POLYTECHNIC OF TURIN

Department of Management and Production Engineering

Master's degree in Engineering and Management – Production Path

Master Thesis on

CUSTOMER ORDER SCHEDULING PROBLEM: REVIEW OF THE STATE OF THE ART

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Abstract

Usually, scheduling relates to the sequencing and timing of jobs in the production system to optimize some performance measures. The most used are makespan, total completion time, and tardiness. Such measures are related to the single job to be produced. However, in a customer-oriented production, the performance measures to optimize and evaluate are not related to the single job, but to customer orders. A customer order may include a set of jobs, and only the status of the complete order is relevant for the customer, and hence for the company.

The customer order scheduling problem, then, differs from the standard scheduling problem only in the performance measures to be optimized. It considers indeed performance measures related to the customer orders, and not to a single job.

The purpose of the thesis is to gain insight into the research domain, summarize the state of the art by tracking trends over time, and enlighten the gaps in the literature that could be a potential interest for future research on Customer Order Scheduling Problem. The literature has been focused mainly on single stage dedicated parallel machines, the most used methodology is heuristics, and the efficiency of the proposed algorithms was tested mostly with computational experiments.

The AI integration with the metaheuristics and a cooperation with Industrials players could improve the quality of solutions and align the research to the real-world by introducing more constraints and multi-objective functions (today not much frequent).

Key-words: order scheduling, customer-centric, order scheduling review paper, customer order scheduling, customer order scheduling problem.

1. Introduction

1.1 Context Overview: From Product to Customer-Centric

In the last ten years, the manufacturing industries have encountered obstacles, as ever-changing customer expectations, fierce competition, globalization, financial crises, and economic downturns. The COVID-19 pandemic and chip shortage represent significant examples of uncertainties or, for some players, new business opportunities to enter the market or conquer a competitive advantage. Therefore, it becomes essential for them to continuously adapt to market demands and integrate the technologies and processes to stay competitive and sustainable.

In other words, manufacturing should increase the resilience of the supply chains to provide the right products requested by the customers at the right time.
To address these challenges and promote and strengthen manufacturing the European Commission foresaw the need for an investment plan called Factories of the Future as part of the Horizon 2020 innovation program, EFFRA (2015).

Playing manufacturing a role in driving innovation by enabling advancements to be implemented in products and services, in 2015 the program of European Factories of the Future Research Association, EFFRA (2015), promoted by the European Commission, outlined both specific objectives for the Factories of the Future initiative:

- To maintain Europe's competitive edge by staying at the forefront of the manufacturing industry.
- To ensure that industry and research work together to implement the program and identify ways to innovate through research.

As evidenced in the EFFRA (2015), and here reported in Figure 1 below, customer-focused manufacturing is identified as one of the six Research and Innovation domains. The research and innovation activities in these domains should aim to accomplish specific and measurable targets, referred to as manufacturing challenges and opportunities.

The Factories of the Future will be able to collect and analyze customer requirements to manufacture the right product, monitor and adjust production scheduling and execution based on consumers' orders and entail coordinating the flow of materials and information along the supply chain. To achieve these objectives, research must facilitate fast interaction among three different actors, industry, researchers, and technological providers, so that, operating in a variable supply network, these actors can reduce their production lead times.

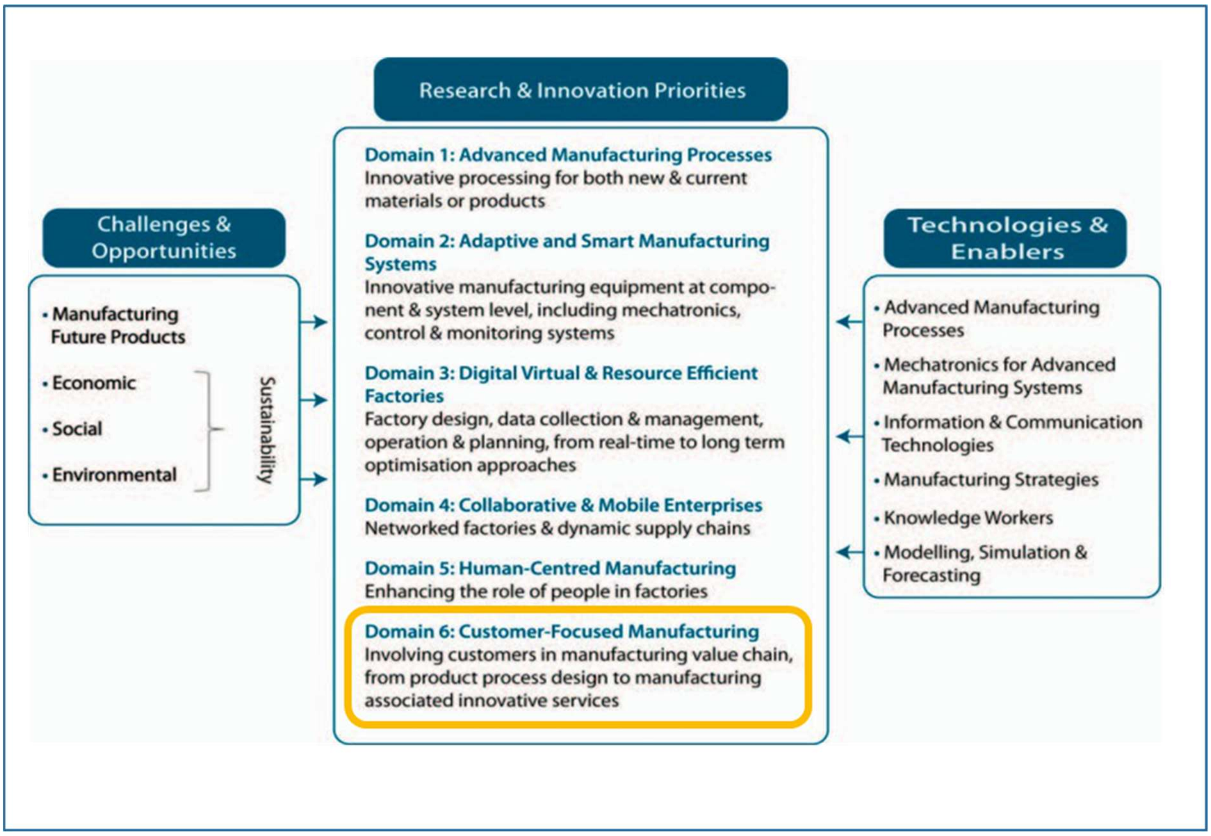

Figure 1 Factories of the Future – RoadMap – EFFRA (2015)

This vision represents a significant challenge for manufacturing companies, that must analyze their supply chains and introduce enhancement actions to make their structure quickly adaptable to the fast-changing customer needs. Each business area should be involved in this process to identify how they can better respond to the demands of their customers.

As reported by Sorgun (2022), manufacturers understand that, in such a competitive environment, “they can no longer rely solely on their product or engage in race-to-thebottom price wars.” Customer-centric manufacturing requires new methods and strategies to meet the changing needs of customers in each phase of the production process.

Traditionally, the customer is considered only at the very end of the chain, whereas in a customer-centric approach, the client demand pulls the material requirement and the production.

Molins and De Mesquita (2019) explained that Industry 4.0 is a new way of managing the supply chain. It involves coordinating smart factories that should give a higher flexibility, making production more responsive to the ever-changing demand.

To achieve this, production scheduling must not only focus on a more efficient allocation of resources but also it must ensure that all the tasks performed are synchronized and coordinated effectively. Figure 2 shows the differences between a traditional supply chain with customer seen only as receiver of the production distribution and a customer-centric supply chain where customer demands pull each phase of the process.

Therefore, resource allocation becomes essential to achieve the desired performance of the supply chain.

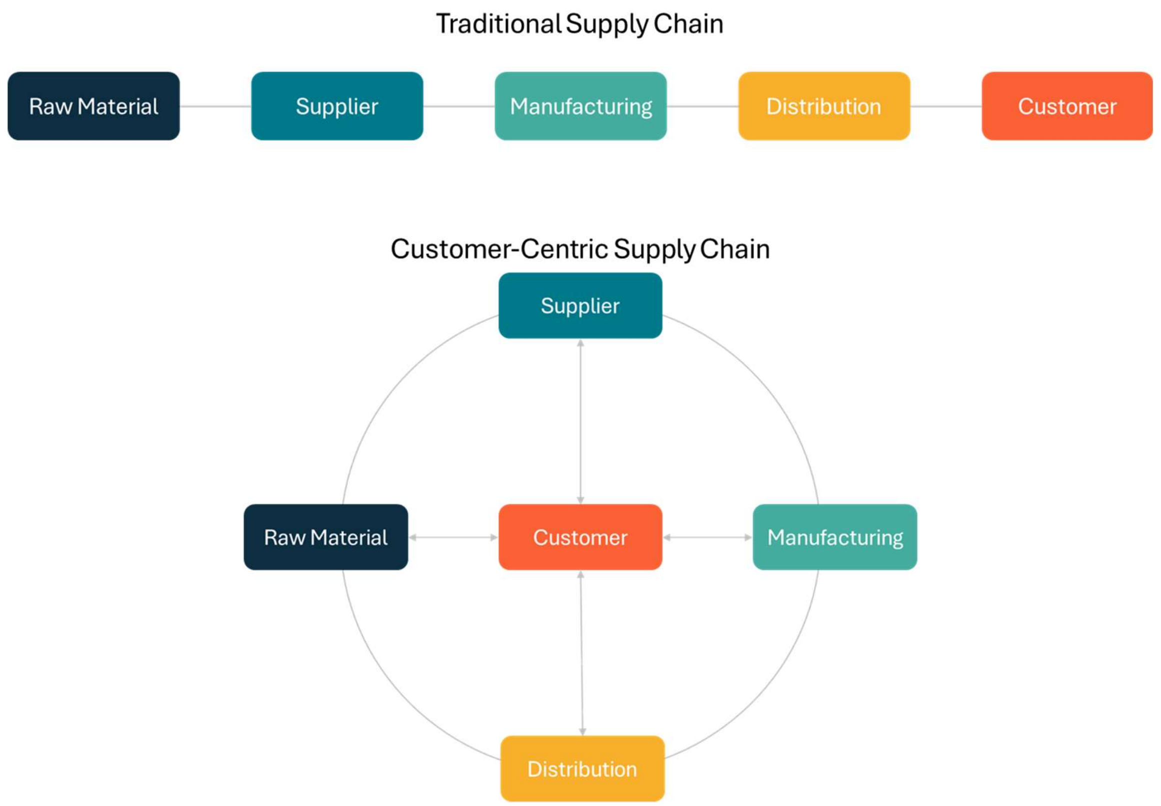

Figure 2. Supply Chain Evolution

1.2 How Can Scheduling Be Customer-Centric?

Scheduling is the process of reserving resources for machines to perform operations according to one or more goals. Efficient scheduling can reduce task completion time, maximize resource efficiency, and ultimately increase profits.

Over the past sixty years, significant efforts have been made in the field of general scheduling theory, which encompasses various models, complex results, and algorithms. Potts and Strusevich (2009) said that a search on the Web for publications with “scheduling” and “machine” as keywords yielded over 200 publications every year since 1996, and more than 300 publications in 2005, 2006, and 2007.

Other more recent review papers on the general scheduling problem highlight that the solutions spectrum for the scheduling problem has been examined with all derivations: it's the case for example of Cheng et al. (2000), Zhang et al. (2019), Xiong et al (2022), Chaudhry and Khan (2016), Komaki et al. (2019), Neufeld et al. (2023), Lee and Loong (2019), Duan et al. (2023), Afshar-Nadjafi (2021), Allahverdi (2015), Del Gallo et al. (2023), (Mathew e Johansson 2023), Allahverdi et al. (2008), Ouelhadj and Petrovic (2009), Molins and De Mesquita (2019), Calis and Bulkan (2015), Yu (2021), Senthilkumar and Narayanan (2010).

The academic world has developed different techniques, mainly heuristics, meta-heuristics, and in more recent years even some AI methods. However, practical articles are very few and the experimental validation seems to be not much used.
For example, Allahverdi focused his studies (1999), (2008), (2015) only on scheduling problems with setup times and found approximately 1000 papers but of these less than 10% had any application in the industrial world.

As evidenced in Mourtzis (2022), the shift from product-centric manufacturing to customer-centric manufacturing has become a reality today. In this context of global competition and fast-paced market, manufacturing companies must respond quickly and accurately to customer orders, which require more personalization and customization of the production. The manufacturing world is expected to establish a more efficient supply chain and improve its operational efficiency.

The purpose of this study is to provide an overview of the research progress on the Customer Order Scheduling problem, without delving into the specifics of the algorithms and techniques involved.

This area of study is relatively new compared to the broader field of scheduling. A search on the internet yields less than 100 papers on this topic, but no peer-reviewed papers.

This thesis wants to show a classification of the literature in Customer Order scheduling, emphasizing some perspective on the status of the research paper.
The review of existing literature reveals several potential areas that are worthy of future research in the Customer Order Scheduling problem.

The work is organized as follows:

- Section 2 presents the notation used in this work.
- Section 3 addresses the adopted strategies, policies, and methods in detail, providing graphical evidence of the results of the investigation.
- Section 4 provides a general analytic analysis of the selected papers.
- Section 5 discusses the conclusions of the work and shows potential future avenues.

2. Scheduling Classification

2.1 Notation

In 1970 Ronald Graham introduced a notation to classify optimal scheduling problems. It consists of three fields: α, β, and γ.

$$\alpha \mid \beta \mid \gamma$$

Figure 3. Graham notation

The first field α describes the layout of the system in terms of number of machines and type. β represents the constraints of the problem such as set-up times, precedencies, or pre-emption. The last field γ is the objective function, the performance measure to maximize.

This work will use Graham notation since it is considered a general standard notation in scheduling and the following abbreviations.

$$n = Number\ of\ orders$$

$$N = Set\ of\ orders\ \{1, \dots, n\}$$

$$m = Number\ of\ machines$$

$$M = Set\ of\ machines\ \{1, \dots, m\}$$

$$j = Number\ of\ jobs$$

$$J = Set\ of\ jobs\ \{1, \dots, j\}$$

$$P_j^n = Processing\ time\ needed\ for\ job\ j\ in\ order\ n$$

$$t = Time$$

2.2 Alpha: Machine Environment

The machine environment depends on the number of execution phases needed to complete a job. As a matter of fact, it's possible to identify

- Single stage job scheduling problems
- Multi-stage job scheduling problems.

In a single stage job, there are four possible machine environments to consider:

- Single machine (indicated as $\alpha = m = 1$): there is only one single machine available, and it can process all the types of jobs.
- Parallel machines:
  - Identical (indicated with $\alpha = P_m$) if all the machines in parallel are identical. This environment is also called Fully Flexible.
  - Uniform (indicated with $\alpha = Q_m$) if all the machines in parallel have uniformly different speeds. Each machine has a speed factor and the time to process job j on machine i is $p_{ij} = \frac{p_j}{s_i}$.
  - Unrelated (noted with $\alpha = R_m$) if the speed of the machine depends on the job and it has no relation with the other parallel machines. The time to complete job j on machine i is p_{ij}.

In multi-stage job scheduling, each job needs to be processed on a certain number of dedicated machines and since each job can only be worked on one machine at a time it is possible to distinguish three main types of machine environments.

- Open shop ($\alpha = O_m$): The number of operations to complete a job is fixed but they can be scheduled in any order.
- Job Shop ($\alpha = J_m$): The number of operations to complete a job is not fixed, but the operations of each job must be processed in a certain order. Each job can have a different processing sequence on different machines.
- Flow Shop ($\alpha = F_m$): The number of operations to complete a job is fixed, and they must be scheduled according to some specified precedencies constraints. It is the case of machines in series.

The subscript letter indicates the number of machines if fixed. E.g. $\alpha = P_2$ indicates a layout of 2 identical machines in parallel.

2.3 Beta: Requirements

The most common constraints taken into consideration in the analyzed papers are explained as follows.

- Release date r_j is the earliest instant at which the job can start being processed.
- Due date d_j is the instant at which the customer expects the order to be completed.
- Process time p_{ij} is the time needed to process job j on machine i.
- Setup time s_{ij} is the time to program the machine i to process job j. If it depends on the previous job completed (sequence-dependent setup time), it is indicated as s_{jk} where k is the previous job.
  It is not taken into consideration in the dedicated machine layout.
- Weight w_j add a degree of importance to the job j.
- Pre-emption $pmtn$ indicates the possibility of interrupting the job processed on a machine. Usually, the machine must finish the job in progress before starting another one.

2.4 Gamma: Objective Function

The third and final field of the Graham Notation specifies the performance metrics to maximize.

The most common objective functions found in the research papers include:

- Minimize the makespan where the makespan is the maximum time to complete all jobs, i.e. the time to complete the last job, $C_{max} = \max C_j$.
- Minimize the order's completion time. C_j is the time to complete job j, therefore the completion time of the order is $\sum C_j$.
- Minimize the weighted completion time of the order, $\sum w_j C_j$.
- Minimize the tardiness of the order, $\sum T_j$ where $T_j = \max\{C_j - d_j; 0\}$ is the tardiness of each job with respect to its due date.
- Minimize the number of tardy jobs, $\sum U_j$ where $U_j = \{1\ if\ the\ job\ is\ tardy; 0\ otherwise\}$.
- Minimize the flow time of the order $\sum F_j$ where $F_j = C_j - r_j$

Figure 4 shows a summary of the most common values for each field of Graham notation. Some researchers aimed to optimize multiple performance metrics.

Description of α, β, γ fields.

α Notation	α Description	β Notation	β Description	γ Notation	γ Description
1	Single machine	ST_{si}	Sequence-independent setup time	C_{max}	Makespan
P	Parallel machines (identical)	SC_{sd}	Sequence-dependent setup cost	E_{max}	Maximum earliness
Q	Parallel machines (uniform)	ST_{sd}	Sequence-dependent setup time	L_{max}	Maximum lateness
R	Parallel machines (unrelated)	$ST_{si,f}$	Sequence-independent family setup time	T_{max}	Maximum tardiness
Fm	m-stage flowshop	$ST_{sd,f}$	Sequence-dependent family setup time	D_{max}	Maximum delivery time
FFm	m-stage flexible (hybrid) flowshop	$SC_{sd,f}$	Sequence-dependent family setup cost	TSC	Total setup/changeover cost
AFm	m-stage assembly flowshop	ST_{psd}	Past-sequence-dependent setup time	TST	Total setup/changeover time
J	Job shop	$Prec$	Precedence constraints	TNS	Total number of setups
FJ	Flexible job shop		Non-zero release date	ΣF_j	Total flowtime
O	Open shop			ΣC_j	Total completion time
				ΣE_j	Total earliness
				ΣT_j	Total tardiness
				ΣW_j	Total waiting time
				ΣU_j	Number of tardy (late) jobs
				$\Sigma w_j C_j$	Total weighted completion time
				$\Sigma w_j F_j$	Total weighted flowtime
				$\Sigma w_j U_j$	Weighted number of tardy jobs
				$\Sigma w_j E_j$	Total weighted earliness
				$\Sigma w_j T_j$	Total weighted tardiness
				$\Sigma w_j W_j$	Total weighted waiting time
				$\Sigma h(E_j)$	Total earliness penalties
				$\Sigma h(T_j)$	Total tardiness penalties
				$TADC$	Total absolute differences in completion times

Figure 4 Description of α, β, Ɣ – Allahverdi (2015)

3. Customer Order Scheduling Problem Literature Review

3.1 Approach to literature review

In today's world technological advancements have led to an increase in the number of products on the market, reflecting the trend the customers are now faced with a range of options and their expectations are changing at a pace. Therefore, businesses need to not only understand but also meet their rapidly evolving expectations and to align their production strategies with the changing needs and desires of their customers.

As reviewed by Li et al. (2022), the customer order scheduling problem (COSP) is a complex issue affecting various industries and different applications, arising for example in the production of semi-finished lenses, in the steel industry, or in systems that offer a mix of products and services that meet the individual needs of customers.

Since over the years, COSP has become a major challenge in manufacturing environments, researchers and practitioners have studied several objective functions to address this challenge in practical applications.

This trend is confirmed by Hoffman et al. (2022), affirming that, in many real-world situations, customers are ordering not only one item but several items of multiple product types. To save transportation costs and distribution execution time, it is advisable to ship all items of an order at once and not separately.

Therefore, in the last two decades, the customer order scheduling problem (COSP) has gained the attention of researchers, representing for the research community a new wave that is significantly different from the interest in the more general scheduling problem, on which tens of thousands of papers have been written over the last 60 years examining every conceivable aspect that can be optimized.

However, despite a limited number of studies, the lack of cooperation between the academic and industrial world is visible.

After conducting a comprehensive search of the literature, it has been determined that only a few review articles exclusively dedicated to the examination of Customer Order scheduling problems exist. As such, the present thesis wants to provide an analysis of the available literature to identify the key trends, gaps, and opportunities for future research in this area.

Before the year 2000, only a few papers on the Customer Order scheduling problem were published. Therefore, the research was narrowed down to the articles published in English from January 1996 to the end of June 2023.

This section wants to show the progress made in literature over the years.
It will start with the methodology used for exploring the state-of-the-art research.
The research has been conducted through

- Scientific databases including ScienceDirect, Scopus, Informs, Google Scholar
- Social networks like ResearchGate, Academia.edu
- Journals as IEEE Explore
- International publishers as Taylor & Francis, Springer, Emerald.

The focus has been on scientific papers and proceedings of reputable conferences for the keywords or a combination of ‘order scheduling’, ‘customer order scheduling’, and ‘order scheduling problem’.

A screening phase of the published works collected followed the first exploration step to see if they were related to the scope of this thesis. Only 62 papers met this criterion.

Figure 5 below illustrates the review process that was used for this dissertation in a graphical format.

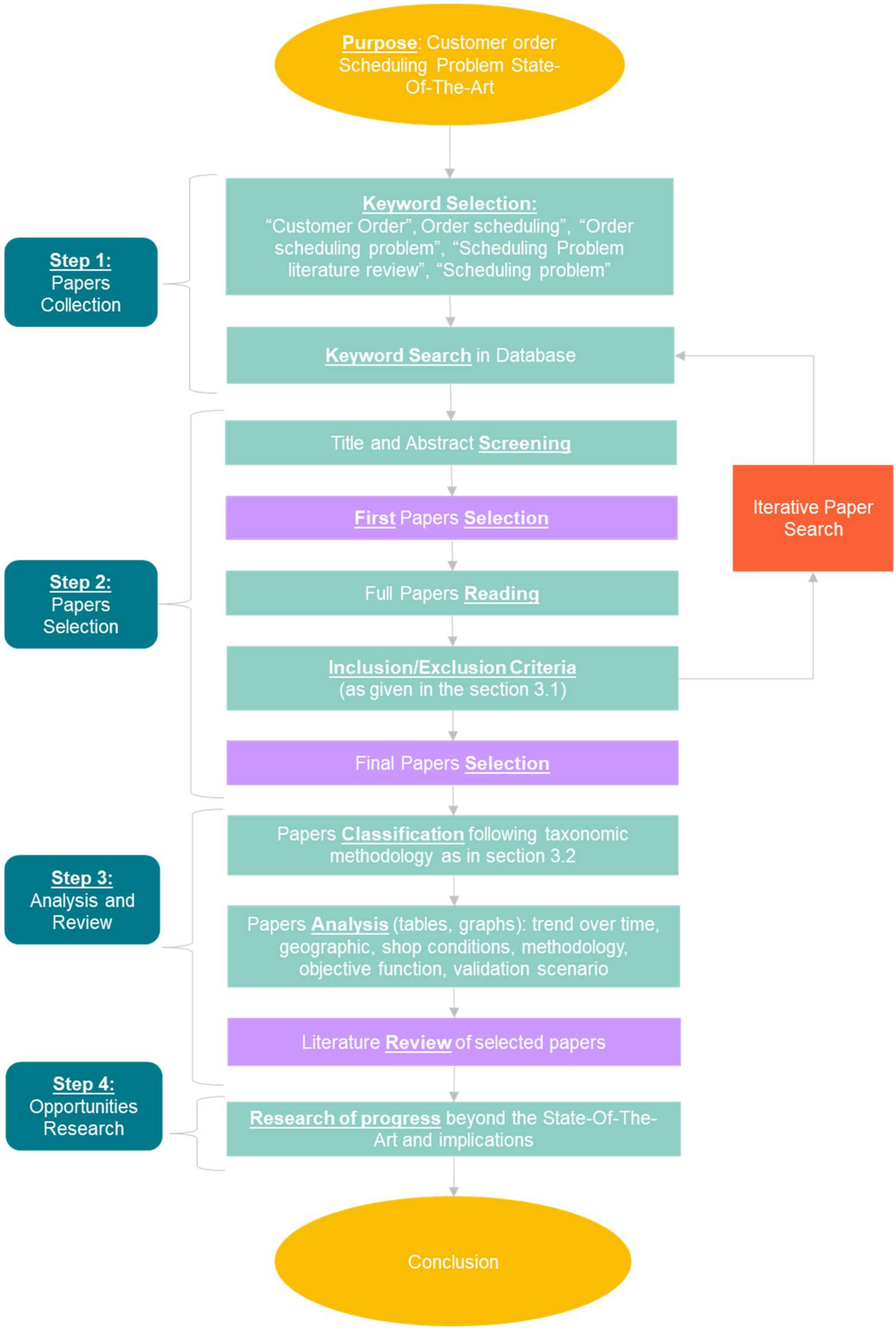

Figure 5. Review Process

3.2 Taxonomic Methodology

Based on the approach proposed by Martinelli et al. (2022), this section presents a taxonomy that considers various aspects of the resulting publications for systematic analysis and classification.

The classification used includes 4 criteria:

1. Shop condition – single machine, parallel machine, job shop, open shop or flow show. Each case entails different ways to manage the order's queue.
2. Methodology – exact, heuristic and their AI evolution (heuristic or metaheuristic models that use AI to explore solutions).
3. Objective function – time related (makespan or completion time) or due date related (lateness, tardiness, tardy orders) both weighted or not.
4. Industrial application – if the paper includes real case study or experimental simulations.

The list of 62 papers is displayed in Chapter 4, divided according to the machine environment.

An overview of the classification criteria presented here can be found in section 3.3.

3.3 Classification Criteria for The Literature Review

The 62 identified papers were analyzed and evaluated answering the following criteria intended to evidence possible trend of the research.

The chart below shows an increasing interest in the Customer Order Scheduling Problem in recent years.

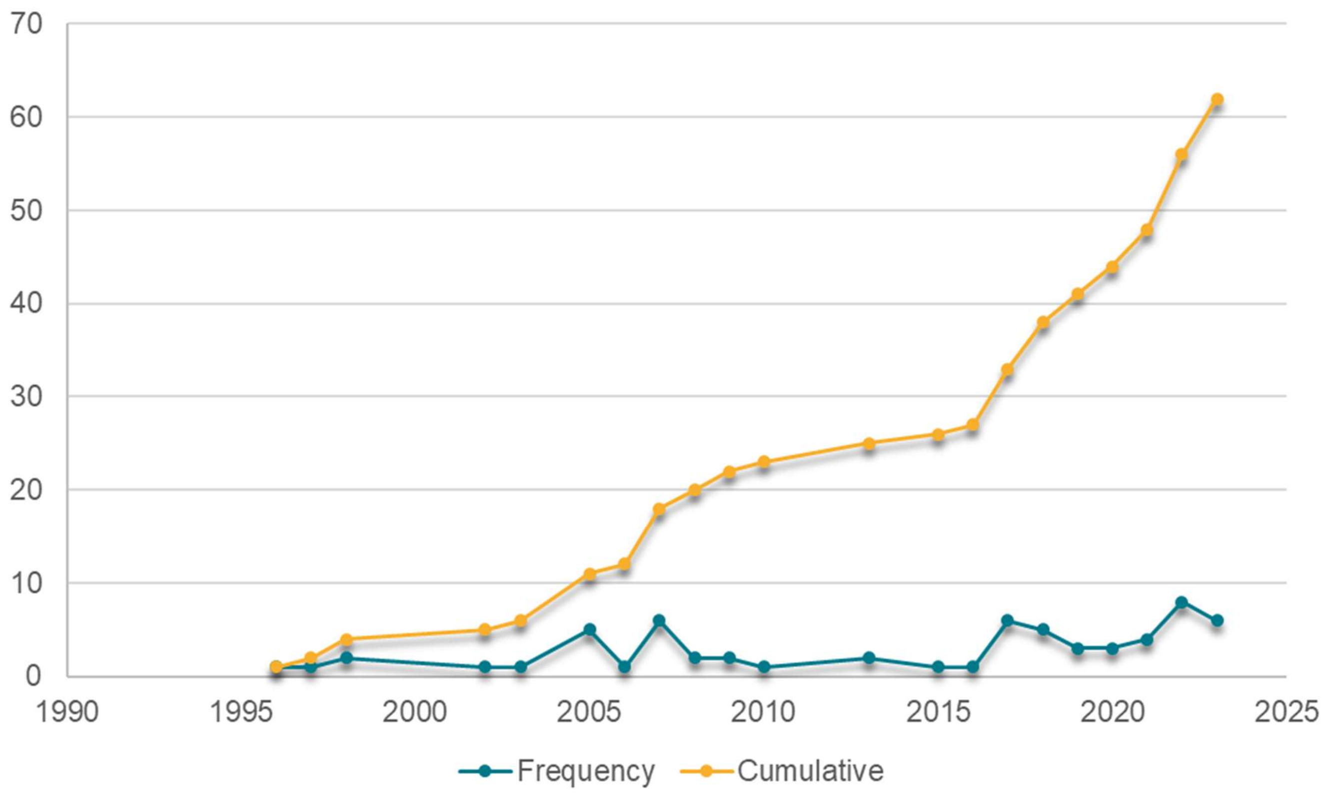

Figure 6. Paper Publication Trend

The geographical distribution of the authors reflects the delocalization of the manufacturing industry and its main players.

The map on the following page puts Taiwan in the 1st place, followed by the US, and then China right behind. Europe doesn't seem to have caught the trend yet, only Germany emerges from the group, but the level of interest is still lower than Brazil. This could be the result of the production offshoring outside the Old Continent in favor of Brazil and China.

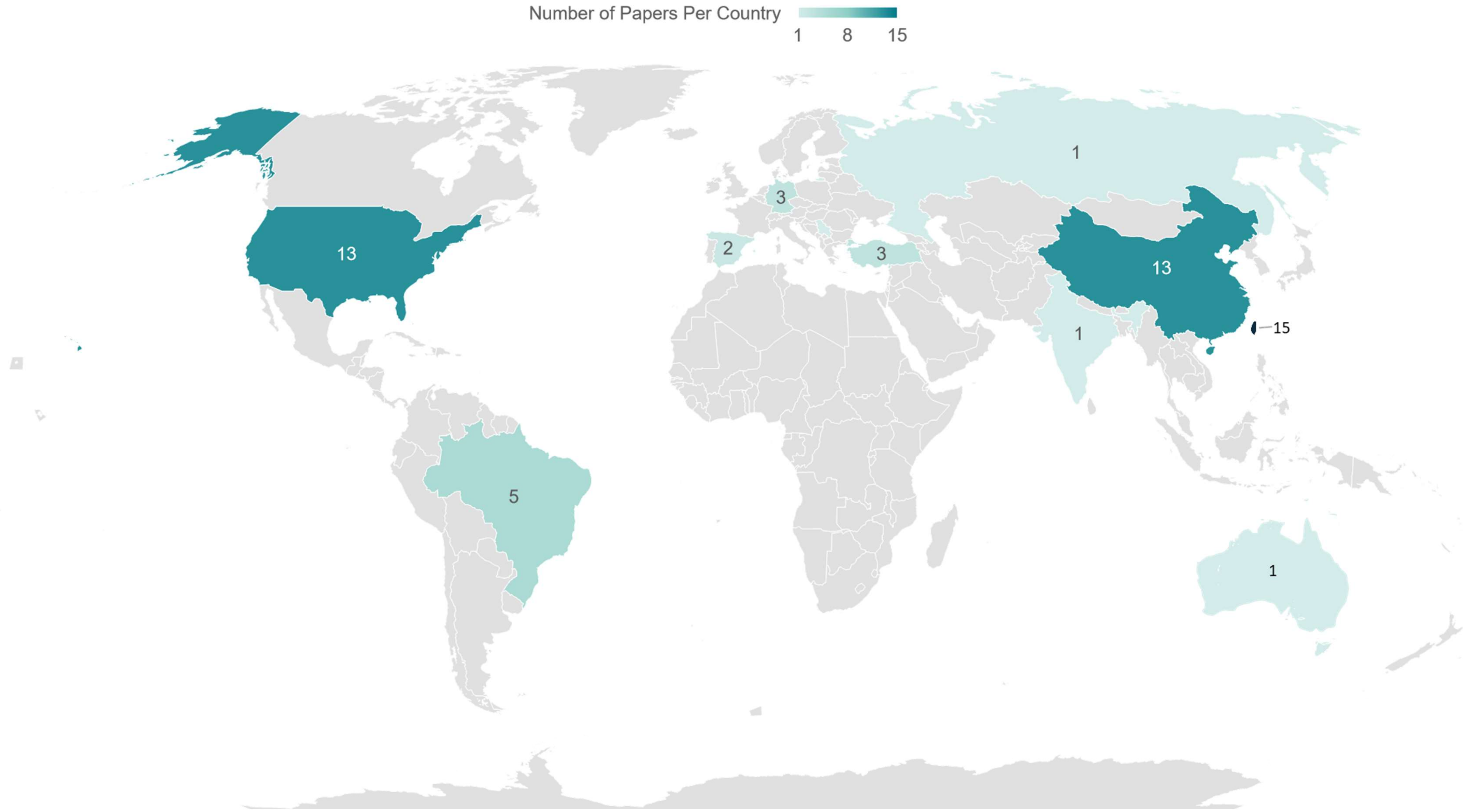

Figure 7. Geographic distribution

The **first classification criterion** used in this review is the Machine Environment. The following chart shows the State of the Art: the use of dedicated machines (different machines for different jobs) is the most examined, followed by a large gap only in the simplest case of a single machine environment.

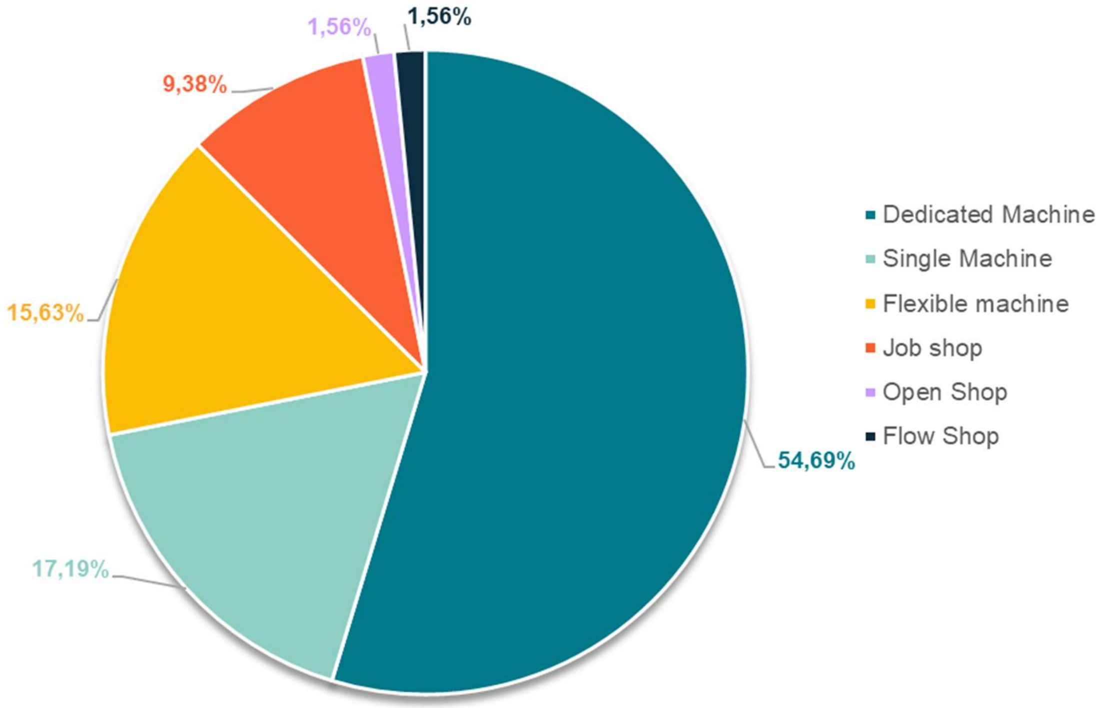

Figure 8. Shop Frequency

The yearly distribution of the paper based on the shop condition doesn't show any trends.

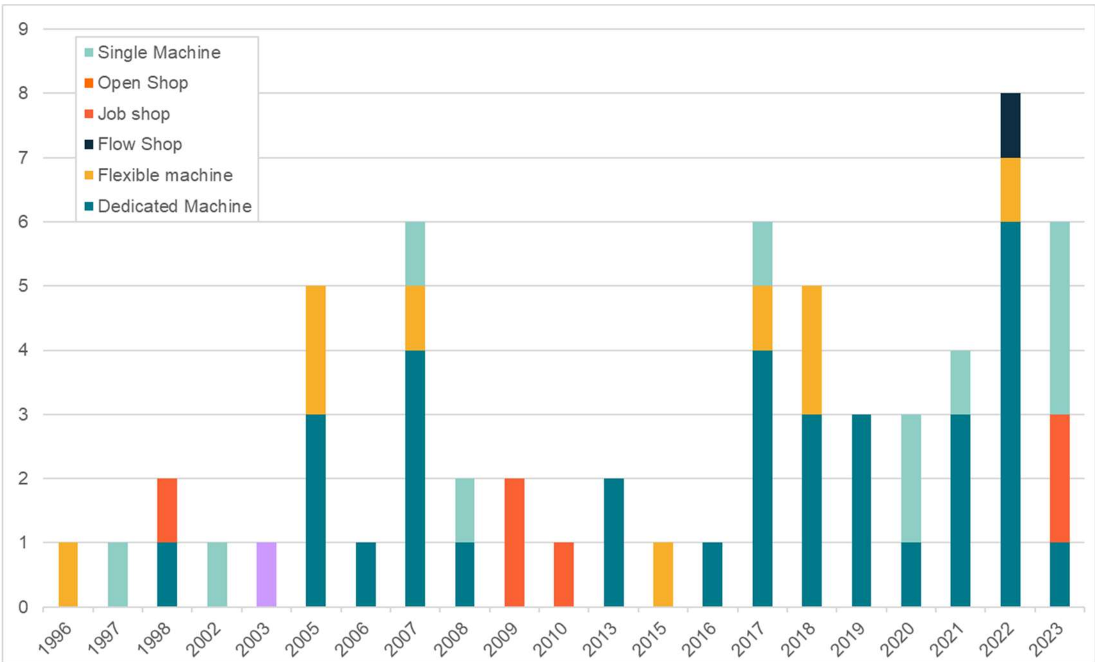

Figure 9. Yearly Distribution of Papers based on Shop Condition

The **second classification criterion** is the Objective Function.

On the graph below it's possible to identify which objective functions have been pursued in the selected studies.

Most of the articles focus on efficiency-related objective functions, mainly minimizing the total completion time of the orders – unweighted $\sum C_j$ or weighted $\sum w_j C_j$.

The other type of objective functions for order scheduling problems are related to the due date of each order: minimize the total tardiness $\sum T_j$ or the number of tardy jobs $\sum U_j$ are the most common goals. They both try to minimize the cost related to the time when the order is completed, compared to its due date.

It is possible to notice that, independently from the machine environment, some papers analyse multi-objective functions.

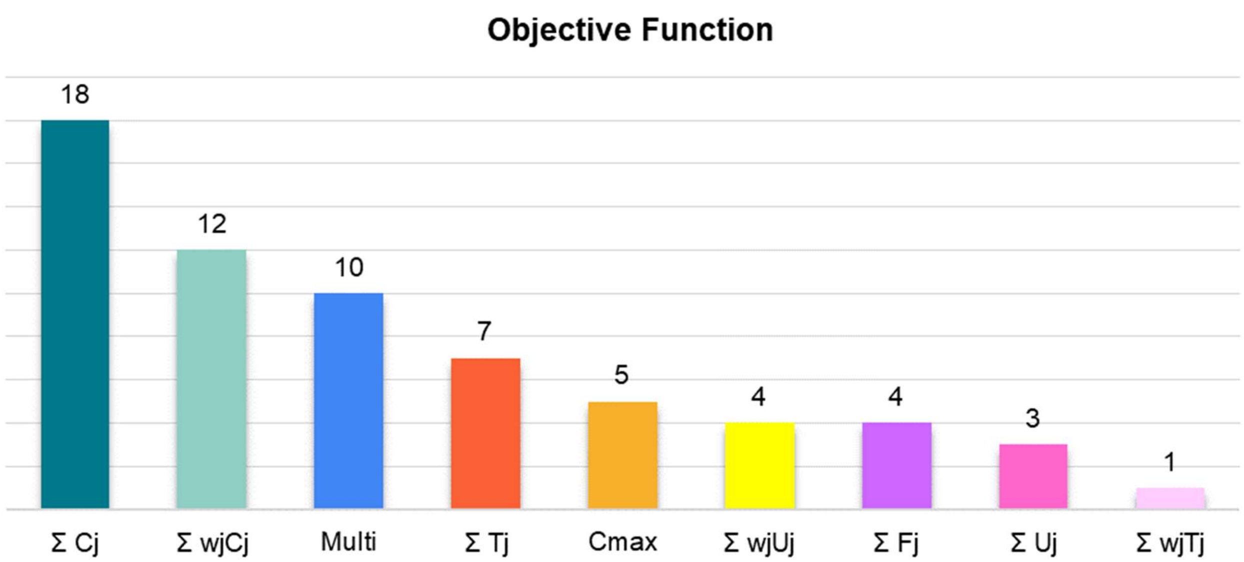

Figure 10. Objective Function Frequency

The **third classification criterion** used to review the literature is related to the type of methodology used.

The studies were categorized based on the type of resolution method used, whether it was a combination or improvement of an existing method, or an adaptation to the proposed problem.

The resolution methods were classified into exact, heuristic, and AI algorithms.

- **Exact**: Find the optimal solution to the proposed problem to minimize costs and maximize operational efficiency.
- **Heuristics**: Find a good but not necessarily optimal solution. When exact methods can be very time-consuming, heuristics are often preferred. In this work "Heuristic" indicates both heuristic techniques which are problem-dependent and metaheuristic algorithms which are problem-independent.
- **AI:** Metaheuristics combined with Machine Learning methodologies to estimate model parameters in the training phase. The integration with ML could improve the solution quality and robustness of metaheuristics.

The following chart presented illustrates the distribution of studies that were analyzed in this research. Most of the papers proposed both Exact and Heuristics Methods (48%), the application of heuristics integrated with ML approaches is still young.

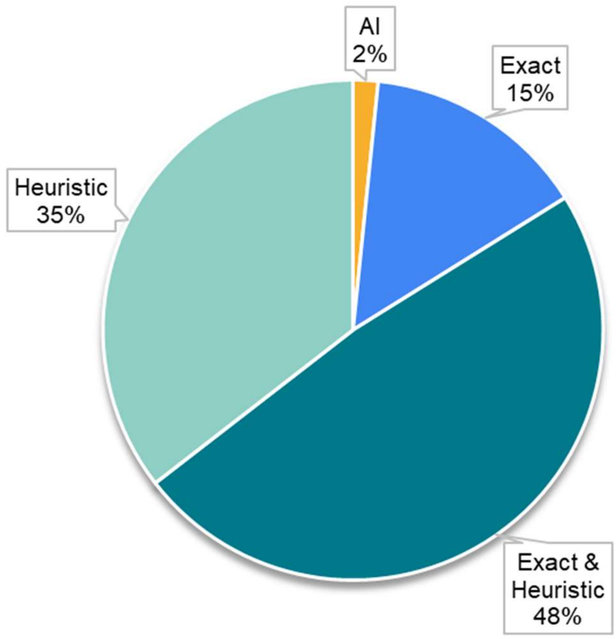

Figure 11. Methodology Percentage Usage

The chart highlights the increasing use of Heuristics. There's only one paper on AI.

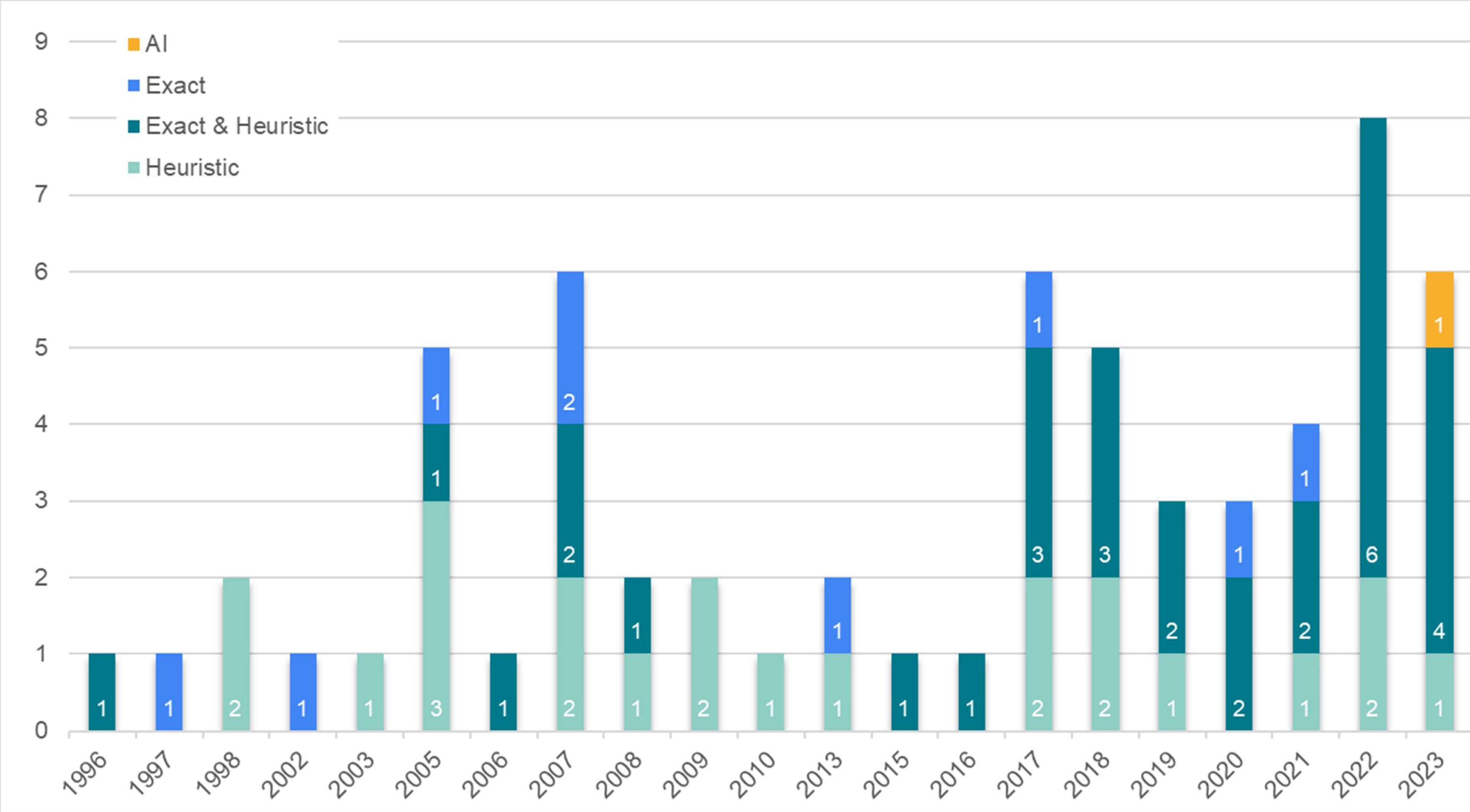

Figure 12. Methodologies Frequency Over The Years

The **fourth and last criterion** used to classify the current research papers was the presence or not of an industrial case study.

A real-world case study could be useful to validate the efficacy and efficiency of the solutions proposed.

To avoid the risk of the research being considered only a theoretical exercise and losing relevance in the industrial world, an assurance strategy needs to be defined.

Research projects that aim to solve customer order scheduling problems are often complicated and involve a significant amount of uncertainty. However, if successful, they have the potential to drive innovation across a wide range of industries. It's important to note that there is a possibility that the proposed solution may not work in a real industrial case due to technical limitations in obtaining data and studying environmental conditions.

For each article included in this study, it was evaluated whether the authors demonstrated their proposal in an industrial validation scenario. It is reasonable to expect that any scientific proposal related to manufacturing should be tested in an industrial setting to demonstrate its practical viability.

As shown in the following figure**Errore. L'origine riferimento non è stata trovata.**, only one of the 62 papers dealt with validation explicitly with real industrial user cases. However, 81% of papers seemed to validate their concepts through computational experiments.

A possible involvement on the industrial applications could help to improve the quality of research.

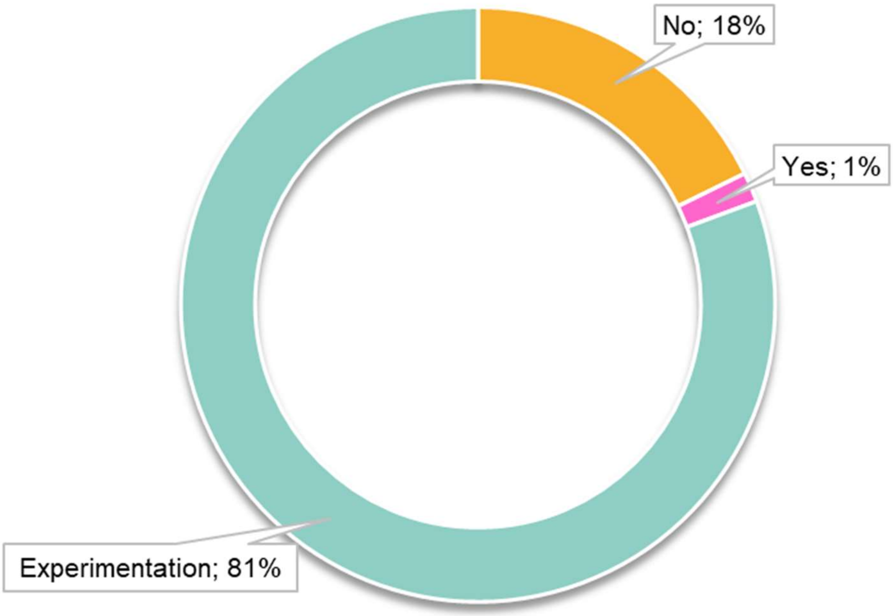

Figure 13. Validation Usage with Industrial Cases

Academia and industry should work together in this direction to face the challenges of Industry 4.0 today and Industry 5.0 tomorrow through the direct involvement of industrial partners in the validation of new ideas. An interactive fertile agora led by an integrated team of academia and industry experts, who are experienced in conducting research, testing hypotheses, and drawing empirical conclusions, should be considered. This will allow both sides to better understand each other's perspectives and work towards better solutions.

4. Customer Order Scheduling Problem Literature Review

This thesis uses 4 classification criteria:

1. Machine Environment
2. Objective Function
3. Algorithm Type
4. Case Study Validation

Firstly, the papers are divided according to their machine environment since it entails different queue systems. The cases considered are:

1. Single Machine
2. Parallel Machine – Flexible or Dedicated
3. Other shop conditions that include Job shop, Flow Shop and Open Shop

Then, in each section the studies are grouped according to the problem type with a particular interest for the objective function.

The third classification criterion can be found in the description of each paper where the algorithms used are nominated with their type (exact or heuristic).

Finally, this work wants to highlight the presence of a validation of the proposed algorithms through real world case study or at least of a quality assessment through experimentation.

4.1 Single Machine Customer Order Scheduling Problem

This section discusses the Customer Order Scheduling problem for a single machine. The research analyzed 11 papers that addressed this issue. After thorough examination and discussions, it was found that various researchers offered solutions to different problems.

Single-machine order scheduling is the simplest machine environment case. Following the guidelines anticipated in the previous chapter, we examined in detail the selected 11 papers. The table below summarizes the research according to the established criteria.

Paper	Problem type	Methodology	Algorithms	Industrial case
Gupta et al.(1997)	$1 \mid s_f \mid F_h(I \mid C_{max})$	Exact	Constructive polynomial time algorithms	No
Ng et al. (2002)	$1 \mid s_f, assembly, GT \mid \Sigma C_j$	Exact	Linear Arrangement problem of graph	No
Erel and Ghosh (2007)	$1 \mid s_f \mid \Sigma\, C_j$	Exact	Dynamic Programming Recursion	No
Hazir et al. (2008)	$1 \mid s_f \mid \Sigma F_j$	Heuristic	SA, GA, ACO, TS	Experimentation
Yue and Wan (2017)	$1 \mid CON, lin, C_{max} \leq K \mid \Sigma(\alpha E_j + \beta T_j + \gamma d + v_j u_j)$	Exact & Heuristic	B&B (e), TS (h)	Experimentation
Macker, et al. (2020)	$1 \mid s_f, assembly \mid \Sigma\, w_j C_j$	Exact & Heuristic	Series Parallel Digraph + Lawler, LB (h)	No
Kovalenko et al. (2020)	$1 \mid s_{jj'}, d_m \mid f(\Sigma\, C_j, \Sigma\, w_j U_j)$	Exact	Axiomatic approach of pareto set reduction	No
Çetinkaya et al. (2021)	$1 \mid s_f, assembly, GT \mid \Sigma C_j$	Exact & Heuristic	MILP (e), TS (h)	Experimentation
Lin et al. (2023)	$1 \mid s_f \mid \alpha \Sigma H_m + (1-\alpha)\, \Sigma\, T_j$	Exact & Heuristic	B&B with LB (e), CSA(h), CSAHH (h)	Experimentation
Li et al (2023)	$1 \mid s_f \mid \alpha \Sigma H_m + \beta \Sigma\, T_j + \gamma \Sigma\, C_j$	Exact & Heuristic	B&B with LB (e), WW (h)	Experimentation
Gupta et al. (2023)	$1 \mid s_f \mid w C_{max} + (1-w) \Sigma H_m$	Exact & Heuristic	MILP (e), B&B (e), local heuristics (h), WW(h)	Experimentation

Table 1. Summary Table for Single Machine Papers

Legend

ACO: Ant Colony Optimization

B&B: Branch & Bound

CSA: Cloudy Theoretical Simulated Annealing

CSAHH: Cloudy Theoretical Simulated Annealing Hyper-Heuristic Algorithm

GA: Genetic Algorithm

LB: Lower Bound

m: Total Number of Customer Orders

MILP: Mixed Integer Linear Programming

SA: Simulated Annealing

TS: Tabu Search

WW: Water-Wave Algorithm

An interesting topic related to single-machine scheduling is exploring issues for orders involving multiple job categories on a single machine. In this scenario, jobs are divided into several classes, and a setup time is needed when a machine switches from one job class to another due to the need to adjust the production equipment or change tools. Thus, the constraint s_f is for sequence-independent setup times where f indicates the family of products. Producers process a class of jobs that come from different orders from different clients, and each client's order will contain multiple classes.

The first paper with this type of setup was written by Gupta et al. (1997). They studied multiple job class scheduling with customer orders to minimize carrying costs and makespan $1|s_f\ |F_h\ (I\ |C_{max})$ where I = carrying costs to store finished jobs included in an order to be completed. The authors proposed two separate polynomial time algorithms for the two hierarchical scheduling problems:

- find a schedule that minimizes carrying cost amongst all schedules that minimize makespan.
- find a schedule that minimizes makespan amongst all schedules that minimize carrying costs.

In 2023 Gupta et al. (2023) explored again the topic using new methodologies. They considered a linear combination of the sum of the holding costs of all orders and the makespan of all jobs, $1\ |s_f|wC_{max} + (1-w)\Sigma H_m$. Initially the authors proposed two exact algorithms – a mixed integer linear programming formulation, a branch-and-bound algorithm – but MILP turned out to be not effective nor efficient to optimality and B&B could not solve instances with 12 or more jobs in a reasonable time. Therefore, they proposed several quicker heuristic algorithms and six variants of the water-wave optimization algorithms to find approximate solutions. The authors assessed the effectiveness and efficiency of the algorithms through computational experiments, but no validation scenario was defined and used to obtain empirical data.

The most studied objective function is ΣC_j.

Ng et al. (2002) studied the problem $1\ |s_f, assembly, GT\ |\Sigma C_j$ where "assembly" means that a job is completed when ready for assembly and "GT" stands for group technology, so the operations in each family are scheduled as batch. The authors supposed to have an instance of the linear arrangement problem of graphs and found optimal solution to minimize the sum of completion times ΣC_j for all orders. They discussed the complexity of the problem and derived that the GT does not affect the discussion. However, they did not specify any validation scenario or empirical data to support their proposal. Furthermore, there is no evidence of the effectiveness of their approach, even with simulation data.

Erel and Ghosh (2007) presented a dynamic programming-based algorithm that could solve approximately the problem of order scheduling on a single machine with setup times $1 \,|s_f\, |\, \Sigma\, C_j$. However, the authors did not define or use a validation scenario to obtain empirical data, nor did they provide any evidence of the effectiveness of the proposed algorithm, even with experimental data.

In the same year Macker et al (2020) provided an optimal solution with Series Parallel Digraph and Lawler's algorithm and a Lower Bound approximation for the problem $1|s_f, assembly\,|\Sigma\, w_j C_j$. No validation scenario was defined and used to obtain empirical data. No evidence of the effectiveness of the proposal even with simulation data was given.

Çetinkaya et al. (2021) aimed to find the optimal schedule to minimize the total completion time with sequence-independent setups ($1\,|s_f, assembly, GT\,|\Sigma C_j$). The authors formulated two exact mixed-integer linear programming (MILP) models with job-based processing approach and a Tabu Search heuristic algorithm. They compared job-based and order-based processing approach with setup and no-setup and demonstrated that job-based approach gives better results when there are setup times. The comparison of the algorithms was done only through computational experiments, but the authors didn't provide a validating industrial case study.

The only study related to Flow Time minimization was conducted by Hazir et al. (2008). The researchers focused on minimizing the sum of customer flow times ΣF_j with independent setup times s_f ($1\,|\,s_f\,|\Sigma F_j$). They applied several heuristics: Ant Colony algorithm, Genetic Algorithm, Tabu Search, and Simulated Annealing algorithm but it emerged that the results were influenced by the problem size not the algorithm chosen. However, Tabu Search and the Ant Colony algorithms provided a faster solution. No validation scenario was defined and used to obtain empirical data. Only experimentation was designed to randomly generate problems to compare the quality of the solutions.

Fast forward to 2017, some papers related to multi-criteria maximization starts to appear.

Yue and Wan (2017) considered a firm that must provide a common due date to its customers while also managing the processing times of their orders. The problem type is $1 \mid CON, lin, C_{max} \leq K \mid \Sigma(\alpha E_j + \beta T_j + \gamma d + v_j u_j)$ where CON indicates common due date assignment method, lin stands for linear consumption of resources,. $C_{max} \leq K$ is the deadline given to complete the orders. The objective of the study was to minimize the total costs of earliness, tardiness, due date assignment, and extra resource consumption (v_j is the cost of unit resource allocated). They propose a B&B algorithm to get an optimal solution, and a heuristic Tabu Search to have an approximate solution. Some computational experiments were implemented to compare the algorithms, but no validation scenario was defined.

Kovalenko et al. (2020) explored the $1 \mid s_{jj'},\ d_m \mid f(\Sigma\, C_j, \Sigma\, w_j U_j)$ with a customer due date and sequence-dependent setup time whenever a product type is switched. Two objective functions were considered: minimize the sum of the order completion times, and the sum of weights of orders that are satisfied before their due dates. The authors proposed an axiomatic approach of Pareto set reduction to maximize this bi-criteria problem. No validation scenario was defined and used to obtain empirical data. There is no evidence of the effectiveness of the proposal, even with experimental data.

Lin et al. (2023) tried to define a schedule that minimizes a linear combination of the total tardiness cost of given orders and the total holding cost related to the delay of an order caused by processing a particular class of job ($1 \mid s_f \mid \Sigma H_m, \Sigma\, T_j$). A branch-and-bound method with a lower bound definition was used for optimal solution, several local heuristics, and several cloudy theoretical simulated annealing hyper-heuristics were proposed to find approximate solutions. Some computational simulations were conducted to assess the quality of the algorithms. However, no validation scenario was defined or used to obtain empirical data.

Finally, Li et al. (2023) delved into the problem $1\,|s_f|\,\alpha\Sigma H_m + \beta\Sigma\, T_j + \gamma\Sigma\, C_j$ and proposed a tri-criteria model by adding the total completion times of all jobs to the linear combination of total holding costs and total tardiness. As in the studies of the previous researchers, a branch-and-bound method was used for optimal solutions, whereas several heuristics and four variants of water wave algorithm were proposed to find approximate solutions. No validation scenario was defined and used to obtain empirical data. The comparison of the algorithms was based solely on computational experiments.

4.2 Parallel Machine Customer Order Scheduling Problem

The single stage parallel machine order scheduling problem (PMOSP) involves scheduling multiple jobs simultaneously on multiple machines. The objective is to determine the allocation of the jobs to the machines and the order in which they should be processed.

The Graham notation defines three types of machines in parallel: identical (P), uniform with different speeds (Q), and completely unrelated (R). Therefore, it is possible to define different types of PMOSP.

As reported in the papers Leung et al. (2005b) and Dauod et al. (2018), it is possible to view parallel machines differently by considering the shop conditions in which they operate. In order scheduling models, the machine environment can be either fully dedicated or flexible.

A fully dedicated environment means that each machine can only process one type of job. On the other hand, a flexible environment allows machines to process more than one type of job. Flexible machines can either be fully flexible, which means they can process all types of jobs, or multi-purpose, which implies that they can only process a specific subset of jobs. It is worth noting that the multi-purpose case lies between the fully flexible and the fully dedicated cases. In this system, each job can only be processed on a specific set of eligible machines, referred to as the job's processing set.

The shop condition implicates a different resolution of the order scheduling problem.

4.2.1 Flexible Machine

The following section presents a perspective on solving the Customer Order Scheduling problem for Flexible machines. The 10 research papers have been summarized in the following table.

Paper	Problem type	Methodology	Algorithms	Industrial case
Blocher and Chhajed (1996)	$P_m \mid \mid \Sigma C_j$	Exact and Heuristic	2-component sequential Heuristics, 2-component dynamic heuristics, MILP (e), LB (e)	Experimentation
Yang and Posner (2005b)	$P_m \mid \mid \Sigma C_{batch}, C_{max}, L_{max}$ $P_m \mid K \mid \Sigma C_{batch}, C_{max}$	Heuristic	SPT models(h), LPT models (h)	No
Yang and Posner (2005)	$P_m \mid \mid \Sigma C_{batch}$	Heuristic	SPT & LPT heuristics with worst-case-bound (h)	Experimentation
Leung et al. (2007b)	$Q_m \mid \Pi \mid \Sigma w_j C_j$ $Q_m \mid \Pi, pmtn \mid \Sigma w_j C_j$	Heuristic	WSPT-LPT (h) with worst-case-bound	Experimentation
Leung et al. (2008)	$P_m \mid \Pi \mid \Sigma w_j C_j$	Exact & Heuristic	2-component sequential Heuristics, 2-component dynamic heuristics, LB + UB (e)	Experimentation
Xu et al. (2015)	$R_m \mid split \mid \Sigma C_j$	Exact & Heuristic	LB (e), UB(e), heuristics with worst-case-bound (h)	Experimentation
Cao et al. (2017)	$P_2 \mid sum, OO \mid C_{max}$	Exact	LB (e), exact algorithms (e)	No
Wu et al. (2018)	$P_m \mid \mid \Sigma U_j$	Exact & Heuristic	B&B (e), PSO (h), GA (h)	Experimentation
Dauod et al. (2018)	$P_m \mid \mid f(C_{max}, delays)$	Exact & Heuristic	Min-max Pareto (e), Math.Model (e), GA (h), LPT (h), LTW(h)	Experimentation
Zhao et al. (2022)	$R_m \mid size \mid C_{max}$	Exact & Heuristic	LB (e), UB (e), Cut-Add PSO (h)	Experimentation

Table 2. Summary Table of Flexible Machine Papers

Legend:

B&B: Branch & Bound

GA: Genetic Algorithm

LB: Lower Bound

LPT: Longest Processing Time

LTW: Least Total Workload

MILP: Mixed Integer Linear Programming

PSO: Particle Swarm Optimization

SPT: Shortest Processing Time

UB: Upper Bound

WSPT: Weighted Shortest Processing Time

The customer order scheduling on a set of parallel identical machines was introduced in 1996 by Blocher and Chhajed (1996). The article aimed to minimize the sum of the lead times of the orders considering all orders available from the start, so the goal was to minimize the sum of the orders' completion times $P_m \mid \mid \Sigma C_j$. To achieve an approximate but effective solution, the authors proposed several heuristics both 2-component sequential (e.g. SOAPT-LPT) and 2-component dynamic ones (greedy). Additionally, a mixed-integer linear programming method and some lower bounds were suggested to obtain the optimal solution and evaluate the effectiveness of the heuristics. No validating real-world scenario was defined or used to collect empirical data.

Yang (2005b) analyzed the complexity of various COSP types with P_m or P_2 , with a capacity constraint on the machines K or not and with the different objective functions separated considering job processed in classes (batch) to minimize the WIP.

These are some of the problems studied:

$P_2| K|C_{max}$, $P_2|K|\Sigma C_{batc}$, $P_m| |\Sigma C_{batch}$, $P_m|p_j = 1| L_{max}$

In the picture below there is a comprehensive table of all the cases analyzed.

The complexity of customer order scheduling problems

Objective	Number of machines	K	Variation	Complexity results	
$C_{B_{\max}}$	2	Yes		Unary NP-complete	Thm. 6.1
$\sum C_{B_i}$	2			Binary NP-complete	Rmrk. 4.1 or Thm. 4.1
$\sum C_{B_i}$	2		π	$O(\bar{p}^2 n \sum_{i \in B} P_i)$	Rmrk. 4.3
$\sum C_{B_i}$	2		J_F	Unary NP-complete	Thm. 4.2
$\sum C_{B_i}$	2		R	Unary NP-complete	Cor. 4.1
$\sum C_{B_i}$	2	Yes		Unary NP-complete	Thm. 6.2
$\sum C_{B_i}$				Unary NP-complete	Rmrk. 5.1
$\sum C_{B_i}$			$p_j = 1$	$O(b \log b)$	Thm. 5.2
$L_{B_{\max}}$			$p_j = 1$	$O(b \log b)$	Thm. 5.3
$\sum w_i C_{B_i}$			$p_j = 1$	Binary NP-complete	Thm. 5.4

Figure 14. Complexity of COS - Yang et al. (2005b)

Yang and Posner (2005) resumed the investigation on one of the heuristics presented by Blocher and Chhajed (1996) for the case $P_m| |\Sigma C_{batc}$ and $P_m| |C_{max}$deriving a worst-case bound for the original heuristic and proposing a new heuristic along with its worst-case bound. The heuristics proposed were compared to LB optimal results through computational study. However, neither of the two methods has been considered to be consistently superior to the other. Besides, in this work no validation scenario was defined or used to collect empirical data.

Ten years later, Xu et al. (2015) proposed three different heuristics with their worst-case-bound to minimize the sum of completion times $\sum C_j$ with unrelated parallel machine and type splitting property. They calculated optimal LB and UB and compared them with computational experiments under various application scenarios for each heuristic to evaluate their effectiveness. However, no validation industrial user case was defined or used to obtain empirical data.

Leung et al. (2007b) studied the minimization of the weighted completion time with uniform machines $\sum w_j C_j$ and an arbitrary Π number of product types, due to its complexity, they proposed several heuristics (both static and dynamic) their worst-case bound for order scheduling on uniform machines, one heuristic for non-pre-emptive scheduling (H_{NP}) and one for pre-emptive scheduling (H_P). No validation scenario was defined and used to obtain empirical data. To evaluate the heuristics empirically, the authors generated a data set to simulate situations with different problem sizes and compare the algorithms with the experimental LB.

Leung et al. (2008) studied the minimization of the weighted completion time with identical machines $\sum w_j C_j$ by modelling sequential two-phase heuristics and dynamic two-phase heuristics with their worst case. The author generated problem instances to compare the algorithms with optimal LB and UB. It was revealed that static WSTP-based heuristics performed better than dynamic ones. No real-world use case.

Cao et al. (2017) considered for the first time an online order OO scheduling problem on two identical machines with same order size sum. In this peculiar context when not all the orders are known at time 0, the authors developed two exact algorithms to minimize the makespan C_{max}. They didn't propose any real-world use case.

The study of C_{max} was continued by Zhao et al. (2022).
They explored the optimization of customer order scheduling and resource allocation in a dynamic environment with parallel machines of varying speeds (unrelated parallel machines) and limited resources $(size)$. Since customer orders arrived stochastically and contained random product amounts, the researchers sought to jointly optimize order scheduling and resource allocation to minimize the makespan C_{max}. To tackle this complex and random problem, they propose the Cut-Add PSO (particle swarm optimization) which uses bound information to cut inferior solutions. The authors simulated empirical data to test the efficiency of the PSO algorithm generated but the study did not define any validation user case scenario.

Dauod et al. (2018) studied the multi-objective optimization of COSP through several heuristics and demonstrated the efficiency with simulations. The authors wanted to minimize the makespan and the order collation delay $delay$ which is the completion time difference between the first and the last job within the same order. The problem can be summarized as $P_m \mid \mid f(C_{max}, delays)$. The context was the mail-order pharmacy automation system, however no real-world data were used to validate the proposed method: genetic algorithm, min-max Pareto approach. Their study focused on both flexible and dedicated machines.

A study that explored a different objective function was the one proposed by Wu et al. (2018). They applied a branch-and-bound algorithm to identify an optimal solution and then a particle swarm optimization algorithm and a genetic algorithm to find a near-optimal solution for the minimization of the number of tardy jobs, $\sum U_j$. Simulations have been carried out to test the efficiency of the different procedures used. However, no validation industrial user case was defined or used to obtain empirical data.

4.2.2 Dedicated Machines

The most common version of the Customer Order Scheduling problem is the dedicated machines variant.

A summary table of all the 35 papers will be presented in the following pages, and after the Legend they will be discussed more in detail.

Paper	Problem type	Methodology	Algorithms	Industrial case
Sung and Yoon (1998)	$R_2\| \|\Sigma w_j C_j$	Heuristic	WSPT-heuristics with worst-case	Experimentation
Leung et al. (2005)	$PD_m\| \|\Sigma C_j$	Heuristic	SPTL (h), ECT(h),TS (h)	Experimentation
Leung et al. (2006)	$PD_m\|d_j = d \|\Sigma U_j$ $PD_m\| \|\Sigma U_j$	Exact & Heuristic	MSMC (h), GMH (h), Exact algorithm (e)	Experimentation
Ahmadi et al. (2005)	$PD_m\| \|\Sigma C_j$	Exact & Heuristic	LB (e), ADH (h), LDH (h), SDH (h)	Experimentation
Huang et al. (2005)	$PD_m\|d_j, size \|\Sigma w_j C_j$	Exact	Optimal lead time (e)	Experimentation
Leung et al. (2007)	$PD_m\|r_j \|\Sigma w_j C_j$ $PD_m\| \|\Sigma w_j C_j$	Exact & Heuristic	Time interval LP based algorithm (e), WSPT(h), WSMP(h), WSMC(h)	Experimentation
Leung et al. (2008)	$PD_m \| \Pi \|\Sigma w_j C_j$	Exact & Heuristics	2-component sequential Heuristics, 2-component dynamic heuristics, LB + UB (e)	Experimentation
Wang and Chen (2007)	$PD_m \| \|\Sigma w_j C_j$	Heuristic	HLP (e)	No
Lin and Kononov (2007)	$PD_m\| \|\Sigma U_j$ $PD_m\| \|\Sigma w_j U_j$	Exact & Heuristic	LPB (e), heuristics (h)	No
Garg et al. (2007)	$PD_m \| OO \|\Sigma w_j C_j$ $PD_m \| OO \|\Sigma F_j$	Exact	LB (e)	No
Wang et al. (2013)	$PD_m \| \|\Sigma w_j C_j$	Exact	Linearization of Quadratic Programming Model (e)	Experimentation

Paper	Problem type	Methodology	Algorithms	Industrial case
Lee (2013)	$PD_m \mid \; \mid \Sigma T_j$	Exact & Heuristic	B&B (e), TPT-EDD (h), MPT-EDD(h), EDD-MCT(h), OMDD (h)	Experimentation
Xu et al. (2016)	$PD_m \mid d_j \mid \Sigma T_j$	Exact & Heuristic	B&B with LB (e), SA (h), PSO (h), MDD (h)	Experimentation
Lin et al. (2017)	$PD_m \mid r_j \mid \Sigma \mathrm{w_j} C_j$	Heuristic	WSPT-heuristics (h), WECT-heuristic (h), IG(h)	Experimentation
Lin et al. (2017b)	$PD_m \mid r_j \mid \Sigma C_j$	Exact & Heuristic	Standard PSO (h), opposite-based PSO (h), PSO with linearly decreasing inertia weight (h), B&B (e)	Experimentation
Shi et al. (2017)	$PD_m \mid \; \mid \Sigma \mathrm{w_j} C_j$	Exact & Heuristic	MILP (e), Hybrid nested Partitions heuristic (h), WSPT(h), WSMP(h), WSMC(h), WSPL(h), WECT(h), local search methods (h)	Experimentation
Framinan and Perez-Gonzalez (2017)	$PD_m \mid \; \mid \Sigma C_j$	Heuristics	Look-ahead Heuristics (h), GSA (h)	Experimentation
Framinan and Perez-Gonzalez (2018)	$PD_m \mid \; \mid \Sigma T_j$	Heuristics	Constructive heuristics (h), Matheuristics (h)	Experimentation
Kung et al. (2018)	$PD_m \mid r_j \mid \Sigma C_j$	Exact & heuristics	B&B (e), SA (h), GA (h)	Experimentation
Daud et al. (2018)	$PD_m \mid \mid C_{max}, delays$	Exact & Heuristic	Min-max Pareto (e), Math.Model (e), GA (h), LPT(h), LTW(h)	Experimentation
Wu et al (2018)	$PD_m \mid \; \mid \alpha \Sigma T_j + \beta \Sigma F_j$	Exact & Heuristic	LB (e), Hybrid IGA(h), PSO (h), SPT-heuristics (h)	Experimentation
Wu et al (2019)	$PD_m \mid r_j \mid \Sigma \mathrm{w_j} C_j$	Exact & Heuristic	LB (e), B&B (e), IGA (h), SPT-heuristics (h)	Experimentation
Lin et al. (2019)	$PD_m \mid r_j \mid \Sigma \mathrm{w_j} \mathrm{U_j}$	Exact & Heuristic	Basic ABC (h), Hydrid ABC (h), B&B (e)	Experimentation

Paper	Problem type	Methodology	Algorithms	Industrial case
Riahi et al. (2019)	$PD_m\| \ \|\Sigma C_j$	Heuristics	PCE (h), PSA (h)	Experimentation
Chen and Li (2020)	$PD_m\|rej, r_j\|D_{max} + \Sigma \mathrm{e_j}$	Exact & Heuristic	IP(e), DP(e), LP-approx (h), Rejection-based algor. (h), Production-Delivery_Rejection Algor. (h)	No
Wu et al. (2021)	$PD_m\| \ \|\Sigma \mathrm{T_j}$	Exact & Heuristics	LB (e), IGA (h)	Experimentation
Shi et al. (2021)	$PD_m\| \ \|\Sigma C_j$	Heuristics	Adaptive Local Search (h)	Experimentation
De Abreu et al. (2022)	$PD_m\|missing\|\Sigma \mathrm{T_j}$	Exact & Heuristic	MILP (e), Constraint Programming Model (e), SR-Q (h), BRKGA (h)	Experimentation
De Athayde Prata et al. (2021)	$PD_m\|s_j\|C_{max}$	Exact	MILP (e)	Experimentation
De Athayde Prata et al. (2022)	$PD_m\|s_j\|C_{max}$	Exact & Heuristic	MILP (e), DDE (h), local search (h)	Experimentation
Li et al. (2022)	$PD_m\| \ \|\Sigma w_j U_j$	Exact & Heuristic	LB (e), B&B (e), Moore's algorithm (h), GA (h), GAHH (h)	Experimentation
De Athayde Prata et al. (2022b)	$PD_m\|d_j\|\Sigma T_j$	Heuristics	SR (h)	Experimentation
Hoffman et al. (2022)	$PD_m\| \ \|\Sigma C_j$	Heuristic	IGA (h), NEH (h), SA (h)	Experimentation
Antonioli et al. (2022)	$PD_m\|s_j, d_j\|\Sigma T_j$	Exact & Heuristic	MILP (e), OMDD (h), SPAM (h), SPAM-JPO (h)	Experimentation
Zipfel et al. (2023)	$PD_m\|s_j, d_j\|\Sigma \mathrm{w_j} T_j$	Exact & Heuristic	LB (e), MILP (e), ILS (h)	Experimentation

Table 3. Summary Table for Dedicated Machine Papers

Legend:

ABC: Artificial Bee Colony

ADH: Assignment Dual Heuristic

ECT: Earliest Completion Time first

EDD-MCT: Earliest Due Date – Maximum Completion Time

DDE. Discrete Differential Evolution

GHM: Generalized Hodgson-Moore Heuristic

GSA: Greedy Search Algorithm

HLP: Hybrid Linear Programming

IGA: Iterative Greedy Algorithm

ILS: Iterated Local Search

JPO: Job-Position Oscillation

LDH: Lagrangian Dual Heuristic

MPT-EDD: Maximum Processing Time – Earliest Due Date

MSMC = Multi-Set Multi-Cover Problem

NEH: Nawaz-Enscore-Ham heuristic

OMDD: Order Scheduling Modified Due Date

PCE: Permutation Construction and Exploration Algorithm

PSA: Perturbative Search Algorithm

SDH: Surrogate Dual Heuristic

SPAM: Same Permutation in All Machines

SPTL: Shortest Processing Time on the machine that currently has the Largest Load

TPT-EDD: Total Processing Time – Earliest Due Date

WSMP: Weighted Shortest Maximum Processing Time First

WSMC: Weighted Smallest Maximum Completion Time First

WSPT: Weighted Shortest Processing Time First

Although the concept of order scheduling in a dedicated machine environment has been around since the early 1990s, the first solution to the problem was not proposed until 1998 by Sung e Yoon (1998). The focus of the study was to minimize the weighted sum of completion time $\sum w_j C_j$ on two independent parallel machines, $R_2| |\Sigma \mathrm{w_j} C_j$. The authors proposed two heuristics incorporating WSPT rule to find approximate solutions. Empirical evaluations of the heuristics' performance were conducted, but real-world industrial scenarios were not used to obtain empirical data.

Huang et al. (2005) studied MTO production. The order scheduling problem was split into three subproblems: resource capacity (in terms of material resources and machine resources bottleneck), order priority, and lead time. The problem can be schematized as $PD_m|d_j, size\ |\Sigma w_j C_j$. The researchers developed an optimal lead time solution. Data from a factory that produces belt pulleys was used to demonstrate the solution. The study aimed to emphasize the importance of quick delivery and on-time delivery to increase a company's market position.

Leung et al. (2007) continued their studies on the minimization of the total weighted completion time $\sum w_j C_j$ started in 2005. They introduced new near-optimal heuristics (WSPT, WSMP, WSMC) for preemptive and non-preemptive cases, with and without release dates. These heuristics used static and dynamic priority rules and obtained the worst-case bounds function of the number of machines and their speed differences. However, no validation scenario was defined and used to retrieve empirical data. Only simulation was done to evaluate and compare the effectiveness of the developed heuristics.

Wang and Cheng (2007) introduced for the same problem $PD_m|\ |\Sigma w_j C_j$an hybrid linear programming (HLP) but they didn't include any real-world validation or computational test to verify the efficiency of the method.

Leung et al. (2008) studied the minimization of the weighted completion time with dedicated machines $\sum w_j C_j$ by modelling sequential two-phase heuristics and dynamic two-phase heuristics with their worst case. The author generated problem instances to compare the algorithms with optimal LB and UB. It was revealed that static WSTP-based heuristics performed better than dynamic ones. No real-world use case.

Fast forward to 2013, Wang et al. (2013) proposed a quadratic formulation to minimize the total weighted completion time $\sum w_j C_j$. The authors, then, transformed the quadratic model into an equivalent mixed-integer linear programming model and eliminated some constraints to reduce the problem size. The achieved linear model was tested with some experimental data, but no real-world scenario was used for its validation.

Lin et al. (2017) conducted a multi-facility order scheduling study with ready times to find an optimal schedule to minimize the total weighted completion time $PD_m| r_j| \sum w_j C_j$. They proposed two types of iterative greedy algorithms and modified five existing heuristics to find near-optimal solutions. The statistical tests showed that the two greedy algorithms performed better for larger order sizes, with one algorithm in particular achieving solutions very close to optimal. No validation scenario was defined or used to obtain empirical data.

Shi et al. (2017) built a mixed-integer linear model for $PD_m \mid \; |\Sigma w_j C_j$. This is the first study to investigate which problem size can be solved optimally using this linear formulation. The authors then developed a hybrid nested partitions heuristic, which is a randomized method combining global and local searches, to provide suitable solutions for large-scale problems within a short period, even for many instances. The algorithms were tested through computational experiments to assess their efficiency in handling problem size. However, no real-world case data was used.

Wu et al. (2019) addressed the challenge of optimizing the total weighted completion time $\sum w_j C_j$ while also considering release dates r_j. To achieve this, they first devised several dominance rules and two lower bounds that could be used within a branch-and-bound methodology to find an exact solution. They then made modifications to five existing heuristics and adopted an iterative greedy (IG) algorithm to find a near-optimal solution. Finally, the team evaluated and compared the performance of the algorithms using one-way analysis of variance and Fisher's least significant differences, although they did not use empirical data from real-world industrial cases.

Garg et al. (2007) relaxed the assumption of knowing all the orders to be scheduled from the beginning. The article focused still on minimizing the weighted completion time $\sum w_j C_j$ with multiple release dates and they introduced the so called “online algorithms” to find an optimal solution and derived a near-optimal algorithm based on linear programming. The authors also proposed lower bounds in both the offline and online settings for the total flow time $\sum F_j$ minimization issue. Finally, they obtained upper bounds and lower bounds for the completion time minimization problem $\sum C_j$. The authors studied the complexity of the solutions, but no computational tests or industrial use cases were used to validate their theories.

The minimization of the total completion time for dedicated machines was studied also by Leung et al. (2005). Firstly, the authors calculated the complexity of $PD_m \mid \mid \Sigma w_j C_j$ and $PD_m \mid \mid \sum w_j T_j$. Then, they proposed two heuristics (SPTL and ECT) and a tabu search metaheuristic to minimize total completion time, $PD_m \mid \mid \sum C_j$, and provided a literature review comparing their results to previous research in the field. The study only used computational data and did not include any real-world use cases.

Ahmadi et al. (2005) examined a make-to-order enterprise (manufacturer of semifinished lenses) that wanted to minimize the weighted sum of customer order delivery times $PD_m \mid \mid \Sigma C_j$. The authors proceed to present an optimal schedule with lower-bounds procedures and some near-optimal heuristic solutions (ADH, LDH, SDH) to minimize average job completion times for dedicated machines. No validation scenario was defined and used to obtain empirical data. Only computational experiments were done to evaluate and compare the effectiveness of the developed methods.

Twelve years later, Framinan and Perez-Gonzalez (2017) proposed a new constructive heuristic that proved to be more effective than the heuristic by Leung et al (2005) to minimize the sum of the completion times of the orders $\sum C_j$. Additionally, the authors developed a greedy search algorithm that was tested to provide higher quality solutions while using less CPU effort than the tabu search method of Leung et al. (2005). The paper did not involve any industry player.

Lin et al. (2017b) focused on the same objective function but introducing different release dates for each order. The problem involves various agents competing for the use of shared processing resources, and each of them aims to minimize their own set of jobs' completion time $PD_m \mid r_j \mid \sum C_j$. The objective of the problem is to find a schedule that satisfies each agent's requirement for its objective function or to minimize a combination of the agents' objective functions. The study proposed three particle swarm optimization algorithms to get near-optimal solutions: a standard PSO, a PSO with a linearly decreasing inertia weight, and an opposite-based PSO method. The proposed artificial algorithms' efficiency was demonstrated through computational tests, although no validation scenario was defined or used to obtain empirical data.

Kung et al. (2018) explored the same problem type, proposing some methods for optimizing the total completion time $\sum C_j$ that considers unequal ready times, $PD_m \mid r_j \mid \Sigma C_j$. The method involves a branch-and-bound approach with two lower bounds to obtain exact solutions. Additionally, the authors suggested four simulated annealing approaches and four heuristic genetic algorithms for obtaining approximate solutions. All the proposed methods were tested experimentally but have not yet been validated using real use cases.

A year later, Riahi et al. (2019) focused on minimizing the sum of completion times $\sum C_j$ without considering release dates, $PD_m \mid \mid \Sigma C_j$. They proposed two algorithms – a constructive search algorithm and a perturbative search algorithm – to achieve this goal. To evaluate the effectiveness of these algorithms, they used the benchmark set generated by Framinan and Perez-Gonzalez (2017). The computational tests demonstrated that the new methods outperformed existing state-of-the-art COSP algorithms. It's worth noting that they did not define or use any validation scenario to collect empirical data.

The same problem was resumed by Shi et al. (2021). They aimed to minimize the total completion time of all the customer orders $PD_m \mid \mid \Sigma C_j$. The study proposed a learning-based two-stage optimization method: for the first step the authors used a learned dispatching rule, then for the second step they devised an adaptive local search. It is worth noting that no validation scenario was defined and used to obtain empirical data. Instead, only computational tests were conducted to evaluate and compare the effectiveness of the developed heuristics.

Hoffman et al. (2022) built upon the previous research which aimed to minimize the total completion time $PD_m|\ |\Sigma C_j$. They developed two iterated greedy algorithms (IGA). Comparison criteria were established, and the results showed that both IGAs outperformed the algorithms from the literature.
Finally, while minimizing the total completion times of the orders De Athayde Prata et al (2022) considered for the first-time sequence-dependent setup times $PD_m \mid s_j \mid \Sigma C_j$. They proposed an exact MILP and iterative algorithms of local search and conducted extensive computational experiments to demonstrate their efficiency.

Only two papers considered the minimization of the makespan.
Dauod et al. (2018) studied the multi-objective optimization of COSP through several heuristics and demonstrated the efficiency with computational experiments. The authors wanted to minimize the delays and the makespan $PD_m \mid \mid C_{max}, delays$ in mail-order pharmacy automation system, however no real-world data were used to validate the proposed method: genetic algorithm, min-max Pareto approach. Their study focused on both flexible and dedicated machines.

De Athayde Prata et al. (2021) considered sequence-dependent setup times and minimized the makespan, $PD_m \mid s_j \mid C_{max}$. They proposed an exact MILP and compared their method to state-of-the-art techniques from related problems with experimental tests. However, they did not define or use a validation scenario to obtain empirical data.

The third most explored objective function for dedicated machines– after weighted completion time and total completion time – is the minimization of total tardiness of the orders.
This problem $PD_m \mid \ \mid \Sigma T_j$ was firstly studied by Lee (2013). He proposed four heuristic algorithms to have near-optimal solutions and used the minimum of their solutions as initial UB for Branch and Bound algorithm. The performances of the algorithms proposed, both heuristics and B&B, were evaluated via numerical experiments, no industrial use case was included to validate the solutions.

Xu et al. (2016) proposed the use of learning effects to minimize total tardiness$\sum T_j$, $PD_m\ |d_j\ |\Sigma T_j$. Firstly, they solve it with a branch-and-bound algorithm that incorporated dominance rules and three lower bounds to get an optimal solution. Then the researchers explored the effectiveness of simulated annealing and particle swarm optimization algorithms. No validation scenario was defined and used to obtain empirical data. Only simulation was done for the comparison of the algorithms.

Framinan and Perez-Gonzalez (2018) resumed the study on minimizing the total tardiness of the orders $\sum T_j$ without considering release dates as constraints $PD_m\ |\ |\Sigma T_j$. The authors proposed a constructive heuristic that takes into account the impact of the unscheduled jobs in the objective function to arrive at a more balanced decision. They also proposed a close-to-optimal metaheuristic strategy that has been shown to provide high-quality solutions even for larger instances. However, this study did not include any real-world data.

Wu et al. (2021) investigated a COSP with various uncertainty factors such as machine breakdowns, unstable operator performance, and changing working conditions. In this scenario, it is assumed that both job processing times and due dates are dependent on the situation. The objective of the study was to find the best schedules that minimize the maximum total tardiness of n customer orders $\sum T_j$ across different possible scenarios, $PD_m\ |\ |\Sigma T_j$. The proposed algorithms were evaluated using statistical methods, and the effectiveness of the developed heuristics was compared through experiments. However, no validation scenario was defined or used to obtain empirical data.

De Abreu et al. (2022) wanted to minimize total tardiness while also accounting for missing operations, $PD_m|\ missing\ |\ \Sigma\ T_j$. The study considered order-dependent setup times; therefore, they were included in processing times. The study highlighted the importance of considering missing operations in industries such as pharmaceuticals – specifically in laboratories for quality control of raw materials, in-process products, and completed goods –, where samples must follow a specific process, therefore orders are not processed on all machines. The study emphasized the importance of correct resource allocation in reducing operational costs and lead times. However, it did not use validation scenarios to obtain empirical data. Instead, computational analysis were used to compare the algorithms.

De Athayde Prata et al. (2022b) presented a size-reduction algorithm (SR) for minimizing the total tardiness $\sum T_j$ while considering due dates d_j, $PD_m \mid d_j \mid \Sigma T_j$. They extended the traditional size-reduction approach, which is based on processing times, to an approach based on due dates. The proposed algorithm tested to find better solutions in lower computational times in comparison with the metaheuristic presented by Framinam and Perez-Gonzalez (2018) which was the best existing algorithm for the problem under study.

In the current literature on customer order scheduling, setup times are often included in the processing times. However, Antonioli et al. (2022) proposed a different approach. They assign each order a sequence-dependent setup time and a due date based on the customer's requirements, $PD_m \mid s_j, d_j \mid \Sigma T_j$. To minimize the total tardiness $\sum T_j$, they developed both a mixed-integer linear programming (MILP) model and a hybrid metaheuristic model. After performing computational tests on all the new and state-of-the-art methods, both the developed methodologies presented better results compared with the algorithms devised by Framinan and Perez-Gonzalez (2018).

Wu et al. (2018) focused on minimizing the linear sum of the maximum tardiness and the total flowtime, $PD_m \mid \mid \Sigma T_j + \Sigma F_j$. To find the optimal solution, they derived several dominance relations and an exact lower bound, whereas they proposed three modified heuristics for finding near-optimal solutions. To solve the problem, they also suggested a particle swarm colony method and a hybrid iterated greedy algorithm. Finally, they conducted a computational experiment to evaluate the performance of all the proposed algorithms. It is worth noting that no validation scenario was defined or used to obtain empirical data.

Zipfel et al. (2023) decided to study the not only the tardiness but the weighted tardiness in the context of additive manufacturing, $PD_m \mid d_j, s_j \mid \Sigma w_j T_j$. The research focused on a company that provides on-demand 3D printing services to its customers. The manufacturer's main objective was to fulfill all customer orders within the given due dates as accurately as possible. To achieve this, the study aimed to minimize the total weighted tardiness of orders $\sum w_j T_j$ while considering the material types of the ordered parts, sequence-dependent setup times and batch processing machines. A mixed-integer programming model and an iterated local search (ILS) are suggested to derive optimal solutions. The effectiveness of these heuristics was evaluated and compared using comprehensive test data through computational experiments. However, no validation scenario was defined and used to obtain empirical data.

Another due-date related objective function that the researchers explored is the minimization of the total number of tardy jobs (weighted or not).
Leung et al. (2006) presented a study with dedicated parallel machine with common due date for the orders focusing on minimizing the maximum lateness L_{max} and the total number of tardy orders $\sum U_j$. In particular, for the $PD_m|d_j = d\ |\Sigma U_j$ they verified that MSMC algorithms can solve also this type of problems, whereas they proposed a GHM heuristic, and an exact algorithm based on bounding strategy for $PD_m|\ |\Sigma U_j$. Several problem instances were generated to prove the effectiveness of the solutions. No validation from industrial cases.

Lin and Kononov (2007) carried forward the research on minimizing the number of tardy orders $\sum U_j$. They proposed a linear programming-based algorithm that could be used for both weighted and unweighted variants of the problem and a heuristic algorithm for the unweighted case. However, they did not define or use any validation scenario to obtain empirical data. There is no evidence of the effectiveness of the proposed algorithm, even with computational data.

Lin et al. (2019) focused on $PD_m\ |\ r_j\ |\Sigma w_j U_j$. The researchers employed artificial algorithms, including four basic bee colony algorithms and four hybrid bee colony algorithms, to search for optimal and approximate solutions. They evaluated the performance of all eight algorithms using statistical analysis of variance and least significant variance. However, the study lacked validation from real data.

Li et al. (2022) relaxed the common assumption of fixed component processing times, ready times, and due dates and instead considered them to be situation dependent.

The objective of the study was to identify a robust sequence of orders that would minimize the sum of the weighted tardy orders $\sum w_j U_j$. The researchers derived dominant properties and a lower bound for the B&B approach to determine an optimal solution. They then evaluated the performance of three variants of Moore's algorithm, a genetic algorithm, and a genetic-algorithm-based hyper-heuristic that incorporated seven proposed low-level heuristics. The GA and GAHH approach resulted efficient in terms of high-quality solutions and effort both for small and large-sized orders. The study did not use any real industrial cases in its evaluation.

A study that differs from the others in terms of constraints and objective function considered is the one by Chen and Li (2020). They were the first to consider rejected orders, rej. The problem aimed to minimize the linear sum of the maximum delivery time for accepted orders D_{max} and the total penalty for rejected orders Σe_j. The problem can be summarized as $PD_m \mid rej, r_j \mid D_{max} + \Sigma e_j$. Three approximation algorithms with their worst-case were presented and analyzed. Furthermore, the researchers presented and tested a dynamic programming algorithm and an approximation algorithm for a scenario where an arbitrary number of machines were involved.

4.3 Customer Order Scheduling Problem for Other Shop

The Customer Order Scheduling literature includes studies in multi-stage environments.

As explained in Chapter 2, it is possible to distinguish three shop conditions:

- Flow shop: all jobs must be worked on a group of m machines according to the same precise sequence of tasks created by a set of precedencies constraints. Material flow is unidirectional.
- Job shop: each job has a different number of operations and different sequences on different machines, so the material flow is multidirectional.
- Open shop: fixed number of operations, no sequence is specified.

The shop's condition has largely been considered in the general scheduling problem. However, when it comes to the customer order scheduling problem, the multi-stage environment is still a relatively new subject. In preparing this State-Of-The-Art thesis, only seven papers were found on the main web sources during the investigation.

The following table wants to summarize the research papers found on shop conditions.

Paper	Problem type	Methodology	Algorithms	Industrial case
Blocher et al. (1998)	$J_m \mid \mid \Sigma C_j$	Heuristic	Dispatching rules (h): EDD, SPT, FCFS, LRO	Experimentation
Ng et al. (2003)	$O_m \mid d_j = d \mid \Sigma w_j U_j$	Heuristic	Minimum vertex cover problem of graphs	No
Hsu and Liu (2009)	$J_m \mid \mid \Sigma F_j$	Heuristic	Dispatching rules (h): MFV	Experimentation
Liu (2009)	$J_m \mid \mid C_{max}$	Heuristic	GA (h)	Experimentation
Liu (2010)	$J_m \mid \mid \Sigma F_j$	Heuristic	Dynamic dispatching rules (h)	Experimentation
Çetinkaya and Yozgat (2022)	$F_2 \mid \mid \Sigma C_j$	Exact & Heuristic	MILP (e), multi-phase heuristic (h), dispatching rules (h)	Experimentation
Mitic et al (2023)	$J_m \mid \mid C_{max}$	Heuristic	GA (h)	Experimentation
Cheng et al. (2023)	$J_m \mid \mid f(Time, Cost, Distance)$	AI	Weighted MOMDP + disjunctive graph + convex hull + PPO and GIN	Experimentation

Table 4. Summary Table for Papers considering Other Shops' conditions.

Legend:

EDD: Earliest Due Date

FCFS: First Come First Served

GA: Genetic Algorithm

GIN: Graph Isomorphism Network

LRO: Least Remaining of Operation

MFV: Minimum Flowtime Variation

MILP: Mixed Integer Linear Programming

MOMDP: Multi-Objective Markov Decision Process

PPO: Proximal Policy Optimization

SPT: Shortest Processing Time

Among the multi-stage shop conditions, the most studied is the job shop.
The first paper relevant to job shop customer order scheduling is by Blocher et al. (1998). The researchers suggested using order-based performance metrics such as order completion time, and order tardiness instead of job-based metrics because the last ones cannot be effective for a shop where orders are made of several jobs, and nothing can be delivered to the customer until the order is completed. The authors explored the impact of various dispatching rules (heuristics) and due date assignment rules for $J_m \mid \mid \Sigma C_j$. No validation scenario was defined and used to obtain empirical data. Only computational tests were done to evaluate and compare the effectiveness of the developed heuristics.

Liu (2009) proposed a technique called lot streaming to efficiently emulate a shop environment. Lot streaming involves splitting a processing job into multiple sub-jobs, allowing for overlapping of successive operations in the same job. Liu's article focused on the effects of lot streaming on the makespan time C_{max} of the problem and applied a genetic algorithm to the issue. However, no validation scenario was defined or used to obtain empirical data. The comparison of algorithms was based solely on experiments.

Mitic et al. (2023) focused their research on solving the COSP in job shops related to the limited capacity of resources and time for small and medium-sized enterprises. The purpose was to minimize the makespan $J_m \mid \mid C_{max}$ and maximize the earnings (essential for SME survival on the market) at the same time: it is the first paper that considers the average cost price of production per unit of time. The authors developed a genetic algorithm to find an optimal solution to build a decision support system to help job shop scheduling and capacity planning. It is also the only article found where the proposed model was based on data dictated by the industrial environment. In addition, the developed algorithm and software solution could be used to address the classic problem of labor scheduling in other industries. The machine-oriented Gantt chart generated optimized machine layout and production planning.

Most of the COSP researchers dealt with focused on reducing the completion time of the batch, Hsu and Liu (2009) instead, concentrated on reducing the stock level of finished goods and increasing the logistic efficiency in a job shop, $J_m \mid \mid \Sigma F_j$. The authors proposed a new dispatching rule MFV (Minimum Flowtime Variation) to decrease the variation in finished time among jobs of the same order. The computational tests demonstrated that the stock level of finished goods decreased by more than 70% with this rule. The JIT (Just-In-Time) methodology was suggested to increase logistic efficiency: most of the jobs of the same order should be finished at the requested time. However, no real-world scenario was defined and used to obtain empirical data.

Liu (2010) examined more in detail the concept of minimizing the customer order flow time in a job shop, which elapses between the release of the first job and the completion of the last job of an order, $J_m \mid \mid \Sigma F_j$. Maximizing this performance metric provides less work-in-process (WIP) and finished good inventory (FGI), increasing the return on investment. Liu realized coordinated scheduling of customer orders, with an order releasing policy and a dynamic dispatching rule for the COS problem, to improve the customer order flow time. Although experimental tests verified the effectiveness of the coordinated scheduling in two order-based job shops, no validation scenario was defined and used to obtain empirical data.

In the years have seen a sprout in the use of AI methodologies also in job shops.
Chen et al. (2023) proposed a multi-objective deep reinforcement learning method for the job shop that was fully trained, and that could be directly applied to solve problems of different sizes without the need for transfer learning. They used a Multi-Objective Markov Decision Process (MOMDP) and a disjunctive graph to represent the order dispatching problem state. They then used a convex hull to determine the weight vector. Finally, they proposed a policy network based on Proximal Policy Optimization (PPO) and Graph Isomorphism Network (GIN) to ensure that the model contained all the information of the problem. The method resulted in superior behaviors than single-objective DLR approaches on larger instances. No real-world scenario was used to obtain empirical data.

Only one paper has been found for each of the others shop environments.

Ng et al. (2003) studied concurrent open shop scheduling, in which two operations of the same job on different machines are allowed to be processed concurrently to minimize the weighted number of tardy jobs $\sum w_j U_j$ and common due dates, $O_m \mid d_j = d \mid \Sigma\, w_j U_j$. The authors gave an approximation algorithm for the minimum vertex cover problem of graphs. No validation scenario was defined and used to obtain empirical data. No evidence of the effectiveness of the proposal even with simulation data.

Çetinkaya and Yozgat (2022) analyzed a situation in which every customer order had several products processed in a two-machine flow shop, $F_2 \mid \mid \Sigma\, C_j$. The hypothesis was that each customer's order for a product could be processed as a batch of identical products. The aim was to construct a schedule of product lots and the sublots' sequence in every job lot by minimizing the sum of completion times of the customer orders. The authors realized a mixed-integer linear programming model and a multi-phase heuristic to respectively provide an optimal or near-optimal solution to the problem. No validation scenario was defined and used to obtain empirical data. Only computational experiments were done to evaluate and compare the effectiveness of the developed heuristics.

Chapter 4 shows that, independently from the machine environment, the Customer Order Scheduling Problem is still young compared to the General Scheduling.
In comparison with it, not as many papers have been found, especially for the multi-stage shop conditions.

Furthermore, there is only one paper in the entire literature that includes an industrial involvement, which could be very useful for the growing industry in founding more practical ways to improve their processes to meet the ever-changing customer demand.

Finally, it is worth noting that only a few researches used Machine Learning methodologies, but all the papers that involved these algorithms found better solutions with respect to traditional exact and heuristic methods. Additionally, the fast computational power of AI algorithms could help solve online scheduling problems which are not much studied yet but very common nowadays in the real-world.

5. Conclusion

In this thesis, the Customer Order Scheduling Problem has been studied through the review of the 62 identified papers. The evolution of this problem has been analyzed from the year 1996 until the most recent paper of 2023, taking into account factors such as shop conditions, objective functions and solution methodologies.

The interest in Customer Order Scheduling Problems has been increasing in the last few years following the trend of customer-centric production instead of a product-centric one to be more competitive in a market dominated by fast-changing demand and globalization.

The papers have been classified according to the constraints of the shop environment, the objective functions studied, the proposed resolution methods, and their use of real-world scenarios.

This study has identified certain gaps in current literature.
Multi-objective problems have started to appear in the most recent years researches, but there is room for further study.
Although different environmental constraints have been reported in the literature, there is still a need to explore more in detail online scheduling, machine breakdowns and resources bottlenecks, delays penalties, which are consequences of unpredictable and dynamic customer demand.

Moreover, a step forward both in terms of solution efficiency and real-time problems, can be taken by leveraging knowledge of reinforcement learning or other AI approaches, which are well-established in the scheduling problem. Reinforcement models can benefit from the experience gained over the years on general scheduling issues (see Appendix A).

The rapid development of Machine Learning Technologies has caught the interest of technology providers as it focuses on appropriate software tools and is a crucial feature for their advancement. Thus, cooperation between the academic world, manufacturing companies, and technology providers could lead to optimal synergy. This approach significantly reduces technical complexity as it only requires an effective connection. It allows for high-quality data without any system inconsistencies, saving both time and money while leaving less room for mistakes.

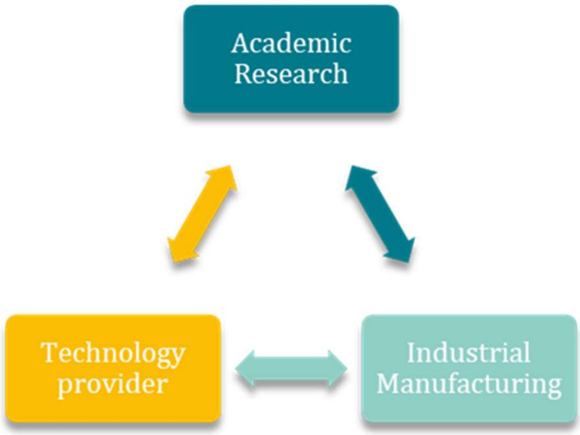

Figure 15. Main players cooperation

Technology providers play a strategic role in the success of the industrial supply chain. Often, they are the ones who introduce new and innovative technologies to academic research and manufacturing companies.

A cooperation between academic world and Industry could bear benefits on all the gaps currently present in the literature.
The use of real cases could help to understand markets, trends, drivers, concepts, and solutions and increase the number of multi-criteria studies coherently with reality.
Moreover, the industry usually deals with a lot of uncertainties, therefore a collaboration between these two players could finance more research that considers real-life constraints.

Furthermore, the validation phase is an important step that demonstrates whether a model or algorithm has been developed in accordance with industry requirements. This phase also ensures that the results produced by the model or algorithm are realistic and in line with the user case, as defined by all partners involved. Therefore, it could be useful to include this step in future research.

From a pure economical perspective, usually, a company may face R&D challenges that are too expensive in terms of qualified labor and required infrastructures. On the other hand, researchers may have difficulty accessing real data due to confidentiality concerns. So, a collaboration could help both the players to reach their goals at a lower cost.

The process of idea exploitation refers to the transfer of technological knowledge or ideas from the research domain to industrial domains. In this context, research is crucial in producing state-of-the-art scientific knowledge that can benefit society.

Bibliography

Afshar-Nadjafi, Behrouz. «Multi-skilling in scheduling problems: A review on models, methods and applications.» *Computers & Industrial Engineering, 151, 107004.*, 2021.

Ahmadi, Reza, Uttsrayan Bagchi, e Thomas A Roemer. «Coordinated scheduling of customer orders for quick response.» *Naval Research Logistics (NRL) 52.6*, 2005: 493-512.

Allahverdi, Ali. «The third comprehensive survey on scheduling problems with setup times/costs.» *European journal of operational research, 246(2)*, 2015: 345-378.

Allahverdi, Ali, C-T. Ng, T.C.E. Cheng, e Mikhail Y. Kovalyov. «A Survey of Scheduling Problems with Setup Times or Costs.» *European journal of operational research 187.3*, 2008: 985-1032.

Allahverdi, Ali, Jatinder Gupta, e Tariq Aldowaisan. «A review of scheduling reasearch involving setup considerations.» *Omega 27.2*, 1999: 219-239.

Antonioli, M., C. Rodrigues, e Bruno De Athayde Prata. «Minimizing total tardiness for the order scheduling problem with sequence-dependent setup times using hybrid matheuristics.» *International Journal of Industrial Engineering Computations 13.2*, 2022: 223-236.

Blocher, James D, e Dilip Chhajed. «The customer order lead-time problem on parallel machines.» *Naval Research Logistics (NRL) 43.5*, 1996: 629-654.

Blocher, James D., Dilip Chhajed, e Mark Leung. «Customer Order Scheduling in a General Job Shop Environment.» *Decision Sciences, Volume 29 Number 4*, 1998.

Calis, Banu, e Serol Bulkan. «A research survey: review of AI solution strategies of job shop scheduling problem.» *Journal of Intelligent Manufacturing 26*, 2015: 961-973.

Cao, Qian, Yi Li, e Guohua Wan. «Online Order Scheduling Problem with the Same Order Size on Two Identical Machines.» *Asia-Pacific Journal of Operational Research*, 2017.

Cetinkaya, Ferda Can, e Gunce Boran Yozgat. «Customer order scheduling with job-based processing and lot streaming in a two-machine flow shop.» *International Journal of Industrial Engineering & Production Research 33.2*, 2022: 1-17.

Çetinkaya, Ferda Can, P. Yeloglu, e H. A. Çatmaka. «Customer order scheduling with job-based processing on a single-machine to minimize the total completion time.» *International Journal of Industrial Engineering Computations 12*, 2021: 273–292.

Chaudhry, Imran Ali, e Abid Ali Khan. «A research survey: review of flexible job shop scheduling techniques.» *International Transactions in Operational Research 23.3*, 2016: 551-591.

Chen, Ren-Xia, e Shi-Sheng Li. «Minimizing maximum delivery completion time for order scheduling with rejection.» *Journal of Combinatorial Optimization. Vol 40*, 2020: 1044–1064.

Chen, Sirui, Yuming Tian, e Lingling An. «Multi-Objective Order Scheduling via Reinforcement Learning.» *Algorithms 16.11 (2023): 495.*, 2023.

Cheng, T. C. Edwin, Jatinder Gupta, e Guoqing Wang. «A review of flowshop scheduling research with setup times.» *Production and operations management 9.3*, 2000: 262-282.

Dauod, H, H Li, D Yoon, e K Srihari. «Multi-objective optimization of the order scheduling problem in mail-order pharmacy automation systems.» *The International Journal of Advanced Manufacturing Technology 99*, 2018: 73-83.

De Abreu, Levi Ribeiro, Bruno de Athayde Prata, Allan Costa Gomes, Stephanie Braga-Santos, e Marcelo Seido Nagano. «A novel BRKGA for the customer order scheduling with missing operations to minimize total tardiness.» *Swarm and Evolutionary Computation 75*, 2022.

De Athayde Prata, Bruno, Carlos Diego Rodrigues, e Jose Manuel Framianan. «A differential evolution algorithm for the customer order scheduling problem with sequence-dependent setup times.» *Expert Systems with Applications 189*, 2022.

De Athayde Prata, Bruno, Carlos Diego Rodrigues, e Jose Manuel Framinan. «Customer order scheduling problem to minimize makespan with sequence-dependent setup times.» *Computers & Industrial Engineering 151 (2021): 106962*, 2021.

De Athayde Prata, Bruno, Giovanni Cordeiro Barroso, e Stephanie Alencar Braga dos Santos. «A size-reduction algorithm for the order scheduling problem with total tardiness minimization.» *Journal of Project Management 7.*, 2022b: 167–176.

Del Gallo, Matteo, Giovanni Mazzuto, Filippo Emanuele Ciarapica, e Maurizio Bevilacqua. «Artificial Intelligence to Solve Production Scheduling Problems in Real Industrial Settings: Systematic Literature Review.» *Electronics 12.23 (2023): 4732.*, 2023.

Duan, Jianguo, Mengting Wang, Qiuglei Zhang, e Jiyun Qin. «Distributed shop scheduling: A comprehensive review on classifications, models and algorithms.» *Mathematical Biosciences and Engineering 20.8*, 2023: 15265-15308.

EFFRA -, European Factories of the Future Reseach Association. *FACTORIES OF THE FUTURE- Multi-annual for the contractua PPP under Horizon 2020.* Innovation programme Horizon 2020, EUROPEAN COMMISSION Directorate-General for Research and Innovation, 2015.

Erel, E., e J.B. Ghosh. «Customer order scheduling on a single machine with family setup times: Complexity and algorithms.» *Applied Mathematics and Computation, 185(1)*, 2007: 11-18.

Framinan, Jose M, e Paz Perez-Gonzalez. «Order scheduling with tardiness objective: Improved approximate solutions.» *European Journal of Operational Research 266.3*, 2018: 840-850.

Framinan, Jose M., e Paz Perez-Gonzalez. «New approximate algorithms for the customer order scheduling problem with total completion time objective.» *Computers & Operations Research 78*, 2017: 181-192.

Garg, Naveen, Amit Kumar, e Vinayaka Pandit. «Order Scheduling Models: Hardness and Algorithms.» *FSTTCS 2007: Foundations of Software Technology and Theoretical Computer Science, 27th International Conference.* New Delhi, India, 2007.

Gupta, Jatinder N.D, Johnny C. Ho, e Jack A.A. Van Der Veen. «Single machine hierarchical scheduling with customer orders and multiple job classes.» *Annals of Operations Research, Volume 70*, 1997: 127-143.

Gupta, Jatinder, et al. «Bicriteria single-machine scheduling with multiple job classes and customer orders.» *Applied Soft Computing 147*, 2023.

Hazir, O., Y. Gunalay, e E. Erel. «Customer order scheduling problem: a comparative metaheuristics study.» *The International Journal of Advanced Manufacturing Technology, 37,*, 2008: 589-598.

Hoffmann, J, J. S. Neufeld, e U. Buscher. «Iterated greedy algorithms for customer order scheduling with dedicated machines.» *IFAC-PapersOnLine, 55(10)*, 2022: 1594-1599.

Hsu, Sheng-Yuan, e C.H. Liu. «Improving the delivery efficiency of the customer order scheduling problem in a job shop.» *Computers & Industrial Engineering 57.3* , 2009: 856-866.

Huang, Pingjuan, Hui Li, e Lidong Han. «Order scheduling problems in make-to-order manufacturing systems.» *IEEE International Conference Mechatronics and Automation, 2005. Vol. 4. IEEE*, 2005.

Kayhan, Behice Melten, e Gokalp Yildiz. «Reinforcement learning applications to machine scheduling problems: a comprehensive literature review.» *Journal of Intelligent Manufacturing* , 2021: 1-25.

Komaki, G.M., Shaya Sheikh, e Behnam Malakooti. «Flow shop scheduling problems with assembly operations: a review and new trends.» *International Journal of Production Research 57.10*, 2019: 2926-2955.

Kovalenko, Yulia, e Aleksey Zakharov. «The Pareto set reduction in bicriteria customer order scheduling on a single machine with setup times.» *Journal of Physics: Conference Series. Vol. 1546. No. 1. IOP Publishing,*, 2020.

Kung, Jan-Yee, et al. «Metaheuristics for order scheduling problem with unequal ready times.» *Discrete Dynamics in Nature and Society, Volume 2018, Article ID 4657368*, 2018.

Lee, Ik Sun. «Minimizing total tardiness for the order scheduling problem.» *International Journal of Production Economics 144.1*, 2013: 128-134.

Lee, Tian-Soon, e Ying-Tai Loong. «A review of scheduling problem and resolution methods in flexible flow shop.» *International Journal of Industrial Engineering Computations 10.1*, 2019: 67-88.

Leung, Joseph Y.-T., Haibing Li, e Michael Pinedo. «Order scheduling in an environment with dedicated resources in parallel.» *Journal of Scheduling. Vol.8*, 2005: 355-386.

Leung, Joseph Y.-T., Haibing Li, e Michael Pinedo. «Scheduling orders for multiple product types to minimize total weighted completion time.» *Discrete Applied Mathematics 155.8*, 2007: 945-970.

Leung, Joseph Y.-T., Haibing Li, e Michael Pinedo. «Scheduling orders for multiple product types with due date related objectives.» *European Journal of Operational Research 168.2.*, 2006: 370-389.

Leung, Joseph Y.-T., Haibing Li, e Michael Pinedo. «Scheduling orders on either dedicated or flexible machines in parallel to minimize total weighted completion time.» *Annals of Operations Research Volume 159*, 2008: 107–123.

Leung, Joseph Y.-T., Haibing Li, Michael Pinedo, e Jiawei Zhang. «Minimizing total weighted completion time when scheduling orders in a flexible environment with uniform machines.» *Information Processing Letters 103*, 2007b: 119-129.

Leung, Joseph, Haibing Li, e Michael Pinedo. «Order scheduling models: an overview.» *1 st International Conference, MISTA'03.* Nottingham, UK, 2005b.

Li, Lung Yu, et al. «A Genetic Hyper-Heuristic for an Order Scheduling Problem with Two Scenario-Dependent Parameters in a Parallel-Machine Environment.» *Mathematics 10.21 (2022): 4146.*, 2022.

Li, Lung-Yu, et al. «Composite heuristics and water wave optimality algorithms for tri-criteria multiple job classes and customer order scheduling on a single machine.» *International Journal of Industrial Engineering Computations 14.2*, 2023: 265-274.

Lin, Bertrand, e Alexander V. Kononov. «Customer order scheduling to minimize the number of late jobs.» *European Journal of Operational Research 183.2*, 2007: 944-948.

Lin, Win Chin, Jianyou Xu, Danyu Bai, I-Hong Chung, Shang Chia Liu, e Chin Chia Wu. «Artificial bee colony algorithms for the order scheduling with release dates.» *Soft Computing 23 (2019).*, 2019: 8677-8688.

Lin, Win Chin, Tzu Hsuan Yang, e Chin Chia Wu. «An Novel Metaheuristic for the Order Scheduling with Ready Times.» *2017 International Conference on Computing Intelligence and Information System (CIIS).* Nanjing, China, 2017. 7-12.

Lin, Win Chin, Yunqiang Yin, Shuenn Ren Cheng, TCE Cheng, Chia Han Wu, e Chin Chia Wu. «Particle swarm optimization and opposite-based particle swarm optimization for two-agent multi-facility customer order scheduling with ready times.» *Applied Soft Computing 52.*, 2017b: 877-884.

Lin, Win-Chin, Xingong Zhang, Xinbo Liu, Kai-Xiang Hu, Shuenn-Ren Cheng, e Ameni Azzouz. «Sequencing single machine multiple-class customer order jobs using heuristics and improved simulated annealing algorithms.» *RAIRO-Operations Research 57.3*, 2023: 1417-1441.

Liu, C-H. «Lot streaming for customer order scheduling problem in job shop environments.» *International Journal of Computer Integrated Manufacturing 22.9*, 2009: 890-907.

Liu, Cheng Hsiang. «A coordinated scheduling system for customer orders scheduling problem in job shop environments.» *Expert Systems with Applications 37.12*, 2010: 7831-7837.

Macker, Alexander, Friedhelm Meyer, e Simon Pukrop. «Approximating Weighted Completion Time for Order Scheduling with Setup Times.» *nternational Conference on Current Trends in Theory and Practice of Informatics. Cham: Springer International Publishing,*, 2020.

Martinelli, Renan, Flavia Cristina Mariano, e Camilla Bertini Martins. «Single machine scheduling in make to order environments: A systematic review.» *Computers & Industrial Engineering, 169, 108190.*, 2022.

Mathew, Ninam Theradapuzha, e Bjorn Johansson. «Production Planning and Scheduling Challenges in the Engineer-to-Order Manufacturing Segment - A Litterature Study.» *International Journal of Innovation, Management and Technology, VOl.14, Num 3, August 2023*, 2023.

Mitic, Predrag, Suzana Petrovic Savic, e Aleksandar Djordjevic. «The Problem of Machine Part Operations Optimal Scheduling in the Production Industry Based on a Customer's Order.» *Applied Sciences 13.19 (2023): 11049.*, 2023.

Molins, Xavier, e Marco Aurelio De Mesquita. *Scheduling in the industry 4.0: a systematic literature review.* Catalunya: Universitat Politecnica, 2019.

Molins, Xavier, e Marco Aurélio De Mesquita. «Scheduling in the industry 4.0: a systematic literature review.» *International Journal of Production Research*, 2019.

Mourtzis, Dimitris. «Advances in Adaptive Scheduling in Industry 4.0.» *Frontiers in Manufacturing Technology 2: 937889*, 2022.

Neufeld, Janis, Sven Schulz, e Udo Buscher. «A systematic review of multi-objective hybrid flow shop scheduling.» *European Journal of Operational Research 309.1*, 2023: 1-23.

Ng, C.T., T.C.E. Cheng, e J.J: Yuan. «Strong NP-hardness of the single machine multi-operation jobs total completion time scheduling problem.» *Information processing letters, 82(4)*, 2002: 187-191.

Ng, Chi To, TC Edwin Cheng, e JJ Yuan. «Concurrent open shop scheduling to minimize the weighted number of tardy jobs.» *Journal of Scheduling 6*, 2003: 405-412.

Ouelhadj, Djamila, e Sanja Petrovic. «A survey of dynamic scheduling in manufacturing systems.» *Journal of scheduling 12*, 2009: 417-431.

Potts, Chris, e Vitaly A. Strusevich. «Fifty Years of Scheduling: A Survey of Milestones.» *Journal of the Operational Research Society 60*, 2009: S41-S68.

Riahi, Vahid, M.A. Hakim Newton, M.M.A. Polash, e Abdul Sattar. «Tailoring customer order scheduling search algorithms.» *Computers & Operations Research 108 (2019).*, 2019: 155-165.

Senthilkumar, Panneerselvam, e Sockalingam Narayanan. «Literature review of single machine scheduling problem with uniform parallel machines.» *Intelligent Information Management*, 2010: 457-474.

Shi, Zhong Shun, Hang Ma, Meiheng Ren, Tao Wu, e Andrew J. Yu. «A learning-based two-stage optimization method for customer order scheduling.» *Computers & Operations Research 136 (2021): 105488*, 2021.

Shi, Zhongshun, Longei Wang, Pai Liu, e Leyuan Shi. «Minimizing completion time for order scheduling: Formulation and heuristic algorithm.» *IEEE Transactions on Automation Science and Engineering 14.4*, 2017: 1558-1569.

Shukla, Brahma Datta, e Pragya Singh Tomar. «A Critical Evaluation and Comparison of Key approaches to Artificial Intelligence in Production Scheduling.» *International Journal of Engineering Sciences & Research Technology*, 2019.

Sorgun, OyKu. *https://www.icrontech.com/blog_item/customer-centric-supply-chain-what-is-it-why-is-it-important-now.* 09 September 2022. https://www.icrontech.com/blog_item/customer-centric-supply-chain-what-is-it-why-is-it-important-now (consultato il giorno January 26/01/24, 2024).

Sung, Chang Sup, e Sang Hum Yoon. «Minimizing total weighted completion time at a pre-assembly stage composed of two feeding machines.» *International Journal of Production Economics 54.3*, 1998: 247-255.

Wang, Guoqing, e TCE Cheng. «Customer order scheduling to minimize total weighted completion time.» *Omega 35.5*, 2007: 623-626.

Wang, Longfei, Zhongshum Shi, e Leyuan Shi. «A novel quadratic formulation for customer order scheduling problem.» *2013 IEEE International Conference on Automation Science and Engineering (CASE).* Madison, WI, USA: IEEE, 2013. 576-580.

Wu, Chin Chia, Danyu Bai, Xingong Zhang, Shuenn Ren Cheng, Jia Cheng Lin, e Zong Lin Wu. «A robust customer order scheduling problem along with scenario-dependent component processing times and due dates.» *Journal of Manufacturing Systems 58 (2021)*, 2021: 291-305.

Wu, Chin Chia, Tzu Hsuan Yang, Chao Chung Kang, Xingong Zhang, I-Hong Chung, e Win Chin Lin. «Using heuristic and iterative greedy algorithms for the total weighted completion time order scheduling with release times.» *Swarm and evolutionary computation 44 (2019).*, 2019: 913-926.

Wu, Chin-Chia, Shang-Chia Liu, Chuanli Zhao, Sheng-Zhi Wang, e Win-Chin Lin. «A Multi-Machine Order Scheduling with Learning Using the Genetic Algorithm and Particle Swarm Optimization.» *The Computer Journal*, 2018: 14-31.

Wu, Chin-Chia, Shang-Chia Liu, Tzu-Yun Lin, Tzu-Hsuan Yang, I-Hong Chung, e Win-Chin Lin. «Bicriterion total flowtime and maximum tardiness minimization for an order scheduling problem.» *Computers & Industrial Engineering 117.*, 2018: 152-163.

Xiong, hegen, Shuangyuan Shi, Danni Ren, e JinJin Hu. «A survey of job shop scheduling problem: The types and models.» *Computers & Operations Research 142 (2022): 105731.*, 2022.

Xu, JianYou, Chin-Chia Wu, Yunqiang Yin, Chuanli Zhao, Yi Chiou, e Win-chin Lin. «An order scheduling problem with position-based learning effect.» *Computers & Operations Research 74*, 2016: 175-186.

Xu, Xiaoyun, Ma Ying, Zhou Zihuan, e Zhao Yaping. «Customer order scheduling on unrelated parallel machines to minimize total completion time.» *IEEE Transactions on Automation Science and Engineering 12.1*, 2015: 244-257.

Yang. «The complexity of customer order scheduling problems on parallel machines.» *Computers & Operations Research 32.7*, 2005b: 1921-1939.

Yang, Jaehwan, e Marc Posner. «Scheduling parallel machines for the customer order problem.» *Journal of Scheduling 8*, 2005: 49-74.

Yu, Yingchen. «A research review on job shop scheduling problem.» *E3S Web of Conferences. Vol. 253. EDP Sciences,*, 2021.

Yue, Q., e G. Wan. «Order scheduling with controllable processing times, common due date and the processing deadline.» *Journal of Systems Science and Systems Engineering, 26*, 2017: 199-218.

Zhang, Jian, Guofu Ding, Yisheng Zou, Shengfeng Qin, e Jianlin Fu. «Review of job shop scheduling research and its new perspectives under Industry 4.0.» *Journal of Intelligent Manufacturing 30* , 2019: 1809-1830.

Zhao, Yaping, Xiang Kong, Xiaoyun Xu, e Endong Xu. «Resource-controlled stochastic customer order scheduling via particle swarm optimization with bound information.» *Industrial Management & Data Systems 122.8*, 2022: 1882-1908.

Zipfel, Benedikt, Janis Neufeld, e Udo Buscher. «An iterated local search for customer order scheduling in additive manufacturing.» *International Journal of Production Research*, 2023: 1-21.

Appendix A. AI strategies in General Scheduling

In this manufacturing ecosystem, characterized by short product life cycles, high product variability, and intense international competition, Artificial intelligence represents an important enabler that can help manufacturing systems quickly respond to changes and predict future anomalies in the production plan.

The AI system can provide decision-making support to ensure efficient and effective production processes.

As demonstrated in the study Del Gallo et al. (2023), a significant area of research in the past decade has been focused on applying Artificial Intelligence to address the general scheduling problem in competitive scenarios that require flexibility and quick responsiveness to changes in production planning.
However, this topic is still relatively new for the Customer Order Scheduling Problem (COSP), and only a few recent papers have suggested utilizing AI approaches to solve this problem.

To address this gap, this section aims to analyze papers that focus on AI solutions for general scheduling problems with the scope of showing potential research areas for COSP. The thesis could be a starting point for researchers and developers in the order scheduling problem domain.

Del Gallo et al. (2023) provided a comprehensive literature review of the AI-based solution strategies for scheduling problems in general.
AI-based scheduling is mainly applicable in the context of smart factories or Industry 4.0, and this research was conducted by analyzing publications from 2011, the year of the advent of Industry 4.0, up to October 2023, resulting in a total of 8291 papers. The article also included a forecast for the end of 2023.

The graphs below designed from the data of Del Gallo et al. (2023) illustrates the increasing trend of research studies covering AI in general scheduling problems from 2011 to 2023, in terms of frequency and cumulative of published papers.

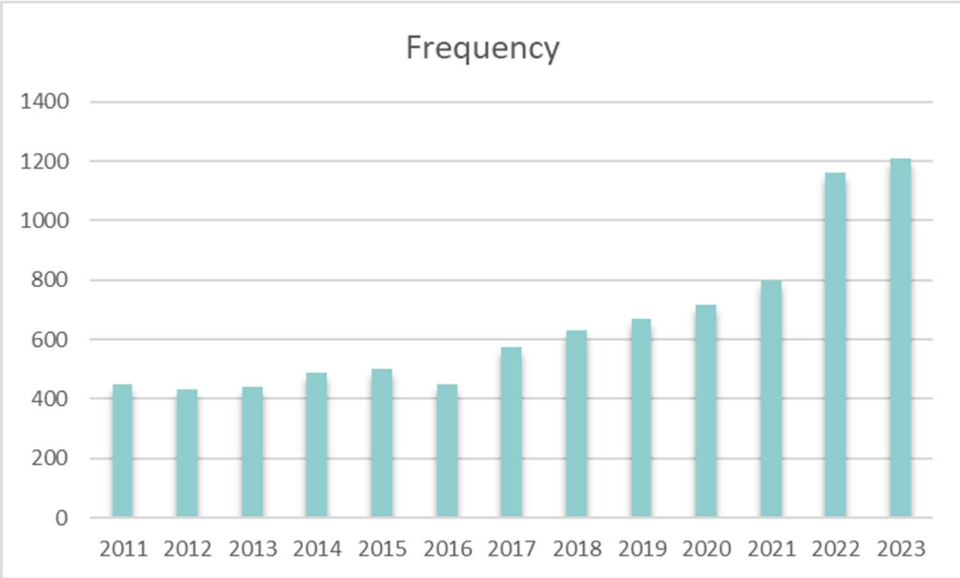

Figure 16. Frequency of papers about AI in General Scheduling

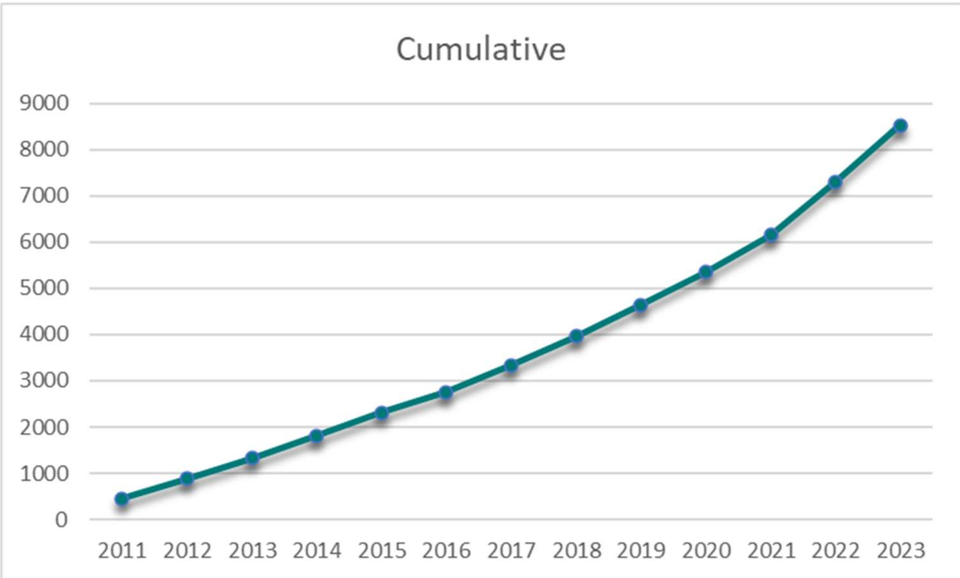

Figure 17. Cumulative of papers about AI in General Scheduling

Del Gallo et al. (2023) have also conducted a specific analysis of the number of papers published on different AI techniques combined with heuristics and metaheuristics. The analysis revealed that the main metaheuristic method used in machine learning architecture is Particle Swarm Optimization (PSO). The main machine learning techniques are Neural Networks (NN), and Reinforcement Learning (RL) for solving general scheduling problems.

Particle Swarm Optimization is a metaheuristic method of search and optimization, inspired by the movement of swarms. The algorithm identifies a new "optimum candidate" in the search space at each iteration, modifying their positions and velocities based on their own and the collective' experiences.

Neural Networks are the central element of deep learning algorithms, a subset of machine learning. It's an artificial system of processing units trained with historical data to optimize overtime the algorithm proposed.

Reinforcement Learning is a machine learning technique in which an agent learns to process an activity through trial-and-error in a dynamic environment. This approach allows to optimize the solutions without being explicitly programmed to do so and without human intervention.

Another analysis presented in the same study aimed to examine if authors used PSO integrated with NN, and/or RL algorithms to solve production scheduling problems in real industrial settings, and what benefits and advantages companies have obtained from using such algorithms. The study found 452 publications about AI methodologies which is about 5% of the total number of papers regarding general scheduling (8291 papers). Their analysis excluded articles that merely illustrated algorithms without any real case application, therefore they conducted the study on benefits of AI only on 31 out of 452 papers.

As mentioned in Del Gall et al. (2023) and Shukla and Tomar (2019), an important aspect that emerged from the analysis is the limited number of publications that explicitly report on the company benefits, compared to the large number of papers found. This could be caused by the difficulty in accessing and sharing company data, as well as the fact that some results of applying a new scheduling plan may not be noticeable in the short term.

From the studies of Del Gallo et al. (2023), it is worth noticing that from 2021 there has been a spread in the use RL which seems to indicate a shift towards this method in solving general schedule problems since RL. As a matter of fact, it was seen that RL algorithms are to be preferred in the more complex situations because they can provide better execution times and flexibility than other algorithms. With RL methods, authors were able to obtain solutions in real or near real-time.

The study of Kayhan and Yildiz (2021) was a comprehensive literature review that examines the applications of RL methods to machine scheduling problems. The RL approach has become popular in production and operations management problems, particularly in decision problems with dynamic environments. It has been shown that the RL approach can model a wide range of problems and deliver better results than conventional methods.

The study analyzed 80 papers, considering different aspects of the problem such as the algorithm used, machine environment, job and machine characteristics, objectives, and benchmark method, and it devised a detailed classification scheme. The papers were then analyzed and interpreted in the context of the machine environment, constraints, and objectives. Through this detailed analysis, the study provides researchers with insights into the field and highlights the trends and deficiencies in literature.

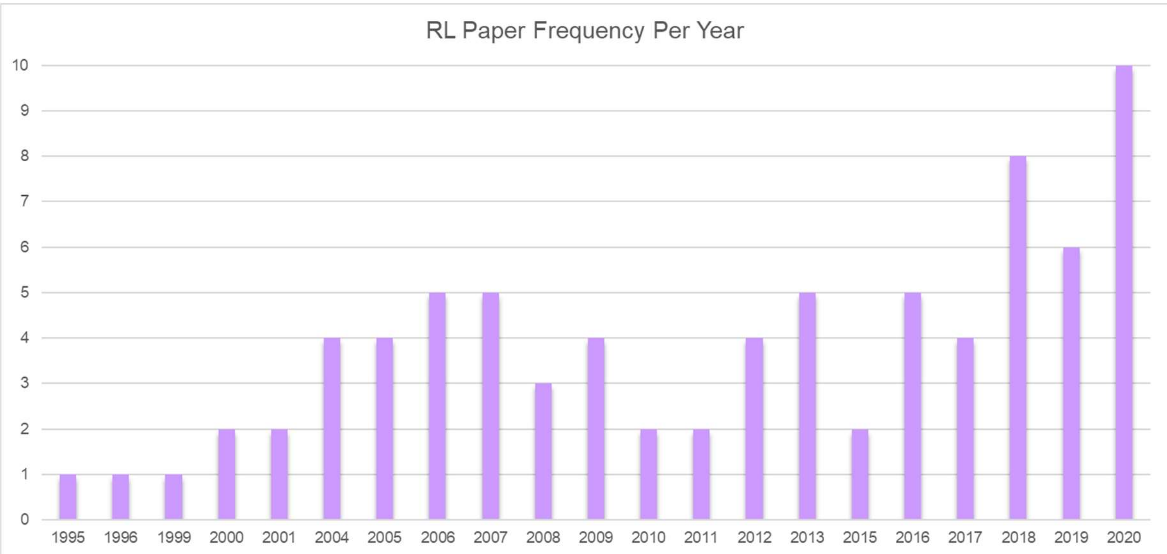

Figure 18. RL Publications Frequency

The frequency of RL publications, as shown in Figure 18**Errore. L'origine riferimento non è stata trovata.**, has increased in recent years, indicating the growing interest of researchers in the application of the RL approach to scheduling problems.

The cumulative function shows the increasing number of papers related to RL. With the advent of Industry 4.0 (2011) especially, the trend has started to grow exponentially.

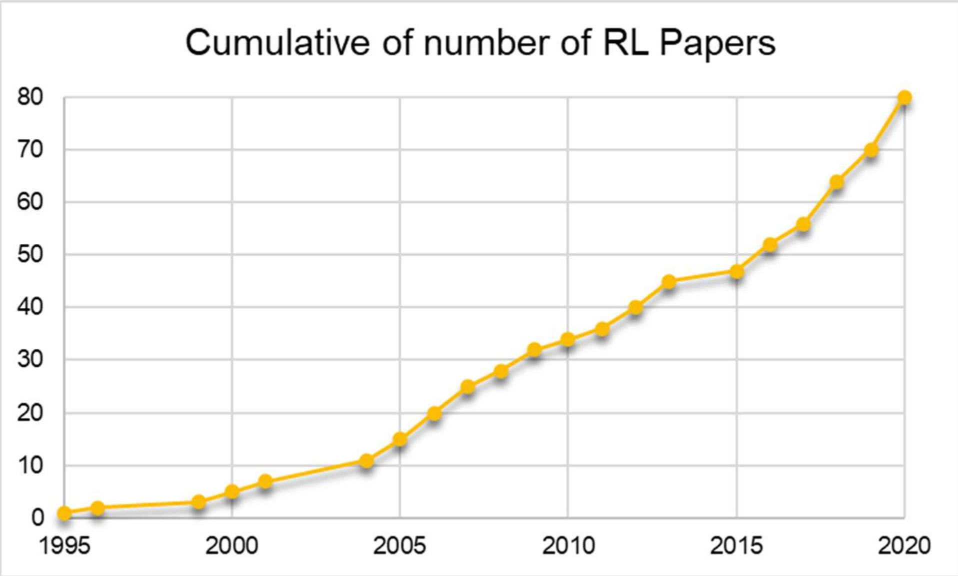

Figure 19. Cumulative of RL Papers 1995-2020

The main RL techniques used to solve general scheduling problems are shown below.

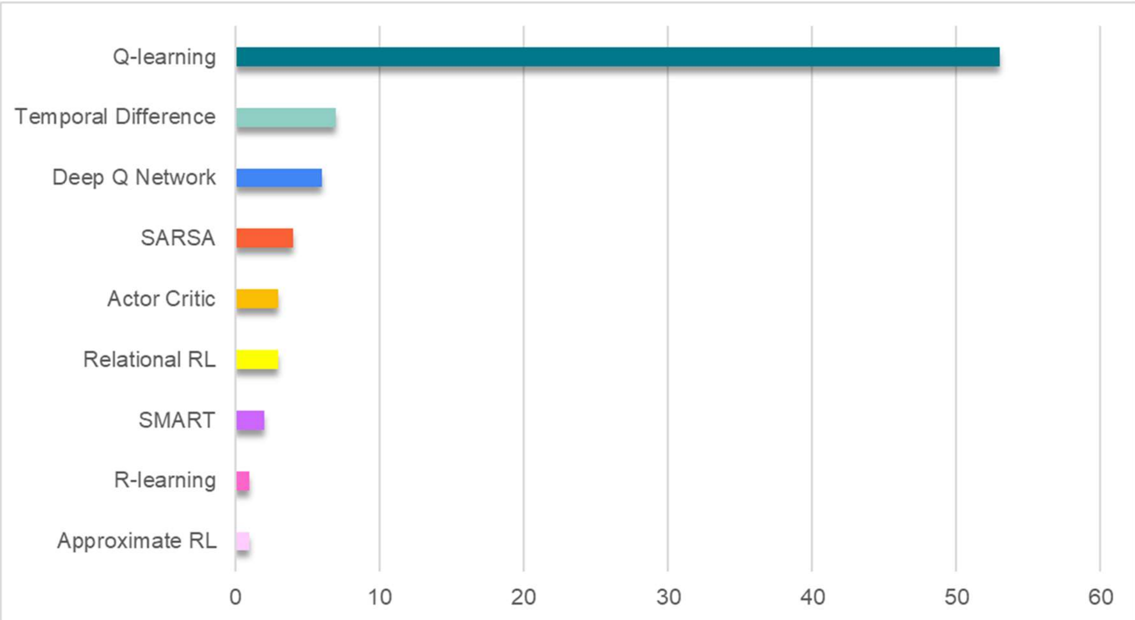

Figure 20. RL technique distribution

Reinforcement Learning employs AI algorithms to make optimal decisions for scheduling problems. It is a versatile and adaptable solution for various scheduling problems, even in different contexts. In the article Kayhan and Yildiz (2021), an in-depth analysis of AI solutions for machine scheduling problems is provided, categorized according to different manufacturing environments as shown in Figure 21.

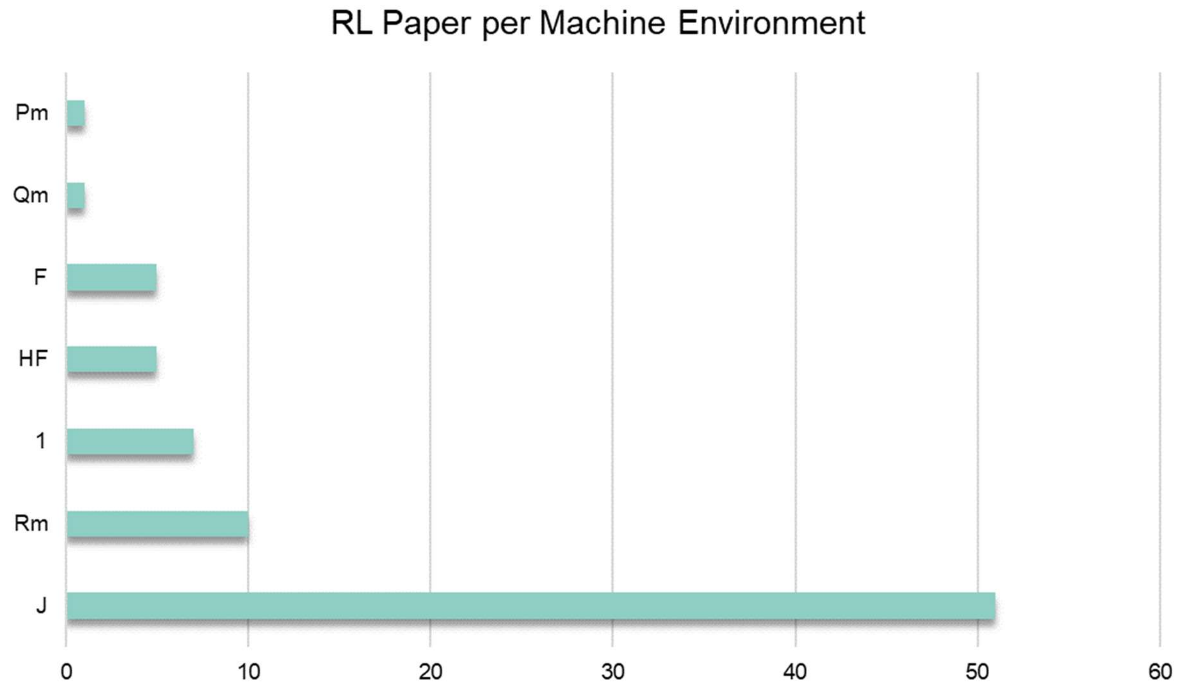

Figure 21. Machine Environment Frequency on RL Papers

The most machine environment solved with RL is the job shop due to its more complicated nature with respect to the others.

The paper Kayhan and Yildiz (2021) showed the RL's potential to solve different scheduling problems both in terms of constraints considered (set-up times, pre-emption, precedencies) and objective functions to maximize (completion time, lateness, tardiness, tardy jobs, flow time, WIP, total costs, makespan).
RL techniques' adaptivity to new situations by learning from feedback from the environment makes this method flexible and suitable also for the customer order scheduling problem.

The study found that RL outperforms dispatching rules for scheduling problems that involve multiple objectives, which is often the case for customer order scheduling problems. Therefore, the application of RL algorithms to these problems should be analyzed further to determine their effectiveness.

Kayhan and Yildiz (2021) suggests that many papers have ignored constraints such as machine breakdown, order cancellation, and setup time, which are frequently encountered in real industrial applications.

Acknowledgments

I would like to dedicate this milestone to my parents who are my biggest supporters.