

POLITECNICO DI TORINO

Master's Degree in ENGINEERING AND MANAGEMENT

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# NEW TECHNOLOGY DIFFUSION IN VERTICAL AND LONGITUDINAL MARKET SEGMENTS

Supervisor

Candidate

**Prof. Marco Cantamessa**

**Giacomo Biondi**

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## Abstract

The process of diffusion of innovations has been thoroughly investigated in the past with a focus on the horizontal segmentation of the adopters. Rogers set the ground in the 1960s with its studies on agricultural communities, developing a framework with five categories of adopters with different acceptance attitudes towards new technologies. The main mathematical model used to describe the process of diffusion of innovation, theorized by Bass in the same period of the past century, analyses homogeneous populations, but is also based on the idea that a part of the customers spontaneously adopts new products, while followers subsequently imitate the behavior of the pioneers under the internal influence of a social network.

Fewer studies in the field of diffusion of innovations are focused on the vertical segmentation of a market. For instance, Christensen, in its groundbreaking work on disrupting innovations, researched on new products and technologies that come in at the bottom of an overlooked or completely new market, but that is not the only possibility in real life (i.e. the introduction of automobiles). Most of the research on the topic is linked to the economic side of new technologies development, explaining how innovation can be an important factor for a firm to differentiate its products from the competition, with positive effects on profitability.

In this thesis, the goal is to integrate the vertical segmentation of a market with the horizontal one, therefore creating a matrix segmentation, in order to study the pace of diffusion of innovation from a mathematical modelling standpoint. Furthermore, the objective is to analyze the saturation of different niches of the market to try and understand if, while some segments still have room for new sales, in some others the stimulation for substitution sales could be needed. Two opposite cases will be taken into account, with new products being introduced from the top or the bottom of a market. Then, the results will be analyzed to find the most relevant implications for managers and practitioners.

# 1. Introduction

## 1.1 Rogers' diffusion of innovation theory

The diffusion of innovation theory, introduced by the seminal work of Everett Rogers in 1962, seeks to explain the reasons why new ideas and technology spread and the ways through which they are able to do so, both in terms of channels and velocity.

This theory analyses the process of adoption both at the individual and aggregate level.

At the aggregate level of analysis, the diffusion of innovation theory suggests the manifestation of an S-shaped cumulative adoption pattern, as the population of individuals acquires the innovation over time. This temporal pattern further proposes that some members of the population adopt earlier than others, reflecting their level of innovativeness.

When looking at single customers and their choices over time, several elements are taken into consideration: the innovation, individuals who already adopted the innovation, individuals who are yet to adopt the innovation, and communication channels that allow these two groups to exchange information.

Rogers describes five characteristics that customers evaluate when deciding whether to adopt an innovation or not:

1. Compatibility: how well does the innovation fit with existing systems, customers' habits and the overall environment?
2. Trialability: is it possible to test the innovation before the purchase?
3. Relative advantage: in what way is this innovation better than the legacy technology?
4. Observability: is it possible to notice the benefits of the new technology, whether from looking at other customers or from firsthand experience?
5. Simplicity: is it easy to understand and implement?

These qualities interact with each other, and customers judge them as a whole, weighing the advantages or disadvantages of the new technology compared to the systems already in place.

According to Rogers, diffusion occurs through a five-step decision-making process, through a series of communication channels over time among the members of a similar social system.

Firstly, individuals need to be aware that a new technology is available. The first time that they are exposed to the innovation, they still lack critical information about it to decide whether to adopt it or not. During this stage, individuals are not engaged enough to actively seek information.

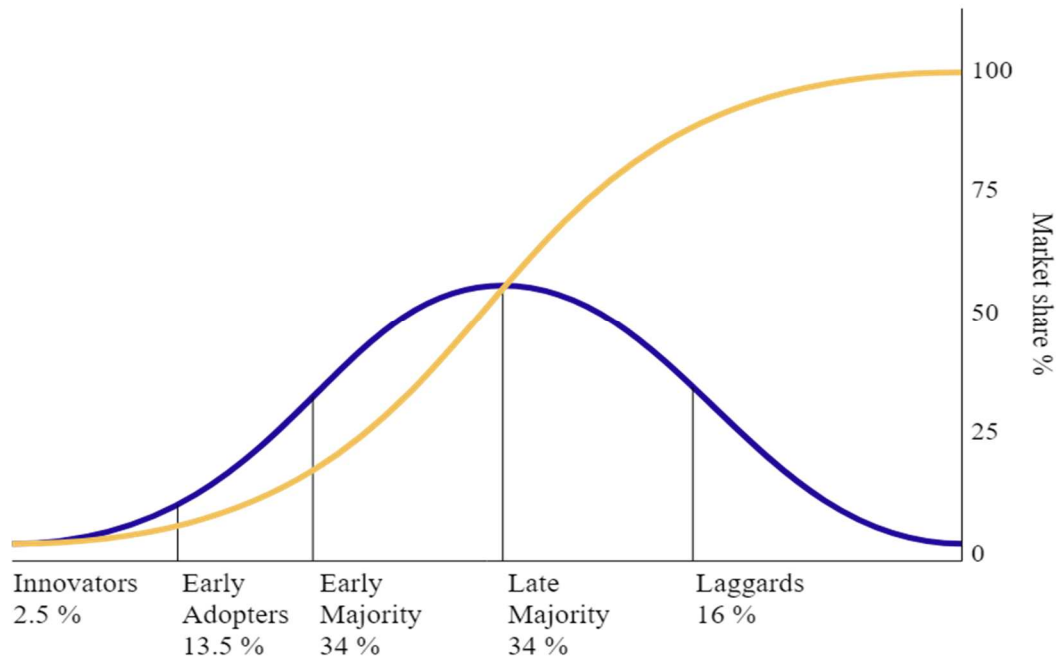
Then, in the phase of persuasion, individuals become interested enough to start actively seeking relevant details about the innovation. They begin to ponder about the idea of change, while weighing the pros and cons of the new technology. When they have acquired all the important information, customers finally make a decision. Due to the intrinsically individualistic nature of this stage, it is the most difficult step on which it is possible to acquire empirical evidence for researchers.

If individuals opt for the adoption of the innovation, they need to implement it afterwards. The new technology is employed to a varying degree depending on the situation. During this stage the individuals also determine the usefulness of the innovation, so that they may search for further information about it.

Finally, the last stage of the process consists of the finalization of the adoption decision. During the phase of confirmation, individuals seek reassurance that the choice of adoption was beneficial and the new technology is useful. The confirmation involves both intrapersonal and interpersonal levels: customers may feel cognitive dissonance, especially in case new negative information is obtained after the adoption; on the other hand, the social network can help reassure individuals and make them feel more comfortable about their decision.

It is important to notice that, while the decision stage is naturally the phase when most individuals reject an innovation, it is possible to encounter failure of adoption in each step of the process.

Another important variable in Rogers' studies is the timeliness of adoption, which guides its distinction of potential adopters into five categories, together with other socioeconomic and personality traits. The categories of adopters are innovators, early adopters, early majority, late majority and laggards, which respectively account for 2.5%, 13.5%, 34%, 34% and 16% of the total potential market, as shown in figure 1.1.



**Fig. 1.1.** Categorization of customers based on the timeliness of their adoption

Innovators are the first customers who adopt new technologies. They are described as the most propense to take risks, which helps them to adopt innovations that could ultimately fail. Moreover, their socioeconomic condition allows them to be exposed to new technologies early on and to influence less innovative segments: in fact, they are thought to be in closer contact with the latest scientific developments and to have a high social status, usually making them be looked upon as opinion leaders; their resources are also good enough to help them financially absorb failed innovations and avoid economic concerns during their decision making process.

Early adopters are less likely to take risks compared to innovators, but they still maintain a high social status, usually paired with good education, financial liquidity and communication skills. Their personality traits, together with their judicious adoption choices, allow them to earn the highest degree of opinion leadership and a central position in their own social networks.

The early majority follows the previous two segments after a certain amount of time, which can be highly varying depending on the case. Their social status is above average, as well as other socioeconomic factors such as education, age, communication and resources. They are relatively more risk-averse, so that their choices can be influenced by the opinions of customers who already adopted the new technology. However, they sometimes can still be regarded as opinion leaders in certain social systems.

Customers in the late majority segment adopt innovations after the average. They are described as more skeptical and they need more information, reassurance from previous adopters and opinion leaders. Their social status and education are below average, and their limited financial availability can be an important factor in determining the rejection of an innovation. They are also less likely to be exposed to new ideas and to be pioneers in the diffusion of a new technology.

Laggards are the last individuals to adopt innovative products. They tend to stick to traditions, usually due to their older age and their aversion towards risks and changes. They are thought to be outcasts, with low social status and little to no influence on other individuals.

As shown by the description of the adopters' segments, not all individuals and groups exert an equal amount of influence over others. Opinion leaders have the most influence during the evaluation stage of the innovation-decision process and on least innovative adopters. Moreover, opinion leaders typically have greater exposure to the mass media and the scientific community, greater contact with change agents, more social experience and exposure, higher socioeconomic status. Research shows that opinion leadership tends to be organized into a hierarchy, with each level in the hierarchy having most influence over other customers in the same group and the ones in the next level below it.

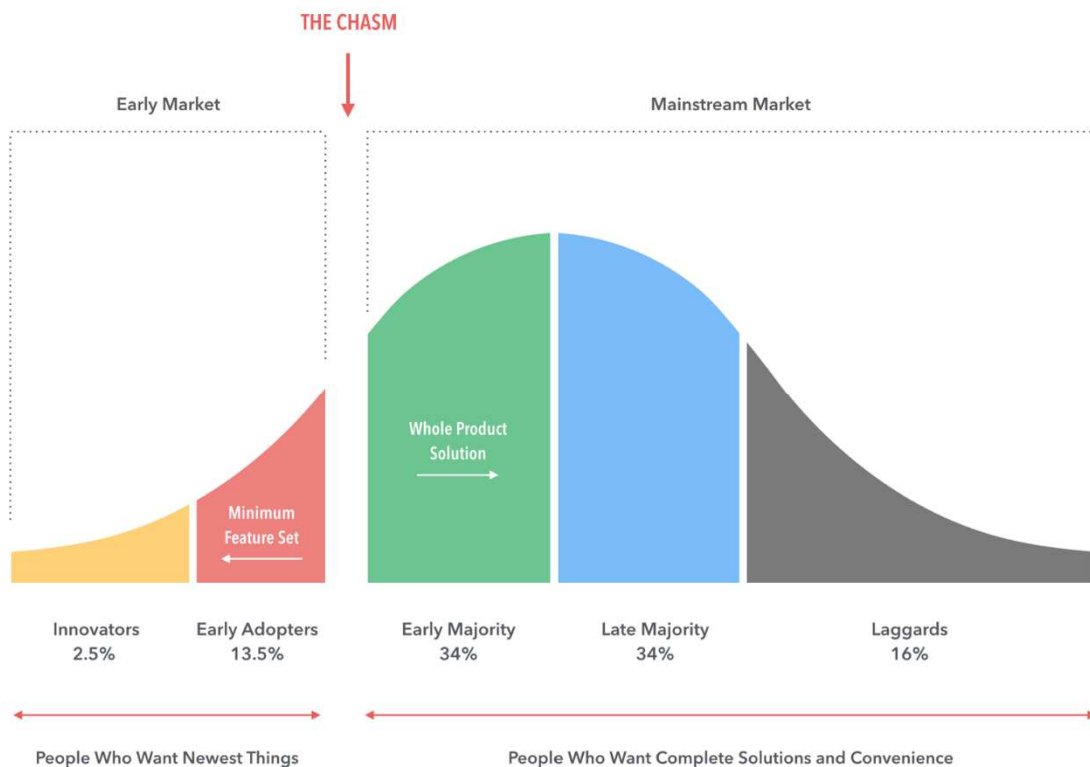
Innovators and early adopters are especially important in the diffusion of innovations, as they are likely to be opinion leaders. They can trigger the diffusion through two different mechanisms: the dissemination of ideas and the imitation of behavior. In fact, through their central position in social systems, innovators and early adopters can spread new ideas and products, broadening the exposure of less innovative segments to the latest technological developments. Additionally, their choices and behavior are imitated by more risk-adverse adopters, who need more guarantees before committing to buy new products.



## 1.2 Expansions on Rogers' theory

After the introduction of Rogers' work, studies on the diffusion of innovations and potential adopters have flourished. Among them, in 1991 Geoffrey A. Moore published "Crossing the Chasm: Marketing and Selling High-Tech Products to Mainstream Customers", a marketing book in which a particular focus is posed on the differences between innovators and early adopters compared to the other segments theorized by Rogers.

The author argues that a chasm is specifically present between the early adopters and the early majority, as their individual traits differences are more evident compared to other subsequent segments. In particular, Moore believes innovators and early adopters are visionaries, technology enthusiasts that have very different expectations on new technologies with respect to pragmatists in the mainstream market. The author therefore introduces marketing suggestions to help firms in avoiding getting stuck in the chasm (shown in figure 1.2).



**Fig. 1.2.** Visual representation of the chasm between early adopters and early majority

Moore believes that firms should focus on one customer segment at a time, tailor-suiting their marketing campaigns on the characteristics required by the target audience, then using each group as a base to spread the new products to the following one.

Firms must understand who the innovators are and engage with them to learn their specific needs, in order to fulfill their expectations on the technical performance of the new technology. Then, ancillary features can be developed when moving to the early adopters' group, while improving the reliability of the innovation and reducing the purchasing risk (e.g. with guarantees, trials etc.). Additionally, other important elements of the innovation's system must be developed, such as a distribution network, complementary products and a starting price, which must just be satisfactory enough for segments that are more focused on the performances of the technology.

The most difficult transition lies in the gap between visionaries and pragmatists, i.e. the early adopters and the early majority. Moore suggests a few actions to help crossing the chasm: firms must work on the ease of use and implementation of the innovation, consolidating the adoption network with complementary products and services. The standard for technical features can be lowered, as customers in the early majority seek better pricing and reliability rather than excellent performance, in contrast with the previous segments.

Other studies have focused on the innovators and early adopters' groups because of their central role in speeding up the diffusion rate of innovations. In particular, the ways through which they are able to spread new ideas have been analyzed.

As previously introduced, the most widely accepted theories assume that early buyers can create pressure to adopt on other potential customers, essentially in two slightly different forms: the dissemination and imitation of ideas. In the first case, the information contagion theory assumes incomplete information that causes high uncertainty for prospective adopters, who in turn seek more guarantees before purchasing. Therefore, the early adopters and innovators' opinion on the innovation, usually disseminated through word-of-mouth, has a significant impact on subsequent purchases. On the other hand, the bandwagon models assume that early adoption of innovations can stimulate a reactive imitative behavior among later adopters.

Many authors have also expanded Rogers' initial work on the definition of peculiar characteristics for each adopters' segment, with a particular focus on addressing the antecedents of customer innovativeness.

Firstly, some factors have been found to be much less relevant for customer innovativeness than initially thought, with age, gender, education as the most notable examples. Moreover, opinion leadership is believed to be decoupled from social status and innovativeness: in fact, it is perfectly possible for very early adopters to be poor communicators with bad leadership, with the opposite case for laggards with good social skills.

Research shows that personality and socioeconomic variables have higher impact on customer innovativeness, such as curiosity, social and communication skills, financial availability and resources. These variables are applicable in general for any kind of innovation, but other factors are better fit to predict domain and market-specific innovativeness and, most importantly, purchasing behavior: for instance, frequency of use of special interest media, product category use and domain-specific resources availability.

One of the most important themes that Rogers' theory and its expansions have not been able to explain is the fact that adopters and rejecters in the same group potentially have the same characteristics. Developed to address limitations in Rogers' theory, agent-based models provide a more dynamic and realistic representation of the diffusion process, considering individuals with diverse characteristics, preferences, and behaviors as opposed to grouping them into macro-categories. These models offer insights into how individual-level interactions influence innovation adoption, capturing the heterogeneity among adopters.

### 1.3 Bass diffusion model

The diffusion of innovations has also been thoroughly studied from a mathematical standpoint, with published work spanning several disciplines, markets and decades.

Diffusion models have been developed to analyze the evolution over time of first purchases of new products by a population, with the goal of better understanding the rate of diffusion of innovations and for forecasting purposes.

The most famous diffusion model was introduced by Frank Bass in 1969 in his work "A new product growth model for consumer durables". It is a "mixed influence" model, in which diffusion is driven by both innovative and imitative effects and the overall market growth follows a generalized logistic curve. Bass aggregates all the segments by Rogers except for innovators and defines them as imitators. Innovators are different from imitators because they are not influenced in the timing of purchase by other members of the social system, while imitators are influenced by the number of previous buyers. Imitators "learn," in some sense, from those who have already bought.

The model is defined by the following hazard function:

$$\lambda(t) = \frac{f(t)}{1 - F(t)} = p + qF(t)$$

Where:

- $\lambda(t)$  is the probability of adopting in period  $t$ , under the condition of not having adopted yet;
- $F(t)$  is the installed base fraction;
- $f(t)$  is the rate of change of the installed base fraction, i.e.  $f(t) = F'(t)$ ;
- $p$  is the coefficient of innovation;
- $q$  is the coefficient of imitation.

Therefore, at any time  $t$ , the fraction of new adopters  $f(t)$  and the cumulative number of adopters  $F(t)$  are equal to:

$$f(t) = \frac{(p+q)^2 e^{-(p+q)t}}{p(1 + \frac{q}{p} e^{-(p+q)t})^2}$$

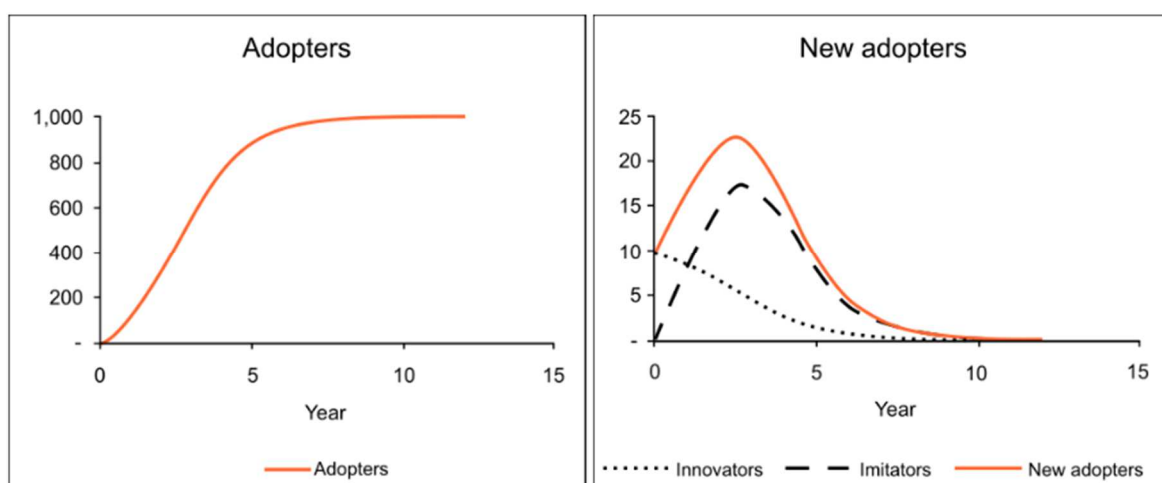
$$F(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}}$$

The coefficient  $p$  is linked to external forces operating on the market, for example advertising. This external influence pushes innovators towards the purchase of new products, regardless of the amount of population that already adopted the new technology.

The coefficient  $q$  is linked to internal forces in the market, that is the social pressure that makes imitators copy the behavior of previous adopters over time.

To find the number of sales (respectively cumulative and instantaneous), it is sufficient to multiply  $F(t)$  and  $f(t)$  by the total market size  $M$ .

In figure 1.3, an example of curves derived from the Bass model is shown, with the distinction between innovators and imitators.



**Fig. 1.3.** On the left-hand side, cumulative sales of an innovation. On the right-hand side, the corresponding instantaneous sales split between innovators and imitators.

The Bass model has had a significant impact in the field of marketing and business management, becoming one of the most cited references for the forecasting of new products' demand thanks to its simplicity and reliability.

For the buildup of his model, the author makes assumptions and simplifications that allow it to suit many different markets and product categories, working very well when looking at aggregate demands.

Over the course of time, other authors have expanded the initial work of Frank Bass to include more features in the model and overcome some of its structural limits (most notably, the bell shape of the new adopters' curve is not compatible with the saddle in sales theorized by Moore).

Some of the most widely spread extensions to the model include Horsky's model, where the market potential depends on the product's price and utility, as well as the available income within the population; the generalized Bass model, where sales are linked to relative variations of price and marketing effort; diffusion models with substitution and additional sales; models with complementary and substitute goods.

#### 1.4 Christensen's disruptive technologies

The term "disruptive technologies" was popularized by Clayton M. Christensen in 1995 in his article "Disruptive Technologies: Catching the Wave".

Christensen identifies three types of innovations: sustaining, efficiency and disruptive innovations. Sustaining innovations are aimed at improving the current products and technologies, meeting the evolving demands of existing customers and, for established companies, keeping the competitive advantage intact. On the other hand, efficiency innovations focus on the optimization of the production processes, in order to enhance the operational efficiency, reduce costs and waste,

finally improving the profit margins; this kind of innovations usually benefit the market-leading firms, too.

Disruptive technologies describe innovations that fundamentally alter existing markets, often challenging incumbents and reshaping the industry landscapes. They can start at the bottom or in a niche of a market, targeting overlooked and underserved segments of customers, whose needs may be quite different from the performances offered by the existing technologies. In low-end or even new markets, customers are less demanding, and the innovations can be designed to be “just enough” to meet their needs. Then, disruptive technologies can gain momentum over time, evolving to meet broader market needs and eventually surpassing established products.

Christensen emphasizes that market-leading firms, often focused on sustaining and efficiency innovations to improve existing products, may overlook disruptive technologies due to their initial limited appeal or lower profit margins. This oversight creates a strategic vulnerability, as disruptive innovations gradually gain traction and new firms erode market share from the incumbents. In fact, disruptive innovations tend to be produced by outsiders and entrepreneurs in startups, rather than existing market-leading companies. Recognizing and responding to disruptive technologies requires a proactive approach, involving adaptability and a willingness to explore innovations that might initially appear inferior to existing solutions.

## 2. Research goal and methodology

As previously seen, the existing literature in the field of the diffusion of innovations shows extensive research on the horizontal segmentation of a market, pioneered by the cornerstone work of Rogers.

Much fewer papers have investigated on the vertical segmentation of a market. Christensen described the way new technologies can be introduced, therefore analyzing when they enter from the top or bottom of the market. Other authors have researched on the innovation efforts of entrant firms, linked to the vertical differentiation of their new products and the related benefits in terms of profits.















However, previous research on vertical segmentation has been applied mainly to firms and their products, avoiding a clustering of customers based on their personal preferences and characteristics.

This attempt to vertically segment the customer base was tangential in Rogers' work and the following extensions to its theory: many authors gave descriptions of the antecedents of innovativeness and the characteristics of Rogers' horizontal segments, from which certain traits can be derived to describe adopters of high, medium or low-end products.















Following these studies, the thesis will start with a matrix segmentation of the population that includes both a horizontal and a vertical clustering, respectively based on Rogers' groups of innovators, early adopters, early majority, late majority and laggards, and on the customers' preference for high, medium or low-end products.

The size of each segment of the matrix will depend on the innovation's type of entry in the market, while keeping the horizontal size fixed following Rogers. Therefore, the expectation is to have a higher density of innovators in the high-end segment and laggards in the low end when the introduction of a new technology comes at the top, vice versa for an innovation's entry from the bottom, as shown in table 2.1 and 2.2.



	Innovators	Early adopters	Early majority	Late majority	Laggards
High end					
Medium end					
Low end					

**Table 2.1.** Population distribution in case of innovation introduction from the top.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end					
Medium end					
Low end					

**Table 2.2.** Population distribution in case of innovation introduction from the bottom.

After completing the population segmentation, the goal of this thesis is to study the diffusion of innovations in a matrix segmented market from a mathematical modelling standpoint.

In case of innovation introduction in a new market, the basis will be provided by one of the many extensions to the Bass model, that is Van den Bulte & Yoshi's version with imitators and influentials. In Van den Bulte & Yoshi's, the important assumption is that influence is asymmetrical in a social system, so that imitators can follow the behavior of people in their cluster as well as influentials, while influentials independently decide whether to adopt or not the new technologies. This mechanism will be replicated in the thesis for each pair of subsequent horizontal segments, in order to simulate a gradual diffusion from the more innovative segments towards the laggards in every vertical cluster of the market.

High, medium and low-end markets will be independent form each other, with dedicated products introduced for each segment (but still pertaining to the same new technology). The demand curve of each cluster will then be aggregated in the total market demand; the comparison with a typical Bass diffusion process will be of particular interest, especially trying to see what happens underneath the macroscopical level reached by the original model.

Moreover, the diffusion of innovations in the model will also depend on the level of performance reached by the new technology. The performance parameter is necessary to simulate the introduction of vertically differentiated products in the market, which will have different attractiveness for the target customers depending on their personal needs and expectations.

Several case studies will be taken into account in the models:

- the introduction of a new technology from the top or bottom of a market;
- innovation entry in a mature, existing market or in a completely new one;
- performance evolution according or discording with the customer expectations.

### 3. New market model build-up

Firstly, we will consider the case of an innovation introduced in a new market, using Van den Bulte and Yoshi's model as a basis. Examples of innovations that create a completely new market could be the invention of automobiles in the early 1900s or the introduction of personal computers in the late 1970s.

#### 3.1 Horizontal segmentation and asymmetrical influence

The model is based on the formulation of Van den Bulte & Yoshi, an expansion of the Bass model. Unlike the great majority of marketing diffusion models, the population is not assumed to be homogeneous in its tendency to be in touch with new developments and to influence the behavior of later adopters. Specifically, the adopters are split into two categories: influentials who are more aware of the latest technological advancements and who affect the other segment of imitators, whose adoptions do not influence the influentials' behavior.

The model includes the dual drivers of adoption, i.e. the external and internal market forces captured by the variables  $p$  and  $q$  of the original Bass model. However, unlike the majority of other extensions, the assumption that all potential adopters are equally affected by both factors falls and the categories of population are not defined by the timeliness of their adoption, but by the drivers behind their decision-making.

In other models, the authors do not set an ex-ante mixture of two segments, with the first adopting independently at rate  $p$  and the second adopting because of social contagion at rate  $qF(t)$ . Bass himself refers to adopters at  $t=0$  as innovators, opposed to imitators who adopt under the social system's pressure, but the definition can only be used ex-post for the previous reasons.

In the original Van den Bulte & Yoshi's model, the set of potential adopters has a constant size  $M$  and consists of two a priori different types of actors, influentials and imitators. These segments make decisions following these hazard function, where the subscripts 1 and 2 respectively denote influentials and imitators:

$$\lambda_1(t) = p_1 + q_1 F_1(t)$$

$$\lambda_2(t) = p_2 + q_2 [w F_1(t) + (1 - w) F_2(t)]$$

Where:

- $\lambda(t)$  is the probability of adoption in period  $t$ , under the condition of not having adopted yet
- $p$  captures the tendency to independently adopt
- $q$  captures the social contagion effect
- $F(t)$  is the cumulative penetration in the segment
- $w$  denotes the relative importance that imitators attach to influentials' versus other imitators' behavior ( $0 \leq w \leq 1$ ).

Therefore, the influentials follow the basic Bass model and their rate of adoption is driven by their own variables  $p_1$  (independent adoption) and  $q_1$  (adoption through imitation).

This set of two hazard functions allows closed-form solutions for the cumulative penetration functions and instantaneous adoption functions of influentials and imitators, assuming  $F_1(0) = 0$  and  $F_2(0) = 0$ .

For influentials, the equations are:

$$F_1(t) = \frac{(1 - e^{-(p_1+q_1)t})}{(1 + \frac{q_1}{p_1} e^{-(p_1+q_1)t})}$$

$$f_1(t) = \frac{(p_1(1 + \frac{q_1}{p_1})^2 e^{-(p_1+q_1)t})}{(1 + \frac{q_1}{p_1} e^{-(p_1+q_1)t})^2}$$

Imitators, on the other hand, do not follow any other standard diffusion model. Their cumulative penetration function is:

$$F_1(t) = 1 + (-p_2q_1 + q_2(p_1w - q_1(1 - w))) \cdot \left( q_1q_2(1 - w)H_1 + e^{(p_2+q_2)t} \left( \frac{p_1 + q_1e^{-(p_1+q_1)t}}{p_1 + q_1} \right)^{\frac{wq_2}{q_1}} \cdot (p_2q_1 + q_2(q_1(1 - w)(1 - H_2) - p_1w)) \right)^{-1}$$

Where:

$$H_1 = {}_2F_1 \left( 1, \frac{wq_2}{q_1}, 1 + \frac{wq_2}{q_1} - \frac{p_2 + q_2}{p_1 + q_1}, \frac{p_1}{p_1 + q_1e^{-(p_1+q_1)t}} \right)$$

$$H_2 = {}_2F_1 \left( 1, \frac{wq_2}{q_1}, 1 + \frac{wq_2}{q_1} - \frac{p_2 + q_2}{p_1 + q_1}, \frac{p_1}{p_1 + q_1} \right)$$

And  ${}_2F_1(1, b, c, k)$  is the Gaussian hypergeometric function:

$${}_2F_1(1, b, c, k) = \sum_{n=0}^{\infty} \frac{\Gamma(b+n)\Gamma(c)}{\Gamma(b)\Gamma(c+n)} k^n$$

This hypergeometric series is convergent for  $|k| < 1$ ;  $k = \pm 1$  if  $c > 1 + b$ . This implies that the closed-form solution for the cumulative penetration function is well defined as long as  $q_1 > 0$ .

Once  $F_1(t)$  and  $F_2(t)$  are known, it is possible to obtain the instantaneous adoption function  $f_2(t)$  for imitators:

$$f_2(t) = [p_2 + q_2[wF_1(t) + (1 - w)F_2(t)]] [1 - F_2(t)]$$

For the purpose of this paper, this model has undergone some changes. The basic idea behind Van den Bulte & Yoshi's model, that is the asymmetrical influence, has been expanded to include all the segments theorized by Rogers. Therefore, in the new model every category act as imitators of the preceding segment and influentials for the following one (except for Innovators, who only influence Early Adopters without imitating anyone, and Laggards, who can only imitate Late Majority without influencing any segment). The resulting set of hazard functions is:

- $\lambda_{innovators}(t) = p_{innovators} + q_{innovators}F_{innovators}(t)$
- $\lambda_{early\ adopters}(t) = p_{early\ adopters} + q_{early\ adopters}[w_{early\ adopters}F_{innovators}(t) + (1 - w_{early\ adopters})F_{early\ adopters}(t)]$
- $\lambda_{early\ majority}(t) = p_{early\ majority} + q_{early\ majority}[w_{early\ majority}F_{early\ adopters}(t) + (1 - w_{early\ majority})F_{early\ majority}(t)]$
- $\lambda_{late\ majority}(t) = p_{late\ majority} + q_{late\ majority}[w_{late\ majority}F_{early\ majority}(t) + (1 - w_{late\ majority})F_{late\ majority}(t)]$
- $\lambda_{laggards}(t) = p_{laggards} + q_{laggards}[w_{laggards}F_{late\ majority}(t) + (1 - w_{laggards})F_{laggards}(t)]$

Moreover, it was not possible to compute values for the Gaussian hypergeometric functions in a consistent way, because the calculation of gamma functions with  $n > 171$  resulted in errors in Excel, as values exceeded the maximum number computable by the program. The solution was approximating the Gaussian hypergeometric function, using its Taylor series representation:

$${}_2F_1(1, b, c, k) = \sum_{n=0}^{\infty} \frac{1 \cdot (b)_n}{(c)_n} \frac{1}{n!} k^n$$

### 3.2 Vertical segmentation and performance

An important addition to the model is the introduction of a vertical segmentation of the adopters, based on their preference towards either high- or low-quality products.

In fact, potential adopters are split into three vertical segments, high-end, medium-end and low-end, with independent adoption patterns, meaning that imitators are not influenced by previous adopters in other vertical clusters. Three new technologies are supposed to be introduced in the market at the same time, differentiated by their level of performances; then, customers decide which product to adopt based on their preference, assuming that they can only choose one.

Following past research on the matter, innovativeness is modelled as slightly higher going from lower to upper segments: imitative adoption, regulated by parameter  $q$ , is favored by strong social systems and connections, which are generally found in upper classes; innovative adoption, linked to parameter  $p$ , is greater thanks to better socioeconomic conditions and accessibility to buy new technologies.

As mentioned before, innovation in the market is not homogeneous: in fact, the new technology is introduced in each vertical segment with different level of performances.

The performances evolve over time following an S-curve, starting with an initial value  $P_{j0}$  at  $t=0$  and approaching the asymptotic limit of the performance  $L_j$ . The evolution rates of the new products are determined by their sensitiveness towards investments,  $b_j$ . Investments are exogenous in the model, and they follow a logistic distribution.

The evolution of performance is therefore modelled by the function:

$$P_{jt} = \frac{L_j}{1 + a_j e^{-b_j CI_{jt}}}$$

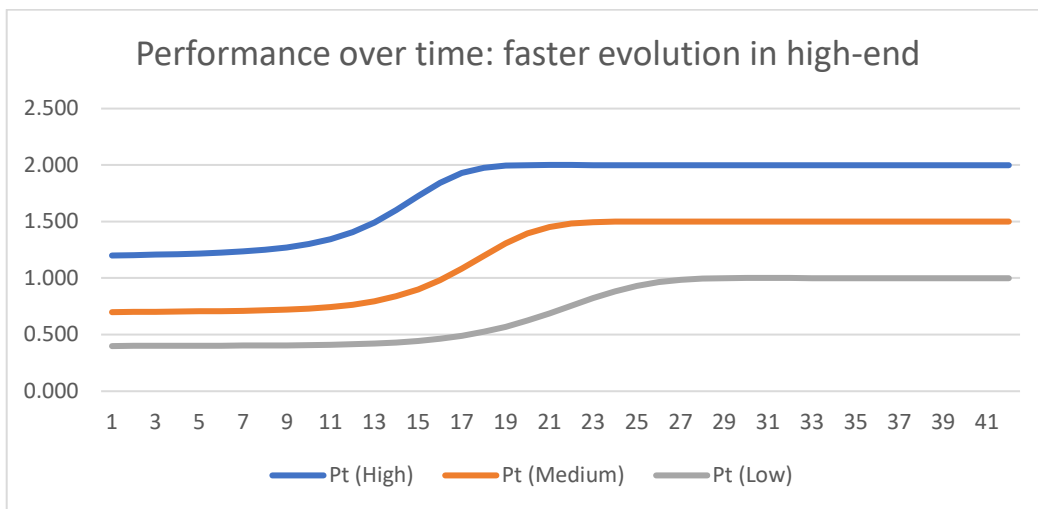
Where:

- $CI_{jt}$  is the cumulated investment in period  $t$
- $a_j = \frac{1}{P_{j0}/L_j} - 1$  is the position parameter of the logistic curve

Parameters  $P_{j0}$ ,  $L_j$  and  $b_j$  are different depending on vertical segment, while the cumulated investments are the same for everyone. It is therefore important to notice that the innovation evolution, as well as the adoption rate, is independent in each cluster.

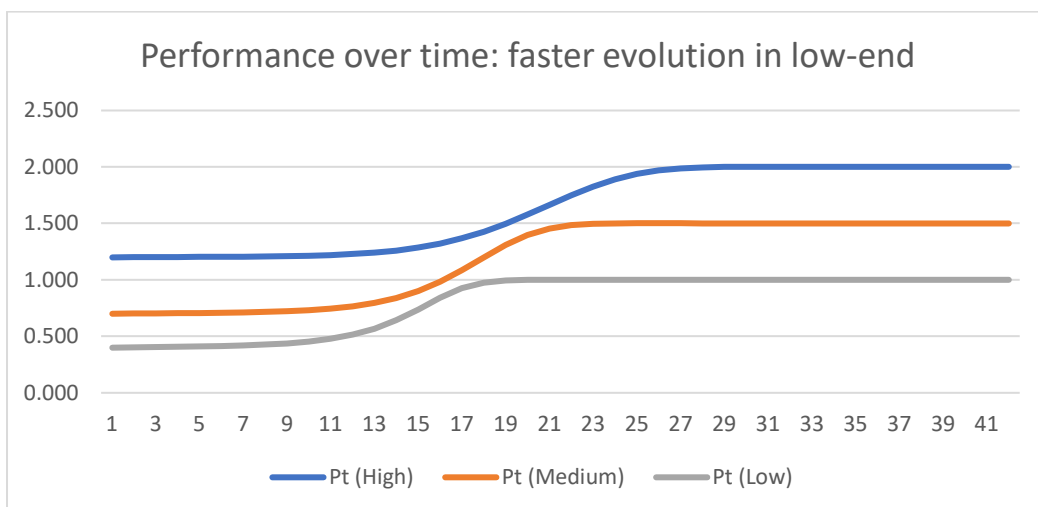
Depending on the values of  $b_j$ , i.e. the sensitivity of the performance evolution towards investments, two cases can be taken into consideration:

1. Performances evolve faster in the high-end market, with  $b_{\text{high}} > b_{\text{medium}} > b_{\text{low}}$



**Fig 3.1.** Performance evolution (faster in high-end market).

2. Performances evolve faster in the low-end market, with  $b_{\text{high}} < b_{\text{medium}} < b_{\text{low}}$



**Fig 3.2.** Performance evolution (faster in low-end market).



Compared to the original Bass model and Van den Bulte & Yoshi's extension, in this model the diffusion of innovation depends on the level of performance of the new technology. Therefore, not only the performance is the discriminating variable used to vertically cluster the adopters, but it is also determining the speed of the diffusion of the innovation.

More specifically, the underlying idea behind the model is that when performances get better, more customers are willing to consider the adoption of the new technology. The available market slowly broadens at the beginning, then undergoes through a "boom" phase in the central part of the evolution S-curves and finally reaches its maximum potential when the performances are stably at their limit.

For simplicity, in the model we start from two performance levels  $P_{10\%}$  and  $P_{90\%}$ , that represent thresholds for which 10% and 90% of the population is potentially interested in adopting the new technology. Then, depending on these values (different for each segment), the position parameters  $a$  and  $b$  of a logistic curve are derived.

The parameters  $a$  and  $b$  are computed as follows:

$$a = e^{P_{10\%}} \left( \frac{1}{0.1} - 1 \right)$$

$$b = \frac{1}{P_{10\%} - P_{90\%}} \ln \left( \frac{\frac{1}{0.9} - 1}{\frac{1}{0.1} - 1} \right)$$

The number of potential adopters  $M_t$  available in period  $t$  is given by:

$$M_t = M \frac{1}{1 + ae^{(-b P_{jt})}}$$

### 3.3 Resulting matrix segmentation

Combining the vertical and horizontal segmentations of the market, we obtain a matrix (see table 3.1) in which potential adopters are defined:

- by their level of innovativeness on the columns, following Rogers' categories;
- by their preference for high, medium or low-end products.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end					
Medium end					
Low end					

**Table 3.1.** Dimensions used for a matrix segmentation of the market

The size of each horizontal segment is fixed, given by Rogers' studies: over the total potential adopters, 2.5% are innovators, 13.5% early adopters, 34% early majority, 34% late majority, 16% laggards. Then, inside each horizontal segment, values are chosen to model how the group is split into high, medium or low-end market, so that the sum of each column gives 100%. Two extreme cases are modelled:

1. Entry of a new technology from the top (table 3.2): innovators and early adopters are preponderantly adopting high-end products, while the low-end market is mainly explored by the following groups;
2. Entry of a new technology from the bottom, à la Christensen (table 3.3): innovative segments purchase low-end products, with upper segments choosing to adopt later on.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	90%	70%	50%	25%	10%
Medium end	7%	20%	30%	50%	30%
Low end	3%	10%	20%	25%	60%

**Table 3.2.** Split of horizontal segments between high, medium or low-end customers. Entry from the top.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	3%	10%	20%	25%	60%
Medium end	7%	20%	30%	50%	30%
Low end	90%	70%	50%	25%	10%

**Table 3.3.** Split of horizontal segments between high, medium or low-end customers. Entry from the bottom.

After this step, every percentage is multiplied by the size of its own horizontal segment. The result is a matrix describing the relative size of each segment over the total market, so that the sum of the whole table gives 100%.

As in previous step, table 3.4 refers to the entry in the market from the top, while table 3.5 is linked to the entry from the bottom:

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	2.3%	9.5%	17.0%	8.5%	1.6%
Medium end	0.2%	2.7%	10.2%	17.0%	4.8%
Low end	0.1%	1.4%	6.8%	8.5%	9.6%

**Table 3.4.** Relative size of each segment over the total market. Entry from the top.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	0.1%	1.4%	6.8%	8.5%	9.6%
Medium end	0.2%	2.7%	10.2%	17.0%	4.8%
Low end	2.3%	9.5%	17.0%	8.5%	1.6%

**Table 3.5.** Relative size of each segment over the total market. Entry from the bottom.

It is interesting to notice that, while innovators and early adopters may be the most important segments for the seeding of ideas and the early diffusion of innovations, in terms of sales they have a relatively low impact over the total market because of their segments' size. Moreover, even in a very unbalanced setting, early majority and late majority have the heaviest weight to tip the scale in the overall market.

### 3.4 Parameters impact on S-curves

The curves of sales in each segment are given by the combination of two factors:

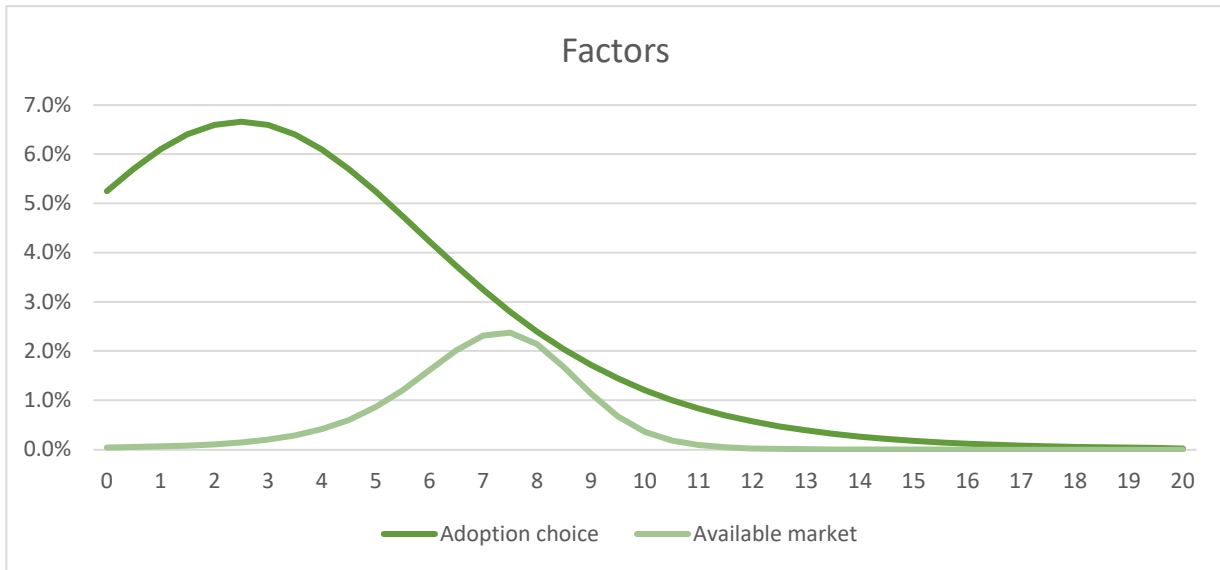
1. Available market effect: the quantity of potential buyers of the new technology is enlarged by improvements in the performances. The available market  $\vartheta$  increases by some percentage points in each period approaching 100%, with the effect becoming more evident when the performance evolution is in the central part of its S-curve.
2. Adoption choice: In each period, customers undergo through the process of deciding whether to adopt the innovation or not. The probability of adoption is given by the closed-form solutions to the hazard functions previously shown.

From a theoretical perspective, this means that in a segment  $j$  with size  $M_j$ , all customers have the same probability of adopting the innovation, defined by their hazard function; however, when the new technology is in its embryonal phase and its performances are still poor, some customers are not even considering the adoption yet, so the actual available market is equal to  $\theta M_j < M_j$ .

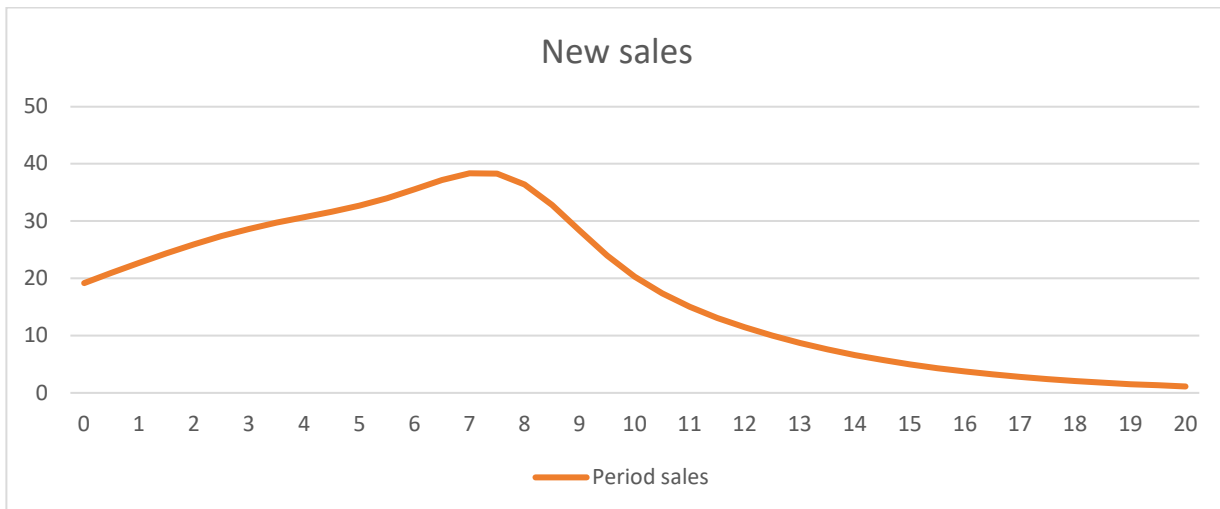
The multiplication between the fraction of people who choose to adopt and the fraction of available market gives the percentage of actual adopters in each period. This result is then multiplied by the maximum size of the segment to finally obtain the sales in every period.

In figures 3.3, an example from the model is shown. The dark green curve represents the percentage of customers who decide to adopt the innovation in period  $t$ , while the light green curve shows the delta available market between consecutive periods  $\theta_t - \theta_{t-1}$ .

The combination of the two effects is clearly noticeable on the curve of new sales in period  $t$ , shown in figure 3.4.



**Fig 3.3.** Factors influencing the sales curve.



**Fig 3.4.** Instantaneous new sales curve.

Firstly, parameters affecting the available market factor will be analyzed. We recall the formulas for the position parameters  $a$  and  $b$  of the logistic curve and the number of potential adopters at time  $t$ :

$$a = e^{aP_{10\%}} \left( \frac{1}{0.1} - 1 \right)$$

$$b = \frac{1}{P_{10\%} - P_{90\%}} \ln \left( \frac{\frac{1}{0.9} - 1}{\frac{1}{0.1} - 1} \right)$$

$$M_{jt} = M_j \frac{1}{1 + ae^{(-b P_{jt})}}$$

We can notice that the only parameters affecting the available market variations are the technologies' level of performance  $P_{jt}$  and the threshold values  $P_{10\%}$  and  $P_{90\%}$ , that respectively represent the performance levels for which 10% and 90% of the customers would consider the adoption.

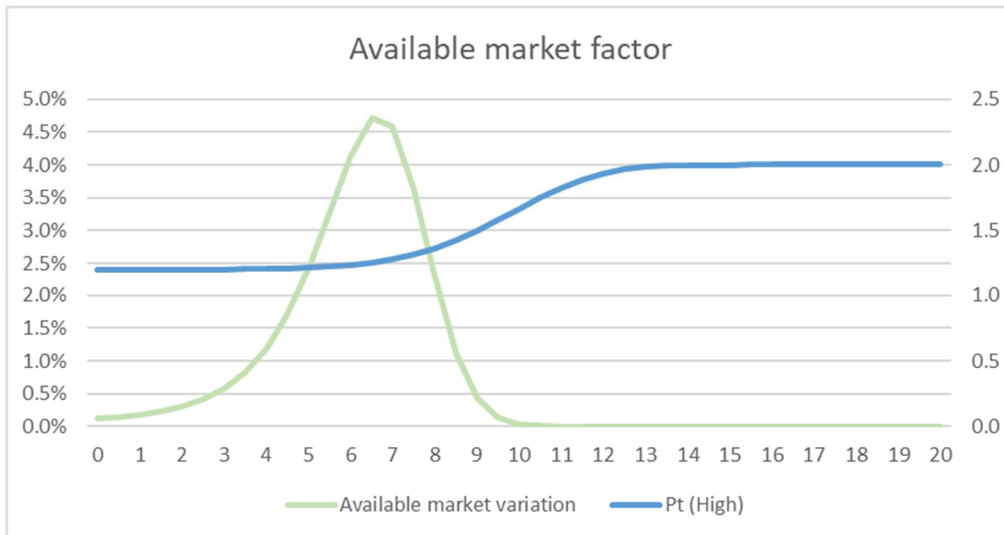
When the performance  $P_{jt}$  evolves in few periods from the starting point  $P_{j0}$  to the limit  $L_j$ , or in case  $P_{10\%}$  and  $P_{90\%}$  are close to each other, the available market variation is abrupt and more noticeable on the actual period sales. In the first case, this happens because the evolution of the new technology is quick and soon allows more customers to be interested in the adoption; in the second case, potential adopters are much more sensitive to the performance level, so that even little variations in  $P_t$  could have significant impact on the available market.

On the other hand, if the performance evolution S-curve is less steep and  $P_{10\%}$  and  $P_{90\%}$  are more distant, the available market increases smoothly and the light green curve seen previously in figure 3.4 has a lower peak, resulting in a less noticeable impact on the actual sales.

In figure 3.5 and 3.6, we can see the difference between choosing values for  $P_{10\%}$  and  $P_{90\%}$  that are relatively closer or further away from each other.



**Fig 3.5.** Available market factor when  $P_{10\%}=0.7$ ;  $P_{90\%}=1.4$ .



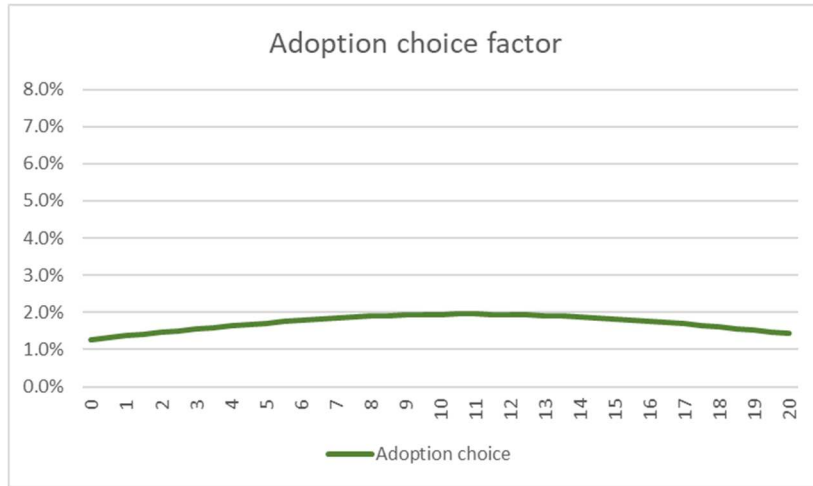
**Fig 3.6.** Available market factor when P10%=1; P90%=1.3.

Regarding the adoption choice factor, the shape of the curve in figure 3.3 is given by parameters  $p$  and  $q$  of each segment (as if we were following a standard Bass model), but also from the adoption pattern of preceding segments. The effect of more innovative segments on less innovative ones is regulated by the parameter  $w$ : the closer it is to 0, the more independent a segment is in its behavior; the closer it is to 1, the more a segment mimics the behavior of the preceding one. However, even if  $w=1$ , the curves of subsequent segments will not be the same, because  $w$  only affects the imitation behavior of a population.

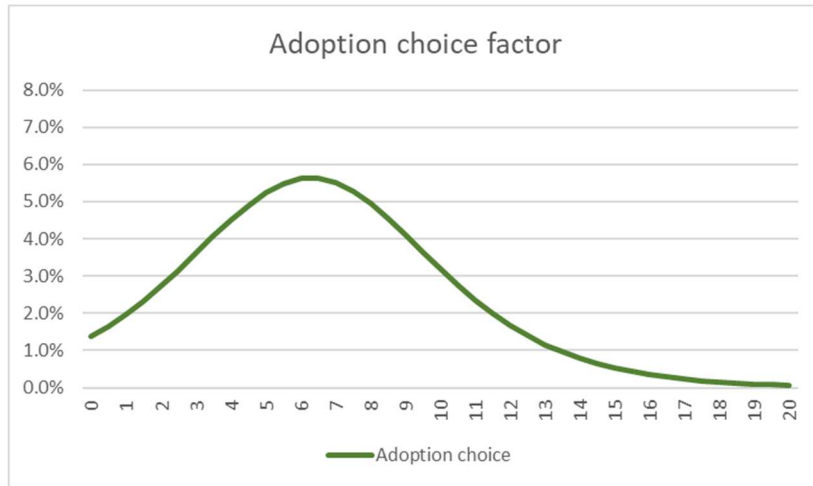
Generally speaking, parameter  $p$  determines how fast a population starts adopting new products and it has an impact on the position of the peak in the curve. On the other hand, parameter  $q$  also affects the speed of adoption, but it has a closer link to the steepness of the curve.

In figures 3.7 and 3.8, the impact of low versus high values for parameter  $q$  is shown, while keeping an average number for parameter  $p$ ; in figure 3.9 and figure 3.10, the same process is repeated for different values of  $p$ , while maintaining a constant  $q$ .

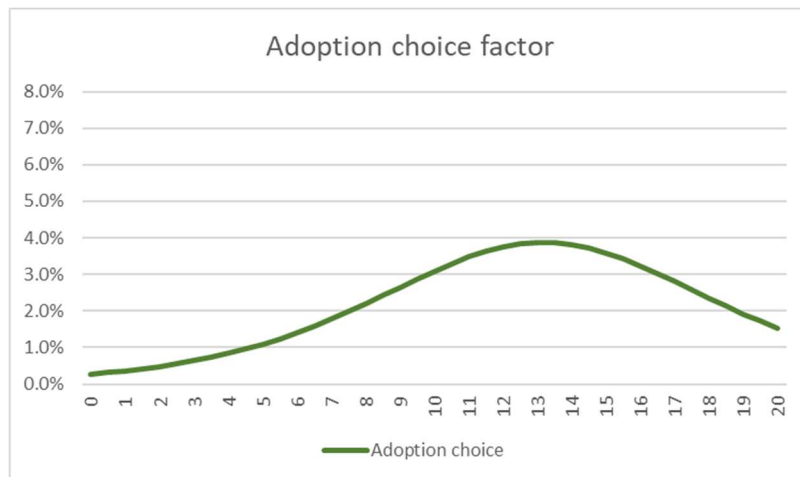




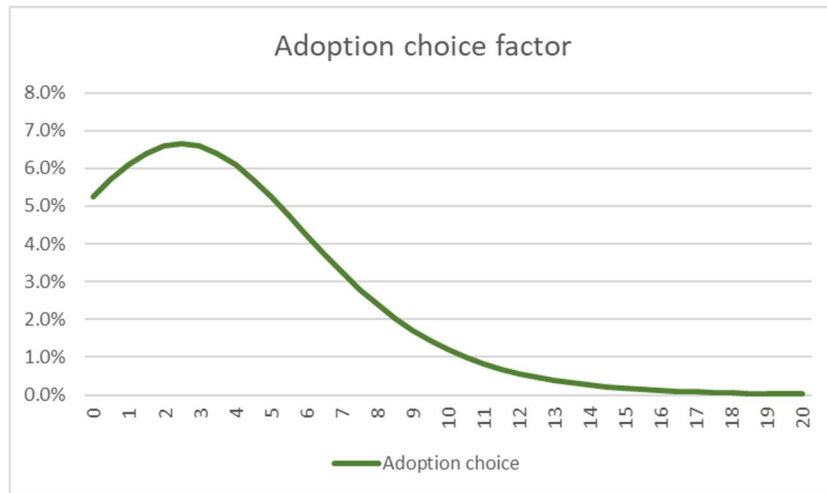
**Fig 3.7.** Adoption choice factor when  $p=0.025$ ;  $q=0.1$ .



**Fig 3.8.** Adoption choice factor when  $p=0.025$ ;  $q=0.4$ .

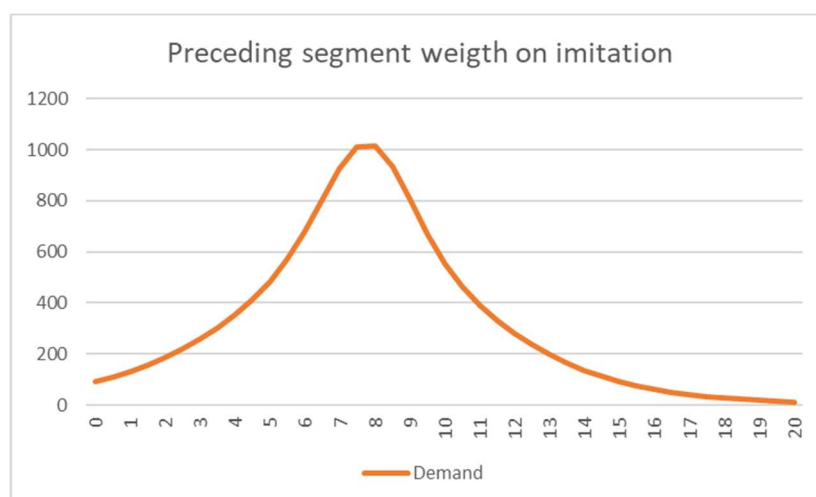


**Fig 3.8.** Adoption choice factor when  $p=0.025$ ;  $q=0.4$ .

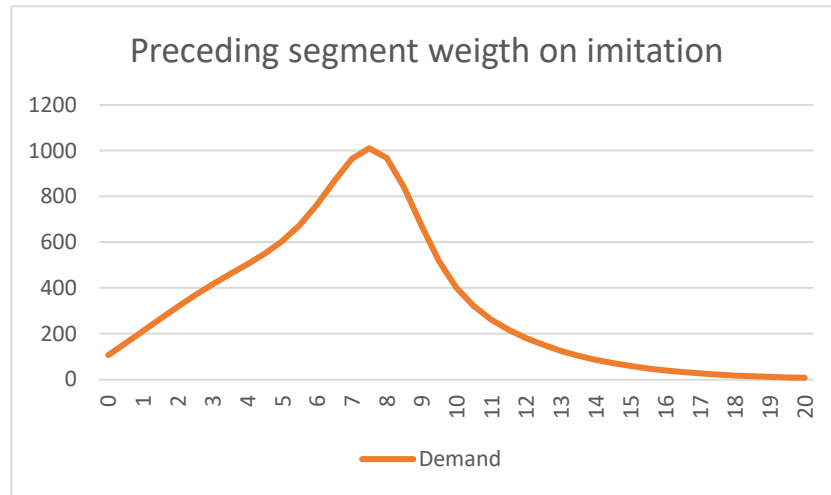


**Fig 3.10.** Adoption choice factor when  $p=0.1$ ;  $q=0.3$ .

Finally, in figures 3.11 and 3.12, the two extreme cases in which  $w=0$  and  $w=1$  are shown. In this example, a population of early adopters is influenced by innovators (they were previously taken as a model for figures 3.3 and 3.4). When  $w$  equals zero, early adopters only imitate customers in their own category; when  $w$  is equal to 1, imitation is only triggered by innovators, so sales increase sooner thanks to their quicker adoption rate. It is important to highlight that the curve in figure 3.12 is not the same as the one in figure 3.4, because parameters  $p$  and  $q$  still remain different.



**Fig 3.11.** Instantaneous sales curve when  $w=0$ .



**Fig 3.12.** Instantaneous sales curve when  $w=0$ .

## 4. Implementation of the new market model

In this section, we will analyze different case studies simulated with the new market model. In particular, we will take into consideration what happens:

- when innovations are introduced in from the top versus the bottom of a market;
- if the performances of the innovations grow faster in the high or low segments.

Therefore, combining these options we get four scenarios, in which:

- customer needs and product development are aligned (introduction from the top and performances fast growth in high segments / introduction from the bottom and performances fast growth in low segments);
- customer needs and product development are misaligned (introduction from the bottom and performances fast growth in high segments / introduction from the top and performances fast growth in low segments).

For all scenarios, the total market size  $M$  has been set at 1,000,000 customers, while values for variables  $q$ ,  $p$  and  $w$  are respectively shown in table 4.1, table 4.2 and table 4.3:

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	0.25	0.4	0.55	0.4	0.35
Medium end	0.2	0.35	0.45	0.35	0.25
Low end	0.15	0.25	0.4	0.25	0.2

**Table 4.1.** Values set for variable  $q$ , by segment.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	0.1	0.05	0.04	0.02	0.007
Medium end	0.08	0.03	0.02	0.01	0.005
Low end	0.06	0.02	0.01	0.005	0.002

**Table 4.2.** Values set for variable p, by segment.

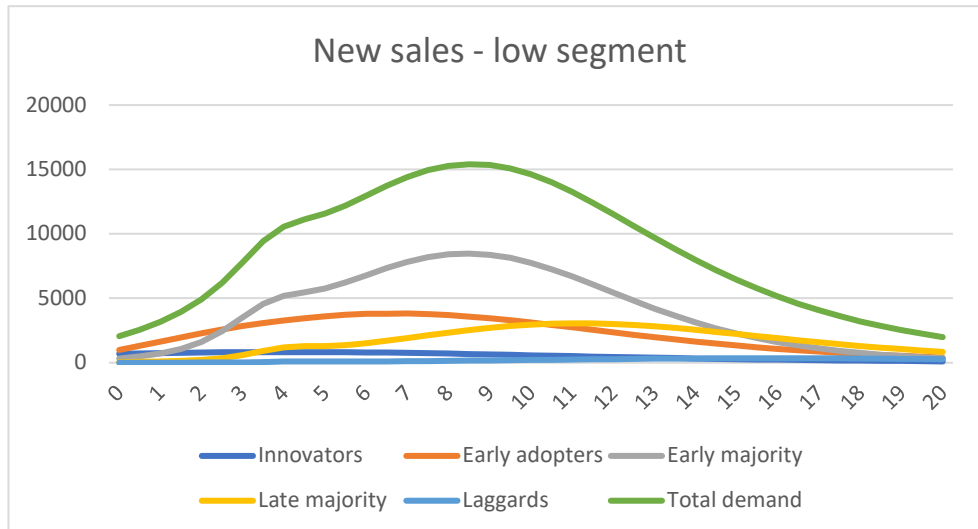
Early adopters	Early majority	Late majority	Laggards
0.7	0.2	0.4	0.3

**Table 4.3.** Values set for variable w, by segment.

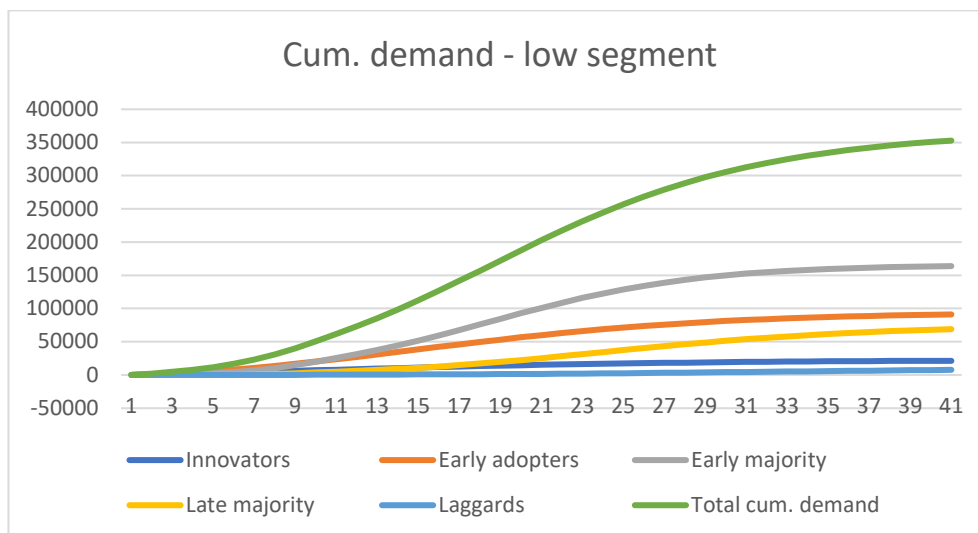
#### 4.1 Introduction from the bottom, performance evolution in low segments

In this case, the innovation is introduced preponderantly in the low segments of the market, where the vast majority of innovators and early adopters lie; the development of the new technology is also aligned with the customers' needs and demand.

Figures 4.1 and 4.2 show the new sales and the cumulated demand in the bottom vertical segment of the market, split by horizontal segment.



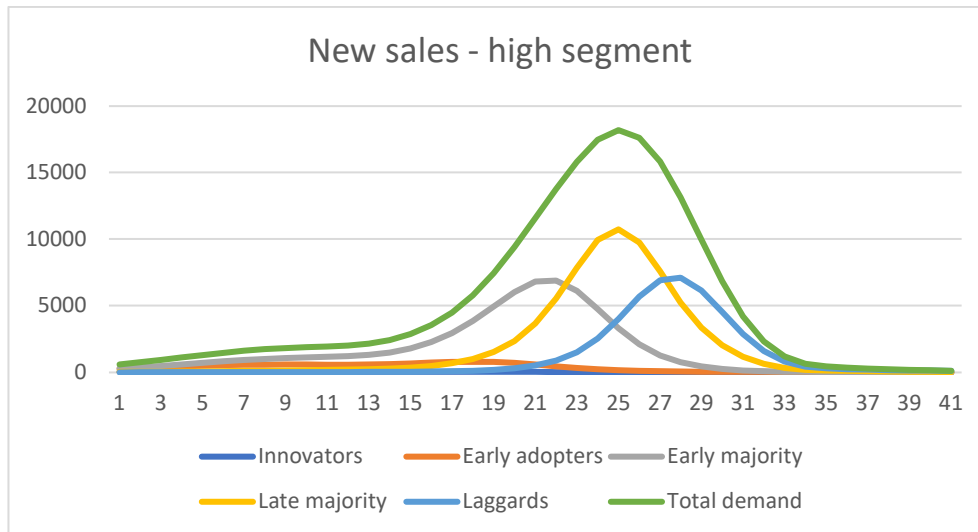
**Fig 4.1.** Instantaneous sales curve in low-end market.



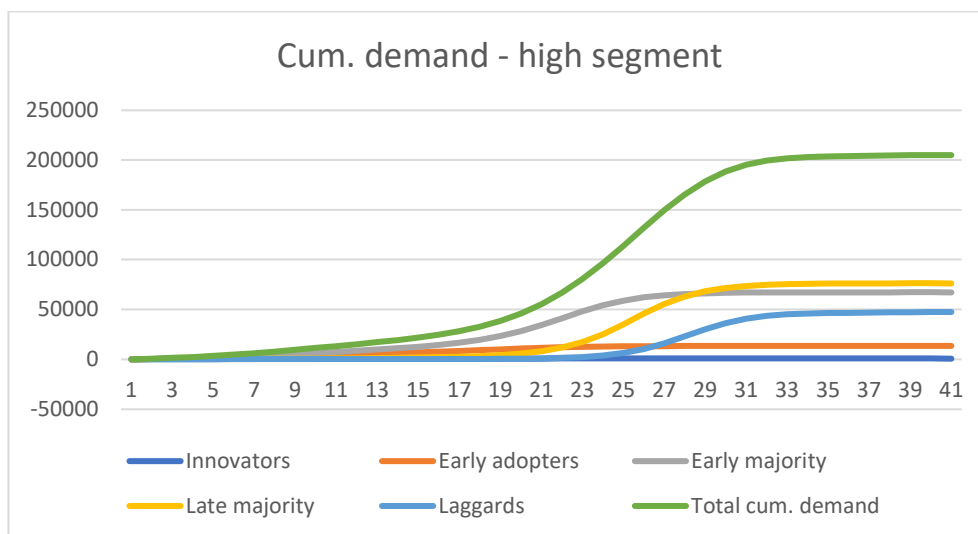
**Fig 4.2.** Cumulated sales curve in low-end market.

It is interesting to notice the bumps in new sales curves for early majority, late majority and laggards, which also reflect on the total demand of the low segment. These are due to an expansion of the available market that comes before the peak of the demand of the specific horizontal segments: the development of the new technology is quick, but less innovative customers are not ready to adopt the innovation yet. This effect is not found in the innovators and early adopters demand, because the performance evolution is in line with their needs and expectations.

The equivalent curves for the upper vertical segment are shown in figures 4.3 and 4.4.



**Fig 4.3.** Instantaneous sales curve in high-end market.

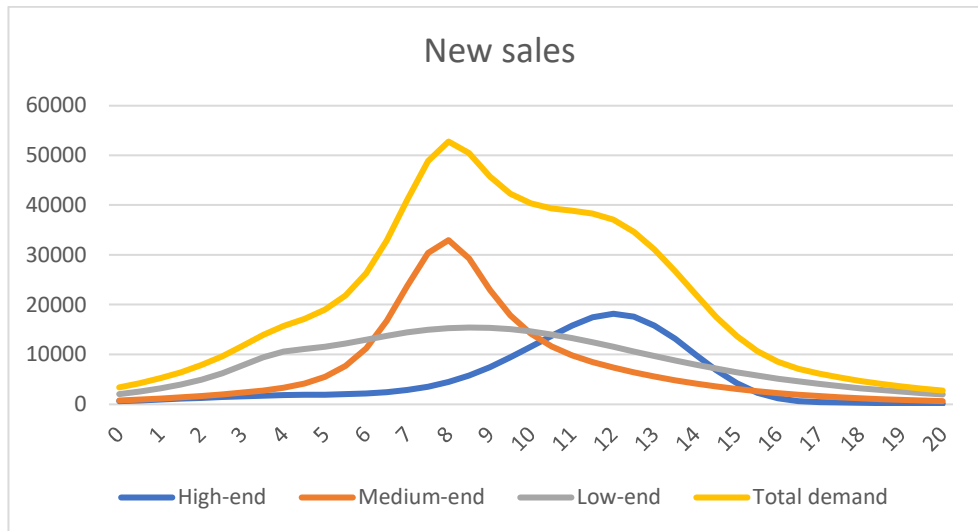


**Fig 4.4.** Instantaneous sales curve in high-end market.

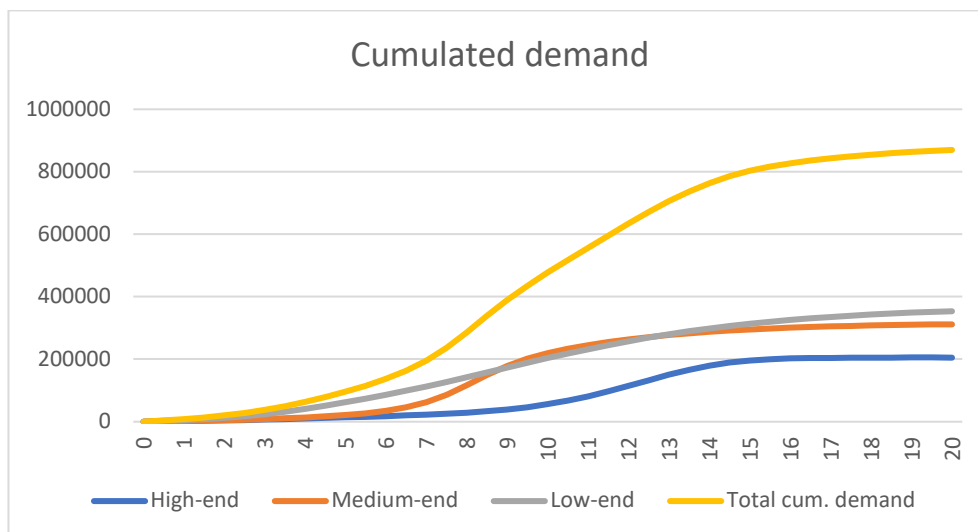
In this case, the situation is opposite to the low-end segment's one. We can see on the total demand curve in figure 4.3 that sales are ramping up right from the beginning, but then they remain stalling for some periods. This is because the product performance is not evolving soon enough, holding back the customers that would like to adopt the innovation; then, when the performance is approaching its upper limit, we can see that sales quickly increase again, as

potential adopters who had been waiting for technological improvements are finally able to adopt the innovation.

Finally, the total sales and cumulated demand are presented in figures 4.5 and 4.6.



**Fig 4.5.** Instantaneous sales curve in the aggregate market.



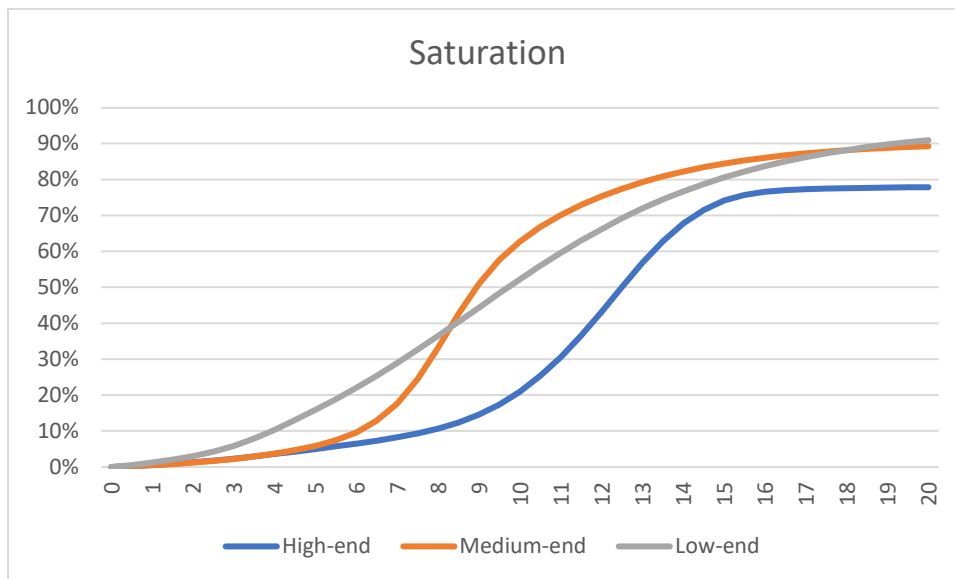
**Fig 4.6.** Cumulated sales curve in the aggregate market.

When looking at the aggregate market, the total demand is equal to the sum of the single segments' sales. The low-end market is the first one to expand, thanks to technological evolution and the beneficial influence of innovators and early adopters, but it never reaches high peaks: customers are less innovative, so their demand is much more spread out than in the other segments. For the medium-end adopters, technological evolution and customer innovativeness are



perfectly aligned, so that both the available market factor and the adoption choice factor peak around  $t=8$ . Finally, the high-end market cannot fully develop until the performance has improved, so that sales remain really low for a longer period than in lower segments.

Saturation (i.e. the ratio between adopters and the total segment's population) is another important parameter that needs to be analyzed (see figure 4.7).



**Fig 4.7.** Saturation levels, by vertical segment.

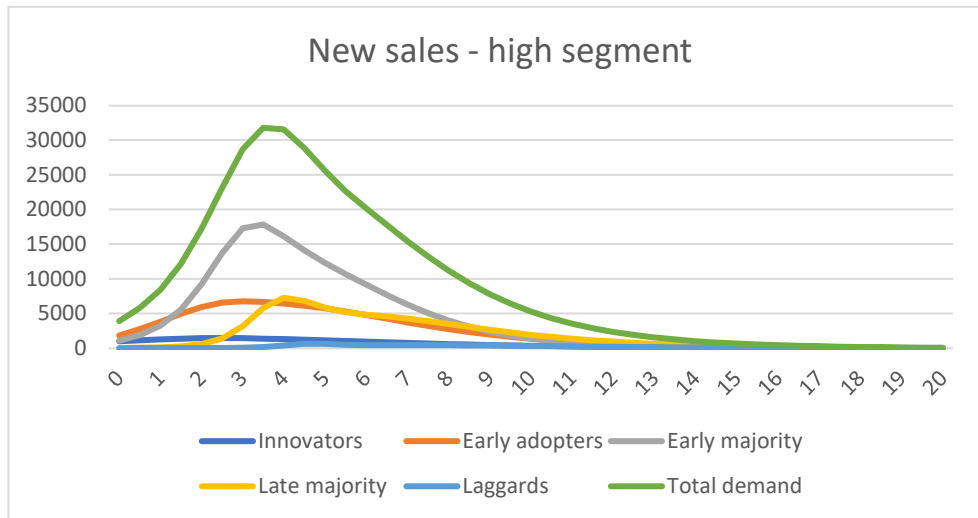
In this graph, the same considerations that were previously done are still holding. It is interesting to look at the difference in the evolution of saturation for low and medium markets: even though the final value is similar, the medium-end market has a much more recognizable S-shape due to aligned performances evolution and customer innovativeness, while the low-end sales are mostly driven by early developments of the new technology and the demand is more stable, spread-out.

## 4.2 Introduction from the top, performance evolution in high segments

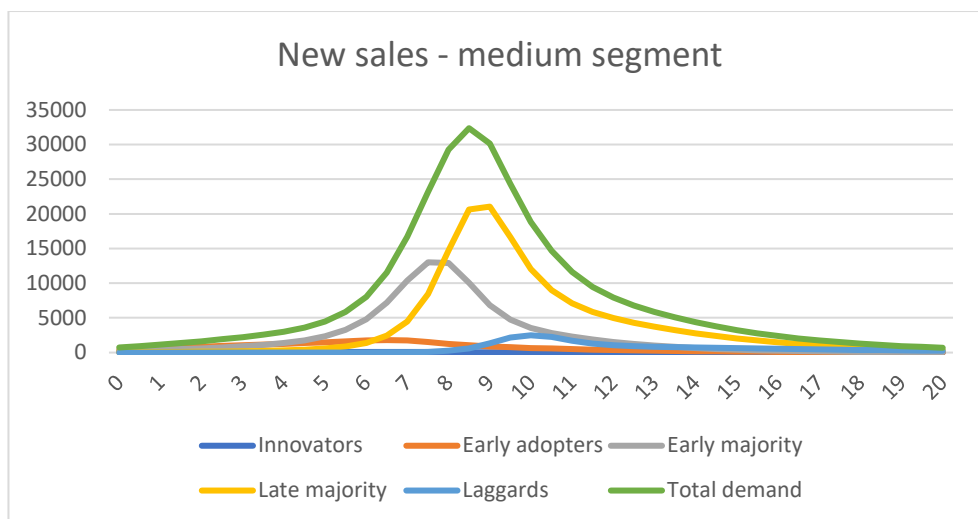
In this case, the innovation is introduced preponderantly in the high-end market, where customers are relatively more innovative than in other vertical segments.

The development of the new technology is aligned with the customers' needs and demand, too.

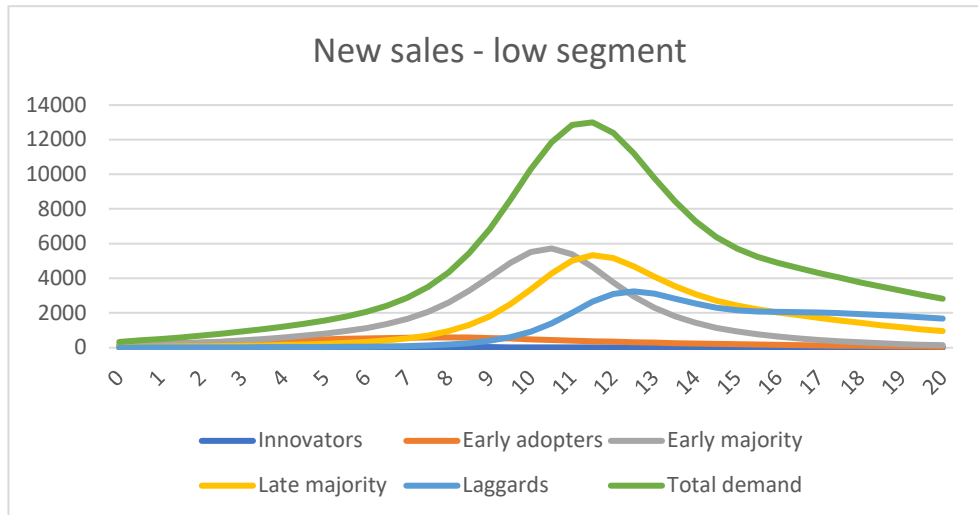
The new sales curves for each vertical segment of the market are presented in figures 4.8, 4.9 and 4.10.



**Fig 4.8.** Instantaneous sales curve in high-end market.



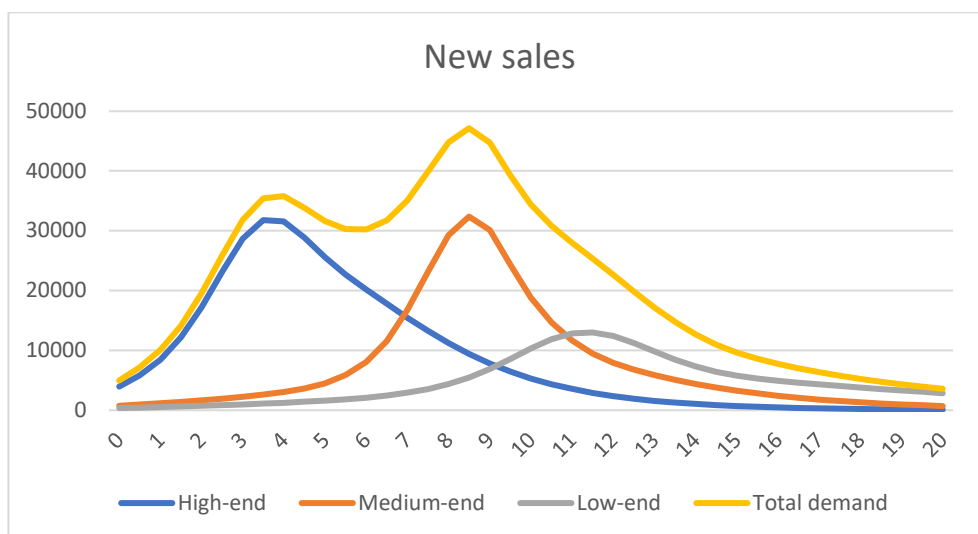
**Fig 4.9.** Instantaneous sales curve in medium-end market.



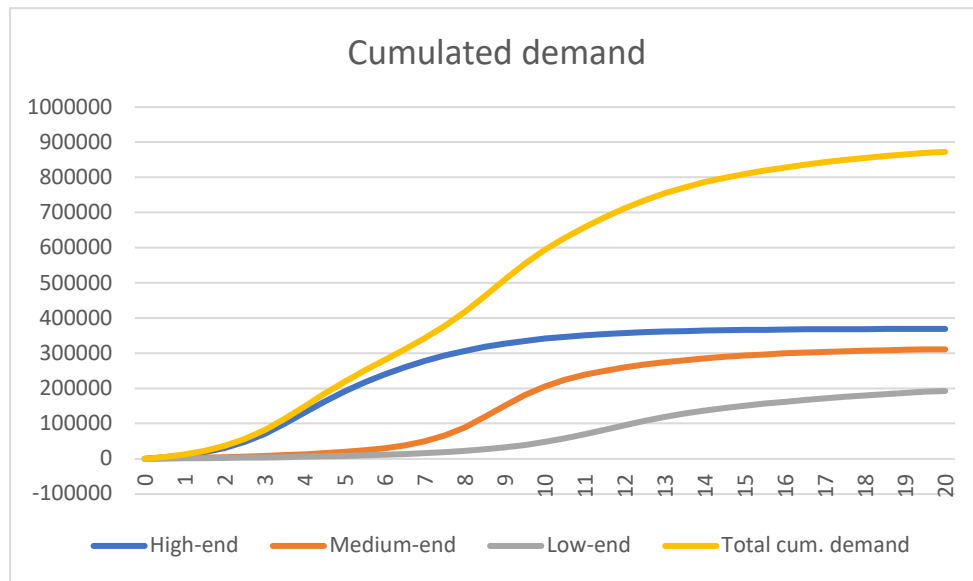
**Fig 4.10.** Instantaneous sales curve in low-end market.

In this case, the main difference from the first scenario is that technological advancements of the innovation and the customer innovativeness are even more aligned, so that expansions in the available market are synchronized with the increasing willingness to adopt in each single segment. This results in higher peaks of demand in every cluster of customers, which are varying for their timeliness.

The aggregate instantaneous demand and cumulated sales for the aggregate market are shown in figures 4.11 and 4.12.



**Fig 4.11.** Instantaneous sales curve in the aggregate market.



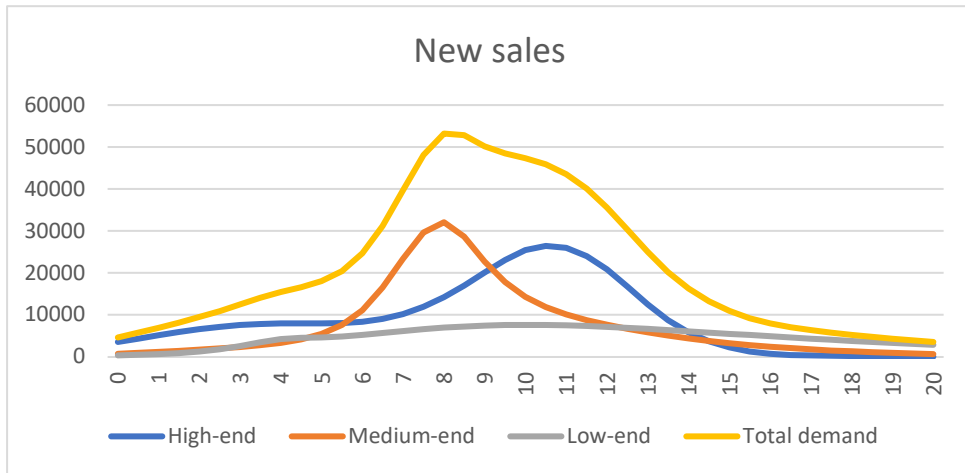
**Fig 4.12.** Cumulated sales curve in the aggregate market.

In figure 4.11, we can clearly see the shape of what Moore called “chasm” in his research. In this model, the effect is explained by a large amount of time passing between the peak of adoption for high-end markets versus the other segments. We can also observe that the demand in each vertical segment follows the typical S-shaped pattern, with sales ramping-up accordingly to the innovativeness of the customers and timeliness of product development.

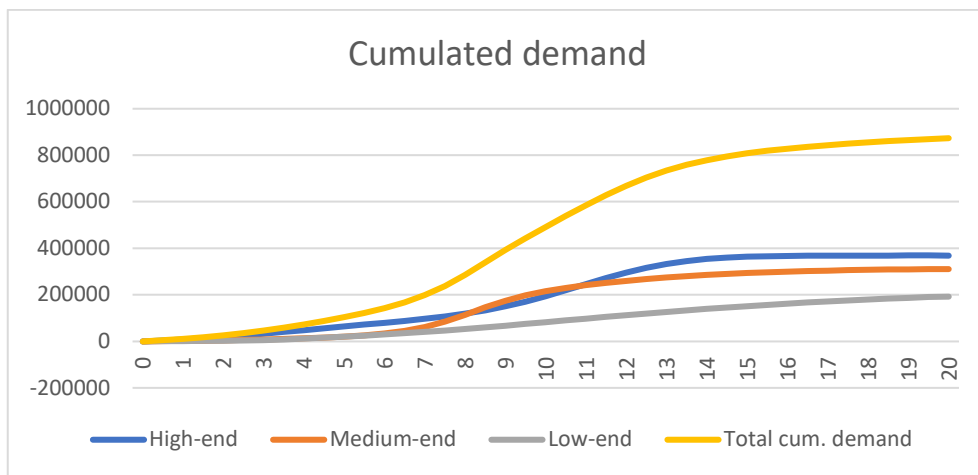
### 4.3 Misalignment between introduction and performance evolution

A misalignment between the market demand and the firms’ offer can occur when products are developed relatively more quickly in the opposite end compared to the launch segment, i.e. performance grows faster in the low-end market with an introduction from the top or it grows faster in the high-end market with an introduction from the bottom.

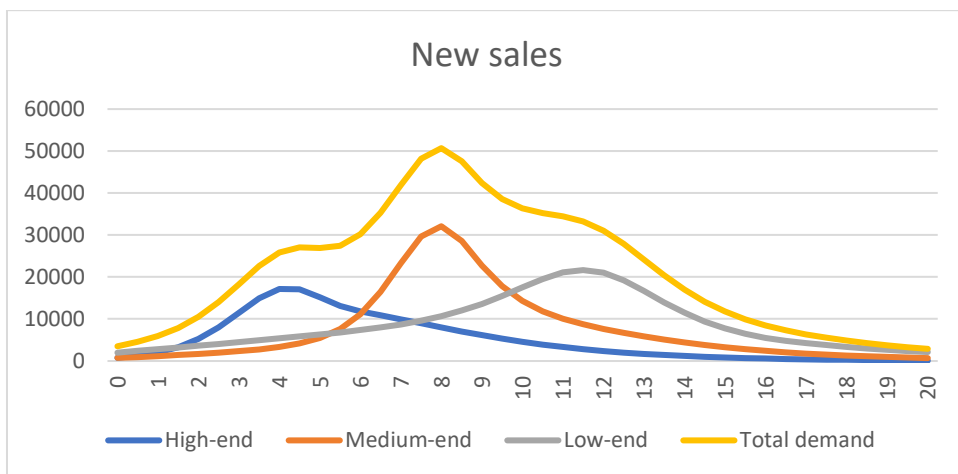
The total sales and cumulated demand for the introduction from the top, with quick evolution in the low-end segment, are presented in figures 4.13 and 4.14. Figures 4.15 and 4.16 show the corresponding graphs for the introduction from the bottom, with quick evolution in the high-end segment.



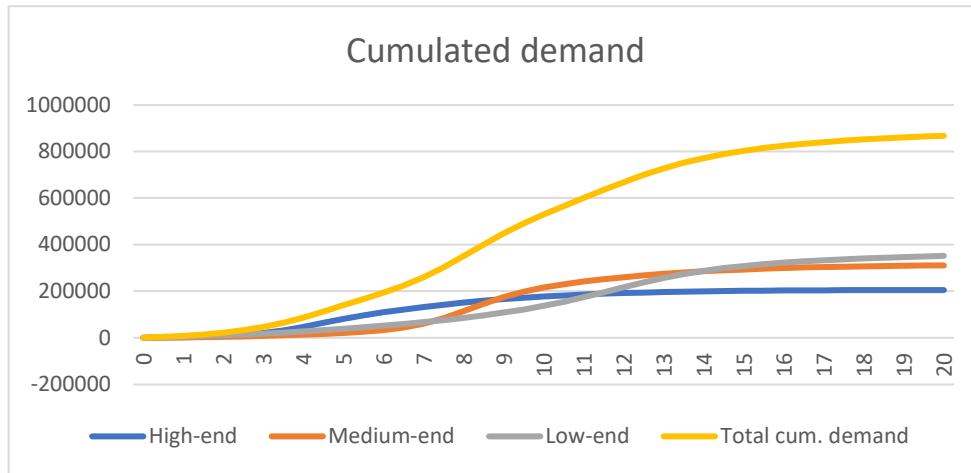
**Fig 4.13.** Instantaneous sales curve in the aggregate market. Introduction from the top, faster performance evolution in the low-end market.



**Fig 4.14.** Cumulated sales curve in the aggregate market. Introduction from the top, faster performance evolution in the low-end market.



**Fig 4.15.** Instantaneous sales curve in the aggregate market. Introduction from the bottom, faster performance evolution in the high-end market.



**Fig 4.16.** Cumulated sales curve in the aggregate market. Introduction from the bottom, faster performance evolution in the high-end market.

The key takeaway, in these scenarios, is that the cumulated demand curve is not too distant from the two cases analyzed in the previous sections. What happens at the micro-level, looking at the vertical segments, is dramatically different in each scenario: the peaks of sales always seem to follow the timing of strong performance improvement, when technologies are in the central part of their evolution S-curve; the segment of introduction, on the other hand, determines the entity of the peaks, that is the maximum period sales reached by the single vertical segments. Nevertheless, when looking at the macro diffusion curve, we always get a final value of around 870 thousand adopters in period  $t=20$ .

This could probably be explained by the fact that, on the long run, what really matters for the diffusion of innovations is the level of performance reached at the end of the technological evolution, not the relative innovativeness of vertical segments or the consequentiality of diffusion through subsequent horizontal segments.

Table 4.4 presents a final summary of the results obtained from the case studies with the new market model.

High-end Medium-end Low-end Total demand

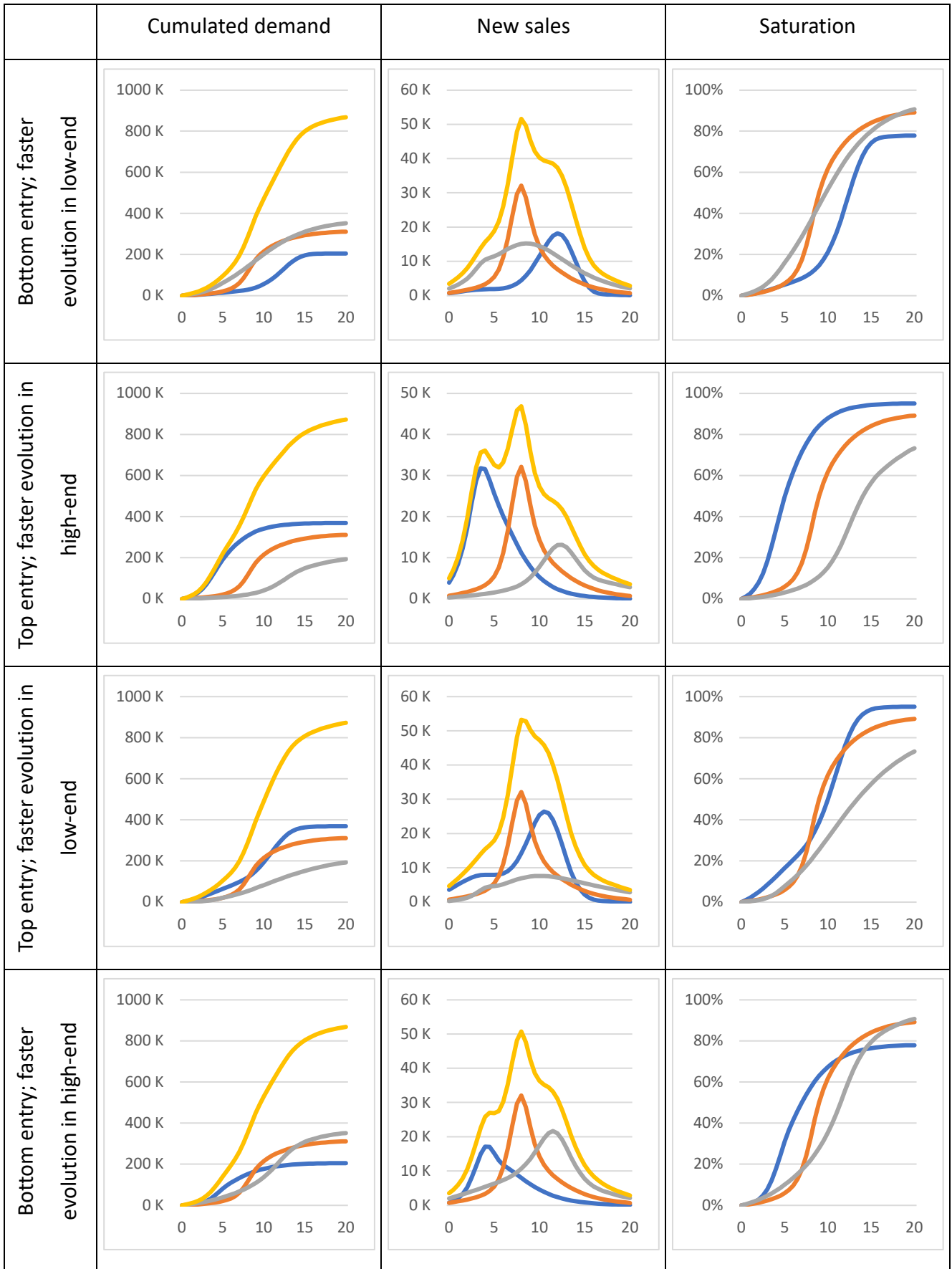


Table 4.4. Scenarios summary with graphs for instantaneous demand, cumulated demand and saturation.

## 5. Existing market model build-up

After the analysis of diffusion of innovations in newly created markets, we now consider the case of new technologies introduced in established markets, e.g. Battery Electric Vehicles in the automotive industry or the 5G network for smartphones.

This model is based on the hypothesis that new technologies enter in a mature market with a fixed installed base, gradually substituting the legacy technology as the level of performance keeps increasing. Therefore, we not only look at the new sales of the innovation in this case, but substitution sales are included too.

In each period, a fraction  $\tau$  of the installed base must be replaced and purchases are needed in order to substitute old products. The period sales will be split into new or legacy technology products, depending on their relative attractiveness to customers.

The horizontal segmentation is the same applied to the new market model, following Rogers' studies. However, in this case no asymmetrical influence in the adoption process is considered, so that each horizontal segment adopts independently from the others.

### 5.1 Vertical segmentation and performance attractiveness

As previously seen for the new market model, the population of potential adopters is split into three vertical segments that have independent adoption patterns and dedicated products introduced simultaneously: high-end, medium-end and low-end segments.

The level of performance of the innovation is not homogeneous in the whole market: in fact, the new technology is introduced in each vertical segment with different levels of performance. Every new product competes with the legacy technology of its own segment, which has a constant level of technical performance.



The performances of the innovation evolve over time following an S-shaped curve, starting with an initial value  $P_{j0}$  at  $t=0$  and approaching the asymptotic limit of the performance  $L_j$ . The evolution rates of the new products are determined by their sensitiveness towards investments,  $b_j$ . As in the previous new market model, investments are also exogenous in existing market model, and they follow a logistic distribution.

The evolution of performance is calculated as follows:

$$P_{jt} = \frac{L_j}{1 + a_j e^{-b_j C I_{jt}}}$$

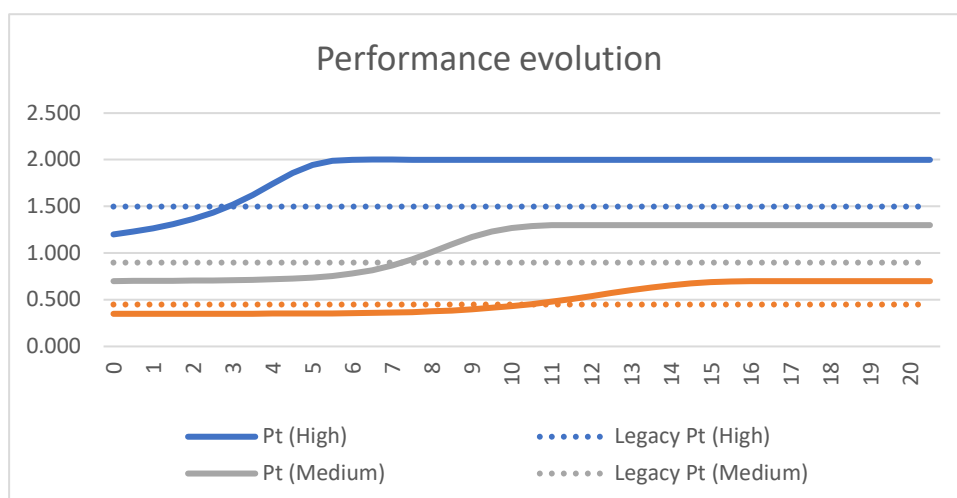
Where:

- $C I_{jt}$  is the cumulated investment in period  $t$
- $a_j = \frac{1}{P_{j0}/L_j} - 1$  is the position parameter of the logistic curve

While the investments are always the same, parameters  $P_{j0}$ ,  $L_j$  and  $b_j$  are different for each vertical segment and they determine the evolution pace of the new products. As previously seen, the innovation evolution, as well as the adoption rate, is independent for each cluster of potential adopters.

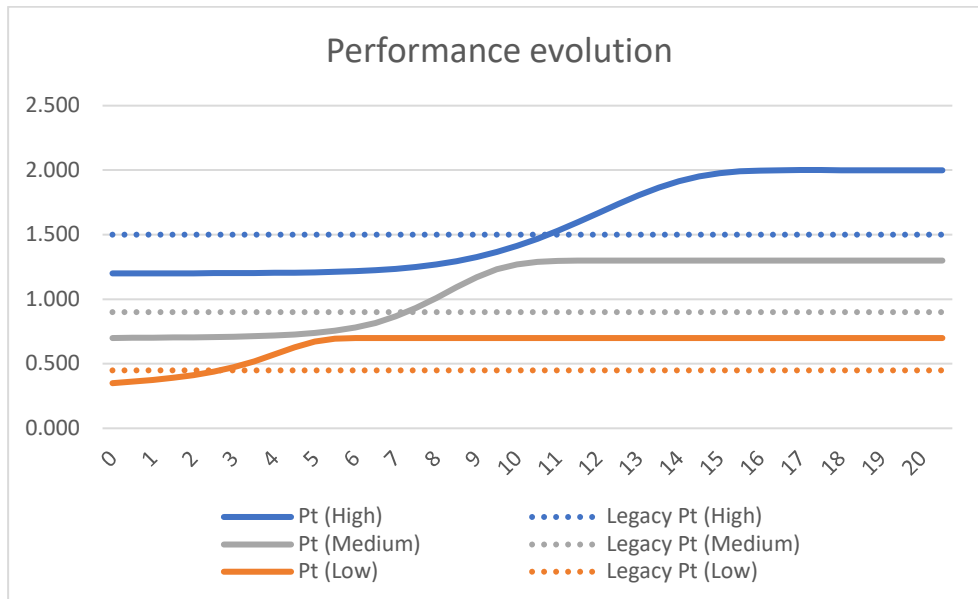
Depending on the values of  $b_j$ , i.e. the sensitivity of the performance evolution towards investments, two cases can be taken into consideration:

1. Performances evolve faster in the high-end market, with  $b_{\text{high}} > b_{\text{medium}} > b_{\text{low}}$



**Fig 5.1.** Performance evolution (faster in high-end market).

2. Performances evolve faster in the low-end market, with  $b_{\text{high}} < b_{\text{medium}} < b_{\text{low}}$



**Fig 5.2.** Performance evolution (faster in low-end market).

In all cases,  $P_{j0}$  is lower for new products than for the existing ones; then, with technical developments, the performances of the innovation improve, while the legacy technology has already reached its limit.

Compared to the new market model, the main difference regarding the vertical differentiation of the products is the introduction of the attractiveness of technologies, which not only depends on their level of performances, but also on their latest installed base.

The attractiveness of a technology, which determines its market share in the new sales, is linked to both technical performances and the current installed base through the following equation:

$$A_{jt} = P_{jt}^{\beta} * (\ln IB_{jt})^{\gamma}$$

Where:

- $P_{jt}$  is the level of performance in period  $t$  for the  $j^{\text{th}}$  technology
- $\beta$  is technical performance factor
- $IB_{jt}$  is the installed base in period  $t$  for the  $j^{\text{th}}$  technology
- $\gamma$  is the installed base factor

In every segment, an initial value above zero of the new technology's installed base was needed as a seed, because of the presence of a logarithm; therefore,  $IB_{j0}$  has been set at 0.2% of each segment's size.

Parameter  $\beta$  regulates the customers' response to technical performance, while parameter  $\gamma$  represents the importance given from customers the network, i.e. the current installed base of the technology.

Finally, the market share of new sales is computed as follows:

$$S_{jt} = \frac{A_{jt}}{\sum_j A_{jt}}$$

Compared to the original Bass model, this model also depends on the level of performance of the innovation. Therefore, not only the technical factor is crucial to vertically cluster the adopters, but it is also determining the speed of the diffusion of the innovation.

However, in the new market model, customers were directly influenced by the performance  $P_{jt}$  in their willingness to adopt the technology, with the network's influence modelled through the variables  $p$  and  $q$ . In this case, customers need to change the existing products periodically, whether more or less frequently depending on the segment, and they split their purchases proportionally to the attractiveness of the technologies, which is only partly impacted by the level of performances. The other factors in the computation of the technologies' attractiveness are  $IB_{jt}$  and  $\gamma$ , which are the variables that capture the network's influence in this model.

As in the previous model, customer innovativeness increases in upper segments. The substitution rate is slightly higher in the top-end market and significantly higher for more innovative segments, as these kinds of customers tend to replace their old products more frequently.

Parameter  $\beta$ , which regulates the potential adopters' sensitivity to the technical characteristics, increases in upper and more innovative segments, as performance levels and quality gain relevance.

On the other hand, parameter  $\gamma$  is higher in less innovative customers, as they are more reluctant to embrace new technologies in order to replace the existing ones.

## 5.2 Resulting matrix segmentation

Also in this case, combining the vertical and horizontal segmentations of the market, we obtain a matrix in which potential adopters are defined:

- by their level of innovativeness on the columns, following Rogers categories;
- by their preference for high, medium or low-end products.

The size of each horizontal segment is fixed, following Rogers' theory: over the total potential adopters, 2.5% are innovators, 13.5% early adopters, 34% early majority, 34% late majority, 16% laggards. Then, inside each horizontal segment, values are chosen to model how the group is split into high, medium or low-end market, so that the sum of each column gives 100%. Two extreme cases are modelled:

1. Entry of a new technology from the top (table 5.1): innovators and early adopters are preponderantly adopting high-end products, while the low-end market is mainly explored by the following groups;
2. Entry of a new technology from the bottom, à la Christensen (table 5.2): innovative segments purchase low-end products, with upper segments choosing to adopt later on.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	90%	70%	50%	25%	10%
Medium end	7%	20%	30%	50%	30%
Low end	3%	10%	20%	25%	60%

**Table 5.1.** Split of horizontal segments between high, medium or low-end customers. Entry from the top.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	3%	10%	20%	25%	60%
Medium end	7%	20%	30%	50%	30%
Low end	90%	70%	50%	25%	10%

**Table 5.2.** Split of horizontal segments between high, medium or low-end customers. Entry from the top.

Then, every percentage is multiplied by the size of its horizontal segment. The result is a matrix describing the relative size of each segment over the total market, so that the sum of the whole table is 100%. Table 5.3 refers to the entry in the market from the top, while table 5.4 shows to the entry from the bottom:

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	2.3%	9.5%	17.0%	8.5%	1.6%
Medium end	0.2%	2.7%	10.2%	17.0%	4.8%
Low end	0.1%	1.4%	6.8%	8.5%	9.6%

**Table 5.3.** Relative size of each segment over the total market. Entry from the top.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	0.1%	1.4%	6.8%	8.5%	9.6%
Medium end	0.2%	2.7%	10.2%	17.0%	4.8%
Low end	2.3%	9.5%	17.0%	8.5%	1.6%

**Table 5.4.** Relative size of each segment over the total market. Entry from the bottom.

In order to be able to compare results in the two scenarios (new vs. existing market), the same values have been chosen for the introduction of a new technology both from the top and from the bottom segments of the market.

### 5.3 Parameters impact on S-curves

The adoption rate in each segment is regulated by:

- The relative weight of parameters  $\beta$  and  $\gamma$ , i.e. the importance attached to performance and network by the customers;
- The timing in which the new technology is becoming technically better than the legacy, determined by the performance evolution S-curve;
- The substitution rate  $\tau$ .

If the customers' sensitivity to the installed base  $\gamma$  is significantly higher than the sensitivity to the performance  $\beta$ , the new technology's attractiveness takes more time to surpass the legacy's one, because the broader installed base is more important than a higher level of performance.

Vice versa, if  $\beta$  is significantly higher than  $\gamma$ , the market share of new sales shifts quickly from the legacy technology in favor of the new one, as soon as the latter becomes technically better.

In figure 5.3, the market share split of new sales is shown when  $\beta$  and  $\gamma$  assume equal values. Then, the same graph is repeated firstly with a high imbalance in favor of the technical performance (figure 5.4), and secondly with a high imbalance in favor of the installed base (figure 5.5).

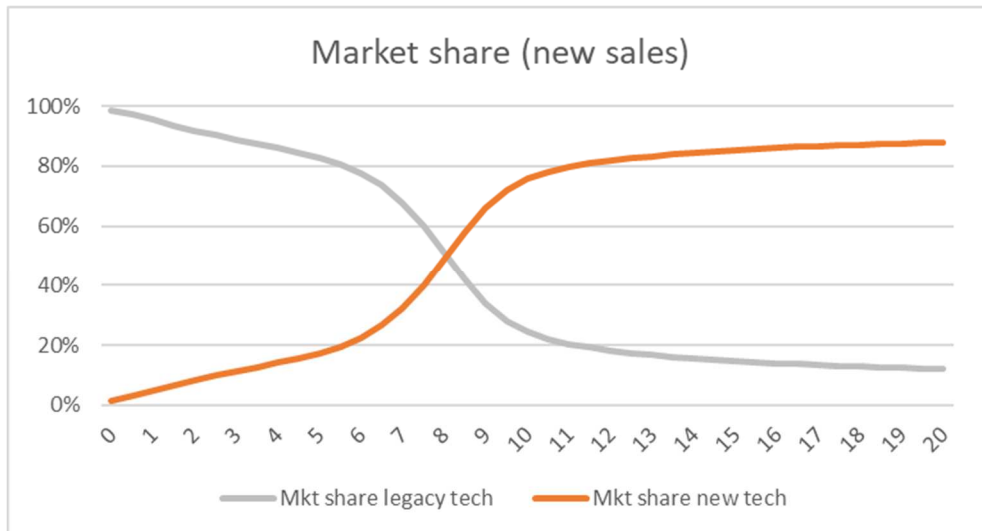


Fig. 5.3. Market share evolution when  $\beta=3.5$  and  $\gamma=3.5$ .

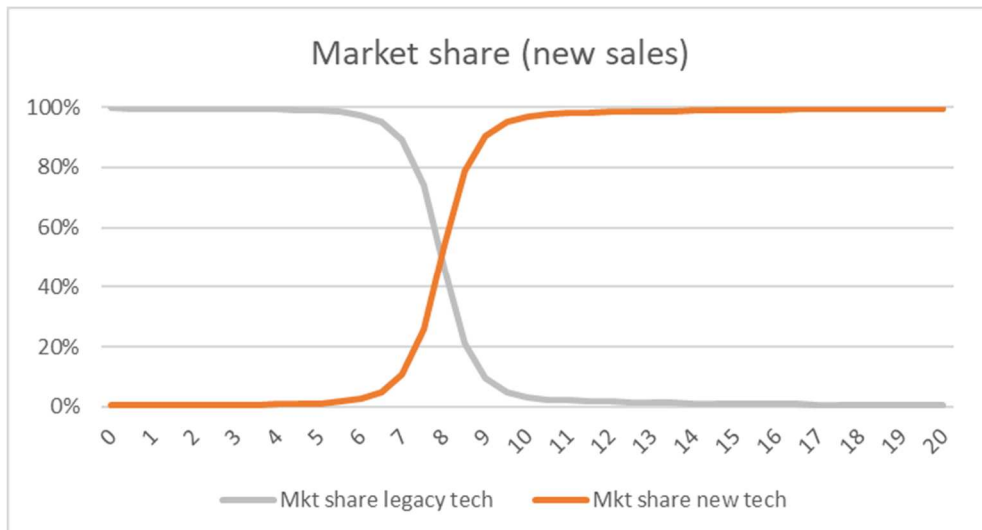


Fig. 5.4. Market share evolution when  $\beta=10$  and  $\gamma=3.5$ .

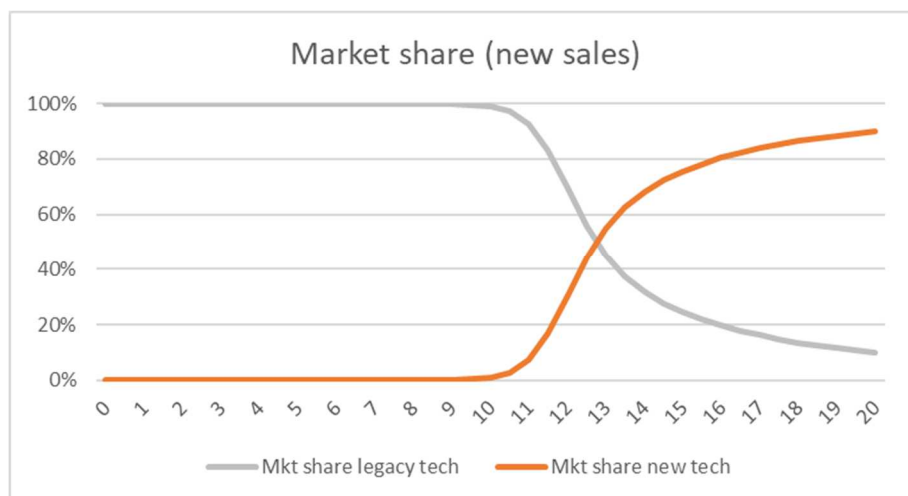


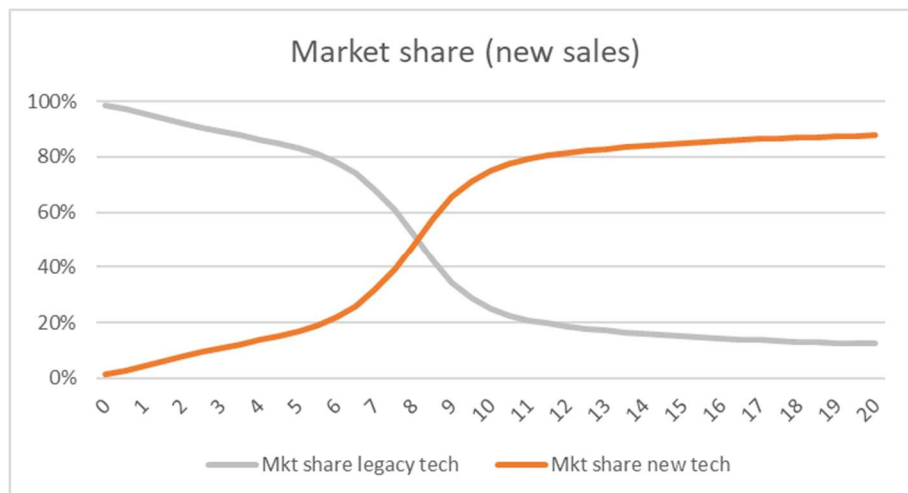
Fig. 5.5. Market share evolution when  $\beta=3.5$  and  $\gamma=6$ .

Finally, we look at the impact of the substitution rate variable  $\tau$ . When the substitution rate  $\tau$  is high, the installed base is replaced more quickly; therefore, the impact of the network, regulated by the variable  $\gamma$ , is lower on sales.

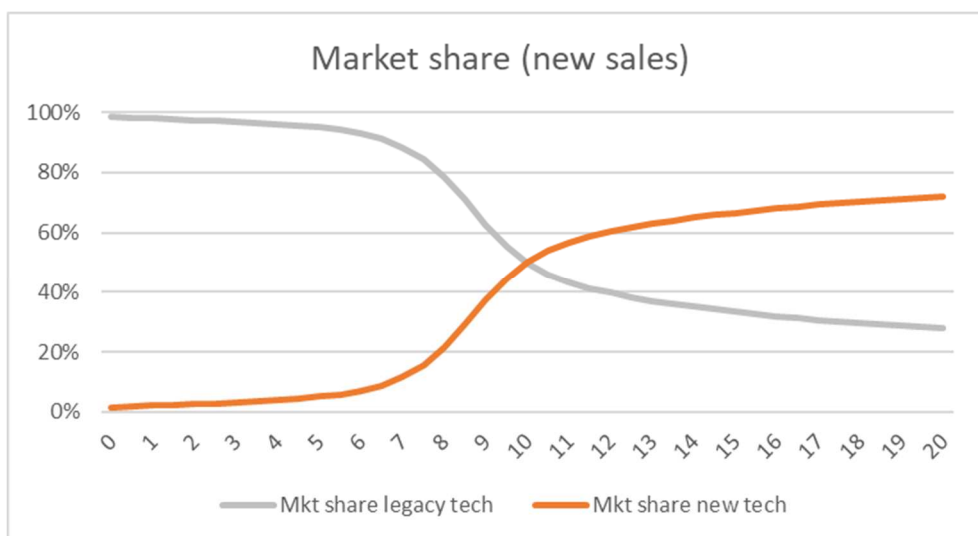
Intuitively,  $\tau$  becomes very low for high value assets and increases with more affordable items (for example,  $\tau$  is around 10% for cars and 40% for smartphones); if  $\tau$  is high, then the products must not be too expensive, so that customers become less risk-averse, and the network effect has less impact.

On the other hand, when  $\tau$  is low, the installed base remains overwhelmingly in favor of the legacy technology, which in turn determines higher attractiveness for the existing technology compared to the new one.

Figure 5.6 represents the market share split of new sales between legacy and new technology when  $\tau=35\%$ ; figure 5.7 shows the same graph for  $\tau=5\%$ .



**Fig. 5.6.** Market share evolution when  $\tau=35\%$ .



**Fig. 5.7.** Market share evolution when  $\tau=5\%$ .



## 6. Implementation of the existing market model

In this section, we will analyze different case studies simulated with the existing market model.

Again, we will consider what happens:

- when innovations are introduced in from the top versus the bottom of a market;
- if the performance of the innovation grows faster in the high or low segments.

Therefore, combining these options we get four scenarios, in which:

- customer needs and product development are aligned (introduction from the top and performances fast growth in high segments / introduction from the bottom and performances fast growth in low segments);
- customer needs and product development are misaligned (introduction from the bottom and performances fast growth in high segments / introduction from the top and performances fast growth in low segments).

For all scenarios, the total market size  $M$  has been set at 1,000,000 customers, while values for variables  $\beta$ ,  $\gamma$  and  $\tau$  are respectively shown in table 14, table 15 and table 16.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	6	4	3	2.5	2
Medium end	5.5	3.5	2.3	2.2	1.7
Low end	5	3	2	2	1.5

**Table 6.1.** Values set for variable  $\beta$ , by segment.

	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	1.7	3	4	4.5	5
Medium end	2	3.5	4.5	5	5.5
Low end	2.5	4	5	5.5	6

**Table 6.2.** Values set for variable  $\gamma$ , by segment.

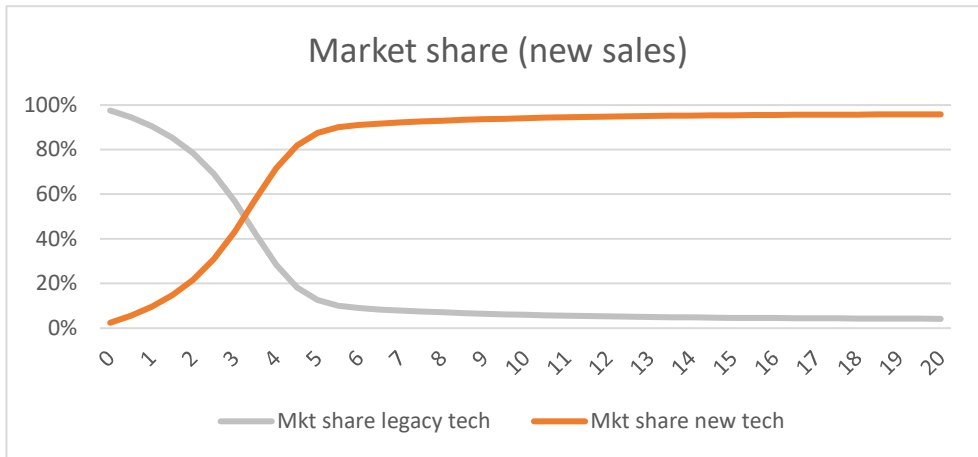
	Innovators	Early adopters	Early majority	Late majority	Laggards
High end	50%	40%	35%	30%	20%
Medium end	45%	37%	32%	27%	15%
Low end	40%	35%	30%	25%	10%

**Table 6.3.** Values set for variable  $\tau$ , by segment.

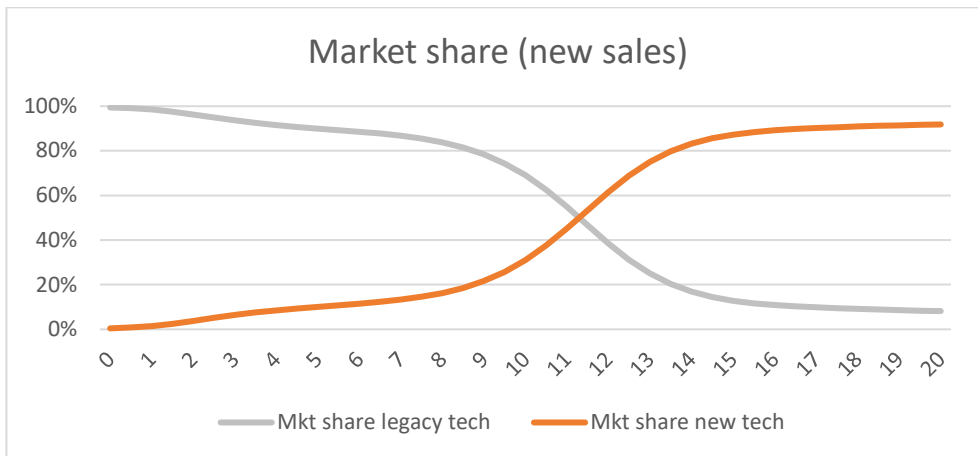
### 6.1 Introduction from the bottom, performance evolution in low segments

Once again, we start our analysis looking at innovations introduced at the bottom of the market; therefore, the vast majority of innovators and early adopters will reside in the low-end segment. In this case, the development of the new technology is also aligned with the customers' needs and expectations.

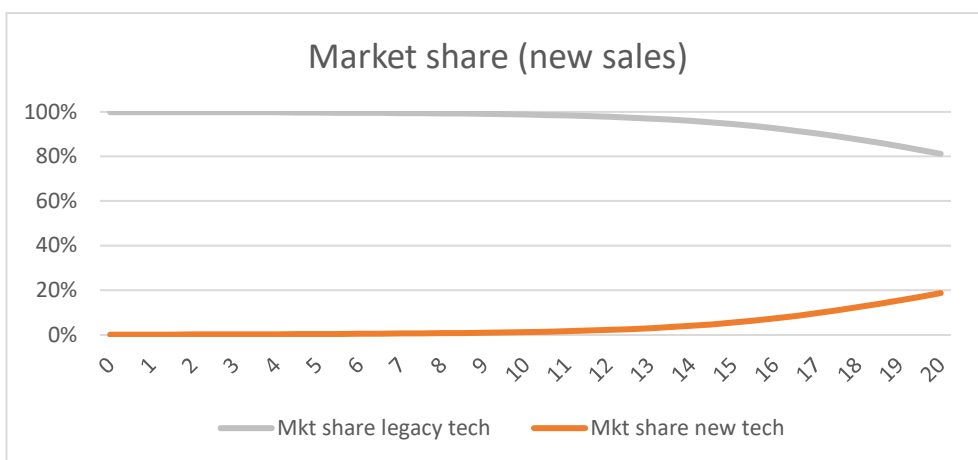
Firstly, the market share split between legacy and new technology sales is presented for low and high-end innovators (figures 6.1 and 6.2); for low and high-end laggards (figures 6.3 and 6.4).



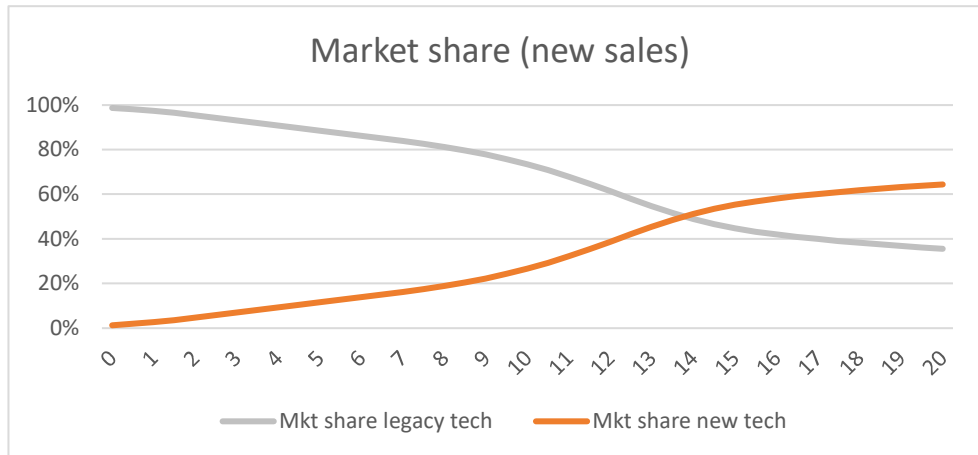
**Fig. 6.1.** Market share split in the low-end innovators segment.



**Fig. 6.2.** Market share split in the high-end innovators segment.



**Fig. 6.3.** Market share split in the low-end laggards segment.

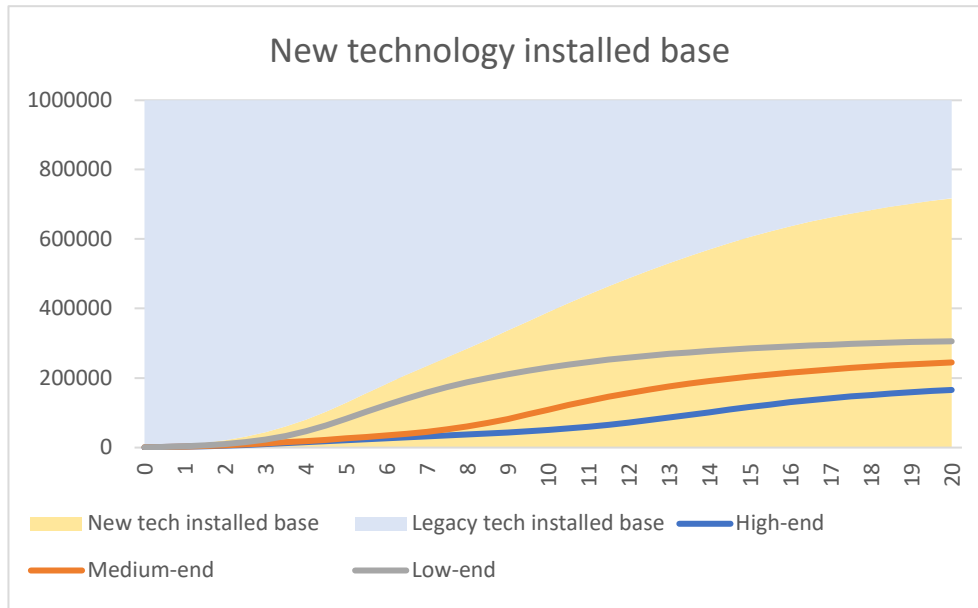


**Fig. 6.4.** Market share split in the high-end laggards segment.

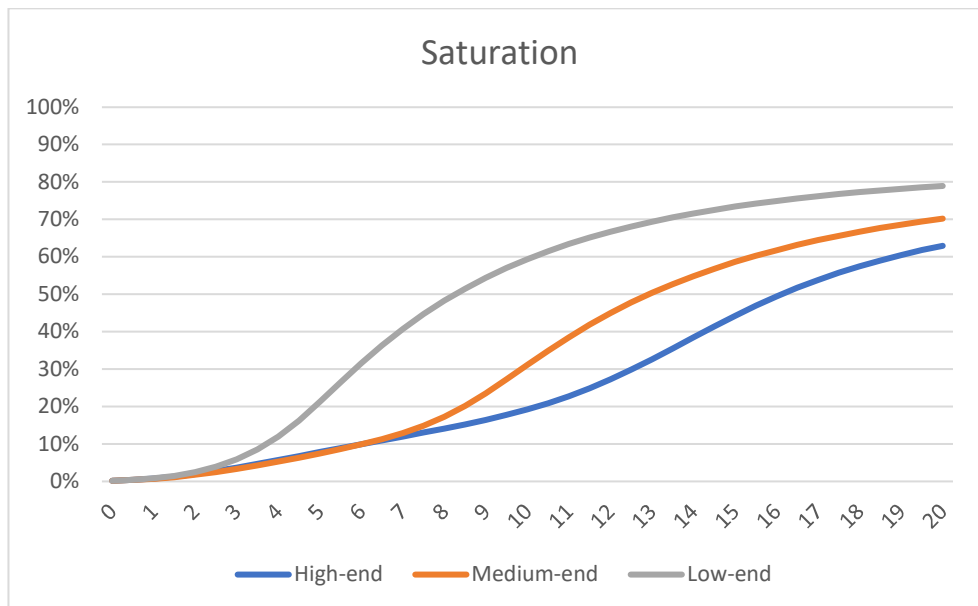
In the first two graphs, we can see populations of adopters who are heavily influenced by the technical developments of the innovation. In both cases, the attractiveness of the new technology becomes higher than the legacy's as soon as the performance becomes better, reflecting then on the sales curve. For high-end innovators, the technical factor is stronger, but the new technology only takes over the existing one towards the final periods of the simulation.

When looking at laggards, the network is much more important, and the potential adopters are very risk adverse. The consequence is that, even if the performances of the new technology have become better than the legacy's, the existing products are still more attractive to this kind of customers. It takes a lot of time for the innovation to slowly penetrate the market. It is interesting to notice that, even if the low-end products' performances develop faster, the market share in that segment never reach the 50% threshold, unlike the top-end laggards.

Now we look into the aggregate market: figure 6.5 shows the penetration of the new technology in the overall installed base; figure 6.6 presents the saturation of each vertical segment.



**Fig. 6.5.** Installed base share of legacy vs. new technology (split by segment).



**Fig. 6.6.** Saturation levels, by segment.

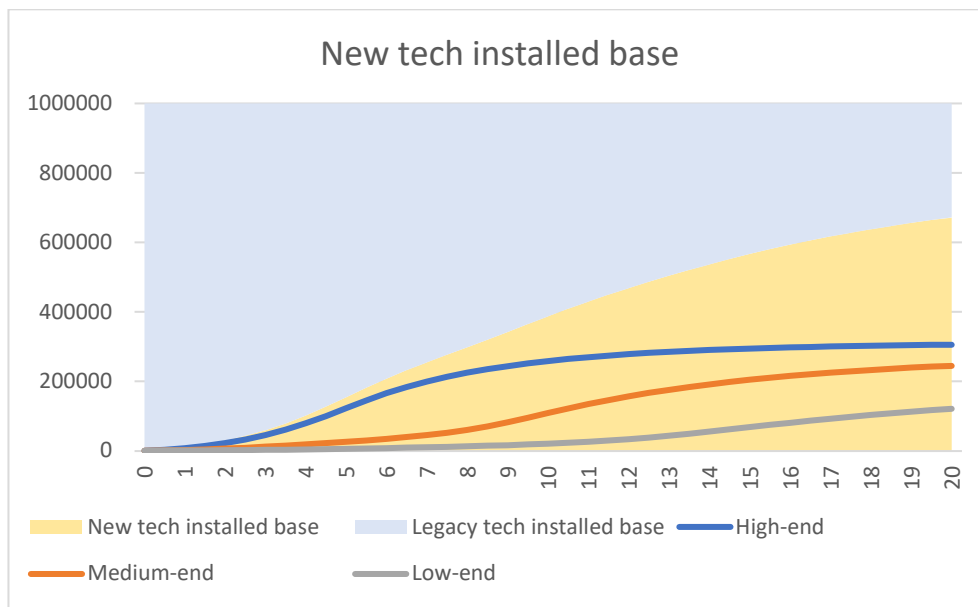
Analyzing the graphs, we can see a less turbulent situation compared to the equivalent case in the new market model. The demand curve is smoother and the saturation curves have a less pronounced S-shape. Moreover, the saturation levels at the end of the simulation are lower. Taking all of this into account, we can assume that the penetration in an existing market is generally slower than in a new one, because of the “stickiness” of the legacy installed base. Even if the

market share is highly in favor of the innovation, it takes several periods to replace all of the old technology, which is especially true when the replacement rate drops near 0%.

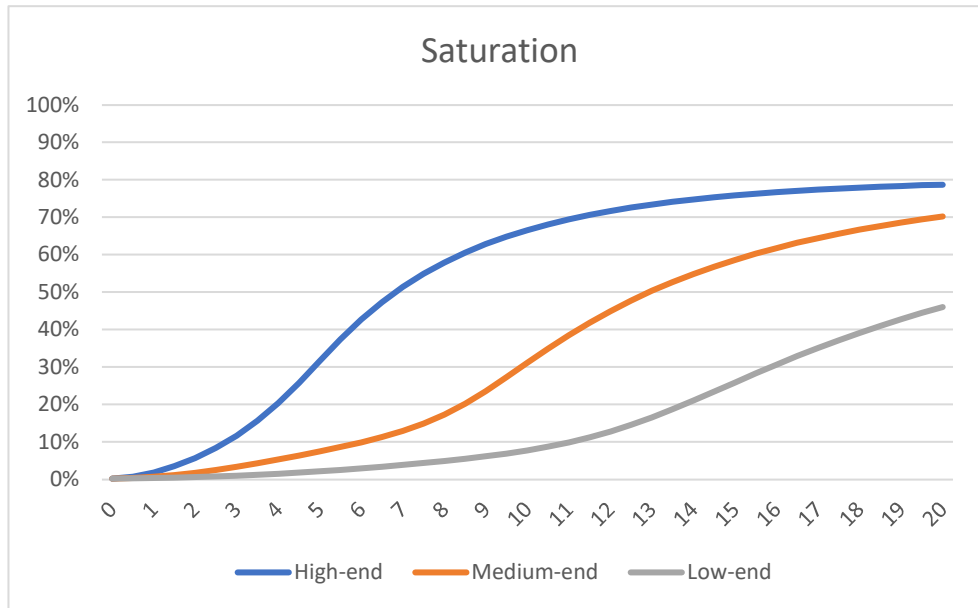
## 6.2 Introduction from the top, performance evolution in high segments

In this scenario, the innovation is introduced in the top-end market, where customers are relatively more innovative and sensitive to quality than in other vertical segments. The development of the new technology is quicker for the upper segments, aligned with the customers' needs and demand.

Figures 6.7 and 6.8 present the penetration of the new technology in the overall installed base and the saturation of each vertical segment.



**Fig. 6.7.** Installed base share of legacy vs. new technology (split by segment).



**Fig. 6.8.** Saturation levels, by segment.

This time, the differences between the first and the second scenario are minimal. The diffusion pattern is very similar, with the top-end customers replacing the low-level ones as the first adopters of the innovation.

From the saturation graph, we can evince a little difference between the segments in which the innovation is introduced: this time, the high-end customers are relatively quicker in their adoption, as shown by the higher steepness of the curve in the early periods.

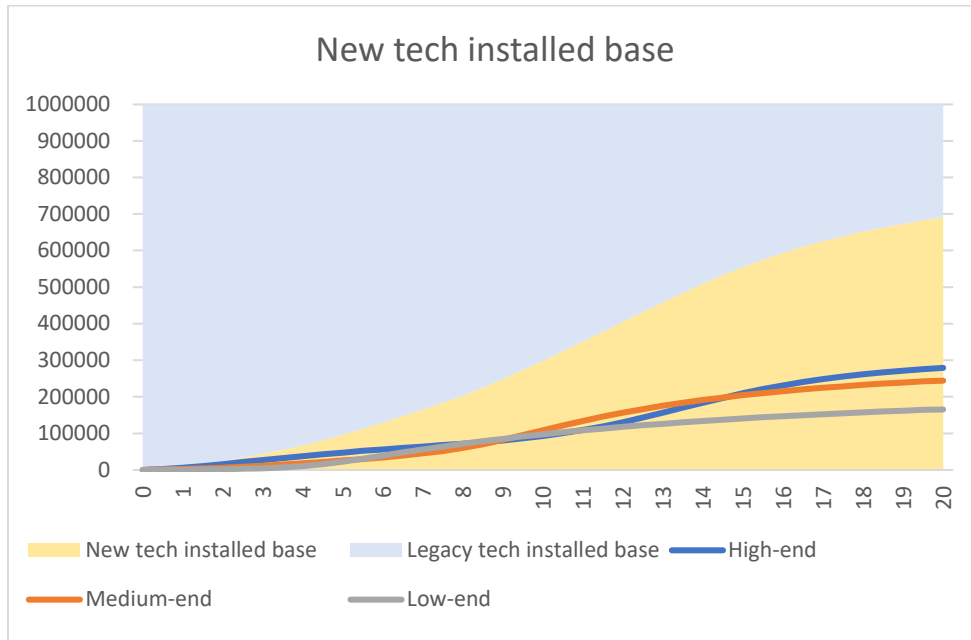
The opposite is true for the latest adopters, with the low-end customers now taking more time than the upper segments in the previous scenario, not even reaching the 50% market penetration at the end of the simulation.

### 6.3 Misalignment between introduction and performance evolution

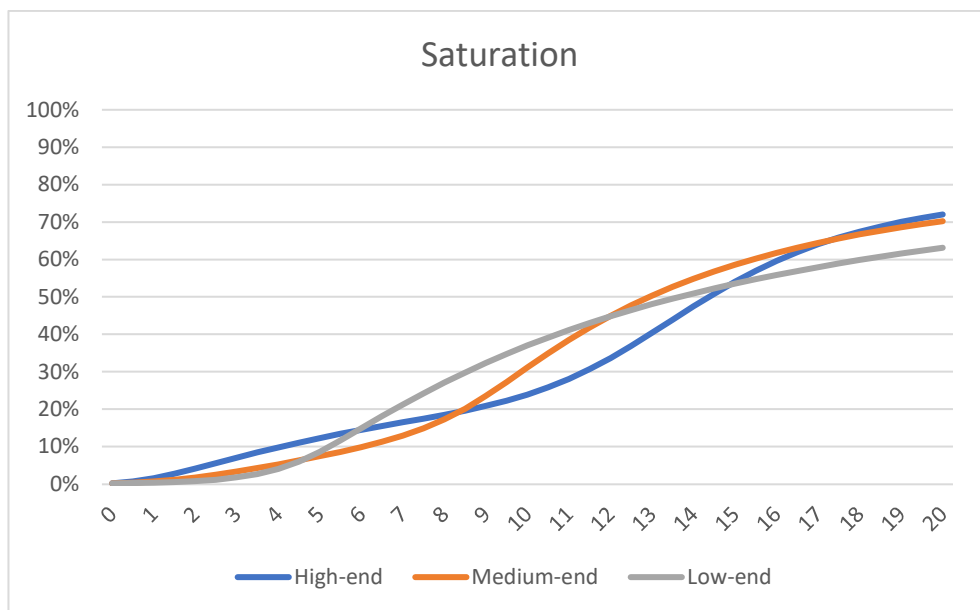
As seen previously in the new market model, a misalignment between the market demand and the firms' offer can occur when products are developed relatively more quickly in the opposite end compared to the launch segment, i.e. performance grows faster in the low-end market with an

introduction from the top or it grows faster in the high-end market with an introduction from the bottom.

The cumulated demand and saturation curves for the introduction from the top, with quick evolution in the low-end segment, are presented in figures 6.9 and 6.10. Figures 6.11 and 6.12 show the corresponding graphs for the introduction from the bottom, with quick evolution in the high-end segment.

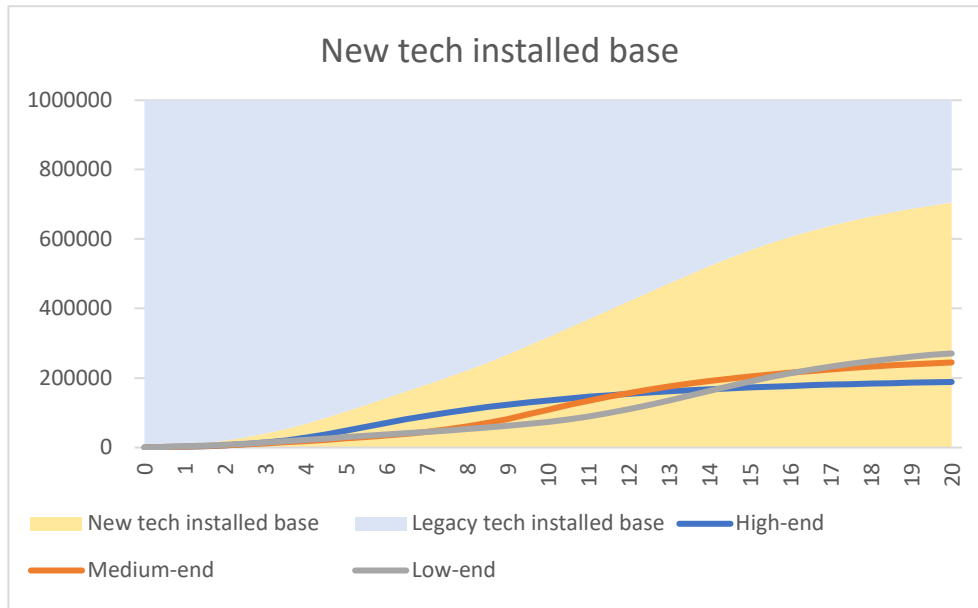


**Fig. 6.9.** Installed base share of legacy vs. new technology (split by segment). Introduction from the top.

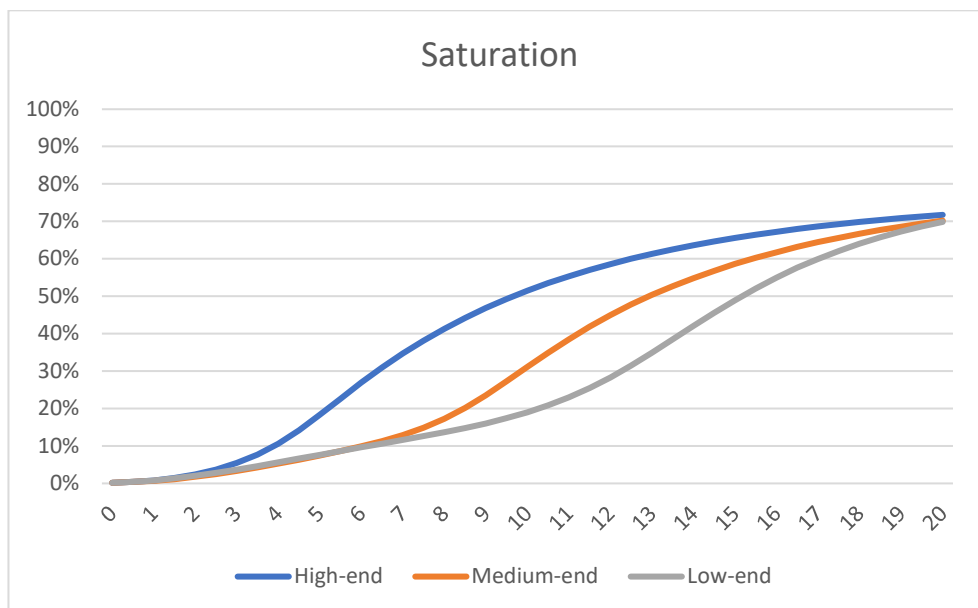


**Fig. 6.10.** Saturation levels, by segment. Introduction from the top.





**Fig. 6.11.** Installed base share of legacy vs. new technology (split by segment). Introduction from the bottom.



**Fig. 6.12.** Saturation levels, by segment. Introduction from the bottom.

Once again, the aggregate demand and the related penetration in the market are not too far away from the cases in which product evolution and customers' needs are aligned, even if the vertical segments' demand varies a lot.

The saturation graph in figure 6.10 is especially interesting: sales in the top-end market quickly ramp up, but they are soon held back by the lack of technical developments; on the other hand,

sales in the bottom are driven by the performance improvements, but they are not sustained by the customers' demand.

Generally speaking, the saturation levels reached at the end of the simulation are below the ones obtained with the new market model; moreover, the aggregate market cumulated demand grows less quickly, especially looking at the central periods, in which all vertical segments are reaching their limit performances. Therefore, evidence seems to point to a negative influence of the legacy technologies on the diffusion of innovations because of the stickiness of the current installed base.

Table 6.4 presents a final summary of the results obtained from the case studies with the existing market model.

New tech installed base   Legacy tech installed base   High-end   Medium-end   Low-end   Total demand

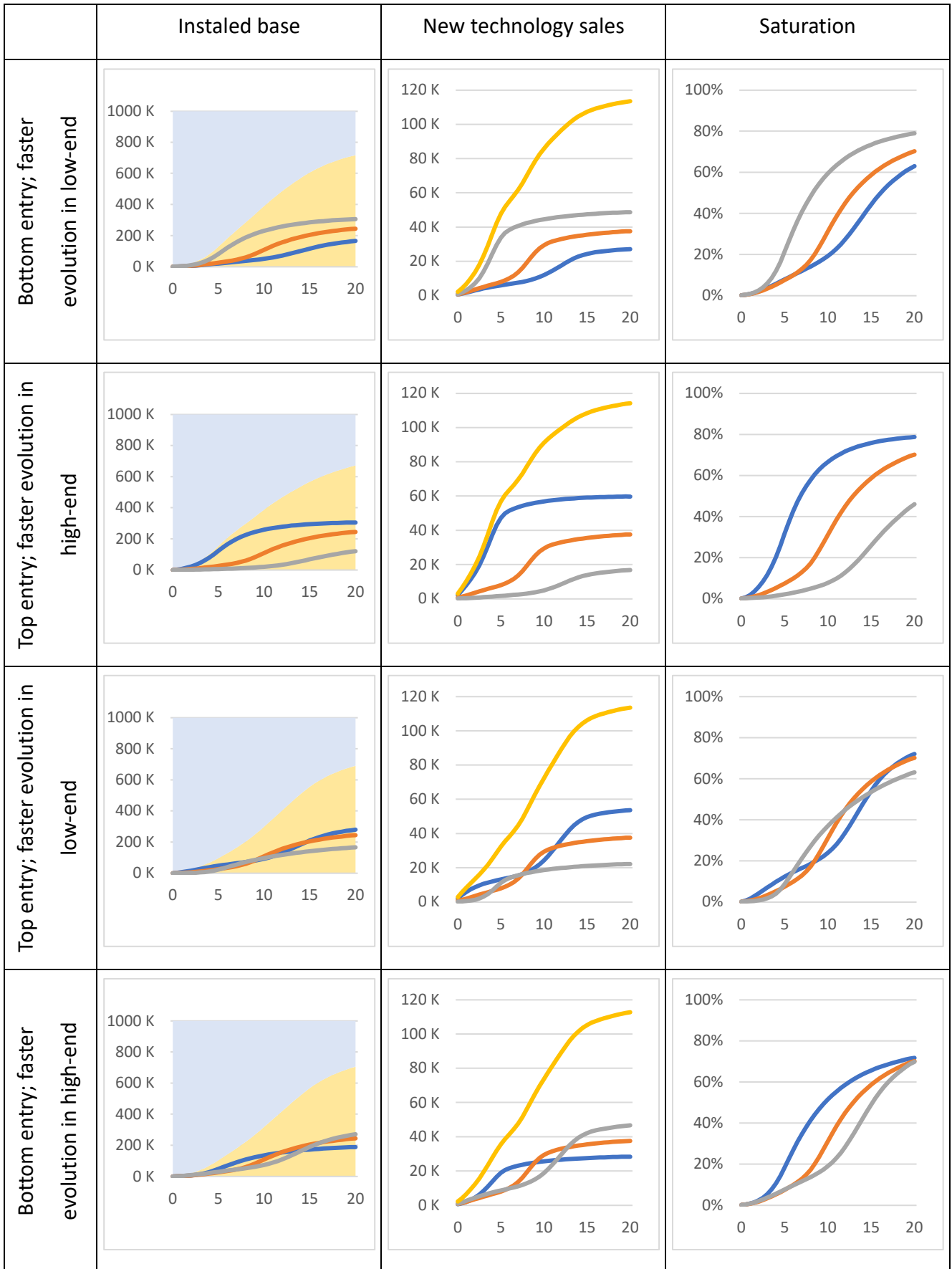


Table 6.4. Scenarios summary with graphs for the installed base, new technology sales and saturation.

## 7. Conclusions

The goal of this thesis was to study the diffusion of innovations in a matrix-segmented market from a mathematical modelling standpoint, integrating in particular the currently used diffusion models with the addition of a vertical segmentation.

Following extensive research on the characteristics of innovations adopters, a matrix segmentation of the population has been created. This clustering includes both a horizontal and a vertical dimension, respectively based on Rogers' groups of innovators, early adopters, early majority, late majority and laggards, and on the customers' preference for high, medium or low-end products. The size of each vertical segment of the matrix depends on the innovation's type of entry in the market, while keeping the horizontal size fixed following Rogers.

After completing the population segmentation, two models have been created for the specific cases in which an innovation creates a completely new market or it is introduced in a currently existing one.

For the new market model, the basis was provided by Van den Bulte & Yoshi's version of the Bass model with imitators and influentials, in which asymmetrical influence in a social system is theorized. This model was expanded with the introduction of the variable of the innovation's performance, and it was implemented in a matrix-segmented market.

In the existing market model, a fixed installed base was hypothesized. Supposing that products in the market need to be replaced periodically, the new technology continuously erodes market share from the legacy technology with the improvement of technical performances, but it is held back by the influence of the systems currently in place.

We analyzed four case studies in the thesis, implementing them in both models. They are the product of the combination of the following situations:

- the introduction of a new technology from the top or bottom of a market;
- performance evolution according or discording with the customer expectations.

The main discovery was that, even in very unbalanced settings in terms of adopters distribution, customer innovativeness, and product evolution, at the macroscopic level all cases seemed very similar with each other. Nevertheless, when looking at the vertical segments' demand curves and saturation levels, the environments were very different and variegated, pointing to the possibility that sales were forced or held back by the timeliness of the innovation development.

This could also explain why, in all its simplicity, the Bass model is still important and useful to these days. The model may not be able to delve too deep into particular cases, especially if looking the microscopic level, but it is able to approximate quite well a broad range of situations and it can be applied to many different industries and markets.

In general, introducing a product from the top of the market yields a slightly higher rate of adoption of the innovation in the early phases, counterbalanced by a longer tail in the instantaneous demand curve caused by less innovative customers in the low-end market. For an introduction from the bottom, the opposite situation holds true: the different vertical segments' demand curves and related peaks are closer to each other, as well as the saturation levels.

Regarding the alignment of performance evolution and customer innovativeness, the diffusion of the innovation follows classical S-shaped curves if the market demand is matched by the firms' offer. On the other hand, in case of misalignment, the demand seems to ramp up or slow down unexpectedly, depending on the type of mismatch between the customers' expectations and the products' technical performances.

Finally, the diffusion of an innovation in an existing market looks slower than in a completely new market, as the competition of the more widely spread legacy technologies reduces the attractiveness of the new products.

## 7.1 Implications for managers and practitioners

It is important to highlight the fact that very different microscopic environment can look very similar when aggregated. This is a dangerous situation for firms, because even if the diffusion of the innovation is not altered too much on the long run, in the short term this could result in strategic errors and misalignment of the offer with respect to the current market demand.

Choosing carefully what to develop is crucial to appeal the target audience at the right time, resulting in a more predictable demand at the microscopic level.

Moreover, investments in the models were not linked to the demand evolution, but in the real world a number of externalities could obviously intervene depending on the sales trends. Seemingly slowing demand could induce managers to cut funds for product development, when it is possible for the event to only be induced by a misalignment between customer needs and product performances. On the other hand, firms could waste investments for customer segments that have actually almost reached their limit saturation.

When looking at the empirical world, we can find cases studies that fit into the description of the previous sections. For instance, let us analyze the video gaming consoles market in the early 1980s.

The first customers in the video gaming industry, particularly during its early days in the 1970s and 1980s, were typically technology enthusiasts, often young and interested in novel electronic entertainment experiences.

During this period, the cost of gaming was relatively high. The gaming hardware, such as arcade cabinets and early home gaming consoles, was a significant investment. Additionally, the cost of purchasing individual game titles or spending time in arcades contributed to a relatively high overall cost of entry.

After this phase, in the early 80's, the market was flooded with a large number of low-quality video games. Many of these titles were rushed to market without proper development and testing, leading to a saturation of poor-quality games. Consumers became disillusioned with the lackluster gaming experiences, and sales began to decline.

Both home console and arcade markets saw an emphasis on producing a high quantity of games rather than ensuring their quality. This flood of mediocre games led to consumer fatigue and a decline in confidence in the industry, ultimately leading to a deep crisis of the sector.

Another possible example of demand and offer misalignment can be represented by the low-cost market for electric vehicles.

Even though the segment is still in the early stages of its development, legacy automakers seem to be struggling to adapt to the target customers' expectations.

Electric vehicles were firstly introduced in the top-end automotive market, pioneered by high-cost market leaders such as Tesla. Early developments have been aligned to the requests of innovators and technology enthusiasts, leading to strong multimedia, neat interior and exterior design, recognizable branding and high-performance engine.

Later developments in the governments' regulations and the extension of recharging infrastructure have slowly started to appeal lower segments of the automotive industry.

Firms have begun to design cars that are specifically designed for this audience, but some key problems still seem unresolved. Key success factors in the low-end market include reliability and affordable cost, at the expense of extra features and performances.

Nevertheless, autonomy ranges still remain insufficient, and they are not compensated by a broad availability of recharging stations. Moreover, target launch prices have not been (or probably will not be) reached by automakers, at least until mass production of batteries is able to cut production costs.

These factors could partly explain a slowing down demand for electric vehicles, which was not predicted in the recent past.

In the opposite situation, when firms' offer and market demand are aligned, the diffusion of innovation seems to happen more smoothly, with consistent and predictable demand.

For example, let us look at the introduction of small hard disk-drives in the computer industry.

In the 1980s and 1990s, mainframe computers dominated the computing landscape, and large, expensive HDDs were the norm. These high-capacity HDDs were suitable for the needs of the established market.

Disruptive innovation occurred with the introduction of smaller, less expensive HDDs that had lower storage capacity. Initially, these smaller drives were not attractive to the mainframe market, as their capacity did not meet the requirements of large-scale computing. However, they found a niche in personal computers and workstations, serving a different set of needs for low-end segments of the market.

As technology advanced, the smaller HDDs underwent rapid improvements in capacity and performance, eventually reaching a point where they not only satisfied the needs of personal computing but also became competitive in higher-end applications. This disruptive innovation led to a shift in the industry, with the once-dominant larger HDDs losing their market share to the smaller, more cost-effective alternatives.

## 7.2 Further research directions

Even though the characteristics of vertical segments of adopters have been inferred in the thesis from past research on the diffusion of innovations field, that is an area that still remains to be thoroughly explored and surely is less studied compared to the horizontal segments of adopters.

Regarding the build-up of the models, the immediate and most useful expansion could be the addition of a link between investments and the demand. This could have a significant impact on the development efforts and ultimately on the performance evolution of the innovation, therefore influencing deeply the purchasing behavior of customers.

Last but not least, the models could be used to study vertical product differentiation in the market. Instead of introducing all products at the same time for all segments, it could be interesting to insert some triggers that induce the launch of a new product the subsequent segment. Then, it could be possible to analyze the link between the diffusion of innovations and the vertical product differentiation, especially regarding the timeliness of introduction in a new segment with respect to the saturation of previous ones.





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