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Thesis

Game theoretical models for the day ahead electricity markets

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Summary

In day-ahead electricity markets, participants bid the amount of energy they are willing to supply as a function of price. A central regulator matches supply and demand and determines the market price for the next day. This thesis work aims to model the dynamics of price formation in day-ahead electricity markets as a game, where the allowable supply functions serve as actions and the competing firms act as players. We propose that the observed states of the auction coincide with the Nash equilibria of this game and derive a system of nonlinear coupled differential equations that these equilibria must satisfy. Furthermore, we solve the system for n firms with affine marginal costs, obtaining the implied supply functions and the resulting market price. In the second part of our research, we apply the developed model to data from the Italian day-ahead electricity market provided by GME. By estimating market demand and firms' affine costs, we determine the implied supply functions and equilibrium prices, extending our simulations to cover the entirety of the last quarter of 2018. The resulting estimates are then discussed and compared to the predictions obtained via Cournot's competition model.

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Mathematics is a game played according to certain simple rules with meaningless marks on paper.

[D. HILBERT]

Chapter 1

Introduction

1.1 Electricity markets

Electric power industries trace back 140 years to the initial development of central station generation/transmission/distribution systems supplying electricity to the public. Their evolution has mirrored technological advancements on both the sides of supply and demand, exploitation of economies of scale, environmental and various other policy constraints, organizational and regulatory innovation, interest group politics and ideology. The first institutions providing electricity at the end of the nineteenth century were, on the general run of things, private, vertically integrated firms supplying single cities or to fractions of them. Governments (both on a local and national scale) exerted supervision of various sorts as a condition for the right to use public right of ways for the distribution infrastructure. The subsequent technological developments and the pressures generated by the scaling economies generally led to expanding enterprises, in parallel with the deployment of high voltage transmission networks, with regulatory bodies maintaining hybrid policies to the regulation of both production and taxation. Furthermore, by the early 1950, together with the broad diffusion of electricity production technologies, a vast array of institutional structures had been put in place, with mixtures of private and government ownership at every level of the distribution network. This mixture of private and public ownership of both the production and distribution infrastructure was, by the second half of the twentieth century, eliminated in favour of total state ownership by some countries such as France, Italy, Ireland and Greece. In these countries, a single, integrated utility owned by the government took on the burden of supplying the national grid. Other forms of regulation included for example Germany, where by the 1990 a small number of integrated regional generation and transmission firms coexisted with municipal distributors. In Japan ten private firms shared regional monopolies, while in Norway municipalities played the central role. Vertical integration was therefore the rule almost everywhere in the world: wholesale electricity markets did not exist and retail customers had only one possible choice of supply. This state of things was destined to change by the end of the twentieth century, when a wave of restructuring of the electricity markets started taking place around the globe.

1.2 Contemporary Restructuring

Arguably the most significant institutional restructuring of the electricity markets has begun in the final decades of the twentieth century, including, among the others, both Europe and a considerable portion of the USA. This contemporary restructuring of the national and international markets has involved a series of changes, including: the separation or unbundling of the previously vertically integrated – through common ownership or regulated long-term contracts – generation, transmission, distribution and retail supply segments of the industry; the deconcentration of and free entry into the generation segment; the reorganization of the transmission/system operations segment; and finally the separation of the physical distribution (delivery) segment from the financial arrangements for retail supply of energy[8]. The fundamental change operated by these restructuring initiatives has been to open the markets to competition, enabling private firms to generate and supply energy, ancillary services and capacity in wholesale markets, while at the same time opening retail supply to competition. Different jurisdictions had different reasons for a restructuring of their electricity supply industries and for the introduction of wholesale markets, nevertheless, the common factor animating these modifications has been the ideal of improving industry performance by means of competition. In this direction new regulations had to account for the fact that electricity as a consumption asset poses a set of characteristic challenges: it has to be available at every time everywhere on the grid and it cannot be significantly stored. Furthermore, production of electricity is subject to stringent capacity constraints in the sense that it is physically impossible to get more than a pre-specified amount of energy from a generation unit in a given time frame. Consequently, direct retail from the producing firms to the costumer is not viable nor feasible in electricity markets, that have therefore experienced a diversification between retail and wholesale, similarly to other energy-based consumption assets such as oil (where a diversification between firms extracting and refining oil and distributors exist). In this scenario, most customers purchase electricity from a distributor through contracts of varying duration, while retailers purchase power from producers on wholesale markets. This general description covers, for example, both the European and the North American markets. In Europe, wholesale markets are decentralized, meaning that the selling and buying parties enter in bilateral transactions then aggregated by the central authority, whilst in North America wholesale markets are centralized: the centralized authority autonomously matches buyers and sellers, determining equilibrium production and price. Indeed, wholesale markets can exist for multiple dates, the most important of which being the spot market since the hourly wholesale price determines the value of energy with hourly granularity. Crucially for the development of this thesis work, in most markets the spot market is, in fact, a day-ahead market and not a true real-time market. This peculiarity is motivated by technological constraints posed by the production plants, that need to program their production with at least a day's advance. Therefore, in this scenario buyers and sellers agree today on the quantities each will buy and sell, consequently determining the price of the next day's specified hour. Evidently, this process is based on a forecast (generally subjected to low variance) of the day-ahead hourly demand, that then needs to be adjusted on the following day by adjustment markets, thus giving rise to "two-settlement" markets in which the price for electricity in a given hour is settled

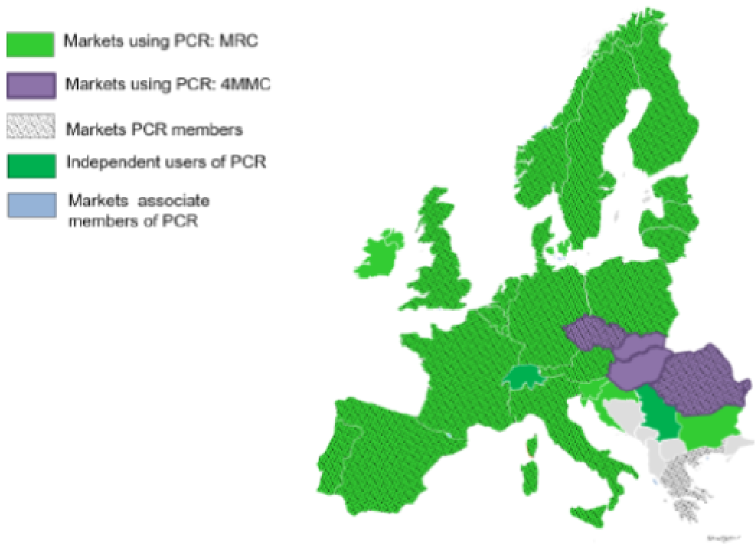


Figure 1.1. Extension of the unified Market Coupling in 2019 served by EUPHEMIA [4].

two times: one in the day ahead and then in the day of adjustment market. Indeed, this dichotomy between the real time adjustment markets and the day ahead ones that set the unitary price has been addressed in two different ways. On one hand the regulators can avoid the presence of an adjustment market entirely by adopting a transmission policy that ensures sufficient transmission capacity to justify the assumption of infinite capacity for actual system operation for the vast majority of hours of the year. This is the case, for example, of the wholesale electricity market for the province of Alberta in Canada [8]. On the other hand, almost any difference between the market model used to set dispatch levels and market prices and the actual operation of the generation units needed to serve demand creates an opportunity for market participants to take actions that raise their profits at the expense of overall market efficiency. Therefore, it is paramount for the regulators to provide systems capable of minimizing those discrepancies. In order to understand the behaviour of a two settlement market, such as the one discussed, it is useful to consider an example. A generation plant sells 50MWh for a given hour of the day in the day ahead market at 60$/MWh, receiving a guaranteed $3000 for the sale. However, if the generation unit owner fails to inject 50 MWh of energy into the grid during the specified delivery hour of the following day, it has to purchase the energy that it fails to deliver at the real time price of the location it is in. If we assume the price at that location to be 70 $/MWh, and the producing firm only delivers 40 MWh of energy over the specified hour, then the firm must purchase the remaining 10 MWh shortfall relative to the day ahead schedule at the locational price of 70$/MWh. In turn, this implies that the final hourly revenue for the considered production plant is of 2300$ deriving from the 50MWh obtained from the day ahead market, minus the 700$ paid for the 10MWh necessary to fulfill the bid production quantity.

1.3 The European model

The specific development of the European model for electricity markets begun officially in 1951 with the creation of the European Coal and Steel Community, but real step towards a unified energy policy begun only in the 1980s, as part of the 1986 Single Act, having as objective to integrate and liberalize some of the key sectors of the European economy. We have previously noted that most electricity industries in Europe were historically national and based on geographical monopolies, and only in the 1990s the European Union and its member states gradually opened these entities to competition. More than 20 years after the first instances of liberalization of the markets, the integration process remains incomplete. Progress in this direction has been hindered by the difficulties in integrating heterogeneous markets in both designs and governance approaches. This is a direct consequence of the initial reluctance to prescribe a unified European strategy common among the member states, in favour of a gradual convergence of different market designs by progressively tightening the rules to drive further convergence through the harmonization of technical rules affecting cross border trades. In this sense, the Price Coupling of Regions project (PCR), an initiative of of eight Power Exchanges (PXs): EPEX SPOT, GME, HEnEx, Nord Pool, OMIE, OPCOM, OTE and TGE covering the electricity markets in Austria, Belgium, Czech Republic, Croatia, Denmark, Estonia, Finland, France, Germany, Hungary, Italy, Ireland, Latvia, Lithuania, Luxembourg, the Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden and UK, has developed a single price coupling algorithm which is commonly known as EUPHEMIA [4]: Pan-European Hybrid Electricity Market Integration Algorithm. EUPHEMIA has been used increasingly since February 2014 to compute both energy allocations and electricity prices across Europe with the aim of maximising the overall welfare and increase pricing transparency. This goal is achieved by means of the Market Coupling principle, which as the name states, is a way to join and integrate different energy markets into one coupled market. In such an institution, demand and offer are no longer limited to the zonal contingencies of territorial scopes, on the contrary electricity transactions can be established between parties belonging to different areas, only subjected to transmission grids constraints. From a financial perspective, market coupling brings as main benefit improved market liquidity, combined with less volatile assets as a consequence of the diversification of production units. Moreover, producing firms also benefit directly from these changes, given that they no longer need to acquire transmission capacity rights to carry out cross border trades, since the cross border exchanges are given directly by the market coupling mechanism. In this sense EUPHEMIA decides the orders to be executed in concordance to the prices to be published such that the *social welfare*, defined as customer surplus+producer surplus + congestion rent is maximal and the power flows necessary comply to the transmission network constraints.

1.4 Outline of the thesis's objectives

In this thesis work we study and adapt a series of game theoretical models to simulate the dynamics of the Italian day ahead electricity market, by leveraging data provided by the GME (Gestore Mercati Energetici), the government agency that manages Italian energy markets. In the next chapters, starting with Chapter 2 we outline the structure of the Italian electricity markets, defining the regulations of both the day ahead and of the consequent settlement markets. The structure of the provided data is also addressed. Chapter 3 introduces the game theoretical elements instrumental to the formulation of both the classical Cournot equilibrium model of competition in quantities and of the supply function equilibrium model commonly used to estimate market prices in electricity markets [10], [1]. In this direction we prove a number of results that are employed in the following chapter. Chapter 4 presents the methodologies employed to preprocess and clean the data, and its subsequent analysis by means of the studied models. Estimates of the unitary market price are provided along with a discussion of the results. The code that executes the simulations is available in the Appendix.

Chapter 2

The Italian electricity market

The creation of the Italian electricity market (mercato elettrico) in 1999 was driven by two fundamental goals: to promote, based on criteria of neutrality, transparency, and objectivity, competition in the activities of electricity production and trading through the establishment of a "marketplace" and to ensure the economic management of adequate availability of dispatching services. The market is subdivided in a spot market: Mercato elettrico a pronti (MPE) and a futures market Mercato elettrico a termine (MTE). Figure 2.1 shows the market's structure with its main constituents

2.1 The spot market

The Italian electricity spot market is articulated in a series of branches:

- day ahead market (Mercato del giorno prima - MGP);
- intraday market (Mercato infragiornaliero - MI);
- dailiy products market (Mercato dei prodotti giornalieri – MPEG);
- dispatch service market (Mercato del servizio di dispacciamento - MSD).

The role and dynamics of each is object of the subsequent sections.

2.1.1 The day-ahead market

In the day-ahead electricity market the majority of electricity trading transactions are hosted [9]. Firms take part to the market by bidding offers in which they state both the amount they are willing to acquire/supply and the relative price. The session of the day-ahead (Meracto del Giorno Prima MGP) market opens at 8:00 a.m. on the ninth day preceding the delivery day and closes at 12:00 p.m. on the day before the delivery day. The results of the MGP are communicated by 12:58 p.m. on the day before the delivery day. The offers are accepted after the market session closes, based on economic merit and in compliance with the transit limits between zones. The MGP is therefore

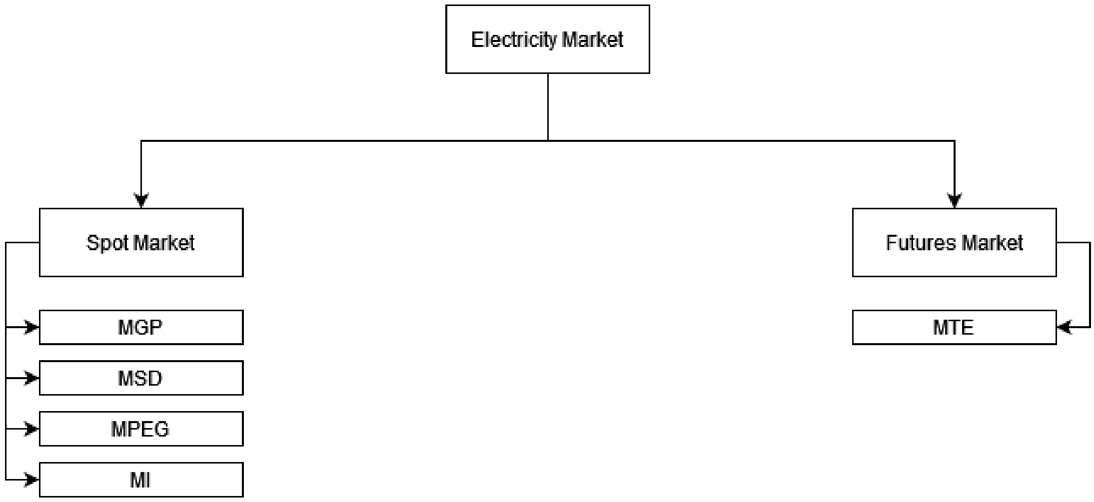


Figure 2.1. Structure of the Italian electricity market, the distinction between spot and futures market is emphasized.

an *auction market* and not a continuous trading market. All the bid and offers both Italian and foreign that are accepted by the MGP are valued by referring to the marginal equilibrium price of the region they belong to. Such a price is determined hourly as the intersection point between the demand and offer curves and varies zonally as a function of saturated transition limits. The unique national price (PUN) is then computed as the weighted average of the quantities sold in each zone. The matching of offer and demand is orchestrated by the GME.

2.1.2 The intraday market

The intraday market allows firms to revise the schedules devised in the day ahead market through buy or sell offers. Negotiations on the intraday market take place in three sessions MI-A and one continuous MI-XBID. In the MI-A sessions, concurrently with the negotiations of the buy and sell offers, the daily interconnection capacity is allocated between all the zones of the Italian and foreign markets involved in Market Coupling. The first session of the MI-A1 exchange takes place after the closing of the MGP, starting from the 12.55 of the day preceding the delivery day and closes at hour 15:00 of the same day. The results of the MI-A1 session are divulged before the 15:30 hour of the day before the delivery date. The MI-A2 sitting starts at 12:55 of the day prior to the delivery day and closes at 22:00 of the same day. The obtained outcomes are divulged prior to the 22:30 of the day before the delivery date. MI-A3 sessions open at 12:55 of the day preceding the delivery date and closes at 10:00 of the same day. The consequent results are divulged on the same day before 10:30. The buy and sell offers are selected on the same basis as in MGP, eith the difference that the accepted offers are remunerated at the zonal price, rather than the PUN. MI-XBID sessions are articulated in three phases, in which, contemporary to the previously cited negotiation, the infra-daily interconnection capacity between all the zones constituting the Italian market is auctioned. It is important to remark that the two auctioning sessions MI-A and MI-XBID take place sequentially in an alternating manner and can not overlap. The role of the main counterparty is played by the GME.

2.1.3 The daily products market

In the daily products market, daily products with mandatory daily delivery are exchanged. To the MPEG can take part all the participants to the electricity market. The MPEG is continuously open, with the modalities prescribed by the regulator. The allowed daily products exchanged in the market include goods with unitary price differential, whose price is determined after the negotiation phase and is the differential, with respect to the unitary national price, of the price that the firms are willing to pay for those products. Moreover, in the market are exchanged also goods at the full unitary price, which is the price directly negotiated in the signed contracts. For both those typologies of products, a number of delivery profiles are available including: baseload (quoted for every day of the week) and peak load, (quoted for weekdays days from Monday to Friday). The GME acts as central counterpart.

2.1.4 The dispatch market

The dispatch market is the instrument employed by Terna S.p.A to manage and control the dispatch network by resolving intra-zonal congestions and real time demand balancing. On the MSD Terna behaves as the central counterpart and accepted offers are remunerated at the bid price (pay-as-bid). The MSD is subdivided into two sub markets: MSD ex-ante and balance market MB. In the MSD ex-ante there is a unique auction held at 12:55 of the day prior to delivery, that closes at 17:00 of the same day. The GME publishes the results of the auction within the 21:45 of the day before the delivery date. In the MSD ex-ante Terna S.p.A. accepts the offers to sell and buy energy with the aim of resolving residual congestions and maintaining the reserve margins. In the MB offers are presented continuously with the aim of regularizing the balance of the Replacement Reserve (RR).

2.2 The futures market

The electricity futures market is the exchange responsible for the negotiation of contracts with bound of delivery and withdraw. All participants to the Italian electricity market have right to take part to the futures market, where negotiations are held continuously. On the MTE the delivery dates for base-load and peak-load contracts are montly, quarterly or yearly. The 'waterfall' mechanism is prescribed for quarterly and yearly contracts. Firms taking part to the market issue proposals defining type and delivery time of the contracts, number of contracts and the price at which they are willing to buy/sell. Monthly contracts, at the end of the negotiation period, are registered by the exchange. ON the futures market can be recorded also off the counter contracts.

2.3 Data

As stated in [9, 8, 2], the greater part of the electricity transactions is harbored by the day ahead market, whose outcomes constitute the unitary national price PUN. The objective of this thesis work is therefore to model the dynamics of day-ahead electricity markets using a game theoretical approach. Chapter 3 follows this theme by providing an overview of the main theoretical instruments object of this essay, namely the Supply Function equilibrium model with its specializations. Indeed, the presented results have been employed to study the behaviour of the Italian MGP, on the basis of data provided by Terna S.p.A. spanning the last three months of 2018. Chapter 4 presents the main assumptions driving the results of the analysis performed and compares the values predicted by the Supply Function model presented in Chapter 3 to Cournot's model.

2.3.1 Overview

The data consists of three comma separated values format files of respective sizes 808, 760 and 786 MB containing records of every bid put forward by the participating firms. The files span the months of October, November and December 2018 and hold a cumulative 16525464 items including all cited markets. Each instance is characterized by the following features:

- 'TR01_PURPOSE_CD', binary categorical with values 'BID' and 'OFF' respectively for demand and offer.
- 'TR01_TYPE_CD', binary, categorical with values 'REG', 'STN'
- 'TR01_STATUS_CD', categorical, with values: 'ACC', 'REJ', 'INC', 'SUB', 'REP' and 'REV' that specify the specific state of the bid/offer.
- 'TR01_MARKET_CD', categorical, specifies the type of market in which the transaction is submitted. It includes all the cited markets managed by GME.
- 'TR01_UNIT_REFERENCE_NO', string containing the identification code for each of the 2530 production units participating in the market.
- 'TR01_MARKET_P_XREF_NO', Nan.
- 'TR01_INTERVAL_NO', integer indicating the hour to which the bet pertains
- 'TR01_BID_OFFER_DATE_DT', string indexing the date in format YYYY\MM\DD,
- 'TR01_TRANSACTION_REFERENCE_NO', unique integer indexing each transaction
- 'TR01_BALANCED_REFERENCE_NO', Nan,
- 'TR01_QUANTITY_NO', float, quantity bid to the market by the producing plant, expressed in MW

- 'TR01_AWARDED_QUANTITY_NO': float, quantity awarded for production by the clearing mechanism
- 'TR01_ENERGY_PRICE_NO': float, energy price associated with the transaction
- 'TR01_MERIT_ORDER_NO': integer, indexes the instance,
- 'TR01_PARTIAL_QTY_ACCEPTED_IN': categorical, 'Y' if accepted, 'N' if rejected,
- 'TR01_ADJ_QUANTITY_NO': float,adjusted quantity by the market
- 'TR01_ADJ_ENERGY_PRICE_NO': float, price tributed by the market to the bid/offer,
- 'TR01_GRID_SUPPLY_POINT_NO': string identifying the contact point to the grid,
- 'TR01_ZONE_CD', string indexing the provenance of the bid/offer,
- 'TR01_AWARDED_PRICE_NO': float, when non zero indicates the prices awarded to the transaction,
- 'TR01_OPERATORE': string indexing each of the 210 firms operating the production plant in the considered interval,
- 'TR01_SUBMITTED_DT': string marking the date and time at which the bid/offer has been submitted with sub second precision,
- 'TR01_BILATERAL_IN', binary char 'Y', 'N'. 'Y' implies that the firm is both seller and producer, 'N' specifies that it is either one or the other,
- 'TR01_SCOPE',string of values: 'RS', 'GR1', 'GR2', 'GR3', 'ACC', 'CA', 'AS', 'GR4'.
- 'TR01_QUARTER_NO', integer between 1 and 4 refers to the trimester of the year,
- 'TR01_BATYPE', NREV, NaN.

As mentioned in the previous section, the analysis presented by this work the only market considered will be the day-ahead, indexed by the 'MGP' string. An instance of the data is displayed in Figure 2.2, where an offer to the intraday market MI5 for 210.154MW at 48.25 euros presented by production unit 'UP_CANDELA_1' belonging to EDINSON S.P.A. has been rejected. From the provided data it is possible to extract a number of statistics describing the composition of the Italian electricity production landscape, conveyed by Table 2.3.1.

```
TR01_PURPOSE_CD                                    BID
TR01_TYPE_CD                                       REG
TR01_STATUS_CD                                     REJ
TR01_MARKET_CD                                     MI5
TR01_UNIT_REFERENCE_NO                     UP_CANDELA_1
TR01_MARKET_P_XREF_NO                              NaN
TR01_INTERVAL_NO                                    23
TR01_BID_OFFER_DATE_DT                        20181016
TR01_TRANSACTION_REFERENCE_NO          995892950733200
TR01_BALANCED_REFERENCE_NO                         NaN
TR01_QUANTITY_NO                               210.154
TR01_AWARDED_QUANTITY_NO                           0.0
TR01_ENERGY_PRICE_NO                             48.25
TR01_MERIT_ORDER_NO                                194
TR01_PARTIAL_QTY_ACCEPTED_IN                         N
TR01_ADJ_QUANTITY_NO                           210.154
TR01_ADJ_ENERGY_PRICE_NO                           NaN
TR01_GRID_SUPPLY_POINT_NO                      PSR_616
TR01_ZONE_CD                                      FOGN
TR01_AWARDED_PRICE_NO                              0.0
TR01_OPERATORE                              EDISON SPA
TR01_SUBMITTED_DT                    20181016055628457
TR01_BILATERAL_IN                                    N
TR01_SCOPE                                         NaN
TR01_QUARTER_NO                                    NaN
TR01_BATYPE                                        NaN
Name: 243, dtype: object
```

Figure 2.2. Sampled output from the considered dataset in date 2018/19/16 with a rejected offer of 210.154 MW. The indexed plant 'UP_CANDELA_1' is a termoelectric central of maximal capacity 360MW owned by EDINSON SPA.

	October	November	December
Gross production	23815.2	23755.4	23324.5
Hydroelectric	3049.5	4772.0	4747.6
Thermoelectric	17523.1	15410.1	14747.1
Geothermic	513.8	496.0	491.9
Eolic	1034.8	2190.6	2442.1
Photovoltaic	1694.1	886.7	895.8

Table 2.1. Gross electrical production over the lat three months of 2018 divided by source. The reliance on thermoelectric sources is evident.

Chapter 3

Models

Having defined the structure of day-ahead wholesale auctions typical of locational marginal pricing markets, we face the problem of finding the natural theoretical framework to understand the strategic behaviour of its participants. In this direction game theoretical models present themselves as natural candidates to study and model the dynamics of day-ahead auctions. In the following, after a brief exposition of some elementary concepts from game theory, the supply function equilibrium model proposed by Klemperer and Meyer[7], with its variations in the formulation given by Newbery and Green [10] and Baldick et. al [2] will be discussed.

3.1 Elements of Game Theory

In this section a series of elementary definitions from game theory is presented with the intent of being foundational material for the following pages. In this context games in strategic form are considered. In strategic form games, for each *player* i belonging to the finite set $\mathcal{V}$, a set of *actions* A_i is available. A_i may be a fairly general structure including for example sets, functions, real numbers and others. The set

$$\chi = \prod_{i \in \mathcal{V}} A_i$$

is denoted *configuration space*. The vector describing the current action selected by each player is $x \in \chi$ and is called *action profile* or *configuration*. Furthermore, each player $i \in \mathcal{V}$ is equipped with a *utility function*, also known as *reward* or *payoff*, denoted with

$$u_i : \chi \to \mathbb{R}.$$

The utility function identifies the payoff $u_i(x)$ that player i gets when each player j plays action $x_j \in A_j$. The following definition formalizes these concepts.

Definition 3.1.1. A strategic form game is a triple $G = (\mathcal{V}, \{A_i\}_{i\in\mathcal{V}}, \{u_i\}_{i\in\mathcal{V}})$, where $\mathcal{V}$ is the set of players, A_i the set of strategies available to each player and $u_i : \chi \to \mathbb{R}$ the utility function for player $i \in \mathcal{V}$.

Following standard notation

$$x_{-i} = x_{\mathcal{V}\setminus\{i\}}$$

denotes the vector obtained from action profile x by removing its $i-$th entry and,

$$u_i(x_i, x_{-i}) = u_i(x),$$

denotes the utility obtained by player i when selecting action x_i while the remainder chooses x_{-i}. A classical example of a strategic game is presented int he following example.

Example (Cournot oligopoly)**.** In this oligopoly the players in the set $\mathcal{V}$ correspond with the set of producers of a certain commodity. Each producer $i \in \mathcal{V}$, with $|\mathcal{V}| \geq 2$, has to choose a strategy $x_i \in [0, \infty)$ that denotes the quantity of the commodity that will be produced and brought to the market; $C_i(x_i)$ denotes the total cost player i has to face when choosing strategy x_i. The price of one unit of the commodity in the market depends on $\sum_{i\in\mathcal{V}} x_i$ and and is computed according to the inverse demand function $\pi : \mathbb{R} \to \mathbb{R}$ that returns the marginal price π as a function of the total quantity demanded. This situation can be modeled by the strategic game $G = (\mathcal{V}, \{A\}_{i\in\mathcal{V}}, \{u\}_{i\in\mathcal{V}})$, where:

- $\{A_i\}_{i\in\mathcal{V}} = [0, \infty)$ and
- for each $i \in \mathcal{V}$ and each $x \in \chi, u_i(x_i, x_{-i}) = \pi(\sum_{j\in\mathcal{V}} x_j)x_i - C_i(x_i)$.

3.1.1 Nash Equilibrium in Strategic Games

In games in strategic form, each player $i \in \mathcal{V}$ acts rationally in choosing the action x_i that maximizes their utility $u_i(x_i, x_{-i})$. Indeed, player's i utility depends on the actions x_{-i} selected by the other players $j \in \mathcal{V} \setminus i$. Therefore, assuming that player i is aware of the set of actions chosen by the other players x_{-i}, and that these actions will not change, the rational behaviour for player i would be to choose

$$x_i = \operatorname*{argmax}_{x_i\in A_i} u_i(x_i, x_{-i}),$$

which is the *best response* that player i can select from A_i knowing that the other players have chosen x_{-i}. This concept can be naturally generalized by defining the best response (BR) function:

$$\mathcal{B}_i(x_{-i}) = \operatorname*{argmax}_{x_i\in A_i} u_i(x_i, x_{-i}),$$

that formalizes the idea that players choose actions with the aim of maximizing their own utilities knowing the actions played by other participants to the game.

Definition 3.1.2. A Nash equilibrium (NE) for the strategic game $G = (\mathcal{V}, \{A_i\}_{i\in\mathcal{V}}, \{u_i\}_{i\in\mathcal{V}})$ is an action configuration $x^* \in \chi$ such that

$$x_i^* \in \mathcal{B}_i(x_i^*), \quad \forall i \in \mathcal{V}. \tag{3.1}$$

In a Nash equilibrium, no player has any incentive to unilaterally deviate from their current action, because the utility obtained with the current action is the best possible given the current actions selected by other players. In general there might be one, several or no NE for a given game in strategic form.

Example (Cournot continued)**.** Let us consider the same $n \geq 2$ firms' oligopoly of the previous example and a demand $D(p) = N - \gamma p$, with $N, \gamma > 0$. Assume that each player (firm) is subjected to quadratic production costs $C_i(x_i) = \frac{1}{2}c_i x_i^2 + a_i x_i$, with $a_i \geq 0$, $c_i > 0$, $\forall i \in \mathcal{V}$ such that each player's utility function has the form:

$$u_i(x_i, x_{-i}) = \pi(\sum_{i\in\mathcal{V}} x_i)x_i - \frac{1}{2}c_i x_i^2 - a_i x_i.$$

The price function π can be made explicit by equating demand and offer

$$\sum_{i\in\mathcal{V}} x_i = N - \gamma p \implies \pi(\sum_{i\in\mathcal{V}} x_i) = \frac{1}{\gamma}\Big(N - \sum_{i\in\mathcal{V}} x_i\Big).$$

First order conditions on the utilities are imposed and the solution, when present, of the resulting system will yield x^*.

$$\frac{\partial}{\partial x_i}\frac{x_i}{\gamma}\Big(N - \sum_{i\in\mathcal{V}} x_i\Big) - \frac{1}{2}c_i x_i^2 - a_i x_i = 0, \quad \forall i \in \mathcal{V},$$

after differenciating, rearranging gives

$$x_i^* = \frac{N - \sum_{i\neq j} x_j^* - a_i\gamma}{2 + \gamma c_i}, \quad \forall i \in \mathcal{V}.$$

It is of importance to recall that to obtain an acceptable configuration the condition $x_i^* \in [0, \infty)$, $\forall i \in \mathcal{V}$ must also be satisfied. For the simplified case of equal linear marginal costs among the firms ($a_i = 0$, $c_i = c_j$, $\forall i, j \in \mathcal{V}$) an explicit solution is available

$$x_i^* = \frac{N}{(n+1) + \gamma c}, \quad \forall i \in \mathcal{V},$$

leading to the following equilibrium utility

$$u_i(x_i^*, x_{-i}^*) = \frac{N^2}{[(n+1) + \gamma c]^2}\left[\frac{1}{\gamma} + \frac{c}{2}\right].$$

The relevance of the model presented in the last two examples will become clear in the following section.

3.2 Supply Function Equilibria

In this section the model of competition in supply functions is presented along with the main results of the theory. Klemperer and Meyer, in a seminal paper [7], argue against the usual notion of firms competing either choosing prices or quantities as strategic variables. Indeed, they reason that a more sensible model of competition should include as strategic variables (a.k.a. actions) supply functions that specify the quantity a firm is willing to supply as a function of price. This framework can be applied to the specific case of day-ahead markets, where firms' bids consist of supply schedules as function of price that we might identify as supply functions. This approach, pioneered by Green and Newbery [10], reinterprets the probability distribution of random shocks to demand in the original work [7] to be an electricity load-duration characteristic. In the next pages the notation presented in [3] is used to study the model in the case of affine demand and affine marginal costs, with the aim of proving a number of useful results for the applications. The rest of this section is organized as follows: Subsection 3.2.1 introduces the notation and the main features of the model, Subsection 3.2.2 reports the fundamental characterization of the Nash equilibria of the game, Subsection 3.2.3 provides an existence theorem and finally Subsection 3.2.4 studies the model in the restricted case of affine supply functions.

3.2.1 Model Description

In the following paragraphs the model is presented using the notation provided in [3], where as in [10] the probability distribution of random shocks to demand in the original work [7] is reinterpreted to be an electricity load-duration characteristic.

Definition 3.2.1 (Affine Demand)**.** Demand $D : \mathbb{R}_+ \times [0,1] \to \mathbb{R}$ is a continuous function of the form:

$$\begin{aligned} D(p,t) &= N(t) - \gamma p, \quad \forall p \in \mathbb{R}_+, \quad \forall t \in [0,1], \\ N(0) &\leq N(t), \quad \forall t \in [0,1] \end{aligned} \tag{3.2}$$

where p is the price, t is the normalized time, $N : [0,1] \to \mathbb{R}_+$ is the load-duration characteristic and $\gamma > 0$ is minus the slope of the demand curve.

Remark 3.2.2. Notably, the assumption of a linear demand-price dependence and linear load-duration characteristic can be restrictive. More complex, continuous, load duration characteristics can be accounted for in this model, however we shall see that the functional form of the load-duration characteristic does not influence the class of Nash equilibria of the game.

The participating firms are identified as elements i of $\mathcal{V}$, with a minimum requirement of $|\mathcal{V}| = n \geq 2$ participants. The second crucial assumption of this work concerns the total variable generation cost function of each firm:

Definition 3.2.3. We call *total variable quadratic generation cost function*, or just *quadratic cost function* of firm $i \in \mathcal{V}$ a function $C_i : [0, \bar{q}_i] \to \mathbb{R}$, $C_i \in \mathbb{C}^2$ of the form:

$$C_i(q_i) = \frac{1}{2} c_i q_i + a_i q_i, \quad \forall q_i \in \mathbb{R}_+. \tag{3.3}$$

with $c_i > 0$, $a_i \geq 0$ and $\bar{q}_i$ maximum production capacity for player i.

Remark 3.2.4. Quadratic cost functions are only a subset of the plethora of possible cost functions. In the following, whenever a result is enunciated it will be specified whether it holds for the class of Definition 3.2.3 or for more general classes.

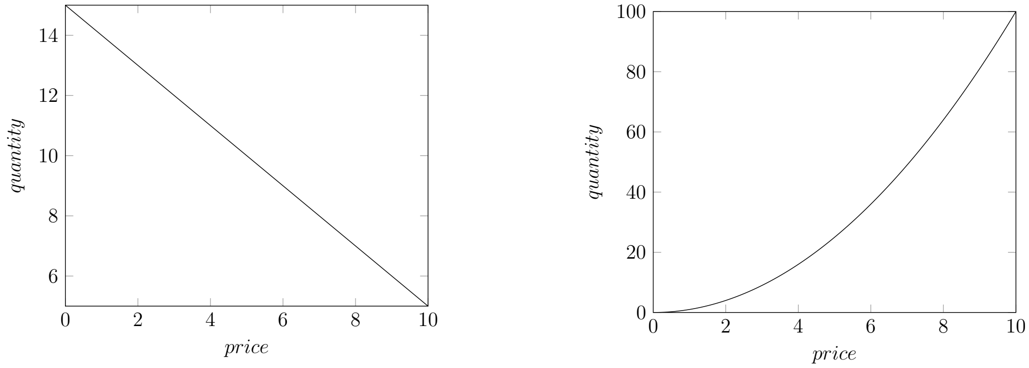


Figure 3.1. *On the left*: plot of an affine demand curve for a given time $t \in [0,1]$ with $\gamma = 1$. *On the right*: example of a supply function on the interval [0,10]

Remark 3.2.5. We can see how the requirement of $c_i > 0$ makes the generic C_i strictly convex, which allows the theory to ignore issues such as minimum-load costs, whereas startup costs can be included with $a_i \neq 0$. The resulting marginal form is therefore:

$$C_i'(q_i) = c_i q_i + a_i,$$

which is an affine function of the supply quantity q_i.

It is of importance to note that in this work a single cost function is assigned to each one of the n participants to the oligopolistic market, despite the varying technologies present in the energy generation portfolio of the single firm. C_i has therefore to be constructed by considering the optimal economic dispatch of the generation portfolio of each participant. Finally, the assumption of affine marginal costs does not capture jumps in marginal cost from, for example, coal to gas technology and does not capture the rapid increase in marginal costs at high output close to the maximum capacity experienced by production plants. However, it does represent the qualitative observation of increasing marginal cost with output.

Following the approach proposed by Green and Newbery [10] it is assumed that each firm bids a supply function into the market: a function that represents the amount of power it is willing to produce at each specified price per unit of energy. The supply function applies throughout the time horizon specified by the load-duration characteristic.

Definition 3.2.6. A differentiable function S_i, $\forall i \in \mathcal{V}$ is called *supply function* if:

$$\begin{aligned} S_i : [\underline{p}, \bar{p}] &\longrightarrow [0, \bar{q}_i] \\ S_i(\underline{p}) \leq S_i(p), &\quad \forall p \in [\underline{p}, \bar{p}], \end{aligned} \tag{3.4}$$

where $\underline{p}$ and $\bar{p}$ are respectively the minimum and maximum allowable prices. The set of supply functions is denoted $\mathbb{S}$.

Remark 3.2.7. The requirement of being non-decreasing for the supply functions: $S_i(\underline{p}) \leq S_i(p)$ stems naturally from the idea that with higher cost should be associated higher energy outputs, while the necessity of differentiability rather than simple continuity will become clear in the following sections.

To retain generality, the supply functions S_i are defined on a closed interval $[\underline{p}, \bar{p}]$, where $\bar{p}$ represents the (possible) market's price cap and $\underline{p}$ is implied by each firm's cost function and have a maximum capacity $\bar{q}$ that represent the maximum amount that firm i can supply to the market. The general theory for capacity constrained supply function models is presented in [3]. For every time t, the market is cleared based on the bid supply functions $S = (S_i)_{i=1,...,n}$ and the market price is obtained.

Definition 3.2.8. Let D be as in (3.2) and S_i, $\forall i \in \mathcal{V}$ as in (3.4). The market price p_t is the unique solution, for each time t of:

$$D(t,p) = N(t) - \gamma p = \sum_{i=1}^{n} S_i(p), \tag{3.5}$$

given by the price function $P(t)$. The set of market prices will be denoted as $\mathbb{P} = \{P(t)|t \in [0,1]\}$.

Remark 3.2.9. The existence of a unique price p_t is guaranteed, in this scenario, by the assumption of continuity of the supply functions whenever the demand function is non negative in $[\underline{p}, \bar{p}]$ along with the fact that demand is continuous and strictly decreasing.

Discontinuous functions over the whole interval $[\underline{p}, \bar{p}]$ would imply a redefinition of the of the notion of a solution and will not be explicitly considered in this thesis. Having defined the overall structure of strategies and market price it is natural to consider the profit accrued by each competing firm.

Definition 3.2.10. Let S_i be the supply function of firm $i \in \mathcal{V}$ and consider the supply functions of the other participating firms denoted by $S_{-i} = (S_j)_{j \neq i}$, the instantaneous profit is:

$$u_{it} = S_i(P(t))P(t) - C_i(S_i(P(t))) \tag{3.6}$$

where $P(t)$ is the price function for every $t \in [0,1]$ resulting from the choice of supply functions by the participating firms $(S_i)_{i=1,...,n}$.

Definition 3.2.11. The *total profit* by firm i over the normalized time interval [0,1] is therefore:

$$u_i(S_i, S_{-i}) = \int_0^1 u_{it} dt = \int_0^1 \Big(S_i(P(t))P(t) - C_i(S_i(P(t))) \Big) dt. \tag{3.7}$$

Definition 3.2.12. An action configuration $S^* = (S^*)_{i=1,...,n}$, where $S^* \in \mathbb{S}$ is a *Nash supplly function equilibrium* (SFE) if:

$$S_j^* \in \underset{S_i \in \mathbb{S}}{\operatorname{argmax}} \{u_i(S_i, S_{-i}^*)\}, \quad \forall i = 1, ..., n, \tag{3.8}$$

with $S_{-j}^* = (S_j^*)_{j \neq i}$.

In other words, a set of supply functions $(S^*)_{i=1,...,n}$ is a SFE if, firm i, given S_{-i}^*, has no incentive in unilaterally modifying its chosen S_i^*. Indeed this is equivalent to what Klemperer and Meyer propose when considering the demand shock and the consequent ex post equilibrium.

3.2.2 Equilibrium conditions as differential equations

Following the ideas of Klemperer and Meyer [7], [10] and [3], it is possible to rewrite the equilibrium conditions of definition 3.2.12 as a set of coupled nonlinear ordinary differential equations. The following derivation uses the notation provided in [3].

Proposition 3.2.13. *Assume demand in the form* (3.2) *and costs as in* (3.3)*. Let* $(S_i^*)_{i \in \mathcal{V}}$ *be a supply function configuration and* $\mathbb{P}$ *as defined in 3.2.8, then if*

$$S_i^*(p) = \Big(p - c_i S_i^*(p) - a_i\Big)\Big(\gamma + \sum_{i \neq j} S_j^{*'}(p)\Big), \quad \forall i \in \mathcal{V}, \ \forall p \in \mathbb{P}, \tag{3.9}$$

then the set $(S_i^*)_{i=1,\dots,n}$ *is a Nash supply function equilibrium.*

Proof. By assuming that for every t each firm $j \neq i$, $i, j \in \mathcal{V}$ has committed to a differentiable supply function S_j and that firm i has committed to supply the residual demand at every given price, it is straightforward to see that firm i will produce the quantity:

$$q_{it} = D(t, p_t) - \sum_{i \neq j} S_j(p_t), \quad \forall t \in [0, 1].$$

The resulting instantaneous profit in p_t for firm i will therefore satisfy:

$$u_{it} = q_{it} p_t - C_i(q_{it})\Big(D(t, p_t) - \sum_{j \neq i} S_j(p_t)\Big), \quad \forall t \in [0{,}1].$$

Having assumed the S_j differentiable, necessary conditions for maximising the instantaneous profit u_{it} under demand D as in (3.2) and costs as in (3.3) are:

$$\begin{aligned}
\frac{d}{dp_t} u_{it} = 0 \iff \frac{d}{dp_t} \ q_{it} p_t - C_i(q_{it})\Big(D(t, p_t) - \sum_{j \neq i} S_j(p_t)\Big) = 0, \\
\iff q_{it} = (p_t - c_i q_{it} - a_i)\Big(\gamma + \sum_{i \neq j} S_j'(p_t)\Big),
\end{aligned}$$

for all $t \in [0, 1]$. These equations can be solved for each t to find a unique optimal p_t and q_{it} for firm i. The quantity q_{it} cannot be immediately identified with a proper supply function S_i. Indeed, if the implicit relationship between q_{it} and p_t is monotonically non decreasing it is possible to define a non decreasing function $S_i : \{p_t | t \in [0{,}1]\} \to [0, \bar{q}_i]$ satisfying:

$$S_i(p_t) = q_{it}, \ \ \forall t \in [0, 1].$$

Applying then the implicit function theorem shows that for each p_t the function S_i is also differentiable, thus concluding the proof. ■

Remark 3.2.14. The derivation carried out in the proof of Proposition 3.2.13 can be generalized to the case of general convex costs C_i, $\forall i \in \mathcal{V}$ and concave demand $D(p, t)$. The resulting equilibrium differential equation reads:

$$S_i^*(p) = \Big(p - C_i'(S_i)\Big)\Big(\frac{d}{dp} D(t, p) + \sum_{i \neq j} S_j^{*'}(p)\Big), \quad \forall i \in \mathcal{V}, \quad \forall p \in \mathbb{P}.$$

Remark 3.2.15. If such a set as in 3.2.13 can be found, then the generic S_i^*, $\forall i \in \mathcal{V}$, achieves the maximum profit per unit time for firm i and each time t, given S_{-i}^*. Consequently, this supply function also maximizes the total profit u_i over the considered time horizon, and, moreover, the supply functions can be calculated without reference to the load-duration characteristic $N(t)$.

3.2.3 Existence Result

The problem of characterizing the conditions for the existence of an SFE in the general case of $n > 2$ capacitated firms with asymmetric cost functions remains unsolved. Nevertheless, in the simpler scenario of symmetric cost functions without capacity constraints, Klemperer and Meyer [7] characterize the conditions for the existence of a SFE and discuss the multiplicity of equilibria. The central theorem of this subsection paraphrases their theory. It applies only to supply functions whose $\bar{q} = \infty$ and $C_i = C_j \;\; \forall i \in \mathcal{V}$, for which equation (3.9) becomes:

$$S^{*'}(p) = \frac{S^*}{p - c(S^*(p)) - a} - \gamma = f(p, S^*) \tag{3.10}$$

A series of intermediate results are required, the proof of which can be found in [7]. To avoid an excess of notation, all the pedices have been dropped as the problem is assumed symmetric.

Lemma 3.2.16. *The locus of points satisfying $f(p, S^*) = 0$ is a differentiable function $S = S^*(p)$ such that:*

- $S(0) = 0$,
- $S^*(p) < C(S'(p))^{-1}, \;\; \forall p > 0$,
- $S^{*'}(p) > 0, \;\; \forall p > 0$,
- $S^{*'}(0) < \frac{1}{C''(0)}$.

Lemma 3.2.17. *A unique differentiable function $S^\infty(p) = (C'^{S(p)})^{-1}$ exists such that $f(p, S^\infty) = \infty$, $S^\infty(0) = 0$ and $0 < S^{\infty'} < \infty$, $\forall p > 0$.*

Lemma 3.2.18. *For every pair of points (p, S) that lay within $f = 0$ and $f = \infty$, $0f(p, S) < \infty$. For all pairs (p, S) in the first quadrant within $f = 0$ and $f = \infty$, $0 > f(p, S) > -\infty$.*

Lemma 3.2.19. *For any $(p_0, S_0) \neq (0{,}0)$, a unique solution to (3.10) exists to which the pair (p_0, S_0) belong. The solution is continuous in an open neighbourhood of (p_0, S_0).*

Lemma 3.2.20. *For any $(p_0, S_0) \neq (0{,}0)$ in the positive quadrant, the unique solution to (3.10) passing through (p_0, S_0) also passes through $(0{,}0)$.*

Proposition 3.2.21. *Let $N(t) \in [0, \infty)$, then a collection of strategies $(S_i^*)_{i=1,\ldots,n}$ is a symmetric SFE if and only if $\forall p \geq 0$, $(S_i^*)_{i=1,\ldots,n}$ satisfies 3.9 and*

$$0 < S_i^{*'}(p) < \infty.$$

Theorem 3.2.22. *Let $N(t) \in [0, \infty)$, then exists a SFE and the set of symmetric equilibria consists either of a single trajectory or of a simply connected set of trajectories.*

As anticipated, finding existence results for the problem (3.9) is generally a complex problem. In the following paragraphs existence (and uniqueness) results will be given for specific classes of supply functions that are deemed sufficiently representative of real world markets.

Example. In the sufficiently idealized scenario with n firms having symmetric quadratic generation costs functions $C(q) = \frac{1}{2}cq^2$, it is possible to provide an exact solution, whose existence is guaranteed by Theorem 3.2.22 when considering a demand $D(p,t) = N(t) - \gamma p$, with $\gamma > 0$.
Recalling 3.9 and avoiding subscripts the differential equations becomes:

$$S^{*'}(p) = \frac{S^*}{p - cS^*} - \gamma,$$

which can be expressed in parametric form with the use of the chain rule as follows:

$$\begin{pmatrix} S' \\ p' \end{pmatrix} = \begin{pmatrix} 1+\gamma c & -\gamma \\ -c & 1 \end{pmatrix} \begin{pmatrix} S \\ p \end{pmatrix},$$

where the superscript refers to differentiation with respect to the time t. It is well known that this ODE admits explicit solutions in the form:

$$\begin{pmatrix} S \\ p \end{pmatrix} = A_1 e^{\lambda_1 t} \begin{pmatrix} v_1 \\ w_1 \end{pmatrix} + A_2 e^{\lambda_2 t} \begin{pmatrix} v_2 \\ w_2 \end{pmatrix}, \quad \lambda_{1/2} = \frac{(2+\gamma c) \pm \sqrt{\gamma^2 c^2 + 4\gamma c}}{2}.$$

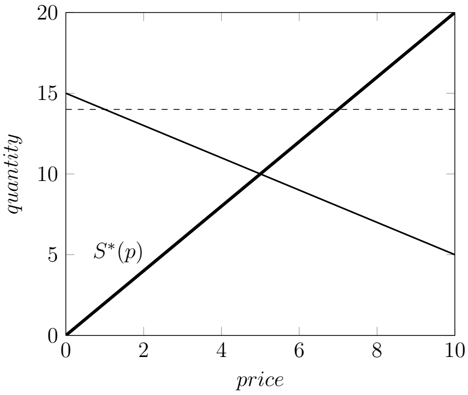
A_1 and A_2 are arbitrary constants chosen to satisfy given initial conditions. Paraphrasing the discussion proposed in [7], if we consider λ_1 to be the largest eigenvalue, $\lambda_1 > 1$ and $\lambda_2 \in (0,1)$ and $(v_1/w_1) < 0$, $(v_2/w_2) > 0$. As $t \to \infty$, $S^* \to 0$ and similarly $p \to 0$, for all A_1, A_2, implying that the solutions must pass through the origin. The single solution having $A_1 = 0$ is the unique linear solution having positive slope at the origin:

$$S^*(p) = \frac{1}{2}\left(-\gamma + \sqrt{\gamma^2 + \frac{4\gamma}{c}} \right) p, \tag{3.11}$$

where the price's coefficient follows directly from:

$$\frac{v_2}{w_2} = \frac{1-\lambda_2}{c} = -\gamma + \sqrt{\gamma^2 + \frac{4\gamma}{c}}.$$

Remark 3.2.23. In accordance with this result it is possible to prove that market prices implied by supply functions are an intermediate between the ones realized by Cournot and Bertrand competitions [7], therefore implying that firms profits also fall between the profits implied by the other models. However, it is not hard to check that firms' expected profits may be higher for specific scenarios in the SFE than in either the stochastic Cournot or the stochastic Bertrand case (in which firms choose quantities and prices based on a forecast of $N(t)$), because only in the SFE do firms adjust optimally to the uncertainty represented by the load duration given their opponent's behavior [6].



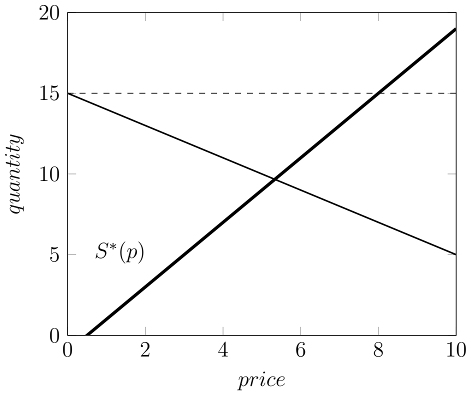


Figure 3.2. *On the left*: Comparison between the optimal amount resulting from Cournot and the optimal strategy resulting from the SFE. *On the right*: Comparison between the optimal quantity prescribed by Cournot's model and the optimal supply function resulting from SFE translated along the price axis to represent a minimum price cap.

3.2.4 Characterization of asymmetric affine SFEs

The assumptions of no capacity constraints and identical linear marginal production costs, while useful to prove existence theorems, fail to convey real world behaviour of markets, prescribing linear supply functions that are implausible due to the presence of fixed generation costs that differ amongst competing firms. In the more general case of asymmetric marginal costs functions, the strategy proposed by the literature [3, 10] to find an SFE is to seek action profiles $(S_i^*)_{i\in\mathcal{V}}$ that are also admissible solutions to (3.9), under some suitable boundary conditions. There are two major problems with this approach: on one hand, in general, a multitude of admissible action profiles solving equation (3.9) is available and, on the other hand, we lack characterizations of stability to discern between observable and unobservable solutions, under the assumption that only stable action profiles will be observed in real world markets. A solution to these two issues, proposed in [2], is to restrict the space of allowable supply functions $\mathbb{S}$, by mandating that they take on a determined affine functional form with respect to price p:

$$S_i(p) = \beta_i(p - \alpha_i), \quad \forall i \in \mathcal{V}, \quad \forall p \in \mathbb{P}, \tag{3.12}$$

where both β_i and α_i are chosen by firm i to be non-negative. This idea stems from the realization that, if we substitute equation (3.12) into (3.9) we obtain

$$\beta_i(p - \alpha_i) = (p - c_i\beta_i(p - \alpha_i) - a_i)\Big(\gamma + \sum_{i\neq j} \beta_j\Big), \quad \forall i \in \mathcal{V}. \tag{3.13}$$

Assuming consistency of the bid supply functions across times, equation (3.13) must be satisfied at every realized value of price p, and therefore it is an identity. The differential problem (3.9) has therefore been reduced to a more tractable algebraic problem in the $\beta_i, \alpha_i, \quad \forall i \in \mathcal{V}$. Indeed, this property holds true for a set of symmetries between parametrized supply functions and costs functions described in [5]. By restricting the set of possible supply functions, Baldick et al. are capable of showing both existence and uniqueness of the resulting SFE for every sub-interval of the form: $[\underline{p}, a_j], ..., [\max_i a_i, \bar{p}]$. This reformulation is necessary to account for the fact that different firms $i, j \in \mathcal{V}$ may have different marginal cost intercept: $a_i \neq a_j$. This in turn implies that, for firms having higher marginal costs, production is profitable only when $p > a_i$ and thus they will not compete for prices below a_i (i.e. their $\beta_i = 0$). The resulting scenario (conveyed for example in Figure 3.4) leads to a family of piece-wise affine supply functions that are not solutions of (3.9) on the global interval $[\underline{p}, \bar{p}]$, but solve (3.9) on every sub interval of the form $[a_i, a_j]$. If the demand D is known, then it is possible to compute every coefficient β_i and limit the analysis only to the sub interval where demand is met, where the set of affine $(S_i)_{i\in\mathcal{V}}$ will be a SFE.

Definition 3.2.24. An *affine supply function* S_i, $\forall i \in \mathcal{V}$ is a function of the form

$$S_i(p) = \beta_i(p - \alpha_i), \quad \forall i \in \mathcal{V}, \tag{3.14}$$

with $\beta_i, \alpha_i \geq 0$. The set of affine supply functions is denoted as $\mathbb{S}^{affine}$.

Proposition 3.2.25. *Assume that the action profile $(S_i^*)_{i\in\mathcal{V}}$ is a SFE in the space of affine supply functions with demand (3.2) and costs (3.3), then the following holds for all firms having $\beta_i > 0$*

$$\beta_i = \frac{\gamma + \sum_{i\neq j} \beta_j}{1 + c_i(\gamma + \sum_{i\neq j} \beta_j)} \tag{3.15}$$

and

$$\alpha_i = a_i, \quad \forall i \in \mathcal{V}.$$

Proof. Supposing that $(S_i^*)_{i\in\mathcal{V}}$ is a SFE, by substituting (3.12) in (3.9) for all the firm biding non zero quantities it is trivial to obtain:

$$\beta_i(p - \alpha_i) = (p - c_i\beta_i(p - \alpha_i) - a_i)\Big(\gamma + \sum_{i\neq j} \beta_j\Big), \quad \forall i \in \mathcal{V}. \tag{3.16}$$

Assuming consistency of the bid supply functions across times, equation (3.16) must be satisfied at every realized value of price p, and therefore equation (3.16) is an identity. Equating coefficients in p, we obtain:

$$\beta_i = \frac{\gamma + \sum_{i\neq j} \beta_j}{1 + c_i(\gamma + \sum_{i\neq j} \beta_j)}, \tag{3.17}$$

while equating the coefficients of the constant terms we get:

$$-\alpha_i\beta_i = -(a_i - c_i\beta_i\alpha_i)\Big(\gamma + \sum_{i\neq j} \beta_j\Big), \quad \forall i \in \mathcal{V}. \tag{3.18}$$

Conditions on α_i can therefore be obtained by substituting (3.17) in (3.18):

$$-\alpha_i(1 - c_i\beta_i)\Big(\gamma + \sum_{i\neq j} \beta_j\Big) = -(a_i - c_i\beta_i\alpha_i)\Big(\gamma + \sum_{i\neq j} \beta_j\Big), \quad \forall i \in \mathcal{V}. \tag{3.19}$$

Rearranging the resulting expression yields $\alpha_i = a_i, \quad \forall i \in \mathcal{V}$, which represents the intuitive idea that for a competing firm bidding supply functions having intercept lower than its minimum marginal costs is never optimal. ■

Remark 3.2.26. In general, for a given interval $[a_i, a_j]$, only a subset of $\mathcal{V}$ of firms having $a_i < a_j$, $i \neq j$, will be submitting non zero supply functions to the market, having β_i that satisfy (3.15). Indeed, $\beta_i > 0$, $\forall i \in \mathcal{V}$ only if the equilibrium price belongs to $[\max a_i \bar{p}]$.

Remark 3.2.27. A fundamental assumption in the proof of Proposition 3.16 is the requirement for competing firms to bid consistently across the time horizon; thus giving rise to a coupling effect that limits the possible equilibria. Without this requirement, there is no limitation to the range of observable equilibria: on one hand firms could behave as Cournot oligopolists at each time throughout the time horizon (possibly leading to higher prices than the SFE), while on the other firms could bid competitively.

The aim of the next few Lemmas is to study the conditions under which (3.15) admits positive solutions, with the idea of completely characterizing the SFE in the space of affine supply functions. To avoid confusion, the single coefficient will be denoted β_i, whereas the vector having as coefficients the β_i will be referred to as β. The following definitions are also useful:

Definition 3.2.28. The subset of players submitting S_i having $\beta_i > 0$ is denoted as $\mathcal{I}$, with $\mathcal{I} \subseteq \mathcal{V}$.

Definition 3.2.29. Given $\gamma > 0$ and β_i slope of the supply function S_i, we define:

$$Z_i = \gamma + \sum_{i \neq j} \beta_j, \quad \forall i = \mathcal{I}. \tag{3.20}$$

Furthermore we provide this third and fourth definitions,

Definition 3.2.30. Given Z as in (3.20), we call

$$\Phi_i(\beta) = \phi_i(Z_i) = \frac{Z_i}{1 + c_i Z_i}, \quad \forall i \in \mathcal{I}. \tag{3.21}$$

Remark 3.2.31. A trivial immediate property of definition 3.2.30 is that every fixed point of $\phi(Z_i)$ satisfies (3.9), as can be seen by just substituting (3.12) into (3.9). The map Φ represents the following strategy for each firm i: each time the firm updates its bid supply function, it chooses the slope of its updated affine supply function slope to maximize profits, given the most recent slopes used by all the other firms.

Definition 3.2.32. Let $A : \mathbb{R}^n \to \mathbb{R}^n$. If $\|A\| \leq 1$ the operator A is called *contraction map*.

In the following lemmas, conditions that Φ has to satisfy in order to be a contraction map will be found. Once those conditions are met, we will be able to apply Banach's fixed point theorem thus showing that Φ converges to the unique entry wise positive solution of (3.17).

Lemma 3.2.33. *If $\beta \geq 0$, then $\forall i = 1, ..., n$, $\frac{\gamma}{1+c_i\gamma} \leq \Phi_i(\beta) = \phi_i(Z_i) < \frac{1}{c_i}$ and the functions ϕ_i are monotonically increasing in Z_i for $\beta \geq 0$.*

Proof. Clearly, when $\beta \geq 0 \rightarrow Z_i \geq \gamma$. For $Z_i = \gamma$, $\Phi_i(\beta) = \phi_i(Z_i) = \phi_i(\gamma) = \frac{\gamma}{1+c_i\gamma}$. Letting $Z_i \rightarrow \infty$, implies $\Phi_i(\beta) = \phi_i(Z_i) \rightarrow \frac{1}{c_i}$. Indeed, $\frac{d\phi_i}{dZ_i} = \frac{1}{(1+c_iZ_i)^2} > 0$, so ϕ_i is strictly monotonically increasing in Z_i whenever $\beta \geq 0$, thus concluding the proof. ■

Indeed, Lemma 3.2.33 bounds β_i at each iteration in the interval:

$$\frac{\gamma}{1+c_i\gamma} \leq \beta_i < \frac{1}{c_i},$$

where $\frac{\gamma}{1+c\gamma}$ represents the optimal slope for a monopolistic scenario where only one firm is competing (with $\beta_i \neq 0$).

Lemma 3.2.34. *The following bound holds:*

$$\left\|\frac{d\Phi}{d\beta}\right\|_2 \leq (n-1)\left\|\frac{\partial\phi}{\partial Z}\Big((11'-I)\beta + 1\gamma\Big)\right\|_2 \tag{3.22}$$

Proof. We begin by noticing that $\Phi(\beta) = \phi(11'-I)\beta + 1\gamma)$, since Z can be rewritten as $Z = (11'-I)\beta + 1\gamma)$. Differenciating Φ with respect to β we obtain:

$$\frac{d\Phi}{d\beta} = \frac{\partial\phi}{\partial Z}\frac{\partial Z}{\partial\beta} = \frac{\partial\phi}{\partial Z}\Big((11'-I)\beta + 1\gamma\Big)(11'-I).$$

To complete the proof it is sufficient to show that $\|(11'-I)\|_2 = n-1$:

$\forall x \ \ |(11'-I)x\|_2 = x'(11'-I)(11'-I)x = x'(11'11'-11'-11'+I)x = x'((n-2)11'+I)x.$

Finally, recalling that $1'x = \sum_i x_i \leq \|x\|_1$ and therefore:

$$\|(11'-I)x\|_2^2 \leq (n-2)\|x\|_1^2 + \|x\|_2^2 \leq (n-2)n\|x\|_2^2 + \|x\|_2^2 = (n-1)^2\|x\|_2^2,$$

where the last inequality follows from the relationship between the 1 and 2 norm of real numbers. Repeating the reasoning with $x = 1$ we obtain the claim. ■

Lemma 3.2.35. *If for all $i \in \mathcal{I}$, $\beta_i \geq \frac{\gamma}{1+\gamma c_i}$, then:*

$$\left\|\frac{d\Phi}{d\beta}\right\|_2 \leq (n-1)\max_i \frac{1}{\left(1 + c_i\gamma\sum_j \frac{1}{1+c_j\gamma}\right)}. \tag{3.23}$$

Proof. By definition 3.2.29, $c_iZ_i = z_i\Big(\gamma + \sum_{j\neq i}\beta_j\Big) \geq c_i\Big(\gamma + \sum_{j\neq i}\frac{\gamma}{1+c_j\gamma}\Big) \geq c_i\gamma\sum_j \frac{1}{1+c_j\gamma}$.

$$\frac{\partial\phi}{\partial Z} = diag\left\{\frac{1}{(1+c_iZ_i)^2}\right\} \implies \left\|\frac{\partial\phi}{\partial Z}\right\|_2 = \max_i\left\{\frac{1}{(1+c_iZ_i)^2}\right\} \leq \max_i \frac{1}{\left(1 + c_i\gamma\sum_j \frac{1}{1+c_j\gamma}\right)^2}$$

Recalling the proof of lemma 3.2.34, it is known that:

$$\frac{d\Phi}{d\beta} = \frac{\partial \phi}{\partial Z}\frac{\partial Z}{\partial \beta} \implies \left\| \frac{d\Phi}{d\beta} \right\| \leq (n-1)\max_i \frac{1}{\left(1 + c_i\gamma \sum_j \frac{1}{1+c_j\gamma}\right)^2}$$

■

The following proposition summarizes the necessary conditions for Φ to be a contraction.

Proposition 3.2.36. *Let* $n < 1 + \min_i \left(1 + c_i\gamma \sum_j \frac{1}{1+c_j\gamma}\right)^2$ *and* $\beta_i \geq \frac{\gamma}{1+c_i\gamma}$ $\forall i \in \mathcal{I}$ *then the map* Φ *is a contraction.*

Proof. By choosing $\beta_i \geq \frac{\gamma}{1+c_i\gamma}$ we know from Lemma 3.2.35 that equation (3.23) holds. To complete the proof:

$$\left\| \frac{d\Phi}{d\beta} \right\|_2 < 1 \iff (n-1)\max_i \frac{1}{\left(1 + c_i\gamma \sum_j \frac{1}{1+c_j\gamma}\right)^2} < 1 \iff n < 1 + \max_i \frac{1}{\left(1 + c_i\gamma \sum_j \frac{1}{1+c_j\gamma}\right)^2}.$$

■

With the results of Proposition 3.2.36 it is possible to enunciate the fundamental characterization of SFE in affine asymmetric uncapacitated supply functions.

Proposition 3.2.37. *Let* S_i *be an affine supply function as in* (3.2.24) *with firm* $i \in \mathcal{V}$ *having affine marginal costs as in* (3.3) *and demand* (3.2)*. Consider an initial component wise non-negative vector* β *and assume that the conditions of Proposition 3.2.36 hold, then exists a unique supply function equilibrium* $(S_i^*)_{i\in\mathcal{V}}$ *that can be reached by iteratively applying strategy* $\Phi(\beta)$ *on every sub interval of the form* $[a_1, a_2], ..., [a_n, \bar{p}]$*.*

Proof. Under the assumptions of Proposition 3.2.36, operator Φ is a contraction map and Banach's fixed point theorem applies. System (3.15) admits therefore a unique positive solution β_i^*, $\forall i \in \mathcal{I}$. Consequently, the resulting non identically zero $(S_i^*)_{i\in\mathcal{I}}$ are strictly increasing and positive supply functions on every interval of the form $[a_i, a_j]$. ■

Remark 3.2.38. Notably, by requiring firm i to bid an affine supply function, the plethora of possible equilibria has reduced to a unique stable equilibrium on every interval $[a_i, a_j]$. Indeed, by proving that the map Φ is a contraction under a reasonable subset of assumptions, it has become clear that for every choice of $\beta \neq \beta^*$ the firms deviating from the equilibrium incur in sub optimal profits, thus implying that the affine equilibrium is stable.

A final remark can be made regarding the assumption $c_i > 0 \quad \forall i \in \mathcal{I}$ that has been ubiquitous in this work. Fehr and Harbord [14] show that for the unique situation of firms having the following costs:

$$a_i \neq a_j \quad \forall i \neq j$$
$$c_1 = 0 \quad \forall i = 1, ..., n,$$

then there is no affine SFE solution, thus motivating the requirement of $c_i > 0 \quad \forall i = 1, ..., n$. Their reasoning goes as follows: first consider an affine function of the form:

$$S_i^{affine}(p) = \beta_i p - \alpha_i,$$

that substituted into (3.9), gives:

$$\beta_i p - \alpha_i = (p - a_i)\Big(\gamma + \sum_{i \neq j} \beta_j\Big).$$

This expression is identically true for all realized prices, therefore, by equating the powers of p it is possible to write:

$$\beta_i = \sum_{i \neq j} \beta_j + \gamma.$$

Considering this expression over all competing firms:

$$\begin{aligned} \sum_{i=1}^{n} \beta_i &= \sum_{i=1}^{n} \sum_{i \neq j} \beta_j + \gamma \\ &= (n-1) \sum_{i=1}^{n} \beta_i + n\gamma \\ 0 &= (n-2) \sum_{i=1}^{n} \beta_i + n\gamma. \end{aligned}$$

This equation has no non-negative solutions for values of β_i and γ, $n \geq 2$. This result is not surprising given that affine SFEs cannot cover the profit-maximising response given constant marginal costs. However, a slight extension of this argument to the more general continuous but nonlinear SFE case shows that the only SFE that can exist in this situation are solutions that are significantly more competitive than the affine SFE. Furthermore, the assumption of firms having constant marginal production costs is unsupported by real world data from day-ahead electricity auctions, where it is clear that production technologies have marginally increasing costs.
Since the uncapapcitated affine supply function is not dependent on the load-duration characteristic $N(t)$, the same β_is will apply for any load-duration. It is therefore possible to estimate the profit over any time interval by considering aggregate demand and price as functions of time.

Lemma 3.2.39. *In the uncapacitated asymmetric SFE having demand 3.2 and costs 3.3 the equilibrium price satisfies*

$$p(t) = \frac{N(t) + \sum_i \beta_i a_i}{\gamma + \sum_i \beta_i}, \quad \forall t \in [0,1]. \tag{3.24}$$

Proof. The proof is straightforward:

$$N(t) - \gamma p = \sum_i \beta_i (p - a_i) \iff N(t) + \sum_i \beta_i a_i = p(t)\Big(\gamma + \sum_i \beta_i\Big).$$

■

3.2.5 Examples of affine supply functions equilibria

To complete this chapter, two examples of applications of the previous model are presented to serve as foundational material for the applications presented in the next chapter. At first an example of two firms competing while subjected to linear marginal costs is considered. Consequently, the second example studies the more delicate case of two firms subjected to affine marginal costs and analyzes the resulting equilibrium.

Linear asymmetric uncapacitated SFE In the following example the linear SFE of two competing firms having asymmetric linear marginal production costs $C_i(S) = \frac{c_i}{2} S^2$ is considered. Firm $i = 1$ is such that $c_1 = 0.5$, while firm $i = 2$ produces subjected to $c_2 = 0.2$, while demand is of the form $D(p) = 1 - p$. A visualization of the results is depicted in Figure 3.3, where the linear supply functions of firms 1 and 2 are dashed. The two slopes β_1 and β_2 have been found by solving directly problem (3.17). To illustrate the convergence properties of strategy Φ, a series of iterations of $\Phi(\beta)$, with initial $\beta_i = \frac{\gamma}{1+c\gamma}$ are also depicted in Figure 3.3. The resulting price is highlighted in red. As previously argued, while useful, the simple approximation of linear marginal costs fails to convey the startup costs characteristic of energy production plants, generally prescribing lower prices than the one observed in real world markets. In the following example a more realistic version is addressed.

Affine asymmetric uncapacitated SFE In switching from linear to affine supply functions an immediate issue arises whenever marginal cost intercepts a_i differ amongst firms. Indeed, as can be seen in Figure 3.4 where $a_1 = 0$ and $a_2 = 0.2$, firm 2 has no incentive in producing below a_2, leaving as sole competitor firm 1. The resulting optimal β_1 is found in the interval $[0, a_2]$ by firm 1 as $\beta = \frac{\gamma}{1+c\gamma}$, solving (3.17). If demand is not intercepted by firm 1 alone in $[0, 0.2]$, the game is repeated with the two participants and a new β_1 is found. Notably, the presence of a number of competing firms gives rise (as a consequence of Proposition 3.2.36) to steeper β_is, thus introducing jumps in the global supply function $S_i^*(p)$, which is defined on the whole interval $[\underline{p}, \bar{p}]$. The notion of SFE as given in Definition 3.8 is therefore altered ex post to be relative to each sub interval $[a_i, \bar{p}]$. This phenomenon is exemplified by Figure 3.4, where $c_1 = 0.1$, $c_2 = 0.3$ and demand is $D = 1 - p$. Indeed, in 0.2 a jump occurs for the supply function of firm 1, due to the competition with firm 2.

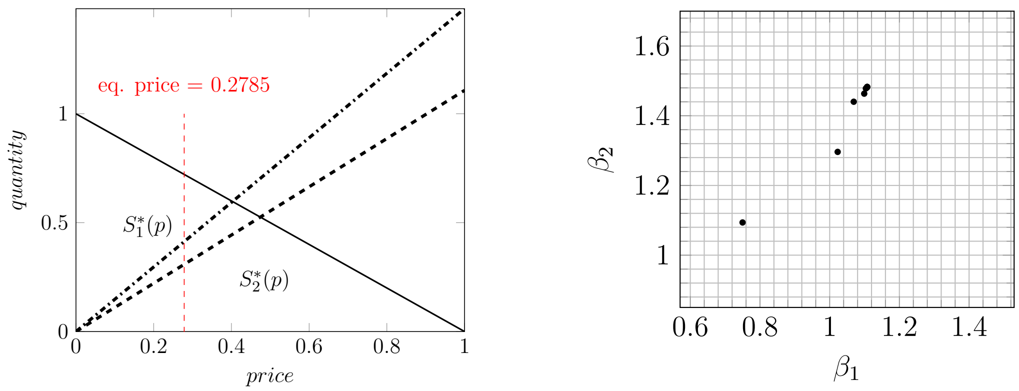


Figure 3.3. *On the left*: Demand (in solid black), supply functions (dashed) and equilibrium price (in red) for the simple duopoly considered in the example. *On the right*: Illustration of the convergence of operator $\Phi(\beta)$.

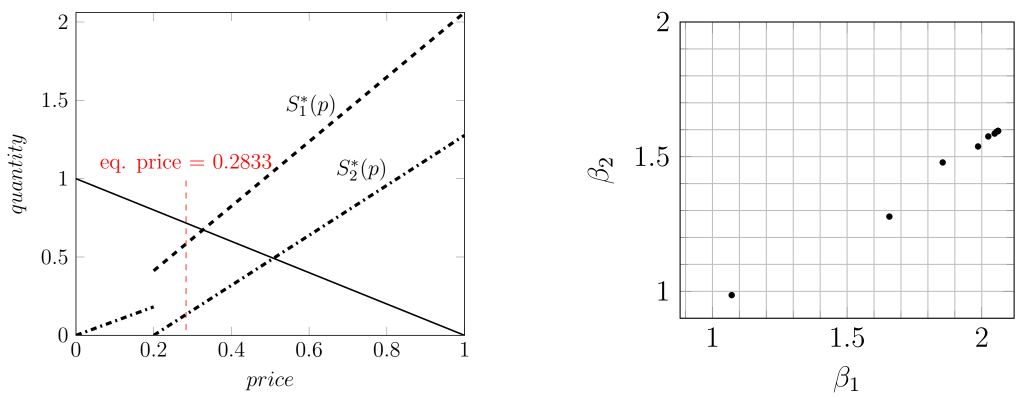


Figure 3.4. *On the left*: Demand (in solid black), supply functions (dashed) and equilibrium price (in red) for the simple duopoly considered in the second example. It is possible to notice the jump in 0.2 of $S_1^*(p)$ as firm 2 enters the auction. The obtained β are a SFE on the sub interval $[0.2, 1]$. *On the right*: Illustration of the convergence of operator $\Phi(\beta)$.

Chapter 4

Data Analysis

This chapter explores an application of the models presented in Chapter 2 to the Italian Day Ahead electricity market. The analysis has been conducted employing data provided by GME and gathered by Terna, the Italian company responsible for the management of the high-voltage electricity grid in Italy. Terna is responsible for maintaining the balance between electricity supply and demand, managing grid operations, and planning for the development and expansion of the grid infrastructure to meet future energy needs. Additionally, Terna has been gathering useful data on the day-ahead exchange managed by Gestore Mercati Energetici (GME), an Italian firm wholly owned by the Ministry of Economy and Finance. Among its many responsibilities GME organizes and manages the natural gas, environmental and electricity markets. In the Italian exchange (known as Italian Power Exchange IPEX), producing firms and distributors commit to the production and retail of electricity wholesale, by participating in one or more of the provided markets the MGD, object of the following analysis. The chapter maintains the notation of the previous and it is structured as follows: Section 2.3.1 delineates the general structure of the obtained data, Section 4.1.2 deals with the problem of estimating demand and describes the methodologies used, Section 4.1.3 outlines the procedure employed to estimate prices and finally Section 4.1.4 illustrates the resulting differences between the estimated and realized price for the considered timeframe.

4.1 Methodologies

With the aim of providing an estimate of the unitary price resulting from the dynamics of the day ahead market, this work applies theoretical results in the form of Cournot and supply functions equilibrium models, as have been introduced in Chapter 3. In this direction, we recall that demand D is assumed to be an affine function of price:

$$D(p,t) = N(t) - \gamma p \quad \forall t \in [0,1], \quad \forall p \in \mathbb{R}_+, \tag{4.1}$$

with $\gamma > 0$, and $N(t)$ the load duration characteristic. Furthermore, each participant to the market is supposed to incur affine marginal production costs of the form:

$$C_i'(q_i) = c_i q_i + a_i \quad \forall i \in \mathcal{V},$$

with $c_i > 0$ and $a_i \geq 0$. Indeed, both assumptions in the form of demand and production costs are reasonable simplifications of the realized market demand and industry costs. Under the recalled assumptions on the functional form of both demand D and production costs C_i, we have considered the models of competition in quantities and supply functions, showing that players competing in quantities display a unique Nash equilibrium of the form:

$$x_i^* = \frac{N - \sum_{i \neq j} x_i^* - a_i \gamma}{2 + \gamma c_i}, \quad \forall i \in \mathcal{V}, \tag{4.2}$$

with x_i^* being the optimal produced quantity at equilibrium by firm $i \in \mathcal{V}$. Competitions in supply functions, on the other hand, has been studied extensively in Chapter 3, where results concerning existence and uniqueness of the Nash equilibrium have been provided for the case of parametrized supply functions of the form:

$$S_i(p) = \beta_i(p - \alpha_i) \quad \forall i \in \mathcal{V}.$$

Indeed, we have proved that for every price interval of the form $[a_i, a_j]$ exists a unique Nash equilibrium, with $a_i = \alpha_i$, for every firm bidding nonzero supply functions. Furthermore, the differential problem characterizing the functional form of the Nash equilibria of the game reduces to an algebraic problem on the β_i, $\forall i \in \mathcal{V}$:

$$\beta_i^* = \frac{\gamma + \sum_{i \neq j} \beta_j^*}{1 + c_i(\gamma + \sum_{i \neq j} \beta_j^*)}, \quad \forall i \in \mathcal{I}. \tag{4.3}$$

System (4.3) has been implemented and numerically solved by means of the SciPy Python module, providing the unique SFE equilibrium in supply functions on every interval of the form $[a_1, a_2]$,...,$[a_i, a_j]$ for firms bidding $\beta_i > 0$. In this application we employ four variations of the cited models:

- Supply Function Equilibrium with linear marginal costs ($a_i = 0, \ \forall i \in \mathcal{V}$),
- Supply Function Equilibrium with affine marginal costs ($a_i > 0, \ \forall i \in \mathcal{V}$),
- Cournot equilibrium with linear marginal costs,
- Cournot equilibirum with affine marginal costs.

For each one of the 2208 datetimes over the three month interval, we have utilized the demand data to derive the corresponding realized market prices. Subsequently, the firms participating in the market for the given time and date have been selected by retaining only those bidding prices within the maximum and minimum realized prices over the horizon. Furthermore, for the scenario in which $a_i > 0$, only firms having $a_i < p$ have been retained. Of the remaining producers, only those bidding quantities greater than the prescribed amount of 100MW have been retained. This threshold, although arbitrary, has been implemented strategically to mitigate the inclusion of negligible producers, thus aligning with the assumption that price formation is influenced by more substantial bids. Finally, the quantity bid by firms outside the competitive scenario has been subtracted to the load duration characteristic. Subsequent to these filtration steps, we have computed supply functions and Cournot quantities for the remaining participants, culminating in the computation of the implied price. In order to evaluate and compare the precision of price predictions generated by the chosen methods, we propose employing the Mean Absolute Error (MAE) as a suitable metric. MAE calculates the average absolute disparity between predicted prices and actual prices, returning smaller values the closer the predicted are to the observed ones. The remainder of this Chapter is organized as follows: Subsection 4.1.1 addresses the techniques employed to clean and prepare the data, Subsection 4.1.2 details the process of estimating the required functional form of demand, Subsection 4.1.3 provides the required parameters with a data based strategy and finally Subsection 4.1.4 shows the resulting equilibrium prices for a single day.

4.1.1 Preprocessing

The process of cleaning and preparing the data has been addressed in a series of sequential steps using the programming language Python. At first, a new dataframe comprised of data exclusively from the day-ahead market has been obtained from the original by indexing feature 'TR01_MARKET_CD' by the string 'MGP'. Subsequently, a new feature called 'DateTime' has been added to serve as index. The feature 'DateTime' groups together bids and offers submitted during the market hours with hourly interval. Furthermore, the columns:

- 'TR01_SCOPE',
- 'TR01_QUARTER_NO',
- 'TR01_BATYPE',
- 'TR01_TRANSACTION_REFERENCE_NO',
- 'TR01_INTERVAL_NO',
- 'TR01_MARKET_P_XREF_NO',
- 'TR01_SUBMITTED_DT',
- 'TR01_BILATERAL_IN' and
- 'TR01_BID_OFFER_DATE_DT',

have been discarded as redundant for the subsequent application of the selected methods. An instance of the selected columns is available at Figure 4.1. The presence of instances having offered at price 0 and awarded quantity greater than 0 has been addressed by substituting with the arbitrary value of 3000 their 'TR01_ENERGY'-'_PRICE_NO' feature. By removing those instances it is possible to accurately extract demand from the data at a zonal level and consequently obtain the realized market price for each zone. The resulting prices are then combined via weighted average as prescribed in [9] and the final unitary price is displayed in Figure 4.1.1.

```
TR01_STATUS_CD                                 REJ
TR01_UNIT_REFERENCE_NO                 UP_CANDELA_1
TR01_QUANTITY_NO                           210.154
TR01_AWARDED_QUANTITY_NO                       0.0
TR01_ENERGY_PRICE_NO                         48.25
TR01_MERIT_ORDER_NO                            194
TR01_PARTIAL_QTY_ACCEPTED_IN                     N
TR01_ADJ_QUANTITY_NO                       210.154
TR01_ZONE_CD                                  FOGN
TR01_AWARDED_PRICE_NO                          0.0
TR01_OPERATORE                          EDISON SPA
DateTime                       2018/19/16 23 00 00
Name: 243, dtype: object
```

Figure 4.1. Sampled output from the considered dataset in date 2018/19/16 with a rejected offer of 210.154 MW. The indexed plant 'UP_CANDELA_1' is a termoelectric central of maximal capacity 360MW owned by EDINSON SPA.

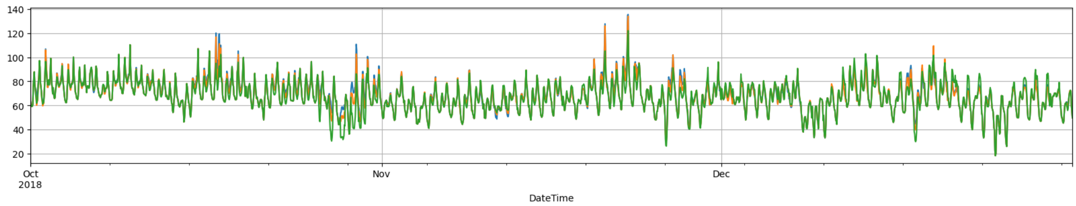


Figure 4.2. *In blue*: market price obtained over the region 'NORD'. *In orange*: market price over all the zones obtained from bids. *In green*: market price over all the zones obtained from offers.

4.1.2 Demand Estimation

One of the main hurdles in the application of game theoretical models to the electricity markets is the problem of reliably estimating demand (see, for example [2]). With the aim of providing a sensible estimate of demand as in the functional form 4.1, the normalized time interval $[0, 1]$, domain of times in the previous exposition, has been considered as discretized in 24 different values, each corresponding to a specific hour of the day for which the day ahead auction takes place on the MGP. Consequently, linear regression coefficients have been estimated for each hour of each day of the considered three months interval, using the utilities provided by the SciKit Learn module [11]. In this direction we identify as covariates, for each hour, the realized prices to predict the implied awarded quantity. The employed module computes the coefficients that minimize the residual sum of squares between the observed quantities and their linear prediction using plain Ordinary Least Squares as implemented by the library SciPy [13]. This method shows effective results, with an average r2 score of 0.773. The resulting set of intercepts constitutes the historical series of $N(t)$ described in Figure 4.7, from which is possible to appreciate the periodic

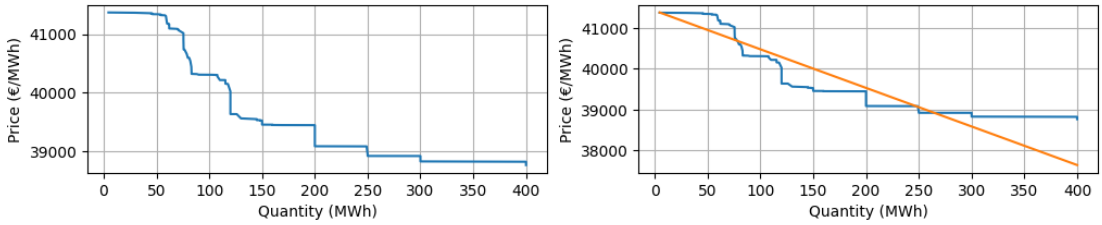


Figure 4.3. *On the left*: demand for the hour 10:00:00 of the 2018-10-16. *On the right*: resulting linear regression.

behaviour of the load duration curve along the time frame considered. Similarly, the resulting set of slopes has been computed and is conveyed in Figure 4.7. Indeed, equation (4.1) assumes a unique value of γ, that this work estimates by considering the averag e value of the cited historical series. The choice of computing the mean rather than other possible statistical measures such as, for example, the median, has been driven by the fact that the observed values of γ present limited positive skewness. Therefore, the difference between the two statistical measures has proven to be negligible for the aim of this work and the former was chosen. In estimating the hourly demand with affine functions approximation errors arise, predominantly due to the step-like behaviour of the realized demand curve that is a direct consequence of market rules [9]. Figure A.1 conveys the difference between the market price and the approximation resulting from the estimate. As can be expected, greater deviations in price estimation correlate with underestimates of γ.

4.1.3 Costs Estimation

The estimation of the affine marginal production cost coefficients $c_i, a_i \forall i \in \mathcal{V}$ assumed by the theory:

$$C_i'(q_i) = c_i q_i + a_i, \quad \forall i \in \mathcal{V}$$

has been obtained directly from the available data at a production unit level, therefore implying that the set of players $\mathcal{V}$ constitutes the set of plants and not of participating firms. The choice of estimating the marginal costs for each production unit as opposed to a firm-level estimate has been driven by the fact that firms take part to the day-ahead market at a plant level, thus implying utilities and costs pertinent to single units. The problem has therefore been subdivided into the estimation of a_i and c_i, $\forall i \in \mathcal{V}$, and is addressed in the next subsections.

Estimation of a_i The $a_i, \forall i \in \mathcal{V}$ represent the minmum price at which it is strategically convenient for firm i to bid a nonzero quantity to the market, and have therefore been estimated accordingly. As can be expected, different production technologies can be distinguished by their marginal cost intercepts. The results of this analysis are displayed in Table 4.1.3 where the mean marginal cost intercept for firms taking part in the 'NORD' region are displayed.

Production tecnology	mean a	mean c
CCGT	25.98	1.931
Hydro	27.73	6.455
Wind	29.10	3.857
Photovoltaic	35.69	1.677
Coal	39.12	0.1223

Table 4.1. Estimated c_i and a_i for different technologies on the location 'NORD' ranked in ascending order of c.

Estimation of c_i The estimation of $c_i, \forall i \in \mathcal{V}$ has proven to be a complex task due to the notable variance in capacity and technologies among plants of the same production type and has therefore been achieved by considering the value implied by the data. To this aim, from equation (3.9), with affine marginal costs (3.3) it is possible to isolate c_i:

$$c_i = \frac{p - a_i}{q_i} - \frac{1}{\gamma + \sum_{i \neq j} S'_j(p)}, \quad \forall i \in \mathcal{V}, \tag{4.4}$$

that implies:

$$\frac{p - a_i}{q_i} > c_i > \frac{p - a_i}{q_i} - \frac{1}{\gamma}. \tag{4.5}$$

The second inequality of equation (4.5) is justified by the fact that $\sum_{i \neq j} S'_j(p) > 0$ whenever at least 2 players take part to the market. In this work we use the first inequality of (4.4) and write

$$c_i \approx \frac{p - a_i}{q_i}, \quad \forall i \in \mathcal{V}. \tag{4.6}$$

In applying approximation (4.6) the maximal quantity bid to the market q_{max}, in conjunction with the mean awarded price $\hat{p}$ have been considered:

$$\hat{c}_i = \frac{\hat{p} - a_i}{q_{max}}, \quad \forall i \in \mathcal{V}. \tag{4.7}$$

This strategy has proven necessary to avoid overestimating c_i whenever a non maximal quantity is bid at a price significantly higher than its actual production costs. The averaged production costs by plant type are depicted in Table 4.1.3.

4.1.4 Price estimation

Having provided sensible estimates of both demand and marginal production costs, it is straightforward to consider the resulting estimated prices. In particular, recalling results presented in Chapter 3, for the Cournot estimate holds:

$$p^* = \frac{N(t) - \sum_{i \in \mathcal{V}} x_i^*}{\gamma},$$

whereas for the supply function equilibirum model holds:

$$p^* = \frac{N(t) + \sum_{i\in\mathcal{V}} a_i\beta_i}{\gamma + \sum_i \beta_i}.$$

Using those two formulas it is possible to produce an estimate of the unitary market price by subtracting to the estimated load duration $N(t)$ the quantity offered to the market by firms behaving as non strategic players (bidding, for example, outside of the market cap, or presenting bids lower than the prescribed threshold). Figure 4.4 conveys the resulting scenario for the last hour of the considered interval, displaying on the left the intersecting curves of estimated demand, cumulative supply functions with $a_i = 0$ and cumulative offer in supply functions with $a_i > 0$. It is possible to notice how the approximation of demand, in this specific instance, tends to overestimate the actual unitary market price, and also how the affine supply functions prescribe higher equilibirum prices as a consequence of the nonzero marginal intercepts. On the right it is possible to appreciate Cournot's cumulative equilibria in both scenarios of $a_i = 0$ and $a_i > 0$. Notably, Cournot's model in competition over quantities tends to overestimate the actual market price.

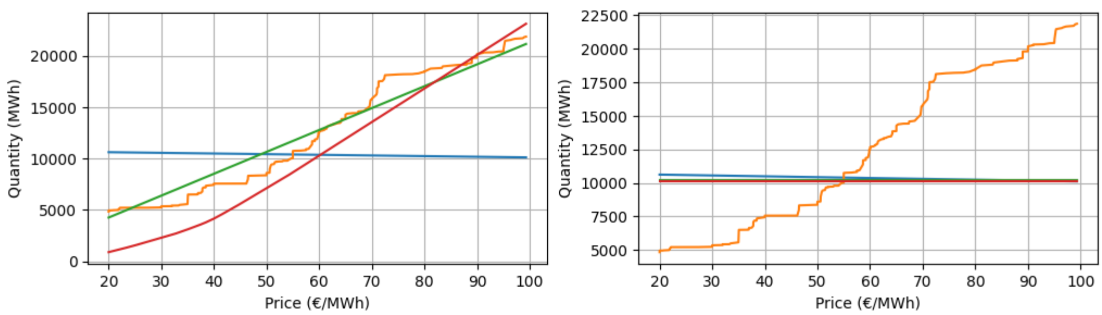


Figure 4.4. Representation of the results for the last timeframe considered by the data. *On the left*: in light blue it is possible to appreciate the linear approximation of demand, in orange the adjusted offer curve. The cumulative supply functions are in green ($a_i = 0$) and red ($a_i \neq 0$). It is possible to notice the composition of affine curves for the affine supply function equilibrium. *On the right*: comparison between the two cumulative equilibria of the Cournot model. In green is displayed the model having $a_i = 0$, in red $a_i \neq 0$.

4.2 Results

The application of the aforementioned estimation procedures to the three month interval of October, November and December 2018 has followed the structure presented in the previous sections, by approximating demand, fitting cost functions, obtaining the implied quantities and supply functions and finally estimating prices. This section presents and discusses the results of those estimates, linking the observed behaviour of the models to other known examples from the literature. Starting with Cournot's equilibrium model, obtained predictions are displayed in Figure 4.5. It is immediately clear that prices result-

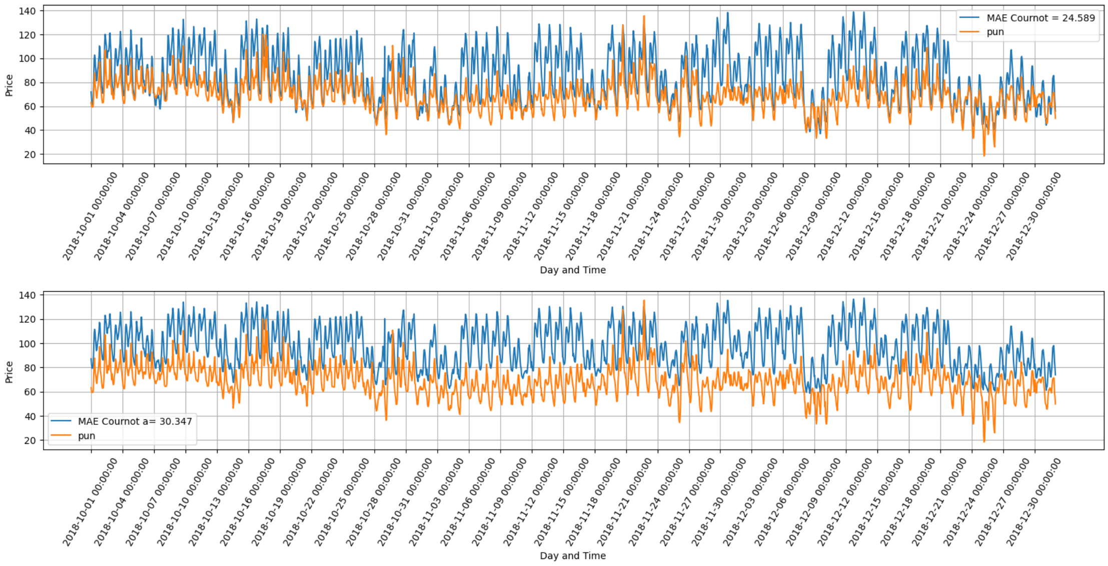


Figure 4.5. Results for the Cournot approximations. *On the top* Estimate with linear marginal costs. *On the bottom*, estimate with affine marginal costs. It is possible to notice how the latter produces higher overestimates of the equilibrium price due to the presence of nonzero marginal cost intercepts.

ing from competition in quantities overestimate the actual unitary equilibrium price, both in the case with affine marginal and linear marginal cost with associated MAE over the three considered months of 24.589 and 30.347 respectively. Indeed, this outcome is in accordance with known results from theory (see for example [6]), where it is prescribed that equilibrium prices deriving from competition in quantities overestimate the actual market prices. The reason of this extremal behaviour is to be identified in a combination of two factors: demand approximation and sensitivity to marginal costs. Indeed, the approximation of demand that has been obtained in this work, as previously stated, relies on an average of the estimated regression slopes of Figure 4.7. This choice, although justified by the limited positive skewness of the coefficient's distribution, still has a negative impact in the estimation process, prescribing, even when applied to real world offers, sensibly higher equilibrium prices. Furthermore, in the case of nonzero marginal intercepts a_i, the increment in marginal costs causes a notable shift in the quantities produced by each competitor, thus driving the prices upwards. This is a direct consequence of the interaction between the elasticity of demand and the increased production costs: with marginally higher costs, firms tend to produce less, therefore giving rise to a cumulative production intercepting the demand curve at higher prices. This dampening effect on production can also be understood under the light of the interaction between quadratic costs and quadratic utilities typical of this model. The situation is the converse for what concerns the supply function equilibrium with linear marginal costs. In this scenario, the resulting prices notably underestimate the final unitary market price, with a MAE of 20.121. This

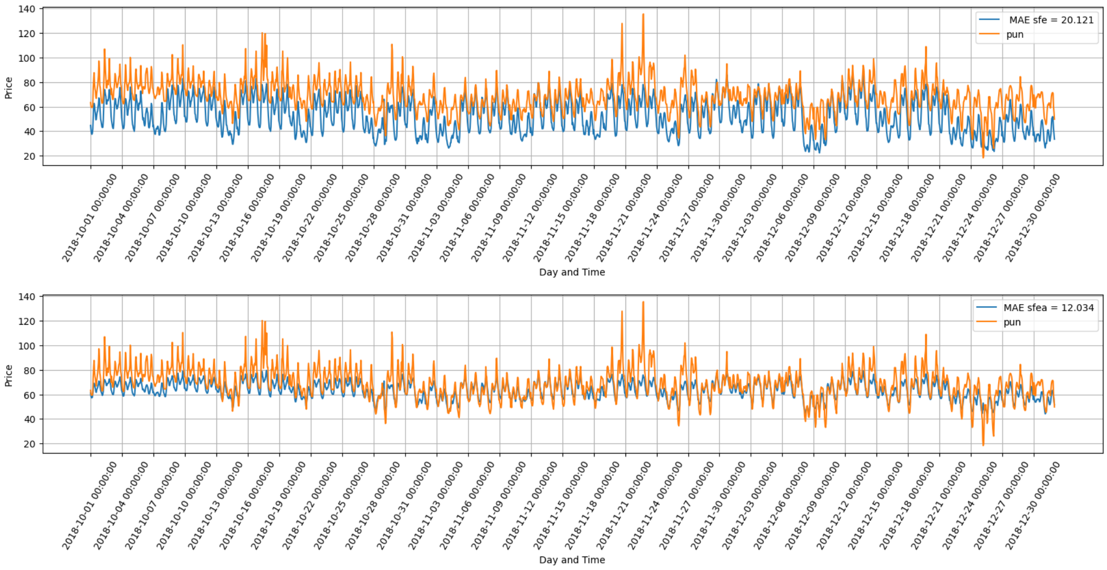


Figure 4.6. Simulation's results for the supply functions are considered. *On the top*: supply function equilibrium with affine marginal costs. *On the bottom*: overall comparison of the selected simulations with the unitary market price.

is indeed a direct consequence of the absence of non zero marginal intercepts, that implies lower production costs, enhancing the utilities of the participating firms. Figure 4.6 conveys the resulting comparison between the affine and linear supply functions estimate. In the case of affine marginal costs, the model predicts higher prices on average, as a direct consequence of the presence of an affine (non zero) intercept. The resulting MAE of 12.034, outperforms all other compared method, prescribing prices that are closer on average to the realized PUN. Interestingly, although both variants of the supply function equilibrium model provide, on average, better estimates than their Cournot counterparts, they struggle to capture significant hourly fluctuations in price. These difficulties stem from the interplay of a number of assumptions behind the competition model. Indeed, both Cournot's model of competition in quantities and supply function equilibrium model are static, in the sense that the only time varying quantity is the load duration characteristic. This is a notable simplification that disregards, for example, the impact of fluctuations in both the prices of coal and natural gas. Indeed, data from Terna [12] shows that a 52.3% of the gross Italian electricity production in 2018 has been achieved trough traditional thermoelectric plants, with a quote of respectively 38.7% from natural gas CCGT plants and 8.6% from coal based production units. This dependence on fossil based production technologies implies that shifts in the futures prices of both coal and natural gas derivatives have an extensive impact on the bidding strategies of the involved firms. Furthermore, the fact that futures market data is available in real time implies that also firms using renewable technologies, that in turn represent 34.5% of the market, are encouraged to raise their prices in response to the expected shift in prices from the increased production costs of thermoelectric plants. This scenario cannot be accounted

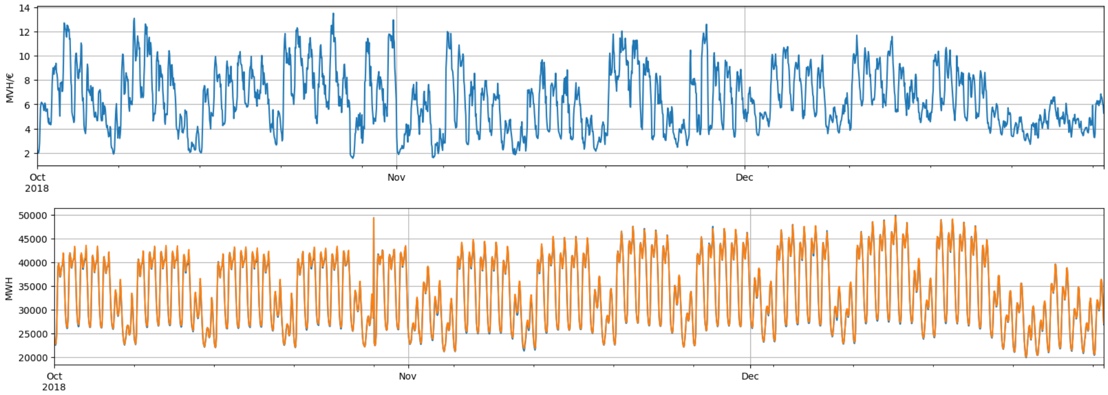


Figure 4.7. *On the top*: gamma estimated for every hour of the considered interval. *On the bottom*: overlap between the intercept of the linear regression of demand and the maximum demand for each time hour of the considered timeframe.

for by the presented models, that assume *constant marginal costs*, and therefore disregard the influence of fluctuations in the price of the necessary raw materials. Another significant simplification assumed by the supply function equilibirum model is the continuity of the prescribed supply functions. In the model this is a necessary requirement in order to obtain a well defined equilibrium price (see Chapter 3), but real world day-ahead markets allow discontinuous step-like schedules, providing an adequate price formation algorithm that can be applied when there are difficulties in finding a unique price [4]. In avoiding the complexities stemming from this resource allocation problem, it is plausible that the presented application underestimates, at least for prices close to the marginal intercept, the actual quantity that a firm is willing to bid to the market. As a final point, it is of importance to recall that this analysis does not account for the production constraints relative to each plant, nor for the transmission constraints of the network. Both these factors will clearly have an impact on the bidding strategy of firms taking part to the day ahead market. The strategy adopted by this work in the selection of strategic participants to the market, as explained in the previous sections, has been to consider only producers bidding an amount greater or equal to 100MW. We expect that this choice actually allows for a reasonable approximation, allowing relatively significant producers to bid and therefore disregarding possible inconsistencies in the production capabilities of small bidders. Nevertheless, even considering the aforementioned limitations, it is possible to argue in favour of the results provided by the supply function equilibirum model due to their notable accordance with realized market prices, that could not be replicated by modelling the day ahead market as a Cournot game.

Chapter 5

Conclusions

In this thesis work, we have presented a comprehensive examination of the Italian electricity market, tracing its origins within the broader context of the European restructuring of electricity markets begun in the late 1990s. Its internal structure, which includes the segmentation into both spot and futures markets, has been discussed and the relevant regulations addressed to emphasize the fundamental role played by the spot day-ahead market, known as the MGP market, in shaping the national unified electricity price. Furthermore, to model the dynamics of the Italian day-ahead market, we have enunciated a number of theoretical concepts from game theory including the Cournot and supply function equilibrium models, enabling us to establish crucial results concerning the existence and characteristics of Nash equilibria that represent observable states of the auction in these strategic games. Moreover, we have employed data provided by Gestore Mercati Energetici to fit the examined models and conduct simulations to observe thee resulting price dynamics. The outcomes of our analysis prove the effectiveness of supply function equilibrium models in modeling the behavior of day-ahead auctions, yielding convincing results and reinforcing the existing literature. In the future, possible extensions of the presented work may consider extending the supply function equilibrium model to account for the possibility of dynamic behaviour of marginal production costs. To this extent, assuming a correlation with the futures price of natural gas could lead to realized prices closer to the ones observed in practice, improving therefore the already convincing performance of the model by capturing the dynamical component of bids in the day ahead electricity markets.

Appendix A

Code Implementation

A.1 Preprocessing

In these snippets we select the necessary modules and libraries, while also defining the path to the employed resources.

In [1]:
```
import xlsxwriter
import statsmodels as sm
from scipy import stats
import warnings
from itertools import product
import matplotlib.pyplot as plt
from tqdm import tqdm
%matplotlib inline
import pandas as pd
import numpy as np
import statsmodels.api as sm
import warnings
from tqdm import tqdm
warnings.filterwarnings("ignore")

#import datetime
from datetime import datetime, timedelta, date
```

In [2]:
```
input_folder = r'C:\Users\Sergi\Documents\dati_tesi\materiale'
#select year and month
y = 2018
m = '10'
```

We select only strategic producers and therefore avoid considering instances having awarded quantity greater than zero at a null price. This is achieved by means of the following function.

In [3]:
```
##function that substitues with $3000$ the BID offers in MGP
#with offered price 0 and awarded quantity $> 0$

def no_indication_of_price(row):
    if  row['TR01_PURPOSE_CD']=='BID' \...
            and row['TR01_STATUS_CD']=='ACC' and \...
            row['TR01_ENERGY_PRICE_NO']==0.0 and\...
            row['TR01_AWARDED_QUANTITY_NO']>0.00:

        return 3000.00
    else:
        return row['TR01_ENERGY_PRICE_NO']
```

In the following snippet we import the csv data on a single dataframe for ease of access and use.

In [4]:
```
df_offers = pd.read_csv(input_folder+\...
    '\TRADEDWH_TR01_GME_OFFERTE_STORICO_'+str(y)+\...
    '_'+str(m)+'.csv', sep ='|',low_memory=False)

df_offers = df_offers.append(pd.read_csv(input_folder+\...
    '\TRADEDWH_TR01_GME_OFFERTE_STORICO_'\...
    +str(y)+'_'+str(11)+'.csv', sep ='|', low_memory=False))

df_offers = df_offers.append(pd.read_csv(input_folder+\...
    '\TRADEDWH_TR01_GME_OFFERTE_STORICO_'\...
    +str(y)+'_'+str(12)+'.csv', sep ='|', low_memory=False))
```

The data contained in the dataframe 'df_offers' includes instances of all the spot markets. In our analysis we are only interested in the day ahead and therefore we select only instances belonging to it.

In [5]:
```
## selects only the offers from the day ahead market and
df_mgp= df_offers[(df_offers['TR01_MARKET_CD']=='MGP')\...
    &(df_offers[ "TR01_STATUS_CD"]!='REP')]

# creates the date time index
df_mgp['DateTime'] = df_mgp.apply(lambda row:\...
datetime.strptime(str\...
    (int(row['TR01_BID_OFFER_DATE_DT'])),'%Y%m%d')
    + timedelta(hours=int(row['TR01_INTERVAL_NO']-1)), axis=1)

df_mgp['TR01_ENERGY_PRICE_NO'] = df_mgp.apply(\...
    no_indication_of_price, axis = 1)
```

In [6]:
```
##dates to compute equilibrium price in time
datetimes = list(set(df_mgp['DateTime']))
datetimes.sort()
```

Here the unitary price PUN is computed following the regulations of [9].

In [7]:
```
##total mean
pun = df_mgp[(df_mgp['TR01_ZONE_CD']=='NORD')&\...
    (df_mgp['TR01_PURPOSE_CD']=='BID')&\...
    (df_mgp['TR01_STATUS_CD']=='ACC')].groupby(['DateTime'])\...
    .agg({'TR01_AWARDED_PRICE_NO':'mean'}).reset_index()\...
    .set_index('DateTime')

### mean over zones for each datetime
pun2 = df_mgp[(df_mgp['TR01_PURPOSE_CD']=='BID')&\...
    (df_mgp['TR01_STATUS_CD']=='ACC')]\...
    .groupby(['DateTime','TR01_ZONE_CD'])\...
    .agg({'TR01_AWARDED_PRICE_NO':'mean'})\...
    .reset_index().set_index('DateTime')

pun2 = pun2.groupby(['DateTime'])\...
['TR01_AWARDED_PRICE_NO'].mean()\...
    .reset_index().set_index('DateTime')

### medium off price
p_off = df_mgp[(df_mgp['TR01_PURPOSE_CD']=='OFF')&\...
    (df_mgp['TR01_STATUS_CD']=='ACC')]\...
    .groupby(['DateTime','TR01_ZONE_CD'])\...
    .agg({'TR01_AWARDED_PRICE_NO':'mean'})\...
    .reset_index().set_index('DateTime')

p_off =p_off.groupby(['DateTime'])['TR01_AWARDED_PRICE_NO']\...
    .mean().reset_index().set_index('DateTime')
```

A.2 Demand estimation

In [8]:
```
##select the datetime
dt = datetimes[24*5]
temp = [d for d in datetimes if d.day == 16]
dt = temp[10]
print(dt)
```

Not all features included in the data are actually necessary to our analysis: the objective of the following code is to isolate only the relevant columns to begin the process of estimating demand.

In [9]:
```
cols = ['TR01_STATUS_CD','TR01_UNIT_REFERENCE_NO',
        'TR01_QUANTITY_NO',
        'TR01_AWARDED_QUANTITY_NO', 'TR01_ENERGY_PRICE_NO',
        'TR01_MERIT_ORDER_NO', 'TR01_PARTIAL_QTY_ACCEPTED_IN',
        'TR01_ZONE_CD', 'TR01_AWARDED_PRICE_NO',
        'TR01_OPERATORE', 'DateTime']

status = ['ACC', 'REJ']
```

```
In [10]:  demand = df_mgp[(df_mgp['TR01_PURPOSE_CD']=='BID')]

          demand_dt = demand[(demand['TR01_STATUS_CD'].isin(status))&\...
              demand['TR01_TYPE_CD']=='REG')\...
              &(demand['DateTime']==dt)][cols]

          demand_dt=demand_dt.sort_values('TR01_MERIT_ORDER_NO')
```

```
In [11]:  x_values_bid = list(np.cumsum(\...
              list(demand_dt['TR01_QUANTITY_NO'])))

          y_values_bid = list(demand_dt['TR01_ENERGY_PRICE_NO'])
```

The following snippet serves as a prototype for the linear regression of demand. In particular, we employ the class provided by SciKit Learn LinearRegression to fit an ordinary least square regression to the polished data.

```
In [12]:  from sklearn.linear_model import LinearRegression
          from sklearn.metrics import r2_score
          clf = LinearRegression(n_jobs=-1)
          clf.fit(X_no_out.reshape(-1,1), y_no_out)

          [clf.intercept_, float(clf.coef_)]
          gamma = -float(clf.coef_)

          D = lambda p: np.maximum(clf.intercept_+clf.coef_*p,\...
              [0]*len(p));
```

A.2.1 Global values estimation

In this section we report the main loop estimating the regression coefficients over the three month interval considered. It works by iterating on three levels from month, to day and finally to hour. It is a generalization of the code presented in the previous sections, extrapolating the time frame demand from the dataframe 'demand' and finding rices and quantities. To avoid outliers, only the values of price and demand lower than 3000 (the amount switched using previous functions) have been considered. The bìobtained values are then stored in the loads_and_date dataframe.

In [13]:
```
from tqdm import tqdm

loads_and_date = pd.DataFrame(\...
    columns=['load_duration', 'gamma'])

for k in tqdm(range(1, 4)):
    for i in range(1, 32):
        temp = [d for d in datetimes if d.day == i and\...
        d.month == k+9]
        for j in range(len(temp)):
            dt = temp[j]
            demand_dt = demand[(demand['TR01_STATUS_CD']\...
            .isin(status))&(demand['TR01_TYPE_CD']=='REG')\...
            &(demand['DateTime']==dt)][cols]
            demand_dt=demand_dt\...
            .sort_values('TR01_MERIT_ORDER_NO')
            x_values_bid = list(np.cumsum(\...
            list(demand_dt['TR01_QUANTITY_NO'])))
            y_values_bid = list(\...
            demand_dt['TR01_ENERGY_PRICE_NO'])
            X= np.array(y_values_bid)
            y= np.array(x_values_bid)
            indices = np.where(X<3000)
            X_no_out = np.array(X[indices])
            y_no_out = y[indices]
            clf = LinearRegression(n_jobs=-1)
            clf.fit(X_no_out.reshape(-1,1), y_no_out)
            loads_and_date.loc[dt] = [float(clf.intercept_),\...
            -float(clf.coef_)]
```

In [14]:
```
all_gammas = np.reshape(loads_and_date['gamma']\...
    .values, (24, 30+31+31))
all_load_duration = np.reshape(\...
    loads_and_date['load_duration']\...
    .values, (24, 30+31+31))
```

In this section a brief analysis of the distribution of the regression slopes is performed. The aggregation has been both hourly and global on the whole scenario. The resulting p-value after a standard Pearson's normal test rejects the normality hypothesis. Further analysis of the global distribution's skewness has shown limited positive values, thus justifying the choice of the sample mean as characterizing factor.

In [15]:
```
from scipy.stats import normaltest

normality_results = normaltest(loads_and_date['gamma'])

fig, axs = plt.subplots(2, 12, figsize=(18, 3))
axs = axs.flatten()

for i in range(1, 25):
    ax = axs[i-1]
    if all_gammas[i-1, 0] != 0:
        data = all_gammas[i-1,:]
        ax.hist(data, bins=12, alpha=0.7)
        norm = normaltest(all_gammas[i-1,:]).pvalue
        ax.set_title('p_v ' + f'{norm:.2f}')
fig.suptitle('Hourly\...
distribution of the gammas over the month of October 2018')
plt.tight_layout()
plt.show()
print(f'Overall mean= {np.mean(all_gammas)}')
print(f'Hourly averages = {np.mean(all_gammas, axis=1)}')
```

In [16]:
```
gamma_un = -float(np.mean(all_gammas))

#choosing a random day and time to
#establish difference between estimates
dt = np.random.choice(datetimes)
```

In [17]:
```
gamma = float(loads_and_date[loads_and_date.index == dt]\...
    ['gamma'].iloc[0])
N = loads_and_date[loads_and_date.index == dt]\...
['load_duration'].iloc[0]

# defining the two regressions
D_unique_gamma = lambda p: np.maximum(N+gamma_un*p, [0]*len(p));
D = lambda p: np.maximum(N+-gamma*p, [0]*len(p));
```

In [18]:
```
demand_dt = demand[(demand['TR01_STATUS_CD']\...
    .isin(status))&(demand['TR01_TYPE_CD']=='REG')\...
    &(demand['DateTime']==dt)][cols]
demand_dt=demand_dt.sort_values('TR01_MERIT_ORDER_NO')
x_values_bid = list(np.cumsum(\...
    list(demand_dt['TR01_QUANTITY_NO'])))
y_values_bid = list(demand_dt\...
    ['TR01_ENERGY_PRICE_NO'])
X = np.array(y_values_bid)
y = np.array(x_values_bid)
indices = np.where(X<3000)
X_no_out = np.array(X[indices])
y_no_out = y[indices]
```

In [19]:
```
game = df_mgp[(df_mgp['TR01_PURPOSE_CD']=='OFF')\...
    &(df_mgp['TR01_TYPE_CD']=='REG')][cols]

game_dt = game[(game['TR01_STATUS_CD']\...
    .isin(status))&(game['DateTime']==dt)]

game_dt=game_dt.sort_values('TR01_MERIT_ORDER_NO')
```

In [20]:
```
x_values_off = list(np.cumsum(\...
    list(game_dt['TR01_QUANTITY_NO'])))
y_values_off = list(game_dt['TR01_ENERGY_PRICE_NO'])

X_off= np.array(y_values_off)
y_off= np.array(x_values_off)

#X = [x for x in X if x < 3000]
#X_no_out = [x for x in X if x < 500]
max_price = 500;
indices = np.where(X_off<max_price)
X_off = np.array(X_off[indices])
y_off = y_off[indices]
```

In [21]:
```
X= np.array(y_values_bid)
y= np.array(x_values_bid)

#X = [x for x in X if x < 3000]
#X_no_out = [x for x in X if x < 500]
indices = np.where(X<max_price)
X_no_out = np.array(X[indices])
y_no_out = y[indices]

#X_no_out = [x for x in X if x < 3000]
#X_no_out = [x for x in X if x < 500]
X_no_out = np.array(X_no_out)
X_no_out.sort()
y_no_out.sort()
y_no_out = np.flip(y_no_out)
```

In [22]:
```
qty_zero_price = game_dt[\...
    game_dt['TR01_ENERGY_PRICE_NO']==0]\...
    ['TR01_AWARDED_QUANTITY_NO'].sum()
print('Quantity sold with zero offer')
qty_zero_price
```

In [23]:
```
eq_price_min =np.min(X_off[D(X_off)<y_off])
eq_price_max = np.max(X_off[D(X_off)>y_off])
eq_price = np.mean([eq_price_min, eq_price_max])
pun_dt = float(pun[pun.index==dt]['TR01_AWARDED_PRICE_NO'])
pun2_dt = float(pun2[pun2.index==dt]['TR01_AWARDED_PRICE_NO'])

eq_price_min_g =np.min(X_off[D_unique_gamma(X_off)<y_off])
eq_price_max_g = np.max(X_off[D_unique_gamma(X_off)>y_off])

print('Equilibrium price with\...
    estimated demand and variable gamma')
print(eq_price_min)
print(eq_price_max)
print('\nPUN')
print(pun_dt)
print(pun2_dt)
print('\nEquilibrium price with\...
    estimated demand and fixed gamma')
print(eq_price_min_g)
print(eq_price_max_g)
```

In [24]:
```
print('Equilibrium quantity')
print(np.min(D(X_off)[D(X_off)<y_off]))
print(np.max(D(X_off)[D(X_off)>y_off]))
```

In [25]:
```
print('Total number of producers participating in MGP')
print('n =',len(set(game_dt['TR01_UNIT_REFERENCE_NO'])))
```

In [26]:
```
game_dt_r = game_dt[game_dt['TR01_ENERGY_PRICE_NO']!=0]
```

In [27]:
```
print('Producers submitting at least one non-zero offer')
print('n =',len(set(game_dt_r['TR01_UNIT_REFERENCE_NO'])))
```

In [28]:
```
min_price = np.min(pun['TR01_AWARDED_PRICE_NO'])
max_price = np.max(pun['TR01_AWARDED_PRICE_NO'])
print(min_price)
print(max_price)
```

In [29]:
```
selected_producers =  list(\...
    set(game_dt[(game_dt['TR01_ENERGY_PRICE_NO']>min_price)&
    (game_dt['TR01_ENERGY_PRICE_NO']<max_price)]\...
    ['TR01_UNIT_REFERENCE_NO']))
```

In [30]:
```
print('Producers submitting strategically relevant offers')
print('n =',len(selected_producers))
```

In [31]:
```
all_game_prod =  list(\...
                set(game[\...
                (game['TR01_ENERGY_PRICE_NO']>min_price)&
                (game['TR01_ENERGY_PRICE_NO']<max_price)]\...
                ['TR01_UNIT_REFERENCE_NO']))
```

In [32]:
```
print('Producers submitting strategically\...
    relevant offers allover the month')
print('n =',len(all_game_prod))
```

In [33]:
```
game_dt_r = game_dt_r[game_dt_r['TR01_UNIT_REFERENCE_NO']\...
    .isin(selected_producers)]

n= len(selected_producers)
print(n)
```

In [34]:
```
##max_quantities =game2[game2['TR01_STATUS_CD']=='ACC']\...
    .groupby(['DateTime','TR01_UNIT_REFERENCE_NO'])\...
    .agg({'TR01_AWARDED_QUANTITY_NO':'sum',\...
    'TR01_ENERGY_PRICE_NO':'max',\...
    'TR01_AWARDED_PRICE_NO':'max'})\...
    .reset_index()

max_quantities = game2[game2['TR01_STATUS_CD']=='ACC']\...
    .groupby(['DateTime','TR01_UNIT_REFERENCE_NO'])\...
    .agg({'TR01_AWARDED_QUANTITY_NO':'sum'})\...
    .reset_index()

max_quantities = max_quantities.groupby(\...
    ['TR01_UNIT_REFERENCE_NO'])\...
    .agg({'TR01_AWARDED_QUANTITY_NO':'max'})\...
    .reset_index()

max_quantities = max_quantities.rename(columns=\...
{'TR01_AWARDED_QUANTITY_NO':'max_mgp_qty'})

game_by_prod_dt = game2[game2['TR01_STATUS_CD']=='ACC']\...
    .groupby(['TR01_UNIT_REFERENCE_NO','DateTime'])\...
    .agg({'TR01_QUANTITY_NO':'sum',\...
    'TR01_AWARDED_QUANTITY_NO':'sum',\...
    'TR01_ENERGY_PRICE_NO':'max',\...
    'TR01_AWARDED_PRICE_NO':'max'})\...
    .reset_index()

game_by_prod_dt = game_by_prod_dt.merge(max_quantities,\...
on='TR01_UNIT_REFERENCE_NO')\...
    .query('TR01_AWARDED_QUANTITY_NO == max_mgp_qty')

game_by_prod = game_by_prod_dt.groupby(\...
['TR01_UNIT_REFERENCE_NO'])\....
    .agg({'max_mgp_qty':'max','TR01_ENERGY_PRICE_NO':'min',\...
    'TR01_AWARDED_PRICE_NO':'min'}).reset_index()
```

In [35]:
```
game_by_prod2 =game2[game2['TR01_STATUS_CD']=='ACC']\...
    .groupby(['TR01_UNIT_REFERENCE_NO'])\...
    .agg({'TR01_ENERGY_PRICE_NO':'min',\...
    'TR01_AWARDED_PRICE_NO':'min'})\...
    .reset_index()

game_by_prod2['mean_price']=(game_by_prod2\...
    ['TR01_ENERGY_PRICE_NO']+\...
    game_by_prod2['TR01_AWARDED_PRICE_NO'])/2

game_by_prod2=game_by_prod2.rename(columns =\...
    {'TR01_AWARDED_PRICE_NO':'a'})
```

In [36]:
```
game_by_prod = game_by_prod.merge(game_by_prod2[\...
['TR01_UNIT_REFERENCE_NO','a']],\...
    on='TR01_UNIT_REFERENCE_NO' )
game_by_prod['mean_price']=game_by_prod\...
    ['TR01_ENERGY_PRICE_NO']+\...
    game_by_prod['TR01_AWARDED_PRICE_NO'])/2
```

In [37]:
```
factor = 0.99
game_by_prod['a'] =factor*game_by_prod['a']

game_by_prod['c'] = 2*game_by_prod['mean_price']/\...
    game_by_prod['max_mgp_qty']

#game_by_prod['c_a'] =2*(game_by_prod['mean_price']-
#game_by_prod['a'])/game_by_prod['max_mgp_qty']
game_by_prod['c_a'] =2*(game_by_prod\...
    ['TR01_AWARDED_PRICE_NO']-\...
    game_by_prod['a'])/game_by_prod['max_mgp_qty']
```

In [38]:
```
print('n producers with c<=0:')
print(len(game_by_prod[game_by_prod['c']<=0]))
print('n producers with c_a <= 0 :')
print(len(game_by_prod[game_by_prod['c_a']<=0]))
```

A.2.2 Finding β_i and Cournot's optimal quantities

Once an estimate of the marginal cost functions and of demand has been established, it is possible to address the problem of solving the algebraic problems concerning the Nash equilibria of Cournot's and supply function equilibirum models. To this end, we employed Python's SciPy module to find the unique solutions to the resulting linear and nonlinear systems using fsolve. Fsolve is a wrap of the MINPACK's Fortran based solver that minimizes the residue's sum via least squares.

In [39]:
```
from scipy.optimize import fsolve

# defining the root problem for the $\beta_i$
#fsolve solves a problem in the form f(x) = 0.
def ang_coeffs(x, c, gamma):
    b = [] # initializing the final array
    for i in range(len(x)):
        if i < len(x):
            k = (gamma + np.sum(x[:i]) + np.sum(x[i+1:]))
        else:
            k = (gamma + np.sum(x[:-1]))
        b.append(k*((1 + c[i]*k)**-1) - x[i])
    return np.array(b)
```

In [40]:
```
## solves the root problem to find cournot's best quantities
def eq_c(x, c, a, gamma, N):
    b = [] # initializing the final array
    for i in range(len(x)):
        if i < len(x):
            k = (N - np.sum(x[:i]) - np.sum(x[i+1:]) - a[i]*gamma)/\...
            (2+gamma*c[i])
        else:
            k = (N - np.sum(x[:-1])-a[i]*gamma)/(2+gamma*c[i])
        b.append(k - x[i])
    return np.array(b)
```

A.2.3 Single frame estimate

The following code aims at applying the presented methods to a single day and hour. We begin by setting the minimum betting quantity to consider strategically relevant producers. The right load duration from the 'loads_and_date' dataframe is selected and the corresponding values of offer and demand obtained to estimate the price implied by demand's approximation. The producers having a_i in the realized price interval are then selected and the number of strategic firms extracted. Their corresponding marginal costs are then used in conjunction with γ to find equilibrium solutions and the final price is thus obtained.

In [41]:
```
min_qty = 100#
```

In [42]:
```
N = loads_and_date[loads_and_date.index == dt]\...
    ['load_duration'].iloc[0]
game_dt = game[(game['TR01_STATUS_CD'].isin(status))\...
    &(game['DateTime']==dt)]
game_dt=game_dt.sort_values('TR01_MERIT_ORDER_NO')

x_values_off = list(np.cumsum(list(game_dt\...
    ['TR01_QUANTITY_NO'])))
y_values_off = list(game_dt['TR01_ENERGY_PRICE_NO'])
X_off= np.array(y_values_off)
y_off= np.array(x_values_off)

indices = np.where(X_off<max_price)
X_off = np.array(X_off[indices])
y_off = y_off[indices]
eq_prices_min =np.min(X_off[D_unique_gamma(X_off)<y_off])
eq_prices_max = np.max(X_off[D_unique_gamma(X_off)>y_off])
eq_prices = np.mean([eq_prices_min, eq_prices_max])

selected_producers =  list(set(game_dt\...
    [(game_dt['TR01_ENERGY_PRICE_NO']>min_price)\...
    &(game_dt['TR01_ENERGY_PRICE_NO']<max_price)]\...
    ['TR01_UNIT_REFERENCE_NO']))

firms = game_by_prod[\...
    (game_by_prod['a']<eq_prices_min)\...
    &(game_by_prod['c_a']>0)\...
    &(game_by_prod['TR01_UNIT_REFERENCE_NO']\...
    .isin(selected_producers))\...
    &(game_by_prod['max_mgp_qty']>min_qty)]\...
    ['TR01_UNIT_REFERENCE_NO']

qty_non_competitive = game_dt[\...
    ~game_dt['TR01_UNIT_REFERENCE_NO'].isin(firms)]\...
['TR01_AWARDED_QUANTITY_NO'].sum()
```

In [43]:
```
c = game_by_prod[game_by_prod\...
    ['TR01_UNIT_REFERENCE_NO'].isin(firms)]['c']
a = np.array(game_by_prod[game_by_prod\...
    ['TR01_UNIT_REFERENCE_NO'].isin(firms)]['a'])
c_a = np.array(game_by_prod[game_by_prod\...
    ['TR01_UNIT_REFERENCE_NO'].isin(firms)]['c_a'])
```

In [44]:
```
beta_initial = np.zeros((len(c), ))+100
betas= fsolve(ang_coeffs,beta_initial,(np.array(c), gamma))
betas_a= fsolve(ang_coeffs,beta_initial,(np.array(c_a), gamma))
```

In [45]:
```
p_sfe = (N-qty_non_competitive)/(np.sum(betas)+gamma)
p_sfe_a = (N-qty_non_competitive+sum(a*betas_a))/\...
    (np.sum(betas_a)+gamma)

q_initial = np.zeros((len(c), ))+10
q_cournot= fsolve(eq_c,q_initial,(np.array(c), gamma,\...
    N-qty_non_competitive))
p_cournot =(N-qty_non_competitive-sum(q_cournot))/gamma
```

In [46]:
```
print('Equilibrium prices:')
print('price sfe =',p_sfe)
print('price sfe with alphas =',p_sfe_a)
print('pun =', pun_dt)
print('eq_price = ', eq_price)
print('price cournot = ', p_cournot)
```

In [47]:
```
N_t = N-qty_non_competitive
D_t = lambda p: N_t-gamma*p;
S = lambda p: sum(betas)*(p);
S_a = lambda p: np.array([np.sum(np.maximum(betas_a*(x-a),0))\...
    for x in p]);
```

A.3 Complete Estimate

The complete estimate of the implied equilibrium prices is performed repeating the specific analysis presented in the previous subsection: extracting the relevant load duration characteristic, selecting demand, offer and computing the equilibrium price implied by the approximate demand. The firms displaying marginal costs within the interval of realized prices are extrapolated and the quantity offered out of the strategic region subtracted to the load duration. The resulting equilibria are then employed to compute the implied market prices, that are stored in the following lists.

In [48]:
```
from tqdm import tqdm
sfe = []
sfea = []
cournot = []
eq_prices_min = {}
eq_prices_max = {}
eq_prices = {}
min_qty = 100# cutoff to the minimum quantity to be considered.
max_price = 500
```

In [49]:
```
for dt in tqdm(datetimes):

    N = loads_and_date[loads_and_date.index == dt]\...
        ['load_duration'].iloc[0]
    game_dt = game[(game['TR01_STATUS_CD'].isin(status))&\...
        (game['DateTime']==dt)]
    game_dt=game_dt.sort_values('TR01_MERIT_ORDER_NO')

    x_values_off = list(np.cumsum(list\...
        (game_dt['TR01_QUANTITY_NO'])))
    y_values_off = list(game_dt['TR01_ENERGY_PRICE_NO'])
    X_off= np.array(y_values_off)
    y_off= np.array(x_values_off)

    indices = np.where(X_off<max_price)
    X_off = np.array(X_off[indices])
    y_off = y_off[indices]
    eq_prices_min[dt] =np.min(X_off\...
        [D_unique_gamma(X_off)<y_off])
    eq_prices_max[dt] = np.max(X_off\...
        [D_unique_gamma(X_off)>y_off])
    eq_prices[dt] = np.mean(\...
        [eq_prices_min[dt], eq_prices_max[dt]])

    selected_producers =  list(set(\...
        game_dt[(game_dt['TR01_ENERGY_PRICE_NO']>min_price)&(
        game_dt['TR01_ENERGY_PRICE_NO']<max_price)]\...
        ['TR01_UNIT_REFERENCE_NO']))
    firms = game_by_prod[(game_by_prod['a']<\...
        eq_prices_min[dt])&(game_by_prod['c_a']>0)&\...
        (game_by_prod['TR01_UNIT_REFERENCE_NO']\...
        .isin(selected_producers))\...
        &(game_by_prod['max_mgp_qty']>min_qty)]\...
        ['TR01_UNIT_REFERENCE_NO']
    qty_non_competitive = game_dt[~game_dt\...
        ['TR01_UNIT_REFERENCE_NO'].isin(firms)]\...
        ['TR01_AWARDED_QUANTITY_NO'].sum()

    c = np.array(game_by_prod[game_by_prod\...
        ['TR01_UNIT_REFERENCE_NO'].isin(firms)]['c'])
    a = np.array(game_by_prod[game_by_prod\...
        ['TR01_UNIT_REFERENCE_NO'].isin(firms)]['a'])
    c_a = np.array(game_by_prod[game_by_prod\...
        ['TR01_UNIT_REFERENCE_NO'].isin(firms)]['c_a'])
```

In [50]:
```
beta_initial = np.zeros((len(c), ))+100
    betas = fsolve(ang_coeffs, beta_initial,\...
    (np.array(c), gamma_un))
    betas_a = fsolve(ang_coeffs, beta_initial,\...
    (np.array(c_a), gamma_un))
    cour = fsolve(eq_c, np.zeros((len(c_a), ))+100,\...
    (np.array(c_a), np.zeros((len(c_a),)), -gamma_un,\...
    N-qty_non_competitive))

    cour_a= fsolve(eq_c, np.zeros((len(c_a), ))+100,\...
    (np.array(c_a),np.zeros((len(c_a),)), -gamma_un,\...
    N-qty_non_competitive))

    p_sfe = (N-qty_non_competitive)/(np.sum(betas)+gamma_un)
    p_sfe_a = (N-qty_non_competitive+np.sum(a*betas_a))/\...
    (np.sum(betas_a)+gamma_un)
    p_cournot = (N-qty_non_competitive-np.sum(cour))/\...
    (-gamma_un)
    p_cournot_a = (N- qty_non_competitve-np.sum(cour_a))/\...
    (-gamma_un)

    sfe.append(p_sfe)
    sfea.append(p_sfe_a)
    cournot.append(p_cournot)
```

A.4 Difference between PUN and estimate

In the following Figure A.1 the difference between the realized PUN and the price implied by the linear approximation of demand is displayed. Notably, prices resulting from the combination of actual offer and approximate demand overestimate the realized market prices. This is consequence of the choice of the unique gamma prescribed by the model, that in this case is the sample mean.

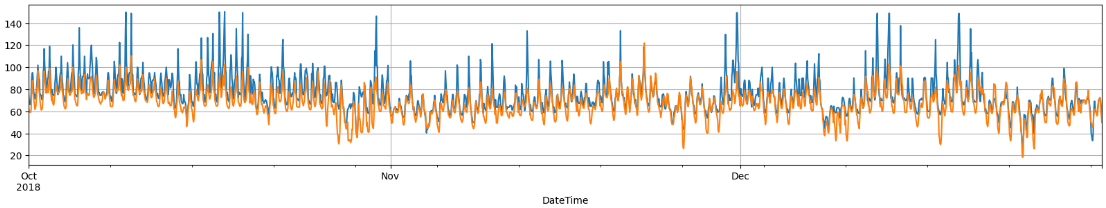


Figure A.1. Difference between PUN in orange and the price implied by the proposed approximation of demand in blue

A.5 Distribution of the regression's slope

Figure A.3 displays the hourly distribution of the estimated linear regression's slope coefficient, while Figure A.2 shows the distribution over the complete interval. The limited skewness justifies the adoption of the sample mean as significant measure.

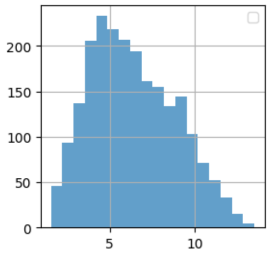


Figure A.2. Estimated γ distribution over the three month interval. The limited skewness justifies the adoption of the sample mean as significant parameter.

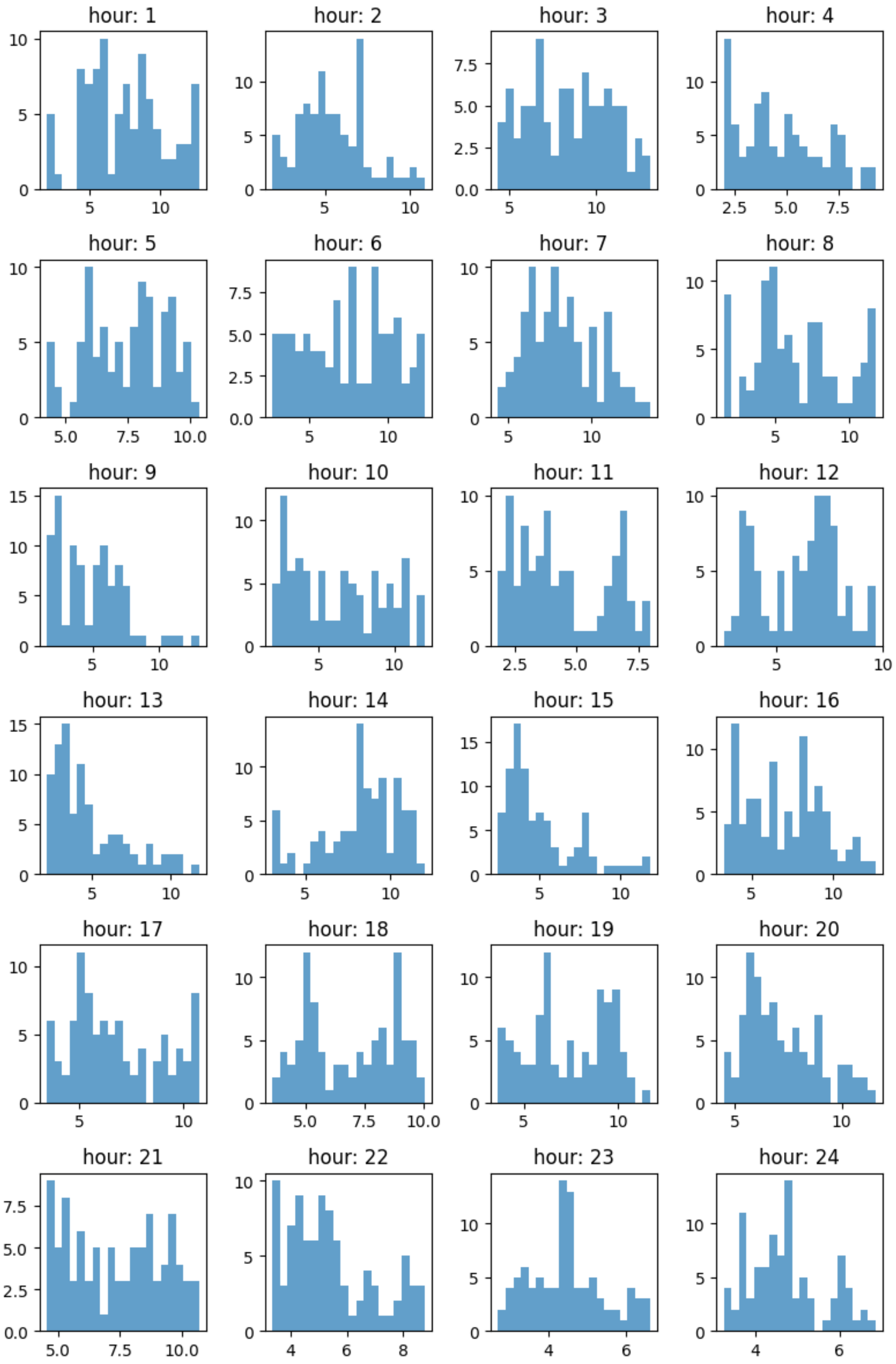


Figure A.3. Hourly distribution of the estimated regression slope.

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