Université Paris-Saclay

Physics of Complex Systems

Predator-Prey model of a technological renewable-based energy transition

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Abstract

The energy transition is a pathway toward transformation of the global energy sector from fossil-based to zero-carbon technologies, in order to cut energy-related CO_2 emissions to limit climate change. Since provision of energy is one of the main ingredient in the economy, transition scenarios must be assessed by a thorough modelling activity. Integrated Assessment Models [IAM], especially analyzed by the International Panel on Climate Change, are one type of models applied to forecast future evolutions and help decision-making.

Contrasting with IAM where the various sectors of the economy are described in detail, we propose, using Lotka-Volterra ecological modelling, to study the introduction of a new energy technology able to mitigate the limits and pollutions of energy provision from natural capital. This well-known dynamical system of differential equations is interpreted in terms of energy investment, using also the notion of Energy Return on (Energy) Invested [EROI], an important question that stays for the moment outside the scope of mainstream modelling. Human beings, and their social buildings, are treated as one predator species, preying energy from the stock of natural capital, in the form of wood from a forest, the lowest level prey. A new species, a renewable energy industry, is introduced acting as a prosumer: a predator, when energy must be invested in industry and devices [e.g. solar panels], and a prey, when energy streams are harvested by the devices and distributed to humans.

In the absence of the new technology, Humans and Wood interact dynamically. Depending on the parameters, several types of trajectories appear, which are analyzed using the tools of dynamical systems. A dynamics of interest is the possibility of a collapse of the human population due to an insufficient energy harvest of biomass. A stable fixed point is reached where Humans disappear, as was (almost) the case for the pre-colonial population of Easter Island. We analyze this fixed point and others, and see how the situation is changed with

the introduction of the industrial species: Could an early development of renewable energy industrial devices, exemplified by solar panels, would have saved the island from population collapse?

To answer this question, once the renewable industry is added in the model, we focus is the effect of the industry growth rate and the renewable energy harvest by humans in the dynamics of the system. Three main scenarios are observed: 1) Wood-Industry coexistence 2) Industry-Humans coexistence 3) Wood-Industry-Humans coexistence. In the former case, industry saves humans without destroying wood however, if EROI is bigger than a critical value and under a threshold value of industry harvest, renewable industry grows until saturating the whole island resources, eliminating wood. For any conditions of renewable industry, there exists a threshold of human initial population under which humans collapse anyhow.

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A

Introduction

To achieve the $2°C$ goal of the Paris Agreement, much of the focus will be on the energy sector, which currently accounts for just under 75% of greenhouse gas (GHG) emissions, generated from the burning of hydrocarbons in the power, industry, transport, and heat sectors. As a result, the decarbonization of the energy sector is the most urgent priority, fossil fuels need to be replaced by low-carbon sources of energy. The most immediate path to reduce emissions (apart from conservation) is by substituting CO_2-free energy for fossil fuels [Creutzig et al., 2017; Louwen et al., 2016; Luderer et al., 2022; Perissi, 2021].

Modeling is a powerful tool to answer and solve some of the open questions in the field, considering global warming, environmental security as well as securing an energy supply as ones of the most significant topics for decision makers. Just as physical models can predict the impact of increased CO_2 on climate, energy-economic models can show the economic and technical impacts of alternative economic strategies for minimizing emissions. In this sense, models that project scenarios are helpful to plan for different possible futures [Nakata, 2004].

Integrated Assessment Model (IAM) [Baumstark et al., 2021] are such a type of global economic models integrating an energy sector among other industrial and household sectors, which have been used to study diverse scenarios of mitigation of greenhouse gas emissions. As physicists, we prefer to develop "as simple as possible" models, to highlight the main characteristic of future trajectory, sacrificing details for handiness and boldness of results [see e.g. Perissi, 2021]. The purpose of these "toy models" then is not to pinpoint future events but to consider the forces which could influence the future along different pathways, using as qualitative observations past events to validate the credibility and highlight the challenges of the proposed models [Bercegol and Benisty, 2022].

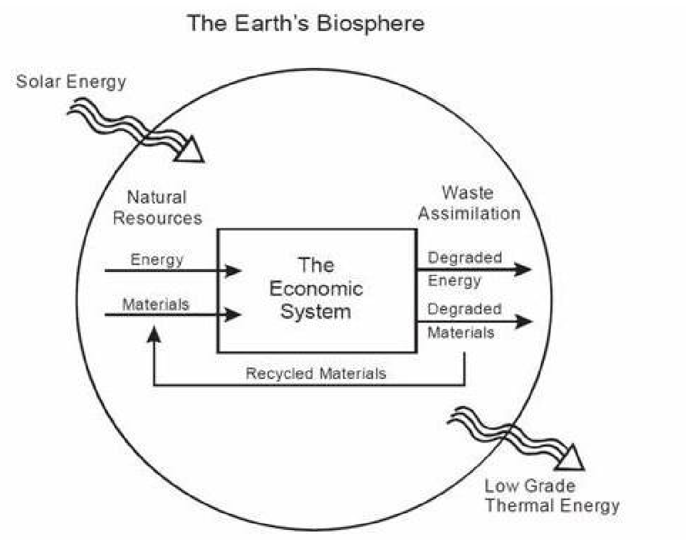


Figure 1: Diagram describing the flow of natural resources through the economy: Valuable resources are procured from nature by the input end of the economy. Recycling of material resources is possible, but only by using up some energy resources [“Diagram”, 2015].

Focusing on the technological challenge and keeping in mind a global, simple scheme represented in Figure 1, the present work presents an ideal and highly simplified model of energy transition using the Lotka-Volterra theory of ecology chains to describe the interactions between energy producers and consumers. Lotka-Volterra dynamical system was previously used in many economic studies (e.g. [Brander and Taylor, 1998]), especially those related to the energy-economy nexus.

Lotka-Volterra model seems also appropriate because it is compatible with the consideration of energy investment. Recent studies of energy technology and industrial processes have outlined the importance of the amount of energy that has to be consumed in building an industrial complex and providing individual energy devices [Hall, 2017]. Lotka-Volterra equations are usually interpreted as describing the evolution of the number of individuals from one species. However they can be interpreted as the amount of energy used to grow the material stock of that species. Thus, when a new energy technology is introduced (as new high tech renewables, e.g. photovoltaic panels [PV panels]), its growth will represent a burden on the previous energy source: the equations will then describe the energy investment needed from other sources to launch the industry. When it has grown, the new energy source will be able to fuel its maintenance and eventual further growth, and concomitantly fuel human needs. Let us now dive into our subject.

CHAPTER

1 Lotka-Volterra model for energy transition

1.1 Derivation of the model

Into the study of population dynamics, the relationship between predators and their prey has been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. Probably the most famous predator–prey models are Lotka–Volterra systems, which are polynomial differential equations of degree two, initially proposed by Alfred J. Lotka in 1925 and Vito Volterra in 1926 in the context of competing species [Jana and Roy, 2021; Kar and Batabyal, 2010; Mittelbach, 1984].

To explain this behaviour, predator-prey models stated a simple system of ordinary differential equations. Considering x_i the densities of the species, the generalized that can be written as follows:

$$\frac{dx_i}{dt} = x_i\left(\alpha_{i0} + \sum_{j=1}^{n} x_j \alpha_{ij}\right), \quad i = 1, \ldots, n \tag{1.1}$$

The applications of these systems have been done in a broad range of fields in the modelling of many natural phenomena, such as chemical reactions, plasma physics, hydrodynamics, optics as well as other problems from social science and economics.

In this work we proposed a new application of this model, inspired by previous publications that have shown the success of bio-inspired design to improve human networks. For instance in [Panyam et al., 2019] such a model has be proposed to develop a robust bio-inspired power grid network as shown in Fig. 1.1.

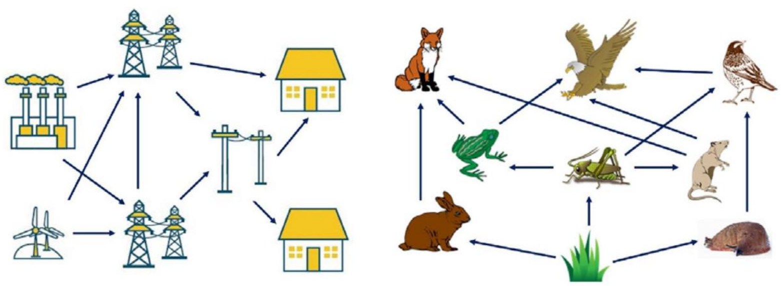


Figure 1.1: Illustrative example of the structurally similar generic topologies of a power grid (left) and a food web (right) [Panyam et al., 2019].

Our first main hypothesis is that the model just transport energy so that no material flows are expressed in the system. By this, we presume that either material recycling is realized, at the expense of more energy consumption, or that materials streams are of second order in the dynamics.

In our dynamical system, we will consider three different species: First, human beings, and their social buildings, are treated as a predator species, preying energy from the stock of natural capital, in the form of biomass simplified as wood from a forest, represented as the lowest level prey, the second species of the model. A third species is introduced, first acting as a predator when energy must be invested in industry and devices [e.g. solar panels], later working also as a prey when energy streams are harvested by the devices and distributed to humans; this species corresponds then to a prosumer, that act like generator at the same time that an consumer of energy. This last assumption appeal to the fact that the renewable sources of energy, necessary to low carbon energy transition, need for their development additional energy investments.

In addition of the past assumptions and to place the problem in a concrete scenario, we make use of the Easter Island case study [Hunt and Lipo, 2009]. We will consider the human population as the consumer component, wood -in representation of biomass- as the open-access renewable resource where energy is taking at the beginning and the industry of a new modern renewable source of energy as an hypothetical technology that is added to the dynamics of the island in order to save the population of a ecological collapse.

Having present these main hypothesis and plugging them in the Easter Island case study, our human-energy network will be modeled using a three-species predator-prey model. Before we introduce the mathematical model and its analysis using non-linear dynamics, we would like to present a brief sketch of the construction of the model that presents the physical interactions between the species of our network:

(i) Presence of three energy reservoirs are taken into account in this study. Here we classify the consumers into two categories, human population and renewable energy industry, and the same happen with the producers, wood and industry. We propose that 'industry' symbolizes both categories, predator and prey whose function is produce as well as consume energy. In the introduced model E_w is the energy storage by a generator that uses wood as resource, E_h the one storage by human society -includying human population- and E_i by the industry.

(ii) In the absence of any consumer, the producers follow the logistic law of growth. Separate carrying capacities K and ε are introduced for wood and industry, respectively. Note that the carrying capacities are limited by the space of the island so that the expansion of one resource prevents the expansion of the other one. Then, the next expression must accomplished:

$$\frac{E_w}{K} + \frac{E_i}{\varepsilon} \leq 1$$

(iii) The effect of the consumption is to reduce the generator growth rate by a term proportional to the energy storage of both components.

(iv) Both consumers (E_h and E_i) are competing internally for the energy of the generator. This competition divides the energy flow induced by humans into two flows, one given by the portion proportional to λ consumed by the industry and other given by a remaining part consumed by humans.

(v) In the absence of any energy generator the consumer's decay rate results in exponential decay.

(vi) The generators contribution to the consumers growth rate is proportional to the available energy storage in both elements of the network.

On the basis of the above assumptions, we will assume that the dynamics of the network described by the magnitudes E_j follow the model given by the following differential equations:

$$
\begin{cases}
\dot{E_w} = rE_w(1 - \frac{E_w}{K} - \frac{E_i}{\varepsilon}) - \alpha_{wh}E_wE_h \\
\dot{E_i} = \gamma E_i(1 - \frac{E_w}{K} - \frac{E_i}{\varepsilon}) + \lambda\alpha_{wh}E_wE_h - \alpha_{ih}E_iE_h \\
\dot{E_h} = -\delta E_h + (1-\lambda)\alpha_{wh}E_wE_h + \alpha_{ih}E_iE_h
\end{cases}
\tag{1.2}
$$

To start the study of the model and to approach first to the most simple behaviour, we will assume that in the interaction between the consumers and generators, the consumption time is negligible so, Type I- functional response is appropriate here.

The proposed model is based on the well-known three-species Lotka-Volterra systems that describes a food chain composed by one prey and two predators as well as one formed by two preys and one predator. However, is important to highlight that our model includes a new specie (renewable industry) that is not described by ecological models due to the fact that behaves as both prey and predator (see Figure 1.2).

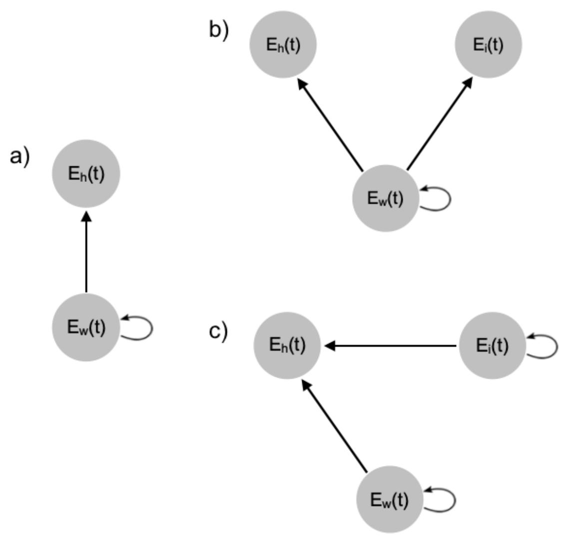


Figure 1.2: a) Schematic diagram of the direct and indirect interactions among four-species predator-prey system. The arrows represent the directions of biomass. Two simple interactions between three species with one renewable resource b) two predator one prey model and c) food chain model

Then, the model simplifies the notion of energy investment at a given time by considering as variables:

- E_w the total energy accumulated in trees and invested in wood. In terms of ecology, it is equivalent to a lowest-level prey.

- E_h the total energy used in building the human society; it is an aggregate which includes food for human beings and energy to build and maintain their dwellings, thus the label "h" could be considered to stand for humans and houses. This term is equivalent to a top-level predator.

- E_i is the total energy used to build and stored in the industrial complex that provides renewable energy harvesters, e.g. solar photovoltaic (PV) panels. This represent both a prey and a mid-level predator.

The parameters $r, \delta, \gamma, K, \varepsilon, \alpha_{ij}, \lambda > 0$ are interpreted as follows:

- r represents the growth rate of the energy production E_w in the absence of consumers for E_i and E_h respectively.

- γ, represents the growth rate of the energy production by the industry, E_i.

- δ is the aggregate depreciation rate of the energy storage by the human population in the absence of producers E_w and E_i.

- K, ε, are the carrying capacities of the environment for wood and industry, respectively, which is determined by the available sustaining resources, including space.

- α_{ij} represents the rate of consumption of the E_i generator by the E_j consumer

- λ represents the portion of the wood energy that human destined to grow renewable industry.

1.2 Estimates of parameters

Let's now to clarify some details about the model. In the system 1.2 of differential equations, the relation between human and wood is rather straightforward and easy to understand. By burning wood, or using it in houses, human maintain or develop both their population and society. We could have applied an efficiency coefficient between the energy withdrawn from wood and that invested in humans; however, since E_h is not used as an energy resource – no one will burn people or houses to get back the original wood energy;– such a coefficient would change nothing to the dynamics.

On the hand, when energy harvested from wood (a fraction λ of that used by humans) is devoted to building the industry, we consider that all this energy is capitalized in E_i. It seems legitimate to consider that the surface of solar energy harvesters (PV panels) will be proportional to E_i, and that the energy delivered by PV panels is proportional to this surface. Thus, the industry delivers solar energy proportionally to E_i, let's call it $k_g E_i$. Hence the term γE_i in the differential equation for industry is the addition of a growth term $k_g E_i$ and a decay term $k_d E_i$.

We can take k_d as the inverse of the lifetime of industrial investments, which we will identify with the lifetime of PV panels. We will quantify this production in "equivalent wood energy", and convert if needed electrical energy delivered by PV panels into "equivalent wood energy" by applying a 1/0.3 inverse efficiency coefficient. These hypotheses allow to express the Energy Return On [Energy] Invested (EROI) of the renewable energy harvesters as a function of the parameters [Hall, 2017]. During the lifetime $1/k_d$ of the PV panels, they will provide $k_g E_i$. The EROI is thus:

$$EROI = \frac{Energy \quad Provided}{Energy \quad Invested} = \frac{\frac{k_g}{k_d} E_i}{E_i} = \frac{k_g}{k_d} = \frac{k_g - k_d + k_d}{k_d} = 1 + \frac{\gamma}{k_d} = 1 + \gamma\tau \tag{1.3}$$

where τ represents the life time of the PV panels. For its part, the Energy Pay Back Time (EPBT) of the solar panels is:

$$EPBT = \frac{\tau}{EROI} = \frac{\frac{1}{k_d}}{1 + \frac{\gamma}{k_d}} = \frac{1}{k_d + \gamma} = \frac{\tau}{1 + \gamma\tau} \tag{1.4}$$

We can also be interested in the numerical values taken by E_w, E_h and E_i, expressed in energy unit. Since the energy depend on the original flow from solar energy, it is naturally expressed as energy unit per unit of surface area, in Joule per meter square, or better on MJ/m^2 to choose a more practical unit.

Also, it is important to remark that the numerical level of our variable is fixed by the carrying capacities, K and ε, being K is the amount of wood energy that can accumulate on the island. Regarding this first capacity, we can express K/S where S is the area of the island: K/S is the maximum energy invested in wood per unit area. Taking 100 tons per ha, i.e. $10 kg/m^2$, and $10 MJ/kg$ for wood (a value representative of an average wood matter with about 50% relative humidity], we obtain for K/S: $100 MJ/m^2$.

On the case of ε, the reasoning should be reversed, using recent PV data. PV panels can turn about 20% of the received solar energy into electrical current, with a global performance ratio of about $70-80\%$. This combines to turn 15% of an average $1700kWh/m^2$ of annual solar flux into electrical current, converted into primary wood energy by applying an $1/0.3$. This translates into an energy production of $850 \times 3.6MJ/m^2 \approx 3000MJ/(m^2 \times year)$. We fix the Life Time of PV panels (+ industry) at 20 years, and consider an EROI of 20, i.e. an EPBT of 1 year. With these hypotheses, the carrying capacity per unit area of PV panels is ε/S: $3000MJ/m^2$ (with S the surface area of the island) [Baumstark et al., 2021; Brockway et al., 2019; Goldschmidt et al., 2021; "Photovoltaics Report - Fraunhofer ISE", n.d.].

Taking this into account, we can also express the other parameters as follows:

$$\gamma = \frac{EROI-1}{\tau} = \frac{19}{20} \approx 1 \quad year^{-1}; \qquad k_d = \frac{1}{\tau} = \frac{1}{20} \quad year^{-1}; \qquad k_g = 1 \quad year^{-1}$$

1.3 Dimensionless model

To simplify the system, we rewrite the model (1.2) introducing the re-scaled variables:

$$\tau = tr, \qquad e_j = \frac{E_j}{K} \qquad for \quad j = w, i, h$$

where τ is the dimensionless time and e_w, e_i, e_h are the dimensionless energies that are obtained dividing by K, the carrying capacity of wood.

Then (1.2) takes the form:

$$\begin{cases} \dot{e_w} = e_w(1 - e_w - \psi e_i - \phi_w e_h) \\ \dot{e_i} = \eta e_i(1 - e_w - \psi e_i - \frac{\phi_i}{\eta} e_h) + \lambda \phi_i e_h e_w \\ \dot{e_h} = e_h(-\theta + (1-\lambda)\phi_w e_w + \phi_i e_i) \end{cases} \tag{1.5}$$

where the parameters are all positive with the re-scaling:

$$\eta = \frac{\gamma}{r}, \qquad \theta = \frac{\delta}{r}, \qquad \psi = \frac{K}{\varepsilon}, \qquad \phi_j = \frac{\alpha_{jh} K}{r} \qquad for \quad j = w, i \tag{1.6}$$

We can rewrite system (1.5) in the matrix form as follows:

$$\begin{bmatrix} \dot{e_w} \\ \dot{e_i} \\ \dot{e_h} \end{bmatrix} = \begin{bmatrix} 1-e_w & -\psi e_w & -\phi_w e_w \\ \lambda\phi_w e_h - \eta e_i & \eta(1-\psi e_i) & -\phi_i e_i \\ (1-\lambda)\phi_w e_h & \phi_i e_h & -\theta \end{bmatrix} \begin{bmatrix} e_w \\ e_i \\ e_h \end{bmatrix} \tag{1.7}$$

Furthermore, it is worth to point the lack of symmetry in the system and the existence of limit cases where symmetry is impose to the system: the first scenario occurs when $\lambda = 0$ where two producers (e_w and e_i) feed one consumer (e_h) (see Figure 1 a); the second scenario happens when $\alpha_{wh} = 0$, the carrying capacity ε tend to infinite and η is negative, in order to have one producer (e_h) and two consumers with different properties (e_i and e_h). In the last case, the subspaces that include e_i, e_h are equivalent and can be obtained from each other, just changing the indices $i \leftrightarrow h$.

In this work, we look for a model that allows to explain the evolution of diverse scenarios on time, for example: the first case of interest consists in the coexistence of a renewable energy generator with a growing human population, the second one starts with the appearance of a new renewable energy technology that coexists with the two existence elements in the network; finally, the last desired scenario is achieved when just the renewable energy source is enough to fulfill the energy requirements of the human population of interested without saturating the carrying capacity of forest.

CHAPTER

2 Wood-human population model

In this chapter we consider the system with no industry. Indeed one goal is to estimate how the investments in renewable energy can modify the equilibrium between human population and the primary energy of wood. Therefore we have first to search the fixed points of the wood-human system such that $\frac{d\mathbf{E}}{dt} = 0$. Then we use the tool of linear stability analysis in order to know their stability as function of the model parameters.

2.1 Linear analysis of the system:

First, considering $e_i = 0$, the system of equations (1.5) form a two species system of differential equations characterizing the evolution of a predator-prey model, as is shown in Fig. (1.2) panel a). Human population, E_h, is the predator while the resource stock, E_w, is the prey.

This scenario reduce the system (1.5) to the equations:

$$\begin{cases} \dot{e_w} = e_w(1 - e_w - \phi_w e_h) \\ \dot{e_h} = e_h(-\theta + \phi_w e_w) \end{cases} \tag{2.1}$$

We can rewrite system (2.1) in the matrix form as follows:

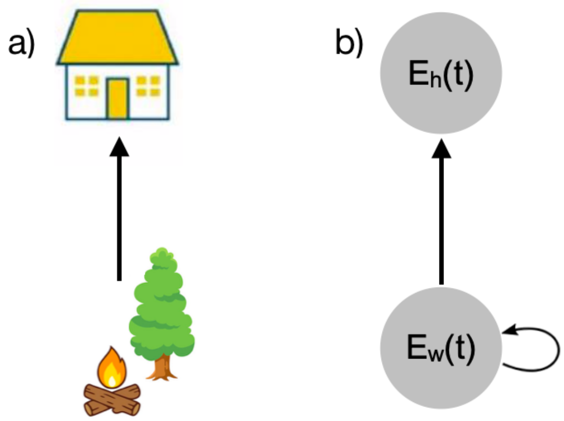


Figure 2.1: Illustrative example of a predator-prey model of two species

$$\begin{bmatrix} \dot{e_w} \\ \dot{e_h} \end{bmatrix} = \begin{bmatrix} 1-e_w & -\phi_w e_w \\ \phi_w e_h & -\theta \end{bmatrix} \begin{bmatrix} e_w \\ e_h \end{bmatrix} \tag{2.2}$$

For the analysis of (2.1) system fixed points we have to consider the solutions that do not change with time, that is, for which $\dot{e}_i = 0$ for $i = w, h$. To study the stability of the fixed points $\mathbf{e}^*$ we will use the linearization of system (2.1) where partial derivatives are evaluated at $\mathbf{e}^*$. This gives us information about the dynamics near the equilibrium point of the original system. To do this, we have to examine the eigenvalues (λ_i) of the Jacobian matrix $\mathbf{J}(\mathbf{e}^*)$: if all (λ_i) have negative real part then $\mathbf{e}^*$ is asymptotically stable, while if some eigenvalues have positive real part then $\mathbf{e}^*$ is not.

In order to analyze the stability, we first compute the Jacobian matrix of partial derivatives:

$$\mathbf{J} = \begin{bmatrix} 1-2e_h-\phi_w e_h & -\phi_w e_w \\ \phi_w e_h & -\theta+\phi_w e_w \end{bmatrix} \tag{2.3}$$

We start analyzing the stability of the trivial fixed point:

$$\mathbf{J}(0,0) = \begin{bmatrix} 1 & 0 \\ 0 & -\theta \end{bmatrix} \tag{2.4}$$

From which the eigenvalues are $\lambda_1 = 1$, $\lambda_2 = -\theta$. We conclude that $\mathbf{e_0^*}$ is a saddle point due to $\lambda_1 > 0$ and $\lambda_2 < 0$. The point will be stable if $\theta < 1$ due to the trace $\tau = 1 - \theta$ will be negative in this case.

Based on physical analysis of the system, the only possible non-trivial fixed points are those in which e_w^* different than zero because human population disappears in absence of wood generator. According to this, the subspaces of (2.1) in which is possible to find fixed points are:

$$\begin{cases} H_1 = \{(e_1, 0)\} \\ H_2 = \{(e_1, e_2)\} \end{cases} \tag{2.5}$$

Analyzing each subspace as was done by the trivial case, we find the fixed points and the stability conditions of (2.1). In order to solve the problem we will make some assumptions, reducing our system to some particular cases:

On H_1, system (2.1) is reduced to the one-dimensional subsystem:

$$\dot{e_w} = e_w(1 - e_w) \tag{2.6}$$

The solution of equation (2.6) is $N(t) = \frac{N_0}{N_0 + (1-N_0)e^{-rt}}$ where $N_0 = N(0)$ is the initial population and r is the intrinsic growth parameter.

Then the non-trivial equilibrium point $\mathbf{e_w^*} = (1, 0)$ is feasible as $e_w^* > 0$. Then, to study the stability, we consider the Jacobian matrix (2.3) evaluated at $\mathbf{e_1^*}$:

$$\mathbf{J}(1,0) = \begin{bmatrix} 1 & \phi_w \\ 0 & -\theta + \phi_w \end{bmatrix} \tag{2.7}$$

With eigenvalues:

$$\begin{cases} \lambda_1 = 1 \\ \lambda_2 = -\theta + \phi_w \end{cases} \tag{2.8}$$

As $\lambda_1 > 0$, the sign of the determinant $\Delta = \theta - \phi_w$ and thus the classification of this fixed point depends on the sign of λ_2. Then $\mathbf{e_1^*}$ is a saddle point if $\theta < \phi_w$ and a spiral if $\theta > \phi_w$; in the case when the fixed point is non-isolated. In this last case, the point is stable if and only if the trace $\tau = -\theta + \phi_w - 1 < 0$ that implies:

$$\phi_w < \theta + 1 \tag{2.9}$$

Notice that, in terms of e_w, Eq. (2.6) has the same form of the Logistic Model $\dot{e_w} = re_w(e_w - \frac{e_w}{K})$ whose fixed points are well known as well as the analytical solution of the model.

On H_2, system (1.5) is reduced to the two-dimensional subsystem 2.1. Analyzing the non-trivial equilibrium point, we find:

$$\mathbf{e_2^*} = \begin{cases} e_w^* = \dfrac{\theta}{\phi_w} \\ e_h^* = \dfrac{\phi_w - \theta}{\phi_w^2} \end{cases} \tag{2.10}$$

First, $\mathbf{e_2^*}$ is feasible when:

$$e_w^* = \frac{\phi_w - \theta}{\phi_w^2} > 0 \Rightarrow \phi_w > \theta \tag{2.11}$$

is accomplished.

On the other hand, evaluating the Jacobian (2.3) at fixed point $\mathbf{e_2^*}$:

$$\mathbf{J}(\frac{\theta}{\phi_w}, \frac{\phi_w - \theta}{\phi_w^2}) = \begin{bmatrix} \dfrac{-\theta}{\phi_w} & -\theta \\ -\theta & 0 \end{bmatrix} \tag{2.12}$$

We find that the corresponding eigenvalues are:

$$\lambda_{1,2} = \frac{\theta(\pm\sqrt{1 - 4\phi_w^2} - 1)}{2} \tag{2.13}$$

It's easy to find the stability conditions given $\tau == \frac{-\theta}{\phi_w}$ which is always negative and thus, the fixed point is stable for any chosen pair of parameters.

On the other hand, to reduce the analysis of the eigenvalues $\lambda_{1,2}$ in order to know the dynamics of $\mathbf{e_2^*}$ we will consider their real and imaginary part. Observing that $\Im(_{1,2}) = \frac{\theta}{2}\sqrt{1 - 4\phi_w^2}$ will be different than zero when:

$$1 \geq 4\phi_w^2 \Longleftrightarrow \frac{1}{2} \geq \phi_w \tag{2.14}$$

We find that the fixed point $\mathbf{e}_2^*$ is a node if this condition is accomplished, otherwise it will be an spiral.

2.2 Harmonic Oscillator Approach

We start writing the model equation in terms of e_w:

$$\begin{aligned}\frac{d^2e_w}{d\tau^2} &= \frac{d}{d\tau}(e_w - e_1^2 - \phi_w e_w e_h)\\ &= \frac{de_w}{d\tau}(1-2e_w - \phi_w e_h) - \theta e_w \frac{de_h}{d\tau}\\ &= \frac{de_w}{d\tau}(1-2e_w - \phi_w e_2) - \phi_w e_w(e_h(-\theta + \phi_w e_w))\end{aligned} \tag{2.15}$$

Now, linearizing around the non-trivial fixed point:

$$\begin{cases} x = e_w - e_w^* \Longrightarrow e_w = x + \frac{\theta}{\phi_w} \\ y = e_h - e_h^* \Longrightarrow e_h = y + \frac{\phi_w - \theta}{\phi_w^2} \end{cases} \tag{2.16}$$

We found the harmonic oscillator expression:

$$\frac{d^2x}{d\tau^2} + \frac{\theta}{\phi_w}\frac{dx}{d\tau} + \frac{\theta}{\phi_w}(\phi_w - \theta)x = 0 \tag{2.17}$$

That follows the form:

$$\frac{d^2x}{d\tau^2} + 2\zeta\omega_0\frac{dx}{d\tau} + \omega_0^2 x = 0 \tag{2.18}$$

From where we have that the natural frequency and the damping coefficient are:

$$\begin{cases} \omega_0 = \sqrt{\frac{\eta}{\beta}} = \sqrt{\frac{\delta}{\alpha_{12}Kr}(\alpha_{12}K - \delta)} \\ 2\zeta\omega_0 = \sqrt{\frac{\eta}{\beta}(\beta - \eta)}\sqrt{\frac{\eta}{\beta}\frac{1}{\beta - \eta}} \Rightarrow \zeta = \frac{1}{2}\sqrt{\frac{r\delta}{\alpha_{12}}\frac{1}{\alpha_{12}K - \delta}} \end{cases} \tag{2.19}$$

The time-series pattern of the model with and without carrying capacity is shown in Figure 2.2. It is possible to identify the damped effect of the carrying capacity in the predator-prey dynamics between the human population and the resource.

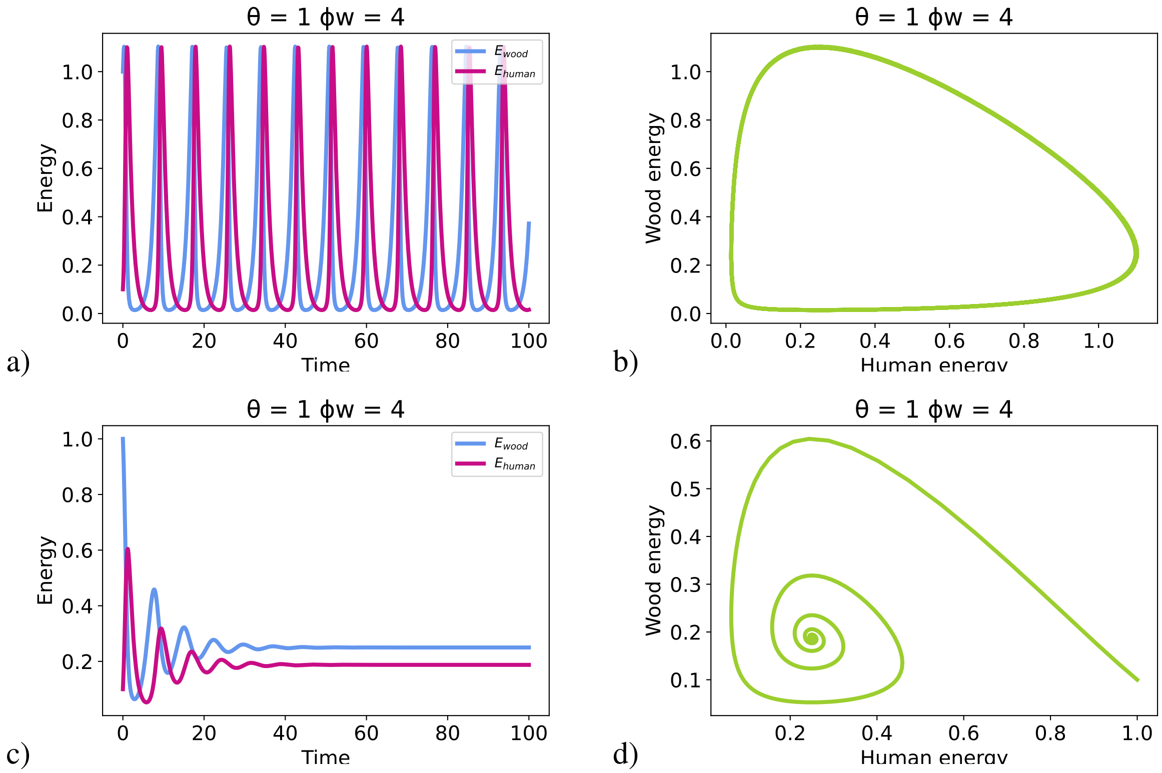


Figure 2.2: Easter Island case base. a) It is possible to observe a cycle between human population and wood for the absence of the resource carrying capacity. b) Phase portrait of a) where a limit cycle is reached. c) After some years of a cycle between humans and wood, the system converges to a steady state of coexistence. d) The phase portrait of c) shows an spiral that confirms the converge of the system.

2.3 Dynamics applied to the population and energy stock interaction

We now characterize the dynamic behaviour of the system before applying to the Easter Island. We assume that the parameter restriction 2.11 is met, implying that an interior steady state exists. In this case, the local behaviour of the system is as follows:

(i) Steady state 1 ($e_w = 0, e_h = 0$) is a saddle point if $\theta > 0$, stable iff $\theta > 1$ and unstable otherwise.

(ii) Steady state 2 ($e_w = 1, e_h = 0$) is a saddle point if $\theta < \phi_w$ and non-isolated if $\theta = \phi_w$. In the last case, is stable iff $\phi_w < \theta + 1$ and unstable otherwise.

(iii) Steady state 3 ($e_w = \frac{\theta}{\phi_w}, e_h = \frac{\phi_w - \theta}{\phi_w^2}$) is a stable node if $\frac{1}{2} \geq \phi_w$ and a stable spiral otherwise.

This propositions are based on the fact that the dynamic of the system if locally linear in the neighborhood of the steady state.

2.3.1 Parameter choice

The first parameter to consider is r, the intrinsic growth or regeneration rate of the resource. We initially assume an intrinsic growth of 0.1 implying that, led to itself, the forest would increase by 10 percent per year in absence of congestion effects. We should consider $\delta = 0.1$ meaning that the population will depreciate by 10 percent per year in absence of the energy stock.

These parameters yields to the time series pattern shown in Figure 2.3 panel c). Based on the simulations we conclude that:

- The steady-state energy resource stock rises if the mortality rate rises or the birth rate falls, *i.e.*, if the human population decreases.
- The steady-state energy population stock rises equiproportionately with an increase in the intrinsic rate if the resource growth, r or if the carrying capacity of the environment rises.

2.3.2 Possible scenarios

This model consider the system of equations:

$$\begin{cases} \dot{e_w} = e_w(1 - e_w - \phi_w e_h) \\ \dot{e_h} = e_h(-\theta + \phi_w e_w) \end{cases} \tag{2.20}$$

with the following fixed points:

$$\begin{cases} \mathbf{e_0^*} = (0,0) \\ \mathbf{e_1^*} = (1,0) \\ \mathbf{e_2^*} = (\frac{\theta}{\phi_w}, \frac{\phi_w - \theta}{\phi_w^2}) \end{cases} \tag{2.21}$$

In Fig. 2.3 four different scenarios with initial conditions ($e_h(0) = e_w(0) = 0.5$) and $\theta = 1$ are presented; the variation of the parameter ϕ_w is then who determines the dynamics. The cases are the following:

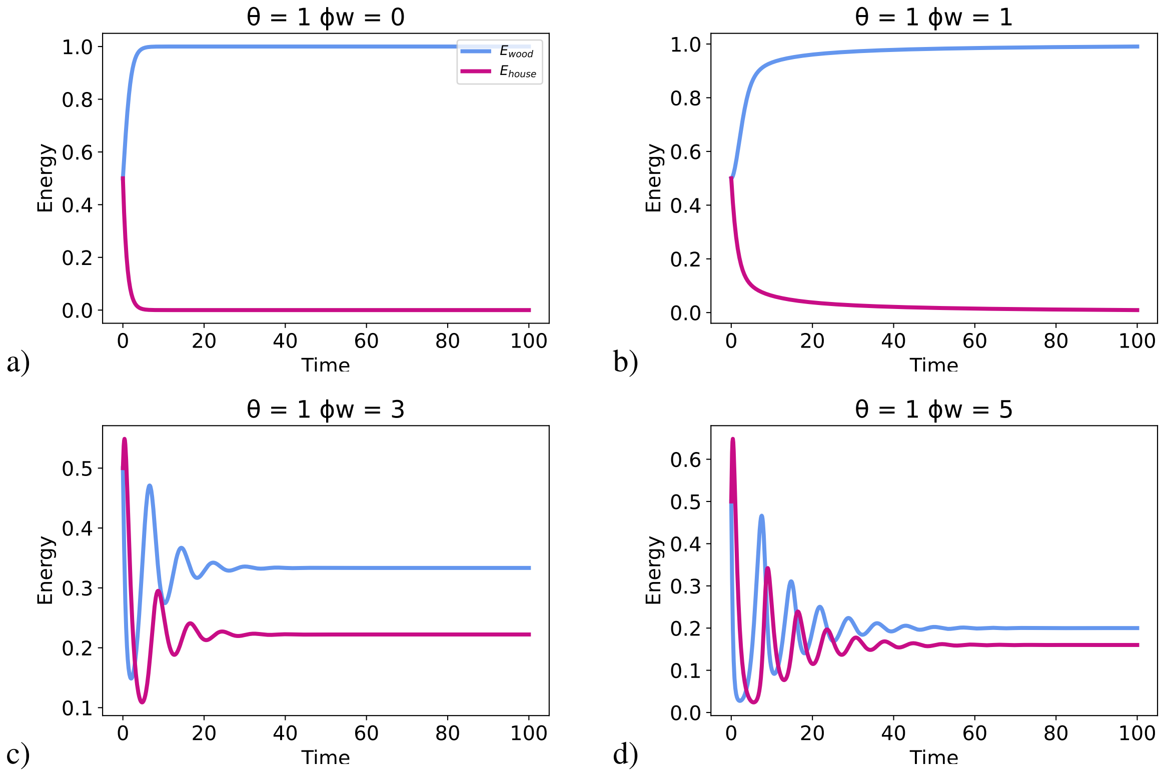


Figure 2.3: Parameter $\theta = 1$ is fixed while parameter ϕ_w varies.

1) When $\phi_w = 0$, wood and humans are independent, wood follows a logistic growth with carrying capacity equal to 1 while humans decays with rate θ.

2) For $\phi_w = 1$, wood and humans interact. The result is a slower decay of human population as well as a slower growth of wood. These two last cases correspond to the fixed point $\mathbf{e_1^*} = (e_w, e_h) = (1, 0)$, which in this case is equal to $\mathbf{e_2^*}$.

3) The parameter ϕ_w fixes wood-human interaction. When $\phi_w > 1$, this avoids wood to reach its carrying capacity but at the same makes humans survive. It is possible to see the formation of oscillations before reaching steady points.

4) Increasing ϕ_w more oscillations are formed. The fixed point is reached later and the steady state values of the 2 species come closer one to the other. This behavior continue as ϕ_w increases. These last two cases correspond to the fixed point $\mathbf{e_2^*} = (e_w, e_h) = (\frac{\theta}{\phi_w}, \frac{\phi_w - \theta}{\phi_w^2})$.

To sum up, for the wood-humans dynamics, the ratio $\frac{\theta}{\phi_w}$ is key to understand behaviour of the system. By their own θ doesn't change significantly the dynamics while the increase of

ϕ_w has as consequence a temporal delay of the fixed point due to the formation of oscillations.

Note that in Fig. 2.3 panel b), where $\theta = \phi_w = 1$ humans collapses while wood reach its carrying capacity. In the next chapter we will explore under what conditions this could be avoided.

CHAPTER 3

Energy-economic models

In this section we will perform a preliminary exploration of the three species model and consider the presence of a technology that allows to harvest solar energy (the environmental resource) in a most efficient way in order to escape from a base case where human population collapses $\theta = \phi_w$. To do that, we will explore as was done for the two species model the stability of the fixed points. The final network is shown in Figure (3.1).

3.1 Linear analysis of the model

As was done in the previous chapter, for the analysis of (1.5) we have to consider the solutions that do not change with time. Observing (1.5), we identify as first equilibrium point the trivial fixed point located at $\mathbf{e_0^*} = (e_w^*, e_i^*, e_h^*) = (0,0,0)$.

In order to analyze its stability, we first compute the Jacobian matrix of partial derivatives:

$$\mathbf{J} = \begin{bmatrix} 1-2e_w-\phi_w e_h-\psi e_i & -\psi e_w & -\phi_w e_w \\ \lambda\phi_w e_h-\eta e_i & \eta(1-2\psi e_i-\frac{\phi_i}{\eta}e_h-e_w) & \lambda\phi_w e_w-\phi_i e_i \\ (1-\lambda)\phi_w e_h & \phi_i e_h & -\theta+(1-\lambda)\phi_w e_w+\phi_i e_i \end{bmatrix} \tag{3.1}$$

Evaluating (3.1) at equilibrium point $\mathbf{e_0^*}$ we find:

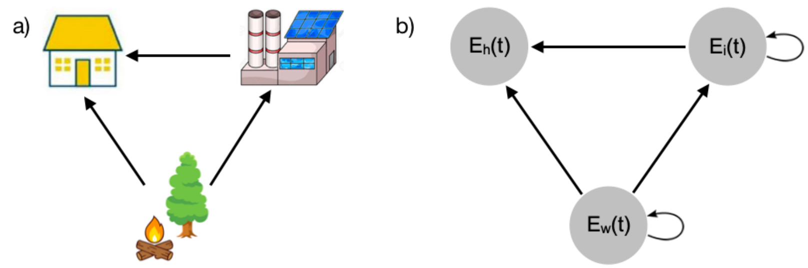


Figure 3.1: Illustrative example of a predator-prey model of three species. An analogy with a human population that harvest natural resources is shown.

$$\mathbf{J}(0,0,0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & -\theta \end{bmatrix} \tag{3.2}$$

From which the eigenvalues $\lambda_1 = 1$, $\lambda_2 = \eta$ and $\lambda_3 = -\theta$. We conclude that $\mathbf{e}_\mathbf{0}^*$ is a saddle point due to $\lambda_3 > 0$ and $\det(\mathbf{J}) = \lambda_1 \lambda_2 \lambda_3 < 0$.

The next fixed points are those that are semi-trivial equilibrium points of system (1.5), where at least one of the entries of $\mathbf{e}^* = (e_w, e_i, e_h)$ is non-trivial. Based on physical analysis of the system, the only possible fixed points are those in which one of the energy generators is different from zero, *i.e.*, $e_w^*, e_i^* \neq 0$, because the human population disappears in absence of a producer.

Thus, the subspaces of (1.5) in which it is possible to find fixed points are:

$$\begin{cases} H_1 = \{(e_w, 0, 0)\} \\ H_2 = \{(0, e_i, 0)\} \\ H_3 = \{(e_w, 0, e_h)\} \\ H_4 = \{(e_w, e_i, 0)\} \\ H_5 = \{(0, e_i, e_h)\} \\ H_6 = \{(e_w, e_i, e_h)\} \end{cases} \tag{3.3}$$

Analyzing each subspace as was done by the trivial equilibrium point, we find the fixed points and the stability conditions of (1.5). In order to solve the problem we will make some assumptions, reducing our system to some particular cases:

(i) On H_1, system (1.5) is reduced again to the Logistic equation. The non-trivial equilibrium point is then $\mathbf{e}_\mathbf{1}^* = (1,0,0)$. Thus, to study the stability, we consider the eigenvalues of the Jacobian matrix (3.1) evaluated at $\mathbf{e}_\mathbf{1}^*$:

$$\begin{cases} \lambda_1 = -1 \\ \lambda_2 = 0 \\ \lambda_3 = -1 + (1-\lambda)\phi_w \end{cases} \tag{3.4}$$

Given that the determinant is equal to zero, this is a non-isolated fixed point.

(ii) On H_2, system (1.5) is reduced again to the Logistic equation. Here, the fixed point is given by $\mathbf{e}_\mathbf{2}^* = (0, 1/\psi, 0)$ with eigenvalues:

$$\begin{cases} \lambda_1 = 0 \\ \lambda_2 = \eta \\ \lambda_3 = \dfrac{\phi_i - \theta\psi}{\psi} \end{cases} \tag{3.5}$$

Once again this is a non-isolated point.

(iii) On H_3, system (1.5) is reduced to a predator-prey system. The non-trivial fixed point is then $\mathbf{e}_\mathbf{3}^* = (\dfrac{\theta}{(1-\lambda)\phi_w}, 0, \dfrac{(1-\lambda)\phi_w - \theta}{(1-\lambda)\phi_w^2})$. As from now on the eigenvalues are difficult to study analytically, we will explore the parameter space using the feasibility conditions. In this case, because both energies should be strictly positive, we find that:

$$\begin{cases} \phi_w < \dfrac{\theta}{(1-\lambda)} \end{cases} \tag{3.6}$$

(iv) On H_4, system (1.5) is a predator-prey model. The non-trivial fixed point is given by: $\mathbf{e}_\mathbf{4}^* = (\dfrac{\phi_i - \psi\theta}{-(1-\lambda)\phi_w\psi + \phi_i}, 0, \dfrac{-(1-\lambda)\phi_w + \theta}{-(1-\lambda)\phi_w\psi + \phi_i})$, then to make the energies strictly positive, we need the conditions:

$$\begin{cases} \phi_i > \psi\theta \\ \phi_w < \dfrac{\theta}{(1-\lambda)} \\ \dfrac{\phi_i}{\phi_w} > (1-\lambda)\psi \end{cases} \tag{3.7}$$

(v) On H_5, system (1.5) is again a predator-prey system. The non-trivial fixed point is: $\mathbf{e_5^*} = (0, \dfrac{\theta}{\phi_i}, \dfrac{\eta\theta}{\phi_i^2}(1 - \dfrac{\psi\theta}{\phi_i}))$. To make the energies positive, the needed condition is:

$$\begin{cases} \phi_i > \psi\theta \end{cases} \tag{3.8}$$

(vi) On H_6, we have to consider the full system of equations. The 3-species coexistence fixed point is given then by:

$$\mathrm{e}_6^* = \begin{cases} e_w^* = \dfrac{\theta(\phi_i - \phi_w\eta)}{\phi_w(-(1-\lambda)\eta\phi_w + \phi_i)} \\ e_i^* = \dfrac{\lambda\phi}{-(1-\lambda)\eta\phi_w + \phi_i} \\ e_h^* = \dfrac{\phi_w(-\eta\phi_w + \lambda\eta\phi_w + \phi_i) - \theta(\phi_i - \phi_w\eta) - \phi_w\psi\lambda\theta}{\phi_w^2(-(1-\lambda)\eta\phi_w + \phi_i)} \end{cases} \tag{3.9}$$

Then to make the energies strictly positive, we need to fulfill the conditions:

$$\begin{cases} \dfrac{\phi_i}{\phi_w} > \eta \\ \dfrac{\phi_i}{\phi_w} > (1-\lambda)\eta \end{cases} \tag{3.10}$$

Which can be reduced to the first inequality.

3.2 Dynamics of the island with presence of renewable energy technology

3.2.1 Parameter choice

Now we should clarify the parameters chosen for the simulations. Summarizing the exploring of parameter space we have made, we have that the values should be of the next order of magnitude:

(i) The carrying capacities are given by $K = 100\ MJ/m^2$ and $\varepsilon = 3000\ MJ/m^2$ as was explained previously in Chapter 2.

(ii) The wood growth rate depends on the biomass growth given by a logistic curve. These curves are in dependence of a variety of variables, such as relative growth rate, net assimilation rate, crop growth rate and biomass duration. Thus, the logistic growth that describes change in the proportion of forest could vary in a range of values that, according to the literature [Dewar, 1990; Hashimotio et al., 2000; Hozumi, 1989; Kopp et al., 2001], is around $10-20\%$ per year. In this work, we will consider $r = 0.1\ year^{-1}$.

(iii) As it was mentioned in Chapter 2, the industry growth rate is fixed in $\gamma \approx 1\ year^{-1}$

(iv) The human depreciation rate in the model represents how the natural mortality of the human population of the island affects the energy consumed by the community. This rate can vary between communities in dependence of underlying factors such as level of health and longevity. For our model, we estimate the depreciation rate as $\delta = 0.1$ per year, *i.e.*, a decrease of 10% of the population per year. Note that the positive terms of the expression $\dot{e}_i$ represent the increase of energy consumed by the community due to the growth of population.

A summary of the parameter values used in the simulations is shown in Table 1.

Parameter	Description	Numerical value	Units
r	Growth rate of wood energy per year	0.1	y^{-1}
γ	Growth rate of industry energy per year	1	y^{-1}
δ	Decrease rate of human population energy	0.1	y^{-1}
K	Carrying capacity of wood	100	MJ/m^2
ε	Carrying capacity of industry	3000	MJ/m^2
α_{wh}	Consumption rate of wood energy by humans	$1/30000$	$m^2/(MJ*y)$
α_{ih}	Consumption rate of industry energy by humans	$1/3000$	$m^2/(MJ*y)$
λ	Consumption percentage of wood energy by industry	0.05	Adimensional

What in terms of the adimensional variables is equivalent to:

Parameter	Equivalence	Numerical value
ψ	$\frac{K}{\varepsilon}$	$1/30$
η	$\frac{\gamma}{r}$	1-12
θ	$\frac{\delta}{r}$	1
ϕ_w	$\frac{\alpha_{wh}K}{r}$	1
ϕ_i	$\frac{\alpha_{ih}K}{r}$	1-12

3.2.2 Possible escenarios

The system is describe by the next system of equations:

$$\begin{cases} \dot{e_w} = e_w(1 - e_w - \psi e_i - \phi_w e_h) \\ \dot{e_i} = \eta e_i(1 - e_w - \psi e_i - \frac{\phi_i}{\eta} e_h) + \lambda \phi_w e_h e_w \\ \dot{e_h} = e_h(-\theta + (1-\lambda)\phi_w e_w + \phi_i e_i) \end{cases} \tag{3.11}$$

with the fixed points:

$$\begin{cases} \mathbf{e_0^*} = (0,0,0) \\ \mathbf{e_1^*} = (1,0,0) \\ \mathbf{e_2^*} = (0,1/\psi,0) \\ \mathbf{e_3^*} = (\dfrac{\theta}{(1-\lambda)\phi_w}, 0, \dfrac{(1-\lambda)\phi_w - \theta}{(1-\lambda)\phi_w^2}) \\ \mathbf{e_4^*} = (e_w, \dfrac{1-e_w}{\psi}, 0) \\ \mathbf{e_5^*} = (0, \dfrac{\theta}{\phi_i}, \dfrac{\eta\theta}{\phi_i^2}(1 - \dfrac{\psi\theta}{\phi_i})) \end{cases} \tag{3.12}$$

Plus the coexistence fixed point:

$$\mathrm{e}_6^* = \begin{cases} e_w^* = \dfrac{\phi_w(-\eta\phi_w + \lambda\eta\phi_w + \phi_i) - \theta(\phi_i - \phi_w\eta) - \phi_w\psi\lambda\theta}{\phi_w^2(-(1-\lambda)\eta\phi_w + \phi_i)} \\ e_i^* = \dfrac{\lambda\phi}{-(1-\lambda)\eta\phi_w + \phi_i} \\ e_h^* = \dfrac{\theta(\phi_i - \phi_w\eta)}{\phi_w(-(1-\lambda)\eta\phi_w + \phi_i)} \end{cases}$$

To understand the effect of industry on the dynamics of wood and human population, we plot in the same graphic and under the same set of parameters two different scenarios: one with the effect of industry ($\lambda = 5/100$) presented in solid lines and another without its effect ($\lambda = 0$) shown in dashed lines.

Let's consider different cases, with variation of initial conditions and a key parameter. In Fig. 3.2 we observe a variation of initial conditions, in particular a decrease of $e_h(0)$.

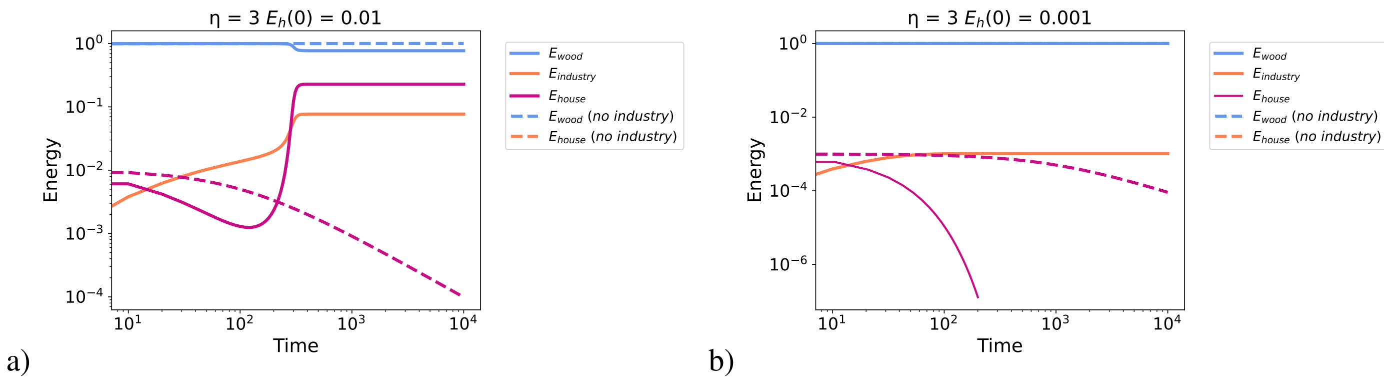
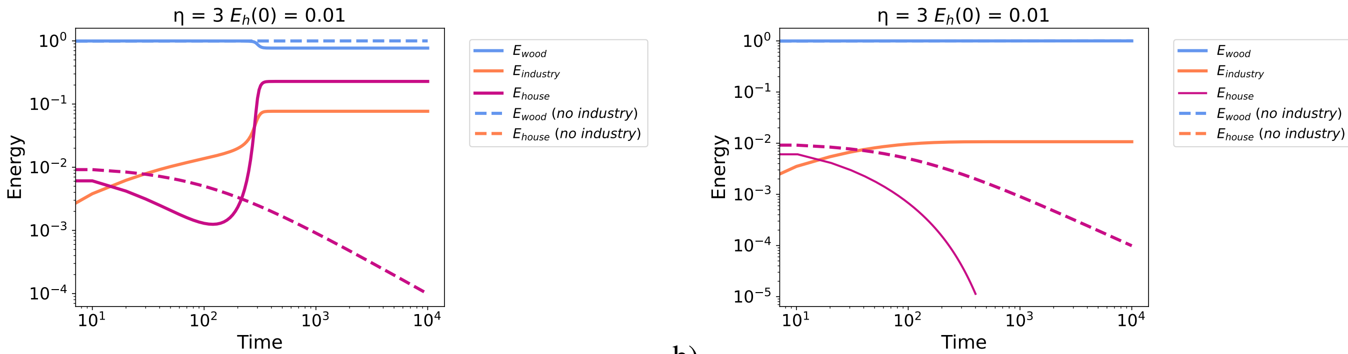


Figure 3.2: Set of parameters: $\phi_w = \theta = 1, \lambda = 5/100, \phi_i = 3.5, \psi = 1/30, \eta = 3$. Initial conditions: $e_w(0) = 1, e_i(0) = 0$ a) $e_h(0) = 0.01$ b) $e_h(0) = 0.001$.

When $e_h(0) = 0.01$, without industry, wood doesn't change its energy storage while human population collapses; however, when industry appears, wood decrease its storage while humans increase its populations until the steady state is reached. It is important to note that steady state correspond to a three species coexistence scenario.

If $e_h(0)$ decreases to 0.001, dynamics changes significantly. With no industry, humans and wood coexistence with a minimum variation of human population over time; once industry appears, wood continue unperturbed, humans go to collapse and industry increase until reach a fixed point. A decrease of human initial population makes a coexistence scenario impossible. This result give us some insight about the existence of a threshold of $e_h(0)$ after which humans survive.

The next case consider same initial conditions but a variation of the parameter $\eta = \frac{\gamma}{r}$ which corresponds to the industry growth rate (see Fig. 3.3). For $\eta = 3$ we have the same dynamics of the above case $e_h(0) = 0.01$: industry allows a coexistence steady state. When $\eta = 1$, with no industry, humans decrease over time until extinction and wood is not affected; once industry appears, wood still is unaffected but humans goes to collapse and industry reach a steady value. This means that the production of energy by industry is key parameter that determines the survival of human population. If industry doesn't produce enough energy while consuming energy from wood, humans cannot survive. From this result we deduce the existence of critical value of η that allows the survival of humans.



a) b)

Figure 3.3: Set of parameters: $\phi_w = \theta = 1, \lambda = 5/100, \phi_i = 3.5, \psi = 1/30$ a) $\eta = 3$ b) $\eta = 1$. Initial conditions: $e_w(0) = 1, e_i(0) = 0, e_h(0) = 0.01$.

We can conclude that in the 3-species system there are many parameters to consider however, we observe that the self-growth rate of industry η determine whether humans will survive or not. Also, the initial human population is decisive for its own survival, no matter how much energy is produced.

This give us some insight in the study of the basin of attraction of the 3-species systems as well as the fixed point that is reached in dependence of the η and ϕ_i values that are chosen. In order to understand this dynamics and having as aim the survival of humans, in the next section we will explore the possible scenarios that are formed under the constrain $\theta = \phi_w$.

3.3 Case $\theta = \phi_w$: How to reach coexistence fixed point?

3.3.1 Parameter choose

For the 3-species model, to study the dependence of the fixed points on the parameter and initial conditions choose, we simplify the system 1.5 with 6 parameters based in the next assumptions:

- $\psi = \frac{K}{\varepsilon} = \frac{100\ MJ}{3000\ MJ}$ based on literature review.
- $\theta = \frac{\delta}{r} = 1$ assuming that the decay rate of humans is equal to the growth rate of wood (in this case $r = 0.1\ y^{-1}$)

- $\phi_w = \frac{\alpha_{wh} K}{r} = 1$. This follows the main assumption $\theta = \phi_w$.
- λ, the portion of energy harvested by humans from wood that is invested to renewable energy growth, is fixed to 5%.

This reduces the number of free parameters to two, η and ϕ_i. Through an analysis of feasibility of the fixed points shown in Eq. 3.12, we find the next conditions:

$$\begin{cases} \phi_i \geq \psi\theta \\ \phi_w \leq \dfrac{\theta}{(1-\lambda)} \\ \dfrac{\phi_i}{\phi_w} \geq (1-\lambda)\psi \\ \dfrac{\phi_i}{\phi_w} \geq \eta > (1-\lambda)\eta \end{cases} \tag{3.13}$$

That, following the assumptions previously made, are equivalent to:

$$\begin{cases} \phi_i \geq \psi > (1-\lambda)\psi \\ \phi_i \geq \eta > (1-\lambda)\eta \end{cases} \tag{3.14}$$

The first assumption is easy to accomplish given that ϕ_i should be greater than $\frac{1}{30}$. The second one implies that the only parameter that should be define with freedom is η however, we should remember that it is define as $\eta = \frac{\gamma}{r}$. As we fixed $r = 0.1\ y^{-1}$, we need to determine the possible values of γ.

Recovering the result of Eq. 1.3 presented in Section 1.2 , we have that:

$$EROI = \frac{Energy \quad Provided}{Energy \quad Invested} = 1 + \gamma\tau \tag{3.15}$$

Then, we can rewrite η as follows:

$$\eta = \frac{EROI - 1}{\tau r} \tag{3.16}$$

We have to fixed the values of EROI for renewable sources of energy. Let's take the particular case of solar panels: EROI has been found between 8.7 and 34.2 for a life-time (τ) of 30 years [Bhandari et al., 2015].

In Fig. 3.4 are shown the possible values of η for the energy sources studied. From now on, we will fix the limit values of η between the minimum hypothetical value ~ 0 (in this case $\eta = 0.1$) and the maximum realistic value $\eta = 12$).

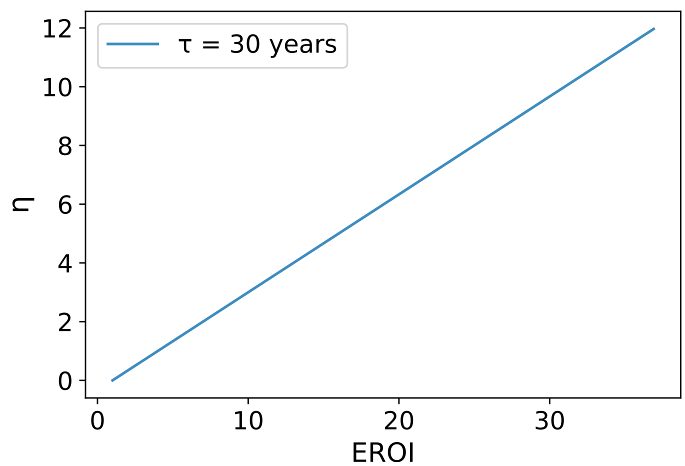


Figure 3.4: η as a function of EROI for solar panels.

3.3.2 Exploration of possible scenarios

Base on the conclusions of the previous sections, we have two parameters (η and ϕ_i) and the human initial condition ($e_h(0)$) to explore. We will start fixing the initial condition to $e_h(0) = 0.01$ varying η and ϕ_i respecting the conditions and limit cases found above.

In Fig. 3.5 two values of η are presenting and three different values of ϕ_i for each case. The first row corresponds to the limit case $\eta = 0.1$: for a small value of ϕ_i (panel a) humans collapses with and without industry. This behavior continue until a critical value of ϕ_i (panel b); for values greater than that human goes to extinction with no industry while a coexistence scenario is reached with it.

For $\eta = 10$, for any value of ϕ_i chosen, humans survive in presence of industry while they collapses without it. The difference is that, the bigger the value of ϕ_i, equivalent to the consumption of renewable energy by humans, human and industry energy storage decreases. Note that for $\phi_i \leq \eta$ (panel c) a human-industry coexistence point is reached due to the collapses of wood.

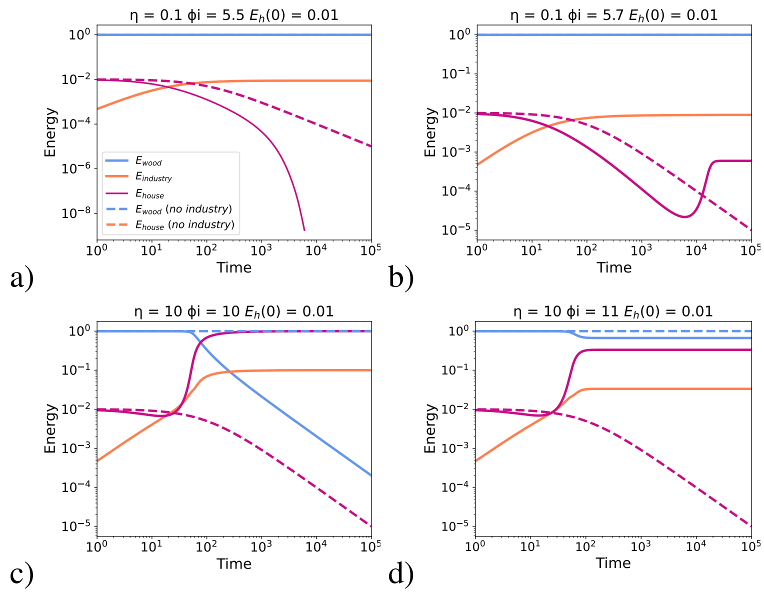


Figure 3.5: Set of parameters: $\phi_w = \theta = 1, \lambda = 5/100, \psi = 1/30$, while ϕ_i and η varies. Initial conditions: $e_w(0) = 1, e_i(0) = 0, e_h(0) = 0.01$

Once again, we observe the presence of a threshold of η after which humans survive. In Fig. 3.6, for $\phi_i = 5.5$, different values of η are displayed. The increase of η implies an increase of human population and two critical values are observed: first, for $\eta = 0.0157$ (panel b), humans survive in presence of industry; second, for $\eta = 5.5$ (panel c), human reaches wood carrying capacity driving it to collapse. Note that, in panel b), at the beginning the wood-industry coexistence point is reached but the steady state corresponds to the three species coexistence fixed point.

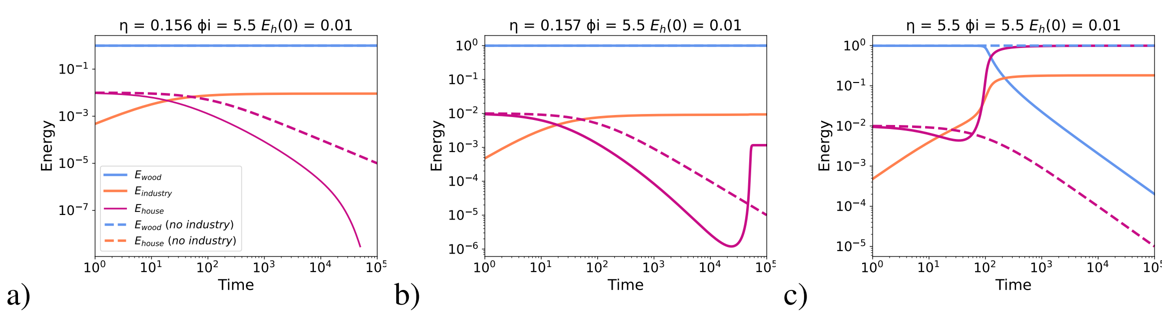


Figure 3.6: Initial conditions: $e_w(0) = 1, e_i(0) = 0, e_h(0) = 0.01$. Set of parameters: $\phi_w = \theta = 1, \lambda = 5/100, \psi = 1/30$ a) $\eta = 0.156$ b) $\eta = 0.157$ a) $\eta = 5.5$

In Fig. 3.7, now the initial conditions of human population varies while the parameters

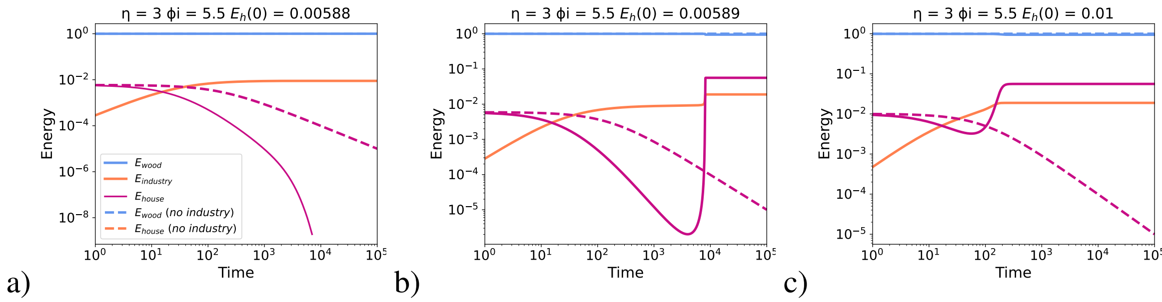


Figure 3.7: Set of parameters: $\phi_w = \theta = 1, \lambda = 5/100, \psi = 1/30, \phi_i = 5.5, \eta = 5.5$. Initial conditions: $e_w(0) = 1, e_i(0) = 0$ a) $e_h(0) = 0.00588$ b) $e_h(0) = 0.00589$ c) $e_h(0) = 0.1$

are fixed ($\eta = 3, \phi_i = 5.5$). For low values of $e_w(0)$, humans collapses with and without industry. When a critical value is overcome (panel b), the coexistence fixed point is reached. While $e_w(0)$ increases, the system goes to the steady state faster, avoiding the intersection between human and industry energy storage.

Now, we change the method of analysis. In Fig. 3.8, we plot the values of e_w, e_i, e_h once the steady state is reached for a range of values of η and ϕ_i with fixed initial conditions. In panel a) it is possible to see that wood survives for all values of η and ϕ_i except for a region where $\eta > \phi_i$. In panel c, where human energy storage is presented, we can observe three regions: first, a blue region where human collapses, a green region located in $\eta > \phi_i$ where human survive with a small population and a yellow region in $\eta < \phi_i$ where humans survive successfully. Observing the steady state of the three species we can identify three regions as is shown in Fig. 3.9 panel b): panel a) corresponds to region I where wood and industry coexist, panel d) corresponds to human-industry coexistence while in panel c) is possible to observe the three species coexistence fixed

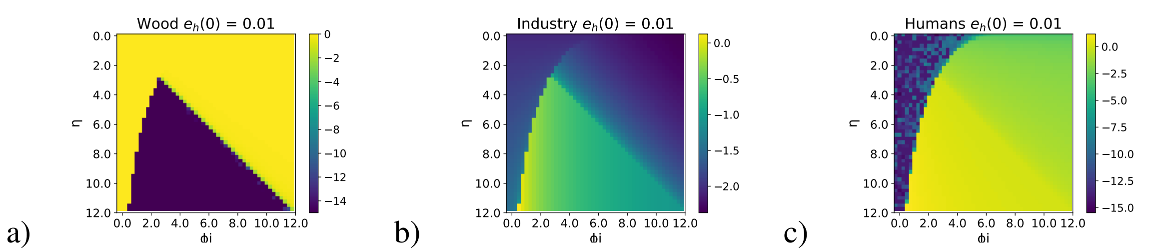


Figure 3.8: Set of parameters: $\phi_w = \theta = 1, \lambda = 5/100, \psi = 1/30$ for different values of η and ϕ_i. Initial conditions: $e_w(0) = 1, e_i(0) = 0.0001, e_h(0) = 0.01$. A log10 scale is used to display steady state values.

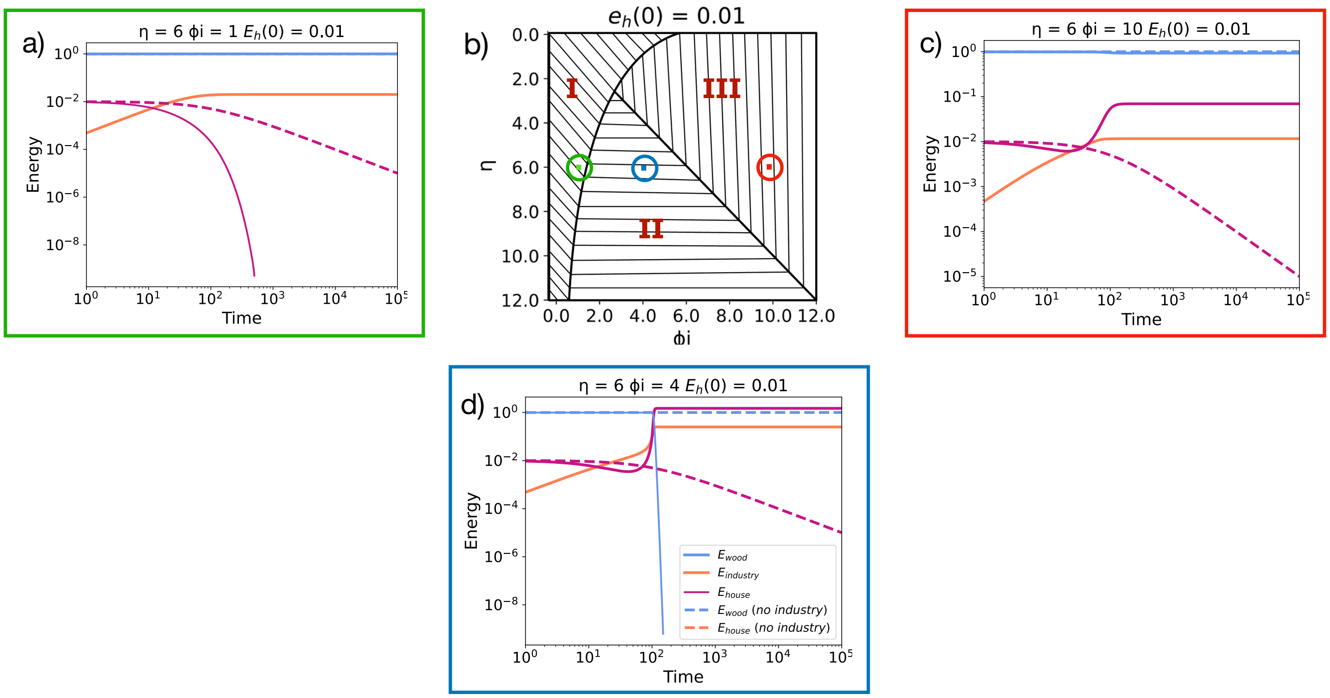


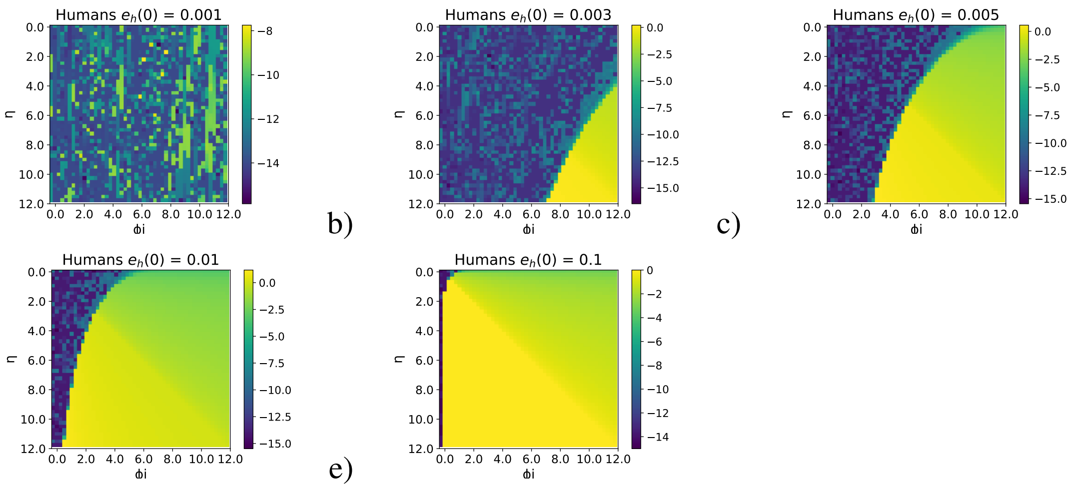
Figure 3.9: Three regions are identify in dependence of ϕ_i and η values as is shown in panel b). a) Region I corresponds to industry-wood coexistence c) Region III corresponds to wood-industry-humans coexistence d) Region II corresponds to industry-human coexistance.

point. Note that these regions corresponds to the three last fixed points given that the constrain $\theta = \phi_w$ makes e_3^* unfeasible -humans become negative-.

In Fig. 3.10 it is possible to see that humans collapses for low initial conditions of human population no matter the parameter choice (panels a). While $e_h(0)$ increases, human survive for values of η and ϕ_i outside the blue region, corresponding to the yellow and green regions in all the cases. Once again it is possible to observe three regions with the singularity that the greater $e_h(0)$, the smallest the blue region is.

In closing, for a fixed initial condition, the greater η, the faster the coexistence fixed point is reached. In the lowest limit case, for $\eta = 0.1$, it is needed a high ϕ_i to escape for collapse; while η increases to the highest limit case, human always survive in presence of industry. The effect of ϕ_i in this limit is reduce the human and industry energy storage.

On the other hand, fixing all the parameters, including θ and ϕ_w, and varying $e_h(0)$ it is



a) b) c) d) e)

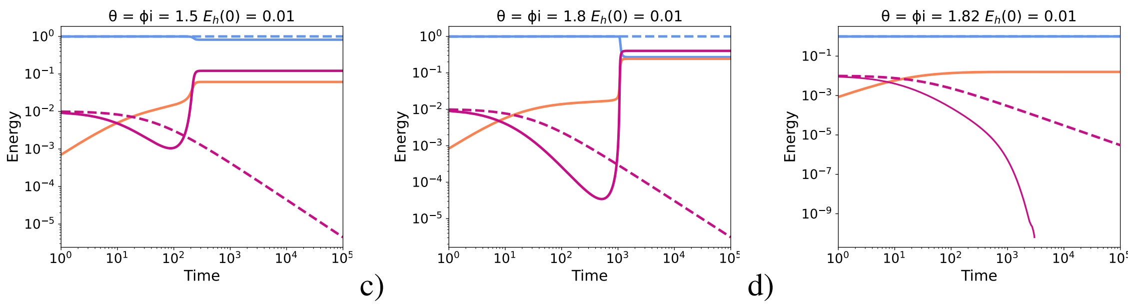
Figure 3.10: Set of parameters: $\phi_w = \theta = 1, \lambda = 5/100, \psi = 1/30$ for different values of η and ϕ_i. Initial conditions: $e_w(0) = 1, e_i(0) = 0$. A log10 scale is used to display steady state values.

possible to see that there is a critical human initial condition that allows human survive in presence of industry (see Fig. 3.7). For $e_h(0)$ above this critical values, the greater $e_h(0)$, the faster the steady state is reached.

3.3.3 Variation of $\theta = \phi_w$ value

Now, we have to study the effects of the $\theta = \phi_w$ value. In Fig. 3.11, the time evolution of the three species system is shown for a fixed set of parameters and a range of $\theta = \phi_w$ values. While the human decay rate and the wood energy harvested by humans increase, the human and industry steady states increase while the wood energy decreases until human population collapses.

Finally, in Fig. 3.12, the steady state values of the system are shown. The blue region, where human population collapses, increases as human decay rate increases. However, for values of η and ϕ_i outside the blue region, a coexistence state is possible. Note that the greater $\theta = \phi_w$ value, the greater the yellow three species coexistence fixed point.



b) c) d)

Figure 3.11: Set of parameters: $\phi_w = \theta = 1, \lambda = 5/100, \psi = 1/30, \phi_i = 5.5, \eta = 3$. Initial conditions: $e_w(0) = 1, e_i(0) = 0, e_h(0) = 0.01$.
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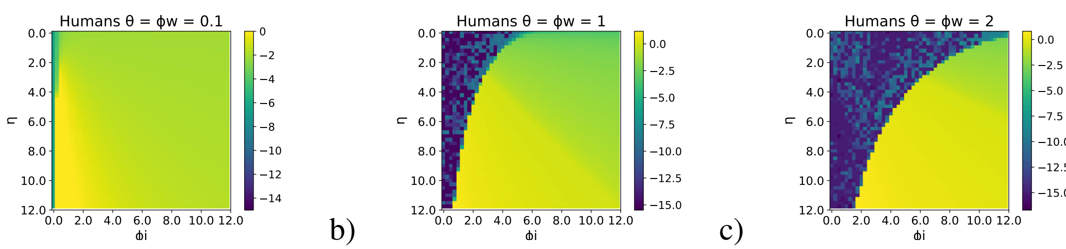

a) b) c)

Figure 3.12: Set of parameters: $\phi_w = \theta = 1, \lambda = 5/100, \psi = 1/30$ for different values of η and ϕ_i. Initial conditions: $e_w(0) = 1, e_i(0) = 0, e_h(0) = 0.01$. A log10 scale is used to display steady state values.

Conclusions

This work presents a simple model of renewable resource growth and population dynamics in presence of a technology that allows energy transitions. The model is employed to provide a fictional scenario for the survival of the population of the Easter Island, case study for the energy transition from an agrarian society to a modern society based on industrial renewables. Two main dynamics are presented, one with the existence of a renewable energy technology and another without its implementation. For the first case and given reasonable parameter values, several scenarios were founded; in one of them, the model generates a cycle in which population grows, the resource is degraded, and population ultimately falls however, this cycle arises because the resource -in this case the forest that provides wood- has a low regeneration rate. When this rate in increase and the resource has a faster growth rate, the model allows a converge toward an steady-state population where humans and forest coexist. This allows to explain how the resource growth rate is highly important for the permanence of a human population on the island.

On the other hand, the second main dynamics presents patterns where the introduction of a technology changes the behaviour of the dynamics in the island. When the human population consumes energy from a second source that has a higher growth rate, even with a slow rate of regeneration of the forest, the community on the island can survive and coexistence with the natural resources. However, to achieve this energy transition and the develop of the civilization that allows the increase of the population in harmony with the resources used to generate energy, is needed an investment of the primary energy source to the development of the renewable technology.

Our analysis give us several important lesson for the modern world. First, the model implies that changes in technology, the environment and human behaviour can create feast and famine cycles that create an unhealthy interaction between human communities and nature due to a lack of sustainable consumption of resources. The case of the Easter Island is just one of the many cases where this lead to the disappearance of a human population. Identifying countries and communities where this behaviour is done could be difficult, but our model can provide of some guidance to identify the key parameters that make cyclical dynamics more likely.

A second lesson is that technology can lead to a more sustainable dynamics between human and nature thanks to a more efficient exploitation of resources. Once is introduce a new source of energy, the human population can avoid the extinction and reach a new steady state of coexistence avoiding cycles due to the abuse of natural resources. The steady-state that is reached and the path to do it can vary from a population to another in dependence of its behaviour and the environmental conditions. To explore several scenarios for a bigger range of parameters is a future goal of this work.

Here is needed to add an important observation derived from the model outputs about the nature and technology relationship. As the model considers solar light per unit area as the main resource shared by both energy producers, the saturation of the island capital used to build and maintain the renewable industry leads forest to extinction. An abuse of the island space destined to solar panels can have a negative impact in natural resources of the island. This possibility has to be considered in the development of renewable industry to create a healthy coexistence of technology and environment.

Third, our analysis of the Easter Island shows that to achieve an energy transition, an investment of energy is needed. The model includes a term related to renewable energy harvested by humans, that depends on the human population but that has effect on the growth rate of the industry, proving that the increasing of the population should lead to a higher investment of the existence energy advantage to generate more and better energy sources. In the present work, the presence of solar panels in the Easter Island allows to the community to reach a coexistence with environment as well as an increase of its number of individuals, being this a fictional scenario that is worth to explore in the actually to avoid environmental and human catastrophe.

The last lesson give us information about the effect of the human initial condition in the dynamics of the island. If the human population is lower than a critical value, no matter how efficient the new industry is, humans will collapse. This result is important and it should be explored in detail to understand the minimum and maximum threshold of human inhabitants that allows communities to survive.

Finally, it is important to add that in the model, non-linearity is needed to analyze the economic development of the island, where sets of different parameters make possible reproduce real and fictional scenarios for human populations and its interaction with natural resources. Increasing the complexity of the model, we can expand our cases of study and with this, the possible applications of the model.

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Diagram describing the flow of natural resources through the economy: Valuable resources are procured from nature by the input end of the economy; the resources flow through the economy, being transformed and manufactured into goods along the way; and invaluable waste and pollution eventually accumulate by the output end. Recycling of material resources is possible, but only by using up some energy resources as well as an additional amount of other material resources; and energy resources, in turn, cannot be recycled at all, but are dissipated as waste heat. (2015). Retrieved July 8, 2022, from https://commons.wikimedia.org/wiki/File:Diagram_of_natural_resource_flows.jpg

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