576,824	0	559,522	7,519,403,000
EBIT	433,242	7,005,178	(944,258,561)	47,847	742,893,322
Retained Earnings	3,170,220	45,925,420	(138,395,413)	220,696	8,908,286,000
Equity Book Value	3,795,071	51,101,946	(64,395,413)	280,161	9,708,286,000
Total Liabilities	5,469,202	55,140,562	381	830,654	9,669,908,000

Table 17 - Main Statistics for 2 - 98 range percentile balance sheet data in 2019. All values are expressed in EUR except for the "Number of Employees" variable.

	mean	std	min	median	max
X1	0.22	0.26	-2.14	0.22	1
X2	0.25	0.23	-4.4	0.21	0.92
X3	0.06	0.1	-2.15	0.04	1.16
X4	0.71	1.03	-0.8	0.35	12.43
X5	1.22	0.65	0	1.12	13.02
Z' Score	2.08	1.03	-0.32	1.95	5.94

Table 18 - Main Statistics for the 2 - 98 range percentile Z' score ratios in 2019.

The distribution of the scores is as follows, enhancing a great number of companies having a Z’ score between 1 and 2:

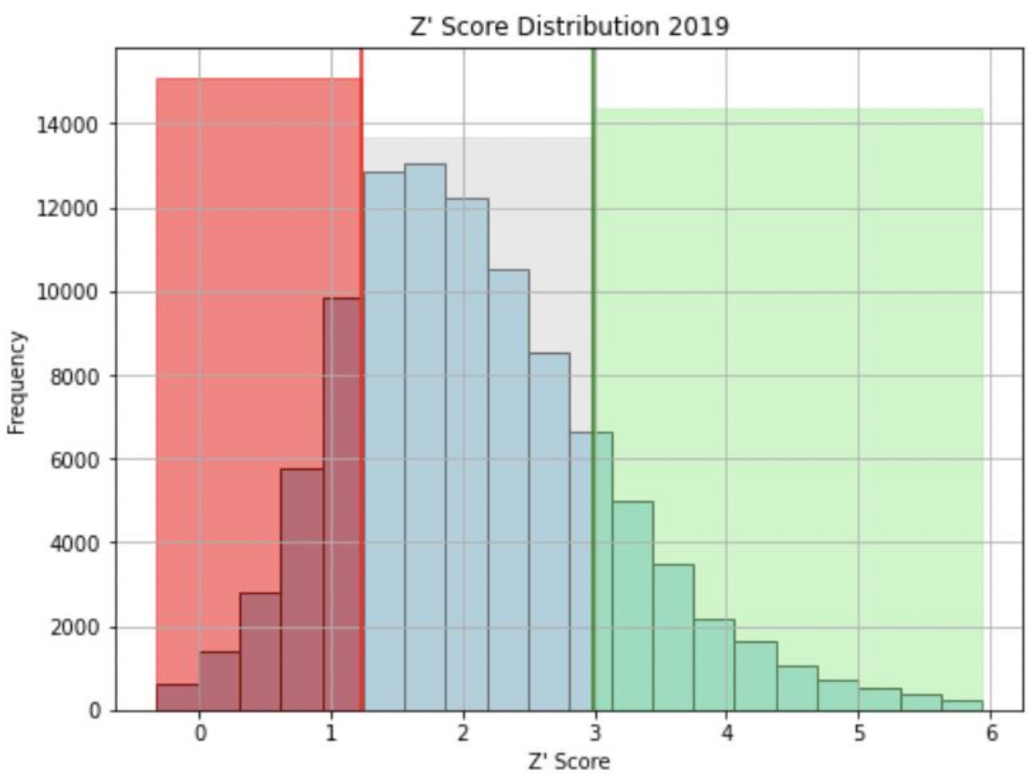


Figure 8 - Z' Score Distribution 2019 2-98 percentile range

2.7 Z’ Score in 2021

The 2021 dataset was initially composed by 279,409 companies. After cleaning the data inside as per the 2019 dataset, the number of companies dropped to 107,480.

As enhanced in I the figure below, also the 2021 dataset seem to contain evident outliers, which will be then removed in the next paragraphs.

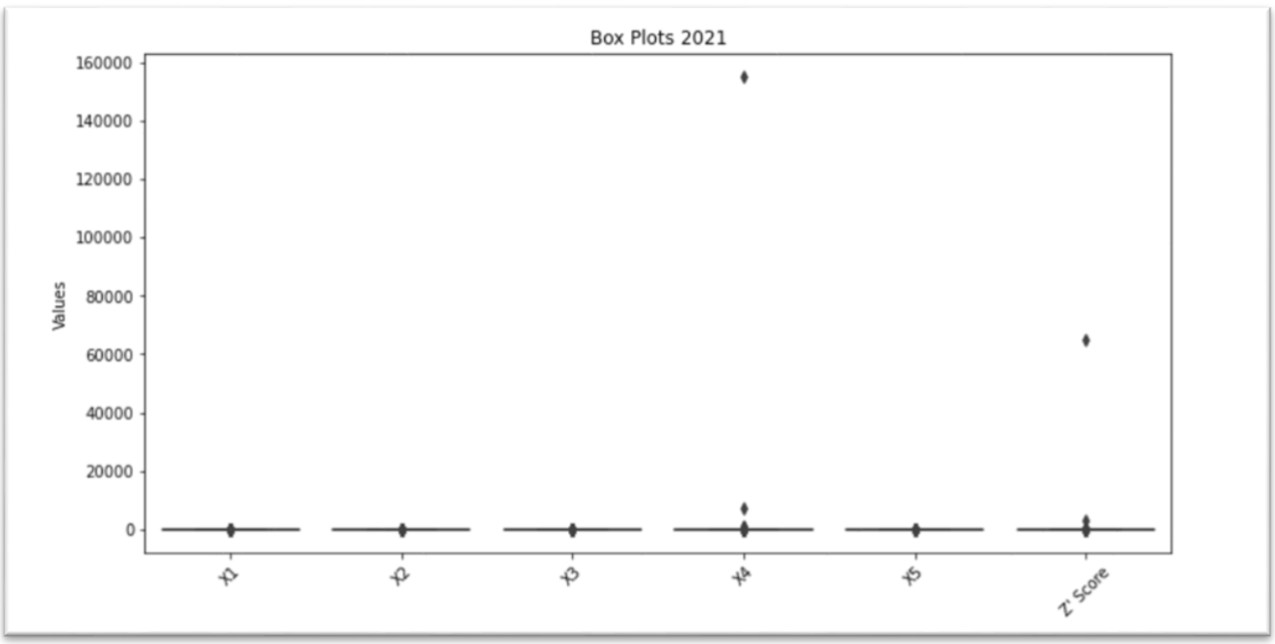


Figure 9 - Variables Box Plots 2021

As highlighted by Table 17 below, the overall situation in 2021 is similar to the 2019 one, with a small increase of 2% companies in the grey zone, to the detriment of the green one.

Zone	**Number of Companies**	**Percentage of Total**
Red	22,530	20.96%
Grey	67,454	62.76%
Green	17,493	16.28%

Table 19 - Z' Score zone distribution 2021.

The process to analyze 2021 companies will be the same as in 2019, by dividing the population in the first and last percentile, and the rest of it, taking out evident outliers in any of the three. In the next chapter these three samples will be compared to the 2019 ones.

The dataset representing the first quantile is formed by 1075 companies, and below are the main characteristics:

	mean	std	min	median	max
Number of Employees	12	46	1	4	1015
Total Revenues	1,477,039	10,780,681	1,012	207,955	252,336,700
Total Assets	2,361,848	13,585,837	1,011	249,145	308,723,000
Current Assets	1,326,337	8,269,507	0	151,110	203,480,000
Current Liabilities	2,439,123	12,933,077	0	307,008	232,880,395
EBIT	(618,732)	2,926,955	(1,004)	(82,704)	243,025,476
Retained Earnings	(1,529,328)	9,613,227	(197,413,711)	(138,018)	1,295,465
Equity Book Value	(1,111,013)	7,818,640	(164,413,711)	(113,000)	23,922,000
Total Liabilities	3,472,861	18,359,241	1,011	420,489	313,593,880

Table 20 - Main Statistics for first percentile balance sheet data in 2021. All values are expressed in EUR except for the "Number of Employees" variable.

	mean	std	min	median	max
X1	-0.92	2.64	-64.78	-0.58	0.99
X2	-1.22	4.09	-115.93	-0.63	0.45
X3	-0.47	0.77	-14.61	-0.33	11.11
X4	-0.35	0.26	-0.99	-0.35	1.08
X5	1.34	3.9	0.02	0.88	100.23
Z' Score	-1.96	2.94	-43.77	-1.11	-0.42

Table 21 - Main Statistics for first percentile Z' score ratios in 2021

As in 2019, also 2021 first percentile companies are mostly affected by a negative Equity Book Value, which is then reflected in the X_4 variable.

The last quantile dataset was initially composed by 1,075 companies, dropping down to 1,065 after removing Z' score entries greater than 100.

	mean	std	min	median	max
Number of Employees	20	219	1	4	7058
Total Revenues	18,064,508	173,894,851	4,001	768,535	4,366,560,000
Total Assets	12,235,320	79,219,942	3,622	715,669	1,753,926,326
Current Assets	6,903,736	41,065,496	2,269	487,661	981,738,174
Current Liabilities	1,740,923	14,555,560	0	89,631	289,108,000
EBIT	2,796,386	35,491,939	(3,609)	89,534	3,868,772,160
Retained Earnings	9,913,068	74,339,076	(2,759,214)	389,757	1,724,957,094
Equity Book Value	10,264,257	74,991,776	(759,214)	446,011	1,735,457,094
Total Liabilities	1,971,063	15,617,779	40	121,199	319,200,000

Table 22 - Main Statistics for last percentile balance sheet data in 2021. All values are expressed in EUR except for the "Number of Employees" variable.

	mean	std	min	median	max
X1	0.49	0.45	-6.58	0.56	1
X2	0.59	0.52	-6.57	0.74	0.99
X3	0.25	0.37	-2.18	0.16	6.24
X4	10.21	17.21	-0.87	6.07	204.72
X5	2.91	4.17	0.02	1.66	67.78
Z' Score	8.81	6.84	5.75	6.95	87.11

Table 23 - Main Statistics for last percentile Z' score ratios in 2021

As shown in Table 23, the Z' score median in this sample equals 6.95, lower of 20 bps compared to 2019. As in 2019 case, the median number of employees in first and last percentile is equal also in 2021. Furthermore, Table 23 highlights some data problems, since the maximum value assumed by X1 is equal to 1, that is almost impossible.

The 2021 middle population (whose Z' score is comprised between the 2nd percentile and the 98th ones), is composed by 100,327 companies, and its main statistics are as follow:

	mean	std	min	median	max
Number of Employees	33	311	1	9	50413
Total Revenues	11,173,923	140,166,975	501	1,377,132	22,469,733,000
Total Assets	12,696,761	179,253,765	975	1,424,152	28,379,000,000
Current Assets	7,496,816	88,051,023	139	984,641	15,569,000,000
Current Liabilities	4,978,069	79,914,135	0	604,651	15,596,000,000
EBIT	668,201	15,138,033	(500)	63,486	21,083,350,474
Retained Earnings	4,537,915	72,094,399	(237,971,000)	283,531	14,950,615,000
Equity Book Value	5,275,523	78,607,964	(68,571,441)	343,052	15,000,615,000
Total Liabilities	7,421,238	116,589,532	254	936,929	21,924,000,000

Table 24 - Main Statistics for 2 - 98 range percentile balance sheet data in 2021. All values are expressed in EUR except for the "Number of Employees" variable.

	mean	std	min	median	max
X1	0.26	0.26	-2.27	0.26	1
X2	0.26	0.24	-3.55	0.23	0.91
X3	0.07	0.1	-2.31	0.05	0.98
X4	0.73	1.03	-0.75	0.38	13.89
X5	1.12	0.61	0	1.02	13.14
Z' Score	2.03	0.99	-0.42	1.91	5.75

Table 25 - Main Statistics for the 2 - 98 range percentile Z' score ratios in 2021.

As shown by the distribution graph below, also in this case as in 2019, most of the companies belonging to the middle dataset, have a Z' score ratio comprised between 1 and 2.

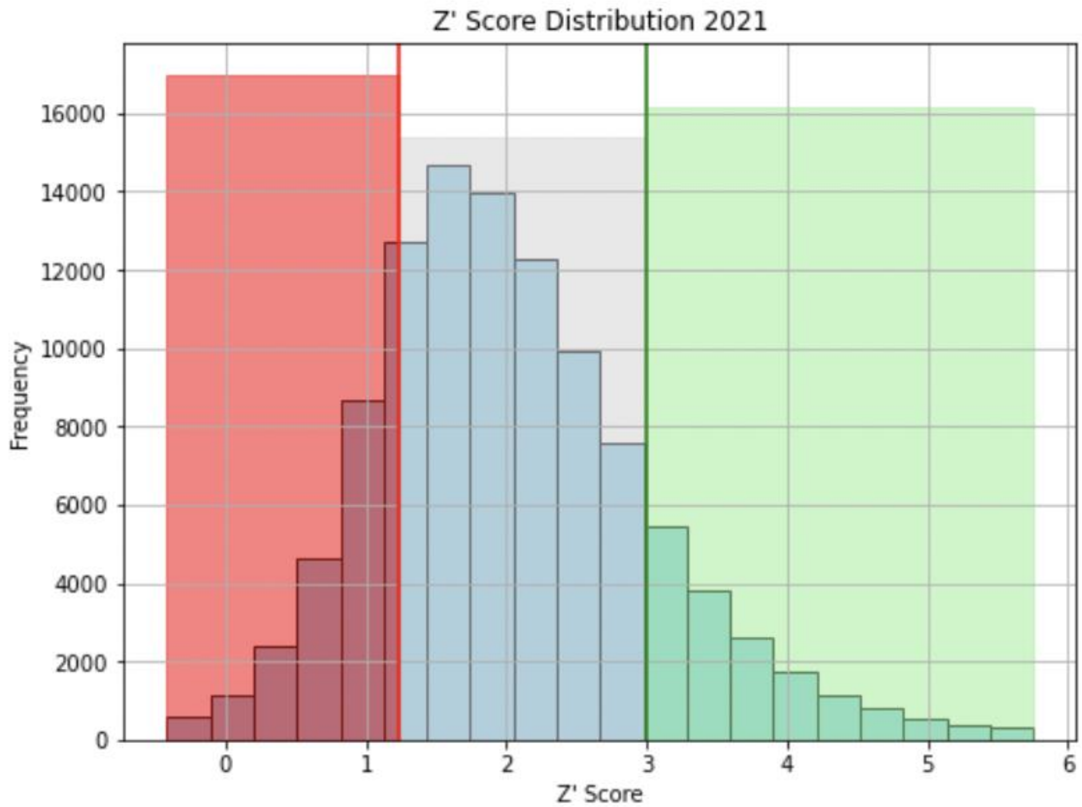


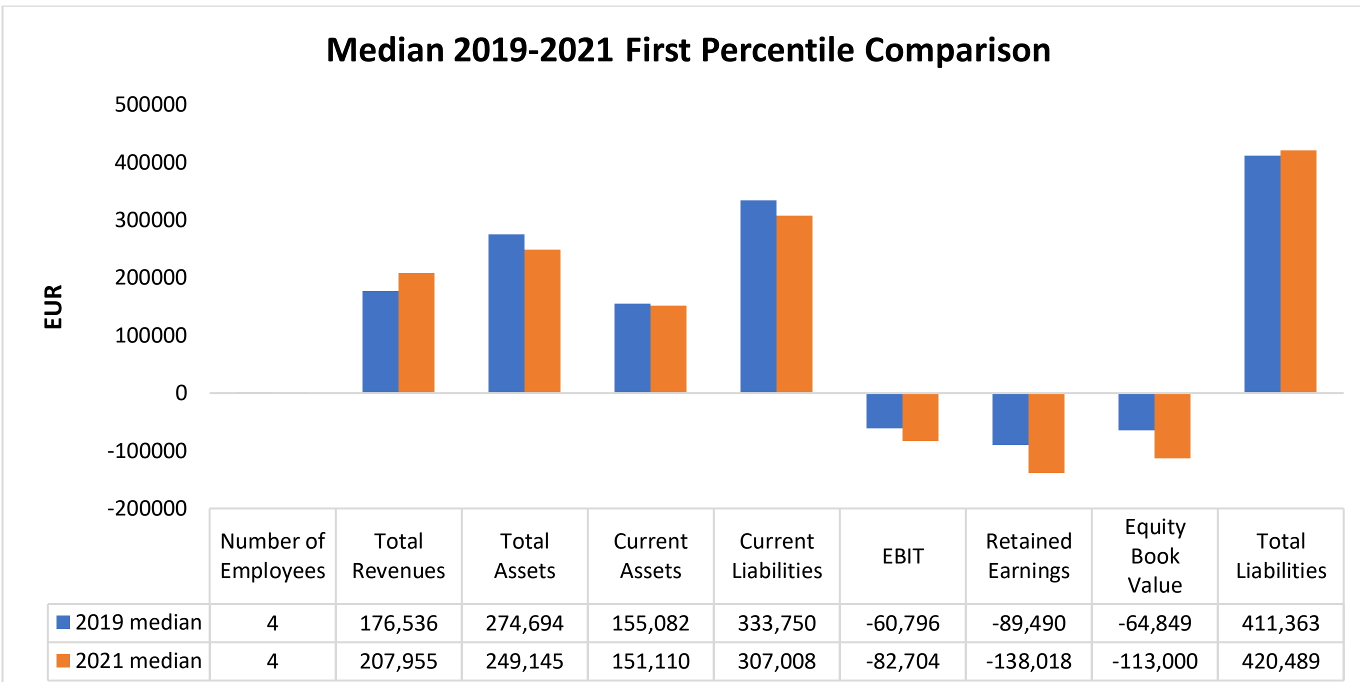
Figure 10 - Z' Score Distribution 2021 2-98 percentile range.

2.8 Results Interpretation

This chapter aims to indagate any possible trend or interpretation among the three samples in the three different percentiles. For the purpose of the analysis, since every dataset may contain outliers, the median will be used to compare the different variables among 2019 and 2021.

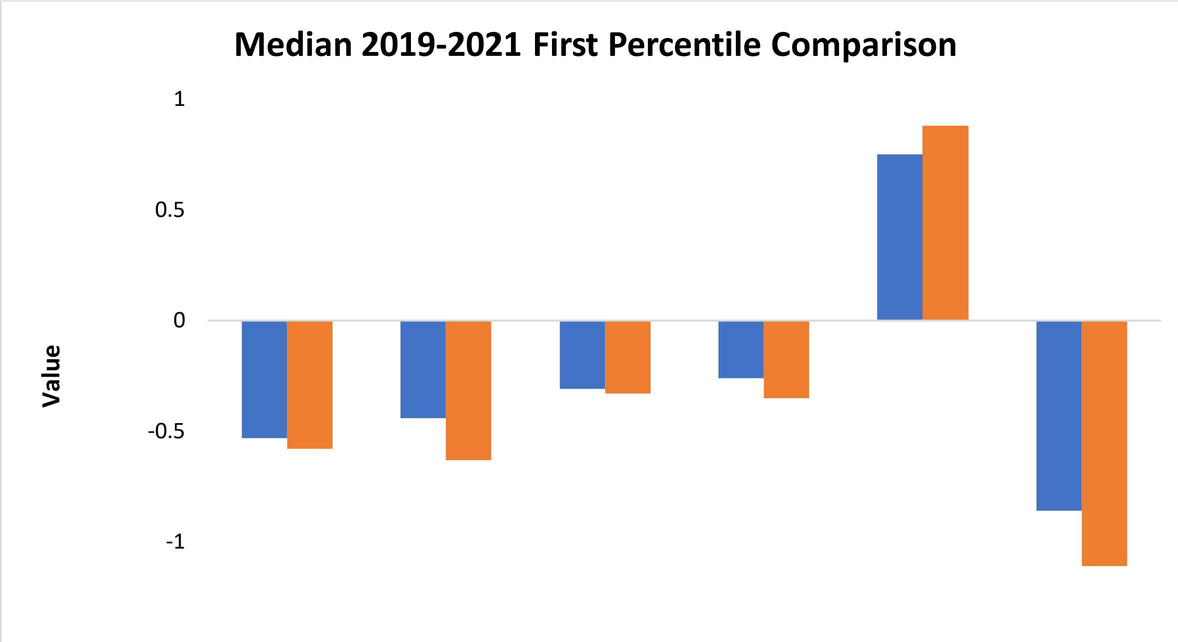
Figure 11 represents comparison among median in 2019 and 2021 for the balance sheet data, as number of employees, revenues, assets, current assets and liabilities, book value of equity, EBIT, retained earnings and total liabilities for the first percentile companies in each year. Figure 12 shows the same, but for the 5 independent variables and the Z’ score.

It is important to denote from Figure 11 that 2021 data are worse for all the fields, except for total revenues, which is then reflected in a better X_5 value in Figure 12.



	Number of Employees	Total Revenues	Total Assets	Current Assets	Current Liabilities	EBIT	Retained Earnings	Equity Book Value	Total Liabilities
2019 median	4	176,536	274,694	155,082	333,750	-60,796	-89,490	-64,849	411,363
2021 median	4	207,955	249,145	151,110	307,008	-82,704	-138,018	-113,000	420,489

Figure 11 - 2019 Median vs 2021 Median Balance Sheet Data First Percentile.

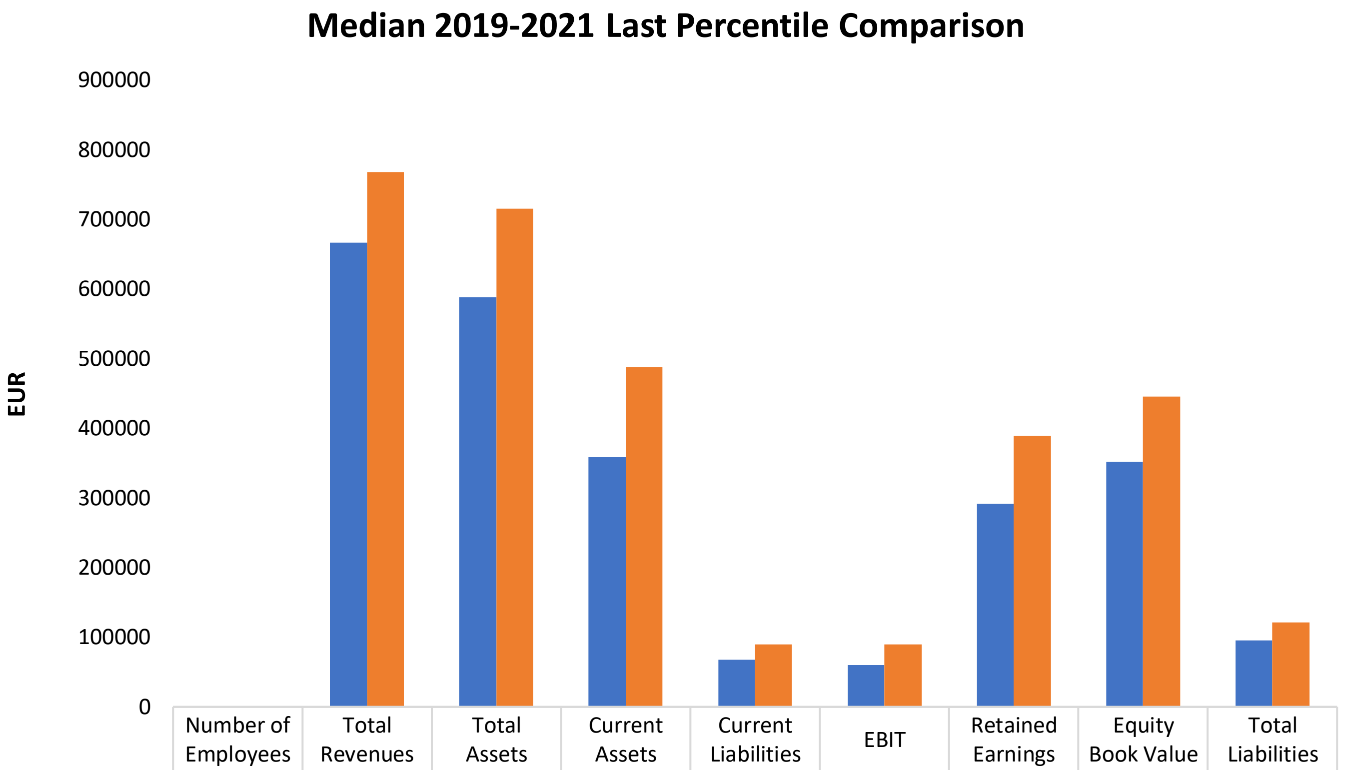


	X1	X2	X3	X4	X5	Z' Score
2019 median	-0.53	-0.44	-0.31	-0.26	0.75	-0.86
2021 median	-0.58	-0.63	-0.33	-0.35	0.88	-1.11

Figure 12 - 2019 Median vs 2021 Median Ratios First Percentile.

In 2021 the Italian economy started to recover from Covid 19 pandemic, with people spending more money, and this is also linked to nowadays inflation, which can be an explanation the 2021 increase in revenues.

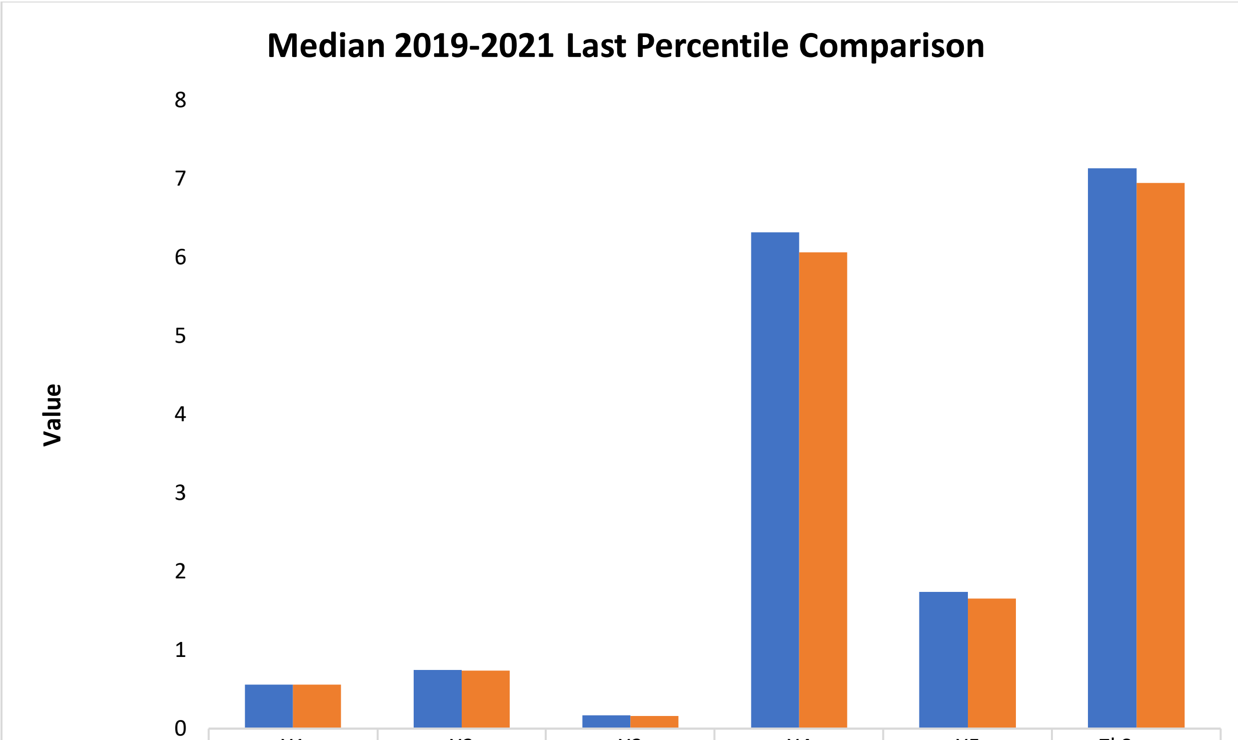
The figure 13 and 14 represents the same as figure 11 and 12, but for the last percentile, the one containing the best companies from a Z' score point of view.



	Number of Employees	Total Revenues	Total Assets	Current Assets	Current Liabilities	EBIT	Retained Earnings	Equity Book Value	Total Liabilities
2019 median	4	667,061	588,261	358,671	67,996	60,170	291,601	351,603	95,710
2021 median	4	768,535	715,669	487,661	89,631	89,534	389,757	446,011	121,199

Figure 13 - 2019 Median vs 2021 Median Balance Sheet Data Last Percentile.

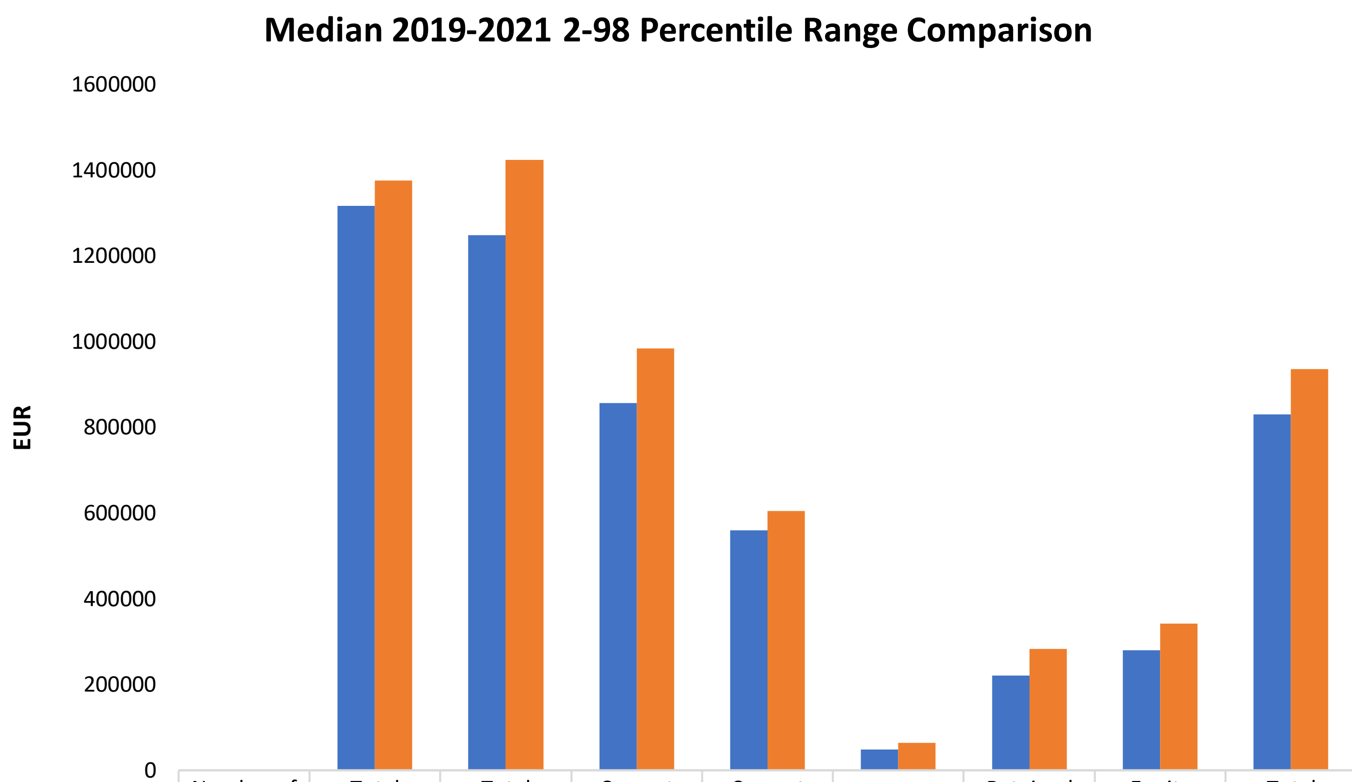
The figures 12 and 13 are interesting because they seem one in contrast with the other. First analyzing Figure 12, it appears that 2021 top companies better performed than 2019 ones. The only bad aspect is in an increase in Total Liabilities, which could explain the worst performance of 2021 in terms of Z' score ratios. Specifically, the median of Z' scores among 2021 is equal to 6.95, lower than 7.14 in 2019.



	X1	X2	X3	X4	X5	Z' Score
2019 median	0.56	0.75	0.17	6.32	1.74	7.14
2021 median	0.56	0.74	0.16	6.07	1.66	6.95

Figure 14 - 2019 Median vs 2021 Median Ratios Last Percentile.

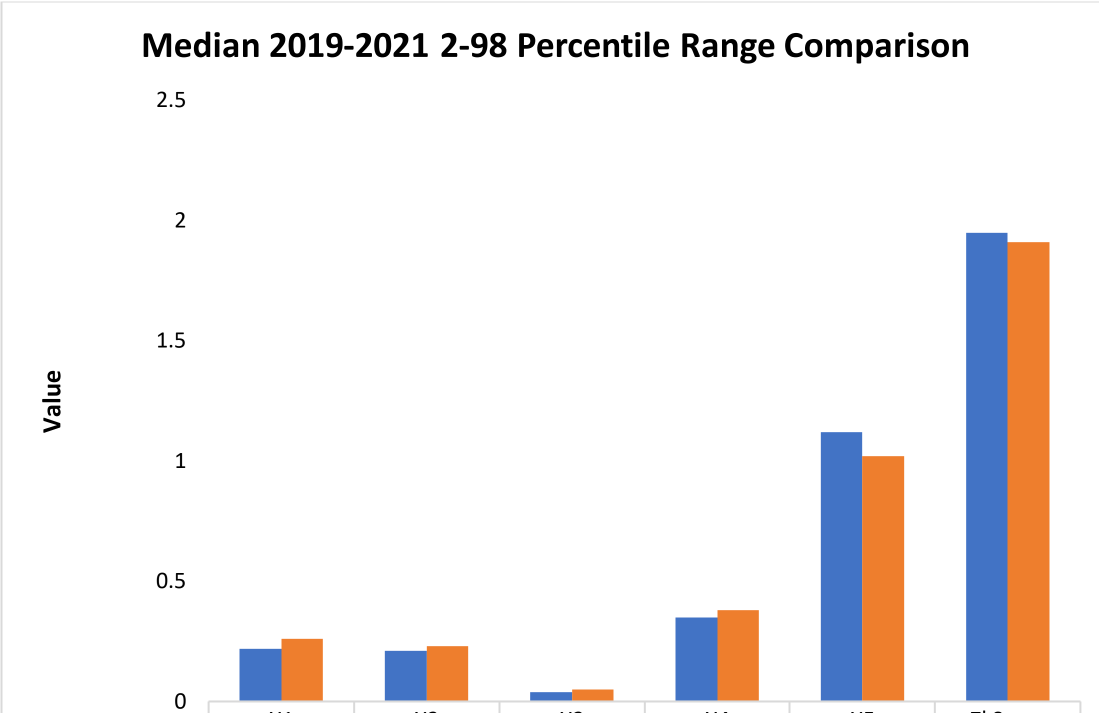
Finally, looking at the majority of the population in Figures 15 and 16, it reveals an overall decrease in the Z' score media from 1.95 to 1.91. This decrease seems to be driven by the variable X5, since it is the only one decreased from 2019. As In the previous case, revenues in 2021 increased from 2019, but at the same time the total assets had a higher rise thanks to an increase in total liabilities, which drops the value of X5 to a lower value compared to two years before.



	Number of Employees	Total Revenues	Total Assets	Current Assets	Current Liabilities	EBIT	Retained Earnings	Equity Book Value	Total Liabilities
2019 median	9	1,316,762	1,249,171	857,790	559,522	47,847	220,696	280,161	830,654
2021 median	9	1,377,132	1,424,152	984,641	604,651	63,486	283,531	343,052	936,929

Figure 15 - 2019 Median vs 2021 Median Balance Sheet Data 2-98 Range Percentile.

In both 2019 and 2021 cases, the Z’ score median is not a good sign, assuming values of 1.95 and 1.91 respectively, far more than 1 point from the green area threshold defined by Altman.



	X1	X2	X3	X4	X5	Z' Score
2019 median	0.22	0.21	0.04	0.35	1.12	1.95
2021 median	0.26	0.23	0.05	0.38	1.02	1.91

Figure 16 - 2019 Median vs 2021 Median Ratios 2-98 Range Percentile.

2.9 Conclusion

While the Z' score effectively captures the situation of the examined firms, it's crucial to underscore the unique economic landscape of Italian companies compared to their American counterparts, as described at the beginning of the chapter. This disparity, along with its implications on financial structure, size, and transparency, necessitates the development of a more nuanced model tailored to our reality. Such refinement is essential to ensure that company scores better reflect the actual experiences of businesses.

From a Z' score point of view, the Italian Manufacturing Industry was not in a good situation neither in 2019 nor 2021. In 2019, 81.52% of the 101,700 companies were marked as insolvent, with 20.72% of them in the so called red area, having a score lower than 1.23. In 2021 the situation got a bit worse, with only 16.28% companies classified as heathy, and 83.72% as not, of which 20.96% in the red zone. In both the years, by looking at Figure 8 and 10, most of the companies in the grey area, have a Z' score nearer to the red one instead of the green one, assuming values within 1.23 and 2.

By comparing the 2019 and 2021 overall, the median of the Z' score lowered from 1.95 to 1.91, due to a decline of the variable X_5 caused by an increase in total assets mostly financed by debt, which did not trigger the same increase in revenues. Looking at the first and last percentile instead, 2021 was a worse year for both from a solvent point of view. For the first percentile group, the Z' score median dropped from -0.86 to -1.11, mostly due to a downturn in the variables X_2 and X_4, caused by a decrease in retained earnings and equity book value respectively. On the contrary, for the last percentile group the Z' score median fell off from 7.14 to 6.95, due to an increase in total liabilities, that on one hand caused an increase in total assets, lowering the X_4 ratio, on the other hand had a direct negative impact on the X_5 variable.

To summarize, the median Z' score decreased from 2019 to 2021 in all the three groups. However, this study cannot conclude if this was due to Covid 19 pandemic or to other factors and cannot even conclude if the Z' scores computed in 2021 were accurate. These two statements are correlated, and one is the cause of the other. For what concern the former, as highlighted at the beginning of this chapter, Z' score was tailored to American companies, a totally different context compared to Italy. For the latter, Covid 19 pandemic was an extra-ordinary situation, where the government helped proactively the firms in the manufacturing industry, as well as in other sectors. One of the helps given by the state was for example as non-repayable contributions at 65% for projects over 30,000 € for investment projects and for the adoption of organizational and social responsibility models (Camera Dei Deputati, 2020). All these supports obviously drugged the system, making the Z' score probably not the most accurate measure for this occasion.

However, what this study revealed, is that the Z' score can be interpreted depending by situation to situation, breaking down the variables to understand from what causes an increase or decrease of the variables.

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